# Rubi 4.16.0 Exponential and Logarithm Integration Test Results

Test results for the 98 problems in "2.1 u (F^(c (a+b x)))^n.m"

Test results for the 93 problems in "2.2 (c+d x) $^m$  (F $^(g (e+f x)))^n (a+b (F<math>^(g (e+f x)))^n$ ) $^p.m$ "

Test results for the 774 problems in "2.3 Exponential functions.m"

Problem 70: Result unnecessarily involves higher level functions.

Problem 71: Result unnecessarily involves higher level functions.

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\begin{split} &\int f^{a+b\,x^2}\,x^9\,\mathrm{d}x\\ &\text{Optimal (type 3, 65 leaves, 1 step):} \\ &\frac{f^{a+b\,x^2}\,\left(24-24\,b\,x^2\,\mathsf{Log}\,[f]+12\,b^2\,x^4\,\mathsf{Log}\,[f]^2-4\,b^3\,x^6\,\mathsf{Log}\,[f]^3+b^4\,x^8\,\mathsf{Log}\,[f]^4\right)}{2\,b^5\,\mathsf{Log}\,[f]^5} \\ &\text{Result (type 4, 24 leaves, 1 step):} \\ &\frac{f^a\,\mathsf{Gamma}\,\big[\,5\,,\,-b\,x^2\,\mathsf{Log}\,[f]\,\big]}{2\,b^5\,\mathsf{Log}\,[f]^5} \end{split}
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# Problem 96: Result unnecessarily involves higher level functions.

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\int f^{a+b\,x^3}\,x^{17}\,\mathrm{d}x
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Optimal (type 3, 78 leaves, 1 step):

$$-\frac{\mathsf{f}^{\mathsf{a}+\mathsf{b}\,\mathsf{x}^3}\,\left(\mathsf{120}-\mathsf{120}\,\mathsf{b}\,\mathsf{x}^3\,\mathsf{Log}\,[\mathsf{f}]\,+\mathsf{60}\,\mathsf{b}^2\,\mathsf{x}^6\,\mathsf{Log}\,[\mathsf{f}]^{\,2}-\mathsf{20}\,\mathsf{b}^3\,\mathsf{x}^9\,\mathsf{Log}\,[\mathsf{f}]^{\,3}+\mathsf{5}\,\mathsf{b}^4\,\mathsf{x}^{12}\,\mathsf{Log}\,[\mathsf{f}]^{\,4}-\mathsf{b}^5\,\mathsf{x}^{15}\,\mathsf{Log}\,[\mathsf{f}]^{\,5}\right)}{3\,\mathsf{b}^6\,\mathsf{Log}\,[\mathsf{f}]^{\,6}}$$

Result (type 4, 24 leaves, 1 step):

$$-\frac{f^{a} \operatorname{Gamma} \left[6, -b x^{3} \operatorname{Log} \left[f\right]\right]}{3 b^{6} \operatorname{Log} \left[f\right]^{6}}$$

# Problem 97: Result unnecessarily involves higher level functions.

$$\int f^{a+b} x^3 x^{14} dx$$

Optimal (type 3, 65 leaves, 1 step):

$$\frac{f^{a+b\,x^3}\,\left(24-24\,b\,x^3\,Log\,[\,f\,]\,+12\,b^2\,x^6\,Log\,[\,f\,]^{\,2}-4\,b^3\,x^9\,Log\,[\,f\,]^{\,3}+b^4\,x^{12}\,Log\,[\,f\,]^{\,4}\right)}{3\,b^5\,Log\,[\,f\,]^{\,5}}$$

Result (type 4, 24 leaves, 1 step):

$$\frac{f^{a} Gamma \left[5, -b x^{3} Log[f]\right]}{3 b^{5} Log[f]^{5}}$$

# Problem 126: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{x}{p}}}{x^6} \, \mathrm{d} x$$

Optimal (type 3, 65 leaves, 1 step):

$$-\frac{f^{a+\frac{b}{x}}\left(24\,x^4-24\,b\,x^3\,Log\,[f]\,+12\,b^2\,x^2\,Log\,[f]^{\,2}-4\,b^3\,x\,Log\,[f]^{\,3}+b^4\,Log\,[f]^{\,4}\right)}{b^5\,x^4\,Log\,[f]^{\,5}}$$

Result (type 4, 22 leaves, 1 step):

$$-\frac{f^{a} \operatorname{Gamma}\left[5, -\frac{b \operatorname{Log}[f]}{x}\right]}{b^{5} \operatorname{Log}[f]^{5}}$$

# Problem 127: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{b}{x}}}{x^7} \, \mathrm{d} x$$

Optimal (type 3, 77 leaves, 1 step):

$$\frac{f^{a+\frac{b}{x}}\left(120\,x^5-120\,b\,x^4\,Log\,[f]\,+60\,b^2\,x^3\,Log\,[f]^{\,2}-20\,b^3\,x^2\,Log\,[f]^{\,3}+5\,b^4\,x\,Log\,[f]^{\,4}-b^5\,Log\,[f]^{\,5}\right)}{b^6\,x^5\,Log\,[f]^{\,6}}$$

Result (type 4, 21 leaves, 1 step):

$$\frac{\textbf{f}^{a} \, \mathsf{Gamma} \left[ \textbf{6,} - \frac{b \, \mathsf{Log}[\textbf{f}]}{x} \right]}{b^{6} \, \mathsf{Log}[\textbf{f}]^{\, 6}}$$

# Problem 139: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} \, \mathrm{d} x$$

Optimal (type 3, 69 leaves, 1 step):

$$-\frac{f^{a+\frac{b}{x^2}}\left(24\,x^8-24\,b\,x^6\,Log\,[f]\,+12\,b^2\,x^4\,Log\,[f]^{\,2}-4\,b^3\,x^2\,Log\,[f]^{\,3}+b^4\,Log\,[f]^{\,4}\right)}{2\,b^5\,x^8\,Log\,[f]^{\,5}}$$

Result (type 4, 24 leaves, 1 step):

$$-\frac{f^{a}\operatorname{Gamma}\left[5,-\frac{b\operatorname{Log}\left[f\right]}{x^{2}}\right]}{2\,b^{5}\operatorname{Log}\left[f\right]^{5}}$$

# Problem 140: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} \, \mathrm{d} x$$

Optimal (type 3, 82 leaves, 1 step):

$$\frac{\textbf{f}^{\text{a}+\frac{b}{x^2}}\left(120\,x^{10}-120\,b\,x^8\,\text{Log}\,\texttt{[f]}+60\,b^2\,x^6\,\text{Log}\,\texttt{[f]}^2-20\,b^3\,x^4\,\text{Log}\,\texttt{[f]}^3+5\,b^4\,x^2\,\text{Log}\,\texttt{[f]}^4-b^5\,\text{Log}\,\texttt{[f]}^5\right)}{2\,b^6\,x^{10}\,\text{Log}\,\texttt{[f]}^6}$$

Result (type 4, 24 leaves, 1 step):

$$\frac{\mathsf{f}^{\mathsf{a}}\,\mathsf{Gamma}\left[\,\mathsf{6}\,\mathsf{,}\,\,-\,\frac{\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{f}\,]}{\mathsf{x}^2}\,\right]}{\,2\,\mathsf{b}^{\mathsf{6}}\,\mathsf{Log}\,[\,\mathsf{f}\,]^{\,\mathsf{6}}}$$

# Problem 165: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,\mathrm{d} x$$

Optimal (type 3, 69 leaves, 1 step):

$$-\frac{f^{a+\frac{b}{x^3}} \left(24 \, x^{12} - 24 \, b \, x^9 \, \text{Log}\, [\, f\,] \, + 12 \, b^2 \, x^6 \, \text{Log}\, [\, f\,]^{\, 2} - 4 \, b^3 \, x^3 \, \text{Log}\, [\, f\,]^{\, 3} + b^4 \, \text{Log}\, [\, f\,]^{\, 4}\right)}{3 \, b^5 \, x^{12} \, \text{Log}\, [\, f\,]^{\, 5}}$$

Result (type 4, 24 leaves, 1 step):

$$-\frac{f^{a} \operatorname{Gamma}\left[5, -\frac{b \operatorname{Log}\left[f\right]}{x^{3}}\right]}{3 b^{5} \operatorname{Log}\left[f\right]^{5}}$$

# Problem 166: Result unnecessarily involves higher level functions.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}} \, \mathrm{d} x$$

Optimal (type 3, 82 leaves, 1 step):

$$\frac{f^{a+\frac{b}{x^3}}\left(120\,x^{15}-120\,b\,x^{12}\,\text{Log}\,[f]\,+60\,b^2\,x^9\,\,\text{Log}\,[f]^{\,2}-20\,b^3\,x^6\,\,\text{Log}\,[f]^{\,3}+5\,b^4\,x^3\,\,\text{Log}\,[f]^{\,4}-b^5\,\,\text{Log}\,[f]^{\,5}\right)}{3\,b^6\,x^{15}\,\,\text{Log}\,[f]^{\,6}}$$

Result (type 4, 24 leaves, 1 step):

$$\frac{f^{a} \operatorname{Gamma}\left[6, -\frac{b \operatorname{Log}[f]}{x^{3}}\right]}{3 b^{6} \operatorname{Log}[f]^{6}}$$

# Problem 255: Result unnecessarily involves higher level functions.

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\int F^{a+b} (c+dx)^2 (c+dx)^{11} dx
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Optimal (type 3, 105 leaves, 1 step):

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2 b<sup>6</sup> d Log[F]<sup>6</sup>
F^{a+b (c+d x)^2} \left(120-120 \ b \ \left(c+d \ x\right)^2 \ Log \left[F\right] \ +60 \ b^2 \ \left(c+d \ x\right)^4 \ Log \left[F\right]^2 -20 \ b^3 \ \left(c+d \ x\right)^6 \ Log \left[F\right]^3 +5 \ b^4 \ \left(c+d \ x\right)^8 \ Log \left[F\right]^4 -b^5 \ \left(c+d \ x\right)^{10} \ Log \left[F\right]^5 \right)
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Result (type 4, 31 leaves, 1 step):

$$-\frac{F^{a} Gamma [6, -b (c + d x)^{2} Log[F]]}{2 b^{6} d Log[F]^{6}}$$

# Problem 256: Result unnecessarily involves higher level functions.

$$\int F^{a+b (c+dx)^2} (c+dx)^9 dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{1}{2 \, b^5 \, d \, \text{Log} \, \lceil \, F \, \rceil^5} F^{a+b \, \, (c+d \, x)^{\, 2}} \, \left( 24 - 24 \, b \, \left( c + d \, x \right)^{\, 2} \, \text{Log} \, [\, F \, ] \, + \, 12 \, b^2 \, \left( c + d \, x \right)^{\, 4} \, \text{Log} \, [\, F \, ] \, ^2 - 4 \, b^3 \, \left( c + d \, x \right)^{\, 6} \, \text{Log} \, [\, F \, ] \, ^3 + b^4 \, \left( c + d \, x \right)^{\, 8} \, \text{Log} \, [\, F \, ] \, ^4 \right)$$

Result (type 4, 31 leaves, 1 step):

$$\frac{F^{a} Gamma [5, -b (c + d x)^{2} Log[F]]}{2 b^{5} d Log[F]^{5}}$$

#### Problem 281: Result unnecessarily involves higher level functions.

$$\int\! F^{a+b\ (c+d\,x)^{\,3}}\,\left(c+d\,x\right)^{\,17}\,\text{d}x$$

Optimal (type 3, 105 leaves, 1 step):

$$-\frac{1}{3 \, b^6 \, d \, \mathsf{Log} \, [\mathsf{F}]^6} \\ \mathsf{F}^{\mathsf{a}+\mathsf{b} \, (\mathsf{c}+\mathsf{d} \, \mathsf{x})^3} \left( 120 - 120 \, \mathsf{b} \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^3 \, \mathsf{Log} \, [\mathsf{F}] \, + 60 \, \mathsf{b}^2 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^6 \, \mathsf{Log} \, [\mathsf{F}]^2 - 20 \, \mathsf{b}^3 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^9 \, \mathsf{Log} \, [\mathsf{F}]^3 + 5 \, \mathsf{b}^4 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^{12} \, \mathsf{Log} \, [\mathsf{F}]^4 - \mathsf{b}^5 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^{15} \, \mathsf{Log} \, [\mathsf{F}]^5 \right) \\ = \frac{1}{3 \, \mathsf{b}^6 \, \mathsf{d} \, \mathsf{Log} \, [\mathsf{F}]^6} \left(\mathsf{Log} \, \mathsf{Log} \, [\mathsf{F}]^4 - \mathsf{b}^5 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^{15} \, \mathsf{Log} \, [\mathsf{F}]^5 \right) \\ = \frac{1}{3 \, \mathsf{b}^6 \, \mathsf{d} \, \mathsf{Log} \, [\mathsf{F}]^6} \left(\mathsf{Log} \, \mathsf{Log} \, [\mathsf{F}]^4 - \mathsf{b}^5 \, \left(\mathsf{c}+\mathsf{d} \, \mathsf{x}\right)^{15} \, \mathsf{Log} \, [\mathsf{F}]^5 \right) \\ = \frac{1}{3 \, \mathsf{b}^6 \, \mathsf{d} \, \mathsf{Log} \, [\mathsf{F}]^6} \left(\mathsf{Log} \, \mathsf{Log} \, [\mathsf{F}]^6 - \mathsf{Log} \, \mathsf{Log} \, \mathsf{Log} \, [\mathsf{F}]^6 - \mathsf{Log} \, \mathsf{Log} \,$$

Result (type 4, 31 leaves, 1 step):

$$-\frac{F^{a} Gamma [6, -b (c + d x)^{3} Log[F]]}{3 b^{6} d Log[F]^{6}}$$

# Problem 282: Result unnecessarily involves higher level functions.

$$\int F^{a+b} (c+dx)^3 (c+dx)^{14} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{1}{3 \, b^5 \, d \, \mathsf{Log} \, [\, F \,]^5} F^{a+b \, \, (c+d \, x)^3} \, \left(24 - 24 \, b \, \left(c + d \, x\right)^3 \, \mathsf{Log} \, [\, F \,] \, + \, 12 \, b^2 \, \left(c + d \, x\right)^6 \, \mathsf{Log} \, [\, F \,]^2 - 4 \, b^3 \, \left(c + d \, x\right)^9 \, \mathsf{Log} \, [\, F \,]^3 + b^4 \, \left(c + d \, x\right)^{12} \, \mathsf{Log} \, [\, F \,]^4 \right)$$

Result (type 4, 31 leaves, 1 step):

$$\frac{F^{a} \operatorname{Gamma} \left[ 5, -b \left( c + d x \right)^{3} \operatorname{Log} \left[ F \right] \right]}{3 b^{5} d \operatorname{Log} \left[ F \right]^{5}}$$

#### Problem 312: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{\left(c+dx\right)^{6}} \, dx$$

Optimal (type 3, 92 leaves, 1 step):

$$-\frac{1}{b^{5} d \left(c+d \, x\right)^{4} Log\left[F\right]^{5}} F^{a+\frac{b}{c+d \, x}} \left(24 \left(c+d \, x\right)^{4}-24 \, b \left(c+d \, x\right)^{3} Log\left[F\right]+12 \, b^{2} \left(c+d \, x\right)^{2} Log\left[F\right]^{2}-4 \, b^{3} \left(c+d \, x\right) Log\left[F\right]^{3}+b^{4} Log\left[F\right]^{4}\right)^{2} Log\left[F\right]^{2} + b^{4} Log\left[F\right]^{4} Log\left[F\right]^{4} + b^{4} Log\left[F\right]^{4} Log\left[F\right]^{4} + b^{4} Log\left[F\right]^{4} Log\left[F\right]^{4} + b^{4} Log$$

Result (type 4, 29 leaves, 1 step):

$$-\frac{\mathsf{F}^{\mathsf{a}}\,\mathsf{Gamma}\left[\mathsf{5},-\frac{\mathsf{b}\,\mathsf{Log}\,[\mathsf{F}]}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]}{\mathsf{b}^{\mathsf{5}}\,\mathsf{d}\,\mathsf{Log}\,[\mathsf{F}]^{\mathsf{5}}}$$

#### Problem 313: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{\left(c+dx\right)^{7}} dx$$

Optimal (type 3, 108 leaves, 1 step):

$$\frac{1}{b^{6} d (c + d x)^{5} Log[F]^{6}} F^{a + \frac{b}{c + d x}} \left(120 (c + d x)^{5} - 120 b (c + d x)^{4} Log[F] + 60 b^{2} (c + d x)^{3} Log[F]^{2} - 20 b^{3} (c + d x)^{2} Log[F]^{3} + 5 b^{4} (c + d x) Log[F]^{4} - b^{5} Log[F]^{5}\right)$$

$$Result (type 4, 28 leaves, 1 step):$$

$$\frac{F^{a} Gamma \left[6, -\frac{b Log[F]}{c + d x}\right]}{b^{6} d Log[F]^{6}}$$

#### Problem 325: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{\left(c+dx\right)^{11}} \, dx$$

Optimal (type 3, 96 leaves, 1 step):

$$-\frac{1}{2\;b^{5}\;d\;\left(c+d\;x\right)^{8}\;Log\left[F\right]^{5}}F^{a+\frac{b}{\left(c+d\;x\right)^{2}}}\left(24\;\left(c+d\;x\right)^{8}-24\;b\;\left(c+d\;x\right)^{6}\;Log\left[F\right]+12\;b^{2}\;\left(c+d\;x\right)^{4}\;Log\left[F\right]^{2}-4\;b^{3}\;\left(c+d\;x\right)^{2}\;Log\left[F\right]^{3}+b^{4}\;Log\left[F\right]^{4}\right)$$

Result (type 4, 31 leaves, 1 step):

$$-\frac{\mathsf{F}^{\mathsf{a}}\,\mathsf{Gamma}\left[\,\mathsf{5}\,\mathsf{,}\,-\,\frac{\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{F}\,]}{\left(\,\mathsf{c}\,+\,\mathsf{d}\,\mathsf{x}\,\right)^{\,2}\,\,\right]}}{\,2\,\,\mathsf{b}^{\mathsf{5}}\,\,\mathsf{d}\,\,\mathsf{Log}\,[\,\mathsf{F}\,]^{\,\mathsf{5}}}$$

# Problem 326: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{\left(c+dx\right)^{13}} \, dx$$

Optimal (type 3, 113 leaves, 1 step):

$$\frac{1}{2 \, b^6 \, d \, \left(c + d \, x\right)^{10} \, \text{Log}[F]^6} \\ F^{a + \frac{b}{\left(c + d \, x\right)^2}} \left( 120 \, \left(c + d \, x\right)^{10} - 120 \, b \, \left(c + d \, x\right)^8 \, \text{Log}[F] + 60 \, b^2 \, \left(c + d \, x\right)^6 \, \text{Log}[F]^2 - 20 \, b^3 \, \left(c + d \, x\right)^4 \, \text{Log}[F]^3 + 5 \, b^4 \, \left(c + d \, x\right)^2 \, \text{Log}[F]^4 - b^5 \, \text{Log}[F]^5 \right)$$

Result (type 4, 31 leaves, 1 step):

$$\frac{F^{a}\,\mathsf{Gamma}\left[\,6\,\text{, }-\frac{b\,\mathsf{Log}\,[\,F\,]\,}{\left(\,c\,+\,d\,\,x\,\right)^{\,2}\,}\right]}{2\,\,b^{6}\,\,d\,\,\mathsf{Log}\,[\,F\,]^{\,6}}$$

# Problem 351: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{(c+d\,x)^3}}}{\left(\,c\,+\,d\,x\right)^{\,16}}\,\mathrm{d}x$$

Optimal (type 3, 96 leaves, 1 step):

$$-\frac{1}{3 \, b^5 \, d \, \left(c + d \, x\right)^{12} \, Log \left[F\right]^5} F^{a + \frac{b}{\left(c + d \, x\right)^3}} \left(24 \, \left(c + d \, x\right)^{12} - 24 \, b \, \left(c + d \, x\right)^9 \, Log \left[F\right] + 12 \, b^2 \, \left(c + d \, x\right)^6 \, Log \left[F\right]^2 - 4 \, b^3 \, \left(c + d \, x\right)^3 \, Log \left[F\right]^3 + b^4 \, Log \left[F\right]^4 \right)^2 + b^4 \, Log \left[F\right]^4 + b^4$$

Result (type 4, 31 leaves, 1 step):

$$-\frac{F^{a} Gamma \left[5, -\frac{b Log[F]}{(c+d x)^{3}}\right]}{3 b^{5} d Log[F]^{5}}$$

# Problem 352: Result unnecessarily involves higher level functions.

$$\int \frac{F^{a+\frac{b}{(c+d\,x)^3}}}{\left(\,c\,+\,d\,\,x\right)^{\,19}}\,\,\mathrm{d}x$$

Optimal (type 3, 113 leaves, 1 step):

$$\frac{1}{3 \, b^6 \, d \, \left(c + d \, x\right)^{15} \, Log [F]^6} \\ F^{a + \frac{b}{\left(c + d \, x\right)^3}} \left(120 \, \left(c + d \, x\right)^{15} - 120 \, b \, \left(c + d \, x\right)^{12} \, Log [F] + 60 \, b^2 \, \left(c + d \, x\right)^9 \, Log [F]^2 - 20 \, b^3 \, \left(c + d \, x\right)^6 \, Log [F]^3 + 5 \, b^4 \, \left(c + d \, x\right)^3 \, Log [F]^4 - b^5 \, Log [F]^5 \right)^{12} + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^4 - b^5 \, Log [F]^5 + 20 \, b^4 \, \left(c + d \, x\right)^{12} \, Log [F]^5 + 20 \, b^4 \, Log [F]^5 + 20 \, b^4 \, Log [F]^5 + 20 \,$$

Result (type 4, 31 leaves, 1 step):

$$\frac{F^{a} \operatorname{Gamma}\left[6, -\frac{b \operatorname{Log}[F]}{(c+d \, x)^{3}}\right]}{3 \, b^{6} \, d \operatorname{Log}[F]^{6}}$$

# Problem 368: Result unnecessarily involves higher level functions.

$$\int F^{a+b \ (c+d \ x)^{\, n}} \ \left( \, c \, + \, d \ x \, \right)^{\, -1+6 \, n} \, \mathrm{d} \! \, x$$

Optimal (type 3, 114 leaves, 1 step):

$$-\frac{1}{b^{6} d \, n \, Log \, [F]^{6}} \\ F^{a+b \, (c+d \, x)^{n}} \left(120 - 120 \, b \, \left(c + d \, x\right)^{n} \, Log \, [F] + 60 \, b^{2} \, \left(c + d \, x\right)^{2 \, n} \, Log \, [F]^{2} - 20 \, b^{3} \, \left(c + d \, x\right)^{3 \, n} \, Log \, [F]^{3} + 5 \, b^{4} \, \left(c + d \, x\right)^{4 \, n} \, Log \, [F]^{4} - b^{5} \, \left(c + d \, x\right)^{5 \, n} \, Log \, [F]^{5} \right) \\ Result (type 4, 32 \, leaves, 1 \, step): \\ -\frac{F^{a} \, Gamma \left[6, -b \, \left(c + d \, x\right)^{n} \, Log \, [F]\right]}{b^{6} \, d \, n \, Log \, [F]^{6}}$$

#### Problem 369: Result unnecessarily involves higher level functions.

$$\int F^{a+b \ (c+d \ x)^{\, n}} \ \left( \, c \, + \, d \, \, x \, \right)^{\, -1+5 \, n} \, \, \mathrm{d} \, x$$

Optimal (type 3, 94 leaves, 1 step):

$$\frac{1}{b^{5} d \, n \, Log \, [F]^{5}} F^{a+b \, (c+d \, x)^{\, n}} \, \left(24 - 24 \, b \, \left(c + d \, x\right)^{\, n} \, Log \, [F] \, + \, 12 \, b^{2} \, \left(c + d \, x\right)^{\, 2 \, n} \, Log \, [F]^{\, 2} - \, 4 \, b^{3} \, \left(c + d \, x\right)^{\, 3 \, n} \, Log \, [F]^{\, 3} + \, b^{4} \, \left(c + d \, x\right)^{\, 4 \, n} \, Log \, [F]^{\, 4} \right)$$

Result (type 4, 31 leaves, 1 step):

$$\frac{F^{a}\;Gamma\left[\,5\,\text{, }-b\;\left(\,c\,+\,d\;x\,\right)^{\,n}\;Log\left[\,F\,\right]\,\right]}{b^{5}\;d\;n\;Log\left[\,F\,\right]^{\,5}}$$

#### Problem 586: Result optimal but 1 more steps used.

$$\int e^{Log\left[\left(d+e\,x\right)^{\,n}\right]^{\,2}}\,\left(d+e\,x\right)^{\,m}\,\mathrm{d}x$$

Optimal (type 4, 76 leaves, 3 steps):

$$\frac{e^{-\frac{\left(1+m\right)^{2}}{4\,n^{2}}\,\sqrt{\mathcal{\pi}}\,\left(\,d\,+\,e\,\,x\,\right)^{\,1+m}\,\left(\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right)^{\,-\,\frac{1+m}{n}}\,\text{Erfi}\left[\,\frac{1+m+2\,n\,\text{Log}\left[\,\left(\,d+e\,x\,\right)^{\,n}\,\right]}{2\,n}\,\right]}{2\,e\,n}$$

Result (type 4, 76 leaves, 4 steps):

$$\frac{e^{-\frac{\left(1+m\right)^{2}}{4\,n^{2}}\,\sqrt{\mathcal{\pi}}\,\left(\,d\,+\,e\,\,x\,\right)^{\,1+m}\,\left(\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right)^{\,-\frac{1+m}{n}}\,\text{Erfi}\left[\,\frac{\,1+m+2\,n\,\text{Log}\left[\,\left(\,d+e\,x\,\right)^{\,n}\,\right]}{\,2\,n}\,\right]}{\,2\,e\,n}$$

### Problem 587: Result valid but suboptimal antiderivative.

Optimal (type 4, 137 leaves, 3 steps):

$$\frac{e^{-\frac{\left(1+m\right)^{2}}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^{\,n}\right)^{-\frac{1+m}{n}}\,\left(d\,g+e\,g\,x\right)^{\,1+m}\,Erfi\!\left[\,\frac{1+m+2\,b\,f\,n\,Log\left[F\right]\,Log\!\left[c\,\left(d+e\,x\right)^{\,n}\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\,\right]}{2\,\sqrt{b}\,\left(e^{-\frac{1}{2}\,b}\right)^{\,n}}$$

Result (type 4, 136 leaves, 4 steps):

$$\frac{ e^{-\frac{\left(1+m\right)^{2}}{4\,b\,f\,n^{2}\,Log\left[F\right]}\,F^{a\,f}\,\sqrt{\pi}\,\left(g\,\left(d+e\,x\right)\right)^{\,1+m}\,\left(c\,\left(d+e\,x\right)^{\,n}\right)^{\,-\frac{1+m}{n}}\,Erfi\left[\,\frac{1+m+2\,b\,f\,n\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\right]}{2\,\sqrt{b}\,\left(g\,\left(d+e\,x\right)^{\,n}\right)^{\,1+m}\,\left(g\,\left(d+e\,x\right)^{\,n$$

## Problem 588: Result optimal but 2 more steps used.

Optimal (type 4, 123 leaves, 3 steps):

$$\frac{ e^{-\frac{9}{4\,b\,f\,n^2\,Log[F]}}\,F^{a\,f}\,g^2\,\sqrt{\pi}\,\left(d+e\,x\right)^3\,\left(c\,\left(d+e\,x\right)^n\right)^{-3/n}\,Erfi\Big[\frac{3+2\,b\,f\,n\,Log[F]\,Log\Big[c\,\left(d+e\,x\right)^n\Big]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^n} \frac{1}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}$$

Result (type 4, 123 leaves, 5 steps):

$$\frac{ e^{-\frac{9}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,g^{2}\,\sqrt{\pi}\,\left(d+e\,x\right)^{3}\,\left(c\,\left(d+e\,x\right)^{n}\right)^{-3/n}\,Erfi\Big[\,\frac{3+2\,b\,f\,n\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}} \Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^{n}\,\left(d+e\,x\right)^{$$

### Problem 589: Result optimal but 2 more steps used.

Optimal (type 4, 115 leaves, 3 steps):

$$\frac{ e^{-\frac{1}{b + n^2 Log[F]}} \, F^{\text{af}} \, g \, \sqrt{\pi} \, \left( d + e \, x \right)^2 \, \left( c \, \left( d + e \, x \right)^n \right)^{-2/n} \, \text{Erfi} \left[ \frac{1 + b \, f \, n \, Log[F] \, Log \left[ c \, \left( d + e \, x \right)^n \right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} \right]}{2 \, \sqrt{b} \, e \, \sqrt{f} \, n \, \sqrt{Log[F]}}$$

Result (type 4, 115 leaves, 5 steps):

$$\frac{\text{e}^{-\frac{1}{b \, \text{f} \, n^2 \, \text{Log}[F]}} \, F^{\text{af}} \, g \, \sqrt{\pi} \, \left( \text{d} + \text{e} \, x \right)^2 \, \left( \text{c} \, \left( \text{d} + \text{e} \, x \right)^n \right)^{-2/n} \, \text{Erfi} \left[ \frac{1 + b \, \text{f} \, n \, \text{Log}[F] \, \text{Log} \left[ \text{c} \, \left( \text{d} + \text{e} \, x \right)^n \right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{\text{Log}[F]}} \right]}{2 \, \sqrt{b} \, e \, \sqrt{f} \, n \, \sqrt{\text{Log}[F]}}$$

# Problem 590: Result optimal but 1 more steps used.

$$\int\! F^{\text{f}\,\left(a+b\,\text{Log}\left[\,c\,\left(\,d+e\,x\,\right)^{\,n}\,\right]^{\,2}\right)}\,\,\text{d}\,x$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{e^{-\frac{1}{4bfn^2 Log[F]}} F^{af} \sqrt{\pi} \left(d + ex\right) \left(c \left(d + ex\right)^n\right)^{-1/n} Erfi\left[\frac{1 + 2bfn Log[F] Log\left[c \left(d + ex\right)^n\right]}{2\sqrt{b} \sqrt{f} n\sqrt{Log[F]}}\right]}{2\sqrt{b} e\sqrt{f} n\sqrt{Log[F]}}$$

Result (type 4, 118 leaves, 4 steps):

$$\frac{ e^{-\frac{1}{4\,b\,f\,n^2\,Log[F]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\Big[\,\frac{1+2\,b\,f\,n\,Log[F]\,Log\big[c\,\left(d+e\,x\right)^n\big]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\,\Big]}{2\,\sqrt{b}\,e\,\sqrt{f}\,n\,\sqrt{Log[F]}}$$

#### Problem 591: Result optimal but 2 more steps used.

$$\int \frac{\mathsf{F}^{\mathsf{f}}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\,\mathsf{n}}\right]^{\,\mathsf{2}}\right)}{\mathsf{d}\,\mathsf{g}+\mathsf{e}\,\mathsf{g}\,\mathsf{x}}\,\mathsf{d}\,\mathsf{x}$$

Optimal (type 4, 67 leaves, 2 steps):

$$\frac{\mathsf{F^{af}}\,\sqrt{\pi}\,\,\mathsf{Erfi}\big[\sqrt{\mathsf{b}}\,\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{Log}\hspace{0.05cm}[\mathsf{F}]}\,\,\mathsf{Log}\hspace{0.05cm}\big[\,\mathsf{c}\,\,\big(\,\mathsf{d}+\mathsf{e}\,\,\mathsf{x}\big)^{\,\mathsf{n}}\,\big]\,\big]}{2\,\sqrt{\mathsf{b}}\,\,\,\mathsf{e}\,\sqrt{\mathsf{f}}\,\,\mathsf{g}\,\mathsf{n}\,\sqrt{\mathsf{Log}\hspace{0.05cm}[\mathsf{F}]}}$$

Result (type 4, 67 leaves, 4 steps):

$$\frac{\mathsf{F^{af}}\,\sqrt{\pi}\,\,\mathsf{Erfi}\big[\,\sqrt{\mathsf{b}}\,\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{Log}\,[\mathsf{F}\,]}\,\,\mathsf{Log}\big[\,\mathsf{c}\,\,\big(\,\mathsf{d}+\mathsf{e}\,\,\mathsf{x}\big)^{\,\mathsf{n}}\,\big]\,\big]}{2\,\sqrt{\mathsf{b}}\,\,\mathsf{e}\,\sqrt{\mathsf{f}}\,\,\mathsf{g}\,\mathsf{n}\,\sqrt{\mathsf{Log}\,[\mathsf{F}\,]}}$$

# Problem 592: Result optimal but 2 more steps used.

$$\int \frac{\mathsf{F}^{\mathsf{f}}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]^{2}\right)}{\left(\mathsf{d}\,\mathsf{g}+\mathsf{e}\,\mathsf{g}\,\mathsf{x}\right)^{2}}\,\mathrm{d}\mathsf{x}$$

Optimal (type 4, 121 leaves, 3 steps):

$$-\frac{\text{e}^{-\frac{1}{4\,\text{bfn}^2\,\text{Log}[F]}}\,F^{\text{af}}\,\sqrt{\pi}\,\left(c\,\left(\text{d}+e\,x\right)^n\right)^{\frac{1}{n}}\,\text{Erfi}\left[\,\frac{1\!-\!2\,\text{bfn}\,\text{Log}[F]\,\,\text{Log}\left[c\,\left(\text{d}\!+\!e\,x\right)^n\right]}{2\,\sqrt{b}\,\,\sqrt{f}\,\,n\,\sqrt{\text{Log}[F]}}\right]}{2\,\sqrt{b}\,\,e\,\sqrt{f}\,\,g^2\,n\,\left(\text{d}+e\,x\right)\,\sqrt{\text{Log}[F]}}$$

Result (type 4, 121 leaves, 5 steps):

$$-\frac{e^{-\frac{1}{4\,b\,f\,n^2\,Log\left[F\right]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^n\right)^{\frac{1}{n}}\,Erfi\Big[\,\frac{1-2\,b\,f\,n\,Log\left[F\right]\,Log\Big[c\,\left(d+e\,x\right)^n\Big]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)\,\sqrt{Log\left[F\right]}}$$

# Problem 593: Result optimal but 2 more steps used.

$$\int \frac{F^{f\left(a+b \log\left[c \left(d+e x\right)^{n}\right]^{2}\right)}}{\left(d g+e g x\right)^{3}} \, dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$-\frac{\text{e}^{-\frac{1}{b \cdot \text{f} \cdot n^2 \cdot \text{Log}[F]}} \, F^{\text{af}} \, \sqrt{\pi} \, \left(\text{c} \, \left(\text{d} + \text{e} \, \text{x}\right)^{\text{n}}\right)^{2/\text{n}} \, \text{Erfi}\left[\frac{1 - \text{b} \cdot \text{f} \, \text{n} \, \text{Log}[F] \, \text{Log}\left[\text{c} \, \left(\text{d} + \text{e} \, \text{x}\right)^{\text{n}}\right]}{\sqrt{b} \, \sqrt{f} \, \, \text{n} \, \sqrt{\text{Log}[F]}}\right]}{2 \, \sqrt{b} \, \, \text{e} \, \sqrt{f} \, \, \text{g}^{\text{3}} \, \text{n} \, \left(\text{d} + \text{e} \, \text{x}\right)^{2} \, \sqrt{\text{Log}[F]}}$$

Result (type 4, 118 leaves, 5 steps):

$$-\frac{\mathrm{e}^{-\frac{1}{b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^{n}\right)^{2/n}\,Erfi\!\left[\frac{1-b\,f\,n\,Log\left[F\right]\,Log\!\left[c\,\left(d+e\,x\right)^{n}\right]}{\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\right]}{2\,\sqrt{b}\,\,e\,\sqrt{f}\,g^{3}\,n\,\left(d+e\,x\right)^{2}\,\sqrt{Log\left[F\right]}}$$

# Problem 594: Result valid but suboptimal antiderivative.

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[ F^{f(a+b\log[c(d+ex)^n]^2)}(g+hx)^m, x \right]$$

Result (type 8, 30 leaves, 0 steps):

CannotIntegrate 
$$\left[\,\mathsf{F}^{\,\mathsf{f}\,\,\left(\,\mathsf{a}\,+\,\mathsf{b}\,\,\mathsf{Log}\,\left[\,\mathsf{c}\,\,\left(\,\mathsf{d}\,+\,\mathsf{e}\,\,x\,\right)\,^{\,\mathsf{n}}\,\right]\,^{\,\mathsf{2}}\,\right)}\,\,\left(\,\mathsf{g}\,+\,\mathsf{h}\,\,x\,\right)^{\,\mathsf{m}}\,,\,\,x\,\right]$$

#### Problem 595: Unable to integrate problem.

$$\Big\lceil F^{f \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^{\, n}\right]^{\, 2}\right)} \, \left(g + h \, x\right)^{\, 3} \, \mathbb{d} x \\$$

Optimal (type 4, 502 leaves, 14 steps):

$$\frac{3 \, e^{-\frac{1}{b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h \, \left(e \, g \, -d \, h\right)^2 \, \sqrt{\pi} \, \left(d \, +e \, x\right)^2 \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-2/n} \, Erfi\left[\frac{1 + b \, f \, n \, Log[F] \, Log\left[c \, \left(d +e \, x\right)^n\right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{e^{-\frac{4}{b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h^3 \, \sqrt{\pi} \, \left(d \, +e \, x\right)^4 \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-4/n} \, Erfi\left[\frac{2 + b \, f \, n \, Log[F] \, Log\left[c \, \left(d +e \, x\right)^n\right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{e^{-\frac{1}{4b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, \left(e \, g \, -d \, h\right)^3 \, \sqrt{\pi} \, \left(d \, +e \, x\right) \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-1/n} \, Erfi\left[\frac{1 + 2 \, b \, f \, n \, Log[F] \, Log\left[c \, \left(d +e \, x\right)^n\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{3 \, e^{-\frac{9}{4b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h^2 \, \left(e \, g \, -d \, h\right) \, \sqrt{\pi} \, \left(d \, +e \, x\right)^3 \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-3/n} \, Erfi\left[\frac{3 + 2 \, b \, f \, n \, Log[F] \, Log[c \, \left(d \, +e \, x\right)^n\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]} - \\ 2 \, \sqrt{b} \, e^4 \, \sqrt{f} \, n \, \sqrt{Log[F]}$$

Result (type 8, 214 leaves, 6 steps):

$$3 \, g^2 \, h \, \text{CannotIntegrate} \left[ F^{f \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]^2\right)} \, x \, , \, x \, \right] \, + \, 3 \, g \, h^2 \, \text{CannotIntegrate} \left[ F^{f \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]^2\right)} \, x^2 \, , \, x \, \right] \, + \\ \\ e^{-\frac{1}{4 \, b \, f \, n^2 \, Log \left[F\right]}} \, F^{a \, f} \, g^3 \, \sqrt{\pi} \, \left(d + e \, x\right) \, \left(c \, \left(d + e \, x\right)^n\right)^{-1/n} \, \text{Erfi} \left[ \frac{1 + 2 \, b \, f \, n \, Log \left[F\right] \, Log \left[c \, \left(d + e \, x\right)^n\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log \left[F\right]}} \right] \\ \\ h^3 \, \text{CannotIntegrate} \left[ F^{f \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]^2\right)} \, x^3 \, , \, x \, \right] \, + \\ \\ 2 \, \sqrt{b} \, e \, \sqrt{f} \, n \, \sqrt{Log \left[F\right]} \right] \\ \\ 2 \, \sqrt{b} \, e \, \sqrt{f} \, n \, \sqrt{Log \left[F\right]}$$

# Problem 596: Unable to integrate problem.

Optimal (type 4, 372 leaves, 11 steps):

$$\frac{e^{-\frac{1}{b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h \, \left(e \, g \, -d \, h\right) \, \sqrt{\pi} \, \left(d \, +e \, x\right)^2 \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-2/n} \, Erfi \left[\frac{1 + b \, f \, n \, Log[F] \, Log[c \, (d +e \, x)^n]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{e^{-\frac{1}{4 \, b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, \left(e \, g \, -d \, h\right)^2 \, \sqrt{\pi} \, \left(d \, +e \, x\right) \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-1/n} \, Erfi \left[\frac{1 + 2 \, b \, f \, n \, Log[F] \, Log[c \, (d +e \, x)^n]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{e^{-\frac{9}{4 \, b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h^2 \, \sqrt{\pi} \, \left(d \, +e \, x\right)^3 \, \left(c \, \left(d \, +e \, x\right)^n\right)^{-3/n} \, Erfi \left[\frac{3 + 2 \, b \, f \, n \, Log[F] \, Log[c \, (d +e \, x)^n]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}$$

#### Result (type 8, 180 leaves, 6 steps):

$$2\,g\,h\,CannotIntegrate\Big[F^{f\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]^{\,2}\right)}\,x,\,x\Big] + h^{2}\,CannotIntegrate\Big[F^{f\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]^{\,2}\right)}\,x^{2},\,x\Big] + \\ \underbrace{e^{-\frac{1}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,g^{2}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^{\,n}\right)^{\,-1/n}\,Erfi\Big[\frac{1+2\,b\,f\,n\,Log\left[F\right)\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\Big]} \\ \underbrace{2\,\sqrt{b}\,\,e\,\sqrt{f}\,\,n\,\sqrt{Log\left[F\right]}}$$

### Problem 597: Unable to integrate problem.

Optimal (type 4, 242 leaves, 8 steps):

$$\frac{e^{-\frac{1}{b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, h \, \sqrt{\pi} \, \left(d + e \, x\right)^2 \, \left(c \, \left(d + e \, x\right)^n\right)^{-2/n} \, Erfi\left[\frac{1 + b \, f \, n \, Log[F] \, Log\left[c \, \left(d + e \, x\right)^n\right]}{\sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{2 \, \sqrt{b} \, e^2 \, \sqrt{f} \, n \, \sqrt{Log[F]}} + \\ \frac{e^{-\frac{1}{4 \, b \, f \, n^2 \, Log[F]}} \, F^{a \, f} \, \left(e \, g - d \, h\right) \, \sqrt{\pi} \, \left(d + e \, x\right) \, \left(c \, \left(d + e \, x\right)^n\right)^{-1/n} \, Erfi\left[\frac{1 + 2 \, b \, f \, n \, Log[F] \, Log\left[c \, \left(d + e \, x\right)^n\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}\right]}{2 \, \sqrt{b} \, \sqrt{f} \, n \, \sqrt{Log[F]}}$$

Result (type 8, 146 leaves, 6 steps):

$$\mathbb{e}^{-\frac{1}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,g\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^{n}\right)^{-1/n}\,Erfi\left[\frac{1+2\,b\,f\,n\,Log\left[E\right]\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\right]\\ +\frac{2\,\sqrt{b}\,\left(e^{-\frac{1}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,\left(e^{-\frac{1}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,g\,\sqrt{\pi}\right)}{2\,\sqrt{b}\,\left(e^{-\frac{1}{4\,b\,f\,n^{2}\,Log\left[F\right]}}\,F^{a\,f}\,g\,\sqrt{\pi}\right)}$$

#### Problem 598: Result optimal but 1 more steps used.

$$\int F^{f(a+b\log[c(d+ex)^n]^2)} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\frac{ e^{-\frac{1}{4\,b\,f\,n^2\,Log[F]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^{\,n}\right)^{\,-1/n}\,Erfi\Big[\,\frac{1+2\,b\,f\,n\,Log[F]\,Log\Big[c\,\left(d+e\,x\right)^{\,n}\Big]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log[F]}}\Big]}{2\,\sqrt{b}\,\left(d+e\,x\right)^{\,n}\,$$

Result (type 4, 118 leaves, 4 steps):

$$\frac{ e^{-\frac{1}{4\,b\,f\,n^2\,Log\left[F\right]}}\,F^{a\,f}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\frac{1+2\,b\,f\,n\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,\sqrt{b}\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}\right]}{2\,\sqrt{b}\,\left(e^{-\frac{1}{4\,b\,f\,n^2\,Log\left[F\right]}}\right)}$$

# Problem 599: Result valid but suboptimal antiderivative.

$$\int \frac{F^{f\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]^{\,2}\right)}}{g+h\,x}\,\mathrm{d}x$$

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\mathsf{F}^{\mathsf{f}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]^{2}\right)}}{\mathsf{g}+\mathsf{h}\,\mathsf{x}},\;\mathsf{x}\right]$$

Result (type 8, 30 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{F^{f(a+b\log[c(d+ex)^n]^2)}}{g+hx}, x\right]$$

# Problem 600: Result valid but suboptimal antiderivative.

$$\int \frac{F^{f\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]^{\, 2}\right)}}{\left(g+h\, x\right)^{\, 2}} \, \mathrm{d} x$$

Optimal (type 8, 30 leaves, 0 steps):

$$\label{eq:continuous_loss} \text{Unintegrable} \Big[ \, \frac{ F^{f \, \left( a + b \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right]^{\, 2} \right)}}{ \left( g + h \, x \right)^{\, 2}} \, \text{, } \, x \, \Big]$$

Result (type 8, 30 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \begin{aligned} & \mathsf{CannotIntegrate} \left[ \, \frac{\mathsf{F}^{\mathsf{f} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \, \right)^{\, \mathsf{n}} \, \right]^{\, \mathsf{2}} \right)}{\left( \mathsf{g} + \mathsf{h} \, \mathsf{x} \, \right)^{\, \mathsf{2}}} \, \text{, } \, \mathsf{x} \, \right] \end{aligned}$$

# Problem 601: Result valid but suboptimal antiderivative.

$$\int\!\frac{F^{f\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]^{\,2}\right)}}{\left(g+h\,x\right)^{\,3}}\,\mathrm{d}x$$

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\mathsf{F}^{\mathsf{f}}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]^{2}\right)}{\left(\mathsf{g}+\mathsf{h}\,\mathsf{x}\right)^{3}},\;\mathsf{x}\right]$$

Result (type 8, 30 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{F^{f(a+b\log[c(d+ex)^n]^2)}}{(g+hx)^3}, x\right]$$

## Problem 602: Result valid but suboptimal antiderivative.

Optimal (type 4, 153 leaves, 4 steps):

$$\frac{1}{2\,b\,e\,\sqrt{f}\,\,n\,\sqrt{\text{Log}\,[F]}}e^{-\frac{\left[1+m+2\,a\,b\,f\,n\,\text{Log}\,[F]\right)^{2}}{4\,b^{2}\,f\,n^{2}\,\text{Log}\,[F]}}\,F^{a^{2}\,f}\,\sqrt{\pi}\,\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^{n}\right)^{-\frac{1+m}{n}}\,\left(d\,g+e\,g\,x\right)^{m}\,\text{Erfi}\,\Big[\frac{1+m+2\,a\,b\,f\,n\,\text{Log}\,[F]\,+2\,b^{2}\,f\,n\,\text{Log}\,[F]\,\,\text{Log}\,\Big[\,c\,\left(d+e\,x\right)^{n}\,\Big]}{2\,b\,\sqrt{f}\,\,n\,\sqrt{\text{Log}\,[F]}}\Big]$$

Result (type 4, 152 leaves, 8 steps):

$$\frac{1}{2\,b\,e\,\sqrt{f}\,\,n\,\sqrt{\text{Log}\,[F\,]}} e^{-\frac{\left(1+m+2\,a\,b\,f\,n\,\text{Log}\,[F\,]\right)^{2}}{4\,b^{2}\,f\,n^{2}\,\text{Log}\,[F]}}\,F^{a^{2}\,f}\,\sqrt{\pi}\,\,\left(d+e\,x\right)\,\left(g\,\left(d+e\,x\right)\right)^{m}\,\left(c\,\left(d+e\,x\right)^{n}\right)^{-\frac{1+m}{n}}\,\text{Erfi}\,\left[\,\frac{1+m+2\,a\,b\,f\,n\,\text{Log}\,[F\,]\,+2\,b^{2}\,f\,n\,\text{Log}\,[F\,]\,\,\text{Log}\,\left[c\,\left(d+e\,x\right)^{n}\right]}{2\,b\,\sqrt{f}\,\,n\,\sqrt{\text{Log}\,[F\,]}}\,\right]$$

#### Problem 603: Result optimal but 4 more steps used.

Optimal (type 4, 133 leaves, 4 steps):

$$\frac{ e^{-\frac{3\left(3+4\,a\,b\,f\,n\,Log\left[F\right]\right)}{4\,b^2\,f\,n^2\,Log\left[F\right]}}\,g^2\,\sqrt{\pi}\,\left(d+e\,x\right)^3\,\left(c\,\left(d+e\,x\right)^n\right)^{-3/n}\,\text{Erfi}\left[\,\frac{\frac{3}{n}+2\,a\,b\,f\,Log\left[F\right]+2\,b^2\,f\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\left[F\right]}}\right]}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}$$

Result (type 4, 133 leaves, 8 steps):

$$\frac{e^{\frac{-3\left(3+4\,a\,b\,f\,n\,Log\left[F\right]\right)}{4\,b^2\,f\,n^2\,Log\left[F\right]}}\,g^2\,\sqrt{\pi}\,\left(d+e\,x\right)^3\,\left(c\,\left(d+e\,x\right)^n\right)^{-3/n}\,Erfi\Big[\,\frac{\frac{3}{n}+2\,a\,b\,f\,Log\left[F\right]+2\,b^2\,f\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\left[F\right]}}\Big]}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}$$

# Problem 604: Result optimal but 4 more steps used.

Optimal (type 4, 122 leaves, 4 steps):

$$e^{-\frac{1+2\,a\,b\,f\,n\,Log\,[F]}{b^2\,f\,n^2\,Log\,[F]}}\,g\,\sqrt{\pi}\,\left(d+e\,x\right)^2\,\left(c\,\left(d+e\,x\right)^n\right)^{-2/n}\,Erfi\left[\,\frac{\frac{1}{n}+a\,b\,f\,Log\,[F]+b^2\,f\,Log\,[F]\,Log\,\left[c\,\left(d+e\,x\right)^n\right]}{b\,\sqrt{f}\,\sqrt{Log\,[F]}}\,\right]$$

Result (type 4, 122 leaves, 8 steps):

$$\frac{e^{\frac{-1+2\,a\,b\,f\,n\,Log\,[F]}{b^2\,f\,n^2\,Log\,[F]}}\,g\,\sqrt{\pi}\,\left(d+e\,x\right)^2\,\left(c\,\left(d+e\,x\right)^n\right)^{-2/n}\,Erfi\Big[\frac{\frac{1}{n}+a\,b\,f\,Log\,[F]\,+b^2\,f\,Log\,[F]\,\,Log\,\left[c\,\left(d+e\,x\right)^n\right]}{b\,\sqrt{f}\,\sqrt{Log\,[F]}}\Big]}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\,[F]}}$$

# Problem 605: Result optimal but 3 more steps used.

$$\int\! F^{f\,\left(a+b\,Log\left[\,c\,\left(\,d+e\,x\,\right)^{\,n}\,\right]\,\right)^{\,2}}\,\text{d} \,x$$

Optimal (type 4, 126 leaves, 4 steps):

$$e^{-\frac{1+4\,a\,b\,f\,n\,Log\,[F]}{4\,b^2\,f\,n^2\,Log\,[F]}}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\frac{1}{n}+2\,a\,b\,f\,Log\,[F]+2\,b^2\,f\,Log\,[F]\,Log\,\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]$$

Result (type 4, 126 leaves, 7 steps):

$$e^{-\frac{1+4\,a\,b\,f\,n\,Log\,[F]}{4\,b^2\,f\,n^2\,Log\,[F]}}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\, Erfi\left[\,\frac{\frac{1}{n}+2\,a\,b\,f\,Log\,[F]+2\,b^2\,f\,Log\,[F]\,\,Log\,\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\,\right] \\ -\frac{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\,[F]}}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\,[F]}}$$

# Problem 606: Result optimal but 4 more steps used.

$$\int \frac{\mathsf{F}^{\mathsf{f} (\mathsf{a}+\mathsf{b} \, \mathsf{Log} \left[\mathsf{c} \, (\mathsf{d}+\mathsf{e} \, \mathsf{x})^{\,\mathsf{n}}\right])^{\,\mathsf{2}}}}{\mathsf{d} \, \mathsf{g} + \mathsf{e} \, \mathsf{g} \, \mathsf{x}} \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 4, 70 leaves, 4 steps):

$$\frac{\sqrt{\pi} \ \mathsf{Erfi} \big[ \mathsf{a} \ \sqrt{\mathsf{f}} \ \sqrt{\mathsf{Log}[\mathsf{F}]} \ + \mathsf{b} \ \sqrt{\mathsf{f}} \ \sqrt{\mathsf{Log}[\mathsf{F}]} \ \mathsf{Log} \big[ \mathsf{c} \ \big( \mathsf{d} + \mathsf{e} \ \mathsf{x} \big)^{\mathsf{n}} \big] \big]}{2 \ \mathsf{b} \ \mathsf{e} \ \sqrt{\mathsf{f}} \ \mathsf{g} \ \mathsf{n} \ \sqrt{\mathsf{Log}[\mathsf{F}]}}$$

Result (type 4, 70 leaves, 8 steps):

$$\frac{\sqrt{\pi} \ \mathsf{Erfi} \big[ \mathsf{a} \ \sqrt{\mathsf{f}} \ \sqrt{\mathsf{Log}[\mathsf{F}]} \ + \mathsf{b} \ \sqrt{\mathsf{f}} \ \sqrt{\mathsf{Log}[\mathsf{F}]} \ \mathsf{Log} \big[ \mathsf{c} \ \big( \mathsf{d} + \mathsf{e} \ \mathsf{x} \big)^{\mathsf{n}} \big] \big]}{2 \, \mathsf{b} \, \mathsf{e} \ \sqrt{\mathsf{f}} \ \mathsf{g} \, \mathsf{n} \ \sqrt{\mathsf{Log}[\mathsf{F}]}}$$

# Problem 607: Result optimal but 4 more steps used.

$$\int \frac{F^{f\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right)^{\, 2}}}{\left(d\, g+e\, g\, x\right)^{\, 2}}\, \mathrm{d} x$$

Optimal (type 4, 128 leaves, 4 steps):

$$-\frac{\frac{\frac{a}{e^{\frac{1}{b\,n}}-\frac{1}{4\,b^{2}\,f\,n^{2}\,Log\,[F]}}\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^{n}\right)^{\frac{1}{n}}\,\text{Erfi}\left[\frac{\frac{1}{n}-2\,a\,b\,f\,Log\,[F]-2\,b^{2}\,f\,Log\,[F]\,Log\,\left[c\,\left(d+e\,x\right)^{n}\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]}{2\,b\,e\,\sqrt{f}\,g^{2}\,n\,\left(d+e\,x\right)\,\sqrt{Log\,[F]}}$$

Result (type 4, 128 leaves, 8 steps):

$$-\frac{e^{\frac{a}{b\,n}-\frac{1}{4\,b^2\,f\,n^2\,Log\,[F]}}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^n\right)^{\frac{1}{n}}\,Erfi\left[\,\frac{\frac{1}{n}-2\,a\,b\,f\,Log\,[F]-2\,b^2\,f\,Log\,[F]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\,\right]}{2\,b\,e\,\sqrt{f}\,g^2\,n\,\left(d+e\,x\right)\,\sqrt{Log\,[F]}}$$

# Problem 608: Result optimal but 4 more steps used.

$$\int \! \frac{F^{f \, \left(a+b \, Log\left[c \, \left(d+e \, x\right)^{\, n}\right]\right)^{\, 2}}}{\left(d \, g+e \, g \, x\right)^{\, 3}} \, \mathrm{d} x$$

Optimal (type 4, 126 leaves, 4 steps):

$$-\frac{e^{-\frac{1-2\,a\,b\,f\,n\,Log[F]}{b^2\,f\,n^2\,Log[F]}}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^n\right)^{2/n}\,Erfi\left[\frac{\frac{1}{n}-a\,b\,f\,Log[F]\,-b^2\,f\,Log[F]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{b\,\sqrt{f}\,\sqrt{Log[F]}}\right]}{2\,b\,e\,\sqrt{f}\,g^3\,n\,\left(d+e\,x\right)^2\,\sqrt{Log\,[F]}}$$

Result (type 4, 126 leaves, 8 steps):

$$-\frac{e^{-\frac{1-2\,a\,b\,f\,n\,Log\,[F]}{b^2\,f\,n^2\,Log\,[F]}}\,\sqrt{\pi}\,\left(c\,\left(d+e\,x\right)^n\right)^{2/n}\,\text{Erfi}\left[\,\frac{\frac{1}{n}-a\,b\,f\,Log\,[F]\,-b^2\,f\,Log\,[F]\,\,Log\,\left[c\,\left(d+e\,x\right)^n\right]}{b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]}{2\,b\,e\,\sqrt{f}\,g^3\,n\,\left(d+e\,x\right)^2\,\sqrt{Log\,[F]}}$$

# Problem 609: Result valid but suboptimal antiderivative.

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[F^{f(a+b\log[c(d+ex)^n])^2}(g+hx)^m,x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$CannotIntegrate\left[\,F^{f\,\left(\,a+b\,Log\left[\,c\,\left(\,d+e\,x\,\right)\,^{\,n}\,\right]\,\right)^{\,2}}\,\left(\,g\,+\,h\,x\,\right)^{\,m}\text{, }x\,\right]$$

# Problem 610: Unable to integrate problem.

Optimal (type 4, 535 leaves, 18 steps):

$$\frac{3 \, e^{-\frac{1+2 \, ab \, f \, n \, \log[F]}{b^3 \, f \, n^2 \, \log[F]}} \, h \, \left(e \, g - d \, h\right)^2 \, \sqrt{\pi} \, \left(d + e \, x\right)^2 \, \left(c \, \left(d + e \, x\right)^n\right)^{-2/n} \, Erfi\left[\frac{\frac{1}{n} + a \, b \, f \, \log[F] + b^2 \, f \, \log[C] \, \log[C] \, (d + e \, x)^n}{b \, \sqrt{f} \, \sqrt{\log[F]}}\right]}{b \, \sqrt{f} \, \sqrt{\log[F]}} + \frac{2 \, b \, e^4 \, \sqrt{f} \, n \, \sqrt{\log[F]}}{e^{-\frac{4 \, (1+a \, b \, f \, n \, \log[F])}{b^3 \, f \, n^2 \, \log[F]}} \, h^3 \, \sqrt{\pi} \, \left(d + e \, x\right)^4 \, \left(c \, \left(d + e \, x\right)^n\right)^{-4/n} \, Erfi\left[\frac{\frac{2}{n} + a \, b \, f \, \log[F] + b^2 \, f \, \log[C] \, (d + e \, x)^n}{b \, \sqrt{f} \, \sqrt{\log[F]}}\right]} + \frac{2 \, b \, e^4 \, \sqrt{f} \, n \, \sqrt{\log[F]}}{2 \, b \, \sqrt{f} \, \sqrt{\log[F]}} + \frac{2 \, b \, e^4 \, \sqrt{f} \, n \, \sqrt{\log[F]}}{2 \, b \, \sqrt{f} \, \sqrt{\log[F]}} + \frac{1}{2 \, b \, e^4 \, \sqrt{f} \, n \, \sqrt{\log[F]}} + \frac{1}{2 \, b \, e^4 \, \sqrt{f} \, n \, \sqrt{\log[F]}}$$

#### Result (type 8, 222 leaves, 9 steps):

# Problem 611: Unable to integrate problem.

Optimal (type 4, 397 leaves, 14 steps):

$$\frac{e^{-\frac{1+2\,a\,b\,f\,n\,log[F]}{b^2\,f\,n^2\,log[F]}}\,h\,\left(e\,g-d\,h\right)\,\sqrt{\pi}\,\left(d+e\,x\right)^2\,\left(c\,\left(d+e\,x\right)^n\right)^{-2/n}\,Erfi\Big[\,\frac{\frac{1}{n}+a\,b\,f\,log[F]+b^2\,f\,log[F]\,log\big[c\,\left(d+e\,x\right)^n\big]}{b\,\sqrt{f}\,\sqrt{log[F]}}\,\Big]}{b\,e^3\,\sqrt{f}\,n\,\sqrt{log[F]}}+\frac{b\,e^3\,\sqrt{f}\,n\,\sqrt{log[F]}}{e^{-\frac{1+4\,a\,b\,f\,n\,log[F]}{4\,b^2\,f\,n^2\,log[F]}}\left(e\,g-d\,h\right)^2\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\Big[\,\frac{\frac{1}{n}+2\,a\,b\,f\,log[F]+2\,b^2\,f\,log[F]\,log\big[c\,\left(d+e\,x\right)^n\big]}{2\,b\,\sqrt{f}\,\sqrt{log[F]}}\Big]}{2\,b\,e^3\,\sqrt{f}\,n\,\sqrt{log[F]}}$$

$$\frac{e^{-\frac{3\,\left(3+4\,a\,b\,f\,n\,log[F]\right)}{4\,b^2\,f\,n^2\,log[F]}}\,h^2\,\sqrt{\pi}\,\left(d+e\,x\right)^3\,\left(c\,\left(d+e\,x\right)^n\right)^{-3/n}\,Erfi\Big[\,\frac{\frac{3}{n}+2\,a\,b\,f\,log[F]+2\,b^2\,f\,log[F]\,log\big[c\,\left(d+e\,x\right)^n\big]}{2\,b\,\sqrt{f}\,\sqrt{log[F]}}\Big]}$$

#### Result (type 8, 188 leaves, 9 steps):

$$2\,g\,h\,CannotIntegrate\Big[F^{f\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}\,x\,\text{, }x\,\Big] \,+\,h^{2}\,CannotIntegrate\Big[F^{f\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}\,x^{2}\,\text{, }x\,\Big] \,+\,\\ \frac{e^{-\frac{1+4\,a\,b\,f\,n\,Log\left[F\right]}{4\,b^{\,2}\,f\,n^{\,2}\,Log\left[F\right]}}\,g^{2}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^{\,n}\right)^{\,-1/n}\,Erfi\Big[\frac{\frac{1}{n}+2\,a\,b\,f\,Log\left[F\right]+2\,b^{\,2}\,f\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\left[F\right]}}\Big]} \\ = \frac{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}$$

# Problem 612: Unable to integrate problem.

Optimal (type 4, 257 leaves, 10 steps):

$$\frac{e^{-\frac{1+2\,a\,b\,f\,n\,Log\,[F]}{b^2\,f\,n^2\,Log\,[F]}}\,h\,\sqrt{\pi}\,\left(d+e\,x\right)^2\,\left(c\,\left(d+e\,x\right)^n\right)^{-2/n}\,Erfi\left[\frac{\frac{1}{n}+a\,b\,f\,Log\,[F]+b^2\,f\,Log\,[F]\,Log\,[c\,\left(d+e\,x\right)^n\right]}{b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]}{2\,b\,e^2\,\sqrt{f}\,n\,\sqrt{Log\,[F]}}+\\\\ \frac{e^{-\frac{1+4\,a\,b\,f\,n\,Log\,[F]}{4\,b^2\,f\,n^2\,Log\,[F]}}\,\left(e\,g-d\,h\right)\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\frac{\frac{1}{n}+2\,a\,b\,f\,Log\,[F]+2\,b^2\,f\,Log\,[F]\,Log\,[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]}\\ 2\,b\,e^2\,\sqrt{f}\,n\,\sqrt{Log\,[F]}$$

Result (type 8, 154 leaves, 9 steps):

$$e^{-\frac{1\cdot4\,a\,b\,f\,n\,\log[F]}{4\,b^2\,f\,n^2\,\log[F]}}\,g\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\frac{\frac{1}{n}+2\,a\,b\,f\,\log[F]+2\,b^2\,f\,\log[F]\,\log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{\log[F]}}\right]\\ +\frac{2\,b\,\sqrt{f}\,\sqrt{\log[F]}}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{\log[F]}}$$

#### Problem 613: Result optimal but 3 more steps used.

$$\left\lceil F^{f\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}\,\text{d}\,x\right.$$

Optimal (type 4, 126 leaves, 4 steps):

$$= \frac{ e^{-\frac{1+4\,a\,b\,f\,n\,Log\left[F\right]}{4\,b^2\,f\,n^2\,Log\left[F\right]}}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\,\frac{\frac{1}{n}+2\,a\,b\,f\,Log\left[F\right]+2\,b^2\,f\,Log\left[F\right]\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\left[F\right]}}\,\right]}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\left[F\right]}}$$

Result (type 4, 126 leaves, 7 steps):

$$\frac{ e^{-\frac{1+4\,a\,b\,f\,n\,Log\,[F]}{4\,b^2\,f\,n^2\,Log\,[F]}}\,\sqrt{\pi}\,\left(d+e\,x\right)\,\left(c\,\left(d+e\,x\right)^n\right)^{-1/n}\,Erfi\left[\frac{\frac{1}{n}+2\,a\,b\,f\,Log\,[F]+2\,b^2\,f\,Log\,[F]\,\,Log\left[c\,\left(d+e\,x\right)^n\right]}{2\,b\,\sqrt{f}\,\sqrt{Log\,[F]}}\right]}{2\,b\,e\,\sqrt{f}\,n\,\sqrt{Log\,[F]}}$$

# Problem 614: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Ff}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]\right)^{2}}{\mathsf{g}+\mathsf{h}\,\mathsf{x}}\,\mathsf{d}\,\mathsf{x}$$

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\mathsf{F}^{\mathsf{f}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]\right)^{2}}}{\mathsf{g}+\mathsf{h}\,\mathsf{x}}\right]$$

Result (type 8, 30 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{F^{f(a+b\log[c(d+ex)^n])^2}}{g+hx}, x\right]$$

# Problem 615: Result valid but suboptimal antiderivative.

$$\int \frac{F^{f\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}}{\left(g+h\,x\right)^{\,2}}\,\mathrm{d}x$$

Optimal (type 8, 30 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\mathsf{F}^{\mathsf{f}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\left(\mathsf{d}+\mathsf{e}\,\mathsf{x}\right)^{\mathsf{n}}\right]\right)^{2}}}{\left(\mathsf{g}+\mathsf{h}\,\mathsf{x}\right)^{2}},\;\mathsf{x}\right]$$

Result (type 8, 30 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{F^{f(a+b\log[c(d+ex)^n])^2}}{(g+hx)^2}, x\right]$$

# Problem 616: Result valid but suboptimal antiderivative.

$$\int \frac{F^{f\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right)^{\, 2}}}{\left(g+h\, x\right)^{\, 3}}\, \mathrm{d} x$$

Optimal (type 8, 30 leaves, 0 steps):

$$\label{eq:unintegrable} Unintegrable \Big[ \, \frac{F^{\,f \, \left( a + b \, Log \left[ c \, \left( d + e \, x \right) \, ^n \right] \, \right)^{\,2}}}{\left( g + h \, x \right)^{\,3}} \text{, } x \, \Big]$$

Result (type 8, 30 leaves, 0 steps):

$$CannotIntegrate \Big[ \, \frac{F^{f \, \left( a + b \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right] \right)^{\, 2}}}{\left( g + h \, x \right)^{\, 3}} \text{, } x \, \Big]$$

# Problem 692: Unable to integrate problem.

$$\int e^{x^x} x^{2x} \left(1 + Log[x]\right) dx$$

Optimal (type 3, 11 leaves, ? steps):

$$e^{x^x} \left(-1 + x^x\right)$$

Result (type 8, 29 leaves, 2 steps):

CannotIntegrate  $\left[e^{x^x} x^{2x}, x\right]$  + CannotIntegrate  $\left[e^{x^x} x^{2x} Log[x], x\right]$ 

# Problem 694: Unable to integrate problem.

$$\int x^{-2-\frac{1}{x}} \left(1 - Log[x]\right) dx$$

Optimal (type 3, 9 leaves, ? steps):

$$-x^{-1/x}$$

Result (type 8, 28 leaves, 2 steps):

CannotIntegrate  $\left[x^{-2-\frac{1}{x}}, x\right]$  - CannotIntegrate  $\left[x^{-2-\frac{1}{x}} Log[x], x\right]$ 

# Test results for the 193 problems in "3.1.2 (d x)^m (a+b log(c x^n))^p.m"

# Test results for the 456 problems in "3.1.4 (f x) $^m$ (d+e x $^r$ ) $^q$ (a+b log(c x $^n$ )) $^p$ .m"

Problem 4: Result valid but suboptimal antiderivative.

$$\int (d + e x) (a + b Log[c x^n]) dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\,b\;d\;n\;x\,-\,\frac{1}{4}\;b\;e\;n\;x^2\,+\,d\;x\;\left(a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)\,+\,\frac{1}{2}\;e\;x^2\;\left(\,a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)$$

Result (type 3, 41 leaves, 2 steps):

$$-\,b\,d\,n\,x\,-\,\frac{1}{4}\,b\,e\,n\,x^2\,+\,\frac{1}{2}\,\left(2\,d\,x\,+\,e\,x^2\right)\,\,\left(a\,+\,b\,Log\,\!\left[\,c\,\,x^n\,\right]\,\right)$$

# Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)\;\left(a+b\;Log\left[\;c\;x^{n}\;\right]\;\right)}{x^{2}}\;\mathrm{d}x$$

Optimal (type 3, 48 leaves, 4 steps):

$$- \, \frac{b \, d \, n}{x} \, - \, \frac{d \, \left( a + b \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right)}{x} \, + \, \frac{e \, \left( a + b \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right)^2}{2 \, b \, n}$$

Result (type 3, 43 leaves, 4 steps):

$$- \, \frac{b \, d \, n}{x} - \frac{1}{2} \, b \, e \, n \, Log \left[ \, x \, \right]^{\, 2} - \left( \frac{d}{x} - e \, Log \left[ \, x \, \right] \, \right) \, \left( a + b \, Log \left[ \, c \, \, x^n \, \right] \, \right)$$

#### Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)\;\left(a+b\;Log\left[\;c\;x^{n}\;\right]\;\right)}{x^{4}}\;\mathrm{d}x$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{b\,d\,n}{9\,x^3} - \frac{b\,e\,n}{4\,x^2} - \frac{d\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,x^3} - \frac{e\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^2}$$

Result (type 3, 48 leaves, 4 steps):

$$-\,\frac{b\,d\,n}{9\,x^3}\,-\,\frac{b\,e\,n}{4\,x^2}\,-\,\frac{1}{6}\,\left(\,\frac{2\,d}{x^3}\,+\,\frac{3\,e}{x^2}\,\right)\,\,\left(\,a\,+\,b\,\,\text{Log}\,\big[\,c\,\,x^n\,\big]\,\right)$$

# Problem 13: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+ex\right)^{2}\left(a+bLog[cx^{n}]\right)}{x} dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$-\frac{1}{4}\,b\,n\,\left(4\,d+e\,x\right)^{2}-\frac{1}{2}\,b\,d^{2}\,n\,Log\left[x\right]^{2}+2\,d\,e\,x\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\\+\frac{1}{2}\,e^{2}\,x^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\\+d^{2}\,Log\left[x\right]\,\left(a+b\,Log\left[c\,x^{n}\right]\right)$$

Result (type 3, 63 leaves, 3 steps):

$$-\,\frac{1}{4}\,b\,n\,\left(4\,d\,+\,e\,x\right)^{\,2}\,-\,\frac{1}{2}\,b\,d^{2}\,n\,Log\left[\,x\,\right]^{\,2}\,+\,\frac{1}{2}\,\left(4\,d\,e\,x\,+\,e^{2}\,x^{2}\,+\,2\,d^{2}\,Log\left[\,x\,\right]\,\right)\,\,\left(a\,+\,b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

#### Problem 14: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{d} + e \; x\right)^2 \; \left(\text{a} + \text{b} \; \text{Log} \left[\, c \; x^n \, \right]\,\right)}{x^2} \, \text{d} x$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{b\,d^{2}\,n}{x}\,-\,b\,\,e^{2}\,n\,\,x\,-\,b\,\,d\,e\,n\,\,Log\,[\,x\,]^{\,2}\,-\,\frac{d^{2}\,\,\left(\,a\,+\,b\,\,Log\,[\,c\,\,x^{n}\,]\,\,\right)}{x}\,+\,e^{2}\,x\,\,\left(\,a\,+\,b\,\,Log\,[\,c\,\,x^{n}\,]\,\,\right)\,+\,2\,\,d\,\,e\,\,Log\,[\,x\,]\,\,\left(\,a\,+\,b\,\,Log\,[\,c\,\,x^{n}\,]\,\,\right)$$

Result (type 3, 61 leaves, 3 steps):

$$-\,\frac{b\;d^2\;n}{x}\,-\,b\;e^2\;n\;x\,-\,b\;d\;e\;n\;Log\,[\,x\,]^{\,2}\,-\,\left(\frac{d^2}{x}\,-\,e^2\;x\,-\,2\;d\;e\;Log\,[\,x\,]\,\right)\,\left(a\,+\,b\;Log\,[\,c\;x^n\,]\,\right)$$

# Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{d} + \mathsf{e} \; \mathsf{x}\right)^{\,2} \; \left(\mathsf{a} + \mathsf{b} \; \mathsf{Log} \left[\,\mathsf{c} \; \mathsf{x}^{\mathsf{n}}\,\right]\,\right)}{\mathsf{x}^{3}} \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 84 leaves, 6 steps):

$$-\frac{b \, n \, \left(d+4 \, e \, x\right)^2}{4 \, x^2} - \frac{1}{2} \, b \, e^2 \, n \, \text{Log} \left[x\right]^2 - \frac{d^2 \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right)}{2 \, x^2} - \frac{2 \, d \, e \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right)}{x} + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, \text{Log} \left[c \, x^n\right]\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left(a+b \, x\right) + e^2 \, \text{Log} \left[x\right] \, \left($$

Result (type 3, 67 leaves, 4 steps):

$$-\,\frac{b\;n\;\left(d+4\;e\;x\right)^{\,2}}{4\;x^{2}}\,-\,\frac{1}{2}\;b\;e^{2}\;n\;Log\left[\,x\,\right]^{\,2}\,-\,\frac{1}{2}\,\left(\,\frac{d^{2}}{x^{2}}\,+\,\frac{4\;d\;e}{x}\,-\,2\;e^{2}\;Log\left[\,x\,\right]\,\right)\;\left(\,a\,+\,b\;Log\left[\,c\;x^{n}\,\right]\,\right)$$

#### Problem 17: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{\,2}\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{5}}\;\mathrm{d}x$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b\,d^2\,n}{16\,x^4} - \frac{2\,b\,d\,e\,n}{9\,x^3} - \frac{b\,e^2\,n}{4\,x^2} - \frac{d^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{4\,x^4} - \frac{2\,d\,e\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,x^3} - \frac{e^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^2}$$

Result (type 3, 74 leaves, 4 steps):

$$-\,\frac{b\;d^2\;n}{16\;x^4}\,-\,\frac{2\;b\;d\;e\;n}{9\;x^3}\,-\,\frac{b\;e^2\;n}{4\;x^2}\,-\,\frac{1}{12}\,\left(\,\frac{3\;d^2}{x^4}\,+\,\frac{8\;d\;e}{x^3}\,+\,\frac{6\;e^2}{x^2}\,\right)\,\left(\,a\,+\,b\;\text{Log}\left[\,c\;x^n\,\right]\,\right)$$

# Problem 18: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{\,2}\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{6}}\;\mathrm{d}x$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b\,d^2\,n}{25\,x^5}\,-\frac{b\,d\,e\,n}{8\,x^4}\,-\frac{b\,e^2\,n}{9\,x^3}\,-\frac{d^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{5\,x^5}\,-\frac{d\,e\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^4}\,-\frac{e^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,x^3}$$

Result (type 3, 74 leaves, 4 steps):

$$-\,\frac{b\;d^2\;n}{25\;x^5}\,-\,\frac{b\;d\;e\;n}{8\;x^4}\,-\,\frac{b\;e^2\;n}{9\;x^3}\,-\,\frac{1}{30}\,\left(\frac{6\;d^2}{x^5}\,+\,\frac{15\;d\;e}{x^4}\,+\,\frac{10\;e^2}{x^3}\right)\,\left(a\,+\,b\;\text{Log}\left[\,c\;x^n\,\right]\,\right)$$

# Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{3}\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{x}\;\mathrm{d}x$$

Optimal (type 3, 122 leaves, 4 steps):

$$-3 b d^{2} e n x - \frac{3}{4} b d e^{2} n x^{2} - \frac{1}{9} b e^{3} n x^{3} - \frac{1}{2} b d^{3} n Log[x]^{2} + 3 d^{2} e x (a + b Log[c x^{n}]) + \frac{3}{2} d e^{2} x^{2} (a + b Log[c x^{n}]) + \frac{1}{3} e^{3} x^{3} (a + b Log[c x^{n}]) + d^{3} Log[x] (a + b Log[c x^{n}])$$

Result (type 3, 94 leaves, 4 steps):

$$-3 \ b \ d^2 \ e \ n \ x - \frac{3}{4} \ b \ d \ e^2 \ n \ x^2 - \frac{1}{9} \ b \ e^3 \ n \ x^3 - \frac{1}{2} \ b \ d^3 \ n \ Log \left[x\,\right]^2 + \frac{1}{6} \ \left(18 \ d^2 \ e \ x + 9 \ d \ e^2 \ x^2 + 2 \ e^3 \ x^3 + 6 \ d^3 \ Log \left[x\,\right]\,\right) \ \left(a + b \ Log \left[c \ x^n\,\right]\right)$$

# Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x\right)^{3}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{x^{2}}\,dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{b\,d^3\,n}{x} - 3\,b\,d\,e^2\,n\,x - \frac{1}{4}\,b\,e^3\,n\,x^2 - \frac{3}{2}\,b\,d^2\,e\,n\,\text{Log}\,[\,x\,]^{\,2} - \frac{d^3\,\left(\,a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x} + \\ 3\,d\,e^2\,x\,\left(\,a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right) + \frac{1}{2}\,e^3\,x^2\,\left(\,a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right) + 3\,d^2\,e\,\text{Log}\,[\,x\,]\,\left(\,a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)$$

Result (type 3, 92 leaves, 3 steps):

$$-\,\frac{b\;d^3\;n}{x}\,-\,3\;b\;d\;e^2\;n\;x\,-\,\frac{1}{4}\;b\;e^3\;n\;x^2\,-\,\frac{3}{2}\;b\;d^2\;e\;n\;Log\left[\,x\,\right]^{\,2}\,-\,\frac{1}{2}\;\left(\,\frac{2\;d^3}{x}\,-\,6\;d\;e^2\;x\,-\,e^3\;x^2\,-\,6\;d^2\;e\;Log\left[\,x\,\right]\,\right)\;\left(\,a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)$$

#### Problem 25: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{\,3}\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{3}}\;\mathrm{d}x$$

Optimal (type 3, 118 leaves, 3 steps):

$$-\frac{b\;d^3\;n}{4\;x^2} - \frac{3\;b\;d^2\;e\;n}{x} - b\;e^3\;n\;x - \frac{3}{2}\;b\;d\;e^2\;n\;Log\left[x\right]^2 - \frac{d^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{2\;x^2} - \frac{3\;d^2\;e\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x} + e^3\;x\;\left(a+b\;Log\left[c\;x^n\right]\right) + 3\;d\;e^2\;Log\left[x\right]\;\left(a+b\;Log\left[c\;x^n\right]\right)$$

Result (type 3, 91 leaves, 3 steps):

$$-\frac{b\,d^3\,n}{4\,x^2}\,-\,\frac{3\,b\,d^2\,e\,n}{x}\,-\,b\,e^3\,n\,x\,-\,\frac{3}{2}\,b\,d\,e^2\,n\,\text{Log}\,[\,x\,]^{\,2}\,-\,\frac{1}{2}\,\left(\frac{d^3}{x^2}\,+\,\frac{6\,d^2\,e}{x}\,-\,2\,e^3\,x\,-\,6\,d\,e^2\,\text{Log}\,[\,x\,]\,\right)\,\left(a\,+\,b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)$$

#### Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{3}\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{x^{4}}\;\mathrm{d}x$$

Optimal (type 3, 126 leaves, 7 steps):

$$-\frac{b\ d^{3}\ n}{9\ x^{3}} - \frac{3\ b\ d^{2}\ e\ n}{4\ x^{2}} - \frac{3\ b\ d\ e^{2}\ n}{x} - \frac{1}{2}\ b\ e^{3}\ n\ Log\left[x\right]^{2} - \frac{d^{3}\ \left(a + b\ Log\left[c\ x^{n}\right]\right)}{3\ x^{3}} - \frac{3\ d^{2}\ e\ \left(a + b\ Log\left[c\ x^{n}\right]\right)}{2\ x^{2}} - \frac{3\ d\ e^{2}\ \left(a + b\ Log\left[c\ x^{n}\right]\right)}{x} + e^{3}\ Log\left[x\right]\ \left(a + b\ Log\left[c\ x^{n}\right]\right)$$

Result (type 3, 98 leaves, 5 steps):

$$-\,\frac{b\,d^3\,n}{9\,x^3}\,-\,\frac{3\,b\,d^2\,e\,n}{4\,x^2}\,-\,\frac{3\,b\,d\,e^2\,n}{x}\,-\,\frac{1}{2}\,b\,e^3\,n\,\text{Log}\,[\,x\,]^{\,2}\,-\,\frac{1}{6}\,\left(\frac{2\,d^3}{x^3}\,+\,\frac{9\,d^2\,e}{x^2}\,+\,\frac{18\,d\,e^2}{x}\,-\,6\,e^3\,\text{Log}\,[\,x\,]\,\right)\,\left(a\,+\,b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)$$

# Problem 29: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{\,3}\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{7}}\;\mathrm{d}x$$

Optimal (type 3, 133 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{36\,x^6} - \frac{3\,b\,d^2\,e\,n}{25\,x^5} - \frac{3\,b\,d\,e^2\,n}{16\,x^4} - \frac{b\,e^3\,n}{9\,x^3} - \frac{d^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{6\,x^6} - \frac{3\,d^2\,e\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{5\,x^5} - \frac{3\,d\,e^2\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{4\,x^4} - \frac{e^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{3\,x^3} - \frac{e^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{6\,x^6} - \frac{e^3\,\left(a+b\,\text{Log}\left[c\,x^$$

Result (type 3, 100 leaves, 4 steps):

$$-\frac{b\ d^3\ n}{36\ x^6} - \frac{3\ b\ d^2\ e\ n}{25\ x^5} - \frac{3\ b\ d\ e^2\ n}{16\ x^4} - \frac{b\ e^3\ n}{9\ x^3} - \frac{1}{60}\ \left(\frac{10\ d^3}{x^6} + \frac{36\ d^2\ e}{x^5} + \frac{45\ d\ e^2}{x^4} + \frac{20\ e^3}{x^3}\right)\ \left(a + b\ Log\left[c\ x^n\right]\right)$$

# Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x\right)^{\,3}\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{8}}\;\mathrm{d}x$$

Optimal (type 3, 133 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{49\,x^7} - \frac{b\,d^2\,e\,n}{12\,x^6} - \frac{3\,b\,d\,e^2\,n}{25\,x^5} - \frac{b\,e^3\,n}{16\,x^4} - \frac{d^3\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)}{7\,x^7} - \frac{d^2\,e\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^6} - \frac{3\,d\,e^2\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)}{5\,x^5} - \frac{e^3\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)}{4\,x^4} - \frac{e^3\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^6} - \frac{e^3\,\left(a\,+\,b\,L$$

Result (type 3, 100 leaves, 4 steps):

$$-\,\frac{b\,d^3\,n}{49\,x^7}\,-\,\frac{b\,d^2\,e\,n}{12\,x^6}\,-\,\frac{3\,b\,d\,e^2\,n}{25\,x^5}\,-\,\frac{b\,e^3\,n}{16\,x^4}\,-\,\frac{1}{140}\,\left(\frac{20\,d^3}{x^7}\,+\,\frac{70\,d^2\,e}{x^6}\,+\,\frac{84\,d\,e^2}{x^5}\,+\,\frac{35\,e^3}{x^4}\right)\,\left(a\,+\,b\,Log\left[\,c\,\,x^n\,\right]\,\right)$$

# Problem 35: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x (d + e x)} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$-\frac{Log\left[1+\frac{d}{e\,x}\right]\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{d}+\frac{b\,n\,PolyLog\left[2,\,-\frac{d}{e\,x}\right]}{d}$$

Result (type 4, 66 leaves, 4 steps):

$$\frac{\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^2}{2\,b\,d\,n}\,-\,\frac{\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\left[1+\frac{e\,x}{d}\,\right]}{d}\,-\,\frac{b\,n\,\text{PolyLog}\left[2\,\text{, }-\frac{e\,x}{d}\,\right]}{d}$$

# Problem 36: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \, Log[c \, x^n]}{x^2 \, (d + e \, x)} \, dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{b\,n}{d\,x}-\frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{d\,x}+\frac{e\,\text{Log}\,\big[\,1+\frac{d}{e\,x}\,\big]\,\,\big(\,a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\big)}{d^2}-\frac{b\,e\,n\,\text{PolyLog}\,\big[\,2,\,\,-\frac{d}{e\,x}\,\big]}{d^2}$$

Result (type 4, 95 leaves, 6 steps):

$$-\frac{b\,n}{d\,x} - \frac{a + b\,\text{Log}\,[\,c\,\,x^n\,]}{d\,x} - \frac{e\,\left(a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^{\,2}}{2\,b\,d^2\,n} + \frac{e\,\left(a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\left[1 + \frac{e\,x}{d}\right]}{d^2} + \frac{b\,e\,n\,\text{PolyLog}\left[\,2\,,\,\,-\frac{e\,x}{d}\,\right]}{d^2}$$

# Problem 37: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^3 (d + e x)} dx$$

Optimal (type 4, 110 leaves, 6 steps):

$$-\frac{b\,n}{4\,d\,x^2} + \frac{b\,e\,n}{d^2\,x} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{2\,d\,x^2} + \frac{e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{d^2\,x} - \frac{e^2\,\text{Log}\,\big[\,1+\frac{d}{e\,x}\,\big]\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{d^3} + \frac{b\,e^2\,n\,\text{PolyLog}\,\big[\,2\,,\,\,-\frac{d}{e\,x}\,\big]}{d^3}$$

Result (type 4, 135 leaves, 7 steps):

$$-\frac{b \, n}{4 \, d \, x^2} + \frac{b \, e \, n}{d^2 \, x} - \frac{a + b \, \text{Log} \left[ c \, x^n \right]}{2 \, d \, x^2} + \frac{e \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right)}{d^2 \, x} + \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right)^2}{2 \, b \, d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3} - \frac{b \, e^2 \, n \, \text{PolyLog} \left[ 2 \, , \, -\frac{e \, x}{d} \right]}{d^3} + \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right)^2}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3} - \frac{b \, e^2 \, n \, \text{PolyLog} \left[ 2 \, , \, -\frac{e \, x}{d} \right]}{d^3 \, n} + \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right)^2}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{b \, e^2 \, n \, \text{PolyLog} \left[ 2 \, , \, -\frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right]}{d^3 \, n} - \frac{e^2 \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] + \frac{e^2 \, \left( a + b \, x \, \text{Log} \left[ c \, x^n \right] \right) \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left[ 1 + \frac{e \, x}{d} \right] \, \text{Log} \left$$

# Problem 38: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^4 (d + e x)} dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$-\frac{b\,n}{9\,d\,x^3} + \frac{b\,e\,n}{4\,d^2\,x^2} - \frac{b\,e^2\,n}{d^3\,x} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{3\,d\,x^3} + \frac{e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,x^2} - \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{d^3\,x} + \frac{e^3\,\text{Log}\left[1+\frac{d}{e\,x}\,\right]\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{d^4} - \frac{b\,e^3\,n\,\text{PolyLog}\left[2\,,\,\,-\frac{d}{e\,x}\,\right]}{d^4}$$

Result (type 4, 173 leaves, 8 steps):

$$-\frac{b\,n}{9\,d\,x^3} + \frac{b\,e\,n}{4\,d^2\,x^2} - \frac{b\,e^2\,n}{d^3\,x} - \frac{a+b\,Log\,[\,c\,\,x^n\,]}{3\,d\,x^3} + \frac{e\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,x^2} - \\ \\ \frac{e^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{d^3\,x} - \frac{e^3\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{2\,b\,d^4\,n} + \frac{e^3\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,Log\,\left[1+\frac{e\,x}{d}\right]}{d^4} + \frac{b\,e^3\,n\,PolyLog\,[\,2\,,\,-\frac{e\,x}{d}\,]}{d^4}$$

# Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a + b \operatorname{Log}\left[c \, x^n\right]\right)}{\left(d + e \, x\right)^2} \, dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$\frac{\frac{3 \, b \, d \, n \, x}{e^3} - \frac{d \, \left(3 \, a + b \, n\right) \, x}{e^3} - \frac{3 \, b \, n \, x^2}{4 \, e^2} - \frac{3 \, b \, d \, x \, Log \left[c \, x^n\right]}{e^3} - \frac{x^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{e \, \left(d + e \, x\right)} + \frac{x^2 \, \left(3 \, a + b \, n + 3 \, b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \left[2 \, , \, -\frac{e \, x}{d}\right]}{e^4}$$

Result (type 4, 151 leaves, 9 steps):

$$-\frac{2 \, a \, d \, x}{e^3} + \frac{2 \, b \, d \, n \, x}{e^3} - \frac{b \, n \, x^2}{4 \, e^2} - \frac{2 \, b \, d \, x \, \text{Log} \, [\, c \, \, x^n \, ]}{e^3} + \frac{x^2 \, \left(a + b \, \text{Log} \, [\, c \, \, x^n \, ] \, \right)}{2 \, e^2} - \frac{d^2 \, x \, \left(a + b \, \text{Log} \, [\, c \, \, x^n \, ] \, \right)}{e^3 \, \left(d + e \, x \, \right)} + \frac{b \, d^2 \, n \, \text{Log} \, [\, d + e \, x \, ]}{e^4} + \frac{3 \, d^2 \, \left(a + b \, \text{Log} \, [\, c \, \, x^n \, ] \, \right) \, \text{Log} \, \left[1 + \frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{e^4} + \frac{3 \, b \, d^2 \, n \, PolyL$$

# Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b \log \left[c x^n\right]\right)}{\left(d + e x\right)^2} dx$$

Optimal (type 4, 98 leaves, 7 steps):

$$-\frac{b\,n\,x}{e^2} + \frac{2\,x\,\left(a + b\,\text{Log}\left[c\,x^n\right]\right)}{e^2} - \frac{x^2\,\left(a + b\,\text{Log}\left[c\,x^n\right]\right)}{e\,\left(d + e\,x\right)} - \frac{d\,\left(2\,a + b\,n + 2\,b\,\text{Log}\left[c\,x^n\right]\right)\,\text{Log}\left[1 + \frac{e\,x}{d}\right]}{e^3} - \frac{2\,b\,d\,n\,\text{PolyLog}\left[2, -\frac{e\,x}{d}\right]}{e^3} + \frac{2\,b\,d\,n\,\text{PolyLog}\left[2, -\frac{e\,x}{d}\right]}{e^3} - \frac{2\,b\,d\,n\,\text{PolyLog}\left[2, -\frac{e\,x}{d}\right]}{e^3} + \frac{2\,b\,d\,n\,\text{PolyLog}\left[2,$$

Result (type 4, 106 leaves, 8 steps):

$$\frac{a\ x}{e^2} - \frac{b\ n\ x}{e^2} + \frac{b\ x\ Log\ [c\ x^n]}{e^2} + \frac{d\ x\ \left(a + b\ Log\ [c\ x^n]\right)}{e^2} - \frac{b\ d\ n\ Log\ [d+e\ x]}{e^3} - \frac{2\ d\ \left(a + b\ Log\ [c\ x^n]\right)\ Log\ \left[1 + \frac{e\ x}{d}\right]}{e^3} - \frac{2\ b\ d\ n\ PolyLog\ \left[2\ ,\ -\frac{e\ x}{d}\right]}{e^3}$$

# Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a + b \log[c x^{n}]\right)}{\left(d + e x\right)^{2}} dx$$

Optimal (type 4, 65 leaves, 3 steps):

$$-\frac{x\,\left(a+b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)}{e\,\left(d+e\,x\right)}\,+\,\frac{\left(\,a+b\,n+b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)\,Log\left[\,1+\frac{e\,x}{d}\,\right]}{e^{2}}\,+\,\frac{b\,n\,PolyLog\left[\,2\,\text{, }-\frac{e\,x}{d}\,\right]}{e^{2}}$$

Result (type 4, 74 leaves, 6 steps):

$$-\frac{x\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{e\,\left(d+e\,x\right)}\,+\,\frac{b\,n\,Log\,[\,d+e\,x\,]}{e^2}\,+\,\frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,Log\,\left[\,1+\frac{e\,x}{d}\,\right]}{e^2}\,+\,\frac{b\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{e\,x}{d}\,\right]}{e^2}$$

# Problem 43: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x (d + e x)^2} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{e\;x\;\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;\mathsf{x}^\mathsf{n}\right]\right)}{\mathsf{d}^2\;\left(\mathsf{d}+\mathsf{e}\;\mathsf{x}\right)}\;-\frac{\mathsf{Log}\left[\mathsf{1}+\frac{\mathsf{d}}{\mathsf{e}\,\mathsf{x}}\right]\;\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;\mathsf{x}^\mathsf{n}\right]\right)}{\mathsf{d}^2}\;+\;\frac{\mathsf{b}\;\mathsf{n}\;\mathsf{Log}\left[\mathsf{d}+\mathsf{e}\;\mathsf{x}\right]}{\mathsf{d}^2}\;+\;\frac{\mathsf{b}\;\mathsf{n}\;\mathsf{PolyLog}\left[\mathsf{2}\,\mathsf{,}\;-\frac{\mathsf{d}}{\mathsf{e}\,\mathsf{x}}\right]}{\mathsf{d}^2}$$

Result (type 4, 102 leaves, 7 steps):

$$-\frac{e\;x\;\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;\mathsf{x}^{\mathsf{n}}\right]\right)}{\mathsf{d}^{2}\;\left(\mathsf{d}+\mathsf{e}\;\mathsf{x}\right)}+\frac{\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;\mathsf{x}^{\mathsf{n}}\right]\right)^{2}}{2\;\mathsf{b}\;\mathsf{d}^{2}\;\mathsf{n}}+\frac{\mathsf{b}\;\mathsf{n}\;\mathsf{Log}\left[\mathsf{d}+\mathsf{e}\;\mathsf{x}\right]}{\mathsf{d}^{2}}-\frac{\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;\mathsf{x}^{\mathsf{n}}\right]\right)\;\mathsf{Log}\left[\mathsf{1}+\frac{\mathsf{e}\;\mathsf{x}}{\mathsf{d}}\right]}{\mathsf{d}^{2}}-\frac{\mathsf{b}\;\mathsf{n}\;\mathsf{PolyLog}\left[\mathsf{2}\;\mathsf{,}\;-\frac{\mathsf{e}\;\mathsf{x}}{\mathsf{d}}\right]}{\mathsf{d}^{2}}$$

## Problem 44: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^2 (d + e x)^2} dx$$

Optimal (type 4, 114 leaves, 7 steps):

$$-\frac{b\,n}{d^{2}\,x}\,-\,\frac{a\,+\,b\,Log\,[\,c\,\,x^{n}\,]}{d^{2}\,x}\,+\,\frac{e^{2}\,x\,\left(\,a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}\,\left(\,d\,+\,e\,\,x\,\right)}\,+\,\frac{2\,e\,Log\,\left[\,1\,+\,\frac{d}{e\,x}\,\right]\,\left(\,a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}}\,-\,\frac{b\,e\,n\,Log\,[\,d\,+\,e\,\,x\,]}{d^{3}}\,-\,\frac{2\,b\,e\,n\,PolyLog\,\left[\,2\,,\,\,-\,\frac{d}{e\,x}\,\right]}{d^{3}}$$

Result (type 4, 134 leaves, 8 steps):

$$-\frac{b\,n}{d^{2}\,x} - \frac{a + b\,\text{Log}\,[\,c\,\,x^{n}\,]}{d^{2}\,x} + \frac{e^{2}\,x\,\left(a + b\,\text{Log}\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}\,\left(d + e\,x\right)} - \frac{e\,\left(a + b\,\text{Log}\,[\,c\,\,x^{n}\,]\,\right)^{2}}{b\,d^{3}\,n} - \\ \frac{b\,e\,n\,\text{Log}\,[\,d + e\,x\,]}{d^{3}} + \frac{2\,e\,\left(a + b\,\text{Log}\,[\,c\,\,x^{n}\,]\,\right)\,\text{Log}\,\left[1 + \frac{e\,x}{d}\,\right]}{d^{3}} + \frac{2\,b\,e\,n\,\text{PolyLog}\,\left[2\,\text{, } - \frac{e\,x}{d}\,\right]}{d^{3}}$$

# Problem 45: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^3 (d + e x)^2} dx$$

Optimal (type 4, 154 leaves, 8 steps):

$$-\frac{b\ n}{4\ d^{2}\ x^{2}} + \frac{2\ b\ e\ n}{d^{3}\ x} - \frac{a\ + b\ Log\ [c\ x^{n}\ ]}{2\ d^{2}\ x^{2}} + \frac{2\ e\ \left(a\ + b\ Log\ [c\ x^{n}\ ]\right)}{d^{3}\ x} - \frac{e^{3}\ x\ \left(a\ + b\ Log\ [c\ x^{n}\ ]\right)}{d^{4}\left(d\ + e\ x\right)} - \frac{3\ b\ e^{2}\ n\ Log\ [d\ + e\ x]}{d^{4}} + \frac{3\ b\ e^{2}\ n\ PolyLog\ [c\ x^{n}\ ]}{d^{4}} + \frac{3\ b\ e^{2}\ n\ PolyLog\ [c\$$

Result (type 4, 178 leaves, 9 steps):

$$-\frac{b\,n}{4\,d^{2}\,x^{2}} + \frac{2\,b\,e\,n}{d^{3}\,x} - \frac{a+b\,\text{Log}\,[\,c\,x^{n}\,]}{2\,d^{2}\,x^{2}} + \frac{2\,e\,\left(a+b\,\text{Log}\,[\,c\,x^{n}\,]\,\right)}{d^{3}\,x} - \frac{e^{3}\,x\,\left(a+b\,\text{Log}\,[\,c\,x^{n}\,]\,\right)}{d^{4}\,\left(d+e\,x\right)} + \frac{3\,e^{2}\,\left(a+b\,\text{Log}\,[\,c\,x^{n}\,]\,\right)^{2}}{2\,b\,d^{4}\,n} + \frac{b\,e^{2}\,n\,\text{Log}\,[\,d+e\,x\,]}{d^{4}} - \frac{3\,e^{2}\,\left(a+b\,\text{Log}\,[\,c\,x^{n}\,]\,\right)\,\text{Log}\,\left[1+\frac{e\,x}{d}\right]}{d^{4}} - \frac{3\,b\,e^{2}\,n\,\text{PolyLog}\,\left[2,-\frac{e\,x}{d}\right]}{d^{4}}$$

#### Problem 46: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a + b \operatorname{Log}\left[c \, x^n\right]\right)}{\left(d + e \, x\right)^3} \, dx$$

Optimal (type 4, 149 leaves, 8 steps):

$$-\frac{3 \, b \, n \, x}{e^3} + \frac{\left(6 \, a + 5 \, b \, n\right) \, x}{2 \, e^3} + \frac{3 \, b \, x \, Log \left[c \, x^n\right]}{e^3} - \frac{x^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{2 \, e \, \left(d + e \, x\right)^2} - \frac{x^2 \, \left(3 \, a + b \, n + 3 \, b \, Log \left[c \, x^n\right]\right)}{2 \, e^2 \, \left(d + e \, x\right)} - \frac{d \, \left(6 \, a + 5 \, b \, n + 6 \, b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{2 \, e^4} - \frac{3 \, b \, d \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^4}$$

Result (type 4, 167 leaves, 11 steps):

$$\begin{split} &\frac{a \, x}{e^3} - \frac{b \, n \, x}{e^3} - \frac{b \, d^2 \, n}{2 \, e^4 \, \left(d + e \, x\right)} - \frac{b \, d \, n \, \text{Log} \left[x\right]}{2 \, e^4} + \frac{b \, x \, \text{Log} \left[c \, x^n\right]}{e^3} + \frac{d^3 \, \left(a + b \, \text{Log} \left[c \, x^n\right]\right)}{2 \, e^4 \, \left(d + e \, x\right)^2} + \\ &\frac{3 \, d \, x \, \left(a + b \, \text{Log} \left[c \, x^n\right]\right)}{e^3 \, \left(d + e \, x\right)} - \frac{5 \, b \, d \, n \, \text{Log} \left[d + e \, x\right]}{2 \, e^4} - \frac{3 \, d \, \left(a + b \, \text{Log} \left[c \, x^n\right]\right) \, \text{Log} \left[1 + \frac{e \, x}{d}\right]}{e^4} - \frac{3 \, b \, d \, n \, \text{PolyLog} \left[2, -\frac{e \, x}{d}\right]}{e^4} \end{split}$$

# Problem 47: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b \log \left[c x^n\right]\right)}{\left(d + e x\right)^3} dx$$

Optimal (type 4, 107 leaves, 4 steps):

Result (type 4, 132 leaves, 9 steps):

$$\frac{b\,d\,n}{2\,e^3\,\left(d+e\,x\right)} + \frac{b\,n\,\text{Log}\,[\,x\,]}{2\,e^3} - \frac{d^2\,\left(\,a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,e^3\,\left(\,d+e\,\,x\right)^2} - \frac{2\,x\,\left(\,a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{e^2\,\left(\,d+e\,\,x\right)} + \frac{3\,b\,n\,\text{Log}\,[\,d+e\,\,x\,]}{2\,e^3} + \frac{\left(\,a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\,\left[\,1+\frac{e\,x}{d}\,\right]}{e^3} + \frac{b\,n\,\text{PolyLog}\,[\,2\,,\,-\frac{e\,x}{d}\,]}{e^3} + \frac{b\,n\,\text{PolyLog}$$

#### Problem 50: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \, Log \, [\, c \, x^n \, ]}{x \, \left(d + e \, x\right)^3} \, dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$-\frac{b\,n}{2\,d^{2}\,\left(d+e\,x\right)}-\frac{b\,n\,Log\,[\,x\,]}{2\,d^{3}}+\frac{a+b\,Log\,[\,c\,\,x^{n}\,]}{2\,d\,\,\left(d+e\,x\right)^{\,2}}-\frac{e\,x\,\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}}-\frac{Log\,[\,1+\frac{d}{e\,x}\,]\,\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}}+\frac{3\,b\,n\,Log\,[\,d+e\,x\,]}{2\,d^{3}}+\frac{b\,n\,PolyLog\,[\,2\,,\,-\frac{d}{e\,x}\,]}{d^{3}}$$

Result (type 4, 156 leaves, 11 steps):

$$-\frac{b\,n}{2\,d^{2}\,\left(d+e\,x\right)}-\frac{b\,n\,Log\,[\,x\,]}{2\,d^{3}}+\frac{a+b\,Log\,[\,c\,\,x^{n}\,]}{2\,d\,\left(d+e\,x\right)^{\,2}}-\frac{e\,x\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{3}\,\left(d+e\,x\right)}+\\ \frac{\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{\,2}}{2\,b\,d^{3}\,n}+\frac{3\,b\,n\,Log\,[\,d+e\,x\,]}{2\,d^{3}}-\frac{\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)\,Log\,\left[\,1+\frac{e\,x}{d}\,\right]}{d^{3}}-\frac{b\,n\,PolyLog\,\left[\,2,\,-\frac{e\,x}{d}\,\right]}{d^{3}}$$

# Problem 51: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log [c x^n]}{x^2 (d + e x)^3} dx$$

#### Optimal (type 4, 171 leaves, 10 steps):

$$-\frac{b\,n}{d^3\,x} + \frac{b\,e\,n}{2\,d^3\,\left(d + e\,x\right)} + \frac{b\,e\,n\,Log\,[\,x\,]}{2\,d^4} - \frac{a + b\,Log\,[\,c\,\,x^n\,]}{d^3\,x} - \frac{e\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,\left(d + e\,x\right)^2} + \\ \frac{2\,e^2\,x\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{d^4\,\left(d + e\,x\right)} + \frac{3\,e\,Log\,[\,1 + \frac{d}{e\,x}\,]\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{d^4} - \frac{5\,b\,e\,n\,Log\,[\,d + e\,x\,]}{2\,d^4} - \frac{3\,b\,e\,n\,PolyLog\,[\,2 , \, -\frac{d}{e\,x}\,]}{d^4}$$

#### Result (type 4, 193 leaves, 11 steps):

$$-\frac{b\,n}{d^3\,x} + \frac{b\,e\,n}{2\,d^3\,\left(d+e\,x\right)} + \frac{b\,e\,n\,\text{Log}\,[\,x\,]}{2\,d^4} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{d^3\,x} - \frac{e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,\left(d+e\,x\right)^2} + \frac{2\,e^2\,x\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{d^4\,\left(d+e\,x\right)} - \frac{3\,e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right) \,\text{Log}\,\left[1+\frac{e\,x}{d}\right]}{2\,b\,d^4\,n} + \frac{3\,b\,e\,n\,\text{PolyLog}\,\left[\,2\,,\,-\frac{e\,x}{d}\,\right]}{d^4}$$

# Problem 52: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^3 (d + e x)^3} dx$$

#### Optimal (type 4, 217 leaves, 11 steps):

#### Result (type 4, 239 leaves, 12 steps):

$$-\frac{b\,n}{4\,d^3\,x^2} + \frac{3\,b\,e\,n}{d^4\,x} - \frac{b\,e^2\,n}{2\,d^4\,\left(d+e\,x\right)} - \frac{b\,e^2\,n\,\text{Log}[x]}{2\,d^5} - \frac{a+b\,\text{Log}[c\,x^n]}{2\,d^3\,x^2} + \frac{3\,e\,\left(a+b\,\text{Log}[c\,x^n]\right)}{d^4\,x} + \frac{e^2\,\left(a+b\,\text{Log}[c\,x^n]\right)}{2\,d^3\,\left(d+e\,x\right)^2} - \frac{3\,e^3\,x\,\left(a+b\,\text{Log}[c\,x^n]\right)}{b\,d^5\,n} + \frac{7\,b\,e^2\,n\,\text{Log}[d+e\,x]}{2\,d^5} - \frac{6\,e^2\,\left(a+b\,\text{Log}[c\,x^n]\right)\,\text{Log}\left[1+\frac{e\,x}{d}\right]}{d^5} - \frac{6\,b\,e^2\,n\,\text{PolyLog}\left[2,-\frac{e\,x}{d}\right]}{d^5}$$

#### Problem 53: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \log[c x^n])}{(d + e x)^4} dx$$

Optimal (type 4, 229 leaves, 10 steps):

$$\frac{10 \, b \, d \, n \, x}{e^5} - \frac{d \, \left(60 \, a + 47 \, b \, n\right) \, x}{6 \, e^5} - \frac{5 \, b \, n \, x^2}{2 \, e^4} - \frac{10 \, b \, d \, x \, Log \left[c \, x^n\right]}{e^5} - \frac{x^5 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{3 \, e \, \left(d + e \, x\right)^3} - \frac{x^4 \, \left(5 \, a + b \, n + 5 \, b \, Log \left[c \, x^n\right]\right)}{6 \, e^2 \, \left(d + e \, x\right)^2} - \frac{x^3 \, \left(20 \, a + 9 \, b \, n + 20 \, b \, Log \left[c \, x^n\right]\right)}{6 \, e^3 \, \left(d + e \, x\right)} + \frac{x^2 \, \left(60 \, a + 47 \, b \, n + 60 \, b \, Log \left[c \, x^n\right]\right)}{12 \, e^4} + \frac{d^2 \, \left(60 \, a + 47 \, b \, n + 60 \, b \, Log \left[c \, x^n\right]\right)}{6 \, e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^6} + \frac{10 \, b \, d^2 \, n \, PolyLog \left[2, -\frac{e$$

Result (type 4, 260 leaves, 15 steps):

$$-\frac{4 \text{ a d } x}{e^5} + \frac{4 \text{ b d n } x}{e^5} - \frac{\text{b n } x^2}{4 \text{ e}^4} - \frac{\text{b d d n } x}{6 \text{ e}^6 \text{ (d + e x)}^2} + \frac{13 \text{ b d}^3 \text{ n}}{6 \text{ e}^6 \text{ (d + e x)}} + \frac{13 \text{ b d}^3 \text{ n}}{6 \text{ e}^6 \text{ (d + e x)}} - \frac{4 \text{ b d x Log[c } x^n]}{e^5} + \frac{x^2 \text{ (a + b Log[c } x^n])}{2 \text{ e}^4} + \frac{d^5 \text{ (a + b Log[c } x^n])}{3 \text{ e}^6 \text{ (d + e x)}^3} - \frac{5 \text{ d}^4 \text{ (a + b Log[c } x^n])}{2 \text{ e}^6 \text{ (d + e x)}^2} - \frac{10 \text{ d}^2 \text{ x (a + b Log[c } x^n])}{e^5} + \frac{47 \text{ b d}^2 \text{ n Log[d + e x]}}{6 \text{ e}^6} + \frac{10 \text{ d}^2 \text{ (a + b Log[c } x^n])}{e^6} + \frac{10 \text{ b d}^2 \text{ n PolyLog[2, -\frac{ex}{d}]}}{e^6}$$

# Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \left(a + b Log[c x^n]\right)}{\left(d + e x\right)^4} dx$$

Optimal (type 4, 183 leaves, 9 steps):

$$-\frac{4 \, b \, n \, x}{e^4} + \frac{\left(12 \, a + 13 \, b \, n\right) \, x}{3 \, e^4} + \frac{4 \, b \, x \, \text{Log}\left[c \, x^n\right]}{e^4} - \frac{x^4 \, \left(a + b \, \text{Log}\left[c \, x^n\right]\right)}{3 \, e \, \left(d + e \, x\right)^3} - \frac{x^3 \, \left(4 \, a + b \, n + 4 \, b \, \text{Log}\left[c \, x^n\right]\right)}{6 \, e^2 \, \left(d + e \, x\right)^2} - \frac{x^2 \, \left(12 \, a + 7 \, b \, n + 12 \, b \, \text{Log}\left[c \, x^n\right]\right)}{6 \, e^3 \, \left(d + e \, x\right)} - \frac{d \, \left(12 \, a + 13 \, b \, n + 12 \, b \, \text{Log}\left[c \, x^n\right]\right)}{3 \, e^5} - \frac{4 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{4 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{e^5} - \frac{2 \, b \, d \, n \, PolyLog\left[2, -\frac{e \, x}{d}\right$$

Result (type 4, 211 leaves, 14 steps):

$$\frac{a\,x}{e^4} - \frac{b\,n\,x}{e^4} + \frac{b\,d^3\,n}{6\,e^5\,\left(d+e\,x\right)^2} - \frac{5\,b\,d^2\,n}{3\,e^5\,\left(d+e\,x\right)} - \frac{5\,b\,d\,n\,\text{Log}\,[x]}{3\,e^5} + \frac{b\,x\,\text{Log}\,[c\,x^n]}{e^4} - \frac{d^4\,\left(a+b\,\text{Log}\,[c\,x^n]\right)}{3\,e^5\,\left(d+e\,x\right)^3} + \frac{2\,d^3\,\left(a+b\,\text{Log}\,[c\,x^n]\right)}{e^5\,\left(d+e\,x\right)^2} + \frac{6\,d\,x\,\left(a+b\,\text{Log}\,[c\,x^n]\right)}{e^4\,\left(d+e\,x\right)} - \frac{4\,d\,\left(a+b\,\text{Log}\,[c\,x^n]\right)\,\text{Log}\,\left[1+\frac{e\,x}{d}\right]}{e^5} - \frac{4\,b\,d\,n\,\text{PolyLog}\,\left[2\,,-\frac{e\,x}{d}\right]}{e^5}$$

# Problem 55: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a + b \log \left[c x^n\right]\right)}{\left(d + e x\right)^4} dx$$

Optimal (type 4, 141 leaves, 5 steps):

$$-\frac{x^{3} \, \left(a+b \, Log \, [c \, x^{n}] \, \right)}{3 \, e \, \left(d+e \, x\right)^{3}} - \frac{x^{2} \, \left(3 \, a+b \, n+3 \, b \, Log \, [c \, x^{n}] \, \right)}{6 \, e^{2} \, \left(d+e \, x\right)^{2}} - \frac{x \, \left(6 \, a+5 \, b \, n+6 \, b \, Log \, [c \, x^{n}] \, \right)}{6 \, e^{3} \, \left(d+e \, x\right)} + \frac{\left(6 \, a+11 \, b \, n+6 \, b \, Log \, [c \, x^{n}] \, \right) \, Log \left[1+\frac{e \, x}{d}\right]}{6 \, e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{e^{4}} + \frac{b \, n \, PolyLog \left[2, -\frac{e \, x}{d$$

Result (type 4, 178 leaves, 12 steps):

$$-\frac{b\,d^{2}\,n}{6\,e^{4}\,\left(d+e\,x\right)^{\,2}}+\frac{7\,b\,d\,n}{6\,e^{4}\,\left(d+e\,x\right)}+\frac{7\,b\,n\,Log\,[\,x\,]}{6\,e^{4}}+\frac{d^{3}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{3\,e^{4}\,\left(d+e\,x\right)^{\,3}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(d+e\,x\right)^{\,2}}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}-\frac{3\,d^{2}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}{2\,e^{4}\,\left(a+b\,Log\,[\,c\,x^{n}\,]\,\right)}-\frac{3\,d^{2}\,$$

### Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x (d + e x)^4} dx$$

Optimal (type 4, 174 leaves, 13 steps):

$$-\frac{b\,n}{6\,d^{2}\,\left(d+e\,x\right)^{\,2}}-\frac{5\,b\,n}{6\,d^{3}\,\left(d+e\,x\right)}-\frac{5\,b\,n\,Log\,[\,x\,]}{6\,d^{4}}+\frac{a+b\,Log\,[\,c\,\,x^{n}\,]}{3\,d\,\left(d+e\,x\right)^{\,3}}+\frac{a+b\,Log\,[\,c\,\,x^{n}\,]}{2\,d^{2}\,\left(d+e\,x\right)^{\,2}}-\frac{e\,x\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{4}\,\left(d+e\,x\right)}-\frac{Log\,[\,1+\frac{d}{e\,x}\,]\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{d^{4}}+\frac{11\,b\,n\,Log\,[\,d+e\,x\,]}{6\,d^{4}}+\frac{b\,n\,PolyLog\,[\,2,\,-\frac{d}{e\,x}\,]}{d^{4}}$$

Result (type 4, 196 leaves, 15 steps):

$$-\frac{b\,n}{6\,d^{2}\,\left(d+e\,x\right)^{\,2}}-\frac{5\,b\,n}{6\,d^{3}\,\left(d+e\,x\right)}-\frac{5\,b\,n\,Log\left[x\right]}{6\,d^{4}}+\frac{a+b\,Log\left[c\,x^{n}\right]}{3\,d\,\left(d+e\,x\right)^{\,3}}+\frac{a+b\,Log\left[c\,x^{n}\right]}{2\,d^{2}\,\left(d+e\,x\right)^{\,2}}-\frac{e\,x\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{d^{4}\,\left(d+e\,x\right)}+\frac{\left(a+b\,Log\left[c\,x^{n}\right]\right)^{\,2}}{2\,b\,d^{4}\,n}+\frac{11\,b\,n\,Log\left[d+e\,x\right]}{6\,d^{4}}-\frac{\left(a+b\,Log\left[c\,x^{n}\right]\right)\,Log\left[1+\frac{e\,x}{d}\right]}{d^{4}}-\frac{b\,n\,PolyLog\left[2,-\frac{e\,x}{d}\right]}{d^{4}}$$

### Problem 60: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^2 (d + e x)^4} dx$$

Optimal (type 4, 211 leaves, 13 steps):

$$-\frac{b\,n}{d^4\,x} + \frac{b\,e\,n}{6\,d^3\,\left(d + e\,x\right)^2} + \frac{4\,b\,e\,n}{3\,d^4\,\left(d + e\,x\right)} + \frac{4\,b\,e\,n\,Log\,[x\,]}{3\,d^5} - \frac{a + b\,Log\,[c\,x^n]}{d^4\,x} - \frac{e\,\left(a + b\,Log\,[c\,x^n]\right)}{3\,d^2\,\left(d + e\,x\right)^3} - \frac{e\,\left(a + b\,Log\,[c\,x^n]\right)}{d^3\,\left(d + e\,x\right)^2} + \frac{3\,e^2\,x\,\left(a + b\,Log\,[c\,x^n]\right)}{d^5\,\left(d + e\,x\right)} + \frac{4\,e\,Log\,\left[1 + \frac{d}{e\,x}\right]\,\left(a + b\,Log\,[c\,x^n]\right)}{d^5} - \frac{13\,b\,e\,n\,Log\,[d + e\,x]}{3\,d^5} - \frac{4\,b\,e\,n\,PolyLog\,\left[2, -\frac{d}{e\,x}\right]}{d^5}$$

Result (type 4, 231 leaves, 14 steps):

$$-\frac{b\,n}{d^4\,x} + \frac{b\,e\,n}{6\,d^3\,\left(d+e\,x\right)^2} + \frac{4\,b\,e\,n}{3\,d^4\,\left(d+e\,x\right)} + \frac{4\,b\,e\,n\,Log\,[x\,]}{3\,d^5} - \frac{a+b\,Log\,[c\,x^n\,]}{d^4\,x} - \frac{e\,\left(a+b\,Log\,[c\,x^n\,]\right)}{3\,d^2\,\left(d+e\,x\right)^3} - \frac{e\,\left(a+b\,Log\,[c\,x^n\,]\right)}{d^3\,\left(d+e\,x\right)^2} + \frac{3\,e^2\,x\,\left(a+b\,Log\,[c\,x^n\,]\right)}{d^5\,\left(d+e\,x\right)} - \frac{2\,e\,\left(a+b\,Log\,[c\,x^n\,]\right)^2}{b\,d^5\,n} - \frac{13\,b\,e\,n\,Log\,[d+e\,x]}{3\,d^5} + \frac{4\,e\,\left(a+b\,Log\,[c\,x^n\,]\right)\,Log\,\left[1+\frac{e\,x}{d}\right]}{d^5} + \frac{4\,b\,e\,n\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{d^5} + \frac{4\,b$$

### Problem 61: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \, Log \, [\, c \, \, x^n \, ]}{x^3 \, \left(d+e \, x\right)^4} \, \, \mathrm{d}x$$

Optimal (type 4, 263 leaves, 14 steps):

$$-\frac{b\,n}{4\,d^4\,x^2} + \frac{4\,b\,e\,n}{d^5\,x} - \frac{b\,e^2\,n}{6\,d^4\,\left(d+e\,x\right)^2} - \frac{11\,b\,e^2\,n}{6\,d^5\,\left(d+e\,x\right)} - \frac{11\,b\,e^2\,n\,\text{Log}[x]}{6\,d^6} - \frac{a+b\,\text{Log}[c\,x^n]}{2\,d^4\,x^2} + \frac{4\,e\,\left(a+b\,\text{Log}[c\,x^n]\right)}{d^5\,x} + \frac{e^2\,\left(a+b\,\text{Log}[c\,x^n]\right)}{3\,d^3\,\left(d+e\,x\right)^3} + \frac{3\,e^2\,\left(a+b\,\text{Log}[c\,x^n]\right)}{2\,d^4\,\left(d+e\,x\right)^2} - \frac{6\,e^3\,x\,\left(a+b\,\text{Log}[c\,x^n]\right)}{d^6\,\left(d+e\,x\right)} - \frac{10\,e^2\,\text{Log}\left[1+\frac{d}{e\,x}\right]\,\left(a+b\,\text{Log}[c\,x^n]\right)}{d^6} + \frac{47\,b\,e^2\,n\,\text{Log}[d+e\,x]}{6\,d^6} + \frac{10\,b\,e^2\,n\,\text{PolyLog}\left[2,-\frac{d}{e\,x}\right]}{d^6}$$

Result (type 4, 285 leaves, 15 steps):

$$-\frac{b\,n}{4\,d^4\,x^2} + \frac{4\,b\,e\,n}{d^5\,x} - \frac{b\,e^2\,n}{6\,d^4\,\left(d+e\,x\right)^2} - \frac{11\,b\,e^2\,n}{6\,d^5\,\left(d+e\,x\right)} - \frac{11\,b\,e^2\,n\,\log\left[x\right]}{6\,d^6} - \frac{a+b\,\log\left[c\,x^n\right]}{2\,d^4\,x^2} + \\ \frac{4\,e\,\left(a+b\,\log\left[c\,x^n\right]\right)}{d^5\,x} + \frac{e^2\,\left(a+b\,\log\left[c\,x^n\right]\right)}{3\,d^3\,\left(d+e\,x\right)^3} + \frac{3\,e^2\,\left(a+b\,\log\left[c\,x^n\right]\right)}{2\,d^4\,\left(d+e\,x\right)^2} - \frac{6\,e^3\,x\,\left(a+b\,\log\left[c\,x^n\right]\right)}{d^6\,\left(d+e\,x\right)} + \\ \frac{5\,e^2\,\left(a+b\,\log\left[c\,x^n\right]\right)^2}{b\,d^6\,n} + \frac{47\,b\,e^2\,n\,\log\left[d+e\,x\right]}{6\,d^6} - \frac{10\,e^2\,\left(a+b\,\log\left[c\,x^n\right]\right)\,\log\left[1+\frac{e\,x}{d}\right]}{d^6} - \frac{10\,b\,e^2\,n\,PolyLog\left[2,-\frac{e\,x}{d}\right]}{d^6}$$

### Problem 62: Result valid but suboptimal antiderivative.

$$\int \frac{x^8 \left(a + b \operatorname{Log}\left[c \, x^n\right]\right)}{\left(d + e \, x\right)^7} \, dx$$

#### Optimal (type 4, 329 leaves, 13 steps):

$$\frac{28 \text{ b d n x}}{e^8} = \frac{d \left(280 \text{ a} + 341 \text{ b n}\right) \text{ x}}{10 \text{ e}^8} = \frac{7 \text{ b n x}^2}{e^7} = \frac{28 \text{ b d x Log[c x}^n]}{e^8} = \frac{x^8 \left(\text{a} + \text{b Log[c x}^n]\right)}{6 \text{ e } \left(\text{d} + \text{e x}\right)^6} = \frac{x^7 \left(8 \text{ a} + \text{b n} + 8 \text{ b Log[c x}^n]\right)}{30 \text{ e}^2 \left(\text{d} + \text{e x}\right)^5} = \frac{x^6 \left(56 \text{ a} + 15 \text{ b n} + 56 \text{ b Log[c x}^n]\right)}{120 \text{ e}^3 \left(\text{d} + \text{e x}\right)^4} = \frac{x^5 \left(168 \text{ a} + 73 \text{ b n} + 168 \text{ b Log[c x}^n]\right)}{180 \text{ e}^4 \left(\text{d} + \text{e x}\right)^3} + \frac{x^2 \left(280 \text{ a} + 341 \text{ b n} + 280 \text{ b Log[c x}^n]\right)}{20 \text{ e}^7} = \frac{x^4 \left(840 \text{ a} + 533 \text{ b n} + 840 \text{ b Log[c x}^n]\right)}{360 \text{ e}^5 \left(\text{d} + \text{e x}\right)^2} = \frac{x^3 \left(840 \text{ a} + 743 \text{ b n} + 840 \text{ b Log[c x}^n]\right)}{90 \text{ e}^6 \left(\text{d} + \text{e x}\right)} + \frac{d^2 \left(280 \text{ a} + 341 \text{ b n} + 280 \text{ b Log[c x}^n]\right) \text{ Log}\left[1 + \frac{\text{e x}}{\text{d}}\right]}{\text{e}^9} + \frac{28 \text{ b d}^2 \text{ n PolyLog}\left[2, -\frac{\text{e x}}{\text{d}}\right]}{\text{e}^9}$$

#### Result (type 4, 394 leaves, 24 steps):

$$-\frac{7 \text{ a d } x}{e^8} + \frac{7 \text{ b d n } x}{e^8} - \frac{b \text{ n } x^2}{4 \text{ e}^7} + \frac{b \text{ d}^7 \text{ n}}{30 \text{ e}^9 \text{ (d + e x)}^5} - \frac{43 \text{ b d}^6 \text{ n}}{120 \text{ e}^9 \text{ (d + e x)}^4} + \frac{167 \text{ b d}^5 \text{ n}}{90 \text{ e}^9 \text{ (d + e x)}^3} - \frac{131 \text{ b d}^4 \text{ n}}{20 \text{ e}^9 \text{ (d + e x)}^2} + \frac{219 \text{ b d}^3 \text{ n}}{10 \text{ e}^9 \text{ (d + e x)}} + \frac{219 \text{ b d}^2 \text{ n Log}[x]}{10 \text{ e}^9} - \frac{7 \text{ b d } x \text{ Log}[c \text{ x}^n]}{e^8} + \frac{x^2 \text{ (a + b Log}[c \text{ x}^n])}{2 \text{ e}^7} - \frac{d^8 \text{ (a + b Log}[c \text{ x}^n])}{6 \text{ e}^9 \text{ (d + e x)}^6} + \frac{8 \text{ d}^7 \text{ (a + b Log}[c \text{ x}^n])}{5 \text{ e}^9 \text{ (d + e x)}^5} - \frac{7 \text{ d}^6 \text{ (a + b Log}[c \text{ x}^n])}{e^9 \text{ (d + e x)}^4} + \frac{56 \text{ d}^5 \text{ (a + b Log}[c \text{ x}^n])}{3 \text{ e}^9 \text{ (d + e x)}^3} - \frac{36 \text{ d}^2 \text{ x (a + b Log}[c \text{ x}^n])}{10 \text{ e}^9} + \frac{341 \text{ b d}^2 \text{ n Log}[d + e \text{ x}]}{10 \text{ e}^9} + \frac{28 \text{ d}^2 \text{ (a + b Log}[c \text{ x}^n]) \text{ Log}[1 + \frac{e \text{ x}}{d}]}{e^9} + \frac{28 \text{ b d}^2 \text{ n PolyLog}[2, -\frac{e \text{ x}}{d}]}{e^9}$$

### Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{x^7 (a + b \log[c x^n])}{(d + e x)^7} dx$$

Optimal (type 4, 285 leaves, 12 steps):

$$-\frac{7 \, b \, n \, x}{e^7} + \frac{\left(140 \, a + 223 \, b \, n\right) \, x}{20 \, e^7} + \frac{7 \, b \, x \, Log \left[c \, x^n\right]}{e^7} - \frac{x^7 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{6 \, e \, \left(d + e \, x\right)^6} - \frac{x^6 \, \left(7 \, a + b \, n + 7 \, b \, Log \left[c \, x^n\right]\right)}{30 \, e^2 \, \left(d + e \, x\right)^5} - \frac{x^5 \, \left(42 \, a + 13 \, b \, n + 42 \, b \, Log \left[c \, x^n\right]\right)}{120 \, e^3 \, \left(d + e \, x\right)^4} - \frac{x^2 \, \left(140 \, a + 153 \, b \, n + 140 \, b \, Log \left[c \, x^n\right]\right)}{40 \, e^6 \, \left(d + e \, x\right)} - \frac{x^4 \, \left(210 \, a + 107 \, b \, n + 210 \, b \, Log \left[c \, x^n\right]\right)}{360 \, e^4 \, \left(d + e \, x\right)^3} - \frac{x^3 \, \left(420 \, a + 319 \, b \, n + 420 \, b \, Log \left[c \, x^n\right]\right)}{360 \, e^5 \, \left(d + e \, x\right)^2} - \frac{d \, \left(140 \, a + 223 \, b \, n + 140 \, b \, Log \left[c \, x^n\right]\right)}{20 \, e^8} - \frac{7 \, b \, d \, n \, PolyLog \left[2 \, , \, -\frac{e \, x}{d}\right]}{e^8} - \frac{20 \, e^8}{20 \, e^8} - \frac{20 \, e^8}{20$$

Result (type 4, 351 leaves, 23 steps):

$$\frac{a\,x}{e^7} - \frac{b\,n\,x}{e^7} - \frac{b\,d^6\,n}{30\,e^8\,\left(d + e\,x\right)^5} + \frac{37\,b\,d^5\,n}{120\,e^8\,\left(d + e\,x\right)^4} - \frac{241\,b\,d^4\,n}{180\,e^8\,\left(d + e\,x\right)^3} + \frac{153\,b\,d^3\,n}{40\,e^8\,\left(d + e\,x\right)^2} - \frac{197\,b\,d^2\,n}{20\,e^8\,\left(d + e\,x\right)} - \frac{197\,b\,d\,n\,Log\,[x]}{20\,e^8} + \frac{b\,x\,Log\,[c\,x^n]\,\right)}{20\,e^8} + \frac{b\,x\,Log\,[c\,x^n]\,\right)}{6\,e^8\,\left(d + e\,x\right)^6} - \frac{7\,d^6\,\left(a + b\,Log\,[c\,x^n]\,\right)}{5\,e^8\,\left(d + e\,x\right)^5} + \frac{21\,d^5\,\left(a + b\,Log\,[c\,x^n]\,\right)}{4\,e^8\,\left(d + e\,x\right)^4} - \frac{35\,d^4\,\left(a + b\,Log\,[c\,x^n]\,\right)}{3\,e^8\,\left(d + e\,x\right)^3} + \frac{25\,d^3\,\left(a + b\,Log\,[c\,x^n]\,\right)}{2\,e^8\,\left(d + e\,x\right)^2} + \frac{21\,d\,x\,\left(a + b\,Log\,[c\,x^n]\,\right)}{e^7\,\left(d + e\,x\right)} - \frac{223\,b\,d\,n\,Log\,[d + e\,x]}{20\,e^8} - \frac{7\,d\,\left(a + b\,Log\,[c\,x^n]\,\right)\,Log\,\left[1 + \frac{e\,x}{d}\right]}{e^8} - \frac{7\,b\,d\,n\,PolyLog\,\left[2\,, -\frac{e\,x}{d}\right]}{e^8} + \frac{21\,d\,x\,\left(a + b\,Log\,[c\,x^n]\,\right)}{e^8} + \frac{223\,b\,d\,n\,Log\,[d + e\,x]}{20\,e^8} - \frac{223\,b\,d\,n\,Log\,[d + e\,x]}{e^8} - \frac{7\,d\,\left(a + b\,Log\,[c\,x^n]\,\right)\,Log\,\left[1 + \frac{e\,x}{d}\right]}{e^8} - \frac{7\,b\,d\,n\,PolyLog\,\left[2\,, -\frac{e\,x}{d}\right]}{e^8} + \frac{223\,b\,d\,n\,Log\,[d + e\,x]}{e^8} - \frac{223\,b\,d\,n\,Log\,[d + e\,x]}{e^8} + \frac{220\,d^3\,n}{e^8} + \frac{220\,d^3\,n}{e^8}$$

# Problem 64: Result valid but suboptimal antiderivative.

$$\int \frac{x^6 \left(a + b \operatorname{Log}\left[c x^n\right]\right)}{\left(d + e x\right)^7} dx$$

Optimal (type 4, 243 leaves, 8 steps):

$$-\frac{x^{6} \left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{6 \, \mathsf{e} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{6}} - \frac{x^{5} \, \left(\mathsf{6} \, \mathsf{a} + \mathsf{b} \, \mathsf{n} + \mathsf{6} \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{30 \, \mathsf{e}^{2} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{5}} - \frac{x^{2} \, \left(20 \, \mathsf{a} + 19 \, \mathsf{b} \, \mathsf{n} + 20 \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{40 \, \mathsf{e}^{5} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{2}} - \frac{x \, \left(20 \, \mathsf{a} + 29 \, \mathsf{b} \, \mathsf{n} + 20 \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{20 \, \mathsf{e}^{6} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)} - \frac{x^{3} \, \left(\mathsf{60} \, \mathsf{a} + 37 \, \mathsf{b} \, \mathsf{n} + \mathsf{60} \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{120 \, \mathsf{e}^{3} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{4}} - \frac{x^{3} \, \left(\mathsf{60} \, \mathsf{a} + 37 \, \mathsf{b} \, \mathsf{n} + \mathsf{60} \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right)}{180 \, \mathsf{e}^{4} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{3}} + \frac{\left(20 \, \mathsf{a} + 49 \, \mathsf{b} \, \mathsf{n} + 20 \, \mathsf{b} \, \mathsf{Log} \, [\mathsf{c} \, \mathsf{x}^{\mathsf{n}} \, ]\right) \, \mathsf{Log} \left[1 + \frac{\mathsf{e} \, \mathsf{x}}{\mathsf{d}}\right]}{\mathsf{e}^{7}} + \frac{\mathsf{b} \, \mathsf{n} \, \mathsf{PolyLog} \left[2, -\frac{\mathsf{e} \, \mathsf{x}}{\mathsf{d}}\right]}{\mathsf{e}^{7}}$$

Result (type 4, 316 leaves, 21 steps):

$$\frac{b\,d^{5}\,n}{30\,e^{7}\,\left(d+e\,x\right)^{5}} - \frac{31\,b\,d^{4}\,n}{120\,e^{7}\,\left(d+e\,x\right)^{4}} + \frac{163\,b\,d^{3}\,n}{180\,e^{7}\,\left(d+e\,x\right)^{3}} - \frac{79\,b\,d^{2}\,n}{40\,e^{7}\,\left(d+e\,x\right)^{2}} + \frac{71\,b\,d\,n}{20\,e^{7}\,\left(d+e\,x\right)} + \frac{71\,b\,d\,n}{20\,e^{7}\,\left(d+e\,x\right)} + \frac{71\,b\,d\,n}{20\,e^{7}\,\left(d+e\,x\right)^{3}} - \frac{15\,d^{4}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{4\,e^{7}\,\left(d+e\,x\right)^{4}} + \frac{20\,d^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{3\,e^{7}\,\left(d+e\,x\right)^{3}} - \frac{15\,d^{4}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{4\,e^{7}\,\left(d+e\,x\right)^{4}} + \frac{20\,d^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{3\,e^{7}\,\left(d+e\,x\right)^{3}} - \frac{15\,d^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2\,e^{7}\,\left(d+e\,x\right)^{3}} - \frac{6\,x\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{e^{6}\,\left(d+e\,x\right)} + \frac{49\,b\,n\,Log\left[d+e\,x\right]}{20\,e^{7}} + \frac{\left(a+b\,Log\left[c\,x^{n}\right]\right)\,Log\left[1+\frac{e\,x}{d}\right]}{e^{7}} + \frac{b\,n\,PolyLog\left[2,\,-\frac{e\,x}{d}\right]}{e^{7}} - \frac{e^{7}\,d^{2}$$

#### Problem 71: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x (d + e x)^7} dx$$

#### Optimal (type 4, 294 leaves, 25 steps):

$$-\frac{b\,n}{30\,d^{2}\,\left(d+e\,x\right)^{\,5}} - \frac{11\,b\,n}{120\,d^{3}\,\left(d+e\,x\right)^{\,4}} - \frac{37\,b\,n}{180\,d^{4}\,\left(d+e\,x\right)^{\,3}} - \frac{19\,b\,n}{40\,d^{5}\,\left(d+e\,x\right)^{\,2}} - \frac{29\,b\,n}{20\,d^{6}\,\left(d+e\,x\right)} - \frac{29\,b\,n}{20\,d^{6}\,\left(d+e\,x\right)^{\,2}} - \frac{29\,b\,n}{20\,d$$

#### Result (type 4, 316 leaves, 27 steps):

$$\frac{b\,n}{30\,d^2\,\left(d+e\,x\right)^5} - \frac{11\,b\,n}{120\,d^3\,\left(d+e\,x\right)^4} - \frac{37\,b\,n}{180\,d^4\,\left(d+e\,x\right)^3} - \frac{19\,b\,n}{40\,d^5\,\left(d+e\,x\right)^2} - \frac{29\,b\,n}{20\,d^6\,\left(d+e\,x\right)} - \frac{29\,b\,n}{20\,d^6\,\left(d+e\,x\right)$$

### Problem 72: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log [c x^n]}{x^2 (d + e x)^7} dx$$

Optimal (type 4, 339 leaves, 22 steps):

$$-\frac{b\,n}{d^{7}\,x} + \frac{b\,e\,n}{30\,d^{3}\,\left(d + e\,x\right)^{\,5}} + \frac{17\,b\,e\,n}{120\,d^{4}\,\left(d + e\,x\right)^{\,4}} + \frac{79\,b\,e\,n}{180\,d^{5}\,\left(d + e\,x\right)^{\,3}} + \frac{53\,b\,e\,n}{40\,d^{6}\,\left(d + e\,x\right)^{\,2}} + \frac{103\,b\,e\,n}{20\,d^{7}\,\left(d + e\,x\right)} + \frac{103\,b\,e\,n\,\log\left[x\right]}{20\,d^{8}} - \frac{20\,d^{8}\,\left(d + e\,x\right)^{\,2}}{20\,d^{8}} - \frac{3\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{4\,d^{4}\,\left(d + e\,x\right)^{\,4}} - \frac{4\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{3\,d^{5}\,\left(d + e\,x\right)^{\,3}} - \frac{5\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{2\,d^{6}\,\left(d + e\,x\right)^{\,2}} + \frac{6\,e^{2}\,x\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{d^{8}\,\left(d + e\,x\right)} + \frac{7\,e\,\log\left[1 + \frac{d}{e\,x}\right]\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{d^{8}} - \frac{223\,b\,e\,n\,\log\left[d + e\,x\right]}{20\,d^{8}} - \frac{7\,b\,e\,n\,PolyLog\left[2, -\frac{d}{e\,x}\right]}{d^{8}}$$

#### Result (type 4, 361 leaves, 23 steps):

$$-\frac{b\,n}{d^{7}\,x} + \frac{b\,e\,n}{30\,d^{3}\,\left(d + e\,x\right)^{5}} + \frac{17\,b\,e\,n}{120\,d^{4}\,\left(d + e\,x\right)^{4}} + \frac{79\,b\,e\,n}{180\,d^{5}\,\left(d + e\,x\right)^{3}} + \frac{53\,b\,e\,n}{40\,d^{6}\,\left(d + e\,x\right)^{2}} + \frac{103\,b\,e\,n}{20\,d^{7}\,\left(d + e\,x\right)} + \frac{103\,b\,e\,n\,\log\left[x\right]}{20\,d^{8}} - \frac{a + b\,\log\left[c\,x^{n}\right]}{d^{7}\,x} - \frac{e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{6\,d^{2}\,\left(d + e\,x\right)^{6}} - \frac{2\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{5\,d^{3}\,\left(d + e\,x\right)^{5}} - \frac{3\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{4\,d^{4}\,\left(d + e\,x\right)^{4}} - \frac{4\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{3\,d^{5}\,\left(d + e\,x\right)^{3}} - \frac{5\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{2\,d^{6}\,\left(d + e\,x\right)^{2}} + \frac{6\,e^{2}\,x\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{d^{8}} - \frac{7\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)}{2\,b\,d^{8}\,n} - \frac{223\,b\,e\,n\,\log\left[d + e\,x\right]}{20\,d^{8}} + \frac{7\,e\,\left(a + b\,\log\left[c\,x^{n}\right]\right)\,\log\left[1 + \frac{e\,x}{d}\right]}{d^{8}} + \frac{7\,b\,e\,n\,PolyLog\left[2, -\frac{e\,x}{d}\right]}{d^{8}} - \frac{103\,b\,e\,n}{20\,d^{8}} + \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} + \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} + \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} + \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} - \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} - \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} - \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} + \frac{103\,b\,e\,n\,\log\left[c\,x^{n}\right]}{20\,d^{8}} - \frac{103\,b\,e$$

## Problem 73: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^3 (d + e x)^7} dx$$

Optimal (type 4, 401 leaves, 23 steps):

$$-\frac{b\,n}{4\,d^{7}\,x^{2}} + \frac{7\,b\,e\,n}{d^{8}\,x} - \frac{b\,e^{2}\,n}{30\,d^{4}\,\left(d+e\,x\right)^{\,5}} - \frac{23\,b\,e^{2}\,n}{120\,d^{5}\,\left(d+e\,x\right)^{\,4}} - \frac{34\,b\,e^{2}\,n}{45\,d^{6}\,\left(d+e\,x\right)^{\,3}} - \frac{14\,b\,e^{2}\,n}{5\,d^{7}\,\left(d+e\,x\right)^{\,2}} - \frac{131\,b\,e^{2}\,n}{10\,d^{8}\,\left(d+e\,x\right)} - \frac{131\,b\,e^{2}\,n\,\log\left[x\right]}{10\,d^{9}} - \frac{10\,d^{9}\,\left(d+e\,x\right)^{\,3}}{10\,d^{9}\,d$$

Result (type 4, 423 leaves, 24 steps):

$$-\frac{b\,n}{4\,d^{7}\,x^{2}} + \frac{7\,b\,e\,n}{d^{8}\,x} - \frac{b\,e^{2}\,n}{30\,d^{4}\,\left(d+e\,x\right)^{5}} - \frac{23\,b\,e^{2}\,n}{120\,d^{5}\,\left(d+e\,x\right)^{4}} - \frac{34\,b\,e^{2}\,n}{45\,d^{6}\,\left(d+e\,x\right)^{3}} - \frac{14\,b\,e^{2}\,n}{5\,d^{7}\,\left(d+e\,x\right)^{2}} - \frac{131\,b\,e^{2}\,n}{10\,d^{8}\,\left(d+e\,x\right)} - \frac{131\,b\,e^{2}\,n\,\log\left[x\right]}{10\,d^{9}} - \frac{a+b\,\log\left[c\,x^{n}\right]}{2\,d^{7}\,x^{2}} + \frac{7\,e\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{2\,d^{7}\,x^{2}} + \frac{e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{6\,d^{3}\,\left(d+e\,x\right)^{6}} + \frac{3\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{5\,d^{4}\,\left(d+e\,x\right)^{5}} + \frac{3\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{2\,d^{5}\,\left(d+e\,x\right)^{4}} + \frac{10\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{3\,d^{6}\,\left(d+e\,x\right)^{3}} + \frac{15\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{2\,d^{7}\,\left(d+e\,x\right)^{2}} - \frac{28\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)}{d^{9}} + \frac{14\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)^{2}}{b\,d^{9}\,n} + \frac{341\,b\,e^{2}\,n\,\log\left[d+e\,x\right]}{10\,d^{9}} - \frac{28\,e^{2}\,\left(a+b\,\log\left[c\,x^{n}\right]\right)\,\log\left[1+\frac{e\,x}{d}\right]}{d^{9}} - \frac{28\,b\,e^{2}\,n\,PolyLog\left[2\,,\,-\frac{e\,x}{d}\right]}{d^{9}}$$

### Problem 86: Result valid but suboptimal antiderivative.

$$\int (d + e x)^{2} (a + b Log[c x^{n}])^{2} dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$2\,b^{2}\,d^{2}\,n^{2}\,x + \frac{1}{2}\,b^{2}\,d\,e\,n^{2}\,x^{2} + \frac{2}{27}\,b^{2}\,e^{2}\,n^{2}\,x^{3} + \frac{b^{2}\,d^{3}\,n^{2}\,Log\,[\,x\,]^{\,2}}{3\,e} - 2\,b\,d^{2}\,n\,x\,\left(a + b\,Log\,[\,c\,\,x^{n}\,]\,\right) - \frac{b\,d\,e\,n\,x^{2}\,\left(a + b\,Log\,[\,c\,\,x^{n}\,]\,\right) - \frac{2}{9}\,b\,e^{2}\,n\,x^{3}\,\left(a + b\,Log\,[\,c\,\,x^{n}\,]\,\right) - \frac{2\,b\,d^{3}\,n\,Log\,[\,x\,]\,\left(a + b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3\,e} + \frac{\left(d + e\,x\right)^{\,3}\,\left(a + b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{\,2}}{3\,e}$$

Result (type 3, 141 leaves, 5 steps):

$$\frac{2 \, b^{2} \, d^{2} \, n^{2} \, x + \frac{1}{2} \, b^{2} \, d \, e \, n^{2} \, x^{2} + \frac{2}{27} \, b^{2} \, e^{2} \, n^{2} \, x^{3} + \frac{b^{2} \, d^{3} \, n^{2} \, Log \, [\, x \, ]^{\, 2}}{3 \, e} - \\ \frac{b \, n \, \left(18 \, d^{2} \, e \, x + 9 \, d \, e^{2} \, x^{2} + 2 \, e^{3} \, x^{3} + 6 \, d^{3} \, Log \, [\, x \, ] \, \right) \, \left(a + b \, Log \, [\, c \, x^{n} \, ] \, \right)}{9 \, e} + \frac{\left(d + e \, x\right)^{\, 3} \, \left(a + b \, Log \, [\, c \, x^{n} \, ] \, \right)^{\, 2}}{3 \, e}$$

## Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log \left[c \, x^n\right]\right)^2}{x \, \left(d+e \, x\right)} \, dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$-\frac{\text{Log}\big[1+\frac{d}{e\,x}\,\big]\,\left(a+b\,\text{Log}\,[c\,x^n\,]\,\right)^2}{d}+\frac{2\,b\,n\,\left(a+b\,\text{Log}\,[c\,x^n\,]\,\right)\,\text{PolyLog}\big[2,-\frac{d}{e\,x}\,\big]}{d}+\frac{2\,b^2\,n^2\,\text{PolyLog}\big[3,-\frac{d}{e\,x}\,\big]}{d}$$

Result (type 4, 98 leaves, 6 steps):

$$\frac{\left(a+b\, \text{Log}\left[\, c\, \, x^{n}\, \right]\,\right)^{\,3}}{3\, b\, d\, n}-\frac{\left(a+b\, \text{Log}\left[\, c\, \, x^{n}\, \right]\,\right)^{\,2}\, \text{Log}\left[\, 1+\frac{e\, x}{d}\, \right]}{d}-\frac{2\, b\, n\, \left(a+b\, \text{Log}\left[\, c\, \, x^{n}\, \right]\,\right)\, \text{PolyLog}\left[\, 2\, ,\, -\frac{e\, x}{d}\, \right]}{d}+\frac{2\, b^{2}\, n^{2}\, \text{PolyLog}\left[\, 3\, ,\, -\frac{e\, x}{d}\, \right]}{d}$$

### Problem 97: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c x^{n}\right]\right)^{2}}{x^{2} \left(d + e x\right)} dx$$

#### Optimal (type 4, 135 leaves, 6 steps):

$$-\frac{2 \, b^2 \, n^2}{d \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d \, x} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d \, x} + \\ \frac{e \, Log \left[1 + \frac{d}{e \, x}\right] \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^2} - \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, \, -\frac{d}{e \, x}\right]}{d^2} - \frac{2 \, b^2 \, e \, n^2 \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^2}$$

#### Result (type 4, 155 leaves, 9 steps):

$$-\frac{2 \, b^2 \, n^2}{d \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d \, x} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d \, x} - \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{3 \, b \, d^2 \, n} + \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^2} + \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^2} - \frac{2 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^2}$$

### Problem 98: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^{\,2}}{x^3\,\,\left(d+e\,x\right)}\,\mathrm{d}x$$

#### Optimal (type 4, 204 leaves, 9 steps):

$$-\frac{\frac{b^{2} \, n^{2}}{4 \, d \, x^{2}}}{\frac{1}{4} \, d \, x^{2}} + \frac{2 \, b^{2} \, e \, n^{2}}{d^{2} \, x} - \frac{b \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{2 \, d \, x^{2}} + \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{d^{2} \, x} - \frac{\left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{2 \, d \, x^{2}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{2} \, x} - \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3}} + \frac{e \, \left(a + b \, Log$$

#### Result (type 4, 226 leaves, 11 steps):

$$-\frac{b^{2} n^{2}}{4 d x^{2}} + \frac{2 b^{2} e n^{2}}{d^{2} x} - \frac{b n \left(a + b \log[c x^{n}]\right)}{2 d x^{2}} + \frac{2 b e n \left(a + b \log[c x^{n}]\right)}{d^{2} x} - \frac{\left(a + b \log[c x^{n}]\right)^{2}}{2 d x^{2}} + \frac{e \left(a + b \log[c x^{n}]\right)^{2}}{d^{2} x} + \frac{e \left(a + b \log[c x^{n}]\right)^{2}}{d^{2} x} + \frac{e \left(a + b \log[c x^{n}]\right)^{2}}{d^{2} x} + \frac{e \left(a + b \log[c x^{n}]\right)^{2}}{d^{3}} + \frac{e^{2} \left(a + b \log[c x^{n}]\right)^{2} \log[1 + \frac{ex}{d}]}{d^{3}} + \frac{e^{2} \left(a + b \log[c x^{n}]\right)^{2}}{d^{3}} + \frac{e^{$$

## Problem 99: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c x^{n}\right]\right)^{2}}{x^{4} \left(d + e x\right)} dx$$

Optimal (type 4, 273 leaves, 12 steps):

$$-\frac{2 \, b^{2} \, n^{2}}{27 \, d \, x^{3}} + \frac{b^{2} \, e \, n^{2}}{4 \, d^{2} \, x^{2}} - \frac{2 \, b^{2} \, e^{2} \, n^{2}}{d^{3} \, x} - \frac{2 \, b \, n \, \left(a + b \, log \left[c \, x^{n}\right]\right)}{9 \, d \, x^{3}} + \frac{b \, e \, n \, \left(a + b \, log \left[c \, x^{n}\right]\right)}{2 \, d^{2} \, x^{2}} - \frac{2 \, b \, e^{2} \, n \, \left(a + b \, log \left[c \, x^{n}\right]\right)}{d^{3} \, x} - \frac{\left(a + b \, log \left[c \, x^{n}\right]\right)^{2}}{3 \, d \, x^{3}} + \frac{e \, \left(a + b \, log \left[c \, x^{n}\right]\right)^{2}}{2 \, d^{2} \, x^{2}} - \frac{e \, a \, b \, log \left[c \, x^{n}\right]}{d^{4}} - \frac{e \, a \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, a \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, b \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, log \left[c \, x^{n}\right]}{2 \, d^{2} \, x^{2}} - \frac{e \, log \left[c \, x^{n}\right]}{2$$

Result (type 4, 295 leaves, 13 steps):

$$-\frac{2\,b^{2}\,n^{2}}{27\,d\,x^{3}} + \frac{b^{2}\,e\,n^{2}}{4\,d^{2}\,x^{2}} - \frac{2\,b^{2}\,e^{2}\,n^{2}}{d^{3}\,x} - \frac{2\,b\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{9\,d\,x^{3}} + \frac{b\,e\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2\,d^{2}\,x^{2}} - \frac{2\,b\,e^{2}\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{3\,d\,x^{3}} - \frac{\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{3\,d\,x^{3}} + \frac{e\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{2\,d^{2}\,x^{2}} - \frac{e^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{d^{3}\,x} - \frac{e^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{3}}{3\,b\,d^{4}\,n} + \frac{e^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}\,Log\left[1+\frac{e\,x}{d}\right]}{d^{4}} + \frac{2\,b\,e^{3}\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,PolyLog\left[2,-\frac{e\,x}{d}\right]}{d^{4}} - \frac{2\,b^{2}\,e^{3}\,n^{2}\,PolyLog\left[3,-\frac{e\,x}{d}\right]}{d^{4}}$$

# Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c x^{n}\right]\right)^{2}}{x \left(d + e x\right)^{2}} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$-\frac{e\,x\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{c}\,\,x^n\,]\,\right)^2}{d^2\,\left(\mathsf{d}+\mathsf{e}\,\,x\right)}\,-\frac{\mathsf{Log}\,\!\left[\,1+\frac{\mathsf{d}}{\mathsf{e}\,x}\,\right]\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{c}\,\,x^n\,]\,\right)^2}{d^2}\,+\,\frac{2\,\mathsf{b}\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{c}\,\,x^n\,]\,\right)\,\mathsf{Log}\,\!\left[\,1+\frac{\mathsf{e}\,x}{\mathsf{d}}\,\right]}{d^2}\,+\,\frac{2\,\mathsf{b}\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{c}\,\,x^n\,]\,\right)\,\mathsf{Log}\,\!\left[\,1+\frac{\mathsf{e}\,x}{\mathsf{d}}\,\right]}{d^2}\,+\,\frac{2\,\mathsf{b}\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\,[\,\mathsf{c}\,\,x^n\,]\,\right)\,\mathsf{PolyLog}\,\!\left[\,2\,,\,-\frac{\mathsf{d}}{\mathsf{e}\,x}\,\right]}{d^2}\,+\,\frac{2\,\mathsf{b}^2\,n^2\,\mathsf{PolyLog}\,\!\left[\,2\,,\,-\frac{\mathsf{e}\,x}{\mathsf{d}}\,\right]}{d^2}\,+\,\frac{2\,\mathsf{b}^2\,n^2\,\mathsf{PolyLog}\,\!\left[\,3\,,\,-\frac{\mathsf{d}}{\mathsf{e}\,x}\,\right]}{d^2}$$

Result (type 4, 170 leaves, 10 steps):

$$-\frac{e \hspace{0.1cm} x \hspace{0.1cm} \left(a + b \hspace{0.1cm} \text{Log} \hspace{0.1cm} [\hspace{0.1cm} c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} \right)^2}{d^2 \hspace{0.1cm} \left(d + e \hspace{0.1cm} x \right)} + \frac{\left(a + b \hspace{0.1cm} \text{Log} \hspace{0.1cm} [\hspace{0.1cm} c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} \right) \hspace{0.1cm} \text{Log} \hspace{0.1cm} [\hspace{0.1cm} c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} \right) \hspace{0.1cm} \text{Log} \hspace{0.1cm} \left[c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} \right)^3 \hspace{0.1cm} + \frac{2 \hspace{0.1cm} b \hspace{0.1cm} n \hspace{0.1cm} \left(a + b \hspace{0.1cm} \text{Log} \hspace{0.1cm} [\hspace{0.1cm} c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} \right) \hspace{0.1cm} \text{PolyLog} \hspace{0.1cm} \left[2, -\frac{e \hspace{0.1cm} x}{d} \right]} - \frac{2 \hspace{0.1cm} b \hspace{0.1cm} n \hspace{0.1cm} \left(a + b \hspace{0.1cm} \text{Log} \hspace{0.1cm} [\hspace{0.1cm} c \hspace{0.1cm} x^n \hspace{0.1cm}] \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[2, -\frac{e \hspace{0.1cm} x}{d} \right]} {d^2} + \frac{2 \hspace{0.1cm} b^2 \hspace{0.1cm} n^2 \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[3, -\frac{e \hspace{0.1cm} x}{d} \right]}{d^2} \hspace{0.1cm} + \hspace{0.1cm} \frac{1}{2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^2 \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[2, -\frac{e \hspace{0.1cm} x}{d} \right]}{d^2} + \frac{2 \hspace{0.1cm} b^2 \hspace{0.1cm} n^2 \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[3, -\frac{e \hspace{0.1cm} x}{d} \right]}{d^2} \hspace{0.1cm} + \hspace{0.1cm} \frac{1}{2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^2 \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[2, -\frac{e \hspace{0.1cm} x}{d} \right]}{d^2} \hspace{0.1cm} + \hspace{0.1cm} \frac{1}{2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^2 \hspace{0.1cm} PolyLog \hspace{0.1cm} \left[2, -\frac{e \hspace{0.1cm} x}{d} \right]}{d^2} \hspace{0.1cm} + \hspace{0.1cm} \frac{1}{2 \hspace{0.1cm} n^2 \hspace{0.1cm} n^$$

### Problem 105: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c x^{n}\right]\right)^{2}}{x^{2} \left(d + e x\right)^{2}} dx$$

Optimal (type 4, 211 leaves, 10 steps):

$$-\frac{2 \, b^2 \, n^2}{d^2 \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^2 \, x} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^2 \, x} + \frac{e^2 \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)} + \frac{2 \, e \, Log \left[1 + \frac{d}{e \, x}\right] \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3} - \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{d}{e \, x}\right]}{d^3} - \frac{2 \, b^2 \, e \, n^2 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, PolyLog \left[3, -\frac{d}{e \, x}$$

Result (type 4, 231 leaves, 12 steps):

$$-\frac{2 \, b^2 \, n^2}{d^2 \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^2 \, x} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^2 \, x} + \frac{e^2 \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)} - \frac{2 \, e \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{3 \, b \, d^3 \, n} - \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} + \frac{2 \, e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{2 \, b^2 \, e \, n^2 \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{4 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[$$

# Problem 106: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log \left[c \, x^n\right]\right)^2}{x^3 \, \left(d+e \, x\right)^2} \, \mathrm{d}x$$

Optimal (type 4, 285 leaves, 12 steps):

$$-\frac{b^{2} \, n^{2}}{4 \, d^{2} \, x^{2}} + \frac{4 \, b^{2} \, e \, n^{2}}{d^{3} \, x} - \frac{b \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{2 \, d^{2} \, x^{2}} + \frac{4 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{d^{3} \, x} - \frac{2 \, d^{2} \, x^{2}}{2 \, d^{2} \, x^{2}} + \frac{2 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{3} \, x} - \frac{2 \, d^{2} \, x^{2}}{2 \, d^{2} \, x^{2}} + \frac{2 \, b \, e^{2} \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{d^{4}} + \frac{2 \, b \, e^{2} \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} + \frac{6 \, b^{2} \, e^{2} \, n^{2} \, PolyLog \left[3, \, -\frac{d}{e \, x}\right]}{d^{4}} +$$

Result (type 4, 304 leaves, 14 steps):

#### Problem 107: Result optimal but 2 more steps used.

$$\int \frac{x^3 \left(a + b \operatorname{Log}\left[c x^n\right]\right)^2}{\left(d + e x\right)^3} \, dx$$

Optimal (type 4, 296 leaves, 17 steps):

$$-\frac{2 \, a \, b \, n \, x}{e^3} + \frac{2 \, b^2 \, n^2 \, x}{e^3} - \frac{2 \, b^2 \, n \, x \, Log[c \, x^n]}{e^3} + \frac{b \, d \, n \, x \, \left(a + b \, Log[c \, x^n]\right)}{e^3 \, \left(d + e \, x\right)} - \frac{d \, \left(a + b \, Log[c \, x^n]\right)^2}{2 \, e^4} + \frac{x \, \left(a + b \, Log[c \, x^n]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log[c \, x^n]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log[c \, x^n]\right)^2}{e^3} + \frac{d^3 \, d \, x \, \left(a + b \, Log[c \, x^n]\right)^2}{e^3} - \frac{b^2 \, d \, n^2 \, Log[d + e \, x]}{e^4} - \frac{5 \, b \, d \, n \, \left(a + b \, Log[c \, x^n]\right) \, Log\left[1 + \frac{e \, x}{d}\right]}{e^4} - \frac{d^3 \, d \, a + b \, Log[c \, x^n]}{e^4} - \frac{d^3 \, d \, a + b \, Log[c \, x^n]\right)^2}{e^4} + \frac{d^3 \, d \, a + b \, Log[c \, x^n]}{e^4} + \frac{d^3 \, d \, a + b \, Log[c \, x^n]}{e^4} - \frac{d^3 \, d \, a + b \, Log[c \, x^n]}{e^4} - \frac{d^3 \, d \, a + b \, Log[c \, x^n]}{e^4} + \frac{d^3 \, d \, a + b$$

#### Result (type 4, 296 leaves, 19 steps):

$$-\frac{2 \, a \, b \, n \, x}{e^3} + \frac{2 \, b^2 \, n^2 \, x}{e^3} - \frac{2 \, b^2 \, n \, x \, Log \left[c \, x^n\right]}{e^3} + \frac{b \, d \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{e^3 \, \left(d + e \, x\right)} - \frac{d \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, e^4} + \frac{x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{e^3} + \frac{d^3 \, \left(a + b \, Log \left[c \, x^n\right]$$

## Problem 108: Result optimal but 2 more steps used.

$$\int \frac{x^2 \left(a + b \operatorname{Log}\left[c x^n\right]\right)^2}{\left(d + e x\right)^3} \, dx$$

Optimal (type 4, 232 leaves, 14 steps):

$$-\frac{b\,n\,x\,\left(a+b\,Log\,[c\,x^{n}]\,\right)}{e^{2}\,\left(d+e\,x\right)} + \frac{\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{2\,e^{3}} - \frac{d^{2}\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{2\,e^{3}\,\left(d+e\,x\right)^{2}} - \frac{2\,x\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{e^{2}\,\left(d+e\,x\right)} + \frac{b^{2}\,n^{2}\,Log\,[d+e\,x]}{e^{3}} + \frac{3\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,Log\,\left[1+\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} - \frac{2\,b^{2}\,n^{2}\,PolyLog\,\left[3,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{3\,b^{2}\,n^{2}\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} - \frac{2\,b^{2}\,n^{2}\,PolyLog\,\left[3,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b^{2}\,n^{2}\,PolyLog\,\left[3,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b^{2}\,n^{2}\,PolyLog\,\left[3,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b^{2}\,n^{2}\,PolyLog\,\left[3,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,x}{d}\right]}{e^{3}} + \frac{2\,b\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2,-\frac{e\,$$

Result (type 4, 232 leaves, 16 steps):

### Problem 109: Result valid but suboptimal antiderivative.

$$\int \frac{x \, \left(a + b \, \text{Log} \left[\, c \, \, x^n \,\right]\,\right)^2}{\left(d + e \, x\right)^3} \, \mathrm{d} x$$

Optimal (type 4, 112 leaves, 4 steps):

$$\frac{b\;n\;x\;\left(\,a\;+\;b\;Log\,\left[\,c\;\,x^{n}\,\right]\,\right)}{d\;e\;\left(\,d\;+\;e\;x\,\right)}\;+\;\frac{x^{2}\;\left(\,a\;+\;b\;Log\,\left[\,c\;\,x^{n}\,\right]\,\right)^{\,2}}{2\;d\;\left(\,d\;+\;e\;x\,\right)^{\,2}}\;-\;\frac{b\;n\;\left(\,a\;+\;b\;n\;+\;b\;Log\,\left[\,c\;\,x^{n}\,\right]\,\right)\;Log\,\left[\,1\;+\;\frac{e\,x}{d}\,\right]}{d\;e^{2}}\;-\;\frac{b^{2}\;n^{2}\;PolyLog\,\left[\,2\;,\;\;-\,\frac{e\,x}{d}\,\right]}{d\;e^{2}}$$

Result (type 4, 176 leaves, 13 steps):

$$\begin{split} & \frac{b \; n \; x \; \left(a + b \; \text{Log}\left[c \; x^n\right]\right)}{d \; e \; \left(d + e \; x\right)} - \frac{\left(a + b \; \text{Log}\left[c \; x^n\right]\right)^2}{2 \; d \; e^2} + \frac{d \; \left(a + b \; \text{Log}\left[c \; x^n\right]\right)^2}{2 \; e^2 \; \left(d + e \; x\right)^2} + \\ & \frac{x \; \left(a + b \; \text{Log}\left[c \; x^n\right]\right)^2}{d \; e \; \left(d + e \; x\right)} - \frac{b^2 \; n^2 \; \text{Log}\left[d + e \; x\right]}{d \; e^2} - \frac{b \; n \; \left(a + b \; \text{Log}\left[c \; x^n\right]\right) \; \text{Log}\left[1 + \frac{e \; x}{d}\right]}{d \; e^2} - \frac{b^2 \; n^2 \; \text{PolyLog}\left[2, -\frac{e \; x}{d}\right]}{d \; e^2} \end{split}$$

# Problem 110: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log \left[c \, x^{n}\right]\right)^{2}}{\left(d+e \, x\right)^{3}} \, \mathrm{d}x$$

Optimal (type 4, 126 leaves, 6 steps):

$$-\frac{b \, n \, x \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{d^{2} \, \left(d + e \, x\right)} - \frac{b \, n \, Log \left[1 + \frac{d}{e \, x}\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{d^{2} \, e} - \frac{\left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{2 \, e \, \left(d + e \, x\right)^{2}} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, PolyLog \left[2, -\frac{d}{e \, x}\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, e} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{2} \, Log \left[d + e \, x\right]}$$

Result (type 4, 145 leaves, 8 steps):

$$-\frac{b \, n \, x \, \left(a + b \, \text{Log}\left[c \, x^n\right]\right)}{d^2 \, \left(d + e \, x\right)} + \frac{\left(a + b \, \text{Log}\left[c \, x^n\right]\right)^2}{2 \, d^2 \, e} - \frac{\left(a + b \, \text{Log}\left[c \, x^n\right]\right)^2}{2 \, e \, \left(d + e \, x\right)^2} + \frac{b^2 \, n^2 \, \text{Log}\left[d + e \, x\right]}{d^2 \, e} - \frac{b \, n \, \left(a + b \, \text{Log}\left[c \, x^n\right]\right) \, \text{Log}\left[1 + \frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^2 \, e} - \frac{b^2 \, n^2 \, n^2$$

## Problem 111: Result optimal but 5 more steps used.

$$\int \frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^{\,2}}{x\,\,\left(d+e\,x\right)^{\,3}}\,\,\mathrm{d}x$$

Optimal (type 4, 257 leaves, 14 steps):

$$\frac{b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^3 \, \left(d + e \, x\right)} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d \, \left(d + e \, x\right)^2} - \frac{e \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} + \frac{3 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyL$$

#### Result (type 4, 257 leaves, 19 steps):

$$\frac{b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^3 \, \left(d + e \, x\right)} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d \, \left(d + e \, x\right)^2} - \frac{e \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d \, \left(d + e \, x\right)^2} - \frac{e \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} + \frac{3 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} + \frac{3 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3,$$

## Problem 112: Result optimal but 4 more steps used.

$$\int \frac{\left(a + b \operatorname{Log}\left[c \, x^{n}\right]\right)^{2}}{x^{2} \, \left(d + e \, x\right)^{3}} \, dx$$

Optimal (type 4, 322 leaves, 16 steps):

$$-\frac{2 \, b^2 \, n^2}{d^3 \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^3 \, x} - \frac{b \, e^2 \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^4 \, \left(d + e \, x\right)} + \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^4} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, x} - \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^4 \, \left(d + e \, x\right)} - \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{b \, d^4 \, n} + \frac{b^2 \, e \, n^2 \, Log \left[d + e \, x\right]}{d^4} - \frac{5 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^4} + \frac{3 \, e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^4} - \frac{5 \, b^2 \, e \, n^2 \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left$$

#### Result (type 4, 322 leaves, 20 steps):

$$-\frac{2 \, b^2 \, n^2}{d^3 \, x} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^3 \, x} - \frac{b \, e^2 \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^4 \, \left(d + e \, x\right)} + \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^4} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^3 \, x} - \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{d^4 \, \left(d + e \, x\right)} - \frac{e \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{b \, d^4 \, n} + \frac{b^2 \, e \, n^2 \, Log \left[d + e \, x\right]}{d^4} - \frac{5 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^4} + \frac{3 \, e \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^4} - \frac{5 \, b^2 \, e \, n^2 \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} - \frac{6 \, b^2 \, e \, n^2 \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[2, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Poly Log \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^4} + \frac{6 \, b \, e \, n \, \left(a + b \, Log \left$$

## Problem 113: Result optimal but 4 more steps used.

$$\int \frac{x^4 \left(a + b \log \left[c x^n\right]\right)^2}{\left(d + e x\right)^4} dx$$

#### Optimal (type 4, 398 leaves, 27 steps):

$$-\frac{2 \text{ a b n x}}{e^4} + \frac{2 \text{ b}^2 \text{ n}^2 \text{ x}}{e^4} - \frac{b^2 \text{ d}^2 \text{ n}^2}{3 \text{ e}^5 \text{ (d + e x)}} - \frac{b^2 \text{ d n}^2 \text{ Log}[x]}{3 \text{ e}^5} - \frac{2 \text{ b}^2 \text{ n x Log}[c \text{ x}^n]}{e^4} + \frac{b \text{ d}^3 \text{ n (a + b Log}[c \text{ x}^n])}{3 \text{ e}^5 \text{ (d + e x)}^2} + \frac{10 \text{ b d n x (a + b Log}[c \text{ x}^n])}{3 \text{ e}^4 \text{ (d + e x)}} - \frac{5 \text{ d (a + b Log}[c \text{ x}^n])^2}{3 \text{ e}^5} + \frac{x \text{ (a + b Log}[c \text{ x}^n])^2}{e^4} - \frac{d^4 \text{ (a + b Log}[c \text{ x}^n])^2}{3 \text{ e}^5 \text{ (d + e x)}^3} + \frac{2 \text{ d}^3 \text{ (a + b Log}[c \text{ x}^n])^2}{e^5 \text{ (d + e x)}^2} + \frac{6 \text{ d x (a + b Log}[c \text{ x}^n])^2}{3 \text{ e}^5 \text{ (d + e x)}^3} - \frac{3 \text{ b}^2 \text{ d n}^2 \text{ Log}[d + e \text{ x}]}{e^5} - \frac{26 \text{ b d n (a + b Log}[c \text{ x}^n]) \text{ Log}[1 + \frac{e \text{ x}}{d}]}{3 \text{ e}^5} - \frac{4 \text{ d (a + b Log}[c \text{ x}^n])^2 \text{ Log}[1 + \frac{e \text{ x}}{d}]}{e^5} + \frac{26 \text{ b d n (a + b Log}[c \text{ x}^n]) \text{ PolyLog}[2, -\frac{e \text{ x}}{d}]}{e^5} + \frac{8 \text{ b}^2 \text{ d n}^2 \text{ PolyLog}[3, -\frac{e \text{ x}}{d}]}{e^5}$$

Result (type 4, 398 leaves, 31 steps):

$$-\frac{2 \ a \ b \ n \ x}{e^4} + \frac{2 \ b^2 \ n^2 \ x}{e^4} - \frac{b^2 \ d^2 \ n^2}{3 \ e^5 \ (d + e \ x)} - \frac{b^2 \ d \ n^2 \ Log[x]}{3 \ e^5} - \frac{2 \ b^2 \ n \ x \ Log[c \ x^n]}{e^4} + \frac{b \ d^3 \ n \ \left(a + b \ Log[c \ x^n]\right)}{3 \ e^5 \ \left(d + e \ x\right)^2} + \frac{10 \ b \ d \ n \ x \ \left(a + b \ Log[c \ x^n]\right)^2}{3 \ e^4 \ \left(d + e \ x\right)} + \frac{5 \ d \ \left(a + b \ Log[c \ x^n]\right)^2}{3 \ e^5} + \frac{x \ \left(a + b \ Log[c \ x^n]\right)^2}{e^4} - \frac{d^4 \ \left(a + b \ Log[c \ x^n]\right)^2}{3 \ e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^2} + \frac{6 \ d \ x \ \left(a + b \ Log[c \ x^n]\right)^2}{3 \ e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^2} + \frac{6 \ d \ x \ \left(a + b \ Log[c \ x^n]\right)^2}{3 \ e^5} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d^3 \ \left(a + b \ Log[c \ x^n]\right)^2}{e^5 \ \left(d + e \ x\right)^3} + \frac{2 \ d$$

### Problem 114: Result optimal but 4 more steps used.

$$\int \frac{x^3 \left(a + b \operatorname{Log}\left[c x^n\right]\right)^2}{\left(d + e x\right)^4} \, dx$$

#### Optimal (type 4, 333 leaves, 24 steps):

$$\frac{b^2 \, d \, n^2}{3 \, e^4 \, \left(d + e \, x\right)} + \frac{b^2 \, n^2 \, \mathsf{Log}[\,x]}{3 \, e^4} - \frac{b \, d^2 \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)}{3 \, e^4 \, \left(d + e \, x\right)^2} - \frac{7 \, b \, n \, x \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)}{3 \, e^3 \, \left(d + e \, x\right)} + \frac{7 \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{6 \, e^4} + \frac{d^3 \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{3 \, e^4 \, \left(d + e \, x\right)^3} - \frac{3 \, d^2 \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{2 \, e^4 \, \left(d + e \, x\right)^2} - \frac{3 \, x \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{e^3 \, \left(d + e \, x\right)} + \frac{2 \, b^2 \, n^2 \, \mathsf{Log}[\,d + e \, x]}{e^4} + \frac{11 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{Log}[\,1 + \frac{e \, x}{d}]}{3 \, e^4} + \frac{\left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2 \, \mathsf{Log}[\,1 + \frac{e \, x}{d}]}{3 \, e^4} + \frac{11 \, b^2 \, n^2 \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{3 \, e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} - \frac{2 \, b^2 \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b^2 \, n^2 \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b^2 \, n^2 \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b^2 \, n^2 \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2 \, , \, -\frac{e \, x}{d}\,]}{e^4} + \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{PolyLog}[\,2$$

#### Result (type 4, 333 leaves, 28 steps):

$$\frac{b^2\,d\,n^2}{3\,\,e^4\,\left(d+e\,x\right)} + \frac{b^2\,n^2\,Log\,[\,x\,]}{3\,\,e^4} - \frac{b\,d^2\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,\,e^4\,\left(d+e\,x\right)^2} - \frac{7\,b\,n\,x\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,e^3\,\left(d+e\,x\right)} + \frac{7\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{6\,e^4} + \frac{2\,b^2\,n^2\,Log\,[\,d+e\,x\,]}{3\,e^4\,\left(d+e\,x\right)^3} - \frac{3\,d^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{2\,e^4\,\left(d+e\,x\right)^2} - \frac{3\,x\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{e^3\,\left(d+e\,x\right)} + \frac{2\,b^2\,n^2\,Log\,[\,d+e\,x\,]}{e^4} + \frac{11\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,Log\,\left[1+\frac{e\,x}{d}\right]}{3\,e^4} + \frac{2\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\,\left[2\,,\,-\frac{e\,x}{d}\right]}{2\,e^4} - \frac{2\,b^2\,n^2\,PolyLog\,\left[3\,,\,-\frac{e\,x}{d}\right]}{e^4} + \frac{2\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\,\left[2\,,\,-\frac{e\,x}{d}\right]}{e^4} + \frac{2\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\,\left[2\,,\,-\frac{e\,x}{d}\right]}{e^4} + \frac{2\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\,\left[2\,,\,-\frac{e\,x}{d}\right]}{e^4} + \frac{2\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\left(a+b\,Log\,[\,c\,\,x^$$

### Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b \log \left[c x^n\right]\right)^2}{\left(d + e x\right)^4} dx$$

#### Optimal (type 4, 161 leaves, 5 steps):

$$\frac{b \, n \, x^2 \, \left(a + b \, \text{Log}\left[c \, x^n\right]\right)}{3 \, d \, e \, \left(d + e \, x\right)^2} + \frac{x^3 \, \left(a + b \, \text{Log}\left[c \, x^n\right]\right)^2}{3 \, d \, \left(d + e \, x\right)^3} + \frac{b \, n \, x \, \left(2 \, a + b \, n + 2 \, b \, \text{Log}\left[c \, x^n\right]\right)}{3 \, d \, e^2 \, \left(d + e \, x\right)} - \frac{b \, n \, \left(2 \, a + 3 \, b \, n + 2 \, b \, \text{Log}\left[c \, x^n\right]\right) \, \text{Log}\left[1 + \frac{e \, x}{d}\right]}{3 \, d \, e^3} - \frac{2 \, b^2 \, n^2 \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{3 \, d \, e^3}$$

#### Result (type 4, 274 leaves, 25 steps):

$$-\frac{b^2\,n^2}{3\,e^3\,\left(d+e\,x\right)} - \frac{b^2\,n^2\,Log\left[x\right]}{3\,d\,e^3} + \frac{b\,d\,n\,\left(a+b\,Log\left[c\,x^n\right]\right)}{3\,e^3\,\left(d+e\,x\right)^2} + \frac{4\,b\,n\,x\,\left(a+b\,Log\left[c\,x^n\right]\right)}{3\,d\,e^2\,\left(d+e\,x\right)} - \frac{2\,\left(a+b\,Log\left[c\,x^n\right]\right)^2}{3\,d\,e^3} - \frac{d^2\,\left(a+b\,Log\left[c\,x^n\right]\right)^2}{3\,e^3\,\left(d+e\,x\right)^3} + \frac{d\,\left(a+b\,Log\left[c\,x^n\right]\right)^2}{3\,e^3\,\left(d+e\,x\right)^2} + \frac{u\,\left(a+b\,Log\left[c\,x^n\right]\right)^2}{u\,e^3\,\left(d+e\,x\right)^2} - \frac{u^2\,\left(a+b\,Log\left[c\,x^n\right]\right)^2}{u\,e^3\,\left(d+e\,x\right)^3} - \frac{u^2\,u$$

## Problem 116: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a + b \log \left[c x^{n}\right]\right)^{2}}{\left(d + e x\right)^{4}} dx$$

#### Optimal (type 4, 210 leaves, 8 steps):

$$\begin{split} &\frac{b^2\,n^2}{3\,d\,e^2\,\left(d+e\,x\right)} - \frac{b\,n\,\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)}{3\,e^2\,\left(d+e\,x\right)^2} + \frac{b\,n\,\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)}{3\,d\,e^2\,\left(d+e\,x\right)} + \frac{\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)^2}{6\,d^2\,e^2} + \\ &\frac{d\,\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)^2}{3\,e^2\,\left(d+e\,x\right)^3} - \frac{\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)^2}{2\,e^2\,\left(d+e\,x\right)^2} - \frac{b\,n\,\left(a+b\,\text{Log}\,\left[c\,x^n\right]\,\right)\,\text{Log}\left[1+\frac{e\,x}{d}\right]}{3\,d^2\,e^2} - \frac{b^2\,n^2\,\text{PolyLog}\left[2,-\frac{e\,x}{d}\right]}{3\,d^2\,e^2} \end{split}$$

#### Result (type 4, 229 leaves, 22 steps):

$$\frac{b^2 \, n^2}{3 \, d \, e^2 \, \left(d + e \, x\right)} + \frac{b^2 \, n^2 \, Log \, [x]}{3 \, d^2 \, e^2} - \frac{b \, n \, \left(a + b \, Log \, [c \, x^n] \, \right)}{3 \, e^2 \, \left(d + e \, x\right)^2} - \frac{b \, n \, x \, \left(a + b \, Log \, [c \, x^n] \, \right)}{3 \, d^2 \, e \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \, [c \, x^n] \, \right)^2}{6 \, d^2 \, e^2} + \frac{d \, \left(a + b \, Log \, [c \, x^n] \, \right)^2}{3 \, e^2 \, \left(d + e \, x\right)^3} - \frac{\left(a + b \, Log \, [c \, x^n] \, \right)^2}{2 \, e^2 \, \left(d + e \, x\right)^2} - \frac{b \, n \, \left(a + b \, Log \, [c \, x^n] \, \right) \, Log \, \left[1 + \frac{e \, x}{d} \, \right]}{3 \, d^2 \, e^2} - \frac{b^2 \, n^2 \, PolyLog \, \left[2 \, , \, -\frac{e \, x}{d} \, \right]}{3 \, d^2 \, e^2}$$

# Problem 117: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c x^{n}\right]\right)^{2}}{\left(d + e x\right)^{4}} dx$$

Optimal (type 4, 203 leaves, 10 steps):

$$-\frac{b^{2} \, n^{2}}{3 \, d^{2} \, e \, \left(d + e \, x\right)} - \frac{b^{2} \, n^{2} \, Log \left[x\right]}{3 \, d^{3} \, e} + \frac{b \, n \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{3 \, d \, e \, \left(d + e \, x\right)^{2}} - \frac{2 \, b \, n \, x \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{3 \, d^{3} \, \left(d + e \, x\right)} - \frac{2 \, b \, n \, Log \left[1 + \frac{d}{e \, x}\right]}{3 \, e^{2} \, \left(d + b \, Log \left[c \, x^{n}\right]\right)} - \frac{\left(a + b \, Log \left[c \, x^{n}\right]\right)^{2}}{3 \, e \, \left(d + e \, x\right)^{3}} + \frac{b^{2} \, n^{2} \, Log \left[d + e \, x\right]}{d^{3} \, e} + \frac{2 \, b^{2} \, n^{2} \, PolyLog \left[2, -\frac{d}{e \, x}\right]}{3 \, d^{3} \, e}$$

Result (type 4, 221 leaves, 12 steps):

$$-\frac{b^2 \, n^2}{3 \, d^2 \, e \, \left(d + e \, x\right)} - \frac{b^2 \, n^2 \, \mathsf{Log} \, [x]}{3 \, d^3 \, e} + \frac{b \, n \, \left(a + b \, \mathsf{Log} \, [c \, x^n] \, \right)}{3 \, d \, e \, \left(d + e \, x\right)^2} - \frac{2 \, b \, n \, x \, \left(a + b \, \mathsf{Log} \, [c \, x^n] \, \right)}{3 \, d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, \mathsf{Log} \, [c \, x^n] \, \right)^2}{3 \, d^3 \, e} - \frac{\left(a + b \, \mathsf{Log} \, [c \, x^n] \, \right)}{3 \, d^3 \, e} - \frac{\left(a + b \, \mathsf{Log} \, [c \, x^n] \, \right)}{3 \, d^3 \, e} - \frac{2 \, b^2 \, n^2 \, \mathsf{PolyLog} \, \left[2, -\frac{e \, x}{d}\right]}{3 \, d^3 \, e}$$

# Problem 118: Result optimal but 7 more steps used.

$$\int \frac{\left(a+b \, Log \left[c \, x^{n}\right]\right)^{2}}{x \, \left(d+e \, x\right)^{4}} \, \mathrm{d}x$$

Optimal (type 4, 351 leaves, 25 steps):

$$\frac{b^2 \, n^2}{3 \, d^3 \, \left(d + e \, x\right)} + \frac{b^2 \, n^2 \, Log \left[x\right]}{3 \, d^4} - \frac{b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{3 \, d^2 \, \left(d + e \, x\right)^2} + \frac{5 \, b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{3 \, d^4 \, \left(d + e \, x\right)} - \frac{5 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{6 \, d^4} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d \, \left(d + e \, x\right)^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d \, \left(d + e \, x\right)^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, b \, d^4 \, n} - \frac{2 \, b^2 \, n^2 \, Log \left[d + e \, x\right]}{d^4} + \frac{11 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{3 \, d^4} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{e \, x}{d}\right]}{3 \, d^4} + \frac{11 \, b^2 \, n^2 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{3 \, d^4} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog$$

Result (type 4, 351 leaves, 32 steps):

$$\frac{b^2 \, n^2}{3 \, d^3 \, \left(d + e \, x\right)} + \frac{b^2 \, n^2 \, Log \left[x\right]}{3 \, d^4} - \frac{b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{3 \, d^2 \, \left(d + e \, x\right)^2} + \frac{5 \, b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{3 \, d^4 \, \left(d + e \, x\right)} - \frac{5 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{6 \, d^4} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d \, \left(d + e \, x\right)^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d \, \left(d + e \, x\right)^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d^4 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3}{3 \, d^4} - \frac{2 \, b^2 \, n^2 \, Log \left[d + e \, x\right]}{d^4} + \frac{11 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{3 \, d^4} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d^4} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2}{3 \, d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^4} + \frac{2 \, b^2 \, n^2 \, PolyLog \left[3, -\frac{e \, x}{$$

## Problem 119: Result optimal but 6 more steps used.

$$\int \frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{x^2\,\left(d+e\,x\right)^4}\,\,\mathrm{d}x$$

#### Optimal (type 4, 420 leaves, 26 steps):

$$-\frac{2 \, b^2 \, n^2}{d^4 \, x} - \frac{b^2 \, e \, n^2}{3 \, d^4 \, \left(d + e \, x\right)} - \frac{b^2 \, e \, n^2 \, Log\left[x\right]}{3 \, d^5} - \frac{2 \, b \, n \, \left(a + b \, Log\left[c \, x^n\right]\right)}{d^4 \, x} + \frac{b \, e \, n \, \left(a + b \, Log\left[c \, x^n\right]\right)}{3 \, d^3 \, \left(d + e \, x\right)^2} - \frac{8 \, b \, e^2 \, n \, x \, \left(a + b \, Log\left[c \, x^n\right]\right)}{3 \, d^5 \, \left(d + e \, x\right)} + \frac{4 \, e \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{3 \, d^5} - \frac{\left(a + b \, Log\left[c \, x^n\right]\right)^2}{3 \, d^2 \, \left(d + e \, x\right)^3} - \frac{e \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{d^3 \, \left(d + e \, x\right)^2} + \frac{3 \, e^2 \, x \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{d^5 \, \left(d + e \, x\right)} - \frac{4 \, e \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{d^5 \, \left(d + e \, x\right)} - \frac{26 \, b \, e \, n \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{3 \, d^5} + \frac{3 \, e^2 \, x \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{d^5 \, \left(d + e \, x\right)} - \frac{26 \, b \, e \, n \, \left(a + b \, Log\left[c \, x^n\right]\right) \, Log\left[1 + \frac{e \, x}{d}\right]}{3 \, d^5} + \frac{4 \, e \, \left(a + b \, Log\left[c \, x^n\right]\right)^2 \, Log\left[1 + \frac{e \, x}{d}\right]}{d^5} - \frac{26 \, b \, e \, n^2 \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} - \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac{3 \, b^2 \, e \, n^2 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^5} + \frac$$

#### Result (type 4, 420 leaves, 32 steps):

$$-\frac{2 \, b^2 \, n^2}{d^4 \, x} - \frac{b^2 \, e \, n^2}{3 \, d^4 \, \left(d + e \, x\right)} - \frac{b^2 \, e \, n^2 \, \mathsf{Log}[\,x]}{3 \, d^5} - \frac{2 \, b \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)}{d^4 \, x} + \frac{b \, e \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)}{3 \, d^3 \, \left(d + e \, x\right)^2} - \frac{8 \, b \, e^2 \, n \, x \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)}{3 \, d^5 \, \left(d + e \, x\right)} + \frac{4 \, e \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{3 \, d^5} - \frac{\left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{3 \, d^2 \, \left(d + e \, x\right)^3} - \frac{e \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{d^3 \, \left(d + e \, x\right)^2} + \frac{3 \, e^2 \, x \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2}{d^5 \, \left(d + e \, x\right)} - \frac{4 \, e \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^3}{3 \, b \, d^5 \, n} + \frac{3 \, b^2 \, e \, n^2 \, \mathsf{Log}[\,d + e \, x]}{d^5} - \frac{26 \, b \, e \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{Log}[\,1 + \frac{e \, x}{d}\,]}{3 \, d^5} + \frac{4 \, e \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right)^2 \, \mathsf{Log}[\,1 + \frac{e \, x}{d}\,]}{d^5} - \frac{26 \, b \, e \, n \, \left(a + b \, \mathsf{Log}[\,c \, x^n]\,\right) \, \mathsf{Log}[\,1 + \frac{e \, x}{d}\,]}{3 \, d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}{d^5} - \frac{8 \, b^2 \, e \, n^2 \, \mathsf{PolyLog}[\,3 \, , \, -\frac{e \, x}{d}\,]}$$

### Problem 120: Result valid but suboptimal antiderivative.

$$\int \frac{x \log [x]^2}{(d + e x)^4} dx$$

Optimal (type 4, 107 leaves, 8 steps):

$$-\frac{x}{3\,d^{2}\,e\,\left(d+e\,x\right)}\,+\,\frac{x\,Log\,[\,x\,]}{3\,d\,e\,\left(d+e\,x\right)^{\,2}}\,+\,\frac{x^{2}\,\left(3\,d+e\,x\right)\,Log\,[\,x\,]^{\,2}}{6\,d^{2}\,\left(d+e\,x\right)^{\,3}}\,-\,\frac{Log\,[\,x\,]\,\,Log\,\left[\,1+\frac{e\,x}{d}\,\right]}{3\,d^{2}\,e^{2}}\,-\,\frac{PolyLog\,[\,2\,,\,-\frac{e\,x}{d}\,]}{3\,d^{2}\,e^{2}}$$

Result (type 4, 157 leaves, 22 steps):

$$\frac{1}{3 \, d \, e^2 \, \left(d + e \, x\right)} + \frac{\text{Log} \left[x\right]}{3 \, d^2 \, e^2} - \frac{\text{Log} \left[x\right]}{3 \, e^2 \, \left(d + e \, x\right)^2} - \frac{x \, \text{Log} \left[x\right]}{3 \, d^2 \, e \, \left(d + e \, x\right)} + \frac{\text{Log} \left[x\right]^2}{3 \, d^2 \, e^2} + \frac{d \, \text{Log} \left[x\right]^2}{3 \, e^2 \, \left(d + e \, x\right)^3} - \frac{\text{Log} \left[x\right]^2}{2 \, e^2 \, \left(d + e \, x\right)^2} - \frac{\text{Log} \left[x\right] \, \text{Log} \left[1 + \frac{e \, x}{d}\right]}{3 \, d^2 \, e^2} - \frac{\text{PolyLog} \left[2, -\frac{e \, x}{d}\right]}{3 \, d^2 \, e^2}$$

## Problem 121: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log \left[c \, x^n\right]\right)^3}{x \, \left(d+e \, x\right)} \, dx$$

Optimal (type 4, 113 leaves, 4 steps):

$$-\frac{Log\big[1+\frac{d}{e\,x}\,\big]\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^3}{d} + \frac{3\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2\,PolyLog\big[2,\,-\frac{d}{e\,x}\,\big]}{d} + \frac{6\,b^2\,n^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\big[3,\,-\frac{d}{e\,x}\,\big]}{d} + \frac{6\,b^3\,n^3\,PolyLog\,\big[4,\,-\frac{d}{e\,x}\,\big]}{d} + \frac{6\,b^3\,n^$$

Result (type 4, 130 leaves, 7 steps):

$$\frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^4}{4\,b\,d\,n} - \frac{\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^3\,Log\left[\,1+\frac{e\,x}{d}\,\right]}{d} - \\ \frac{3\,b\,n\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)^2\,PolyLog\left[\,2\,,\,-\frac{e\,x}{d}\,\right]}{d} + \frac{6\,b^2\,n^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\left[\,3\,,\,-\frac{e\,x}{d}\,\right]}{d} - \frac{6\,b^3\,n^3\,PolyLog\left[\,4\,,\,-\frac{e\,x}{d}\,\right]}{d} + \frac{6\,b^2\,n^2\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)\,PolyLog\left[\,3\,,\,-\frac{e\,x}{d}\,\right]}{d} - \frac{6\,b^3\,n^3\,PolyLog\left[\,4\,,\,-\frac{e\,x}{d}\,\right]}{d} + \frac{6\,b^3\,n^3\,PolyLog\left[\,4\,,\,-\frac{e\,x}{d}\,$$

### Problem 122: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log \left[c \, x^{n}\right]\right)^{3}}{x \, \left(d+e \, x\right)^{2}} \, \mathrm{d}x$$

Optimal (type 4, 217 leaves, 9 steps):

$$-\frac{e\,x\,\left(a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{3}}{d^{2}\,\left(d\,+\,e\,\,x\right)}-\frac{Log\,\left[1\,+\,\frac{d}{e\,x}\,\right]\,\left(a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{3}}{d^{2}}+\frac{3\,b\,n\,\left(a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{2}\,Log\,\left[1\,+\,\frac{e\,x}{d}\right]}{d^{2}}+\frac{3\,b\,n\,\left(a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)^{2}\,PolyLog\,\left[2\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{2}\,n^{2}\,\left(a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)\,PolyLog\,\left[3\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^{2}}+\frac{6\,b^{3}\,n^{3}\,PolyLog\,\left[4\,,\,-\,\frac{d}{e\,x}\,\right]}{d^$$

Result (type 4, 234 leaves, 12 steps):

$$-\frac{e \times \left(a + b \, \text{Log}\left[c \, x^{n}\right]\right)^{3}}{d^{2} \left(d + e \, x\right)} + \frac{\left(a + b \, \text{Log}\left[c \, x^{n}\right]\right)^{4}}{4 \, b \, d^{2} \, n} + \frac{3 \, b \, n \, \left(a + b \, \text{Log}\left[c \, x^{n}\right]\right)^{2} \, \text{Log}\left[1 + \frac{e \, x}{d}\right]}{d^{2}} - \frac{\left(a + b \, \text{Log}\left[c \, x^{n}\right]\right)^{3} \, \text{Log}\left[1 + \frac{e \, x}{d}\right]}{d^{2}} + \frac{6 \, b^{2} \, n^{2} \, \left(a + b \, \text{Log}\left[c \, x^{n}\right]\right) \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^{2}} - \frac{3 \, b \, n \, \left(a + b \, \text{Log}\left[c \, x^{n}\right]\right)^{2} \, \text{PolyLog}\left[2, -\frac{e \, x}{d}\right]}{d^{2}} - \frac{6 \, b^{3} \, n^{3} \, \text{PolyLog}\left[3, -\frac{e \, x}{d}\right]}{d^{2}} - \frac{6 \, b^{3} \, n^{3} \, \text{PolyLog}\left[4, -\frac{e \, x}{d}\right]}{d^{2}}$$

### Problem 123: Result optimal but 6 more steps used.

$$\int \frac{\left(\,a\,+\,b\,\,Log\,[\,c\,\,x^{n}\,]\,\,\right)^{\,3}}{x\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,3}}\,\,\mathrm{d}\,x$$

Optimal (type 4, 361 leaves, 18 steps):

$$\frac{3 \, b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^3 \, \left(d + e \, x\right)} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3}{2 \, d^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3}{2 \, d \, \left(d + e \, x\right)^2} - \frac{e \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^4}{4 \, b \, d^3 \, n} - \frac{3 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{2 \, d^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{3 \, b^3 \, n^3 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{9 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{3 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{9 \, b^3 \, n^3 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^3 \, n^3 \, PolyLog \left[4,$$

Result (type 4, 361 leaves, 24 steps):

$$\frac{3 \, b \, e \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^3 \, \left(d + e \, x\right)} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3}{2 \, d^3} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3}{2 \, d \, \left(d + e \, x\right)^2} - \frac{e \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{d^3 \, \left(d + e \, x\right)} + \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^4}{4 \, b \, d^3 \, n} - \frac{3 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{2 \, d^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^3 \, Log \left[1 + \frac{e \, x}{d}\right]}{d^3} - \frac{3 \, b^3 \, n^3 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} + \frac{9 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{3 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^3 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^3} - \frac{9 \, b^3 \, n^3 \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} + \frac{6 \, b^2 \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[3, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d^3} - \frac{6 \, b^3 \, n^3 \, PolyLog \left[4, -\frac{e \, x}{d}\right]}{d$$

### Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{x^3}\,\mathrm{d}x$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b d n}{4 x^{2}}-\frac{d (a + b Log[c x^{n}])}{2 x^{2}}+\frac{e (a + b Log[c x^{n}])^{2}}{2 b n}$$

Result (type 3, 47 leaves, 3 steps):

$$-\,\frac{b\,d\,n}{4\,x^2}\,-\,\frac{1}{2}\,b\,e\,n\,Log\,[\,x\,]^{\,2}\,-\,\frac{1}{2}\,\left(\frac{d}{x^2}\,-\,2\,e\,Log\,[\,x\,]\,\right)\,\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)$$

#### Problem 176: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^5}\,\mathrm{d}x$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{b\,d\,n}{16\,x^4} - \frac{b\,e\,n}{4\,x^2} - \frac{d\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{4\,x^4} - \frac{e\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{2\,x^2}$$

Result (type 3, 47 leaves, 4 steps):

$$-\frac{b\,d\,n}{16\,x^4} - \frac{b\,e\,n}{4\,x^2} - \frac{1}{4}\,\left(\frac{d}{x^4} + \frac{2\,e}{x^2}\right)\,\left(a + b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

### Problem 179: Result valid but suboptimal antiderivative.

$$\left\lceil \left(d+e\;x^2\right)\; \left(a+b\;Log\left[\;c\;x^n\;\right]\right)\; \text{d}x\right.$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\,b\,d\,n\,x\,-\,\frac{1}{9}\,b\,e\,n\,x^3\,+\,d\,x\,\left(a\,+\,b\,Log\left[\,c\,\,x^n\,\right]\,\right)\,+\,\frac{1}{3}\,e\,x^3\,\left(a\,+\,b\,Log\left[\,c\,\,x^n\,\right]\,\right)$$

Result (type 3, 41 leaves, 2 steps):

$$-\,b\,d\,n\,x\,-\,\frac{1}{9}\,b\,e\,n\,x^3\,+\,\frac{1}{3}\,\left(3\,d\,x\,+\,e\,x^3\right)\,\,\left(a\,+\,b\,Log\,\!\left[\,c\,\,x^n\,\right]\,\right)$$

#### Problem 180: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^2}\;\mathrm{d}x$$

Optimal (type 3, 44 leaves, 2 steps):

$$-\frac{b d n}{x} - b e n x - \frac{d \left(a + b \log \left[c x^{n}\right]\right)}{x} + e x \left(a + b \log \left[c x^{n}\right]\right)$$

Result (type 3, 37 leaves, 2 steps):

$$-\,\frac{b\;d\;n}{x}-b\;e\;n\;x-\left(\frac{d}{x}-e\;x\right)\;\left(a+b\;\text{Log}\left[\;c\;x^n\;\right]\;\right)$$

## Problem 181: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^4}\;\mathrm{d}x$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{b d n}{9 x^3} - \frac{b e n}{x} - \frac{d (a + b Log[c x^n])}{3 x^3} - \frac{e (a + b Log[c x^n])}{x}$$

Result (type 3, 45 leaves, 4 steps):

$$-\,\frac{b\;d\;n}{9\;x^3}\,-\,\frac{b\;e\;n}{x}\,-\,\frac{1}{3}\;\left(\,\frac{d}{x^3}\,+\,\frac{3\;e}{x}\,\right)\;\left(\,a\,+\,b\;\text{Log}\left[\,c\;x^n\,\right]\,\right)$$

## Problem 182: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{x^6}\,\mathrm{d}x$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{b\,d\,n}{25\,x^5} - \frac{b\,e\,n}{9\,x^3} - \frac{d\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{5\,x^5} - \frac{e\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{3\,x^3}$$

Result (type 3, 48 leaves, 4 steps):

$$-\,\frac{b\,d\,n}{25\,x^5} - \frac{b\,e\,n}{9\,x^3} - \frac{1}{15}\,\left(\frac{3\,d}{x^5} + \frac{5\,e}{x^3}\right)\,\left(a + b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

#### Problem 186: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x}\,\text{d}x$$

Optimal (type 3, 89 leaves, 3 steps):

$$-\frac{1}{2} b d e n x^{2} - \frac{1}{16} b e^{2} n x^{4} - \frac{1}{2} b d^{2} n Log[x]^{2} + d e x^{2} \left(a + b Log[c x^{n}]\right) + \frac{1}{4} e^{2} x^{4} \left(a + b Log[c x^{n}]\right) + d^{2} Log[x] \left(a + b Log[c x^{n}]\right)$$

Result (type 3, 73 leaves, 3 steps):

$$-\,\frac{1}{2}\,b\,d\,e\,n\,x^2\,-\,\frac{1}{16}\,b\,e^2\,n\,x^4\,-\,\frac{1}{2}\,b\,d^2\,n\,Log\,[\,x\,]^{\,2}\,+\,\frac{1}{4}\,\left(4\,d\,e\,x^2\,+\,e^2\,x^4\,+\,4\,d^2\,Log\,[\,x\,]\,\right)\,\left(a\,+\,b\,Log\,[\,c\,x^n\,]\,\right)$$

### Problem 187: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^2\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^3}\;\mathrm{d}x$$

Optimal (type 3, 91 leaves, 7 steps):

$$-\frac{b\ d^{2}\ n}{4\ x^{2}}-\frac{1}{4}\ b\ e^{2}\ n\ x^{2}-b\ d\ e\ n\ Log\left[x\right]^{2}-\frac{d^{2}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{2\ x^{2}}+\frac{1}{2}\ e^{2}\ x^{2}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)+2\ d\ e\ Log\left[x\right]\ \left(a+b\ Log\left[c\ x^{n}\right]\right)$$

Result (type 3, 71 leaves, 7 steps):

$$-\,\frac{b\,d^2\,n}{4\,x^2}\,-\,\frac{1}{4}\,b\,\,e^2\,n\,\,x^2\,-\,b\,\,d\,\,e\,\,n\,\,Log\,[\,x\,]^{\,2}\,-\,\frac{1}{2}\,\left(\frac{d^2}{x^2}\,-\,e^2\,\,x^2\,-\,4\,\,d\,\,e\,\,Log\,[\,x\,]\,\right)\,\,\left(a\,+\,b\,\,Log\,[\,c\,\,x^n\,]\,\right)$$

## Problem 188: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^2\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^5}\;\mathrm{d}x$$

Optimal (type 3, 90 leaves, 7 steps):

$$-\frac{b\,d^{2}\,n}{16\,x^{4}}-\frac{b\,d\,e\,n}{2\,x^{2}}-\frac{1}{2}\,b\,e^{2}\,n\,Log\,[\,x\,]^{\,2}-\frac{d^{2}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{4\,x^{4}}-\frac{d\,e\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{x^{2}}+e^{2}\,Log\,[\,x\,]\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)$$

Result (type 3, 73 leaves, 5 steps):

$$-\,\frac{b\,\,d^2\,n}{16\,\,x^4}\,-\,\frac{b\,\,d\,e\,\,n}{2\,\,x^2}\,-\,\frac{1}{2}\,\,b\,\,e^2\,\,n\,\,Log\,[\,x\,]^{\,2}\,-\,\frac{1}{4}\,\left(\frac{d^2}{x^4}\,+\,\frac{4\,\,d\,\,e}{x^2}\,-\,4\,\,e^2\,\,Log\,[\,x\,]\,\,\right)\,\,\left(a\,+\,b\,\,Log\,[\,c\,\,x^n\,]\,\right)$$

### Problem 191: Result valid but suboptimal antiderivative.

$$\int (d + e x^2)^2 (a + b Log[c x^n]) dx$$

Optimal (type 3, 86 leaves, 2 steps):

$$-\,b\,\,d^2\,\,n\,\,x\,-\,\frac{2}{9}\,\,b\,\,d\,\,e\,\,n\,\,x^3\,-\,\frac{1}{25}\,\,b\,\,e^2\,\,n\,\,x^5\,+\,d^2\,\,x\,\,\left(\,a\,+\,b\,\,\text{Log}\,\left[\,c\,\,x^n\,\right]\,\right)\,\,+\,\frac{2}{3}\,\,d\,\,e\,\,x^3\,\,\left(\,a\,+\,b\,\,\text{Log}\,\left[\,c\,\,x^n\,\right]\,\right)\,\,+\,\frac{1}{5}\,\,e^2\,\,x^5\,\,\left(\,a\,+\,b\,\,\text{Log}\,\left[\,c\,\,x^n\,\right]\,\right)$$

Result (type 3, 68 leaves, 2 steps):

$$-\,b\,\,d^2\,\,n\,x\,-\,\frac{2}{9}\,\,b\,\,d\,\,e\,\,n\,\,x^3\,-\,\frac{1}{25}\,\,b\,\,e^2\,\,n\,\,x^5\,+\,\frac{1}{15}\,\,\left(15\,\,d^2\,\,x\,+\,10\,\,d\,\,e\,\,x^3\,+\,3\,\,e^2\,\,x^5\right)\,\,\left(a\,+\,b\,\,Log\left[\,c\,\,x^n\,\right]\,\right)$$

### Problem 192: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^2\;\left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\mathsf{c}\;x^n\right]\right)}{\mathsf{x}^2}\;\mathrm{d}\mathsf{x}$$

Optimal (type 3, 83 leaves, 2 steps):

$$-\frac{b\,d^2\,n}{x} - 2\,b\,d\,e\,n\,x - \frac{1}{9}\,b\,e^2\,n\,x^3 - \frac{d^2\,\left(a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x} + 2\,d\,e\,x\,\left(a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right) + \frac{1}{3}\,e^2\,x^3\,\left(a + b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)$$

Result (type 3, 66 leaves, 2 steps):

$$-\,\frac{b\;d^2\;n}{x}\,-\,2\;b\;d\;e\;n\;x\,-\,\frac{1}{9}\;b\;e^2\;n\;x^3\,-\,\frac{1}{3}\;\left(\frac{3\;d^2}{x}\,-\,6\;d\;e\;x\,-\,e^2\;x^3\right)\;\left(a\,+\,b\;\text{Log}\left[\,c\;x^n\,\right]\,\right)$$

### Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^2\,\left(a+b\,Log\left[c\,x^n\right]\right)}{x^4}\,\mathrm{d}x$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{b\;d^2\;n}{9\;x^3}\;-\;\frac{2\;b\;d\;e\;n}{x}\;-\;b\;e^2\;n\;x\;-\;\frac{d^2\;\left(\,a\;+\;b\;Log\,[\,c\;x^n\,]\,\,\right)}{3\;x^3}\;-\;\frac{2\;d\;e\;\left(\,a\;+\;b\;Log\,[\,c\;x^n\,]\,\,\right)}{x}\;+\;e^2\;x\;\left(\,a\;+\;b\;Log\,[\,c\;x^n\,]\,\,\right)$$

Result (type 3, 65 leaves, 2 steps):

$$-\,\frac{b\;d^2\;n}{9\;x^3}\,-\,\frac{2\;b\;d\;e\;n}{x}\,-\,b\;e^2\;n\;x\,-\,\frac{1}{3}\,\left(\frac{d^2}{x^3}\,+\,\frac{6\;d\;e}{x}\,-\,3\;e^2\;x\right)\;\left(a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)$$

## Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^2\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^6}\;\mathrm{d}x$$

Optimal (type 3, 91 leaves, 4 steps):

$$-\frac{b\ d^{2}\ n}{25\ x^{5}}-\frac{2\ b\ d\ e\ n}{9\ x^{3}}-\frac{b\ e^{2}\ n}{x}-\frac{d^{2}\ \left(a+b\ Log\ [c\ x^{n}\ ]\right)}{5\ x^{5}}-\frac{2\ d\ e\ \left(a+b\ Log\ [c\ x^{n}\ ]\right)}{3\ x^{3}}-\frac{e^{2}\ \left(a+b\ Log\ [c\ x^{n}\ ]\right)}{x}$$

Result (type 3, 72 leaves, 4 steps):

$$-\,\frac{b\,d^2\,n}{25\,x^5}\,-\,\frac{2\,b\,d\,e\,n}{9\,x^3}\,-\,\frac{b\,e^2\,n}{x}\,-\,\frac{1}{15}\,\left(\frac{3\,d^2}{x^5}\,+\,\frac{10\,d\,e}{x^3}\,+\,\frac{15\,e^2}{x}\right)\,\left(a\,+\,b\,Log\left[\,c\,\,x^n\,\right]\,\right)$$

#### Problem 195: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{d} + \mathsf{e} \; x^2\right)^2 \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \, [\, \mathsf{c} \; x^n \, ] \, \right)}{x^8} \, \mathrm{d} x$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{49\,x^{7}}-\frac{2\,b\,d\,e\,n}{25\,x^{5}}-\frac{b\,e^{2}\,n}{9\,x^{3}}-\frac{d^{2}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{7\,x^{7}}-\frac{2\,d\,e\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{5\,x^{5}}-\frac{e^{2}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3\,x^{3}}$$

Result (type 3, 74 leaves, 4 steps):

$$-\frac{b\;d^2\;n}{49\;x^7}\;-\;\frac{2\;b\;d\;e\;n}{25\;x^5}\;-\;\frac{b\;e^2\;n}{9\;x^3}\;-\;\frac{1}{105}\;\left(\frac{15\;d^2}{x^7}\;+\;\frac{42\;d\;e}{x^5}\;+\;\frac{35\;e^2}{x^3}\right)\;\left(a\;+\;b\;Log\left[\;c\;x^n\;\right]\;\right)$$

### Problem 199: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{x}\,\mathrm{d}x$$

Optimal (type 3, 130 leaves, 5 steps):

$$-\frac{3}{4} b d^{2} e n x^{2} - \frac{3}{16} b d e^{2} n x^{4} - \frac{1}{36} b e^{3} n x^{6} - \frac{1}{2} b d^{3} n Log[x]^{2} + \frac{3}{2} d^{2} e x^{2} (a + b Log[c x^{n}]) + \frac{3}{4} d e^{2} x^{4} (a + b Log[c x^{n}]) + \frac{1}{6} e^{3} x^{6} (a + b Log[c x^{n}]) + d^{3} Log[x] (a + b Log[c x^{n}])$$

Result (type 3, 100 leaves, 5 steps):

$$-\frac{3}{4} b d^{2} e n x^{2} - \frac{3}{16} b d e^{2} n x^{4} - \frac{1}{36} b e^{3} n x^{6} - \frac{1}{2} b d^{3} n Log[x]^{2} + \frac{1}{12} \left(18 d^{2} e x^{2} + 9 d e^{2} x^{4} + 2 e^{3} x^{6} + 12 d^{3} Log[x]\right) \left(a + b Log[c x^{n}]\right)$$

#### Problem 200: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^3}\;\mathrm{d}x$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b\ d^3\ n}{4\ x^2} - \frac{3}{4}\ b\ d\ e^2\ n\ x^2 - \frac{1}{16}\ b\ e^3\ n\ x^4 - \frac{3}{2}\ b\ d^2\ e\ n\ \text{Log}\,[\,x\,]^{\,2} - \frac{d^3\ \left(\,a + b\ \text{Log}\,[\,c\ x^n\,]\,\right)}{2\ x^2} + \\ \frac{3}{2}\ d\ e^2\ x^2\ \left(\,a + b\ \text{Log}\,[\,c\ x^n\,]\,\right) + \frac{1}{4}\ e^3\ x^4\ \left(\,a + b\ \text{Log}\,[\,c\ x^n\,]\,\right) + 3\ d^2\ e\ \text{Log}\,[\,x\,]\ \left(\,a + b\ \text{Log}\,[\,c\ x^n\,]\,\right)$$

Result (type 3, 100 leaves, 7 steps):

$$-\,\frac{b\,d^3\,n}{4\,x^2}\,-\,\frac{3}{4}\,b\,d\,e^2\,n\,x^2\,-\,\frac{1}{16}\,b\,e^3\,n\,x^4\,-\,\frac{3}{2}\,b\,d^2\,e\,n\,\text{Log}\,[\,x\,]^{\,2}\,-\,\frac{1}{4}\,\left(\frac{2\,d^3}{x^2}\,-\,6\,d\,e^2\,x^2\,-\,e^3\,x^4\,-\,12\,d^2\,e\,\text{Log}\,[\,x\,]\,\right)\,\left(a+b\,\text{Log}\,[\,c\,x^n\,]\,\right)$$

### Problem 201: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^5}\;dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b\;d^3\;n}{16\;x^4} - \frac{3\;b\;d^2\;e\;n}{4\;x^2} - \frac{1}{4}\;b\;e^3\;n\;x^2 - \frac{3}{2}\;b\;d\;e^2\;n\;Log\left[x\right]^2 - \frac{d^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{4\;x^4} - \\ \frac{3\;d^2\;e\;\left(a+b\;Log\left[c\;x^n\right]\right)}{2\;x^2} + \frac{1}{2}\;e^3\;x^2\;\left(a+b\;Log\left[c\;x^n\right]\right) + 3\;d\;e^2\;Log\left[x\right]\;\left(a+b\;Log\left[c\;x^n\right]\right)$$

Result (type 3, 99 leaves, 7 steps):

$$-\,\frac{b\,d^3\,n}{16\,x^4}\,-\,\frac{3\,b\,d^2\,e\,n}{4\,x^2}\,-\,\frac{1}{4}\,b\,e^3\,n\,x^2\,-\,\frac{3}{2}\,b\,d\,e^2\,n\,\text{Log}\,[\,x\,]^{\,2}\,-\,\frac{1}{4}\,\left(\frac{d^3}{x^4}\,+\,\frac{6\,d^2\,e}{x^2}\,-\,2\,e^3\,x^2\,-\,12\,d\,e^2\,\text{Log}\,[\,x\,]\,\right)\,\left(a\,+\,b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)$$

#### Problem 204: Result valid but suboptimal antiderivative.

$$\int \left(d+e\;x^2\right)^3\;\left(a+b\;Log\left[\,c\;x^n\,\right]\,\right)\;\mathrm{d}x$$

Optimal (type 3, 121 leaves, 2 steps):

$$\begin{split} &-b\;d^3\;n\;x - \frac{1}{3}\;b\;d^2\;e\;n\;x^3 - \frac{3}{25}\;b\;d\;e^2\;n\;x^5 - \frac{1}{49}\;b\;e^3\;n\;x^7 + d^3\;x\;\left(a+b\;Log\left[c\;x^n\right]\right) \;+\\ &-b\;d^2\;e\;x^3\;\left(a+b\;Log\left[c\;x^n\right]\right) + \frac{3}{5}\;d\;e^2\;x^5\;\left(a+b\;Log\left[c\;x^n\right]\right) + \frac{1}{7}\;e^3\;x^7\;\left(a+b\;Log\left[c\;x^n\right]\right) \end{split}$$

Result (type 3, 94 leaves, 2 steps):

$$-\,b\,\,d^3\,\,n\,x\,-\,\frac{1}{3}\,\,b\,\,d^2\,e\,\,n\,\,x^3\,-\,\frac{3}{25}\,\,b\,\,d\,\,e^2\,\,n\,\,x^5\,-\,\frac{1}{49}\,\,b\,\,e^3\,\,n\,\,x^7\,+\,\frac{1}{35}\,\,\left(35\,d^3\,x\,+\,35\,d^2\,e\,\,x^3\,+\,21\,d\,\,e^2\,\,x^5\,+\,5\,\,e^3\,\,x^7\right)\,\,\left(a\,+\,b\,\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

# Problem 205: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^2}\;dx$$

Optimal (type 3, 118 leaves, 2 steps):

$$-\frac{b\ d^{3}\ n}{x} - 3\ b\ d^{2}\ e\ n\ x - \frac{1}{3}\ b\ d\ e^{2}\ n\ x^{3} - \frac{1}{25}\ b\ e^{3}\ n\ x^{5} - \frac{d^{3}\ \left(a + b\ Log\left[c\ x^{n}\right]\right)}{x} + 3\ d^{2}\ e\ x\ \left(a + b\ Log\left[c\ x^{n}\right]\right) + d\ e^{2}\ x^{3}\ \left(a + b\ Log\left[c\ x^{n}\right]\right) + \frac{1}{5}\ e^{3}\ x^{5}\ \left(a + b\ Log\left[c\ x^{n}\right]\right) + \frac{1}{5}\ e^{3}\ x^{5} + \frac{1}{5}\ x^{$$

Result (type 3, 92 leaves, 2 steps):

$$-\,\frac{b\;d^3\;n}{x}\,-\,3\;b\;d^2\;e\;n\;x\,-\,\frac{1}{3}\;b\;d\;e^2\;n\;x^3\,-\,\frac{1}{25}\;b\;e^3\;n\;x^5\,-\,\frac{1}{5}\;\left(\frac{5\;d^3}{x}\,-\,15\;d^2\;e\;x\,-\,5\;d\;e^2\;x^3\,-\,e^3\;x^5\right)\;\left(a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)$$

## Problem 206: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{d} + e \; x^2\right)^3 \; \left(\text{a} + \text{b} \; \text{Log} \left[\text{c} \; x^n\right]\right)}{x^4} \; \text{d} x$$

Optimal (type 3, 121 leaves, 3 steps):

$$-\frac{b\,d^{3}\,n}{9\,x^{3}}-\frac{3\,b\,d^{2}\,e\,n}{x}-3\,b\,d\,e^{2}\,n\,x-\frac{1}{9}\,b\,e^{3}\,n\,x^{3}-\frac{d^{3}\,\left(a+b\,Log\,\left[c\,x^{n}\,\right]\,\right)}{3\,x^{3}}-\frac{3\,d^{2}\,e\,\left(a+b\,Log\,\left[c\,x^{n}\,\right]\,\right)}{x}+3\,d\,e^{2}\,x\,\left(a+b\,Log\,\left[c\,x^{n}\,\right]\,\right)+\frac{1}{3}\,e^{3}\,x^{3}\,\left(a+b\,Log\,\left[c\,x^{n}\,\right]\,\right)$$

Result (type 3, 91 leaves, 3 steps):

$$-\,\frac{b\;d^3\;n}{9\;x^3}\,-\,\frac{3\;b\;d^2\;e\;n}{x}\,-\,3\;b\;d\;e^2\;n\;x\,-\,\frac{1}{9}\;b\;e^3\;n\;x^3\,-\,\frac{1}{3}\;\left(\frac{d^3}{x^3}\,+\,\frac{9\;d^2\;e}{x}\,-\,9\;d\;e^2\;x\,-\,e^3\;x^3\right)\;\left(a\,+\,b\;Log\left[\,c\;x^n\,\right]\,\right)$$

### Problem 207: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{x^6}\,\mathrm{d}x$$

Optimal (type 3, 118 leaves, 2 steps):

$$-\frac{b\ d^{3}\ n}{25\ x^{5}}-\frac{b\ d^{2}\ e\ n}{3\ x^{3}}-\frac{3\ b\ d\ e^{2}\ n}{x}-b\ e^{3}\ n\ x-\frac{d^{3}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{5\ x^{5}}-\frac{d^{2}\ e\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{x^{3}}-\frac{3\ d\ e^{2}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{x}+e^{3}\ x\ \left(a+b\ Log\left[c\ x^{n}\right]\right)$$

Result (type 3, 91 leaves, 2 steps):

$$-\frac{b\;d^3\;n}{25\;x^5}\;-\;\frac{b\;d^2\;e\;n}{3\;x^3}\;-\;\frac{3\;b\;d\;e^2\;n}{x}\;-\;b\;e^3\;n\;x\;-\;\frac{1}{5}\;\left(\frac{d^3}{x^5}\;+\;\frac{5\;d^2\;e}{x^3}\;+\;\frac{15\;d\;e^2}{x}\;-\;5\;e^3\;x\right)\;\left(a\;+\;b\;Log\left[\;c\;x^n\;\right]\;\right)$$

#### Problem 208: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^3\,\left(a+b\,Log\,[\,c\,x^n\,]\,\right)}{v^8}\,dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{49\,x^7} - \frac{3\,b\,d^2\,e\,n}{25\,x^5} - \frac{b\,d\,e^2\,n}{3\,x^3} - \frac{b\,e^3\,n}{x} - \frac{d^3\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x} - \frac{3\,d^2\,e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{5\,x^5} - \frac{d\,e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x^3} - \frac{e^3\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x} - \frac{e^3\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{x^3} - \frac{e^3\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,$$

Result (type 3, 98 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{49\,x^7}-\frac{3\,b\,d^2\,e\,n}{25\,x^5}-\frac{b\,d\,e^2\,n}{3\,x^3}-\frac{b\,e^3\,n}{x}-\frac{1}{35}\left(\frac{5\,d^3}{x^7}+\frac{21\,d^2\,e}{x^5}+\frac{35\,d\,e^2}{x^3}+\frac{35\,e^3}{x}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

### Problem 209: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^2\right)^3\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^{10}}\;\mathrm{d}x$$

Optimal (type 3, 133 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{81\,x^9} - \frac{3\,b\,d^2\,e\,n}{49\,x^7} - \frac{3\,b\,d\,e^2\,n}{25\,x^5} - \frac{b\,e^3\,n}{9\,x^3} - \frac{d^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{9\,x^9} - \frac{3\,d^2\,e\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{7\,x^7} - \frac{3\,d\,e^2\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{5\,x^5} - \frac{e^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{3\,x^3} + \frac{e^3\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)}{25\,x^5} - \frac{e^3\,\left(a+b\,\text{Log}\left[c$$

Result (type 3, 100 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{81\,x^9}\,-\,\frac{3\,b\,d^2\,e\,n}{49\,x^7}\,-\,\frac{3\,b\,d\,e^2\,n}{25\,x^5}\,-\,\frac{b\,e^3\,n}{9\,x^3}\,-\,\frac{1}{315}\,\left(\frac{35\,d^3}{x^9}\,+\,\frac{135\,d^2\,e}{x^7}\,+\,\frac{189\,d\,e^2}{x^5}\,+\,\frac{105\,e^3}{x^3}\right)\,\left(a+b\,Log\left[\,c\,\,x^n\,\right]\,\right)$$

### Problem 214: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c x^n]}{x^3 (d + e x^2)} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$-\frac{b\,n}{4\,d\,x^2}\,-\,\frac{a\,+\,b\,Log\,[\,c\,\,x^n\,]}{2\,d\,x^2}\,+\,\frac{e\,Log\,\big[\,1\,+\,\frac{d}{e\,x^2}\,\big]\,\,\big(\,a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\big)}{2\,d^2}\,-\,\frac{b\,e\,n\,PolyLog\,\big[\,2\,,\,\,-\,\frac{d}{e\,x^2}\,\big]}{4\,d^2}$$

Result (type 4, 109 leaves, 6 steps):

$$-\frac{b\,n}{4\,d\,x^2}\,-\,\frac{a\,+\,b\,Log\,[\,c\,\,x^n\,]}{2\,d\,x^2}\,-\,\frac{e\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)^{\,2}}{2\,b\,d^2\,n}\,+\,\frac{e\,\left(a\,+\,b\,Log\,[\,c\,\,x^n\,]\,\right)\,Log\,\left[\,1\,+\,\frac{e\,x^2}{d}\,\right]}{2\,d^2}\,+\,\frac{b\,e\,n\,PolyLog\,\left[\,2\,,\,\,-\,\frac{e\,x^2}{d}\,\right]}{4\,d^2}$$

### Problem 215: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \, Log \, [\, c \, x^n \, ]}{x^5 \, \left(d + e \, x^2\right)} \, dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$-\frac{b\,n}{16\,d\,x^4} + \frac{b\,e\,n}{4\,d^2\,x^2} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{4\,d\,x^4} + \frac{e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,x^2} - \frac{e^2\,\text{Log}\,\big[\,1 + \frac{d}{e\,x^2}\,\big]\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,d^3} + \frac{b\,e^2\,n\,\text{PolyLog}\,\big[\,2\,,\,\,-\frac{d}{e\,x^2}\,\big]}{4\,d^3}$$

Result (type 4, 149 leaves, 7 steps):

$$-\frac{b\,n}{16\,d\,x^4} + \frac{b\,e\,n}{4\,d^2\,x^2} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{4\,d\,\,x^4} + \frac{e\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{2\,d^2\,x^2} + \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^2}{2\,b\,d^3\,n} - \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\,\left[1+\frac{e\,x^2}{d}\right]}{2\,d^3} - \frac{b\,e^2\,n\,\text{PolyLog}\,\left[2\,,\,-\frac{e\,x^2}{d}\right]}{4\,d^3} + \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^2}{2\,d^3\,n} - \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\,\left[1+\frac{e\,x^2}{d}\right]}{2\,d^3} - \frac{b\,e^2\,n\,\text{PolyLog}\,\left[2\,,\,-\frac{e\,x^2}{d}\right]}{4\,d^3} + \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^2}{2\,d^3\,n} - \frac{e^2\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\,\left[1+\frac{e\,x^2}{d}\right]}{2\,d^3} - \frac{e^2\,n\,\text{PolyLog}\,\left[2\,,\,-\frac{e\,x^2}{d}\right]}{4\,d^3} - \frac{e^2\,n\,\text{PolyLog}\,\left[2\,,\,-\frac{e\,x^2}{d}\right]}{2\,d^3} - \frac{e^2\,n\,\text{PolyLog}\,$$

# Problem 219: Result optimal but 1 more steps used.

$$\int \frac{a+b \, Log \, [\, c \, \, x^n \, ]}{x^2 \, \left(d+e \, \, x^2\right)} \, \, \mathrm{d}x$$

Optimal (type 4, 134 leaves, 7 steps):

$$-\frac{b\,n}{d\,x} - \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{d\,x} - \frac{\sqrt{e}\,\,\text{ArcTan}\,\big[\,\frac{\sqrt{e}\,\,x}{\sqrt{d}}\,\big]\,\,\big(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\big)}{d^{3/2}} + \frac{i\,\,b\,\sqrt{e}\,\,n\,\text{PolyLog}\,\big[\,2,\,\,-\frac{i\,\,\sqrt{e}\,\,x}{\sqrt{d}}\,\big]}{2\,d^{3/2}} - \frac{i\,\,b\,\sqrt{e}\,\,n\,\text{PolyLog}\,\big[\,2,\,\,\frac{i\,\,\sqrt{e}\,\,x}{\sqrt{d}}\,\big]}{2\,d^{3/2}}$$

Result (type 4, 134 leaves, 8 steps):

$$-\frac{b\,n}{d\,x}\,-\,\frac{a\,+\,b\,\text{Log}\,[\,c\,\,x^{n}\,]}{d\,x}\,-\,\frac{\sqrt{e}\,\,\text{ArcTan}\,\big[\,\frac{\sqrt{e}\,\,x}{\sqrt{d}}\,\big]\,\,\big(a\,+\,b\,\text{Log}\,[\,c\,\,x^{n}\,]\,\big)}{d^{3/2}}\,+\,\frac{\dot{\mathbb{1}}\,\,b\,\sqrt{e}\,\,\,n\,\text{PolyLog}\,\big[\,2\,,\,\,-\,\frac{\dot{\mathbb{1}}\,\sqrt{e}\,\,x}{\sqrt{d}}\,\big]}{2\,d^{3/2}}\,-\,\frac{\dot{\mathbb{1}}\,\,b\,\sqrt{e}\,\,\,n\,\text{PolyLog}\,\big[\,2\,,\,\,\frac{\dot{\mathbb{1}}\,\sqrt{e}\,\,x}{\sqrt{d}}\,\big]}{2\,d^{3/2}}$$

## Problem 225: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \, Log \, [\, c \, \, x^n \, ]}{x^3 \, \left(d+e \, x^2\right)^2} \, \mathrm{d}x$$

Optimal (type 4, 126 leaves, 5 steps):

$$-\frac{b\,n}{2\,d^{2}\,x^{2}}+\frac{a+b\,Log\,[\,c\,\,x^{n}\,]}{2\,d\,\,x^{2}\,\left(d+e\,x^{2}\right)}-\frac{4\,a-b\,n+4\,b\,Log\,[\,c\,\,x^{n}\,]}{4\,d^{2}\,x^{2}}+\frac{e\,Log\,[\,1+\frac{d}{e\,x^{2}}\,]\,\left(4\,a-b\,n+4\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{4\,d^{3}}-\frac{b\,e\,n\,PolyLog\,[\,2\,,\,-\frac{d}{e\,x^{2}}\,]}{2\,d^{3}}$$

Result (type 4, 159 leaves, 7 steps):

$$-\frac{b\,n}{2\,d^2\,x^2} + \frac{a+b\,\text{Log}\,[\,c\,\,x^n\,]}{2\,d\,\,x^2\,\,\left(d+e\,x^2\right)} - \frac{4\,a-b\,n+4\,b\,\text{Log}\,[\,c\,\,x^n\,]}{4\,d^2\,\,x^2} - \\ \\ \frac{e\,\left(4\,a-b\,n+4\,b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)^2}{16\,b\,d^3\,n} + \frac{e\,\left(4\,a-b\,n+4\,b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)\,\text{Log}\left[1+\frac{e\,x^2}{d}\right]}{4\,d^3} + \frac{b\,e\,n\,\text{PolyLog}\left[2\,\text{,}\,-\frac{e\,x^2}{d}\right]}{2\,d^3}$$

## Problem 229: Result optimal but 1 more steps used.

$$\int \frac{a + b \log[c x^n]}{x^2 (d + e x^2)^2} dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$-\frac{\frac{3 \, b \, n}{2 \, d^{2} \, x}+\frac{a+b \, Log \left[c \, x^{n}\right]}{2 \, d \, x \, \left(d+e \, x^{2}\right)}-\frac{\frac{3 \, a-b \, n+3 \, b \, Log \left[c \, x^{n}\right]}{2 \, d^{2} \, x}-\frac{2 \, d^{2} \, x}{2 \, d^{2} \, x}$$

$$-\frac{\sqrt{e} \, \, ArcTan \left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right] \, \left(3 \, a-b \, n+3 \, b \, Log \left[c \, x^{n}\right]\right)}{2 \, d^{5/2}}+\frac{3 \, \dot{i} \, b \, \sqrt{e} \, \, n \, PolyLog \left[2,-\frac{\dot{i} \, \sqrt{e} \, x}{\sqrt{d}}\right]}{4 \, d^{5/2}}-\frac{3 \, \dot{i} \, b \, \sqrt{e} \, \, n \, PolyLog \left[2,-\frac{\dot{i} \, \sqrt{e} \, x}{\sqrt{d}}\right]}{4 \, d^{5/2}}$$

#### Result (type 4, 183 leaves, 9 steps):

$$-\frac{\frac{3 \, b \, n}{2 \, d^2 \, x} + \frac{a + b \, \text{Log} \left[ c \, x^n \right]}{2 \, d \, x \, \left( d + e \, x^2 \right)} - \frac{3 \, a - b \, n + 3 \, b \, \text{Log} \left[ c \, x^n \right]}{2 \, d^2 \, x} - \frac{\sqrt{e} \, \text{ArcTan} \left[ \frac{\sqrt{e} \, x}{\sqrt{d}} \right] \, \left( 3 \, a - b \, n + 3 \, b \, \text{Log} \left[ c \, x^n \right] \right)}{2 \, d^{5/2}} + \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, - \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{d}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, \text{PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]}{4 \, d^{5/2}} - \frac{3 \, \dot{\mathbb{1}} \, b \, \sqrt{e} \, n \, PolyLog} \left[ 2 \, , \, \frac{\dot{\mathbb{1}} \, \sqrt{e} \, \, x}{\sqrt{e}} \right]} \right]$$

#### Problem 235: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \; Log \, [\, c \; x^n \, ]}{x^3 \; \left(d + e \; x^2\right)^3} \; \mathrm{d} x$$

#### Optimal (type 4, 162 leaves, 6 steps):

$$-\frac{3 \, b \, n}{4 \, d^3 \, x^2} + \frac{a + b \, \text{Log} \, [\, c \, x^n \, ]}{4 \, d \, x^2 \, \left(d + e \, x^2\right)^2} + \frac{6 \, a - b \, n + 6 \, b \, \text{Log} \, [\, c \, x^n \, ]}{8 \, d^2 \, x^2 \, \left(d + e \, x^2\right)} - \\ \frac{12 \, a - 5 \, b \, n + 12 \, b \, \text{Log} \, [\, c \, x^n \, ]}{8 \, d^3 \, x^2} + \frac{e \, \text{Log} \, \left[1 + \frac{d}{e \, x^2} \, \right] \, \left(12 \, a - 5 \, b \, n + 12 \, b \, \text{Log} \, [\, c \, x^n \, ]\,\right)}{8 \, d^4} - \frac{3 \, b \, e \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{d}{e \, x^2} \, \right]}{4 \, d^4}$$

#### Result (type 4, 195 leaves, 8 steps):

$$-\frac{3 \, b \, n}{4 \, d^3 \, x^2} + \frac{a + b \, \text{Log} \, [\, c \, \, x^n \, ]}{4 \, d \, x^2 \, \left(d + e \, x^2\right)^2} + \frac{6 \, a - b \, n + 6 \, b \, \text{Log} \, [\, c \, \, x^n \, ]}{8 \, d^2 \, x^2 \, \left(d + e \, x^2\right)} - \frac{12 \, a - 5 \, b \, n + 12 \, b \, \text{Log} \, [\, c \, \, x^n \, ]}{8 \, d^3 \, x^2} - \frac{e \, \left(12 \, a - 5 \, b \, n + 12 \, b \, \text{Log} \, [\, c \, \, x^n \, ]\,\right) \, \text{Log} \, \left[1 + \frac{e \, x^2}{d}\right]}{96 \, b \, d^4 \, n} + \frac{3 \, b \, e \, n \, \text{PolyLog} \, \left[2 \, , \, -\frac{e \, x^2}{d}\right]}{4 \, d^4}$$

## Problem 239: Result optimal but 1 more steps used.

$$\int \frac{a + b \log[c x^n]}{x^2 (d + e x^2)^3} dx$$

Optimal (type 4, 219 leaves, 9 steps):

$$-\frac{15 \text{ b n}}{8 \text{ d}^3 \text{ x}} + \frac{\text{a} + \text{b Log[c } \text{x}^n]}{4 \text{ d x } \left(\text{d} + \text{e } \text{x}^2\right)^2} + \frac{5 \text{ a} - \text{b n} + 5 \text{ b Log[c } \text{x}^n]}{8 \text{ d}^2 \text{ x } \left(\text{d} + \text{e } \text{x}^2\right)} - \frac{15 \text{ a} - 8 \text{ b n} + 15 \text{ b Log[c } \text{x}^n]}{8 \text{ d}^3 \text{ x}} - \frac{\sqrt{\text{e}} \text{ ArcTan} \left[\frac{\sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right] \left(15 \text{ a} - 8 \text{ b n} + 15 \text{ b Log[c } \text{x}^n]\right)}{8 \text{ d}^{7/2}} + \frac{15 \text{ i b } \sqrt{\text{e}} \text{ n PolyLog[2, } \frac{\text{i } \sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right]}{16 \text{ d}^{7/2}} - \frac{15 \text{ i b } \sqrt{\text{e}} \text{ n PolyLog[2, } \frac{\text{i } \sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right]}{16 \text{ d}^{7/2}}$$

#### Result (type 4, 219 leaves, 10 steps):

$$-\frac{\frac{15 \text{ b n}}{8 \text{ d}^3 \text{ x}} + \frac{\text{a} + \text{b Log[c } \text{x}^n]}{4 \text{ d } \text{x} \left(\text{d} + \text{e } \text{x}^2\right)^2} + \frac{5 \text{ a} - \text{b n} + 5 \text{ b Log[c } \text{x}^n]}{8 \text{ d}^2 \text{ x} \left(\text{d} + \text{e } \text{x}^2\right)} - \frac{15 \text{ a} - 8 \text{ b n} + 15 \text{ b Log[c } \text{x}^n]}{8 \text{ d}^3 \text{ x}} - \frac{\sqrt{\text{e}} \text{ ArcTan} \left[\frac{\sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right] \left(15 \text{ a} - 8 \text{ b n} + 15 \text{ b Log[c } \text{x}^n]\right)}{8 \text{ d}^{7/2}} + \frac{15 \text{ i} \text{ b} \sqrt{\text{e}} \text{ n PolyLog[2, } \frac{\text{i} \sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right]}{16 \text{ d}^{7/2}} - \frac{15 \text{ i} \text{ b} \sqrt{\text{e}} \text{ n PolyLog[2, } \frac{\text{i} \sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right]}{16 \text{ d}^{7/2}}$$

#### Problem 359: Result valid but suboptimal antiderivative.

$$\int \left( f x \right)^{-1+m} \left( d + e x^{m} \right)^{3} \left( a + b Log \left[ c x^{n} \right] \right)^{2} dx$$

#### Optimal (type 3, 372 leaves, 7 steps):

$$\frac{2 \, b^{2} \, d^{3} \, n^{2} \, x \, \left( \, f \, x \, \right)^{-1+m}}{m^{3}} \, + \, \frac{3 \, b^{2} \, d^{2} \, e \, n^{2} \, x^{1+m} \, \left( \, f \, x \, \right)^{-1+m}}{4 \, m^{3}} \, + \, \frac{2 \, b^{2} \, d \, e^{2} \, n^{2} \, x^{1+2\,m} \, \left( \, f \, x \, \right)^{-1+m}}{9 \, m^{3}} \, + \, \frac{b^{2} \, e^{3} \, n^{2} \, x^{1+3\,m} \, \left( \, f \, x \, \right)^{-1+m}}{32 \, m^{3}} \, + \, \frac{b^{2} \, d^{4} \, n^{2} \, x^{1-m} \, \left( \, f \, x \, \right)^{-1+m} \, Log \left[ \, x \, \right]^{2}}{4 \, e \, m} \, - \\ \frac{2 \, b \, d^{3} \, n \, x \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{m^{2}} \, - \, \frac{3 \, b \, d^{2} \, e \, n \, x^{1+m} \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{2 \, e \, m} \, + \, \frac{2 \, b \, d \, e^{2} \, n^{2} \, x^{1+3\,m} \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{4 \, e \, m} \, - \\ \frac{b \, e^{3} \, n \, x^{1+3\,m} \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{3 \, m^{2}} \, - \, \frac{b \, d^{4} \, n \, x^{1-m} \, \left( \, f \, x \, \right)^{-1+m} \, Log \left[ \, x \, \right] \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{2 \, e \, m} \, + \, \frac{x^{1-m} \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, d \, + \, e \, x^{m} \, \right)^{4} \, \left( \, a \, + \, b \, Log \left[ \, c \, \, x^{n} \, \right] \right)}{4 \, e \, m} \, - \, \frac{a \, b \, d^{2} \, n \, x^{1+3\,m} \, \left( \, f \, x \, \right)^{-1+m} \, \left( \, f$$

#### Result (type 3, 294 leaves, 7 steps):

$$\frac{2 \, b^2 \, d^3 \, n^2 \, x \, \left( \, f \, x \, \right)^{-1+m}}{m^3} + \frac{3 \, b^2 \, d^2 \, e \, n^2 \, x^{1+m} \, \left( \, f \, x \, \right)^{-1+m}}{4 \, m^3} + \frac{2 \, b^2 \, d \, e^2 \, n^2 \, x^{1+2 \, m} \, \left( \, f \, x \, \right)^{-1+m}}{9 \, m^3} + \frac{b^2 \, e^3 \, n^2 \, x^{1+3 \, m} \, \left( \, f \, x \, \right)^{-1+m}}{32 \, m^3} + \frac{b^2 \, d^4 \, n^2 \, x^{1-m} \, \left( \, f \, x \, \right)^{-1+m} \, Log \left[ \, x \, \right]^2}{4 \, e \, m} - \frac{b \, n \, x^{1-m} \, \left( \, f \, x \, \right)^{-1+m} \, Log \left[ \, f \, x \, \right)^{-1+m} \, \left( \,$$

#### Problem 360: Result valid but suboptimal antiderivative.

$$\int \left( \textbf{f} \, \textbf{x} \right)^{-\textbf{1}+\textbf{m}} \, \left( \textbf{d} + \textbf{e} \, \textbf{x}^{\textbf{m}} \right)^{\textbf{2}} \, \left( \textbf{a} + \textbf{b} \, \textbf{Log} \left[ \, \textbf{c} \, \, \textbf{x}^{\textbf{n}} \, \right] \, \right)^{\textbf{2}} \, \text{d}\textbf{x}$$

Optimal (type 3, 298 leaves, 7 steps):

$$\frac{2 \ b^{2} \ d^{2} \ n^{2} \ x \ \left(f \ x\right)^{-1+m}}{m^{3}} + \frac{b^{2} \ d \ e \ n^{2} \ x^{1+m} \ \left(f \ x\right)^{-1+m}}{2 \ m^{3}} + \frac{2 \ b^{2} \ e^{2} \ n^{2} \ x^{1+2 \ m} \ \left(f \ x\right)^{-1+m}}{27 \ m^{3}} + \frac{b^{2} \ d^{3} \ n^{2} \ x^{1-m} \ \left(f \ x\right)^{-1+m} \ Log \left[x\right]^{2}}{3 \ e \ m} - \frac{2 \ b \ d^{2} \ n \ x \ \left(f \ x\right)^{-1+m} \ \left(a + b \ Log \left[c \ x^{n}\right]\right)}{m^{2}} - \frac{b \ d \ e \ n \ x^{1+m} \ \left(f \ x\right)^{-1+m} \ \left(a + b \ Log \left[c \ x^{n}\right]\right)}{m^{2}} - \frac{2 \ b \ e^{2} \ n \ x^{1+2 \ m} \ \left(f \ x\right)^{-1+m} \ \left(a + b \ Log \left[c \ x^{n}\right]\right)}{9 \ m^{2}} - \frac{2 \ b \ d^{3} \ n \ x^{1-m} \ \left(f \ x\right)^{-1+m} \ \left(a + b \ Log \left[c \ x^{n}\right]\right)}{3 \ e \ m} - \frac{2 \ b \ d^{3} \ n \ x^{1-m} \ \left(f \ x\right)^{-1+m} \ \left(a + b \ Log \left[c \ x^{n}\right]\right)}{3 \ e \ m}$$

Result (type 3, 245 leaves, 7 steps):

$$\frac{2\;b^{2}\;d^{2}\;n^{2}\;x\;\left(f\;x\right)^{-1+m}}{m^{3}}\;+\;\frac{b^{2}\;d\;e\;n^{2}\;x^{1+m}\;\left(f\;x\right)^{-1+m}}{2\;m^{3}}\;+\;\frac{2\;b^{2}\;e^{2}\;n^{2}\;x^{1+2\;m}\;\left(f\;x\right)^{-1+m}}{27\;m^{3}}\;+\;\frac{b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(f\;x\right)^{-1+m}\;Log\left[x\right]^{2}}{3\;e\;m}\;-\\ \frac{b\;n\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(\frac{18\;d^{2}\;e\;x^{m}}{m}\;+\;\frac{9\;d\;e^{2}\;x^{2\;m}}{m}\;+\;\frac{2\;e^{3}\;x^{3\;m}}{m}\;+\;6\;d^{3}\;Log\left[x\right]\right)\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)}{9\;e\;m}\;+\;\frac{x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\right)^{2}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(f\;x\right)^{-1+m}\;\left(d\;+\;e\;x^{m}\right)^{3}}{3\;e\;m}\;+\;\frac{2\;b^{2}\;d^{3}\;n^{2}\;x^{1-m}\;\left(f\;x\right)^{-1+m}\;\left(f$$

#### Problem 361: Result valid but suboptimal antiderivative.

$$\int \left(\,f\,x\,\right)^{\,-1+m} \, \left(\,d\,+\,e\,\,x^m\,\right) \, \, \left(\,a\,+\,b\,\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)^{\,2} \, \, \text{d}\,x$$

Optimal (type 3, 226 leaves, 7 steps):

$$\frac{2 \, b^{2} \, d \, n^{2} \, x \, \left( \, f \, x \right)^{\, -1 + m}}{m^{3}} \, + \, \frac{b^{2} \, e \, n^{2} \, x^{1 + m} \, \left( \, f \, x \right)^{\, -1 + m}}{4 \, m^{3}} \, + \, \frac{b^{2} \, d^{2} \, n^{2} \, x^{1 - m} \, \left( \, f \, x \right)^{\, -1 + m} \, Log \left[ \, x \right]^{\, 2}}{2 \, e \, m} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{b \, d^{2} \, n \, x^{1 - m} \, \left( \, f \, x \right)^{\, -1 + m} \, Log \left[ \, x \, \right] \, \left( \, a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{e \, m} \, + \, \frac{x^{1 - m} \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + e \, x^{m} \right)^{\, 2} \, \left( \, a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{2 \, e \, m} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + e \, x^{m} \right)^{\, 2} \, \left( \, a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{2 \, e \, m} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \, \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \, \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \, \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{m^{2}} \, - \, \frac{2 \, b \, d \, n \, x \, \left( \, f \, x \, \right)^{\, -1 + m} \, \left( \, d \, + b \, Log \left[ \, c \, \, x^{$$

Result (type 3, 195 leaves, 7 steps):

$$\frac{2 \, b^{2} \, d \, n^{2} \, x \, \left( f \, x \right)^{-1+m}}{m^{3}} + \frac{b^{2} \, e \, n^{2} \, x^{1+m} \, \left( f \, x \right)^{-1+m}}{4 \, m^{3}} + \frac{b^{2} \, d^{2} \, n^{2} \, x^{1-m} \, \left( f \, x \right)^{-1+m} \, Log \left[ \, x \, \right]^{\, 2}}{2 \, e \, m} - \\ \frac{b \, n \, x^{1-m} \, \left( f \, x \right)^{-1+m} \, \left( \frac{4 \, d \, e \, x^{m}}{m} + \frac{e^{2} \, x^{2\,m}}{m} + 2 \, d^{2} \, Log \left[ \, x \, \right] \, \right) \, \left( a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)}{2 \, e \, m} + \frac{x^{1-m} \, \left( f \, x \right)^{-1+m} \, \left( d + e \, x^{m} \right)^{\, 2} \, \left( a + b \, Log \left[ \, c \, \, x^{n} \, \right] \, \right)^{\, 2}}{2 \, e \, m}$$

## Problem 371: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \ x^{r}\right) \ \left(a+b \ Log \left[c \ x^{n}\right]\right)}{x^{3}} \ dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\,\frac{b\;d\;n}{4\;x^2}\,-\,\frac{b\;e\;n\;x^{-2+r}}{\left(2-r\right)^2}\,-\,\frac{d\;\left(a+b\;Log\,[\,c\;x^n\,]\,\right)}{2\;x^2}\,-\,\frac{e\;x^{-2+r}\;\left(a+b\;Log\,[\,c\;x^n\,]\,\right)}{2-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\,\frac{b\,d\,n}{4\,x^2}\,-\,\frac{b\,e\,n\,x^{-2+r}}{\left(2\,-\,r\right)^{\,2}}\,-\,\frac{1}{2}\,\left(\frac{d}{x^2}\,+\,\frac{2\,e\,x^{-2+r}}{2\,-\,r}\right)\,\left(a\,+\,b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

## Problem 372: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^{n}\right)\;\left(a+b\;Log\left[\,c\;x^{n}\,\right]\,\right)}{x^{5}}\;\mathrm{d}x$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\,\frac{b\,d\,n}{16\,x^4}\,-\,\frac{b\,e\,n\,x^{-4+r}}{(4-r)^{\,2}}\,-\,\frac{d\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{4\,x^4}\,-\,\frac{e\,x^{-4+r}\,\left(a+b\,\text{Log}\,[\,c\,\,x^n\,]\,\right)}{4-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\,\frac{b\,d\,n}{16\,x^4}\,-\,\frac{b\,e\,n\,x^{-4+r}}{\left(4-r\right)^{\,2}}\,-\,\frac{1}{4}\,\left(\frac{d}{x^4}\,+\,\frac{4\,e\,x^{-4+r}}{4-r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

## Problem 375: Result valid but suboptimal antiderivative.

$$\int \left(d + e \, x^r\right) \, \left(a + b \, Log\left[c \, x^n\right]\right) \, dx$$

Optimal (type 3, 57 leaves, 3 steps):

$$- \, b \, d \, n \, x - \frac{b \, e \, n \, x^{1+r}}{\left(1 + r\right)^2} + d \, x \, \left(a + b \, Log\left[\, c \, \, x^n\,\right]\,\right) \, + \, \frac{e \, x^{1+r} \, \left(a + b \, Log\left[\, c \, \, x^n\,\right]\,\right)}{1 + r}$$

Result (type 3, 49 leaves, 3 steps):

$$-\,b\,\,d\,n\,\,x\,-\,\,\frac{b\,e\,\,n\,\,x^{1+r}}{\left(\,1\,+\,r\,\right)^{\,2}}\,+\,\,\left(d\,\,x\,+\,\,\frac{e\,\,x^{1+r}}{1\,+\,r}\,\right)\,\,\left(\,a\,+\,b\,\,\text{Log}\,\big[\,c\,\,x^{n}\,\big]\,\,\right)$$

### Problem 376: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^{r}\right)\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{x^{2}}\,\mathrm{d}x$$

Optimal (type 3, 67 leaves, 4 steps):

$$-\frac{b\,d\,n}{x} - \frac{b\,e\,n\,x^{-1+r}}{\left(1-r\right)^2} - \frac{d\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{x} - \frac{e\,x^{-1+r}\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{1-r}$$

Result (type 3, 58 leaves, 4 steps):

$$-\,\frac{b\,d\,n}{x}\,-\,\frac{b\,e\,n\,x^{-1+r}}{\left(1-r\right)^{\,2}}\,-\,\left(\frac{d}{x}\,+\,\frac{e\,x^{-1+r}}{1-r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^{n}\,\right]\,\right)$$

Problem 377: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^r\right)\;\left(a+b\;Log\left[c\;x^n\right]\right)}{x^4}\;\mathrm{d}x$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\,\frac{b\;d\;n}{9\;x^3}\,-\,\frac{b\;e\;n\;x^{-3+r}}{\left(3-r\right)^2}\,-\,\frac{d\;\left(a+b\;Log\,[\,c\;x^n\,]\,\right)}{3\;x^3}\,-\,\frac{e\;x^{-3+r}\;\left(a+b\;Log\,[\,c\;x^n\,]\,\right)}{3-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\,\frac{b\,d\,n}{9\,x^3}\,-\,\frac{b\,e\,n\,x^{-3+r}}{\left(3-r\right)^2}\,-\,\frac{1}{3}\,\left(\frac{d}{x^3}\,+\,\frac{3\,e\,x^{-3+r}}{3-r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

Problem 378: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^{r}\right)\;\left(a+b\;Log\left[\;c\;x^{n}\;\right]\right)}{x^{6}}\;\mathrm{d}x$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\,\frac{b\,d\,n}{25\,x^5}\,-\,\frac{b\,e\,n\,x^{-5+r}}{(5-r)^2}\,-\,\frac{d\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{5\,x^5}\,-\,\frac{e\,x^{-5+r}\,\left(a+b\,Log\,[\,c\,\,x^n\,]\,\right)}{5-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\,\frac{b\;d\;n}{25\;x^5}\,-\,\frac{b\;e\;n\;x^{-5+r}}{(5-r)^{\;2}}\,-\,\frac{1}{5}\;\left(\frac{d}{x^5}\,+\,\frac{5\;e\;x^{-5+r}}{5-r}\right)\;\left(a+b\;\text{Log}\left[\,c\;x^n\,\right]\,\right)$$

Problem 382: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{d} + \mathsf{e} \; \mathsf{x}^{\mathsf{r}}\right)^{\,2} \; \left(\mathsf{a} + \mathsf{b} \; \mathsf{Log}\left[\mathsf{c} \; \mathsf{x}^{\mathsf{n}}\right]\right)}{\mathsf{x}} \; \mathrm{d} \mathsf{x}$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{2 \, b \, d \, e \, n \, x^{r}}{r^{2}} - \frac{b \, e^{2} \, n \, x^{2 \, r}}{4 \, r^{2}} - \frac{1}{2} \, b \, d^{2} \, n \, Log \left[x\right]^{2} + \frac{2 \, d \, e \, x^{r} \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{r} + \frac{e^{2} \, x^{2 \, r} \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{2 \, r} + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, Log \left[x\right]$$

Result (type 3, 87 leaves, 5 steps):

$$-\,\frac{2\,b\,d\,e\,n\,x^{r}}{r^{2}}\,-\,\frac{b\,e^{2}\,n\,x^{2\,r}}{4\,r^{2}}\,-\,\frac{1}{2}\,b\,d^{2}\,n\,Log\left[\,x\,\right]^{\,2}\,+\,\frac{1}{2}\,\left(\frac{4\,d\,e\,x^{r}}{r}\,+\,\frac{e^{2}\,x^{2\,r}}{r}\,+\,2\,d^{2}\,Log\left[\,x\,\right]\,\right)\,\left(\,a\,+\,b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

## Problem 383: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \ x^{n}\right)^{2} \, \left(a+b \, Log \left[c \ x^{n}\right]\right)}{x^{3}} \, dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{4\,x^{2}}\,-\frac{b\,e^{2}\,n\,x^{-2\,(1-r)}}{4\,\left(1-r\right)^{\,2}}\,-\frac{2\,b\,d\,e\,n\,x^{-2+r}}{\left(2-r\right)^{\,2}}\,-\frac{d^{2}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\,\right)}{2\,x^{2}}\,-\frac{e^{2}\,x^{-2\,(1-r)}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\,\right)}{2\,\left(1-r\right)}\,-\frac{2\,d\,e\,x^{-2+r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\,\right)}{2-r}$$

Result (type 3, 114 leaves, 4 steps):

$$-\,\frac{b\,d^{2}\,n}{4\,x^{2}}\,-\,\frac{b\,e^{2}\,n\,x^{-2\,\,(1-r)}}{4\,\,\left(1-r\right)^{\,2}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-2+r}}{\left(2-r\right)^{\,2}}\,-\,\frac{1}{2}\,\left(\frac{d^{2}}{x^{2}}\,+\,\frac{e^{2}\,x^{-2\,\,(1-r)}}{1-r}\,+\,\frac{4\,d\,e\,x^{-2+r}}{2-r}\right)\,\,\left(a+b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

### Problem 384: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{n}\right)^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{v^{5}}\,dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{16\,x^{4}}\,-\,\frac{b\,e^{2}\,n\,x^{-2\,\,(2-r)}}{4\,\left(2-r\right)^{2}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-4+r}}{\left(4-r\right)^{2}}\,-\,\frac{d^{2}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{4\,x^{4}}\,-\,\frac{e^{2}\,x^{-2\,\,(2-r)}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{2\,\left(2-r\right)}\,-\,\frac{2\,d\,e\,x^{-4+r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{4-r}$$

Result (type 3, 115 leaves, 4 steps):

$$-\frac{\text{b d}^2 \, \text{n}}{16 \, \text{x}^4} - \frac{\text{b e}^2 \, \text{n } \, \text{x}^{-2 \, (2-r)}}{4 \, \left(2-r\right)^2} - \frac{2 \, \text{b d e n } \, \text{x}^{-4+r}}{\left(4-r\right)^2} - \frac{1}{4} \left(\frac{\text{d}^2}{\text{x}^4} + \frac{2 \, \text{e}^2 \, \text{x}^{-2 \, (2-r)}}{2-r} + \frac{8 \, \text{d e } \, \text{x}^{-4+r}}{4-r}\right) \, \left(\text{a + b Log} \left[\text{c } \, \text{x}^{\text{n}}\right]\right)$$

### Problem 387: Result valid but suboptimal antiderivative.

$$\int (d + e x^{r})^{2} (a + b Log[c x^{n}]) dx$$

Optimal (type 3, 113 leaves, 2 steps):

$$-b \ d^2 \ n \ x - \frac{2 \ b \ d \ e \ n \ x^{1+r}}{\left(1+r\right)^2} - \frac{b \ e^2 \ n \ x^{1+2 \ r}}{\left(1+2 \ r\right)^2} + d^2 \ x \ \left(a + b \ Log\left[c \ x^n\right]\right) + \frac{2 \ d \ e \ x^{1+r} \ \left(a + b \ Log\left[c \ x^n\right]\right)}{1+r} + \frac{e^2 \ x^{1+2 \ r} \ \left(a + b \ Log\left[c \ x^n\right]\right)}{1+2 \ r}$$

Result (type 3, 95 leaves, 2 steps):

$$-\,b\,\,d^2\,\,n\,\,x\,-\,\,\frac{2\,\,b\,\,d\,e\,\,n\,\,x^{1+r}}{\left(\,1\,+\,r\,\right)^{\,\,2}}\,-\,\,\frac{b\,\,e^2\,\,n\,\,x^{1+2\,\,r}}{\left(\,1\,+\,2\,\,r\,\right)^{\,\,2}}\,+\,\,\left(d^2\,\,x\,+\,\,\frac{2\,\,d\,\,e\,\,x^{1+r}}{1\,+\,r}\,+\,\,\frac{e^2\,\,x^{1+2\,\,r}}{1\,+\,2\,\,r}\right)\,\,\left(\,a\,+\,b\,\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

## Problem 388: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \, x^{n}\right)^{2} \, \left(a+b \, Log\left[c \, x^{n}\right]\right)}{x^{2}} \, dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$-\frac{b\ d^{2}\ n}{x}-\frac{2\ b\ d\ e\ n\ x^{-1+r}}{\left(1-r\right)^{2}}-\frac{b\ e^{2}\ n\ x^{-1+2\ r}}{\left(1-2\ r\right)^{2}}-\frac{d^{2}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{x}-\frac{2\ d\ e\ x^{-1+r}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{1-r}-\frac{e^{2}\ x^{-1+2\ r}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{1-2\ r}$$

Result (type 3, 104 leaves, 3 steps):

$$-\,\frac{b\;d^2\;n}{x}\,-\,\frac{2\;b\;d\;e\;n\;x^{-1+r}}{\left(1-r\right)^2}\,-\,\frac{b\;e^2\;n\;x^{-1+2\;r}}{\left(1-2\;r\right)^2}\,-\,\left(\frac{d^2}{x}\,+\,\frac{2\;d\;e\;x^{-1+r}}{1-r}\,+\,\frac{e^2\;x^{-1+2\;r}}{1-2\;r}\right)\;\left(a+b\;\text{Log}\left[\;c\;x^n\;\right]\,\right)$$

### Problem 389: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{n}\right)^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x^{4}}\,\mathrm{d}x$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{9\,x^{3}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-3+r}}{\left(3-r\right)^{2}}\,-\,\frac{b\,e^{2}\,n\,x^{-3+2\,r}}{\left(3-2\,r\right)^{2}}\,-\,\frac{d^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{3\,x^{3}}\,-\,\frac{2\,d\,e\,x^{-3+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{3-r}\,-\,\frac{e^{2}\,x^{-3+2\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{3-2\,r}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{9\,x^{3}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-3+r}}{\left(3-r\right)^{\,2}}\,-\,\frac{b\,e^{2}\,n\,x^{-3+2\,r}}{\left(3-2\,r\right)^{\,2}}\,-\,\frac{1}{3}\,\left(\frac{d^{2}}{x^{3}}\,+\,\frac{6\,d\,e\,x^{-3+r}}{3-r}\,+\,\frac{3\,e^{2}\,x^{-3+2\,r}}{3-2\,r}\right)\,\left(a+b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

# Problem 390: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{d} + \mathsf{e} \; \mathsf{x}^\mathsf{r}\right)^2 \; \left(\mathsf{a} + \mathsf{b} \; \mathsf{Log}\left[\mathsf{c} \; \mathsf{x}^\mathsf{n}\right]\right)}{\mathsf{x}^\mathsf{6}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b\,d^{2}\,n}{25\,x^{5}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-5+r}}{\left(5-r\right)^{\,2}}\,-\,\frac{b\,e^{2}\,n\,x^{-5+2\,r}}{\left(5-2\,r\right)^{\,2}}\,-\,\frac{d^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\,\right)}{5\,x^{5}}\,-\,\frac{2\,d\,e\,x^{-5+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\,\right)}{5-r}\,-\,\frac{e^{2}\,x^{-5+2\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\,\right)}{5-2\,r}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{b\ d^{2}\ n}{25\ x^{5}}-\frac{2\ b\ d\ e\ n\ x^{-5+r}}{\left(5-r\right)^{2}}-\frac{b\ e^{2}\ n\ x^{-5+2}\ r}{\left(5-2\ r\right)^{2}}-\frac{1}{5}\left(\frac{d^{2}}{x^{5}}+\frac{10\ d\ e\ x^{-5+r}}{5-r}+\frac{5\ e^{2}\ x^{-5+2}\ r}{5-2\ r}\right)\ \left(a+b\ Log\left[c\ x^{n}\right]\right)$$

# Problem 391: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \, x^{n}\right)^{2} \, \left(a+b \, Log \left[c \, x^{n}\right]\right)}{x^{8}} \, dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b\;d^{2}\;n}{49\;x^{7}}\;-\;\frac{2\;b\;d\;e\;n\;x^{-7+r}}{\left(7\;-\;r\right)^{\;2}}\;-\;\frac{b\;e^{2}\;n\;x^{-7+2\;r}}{\left(7\;-\;2\;r\right)^{\;2}}\;-\;\frac{d^{2}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\;\right)}{7\;x^{7}}\;-\;\frac{2\;d\;e\;x^{-7+r}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\;\right)}{7\;-\;r}\;-\;\frac{e^{2}\;x^{-7+2\;r}\;\left(a\;+\;b\;Log\left[c\;x^{n}\right]\;\right)}{7\;-\;2\;r}$$

Result (type 3, 109 leaves, 4 steps):

$$-\,\frac{b\,d^{2}\,n}{49\,x^{7}}\,-\,\frac{2\,b\,d\,e\,n\,x^{-7+r}}{\left(7\,-\,r\right)^{\,2}}\,-\,\frac{b\,e^{2}\,n\,x^{-7+2\,r}}{\left(7\,-\,2\,r\right)^{\,2}}\,-\,\frac{1}{7}\,\left(\frac{d^{2}}{x^{7}}\,+\,\frac{14\,d\,e\,x^{-7+r}}{7\,-\,r}\,+\,\frac{7\,e^{2}\,x^{-7+2\,r}}{7\,-\,2\,r}\right)\,\left(a\,+\,b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

### Problem 395: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \, x^r\right)^3 \, \left(a+b \, Log \left[c \, x^n\right]\right)}{v} \, dx$$

Optimal (type 3, 152 leaves, 5 steps):

$$-\frac{\frac{3 \ b \ d^{2} \ e \ n \ x^{r}}{r^{2}}-\frac{\frac{3 \ b \ d \ e^{2} \ n \ x^{2 \ r}}{4 \ r^{2}}-\frac{b \ e^{3} \ n \ x^{3 \ r}}{9 \ r^{2}}-\frac{1}{2} \ b \ d^{3} \ n \ Log [x]^{2}+\frac{3 \ d^{2} \ e \ x^{r} \ \left(a+b \ Log [c \ x^{n}]\right)}{r}+\frac{3 \ d \ e^{2} \ x^{2 \ r} \ \left(a+b \ Log [c \ x^{n}]\right)}{2 \ r}+\frac{e^{3} \ x^{3 \ r} \ \left(a+b \ Log [c \ x^{n}]\right)}{3 \ r}+d^{3} \ Log [x] \ \left(a+b \ Log [c \ x^{n}]\right)$$

Result (type 3, 124 leaves, 5 steps):

$$-\,\frac{3\,b\,d^{2}\,e\,n\,x^{r}}{r^{2}}\,-\,\frac{3\,b\,d\,e^{2}\,n\,x^{2\,r}}{4\,r^{2}}\,-\,\frac{b\,e^{3}\,n\,x^{3\,r}}{9\,r^{2}}\,-\,\frac{1}{2}\,b\,d^{3}\,n\,Log\left[x\,\right]^{\,2}\,+\,\frac{1}{6}\,\left(\frac{18\,d^{2}\,e\,x^{r}}{r}\,+\,\frac{9\,d\,e^{2}\,x^{2\,r}}{r}\,+\,\frac{2\,e^{3}\,x^{3\,r}}{r}\,+\,6\,d^{3}\,Log\left[x\,\right]\,\right)\,\left(a\,+\,b\,Log\left[c\,x^{n}\,\right]\right)$$

### Problem 396: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{n}\right)^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x^{3}}\,dx$$

#### Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{4\,x^{2}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-2}\,(^{1-r})}{4\,\left(1-r\right)^{2}}-\frac{3\,b\,d^{2}\,e\,n\,x^{-2+r}}{\left(2-r\right)^{2}}-\frac{b\,e^{3}\,n\,x^{-2+3\,r}}{\left(2-3\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2\,x^{2}}-\frac{3\,d^{2}\,e\,x^{-2+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2\,\left(1-r\right)}-\frac{3\,d^{2}\,e\,x^{-2+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2-r}-\frac{e^{3}\,x^{-2+3\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{2-3\,r}$$

#### Result (type 3, 161 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{4\,x^{2}}\,-\,\frac{3\,b\,d\,e^{2}\,n\,x^{-2}\,(^{1-r})}{4\,\left(1-r\right)^{\,2}}\,-\,\frac{3\,b\,d^{2}\,e\,n\,x^{-2+r}}{\left(2-r\right)^{\,2}}\,-\,\frac{b\,e^{3}\,n\,x^{-2+3\,r}}{\left(2-3\,r\right)^{\,2}}\,-\,\frac{1}{2}\,\left(\frac{d^{3}}{x^{2}}\,+\,\frac{3\,d\,e^{2}\,x^{-2}\,(^{1-r})}{1-r}\,+\,\frac{6\,d^{2}\,e\,x^{-2+r}}{2-r}\,+\,\frac{2\,e^{3}\,x^{-2+3\,r}}{2-3\,r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^{n}\,\right]\,\right)$$

### Problem 397: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^{r}\right)^{3}\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{x^{5}}\;\mathrm{d}x$$

#### Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{16\,x^{4}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-2}\,{}^{(2-r)}}{4\,\left(2-r\right)^{2}}-\frac{3\,b\,d^{2}\,e\,n\,x^{-4+r}}{\left(4-r\right)^{2}}-\frac{b\,e^{3}\,n\,x^{-4+3\,r}}{\left(4-3\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{4\,x^{4}}-\frac{3\,d\,e^{2}\,x^{-2}\,{}^{(2-r)}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{2\,\left(2-r\right)}-\frac{3\,d^{2}\,e\,x^{-4+r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{4-r}-\frac{e^{3}\,x^{-4+3\,r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{4-3\,r}$$

#### Result (type 3, 161 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{16\,x^4} - \frac{3\,b\,d\,e^2\,n\,x^{-2\,\,(2-r)}}{4\,\,\left(2-r\right)^2} - \frac{3\,b\,d^2\,e\,n\,x^{-4+r}}{\left(4-r\right)^2} - \frac{b\,e^3\,n\,x^{-4+3\,r}}{\left(4-3\,r\right)^2} - \frac{1}{4}\,\left(\frac{d^3}{x^4} + \frac{6\,d\,e^2\,x^{-2\,\,(2-r)}}{2-r} + \frac{12\,d^2\,e\,x^{-4+r}}{4-r} + \frac{4\,e^3\,x^{-4+3\,r}}{4-3\,r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

# Problem 400: Result valid but suboptimal antiderivative.

$$\int (d + e x^{n})^{3} (a + b Log[c x^{n}]) dx$$

#### Optimal (type 3, 169 leaves, 2 steps):

$$-b\,d^{3}\,n\,x\,-\,\frac{3\,b\,d^{2}\,e\,n\,x^{1+r}}{\left(1+r\right)^{\,2}}\,-\,\frac{3\,b\,d\,e^{2}\,n\,x^{1+2\,r}}{\left(1+2\,r\right)^{\,2}}\,-\,\frac{b\,e^{3}\,n\,x^{1+3\,r}}{\left(1+3\,r\right)^{\,2}}\,+\,d^{3}\,x\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,+\,\frac{3\,d^{2}\,e\,x^{1+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1+r}\,+\,\frac{3\,d\,e^{2}\,x^{1+2\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1+2\,r}\,+\,\frac{e^{3}\,x^{1+3\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1+3\,r}$$

#### Result (type 3, 141 leaves, 2 steps):

$$-\,b\,\,d^{3}\,\,n\,\,x\,-\,\,\frac{3\,\,b\,\,d^{2}\,e\,\,n\,\,x^{1+r}}{\left(1\,+\,r\right)^{\,2}}\,-\,\,\frac{3\,\,b\,\,d\,\,e^{2}\,\,n\,\,x^{1+2\,\,r}}{\left(1\,+\,2\,\,r\right)^{\,2}}\,-\,\,\frac{b\,\,e^{3}\,\,n\,\,x^{1+3\,\,r}}{\left(1\,+\,3\,\,r\right)^{\,2}}\,+\,\,\left(d^{3}\,\,x\,+\,\,\frac{3\,\,d^{2}\,e\,\,x^{1+r}}{1\,+\,r}\,+\,\,\frac{3\,\,d\,\,e^{2}\,\,x^{1+2\,\,r}}{1\,+\,2\,\,r}\,+\,\,\frac{e^{3}\,\,x^{1+3\,\,r}}{1\,+\,3\,\,r}\right)\,\,\left(a\,+\,b\,\,\text{Log}\left[\,c\,\,x^{n}\,\,\right]\,\right)$$

### Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e \ x^{n}\right)^{3} \, \left(a+b \, Log \left[c \ x^{n}\right]\right)}{x^{2}} \, dx$$

#### Optimal (type 3, 179 leaves, 3 steps):

$$-\frac{b\,d^{3}\,n}{x}-\frac{3\,b\,d^{2}\,e\,n\,x^{-1+r}}{\left(1-r\right)^{2}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-1+2\,r}}{\left(1-2\,r\right)^{2}}-\frac{b\,e^{3}\,n\,x^{-1+3\,r}}{\left(1-3\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x}-\frac{3\,d^{2}\,e\,x^{-1+r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1-r}-\frac{3\,d\,e^{2}\,x^{-1+2\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1-2\,r}-\frac{e^{3}\,x^{-1+3\,r}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{1-3\,r}$$

#### Result (type 3, 150 leaves, 3 steps):

$$-\,\frac{b\;d^3\;n}{x}\,-\,\frac{3\;b\;d^2\;e\;n\;x^{-1+r}}{\left(1-r\right)^2}\,-\,\frac{3\;b\;d\;e^2\;n\;x^{-1+2\;r}}{\left(1-2\;r\right)^2}\,-\,\frac{b\;e^3\;n\;x^{-1+3\;r}}{\left(1-3\;r\right)^2}\,-\,\left(\frac{d^3}{x}\,+\,\frac{3\;d^2\;e\;x^{-1+r}}{1-r}\,+\,\frac{3\;d\;e^2\;x^{-1+2\;r}}{1-2\;r}\,+\,\frac{e^3\;x^{-1+3\;r}}{1-3\;r}\right)\;\left(a+b\;\text{Log}\left[\;c\;x^n\right]\;\right)$$

### Problem 402: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{n}\right)^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x^{4}}\,dx$$

#### Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{9\,x^{3}}-\frac{b\,e^{3}\,n\,x^{-3\,\,(1-r)}}{9\,\left(1-r\right)^{2}}-\frac{3\,b\,d^{2}\,e\,n\,x^{-3+r}}{\left(3-r\right)^{2}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-3+2\,r}}{\left(3-2\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3\,x^{3}}-\frac{e^{3}\,x^{-3\,\,(1-r)}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3\,\left(1-r\right)}-\frac{3\,d^{2}\,e\,x^{-3+r}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3-r}-\frac{3\,d\,e^{2}\,x^{-3+2\,r}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3-2\,r}$$

Result (type 3, 160 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{9\,x^3}\,-\,\frac{b\,e^3\,n\,x^{-3\,\,(1-r)}}{9\,\left(1-r\right)^2}\,-\,\frac{3\,b\,d^2\,e\,n\,x^{-3+r}}{\left(3-r\right)^2}\,-\,\frac{3\,b\,d\,e^2\,n\,x^{-3+2\,r}}{\left(3-2\,r\right)^2}\,-\,\frac{1}{3}\,\left(\frac{d^3}{x^3}\,+\,\frac{e^3\,x^{-3\,\,(1-r)}}{1-r}\,+\,\frac{9\,d^2\,e\,x^{-3+r}}{3-r}\,+\,\frac{9\,d\,e^2\,x^{-3+2\,r}}{3-2\,r}\right)\,\left(a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right)$$

### Problem 403: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\;x^{n}\right)^{3}\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{x^{6}}\;\mathrm{d}x$$

#### Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{b\ d^{3}\ n}{25\ x^{5}} - \frac{3\ b\ d^{2}\ e\ n\ x^{-5+r}}{(5-r)^{2}} - \frac{3\ b\ d\ e^{2}\ n\ x^{-5+2}\ r}{\left(5-2\ r\right)^{2}} - \frac{b\ e^{3}\ n\ x^{-5+3}\ r}{\left(5-3\ r\right)^{2}} - \frac{d^{3}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{5\ x^{5}} - \frac{3\ d^{2}\ e\ x^{-5+r}\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{5-r} - \frac{3\ d\ e^{2}\ x^{-5+2}\ r\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{5-2\ r} - \frac{e^{3}\ x^{-5+3}\ r\ \left(a+b\ Log\left[c\ x^{n}\right]\right)}{5-3\ r}$$

#### Result (type 3, 155 leaves, 4 steps):

$$-\frac{b\;d^3\;n}{25\;x^5} - \frac{3\;b\;d^2\;e\;n\;x^{-5+r}}{\left(5-r\right)^2} - \frac{3\;b\;d\;e^2\;n\;x^{-5+2\;r}}{\left(5-2\;r\right)^2} - \frac{b\;e^3\;n\;x^{-5+3\;r}}{\left(5-3\;r\right)^2} - \frac{1}{5}\;\left(\frac{d^3}{x^5} + \frac{15\;d^2\;e\;x^{-5+r}}{5-r} + \frac{15\;d\;e^2\;x^{-5+2\;r}}{5-2\;r} + \frac{5\;e^3\;x^{-5+3\;r}}{5-3\;r}\right)\;\left(a+b\;\text{Log}\left[c\;x^n\right]\right)$$

### Problem 404: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{r}\right)^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x^{8}}\,dx$$

#### Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{49\,x^{7}}-\frac{3\,b\,d^{2}\,e\,n\,x^{-7+r}}{(7-r)^{2}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-7+2\,r}}{\left(7-2\,r\right)^{2}}-\frac{b\,e^{3}\,n\,x^{-7+3\,r}}{\left(7-3\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{7\,x^{7}}-\frac{3\,d^{2}\,e\,x^{-7+r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{7-r}-\frac{3\,d\,e^{2}\,x^{-7+2\,r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{7-2\,r}-\frac{e^{3}\,x^{-7+3\,r}\,\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)}{7-3\,r}$$

#### Result (type 3, 155 leaves, 4 steps):

$$-\frac{b\,d^3\,n}{49\,x^7}\,-\,\frac{3\,b\,d^2\,e\,n\,x^{-7+r}}{\left(7-r\right)^2}\,-\,\frac{3\,b\,d\,e^2\,n\,x^{-7+2\,r}}{\left(7-2\,r\right)^2}\,-\,\frac{b\,e^3\,n\,x^{-7+3\,r}}{\left(7-3\,r\right)^2}\,-\,\frac{1}{7}\,\left(\frac{d^3}{x^7}\,+\,\frac{21\,d^2\,e\,x^{-7+r}}{7-r}\,+\,\frac{21\,d\,e^2\,x^{-7+2\,r}}{7-2\,r}\,+\,\frac{7\,e^3\,x^{-7+3\,r}}{7-3\,r}\right)\,\left(a+b\,\text{Log}\left[c\,x^n\right]\right)$$

### Problem 405: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{r}\right)^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x^{10}}\,dx$$

Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b\,d^{3}\,n}{81\,x^{9}}-\frac{b\,e^{3}\,n\,x^{-3}\,(^{3-r})}{9\,\left(3-r\right)^{2}}-\frac{3\,b\,d^{2}\,e\,n\,x^{-9+r}}{\left(9-r\right)^{2}}-\frac{3\,b\,d\,e^{2}\,n\,x^{-9+2\,r}}{\left(9-2\,r\right)^{2}}-\frac{d^{3}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{9\,x^{9}}-\frac{e^{3}\,x^{-3}\,(^{3-r})\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{3\,\left(3-r\right)}-\frac{3\,d^{2}\,e\,x^{-9+r}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{9-r}-\frac{3\,d\,e^{2}\,x^{-9+2\,r}\,\left(a+b\,Log\,[\,c\,\,x^{n}\,]\,\right)}{9-2\,r}$$

Result (type 3, 161 leaves, 4 steps):

$$-\frac{b\ d^{3}\ n}{81\ x^{9}}-\frac{b\ e^{3}\ n\ x^{-3\ (3-r)}}{9\ \left(3-r\right)^{2}}-\frac{3\ b\ d^{2}\ e\ n\ x^{-9+r}}{\left(9-r\right)^{2}}-\frac{3\ b\ d\ e^{2}\ n\ x^{-9+2\ r}}{\left(9-2\ r\right)^{2}}-\frac{1}{9}\left(\frac{d^{3}}{x^{9}}+\frac{3\ e^{3}\ x^{-3\ (3-r)}}{3-r}+\frac{27\ d^{2}\ e\ x^{-9+r}}{9-r}+\frac{27\ d\ e^{2}\ x^{-9+2\ r}}{9-2\ r}\right)\ \left(a+b\ Log\left[c\ x^{n}\right]\right)$$

### Problem 421: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{n}\right)^{3}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x}\,\mathrm{d}x$$

Optimal (type 3, 152 leaves, 5 steps):

$$-\frac{3 b d^{2} e n x^{r}}{r^{2}} - \frac{3 b d e^{2} n x^{2} r}{4 r^{2}} - \frac{b e^{3} n x^{3} r}{9 r^{2}} - \frac{1}{2} b d^{3} n Log[x]^{2} + \frac{3 d^{2} e x^{r} (a + b Log[c x^{n}])}{r} + \frac{3 d e^{2} x^{2} r (a + b Log[c x^{n}])}{2 r} + \frac{e^{3} x^{3} r (a + b Log[c x^{n}])}{3 r} + d^{3} Log[x] (a + b Log[c x^{n}])$$

Result (type 3, 124 leaves, 5 steps):

$$-\frac{3 \, b \, d^2 \, e \, n \, x^r}{r^2} - \frac{3 \, b \, d \, e^2 \, n \, x^2 \, r}{4 \, r^2} - \frac{b \, e^3 \, n \, x^3 \, r}{9 \, r^2} - \frac{1}{2} \, b \, d^3 \, n \, \text{Log} \left[x\right]^2 + \frac{1}{6} \left(\frac{18 \, d^2 \, e \, x^r}{r} + \frac{9 \, d \, e^2 \, x^2 \, r}{r} + \frac{2 \, e^3 \, x^3 \, r}{r} + 6 \, d^3 \, \text{Log} \left[x\right]\right) \left(a + b \, \text{Log} \left[c \, x^n\right]\right)$$

### Problem 422: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^{r}\right)^{2}\,\left(a+b\,Log\left[c\,x^{n}\right]\right)}{x}\,dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{2 \, b \, d \, e \, n \, x^{r}}{r^{2}} - \frac{b \, e^{2} \, n \, x^{2 \, r}}{4 \, r^{2}} - \frac{1}{2} \, b \, d^{2} \, n \, Log \left[x\right]^{2} + \frac{2 \, d \, e \, x^{r} \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{r} + \frac{e^{2} \, x^{2 \, r} \, \left(a + b \, Log \left[c \, x^{n}\right]\right)}{2 \, r} + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, Log \left[x\right] \, \left(a + b \, Log \left[c \, x^{n}\right]\right) + d^{2} \, Log \left[x\right] \, Log \left[x\right]$$

Result (type 3, 87 leaves, 5 steps):

$$-\,\frac{2\,b\,d\,e\,n\,x^{r}}{r^{2}}\,-\,\frac{b\,e^{2}\,n\,x^{2\,r}}{4\,r^{2}}\,-\,\frac{1}{2}\,b\,d^{2}\,n\,Log\left[\,x\,\right]^{\,2}\,+\,\frac{1}{2}\,\left(\,\frac{4\,d\,e\,x^{r}}{r}\,+\,\frac{e^{2}\,x^{2\,r}}{r}\,+\,2\,d^{2}\,Log\left[\,x\,\right]\,\right)\,\left(\,a\,+\,b\,Log\left[\,c\,\,x^{n}\,\right]\,\right)$$

# Problem 454: Result valid but suboptimal antiderivative.

$$\int \frac{(f+gx)(a+b\log[cx^n])}{(d+ex)^3} dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$\frac{b \left(e \, f - d \, g\right) \, n}{2 \, d \, e^2 \, \left(d + e \, x\right)} + \frac{b \, f^2 \, n \, Log\left[x\right]}{2 \, d^2 \, \left(e \, f - d \, g\right)} - \frac{\left(f + g \, x\right)^2 \, \left(a + b \, Log\left[c \, x^n\right]\right)}{2 \, \left(e \, f - d \, g\right) \, \left(d + e \, x\right)^2} - \frac{b \, \left(e \, f + d \, g\right) \, n \, Log\left[d + e \, x\right]}{2 \, d^2 \, e^2}$$

Result (type 3, 151 leaves, 7 steps):

$$\frac{b \ \left(e \ f - d \ g\right) \ n}{2 \ d \ e^2 \ \left(d + e \ x\right)} + \frac{b \ \left(e \ f - d \ g\right) \ n \ Log \left[x\right]}{2 \ d^2 \ e^2} - \frac{\left(e \ f - d \ g\right) \ \left(a + b \ Log \left[c \ x^n\right]\right)}{2 \ e^2 \ \left(d + e \ x\right)^2} + \frac{g \ x \ \left(a + b \ Log \left[c \ x^n\right]\right)}{d \ e \ \left(d + e \ x\right)} - \frac{b \ g \ n \ Log \left[d + e \ x\right]}{d \ e^2} - \frac{b \ \left(e \ f - d \ g\right) \ n \ Log \left[d + e \ x\right]}{2 \ d^2 \ e^2}$$

### Problem 455: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{\left(d+e\,x\right)^{3}}\,\mathrm{d}x$$

Optimal (type 4, 202 leaves, 8 steps):

$$-\frac{b \left(e \, f - d \, g\right) \, n \, x \, \left(a + b \, Log \left[c \, x^n\right]\right)}{d^2 \, e \, \left(d + e \, x\right)} + \frac{f^2 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, d^2 \, \left(e \, f - d \, g\right)} - \frac{\left(f + g \, x\right)^2 \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{2 \, \left(e \, f - d \, g\right) \, \left(d + e \, x\right)^2} + \frac{b^2 \, \left(e \, f - d \, g\right) \, n^2 \, Log \left[d + e \, x\right]}{d^2 \, e^2} - \frac{b \, \left(e \, f + d \, g\right) \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{b^2 \, \left(e \, f + d \, g\right) \, n^2 \, PolyLog \left[2, -\frac{e \, x}{d}\right]}{d^2 \, e^2}$$

Result (type 4, 278 leaves, 13 steps):

$$-\frac{b\;\left(e\;f-d\;g\right)\;n\;x\;\left(a+b\;Log\left[c\;x^{n}\right]\right)}{d^{2}\;e\;\left(d+e\;x\right)} + \frac{\left(e\;f-d\;g\right)\;\left(a+b\;Log\left[c\;x^{n}\right]\right)^{2}}{2\;d^{2}\;e^{2}} - \frac{\left(e\;f-d\;g\right)\;\left(a+b\;Log\left[c\;x^{n}\right]\right)^{2}}{2\;e^{2}\left(d+e\;x\right)^{2}} + \\ \frac{g\;x\;\left(a+b\;Log\left[c\;x^{n}\right]\right)^{2}}{d\;e\;\left(d+e\;x\right)} + \frac{b^{2}\;\left(e\;f-d\;g\right)\;n^{2}\;Log\left[d+e\;x\right]}{d^{2}\;e^{2}} - \frac{2\;b\;g\;n\;\left(a+b\;Log\left[c\;x^{n}\right]\right)\;Log\left[1+\frac{e\;x}{d}\right]}{d\;e^{2}} - \\ \frac{b\;\left(e\;f-d\;g\right)\;n\;\left(a+b\;Log\left[c\;x^{n}\right]\right)\;Log\left[1+\frac{e\;x}{d}\right]}{d^{2}\;e^{2}} - \frac{2\;b^{2}\;g\;n^{2}\;PolyLog\left[2,-\frac{e\;x}{d}\right]}{d\;e^{2}} - \frac{b^{2}\;\left(e\;f-d\;g\right)\;n^{2}\;PolyLog\left[2,-\frac{e\;x}{d}\right]}{d^{2}\;e^{2}}$$

Problem 456: Result valid but suboptimal antiderivative.

$$\int \frac{(f+gx) (a+b \log [cx^n])^3}{(d+ex)^3} dx$$

Optimal (type 4, 295 leaves, 11 steps):

$$-\frac{3 \ b \ \left(e \ f-d \ g\right) \ n \ x \ \left(a+b \ Log \left[c \ x^n\right]\right)^2}{2 \ d^2 \ e \ \left(d+e \ x\right)} + \frac{f^2 \ \left(a+b \ Log \left[c \ x^n\right]\right)^3}{2 \ d^2 \ \left(e \ f-d \ g\right)} - \frac{\left(f+g \ x\right)^2 \ \left(a+b \ Log \left[c \ x^n\right]\right)^3}{2 \ \left(e \ f-d \ g\right) \ \left(d+e \ x\right)^2} + \frac{3 \ b^2 \ \left(e \ f-d \ g\right) \ n^2 \ \left(a+b \ Log \left[c \ x^n\right]\right)^3}{2 \ d^2 \ e^2} + \frac{3 \ b^3 \ \left(e \ f-d \ g\right) \ n^3 \ PolyLog \left[2,-\frac{e \ x}{d}\right]}{d^2 \ e^2} - \frac{3 \ b^2 \ \left(e \ f+d \ g\right) \ n^2 \ \left(a+b \ Log \left[c \ x^n\right]\right)^2 \ Log \left[1+\frac{e \ x}{d}\right]}{d^2 \ e^2} + \frac{3 \ b^3 \ \left(e \ f+d \ g\right) \ n^3 \ PolyLog \left[3,-\frac{e \ x}{d}\right]}{d^2 \ e^2}$$

Result (type 4, 408 leaves, 17 steps):

$$-\frac{3 \, b \, \left(e \, f - d \, g\right) \, n \, x \, \left(a + b \, Log\left[c \, x^n\right]\right)^2}{2 \, d^2 \, e \, \left(d + e \, x\right)} + \frac{\left(e \, f - d \, g\right) \, \left(a + b \, Log\left[c \, x^n\right]\right)^3}{2 \, d^2 \, e^2} - \frac{\left(e \, f - d \, g\right) \, \left(a + b \, Log\left[c \, x^n\right]\right)^3}{2 \, e^2 \, \left(d + e \, x\right)^2} + \frac{g \, x \, \left(a + b \, Log\left[c \, x^n\right]\right)^3}{d \, e \, \left(d + e \, x\right)} + \frac{3 \, b^2 \, \left(e \, f - d \, g\right) \, n^2 \, \left(a + b \, Log\left[c \, x^n\right]\right) \, Log\left[1 + \frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b \, g \, n \, \left(a + b \, Log\left[c \, x^n\right]\right)^2 \, Log\left[1 + \frac{e \, x}{d}\right]}{d \, e^2} - \frac{3 \, b \, g \, n \, \left(a + b \, Log\left[c \, x^n\right]\right)^2 \, Log\left[1 + \frac{e \, x}{d}\right]}{d \, e^2} - \frac{3 \, b \, g \, n^2 \, \left(a + b \, Log\left[c \, x^n\right]\right)^2 \, Log\left[1 + \frac{e \, x}{d}\right]}{d \, e^2} - \frac{3 \, b \, g \, n^3 \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{d \, e^2} - \frac{6 \, b^2 \, g \, n^2 \, \left(a + b \, Log\left[c \, x^n\right]\right) \, PolyLog\left[2, -\frac{e \, x}{d}\right]}{d \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, PolyLog\left[3, -\frac{e \, x}{d}\right]}{d^2 \, e^2} - \frac{3 \, b^3 \, \left(e \, f - d \, g\right) \, n^3 \, P$$

# Test results for the 249 problems in "3.1.5 u (a+b log(c x^n))^p.m"

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c \, x^{n}\right]\right)^{2} \log \left[1 + e \, x\right]}{x^{2}} \, \mathrm{d}x$$

Optimal (type 4, 203 leaves, 10 steps):

$$2 \, b^2 \, e \, n^2 \, Log \, [x] \, - 2 \, b \, e \, n \, Log \, \Big[ \, 1 \, + \, \frac{1}{e \, x} \, \Big] \, \left( a \, + \, b \, Log \, \Big[ \, c \, x^n \, \Big] \, \right) \, - e \, Log \, \Big[ \, 1 \, + \, \frac{1}{e \, x} \, \Big] \, \left( a \, + \, b \, Log \, \Big[ \, c \, x^n \, \Big] \, \right)^2 \, - \\ 2 \, b^2 \, e \, n^2 \, Log \, [1 \, + \, e \, x] \, - \, \frac{2 \, b^2 \, n^2 \, Log \, [1 \, + \, e \, x]}{x} \, - \, \frac{2 \, b \, n \, \left( a \, + \, b \, Log \, [c \, x^n] \, \right) \, Log \, [1 \, + \, e \, x]}{x} \, - \, \frac{\left( a \, + \, b \, Log \, [c \, x^n] \, \right)^2 \, Log \, [1 \, + \, e \, x]}{x} \, + \\ 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 2 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b \, e \, n \, \left( a \, + \, b \, Log \, [c \, x^n] \, \right) \, PolyLog \, \Big[ \, 2 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog \, \Big[ \, 3 \, , \, - \, \frac{1}{e \, x} \, \Big] \, + 2 \, b^2 \, e \, n^2 \, PolyLog$$

#### Result (type 4, 220 leaves, 15 steps):

$$2 b^{2} e n^{2} Log[x] + e \left(a + b Log[c x^{n}]\right)^{2} + \frac{e \left(a + b Log[c x^{n}]\right)^{3}}{3 b n} - 2 b^{2} e n^{2} Log[1 + e x] - \frac{2 b^{2} n^{2} Log[1 + e x]}{x} - 2 b e n \left(a + b Log[c x^{n}]\right) Log[1 + e x] - \frac{2 b n \left(a + b Log[c x^{n}]\right) Log[1 + e x]}{x} - e \left(a + b Log[c x^{n}]\right)^{2} Log[1 + e x] - \frac{\left(a + b Log[c x^{n}]\right)^{2} Log[1 + e x]}{x} - 2 b^{2} e n^{2} PolyLog[2, -e x] - 2 b e n \left(a + b Log[c x^{n}]\right) PolyLog[2, -e x] + 2 b^{2} e n^{2} PolyLog[3, -e x]$$

### Problem 16: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \, x^n\right]\right)^2 \log \left[1+e \, x\right]}{x^3} \, \mathrm{d}x$$

Optimal (type 4, 287 leaves, 14 steps):

$$-\frac{7 \, b^2 \, e \, n^2}{4 \, x} - \frac{1}{4} \, b^2 \, e^2 \, n^2 \, \text{Log}[x] - \frac{3 \, b \, e \, n \, \left(a + b \, \text{Log}[c \, x^n] \right)}{2 \, x} + \frac{1}{2} \, b \, e^2 \, n \, \text{Log}[1 + \frac{1}{e \, x}] \, \left(a + b \, \text{Log}[c \, x^n] \right) - \frac{e \, \left(a + b \, \text{Log}[c \, x^n] \right)^2}{2 \, x} + \frac{1}{2} \, e^2 \, \text{Log}[1 + \frac{1}{e \, x}] \, \left(a + b \, \text{Log}[c \, x^n] \right)^2 + \frac{1}{4} \, b^2 \, e^2 \, n^2 \, \text{Log}[1 + e \, x] - \frac{b^2 \, n^2 \, \text{Log}[1 + e \, x]}{4 \, x^2} - \frac{b \, n \, \left(a + b \, \text{Log}[c \, x^n] \right) \, \text{Log}[1 + e \, x]}{2 \, x^2} - \frac{\left(a + b \, \text{Log}[c \, x^n] \right)^2 \, \text{Log}[1 + e \, x]}{2 \, x^2} - \frac{\left(a + b \, \text{Log}[c \, x^n] \right)^2 \, \text{Log}[1 + e \, x]}{2 \, x^2} - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[2, -\frac{1}{e \, x}] - b \, e^2 \, n \, \left(a + b \, \text{Log}[c \, x^n] \right) \, \text{PolyLog}[2, -\frac{1}{e \, x}] - b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{e \, x}] - \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog}[3, -\frac{1}{2} \, a^2 \, a^2$$

### Result (type 4, 310 leaves, 19 steps):

$$-\frac{7 \, b^2 \, e \, n^2}{4 \, x} - \frac{1}{4} \, b^2 \, e^2 \, n^2 \, \text{Log} \, [x] - \frac{3 \, b \, e \, n \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right)}{2 \, x} - \frac{1}{4} \, e^2 \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right)^2 - \frac{e \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right)^2}{2 \, x} - \frac{e^2 \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right)^3}{6 \, b \, n} + \frac{1}{4} \, b^2 \, e^2 \, n^2 \, \text{Log} \, [1 + e \, x] - \frac{b^2 \, n^2 \, \text{Log} \, [1 + e \, x]}{4 \, x^2} + \frac{1}{2} \, b \, e^2 \, n \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right) \, \text{Log} \, [1 + e \, x] - \frac{b \, n \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right) \, \text{Log} \, [1 + e \, x]}{2 \, x^2} + \frac{1}{2} \, e^2 \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right)^2 \, \text{Log} \, [1 + e \, x] - \frac{\left(a + b \, \text{Log} \, [c \, x^n] \, \right)^2 \, \text{Log} \, [1 + e \, x]}{2 \, x^2} + \frac{1}{2} \, b^2 \, e^2 \, n^2 \, \text{PolyLog} \, [2, -e \, x] + b \, e^2 \, n \, \left(a + b \, \text{Log} \, [c \, x^n] \, \right) \, \text{PolyLog} \, [2, -e \, x] - b^2 \, e^2 \, n^2 \, \text{PolyLog} \, [3, -e \, x]$$

### Problem 22: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \, x^n\right]\right)^3 \log \left[1+e \, x\right]}{x^2} \, \mathrm{d}x$$

Optimal (type 4, 342 leaves, 14 steps):

$$6\,b^3\,e\,n^3\,Log[\,x]\,-6\,b^2\,e\,n^2\,Log\big[\,1\,+\,\frac{1}{e\,x}\,\big]\,\,\left(a\,+\,b\,Log\big[\,c\,\,x^n\,\big]\,\right)\,-3\,b\,e\,n\,Log\big[\,1\,+\,\frac{1}{e\,x}\,\big]\,\,\left(a\,+\,b\,Log\big[\,c\,\,x^n\,\big]\,\right)^2\,-\,e\,Log\big[\,1\,+\,\frac{1}{e\,x}\,\big]\,\,\left(a\,+\,b\,Log\big[\,c\,\,x^n\,\big]\,\right)^3\,-\, \\ 6\,b^3\,e\,n^3\,Log[\,1\,+\,e\,x]\,-\,\frac{6\,b^3\,n^3\,Log[\,1\,+\,e\,x]}{x}\,-\,\frac{6\,b^2\,n^2\,\,\big(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)\,Log[\,1\,+\,e\,x]}{x}\,-\,\frac{3\,b\,n\,\,\big(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^2\,Log[\,1\,+\,e\,x]}{x}\,-\,\frac{3\,b\,n\,\,\big(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^2\,Log[\,1\,+\,e\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,e\,\,x]}{x}\,-\,\frac{(a\,+\,b\,Log[\,c\,\,x^n\,]\,\big)^3\,Log[\,1\,+\,$$

Result (type 4, 360 leaves, 22 steps):

$$6\,b^3\,e\,n^3\,Log[x] + 3\,b\,e\,n\,\left(a + b\,Log[c\,x^n]\right)^2 + e\,\left(a + b\,Log[c\,x^n]\right)^3 + \frac{e\,\left(a + b\,Log[c\,x^n]\right)^4}{4\,b\,n} - 6\,b^3\,e\,n^3\,Log[1 + e\,x] - \frac{6\,b^3\,n^3\,Log[1 + e\,x]}{x} - 6\,b^2\,e\,n^2\,\left(a + b\,Log[c\,x^n]\right)\,Log[1 + e\,x] - \frac{6\,b^2\,n^2\,\left(a + b\,Log[c\,x^n]\right)\,Log[1 + e\,x]}{x} - 3\,b\,e\,n\,\left(a + b\,Log[c\,x^n]\right)^2\,Log[1 + e\,x] - \frac{3\,b\,n\,\left(a + b\,Log[c\,x^n]\right)^2\,Log[1 + e\,x]}{x} - e\,\left(a + b\,Log[c\,x^n]\right)^3\,Log[1 + e\,x] - \frac{\left(a + b\,Log[c\,x^n]\right)^3\,Log[1 + e\,x]}{x} - \frac{6\,b^3\,e\,n^3\,PolyLog[2, -e\,x] - 6\,b^2\,e\,n^2\,\left(a + b\,Log[c\,x^n]\right)\,PolyLog[2, -e\,x] - 3\,b\,e\,n\,\left(a + b\,Log[c\,x^n]\right)^2\,PolyLog[2, -e\,x] + 6\,b^3\,e\,n^3\,PolyLog[3, -e\,x] + 6\,b^2\,e\,n^2\,\left(a + b\,Log[c\,x^n]\right)\,PolyLog[3, -e\,x] - 6\,b^3\,e\,n^3\,PolyLog[4, -e\,x]$$

### Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \mathsf{Log}\, [\, c\,\, x^n\, ]\,\right)^3 \, \mathsf{Log}\, [\, 1+e\,\, x\, ]}{x^3} \, \, \mathrm{d} x$$

Optimal (type 4, 470 leaves, 22 steps):

$$-\frac{45 \, b^3 \, e^n^3}{8 \, x} - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}[\,x] \, - \frac{21 \, b^2 \, e^n^2 \, \left(a + b \, \text{Log}[\,c \, x^n]\,\right)}{4 \, x} + \frac{3}{4} \, b^2 \, e^2 \, n^2 \, \text{Log}\left[1 + \frac{1}{e \, x}\right] \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right) - \frac{9 \, b \, e^n \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right)^2}{4 \, x} + \frac{3}{4} \, b^2 \, e^2 \, n^2 \, \text{Log}\left[1 + \frac{1}{e \, x}\right] \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right) - \frac{9 \, b \, e^n \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right)^2}{4 \, x} + \frac{3}{4} \, b^2 \, e^2 \, n^2 \, \text{Log}\left[1 + \frac{1}{e \, x}\right] \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right)^3 + \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{Log}\left[1 + e \, x\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{PolyLog}\left[2, -\frac{1}{e \, x}\right] - \frac{3}{2} \, b^2 \, e^2 \, n^2 \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right) \, \text{PolyLog}\left[2, -\frac{1}{e \, x}\right] - \frac{3}{2} \, b \, e^2 \, n \, \left(a + b \, \text{Log}\left[\,c \, x^n\right]\,\right)^2 \, \text{PolyLog}\left[2, -\frac{1}{e \, x}\right] - \frac{3}{8} \, b^3 \, e^2 \, n^3 \, \text{PolyLog}\left[3, -\frac{1}{e \, x}\right] - 3 \, b^3 \, e^2 \, n^3 \, \text{PolyLog}\left[3, -\frac{1}{e \, x}\right] - 3 \, b^3 \, e^2 \, n^3 \, \text{PolyLog}\left[4, -\frac{1}{e \, x}\right]$$

#### Result (type 4, 499 leaves, 30 steps):

$$-\frac{45\,b^3\,e\,n^3}{8\,x} - \frac{3}{8}\,b^3\,e^2\,n^3\,\text{Log}[x] - \frac{21\,b^2\,e\,n^2\,\left(a + b\,\text{Log}[c\,x^n]\right)}{4\,x} - \frac{3}{8}\,b\,e^2\,n\,\left(a + b\,\text{Log}[c\,x^n]\right)^2 - \frac{9\,b\,e\,n\,\left(a + b\,\text{Log}[c\,x^n]\right)^2}{4\,x} - \frac{1}{4}\,e^2\,\left(a + b\,\text{Log}[c\,x^n]\right)^3 - \frac{e\,\left(a + b\,\text{Log}[c\,x^n]\right)^3}{2\,x} - \frac{e^2\,\left(a + b\,\text{Log}[c\,x^n]\right)^4}{8\,b\,n} + \frac{3}{8}\,b^3\,e^2\,n^3\,\text{Log}[1 + e\,x] - \frac{3\,b^3\,n^3\,\text{Log}[1 + e\,x]}{8\,x^2} + \frac{3}{4}\,b^2\,e^2\,n^2\,\left(a + b\,\text{Log}[c\,x^n]\right)\,\text{Log}[1 + e\,x] - \frac{3\,b^2\,n^2\,\left(a + b\,\text{Log}[c\,x^n]\right)\,\text{Log}[1 + e\,x]}{4\,x^2} + \frac{3}{4}\,b\,e^2\,n\,\left(a + b\,\text{Log}[c\,x^n]\right)^2\,\text{Log}[1 + e\,x] - \frac{3\,b^3\,n^3\,\text{Log}[1 + e\,x]}{4\,x^2} + \frac{3}{4}\,b\,e^2\,n\,\left(a + b\,\text{Log}[c\,x^n]\right)^3\,\text{Log}[1 + e\,x] - \frac{3\,b^3\,n^3\,\text{Log}[1 + e\,x]}{2\,x^2} + \frac{3}{4}\,b^3\,e^2\,n^3\,\text{PolyLog}[2, -e\,x] + \frac{3}{2}\,b^2\,e^2\,n^2\,\left(a + b\,\text{Log}[c\,x^n]\right)^3\,\text{PolyLog}[2, -e\,x] + \frac{3}{2}\,b\,e^2\,n\,\left(a + b\,\text{Log}[c\,x^n]\right)^2\,\text{PolyLog}[2, -e\,x] - \frac{3}{2}\,b^3\,e^2\,n^3\,\text{PolyLog}[3, -e\,x] - 3\,b^2\,e^2\,n^2\,\left(a + b\,\text{Log}[c\,x^n]\right)^3\,\text{PolyLog}[3, -e\,x] + 3\,b^3\,e^2\,n^3\,\text{PolyLog}[4, -e\,x]$$

### Problem 39: Result optimal but 2 more steps used.

$$\int \frac{\left(a + b \log[c x^n]\right)^2 \log\left[d\left(\frac{1}{d} + f x^2\right)\right]}{x^4} dx$$

Optimal (type 4, 543 leaves, 22 steps):

$$-\frac{52\,b^2\,d\,f\,n^2}{27\,x} - \frac{4}{27}\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{ArcTan}\big[\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{16\,b\,d\,f\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)}{9\,x} - \frac{4}{9}\,b\,d^{3/2}\,f^{3/2}\,n\,\text{ArcTan}\big[\sqrt{d}\,\,\sqrt{f}\,\,x\big]\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big) - \frac{2\,d\,f\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)^2}{3\,x} + \frac{1}{3}\,\,\big(-d\big)^{3/2}\,f^{3/2}\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2\,b^2\,n^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{3\,x} - \frac{2\,b\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{Log}\,\big[1+d\,f\,x^2\big]}{9\,x^3} - \frac{(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{9\,x^3} - \frac{(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{3\,x^3} - \frac{2\,b\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{PolyLog}\,\big[2,\,-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] + \frac{2}{3}\,b\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{PolyLog}\,\big[2,\,-i\,\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{PolyLog}\,\big[2,\,-i\,\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,\,-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,\,\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,\,\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,\,\sqrt{-d}\,\,\sqrt{f}\,\,x\big]$$

#### Result (type 4, 543 leaves, 24 steps):

$$-\frac{52\,b^2\,d\,f\,n^2}{27\,x} - \frac{4}{27}\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{ArcTan}\big[\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{16\,b\,d\,f\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)}{9\,x} - \frac{4}{9}\,b\,d^{3/2}\,f^{3/2}\,n\,\text{ArcTan}\big[\sqrt{d}\,\,\sqrt{f}\,\,x\big]\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big) - \frac{2\,d\,f\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)^2}{3\,x} + \frac{1}{3}\,\,\big(-d\big)^{3/2}\,f^{3/2}\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2\,b^2\,n^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{27\,x^3} - \frac{2\,b\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{Log}\,\big[1+d\,f\,x^2\big]}{9\,x^3} - \frac{(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{9\,x^3} - \frac{(a+b\,\text{Log}\,[c\,x^n]\,\big)^2\,\text{Log}\,\big[1+d\,f\,x^2\big]}{3\,x^3} - \frac{2\,b\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n\,\,\big(a+b\,\text{Log}\,[c\,x^n]\,\big)\,\text{PolyLog}\,\big[2,-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] + \frac{2}{9}\,\dot{b}\,\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[2,-\dot{a}\,\,\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{9}\,\dot{b}\,\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[2,-\dot{a}\,\,\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{9}\,\dot{b}\,\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[2,-\dot{a}\,\,\sqrt{d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{9}\,\dot{b}\,\,b^2\,d^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,\text{PolyLog}\,\big[3,-\sqrt{-d}\,\,\sqrt{f}\,\,x\big] - \frac{2}{3}\,b^2\,\,\big(-d\big)^{3/2}\,f^{3/2}\,n^2\,n^2\,\text{PolyLog}\,\big$$

### Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log[c x^n]\right)^2 \log[d (e + f x)^m]}{x^2} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$\frac{2\,b^2\,f\,m\,n^2\,Log\,[\,x\,]}{e} - \frac{2\,b\,f\,m\,n\,Log\,[\,1 + \frac{e}{f\,x}\,]\,\,\left(\,a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{e} - \frac{f\,m\,Log\,[\,1 + \frac{e}{f\,x}\,]\,\,\left(\,a + b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{e} - \frac{2\,b^2\,f\,m\,n^2\,Log\,[\,e + f\,x\,]}{e} - \frac{2\,b^2\,f\,m\,n^2\,Log\,[\,e + f\,x\,]}{e} - \frac{2\,b^2\,f\,m\,n^2\,Log\,[\,e + f\,x\,]^{\,m}\,]}{x} + \frac{2\,b^2\,f\,m\,n^2\,PolyLog\,[\,2 \,,\, -\frac{e}{f\,x}\,]}{e} + \frac{2\,b^2\,f\,m\,n^2\,PolyLog\,[\,2 \,,\, -\frac{e}{f\,x}\,]}{e} + \frac{2\,b^2\,f\,m\,n^2\,PolyLog\,[\,3 \,,\, -\frac{e}{f\,x}\,]}{e} + \frac{2\,b^2\,f\,m\,n^2\,PolyLog\,[$$

#### Result (type 4, 283 leaves, 15 steps):

$$\frac{2\,b^{2}\,f\,m\,n^{2}\,Log\left[x\right]}{e} + \frac{f\,m\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{2}}{e} + \frac{f\,m\,\left(a+b\,Log\left[c\,x^{n}\right]\right)^{3}}{3\,b\,e\,n} - \frac{2\,b^{2}\,f\,m\,n^{2}\,Log\left[e+f\,x\right]}{e} - \frac{2\,b^{2}\,n^{2}\,Log\left[d\,\left(e+f\,x\right)^{m}\right]}{x} - \frac{2\,b^{2}\,n^{2}\,Log\left[d\,\left(e+f\,x\right)^{m}\right]}{x} - \frac{2\,b^{2}\,m\,n^{2}\,Log\left[d\,\left(e+f\,x\right)^{m}\right]}{e} - \frac{2\,b\,f\,m\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,Log\left[1+\frac{f\,x}{e}\right]}{e} - \frac{2\,b^{2}\,f\,m\,n^{2}\,PolyLog\left[2,-\frac{f\,x}{e}\right]}{e} - \frac{2\,b\,f\,m\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,PolyLog\left[2,-\frac{f\,x}{e}\right]}{e} + \frac{2\,b^{2}\,f\,m\,n^{2}\,PolyLog\left[3,-\frac{f\,x}{e}\right]}{e} - \frac{2\,b\,f\,m\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,PolyLog\left[2,-\frac{f\,x}{e}\right]}{e} + \frac{2\,b\,f\,m\,n^{2}\,PolyLog\left[3,-\frac{f\,x}{e}\right]}{e} - \frac{2\,b\,f\,m\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,PolyLog\left[2,-\frac{f\,x}{e}\right]}{e} + \frac{2\,b\,f\,m\,n^{2}\,PolyLog\left[3,-\frac{f\,x}{e}\right]}{e} - \frac{2\,b\,f\,m\,n\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\,PolyLog\left[2,-\frac{f\,x}{e}\right]}{e} + \frac{2\,b\,f\,m\,n^{2}\,PolyLog\left[3,-\frac{f\,x}{e}\right]}{e} + \frac{2\,b\,f\,m\,n^{2}\,PolyLog\left[3,-\frac{f\,x}{e}\right]}{$$

### Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log[c x^n]\right)^2 \log[d (e + f x)^m]}{x^3} dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$-\frac{7 \, b^2 \, f \, m \, n^2}{4 \, e \, x} - \frac{b^2 \, f^2 \, m \, n^2 \, Log \left[x\right]}{4 \, e^2} - \frac{3 \, b \, f \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{2 \, e \, x} + \frac{b \, f^2 \, m \, n \, Log \left[1 + \frac{e}{f \, x}\right] \, \left(a + b \, Log \left[c \, x^n\right]\right)}{2 \, e^2} + \frac{2 \, e \, x}{2 \, e \, x} + \frac{f^2 \, m \, Log \left[1 + \frac{e}{f \, x}\right] \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{4 \, e^2} + \frac{b^2 \, f^2 \, m \, n^2 \, Log \left[e + f \, x\right]}{4 \, e^2} - \frac{b^2 \, n^2 \, Log \left[d \, \left(e + f \, x\right)^m\right]}{4 \, x^2} - \frac{b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[d \, \left(e + f \, x\right)^m\right]}{2 \, x^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[2, -\frac{e}{f \, x}\right]}{2 \, e^2} - \frac{b \, f^2 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2} - \frac{b^2 \, f^2 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{e^2}$$

Result (type 4, 385 leaves, 19 steps):

$$-\frac{7\,b^{2}\,f\,m\,n^{2}}{4\,e\,x} - \frac{b^{2}\,f^{2}\,m\,n^{2}\,Log\,[x]}{4\,e^{2}} - \frac{3\,b\,f\,m\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)}{2\,e\,x} - \frac{f^{2}\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{4\,e^{2}} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{2\,e\,x} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{2\,e\,x} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}}{2\,e\,x} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}\,Log\,\left[d\,\left(e+f\,x\right)^{m}\right]}{2\,e\,x} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,Log\,\left[d\,\left(e+f\,x\right)^{m}\right]}{2\,x^{2}} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,Log\,\left[d\,\left(e+f\,x\right)^{m}\right]}{2\,x^{2}} - \frac{f\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,Log\,\left[d\,\left(e+f\,x\right)^{m}\right]}{2\,e^{2}} + \frac{f^{2}\,m\,\left(a+b\,Log\,[c\,x^{n}]\,\right)^{2}\,Log\,\left[1+\frac{f\,x}{e}\right]}{2\,e^{2}} + \frac{b\,f^{2}\,m\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2\,,\,-\frac{f\,x}{e}\right]}{2\,e^{2}} - \frac{b^{2}\,f^{2}\,m\,n^{2}\,PolyLog\,\left[3\,,\,-\frac{f\,x}{e}\right]}{e^{2}} + \frac{b\,f^{2}\,m\,n\,\left(a+b\,Log\,[c\,x^{n}]\,\right)\,PolyLog\,\left[2\,,\,-\frac{f\,x}{e}\right]}{e^{2}} - \frac{b^{2}\,f^{2}\,m\,n^{2}\,PolyLog\,\left[3\,,\,-\frac{f\,x}{e}\right]}{e^{2}} + \frac{b^{2}\,f^{2}\,m\,n^{2}\,PolyLo$$

### Problem 84: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log[c x^n]\right)^2 \log[d (e + f x)^m]}{x^4} dx$$

#### Optimal (type 4, 420 leaves, 19 steps):

$$-\frac{19 \, b^2 \, f \, m \, n^2}{108 \, e \, x^2} + \frac{26 \, b^2 \, f^2 \, m \, n^2}{27 \, e^2 \, x} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, Log [x]}{27 \, e^3} - \frac{5 \, b \, f \, m \, n \, \left(a + b \, Log [c \, x^n]\right)}{18 \, e \, x^2} + \frac{8 \, b \, f^2 \, m \, n \, \left(a + b \, Log [c \, x^n]\right)}{9 \, e^2 \, x} - \frac{2 \, b \, f^3 \, m \, n \, Log \left[1 + \frac{e}{f \, x}\right] \, \left(a + b \, Log [c \, x^n]\right)}{9 \, e^3} - \frac{f \, m \, \left(a + b \, Log [c \, x^n]\right)^2}{6 \, e \, x^2} + \frac{f^2 \, m \, \left(a + b \, Log [c \, x^n]\right)^2}{3 \, e^2 \, x} - \frac{f^3 \, m \, Log \left[1 + \frac{e}{f \, x}\right] \, \left(a + b \, Log [c \, x^n]\right)^2}{3 \, e^3} - \frac{2 \, b \, n \, \left(a + b \, Log [c \, x^n]\right)^2}{3 \, e^3} - \frac{2 \, b \, n \, \left(a + b \, Log [c \, x^n]\right) \, Log \left[d \, \left(e + f \, x\right)^m\right]}{9 \, x^3} - \frac{\left(a + b \, Log [c \, x^n]\right)^2 \, Log \left[d \, \left(e + f \, x\right)^m\right]}{3 \, x^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[2, -\frac{e}{f \, x}\right]}{3 \, e^3} + \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log [c \, x^n]\right) \, PolyLog \left[2, -\frac{e}{f \, x}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{e}{f \, x}\right]}{3 \, e^3}$$

#### Result (type 4, 462 leaves, 22 steps):

$$-\frac{19 \, b^2 \, f \, m \, n^2}{108 \, e \, x^2} + \frac{26 \, b^2 \, f^2 \, m \, n^2}{27 \, e^2 \, x} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, Log \left[x\right]}{27 \, e^3} - \frac{5 \, b \, f \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{18 \, e \, x^2} + \frac{8 \, b \, f^2 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{9 \, e^2 \, x} + \frac{f^3 \, m \, \left(a + b \, Log \left[c \, x^n\right]\right)^2}{9 \, e^3} - \frac{f^3 \, m \, \left(a + b \, Log \left[c \, x^n\right]\right)^3}{9 \, b^3 \, n} - \frac{2 \, b^2 \, f^3 \, m \, n^2 \, Log \left[e + f \, x\right]}{27 \, e^3} - \frac{2 \, b^2 \, n^2 \, Log \left[d \, \left(e + f \, x\right)^m\right]}{27 \, x^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[d \, \left(e + f \, x\right)^m\right]}{9 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[d \, \left(e + f \, x\right)^m\right]}{9 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \left[1 + \frac{f \, x}{e}\right]}{9 \, e^3} - \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[2, -\frac{f \, x}{e}\right]}{9 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{f \, x}{e}\right]}{9 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} - \frac{2 \, b \, f^3 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} + \frac{2 \, b^2 \, f^3 \, m \, n^2 \, PolyLog \left[3, -\frac{f \, x}{e}\right]}{3 \, e^3} +$$

### Problem 88: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c x^{n}\right]\right)^{3} \log\left[d \left(e + f x\right)^{m}\right]}{x^{2}} dx$$

Optimal (type 4, 411 leaves, 14 steps):

$$\frac{6\,b^3\,f\,m\,n^3\,Log\,[\,x\,]}{e} - \frac{6\,b^2\,f\,m\,n^2\,Log\,[\,1 + \frac{e}{f\,x}\,]\,\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)}{e} - \frac{3\,b\,f\,m\,n\,Log\,[\,1 + \frac{e}{f\,x}\,]\,\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)^2}{e} - \frac{f\,m\,Log\,[\,1 + \frac{e}{f\,x}\,]\,\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)^3}{e} - \frac{6\,b^3\,f\,m\,n^3\,Log\,[\,e + f\,x\,]}{e} - \frac{6\,b^3\,f\,m\,n^3\,Log\,[\,d\,\,\left(e + f\,x\right)^m\,]}{x} - \frac{6\,b^2\,n^2\,\,\left(a + b\,Log\,[\,c\,\,x^n\,]\,\right)\,Log\,[\,d\,\,\left(e + f\,x\right)^m\,]}{x} - \frac{3\,b\,f\,m\,n\,Log\,[\,d\,\,\left(e + f\,x\right)^m\,]}{x} - \frac{6\,b^3\,f\,m\,n^3\,PolyLog\,[\,2 \,, \, - \frac{e}{f\,x}\,]}{x} + \frac{6\,b^3\,f\,m\,n^3\,PolyLog\,[\,2 \,, \, - \frac{e}{f\,x}\,]}{e} + \frac{6\,b^3\,f\,m\,n^3\,PolyL$$

#### Result (type 4, 459 leaves, 22 steps):

$$\frac{6\,b^3\,f\,m\,n^3\,Log[\,x]}{e} + \frac{3\,b\,f\,m\,n\,\left(a+b\,Log[\,c\,x^n]\,\right)^2}{e} + \frac{f\,m\,\left(a+b\,Log[\,c\,x^n]\,\right)^3}{e} + \frac{f\,m\,\left(a+b\,Log[\,c\,x^n]\,\right)^4}{e} - \frac{6\,b^3\,f\,m\,n^3\,Log[\,e+f\,x]}{e} - \frac{6\,b^3\,f\,m\,n^3\,Log[\,d\,\left(e+f\,x\right)^m]}{e} - \frac{6\,b^3\,f\,m\,n^3\,Log[\,$$

# Problem 89: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \mathsf{Log}\left[c \, x^n\right]\right)^3 \, \mathsf{Log}\left[d \, \left(e+f \, x\right)^m\right]}{x^3} \, \mathrm{d} x$$

Optimal (type 4, 555 leaves, 22 steps):

$$\frac{45\,b^3\,fm\,n^3}{8\,e\,x} \quad \frac{3\,b^3\,f^2\,m\,n^3\,\log[x]}{8\,e^2} \quad \frac{21\,b^2\,fm\,n^2\,\left(a+b\,\log[c\,x^n]\right)}{4\,e\,x} + \frac{3\,b^2\,f^2\,m\,n^2\,\log\left[1+\frac{e}{f\,x}\right]\,\left(a+b\,\log[c\,x^n]\right)}{4\,e^2} \\ \frac{9\,b\,fm\,n\,\left(a+b\,\log[c\,x^n]\right)^2}{4\,e\,x} + \frac{3\,b\,f^2\,m\,n\,\log\left[1+\frac{e}{f\,x}\right]\,\left(a+b\,\log[c\,x^n]\right)^2}{4\,e^2} - \frac{f\,m\,\left(a+b\,\log[c\,x^n]\right)^3}{2\,e\,x} + \frac{f^2\,m\,\log\left[1+\frac{e}{f\,x}\right]\,\left(a+b\,\log[c\,x^n]\right)^3}{2\,e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,\log[e+f\,x]}{8\,e^2} - \frac{3\,b^3\,n^3\,\log[d\,\left(e+f\,x\right)^m]}{8\,x^2} - \frac{3\,b^2\,n^2\,\left(a+b\,\log[c\,x^n]\right)\,\log[d\,\left(e+f\,x\right)^m]}{4\,x^2} - \frac{3\,b^3\,n^3\,\log[d\,\left(e+f\,x\right)^m]}{2\,x^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[2,-\frac{e}{f\,x}\right]}{4\,e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[2,-\frac{e}{f\,x}\right]}{2\,e^2} - \frac{3\,b^2\,f^2\,m\,n^2\,\left(a+b\,\log[c\,x^n]\right)\,polyLog\left[3,-\frac{e}{f\,x}\right]}{2\,e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[3,-\frac{e}{f\,x}\right]}{2\,e^2} - \frac{3\,b^2\,f^2\,m\,n^2\,\left(a+b\,\log[c\,x^n]\right)\,polyLog\left[3,-\frac{e}{f\,x}\right]}{e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[3,-\frac{e}{f\,x}\right]}{2\,e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} \\ \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2} - \frac{3\,b^3\,f^2\,m\,n^3\,polyLog\left[4,-\frac{e}{f\,x}\right]}{e^2}$$

#### Result (type 4, 614 leaves, 30 steps):

$$\frac{45 \, b^3 \, fm \, n^3}{8 \, ex} \, \frac{3 \, b^3 \, f^2 \, m \, n^3 \, Log[x]}{4 \, ex} \, \frac{21 \, b^2 \, fm \, n^2 \, \left(a + b \, Log[c \, x^n]\right)}{4 \, ex} \, \frac{3 \, b \, f^2 \, m \, n \, \left(a + b \, Log[c \, x^n]\right)^2}{8 \, ex} \, \frac{9 \, b \, fm \, n \, \left(a + b \, Log[c \, x^n]\right)^3}{2 \, ex} \, \frac{4 \, ex}{8 \, be^2} \, \frac{3 \, b^3 \, f^2 \, m \, n^3 \, Log[e + fx]}{8 \, be^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, x^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, be^2} \, \frac{3 \, b^3 \, f^2 \, m \, n^3 \, Log[e + fx]}{8 \, be^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, x^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e + fx\right)^m]}{8 \, ae^2} \, - \frac{3 \, b^3 \, n^3 \, Log[d \, \left(e$$

# Problem 103: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c x^{n}\right]\right)^{2} \log \left[d \left(e+f x^{2}\right)^{m}\right]}{x^{5}} dx$$

Optimal (type 4, 356 leaves, 15 steps):

$$-\frac{7 \ b^{2} \ fm \ n^{2}}{32 \ ex^{2}} - \frac{b^{2} \ f^{2} \ m \ n^{2} \ Log[x]}{16 \ e^{2}} - \frac{3 \ b \ fm \ n \ (a + b \ Log[c \ x^{n}])}{8 \ ex^{2}} + \frac{b \ f^{2} \ m \ n \ Log[1 + \frac{e}{f x^{2}}] \ (a + b \ Log[c \ x^{n}])}{8 \ e^{2}} - \frac{fm \ (a + b \ Log[c \ x^{n}])^{2}}{4 \ ex^{2}} + \frac{f^{2} \ m \ Log[1 + \frac{e}{f x^{2}}] \ (a + b \ Log[c \ x^{n}])^{2}}{32 \ e^{2}} + \frac{b^{2} \ f^{2} \ m \ n^{2} \ Log[e + f \ x^{2}]}{32 \ x^{4}} - \frac{b \ n \ (a + b \ Log[c \ x^{n}]) \ Log[d \ (e + f \ x^{2})^{m}]}{8 \ x^{4}} - \frac{b^{2} \ f^{2} \ m \ n^{2} \ PolyLog[2, -\frac{e}{f x^{2}}]}{8 \ e^{2}} - \frac{b \ f^{2} \ m \ n \ (a + b \ Log[c \ x^{n}]) \ PolyLog[2, -\frac{e}{f x^{2}}]}{8 \ e^{2}} - \frac{b^{2} \ f^{2} \ m \ n^{2} \ PolyLog[3, -\frac{e}{f \ x^{2}}]}{8 \ e^{2}}$$

#### Result (type 4, 408 leaves, 20 steps):

$$\frac{7 \ b^2 \ fm \ n^2}{32 \ ex^2} - \frac{b^2 \ f^2 \ m \ n^2 \ Log[x]}{16 \ e^2} - \frac{3 \ b \ fm \ n \ \left(a + b \ Log[c \ x^n]\right)}{8 \ ex^2} - \frac{f^2 \ m \ \left(a + b \ Log[c \ x^n]\right)^2}{8 \ e^2} - \frac{fm \ \left(a + b \ Log[c \ x^n]\right)^2}{4 \ ex^2} - \frac{f^2 \ m \ \left(a + b \ Log[c \ x^n]\right)^2}{4 \ ex^2} - \frac{fm \ \left(a + b \ Log[c \ x^n]\right)^2}{4 \ ex^2} - \frac{fm \ \left(a + b \ Log[c \ x^n]\right)^2}{8 \ e^2} - \frac{fm \ \left(a + b \ Log[c \ x^n]\right)^2}{8 \ x^4} - \frac{f^2 \ m \ \left(a + b \ Log[c \ x^n]\right) \ Log[d \ \left(e + f \ x^2\right)^m]}{8 \ x^4} - \frac{\left(a + b \ Log[c \ x^n]\right)^2 \ Log[d \ \left(e + f \ x^2\right)^m]}{8 \ e^2} + \frac{f^2 \ m \ \left(a + b \ Log[c \ x^n]\right)^2 \ Log[1 + \frac{fx^2}{e}]}{4 \ e^2} + \frac{b \ f^2 \ m \ n \ \left(a + b \ Log[c \ x^n]\right) \ PolyLog[2, -\frac{fx^2}{e}]}{8 \ e^2} - \frac{b^2 \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n \ \left(a + b \ Log[c \ x^n]\right) \ PolyLog[2, -\frac{fx^2}{e}]}{8 \ e^2} - \frac{b^2 \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n \ a + b \ Log[c \ x^n]\right) \ PolyLog[2, -\frac{fx^2}{e}]}{8 \ e^2} - \frac{b^2 \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{e}]}{8 \ e^2} + \frac{b \ f^2 \ m \ n^2 \ PolyLog[3, -\frac{fx^2}{$$

### Problem 107: Result optimal but 2 more steps used.

$$\int \frac{\left(a + b \log[c x^n]\right)^2 \log\left[d \left(e + f x^2\right)^m\right]}{x^4} dx$$

Optimal (type 4, 571 leaves, 22 steps):

$$-\frac{52\,b^{2}\,f\,m\,n^{2}}{27\,e\,x} - \frac{4\,b^{2}\,f^{3/2}\,m\,n^{2}\,ArcTan\Big[\frac{\sqrt{f}\,x}{\sqrt{e}}\Big]}{27\,e^{3/2}} - \frac{16\,b\,f\,m\,n\,\left(a+b\,Log[c\,x^{n}]\right)}{9\,e\,x} - \frac{4\,b\,f^{3/2}\,m\,n\,ArcTan\Big[\frac{\sqrt{f}\,x}{\sqrt{e}}\Big]}{9\,e^{3/2}} - \frac{2\,f\,m\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[1-\frac{\sqrt{f}\,x}{\sqrt{-e}}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{f^{3/2}\,m\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[1+\frac{\sqrt{f}\,x}{\sqrt{-e}}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b\,n\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[d\,\left(e+f\,x^{2}\right)^{m}\Big]}{9\,x^{3}} - \frac{2\,b\,n\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[d\,\left(e+f\,x^{2}\right)^{m}\Big]}{3\,x^{3}} - \frac{2\,b\,f^{3/2}\,m\,n\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[d\,\left(e+f\,x^{2}\right)^{m}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b\,f^{3/2}\,m\,n\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[d\,\left(e+f\,x^{2}\right)^{m}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b\,f^{3/2}\,m\,n\,\left(a+b\,Log[c\,x^{n}]\right)^{2}\,Log\Big[d\,\left(e+f\,x^{2}\right)^{m}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b\,f^{3/2}\,m\,n^{2}\,PolyLog\Big[2,\,\frac{\sqrt{f}\,x}{\sqrt{-e}}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b^{2}\,f^{3/2}\,m\,n^{2}\,PolyLog\Big[3,\,\frac{\sqrt{f}\,x}{\sqrt{-e}}\Big]}{3\,\left(-e\right)^{3/2}} - \frac{2\,b^{2}\,f^{3/2}\,m\,n^{2}\,PolyLog\Big[3,\,\frac{\sqrt{f}\,x}{\sqrt{-e}}\Big]}{3$$

#### Result (type 4, 571 leaves, 24 steps):

$$\frac{52 \, b^2 \, f \, m \, n^2}{27 \, e \, x} - \frac{4 \, b^2 \, f^{3/2} \, m \, n^2 \, ArcTan \Big[ \frac{\sqrt{f} \, x}{\sqrt{e}} \Big]}{27 \, e^{3/2}} - \frac{16 \, b \, f \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)}{9 \, e \, x} - \frac{4 \, b \, f^{3/2} \, m \, n \, ArcTan \Big[ \frac{\sqrt{f} \, x}{\sqrt{e}} \Big]}{9 \, e^{3/2}} - \frac{2 \, f \, m \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \Big[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, (-e)^{3/2}} - \frac{f^{3/2} \, m \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \Big[1 + \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, (-e)^{3/2}} - \frac{2 \, b \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \Big[d \, \left(e + f \, x^2\right)^m \Big]}{9 \, x^3} - \frac{\left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \Big[d \, \left(e + f \, x^2\right)^m \Big]}{3 \, x^3} - \frac{2 \, b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right) \, Log \Big[d \, \left(e + f \, x^2\right)^m \Big]}{3 \, x^3} - \frac{2 \, b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \Big[d \, \left(e + f \, x^2\right)^m \Big]}{3 \, x^3} - \frac{2 \, b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \Big[d \, \left(e + f \, x^2\right)^m \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, PolyLog \Big[2, \, -\frac{i \, \sqrt{f} \, x}{\sqrt{e}} \Big]}{9 \, e^{3/2}} - \frac{2 \, b \, f^{3/2} \, m \, n^2 \, PolyLog \Big[2, \, \frac{i \, \sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}} - \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, PolyLog \Big[3, \, \frac{\sqrt{f} \, x}{\sqrt{-e}} \Big]}{3 \, \left(-e\right)^{3/2}}$$

# Problem 114: Result optimal but 3 more steps used.

$$\int \frac{\left(a + b \log[c x^n]\right)^3 \log\left[d \left(e + f x^2\right)^m\right]}{x^4} dx$$

Optimal (type 4, 1007 leaves, 36 steps):

$$\frac{160 \, b^3 \, f \, m \, n^3}{27 \, e \, x} - \frac{4 \, b^3 \, f^{3/2} \, m \, n^3 \, Arc Tan \left[\frac{\sqrt{f} \, x}{\sqrt{e}}\right]}{27 \, e \, x} - \frac{52 \, b^2 \, f \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{9 \, e \, x} - \frac{4 \, b^2 \, f^{3/2} \, m \, n^2 \, Arc Tan \left[\frac{\sqrt{f} \, x}{\sqrt{e}}\right]}{9 \, e^{3/2}} + \frac{b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, e \, x} + \frac{b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{b \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 + \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{3 \, (-e)^{3/2}}{3 \, (-e)^{3/2}} + \frac{3 \, (-e)^{3/2}}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[d \, \left(e + f \, x^2\right)^n\right]}{3 \, (a + b \, Log \left[c \, x^n\right])^3 \, Log \left[d \, \left(e + f \, x^2\right)^n\right]} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2, -\frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[$$

Result (type 4, 1007 leaves, 39 steps):

$$\frac{160 \, b^3 \, f \, m \, n^3}{27 \, e \, x} = \frac{4 \, b^3 \, f^{3/2} \, m \, n^3 \, ArcTan \left[\frac{\sqrt{f} \, x}{\sqrt{e}}\right]}{37 \, e \, x} = \frac{52 \, b^2 \, f \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right)}{9 \, e \, x} = \frac{4 \, b^3 \, f^{3/2} \, m \, n^2 \, ArcTan \left[\frac{\sqrt{f} \, x}{\sqrt{e}}\right]}{9 \, e^{3/2}} + \frac{9 \, e^{3/2} \, m \, n^2 \, ArcTan \left[\frac{\sqrt{f} \, x}{\sqrt{e}}\right]}{3 \, e \, x} + \frac{9 \, e^{3/2} \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{57 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{57 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^2 \, Log \left[1 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{57 \, m \, n \, \left(a + b \, Log \left[c \, x^n\right]\right)^3 \, Log \left[1 + \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{3 \, (-e)^{3/2}}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^2 \, \left(a + b \, Log \left[c \, x^n\right]\right) \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^2 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{3 \, (-e)^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[2 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[3 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[3 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[4 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[4 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[4 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[4 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9 \, e^{3/2}} + \frac{2 \, b^3 \, f^{3/2} \, m \, n^3 \, PolyLog \left[4 - \frac{\sqrt{f} \, x}{\sqrt{-e}}\right]}{9$$

# Test results for the 314 problems in "3.2.1 (f+g x) $^m$ (A+B log(e ((a+b x) over (c+d x)) $^n$ ) $^p$ .m"

### Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{a g + b g x} dx$$

Optimal (type 4, 84 leaves, 5 steps):

$$-\frac{\text{Log}\left[-\frac{\text{bc-ad}}{\text{d(a+bx)}}\right] \, \left(\text{A} + \text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)}{\text{bg}} + \frac{\text{BnPolyLog}\left[\text{2, 1} + \frac{\text{bc-ad}}{\text{d(a+bx)}}\right]}{\text{bg}}$$

Result (type 4, 126 leaves, 9 steps):

$$-\frac{B\,n\,Log\!\left[g\,\left(a+b\,x\right)\,\right]^{2}}{2\,b\,g}+\frac{\left(A+B\,Log\!\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[a\,g+b\,g\,x\right]}{b\,g}+\frac{B\,n\,Log\!\left[\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\right]\,Log\left[a\,g+b\,g\,x\right]}{b\,g}+\frac{B\,n\,PolyLog\!\left[2,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b\,g}$$

# Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{2}} dx$$

Optimal (type 3, 67 leaves, 2 steps):

$$-\frac{B n}{b g^2 (a + b x)} - \frac{\left(c + d x\right) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{\left(b c - a d\right) g^2 (a + b x)}$$

Result (type 3, 108 leaves, 4 steps):

$$-\frac{B\,n}{b\,g^2\,\left(a+b\,x\right)}-\frac{B\,d\,n\,Log\left[\,a+b\,x\,\right]}{b\,\left(b\,c-a\,d\right)\,g^2}-\frac{A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{b\,g^2\,\left(a+b\,x\right)}+\frac{B\,d\,n\,Log\left[\,c+d\,x\,\right]}{b\,\left(b\,c-a\,d\right)\,g^2}$$

### Problem 10: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^4 \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 396 leaves, 8 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g^4 \, n \, \left(a + b \, x\right)^4 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{10 \, b \, d} + \frac{g^4 \, \left(a + b \, x\right)^5 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{5 \, b} + \frac{B \, \left(b \, c - a \, d\right)^2 \, g^4 \, n \, \left(a + b \, x\right)^3 \, \left(4 \, A + B \, n + 4 \, B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{30 \, b \, d^2} - \frac{B \, \left(b \, c - a \, d\right)^3 \, g^4 \, n \, \left(a + b \, x\right)^2 \, \left(12 \, A + 7 \, B \, n + 12 \, B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{60 \, b \, d^3} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^4 \, n \, \left(a + b \, x\right) \, \left(12 \, A + 13 \, B \, n + 12 \, B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{30 \, b \, d^4} + \frac{B \, \left(b \, c - a \, d\right)^5 \, g^4 \, n \, \left(12 \, A + 25 \, B \, n + 12 \, B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right) \, Log\left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)}\right]}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}\right)}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}\right)}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}\right)}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}\right)}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}\right)}{5 \, B \, d^5} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog\left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + a \, d\right)^5 \, B^2}\right)}$$

Result (type 4, 602 leaves, 27 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,n\,x}{5\,d^{4}} + \frac{13\,B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,n^{2}\,x}{30\,d^{4}} - \frac{7\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,n^{2}\,\left(a+b\,x\right)^{2}}{60\,b\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,n^{2}\,\left(a+b\,x\right)^{3}}{30\,b\,d^{2}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{5\,b\,d^{4}} - \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,n\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{5\,b\,d^{3}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,n\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{15\,b\,d^{2}} - \frac{B\,\left(b\,c-a\,d\right)\,g^{4}\,n\,\left(a+b\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{10\,b\,d} + \frac{10\,b\,d}{10\,b\,d} + \frac{g^{4}\,\left(a+b\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{5\,b\,d^{5}} - \frac{5\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,n^{2}\,Log\left[c+d\,x\right]}{6\,b\,d^{5}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,n^{2}\,Log\left[c+d\,x\right]}{5\,b\,d^{5}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,n^{2}\,Log\left[c+d\,x\right]^{2}}{5\,b\,d^{5}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,B^{2$$

### Problem 11: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^{3} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 335 leaves, 7 steps):

$$\frac{B \left( b \, c - a \, d \right) \, g^{3} \, n \, \left( a + b \, x \right)^{3} \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)}{6 \, b \, d} + \frac{g^{3} \, \left( a + b \, x \right)^{4} \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)^{2}}{4 \, b} + \frac{g^{3} \, \left( a + b \, x \right)^{2} \, \left( 3 \, A + B \, n + 3 \, B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)}{4 \, b} + \frac{g^{3} \, \left( a + b \, x \right)^{2} \, \left( 3 \, A + B \, n + 3 \, B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)}{12 \, b \, d^{2}} + \frac{g^{3} \, \left( a + b \, x \right)^{2} \, \left( a + b \, x \right) \, \left( a + b \, x \right)^{n} \right] \right)}{12 \, b \, d^{3}} + \frac{g^{3} \, \left( a + b \, x \right)^{2} \, \left( a + b \, x \right)^{2} \, \left( a + b \, x \right)^{2} \, \left( a + b \, x \right) \, \left( a + b \, x$$

#### Result (type 4, 512 leaves, 23 steps):

$$-\frac{A\ B\ \left(b\ c-a\ d\right)^{3}\ g^{3}\ n\ x}{2\ d^{3}} - \frac{5\ B^{2}\ \left(b\ c-a\ d\right)^{3}\ g^{3}\ n^{2}\ x}{12\ d^{3}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{2}\ g^{3}\ n^{2}\ \left(a+b\ x\right)^{2}}{12\ b\ d^{2}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ g^{3}\ n\ \left(a+b\ x\right)\ Log\left[e\ \left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]}{2\ b\ d^{3}} + \frac{B\ \left(b\ c-a\ d\right)^{2}\ g^{3}\ n\ \left(a+b\ x\right)^{2}}{12\ b\ d^{2}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ g^{3}\ n\ \left(a+b\ x\right)\ Log\left[e\ \left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]}{2\ b\ d^{3}} + \frac{B\ \left(b\ c-a\ d\right)^{2}\ g^{3}\ n\ \left(a+b\ x\right)^{3}\ \left(A+B\ Log\left[e\ \left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]\right)}{6\ b\ d} + \frac{B\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]}{12\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{4\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ PolyLog\left[2,\ \frac{b\ (c+d\ x)}{b\ c-a\ d}\right)^{4}\ g^{3}\ n^{2}\ Log\left[2,\ \frac{b\ (c+d\ x)}{b\ c-a\ d}\right]}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{4\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ PolyLog\left[2,\ \frac{b\ (c+d\ x)}{b\ c-a\ d}\right]}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ g^{3}\ n^{2}\ Log\left[c+d\ x\right]^{2}}{2\ b\ d^{4}} + \frac{B^{2}\ \left(b\ c-a\ d$$

# Problem 12: Result valid but suboptimal antiderivative.

$$\int \left(a g + b g x\right)^{2} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 274 leaves, 6 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g^{2} \ n \left(a + b \ x\right)^{2} \left(A + B \ Log\left[e \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{3 \ b \ d} + \frac{g^{2} \left(a + b \ x\right)^{3} \left(A + B \ Log\left[e \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)^{2}}{3 \ b} + \frac{B \left(b \ c - a \ d\right)^{2} \ g^{2} \ n \left(a + b \ x\right) \left(2 \ A + B \ n + 2 \ B \ Log\left[e \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{3 \ b \ d^{3}} + \frac{3 \ b \ d^{3}}{3 \ b \ d^{3}}$$

Result (type 4, 420 leaves, 19 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,x}{3\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,x}{3\,d^{2}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b\,d^{2}} - \frac{B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,d} + \frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{3\,b\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]}{b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,Log\left[c+d\,x\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,n^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right$$

# Problem 13: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 196 leaves, 5 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{b\,d}+\frac{g\,\left(a+b\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{2\,b}-\\ \frac{B\left(b\,c-a\,d\right)^{\,2}\,g\,n\,\left(A+B\,n+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\,\right]}{b\,d^{\,2}}-\frac{B^{\,2}\,\left(b\,c-a\,d\right)^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,2\,\text{, }\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{b\,d^{\,2}}$$

Result (type 4, 309 leaves, 15 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,n\,x}{d} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{b\,d} + \\ \frac{g\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{2\,b} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\left[c+d\,x\right]}{b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{b\,d^2} + \\ \frac{B\,\left(b\,c-a\,d\right)^2\,g\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{b\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\left[c+d\,x\right]^2}{2\,b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b\,d^2}$$

### Problem 14: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log \left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]\right)^{2}}{a g + b g x} dx$$

#### Optimal (type 4, 138 leaves, 4 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right]\right)^2\mathsf{Log}\!\left[\mathsf{1}-\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\right]}{\mathsf{b}\,\mathsf{g}} + \frac{2\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right]\right)\mathsf{PolyLog}\!\left[\mathsf{2},\,\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\right]}{\mathsf{b}\,\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{n}^2\,\mathsf{PolyLog}\!\left[\mathsf{3},\,\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\right]}{\mathsf{b}\,\mathsf{g}}$$

#### Result (type 4, 789 leaves, 45 steps):

$$\frac{A \, B \, n \, Log \left[g \, \left(a + b \, x \right)^{\, 2} \right]^{2}}{b \, g} + \frac{B^{\, 2} \, n^{2} \, Log \left[g \, \left(a + b \, x \right)^{\, 3} \right]}{3 \, b \, g} - \frac{B^{\, 2} \, n^{2} \, Log \left[g \, \left(a + b \, x \right)^{\, 2} \right] \, Log \left[-c - d \, x \right]}{b \, g} + \frac{B^{\, 2} \, n \, Log \left[\left(a + b \, x \right)^{\, n}\right] \, Log \left[\left(a + b \, x \right)^{\, n}\right] \, Log \left[\left(a + b \, x \right)^{\, n}\right] \, Log \left[-c - d \, x \right]}{b \, g} + \frac{B^{\, 2} \, Log \left[g \, \left(a + b \, x \right)^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right] \, Log \left[\left(a + b \, x \right)^{\, n}\right]^{\, 2} \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\frac{b \, \left(c + d \, x \right)^{\, - n}}{b \, c - a \, d}\right]}{b \, g} + \frac{B^{\, 2} \, Log \left[\left(a + b \, x \right)^{\, n}\right] \, Log \left[\left(a + b \, x$$

Problem 15: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{2}} dx$$

Optimal (type 3, 136 leaves, 3 steps):

$$-\frac{2\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}\,-\frac{2\,B\,n\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}\,-\frac{\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}$$

Result (type 4, 512 leaves, 24 steps):

$$-\frac{2 \, B^2 \, n^2}{b \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, B^2 \, d \, n^2 \, Log \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{B^2 \, d \, n^2 \, Log \left[a + b \, x\right]^2}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, B \, d \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, n^2 \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d$$

Problem 16: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{3}} dx$$

Optimal (type 3, 288 leaves, 7 steps):

$$\frac{2\,B^{2}\,d\,n^{2}\,\left(\,c\,+\,d\,x\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)}\,-\,\frac{\,b\,B^{2}\,n^{2}\,\left(\,c\,+\,d\,x\,\right)^{\,2}}{\,4\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}\,+\,\frac{\,2\,B\,d\,n\,\left(\,c\,+\,d\,\,x\,\right)\,\left(\,A\,+\,B\,\,Log\left[\,e\,\left(\,\frac{a\,+\,b\,\,x}{c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)}{\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)}\,-\,\frac{\,b\,\,B\,n\,\left(\,c\,+\,d\,\,x\,\right)\,\left(\,A\,+\,B\,\,Log\left[\,e\,\left(\,\frac{a\,+\,b\,\,x}{c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)^{\,2}}{\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}\,+\,\frac{\,d\,\left(\,c\,+\,d\,\,x\,\right)\,\left(\,A\,+\,B\,\,Log\left[\,e\,\left(\,\frac{a\,+\,b\,\,x}{c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)^{\,2}}{\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}\,-\,\frac{\,b\,\left(\,c\,+\,d\,\,x\,\right)^{\,2}\,\left(\,A\,+\,B\,\,Log\left[\,e\,\left(\,\frac{a\,+\,b\,\,x}{c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)^{\,2}}{\,2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}$$

Result (type 4, 626 leaves, 28 steps):

$$-\frac{B^2 \, n^2}{4 \, b \, g^3 \, \left(a + b \, x\right)^2} + \frac{3 \, B^2 \, d \, n^2}{2 \, b \, \left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)} + \frac{3 \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(b \, c - a \, d\right)^2 \, g^3}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(b \, c - a \, d\right)^2 \, g^3}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A + B \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A + B \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A + B \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, Log \left[a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[b \, \left(a + b \, x\right]^n \left[b \, \left(a + b \, x\right]^n \right]\right)}{b \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{A \, B^2 \, d^2 \, n^2 \, Log \left[a + b \, x\right]^n \left[$$

# Problem 17: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{4}} dx$$

Optimal (type 3, 448 leaves, 9 steps):

$$-\frac{2 \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} + \frac{b \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{2 \, b^2 \, B^2 \, n^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} - \frac{2 \, B \, d^2 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} + \frac{b \, B \, d \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{2 \, b^2 \, B \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{9 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} - \frac{d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} - \frac{b^2 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3}$$

Result (type 4, 736 leaves, 32 steps):

$$-\frac{2\,B^{2}\,n^{2}}{27\,b\,g^{4}\,\left(a+b\,x\right)^{3}} + \frac{5\,B^{2}\,d\,n^{2}}{18\,b\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{11\,B^{2}\,d^{2}\,n^{2}}{9\,b\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,\left(a+b\,x\right)} - \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]}{9\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{2\,B\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{9\,b\,g^{4}\,\left(a+b\,x\right)^{3}} + \frac{B\,d\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{2\,B\,d^{2}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,\left(a+b\,x\right)} - \frac{2\,B\,d^{3}\,n\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{2\,B\,d^{3}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{2\,B^{2}\,d^{3}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B\,d^{3}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[c+d\,x\right]}{9\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{2\,B^{2}\,d^{3}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B\,d^{3}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]\,Log\left[a+b\,x\right]}{9\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{2\,B^{2}\,d^{3}\,n^{2}\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B\,d^{3}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]}{9\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B\,d^{3}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B^{2}\,d^{3}\,n^$$

# Problem 18: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{5}} dx$$

Optimal (type 3, 615 leaves, 11 steps):

$$\frac{2 \, B^2 \, d^3 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^2} + \frac{2 \, b^2 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^3}{9 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, B^2 \, n^2 \, \left(c + d \, x\right)^4}{4 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} + \frac{2 \, B \, d^3 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{3 \, b \, B \, d^2 \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} + \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{8 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^4} + \frac{d^3 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(a + b \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^3}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^3}{\left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^3}{\left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^3}{\left(a + b \, x\right)^3} - \frac{b^3 \, d^3 \,$$

Result (type 4, 826 leaves, 36 steps):

$$-\frac{B^2 \, n^2}{32 \, b \, g^5 \, \left(a + b \, x\right)^4} + \frac{7 \, B^2 \, d \, n^2}{72 \, b \, \left(b \, c - a \, d\right) \, g^5 \, \left(a + b \, x\right)^3} - \frac{13 \, B^2 \, d^2 \, n^2}{48 \, b \, \left(b \, c - a \, d\right)^2 \, g^5 \, \left(a + b \, x\right)^2} + \frac{25 \, B^2 \, d^3 \, n^2}{24 \, b \, \left(b \, c - a \, d\right)^3 \, g^5 \, \left(a + b \, x\right)} + \frac{25 \, B^2 \, d^4 \, n^2 \, Log \left[a + b \, x\right]}{24 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{B^2 \, d^4 \, n^2 \, Log \left[a + b \, x\right]^2}{4 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{B^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{8 \, b \, g^5 \, \left(a + b \, x\right)^4} + \frac{B \, d \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{6 \, b \, \left(b \, c - a \, d\right) \, g^5 \, \left(a + b \, x\right)^3} - \frac{B \, d^3 \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{8 \, b \, g^5 \, \left(a + b \, x\right)^4} + \frac{B \, d^4 \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{6 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{B \, d^4 \, n \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{B^2 \, d^4 \, n^2 \, Log \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, Log \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, Log \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, Log \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^2 \, d^4 \, n^2 \, PolyLog \left[c + d \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^4 \, g^5}$$

### Problem 19: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]},\,x\right]$$

Result (type 8, 106 leaves, 2 steps):

$$a^{2} g^{2} CannotIntegrate \left[\frac{1}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + \\ 2 a b g^{2} CannotIntegrate \left[\frac{x}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x$$

# Problem 20: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]} dx$$

Optimal (type 8, 35 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

a g CannotIntegrate 
$$\left[\frac{1}{A+B \log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^n\right]}, x\right] + b g CannotIntegrate \left[\frac{x}{A+B \log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^n\right]}, x\right]$$

# Problem 21: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a g + b g x\right) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)}, x\right]$$

### Problem 22: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$\frac{e^{\frac{A}{B\,n}}\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{\frac{1}{n}}\,\left(c+d\,x\right)\,\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \, \frac{1}{ \left( \text{a g} + \text{b g x} \right)^2 \, \left( \text{A} + \text{B Log} \Big[ \, \text{e} \, \left( \frac{\text{a+b x}}{\text{c+d x}} \right)^n \, \Big] \, \right) } \, \text{, } \, \text{x} \, \Big]$$

### Problem 23: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 4, 197 leaves, 7 steps):

$$\frac{b\stackrel{2A}{\in^{\frac{2A}{B\,n}}}\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{2/n}\,\left(c+d\,x\right)^{2}\,\text{ExpIntegralEi}\left[-\frac{2\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,n\,\left(a+b\,x\right)^{2}}-\frac{d\,e^{\frac{A}{B\,n}}\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{\frac{1}{n}}\,\left(c+d\,x\right)\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,n\,\left(a+b\,x\right)}$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^3\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

### Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2},\,x\right]$$

Result (type 8, 106 leaves, 2 steps):

$$a^2 \, g^2 \, \text{CannotIntegrate} \, \Big[ \, \frac{1}{ \left( A + B \, \text{Log} \, \Big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \Big] \, \right)^2} \, \text{, } \, x \, \Big] \, + \\$$

2 a b g² CannotIntegrate 
$$\left[\frac{x}{\left(A+B \log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^n\right]\right)^2}$$
,  $x\right] + b^2 g^2$  CannotIntegrate  $\left[\frac{x^2}{\left(A+B \log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^n\right]\right)^2}$ ,  $x\right]$ 

# Problem 25: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 35 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{\left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\Big( \text{A} + \text{B Log} \Big[ \text{e} \left( \frac{\text{a+b x}}{\text{c+d x}} \right)^{\text{n}} \Big] \Big)^2} \text{, } \text{x} \Big] + \text{b g CannotIntegrate} \Big[ \frac{\text{x}}{\Big( \text{A} + \text{B Log} \Big[ \text{e} \left( \frac{\text{a+b x}}{\text{c+d x}} \right)^{\text{n}} \Big] \Big)^2} \text{, } \text{x} \Big]$$

### Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}\,d\!\!\mid x$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag+bgx\right)\left(A+BLog\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)^{2}},x\right]$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right) \left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}, x\right]$$

### Problem 27: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$-\frac{e^{\frac{A}{B\,n}}\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right)^{\frac{1}{n}}\,\left(c+d\,x\right)\,\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]}{B\,n}\right]}{B^2\,\left(b\,c-a\,d\right)\,g^2\,n^2\,\left(a+b\,x\right)}-\frac{c+d\,x}{B\,\left(b\,c-a\,d\right)\,g^2\,n\,\left(a+b\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^2\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}$$
,  $x\right]$ 

### Problem 28: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 4, 314 leaves, 9 steps):

$$-\frac{2 \ b \ e^{\frac{2A}{8n}} \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \ \left(c+dx\right)^2 \ ExpIntegralEi\left[-\frac{2 \left(A+B \ Log\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{B \ n}\right]}{B^2 \ \left(b \ c-a \ d\right)^2 \ g^3 \ n^2 \ \left(a+b \ x\right)^2} + \frac{d \ e^{\frac{A}{8n}} \left(e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right)^{\frac{1}{n}} \left(c+d \ x\right) \ ExpIntegralEi\left[-\frac{A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]}{B \ n}\right]}{B^2 \ \left(b \ c-a \ d\right)^2 \ g^3 \ n^2 \ \left(a+b \ x\right)} + \frac{d \ e^{\frac{A}{8n}} \left(e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right)^{\frac{1}{n}} \left(c+d \ x\right) \ ExpIntegralEi\left[-\frac{A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]}{B \ n}\right]}{B \ \left(b \ c-a \ d\right)^2 \ g^3 \ n^2 \ \left(a+b \ x\right)} + \frac{d \ e^{\frac{A}{8n}} \left(e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right)^{\frac{1}{n}} \left(c+d \ x\right) \ ExpIntegralEi\left[-\frac{A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]}{B \ n}\right]}{B^2 \ \left(b \ c-a \ d\right)^2 \ g^3 \ n^2 \ \left(a+b \ x\right)} + \frac{d \ e^{\frac{A}{8n}} \left(e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right)^{\frac{1}{n}} \left(c+d \ x\right) \ ExpIntegralEi\left[-\frac{A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]}{B \ n}\right]}{B^2 \ \left(b \ c-a \ d\right)^2 \ g^3 \ n^2 \ \left(a+b \ x\right)} + \frac{d \ e^{\frac{A}{8n}} \left(e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right)^{\frac{1}{n}} \left(c+d \ x\right) \ ExpIntegralEi\left[-\frac{A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]}{B \ n}\right]}$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag + bg x\right)^{3}\left(A + BLog\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)^{2}}, x\right]$$

### Problem 33: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[ e \left( \frac{a+b x}{c+d x} \right)^{n} \right]}{c g + d g x} dl x$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,\frac{b\,c-a\,d}{b\,(c+d\,x)}\,\right]}{d\,g}\,-\,\frac{B\,n\,PolyLog\left[\,2\,\text{, }\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\,\right]}{d\,g}$$

Result (type 4, 128 leaves, 9 steps):

$$\frac{B\,n\,Log\!\left[g\,\left(c+d\,x\right)\,\right]^2}{2\,d\,g} - \frac{B\,n\,Log\!\left[-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]\,Log\,\left[c\,g+d\,g\,x\right]}{d\,g} + \frac{\left(A+B\,Log\!\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\,\left[c\,g+d\,g\,x\right]}{d\,g} - \frac{B\,n\,PolyLog\!\left[2\,,\,\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\right]}{d\,g}$$

# Problem 34: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e^{\left(\frac{a+b x}{c+d x}\right)^{n}} \right]}{\left( c g + d g x \right)^{2}} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{A \left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(c+d\,x\right)} - \frac{B\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(c+d\,x\right)} + \frac{B\,\left(a+b\,x\right)\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{\left(b\,c-a\,d\right)\,g^2\,\left(c+d\,x\right)}$$

Result (type 3, 107 leaves, 4 steps):

$$\frac{B\,n}{d\,g^2\,\left(c+d\,x\right)} + \frac{b\,B\,n\,Log\,\left[\,a+b\,x\,\right]}{d\,\left(\,b\,c-a\,d\,\right)\,g^2} - \frac{A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{d\,g^2\,\left(\,c+d\,x\,\right)} - \frac{b\,B\,n\,Log\,\left[\,c+d\,x\,\right]}{d\,\left(\,b\,c-a\,d\,\right)\,g^2}$$

# Problem 38: Result valid but suboptimal antiderivative.

$$\int (c g + d g x)^4 \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 544 leaves, 19 steps):

$$\frac{13 \, B^{2} \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, n^{2} \, x}{30 \, b^{4}} + \frac{7 \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{4} \, n^{2} \, \left(c + d \, x\right)^{2}}{60 \, b^{3} \, d} + \frac{B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{4} \, n^{2} \, \left(c + d \, x\right)^{3}}{30 \, b^{2} \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, n \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{5 \, b^{3} \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^{2} \, g^{4} \, n \, \left(c + d \, x\right)^{3} \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{15 \, b^{2} \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^{2} \, g^{4} \, n \, \left(c + d \, x\right)^{3} \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{10 \, b \, d} + \frac{g^{4} \, \left(c + d \, x\right)^{5} \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)^{2}}{5 \, d} + \frac{13 \, B^{2} \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, n^{2} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{30 \, b^{5} \, d} + \frac{2 \, B \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, n \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right) \, Log\left[1 - \frac{b \, \left(c + d \, x\right)}{d \, \left(a + b \, x\right)}\right]}{5 \, b^{5} \, d} - \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, n^{2} \, PolyLog\left[2, \, \frac{b \, \left(c + d \, x\right)}{d \, \left(a + b \, x\right)}\right]}{5 \, b^{5} \, d}$$

#### Result (type 4, 634 leaves, 27 steps):

$$-\frac{2 \, A \, B \, \left(b \, c - a \, d\right)^4 \, g^4 \, n \, x}{5 \, b^4} + \frac{13 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^4 \, n^2 \, x}{30 \, b^4} + \frac{7 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^4 \, n^2 \, \left(c + d \, x\right)^2}{600 \, b^3 \, d} + \frac{B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, Log \left[a + b \, x\right]}{30 \, b^5 \, d} + \frac{B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, Log \left[a + b \, x\right]^2}{5 \, b^5 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{5 \, b^5} - \frac{13 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^4 \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^3 \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^2 \, g^4 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{15 \, b^2 \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^5 \, g^4 \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^5 \, d} + \frac{g^4 \, \left(c + d \, x\right)^5 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^5 \, d} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{5 \, b^5 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{5 \, b^5 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{5 \, b^5 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^5 \, g^4 \, n^2 \, PolyLog \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{5 \, b^5 \, d}$$

# Problem 39: Result valid but suboptimal antiderivative.

$$\int (c g + d g x)^{3} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 454 leaves, 15 steps):

$$\frac{5 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, n^2 \, x}{12 \, b^3} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, n^2 \, \left(c + d \, x\right)^2}{12 \, b^2 \, d} - \frac{B \, \left(b \, c - a \, d\right)^3 \, g^3 \, n \, \left(a + b \, x\right) \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^4} - \frac{B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, b^2 \, d} - \frac{B \, \left(b \, c - a \, d\right) \, g^3 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{6 \, b \, d} + \frac{6 \, b \, d}{6 \, b \, d} + \frac{g^3 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{4 \, d} + \frac{5 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, Log \left[\frac{a + b \, x}{c + d \, x}\right]}{12 \, b^4 \, d} + \frac{11 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, Log \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, n^2 \, PolyLog \left[c + d \, x\right$$

#### Result (type 4, 544 leaves, 23 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)^3\,g^3\,n\,x}{2\,b^3} + \frac{5\,B^2\,\left(b\,c-a\,d\right)^3\,g^3\,n^2\,x}{12\,b^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^3\,n^2\,\left(c+d\,x\right)^2}{12\,b^2\,d} + \frac{5\,B^2\,\left(b\,c-a\,d\right)^4\,g^3\,n^2\,Log\,[a+b\,x]}{12\,b^4\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^3\,n^2\,Log\,[a+b\,x]^2}{12\,b^4\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^3\,n\,\left(a+b\,x\right)\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{2\,b^4\,d} - \frac{B\,\left(b\,c-a\,d\right)^2\,g^3\,n\,\left(c+d\,x\right)^2\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} - \frac{B\,\left(b\,c-a\,d\right)^4\,g^3\,n\,Log\,[a+b\,x]\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,\left(c+d\,x\right)^4\,\left(A+B\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,d} + \frac{g^3\,n^2\,Log\,\left[c+d\,x\right]}{2\,b^4\,d} + \frac{g^3\,n^2\,Log\,\left[c+d\,x\right]}{2\,b^4\,$$

### Problem 40: Result valid but suboptimal antiderivative.

$$\int \left(c g + d g x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 361 leaves, 11 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} g^{2} \ n^{2} \ x}{3 \ b^{2}} - \frac{2 \ B \left(b \ c-a \ d\right)^{2} g^{2} \ n \ \left(a+b \ x\right) \ \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b^{3}} - \frac{B \left(b \ c-a \ d\right) \ g^{2} \ n \ \left(c+d \ x\right)^{2} \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b \ d} + \frac{g^{2} \left(c+d \ x\right)^{3} \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{3 \ b^{3} \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ Log\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ Log\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3$$

#### Result (type 4, 454 leaves, 19 steps):

$$-\frac{2 \, A \, B \, \left(b \, c - a \, d\right)^2 \, g^2 \, n \, x}{3 \, b^2} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, g^2 \, n^2 \, x}{3 \, b^2} + \frac{B^2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, n^2 \, Log \left[a + b \, x\right]}{3 \, b^3 \, d} + \frac{B^2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, n^2 \, Log \left[a + b \, x\right]^2}{3 \, b^3 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^2 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, b^3} - \frac{B \, \left(b \, c - a \, d\right) \, g^2 \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b \, d} - \frac{2 \, B \, \left(b \, c - a \, d\right)^3 \, g^2 \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^3 \, d} + \frac{g^2 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, b^3 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, n^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b^3 \, d} - \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, n^2 \, PolyLog \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{3 \, b^3 \, d}$$

### Problem 41: Result valid but suboptimal antiderivative.

$$\int \left(c g + d g x\right) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{b^{2}}+\frac{g\left(\,c+d\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{2\,d}+\\ -\frac{B^{2}\,\left(b\,c-a\,d\right)^{\,2}\,g\,n^{2}\,Log\left[\,c+d\,x\,\right]}{b^{2}\,d}+\frac{B\,\left(b\,c-a\,d\right)^{\,2}\,g\,n\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,1-\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\,\right]}{b^{2}\,d}-\frac{B^{2}\,\left(b\,c-a\,d\right)^{\,2}\,g\,n^{2}\,PolyLog\left[\,2\,,\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\,\right]}{b^{2}\,d}$$

Result (type 4, 307 leaves, 15 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,n\,x}{b} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\,[\,a+b\,x\,]^2}{2\,b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]}{b^2} - \frac{B\,\left(b\,c-a\,d\right)^2\,g\,n\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)}{b^2\,d} + \frac{g\,\left(c+d\,x\right)^2\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2}{2\,d} + \frac{g\,\left(c+d\,x\right)^2\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2}{2\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\,[\,c+d\,x\,]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,PolyLog\,\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{b^2\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,PolyLog\,\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{$$

# Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{c g + d g x} dx$$

### Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\,\big)^{\,2}\,\mathsf{Log}\big[\,\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\mathsf{g}}\,-\frac{2\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\right)\,\mathsf{PolyLog}\big[\,\mathsf{2}\,,\,\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\mathsf{g}}\,+\frac{2\,\mathsf{B}^{\,2}\,\mathsf{n}^{\,2}\,\mathsf{PolyLog}\big[\,\mathsf{3}\,,\,\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\mathsf{g}}$$

### Result (type 4, 782 leaves, 45 steps):

$$\frac{B^{2} \log \left[ \left( a + b \, x \right)^{n} \right]^{2} \log \left[ \frac{b \, (c + d \, x)}{b \, c - a \, d} \right]}{d \, g} = \frac{B^{2} \log \left[ \left( a + b \, x \right)^{n} \right]^{2} \log \left[ g \, \left( c + d \, x \right) \right]^{2}}{d \, g} + \frac{A \, B \, n \, \log \left[ g \, \left( c + d \, x \right) \right]^{2}}{d \, g} = \frac{B^{2} \, n^{2} \, \log \left[ a + b \, x \right] \, \log \left[ g \, \left( c + d \, x \right) \right]^{2}}{d \, g} + \frac{B^{2} \, n^{2} \, \log \left[ g \, \left( c + d \, x \right) \right]^{3}}{d \, g} = \frac{2 \, B^{2} \, n \, \log \left[ a + b \, x \right] \, \log \left[ \left( c + d \, x \right) \right]^{2}}{d \, g} + \frac{B^{2} \, n^{2} \, \log \left[ g \, \left( c + d \, x \right) \right]^{3}}{3 \, d \, g} = \frac{2 \, B^{2} \, n \, \log \left[ a + b \, x \right] \, \log \left[ \left( c + d \, x \right) \right] \, \log \left[ \left( c + d \, x \right) \right]^{n}}{d \, g} = \frac{B^{2} \, Log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, Log \left[ \left( c + d \, x \right) \right]^{n}}{d \, g} + \frac{B^{2} \, Log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, Log \left[ \left( c + d \, x \right) \right]^{n}}{d \, g} + \frac{2 \, B^{2} \, n \, Log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, Log \left[ \left( c + d \, x \right) \right]^{n}}{d \, g} + \frac{2 \, B^{2} \, n \, Log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \left( Log \left[ \left( a + b \, x \right) \right] - Log \left[ e \, \left( \frac{a - b \, x}{c + d \, x} \right) \right] \, Log \left[ c \, g + d \, g \, x \right]}{d \, g} + \frac{2 \, B^{2} \, n \, Log \left[ \left( \frac{a + b \, x}{c + d \, x} \right) \right] \, Log \left[ \left( a + b \, x \right)^{n} \right] - Log \left[ \left( \frac{a - b \, x}{c + d \, x} \right) \, Log \left[ \left( c + d \, x \right)^{-n} \right]$$

### Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(c g + d g x\right)^{2}} dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$-\frac{2\,A\,B\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)}\,+\,\frac{2\,B^{2}\,n^{2}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)}\,-\,\frac{2\,B^{2}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)}\,+\,\frac{\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)}$$

Result (type 4, 514 leaves, 24 steps):

$$-\frac{2 \, B^2 \, n^2}{d \, g^2 \, \left(c + d \, x\right)} - \frac{2 \, b \, B^2 \, n^2 \, Log \left[a + b \, x\right]}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{b \, B^2 \, n^2 \, Log \left[a + b \, x\right]^2}{d \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, g^2 \, \left(c + d \, x\right)} + \frac{2 \, b \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{\left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{$$

# Problem 44: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(c g + d g x\right)^{3}} dx$$

Optimal (type 3, 317 leaves, 8 steps):

$$-\frac{B^2\,d\,n^2\,\left(a+b\,x\right)^2}{4\,\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)^2} - \frac{2\,A\,b\,B\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} + \frac{2\,b\,B^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} - \frac{2\,b\,B^2\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} + \frac{B\,d\,n\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{2\,\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)^2} + \frac{B\,d\,n\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{2\,\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2}{\left(a+b\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\left(a+b\,x\right)^2$$

Result (type 4, 626 leaves, 28 steps):

$$-\frac{B^2 \, n^2}{4 \, d \, g^3 \, \left(c + d \, x\right)^2} - \frac{3 \, b \, B^2 \, n^2}{2 \, d \, \left(b \, c - a \, d\right) \, g^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, B^2 \, n^2 \, Log \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B^2 \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, d \, \left(b \, c - a \, d\right)^2 \, g^3} + \frac{B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, g^3 \, \left(c + d \, x\right)^2} + \frac{b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right) \, g^3 \, \left(c + d \, x\right)} + \frac{b^2 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B^2 \, n^2 \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \, B \, n \, \left(a \, b \, b \, n \, \left(a \, b \, c - a \, d\right)^2 \, g^3}{d \, \left(b \, c - a \, d\right)^2 \, g^3} - \frac{b^2 \,$$

### Problem 45: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]\right)^{2}}{\left(c g + d g x\right)^{4}} dx$$

Optimal (type 3, 429 leaves, 6 steps):

$$\frac{2\,B^{2}\,d^{2}\,n^{2}\,\left(a+b\,x\right)^{3}}{27\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)^{3}} - \frac{b\,B^{2}\,d\,n^{2}\,\left(a+b\,x\right)^{2}}{2\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)^{2}} + \frac{2\,b^{2}\,B^{2}\,n^{2}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)} - \frac{2\,B\,d^{2}\,n\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)^{2}} + \frac{b\,B\,d\,n\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)^{2}} - \frac{2\,b^{2}\,B\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(c+d\,x\right)} - \frac{\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,d\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{b^{3}\,B^{2}\,n^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{3\,d\,\left(b\,c-a\,d\right)^{3}\,g^{4}}$$

Result (type 4, 736 leaves, 32 steps):

$$\frac{2 \, B^2 \, n^2}{27 \, d \, g^4 \, \left(c + d \, x\right)^3} - \frac{5 \, b \, B^2 \, n^2}{18 \, d \, \left(b \, c - a \, d\right) \, g^4 \, \left(c + d \, x\right)^2} - \frac{11 \, b^2 \, B^2 \, n^2}{9 \, d \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(c + d \, x\right)} - \frac{11 \, b^3 \, B^2 \, n^2 \, Log \left[a + b \, x\right]}{9 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, B \, n \, \left(a + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d \, \left(b \, c - a \, d\right) \, g^4 \, \left(c + d \, x\right)^2} + \frac{b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d \, \left(b \, c - a \, d\right) \, g^4 \, \left(c + d \, x\right)^2} + \frac{2 \, b^2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, b^3 \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{3 \, d \, \left(b \, c - a \, d\right)^3 \, g^4}$$

#### Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(c g + d g x\right)^{5}} dx$$

Optimal (type 3, 536 leaves, 5 steps):

$$-\frac{B^2\,d^3\,n^2\,\left(a+b\,x\right)^4}{32\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^4} + \frac{2\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)^3}{9\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^3} - \frac{3\,b^2\,B^2\,d\,n^2\,\left(a+b\,x\right)^2}{4\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^2} + \frac{2\,b^3\,B^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)} + \frac{2\,b^3\,B^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)} + \frac{B\,d^3\,n\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^4} - \frac{2\,b\,B\,d^2\,n\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{3\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^3} + \frac{3\,b^2\,B\,d\,n\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^2} - \frac{2\,b\,B\,d^2\,n\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{3\,\left(b\,c-a\,d\right)^4\,g^5\,\left(c+d\,x\right)^3} + \frac{b^4\,B\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{4\,d\,\left(b\,c-a\,d\right)^4\,g^5} - \frac{b^4\,B^2\,n^2\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^2}{4\,d\,\left(b\,c-a\,d\right)^4\,g^5}$$

Result (type 4, 826 leaves, 36 steps):

$$-\frac{B^2 \, n^2}{32 \, d \, g^5 \, \left(c + d \, x\right)^4} - \frac{7 \, b \, B^2 \, n^2}{72 \, d \, \left(b \, c - a \, d\right) \, g^5 \, \left(c + d \, x\right)^3} - \frac{13 \, b^2 \, B^2 \, n^2}{48 \, d \, \left(b \, c - a \, d\right)^2 \, g^5 \, \left(c + d \, x\right)^2} - \frac{25 \, b^3 \, B^2 \, n^2}{24 \, d \, \left(b \, c - a \, d\right)^3 \, g^5 \, \left(c + d \, x\right)} - \frac{25 \, b^4 \, B^2 \, n^2 \, Log \left[a + b \, x\right]}{24 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{8 \, d \, g^5 \, \left(c + d \, x\right)^4} + \frac{b \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{B^3 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{8 \, d \, g^5 \, \left(c + d \, x\right)^4} + \frac{b^4 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \left(b \, c - a \, d\right)^3 \, g^5 \, \left(c + d \, x\right)} + \frac{b^4 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B \, n \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B \, n \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} - \frac{b^4 \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B^2 \, n^2 \, PolyLog \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B^2 \, n^2 \, PolyLog \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B^2 \, n^2 \, PolyLog \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B^2 \, n^2 \, PolyLog \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^4 \, g^5} + \frac{b^4 \, B^2 \, n^2 \, PolyLog \left[a + b \, x\right]}{2 \, d \, \left(b \, c -$$

#### Problem 47: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c g + d g x\right)^{2}}{A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(c g + d g x\right)^{2}}{A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]}, x\right]$$

Result (type 8, 106 leaves, 2 steps):

$$c^{2} g^{2} CannotIntegrate \left[\frac{1}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + \\ 2 c d g^{2} CannotIntegrate \left[\frac{x}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{A+B Log \left[e\left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]}, x\right] + d^{2} g^{2} CannotIntegrat$$

### Problem 48: Result valid but suboptimal antiderivative.

$$\int \frac{c g + d g x}{A + B Log \left[e \left(\frac{a+b x}{c+d x}\right)^{n}\right]} dx$$

Optimal (type 8, 35 leaves, 0 steps):

Unintegrable 
$$\left[\frac{c g + d g x}{A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

c g CannotIntegrate 
$$\left[\frac{1}{A + B \log\left[e\left(\frac{a+b \cdot x}{c+d \cdot x}\right)^{n}\right]}, x\right] + d g CannotIntegrate \left[\frac{x}{A + B \log\left[e\left(\frac{a+b \cdot x}{c+d \cdot x}\right)^{n}\right]}, x\right]$$

### Problem 49: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c g + d g x\right) \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(c g + d g x\right) \left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)}, x\right]$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(c g + d g x\right) \left(A + B Log \left[e \left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)}, x\right]$$

#### Problem 50: Unable to integrate problem.

$$\int \frac{1}{\left(c g + d g x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 4, 96 leaves, 3 steps):

$$\frac{\mathrm{e}^{-\frac{A}{B\,n}}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^n\right)^{-1/n}\,\mathsf{ExpIntegralEi}\left[\frac{\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^n\right]}{\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{g}^2\,\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \, \frac{1}{ \left( c \; g + d \; g \; x \right)^2 \, \left( A + B \, \text{Log} \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{\, n} \, \right] \, \right) } \, \text{, } \, x \, \Big]$$

#### Problem 51: Unable to integrate problem.

$$\int \frac{1}{\left(c g + d g x\right)^{3} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 4, 199 leaves, 7 steps):

$$\frac{b e^{-\frac{A}{Bn}} \left(a + b x\right) \left(e \left(\frac{a + b x}{c + d x}\right)^{n}\right)^{-1/n} ExpIntegralEi\left[\frac{A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]}{B n}\right]}{B \left(b c - a d\right)^{2} g^{3} n \left(c + d x\right)} - \frac{d e^{-\frac{2A}{Bn}} \left(a + b x\right)^{2} \left(e \left(\frac{a + b x}{c + d x}\right)^{n}\right)^{-2/n} ExpIntegralEi\left[\frac{2 \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)}{B n}\right]}{B \left(b c - a d\right)^{2} g^{3} n \left(c + d x\right)^{2}}$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(c\,g+d\,g\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

#### Problem 52: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c g + d g x\right)^{2}}{\left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(c g + d g x\right)^{2}}{\left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}, x\right]$$

Result (type 8, 106 leaves, 2 steps):

$$c^2 \, g^2 \, \text{CannotIntegrate} \, \Big[ \, \frac{1}{ \left( A + B \, \text{Log} \, \Big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \Big] \, \right)^2} \, \text{, } \, x \, \Big] \, + \\$$

$$2 \text{ c d } g^2 \text{ CannotIntegrate} \Big[ \frac{x}{\left( A + B \text{ Log} \Big[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \Big] \right)^2}, \text{ } x \Big] + d^2 g^2 \text{ CannotIntegrate} \Big[ \frac{x^2}{\left( A + B \text{ Log} \Big[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \Big] \right)^2}, \text{ } x \Big]$$

### Problem 53: Result valid but suboptimal antiderivative.

$$\int \frac{c g + d g x}{\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 35 leaves, 0 steps):

Unintegrable 
$$\left[\frac{c g + d g x}{\left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

$$c \ g \ Cannot Integrate \Big[ \frac{1}{\Big( A + B \ Log \Big[ e \ \Big( \frac{a + b \ x}{c + d \ x} \Big)^n \Big] \Big)^2} \text{, } x \Big] + d \ g \ Cannot Integrate \Big[ \frac{x}{\Big( A + B \ Log \Big[ e \ \Big( \frac{a + b \ x}{c + d \ x} \Big)^n \Big] \Big)^2} \text{, } x \Big]$$

#### Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c g + d g x\right) \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(c g + d g x\right) \left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}, x\right]$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(c g + d g x\right) \left(A + B Log\left[e \left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}, x\right]$$

## Problem 55: Unable to integrate problem.

$$\int \frac{1}{\left(c g + d g x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 4, 154 leaves, 4 steps):

$$\frac{e^{-\frac{A}{B\,n}}\,\left(a+b\,x\right)\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{-1/n}\,\text{ExpIntegralEi}\!\left[\frac{A+B\,\text{Log}\!\left[e\,\left(\frac{a-b\,x}{c+d\,x}\right)^{n}\right]}{B\,n}\right]}{B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n^{2}\,\left(c+d\,x\right)}-\frac{a+b\,x}{B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\!\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(c\,g+d\,g\,x\right)^{2}\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}},\,x\right]$$

#### Problem 56: Unable to integrate problem.

$$\int \frac{1}{\left(c g + d g x\right)^{3} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 4, 256 leaves, 10 steps):

$$\frac{b \, e^{-\frac{A}{B\,n}} \, \left(a + b\,x\right) \, \left(e \, \left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right)^{-1/n} \, \text{ExpIntegralEi} \left[\frac{A + B \, \text{Log}\left[e \, \left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]}{B\,n}\right]}{B\,n} - \frac{2\,d \, e^{-\frac{2A}{B\,n}} \, \left(a + b\,x\right)^{2} \, \left(e \, \left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right)^{-2/n} \, \text{ExpIntegralEi} \left[\frac{2 \, \left(A + B \, \text{Log}\left[e \, \left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]\right)}{B\,n}\right]}{B\,n} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2} \, \left(c + d\,x\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, g^{3} \, n^{2}} - \frac{a + b\,x}{B\,\left(b\,c - a\,d\right)^{2} \, n^{2}} - \frac{a +$$

Result (type 8, 37 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(c g + d g x\right)^{3} \left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}, x\right]$$

### Problem 57: Result valid but suboptimal antiderivative.

$$\int (f + g x)^4 \left( A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 364 leaves, 3 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, \left( b \, c - a \, d \right) \, g \, \left( a^3 \, d^3 \, g^3 - a^2 \, b \, d^2 \, g^2 \, \left( 5 \, d \, f - c \, g \right) \, + a \, b^2 \, d \, g \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \, - b^3 \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, n \, x \, - \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, \left( a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) \, + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, n \, x^2}{10 \, b^3 \, d^3} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^3 \, \left( 5 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, n \, x^3}{15 \, b^2 \, d^2} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^4 \, n \, x^4}{5 \, b^5 \, g} \, + \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, b^5 \, g} \, + \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f -$$

Result (type 3, 348 leaves, 4 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, g \, \left( 10 \, a \, b^3 \, d^4 \, f^3 - 10 \, a^2 \, b^2 \, d^4 \, f^2 \, g + 5 \, a^3 \, b \, d^4 \, f \, g^2 - a^4 \, d^4 \, g^3 - b^4 \, c \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, n \, x \, - \\ \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, \left( a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, n \, x^2}{10 \, b^3 \, d^3} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^3 \, \left( 5 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, n \, x^3}{15 \, b^2 \, d^2} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^4 \, n \, x^4}{5 \, b^5 \, g} \, + \frac{\left( f + g \, x \right)^5 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{5 \, g^5 \, g} \, + \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, + \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, n \, Log \left[$$

### Problem 58: Result optimal but 1 more steps used.

$$\int (f + g x)^{3} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right) dx$$

Optimal (type 3, 235 leaves, 3 steps):

$$\frac{B \left( b \ c - a \ d \right) \ g \left( a^2 \ d^2 \ g^2 - a \ b \ d \ g \left( 4 \ d \ f - c \ g \right) + b^2 \left( 6 \ d^2 \ f^2 - 4 \ c \ d \ f \ g + c^2 \ g^2 \right) \right) \ n \ x}{4 \ b^3 \ d^3} - \frac{B \left( b \ c - a \ d \right) \ g^2 \left( 4 \ b \ d \ f - b \ c \ g - a \ d \ g \right) \ n \ x^2}{8 \ b^2 \ d^2} - \frac{B \left( b \ c - a \ d \right) \ g^3 \ n \ x^3}{4 \ b^4 \ g} + \frac{\left( f + g \ x \right)^4 \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)}{4 \ g} + \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g} - \frac{B \left( d \ f - c \ g \right)^4 \ n \ Log \left[ c + d \ x \right]}{4 \ d^4 \ g}$$

Result (type 3, 235 leaves, 4 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \left(a^2 \ d^2 \ g^2 - a \ b \ d \ g \ \left(4 \ d \ f - c \ g\right) + b^2 \left(6 \ d^2 \ f^2 - 4 \ c \ d \ f \ g + c^2 \ g^2\right)\right) \ n \ x}{4 \ b^3 \ d^3} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ b^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ b^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ n \ x^2}{4 \ d^4 \ g} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ d^2 \ d^2 + d^2 \ d^2 \ d^2 \ d^2 + d^2 \ d^2 + d^2 \ d^2 \ d^2 + d^2 \ d^2 \ d^2 + d^2 \ d^2 \ d^2 + d^2 + d^2 \ d^2 + d^2$$

#### Problem 59: Result optimal but 1 more steps used.

$$\int \left(f+g\,x\right)^2\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 157 leaves, 3 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, x}{3 \, b^2 \, d^2} - \frac{B \, \left(b \, c - a \, d\right) \, g^2 \, n \, x^2}{6 \, b \, d} - \\ \frac{B \, \left(b \, f - a \, g\right)^3 \, n \, Log \, [\, a + b \, x\,]}{3 \, b^3 \, g} + \frac{\left(f + g \, x\right)^3 \, \left(A + B \, Log \, \left[\, e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\,\right]\,\right)}{3 \, g} + \frac{B \, \left(d \, f - c \, g\right)^3 \, n \, Log \, [\, c + d \, x\,]}{3 \, d^3 \, g}$$

Result (type 3, 157 leaves, 4 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, x}{3 \, b^2 \, d^2} - \frac{B \, \left(b \, c - a \, d\right) \, g^2 \, n \, x^2}{6 \, b \, d} - \\ \frac{B \, \left(b \, f - a \, g\right)^3 \, n \, Log \left[a + b \, x\right]}{3 \, b^3 \, g} + \frac{\left(f + g \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, g} + \frac{B \, \left(d \, f - c \, g\right)^3 \, n \, Log \left[c + d \, x\right]}{3 \, d^3 \, g}$$

#### Problem 60: Result optimal but 1 more steps used.

$$\int (f + g x) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right) dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \ n \ x}{2 \ b \ d} - \frac{B \left(b \ f - a \ g\right)^2 \ n \ Log \left[a + b \ x\right]}{2 \ b^2 \ g} + \frac{\left(f + g \ x\right)^2 \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{2 \ g} + \frac{B \left(d \ f - c \ g\right)^2 \ n \ Log \left[c + d \ x\right]}{2 \ d^2 \ g}$$

Result (type 3, 115 leaves, 4 steps):

$$-\frac{B\,\left(b\,c-a\,d\right)\,g\,n\,x}{2\,b\,d}\,-\,\frac{B\,\left(b\,f-a\,g\right)^{\,2}\,n\,Log\,[\,a+b\,x\,]}{2\,b^{\,2}\,g}\,+\,\frac{\left(f+g\,x\right)^{\,2}\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{2\,g}\,+\,\frac{B\,\left(d\,f-c\,g\right)^{\,2}\,n\,Log\,[\,c+d\,x\,]}{2\,d^{\,2}\,g}$$

# Problem 62: Result optimal but 2 more steps used.

$$\frac{A + B Log \left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]}{f + g x} dx$$

Optimal (type 4, 147 leaves, 7 steps):

$$-\frac{B\,n\,\text{Log}\!\left[-\frac{g\,(a+b\,x)}{b\,f-a\,g}\right]\,\text{Log}\,[\,f+g\,x\,]}{g} + \frac{\left(A+B\,\text{Log}\!\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,\text{Log}\,[\,f+g\,x\,]}{g} + \frac{g}{g} \\ -\frac{B\,n\,\text{Log}\!\left[-\frac{g\,(c+d\,x)}{d\,f-c\,g}\right]\,\text{Log}\,[\,f+g\,x\,]}{g} - \frac{B\,n\,\text{PolyLog}\!\left[\,2\,,\,\frac{b\,(f+g\,x)}{b\,f-a\,g}\right]}{g} + \frac{B\,n\,\text{PolyLog}\!\left[\,2\,,\,\frac{d\,(f+g\,x)}{d\,f-c\,g}\right]}{g} \\ -\frac{B\,n\,\text{PolyLog}\!\left[\,2\,,\,\frac{b\,(f+g\,x)}{b\,f-a\,g}\right]}{g} + \frac{B\,n\,\text{PolyLog}\!\left[\,2\,,\,\frac{d\,(f+g\,x)}{d\,f-c\,g}\right]}{g} +$$

Result (type 4, 147 leaves, 9 steps):

$$-\frac{B \, n \, Log \left[-\frac{g \, (a+b \, x)}{b \, f-a \, g}\right] \, Log \, [\, f+g \, x\,]}{g} + \frac{\left(A+B \, Log \left[\, e \, \left(\frac{a+b \, x}{c+d \, x}\right)^n\,\right]\,\right) \, Log \, [\, f+g \, x\,]}{g} + \frac{g}{g} + \frac{B \, n \, Log \left[-\frac{g \, (c+d \, x)}{d \, f-c \, g}\right] \, Log \, [\, f+g \, x\,]}{g} - \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{b \, (f+g \, x)}{b \, f-a \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d \, (f+g \, x)}{d \, f-c \, g}\,\right]}{g} + \frac{B \, n \, Poly Log \left[\, 2\,, \, \frac{d$$

## Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( f + g x \right)^{2}} dx$$

Optimal (type 3, 91 leaves, 3 steps):

$$\frac{\left(\mathsf{a} + \mathsf{b} \; \mathsf{x}\right) \; \left(\mathsf{A} + \mathsf{B} \; \mathsf{Log}\left[\; \mathsf{e} \; \left(\frac{\mathsf{a} + \mathsf{b} \; \mathsf{x}}{\mathsf{c} + \mathsf{d} \; \mathsf{x}}\right)^{\; \mathsf{n}}\;\right] \right)}{\left(\mathsf{b} \; \mathsf{f} - \mathsf{a} \; \mathsf{g}\right) \; \left(\mathsf{f} + \mathsf{g} \; \mathsf{x}\right)} + \frac{\mathsf{B} \; \left(\mathsf{b} \; \mathsf{c} - \mathsf{a} \; \mathsf{d}\right) \; \mathsf{n} \; \mathsf{Log}\left[\; \frac{\mathsf{f} + \mathsf{g} \; \mathsf{x}}{\mathsf{c} + \mathsf{d} \; \mathsf{x}}\right]}{\left(\mathsf{b} \; \mathsf{f} - \mathsf{a} \; \mathsf{g}\right) \; \left(\mathsf{d} \; \mathsf{f} - \mathsf{c} \; \mathsf{g}\right)}$$

Result (type 3, 119 leaves, 4 steps):

$$\frac{b \, B \, n \, Log \left[a + b \, x\right]}{g \, \left(b \, f - a \, g\right)} - \frac{A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{g \, \left(f + g \, x\right)} - \frac{B \, d \, n \, Log \left[c + d \, x\right]}{g \, \left(d \, f - c \, g\right)} + \frac{B \, \left(b \, c - a \, d\right) \, n \, Log \left[f + g \, x\right]}{\left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right)}$$

## Problem 64: Result optimal but 1 more steps used.

$$\int \frac{A + B \log \left[ e^{\left(\frac{a + b x}{c + d x}\right)^{n}} \right]}{\left(f + g x\right)^{3}} dx$$

Optimal (type 3, 190 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n}{2 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, n \, Log \left[a + b \, x\right]}{2 \, g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, n \, Log \left[c + d \, x\right]}{2 \, g \, \left(d \, f - c \, g\right)^2} + \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, Log \left[f + g \, x\right]}{2 \, \left(d \, f - c \, g\right)^2}$$

Result (type 3, 190 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n}{2 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, n \, Log \left[a + b \, x\right]}{2 \, g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, n \, Log \left[c + d \, x\right]}{2 \, g \, \left(d \, f - c \, g\right)^2} + \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, Log \left[f + g \, x\right]}{2 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

### Problem 65: Result optimal but 1 more steps used.

$$\int \frac{A + B \log \left[ e^{\left(\frac{a + b x}{c + d x}\right)^{n}} \right]}{\left(f + g x\right)^{4}} dx$$

Optimal (type 3, 283 leaves, 3 steps):

$$-\frac{B \left(b \, c-a \, d\right) \, n}{6 \, \left(b \, f-a \, g\right) \, \left(d \, f-c \, g\right) \, \left(f+g \, x\right)^2} - \frac{B \left(b \, c-a \, d\right) \, \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right) \, n}{3 \, \left(b \, f-a \, g\right)^2 \, \left(d \, f-c \, g\right)^2 \, \left(f+g \, x\right)} + \frac{b^3 \, B \, n \, Log \left[a+b \, x\right]}{3 \, g \, \left(b \, f-a \, g\right)^3} - \frac{A+B \, Log \left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^n\right]}{3 \, g \, \left(f+g \, x\right)^3} - \frac{B \, d^3 \, n \, Log \left[c+d \, x\right]}{3 \, g \, \left(d \, f-c \, g\right)^3} + \frac{B \, \left(b \, c-a \, d\right) \, \left(a^2 \, d^2 \, g^2-a \, b \, d \, g \, \left(3 \, d \, f-c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2-3 \, c \, d \, f \, g+c^2 \, g^2\right)\right) \, n \, Log \left[f+g \, x\right]}{3 \, \left(b \, f-a \, g\right)^3 \, \left(d \, f-c \, g\right)^3}$$

Result (type 3, 283 leaves, 4 steps):

$$-\frac{B \left(b \, c-a \, d\right) \, n}{6 \, \left(b \, f-a \, g\right) \, \left(d \, f-c \, g\right) \, \left(f+g \, x\right)^2} - \frac{B \left(b \, c-a \, d\right) \, \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right) \, n}{3 \, \left(b \, f-a \, g\right)^2 \, \left(d \, f-c \, g\right)^2 \, \left(f+g \, x\right)} + \frac{b^3 \, B \, n \, Log \left[a+b \, x\right]}{3 \, g \, \left(b \, f-a \, g\right)^3} - \frac{A+B \, Log \left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^n\right]}{3 \, g \, \left(f+g \, x\right)^3} - \frac{B \, d^3 \, n \, Log \left[c+d \, x\right]}{3 \, g \, \left(d \, f-c \, g\right)^3} + \frac{B \, \left(b \, c-a \, d\right) \, \left(a^2 \, d^2 \, g^2-a \, b \, d \, g \, \left(3 \, d \, f-c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2-3 \, c \, d \, f \, g+c^2 \, g^2\right)\right) \, n \, Log \left[f+g \, x\right]}{3 \, \left(b \, f-a \, g\right)^3 \, \left(d \, f-c \, g\right)^3}$$

## Problem 66: Result optimal but 1 more steps used.

$$\int \frac{A + B \log \left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]}{\left(f + g x\right)^{5}} dx$$

Optimal (type 3, 388 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n}{12 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n}{8 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(f + g \, x\right)^2} - \frac{B \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(3 \, d \, f - c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, n}{4 \, \left(b \, f - a \, g\right)^3 \, \left(d \, f - c \, g\right)^3 \, \left(f + g \, x\right)} + \frac{b^4 \, B \, n \, Log \left[a + b \, x\right]}{4 \, g \, \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{4 \, g \, \left(f + g \, x\right)^4} - \frac{B \, d^4 \, n \, Log \left[c + d \, x\right]}{4 \, g \, \left(d \, f - c \, g\right)^4} - \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \, \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, n \, Log \left[f + g \, x\right]}{4 \, \left(b \, f - a \, g\right)^4 \, \left(d \, f - c \, g\right)^4}$$

Result (type 3, 388 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n}{12 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n}{8 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(f + g \, x\right)^2} - \frac{B \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(3 \, d \, f - c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, n}{4 \, \left(b \, f - a \, g\right)^3 \, \left(d \, f - c \, g\right)^3 \, \left(f + g \, x\right)} + \frac{b^4 \, B \, n \, Log \left[a + b \, x\right]}{4 \, g \, \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{4 \, g \, \left(f + g \, x\right)^4} - \frac{B \, d^4 \, n \, Log \left[c + d \, x\right]}{4 \, g \, \left(d \, f - c \, g\right)^4} - \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \, \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, n \, Log \left[f + g \, x\right]}{4 \, \left(b \, f - a \, g\right)^4 \, \left(d \, f - c \, g\right)^4}$$

## Problem 67: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{3} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 923 leaves, 15 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^3 \, g^3 \, n^2 \, x}{6 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^2 \, g^2 \, \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, n^2 \, x}{4 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^2 \, g^3 \, n^2 \, \left( c + d \, x \right)^2}{2 \, b^4 \, d^3} - \frac{1}{2 \, b^4 \, d^3} \\ B \left( b \, c - a \, d \right) \, g \left( a^2 \, d^2 \, g^2 - 2 \, a \, b \, d \, g \, \left( 2 \, d \, f - c \, g \right) + b^2 \left( 6 \, d^2 \, f^2 - 8 \, c \, d \, f \, g + 3 \, c^2 \, g^2 \right) \right) \, n \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) - \frac{B \left( b \, c - a \, d \right) \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, n \, \left( c + d \, x \right)^2 \left( A + B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) - \frac{B \left( b \, c - a \, d \right) \, g^3 \, n \, \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ e \left( \frac{a - b \, x}{c + d \, x} \right)^n \right] \right)}{4 \, b^2 \, d^4} - \frac{6 \, b \, d^4}{6 \, d^4} - \frac{1}{2 \, b^4 \, d^4} -$$

Result (type 4, 1060 leaves, 31 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,\left(a^2\,d^2\,g^2-a\,b\,d\,g\,\left(4\,d\,f-c\,g\right)+b^2\,\left(6\,d^2\,f^2-4\,c\,d\,f\,g+c^2\,g^2\right)\right)\,n\,x}{2\,b^3\,d^3} - \frac{2\,b^3\,d^3}{6\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,n^2\,x}{4\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^3\,n^2\,x^2}{4\,b^3\,d^3} - \frac{12\,b^2\,d^2}{4\,b^4\,g^2} - \frac{a^3\,B^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,b^4\,g^2} - \frac{a^3\,B^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,b^4\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,b^4\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,b^4\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,n^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,n^2\,\log\left[a+b\,x\right]}{4\,b^4\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,\left(a^2\,d^2\,g^2-a\,b\,d\,g\,\left(4\,d\,f-c\,g\right)+b^2\,\left(6\,d^2\,f^2-4\,c\,d\,f\,g+c^2\,g^2\right)\right)\,n\,\left(a+b\,x\right)\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{4\,b^2\,d^2} - \frac{2\,b^4\,d^3}{6\,b\,d} - \frac{2\,b^4\,g}{4\,g} - \frac{B\,\left(b\,c-a\,d\right)\,g^3\,n\,x^3\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d^2} - \frac{B\,\left(b\,c-a\,d\right)\,g^3\,n\,x^3\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d^2} + \frac{4\,g}{4\,g} - \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d^2} - \frac{B\,\left(b\,c-a\,d\right)\,g^3\,n\,x^3\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,b^4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,b^4\,g} + \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,g^3\,n^2\,x^2}{4\,g} +$$

### Problem 68: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{2} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 565 leaves, 12 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} g^{2} n^{2} x}{3 \ b^{2} d^{2}} = \frac{2 \ B \left(b \ c-a \ d\right) \ g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ n \left(a+b \ x\right) \ \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b^{3} \ d^{2}} = \frac{B \left(b \ c-a \ d\right) \ g^{2} \ n \left(c+d \ x\right)^{2} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b^{3} \ d^{3}} = \frac{\left(b \ f-a \ g\right)^{3} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{3 \ b^{3} \ g} + \frac{\left(f+g \ x\right)^{3} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{3 \ g} + \frac{1}{3 \ g} + \frac{1}{3 \ g^{3} \ g^{3}} = \frac{1}{3 \ b^{3} \ d^{3}} = \frac{1}{3 \ b^{3} \ d^{3}} + \frac{1}{3 \ b^{3} \ d^{$$

#### Result (type 4, 699 leaves, 27 steps):

$$\frac{2 \, A \, B \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, x}{3 \, b^{2} \, d^{2}} + \frac{B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, n^{2} \, x}{3 \, b^{3} \, d} + \frac{a^{2} \, B^{2} \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(a + b \, x\right) \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{3 \, b^{3} \, g} + \frac{B^{2} \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(a + b \, x\right) \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{3 \, b^{3} \, g} - \frac{3 \, b^{3} \, d^{2}}{3 \, b^{3} \, g} + \frac{2 \, B \, \left(b \, f - a \, g\right)^{3} \, n \, Log\left[a + b \, x\right] \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, b \, d} + \frac{2 \, B \, \left(b \, f - a \, g\right)^{3} \, n \, Log\left[a + b \, x\right] \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, b \, d} + \frac{2 \, B^{2} \, c^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b \, d^{3}} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, d^{3}} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, d^{3}} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, Log\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, PolyLog\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, PolyLog\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, PolyLog\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g\right)^{3} \, n^{2} \, PolyLog\left[c + d \, x\right]}{3 \, b^{3} \, g} + \frac{2 \, B^{2} \, \left(d \, f - c \, g$$

### Problem 69: Result valid but suboptimal antiderivative.

$$\int (f + g x) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 290 leaves, 9 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{2}\,d} - \frac{\left(b\,f-a\,g\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{2\,b^{2}\,g} + \\ \frac{\left(f+g\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{2\,g} + \frac{B\left(b\,c-a\,d\right)\,\left(2\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{b^{2}\,d^{2}} + \\ \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g\,n^{2}\,Log\left[c+d\,x\right]}{b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,n^{2}\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b^{2}\,d^{2}} + \\ \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,g\,n^{2}\,Log\left[c+d\,x\right]}{b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)\,n^{2}\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b^{2}\,d^{2}} + \\ \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,g\,n^{2}\,Log\left[c+d\,x\right]}{b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)\,n^{2}\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b^{2}\,d^{2}} + \\ \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,g\,n^{2}\,Log\left[c+d\,x\right]}{b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)\,n^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,d\,g\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,d\,x\right)^{2}\,Rog\left[c+d\,x\right]}{b^{2}\,Rog\left[c+d\,x\right]} + \frac{B^{2}\,\left(b\,c-a\,a\,x\right)^{2}\,Rog\left[c+d\,x\right]}{$$

Result (type 4, 481 leaves, 23 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,n\,x}{b\,d} + \frac{B^2\,\left(b\,f-a\,g\right)^2\,n^2\,Log\,[\,a+b\,x\,]^2}{2\,b^2\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,n\,\left(a+b\,x\right)\,Log\,[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,]}{b^2\,d} - \frac{B\,\left(b\,f-a\,g\right)^2\,n\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)}{b^2\,g} + \frac{\left(f+g\,x\right)^2\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2}{2\,g} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,n^2\,Log\,[\,c+d\,x\,]}{b^2\,d^2} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2\,Log\,[\,c+d\,x\,]}{d^2\,g} + \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2\,Log\,[\,c+d\,x\,]}{b^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2\,Log\,[\,c+d\,x\,]^2}{d^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2\,Log\,[\,c+d\,x\,]^2}{b^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2\,PolyLog\,[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,]}{d^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,n^2$$

#### Problem 70: Result valid but suboptimal antiderivative.

$$\int \left( A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 135 leaves, 6 steps):

$$\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\;\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\right]\,\right)^{\,\mathsf{2}}}{\mathsf{b}}\;+\;\frac{2\;\mathsf{B}\;\left(\mathsf{b}\;\mathsf{c}-\mathsf{a}\;\mathsf{d}\right)\;\mathsf{n}\;\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\right]\,\right)\;\mathsf{Log}\left[\,\frac{\mathsf{b}\;\mathsf{c}-\mathsf{a}\;\mathsf{d}}{\mathsf{b}\;(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\right]}{\mathsf{b}\;\mathsf{d}}\;+\;\frac{2\;\mathsf{B}^{\,\mathsf{2}}\;\left(\mathsf{b}\;\mathsf{c}-\mathsf{a}\;\mathsf{d}\right)\;\mathsf{n}^{\,\mathsf{2}}\;\mathsf{PolyLog}\left[\,\mathsf{2}\;,\;\frac{\mathsf{d}\;\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{b}\;\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,\right]}{\mathsf{b}\;\mathsf{d}}\;$$

Result (type 4, 275 leaves, 20 steps):

$$-\frac{a \, B^2 \, n^2 \, Log \, [\, a + b \, x \, ]^{\, 2}}{b} + \frac{2 \, a \, B \, n \, Log \, [\, a + b \, x \, ] \, \left(A + B \, Log \, \left[\, e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{\, n} \, \right]\right)}{b} + x \, \left(A + B \, Log \, \left[\, e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{\, n} \, \right]\right)^2 + \\ \frac{2 \, B^2 \, c \, n^2 \, Log \, \left[\, -\frac{d \, (a + b \, x)}{b \, c - a \, d} \, \right] \, Log \, [\, c + d \, x \, ]}{d} - \frac{2 \, B \, c \, n \, \left(A + B \, Log \, \left[\, e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{\, n} \, \right]\right) \, Log \, [\, c + d \, x \, ]}{d} - \frac{B^2 \, c \, n^2 \, Log \, [\, c + d \, x \, ]^2}{d} + \\ \frac{2 \, a \, B^2 \, n^2 \, Log \, \left[\, a + b \, x \, \right] \, Log \, \left[\, b \, (c + d \, x) \, \right]}{b \, b \, c - a \, d} + \frac{2 \, a \, B^2 \, n^2 \, PolyLog \, \left[\, 2 \, , \, -\frac{d \, (a + b \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \, B^2 \, c \, n^2 \, PolyLog \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{d} + \frac{2 \,$$

### Problem 71: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{f + gx} dx$$

Optimal (type 4, 297 leaves, 9 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\mathsf{n}}\right]\right)^{2}\,\mathsf{Log}\!\left[\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}}}{\mathsf{g}} + \frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\mathsf{n}}\right]\right)^{2}\,\mathsf{Log}\!\left[\mathsf{1}-\frac{(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g})\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g})\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} - \frac{2\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\mathsf{n}}\right]\right)\mathsf{PolyLog}\!\left[\mathsf{2},\,\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} + \frac{2\,\mathsf{B}^{2}\,\mathsf{n}^{2}\,\mathsf{PolyLog}\!\left[\mathsf{3},\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} - \frac{2\,\mathsf{B}^{2}\,\mathsf{n}^{2}\,\mathsf{PolyLog}\!\left[\mathsf{3},\,\frac{(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g})\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g})\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}}$$

Result (type 4, 2233 leaves, 43 steps):

$$\frac{2\,\text{AB}\,\text{n}\,\text{Log}\left[-\frac{g\,(2a+b\,x)}{b\,f-a\,g}\right]\,\text{Log}\left[f+g\,x\right]}{g} = \frac{g^2\,\text{Log}\left[\left(a+b\,x\right)^n\right]^2\,\text{Log}\left[f+g\,x\right]}{g} + \frac{\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+a\,x}\right]\right)\right)^2\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n^2\,\text{Log}\left[a+b\,x\right]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n^2\,\text{Log}\left[a+b\,x\right]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n^2\,\text{Log}\left[a+b\,x\right]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n\,(\,n\,\text{Log}\left[a+b\,x\right]\,-\,\text{Log}\left[\left(a+b\,x\right)^n\right]\right)\,\text{Log}\left[-\frac{g\,(c+d\,x)}{d\,f-c\,g}\right]\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n\,\text{Log}\left[-\frac{g\,(a+b\,x)}{b\,f-a\,g}\right]\,\left(\text{Log}\left[\left(a+b\,x\right)^n\right]\right)\,\text{Log}\left[e\,\left(\frac{a+b\,x}{a\,f-c\,g}\right)\right]\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n\,\text{Log}\left[-\frac{g\,(a+b\,x)}{b\,f-a\,g}\right]\,\left(\text{Log}\left[\left(a+b\,x\right)^n\right]-\text{Log}\left[e\,\left(\frac{a+b\,x}{a\,f-c\,g}\right)\right]+\text{Log}\left[\left(c+d\,x\right)^{-n}\right]\right)\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n\,\text{Log}\left[-\frac{g\,(a+b\,x)}{b\,f-a\,g}\right]\,\left(\text{Log}\left[\left(a+b\,x\right)^n\right]-\text{Log}\left[e\,\left(\frac{a+b\,x}{a\,f-c\,g}\right)\right]+\text{Log}\left[\left(c+d\,x\right)^{-n}\right]\right)\,\text{Log}\left[f+g\,x\right]}{g} + \frac{2\,B^2\,n\,\text{Log}\left[-\frac{g\,(a+b\,x)}{a\,f-c\,g}\right]}{g} + \frac{B^2\,\text{Log}\left[\left(a+b\,x\right)^n\right]^2\,\text{Log}\left[\frac{b\,(f+g\,x)}{b\,f-a\,g}\right]}{g} + \frac{B^2\,\text{Log}\left[\left(a+b\,x\right)^n\right]^2\,\text{Log}\left[\frac{b\,(f+g\,x)}{a\,f-a\,g}\right]}{g} + \frac{B^$$

$$\frac{B^{2} n^{2} \left( log \left[ -\frac{d(a + b \times a)}{b + c + ad} \right] - log \left[ -\frac{(b(a + b \times b))}{b + c + ad} \right] \right) \left( log \left[ c + d \times a \right] + log \left[ -\frac{(b(a + b \times a))}{b + c + ad} \right] \right)}{g} \\ = 2B^{2} n^{2} \left( log \left[ f + g \times a \right] - log \left[ -\frac{(b(a + ad))}{(d + c \times b)} \right] \right) Polylog \left[ 2, -\frac{d(a + b \times a)}{b + c + ad} \right] \\ = 2B^{2} n^{2} \left( log \left[ f + g \times a \right] - log \left[ -\frac{(b(a + ad))}{(d + ag)} \right] Polylog \left[ 2, -\frac{b(c + ad))}{b + c + ad} \right] \\ = 2B^{2} n^{2} \left( log \left[ f + g \times a \right] - log \left[ -\frac{(b(a + ad))}{(b + ag)} \right] Polylog \left[ 2, -\frac{b(c + ad))}{b + c + ad} \right] \\ = 2B^{2} n^{2} log \left[ -\frac{(b(a + ad))}{(d + c \times g)} \right] Polylog \left[ 2, -\frac{b(c + ad))}{b + ag} \right] \\ = 2B^{2} n^{2} log \left[ -\frac{(b(a + ad))}{(d + c \times g)} \right] Polylog \left[ 2, -\frac{b(a + ad))}{b + ag} \right] \\ = 2B^{2} n^{2} log \left[ -\frac{(b(a + ad))}{(b + ag)} \right] Polylog \left[ 2, -\frac{b(a + ad))}{(b + ag)} \right] Polylog \left[ 2, -\frac{(d + c \times g))}{(b + ag)} \right] Polylog \left[ 2, -\frac{(d + c \times g))}{(b + ag)} \right] \\ = 2B^{2} n^{2} log \left[ -\frac{(b(a + ad))}{(b + ag)} \right] Polylog \left[ 2, -\frac{(b(a + ad))}{(b +$$

## Problem 72: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(f + gx\right)^{2}} dx$$

Optimal (type 4, 206 leaves, 4 steps):

$$\frac{\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(b\,f-a\,g\right)\,\left(f+g\,x\right)} + \frac{2\,B\,\left(b\,c-a\,d\right)\,n\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,1-\frac{\left(d\,f-c\,g\right)\,\left(a+b\,x\right)}{\left(b\,f-a\,g\right)\,\left(c+d\,x\right)}\,\right]}{\left(b\,f-a\,g\right)\,\left(d\,f-c\,g\right)} + \frac{2\,B^2\,\left(b\,c-a\,d\right)\,n^2\,PolyLog\left[\,2\,,\,\frac{\left(d\,f-c\,g\right)\,\left(a+b\,x\right)}{\left(b\,f-a\,g\right)\,\left(c+d\,x\right)}\,\right]}{\left(b\,f-a\,g\right)} + \frac{2\,B^2\,\left(b\,c-a\,d\right)\,n^2\,PolyLog\left[\,2\,,\,\frac{\left(d\,f-c\,g\right)\,\left(a+b\,x\right)}{\left(b\,f-a\,g\right)}\,\right]}{\left(b\,f-a\,g\right)} + \frac{2\,B^2\,\left(b\,c-a\,d\right)\,n^2\,PolyLog\left[\,2\,,\,\frac{\left(d\,f-c\,g\right)\,\left(a+b\,x\right)}{\left(b\,f-a\,g\right)}\,\right]}{\left(b\,f-a\,g\right)} + \frac{2\,B^2\,\left(b\,f-a\,g\right)\,n^2\,PolyLog\left[\,2\,,\,\frac{\left(d\,f-c\,g\right)\,\left(a+b\,x\right)}{\left(b\,f-a\,g\right)}\,\right]}{\left(b\,f-a\,g\right)} + \frac{2\,B^2\,\left(b\,f-a\,g\right)\,n^2\,PolyLog\left[\,2\,,\,\frac{\left(d\,f-c\,g\right)\,n^2\,PolyLog\left[\,2\,g\right]}{\left(b\,f-a\,g\right)}\,\right]}$$

Result (type 4, 657 leaves, 29 steps):

$$-\frac{b\,B^{2}\,n^{2}\,Log\,[\,a+b\,x\,]^{\,2}}{g\,\left(b\,f-a\,g\right)} + \frac{2\,b\,B\,n\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{g\,\left(b\,f-a\,g\right)} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{g\,\left(f+g\,x\right)} + \frac{2\,B^{\,2}\,d\,n^{\,2}\,Log\,[\,c+d\,x\,]}{g\,\left(d\,f-c\,g\right)} - \frac{2\,B^{\,2}\,d\,n^{\,2}\,Log\,[\,c+d\,x\,]}{g\,\left(d\,f-c\,g\right)} - \frac{B^{\,2}\,d\,n^{\,2}\,Log\,[\,c+d\,x\,]^{\,2}}{g\,\left(d\,f-c\,g\right)} + \frac{2\,b\,B^{\,2}\,n^{\,2}\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{g\,\left(b\,f-a\,g\right)} - \frac{2\,B^{\,2}\,d\,n^{\,2}\,Log\,[\,c+d\,x\,]^{\,2}}{g\,\left(d\,f-c\,g\right)} + \frac{2\,B\,B^{\,2}\,n^{\,2}\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{g\,\left(b\,f-a\,g\right)} - \frac{2\,B^{\,2}\,\left(b\,c-a\,d\right)\,n^{\,2}\,Log\,[\,c+d\,x\,]}{g\,\left(b\,f-a\,g\right)} + \frac{2\,B^{\,2}\,\left(b\,c-a\,d\right)\,n^{\,$$

#### Problem 73: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(f + gx\right)^{3}} dx$$

Optimal (type 4, 389 leaves, 9 steps):

$$\frac{B \; \left( b \; c - a \; d \right) \; g \; n \; \left( a + b \; x \right) \; \left( A + B \; Log \left[ e \; \left( \frac{a + b \; x}{c + d \; x} \right)^n \right] \right)}{\left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right) \; \left( f + g \; x \right)} + \frac{b^2 \; \left( A + B \; Log \left[ e \; \left( \frac{a + b \; x}{c + d \; x} \right)^n \right] \right)^2}{2 \; g \; \left( b \; f - a \; g \right)^2} - \frac{\left( A + B \; Log \left[ e \; \left( \frac{a + b \; x}{c + d \; x} \right)^n \right] \right)^2}{2 \; g \; \left( f + g \; x \right)^2} + \frac{B^2 \; \left( b \; c - a \; d \right)^2 \; g \; n^2 \; Log \left[ \frac{f + g \; x}{c + d \; x} \right]}{\left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right)^2} + \frac{B \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; f - b \; c \; g - a \; d \; g \right) \; n \; \left( A + B \; Log \left[ e \; \left( \frac{a + b \; x}{c + d \; x} \right)^n \right] \right) \; Log \left[ 1 - \frac{\left( d \; f - c \; g \right) \; \left( a + b \; x \right)}{\left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right)^2} + \frac{B^2 \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; f - b \; c \; g - a \; d \; g \right) \; n^2 \; PolyLog \left[ 2 , \; \frac{\left( d \; f - c \; g \right) \; \left( a + b \; x \right)}{\left( b \; f - a \; g \right) \; \left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right)^2} + \frac{B^2 \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; f - b \; c \; g - a \; d \; g \right) \; n^2 \; PolyLog \left[ 2 , \; \frac{\left( d \; f - c \; g \right) \; \left( a + b \; x \right)}{\left( b \; f - a \; g \right) \; \left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right)^2} \right)} \right]}{\left( b \; f - a \; g \right)^2 \; \left( d \; f - c \; g \right)^2}$$

Result (type 4, 941 leaves, 33 steps):

$$\frac{b \, B^2 \, (b \, c - a \, d) \, n^2 \, Log \, [a + b \, x]}{(b \, f - a \, g)^2 \, (d \, f - c \, g)} - \frac{b^2 \, B^2 \, n^2 \, Log \, [a + b \, x]^2}{(b \, f - a \, g)^2} - \frac{B \, (b \, c - a \, d) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right)\right)}{(b \, f - a \, g)^2 \, (d \, f - c \, g)} + \frac{b^2 \, B \, n \, Log \, \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right)}{g \, (b \, f - a \, g)^2} - \frac{A + B \, Log \, \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right)\right)^2}{(b \, f - a \, g)^2} - \frac{B^2 \, d \, (b \, c - a \, d) \, n^2 \, Log \, [c + d \, x]}{(b \, f - a \, g)} + \frac{B^2 \, d^2 \, n^2 \, Log \, \left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \, [c + d \, x]}{g \, (d \, f - c \, g)^2} + \frac{B^2 \, d^2 \, n^2 \, Log \, [c + d \, x]}{g \, (d \, f - c \, g)^2} + \frac{B^2 \, d^2 \, n^2 \, Log \, [c + d \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, d^2 \, n^2 \, Log \, [c + d \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, (2 \, b \, d \, f - b \, c \, g - a \, d \, g) \, n^2 \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, (2 \, b \, d \, f - b \, c \, g - a \, d \, g) \, n^2 \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, (2 \, b \, d \, f - b \, c \, g - a \, d \, g) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a + b \, x}{b \, c - a \, d}\right)^n\right] \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a \, b \, x}{b \, c - a \, d}\right)^n\right] \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a \, b \, x}{b \, c - a \, d}\right)^n\right] \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a \, b \, x}{b \, c - a \, d}\right)^n\right] \, Log \, [f + g \, x]}{g \, (b \, f - a \, g)^2 \, (d \, f - c \, g)^2} + \frac{B^2 \, (b \, c - a \, d) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \, \left(A + B \, Log \, \left[e \, \left(\frac{a \, b \, x}{b \,$$

#### Problem 74: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]\right)^{2}}{\left(f + g x\right)^{4}} dx$$

Optimal (type 4, 747 leaves, 12 steps):

$$\frac{B^{2} \left(b \, c-a \, d\right)^{2} g^{2} \, n^{2} \left(c+d \, x\right)}{3 \left(b \, f-a \, g\right)^{2} \left(d \, f-c \, g\right)^{3} \left(f+g \, x\right)}{3 \left(b \, f-a \, g\right)^{2} \left(d \, f-c \, g\right)^{3} \left(f+g \, x\right)} - \frac{B \left(b \, c-a \, d\right) \, g^{2} \, n \left(c+d \, x\right)^{2} \left(A+B \, Log\left[e \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3} \left(d \, f-c \, g\right)^{3} \left(f+g \, x\right)^{2}} + \frac{2B \left(b \, c-a \, d\right) \, g \left(3 \, b \, d \, f-b \, c \, g-2 \, a \, d \, g\right) \, n \left(a+b \, x\right) \, \left(A+B \, Log\left[e \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{2} \left(f+g \, x\right)} + \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g^{2} \, n^{2} \, Log\left[\frac{a+b \, x}{c+d \, x}\right]^{n}\right)}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g^{2} \, n^{2} \, Log\left[\frac{a+b \, x}{c+d \, x}\right]}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{2B^{2} \left(b \, c-a \, d\right)^{2} \, g \left(3 \, b \, d \, f-b \, c \, g-2 \, a \, d \, g\right) \, n^{2} \, Log\left[\frac{a+g \, x}{c+d \, x}\right]}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left(b \, f-a \, g\right)^{3} \left(d \, f-c \, g\right)^{3}} + \frac{1}{3 \left$$

Result (type 4, 1427 leaves, 37 steps):

$$\frac{B^2 \left(b \, c - a \, d\right)^2 g \, n^2}{3 \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^2 \left(f \, f + g \, x\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)} + \frac{2 \, b \, B^2 \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \left(2 \, b \, g \, [a + b \, x] - b \, a \, g\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^2} + \frac{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, b \, B^2 \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n \left(A + B \, Log \left[e \left(\frac{a \cdot b \, x}{c \cdot d \, x}\right)^n\right)\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^2} + \frac{2 \, b \, B \, b \, b \, c \, g \, a \, d \, g\right) \, n \left(A + B \, Log \left[e \left(\frac{a \cdot b \, x}{c \cdot d \, x}\right)^n\right)\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, b \, B \, b \, b \, c \, g \, a \, d \, g\right) \, n \left(A + B \, Log \left[e \left(\frac{a \cdot b \, x}{c \cdot d \, x}\right)^n\right)\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^2 \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - c \, g\right)^3}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, b \, c \, g\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, c \, d \, d\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, c \, d \, d\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, c \, d \, d\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, c \, d \, d\right)}{3 \left(d \, f - c \, g\right)^3} + \frac{2 \, B^3 \, d^3 \, n \left(A + B \, Log \left[e \, \left(\frac{a \cdot b \, x}{c \cdot d \, x}\right)^n\right) \, Log \left[e \, d \, d \, d \, d \, c \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, d \, d \, c \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, d \, d \, d \, c \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{2 \, B^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, d \, d \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{2 \, b^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, d \, d \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{2 \, b^2 \, d^3 \, n^2 \, Log \left[e \, d \, d \, d \, d \, d \, c \, g\right)}{3 \, g \left(d \, f - c \, g\right)^3} + \frac{$$

### Problem 75: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(f + gx\right)^{5}} dx$$

Optimal (type 4, 1208 leaves, 15 steps):

$$\frac{B^2 \left( b\, c-a\, d \right)^2 g^3 n^2 \left( c+d\, x \right)^2}{12 \left( b\, f-a\, g \right)^3 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)^2} - \frac{B^2 \left( b\, c-a\, d \right)^3 g^3 n^2 \left( c+d\, x \right)}{6 \left( b\, f-a\, g \right)^3 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)} + \frac{B^2 \left( b\, c-a\, d \right)^2 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 \left( c+d\, x \right)}{4 \left( b\, f-a\, g \right)^3 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)} + \frac{B^2 \left( b\, c-a\, d \right)^2 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 \left( c+d\, x \right)}{4 \left( b\, f-a\, g \right)^3 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)} + \frac{B^2 \left( b\, c-a\, d \right)^2 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n \left( c+d\, x \right)^2 \left( A+B\, Log \left[ e\, \left( \frac{a+b\, x}{c+d\, x} \right)^n \right] \right)}{4 \left( b\, f-a\, g \right)^2 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)^2} + \frac{B^2 \left( b\, c-a\, d \right) g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n \left( c+d\, x \right)^2 \left( A+B\, Log \left[ e\, \left( \frac{a+b\, x}{c+d\, x} \right)^n \right] \right)}{4 \left( b\, f-a\, g \right)^2 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)^2} + \frac{A \left( b\, f-a\, g \right)^2 \left( d\, f-c\, g \right)^4 \left( f+g\, x \right)^2}{4 \left( b\, f-a\, g \right)^3 \left( f+g\, x \right)} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{a+b\, x}{c+d\, x} \right)}{4 \left( b\, f-a\, g \right)^4} - \frac{A^2 B\, Log \left[ e\, \left( \frac{a+b\, x}{c+d\, x} \right)^n \right] \right)^2}{4 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^4 g^3 n^2 Log \left( \frac{a+b\, x}{c+d\, x} \right)}{4 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^4 g^3 n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{6 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{4 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{4 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{4 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{2 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g \right)^4} + \frac{B^2 \left( b\, c-a\, d \right)^3 g^2 \left( 4\, b\, d\, f-b\, c\, g-3\, a\, d\, g \right) n^2 Log \left( \frac{f+g\, x}{c+d\, x} \right)}{2 \left( b\, f-a\, g \right)^4 \left( d\, f-c\, g$$

Result (type 4, 1968 leaves, 41 steps):

$$\frac{B^2 \left(b \, c - a \, d\right)^2 g \, n^2}{12 \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^2 \left(f + g \, x\right)^2} = \frac{5 \, B^2 \left(b \, c - a \, d\right)^2 g \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, n^2}{12 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3 \left(d$$

$$\frac{B^2 \, d^4 \, n^2 \, \text{Log} \left[ - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \text{Log} \left[ c + d \, x \right]}{2 \, g \, \left( d \, f - c \, g \right)^4} - \frac{B \, d^4 \, n \, \left( A + B \, \text{Log} \left[ e \, \left( \frac{a \, b \, x}{c \, d \, x} \right)^n \right] \, \text{Log} \left[ c + d \, x \right]}{2 \, g \, \left( d \, f - c \, g \right)^4} + \frac{B^2 \, d^4 \, n^2 \, \text{Log} \left[ \frac{b \, (c \, d \, x)}{b \, c - a \, d} \right]}{2 \, g \, \left( b \, f - a \, g \right)^2 \, g \, \left( b \, d - b \, c \, g - a \, d \, g \right)^2 \, n^2 \, \text{Log} \left[ f + g \, x \right]}{4 \, \left( b \, f - a \, g \right)^4} + \frac{B^2 \, \left( b \, c - a \, d \right)^2 \, g \, \left( 2 \, b \, d \, f - b \, c \, g - a \, d \, g \right)^2 \, n^2 \, \text{Log} \left[ f + g \, x \right]}{4 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, g \, \left( 3 \, d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4}{4 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, g \, \left( 3 \, d \, f - c \, g \right)^2 \, h^2 \, d \, d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4}{4 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{2 \, B^2 \, \left( b \, c - a \, d \, g \right) \, \left( 3 \, d \, f - c \, g \right)^2 \, h^2 \, d \, d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4}{3 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{1}{2 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{1}{2 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4} + \frac{1}{2 \, \left( b \, f - a \, g \right)^4 \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)^4} \, \left( d \, f - c \, g \right)^4 \, \left( d \, f - c \, g \right)$$

## Problem 76: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f + g x\right)^{2}}{A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+gx\right)^{2}}{A+B \log \left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$f^{2} \, \text{CannotIntegrate} \big[ \, \frac{1}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, 2 \, f \, g \, \text{CannotIntegrate} \big[ \, \frac{x}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \big] } \,, \, \, x \, \big] \, + \, g^{2} \, \text{CannotIntegrate} \big[ \, \frac{x^{2}}{A + B \, \text{Log} \big[ \, e$$

#### Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f+gx}{A+B Log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$f \ Cannot Integrate \Big[ \frac{1}{A + B \ Log \Big[ e \ \Big( \frac{a + b \ x}{c + d \ x} \Big)^n \Big]}, \ x \Big] + g \ Cannot Integrate \Big[ \frac{x}{A + B \ Log \Big[ e \ \Big( \frac{a + b \ x}{c + d \ x} \Big)^n \Big]}, \ x \Big]$$

### Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{1}{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]} dx$$

Optimal (type 8, 26 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{A + B \log \left[e^{\left(\frac{a+b \times}{c+d \times}\right)^{n}}\right]}, x\right]$$

Result (type 8, 26 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{A + B \log \left[e^{\left(\frac{a+b \cdot x}{c+d \cdot x}\right)^{n}}\right]}, x\right]$$

## Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}\,\,\mathrm{d}x$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

#### Problem 80: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^{2}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^2\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

#### Problem 81: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{3} \left(A + B \log\left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^{3}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}$$
,  $x\right]$ 

### Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f + g x\right)^{2}}{\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+g\,x\right)^2}{\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2},\,x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$f^{2}\, \text{CannotIntegrate}\, \Big[\, \frac{1}{\left(A + B\, \text{Log}\, \Big[\, e\, \left(\frac{a + b\, x}{c + d\, x}\right)^{\, n}\, \Big]\, \right)^{\, 2}}\, \text{, } x\, \Big] \, + \\$$

$$2 \text{ f g CannotIntegrate} \Big[ \frac{x}{\left( A + B \text{ Log} \Big[ \text{ e} \left( \frac{\text{a+b x}}{\text{c+d x}} \right)^n \Big] \right)^2} \text{, } x \Big] + \text{g}^2 \text{ CannotIntegrate} \Big[ \frac{x^2}{\left( A + B \text{ Log} \Big[ \text{ e} \left( \frac{\text{a+b x}}{\text{c+d x}} \right)^n \Big] \right)^2} \text{, } x \Big]$$

#### Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f+gx}{\left(A+B\log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)^{2}}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$\label{eq:fcannotIntegrate} \begin{split} &\text{f CannotIntegrate} \big[ \, \frac{1}{\left( \text{A} + \text{B Log} \big[ \, \text{e} \, \left( \frac{\text{a} + \text{b} \, x}{\text{c} + \text{d} \, x} \right)^n \, \big] \, \right)^2} \text{, } x \, \big] \, + \, \text{g CannotIntegrate} \big[ \, \frac{x}{\left( \text{A} + \text{B Log} \big[ \, \text{e} \, \left( \frac{\text{a} + \text{b} \, x}{\text{c} + \text{d} \, x} \right)^n \, \big] \, \right)^2} \text{, } x \, \big] \end{split}$$

### Problem 84: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 26 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}, x\right]$$

Result (type 8, 26 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}, x\right]$$

### Problem 85: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right) \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2},\,x\right]$$

### Problem 86: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}$$
,  $x\right]$ 

### Problem 87: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{3} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2},\,x\right]$$

#### Problem 92: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{a g + b g x} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{Log\left[-\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b\,g}+\frac{B\,PolyLog\left[2,\,1+\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]}{b\,g}$$

Result (type 4, 120 leaves, 10 steps):

$$-\frac{B \ Log \left[g \ \left(a+b \ x\right)\right]^2}{2 \ b \ g}+\frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \left[a \ g+b \ g \ x\right]}{b \ g}+\frac{B \ Log \left[\frac{b \ (c+d \ x)}{b \ c-a \ d}\right] \ Log \left[a \ g+b \ g \ x\right]}{b \ g}+\frac{B \ Poly Log \left[2, \ -\frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{b \ g}$$

### Problem 93: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^2} dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$- \; \frac{B}{b \; g^2 \; \left(a + b \; x\right)} \; - \; \frac{\left(\,c \, + d \; x\,\right) \; \left(A \, + \, B \; Log\left[\,\frac{e \; \left(a + b \; x\,\right)}{c + d \; x}\,\,\right]\,\right)}{\left(\,b \; c \, - \, a \; d\,\right) \; g^2 \; \left(\,a \, + \, b \; x\,\right)}$$

Result (type 3, 102 leaves, 4 steps):

$$-\frac{B}{b\,g^2\,\left(a+b\,x\right)}-\frac{B\,d\,Log\,[\,a+b\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}-\frac{A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]}{b\,g^2\,\left(a+b\,x\right)}+\frac{B\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}$$

### Problem 97: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^4 \left( A + B Log \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 365 leaves, 8 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g^{4} \left(a + b \ x\right)^{4} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{10 \ b \ d} + \frac{g^{4} \left(a + b \ x\right)^{5} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^{2}}{5 \ b} + \frac{B \left(b \ c - a \ d\right)^{2} g^{4} \left(a + b \ x\right)^{3} \left(4 \ A + B + 4 \ B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{30 \ b \ d^{2}} + \frac{B \left(b \ c - a \ d\right)^{4} g^{4} \left(a + b \ x\right) \left(12 \ A + 13 \ B + 12 \ B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{30 \ b \ d^{4}} + \frac{B \left(b \ c - a \ d\right)^{5} g^{4} \ Log\left[\frac{b \ c - a \ d}{b \ (c + d \ x)}\right]}{30 \ b \ d^{5}} + \frac{2 \ B^{2} \left(b \ c - a \ d\right)^{5} g^{4} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{5 \ b \ d^{5}}$$

Result (type 4, 557 leaves, 28 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^4\,g^4\,x}{5\,d^4} + \frac{13\,B^2\,\left(b\,c-a\,d\right)^4\,g^4\,x}{30\,d^4} - \frac{7\,B^2\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^2}{60\,b\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3}{30\,b\,d^2} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^4\,g^4\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{5\,b\,d^4} - \frac{B\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{15\,b\,d^2} - \frac{B\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b\,d} + \frac{2\,B\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b\,d} - \frac{B\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b\,d} - \frac{B\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{10\,b\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{10\,b\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{10\,b\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{10\,b\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{10\,b\,d^5} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[$$

### Problem 98: Result valid but suboptimal antiderivative.

$$\int \left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2} dx$$

Optimal (type 4, 309 leaves, 7 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g^{3} \left(a + b \ x\right)^{3} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{6 \ b \ d} + \frac{g^{3} \left(a + b \ x\right)^{4} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^{2}}{4 \ b} + \frac{B \left(b \ c - a \ d\right)^{2} \ g^{3} \left(a + b \ x\right)^{2} \left(3 \ A + B + 3 \ B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b \ d^{2}} - \frac{B \left(b \ c - a \ d\right)^{3} \ g^{3} \left(a + b \ x\right) \left(6 \ A + 5 \ B + 6 \ B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b \ d^{3}} - \frac{B \left(b \ c - a \ d\right)^{4} \ g^{3} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{12 \ b \ d^{4}}$$

Result (type 4, 474 leaves, 24 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,x}{2\,d^{3}} - \frac{5\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,x}{12\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}}{12\,b\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{2\,b\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}}{12\,b\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{6\,b\,d} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{6\,b\,d} + \frac{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]}{12\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]}{4\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]^{2}}{4\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]^{2}}{2\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,d^{4}\,B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{2\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,d^{4}\,B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{2\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,d^{4}\,B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,d^$$

### Problem 99: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^{2} \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)^{2} dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,b\,d}+\frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{3\,b}+\frac{B\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(a+b\,x\right)\,\left(2\,A+B+2\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,b\,d^{2}}+\frac{B\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\left(2\,A+3\,B+2\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,b\,d^{3}}+\frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{3\,b\,d^{3}}$$

Result (type 4, 389 leaves, 20 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{3\,b\,d^{2}} - \frac{B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b\,d^{3}} + \frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{3\,b\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2}\,B^{2$$

#### Problem 100: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)^{2} dx$$

#### Optimal (type 4, 180 leaves, 5 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b\,d}+\frac{g\,\left(a+b\,x\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{\,2}}{2\,b}-\\ \frac{B\,\left(b\,c-a\,d\right)^{\,2}\,g\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b\,d^{\,2}}-\frac{B^{\,2}\,\left(b\,c-a\,d\right)^{\,2}\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b\,d^{\,2}}$$

#### Result (type 4, 285 leaves, 16 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,x}{d} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{b\,d} + \\ \frac{g\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{2\,b} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[c+d\,x\right]}{b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{b\,d^2} + \\ \frac{B\,\left(b\,c-a\,d\right)^2\,g\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,Log\left[c+d\,x\right]}{b\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[c+d\,x\right]^2}{2\,b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b\,d^2}$$

#### Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{a g + b g x} dx$$

Optimal (type 4, 128 leaves, 4 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\frac{\mathsf{e}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\big]\right)^2\,\mathsf{Log}\big[1-\frac{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}} + \frac{2\,\mathsf{B}\,\,\Big(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\frac{\mathsf{e}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\big]\Big)\,\mathsf{PolyLog}\big[2\,,\,\,\frac{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\big[3\,,\,\,\frac{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}}$$

Result (type 4, 728 leaves, 46 steps):

$$\frac{A\,B\,Log\big[g\,\left(a+b\,x\right)\big]^2}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\right)\big]^3}{3\,b\,g} - \frac{B^2\,Log\big[a+b\,x\big]^2\,Log\big[-c-d\,x\big]}{b\,g} + \frac{2\,B^2\,Log\big[a+b\,x\big]\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[-c-d\,x\big]}{b\,g} - \frac{b\,g}{b\,g} - \frac{b\,g\,Log\big[g\,\left(a+b\,x\right)\big]\,Log\big[\frac{1}{c+d\,x}\big]^2}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[\frac{1}{c+d\,x}\big]^2}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[\frac{1}{c+d\,x}\big]^2}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[\frac{b\,(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[\frac{b\,(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[\frac{1}{c+d\,x}\big]^2}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} - \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[g\,\left(a+b\,x\big)\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} - \frac{B^2\,Log\big[\frac{e\,(a+b\,x)}{b\,c-a\,d}\big]\,Log\big[a\,g+b\,g\,x\big]}{b\,g} + \frac{B^2\,Log\big[\frac{e\,(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{B^2\,Log\big[\frac{e\,(a+b\,x$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(a g + b g x\right)^{2}} dx$$

Optimal (type 3, 126 leaves, 3 steps):

$$-\frac{2 \ B^2 \ \left(c+d \ x\right)}{\left(b \ c-a \ d\right) \ g^2 \ \left(a+b \ x\right)} - \frac{2 \ B \ \left(c+d \ x\right) \ \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{\left(b \ c-a \ d\right) \ g^2 \ \left(a+b \ x\right)} - \frac{\left(c+d \ x\right) \ \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^2}{\left(b \ c-a \ d\right) \ g^2 \ \left(a+b \ x\right)}$$

Result (type 4, 470 leaves, 26 steps):

$$\frac{2 \, B^2}{b \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{B^2 \, d \, Log \, \left[a + b \, x\right]^2}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, B \, d \, Log \, \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B^2 \, d \, Log \, \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{2 \, B^2 \, d \, Log \, \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \, \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, Log \, \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B \, d \, Log \, \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \, \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right] \, Log \, \left[\frac{e \, (a + b \, x)}{b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right] \, Log \, \left[\frac{e \, (a + b \, x)}{b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right] \, Log \, \left[\frac{e \, (a + b \, x)}{b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right] \, Log \, \left[\frac{e \, (a + b \, x)}{b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{2 \, B^2 \, d \, Log \, \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right)$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(a g + b g x\right)^{3}} dx$$

Optimal (type 3, 268 leaves, 7 steps):

$$\frac{2\,B^{2}\,d\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,B^{2}\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{2\,B\,d\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,B\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{b\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^{2}\,\left(a+b\,x\right)^{2}}{2\,\left(a+b\,x\right)^{2}} + \frac{d\,\left(c+d\,x\right)^$$

Result (type 4, 577 leaves, 30 steps):

$$-\frac{B^{2}}{4 \ b \ g^{3} \ \left(a + b \ x\right)^{2}} + \frac{3 \ B^{2} \ d}{2 \ b \ \left(b \ c - a \ d\right) \ g^{3} \ \left(a + b \ x\right)} + \frac{3 \ B^{2} \ d^{2} \ Log \left[a + b \ x\right]}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B^{2} \ d^{2} \ Log \left[a + b \ x\right]^{2}}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B \ d^{2} \ Log \left[a + b \ x\right]^{2}}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B \ d^{2} \ Log \left[a + b \ x\right]^{2}}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B \ d^{2} \ Log \left[a + b \ x\right]^{2}}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B \ d^{2} \ Log \left[a + b \ x\right]^{2}}{b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{B \ d^{2} \ Log \left[a + b \ x\right]^{2}}{b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3}} - \frac{\left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]}{2 \ b \ \left(b \$$

## Problem 104: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{\left(ag + bg x\right)^4} dx$$

Optimal (type 3, 418 leaves, 9 steps):

$$-\frac{2\,B^{2}\,d^{2}\,\left(\,c\,+\,d\,x\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)} + \frac{b\,B^{\,2}\,d\,\left(\,c\,+\,d\,\,x\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}} - \frac{2\,b^{\,2}\,B^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}}{27\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \\ \frac{2\,B\,d^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)} + \frac{b\,B\,d\,\left(\,c\,+\,d\,\,x\,\right)^{\,2}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}} - \frac{2\,b^{\,2}\,B\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)}{9\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \\ \frac{d^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)} + \frac{b\,d\,\left(\,c\,+\,d\,\,x\,\right)^{\,2}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)} - \frac{b^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,4}\,\left(\,a\,+\,b\,\,x\,\right)} - \frac{b^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{9\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{9\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{9\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{9\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{9\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,$$

Result (type 4, 680 leaves, 34 steps):

$$\frac{2 \, B^2}{27 \, b \, g^4 \, \left(a + b \, x\right)^3} + \frac{5 \, B^2 \, d}{18 \, b \, \left(b \, c - a \, d\right) \, g^4 \, \left(a + b \, x\right)^2} - \frac{11 \, B^2 \, d^2}{9 \, b \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(a + b \, x\right)} - \frac{11 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{9 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B \, d^3 \, \left(a + b \, x\right)^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B \, d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B \, d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B \, d^3 \, \left(b \, c - a \, d\right)^3 \, g^4}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{2 \, B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, B \, d^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, B \, d^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{2 \, B \, d^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c -$$

Problem 105: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(ag + bgx\right)^{5}} dx$$

Optimal (type 3, 575 leaves, 11 steps):

$$\frac{2 \, B^2 \, d^3 \, \left(\,c + d\,x\,\right)}{\left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)} - \frac{3 \, b \, B^2 \, d^2 \, \left(\,c + d\,x\,\right)^2}{4 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^2} + \frac{2 \, b^2 \, B^2 \, d \, \left(\,c + d\,x\,\right)^3}{9 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^3} - \frac{b^3 \, B^2 \, \left(\,c + d\,x\,\right)^4}{32 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{9 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^3} - \frac{b^3 \, B^2 \, \left(\,c + d\,x\,\right)^4}{32 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{9 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^3} - \frac{b^3 \, B^2 \, \left(\,c + d\,x\,\right)^4 \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{8 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^3} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{3 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^3} - \frac{b^3 \, B^2 \, \left(\,c + d\,x\,\right)^4 \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{8 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{\left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{8 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{8 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,A + B \, Log\left[\frac{e \, (a + b\,x)}{c + d\,x}\right]\right)}{8 \, \left(\,b \, c - a \, d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,a + b\,x\,\right)}{8 \, \left(\,b \, c - a \,d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,a + b\,x\,\right)}{8 \, \left(\,b \, c - a \,d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,a + b\,x\,\right)}{8 \, \left(\,b \, c - a \,d\,\right)^4 \, g^5 \, \left(\,a + b\,x\,\right)^4} + \frac{2 \, B \, d^3 \, \left(\,c + d\,x\,\right) \, \left(\,a + b\,x\,\right) \, \left(\,a + b\,x\,\right) \, \left(\,a + b\,x\,\right)^4}{8 \, \left(\,a + b\,x\,\right)^3} + \frac{2 \, B \, d^3 \, \left(\,a + b\,$$

#### Result (type 4, 763 leaves, 38 steps):

$$\frac{B^{2}}{32 \, b \, g^{5} \, \left(a + b \, x\right)^{4}} + \frac{7 \, B^{2} \, d}{72 \, b \, \left(b \, c - a \, d\right) \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{13 \, B^{2} \, d^{2}}{48 \, b \, \left(b \, c - a \, d\right)^{2} \, g^{5} \, \left(a + b \, x\right)^{2}} + \frac{25 \, B^{2} \, d^{3}}{24 \, b \, \left(b \, c - a \, d\right)^{3} \, g^{5} \, \left(a + b \, x\right)} + \frac{25 \, B^{2} \, d^{3}}{24 \, b \, \left(b \, c - a \, d\right)^{3} \, g^{5} \, \left(a + b \, x\right)} + \frac{25 \, B^{2} \, d^{4} \, Log \left[a + b \, x\right]}{24 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B^{2} \, d^{4} \, Log \left[a + b \, x\right]^{2}}{4 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{8 \, b \, g^{5} \, \left(a + b \, x\right)^{4}} + \frac{B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{6 \, b \, \left(b \, c - a \, d\right) \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{B \, d^{4} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b \, \left(b \, c - a \, d\right) \, g^{5} \, \left(a + b \, x\right)^{2}} + \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, d^{4} \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, d^{4} \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} - \frac{B \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} + \frac{B^{2} \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} + \frac{B^{2} \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} + \frac{B^{2} \, d^{4} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{2 \, b \, \left(b \, c - a \, d\right)^{4} \, g^{5}} + \frac{B^{2} \, d^{4} \, Log \left[a + b$$

### Problem 109: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{A + B \log\left[\frac{e (a + b x)}{c + d x}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(ag+bgx\right)^{2}}{A+BLog\left[\frac{e(a+bx)}{c+dx}\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$a^2 \, g^2 \, \text{CannotIntegrate} \big[ \, \frac{1}{A + B \, \text{Log} \big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \big]} \,, \, \, x \, \big] \, + \, 2 \, a \, b \, g^2 \, \text{CannotIntegrate} \big[ \, \frac{x}{A + B \, \text{Log} \big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \big]} \,, \, \, x \, \big] \, + \, b^2 \, g^2 \, \text{CannotIntegrate} \big[ \, \frac{x^2}{A + B \, \text{Log} \big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \big]} \,, \, \, x \, \big]$$

#### Problem 110: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{A + B Log\left[\frac{e (a+b x)}{c+d x}\right]}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})}{\mathsf{c} + \mathsf{d} \, \mathsf{x}} \Big]} \text{, } \mathsf{x} \, \Big] + \mathsf{b} \, \mathsf{g} \, \mathsf{CannotIntegrate} \Big[ \frac{\mathsf{x}}{\mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})}{\mathsf{c} + \mathsf{d} \, \mathsf{x}} \Big]} \text{, } \mathsf{x} \, \Big]$$

#### Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}\,\mathrm{d}x$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag + bg x\right)\left(A + B Log\left[\frac{e(a+bx)}{c+dx}\right]\right)}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)}, x\right]$$

# Problem 112: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)} dx$$

Optimal (type 4, 50 leaves, 3 steps):

$$\frac{e \ e^{A/B} \ ExpIntegralEi\left[-\frac{A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{B}\right]}{B \ \left(b \ c \ -a \ d\right) \ g^2}$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag+bgx\right)^{2}\left(A+BLog\left[\frac{e(a+bx)}{c+dx}\right]\right)}, x\right]$$

#### Problem 113: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)} dx$$

Optimal (type 4, 107 leaves, 7 steps):

$$\frac{b \ e^2 \ e^{\frac{zA}{B}} \ ExpIntegralEi\left[-\frac{2\left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{B}\right]}{B \ \left(b \ c-a \ d\right)^2 g^3} - \frac{d \ e \ e^{A/B} \ ExpIntegralEi\left[-\frac{A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{B}\right]}{B \ \left(b \ c-a \ d\right)^2 g^3}$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag+bgx\right)^3\left(A+BLog\left[\frac{e(a+bx)}{c+dx}\right]\right)}$$
,  $x\right]$ 

#### Problem 114: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2},\,x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$a^2 \, g^2 \, \text{CannotIntegrate} \, \Big[ \, \frac{1}{ \Big( A \, + \, B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \Big)^2} \, \text{, } x \, \Big] \, + \, \frac{1}{c + d \, x} \, \Big] \, \Big] \,$$

$$2 \text{ a b } g^2 \text{ CannotIntegrate} \Big[ \frac{x}{\left( A + B \text{ Log} \Big[ \frac{e \cdot (a + b \cdot x)}{c + d \cdot x} \Big] \right)^2} \text{, } x \Big] + b^2 g^2 \text{ CannotIntegrate} \Big[ \frac{x^2}{\left( A + B \text{ Log} \Big[ \frac{e \cdot (a + b \cdot x)}{c + d \cdot x} \Big] \right)^2} \text{, } x \Big]$$

# Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^{2}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{\left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)^2}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\left( \text{A} + \text{B Log} \Big[ \frac{e \cdot (\text{a} + \text{b x})}{c + \text{d x}} \Big] \right)^2} \text{, x} \Big] + \text{b g CannotIntegrate} \Big[ \frac{x}{\left( \text{A} + \text{B Log} \Big[ \frac{e \cdot (\text{a} + \text{b x})}{c + \text{d x}} \Big] \right)^2} \text{, x} \Big]$$

# Problem 116: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag+bgx\right)\left(A+BLog\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}},x\right]$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{\,2}}$$
,  $x\right]$ 

# Problem 117: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 4, 103 leaves, 4 steps):

$$-\frac{\text{e } \text{ } \text{ } \text{ } \text{ExpIntegralEi} \left[-\frac{\text{A}+\text{B Log}\left[\frac{\text{e } (\text{a}+\text{b x})}{\text{c}+\text{d x}}\right]}{\text{B}}\right]}{\text{B}^2 \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ } \text{g}^2} - \frac{\text{c } + \text{d } \text{x}}{\text{B} \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ } \text{g}^2 \left(\text{a } + \text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\frac{\text{e } (\text{a}+\text{b } \text{x})}{\text{c}+\text{d } \text{x}}\right]\right)}{\text{B} \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ } \text{g}^2 \left(\text{a } + \text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\frac{\text{e } (\text{a}+\text{b } \text{x})}{\text{c}+\text{d } \text{x}}\right]\right)}{\text{ExpIntegralEi}}\right)$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag + bg x\right)^{2}\left(A + BLog\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}, x\right]$$

### Problem 118: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 4, 212 leaves, 9 steps):

$$-\frac{2 \text{ b } e^2 \text{ } e^{\frac{2A}{B}} \text{ ExpIntegralEi} \Big[ -\frac{2 \left( A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right] \right)}{B} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B^2 \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B^2 \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ ExpIntegralEi} \Big[ -\frac{A + B \text{ Log} \left[ \frac{e \left( a + b \cdot x \right)}{c + d \cdot x} \right]}{B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \text{ } e \text{ } e^{A/B} \text{ } e \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left( b \text{ } c - a \text{ } d \right)^2 g^3} + \frac{d \text{ } e^{A/B} \text{ } e^{A/B} \text{ } e^{A/B} \Big]}{B \left($$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag+bgx\right)^{3}\left(A+BLog\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}},x\right]$$

# Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e (a+bx)^2}{(c+dx)^2}\right]}{a g + b g x} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{Log\left[-\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\,\right]\right)}{b\,g}+\frac{2\,B\,PolyLog\left[\,2\,,\,\,1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]}{b\,g}$$

Result (type 4, 122 leaves, 10 steps):

$$-\frac{B \, Log \left[g \, \left(a+b \, x\right)\,\right]^2}{b \, g} + \frac{\left(A+B \, Log \left[\frac{e \, \left(a+b \, x\right)^2}{\left(c+d \, x\right)^2}\right]\right) \, Log \left[a \, g+b \, g \, x\right]}{b \, g} + \frac{2 \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right] \, Log \left[a \, g+b \, g \, x\right]}{b \, g} + \frac{2 \, B \, Poly Log \left[2 \, , \, -\frac{d \, \left(a+b \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, B \, Log \left[\frac{b \, \left(c+d \, x\right)}{b \, c-a \, d}\right]}{b \, g} + \frac{d \, Lo$$

# Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e \cdot (a+b \cdot x)^2}{(c+d \cdot x)^2}\right]}{\left(a \cdot g + b \cdot g \cdot x\right)^2} dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$-\frac{2 B}{b g^2 (a+b x)} - \frac{\left(c+d x\right) \left(A+B Log\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)}{\left(b c-a d\right) g^2 (a+b x)}$$

Result (type 3, 105 leaves, 4 steps):

$$-\,\frac{2\,B}{b\,g^2\,\left(a+b\,x\right)}\,-\,\frac{2\,B\,d\,Log\,[\,a+b\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}\,-\,\frac{A+B\,Log\,\left[\,\frac{e\,\,(a+b\,x)^{\,2}}{\left(c+d\,x\right)^{\,2}}\,\right]}{b\,g^2\,\left(a+b\,x\right)}\,+\,\frac{2\,B\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}$$

## Problem 128: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^4\,\left[A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\,\right]\,\right]^2\,\mathrm{d}x$$

Optimal (type 4, 377 leaves, 8 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ g^{4} \ \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)}{5 \ b \ d} + \frac{g^{4} \ \left(a+b \ x\right)^{5} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{5 \ b} + \frac{2 \ B \left(b \ c-a \ d\right)^{2} \ g^{4} \ \left(a+b \ x\right)^{3} \ \left(2 \ A+B+2 \ B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)}{15 \ b \ d^{3}} + \frac{g^{4} \ \left(a+b \ x\right)^{5} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)}{2 \ b \ d^{3}} + \frac{2 \ B \left(b \ c-a \ d\right)^{4} \ g^{4} \ \left(a+b \ x\right) \ \left(6 \ A+13 \ B+6 \ B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)}{15 \ b \ d^{5}} + \frac{2 \ B \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ g^{4} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{5 \ b \ d^{5}} + \frac{2 \ B \ B^{2} \left(b \ c-a \ d\right)^{5} \ B^{2} \ B^{2} \left(b \ c-a \ d\right)^{5} \ B^{2} \ B^{2} \left(b \ c-a \ d\right)^{5} \ B^{2} \ B^{2} \left(b \ c-a \ d\right)^{5} \ B^{2} \ B^{2} \left(b \ c-a \ d\right)^{5} \ B$$

Result (type 4, 569 leaves, 28 steps):

$$\frac{4\,A\,B\,\left(b\,c-a\,d\right)^4\,g^4\,x}{5\,d^4} + \frac{26\,B^2\,\left(b\,c-a\,d\right)^4\,g^4\,x}{15\,d^4} - \frac{7\,B^2\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^2}{15\,b\,d^3} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3}{15\,b\,d^2} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3}{15\,b\,d^2} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^4\,g^4\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]}{5\,b\,d^4} - \frac{2\,B\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)}{5\,b\,d^3} + \frac{5\,b\,d^3}{15\,b^3} + \frac{4\,B\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)}{15\,b\,d^2} - \frac{B\,\left(b\,c-a\,d\right)\,g^4\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)}{5\,b\,d} + \frac{5\,b\,d^4}{15\,b^3} + \frac{5\,b\,d^5}{15\,b^3} + \frac{8\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{5\,b\,d^5} - \frac{4\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{5\,b\,d^5} + \frac{8\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{5\,b\,d^5} + \frac{8\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} + \frac{6\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,B^2\,B^2\,\left(b\,c-a\,d\right)^5\,B^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}$$

## Problem 129: Result valid but suboptimal antiderivative.

$$\int (ag + bg x)^{3} \left(A + B Log \left[\frac{e(a + bx)^{2}}{(c + dx)^{2}}\right]\right)^{2} dx$$

Optimal (type 4, 319 leaves, 7 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right)^{3} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)}{3 \ b \ d} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{4 \ b} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)^{2}}{(c+d \ x)^{2}}\right]\right)^{2}}{3 \ b \ d^{4}} + \frac{g^{3} \left(a+b \ x\right)^{4} \left(a+b$$

Result (type 4, 470 leaves, 24 steps):

$$-\frac{A\ B\ \left(b\ C-a\ d\right)^{3}\ g^{3}\ x}{d^{3}}-\frac{5\ B^{2}\ \left(b\ C-a\ d\right)^{3}\ g^{3}\ x}{3\ d^{3}}+\frac{B^{2}\ \left(b\ C-a\ d\right)^{2}\ g^{3}\ \left(a+b\ x\right)^{2}}{3\ b\ d^{2}}-\frac{B^{2}\ \left(b\ C-a\ d\right)^{3}\ g^{3}\ \left(a+b\ x\right)\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ d^{3}}+\frac{B\ \left(b\ C-a\ d\right)^{2}\ g^{3}\ \left(a+b\ x\right)^{2}}{3\ b\ d^{2}}-\frac{B\ \left(b\ C-a\ d\right)^{3}\ g^{3}\ \left(a+b\ x\right)\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ d^{3}}+\frac{B\ \left(b\ C-a\ d\right)^{2}\ g^{3}\ \left(a+b\ x\right)^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]\right)}{3\ b\ d^{4}}+\frac{B\ \left(b\ C-a\ d\right)^{4}\ g^{3}\ Log\left[c+d\ x\right]}{3\ b\ d^{4}}-\frac{2\ B^{2}\ \left(b\ C-a\ d\right)^{4}\ g^{3}\ Log\left[-\frac{d\ (a+b\ x)}{b\ C-a\ d}\right]\ Log\left[c+d\ x\right]}{b\ d^{4}}+\frac{B\ \left(b\ C-a\ d\right)^{4}\ g^{3}\ Log\left[c+d\ x\right]^{2}}{b\ d^{4}}-\frac{2\ B^{2}\ \left(b\ C-a\ d\right)^{4}\ g^{3}\ PolyLog\left[2,\frac{b\ (c+d\ x)}{b\ C-a\ d}\right]}{b\ d^{4}}$$

## Problem 130: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)^2}{\left(\,c+d\,x\right)^2}\,\right]\,\right)^2\,\mathrm{d}x$$

#### Optimal (type 4, 255 leaves, 6 steps):

$$-\frac{2 \ B \ \left(b \ C - a \ d\right) \ g^{2} \ \left(a + b \ x\right)^{2} \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]\right)}{3 \ b \ d} + \frac{g^{2} \ \left(a + b \ x\right)^{3} \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]\right)^{2}}{3 \ b} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{2} \ g^{2} \ \left(a + b \ x\right) \ \left(A + B + B \ Log\left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]\right)}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{2} \ g^{2} \ \left(a + b \ x\right) \ \left(A + B + B \ Log\left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]\right)}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ PolyLog\left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{3 \ b \ d^{3}} + \frac{4 \ B \ \left(b \ C - a \ d\right)^{3} \ g^{2} \ \left(a + b \ x\right) \ \left$$

### Result (type 4, 397 leaves, 20 steps):

$$\frac{4\,A\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} + \frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} + \frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]}{3\,b\,d^{2}} - \frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b\,d^{3}} + \frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,b\,d^{3}} - \frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{b\,d^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{3\,b\,d^{3}}{3\,b\,d^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]^{2}}{3\,b\,d^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{1}{10\,a^{3}} + \frac{1}{10\,a^{3}$$

## Problem 131: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\,\Big[\,\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\,\Big]\,\right)^{2}\,\mathrm{d}x$$

#### Optimal (type 4, 188 leaves, 5 steps):

$$-\frac{2 \text{ B } \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ g } \left(\text{a}+\text{b } \text{x}\right) \cdot \left(\text{A}+\text{B } \text{Log}\left[\frac{\text{e } \left(\text{a}+\text{b } \text{x}\right)^{2}}{\left(\text{c}+\text{d } \text{x}\right)^{2}}\right]\right)}{\text{b } \text{d}} + \frac{\text{g } \left(\text{a}+\text{b } \text{x}\right)^{2} \cdot \left(\text{A}+\text{B } \text{Log}\left[\frac{\text{e } \left(\text{a}+\text{b } \text{x}\right)^{2}}{\left(\text{c}+\text{d } \text{x}\right)^{2}}\right]\right)^{2}}{2 \text{ b}} - \frac{2 \text{ B } \left(\text{b } \text{c}-\text{a } \text{d}\right)^{2} \text{ g } \left(\text{A}+\text{2 B } +\text{B } \text{Log}\left[\frac{\text{e } \left(\text{a}+\text{b } \text{x}\right)^{2}}{\left(\text{c}+\text{d } \text{x}\right)^{2}}\right]\right) \text{ Log}\left[\frac{\text{b } \text{c}-\text{a } \text{d}}{\text{b } \left(\text{c}+\text{d } \text{x}\right)}\right]}{\text{b } \text{d}^{2}} - \frac{4 \text{ B}^{2} \cdot \left(\text{b } \text{c}-\text{a } \text{d}\right)^{2} \text{ g PolyLog}\left[\text{2, } \frac{\text{d } \left(\text{a}+\text{b } \text{x}\right)}{\text{b } \left(\text{c}+\text{d } \text{x}\right)}\right]}{\text{b } \text{d}^{2}}$$

#### Result (type 4, 291 leaves, 16 steps):

$$-\frac{2\,A\,B\,\left(b\,c-a\,d\right)\,g\,x}{d} - \frac{2\,B^2\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]}{b\,d} + \\ \frac{g\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)^2}{2\,b} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[c+d\,x\right]}{b\,d^2} - \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{b\,d^2} + \\ \frac{2\,B\,\left(b\,c-a\,d\right)^2\,g\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)\,Log\left[c+d\,x\right]}{b\,d^2} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[c+d\,x\right]^2}{b\,d^2} - \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,PolyLog\left[2,\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\right]}{b\,d^2}$$

## Problem 132: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}}{a g + b g x} dx$$

#### Optimal (type 4, 132 leaves, 4 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\frac{\mathsf{e}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})^2}{(\mathsf{c}+\mathsf{d}\,\mathsf{x})^2}\big]\right)^2\,\mathsf{Log}\big[1-\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}}+\frac{4\,\mathsf{B}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\frac{\mathsf{e}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})^2}{(\mathsf{c}+\mathsf{d}\,\mathsf{x})^2}\big]\right)\,\mathsf{PolyLog}\big[2\,\text{,}\,\,\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}}+\frac{8\,\mathsf{B}^2\,\mathsf{PolyLog}\big[3\,\text{,}\,\,\frac{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}\big]}{\mathsf{b}\,\mathsf{g}}$$

Result (type 4, 749 leaves, 46 steps):

Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}}{\left(ag + bgx\right)^{2}} dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$-\frac{8\,B^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}-\frac{4\,B\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]\right)}{\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}-\frac{\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]\right)^{2}}{\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}$$

Result (type 4, 480 leaves, 26 steps):

$$-\frac{8 \, B^2}{b \, g^2 \, \left(a + b \, x\right)} - \frac{8 \, B^2 \, d \, Log \left[a + b \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B^2 \, d \, Log \left[a + b \, x\right]^2}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{4 \, B \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{b \, g^2 \, \left(a + b \, x\right)} - \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{4 \, B \, d \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} - \frac{8 \, B^2 \, d \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{e \, (a + b \, x)}{(c + d \, x)^2}\right]}{b \, \left(b \, c - a \, d\right) \, g^2} + \frac{4 \, B \, d \, Log \left[a + b \, x\right]$$

Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}}{\left(a g + b g x\right)^{3}} dx$$

Optimal (type 3, 272 leaves, 7 steps):

$$\frac{8 \, B^2 \, d \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b \, B^2 \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)^2} + \frac{4 \, B \, d \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b \, B \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)^2}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)^2}{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)}$$

Result (type 4, 579 leaves, 30 steps):

$$-\frac{B^{2}}{b\ g^{3}\ \left(a+b\ x\right)^{2}} + \frac{6\ B^{2}\ d}{b\ \left(b\ c-a\ d\right)\ g^{3}\ \left(a+b\ x\right)} + \frac{6\ B^{2}\ d^{2}\ Log\left[a+b\ x\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{2\ B^{2}\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{B\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]\right)}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{B\ d^{2}\ Log\left[a+b\ x\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{B\ d^{2}\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{B\ d^{2}\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ d^{2}\ Log\left[a+b\ x\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ Log\left[\frac{e\ (a+b\ x)^{2}}{(c+d\ x)^{2}}\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ g^{3}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}} - \frac{A\ B\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}}{b\ \left(b\ c-a\ d\right)^{2}\ B\ d^{2}\ Log\left[a+b\ x\right]^{2}} + \frac{A\ B\ B\ d\ B\ d\ B\ d\ B\ d\ B\ d^$$

# Problem 135: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}}{\left(ag + bgx\right)^{4}} dx$$

Optimal (type 3, 429 leaves, 9 steps):

$$-\frac{8 \, B^2 \, d^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} + \frac{2 \, b \, B^2 \, d \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{8 \, b^2 \, B^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} - \frac{4 \, B \, d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^2}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} + \frac{2 \, b \, B \, d \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^2}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} - \frac{4 \, b^2 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^3}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{4 \, b^2 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^3}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{4 \, b^2 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^3}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{4 \, b^2 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^3}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} - \frac{4 \, b^2 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{\left(c + d \, x\right)^3}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)}$$

Result (type 4, 692 leaves, 34 steps):

$$-\frac{8 \, B^2}{27 \, b \, g^4 \, \left(a + b \, x\right)^3} + \frac{10 \, B^2 \, d}{9 \, b \, \left(b \, c - a \, d\right) \, g^4 \, \left(a + b \, x\right)^2} - \frac{44 \, B^2 \, d^2}{9 \, b \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(a + b \, x\right)} - \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{9 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{4 \, B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{4 \, B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^2 \, \left(a + b \, x\right)^2}{3 \, b \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(a + b \, x\right)} - \frac{4 \, B \, d^3 \, Log \left[a + b \, x\right] \, \left(a + b \, x\right)^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b \, x\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{4 \, B \, d^3 \, \left(a + b$$

# Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\, \frac{e \, \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})^{\, 2}}{\left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)^{\, 2}}\,\right]\,\right)^{\, 2}}{\left(\mathsf{a} \, \mathsf{g} + \mathsf{b} \, \mathsf{g} \, \mathsf{x}\right)^{\, 5}} \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 587 leaves, 11 steps):

$$\frac{8 \, B^2 \, d^3 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{3 \, b \, B^2 \, d^2 \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^2} + \frac{8 \, b^2 \, B^2 \, d \, \left(c + d \, x\right)^3}{9 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, B^2 \, \left(c + d \, x\right)^4}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} + \frac{4 \, B \, d^3 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{3 \, b \, B \, d^2 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} + \frac{4 \, b^2 \, B \, d \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, B \, \left(c + d \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} + \frac{d^3 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}\right)}{\left(b \, c - a \, d\right)^4 \, g^5 \, \left(a + b \, x\right)^3} - \frac{b^3 \, \left(c +$$

### Result (type 4, 757 leaves, 38 steps):

$$-\frac{B^{2}}{8 \ b \ g^{5} \ (a + b \ x)^{4}} + \frac{7 \ B^{2} \ d}{18 \ b \ (b \ c - a \ d)} \frac{13 \ B^{2} \ d^{2}}{12 \ b \ (b \ c - a \ d)^{2} \ g^{5} \ (a + b \ x)^{2}} + \frac{25 \ B^{2} \ d^{3}}{6 \ b \ (b \ c - a \ d)^{3} \ g^{5} \ (a + b \ x)} + \frac{25 \ B^{2} \ d^{3}}{6 \ b \ (b \ c - a \ d)^{3} \ g^{5} \ (a + b \ x)} + \frac{25 \ B^{2} \ d^{4} \ Log \left[a + b \ x\right]^{2}}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{B \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{4 \ b \ g^{5} \ (a + b \ x)^{4}} + \frac{B \ d \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{3 \ b \ (b \ c - a \ d)^{2} \ g^{5} \ (a + b \ x)^{3}} - \frac{B \ d^{3} \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{b \ (b \ c - a \ d)^{2} \ g^{5} \ (a + b \ x)} + \frac{B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{b \ (b \ c - a \ d)^{2} \ g^{5} \ (a + b \ x)} + \frac{B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right])}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (A + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ (a + B \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (a + b \ Log \left[\frac{e \ (a + b \ x)^{2}}{(c + d \ x)^{2}}\right]}{b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{A \ B \ d^{4} \ Log \left[a + b \ x\right] \ (a + b \ x)^{2} \ B^{4} \ (a + b \ Log \left[a + b \ x\right] \ (a + b \ x)^{2} \ (a + b \ x)^{2$$

# Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]} dx$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]}$$
,  $x\right]$ 

Result (type 8, 103 leaves, 2 steps):

$$a^{2} g^{2} CannotIntegrate \Big[ \frac{1}{A + B Log \Big[ \frac{e (a+b x)^{2}}{(c+d x)^{2}} \Big]}, x \Big] + 2 a b g^{2} CannotIntegrate \Big[ \frac{x}{A + B Log \Big[ \frac{e (a+b x)^{2}}{(c+d x)^{2}} \Big]}, x \Big] + b^{2} g^{2} CannotIntegrate \Big[ \frac{x^{2}}{A + B Log \Big[ \frac{e (a+b x)^{2}}{(c+d x)^{2}} \Big]}, x \Big]$$

# Problem 138: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B Log \left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{A + B Log \left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})^2}{(\mathsf{c} + \mathsf{d} \, \mathsf{x})^2} \Big] }, \, \, \mathsf{x} \, \Big] + \mathsf{b} \, \mathsf{g} \, \mathsf{CannotIntegrate} \Big[ \frac{\mathsf{x}}{\mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})^2}{(\mathsf{c} + \mathsf{d} \, \mathsf{x})^2} \Big] }, \, \, \mathsf{x} \, \Big]$$

## Problem 139: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)}\,d!x$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag+bgx\right)\left(A+BLog\left[\frac{e(a+bx)^{2}}{\left(c+dx\right)^{2}}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]\right)}$$
,  $x\right]$ 

# Problem 140: Unable to integrate problem.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)}\,dlx$$

Optimal (type 4, 91 leaves, 3 steps):

$$\frac{e^{\frac{A}{2B}}\sqrt{\frac{e\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}}\left(c+d\,x\right)\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[\frac{e\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]}{2\,B}\right]}{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[ \frac{1}{\left( a\,g + b\,g\,x \right)^2 \left( A + B\,Log \left[ \frac{e\,(a+b\,x)^2}{(c+d\,x)^2} \right] \right)} \,$$
,  $x \right]$ 

# Problem 141: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$\frac{b \; e \; e^{A/B} \; ExpIntegralEi\left[-\frac{A+B \; Log\left[\frac{e \; (a+b \; x)^2}{\left(c+d \; x\right)^2}\right]}{B}\right]}{2 \; B \; \left(b \; c-a \; d\right)^2 \; g^3} \; - \; \frac{d \; e^{\frac{A}{2 \; B}} \; \sqrt{\frac{e \; (a+b \; x)^2}{\left(c+d \; x\right)^2}} \; \left(c+d \; x\right) \; ExpIntegralEi\left[-\frac{A+B \; Log\left[\frac{e \; (a+b \; x)^2}{\left(c+d \; x\right)^2}\right]}{2 \; B}\right]}{2 \; B \; \left(b \; c-a \; d\right)^2 \; g^3 \; \left(a+b \; x\right)}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \, \frac{1}{ \left( \text{a g} + \text{b g x} \right)^3 \, \left( \text{A} + \text{B Log} \Big[ \, \frac{\text{e} \, \left( \text{a} + \text{b x} \, \right)^2}{\left( \text{c} + \text{d x} \, \right)^2} \, \Big] \, \right) } \, , \, \, x \, \Big]$$

# Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[\frac{e\cdot(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}\,\mathrm{d}x$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)^2}$$
,  $x\right]$ 

Result (type 8, 103 leaves, 2 steps):

$$a^{2} g^{2} \, \text{CannotIntegrate} \Big[ \frac{1}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \\ \\ 2 \, a \, b \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \, \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big] \right)^{\, 2}}$$

# Problem 143: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{\left(A + B Log \left[\frac{e \cdot (a + b x)^2}{(c + d x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\left( \text{A} + \text{B Log} \Big[ \frac{e \cdot (\text{a} + \text{b x})^2}{\left( \text{c} + \text{d x} \right)^2} \Big] \right)^2} \text{, } \text{x} \Big] + \text{b g CannotIntegrate} \Big[ \frac{\text{x}}{\left( \text{A} + \text{B Log} \Big[ \frac{e \cdot (\text{a} + \text{b x})^2}{\left( \text{c} + \text{d x} \right)^2} \Big] \right)^2} \text{, } \text{x} \Big]$$

## Problem 144: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)^{\,2}}$$
,  $x\right]$ 

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \frac{1}{ \left( \text{a g} + \text{b g x} \right) \ \left( \text{A} + \text{B Log} \Big[ \frac{e \cdot (\text{a+b x})^2}{(\text{c+d x})^2} \Big] \right)^2 } \text{, x} \Big]$$

# Problem 145: Unable to integrate problem.

$$\int \frac{1}{\left(ag + bg x\right)^{2} \left(A + B Log\left[\frac{e(a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$-\frac{e^{\frac{A}{2\,B}}\,\sqrt{\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}}\,\,\left(c+d\,x\right)\,\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]}{2\,B}\right]}{4\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}-\frac{c+d\,x}{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,\left(A+B\,\text{Log}\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^2\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}$$
,  $x\right]$ 

## Problem 146: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 4, 263 leaves, 9 steps):

$$-\frac{b \ e \ e^{A/B} \ ExpIntegralEi}{2 \ B^2 \ (b \ c - a \ d)^2 \ g^3} + \frac{d \ e^{\frac{A}{2 \, B}} \sqrt{\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}} \ \left(c + d \ x\right) \ ExpIntegralEi}{4 \ B^2 \ (b \ c - a \ d)^2 \ g^3 \ \left(a + b \ x\right)} \\ -\frac{d \ (c + d \ x)}{2 \ B} + \frac{d \ e^{\frac{A}{2 \, B}} \sqrt{\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}} \ \left(c + d \ x\right) \ ExpIntegralEi}{4 \ B^2 \ (b \ c - a \ d)^2 \ g^3 \ \left(a + b \ x\right)} \\ -\frac{d \ (c + d \ x)}{2 \ B} + \frac{d \ e^{\frac{A}{2 \, B}} \sqrt{\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}} \ \left(c + d \ x\right) \ ExpIntegralEi} \left[-\frac{A+B \ Log\left[\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}\right]}{2 \ B} \right]} \\ -\frac{d \ (c + d \ x)}{2 \ B \ (b \ c - a \ d)^2 \ g^3 \ (a + b \ x)} \left(A + B \ Log\left[\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}\right]\right)}{2 \ B \ (b \ c - a \ d)^2 \ g^3 \ (a + b \ x)^2 \ \left(A + B \ Log\left[\frac{e \ (a+b \ x)^2}{(c+d \ x)^2}\right]\right)}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}}, x\right]$$

# Problem 147: Result valid but suboptimal antiderivative.

$$\left[ \left( a+b\;x\right) ^{4}\;\left( A+B\;Log\left[ e\;\left( a+b\;x\right) ^{n}\;\left( c+d\;x\right) ^{-n}\right] \right)\;\text{d}x$$

Optimal (type 3, 171 leaves, 3 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 \, n \, x}{5 \, d^4} - \frac{B \left(b \, c - a \, d\right)^3 \, n \, \left(a + b \, x\right)^2}{10 \, b \, d^3} + \frac{B \left(b \, c - a \, d\right)^2 \, n \, \left(a + b \, x\right)^3}{15 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right)^4}{20 \, b \, d} - \frac{B \left(b \, c - a \, d\right)^5 \, n \, Log \left[c + d \, x\right]}{5 \, b \, d^5} + \frac{\left(a + b \, x\right)^5 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{5 \, b} + \frac{\left(a + b \, x\right)^5 \, \left(a + b \, x\right)^5 \, \left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6}{5 \, b} + \frac{\left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6}{5 \, b} + \frac{\left(a + b \, x\right)^6 \, \left(a + b \, x\right)^6 \,$$

Result (type 3, 183 leaves, 5 steps):

$$\frac{B \, \left( b \, c - a \, d \right)^4 \, n \, x}{5 \, d^4} \, - \, \frac{B \, \left( b \, c - a \, d \right)^3 \, n \, \left( a + b \, x \right)^2}{10 \, b \, d^3} \, + \, \frac{B \, \left( b \, c - a \, d \right)^2 \, n \, \left( a + b \, x \right)^3}{15 \, b \, d^2} \, - \\ \frac{B \, \left( b \, c - a \, d \right) \, n \, \left( a + b \, x \right)^4}{20 \, b \, d} \, + \, \frac{A \, \left( a + b \, x \right)^5}{5 \, b} \, - \, \frac{B \, \left( b \, c - a \, d \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]}{5 \, b} \, - \, \frac{B \, \left( b \, c - a \, d \right)^5 \, n \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b \, d^5} \, + \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a + b \, x \right)^5 \, Log \left[ c + d \, x \right]}{5 \, b} \, - \, \frac{B \, \left( a +$$

# Problem 148: Result valid but suboptimal antiderivative.

$$\left[\left(a+b\,x\right)^3\,\left(A+B\,\text{Log}\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)\,\text{d}x$$

Optimal (type 3, 142 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{3} \, n \, x}{4 \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{2} \, n \, \left(a + b \, x\right)^{2}}{8 \, b \, d^{2}} - \frac{B \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right)^{3}}{12 \, b \, d} + \frac{B \left(b \, c - a \, d\right)^{4} \, n \, Log \left[c + d \, x\right]}{4 \, b \, d^{4}} + \frac{\left(a + b \, x\right)^{4} \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{4 \, b} + \frac{\left(a + b \, x\right)^{4} \, \left(a + b \, x\right)^{4} \, \left(a + b \, x\right)^{6} \, \left(c + d \, x\right)^{-6}}{4 \, b} + \frac{\left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(c + d \, x\right)^{-6}}{4 \, b} + \frac{\left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(c + d \, x\right)^{-6}}{4 \, b} + \frac{\left(a + b \, x\right)^{6} \, \left(a + b$$

Result (type 3, 154 leaves, 5 steps):

$$-\frac{B \left(b \, c - a \, d\right)^3 \, n \, x}{4 \, d^3} + \frac{B \left(b \, c - a \, d\right)^2 \, n \, \left(a + b \, x\right)^2}{8 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right)^3}{12 \, b \, d} + \frac{A \left(a + b \, x\right)^4}{4 \, b} + \frac{B \left(b \, c - a \, d\right)^4 \, n \, Log \left[c + d \, x\right]}{4 \, b \, d^4} + \frac{B \left(a + b \, x\right)^4 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{4 \, b}$$

# Problem 149: Result valid but suboptimal antiderivative.

$$\int (a + b x)^{2} (A + B Log[e (a + b x)^{n} (c + d x)^{-n}]) dx$$

Optimal (type 3, 113 leaves, 3 steps):

$$\frac{B \, \left( b \, c - a \, d \right)^2 \, n \, x}{3 \, d^2} \, - \, \frac{B \, \left( b \, c - a \, d \right) \, n \, \left( a + b \, x \right)^2}{6 \, b \, d} \, - \, \frac{B \, \left( b \, c - a \, d \right)^3 \, n \, Log \left[ c + d \, x \right]}{3 \, b \, d^3} \, + \, \frac{\left( a + b \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right] \right)}{3 \, b} \, d^3 \, d^2 \, d^2$$

Result (type 3, 125 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{2} \, n \, x}{3 \, d^{2}} - \frac{B \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right)^{2}}{6 \, b \, d} + \frac{A \left(a + b \, x\right)^{3}}{3 \, b} - \frac{B \left(b \, c - a \, d\right)^{3} \, n \, Log \left[c + d \, x\right]}{3 \, b \, d^{3}} + \frac{B \left(a + b \, x\right)^{3} \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b \, x\right)^{n}}{3 \, b} + \frac{B \left(a + b$$

# Problem 150: Result valid but suboptimal antiderivative.

$$\left[\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 84 leaves, 3 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, n \, x}{2 \, d} + \frac{B \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[\, c + d \, x\,\right]}{2 \, b \, d^2} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, -n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, -n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, -n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, -n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\, e \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}\,\right]\,\right)}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n} \, \left(c + d \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}{2 \, b} + \frac{\left(a + b \, x\right)^2 \, \left(a + b \, x\right)^{\, n}}$$

Result (type 3, 96 leaves, 5 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, n \, x}{2 \, d} + \frac{A \, \left(a + b \, x\right)^2}{2 \, b} + \frac{B \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[c + d \, x\right]}{2 \, b \, d^2} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^2 \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{2 \, b} + \frac{B \, \left(a + b \, x\right)^n \, \left(c + d \,$$

# Problem 151: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( a + b x \right)^{n} \left( c + d x \right)^{-n} \right]}{a + b x} dx$$

Optimal (type 4, 79 leaves, 5 steps):

$$-\frac{Log\left[-\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)}{b}\,+\,\frac{B\,n\,PolyLog\left[\,2\,\text{, }1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]}{b}$$

Result (type 4, 87 leaves, 7 steps):

$$\frac{A\;Log\left[\,a\,+\,b\;x\,\right]}{b}\;-\;\frac{B\;Log\left[\,-\,\frac{b\;c-a\;d}{d\;(a+b\;x)}\,\right]\;Log\left[\,e\;\left(\,a\,+\,b\;x\,\right)^{\,n}\;\left(\,c\,+\,d\;x\,\right)^{\,-\,n}\,\right]}{b}\;+\;\frac{B\;n\;PolyLog\left[\,2\,,\;1\,+\,\frac{b\;c-a\;d}{d\;(a+b\;x)}\,\right]}{b}$$

# Problem 152: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]}{\left(a + b x\right)^{2}} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$-\frac{B\,n}{b\,\left(a+b\,x\right)}\,-\,\frac{B\,d\,n\,Log\left[\,a+b\,x\,\right]}{b\,\left(\,b\,c\,-\,a\,d\,\right)}\,+\,\frac{B\,d\,n\,Log\left[\,c\,+\,d\,x\,\right]}{b\,\left(\,b\,c\,-\,a\,d\,\right)}\,-\,\frac{A\,+\,B\,Log\left[\,e\,\left(\,a+b\,x\,\right)^{\,n}\,\left(\,c\,+\,d\,x\,\right)^{\,-\,n}\,\right]}{b\,\left(\,a+b\,x\,\right)}$$

Result (type 3, 72 leaves, 4 steps):

$$-\frac{A}{b\left(a+b\,x\right)}-\frac{B\,n}{b\left(a+b\,x\right)}-\frac{B\,\left(c+d\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)\,\left(a+b\,x\right)}$$

# Problem 153: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]}{\left(a + b x\right)^{3}} dx$$

Optimal (type 3, 137 leaves, 3 steps):

$$-\frac{\,B\,n\,}{4\,b\,\left(a+b\,x\right)^{\,2}}+\frac{\,B\,d\,n\,}{2\,b\,\left(b\,c-a\,d\right)\,\left(a+b\,x\right)}+\frac{\,B\,d^{\,2}\,n\,Log\,[\,a+b\,x\,]}{\,2\,b\,\left(b\,c-a\,d\right)^{\,2}}-\frac{\,B\,d^{\,2}\,n\,Log\,[\,c+d\,x\,]}{\,2\,b\,\left(b\,c-a\,d\right)^{\,2}}-\frac{\,A+B\,Log\,\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\,\right]}{\,2\,b\,\left(a+b\,x\right)^{\,2}}$$

Result (type 3, 149 leaves, 5 steps):

$$-\frac{A}{2 \ b \ \left(a + b \ x\right)^2} - \frac{B \ n}{4 \ b \ \left(a + b \ x\right)^2} + \frac{B \ d \ n}{2 \ b \ \left(b \ c - a \ d\right) \ \left(a + b \ x\right)} + \frac{B \ d^2 \ n \ Log \left[a + b \ x\right]}{2 \ b \ \left(b \ c - a \ d\right)^2} - \frac{B \ d^2 \ n \ Log \left[c + d \ x\right]}{2 \ b \ \left(b \ c - a \ d\right)^2} - \frac{B \ Log \left[e \ \left(a + b \ x\right)^n \ \left(c + d \ x\right)^{-n}\right]}{2 \ b \ \left(a + b \ x\right)^2}$$

## Problem 154: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log[e(a + bx)^n(c + dx)^{-n}]}{(a + bx)^4} dx$$

Optimal (type 3, 166 leaves, 3 steps):

$$-\frac{B\,n}{9\,b\,\left(a+b\,x\right)^{\,3}} + \frac{B\,d\,n}{6\,b\,\left(b\,c-a\,d\right)\,\left(a+b\,x\right)^{\,2}} - \frac{B\,d^{\,2}\,n}{3\,b\,\left(b\,c-a\,d\right)^{\,2}\,\left(a+b\,x\right)} - \frac{B\,d^{\,3}\,n\,Log\left[a+b\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{\,3}} + \frac{B\,d^{\,3}\,n\,Log\left[c+d\,x\right]}{3\,b\,\left(b\,c-a\,d\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{B\,d^{\,3}\,n\,Log\left[a+b\,x\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,n}\right]}{3\,b\,\left(a+b\,x\right)^{\,3}} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,n}}{3\,a\,x} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}}{3\,a\,x} - \frac{A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(a+b\,x\right)^{\,n}}{3$$

Result (type 3, 178 leaves, 5 steps):

$$-\frac{A}{3 \ b \ (a + b \ x)^3} - \frac{B \ n}{9 \ b \ (a + b \ x)^3} + \frac{B \ d \ n}{6 \ b \ (b \ c - a \ d) \ (a + b \ x)^2} - \frac{B \ d^3 \ n \ Log \left[a + b \ x\right)^2}{3 \ b \ (b \ c - a \ d)^3} + \frac{B \ d^3 \ n \ Log \left[c + d \ x\right]}{3 \ b \ (b \ c - a \ d)^3} - \frac{B \ Log \left[e \ (a + b \ x)^n \ (c + d \ x)^{-n}\right]}{3 \ b \ (a + b \ x)^3}$$

## Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log[e(a + bx)^n(c + dx)^{-n}]}{(a + bx)^5} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$-\frac{B\,n}{16\,b\,\left(a+b\,x\right)^4} + \frac{B\,d\,n}{12\,b\,\left(b\,c-a\,d\right)\,\left(a+b\,x\right)^3} - \frac{B\,d^2\,n}{8\,b\,\left(b\,c-a\,d\right)^2\,\left(a+b\,x\right)^2} + \\ \frac{B\,d^3\,n}{4\,b\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{B\,d^4\,n\,Log\,[\,a+b\,x\,]}{4\,b\,\left(b\,c-a\,d\right)^4} - \frac{B\,d^4\,n\,Log\,[\,c+d\,x\,]}{4\,b\,\left(b\,c-a\,d\right)^4} - \frac{A+B\,Log\,[\,e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\,]}{4\,b\,\left(a+b\,x\right)^4}$$

Result (type 3, 207 leaves, 5 steps):

$$-\frac{A}{4 \ b \ (a + b \ x)^4} - \frac{B \ n}{16 \ b \ (a + b \ x)^4} + \frac{B \ d \ n}{12 \ b \ (b \ c - a \ d) \ (a + b \ x)^3} - \frac{B \ d^2 \ n}{8 \ b \ (b \ c - a \ d)^2 \ (a + b \ x)^2} + \frac{B \ d^4 \ n \ Log \left[a + b \ x\right]}{4 \ b \ (b \ c - a \ d)^4} - \frac{B \ d^4 \ n \ Log \left[c + d \ x\right]}{4 \ b \ (b \ c - a \ d)^4} - \frac{B \ Log \left[e \ (a + b \ x)^n \ (c + d \ x)^{-n}\right]}{4 \ b \ (a + b \ x)^4}$$

## Problem 156: Result valid but suboptimal antiderivative.

$$\left[ \left( a+b\;x\right) ^{3}\;\left( A+B\;Log\left[ \,e\,\left( \,a+b\;x\right) ^{\,n}\;\left( \,c+d\;x\right) ^{\,-n}\,\right] \,\right) ^{\,2}\;\mathrm{d}x\right.$$

Optimal (type 4, 322 leaves, 8 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ n \left(a+b \ x\right)^{3} \left(A+B \ Log\left[e \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)}{6 b \ d} + \frac{\left(a+b \ x\right)^{4} \left(A+B \ Log\left[e \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)^{2}}{4 b} + \frac{\left(a+b \ x\right)^{4} \left(A+B \ Log\left[e \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)^{2}}{4 b} + \frac{B \left(b \ c-a \ d\right)^{2} n \left(a+b \ x\right)^{2} \left(3 \ A+B \ n+3 \ B \ Log\left[e \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)}{12 b \ d^{2}} - \frac{B \left(b \ c-a \ d\right)^{3} n \left(a+b \ x\right) \left(6 \ A+5 \ B \ n+6 \ B \ Log\left[e \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)}{12 b \ d^{4}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} n^{2} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{2 b \ d^{4}}$$

Result (type 4, 542 leaves, 21 steps):

$$-\frac{A\ B\ \left(b\ c-a\ d\right)^{3}\ n\ x}{2\ d^{3}} - \frac{5\ B^{2}\ \left(b\ c-a\ d\right)^{3}\ n^{2}\ x}{12\ d^{3}} + \frac{A\ B\ \left(b\ c-a\ d\right)^{2}\ n\ \left(a+b\ x\right)^{2}}{4\ b\ d^{2}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{2}\ n^{2}\ \left(a+b\ x\right)^{2}}{12\ b\ d^{2}} - \frac{A\ B\ \left(b\ c-a\ d\right)^{3}\ n\ \left(a+b\ x\right)^{3}}{4\ b} + \frac{A^{2}\ \left(a+b\ x\right)^{4}}{4\ b} + \frac{A\ B\ \left(b\ c-a\ d\right)^{4}\ n\ Log\left[c+d\ x\right]}{12\ b\ d^{4}} + \frac{11\ B^{2}\ \left(b\ c-a\ d\right)^{4}\ n^{2}\ Log\left[c+d\ x\right]}{12\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ n\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{4\ b\ d^{2}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ n\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{4\ b\ d^{2}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{4\ b\ d^{2}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n\ Log\left[e\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n\ Log\left[e\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n\ Log\left[e\ \left(a+b\ x\right)^{n}\ \left(c+d\ x\right)^{-n}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{2\ b\ d^{4}} - \frac{B^{2}\ \left(b\ c-a\ d\right)^{n}\ n^{2}\ n^{2}$$

### Problem 157: Result valid but suboptimal antiderivative.

$$\int \left(\,a\,+\,b\,\,x\,\right)^{\,2}\,\,\left(\,A\,+\,B\,\,Log\,\left[\,e\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,n}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,-\,n}\,\right]\,\right)^{\,2}\,\,\mathrm{d}x$$

#### Optimal (type 4, 263 leaves, 7 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right)^{2} \, \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, b \, d} + \frac{\left(a + b \, x\right)^{3} \, \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)^{2}}{3 \, b} + \frac{B \, \left(b \, c - a \, d\right)^{2} \, n \, \left(a + b \, x\right) \, \left(2 \, A + B \, n + 2 \, B \, Log\left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, b \, d^{3}} + \frac{B \, \left(b \, c - a \, d\right)^{3} \, n \, Log\left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)}\right] \, \left(2 \, A + 3 \, B \, n + 2 \, B \, Log\left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, n^{2} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{3 \, b \, d^{3}}$$

#### Result (type 4, 427 leaves, 18 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,n\,x}{3\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,n^{2}\,x}{3\,d^{2}} - \frac{A\,B\,\left(b\,c-a\,d\right)\,n\,\left(a+b\,x\right)^{2}}{3\,b\,d} + \frac{A^{2}\,\left(a+b\,x\right)^{3}}{3\,b} - \frac{2\,A\,B\,\left(b\,c-a\,d\right)^{3}\,n\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,n^{2}\,Log\left[c+d\,x\right]}{b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,n\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d} + \frac{2\,A\,B\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,n\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,n^{2}\,PolyLog\left[2\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,n^{2}\,PolyLog\left[2\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{n}\right]^{2}}{3\,b\,d^{3}} + \frac{2\,$$

# Problem 158: Result valid but suboptimal antiderivative.

$$\int \left( a + b \, x \right) \, \left( A + B \, Log \left[ \, e \, \left( \, a + b \, x \right)^{\, n} \, \left( \, c + d \, x \right)^{\, -n} \, \right] \, \right)^{\, 2} \, \mathrm{d}x$$

Optimal (type 4, 195 leaves, 6 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}{b\,d}+\frac{\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{2}}{2\,b}}{2\,b}$$

$$-\frac{B\left(b\,c-a\,d\right)^{2}\,n\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,n+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}{b\,d^{2}}-\frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,n^{2}\,PolyLog\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b\,d^{2}}$$

Result (type 4, 308 leaves, 15 steps):

$$-\frac{A\ B\ \left(b\ c-a\ d\right)\ n\ x}{d} + \frac{A^2\ \left(a+b\ x\right)^2}{2\ b} + \frac{A\ B\ \left(b\ c-a\ d\right)^2\ n\ Log\left[c+d\ x\right]}{b\ d^2} + \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ Log\left[c+d\ x\right]}{b\ d^2} - \\ \frac{B^2\ \left(b\ c-a\ d\right)\ n\ \left(a+b\ x\right)\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]}{b} - \\ \frac{B^2\ \left(b\ c-a\ d\right)^2\ n\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]}{b\ d^2} + \frac{A\ B\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]}{b} - \\ \frac{B^2\ \left(b\ c-a\ d\right)^2\ n\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]}{b\ d^2} - \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{2\ b} - \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} - \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} - \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} - \frac{B^2\ \left(b\ c-a\ d\right)^2\ n^2\ PolyLog\left[2,\ \frac{d\ (a+b\ x)}{b\ (c+d\ x)}\right]}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]^2}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right)^2\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^n\ \left(a+b\ x\right)^n\ \left(a+b\ x\right)^n}{b\ d^2} + \frac{B^2\ \left(a+b\ x\right$$

### Problem 159: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e^{\left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{a + b x} dx$$

Optimal (type 4, 131 leaves, 5 steps):

Result (type 4, 227 leaves, 10 steps):

$$\frac{A^2 \, Log \, [\, a \, + \, b \, x \,]}{b} - \frac{2 \, A \, B \, Log \, \left[ \, - \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right] \, Log \, \left[ \, e \, \left( \, a \, + \, b \, x \, \right)^{\, n} \, \left( \, c \, + \, d \, x \, \right)^{\, - n} \, \right]}{b} - \frac{B^2 \, Log \, \left[ \, - \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right] \, Log \, \left[ \, e \, \left( \, a \, + \, b \, x \, \right)^{\, n} \, \left( \, c \, + \, d \, x \, \right)^{\, - n} \, \right]}{b} + \frac{2 \, B^2 \, n \, Log \, \left[ \, e \, \left( \, a \, + \, b \, x \, \right)^{\, n} \, \left( \, c \, + \, d \, x \, \right)^{\, - n} \, \right] \, PolyLog \, \left[ \, 2 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^2 \, n^2 \, PolyLog \, \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{b} + \frac{2 \, B^$$

## Problem 160: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}}{\left(\,a+b\,x\right)^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 3, 129 leaves, 4 steps):

$$-\frac{2 \ B^2 \ n^2 \ \left(c+d \ x\right)}{\left(b \ c-a \ d\right) \ \left(a+b \ x\right)} - \frac{2 \ B \ n \ \left(c+d \ x\right) \ \left(A+B \ Log\left[e \ \left(a+b \ x\right)^n \ \left(c+d \ x\right)^{-n}\right]\right)}{\left(b \ c-a \ d\right) \ \left(a+b \ x\right)} - \frac{\left(c+d \ x\right) \ \left(A+B \ Log\left[e \ \left(a+b \ x\right)^n \ \left(c+d \ x\right)^{-n}\right]\right)^2}{\left(b \ c-a \ d\right) \ \left(a+b \ x\right)}$$

Result (type 3, 189 leaves, 7 steps):

$$-\frac{A^{2}}{b\;\left(a+b\;x\right)}-\frac{2\;A\;B\;n}{b\;\left(a+b\;x\right)}-\frac{2\;B^{2}\;n^{2}}{b\;\left(a+b\;x\right)}-\frac{2\;A\;B\;\left(c+d\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;c-a\;d\right)\;\left(a+b\;x\right)}-\frac{2\;A\;B\;\left(c+d\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;c-a\;d\right)\;\left(a+b\;x\right)}-\frac{B^{2}\;\left(c+d\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]^{2}}{\left(b\;c-a\;d\right)\;\left(a+b\;x\right)}$$

### Problem 161: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{\left(a + b x\right)^{3}} dx$$

Optimal (type 3, 274 leaves, 8 steps):

$$\begin{split} &\frac{2\,B^2\,d\,n^2\,\left(\,c + d\,x\,\right)}{\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{\,b\,B^2\,n^2\,\left(\,c + d\,x\,\right)^{\,2}}{4\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,2}} + \\ &\frac{2\,B\,d\,n\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)}{\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{\,b\,B\,n\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{\,b\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,n}\,\right)^{\,2}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,b\,\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,c + d\,x\,\right)\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}} + \\ &\frac{\,d\,\left(\,a +$$

Result (type 3, 411 leaves, 12 steps):

$$-\frac{A^{2}}{2\;b\;\left(a+b\,x\right)^{2}}-\frac{A\,B\,n}{2\;b\;\left(a+b\,x\right)^{2}}+\frac{A\,B\,d\,n}{b\;\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}+\frac{2\,B^{2}\,d\,n^{2}}{b\;\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{b\,B^{2}\,n^{2}\,\left(c+d\,x\right)^{2}}{4\;\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)^{2}}+\frac{A\,B\,d^{2}\,n\,Log\left[c+d\,x\right]}{4\;\left(b\,c-a\,d\right)^{2}}-\frac{A\,B\,d^{2}\,n\,Log\left[c+d\,x\right]}{b\;\left(b\,c-a\,d\right)^{2}}-\frac{A\,B\,Log\left[e\;\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{b\;\left(a+b\,x\right)^{2}}+\frac{2\,B^{2}\,d\,n\,\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)}-\frac{b\,B^{2}\,n\,\left(c+d\,x\right)^{-n}\left[c+d\,x\right)^{-n}\left[c+d\,x\right]^{-n}}{b\;\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)}+\frac{B^{2}\,d\,\left(c+d\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]^{2}}{\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)}-\frac{b\,B^{2}\,\left(c+d\,x\right)^{2}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{2\;\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)^{2}}+\frac{B^{2}\,d\,\left(c+d\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]^{2}}{\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)}-\frac{b\,B^{2}\,\left(c+d\,x\right)^{2}\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{2\;\left(b\,c-a\,d\right)^{2}\,\left(a+b\,x\right)^{2}}$$

## Problem 162: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{\left(a + b x\right)^{4}} dx$$

Optimal (type 3, 427 leaves, 10 steps):

$$-\frac{2 \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^2} - \frac{2 \, b^2 \, B^2 \, n^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^3} - \frac{2 \, B \, d^2 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, B \, d \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]} - \frac{d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, d \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, d \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, d \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, d \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)} + \frac{b \, d \, n \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^3} + \frac{b \, d \, n \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right)^3}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right)^3} + \frac{b \, d \, n \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n} \, \left(c + d \, x\right)^{-n}\right)^3}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}} + \frac{b \, d \, n \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n} \, \left(c + d \, x\right)^{-n}}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}} + \frac{b \, d \, n \, \left(c + d \, x\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}} + \frac{b \, d \, n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^n \, \left(c + d \, x\right)^{-n}}{\left(b \, c - a \, d\right)^3 \, \left(a + b \, x\right)^n \, \left(c$$

Result (type 4, 730 leaves, 26 steps):

$$-\frac{A^2}{3 \ b \ (a+b \ x)^3} - \frac{2 \ A \ B \ n}{9 \ b \ (a+b \ x)^3} - \frac{2 \ B^2 \ n^2}{27 \ b \ (a+b \ x)^3} + \frac{A \ B \ d \ n}{3 \ b \ (b \ c - a \ d) \ (a+b \ x)^2} + \frac{5 \ B^2 \ d \ n^2}{3 \ b \ (b \ c - a \ d) \ (a+b \ x)} - \frac{2 \ A \ B \ d^3 \ n \ Log \left[a+b \ x\right]}{3 \ b \ (b \ c - a \ d)^2 \ (a+b \ x)} - \frac{2 \ A \ B \ d^3 \ n \ Log \left[a+b \ x\right]}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ A \ B \ d^3 \ n \ Log \left[a+b \ x\right]}{3 \ b \ (b \ c - a \ d)^3} - \frac{3 \ b \ (b \ c - a \ d)^3}{3 \ b \ (a+b \ x)^n \ (c+d \ x)^{-n}} - \frac{2 \ A \ B \ Log \left[a+b \ x\right)^n \ (c+d \ x)^{-n}}{3 \ b \ (a+b \ x)^3} - \frac{2 \ A \ B \ Log \left[a+b \ x\right)^n \ (c+d \ x)^{-n}}{3 \ b \ (a+b \ x)^3} - \frac{2 \ B^2 \ d^3 \ n^2 \ Log \left[a+b \ x\right)^n \ (c+d \ x)^{-n}}{3 \ b \ (a+b \ x)^n \ (c+d \ x)^{-n}} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^{-n}}{3 \ b \ (b \ c - a \ d)^3} + \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^{-n}}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^{-n}}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^{-n}}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^{-n}}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n \ Log \left[a+b \ x\right)^n \ (c+d \ x)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n^2 \ Poly Log \left[a+b \ x\right)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n^2 \ Poly Log \left[a+b \ x\right)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n^2 \ Poly Log \left[a+b \ x\right)^n \ (c+d \ x)^n}{3 \ b \ (b \ c - a \ d)^3} - \frac{2 \ B^2 \ d^3 \ n^2 \ Poly Log \left[a+b \ x\right)^n \ (c+d \ x)^n}{3 \ b \ (a+b \ x)^n \ (c+d \ x)^n} - \frac{2 \ B^2 \ d^3 \ n^2 \ Poly Log \left[a$$

# Problem 163: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{\left(a + b x\right)^{5}} dx$$

#### Optimal (type 3, 587 leaves, 12 steps):

$$\frac{2 \, B^2 \, d^3 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} - \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^2} + \frac{2 \, b^2 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^3}{9 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} - \frac{b^3 \, B^2 \, n^2 \, \left(c + d \, x\right)^4}{32 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^4} + \frac{2 \, B \, d^3 \, n \, \left(c + d \, x\right) \, \left(a + b \, x\right)^3}{9 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} - \frac{3 \, b \, B \, d^2 \, n \, \left(c + d \, x\right)^2 \, \left(a + b \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} - \frac{3 \, b \, B \, d^2 \, n \, \left(c + d \, x\right)^2 \, \left(a + b \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} - \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(a + b \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{8 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^4} + \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^4 \, \left(a + b \, x\right)^5}{2 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^5} + \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^4 \, \left(a + b \, x\right)^5}{2 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^5} + \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^4 \, \left(a + b \, x\right)^6 \, \left(c + d \, x\right)^6 \, \left(c + d$$

Result (type 4, 843 leaves, 29 steps):

$$\frac{A^2}{4 \ b \ (a + b \ x)^4} - \frac{A \ B \ n}{8 \ b \ (a + b \ x)^4} - \frac{B^2 \ n^2}{32 \ b \ (a + b \ x)^4} + \frac{A \ B \ d \ n}{6 \ b \ (b \ c - a \ d)} + \frac{7 \ B^2 \ d \ n^2}{72 \ b \ (b \ c - a \ d)} - \frac{A \ B \ d^2 \ n}{4 \ b \ (b \ c - a \ d)^2 \ (a + b \ x)^2} - \frac{A \ B \ d^3 \ n}{4 \ b \ (b \ c - a \ d)^2 \ (a + b \ x)^2} + \frac{25 \ B^2 \ d^3 \ n^2}{24 \ b \ (b \ c - a \ d)^3 \ (a + b \ x)} + \frac{A \ B \ d^4 \ n \ Log \left[a + b \ x\right]}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]}{24 \ b \ (b \ c - a \ d)^4} - \frac{A \ B \ d^4 \ n \ Log \left[a + b \ x\right]}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]}{24 \ b \ (b \ c - a \ d)^4} + \frac{2 \ b \ (b \ c - a \ d)^4}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]}{24 \ b \ (b \ c - a \ d)^4} + \frac{2 \ b \ (a + b \ x)^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{2 \ b \ (b \ c - a \ d)^4} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{8 \ b \ (a + b \ x)^n \ (c + d \ x)^{-n}} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^{-n}}{8 \ b \ (a + b \ x)^n \ (c + d \ x)^{-n}} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^n \ (c + d \ x)^{-n}}{8 \ b \ (a + b \ x)^n \ (c + d \ x)^{-n}} + \frac{13 \ B^2 \ d^4 \ n^2 \ Log \left[a + b \ x\right]^n \ (c + d \ x)^n \ (c + d \$$

# Problem 164: Result valid but suboptimal antiderivative.

$$\left[ \left( a + b x \right)^{3} \left( A + B Log \left[ e \left( a + b x \right)^{n} \left( c + d x \right)^{-n} \right] \right)^{3} dx$$

Optimal (type 4, 809 leaves, 27 steps):

$$-\frac{B^{3} \left(b \, c - a \, d\right)^{3} \, n^{3} \, x}{4 \, d^{3}} - \frac{B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{4 \, b \, d^{4}} + \frac{3 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, Log\left[c + d \, x\right]}{2 \, b \, d^{4}} - \frac{7 \, B^{2} \left(b \, c - a \, d\right)^{3} \, n^{2} \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)}{4 \, b \, d^{3}} + \frac{b \, B^{2} \left(b \, c - a \, d\right)^{2} \, n^{2} \left(c + d \, x\right)^{2} \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)}{4 \, d^{4}} - \frac{9 \, B^{2} \left(b \, c - a \, d\right)^{4} \, n^{2} \, Log\left[\frac{b \, c - a \, d}{b \, (c + d \, x)}\right] \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)}{2 \, d^{4}} - \frac{9 \, B^{2} \left(b \, c - a \, d\right)^{4} \, n^{2} \, Log\left[\frac{b \, c - a \, d}{b \, (c + d \, x)^{2}} \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)^{2}}{4 \, b^{3}} - \frac{9 \, B^{3} \left(b \, c - a \, d\right)^{2} \, n \, \left(c + d \, x\right)^{2} \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)^{2}}{8 \, d^{4}} - \frac{9 \, B^{3} \left(b \, c - a \, d\right)^{2} \, n \, \left(c + d \, x\right)^{2} \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right)^{2}}{4 \, d^{4}} + \frac{4 \, b \, d^{4}}{4 \, b^{4}} + \frac{4 \, b \, d^{4}}{4 \, b^{4}} + \frac{4 \, b \, d^{4}}{4 \, b^{4}} + \frac{9 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{3 \, B^{2} \left(b \, c - a \, d\right)^{4} \, n^{2} \, \left(A + B \, Log\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right) \, Log\left[1 - \frac{b \, \left(c + d \, x\right)^{-n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{4 \, b \, d^{4}}{4 \, b^{4}} + \frac{9 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{3 \, B^{2} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[e \, \left(a + b \, x\right)^{n} \left(c + d \, x\right)^{-n}\right]\right) \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{3 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{3 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \, \left(a + b \, x\right)^{n}}\right] - \frac{3 \, B^{3} \left(b \, c - a \, d\right)^{4} \, n^{3} \, PolyLog\left[2, \frac{d \, \left(a + b \, x\right)^{n}}{d \,$$

Result (type 4, 1203 leaves, 56 steps):

$$\frac{3A^2B \left( bc - ad \right)^3 n x}{4 \, d^3} - \frac{5AB^2 \left( bc - ad \right)^3 n^2 x}{4 \, d^3} - \frac{B^3 \left( bc - ad \right)^3 n^3 x}{4 \, d^3} + \frac{3A^2B \left( bc - ad \right)^2 n \left( a + bx \right)^2}{8 \, bd^2} + \frac{AB^2 \left( bc - ad \right)^2 n^2 \left( a + bx \right)^2}{4 \, bd^2} - \frac{A^2B \left( bc - ad \right)^4 n \left( a + bx \right)^3}{4 \, bd} + \frac{A^3 \left( a + bx \right)^4}{4 \, b} + \frac{3A^2B \left( bc - ad \right)^4 n Log \left[ c + dx \right]}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4 n Log \left[ c + dx \right]}{4 \, bd^4} + \frac{3A^3 \left( a + bx \right)^4 n \left( a + bx \right) Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^3} - \frac{3AB^2 \left( bc - ad \right)^3 n \left( a + bx \right) Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^3} + \frac{3AB^2 \left( bc - ad \right)^2 n \left( a + bx \right)^2 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^2} + \frac{3AB^2 \left( bc - ad \right)^2 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^2} + \frac{3AB^2 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^2} + \frac{3AB^2 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^2} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^2} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^4} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]^2}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]^2}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx \right)^n \left( c + dx \right)^{-n} \right]^2}{4 \, bd^3} + \frac{3AB^3 \left( bc - ad \right)^3 n \left( a + bx \right)^3 Log \left[ c \left( a + bx$$

### Problem 165: Result valid but suboptimal antiderivative.

$$\int \left(\,a\,+\,b\,\,x\,\right)^{\,2}\,\,\left(\,A\,+\,B\,\,Log\left[\,e\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,n}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,-\,n}\,\right]\,\right)^{\,3}\,\,\mathrm{d}x$$

Optimal (type 4, 614 leaves, 17 steps):

$$\frac{B^{3} \left(b \, c-a \, d\right)^{3} \, n^{3} \, Log \left[c+d \, x\right]}{b \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{2} \, n^{2} \, \left(a+b \, x\right) \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right)}{b \, d^{2}} + \frac{4 \, B^{2} \left(b \, c-a \, d\right)^{3} \, n^{2} \, Log \left[\frac{b \, c-a \, d}{b \, \left(c+d \, x\right)}\right] \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right)}{b \, d^{3}} + \frac{2 \, B \, \left(b \, c-a \, d\right)^{2} \, n \, \left(a+b \, x\right) \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right)^{2}}{b \, d^{3}} + \frac{B \, \left(b \, c-a \, d\right)^{3} \, n \, Log \left[\frac{b \, c-a \, d}{b \, \left(c+d \, x\right)}\right] \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right)^{2}}{b \, d^{3}} + \frac{\left(a+b \, x\right)^{3} \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right)^{2}}{b \, d^{3}} + \frac{B^{2} \, \left(b \, c-a \, d\right)^{3} \, n^{2} \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right) \, Log \left[1-\frac{b \, \left(c+d \, x\right)}{d \, \left(a+b \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{2} \, \left(b \, c-a \, d\right)^{3} \, n^{2} \, \left(A+B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]\right) \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} - \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \, c-a \, d\right)^{3} \, n^{3} \, PolyLog \left[2, \frac{d \, \left(a+b \, x\right)}{b \, \left(c+d \, x\right)}\right]}{b \, d^{3}} + \frac{2 \, B^{3} \, \left(b \,$$

#### Result (type 4, 915 leaves, 40 steps):

$$\frac{A^{2} B \left(b c-a d\right)^{2} n x}{d^{2}} + \frac{A B^{2} \left(b c-a d\right)^{2} n^{2} x}{d^{2}} - \frac{A^{2} B \left(b c-a d\right)^{3} n^{2} Log \left[c+d x\right]}{2 b d} + \frac{A^{3} \left(a+b x\right)^{3}}{3 b} - \frac{A^{2} B \left(b c-a d\right)^{3} n Log \left[c+d x\right]}{b d^{3}} - \frac{3 A B^{2} \left(b c-a d\right)^{3} n^{2} Log \left[c+d x\right]}{b d^{3}} - \frac{B^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(a+b x\right)^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(a+b x\right)^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(a+b x\right)^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]}{b (c-a d)^{3} n Log \left[b c-a d\right)^{3} n Log \left[c+d x\right]^{-n}}{b d^{3}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n Log \left[b c-a d\right)^{3} n^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n Log \left[b c-a d\right]^{3} n Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(b c-a d\right)^{3} n^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}}{b (c-a d)^{3} n^{3} Log \left[c+d x\right]^{-n}} + \frac{A^{3} \left(a+b x\right)^{3} Log \left[c+d x\right]^{-n}$$

# Problem 166: Result valid but suboptimal antiderivative.

$$\left[ \, \left( \, a \, + \, b \, \, x \, \right) \, \, \left( \, A \, + \, B \, \, Log \left[ \, e \, \, \left( \, a \, + \, b \, \, x \, \right) \, ^{n} \, \, \left( \, c \, + \, d \, \, x \, \right) \, ^{-n} \, \right] \, \right)^{\, 3} \, \, \mathrm{d} \, x$$

#### Optimal (type 4, 376 leaves, 11 steps):

$$\frac{3 \, B^2 \, \left( b \, c - a \, d \right)^2 \, n^2 \, Log \left[ \frac{b \, c - a \, d}{b \, \left( c + d \, x \right)^n} \right] \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right] \right)}{b \, d^2} - \frac{3 \, B \, \left( b \, c - a \, d \right) \, n \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right] \right)^2}{2 \, b \, d} - \frac{3 \, B \, \left( b \, c - a \, d \right)^2 \, n \, Log \left[ \frac{b \, c - a \, d}{b \, \left( c + d \, x \right)} \right] \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right] \right)^2}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left( b \, c - a \, d \right)^2 \, n^3 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left( b \, c - a \, d \right)^2 \, n^3 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{b \, d^2} + \frac{3 \, B^3 \, \left( b \, c - a \, d \right)^2 \, n^3 \, PolyLog \left[ 3 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{b \, d^2}$$

#### Result (type 4, 700 leaves, 27 steps):

$$\frac{3 \, A^2 \, B \, \left(b \, c - a \, d\right) \, n \, x}{2 \, d} + \frac{A^3 \, \left(a + b \, x\right)^2}{2 \, b} + \frac{3 \, A^2 \, B \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[c + d \, x\right]}{2 \, b \, d^2} + \frac{3 \, A^2 \, B \, \left(b \, c - a \, d\right)^2 \, n^2 \, Log \left[c + d \, x\right]}{b \, d^2} - \frac{3 \, A^3 \, B^2 \, \left(b \, c - a \, d\right) \, n \, \left(a + b \, x\right) \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{b \, d} + \frac{3 \, A^2 \, B \, \left(a + b \, x\right)^2 \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)^n}\right] \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^2 \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^2 \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{b \, d^2} - \frac{3 \, A \, B^2 \, \left(a + b \, x\right)^2 \, Log \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^2}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)^{-n}}\right]^2}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)^{-n}}\right]^2}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^2 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]^3}{2 \, b \, d^2} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 \, n^3 \, PolyLog \left[c \, \left(a + b \, x\right)^n \, \left(c + d$$

## Problem 167: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[e^{\left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{3}}{a + b x} dx$$

Optimal (type 4, 186 leaves, 6 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)^3\,\mathsf{Log}\left[1-\frac{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\mathsf{b}} + \frac{3\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)^2\,\mathsf{PolyLog}\left[2,\,\frac{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\mathsf{b}} \\ \frac{6\,\mathsf{B}^2\,\mathsf{n}^2\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)\,\mathsf{PolyLog}\left[3,\,\frac{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\mathsf{b}} + \frac{6\,\mathsf{B}^3\,\mathsf{n}^3\,\mathsf{PolyLog}\left[4,\,\frac{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\mathsf{b}} \\$$

Result (type 4, 424 leaves, 14 steps):

$$\frac{A^{3} \, Log \, [\, a + b \, x \, ]}{b} - \frac{3 \, A^{2} \, B \, Log \, \left[ -\frac{b \, c - a \, d}{d \, (a + b \, x)} \right] \, Log \, \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right]}{b} - \frac{3 \, A \, B^{2} \, Log \, \left[ -\frac{b \, c - a \, d}{d \, (a + b \, x)} \right] \, Log \, \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right]^{2}}{b} - \frac{B^{3} \, Log \, \left[ -\frac{b \, c - a \, d}{d \, (a + b \, x)} \right] \, Log \, \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right]^{3}}{b} + \frac{3 \, A^{2} \, B \, n \, PolyLog \, \left[ 2 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, A \, B^{2} \, n \, Log \, \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right]^{2} \, PolyLog \, \left[ 2 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{2} \, Log \, \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right] \, PolyLog \, \left[ 3 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \, 1 + \frac{b \, c - a \, d}{d \, (a + b \, x)} \right]}{b} + \frac{6 \, B^{3} \, n^{3} \, PolyLog \, \left[ 4 \, , \,$$

### Problem 168: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{3}}{\left(a + b x\right)^{2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\begin{split} & \frac{6 \, B^3 \, n^3 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right) \, \left(a + b \, x\right)} - \frac{6 \, B^2 \, n^2 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{\left(b \, c - a \, d\right) \, \left(a + b \, x\right)} - \\ & \frac{3 \, B \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{\left(b \, c - a \, d\right) \, \left(a + b \, x\right)} - \frac{\left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{\left(b \, c - a \, d\right) \, \left(a + b \, x\right)} \end{split}$$

Result (type 3, 360 leaves, 11 steps):

$$-\frac{A^{3}}{b\;\left(a+b\,x\right)}-\frac{3\,A^{2}\,B\,n}{b\;\left(a+b\,x\right)}-\frac{6\,A\,B^{2}\,n^{2}}{b\;\left(a+b\,x\right)}-\frac{6\,B^{3}\,n^{3}}{b\;\left(a+b\,x\right)}-\frac{3\,A^{2}\,B\;\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{6\,B^{3}\,n^{3}}{b\;\left(a+b\,x\right)}-\frac{3\,A^{2}\,B\;\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{6\,B^{3}\,n^{2}\,\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{6\,B^{3}\,n^{2}\,\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{3\,B^{3}\,n^{2}\,\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]^{2}}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}-\frac{B^{3}\,\left(c+d\,x\right)\,Log\left[e\;\left(a+b\,x\right)^{n}\;\left(c+d\,x\right)^{-n}\right]^{3}}{\left(b\,c-a\,d\right)\;\left(a+b\,x\right)}$$

# Problem 169: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}}{\left(\,a+b\,x\right)^{\,3}}\,\,\mathrm{d} x$$

### Optimal (type 3, 390 leaves, 10 steps):

$$\frac{6\,B^3\,d\,n^3\,\left(\,c + d\,x\,\right)}{\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,-}} - \frac{3\,b\,B^3\,n^3\,\left(\,c + d\,x\,\right)^{\,2}}{8\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)^{\,2}} + \\ \frac{6\,B^2\,d\,n^2\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)}{\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{3\,b\,B^2\,n^2\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)}{4\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} + \\ \frac{3\,B\,d\,n\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{3\,b\,B\,n\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,2}}{4\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} + \\ \frac{d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,3}}{\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{b\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,3}}{2\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} + \\ \frac{d\,\left(\,c + d\,x\,\right)\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,3}}{2\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} - \frac{b\,\left(\,c + d\,x\,\right)^{\,2}\,\left(\,A + B\,Log\left[\,e\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,c + d\,x\,\right)^{\,-n}\,\right]\,\right)^{\,3}}{2\,\left(\,b\,c - a\,d\,\right)^{\,2}\,\left(\,a + b\,x\,\right)} + \frac{b\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}} + \frac{b\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}} + \frac{b\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}}{2\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,x\,\right)^{\,n}} + \frac{b\,\left(\,a + b\,x\,\right)^{\,n}\,\left(\,a + b\,$$

Result (type 3, 811 leaves, 21 steps):

$$-\frac{A^3}{2\ b\ (a+b\,x)^2} - \frac{3\,A^2\,B\,n}{4\,b\ (a+b\,x)^2} + \frac{3\,A^2\,B\,d\,n}{2\,b\ (b\,c-a\,d)\ (a+b\,x)} + \frac{6\,A\,B^2\,d\,n^2}{b\,(b\,c-a\,d)\ (a+b\,x)} + \frac{6\,B^3\,d\,n^3}{b\,(b\,(c-a\,d)\ (a+b\,x)} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{b\,(b\,c-a\,d)\ (a+b\,x)} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{b\,(b\,c-a\,d)^2} - \frac{3\,a\,b\,B^3\,n^3\,(c+d\,x)^2}{8\,(b\,c-a\,d)^2} + \frac{3\,A^2\,B\,d^2\,n\,a\,(c+b\,x)}{2\,b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{2\,b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{2\,b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{2\,b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)}{2\,b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)^{-n}}{2\,b\,(a+b\,x)^2} + \frac{6\,B^3\,d\,n^2\,(c+d\,x)}{b\,(b\,c-a\,d)^2} - \frac{3\,A^2\,B\,d^2\,n\,a\,(c+d\,x)^{-n}}{a\,b\,a^2\,n\,a\,(c+d\,x)^{-n}} + \frac{6\,B^3\,d\,n^2\,(c+d\,x)^{-n}}{a\,b\,a^2\,n\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}} + \frac{6\,B^3\,d\,n^2\,(c+d\,x)^{-n}}{a\,b\,a^2\,n\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}} + \frac{3\,B^3\,d\,n\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} + \frac{3\,B^3\,a\,n\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} - \frac{3\,A\,b\,B^2\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} - \frac{3\,A\,b\,B^2\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} - \frac{3\,A\,b\,B^2\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} - \frac{3\,A\,b\,B^2\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^2\,a\,(c+d\,x)^{-n}}{a\,(b\,c-a\,d)^2\,(a+b\,x)^2} - \frac{3\,A\,b\,B^2\,(c+d\,x)^2\,a\,(c+$$

Problem 170: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}}{\left(\,a+b\,x\right)^{\,4}}\,\mathrm{d}x$$

Optimal (type 3, 611 leaves, 13 steps):

$$-\frac{6\,B^3\,d^2\,n^3\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{3\,b\,B^3\,d\,n^3\,\left(c+d\,x\right)^2}{4\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^2} - \frac{2\,b^2\,B^3\,n^3\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} - \frac{6\,B^2\,d^2\,n^2\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{3\,b\,B^3\,d\,n^3\,\left(c+d\,x\right)^2}{4\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^2} - \frac{2\,b^2\,B^3\,n^3\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} - \frac{6\,B^2\,d^2\,n^2\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B^2\,n^2\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)}{9\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} - \frac{2\,b^2\,B^2\,n^2\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^2}{2\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^2} - \frac{2\,b^2\,B^2\,n^2\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^2}{2\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^2} - \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^2}{2\,\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^2} - \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,x\right)^3} + \frac{2\,b^2\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^3\,\left(a+b\,$$

Result (type 4, 1876 leaves, 66 steps):

Problem 171: Result unnecessarily involves higher level functions and more than twice size of optimal

#### antiderivative.

$$\int \frac{\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}}{\left(\,a+b\,x\right)^{\,5}}\,\mathrm{d}x$$

#### Optimal (type 3, 830 leaves, 16 steps):

$$\frac{6 \, B^3 \, d^3 \, n^3 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} - \frac{9 \, b \, B^3 \, d^2 \, n^3 \, \left(c + d \, x\right)^2}{8 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^2} + \frac{2 \, b^2 \, B^3 \, d \, n^3 \, \left(c + d \, x\right)^3}{9 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^4} - \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^4}{128 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^4} + \frac{6 \, B^2 \, d^3 \, n^2 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{\left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} - \frac{9 \, b \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{2 \, b^2 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} - \frac{9 \, b \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{32 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^4} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{4 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, \left(a + b \, x\right)^3} + \frac{3 \, b^3 \, B^3 \, n^3 \, \left(c + d \, x\right)^3 \, \left(a + B$$

### Result (type 4, 2173 leaves, 93 steps):

$$-\frac{A^{3}}{4 \ b \ (a + b \ x)^{4}} - \frac{3 \ A^{2} \ B \ n}{16 \ b \ (a + b \ x)^{4}} - \frac{3 \ A \ B^{2} \ n^{2}}{32 \ b \ (a + b \ x)^{4}} - \frac{3 \ B^{3} \ n^{3}}{128 \ b \ (a + b \ x)^{4}} + \frac{A^{2} \ B \ d \ n}{4 \ b \ (b \ c - a \ d) \ (a + b \ x)^{3}} + \frac{7 \ A \ B^{2} \ d \ n^{2}}{24 \ b \ (b \ c - a \ d) \ (a + b \ x)^{3}} + \frac{3 \ A^{2} \ B \ d \ n}{24 \ b \ (b \ c - a \ d) \ (a + b \ x)^{3}} + \frac{3 \ A^{2} \ B \ d \ n}{24 \ b \ (b \ c - a \ d)^{2} \ (a + b \ x)^{2}} - \frac{3 \ A^{2} \ B \ d^{2} \ n^{2}}{16 \ b \ (b \ c - a \ d)^{2} \ (a + b \ x)^{2}} - \frac{79 \ B^{3} \ d^{2} \ n^{3}}{192 \ b \ (b \ c - a \ d)^{2} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{3} \ n}{4 \ b \ (b \ c - a \ d)^{3} \ (a + b \ x)} + \frac{25 \ A \ B^{2} \ d^{3} \ n^{2}}{8 \ b \ (b \ c - a \ d)^{3} \ (a + b \ x)} + \frac{451 \ B^{3} \ d^{3} \ n^{3}}{96 \ b \ (b \ c - a \ d)^{3} \ (a + b \ x)} - \frac{3 \ b \ B^{3} \ d^{2} \ n^{3} \ (c + d \ x)^{2}}{16 \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)}{4 \ b \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)}{4 \ b \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4} \ (a + b \ x)^{2}} + \frac{3 \ A^{2} \ B \ d^{4} \ n \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}}{4 \ b \ (b \ c - a \ d)^{4}} + \frac{3 \ A^{2} \ B \ d^{4} \ n^{2} \ Log \ [a + b \ x)^{2}$$

$$\frac{AB^{2} d \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{2b \, (b \, c-a \, d) \, (a+b \, x)^{\, 3}} + \frac{7\, B^{3} \, d^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{24 \, b \, (b \, c-a \, d) \, (a+b \, x)^{\, 3}} + \frac{3\, AB^{\, 2} \, d^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{4b \, (b \, c-a \, d)^{\, 2} \, (a+b \, x)^{\, 2}} + \frac{3\, AB^{\, 2} \, d^{\, 3} \, (c+d \, x) \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, B^{\, 2} \, d^{\, 2} \, n^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]} + \frac{3\, AB^{\, 2} \, d^{\, 3} \, n \, (c+d \, x) \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, B^{\, 2} \, d^{\, 2} \, n^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]} - \frac{3\, AB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, B^{\, 2} \, d^{\, 4} \, n^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]} - \frac{3\, AB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, B^{\, 2} \, d^{\, 4} \, n^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]} - \frac{3\, AB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, b^{\, 2} \, d^{\, 4} \, n^{\, 2} \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]} + \frac{3\, AB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, b^{\, 2} \, b \, (b\, c-a\, d)^{\, 4}} + \frac{3\, AB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, b \, (b\, c-a\, d)^{\, 4}} + \frac{3\, BB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]}{3a \, b^{\, 2} \, b \, (b\, c-a\, d)^{\, 4}} + \frac{3\, BB^{\, 2} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}}{3a \, b^{\, 3} \, d \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}} + \frac{3\, B^{\, 3} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}}{3a \, b^{\, 3} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}} + \frac{3\, B^{\, 3} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}}{3a \, b^{\, 3} \, d^{\, 4} \, n \, Log \left[e \, (a+b \, x)^{\, n} \, (c+d \, x)^{\, -n}\right]^{\, 2}} + \frac{3\, B^{\, 3} \, d^{\, 4} \, n$$

# Problem 172: Unable to integrate problem.

$$\int \frac{1}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathsf{x}\right)^{\,2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\mathsf{a} + \mathsf{b}\,\mathsf{x}\right)^{\,\mathsf{n}}\,\left(\mathsf{c} + \mathsf{d}\,\mathsf{x}\right)^{\,-\mathsf{n}}\,\right]\,\right)}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 4, 96 leaves, 4 steps):

$$\frac{e^{\frac{A}{B\,n}}\,\left(c+d\,x\right)\,\left(e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)}$$

Result (type 8, 38 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^{2}\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}$$
,  $x\right]$ 

Problem 177: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e \cdot (c + d x)}{a + b x}\right]}{a g + b g x} dx$$

Optimal (type 4, 81 leaves, 5 steps):

$$-\frac{Log\left[-\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right]\right)}{b\,g}\,-\,\frac{B\,PolyLog\left[2\,\text{, }1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{b\,g}$$

Result (type 4, 122 leaves, 10 steps):

$$\frac{B \ Log \left[g \ \left(a+b \ x\right) \ \right]^2}{2 \ b \ g} - \frac{B \ Log \left[\frac{b \ \left(c+d \ x\right)}{b \ c-a \ d} \right] \ Log \left[a \ g+b \ g \ x\right]}{b \ g} + \frac{\left(A+B \ Log \left[\frac{e \ \left(c+d \ x\right)}{a+b \ x} \right]\right) \ Log \left[a \ g+b \ g \ x\right]}{b \ g} - \frac{B \ Poly Log \left[2 \text{, } -\frac{d \ \left(a+b \ x\right)}{b \ c-a \ d} \right]}{b \ g}$$

Problem 178: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e \cdot (c + d x)}{a + b x}\right]}{\left(a g + b g x\right)^{2}} dx$$

Optimal (type 3, 64 leaves, 3 steps):

$$-\frac{A-B}{b\ g^2\ \left(a+b\ x\right)}\ -\frac{B\ \left(c+d\ x\right)\ Log\left[\frac{e\ \left(c+d\ x\right)}{a+b\ x}\right]}{\left(b\ c-a\ d\right)\ g^2\ \left(a+b\ x\right)}$$

Result (type 3, 101 leaves, 4 steps):

$$\frac{B}{b g^2 (a+bx)} + \frac{B d Log[a+bx]}{b (b c-a d) g^2} - \frac{B d Log[c+dx]}{b (b c-a d) g^2} - \frac{A+B Log\left[\frac{e (c+dx)}{a+bx}\right]}{b g^2 (a+bx)}$$

Problem 182: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^4 \left(A + B Log \left[\frac{e (c + d x)}{a + b x}\right]\right)^2 dx$$

Optimal (type 4, 503 leaves, 19 steps):

### Result (type 4, 557 leaves, 28 steps):

$$-\frac{2 \text{ A B } \left(b \text{ C } - \text{ a } \text{ d}\right)^4 g^4 \text{ x }}{5 \text{ d}^4} + \frac{13 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^4 g^4 \text{ x }}{30 \text{ d}^4} - \frac{7 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^3 g^4 \left(a + b \text{ x}\right)^2}{60 \text{ b } \text{ d}^3} + \frac{B^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^2 g^4 \left(a + b \text{ x}\right)^3}{30 \text{ b } \text{ d}^2} - \frac{5 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right]}{60 \text{ b } \text{ d}^5} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^2 g^4 \left(a + b \text{ x}\right)^3}{30 \text{ b } \text{ d}^2} - \frac{5 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right]}{60 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^2 g^4 \left(a + b \text{ x}\right)^3}{30 \text{ b } \text{ d}^2} - \frac{5 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right]}{60 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right]}{5 \text{ b } \text{ d}^5} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^4 g^4 \left(a + b \text{ x}\right) \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]}{60 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^4 g^4 \left(a + b \text{ x}\right) \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]}{60 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^4 g^4 \left(a + b \text{ x}\right) \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]}{60 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^2 g^4 \left(a + b \text{ x}\right)^3 \left(A + B \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]\right)}{15 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right)^4 \left(A + B \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]\right)}{10 \text{ b } \text{ d}^3} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ Log} \left[\text{C } + \text{ d } \text{ x}\right)^3 \left(A + B \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]\right)}{10 \text{ b } \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ x}}\right]} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ PolyLog} \left[2, \frac{b \text{ (c+d x)}}{b \text{ c-a} \text{ d}}\right]}{10 \text{ b } \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ Log}}\right]} + \frac{2 \text{ B}^2 \left(b \text{ C } - \text{ a } \text{ d}\right)^5 g^4 \text{ PolyLog} \left[2, \frac{b \text{ (c+d x)}}{b \text{ c-a} \text{ d}}\right]}{10 \text{ B} \text{ Log} \left[\frac{e \text{ (c+d x)}}{a + b \text{ Log$$

# Problem 183: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^{3} \left(A + B Log \left[\frac{e (c + d x)}{a + b x}\right]\right)^{2} dx$$

Optimal (type 4, 420 leaves, 15 steps):

$$-\frac{5 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, x}{12 \, d^3} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)^2}{12 \, b \, d^2} + \frac{11 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, Log \left[a + b \, x\right]}{12 \, b \, d^4} + \frac{5 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, Log \left[\frac{c + d \, x}{a + b \, x}\right]}{12 \, b \, d^4} - \frac{B \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log \left[\frac{e \, (c + d \, x)}{a + b \, x}\right]\right)}{4 \, b \, d^2} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, \left(a + b \, x\right)^3 \, \left(A + B \, Log \left[\frac{e \, (c + d \, x)}{a + b \, x}\right]\right)}{6 \, b \, d} + \frac{B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[\frac{e \, (c + d \, x)}{a + b \, x}\right]\right)}{2 \, d^4} + \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, \left(A + B \, Log \left[\frac{e \, (c + d \, x)}{a + b \, x}\right]\right)}{2 \, b \, d^4} - \frac{B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, PolyLog \left[2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{2 \, b \, d^4}$$

Result (type 4, 474 leaves, 24 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,x}{2\,d^{3}} - \frac{5\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,x}{12\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}}{12\,b\,d^{2}} + \frac{11\,B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]}{12\,b\,d^{4}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]}{2\,b\,d^{4}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]^{2}}{4\,b\,d^{4}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]}{2\,b\,d^{3}} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{4\,b\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{6\,b\,d} - \frac{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,Log\left[c+d\,x\right]\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{4\,b} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{4\,b} - \frac{g^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{4\,b} - \frac{g^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,\left(a+b\,x\right)^{4}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{a+b\,x}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{a+b\,x}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{a+b\,x}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{a+b\,x}\right]}{2\,b\,d^{4}} + \frac{g^{3}\,\left(a+b\,x\right)^{4}\,g^{4}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{a+b\,x}\right]}{2\,b$$

#### Problem 184: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^{2} \left(A + B Log \left[\frac{e (c + d x)}{a + b x}\right]\right)^{2} dx$$

#### Optimal (type 4, 335 leaves, 11 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} g^{2} \ x}{3 \ d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ Log \left[a+b \ x\right]}{b \ d^{3}} - \frac{B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ Log \left[\frac{c+d \ x}{a+b \ x}\right]}{3 \ b \ d^{3}} + \frac{B \left(b \ c-a \ d\right)^{2} \left(a+b \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (c+d \ x)}{a+b \ x}\right]\right)}{3 \ b \ d^{3}} - \frac{2 \ B \left(b \ c-a \ d\right)^{2} g^{2} \left(c+d \ x\right) \left(A+B \ Log \left[\frac{e \ (c+d \ x)}{a+b \ x}\right]\right)}{3 \ d^{3}} + \frac{g^{2} \left(a+b \ x\right)^{3} \left(A+B \ Log \left[\frac{e \ (c+d \ x)}{a+b \ x}\right]\right)}{3 \ b \ d^{3}} - \frac{2 \ B \left(b \ c-a \ d\right)^{3} g^{2} \left(A+B \ Log \left[\frac{e \ (c+d \ x)}{a+b \ x}\right]\right)}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ PolyLog \left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \$$

#### Result (type 4, 389 leaves, 20 steps):

$$-\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]}{3\,b\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[c+d\,x\right]}{3\,b\,d^{3}} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)^{2}}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^{3}\,g^{2}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b\,d^{3}} + \frac{2\,B^{2}\,\left(a+b\,x\right)^$$

### Problem 185: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left(A + B Log \left[\frac{e(c + d x)}{a + b x}\right]\right)^{2} dx$$

Optimal (type 4, 202 leaves, 7 steps):

$$\frac{B^2 \left(b \ c - a \ d\right)^2 g \ Log \left[a + b \ x\right]}{b \ d^2} + \frac{B \left(b \ c - a \ d\right) g \left(c + d \ x\right) \left(A + B \ Log \left[\frac{e \ (c + d \ x)}{a + b \ x}\right]\right)}{d^2} + \frac{g \left(a + b \ x\right)^2 \left(A + B \ Log \left[\frac{e \ (c + d \ x)}{a + b \ x}\right]\right)^2}{2 \ b} \\ = \frac{B \left(b \ c - a \ d\right)^2 g \left(A + B \ Log \left[\frac{e \ (c + d \ x)}{a + b \ x}\right]\right) \ Log \left[1 - \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{b \ d^2} - \frac{B^2 \left(b \ c - a \ d\right)^2 g \ PolyLog \left[2, \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{b \ d^2}$$

Result (type 4, 284 leaves, 16 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,x}{d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]}{b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,\left[\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]\,Log\,[\,c+d\,x\,]}{b\,d^2} + \\ \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]^2}{2\,b\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]}{b\,d} - \frac{B\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)}{b\,d^2} + \\ \frac{g\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{2\,b} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,PolyLog\left[\,2\,,\frac{b\,(c+d\,x)}{b\,c-a\,d}\,\right]}{b\,d^2} + \\ \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)}{b\,d^2} + \\ \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,PolyLog\left[\,2\,,\frac{b\,(c+d\,x)}{b\,c-a\,d}\,\right]}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,PolyLog\left[\,2\,,\frac{b\,(c+d\,x)}{b\,c-a\,d}\,\right]}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\right)^2}{b\,d^2} + \\ \frac{B^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x}\,\right]\,\left(A+B\,Log\left[\,\frac{e\,(c+d\,x)}{a+b\,x$$

# Problem 186: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e \cdot (c + d x)}{a + b x}\right]\right)^{2}}{a g + b g x} dx$$

Optimal (type 4, 128 leaves, 4 steps):

$$-\frac{\text{Log}\left[-\frac{b \text{ c-a d}}{d \text{ (a+b x)}}\right] \left(A + B \text{ Log}\left[\frac{e \text{ (c+d x)}}{a+b \text{ x}}\right]\right)^2}{b \text{ g}} - \frac{2 \text{ B}\left(A + B \text{ Log}\left[\frac{e \text{ (c+d x)}}{a+b \text{ x}}\right]\right) \text{ PolyLog}\left[2, \frac{b \text{ (c+d x)}}{d \text{ (a+b x)}}\right]}{b \text{ g}} + \frac{2 \text{ B}^2 \text{ PolyLog}\left[3, \frac{b \text{ (c+d x)}}{d \text{ (a+b x)}}\right]}{b \text{ g}}$$

Result (type 4, 719 leaves, 47 steps):

$$\frac{A \, B \, Log \left[g \, \left(a + b \, x\right)\right]^{2}}{b \, g} + \frac{B^{2} \, Log \left[g \, \left(a + b \, x\right)\right]^{3}}{3 \, b \, g} - \frac{B^{2} \, Log \left[\frac{1}{a + b \, x}\right]^{2} \, Log \left[c + d \, x\right]}{b \, g} - \frac{2 \, B^{2} \, Log \left[\frac{1}{a + b \, x}\right] \, Log \left[g \, \left(a + b \, x\right)\right] \, Log \left[c + d \, x\right]}{b \, g} - \frac{B^{2} \, Log \left[g \, \left(a + b \, x\right)\right] \, Log \left[c + d \, x\right]^{2}}{b \, g} + \frac{B^{2} \, Log \left[-\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d}\right] \, Log \left[c + d \, x\right]^{2}}{b \, g} - \frac{B^{2} \, Log \left[g \, \left(a + b \, x\right)\right] \, Log \left[c + d \, x\right]^{2}}{b \, g} + \frac{B^{2} \, Log \left[\frac{b \, \left(c + d \, x\right)}{b \, c - a \, d}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{B^{2} \, Log \left[g \, \left(a + b \, x\right)\right]^{2} \, Log \left[\frac{b \, \left(c + d \, x\right)}{b \, c - a \, d}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{B^{2} \, Log \left[g \, \left(a + b \, x\right)\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, Log \left[\frac{e \, \left(c + d \, x\right)}{b \, c - a \, d}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, Log \left[\frac{e \, \left(c + d \, x\right)}{a + b \, x}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, Log \left[\frac{e \, \left(c + d \, x\right)}{a + b \, x}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{B^{2} \, Log \left[\frac{e \, \left(c + d \, x\right)}{a + b \, x}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{B^{2} \, Log \left[\frac{e \, \left(c + d \, x\right)}{a + b \, x}\right] \, Log \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} + \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, PolyLog \left[a \, g + b \, g \, x\right]}{b \, g} - \frac{A \, B \, Po$$

Problem 187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B Log\left[\frac{e(c+dx)}{a+bx}\right]\right)^{2}}{\left(ag + bgx\right)^{2}} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{2\,A\,B\,\left(c\,+\,d\,x\right)}{\left(b\,c\,-\,a\,d\right)\,g^{2}\,\left(a\,+\,b\,x\right)}\,-\,\frac{2\,B^{2}\,\left(c\,+\,d\,x\right)}{\left(b\,c\,-\,a\,d\right)\,g^{2}\,\left(a\,+\,b\,x\right)}\,+\,\frac{2\,B^{2}\,\left(c\,+\,d\,x\right)\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)}{a\,+\,b\,x}\right]}{\left(b\,c\,-\,a\,d\right)\,g^{2}\,\left(a\,+\,b\,x\right)}\,-\,\frac{\left(c\,+\,d\,x\right)\,\left(A\,+\,B\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)}{a\,+\,b\,x}\right]\right)^{2}}{\left(b\,c\,-\,a\,d\right)\,g^{2}\,\left(a\,+\,b\,x\right)}$$

Result (type 4, 470 leaves, 26 steps):

$$-\frac{2\,B^{2}}{b\,g^{2}\,\left(a+b\,x\right)} - \frac{2\,B^{2}\,d\,Log\,[\,a+b\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{B^{2}\,d\,Log\,[\,a+b\,x\,]^{\,2}}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{2\,B^{2}\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{2\,B^{2}\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{2\,B^{2}\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{2\,B^{2}\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{2\,B^{2}\,d\,Log\,[\,a+b\,x\,]\,\,Log\,\left[\,\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\,\right]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{2\,B\,\left(A+B\,Log\,\left[\,\frac{e\,\left(c+d\,x\right)}{a+b\,x}\,\right]\,\right)}{b\,g^{2}\,\left(a+b\,x\right)} + \frac{2\,B\,d\,Log\,[\,a+b\,x\,]\,\,\left(A+B\,Log\,\left[\,\frac{e\,\left(c+d\,x\right)}{a+b\,x}\,\right]\,\right)}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{2\,B^{2}\,d\,PolyLog\,[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\,\right]}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{2\,B^{2}\,d\,PolyLog\,[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\,\right)}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{2\,B^{2}\,d\,PolyLog\,[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\,\right)}{b\,\left(a+b\,x\right)} - \frac{2\,B^{2}\,d\,PolyLog\,[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\,\right)}{b\,\left(a+b\,x\right)} - \frac{2\,B^{2}\,d\,PolyLog\,[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\,\right)}{b\,\left(a+b\,x\right)} - \frac{2\,B^{2$$

## Problem 188: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log \left[\frac{e(c+dx)}{a+bx}\right]\right)^{2}}{\left(ag + bgx\right)^{3}} dx$$

Optimal (type 3, 296 leaves, 8 steps):

Result (type 4, 578 leaves, 30 steps):

$$-\frac{B^{2}}{4 \ b \ g^{3} \ (a + b \ x)^{2}} + \frac{3 \ B^{2} \ d}{2 \ b \ (b \ c - a \ d) \ g^{3} \ (a + b \ x)} + \frac{3 \ B^{2} \ d^{2} \ Log \ [a + b \ x]}{2 \ b \ (b \ c - a \ d)^{2} \ g^{3}} - \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]^{2}}{2 \ b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [c + d \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} - \frac{B^{2} \ d^{2} \ Log \ [c + d \ x]}{2 \ b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} - \frac{B^{2} \ d^{2} \ Log \ [c + d \ x]}{2 \ b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} - \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^{2} \ g^{3}} + \frac{B^{2} \ d^{2} \ Log \ [a + b \ x]}{$$

Problem 189: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^{2}}{\left(ag + bgx\right)^{4}} dx$$

Optimal (type 3, 399 leaves, 6 steps):

$$-\frac{2\,B^{2}\,d^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)} + \frac{b\,B^{2}\,d\,\left(c+d\,x\right)^{2}}{2\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{2\,b^{2}\,B^{2}\,\left(c+d\,x\right)^{3}}{27\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{3}} + \frac{B^{2}\,d^{3}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]^{2}}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} + \frac{2\,B\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)} - \frac{b\,B\,d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{2\,B\,d^{3}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,\left(b\,c-a\,d\right)^{3}\,g^{4}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}{3\,b\,g^{4}$$

#### Result (type 4, 680 leaves, 34 steps):

$$-\frac{2 \, B^2}{27 \, b \, g^4 \, \left(a + b \, x\right)^3} + \frac{5 \, B^2 \, d}{18 \, b \, \left(b \, c - a \, d\right) \, g^4 \, \left(a + b \, x\right)^2} - \frac{11 \, B^2 \, d^2}{9 \, b \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(a + b \, x\right)} - \frac{11 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{9 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[c + d \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{a \, b \, c - a \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{a \, b \, c \, d}\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{B^2 \, d^3 \, Log \left[a +$$

#### Problem 190: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log \left[\frac{e (c+dx)}{a+bx}\right]\right)^{2}}{\left(ag + bgx\right)^{5}} dx$$

#### Optimal (type 3, 498 leaves, 5 steps):

$$\frac{2\,B^{2}\,d^{3}\,\left(\,c\,+\,d\,x\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{5}\,\left(\,a\,+\,b\,\,x\,\right)} - \frac{3\,b\,B^{2}\,d^{2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,2}}{4\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}} + \frac{2\,b^{\,2}\,B^{\,2}\,d\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}}{9\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{b^{\,3}\,B^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}}{32\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{4\,b\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{2\,b^{\,2}\,B^{\,2}\,d\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}}{4\,b\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{4\,b\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{4\,b\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}} - \frac{B^{\,2}\,d^{\,4}\,Log\left[\frac{c\,+d\,\,x}{a\,+\,b\,\,x}\right]^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,4}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)$$

Result (type 4, 763 leaves, 38 steps):

$$-\frac{B^{2}}{32 \ b \ g^{5} \ (a + b \ x)^{4}} + \frac{7 \ B^{2} \ d}{72 \ b \ (b \ c - a \ d)} \frac{13 \ B^{2} \ d^{2}}{48 \ b \ (b \ c - a \ d)^{2} \ g^{5} \ (a + b \ x)^{2}} + \frac{25 \ B^{2} \ d^{3}}{24 \ b \ (b \ c - a \ d)^{3} \ g^{5} \ (a + b \ x)} + \frac{25 \ B^{2} \ d^{4} \ b \ (b \ c - a \ d)^{3} \ g^{5} \ (a + b \ x)}{24 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{B^{2} \ d^{4} \ Log \left[ a + b \ x \right]^{2}}{4 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{25 \ B^{2} \ d^{4} \ Log \left[ c + d \ x \right]}{24 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ c + d \ x \right]}{24 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{B^{2} \ d^{4} \ Log \left[ a + b \ x \right] \ Log \left[ \frac{b \ (c + d \ x)}{b \ c - a \ d} \right]}{24 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ c + d \ x \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{B \ d^{4} \ Log \left[ \frac{e \ (c + d \ x)}{b \ c - a \ d} \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B \ d^{4} \ Log \left[ \frac{e \ (c + d \ x)}{a + b \ x} \right]}{8 \ b \ g^{5} \ (a + b \ x)^{4}} - \frac{B \ d \ (A + B \ Log \left[ \frac{e \ (c + d \ x)}{a + b \ x} \right])}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{8 \ b \ g^{5} \ (a + b \ x)^{4}} - \frac{B \ d \ (A + B \ Log \left[ \frac{e \ (c + d \ x)}{a + b \ x} \right])}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{8 \ b \ g^{5} \ (a + b \ x)^{4}} - \frac{B \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \ (c + d \ x) \right]}{2 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{B^{2} \ d^{4} \ Log \left[ e \$$

#### Problem 191: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{A + B \log\left[\frac{e (c + d x)}{a + b x}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a g + b g x\right)^{2}}{A + B Log\left[\frac{e (c+d x)}{a+b x}\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$a^2 \ g^2 \ Cannot Integrate \Big[ \frac{1}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + 2 \ a \ b \ g^2 \ Cannot Integrate \Big[ \frac{x}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x} \Big]}, \ x \Big] + b^2 \ g^2 \ Cannot Integrate \Big[ \frac{x^2}{A + B \ Log \Big[ \frac{e \ (c + d \ x)}{a + b \ x$$

## Problem 192: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B Log \left[\frac{e (c+dx)}{a+bx}\right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{A + B Log\left[\frac{e (c+d x)}{a+b x}\right]}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)}{a + b \, x} \Big]} \text{, } x \Big] + \text{b g CannotIntegrate} \Big[ \frac{x}{A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)}{a + b \, x} \Big]} \text{, } x \Big]$$

## Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (c+d x)}{a+b x}\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(c+d\,x\right)}{a+b\,x}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (c + d x)}{a + b x}\right]\right)}, x\right]$$

### Problem 194: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e \cdot (c + d x)}{a + b x}\right]\right)} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$-\frac{e^{-\frac{A}{B}} \text{ ExpIntegralEi}\left[\frac{A+B \text{ Log}\left[\frac{e \cdot (c+d \cdot x)}{a+b \cdot x}\right]}{B}\right]}{B \text{ (b c - a d) e g}^2}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \, \frac{1}{ \left( \text{a g} + \text{b g x} \right)^2 \, \left( \text{A} + \text{B Log} \Big[ \, \frac{\text{e (c+d \, x)}}{\text{a+b \, x}} \, \Big] \, \right) } \, \text{, x} \, \Big]$$

# Problem 195: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (c+d x)}{a+b x}\right]\right)} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\frac{d \, e^{-\frac{A}{B}} \, \text{ExpIntegralEi} \left[ \, \frac{A+B \, \text{Log} \left[ \frac{e \, (c+d \, x)}{a+b \, x} \right]}{B} \, \right]}{B \, \left( b \, c \, -a \, d \, \right)^{\, 2} \, e \, g^{3}} \, - \, \frac{b \, e^{-\frac{2 \, A}{B}} \, \, \text{ExpIntegralEi} \left[ \, \frac{2 \, \left( A+B \, \text{Log} \left[ \frac{e \, (c+d \, x)}{a+b \, x} \right] \right)}{B} \, \right]}{B \, \left( b \, c \, -a \, d \, \right)^{\, 2} \, e^{2} \, g^{3}}$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log \left[\frac{e (c+dx)}{a+b x}\right]\right)}, x\right]$$

#### Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{\left(A + B Log\left[\frac{e (c + d x)}{a + b x}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)^2},\,x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$a^{2} g^{2} CannotIntegrate \left[\frac{1}{\left(A+B Log\left[\frac{e^{-(c+d \, x)}}{a+b \, x}\right]\right)^{2}}, \, x\right] + \\ 2 a b g^{2} CannotIntegrate \left[\frac{x}{\left(A+B Log\left[\frac{e^{-(c+d \, x)}}{a+b \, x}\right]\right)^{2}}, \, x\right] + b^{2} g^{2} CannotIntegrate \left[\frac{x^{2}}{\left(A+B Log\left[\frac{e^{-(c+d \, x)}}{a+b \, x}\right]\right)^{2}}, \, x\right]$$

# Problem 197: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right)^{2}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{\left(A + B Log \left[\frac{e \cdot (c + d x)}{a + b x}\right]\right)^2}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\left( A + B \, \text{Log} \left[ \frac{e \, (c + d \, x)}{a + b \, x} \right] \right)^2} \text{, } x \, \Big] + b \, g \, \text{CannotIntegrate} \Big[ \frac{x}{\left( A + B \, \text{Log} \left[ \frac{e \, (c + d \, x)}{a + b \, x} \right] \right)^2} \text{, } x \, \Big]$$

# Problem 198: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (c+d x)}{a+b x}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag+bgx\right)\left(A+BLog\left[\frac{e\cdot(c+dx)}{a+bx}\right]\right)^{2}},x\right]$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (c+d x)}{a+b x}\right]\right)^2}, x\right]$$

# Problem 199: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (c + d x)}{a + b x}\right]\right)^{2}} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$-\frac{e^{-\frac{A}{B}}\,\text{ExpIntegralEi}\left[\frac{A+B\,\text{Log}\left[\frac{e\,(c+d\,x)}{a\,\text{s}\,\text{h}\,x}\right]}{B}\right]}{B^2\,\left(b\,\,c-a\,\,d\right)\,e\,g^2} + \frac{c+d\,x}{B\,\left(b\,\,c-a\,\,d\right)\,g^2\,\left(a+b\,\,x\right)\,\left(A+B\,\text{Log}\left[\frac{e\,(c+d\,x)}{a+b\,x}\right]\right)}$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[ \frac{1}{\left( a g + b g x \right)^{2} \left( A + B Log \left[ \frac{e \cdot (c + d x)}{a + b x} \right] \right)^{2}}, x \right]$$

# Problem 200: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (c + d x)}{a + b x}\right]\right)^{2}} dx$$

Optimal (type 4, 159 leaves, 10 steps):

$$\frac{d \, e^{-\frac{A}{B}} \, \text{ExpIntegralEi} \left[ \, \frac{A+B \, \text{Log} \left[ \frac{e \, \left( c+d \, x \right)}{B \, \text{a.b.x}} \right]}{B} \, \right]}{B^2 \, \left( b \, c - a \, d \, \right)^2 \, e \, g^3} \, - \, \frac{2 \, b \, e^{-\frac{2 \, A}{B}} \, \text{ExpIntegralEi} \left[ \, \frac{2 \, \left( A+B \, \text{Log} \left[ \frac{e \, \left( c+d \, x \right)}{a+b \, x} \right] \right)}{B} \, \right]}{B^2 \, \left( b \, c - a \, d \, \right)^2 \, e^2 \, g^3} \, + \, \frac{c + d \, x}{B \, \left( b \, c - a \, d \, \right) \, g^3 \, \left( a + b \, x \, \right)^2 \, \left( A+B \, \text{Log} \left[ \frac{e \, \left( c+d \, x \right)}{a+b \, x} \right] \right)}{B^2 \, \left( b \, c - a \, d \, \right)^2 \, e^2 \, g^3} \, + \, \frac{c + d \, x}{B \, \left( b \, c - a \, d \, \right) \, g^3 \, \left( a + b \, x \, \right)^2 \, \left( A+B \, \text{Log} \left[ \frac{e \, \left( c+d \, x \right)}{a+b \, x} \right] \right)}{B^2 \, \left( b \, c - a \, d \, \right)^2 \, e^2 \, g^3} \, + \, \frac{c + d \, x}{B \, \left( b \, c - a \, d \, \right)^2 \, e^2 \, g^3} \, + \, \frac{c + d \, x}{B \, \left( b \, c - a \, d \, \right)^2 \, e^2 \, g^3}$$

Result (type 8, 34 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag + bg x\right)^{3}\left(A + B Log\left[\frac{e(c+dx)}{a+bx}\right]\right)^{2}}, x\right]$$

# Problem 205: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e (c+dx)^{2}}{(a+bx)^{2}}\right]}{a g + b g x} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{\text{Log}\left[-\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]\,\left(A+B\,\text{Log}\left[\frac{e\,\left(c+d\,x\right)^{\,2}}{\left(a+b\,x\right)^{\,2}}\,\right]\right)}{b\,g}-\frac{2\,B\,\text{PolyLog}\left[\,2\,\text{, }1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\,\right]}{b\,g}$$

Result (type 4, 121 leaves, 10 steps):

$$\frac{B \ Log \left[g \ \left(a+b \ x\right) \ \right]^2}{b \ g} - \frac{2 \ B \ Log \left[\frac{b \cdot (c+d \ x)}{b \ c-a \ d} \ \right] \ Log \left[a \ g+b \ g \ x\right]}{b \ g} + \frac{\left(A+B \ Log \left[\frac{e \cdot (c+d \ x)^2}{(a+b \ x)^2} \ \right]\right) \ Log \left[a \ g+b \ g \ x\right]}{b \ g} - \frac{2 \ B \ PolyLog \left[2 \ , \ -\frac{d \cdot (a+b \ x)}{b \ c-a \ d} \ \right]}{b \ g}$$

## Problem 206: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e \cdot (c + d \cdot x)^{2}}{(a + b \cdot x)^{2}}\right]}{\left(a \cdot g + b \cdot g \cdot x\right)^{2}} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$-\frac{A\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}\,+\,\frac{2\,B\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}\,-\,\frac{B\,\left(c+d\,x\right)\,Log\left[\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\right]}{\left(b\,c-a\,d\right)\,g^2\,\left(a+b\,x\right)}$$

Result (type 3, 105 leaves, 4 steps):

$$\frac{2\,B}{b\,g^2\,\left(a+b\,x\right)}\,+\,\frac{2\,B\,d\,Log\,[\,a+b\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}\,-\,\frac{2\,B\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^2}\,-\,\frac{A+B\,Log\,\left[\,\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,\right]}{b\,g^2\,\left(a+b\,x\right)}$$

#### Problem 210: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^4\,\left[A+B\,Log\,\left[\,\frac{e\,\left(\,c+d\,x\,\right)^{\,2}}{\left(\,a+b\,x\,\right)^{\,2}}\,\right]\,\right]^2\,\mathrm{d}x$$

Optimal (type 4, 515 leaves, 19 steps):

$$\frac{26\,B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,x}{15\,d^{4}} - \frac{7\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{2}}{15\,b\,d^{3}} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,\left(a+b\,x\right)^{3}}{15\,b\,d^{2}} - \frac{10\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,Log\left[a+b\,x\right]}{3\,b\,d^{5}} - \frac{26\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]}{15\,b\,d^{5}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{5\,b\,d^{5}} - \frac{4\,B\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{15\,b\,d^{5}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{5\,b\,d^{5}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{15\,b\,d^{5}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{5\,b\,d^{5}} + \frac{2\,B\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{5\,b\,d^{5}} + \frac{2\,B\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{5\,b\,d^{5}} + \frac{2\,B\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,B^{2}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{5\,b\,d^{5}} + \frac{2\,B\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,B^{2}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{5\,b\,d^{5}} + \frac{2\,B\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,B^{2}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{5\,b\,d^{5}} + \frac{2\,B\,B^{2}\,\left(b\,c-a\,d\right)^{5}\,B^{2}\,PolyLog\left[2,\frac{d\,(a+b\,x)}{b\,(c+d\,x$$

#### Result (type 4, 569 leaves, 28 steps):

$$-\frac{4\,A\,B\,\left(b\,c-a\,d\right)^4\,g^4\,x}{5\,d^4} + \frac{26\,B^2\,\left(b\,c-a\,d\right)^4\,g^4\,x}{15\,d^4} - \frac{7\,B^2\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^2}{15\,b\,d^3} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3}{15\,b\,d^2} - \frac{16\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]}{3\,b\,d^5} + \frac{8\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{5\,b\,d^5} - \frac{4\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]^2}{5\,b\,d^5} - \frac{4\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]}{5\,b\,d^5} - \frac{2\,B\,\left(b\,c-a\,d\right)^3\,g^4\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} - \frac{4\,B\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{B\,\left(b\,c-a\,d\right)\,g^4\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{4\,B\,\left(b\,c-a\,d\right)^5\,g^4\,Log\left[c+d\,x\right]\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} - \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} - \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} - \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^5\,g^4\,PolyLog\left[2\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{5\,b\,d^5} - \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(a+b\,x\right)^5\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)^2}{5\,b\,d^5} + \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}{5\,b\,d^5} + \frac{2\,B^2\,\left(a+b\,x\right)^3\,\left(a+b\,x\right)$$

# Problem 211: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\,\left[\,\frac{e\,\left(c+d\,x\right)^{\,2}}{\left(a+b\,x\right)^{\,2}}\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 422 leaves, 15 steps):

$$-\frac{5 \ B^{2} \ \left(b \ c-a \ d\right)^{3} \ g^{3} \ x}{3 \ d^{3}} + \frac{B^{2} \ \left(b \ c-a \ d\right)^{2} \ g^{3} \ \left(a+b \ x\right)^{2}}{3 \ b \ d^{2}} + \frac{11 \ B^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ Log \left[a+b \ x\right]}{3 \ b \ d^{4}} + \frac{5 \ B^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ Log \left[\frac{c+d \ x}{a+b \ x}\right]}{3 \ b \ d^{4}} - \frac{13 \ b^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ b \ d^{4}}{3 \ b \ d^{4}} + \frac{11 \ B^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ Log \left[a+b \ x\right]}{3 \ b \ d^{4}} - \frac{13 \ b^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ \left(a+b \ x\right)^{2} \ \left(a+b \ x\right)^{2} \ \left(a+b \ x\right)^{2} \ \left(a+b \ x\right)^{2} \left(a+b \ x\right)^{2} \left(a+b \ x\right)^{3} \ \left(a+b \ x\right)^{2} \left(a+b \ x\right)^{2} + \frac{11 \ b^{2} \ \left(b \ c-a \ d\right)^{4} \ g^{3} \ \left(a+b \ x\right)^{3} \ \left(a+b \ x\right$$

Result (type 4, 469 leaves, 24 steps):

$$\frac{A \ B \ \left(b \ c - a \ d\right)^{3} \ g^{3} \ x}{d^{3}} - \frac{5 \ B^{2} \ \left(b \ c - a \ d\right)^{3} \ g^{3} \ x}{3 \ d^{3}} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ g^{3} \ \left(a + b \ x\right)^{2}}{3 \ b \ d^{2}} + \frac{11 \ B^{2} \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right]}{3 \ b \ d^{4}} - \frac{2 \ B^{2} \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right]}{b \ d^{4}} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right]^{2}}{b \ d^{3}} - \frac{B \ \left(b \ c - a \ d\right)^{2} \ g^{3} \ \left(a + b \ x\right) \ Log \left[\frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}}\right]}{b \ d^{3}} - \frac{B \ \left(b \ c - a \ d\right)^{2} \ g^{3} \ \left(a + b \ x\right)^{2} \ \left(A + B \ Log \left[\frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}}\right]\right)}{3 \ b \ d} - \frac{B \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right] \ \left(A + B \ Log \left[\frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}}\right]\right)}{3 \ b \ d^{4}} - \frac{B \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right] \ \left(A + B \ Log \left[\frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}}\right]\right)}{4 \ b} - \frac{B \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ Log \left[c + d \ x\right] \ \left(A + B \ Log \left[\frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}}\right]\right)}{b \ d^{4}} - \frac{B^{2} \ \left(b \ c - a \ d\right)^{4} \ g^{3} \ PolyLog \left[c + d \ x\right]}{b \ d^{4}}$$

#### Problem 212: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\,\left[\,\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,\right]\,\right)^2\,dlx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,d^{2}}-\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[\,a+b\,x\right]}{b\,d^{3}}-\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[\,\frac{c+d\,x}{a+b\,x}\right]}{3\,b\,d^{3}}+\\ \frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(\,a+b\,x\right)^{2}\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)}{3\,b\,d}-\frac{4\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(\,c+d\,x\right)\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)}{3\,d^{3}}+\\ \frac{g^{2}\,\left(\,a+b\,x\right)^{3}\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)^{2}}{3\,b\,d^{3}}-\frac{4\,B\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)}{3\,b\,d^{3}}+\\ \frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)^{2}}{3\,b\,d^{3}}+\frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,PolyLog\left[\,2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{3\,b\,d^{3}}+\\ \frac{g^{2}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\,\right]\right)^{2}}{3\,b\,d^{3}}+\frac{g^{2}\,B^{2}\,\left(a+b\,x\right)^{3}\,B^{2}\,$$

Result (type 4, 397 leaves, 20 steps):

$$-\frac{4 \, A \, B \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, x}{3 \, d^{2}} + \frac{4 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, x}{3 \, d^{2}} - \frac{4 \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right]}{b \, d^{3}} + \frac{8 \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[-\frac{d \, (a+b \, x)}{b \, c - a \, d}\right] \, Log \left[c + d \, x\right]}{3 \, b \, d^{3}} - \frac{4 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, \left(a + b \, x\right) \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]}{3 \, b \, d^{3}} + \frac{2 \, B \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(A + B \, Log \left[\frac{e \, (c+d \, x)^{2}}{(a+b \, x)^{2}}\right]\right)}{3 \, b \, d^{3}} + \frac{2 \, B \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, g^{2} \, Log \left[c + d \, x\right] \, \left(a + b \, x\right)^{2} \, \left($$

## Problem 213: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left(A + B Log \left[\frac{e (c + d x)^{2}}{(a + b x)^{2}}\right]\right)^{2} dx$$

Optimal (type 4, 211 leaves, 7 steps):

$$\frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\,a+b\,x\,\right]}{b\,d^2} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(\,c+d\,x\right)\,\left(\,A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,\right]\,\right)}{d^2} + \frac{g\,\left(\,a+b\,x\,\right)^2\,\left(\,A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,\right]\,\right)^2}{2\,b} + \frac{2\,B\,\left(\,b\,c-a\,d\,\right)^2\,g\,\left(\,A+B\,Log\left[\,\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\,\right]\,\right)^2}{2\,b} + \frac{2\,B\,\left(\,b\,c-a\,d\,\right)^2\,g\,PolyLog\left[\,2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{b\,d^2} + \frac{2\,B\,\left(\,a+b\,x\right)^2\,g\,PolyLog\left[\,2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{b\,d^2} + \frac{2\,B\,\left(\,a+b\,x\right)^2\,g\,PolyLog\left[$$

Result (type 4, 291 leaves, 16 steps):

# Problem 214: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e \cdot (c + d \times)^{2}}{(a + b \times)^{2}}\right]\right)^{2}}{a \cdot g + b \cdot g \cdot x} dx$$

Optimal (type 4, 132 leaves, 4 steps):

$$-\frac{\text{Log}\left[-\frac{\text{b c-a d}}{\text{d } (\text{a+b x})}\right] \left(\text{A} + \text{B Log}\left[\frac{\text{e } (\text{c+d x})^2}{(\text{a+b x})^2}\right]\right)^2}{\text{b g}} - \frac{4 \text{ B } \left(\text{A} + \text{B Log}\left[\frac{\text{e } (\text{c+d x})^2}{(\text{a+b x})^2}\right]\right) \text{ PolyLog}\left[2, \frac{\text{b } (\text{c+d x})}{\text{d } (\text{a+b x})}\right]}{\text{b g}} + \frac{8 \text{ B}^2 \text{ PolyLog}\left[3, \frac{\text{b } (\text{c+d x})}{\text{d } (\text{a+b x})}\right]}{\text{b g}}$$

Result (type 4, 740 leaves, 46 steps):

$$\frac{2\,A\,B\,Log\big[g\,\left(a+b\,x\right)\big]^2}{b\,g} + \frac{4\,B^2\,Log\big[g\,\left(a+b\,x\right)\big]^3}{3\,b\,g} - \frac{B^2\,Log\big[\frac{1}{(a+b\,x)^2}\big]^2\,Log\big[c+d\,x\big]}{b\,g} - \frac{4\,B^2\,Log\big[\frac{1}{(a+b\,x)^2}\big]\,Log\big[g\,\left(a+b\,x\right)\big]\,Log\big[c+d\,x\big]}{b\,g} \\ \frac{4\,B^2\,Log\big[g\,\left(a+b\,x\right)\big]^2\,Log\big[c+d\,x\big]}{b\,g} + \frac{B^2\,Log\big[\frac{1}{(a+b\,x)^2}\big]^2\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{4\,B^2\,Log\big[g\,\left(a+b\,x\right)\big]^2\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{B^2\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{B^2\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B^2\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]\,Log\,[a\,g+b\,g\,x]}{b\,g} + \frac{A\,B\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B\,Log\big[\frac{b\cdot(c+d\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B\,B\,Log\big[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\big] Log\,[a\,g+b\,g\,x]}{b\,g} + \frac{A\,B\,B\,Log\big[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\big] Log\,[a\,g+b\,g\,x]}{b\,g} + \frac{A\,B\,B\,Log\big[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\big] Log\,[a\,g+b\,g\,x]}{b\,g} - \frac{A\,B\,B\,DoyLog\big[2\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} - \frac{A\,B\,B\,DoyLog\big[2\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B^2\,Log\big[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\big] Log\,[a\,g+b\,g\,x]^2}{b\,g} - \frac{A\,B\,B\,DoyLog\big[2\,,\,-\frac{d\cdot(a+b\,x)}{(a+b\,x)^2}\big] PolyLog\big[2\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} + \frac{A\,B^2\,Log\big[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\big] PolyLog\big[2\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} - \frac{B\,B^2\,PolyLog\big[3\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b\,g} - \frac{B\,B^2\,PolyLog\big[3\,,\,-\frac{d\cdot(a+b\,x)}{b\,c-a\,d}\big]}{b$$

Problem 215: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e \cdot (c + d x)^{2}}{(a + b x)^{2}}\right]\right)^{2}}{\left(a g + b g x\right)^{2}} dx$$

Optimal (type 3, 157 leaves, 4 steps):

$$\frac{4 \text{ A B } \left(\text{c} + \text{d x}\right)}{\left(\text{b c} - \text{a d}\right) \text{ g}^{2} \left(\text{a} + \text{b x}\right)} - \frac{8 \text{ B}^{2} \left(\text{c} + \text{d x}\right)}{\left(\text{b c} - \text{a d}\right) \text{ g}^{2} \left(\text{a} + \text{b x}\right)} + \frac{4 \text{ B}^{2} \left(\text{c} + \text{d x}\right) \text{ Log}\left[\frac{e \cdot (\text{c} + \text{d x})^{2}}{(\text{a} + \text{b x})^{2}}\right]}{\left(\text{b c} - \text{a d}\right) \text{ g}^{2} \left(\text{a} + \text{b x}\right)} - \frac{\left(\text{c} + \text{d x}\right) \left(\text{A} + \text{B Log}\left[\frac{e \cdot (\text{c} + \text{d x})^{2}}{(\text{a} + \text{b x})^{2}}\right]\right)^{2}}{\left(\text{b c} - \text{a d}\right) \text{ g}^{2} \left(\text{a} + \text{b x}\right)}$$

Result (type 4, 480 leaves, 26 steps):

$$-\frac{8\,B^{2}}{b\,g^{2}\,\left(a+b\,x\right)} - \frac{8\,B^{2}\,d\,Log\,[\,a+b\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{4\,B^{2}\,d\,Log\,[\,a+b\,x\,]^{\,2}}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{8\,B^{2}\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{8\,B^{2}\,d\,Log\,\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{4\,B\,d\,Log\,[\,c+d\,x\,]}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{8\,B^{2}\,d\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b\,\left(b\,c-a\,d\right)\,g^{2}} + \frac{4\,B\,d\,Log\,\left[\frac{e\,(c+d\,x)^{\,2}}{(a+b\,x)^{\,2}}\right]\right)}{b\,g^{2}\,\left(a+b\,x\right)} + \frac{4\,B\,d\,Log\,\left[\frac{e\,(c+d\,x)^{\,2}}{(a+b\,x)^{\,2}}\right]\right)}{b\,\left(b\,c-a\,d\right)\,g^{2}} - \frac{4\,B\,d\,Log\,\left[\,c+d\,x\,\right]}{b\,(a+b\,x)^{\,2}} - \frac{4\,B\,d\,Log\,\left[\,c+d\,x\,\right]}{b\,(a+b\,x)^{\,2}}$$

# Problem 216: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[\frac{e \cdot (c + d x)^{2}}{(a + b x)^{2}}\right]\right)^{2}}{\left(a g + b g x\right)^{3}} dx$$

#### Optimal (type 3, 299 leaves, 8 steps):

$$-\frac{4\,A\,B\,d\,\left(c\,+\,d\,x\right)}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)} + \frac{8\,B^{\,2}\,d\,\left(c\,+\,d\,x\right)}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)} - \frac{b\,B^{\,2}\,\left(c\,+\,d\,x\right)^{\,2}}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)^{\,2}} - \frac{4\,B^{\,2}\,d\,\left(c\,+\,d\,x\right)\,\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)^{\,2}}{\left(a\,+\,b\,x\right)^{\,2}}\right]}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)} + \frac{b\,B\,\left(c\,+\,d\,x\right)^{\,2}\,\left(A\,+\,B\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)^{\,2}}{\left(a\,+\,b\,x\right)^{\,2}}\right]\right)^{\,2}}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)} - \frac{b\,\left(c\,+\,d\,x\right)^{\,2}\,\left(A\,+\,B\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)^{\,2}}{\left(a\,+\,b\,x\right)^{\,2}}\right]\right)^{\,2}}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)} - \frac{b\,\left(c\,+\,d\,x\right)^{\,2}\,\left(A\,+\,B\,Log\left[\frac{e\,\left(c\,+\,d\,x\right)^{\,2}}{\left(a\,+\,b\,x\right)^{\,2}}\right]\right)^{\,2}}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,3}\,\left(a\,+\,b\,x\right)}$$

#### Result (type 4, 578 leaves, 30 steps):

$$\frac{B^2}{b \ g^3 \ (a + b \ x)^2} + \frac{6 \ B^2 \ d}{b \ (b \ c - a \ d) \ g^3 \ (a + b \ x)} + \frac{6 \ B^2 \ d^2 \ Log \ [a + b \ x]}{b \ (b \ c - a \ d)^2 \ g^3} - \frac{2 \ B^2 \ d^2 \ Log \ [a + b \ x]^2}{b \ (b \ c - a \ d)^2 \ g^3} - \frac{6 \ B^2 \ d^2 \ Log \ [c + d \ x]^2}{b \ (b \ c - a \ d)^2 \ g^3} - \frac{6 \ B^2 \ d^2 \ Log \ [c + d \ x]^2}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{4 \ B^2 \ d^2 \ Log \ [a + b \ x] \ Log \ \left[\frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{4 \ B^2 \ d^2 \ Log \ [a + b \ x] \ Log \ \left[\frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{B \ (a + B \ Log \ \left[\frac{e \ (c + d \ x)^2}{(a + b \ x)^2}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{2 \ B \ d^2 \ Log \ [a + b \ x] \ (A + B \ Log \ \left[\frac{e \ (c + d \ x)^2}{(a + b \ x)^2}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{2 \ B \ d^2 \ Log \ [a + b \ x] \ (A + B \ Log \ \left[\frac{e \ (c + d \ x)^2}{(a + b \ x)^2}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{2 \ B \ d^2 \ Log \ [a + b \ x] \ (A + B \ Log \ \left[\frac{e \ (c + d \ x)^2}{(a + b \ x)^2}\right]}{b \ (b \ c - a \ d)^2 \ g^3} + \frac{4 \ B^2 \ d^2 \ Poly Log \ [a + b \ x]}{b \ (b \ c - a \ d)^2 \ g^3}$$

# Problem 217: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[\frac{e(c+dx)^{2}}{(a+bx)^{2}}\right]\right)^{2}}{\left(ag + bgx\right)^{4}} dx$$

#### Optimal (type 3, 407 leaves, 6 steps):

$$-\frac{8 \, B^2 \, d^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} + \frac{2 \, b \, B^2 \, d \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{8 \, b^2 \, B^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} + \frac{4 \, B^2 \, d^3 \, Log \left[\frac{c + d \, x}{a + b \, x}\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{4 \, B \, d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^2} - \frac{2 \, b \, B \, d \, \left(c + d \, x\right)^2 \, \left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{9 \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)^3} - \frac{4 \, B \, d^3 \, Log \left[\frac{c + d \, x}{a + b \, x}\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4 \, \left(a + b \, x\right)} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(A + B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]\right)}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} - \frac{\left(a \, B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a + b \, x\right)^2}\right]}{3 \, b \, \left(a \, B \, Log \left[\frac{e \, \left(c + d \, x\right)^2}{\left(a$$

#### Result (type 4, 692 leaves, 34 steps):

$$\frac{8 \, B^2}{27 \, b \, g^4 \, \left(a + b \, x\right)^3} + \frac{10 \, B^2 \, d}{9 \, b \, \left(b \, c - a \, d\right) \, g^4 \, \left(a + b \, x\right)^2} - \frac{44 \, B^2 \, d^3}{9 \, b \, \left(b \, c - a \, d\right)^2 \, g^4 \, \left(a + b \, x\right)} - \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{9 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{4 \, B^2 \, d^3 \, Log \left[a + b \, x\right]^2}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a \, d\right)^3 \, g^4} + \frac{44 \, B^2 \, d^3 \, Log \left[a + b \, x\right]}{3 \, b \, \left(b \, c - a$$

# Problem 218: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[\frac{e \cdot (c + d x)^{2}}{(a + b x)^{2}}\right]\right)^{2}}{\left(a g + b g x\right)^{5}} dx$$

Optimal (type 3, 501 leaves, 5 steps):

$$\frac{8\,B^{2}\,d^{3}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)} - \frac{3\,b\,B^{2}\,d^{2}\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)^{2}} + \frac{8\,b^{2}\,B^{2}\,d\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)^{3}} - \frac{b^{3}\,B^{2}\,\left(c+d\,x\right)^{4}}{8\,\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)^{4}} - \frac{B^{2}\,d^{4}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]^{2}}{b\,\left(b\,c-a\,d\right)^{4}\,g^{5}} + \frac{4\,B\,d^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)} + \frac{3\,b\,B\,d^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)^{2}} - \frac{4\,b^{2}\,B\,d\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{3\,\left(b\,c-a\,d\right)^{4}\,g^{5}\,\left(a+b\,x\right)^{3}} + \frac{b^{3}\,B\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{b\,\left(b\,c-a\,d\right)^{4}\,g^{5}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{4\,b\,g^{5}\,\left(a+b\,x\right)^{4}} + \frac{B\,d^{4}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)}{b\,\left(b\,c-a\,d\right)^{4}\,g^{5}} - \frac{\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{2}}{(a+b\,x)^{2}}\right]\right)^{2}}{4\,b\,g^{5}\,\left(a+b\,x\right)^{4}} + \frac{B^{2}\,d^{4}\,Log\left[\frac{c+d\,x}{a+b\,x}\right]^{2}}{b\,\left(b\,c-a\,d\right)^{4}\,g^{5}} - \frac{A^{2}\,B^$$

Result (type 4, 758 leaves, 38 steps):

$$-\frac{B^{2}}{8 \ b \ g^{5} \ (a + b \ x)^{4}} + \frac{7 \ B^{2} \ d}{18 \ b \ (b \ c - a \ d)} \frac{13 \ B^{2} \ d^{2}}{18 \ b \ (b \ c - a \ d)^{2}} \frac{13 \ B^{2} \ d^{2}}{18 \ b \ (b \ c - a \ d)^{2}} \frac{13 \ B^{2} \ d^{2}}{18 \ b \ (b \ c - a \ d)^{2}} \frac{13 \ B^{2} \ d^{2}}{18 \ b \ (b \ c - a \ d)^{3}} \frac{1}{9^{5} \ (a + b \ x)^{2}} + \frac{25 \ B^{2} \ d^{3}}{6 \ b \ (b \ c - a \ d)^{3} \ g^{5} \ (a + b \ x)} + \frac{25 \ B^{2} \ d^{4} \ Log \left[a + b \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{25 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{25 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} + \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right]}{6 \ b \ (b \ c - a \ d)^{4} \ g^{5}} - \frac{2 \ B^{2} \ d^{4} \ Log \left[c + d \ x\right$$

# Problem 219: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2}}{A + B Log\left[\frac{e (c+d x)^{2}}{(a+b x)^{2}}\right]} dx$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a g + b g x\right)^{2}}{A + B Log\left[\frac{e (c+d x)^{2}}{(a+b x)^{2}}\right]}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$a^{2} g^{2} \ \text{CannotIntegrate} \Big[ \frac{1}{A + B \ \text{Log} \Big[ \frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}} \Big]}, \ x \Big] + 2 \ a \ b \ g^{2} \ \text{CannotIntegrate} \Big[ \frac{x}{A + B \ \text{Log} \Big[ \frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}} \Big]}, \ x \Big] + b^{2} \ g^{2} \ \text{CannotIntegrate} \Big[ \frac{x^{2}}{A + B \ \text{Log} \Big[ \frac{e \ (c + d \ x)^{2}}{(a + b \ x)^{2}} \Big]}, \ x \Big]$$

# Problem 220: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B Log \left[\frac{e (c + d x)^{2}}{(a + b x)^{2}}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{A + B Log \left[\frac{e (c+d x)^2}{(a+b x)^2}\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$\text{a g CannotIntegrate}\Big[\frac{1}{\mathsf{A} + \mathsf{B} \, \mathsf{Log}\Big[\frac{e \, (c + d \, x)^2}{(a + b \, x)^2}\Big]} \text{, } x\Big] + \text{b g CannotIntegrate}\Big[\frac{x}{\mathsf{A} + \mathsf{B} \, \mathsf{Log}\Big[\frac{e \, (c + d \, x)^2}{(a + b \, x)^2}\Big]} \text{, } x\Big]$$

### Problem 221: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a\,g + b\,g\,x\right)\,\left(A + B\,Log\left[\frac{e\,(c + d\,x)^{\,2}}{\left(a + b\,x\right)^{\,2}}\right]\right)}\,\mathrm{d}x$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(ag+bgx\right)\left(A+BLog\left[\frac{e\left(c+dx\right)^{2}}{\left(a+bx\right)^{2}}\right]\right)},x\right]$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{(a+b\,x)^2}\right]\right)}$$
,  $x\right]$ 

# Problem 222: Unable to integrate problem.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\right]\right)}\,d\!\!\mid\! x$$

Optimal (type 4, 91 leaves, 3 steps):

$$-\frac{\text{e}^{-\frac{A}{2B}}\left(c+d\,x\right)\,\text{ExpIntegralEi}\left[\frac{A+B\,\text{Log}\left[\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\right]}{2\,B}\right]}{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,\sqrt{\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}}}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^2\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{\left(a+b\,x\right)^2}\right]\right)}$$
,  $x\right]$ 

## Problem 223: Unable to integrate problem.

$$\int \frac{1}{\left(ag + bg x\right)^{3} \left(A + B Log\left[\frac{e(c+dx)^{2}}{(a+bx)^{2}}\right]\right)} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$\frac{d \, e^{-\frac{A}{2\,B}} \, \left(c + d\,x\right) \, \text{ExpIntegralEi} \left[\frac{A + B \, \text{Log}\left[\frac{e\,\left(c + d\,x\right)^2}{\left(a + b\,x\right)^2}\right]}{2\,B}\right]}{2\,B \, \left(b\,c - a\,d\right)^2 \, g^3 \, \left(a + b\,x\right) \, \sqrt{\frac{e\,\left(c + d\,x\right)^2}{\left(a + b\,x\right)^2}}} \, \\ - \, \frac{b \, e^{-\frac{A}{B}} \, \text{ExpIntegralEi} \left[\frac{A + B \, \text{Log}\left[\frac{e\,\left(c + d\,x\right)^2}{\left(a + b\,x\right)^2}\right]}{B}\right]}{2\,B \, \left(b\,c - a\,d\right)^2 \, e\,g^3}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(c+d\,x)^2}{\left(a+b\,x\right)^2}\right]\right)}$$
,  $x\right]$ 

# Problem 224: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^2}{\left(A+B\,Log\left[\frac{e\cdot(c+d\,x)^2}{(a+b\,x)^2}\right]\right)^2}\,\mathrm{d}x$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(a g + b g x\right)^{2}}{\left(A + B Log \left[\frac{e (c+d x)^{2}}{(a+b x)^{2}}\right]\right)^{2}}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$a^{2} g^{2} \, \text{CannotIntegrate} \Big[ \frac{1}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \\ 2 \, a \, b \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}, \, x \Big] \, + \, b^{2} \, g^{2} \, \text{CannotIntegrate} \Big[ \frac{x^{2}}{\left( A + B \, \text{Log} \Big[ \frac{e \, (c + d \, x)^{\, 2}}{(a + b \, x)^{\, 2}} \Big] \right)^{\, 2}}$$

## Problem 225: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (c + d x)^{2}}{(a + b x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 34 leaves, 0 steps):

Unintegrable 
$$\left[\frac{a g + b g x}{\left(A + B Log \left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$\text{a g CannotIntegrate} \Big[ \frac{1}{\left( \mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{c} + \mathsf{d} \, \mathsf{x})^2}{\left( \mathsf{a} + \mathsf{b} \, \mathsf{x} \right)^2} \Big] \right)^2} \text{, } \, \mathsf{x} \, \Big] + \mathsf{b} \, \mathsf{g} \, \mathsf{CannotIntegrate} \Big[ \frac{\mathsf{x}}{\left( \mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \frac{e \, (\mathsf{c} + \mathsf{d} \, \mathsf{x})^2}{\left( \mathsf{a} + \mathsf{b} \, \mathsf{x} \right)^2} \Big] \right)^2} \text{, } \, \mathsf{x} \, \Big]$$

### Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(c+d\,x)^{\,2}}{\left(a+b\,x\right)^{\,2}}\right]\right)^{\,2}}\,\mathrm{d}x$$

Optimal (type 8, 36 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\right]\right)^{2}}$$
,  $x\right]$ 

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \frac{1}{ \left( \text{a g} + \text{b g x} \right) \ \left( \text{A} + \text{B Log} \Big[ \frac{e \ (c + \text{d x})^2}{(a + \text{b x})^2} \Big] \right)^2 } \text{, x} \Big]$$

### Problem 227: Unable to integrate problem.

$$\int \frac{1}{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2}\right]\right)^2}\,d\!\!\mid \! x$$

Optimal (type 4, 147 leaves, 4 steps):

$$-\frac{\text{e}^{-\frac{A}{2\,B}}\left(c+d\,x\right)\,\text{ExpIntegralEi}\left[\frac{A+B\,\text{Log}\left[\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\right]}{2\,B}\right]}{4\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,\sqrt{\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}}}+\frac{c+d\,x}{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,\left(A+B\,\text{Log}\left[\frac{e\,\left(c+d\,x\right)^{2}}{\left(a+b\,x\right)^{2}}\right]\right)}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(ag+bgx\right)^{2}\left(A+BLog\left[\frac{e\left(c+dx\right)^{2}}{\left(a+bx\right)^{2}}\right]\right)^{2}},x\right]$$

### Problem 228: Unable to integrate problem.

$$\int \frac{1}{\left(a g + b g x\right)^3 \left(A + B Log\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2} dx$$

Optimal (type 4, 206 leaves, 10 steps):

$$\frac{d \, e^{-\frac{A}{2\,B}} \, \left(c + d\,x\right) \, \text{ExpIntegralEi} \left[\frac{\frac{A + B \, \text{Log}\left[\frac{e\left(c + d\,x\right)^{2}}{\left(a + b\,x\right)^{2}}\right]}{2\,B}\right]}{2\,B} \, - \, \frac{b \, e^{-\frac{A}{B}} \, \text{ExpIntegralEi} \left[\frac{A + B \, \text{Log}\left[\frac{e\left(c + d\,x\right)^{2}}{\left(a + b\,x\right)^{2}}\right]}{B}\right]}{2\,B \, \left(b \, c - a \, d\right)^{2} \, e \, g^{3}} \, + \, \frac{c + d\,x}{2\,B \, \left(b \, c - a \, d\right) \, g^{3} \, \left(a + b\,x\right)^{2} \, \left(A + B \, \text{Log}\left[\frac{e\left(c + d\,x\right)^{2}}{\left(a + b\,x\right)^{2}}\right]\right)}{2\,B^{2} \, \left(a + b\,x\right)^{2} \, \left(A + B \, \text{Log}\left[\frac{e\left(c + d\,x\right)^{2}}{\left(a + b\,x\right)^{2}}\right]\right)}$$

Result (type 8, 36 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a g + b g x\right)^3 \left(A + B Log\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}, x\right]$$

# Problem 229: Unable to integrate problem.

$$\int \frac{1}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathsf{x}\right)^{\,2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\,\mathsf{a} + \mathsf{b}\,\mathsf{x}\right)^{\,\mathsf{n}}\,\left(\,\mathsf{c} + \mathsf{d}\,\mathsf{x}\right)^{\,-\mathsf{n}}\,\right]\,\right)}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 4, 96 leaves, 4 steps):

$$\frac{e^{\frac{A}{B\,n}}\,\left(c+d\,x\right)\,\left(e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left[-\frac{A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)}$$

Result (type 8, 38 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(a\,g+b\,g\,x\right)^{2}\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}$$
,  $x\right]$ 

### Problem 230: Result valid but suboptimal antiderivative.

$$\int (f + g x)^4 \left( A + B Log \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 355 leaves, 3 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, \left( b \, c - a \, d \right) \, g \, \left( a^3 \, d^3 \, g^3 - a^2 \, b \, d^2 \, g^2 \, \left( 5 \, d \, f - c \, g \right) \, + a \, b^2 \, d \, g \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \, - b^3 \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, x \, - \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, \left( a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) \, + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, x^2}{10 \, b^3 \, d^3} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^3 \, \left( 5 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, x^3}{15 \, b^2 \, d^2} \, - \frac{B \, \left( b \, c - a \, d \right) \, g^4 \, x^4}{20 \, b \, d} \, - \frac{B \, \left( b \, f - a \, g \right)^5 \, Log \, [a + b \, x]}{5 \, b^5 \, g} \, + \frac{\left( f + g \, x \right)^5 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{5 \, g} \, + \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g} \, - \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \,$$

Result (type 3, 339 leaves, 4 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, g \, \left( 10 \, a \, b^3 \, d^4 \, f^3 - 10 \, a^2 \, b^2 \, d^4 \, f^2 \, g + 5 \, a^3 \, b \, d^4 \, f \, g^2 - a^4 \, d^4 \, g^3 - b^4 \, c \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, x \, - \\ \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, \left( a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, x^2}{10 \, b^3 \, d^3} - \frac{B \, \left( b \, c - a \, d \right) \, g^3 \, \left( 5 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, x^3}{15 \, b^2 \, d^2} - \frac{B \, \left( b \, c - a \, d \right) \, g^4 \, x^4}{20 \, b \, d} - \frac{B \, \left( b \, f - a \, g \right)^5 \, Log \, [a + b \, x]}{5 \, b^5 \, g} + \frac{\left( f + g \, x \right)^5 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{5 \, g} + \frac{B \, \left( d \, f - c \, g \right)^5 \, Log \, [c + d \, x]}{5 \, d^5 \, g}$$

#### Problem 231: Result optimal but 1 more steps used.

$$\int (f + g x)^{3} \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right) dx$$

Optimal (type 3, 227 leaves, 3 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \left(a^2 \ d^2 \ g^2 - a \ b \ d \ g \ \left(4 \ d \ f - c \ g\right) + b^2 \left(6 \ d^2 \ f^2 - 4 \ c \ d \ f \ g + c^2 \ g^2\right)\right) \ x}{4 \ b^3 \ d^3} - \frac{B \left(b \ c - a \ d\right) \ g^2 \left(4 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ x^2}{8 \ b^2 \ d^2} - \frac{B \left(b \ c - a \ d\right) \ g^3 \ x^3}{12 \ b \ d} - \frac{B \left(b \ f - a \ g\right)^4 \ Log \left[a + b \ x\right]}{4 \ b^4 \ g} + \frac{\left(f + g \ x\right)^4 \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{4 \ g} + \frac{B \left(d \ f - c \ g\right)^4 \ Log \left[c + d \ x\right]}{4 \ d^4 \ g}$$

Result (type 3, 227 leaves, 4 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ g \left(a^2 \ d^2 \ g^2-a \ b \ d \ g \ \left(4 \ d \ f-c \ g\right) \ +b^2 \left(6 \ d^2 \ f^2-4 \ c \ d \ f \ g+c^2 \ g^2\right)\right) \ x}{4 \ b^3 \ d^3} -\frac{B \left(b \ c-a \ d\right) \ g^2 \left(4 \ b \ d \ f-b \ c \ g-a \ d \ g\right) \ x^2}{8 \ b^2 \ d^2} -\frac{B \left(b \ c-a \ d\right) \ g^3 \ x^3}{12 \ b \ d} -\frac{B \left(b \ f-a \ g\right)^4 \ Log \left[a+b \ x\right]}{4 \ b^4 \ g} +\frac{\left(f+g \ x\right)^4 \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{4 \ g} +\frac{B \left(d \ f-c \ g\right)^4 \ Log \left[c+d \ x\right]}{4 \ d^4 \ g}$$

# Problem 232: Result optimal but 1 more steps used.

$$\int \left(f+g\,x\right)^{\,2}\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c\,+d\,x}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 150 leaves, 3 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \left(3 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ x}{3 \ b^{2} \ d^{2}} - \frac{B \left(b \ c - a \ d\right) \ g^{2} \ x^{2}}{6 \ b \ d} - \frac{B \left(b \ f - a \ g\right)^{3} \ Log \left[a + b \ x\right]}{3 \ b^{3} \ g} + \frac{\left(f + g \ x\right)^{3} \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{3 \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g}$$

Result (type 3, 150 leaves, 4 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \left(3 \ b \ d \ f - b \ c \ g - a \ d \ g\right) \ x}{3 \ b^{2} \ d^{2}} - \frac{B \left(b \ c - a \ d\right) \ g^{2} \ x^{2}}{6 \ b \ d} - \frac{B \left(b \ f - a \ g\right)^{3} \ Log \left[a + b \ x\right]}{3 \ b^{3} \ g} + \frac{\left(f + g \ x\right)^{3} \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{3 \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3} \ Log \left[c + d \ x\right]}{3 \ d^{3} \ g} + \frac{B \left(d \ f - c \ g\right)^{3}$$

#### Problem 233: Result optimal but 1 more steps used.

$$\int \left( f + g \; x \right) \; \left( A + B \; Log \left[ \; \frac{e \; \left( a + b \; x \right)}{c + d \; x} \; \right] \right) \; \mathrm{d}x$$

Optimal (type 3, 109 leaves, 3 steps):

$$-\frac{B (b c - a d) g x}{2 b d} - \frac{B (b f - a g)^{2} Log[a + b x]}{2 b^{2} g} + \frac{(f + g x)^{2} (A + B Log[\frac{e (a + b x)}{c + d x}])}{2 g} + \frac{B (d f - c g)^{2} Log[c + d x]}{2 d^{2} g}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{ B \left( b \ c - a \ d \right) \ g \ x}{2 \ b \ d} - \frac{ B \left( b \ f - a \ g \right)^2 \ Log \left[ a + b \ x \right] }{2 \ b^2 \ g} + \frac{ \left( f + g \ x \right)^2 \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ g} + \frac{B \left( d \ f - c \ g \right)^2 \ Log \left[ c + d \ x \right]}{2 \ d^2 \ g}$$

#### Problem 235: Result optimal but 3 more steps used.

$$\int \frac{A + B Log\left[\frac{e (a+bx)}{c+dx}\right]}{f + gx} dx$$

Optimal (type 4, 140 leaves, 7 steps):

$$-\frac{B \, Log \left[-\frac{g \, (a+b \, X)}{b \, f-a \, g}\right] \, Log \, [f+g \, X]}{g} + \frac{\left(A+B \, Log \left[\frac{e \, (a+b \, X)}{c+d \, X}\right]\right) \, Log \, [f+g \, X]}{g} + \frac{B \, Log \left[-\frac{g \, (c+d \, X)}{d \, f-c \, g}\right] \, Log \, [f+g \, X]}{g} - \frac{B \, PolyLog \left[2, \frac{b \, (f+g \, X)}{b \, f-a \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyLog \left[2, \frac{d \, (f+g \, X)}{d \, f-c \, g}\right]}{g} + \frac{B \, PolyL$$

Result (type 4, 140 leaves, 10 steps):

$$-\frac{B\,Log\left[-\frac{g\,(a+b\,X)}{b\,f-a\,g}\right]\,Log\,[\,f+g\,X\,]}{g} + \frac{\left(A+B\,Log\left[\frac{e\,(a+b\,X)}{c+d\,X}\right]\right)\,Log\,[\,f+g\,X\,]}{g} + \frac{B\,Log\left[-\frac{g\,(c+d\,X)}{d\,f-c\,g}\right]\,Log\,[\,f+g\,X\,]}{g} - \frac{B\,PolyLog\left[\,2\,,\,\frac{b\,(f+g\,X)}{b\,f-a\,g}\right]}{g} + \frac{B\,PolyLog\left[\,2\,,\,\frac{d\,(f+g\,X)}{d\,f-c\,g}\right]}{g} + \frac{B\,Po$$

## Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(f + gx\right)^2} dx$$

Optimal (type 3, 87 leaves, 3 steps):

$$\frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]\right)}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{f} + \mathsf{g} \, \mathsf{x}\right)} + \frac{\mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \mathsf{Log}\left[\frac{\mathsf{f} + \mathsf{g} \, \mathsf{x}}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right)}$$

Result (type 3, 113 leaves, 4 steps):

$$\frac{b \ B \ Log \left[\,a + b \ x\,\right]}{g \ \left(\,b \ f - a \ g\,\right)} - \frac{A + B \ Log \left[\,\frac{e \ \left(a + b \ x\,\right)}{c + d \ x}\,\right]}{g \ \left(\,f + g \ x\,\right)} - \frac{B \ d \ Log \left[\,c + d \ x\,\right]}{g \ \left(\,d \ f - c \ g\,\right)} + \frac{B \ \left(\,b \ c - a \ d\,\right) \ Log \left[\,f + g \ x\,\right]}{\left(\,b \ f - a \ g\,\right) \ \left(\,d \ f - c \ g\,\right)}$$

### Problem 237: Result optimal but 1 more steps used.

$$\int \frac{A + B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{\left(f + g \, x\right)^3} \, \mathrm{d}x$$

Optimal (type 3, 183 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{2 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, Log \left[a + b \, x\right]}{2 \, g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, Log \left[c + d \, x\right]}{2 \, g \, \left(d \, f - c \, g\right)^2} + \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, Log \left[f + g \, x\right]}{2 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

Result (type 3, 183 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{2 \, \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, Log \left[a + b \, x\right]}{2 \, g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, Log \left[c + d \, x\right]}{2 \, g \, \left(d \, f - c \, g\right)^2} + \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, Log \left[f + g \, x\right]}{2 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

## Problem 238: Result optimal but 1 more steps used.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(f + gx\right)^4} dx$$

Optimal (type 3, 275 leaves, 3 steps):

$$-\frac{B \left(b \, c-a \, d\right)}{6 \left(b \, f-a \, g\right) \left(d \, f-c \, g\right) \left(f+g \, x\right)^2} - \frac{B \left(b \, c-a \, d\right) \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right)}{3 \left(b \, f-a \, g\right)^2 \left(f+g \, x\right)} + \frac{b^3 \, B \, Log \left[a+b \, x\right]}{3 \, g \left(b \, f-a \, g\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{B \left(b \, c-a \, d\right) \left(a^2 \, d^2 \, g^2-a \, b \, d \, g \left(3 \, d \, f-c \, g\right)^3\right)}{3 \, g \left(d \, f-c \, g\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac$$

Result (type 3, 275 leaves, 4 steps):

$$-\frac{B \left(b \, c-a \, d\right)}{6 \left(b \, f-a \, g\right) \left(d \, f-c \, g\right) \left(f+g \, x\right)^2} - \frac{B \left(b \, c-a \, d\right) \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right)}{3 \left(b \, f-a \, g\right)^2 \left(d \, f-c \, g\right)^2 \left(f+g \, x\right)} + \frac{b^3 \, B \, Log \left[a+b \, x\right]}{3 \, g \left(b \, f-a \, g\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{B \, d^3 \, Log \left[c+d \, x\right]}{3 \, g \left(d \, f-c \, g\right)^3} + \frac{B \left(b \, c-a \, d\right) \left(a^2 \, d^2 \, g^2-a \, b \, d \, g \left(3 \, d \, f-c \, g\right)+b^2 \left(3 \, d^2 \, f^2-3 \, c \, d \, f \, g+c^2 \, g^2\right)\right) \, Log \left[f+g \, x\right]}{3 \left(b \, f-a \, g\right)^3 \left(d \, f-c \, g\right)^3}$$

# Problem 239: Result optimal but 1 more steps used.

$$\int \frac{A + B Log\left[\frac{e (a+bx)}{c+dx}\right]}{\left(f + gx\right)^5} dx$$

Optimal (type 3, 379 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{12 \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)^3}{4 \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(f + g \, x\right)^2} \\ -\frac{B \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(3 \, d \, f - c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right)}{4 \, \left(b \, f - a \, g\right)^3 \, \left(d \, f - c \, g\right)^3 \, \left(f + g \, x\right)} \\ +\frac{b^4 \, B \, Log \left[a + b \, x\right]}{4 \, g \, \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{4 \, g \, \left(f + g \, x\right)^4} - \frac{B \, d^4 \, Log \left[c + d \, x\right]}{4 \, g \, \left(d \, f - c \, g\right)^4} - \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \, \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, Log \left[f + g \, x\right)}{4 \, \left(b \, f - a \, g\right)^4 \, \left(d \, f - c \, g\right)^4}$$

Result (type 3, 379 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{12 \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right)}{8 \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(f + g \, x\right)^2} - \frac{B \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(3 \, d \, f - c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right)}{4 \left(b \, f - a \, g\right)^3 \, \left(d \, f - c \, g\right)^3 \, \left(f + g \, x\right)} + \frac{b^4 \, B \, Log \left[a + b \, x\right]}{4 \, g \, \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[\frac{e \cdot (a + b \, x)}{c + d \, x}\right]}{4 \, g \, \left(f + g \, x\right)^4} - \frac{B \, d^4 \, Log \left[c + d \, x\right]}{4 \, g \, \left(d \, f - c \, g\right)^4} - \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \, \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, Log \left[f + g \, x\right]}{4 \, \left(b \, f - a \, g\right)^4 \, \left(d \, f - c \, g\right)^4}$$

#### Problem 240: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{3} \left[ A + B Log \left[ \frac{e (a + b x)}{c + d x} \right] \right]^{2} dx$$

Optimal (type 4, 874 leaves, 15 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^3 \, g^3 \, x}{6 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^2 \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, x}{4 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^2 \, g^3 \left( c + d \, x \right)^2}{12 \, b^2 \, d^4} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{4 \, b^4 \, d^4} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{4 \, b^4 \, d^4} - \frac{1}{2 \, b^4 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{4 \, b^4 \, d^4} - \frac{1}{2 \, b^4 \, d^3} + \frac{B \left( b \, c - a \, d \right)^3 \, g^2 \left( 4 \, b \, d \, f - 3 \, b \, c \, g - a \, d \, g \right) \, \left( c + d \, x \right)^2 \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{4 \, b^2 \, d^4} - \frac{B \left( b \, c - a \, d \right) \, g^3 \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{4 \, b^2 \, d^4} - \frac{1}{2 \, b^4 \, d^4} - \frac{1}{2 \, b^4 \, d^4} + \frac{1}{2 \, b^4 \, d^4} - \frac{1}{2 \, b$$

Result (type 4, 994 leaves, 33 steps):

$$\frac{B^{2} \left( b \, c - a \, d \right)^{2} \left( b \, c + a \, d \right) \, g^{3} \, x}{6 \, b^{3} \, d^{3}} + \frac{B^{2} \left( b \, c - a \, d \right)^{2} g^{2} \left( 4 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, x}{4 \, b^{3} \, d^{3}} - \frac{4 \, b^{3} \, d^{3}}{2 \, b^{3} \, d^{3}} + \frac{B^{2} \left( b \, c - a \, d \right)^{2} g^{2} \, x^{2} - 12 \, b^{2} \, d^{2}}{12 \, b^{2} \, d^{2}} - \frac{12 \, b^{2} \, d^{2}}{12 \, b^{2} \, d^{2}} - \frac{12 \, b^{2} \, d^{2}}{12 \, b^{2} \, d^{2}} - \frac{12 \, b^{2} \, d^{2}}{4 \, b^{4} \, g} - \frac{B^{2} \left( b \, c - a \, d \right) \, g^{2} \left( 4 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, Log \left( a + b \, x \right)^{2}}{4 \, b^{4} \, g} - \frac{B^{2} \left( b \, c - a \, d \right) \, g^{2} \left( a \, b \, d \, f - c \, g \right)^{2} \, b^{2} \left( b \, d \, f - b \, c \, g - a \, d \, g \right) \, Log \left( a + b \, x \right)^{2}}{4 \, b^{4} \, g} - \frac{B^{2} \left( b \, c - a \, d \right) \, g^{2} \left( a \, b \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, Log \left( a + b \, x \right)^{2}}{4 \, b^{4} \, g} - \frac{B^{2} \left( b \, c - a \, d \, g \right) \, g^{2} \left( a \, b \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \left( a \, d \, f - c \, g \right)^{2} \, b^{2} \, d^{2} \, d^{2}$$

# Problem 241: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{2} \left( A + B Log \left[ \frac{e (a + b x)}{c + d x} \right] \right)^{2} dx$$

Optimal (type 4, 532 leaves, 12 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} g^{2} \ x}{3 \ b^{2} \ d^{2}} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} g^{2} \ Log \left[\frac{a+b \ x}{c+d \ x}\right]}{3 \ b^{3} \ d^{3}} - \frac{2 \ B \left(b \ c-a \ d\right) g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \left(a+b \ x\right) \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{3 \ b^{3} \ d^{2}} - \frac{B \left(b \ c-a \ d\right) g^{2} \left(c+d \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{3 \ b \ d^{3}} + \frac{1}{3 \ b^{3} \ d^{3}} + \frac{1}{3 \ b^{3} \ d^{3}} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2} + \frac{\left(f+g \ x\right)^{3} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{3 \ b^{3} \ g} + \frac{\left(f+g \ x\right)^{3} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d \ f-2 \ b \ c \ g-a \ d \ g\right) \ Log \left[c+d \ x\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d^{2} \ c-a \ d \ d^{2} \ g^{2}\right) \ PolyLog \left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d^{2} \ c-a \ d \ d^{2} \ g^{2}\right) \ PolyLog \left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} g \left(3 \ b \ d^{2} \ c-a \ d \ d^{2} \ g^{2}\right) \ PolyLog \left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{3 \ b^{3} \ d^{3}} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{2} \left(a \ b \ c-a \ d\ d\right)^{2} \left(a \ b \ d^{2} \ d^{2}\right) + \frac{2 \ b^{2} \left(a \ b \ d^$$

#### Result (type 4, 649 leaves, 29 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^2 g^2 \, x}{3 \, b^2 \, d^2} - \frac{2 \, A \, B \, \left( b \, c - a \, d \right) \, g \, \left( 3 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, x}{3 \, b^2 \, d^2} + \frac{a^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^2 \, Log \left[ a + b \, x \right]}{3 \, b^3 \, d} + \frac{B^2 \left( b \, f - a \, g \right)^3 \, Log \left[ a + b \, x \right]^2}{3 \, b^3 \, g} - \frac{2 \, B^2 \, \left( b \, c - a \, d \right) \, g \, \left( 3 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, \left( a + b \, x \right) \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]}{3 \, b^3 \, g} - \frac{3 \, b^3 \, d^2}{3 \, b^3 \, g} + \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, x^2 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{3 \, b \, d} - \frac{2 \, B \, \left( b \, f - a \, g \right)^3 \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{3 \, b \, d^3 \, g} + \frac{2 \, B^2 \, \left( b \, c - a \, d \right) \, g^2 \, \left( b \, c - a \, d \right)^2 \, g \, \left( 3 \, b \, d \, f - b \, c \, g - a \, d \, g \right) \, Log \left[ c + d \, x \right]}{3 \, b^3 \, d^3} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, g} + \frac{2 \, B \, \left( d \, f - c \, g \right)^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right) \, Log \left[ c + d \, x \right]}{3 \, d^3 \, g} + \frac{2 \, B^2 \, \left( b \, f - a \, g \right)^3 \, Log \left[ c + d \, x \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( b \, f - a \, g \right)^3 \, Log \left[ c + d \, x \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( b \, f - a \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{3 \, d^3 \, g} - \frac{2 \, B^2 \, \left( d \, f - c \, g \right)^3 \, PolyLog \left[ 2 \, - \frac{d \, (a + b \, x)}{b$$

### Problem 242: Result valid but suboptimal antiderivative.

$$\int (f + g x) \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)^{2} dx$$

Optimal (type 4, 270 leaves, 9 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ g \left(a+b \ x\right) \ \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{b^2 \ d} + \\ \frac{B \left(b \ c-a \ d\right) \ \left(2 \ b \ d \ f-b \ c \ g-a \ d \ g\right) \ Log\left[\frac{b \ c-a \ d}{b \ (c+d \ x)}\right] \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{b^2 \ d^2} - \frac{\left(b \ f-a \ g\right)^2 \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^2}{2 \ b^2 \ g} + \\ \frac{\left(f+g \ x\right)^2 \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^2}{b^2 \ d^2} + \frac{B^2 \left(b \ c-a \ d\right) \ \left(2 \ b \ d \ f-b \ c \ g-a \ d \ g\right) \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{b^2 \ d^2}$$

#### Result (type 4, 444 leaves, 25 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,g\,x}{b\,d} + \frac{B^2\,\left(b\,f-a\,g\right)^2\,Log\,[\,a+b\,x\,]^{\,2}}{2\,b^2\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{b^2\,d} - \frac{B\,\left(b\,f-a\,g\right)^2\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^2\,g} + \frac{\left(f+g\,x\right)^2\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{2\,g} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[\,c+d\,x\,]}{b^2\,d^2} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,Log\,[\,c+d\,x\,]}{d^2\,g} + \frac{B\,\left(d\,f-c\,g\right)^2\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{d^2\,g} + \frac{B^2\,\left(d\,f-c\,g\right)^2\,Log\,[\,c+d\,x\,]}{2\,d^2\,g} - \frac{B^2\,\left(b\,f-a\,g\right)^2\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,PolyLog\,[\,2\,,\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,PolyLog\,[\,2\,,\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)}{d^2\,g} - \frac{B^2\,\left(d\,f-c\,g\right)^2\,PolyLog\,[\,2\,,\,\frac{b\,(c+d\,x)}{b\,c$$

## Problem 243: Result valid but suboptimal antiderivative.

$$\int \left( A + B \log \left[ \frac{e \left( a + b x \right)}{c + d x} \right] \right)^{2} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{2 \text{ B } \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ Log}\left[\frac{\text{b } \text{c}-\text{a } \text{d}}{\text{b } \text{ (c+d } \text{x})}\right] \left(\text{A}+\text{B } \text{Log}\left[\frac{\text{e } (\text{a}+\text{b } \text{x})}{\text{c+d } \text{x}}\right]\right)}{\text{b } \text{d}}+\frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B } \text{Log}\left[\frac{\text{e } (\text{a}+\text{b } \text{x})}{\text{c+d } \text{x}}\right]\right)^2}{\text{b } \text{d}}+\frac{2 \text{ B}^2 \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ PolyLog}\left[2\text{, }\frac{\text{d } (\text{a}+\text{b } \text{x})}{\text{b } (\text{c}+\text{d } \text{x})}\right]}{\text{b } \text{d}}$$

Result (type 4, 246 leaves, 22 steps):

$$-\frac{a \ B^{2} \ Log \left[a + b \ x\right]^{2}}{b} + \frac{2 \ a \ B \ Log \left[a + b \ x\right] \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b} + x \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^{2} + \\ \frac{2 \ B^{2} \ c \ Log \left[-\frac{d \ (a + b \ x)}{b \ c - a \ d}\right] \ Log \left[c + d \ x\right]}{d} - \frac{2 \ B \ c \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right) \ Log \left[c + d \ x\right]}{d} - \frac{B^{2} \ c \ Log \left[c + d \ x\right]^{2}}{d} + \\ \frac{2 \ a \ B^{2} \ Log \left[a + b \ x\right] \ Log \left[\frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{b \ b \ c - a \ d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d} + \frac{2 \ B^{2} \ c \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{$$

## Problem 244: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{f + gx} dx$$

#### Optimal (type 4, 277 leaves, 9 steps):

$$-\frac{\text{Log}\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{\mathsf{g}} + \frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2\,\mathsf{Log}\left[1-\frac{(d\,f-c\,g)\,(a+b\,x)}{(b\,f-a\,g)\,(c+d\,x)}\right]}{\mathsf{g}} - \frac{2\,\mathsf{B}\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,\mathsf{PolyLog}\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{\mathsf{g}} \\ = \frac{2\,\mathsf{B}\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,\mathsf{PolyLog}\left[2,\,\frac{(d\,f-c\,g)\,(a+b\,x)}{b\,(c+d\,x)}\right]}{\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{\mathsf{g}} - \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{(d\,f-c\,g)\,(a+b\,x)}{(b\,f-a\,g)\,(c+d\,x)}\right]}{\mathsf{g}} \\ = \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{(d\,f-c\,g)\,(a+b\,x)}{(b\,f-a\,g)\,(c+d\,x)}\right]}{\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{\mathsf{g}} - \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{(d\,f-c\,g)\,(a+b\,x)}{(b\,f-a\,g)\,(c+d\,x)}\right]}{\mathsf{g}} \\ = \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{(d\,f-c\,g)\,(a+b\,x)}{(b\,f-a\,g)\,(c+d\,x)}\right]}{\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{\mathsf{g}} + \frac{2\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{d\,(a+b\,x)}$$

#### Result (type 4, 1998 leaves, 41 steps):

$$\frac{B^2 Log[a+bx]^2 Log[f+gx]}{g} = \frac{2AB Log\left[-\frac{g(a+bx)}{bf-ag}\right] Log[f+gx]}{g} - \frac{B^2 Log\left[\frac{1}{c+dx}\right]^2 Log[f+gx]}{g} + \frac{g}{g}$$

$$2B^2 Log\left[-\frac{g(a+bx)}{bf-ag}\right] \left(Log[a+bx] + Log\left[\frac{1}{c+dx}\right] - Log\left[\frac{e(a+bx)}{c+dx}\right]\right) Log[f+gx]}{g} + \frac{(A+B Log\left[\frac{e(a+bx)}{c+dx}\right])^2 Log[f+gx]}{g} + \frac{g}{g}$$

$$2B^2 Log\left[-\frac{d(a+bx)}{bc-ad}\right] Log[c+dx] Log[f+gx] - \frac{2B^2 Log\left[-\frac{g(a+bx)}{bf-ag}\right] \left(Log\left[\frac{1}{c+dx}\right] + Log[c+dx]\right) Log[f+gx]}{g} + \frac{g}{g}$$

$$2B^2 Log[a+bx] Log\left[\frac{b(c+dx)}{bc-ad}\right] Log[f+gx] - \frac{2B^2 Log\left[-\frac{g(a+bx)}{bf-ag}\right] \left(Log\left[\frac{1}{c+dx}\right] + Log[c+dx]\right) Log[f+gx]}{g} - \frac{g}{g}$$

$$2B^2 Log[a+bx] Log\left[\frac{b(c+dx)}{bc-ad}\right] Log[f+gx] + \frac{2AB Log\left[-\frac{g(c+dx)}{df-cg}\right] Log[f+gx]}{g} - \frac{g}{g}$$

$$2B^2 Log[a+bx] + Log\left[\frac{1}{c+dx}\right] - Log\left[\frac{e(a+bx)}{c+dx}\right]\right) Log\left[-\frac{g(c+dx)}{df-cg}\right] Log[f+gx] + \frac{g}{g}$$

$$B^2 Log\left[\frac{1}{c+dx}\right]^2 Log\left[\frac{d(f+gx)}{df-cg}\right] + \frac{g}{bc-ad}\right] + Log\left[\frac{bf-ag}{(bc-ad)} \frac{(c+gx)}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(c+gx)}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(c+gx)}{(bc-ad)}\right]^2 - \frac{g}{g}$$

$$B^2 \left(Log\left[\frac{b(c+dx)}{bc-ad}\right] - Log\left[-\frac{g(c+dx)}{df-cg}\right] - Log\left[-\frac{(bc-ad)}{(bc-ad)} \frac{(f+gx)}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(c+gx)}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(c+gx)}{(bc-ad)}\right]^2 - \frac{g}{g}$$

$$B^2 \left(Log\left[-\frac{d(a+bx)}{bc-ad}\right] + Log\left[-\frac{df-cg}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(f+gx)}{(bc-ad)} \frac{(f+gx)}{(bc-ad)} \frac{(bf-ag)}{(bc-ad)} \frac{(f+gx)}{(bc-ad)} \frac{(bf-ag)}{(df-cg)} \frac{(bf-ag)}{(af-cg)} \frac{(bf-ag)}{(a$$

$$\frac{2 \, B^2 \, \text{Log} [a + b \, x] \, \text{PolyLog} [2, -\frac{g \, (a + b \, x)}{b \, f - a \, g}]}{g} + \frac{2 \, B^2 \, \left( \text{Log} [f + g \, x] - \text{Log} \left[ \frac{(b \, c - a \, d) \, (f + g \, x)}{(b \, f - a \, g) \, (c + d \, x)} \right] \, \text{PolyLog} [2, \frac{b \, (c + d \, x)}{b \, c - a \, d}]}{g} \\ = \frac{2 \, B^2 \, \text{Log} \left[ -\frac{1}{d \, f - c \, g} \right] \, PolyLog \left[ 2, -\frac{g \, (c \, d \, x)}{(d \, f - c \, g) \, (a \, b \, x)} \right]}{g} - 2 \, B^2 \, \text{Log} \left[ -\frac{(b \, c - a \, d) \, (f \, g \, x)}{(d \, f - c \, g) \, (a \, b \, x)}} \right] \, PolyLog \left[ 2, \frac{g \, (a \, b \, x)}{b \, (f \, g \, x)} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ -\frac{(b \, c - a \, d) \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)}} \right]}{g} \, PolyLog \left[ 2, \frac{g \, (c \, d \, x)}{d \, (f \, g \, x)} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{(b \, c - a \, d) \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)} \right]}{g} \, PolyLog \left[ 2, \frac{b \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)}} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{(b \, c \, a \, d) \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)} \right]}{g} \, PolyLog \left[ 2, \frac{b \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{(b \, c \, a \, d) \, (f \, g \, x)}{(b \, f - a \, g) \, (c \, d \, x)} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, \left( \text{Log} \left[ \frac{1}{c \, d \, x} \right] + \text{Log} \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \, B^2 \, PolyLog \left[ \frac{1}{c \, d \, x} \right]}{g} + \frac{2 \,$$

#### Problem 245: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{\left(f + g \, x\right)^2} \, dx$$

Optimal (type 4, 196 leaves, 4 steps):

$$\frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]\right)^2}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{f} + \mathsf{g} \, \mathsf{x}\right)} + \frac{2 \, \mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, (\mathsf{a} + \mathsf{b} \, \mathsf{x})}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]\right) \, \mathsf{Log}\left[1 - \frac{\left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right) \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)}\right]} + \frac{2 \, \mathsf{B}^2 \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \mathsf{PolyLog}\left[2, \frac{\left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right) \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)}\right]}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right)} + \frac{2 \, \mathsf{B}^2 \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \mathsf{PolyLog}\left[2, \frac{\left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right) \, \left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right)}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right)}\right)}{\left(\mathsf{b} \, \mathsf{f} - \mathsf{a} \, \mathsf{g}\right) \, \left(\mathsf{d} \, \mathsf{f} - \mathsf{c} \, \mathsf{g}\right)}$$

Result (type 4, 612 leaves, 32 steps):

$$\frac{b \, B^2 \, Log \, [a+b \, x]^2}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, b \, B \, Log \, [a+b \, x] \, \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{g \, \left(b \, f-a \, g\right)} - \frac{\left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^2}{g \, \left(f+g \, x\right)} + \frac{2 \, B^2 \, d \, Log \left[-\frac{d \, (a+b \, x)}{b \, c-a \, d}\right] \, Log \, [c+d \, x]}{g \, \left(d \, f-c \, g\right)} - \frac{2 \, B \, d \, \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right) \, Log \, [c+d \, x]^2}{g \, \left(d \, f-c \, g\right)} + \frac{2 \, b \, B^2 \, Log \, [a+b \, x] \, Log \left[\frac{b \, (c+d \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} - \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, Log \left[-\frac{g \, (a+b \, x)}{b \, f-a \, g}\right] \, Log \, [f+g \, x]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, b \, B^2 \, Log \, [a+b \, x] \, Log \left[\frac{b \, (c+d \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right) \, \left(b \, f-a \, g\right) \, \left(d \, f-c \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, Log \, \left[-\frac{g \, (c+d \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, b \, B^2 \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, f-a \, g}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, d \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, d \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, d \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, d \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, PolyLog \, \left[2, -\frac{d \, (a+b \, x)}{b \, c-a \, d}\right]}{g \, \left(b \, f-a \, g\right)} + \frac{2 \, B^2 \, \left(b \, c-a \, d\right) \, Poly$$

#### Problem 246: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(f + gx\right)^{3}} dx$$

Optimal (type 4, 369 leaves, 9 steps):

$$\frac{B \left(b \, c - a \, d\right) \, g \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{2 \, g \, \left(b \, f - a \, g\right)^2} - \frac{\left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{2 \, g \, \left(f + g \, x\right)^2} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, g \, Log\left[\frac{f + g \, x}{c + d \, x}\right]}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, \left(d \, f - c \, g\right)^2}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{B^2 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, PolyLog\left[2, \, \frac{\left(d \, f - c \, g\right) \, \left(a + b \, x\right)}{\left(b \, f - a \, g\right) \, \left(c + d \, x\right)}\right]} + \frac{B^2 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, PolyLog\left[2, \, \frac{\left(d \, f - c \, g\right) \, \left(a + b \, x\right)}{\left(b \, f - a \, g\right) \, \left(c + d \, x\right)}\right]} + \frac{B^2 \, \left(b \, c - a \, d\right) \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

Result (type 4, 883 leaves, 36 steps):

$$\frac{b \, B^2 \, (b \, c - a \, d) \, Log \left[ a + b \, x \right]}{\left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)} - \frac{b^2 \, B^2 \, Log \left[ a + b \, x \right]^2}{2 \, g \, \left( b \, f - a \, g \right)^2} - \frac{B \, (b \, c - a \, d) \, \left( d \, f - c \, g \right) \, \left( f + g \, x \right)}{\left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)} + \frac{b^2 \, B \, Log \left[ a + b \, x \right]}{\left( b \, f - a \, g \right)^2} - \frac{A \, B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]}{2 \, g \, \left( f \, f \, g \, x \right)^2} - \frac{B^2 \, d \, \left( b \, c - a \, d \right) \, Log \left[ c + d \, x \right]}{\left( b \, f - a \, g \right) \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, Log \left[ c + d \, x \right]}{g \, \left( d \, f - c \, g \right)^2} - \frac{B^2 \, d \, \left( b \, f - a \, g \right) \, \left( d \, f - c \, g \right)^2}{\left( b \, f - a \, g \right) \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f - c \, g \right)^2} + \frac{B^2 \, d^2 \, Log \left[ c + d \, x \right]}{g \, \left( b \, f - a \, g \right)^2 \, \left( d \, f -$$

#### Problem 247: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(f + g x\right)^{4}} dx$$

Optimal (type 4, 714 leaves, 12 steps):

$$\frac{B^2 \left(b\,c-a\,d\right)^2 g^2 \left(c+d\,x\right)}{3 \left(b\,f-a\,g\right)^2 \left(d\,f-c\,g\right)^3 \left(f+g\,x\right)} + \frac{B^2 \left(b\,c-a\,d\right)^3 g^2 \, Log\left[\frac{a+b\,x}{c+d\,x}\right]}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3} - \frac{B \left(b\,c-a\,d\right) g^2 \left(c+d\,x\right)^2 \left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3 \left(b\,f-a\,g\right)^3 \left(f+g\,x\right)^2} + \frac{2\,B \left(b\,c-a\,d\right) g \left(3\,b\,d\,f-b\,c\,g-2\,a\,d\,g\right) \left(a+b\,x\right) \left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^2 \left(f+g\,x\right)} + \frac{b^3 \left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3 g \left(b\,f-a\,g\right)^3} - \frac{\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3 g \left(f+g\,x\right)^3} - \frac{B^2 \left(b\,c-a\,d\right)^3 g^2 \, Log\left[\frac{f+g\,x}{c+d\,x}\right]}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3 \left(d\,f-c\,g\right)^3} + \frac{2\,B^2 \left(b\,c-a\,d\right)^2 g \left(3\,b\,d\,f-b\,c\,g-2\,a\,d\,g\right) \, Log\left[\frac{f+g\,x}{c+d\,x}\right]}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3} + \frac{1}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3} - \frac{\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3} - \frac{2\,B^2 \left(b\,c-a\,d\right) \left(a^2\,d^2\,g^2-a\,b\,d\,g\left(3\,d\,f-c\,g\right)+b^2 \left(3\,d^2\,f^2-3\,c\,d\,f\,g+c^2\,g^2\right)\right) \left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right) \, Log\left[1-\frac{\left(d\,f-c\,g\right) \left(a+b\,x\right)}{\left(b\,f-a\,g\right) \left(c+d\,x\right)}\right] + \frac{2\,B^2 \left(b\,c-a\,d\right) \left(a^2\,d^2\,g^2-a\,b\,d\,g\left(3\,d\,f-c\,g\right)+b^2 \left(3\,d^2\,f^2-3\,c\,d\,f\,g+c^2\,g^2\right)\right) \, PolyLog\left[2,\frac{\left(d\,f-c\,g\right) \left(a+b\,x\right)}{\left(b\,f-a\,g\right) \left(c+d\,x\right)}\right]}{3 \left(b\,f-a\,g\right)^3 \left(d\,f-c\,g\right)^3}$$

Result (type 4, 1356 leaves, 40 steps):

$$\frac{B^2 \left(b \, c - a \, d\right)^2 \, g}{3 \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^2 \left(f \, + g \, x\right)}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3} \frac{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3 \left(d \, f - c \, g\right)^3}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3 \left(d \, f - c \, g\right)^3} \frac{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3}{3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)} \frac{2 \, B \left(b \, c - a \, d\right) \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, g \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right) \left(d \, f - c \, g\right) \left(d \, f - c \, g\right)^2 \left(d \, f - c \, g\right)^2 \left(f + g \, x\right)} \frac{2 \, B \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, g \left(b \, f - a \, g\right)^3} \frac{2 \, B^2 \, d^2 \left(b \, c - a \, d\right) \, Log\left[c + d \, x\right]}{3 \, g \left(b \, f - a \, g\right)^3} \frac{2 \, B^2 \, d^2 \left(b \, c - a \, d\right) \, Log\left[c + d \, x\right]}{3 \, g \left(b \, f - a \, g\right)^3} \frac{2 \, B^3 \, d^2 \left(b \, c - a \, d\right) \, Log\left[c + d \, x\right]}{3 \, g \left(d \, f - c \, g\right)^3} \frac{2 \, B^3 \, d^3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3}{3 \, g \left(d \, f - c \, g\right)^3} \frac{2 \, B^3 \, d^3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3}{3 \, g \left(d \, f - c \, g\right)^3} \frac{2 \, B^3 \, d^3 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3}{3 \, g \left(d \, f - c \, g\right)^3} \frac{3 \, g \, d \, f - c \, g\right)^3}{3 \, g \left(d \, f - c \, g\right)^3} \frac{2 \, B^3 \, d^3 \left(b \, f - a \, g\right)^3 \, d^3 \, d^3 \, d^3 \, d^3 \, g \, d^3 \, d^3 \, g \, d^3 \, g$$

## Problem 248: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(f + gx\right)^{5}} dx$$

Optimal (type 4, 1159 leaves, 15 steps):

$$-\frac{B^2 \left(b \, c - a \, d\right)^2 g^3 \left(c + d \, x\right)^2}{12 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^2} - \frac{B^2 \left(b \, c - a \, d\right)^3 g^3 \left(c + d \, x\right)}{6 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)} + \frac{B^2 \left(b \, c - a \, d\right)^2 g^2 \left(4 \, b \, d \, f - b \, c \, g - 3 \, a \, d \, g\right) \left(c + d \, x\right)}{6 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)} - \frac{B^2 \left(b \, c - a \, d\right)^4 g^3 \log \left[\frac{a \cdot b \, x}{c \cdot d \, x}\right]}{6 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4} + \frac{B^2 \left(b \, c - a \, d\right)^3 g^2 \left(4 \, b \, d \, f - b \, c \, g - 3 \, a \, d \, g\right) \log \left[\frac{a \cdot b \, x}{c \cdot d \, x}\right]}{4 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)} - \frac{B \left(b \, c - a \, d\right)^2 g^2 \left(4 \, b \, d \, f - b \, c \, g - 3 \, a \, d \, g\right) \left(c + d \, x\right)^2 \left(A + B \, \log \left[\frac{e \cdot (a + b \, x)}{c \cdot d \, x}\right]\right)}{4 \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) g^2 \left(4 \, b \, d \, f - b \, c \, g - 3 \, a \, d \, g\right) \left(c + d \, x\right)^2 \left(A + B \, \log \left[\frac{e \cdot (a + b \, x)}{c \cdot d \, x}\right]\right)}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) g^2 \left(4 \, b \, d \, f - b \, c \, g\right)^4 \left(f + g \, x\right)^2}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^3} + \frac{A \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^2}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4 \left(f + g \, x\right)^2} + \frac{B^2 \left(b \, c - a \, d\right) g^2 \left(4 \, b \, d \, f - c \, g\right)^4 \left(6 \, f - c \, g\right)^4 \left(f + g \, x\right)^2}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^3 \left(f + g \, x\right)} + \frac{B^2 \left(b \, c - a \, d\right)^3 g^2 \left(4 \, b \, d \, f - c \, g\right)^4 \left(6 \, f - c \, g\right)^4}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4} + \frac{B^2 \left(b \, c - a \, d\right)^4 g^3 \log \left[\frac{f \cdot g \, x}{c \cdot d \, x}\right]}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4} + \frac{B^2 \left(b \, c - a \, d\right)^3 g^2 \left(4 \, b \, d \, f - c \, g\right)^4 \left(d \, f - c \, g\right)^4}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4} + \frac{B^2 \left(b \, c - a \, d\right)^2 g \left(3 \, a^2 \, d^2 \, g^2 - 2 \, a \, b \, dg \left(4 \, d \, f - c \, g\right)^4 \left(4 \, f - c \, g\right)^4}{4 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4} - \frac{B \left(b \, c - a \, d\right)^2 \left(2 \, b \, d^2 \, g^2 - 2 \, a \, b \, dg \left(4 \, d \, f - c \, g\right)^4 \left(4$$

Result (type 4, 1881 leaves, 44 steps):

$$\frac{B^2 \left(bc-ad\right)^2 g}{12 \left(bf-ag\right)^2 \left(df-cg\right)^2 \left(f+gx\right)^2} - \frac{5B^2 \left(bc-ad\right)^2 g \left(2bdf-bcg-adg\right)}{12 \left(bf-ag\right)^3 \left(df-cg\right)^2 \left(f+gx\right)^2} - \frac{b^2 B^2 \left(bc-ad\right) \left(2bdf-bcg-adg\right) \left(df-cg\right)^2 \left(f+gx\right)^2}{12 \left(bf-ag\right)^4 \left(df-cg\right)^4} - \frac{bB^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3f^2 - cg\right)^3\right) \log \left(a+bx\right)}{6 \left(bf-ag\right)^4 \left(df-cg\right)^2} - \frac{bB^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \log \left(a+bx\right)}{2 \left(bf-ag\right)^4 \left(df-cg\right)^2} - \frac{bB^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right)}{2 \left(bf-ag\right)^4 \left(bf-ag\right)^4 - 6 \left(bf-ag\right) \left(df-cg\right) \left(f+gx\right)^2} - \frac{4 \left(bf-ag\right)^2 \left(df-cg\right)^2 \left(f+gx\right)^2}{4 \left(bf-ag\right)^4 - 6 \left(bf-ag\right) \left(df-cg\right)^3 \left(f+gx\right)} - \frac{b^2 B^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \left(A + B \log \left[\frac{a+bxy}{c+dx}\right]\right)}{2 \left(bf-ag\right)^4 - 2 \left(bf-ag\right)^3 \left(df-cg\right)^3} - \frac{b^2 B^2 \left(bc-ad\right) \left(2bdf-bcg-adg\right) \log \left(c+dx\right)}{2 \left(bf-ag\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(2bdf-bcg-adg\right) \log \left(c+dx\right)}{4 \left(bf-ag\right)^2 \left(df-cg\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \log \left(c+dx\right)}{2 \left(bf-ag\right)^3 \left(df-cg\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(2bdf-bcg-adg\right) \log \left(c+dx\right)}{4 \left(bf-ag\right)^2 \left(df-cg\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(a^2 d^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \log \left(c+dx\right)}{2 \left(bf-ag\right)^3 \left(df-cg\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(a^2 b^2 g^2 - abdg \left(a^2 b^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \log \left(c+dx\right)}{2 \left(bf-ag\right)^4 \left(af-cg\right)^4} - \frac{b^2 B^2 \left(bc-ad\right) \left(a^2 b^2 g^2 - abdg \left(3df-cg\right) + b^2 \left(3d^2 f^2 - 3cdf g + c^2 g^2\right)\right) \log \left(c+dx\right)}{2 \left(af-cg\right)^4} - \frac{b^2 B^2 \left(af-cg\right)^4}{2 \left(a$$

## Problem 249: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Log}\left[\frac{1+x}{-1+x}\right]}{x^2} \, \mathrm{d}x$$

Optimal (type 3, 35 leaves, 3 steps):

$$2 \log \left[-\frac{x}{1-x}\right] - \frac{\left(1+x\right) \log \left[-\frac{1+x}{1-x}\right]}{x}$$

Result (type 3, 34 leaves, 4 steps):

$$2 Log[x] - 2 Log[1 + x] - \frac{(1 - x) Log[-\frac{1+x}{1-x}]}{x}$$

## Problem 250: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f + g x\right)^{2}}{A + B Log\left[\frac{e (a + b x)}{c + d x}\right]} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+g\,x\right)^2}{A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]},\,x\right]$$

Result (type 8, 88 leaves, 2 steps):

$$f^2 \, \text{CannotIntegrate} \Big[ \, \frac{1}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, 2 \, f \, g \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate} \Big[ \, \frac{x^2}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \, \text{, } x \, \Big] \, + \, g^2 \, \text{CannotIntegrate}$$

# Problem 251: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{A + B \log \left[\frac{e \cdot (a + b x)}{c + d x}\right]} dx$$

Optimal (type 8, 29 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f+gx}{A+B Log\left[\frac{e(a+bx)}{c+dx}\right]}, x\right]$$

Result (type 8, 53 leaves, 2 steps):

$$\label{eq:fcannotIntegrate} \begin{split} &\text{f CannotIntegrate} \Big[ \frac{1}{A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \text{, } x \Big] + g \, \text{CannotIntegrate} \Big[ \frac{x}{A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)}{c + d \, x} \Big]} \text{, } x \Big] \end{split}$$

## Problem 252: Result valid but suboptimal antiderivative.

$$\int \frac{1}{A + B \log \left[ \frac{e (a+bx)}{c+dx} \right]} dx$$

Optimal (type 8, 23 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{A + B \log \left[\frac{e (a+b x)}{c+d x}\right]}, x\right]$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{A + B \log \left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]}, x\right]$$

## Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right) \left(A + B Log\left[\frac{e (a + b x)}{c + d x}\right]\right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)},\,x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \text{CannotIntegrate} \Big[ \, \frac{1}{ \left( \texttt{f} + \texttt{g} \, \texttt{x} \right) \, \left( \texttt{A} + \texttt{B} \, \texttt{Log} \left[ \, \frac{\texttt{e} \, \left( \texttt{a} + \texttt{b} \, \texttt{x} \right)}{\texttt{c} + \texttt{d} \, \texttt{x}} \, \right] \, \right)} \, , \, \, \texttt{x} \, \Big]$$

## Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[\frac{e (a+bx)}{c+dx}\right]\right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^2\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 31 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}$$
,  $x\right]$ 

# Problem 255: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{3} \left(A + B Log\left[\frac{e (a+bx)}{c+dx}\right]\right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)},\,x\right]$$

Result (type 8, 31 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+gx\right)^{3}\left(A+B\log\left[\frac{e(a+bx)}{c+dx}\right]\right)},x\right]$$

### Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f + g x\right)^{2}}{\left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+gx\right)^{2}}{\left(A+B\log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}, x\right]$$

Result (type 8, 88 leaves, 2 steps):

$$f^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{1}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, 2 \, f \, g \, \text{CannotIntegrate} \, \Big[ \, \frac{x}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \, \Big[ \, \frac{e \, (a + b \, x)}{c + d \, x} \, \Big] \, \right)^{2}} \, \text{, } x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \, \Big[ \, \frac{x$$

## Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left(A + B \operatorname{Log}\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]\right)^{2}} dx$$

Optimal (type 8, 29 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f + g x}{\left(A + B Log \left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

Result (type 8, 53 leaves, 2 steps):

$$f \ Cannot Integrate \left[ \frac{1}{\left( A + B \ Log \left[ \frac{e \ (a+b \ x)}{c+d \ x} \right] \right)^2}, \ x \right] + g \ Cannot Integrate \left[ \frac{x}{\left( A + B \ Log \left[ \frac{e \ (a+b \ x)}{c+d \ x} \right] \right)^2}, \ x \right]$$

## Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 8, 23 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(A + B \log \left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(A + B Log\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

### Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right) \left(A + B Log\left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,\text{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2},\,x\right]$$

Result (type 8, 31 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}$$
,  $x\right]$ 

## Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^2\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2},\,x\right]$$

Result (type 8, 31 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^{2}\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}},\,x\right]$$

# Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{3} \left(A + B Log\left[\frac{e (a + b x)}{c + d x}\right]\right)^{2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2},\,x\right]$$

Result (type 8, 31 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}$$
,  $x\right]$ 

## Problem 262: Result valid but suboptimal antiderivative.

$$\int \left(f+g\,x\right)^4\,\left(A+B\,Log\left[\,\frac{e\,\left(\,a+b\,x\right)^{\,2}}{\left(\,c+d\,x\right)^{\,2}}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 357 leaves, 3 steps):

$$\frac{1}{5 \, b^4 \, d^4} 2 \, B \, \left( b \, c - a \, d \right) \, g \, \left( a^3 \, d^3 \, g^3 - a^2 \, b \, d^2 \, g^2 \, \left( 5 \, d \, f - c \, g \right) + a \, b^2 \, d \, g \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) - b^3 \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, x - b^2 \, d^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, x^2 - b^3 \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, x - b^3 \, d^3 \,$$

Result (type 3, 341 leaves, 4 steps):

$$\frac{1}{5 \, b^4 \, d^4} 2 \, B \, g \, \left( 10 \, a \, b^3 \, d^4 \, f^3 - 10 \, a^2 \, b^2 \, d^4 \, f^2 \, g + 5 \, a^3 \, b \, d^4 \, f \, g^2 - a^4 \, d^4 \, g^3 - b^4 \, c \, \left( 10 \, d^3 \, f^3 - 10 \, c \, d^2 \, f^2 \, g + 5 \, c^2 \, d \, f \, g^2 - c^3 \, g^3 \right) \right) \, x - \\ \frac{B \, \left( b \, c - a \, d \right) \, g^2 \, \left( a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left( 5 \, d \, f - c \, g \right) + b^2 \, \left( 10 \, d^2 \, f^2 - 5 \, c \, d \, f \, g + c^2 \, g^2 \right) \right) \, x^2}{5 \, b^3 \, d^3} - \\ \frac{B \, \left( b \, c - a \, d \right) \, g^4 \, x^4}{10 \, b \, d} - \frac{2 \, B \, \left( b \, f - a \, g \right)^5 \, Log \left[ a + b \, x \right]}{5 \, b^5 \, g} + \frac{\left( f + g \, x \right)^5 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)^2}{(c + d \, x)^2} \right] \right)}{5 \, g} + \frac{2 \, B \, \left( d \, f - c \, g \right)^5 \, Log \left[ c + d \, x \right]}{5 \, d^5 \, g}$$

## Problem 263: Result optimal but 1 more steps used.

$$\int \left(f+g\,x\right)^3\,\left(A+B\,Log\,\Big[\,\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(\,c+d\,x\right)^{\,2}}\,\Big]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 229 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(4 \, d \, f - c \, g\right) \, + b^2 \, \left(6 \, d^2 \, f^2 - 4 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, x}{2 \, b^3 \, d^3} \\ -\frac{B \left(b \, c - a \, d\right) \, g^3 \, x^3}{6 \, b \, d} - \frac{B \left(b \, f - a \, g\right)^4 \, Log \left[a + b \, x\right]}{2 \, b^4 \, g} + \frac{\left(f + g \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{4 \, g} + \frac{B \left(d \, f - c \, g\right)^4 \, Log \left[c + d \, x\right]}{2 \, d^4 \, g}$$

Result (type 3, 229 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(4 \, d \, f - c \, g\right) \, + b^2 \, \left(6 \, d^2 \, f^2 - 4 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, x}{2 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right) \, g^2 \, \left(4 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, x^2}{4 \, b^2 \, d^2} + \frac{\left(f + g \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{4 \, g} + \frac{B \left(d \, f - c \, g\right)^4 \, Log \left[c + d \, x\right]}{2 \, d^4 \, g}$$

### Problem 264: Result optimal but 1 more steps used.

$$\int \left(f+g\,x\right)^2\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,\right]\right)\,\mathrm{d}x$$

Optimal (type 3, 152 leaves, 3 steps):

$$-\frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(3\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,x}{3\,b^{2}\,d^{2}} - \frac{B\,\left(b\,c-a\,d\right)\,g^{2}\,x^{2}}{3\,b\,d} - \\ \frac{2\,B\,\left(b\,f-a\,g\right)^{3}\,Log\,[\,a+b\,x\,]}{3\,b^{3}\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(\,a+b\,x\,)^{\,2}}{(\,c+d\,x\,)^{\,2}}\right]\right)}{3\,g} + \frac{2\,B\,\left(d\,f-c\,g\right)^{\,3}\,Log\,[\,c+d\,x\,]}{3\,d^{\,3}\,g}$$

Result (type 3, 152 leaves, 4 steps):

$$\begin{split} & \frac{2 \, B \, \left(b \, c - a \, d\right) \, g \, \left(3 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, x}{3 \, b^2 \, d^2} - \frac{B \, \left(b \, c - a \, d\right) \, g^2 \, x^2}{3 \, b \, d} - \\ & \frac{2 \, B \, \left(b \, f - a \, g\right)^3 \, Log \, [\, a + b \, x \, ]}{3 \, b^3 \, g} + \frac{\left(f + g \, x\right)^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{3 \, g} + \frac{2 \, B \, \left(d \, f - c \, g\right)^3 \, Log \, [\, c + d \, x \, ]}{3 \, d^3 \, g} \end{split}$$

### Problem 265: Result optimal but 1 more steps used.

$$\int (f + g x) \left(A + B Log \left[\frac{e (a + b x)^{2}}{(c + d x)^{2}}\right]\right) dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ g \ x}{b \ d} - \frac{B \left(b \ f - a \ g\right)^2 \ Log \left[a + b \ x\right]}{b^2 \ g} + \frac{\left(f + g \ x\right)^2 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)^2}{(c + d \ x)^2}\right]\right)}{2 \ g} + \frac{B \left(d \ f - c \ g\right)^2 \ Log \left[c + d \ x\right]}{d^2 \ g}$$

Result (type 3, 104 leaves, 4 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, g \, x}{b \, d} \, - \, \frac{B \, \left(b \, f - a \, g\right)^2 \, Log \left[\, a + b \, x\,\right]}{b^2 \, g} \, + \, \frac{\left(f + g \, x\right)^2 \, \left(A + B \, Log\left[\, \frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\,\right]\,\right)}{2 \, g} \, + \, \frac{B \, \left(d \, f - c \, g\right)^2 \, Log \left[\, c + d \, x\,\right]}{d^2 \, g} \, d^2 \, g^2 \, d^2 \, d^2 \, g^2 \, d^2 \, d^2 \, g^2 \, d^2 \, d^$$

# Problem 267: Result optimal but 3 more steps used.

$$\int \frac{A + B Log \left[\frac{e (a+bx)^2}{(c+dx)^2}\right]}{f + gx} dx$$

#### Optimal (type 4, 144 leaves, 7 steps):

$$-\frac{2 \, B \, Log \left[-\frac{g \, (a+b \, x)}{b \, f-a \, g}\right] \, Log \, [\, f+g \, x\,]}{g} + \frac{\left(A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]\right) \, Log \, [\, f+g \, x\,]}{g} + \frac{g}{g} \\ -\frac{2 \, B \, Log \left[-\frac{g \, (c+d \, x)}{d \, f-c \, g}\right] \, Log \, [\, f+g \, x\,]}{g} - \frac{2 \, B \, PolyLog \left[2, \frac{b \, (f+g \, x)}{b \, f-a \, g}\right]}{g} + \frac{2 \, B \, PolyLog \left[2, \frac{d \, (f+g \, x)}{d \, f-c \, g}\right]}{g}$$

#### Result (type 4, 144 leaves, 10 steps):

$$-\frac{2\,B\,Log\left[-\frac{g\,\left(a+b\,X\right)}{b\,f-a\,g}\right]\,Log\,[\,f+g\,X\,]}{g}\,+\,\frac{\left(A+B\,Log\left[\frac{e\,\left(a+b\,X\right)^{\,2}}{\left(c+d\,X\right)^{\,2}}\right]\right)\,Log\,[\,f+g\,X\,]}{g}\,+\,\frac{g}{g}\,+\,\frac{2\,B\,Log\left[-\frac{g\,\left(c+d\,X\right)}{d\,f-c\,g}\right]\,Log\,[\,f+g\,X\,]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{d\,f-c\,g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left[\,2\,,\,\frac{d\,\left(f+g\,X\right)}{g}\right]}{g}\,+\,\frac{2\,B\,PolyLog\left$$

### Problem 268: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]}{\left(f + gx\right)^{2}} dx$$

#### Optimal (type 3, 90 leaves, 3 steps):

$$\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^{\,2}}{\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{\,2}}\right]\right)}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{f}+\mathsf{g}\,\mathsf{x}\right)}+\frac{2\,\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{Log}\left[\frac{\mathsf{f}+\mathsf{g}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)}$$

#### Result (type 3, 117 leaves, 4 steps):

$$\frac{2\,b\,B\,Log\,[\,a\,+\,b\,\,x\,]}{g\,\left(b\,f\,-\,a\,g\right)}\,-\,\frac{A\,+\,B\,Log\,\left[\,\frac{e\,\,(\,a\,+\,b\,\,x\,)^{\,2}}{(\,c\,+\,d\,\,x\,)^{\,2}}\,\right]}{g\,\left(f\,+\,g\,x\right)}\,-\,\frac{2\,B\,d\,Log\,[\,c\,+\,d\,\,x\,]}{g\,\left(d\,f\,-\,c\,g\right)}\,+\,\frac{2\,B\,\left(b\,c\,-\,a\,d\right)\,\,Log\,[\,f\,+\,g\,\,x\,]}{\left(b\,f\,-\,a\,g\right)\,\,\left(d\,f\,-\,c\,g\right)}$$

## Problem 269: Result optimal but 1 more steps used.

$$\int \frac{A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]}{\left(f + gx\right)^{3}} dx$$

#### Optimal (type 3, 175 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{\left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, Log \left[a + b \, x\right]}{g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, Log \left[c + d \, x\right]}{g \, \left(d \, f - c \, g\right)^2} + \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, Log \left[f + g \, x\right]}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

Result (type 3, 175 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{\left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)} + \frac{b^2 \, B \, Log \left[a + b \, x\right]}{g \, \left(b \, f - a \, g\right)^2} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{2 \, g \, \left(f + g \, x\right)^2} - \frac{B \, d^2 \, Log \left[c + d \, x\right]}{g \, \left(d \, f - c \, g\right)^2} + \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, Log \left[f + g \, x\right]}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2}$$

Problem 270: Result optimal but 1 more steps used.

$$\int \frac{A + B Log\left[\frac{e (a+bx)^2}{(c+dx)^2}\right]}{\left(f + gx\right)^4} dx$$

Optimal (type 3, 277 leaves, 3 steps):

$$-\frac{B \left(b \, c-a \, d\right)}{3 \left(b \, f-a \, g\right) \left(d \, f-c \, g\right) \left(f+g \, x\right)^2} - \frac{2 \, B \left(b \, c-a \, d\right) \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right)}{3 \left(b \, f-a \, g\right)^2 \left(d \, f-c \, g\right)^2 \left(f+g \, x\right)} + \frac{2 \, b^3 \, B \, Log \left[a+b \, x\right]}{3 \, g \left(b \, f-a \, g\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{2 \, B \, d^3 \, Log \left[c+d \, x\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{2 \, B \, d^3 \, Log \left[c+d \, x\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+g \, x)^3}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+g \, x)^3}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+g \, x)^3}\right]}{3 \, g \left(f+g \, x\right)^3} - \frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+g \, x)^3}\right]}{3 \, g \left(f+g \, x\right)^3}$$

Result (type 3, 277 leaves, 4 steps):

$$-\frac{B \left(b \, c-a \, d\right)}{3 \left(b \, f-a \, g\right) \left(d \, f-c \, g\right) \left(f+g \, x\right)^2} -\frac{2 \, B \left(b \, c-a \, d\right) \left(2 \, b \, d \, f-b \, c \, g-a \, d \, g\right)}{3 \left(b \, f-a \, g\right)^2 \left(d \, f-c \, g\right)^2 \left(f+g \, x\right)} + \frac{2 \, b^3 \, B \, Log \left[a+b \, x\right]}{3 \, g \left(b \, f-a \, g\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{2 \, B \, d^3 \, Log \left[c+d \, x\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{2 \, B \, d^3 \, Log \left[c+d \, x\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3} -\frac{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}{3 \, g \left(f+g \, x\right)^3}$$

Problem 271: Result optimal but 1 more steps used.

$$\int \frac{A + B Log\left[\frac{e \cdot (a+b \cdot x)^2}{(c+d \cdot x)^2}\right]}{\left(f + g \cdot x\right)^5} dx$$

Optimal (type 3, 381 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{6 \left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right) \, \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right)}{4 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2 \, \left(f + g \, x\right)^2} - \frac{B \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \, \left(3 \, d \, f - c \, g\right) + b^2 \, \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right)}{2 \, \left(b \, f - a \, g\right)^3 \, \left(d \, f - c \, g\right)^3 \, \left(f + g \, x\right)} + \frac{b^4 \, B \, Log \left[a + b \, x\right]}{2 \, g \, \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]}{4 \, g \, \left(f + g \, x\right)^4} - \frac{B \, d^4 \, Log \left[c + d \, x\right]}{2 \, g \, \left(d \, f - c \, g\right)^4} - \frac{B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \, \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \, \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, Log \left[f + g \, x\right]}{2 \, \left(b \, f - a \, g\right)^4 \, \left(d \, f - c \, g\right)^4}$$

Result (type 3, 381 leaves, 4 steps):

$$-\frac{B \left(b \, c - a \, d\right)}{6 \left(b \, f - a \, g\right) \left(d \, f - c \, g\right) \left(f + g \, x\right)^3} - \frac{B \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right)}{4 \left(b \, f - a \, g\right)^2 \left(d \, f - c \, g\right)^2 \left(f + g \, x\right)^2} - \\ \frac{B \left(b \, c - a \, d\right) \left(a^2 \, d^2 \, g^2 - a \, b \, d \, g \left(3 \, d \, f - c \, g\right) + b^2 \left(3 \, d^2 \, f^2 - 3 \, c \, d \, f \, g + c^2 \, g^2\right)\right)}{2 \left(b \, f - a \, g\right)^3 \left(d \, f - c \, g\right)^3 \left(f + g \, x\right)} + \frac{b^4 \, B \, Log \left[a + b \, x\right]}{2 \, g \left(b \, f - a \, g\right)^4} - \frac{A + B \, Log \left[\frac{e \cdot (a + b \, x)^2}{(c + d \, x)^2}\right]}{4 \, g \left(f + g \, x\right)^4} - \\ \frac{B \, d^4 \, Log \left[c + d \, x\right]}{2 \, g \left(d \, f - c \, g\right)^4} - \frac{B \left(b \, c - a \, d\right) \left(2 \, b \, d \, f - b \, c \, g - a \, d \, g\right) \left(2 \, a \, b \, d^2 \, f \, g - a^2 \, d^2 \, g^2 - b^2 \left(2 \, d^2 \, f^2 - 2 \, c \, d \, f \, g + c^2 \, g^2\right)\right) \, Log \left[f + g \, x\right]}{2 \left(b \, f - a \, g\right)^4 \left(d \, f - c \, g\right)^4}$$

# Problem 272: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{3} \left(A + B Log \left[\frac{e (a + b x)^{2}}{(c + d x)^{2}}\right]\right)^{2} dx$$

Optimal (type 4, 869 leaves, 15 steps):

$$\frac{2\,B^2\,\left(b\,c-a\,d\right)^3\,g^3\,x}{3\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^2\,\left(4\,b\,d\,f-3\,b\,c\,g-a\,d\,g\right)\,x}{b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^3\,\left(c+d\,x\right)^2}{3\,b^2\,d^4} - \frac{1}{b^4\,d^3}$$

$$B\,\left(b\,c-a\,d\right)\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(2\,d\,f-c\,g\right) + b^2\,\left(6\,d^2\,f^2-8\,c\,d\,f\,g+3\,c^2\,g^2\right)\right)\,\left(a+b\,x\right)\,\left[A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right] - \frac{B\,\left(b\,c-a\,d\right)\,g^2\,\left(4\,b\,d\,f-3\,b\,c\,g-a\,d\,g\right)\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)}{2\,b^2\,d^4} - \frac{B\,\left(b\,c-a\,d\right)\,g^3\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right)}{3\,b^4\,d^4} - \frac{3\,b\,d^4}{3\,b^4\,d^4} - \frac{1}{b^4\,d^4}$$

$$B\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,\left(2\,a\,b\,d^2\,f\,g-a^2\,d^2\,g^2-b^2\,\left(2\,d^2\,f^2-2\,c\,d\,f\,g+c^2\,g^2\right)\right)\,\left[A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\right]\right]\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right] + \frac{2\,B^2\,\left(b\,c-a\,d\right)^4\,g^3\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{3\,b^4\,d^4} + \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,\left(4\,b\,d\,f-3\,b\,c\,g-a\,d\,g\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{b^4\,d^4} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^4\,g^3\,Log\left[c+d\,x\right]}{3\,b^4\,d^4} + \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,\left(4\,b\,d\,f-3\,b\,c\,g-a\,d\,g\right)\,Log\left[c+d\,x\right]}{b^4\,d^4} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(2\,d\,f-c\,g\right) + b^2\,\left(6\,d^2\,f^2-8\,c\,d\,f\,g+3\,c^2\,g^2\right)\right)\,Log\left[c+d\,x\right]}{b^4\,d^4} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(2\,d\,f-c\,g\right) + b^2\,\left(6\,d^2\,f^2-2\,c\,d\,f\,g+c^2\,g^2\right)\right)\,Log\left[c+d\,x\right]}{b^4\,d^4} + \frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,g\,\left(a^2\,d^2\,g^2-2\,a\,b\,d\,$$

Result (type 4, 973 leaves, 33 steps):

$$\frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,\left(b\,c+a\,d\right)\,g^3\,x}{3\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,x}{b^3\,d^3} - \frac{3\,B^2\,\left(b\,c-a\,d\right)\,g\,\left(a^2\,d^2\,g^2-a\,b\,d\,g\,\left(4\,d\,f-c\,g\right) + b^2\,\left(6\,d^2\,f^2-4\,c\,d\,f\,g+c^2\,g^2\right)\right)\,x}{b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^3\,x^2}{3\,b^2\,d^2} - \frac{3\,b^2\,d^2}{3\,b^2\,d^2} - \frac{3\,b^2\,d^2}{3\,b^3\,d^3} - \frac{3\,b^2\,d^2}{b^4\,g^2} - \frac{2\,B^2\,\left(b\,c-a\,d\right)\,g^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,Log\left(a+b\,x\right)}{b^4\,d^2} + \frac{B^2\,\left(b\,f-a\,g\right)^4\,Log\left(a+b\,x\right)^2}{b^4\,g} - \frac{B^2\,\left(b\,c-a\,d\right)\,g\,\left(a^2\,d^2\,g^2-a\,b\,d\,g\,\left(4\,d\,f-c\,g\right) + b^2\,\left(6\,d^2\,f^2-4\,c\,d\,f\,g+c^2\,g^2\right)\right)\,\left(a+b\,x\right)\,Log\left(\frac{a\,(a+b\,x)^2}{(c+d\,x)^2}\right)}{b^4\,g} - \frac{B\,\left(b\,c-a\,d\right)\,g^2\,\left(4\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,x^2\,\left(A+B\,Log\left(\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right)\right)}{b^4\,d^3} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d}{3\,b\,d} - \frac{3\,b\,d^2}{3\,b\,d} - \frac{3\,b\,d^2}{3\,b\,d^2} - \frac{3\,b\,d^2}$$

#### Problem 273: Result valid but suboptimal antiderivative.

$$\int (f + g x)^{2} \left(A + B Log \left[\frac{e (a + b x)^{2}}{(c + d x)^{2}}\right]\right)^{2} dx$$

Optimal (type 4, 542 leaves, 12 steps):

$$\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,b^{2}\,d^{2}} - \frac{4\,B\,\left(b\,c-a\,d\right)\,g\,\left(3\,b\,d\,f-2\,b\,c\,g-a\,d\,g\right)\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b^{3}\,d^{2}} - \frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b\,d^{3}} - \frac{\left(b\,f-a\,g\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,b^{3}\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,g} + \frac{1}{3\,g} + \frac{1}{3\,b^{3}\,d^{3}} + \frac{1}$$

#### Result (type 4, 659 leaves, 29 steps):

$$\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,x}{3\,b^{2}\,d^{2}} - \frac{4\,A\,B\,\left(b\,c-a\,d\right)\,g\,\left(3\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,x}{3\,b^{2}\,d^{2}} + \frac{4\,a^{2}\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,Log\left[a+b\,x\right]}{3\,b^{3}\,d} + \frac{4\,B^{2}\,\left(b\,f-a\,g\right)^{3}\,Log\left[a+b\,x\right]^{2}}{3\,b^{3}\,g} - \frac{4\,B^{2}\,\left(b\,c-a\,d\right)\,g\,\left(3\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]}{3\,b^{3}\,g} - \frac{2\,B\,\left(b\,f-a\,g\right)^{3}\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b^{3}\,g} - \frac{4\,B\,\left(b\,f-a\,g\right)^{3}\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b^{3}\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,b^{3}\,g} - \frac{4\,B^{2}\,c^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,Log\left[c+d\,x\right]}{3\,b^{3}\,g} + \frac{8\,B^{2}\,\left(b\,f-a\,g\right)^{3}\,Log\left[a+b\,x\right]\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,b^{3}\,g} + \frac{4\,B^{2}\,\left(d\,f-c\,g\right)^{3}\,Log\left[c+d\,x\right]}{3\,d^{3}\,g} - \frac{3\,d^{3}\,g}{3\,d^{3}\,g} - \frac{4\,B^{2}\,\left(b\,f-a\,g\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)\,Log\left[c+d\,x\right]}{3\,b^{3}\,g} + \frac{4\,B^{2}\,\left(d\,f-c\,g\right)^{3}\,Log\left[c+d\,x\right]}{3\,d^{3}\,g} - \frac{3\,d^{3}\,g}{3\,d^{3}\,g} - \frac{3\,d^{3}\,g}{3\,d^{3}\,g} - \frac{3\,d^{3}\,g}{3\,d^{3}\,g} - \frac{3\,B^{2}\,\left(b\,f-a\,g\right)^{3}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,d^{3}\,g} - \frac{8\,B^{2}\,\left(b\,f-a\,g\right)^{3}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,d^{3}\,g} - \frac{3\,d^{3}\,g}{3\,d^{3}\,g} - \frac{3\,d^{$$

### Problem 274: Result valid but suboptimal antiderivative.

$$\int \left(f+g\,x\right) \, \left(A+B\,Log\, \left[\,\frac{e\,\left(\,a+b\,x\,\right)^{\,2}}{\left(\,c+d\,x\,\right)^{\,2}}\,\right]\,\right)^{2} \, \mathrm{d}x$$

Optimal (type 4, 281 leaves, 9 steps):

$$-\frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{\,2}}{(c+d\,x)^{\,2}}\right]\right)}{b^{2}\,d}-\frac{\left(b\,f-a\,g\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{\,2}}{(c+d\,x)^{\,2}}\right]\right)^{\,2}}{2\,b^{2}\,g}+\\ \frac{\left(f+g\,x\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{\,2}}{(c+d\,x)^{\,2}}\right]\right)^{\,2}}{2\,g}+\frac{2\,B\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{\,2}}{(c+d\,x)^{\,2}}\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b^{\,2}\,d^{\,2}}+\\ \frac{4\,B^{\,2}\,\left(b\,c-a\,d\right)^{\,2}\,g\,Log\left[c+d\,x\right]}{b^{\,2}\,d^{\,2}}+\frac{4\,B^{\,2}\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,f-b\,c\,g-a\,d\,g\right)\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{b^{\,2}\,d^{\,2}}$$

Result (type 4, 450 leaves, 25 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)\,g\,x}{b\,d} + \frac{2\,B^2\,\left(b\,f-a\,g\right)^2\,Log\,[a+b\,x]^2}{b^2\,g} - \frac{2\,B^2\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,Log\,\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]}{b^2\,d} - \frac{2\,B\,\left(b\,f-a\,g\right)^2\,Log\,[a+b\,x]\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)}{b^2\,g} + \frac{\left(f+g\,x\right)^2\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}{2\,g} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\,[c+d\,x]}{b^2\,d^2} - \frac{4\,B^2\,\left(d\,f-c\,g\right)^2\,Log\,[c+d\,x]}{d^2\,g} + \frac{2\,B\,\left(d\,f-c\,g\right)^2\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)\,Log\,[c+d\,x]}{d^2\,g} + \frac{2\,B^2\,\left(d\,f-c\,g\right)^2\,Log\,[c+d\,x]}{d^2\,g} - \frac{4\,B^2\,\left(b\,f-a\,g\right)^2\,Log\,[a+b\,x]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^2\,g} - \frac{4\,B^2\,\left(d\,f-c\,g\right)^2\,PolyLog\,\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^2\,g} - \frac{4\,B^2\,\left(d\,f-c\,g$$

## Problem 275: Result valid but suboptimal antiderivative.

$$\int \left( A + B Log \left[ \frac{e \left( a + b x \right)^{2}}{\left( c + d x \right)^{2}} \right] \right)^{2} dx$$

Optimal (type 4, 129 leaves, 6 steps):

$$\frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^{2}}{\left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)^{2}}\right]\right)^{2}}{\mathsf{b}} + \frac{4 \, \mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^{2}}{\left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)^{2}}\right]\right) \, \mathsf{Log}\left[\frac{\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}}{\mathsf{b} \, \left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)}\right]}{\mathsf{b} \, \mathsf{d}} + \frac{8 \, \mathsf{B}^{2} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \mathsf{PolyLog}\left[2, \, \frac{\mathsf{d} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\mathsf{b} \, \left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right)}\right]}{\mathsf{b} \, \mathsf{d}} + \frac{\mathsf{b} \, \mathsf{d} \, \mathsf{$$

Result (type 4, 252 leaves, 22 steps):

$$-\frac{4 \, a \, B^2 \, Log \, [\, a + b \, x \, ]^{\, 2}}{b} + \frac{4 \, a \, B \, Log \, [\, a + b \, x \, ] \, \left(A + B \, Log \, \left[\frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}}\right]\right)}{b} + x \, \left(A + B \, Log \, \left[\frac{e \, (a + b \, x)^{\, 2}}{\left(c + d \, x\right)^{\, 2}}\right]\right)^2 + \\ \frac{8 \, B^2 \, c \, Log \, \left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B \, c \, \left(A + B \, Log \, \left[\frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}}\right]\right) \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]^{\, 2}}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\, c + d \, x \, ]}{d} - \frac{4 \, B^2 \, c \, Log \, [\,$$

# Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}}{f + gx} dx$$

#### Optimal (type 4, 285 leaves, 9 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})^2}{(\mathsf{c}+\mathsf{d}\,\mathsf{x})^2}\right]\right)^2\,\mathsf{Log}\left[\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} + \frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})^2}{(\mathsf{c}+\mathsf{d}\,\mathsf{x})^2}\right]\right)^2\,\mathsf{Log}\left[1-\frac{(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g})\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g})\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} - \frac{\mathsf{4}\,\mathsf{B}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})^2}{(\mathsf{c}+\mathsf{d}\,\mathsf{x})^2}\right]\right)\,\mathsf{PolyLog}\left[2,\,\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} + \frac{\mathsf{8}\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}} - \frac{\mathsf{8}\,\mathsf{B}^2\,\mathsf{PolyLog}\left[3,\,\frac{(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g})\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g})\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\right]}{\mathsf{g}}$$

#### Result (type 4, 2126 leaves, 44 steps):

$$\frac{4\,A\,B\,Log\left[-\frac{g\,(a+b\,X)}{b\,f-a\,g}\right)\,Log\,[\,f+g\,X]}{g} - \frac{B^2\,Log\left[\,\left(\,a+b\,X\,\right)^{\,2}\,\right]^{\,2}\,Log\,[\,f+g\,X]}{g} - \frac{B^2\,Log\left[\,\frac{1}{(c+d\,X)^{\,2}}\,\right]^{\,2}\,Log\,[\,f+g\,X]}{g} + \frac{g}{g} + \frac{4\,B\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{b\,f-a\,g}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g}{g} + \frac{g\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{(c+d\,X)^{\,2}}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{(c+d\,X)^{\,2}}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{(c+d\,X)^{\,2}}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{(c+d\,X)^{\,2}}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{e\,(a+b\,X)^{\,2}}{b\,f-a\,g}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{b\,(c+d\,X)}{b\,f-a\,g}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{b\,(c+d\,X)}{b\,f-a\,g}\,\right]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\left[\,\frac{b\,(c+d\,X)}{d\,f-c\,g}\,\right]\,Log\,[\,f+g\,X]}{g} - \frac{g\,Log\,[\,a+b\,X] - Log\left[\,(\,a+b\,X)^{\,2}\,\right]\,Log\,[\,\,a+b\,X] - Log\left[\,(\,a+b\,X)^{\,2}\,\right]\,Log\,[\,f+g\,X]}{g} - \frac{g\,Log\,[\,a+b\,X] - Log\,[\,\,(\,a+b\,X)^{\,2}\,]\,Log\,[\,f+g\,X]}{g} + \frac{g\,Log\,[\,\,a+b\,X] - Log\,[\,\,(\,a+b\,X)^{\,2}\,]\,Log\,[\,\,a+b\,X] - Log\,[\,\,\,a+b\,X] - Log\,[\,\,\,a+b\,X] - Log\,[\,\,\,a+b\,X] - Log\,[\,\,\,a+b\,X] - Log\,[\,\,a+b\,X] - Log$$

$$\frac{4B^2 \left( \log \left[ \frac{b(c+dx)}{b(c+ad)} - \log \left[ -\frac{a(c+dx)}{d(c+g)} \right] \right) \left( \log \left[ a + b \, x \right] + \log \left[ -\frac{(d+c,g)}{(d+c,g)} + (b+c) \right] \right)^2}{g} } {8}$$

$$\frac{4B^2 \left( \log \left[ -\frac{d(a+bx)}{b(a+d)} + \log \left[ -\frac{(d+c,g)}{(d+g)} - \log \left[ -\frac{(d+c,g)}{(b(c+d))} + (b(a+g)) + (cdx)} \right] \right)}{g}$$

$$\frac{4B^2 \left( \log \left[ -\frac{d(a+bx)}{d(d+g)} + \log \left[ -\frac{(a+bx)}{(d+g)} + \log \left[ -\frac{(b+c)}{(b(c+d))} + \log \left[ -\frac{(b+c)}{(b+g)} + (cdx)} \right] \right) \right)}{g}$$

$$\frac{4B^2 \left( \log \left[ -\frac{d(a+bx)}{a(d+d)} - \log \left[ -\frac{(a+bx)}{a(d+d)} \right] \right) \right) \left( \log \left[ c + d \, x \right] + \log \left[ -\frac{(b+c)}{(b+g)} + \log \left[ -\frac{(b+c)}{(d+g)} + \log \left$$

# Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}}{\left(f + g x\right)^{2}} dl x$$

Optimal (type 4, 200 leaves, 4 steps):

$$\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^{2}}{\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{2}}\right]\right)^{2}}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{f}+\mathsf{g}\,\mathsf{x}\right)}+\frac{4\,\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^{2}}{\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{2}}\right]\right)\,\mathsf{Log}\left[\mathsf{1}-\frac{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)}+\frac{8\,\mathsf{B}^{2}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{PolyLog}\left[\mathsf{2},\,\frac{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]}+\frac{\mathsf{B}\,\mathsf{B}^{2}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{PolyLog}\left[\mathsf{2},\,\frac{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)}\right]}$$

Result (type 4, 620 leaves, 32 steps):

$$-\frac{4 \, b \, B^2 \, Log \left[a + b \, x\right]^2}{g \, \left(b \, f - a \, g\right)} + \frac{4 \, b \, B \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{g \, \left(f + g \, x\right)} - \frac{\left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)^2}{g \, \left(f + g \, x\right)} + \frac{8 \, B^2 \, d \, Log \left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \left[c + d \, x\right]}{g \, \left(d \, f - c \, g\right)} - \frac{4 \, B \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right) \, Log \left[c + d \, x\right]}{g \, \left(d \, f - c \, g\right)} + \frac{8 \, b \, B^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} - \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, Log \left[-\frac{g \, (a + b \, x)}{b \, f - a \, g}\right] \, Log \left[f + g \, x\right]}{g \, \left(b \, f - a \, g\right)} + \frac{4 \, B \, b \, B^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} - \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, Log \left[-\frac{g \, (a + b \, x)}{b \, f - a \, g}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, b \, B^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} - \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, Log \left[-\frac{g \, (a + b \, x)}{b \, f - a \, g}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, b \, B^2 \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, Log \left[-\frac{g \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, b \, B^2 \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, b \, B^2 \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{8 \, B^2 \, \left(b \, c - a \, d\right) \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)} + \frac{d \, B^2 \, \left(b \, c - a \, d\right) \, PolyLog \left[2 \, d \, - \frac{d \, (a + b \, x)}$$

## Problem 278: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}}{\left(f + gx\right)^{3}} dx$$

Optimal (type 4, 381 leaves, 9 steps):

$$\frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)\,\left(f+g\,x\right)} + \frac{b^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}{2\,g\,\left(b\,f-a\,g\right)^2} - \frac{\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}{2\,g\,\left(f+g\,x\right)^2} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{4\,B^2\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(b\,c-a\,d\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(a\,f-a\,g\right)^2\,g\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{\left(b\,f-a\,g\right)^2\,\left(d\,f-c\,g\right)^2} + \frac{2\,B\,\left(a\,f-a\,g\right)^2\,$$

Result (type 4, 899 leaves, 36 steps):

$$\frac{4 \, b \, B^2 \, \left(b \, c - a \, d\right) \, Log[\, a + b \, x\,]}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)} - \frac{2 \, b^2 \, B^2 \, Log[\, a + b \, x\,]^2}{g \, \left(b \, f - a \, g\right)^2} - \frac{2 \, B \, \left(b \, c - a \, d\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{\left(b \, f - a \, g\right)^2 \, \left(c + d \, x\right)^2} - \frac{4 \, B^2 \, d \, \left(b \, c - a \, d\right) \, Log[\, c + d \, x\,]}{\left(b \, f - a \, g\right) \, \left(d \, f - c \, g\right)} + \frac{4 \, B^2 \, d^2 \, Log\left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log\left[\, c + d \, x\,\right]}{g \, \left(d \, f - c \, g\right)^2} - \frac{2 \, B \, d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)^2}{g \, \left(d \, f - c \, g\right)^2} + \frac{4 \, B^2 \, d \, \left(b \, c - a \, d\right) \, Log\left[\, c + d \, x\,\right]}{g \, \left(d \, f - c \, g\right)^2} - \frac{2 \, B \, d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)^2}{(c + d \, x)^2}\right]\right)}{g \, \left(d \, f - c \, g\right)^2} + \frac{4 \, B^2 \, B^2 \, Log\left[\, a + b \, x\,\right] \, Log\left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)^2 \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{4 \, B^2 \, B^2 \, Log\left[\, a + b \, x\,\right] \, Log\left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{g \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{4 \, B^2 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g\right)^2 \, \left(d \, f - c \, g\right)^2}{g \, \left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{4 \, B^2 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d\, g\right) \, \left(d \, f - c \, g\right)^2}{g \, \left(d \, f - c \, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d\, g\right) \, \left(d \, f - c \, g\right)^2}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d\, g\right) \, \left(d \, f - c \, g\right)^2}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d\, g\right) \, \left(d \, f - c \, g\right)^2}{\left(b \, f - a \, g\right)^2 \, \left(d \, f - c \, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, f - b \, c \, g - a \, d\, g\right) \, \left(d \, f - c \, g\right)^2}{\left(b \, f - c \, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\, g\right) \, \left(2 \, b \, d \, f - b \, c\, g - a \, d\, g\right) \, \left(d \, f - c\, g\right)^2}{\left(b \, f - c\, g\right)^2} + \frac{2 \, B \, \left(b \, c - a \, d\, g\right) \, \left(2 \, b \, d \, f - b \, c\, g - a \, d\,$$

## Problem 279: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \, Log\left[\frac{e \, (a + b \, x)^{\, 2}}{\left(c + d \, x\right)^{\, 2}}\right]\right)^{\, 2}}{\left(f + g \, x\right)^{\, 4}} \, \mathrm{d}x$$

Optimal (type 4, 724 leaves, 12 steps):

$$\frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\left(c+d\,x\right)}{3\,\left(b\,f-a\,g\right)^{2}\,\left(d\,f-c\,g\right)^{3}\,\left(f+g\,x\right)} - \frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,\left(b\,f-a\,g\right)\,\left(f+g\,x\right)} + \frac{4\,B\,\left(b\,c-a\,d\right)\,g\,\left(3\,b\,d\,f-b\,c\,g-2\,a\,d\,g\right)\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)}{3\,\left(b\,f-a\,g\right)^{3}\,\left(d\,f-c\,g\right)^{3}\,\left(f+g\,x\right)} + \frac{b^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,g\,\left(b\,f-a\,g\right)^{3}} - \frac{\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{(c+d\,x)^{2}}\right]\right)^{2}}{3\,g\,\left(f+g\,x\right)^{3}} + \frac{4\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{3\,\left(b\,f-a\,g\right)^{3}\,\left(d\,f-c\,g\right)^{3}} + \frac{8\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g\,\left(3\,b\,d\,f-b\,c\,g-2\,a\,d\,g\right)\,Log\left[\frac{f+g\,x}{c+d\,x}\right]}{3\,\left(b\,f-a\,g\right)^{3}\,\left(d\,f-c\,g\right)^{3}} + \frac{1}{3\,\left(b\,f-a\,g\right)^{3}\,\left(d\,f-c\,g\right)^{3}} + \frac{1}{3\,\left(b\,f-a\,g\right)^{3}\,\left(d\,f-c$$

Result (type 4, 1369 leaves, 40 steps):

# Problem 280: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}}{\left(f + gx\right)^{5}} dx$$

Optimal (type 4, 1154 leaves, 15 steps):

$$-\frac{B^2 \left(bc-ad\right)^2 g^3 \left(c+dx\right)^2}{3 \left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)^2} - \frac{2 B^2 \left(bc-ad\right)^3 g^3 \left(c+dx\right)}{3 \left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)} + \frac{B^2 \left(bc-ad\right)^2 g^2 \left(4 b d f-b c g-3 a d g\right) \left(c+dx\right)}{\left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)} + \frac{B \left(bc-ad\right)^3 g^2 \left(4 b d f-b c g-3 a d g\right) \left(c+dx\right)}{\left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)} + \frac{B \left(bc-ad\right)^3 \left(df-cg\right)^4 \left(f+gx\right)}{\left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)} + \frac{B \left(bc-ad\right) g^2 \left(4 b d f-b c g-3 a d g\right) \left(c+dx\right)^2 \left(A + B Log\left[\frac{e \left(a+bx\right)^2}{\left(c+dx\right)^2}\right]\right)}{3 \left(bf-ag\right)^3 \left(df-cg\right)^4 \left(f+gx\right)^3} - \frac{B \left(bc-ad\right) g^2 \left(4 b d f-b c g-3 a d g\right) \left(c+dx\right)^2 \left(A + B Log\left[\frac{e \left(a+bx\right)^2}{\left(c+dx\right)^2}\right]\right)}{2 \left(bf-ag\right)^3 \left(df-cg\right)^4 \left(df-cg\right)^4 \left(df-cg\right)^4 + \frac{b^4 \left(A + B Log\left[\frac{e \left(a+bx\right)^2}{\left(c+dx\right)^2}\right]\right)^2}{4 g \left(bf-ag\right)^4} - \frac{\left(A + B Log\left[\frac{e \left(a+bx\right)^2}{\left(c+dx\right)^2}\right]\right)^2}{3 \left(bf-ag\right)^4 \left(df-cg\right)^4} - \frac{2 B^2 \left(bc-ad\right)^4 g^3 Log\left[\frac{a+bx}{c+dx}\right]}{3 \left(bf-ag\right)^4 \left(df-cg\right)^4} + \frac{B^2 \left(bc-ad\right)^3 g^2 \left(4 b d f-b c g-3 a d g\right) Log\left[\frac{f+gx}{c+dx}\right]}{3 \left(bf-ag\right)^4 \left(df-cg\right)^4} - \frac{2 B^2 \left(bc-ad\right)^3 g^2 \left(4 b d f-b c g-3 a d g\right) Log\left[\frac{f+gx}{c+dx}\right]}{\left(bf-ag\right)^4 \left(df-cg\right)^4} + \frac{2 B^2 \left(bc-ad\right)^4 g^3 Log\left[\frac{f+gx}{c+dx}\right]}{3 \left(bf-ag\right)^4 \left(df-cg\right)^4} - \frac{1}{\left(bf-ag\right)^4 \left(df-cg\right)^4} - \frac{1}{\left(bf-a$$

Result (type 4, 1854 leaves, 44 steps):

$$\begin{array}{c} & 8^2 \left( bc - ad \right)^2 g \\ & 3 \left( bf - ag \right)^2 \left( df - cg \right)^2 \left( f + gx \right)^2 \\ & 3 \left( bf - ag \right)^2 \left( df - cg \right)^3 \left( f + cg \right)^3 \left( f + cg \right)^3 \left( f + gx \right) \\ & 3 \left( bf - ag \right)^4 \left( df - cg \right)^2 \\ & 5 \left( bc - ad \right) \left( 2bdf - bcg - adg \right) \log(a + bx) \\ & 2b^2 \left( bc - ad \right) \left( 2bdf - bcg - adg \right) \log(a + bx) \\ & 2b^2 \left( bc - ad \right) \left( 2bdf - bcg - adg \right) \log(a + bx) \\ & 2b^2 \left( bc - ad \right) \left( 2bdf - bcg - adg \right) \log(a + bx) \\ & 2b^2 \left( bc - ad \right) \left( ab - ag \right)^4 \left( df - cg \right)^2 \\ & 2b^2 \left( bc - ad \right) \left( ab - ag \right)^4 \left( df - cg \right)^2 \\ & 2b^2 \left( bc - ad \right) \left( ab - ag \right)^4 \left( df - cg \right)^2 \\ & 3 \left( bf - ag \right)^4 \left( df - cg \right)^2 \\ & 3 \left( bf - ag \right)^4 \left( df - cg \right)^2 \left( d$$

## Problem 281: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f+g\,x\right)^2}{A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]}\,\mathrm{d}x$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+g\,x\right)^{2}}{A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]},\,x\right]$$

Result (type 8, 94 leaves, 2 steps):

$$f^{2} \, \text{CannotIntegrate} \Big[ \, \frac{1}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, \, x \, \Big] \, + \, 2 \, f \, g \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A + B \, \text{Log} \Big[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \Big]}, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x}{A +$$

## Problem 282: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{A + B Log \left[\frac{e (a+b x)^2}{(c+d x)^2}\right]} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f+gx}{A+B Log\left[\frac{e(a+bx)^{2}}{(c+dx)^{2}}\right]}, x\right]$$

Result (type 8, 57 leaves, 2 steps):

$$\label{eq:fcannotIntegrate} \text{f CannotIntegrate} \Big[ \frac{1}{A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big]} \text{, } x \, \Big] \, + \, g \, \text{CannotIntegrate} \Big[ \frac{x}{A + B \, \text{Log} \Big[ \frac{e \, (a + b \, x)^{\, 2}}{(c + d \, x)^{\, 2}} \Big]} \text{, } x \, \Big]$$

## Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{1}{A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]} dx$$

Optimal (type 8, 25 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{A+B \, Log \left[\frac{e \, (a+b \, x)^2}{(c+d \, x)^2}\right]}$$
,  $x\right]$ 

Result (type 8, 25 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{A + B Log\left[\frac{e \cdot (a+b \cdot x)^2}{(c+d \cdot x)^2}\right]}, x\right]$$

### Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right) \left(A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 33 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)}$$
,  $x\right]$ 

## Problem 285: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^{2}\left(A+B\,Log\left[\frac{e\,(a+b\,x)^{2}}{\left(c+d\,x\right)^{2}}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 33 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \text{CannotIntegrate} \Big[ \, \frac{1}{ \left( \, f + g \, x \right)^{\, 2} \, \left( A + B \, Log \left[ \, \frac{e \, \left( \, a + b \, x \, \right)^{\, 2}}{\left( \, c + d \, x \, \right)^{\, 2}} \, \right] \, \right)} \, , \, \, x \, \Big]$$

## Problem 286: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{\left(c+d\,x\right)^2}\right]\right)}$$
,  $x\right]$ 

Result (type 8, 33 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^{3}\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{2}}{\left(c+d\,x\right)^{2}}\right]\right)}$$
,  $x\right]$ 

## Problem 287: Result valid but suboptimal antiderivative.

$$\int \frac{\left(f + g x\right)^{2}}{\left(A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(f+gx\right)^{2}}{\left(A+B\log\left[\frac{e\cdot(a+b\cdot x)^{2}}{\left(c+d\cdot x\right)^{2}}\right]\right)^{2}}, x\right]$$

Result (type 8, 94 leaves, 2 steps):

$$f^{2} \, \text{CannotIntegrate} \Big[ \, \frac{1}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, 2 \, f \, g \, \text{CannotIntegrate} \Big[ \, \frac{x}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right)^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \right]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \Big]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \Big]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \Big]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{(c + d \, x)^{2}} \, \right] \, \Big]^{2}} \, , \, \, x \, \Big] \, + \, g^{2} \, \text{CannotIntegrate} \Big[ \, \frac{x^{2}}{\left( A + B \, \text{Log} \left[ \, \frac{e \, (a + b \, x)^{2}}{$$

## Problem 288: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left(A + B Log \left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{f+gx}{\left(A+B \log \left[\frac{e^{-(a+bx)^2}}{(c+dx)^2}\right]\right)^2}, x\right]$$

Result (type 8, 57 leaves, 2 steps):

$$\label{eq:fcannotIntegrate} \begin{split} &\text{f CannotIntegrate} \, \big[ \, \frac{1}{\left( A + B \, \text{Log} \, \big[ \, \frac{e \, (a + b \, x)^2}{\left( c + d \, x \right)^2} \, \big] \, \right)^2} \text{, } x \, \big] \, \\ & + g \, \text{CannotIntegrate} \, \big[ \, \frac{x}{\left( A + B \, \text{Log} \, \big[ \, \frac{e \, (a + b \, x)^2}{\left( c + d \, x \right)^2} \, \big] \, \right)^2} \text{, } x \, \big] \end{split}$$

## Problem 289: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 25 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(A + B \log \left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 25 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(A+B\ Log\left[\frac{e\ (a+b\ x)^2}{(c+d\ x)^2}\right]\right)^2}$$
,  $x\right]$ 

### Problem 290: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right) \left(A + B Log\left[\frac{e (a+bx)^{2}}{(c+dx)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{\left(c+d\,x\right)^2}\right]\right)^2},\,x\right]$$

Result (type 8, 33 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{\left(c+d\,x\right)^2}\right]\right)^2}$$
,  $x\right]$ 

## Problem 291: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)^{2}}{(c+d x)^{2}}\right]\right)^{2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)^{\,2}}{\left(c+d\,x\right)^{\,2}}\right]\right)^{\,2}}$$
,  $x\right]$ 

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate} \Big[ \frac{1}{\left( f + g \, x \right)^2 \, \left( A + B \, \text{Log} \left[ \frac{e \, (a + b \, x)^2}{\left( c + d \, x \right)^2} \right] \right)^2 } \text{, } x \Big]$$

### Problem 292: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(f + g x\right)^3 \left(A + B Log\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 8, 33 leaves, 0 steps):

Unintegrable 
$$\left[\frac{1}{\left(f+g\,x\right)^3\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{(c+d\,x)^2}\right]\right)^2}$$
,  $x\right]$ 

Result (type 8, 33 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{1}{\left(f+g\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)^2}{\left(c+d\,x\right)^2}\right]\right)^2}$$
,  $x\right]$ 

### Problem 293: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)^4\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 365 leaves, 3 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, \left( b \, c - a \, d \right) \, h \, \left( a^3 \, d^3 \, h^3 - a^2 \, b \, d^2 \, h^2 \, \left( 5 \, d \, g - c \, h \right) \, + a \, b^2 \, d \, h \, \left( 10 \, d^2 \, g^2 - 5 \, c \, d \, g \, h + c^2 \, h^2 \right) \, - b^3 \, \left( 10 \, d^3 \, g^3 - 10 \, c \, d^2 \, g^2 \, h + 5 \, c^2 \, d \, g \, h^2 - c^3 \, h^3 \right) \right) \, n \, x - b^2 \, d^2 \, d^2 \, h^2 - a \, b \, d \, h \, \left( 5 \, d \, g - c \, h \right) \, + b^2 \, \left( 10 \, d^2 \, g^2 - 5 \, c \, d \, g \, h + c^2 \, h^2 \right) \right) \, n \, x^2 \, d^2 \, d^2$$

Result (type 3, 377 leaves, 5 steps):

$$\frac{1}{5 \, b^4 \, d^4} B \, \left( b \, c - a \, d \right) \, h \, \left( a^3 \, d^3 \, h^3 - a^2 \, b \, d^2 \, h^2 \, \left( 5 \, d \, g - c \, h \right) \, + \, a \, b^2 \, d \, h \, \left( 10 \, d^2 \, g^2 - 5 \, c \, d \, g \, h + c^2 \, h^2 \right) \, - \, b^3 \, \left( 10 \, d^3 \, g^3 - 10 \, c \, d^2 \, g^2 \, h + 5 \, c^2 \, d \, g \, h^2 - c^3 \, h^3 \right) \right) \, n \, x - \, b^4 \, d^4 \, d$$

#### Problem 294: Result valid but suboptimal antiderivative.

$$\left\lceil \left(g + h \, x\right)^3 \, \left(A + B \, Log\left[\,e \, \left(\,a + b \, x\,\right)^{\,n} \, \left(\,c + d \, x\right)^{\,-n}\,\right]\,\right) \, \mathbb{d} x$$

Optimal (type 3, 236 leaves, 3 steps):

$$-\frac{B \left(b \ c-a \ d\right) \ h \left(a^{2} \ d^{2} \ h^{2}-a \ b \ d \ h \left(4 \ d \ g-c \ h\right) \ +b^{2} \left(6 \ d^{2} \ g^{2}-4 \ c \ d \ g \ h+c^{2} \ h^{2}\right)\right) \ n \ x}{4 \ b^{3} \ d^{3}} -\frac{B \left(b \ c-a \ d\right) \ h^{2} \left(4 \ b \ d \ g-b \ c \ h-a \ d \ h\right) \ n \ x^{2}}{8 \ b^{2} \ d^{2}} -\frac{B \left(b \ c-a \ d\right) \ h^{3} \ n \ x^{3}}{4 \ b^{4} \ h} +\frac{B \left(d \ g-c \ h\right)^{4} \ n \ Log \left[c+d \ x\right]}{4 \ d^{4} \ h} +\frac{\left(g+h \ x\right)^{4} \left(A+B \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)}{4 \ h} +\frac{B \left(d \ g-c \ h\right)^{4} \ n \ Log \left[c+d \ x\right]}{4 \ d^{4} \ h} +\frac{\left(g+h \ x\right)^{4} \left(A+B \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]\right)}{4 \ h} +\frac{B \left(d \ g-c \ h\right)^{4} \ h \ h}{4 \ h} +\frac{B \left(d \ g-c \ h\right)^{4} \ h \ h}{4 \ h} +\frac{B \left(d \ g-c \ h\right)^{4} \ h}{4 \ h} +\frac{B \left(d$$

Result (type 3, 248 leaves, 5 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ h \left(a^2 \ d^2 \ h^2 - a \ b \ d \ h \left(4 \ d \ g - c \ h\right) + b^2 \left(6 \ d^2 \ g^2 - 4 \ c \ d \ g \ h + c^2 \ h^2\right)\right) \ n \ x}{4 \ b^3 \ d^3} - \frac{B \left(b \ c - a \ d\right) \ h^2 \left(4 \ b \ d \ g - b \ c \ h - a \ d \ h\right) \ n \ x^2}{8 \ b^2 \ d^2} - \frac{B \left(b \ c - a \ d\right) \ h^3 \ n \ x^3}{12 \ b \ d} + \frac{A \left(g + h \ x\right)^4}{4 \ h} - \frac{B \left(b \ g - a \ h\right)^4 \ n \ Log \left[a + b \ x\right]}{4 \ b^4 \ h} + \frac{B \left(d \ g - c \ h\right)^4 \ n \ Log \left[c + d \ x\right]}{4 \ d^4 \ h} + \frac{B \left(g + h \ x\right)^4 \ Log \left[e \ \left(a + b \ x\right)^n \left(c + d \ x\right)^{-n}\right]}{4 \ h}$$

#### Problem 295: Result valid but suboptimal antiderivative.

$$\int (g+hx)^2 (A+BLog[e(a+bx)^n(c+dx)^{-n}]) dx$$

Optimal (type 3, 158 leaves, 3 steps):

$$-\frac{B \, \left(b \, c - a \, d\right) \, h \, \left(3 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n \, x}{3 \, b^2 \, d^2} - \frac{B \, \left(b \, c - a \, d\right) \, h^2 \, n \, x^2}{6 \, b \, d} - \frac{B \, \left(b \, g - a \, h\right)^3 \, n \, Log \left[a + b \, x\right]}{3 \, b^3 \, h} + \frac{B \, \left(d \, g - c \, h\right)^3 \, n \, Log \left[c + d \, x\right]}{3 \, d^3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{3 \, h} + \frac{\left(g + h \, x\right)^3 \, \left(a + B \, Log \left[e$$

Result (type 3, 170 leaves, 5 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ h \left(3 \ b \ d \ g - b \ c \ h - a \ d \ h\right) \ n \ x}{3 \ b^2 \ d^2} - \frac{B \left(b \ c - a \ d\right) \ h^2 \ n \ x^2}{6 \ b \ d} + \frac{A \left(g + h \ x\right)^3}{3 \ h} - \frac{B \left(b \ g - a \ h\right)^3 \ n \ Log \left[a + b \ x\right]}{3 \ b^3 \ h} + \frac{B \left(d \ g - c \ h\right)^3 \ n \ Log \left[c + d \ x\right]}{3 \ d^3 \ h} + \frac{B \left(g + h \ x\right)^3 \ Log \left[e \ \left(a + b \ x\right)^n \ \left(c + d \ x\right)^{-n}\right]}{3 \ h}$$

## Problem 296: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 116 leaves, 3 steps):

$$-\frac{\text{B } \left(\text{b } \text{c }-\text{a } \text{d }\right) \text{ h } \text{n } \text{x}}{\text{2 b } \text{d}}-\frac{\text{B } \left(\text{b } \text{g }-\text{a } \text{h}\right)^2 \text{ n } \text{Log } \left[\text{a }+\text{b } \text{x}\right]}{\text{2 b }^2 \text{ h}}+\frac{\text{B } \left(\text{d } \text{g }-\text{c } \text{h}\right)^2 \text{ n } \text{Log } \left[\text{c }+\text{d } \text{x}\right]}{\text{2 d }^2 \text{ h}}+\frac{\left(\text{g }+\text{h } \text{x}\right)^2 \left(\text{A }+\text{B } \text{Log } \left[\text{e } \left(\text{a }+\text{b } \text{x}\right)^n \left(\text{c }+\text{d } \text{x}\right)^{-n}\right]\right)}{\text{2 h }}$$

Result (type 3, 128 leaves, 5 steps):

$$-\frac{B\,\left(b\,c-a\,d\right)\,h\,n\,x}{2\,b\,d} + \frac{A\,\left(g+h\,x\right)^{\,2}}{2\,h} - \frac{B\,\left(b\,g-a\,h\right)^{\,2}\,n\,Log\,\left[a+b\,x\right]}{2\,b^{\,2}\,h} + \frac{B\,\left(d\,g-c\,h\right)^{\,2}\,n\,Log\,\left[c+d\,x\right]}{2\,d^{\,2}\,h} + \frac{B\,\left(g+h\,x\right)^{\,2}\,Log\,\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]}{2\,h} + \frac{B\,\left(g+h\,x\right)^{\,2}\,Log\,\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,n}\right]}{2\,h} + \frac{B\,\left(g+h\,x\right)^{\,n}\,Log\,\left[e\,\left(a+b\,x\right)^{\,n}\,Log\,\left(a+b\,x\right)^{\,n}\right]}{2\,h} + \frac{B\,\left(g+h\,x\right)^{\,n}\,Log\,\left[e\,\left(a+b\,x\right)^{\,n}\,Log\,\left(a+b\,x\right)^{\,n}\right]}{2\,h} + \frac{B\,\left(a+b\,x\right)^{\,n}\,Log\,\left(a+b\,x\right)^{\,n}\,Log\,\left(a+b\,x\right)^{\,n}}{2\,h} + \frac{B\,\left(a+b\,x\right)^{\,n}\,Log\,\left(a+b\,x\right)^{\,n}}{2\,h} + \frac{B\,$$

## Problem 298: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log[e(a + bx)^n(c + dx)^{-n}]}{g + hx} dx$$

Optimal (type 4, 148 leaves, 7 steps):

$$-\frac{B\,n\,Log\left[-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]\,Log\,[g+h\,x]}{h} + \frac{B\,n\,Log\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,Log\,[g+h\,x]}{h} + \\ \frac{\left(A+B\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]\right)\,Log\,[g+h\,x]}{h} - \frac{B\,n\,PolyLog\left[2,\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h} + \frac{B\,n\,PolyLog\left[2,\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h} + \frac{B\,n\,PolyLog\left[2,\frac{d\,(g+h\,x)}{d\,g-c\,$$

Result (type 4, 156 leaves, 9 steps):

$$\frac{A \ Log \left[g+h \ x\right]}{h} - \frac{B \ n \ Log \left[-\frac{h \ (a+b \ x)}{b \ g-a \ h}\right] \ Log \left[g+h \ x\right]}{h} + \frac{B \ n \ Log \left[-\frac{h \ (c+d \ x)}{d \ g-c \ h}\right] \ Log \left[g+h \ x\right]}{h} + \frac{B \ n \ Log \left[-\frac{h \ (c+d \ x)}{d \ g-c \ h}\right] \ Log \left[g+h \ x\right]}{h} + \frac{B \ n \ PolyLog \left[2,\frac{b \ (g+h \ x)}{b \ g-a \ h}\right]}{h} + \frac{B \ n \ PolyLog \left[2,\frac{d \ (g+h \ x)}{d \ g-c \ h}\right]}{h}$$

### Problem 299: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]}{\left(g + h x\right)^{2}} dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{b \ B \ n \ Log \left[a + b \ x\right]}{h \ \left(b \ g - a \ h\right)} - \frac{B \ d \ n \ Log \left[c + d \ x\right]}{h \ \left(d \ g - c \ h\right)} - \frac{A + B \ Log \left[e \ \left(a + b \ x\right)^n \ \left(c + d \ x\right)^{-n}\right]}{h \ \left(g + h \ x\right)} + \frac{B \ \left(b \ c - a \ d\right) \ n \ Log \left[g + h \ x\right]}{\left(b \ g - a \ h\right) \ \left(d \ g - c \ h\right)}$$

Result (type 3, 132 leaves, 6 steps):

$$-\frac{A}{h\ \left(g+h\ x\right)}-\frac{B\ \left(b\ c-a\ d\right)\ n\ Log\left[c+d\ x\right]}{\left(b\ g-a\ h\right)\ \left(d\ g-c\ h\right)}+\frac{B\ \left(a+b\ x\right)\ Log\left[e\ \left(a+b\ x\right)^n\ \left(c+d\ x\right)^{-n}\right]}{\left(b\ g-a\ h\right)\ \left(d\ g-a\ h\right)\ \left(d\ g-c\ h\right)}+\frac{B\ \left(b\ c-a\ d\right)\ n\ Log\left[g+h\ x\right]}{\left(b\ g-a\ h\right)\ \left(d\ g-c\ h\right)}$$

#### Problem 300: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log[e(a + bx)^n(c + dx)^{-n}]}{(g + hx)^3} dx$$

Optimal (type 3, 191 leaves, 3 steps):

$$-\frac{B \left(b c - a d\right) n}{2 \left(b g - a h\right) \left(d g - c h\right) \left(g + h x\right)} + \frac{b^2 B n Log [a + b x]}{2 h \left(b g - a h\right)^2} - \frac{B d^2 n Log [c + d x]}{2 h \left(d g - c h\right)^2} - \frac{A + B Log [e \left(a + b x\right)^n \left(c + d x\right)^{-n}]}{2 h \left(g + h x\right)^2} + \frac{B \left(b c - a d\right) \left(2 b d g - b c h - a d h\right) n Log [g + h x]}{2 \left(b g - a h\right)^2 \left(d g - c h\right)^2}$$

Result (type 3, 203 leaves, 5 steps):

$$-\frac{A}{2\;h\;\left(g+h\;x\right)^{\;2}}-\frac{B\;\left(b\;c-a\;d\right)\;n}{2\;\left(b\;g-a\;h\right)\;\left(d\;g-c\;h\right)\;\left(g+h\;x\right)}+\frac{b^{2}\;B\;n\;Log\left[a+b\;x\right]}{2\;h\;\left(b\;g-a\;h\right)^{\;2}}-\\ \frac{B\;d^{2}\;n\;Log\left[c+d\;x\right]}{2\;h\;\left(d\;g-c\;h\right)^{\;2}}-\frac{B\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{2\;h\;\left(g+h\;x\right)^{\;2}}+\frac{B\;\left(b\;c-a\;d\right)\;\left(2\;b\;d\;g-b\;c\;h-a\;d\;h\right)\;n\;Log\left[g+h\;x\right]}{2\;\left(b\;g-a\;h\right)^{\;2}\left(d\;g-c\;h\right)^{\;2}}$$

### Problem 301: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]}{\left(g + h x\right)^{4}} dx$$

Optimal (type 3, 284 leaves, 3 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, n}{6 \, \left(b \, g - a \, h\right) \, \left(d \, g - c \, h\right) \, \left(g + h \, x\right)^2} - \frac{B \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n}{3 \, \left(b \, g - a \, h\right)^2 \, \left(g + h \, x\right)} + \frac{b^3 \, B \, n \, Log \left[a + b \, x\right]}{3 \, h \, \left(b \, g - a \, h\right)^3} - \frac{B \, d^3 \, n \, Log \left[c + d \, x\right]}{3 \, h \, \left(d \, g - c \, h\right)^3} - \frac{A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]}{3 \, h \, \left(d \, g - c \, h\right)} + \frac{B \, \left(b \, c - a \, d\right) \, \left(a^2 \, d^2 \, h^2 - a \, b \, d \, h \, \left(3 \, d \, g - c \, h\right) + b^2 \, \left(3 \, d^2 \, g^2 - 3 \, c \, d \, g \, h + c^2 \, h^2\right)\right) \, n \, Log \left[g + h \, x\right]}{3 \, \left(b \, g - a \, h\right)^3 \, \left(d \, g - c \, h\right)^3}$$

Result (type 3, 296 leaves, 5 steps):

$$-\frac{A}{3\,h\,\left(g+h\,x\right)^3} - \frac{B\,\left(b\,c-a\,d\right)\,n}{6\,\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)\,\left(g+h\,x\right)^2} - \frac{B\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,g-b\,c\,h-a\,d\,h\right)\,n}{3\,\left(b\,g-a\,h\right)^2\,\left(g+h\,x\right)} + \frac{b^3\,B\,n\,Log\,[\,a+b\,x\,]}{3\,h\,\left(b\,g-a\,h\right)^3} - \frac{B\,d^3\,n\,Log\,[\,c+d\,x\,]}{3\,h\,\left(d\,g-c\,h\right)^3} - \frac$$

## Problem 302: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log[e (a + b x)^n (c + d x)^{-n}]}{(g + h x)^5} dx$$

#### Optimal (type 3, 389 leaves, 3 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ n}{12 \left(b \ g - a \ h\right) \ \left(d \ g - c \ h\right) \ \left(g + h \ x\right)^3} - \frac{B \left(b \ c - a \ d\right) \ \left(2 \ b \ d \ g - b \ c \ h - a \ d \ h\right) \ n}{8 \left(b \ g - a \ h\right)^2 \ \left(d \ g - c \ h\right)^2 \ \left(g + h \ x\right)^2} - \frac{B \left(b \ c - a \ d\right) \ \left(a^2 \ d^2 \ h^2 - a \ b \ d \ h \ \left(3 \ d \ g - c \ h\right) + b^2 \ \left(3 \ d^2 \ g^2 - 3 \ c \ d \ g \ h + c^2 \ h^2\right)\right) \ n}{4 \left(b \ g - a \ h\right)^3 \ \left(d \ g - c \ h\right) + b^2 \left(3 \ d^2 \ g^2 - 3 \ c \ d \ g \ h + c^2 \ h^2\right)\right) \ n} + \frac{b^4 \ B \ n \ Log \left[a + b \ x\right]}{4 \ h \ \left(b \ g - a \ h\right)^4} - \frac{B \ d^4 \ n \ Log \left[c + d \ x\right]}{4 \ h \ \left(d \ g - c \ h\right)^4} - \frac{A + B \ Log \left[e \ \left(a + b \ x\right)^n \ \left(c + d \ x\right)^{-n}\right]}{4 \ h \ \left(g + h \ x\right)^4} - \frac{B \ \left(b \ c - a \ d\right) \ \left(2 \ b \ d \ g - b \ c \ h - a \ d \ h\right) \ \left(2 \ a \ b \ d^2 \ g \ h - a^2 \ d^2 \ h^2 - b^2 \ \left(2 \ d^2 \ g^2 - 2 \ c \ d \ g \ h + c^2 \ h^2\right)\right) \ n \ Log \left[g + h \ x\right]}{4 \ \left(b \ g - a \ h\right)^4 \ \left(d \ g - c \ h\right)^4}$$

#### Result (type 3, 401 leaves, 5 steps):

$$-\frac{A}{4\,h\,\left(g+h\,x\right)^4} - \frac{B\,\left(b\,c-a\,d\right)\,n}{12\,\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)\,\left(g+h\,x\right)^3} - \frac{B\,\left(b\,c-a\,d\right)\,\left(2\,b\,d\,g-b\,c\,h-a\,d\,h\right)\,n}{8\,\left(b\,g-a\,h\right)^2\,\left(d\,g-c\,h\right)^2\,\left(g+h\,x\right)^2} - \frac{B\,\left(b\,c-a\,d\right)\,\left(d\,g-c\,h\right)\,\left(d\,g-c\,h\right)\,\left(g+h\,x\right)^3}{4\,\left(b\,g-a\,h\right)^3\,\left(d\,g-c\,h\right)^3\,\left(g+h\,x\right)} + \frac{b^4\,B\,n\,Log\left[a+b\,x\right]}{4\,h\,\left(b\,g-a\,h\right)^4} - \frac{B\,d^4\,n\,Log\left[c+d\,x\right]}{4\,h\,\left(d\,g-c\,h\right)^4} - \frac{B\,d^4\,n\,Log\left[c+d\,$$

## Problem 303: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

#### Optimal (type 4, 570 leaves, 13 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^2 \, h^2 \, n^2 \, x}{3 \, b^2 \, d^2} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^2 \, n^2 \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{3 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^2 \, n^2 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^2 \, n^2 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^3 \, n^3}{3 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^3 \, n^3}{3 \, b^3 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^3 \, h^3 \, h^3 \, n^3}{3 \, b^3 \, d^3} + \frac{B \left( b \, c - a \, d \right)^3 \, h^3 \, h^3 \, n^3}{3 \, b^3 \, d^3} + \frac{B \left( b \, c - a \, d \right)^3 \, h^3 \, h^3 \, n^3}{3 \, b^3 \, h^3} + \frac{1}{3 \, b^3 \, d^3} + \frac{1}{3 \, b^3 \, d^3} + \frac{1}{3 \, b^3 \, d^3} + \frac{1}{3 \, b^3 \, h^3} + \frac{1}{3 \, b^3 \, h^3} + \frac{1}{3 \, b^3 \, d^3} + \frac{1}{3 \, b^3 \, d$$

#### Result (type 4, 697 leaves, 23 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)\,h\,\left(3\,b\,d\,g-b\,c\,h-a\,d\,h\right)\,n\,x}{3\,b^2\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,h^2\,n^2\,x}{3\,b^2\,d^2} - \frac{A\,B\,\left(b\,c-a\,d\right)\,h^2\,n\,x^2}{3\,b\,d} + \frac{A^2\,\left(g+h\,x\right)^3}{3\,h} - \frac{2\,A\,B\,\left(b\,g-a\,h\right)^3\,n\,Log\left[a+b\,x\right]}{3\,b^3\,h} + \frac{a^2\,B^2\,\left(b\,c-a\,d\right)\,h^2\,n^2\,Log\left[a+b\,x\right]}{3\,b^3\,d} + \frac{2\,A\,B\,\left(d\,g-c\,h\right)^3\,n\,Log\left[c+d\,x\right]}{3\,d^3\,h} - \frac{B^2\,c^2\,\left(b\,c-a\,d\right)\,h^2\,n^2\,Log\left[c+d\,x\right]}{3\,b\,d^3} + \frac{2\,B^2\,\left(b\,c-a\,d\right)\,h^2\,n^2\,Log\left[c+d\,x\right]}{3\,b^3\,d^3} + \frac{2\,B^2\,\left(b\,c-a\,d\right)\,h^2\,n\,x^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b\,d} - \frac{2\,B^2\,\left(b\,c-a\,d\right)\,h\,\left(3\,b\,d\,g-b\,c\,h-a\,d\,h\right)\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b^3\,d^2} + \frac{2\,A\,B\,\left(g+h\,x\right)^3\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,h} + \frac{2\,B^2\,\left(b\,g-a\,h\right)^3\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b^3\,h} + \frac{2\,B^2\,\left(d\,g-c\,h\right)^3\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,d^3\,h} + \frac{2\,B^2\,\left(d\,g-c\,h\right)^3\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b^3\,h} + \frac{2\,B^2\,\left(d\,g-c\,h\right)^3\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b^3\,h} + \frac{2\,B^2\,\left(d\,g-c\,h\right)^3\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{3\,b^3\,h} + \frac{2\,B^2\,\left(d\,g-c\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{3\,b^3\,h} + \frac{2\,B^2\,\left(b\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{3\,b^3\,h} + \frac{B^2\,\left(b\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{3\,b^3\,h} + \frac{B^2\,\left(b\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{3\,b^3\,h} + \frac{B^2\,\left(b\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{3\,b^3\,h} + \frac{B^2\,\left(a\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right)}\right]}{3\,b^3\,h} + \frac{B^2\,\left(a\,g-a\,h\right)^3\,n^2\,PolyLog\left[2\,\frac{1+\frac{b\,c-a\,d}{d\,\left(a+b\,x\right$$

### Problem 304: Result valid but suboptimal antiderivative.

$$\left\lceil \left(g+h\,x\right)\, \left(A+B\,Log\left[\,e\, \left(\,a+b\,x\right)^{\,n}\, \left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}\, \text{d}x\right.$$

Optimal (type 4, 294 leaves, 10 steps):

$$\frac{B^{2} \; \left(b \; c-a \; d\right)^{2} \; h \; n^{2} \; Log \left[c+d \; x\right]}{b^{2} \; d^{2}} - \frac{B \; \left(b \; c-a \; d\right) \; h \; n \; \left(a+b \; x\right) \; \left(A+B \; Log \left[e \; \left(a+b \; x\right)^{n} \; \left(c+d \; x\right)^{-n}\right]\right)}{b^{2} \; d} + \frac{B \; \left(b \; c-a \; d\right) \; \left(2 \; b \; d \; g-b \; c \; h-a \; d \; h\right) \; n \; Log \left[\frac{b \; c-a \; d}{b \; \left(c+d \; x\right)}\right] \; \left(A+B \; Log \left[e \; \left(a+b \; x\right)^{n} \; \left(c+d \; x\right)^{-n}\right]\right)}{b^{2} \; d^{2}} - \frac{\left(b \; g-a \; h\right)^{2} \; \left(A+B \; Log \left[e \; \left(a+b \; x\right)^{n} \; \left(c+d \; x\right)^{-n}\right]\right)^{2}}{2 \; b^{2} \; h} + \frac{\left(g+h \; x\right)^{2} \; \left(A+B \; Log \left[e \; \left(a+b \; x\right)^{n} \; \left(c+d \; x\right)^{-n}\right]\right)^{2}}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(b \; c-a \; d\right) \; \left(2 \; b \; d \; g-b \; c \; h-a \; d \; h\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(b \; c-a \; d\right) \; \left(2 \; b \; d \; g-b \; c \; h-a \; d \; h\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(b \; c-a \; d\right) \; \left(2 \; b \; d \; g-b \; c \; h-a \; d \; h\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(c+d \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; c-a \; d\right) \; n^{2} \; PolyLog \left[2, \; \frac{d \; \left(a+b \; x\right)}{b \; \left(a+b \; x\right)}\right]}{b^{2} \; d^{2}} + \frac{B^{2} \; \left(a+b \; x\right)}{b^{2} \; d^$$

#### Result (type 4, 449 leaves, 20 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,h\,n\,x}{b\,d} + \frac{A^2\,\left(g+h\,x\right)^2}{2\,h} - \frac{A\,B\,\left(b\,g-a\,h\right)^2\,n\,Log\left[a+b\,x\right]}{b^2\,h} + \frac{A\,B\,\left(d\,g-c\,h\right)^2\,n\,Log\left[c+d\,x\right]}{d^2\,h} + \frac{B^2\,\left(b\,c-a\,d\right)\,h\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{b^2\,d} + \frac{A\,B\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{h} + \frac{B^2\,\left(b\,g-a\,h\right)^2\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{h} + \frac{B^2\,\left(b\,g-a\,h\right)^2\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{b^2\,h} + \frac{B^2\,\left(d\,g-c\,h\right)^2\,n\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{d^2\,h} + \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{b^2\,h} + \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{d^2\,h} - \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{d^2\,h} - \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{b^2\,h} + \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{d^2\,h} - \frac{B^2\,\left(g+h\,x\right)^2\,Log\left[e\,\left(a+b\,x\right)^n\,\left(c+d\,x\right)^{-n}\right]}{b^2\,h} + \frac{B^2\,\left(g+h\,x\right)^2\,Log$$

#### Problem 305: Result valid but suboptimal antiderivative.

$$\left\lceil \, \left( A + B \, Log \left[ \, e \, \left( \, a + b \, x \right)^{\, n} \, \left( \, c + d \, x \right)^{\, -n} \, \right] \, \right)^{\, 2} \, \mathrm{d} x \right.$$

Optimal (type 4, 137 leaves, 6 steps):

$$\frac{2\,B\,\left(b\,c-a\,d\right)\,n\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}{b\,d}+\\ \\ \frac{\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{2}}{b}+\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,n^{2}\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b\,d}$$

Result (type 4, 195 leaves, 10 steps):

$$A^{2} x - \frac{2 A B \left(b c - a d\right) n Log[c + d x]}{b d} + \frac{2 A B \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]}{b} + \frac{2 B^{2} \left(b c - a d\right) n Log[\frac{b c - a d}{b \left(c + d x\right)}\right] Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]^{2}}{b} + \frac{2 B^{2} \left(b c - a d\right) n^{2} PolyLog[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right) Log[e \left(a + b x\right)^{n} \left(c + d x\right)^{n}\right)^{2}}{b d} + \frac{B^{2} \left(a + b x\right)^{n} \left(a + b x\right)^{n}}{b d} + \frac{B^{2} \left(a + b x\right$$

## Problem 306: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A+B \, Log\left[\, e\, \left(\, a+b\, x\,\right)^{\, n} \, \left(\, c+d\, x\,\right)^{\, -n}\,\right]\,\right)^{\, 2}}{g+h\, x} \, \mathrm{d}x$$

#### Optimal (type 4, 301 leaves, 10 steps):

$$\frac{\text{Log}\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{2}}{h}}{h} + \frac{\left(A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{2}\,\text{Log}\left[1-\frac{\left(d\,g-c\,h\right)\,\left(a+b\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{h} - \frac{2\,B\,n\,\left(A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)\,\text{PolyLog}\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{h} + \frac{2\,B^{2}\,n^{2}\,\text{PolyLog}\left[3,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{h} - \frac{2\,B^{2}\,n^{2}\,\text{PolyLog}\left[3,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{h} + \frac{2\,B^{2}\,n^{2}\,n^{2}\,\text{PolyLog}\left[3,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{h} + \frac{2\,B^{2}\,n$$

#### Result (type 4, 473 leaves, 16 steps):

$$-\frac{B^{2} \, Log \left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]^{2}}{h} + \frac{A^{2} \, Log \left[g+h \, x\right]}{h} - \frac{2 \, A \, B \, n \, Log \left[-\frac{h \, (a+b \, x)}{b \, g-a \, h}\right] \, Log \left[g+h \, x\right]}{h} + \frac{2 \, A \, B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right] \, Log \left[g+h \, x\right]}{h} + \frac{2 \, A \, B \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right] \, Log \left[g+h \, x\right]}{h} + \frac{B^{2} \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right]^{2} \, Log \left[\frac{(b \, c-a \, d) \, (g+h \, x)}{(b \, g-a \, h) \, (c+d \, x)}\right]}{h} + \frac{2 \, A \, B \, n \, Poly Log \left[2, \, \frac{d \, (g+h \, x)}{d \, g-c \, h}\right]}{h} - \frac{2 \, B^{2} \, n \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right] \, Poly Log \left[2, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right] \, Poly Log \left[2, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n \, Log \left[e \, \left(a+b \, x\right)^{n} \, \left(c+d \, x\right)^{-n}\right] \, Poly Log \left[3, \, 1-\frac{(b \, c-a \, d) \, (g+h \, x)}{(b \, g-a \, h) \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1-\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{h} + \frac{2 \, B^{2} \, n^{2} \, Poly Log \left[3, \, 1$$

### Problem 307: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{\left(g + h x\right)^{2}} dx$$

Optimal (type 4, 208 leaves, 5 steps):

$$\frac{ \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right] \right)^{2}}{ \left( b \, g - a \, h \right) \, \left( g + h \, x \right)} + \\ \frac{ 2 \, B \, \left( b \, c - a \, d \right) \, n \, \left( A + B \, Log \left[ e \, \left( a + b \, x \right)^{n} \, \left( c + d \, x \right)^{-n} \right] \right) \, Log \left[ 1 - \frac{\left( d \, g - c \, h \right) \, \left( a + b \, x \right)}{\left( b \, g - a \, h \right) \, \left( c + d \, x \right)} \right]}{ \left( b \, g - a \, h \right) \, \left( d \, g - c \, h \right)} + \frac{ 2 \, B^{2} \, \left( b \, c - a \, d \right) \, n^{2} \, PolyLog \left[ 2 \, , \, \frac{\left( d \, g - c \, h \right) \, \left( a + b \, x \right)}{\left( b \, g - a \, h \right) \, \left( b \, g - a \, h \right)} \right]} \\ + \frac{ \left( b \, g - a \, h \right) \, \left( d \, g - c \, h \right) }{ \left( b \, g - a \, h \right) \, \left( d \, g - c \, h \right)}$$

Result (type 4, 343 leaves, 10 steps):

$$-\frac{A^{2}}{h\;\left(g+h\;x\right)}-\frac{2\;A\;B\;\left(b\;c-a\;d\right)\;n\;Log\left[c+d\;x\right]}{\left(b\;g-a\;h\right)\;\left(d\;g-c\;h\right)}+\frac{2\;A\;B\;\left(a+b\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;g-a\;h\right)\;\left(g+h\;x\right)}+\frac{B^{2}\;\left(a+b\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]^{2}}{\left(b\;g-a\;h\right)\;\left(g+h\;x\right)}+\frac{2\;B^{2}\;\left(b\;c-a\;d\right)\;n\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;g-a\;h\right)\;\left(d\;g-c\;h\right)}+\frac{B^{2}\;\left(a+b\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]^{2}}{\left(b\;g-a\;h\right)\;\left(g+h\;x\right)}+\frac{2\;B^{2}\;\left(b\;c-a\;d\right)\;n\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;g-a\;h\right)\;\left(c+d\;x\right)}+\frac{2\;B^{2}\;\left(b\;c-a\;d\right)\;n^{2}\;PolyLog\left[2,\;1-\frac{\left(b\;c-a\;d\right)\;\left(g+h\;x\right)}{\left(b\;g-a\;h\right)\;\left(c+d\;x\right)}\right]}{\left(b\;g-a\;h\right)}$$

## Problem 308: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log \left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{\left(g + h x\right)^{3}} dx$$

Optimal (type 4, 393 leaves, 10 steps):

$$\frac{B \; \left( b \; c - a \; d \right) \; h \; n \; \left( a + b \; x \right) \; \left( A + B \; Log \left[ e \; \left( a + b \; x \right)^{n} \; \left( c + d \; x \right)^{-n} \right] \right)}{\left( b \; g - a \; h \right)^{2} \; \left( d \; g - c \; h \right) \; \left( g + h \; x \right)} + \frac{b^{2} \; \left( A + B \; Log \left[ e \; \left( a + b \; x \right)^{n} \; \left( c + d \; x \right)^{-n} \right] \right)^{2}}{2 \; h \; \left( b \; g - a \; h \right)^{2} \; \left( d \; g - c \; h \right) \; \left( g + h \; x \right)^{2}} + \frac{B \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; g - b \; c \; h - a \; d \; h \right) \; n \; \left( A + B \; Log \left[ e \; \left( a + b \; x \right)^{n} \; \left( c + d \; x \right)^{-n} \right] \right) \; Log \left[ 1 - \frac{\left( d \; g - c \; h \right) \; \left( a + b \; x \right)}{\left( b \; g - a \; h \right) \; \left( b \; g - a \; h \right)^{2}} + \frac{B \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; g - b \; c \; h - a \; d \; h \right) \; n \; \left( A + B \; Log \left[ e \; \left( a + b \; x \right)^{n} \; \left( c + d \; x \right)^{-n} \right] \right) \; Log \left[ 1 - \frac{\left( d \; g - c \; h \right) \; \left( a + b \; x \right)}{\left( b \; g - a \; h \right) \; \left( b \; g - a \; h \right)^{2}} \; \left( b \; g - a \; h \right)^{2} \; \left( d \; g - c \; h \right)^{2}} + \frac{B^{2} \; \left( b \; c - a \; d \right) \; \left( 2 \; b \; d \; g - b \; c \; h - a \; d \; h \right) \; n^{2} \; PolyLog \left[ 2 , \; \frac{\left( d \; g - c \; h \right) \; \left( a + b \; x \right)}{\left( b \; g - a \; h \right) \; \left( c + d \; x \right)^{-n}} \right] \right) \; Log \left[ 1 - \frac{\left( d \; g - c \; h \right) \; \left( a + b \; x \right)}{\left( b \; g - a \; h \right)^{2}} \; \left( d \; g - c \; h \right)^{2}} \right] \; \left( b \; g - a \; h \right)^{2} \; \left( d \; g - c \; h \right)^{2}$$

Result (type 4, 968 leaves, 29 steps):

$$-\frac{A^2}{2\;h\;(g+h\,x)^2} - \frac{AB\;(b\,c-a\,d)\;n}{(b\,g-a\,h)\;(d\,g-c\,h)\;(g+h\,x)} + \frac{Ab^2\,B\,n\,Log\,[a+b\,x]}{h\;(b\,g-a\,h)^2} - \frac{AB\;d^2\,n\,Log\,[c+d\,x]}{h\;(d\,g-c\,h)^2} - \frac{B^2\;(b\,c-a\,d)^2\,h\,n^2\,Log\,[c+d\,x]}{(b\,g-a\,h)^2\;(d\,g-c\,h)^2} - \frac{B^2\;(b\,c-a\,d)^2\,h\,n^2\,Log\,[c+d\,x]}{(b\,g-a\,h)^2\;(d\,g-c\,h)^2} - \frac{AB\,B\,Log\,[a+b\,x]^n\;(c+d\,x)^{-n}]}{(b\,g-a\,h)^2\;(d\,g-c\,h)} + \frac{B^2\;(b\,c-a\,d)\,h\,n\;(a+b\,x)\;Log\,[e\;(a+b\,x)^n\;(c+d\,x)^{-n}]}{(b\,g-a\,h)^2\;(d\,g-c\,h)} - \frac{b^2\,B^2\,n\,Log\,[-\frac{b\,c-a\,d}{d\;(a+b\,x)}]\;Log\,[e\;(a+b\,x)^n\;(c+d\,x)^{-n}]}{(b\,g-a\,h)^2} + \frac{B^2\;d^2\,n\,Log\,[-\frac{b\,c-a\,d}{d\;(a+b\,x)}]\;Log\,[e\;(a+b\,x)^n\;(c+d\,x)^{-n}]}{(b\,g-a\,h)^2\;(d\,g-c\,h)^2} + \frac{AB\;(b\,c-a\,d)\;(2\,b\,d\,g-b\,c\,h-a\,d\,h)\;n\,Log\,[g+h\,x]}{(b\,g-a\,h)^2\;(d\,g-c\,h)^2} + \frac{B^2\;(b\,c-a\,d)^2\,h\,n^2\,Log\,[g+h\,x]}{(b\,g-a\,h)^2\;(d\,g-c\,h)^2} + \frac{B$$

#### Problem 309: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)^2\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}\,\,\mathrm{d}x$$

Optimal (type 4, 875 leaves, 19 steps):

$$\frac{B^{3} \left(b \, c - a \, d\right)^{3} h^{2} n^{3} \, Log \left[c + d \, x\right]}{b^{3} d^{3}} + \frac{B^{2} \left(b \, c - a \, d\right)^{2} h^{2} \, n^{2} \, \left(a + b \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{b^{3} d^{3}} - \frac{2B^{2} \left(b \, c - a \, d\right)^{2} h \, \left(3 \, b \, d \, g - 2 \, b \, c \, h - a \, d \, h\right) \, n^{2} \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)^{-n}}\right] \left(A + B \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)}{b^{3} d^{3}} - \frac{B \, \left(b \, c - a \, d\right) \, h \, \left(3 \, b \, d \, g - 2 \, b \, c \, h - a \, d \, h\right) \, n \, \left(a + b \, x\right) \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)^{2}}{b^{3} d^{3}} - \frac{B \, \left(b \, c - a \, d\right) \, h^{2} \, n \, \left(c + d \, x\right)^{2} \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^{n} \, \left(c + d \, x\right)^{-n}\right]\right)^{2}}{b^{3} \, d^{3}} + \frac{1}{b^{3} \, d^{3}} - \frac{1}{b^{3} \, d^{3}} - \frac{1}{b^{3} \, d^{3}} - \frac{1}{b^{3} \, d^{3}} + \frac{1}{b^{3} \, d^{3}} + \frac{1}{b^{3} \, d^{3}} - \frac{1}{b$$

Result (type 4, 1640 leaves, 53 steps):

$$\frac{A^2 B \left( b c - a d \right) h \left( 3b d g - b c h - a d h \right) n x}{b^2 d^2} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 x}{b^2 d^2} , \frac{A^2 B \left( b c - a d \right) h^2 n^2 Log \left( a + b x \right)}{b^2 d^3} , \frac{A^2 B \left( b g - a h \right)^3 n Log \left( a - b x \right)}{b^3 h} , \frac{A^2 B \left( b g - a h \right)^2 n Log \left( a - b x \right)}{b^3 d} , \frac{A^2 B \left( b g - a h \right)^3 n Log \left( c - d x \right)}{b^3 h} , \frac{A^2 B \left( b g - a h \right)^3 n Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^2 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n}}{b^3 d} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - d x \right)^{-n} \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^3 h^2 n^3 Log \left( c - a b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 n^2 Log \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 n^2 Log \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( b c - a d \right)^2 h^2 n^2 n^2 Log \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( a - b x \right)^n \left( c - d x \right)^{-n}}{b^3 h} , \frac{A^2 B \left( a -$$

## Problem 310: Result valid but suboptimal antiderivative.

$$\left\lceil \left(g+h\,x\right)\, \left(A+B\,Log\left[\,e\,\left(\,a+b\,x\,\right)^{\,n}\, \left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}\, \text{d}x$$

Optimal (type 4, 466 leaves, 13 steps):

$$-\frac{3 \, B^2 \, \left(b \, c - a \, d\right)^2 h \, n^2 \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)^n} \right] \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)}{b^2 \, d^2} \\ -\frac{3 \, B \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n \, Log \left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)}\right] \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^2}{2 \, b^2 \, d^2} \\ -\frac{\left(b \, g - a \, h\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{2 \, b^2 \, d^2} \\ -\frac{\left(g + h \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{2 \, b} + \\ -\frac{\left(g + h \, x\right)^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right)^3}{2 \, h} - \frac{3 \, B^3 \, \left(b \, c - a \, d\right)^2 h \, n^3 \, PolyLog \left[2, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{b^2 \, d^2} \\ + \\ -\frac{3 \, B^2 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^2 \, \left(A + B \, Log \left[e \, \left(a + b \, x\right)^n \, \left(c + d \, x\right)^{-n}\right]\right) \, PolyLog \left[2, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \, h\right) \, n^3 \, PolyLog \left[3, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]} \\ -\frac{3 \, B^3 \, \left(b \, c - a \, d\right) \, \left(2 \, b \, d \, g - b \, c \, h - a \, d \,$$

Result (type 4, 1030 leaves, 35 steps):

$$\frac{3A^2B \left(bc-ad\right)hnx}{2bd} + \frac{A^3 \left(g+hx\right)^2}{2h} - \frac{3A^2B \left(bg-ah\right)^2 n Log \left[a+bx\right]}{2b^2h} + \frac{3A^2B \left(dg-ch\right)^2 n Log \left[c+dx\right]}{2d^2h} + \frac{3A^2B \left(bc-ad\right)^2 h n^2 Log \left[c+dx\right]}{2b^2} + \frac{3A^2B \left(bc-ad\right)hn \left(a+bx\right) Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{b^2 d^2} + \frac{3A^2B \left(g+hx\right)^2 Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{2h} + \frac{3A^2B \left(g+hx\right)^2 Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{2h} + \frac{3A^2B \left(bg-ah\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{2h} + \frac{3A^2B \left(dg-ch\right)^2 n Log \left[\frac{bc-ad}{b \left(c-dx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{2h} + \frac{3B^3 \left(bg-ah\right)^2 n Log \left[\frac{bc-ad}{b \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]}{2b^2 d} + \frac{3B^3 \left(bg-ah\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^2}{2b^2 d} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^2}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^2}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^2}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[\frac{bc-ad}{d \left(a+bx\right)}\right] Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^2}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right]^3}{2b^2 h} + \frac{3B^3 \left(dg-ch\right)^2 n Log \left[e \left(a+bx\right)^n \left(c+dx\right)^{-n}\right$$

## Problem 311: Result valid but suboptimal antiderivative.

$$\int \left(A + B \, Log \left[\,e \, \left(\,a + b \, x\,\right)^{\,n} \, \left(\,c + d \,x\,\right)^{\,-n}\,\right]\,\right)^{\,3} \, \mathrm{d} x$$

Optimal (type 4, 203 leaves, 6 steps):

$$\frac{3 \text{ B } \left(\text{b } \text{c}-\text{a } \text{d}\right) \text{ n Log}\left[\frac{\text{b } \text{c}-\text{a } \text{d}}{\text{b } \left(\text{c}+\text{d } \text{x}\right)}\right] \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{2}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{-\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{A}+\text{B Log}\left[\text{e } \left(\text{a}+\text{b } \text{x}\right)^{\text{n}} \left(\text{c}+\text{d } \text{x}\right)^{\text{n}}\right]\right)^{3}}{\text{b } \text{d}} + \frac{\left(\text{a}+\text{b } \text{x}\right) \left(\text{a}+\text{b } \text{b } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{x}\right)^{3}} + \frac{\left(\text{a}+\text{b } \text{b}\right) \left(\text{b} \left(\text{c}+\text{d } \text{c}\right)^{\text{n}} \left(\text{c}+\text{d } \text{c}\right)^{\text{n}}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}\right)^{3}}{\text{b } \left(\text{c}+\text{d } \text{c}\right)^{3}} + \frac{\left(\text{a}+\text{b}\right) \left(\text{c}+\text{d } \text{c}$$

Result (type 4, 408 leaves, 14 steps):

$$A^{3} x - \frac{3 A^{2} B \left(b c - a d\right) n Log [c + d x]}{b d} + \frac{3 A^{2} B \left(a + b x\right) Log [e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]}{b} + \frac{6 A B^{2} \left(b c - a d\right) n Log \left[\frac{b c - a d}{b \left(c + d x\right)}\right] Log [e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]}{b d} + \frac{3 A B^{2} \left(a + b x\right) Log [e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]^{2}}{b} + \frac{3 A B^{2} \left(a + b x\right) Log [e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]^{2}}{b} + \frac{3 A B^{2} \left(a + b x\right) Log [e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}]^{2}}{b} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(b c - a d\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(a + b x\right) n^{2} PolyLog [2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right)}{b d} + \frac{6 A B^{2} \left(a + b x\right) n^{2$$

### Problem 312: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A+B \, Log\left[\, e\, \left(\, a+b\, x\,\right)^{\, n} \, \left(\, c+d\, x\,\right)^{\, -n}\,\right]\,\right)^{\, 3}}{g+h\, x} \, \mathrm{d} x$$

#### Optimal (type 4, 425 leaves, 12 steps):

$$\frac{Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{3}}{h}}{h} + \frac{\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{3}\,Log\left[1-\frac{(d\,g-c\,h)\,(a+b\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]}{h} - \frac{3\,B\,n\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)^{2}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{h} + \frac{6\,B^{2}\,n^{2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{h} + \frac{6\,B^{2}\,n^{2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{h} + \frac{6\,B^{3}\,n^{3}\,PolyLog\left[4,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{h} + \frac{6\,B^{3}\,n^{3}\,PolyLog\left[4,\,\frac{d\,$$

Result (type 4, 921 leaves, 25 steps):

$$\frac{3 \, A \, B^2 \, Log \left[ \frac{b \, C + d \, d}{b \, (c + d \, x)} \right] \, Log \left[ e \, \left( a + b \, x \right)^n \, \left( c + d \, x \right)^{-n} \right]^2}{h} + \frac{A^3 \, Log \left[ g + h \, x \right]}{h} + \frac{A^3 \, Log \left[$$

## Problem 313: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{3}}{\left(g + h x\right)^{2}} dx$$

Optimal (type 4, 302 leaves, 6 steps):

$$\frac{\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,3}}{\left(b\,g-a\,h\right)\,\left(g+h\,x\right)} + \frac{3\,B\,\left(b\,c-a\,d\right)\,n\,\left(A+B\,Log\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}\,Log\left[\,1-\frac{\left(d\,g-c\,h\right)\,\left(a+b\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\,\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{\,2}\,\left(b\,c-a\,d\right)\,n^{\,2}\,\left(A+B\,Log\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\,\right]\,\right)\,PolyLog\left[\,2\,,\,\frac{\left(d\,g-c\,h\right)\,\left(a+b\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\,\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} - \frac{6\,B^{\,3}\,\left(b\,c-a\,d\right)\,n^{\,3}\,PolyLog\left[\,3\,,\,\frac{\left(d\,g-c\,h\right)\,\left(a+b\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\,\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)}$$

Result (type 4, 650 leaves, 14 steps):

$$-\frac{A^{3}}{h\left(g+h\,x\right)} - \frac{3\,A^{2}\,B\,\left(b\,c-a\,d\right)\,n\,Log\left[c+d\,x\right)}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{3\,A^{2}\,B\,\left(a+b\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]}{\left(b\,g-a\,h\right)\,\left(g+h\,x\right)} + \frac{3\,A^{2}\,B\,\left(a+b\,x\right)\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]^{2}}{\left(b\,g-a\,h\right)\,\left(g+h\,x\right)} + \frac{3\,A^{2}\,B\,\left(b\,c-a\,d\right)\,n\,Log\left[g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(g+h\,x\right)} + \frac{6\,A\,B^{2}\,\left(b\,c-a\,d\right)\,n\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\,Log\left[\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{3\,A^{2}\,B\,\left(b\,c-a\,d\right)\,n\,Log\left[g+h\,x\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,A\,B^{2}\,\left(b\,c-a\,d\right)\,n\,Log\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\,Log\left[\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,A\,B^{2}\,\left(b\,c-a\,d\right)\,n^{2}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,A\,B^{2}\,\left(b\,c-a\,d\right)\,n^{2}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{2}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} - \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[3\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[3\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[3\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(d\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-a\,d\right)\,n^{3}\,PolyLog\left[2\,,\,1-\frac{\left(b\,c-a\,d\right)\,\left(g+h\,x\right)}{\left(b\,g-a\,h\right)\,\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)\,\left(a\,g-c\,h\right)} + \frac{6\,B^{3}\,\left(b\,c-$$

### Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A+B \ Log\left[\left.e \ \left(a+b \ x\right)^n \ \left(c+d \ x\right)^{-n}\right]\right)^3}{\left(g+h \ x\right)^3} \ \mathrm{d} x$$

Optimal (type 4, 629 leaves, 13 steps):

$$\frac{3\,B\,\left(b\,c-a\,d\right)\,h\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)^{\,2}}{2\,\left(b\,g-a\,h\right)^{\,2}\,\left(d\,g-c\,h\right)\,\left(g+h\,x\right)} + \frac{b^{\,2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)^{\,3}}{2\,h\,\left(b\,g-a\,h\right)^{\,2}} - \frac{\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)^{\,3}}{2\,h\,\left(g+h\,x\right)^{\,2}} + \frac{3\,B^{\,2}\,\left(b\,c-a\,d\right)^{\,2}\,h\,n^{\,2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)\,Log\left[1-\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} + \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,h\,n^{\,2}\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)\,Log\left[1-\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(b\,g-a\,h\right)^{\,2}\left(b\,g-a\,h\right)^{\,2}\left(b\,g-a\,h\right)^{\,2}} + \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,h\,n^{\,3}\,PolyLog\left[2\,,\,\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} + \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,h\,n^{\,3}\,PolyLog\left[2\,,\,\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\cdot\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(2\,b\,d\,g-b\,c\,h-a\,d\,h\right)^{\,3}\,PolyLog\left[3\,,\,\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\cdot\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(2\,b\,d\,g-b\,c\,h-a\,d\,h\right)^{\,3}\,PolyLog\left[3\,,\,\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\cdot\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(2\,b\,d\,g-b\,c\,h-a\,d\,h\right)^{\,3}\,PolyLog\left[3\,,\,\frac{(d\,g-c\,h)\cdot\left(a+b\,x\right)}{(b\,g-a\,h)\cdot\left(c+d\,x\right)}\right]}{\left(b\,g-a\,h\right)^{\,2}\left(d\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(b\,g-a\,h\right)^{\,2}\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(b\,g-a\,h\right)^{\,2}\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(b\,c-a\,d\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(b\,g-a\,h\right)^{\,2}\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)^{\,2}} - \frac{3\,B^{\,3}\,\left(a\,g-c\,h\right)^{\,2}}{\left(a\,g-c\,h\right)$$

Result (type 4, 2207 leaves, 49 steps):

$$-\frac{A^{3}}{2\;h\;\left(g+h\;x\right)^{2}}-\frac{3\;A^{2}\;B\;\left(b\;c-a\;d\right)\;n}{2\;\left(b\;g-a\;h\right)\;\left(d\;g-c\;h\right)\;\left(g+h\;x\right)}+\frac{3\;A^{2}\;b^{2}\;B\;n\;Log\left[a+b\;x\right]}{2\;h\;\left(b\;g-a\;h\right)^{2}}-\frac{3\;A^{2}\;B\;d^{2}\;n\;Log\left[c+d\;x\right]}{2\;h\;\left(d\;g-c\;h\right)^{2}}-\frac{3\;A^{2}\;B\;d^{2}\;n\;Log\left[c+d\;x\right]}{2\;h\;\left(d\;g-c\;h\right)^{2}}-\frac{3\;A^{2}\;B\;d^{2}\;n\;Log\left[c+d\;x\right]}{2\;h\;\left(d\;g-c\;h\right)^{2}}+\frac{3\;A\;B^{2}\;\left(b\;c-a\;d\right)\;h\;n\;\left(a+b\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;g-a\;h\right)^{2}\;\left(d\;g-c\;h\right)^{2}}-\frac{3\;A^{2}\;B\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{2\;h\;\left(g+h\;x\right)^{2}}+\frac{3\;A\;B^{2}\;\left(b\;c-a\;d\right)\;h\;n\;\left(a+b\;x\right)\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{\left(b\;g-a\;h\right)^{2}\;\left(d\;g-c\;h\right)}-\frac{3\;A^{2}\;B\;Log\left[e\;\left(a+b\;x\right)^{n}\;\left(c+d\;x\right)^{-n}\right]}{2\;h\;\left(g+h\;x\right)^{2}}-\frac{3\;A^{2}\;B\;d^{2}\;n\;Log\left[c+d\;x\right]}{\left(b\;g-a\;h\right)^{2}\;\left(d\;g-c\;h\right)}$$

$$\frac{3AB^2 B \cap \log \left[-\frac{b \cdot c + d}{c \left(a + b \cdot x\right)^n \left(c + d \cdot x\right)^{-n}\right]}{h \left(b \cdot g - a h\right)^2} + \frac{3AB^2 d^2 \cap \log \left[-\frac{b \cdot c + d}{h \left(d \cdot g - ch\right)^2}\right]}{h \left(d \cdot g - ch\right)^2} + \frac{3AB^2 \log \left[e \left(a + b \cdot x\right)^n \left(c + d \cdot x\right)^{-n}\right]}{h \left(d \cdot g - ch\right)^2} + \frac{3AB^2 \log \left[e \left(a + b \cdot x\right)^n \left(c + d \cdot x\right)^{-n}\right]^2}{2h \left(g - ch\right)^2} + \frac{2h \left(g - ch\right)^2}{2h \left(g - ch\right)} + \frac{2h \left(g - ch\right)^2}{2h \left(g - ch\right)^2} + \frac{2h \left(g - ch\right)^2}{$$

$$\frac{3 \, b^2 \, B^3 \, n^3 \, PolyLog \left[ \, 3 \, , \, 1 \, + \, \frac{b \, c - a \, d}{d \, (a + b \, x)} \, \right]}{h \, \left( b \, g - a \, h \right)^2} - \frac{3 \, B^3 \, d^2 \, n^3 \, PolyLog \left[ \, 3 \, , \, 1 \, - \, \frac{b \, c - a \, d}{b \, (c + d \, x)} \, \right]}{h \, \left( d \, g - c \, h \right)^2} + \\ \frac{3 \, B^3 \, \left( b \, c - a \, d \right) \, \left( 2 \, b \, d \, g - b \, c \, h - a \, d \, h \right) \, n^3 \, PolyLog \left[ \, 3 \, , \, 1 \, - \, \frac{b \, c - a \, d}{b \, (c + d \, x)} \, \right]}{\left( b \, g - a \, h \right)^2 \, \left( d \, g - c \, h \right)^2} - \frac{3 \, B^3 \, \left( b \, c - a \, d \right) \, \left( 2 \, b \, d \, g - b \, c \, h - a \, d \, h \right) \, n^3 \, PolyLog \left[ \, 3 \, , \, 1 \, - \, \frac{\left( b \, c - a \, d \right) \, \left( g + h \, x \right)}{\left( b \, g - a \, h \right)^2 \, \left( d \, g - c \, h \right)^2} \right]}{\left( b \, g - a \, h \right)^2 \, \left( d \, g - c \, h \right)^2}$$

# Test results for the 263 problems in "3.2.2 (f+gx)^m (h+ix)^g ( $A+B\log(e(a+bx))$ over (c+d) x))^n))^p.m"

### Problem 1: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\,\big[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\big]\right)\,\mathrm{d}x$$

#### Optimal (type 3, 212 leaves, 5 steps):

$$-\frac{\frac{\mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right)^4 \, \mathsf{g}^3 \, \mathsf{i} \, \mathsf{x}}{20 \, \mathsf{b} \, \mathsf{d}^3} + \frac{\mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right)^3 \, \mathsf{g}^3 \, \mathsf{i} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^2}{40 \, \mathsf{b}^2 \, \mathsf{d}^2} - \frac{\mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right)^2 \, \mathsf{g}^3 \, \mathsf{i} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^3}{60 \, \mathsf{b}^2 \, \mathsf{d}} + \\ \frac{\mathsf{g}^3 \, \mathsf{i} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^4 \, \left(\mathsf{c} + \mathsf{d} \, \mathsf{x}\right) \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]\right)}{\mathsf{5} \, \mathsf{b}} + \frac{\left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right) \, \mathsf{g}^3 \, \mathsf{i} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)^4 \, \left(\mathsf{A} - \mathsf{B} + \mathsf{B} \, \mathsf{Log}\left[\frac{\mathsf{e} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{x}\right)}{\mathsf{c} + \mathsf{d} \, \mathsf{x}}\right]\right)}{20 \, \mathsf{b}^2} + \frac{\mathsf{B} \, \left(\mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d}\right)^5 \, \mathsf{g}^3 \, \mathsf{i} \, \mathsf{Log}\left[\mathsf{c} + \mathsf{d} \, \mathsf{x}\right]}{20 \, \mathsf{b}^2 \, \mathsf{d}^4}$$

### Result (type 3, 232 leaves, 10 steps):

$$-\frac{B \left(b \ c - a \ d\right)^4 g^3 \ i \ x}{20 \ b \ d^3} + \frac{B \left(b \ c - a \ d\right)^3 g^3 \ i \ \left(a + b \ x\right)^2}{40 \ b^2 \ d^2} - \frac{B \left(b \ c - a \ d\right)^2 g^3 \ i \ \left(a + b \ x\right)^3}{60 \ b^2 \ d} - \frac{B \left(b \ c - a \ d\right) g^3 \ i \ \left(a + b \ x\right)^4}{20 \ b^2} + \frac{20 \ b^2}{4 \ b^2} + \frac{\left(b \ c - a \ d\right)^3 g^3 \ i \ \left(a + b \ x\right)^5 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{5 \ b^2} + \frac{B \left(b \ c - a \ d\right)^5 g^3 \ i \ Log\left[c + d \ x\right]}{20 \ b^2 \ d^4}$$

## Problem 2: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 180 leaves, 5 steps):

$$\frac{B \left(b \ c - a \ d\right)^{3} \ g^{2} \ i \ x}{12 \ b \ d^{2}} - \frac{B \left(b \ c - a \ d\right)^{2} \ g^{2} \ i \ \left(a + b \ x\right)^{2}}{24 \ b^{2} \ d} + \frac{g^{2} \ i \ \left(a + b \ x\right)^{3} \ \left(c + d \ x\right) \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{4 \ b}}{4 \ b} + \frac{\left(b \ c - a \ d\right)^{2} \ g^{2} \ i \ \left(a + b \ x\right)^{3} \ \left(c + d \ x\right) \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b^{2}} + \frac{B \left(b \ c - a \ d\right)^{4} \ g^{2} \ i \ Log\left[c + d \ x\right]}{12 \ b^{2} \ d^{3}}$$

Result (type 3, 200 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 \, g^2 \, i \, x}{12 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right)^2 \, g^2 \, i \, \left(a + b \, x\right)^2}{24 \, b^2 \, d} - \frac{B \left(b \, c - a \, d\right) \, g^2 \, i \, \left(a + b \, x\right)^3}{12 \, b^2} + \frac{12 \, b^2}{\left(b \, c - a \, d\right) \, g^2 \, i \, \left(a + b \, x\right)^3 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^2} + \frac{d \, g^2 \, i \, \left(a + b \, x\right)^4 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, b^2} - \frac{B \left(b \, c - a \, d\right)^4 \, g^2 \, i \, Log\left[c + d \, x\right]}{12 \, b^2 \, d^3}$$

### Problem 3: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right) \,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right) \,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right) \,\mathrm{d}x$$

Optimal (type 3, 140 leaves, 5 steps):

$$-\frac{B\left(b\,c-a\,d\right)^{2}\,g\,i\,x}{6\,b\,d}+\frac{g\,i\,\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,b}+\frac{\left(b\,c-a\,d\right)\,g\,i\,\left(a+b\,x\right)^{2}\,\left(A-B+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{6\,b^{2}}+\frac{B\left(b\,c-a\,d\right)^{3}\,g\,i\,Log\left[c+d\,x\right]}{6\,b^{2}}$$

Result (type 3, 294 leaves, 13 steps):

$$a \ A \ c \ g \ i \ x - \frac{1}{3} \ b \ B \ \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) \ d \ g \ i \ x - \frac{B \ \left( b \ c - a \ d \right) \ \left( b \ c + a \ d \right) \ g \ i \ x}{2 \ b \ d} - \frac{1}{6} \ B \ \left( b \ c - a \ d \right) \ g \ i \ x^2 + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} - \frac{a^2 \ B \ \left( b \ c + a \ d \right) \ g \ i \ Log \left[ a + b \ x \right)}{2 \ b^2} + \frac{a \ B \ c \ g \ i \ \left( a + b \ x \right) \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right]}{b} + \frac{1}{2} \ \left( b \ c + a \ d \right) \ g \ i \ x^2 \ \left( A + B \ Log \left[ \frac{e \ \left( a + b \ x \right)}{c + d \ x} \right] \right) + \frac{1}{2} \ b \ d \ g \ i \ x^3 \ \left( A + B \ Log \left[ \frac{e \ \left( a + b \ x \right)}{c + d \ x} \right] \right) - \frac{b \ B \ c^3 \ g \ i \ Log \left[ c + d \ x \right]}{3 \ d^2} - \frac{a \ B \ c \ \left( b \ c - a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{b \ d} + \frac{B \ c^2 \ \left( b \ c + a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{2 \ d^2} - \frac{a \ B \ c \ \left( b \ c - a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{b \ d} + \frac{B \ c^2 \ \left( b \ c + a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{2 \ d^2} - \frac{a \ B \ c \ \left( b \ c - a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{b \ d} + \frac{B \ c^2 \ \left( b \ c + a \ d \right) \ g \ i \ Log \left[ c + d \ x \right]}{2 \ d^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{2 \ d^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g \ i \ Log \left[ a + b \ x \right]}{3 \ b^2} + \frac{a^3 \ B \ d \ g$$

## Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right) \, \left(\mathbf{A} + \mathbf{B} \, \mathsf{Log}\left[\frac{\mathbf{e} \, (\mathbf{a} + \mathbf{b} \, \mathbf{x})}{\mathbf{c} + d \, \mathbf{x}}\right]\right)}{\mathbf{a} \, \mathbf{g} + \mathbf{b} \, \mathbf{g} \, \mathbf{x}} \, d\mathbf{x}$$

Optimal (type 4, 133 leaves, 6 steps):

$$\frac{\text{i} \left(\text{c} + \text{d} \, \text{x}\right) \, \left(\text{A} + \text{B} \, \text{Log}\left[\frac{\text{e} \, \left(\text{a} + \text{b} \, \text{x}\right)}{\text{c} + \text{d} \, \text{x}}\right]\right)}{\text{b} \, \text{g}} - \frac{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{i} \, \text{Log}\left[-\frac{\text{b} \, \text{c} - \text{a} \, \text{d}}{\text{d} \, \left(\text{a} + \text{b} \, \text{x}\right)}\right] \left(\text{A} - \text{B} + \text{B} \, \text{Log}\left[\frac{\text{e} \, \left(\text{a} + \text{b} \, \text{x}\right)}{\text{c} + \text{d} \, \text{x}}\right]\right)}{\text{b}^{2} \, \text{g}} + \frac{\text{B} \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{i} \, \text{PolyLog}\left[\text{2, 1} + \frac{\text{b} \, \text{c} - \text{a} \, \text{d}}{\text{d} \, \left(\text{a} + \text{b} \, \text{x}\right)}\right]}{\text{b}^{2} \, \text{g}}$$

Result (type 4, 213 leaves, 14 steps):

$$\frac{\text{Adix}}{\text{bg}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iLog[a+bx]}^2}{2\text{b}^2\text{g}} + \frac{\text{Bdi}\left(\text{a+bx}\right)\text{Log}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]}{\text{b}^2\text{g}} + \frac{\left(\text{bc-ad}\right)\text{iLog[a+bx]}\left(\text{A+BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iLog[c+dx]}}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{iLog[a+bx]}\left(\text{A+BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iLog[c+dx]}}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{iPolyLog}\left[2, -\frac{\text{d}\cdot(\text{a+bx})}{\text{bc-ad}}\right]}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iPolyLog}\left[2, -\frac{\text{d}\cdot(\text{a+bx})}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iPolyLog}\left[2, -\frac{\text{d}\cdot(\text{a+bx})}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iPolyLog}\left[2, -\frac{\text{d}\cdot(\text{a+bx})}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iPolyLog}\left[2, -\frac{\text{d}\cdot(\text{a+bx})}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{iPoly$$

## Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right) \, \left(\mathbf{A} + \mathbf{B} \, \mathsf{Log}\left[\frac{\mathbf{e} \, (\mathbf{a} + \mathbf{b} \, \mathbf{x})}{\mathbf{c} + d \, \mathbf{x}}\right]\right)}{\left(\mathbf{a} \, \mathbf{g} + \mathbf{b} \, \mathbf{g} \, \mathbf{x}\right)^{2}} \, d\mathbf{x}$$

Optimal (type 4, 142 leaves, 5 steps):

$$-\frac{\text{Bi}\left(\text{c}+\text{d}\,\text{x}\right)}{\text{b}\,\text{g}^{2}\,\left(\text{a}+\text{b}\,\text{x}\right)}-\frac{\text{i}\left(\text{c}+\text{d}\,\text{x}\right)\,\left(\text{A}+\text{B}\,\text{Log}\left[\frac{\text{e}\,\left(\text{a}+\text{b}\,\text{x}\right)}{\text{c}+\text{d}\,\text{x}}\right]\right)}{\text{b}\,\text{g}^{2}\,\left(\text{a}+\text{b}\,\text{x}\right)}-\frac{\text{d}\,\text{i}\left(\text{A}+\text{B}\,\text{Log}\left[\frac{\text{e}\,\left(\text{a}+\text{b}\,\text{x}\right)}{\text{c}+\text{d}\,\text{x}}\right]\right)\,\text{Log}\left[1-\frac{\text{b}\,\left(\text{c}+\text{d}\,\text{x}\right)}{\text{d}\,\left(\text{a}+\text{b}\,\text{x}\right)}\right]}{\text{b}^{2}\,\text{g}^{2}}+\frac{\text{B}\,\text{d}\,\text{i}\,\text{PolyLog}\left[2,\frac{\text{b}\,\left(\text{c}+\text{d}\,\text{x}\right)}{\text{d}\,\left(\text{a}+\text{b}\,\text{x}\right)}\right]}{\text{b}^{2}\,\text{g}^{2}}$$

Result (type 4, 221 leaves, 15 steps):

$$- \frac{B \left( b \, c - a \, d \right) \, i}{b^2 \, g^2 \, \left( a + b \, x \right)} - \frac{B \, d \, i \, Log \left[ a + b \, x \right]^2}{b^2 \, g^2} - \frac{B \, d \, i \, Log \left[ a + b \, x \right]^2}{2 \, b^2 \, g^2} - \frac{\left( b \, c - a \, d \right) \, i \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{b^2 \, g^2 \, \left( a + b \, x \right)} + \frac{B \, d \, i \, Log \left[ c + d \, x \right]}{b^2 \, g^2} + \frac{B \, d \, i \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c + d \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^2 \, g^2} + \frac{B \, d \, i \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{$$

## Problem 7: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\frac{\text{e}\left(\text{a}+\text{bx}\right)}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 3, 85 leaves, 2 steps):

$$-\frac{\text{Bi} (c + d x)^{2}}{4 (b c - a d) g^{3} (a + b x)^{2}} - \frac{\text{i} (c + d x)^{2} (A + B Log \left[\frac{e (a + b x)}{c + d x}\right])}{2 (b c - a d) g^{3} (a + b x)^{2}}$$

Result (type 3, 191 leaves, 10 steps):

$$-\frac{B \left(b \ c - a \ d\right) \ i}{4 \ b^{2} \ g^{3} \ \left(a + b \ x\right)^{2}} - \frac{B \ d \ i}{2 \ b^{2} \ g^{3} \ \left(a + b \ x\right)} - \frac{B \ d^{2} \ i \ Log \left[a + b \ x\right]}{2 \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} - \frac{\left(b \ c - a \ d\right) \ i \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b^{2} \ g^{3} \ \left(a + b \ x\right)} + \frac{B \ d^{2} \ i \ Log \left[c + d \ x\right]}{2 \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} - \frac{b^{2} \ g^{3} \ \left(a + b \ x\right)}{b^{2} \ g^{3} \ \left(a + b \ x\right)} + \frac{B \ d^{2} \ i \ Log \left[c + d \ x\right]}{2 \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} - \frac{b^{2} \ g^{3} \ \left(a + b \ x\right)}{b^{2} \ g^{3} \ \left(a + b \ x\right)} + \frac{b^{2} \ g^{3} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} - \frac{b^{2} \ g^{3} \ \left(a + b \ x\right)}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}} + \frac{b^{2} \ \left(b \ c - a \ d\right) \ g^{3}}{b^{2} \ \left(b \ c - a \ d\right)$$

## Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\frac{\text{e} \cdot (\text{a+bx})}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^4} \, dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$\frac{\text{Bdi} \left(\text{c} + \text{dx}\right)^{2}}{4 \, \left(\text{bc} - \text{ad}\right)^{2} \, \text{g}^{4} \, \left(\text{a} + \text{bx}\right)^{2}} - \frac{\text{bBi} \left(\text{c} + \text{dx}\right)^{3}}{9 \, \left(\text{bc} - \text{ad}\right)^{2} \, \text{g}^{4} \, \left(\text{a} + \text{bx}\right)^{3}} + \frac{\text{di} \left(\text{c} + \text{dx}\right)^{2} \left(\text{A} + \text{BLog}\left[\frac{\text{e} \cdot \left(\text{a} + \text{bx}\right)}{\text{c} + \text{dx}}\right]\right)}{2 \, \left(\text{bc} - \text{ad}\right)^{2} \, \text{g}^{4} \, \left(\text{a} + \text{bx}\right)^{2}} - \frac{\text{bi} \left(\text{c} + \text{dx}\right)^{3} \, \left(\text{A} + \text{BLog}\left[\frac{\text{e} \cdot \left(\text{a} + \text{bx}\right)}{\text{c} + \text{dx}}\right]\right)}{3 \, \left(\text{bc} - \text{ad}\right)^{2} \, \text{g}^{4} \, \left(\text{a} + \text{bx}\right)^{3}}$$

Result (type 3, 225 leaves, 10 steps):

$$-\frac{B \left(b c - a d\right) i}{9 b^{2} g^{4} \left(a + b x\right)^{3}} - \frac{B d i}{12 b^{2} g^{4} \left(a + b x\right)^{2}} + \frac{B d^{2} i}{6 b^{2} \left(b c - a d\right) g^{4} \left(a + b x\right)} + \frac{B d^{2} i}{6 b^{2} \left(b c - a d\right) g^{4} \left(a + b x\right)} + \frac{B d^{3} i Log [a + b x]}{6 b^{2} \left(b c - a d\right) i \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 b^{2} g^{4} \left(a + b x\right)^{3}} - \frac{d i \left(A + B Log \left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^{2} g^{4} \left(a + b x\right)^{2}} - \frac{B d^{3} i Log [c + d x]}{6 b^{2} \left(b c - a d\right)^{2} g^{4}}$$

## Problem 9: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\frac{\text{e}\left(\text{a} + \text{bx}\right)}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

Optimal (type 3, 269 leaves, 5 steps):

$$-\frac{B\,d^{2}\,\mathbf{i}\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{2}}+\frac{2\,b\,B\,d\,\mathbf{i}\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{3}}-\frac{b^{2}\,B\,\mathbf{i}\,\left(c+d\,x\right)^{4}}{16\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{4}}-\\ \frac{d^{2}\,\mathbf{i}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{3}}+\frac{2\,b\,d\,\mathbf{i}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{3}}-\frac{b^{2}\,\mathbf{i}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{4\,\left(b\,c-a\,d\right)^{3}\,g^{5}\,\left(a+b\,x\right)^{4}}$$

Result (type 3, 257 leaves, 10 steps):

$$-\frac{B \left(b c - a d\right) i}{16 b^{2} g^{5} \left(a + b x\right)^{4}} - \frac{B d i}{36 b^{2} g^{5} \left(a + b x\right)^{3}} + \frac{B d^{2} i}{24 b^{2} \left(b c - a d\right) g^{5} \left(a + b x\right)^{2}} - \frac{B d^{3} i}{12 b^{2} \left(b c - a d\right)^{2} g^{5} \left(a + b x\right)} - \frac{B d^{3} i}{12 b^{2} \left(b c - a d\right)^{2} g^{5} \left(a + b x\right)} - \frac{B d^{4} i Log [a + b x]}{12 b^{2} \left(b c - a d\right)^{3} g^{5}} - \frac{\left(b c - a d\right) i \left(A + B Log \left[\frac{e \cdot (a + b x)}{c + d x}\right]\right)}{4 b^{2} g^{5} \left(a + b x\right)^{4}} - \frac{d i \left(A + B Log \left[\frac{e \cdot (a + b x)}{c + d x}\right]\right)}{3 b^{2} g^{5} \left(a + b x\right)^{3}} + \frac{B d^{4} i Log [c + d x]}{12 b^{2} \left(b c - a d\right)^{3} g^{5}}$$

## Problem 10: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,3}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 423 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{5} g^{3} \, i^{2} \, x}{60 \, b^{2} \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{4} g^{3} \, i^{2} \, \left(c + d \, x\right)^{2}}{120 \, b \, d^{4}} - \frac{19 \, B \left(b \, c - a \, d\right)^{3} g^{3} \, i^{2} \, \left(c + d \, x\right)^{3}}{180 \, d^{4}} + \frac{130 \, B \left(b \, c - a \, d\right)^{2} g^{3} \, i^{2} \, \left(c + d \, x\right)^{4}}{120 \, d^{4}} - \frac{b^{2} \, B \left(b \, c - a \, d\right) g^{3} \, i^{2} \, \left(c + d \, x\right)^{5}}{30 \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{60 \, b^{3} \, d^{4}} - \frac{\left(b \, c - a \, d\right)^{3} g^{3} \, i^{2} \, \left(c + d \, x\right)^{3} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, d^{4}} + \frac{3 \, b \, \left(b \, c - a \, d\right)^{2} g^{3} \, i^{2} \, \left(c + d \, x\right)^{4} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^{4}} - \frac{3 \, b^{2} \, \left(b \, c - a \, d\right) g^{3} \, i^{2} \, \left(c + d \, x\right)^{5} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, d^{4}} + \frac{b^{3} g^{3} \, i^{2} \, \left(c + d \, x\right)^{6} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} +$$

#### Result (type 3, 330 leaves, 14 steps):

$$-\frac{B \left(b \ c-a \ d\right)^{5} g^{3} \ i^{2} \ x}{60 \ b^{2} \ d^{3}} + \frac{B \left(b \ c-a \ d\right)^{4} g^{3} \ i^{2} \left(a+b \ x\right)^{2}}{120 \ b^{3} \ d^{2}} - \frac{B \left(b \ c-a \ d\right)^{3} g^{3} \ i^{2} \left(a+b \ x\right)^{3}}{180 \ b^{3} \ d} - \frac{7 \ B \left(b \ c-a \ d\right)^{2} g^{3} \ i^{2} \left(a+b \ x\right)^{4} \left(a+b \$$

## Problem 11: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 337 leaves, 5 steps):

$$-\frac{B \left(b \, c - a \, d\right)^4 g^2 \, \mathbf{i}^2 \, x}{30 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^2}{60 \, b \, d^3} + \frac{B \left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^3}{10 \, d^3} - \frac{b \, B \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^4}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{30 \, b^3 \, d^3} + \frac{\left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, d^3} - \frac{b \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)$$

#### Result (type 3, 296 leaves, 14 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 g^2 \, \mathbf{i}^2 \, x}{30 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^2}{60 \, b^3 \, d} - \frac{B \left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^3}{10 \, b^3} - \frac{B \, d \, \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^4}{20 \, b^3} + \frac{\left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^3} + \frac{d \, \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^3} + \frac{d^2 \, g^2 \, \mathbf{i}^2 \, \left(a + b \, x\right)^5 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, b^3} - \frac{B \, \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3}$$

## Problem 12: Result valid but suboptimal antiderivative.

$$\int \left(a\;g+b\;g\;x\right)\;\left(c\;i+d\;i\;x\right)^{\;2}\;\left(A+B\;Log\,\big[\,\frac{e\;\left(a+b\;x\right)}{c+d\;x}\,\big]\right)\;\mathrm{d}x$$

Optimal (type 3, 239 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^2 \, x}{12 \, b^2 \, d} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^2 \, \left(c + d \, x\right)^2}{24 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3}{12 \, d^2} + \frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^2 \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{12 \, b^3 \, d^2} - \frac{\left(b \, c - a \, d\right)^3 g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, d^2} + \frac{b \, g \, \mathbf{i}^2 \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^2} + \frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^2 \, Log\left[c + d \, x\right]}{12 \, b^3 \, d^2}$$

Result (type 3, 200 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^2 \, x}{12 \, b^2 \, d} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^2 \, \left(c + d \, x\right)^2}{24 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3}{12 \, d^2} + \\ \frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^2 \, Log \left[a + b \, x\right]}{12 \, b^3 \, d^2} - \frac{\left(b \, c - a \, d\right) g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, d^2} + \frac{b \, g \, \mathbf{i}^2 \, \left(c + d \, x\right)^4 \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^2}$$

## Problem 14: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^2 \, \left(\text{A} + \text{BLog}\left[\frac{\text{e} \, (\text{a+b} \, x)}{\text{c+d} \, x}\right]\right)}{\text{ag+bg} \, x} \, \mathrm{d} x$$

Optimal (type 4, 276 leaves, 10 steps):

$$-\frac{B \ d \ \left(b \ c - a \ d\right) \ i^{2} \ x}{2 \ b^{2} \ g} - \frac{B \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[\frac{a + b \ x}{c + d \ x}\right]}{2 \ b^{3} \ g} + \frac{d \ \left(b \ c - a \ d\right) \ i^{2} \ \left(a + b \ x\right) \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ g} + \frac{i^{2} \ \left(c + d \ x\right)^{2} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ b \ g} + \frac{3 \ B \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{2 \ b \ g} + \frac{a \ b^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ PolyLog\left[c + d \ x\right]}{b^{3} \ g} + \frac{a^{2} \ \left(c + d \ x\right)^{2} \left(a + b \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{a^{2} \ \left(c + d \ x\right)^{2} \left(a + b \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{a^{2} \ \left(c + d \ x\right)^{2} \left(a + b \ x\right)^{2} \left(a + b \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{a^{2} \ \left(c + d \ x\right)^{2} \left(a + b \ x\right)^{2} \left(a + b \ x\right)^{2} \left(a + b \ x\right)}{b^{3} \ g} + \frac{a^{2} \ \left(c + d \ x\right)^{2} \left(a + b \ x\right)$$

Result (type 4, 354 leaves, 19 steps):

$$\frac{A\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,x}{b^{2}\,g} - \frac{B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,x}{2\,b^{2}\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[a+b\,x\right]}{2\,b^{3}\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[a+b\,x\right]}{2\,b^{3}\,g} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b^{3}\,g} + \frac{\mathbf{i}^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,b\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[c+d\,x\right]}{b^{3}\,g} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[c+d\,x\right]}{b\,c-a\,d} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,Log\left[\frac{e\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,PolyLog\left[2,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(a\,a+b\,x\right)^{2}\,PolyLog\left[2,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(a\,a+b\,x\right)^{2}\,PolyLog\left[2,-$$

## Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A}+\text{BLog}\left[\frac{\text{e}\left(\text{a}+\text{b}\,\text{x}\right)}{\text{c+d}\,\text{x}}\right]\right)}{\left(\text{ag+bgx}\right)^{2}} \, dx$$

Optimal (type 4, 247 leaves, 8 steps):

$$-\frac{B\left(b\;c-a\;d\right)\;\mathbf{i}^{2}\;\left(c+d\;x\right)}{b^{2}\;g^{2}\;\left(a+b\;x\right)} + \frac{d^{2}\;\mathbf{i}^{2}\;\left(a+b\;x\right)\;\left(A+B\;Log\left[\frac{e\;(a+b\;x)}{c+d\;x}\right]\right)}{b^{3}\;g^{2}} - \frac{\left(b\;c-a\;d\right)\;\mathbf{i}^{2}\;\left(c+d\;x\right)\;\left(A+B\;Log\left[\frac{e\;(a+b\;x)}{c+d\;x}\right]\right)}{b^{2}\;g^{2}\;\left(a+b\;x\right)} - \frac{B\;d\;\left(b\;c-a\;d\right)\;\mathbf{i}^{2}\;Log\left[c+d\;x\right]}{b^{3}\;g^{2}} - \frac{2\;d\;\left(b\;c-a\;d\right)\;\mathbf{i}^{2}\;\left(A+B\;Log\left[\frac{e\;(a+b\;x)}{c+d\;x}\right]\right)\;Log\left[1-\frac{b\;(c+d\;x)}{d\;(a+b\;x)}\right]}{b^{3}\;g^{2}} + \frac{2\;B\;d\;\left(b\;c-a\;d\right)\;\mathbf{i}^{2}\;PolyLog\left[2\,,\,\frac{b\;(c+d\;x)}{d\;(a+b\;x)}\right]}{b^{3}\;g^{2}} + \frac{2\;B\;d\;(b\;c-a\;d)\;\mathbf{i}^{2}\;PolyLog\left[2\,,\,\frac{b\;(c+d\;x)}{d\;(a+b\;x)}\right]}{b^{3}\;g^{2}} + \frac{2\;B\;d\;(b\;c-a\;d)\;\mathbf{i}^{2}\;PolyLog\left[2\,,\,\frac{b\;(c+d\;x$$

Result (type 4, 313 leaves, 18 steps):

$$\frac{\frac{A}{b^2} \frac{d^2 \, i^2 \, x}{b^2 \, g^2} - \frac{B}{b^3} \frac{\left(b \, c - a \, d\right)^2 \, i^2}{b^3 \, g^2} - \frac{B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Log \left[a + b \, x\right]}{b^3 \, g^2} + \frac{B \, d^2 \, i^2 \, \left(a + b \, x\right) \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{b^3 \, g^2} - \frac{\left(b \, c - a \, d\right)^2 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^3 \, g^2} + \frac{2 \, d \, \left(b \, c - a \, d\right) \, i^2 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, g^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, B^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, B^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, B^2} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Poly Log \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^3 \, B^2} + \frac{2 \,$$

## Problem 16: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A}+\text{BLog}\left[\frac{\text{e}\left(\text{a}+\text{bx}\right)}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

#### Optimal (type 4, 230 leaves, 7 steps):

$$-\frac{B\,d\,i^{2}\,\left(c+d\,x\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{B\,i^{2}\,\left(c+d\,x\right)^{2}}{4\,b\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{d\,i^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \\ \\ \frac{i^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,b\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{d^{2}\,i^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,Log\left[1-\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{3}\,g^{3}} + \frac{B\,d^{2}\,i^{2}\,PolyLog\left[2\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{3}\,g^{3}}$$

#### Result (type 4, 338 leaves, 19 steps):

$$-\frac{B\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}}{4\,b^{3}\,g^{3}\,\left(a+b\,x\right)^{2}}-\frac{3\,B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}}{2\,b^{3}\,g^{3}\,\left(a+b\,x\right)}-\frac{3\,B\,d^{2}\,\mathbf{i}^{2}\,Log\,[a+b\,x]}{2\,b^{3}\,g^{3}}-\frac{B\,d^{2}\,\mathbf{i}^{2}\,Log\,[a+b\,x]}{2\,b^{3}\,g^{3}}-\frac{B\,d^{2}\,\mathbf{i}^{2}\,Log\,[a+b\,x]^{2}}{2\,b^{3}\,g^{3}}-\frac{\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,b^{3}\,g^{3}\,\left(a+b\,x\right)^{2}}-\frac{2\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)}+\frac{d^{2}\,\mathbf{i}^{2}\,Log\,[a+b\,x]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{3}\,g^{3}}+\frac{B\,d^{2}\,\mathbf{i}^{2}\,Log\,[c+d\,x]}{b\,c-a\,d}+\frac{B\,d^{2}\,\mathbf{i}^{2}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{3}\,g^{3}}$$

### Problem 17: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\operatorname{Ci} + \operatorname{dix}\right)^{2} \left(\operatorname{A} + \operatorname{B}\operatorname{Log}\left[\frac{\operatorname{e}\left(\operatorname{a} + \operatorname{bx}\right)}{\operatorname{c} + \operatorname{dx}}\right]\right)}{\left(\operatorname{a}\operatorname{g} + \operatorname{b}\operatorname{gx}\right)^{4}} \, \mathrm{d}x$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\,\frac{\,B\,\,i^{\,2}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}}{\,9\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,g^{\,4}\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}\,-\,\frac{\,i^{\,2}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\,\left(\,A\,+\,B\,\,Log\left[\,\frac{e\,\,(a+b\,\,x)}{c+d\,\,x}\,\right]\,\right)}{\,3\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,g^{\,4}\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}$$

Result (type 3, 287 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{2} \, i^{2}}{9 \, b^{3} \, g^{4} \, \left(a + b \, x\right)^{3}} - \frac{B \, d \, \left(b \, c - a \, d\right) \, i^{2}}{3 \, b^{3} \, g^{4} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{2} \, i^{2}}{3 \, b^{3} \, g^{4} \, \left(a + b \, x\right)} - \frac{B \, d^{3} \, i^{2} \, Log \left[a + b \, x\right]}{3 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{4}} - \frac{\left(b \, c - a \, d\right)^{2} \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^{3} \, g^{4} \, \left(a + b \, x\right)^{3}} - \frac{d \, \left(b \, c - a \, d\right) \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^{3} \, g^{4} \, \left(a + b \, x\right)} - \frac{d^{2} \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^{3} \, g^{4} \, \left(a + b \, x\right)} + \frac{B \, d^{3} \, i^{2} \, Log \left[c + d \, x\right]}{3 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{4}}$$

## Problem 18: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{2} \, \left(A + B \, \mathsf{Log}\left[\frac{e \, (a + b \, \mathbf{x})}{c + d \, \mathbf{x}}\right]\right)}{\left(a \, g + b \, g \, \mathbf{x}\right)^{5}} \, \mathrm{d}\mathbf{x}$$

Optimal (type 3, 181 leaves, 5 steps):

$$\frac{B d i^{2} (c + d x)^{3}}{9 (b c - a d)^{2} g^{5} (a + b x)^{3}} - \frac{b B i^{2} (c + d x)^{4}}{16 (b c - a d)^{2} g^{5} (a + b x)^{4}} + \frac{d i^{2} (c + d x)^{3} (A + B Log \left[\frac{e (a + b x)}{c + d x}\right])}{3 (b c - a d)^{2} g^{5} (a + b x)^{3}} - \frac{b i^{2} (c + d x)^{4} (A + B Log \left[\frac{e (a + b x)}{c + d x}\right])}{4 (b c - a d)^{2} g^{5} (a + b x)^{4}}$$

Result (type 3, 325 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{2} \, i^{2}}{16 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{4}} - \frac{5 \, B \, d \, \left(b \, c - a \, d\right) \, i^{2}}{36 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{B \, d^{2} \, i^{2}}{24 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{2}} + \frac{B \, d^{3} \, i^{2}}{12 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{5} \, \left(a + b \, x\right)} + \frac{B \, d^{4} \, i^{2} \, Log \left[a + b \, x\right]}{12 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{5}} - \frac{\left(b \, c - a \, d\right)^{2} \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{4}} - \frac{2 \, d \, \left(b \, c - a \, d\right) \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{d^{2} \, i^{2} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{4} \, i^{2} \, Log \left[c + d \, x\right]}{12 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{5}}$$

## Problem 19: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A}+\text{BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^{6}} \, dx$$

Optimal (type 3, 281 leaves, 5 steps):

$$-\frac{B\,d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{3}}+\frac{b\,B\,d\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{4}}{8\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b^{2}\,B\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{5}}{25\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}-\\ \frac{d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}+\frac{b\,d\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{5\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}$$

### Result (type 3, 359 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{2} \, i^{2}}{25 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{5}} - \frac{3 \, B \, d \, \left(b \, c - a \, d\right) \, i^{2}}{40 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{4}} - \frac{B \, d^{2} \, i^{2}}{90 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{3}} + \frac{B \, d^{3} \, i^{2}}{60 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{6} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{4} \, i^{2}}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{6} \, \left(a + b \, x\right)} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{4} \, i^{2}}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{6} \, \left(a + b \, x\right)} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{B \, d^{5} \, i^{2} \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(a + b \, x\right)^{3}} - \frac{B \, d^{5} \, i^{$$

### Problem 20: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,3}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)\,\mathrm{d}x$$

#### Optimal (type 3, 457 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{3} \, x}{140 \, b^{3} \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{5} g^{3} \, i^{3} \, \left(c + d \, x\right)^{2}}{280 \, b^{2} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{4} g^{3} \, i^{3} \, \left(c + d \, x\right)^{3}}{420 \, b \, d^{4}} - \frac{17 \, B \left(b \, c - a \, d\right)^{3} g^{3} \, i^{3} \, \left(c + d \, x\right)^{4}}{280 \, d^{4}} + \frac{b \, B \left(b \, c - a \, d\right)^{2} g^{3} \, i^{3} \, \left(c + d \, x\right)^{5}}{14 \, d^{4}} - \frac{b^{2} \, B \left(b \, c - a \, d\right) g^{3} \, i^{3} \, \left(c + d \, x\right)^{6}}{42 \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{7} g^{3} \, i^{3} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{140 \, b^{4} \, d^{4}} - \frac{\left(b \, c - a \, d\right)^{3} g^{3} \, i^{3} \, \left(c + d \, x\right)^{4} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^{4}} + \frac{3 \, b \, \left(b \, c - a \, d\right)^{2} g^{3} \, i^{3} \, \left(c + d \, x\right)^{5} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, d^{4}} - \frac{b^{2} \, \left(b \, c - a \, d\right)^{3} g^{3} \, i^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^{4}} + \frac{b^{3} g^{3} \, i^{3} \, \left(c + d \, x\right)^{7} \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{7 \, d^{4}} + \frac{B \, \left(b \, c - a \, d\right)^{7} g^{3} \, i^{3} \, Log\left[c + d \, x\right]}{140 \, b^{4} \, d^{4}}$$

### Result (type 3, 416 leaves, 18 steps):

$$-\frac{B \left(b \ c - a \ d\right)^{6} g^{3} \ i^{3} \ x}{140 \ b^{3} \ d^{3}} + \frac{B \left(b \ c - a \ d\right)^{5} g^{3} \ i^{3} \left(a + b \ x\right)^{2}}{280 \ b^{4} \ d^{2}} - \frac{B \left(b \ c - a \ d\right)^{4} g^{3} \ i^{3} \left(a + b \ x\right)^{3}}{420 \ b^{4} \ d} - \frac{17 \ B \left(b \ c - a \ d\right)^{3} g^{3} \ i^{3} \left(a + b \ x\right)^{4}}{280 \ b^{4}} - \frac{B \ d \left(b \ c - a \ d\right)^{2} g^{3} \ i^{3} \left(a + b \ x\right)^{5}}{14 \ b^{4}} - \frac{B \ d^{2} \left(b \ c - a \ d\right)^{3} g^{3} \ i^{3} \left(a + b \ x\right)^{6}}{42 \ b^{4}} + \frac{\left(b \ c - a \ d\right)^{3} g^{3} \ i^{3} \left(a + b \ x\right)^{4} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{4 \ b^{4}} + \frac{3 \ d \left(b \ c - a \ d\right)^{2} g^{3} \ i^{3} \left(a + b \ x\right)^{5} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{5 \ b^{4}} + \frac{d^{3} g^{3} \ i^{3} \left(a + b \ x\right)^{7} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{7 \ b^{4}} + \frac{B \left(b \ c - a \ d\right)^{7} g^{3} \ i^{3} \ Log\left[c + d \ x\right]}{140 \ b^{4}}$$

## Problem 21: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\,\big[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\big]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 371 leaves, 5 steps):

$$-\frac{B \left(b \ c-a \ d\right)^{5} g^{2} \ i^{3} \ x}{60 \ b^{3} \ d^{2}} - \frac{B \left(b \ c-a \ d\right)^{4} g^{2} \ i^{3} \left(c+d \ x\right)^{2}}{120 \ b^{2} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{3} g^{2} \ i^{3} \left(c+d \ x\right)^{3}}{180 \ b \ d^{3}} + \frac{7 \ B \left(b \ c-a \ d\right)^{2} g^{2} \ i^{3} \left(c+d \ x\right)^{4}}{120 \ d^{3}} - \frac{b \ B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \left(c+d \ x\right)^{4}}{120 \ d^{3}} - \frac{b \ B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \left(c+d \ x\right)^{4} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{60 \ b^{4} \ d^{3}} - \frac{2 \ b \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \left(c+d \ x\right)^{6} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ Log\left[c+d \ x\right]}{10 \ b^{4} \ b$$

Result (type 3, 330 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{5} g^{2} \, \mathbf{i}^{3} \, x}{60 \, b^{3} \, d^{2}} - \frac{B \left(b \, c - a \, d\right)^{4} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{2}}{120 \, b^{2} \, d^{3}} - \frac{B \left(b \, c - a \, d\right)^{3} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{3}}{180 \, b \, d^{3}} + \frac{7 \, B \left(b \, c - a \, d\right)^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{4}}{120 \, d^{3}} - \frac{b \, B \left(b \, c - a \, d\right)^{6} g^{2} \, \mathbf{i}^{3} \, Log \left[a + b \, x\right]}{60 \, b^{4} \, d^{3}} + \frac{\left(b \, c - a \, d\right)^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{4} \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^{3}} - \frac{2 \, b \, \left(b \, c - a \, d\right) g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{5} \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, d^{3}} + \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{6 \, d^{3}}$$

## Problem 22: Result valid but suboptimal antiderivative.

$$\int \left( a\;g + b\;g\;x \right)\; \left( c\;i + d\;i\;x \right)^{\;3}\; \left( A + B\;Log\, \left[\;\frac{e\;\left( a + b\;x \right)}{c + d\;x}\;\right] \right)\;\mathrm{d}x$$

Optimal (type 3, 271 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^3 \, x}{20 \, b^3 \, d} + \frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^3 \, \left(c + d \, x\right)^2}{40 \, b^2 \, d^2} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^3 \, \left(c + d \, x\right)^3}{60 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, \left(c + d \, x\right)^4}{20 \, d^2} + \frac{B \left(b \, c - a \, d\right)^5 g \, \mathbf{i}^3 \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{20 \, b^4 \, d^2} - \frac{b \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^2} + \frac{b \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)^5 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right)^5 g \, \mathbf{i}^3 \, Log\left[c + d \, x\right]}{20 \, b^4 \, d^2}$$

Result (type 3, 232 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^3 \, x}{20 \, b^3 \, d} + \frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^3 \, \left(c + d \, x\right)^2}{40 \, b^2 \, d^2} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^3 \, \left(c + d \, x\right)^3}{60 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, \left(c + d \, x\right)^4}{20 \, d^2} + \frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^3 \, \left(c + d \, x\right)^4}{40 \, b^2 \, d^2} - \frac{\left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d^2} + \frac{b \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)^5 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, d^2}$$

## Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{3} \, \left(A + B \, Log\left[\frac{e \, (a + b \, \mathbf{x})}{c + d \, \mathbf{x}}\right]\right)}{a \, g + b \, g \, \mathbf{x}} \, d\mathbf{x}$$

Optimal (type 4, 356 leaves, 14 steps):

$$\frac{5 \, B \, d \, \left(b \, c - a \, d\right)^2 \, i^3 \, x}{6 \, b^3 \, g} \frac{B \, \left(b \, c - a \, d\right) \, i^3 \, \left(c + d \, x\right)^2}{6 \, b^2 \, g} \frac{5 \, B \, \left(b \, c - a \, d\right)^3 \, i^3 \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{6 \, b^4 \, g} + \\ \frac{d \, \left(b \, c - a \, d\right)^2 \, i^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^4 \, g} + \frac{\left(b \, c - a \, d\right) \, i^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^2 \, g} + \frac{i^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b \, g} \\ \frac{11 \, B \, \left(b \, c - a \, d\right)^3 \, i^3 \, Log\left[c + d \, x\right]}{6 \, b^4 \, g} - \frac{\left(b \, c - a \, d\right)^3 \, i^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log\left[1 - \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, i^3 \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g}$$

#### Result (type 4, 436 leaves, 23 steps):

## Problem 25: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^3\,\left(A + B\,\mathsf{Log}\left[\frac{e\,(a+b\,\mathbf{x})}{c+d\,\mathbf{x}}\right]\right)}{\left(a\,g + b\,g\,\mathbf{x}\right)^2}\,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 373 leaves, 11 steps):

$$-\frac{B\,d^{2}\,\left(b\,c-a\,d\right)\,\mathbf{i}^{3}\,x}{2\,b^{3}\,g^{2}}\,-\frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)}{b^{3}\,g^{2}\,\left(a+b\,x\right)}\,-\frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{2\,b^{4}\,g^{2}}\,+\frac{2\,d^{2}\,\left(b\,c-a\,d\right)\,\mathbf{i}^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b^{4}\,g^{2}}\,-\frac{\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,b^{2}\,g^{2}}\,-\frac{5\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\left[c+d\,x\right]}{2\,b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\left[c+d\,x\right]}{2\,b^{4}\,g^{2}}\,-\frac{5\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\left[c+d\,x\right]}{2\,b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,a}{d\,a}\right]}{b^{4}\,g^{2}}\,-\frac{3\,B\,d\,\left(b\,a\,a-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,a}{d\,a}\right]}{b^{4}\,B^{2}}\,-\frac{3\,B\,d\,\left(b\,a-a\,a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\left[2\,\frac{b\,a}{d\,a}\right]}{b^{4}\,B^{2}}\,-\frac{3\,B\,d\,\left(a+b\,a-$$

Result (type 4, 521 leaves, 22 steps):

$$\frac{A\,d^{2}\,\left(3\,b\,c-2\,a\,d\right)\,\mathbf{i}^{3}\,x}{b^{3}\,g^{2}} - \frac{B\,d^{2}\,\left(b\,c-a\,d\right)\,\mathbf{i}^{3}\,x}{2\,b^{3}\,g^{2}} - \frac{B\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{3}}{b^{4}\,g^{2}\,\left(a+b\,x\right)} - \frac{a^{2}\,B\,d^{3}\,\mathbf{i}^{3}\,Log\,[a+b\,x]}{2\,b^{4}\,g^{2}} - \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[a+b\,x]}{b^{4}\,g^{2}} - \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[a+b\,x]}{b^{4}\,g^{2}} - \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[a+b\,x]}{b^{4}\,g^{2}} - \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[a+b\,x]}{b^{4}\,g^{2}} - \frac{\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{4}\,g^{2}} + \frac{B\,d^{2}\,\left(3\,b\,c-2\,a\,d\right)\,\mathbf{i}^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{b^{4}\,g^{2}} - \frac{B\,d\,\left(3\,b\,c-2\,a\,d\right)\,\left(b\,c-a\,d\right)\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,Log\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,PolyLog\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,PolyLog\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,PolyLog\,[c+d\,x]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a$$

## Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{3} \left(\text{A}+\text{BLog}\left[\frac{\text{e}\left(\text{a}+\text{bx}\right)}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 4, 345 leaves, 9 steps):

$$-\frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, \left(c + d \, x\right)}{b^{3} \, g^{3} \, \left(a + b \, x\right)} - \frac{B \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{2}}{4 \, b^{2} \, g^{3} \, \left(a + b \, x\right)^{2}} + \frac{d^{3} \, \mathbf{i}^{3} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^{4} \, g^{3}} - \frac{2 \, d \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^{3} \, g^{3} \, \left(a + b \, x\right)} - \frac{\left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{2} \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{2} \, g^{3} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, Log\left[c + d \, x\right]}{b^{4} \, g^{3}} - \frac{B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, PolyLog\left[2, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^{4} \, g^{3}} - \frac{B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, PolyLog\left[2, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^{4} \, g^{3}} - \frac{B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, PolyLog\left[2, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^{4} \, g^{3}} - \frac{B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, B \, d^{2} \, \left(b \, c - a \, d\right) \, \mathbf{i}^{3} \, \mathbf{i}^{3} \, \mathbf{i}^{3} + \mathbf{i}^{3} \, \mathbf{i}$$

Result (type 4, 442 leaves, 22 steps):

$$\frac{A\,d^3\,i^3\,x}{b^3\,g^3} - \frac{B\,\left(b\,c - a\,d\right)^3\,i^3}{4\,b^4\,g^3\,\left(a + b\,x\right)^2} - \frac{5\,B\,d\,\left(b\,c - a\,d\right)^2\,i^3}{2\,b^4\,g^3\,\left(a + b\,x\right)} - \frac{5\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,Log\left[a + b\,x\right]}{2\,b^4\,g^3} - \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,Log\left[a + b\,x\right]^2}{2\,b^4\,g^3} + \frac{B\,d^3\,i^3\,\left(a + b\,x\right)\,Log\left[\frac{e\,(a + b\,x)}{c + d\,x}\right]}{b^4\,g^3} - \frac{\left(b\,c - a\,d\right)^3\,i^3\,\left(A + B\,Log\left[\frac{e\,(a + b\,x)}{c + d\,x}\right]\right)}{2\,b^4\,g^3\,\left(a + b\,x\right)^2} - \frac{3\,d\,\left(b\,c - a\,d\right)^3\,i^3\left(A + B\,Log\left[\frac{e\,(a + b\,x)}{c + d\,x}\right]\right)}{b^4\,g^3\left(a + b\,x\right)} + \frac{3\,d^2\,\left(b\,c - a\,d\right)\,i^3\,Log\left[a + b\,x\right]\,\left(A + B\,Log\left[\frac{e\,(a + b\,x)}{c + d\,x}\right]\right)}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,Log\left[c + d\,x\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,Log\left[a + b\,x\right]\,Log\left[\frac{b\,(c + d\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} + \frac{3\,B\,d^2\,\left(b\,c - a\,d\right)\,i^3\,PolyLog\left[2\,, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^4\,g^3} +$$

## Problem 27: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \left(\text{A} + \text{BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^4} \, dx$$

#### Optimal (type 4, 310 leaves, 9 steps):

$$-\frac{B\ d^{2}\ i^{3}\ \left(c+d\ x\right)}{b^{3}\ g^{4}\ \left(a+b\ x\right)} - \frac{B\ d\ i^{3}\ \left(c+d\ x\right)^{2}}{4\ b^{2}\ g^{4}\ \left(a+b\ x\right)^{2}} - \frac{B\ i^{3}\ \left(c+d\ x\right)^{3}}{9\ b\ g^{4}\ \left(a+b\ x\right)^{3}} - \frac{d^{2}\ i^{3}\ \left(c+d\ x\right)\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{b^{3}\ g^{4}\ \left(a+b\ x\right)} - \frac{d\ i^{3}\ \left(c+d\ x\right)^{2}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{2\ b^{2}\ g^{4}\ \left(a+b\ x\right)^{2}} - \frac{d^{3}\ i^{3}\ \left(a+b\ x\right)^{2}}{2\ b^{2}\ g^{4}\ \left(a+b\ x\right)^{2}} - \frac{d^{3}\ i^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{b^{4}\ g^{4}} - \frac{d^{3}\ i^{3}\ PolyLog\left[2,\ \frac{b\ (c+d\ x)}{d\ (a+b\ x)}\right]}{b^{4}\ g^{4}}$$

#### Result (type 4, 424 leaves, 23 steps):

$$\frac{B \left(b \, c - a \, d\right)^{3} \, i^{3}}{9 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{3}} - \frac{7 \, B \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3}}{12 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{2}} - \frac{11 \, B \, d^{2} \, \left(b \, c - a \, d\right) \, i^{3}}{6 \, b^{4} \, g^{4} \, \left(a + b \, x\right)} - \frac{11 \, B \, d^{3} \, i^{3} \, Log \left[a + b \, x\right]}{6 \, b^{4} \, g^{4}} - \frac{B \, d^{3} \, i^{3} \, Log \left[a + b \, x\right]^{2}}{2 \, b^{4} \, g^{4}} - \frac{\left(b \, c - a \, d\right)^{2} \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{3}} - \frac{3 \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{2}} - \frac{3 \, d^{2} \, \left(b \, c - a \, d\right) \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{2}} + \frac{2 \, b^{4} \, g^{4} \, \left(a + b \, x\right)^{2}}{6 \, b^{4} \, g^{4}} + \frac{B \, d^{3} \, i^{3} \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{b^{4} \, g^{4}} + \frac{B \, d^{3} \, i^{3} \, PolyLog \left[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}\right]}{b^{4} \, g^{4}}$$

## Problem 28: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^3\,\left(A + B\,\mathsf{Log}\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(a\,g + b\,g\,x\right)^5}\,\mathrm{d}x$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\,\frac{\,B\,\,i^{3}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}}{\,16\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,g^{5}\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}}\,-\,\frac{\,i^{3}\,\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\,\left(\,A\,+\,B\,\,Log\left[\,\frac{e\,\,(a\,+\,b\,\,x)}{c\,+\,d\,\,x}\,\right]\,\right)}{\,4\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,g^{5}\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}}$$

Result (type 3, 373 leaves, 18 steps):

$$-\frac{\frac{B\ \left(b\ c-a\ d\right)^{3}\ i^{3}}{16\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{4}}-\frac{B\ d\ \left(b\ c-a\ d\right)^{2}\ i^{3}}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{3}}-\frac{3\ B\ d^{2}\ \left(b\ c-a\ d\right)\ i^{3}}{8\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{2}}-\frac{B\ d^{3}\ i^{3}}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)}-\frac{B\ d^{3}\ i^{3}}{4\ b^{4}\ \left(b\ c-a\ d\right)\ g^{5}}-\frac{\left(b\ c-a\ d\right)^{3}\ i^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{4}}-\frac{d\ \left(b\ c-a\ d\right)^{3}\ i^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{4}}-\frac{d\ d^{3}\ i^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)^{4}}-\frac{d\ d^{3}\ i^{3}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ b^{4}\ g^{5}\ \left(a+b\ x\right)}+\frac{d\ d^{4}\ i^{3}\ Log\left[c+d\ x\right]}{4\ b^{4}\ \left(b\ c-a\ d\right)\ g^{5}}$$

### Problem 29: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^3\,\left(A + B\,\mathsf{Log}\left[\frac{e\,(a+b\,\mathbf{x})}{c+d\,\mathbf{x}}\right]\right)}{\left(a\,g + b\,g\,\mathbf{x}\right)^6}\,\mathrm{d}\mathbf{x}$$

Optimal (type 3, 181 leaves, 5 steps):

$$\frac{\text{B d i}^{3} \left(\text{c} + \text{d x}\right)^{4}}{16 \left(\text{b c} - \text{a d}\right)^{2} g^{6} \left(\text{a} + \text{b x}\right)^{4}} - \frac{\text{b B i}^{3} \left(\text{c} + \text{d x}\right)^{5}}{25 \left(\text{b c} - \text{a d}\right)^{2} g^{6} \left(\text{a} + \text{b x}\right)^{5}} + \frac{\text{d i}^{3} \left(\text{c} + \text{d x}\right)^{4} \left(\text{A + B Log}\left[\frac{e \cdot (\text{a} + \text{b x})}{c + \text{d x}}\right]\right)}{4 \left(\text{b c} - \text{a d}\right)^{2} g^{6} \left(\text{a} + \text{b x}\right)^{4}} - \frac{\text{b i}^{3} \left(\text{c} + \text{d x}\right)^{5} \left(\text{A + B Log}\left[\frac{e \cdot (\text{a} + \text{b x})}{c + \text{d x}}\right]\right)}{5 \left(\text{b c} - \text{a d}\right)^{2} g^{6} \left(\text{a} + \text{b x}\right)^{5}}$$

Result (type 3, 409 leaves, 18 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{3} \, i^{3}}{25 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{5}} - \frac{11 \, B \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3}}{80 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{4}} - \frac{3 \, B \, d^{2} \, \left(b \, c - a \, d\right) \, i^{3}}{20 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{3}} - \frac{B \, d^{3} \, i^{3}}{40 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{2}} + \frac{B \, d^{5} \, i^{3} \, Log \left[a + b \, x\right]}{20 \, b^{4} \, \left(b \, c - a \, d\right)^{2} \, g^{6}} - \frac{\left(b \, c - a \, d\right)^{3} \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{5 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{5}} - \frac{3 \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{3}} - \frac{d^{3} \, i^{3} \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4} \, g^{6} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{5} \, i^{3} \, Log \left[c + d \, x\right]}{20 \, b^{4} \, \left(b \, c - a \, d\right)^{2} \, g^{6}}$$

## Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \left(\text{A}+\text{BLog}\left[\frac{\text{e}\cdot(\text{a}+\text{bx})}{\text{c+dx}}\right]\right)}{\left(\text{ag+bgx}\right)^7} \, \text{d}x$$

Optimal (type 3, 281 leaves, 5 steps):

$$-\frac{\text{B d}^{2} \, \mathbf{i}^{3} \, \left(\text{c} + \text{d} \, \mathbf{x}\right)^{4}}{16 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{4}} + \frac{2 \, \text{b B d i}^{3} \, \left(\text{c} + \text{d x}\right)^{5}}{25 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{5}} - \frac{\text{b}^{2} \, \text{B i}^{3} \, \left(\text{c} + \text{d x}\right)^{6}}{36 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{6}} - \frac{\text{d}^{2} \, \mathbf{i}^{3} \, \left(\text{c} + \text{d x}\right)^{4} \, \left(\text{A} + \text{B Log}\left[\frac{\mathbf{e} \, \left(\text{a} + \text{b x}\right)}{\mathbf{c} + \text{d x}}\right]\right)}{4 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{5}} + \frac{2 \, \text{b d i}^{3} \, \left(\text{c} + \text{d x}\right)^{5} \, \left(\text{A} + \text{B Log}\left[\frac{\mathbf{e} \, \left(\text{a} + \text{b x}\right)}{\mathbf{c} + \text{d x}}\right]\right)}{5 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{5}} - \frac{\text{b}^{2} \, \mathbf{i}^{3} \, \left(\text{c} + \text{d x}\right)^{6} \, \left(\text{A} + \text{B Log}\left[\frac{\mathbf{e} \, \left(\text{a} + \text{b x}\right)}{\mathbf{c} + \text{d x}}\right]\right)}{6 \, \left(\text{b c} - \text{a d}\right)^{3} \, \mathbf{g}^{7} \, \left(\text{a} + \text{b x}\right)^{6}}$$

Result (type 3, 445 leaves, 18 steps):

$$\frac{ B \left( b \ c - a \ d \right)^{3} \ i^{3}}{36 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{6}} - \frac{13 \ B \ d \left( b \ c - a \ d \right)^{2} \ i^{3}}{150 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{5}} - \frac{19 \ B \ d^{2} \ \left( b \ c - a \ d \right) \ i^{3}}{240 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{4}} - \frac{B \ d^{3} \ i^{3}}{180 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{3}} + \\ \frac{B \ d^{4} \ i^{3}}{120 \ b^{4} \ \left( b \ c - a \ d \right) \ g^{7} \ \left( a + b \ x \right)^{2}} - \frac{B \ d^{5} \ i^{3}}{60 \ b^{4} \ \left( b \ c - a \ d \right)^{2} \ g^{7} \ \left( a + b \ x \right)} - \frac{B \ d^{6} \ i^{3} \ Log \left[ a + b \ x \right]}{60 \ b^{4} \ \left( b \ c - a \ d \right)^{3} \ g^{7}} - \frac{\left( b \ c - a \ d \right)^{3} \ i^{3} \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{60 \ b^{4} \ \left( b \ c - a \ d \right)^{2} \ g^{7} \ \left( a + b \ x \right)^{6}} - \frac{3 \ d^{2} \ \left( b \ c - a \ d \right) \ i^{3} \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{4 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{4}} - \frac{d^{3} \ i^{3} \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{3 \ b^{4} \ g^{7} \ \left( a + b \ x \right)^{3}} + \frac{B \ d^{6} \ i^{3} \ Log \left[ c + d \ x \right]}{60 \ b^{4} \ \left( b \ c - a \ d \right)^{3} \ g^{7}}$$

### Problem 31: Result valid but suboptimal antiderivative.

$$\left(\frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{c\,i+d\,i\,x}\,dx$$

Optimal (type 4, 252 leaves, 6 steps):

$$\frac{g^3 \left(a+b\,x\right)^3 \left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{3\,d\,\mathbf{i}} - \frac{\left(b\,c-a\,d\right)\,g^3 \left(a+b\,x\right)^2 \left(3\,A+B+3\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{6\,d^2\,\mathbf{i}} + \frac{\left(b\,c-a\,d\right)^2\,g^3 \left(a+b\,x\right) \,\left(6\,A+5\,B+6\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{6\,d^3\,\mathbf{i}} + \frac{\left(b\,c-a\,d\right)^3\,g^3\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right] \left(6\,A+11\,B+6\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{6\,d^4\,\mathbf{i}} + \frac{B\,\left(b\,c-a\,d\right)^3\,g^3\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^4\,\mathbf{i}}$$

Result (type 4, 408 leaves, 23 steps):

$$\frac{A\,b\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,x}{d^{3}\,i} + \frac{5\,b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,x}{6\,d^{3}\,i} - \frac{B\,\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)^{2}}{6\,d^{2}\,i} + \\ \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{d^{3}\,i} - \frac{\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,d^{2}\,i} + \frac{g^{3}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,d\,i} - \\ \frac{11\,B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,Log\left[c+d\,x\right]}{6\,d^{4}\,i} - \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,Log\left[i\,\left(c+d\,x\right)\right]^{2}}{2\,d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c\,i+d\,i\,x\right]}{d^{4}\,i} - \\ \frac{\left(b\,c-a\,d\right)^{3}\,g^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,Log\left[c\,i+d\,i\,x\right]}{d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^{4}\,i}$$

## Problem 32: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\;g+b\;g\;x\right)^{\;2}\;\left(A+B\;Log\left[\frac{e\;(a+b\;x)}{c+d\;x}\right]\right)}{c\;i+d\;i\;x}\;\text{d}\,x$$

Optimal (type 4, 198 leaves, 5 steps):

$$\frac{g^2 \left(a+b\,x\right)^2 \left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,d\,i} - \frac{\left(b\,c-a\,d\right)\,g^2 \left(a+b\,x\right) \, \left(2\,A+B+2\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,d^2\,i} - \frac{\left(b\,c-a\,d\right)^2\,g^2\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right] \left(2\,A+3\,B+2\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,d^3\,i} - \frac{B\,\left(b\,c-a\,d\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,x}{b\,x}\right]}{d^3\,i} - \frac{B\,\left(a+b\,x\right)^2\,g^2\,PolyLog\left[2\,\frac{d\,x}{b\,x}$$

Result (type 4, 329 leaves, 19 steps):

$$\frac{ \text{A b } \left( \text{b c - a d} \right) \, \text{g}^2 \, \text{x}}{ \text{d}^2 \, \text{i}} - \frac{ \text{b B } \left( \text{b c - a d} \right) \, \text{g}^2 \, \text{x}}{ 2 \, \text{d}^2 \, \text{i}} - \frac{ \text{B } \left( \text{b c - a d} \right) \, \text{g}^2 \, \left( \text{a + b x} \right) \, \text{Log} \left[ \frac{\text{e } \left( \text{a + b x} \right)}{\text{c + d x}} \right]}{ \text{2 d i}} + \frac{ \text{g}^2 \, \left( \text{a + b x} \right)^2 \, \left( \text{A + B Log} \left[ \frac{\text{e } \left( \text{a + b x} \right)}{\text{c + d x}} \right] \right) }{ \text{2 d i}} + \frac{ \text{B } \left( \text{b c - a d} \right)^2 \, \text{g}^2 \, \text{Log} \left[ \text{i} \, \left( \text{c + d x} \right) \right]^2}{ \text{2 d}^3 \, \text{i}} - \frac{ \text{B } \left( \text{b c - a d} \right)^2 \, \text{g}^2 \, \text{Log} \left[ -\frac{\text{d } \left( \text{a + b x} \right)}{\text{b c - a d}} \right] \, \text{Log} \left[ \text{c i + d i x} \right] }{ \text{d}^3 \, \text{i}} + \frac{ \left( \text{b c - a d} \right)^2 \, \text{g}^2 \, \text{PolyLog} \left[ 2, \frac{\text{b } \left( \text{c + d x} \right)}{\text{b c - a d}} \right] }{ \text{d}^3 \, \text{i}} \right) }{ \text{d}^3 \, \text{i}}$$

## Problem 33: Result valid but suboptimal antiderivative.

$$\int \frac{\left(ag + bgx\right) \left(A + B Log\left[\frac{e(a+bx)}{c+dx}\right]\right)}{ci + dix} dx$$

Optimal (type 4, 125 leaves, 4 steps):

$$\frac{g\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]\right)}{\mathsf{d}\,\mathsf{i}} + \frac{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{g}\,\mathsf{Log}\left[\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]\left(\mathsf{A}+\mathsf{B}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]\right)}{\mathsf{d}^2\,\mathsf{i}} + \frac{\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathsf{g}\,\mathsf{PolyLog}\left[\mathsf{2},\,\frac{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]}{\mathsf{d}^2\,\mathsf{i}}$$

Result (type 4, 213 leaves, 14 steps):

$$\frac{A \ b \ g \ x}{d \ i} + \frac{B \ g \ \left(a + b \ x\right) \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]}{d \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[-\frac{d \ (a + b \ x)}{b \ c - a \ d}\right] \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{\left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ PolyLog\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ Log\left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c$$

## Problem 34: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e (a+bx)}{c+dx}\right]}{c i + d i x} dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$-\frac{\text{Log}\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \, \left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{\mathsf{d} \, \mathbf{i}} - \frac{\mathsf{B} \, \mathsf{PolyLog}\left[\mathsf{2}, \, \frac{d \, (a+b \, x)}{b \, (c+d \, x)}\right]}{\mathsf{d} \, \mathbf{i}}$$

Result (type 4, 122 leaves, 10 steps):

$$\frac{B \, Log \left[ \, \mathbf{i} \, \left( c + d \, x \right) \, \right]^2}{2 \, d \, \mathbf{i}} \, - \, \frac{B \, Log \left[ \, - \, \frac{d \, \left( a + b \, x \right)}{b \, c - a \, d} \, \right] \, Log \left[ \, c \, \, \mathbf{i} + d \, \, \mathbf{i} \, \, x \, \right]}{d \, \mathbf{i}} \, + \, \frac{\left( A + B \, Log \left[ \, \frac{e \, \left( a + b \, x \right)}{c + d \, x} \, \right] \right) \, Log \left[ \, c \, \, \mathbf{i} + d \, \, \mathbf{i} \, \, x \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d \, \mathbf{i}} \, - \, \frac{B \, PolyLog \left[ \, 2 \, , \, \frac{b \, \left( c + d \, x \right)}{b \, c - a \, d} \, \right]}{d$$

## Problem 35: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e \cdot (a+b \cdot x)}{c+d \cdot x}\right]}{\left(a \cdot g + b \cdot g \cdot x\right) \left(c \cdot i + d \cdot i \cdot x\right)} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{\left(A + B \log\left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{2 B \left(b c - a d\right) g i}$$

Result (type 4, 304 leaves, 20 steps):

$$-\frac{B \ Log \ [a+b \ x]^2}{2 \ (b \ c-a \ d) \ g \ i} + \frac{Log \ [a+b \ x] \ \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{(b \ c-a \ d) \ g \ i} + \frac{B \ Log \left[-\frac{d \ (a+b \ x)}{b \ c-a \ d}\right] \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \ [c+d \ x]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{(b \ c-a \ d) \ g \ i} - \frac{\left(A+B \$$

Problem 36: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B Log\left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{2}\left(ci + dix\right)} dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$-\frac{\left(b\;B\;\left(c\;+\;d\;x\right)}{\left(b\;c\;-\;a\;d\right)^{\;2}\;g^{\;2}\;\mathbf{i}\;\left(a\;+\;b\;x\right)}\;+\;\frac{\left(B\;d\;Log\left[\frac{a\;+\;b\;x}{c\;+\;d\;x}\right]^{\;2}}{\;2\;\left(b\;c\;-\;a\;d\right)^{\;2}\;g^{\;2}\;\mathbf{i}}\;-\;\frac{b\;\left(c\;+\;d\;x\right)\;\left(A\;+\;B\;Log\left[\frac{e\;\left(a\;+\;b\;x\right)}{c\;+\;d\;x}\right]\right)}{\left(b\;c\;-\;a\;d\right)^{\;2}\;g^{\;2}\;\mathbf{i}\;\left(a\;+\;b\;x\right)}\;-\;\frac{d\;Log\left[\frac{a\;+\;b\;x}{c\;+\;d\;x}\right]\;\left(A\;+\;B\;Log\left[\frac{e\;\left(a\;+\;b\;x\right)}{c\;+\;d\;x}\right]\right)}{\left(b\;c\;-\;a\;d\right)^{\;2}\;g^{\;2}\;\mathbf{i}}\;\left(b\;c\;-\;a\;d\right)^{\;2}\;g^{\;2}\;\mathbf{i}\;\left(a\;+\;b\;x\right)\;$$

Result (type 4, 437 leaves, 24 steps):

$$-\frac{B}{\left(b\,c-a\,d\right)}\frac{B}{g^{2}\,\mathbf{i}\,\left(a+b\,x\right)} - \frac{B\,d\,Log\left[a+b\,x\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,Log\left[a+b\,x\right]^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{\left(b\,c-a\,d\right)\,g^{2}\,\mathbf{i}\,\left(a+b\,x\right)} - \frac{A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{d\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,PolyLog\left[2,-\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,PolyLog\left[2,-\frac{b\,(c+d\,x)}{b\,c$$

Problem 37: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{3} \left(ci + dix\right)} dx$$

Optimal (type 3, 255 leaves, 7 steps):

$$-\frac{B\left(c+d\,x\right)^{\,2}\,\left(b-\frac{4\,d\,\left(a+b\,x\right)}{c+d\,x}\right)^{\,2}}{4\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)^{\,2}}-\frac{B\,d^{\,2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{\,2}}{2\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}}+\\\\ -\frac{2\,b\,d\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)}-\frac{b^{\,2}\,\left(c+d\,x\right)^{\,2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)^{\,2}}+\frac{d^{\,2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}}$$

#### Result (type 4, 535 leaves, 28 steps):

$$-\frac{B}{4 \; \left(b \; c-a \; d\right) \; g^{3} \; i \; \left(a+b \; x\right)^{2}}{4 \; \left(b \; c-a \; d\right)^{2} \; g^{3} \; i \; \left(a+b \; x\right)} + \frac{3 \; B \; d^{2} \; Log \left[a+b \; x\right]}{2 \; \left(b \; c-a \; d\right)^{3} \; g^{3} \; i} - \frac{B \; d^{2} \; Log \left[a+b \; x\right]^{2}}{2 \; \left(b \; c-a \; d\right)^{3} \; g^{3} \; i} - \frac{A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right]}{2 \; \left(b \; c-a \; d\right) \; g^{3} \; i \; \left(a+b \; x\right)^{2}} + \frac{d^{2} \; Log \left[a+b \; x\right] \; \left(A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right]\right)}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} - \frac{3 \; B \; d^{2} \; Log \left[c+d \; x\right]}{2 \; \left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} - \frac{A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right] \; Log \left[c+d \; x\right]}{2 \; \left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} - \frac{A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right] \; Log \left[\frac{e \; (a+b \; x)}{b \; c-a \; d}\right] \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right]}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i}}{\left(b \; c-a \; d\right)^{3} \; g^{3} \; i} + \frac{B \; d^{2} \; PolyLog \left[2, \; -\frac{d \; (a+b \; x)}{b \; c-a \; d}\right$$

## Problem 38: Result unnecessarily involves higher level functions.

$$\int \frac{A + B Log\left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^4 \left(ci + dix\right)} dx$$

#### Optimal (type 3, 373 leaves, 8 steps):

$$-\frac{3 \, b \, B \, d^{2} \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{2}} - \frac{b^{3} \, B \, \left(c + d \, x\right)^{3}}{9 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{3}} + \frac{B \, d^{3} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{2}}{2 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, d^{2} \, \mathbf{i}}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}} - \frac{3 \, b \, d^{2} \, \left(c +$$

### Result (type 4, 620 leaves, 32 steps):

$$\frac{B}{9} \frac{B}{(b \, c - a \, d)} \frac{B}{g^4 \, i} \frac{5 \, B \, d}{12 \, (b \, c - a \, d)^2 \, g^4 \, i} \frac{12 \, (b \, c - a \, d)^2 \, g^4 \, i}{12 \, (b \, c - a \, d)^2 \, g^4 \, i} \frac{13 \, B \, d^2}{6 \, (b \, c - a \, d)^3 \, g^4 \, i} \frac{13 \, B \, d^3 \, Log \left[a + b \, x\right]}{6 \, (b \, c - a \, d)^4 \, g^4 \, i} + \frac{B \, d^3 \, Log \left[a + b \, x\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^2 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^2 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^2 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^3 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^3 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \, c - a \, d)^4 \, g^4 \, i} \frac{A \, B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{2 \, (b \,$$

## Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,3}\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 4, 341 leaves, 9 steps):

$$\frac{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)}{d^3 \, i^2 \, \left(c + d \, x\right)} - \frac{\left(6 \, A + 5 \, B\right) \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right)}{2 \, d^3 \, i^2 \, \left(c + d \, x\right)} - \frac{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{d^3 \, i^2 \, \left(c + d \, x\right)} + \frac{g^3 \, \left(a + b \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d \, i^2 \, \left(c + d \, x\right)} - \frac{\left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)^2 \, \left(3 \, A + B + 3 \, B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, i^2 \, \left(c + d \, x\right)}{2 \, d^2 \,$$

Result (type 4, 519 leaves, 22 steps):

$$\frac{ A \, b^2 \, \left( 2 \, b \, c - 3 \, a \, d \right) \, g^3 \, x }{ d^3 \, i^2 } - \frac{ b^2 \, B \, \left( b \, c - a \, d \right) \, g^3 \, x }{ 2 \, d^3 \, i^2 } - \frac{ B \, \left( b \, c - a \, d \right)^3 \, g^3 }{ d^4 \, i^2 \, \left( c + d \, x \right) } - \frac{ a^2 \, b \, B \, g^3 \, Log \left[ a + b \, x \right] }{ 2 \, d^2 \, i^2 } - \frac{ b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ a + b \, x \right] }{ d^4 \, i^2 } - \frac{ b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ a + b \, x \right] }{ d^4 \, i^2 } - \frac{ b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ a + b \, x \right] }{ d^4 \, i^2 } - \frac{ b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ a + b \, x \right] }{ d^4 \, i^2 } + \frac{ b^3 \, B \, c^2 \, g^3 \, Log \left[ c + d \, x \right] }{ 2 \, d^4 \, i^2 } + \frac{ b^3 \, B \, c^2 \, g^3 \, Log \left[ c + d \, x \right] }{ 2 \, d^4 \, i^2 } + \frac{ b^3 \, B \, c^2 \, g^3 \, Log \left[ c + d \, x \right] }{ 2 \, d^4 \, i^2 } + \frac{ b^3 \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ 3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c - d \, x \right] }{ d^4 \, i^2 } + \frac{ b^3 \, B \, c^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } + \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ 3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } + \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, Log \left[ c + d \, x \right] }{ d^4 \, i^2 } - \frac{ a^3 \, b \, B \, \left( b \, c - a \, d \right)^$$

## Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 4, 260 leaves, 8 steps):

$$-\frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}+\frac{\left(2\,A+B\right)\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}+\frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]}{d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}+\\ \frac{g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}+\frac{b\,\left(b\,c-a\,d\right)\,g^{2}\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(2\,A+B+2\,B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d^{3}\,\mathbf{i}^{2}}+\frac{2\,b\,B\,\left(b\,c-a\,d\right)\,g^{2}\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,\mathbf{i}^{2}}$$

Result (type 4, 336 leaves, 18 steps):

$$\frac{A \, b^2 \, g^2 \, x}{d^2 \, i^2} + \frac{B \, \left(b \, c - a \, d\right)^2 \, g^2}{d^3 \, i^2 \, \left(c + d \, x\right)} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[a + b \, x\right]}{d^3 \, i^2} + \frac{b \, B \, g^2 \, \left(a + b \, x\right) \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{d^2 \, i^2} - \frac{\left(b \, c - a \, d\right)^2 \, g^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^3 \, i^2} - \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]}{d^3 \, i^2} - \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]}{d^3 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, PolyLog \left[2, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{d^3 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, PolyLog \left[2, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{d^3 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, PolyLog \left[2, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{d^3 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, PolyLog \left[2, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{d^3 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{b \, B \, \left(b \, c - a \, d\right) \,$$

### Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(c\,i+d\,i\,x\right)^{2}}\,\mathrm{d}x$$

Optimal (type 4, 160 leaves, 7 steps):

$$-\frac{A\,g\,\left(a+b\,x\right)}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,+\,\frac{B\,g\,\left(a+b\,x\right)}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,-\,\frac{B\,g\,\left(a+b\,x\right)\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,-\,\frac{b\,g\,Log\left[\frac{b\,c+a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,B\,x}{b\,x}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,PolyLog\left[2\,,\,\frac{d\,B\,x}{b\,x}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,-\,\frac{b\,B\,x}{b^{2}}\,$$

Result (type 4, 222 leaves, 15 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g}{d^2 \, i^2 \, \left(c + d \, x\right)} - \frac{b \, B \, g \, Log \, [\, a + b \, x\,]}{d^2 \, i^2} + \frac{\left(b \, c - a \, d\right) \, g \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^2 \, i^2} + \frac{b \, B \, g \, Log \, [\, c + d \, x\,]}{d^2 \, i^2} - \frac{b \, B \, g \, Log \, [\, c + d \, x\,]}{d^2 \, i^2} - \frac{b \, B \, g \, Log \, [\, c + d \, x\,]}{d^2 \, i^2} + \frac{b \, B \, g \, Log \, [\, c + d \, x\,]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,}\right]}{d^2 \, i^2} - \frac{b \, B \, g \, Poly Log \, \left[\, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d\,$$

### Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 98 leaves, 3 steps):

$$\frac{A \left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^2\,\left(c+d\,x\right)}\,-\,\,\frac{B\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^2\,\left(c+d\,x\right)}\,+\,\,\frac{B\,\left(a+b\,x\right)\,\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^2\,\left(c+d\,x\right)}$$

Result (type 3, 101 leaves, 4 steps):

$$\frac{B}{\text{d}\,\mathbf{i}^2\,\left(c+\text{d}\,x\right)} + \frac{\text{b}\,B\,\text{Log}\,[\,a+\text{b}\,x\,]}{\text{d}\,\left(\text{b}\,c-\text{a}\,\text{d}\right)\,\,\mathbf{i}^2} - \frac{A+B\,\text{Log}\left[\,\frac{e\,\,(a+\text{b}\,x)}{c+\text{d}\,x}\,\right]}{\text{d}\,\mathbf{i}^2\,\left(c+\text{d}\,x\right)} - \frac{\text{b}\,B\,\text{Log}\,[\,c+\text{d}\,x\,]}{\text{d}\,\left(\text{b}\,c-\text{a}\,\text{d}\right)\,\,\mathbf{i}^2}$$

Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(a g + b g x\right) \left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$-\frac{\text{A d } \left(\text{a + b x}\right)}{\left(\text{b c - a d}\right)^2 \text{g } \mathbf{i}^2 \, \left(\text{c + d x}\right)} + \frac{\text{B d } \left(\text{a + b x}\right)}{\left(\text{b c - a d}\right)^2 \, \text{g } \mathbf{i}^2 \, \left(\text{c + d x}\right)} - \frac{\text{B d } \left(\text{a + b x}\right) \, \text{Log}\left[\frac{\text{e } (\text{a + b x})}{\text{c + d x}}\right]}{\left(\text{b c - a d}\right)^2 \, \text{g } \mathbf{i}^2 \, \left(\text{c + d x}\right)} + \frac{\text{b } \left(\text{A + B Log}\left[\frac{\text{e } (\text{a + b x})}{\text{c + d x}}\right]\right)^2}{2 \, \text{B } \left(\text{b c - a d}\right)^2 \, \text{g } \mathbf{i}^2}$$

Result (type 4, 432 leaves, 24 steps):

$$-\frac{B}{\left(b\,c-a\,d\right)\,g\,i^{2}\,\left(c+d\,x\right)} - \frac{b\,B\,Log\,[\,a+b\,x\,]}{\left(b\,c-a\,d\right)^{2}\,g\,i^{2}} - \frac{b\,B\,Log\,[\,a+b\,x\,]^{\,2}}{2\,\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} + \frac{A+B\,Log\,\left[\,\frac{e\,(a+b\,x)}{c+d\,x}\,\right]}{\left(b\,c-a\,d\right)\,g\,i^{2}\,\left(c+d\,x\right)} + \frac{b\,B\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} + \frac{b\,B\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} + \frac{b\,B\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} - \frac{b\,\left(A+B\,Log\,\left[\,\frac{e\,(a+b\,x)}{c+d\,x}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} - \frac{b\,\left(A+B\,Log\,\left[\,\frac{e\,(a+b\,x)}{c+d\,x}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,PolyLog\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,PolyLog\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} + \frac{b\,B$$

Problem 44: Result unnecessarily involves higher level functions.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{2} \left(ci + dix\right)^{2}} dx$$

Optimal (type 3, 261 leaves, 4 steps):

$$-\frac{B\ d^{2}\ \left(a+b\ x\right)}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}\ \left(c+d\ x\right)} - \frac{b^{2}\ B\ \left(c+d\ x\right)}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}\ \left(a+b\ x\right)} + \frac{b\ B\ d\ Log\left[\frac{a+b\ x}{c+d\ x}\right]^{2}}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}} + \\ \frac{d^{2}\ \left(a+b\ x\right)\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}} - \frac{b^{2}\ \left(c+d\ x\right)\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}} - \frac{2\ b\ d\ Log\left[\frac{a+b\ x}{c+d\ x}\right]\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{\left(b\ c-a\ d\right)^{3}\ g^{2}\ \mathbf{i}^{2}}$$

### Result (type 4, 462 leaves, 28 steps):

$$-\frac{b \, B}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(a + b \, x\right)} + \frac{B \, d}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(c + d \, x\right)} + \frac{b \, B \, d \, Log \left[a + b \, x\right]^2}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} - \frac{b \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(a + b \, x\right)} - \frac{d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(a + b \, x\right)} - \frac{2 \, b \, B \, d \, Log \left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} + \frac{2 \, b \, d \, log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^3$$

## Problem 45: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(a g + b g x\right)^{3} \left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 364 leaves, 8 steps):

$$\frac{B \, d^3 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{3 \, b^2 \, B \, d \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)} - \frac{b^3 \, B \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)^2} - \frac{3 \, b \, B \, d^2 \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(a + b \, x\right) \, \left(a + b \, x\right) \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{d^3 \, \left(a + b \, x\right) \, \left(a$$

Result (type 4, 630 leaves, 32 steps):

$$-\frac{b\ B}{4\ (b\ c-a\ d)^2\ g^3\ i^2\ (a+b\ x)^2} + \frac{5\ b\ B\ d}{2\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)} - \frac{B\ d^2}{(b\ c-a\ d)^3\ g^3\ i^2\ (c+d\ x)} + \frac{3\ b\ B\ d^2\ Log\ [a+b\ x]}{2\ (b\ c-a\ d)^4\ g^3\ i^2} - \frac{3\ b\ B\ d^2\ Log\ [a+b\ x]^2}{2\ (b\ c-a\ d)^4\ g^3\ i^2} - \frac{b\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{2\ (b\ c-a\ d)^2\ g^3\ i^2\ (a+b\ x)^2} + \frac{2\ b\ d\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{(b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)} + \frac{d^2\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{(b\ c-a\ d)^3\ g^3\ i^2\ (c+d\ x)} + \frac{3\ b\ B\ d^2\ Log\ [c+d\ x]}{(b\ c-a\ d)^3\ g^3\ i^2\ (c+d\ x)} + \frac{3\ b\ B\ d^2\ Log\ [c+d\ x]}{(b\ c-a\ d)^4\ g^3\ i^2} - \frac{3\ b\ B\ d^2\ Log\ [c+d\ x]}{(b\ c-a\ d)^4\ g^3\ i^2} + \frac{3\ b\ B\ d^2\ Log\ [c+d\ x]}{(b\ c-a\ d)^4\ g^3\ i^2} + \frac{3\ b\ B\ d^2\ PolyLog\ [2\ , -\frac{d\ (a+b\ x)}{b\ c-a\ d}]}{(b\ c-a\ d)^4\ g^3\ i^2} + \frac{3\ b\ B\ d^2\ PolyLog\ [2\ , -\frac{d\ (a+b\ x)}{b\ c-a\ d}]}{(b\ c-a\ d)^4\ g^3\ i^2} + \frac{3\ b\ B\ d^2\ PolyLog\ [2\ , -\frac{d\ (a+b\ x)}{b\ c-a\ d}]}{(b\ c-a\ d)^4\ g^3\ i^2}$$

## Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{4}\left(ci + dix\right)^{2}} dx$$

#### Optimal (type 3, 457 leaves, 4 steps):

$$-\frac{B\,d^{4}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(c+d\,x\right)} - \frac{6\,b^{2}\,B\,d^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} + \frac{b^{3}\,B\,d\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{2}} - \frac{b^{4}\,B\,\left(c+d\,x\right)^{3}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} + \frac{2\,b\,B\,d^{3}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(c+d\,x\right)} - \frac{b^{4}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(c+d\,x\right)} - \frac{6\,b^{2}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} + \frac{b^{3}\,B\,d\,\left(c+d\,x\right)^{2}\,\left(a+b\,x\right)^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} - \frac{b^{4}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} - \frac{a\,b^{4}\,d^{3}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}} + \frac{b^{4}\,d^{4}\,\left(a+b\,x\right)^{3}\,d^{4}\,d^{4}\,\left(a+b\,x\right)^{2}\,d^{4$$

Result (type 4, 705 leaves, 36 steps):

$$\frac{b\,B}{9\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{3}} + \frac{2\,b\,B\,d}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{2}} - \frac{13\,b\,B\,d^{2}}{3\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} + \frac{B\,d^{3}}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i^{2}\,\left(c+d\,x\right)} - \frac{10\,b\,B\,d^{3}\,Log\left[a+b\,x\right]}{3\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,i^{2}} - \frac{b\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{3}} + \frac{b\,d\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,i^{2}} - \frac{3\,b\,d^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{3}} + \frac{b\,d\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{4}\,i^{2}\,\left(a+b\,x\right)^{2}} - \frac{3\,b\,d^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} - \frac{d^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i^{2}\,\left(a+b\,x\right)} - \frac{d^{3}\,d^{2}$$

### Problem 47: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(c\,i+d\,i\,x\right)^3}\,\mathrm{d}x$$

#### Optimal (type 4, 361 leaves, 9 steps):

$$-\frac{3 \, B \, \left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)^2}{4 \, d^2 \, i^3 \, \left(c + d \, x\right)^2} - \frac{3 \, b \, B \, \left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)}{d^3 \, i^3 \, \left(c + d \, x\right)} + \frac{b \, \left(3 \, A + B\right) \, \left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)}{d^3 \, i^3 \, \left(c + d \, x\right)} + \frac{3 \, b \, B \, \left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{c + d \, x} + \frac{g^3 \, \left(a + b \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^3 \, i^3 \, \left(c + d \, x\right)} + \frac{\left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)^2 \, \left(3 \, A + B + 3 \, B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{2 \, d^2 \, i^3 \, \left(c + d \, x\right)^2}{2 \, d^2 \, i^3 \, \left(c + d \, x\right)^2} + \frac{b^2 \, \left(b \, c - a \, d\right) \, g^3 \, PolyLog\left[2, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^4 \, i^3}$$

### Result (type 4, 442 leaves, 22 steps):

$$\frac{A \, b^3 \, g^3 \, x}{d^3 \, i^3} - \frac{B \, \left(b \, c - a \, d\right)^3 \, g^3}{4 \, d^4 \, i^3 \, \left(c + d \, x\right)^2} + \frac{5 \, b \, B \, \left(b \, c - a \, d\right)^2 \, g^3}{2 \, d^4 \, i^3 \, \left(c + d \, x\right)} + \frac{5 \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, Log \left[a + b \, x\right]}{2 \, d^4 \, i^3} + \frac{b^2 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^4 \, i^3} + \frac{\left(b \, c - a \, d\right)^3 \, g^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^4 \, i^3 \, \left(c + d \, x\right)^2} - \frac{3 \, b \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^3 \, \left(c + d \, x\right)} - \frac{7 \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^3} - \frac{3 \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{d^4 \, i^3} - \frac{3 \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^3} + \frac{3 \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, PolyLog \left[2, \frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{2 \, d^4 \, i^3}$$

## Problem 48: Result valid but suboptimal antiderivative.

$$\frac{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)}{\left(c i + d i x\right)^{3}} dx$$

Optimal (type 4, 251 leaves, 8 steps):

$$\frac{B\,g^{2}\,\left(a+b\,x\right)^{2}}{4\,d\,i^{3}\,\left(c+d\,x\right)^{2}} - \frac{A\,b\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,i^{3}\,\left(c+d\,x\right)} + \frac{b\,B\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,i^{3}\,\left(c+d\,x\right)} - \frac{b\,B\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{d^{2}\,i^{3}\,\left(c+d\,x\right)} - \frac{g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{d^{2}\,i^{3}\,\left(c+d\,x\right)} - \frac{g^{2}\,g^{2}\,Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{d^{3}\,i^{3}} - \frac{b^{2}\,B\,g^{2}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{d^{3}\,i^{3}} - \frac{g^{2}\,g^{2}\,Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{d^{3}\,i^{3}} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{d^{3}\,i^{3}} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]} - \frac{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}{g^{2}\,Log\left[\frac{a\,(a+b\,x)}{b\,(c+d\,x)}\right]}$$

Result (type 4, 340 leaves, 19 steps):

$$\frac{B \left(b \ c - a \ d\right)^{2} g^{2}}{4 \ d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}} - \frac{3 \ b \ B \left(b \ c - a \ d\right) \ g^{2}}{2 \ d^{3} \ i^{3} \ \left(c + d \ x\right)} - \frac{3 \ b^{2} \ B \ g^{2} \ Log \left[a + b \ x\right]}{2 \ d^{3} \ i^{3}} - \frac{\left(b \ c - a \ d\right)^{2} g^{2} \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}} + \frac{2 \ b \left(b \ c - a \ d\right)^{2} g^{2} \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{d^{3} \ i^{3} \ \left(c + d \ x\right)} + \frac{3 \ b^{2} \ B \ g^{2} \ Log \left[c + d \ x\right]}{2 \ d^{3} \ i^{3}} - \frac{b^{2} \ B \ g^{2} \ Log \left[-\frac{d \ (a + b \ x)}{b \ c - a \ d}\right] \ Log \left[c + d \ x\right]}{d^{3} \ i^{3}} + \frac{b^{2} \ B \ g^{2} \ Log \left[c + d \ x\right]^{2}}{2 \ d^{3} \ i^{3}} - \frac{b^{2} \ B \ g^{2} \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^{3} \ i^{3}}$$

## Problem 49: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{3}}\,\mathrm{d}x$$

Optimal (type 3, 85 leaves, 2 steps):

$$-\frac{\text{B g } \left(\text{a + b x}\right)^{2}}{\text{4 } \left(\text{b c - a d}\right) \, \mathbf{i}^{3} \, \left(\text{c + d x}\right)^{2}} + \frac{\text{g } \left(\text{a + b x}\right)^{2} \, \left(\text{A + B Log}\left[\frac{\text{e } \left(\text{a + b x}\right)}{\text{c + d x}}\right]\right)}{\text{2 } \left(\text{b c - a d}\right) \, \mathbf{i}^{3} \, \left(\text{c + d x}\right)^{2}}$$

Result (type 3, 191 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g}{4 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} + \frac{b \, B \, g}{2 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{b^2 \, B \, g \, Log \left[a + b \, x\right]}{2 \, d^2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3} + \frac{\left(b \, c - a \, d\right) \, g \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} - \frac{b \, g \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{b^2 \, B \, g \, Log \left[c + d \, x\right]}{2 \, d^2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3}$$

## Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(a g + b g x\right) \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 243 leaves, 4 steps):

$$-\frac{B\left(4\,b-\frac{d\,\left(a+b\,x\right)}{c+d\,x}\right)^{2}}{4\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}-\frac{b^{2}\,B\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}+\frac{d^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}}-\frac{2\,b\,d\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}\,\left(c+d\,x\right)}+\frac{b^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}$$

Result (type 4, 535 leaves, 28 steps):

$$-\frac{B}{4 \; \left(b \; c-a \; d\right) \; g \; i^{3} \; \left(c+d \; x\right)^{2}} - \frac{3 \; b \; B}{2 \; \left(b \; c-a \; d\right)^{2} \; g \; i^{3} \; \left(c+d \; x\right)} - \frac{3 \; b^{2} \; B \; Log \left[a+b \; x\right]}{2 \; \left(b \; c-a \; d\right)^{3} \; g \; i^{3}} - \frac{b^{2} \; B \; Log \left[a+b \; x\right]^{2}}{2 \; \left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right]}{2 \; \left(b \; c-a \; d\right)^{2} \; g \; i^{3} \; \left(c+d \; x\right)^{2}} + \frac{b^{2} \; Log \left[a+b \; x\right] \; \left(A+B \; Log \left[\frac{e \; (a+b \; x)}{c+d \; x}\right]\right)}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{3 \; b^{2} \; B \; Log \left[c+d \; x\right]}{2 \; \left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} - \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} - \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; Log \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-a \; d\right)^{3} \; g \; i^{3}} + \frac{b^{2} \; B \; PolyLog \left[c+d \; x\right]}{\left(b \; c-$$

## Problem 52: Result unnecessarily involves higher level functions.

$$\int \frac{A + B Log \left[\frac{e (a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{2} \left(ci + dix\right)^{3}} dx$$

Optimal (type 3, 365 leaves, 4 steps):

$$\frac{B\,d^{3}\,\left(a+b\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} - \frac{3\,b\,B\,d^{2}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{b^{3}\,B\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(a+b\,x\right)} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3\,b^{2}\,B\,d\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3$$

Result (type 4, 631 leaves, 32 steps):

$$-\frac{b^2 \, B}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(a + b \, x\right)} + \frac{B \, d}{4 \, \left(b \, c - a \, d\right)^2 \, g^2 \, i^3 \, \left(c + d \, x\right)^2} + \frac{5 \, b \, B \, d}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} + \frac{3 \, b^2 \, B \, d \, Log \left[a + b \, x\right]}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B \, d \, Log \left[a + b \, x\right]}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{b^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(a + b \, x\right)} - \frac{d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)^2} - \frac{2 \, b \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, PolyLog \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac$$

# Problem 53: Result unnecessarily involves higher level functions.

$$\int \frac{A + B Log\left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^{3} \left(ci + dix\right)^{3}} dx$$

Optimal (type 3, 463 leaves, 5 steps):

$$-\frac{B\,d^{4}\,\left(a+b\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} + \frac{4\,b\,B\,d^{3}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{4\,b^{3}\,B\,d\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(a+b\,x\right)} - \frac{b^{4}\,B\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(a+b\,x\right)^{2}} - \frac{3\,b^{2}\,B\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{d^{4}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} - \frac{4\,b\,d^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{4\,b^{3}\,d\,\left(c+d\,x\right)^{2}}{2\,\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}} + \frac{6\,b^{2}\,d^{2}\,Log\left[\frac{a+b\,x}{c$$

Result (type 4, 673 leaves, 36 steps):

$$\frac{b^2 \, B}{4 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^3 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(a + b \, x\right)} + \frac{7 \, b^2 \, B \, d}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(a + b \, x\right)} - \frac{B \, d^2}{4 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} - \frac{3 \, b^2 \, B \, d^2 \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3} - \frac{b^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^3 \, \left(a + b \, x\right)^2} + \frac{3 \, b^2 \, d \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{3 \, b \, d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{3 \, b \, d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^3} + \frac{d^2 \, b^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3} - \frac{d^2 \, b^2 \,$$

# Problem 54: Result unnecessarily involves higher level functions.

$$\int \frac{A + B Log\left[\frac{e(a+bx)}{c+dx}\right]}{\left(ag + bgx\right)^4 \left(ci + dix\right)^3} dx$$

#### Optimal (type 3, 563 leaves, 8 steps):

$$\frac{B\,d^{5}\,\left(a+b\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)^{2}} - \frac{5\,b\,B\,d^{4}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,B\,d^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)^{2}} + \frac{5\,b^{4}\,B\,d\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)^{2}} - \frac{b^{5}\,B\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)^{3}} + \frac{5\,b^{2}\,B\,d^{3}\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}} - \frac{d^{5}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} + \frac{5\,b\,d^{4}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} + \frac{5\,b^{4}\,d\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)} + \frac{5\,b^{4}\,d\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)} + \frac{5\,b^{4}\,d\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)} + \frac{5\,b^{4}\,d\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} - \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(c+d\,x\right)} + \frac{10\,b^{3}\,d^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)} + \frac{10\,b^{3}\,d^{2}\,d^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{6}\,g^{4}\,i^{3}\,\left(a+b\,x\right)} + \frac{10\,b^{3}\,d^{2}\,d^{$$

Result (type 4, 825 leaves, 40 steps):

$$-\frac{b^2 \, B}{9 \, \left(b \, c - a \, d\right)^3 \, g^4 \, i^3 \, \left(a + b \, x\right)^3} + \frac{11 \, b^2 \, B \, d}{12 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} - \frac{47 \, b^2 \, B \, d^2}{6 \, \left(b \, c - a \, d\right)^5 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{B \, d^3}{4 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i^3 \, \left(c + d \, x\right)^2} + \frac{9 \, b \, B \, d^3}{4 \, \left(b \, c - a \, d\right)^5 \, g^4 \, i^3 \, \left(c + d \, x\right)} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[a + b \, x\right]}{3 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{b^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(c + d \, x\right)} - \frac{b^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{b^2 \, d \, A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} - \frac{d^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{4 \, b \, d^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B \, d^3 \, Log \left[c + d \, x\right]}{2 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \,$$

## Problem 55: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 539 leaves, 11 steps):

$$\frac{3 \, B^2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i \, x}{10 \, b \, d^3} - \frac{3 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i \, \left(c + d \, x\right)^2}{20 \, d^4} + \frac{b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, i \, \left(c + d \, x\right)^3}{30 \, d^4} - \frac{B \, \left(b \, c - a \, d\right)^2 \, g^3 \, i \, \left(a + b \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{30 \, b^2 \, d} - \frac{B \, \left(b \, c - a \, d\right) \, g^3 \, i \, \left(a + b \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{10 \, b^2} + \frac{\left(b \, c - a \, d\right) \, g^3 \, i \, \left(a + b \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{5 \, b} + \frac{B \, \left(b \, c - a \, d\right)^3 \, g^3 \, i \, \left(a + b \, x\right)^2 \, \left(3 \, A + B + 3 \, B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{c + d \, x} - \frac{B \, \left(b \, c - a \, d\right)^4 \, g^3 \, i \, \left(a + b \, x\right) \, \left(6 \, A + 5 \, B + 6 \, B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{60 \, b^2 \, d^4} - \frac{B^2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i \, Log\left[c + d \, x\right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i \, Log\left[c + d \, x\right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i \, PolyLog\left[c, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{10 \, b^2 \, d^4}$$

Result (type 4, 622 leaves, 54 steps):

$$-\frac{A \ B \ \left(b \ c - a \ d\right)^4 \ g^3 \ i \ x}{10 \ b \ d^3} + \frac{B^2 \ \left(b \ c - a \ d\right)^4 \ g^3 \ i \ x}{600 \ b \ d^3} - \frac{B^2 \ \left(b \ c - a \ d\right)^3 \ g^3 \ i \ \left(a + b \ x\right)^2}{30 \ b^2 \ d^2} + \frac{B^2 \ \left(b \ c - a \ d\right)^2 \ g^3 \ i \ \left(a + b \ x\right)^3}{30 \ b^2 \ d} - \frac{B^2 \ \left(b \ c - a \ d\right)^3 \ g^3 \ i \ \left(a + b \ x\right)^2 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{10 \ b^2 \ d^3} + \frac{B \ \left(b \ c - a \ d\right)^3 \ g^3 \ i \ \left(a + b \ x\right)^2 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{20 \ b^2 \ d^2} - \frac{B \ \left(b \ c - a \ d\right)^2 \ g^3 \ i \ \left(a + b \ x\right)^3 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{30 \ b^2 \ d} - \frac{B \ \left(b \ c - a \ d\right)^3 \ g^3 \ i \ \left(a + b \ x\right)^4 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{30 \ b^2 \ d} - \frac{B \ \left(b \ c - a \ d\right)^3 \ g^3 \ i \ \left(a + b \ x\right)^4 \ \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^2}{4 \ b^2} + \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]}{4 \ b^2} + \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]}{10 \ b^2 \ d^4} - \frac{B \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} - \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10 \ b^2 \ d^4} + \frac{B^2 \ \left(b \ c - a \ d\right)^5 \ g^3 \ i \ Log \left[c + d \ x\right]^2}{10$$

### Problem 56: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 450 leaves, 10 steps):

$$-\frac{B^2 \left(b \ c - a \ d\right)^3 \ g^2 \ i \ x}{3 \ b \ d^2} + \frac{B^2 \left(b \ c - a \ d\right)^2 \ g^2 \ i \ \left(c + d \ x\right)^2}{12 \ d^3} - \frac{B \left(b \ c - a \ d\right)^2 \ g^2 \ i \ \left(a + b \ x\right)^2 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b^2 \ d} - \frac{B \left(b \ c - a \ d\right) \ g^2 \ i \ \left(a + b \ x\right)^3 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b^2} + \frac{\left(b \ c - a \ d\right) \ g^2 \ i \ \left(a + b \ x\right)^3 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^2}{12 \ b^2} + \frac{g^2 \ i \ \left(a + b \ x\right)^3 \left(a + b \ x\right) \left(2 \ A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b^2 \ d^2} + \frac{B \left(b \ c - a \ d\right)^3 \ g^2 \ i \ \left(a + b \ x\right) \left(2 \ A + B \ + 2 \ B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{12 \ b^2 \ d^2} + \frac{B^2 \left(b \ c - a \ d\right)^4 \ g^2 \ i \ Log\left[c + d \ x\right]}{6 \ b^2 \ d^3} + \frac{B^2 \left(b \ c - a \ d\right)^4 \ g^2 \ i \ Poly Log\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{6 \ b^2 \ d^3}$$

Result (type 4, 537 leaves, 46 steps):

$$\frac{A\,B\,\left(b\,c\,-a\,d\right)^{3}\,g^{2}\,i\,x}{6\,b\,d^{2}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{3}\,g^{2}\,i\,x}{12\,b\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{2}\,g^{2}\,i\,\left(a+b\,x\right)^{2}}{12\,b^{2}\,d} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{3}\,g^{2}\,i\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{6\,b^{2}} - \frac{B\,\left(b\,c\,-a\,d\right)^{2}\,g^{2}\,i\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{6\,b^{2}} + \frac{B\,\left(b\,c\,-a\,d\right)^{2}\,g^{2}\,i\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b^{2}} + \frac{\left(b\,c\,-a\,d\right)\,g^{2}\,i\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]}{6\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,Log\left[c+d\,x\right]^{2}}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g^{2}\,i\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{12\,b^{2}\,d^{3}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^$$

### Problem 57: Result valid but suboptimal antiderivative.

$$\int \left( a\;g + b\;g\;x \right) \; \left( c\;i + d\;i\;x \right) \; \left( A + B\;Log\left[ \;\frac{e\;\left( a + b\;x \right)}{c + d\;x} \;\right] \right)^2 \; \mathrm{d}x$$

Optimal (type 4, 343 leaves, 9 steps):

$$\frac{B^{2} \left(b \, c-a \, d\right)^{2} \, g \, i \, x}{3 \, b \, d} - \frac{B \left(b \, c-a \, d\right)^{2} \, g \, i \, \left(a+b \, x\right) \, \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{3 \, b^{2} \, d} - \frac{B \left(b \, c-a \, d\right) \, g \, i \, \left(a+b \, x\right)^{2} \, \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{3 \, b^{2}} + \frac{\left(b \, c-a \, d\right) \, g \, i \, \left(a+b \, x\right)^{2} \, \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{3 \, b} - \frac{\left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{3 \, b^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g \, i \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}}$$

Result (type 4, 1214 leaves, 78 steps):

$$\frac{2}{3} \text{AbB} \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) \text{dgi} x + \frac{B^2 \left( bc - ad \right)^2 \text{gi} x}{3bd} - \frac{AB \left( bc - ad \right) \left( bc + ad \right) \text{gi} x}{bd} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{3b^2} - \frac{a^3 B^2 \text{dgi} \log \left[ a + bx \right)^2}{3b^2} + \frac{a^2 B^2 \left( bc + ad \right) \text{gi} \log \left[ a + bx \right)^2}{2b^2} - \frac{B^2 \left( bc - ad \right) \left( bc + ad \right) \text{gi} \left( a + bx \right) \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{3b^2} - \frac{3b^2 d}{3b^2} - \frac{a^3 B^2 \text{dgi} \log \left[ a + bx \right]}{2b^2} + \frac{a^2 B^2 \left( bc + ad \right) \text{gi} \log \left[ a + bx \right)^2}{2b^2} - \frac{B^2 \left( bc - ad \right) \left( bc + ad \right) \text{gi} \left( a + bx \right) \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{3b^2} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{bc - ad} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{bc - ad} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{bc - ad} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{bc - ad} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ a + bx \right]}{bc - ad} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad \right)}}{bc - ad } + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad } + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad } + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{bc - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( bc - ad \right) \text{gi} \log \left[ \frac{e \left( a + bx \right)}{c + dx} \right]}{ac - ad }} + \frac{a^2 B^2 \left( \frac{e \left( a + bx \right)}{c + dx} \right)}{ac - ad }$$

# Problem 58: Result valid but suboptimal antiderivative.

$$\int \left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right) \,\left(A + B\,\mathsf{Log}\left[\,\frac{e\,\left(a + b\,\mathbf{x}\right)}{c + d\,\mathbf{x}}\,\right]\,\right)^2\,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 203 leaves, 7 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,\mathbf{i}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b^{2}}+\frac{\mathbf{i}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{2\,d}+\\ \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}\,Log\left[c+d\,x\right]}{b^{2}\,d}+\frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)\,Log\left[1-\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{2}\,d}-\frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}\,PolyLog\left[2\,\mathbf{,}\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{2}\,d}$$

Result (type 4, 283 leaves, 16 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,i\,x}{b} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,a+b\,x\,]^{\,2}}{2\,b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)\,i\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{b^2} - \frac{B\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^2\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,c+d\,x\,]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,a+b\,x\,]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,PolyLog\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^2\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,c+d\,x\,]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,Log\,[\,a+b\,x\,]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,PolyLog\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,$$

### Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \, \left(\text{A} + \text{BLog}\left[\frac{\text{e}\, \left(\text{a+b}\, x\right)}{\text{c+d}\, x}\right]\right)^2}{\text{ag+bg}\, x} \, \text{d}\, x$$

Optimal (type 4, 286 leaves, 8 steps):

$$\frac{2\,B\,\left(b\,c-a\,d\right)\,i\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b^{2}\,g}+\frac{d\,i\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b^{2}\,g}-\frac{\left(b\,c-a\,d\right)\,i\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}\,Log\left[1-\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{2}\,g}+\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,i\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{b^{2}\,g}+\frac{2\,B\,\left(b\,c-a\,d\right)\,i\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b^{2}\,g}+\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,i\,PolyLog\left[3,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{2}\,g}+\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,i\,PolyLog\left[3,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{2}\,g}$$

Result (type 4, 644 leaves, 39 steps):

$$-\frac{a\,B^{2}\,d\,i\,Log\,[\,a+b\,x\,]^{\,2}}{b^{2}\,g} - \frac{A\,B\,\left(b\,c-a\,d\right)\,i\,Log\,[\,a+b\,x\,]^{\,2}}{b^{2}\,g} - \frac{B^{2}\,\left(b\,c-a\,d\right)\,i\,Log\,\left[-\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]^{\,2}}{b^{2}\,g} - \frac{B^{2}\,\left(b\,c-a\,d\right)\,i\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]^{\,2}}{b^{2}\,g} + \frac{2\,a\,B\,d\,i\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{\,2}}{b^{\,2}\,g} + \frac{\left(b\,c-a\,d\right)\,i\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{\,2}}{b^{\,2}\,g} + \frac{2\,B^{\,2}\,c\,i\,Log\,\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\,[\,c+d\,x\,]}{b^{\,2}\,g} + \frac{2\,a\,B^{\,2}\,d\,i\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{c+d\,x}\right]\right)^{\,2}}{b^{\,2}\,g} + \frac{2\,a\,B^{\,2}\,d\,i\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{\,2}\,g} + \frac{2\,a\,B^{\,2}\,d\,i\,Log\,[\,a+b\,x\,]\,Log\,\left[\frac{b\,(c+d\,$$

# Problem 60: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\frac{\text{e} (\text{a+bx})}{\text{c+dx}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{2}} \, dx$$

Optimal (type 4, 241 leaves, 7 steps):

$$-\frac{2\,B^{2}\,\mathbf{i}\,\left(c+d\,x\right)}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{2\,B\,\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{b\,g^{2}}-\frac{\mathbf{i}\,\left(a+b\,x\right)\,\left(a$$

Result (type 4, 705 leaves, 43 steps):

$$-\frac{2 \, B^2 \left( b \, C - a \, d \right) \, i}{b^2 \, g^2 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d \, i \, Log \left[ a + b \, x \right]}{b^2 \, g^2} - \frac{b^2 \, g^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ a + b \, x \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ c + d \, x \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \, g^2} + \frac{B^2 \, d \, i \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]^2}{b^2 \,$$

Problem 61: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\frac{e\,(\mathsf{a} + \mathsf{b}\,\mathbf{x})}{c + d\,\mathbf{x}}\right]\right)^2}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^3}\,\mathrm{d}\mathbf{x}$$

Optimal (type 3, 141 leaves, 3 steps):

$$-\frac{B^{2} i \left(c+d x\right)^{2}}{4 \left(b c-a d\right) g^{3} \left(a+b x\right)^{2}}-\frac{B i \left(c+d x\right)^{2} \left(A+B Log\left[\frac{e \cdot (a+b \cdot x)}{c+d \cdot x}\right]\right)}{2 \left(b c-a d\right) g^{3} \left(a+b \cdot x\right)^{2}}-\frac{i \left(c+d \cdot x\right)^{2} \left(A+B Log\left[\frac{e \cdot (a+b \cdot x)}{c+d \cdot x}\right]\right)^{2}}{2 \left(b \cdot c-a d\right) g^{3} \left(a+b \cdot x\right)^{2}}$$

Result (type 4, 639 leaves, 58 steps):

$$-\frac{B^2 \left(b \ c-a \ d\right) \ i}{4 \ b^2 \ g^3 \left(a+b \ x\right)^2} - \frac{B^2 \ d \ i}{2 \ b^2 \ g^3 \left(a+b \ x\right)} - \frac{B^2 \ d^2 \ i \ Log \left[a+b \ x\right]}{2 \ b^2 \left(b \ c-a \ d\right) \ g^3} + \frac{B^2 \ d^2 \ i \ Log \left[a+b \ x\right]^2}{2 \ b^2 \left(b \ c-a \ d\right) \ g^3} - \frac{B \left(b \ c-a \ d\right) \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{2 \ b^2 \ g^3 \left(a+b \ x\right)^2} - \frac{B \ d^2 \ i \ Log \left[a+b \ x\right] \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{b^2 \ g^3 \left(a+b \ x\right)} - \frac{B \ d^2 \ i \ Log \left[a+b \ x\right] \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{b^2 \ g^3 \left(a+b \ x\right)^2} - \frac{\left(b \ c-a \ d\right) \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^2}{2 \ b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} + \frac{B \ d^2 \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^2}{b^2 \ (b \ c-a \ d) \ g^3} + \frac{B^2 \ d^2 \ i \ Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} + \frac{B \ d^2 \ i \ \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} + \frac{B^2 \ d^2 \ i \ Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2 \ (b \ c-a \ d) \ g^3} - \frac{B^2 \ d^2 \ i \ Poly Log \left[c+d \ x\right]}{b^2$$

Problem 62: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A}+\text{BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)^2}{\left(\text{ag+bgx}\right)^4} \, dx$$

Optimal (type 3, 287 leaves, 7 steps):

$$\frac{B^2\,d\,\mathbf{i}\,\left(c+d\,x\right)^2}{4\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^2} - \frac{2\,b\,B^2\,\mathbf{i}\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3} + \frac{B\,d\,\mathbf{i}\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^2} - \\ \frac{2\,b\,B\,\mathbf{i}\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^2} + \frac{d\,\mathbf{i}\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{2\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^2} - \frac{b\,\mathbf{i}\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3\,\left(b\,c-a\,d\right)^2\,g^4\,\left(a+b\,x\right)^3}$$

Result (type 4, 741 leaves, 66 steps):

$$-\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,i}{27\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} + \frac{B^{2}\,d\,i}{36\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} + \frac{5\,B^{2}\,d^{2}\,i}{18\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{5\,B^{2}\,d^{3}\,i\,Log\left[a+b\,x\right]}{18\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{B^{2}\,d^{3}\,i\,Log\left[a+b\,x\right]^{2}}{9\,b^{2}\,g^{4}} - \frac{2\,B\,\left(b\,c-a\,d\right)\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{9\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{B\,d\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{6\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} + \frac{B\,d^{2}\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{B\,d^{3}\,i\,Log\left[a+b\,x\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{\left(b\,c-a\,d\right)\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{d\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{b\,d^{3}\,i\,Log\left[a+b\,x\right]}{2\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{d\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} - \frac{5\,B^{2}\,d^{3}\,i\,Log\left[c+d\,x\right]}{2\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} - \frac{B\,d^{3}\,i\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{5\,B^{2}\,d^{3}\,i\,Log\left[c+d\,x\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,Log\left[a+b\,x\right]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,Log\left[a+b\,x\right]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,PolyLog\left[a+b\,x\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,PolyLog\left[a+b\,x\right]}{3\,b$$

### Problem 63: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A}+\text{BLog}\left[\frac{\text{e}\cdot(\text{a}+\text{b}\cdot\text{x})}{\text{c+d}\cdot\text{x}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

Optimal (type 3, 445 leaves, 9 steps):

$$-\frac{B^2\,d^2\,\mathbf{i}\,\left(c+d\,x\right)^2}{4\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^2} + \frac{4\,b\,B^2\,d\,\mathbf{i}\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^3} - \frac{b^2\,B^2\,\mathbf{i}\,\left(c+d\,x\right)^4}{32\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^4} - \\ \frac{B\,d^2\,\mathbf{i}\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^2} + \frac{4\,b\,B\,d\,\mathbf{i}\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{9\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^3} - \frac{b^2\,B\,\mathbf{i}\,\left(c+d\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{8\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^4} - \\ \frac{d^2\,\mathbf{i}\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{2\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^3} + \frac{2\,b\,d\,\mathbf{i}\,\left(c+d\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^3} - \frac{b^2\,\mathbf{i}\,\left(c+d\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{4\,\left(b\,c-a\,d\right)^3\,g^5\,\left(a+b\,x\right)^4} - \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2} + \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^3}{4\,\left(a+b\,x\right)^3} - \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2} - \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}\right)^2} - \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}\right)^2} + \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}\right)^2} - \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}\right)^2} + \frac{b^2\,\mathbf{i}\,\left(a+b\,x\right)^4}{4\,\left(a+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right$$

Result (type 4, 826 leaves, 74 steps):

$$-\frac{B^{2} \left(b \ c-a \ d\right) \ i}{32 \ b^{2} \ g^{5} \ \left(a+b \ x\right)^{4}} + \frac{5 \ B^{2} d \ i}{216 \ b^{2} \ g^{5} \ \left(a+b \ x\right)^{3}} + \frac{B^{2} d^{2} \ i}{144 \ b^{2} \left(b \ c-a \ d\right) \ g^{5} \left(a+b \ x\right)^{2}} - \frac{13 \ B^{2} d^{3} \ i}{72 \ b^{2} \left(b \ c-a \ d\right)^{2} \ g^{5} \left(a+b \ x\right)} - \frac{13 \ B^{2} d^{4} \ i \ Log \left[a+b \ x\right)}{72 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} + \frac{B^{2} d^{4} \ i \ Log \left[a+b \ x\right]^{3}}{12 \ b^{2} \left(b \ c-a \ d\right) \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{8 \ b^{2} \ g^{5} \left(a+b \ x\right)^{4}} - \frac{B \ d \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{18 \ b^{2} \ g^{5} \left(a+b \ x\right)^{3}} + \frac{B \ d^{2} \ i \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{12 \ b^{2} \left(b \ c-a \ d\right) \ g^{5} \left(a+b \ x\right)^{2}} - \frac{B \ d^{4} \ i \ Log \left[a+b \ x\right] \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B \ d^{4} \ i \ Log \left[a+b \ x\right] \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{12 \ b^{2} \left(a+b \ x\right)^{4}} - \frac{B \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(b \ c-a \ d\right)^{3} \ g^{5}} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left(a+b \ x\right)} - \frac{B^{2} \ d^{4} \ i \ Log \left[a+b \ x\right]}{12 \ b^{2} \left$$

### Problem 64: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^2\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 711 leaves, 17 steps):

$$\frac{3 B^{2} \left(b \, c-a \, d\right)^{5} g^{3} \, i^{2} \, x}{20 b^{2} d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{2} g^{3} \, i^{2} \left(a+b \, x\right)^{4}}{60 b^{3}} - \frac{3 B^{2} \left(b \, c-a \, d\right)^{4} g^{3} \, i^{2} \left(c+d \, x\right)^{2}}{40 b \, d^{4}} + \frac{B^{2} \left(b \, c-a \, d\right)^{3} g^{3} \, i^{2} \left(c+d \, x\right)^{3}}{60 \, d^{4}} - \frac{B \left(b \, c-a \, d\right)^{3} g^{3} \, i^{2} \left(a+b \, x\right)^{3} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{90 \, b^{3} \, d} - \frac{B \left(b \, c-a \, d\right)^{2} g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3}} - \frac{B \left(b \, c-a \, d\right)^{2} g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{15 \, b^{2}} + \frac{\left(b \, c-a \, d\right)^{2} g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{60 \, b^{3}} + \frac{\left(b \, c-a \, d\right)^{2} g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{60 \, b^{3}} + \frac{\left(b \, c-a \, d\right)^{3} g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{15 \, b^{2}} + \frac{g^{3} \, i^{2} \left(a+b \, x\right)^{4} \left(c+d \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{6 \, b} + \frac{B \left(b \, c-a \, d\right)^{4} g^{3} \, i^{2} \left(a+b \, x\right)^{2} \left(3A+B+3 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{180 \, b^{3} \, d^{3}} - \frac{B \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(6A+11 \, B+6 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3} \, d^{4}} - \frac{B^{2} \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(6A+11 \, B+6 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3} \, d^{4}} - \frac{B^{2} \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(6A+11 \, B+6 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3} \, d^{4}} - \frac{B^{2} \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(6A+11 \, B+6 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3} \, d^{4}} - \frac{B^{2} \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(6A+11 \, B+6 \, B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{20 \, b^{3} \, d^{4}} - \frac{B^{2} \left(b \, c-a \, d\right)^{6} g^{3} \, i^{2} \, Log\left[\frac$$

Result (type 4, 790 leaves, 86 steps):

$$-\frac{A\ B\ (b\ c-a\ d)^5\ g^3\ i^2\ x}{30\ b^2\ d^3} + \frac{B^2\ (b\ c-a\ d)^5\ g^3\ i^2\ x}{45\ b^2\ d^3} - \frac{7\ B^2\ (b\ c-a\ d)^4\ g^3\ i^2\ (a+b\ x)^2}{360\ b^3\ d^2} + \frac{B^2\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3}{60\ b^3\ d} + \frac{B^2\ (b\ c-a\ d)^2\ g^3\ i^2\ (a+b\ x)^4}{60\ b^3} - \frac{B^2\ (b\ c-a\ d)^5\ g^3\ i^2\ (a+b\ x)\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]}{60\ b^3\ d^3} + \frac{B\ (b\ c-a\ d)^4\ g^3\ i^2\ (a+b\ x)^2\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{60\ b^3\ d^2} - \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d} - \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d} - \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d} - \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d} - \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^3\ g^3\ i^2\ (a+b\ x)^3\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right])}{90\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]}{90\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c+d\ x\right]^2}{30\ b^3\ d^4} + \frac{B\ (b\ c-a\ d)^6\ g^3\ i^2\ Log\left[c$$

## Problem 65: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 761 leaves, 15 steps):

$$-\frac{B^{2} \left(b \, c-a \, d\right)^{4} \, g^{2} \, i^{2} \, x}{10 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} \, g^{2} \, i^{2} \left(c+d \, x\right)^{2}}{20 \, b \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{2} \, g^{2} \, i^{2} \left(c+d \, x\right)^{3}}{30 \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[\frac{a+b \, x}{c+d \, x}\right]}{30 \, b^{3} \, d^{3}} - \frac{B \left(b \, c-a \, d\right)^{3} \, g^{2} \, i^{2} \left(a+b \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{30 \, b^{3} \, d} - \frac{B \left(b \, c-a \, d\right)^{2} \, g^{2} \, i^{2} \left(a+b \, x\right)^{3} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{15 \, b^{3}} - \frac{B \left(b \, c-a \, d\right)^{3} \, g^{2} \, i^{2} \left(c+d \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{5 \, b \, d^{3}} + \frac{B \left(b \, c-a \, d\right)^{2} \, g^{2} \, i^{2} \left(c+d \, x\right)^{3} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{15 \, d^{3}} - \frac{b \, B \left(b \, c-a \, d\right) \, g^{2} \, i^{2} \left(c+d \, x\right)^{4} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{10 \, d^{3}} + \frac{10 \, d^{3}}{10 \, b^{3}} + \frac{\left(b \, c-a \, d\right)^{2} \, g^{2} \, i^{2} \left(a+b \, x\right)^{3} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{10 \, b^{3}} + \frac{B \left(b \, c-a \, d\right)^{4} \, g^{2} \, i^{2} \left(a+b \, x\right)^{3} \left(c+d \, x\right) \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{10 \, b^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5} \, g^{2} \, i^{2} \, Log \left[c+d \, x\right]}{10 \, b^{3} \, d^{3}} + \frac{B^{2} \left(b \, c-a \, d\right)^{5$$

#### Result (type 4, 666 leaves, 74 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)^4\,g^2\,i^2\,x}{15\,b^2\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i^2\,x}{15\,b^2\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,i^2\,\left(a+b\,x\right)^2}{20\,b^3\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^2\,i^2\,\left(a+b\,x\right)^3}{30\,b^3} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i^2\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{15\,b^3\,d^2} - \frac{B\,\left(b\,c-a\,d\right)^3\,g^2\,i^2\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{30\,b^3\,d} - \frac{B\,\left(b\,c-a\,d\right)^2\,g^2\,i^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{5\,b^3} - \frac{B\,d\,\left(b\,c-a\,d\right)^2\,g^2\,i^2\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b^3} + \frac{\left(b\,c-a\,d\right)^2\,g^2\,i^2\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{3\,b^3} + \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{15\,b^3\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,\left(a+b\,x\right)^5\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^2}{15\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,Log\left[c+d\,x\right]}{15\,b^3\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,Log\left[c+d\,x\right]^2}{30\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{15\,b^3\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,Log\left[c+d\,x\right]^2}{15\,b^3\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,Log\left[c+d\,x\right]^2}{30\,b^3\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{15\,b^3\,d^3} - \frac{B\,(b\,c-a\,d)^5\,g^2\,i^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{15\,b^3\,d^3} - \frac{B\,(b\,c-a\,d)^5\,g^2\,i^2\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,$$

# Problem 66: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 589 leaves, 14 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{3} g \ i^{2} x}{12 b^{2} d} + \frac{B^{2} \left(b \ c-a \ d\right)^{2} g \ i^{2} \left(c+d \ x\right)^{2}}{12 b^{2} d} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} Log \left[\frac{a+b \ x}{c+d \ x}\right]}{6 b^{3} d} - \frac{B \left(b \ c-a \ d\right)^{3} g \ i^{2} \left(a+b \ x\right) \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{6 b^{3} d} - \frac{B \left(b \ c-a \ d\right)^{2} g \ i^{2} \left(a+b \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{4 b d^{2}} - \frac{B \left(b \ c-a \ d\right)^{2} g \ i^{2} \left(c+d \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{12 b^{3}} - \frac{B \left(b \ c-a \ d\right)^{2} g \ i^{2} \left(a+b \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{12 b^{3}} + \frac{\left(b \ c-a \ d\right)^{2} g \ i^{2} \left(a+b \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{12 b^{3}} + \frac{\left(b \ c-a \ d\right)^{2} g \ i^{2} \left(a+b \ x\right)^{2} \left(A+B \ Log \left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{4 b} - \frac{B \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ Log \left[c+d \ x\right]}{6 b^{3} d^{2}} - \frac{B^{2} \left(b \ c-a \ d\right)^{4}$$

#### Result (type 4, 570 leaves, 46 steps):

$$\frac{A\,B\,\left(b\,c\,-a\,d\right)^{3}\,g\,i^{2}\,x}{6\,b^{2}\,d} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{3}\,g\,i^{2}\,x}{12\,b^{2}\,d} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{2}\,g\,i^{2}\,\left(c\,+d\,x\right)^{2}}{12\,b\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{12\,b^{3}\,d^{2}} - \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]^{2}}{12\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{12\,b^{3}\,d^{2}} - \frac{B\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d} + \frac{B\,\left(b\,c\,-a\,d\right)^{2}\,g\,i^{2}\,\left(c\,+d\,x\right)^{2}\,\left(A\,+B\,Log\left[\frac{e\,(a\,+b\,x)}{c\,+d\,x}\right]\right)}{12\,b\,d^{2}} - \frac{B\,\left(b\,c\,-a\,d\right)\,g\,i^{2}\,\left(c\,+d\,x\right)^{3}\,\left(A\,+B\,Log\left[\frac{e\,(a\,+b\,x)}{c\,+d\,x}\right]\right)}{6\,b^{3}\,d^{2}} + \frac{B\,g\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B\,g\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,Log\left[a\,+b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,PolyLog\left[a\,-b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,PolyLog\left[a\,-b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,PolyLog\left[a\,-b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c\,-a\,d\right)^{4}\,g\,i^{2}\,PolyLog\left[a\,-b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,a\,-a\,d\right)^{4}\,g\,i^{2}\,PolyLog\left[a\,-b\,x\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}$$

### Problem 67: Result valid but suboptimal antiderivative.

$$\int (c i + d i x)^{2} \left( A + B Log \left[ \frac{e (a + b x)}{c + d x} \right] \right)^{2} dx$$

Optimal (type 4, 334 leaves, 11 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} \ i^{2} \ x}{3 \ b^{2}} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ Log\left[\frac{a+b \ x}{c+d \ x}\right]}{3 \ b^{3} \ d} - \frac{2 \ B \left(b \ c-a \ d\right)^{2} \ i^{2} \left(a+b \ x\right) \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{3 \ b^{3}} - \frac{B \left(b \ c-a \ d\right)^{2} \ i^{2} \left(c+d \ x\right)^{3} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{3 \ d} - \frac{2 \ B \left(b \ c-a \ d\right)^{3} \ i^{2} \left(c+d \ x\right)^{3} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{3 \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ Log\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ Log\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ Log\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ PolyLog\left[2, \frac{b \ (c+d \ x)}{d \ (a+b \ x)}\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ a^{2} \ b^{2} \$$

#### Result (type 4, 420 leaves, 20 steps):

$$-\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,x}{3\,b^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,x}{3\,b^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,Log\,[\,a+b\,x\,]}{3\,b^{3}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,Log\,[\,a+b\,x\,]^{2}}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{3\,b^{3}} - \frac{B\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,b\,d} - \frac{2\,B\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,Log\,[\,a+b\,x\,]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{3\,d} + \frac{\mathbf{i}^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{3\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,Log\,[\,c+d\,x\,]}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,Log\,[\,a+b\,x\,]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{2}\,PolyLog\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right$$

# Problem 68: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^2 \left(\text{A} + \text{BLog}\left[\frac{\text{e}\cdot(\text{a}+\text{b}\cdot\text{x})}{\text{c+d}\cdot\text{x}}\right]\right)^2}{\text{ag+bgx}} \, dx$$

Optimal (type 4, 535 leaves, 15 steps):

$$-\frac{B \ d \ \left(b \ c - a \ d\right) \ i^{2} \ \left(a + b \ x\right) \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{2 \ B \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[\frac{b \ c - a \ d}{b \ (c + d \ x)}\right] \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{d \ \left(b \ c - a \ d\right) \ i^{2} \ \left(a + b \ x\right) \ \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{b^{3} \ g} + \frac{i^{2} \ \left(c + d \ x\right)^{2} \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^{2}}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2} \ Log\left[c + d \ x\right]}{b^{3} \ g} + \frac{B^{2} \ \left(b \ c - a \ d\right)^{2} \ i^{2}$$

Result (type 4, 1676 leaves, 86 steps):

# Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^2 \left(\text{A}+\text{BLog}\left[\frac{\text{e}\cdot(\text{a+bx})}{\text{c+dx}}\right]\right)^2}{\left(\text{ag+bgx}\right)^2} \, \mathrm{d}x$$

Optimal (type 4, 442 leaves, 11 steps):

$$-\frac{2 \, B^2 \, \left(b \, c - a \, d\right) \, i^2 \, \left(c + d \, x\right)}{b^2 \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, B \, \left(b \, c - a \, d\right) \, i^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^2 \, g^2 \, \left(a + b \, x\right)} + \frac{2 \, B \, d \, \left(b \, c - a \, d\right) \, i^2 \, Log\left[\frac{b \, c - a \, d}{b \, \left(c + d \, x\right)}\right] \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^3 \, g^2} + \frac{d^2 \, i^2 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{b^3 \, g^2} - \frac{\left(b \, c - a \, d\right) \, i^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{b^2 \, g^2 \, \left(a + b \, x\right)} - \frac{2 \, d \, \left(b \, c - a \, d\right) \, i^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2 \, Log\left[1 - \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^3 \, g^2} + \frac{2 \, B^2 \, d \, \left(b \, c - a \, d\right) \, i^2 \, PolyLog\left[2, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^3 \, g^2} + \frac{4 \, B^2 \, d \, \left(b \, c - a \, d\right) \, i^2 \, PolyLog\left[3, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^3 \, g^2} + \frac{4 \, B^2 \, d \, \left(b \, c - a \, d\right) \, i^2 \, PolyLog\left[3, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^3 \, g^2}$$

Result (type 4, 1219 leaves, 65 steps):

$$\frac{28^2 \left(bc - ad\right)^2 i^2}{b^3 g^2 \left(a + bx\right)} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]^2}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]^2}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]^2}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]^2}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[\frac{e \cdot (a + bx)}{c \cdot dx}]^2}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[\frac{e \cdot (a + bx)}{c \cdot dx}]^2}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} - \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(A + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(a + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \left(a + B \log[\frac{e \cdot (a + bx)}{c \cdot dx}]\right)}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[c + dx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[c + dx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx] \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx] \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx] \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc - ad\right) i^2 \log[a + bx]}{b^3 g^2} + \frac{28^2 d \left(bc -$$

## Problem 70: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A+BLog}\left[\frac{\text{e}\left(\text{a+bx}\right)}{\text{c+dx}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 4, 387 leaves, 10 steps):

$$-\frac{2\,B^{2}\,d\,\,i^{2}\,\left(c+d\,x\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{B^{2}\,\,i^{2}\,\left(c+d\,x\right)^{2}}{4\,b\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{2\,B\,d\,\,i^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{B\,i^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{d\,i^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{i^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{2\,b\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{d\,i^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{b^{2}\,g^{3}\,\left(a+b\,x\right)} - \frac{2\,B\,d^{2}\,i^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{3}\,g^{3}} + \frac{2\,B^{2}\,d^{2}\,i^{2}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{3}\,g^{3}} + \frac{2\,B^{2}\,d^{2}\,i^{2}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{$$

Result (type 4, 932 leaves, 73 steps):

$$\frac{B^{2} \left(b \ c - a \ d\right)^{2} i^{2}}{4 b^{3} g^{3} \left(a + b \ x\right)^{2}} - \frac{5 B^{2} d \left(b \ c - a \ d\right)^{2} i^{2}}{2 b^{3} g^{3} \left(a + b \ x\right)} - \frac{5 B^{2} d^{2} i^{2} \log \left[a + b \ x\right]}{2 b^{3} g^{3}} - \frac{A B d^{2} i^{2} \log \left[a + b \ x\right]^{2}}{b^{3} g^{3}} + \frac{3 B^{2} d^{2} i^{2} \log \left[a + b \ x\right]^{2}}{2 b^{3} g^{3}} - \frac{B^{2} d^{2} i^{2} \log \left[a + b \ x\right] \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]^{2}}{b^{3} g^{3}} - \frac{B^{2} d^{2} i^{2} \log \left[a + b \ x\right] \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]^{2}}{b^{3} g^{3}} - \frac{B \left(b \ c - a \ d\right)^{2} i^{2} \left(A + B \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]\right)}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3} \left(a + b \ x\right)^{2}} - \frac{3 B d^{2} i^{2} \log \left[a + b \ x\right] \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]}{b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3} \left(a + b \ x\right)^{2}} - \frac{3 B d^{2} i^{2} \log \left[a + b \ x\right] \left(A + B \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]\right)}{b^{3} g^{3}} - \frac{\left(b \ c - a \ d\right)^{2} i^{2} \left(A + B \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]\right)^{2}}{2 b^{3} g^{3} \left(a + b \ x\right)^{2}} - \frac{2 d \left(b \ c - a \ d\right)^{2} i^{2} \left(A + B \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]\right)^{2}}{2 b^{3} g^{3} \left(a + b \ x\right)^{2}} - \frac{2 d^{2} i^{2} \log \left[a + b \ x\right] \left(A + B \log \left[\frac{e \left(a + b \ x\right)}{c + d \ x}\right]\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^{3} \left(a + b \ x\right)^{2}}{2 b^{3} g^{3}} - \frac{2 b^{3} g^$$

Problem 71: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A} + \text{BLog}\left[\frac{\text{e} \cdot (\text{a} + \text{bx})}{\text{c+dx}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{4}} \, dx$$

Optimal (type 3, 147 leaves, 3 steps):

$$-\frac{2\,B^{2}\,i^{2}\,\left(c+d\,x\right)^{\,3}}{27\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{\,3}}\,-\frac{2\,B\,i^{2}\,\left(c+d\,x\right)^{\,3}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{9\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{\,3}}\,-\frac{i^{2}\,\left(c+d\,x\right)^{\,3}\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{\,2}}{3\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{\,3}}$$

Result (type 4, 827 leaves, 92 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, i^2}{27 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B^2 \, d \, \left( b \, c - a \, d \right) \, i^2}{9 \, b^3 \, g^4 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^2 \, i^2}{9 \, b^3 \, g^4 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ a + b \, x \right)}{9 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} + \frac{B^2 \, d^3 \, i^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B \, d \, \left( b \, c - a \, d \right) \, i^2 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B \, d \, \left( b \, c - a \, d \right) \, i^2 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^2} - \frac{2 \, B \, d^3 \, i^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^2} - \frac{2 \, B \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] - \frac{2 \, B^2 \, d^3 \, i^2 \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right]} - \frac{2 \, B^2 \, d^3 \, i^2 \,$$

Problem 72: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^2 \left(\text{A} + \text{BLog}\left[\frac{\text{e}\cdot(\text{a}+\text{b}\cdot\text{x})}{\text{c}+\text{d}\cdot\text{x}}\right]\right)^2}{\left(\text{ag+bgx}\right)^5} \, \text{dlx}$$

Optimal (type 3, 299 leaves, 7 steps):

$$\frac{2\,B^{2}\,d\,i^{2}\,\left(\,c\,+\,d\,x\,\right)^{\,3}}{27\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}\,-\,\frac{\,b\,B^{2}\,i^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}}{32\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}}\,+\,\frac{2\,B\,d\,i^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)}{9\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}\,-\,\frac{\,b\,B\,i^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}\,-\,\frac{\,b\,i^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{3\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}\,-\,\frac{\,b\,i^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\left(\,A\,+\,B\,Log\left[\,\frac{e\,\left(\,a\,+\,b\,\,x\,\right)}{c\,+\,d\,\,x}\,\right]\,\right)^{\,2}}{4\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}}$$

Result (type 4, 920 leaves, 104 steps):

$$-\frac{B^2 \left(b \ c - a \ d\right)^2 \ i^2}{32 \ b^3 \ g^5 \ \left(a + b \ x\right)^4} - \frac{11 \ B^2 \ d \left(b \ c - a \ d\right) \ i^2}{216 \ b^3 \ g^5 \ \left(a + b \ x\right)^3} + \frac{5 \ B^2 \ d^2 \ i^2}{144 \ b^3 \ g^5 \ \left(a + b \ x\right)^2} + \frac{7 \ B^2 \ d^3 \ i^2}{72 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)} + \frac{7 \ B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]}{72 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^3}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ Log \left[a + b \ x\right]^2}{12 \ b^3 \ \left(b \$$

# Problem 73: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a} + \mathsf{b}\,\mathbf{x}\right)}{\mathsf{c} + \mathsf{d}\,\mathbf{x}}\right]\right)^{2}}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^{6}}\,\mathrm{d}\mathbf{x}$$

Optimal (type 3, 463 leaves, 9 steps):

$$-\frac{2\,B^{2}\,d^{2}\,\mathbf{i}^{2}\,\left(\,c\,+\,d\,x\,\right)^{\,3}}{27\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{16\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{2\,b^{\,2}\,B^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}}{125\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{2\,B\,d^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)}{4\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} + \frac{b\,\,B\,d\,\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)}{4\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{2\,b^{\,2}\,B\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)}{25\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{d^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,3}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,4}} - \frac{b\,\,d\,\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,4}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}\,\left(\,A\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}\,\left(\,a\,+\,B\,\,Log\left[\frac{e\,\,(a+b\,x)}{c+d\,x}\right]\,\right)^{\,2}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)^{\,5}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}}{2\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,3}\,g^{\,6}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,5}}{2\,\left(\,a\,\,a\,\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,\,a\,\,b\,\,x\,\right)^{\,5}}{2\,\left(\,a\,\,a\,\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\left(\,a\,\,a\,\,b\,\,x\,\right)^{\,5}}{2\,\left(\,a\,\,a\,\,b\,\,x\,\right)^{\,5}} - \frac{b^{\,2}\,\mathbf{i}^{\,2}\,\mathbf{i}^{\,2}\,\mathbf{i}^{\,2}\,\mathbf{i}^{\,2}\,\mathbf{i}^{$$

Result (type 4. 1009 leaves, 116 steps):

$$-\frac{2 \, B^2 \, \left(b \, c - a \, d\right)^2 \, i^2}{125 \, b^3 \, g^6 \, \left(a + b \, x\right)^5} - \frac{7 \, B^2 \, d \, \left(b \, c - a \, d\right)^2}{400 \, b^3 \, g^6 \, \left(a + b \, x\right)^4} + \frac{43 \, B^2 \, d^2 \, i^2}{2700 \, b^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{13 \, B^2 \, d^3 \, i^2}{1800 \, b^3 \, \left(b \, c - a \, d\right) \, g^6 \, \left(a + b \, x\right)^2} - \frac{47 \, B^2 \, d^4 \, i^2}{900 \, b^3 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)} - \frac{47 \, B^2 \, d^5 \, i^2 \, Log \left[a + b \, x\right]}{900 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6} + \frac{B^2 \, d^5 \, i^2 \, Log \left[a + b \, x\right]^2}{30 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6} - \frac{2 \, B \, \left(b \, c - a \, d\right)^2 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{25 \, b^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{B \, d^2 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{45 \, b^3 \, g^6 \, \left(a + b \, x\right)^3} + \frac{B \, d^3 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{30 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{45 \, b^3 \, g^6 \, \left(a + b \, x\right)^3} + \frac{B \, d^3 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{30 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6 \, \left(a + b \, x\right)^3} - \frac{B \, d^4 \, i^2 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{15 \, b^3 \, \left(b \, c - a \, d\right)^3 \, g^6} - \frac{B \, d^5 \, i^2 \, Log \left[e \, d \, d \, x\right)}{15 \, b^3$$

# Problem 74: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,3}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\left[\,\frac{e\,\left(\,a+b\,x\right)}{c+d\,x}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 1089 leaves, 22 steps):

$$\frac{5B^{2} \left(bc-ad\right)^{6} g^{3} i^{3} \times }{84b^{3} d^{3}} + \frac{B^{2} \left(bc-ad\right)^{3} g^{3} i^{3} \left(a+bx\right)^{4}}{140b^{4}} - \frac{29B^{2} \left(bc-ad\right)^{5} g^{3} i^{3} \left(c+dx\right)^{2}}{840b^{2} d^{4}} + \frac{47B^{2} \left(bc-ad\right)^{4} g^{3} i^{3} \left(c+dx\right)^{3}}{1260b^{4}} - \frac{13B^{2} \left(bc-ad\right)^{3} g^{3} i^{3} \left(c+dx\right)^{4}}{420d^{4}} + \frac{bB^{2} \left(bc-ad\right)^{2} g^{3} i^{3} \left(c+dx\right)^{5}}{1050d^{4}} - \frac{1260b^{4}}{420d^{4}} - \frac{210b^{4} d^{4}}{210b^{4}} - \frac{120b^{4} d^{4}}{35b^{3}} - \frac{1260b^{4} d^{4}}{35b^{3}} -$$

Result (type 4, 896 leaves, 122 steps):

$$-\frac{A\ B\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ x}{70\ b^{3}\ d^{3}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ x}{70\ b^{3}\ d^{3}} - \frac{3\ B^{2}\ (b\ c-a\ d)^{5}\ g^{3}\ i^{3}\ (a+b\ x)^{2}}{280\ b^{4}\ d^{2}} + \frac{11\ B^{2}\ (b\ c-a\ d)^{4}\ g^{3}\ i^{3}\ (a+b\ x)^{3}}{1260\ b^{4}\ d} + \frac{B^{2}\ (b\ c-a\ d)^{3}\ g^{3}\ i^{3}\ (a+b\ x)^{4}}{42\ b^{4}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ (a+b\ x)^{5}}{1260\ b^{4}\ d} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ (a+b\ x)^{5}}{120\ b^{4}\ d^{3}} - \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ (a+b\ x)\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]}{120\ b^{4}\ d^{3}} - \frac{B\ (b\ c-a\ d)^{5}\ g^{3}\ i^{3}\ (a+b\ x)^{5}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d} - \frac{17\ B\ (b\ c-a\ d)^{3}\ g^{3}\ i^{3}\ (a+b\ x)^{4}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d} - \frac{B\ d^{2}\ (b\ c-a\ d)^{3}\ g^{3}\ i^{3}\ (a+b\ x)^{6}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d} - \frac{B\ d^{2}\ (b\ c-a\ d)^{3}\ g^{3}\ i^{3}\ (a+b\ x)^{6}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d} + \frac{B\ d^{2}\ (b\ c-a\ d)^{2}\ g^{3}\ i^{3}\ (a+b\ x)^{6}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d} + \frac{B\ d^{2}\ (b\ c-a\ d)^{2}\ g^{3}\ i^{3}\ (a+b\ x)^{6}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d^{4}} + \frac{B\ d^{2}\ (b\ c-a\ d)^{2}\ g^{3}\ i^{3}\ (a+b\ x)^{6}\ (A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{120\ b^{4}\ d^{4}\ d^{$$

# Problem 75: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\,\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 908 leaves, 20 steps):

$$\frac{7\,B^2\,\left(b\,c-a\,d\right)^5\,g^2\,i^3\,x}{180\,b^3\,d^2} - \frac{7\,B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)^2}{360\,b^2\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,i^3\,\left(c+d\,x\right)^3}{60\,b\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^6\,g^2\,i^3\,\log\left[\frac{a+b\,x}{c+d\,x}\right]}{36\,b^4\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)^2\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{60\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)^2\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{60\,d^3} - \frac{B\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)^2\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{30\,b^4} - \frac{B\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)^2\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b^2\,d^3} + \frac{B\,\left(b\,c-a\,d\right)^3\,g^2\,i^3\,\left(c+d\,x\right)^3\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{60\,d^3} + \frac{7\,B\,\left(b\,c-a\,d\right)^2\,g^2\,i^3\,\left(c+d\,x\right)^4\,\left(A+B\,\log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{60\,d^3} - \frac{60\,d^3}{60\,d^3} + \frac{10\,b^2\,d^3}{60\,d^3} + \frac{10\,b^2\,d^3\,d^3}{60\,d^3} + \frac{10\,b^2\,d^3\,d^3}{60\,d^3} + \frac{10\,b^2\,d^3\,d^3}{60\,d^3} + \frac{10\,b^2\,d^3\,d^3}{6$$

Result (type 4, 825 leaves, 86 steps):

$$\frac{A B \left(b c - a d\right)^{5} g^{2} i^{3} x}{30 b^{3} d^{2}} - \frac{B^{2} \left(b c - a d\right)^{5} g^{2} i^{3} x}{45 b^{3} d^{2}} - \frac{7 B^{2} \left(b c - a d\right)^{4} g^{2} i^{3} \left(c + d x\right)^{2}}{360 b^{2} d^{3}} - \frac{B^{2} \left(b c - a d\right)^{3} g^{2} i^{3} \left(c + d x\right)^{3}}{60 b d^{3}} + \frac{B^{2} \left(b c - a d\right)^{6} g^{2} i^{3} \left(c + d x\right)^{4}}{60 d^{3}} - \frac{B^{2} \left(b c - a d\right)^{6} g^{2} i^{3} \left(b c + d x\right)^{2}}{60 b^{4} d^{3}} - \frac{B^{2} \left(b c - a d\right)^{5} g^{2} i^{3} \left(a + b x\right) Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]}{30 b^{4} d^{2}} - \frac{B^{2} \left(b c - a d\right)^{5} g^{2} i^{3} \left(a + b x\right) Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]}{30 b^{4} d^{2}} - \frac{B^{2} \left(b c - a d\right)^{6} g^{2} i^{3} \left(c + d x\right)^{2} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{90 b d^{3}} - \frac{B \left(b c - a d\right)^{2} g^{2} i^{3} \left(c + d x\right)^{3} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{90 b d^{3}} - \frac{b B \left(b c - a d\right)^{2} g^{2} i^{3} \left(c + d x\right)^{5} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{15 d^{3}} - \frac{B \left(b c - a d\right)^{6} g^{2} i^{3} Log\left[a + b x\right] \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{4 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{4} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{4 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{4} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{4} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{4 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{4} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(A + B Log\left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^{2}}{6 d^{3}} - \frac{b^{2} g^{2} i^{3} \left(c + d x\right)^{6} \left(a - a d\right)^{6} g^{2} i^{3} log\left[a + b x\right] \left(a - a d\right)^{6} g^{2} i^{3} log\left[a + b x\right] log\left[\frac{e \left(a + b x\right)}{c + d x}\right] log\left[\frac$$

### Problem 76: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^3\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)}{c+d\,x}\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 730 leaves, 19 steps):

$$\frac{B^{2} \left(b \, c-a \, d\right)^{4} g \, i^{3} \, x}{60 \, b^{3} \, d} + \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i^{3} \left(c+d \, x\right)^{2}}{30 \, b^{2} \, d^{2}} + \frac{B^{2} \left(b \, c-a \, d\right)^{2} g \, i^{3} \left(c+d \, x\right)^{3}}{30 \, b \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{5} g \, i^{3} \, Log \left[\frac{a+b \, x}{c+d \, x}\right]}{12 \, b^{4} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{4} g \, i^{3} \left(a+b \, x\right) \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{10 \, b^{4}} - \frac{10 \, b^{4} \, d}{2} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i^{3} \left(a+b \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{10 \, b^{4}} + \frac{3 \, B \left(b \, c-a \, d\right)^{3} g \, i^{3} \left(c+d \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{10 \, b^{4}} + \frac{10 \, d^{2}}{20 \, b^{2}} + \frac{10 \, b^{3}}{20 \, b^{2}} + \frac{10 \, b^{2}}{20 \, b^{2}} + \frac{10 \, b^{2}$$

#### Result (type 4, 655 leaves, 54 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)^{4}\,g\,i^{3}\,x}{10\,b^{3}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{3}\,x}{60\,b^{3}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{2}}{30\,b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{3}}{30\,b^{2}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[a+b\,x\right]}{60\,b^{4}\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{10\,b^{4}\,d} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{20\,b^{2}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b^{4}\,d} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{20\,b^{2}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g\,i^{3}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{10\,b^{4}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{5\,d^{2}} - \frac{\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[a+b\,x\right]\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{4\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[c+d\,x\right)^{5}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{5\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[c+d\,x\right]}{10\,b^{4}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{10\,b^{4}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,Log\left[a+b\,x\right]}{10\,b^{4}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g\,i^{3}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{10\,b^{4}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^$$

### Problem 77: Result valid but suboptimal antiderivative.

$$\int \left( c \, \operatorname{\textbf{i}} + d \, \operatorname{\textbf{i}} \, x \right)^3 \, \left( A + B \, Log \left[ \, \frac{e \, \left( a + b \, x \right)}{c + d \, x} \, \right] \, \right)^2 \, \mathrm{d} x$$

Optimal (type 4, 420 leaves, 15 steps):

$$\frac{5 \, B^{2} \, \left(b \, c - a \, d\right)^{3} \, i^{3} \, x}{12 \, b^{3}} + \frac{B^{2} \, \left(b \, c - a \, d\right)^{2} \, i^{3} \, \left(c + d \, x\right)^{2}}{12 \, b^{2} \, d} + \frac{5 \, B^{2} \, \left(b \, c - a \, d\right)^{4} \, i^{3} \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{12 \, b^{4} \, d} - \frac{B \, \left(b \, c - a \, d\right)^{3} \, i^{3} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4}} - \frac{B \, \left(b \, c - a \, d\right)^{2} \, i^{3} \, \left(c + d \, x\right)^{2} \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, b^{2} \, d} + \frac{B \, \left(b \, c - a \, d\right)^{4} \, i^{3} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, d} + \frac{11 \, B^{2} \, \left(b \, c - a \, d\right)^{4} \, i^{3} \, Log\left[c + d \, x\right]}{2 \, b^{4} \, d} + \frac{B \, \left(b \, c - a \, d\right)^{4} \, i^{3} \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, b^{4} \, d} - \frac{B^{2} \, \left(b \, c - a \, d\right)^{4} \, i^{3} \, PolyLog\left[2, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{2 \, b^{4} \, d} + \frac{12 \, b^{4} \, d}{2 \, b^{4} \, d} +$$

#### Result (type 4, 503 leaves, 24 steps):

$$-\frac{A\ B\ \left(b\ c-a\ d\right)^{3}\ i^{3}\ x}{2\ b^{3}} + \frac{5\ B^{2}\ \left(b\ c-a\ d\right)^{3}\ i^{3}\ x}{12\ b^{3}} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{2}\ i^{3}\ \left(c+d\ x\right)^{2}}{12\ b^{2}\ d} + \frac{5\ B^{2}\ \left(b\ c-a\ d\right)^{4}\ i^{3}\ Log\left[a+b\ x\right]}{12\ b^{4}\ d} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ i^{3}\ x}{12\ b^{4}\ d} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{3}\ i^{3}\ \left(a+b\ x\right)\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]}{2\ b^{4}\ d} - \frac{B\ \left(b\ c-a\ d\right)^{2}\ i^{3}\ \left(c+d\ x\right)^{2}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ b^{2}\ d} - \frac{B\ \left(b\ c-a\ d\right)^{4}\ i^{3}\ Log\left[a+b\ x\right]\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{2\ b^{4}\ d} + \frac{i^{3}\ \left(c+d\ x\right)^{4}\ \left(A+B\ Log\left[\frac{e\ (a+b\ x)}{c+d\ x}\right]\right)}{4\ d} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ i^{3}\ Log\left[a+b\ x\right]\ Log\left[\frac{b\ (c+d\ x)}{b\ c-a\ d}\right]}{2\ b^{4}\ d} + \frac{B^{2}\ \left(b\ c-a\ d\right)^{4}\ i^{3}\ PolyLog\left[2\ ,\ -\frac{d\ (a+b\ x)}{b\ c-a\ d}\right]}{2\ b^{4}\ d}$$

# Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\operatorname{ci} + \operatorname{dix}\right)^{3} \left(\operatorname{A} + \operatorname{B}\operatorname{Log}\left[\frac{\operatorname{e}\left(\operatorname{a} + \operatorname{b} x\right)}{\operatorname{c} + \operatorname{d} x}\right]\right)^{2}}{\operatorname{a} \operatorname{g} + \operatorname{b} \operatorname{g} x} \, \mathrm{d} x$$

Optimal (type 4, 712 leaves, 26 steps):

$$\frac{B^2 d \left(b \, c - a \, d\right)^2 i^3 \, x}{3 \, b^3 \, g} + \frac{B^2 \left(b \, c - a \, d\right)^3 i^3 \, Log \left[\frac{a + b \, x}{c + d \, x}\right]}{3 \, b^4 \, g} - \frac{5 \, B \, d \left(b \, c - a \, d\right)^2 i^3 \, \left(a + b \, x\right) \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^4 \, g} - \frac{B \left(b \, c - a \, d\right)^3 i^3 \, Log \left[\frac{b \, c - a \, d}{b \, (c + d \, x)}\right] \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{3 \, b^2 \, g} + \frac{2 \, B \left(b \, c - a \, d\right)^3 i^3 \, Log \left[\frac{b \, c - a \, d}{b \, (c + d \, x)}\right] \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{b^4 \, g} + \frac{d \left(b \, c - a \, d\right)^3 i^3 \, Log \left[\frac{b \, c - a \, d}{b \, (c + d \, x)^2} \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)\right)}{2 \, b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, Log \left[c + d \, x\right]}{2 \, b^2 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, Log \left[c + d \, x\right]}{b^4 \, g} + \frac{5 \, B \left(b \, c - a \, d\right)^3 i^3 \, \left(A + B \, Log \left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log \left[1 - \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{3 \, b^4 \, g} + \frac{3 \, b^4 \, g}{3 \, b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{3 \, b^4 \, g} + \frac{3 \, b^4 \, g}{3 \, b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{3 \, b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{3 \, b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2 \left(b \, c - a \, d\right)^3 i^3 \, PolyLog \left[3, \, \frac{d \, (a + b \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{2 \, B^2$$

Result (type 4, 1868 leaves, 106 steps):

## Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{3}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\frac{e\,\left(\mathsf{a} + \mathsf{b}\,\mathbf{x}\right)}{c + d\,\mathbf{x}}\right]\right)^{2}}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^{2}}\,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 692 leaves, 17 steps):

Result (type 4, 1751 leaves, 90 steps):

$$\frac{\text{AB } d^3 \left( \text{bc} - \text{ad} \right) \frac{12}{\text{b}^2} \left( 2 \left( \text{bc} - \text{ad} \right)^2 \frac{12}{\text{b}^2} \left( \text{bc} - \text{ad} \right)^2 \frac{12}{\text{b}^2} \left( 2 \left( \text{bc} - \text{ad} \right)^2 \frac{12}{\text{b}^2} \left( \text{bc} - \text{ad} \right)$$

## Problem 80: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{3}\,\left(A + B\,\mathsf{Log}\left[\frac{e\,(a + b\,\mathbf{x})}{c + d\,\mathbf{x}}\right]\right)^{2}}{\left(a\,g + b\,g\,\mathbf{x}\right)^{3}}\,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 604 leaves, 13 steps):

$$\frac{4\,B^{2}\,d\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)} - \frac{B^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)^{2}}{4\,b^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{4\,B\,d\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)} - \frac{B\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{4}\,g^{3}} + \frac{2\,B\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{b^{4}\,g^{3}} + \frac{2\,d\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{b^{4}\,g^{3}} - \frac{\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{b^{4}\,g^{3}} - \frac{\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{b^{4}\,g^{3}} + \frac{2\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{b^{4}\,g^{3}} + \frac{2\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{b\,(c+d\,x)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{b\,(a+b\,x)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,$$

Result (type 4, 1412 leaves, 95 steps):

$$\frac{B^2 \left( b \, C - a \, d \right)^3 \, i^3}{4b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{9 \, B^2 \, d \, \left( b \, C - a \, d \right)^2 \, i^3}{2b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{9 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a + b \, x \right]^2}{2b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a + b \, x \right]^2}{b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a + b \, x \right]^2}{2b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a - b \, x \right]^2}{2b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a - b \, x \right]^2}{2b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a - b \, x \right]^2}{2b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a - b \, x \right]^2}{b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a - b \, x \right]^2}{2b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3 \, \left( a + b \, x \right)^2} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^3 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^3 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a + b \, x \right] \, \left( a + B \, \log \left[ \frac{e \, \left( a + b \, x \right)}{c + d \, x} \right] \right)}{b^4 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log \left[ a + b \, x \right] \, \log \left[ c + d \, x \right]}{b^2 \, g^3} = \frac{3 \, B^2 \, d^2 \, \left( b \, C - a \, d \right)^2 \, i^3 \, \log$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{3} \left(\text{A+BLog}\left[\frac{\text{e}\left(\text{a+bx}\right)}{\text{c+dx}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

Optimal (type 3, 147 leaves, 3 steps):

$$-\frac{B^{2} \, \mathbf{i}^{3} \, \left(c+d\,x\right)^{4}}{32 \, \left(b\,c-a\,d\right) \, g^{5} \, \left(a+b\,x\right)^{4}} - \frac{B \, \mathbf{i}^{3} \, \left(c+d\,x\right)^{4} \, \left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{8 \, \left(b\,c-a\,d\right) \, g^{5} \, \left(a+b\,x\right)^{4}} - \frac{\mathbf{i}^{3} \, \left(c+d\,x\right)^{4} \, \left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{4 \, \left(b\,c-a\,d\right) \, g^{5} \, \left(a+b\,x\right)^{4}}$$

Result (type 4, 970 leaves, 130 steps):

$$\frac{-B^2 \left( b \ c - a \ d \right)^3 \ i^3}{32 \ b^4 \ g^5 \ (a + b \ x )^4} - \frac{B^2 \ d \left( b \ c - a \ d \right)^2 \ i^3}{8 \ b^4 \ g^5 \ (a + b \ x )^3} - \frac{3B^2 \ d^2 \left( b \ c - a \ d \right) \ i^3}{16 \ b^4 \ g^5 \ (a + b \ x )^2} - \frac{B^2 \ d^3 \ i^3}{8 \ b^4 \ g^5 \ (a + b \ x )} - \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right]}{8 \ b^4 \ \left( b \ c - a \ d \right) \ g^5} + \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right)^2}{4 \ b^4 \ \left( b \ c - a \ d \right)^3 \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)} - \frac{B \ d \ \left( b \ c - a \ d \right)^2 \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d \ \left( b \ c - a \ d \right)^2 \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d^3 \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( b \ c - a \ d \right) \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)^2}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( a + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)^2}{2 \ b^4 \ g^5 \ \left( a + b \ x \right)^3} - \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ \left( a + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)^2}{2 \ b^4 \ \left( b \ c - a \ d \right) \ g^5} - \frac{B^2 \ d^4 \ i^3 \ Log \left[ a + b \ x \right] \ Log \left[ a + b \ x \right] \ \left( a + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right) \ Log \left[ a + b \ x \right] \ \left( a + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right) \ Log \left[ a + b \$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{3} \, \left(A + B \, \mathsf{Log}\left[\frac{e \, (a + b \, \mathbf{x})}{c + d \, \mathbf{x}}\right]\right)^{2}}{\left(a \, g + b \, g \, \mathbf{x}\right)^{6}} \, \mathrm{d}\mathbf{x}$$

Optimal (type 3, 299 leaves, 7 steps):

$$\frac{B^2 \, d \, \mathbf{i}^3 \, \left(c + d \, x\right)^4}{32 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^4} - \frac{2 \, b \, B^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)^5}{125 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^5} + \frac{B \, d \, \mathbf{i}^3 \, \left(c + d \, x\right)^4 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{8 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^4} - \frac{2 \, b \, B \, \mathbf{i}^3 \, \left(c + d \, x\right)^5 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{4 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^4} - \frac{b \, \mathbf{i}^3 \, \left(c + d \, x\right)^5 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{4 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^4} - \frac{b \, \mathbf{i}^3 \, \left(c + d \, x\right)^5 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{5 \, \left(b \, c - a \, d\right)^2 \, g^6 \, \left(a + b \, x\right)^5}$$

Result (type 4, 1061 leaves, 146 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right)^3 \, i^3}{125 \, b^4 \, g^6 \, \left( a + b \, x \right)^5} - \frac{39 \, B^2 \, d \, \left( b \, c - a \, d \right)^2 \, i^3}{800 \, b^4 \, g^6 \, \left( a + b \, x \right)^3} - \frac{7 \, B^2 \, d^2 \, \left( b \, c - a \, d \right) \, i^3}{200 \, b^4 \, g^6 \, \left( a + b \, x \right)^2} + \frac{11 \, B^2 \, d^3 \, i^3}{200 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6} + \frac{200 \, b^4 \, \left( b \, c - a \, d \right)^2 \, i^3}{200 \, b^4 \, \left( b \, c - a \, d \right)^3 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)} + \frac{11 \, B \, d \, b^2 \, b^2 \, \left( b \, c - a \, d \right)^2 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{200 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6} + \frac{20 \, b^4 \, \left( b \, c - a \, d \right)^3 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{200 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6} + \frac{25 \, b^4 \, g^6 \, \left( a + b \, x \right)^5}{200 \, b^4 \, \left( a + b \, x \right)^3} + \frac{B \, d^4 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{10 \, b^4 \, g^6 \, \left( a + b \, x \right)^3} + \frac{B \, d^3 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{20 \, b^4 \, g^6 \, \left( a + b \, x \right)^2} + \frac{B \, d^4 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{10 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6} + \frac{B \, d^4 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{10 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6 \, \left( a + b \, x \right)^3} + \frac{B \, d^4 \, i^3 \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{10 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6 \, \left( a + b \, x \right)^3} + \frac{B \, d^5 \, i^3 \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ \frac{e \, (a + b \, x)}{c + d \, x} \right] \right)}{10 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6 \, \left( a + b \, x \right)^3} + \frac{B \, d^5 \, i^3 \, Log \left[ a + b \, x \right] \, d^2 \, \left( b \, c - a \, d \right)^2 \, g^6}{10 \, b^4 \, \left( b \, c - a \, d \right)^2 \, g^6 \, \left( a + b \, x \right)^3}$$

## Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \, \left(\text{A}+\text{BLog}\left[\frac{\text{e}\, (\text{a+b}\, x)}{\text{c+d}\, x}\right]\right)^2}{\left(\text{ag+bg}\, x\right)^7} \, \mathrm{d}x$$

Optimal (type 3, 463 leaves, 9 steps):

$$-\frac{B^2 \ d^2 \ i^3 \ \left(c + d \ x\right)^4}{32 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^4} + \frac{4 \ b \ B^2 \ d \ i^3 \ \left(c + d \ x\right)^5}{125 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^5} - \frac{b^2 \ B^2 \ i^3 \ \left(c + d \ x\right)^6}{108 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^6} - \frac{B \ d^2 \ i^3 \ \left(c + d \ x\right)^4 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{25 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^5} - \frac{b^2 \ B \ i^3 \ \left(c + d \ x\right)^6 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{18 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^6} - \frac{d^2 \ i^3 \ \left(c + d \ x\right)^6 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{25 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^5} - \frac{b^2 \ B \ i^3 \ \left(c + d \ x\right)^6 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{18 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^6} - \frac{d^2 \ i^3 \ \left(c + d \ x\right)^6 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{5 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^5} - \frac{b^2 \ i^3 \ \left(c + d \ x\right)^6 \left(A + B \ Log\left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)^2}{6 \ \left(b \ c - a \ d\right)^3 \ g^7 \ \left(a + b \ x\right)^6}$$

Result (type 4, 1152 leaves, 162 steps):

$$\frac{B^2 \left( b \ c - a \ d \right)^3 \ i^3}{108 \ b^4 \ g^7 \ \left( a + b \ x \right)^6} - \frac{53 \ B^2 \ d \left( b \ c - a \ d \right)^2 \ i^3}{2250 \ b^4 \ g^7 \ \left( a + b \ x \right)^5} - \frac{73 \ B^2 \ d^2 \left( b \ c - a \ d \right) \ i^3}{7200 \ b^4 \ g^7 \ \left( a + b \ x \right)^4} + \frac{53 \ B^2 \ d^3 \ i^3}{5400 \ b^4 \ g^7 \ \left( a + b \ x \right)^3} - \frac{23 \ B^2 \ d^4 \ i^3}{3600 \ b^4 \ \left( b \ c - a \ d \right) \ g^7 \ \left( a + b \ x \right)^2} - \frac{37 \ B^2 \ d^6 \ i^3 \ Log \left[ a + b \ x \right]^4}{1800 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} + \frac{B^2 \ d^6 \ i^3 \ Log \left[ a + b \ x \right]^2}{60 \ b^4 \ \left( b \ c - a \ d \right)^3 \ i^3 \ \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)} - \frac{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6}{1800 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6}{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6} - \frac{13 \ B \ d^4 \ \left( b \ c - a \ d \right)^3 \ g^7}{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6} - \frac{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6}{18 \ b^4 \ g^7 \ \left( a + b \ x \right)^6} - \frac{13 \ B \ d^4 \ g^7 \ \left( a + b \ x \right)^5}{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4} - \frac{19 \ B \ d^2 \ \left( b \ c - a \ d \right) \ i^3 \left( A + B \ Log \left[ \frac{e \ (a + b \ x)}{c + d \ x} \right] \right)}{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4}{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4} - \frac{90 \ b^4 \ g^7 \ \left( a + b \ x \right)^3}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4}{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^3} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4}{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^3} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^3}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^4}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^3}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ g^7 \ \left( a + b \ x \right)^3}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7}{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7} - \frac{120 \ b^4 \ \left( b \ c - a \ d \right)^3 \ g^7}{120 \ b^4 \ \left( b \ c - a \ d$$

## Problem 84: Result valid but suboptimal antiderivative.

$$\int \frac{\left(ag + bgx\right)^3 \left(A + B Log\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{ci + dix} dx$$

Optimal (type 4, 718 leaves, 25 steps):

$$\frac{b B^2 \left(b C - a d\right)^2 g^3 x}{3 d^3 i} + \frac{B^2 \left(b C - a d\right)^3 g^3 Log \left[\frac{a + b x}{c + d x}\right]}{3 d^4 i} + \frac{7 B \left(b C - a d\right)^2 g^3 \left(a + b x\right) \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{3 d^3 i} - \frac{b^2 B \left(b C - a d\right) g^3 \left(C + d x\right)^2 \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{3 d^4 i} + \frac{6 B \left(b C - a d\right)^3 g^3 Log \left[\frac{b C - a d}{b \left(c + d x\right)}\right] \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{d^4 i} + \frac{3 \left(b C - a d\right)^2 g^3 \left(a + b x\right) \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{d^3 i} - \frac{3 b^2 \left(b C - a d\right) g^3 \left(C + d x\right)^2 \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)}{2 d^4 i} + \frac{b^3 g^3 \left(C + d x\right)^3 \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^2}{3 d^4 i} + \frac{\left(b C - a d\right)^3 g^3 Log \left[\frac{b C - a d}{b \left(c - d x\right)}\right] \left(A + B Log \left[\frac{e \left(a + b x\right)}{c + d x}\right]\right)^2}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 Log \left[C + d x\right]}{d^4 i} + \frac{2 B \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{2 B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{B^2 \left(b C - a d\right)^3 g^3 PolyLog \left[2, \frac{d \left(a + b x\right)}{b \left(c + d x\right)}\right]}{d^4 i} - \frac{B^2 \left$$

Result (type 4, 1828 leaves, 106 steps):

## Problem 85: Result valid but suboptimal antiderivative.

$$\left(\frac{\left(a g + b g x\right)^{2} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{c i + d i x}\right) dx$$

Optimal (type 4, 536 leaves, 15 steps):

$$\frac{B \left(b \, c - a \, d\right) \, g^{2} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^{2} \, i} - \frac{4 \, B \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, Log\left[\frac{b \, c - a \, d}{b \, (c + d \, x)}\right] \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{d^{3} \, i} - \frac{2 \, \left(b \, c - a \, d\right) \, g^{2} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^{2}}{d^{3} \, i} + \frac{b^{2} \, g^{2} \, \left(c + d \, x\right)^{2} \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^{2}}{2 \, d^{3} \, i} - \frac{\left(b \, c - a \, d\right)^{2} \, g^{2} \, Log\left[\frac{b \, c - a \, d}{b \, (c + d \, x)}\right] \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^{2}}{d^{3} \, i} + \frac{B \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right) \, Log\left[1 - \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{d^{3} \, i} - \frac{4 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} - \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{2} \, g^{2} \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right)^{$$

Result (type 4, 1666 leaves, 86 steps):

$$\frac{A b B \left[b c - a d\right] g^2 x}{d^2 i} = \frac{a B^2 \left\{b c - a d\right] g^2 \log \left[a + b x\right]^2}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a + b x\right] \left[a - d\right]^2}{d^3 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]^2}{d^3 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^3 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[c - d x\right]^2}{d^2 i} = \frac{B^2 \left\{b c - a d\right]^2 g^2 \log \left[a - b x\right]^2 \log \left[a - b x\right]^2$$

# Problem 86: Result valid but suboptimal antiderivative.

$$\left(\frac{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{c i + d i x}\right)^{2} dx$$

Optimal (type 4, 283 leaves, 9 steps):

$$\frac{2\,B\,\left(b\,c-a\,d\right)\,g\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d^{2}\,\mathbf{i}} + \frac{g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{d\,\mathbf{i}} + \frac{\left(b\,c-a\,d\right)\,g\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{d^{2}\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d^{2}\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)}{d^{2}\,\mathbf{i}} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,g\,PolyLog\left[3,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,PolyLog\left[3,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}} + \frac{2\,B\,\left(a+b\,x\right)\,g\,\left(a+b\,$$

Result (type 4, 1072 leaves, 68 steps):

$$\frac{a \, B^2 \, g \, Log \left[ a + b \, x \right]^2}{di} + \frac{B^2 \, \left( b \, c - a \, d \right) \, g \, Log \left[ a + b \, x \right] \, Log \left[ \frac{1}{c + d \, x} \right]^2}{d^2i} - \frac{B^2 \, \left( b \, c - a \, d \right) \, g \, Log \left[ - \frac{d \, \left( a \, b \, b \, x \right)}{b \, c + d \, x} \right]^2}{d^2i} + \frac{1}{b \, c - a \, d^2i} + \frac{1}{b$$

### Problem 87: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{c i + d i x} dx$$

Optimal (type 4, 127 leaves, 4 steps):

$$-\frac{\text{Log}\big[\frac{\text{b}\,\text{c}-\text{a}\,\text{d}}{\text{b}\,\,(\text{c}+\text{d}\,\text{x})}\big]\,\,\Big(\text{A}+\text{B}\,\text{Log}\big[\frac{\text{e}\,\,(\text{a}+\text{b}\,\text{x})}{\text{c}+\text{d}\,\text{x}}\big]\Big)^2}{\text{d}\,\text{i}} -\frac{2\,\text{B}\,\,\Big(\text{A}+\text{B}\,\text{Log}\big[\frac{\text{e}\,\,(\text{a}+\text{b}\,\text{x})}{\text{c}+\text{d}\,\text{x}}\big]\Big)\,\,\text{PolyLog}\big[2\,\text{,}\,\,\frac{\text{d}\,\,(\text{a}+\text{b}\,\text{x})}{\text{b}\,\,(\text{c}+\text{d}\,\text{x})}\big]}{\text{d}\,\,\text{i}} +\frac{2\,\text{B}^2\,\,\text{PolyLog}\big[3\,\text{,}\,\,\frac{\text{d}\,\,(\text{a}+\text{b}\,\text{x})}{\text{b}\,\,(\text{c}+\text{d}\,\text{x})}\big]}{\text{d}\,\,\text{i}}$$

Result (type 4, 721 leaves, 46 steps):

$$\frac{B^2 \log \left[ a + b \, x \right] \, \log \left[ \frac{1}{c + d \, x} \right]^2}{d \, i} + \frac{B^2 \log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \log \left[ \frac{1}{c + d \, x} \right]^2}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]^2 \log \left[ \frac{b \, (c + d \, x)}{b \, c - a \, d} \right]}{d \, i} - \frac{B^2 \log \left[ a + b \, x \right] \, 2 \log \left[ a + b \, x \right]^2 \log \left[ a + b \, x \right]^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] - \frac{B^2 \log \left[ a + b \, x \right]^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right] \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x \right]}{d \, i} + \frac{B^2 \log \left[ a + b \, x$$

Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]\right)^{2}}{\left(a \cdot g + b \cdot g \cdot x\right) \cdot \left(c \cdot i + d \cdot i \cdot x\right)} \, dx$$

Optimal (type 3, 44 leaves, 3 steps):

$$\frac{\left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\,\frac{\mathsf{e}\,\,(\mathsf{a} + \mathsf{b}\,\mathsf{x})}{\mathsf{c} + \mathsf{d}\,\mathsf{x}}\,\right]\,\right)^{\,3}}{\mathsf{3}\,\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c} - \mathsf{a}\,\mathsf{d}\right)\,\mathsf{g}\,\mathsf{i}}$$

Result (type 4, 1163 leaves, 61 steps):

$$\frac{A\,B\,Log\,[\,a + b\,x\,]^{\,2}}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,[\,a + b\,x\,]\,Log\,\left[\,\frac{1}{c + d\,x}\,\right]^{\,2}}{\left(b\,c - a\,d\right)\,g\,i} - \frac{B^{2}\,Log\,\left[\,-\frac{d\,(a + b\,x)}{b\,c - a\,d}\,\right]\,Log\,\left[\,\frac{1}{c + d\,x}\,\right]^{\,2}}{\left(b\,c - a\,d\right)\,g\,i} - \frac{B^{2}\,Log\,[\,a + b\,x\,]\,Log\,\left[\,\frac{e\,(a + b\,x)}{c + d\,x}\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{Log\,[\,a + b\,x\,]\,Log\,\left[\,\frac{e\,(a + b\,x)}{c + d\,x}\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,[\,a + b\,x\,]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{2\,A\,B\,Log\,\left[\,-\frac{d\,(a + b\,x)}{b\,c - a\,d}\,\right]\,Log\,[\,c + d\,x\,]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,[\,a + b\,x\,]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{2\,A\,B\,Log\,\left[\,-\frac{d\,(a + b\,x)}{b\,c - a\,d}\,\right]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{2\,B^{2}\,Log\,[\,a + b\,x\,]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{2\,B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{2\,B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,c + d\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a + b\,x\,\right]}{\left(b\,c - a\,d\right)\,g\,i} + \frac{B^{2}\,Log\,\left[\,a + b\,x\,\right]\,Log\,\left[\,a +$$

## Problem 89: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)} dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$-\frac{2 \, b \, B^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{2 \, b \, B \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)^2}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{d \, \left(A + B \, Log\left[\frac{e \, \left(a + b \, x\right)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i}}$$

Result (type 4, 1684 leaves, 87 steps):

$$\frac{2 \, B^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{2 \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{A \, B \, d \, Log \left[a + b \, x\right]^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{B^2 \, d \, Log \left[a + b \, x\right] \, Log \left[\frac{1}{c + d \, x}\right]^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^$$

Problem 90: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(a g + b g x\right)^{3} \left(c i + d i x\right)} dx$$

Optimal (type 3, 343 leaves, 9 steps):

$$\frac{4 \, b \, B^2 \, d \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b^2 \, B^2 \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{4 \, b \, B \, d \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b^2 \, B \, \left(c + d \, x\right)^2 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b^2 \, \left(c + d \, x\right)^2 \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{d^2 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^3} + \frac{d^2 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^3} + \frac{d^2 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^3} + \frac{d^2 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^2 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b \, x\right)^3}{3 \, B \, \left(a + b \, x\right)^3} + \frac{d^3 \, \left(a + b \, x\right)^3 \, d^3 \, \mathbf{i} \, \left(a + b$$

Result (type 4, 1899 leaves, 117 steps):

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]\right)^{2}}{\left(a \cdot g + b \cdot g \cdot x\right)^{4} \cdot \left(c \cdot i + d \cdot i \cdot x\right)} dx$$

Optimal (type 3, 507 leaves, 11 steps):

$$-\frac{6 \, b \, B^2 \, d^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, B^2 \, d \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^2} - \frac{2 \, b^3 \, B^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{6 \, b \, B \, d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, B \, d \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{2 \, b^3 \, B \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{9 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{3 \, b \, d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} - \frac{d^3 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^4 \, g^4 \, i}$$

Result (type 4, 2044 leaves, 151 steps):

### Problem 92: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{3} \left(A + B Log\left[\frac{e (a+b x)}{c+d x}\right]\right)^{2}}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 4, 722 leaves, 18 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)}{d^{3}\,i^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)}{d^{3}\,i^{2}\,\left(c+d\,x\right)} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{d^{3}\,i^{2}\,\left(c+d\,x\right)} - \frac{b\,B\,\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{d^{3}\,i^{2}} - \frac{6\,b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{d^{4}\,i^{2}} - \frac{3\,b\,\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{d^{3}\,i^{2}} - \frac{\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{d^{3}\,i^{2}} + \frac{b\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,Log\left[c+d\,x\right]}{d^{4}\,i^{2}} + \frac{b\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,PolyLog\left[c+d\,x\right]}{d^{4}\,i^{2}} + \frac{b\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,PolyLog\left[c+d$$

#### Result (type 4, 2224 leaves, 119 steps):

$$\frac{A \, b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, x}{d^3 \, i^2} + \frac{2 \, B^2 \, \left(b \, c - a \, d\right)^3 \, g^3}{d^4 \, i^2 \, \left(c + d \, x\right)} + \frac{2 \, b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, Log \left[a + b \, x\right]^2}{d^4 \, i^2} + \frac{a^2 \, b \, B^2 \, g^3 \, Log \left[a + b \, x\right]^2}{2 \, d^2 \, i^2} + \frac{a \, b \, B^2 \, \left(2 \, b \, c - a \, d\right) \, g^3 \, Log \left[a + b \, x\right]^2}{d^3 \, i^2} + \frac{b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, Log \left[a + b \, x\right]^2}{d^4 \, i^2} + \frac{a^2 \, b \, B^2 \, g^3 \, Log \left[a + b \, x\right]^2}{2 \, d^2 \, i^2} + \frac{a \, b \, B^2 \, \left(2 \, b \, c - a \, d\right) \, g^3 \, Log \left[a + b \, x\right]^2}{d^4 \, i^2} + \frac{b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{a \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \left[\frac{a \, (a + b \, x)}{b \, c - a \, d}\right] \, d^4 \, i^2} + \frac{a^2 \, b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, Log \left[a + b \, x\right] \, Log \left[\frac{a \, (a + b \, x)}{b \, c - a \, d}\right] \, Log \left[\frac{a \, (a + b \, x)}{b \, c - a \, d}\right] \, d^4 \, i^2} - \frac{a^2 \, b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2 \, \left(c + d \, x\right)} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a + b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[\frac{a \, (a \, b \, x)}{c + d \, x}\right]\right)}{d^4 \, i^2} - \frac{a^2 \, b \, B \, g^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Lo$$

Problem 93: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,2}\,\left(A+B\,Log\left[\,\frac{e\,\left(a+b\,x\right)\,}{c+d\,x}\,\right]\,\right)^{\,2}}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}}\,\mathrm{d}x$$

Optimal (type 4, 469 leaves, 12 steps):

$$-\frac{2\,A\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]}{d^{2}\,i^{2}\,\left(c+d\,x\right)} + \frac{2\,b\,B\,\left(b\,c-a\,d\right)\,g^{2}\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{d^{3}\,i^{2}} + \frac{b\,g^{2}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{d^{2}\,i^{2}} + \frac{b\,g^{2}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{d^{2}\,i^{2}\,\left(c+d\,x\right)} + \frac{2\,b\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{2\,b\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{4\,b\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{4\,b\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,g^{2}\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{4\,b\,B^{2}\,\left(a+b\,x\right)\,g^{2}\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{4\,b\,B^{2}\,\left(a+b\,x\right)\,g^{2}\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} + \frac{4\,b\,B^{2}\,\left(a+b\,x\right)\,g^{2}\,PolyLog\left[3,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,i^{2}} +$$

Result (type 4, 1681 leaves, 94 steps):

$$\frac{28^{2} \left( b c - a d \right)^{2} g^{2}}{d^{2} i^{2} \left( c + d x \right)} = \frac{d^{2} i^{2}}{d^{2} i^{2}} = \frac{d^{2} i^{2}}{d^{2} i^{2}}$$

### Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right) \left(A + B Log\left[\frac{e (a + b x)}{c + d x}\right]\right)^{2}}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 4, 261 leaves, 9 steps):

$$\frac{2\,A\,B\,g\,\left(a+b\,x\right)}{d\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,g\,\left(a+b\,x\right)}{d\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} + \frac{2\,B^{2}\,g\,\left(a+b\,x\right)\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]}{d\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{d\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{b\,g\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{2}}{d^{2}\,\,\mathbf{i}^{2}} - \frac{2\,b\,B\,g\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)\,PolyLog\left[2,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\,\mathbf{i}^{2}} + \frac{2\,b\,B^{2}\,g\,PolyLog\left[3,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\,\mathbf{i}^{2}}$$

Result (type 4, 1060 leaves, 72 steps):

$$\frac{2B^2 \left(bc - ad\right) g}{d^2 i^2 \left(c + dx\right)} + \frac{2bB^2 g Log [a + bx]}{d^2 i^2} + \frac{bB^2 g Log [a + bx]^2}{d^2 i^2} - \frac{bB^2 g Log [a + bx] Log \left[\frac{1}{c + dx}\right]^2}{d^2 i^2} + \frac{bB^2 g Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log [a + bx]}{d^2 i^2} + \frac{bB^2 g Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log [a + bx]^2}{d^2 i^2} + \frac{bB^2 g Log \left[-\frac{d (a + bx)}{c + dx}\right]}{d^2 i^2 \left(c + dx\right)} - \frac{2bBg Log [a + bx]^2 Log \left[c + dx\right]}{d^2 i^2} - \frac{2bBg Log [a + bx]^2 Log \left[c + dx\right]}{d^2 i^2} + \frac{2bBg Log [a + bx]^2 Log \left[c + dx\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} + \frac{bB^2 g Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} + \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} + \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg Log \left[-\frac{d (a + bx)}{bc - ad}\right]}{d^2 i^2} - \frac{2bBg L$$

Problem 95: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 152 leaves, 4 steps):

$$-\frac{2\,A\,B\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,+\,\frac{2\,B^2\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,-\,\frac{2\,B^2\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,+\,\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\frac{\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right]\right)^2}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}$$

Result (type 4, 472 leaves, 26 steps):

$$-\frac{2\,B^{2}}{d\,\,\mathbf{i}^{2}\,\left(\,c\,+\,d\,\,x\,\right)} - \frac{2\,b\,\,B^{2}\,\,Log\,\left[\,a\,+\,b\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} - \frac{b\,\,B^{2}\,\,Log\,\left[\,a\,+\,b\,\,x\,\right]^{\,2}}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} + \frac{2\,B\,\,\left(\,A\,+\,B\,\,Log\,\left[\,\frac{e\,\,(a\,+\,b\,\,x)}{c\,+\,d\,\,x}\,\right]\,\right)}{d\,\,\mathbf{i}^{2}\,\left(\,c\,+\,d\,\,x\,\right)} + \frac{2\,b\,\,B\,\,Log\,\left[\,c\,+\,d\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} + \frac{2\,b\,\,B^{2}\,\,Log\,\left[\,c\,+\,d\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} + \frac{2\,b\,\,B^{2}\,\,Log\,\left[\,c\,+\,d\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} + \frac{2\,b\,\,B^{2}\,\,Log\,\left[\,c\,+\,d\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} - \frac{2\,b\,\,B^{2}\,\,PolyLog\,\left[\,c\,+\,d\,\,x\,\right]}{d\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{2}} - \frac{2\,b\,\,B^{2}\,\,PolyL$$

Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{\left(ag + bgx\right) \left(ci + dix\right)^{2}} dx$$

Optimal (type 3, 214 leaves, 7 steps):

$$\frac{2\,A\,B\,d\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{\,2}\,g\,\mathbf{i}^{\,2}\,\left(c+d\,x\right)}\,-\,\frac{2\,B^{\,2}\,d\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{\,2}\,g\,\mathbf{i}^{\,2}\,\left(c+d\,x\right)}\,+\,\frac{2\,B^{\,2}\,d\,\left(a+b\,x\right)\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{\,2}\,g\,\mathbf{i}^{\,2}\,\left(c+d\,x\right)}\,-\,\frac{d\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{\,2}}{\left(b\,c-a\,d\right)^{\,2}\,g\,\mathbf{i}^{\,2}\,\left(c+d\,x\right)}\,+\,\frac{b\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^{\,3}}{3\,B\,\left(b\,c-a\,d\right)^{\,2}\,g\,\mathbf{i}^{\,2}}$$

Result (type 4, 1687 leaves, 87 steps):

$$\frac{2 \, B^2}{\left(b \, C - a \, d\right)} \, g \, i^2 \, \left(c + d \, x\right) + \frac{2 \, b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left(b \, C - a \, d\right)^2 \, g \, i^2} + \frac{b \, B^2 \, \log\left(a + b \, x\right)}{\left$$

Problem 97: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 365 leaves, 10 steps):

$$-\frac{2 \, A \, B \, d^{2} \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} + \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, b^{2} \, B^{2} \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{$$

Result (type 4, 1521 leaves, 113 steps):

$$\frac{2 \, b \, B^2}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, i^2 \, \left(a + b \, x\right)} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b \, B^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^2} \frac{2 \, b^2 \, d$$

Problem 98: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{\left(ag + bgx\right)^{3} \left(ci + dix\right)^{2}} dx$$

Optimal (type 3, 523 leaves, 12 steps):

$$\frac{2\,A\,B\,d^{3}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,d^{3}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(c+d\,x\right)} + \frac{6\,b^{2}\,B^{2}\,d\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,B^{2}\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{2\,B^{2}\,d^{3}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(c+d\,x\right)} + \frac{6\,b^{2}\,B\,d\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,B\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} - \frac{b^{3}\,B\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} - \frac{b^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{3}}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(A+B\,Log\left[\frac{e\,(a+b\,x)}{c+d\,x}\right]\right)^{3}}{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}}$$

Result (type 4, 2071 leaves, 143 steps):

Problem 99: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{\left(ag + bgx\right)^4 \left(ci + dix\right)^2} dx$$

Optimal (type 3, 682 leaves, 14 steps):

$$\frac{2 \, A \, B \, d^4 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(c + d \, x\right)} - \frac{12 \, b^2 \, B^2 \, d^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(c + d \, x\right)} + \frac{b^3 \, B^2 \, d \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^3} - \frac{2 \, b^4 \, B^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(c + d \, x\right)} - \frac{12 \, b^2 \, B \, d^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^3} - \frac{2 \, b^4 \, B \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} - \frac{2 \, b^4 \, B \, d^2 \, \left(c + d \, x\right) \, \left(a + b \, B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} - \frac{2 \, b^4 \, B \, d^2 \, \left(c + d \, x\right)^3}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} - \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} - \frac{2 \, b^4 \, B \, d^2 \, \left(c + d \, x\right)^3}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} - \frac{2 \, b^4 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^2} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^3} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^2}{\left(b \, c - a \, d\right)^5 \, g^4 \, i^2 \, \left(a + b \, x\right)^3} + \frac{2 \, b^3 \, B \, d \, \left(c + d \, x\right)^3 \, \left(a + b \, a$$

Result (type 4, 2222 leaves, 177 steps):

$$\frac{2 \, b \, B^2}{27 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (a + b \, x)^3}{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (a + b \, x)^2} \, \frac{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (a + b \, x)}{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (a + b \, x)} \, \frac{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (c + b \, x)}{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (c + b \, x)} \, \frac{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2 \, (c + b \, x)}{9 \, (b \, c - a \, d)^2 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{4 \, b \, B^2 \, d^3 \, \log (a + b \, x) \, \log \left[\frac{1}{c + b \, x}\right]}{(b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^2 \, d^3 \, \log (a + b \, x)}{9 \, (b \, c - a \, d)^3 \, g^2 \, i^2} \, \frac{10 \, b \, B^3 \, d^3 \, (a + b \, \log \left[\frac{a \, a \, b \, x}{c \, c \, d \, x}\right]}{9 \, (b \, c - a \, d)^3 \, g^4 \, i^2} \, (a + b \, x)} \, \frac{10 \, b \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (b \, c - a \, d)^3 \, g^4 \, i^2} \, (a + b \, x)} \, \frac{10 \, b \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (b \, c - a \, d)^3 \, g^4 \, i^2} \, (a + b \, x)} \, \frac{10 \, b \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (b \, c \, a \, d)^3 \, g^4 \, i^2} \, (a + b \, x)} \, \frac{10 \, b \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (b \, c \, a \, d)^3 \, g^4 \, i^2} \, (a \, b \, x)} \, \frac{10 \, b \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (b \, c \, a \, d)^3 \, g^4 \, i^2} \, (a \, b \, x)^3} \, \frac{10 \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (a \, b \, c \, a^3 \, b^3 \, g^4 \, i^2} \, \frac{10 \, b^3 \, a^3 \, b^3 \, a^3}{9 \, (a \, b \, b \, a^3 \, b^3 \, a^3} \, \frac{10 \, c \, a^3 \, b^3 \, g^4 \, i^2}{9 \, (a \, b \, a^3 \, b^3 \, b^$$

### Problem 100: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^2}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^3}\,\mathrm{d}x$$

Optimal (type 4, 635 leaves, 14 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right)^{2}}{4 \ d^{2} \ i^{3} \left(c+d \ x\right)^{2}} - \frac{4 \ b \ B \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right)}{d^{3} \ i^{3} \left(c+d \ x\right)} + \frac{4 \ b \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right)}{d^{3} \ i^{3} \left(c+d \ x\right)} - \frac{4 \ b \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right) \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]}{d^{3} \ i^{3} \left(c+d \ x\right)} - \frac{B \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right)^{2} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{d^{3} \ i^{3} \left(c+d \ x\right)^{2}} + \frac{2 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ Log\left[\frac{b \ c-a \ d}{b \ (c+d \ x)}\right] \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{d^{3} \ i^{3}} + \frac{2 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ \left(a+b \ x\right)^{2} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)}{d^{3} \ i^{3} \left(c+d \ x\right)^{2}} + \frac{2 \ b \left(b \ c-a \ d\right) \ g^{3} \left(a+b \ x\right) \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right)^{2}}{d^{3} \ i^{3} \left(c+d \ x\right)} + \frac{2 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{3} \ i^{3} \left(c+d \ x\right)} + \frac{6 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \left(A+B \ Log\left[\frac{e \ (a+b \ x)}{c+d \ x}\right]\right) \ PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} - \frac{6 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ PolyLog\left[3, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ b$$

Result (type 4, 1890 leaves, 124 steps):

$$\frac{B^{2}(bc-ad)^{2}g^{3}}{4d^{4}1^{3}(c+dx)^{2}} = 2d^{4}1^{3}(c+dx) + 2d^{4}1^{3}(c+dx) + 2d^{4}1^{3}(c+dx)^{2} + 2d^{4}1^{$$

### Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,Log\left[\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right]\right)^2}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^3}\,\mathrm{d}x$$

Optimal (type 4, 410 leaves, 11 steps):

$$-\frac{B^{2} g^{2} \left(a+b \, x\right)^{2}}{4 \, d \, i^{3} \left(c+d \, x\right)^{2}} + \frac{2 \, A \, b \, B \, g^{2} \left(a+b \, x\right)}{d^{2} \, i^{3} \left(c+d \, x\right)} - \frac{2 \, b \, B^{2} \, g^{2} \left(a+b \, x\right)}{d^{2} \, i^{3} \left(c+d \, x\right)} + \frac{2 \, b \, B^{2} \, g^{2} \left(a+b \, x\right) \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{d^{2} \, i^{3} \left(c+d \, x\right)} + \frac{B \, g^{2} \left(a+b \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)}{d^{2} \, i^{3} \left(c+d \, x\right)} - \frac{g^{2} \left(a+b \, x\right)^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{2 \, d \, i^{3} \left(c+d \, x\right)^{2}} - \frac{b \, g^{2} \left(a+b \, x\right) \, \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{d^{2} \, i^{3} \left(c+d \, x\right)} - \frac{b^{2} \, g^{2} \, Log \left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right] \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{d^{3} \, i^{3}} - \frac{2 \, b^{2} \, B \, g^{2} \left(A+B \, Log \left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right) \, PolyLog \left[2, \frac{d \, (a+b \, x)}{b \, (c+d \, x)}\right]}{d^{3} \, i^{3}} + \frac{2 \, b^{2} \, B^{2} \, g^{2} \, PolyLog \left[3, \frac{d \, (a+b \, x)}{b \, (c+d \, x)}\right]}{d^{3} \, i^{3}}$$

Result (type 4, 1328 leaves, 102 steps):

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a\;g+b\;g\;x\right)\;\left(A+B\;Log\left[\frac{e\;(a+b\;x)}{c+d\;x}\right]\right)^2}{\left(c\;i+d\;i\;x\right)^3}\,\mathrm{d}x$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{B^2 g \left(a+b x\right)^2}{4 \left(b c-a d\right) i^3 \left(c+d x\right)^2}-\frac{B g \left(a+b x\right)^2 \left(A+B Log\left[\frac{e \left(a+b x\right)}{c+d x}\right]\right)}{2 \left(b c-a d\right) i^3 \left(c+d x\right)^2}+\frac{g \left(a+b x\right)^2 \left(A+B Log\left[\frac{e \left(a+b x\right)}{c+d x}\right]\right)^2}{2 \left(b c-a d\right) i^3 \left(c+d x\right)^2}$$

Result (type 4, 634 leaves, 58 steps):

$$\frac{B^2 \left(b \ c - a \ d\right) \ g}{4 \ d^2 \ i^3 \ \left(c + d \ x\right)^2} - \frac{b \ B^2 \ g}{2 \ d^2 \ i^3 \ \left(c + d \ x\right)} - \frac{b^2 \ B^2 \ g \ Log \left[a + b \ x\right]}{2 \ d^2 \ \left(b \ c - a \ d\right) \ i^3} - \frac{b^2 \ B^2 \ g \ Log \left[a + b \ x\right]^2}{2 \ d^2 \ \left(b \ c - a \ d\right) \ i^3} - \frac{B \left(b \ c - a \ d\right) \ g \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{2 \ d^2 \ i^3 \ \left(c + d \ x\right)^2} + \frac{b^2 \ B \ g \ Log \left[a + b \ x\right] \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{d^2 \ i^3 \ \left(c + d \ x\right)} + \frac{b^2 \ B \ g \ Log \left[a + b \ x\right] \left(A + B \ Log \left[\frac{e \ (a + b \ x)}{c + d \ x}\right]\right)}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} - \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} - \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ Log \left[c + d \ x\right]}{d^2 \ (b \ c - a \ d) \ i^3} + \frac{b^2 \ B^2$$

## Problem 103: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B Log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(ci+dix\right)^{3}} dx$$

#### Optimal (type 3, 296 leaves, 8 steps):

$$-\frac{B^{2} d \left(a+b \, x\right)^{2}}{4 \left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)^{2}} - \frac{2 \, A \, b \, B \left(a+b \, x\right)}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{2 \, b \, B^{2} \left(a+b \, x\right)}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} - \frac{2 \, b \, B^{2} \left(a+b \, x\right) \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(A+B \, Log\left[\frac{e \, (a+b \, x)}{c+d \, x}\right]\right)^{2}}{\left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2}} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2} \left(a+b \, x\right)^{2}}{\left(b \, c-a \, d\right)^{2} i^{3} \left(c+d \, x\right)^{2}} + \frac{B \, d \, \left(a+b \, x\right)^{2} \left(a+b$$

#### Result (type 4, 577 leaves, 30 steps):

$$\frac{B^2}{4 \text{ d } i^3 \text{ } \left( c + d \, x \right)^2} - \frac{3 \text{ b } B^2}{2 \text{ d } \left( \text{ b } c - \text{ a } d \right) \text{ } i^3 \text{ } \left( c + d \, x \right)} - \frac{3 \text{ b}^2 \text{ B}^2 \text{ Log} \left[ \text{ a} + \text{ b } \, x \right]}{2 \text{ d } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3} - \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ a} + \text{ b } \, x \right]^2}{2 \text{ d } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3} + \frac{b \text{ B } \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right) \text{ } i^3 \text{ } \left( c + d \, x \right)} + \frac{b^2 \text{ B } \text{ Log} \left[ \text{ a} + \text{ b} \, x \right] \text{ } \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right) \text{ } i^3 \text{ } \left( c + d \, x \right)} + \frac{b^2 \text{ B } \text{ Log} \left[ \text{ a} + \text{ b} \, x \right] \text{ } \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3} - \frac{2 \text{ d } i^3 \text{ } \left( \text{ c} + d \, x \right)^2}{2 \text{ d } i^3 \text{ } \left( \text{ c} + d \, x \right)^2} + \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3} - \frac{b^2 \text{ B} \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} - \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3} - \frac{b^2 \text{ B} \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} - \frac{b^2 \text{ B} \left( \text{ A} + \text{ B } \text{ Log} \left[ \frac{e \text{ } \left( \text{ a} + \text{ b} \, x \right)}{c + d \, x} \right] \right)}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} - \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} - \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} + \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} + \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} + \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d \, x \right]}{d \text{ } \left( \text{ b } c - \text{ a } d \right)^2 \text{ } i^3}} + \frac{b^2 \text{ B}^2 \text{ Log} \left[ \text{ c} + d$$

Problem 104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(ag + bgx\right)\left(ci + dix\right)^{3}} dx$$

Optimal (type 3, 375 leaves, 15 steps):

$$\frac{B^2 \, d^2 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} + \frac{4 \, A \, b \, B \, d \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{4 \, b \, B^2 \, d \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{4 \, b \, B^2 \, d \, \left(a + b \, x\right) \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{B \, d^2 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{b^2 \, \left(A + B \, Log\left[\frac{e \, (a + b \, x)}{c + d \, x}\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3} + \frac{b^2 \, \left(a + b \, x\right)^2 \, \left(a + b \,$$

Result (type 4, 1899 leaves, 117 steps):

$$\frac{B^2}{4 \left(bc-ad\right)^2 g^{\frac{1}{3}} \left(c+dx\right)^2} + \frac{7 b^2 B^2}{2 \left(bc-ad\right)^2 g^{\frac{1}{3}} \left(c+dx\right)^2} + \frac{7 b^2 B^2 \log [a+bx]}{2 \left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{Ab^2 B \log [a-bx]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} + \frac{3b^2 B^2 \log [a+bx] \log \left[\frac{a(a+bx)}{c+dx}\right]^2}{2 \left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 B^2 \log \left[-\frac{bc-ad}{c+dx}\right]^2}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 \log \left[-\frac{bc-ad}{c+dx}\right]}{\left(bc-ad\right)^3 g^{\frac{1}{3}}} - \frac{b^2 \log \left[-\frac{bc-ad}{c+dx}\right]}{\left(bc-ad\right)^3$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)}{c+dx}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 525 leaves, 12 steps):

$$-\frac{B^2\,d^3\,\left(a+b\,x\right)^2}{4\,\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} - \frac{6\,A\,b\,B\,d^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{6\,b\,B^2\,d^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(c+d\,x\right)} - \frac{2\,b^3\,B^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(a+b\,x\right)} - \frac{4\,b\,B\,d^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(c+d\,x\right)} - \frac{2\,b^3\,B\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,\mathbf{i}^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B\,\left(c+d\,x\right)$$

Result (type 4, 2071 leaves, 143 steps):

362

Problem 106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[\frac{e (a+bx)}{c+dx}\right]\right)^2}{\left(a g + b g x\right)^3 (c i + d i x)^3} dx$$

Optimal (type 3, 685 leaves, 14 steps):

$$\frac{B^2\,d^4\,\left(\,a+b\,x\,\right)^{\,2}}{4\,\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,c+d\,x\,\right)^{\,2}} + \frac{8\,A\,b\,B\,d^3\,\left(\,a+b\,x\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,c+d\,x\,\right)} - \frac{8\,b\,B^2\,d^3\,\left(\,a+b\,x\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,c+d\,x\,\right)} + \frac{8\,b^3\,B^2\,d\,\left(\,c+d\,x\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} - \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} - \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,A+B\,Log\left[\frac{e\,\left(\,a+b\,x\,\right)}{c+d\,x}\,\right]\,\right)}{2\,\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} + \frac{B\,b^3\,B^2\,d\,\left(\,c+d\,x\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} - \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,A+B\,Log\left[\frac{e\,\left(\,a+b\,x\,\right)}{c+d\,x}\,\right]\,\right)}{2\,\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} + \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,A+B\,Log\left[\frac{e\,\left(\,a+b\,x\,\right)}{c+d\,x}\,\right]\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} + \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,A+B\,Log\left[\frac{e\,\left(\,a+b\,x\,\right)}{c+d\,x}\,\right]\,\right)}{\left(\,b\,\,c-a\,\,d\,\right)^{\,5}\,g^3\,\,i^3\,\left(\,a+b\,x\,\right)} + \frac{B\,d^4\,\left(\,a+b\,x\,\right)^{\,2}\,\left(\,a+b\,x\,\right)$$

Result (type 4, 1921 leaves, 173 steps):

# Problem 107: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[\frac{e(a+bx)}{c+dx}\right]\right)^{2}}{\left(ag + bgx\right)^{4} \left(ci + dix\right)^{3}} dx$$

Optimal (type 3, 851 leaves, 16 steps):

$$\frac{B^2\,d^5\,\left(\,a+b\,x\,\right)^{\,2}}{4\,\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,c+d\,x\,\right)^{\,2}} + \frac{10\,a\,b\,B\,d^{\,4}\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,c+d\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,c+d\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,a+b\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,3}\,\left(\,c+d\,x\,\right)} + \frac{10\,b\,B^2\,d^4\,\left(\,a+b\,x\,\right)}{\left(\,b\,c-a\,d\,\right)^{\,6}\,g^{\,4}\,i^{\,$$

Result (type 4, 2454 leaves, 207 steps):

$$\frac{2 \, b^2 \, B^2}{27 \, \left( b \, c - a \, d \, \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \, \right)^3}{36 \, \left( b \, c - a \, d \, \right)^4 \, g^4 \, i^3 \, \left( a + b \, x \, \right)^2} - \frac{319 \, b^2 \, B^2 \, d^2}{18 \, \left( b \, c - a \, d \, \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \, \right)} - \frac{B^2 \, d^3}{4 \, \left( b \, c - a \, d \, \right)^4 \, g^4 \, i^3 \, \left( c + d \, x \, \right)^2} - \frac{19 \, b \, B^2 \, d^3}{4 \, \left( b \, c - a \, d \, \right)^5 \, g^4 \, i^3 \, \left( c + d \, x \, \right)^2} - \frac{19 \, b \, B^2 \, d^3}{4 \, \left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3 \, \left( c + d \, x \, \right)^2} - \frac{245 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right]}{9 \, \left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} + \frac{10 \, a \, b^2 \, B \, d^3 \, Log \left[ a + b \, x \, \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{1}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a + b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a + b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right] \, Log \left[ \frac{a \, a \, b \, x}{c + d \, x} \right]^2}{\left( b \, c - a \, d \, \right)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \, \right$$

$$\frac{6 \, b^2 \, d^2 \, \left( A + B \, Log \left[ \frac{e_1(a + b \times 1)}{c + d \times} \right]^2}{\left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \right)} - \frac{d^3 \, \left( A + B \, Log \left[ \frac{e_1(a + b \times 1)}{c + d \times} \right]^2}{2 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( c + d \, x \right)} - \frac{d^3 \, \left( A + B \, Log \left[ \frac{e_1(a + b \times 1)}{c + d \times} \right]^2}{2 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( c + d \, x \right)} - \frac{10 \, b^2 \, d^3 \, Log \left[ a + b \, x \right] \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3}{2 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} - \frac{20 \, A \, b^2 \, B \, d^3 \, Log \left[ c + d \, x \right]}{20 \, A \, b^2 \, B \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, A \, b^2 \, B \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^3 \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]} - \frac{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^3 \, d^3 \, \left( A + B \, Log \left[ \frac{e_1(a + b \, x)}{c + d \, x} \right] \right) \, Log \left[ c + d \, x \right]} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^3 \, d^3 \, \left( A + B \, Log \left[ \frac{e_1(a + b \, x)}{c + d \, x} \right] \right) \, Log \left[ c + d \, x \right]} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]}{20 \, b^2 \, B^3 \, d^3 \, \left( A + B \, Log \left[ \frac{e_1(a + b \, x)}{c + d \, x} \right] \right) \, Log \left[ c + d \, x \right]} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2} - \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x \right]^2}{20 \, b^2 \, B^2 \, d^3 \, Log \left[ c + d \, x$$

# Problem 108: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,\mathrm{d}x$$

Optimal (type 3, 223 leaves, 5 steps):

$$-\frac{B \left(b \, c - a \, d\right)^4 g^3 \, i \, n \, x}{20 \, b \, d^3} + \frac{B \left(b \, c - a \, d\right)^3 g^3 \, i \, n \, \left(a + b \, x\right)^2}{40 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^2 g^3 \, i \, n \, \left(a + b \, x\right)^3}{60 \, b^2 \, d} + \frac{g^3 \, i \, \left(a + b \, x\right)^4 \left(c + d \, x\right) \, \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b} + \frac{\left(b \, c - a \, d\right) \, g^3 \, i \, \left(a + b \, x\right)^4 \, \left(A - B \, n + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{20 \, b^2} + \frac{B \left(b \, c - a \, d\right)^5 g^3 \, i \, n \, Log\left[c + d \, x\right]}{20 \, b^2 \, d^4}$$

Result (type 3, 243 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right)^4 g^3 \, i \, n \, x}{20 \, b \, d^3} + \frac{B \left(b \, c - a \, d\right)^3 g^3 \, i \, n \, \left(a + b \, x\right)^2}{40 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^2 g^3 \, i \, n \, \left(a + b \, x\right)^3}{60 \, b^2 \, d} - \frac{B \left(b \, c - a \, d\right) g^3 \, i \, n \, \left(a + b \, x\right)^4}{20 \, b^2} + \frac{\left(b \, c - a \, d\right) g^3 \, i \, n \, \left(a + b \, x\right)^4}{4 \, b^2} + \frac{d \, g^3 \, i \, \left(a + b \, x\right)^5 \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^2} + \frac{B \left(b \, c - a \, d\right)^5 g^3 \, i \, n \, Log\left[c + d \, x\right]}{20 \, b^2 \, d^4}$$

### Problem 109: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right)^2 \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right) \, \left( A + B \, Log \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \right] \right) \, \mathrm{d}x$$

Optimal (type 3, 190 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{3} g^{2} \, i \, n \, x}{12 \, b \, d^{2}} - \frac{B \left(b \, c - a \, d\right)^{2} g^{2} \, i \, n \, \left(a + b \, x\right)^{2}}{24 \, b^{2} \, d} + \frac{g^{2} \, i \, \left(a + b \, x\right)^{3} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{4 \, b} + \frac{\left(b \, c - a \, d\right) \, g^{2} \, i \, \left(a + b \, x\right)^{3} \, \left(A - B \, n + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{12 \, b^{2}} - \frac{B \left(b \, c - a \, d\right)^{4} \, g^{2} \, i \, n \, Log\left[c + d \, x\right]}{12 \, b^{2} \, d^{3}}$$

Result (type 3, 210 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 \, g^2 \, i \, n \, x}{12 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right)^2 \, g^2 \, i \, n \, \left(a + b \, x\right)^2}{24 \, b^2 \, d} - \frac{B \left(b \, c - a \, d\right) \, g^2 \, i \, n \, \left(a + b \, x\right)^3}{12 \, b^2} + \\ \frac{\left(b \, c - a \, d\right) \, g^2 \, i \, \left(a + b \, x\right)^3 \, \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^2} + \frac{d \, g^2 \, i \, \left(a + b \, x\right)^4 \, \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, b^2} - \frac{B \left(b \, c - a \, d\right)^4 \, g^2 \, i \, n \, Log\left[c + d \, x\right]}{12 \, b^2 \, d^3}$$

# Problem 110: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right) \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, \mathrm{d} x$$

Optimal (type 3, 149 leaves, 5 steps):

$$-\frac{B\left(b\,c-a\,d\right)^{2}\,g\,i\,n\,x}{6\,b\,d} + \frac{g\,i\,\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b} + \\ \frac{\left(b\,c-a\,d\right)\,g\,i\,\left(a+b\,x\right)^{2}\,\left(A-B\,n+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{6\,b^{2}} + \frac{B\left(b\,c-a\,d\right)^{3}\,g\,i\,n\,Log\left[c+d\,x\right]}{6\,b^{2}}$$

Result (type 3, 311 leaves, 13 steps):

$$a \, A \, c \, g \, i \, x \, - \, \frac{1}{3} \, b \, B \, \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) \, d \, g \, i \, n \, x \, - \, \frac{B \, \left( b \, c \, - \, a \, d \right) \, \left( b \, c \, + \, a \, d \right) \, g \, i \, n \, x}{2 \, b \, d} \, - \, \frac{1}{6} \, B \, \left( b \, c \, - \, a \, d \right) \, g \, i \, n \, x^2 \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{3 \, b^2} \, - \, \frac{a^2 \, B \, \left( b \, c \, + \, a \, d \right) \, g \, i \, n \, Log \left[ a \, + \, b \, x \right)}{2 \, b^2} \, + \, \frac{a \, B \, c \, g \, i \, \left( a \, + \, b \, x \right) \, Log \left[ e \, \left( \frac{a \, + \, b \, x}{c \, + \, d \, x} \right)^n \right]}{b} \, + \, \frac{1}{2} \, \left( b \, c \, + \, a \, d \right) \, g \, i \, x^2 \, \left( A \, + \, B \, Log \left[ e \, \left( \frac{a \, + \, b \, x}{c \, + \, d \, x} \right)^n \right] \right) \, + \, \frac{1}{3} \, b \, d \, g \, i \, x^3 \, \left( A \, + \, B \, Log \left[ e \, \left( \frac{a \, + \, b \, x}{c \, + \, d \, x} \right)^n \right] \right) \, - \, \frac{b \, B \, c^3 \, g \, i \, n \, Log \left[ c \, + \, d \, x \right]}{3 \, d^2} \, - \, \frac{a \, B \, c \, \left( b \, c \, - \, a \, d \right) \, g \, i \, n \, Log \left[ c \, + \, d \, x \right]}{b \, d} \, + \, \frac{B \, c^2 \, \left( b \, c \, + \, a \, d \right) \, g \, i \, n \, Log \left[ c \, + \, d \, x \right]}{2 \, d^2} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{3 \, d^2} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{3 \, b^2} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{b \, d} \, + \, \frac{a^3 \, B \, d \, g \, i \, n \, Log \left[ a \, + \, b \, x \right]}{$$

# Problem 112: Result valid but suboptimal antiderivative.

$$\frac{\left(c i + d i x\right) \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)}{a g + b g x} dx$$

Optimal (type 4, 141 leaves, 6 steps):

$$\frac{\text{i} \left(\text{c} + \text{d} \, \text{x}\right) \, \left(\text{A} + \text{B} \, \text{Log}\left[\,\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^{\, n}\,\right]\,\right)}{\text{b} \, \text{g}} - \frac{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{i} \, \text{Log}\left[\,-\frac{\text{b} \, \text{c} - \text{a} \, \text{d}}{\text{d} \, \left(\text{a} + \text{b} \, \text{x}\right)}\,\right] \, \left(\text{A} - \text{B} \, \text{n} + \text{B} \, \text{Log}\left[\,\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^{\, n}\,\right]\,\right)}{\text{b}^{2} \, \text{g}} + \frac{\text{B} \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{i} \, \text{n} \, \text{PolyLog}\left[\,\text{2} \, , \, \, \text{1} + \frac{\text{b} \, \text{c} - \text{a} \, \text{d}}{\text{d} \, \left(\text{a} + \text{b} \, \text{x}\right)}\,\right]}{\text{b}^{2} \, \text{g}}$$

Result (type 4, 223 leaves, 13 steps):

$$\frac{\text{Adix}}{\text{bg}} - \frac{\text{B}\left(\text{bc-ad}\right)\text{inLog[a+bx]}^2}{2\text{b}^2\text{g}} + \frac{\text{Bdi}\left(\text{a+bx}\right)\text{Log}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]}{\text{b}^2\text{g}} + \frac{\left(\text{bc-ad}\right)\text{iLog[a+bx]}\left(\text{A+BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)}{\text{b}^2\text{g}} - \frac{\left(\text{bc-ad}\right)\text{inLog[c+dx]}}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{inLog[a+bx]}\text{Log}\left[\frac{\text{b}\left(\text{c+dx}\right)}{\text{bc-ad}}\right]}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{inPolyLog}\left[\text{2,} - \frac{\text{d}\left(\text{a+bx}\right)}{\text{bc-ad}}\right]}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{inPolyLog}\left[\text{2,} - \frac{\text{d}\left(\text{a+bx}\right)}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{bc-ad}\right)\text{inPolyLog}\left[\text{2,} - \frac{\text{d}\left(\text{a+bx}\right)}{\text{b}^2\text{g}}\right]}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}\left(\text{a+bx}\right)\text{g}}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}\left(\text{a+bx}\right)\text{g}}{\text{b}^2\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{g}} + \frac{\text{B}\left(\text{a+bx}\right)\text{g}}{\text{$$

# Problem 113: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A+BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)}{\left(\text{ag+bgx}\right)^{2}} \, dx$$

Optimal (type 4, 150 leaves, 5 steps):

$$-\frac{\text{Bin}\left(\text{c}+\text{d}\,\text{x}\right)}{\text{b}\,\text{g}^{2}\,\left(\text{a}+\text{b}\,\text{x}\right)}-\frac{\text{i}\left(\text{c}+\text{d}\,\text{x}\right)\,\left(\text{A}+\text{B}\,\text{Log}\left[\text{e}\left(\frac{\text{a}+\text{b}\,\text{x}}{\text{c}+\text{d}\,\text{x}}\right)^{\text{n}}\right]\right)}{\text{b}\,\text{g}^{2}\,\left(\text{a}+\text{b}\,\text{x}\right)}-\frac{\text{di}\left(\text{A}+\text{B}\,\text{Log}\left[\text{e}\left(\frac{\text{a}+\text{b}\,\text{x}}{\text{c}+\text{d}\,\text{x}}\right)^{\text{n}}\right]\right)\,\text{Log}\left[1-\frac{\text{b}\,\left(\text{c}+\text{d}\,\text{x}\right)}{\text{d}\,\left(\text{a}+\text{b}\,\text{x}\right)}\right]}{\text{b}^{2}\,\text{g}^{2}}+\frac{\text{B}\,\text{din}\,\text{PolyLog}\left[2,\frac{\text{b}\,\left(\text{c}+\text{d}\,\text{x}\right)}{\text{d}\,\left(\text{a}+\text{b}\,\text{x}\right)}\right]}{\text{b}^{2}\,\text{g}^{2}}$$

Result (type 4, 233 leaves, 14 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,i\,n}{b^{2}\,g^{2}\,\left(a+b\,x\right)} - \frac{B\,d\,i\,n\,Log\,[\,a+b\,x\,]}{b^{2}\,g^{2}} - \frac{B\,d\,i\,n\,Log\,[\,a+b\,x\,]^{\,2}}{2\,b^{2}\,g^{2}} - \frac{\left(b\,c-a\,d\right)\,i\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{b^{2}\,g^{2}\,\left(a+b\,x\right)} + \frac{B\,d\,i\,n\,Log\,[\,c+d\,x\,]}{b^{2}\,g^{2}} + \frac{B\,d\,i\,n\,Log\,[\,c+d\,x\,]}{b^{2}\,g^{2}} + \frac{B\,d\,i\,n\,Log\,[\,a+b\,x\,]\,Log\,\left[\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\,\right]}{b^{2}\,g^{2}} + \frac{B\,d\,i\,n\,PolyLog\,\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{b^{2}\,g^{2}}$$

# Problem 114: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right) \, \left(A + B \, \mathsf{Log}\left[\, e \, \left(\frac{a + b \, \mathbf{x}}{c + d \, \mathbf{x}}\right)^{\, n}\, \right]\,\right)}{\left(a \, g + b \, g \, \mathbf{x}\right)^{\, 3}} \, \mathrm{d}\mathbf{x}$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{\text{Bin}\left(\text{c}+\text{d}\,\text{x}\right)^{2}}{4\,\left(\text{bc}-\text{ad}\right)\,\text{g}^{3}\,\left(\text{a}+\text{bx}\right)^{2}}-\frac{\text{i}\,\left(\text{c}+\text{d}\,\text{x}\right)^{2}\,\left(\text{A}+\text{BLog}\left[\,\text{e}\,\left(\frac{\text{a}+\text{b}\,\text{x}}{\text{c}+\text{d}\,\text{x}}\right)^{\,\text{n}}\,\right]\,\right)}{2\,\left(\text{bc}-\text{ad}\right)\,\text{g}^{3}\,\left(\text{a}+\text{bx}\right)^{2}}$$

Result (type 3, 201 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, i \, n}{4 \, b^2 \, g^3 \, \left(a + b \, x\right)^2} - \frac{B \, d \, i \, n}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B \, d^2 \, i \, n \, Log \left[a + b \, x\right]}{2 \, b^2 \, \left(b \, c - a \, d\right) \, g^3} - \frac{\left(b \, c - a \, d\right) \, i \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{d \, i \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)} + \frac{B \, d^2 \, i \, n \, Log \left[c + d \, x\right]}{2 \, b^2 \, \left(b \, c - a \, d\right) \, g^3}$$

# Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \, \left(\text{A} + \text{B} \, \text{Log}\left[\,\text{e} \, \left(\frac{\text{a+b} \, x}{\text{c+d} \, x}\right)^{\, \text{n}}\,\right]\,\right)}{\left(\text{ag+bg}\, x\right)^{\, 4}} \, \, \mathrm{d} x$$

Optimal (type 3, 181 leaves, 5 steps):

$$\frac{\text{Bdin} \left(c + \text{dx}\right)^{2}}{4 \, \left(b \, c - \text{ad}\right)^{2} g^{4} \, \left(a + b \, x\right)^{2}} - \frac{b \, \text{Bin} \left(c + \text{dx}\right)^{3}}{9 \, \left(b \, c - \text{ad}\right)^{2} g^{4} \, \left(a + b \, x\right)^{3}} + \frac{\text{di} \left(c + \text{dx}\right)^{2} \left(A + B \, \text{Log}\left[e \left(\frac{a + b \, x}{c + \text{dx}}\right)^{n}\right]\right)}{2 \, \left(b \, c - \text{ad}\right)^{2} g^{4} \, \left(a + b \, x\right)^{3}} - \frac{b \, \text{i} \left(c + \text{dx}\right)^{3} \, \left(A + B \, \text{Log}\left[e \left(\frac{a + b \, x}{c + \text{dx}}\right)^{n}\right]\right)}{3 \, \left(b \, c - \text{ad}\right)^{2} g^{4} \, \left(a + b \, x\right)^{3}}$$

Result (type 3, 236 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, i \, n}{9 \, b^2 \, g^4 \, \left(a + b \, x\right)^3} - \frac{B \, d \, i \, n}{12 \, b^2 \, g^4 \, \left(a + b \, x\right)^2} + \frac{B \, d^2 \, i \, n}{6 \, b^2 \, \left(b \, c - a \, d\right) \, g^4 \, \left(a + b \, x\right)} + \\ \frac{B \, d^3 \, i \, n \, Log \left[a + b \, x\right]}{6 \, b^2 \, \left(b \, c - a \, d\right) \, i \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^2 \, g^4 \, \left(a + b \, x\right)^3} - \frac{d \, i \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^4 \, \left(a + b \, x\right)^2} - \frac{B \, d^3 \, i \, n \, Log \left[c + d \, x\right]}{6 \, b^2 \, \left(b \, c - a \, d\right)^2 \, g^4}$$

# Problem 116: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

Optimal (type 3, 281 leaves, 5 steps):

$$-\frac{B\,d^{2}\,i\,n\,\left(c+d\,x\right)^{\,2}}{4\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,2}}+\frac{2\,b\,B\,d\,i\,n\,\left(c+d\,x\right)^{\,3}}{9\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,3}}-\frac{b^{2}\,B\,i\,n\,\left(c+d\,x\right)^{\,4}}{16\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,4}}-\\ \frac{d^{2}\,i\,\left(c+d\,x\right)^{\,2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,2}}+\frac{2\,b\,d\,i\,\left(c+d\,x\right)^{\,3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{3\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,3}}-\frac{b^{2}\,i\,\left(c+d\,x\right)^{\,4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{4\,\left(b\,c-a\,d\right)^{\,3}\,g^{5}\,\left(a+b\,x\right)^{\,4}}$$

#### Result (type 3, 269 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, i \, n}{16 \, b^2 \, g^5 \, \left(a + b \, x\right)^4} - \frac{B \, d \, i \, n}{36 \, b^2 \, g^5 \, \left(a + b \, x\right)^3} + \frac{B \, d^2 \, i \, n}{24 \, b^2 \, \left(b \, c - a \, d\right) \, g^5 \, \left(a + b \, x\right)^2} - \frac{B \, d^3 \, i \, n}{12 \, b^2 \, \left(b \, c - a \, d\right)^2 \, g^5 \, \left(a + b \, x\right)} - \frac{B \, d^3 \, i \, n}{12 \, b^2 \, \left(b \, c - a \, d\right)^2 \, g^5 \, \left(a + b \, x\right)} - \frac{B \, d^4 \, i \, n \, Log \left[a + b \, x\right]}{12 \, b^2 \, \left(b \, c - a \, d\right)^3 \, g^5} - \frac{\left(b \, c - a \, d\right) \, i \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, b^2 \, g^5 \, \left(a + b \, x\right)^4} - \frac{d \, i \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^2 \, g^5 \, \left(a + b \, x\right)^3} + \frac{B \, d^4 \, i \, n \, Log \left[c + d \, x\right]}{12 \, b^2 \, \left(b \, c - a \, d\right)^3 \, g^5}$$

# Problem 117: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right)^3 \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, \mathrm{d}x$$

Optimal (type 3, 442 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{5} g^{3} \, i^{2} \, n \, x}{60 \, b^{2} \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{4} g^{3} \, i^{2} \, n \, \left(c + d \, x\right)^{2}}{120 \, b \, d^{4}} - \frac{19 \, B \left(b \, c - a \, d\right)^{3} g^{3} \, i^{2} \, n \, \left(c + d \, x\right)^{3}}{180 \, d^{4}} + \frac{13 \, b \, B \left(b \, c - a \, d\right)^{2} g^{3} \, i^{2} \, n \, \left(c + d \, x\right)^{4}}{120 \, b^{4}} - \frac{b^{2} \, B \left(b \, c - a \, d\right) g^{3} \, i^{2} \, n \, \left(c + d \, x\right)^{5}}{30 \, d^{4}} - \frac{\left(b \, c - a \, d\right)^{3} g^{3} \, i^{2} \left(c + d \, x\right)^{3} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{30 \, d^{4}} - \frac{3 \, b^{2} \left(b \, c - a \, d\right) g^{3} \, i^{2} \left(c + d \, x\right)^{5} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, d^{4}} + \frac{b^{3} \, g^{3} \, i^{2} \left(c + d \, x\right)^{6} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c + d \, x\right]}{60 \, b^{3} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, i^{2} \, n \, Log\left[c +$$

#### Result (type 3, 345 leaves, 14 steps):

$$-\frac{B\left(b\,c-a\,d\right)^{5}g^{3}\,\mathbf{i}^{2}\,n\,x}{60\,b^{2}\,d^{3}} + \frac{B\left(b\,c-a\,d\right)^{4}g^{3}\,\mathbf{i}^{2}\,n\,\left(a+b\,x\right)^{2}}{120\,b^{3}\,d^{2}} - \frac{B\left(b\,c-a\,d\right)^{3}g^{3}\,\mathbf{i}^{2}\,n\,\left(a+b\,x\right)^{3}}{180\,b^{3}\,d} - \frac{7\,B\left(b\,c-a\,d\right)^{2}g^{3}\,\mathbf{i}^{2}\,n\,\left(a+b\,x\right)^{4}}{120\,b^{3}} - \frac{B\,d\left(b\,c-a\,d\right)g^{3}\,\mathbf{i}^{2}\,n\,\left(a+b\,x\right)^{5}}{30\,b^{3}} + \frac{\left(b\,c-a\,d\right)^{2}g^{3}\,\mathbf{i}^{2}\left(a+b\,x\right)^{4}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,b^{3}} + \frac{2\,d\left(b\,c-a\,d\right)g^{3}\,\mathbf{i}^{2}\left(a+b\,x\right)^{5}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{5\,b^{3}} + \frac{d^{2}g^{3}\,\mathbf{i}^{2}\left(a+b\,x\right)^{6}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{6\,b^{3}} + \frac{B\left(b\,c-a\,d\right)^{6}g^{3}\,\mathbf{i}^{2}\,n\,Log\left[c+d\,x\right]}{60\,b^{3}\,d^{4}}$$

### Problem 118: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right)^2 \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, \mathrm{d}x$$

Optimal (type 3, 352 leaves, 5 steps):

$$-\frac{B \left(b \, c - a \, d\right)^4 g^2 \, \mathbf{i}^2 \, n \, x}{30 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^2}{60 \, b \, d^3} + \frac{B \left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^3}{10 \, d^3} - \frac{b \, B \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^4}{20 \, d^3} + \frac{\left(b \, c - a \, d\right)^2 g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d^3} - \frac{b \, \left(b \, c - a \, d\right) g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d^3} + \frac{b^2 \, g^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)^5 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, n \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3} - \frac{B \left(b \, c - a \, d\right)^5 g^2 \, \mathbf{i}^2 \, n \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3}$$

Result (type 3, 310 leaves, 14 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 g^2 \, i^2 \, n \, x}{30 \, b^2 \, d^2} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, i^2 \, n \, \left(a + b \, x\right)^2}{60 \, b^3 \, d} - \frac{B \left(b \, c - a \, d\right)^2 g^2 \, i^2 \, n \, \left(a + b \, x\right)^3}{10 \, b^3} - \frac{B \, d \, \left(b \, c - a \, d\right) g^2 \, i^2 \, n \, \left(a + b \, x\right)^4}{20 \, b^3} + \frac{\left(b \, c - a \, d\right)^2 g^2 \, i^2 \, \left(a + b \, x\right)^3 \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^3} + \frac{d \, \left(b \, c - a \, d\right) g^2 \, i^2 \, \left(a + b \, x\right)^4 \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^3} + \frac{d^2 \, g^2 \, i^2 \, \left(a + b \, x\right)^5 \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^3} - \frac{B \, \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n \, Log\left[c + d \, x\right]}{30 \, b^3 \, d^3}$$

# Problem 119: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right) \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^2 \, \left( A + B \, \mathsf{Log} \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \right] \, \right) \, \mathrm{d} x$$

Optimal (type 3, 250 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^2 \, n \, x}{12 \, b^2 \, d} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^2}{24 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^3}{12 \, d^2} - \frac{\left(b \, c - a \, d\right) g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d^2} + \frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^2 \, n \, Log \left[\frac{a + b \, x}{c + d \, x}\right]}{12 \, b^3 \, d^2} + \frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^2 \, n \, Log \left[c + d \, x\right]}{12 \, b^3 \, d^2}$$

Result (type 3, 210 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^2 \, n \, x}{12 \, b^2 \, d} + \frac{B \left(b \, c - a \, d\right)^2 \, g \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^2}{24 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) \, g \, \mathbf{i}^2 \, n \, \left(c + d \, x\right)^3}{12 \, d^2} + \\ \frac{B \left(b \, c - a \, d\right)^4 \, g \, \mathbf{i}^2 \, n \, \text{Log} \left[a + b \, x\right]}{12 \, b^3 \, d^2} - \frac{\left(b \, c - a \, d\right) \, g \, \mathbf{i}^2 \, \left(c + d \, x\right)^3 \, \left(A + B \, \text{Log} \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d^2} + \frac{b \, g \, \mathbf{i}^2 \, \left(c + d \, x\right)^4 \, \left(A + B \, \text{Log} \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, d^2}$$

# Problem 121: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{2} \, \left(A + B \, \mathsf{Log}\left[e \, \left(\frac{a + b \, \mathbf{x}}{c + d \, \mathbf{x}}\right)^{n}\right]\right)}{a \, g + b \, g \, \mathbf{x}} \, d\mathbf{x}$$

Optimal (type 4, 289 leaves, 10 steps):

### Result (type 4, 369 leaves, 18 steps):

$$\frac{A\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,x}{b^{2}\,g} - \frac{B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,n\,x}{2\,b^{2}\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[a+b\,x\right]}{2\,b^{3}\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[g\,\left(a+b\,x\right)\right]^{2}}{2\,b^{3}\,g} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{b^{3}\,g} + \frac{\mathbf{i}^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,b\,g} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[c+d\,x\right]}{b^{3}\,g} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[c+d\,x\right]}{b\,c-a\,d} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(b\,(c-a\,d\,x)^{2}\,\mathbf{i}^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(b\,(c-a\,d\,x)^{2}\,a\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(b\,(c-a\,d\,x)^{2}\,a\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} + \frac{B\,\left(b\,(c-a\,d\,x)^{2}\,a\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]$$

# Problem 122: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,e\,\left(\frac{\mathsf{a} + \mathsf{b}\,\mathbf{x}}{\mathsf{c} + d\,\mathbf{x}}\right)^{\,n}\,\right]\,\right)}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^{\,2}}\,\mathrm{d}\mathbf{x}$$

### Optimal (type 4, 259 leaves, 8 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,n\,\left(c+d\,x\right)}{b^{2}\,g^{2}\,\left(a+b\,x\right)}+\frac{d^{2}\,\mathbf{i}^{2}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{2}}-\frac{\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{2}\,g^{2}\,\left(a+b\,x\right)}-\frac{B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,n\,Log\left[c+d\,x\right]}{b^{3}\,g^{2}}-\frac{2\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[1-\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{3}\,g^{2}}+\frac{2\,B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,n\,PolyLog\left[2,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{3}\,g^{2}}$$

### Result (type 4, 327 leaves, 17 steps):

$$\frac{A\,d^{2}\,i^{2}\,x}{b^{2}\,g^{2}} - \frac{B\,\left(b\,c - a\,d\right)^{2}\,i^{2}\,n}{b^{3}\,g^{2}\,\left(a + b\,x\right)} - \frac{B\,d\,\left(b\,c - a\,d\right)\,i^{2}\,n\,Log\,[\,a + b\,x\,]}{b^{3}\,g^{2}} - \frac{B\,d\,\left(b\,c - a\,d\right)\,i^{2}\,n\,Log\,[\,a + b\,x\,]^{2}}{b^{3}\,g^{2}} + \\ \frac{B\,d^{2}\,i^{2}\,\left(a + b\,x\right)\,Log\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{b^{3}\,g^{2}} - \frac{\left(b\,c - a\,d\right)^{2}\,i^{2}\,\left(A + B\,Log\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\right)}{b^{3}\,g^{2}} + \frac{2\,d\,\left(b\,c - a\,d\right)\,i^{2}\,Log\,[\,a + b\,x\,]\,\left(A + B\,Log\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\right)}{b^{3}\,g^{2}} + \\ \frac{2\,B\,d\,\left(b\,c - a\,d\right)\,i^{2}\,n\,Log\,[\,a + b\,x\,]\,Log\left[\,\frac{b\,(c + d\,x)}{b\,c - a\,d}\,\right]}{b^{3}\,g^{2}} + \frac{2\,B\,d\,\left(b\,c - a\,d\right)\,i^{2}\,n\,PolyLog\left[\,2\,,\,-\frac{d\,(a + b\,x)}{b\,c - a\,d}\,\right]}{b^{3}\,g^{2}}$$

# Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{2} \, \left(A + B \, \mathsf{Log}\left[e \, \left(\frac{\mathsf{a} + \mathsf{b} \, \mathbf{x}}{\mathsf{c} + \mathsf{d} \, \mathbf{x}}\right)^{n}\right]\right)}{\left(\mathsf{a} \, \mathsf{g} + \mathsf{b} \, \mathsf{g} \, \mathsf{x}\right)^{3}} \, \mathrm{d} \mathbf{x}$$

Optimal (type 4, 242 leaves, 7 steps):

$$-\frac{B\,d\,i^{2}\,n\,\left(c+d\,x\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)}-\frac{B\,i^{2}\,n\,\left(c+d\,x\right)^{2}}{4\,b\,g^{3}\,\left(a+b\,x\right)^{2}}-\frac{d\,i^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{2}\,g^{3}\,\left(a+b\,x\right)}-\frac{i^{2}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{3}}-\frac{d^{2}\,i^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[1-\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{3}\,g^{3}}+\frac{B\,d^{2}\,i^{2}\,n\,PolyLog\left[2\,,\,\frac{b\,\left(c+d\,x\right)}{d\,\left(a+b\,x\right)}\right]}{b^{3}\,g^{3}}$$

#### Result (type 4, 354 leaves, 18 steps):

$$-\frac{B\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,n}{4\,b^{3}\,g^{3}\,\left(a+b\,x\right)^{2}}-\frac{3\,B\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,n}{2\,b^{3}\,g^{3}\,\left(a+b\,x\right)}-\frac{3\,B\,d^{2}\,\mathbf{i}^{2}\,n\,Log\left[a+b\,x\right]}{2\,b^{3}\,g^{3}}-\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,Log\left[a+b\,x\right]^{2}}{2\,b^{3}\,g^{3}}-\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,Log\left[a+b\,x\right]^{2}}{2\,b^{3}\,g^{3}}-\frac{\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,b^{3}\,g^{3}\,\left(a+b\,x\right)^{2}}-\frac{2\,d\,\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)}+\frac{d^{2}\,\mathbf{i}^{2}\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{3}}+\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,Log\left[c+d\,x\right]}{b\,c-a\,d}+\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,Log\left[a+b\,x\right]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g^{3}}+\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{3}\,g^{3}}$$

# Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \, \mathbf{i} + d \, \mathbf{i} \, \mathbf{x}\right)^{2} \, \left(A + B \, \mathsf{Log}\left[e \, \left(\frac{a + b \, \mathbf{x}}{c + d \, \mathbf{x}}\right)^{n}\right]\right)}{\left(a \, g + b \, g \, \mathbf{x}\right)^{4}} \, \mathrm{d}\mathbf{x}$$

Optimal (type 3, 93 leaves, 2 steps):

$$-\,\frac{\text{Bi}^2\,n\,\left(c\,+\,d\,x\right)^3}{9\,\left(b\,c\,-\,a\,d\right)\,g^4\,\left(a\,+\,b\,x\right)^3}\,-\,\frac{\text{i}^2\,\left(c\,+\,d\,x\right)^3\,\left(A\,+\,B\,Log\left[\,e\,\left(\frac{a+b\,x}{c\,+\,d\,x}\right)^{\,n}\,\right]\,\right)}{3\,\left(b\,c\,-\,a\,d\right)\,g^4\,\left(a\,+\,b\,x\right)^3}$$

Result (type 3, 301 leaves, 14 steps):

$$\frac{B \left( b \ c - a \ d \right)^{2} \ i^{2} \ n}{9 \ b^{3} \ g^{4} \ \left( a + b \ x \right)^{3}} - \frac{B \ d \ \left( b \ c - a \ d \right) \ i^{2} \ n}{3 \ b^{3} \ g^{4} \ \left( a + b \ x \right)^{2}} - \frac{B \ d^{2} \ i^{2} \ n}{3 \ b^{3} \ g^{4} \ \left( a + b \ x \right)} - \frac{B \ d^{3} \ i^{2} \ n \ Log \left[ a + b \ x \right]}{3 \ b^{3} \ \left( b \ c - a \ d \right) \ g^{4}} - \frac{\left( b \ c - a \ d \right)^{2} \ i^{2} \ \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^{n} \right] \right)}{3 \ b^{3} \ g^{4} \ \left( a + b \ x \right)^{3}} - \frac{d \ \left( b \ c - a \ d \right) \ i^{2} \ \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^{n} \right] \right)}{b^{3} \ g^{4} \ \left( a + b \ x \right)} + \frac{B \ d^{3} \ i^{2} \ n \ Log \left[ c + d \ x \right]}{3 \ b^{3} \ \left( b \ c - a \ d \right) \ g^{4}}$$

# Problem 125: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A+BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{11}\right]\right)}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

Optimal (type 3, 189 leaves, 5 steps):

$$\frac{\text{B d i}^{2} \text{ n } \left(\text{c} + \text{d x}\right)^{3}}{9 \, \left(\text{b c} - \text{a d}\right)^{2} \, g^{5} \, \left(\text{a} + \text{b x}\right)^{3}} - \frac{\text{b B i}^{2} \, \text{n } \left(\text{c} + \text{d x}\right)^{4}}{16 \, \left(\text{b c} - \text{a d}\right)^{2} \, g^{5} \, \left(\text{a} + \text{b x}\right)^{4}} + \frac{\text{d i}^{2} \, \left(\text{c} + \text{d x}\right)^{3} \, \left(\text{A} + \text{B Log}\left[\text{e}\left(\frac{\text{a} + \text{b x}}{\text{c} + \text{d x}}\right)^{\text{n}}\right]\right)}{3 \, \left(\text{b c} - \text{a d}\right)^{2} \, g^{5} \, \left(\text{a} + \text{b x}\right)^{3}} - \frac{\text{b i}^{2} \, \left(\text{c} + \text{d x}\right)^{4} \, \left(\text{A} + \text{B Log}\left[\text{e}\left(\frac{\text{a} + \text{b x}}{\text{c} + \text{d x}}\right)^{\text{n}}\right]\right)}{4 \, \left(\text{b c} - \text{a d}\right)^{2} \, g^{5} \, \left(\text{a} + \text{b x}\right)^{4}}$$

Result (type 3, 340 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{2} \, \mathbf{i}^{2} \, n}{16 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{4}} - \frac{5 \, B \, d \, \left(b \, c - a \, d\right) \, \mathbf{i}^{2} \, n}{36 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{B \, d^{2} \, \mathbf{i}^{2} \, n}{24 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{2}} + \frac{B \, d^{3} \, \mathbf{i}^{2} \, n}{12 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{5} \, \left(a + b \, x\right)} + \frac{B \, d^{4} \, \mathbf{i}^{2} \, n \, Log \left[a + b \, x\right]}{12 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{5}} - \frac{\left(b \, c - a \, d\right)^{2} \, \mathbf{i}^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{4 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{4}} - \frac{2 \, d \, \left(b \, c - a \, d\right) \, \mathbf{i}^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{3}} - \frac{d^{2} \, \mathbf{i}^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{2 \, b^{3} \, g^{5} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{4} \, \mathbf{i}^{2} \, n \, Log \left[c + d \, x\right]}{12 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{5}}$$

### Problem 126: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a} + \mathsf{b}\,\mathbf{x}}{\mathsf{c} + \mathsf{d}\,\mathbf{x}}\right)^{\,\mathsf{n}}\,\right]\,\right)}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^{\,\mathsf{6}}}\,\,\mathrm{d}\mathbf{x}$$

Optimal (type 3, 293 leaves, 5 steps):

$$-\frac{B\,d^{2}\,\mathbf{i}^{2}\,n\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{3}}+\frac{b\,B\,d\,\mathbf{i}^{2}\,n\,\left(c+d\,x\right)^{4}}{8\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b^{2}\,B\,\mathbf{i}^{2}\,n\,\left(c+d\,x\right)^{5}}{25\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}-\\ \frac{d^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{3}}+\frac{b\,d\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{5\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}$$

Result (type 3, 375 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{2} \, \mathbf{i}^{2} \, n}{25 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{5}} - \frac{3 \, B \, d \, \left(b \, c - a \, d\right) \, \mathbf{i}^{2} \, n}{40 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{4}} - \frac{B \, d^{2} \, \mathbf{i}^{2} \, n}{90 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{3}} + \frac{B \, d^{3} \, \mathbf{i}^{2} \, n}{60 \, b^{3} \, \left(b \, c - a \, d\right) \, g^{6} \, \left(a + b \, x\right)^{2}} - \frac{B \, d^{4} \, \mathbf{i}^{2} \, n}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{2} \, g^{6} \, \left(a + b \, x\right)} - \frac{B \, d^{5} \, \mathbf{i}^{2} \, n \, Log \left[a + b \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}} - \frac{\left(b \, c - a \, d\right)^{2} \, \mathbf{i}^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{3}} - \frac{d^{2} \, \mathbf{i}^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{3 \, b^{3} \, g^{6} \, \left(a + b \, x\right)^{3}} + \frac{B \, d^{5} \, \mathbf{i}^{2} \, n \, Log \left[c + d \, x\right]}{30 \, b^{3} \, \left(b \, c - a \, d\right)^{3} \, g^{6}}$$

# Problem 127: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,3}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 477 leaves, 5 steps):

$$\frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, \mathbf{i}^{3} \, n \, x}{140 \, b^{3} \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{5} g^{3} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{2}}{280 \, b^{2} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{4} g^{3} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{3}}{420 \, b \, d^{4}} - \frac{17 \, B \left(b \, c - a \, d\right)^{3} g^{3} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{4}}{280 \, d^{4}} + \frac{b \, B \left(b \, c - a \, d\right)^{2} g^{3} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{4}}{420 \, b \, d^{4}} - \frac{b^{2} \, B \left(b \, c - a \, d\right)^{3} g^{3} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{4} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{4d^{4}} + \frac{3 \, b \, \left(b \, c - a \, d\right)^{2} g^{3} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{5} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{42 \, d^{4}} - \frac{b^{2} \left(b \, c - a \, d\right)^{3} g^{3} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{4} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{2 \, d^{4}} + \frac{b^{3} g^{3} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{7} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{140 \, b^{4} \, d^{4}} + \frac{B \left(b \, c - a \, d\right)^{7} g^{3} \, \mathbf{i}^{3} \, n \, Log\left[c + d \, x\right]}{140 \, b^{4} \, d^{4}}$$

#### Result (type 3, 435 leaves, 18 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{6} g^{3} \, \mathbf{i}^{3} \, n \, x}{140 \, b^{3} \, d^{3}} + \frac{B \left(b \, c - a \, d\right)^{5} g^{3} \, \mathbf{i}^{3} \, n \, \left(a + b \, x\right)^{2}}{280 \, b^{4} \, d^{2}} - \frac{B \left(b \, c - a \, d\right)^{4} g^{3} \, \mathbf{i}^{3} \, n \, \left(a + b \, x\right)^{3}}{420 \, b^{4} \, d} - \frac{17 \, B \left(b \, c - a \, d\right)^{3} g^{3} \, \mathbf{i}^{3} \, n \, \left(a + b \, x\right)^{4}}{14 \, b^{4}} - \frac{B \, d \left(b \, c - a \, d\right)^{2} g^{3} \, \mathbf{i}^{3} \, n \, \left(a + b \, x\right)^{5}}{42 \, b^{4}} + \frac{B \, d^{2} \left(b \, c - a \, d\right) g^{3} \, \mathbf{i}^{3} \, n \, \left(a + b \, x\right)^{6}}{42 \, b^{4}} + \frac{\left(b \, c - a \, d\right)^{3} g^{3} \, \mathbf{i}^{3} \, \left(a + b \, x\right)^{4} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{4 \, b^{4}} + \frac{3 \, d \, \left(b \, c - a \, d\right)^{2} g^{3} \, \mathbf{i}^{3} \, \left(a + b \, x\right)^{5} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{5 \, b^{4}} + \frac{d^{3} g^{3} \, \mathbf{i}^{3} \, \left(a + b \, x\right)^{7} \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{7 \, b^{4}} + \frac{B \left(b \, c - a \, d\right)^{7} g^{3} \, \mathbf{i}^{3} \, n \, Log\left[c + d \, x\right]}{140 \, b^{4} \, d^{4}}$$

# Problem 128: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,\mathrm{d}x$$

Optimal (type 3, 387 leaves, 5 steps):

$$-\frac{B \left(b \ c-a \ d\right)^{5} g^{2} \ i^{3} \ n \ x}{60 \ b^{3} \ d^{2}} - \frac{B \left(b \ c-a \ d\right)^{4} g^{2} \ i^{3} \ n \ \left(c+d \ x\right)^{2}}{120 \ b^{2} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{3} g^{2} \ i^{3} \ n \ \left(c+d \ x\right)^{3}}{120 \ d^{3}} + \frac{7 \ B \left(b \ c-a \ d\right)^{2} g^{2} \ i^{3} \ n \ \left(c+d \ x\right)^{4}}{120 \ d^{3}} - \frac{b \left(b \ c-a \ d\right)^{2} g^{2} \ i^{3} \ n \ \left(c+d \ x\right)^{4} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{4 \ d^{3}} - \frac{2 \ b \left(b \ c-a \ d\right) g^{2} \ i^{3} \ \left(c+d \ x\right)^{5} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{5 \ d^{3}} + \frac{b^{2} g^{2} \ i^{3} \left(c+d \ x\right)^{6} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ n \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} - \frac{B \left(b \ c-a \ d\right)^{6} g^{2} \ i^{3} \ n \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} + \frac{120 \ d^{3} \ n \ Log\left[c+d \ x\right]}{60 \ b^{4} \ d^{3}} + \frac{120 \ d^{3} \ n^{3} \ n$$

Result (type 3, 345 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right)^{5} g^{2} \, \mathbf{i}^{3} \, n \, x}{60 \, b^{3} \, d^{2}} - \frac{B \left(b \, c - a \, d\right)^{4} g^{2} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{2}}{120 \, b^{2} \, d^{3}} - \frac{B \left(b \, c - a \, d\right)^{3} g^{2} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{3}}{180 \, b \, d^{3}} + \frac{7 \, B \left(b \, c - a \, d\right)^{2} g^{2} \, \mathbf{i}^{3} \, n \, \left(c + d \, x\right)^{4}}{120 \, d^{3}} - \frac{b \, B \left(b \, c - a \, d\right)^{6} g^{2} \, \mathbf{i}^{3} \, n \, Log \left[a + b \, x\right]}{60 \, b^{4} \, d^{3}} + \frac{\left(b \, c - a \, d\right)^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{4} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{4 \, d^{3}} - \frac{2 \, b \, \left(b \, c - a \, d\right) g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{5} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{5 \, d^{3}} + \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{6 \, d^{3}} - \frac{b^{2} g^{2} \, \mathbf{i}^{3} \, \left(c + d \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6} \, \left(a + b \, x\right)^{6}$$

### Problem 129: Result valid but suboptimal antiderivative.

$$\int \left( a g + b g x \right) \left( c i + d i x \right)^{3} \left( A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right] \right) dx$$

Optimal (type 3, 283 leaves, 5 steps):

$$\begin{split} &\frac{B\,\left(b\,c-a\,d\right)^{4}\,g\,\,\mathbf{i}^{3}\,n\,x}{20\,b^{3}\,d} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g\,\,\mathbf{i}^{3}\,n\,\left(c+d\,x\right)^{2}}{40\,b^{2}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,g\,\,\mathbf{i}^{3}\,n\,\left(c+d\,x\right)^{3}}{60\,b\,d^{2}} - \\ &\frac{B\,\left(b\,c-a\,d\right)\,g\,\,\mathbf{i}^{3}\,n\,\left(c+d\,x\right)^{4}}{20\,d^{2}} - \frac{\left(b\,c-a\,d\right)\,g\,\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,d^{2}} + \\ &\frac{b\,g\,\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{5\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{5}\,g\,\,\mathbf{i}^{3}\,n\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{20\,b^{4}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{5}\,g\,\,\mathbf{i}^{3}\,n\,Log\left[c+d\,x\right]}{20\,b^{4}\,d^{2}} \end{split}$$

Result (type 3, 243 leaves, 10 steps):

$$\frac{B \left(b \, c - a \, d\right)^4 g \, \mathbf{i}^3 \, n \, x}{20 \, b^3 \, d} + \frac{B \left(b \, c - a \, d\right)^3 g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^2}{40 \, b^2 \, d^2} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^3}{60 \, b \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4}{20 \, d^2} + \frac{B \left(b \, c - a \, d\right)^2 g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^3}{4 \, d^2} - \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4}{20 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a \, a \, b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right) g \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^4 \left(A + B \, Log\left[e \left(\frac{a \, a \, b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, d^2} + \frac{B \left(b \, c - a \, d\right$$

### Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^3\,\left(A + B\,\mathsf{Log}\left[e\,\left(\frac{a + b\,\mathbf{x}}{c + d\,\mathbf{x}}\right)^n\right]\right)}{a\,g + b\,g\,\mathbf{x}}\,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 373 leaves, 14 steps):

$$-\frac{5 \, B \, d \, \left(b \, c - a \, d\right)^2 \, \mathbf{i}^3 \, n \, x}{6 \, b^3 \, g} - \frac{B \, \left(b \, c - a \, d\right) \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^2}{6 \, b^2 \, g} + \frac{d \, \left(b \, c - a \, d\right)^2 \, \mathbf{i}^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^4 \, g} + \frac{\left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b \, g} - \frac{5 \, B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]}{6 \, b^4 \, g} - \frac{11 \, B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, Log\left[c + d \, x\right]}{b^4 \, g} - \frac{\left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, PolyLog\left[2, \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)}\right]}{b^4 \, g} + \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \,$$

Result (type 4, 455 leaves, 22 steps):

$$\frac{A\,d\,\left(b\,c-a\,d\right)^{\,2}\,i^{\,3}\,x}{b^{\,3}\,g} - \frac{5\,B\,d\,\left(b\,c-a\,d\right)^{\,2}\,i^{\,3}\,n\,x}{6\,b^{\,3}\,g} - \frac{B\,\left(b\,c-a\,d\right)\,i^{\,3}\,n\,\left(c+d\,x\right)^{\,2}}{6\,b^{\,2}\,g} - \frac{5\,B\,\left(b\,c-a\,d\right)^{\,3}\,i^{\,3}\,n\,\text{Log}\left[\,a+b\,x\,\right)}{6\,b^{\,4}\,g} - \frac{B\,d\,\left(b\,c-a\,d\right)^{\,2}\,i^{\,3}\,\left(a+b\,x\right)\,\text{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]}{b^{\,4}\,g} + \frac{B\,d\,\left(b\,c-a\,d\right)^{\,2}\,i^{\,3}\,\left(a+b\,x\right)\,\text{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]}{b^{\,4}\,g} + \frac{\left(b\,c-a\,d\right)\,i^{\,3}\,\left(c+d\,x\right)^{\,2}\,\left(A+B\,\text{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{2\,b^{\,2}\,g} + \frac{i^{\,3}\,\left(c+d\,x\right)^{\,3}\,\left(A+B\,\text{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{b^{\,4}\,g} + \frac{B\,\left(b\,c-a\,d\right)^{\,3}\,i^{\,3}\,n\,\text{Log}\left[\,c+d\,x\right]}{b^{\,4}\,g} + \frac{\left(b\,c-a\,d\right)^{\,3}\,i^{\,3}\,\left(A+B\,\text{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)\,\text{Log}\left[\,a\,g+b\,g\,x\right]}{b^{\,4}\,g} + \frac{B\,\left(b\,c-a\,d\right)^{\,3}\,i^{\,3}\,n\,\text{PolyLog}\left[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b^{\,4}\,g} + \frac{B\,\left(a\,b\,c-a\,d\right)^{\,3}\,i^{\,3}\,n\,\text{PolyLog}\left[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{b^{\,4}\,g} + \frac{B\,\left(a\,b\,c-a\,d\right)^{\,3}\,i^{\,3}\,n\,\text{PolyLog}\left[\,2\,,\,-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]}{a^{\,2}\,b^{\,2}\,a^{\,2}\,$$

### Problem 132: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \, \left(\text{A}+\text{BLog}\left[\,\text{e}\,\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\,\text{n}}\,\right]\,\right)}{\left(\text{ag+bgx}\right)^2} \, \text{d}x$$

Optimal (type 4, 390 leaves, 11 steps):

$$\frac{B \, d^2 \, \left( b \, c - a \, d \right) \, \mathbf{i}^3 \, n \, x}{2 \, b^3 \, g^2} - \frac{B \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, \left( c + d \, x \right)}{b^3 \, g^2 \, \left( a + b \, x \right)} + \frac{2 \, d^2 \, \left( b \, c - a \, d \right) \, \mathbf{i}^3 \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^4 \, g^2} - \frac{\left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, \left( c + d \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{2 \, b^2 \, g^2} - \frac{B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{2 \, b^4 \, g^2} - \frac{B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{2 \, b^4 \, g^2} - \frac{B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, Log \left[ \frac{a + b \, x}{c + d \, x} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, \mathbf{i}^3 \, n \, PolyLog \left[ 2 \, , \, \frac{b \, (c + d \, x)}{d \, (a + b \, x)} \right]}{b^4 \, g^2} + \frac{B \, B \, d \, \left( b \, c - a \, d \, d \, \right)^2 \, B \, d \, \left( b \, c$$

Result (type 4, 543 leaves, 21 steps):

$$\frac{A\,d^{2}\,\left(3\,b\,c-2\,a\,d\right)\,\mathbf{i}^{3}\,x}{b^{3}\,g^{2}} - \frac{B\,d^{2}\,\left(b\,c-a\,d\right)\,\mathbf{i}^{3}\,n\,x}{2\,b^{3}\,g^{2}} - \frac{B\,\left(b\,c-a\,d\right)^{3}\,\mathbf{i}^{3}\,n}{b^{4}\,g^{2}\,\left(a+b\,x\right)} - \frac{a^{2}\,B\,d^{3}\,\mathbf{i}^{3}\,n\,\text{Log}\left[a+b\,x\right]}{2\,b^{4}\,g^{2}} - \frac{B\,d^{2}\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,n\,\text{Log}\left[a+b\,x\right]^{2}}{2\,b^{4}\,g^{2}} + \frac{B\,d^{2}\,\left(3\,b\,c-2\,a\,d\right)\,\mathbf{i}^{3}\,\left(a+b\,x\right)\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{b^{4}\,g^{2}} + \frac{d^{3}\,\mathbf{i}^{3}\,x^{2}\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{4}\,g^{2}} + \frac{3\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{Log}\left[a+b\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{4}\,g^{2}} + \frac{3\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{Log}\left[a+b\,x\right]\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{4}\,g^{2}} + \frac{3\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{n}\,\text{Log}\left[c+d\,x\right]}{b^{4}\,g^{2}} + \frac{B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{n}\,\text{Log}\left[c+d\,x\right]}{b^{4}\,g^{2}} + \frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{n}\,\text{Log}\left[c+d\,x\right]}{b^{4}\,g^{2}} + \frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{n}\,\text{Log}\left[c+d\,x\right]}{b^{2}\,c-a\,d}} + \frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\text{n}\,\text{PolyLog}\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{2}\,c-a\,d}} + \frac{3\,B\,d\,\left(b\,c-a\,d\right)^{2}\,\mathbf{i}^{3}\,\mathbf$$

# Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)}{\left(\text{ag+bgx}\right)^3} \, dx$$

Optimal (type 4, 361 leaves, 9 steps):

$$-\frac{2\,B\,d\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,n\,\left(c+d\,x\right)}{b^3\,g^3\,\left(a+b\,x\right)} - \frac{B\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,n\,\left(c+d\,x\right)^2}{4\,b^2\,g^3\,\left(a+b\,x\right)^2} + \frac{d^3\,\,\mathbf{i}^3\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^4\,g^3} - \frac{2\,d\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^3\,\left(a+b\,x\right)} - \frac{\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,b^2\,g^3\,\left(a+b\,x\right)^2} - \frac{B\,d^2\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,n\,Log\left[c+d\,x\right)}{b^4\,g^3} - \frac{B\,d^2\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,n\,PolyLog\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{b^4\,g^3} - \frac{B\,d^2\,\left(b\,c-a\,d\right)\,\,\mathbf{i}^3\,n\,PolyLog\left[e\,\left(\frac{b\,(c+d\,x)}{c+d\,x}\right)^n\right]}{b^4\,g^3} - \frac{B\,d^2\,\left(a+b\,x\right)^2}{b^4\,g^3} - \frac{B\,d^2\,\left(a+b\,x\right)^2}{b^4\,$$

Result (type 4, 461 leaves, 21 steps):

$$\frac{A\,d^{3}\,i^{3}\,x}{b^{3}\,g^{3}} - \frac{B\,\left(b\,c - a\,d\right)^{3}\,i^{3}\,n}{4\,b^{4}\,g^{3}\,\left(a + b\,x\right)^{2}} - \frac{5\,B\,d\,\left(b\,c - a\,d\right)^{2}\,i^{3}\,n}{2\,b^{4}\,g^{3}\,\left(a + b\,x\right)} - \frac{5\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,Log\left[a + b\,x\right]}{2\,b^{4}\,g^{3}} - \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,Log\left[a + b\,x\right]^{2}}{2\,b^{4}\,g^{3}} + \frac{B\,d^{3}\,i^{3}\,\left(a + b\,x\right)\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]}{b^{4}\,g^{3}} - \frac{\left(b\,c - a\,d\right)^{3}\,i^{3}\,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]\right)}{2\,b^{4}\,g^{3}\,\left(a + b\,x\right)^{2}} - \frac{3\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,Log\left[a + b\,x\right]\,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]\right)}{b^{4}\,g^{3}\,\left(a + b\,x\right)} + \frac{3\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,Log\left[a + b\,x\right]\,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{n}\right]\right)}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,Log\left[c + d\,x\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{4}\,g^{3}} + \frac{3\,B\,d^{2}\,\left(b\,c - a\,d\right)\,i^{3}\,n\,PolyLog\left[2, -\frac{d\,(a + b\,x)}{b\,c - a\,d}\right]}{b^{2$$

# Problem 134: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^3\,\left(A + B\,\mathsf{Log}\left[\,e\,\left(\frac{a + b\,\mathbf{x}}{c + d\,\mathbf{x}}\right)^{\,n}\,\right]\,\right)}{\left(a\,g + b\,g\,\mathbf{x}\right)^4}\,\mathrm{d}\mathbf{x}$$

### Optimal (type 4, 326 leaves, 9 steps):

$$-\frac{B\,d^{2}\,i^{3}\,n\,\left(c+d\,x\right)}{b^{3}\,g^{4}\,\left(a+b\,x\right)}-\frac{B\,d\,i^{3}\,n\,\left(c+d\,x\right)^{2}}{4\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}}-\frac{B\,i^{3}\,n\,\left(c+d\,x\right)^{3}}{9\,b\,g^{4}\,\left(a+b\,x\right)^{3}}-\frac{d^{2}\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{4}\,\left(a+b\,x\right)}-\frac{d\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}}-\frac{d^{2}\,i^{3}\,\left(c+d\,x\right)^{3}}{9\,b\,g^{4}\,\left(a+b\,x\right)^{3}}-\frac{d^{2}\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{4}\,\left(a+b\,x\right)}+\frac{B\,d^{3}\,i^{3}\,n\,PolyLog\left[2,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{4}\,g^{4}}$$

### Result (type 4, 444 leaves, 22 steps):

$$\frac{ \left[ \begin{array}{c} \frac{B \left( b \, c - a \, d \right)^3 \, i^3 \, n}{9 \, b^4 \, g^4 \, \left( a + b \, x \right)^3} - \frac{7 \, B \, d \, \left( b \, c - a \, d \right)^2 \, i^3 \, n}{12 \, b^4 \, g^4 \, \left( a + b \, x \right)^2} - \frac{11 \, B \, d^2 \, \left( b \, c - a \, d \right) \, i^3 \, n}{6 \, b^4 \, g^4 \, \left( a + b \, x \right)} - \frac{11 \, B \, d^3 \, i^3 \, n \, Log \left[ a + b \, x \right)}{6 \, b^4 \, g^4} - \frac{2 \, b^4 \, g^4}{2 \, b^4 \, g^4} - \frac{2 \, b^4 \, g^4}{2 \, b^4 \, g^4} - \frac{\left( b \, c - a \, d \right)^2 \, i^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{3 \, b^4 \, g^4 \, \left( a + b \, x \right)^3} - \frac{3 \, d \, \left( b \, c - a \, d \right)^2 \, i^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{2 \, b^4 \, g^4 \, \left( a + b \, x \right)^2} - \frac{3 \, d^2 \, \left( b \, c - a \, d \right) \, i^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{2 \, b^4 \, g^4 \, \left( a + b \, x \right)^2} + \frac{2 \, b^4 \, g^4 \, \left( a + b \, x \right)}{2 \, b^4 \, g^4} + \frac{3 \, i^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c + d \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \right]}{b^4 \, g^4} + \frac{B \, d^3 \, i^3 \, n \, PolyLog \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c \,$$

# Problem 135: Result valid but suboptimal antiderivative.

$$\frac{\left( a g + b g x \right)^{3} \left( A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right] \right)}{c i + d i x} dx$$

#### Optimal (type 4, 269 leaves, 6 steps):

$$\frac{g^{3} \left(a+b\,x\right)^{3} \left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,d\,i} - \frac{\left(b\,c-a\,d\right)\,g^{3} \left(a+b\,x\right)^{2} \left(3\,A+B\,n+3\,B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{6\,d^{2}\,i} + \frac{\left(b\,c-a\,d\right)^{2}g^{3} \left(a+b\,x\right) \left(6\,A+5\,B\,n+6\,B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{6\,d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{3}g^{3} \left(6\,A+11\,B\,n+6\,B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right) \,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{6\,d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}g^{3}\,n\,PolyLog\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}g^{3}\,n\,PolyLog\left[2,\,\frac{d\,a+b\,x}{b\,\left(c+d\,x\right)}\right]}{d^{4}\,i} + \frac{B\,\left(a\,a+b\,x}{b\,\left(c+d\,x\right)}\right)}{d^{4}\,i} + \frac{B\,\left($$

#### Result (type 4, 426 leaves, 22 steps):

$$\frac{A\,b\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,x}{d^{3}\,i} + \frac{5\,b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,n\,x}{6\,d^{3}\,i} - \frac{B\,\left(b\,c-a\,d\right)\,g^{3}\,n\,\left(a+b\,x\right)^{2}}{6\,d^{2}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{3}\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d^{3}\,i} - \frac{\left(b\,c-a\,d\right)\,g^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,d^{2}\,i} + \frac{g^{3}\,\left(a+b\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,d\,i} + \frac{11\,B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,n\,Log\left[c+d\,x\right]}{6\,d^{4}\,i} - \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,n\,Log\left[i\,\left(c+d\,x\right)\right]^{2}}{2\,d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,n\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c\,i+d\,i\,x\right]}{d^{4}\,i} - \frac{\left(b\,c-a\,d\right)^{3}\,g^{3}\,n\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,n\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^{4}\,i} + \frac{B\,\left(b\,c-a\,d\right)^{3}\,g^{3}\,$$

# Problem 136: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{2} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)}{c i + d i x} dx$$

### Optimal (type 4, 211 leaves, 5 steps):

$$\frac{g^{2} \left(a+b\,x\right)^{2} \left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,d\,\mathbf{i}} - \frac{\left(b\,c-a\,d\right)\,g^{2} \left(a+b\,x\right) \,\left(2\,A+B\,n+2\,B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,d^{2}\,\mathbf{i}} - \frac{\left(b\,c-a\,d\right)^{2}\,g^{2} \left(2\,A+3\,B\,n+2\,B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,d^{3}\,\mathbf{i}} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2\,\frac{d\,(a+b\,x)}{b\,(c+d\,x)}\right]}{d^{3}\,\mathbf{i}} - \frac{B\,\left(a+b\,x\right)^{2}\,g^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,n\,PolyLog\left[a+b\,x\right)^{2}\,$$

Result (type 4, 343 leaves, 18 steps):

$$-\frac{A\,b\,\left(b\,c-a\,d\right)\,g^{2}\,x}{d^{2}\,i} - \frac{b\,B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,x}{2\,d^{2}\,i} - \frac{B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d^{2}\,i} + \frac{g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,d^{3}\,i} + \frac{3\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,Log\left[i\,\left(c+d\,x\right)\right]^{2}}{2\,d^{3}\,i} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,Log\left[-\frac{d\,\left(a+b\,x\right)}{b\,c-a\,d}\right]\,Log\left[c\,i+d\,i\,x\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{d^{3}\,i} - \frac{B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,\left(c+d\,x\right)}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,c-a\,d}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,c-a\,d}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,c-a\,d}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,PolyLog\left[2,\frac{b\,c-a\,d}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{\left(b\,c-a\,d\right)^{2}\,n$$

# Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{c\,i+d\,i\,x}\,\mathrm{d}x$$

Optimal (type 4, 134 leaves, 4 steps):

$$\frac{g\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\,\right)}{\mathsf{d}\,\mathsf{i}}}{\mathsf{d}\,\mathsf{i}} + \frac{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,g\left(\mathsf{A}+\mathsf{B}\,\mathsf{n}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\,\right)\,\mathsf{Log}\big[\,\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}^2\,\mathsf{i}} + \frac{\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,g\,\mathsf{n}\,\mathsf{PolyLog}\big[\,\mathsf{2}\,,\,\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}^2\,\mathsf{i}}$$

Result (type 4, 223 leaves, 13 steps):

$$\frac{A \ b \ g \ x}{d \ i} + \frac{B \ g \ \left(a + b \ x\right) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{d \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ n \ Log \left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ n \ Log \left[-\frac{d \ (a + b \ x)}{b \ c - a \ d}\right] \ Log \left[c + d \ x\right]}{d^2 \ i} - \frac{\left(b \ c - a \ d\right) \ g \ n \ Log \left[c + d \ x\right]}{d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ n \ Log \left[-\frac{d \ (a + b \ x)}{b \ c - a \ d}\right] \ Log \left[c + d \ x\right]}{d^2 \ i} - \frac{B \ \left(b \ c - a \ d\right) \ g \ n \ Log \left[c + d \ x\right]^2}{2 \ d^2 \ i} + \frac{B \ \left(b \ c - a \ d\right) \ g \ n \ PolyLog \left[2, \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^2 \ i}$$

# Problem 138: Result valid but suboptimal antiderivative.

$$\int \frac{A + B Log \left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]}{c i + d i x} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\mathsf{n}}\right]\right)\,\mathsf{Log}\!\left[\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]}{\mathsf{d}\,\mathsf{i}}-\frac{\mathsf{B}\,\mathsf{n}\,\mathsf{PolyLog}\!\left[\mathsf{2},\,\frac{\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\mathsf{b}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\right]}{\mathsf{d}\,\mathsf{i}}$$

Result (type 4, 128 leaves, 9 steps):

$$\frac{B\,n\,\text{Log}\big[\text{i}\,\big(\text{c}+\text{d}\,\text{x}\big)\,\big]^2}{2\,\text{d}\,\text{i}} - \frac{B\,n\,\text{Log}\big[-\frac{\text{d}\,(\text{a}+\text{b}\,\text{x})}{\text{b}\,\text{c}-\text{a}\,\text{d}}\,\big]\,\,\text{Log}\,[\text{c}\,\text{i}+\text{d}\,\text{i}\,\text{x}]}{\text{d}\,\text{i}} + \frac{\left(A+B\,\text{Log}\big[\text{e}\,\left(\frac{\text{a}+\text{b}\,\text{x}}{\text{c}+\text{d}\,\text{x}}\right)^n\big]\right)\,\,\text{Log}\,[\text{c}\,\text{i}+\text{d}\,\text{i}\,\text{x}]}{\text{d}\,\text{i}} - \frac{B\,n\,\text{PolyLog}\big[2,\,\frac{\text{b}\,(\text{c}+\text{d}\,\text{x})}{\text{b}\,\text{c}-\text{a}\,\text{d}}\big]}{\text{d}\,\text{i}}$$

Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right) \left( c i + d i x \right)} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{\left(A + B \log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)^{2}}{2 B \left(b c - a d\right) g i n}$$

Result (type 4, 316 leaves, 18 steps):

$$-\frac{B\,n\,Log\,[\,a+b\,x\,]^{\,2}}{2\,\left(b\,c-a\,d\right)\,g\,i} + \frac{Log\,[\,a+b\,x\,]\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c-a\,d\right)\,g\,i} + \frac{B\,n\,Log\,\left[\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,}{\left(b\,c-a\,d\,g\,g\,i} - \frac{\left(A+B\,Log\,\left[\,e\,\left($$

Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[ e^{\left(\frac{a + b x}{c + d x}\right)^{n}} \right]}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$-\frac{b\,B\,n\,\left(c\,+\,d\,x\right)}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,2}\,\mathbf{i}\,\left(a\,+\,b\,x\right)}\,-\,\frac{b\,\left(c\,+\,d\,x\right)\,\left(A\,+\,B\,Log\left[\,e\,\left(\frac{a\,+\,b\,x}{c\,+\,d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,2}\,\mathbf{i}\,\left(a\,+\,b\,x\right)}\,-\,\frac{d\,\left(A\,+\,B\,Log\left[\,e\,\left(\frac{a\,+\,b\,x}{c\,+\,d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,\frac{a\,+\,b\,x}{c\,+\,d\,x}\,\right]}{\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,2}\,\mathbf{i}}\,+\,\frac{B\,d\,n\,Log\left[\,\frac{a\,+\,b\,x}{c\,+\,d\,x}\,\right]^{\,2}}{2\,\left(b\,c\,-\,a\,d\right)^{\,2}\,g^{\,2}\,\mathbf{i}}$$

Result (type 4, 455 leaves, 22 steps):

$$-\frac{B\,n}{\left(b\,c-a\,d\right)\,g^{2}\,\mathbf{i}\,\left(a+b\,x\right)} - \frac{B\,d\,n\,Log\,[\,a+b\,x\,]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,n\,Log\,[\,a+b\,x\,]^{2}}{2\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{\left(b\,c-a\,d\right)\,g^{2}\,\mathbf{i}\,\left(a+b\,x\right)} - \frac{d\,Log\,[\,a+b\,x\,]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,n\,Log\,[\,c+d\,x\,]}{2\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,n\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{d\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,n\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,n\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{d\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} + \frac{B\,d\,n\,Log\,\left[\,c+d\,x\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,n\,PolyLog\,\left[\,2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,\mathbf{i}} - \frac{B\,d\,n\,Poly$$

# Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{3} \left( c i + d i x \right)} dx$$

### Optimal (type 3, 266 leaves, 7 steps):

$$-\frac{B\,n\,\left(c+d\,x\right)^{\,2}\,\left(b-\frac{4\,d\,\left(a+b\,x\right)}{c+d\,x}\right)^{\,2}}{4\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)^{\,2}} + \frac{2\,b\,d\,\left(c+d\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)} - \frac{b^{\,2}\,\left(c+d\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{2\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}\,\left(a+b\,x\right)^{\,2}} + \frac{d^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\left[\,\frac{a+b\,x}{c+d\,x}\,\right]}{\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}} - \frac{B\,d^{\,2}\,n\,Log\left[\,\frac{a+b\,x}{c+d\,x}\,\right]^{\,2}}{2\,\left(b\,c-a\,d\right)^{\,3}\,g^{\,3}\,\mathbf{i}}$$

#### Result (type 4, 557 leaves, 26 steps):

$$-\frac{B\,n}{4\,\left(b\,c-a\,d\right)\,g^3\,i\,\left(a+b\,x\right)^2} + \frac{3\,B\,d\,n}{2\,\left(b\,c-a\,d\right)^2\,g^3\,i\,\left(a+b\,x\right)} + \frac{3\,B\,d^2\,n\,Log\left[a+b\,x\right]}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{B\,d^2\,n\,Log\left[a+b\,x\right]^2}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{2\,\left(b\,c-a\,d\right)\,g^3\,i\,\left(a+b\,x\right)^2} + \frac{d\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^2\,g^3\,i\,\left(a+b\,x\right)} + \frac{d^2\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{3\,B\,d^2\,n\,Log\left[c+d\,x\right]}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i} + \frac{B\,d^2\,n\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,n\,Log\left[a+b\,x\right]}{\left(b\,c-a\,d\right)^3\,g^3\,i} - \frac{d^2\,n\,Log\left[a+b\,x\right]}{\left(b$$

# Problem 142: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{4} \left( c i + d i x \right)} dx$$

Optimal (type 3, 389 leaves, 8 steps):

$$-\frac{3 \, b \, B \, d^{2} \, n \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{2}} - \frac{b^{3} \, B \, n \, \left(c + d \, x\right)^{3}}{9 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{3}} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{3 \, b \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{3 \, b^{2} \, d \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{3 \, b^{2} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{3}} - \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i} \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d \, \left(c + d \, x\right)^{2} \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} - \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right) \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right)^{2} \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right)^{2} \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right)^{2} \, \left(a + b \, x\right)^{2} \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d^{2} \, \left(c + d \, x\right)^{2} \, \left(a + b \, x\right)^{2} \, \left(a + b \, x\right)^{2}}{4 \, \left(a + b \, x\right)^{2}} + \frac{3 \, b^{2} \, d^{2} \, \left(c$$

Result (type 4, 646 leaves, 30 steps):

$$\frac{B \, \text{n}}{9 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{g}^4 \, \text{i} \, \left(\text{a} + \text{b} \, \text{x}\right)^3} + \frac{5 \, \text{B} \, \text{d} \, \text{n}}{12 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^2 \, \text{g}^4 \, \text{i} \, \left(\text{a} + \text{b} \, \text{x}\right)^2} - \frac{11 \, \text{B} \, \text{d}^2 \, \text{n}}{6 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^3 \, \text{g}^4 \, \text{i} \, \left(\text{a} + \text{b} \, \text{x}\right)} - \frac{11 \, \text{B} \, \text{d}^3 \, \text{n} \, \text{Log} \left[\text{a} + \text{b} \, \text{x}\right)}{6 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} - \frac{A + B \, \text{Log} \left[\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]}{3 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right) \, \text{g}^4 \, \text{i} \, \left(\text{a} + \text{b} \, \text{x}\right)^3} + \frac{d \, \left(\text{A} + B \, \text{Log} \left[\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]\right)}{2 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} - \frac{d^2 \, \left(\text{A} + B \, \text{Log} \left[\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]\right)}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} + \frac{d^3 \, \text{n} \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]\right)}{6 \, \left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} - \frac{B \, \text{d}^3 \, \text{n} \, \text{Log} \left[\text{c} + \, \text{d} \, \text{x}\right]}{b \, \text{c} - \text{a} \, \text{d}}\right)^4 \, \text{g}^4 \, \text{i}} - \frac{B \, \text{d}^3 \, \text{n} \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]\right) \, \text{Log} \left[\text{c} + \, \text{d} \, \text{x}\right]}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} - \frac{B \, \text{d}^3 \, \text{n} \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right] \, \text{Log} \left[\text{c} + \, \text{d} \, \text{x}\right]}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} + \frac{d^3 \, \left(\text{A} + B \, \text{Log} \left[\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{c} + \text{d} \, \text{x}}\right)^n\right]\right) \, \text{Log} \left[\text{c} + \, \text{d} \, \text{x}\right]}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} + \frac{d^3 \, \text{n} \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{d}}\right)^n\right] \, \text{Log} \left[\text{c} + \, \text{d} \, \text{d}\right)}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} + \frac{d^3 \, \left(\text{A} + B \, \text{Log} \left[\text{e} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{d}}\right)^n\right] \, \text{Log} \left[\text{c} + \, \text{d} \, \text{d}\right)}{\left(\text{b} \, \text{c} - \text{a} \, \text{d}\right)^4 \, \text{g}^4 \, \text{i}} + \frac{d^3 \, \text{n} \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{b} \, \text{x}}{\text{d}}\right)^n\right] \, \text{Log} \left[\text{c} \, \left(\frac{\text{a} + \text{$$

# Problem 143: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(\,c\,\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 4, 359 leaves, 9 steps):

$$\frac{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, \left(a + b \, x\right)}{d^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} - \frac{\left(b \, c - a \, d\right)^2 \, g^3 \, \left(6 \, A + 5 \, B \, n\right) \, \left(a + b \, x\right)}{2 \, d^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} - \frac{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{d^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{g^3 \, \left(a + b \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d \, \mathbf{i}^2 \, \left(c + d \, x\right)} - \frac{\left(b \, c - a \, d\right) \, g^3 \, \left(a + b \, x\right)^2 \, \left(3 \, A + B \, n + 3 \, B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} - \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)}{2 \, d^2 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{2 \, d^2 \, \mathbf$$

Result (type 4, 541 leaves, 21 steps):

$$-\frac{A \, b^2 \, \left(2 \, b \, c - 3 \, a \, d\right) \, g^3 \, x}{d^3 \, i^2} - \frac{b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, n \, x}{2 \, d^3 \, i^2} - \frac{B \, \left(b \, c - a \, d\right)^3 \, g^3 \, n}{d^4 \, i^2 \, \left(c + d \, x\right)} - \frac{a^2 \, b \, B \, g^3 \, n \, Log \left[a + b \, x\right]}{2 \, d^2 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[a + b \, x\right]}{d^4 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[a + b \, x\right]}{d^4 \, i^2} - \frac{b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[a + b \, x\right]}{d^4 \, i^2} - \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} - \frac{3 \, b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{3 \, b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{2 \, d^4 \, i^2} - \frac{3 \, b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} - \frac{3 \, b \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, c^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c + d \, x\right]}{d^4 \, i^2} + \frac{b^3 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, Log \left[c +$$

# Problem 144: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(\,c\,\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 4, 275 leaves, 8 steps):

$$-\frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)}+\frac{\left(b\,c-a\,d\right)\,g^{2}\,\left(2\,A+B\,n\right)\,\left(a+b\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)}+\frac{2\,B\,\left(b\,c-a\,d\right)\,g^{2}\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d\,i^{2}\,\left(c+d\,x\right)}+\frac{d^{2}\,i^{2}\,\left(c+d\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)}+\frac{d^{2}\,i^{2}\,\left(c+d\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)}+\frac{d^{2}\,i^{2}\,\left(c+d\,x\right)}{d^{2}\,i^{2}\,\left(c+d\,x\right)}+\frac{d^{2}\,i^{2}\,\left(c+d\,x\right)}{d^{2}\,i^{2}}+\frac{d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}}{d^{2}\,i^{2}\,d^{2}\,i^{2}}+\frac{d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}}{d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}}+\frac{d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,d^{2}\,i^{2}\,d^{2}\,i^{2}\,d^{2}\,d^{2}\,d^{2}\,i^{2}\,d^{2}\,$$

Result (type 4, 351 leaves, 17 steps):

$$\frac{A \, b^2 \, g^2 \, x}{d^2 \, i^2} + \frac{B \, \left(b \, c - a \, d\right)^2 \, g^2 \, n}{d^3 \, i^2 \, \left(c + d \, x\right)} + \frac{b \, B \, \left(b \, c - a \, d\right) \, g^2 \, n \, Log \left[a + b \, x\right]}{d^3 \, i^2} + \frac{b \, B \, g^2 \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{d^2 \, i^2} - \frac{\left(b \, c - a \, d\right)^2 \, g^2 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d^3 \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, n \, Log \left[c + d \, x\right]}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, n \, Log \left[-\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d}\right] \, Log \left[c + d \, x\right]}{d^3 \, i^2} - \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, n \, Log \left[c + d \, x\right]^2}{d^3 \, i^2} + \frac{2 \, b \, B \, \left(b \, c - a \, d\right) \, g^2 \, n \, Poly Log \left[2, \, \frac{b \, \left(c + d \, x\right)}{b \, c - a \, d}\right]}{d^3 \, i^2}$$

# Problem 145: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right) \left(A + B Log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 4, 168 leaves, 7 steps):

$$-\frac{A\,g\,\left(a+b\,x\right)}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,+\,\frac{B\,g\,n\,\left(a+b\,x\right)}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,-\,\frac{B\,g\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d\,\mathbf{i}^{2}\,\left(c+d\,x\right)}\,-\,\frac{b\,g\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}\,-\,\frac{b\,B\,g\,n\,PolyLog\left[2\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{2}\,\mathbf{i}^{2}}$$

Result (type 4, 234 leaves, 14 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g \, n}{d^2 \, \dot{\imath}^2 \, \left(c + d \, x\right)} - \frac{b \, B \, g \, n \, Log \left[a + b \, x\right]}{d^2 \, \dot{\imath}^2} + \frac{\left(b \, c - a \, d\right) \, g \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d^2 \, \dot{\imath}^2} + \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]^2}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, PolyLog \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]^2}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log \left[c + d \, x\right]}{d^2 \, \dot{\imath}^2} - \frac{b \, B \, g \, n \, Log$$

# Problem 146: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( c i + d i x \right)^{2}} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{A\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)}-\frac{B\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)}+\frac{B\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)}$$

Result (type 3, 107 leaves, 4 steps):

$$\frac{B\,n}{d\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)} + \frac{b\,B\,n\,\mathsf{Log}\,[\,\mathsf{a}+\mathsf{b}\,\mathsf{x}\,]}{d\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathbf{i}^2} - \frac{A+B\,\mathsf{Log}\,\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^n\,\big]}{d\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)} - \frac{b\,B\,n\,\mathsf{Log}\,[\,\mathsf{c}+\mathsf{d}\,\mathsf{x}\,]}{d\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\mathbf{i}^2}$$

Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right) \left( c i + d i x \right)^{2}} dx$$

Optimal (type 3, 166 leaves, 5 steps):

$$-\frac{\text{Ad}\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\mathsf{g}\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}+\frac{\text{Bd}\,\mathsf{n}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\mathsf{g}\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}-\frac{\text{Bd}\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\mathsf{Log}\!\left[\mathsf{e}\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right]}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\mathsf{g}\,\mathbf{i}^2\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}+\frac{\mathsf{b}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right]\right)^2}{2\,\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\mathsf{g}\,\mathbf{i}^2\,\mathsf{n}^2}$$

Result (type 4, 450 leaves, 22 steps):

$$-\frac{B\,n}{\left(b\,c-a\,d\right)\,g\,i^{2}\,\left(c+d\,x\right)} - \frac{b\,B\,n\,Log\,[\,a+b\,x\,]}{\left(b\,c-a\,d\right)^{2}\,g\,i^{2}} - \frac{b\,B\,n\,Log\,[\,a+b\,x\,]^{\,2}}{2\,\left(b\,c-a\,d\right)^{\,2}\,g\,i^{2}} + \frac{A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{\left(b\,c-a\,d\right)\,g\,i^{\,2}\,\left(c+d\,x\right)} + \frac{b\,B\,n\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} + \frac{b\,B\,n\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} + \frac{b\,B\,n\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} + \frac{b\,B\,n\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,n\,Log\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,n\,PolyLog\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,n\,PolyLog\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} - \frac{b\,B\,n\,PolyLog\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g\,i^{\,2}} + \frac{b\,B\,n\,PolyLog\,[\,c+d\,x\,]}{\left(b\,c-a\,d\right)^{\,2}\,g$$

Problem 148: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{2} \left( c i + d i x \right)^{2}} dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$-\frac{B\,d^{2}\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}-\frac{b^{2}\,B\,n\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}\,\left(a+b\,x\right)}+\frac{d^{2}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}-\frac{b^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}\,\left(c+d\,x\right)}-\frac{2\,b\,d\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}}+\frac{b\,B\,d\,n\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,\mathbf{i}^{2}}$$

Result (type 4, 482 leaves, 26 steps):

$$-\frac{b\,B\,n}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{2}\,\left(a+b\,x\right)} + \frac{B\,d\,n}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{2}\,\left(c+d\,x\right)} + \frac{b\,B\,d\,n\,Log\,[\,a+b\,x\,]^{\,2}}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{b\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{2}\,\left(a+b\,x\right)} - \frac{d\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{2}\,\left(a+b\,x\right)} - \frac{2\,b\,B\,d\,n\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} + \frac{2\,b\,d\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,\left[\,c+d\,x\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} + \frac{2\,b\,d\,\left(A+B\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,\left[\,c+d\,x\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} + \frac{b\,B\,d\,n\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,\left[\,c+d\,x\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} + \frac{b\,B\,d\,n\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)\,Log\,\left[\,c+d\,x\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} + \frac{b\,B\,d\,n\,Log\,\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{2\,b\,B\,d\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{2\,b\,B\,d\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{2\,b\,B\,d\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{2\,b\,B\,d\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\,d\right)^{3}\,g^{2}\,i^{2}} - \frac{2\,b\,B\,d\,n\,PolyLog\,\left[\,2\,,\,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\,\right]}{\left(b\,c-a\,d\,d\right)^{3}\,$$

# Problem 149: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{3} \left( c i + d i x \right)^{2}} dx$$

### Optimal (type 3, 380 leaves, 8 steps):

$$\frac{B \, d^3 \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{3 \, b^2 \, B \, d \, n \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)} - \frac{b^3 \, B \, n \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)} - \frac{d^3 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e\, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(c + d \, x\right)} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e\, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)} - \frac{b^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e\, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2 \, \left(a + b \, x\right)} - \frac{3 \, b \, B \, d^2 \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{3 \, b \, B \, d^2 \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2} - \frac{3 \, b \, B \, d^2 \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, \mathbf{i}^2}$$

### Result (type 4, 656 leaves, 30 steps):

$$\frac{b \, B \, n}{4 \, \left(b \, c - a \, d\right)^2 \, g^3 \, i^2 \, \left(a + b \, x\right)^2} + \frac{5 \, b \, B \, d \, n}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, i^2 \, \left(a + b \, x\right)} - \frac{B \, d^2 \, n}{\left(b \, c - a \, d\right)^3 \, g^3 \, i^2 \, \left(c + d \, x\right)} + \frac{3 \, b \, B \, d^2 \, n \, Log \left[a + b \, x\right]}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{3 \, b \, B \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i^2} - \frac{b \, d^2 \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^3 \, i$$

# Problem 150: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]}{\left(ag + bgx\right)^{4} \left(ci + dix\right)^{2}} dx$$

Optimal (type 3, 477 leaves, 4 steps):

$$-\frac{B\,d^{4}\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{6\,b^{2}\,B\,d^{2}\,n\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)} + \frac{b^{3}\,B\,d\,n\,\left(c+d\,x\right)^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} - \frac{b^{4}\,B\,n\,\left(c+d\,x\right)^{3}}{9\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{3}} + \frac{d^{4}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{9\,\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{3}} + \frac{d^{4}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{6\,b^{2}\,d^{2}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{2\,b^{3}\,d\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{2\,b\,B\,d^{3}\,n\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{\left(b\,c-a\,d\right)^{5}\,g^{4}\,\mathbf{i}^{2}}$$

Result (type 4, 735 leaves, 34 steps):

$$\frac{b\,B\,n}{9\,\left(b\,c-a\,d\right)^2\,g^4\,i^2\,\left(a+b\,x\right)^3} + \frac{2\,b\,B\,d\,n}{3\,\left(b\,c-a\,d\right)^3\,g^4\,i^2\,\left(a+b\,x\right)^2} - \frac{13\,b\,B\,d^2\,n}{3\,\left(b\,c-a\,d\right)^4\,g^4\,i^2\,\left(a+b\,x\right)} + \frac{B\,d^3\,n}{\left(b\,c-a\,d\right)^4\,g^4\,i^2\,\left(c+d\,x\right)} - \frac{10\,b\,B\,d^3\,n\,Log\left[a+b\,x\right]}{3\,\left(b\,c-a\,d\right)^5\,g^4\,i^2} + \frac{2\,b\,B\,d^3\,n\,Log\left[a+b\,x\right]^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} - \frac{b\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{3\,\left(b\,c-a\,d\right)^2\,g^4\,i^2\,\left(a+b\,x\right)^3} + \frac{b\,d\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^3\,g^4\,i^2\,\left(a+b\,x\right)^2} - \frac{3\,b\,d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{3\,\left(b\,c-a\,d\right)^2\,g^4\,i^2\,\left(a+b\,x\right)^3} + \frac{b\,d\,d^3\,Log\left[a+b\,x\right]^2}{\left(b\,c-a\,d\right)^3\,g^4\,i^2\,\left(a+b\,x\right)^2} - \frac{3\,b\,d^3\,g^4\,i^2\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^4\,g^4\,i^2\,\left(c+d\,x\right)} - \frac{4\,b\,d^3\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} + \frac{10\,b\,d^3\,n\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} - \frac{4\,b\,B\,d^3\,n\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} + \frac{4\,b\,d^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} - \frac{2\,b\,B\,d^3\,n\,Log\left[c+d\,x\right]^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} - \frac{4\,b\,B\,d^3\,n\,PolyLog\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^5\,g^4\,i^2} - \frac{4\,b\,B\,d^3\,n\,PolyLog\left[2,\frac{b\,(c+d\,x)}{$$

# Problem 151: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}}\,\,\mathrm{d}x$$

Optimal (type 4, 382 leaves, 9 steps):

$$-\frac{3 \ B \ \left(b \ C - a \ d\right) \ g^{3} \ n \ \left(a + b \ x\right)^{2}}{4 \ d^{2} \ i^{3} \ \left(c + d \ x\right)^{2}} - \frac{3 \ b \ B \ \left(b \ C - a \ d\right) \ g^{3} \ n \ \left(a + b \ x\right)}{d^{3} \ i^{3} \ \left(c + d \ x\right)} + \frac{b \ \left(b \ C - a \ d\right) \ g^{3} \ \left(3 \ A + B \ n\right) \ \left(a + b \ x\right)}{d^{3} \ i^{3} \ \left(c + d \ x\right)} + \frac{3 \ b \ B \ \left(b \ C - a \ d\right) \ g^{3} \ \left(a + b \ x\right)^{2} \ \left(a + b \ x\right)^{2} \left(3 \ A + B \ n + 3 \ B \ Log\left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{d^{3} \ i^{3} \ \left(c + d \ x\right)} + \frac{d \ i^{3} \ \left(a + b \ x\right)^{3} \ \left(A + B \ Log\left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}} + \frac{d \ i^{3} \ \left(c + d \ x\right)^{2}}{d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}} + \frac{d \ b^{2} \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{d^{4} \ i^{3}} + \frac{d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}}{d^{4} \ i^{3}} + \frac{d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}}{d^{4} \ i^{3}} + \frac{d^{3} \ b^{3} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{d^{4} \ i^{3}} + \frac{d^{3} \ i^{3} \ \left(c + d \ x\right)^{2}}{d^{4} \ i^{3}} + \frac{d^{3} \ b^{3} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{d^{4} \ i^{3}} + \frac{d^{3} \ b^{3} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{d^{4} \ i^{3}} + \frac{d^{3} \ b^{3} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog\left[2, \ \frac{d \ (a + b \ x)}{b \ (c + d \ x)}\right]}{d^{4} \ i^{3}} + \frac{d^{3} \ b^{3} \ b^{$$

Result (type 4, 461 leaves, 21 steps):

$$\frac{A \ b^{3} \ g^{3} \ x}{d^{3} \ i^{3}} - \frac{B \ \left(b \ c - a \ d\right)^{3} \ g^{3} \ n}{4 \ d^{4} \ i^{3} \ \left(c + d \ x\right)^{2}} + \frac{5 \ b \ B \ \left(b \ c - a \ d\right)^{2} \ g^{3} \ n}{2 \ d^{4} \ i^{3} \ \left(c + d \ x\right)} + \frac{5 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[a + b \ x\right)}{2 \ d^{4} \ i^{3}} + \frac{5 \ b \ B \ \left(b \ c - a \ d\right)^{3} \ g^{3} \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{2 \ d^{4} \ i^{3}} + \frac{\left(b \ c - a \ d\right)^{3} \ g^{3} \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{2 \ d^{4} \ i^{3}} - \frac{3 \ b \ \left(b \ c - a \ d\right)^{2} \ g^{3} \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^{n}\right]\right)}{d^{4} \ i^{3}} - \frac{7 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]^{2}}{d^{4} \ i^{3}} + \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[2, \ \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]^{2}}{d^{4} \ i^{3}} + \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[2, \ \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]^{2}}{d^{4} \ i^{3}} + \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[2, \ \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[c + d \ x\right]^{2}}{d^{4} \ i^{3}} + \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[2, \ \frac{b \ (c + d \ x)}{b \ c - a \ d}\right]}{d^{4} \ i^{3}} - \frac{3 \ b^{2} \ B \ \left(b \ c - a \ d\right) \ g^{3} \ n \ Log \left[a \ b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ g^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ p^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right) \ p^{3} \ n \ PolyLog \left[a \ b \ c - a \ d\right)$$

# Problem 152: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(c\,i+d\,i\,x\right)^{3}}\,\mathrm{d}x$$

Optimal (type 4, 263 leaves, 8 steps):

$$\frac{B\,g^{2}\,n\,\left(a+b\,x\right)^{2}}{4\,d\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} - \frac{A\,b\,g^{2}\,\left(a+b\,x\right)}{d^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{b\,B\,g^{2}\,n\,\left(a+b\,x\right)}{d^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{b\,B\,g^{2}\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{g^{2}\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,\mathbf{i}^{3}} - \frac{b^{2}\,B\,g^{2}\,n\,PolyLog\left[2\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^{3}\,\mathbf{i}^{3}}$$

Result (type 4, 356 leaves, 18 steps):

$$\frac{B \left(b \, c - a \, d\right)^{2} g^{2} \, n}{4 \, d^{3} \, i^{3} \, \left(c + d \, x\right)^{2}} - \frac{3 \, b \, B \, \left(b \, c - a \, d\right) \, g^{2} \, n}{2 \, d^{3} \, i^{3} \, \left(c + d \, x\right)} - \frac{3 \, b^{2} \, B \, g^{2} \, n \, Log \left[a + b \, x\right]}{2 \, d^{3} \, i^{3}} - \frac{\left(b \, c - a \, d\right)^{2} \, g^{2} \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{2 \, d^{3} \, i^{3} \, \left(c + d \, x\right)} + \frac{2 \, b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{2 \, d^{3} \, i^{3}} - \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{2 \, d^{3} \, i^{3}} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{2 \, d^{3} \, i^{3}} - \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{2 \, d^{3} \, i^{3}} - \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{2 \, d^{3} \, i^{3}} - \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} - \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \, x\right]^{2}}{b \, c - a \, d} + \frac{b^{2} \, B \, g^{2} \, n \, Log \left[c + d \,$$

# Problem 153: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right) \left(A + B Log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)}{\left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{\text{Bgn}\left(\mathsf{a}+\mathsf{bx}\right)^{2}}{4\left(\mathsf{bc-ad}\right)\,\mathbf{i}^{3}\left(\mathsf{c}+\mathsf{dx}\right)^{2}}+\frac{\mathsf{g}\left(\mathsf{a}+\mathsf{bx}\right)^{2}\left(\mathsf{A}+\mathsf{BLog}\left[\mathsf{e}\left(\frac{\mathsf{a}+\mathsf{bx}}{\mathsf{c}+\mathsf{dx}}\right)^{\mathsf{n}}\right]\right)}{2\left(\mathsf{bc-ad}\right)\,\mathbf{i}^{3}\left(\mathsf{c}+\mathsf{dx}\right)^{2}}$$

Result (type 3, 201 leaves, 10 steps):

$$-\frac{B \left(b \, c - a \, d\right) \, g \, n}{4 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} + \frac{b \, B \, g \, n}{2 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{b^2 \, B \, g \, n \, Log \left[a + b \, x\right]}{2 \, d^2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3} + \frac{\left(b \, c - a \, d\right) \, g \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{b \, g \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d^2 \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{b^2 \, B \, g \, n \, Log \left[c + d \, x\right]}{2 \, d^2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3}$$

# Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right) \left( c i + d i x \right)^{3}} dx$$

Optimal (type 3, 254 leaves, 4 steps):

$$-\frac{B\,n\,\left(4\,b-\frac{d\,\left(a+b\,x\right)}{c+d\,x}\right)^{2}}{4\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}+\frac{d^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}}-\\\\ \frac{2\,b\,d\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}\,\left(c+d\,x\right)}+\frac{b^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}{\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}-\frac{b^{2}\,B\,n\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{3}\,g\,\mathbf{i}^{3}}$$

Result (type 4, 557 leaves, 26 steps):

$$\frac{B\,n}{4\,\left(b\,c-a\,d\right)\,g\,i^3\,\left(c+d\,x\right)^2} - \frac{3\,b\,B\,n}{2\,\left(b\,c-a\,d\right)^2\,g\,i^3\,\left(c+d\,x\right)} - \frac{3\,b^2\,B\,n\,Log\left[a+b\,x\right]}{2\,\left(b\,c-a\,d\right)^3\,g\,i^3} - \frac{b^2\,B\,n\,Log\left[a+b\,x\right]^2}{2\,\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{2\,\left(b\,c-a\,d\right)\,g\,i^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^2\,g\,i^3\,\left(c+d\,x\right)} + \frac{b^2\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{3\,b^2\,B\,n\,Log\left[c+d\,x\right]}{2\,\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{b^2\,B\,n\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g\,i^3} - \frac{b^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g\,i^3} - \frac{b^2\,B\,n\,Log\left[c+d\,x\right]}{\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{b^2\,B\,n\,Log\left[a+b\,x\right]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{b^2\,B\,n\,PolyLog\left[2\,,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{\left(b\,c-a\,d\right)^3\,g\,i^3} + \frac{b^2\,B\,n\,PolyLog\left[2\,,$$

# Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]}{\left(ag + bgx\right)^{2} \left(ci + dix\right)^{3}} dx$$

### Optimal (type 3, 381 leaves, 4 steps):

$$\frac{B\,d^{3}\,n\,\left(a+b\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} - \frac{3\,b\,B\,d^{2}\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{b^{3}\,B\,n\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(a+b\,x\right)} - \frac{d^{3}\,\left(a+b\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} + \frac{3\,b\,d^{2}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} - \frac{b^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}} + \frac{3\,b^{2}\,B\,d\,n\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^{2}}{2\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,\mathbf{i}^{3}}$$

### Result (type 4, 657 leaves, 30 steps):

$$-\frac{b^2 \, B \, n}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(a + b \, x\right)} + \frac{B \, d \, n}{4 \, \left(b \, c - a \, d\right)^2 \, g^2 \, i^3 \, \left(c + d \, x\right)^2} + \frac{5 \, b \, B \, d \, n}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} + \frac{3 \, b^2 \, B \, d \, n \, Log \left[a + b \, x\right]}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B \, d \, n \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{b^2 \, A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)^2} - \frac{2 \, b \, d \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, d \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, d \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)} - \frac{3 \, b^2 \, d \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[a \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} - \frac{3 \, b^2 \, B \, d \, n \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, d \, d \, Rog \left[a + b \, x\right] \, \left(a + B \, Log \left[a + b \, x\right] \, \left(a +$$

# Problem 157: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{3} \left( c i + d i x \right)^{3}} dx$$

Optimal (type 3, 483 leaves, 5 steps):

$$-\frac{B\,d^{4}\,n\,\left(a+b\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)^{2}} + \frac{4\,b\,B\,d^{3}\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(c+d\,x\right)} + \frac{4\,b^{3}\,B\,d\,n\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(a+b\,x\right)} - \frac{b^{4}\,B\,n\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{5}\,g^{3}\,\mathbf{i}^{3}\,\left(a+b\,x\right)^{2}} + \frac{d^{4}\,d\,a+b\,x^{2}$$

Result (type 4, 701 leaves, 34 steps):

$$-\frac{b^2\,B\,n}{4\,\left(b\,c-a\,d\right)^3\,g^3\,i^3\,\left(a+b\,x\right)^2} + \frac{7\,b^2\,B\,d\,n}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(a+b\,x\right)} - \frac{B\,d^2\,n}{4\,\left(b\,c-a\,d\right)^3\,g^3\,i^3\,\left(c+d\,x\right)^2} - \frac{7\,b\,B\,d^2\,n}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} - \frac{3\,b^2\,B\,d^2\,n\,Log\left[a+b\,x\right]^2}{\left(b\,c-a\,d\right)^5\,g^3\,i^3} - \frac{b^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i^3\,\left(a+b\,x\right)^2} + \frac{3\,b^2\,d\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(a+b\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i^3\,\left(c+d\,x\right)^2} + \frac{3\,b\,d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)^2} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^3\,g^3\,i^3\,\left(c+d\,x\right)^2} + \frac{3\,b\,d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)^2} + \frac{3\,b\,d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^4\,g^3\,i^3\,\left(c+d\,x\right)} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{2\,\left(b\,c-a\,d\right)^5\,g^3\,i^3} + \frac{d^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n$$

# Problem 158: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( a g + b g x \right)^{4} \left( c i + d i x \right)^{3}} dx$$

Optimal (type 3, 587 leaves, 8 steps):

$$\frac{B \, d^{5} \, n \, \left(a + b \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(c + d \, x\right)^{2}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(c + d \, x\right)} - \frac{10 \, b^{3} \, B \, d^{2} \, n \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(c + d \, x\right)^{2}} + \frac{5 \, b^{4} \, B \, d \, n \, \left(c + d \, x\right)^{2}}{4 \, \left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)^{2}} - \frac{b^{5} \, B \, n \, \left(c + d \, x\right)^{3}}{9 \, \left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)^{2}} - \frac{b^{5} \, B \, n \, \left(c + d \, x\right)^{3}}{9 \, \left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)^{2}} - \frac{b^{5} \, B \, n \, \left(c + d \, x\right)^{3}}{9 \, \left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)^{2}} - \frac{b^{5} \, b \, d^{4} \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(c + d \, x\right)} - \frac{10 \, b^{3} \, d^{2} \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3} \, \left(a + b \, x\right)^{3}} - \frac{10 \, b^{2} \, d^{3} \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a + b \, x}{c + d \, x}\right]^{n}}{\left(b \, c - a \, d\right)^{6} \, g^{4} \, i^{3}} + \frac{5 \, b^{2} \, B \, d^{3} \, n \, Log\left[\frac{a \, b \, x}{c + d \, x}\right]^{n}}{\left(b \, c$$

#### Result (type 4, 859 leaves, 38 steps):

$$\frac{b^2 \, B \, n}{9 \, (b \, c - a \, d)^3 \, g^4 \, i^3 \, (a + b \, x)^3} + \frac{11 \, b^2 \, B \, d \, n}{12 \, (b \, c - a \, d)^4 \, g^4 \, i^3 \, (a + b \, x)^2} - \frac{47 \, b^2 \, B \, d^2 \, n}{6 \, (b \, c - a \, d)^5 \, g^4 \, i^3 \, (a + b \, x)} + \frac{B \, d^3 \, n}{4 \, (b \, c - a \, d)^4 \, g^4 \, i^3 \, (c + d \, x)^2} + \frac{9 \, b \, B \, d^3 \, n}{4 \, (b \, c - a \, d)^5 \, g^4 \, i^3 \, (a + b \, x)} + \frac{5 \, b^2 \, B \, d^3 \, n \, Log \left[a + b \, x\right]^2}{(b \, c - a \, d)^5 \, g^4 \, i^3 \, (c + d \, x)} + \frac{5 \, b^2 \, B \, d^3 \, n \, Log \left[a + b \, x\right]^2}{(b \, c - a \, d)^6 \, g^4 \, i^3} - \frac{b^2 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, (b \, c - a \, d)^6 \, g^4 \, i^3 \, (a + b \, x)} + \frac{5 \, b^2 \, B \, d^3 \, n \, Log \left[a + b \, x\right]^2}{(b \, c - a \, d)^6 \, g^4 \, i^3} - \frac{b^2 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, (b \, c - a \, d)^6 \, g^4 \, i^3 \, (a + b \, x)} + \frac{4 \, b \, d^3 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, (b \, c - a \, d)^6 \, g^4 \, i^3 \, (a + b \, x)} + \frac{3 \, b^2 \, d \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, (b \, c - a \, d)^6 \, g^4 \, i^3} - \frac{4 \, b \, d^3 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} - \frac{10 \, b^2 \, d^3 \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, d^3 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, d^3 \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{(b \, c - a \, d)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B \, d^3 \, n$$

### Problem 159: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2\,\mathrm{d}x$$

Optimal (type 4, 584 leaves, 11 steps):

$$\frac{3 \, B^2 \, \left( b \, c - a \, d \right)^4 \, g^3 \, i \, n^2 \, x}{10 \, b \, d^3} - \frac{3 \, B^2 \, \left( b \, c - a \, d \right)^3 \, g^3 \, i \, n^2 \, \left( c + d \, x \right)^2}{20 \, d^4} + \frac{b \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, i \, n^2 \, \left( c + d \, x \right)^3}{30 \, d^4} - \frac{B \, \left( b \, c - a \, d \right)^2 \, g^3 \, i \, n \, \left( a + b \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{30 \, b^2 \, d} - \frac{B \, \left( b \, c - a \, d \right) \, g^3 \, i \, n \, \left( a + b \, x \right)^4 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{10 \, b^2} + \frac{g^3 \, i \, \left( a + b \, x \right)^4 \, \left( c + d \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{5 \, b} + \frac{B \, \left( b \, c - a \, d \right)^3 \, g^3 \, i \, n \, \left( a + b \, x \right)^2 \, \left( 3 \, A + B \, n + 3 \, B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{5 \, b} - \frac{B \, \left( b \, c - a \, d \right)^3 \, g^3 \, i \, n \, \left( a + b \, x \right)^2 \, \left( 3 \, A + B \, n + 3 \, B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{60 \, b^2 \, d^3} - \frac{B \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n \, \left( 6 \, A + 11 \, B \, n + 6 \, B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{60 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, c - a \, d \right)^5 \, g^3 \, i \, n^2 \, PolyLog \left[ 2 , \, \frac{d \, \left( a + b \, x$$

#### Result (type 4, 670 leaves, 52 steps):

$$\frac{A \, B \, \left( b \, C - a \, d \right)^4 \, g^3 \, i \, n \, x}{10 \, b \, d^3} + \frac{B^2 \, \left( b \, C - a \, d \right)^4 \, g^3 \, i \, n^2 \, x}{600 \, b \, d^3} - \frac{B^2 \, \left( b \, C - a \, d \right)^3 \, g^3 \, i \, n^2 \, \left( a + b \, x \right)^2}{30 \, b^2 \, d^2} + \frac{B^2 \, \left( b \, C - a \, d \right)^3 \, g^3 \, i \, n^2 \, \left( a + b \, x \right)^3}{30 \, b^2 \, d} - \frac{B^2 \, \left( b \, C - a \, d \right)^3 \, g^3 \, i \, n \, \left( a + b \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{10 \, b^2 \, d^3} - \frac{B \, \left( b \, C - a \, d \right)^3 \, g^3 \, i \, n \, \left( a + b \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{20 \, b^2 \, d^2} - \frac{B \, \left( b \, C - a \, d \right)^2 \, g^3 \, i \, n \, \left( a + b \, x \right)^4 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{10 \, b^2} - \frac{B \, \left( b \, C - a \, d \right) \, g^3 \, i \, n \, \left( a + b \, x \right)^4 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{10 \, b^2} + \frac{d \, g^3 \, i \, \left( a + b \, x \right)^5 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{10 \, b^2} + \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{12 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)^5 \, g^3 \, i \, n^2 \, Log \left[ c + d \, x \right]}{10 \, b^2 \, d^4} - \frac{B^2 \, \left( b \, C - a \, d \right)$$

# Problem 160: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^2\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2\,\mathrm{d}x$$

Optimal (type 4, 487 leaves, 10 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^3 g^2 \, i \, n^2 \, x}{3 \, b \, d^2} + \frac{B^2 \left( b \, c - a \, d \right)^2 g^2 \, i \, n^2 \left( c + d \, x \right)^2}{12 \, d^3} - \frac{B \left( b \, c - a \, d \right)^2 g^2 \, i \, n \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{12 \, b^2} + \frac{\left( b \, c - a \, d \right) g^2 \, i \, \left( a + b \, x \right)^3 \left( A + B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{12 \, b^2} + \frac{\left( b \, c - a \, d \right) g^2 \, i \, n \, \left( a + b \, x \right)^3 \left( A + B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{4 \, b} + \frac{B \left( b \, c - a \, d \right)^3 g^2 \, i \, n \, \left( a + b \, x \right) \left( 2 \, A + B \, n + 2 \, B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{12 \, b^2} + \frac{B \left( b \, c - a \, d \right)^3 g^2 \, i \, n \, \left( a + b \, x \right) \left( 2 \, A + B \, n + 2 \, B \, Log \left[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{12 \, b^2} + \frac{B \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, Log \left[ c + d \, x \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3} + \frac{B^2 \left( b \, c - a \, d \right)^4 g^2 \, i \, n^2 \, PolyLog \left[ 2, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)} \right]}{6 \, b^2 \, d^3$$

#### Result (type 4, 578 leaves, 44 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)^3\,g^2\,i\,n\,x}{6\,b\,d^2} - \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,i\,n^2\,x}{12\,b\,d^2} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,g^2\,i\,n^2\,\left(a+b\,x\right)^2}{12\,b^2\,d} + \\ \frac{B^2\,\left(b\,c-a\,d\right)^3\,g^2\,i\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{6\,b^2\,d^2} - \frac{B\,\left(b\,c-a\,d\right)^2\,g^2\,i\,n\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{12\,b^2\,d} - \\ \frac{B\,\left(b\,c-a\,d\right)\,g^2\,i\,n\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{6\,b^2} + \frac{\left(b\,c-a\,d\right)\,g^2\,i\,\left(a+b\,x\right)^3\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{3\,b^2} + \\ \frac{d\,g^2\,i\,\left(a+b\,x\right)^4\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{4\,b^2} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,Log\left[c+d\,x\right]}{12\,b^2\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,Log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,Log\left[c+d\,x\right]}{6\,b^2\,d^3} - \\ \frac{B\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[c+d\,x\right]}{12\,b^2\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,Log\left[c+d\,x\right]^2}{12\,b^2\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,PolyLog\left[2,\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^2\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,Log\left[c+d\,x\right]^2}{6\,b^2\,d^3} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,PolyLog\left[2,\,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^2\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,g^2\,i\,n^2\,Log\left[c+d\,x\right]^2}{6\,b^2\,d^3} - \frac{B^2\,\left(b\,c-a\,d\right)^4$$

### Problem 161: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) (c i + d i x) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 372 leaves, 9 steps):

$$\frac{B^{2} \left(b \, c-a \, d\right)^{2} g \, i \, n^{2} \, x}{3 \, b \, d} - \frac{B \left(b \, c-a \, d\right)^{2} g \, i \, n \, \left(a+b \, x\right) \, \left(A+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)}{3 \, b^{2} \, d} - \frac{B \left(b \, c-a \, d\right) \, g \, i \, n \, \left(a+b \, x\right)^{2} \, \left(A+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)}{3 \, b^{2}} + \frac{g \, i \, \left(a+b \, x\right)^{2} \, \left(c+d \, x\right) \, \left(A+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)^{2}}{3 \, b} - \frac{B \left(b \, c-a \, d\right)^{3} g \, i \, n \, \left(A+B \, n+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right) \, Log\left[\frac{b \, c-a \, d}{b \, (c+d \, x)}\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c-a \, d\right)^{3} g \, i \, n^{2} \, Log\left[c+d \, x\right]}{3 \, b^{2} \, Log\left[c+d \, x\right]} - \frac{B^{2} \left(b \, c-a \, d\right)^{3}$$

Result (type 4, 1323 leaves, 72 steps):

Result (type 4, 1323 leaves, 72 steps): 
$$-\frac{2}{a} \text{ Ab B} \left( \frac{a^2}{b^2} - \frac{c^2}{c^2} \right) \text{ dg in } x - \frac{AB \left( bc - ad \right) \left( bc + ad \right) \text{ gin } x}{bb} + \frac{B^2 \left( bc - ad \right)^2 \text{ gin}^2 x}{3bd} + \frac{a^2 B^2 \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right]^2}{3b^2} - \frac{b}{b}$$

$$\frac{a^3 B^2 d \text{ gin}^2 \text{ Log} \left[ a + bx \right]^2}{3b^2} + \frac{a^2 B^2 \left( bc + ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right]^2}{2b^2} - \frac{B^2 \left( bc - ad \right) \left( bc + ad \right) \text{ gin} \left( a + bx \right) \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right]}{3b^2} + \frac{a^2 B^2 \left( bc - ad \right) \left( a + bx \right) \left( a + b \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{3b^2} + \frac{2a^2 B \text{ cgin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{c + d} + \frac{2a^3 B \text{ dgin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{c + d} + \frac{a^2 B \left( bc - ad \right) \text{ gin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin} \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( A + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B^2 \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B^2 \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left( \frac{a + bx}{c + dx} \right)^n \right] \right)}{ab^2} + \frac{a^2 B^2 \left( bc - ad \right) \text{ gin}^2 \text{ Log} \left[ a + bx \right] \left( a + B \text{ Log} \left[ e \left($$

# Problem 162: Result valid but suboptimal antiderivative.

$$\int (c i + d i x) \left( A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,i\,n\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^2}+\frac{i\,\left(c+d\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{2\,d}+\\\\ \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,Log\left[c+d\,x\right]}{b^2\,d}+\frac{B\,\left(b\,c-a\,d\right)^2\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[1-\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^2\,d}-\frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,PolyLog\left[2,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^2\,d}$$

Result (type 4, 307 leaves, 15 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)\,i\,n\,x}{b} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,Log\,[\,a+b\,x\,]^{\,2}}{2\,b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)\,i\,n\,\left(a+b\,x\right)\,Log\,\big[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\big]}{b^2} - \frac{B^2\,\left(b\,c-a\,d\right)\,i\,n\,\left(a+b\,x\right)\,Log\,\big[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\big]}{b^2} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n\,Log\,\big[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\big]\right)}{b^2\,d} + \frac{i\,\left(c+d\,x\right)^2\,\left(A+B\,Log\,\big[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\big]\right)^2}{2\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,Log\,\big[\,c+d\,x\big]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,PolyLog\,\big[\,2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\big]}{b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^2\,i\,n^2\,PolyLog\,\big[\,2,\,-\frac{d\,(a+$$

### Problem 163: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\text{ag+bgx}} \, dx$$

Optimal (type 4, 306 leaves, 8 steps):

Result (type 4, 692 leaves, 36 steps):

$$\frac{\mathsf{AB} \left( \mathsf{b} \, \mathsf{c} - \mathsf{a} \, \mathsf{d} \right) \, \mathsf{in} \, \mathsf{Log} [\, \mathsf{a} + \mathsf{b} \, \mathsf{x} \, \mathsf{l}^{\, 2} \, \mathsf{c} \, \mathsf{b}^{\, 2} \, \mathsf{g} \, \mathsf{g} \, \mathsf{b}^{\, 2} \, \mathsf{g} \, \mathsf{g} \, \mathsf{b}^{\, 2} \, \mathsf{g} \, \mathsf{g} \, \mathsf{b}^{\, 2} \, \mathsf{g} \, \mathsf{g}$$

### Problem 164: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \, \left(\text{A}+\text{BLog}\left[\,\text{e}\,\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\,\text{n}}\,\right]\,\right)^{\,2}}{\left(\text{ag+bgx}\right)^{\,2}} \, \text{d}x}{\left(\text{ag+bgx}\right)^{\,2}}$$

Optimal (type 4, 261 leaves, 7 steps):

$$-\frac{2\,B^{2}\,\text{i}\,n^{2}\,\left(c+d\,x\right)}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{2\,B\,\text{i}\,n\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(c+d\,x\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b\,g^{2}\,\left(a+b\,x\right)}-\frac{i\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)}{b\,g^{2}\,g^{2}}-\frac{i\,\left(a+b\,x\right)\,\left(a$$

Result (type 4, 766 leaves, 40 steps):

$$\frac{2 \, B^2 \, \left( b \, C - a \, d \right) \, i \, n^2}{b^2 \, g^2 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d \, i \, n^2 \, Log \left[ a + b \, x \right]}{b^2 \, g^2} - \frac{B^2 \, d \, i \, n \, Log \left[ a + b \, x \right]}{b^2 \, g^2} + \frac{B^2 \, d \, i \, n^2 \, Log \left[ a + b \, x \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]^2}{b^2 \, g^2} - \frac{B^2 \, d \, i \, Log \left[ a + b \, x \right] \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]^2}{b^2 \, g^2} - \frac{2 \, B \, \left( b \, C - a \, d \right) \, i \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} - \frac{2 \, B \, d \, i \, n \, Log \left[ a + b \, x \right]}{b^2 \, g^2} - \frac{2 \, B \, \left( b \, C - a \, d \right) \, i \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, Log \left[ a + b \, x \right]}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( a + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( a + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]}{b^2 \, g^2} + \frac{2 \, B \, d \, i \, n \, \left( a + B \, Log \left[ e \, \left$$

Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A} + \text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 3, 151 leaves, 3 steps):

$$-\frac{B^{2} i n^{2} \left(c+d \, x\right)^{2}}{4 \, \left(b \, c-a \, d\right) \, g^{3} \, \left(a+b \, x\right)^{2}}-\frac{B \, i n \, \left(c+d \, x\right)^{2} \, \left(A+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)}{2 \, \left(b \, c-a \, d\right) \, g^{3} \, \left(a+b \, x\right)^{2}}-\frac{i \, \left(c+d \, x\right)^{2} \, \left(A+B \, Log\left[e \, \left(\frac{a+b \, x}{c+d \, x}\right)^{n}\right]\right)^{2}}{2 \, \left(b \, c-a \, d\right) \, g^{3} \, \left(a+b \, x\right)^{2}}$$

Result (type 4, 691 leaves, 54 steps):

$$-\frac{B^2 \left(b \, c - a \, d\right) \, i \, n^2}{4 \, b^2 \, g^3 \, \left(a + b \, x\right)^2} - \frac{B^2 \, d \, i \, n^2}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[a + b \, x\right]}{2 \, b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, b^2 \, \left(b \, c - a \, d\right) \, i \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B \, d^2 \, i \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B \, d^2 \, i \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} - \frac{\left(b \, c - a \, d\right) \, i \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)^2} - \frac{d \, i \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, b^2 \, g^3 \, \left(a + b \, x\right)^2} - \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c - d \, x\right]}{b^2 \, a \, d} - \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{B^2 \, d^2 \, i \, n^2 \, Log \left[c + d \, x\right]}{b^2 \, \left(b \, c - a \, d\right) \, g^3} + \frac{$$

Problem 166: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A+BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{4}} \, dx$$

Optimal (type 3, 307 leaves, 7 steps):

Result (type 4, 800 leaves, 62 steps):

$$-\frac{2\,B^{2}\,\left(b\,c-a\,d\right)\,i\,n^{2}}{27\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{3}} + \frac{B^{2}\,d\,i\,n^{2}}{36\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} + \frac{5\,B^{2}\,d^{2}\,i\,n^{2}}{18\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{5\,B^{2}\,d^{3}\,i\,n^{2}\,Log\left[a+b\,x\right]}{18\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{B^{2}\,d^{3}\,i\,n^{2}\,Log\left[a+b\,x\right]^{2}}{6\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{2\,B\,\left(b\,c-a\,d\right)^{2}\,g^{4}}{6\,b^{2}\,\left(a+b\,x\right)^{3}} - \frac{B\,d\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{6\,b^{2}\,g^{4}\,\left(a+b\,x\right)^{2}} + \frac{B\,d^{2}\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{B\,d^{2}\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{B\,d^{2}\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{B\,d^{2}\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)} + \frac{B\,d^{2}\,i\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} - \frac{B\,d^{3}\,i\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{4}} + \frac{B^{2}\,d^{3}\,i\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b^{2}\,\left($$

## Problem 167: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\text{ci+dix}\right) \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{5}} \, dx$$

#### Optimal (type 3, 475 leaves, 9 steps):

$$-\frac{B^2 d^2 i n^2 (c + d x)^2}{4 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B^2 d i n^2 (c + d x)^3}{27 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 B^2 i n^2 (c + d x)^4}{32 (b c - a d)^3 g^5 (a + b x)^4} - \frac{B d^2 i n (c + d x)^2 (A + B Log[e (\frac{a + b x}{c + d x})^n])}{2 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B d i n (c + d x)^3 (A + B Log[e (\frac{a + b x}{c + d x})^n])}{9 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 B i n (c + d x)^4 (A + B Log[e (\frac{a + b x}{c + d x})^n])}{8 (b c - a d)^3 g^5 (a + b x)^4} - \frac{d^2 i (c + d x)^2 (A + B Log[e (\frac{a + b x}{c + d x})^n])^2}{2 (b c - a d)^3 g^5 (a + b x)^2} + \frac{2 b d i (c + d x)^3 (A + B Log[e (\frac{a + b x}{c + d x})^n])^2}{3 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 i (c + d x)^4 (A + B Log[e (\frac{a + b x}{c + d x})^n])^2}{4 (b c - a d)^3 g^5 (a + b x)^4}$$

Result (type 4, 892 leaves, 70 steps):

$$\frac{B^2 \left( b \ c - a \ d \right) \ i \ n^2}{32 \ b^2 \ g^5 \ \left( a + b \ x \right)^4} + \frac{5 \ B^2 \ d \ i \ n^2}{216 \ b^2 \ g^5 \ \left( a + b \ x \right)^3} + \frac{B^2 \ d^2 \ i \ n^2}{144 \ b^2 \ \left( b \ c - a \ d \right) \ g^5 \ \left( a + b \ x \right)^2} - \frac{13 \ B^2 \ d^3 \ i \ n^2}{72 \ b^2 \ \left( b \ c - a \ d \right)^2 \ g^5 \ \left( a + b \ x \right)} - \frac{13 \ B^2 \ d^4 \ i \ n^2 \ Log \left[ a + b \ x \right)}{72 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} + \frac{B \ d^3 \ i \ n^2 \ Log \left[ a + b \ x \right)^3}{8 \ b^2 \ g^5 \ \left( a + b \ x \right)^3} + \frac{B \ d^3 \ i \ n \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B \ d^4 \ i \ n \ Log \left[ a + b \ x \right)^4}{8 \ b^2 \ g^5 \ \left( a + b \ x \right)^3} + \frac{B \ d^3 \ i \ n \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B \ d^4 \ i \ n \ Log \left[ a + b \ x \right) \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B \ d^4 \ i \ n \ Log \left[ a + b \ x \right) \left( A + B \ Log \left[ e \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{6 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{B^2 \ d^4 \ i \ n^2 \ Log \left[ c + d \ x \right]}{12 \ b^2 \ \left( b \ c - a \ d \right)^3 \ g^5} - \frac{$$

### Problem 168: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^3\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^2\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2\,\mathrm{d}x$$

#### Optimal (type 4, 766 leaves, 17 steps):

$$\frac{3B^{2}\left(b\,c-a\,d\right)^{5}g^{3}\,i^{2}\,n^{2}\,x}{20\,b^{2}d^{3}} + \frac{B^{2}\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\,n^{2}\left(a+b\,x\right)^{4}}{60\,b^{3}} - \frac{3B^{2}\left(b\,c-a\,d\right)^{4}g^{3}\,i^{2}\,n^{2}\left(c+d\,x\right)^{2}}{40\,b\,d^{4}} + \frac{B^{2}\left(b\,c-a\,d\right)^{3}g^{3}\,i^{2}\,n^{2}\left(c+d\,x\right)^{3}}{60\,d^{4}} - \frac{B\left(b\,c-a\,d\right)^{3}g^{3}\,i^{2}\,n\left(a+b\,x\right)^{3}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{90\,b^{3}\,d} - \frac{90\,b^{3}\,d}{15\,b^{2}} + \frac{B\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\,n\left(a+b\,x\right)^{4}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{20\,b^{3}} + \frac{B\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\left(a+b\,x\right)^{4}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\left(a+b\,x\right)^{4}\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\left(a+b\,x\right)^{4}\left(c+d\,x\right)\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{2}g^{3}\,i^{2}\left(a+b\,x\right)^{4}\left(c+d\,x\right)\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{4}g^{3}\,i^{2}\,n\left(a+b\,x\right)^{4}\left(c+d\,x\right)\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{4}g^{3}\,i^{2}\,n\left(a+b\,x\right)^{2}\left(3A+B\,n+3\,B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{15\,b^{2}} + \frac{\left(b\,c-a\,d\right)^{4}g^{3}\,i^{2}\,n\left(a+b\,x\right)^{2}\left$$

#### Result (type 4, 848 leaves, 83 steps):

$$\frac{A \ B \ (b \ c - a \ d)^5 \ g^3 \ i^2 \ n \ x}{300 b^2 \ d^3} + \frac{B^2 \ (b \ c - a \ d)^5 \ g^3 \ i^2 \ n^2 \ x}{45 b^2 \ d^3} - \frac{7 \ B^2 \ (b \ c - a \ d)^4 \ g^3 \ i^2 \ n^2 \ (a + b \ x)^2}{360 b^3 \ d^2} + \frac{B^2 \ (b \ c - a \ d)^3 \ g^3 \ i^2 \ n^2 \ (a + b \ x)^3}{60 b^3 \ d} + \frac{B^2 \ (b \ c - a \ d)^2 \ g^3 \ i^2 \ n^2 \ (a + b \ x)^4}{60 b^3} - \frac{B^2 \ (b \ c - a \ d)^5 \ g^3 \ i^2 \ n \ (a + b \ x) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{30 b^3 \ d^3} + \frac{B^2 \ (b \ c - a \ d)^2 \ g^3 \ i^2 \ n \ (a + b \ x)^4}{60 b^3} - \frac{B \ (b \ c - a \ d)^3 \ g^3 \ i^2 \ n \ (a + b \ x)^3 \ (a + b \ x) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{90 b^3 \ d} - \frac{B \ (b \ c - a \ d)^3 \ g^3 \ i^2 \ n \ (a + b \ x)^5 \ (A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right])}{15 b^3} + \frac{(b \ c - a \ d)^2 \ g^3 \ i^2 \ (a + b \ x)^4 \ (A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right])}{4 b^3} + \frac{2 \ d \ (b \ c - a \ d) \ g^3 \ i^2 \ n^2 \ (a + b \ x)^5 \ (A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right])}{5 b^3} + \frac{d^2 \ g^3 \ i^2 \ (a + b \ x)^6 \ (A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right])}{6 b^3} + \frac{d^2 \ g^3 \ i^2 \ (a + b \ x)^6 \ (A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right])}{90 b^3 \ d^4} + \frac{d^3 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]}{90 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^2}{30 b^3 \ d^4} - \frac{B^2 \ (b \ c - a \ d)^6 \ g^3 \ i^2 \ n^2 \ Log \left[c + d \ x\right]^$$

### Problem 169: Result valid but suboptimal antiderivative.

$$\int \left(a g + b g x\right)^{2} \left(c i + d i x\right)^{2} \left(A + B Log\left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 819 leaves, 15 steps):

$$-\frac{B^{2} \left(b \, c - a \, d\right)^{4} g^{2} \, i^{2} \, n^{2}}{10 \, b^{2} \, d^{2}} - \frac{B^{2} \left(b \, c - a \, d\right)^{3} g^{2} \, i^{2} \, n^{2} \left(c + d \, x\right)^{2}}{20 \, b \, d^{3}} + \frac{B^{2} \left(b \, c - a \, d\right)^{2} g^{2} \, i^{2} \, n^{2} \left(c + d \, x\right)^{3}}{30 \, d^{3}} - \frac{B \left(b \, c - a \, d\right)^{3} g^{2} \, i^{2} \, n^{2} \left(a + b \, x\right)^{2} \left(A + B \, Log\left[e^{\left(\frac{a + b \, x}{c + d \, x}\right)^{3}}\right]\right)}{30 \, b^{3} \, d} - \frac{B \left(b \, c - a \, d\right)^{2} g^{2} \, i^{2} \, n^{2} \left(a + b \, x\right)^{3} \left(A + B \, Log\left[e^{\left(\frac{a + b \, x}{c + d \, x}\right)^{3}}\right]\right)}{15 \, b^{3}} - \frac{B \left(b \, c - a \, d\right)^{3} g^{2} \, i^{2} \, n^{2} \left(c + d \, x\right)^{2} \left(A + B \, Log\left[e^{\left(\frac{a + b \, x}{c + d \, x}\right)^{3}}\right]\right)}{15 \, b^{3}} + \frac{4 \, B \left(b \, c - a \, d\right)^{2} g^{2} \, i^{2} \, n^{2} \left(c + d \, x\right)^{3} \left(A + B \, Log\left[e^{\left(\frac{a + b \, x}{c + d \, x}\right)^{3}}\right]\right)}{15 \, b^{3}} - \frac{15 \, d^{3}}{15 \, d^{3}} + \frac{10 \, d^{3}}{10 \, d^{3}} \left(c + d \, x\right)^{3} \left(A + B \, Log\left[e^{\left(\frac{a + b \, x}{c + d \, x}\right)^{3}}\right]\right)^{2}}{10 \, b^{2}} + \frac{10 \, d^{3}}{30 \, b^{3}} + \frac{10 \, b^{3}}{30 \, b^{3}} + \frac{10 \, b^{3}}{30 \, b^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3} \, d^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} + \frac{10 \, b^{3} \, d^{3} \, d^{3}}{10 \, b^{3} \, d^{3}} +$$

#### Result (type 4, 714 leaves, 71 steps):

$$\frac{A B \left(b \, c - a \, d\right)^4 g^2 \, i^2 \, n \, x}{15 \, b^2 \, d^2} - \frac{B^2 \left(b \, c - a \, d\right)^4 g^2 \, i^2 \, n^2 \, x}{15 \, b^2 \, d^2} + \frac{B^2 \left(b \, c - a \, d\right)^3 g^2 \, i^2 \, n^2 \left(a + b \, x\right)^2}{20 \, b^3 \, d} + \frac{B^2 \left(b \, c - a \, d\right)^2 g^2 \, i^2 \, n^2 \left(a + b \, x\right)^3}{30 \, b^3} + \frac{B^2 \left(b \, c - a \, d\right)^4 g^2 \, i^2 \, n \, \left(a + b \, x\right) \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^2} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, i^2 \, n \, \left(a + b \, x\right)^2 \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{30 \, b^3 \, d} - \frac{B \left(b \, c - a \, d\right)^3 g^2 \, i^2 \, n \, \left(a + b \, x\right)^4 \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{10 \, b^3} + \frac{\left(b \, c - a \, d\right)^2 g^2 \, i^2 \, n \, \left(a + b \, x\right)^3 \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{5 \, b^3} + \frac{d \left(b \, c - a \, d\right) g^2 \, i^2 \, n \, \left(a + b \, x\right)^4 \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^3} + \frac{d \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{15 \, b^3 \, d^3} + \frac{d^2 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a \, b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^2 \, i^2 \, n^2 \, Log \left[e \left(\frac{a \, b \, x}{c + d \, x}\right)^n\right]}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c - a \, d\right)^5 g^3 \, d^3}{15 \, b^3 \, d^3} + \frac{d^3 \left(b \, c$$

# Problem 170: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2\,\mathrm{d}x$$

Optimal (type 4, 635 leaves, 14 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{3} g \ i^{2} \ n^{2} }{12 \ b^{2} \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{2} g \ i^{2} \ n^{2} \left(c+d \ x\right)^{2}}{12 \ b \ d^{2}} - \frac{B \left(b \ c-a \ d\right)^{3} g \ i^{2} \ n \left(a+b \ x\right) \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{6 \ b^{3} \ d} - \frac{B \left(b \ c-a \ d\right)^{2} g \ i^{2} \ n \left(a+b \ x\right)^{2} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{6 \ b^{3}} + \frac{B \left(b \ c-a \ d\right)^{2} g \ i^{2} \ n \left(c+d \ x\right)^{3} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{4 \ b \ d^{2}} - \frac{B \left(b \ c-a \ d\right) g \ i^{2} \ n \left(c+d \ x\right)^{3} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{6 \ d^{2}} + \frac{\left(b \ c-a \ d\right) g \ i^{2} \ n \left(c+d \ x\right)^{2} \left(c+d \ x\right) \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{6 \ b^{2}} + \frac{\left(b \ c-a \ d\right) g \ i^{2} \left(a+b \ x\right)^{2} \left(c+d \ x\right) \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{6 \ b^{2}} + \frac{g \ i^{2} \left(a+b \ x\right)^{2} \left(c+d \ x\right)^{2} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{6 \ b^{2}} + \frac{g \ i^{2} \left(a+b \ x\right)^{2} \left(c+d \ x\right)^{2} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{6 \ b^{2}} + \frac{g \ i^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} \left(a+b \ x\right)^{2} \left(c+d \ x\right) \left(a+b \ a \ b \ b \ b \ a^{2} \right)}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} \ n^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} PolyLog\left[2, \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{6 \ b^{3} \ d^{2}} + \frac{g^{2} \left(b \ c-a \ d\right)^{4} g \ i^{2} PolyLog\left[2, \frac{d \ (a$$

#### Result (type 4, 614 leaves, 44 steps):

$$\frac{A\,B\,\left(b\,c-a\,d\right)^{3}\,g\,i^{2}\,n\,x}{6\,b^{2}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g\,i^{2}\,n^{2}\,x}{12\,b^{2}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{2}}{12\,b\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,Log\left[a+b\,x\right]}{12\,b^{3}\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{12\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g\,i^{2}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{6\,b^{3}\,d} + \frac{B\,\left(b\,c-a\,d\right)^{2}\,g\,i^{2}\,n\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{12\,b^{3}\,d^{2}} + \frac{B\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n\,Log\left[a+b\,x\right]\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{12\,b^{2}} - \frac{\left(b\,c-a\,d\right)\,g\,i^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{6\,d^{2}} + \frac{b\,g\,i^{2}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{4\,d^{2}} - \frac{\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,Log\left[c+d\,x\right]}{3\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,Log\left[a+b\,x\right]\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{6\,b^{3}\,d^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{4}\,g\,i^{2}\,n^{2}\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{6\,b^{3}\,d^{2}} +$$

# Problem 171: Result valid but suboptimal antiderivative.

$$\int \left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^2 \,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\right)^2 \,\mathrm{d}\mathbf{x}$$

Optimal (type 4, 361 leaves, 11 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right)^{2} \ i^{2} \ n^{2} \ x}{3 \ b^{2}} - \frac{2 \ B \left(b \ c-a \ d\right)^{2} \ i^{2} \ n \left(a+b \ x\right) \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b^{3}} - \frac{B \left(b \ c-a \ d\right) \ i^{2} \ n \left(c+d \ x\right)^{2} \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{3 \ b \ d} - \frac{i^{2} \left(c+d \ x\right)^{3} \left(A+B \ Log\left[e \left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{3 \ b^{3} \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ Log\left[\frac{a+b \ x}{c+d \ x}\right]}{3 \ b^{3} \ d} + \frac{B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ Log\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ PolyLog\left[c+d \ x\right]}{3 \ b^{3} \ d} + \frac{2 \ B^{2} \left(b \ c-a \ d\right)^{3} \ i^{2} \ n^{2} \ p^{2} \$$

Result (type 4, 454 leaves, 19 steps):

$$\frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,x}{3\,b^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n^{2}\,x}{3\,b^{2}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{Log}\left[a+b\,x\right]}{3\,b^{3}\,d} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{Log}\left[a+b\,x\right]^{2}}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,\left(a+b\,x\right)\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{3\,b^{3}} - \frac{B\,\left(b\,c-a\,d\right)\,i^{2}\,n\,\left(c+d\,x\right)^{2}\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b\,d} - \frac{2\,B\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n\,\text{Log}\left[a+b\,x\right]\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,b^{3}\,d} + \frac{i^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{Log}\left[c+d\,x\right]}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{Log}\left[a+b\,x\right]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{PolyLog}\left[2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{PolyLog}\left[2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{PolyLog}\left[2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{PolyLog}\left[2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,\text{PolyLog}\left[2,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{3\,b^{3}\,d} + \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2}\,n^{2}\,B^{2}\,\left(b\,c-a\,d\right)^{3}\,i^{2$$

# Problem 172: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^2 \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^2}{\text{ag+bgx}} \, dx$$

Optimal (type 4, 572 leaves, 15 steps):

$$\frac{B \ d \ (b \ c - a \ d) \ i^2 \ n \ (a + b \ x) \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)^2}{b^3 \ g} + \frac{d \ (b \ c - a \ d) \ i^2 \ (a + b \ x) \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)^2}{b^3 \ g} + \frac{i^2 \ (c + d \ x)^2 \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)^2 + 2 \ B \ \left(b \ c - a \ d\right)^2 \ i^2 \ n \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right) \ Log \left[\frac{b \ c - a \ d}{b \ (c + d \ x)}\right] + \frac{b^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ Log \left[c + d \ x\right]}{b^3 \ g} + \frac{B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ Log \left[c + d \ x\right]}{b^3 \ g} + \frac{B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ Log \left[c + d \ x\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[2, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \ (c + d \ x)}{d \ (a + b \ x)}\right]}{b^3 \ g} + \frac{2 \ B^2 \ \left(b \ c - a \ d\right)^2 \ i^2 \ n^2 \ PolyLog \left[3, \frac{b \$$

#### Result (type 4, 1790 leaves, 82 steps):

$$\frac{ABd\left(bc-ad\right)i^2nx}{b^2g} - \frac{aB^2d\left(bc-ad\right)i^2n^2Log[a+bx]^2}{b^3g} + \frac{B^2\left(bc-ad\right)^2i^2n^2Log[a+bx]^2}{2b^3g} - \frac{2b^3g}{2b^3g} - \frac{AB\left(bc-ad\right)^2i^2nLog\left[g\left(a+bx\right)\right]^2}{b^3g} + \frac{B^2\left(bc-ad\right)^2i^2n^2Log\left[g\left(a+bx\right)\right]^3}{3b^3g} - \frac{B^2\left(bc-ad\right)^2i^2n^2Log\left[g\left(a+bx\right)\right]^2Log\left[-c-dx\right]}{b^3g} + \frac{B^2\left(bc-ad\right)^2i^2nLog\left[g\left(a+bx\right)\right]^2Log\left[-c-dx\right]}{b^3g} - \frac{B^2\left(bc-ad\right)^2i^2Log\left[\left(a+bx\right)\right]^2Log\left[-c-dx\right]}{b^3g} - \frac{B^2\left(bc-ad\right)^2i^2Log\left[a+bx\right)\left(A+BLog\left[e\left(\frac{a+bx}{c+dx}\right)\right]\right)}{b^3g} - \frac{B^2\left(bc-ad\right)^2i^2Log\left[-\frac{a+bx}{c+dx}\right]}{b^3g} - \frac{B^2\left(bc-ad$$

$$\frac{\left(b\,c-a\,d\right)^{2}\,i^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}\,Log\left[a\,g+b\,g\,x\right]}{b^{3}\,g} + \frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,Log\left[a\,g+b\,g\,x\right]}{b^{3}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,\left(Log\left[\left(a+b\,x\right)^{n}\right] - Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right] + Log\left[\left(c+d\,x\right)^{-n}\right]\right)\,Log\left[a\,g+b\,g\,x\right]}{b^{3}\,g} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,Log\left[a\,g+b\,g\,x\right]^{2}}{b^{3}\,g} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,Log\left[a\,g+b\,g\,x\right]^{2}}{b^{3}\,g} + \frac{2\,A\,B\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,Log\left[a\,g+b\,g\,x\right]^{2}}{b^{3}\,g} + \frac{2\,a\,B^{2}\,d\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(a+b\,x\right)^{n}\right]\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(a+b\,x\right)^{n}\right]\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]\right)\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{3}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]\right)\,PolyLog\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]\right)\,PolyLog\left[3,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,g} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,a^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,a^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]}{b^{2}\,n\,Log\left[\left(c+d\,x\right)^{-n}\right]} - \frac{2\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,i^{2}\,n\,Log\left[\left($$

# Problem 173: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{2}} \, dx$$

Optimal (type 4, 472 leaves, 11 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right) \, \mathbf{i}^2 \, n^2 \, \left( c + d \, x \right)}{b^2 \, g^2 \, \left( a + b \, x \right)} - \frac{2 \, B \, \left( b \, c - a \, d \right) \, \mathbf{i}^2 \, n \, \left( c + d \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{b^2 \, g^2 \, \left( a + b \, x \right)} + \frac{b^2 \, g^2 \, \left( a + b \, x \right)}{b^3 \, g^2} - \frac{\left( b \, c - a \, d \right) \, \mathbf{i}^2 \, \left( c + d \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{b^2 \, g^2 \, \left( a + b \, x \right)} + \frac{2 \, B \, d \, \left( b \, c - a \, d \right) \, \mathbf{i}^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, Log \left[ \frac{b \, c - a \, d}{b \, \left( c + d \, x \right)} \right]}{b^3 \, g^2} - \frac{2 \, d \, \left( b \, c - a \, d \right) \, \mathbf{i}^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, Log \left[ 1 - \frac{b \, \left( c + d \, x \right)}{d \, \left( a + b \, x \right)} \right]}{d \, \left( a + b \, x \right)} + \frac{2 \, B^2 \, d \, \left( b \, c - a \, d \right) \, \mathbf{i}^2 \, n^2 \, PolyLog \left[ 2 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{b^3 \, g^2} + \frac{4 \, B^2 \, d \, \left( b \, c - a \, d \, \right) \, \mathbf{i}^2 \, n^2 \, PolyLog \left[ 3 \, , \, \frac{b \, \left( c + d \, x \right)}{d \, \left( a + b \, x \right)} \right]}{b^3 \, g^2}$$

Result (type 4, 1309 leaves, 60 steps):

$$\frac{2\,B^2\,\left(b\,c-a\,d\right)^2\,i^2\,n^2}{b^3\,g^2\,\left(a+b\,x\right)} = \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)\,i^2\,n^2\,\log[a+b\,x]}{b^3\,g^2} = \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)\,i^2\,n^2\,\log[a+b\,x]^2}{b^3\,g^2} = \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)\,i^2\,\log\left[-\frac{b\,c+a\,d}{d\,(a+b\,x)}\right]\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]^2}{b^3\,g^2} = \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)\,i^2\,\log\left[-\frac{b\,c+a\,d}{d\,(a+b\,x)}\right]\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]^2}{b^3\,g^2} = \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)\,i^2\,\log\left[-\frac{b\,c+a\,d}{d\,(a+b\,x)}\right]\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]^2}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2\,\left(a+b\,x\right)} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} = \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d\,d\,\left(b\,c-a\,d\right)^2\,i^2\,\left(B\,d\,a\right)^2}{b^3\,g^2} + \frac{2\,B\,d$$

# Problem 174: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{2} \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 4, 417 leaves, 10 steps):

$$-\frac{2 \, B^2 \, d \, i^2 \, n^2 \, \left(c + d \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{B^2 \, i^2 \, n^2 \, \left(c + d \, x\right)^2}{4 \, b \, g^3 \, \left(a + b \, x\right)^2} - \frac{2 \, B \, d \, i^2 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, g^3 \, \left(a + b \, x\right)} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \, g^3 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \, g^2 \, g^2 \, \left(a + b \, x\right)^2}{b^2 \, \left(a + b \, x\right)^2} - \frac{b^2 \,$$

Result (type 4, 1003 leaves, 68 steps):

$$\frac{B^2 \left(b \, c - a \, d\right)^2 \, i^2 \, n^2}{4 \, b^3 \, g^3 \, \left(a + b \, x\right)^2} = \frac{5 \, B^2 \, d \, \left(b \, c - a \, d\right)^2 \, i^2 \, n^2}{2 \, b^3 \, g^3} = \frac{5 \, B^2 \, d \, \left(b \, c - a \, d\right)^2 \, i^2 \, n^2}{2 \, b^3 \, g^3} = \frac{5 \, B^2 \, d^2 \, i^2 \, Log \left[-\frac{b \, c - a \, d}{d \, \left(a + b \, x\right)}\right] \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, Log \left[-\frac{b \, c - a \, d}{d \, \left(a + b \, x\right)}\right] \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} = \frac{B^2 \, d^2 \, i^2 \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} = \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} = \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3 \, \left(a + b \, x\right)} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} = \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^3 \, g^3} + \frac{B^2 \, d^2 \, i^2 \, n^2 \, Log \left[e \, \left(\frac{a - b$$

Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a} + \mathsf{b}\,\mathsf{x}}{\mathsf{c} + d\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\right]\,\right)^{\,\mathsf{2}}}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathsf{x}\right)^{\,\mathsf{4}}}\,\mathrm{d}\!\!1\,\mathsf{x}$$

Optimal (type 3, 157 leaves, 3 steps):

$$-\frac{2\,B^{2}\,\mathbf{i}^{2}\,n^{2}\,\left(c+d\,x\right)^{3}}{27\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{3}}\,-\frac{2\,B\,\mathbf{i}^{2}\,n\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{9\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{3}}\,-\frac{\mathbf{i}^{2}\,\left(\,c+d\,x\right)^{3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{2}}{3\,\left(b\,c-a\,d\right)\,g^{4}\,\left(a+b\,x\right)^{3}}$$

Result (type 4, 889 leaves, 86 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, i^2 \, n^2}{27 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B^2 \, d \, \left( b \, c - a \, d \right) \, i^2 \, n^2}{9 \, b^3 \, g^4 \, \left( a + b \, x \right)^2} - \frac{2 \, B^2 \, d^2 \, i^2 \, n^2}{9 \, b^3 \, g^4 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)}{9 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} + \frac{B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B \, d \, \left( b \, c - a \, d \right) \, g^4}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B \, d^3 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B \, d^3 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^2}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B \, d^3 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^3}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^2} - \frac{2 \, B \, d^3 \, i^2 \, n \, \left( a + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B \, d^3 \, i^2 \, n \, Log \left[ a + b \, x \right)^3}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B \, d^3 \, i^2 \, n \, \left( a + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)} - \frac{2 \, B \, d^3 \, i^2 \, n \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, g^4 \, \left( a + b \, x \right)^3} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ c + d \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right) \, Log \left[ c + d \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ e \, d \, a \, b \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ e \, d \, a \, b \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3 \, i^2 \, n^2 \, Log \left[ e \, d \, a \, b \, x \right]}{3 \, b^3 \, \left( b \, c - a \, d \right) \, g^4} - \frac{2 \, B^2 \, d^3$$

Problem 176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a} + \mathsf{b}\,\mathsf{x}}{\mathsf{c} + d\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\right]\,\right)^{\,2}}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathsf{x}\right)^{\,\mathsf{5}}}\,\mathrm{d}\!\!1\,\mathsf{x}$$

Optimal (type 3, 319 leaves, 7 steps):

$$\frac{2\,B^{2}\,d\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{3}}{27\,\left(b\,c-a\,d\right)^{2}\,g^{5}\,\left(a+b\,x\right)^{3}}-\frac{b\,B^{2}\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{4}}{32\,\left(b\,c-a\,d\right)^{2}\,g^{5}\,\left(a+b\,x\right)^{4}}+\frac{2\,B\,d\,i^{2}\,n\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{9\,\left(b\,c-a\,d\right)^{2}\,g^{5}\,\left(a+b\,x\right)^{3}}-\frac{b\,B\,i^{2}\,n\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{3\,\left(b\,c-a\,d\right)^{2}\,g^{5}\,\left(a+b\,x\right)^{3}}-\frac{b\,i^{2}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{4\,\left(b\,c-a\,d\right)^{2}\,g^{5}\,\left(a+b\,x\right)^{4}}$$

Result (type 4, 989 leaves, 98 steps):

$$\frac{B^2 \left(b \ c - a \ d\right)^2 \ i^2 \ n^2}{32 \ b^3 \ g^5 \ \left(a + b \ x\right)^4} - \frac{11 \ B^2 \ d \ \left(b \ c - a \ d\right)^2 i^2 \ n^2}{216 \ b^3 \ g^5 \ \left(a + b \ x\right)^3} + \frac{5 \ B^2 \ d^2 \ i^2 \ n^2}{144 \ b^3 \ g^5 \ \left(a + b \ x\right)^2} + \frac{7 \ B^2 \ d^3 \ i^2 \ n^2}{72 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)} + \frac{7 \ B^2 \ d^4 \ i^2 \ n^2 \ Log \left[a + b \ x\right]}{72 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B \left(b \ c - a \ d\right)^2 i^2 \ n \ \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{8 \ b^3 \ g^5 \ \left(a + b \ x\right)^4} + \frac{5 \ B^2 \ d^2 \ i^2 \ n^2}{8 \ b^3 \ g^5 \ \left(a + b \ x\right)^3} - \frac{5 \ B \ d \ \left(b \ c - a \ d\right) \ i^2 \ n \ \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{8 \ b^3 \ g^5 \ \left(a + b \ x\right)^4} + \frac{B \ d^3 \ i^2 \ n \ \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)} + \frac{B \ d^3 \ i^2 \ n \ \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)} + \frac{B \ d^4 \ i^2 \ n \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)^3} - \frac{B \ d^4 \ i^2 \ n \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)^3} - \frac{B \ d^4 \ i^2 \ n \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right) \ g^5 \ \left(a + b \ x\right)^3} - \frac{B \ d^4 \ i^2 \ n \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{6 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{2 \ d \ \left(b \ c - a \ d\right)^2 \left(A + B \ Log \left[e \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{3 \ b^3 \ g^5 \ \left(a + b \ x\right)^3} - \frac{2 \ d \ \left(b \ c - a \ d\right)^2 \ g^5}{2 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ n^2 \ Log \left[c + d \ x\right]}{6 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} - \frac{B^2 \ d^4 \ i^2 \ n^2 \ Log \left[a + b \ x\right] \ Log \left[a + b \ x\right] \ Log \left[a + b \ x\right]}{6 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ n^2 \ Log \left[a + b \ x\right] \ Log \left[a + b \ x\right]}{6 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2 \ n^2 \ Log \left[a + b \ x\right]}{6 \ b^3 \ \left(b \ c - a \ d\right)^2 \ g^5} + \frac{B^2 \ d^4 \ i^2$$

Problem 177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c\,\mathbf{i} + d\,\mathbf{i}\,\mathbf{x}\right)^{2}\,\left(\mathsf{A} + \mathsf{B}\,\mathsf{Log}\left[\,e\,\left(\frac{\mathsf{a} + \mathsf{b}\,\mathbf{x}}{\mathsf{c} + d\,\mathbf{x}}\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\mathsf{a}\,\mathsf{g} + \mathsf{b}\,\mathsf{g}\,\mathbf{x}\right)^{\,6}}\,\mathrm{d}\mathbf{x}$$

Optimal (type 3, 493 leaves, 9 steps):

$$-\frac{2\,B^{2}\,d^{2}\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{3}}{27\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{3}}+\frac{b\,B^{2}\,d\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{4}}{16\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{2\,b^{2}\,B^{2}\,i^{2}\,n^{2}\,\left(c+d\,x\right)^{5}}{125\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}-\frac{2\,B\,d^{2}\,i^{2}\,n\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}+\frac{b\,B\,d\,i^{2}\,n\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{2\,b^{2}\,B\,i^{2}\,n\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{25\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}-\frac{d^{2}\,i^{2}\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{4\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b\,d\,i^{2}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}-\frac{b^{2}\,i^{2}\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{2\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{4}}-\frac{b\,d\,i^{2}\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{5\,\left(b\,c-a\,d\right)^{3}\,g^{6}\,\left(a+b\,x\right)^{5}}$$

#### Result (type 4, 1085 leaves, 110 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, i^2 \, n^2}{125 \, b^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{7 \, B^2 \, d \, \left( b \, c - a \, d \right) \, i^2 \, n^2}{400 \, b^3 \, g^6 \, \left( a + b \, x \right)^4} + \frac{43 \, B^2 \, d^2 \, i^2 \, n^2}{2700 \, b^3 \, g^6 \, \left( a + b \, x \right)^3} - \frac{13 \, B^2 \, d^3 \, i^2 \, n^2}{1800 \, b^3 \, \left( b \, c - a \, d \right) \, g^6 \, \left( a + b \, x \right)^2} - \frac{47 \, B^2 \, d^5 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^4}{2700 \, b^3 \, g^6 \, \left( a + b \, x \right)^3} - \frac{1800 \, b^3 \, \left( b \, c - a \, d \right) \, g^6 \, \left( a + b \, x \right)^2}{2900 \, b^3 \, \left( b \, c - a \, d \right)^2 \, g^6 \, \left( a + b \, x \right)} - \frac{47 \, B^2 \, d^5 \, i^2 \, n^2 \, Log \left[ a + b \, x \right)^3}{9900 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6} + \frac{B^2 \, d^5 \, i^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{30 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6} - \frac{2 \, B \, \left( b \, c - a \, d \right)^2 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{2900 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6} + \frac{B^2 \, d^5 \, i^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{30 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6} - \frac{2 \, B \, \left( b \, c - a \, d \right)^2 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{25 \, b^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{B \, d^4 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{45 \, b^3 \, g^6 \, \left( a + b \, x \right)^3} + \frac{B \, d^3 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{30 \, b^3 \, \left( b \, c - a \, d \right) \, g^6 \, \left( a + b \, x \right)^2} - \frac{B \, d^4 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{15 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{B \, d^4 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{15 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{B \, d^4 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{15 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{B \, d^4 \, i^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{15 \, b^3 \, \left( b \, c - a \, d \right)^3 \, g^6 \, \left( a + b \, x \right)^5} - \frac{B \, d^3 \, i^2 \, n^2 \, Log \left[ a \, d \, b \, x \right)^3}{15 \, b^3 \, \left( a \, b \,$$

# Problem 178: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^{3} (c i + d i x)^{3} (A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]^{2} dx$$

Optimal (type 4, 1172 leaves, 22 steps):

$$\frac{5B^2 \left(bc-ad\right)^6 g^3 i^3 n^2 x}{84 b^3 d^3} + \frac{B^2 \left(bc-ad\right)^3 g^3 i^3 n^2 \left(a+bx\right)^4}{140 b^4} - \frac{29B^2 \left(bc-ad\right)^5 g^3 i^3 n^2 \left(c+dx\right)^2}{840 b^2 d^4} + \frac{47B^2 \left(bc-ad\right)^4 g^3 i^3 n^2 \left(c+dx\right)^3}{1260 b^4} - \frac{13B^2 \left(bc-ad\right)^3 g^3 i^3 n^2 \left(c+dx\right)^4}{420 d^4} + \frac{bB^2 \left(bc-ad\right)^2 g^3 i^3 n^2 \left(c+dx\right)^5}{105 d^4} - \frac{105 d^4}{420 d^4} + \frac{105 d^4}{105 d^4} + \frac{105 d^4}{105 d^4} + \frac{100 b^4}{105 d^4} + \frac{1$$

Result (type 4, 961 leaves, 118 steps):

$$-\frac{A\ B\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n\ x}{70\ b^{3}\ d^{3}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}}{70\ b^{3}\ d^{3}} - \frac{3\ B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{2}}{280\ b^{4}\ d^{2}} + \frac{11\ B^{2}\ (b\ c-a\ d)^{4}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{3}}{1260\ b^{4}\ d} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{2}}{1260\ b^{4}\ d^{2}} + \frac{11\ B^{2}\ (b\ c-a\ d)^{4}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{3}}{1260\ b^{4}\ d^{3}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{2}}{1260\ b^{4}\ d^{3}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{2}\ (a+b\ x)^{4}}{105\ b^{4}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{5}\ (a+b\ x)\ bog\left[e\left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]}{70\ b^{4}\ d^{4}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{5}\ (a+b\ x)\ bog\left[e\left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]}{105\ b^{4}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n\ (a+b\ x)^{3}\ (a+b\ x)^{3}\ (a+b\ x)\ bog\left[e\left(\frac{a+b\ x}{c+d\ x}\right)^{n}\right]}{100\ b^{4}\ d^{4}} + \frac{B^{2}\ (b\ c-a\ d)^{6}\ g^{3}\ i^{3}\ n^{2}\ (a+b\ x)^{3}\ (a+b$$

# Problem 179: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)^{\,2}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 4, 976 leaves, 20 steps):

$$\frac{7 \, B^2 \, \left( b \, c - a \, d \right)^5 \, g^2 \, i^3 \, n^2 \, x}{180 \, b^3 \, d^2} = \frac{7 \, B^2 \, \left( b \, c - a \, d \right)^4 \, g^2 \, i^3 \, n^2 \, \left( c + d \, x \right)^4}{360 \, b^2 \, d^3} = \frac{8^2 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n^2 \, \left( c + d \, x \right)^3}{60 \, b^3} + \frac{8^2 \, \left( b \, c - a \, d \right)^2 \, g^2 \, i^3 \, n^2 \, \left( c + d \, x \right)^4}{60 \, b^3} = \frac{8 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n^2 \, \left( c + d \, x \right)^3}{60 \, b^4} + \frac{8 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n \, \left( a + b \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right) \right)}{30 \, b^4} = \frac{8 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n \, \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right) \right)}{30 \, b^4} = \frac{8 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n \, \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right) \right)}{30 \, b^4} = \frac{8 \, \left( b \, c - a \, d \right)^3 \, g^2 \, i^3 \, n \, \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right) \right)}{45 \, b^3} = \frac{45 \, b^3}{30 \, b^4} = \frac{7 \, B \, \left( b \, c - a \, d \right)^2 \, g^2 \, i^3 \, n \, \left( c + d \, x \right)^3 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right) \right)}{45 \, b^3} = \frac{15 \, d^3}{30 \, b^4} =$$

Result (type 4, 886 leaves, 83 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)^{5}\,g^{2}\,i^{3}\,n\,x}{30\,b^{3}\,d^{2}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{2}\,i^{3}\,n^{2}\,x}{45\,b^{3}\,d^{2}} - \frac{7\,B^{2}\,\left(b\,c-a\,d\right)^{4}\,g^{2}\,i^{3}\,n^{2}\,\left(c+d\,x\right)^{2}}{360\,b^{2}\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{3}\,g^{2}\,i^{3}\,n^{2}\,\left(c+d\,x\right)^{3}}{60\,b\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,n^{2}\,\left(c+d\,x\right)^{4}}{60\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]}{45\,b^{4}\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]^{4}}{60\,b^{4}\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]}{45\,b^{4}\,d^{3}} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{60\,b^{4}\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{5}\,g^{2}\,i^{3}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{30\,b^{4}\,d^{3}} - \frac{30\,b^{4}\,d^{3}}{90\,b^{4}\,d^{2}} - \frac{90\,b^{4}\,d^{2}}{90\,b^{4}\,d^{2}} + \frac{7\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,n\,\left(c+d\,x\right)^{3}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{90\,b^{4}\,d^{3}} - \frac{b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,n\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{90\,b^{4}\,d^{3}} - \frac{b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,n\,\left(c+d\,x\right)^{5}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{15\,d^{3}} - \frac{b\,B\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,n\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,d^{3}} - \frac{b\,B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,i^{3}\,\left(c+d\,x\right)^{4}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{4\,d^{3}} - \frac{b\,B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]}{6\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,n^{2}\,Log\left[a+b\,x\right]}{30\,b^{4}\,d^{3}} - \frac{B^{2}\,\left(b\,c-a\,d\right)^{6}\,g^{2}\,i^{3}\,$$

# Problem 180: Result valid but suboptimal antiderivative.

$$\int \left(a\,g+b\,g\,x\right)\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^3\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 4, 786 leaves, 19 steps):

$$\frac{B^2 \left( b \, c - a \, d \right)^4 g \, i^3 \, n^2 \, x}{600 \, b^3 \, d} + \frac{B^2 \left( b \, c - a \, d \right)^3 g \, i^3 \, n^2 \left( c + d \, x \right)^2}{300 \, b^2 \, d^2} + \frac{B^2 \left( b \, c - a \, d \right)^2 g \, i^3 \, n^2 \left( c + d \, x \right)^3}{300 \, b \, d^2} - \frac{B \left( b \, c - a \, d \right)^4 g \, i^3 \, n \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{100 \, b^4 \, d} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, n \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{100 \, b^4 \, d^2} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, n \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{100 \, b^4 \, d^2} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, n \, \left( c + d \, x \right)^3 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{300 \, b \, d^2} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, n \, \left( c + d \, x \right)^3 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{300 \, b \, d^2} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, n \, \left( c + d \, x \right)^3 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{100 \, d^2} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{200 \, b^4} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{100 \, b^3} + \frac{B \left( b \, c - a \, d \right)^3 g \, i^3 \, \left( a + b \, x \right)^2 \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{100 \, b^3} + \frac{B \left( b \, c - a \, d \right)^5 g \, i^3 \, n \, \left( A + B \, n + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{100 \, b^4 \, c^2} + \frac{B \left( b \, c - a \, d \right)^5 g \, i^3 \, n \, \left( A + B \, n + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{100 \, b^4 \, c^2} + \frac{B \left( b \, c - a \, d \right)^5 g \, i^3 \, n \, \left( A + B \, n + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{100 \, b^4 \, d^2} + \frac{B \left( b \, c - a \, d \right)^5 g \, i^3 \, n \, \left( A + B \, n + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{100 \, b^4 \, d^2} + \frac{B \left( b \, c - a \, d \right)^5 g \, i^3 \, n \, \left( A + B \, b \, c - a \, d \right)^5 g \, i^3 \, n^2 \, PolyLog \left[$$

#### Result (type 4, 706 leaves, 52 steps):

$$\frac{A \ B \ (b \ c - a \ d)^4 \ g \ i^3 \ n^2 \ }{10 \ b^3 \ d} + \frac{B^2 \ (b \ c - a \ d)^4 \ g \ i^3 \ n^2 \ }{60 \ b^3 \ d} + \frac{B^2 \ (b \ c - a \ d)^3 \ g \ i^3 \ n^2 \ (c + d \ x)^2}{30 \ b^2 \ d^2} + \frac{B^2 \ (b \ c - a \ d)^2 \ g \ i^3 \ n^2 \ (c + d \ x)^3}{30 \ b \ d^2} + \frac{B^2 \ (b \ c - a \ d)^2 \ g \ i^3 \ n^2 \ (c + d \ x)^3}{30 \ b \ d^2} + \frac{B^2 \ (b \ c - a \ d)^4 \ g \ i^3 \ n \ (a + b \ x) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{10 \ b^4 \ d} + \frac{B^2 \ (b \ c - a \ d)^3 \ g \ i^3 \ n \ (c + d \ x) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{10 \ b^4 \ d^2} + \frac{B \ (b \ c - a \ d)^2 \ g \ i^3 \ n \ (c + d \ x) \ (a + b \ x) \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]}{10 \ b^4 \ d^2} - \frac{B \ (b \ c - a \ d)^2 \ g \ i^3 \ n \ Log \left[a + b \ x\right] \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{10 \ b^4 \ d^2} - \frac{B \ (b \ c - a \ d)^5 \ g \ i^3 \ n \ Log \left[a + b \ x\right] \ \left(A + B \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{10 \ b^4 \ d^2} - \frac{B^2 \ (b \ c - a \ d)^5 \ g \ i^3 \ n^2 \ Log \left[a + b \ x\right] \ Log \left[e \ \left(\frac{a + b \ x}{c + d \ x}\right)^n\right]\right)}{10 \ b^4 \ d^2} - \frac{B^2 \ (b \ c - a \ d)^5 \ g \ i^3 \ n^2 \ Log \left[a + b \ x\right] \ Log \left[\frac{b \ (c - a \ d)^5 \ g \ i^3 \ n^2 \ PolyLog \left[2, -\frac{d \ (a + b \ x)}{b \ c - a \ d}\right]}{10 \ b^4 \ d^2}$$

# Problem 181: Result valid but suboptimal antiderivative.

$$\int (c i + d i x)^{3} \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 4, 454 leaves, 15 steps):

$$\frac{5 \, B^2 \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n^2 \, x}{12 \, b^3} + \frac{B^2 \, \left(b \, c - a \, d\right)^2 \, \mathbf{i}^3 \, n^2 \, \left(c + d \, x\right)^2}{12 \, b^2 \, d} - \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, \left(a + b \, x\right) \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^4} - \frac{B \, \left(b \, c - a \, d\right)^3 \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{4 \, b^2 \, d} - \frac{B \, \left(b \, c - a \, d\right) \, \mathbf{i}^3 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{6 \, b \, d} + \frac{11 \, B^2 \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, Log \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{11 \, B^2 \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, Log \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, Log \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, PolyLog \left[c + d \, x\right]}{12 \, b^4 \, d} + \frac{B \, \left(b \, c - a \, d\right)^4 \, \mathbf{i}^3 \, n^2 \, Poly$$

#### Result (type 4, 544 leaves, 23 steps):

$$-\frac{A\,B\,\left(b\,c-a\,d\right)^3\,\mathbf{i}^3\,n\,x}{2\,b^3} + \frac{5\,B^2\,\left(b\,c-a\,d\right)^3\,\mathbf{i}^3\,n^2\,x}{12\,b^3} + \frac{B^2\,\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,n^2\,\left(c+d\,x\right)^2}{12\,b^2\,d} + \frac{5\,B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{Log}\left[a+b\,x\right]}{12\,b^4\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{Log}\left[a+b\,x\right]}{12\,b^4\,d} + \frac{B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{Log}\left[a+b\,x\right]^2}{2\,b^4\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^3\,\mathbf{i}^3\,n\,\left(a+b\,x\right)\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{4\,b^2\,d} - \frac{B\,\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,n\,\left(c+d\,x\right)^2\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} - \frac{B\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n\,\text{Log}\left[a+b\,x\right]\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} + \frac{\mathbf{i}^3\,\left(c+d\,x\right)^4\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} - \frac{B\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{Log}\left[a+b\,x\right]\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} + \frac{\mathbf{i}^3\,\left(c+d\,x\right)^4\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{4\,b^2\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{Log}\left[a+b\,x\right]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{2\,b^4\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\text{PolyLog}\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{2\,b^4\,d} - \frac{B^2\,\left(b\,c-a\,d\right)^4\,\mathbf{i}^3\,n^2\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^2\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i}^3\,n^3\,\mathbf{i$$

### Problem 182: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{3} \left(\text{A+BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\text{ag+bgx}} dx$$

Optimal (type 4, 762 leaves, 26 steps):

$$\frac{8^2 d \left( b c - a d \right)^2 \frac{13}{10} n^2 \chi}{3b^2 g} = \frac{58 d \left( b c - a d \right)^2 \frac{13}{10} \left( a + b x \right) \left( a + b x b \left[ \left( \frac{a + b x}{c + d x} \right)^2 \right)}{3b^2 g} = \frac{3b^2 g}{3b^2 g}$$

$$\frac{d \left( b c - a d \right)^2 \frac{13}{10} \left( a + b x \right) \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \left( b c - a d \right) \frac{13}{10} \left( c - d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( a + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( a + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( c + d x \right)^2 \left( a + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right) \right)^2 + \frac{13}{10} \left( a + b x \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \frac{13}{10} \left( a + b x \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \frac{13}{10} \left( a + b x \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \frac{13}{10} \left( a + b x \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^2 + \frac{13}{10} \left( a + b x \log \left[ e \left( \frac{a + b x}{c + d x} \right)^2 \right)^$$

### Problem 183: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{3} \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{2}} \, dx$$

Optimal (type 4, 739 leaves, 17 steps):

$$\frac{28^{2} \left( bc - ad \right)^{2} 2^{3} n^{2} \left( c + dx \right)}{b^{2} g^{2} \left( a \cdot bx \right)} \frac{Bd^{2} \left( bc - ad \right)^{2} 1^{3} \left( a + bx \right)}{b^{2} g^{2}} \frac{b^{3} g^{2} \left( a \cdot bx \right)}{b^{2} g^{2}} \frac{b^{3} g^{2} \left( a \cdot bx \right)}{b^{4} g^{2}} \frac{b^{4} g^{2}}{b^{4} g^{2}}{b^{4} g^{2}} \frac{b^{4} g^{2}}{b^{4} g^{2}} \frac{b^{4}$$

 $b^4 g^2$ 

$$\frac{2\,B^2\,c\,d\,\left(3\,b\,c-2\,a\,d\right)\,\frac{13}{b}\,n^2\,Log\left[-\,\frac{d\,(a\,b\,b\,x)}{b\,c-a\,d}\,\right]\,Log\left[c\,+\,d\,x\right]}{b^3\,g^2} - \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)^2\,\frac{13}{b}\,n^2\,Log\left[-\,\frac{d\,(a\,b\,b\,x)}{b\,c-a\,d}\,\right]\,Log\left[c\,+\,d\,x\right]}{b^4\,g^2} + \frac{B\,c^2\,d\,i^3\,n\,\left(A\,+\,B\,Log\left[e\,\left(\frac{a\,b\,b\,x}{c\,c\,d\,x}\right)^n\right]\right)\,Log\left[c\,+\,d\,x\right]}{b^2\,g^2} - \frac{2\,B\,c\,d\,\left(3\,b\,c-2\,a\,d\right)\,\,i^3\,n\,\left(A\,+\,B\,Log\left[e\,\left(\frac{a\,b\,b\,x}{c\,c\,d\,x}\right)^n\right]\right)\,Log\left[c\,+\,d\,x\right]}{b^3\,g^2} + \frac{2\,B\,d\,\left(b\,c-a\,d\right)^2\,i^3\,n\,\left(A\,+\,B\,Log\left[e\,\left(\frac{a\,b\,b\,x}{c\,c\,d\,x}\right)^n\right]\right)\,Log\left[c\,+\,d\,x\right]}{b^4\,g^2} + \frac{B^2\,c^2\,d\,i^3\,n^2\,Log\left[c\,+\,d\,x\right]^2}{2\,b^2\,g^2} - \frac{B^2\,c\,d\,\left(3\,b\,c-2\,a\,d\right)\,i^3\,n^2\,Log\left[c\,+\,d\,x\right]^2}{b^3\,g^2} + \frac{B^2\,d\,\left(b\,c-a\,d\right)^2\,i^3\,n\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,a\,d}\right]}{b^4\,g^2} - \frac{a^2\,B^2\,d^3\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,Log\left[a\,+\,b\,x\right]\,Log\left[\frac{b\,(c\,c\,d\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,PolyLog\left[2\,,\,-\frac{d\,(a\,b\,b\,x)}{b\,c\,-a\,d}\right]}{b^4\,g^2} + \frac{2\,B^2\,d\,\left(b\,c\,-a\,d\right)^2\,i^3\,n^2\,$$

### Problem 184: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^{3} \left(\text{A}+\text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}}{\left(\text{ag+bgx}\right)^{3}} \, dx$$

Optimal (type 4, 644 leaves, 13 steps):

$$-\frac{4\,B^{2}\,d\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,\left(c+d\,x\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)} - \frac{B^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,\left(c+d\,x\right)^{2}}{4\,b^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} - \frac{4\,B\,d\,\left(b\,c-a\,d\right)\,i^{3}\,n\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{b^{3}\,g^{3}\,\left(a+b\,x\right)} - \frac{B\,\left(b\,c-a\,d\right)\,i^{3}\,n\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,b^{2}\,g^{3}\,\left(a+b\,x\right)^{2}} + \frac{d^{3}\,i^{3}\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b^{4}\,g^{3}} - \frac{2\,d\,\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b^{3}\,g^{3}\,\left(a+b\,x\right)} - \frac{\left(b\,c-a\,d\right)\,i^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{b^{3}\,g^{3}\,\left(a+b\,x\right)} + \frac{2\,B\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{b^{4}\,g^{3}} + \frac{2\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,PolyLog\left[2,\,\frac{d\,(a+b\,x)}{b\,\left(c+d\,x\right)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{d\,(a-b\,x)}\right]}{b^{4}\,g^{3}} + \frac{6\,B^{2}\,d^{2}\,\left(b\,c-a\,d\right)\,i^{3}\,n^{2}\,PolyLog\left[3,\,\frac{b\,(c+d\,x)}{d\,(a-b\,x)}\right]}{b^{2}\,p^{2}\,p^{2}\,p^{2}\,p^{2}\,p^{2}\,p^{2}\,p^{2}\,p^{2}$$

Result (type 4, 1512 leaves, 88 steps):

$$\frac{B^2 \left( b \ c \ a \ d \right)^3 \ s^3 \ n^2}{4 \ b^4 \ g^3 \ (a + b \ x)^2} \frac{9 B^2 \ d \ (b \ c \ a \ d \right) \ i^3 \ n^2}{2 \ b^4 \ g^3} \frac{9 B^2 \ d \ (b \ c \ a \ d \right) \ i^3 \ n^2 \ log [a + b \ x]^2}{2 \ b^4 \ g^3} \frac{3 \ n^2 \ log [a + b \ x]^2}{2 \ b^4 \ g^3} \frac{b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ B^2 \ d^2 \ (b \ c - a \ d ) \ i^3 \ n^2 \ log [a + b \ x]^2}{2 \ b^4 \ g^3} \frac{3 B^2 \ d^2 \ (b \ c - a \ d ) \ i^3 \ log [a - b \ x]^2}{2 \ b^4 \ g^3} \frac{b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a^3 \ x \ (a + B \ log \left[e \left(\frac{a \cdot b x}{c \cdot d x}\right)^n\right)}{b^4 \ g^3} - \frac{b^4 \ g^3}{b^4 \ g^3} \frac{a \ b^4 \ g^3}{2 \ b^4 \ g^3} \frac{a^3 \ x \ (a + B \ log \left[e \left(\frac{a \cdot b x}{c \cdot d x}\right)^n\right)}{b^4 \ g^3} + \frac{a^3 \ i^3 \ x \ (a + B \ log \left[e \left(\frac{a \cdot b x}{c \cdot d x}\right)^n\right)}{b^3 \ g^3} \frac{a^3 \ a^3 \$$

## Problem 185: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\text{ci+dix}\right)^3 \left(\text{A} + \text{BLog}\left[\text{e}\left(\frac{\text{a}+\text{bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^2}{\left(\text{ag+bgx}\right)^4} \, dx$$

Optimal (type 4, 561 leaves, 13 steps):

$$-\frac{2 \, B^2 \, d^2 \, i^3 \, n^2 \, \left(c + d \, x\right)}{b^3 \, g^4 \, \left(a + b \, x\right)} - \frac{B^2 \, d \, i^3 \, n^2 \, \left(c + d \, x\right)^2}{4 \, b^2 \, g^4 \, \left(a + b \, x\right)^2} - \frac{2 \, B^2 \, i^3 \, n^2 \, \left(c + d \, x\right)^3}{27 \, b \, g^4 \, \left(a + b \, x\right)^3} - \frac{2 \, B \, d^2 \, i^3 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^3 \, g^4 \, \left(a + b \, x\right)} - \frac{2 \, B \, i^3 \, n \, \left(c + d \, x\right)^3}{27 \, b \, g^4 \, \left(a + b \, x\right)} - \frac{2 \, B \, i^3 \, n \, \left(c + d \, x\right)^3}{27 \, b^3 \, g^4 \, \left(a + b \, x\right)} - \frac{2 \, B \, i^3 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^4 \, \left(a + b \, x\right)^2} - \frac{2 \, B \, i^3 \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, b^2 \, g^4 \, \left(a + b \, x\right)^2} - \frac{1^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b \, g^4 \, \left(a + b \, x\right)^3} - \frac{d^3 \, i^3 \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2 \, Log \left[1 - \frac{b \, \left(c + d \, x\right)}{d \, \left(a + b \, x\right)}\right]}{2 \, b^4 \, g^4} + \frac{2 \, B \, d^3 \, i^3 \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{b^4 \, g^4} + \frac{2 \, B^2 \, d^3 \, i^3 \, n^2 \, PolyLog \left[3, \, \frac{b \, \left(c + d \, x\right)}{d \, \left(a + b \, x\right)}\right]}{b^4 \, g^4}$$

Result (type 4, 1170 leaves, 100 steps):

$$\frac{2 \, B^2 \, \left(b \, C - a \, d\right)^3 \, i^3 \, n^2}{27 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{17 \, B^2 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, n^2}{36 \, b^4 \, g^4 \, \left(a + b \, x\right)^2} - \frac{49 \, B^2 \, d^2 \, \left(b \, C - a \, d\right) \, i^3 \, n^2}{18 \, b^4 \, g^4 \, \left(a + b \, x\right)} - \frac{18 \, b^4 \, g^4 \, \left(a + b \, x\right)}{18 \, b^4 \, g^4 \, \left(a + b \, x\right)} - \frac{18 \, b^4 \, g^4 \, \left(a + b \, x\right)}{18 \, b^4 \, g^4} - \frac{18 \, b^4 \, g^4 \, \left(a + b \, x\right)}{6 \, b^4 \, g^4} - \frac{11 \, B^2 \, d^3 \, i^3 \, n^2 \, Log \left[a + b \, x\right]^2}{6 \, b^4 \, g^4} - \frac{B^2 \, d^3 \, i^3 \, Log \left[a - \frac{b \, x \, a}{d \, a \, b \, x}\right] \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]^2}{b^4 \, g^4} - \frac{B^2 \, d^3 \, i^3 \, n \, \left(a + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right] \, - \frac{2}{3} \, B \, d \, \left(b \, C - a \, d\right)^3 \, i^3 \, n \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{9 \, b^4 \, g^4} - \frac{9 \, b^4 \, g^4 \, \left(a + b \, x\right)^3}{9 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{7 \, B \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, n \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)} - \frac{11 \, B \, d^3 \, i^3 \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{3 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{3 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{3 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{3 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, \left(A + B \, Log \left[e \, \left(\frac{a - b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, b^4 \, g^4 \, \left(a + b \, x\right)^3} - \frac{3 \, d \, \left(b \, C - a \, d\right)^2 \, i^3 \, n^2 \, Log \left[c + d \, x\right]}{18 \, b^4 \, g^4} - \frac{3 \, d^3 \, i^3 \, n^2 \, Log \left[c + d \, x\right]}{3 \, b^4 \, g^4} - \frac{3 \, d^3 \, g^4 \, \left(a + b \, x\right)}{3 \, b^4 \, g^4} - \frac{3 \, d^3 \, g^4 \, \left(a + b \, x\right)}{3 \, b^4 \, g^4} - \frac{3 \, d^3 \, g^4 \, \left(a + b \, x\right)}{3 \, b^4 \, g^4} - \frac{3 \, d^3 \, g^4 \, \left(a + b \, x\right)}{3 \, b^4 \, g^4} - \frac{3 \, d^3 \, g^3 \, n^2 \, Log \left[c + d \,$$

# Problem 186: Result valid but suboptimal antiderivative.

$$\left(\frac{\left(a g + b g x\right)^{3} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{c i + d i x}\right)^{3} dx$$

Optimal (type 4, 768 leaves, 25 steps):

$$\frac{b \, B^2 \, \left(b \, c - a \, d\right)^2 \, g^3 \, n^2 \, x}{3 \, d^3 \, i} + \frac{7 \, B \, \left(b \, c - a \, d\right)^2 \, g^3 \, n \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d^3 \, i} - \frac{b^2 \, B \, \left(b \, c - a \, d\right) \, g^3 \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, d^4 \, i} + \frac{3 \, d^3 \, i}{3 \, d^3 \, i} - \frac{3 \, b^2 \, \left(b \, c - a \, d\right) \, g^3 \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4 \, i} + \frac{b^3 \, g^3 \, n^2 \, Log\left[e\left(\frac{a \, b \, x}{c + d \, x}\right)^n\right]}{3 \, d^4$$

#### Result (type 4, 1952 leaves, 101 steps):

$$\frac{5 \, A \, b \, B \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, x}{3 \, d^3 \, i} + \frac{b \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n^2 \, x}{3 \, d^3 \, i} - \frac{a \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n^2 \, Log \left[ a + b \, x \right]^2}{d^3 \, i} + \frac{5 \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, \left( a + b \, x \right) \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]}{3 \, d^3 \, i} - \frac{a \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right]^2}{d^3 \, i} + \frac{5 \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, \left( a + b \, x \right) \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right]}{3 \, d^3 \, i} - \frac{a \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i} + \frac{a \, B \, b \, b \, c \, \left( a + b \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, d \right)^2 \, g^3 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, d \, a \, b \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, d \, a \, b \, x \right)^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a \, b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, a \, b \, x \right)^2 \, \left( a \, b \, c \, b \, a \, d \, a \, b \, x \right)^2}{d^3 \, i} + \frac{a \, B^2 \, \left( b \, c - a \, d \, a \, b \, x \right)^2 \, \left( a \, b \, x \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b \, a \, b \, x \right)^2 \, \left( a \, b$$

$$\frac{b\, B^2\, c\, (b\, c\, -a\, d)^2\, g^3\, n^2\, log\, [c\, +d\, x]^2\, }{d^4\, i} = \frac{5\, B^2\, (b\, c\, -a\, d)^3\, g^3\, n^2\, log\, [a\, +b\, x]\, log\, \left[\frac{b\, [c\, a\, d\, x]}{b\, c\, a\, d}\right]}{6\, d^4\, i} = \frac{6\, d^4\, i}{d^4\, i} = \frac$$

# Problem 187: Result valid but suboptimal antiderivative.

$$\int \frac{\left(ag + bg x\right)^{2} \left(A + B Log\left[e\left(\frac{a + bx}{c + dx}\right)^{n}\right]\right)^{2}}{c i + d i x} dx$$

Optimal (type 4, 573 leaves, 15 steps):

$$\frac{B \left( b \, c - a \, d \right) \, g^{2} \, n \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)}{d^{2} \, i} - \frac{2 \, \left( b \, c - a \, d \right) \, g^{2} \, \left( a + b \, x \right) \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)^{2}}{d^{3} \, i} + \frac{b^{2} \, g^{2} \, \left( c + d \, x \right)^{2} \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)^{2}}{2 \, d^{3} \, i} + \frac{b^{2} \, g^{2} \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)^{2} \, Log \left[ \frac{b \, c - a \, d}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{b^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right)^{2} \, Log \left[ \frac{b \, c - a \, d}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{b^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^{n} \right] \right) \, Log \left[ 1 - \frac{b \, \left( c + d \, x \right)}{d \, \left( a + b \, x \right)} \right]}{d^{3} \, i} - \frac{4 \, B^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 2 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} - \frac{2 \, B \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 2 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} - \frac{2 \, B^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 2 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 3 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 3 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 3 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 3 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \, d \right)^{2} \, g^{2} \, n^{2} \, PolyLog \left[ 3 \, , \, \frac{d \, \left( a + b \, x \right)}{b \, \left( c + d \, x \right)} \right]}{d^{3} \, i} + \frac{2 \, B^{2} \, \left( b \, c - a \, d \, d \, d \, d \, d$$

Result (type 4, 1780 leaves, 82 steps):

$$\frac{Ab\,B\,\left(b\,c-a\,d\right)\,g^{2}\,n\,x}{d^{2}\,i} + \frac{a\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n\,2\,\log\left[a+b\,x\right]^{2}}{d^{2}\,i} - \frac{B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n\,\left(a+b\,x\right)\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{d^{2}\,i} - \frac{b\,\left(b\,c-a\,d\right)\,g^{2}\,x\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{d^{2}\,i} + \frac{g^{2}\,\left(a+b\,x\right)^{2}\,\left(A+B\,\log\left[e\,\left(\frac{a-b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{d^{2}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n\,2\,\log\left[c+d\,x\right]}{d^{2}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n^{2}\,\log\left[c+d\,x\right]}{d^{3}\,i} - \frac{b\,\left(b\,c-a\,d\right)\,g^{2}\,n^{2}\,\log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,\log\left[c+d\,x\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\log\left[c+d\,x\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\log\left[c+d\,x\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n\,\left(A+B\,\log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)\log\left[c+d\,x\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[c+d\,x\right]^{2}}{2\,d^{3}\,i} - \frac{2\,a\,B^{2}\,\left(b\,c-a\,d\right)\,g^{2}\,n^{2}\,\log\left[a+b\,x\right]\,\log\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[i\,\left(c+d\,x\right)\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[\left(c+d\,x\right)^{2}\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[\left(c+d\,x\right)^{2}\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[\left(c+d\,x\right)^{2}\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{n}\right]^{2}\,\log\left[\left(c+d\,x\right)^{2}\right]^{2}}{d^{3}\,i} + \frac{B^{2}\,\left(b\,c-a\,d\right)^{2}\,g^{2}\,n^{2}\,\log\left[\left(a+b\,x\right)^{2}\right]^{2}\,\log\left[\left(a+b\,x\right)^{2}\right]^{2}}{d^{3}\,i}$$

$$\frac{2\,\mathsf{A}\,\mathsf{B}\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\mathsf{n}\,\mathsf{Log}\!\left[-\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}\right]\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]}{\mathsf{d}^3\,\mathsf{i}} + \frac{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right]\right)^2\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]}{\mathsf{d}^3\,\mathsf{i}} + \frac{2\,\mathsf{B}^2\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\mathsf{n}\,\mathsf{Log}\!\left[-\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}\right]\,\mathsf{Log}\!\left[\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\right] - \mathsf{Log}\!\left[\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^\mathsf{n}\right] + \mathsf{Log}\!\left[\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]}{\mathsf{d}^3\,\mathsf{i}} - \frac{\mathsf{B}^2\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\mathsf{n}\,\mathsf{Log}\!\left[\left(\frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}\right)\right]\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]^2}{\mathsf{d}^3\,\mathsf{i}} + \frac{\mathsf{B}^2\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\mathsf{n}\,\mathsf{Log}\!\left[\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{x}}\right)^\mathsf{n}\right]\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]^2}{\mathsf{d}^3\,\mathsf{i}} - \frac{\mathsf{B}^2\,\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)^2\,\mathsf{g}^2\,\mathsf{n}\,\mathsf{Log}\!\left[\left(\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{x}}\right)^\mathsf{n}\right]\,\mathsf{Log}\!\left[\mathsf{c}\,\mathsf{i}+\mathsf{d}\,\mathsf{i}\,\mathsf{x}\right]^2}{\mathsf{d}^3\,\mathsf{i}} - \frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \frac{\mathsf{d}\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}} - \mathsf{d}\,\mathsf{d}^3\,\mathsf{i}$$

# Problem 188: Result valid but suboptimal antiderivative.

$$\bigcap \frac{\left( \text{ag+bgx} \right) \ \left( \text{A+BLog} \left[ \text{e} \left( \frac{\text{a+bx}}{\text{c+dx}} \right)^{\text{n}} \right] \right)^{2}}{\text{ci+dix}} \, \text{dx}$$

Optimal (type 4, 303 leaves, 9 steps):

$$\frac{g\left(a+b\,x\right)\,\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{d\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,n\,\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} + \frac{\left(b\,c-a\,d\right)\,g\,\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2\,Log\left[\frac{b\,c-a\,d}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,n^2\,PolyLog\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,n\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,PolyLog\left[2,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} - \frac{2\,B^2\,\left(b\,c-a\,d\right)\,g\,n^2\,PolyLog\left[3,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} + \frac{2\,B\,\left(b\,c-a\,d\right)\,g\,n^2\,PolyLog\left[3,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\right]}{d^2\,\mathbf{i}} + \frac{2\,B\,\left(a+b\,x\right)}{b\,\left(a+b\,x\right)} + \frac{2\,B\,\left(a+$$

Result (type 4, 1156 leaves, 65 steps):

$$\frac{a \, b^2 \, g \, n^2 \, Log \, [a + b \, x]^2}{di} + \frac{2 \, a \, B \, g \, n \, Log \, [a + b \, x]}{di} + \frac{di}{di} + \frac{di}{bc - ad} \, g \, n \, Log \, [-\frac{d \, (a + b \, x)}{bc - ad}] \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, b \, B^2 \, c \, g \, n^2 \, Log \, [-\frac{d \, (a + b \, x)}{bc - ad}] \, Log \, [c + d \, x]}{d^2 \, i} + \frac{B^2 \, \left(b \, c - ad\right) \, g \, Log \, [\left(a + b \, x\right)^n]^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, b \, B^2 \, c \, g \, n^2 \, Log \, [\left(a + b \, x\right)^n]^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, b \, B^2 \, c \, g \, n^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, \left(b \, c - ad\right) \, g \, \left(A + B \, Log \, \left[\left(\frac{a \, b \, x}{c \, c \, x}\right)^n\right]\right)^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, c \, g \, n^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, c \, g \, n^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, c \, g \, n^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, (b \, c - ad) \, g \, n^2 \, Log \, [c + d \, x]}{d^2 \, i} + \frac{2 \, a \, B^2 \, g \, n^2 \, Log \, [a + b \, x] \, Log \, [c + d \, x]^2}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, [a + b \, x] \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, [a + b \, x] \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(a + b \, x\right) \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(a + b \, x\right) \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(a + b \, x\right) \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \, g \, n^2 \, Log \, \left(c + d \, x\right)^{-2}}{d^2 \, i} + \frac{2 \, B^2 \, \left(b \, c - ad\right) \,$$

Problem 189: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B Log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{c i + d i x} dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\,\big)^{\,2}\,\mathsf{Log}\big[\,\frac{\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\mathsf{i}}\,-\frac{2\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\big[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,\mathsf{n}}\,\big]\right)\,\mathsf{PolyLog}\big[\,\mathsf{2}\,,\,\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\,\mathsf{i}}\,+\frac{2\,\mathsf{B}^{\,2}\,\mathsf{n}^{\,2}\,\mathsf{PolyLog}\big[\,\mathsf{3}\,,\,\,\frac{\mathsf{d}\,\,(\mathsf{a}+\mathsf{b}\,\mathsf{x})}{\mathsf{b}\,\,(\mathsf{c}+\mathsf{d}\,\mathsf{x})}\,\big]}{\mathsf{d}\,\,\mathsf{i}}\,$$

Result (type 4, 782 leaves, 45 steps):

$$\frac{B^2 \log \left[ \left( a + b \, x \right)^n \right]^2 \log \left[ \frac{b \, (c + d \, x)}{b \, c - a \, d} \right]}{d \, i} = \frac{B^2 \log \left[ \left( a + b \, x \right)^n \right]^2 \log \left[ \, i \, \left( c + d \, x \right) \right]}{d \, i} + \frac{A \, B \, n \, \log \left[ \, i \, \left( c + d \, x \right) \right]^2}{d \, i} = \frac{B^2 \, n^2 \, \log \left[ \, a + b \, x \right] \, \log \left[ \, i \, \left( c + d \, x \right) \right]^2}{d \, i} + \frac{B^2 \, n^2 \, \log \left[ \, i \, \left( c + d \, x \right) \right]^3}{3 \, d \, i} = \frac{2 \, B^2 \, n \, \log \left[ \, a + b \, x \right] \, \log \left[ \, i \, \left( c + d \, x \right) \right] \, \log \left[ \, \left( c + d \, x \right) \right]^2}{d \, i} + \frac{B^2 \, \log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \log \left[ \, \left( c + d \, x \right) - n \right]^2}{3 \, d \, i} = \frac{2 \, B^2 \, n \, \log \left[ \, \left( c + d \, x \right) - n \right]}{b \, c - a \, d} + \frac{B^2 \, \log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \log \left[ \, \left( c + d \, x \right) - n \right]^2}{d \, i} - \frac{2 \, A \, B \, n \, \log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \log \left[ \, c \, i + d \, i \, x \right]}{d \, i} + \frac{2 \, B^2 \, n \, \log \left[ -\frac{d \, (a + b \, x)}{b \, c - a \, d} \right] \, \left( \log \left[ \, \left( a + b \, x \right)^n \right] - \log \left[ e \, \left( \frac{a - b \, x}{c + d \, x} \right)^n \right] + \log \left[ \, \left( c + d \, x \right)^{-n} \right] \right) \, \log \left[ c \, i + d \, i \, x \right]}{d \, i} + \frac{2 \, B^2 \, n \, \log \left[ \, \left( \frac{a \, b \, b \, x}{c + d \, x} \right) \right] \, \left( \log \left[ \, \left( a + b \, x \right)^n \right] - \log \left[ \, \left( \frac{a \, b \, x}{c + d \, x} \right)^n \right] + \log \left[ \, \left( c + d \, x \right)^{-n} \right] \right) \, \log \left[ c \, i + d \, i \, x \right]}{d \, i} + \frac{2 \, B^2 \, n \, \log \left[ \, \left( \frac{a \, b \, b \, x}{c + d \, x} \right) \right] \, \log \left[ \, \left( a \, b \, x \right)^n \right] - \log \left[ \, \left( \frac{a \, b \, b \, x}{c + d \, x} \right) \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] + \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \right) \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right] \, \log \left[ \, \left( c \, d \, x \right)^{-n} \right]$$

Problem 190: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right) \left(c i + d i x\right)} dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$\frac{\left(A+B \, Log\left[\, e\, \left(\frac{a+b\, x}{c+d\, x}\right)^{\, n}\,\right]\,\right)^{\, 3}}{3\, B\, \left(\, b\, c\, -\, a\, d\,\right)\, g\, i\, n}$$

Result (type 4, 1237 leaves, 59 steps):

$$\frac{AB \, n \, Log \left[a + b \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} - \frac{B^{2} \, Log \left[-\frac{b \, c - a \, d}{d \, (a + b \, x)}\right] \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} - \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[\left(a + b \, x\right)^{n}\right]^{2} \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[\left(a + b \, x\right)^{n}\right]^{2} \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[\left(a + b \, x\right)^{n}\right]^{2} \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[\left(a + b \, x\right)^{n}\right]^{2} \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[\left(a + b \, x\right)^{n}\right]^{2} \, Log \left[c + d \, x\right]}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[b \, c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[b \, c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[c + d \, x\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[\left(c + d \, x\right)^{-n}\right]^{2}}{\left(b \, c - a \, d\right) \, g \, i} + \frac{B^{2} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right] \, Log \left[a + b$$

Problem 191: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a + b x}{c + d x}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)} dx$$

Optimal (type 3, 199 leaves, 7 steps):

$$-\frac{2 \, b \, B^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{2 \, b \, B \, n \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3}{\left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{d \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^2 \, g^2 \, \mathbf{i} \, n}$$

Result (type 4, 1800 leaves, 83 steps):

$$\frac{2 \, B^2 \, n^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{2 \, B^2 \, d \, n^2 \, Log \left[a + b \, x\right]}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{A \, B \, d \, n \, Log \left[a + b \, x\right]^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} + \frac{B^2 \, d \, n^2 \, Log \left[a + b \, x\right]^2}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(b \, c - ad\right)^2 \, g^2 \, i}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \, b \, x \, x\right)^n \, 1}{\left(b \, c - ad\right)^2 \, g^2 \, i} - \frac{\left(a \,$$

Problem 192: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(ag + bgx\right)^{3} \left(ci + dix\right)} dx$$

Optimal (type 3, 369 leaves, 9 steps):

$$\frac{4 \, b \, B^2 \, d \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b^2 \, B^2 \, n^2 \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{4 \, b \, B \, d \, n \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} - \frac{b^2 \, B \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)} + \frac{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, \left(a + b \, x\right)^2} + \frac{d^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, n} + \frac{d^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g^3 \, \mathbf{i} \, n}$$

Result (type 4, 2025 leaves, 111 steps):

# Problem 193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log \left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(a g + b g x\right)^{4} \left(c i + d i x\right)} dx$$

Optimal (type 3, 543 leaves, 11 steps):

$$-\frac{6 \, b \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^2} - \frac{2 \, b^3 \, B^2 \, n^2 \, \left(c + d \, x\right)^3}{27 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{6 \, b \, B \, d^2 \, n \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^2} + \frac{3 \, b^2 \, B \, d \, n \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{9 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{2 \, b^3 \, B \, n \, \left(c + d \, x\right)^3 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{9 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{3 \, b \, d^2 \, \left(c + d \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{\left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} - \frac{d^3 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, B \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, n} + \frac{3 \, b^2 \, d \, n \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, x\right)^3}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^2 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(b \, c - a \, d\right)^4 \, g^4 \, i \, \left(a + b \, x\right)^3} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{3 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3} + \frac{3 \, b^2 \, d \, \left(c + d \, x\right)^3 \, \left(a + b \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3}{3 \, \left(a + b \, Log\left[e \, \left(\frac{a$$

#### Result (type 4, 2180 leaves, 143 steps):

$$-\frac{2\,B^{2}\,n^{2}}{27\,\left(b\,c-a\,d\right)\,g^{4}\,i\,\left(a+b\,x\right)^{3}} + \frac{19\,B^{2}\,d\,n^{2}}{36\,\left(b\,c-a\,d\right)^{2}\,g^{4}\,i\,\left(a+b\,x\right)^{2}} - \frac{85\,B^{2}\,d^{2}\,n^{2}}{18\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)} - \frac{85\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]}{18\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{A\,B\,d^{3}\,n\,Log\left[a+b\,x\right]^{2}}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{6\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{B^{2}\,d^{3}\,Log\left[\left(\frac{a+b\,x}{d\,(a+b\,x)}\right)^{3}\right)\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]^{2}}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{11\,B^{2}\,d^{3}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{6\,\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{B^{2}\,d^{3}\,Log\left[\left(\frac{a+b\,x}{d\,(a+b\,x)}\right)^{3}\right)\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]^{2}}{\left(b\,c-a\,d\right)^{4}\,g^{4}\,i} + \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)}{9\,\left(b\,c-a\,d\right)\,g^{4}\,i\,\left(a+b\,x\right)^{3}} + \frac{5\,B\,d\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)}{6\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)^{2}} - \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)} - \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)} - \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)^{2}}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)^{2}} + \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)\right)^{2}}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)} - \frac{11\,B\,d^{2}\,n\,\left(A+B\,Log\left[\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)}{3\,\left(b\,c-a\,d\right)^{3}\,g^{4}\,i\,\left(a+b\,x\right)} - \frac{11\,B\,d^{2}\,n\,$$

$$\frac{d^3 \left( A + B \, Log \left[ e \, \left( \frac{a \cdot b \cdot x}{c \cdot a \cdot d} \right)^n \right] \right)^2 \, Log \left[ c + d \, x \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} + \frac{A \, B \, d^3 \, n \, Log \left[ c + d \, x \right]^2}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} + \frac{11 \, B^2 \, d^3 \, n^2 \, Log \left[ c + d \, x \right]^2}{6 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{B^2 \, d^3 \, n \, Log \left[ e \, d \, x \right]^2}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} + \frac{B^2 \, d^3 \, n^2 \, Log \left[ c + d \, x \right]^3}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, A \, B \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c \cdot d \, x)}{b \, c - a \, d} \right]}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, A \, B \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c \cdot d \, x)}{b \, c - a \, d} \right]}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, A \, B \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c \cdot d \, x)}{b \, c - a \, d} \right]}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, A \, B \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c \cdot d \, x)}{b \, c - a \, d} \right]}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, A \, B \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \frac{b \, (c \cdot d \, x)}{b \, c - a \, d} \right]}{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ c + d \, x \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{3 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \left( c + d \, x \right)^{-n} \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} + \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \left( c + d \, x \right)^{-n} \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \left( c + d \, x \right)^{-n} \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \left( c + d \, x \right)^{-n} \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right] \, Log \left[ \left( a + b \, x \right)^n \right] \, PolyLog \left[ a + b \, x \right] \, Log \left[ \left( c + d \, x \right)^{-n} \right]}{\left( b \, c - a \, d \right)^4 \, g^4 \, i} - \frac{2 \, B^2 \, d^3 \, n \, Log \left[ \left( a + b \, x \right)^n \right] \, PolyLog \left[ a + b \, x \right] \, Log \left[ \left( a + b \, x \right)^n \right] \, PolyLog \left[ a + b \, x \right] \, Log \left[ \left( a + b \, x \right)^n$$

#### Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^2}{\left(\,c\,\,\mathbf{i}+d\,\mathbf{i}\,x\,\right)^2}\,\mathrm{d}\,x$$

Optimal (type 4, 770 leaves, 18 steps):

$$\frac{2A8 \left( bc - ad \right)^2 g^3 n \left( a + bx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{2B^2 \left( bc - ad \right)^2 g^3 n \left( a + bx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^2 \left( c + dx \right)} \frac{d^3 \, i^2 \left( c + dx \right)}{d^3 \, i^3 \left( c + dx \right)^2 \left( a + B \, Log \left[ e \left( \frac{a \, bx \, x}{c \, cd \, x} \right)^n \right) \right)}{2 \, b^3 \, g^3 \left( c + dx \right)^2 \left( a + B \, Log \left[ e \left( \frac{a \, bx \, x}{c \, cd \, x} \right)^n \right) \right)} \frac{d^3 \, i^3 \left( c + dx \right)^2 \left( a + B \, Log \left[ e \left( \frac{a \, bx \, x}{c \, cd \, x} \right)^n \right) \right)}{2 \, d^4 \, i^2} \frac{d^4 \, i^2}{2} \frac{d^4$$

$$\frac{3 \, b \, b^2 \, (b \, c \, - \, a \, d)^2 \, g^3 \, \log \left[ \, (a \, + \, b \, x)^n \right]^2 \, \log \left[ \, (c \, + \, d \, x) \right]}{d^4 \, i^2} = \frac{d^4 \, i^2}{d^4 \, i^2} = \frac{$$

# Problem 195: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\,c\,\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,2}}\,\mathrm{d}x$$

Optimal (type 4, 500 leaves, 12 steps):

$$-\frac{2 \, A \, B \, \left(b \, c - a \, d\right) \, g^{2} \, n \, \left(a + b \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{2 \, B^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, \left(a + b \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{2} \, i^{2} \, \left(c + d \, x\right)} + \frac{d^{2} \, i^{2} \, \left(c + d \, x\right)}{d^{3} \, i^{2}} + \frac{d^{2} \, i^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[2, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, \left(b \, c - a \, d\right) \, g^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{3} \, i^{2}} + \frac{d^{2} \, b^{2} \, n^{2} \, PolyLog \left[3, \, \frac{d \, \left(a + b \, x\right)}{b \, \left(c + d \, x\right)}\right]}{d^{2} \, p^{2} \, Poly$$

Result (type 4, 1807 leaves, 89 steps):

$$\frac{2 \, B^2 \, \left( b \, c - a \, d \right)^2 \, g^2 \, n^2}{d^3 \, i^2 \, \left( c + d \, x \right)} - \frac{2 \, b \, B^2 \, \left( b \, c - a \, d \right)}{d^3 \, i^2} \, \frac{g^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{d^3 \, i^2} - \frac{b \, B^2 \, \left( b \, c - a \, d \right)}{d^3 \, i^2} \, \frac{g^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{d^3 \, i^2} + \frac{2 \, B \, \left( b \, c - a \, d \right)^2 \, g^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, a \, b \, B \, g^2 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, a \, b \, B \, g^2 \, n \, Log \left[ a + b \, x \right] \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{b^2 \, g^2 \, x \, \left( A + B \, Log \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \right] \right)^2}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^2 \, n \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{4 \, A \, b \, B \, \left( b \, c - a \, d \right) \, g^2 \, n \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^2 \, n \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c + d \, x \right)} + \frac{2 \, b^2 \, B^2 \, c \, g^2 \, n^2 \, Log \left[ c \, d \, x \right]}{d^3 \, i^2 \, \left( c$$

$$\frac{b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[c + d \, x]^2}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[a + b \, x] \, \text{Log}[c + d \, x]^2}{d^3 \, i^2} - \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[c + d \, x]^3}{d^3 \, i^2} - \frac{2 \, a \, b \, B^2 \, g^2 \, n^2 \, \text{Log}[a + b \, x] \, \text{Log}[\frac{b \, (c + d \, x)}{b \, c - a \, d)}}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[a + b \, x] \, \text{Log}[\frac{b \, (c + d \, x)}{b \, c - a \, d)}}{d^3 \, i^2} - \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[a + b \, x] \, \text{Log}[\frac{b \, (c + d \, x)}{b \, c - a \, d)}}{d^3 \, i^2} - \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{Log}[a + b \, x] \, \text{Log}[\frac{b \, (c + d \, x)}{b \, c - a \, d)}}{d^3 \, i^2} + \frac{4 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a + b \, x] \, \text{Log}[(c + d \, x)^{-n}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a + b \, x] \, \text{Log}[(c + d \, x)^{-n}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a + b \, x] \, \text{Log}[(c + d \, x)^{-n}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a + b \, x] \, \text{Log}[(c + d \, x)^{-n}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a + b \, x] \, \text{Log}[(c + d \, x)^{-n}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{Log}[a \, (a + b \, x)^n] - \text{Log}[a \, (a + b \, x)^n] - \text{Log}[a \, (a \, b \, x)^n] - \text{Log}[a \, (a \, b \, x)^n] + \text{Log}[(c \, c \, d \, x)^{-n}]^2}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{PolyLog}[2, -\frac{d \, (a \, b \, x)}{b \, c - a \, d)}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n \, \text{PolyLog}[2, -\frac{d \, (a \, b \, x)}{b \, c - a \, d)}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{PolyLog}[2, -\frac{d \, (a \, b \, x)}{b \, c - a \, d)}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{PolyLog}[2, -\frac{b \, (c \, d \, x)}{b \, c - a \, d)}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \, \text{PolyLog}[2, -\frac{b \, (c \, d \, x)}{b \, c - a \, d)}]}{d^3 \, i^2} + \frac{2 \, b \, B^2 \, (b \, c - a \, d) \, g^2 \, n^2 \,$$

#### Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right) \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(c i + d i x\right)^{2}} dx$$

#### Optimal (type 4, 282 leaves, 9 steps):

$$\frac{2\,A\,B\,g\,n\,\left(a+b\,x\right)}{d\,i^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,g\,n^{2}\,\left(a+b\,x\right)}{d\,i^{2}\,\left(c+d\,x\right)} + \frac{2\,B^{2}\,g\,n\,\left(a+b\,x\right)\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{d\,i^{2}\,\left(c+d\,x\right)} - \frac{g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{d\,i^{\,2}\,\left(c+d\,x\right)} - \frac{b\,g\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)}{d^{\,2}\,i^{\,2}} - \frac{g\,\left(a+b\,x\right)\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)}{d\,i^{\,2}\,\left(c+d\,x\right)} + \frac{2\,b\,B^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,3\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{d^{\,2}\,i^{\,2}} - \frac{2\,b\,B\,g\,n\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)}{d^{\,2}\,i^{\,2}} + \frac{2\,b\,B^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,3\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{d^{\,2}\,i^{\,2}} - \frac{2\,b\,B\,g\,n\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)}{d^{\,2}\,i^{\,2}} + \frac{2\,b\,B^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,3\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{d^{\,2}\,i^{\,2}} - \frac{2\,b\,B\,g\,n\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\right)}{d^{\,2}\,i^{\,2}} + \frac{2\,b\,B^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,3\,,\,\frac{d\,\left(a+b\,x\right)}{b\,\left(c+d\,x\right)}\,\right]}{d^{\,2}\,i^{\,2}} + \frac{2\,b\,B^{\,2}\,g\,n^{\,2}\,PolyLog\left[\,3\,,\,\frac{d\,\left(a+b\,x\right)}$$

Result (type 4, 1157 leaves, 69 steps):

$$\frac{2\,B^2\,\left(b\,c-a\,d\right)\,g\,n^2}{d^2\,i^2\,\left(c+d\,x\right)} + \frac{2\,b\,B^2\,g\,n^2\,Log\left[a+b\,x\right]}{d^2\,i^2} + \frac{b\,B^2\,g\,n^2\,Log\left[a+b\,x\right]^2}{d^2\,i^2} + \frac{2\,B\,B\,g\,n\,Log\left[a+b\,x\right]^2}{d^2\,i^2} + \frac{2\,B\,B\,g\,n\,Log\left[a+b\,x\right]}{d^2\,i^2\,\left(c+d\,x\right)} + \frac{\left(b\,c-a\,d\right)\,g\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{d^2\,i^2\,\left(c+d\,x\right)} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]}{d^2\,i^2\,\left(c+d\,x\right)} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]}{d^2\,i^2\,\left(c+d\,x\right)} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[c+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,n\,Log\left[e+d\,x\right]^2}{d^2\,i^2} + \frac{2\,b\,B\,g\,g\,n\,Log\left[e+d\,x\right]^2$$

Problem 197: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+b x}{c+d x}\right)^{n}}\right]\right)^{2}}{\left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$-\frac{2\,A\,B\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^{2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,+\,\frac{2\,B^{2}\,n^{2}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^{2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,-\,\frac{2\,B^{2}\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,n}\,\right]}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^{2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}\,+\,\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,n}\,\right]\right)^{2}}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathbf{i}^{2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}$$

Result (type 4, 514 leaves, 24 steps):

$$-\frac{2 \, B^2 \, n^2}{d \, i^2 \, \left(c + d \, x\right)} - \frac{2 \, b \, B^2 \, n^2 \, Log \left[a + b \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} - \frac{b \, B^2 \, n^2 \, Log \left[a + b \, x\right]^2}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d \, i^2 \, \left(c + d \, x\right)} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right) \, i^2} + \frac{2 \, b \, B^2 \, n^2 \, PolyLog \left[c + d \, x\right]}{d \,$$

Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right) \left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 231 leaves, 7 steps):

$$\begin{split} &\frac{2\,A\,B\,d\,n\,\left(\,a\,+\,b\,\,x\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g\,\,\mathbf{i}^{\,2}\,\left(\,c\,+\,d\,\,x\,\right)} \,-\, \frac{2\,\,B^{\,2}\,d\,\,n^{\,2}\,\,\left(\,a\,+\,b\,\,x\,\right)}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g\,\,\mathbf{i}^{\,2}\,\,\left(\,c\,+\,d\,\,x\,\right)} \,\,+\, \\ &\frac{2\,B^{\,2}\,d\,n\,\,\left(\,a\,+\,b\,\,x\,\right)\,\,Log\left[\,e\,\,\left(\frac{\,a\,+\,b\,\,x\,}{\,c\,+\,d\,\,x}\,\right)^{\,n}\,\right]}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g\,\,\mathbf{i}^{\,2}\,\,\left(\,c\,+\,d\,\,x\,\right)} \,-\, \frac{d\,\,\left(\,a\,+\,b\,\,x\,\right)\,\,\left(\,A\,+\,B\,\,Log\left[\,e\,\,\left(\frac{\,a\,+\,b\,\,x\,}{\,c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g\,\,\mathbf{i}^{\,2}\,\,\left(\,c\,+\,d\,\,x\,\right)} \,+\, \frac{b\,\,\left(\,A\,+\,B\,\,Log\left[\,e\,\,\left(\frac{\,a\,+\,b\,\,x\,}{\,c\,+\,d\,\,x}\,\right)^{\,n}\,\right]\,\right)^{\,3}}{3\,\,B\,\,\left(\,b\,\,c\,-\,a\,\,d\,\right)^{\,2}\,g\,\,\mathbf{i}^{\,2}\,\,n} \end{split}$$

Result (type 4, 1803 leaves, 83 steps):

$$\frac{2 \, B^2 \, n^2}{\left(b \, c \, a \, d\right)^2 \, g^2 \, \left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{a \, b \, B \, \ln \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, n^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \, c \, a \, d\right)^2 \, g^2} \, \frac{b \, B^2 \, \log \left[a \, + b \, x\right]^2}{\left(b \,$$

Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)^{2}} dx$$

Optimal (type 3, 392 leaves, 10 steps):

$$-\frac{2 \, A \, B \, d^{2} \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} + \frac{2 \, B^{2} \, d^{2} \, n^{2} \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, b^{2} \, B^{2} \, n^{2} \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^{2} \, n \, \left(a + b \, x\right) \, Log\left[e\left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right)^{3} \, g^{2} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{2 \, B^{2} \, d^$$

Result (type 4, 1621 leaves, 107 steps):

$$\frac{2 \, b \, B^2 \, n^2}{\left(b \, c \, - a \, d\right)^2 \, g^2 \, i^2 \, \left(a \, b \, x\right)}{\left(b \, c \, - a \, d\right)^2 \, g^2 \, i^2 \, \left(a \, b \, x\right)} \, \left(b \, c \, - a \, d\right)^2 \, g^2 \, i^2 \, \left(c \, + d \, x\right)} \, \frac{4 \, b \, B^2 \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B \, d \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B \, d \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B \, n \, \left(A \, + \, B \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)\right)}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B \, n \, \left(A \, + \, B \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)\right)}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2 \, \left(a \, b \, x\right)} \, \frac{2 \, B \, d \, n \, \left(A \, + \, B \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)\right)^2}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2 \, \left(a \, b \, x\right)} \, \frac{2 \, B \, d \, n \, \left(A \, + \, B \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)\right)^2}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2 \, \left(a \, b \, x\right)} \, \frac{b \, \left(A \, + \, B \, \log\left[a \, \left(\frac{a \, b \, x}{c \, c \, d \, x}\right)^n\right)\right)^2}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B \, d \, n \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, \left(A \, B \, B \, d \, n \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B^2 \, d \, n \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B^2 \, d \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{b \, d \, n^2 \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B^2 \, d \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B^2 \, d \, \log\left[a \, b \, x\right]}{\left(b \, c \, - a \, d\right)^3 \, g^2 \, i^2} \, \frac{2 \, b \, B^2 \, d \, \log\left[a \, b \, x\right]$$

Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]\right)^{2}}{\left(ag + bgx\right)^{3}\left(ci + dix\right)^{2}} dx$$

Optimal (type 3, 560 leaves, 12 steps):

$$\frac{2\,A\,B\,d^{3}\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} - \frac{2\,B^{2}\,d^{3}\,n^{2}\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} + \frac{6\,b^{2}\,B^{2}\,d\,n^{2}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,B^{2}\,n^{2}\,\left(c+d\,x\right)^{2}}{4\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{2\,B^{2}\,d^{3}\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(c+d\,x\right)} + \frac{6\,b^{2}\,B\,d\,n\,\left(c+d\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,B\,n\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,B\,n\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}}{\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)} - \frac{b^{3}\,\left(c+d\,x\right)^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{3}}{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{3}}{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(b\,c-a\,d\right)^{4}\,g^{3}\,\,\mathbf{i}^{2}\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x\right)^{2}}{B\,\left(a+b\,x\right)^{2}} + \frac{b\,d^{2}\,\left(a+b\,x$$

Result (type 4, 2207 leaves, 135 steps):

$$\frac{bB^2n^2}{4 \left(bc-ad\right)^2g^3i^2\left(a+bx\right)^2} + \frac{11bB^2dn^2}{2 \left(bc-ad\right)^3g^3i^2\left(a+bx\right)} + \frac{2B^2d^2n^2}{\left(bc-ad\right)^3g^3i^2\left(c+dx\right)} + \frac{15bB^2d^2n^2 \log[a+bx]}{2 \left(bc-ad\right)^4g^3i^2} - \frac{3bB^2d^2 \log[a+bx]}{2 \left(bc-ad\right)^3g^3i^2 \left(a+bx\right)^2} - \frac{3bB^2d^2 \log[a+bx]}{2 \left(bc-ad\right)^4g^3i^2} - \frac{3bB^2d^2 \log[a+bx]}{2 \left(bc-ad\right)^3g^3i^2 \left(a+bx\right)^2} - \frac{3bB^2d^2 \log[a+bx]}{2 \left(bc-ad\right)^4g^3i^2} - \frac{3bB^2d^2 \log[a$$

$$\frac{6 \, A \, b \, B \, d^2 \, n \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a+b \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a+b \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a+b \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{3 \, b \, B^2 \, d^2 \, n^2 \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c + d \, x \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c+d \, x)}{b \, c - a \, d} \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Log} \left[ \left( c \, - a \, d \right)^4 \, g^3 \, i^2 \right]}{\left( b \, c \, - a \, d \right)^4 \, g^3 \, i^2} + \frac{6 \, b \, B^2 \, d^2 \, n \, \text{Lo$$

# Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right)^{4} \left(c i + d i x\right)^{2}} dx$$

#### Optimal (type 3, 729 leaves, 14 steps):

$$-\frac{2\,A\,B\,d^4\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(c+d\,x\right)} + \frac{2\,B^2\,d^4\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(c+d\,x\right)} - \frac{12\,b^2\,B^2\,d^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} + \frac{b^3\,B^2\,d\,n^2\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B^2\,n^2\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} - \frac{2\,b^2\,B^2\,n^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} + \frac{b^3\,B^2\,d\,n^2\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B^2\,n^2\,\left(c+d\,x\right)^3}{27\,\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} + \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)} + \frac{2\,b^3\,B\,d\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^2\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^2}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3\,\left(a+b\,x\right)^3}{\left(b\,c-a\,d\right)^5\,g^4\,i^2\,\left(a+b\,x\right)^3} - \frac{2\,b^4\,B\,n\,\left(c+d\,x\right)^3}{\left$$

#### Result (type 4, 2368 leaves, 167 steps):

$$-\frac{2 \, b \, B^{2} \, n^{2}}{27 \, \left(b \, c - a \, d\right)^{2} \, g^{4} \, \mathbf{i}^{2} \, \left(a + b \, x\right)^{3}} + \frac{7 \, b \, B^{2} \, d \, n^{2}}{9 \, \left(b \, c - a \, d\right)^{3} \, g^{4} \, \mathbf{i}^{2} \, \left(a + b \, x\right)^{2}} - \frac{92 \, b \, B^{2} \, d^{2} \, n^{2}}{9 \, \left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}^{2} \, \left(a + b \, x\right)} - \frac{2 \, B^{2} \, d^{3} \, n^{2}}{\left(b \, c - a \, d\right)^{4} \, g^{4} \, \mathbf{i}^{2} \, \left(c + d \, x\right)} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} + \frac{4 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]^{2}}{3 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} + \frac{4 \, b \, B^{2} \, d^{3} \, Log \left[a + b \, x\right]^{2}}{\left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} + \frac{4 \, b \, B^{2} \, d^{3} \, Log \left[a + b \, x\right]^{2}}{\left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{4 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{4 \, b \, B^{2} \, d^{3} \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right]}{\left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{2} \, d^{3} \, n^{2} \, Log \left[a + b \, x\right]}{9 \, \left(b \, c - a \, d\right)^{5} \, g^{4} \, \mathbf{i}^{2}} - \frac{110 \, b \, B^{$$

Problem 202: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a g + b g x\right)^{3} \left(A + B Log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(c i + d i x\right)^{3}} dx$$

Optimal (type 4, 676 leaves, 14 steps):

$$\frac{B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \left(a+b \ x\right)^{2}}{4 \ d^{2} \ i^{3} \left(c+d \ x\right)^{2}} - \frac{4 \ A \ b \ B \left(b \ c-a \ d\right) \ g^{3} \ n \left(a+b \ x\right)}{d^{3} \ i^{3} \left(c+d \ x\right)} + \frac{4 \ b \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \left(a+b \ x\right)}{d^{3} \ i^{3} \left(c+d \ x\right)} - \frac{4 \ b \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n \left(a+b \ x\right) \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]}{d^{3} \ i^{3} \left(c+d \ x\right)} + \frac{B \left(b \ c-a \ d\right) \ g^{3} \ n \left(a+b \ x\right)^{2} \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{2 \ d^{2} \ i^{3} \left(c+d \ x\right)^{2}} + \frac{B^{2} \ g^{3} \left(a+b \ x\right) \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)^{2}}{d^{3} \ i^{3} \left(c+d \ x\right)^{2}} + \frac{2 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ n \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right)}{2 \ d^{3} \ i^{3} \left(c+d \ x\right)^{2}} + \frac{2 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ n \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right) \ Log\left[\frac{b \ c-a \ d}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{3 \ b^{2} \left(b \ c-a \ d\right) \ g^{3} \ n \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right) \ Log\left[\frac{b \ c-a \ d}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{2 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B \left(b \ c-a \ d\right) \ g^{3} \ n \left(A+B \ Log\left[e\left(\frac{a+b \ x}{c+d \ x}\right)^{n}\right]\right) \ PolyLog\left[2, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]} - \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \ PolyLog\left[3, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \ PolyLog\left[3, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \ PolyLog\left[3, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ B^{2} \left(b \ c-a \ d\right) \ g^{3} \ n^{2} \ PolyLog\left[3, \ \frac{d \ (a+b \ x)}{b \ (c+d \ x)}\right]}{d^{4} \ i^{3}} + \frac{6 \ b^{2} \ b^{$$

Result (type 4, 2026 leaves, 117 steps):

$$\frac{B^{2} \left( b \, c - a \, d \right)^{3} g^{3} \, n^{2}}{4 \, d^{4} \, i^{3} \left( c + d \, x \right)^{2}} - \frac{9 \, b \, B^{2} \left( b \, c - a \, d \right)^{2} g^{3} \, n^{2}}{2 \, d^{4} \, i^{3}} - \frac{9 \, b^{2} \, B^{2} \left( b \, c - a \, d \right)^{2} g^{3} \, n^{2}}{2 \, d^{4} \, i^{3}} - \frac{9 \, b^{2} \, B^{2} \left( b \, c - a \, d \right)^{2} g^{3} \, n^{2} \, Log \left[ a + b \, x \right]^{2}}{2 \, d^{4} \, i^{3}} - \frac{3 \, b^{2} \, B^{2} \, g^{3} \, n^{2} \, Log \left[ a + b \, x \right]^{2}}{2 \, d^{4} \, i^{3}} - \frac{2 \, d^{4} \, i^{3}}{2 \, d^{4} \, i^{3}} - \frac{2 \, d^{4} \,$$

$$\frac{3 \, b^2 \, (b \, c - a \, d) \, g^3 \, \left( A + B \, tog \left[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^2 \right)^2 \, tog \left[ c + d \, x \right]}{d^4 \, i^3} - \frac{3 \, A \, b^2 \, B \, \left( b \, c - a \, d \right) \, g^3 \, n \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ c + d \, x \right]^2}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ a + b \, x \right] \, tog \left[ \frac{b \, (c + d \, x)^2}{b \, c - a \, d} \right]}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ a + b \, x \right] \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} - \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n \, tog \left[ c + d \, x \right] \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n \, tog \left[ a + b \, x \right] \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n \, tog \left[ a + b \, x \right] \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ a + b \, x \right] \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3} + \frac{3 \, b^2 \, B^2 \, \left( b \, c - a \, d \right) \, g^3 \, n^2 \, tog \left[ \left( c + d \, x \right)^{-n} \right]}{d^4 \, i^3$$

## Problem 203: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,2}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\,c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,3}}\,\mathrm{d}x$$

Optimal (type 4, 441 leaves, 11 steps):

$$\frac{B^2 \, g^2 \, n^2 \, \left(a + b \, x\right)^2}{4 \, d \, i^3 \, \left(c + d \, x\right)^2} + \frac{2 \, A \, b \, B \, g^2 \, n \, \left(a + b \, x\right)}{d^2 \, i^3 \, \left(c + d \, x\right)} - \frac{2 \, b \, B^2 \, g^2 \, n^2 \, \left(a + b \, x\right)}{d^2 \, i^3 \, \left(c + d \, x\right)} + \frac{2 \, b \, B^2 \, g^2 \, n \, \left(a + b \, x\right) \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{d^2 \, i^3 \, \left(c + d \, x\right)} + \frac{B \, g^2 \, n \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{d^2 \, i^3 \, \left(c + d \, x\right)^2} - \frac{g^2 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{d^3 \, i^3} - \frac{b \, g^2 \, \left(a + b \, x\right) \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)}{b \, (c + d \, x)}\right]}{d^3 \, i^3} + \frac{2 \, b^2 \, B^2 \, g^2 \, n^2 \, PolyLog\left[3, \, \frac{d \, (a + b \, x)$$

Result (type 4, 1435 leaves, 97 steps):

$$\frac{B^2 \left( b \, C - a \, d \right)^2 \, g^2 \, n^2}{4 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{5 \, b^2 \, B^2 \, \left( b \, C - a \, d \right) \, g^2 \, n^2 \, \left( b \, c \, \left( a \, d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{3 \, b^2 \, B^2 \, g^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{2 \, d^3 \, i^3} + \frac{B \left( b \, C - a \, d \right) \, g^2 \, n \, \left( A + B \, Log \left[ e \, \left( \frac{a \cdot b \, x}{c \cdot d \, x} \right)^n \right) \right)}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)} - \frac{3 \, b^2 \, B \, g^2 \, n \, Log \left[ a + b \, x \right]}{3 \, d^3 \, i^3} + \frac{3 \, b^2 \, B^2 \, g^2 \, \left( A + B \, Log \left[ e \, \left( \frac{a \cdot b \, x}{c \cdot d \, x} \right)^n \right) \right)^2}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)} - \frac{3 \, b^2 \, B \, g^2 \, n \, Log \left[ a + b \, x \right]}{3 \, i^3} + \frac{3 \, b^2 \, B^2 \, g^2 \, n^2 \, Log \left[ a + b \, x \right]^2}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{2 \, d^3 \, i^3}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2} + \frac{2 \, d^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^2 \, n \, Log \left[ c + d \, x \right]}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3 \, \left( c + d \, x \right)^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^2 \, n \, Log \left[ c + d \, x \right]^2}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3}{2 \, d^3 \, i^3} + \frac{2 \, d^3 \, g^3 \, i^3 \, d^3 \, i^3}{2 \, d^3 \,$$

Problem 204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a\,g + b\,g\,x\right)\,\left(A + B\,Log\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\,c\,\,\mathbf{i} + d\,\mathbf{i}\,x\right)^{\,3}}\,\,\mathrm{d}x$$

Optimal (type 3, 151 leaves, 3 steps):

$$\frac{B^2 g \, n^2 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} - \frac{B \, g \, n \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} + \frac{g \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, \left(b \, c - a \, d\right) \, \mathbf{i}^3 \, \left(c + d \, x\right)^2}$$

Result (type 4, 686 leaves, 54 steps):

$$\frac{B^2 \left(b \ c-a \ d\right) \ g \ n^2}{4 \ d^2 \ i^3 \left(c+d \ x\right)^2} - \frac{b \ B^2 \ g \ n^2}{2 \ d^2 \ i^3 \left(c+d \ x\right)} - \frac{b^2 \ B^2 \ g \ n^2 \ Log \left[a+b \ x\right]}{2 \ d^2 \left(b \ c-a \ d\right) \ i^3} - \frac{b^2 \ B^2 \ g \ n^2 \ Log \left[a+b \ x\right]^2}{2 \ d^2 \left(b \ c-a \ d\right) \ i^3} - \frac{B \left(b \ c-a \ d\right) \ g \ n \left(A+B \ Log \left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]\right)}{2 \ d^2 \ i^3 \left(c+d \ x\right)^2} + \frac{b^2 \ B \ g \ n \ Log \left[a+b \ x\right] \left(A+B \ Log \left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]\right)}{d^2 \ i^3 \left(c+d \ x\right)} + \frac{b^2 \ B \ g \ n \ Log \left[a+b \ x\right] \left(A+B \ Log \left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]\right)}{d^2 \ i^3 \left(c+d \ x\right)^2} + \frac{b^2 \ B^2 \ g \ n^2 \ Log \left[c+d \ x\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Log \left[c+d \ x\right]}{d^2 \ (b \ c-a \ d) \ i^3} - \frac{b^2 \ B \ g \ n \left(A+B \ Log \left[e \left(\frac{a+b \ x}{c+d \ x}\right)^n\right]\right)^2}{2 \ d^2 \ (b \ c-a \ d) \ i^3} - \frac{b^2 \ B \ g \ n^2 \ Log \left[c+d \ x\right]}{2 \ d^2 \ (b \ c-a \ d) \ i^3} - \frac{b^2 \ B \ g \ n^2 \ Log \left[c+d \ x\right]}{2 \ d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Log \left[c+d \ x\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3} + \frac{b^2 \ B^2 \ g \ n^2 \ Poly Log \left[2, \frac{d \ (a+b \ x)}{b \ c-a \ d}\right]}{d^2 \ (b \ c-a \ d) \ i^3}$$

## Problem 205: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \log\left[e^{\left(\frac{a+bx}{c+dx}\right)^{n}}\right]\right)^{2}}{\left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 317 leaves, 8 steps):

$$-\frac{B^2\,d\,n^2\,\left(a+b\,x\right)^2}{4\,\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} - \frac{2\,A\,b\,B\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{2\,b\,B^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} - \frac{2\,b\,B^2\,n\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{B\,d\,n\,\left(a+b\,x\right)^2\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{2\,\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)\,\left(a+b\,x\right)}{\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(b\,c-a\,d\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2}{\left(a+b\,x\right)^2\,\mathbf{i}^3\,\left(c+d\,x\right)^2} + \frac{b\,\left(a+b\,x\right)^2\,\mathbf{i}^3\,\mathbf$$

Result (type 4, 626 leaves, 28 steps):

$$-\frac{B^{2} \, n^{2}}{4 \, d \, i^{3} \, \left(c + d \, x\right)^{2}} - \frac{3 \, b \, B^{2} \, n^{2}}{2 \, d \, \left(b \, c - a \, d\right) \, i^{3} \, \left(c + d \, x\right)} - \frac{3 \, b^{2} \, B^{2} \, n^{2} \, Log \left[a + b \, x\right]}{2 \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} - \frac{b^{2} \, B^{2} \, n^{2} \, Log \left[a + b \, x\right]^{2}}{2 \, d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{B \, n \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{d \, \left(b \, c - a \, d\right) \, i^{3} \, \left(c + d \, x\right)} + \frac{b^{2} \, B \, n \, Log \left[a + b \, x\right] \, \left(A + B \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{b^{2} \, B^{2} \, n^{2} \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]\right)}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{b^{2} \, B^{2} \, n^{2} \, Log \left[-\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d\right)^{2} \, i^{3}}\right] \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{b^{2} \, B^{2} \, n^{2} \, Log \left[-\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d\right)^{2} \, i^{3}}\right] \, Log \left[c + d \, x\right]}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{b^{2} \, B^{2} \, n^{2} \, Log \left[a + b \, x\right] \, Log \left[\frac{b \, \left(c + d \, x\right)}{b \, c - a \, d\right)^{2}} + \frac{b^{2} \, B^{2} \, n^{2} \, Poly Log \left[2, -\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d\right)^{2}}\right]}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}} + \frac{b^{2} \, B^{2} \, n^{2} \, Poly Log \left[2, -\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d\right)^{2}} + \frac{b^{2} \, B^{2} \, n^{2} \, Poly Log \left[2, -\frac{d \, \left(a + b \, x\right)}{b \, c - a \, d\right)^{2}}\right]}{d \, \left(b \, c - a \, d\right)^{2} \, i^{3}}$$

Problem 206: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right) \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 402 leaves, 15 steps):

$$\frac{B^2 \, d^2 \, n^2 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)^2} + \frac{4 \, A \, b \, B \, d \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{4 \, b \, B^2 \, d \, n^2 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{4 \, b \, B^2 \, d \, n^2 \, \left(a + b \, x\right) \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} - \frac{B \, d^2 \, n \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{d^2 \, \left(a + b \, x\right)^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^2}{2 \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{b^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)} + \frac{b^2 \, \left(A + B \, Log\left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)^3}{3 \, B \, \left(b \, c - a \, d\right)^3 \, g \, \mathbf{i}^3 \, \left(c + d \, x\right)}$$

Result (type 4, 2025 leaves, 111 steps):

$$\frac{8^{3}n^{2}}{4\left(bc-ad\right)^{2}g^{13}\left(c+dx\right)^{2}} \cdot \frac{7b^{2}n^{2}n^{2}}{2\left(bc-ad\right)^{2}g^{13}} \cdot \frac{Ab^{2}Bn\log[a+bx]}{2\left(bc-ad\right)^{2}g^{13}} \cdot \frac{Ab^{2}Bn\log[a+bx]}{2\left(bc-ad\right)^{2}g^{13}} \cdot \frac{3b^{2}B^{2}\log[a+bx]}{2\left(bc-ad\right)^{2}g^{13}} \cdot \frac{Bn\left(A+B\log\left[e\left(\frac{a+bx}{c+dx}\right)^{2}\right)}{2\left(bc-ad\right)^{2}g^{13}} \cdot \frac{Bn\left(A+B\log\left[e\left(\frac{a+$$

# Problem 207: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right)^{2} \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 562 leaves, 12 steps):

$$-\frac{B^2\,d^3\,n^2\,\left(a+b\,x\right)^2}{4\,\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)^2} - \frac{6\,A\,b\,B\,d^2\,n\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)} + \frac{6\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(c+d\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{6\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{6\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{6\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{6\,b\,B^2\,d^2\,n^2\,\left(a+b\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,i^3\,\left(a+b\,x\right)} - \frac{2\,b^3\,B^2\,n^2\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)^4\,g^2\,$$

Result (type 4, 2207 leaves, 135 steps):

$$\frac{2 \, b^2 \, B^2 \, n^2}{ \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(a + b \, x\right)^2} - \frac{B^2 \, d \, n^2}{4 \, \left(b \, c - a \, d\right)^2 \, g^2 \, i^3 \, \left(c + d \, x\right)^2} - \frac{2 \, \left(b \, c - a \, d\right)^3 \, g^2 \, i^3 \, \left(c + d \, x\right)}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, n^2 \, Log \left[a + b \, x\right]^2}{2 \, \left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, n^2 \, Log \left[a + b \, x\right]^2}{\left(b \, c - a \, d\right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, Log \left[a + b \, x\right] \, Log \left[a + b \, x\right$$

$$\frac{b^2 \, B^2 \, d \, n^2 \, \text{Log} \left[ \, c + d \, x \, \right]^3}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} - \frac{6 \, A \, b^2 \, B \, d \, n \, \text{Log} \left[ \, \frac{b \, (c + d \, x)}{b \, c - a \, d \, } \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, n^2 \, \text{Log} \left[ \, \frac{b \, (c + d \, x)}{b \, c - a \, d \, } \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{3 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, (c + d \, x)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, (c + d \, x)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, (c + d \, x)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]^2}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right]}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n^2 \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d \, \right)}}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n^2 \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d \, \right)}}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d \, \right)}}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d \, \right)}}{\left( \, b \, c - a \, d \, \right)^4 \, g^2 \, i^3} + \frac{6 \, b^2 \, B^2 \, d \, n \, \text{Log} \left[ \, \left( \, c + d \, x \, \right)^{-n} \right] \, \text{PolyLog} \left[ 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d \, \right)}}{\left( \, b \, c -$$

Problem 208: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right)^{3} \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 732 leaves, 14 steps):

$$\frac{B^2 \, d^4 \, n^2 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{8 \, A \, b \, B \, d^3 \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} - \frac{8 \, b \, B^2 \, d^3 \, n^2 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{8 \, b^3 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)^2} + \frac{8 \, b^3 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)} + \frac{6 \, b^4 \, B^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)} + \frac{8 \, b^3 \, B^2 \, d \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)^2} + \frac{8 \, b^3 \, B \, d \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)^2}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{8 \, b^3 \, B \, d \, n \, \left(c + d \, x\right) \, \left(A + B \, Log \left[e \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]\right)}{\left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)} - \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)^2} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(a + b \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c - a \, d\right)^5 \, g^3 \, i^3 \, \left(c + d \, x\right)} + \frac{1}{2 \, \left(b \, c$$

Result (type 4, 2041 leaves, 163 steps):

# Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \log\left[e\left(\frac{a + b x}{c + d x}\right)^{n}\right]\right)^{2}}{\left(a g + b g x\right)^{4} \left(c i + d i x\right)^{3}} dx$$

Optimal (type 3, 908 leaves, 16 steps):

$$\frac{B^2 \, d^5 \, n^2 \, \left(a + b \, x\right)^2}{4 \, \left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(c + d \, x\right)^2}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(c + d \, x\right)} + \frac{10 \, b \, B^2 \, d^4 \, n^2 \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(c + d \, x\right)} + \frac{20 \, b^3 \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{20 \, b^3 \, B^2 \, d^2 \, n^2 \, \left(c + d \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right)}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^n\right]}{\left(b \, c - a \, d\right)^6 \, g^4 \, i^3 \, \left(a + b \, x\right)^2} + \frac{10 \, b \, B^2 \, d^4 \, n \, \left(a + b \, x\right) \, Log \left[e \, \left(\frac{a + b \, x}{c + d \, x}\right)^$$

#### Result (type 4, 2610 leaves, 195 steps):

$$\frac{2 \, b^2 \, B^2 \, n^2}{27 \, \left( b \, c - a \, d \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \right)^3}{36 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i^3 \, \left( a + b \, x \right)^2} - \frac{319 \, b^2 \, B^2 \, d^2 \, n^2}{18 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \right)} - \frac{B^2 \, d^3 \, n^2}{4 \, \left( b \, c - a \, d \right)^4 \, g^4 \, i^3 \, \left( c + d \, x \right)^2} - \frac{19 \, b \, B^2 \, d^3 \, n^2}{4 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \right)} - \frac{245 \, b^2 \, B^2 \, d^3 \, n^2 \, Log \left[ a + b \, x \right]}{9 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, A \, b^2 \, B \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{\left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n^2 \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^6 \, g^4 \, i^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, Log \left[ a + b \, x \right] \, Log \left[ a + b \, x \right] \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{3 \, \left( b \, c - a \, d \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, B^2 \, d^3 \, n \, Log \left[ a + b \, x \right]^2}{9 \, \left( b \, c - a \, d \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, b^2 \, B^2 \, d^3 \, n \, \left( a + b \, Log \left[ a + b \, x \right]^3}{9 \, \left( b \, c - a \, d \right)^3 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, b^2 \, b^2 \, a^3 \, n \, \left( a + b \, Log \left[ a + b \, x \right]^3}{9 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, b^2 \, b^2 \, a^3 \, n \, \left( a + b \, b \, Log \left[ a + b \, x \right]^3}{9 \, \left( b \, c - a \, d \right)^5 \, g^4 \, i^3 \, \left( a + b \, x \right)^3} + \frac{10 \, b^2 \, b^2 \, b^2 \, a^3$$

$$\frac{d^3 \left(A + B \log\left[e\left(\frac{a + b \times x}{c + c + d \times s}\right)^{-1}\right)^2}{2 \left(b \cdot c - ad\right)^6 g^4 i^3 \left(c + d \times x\right)^2 - \left(b \cdot c - ad\right)^6 g^4 i^3 \left(c + d \times x\right)^2 - \left(b \cdot c - ad\right)^6 g^4 i^3 \left(c + d \times x\right)^2 - \left(b \cdot c - ad\right)^6 g^4 i^3 \left(c + d \times x\right)^2 - \left(b \cdot c - ad\right)^6 g^4 i^3 \left(c + d \times x\right)^2 - \left(b \cdot c - ad\right)^6 g^4 i^3 - \left(b \cdot c - a$$

# Problem 210: Unable to integrate problem.

$$\int \left(a\,g+b\,g\,x\right)^{m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{-2-m}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\,\right]\,\right)^{p}\,\mathrm{d}x$$

Optimal (type 4, 189 leaves, 3 steps):

$$\left( e^{-\frac{A \left( 1+m \right)}{B \, n}} \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( e \, \left( \frac{a+b \, x}{c+d \, x} \right)^{n} \right)^{-\frac{1+m}{n}} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{-m} \, \text{Gamma} \left[ 1+p \text{, } -\frac{\left( 1+m \right) \, \left( A+B \, \text{Log} \left[ e \, \left( \frac{a+b \, x}{c+d \, x} \right)^{n} \right] \right)}{B \, n} \right]^{-1+m} \, \left( \left( b \, c-a \, d \right) \, \mathbf{i}^{\, 2} \, \left( 1+m \right) \, \left( c+d \, x \right) \right)^{-1+m} \, \left( c+d \, x \right) \right)$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \left( a \ g + b \ g \ x \right)^m \ \left( c \ i + d \ i \ x \right)^{-2-m} \ \left( A + B \ Log \left[ e \ \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)^p \text{, } x \right]$$

## Problem 211: Unable to integrate problem.

$$\int \left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{m}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\,\right]\,\right)^{p}\,\mathrm{d}x$$

Optimal (type 4, 190 leaves, 3 steps):

$$-\left(\left(e^{\frac{A\left(1+m\right)}{B\,n}}\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{\frac{1+m}{n}}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{2+m}\,Gamma\left[\mathbf{1}+p,\,\frac{\left(\mathbf{1}+m\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{B\,n}\right)^{-p}\right)\right)$$

$$\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{p}\left(\frac{\left(\mathbf{1}+m\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{B\,n}\right)^{-p}\right)\left/\left(\left(b\,c-a\,d\right)\,\mathbf{i}^{2}\,\left(\mathbf{1}+m\right)\,\left(c+d\,x\right)\right)\right)$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \left. \left( a \ g + b \ g \ x \right)^{-2-m} \ \left( c \ i + d \ i \ x \right)^m \ \left( A + B \ Log \left[ e \ \left( \frac{a + b \ x}{c + d \ x} \right)^n \right] \right)^p \text{, } x \right]$$

# Problem 212: Unable to integrate problem.

$$\int \left( a \, g + b \, g \, x \right)^m \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{-2-m} \, \left( A + B \, Log \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \right] \, \right)^3 \, \mathrm{d} x$$

Optimal (type 3, 292 leaves, 4 steps):

$$-\frac{6\,B^{3}\,n^{3}\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(i\,\left(c+d\,x\right)\right)^{\,-m}}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{\,4}\,\left(c+d\,x\right)} + \frac{6\,B^{2}\,n^{2}\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(i\,\left(c+d\,x\right)\right)^{\,-m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{\,3}\,\left(c+d\,x\right)} - \frac{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{\,3}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{\,2}\,\left(c+d\,x\right)} + \frac{\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(i\,\left(c+d\,x\right)\right)^{\,-m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)^{\,3}}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{\,2}\,\left(c+d\,x\right)}$$

Result (type 8, 281 leaves, 6 steps):

$$\frac{A^{3} \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(b \, c - a \, d\right) \, g \, \mathbf{i} \, \left(\mathbf{1} + m\right)} \, - \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(b \, c - a \, d\right) \, g \, \mathbf{i} \, \left(\mathbf{1} + m\right)^{\, 2}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, \mathbf{1} + m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, -\mathbf{1} - m}}{\left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{\, -\mathbf{1} - m}} \, + \, \frac{3 \, A^{2} \, B \, n \, \left(a \, g + b \, g \, x\right)^{\, -\mathbf{1} - m}}{$$

3 A B<sup>2</sup> CannotIntegrate 
$$\left[\left(a\,g+b\,g\,x\right)^{\,m}\,\left(c\,i+d\,i\,x\right)^{\,-2-m}\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]^{\,2}$$
,  $x\,\right]$  +

$$B^{3} \text{ CannotIntegrate} \left[ \left( a \, g + b \, g \, x \right)^{m} \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{-2-m} \, \mathsf{Log} \left[ e \, \left( \frac{\mathsf{a} + b \, x}{\mathsf{c} + d \, x} \right)^{\mathsf{n}} \right]^{3}, \, x \right] \\ + \frac{3 \, \mathsf{A}^{2} \, \mathsf{B} \, \left( \mathsf{a} \, g + b \, g \, x \right)^{1+m} \, \left( \mathsf{c} \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{-1-m} \, \mathsf{Log} \left[ e \, \left( \frac{\mathsf{a} + b \, x}{\mathsf{c} + d \, x} \right)^{\mathsf{n}} \right]^{2}}{\left( b \, \mathsf{c} - \mathsf{a} \, d \right) \, \mathsf{g} \, \mathbf{i} \, \left( 1 + \mathsf{m} \right)}$$

## Problem 213: Unable to integrate problem.

$$\int \left(a\,g+b\,g\,x\right)^m\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{-2-m}\,\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\,\right]\,\right)^2\,\mathrm{d}x$$

Optimal (type 3, 210 leaves, 3 steps):

$$\frac{2\;B^2\;n^2\;\left(\,a\,+\,b\;x\,\right)\;\left(\,g\;\left(\,a\,+\,b\;x\,\right)\,\right)^{\,m}\;\left(\,\mathbf{i}\;\left(\,c\,+\,d\;x\,\right)\,\right)^{\,-\,m}}{\left(\,b\;c\,-\,a\;d\,\right)\;\,\mathbf{i}^2\;\left(\,\mathbf{1}\,+\,m\,\right)^{\,3}\;\left(\,c\,+\,d\;x\,\right)}\;-$$

$$\frac{2\,B\,n\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{\,-m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(1+m\right)^{\,2}\,\left(c+d\,x\right)} + \frac{\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{\,-m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)^{\,2}}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{\,2}\,\left(1+m\right)\,\left(c+d\,x\right)}$$

Result (type 8, 224 leaves, 6 steps):

$$B^{2} \, \text{CannotIntegrate} \left[ \, \left( \, a \, g + b \, g \, x \, \right)^{\, m} \, \left( \, c \, \mathbf{i} + d \, \mathbf{i} \, x \, \right)^{\, -2 - m} \, Log \left[ \, e \, \left( \, \frac{a + b \, x}{c + d \, x} \, \right)^{\, n} \, \right]^{\, 2} \, , \, \, x \, \right] \, + \, \frac{2 \, A \, B \, \left( \, a \, g + b \, g \, x \, \right)^{\, 1 + m} \, \left( \, c \, \mathbf{i} + d \, \mathbf{i} \, x \, \right)^{\, -1 - m} \, Log \left[ \, e \, \left( \, \frac{a + b \, x}{c + d \, x} \, \right)^{\, n} \, \right]}{\left( \, b \, c - a \, d \, \right) \, g \, \mathbf{i} \, \left( \, \mathbf{1} + m \, \right)}$$

## Problem 214: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right)^m \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{-2-m} \, \left( A + B \, Log \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \right] \, \right) \, \mathrm{d} x$$

Optimal (type 3, 128 leaves, 2 steps):

$$-\frac{B\,n\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{\,-m}}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{\,2}\,\left(\mathbf{1}+m\right)^{\,2}\,\left(c+d\,x\right)}+\frac{\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{\,m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{\,-m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\right]\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{\,2}\,\left(\mathbf{1}+m\right)\,\,\left(c+d\,x\right)}$$

Result (type 3, 168 leaves, 6 steps):

$$\frac{A \left(a\,g+b\,g\,x\right)^{\,\mathbf{1}+\mathbf{m}}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,\mathbf{-1}-\mathbf{m}}}{\left(b\,c-a\,d\right)\,g\,\mathbf{i}\,\left(\mathbf{1}+\mathbf{m}\right)} - \frac{B\,n\,\left(a\,g+b\,g\,x\right)^{\,\mathbf{1}+\mathbf{m}}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,\mathbf{-1}-\mathbf{m}}}{\left(b\,c-a\,d\right)\,g\,\mathbf{i}\,\left(\mathbf{1}+\mathbf{m}\right)^{\,2}} + \frac{B\,\left(a\,g+b\,g\,x\right)^{\,\mathbf{1}+\mathbf{m}}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,\mathbf{-1}-\mathbf{m}}\,\mathsf{Log}\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]}{\left(b\,c-a\,d\right)\,g\,\mathbf{i}\,\left(\mathbf{1}+\mathbf{m}\right)}$$

# Problem 215: Unable to integrate problem.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,-2-m}}{A+B\,Log\!\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}\,\mathrm{d}x$$

Optimal (type 4, 125 leaves, 3 steps):

$$\frac{\mathrm{e}^{-\frac{A\,\left(1+m\right)}{B\,n}}\,\left(\,a\,+\,b\,\,x\,\right)\,\left(\,g\,\left(\,a\,+\,b\,\,x\,\right)\,\right)^{\,m}\,\left(\,e\,\left(\,\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right)^{\,-\frac{1+m}{n}}\,\left(\,\mathbf{i}\,\left(\,c\,+\,d\,\,x\,\right)\,\right)^{\,-m}\,\,\text{ExpIntegralEi}\left[\,\frac{\,\left(\,1+m\right)\,\left(\,A+B\,\,\text{Log}\left[\,e\,\left(\,\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)}{B\,n}\,\right]}{B\,\left(\,b\,\,c\,-\,a\,\,d\,\right)\,\,\mathbf{i}^{\,2}\,\,n\,\left(\,c\,+\,d\,\,x\,\right)}$$

Result (type 8, 51 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^{m}\,\left(c\,i+d\,i\,x\right)^{-2-m}}{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]},\,x\right]$$

## Problem 216: Unable to integrate problem.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{m}\,\left(c\,i+d\,i\,x\right)^{-2-m}}{\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{\,n}\,\right]\,\right)^{\,2}}\,\mathrm{d}x$$

Optimal (type 4, 206 leaves, 4 steps):

$$\left( e^{-\frac{A \left( 1+m \right)}{B \, n}} \left( 1+m \right) \, \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( e \, \left( \frac{a+b \, x}{c+d \, x} \right)^{\, n} \right)^{-\frac{1+m}{n}} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{-m} \, \text{ExpIntegralEi} \left[ \, \frac{\left( 1+m \right) \, \left( A+B \, \text{Log} \left[ e \, \left( \frac{a+b \, x}{c+d \, x} \right)^{\, n} \right] \right)}{B \, n} \right] \right) / \left( B^2 \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^2 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{-m}}{B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n} \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{-m}}{B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^2 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{-m}}{B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^2 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{\, m} \, \left( \mathbf{i} \, \left( c+d \, x \right) \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right) \, \left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right) \, \left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m} \, \left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)^{\, m}}{B \, n} \right) - \frac{\left( a+b \, x \right)$$

Result (type 8. 51 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \begin{aligned} &\text{CannotIntegrate} \big[ \, \frac{ \left( \text{ag} + \text{bgx} \right)^m \, \left( \text{ci} + \text{dix} \right)^{-2-m}}{ \left( \text{A} + \text{BLog} \big[ \text{e} \, \left( \frac{\text{a+bx}}{\text{c+dx}} \right)^n \big] \, \right)^2} \text{, } x \, \big] \end{aligned}$$

# Problem 217: Unable to integrate problem.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{\,m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,-2-m}}{\left(A+B\,Log\left[\,e\,\left(\frac{a+b\,x}{c-d\,x}\right)^{\,n}\,\right]\,\right)^{\,3}}\,\mathrm{d}x$$

Optimal (type 4, 295 leaves, 5 steps):

$$\left( e^{-\frac{A \left( 1+m \right)}{B \, n}} \left( 1+m \right)^2 \left( a+b \, x \right) \left( g \left( a+b \, x \right) \right)^m \left( e \left( \frac{a+b \, x}{c+d \, x} \right)^n \right)^{-\frac{1+m}{n}} \left( \mathbf{i} \left( c+d \, x \right) \right)^{-m} \\ \mathrm{ExpIntegralEi} \left[ \frac{\left( 1+m \right) \left( A+B \, Log \left[ e \left( \frac{a+b \, x}{c+d \, x} \right)^n \right] \right)}{B \, n} \right] \right) \right) \\ \left( 2 \, B^3 \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \left( g \, \left( a+b \, x \right) \right)^m \left( \mathbf{i} \left( c+d \, x \right) \right)^{-m}}{2 \, B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \left( g \, \left( a+b \, x \right) \right)^m \left( \mathbf{i} \left( c+d \, x \right) \right)^{-m}}{2 \, B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \left( g \, \left( a+b \, x \right) \right)^m \left( \mathbf{i} \left( c+d \, x \right) \right)^{-m}}{2 \, B \, \left( b \, c-a \, d \right) \, \mathbf{i}^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \left( a+b \, x \right) \left($$

Result (type 8, 51 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^{m}\,\left(c\,i+d\,i\,x\right)^{-2-m}}{\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{3}},\,x\right]$$

# Problem 218: Unable to integrate problem.

$$\int \left( a\;g + b\;g\;x \right)^{-2-m}\; \left( c\;\mathbf{i} + d\;\mathbf{i}\;x \right)^{m}\; \left( A + B\;Log\left[ e\; \left( \frac{a + b\;x}{c + d\;x} \right)^{n} \right] \right)^{3}\;\mathrm{d}x$$

Optimal (type 3, 309 leaves, 4 steps):

$$-\frac{6\,B^{3}\,n^{3}\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(i\,\left(c+d\,x\right)\right)^{2+m}}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{4}\,\left(c+d\,x\right)}-\frac{6\,B^{2}\,n^{2}\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(i\,\left(c+d\,x\right)\right)^{2+m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{3}\,\left(c+d\,x\right)}-\frac{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{3}\,\left(c+d\,x\right)}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{2}\,\left(c+d\,x\right)}-\frac{\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(i\,\left(c+d\,x\right)\right)^{2+m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{3}}{\left(b\,c-a\,d\right)\,i^{2}\,\left(1+m\right)^{2}\,\left(c+d\,x\right)}$$

Result (type 8, 282 leaves, 6 steps):

$$-\frac{A^{3} \left(a\,g+b\,g\,x\right)^{-1-m} \left(c\,i+d\,i\,x\right)^{1+m}}{\left(b\,c-a\,d\right)\,g\,i\,\left(1+m\right)} - \frac{3\,A^{2}\,B\,n\,\left(a\,g+b\,g\,x\right)^{-1-m}\,\left(c\,i+d\,i\,x\right)^{1+m}}{\left(b\,c-a\,d\right)\,g\,i\,\left(1+m\right)^{2}} + \\ 3\,A\,B^{2}\,CannotIntegrate\,\left[\,\left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,i+d\,i\,x\right)^{m}\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\,\right]^{2}\text{, }x\,\right] + \\ B^{3}\,CannotIntegrate\,\left[\,\left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,i+d\,i\,x\right)^{m}\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\,\right]^{3}\text{, }x\,\right] - \frac{3\,A^{2}\,B\,\left(a\,g+b\,g\,x\right)^{-1-m}\,\left(c\,i+d\,i\,x\right)^{1+m}\,Log\,\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\,\right]}{\left(b\,c-a\,d\right)\,g\,i\,\left(1+m\right)}$$

#### Problem 219: Unable to integrate problem.

$$\int \left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}\,\mathrm{d}x$$

Optimal (type 3, 223 leaves, 3 steps):

$$-\frac{2\;B^2\;n^2\;\left(\,a\,+\,b\;x\,\right)\;\left(\,g\;\left(\,a\,+\,b\;x\,\right)\,\right)^{\,-\,2\,-\,m}\;\left(\,\mathbf{i}\;\left(\,c\,+\,d\;x\,\right)\,\right)^{\,2\,+\,m}}{\left(\,b\;c\,-\,a\;d\,\right)\;\,\mathbf{i}^{\,2}\;\left(\,\mathbf{1}\,+\,m\,\right)^{\,3}\;\left(\,c\,+\,d\;x\,\right)}\;-$$

$$\frac{2\,B\,n\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{g}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\right)^{-2-\mathsf{m}}\,\left(\mathsf{i}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)\right)^{2+\mathsf{m}}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,n}\,\right]\,\right)}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathsf{i}^{\,2}\,\left(\mathsf{1}+\mathsf{m}\right)^{\,2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}-\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\,\left(\mathsf{g}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)\right)^{-2-\mathsf{m}}\,\left(\mathsf{i}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)\right)^{2+\mathsf{m}}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\,n}\,\right]\right)^{\,2}}{\left(\mathsf{b}\,\mathsf{c}-\mathsf{a}\,\mathsf{d}\right)\,\,\mathsf{i}^{\,2}\,\left(\mathsf{1}+\mathsf{m}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}$$

Result (type 8, 225 leaves, 6 steps):

$$-\frac{{{\mathsf{A}}^{2} \; \left(\mathsf{a} \; \mathsf{g} + \mathsf{b} \; \mathsf{g} \; \mathsf{x}\right)^{\; -1 - \mathsf{m} \; \left(\mathsf{c} \; \mathsf{i} + \mathsf{d} \; \mathsf{i} \; \mathsf{x}\right)^{\; 1 + \mathsf{m}}}{\left(\mathsf{b} \; \mathsf{c} - \mathsf{a} \; \mathsf{d}\right) \; \mathsf{g} \; \mathsf{i} \; \left(\mathsf{1} + \mathsf{m}\right)} \; - \; \frac{\mathsf{2} \; \mathsf{A} \; \mathsf{B} \; \mathsf{n} \; \left(\mathsf{a} \; \mathsf{g} + \mathsf{b} \; \mathsf{g} \; \mathsf{x}\right)^{\; -1 - \mathsf{m} \; } \left(\mathsf{c} \; \mathsf{i} + \mathsf{d} \; \mathsf{i} \; \mathsf{x}\right)^{\; 1 + \mathsf{m}}}{\left(\mathsf{b} \; \mathsf{c} - \mathsf{a} \; \mathsf{d}\right) \; \mathsf{g} \; \mathsf{i} \; \left(\mathsf{1} + \mathsf{m}\right)^{\; 2}} \; + \; \frac{\mathsf{3} \; \mathsf{a} \; \mathsf{n} \; \mathsf{n}$$

$$B^{2} \text{ CannotIntegrate} \left[ \left( a \, g + b \, g \, x \right)^{-2-m} \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{m} \, \mathsf{Log} \left[ e \, \left( \frac{\mathsf{a} + b \, x}{\mathsf{c} + d \, x} \right)^{\mathsf{n}} \right]^{2} \text{, } x \right] \\ - \frac{2 \, \mathsf{A} \, \mathsf{B} \, \left( \mathsf{a} \, g + b \, g \, x \right)^{-1-m} \, \left( \mathsf{c} \, \mathbf{i} + d \, \mathbf{i} \, x \right)^{1+m} \, \mathsf{Log} \left[ e \, \left( \frac{\mathsf{a} + b \, x}{\mathsf{c} + d \, x} \right)^{\mathsf{n}} \right]}{\left( b \, \mathsf{c} - \mathsf{a} \, d \right) \, \mathsf{g} \, \mathbf{i} \, \left( 1 + \mathsf{m} \right)}$$

#### Problem 220: Result valid but suboptimal antiderivative.

$$\int \left( a \, g + b \, g \, x \right)^{-2-m} \, \left( c \, \mathbf{i} + d \, \mathbf{i} \, x \right)^m \, \left( A + B \, Log \left[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \right] \, \right) \, \mathrm{d} x$$

Optimal (type 3, 137 leaves, 2 steps):

$$-\frac{B\,n\,\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{2+m}}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(\mathbf{1}+m\right)^{2}\,\left(c+d\,x\right)}-\frac{\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{2+m}\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{\left(b\,c-a\,d\right)\,\,\mathbf{i}^{2}\,\left(\mathbf{1}+m\right)\,\left(c+d\,x\right)}$$

Result (type 3, 170 leaves, 6 steps):

$$-\frac{A \left(a \, g + b \, g \, x\right)^{-1 - m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{1 + m}}{\left(b \, c - a \, d\right) \, g \, \mathbf{i} \, \left(1 + m\right)} - \frac{B \, n \, \left(a \, g + b \, g \, x\right)^{-1 - m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{1 + m}}{\left(b \, c - a \, d\right) \, g \, \mathbf{i} \, \left(1 + m\right)^{2}} - \frac{B \, \left(a \, g + b \, g \, x\right)^{-1 - m} \, \left(c \, \mathbf{i} + d \, \mathbf{i} \, x\right)^{1 + m} \, Log\left[e \left(\frac{a + b \, x}{c + d \, x}\right)^{n}\right]}{\left(b \, c - a \, d\right) \, g \, \mathbf{i} \, \left(1 + m\right)}$$

## Problem 221: Unable to integrate problem.

$$\int \frac{\left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,i+d\,i\,x\right)^{m}}{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}\,\mathrm{d}x$$

Optimal (type 4, 128 leaves, 3 steps):

$$\frac{e^{\frac{A\left(1+m\right)}{B\,n}}\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{\frac{1+m}{n}}\left(i\,\left(c+d\,x\right)\right)^{2+m}\,\text{ExpIntegralEi}\left[-\frac{\left(1+m\right)\,\left(A+B\,\text{Log}\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{B\,n}\right]}{B\,\left(b\,c-a\,d\right)\,i^{2}\,n\,\left(c+d\,x\right)}$$

Result (type 8, 51 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,i+d\,i\,x\right)^{m}}{A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]},\,x\right]$$

## Problem 222: Unable to integrate problem.

$$\int \frac{\left(\text{ag} + \text{bgx}\right)^{-2-\text{m}} \left(\text{ci} + \text{dix}\right)^{\text{m}}}{\left(\text{A} + \text{BLog}\left[\text{e}\left(\frac{\text{a+bx}}{\text{c+dx}}\right)^{\text{n}}\right]\right)^{2}} \, \text{d}x$$

Optimal (type 4, 214 leaves, 4 steps):

$$-\left(\left(\frac{e^{\frac{A(1+m)}{Bn}}}{e^{\frac{Bn}{Bn}}}\left(1+m\right)\left(a+b\,x\right)\left(g\left(a+b\,x\right)\right)^{-2-m}\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right)^{\frac{1+m}{n}}\left(i\,\left(c+d\,x\right)\right)^{2+m}\,\text{ExpIntegralEi}\left[-\frac{\left(1+m\right)\left(A+B\,\text{Log}\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{B\,n}\right]\right)\right/$$

$$\left(B^{2}\left(b\,c-a\,d\right)\,i^{2}\,n^{2}\left(c+d\,x\right)\right) - \frac{\left(a+b\,x\right)\left(g\left(a+b\,x\right)\right)^{-2-m}\left(i\,\left(c+d\,x\right)\right)^{2+m}}{B\left(b\,c-a\,d\right)\,i^{2}\,n\left(c+d\,x\right)\left(A+B\,\text{Log}\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}$$

Result (type 8, 51 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\left(a\,g+b\,g\,x\right)^{-2-m}\,\left(c\,i+d\,i\,x\right)^{m}}{\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)^{2}},\,x\right]$$

## Problem 223: Unable to integrate problem.

$$\int \frac{\left(a g + b g x\right)^{-2-m} \left(c i + d i x\right)^{m}}{\left(A + B Log\left[e\left(\frac{a+b x}{c+d x}\right)^{n}\right]\right)^{3}} dx$$

Optimal (type 4, 306 leaves, 5 steps):

$$\left( e^{\frac{A \, (1+m)}{B \, n}} \, \left( 1+m \right)^2 \, \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \, \right)^{-2-m} \, \left( e \, \left( \frac{a+b \, x}{c+d \, x} \right)^n \right)^{\frac{1+m}{n}} \, \left( i \, \left( c+d \, x \right) \, \right)^{2+m} \, \text{ExpIntegralEi} \left[ -\frac{\left( 1+m \right) \, \left( A+B \, Log \left[ e \, \left( \frac{a+b \, x}{c+d \, x} \right)^n \right] \right)}{B \, n} \right] \right) \\ \left( 2 \, B^3 \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right) - \frac{\left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( g \, \left( a+b \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right) \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}}{2 \, B \, \left( b \, c-a \, d \right) \, i^2 \, n^3 \, \left( c+d \, x \right)^{-2-m} \, \left( i \, \left( c+d \, x \right) \right)^{2+m}} + \frac{\left( 1+m \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right) \, \left( a+b \, x \right)^{2+m} \, \left( a+b \, x \right)^{2+m}}{2 \, B \, \left( a+b \, x \right)^{2+m} \, \left( a+b \, x \right)^{2+m}} \right)^{2+m}} \right)$$

Result (type 8, 51 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \begin{aligned} &\text{CannotIntegrate} \big[ \; \frac{ \left( \text{ag} + \text{bgx} \right)^{-2-\text{m}} \; \left( \text{ci} + \text{dix} \right)^{\text{m}} }{ \left( \text{A} + \text{BLog} \big[ \text{e} \; \left( \frac{\text{a+bx}}{\text{c+dx}} \right)^{\text{n}} \big] \right)^{3}} \text{, } \text{x} \, \big] \end{aligned}$$

#### Problem 226: Unable to integrate problem.

$$\int \left(a\,g+b\,g\,x\right)^{\,m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,-2-m}\,\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,p}\,\mathrm{d}x$$

Optimal (type 4, 193 leaves, 4 steps):

$$\left( e^{-\frac{A\left(1+m\right)}{B\,n}} \left( a+b\,x \right) \, \left( g\, \left( a+b\,x \right) \right)^{\,m} \, \left( i\, \left( c+d\,x \right) \right)^{-m} \, \left( e\, \left( a+b\,x \right)^{\,n} \, \left( c+d\,x \right)^{-n} \right)^{-\frac{1+m}{n}} Gamma \left[ 1+p, -\frac{\left( 1+m \right) \, \left( A+B\,Log \left[ e\, \left( a+b\,x \right)^{\,n} \, \left( c+d\,x \right)^{-n} \right] \right)}{B\,n} \right)^{-p} \right) \\ \left( A+B\,Log \left[ e\, \left( a+b\,x \right)^{\,n} \, \left( c+d\,x \right)^{-n} \right] \right)^{\,p} \left( -\frac{\left( 1+m \right) \, \left( A+B\,Log \left[ e\, \left( a+b\,x \right)^{\,n} \, \left( c+d\,x \right)^{-n} \right] \right)}{B\,n} \right)^{-p} \right) \right/ \, \left( \left( b\,c-a\,d \right) \, i^2 \, \left( 1+m \right) \, \left( c+d\,x \right) \right)^{\,p} \right)^{-p} d^{-p} d$$

Result (type 8, 52 leaves, 0 steps):

$$CannotIntegrate\left[\,\left(a\,g+b\,g\,x\right)^{\,m}\,\left(c\,\mathbf{i}+d\,\mathbf{i}\,x\right)^{\,-2-m}\,\left(A+B\,Log\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,p}\text{, }x\,\right]$$

## Problem 227: Unable to integrate problem.

$$\left[ \left( a\;g + b\;g\;x \right)^{-2-m} \; \left( c\;i + d\;i\;x \right)^{m} \; \left( A + B\;Log\left[ e\; \left( a + b\;x \right)^{n} \; \left( c + d\;x \right)^{-n} \right] \right)^{p} \; \mathrm{d}x \right]$$

Optimal (type 4, 194 leaves, 4 steps):

$$-\left(\left[e^{\frac{A\left(1+m\right)}{B\,n}}\left(a+b\,x\right)\,\left(g\,\left(a+b\,x\right)\right)^{-2-m}\,\left(\mathbf{i}\,\left(c+d\,x\right)\right)^{2+m}\,\left(e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right)^{\frac{1+m}{n}}\mathsf{Gamma}\left[\mathbf{1}+p,\,\frac{\left(\mathbf{1}+m\right)\,\left(A+B\,\mathsf{Log}\left[e\,\left(a+b\,x\right)^{n}\,\left(c+d\,x\right)^{-n}\right]\right)}{B\,n}\right)^{-p}\right)\right)\right)$$

Result (type 8, 52 leaves, 0 steps):

CannotIntegrate  $\left[\left(ag+bgx\right)^{-2-m}\left(ci+dix\right)^{m}\left(A+BLog\left[e\left(a+bx\right)^{n}\left(c+dx\right)^{-n}\right]\right)^{p}$ ,  $x\right]$ 

## Problem 249: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \log\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{4}}{\left(f + g x\right) \left(a h + b h x\right)} dx$$

Optimal (type 4, 361 leaves, 8 steps):

$$-\frac{\left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right)^{4} \, Log \left[1-\frac{\left(b\, f-a\, g\right) \, \left(c+d\, x\right)}{\left(d\, f-c\, g\right) \, \left(a+b\, x\right)}\right]}{\left(b\, f-a\, g\right) \, h}}{\left(b\, f-a\, g\right) \, h} + \\ \frac{4\, B\, n\, \left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right)^{3} \, PolyLog \left[2,\, \frac{\left(b\, f-a\, g\right) \, \left(c+d\, x\right)}{\left(d\, f-c\, g\right) \, \left(a+b\, x\right)}\right]}{\left(d\, f-c\, g\right) \, \left(a+b\, x\right)}} + \frac{12\, B^{2} \, n^{2} \, \left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right)^{2} \, PolyLog \left[3,\, \frac{\left(b\, f-a\, g\right) \, \left(c+d\, x\right)}{\left(d\, f-c\, g\right) \, \left(a+b\, x\right)}\right]}{\left(b\, f-a\, g\right) \, h} + \frac{24\, B^{4} \, n^{4} \, PolyLog \left[5,\, \frac{\left(b\, f-a\, g\right) \, \left(c+d\, x\right)}{\left(d\, f-c\, g\right) \, \left(a+b\, x\right)}\right]}{\left(b\, f-a\, g\right) \, h}$$

Result (type 4, 1021 leaves, 20 steps):

$$\frac{A^4 \log[a + b \, x]}{(b \, f - a \, g) \, h} - \frac{A^4 \log[f + g \, x]}{(b \, f - a \, g) \, h} - \frac{4 \, A^3 \, B \, \log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}] \, \log[-\frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}]}{(b \, f - a \, g) \, h} - \frac{4 \, A^3 \, B \, \log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, \log[-\frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}]}{(b \, f - a \, g) \, h} - \frac{4 \, A \, B^3 \, \log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^3 \, \log[-\frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}]}{(b \, f - a \, g) \, h} + \frac{4 \, A^3 \, B \, n \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}{(b \, f - a \, g) \, h} + \frac{4 \, A^3 \, B \, n \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}{(b \, f - a \, g) \, h} + \frac{12 \, A \, B^3 \, n \, Log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}}{(b \, f - a \, g) \, h} + \frac{4 \, A^3 \, B \, n \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}}{(b \, f - a \, g) \, h} + \frac{12 \, A \, B^3 \, n \, Log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}} + \frac{4 \, A^3 \, B \, n \, Log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}}} + \frac{12 \, A \, B^3 \, n \, Log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, PolyLog[2, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}} + \frac{12 \, A^3 \, B \, n \, Log[e \, (a + b \, x)^n \, (c + d \, x)^{-n}]^2 \, PolyLog[3, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}} + \frac{12 \, B^3 \, n^3 \, PolyLog[6, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}} + \frac{12 \, B^3 \, n^3 \, PolyLog[6, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}} + \frac{12 \, B^3 \, n^3 \, PolyLog[6, 1 + \frac{(b \, c - a \, d) \, (f + g \, x)}{(d \, f - c \, g) \, (a + b \, x)}} + \frac{12 \, B^3 \, n^3 \, PolyLog[6, 1 + \frac{(b \, c - a \, d) \, (f + g \,$$

#### Problem 250: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\,\mathsf{e}\,\left(\,\mathsf{a} + \mathsf{b}\,\mathsf{x}\,\right)^{\,\mathsf{n}}\,\left(\,\mathsf{c} + \mathsf{d}\,\mathsf{x}\,\right)^{\,-\mathsf{n}}\,\right]\,\right)^{\,3}}{\left(\,\mathsf{f} + \mathsf{g}\,\mathsf{x}\,\right)\,\left(\,\mathsf{a}\,\mathsf{h} + \mathsf{b}\,\mathsf{h}\,\mathsf{x}\,\right)} \,\,\mathrm{d} \,\mathsf{x}$$

Optimal (type 4, 282 leaves, 7 steps):

$$-\frac{\left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right)^{3} \, Log \left[1-\frac{(b\, f-a\, g) \, (c+d\, x)}{(d\, f-c\, g) \, (a+b\, x)}\right]}{\left(b\, f-a\, g\right)\, h}+\frac{3\, B\, n\, \left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right)^{2} \, PolyLog \left[2,\, \frac{(b\, f-a\, g) \, (c+d\, x)}{(d\, f-c\, g) \, (a+b\, x)}\right]}{\left(b\, f-a\, g\right)\, h}}{\left(b\, f-a\, g\right)\, h}$$

$$-\frac{6\, B^{2} \, n^{2} \, \left(A+B \, Log \left[e\, \left(a+b\, x\right)^{n} \, \left(c+d\, x\right)^{-n}\right]\right) \, PolyLog \left[3,\, \frac{(b\, f-a\, g) \, (c+d\, x)}{(d\, f-c\, g) \, (a+b\, x)}\right]}{\left(d\, f-c\, g\right)\, \left(a+b\, x\right)}}+\frac{6\, B^{3} \, n^{3} \, PolyLog \left[4,\, \frac{(b\, f-a\, g) \, (c+d\, x)}{(d\, f-c\, g) \, (a+b\, x)}\right]}{\left(b\, f-a\, g\right)\, h}}{\left(b\, f-a\, g\right)\, h}$$

Result (type 4, 656 leaves, 15 steps):

$$\frac{A^{3} \ Log \left[a+b \ x\right]}{\left(b \ f-a \ g\right) \ h} - \frac{A^{3} \ Log \left[f+g \ x\right]}{\left(b \ f-a \ g\right) \ h} - \frac{3 \ A^{2} \ B \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right] \ Log \left[-\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{3 \ A^{2} \ B \ n \ Poly Log \left[2, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{3 \ A^{2} \ B \ n \ Poly Log \left[2, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{3 \ A^{2} \ B \ n \ Poly Log \left[2, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{2} \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right]^{2} \ Poly Log \left[2, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{2} \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right] \ Poly Log \left[3, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{2} \ Log \left[e \ \left(a+b \ x\right)^{n} \left(c+d \ x\right)^{-n}\right] \ Poly Log \left[3, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{3} \ Poly Log \left[4, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{3} \ Poly Log \left[4, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{3} \ Poly Log \left[4, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{3} \ Poly Log \left[4, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h} + \frac{6 \ B^{3} \ n^{3} \ Poly Log \left[4, \ 1+\frac{(b \ c-a \ d) \ (f+g \ x)}{(d \ f-c \ g) \ (a+b \ x)}\right]}{\left(b \ f-a \ g\right) \ h}}$$

## Problem 251: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\mathsf{A} + \mathsf{B} \, \mathsf{Log}\left[\mathsf{e}\, \left(\mathsf{a} + \mathsf{b}\, \mathsf{x}\right)^{\mathsf{n}} \, \left(\mathsf{c} + \mathsf{d}\, \mathsf{x}\right)^{-\mathsf{n}}\right]\right)^{2}}{\left(\mathsf{f} + \mathsf{g}\, \mathsf{x}\right) \, \left(\mathsf{a}\, \mathsf{h} + \mathsf{b}\, \mathsf{h}\, \mathsf{x}\right)} \, \, \mathrm{d} \mathsf{x}$$

Optimal (type 4, 203 leaves, 6 steps):

$$-\frac{\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]\right)^{\,2}\,Log\left[1-\frac{(b\,f-a\,g)\,\,\left(c+d\,x\right)}{(d\,f-c\,g)\,\,\left(a+b\,x\right)}\right]}{\left(b\,f-a\,g\right)\,h}+\\\\ \frac{2\,B\,n\,\left(A+B\,Log\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\right]\right)\,PolyLog\left[2\,\text{,}\,\,\frac{(b\,f-a\,g)\,\,\left(c+d\,x\right)}{(d\,f-c\,g)\,\,\left(a+b\,x\right)}\right]}{\left(b\,f-a\,g\right)\,h}+\\ \frac{2\,B^2\,n^2\,PolyLog\left[3\,\text{,}\,\,\frac{(b\,f-a\,g)\,\,\left(c+d\,x\right)}{(d\,f-c\,g)\,\,\left(a+b\,x\right)}\right]}{\left(b\,f-a\,g\right)\,h}$$

Result (type 4, 371 leaves, 11 steps):

$$\frac{A^{2} \, Log \left[\, a + b \, x \,\right]}{\left(\, b \, f - a \, g \,\right) \, h} - \frac{A^{2} \, Log \left[\, f + g \, x \,\right]}{\left(\, b \, f - a \, g \,\right) \, h} - \frac{2 \, A \, B \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, n} \, \left(\, b \, f - a \, g \,\right) \, h$$

## Problem 252: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log[e(a+bx)^n(c+dx)^{-n}]}{(f+gx)(ah+bhx)} dx$$

Optimal (type 4, 123 leaves, 5 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\,\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^{\,\mathsf{n}}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{\,\mathsf{-n}}\,\right]\,\right)\,\,\mathsf{Log}\left[\,\mathsf{1}-\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\,\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}+\frac{\mathsf{B}\,\mathsf{n}\,\mathsf{PolyLog}\left[\,\mathsf{2}\,\mathsf{,}\,\,\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\,\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}$$

Result (type 4, 163 leaves, 8 steps):

$$\frac{A \, Log \, [\, a+b \, x\,]}{\left(b \, f-a \, g\right) \, h} - \frac{A \, Log \, [\, f+g \, x\,]}{\left(b \, f-a \, g\right) \, h} - \frac{B \, Log \, \left[\, e \, \left(\, a+b \, x\,\right)^{\, n} \, \left(\, c+d \, x\,\right)^{\, n} \, \left(\, c+d \, x\,\right)^{\, n}\, \right] \, Log \, \left[\, -\frac{\left(b \, c-a \, d\right) \, \left(f+g \, x\right)}{\left(d \, f-c \, g\right) \, \left(a+b \, x\right)}\, \right]}{\left(b \, f-a \, g\right) \, h} + \frac{B \, n \, PolyLog \, \left[\, 2\,, \, 1+\frac{\left(b \, c-a \, d\right) \, \left(f+g \, x\right)}{\left(d \, f-c \, g\right) \, \left(a+b \, x\right)}\, \right]}{\left(b \, f-a \, g\right) \, h}$$

## Problem 253: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{1}{\left(f+g\,x\right)\,\left(a\,h+b\,h\,x\right)\,\left(A+B\,Log\left[\,e\,\left(a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)}\,\,\mathrm{d}x$$

Optimal (type 9, 81 leaves, 1 step):

$$Subst \Big[ \text{Unintegrable} \Big[ \frac{1}{\Big( f + g \, x \Big) \, \Big( a \, h + b \, h \, x \Big) \, \Big( A + B \, Log \Big[ e \, \Big( \frac{a + b \, x}{c + d \, x} \Big)^n \Big] \Big)}, \, x \Big], \, e \, \Big( \frac{a + b \, x}{c + d \, x} \Big)^n, \, e \, \Big( a + b \, x \Big)^n \, \Big( c + d \, x \Big)^{-n} \Big]$$

Result (type 8, 102 leaves, 2 steps):

$$\frac{\text{b CannotIntegrate}\Big[\frac{1}{(a+b\,x)\,\left(A+B\,\text{Log}\big[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\big]\right)}\text{, }x\Big]}{\left(b\,f-a\,g\right)\,h}-\frac{g\,\text{CannotIntegrate}\Big[\frac{1}{(f+g\,x)\,\left(A+B\,\text{Log}\big[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\big]\right)}\text{, }x\Big]}{\left(b\,f-a\,g\right)\,h}$$

# Problem 254: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{1}{\left(\texttt{f} + \texttt{g} \, \texttt{x}\right) \, \left(\texttt{a} \, \texttt{h} + \texttt{b} \, \texttt{h} \, \texttt{x}\right) \, \left(\texttt{A} + \texttt{B} \, \texttt{Log}\left[\,\texttt{e} \, \left(\texttt{a} + \texttt{b} \, \texttt{x}\right)^{\, \texttt{n}} \, \left(\texttt{c} + \texttt{d} \, \texttt{x}\right)^{\, -n}\,\right]\,\right)^{\, 2}} \, \, \mathbb{d} \, \texttt{x}$$

Optimal (type 9, 81 leaves, 1 step):

$$Subst \Big[ Unintegrable \Big[ \frac{1}{\Big( f + g \, x \Big) \, \Big( a \, h + b \, h \, x \Big) \, \left( A + B \, Log \Big[ e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \Big] \right)^2} \text{, } x \Big] \text{, } e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \, \left( c + d \, x \right)^{-n} \Big]$$

Result (type 8, 102 leaves, 2 steps):

$$\frac{\text{b CannotIntegrate} \Big[ \frac{1}{\left( \text{a+b x} \right) \, \left( \text{A+B Log} \left[ \text{e } \left( \text{a+b x} \right)^{n} \, \left( \text{c+d x} \right)^{-n} \right] \right)^{2}} \, , \, \, x \Big]}{\left( \text{b f-a g} \right) \, h} \, - \, \frac{\text{g CannotIntegrate} \Big[ \frac{1}{\left( \text{f+g x} \right) \, \left( \text{A+B Log} \left[ \text{e } \left( \text{a+b x} \right)^{n} \, \left( \text{c+d x} \right)^{-n} \right] \right)^{2}} \, , \, \, x \Big]}{\left( \text{b f-a g} \right) \, h}$$

## Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{3}}{a f h + b g h x^{2} + h \left(b f x + a g x\right)} dx$$

Optimal (type 4, 282 leaves, 8 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)^3\,\mathsf{Log}\left[\mathsf{1}-\frac{(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g})\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}+\frac{3\,\mathsf{B}\,\mathsf{n}\,\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)^2\,\mathsf{PolyLog}\left[\mathsf{2}\,,\,\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}+\frac{6\,\mathsf{B}^3\,\mathsf{n}^3\,\mathsf{PolyLog}\left[\mathsf{4}\,,\,\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}}\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}$$

Result (type 4, 656 leaves, 17 steps):

$$\frac{A^{3} \, Log \left[\, a + b \, x\,\right]}{\left(\, b \, f - a \, g\,\right) \, h} - \frac{A^{3} \, Log \left[\, f + g \, x\,\right]}{\left(\, b \, f - a \, g\,\right) \, h} - \frac{3 \, A^{2} \, B \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right] \, Log \left[\, - \frac{\left(\, b \, c - a \, d\,\right) \, \left(\, f + g \, x\,\right)}{\left(\, d \, f - c \, g\,\right) \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, Log \left[\, e \, \left(\, a + b \, x\,\right)^{\, n} \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, PolyLog \left[\, 2 \, , \, \, 1 \, + \, \frac{\left(\, b \, c - a \, d\,\right) \, \left(\, f + g \, x\,\right)}{\left(\, d \, f - c \, g\,\right) \, \left(\, a + b \, x\,\right)^{\, n}} \, + \, \frac{\left(\, b \, c - a \, d\,\right) \, \left(\, c + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, PolyLog \left[\, 2 \, , \, \, 1 \, + \, \frac{\left(\, b \, c - a \, d\,\right) \, \left(\, f + g \, x\,\right)}{\left(\, d \, f - c \, g\,\right) \, \left(\, a + b \, x\,\right)^{\, n}} \, + \, \frac{\left(\, b \, c \, - a \, d\,\right) \, \left(\, c \, + d \, x\,\right)^{\, - n}\,\right]^{\, 2} \, PolyLog \left[\, 2 \, , \, \, 1 \, + \, \frac{\left(\, b \, c \, - a \, d\,\right) \, \left(\, f + g \, x\,\right)}{\left(\, d \, f - c \, g\,\right) \, \left(\, a \, f \, b \, x\,\right)^{\, n}} \, + \, \frac{\left(\, b \, c \, - a \, d\,\right) \, \left(\, c \, + d \, x\,\right)^{\, n}\,\right]^{\, 2} \, PolyLog \left[\, 2 \, , \, \, 1 \, + \, \frac{\left(\, b \, c \, - a \, d\,\right) \, \left(\, f \, + g \, x\,\right)}{\left(\, d \, f - c \, g\,\right) \, \left(\, a \, + b \, x\,\right)^{\, n}} \, + \, \frac{\left(\, b \, c \, - a \, d\,\right) \, \left(\, c \, + d \, x\,\right)^{\,$$

#### Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(a + b x\right)^{n} \left(c + d x\right)^{-n}\right]\right)^{2}}{a f h + b g h x^{2} + h \left(b f x + a g x\right)} dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$-\frac{\left(A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}\,Log\left[\,1-\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B\,n\,\left(\,A+B\,Log\left[\,e\,\left(\,a+b\,x\right)^{\,n}\,\left(\,c+d\,x\right)^{\,-n}\,\right]\,\right)\,PolyLog\left[\,2\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}\,\right]}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,b\,f-a\,g\right)\,\left(\,a+b\,x\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\left(\,a+b\,x\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\left(\,c+d\,x\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,,\,\frac{\left(\,b\,f-a\,g\right)\,\,}{\left(\,d\,f-c\,g\right)\,\,}}}{\left(\,b\,f-a\,g\right)\,h}}{+\frac{2\,B^{2}\,n^{2}\,PolyLog\left[\,3\,h^{2}\,n^{2}\,h^{2}\,h^{2}\,h^{2}\,h$$

Result (type 4, 371 leaves, 13 steps):

$$\frac{A^{2} \, Log \left[\, a + b \, x \,\right]}{\left(\, b \, f - a \, g \,\right) \, h} - \frac{A^{2} \, Log \left[\, f + g \, x \,\right]}{\left(\, b \, f - a \, g \,\right) \, h} - \frac{2 \, A \, B \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, - \frac{\left(\, b \, c - a \, d \,\right) \, \left(\, f + g \, x \,\right)}{\left(\, d \, f - c \, g \,\right) \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, - n} \right] \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, \left(\, c + d \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[\, e \, \left(\, a + b \, x \,\right)^{\, n} \, Log \left[$$

## Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( a + b x \right)^{n} \left( c + d x \right)^{-n} \right]}{a f h + b g h x^{2} + h \left( b f x + a g x \right)} dx$$

Optimal (type 4, 123 leaves, 6 steps):

$$-\frac{\left(\mathsf{A}+\mathsf{B}\,\mathsf{Log}\!\left[\mathsf{e}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)^\mathsf{n}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)^{-\mathsf{n}}\right]\right)\,\mathsf{Log}\!\left[\mathsf{1}-\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}+\frac{\mathsf{B}\,\mathsf{n}\,\mathsf{PolyLog}\!\left[\mathsf{2}\,\mathsf{,}\,\frac{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{x}\right)}{\left(\mathsf{d}\,\mathsf{f}-\mathsf{c}\,\mathsf{g}\right)\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{x}\right)}\right]}{\left(\mathsf{b}\,\mathsf{f}-\mathsf{a}\,\mathsf{g}\right)\,\mathsf{h}}$$

Result (type 4, 163 leaves, 10 steps):

$$\frac{A \, Log \, [\, a \, + \, b \, \, x \,]}{\left(b \, f - a \, g\right) \, h} \, - \, \frac{A \, Log \, [\, f \, + \, g \, x \,]}{\left(b \, f - a \, g\right) \, h} \, - \, \frac{B \, Log \left[\, e \, \left(\, a \, + \, b \, \, x \,\right)^{\, n} \, \left(\, c \, + \, d \, x \,\right)^{\, - n} \right] \, Log \left[\, - \, \frac{\left(\, b \, c - a \, d\right) \, \left(\, f + g \, x \,\right)}{\left(\, d \, f - c \, g\right) \, \left(\, a + b \, x \,\right)} \, \right]}{\left(\, b \, f - a \, g\right) \, h} \, + \, \frac{B \, n \, PolyLog \left[\, 2 \,, \, 1 \, + \, \frac{\left(\, b \, c - a \, d\right) \, \left(\, f + g \, x \,\right)}{\left(\, d \, f - c \, g\right) \, \left(\, a + b \, x \,\right)} \, \right]}{\left(\, b \, f - a \, g\right) \, h}$$

# Problem 262: Rubi result verified and simpler than optimal antiderivative.

$$\int \! \frac{1}{\left(a\,f\,h + b\,g\,h\,x^2 + h\,\left(b\,f\,x + a\,g\,x\right)\,\right)\,\left(A + B\,Log\left[\,e\,\left(a + b\,x\right)^{\,n}\,\left(c + d\,x\right)^{\,-n}\,\right]\,\right)}\,\,\mathrm{d}x$$

Optimal (type 9, 82 leaves, 3 steps):

$$\frac{\text{Subst} \left[ \text{Unintegrable} \left[ \frac{1}{(a+b\,x) \, \left( f+g\,x \right) \, \left( A+B\,\text{Log} \left[ e \left( \frac{a+b\,x}{c+d\,x} \right)^n \right] \right)} \,,\,\, x \right] \,,\,\, e \, \left( \frac{a+b\,x}{c+d\,x} \right)^n \,,\,\, e \, \left( a+b\,x \right)^n \, \left( c+d\,x \right)^{-n} \right]}{h}$$

Result (type 8, 102 leaves, 4 steps):

$$\frac{\text{b CannotIntegrate}\Big[\frac{1}{(a+b\,x)\,\left(A+B\,\text{Log}\big[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\big]\right)}\,\,\text{, }x\Big]}{\left(b\,\,f-a\,\,g\right)\,\,h}\,-\,\frac{g\,\,\text{CannotIntegrate}\,\Big[\,\frac{1}{(f+g\,x)\,\left(A+B\,\text{Log}\big[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{\,-n}\big]\right)}\,\,\text{, }x\Big]}{\left(b\,\,f-a\,\,g\right)\,\,h}$$

## Problem 263: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{1}{\left(a\,f\,h + b\,g\,h\,x^2 + h\,\left(b\,f\,x + a\,g\,x\right)\,\right)\,\,\left(A + B\,Log\left[\,e\,\left(a + b\,x\right)^{\,n}\,\left(\,c + d\,x\right)^{\,-n}\,\right]\,\right)^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 9, 82 leaves, 3 steps):

$$\frac{\text{Subst} \Big[ \text{Unintegrable} \Big[ \, \frac{1}{(\mathsf{a} + \mathsf{b} \, \mathsf{x}) \, \left( \mathsf{f} + \mathsf{g} \, \mathsf{x} \right) \, \left( \mathsf{A} + \mathsf{B} \, \mathsf{Log} \Big[ \mathsf{e} \, \Big( \frac{\mathsf{a} + \mathsf{b} \, \mathsf{x}}{\mathsf{c} + \mathsf{d} \, \mathsf{x}} \Big)^n \Big] \, \mathsf{e} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{x} \right)^n \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right)^{-n} \Big]}{\mathsf{b}}$$

Result (type 8, 102 leaves, 4 steps):

$$\frac{\text{b CannotIntegrate}\Big[\frac{1}{(a+b\,x)\,\left(A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)^{\,2}}\,\text{, }x\Big]}{\left(b\,f-a\,g\right)\,h}-\frac{g\,\text{CannotIntegrate}\Big[\frac{1}{(f+g\,x)\,\left(A+B\,\text{Log}\left[e\,\left(a+b\,x\right)^{\,n}\,\left(c+d\,x\right)^{-n}\right]\right)^{\,2}}\,\text{, }x\Big]}{\left(b\,f-a\,g\right)\,h}$$

# Test results for the 108 problems in "3.2.3 u log(e (f (a+b x) $^p$ ) (c+d x) $^q$ ) $^r$ .

#### Problem 1: Result valid but suboptimal antiderivative.

$$\int \left(f + \frac{g}{x}\right)^3 \, \left(A + B \, \text{Log}\left[\,e\, \left(\frac{a + b \, x}{c + d \, x}\right)^n\,\right]\,\right) \, \text{d}x$$

Optimal (type 4, 404 leaves, 16 steps):

Result (type 4, 385 leaves, 20 steps):

$$-\frac{B\left(b\,c-a\,d\right)\,g^{3}\,n}{2\,a\,c\,x} + A\,f^{3}\,x + \frac{3\,B\left(b\,c-a\,d\right)\,f\,g^{2}\,n\,Log\left[x\right]}{a\,c} - \frac{1}{2}\,B\left(\frac{b^{2}}{a^{2}} - \frac{d^{2}}{c^{2}}\right)\,g^{3}\,n\,Log\left[x\right] - \frac{3\,b\,B\,f\,g^{2}\,n\,Log\left[a+b\,x\right]}{a} + \frac{b^{2}\,B\,g^{3}\,n\,Log\left[a+b\,x\right]}{2\,a^{2}} - 3\,B\,f^{2}\,g\,n\,Log\left[x\right]\,Log\left[1+\frac{b\,x}{a}\right] + \frac{B\,f^{3}\,\left(a+b\,x\right)\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{b} - \frac{g^{3}\,\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right)}{2\,x^{2}} - \frac{3\,B\,d\,f\,g^{2}\,n\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]}{x} + 3\,f^{2}\,g\,Log\left[x\right]\,\left(A+B\,Log\left[e\left(\frac{a+b\,x}{c+d\,x}\right)^{n}\right]\right) - \frac{B\,\left(b\,c-a\,d\right)\,f^{3}\,n\,Log\left[c+d\,x\right]}{b\,d} + \frac{3\,B\,d\,f\,g^{2}\,n\,Log\left[c+d\,x\right]}{c} - \frac{B\,d^{2}\,g^{3}\,n\,Log\left[c+d\,x\right]}{2\,c^{2}} + 3\,B\,f^{2}\,g\,n\,Log\left[x\right]\,Log\left[1+\frac{d\,x}{c}\right] - 3\,B\,f^{2}\,g\,n\,PolyLog\left[2,-\frac{b\,x}{a}\right] + 3\,B\,f^{2}\,g\,n\,PolyLog\left[2,-\frac{d\,x}{c}\right]$$

# Problem 2: Result valid but suboptimal antiderivative.

$$\int \left( f + \frac{g}{x} \right)^2 \left( A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 4, 263 leaves, 13 steps):

$$A f^2 x - 2 B f g n Log[x] Log[1 + \frac{b x}{a}] + \frac{B f^2 (a + b x) Log[e(\frac{a + b x}{c + d x})^n]}{b} + \frac{(b c - a d) g^2 (a + b x) (A + B Log[e(\frac{a + b x}{c + d x})^n])}{a (c + d x) (a - \frac{c (a + b x)}{c + d x})} + \frac{2 f g Log[x] (A + B Log[e(\frac{a + b x}{c + d x})^n]) - \frac{B (b c - a d) f^2 n Log[c + d x]}{b d} + 2 B f g n Log[x] Log[1 + \frac{d x}{c}] + \frac{B (b c - a d) g^2 n Log[a - \frac{c (a + b x)}{c + d x}]}{a c} - 2 B f g n PolyLog[2, -\frac{b x}{a}] + 2 B f g n PolyLog[2, -\frac{d x}{c}]$$

Result (type 4, 242 leaves, 16 steps):

$$A f^2 x + \frac{B \left(b c - a d\right) g^2 n Log[x]}{a c} - \frac{b B g^2 n Log[a + b x]}{a} - 2 B f g n Log[x] Log[1 + \frac{b x}{a}] + \frac{B f^2 \left(a + b x\right) Log[e\left(\frac{a + b x}{c + d x}\right)^n]}{b} - \frac{g^2 \left(A + B Log[e\left(\frac{a + b x}{c + d x}\right)^n]\right)}{x} + 2 f g Log[x] \left(A + B Log[e\left(\frac{a + b x}{c + d x}\right)^n]\right) - \frac{B \left(b c - a d\right) f^2 n Log[c + d x]}{b d} + \frac{B d g^2 n Log[c + d x]}{c} + 2 B f g n Log[x] Log[1 + \frac{d x}{c}] - 2 B f g n PolyLog[2, -\frac{b x}{a}] + 2 B f g n PolyLog[2, -\frac{d x}{c}]$$

## Problem 3: Result optimal but 2 more steps used.

$$\int \left(f + \frac{g}{x}\right) \left(A + B Log \left[e \left(\frac{a + b x}{c + d x}\right)^{n}\right]\right) dx$$

Optimal (type 4, 143 leaves, 10 steps):

$$A f x - B g n Log[x] Log[1 + \frac{b x}{a}] + \frac{B f (a + b x) Log[e (\frac{a + b x}{c + d x})^n]}{b} + g Log[x] (A + B Log[e (\frac{a + b x}{c + d x})^n]) - \frac{B (b c - a d) f n Log[c + d x]}{b d} + B g n Log[x] Log[1 + \frac{d x}{c}] - B g n PolyLog[2, -\frac{b x}{a}] + B g n PolyLog[2, -\frac{d x}{c}]$$

Result (type 4, 143 leaves, 12 steps):

$$A f x - B g n Log[x] Log[1 + \frac{b x}{a}] + \frac{B f (a + b x) Log[e (\frac{a + b x}{c + d x})^n]}{b} + g Log[x] (A + B Log[e (\frac{a + b x}{c + d x})^n]) - \frac{B (b c - a d) f n Log[c + d x]}{b d} + B g n Log[x] Log[1 + \frac{d x}{c}] - B g n PolyLog[2, -\frac{b x}{a}] + B g n PolyLog[2, -\frac{d x}{c}]$$

# Problem 4: Result optimal but 2 more steps used.

$$\int \frac{A + B Log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{f + \frac{g}{x}} dx$$

Optimal (type 4, 217 leaves, 12 steps):

$$\frac{A\,x}{f} + \frac{B\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{b\,f} - \frac{B\,\left(b\,c-a\,d\right)\,n\,Log\left[c+d\,x\right]}{b\,d\,f} + \frac{B\,g\,n\,Log\left[\frac{f\,(a+b\,x)}{a\,f-b\,g}\right]\,Log\left[g+f\,x\right]}{f^2} - \frac{g\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[g+f\,x\right]}{f^2} - \frac{B\,g\,n\,Log\left[\frac{f\,(c+d\,x)}{c\,f-d\,g}\right]\,Log\left[g+f\,x\right]}{f^2} + \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{b\,(g+f\,x)}{a\,f-b\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g\,x)}{c\,f-d\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g\,x)}{c\,f-d\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g\,x)}{c\,f-d\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g$$

Result (type 4, 217 leaves, 14 steps):

$$\frac{A\,x}{f} + \frac{B\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{b\,f} - \frac{B\,\left(b\,c-a\,d\right)\,n\,Log\left[c+d\,x\right]}{b\,d\,f} + \frac{B\,g\,n\,Log\left[\frac{f\,(a+b\,x)}{a\,f-b\,g}\right]\,Log\left[g+f\,x\right]}{f^2} - \frac{g\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[g+f\,x\right]}{f^2} - \frac{B\,g\,n\,Log\left[\frac{f\,(c+d\,x)}{c\,f-d\,g}\right]\,Log\left[g+f\,x\right]}{f^2} + \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{b\,(g+f\,x)}{a\,f-b\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^2} - \frac{B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g$$

#### Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e^{\left(\frac{a + b x}{c + d x}\right)^{n}} \right]}{\left(f + \frac{g}{x}\right)^{2}} dx$$

Optimal (type 4, 322 leaves, 15 steps):

$$\frac{A\,x}{f^2} + \frac{B\,\left(a+b\,x\right)\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]}{b\,f^2} - \frac{g^2\,\left(a+b\,x\right)\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)}{f^2\left(a\,f-b\,g\right)\,\left(g+f\,x\right)} - \frac{B\,\left(b\,c-a\,d\right)\,n\,Log\left[c+d\,x\right]}{b\,d\,f^2} + \\ \frac{2\,B\,g\,n\,Log\left[\frac{f\,(a+b\,x)}{a\,f-b\,g}\right]\,Log\left[g+f\,x\right]}{f^3} - \frac{2\,g\,\left(A+B\,Log\left[e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right]\right)\,Log\left[g+f\,x\right]}{f^3} - \frac{2\,B\,g\,n\,Log\left[\frac{f\,(c+d\,x)}{c\,f-d\,g}\right]\,Log\left[g+f\,x\right]}{f^3} + \\ \frac{B\,\left(b\,c-a\,d\right)\,g^2\,n\,Log\left[\frac{g+f\,x}{c+d\,x}\right]}{f^2\left(a\,f-b\,g\right)\,\left(c\,f-d\,g\right)} + \frac{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{b\,(g+f\,x)}{a\,f-b\,g}\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^3} + \\ \frac{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^3} + \frac{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{b\,(g+f\,x)}{a\,f-b\,g}\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^3} + \\ \frac{1}{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+f\,x)}{c\,f-d\,g}\right]}{f^3} + \frac{1}{2\,B\,g\,n\,PolyLog\left[2\,,\,-\frac{d\,(g+g\,x)}{c\,f-d\,g}\right]}{f^3} +$$

Result (type 4, 352 leaves, 18 steps):

$$\frac{A\,x}{f^2} - \frac{b\,B\,g^2\,n\,Log\,[\,a + b\,x\,]}{f^3\,\left(a\,f - b\,g\right)} + \frac{B\,\left(a + b\,x\right)\,Log\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{b\,f^2} - \frac{g^2\,\left(A + B\,Log\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\right)}{f^3\,\left(g + f\,x\right)} - \frac{B\,\left(b\,c - a\,d\right)\,n\,Log\,[\,c + d\,x\,]}{b\,d\,f^2} + \frac{B\,\left(b\,c - a\,d\right)\,g^2\,n\,Log\,[\,g + f\,x\,]}{f^3\,\left(c\,f - d\,g\right)} + \frac{2\,B\,g\,n\,Log\,\left[\frac{f\,(a + b\,x)}{a\,f - b\,g}\right]\,Log\,[\,g + f\,x\,]}{f^3} - \frac{2\,B\,g\,n\,Log\,\left[\frac{e\,(a + b\,x)}{c\,f - d\,g}\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,g + f\,x\,\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]\,hog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}{f^3} - \frac{2\,B\,g\,n\,PolyLog\,\left[\,e\,\left(\frac{a + b\,x}{c + d\,x}\right)^{\,n}\,\right]}$$

## Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \log \left[ e \left( \frac{a + b x}{c + d x} \right)^{n} \right]}{\left( f + \frac{g}{x} \right)^{3}} dx$$

Optimal (type 4, 531 leaves, 18 steps):

$$\frac{A\,x}{f^3} + \frac{B\,\left(b\,c - a\,d\right)\,g^3\,n}{2\,f^3\,\left(a\,f - b\,g\right)\,\left(c\,f - d\,g\right)\,\left(g + f\,x\right)} - \frac{b^2\,B\,g^3\,n\,Log\left[a + b\,x\right]}{2\,f^4\,\left(a\,f - b\,g\right)^2} + \frac{B\,\left(a + b\,x\right)\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]}{b\,f^3} + \frac{g^3\,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\right)}{2\,f^4\,\left(g + f\,x\right)^2} - \frac{3\,g^2\,\left(a + b\,x\right)\,\left(A + B\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\right)}{f^3\,\left(a\,f - b\,g\right)\,\left(g + f\,x\right)} - \frac{B\,\left(b\,c - a\,d\right)\,n\,Log\left[c + d\,x\right]}{b\,d\,f^3} + \frac{B\,d^2\,g^3\,n\,Log\left[c + d\,x\right]}{2\,f^4\,\left(c\,f - d\,g\right)^2} + \frac{B\,\left(b\,c - a\,d\right)\,g^3\,\left(b\,c\,f + a\,d\,f - 2\,b\,d\,g\right)\,n\,Log\left[g + f\,x\right]}{f^3\,\left(a\,f - b\,g\right)^2\,\left(c\,f - d\,g\right)^2} + \frac{3\,B\,g\,n\,Log\left[\frac{f\,(a + b\,x)}{a\,f - b\,g}\right]\,Log\left[g + f\,x\right]}{f^4} - \frac{3\,B\,g\,n\,Log\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\,hog\left[g + f\,x\right]}{f^4} + \frac{3\,B\,g\,n\,Log\left[\frac{g + f\,x}{c + d\,x}\right]}{f^3\,\left(a\,f - b\,g\right)\,\left(c\,f - d\,g\right)} + \frac{3\,B\,g\,n\,PolyLog\left[2, -\frac{b\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\left[2, -\frac{d\,(g + f\,x)}{c\,f - d\,g}\right]}{f^4} + \frac{3\,B\,g\,n\,PolyLog\left[2, -\frac{d\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} + \frac{3\,B\,g\,n\,PolyLog\left[2, -\frac{b\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\left[2, -\frac{d\,(g + f\,x)}{c\,f - d\,g}\right]}{f^4} + \frac{1}{f^4} + \frac{1}{f$$

Result (type 4, 562 leaves, 22 steps):

$$\frac{A\,x}{f^3} + \frac{B\,\left(b\,c - a\,d\right)\,g^3\,n}{2\,f^3\,\left(a\,f - b\,g\right)\,\left(c\,f - d\,g\right)\,\left(g + f\,x\right)}{2\,f^4\,\left(a\,f - b\,g\right)^2} - \frac{b^2\,B\,g^3\,n\,Log\,[a + b\,x]}{f^4\,\left(a\,f - b\,g\right)} + \frac{B\,\left(a + b\,x\right)\,Log\,\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]}{b\,f^3} + \frac{g^3\,\left(A + B\,Log\,\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\right)}{f^4\,\left(g + f\,x\right)} - \frac{3\,g^2\,\left(A + B\,Log\,\left[e\,\left(\frac{a + b\,x}{c + d\,x}\right)^n\right]\right)}{f^4\,\left(g + f\,x\right)} - \frac{B\,\left(b\,c - a\,d\right)\,n\,Log\,[c + d\,x]}{b\,d\,f^3} + \frac{B\,d^2\,g^3\,n\,Log\,[c + d\,x]}{2\,f^4\,\left(c\,f - d\,g\right)^2} + \frac{3\,B\,d\,g^2\,n\,Log\,[c + d\,x]}{f^4\,\left(c\,f - d\,g\right)} + \frac{3\,B\,d\,g^2\,n\,Log\,[c + d\,x]}{f^4\,\left(c\,f - d\,g\right)} + \frac{3\,B\,g\,n\,Log\,\left[\frac{f\,(a + b\,x)}{a\,f - b\,g}\right]\,Log\,[g + f\,x]}{f^4\,\left(c\,f - d\,g\right)} + \frac{3\,B\,g\,n\,Log\,\left[\frac{f\,(a + b\,x)}{a\,f - b\,g}\right]\,Log\,[g + f\,x]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\,\left[2\,, -\frac{b\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\,\left[2\,, -\frac{d\,(g + f\,x)}{c\,f - d\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\,\left[2\,, -\frac{d\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\,\left[2\,, -\frac{d\,(g + f\,x)}{a\,f - b\,g}\right]}{f^4} - \frac{3\,B\,g\,n\,PolyLog\,\left[2\,, -\frac{d\,(g + f\,x)}{c\,f - d\,g}\right]}{f^4} - \frac{1}{f^4} + \frac{1}{f^4} - \frac{1}$$

# Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[e\left(f\left(a+b\,x\right)^{p}\left(c+d\,x\right)^{q}\right)^{r}\right]^{2}}{g+h\,x}\,dx$$

Optimal (type 4, 1471 leaves, ? steps):

$$\frac{pq\, n^2 \log \left[ -\frac{b(x+d-1)}{(g+h)} \log \left( -\frac{b(x+h)}{(g+h)} \right) - p^2 \log (g+hx) - 2p\, q^2 \log (g+hx) - \log (c+dx) \log [g+hx] - h}{h} }{h}$$

$$\frac{q^2 n^2 \log (c+dx)^2 \log [g+hx]}{h} - 2p\, \log (g+hx) - 2p\, q^2 \log (g+hx) - h}{h}$$

$$\frac{2q\, n^2 \log (c+dx)^2 \log [g+hx]}{h} - 2p\, q^2 \log (g+hx) - 2p\, q^2 \log (g+hx) - h}{h}$$

$$\frac{2q\, n^2 \log (c+dx) \log \left[ g \left( f \left( a+bx \right)^p \left( c+dx \right)^q \right)^r \right] \log [g+hx]}{h} - p\, q^2 \log \left[ \frac{b(c+dx)}{bg+h} \right] \log \left[ \frac{b(c$$

 $\frac{Log\left[\left(c+d\,x\right)^{\,q\,r}\right]^{\,2}\,Log\left[g+h\,x\right]}{h}\,+\,\frac{1}{h}2\,p\,r\,Log\left[-\,\frac{h\,\left(a+b\,x\right)}{b\,g-a\,h}\,\right]\,\left(Log\left[\left(a+b\,x\right)^{\,p\,r}\right]\,+\,Log\left[\left(c+d\,x\right)^{\,q\,r}\right]\,-\,Log\left[e\,\left(f\,\left(a+b\,x\right)^{\,p}\,\left(c+d\,x\right)^{\,q}\right)^{\,r}\right]\right)\,Log\left[g+h\,x\right]\,+\,Log\left[\left(c+d\,x\right)^{\,q\,r}\right]$ 

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\frac{1}{h} 2 \operatorname{qr} \operatorname{Log} \left[ -\frac{h \left(c + d \, X\right)}{d \, g - c \, h} \right] \left( \operatorname{Log} \left[ \left(a + b \, X\right)^{p \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] - \operatorname{Log} \left[ e \left(f \left(a + b \, X\right)^{p} \left(c + d \, X\right)^{q}\right)^{r} \right] \right) \operatorname{Log} \left[ g + h \, X \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname{Log} \left[ \left(c + d \, X\right)^{q \, r} \right] + \operatorname
        \frac{\text{Log}\left[\text{e}\left(\text{f}\left(\text{a}+\text{b}\,\text{x}\right)^{\text{p}}\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}}\right)^{\text{r}}\right]^{\text{2}}\,\text{Log}\left[\text{g}+\text{h}\,\text{x}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{a}+\text{b}\,\text{x}\right)^{\text{p}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{b}\cdot(\text{g}+\text{h}\,\text{x})}{\text{b}\,\text{g}-\text{a}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{c}+\text{d}\,\text{x}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\cdot(\text{g}+\text{h}\,\text{x})}{\text{d}\,\text{g}-\text{c}\,\text{h}}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\,\text{g}-\text{c}\,\text{h}}\text{d}\,\text{s}\right]}{\text{h}}}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\,\text{g}-\text{c}\,\text{h}}{\text{s}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]^{\text{2}}\,\text{Log}\left[\frac{\text{d}\,\text{g}-\text{c}\,\text{h}}{\text{s}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{s}\right)^{\text{q}\,\text{r}}\right]}{\text{h}}+\frac{\text{Log}\left[\left(\text{d}\,\text{s}\right)^{\text{q}\,\text{r
           p \ q \ r^2 \ \left( Log \Big[ \frac{b \ (c+d \ x)}{b \ c-a \ d} \Big] \ + \ Log \Big[ \frac{b \ g-a \ h}{b \ (g+h \ x)} \Big] \ - \ Log \Big[ \frac{(b \ g-a \ h) \ (c+d \ x)}{(b \ c-a \ d) \ (g+h \ x)} \Big] \right) \ Log \Big[ - \frac{(b \ c-a \ d) \ (g+h \ x)}{(d \ g-c \ h) \ (a+b \ x)} \Big]^2 
          p \ q \ r^2 \ \left( Log \left[ \frac{b \ (c+d \ x)}{b \ c-a \ d} \right] - Log \left[ - \frac{h \ (c+d \ x)}{d \ g-c \ h} \right] \right) \ \left( Log \left[ \ a + b \ x \right] \ + Log \left[ - \frac{(b \ c-a \ d) \ (g+h \ x)}{(d \ g-c \ h) \ (a+b \ x)} \right] \right)^2
          p \ q \ r^2 \ \left( Log \left[ -\frac{d \ (a+b \ x)}{b \ c-a \ d} \right] \ + \ Log \left[ \frac{d \ g-c \ h}{d \ (g+h \ x)} \right] \ - \ Log \left[ -\frac{(d \ g-c \ h) \ (a+b \ x)}{(b \ c-a \ d) \ (g+h \ x)} \right] \right) \ Log \left[ \frac{(b \ c-a \ d) \ (g+h \ x)}{(b \ g-a \ h) \ (c+d \ x)} \right]^2
           p q r^2 \left( Log \left[ -\frac{d \cdot (a+b \cdot x)}{b \cdot c-a \cdot d} \right] - Log \left[ -\frac{h \cdot (a+b \cdot x)}{b \cdot g-a \cdot h} \right] \right) \left( Log \left[ c + d \cdot x \right] + Log \left[ \frac{(b \cdot c-a \cdot d) \cdot (g+h \cdot x)}{(b \cdot g-a \cdot h) \cdot (c+d \cdot x)} \right] \right)^2 \\ = 2 p q r^2 \left( Log \left[ g + h \cdot x \right] - Log \left[ -\frac{(b \cdot c-a \cdot d) \cdot (g+h \cdot x)}{(d \cdot g-c \cdot h) \cdot (a+b \cdot x)} \right] \right) PolyLog \left[ 2 , -\frac{d \cdot (a+b \cdot x)}{b \cdot c-a \cdot d} \right] 
          \frac{2 \text{ p q r}^2 \text{ Log} \left[-\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{d g-c h}) \cdot (\text{a+b x})}\right] \text{ PolyLog} \left[2, -\frac{(\text{d g-c h}) \cdot (\text{a+b x})}{(\text{b c-a d}) \cdot (\text{g+h x})}\right]}{(\text{b c-a d}) \cdot (\text{g+h x})} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{d} \cdot (\text{g+h x})} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right] \text{ PolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{c+d x})}{\text{d} \cdot (\text{g+h x})}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{g+h x})}{(\text{b g-a h}) \cdot (\text{c+d x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{b c-a h})}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{b c-a h})}{\text{d} \cdot (\text{g+h x})}\right]}\right]} \\ + \frac{2 \text{ p q r}^2 \text{ Log} \left[\frac{(\text{b c-a d}) \cdot (\text{b c-a h})}{(\text{b c-a h}) \cdot (\text{c+d x})}\right]}{\text{HolyLog} \left[2, \frac{\text{h} \cdot (\text{b c-a h})}{\text{d} \cdot (\text{c+d x})}\right]} \\ + \frac{2 \text{ p 
          2\,p\,q\,r^2\,Log\left[\,\frac{(b\,c-a\,d)\ (g+h\,x)}{(b\,g-a\,h)\ (c+d\,x)}\,\right]\,PolyLog\left[\,2\,\text{, }\,\frac{(b\,g-a\,h)\ (c+d\,x)}{(b\,c-a\,d)\ (g+h\,x)}\,\right]\\ -2\,p\,r\,\left(\,q\,r\,Log\left[\,c\,+d\,x\,\right]\,-Log\left[\,\left(\,c\,+d\,x\,\right)^{\,q\,r}\,\right]\,\right)\,PolyLog\left[\,2\,\text{, }\,\frac{b\,(g+h\,x)}{b\,g-a\,h}\,\left(\,c\,+d\,x\,\right)^{\,q\,r}\,\right]\\ -2\,p\,r\,\left(\,q\,r\,Log\left[\,c\,+d\,x\,\right]\,-Log\left[\,\left(\,c\,+d\,x\,\right)^{\,q\,r}\,\right]\,\right)\,PolyLog\left[\,2\,\text{, }\,\frac{b\,(g+h\,x)}{b\,g-a\,h}\,\left(\,c\,+d\,x\,\right)^{\,q\,r}\,\right]
          2\,p\,r\,\left(\text{Log}\left[\,\left(\,a\,+\,b\,\,x\,\right)^{\,p\,r}\,\right]\,+\,\text{Log}\left[\,\left(\,c\,+\,d\,\,x\,\right)^{\,q\,r}\,\right]\,-\,\text{Log}\left[\,e\,\,\left(\,f\,\left(\,a\,+\,b\,\,x\,\right)^{\,p}\,\left(\,c\,+\,d\,\,x\,\right)^{\,q}\,\right)^{\,r}\,\right]\,\right)\,\,\text{PolyLog}\left[\,2\,\text{,}\,\,\frac{b\,\,\left(\,g\,+\,h\,\,x\,\right)}{b\,g\,-\,a\,h}\,\right]
          \frac{2 \, p \, q \, r^2 \, \left( \text{Log} \left[ \, c + d \, x \, \right] \, + \, \text{Log} \left[ \, \frac{\left( b \, c - a \, d \right) \, \left( g + h \, x \right)}{\left( b \, g - a \, h \right) \, \left( c + d \, x \right)} \, \right] \right) \, \text{PolyLog} \left[ \, 2 \, , \, \, \frac{b \, \left( g + h \, x \right)}{b \, g - a \, h} \, \right] }{b \, g - a \, h} \, \left[ \, \frac{2 \, q \, r \, \left( p \, r \, \, \text{Log} \left[ \, a + b \, x \, \right] \, - \, \text{Log} \left[ \, \left( a + b \, x \, \right)^{\, p \, r} \, \right] \right) \, \text{PolyLog} \left[ \, 2 \, , \, \, \frac{d \, \left( g + h \, x \right)}{d \, g - c \, h} \, \right] }{c \, g - c \, h} \, \left[ \, \frac{d \, \left( g + h \, x \, \right)}{d \, g - c \, h} \, \right] \right] \, \left[ \, \frac{d \, \left( g + h \, x \, \right)}{d \, g - c \, h} \, \right] 
          2\,q\,r\,\left(\text{Log}\left[\,\left(a+b\,x\right)^{\,p\,r}\,\right]\,+\,\text{Log}\left[\,\left(c+d\,x\right)^{\,q\,r}\,\right]\,-\,\text{Log}\left[\,e\,\left(f\,\left(a+b\,x\right)^{\,p}\,\left(c+d\,x\right)^{\,q}\right)^{\,r}\,\right]\,\right)\,\,\text{PolyLog}\left[\,2\,\text{,}\,\,\frac{d\,\left(g+h\,x\right)}{d\,g-c\,h}\,\right]
          2 p q r^2 \left( Log[a+bx] + Log\left[-\frac{(bc-ad) \cdot (g+hx)}{(dg-ch) \cdot (a+bx)}\right] \right) PolyLog\left[2, \frac{d \cdot (g+hx)}{dg-ch}\right] - 2 p q r^2 PolyLog\left[3, -\frac{d \cdot (a+bx)}{bc-ad}\right]
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     2 p^2 r^2 PolyLog [3, -\frac{h(a+bx)}{haa}]
                                                                                                                                                                                                                                                                                                                                                                                                              \frac{2 \operatorname{q}^2 \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{h} \cdot (\operatorname{c} + \operatorname{d} x)}{\operatorname{d} \operatorname{g} - \operatorname{c} \operatorname{h}} \right]}{\operatorname{d} \operatorname{g} - \operatorname{c} \operatorname{h}} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{h} \cdot (\operatorname{a} + \operatorname{b} x)}{\operatorname{b} \cdot (\operatorname{g} + \operatorname{h} x)} \right]}{\operatorname{b} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{c} \operatorname{h} \cdot (\operatorname{a} + \operatorname{b} x)}{\operatorname{b} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{b} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{c} \operatorname{h} \cdot (\operatorname{a} + \operatorname{b} x)}{\operatorname{b} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{c} \operatorname{h} \cdot (\operatorname{d} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{c} \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{r}^2 \operatorname{PolyLog} \left[ 3 \text{, } -\frac{\operatorname{d} \operatorname{d} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \right]}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{d} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} \operatorname{q} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{p} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)}{\operatorname{g} - \operatorname{g} - \operatorname{h} \cdot (\operatorname{g} - \operatorname{h} x)} \frac{2 \operatorname{g} - \operatorname{g}
          2 p q r^2 PolyLog \left[ 3, \frac{b (c+d x)}{b c-a d} \right]
```

$$\frac{2 p q r^2 PolyLog \left[3, \frac{h \cdot (c + d \cdot x)}{d \cdot (g + h \cdot x)}\right]}{h} - \frac{2 p q r^2 PolyLog \left[3, \frac{(b \cdot g - a \cdot h) \cdot (c + d \cdot x)}{(b \cdot c - a \cdot d) \cdot (g + h \cdot x)}\right]}{h} + \frac{2 p q r^2 PolyLog \left[3, \frac{b \cdot (g + h \cdot x)}{b \cdot g - a \cdot h}\right]}{h} + \frac{2 p q r^2 PolyLog \left[3, \frac{d \cdot (g + h \cdot x)}{b \cdot g - a \cdot h}\right]}{h}$$

# Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{Log \left[ e \left( f \left( a + b \, x \right)^p \left( c + d \, x \right)^q \right)^r \right]^2}{\left( g + h \, x \right)^2} \, \mathrm{d} x$$

Optimal (type 4, 832 leaves, 31 steps):

$$\frac{2 \, b \, p \, q \, r^2 \, Log[-\frac{d \, (a + b \, x)}{b \, c - a \, d}] \, Log[c + d \, x]}{h \, (b \, g - a \, h)} + \frac{2 \, d \, p \, q \, r^2 \, Log[a + b \, x] \, Log[e \, (f \, (a + b \, x)^p \, (c + d \, x)^q)^r])}{h \, (b \, g - a \, h)} - \frac{2 \, b \, p \, r \, Log[a + b \, x] \, (p \, r \, Log[a + b \, x] + q \, r \, Log[c + d \, x] - Log[e \, (f \, (a + b \, x)^p \, (c + d \, x)^q)^r])}{h \, (b \, g - a \, h)} - \frac{2 \, d \, q \, r \, Log[c + d \, x] \, + q \, r \, Log[c + d \, x] - Log[e \, (f \, (a + b \, x)^p \, (c + d \, x)^q)^r])}{h \, (d \, g - c \, h)} - \frac{1}{h \, (d \, g - c \, h)} + \frac{2 \, b \, p \, r \, (p \, r \, Log[a + b \, x] + q \, r \, Log[c + d \, x] - Log[e \, (f \, (a + b \, x)^p \, (c + d \, x)^q)^r]) \, Log[g + h \, x]}{h \, (b \, g - a \, h)} + \frac{2 \, d \, q \, r \, (p \, r \, Log[c + d \, x] - Log[e \, (f \, (a + b \, x)^p \, (c + d \, x)^q)^r]) \, Log[g + h \, x]}{h \, (d \, g - c \, h)} - \frac{2 \, d \, p \, q \, r^2 \, Log[a + b \, x] \, Log[\frac{b \, (g + h \, x)}{b \, g - a \, h}]}{h \, (d \, g - c \, h)} + \frac{2 \, d \, p \, q \, r^2 \, Log[a + b \, x] \, Log[1 + \frac{b \, g - a \, h}{h \, (a + b \, x)^p}]}{h \, (d \, g - c \, h)} - \frac{2 \, d \, p \, q \, r^2 \, Log[c + d \, x] \, Log[1 + \frac{d \, g - c \, h}{h \, (c + d \, x)}]}{h \, (d \, g - c \, h)} + \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}]}{h \, (d \, g - c \, h)} - \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{h \, (a + b \, x)}{b \, g - a \, h}]}{h \, (d \, g - c \, h)} + \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}]}{h \, (d \, g - c \, h)} - \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{h \, (a + b \, x)}{b \, g - a \, h}]}{h \, (d \, g - c \, h)} + \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}]}{h \, (d \, g - c \, h)} - \frac{2 \, b \, p \, q \, r^2 \, PolyLog[2, -\frac{h \, (a + b \, x)}{d \, g - c \, h}]}{h \, (b \, g - a \, h)} + \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{d \, (a + b \, x)}{b \, c - a \, d}]}{h \, (b \, g - a \, h)} - \frac{2 \, b \, p \, q \, r^2 \, PolyLog[2, -\frac{h \, (a + b \, x)}{d \, g - c \, h}}{h \, (b \, g - a \, h)} + \frac{2 \, d \, p \, q \, r^2 \, PolyLog[2, -\frac{h \, (a + b \, x)}{b \, c - a \, d}]}{h \, (b \, g - a \, h)} - \frac{2 \, b \, p \, q \, r^2 \, PolyLog[2, -\frac{$$

Result (type 4, 878 leaves, 35 steps):

$$\frac{b \, p^2 \, r^2 \, Log[a + b \, x]^2}{h \, (b \, g - a \, h)} + \frac{2 \, b \, p \, q \, r^2 \, Log\left[-\frac{d \, (a + b \, x)}{b \, c - a \, d}\right] \, Log[c + d \, x)}{h \, (b \, g - a \, h)} + \frac{d \, q^2 \, r^2 \, Log[c + d \, x]^2}{h \, (d \, g - c \, h)} + \frac{2 \, d \, p \, q \, r^2 \, Log[a + b \, x] \, Log\left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{h \, (d \, g - c \, h)} - \frac{2 \, b \, p \, r \, Log[a + b \, x] \, (p \, r \, Log[a + b \, x] + q \, r \, Log[c + d \, x] - Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(b \, g - a \, h\right)} - \frac{2 \, d \, q \, r \, Log[c + d \, x] + q \, r \, Log[c + d \, x] - Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(d \, g - c \, h\right)} - \frac{Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(g + h \, x\right)} + \frac{2 \, d \, p \, r \, \left(p \, r \, Log[a + b \, x] + q \, r \, Log[c + d \, x] - Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(b \, g - a \, h\right)} + \frac{2 \, d \, q \, r \, \left(p \, r \, Log[a + b \, x] + q \, r \, Log[c + d \, x] - Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(d \, g - c \, h\right)} - \frac{2 \, d \, q \, r \, \left(p \, r \, Log[a + b \, x] + q \, r \, Log\left[e \, \left(f \, \left(a + b \, x\right)^p \, \left(c + d \, x\right)^q\right)^r\right]\right)}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log[a + b \, x] \, Log\left[\frac{b \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} - \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{b \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g + h \, x)}{b \, g - a \, h}\right]}{h \, \left(d \, g - c \, h\right)} + \frac{2 \, d \, p \, q \, r^2 \, Log\left[c + d \, x\right] \, Log\left[\frac{d \, (g \, h \, x)}{d \, g - c \, h}\right]}{h \, \left(d \, g - c \, h\right)} +$$

## Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[e\left(f\left(a+b\,x\right)^{p}\left(c+d\,x\right)^{q}\right)^{r}\right]^{2}}{\left(g+h\,x\right)^{3}}\,dx$$

Optimal (type 4, 1304 leaves, 43 steps):

$$\frac{b \, d \, p \, q \, r^2 \, Log(a + b \, x)}{h \, (b \, g - a \, h)} \frac{d \, p \, q \, r^2 \, Log(a + b \, x)}{h \, (b \, g - a \, h)^2 \, (g + b \, x)} \frac{b \, p^2 \, r^2 \, (a + b \, x) \, Log(a + b \, x)}{(b \, g - a \, h)^2 \, (g + b \, x)} \frac{b \, d \, p \, q \, r^2 \, Log(c + d \, x)}{(b \, g - a \, h)^2 \, (g + b \, x)} \frac{d \, p \, q \, r^2 \, Log(c + d \, x)}{(d \, g - c \, h)^2 \, (g + b \, x)} \frac{d \, p \, q \, r^2 \, Log(c + d \, x)}{(d \, g - c \, h)^2 \, (g + b \, x)} \frac{b^2 \, p \, q \, r^2 \, Log\left[-\frac{d \, (a + b \, x)^2}{b \, c - a \, d}\right]}{h \, (b \, g - a \, h)^2} \frac{d^2 \, p \, q \, r^2 \, Log(a + b \, x) \, Log\left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{h \, (b \, g - a \, h)^2 \, (g + b \, x)} \frac{d^2 \, p \, q \, r^2 \, Log(a + b \, x) \, Log\left[\frac{b \, (c + d \, x)}{b \, c - a \, d}\right]}{h \, (b \, g - a \, h)^2 \, (g + b \, x)} \frac{d^2 \, p \, q \, r^2 \, Log(a + b \, x) \, q \, r \, Log\left[e \, \left(f \, (a + b \, x)^2 \, \left(c + d \, x\right)^4\right)^7\right]\right)}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, p \, Log(a + b \, x) \, q \, r \, Log\left[e \, \left(f \, (a + b \, x)^2 \, \left(c + d \, x\right)^4\right)^7\right]\right)}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, Log\left[a + b \, x\right] \, q \, r \, Log\left[c + d \, x\right] \, - Log\left[e \, \left(f \, (a + b \, x\right)^2 \, \left(c + d \, x\right)^4\right)^7\right]\right)}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, Log\left[a + b \, x\right] \, q \, r \, Log\left[c + d \, x\right] \, - Log\left[e \, \left(f \, (a + b \, x\right)^2 \, \left(c + d \, x\right)^4\right)^7\right]\right)}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, Log\left[a + b \, x\right] \, q \, r \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right]}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, Log\left[a + b \, x\right] \, q \, r \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right]}{h \, (b \, g - a \, h)^2} \frac{d^2 \, q \, r \, Log\left[a + b \, x\right] \, q \, Log\left[a + b \, x\right]$$

Result (type 4, 1362 leaves, 47 steps):

$$\frac{b \, d \, p \, q \, r^2 \, Log \left[ a + b \, x \right]}{h \, \left( b \, g - a \, h \right)} \, \frac{d \, p \, q \, r^2 \, Log \left[ a + b \, x \right]}{h \, \left( b \, g - c \, h \right)} \, \frac{b \, p^2 \, r^2 \, \left( a + b \, x \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{b \, d \, p \, q \, r^2 \, Log \left[ c + d \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{b \, d \, p \, q \, r^2 \, Log \left[ c + d \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{b \, d \, p \, q \, r^2 \, Log \left[ c + d \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{b^2 \, p \, q \, r^2 \, Log \left[ c + d \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{b^2 \, p \, q \, r^2 \, Log \left[ c + d \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, q^2 \, Log \left[ c + d \, x \right]^2}{2 \, h \, \left( d \, g - c \, h \right)^2} \, \frac{d^2 \, q \, q^2 \, Log \left[ c + d \, x \right]^2}{2 \, h \, \left( d \, g - c \, h \right)^2} \, \frac{d^2 \, q \, q^2 \, Log \left[ c + d \, x \right]^2}{h \, \left( d \, g - c \, h \right)^2} \, \frac{d^2 \, q \, q^2 \, Log \left[ c + d \, x \right]^2}{h \, \left( d \, g - c \, h \right)^2} \, \frac{d^2 \, q \, q^2 \, Log \left[ c + d \, x \right] \, Log \left[ e \, \left( f \, \left( a + b \, x \right)^p \, \left( c + d \, x \right)^q \, r^p \right) \right)}{h \, \left( d \, g - c \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ c + d \, x \right] \, - Log \left[ e \, \left( f \, \left( a + b \, x \right)^p \, \left( c + d \, x \right)^q \, r^p \right) \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ c + d \, x \right] \, - Log \left[ e \, \left( f \, \left( a + b \, x \right)^p \, \left( c + d \, x \right)^q \, r^p \right) \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ c + d \, x \right] \, - Log \left[ e \, \left( f \, \left( a + b \, x \right)^p \, \left( c + d \, x \right)^q \, r^p \right) \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ c + d \, x \right] \, d^2 \, r^2 \, Log \left[ a + b \, x \right]}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, \left( c + d \, x \right)^q \, r^2 \, \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, \left( c + d \, x \right)^q \, r^2 \, \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, \left( c + d \, x \right)^q \, r^2 \, \right)}{h \, \left( b \, g - a \, h \right)^2} \, \frac{d^2 \, q \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ a + b \, x \right] \, r^2 \, Log \left[ a + b \, x \right] \, r^$$

# Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[e\left(f\left(a+b\,x\right)^{p}\left(c+d\,x\right)^{q}\right)^{r}\right]^{2}}{\left(g+h\,x\right)^{4}}\,\mathrm{d}x$$

Optimal (type 4, 1957 leaves, 57 steps):

$$\frac{b^2 p^2 r^2}{3h \left( bg - ah \right)^2 \left( g + hx \right)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^2 d p q r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^2 d p q r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^2 d p q r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^2 \left( bg - ch \right)^2} = \frac{b^3 p^2 r^2 \log (a + bx)}{3h \left( bg - ah \right)^3} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p^2 r^2 \log (a + bx)}{b^3 p^2 r^2 \log (a + bx)} = \frac{b^3 p$$

$$\frac{2 \, b^{3} \, p^{2} \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{b \, g - a \, h}{h \, (a + b \, x)} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, d^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{d \, (a + b \, x)}{b \, c - a \, d} \, \right]}{3 \, h \, \left( d \, g - c \, h \right)^{3}} - \frac{2 \, d^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (a + b \, x)}{b \, g - a \, h} \, \right]}{3 \, h \, \left( d \, g - c \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} - \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{b \, (c + d \, x)}{b \, c - a \, d} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \, h \, \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a \, h \, h \, \right)^{3}} + \frac{2 \, b^{3} \, p \, q \, r^{2} \, PolyLog \left[ \, 2 \, , \, - \frac{h \, (c + d \, x)}{d \, g - c \, h} \, \right]}{3 \, h \, \left( b \, g - a$$

#### Result (type 4, 2013 leaves, 61 steps):

$$\frac{b^2 p^2 r^2}{3h \left( bg - ah \right)^2 \left( g + hx \right)}{3h \left( bg - ah \right)} \frac{3h \left( bg - ah \right) \left( g - ch \right) \left( g + hx \right)}{3h \left( bg - ah \right)^2 \left( g + hx \right)} \frac{3h \left( bg - ah \right) \left( g - ch \right) \left( g - ch \right)}{3h \left( bg - ah \right)^2 \left( g - ch \right)} \frac{3h \left( bg - ah \right)^3}{3h \left( bg - ah \right)} \frac{3h \left( bg - ah \right) \left( g - ch \right)^2}{3h \left( bg - ah \right)^2 \left( g - ch \right)} \frac{b^2 q^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right) \left( g + bx \right)} + \frac{dp q^2 Log \left( a + bx \right)}{3h \left( dg - ch \right)^2 \left( g + bx \right)} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( dg - ch \right)^2 \left( g + bx \right)} + \frac{2b^2 p^2 r^2 \left( a + bx \right) Log \left( a + bx \right)}{3h \left( bg - ah \right)^3 \left( bg - ah \right) \left( g + bx \right)^2} + \frac{3h \left( dg - ch \right)^2 \left( g + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)^2}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 r^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( bg - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( ag - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( ag - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( ag - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h \left( ag - ah \right)^3} + \frac{2d^2 q^2 q^2 Log \left( a + bx \right)}{3h$$

$$\frac{2\,d^{3}\,q\,r\,\left(p\,r\,Log\,[\,a+b\,x\,]+q\,r\,Log\,[\,c+d\,x\,]\,-Log\,\left[\,e\,\left(f\,\left(\,a+b\,x\,\right)^{\,p}\,\left(\,c+d\,x\,\right)^{\,q}\,\right)^{\,r}\,\right]\right)\,Log\,[\,g+h\,x\,]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} - \frac{2\,b^{3}\,p^{2}\,r^{2}\,Log\,[\,a+b\,x\,]\,Log\,\left[\,\frac{b\,\left(g+h\,x\,\right)}{b\,g-a\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,b^{3}\,p\,q\,r^{2}\,Log\,[\,c+d\,x\,]\,Log\,\left[\,\frac{d\,\left(g+h\,x\,\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,d^{3}\,q^{2}\,r^{2}\,Log\,[\,c+d\,x\,]\,Log\,\left[\,\frac{d\,\left(g+h\,x\,\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} + \frac{2\,d^{3}\,p\,q\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{d\,\left(a+b\,x\,\right)}{b\,c-a\,d}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,b^{3}\,p^{2}\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(a+b\,x\,\right)}{b\,g-a\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,d^{3}\,p\,q\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(a+b\,x\,\right)}{b\,g-a\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} + \frac{2\,b^{3}\,p\,q\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{b\,g-a\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,d^{3}\,q^{2}\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{b\,g-a\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} + \frac{2\,d^{3}\,p\,q\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(b\,g-a\,h\right)^{\,3}} - \frac{2\,d^{3}\,q^{2}\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} + \frac{2\,d^{3}\,p\,q\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} - \frac{2\,d^{3}\,q^{2}\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{\,3}} + \frac{2\,d^{3}\,q^{2}\,r^{2}\,PolyLog\,\left[\,2\,,\,-\frac{h\,\left(c+d\,x\right)}{d\,g-c\,h}\,\right]}{3\,h\,\left(d\,g-c\,h\right)^{$$

## Problem 43: Result optimal but 1 more steps used.

$$\int \frac{\left(a + b \operatorname{Log}\left[\frac{\sqrt{1 - c x}}{\sqrt{1 + c x}}\right]\right)^{n}}{1 - c^{2} x^{2}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{\left(a+b Log\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^{1+n}}{b c \left(1+n\right)}$$

Result (type 3, 42 leaves, 3 steps):

$$-\frac{\left(a+b \log \left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^{1+n}}{b c \left(1+n\right)}$$

# Problem 47: Result optimal but 1 more steps used.

$$\int \; \frac{1}{\left(1-c^2\;x^2\right)\; \left(\mathsf{a}+\mathsf{b}\;\mathsf{Log}\left[\frac{\sqrt{1-c\;x}}{\sqrt{1+c\;x}}\right]\right)} \; \mathrm{d}x$$

Optimal (type 3, 34 leaves, 2 steps):

$$-\frac{\mathsf{Log}\left[\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\frac{\sqrt{\mathsf{1-c}\,\mathsf{x}}}{\sqrt{\mathsf{1+c}\,\mathsf{x}}}\right]\right]}{\mathsf{b}\,\mathsf{c}}$$

Result (type 3, 34 leaves, 3 steps):

$$-\frac{\mathsf{Log}\left[\,\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\,\frac{\sqrt{1-\mathsf{c}\,\mathsf{x}}}{\sqrt{1+\mathsf{c}\,\mathsf{x}}}\,\right]\,\right]}{\mathsf{b}\,\mathsf{c}}$$

# Problem 48: Result optimal but 1 more steps used.

$$\int \frac{1}{\left(1 - c^2 x^2\right) \left(a + b Log\left[\frac{\sqrt{1 - c x}}{\sqrt{1 + c x}}\right]\right)^2} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{1}{b c \left(a + b Log \left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)}$$

Result (type 3, 34 leaves, 3 steps):

$$\frac{1}{b c \left(a + b Log\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)}$$

## Problem 49: Result optimal but 1 more steps used.

$$\int \frac{1}{\left(1-c^2 \, x^2\right) \, \left(\mathsf{a}+\mathsf{b} \, \mathsf{Log}\left[\frac{\sqrt{1-c \, x}}{\sqrt{1+c \, x}}\right]\right)^3} \, \mathrm{d} x$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{1}{2 b c \left(a + b Log \left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2}$$

Result (type 3, 37 leaves, 3 steps):

$$\frac{1}{2 b c \left(a + b log \left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2}$$

## Problem 74: Unable to integrate problem.

$$\int \left( \frac{1}{\left(c+d\,x\right)\,\left(-a+c+\left(-b+d\right)\,x\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]} + \frac{Log\left[1-\frac{a+b\,x}{c+d\,x}\right]}{\left(a+b\,x\right)\,\left(c+d\,x\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^2} \right) \, \mathrm{d}x$$

Optimal (type 3, 45 leaves, ? steps):

$$- \; \frac{ \; Log \left[ 1 - \frac{a + b \, x}{c + d \, x} \right] }{ \left( b \; c \; - \; a \; d \right) \; Log \left[ \frac{a + b \, x}{c + d \, x} \right] }$$

Result (type 8, 152 leaves, 3 steps):

$$\frac{\text{b CannotIntegrate} \left[ \frac{\text{Log} \left[ 1 - \frac{a + b \, x}{c + d \, x} \right]}{\text{b c - a d}} \right], \, x \right]}{\text{b c - a d}} - \frac{\text{d CannotIntegrate} \left[ \frac{\text{Log} \left[ 1 - \frac{a + b \, x}{c + d \, x} \right]}{\text{(c+d \, x) Log} \left[ \frac{a + b \, x}{c + d \, x} \right]^2} \right]}{\text{b c - a d}} + \text{Unintegrable} \left[ \frac{1}{\left( c + d \, x \right) \, \left( -a + c + \left( -b + d \right) \, x \right) \, \text{Log} \left[ \frac{a + b \, x}{c + d \, x} \right]}}{\text{b c - a d}} \right], \, x \right]$$

## Problem 75: Unable to integrate problem.

$$\int \left( -\frac{1}{\left(a+b\,x\right)\,\left(a-c+\left(b-d\right)\,x\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]} + \frac{Log\left[1-\frac{c+d\,x}{a+b\,x}\right]}{\left(a+b\,x\right)\,\left(c+d\,x\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]^2} \right) \,\mathrm{d}x$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{Log\left[1-\frac{c+d\,x}{a+b\,x}\right]}{\left(b\,c-a\,d\right)\,Log\left[\frac{a+b\,x}{c+d\,x}\right]}$$

Result (type 8, 154 leaves, 3 steps):

$$\frac{b \, \text{CannotIntegrate} \Big[ \, \frac{\text{Log} \left[ 1 - \frac{c \cdot d \, x}{a \cdot b \, x} \right]^2}{(a + b \, x) \, \text{Log} \left[ \frac{a \cdot b \, x}{c \cdot d \, x} \right]^2} \,, \, \, x \, \Big]}{b \, c - a \, d} - \frac{d \, \text{CannotIntegrate} \Big[ \, \frac{\text{Log} \left[ 1 - \frac{c \cdot d \, x}{a \cdot b \, x} \right]}{(c + d \, x) \, \text{Log} \left[ \frac{a \cdot b \, x}{c \cdot d \, x} \right]^2} \,, \, \, x \, \Big]}{b \, c - a \, d} - \text{Unintegrable} \Big[ \, \frac{1}{\left( a + b \, x \right) \, \left( a - c + \left( b - d \right) \, x \right) \, \text{Log} \left[ \frac{a + b \, x}{c \cdot d \, x} \right]} \,, \, \, x \, \Big]}$$

## Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[e\left(\frac{a+bx}{c+dx}\right)^{n}\right]}{f-gx^{2}} dx$$

Optimal (type 4, 291 leaves, 7 steps):

$$\frac{\text{Log}\Big[e\left(\frac{\mathsf{a}+\mathsf{b}\,x}{\mathsf{c}+\mathsf{d}\,x}\right)^n\Big]\,\,\text{Log}\Big[1-\frac{\left(\mathsf{d}\,\sqrt{\mathsf{f}}\,-\mathsf{c}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{a}+\mathsf{b}\,x)}{\left(\mathsf{b}\,\sqrt{\mathsf{f}}\,-\mathsf{a}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{c}+\mathsf{d}\,x)}\Big]}{2\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{g}}}-\frac{\text{Log}\Big[e\left(\frac{\mathsf{a}+\mathsf{b}\,x}{\mathsf{c}+\mathsf{d}\,x}\right)^n\Big]\,\,\text{Log}\Big[1-\frac{\left(\mathsf{d}\,\sqrt{\mathsf{f}}\,+\mathsf{c}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{a}+\mathsf{b}\,x)}{\left(\mathsf{b}\,\sqrt{\mathsf{f}}\,+\mathsf{a}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{c}+\mathsf{d}\,x)}\Big]}{2\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{g}}}+\frac{\mathsf{n}\,\,\mathsf{PolyLog}\Big[2\,\mathsf{n}\,\,\frac{\left(\mathsf{d}\,\sqrt{\mathsf{f}}\,-\mathsf{c}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{a}+\mathsf{b}\,x)}{\left(\mathsf{b}\,\sqrt{\mathsf{f}}\,-\mathsf{a}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{c}+\mathsf{d}\,x)}\Big]}{2\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{g}}}-\frac{\mathsf{n}\,\,\mathsf{PolyLog}\Big[2\,\mathsf{n}\,\,\frac{\left(\mathsf{d}\,\sqrt{\mathsf{f}}\,+\mathsf{c}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{a}+\mathsf{b}\,x)}{\left(\mathsf{b}\,\sqrt{\mathsf{f}}\,+\mathsf{a}\,\sqrt{\mathsf{g}}\right)\,\,(\mathsf{c}+\mathsf{d}\,x)}\Big]}{2\,\sqrt{\mathsf{f}}\,\,\sqrt{\mathsf{g}}}$$

#### Result (type 4, 468 leaves, 18 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{g} \ x}{\sqrt{f}}\right] \left(n \ \text{Log}\left[a + b \ x\right] - \text{Log}\left[e\left(\frac{a + b \ x}{c + d \ x}\right)^n\right] - n \ \text{Log}\left[c + d \ x\right]\right)}{\sqrt{f} \ \sqrt{g}} \\ -\frac{n \ \text{Log}\left[c + d \ x\right] \ \text{Log}\left[\frac{b \left(\sqrt{f} - \sqrt{g} \ x\right)}{b \sqrt{f} + a \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{Log}\left[a + b \ x\right] \ \text{Log}\left[\frac{b \left(\sqrt{f} + \sqrt{g} \ x\right)}{b \sqrt{f} - a \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} - \frac{n \ \text{Log}\left[c + d \ x\right] \ \text{Log}\left[\frac{d \left(\sqrt{f} + \sqrt{g} \ x\right)}{b \sqrt{f} - c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{Log}\left[a + b \ x\right] \ \text{Log}\left[\frac{b \left(\sqrt{f} + \sqrt{g} \ x\right)}{b \sqrt{f} - a \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} - \frac{n \ \text{Log}\left[c + d \ x\right] \ \text{Log}\left[\frac{d \left(\sqrt{f} + \sqrt{g} \ x\right)}{d \sqrt{f} - c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac{n \ \text{PolyLog}\left[2, \frac{\sqrt{g} \ (c + d \ x)}{d \sqrt{f} + c \sqrt{g}}\right]}{2 \ \sqrt{f} \ \sqrt{g}} + \frac$$

#### Problem 85: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Log}\left[\mathsf{e}\left(\frac{\mathsf{a}+\mathsf{b}\,\mathsf{x}}{\mathsf{c}+\mathsf{d}\,\mathsf{x}}\right)^{\mathsf{n}}\right]}{\mathsf{f}+\mathsf{g}\,\mathsf{x}+\mathsf{h}\,\mathsf{x}^{2}}\,\mathrm{d}\mathsf{x}$$

#### Optimal (type 4, 401 leaves, 7 steps):

$$\frac{ \text{Log} \Big[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \Big] \, \text{Log} \Big[ 1 - \frac{2 \, \left( d^2 \, f - c \, d \, g + c^2 \, h \right) \, \left( a + b \, x \right)}{\left( 2 \, b \, d \, f - b \, c \, g - a \, d \, g + 2 \, a \, c \, h - \left( b \, c - a \, d \right) \, \sqrt{g^2 - 4 \, f \, h} \, \right) \, \left( c + d \, x \right)} + \frac{ \text{Log} \Big[ e \left( \frac{a + b \, x}{c + d \, x} \right)^n \Big] \, \text{Log} \Big[ 1 - \frac{2 \, \left( d^2 \, f - c \, d \, g + c^2 \, h \right) \, \left( a + b \, x \right)}{\left( 2 \, b \, d \, f - b \, c \, g - a \, d \, g + 2 \, a \, c \, h + \left( b \, c - a \, d \right) \, \sqrt{g^2 - 4 \, f \, h} \, \right) \, \left( c + d \, x \right)} \Big] }{ \sqrt{g^2 - 4 \, f \, h}} + \frac{ n \, \text{PolyLog} \Big[ 2 \, , \, \frac{2 \, \left( d^2 \, f - c \, d \, g + c^2 \, h \right) \, \left( a + b \, x \right)}{\left( 2 \, b \, d \, f - b \, c \, g - a \, d \, g + 2 \, a \, c \, h - \left( b \, c - a \, d \right) \, \sqrt{g^2 - 4 \, f \, h} \, \right) \, \left( c + d \, x \right)} }{ \sqrt{g^2 - 4 \, f \, h}} + \frac{n \, \text{PolyLog} \Big[ 2 \, , \, \frac{2 \, \left( d^2 \, f - c \, d \, g + c^2 \, h \right) \, \left( a + b \, x \right)}{\left( 2 \, b \, d \, f - b \, c \, g - a \, d \, g + 2 \, a \, c \, h + \left( b \, c - a \, d \, y \, \sqrt{g^2 - 4 \, f \, h} \, \right)} }{ \sqrt{g^2 - 4 \, f \, h}} \Big] \, \left( c + d \, x \right)}$$

Result (type 4, 545 leaves, 19 steps):

$$\frac{2 \, \text{ArcTanh} \big[ \frac{g + 2 \, h \, x}{\sqrt{g^2 - 4 \, f \, h}} \big] \, \left( n \, \text{Log} \, \big[ \, a + b \, x \, \big] \, - \text{Log} \, \big[ \, e \, \left( \frac{a + b \, x}{c + d \, x} \right)^n \big] \, - n \, \text{Log} \, \big[ \, c + d \, x \, \big]}{\sqrt{g^2 - 4 \, f \, h}} \, + \, \frac{n \, \text{Log} \, \big[ \, a + b \, x \, \big] \, \text{Log} \, \big[ \, - \frac{b \, \big[ g - \sqrt{g^2 - 4 \, f \, h} \, + 2 \, h \, x \big]}{2 \, a \, h - b \, \big[ g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}} \, - \, \frac{n \, \text{Log} \, \big[ \, a + b \, x \, \big] \, \text{Log} \, \big[ \, - \frac{b \, \big[ g + \sqrt{g^2 - 4 \, f \, h} \, + 2 \, h \, x \big]}{2 \, a \, h - b \, \big[ g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}}{\sqrt{g^2 - 4 \, f \, h}} \, - \, \frac{n \, \text{Log} \, \big[ \, a + b \, x \, \big] \, \text{Log} \, \big[ \, - \frac{b \, \big[ g + \sqrt{g^2 - 4 \, f \, h} \, + 2 \, h \, x \big]}{2 \, a \, h - b \, \big[ g + \sqrt{g^2 - 4 \, f \, h}} \, \big]}}{\sqrt{g^2 - 4 \, f \, h}} \, + \, \frac{n \, \text{Log} \, \big[ \, c + d \, x \, \big] \, \text{Log} \, \big[ \, - \frac{d \, \big[ g + \sqrt{g^2 - 4 \, f \, h} \, + 2 \, h \, x \big]}{2 \, c \, h - d \, \big[ g + \sqrt{g^2 - 4 \, f \, h}} \, \big]}}{\sqrt{g^2 - 4 \, f \, h}} \, + \, \frac{n \, \text{Log} \, \big[ \, c + d \, x \, \big] \, \text{Log} \, \big[ \, - \frac{d \, \big[ \, g + \sqrt{g^2 - 4 \, f \, h} \, + 2 \, h \, x \big]}{2 \, c \, h - d \, \big[ \, g + \sqrt{g^2 - 4 \, f \, h}} \, \big]}}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g + \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, - \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \, 2 \, \frac{2 \, h \, (c + d \, x)}{2 \, c \, h - d \, \big[ \, g - \sqrt{g^2 - 4 \, f \, h}} \, \big]}{\sqrt{g^2 - 4 \, f \, h}}} \, + \, \frac{n \, \text{PolyLog} \, \big[ \,$$

## Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}\left[\frac{(b\,e-a\,f)\cdot(c+d\,x)}{(d\,e-c\,f)\cdot(a+b\,x)}\right]^2}{e+f\,x} \, \text{d}x$$

#### Optimal (type 4, 322 leaves, 9 steps):

$$-\frac{\text{Log}\left[-\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]\,\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2}{\text{f}} + \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,\text{Log}\left[1-\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{\text{f}} - \frac{2\,\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{\text{f}} + \frac{2\,\text{PolyLog}\left[3,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{\text{f}} - \frac{2\,\text{PolyLog}\left[3,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{\text{f}} + \frac{2\,\text{PolyLog}\left[3,\,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{\text{f}} + \frac{2\,\text{PolyLog}\left[3,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{\text{f}} + \frac{2\,\text{PolyLog}\left[3,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{d\,(a+b\,x)}\right]}{\text{f}} + \frac{2\,\text{PolyLog}\left[3,\,\frac{(b\,e-a\,f)\,(c+d\,x)}{d\,(a+b\,x)}\right]}{$$

#### Result (type 4, 334 leaves, 7 steps):

$$-\frac{\text{Log}\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2}{f} + \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,\text{Log}\left[\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{f} + \frac{2\,\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(e+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(e+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(e+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(e+d\,x)}\right]}{f} - \frac{2\,\text{PolyLog}\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(e+d\,x)}\right]}{f} - \frac{2\,\text{P$$

## Problem 108: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[\frac{(b\,e-a\,f)\cdot(c+d\,x)}{(d\,e-c\,f)\cdot(a+b\,x)}\right]\,Log\left[\frac{b\cdot(e+f\,x)}{b\,e-a\,f}\right]}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,\mathrm{d}x$$

#### Optimal (type 4, 433 leaves, 10 steps):

$$-\frac{\text{Log}\left[-\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]\,\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2}{2\,\left(b\,c-a\,d\right)} - \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,\text{Log}\left[\frac{b\,(e+f\,x)}{b\,e-a\,f}\right]}{2\,\left(b\,c-a\,d\right)} + \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,\text{Log}\left[1 - \frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{2\,\left(b\,c-a\,d\right)} - \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{2\,\left(b\,c-a\,d\right)} + \frac{\text{Log}\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]}{2\,\left(b\,c-a\,d\right)} - \frac{\text{PolyLog}\left[3,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b\,c-a\,d} - \frac{\text{PolyLog}\left[3,\frac{(b\,e-a\,f)\,(c+d\,x)}{d\,(a+b\,x)}\right]}{b\,c-a\,d} - \frac{\text{PolyLog}\left[3,\frac{(b\,e-a\,f$$

#### Result (type 4, 445 leaves, 8 steps):

$$-\frac{Log\left[\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]\,Log\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2}{2\,\left(b\,c-a\,d\right)} - \frac{Log\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,Log\left[\frac{b\,(e+f\,x)}{b\,e-a\,f}\right]}{2\,\left(b\,c-a\,d\right)} + \frac{Log\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]^2\,Log\left[\frac{(b\,e-a\,f)}{b\,(c+d\,x)}\right]}{2\,\left(b\,c-a\,d\right)} + \frac{Log\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]\,PolyLog\left[2\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b\,c-a\,d} - \frac{Log\left[\frac{(b\,e-a\,f)\,(c+d\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{b\,c-a\,d} - \frac{PolyLog\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{b\,c-a\,d} + \frac{PolyLog\left[3\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b\,c-a\,d} - \frac{PolyLog\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{b\,c-a\,d} + \frac{PolyLog\left[3\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b\,c-a\,d} - \frac{PolyLog\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{b\,c-a\,d} + \frac{PolyLog\left[3\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b\,c-a\,d} - \frac{PolyLog\left[3\,,\,1-\frac{(b\,c-a\,d)\,(e+f\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]}{b\,c-a\,d} + \frac{PolyLog\left[3\,,\,1-\frac{b\,c-a\,d}{b\,(c+d\,x)}\right]}{b\,c-a\,d} + \frac{PolyLog\left[3\,,\,1-\frac{b\,c-a\,d}{b\,$$

# Test results for the 547 problems in "3.3 u (a+b log(c (d+e x)^n))^p.m"

#### Problem 44: Result valid but suboptimal antiderivative.

$$\int \left(f+g\,x\right)^3\,\left(a+b\,\text{Log}\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]\,\right)^2\,\text{d}x$$

#### Optimal (type 3, 365 leaves, 8 steps):

$$\frac{2 \, b^2 \, \left(e \, f - d \, g\right)^3 \, n^2 \, x}{e^3} + \frac{3 \, b^2 \, g \, \left(e \, f - d \, g\right)^2 \, n^2 \, \left(d + e \, x\right)^2}{4 \, e^4} + \frac{2 \, b^2 \, g^2 \, \left(e \, f - d \, g\right) \, n^2 \, \left(d + e \, x\right)^3}{9 \, e^4} + \frac{b^2 \, \left(e \, f - d \, g\right)^4 \, n^2 \, \text{Log} \left[d + e \, x\right]^2}{4 \, e^4 \, g} - \frac{2 \, b \, \left(e \, f - d \, g\right)^3 \, n \, \left(d + e \, x\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{e^4} - \frac{3 \, b \, g \, \left(e \, f - d \, g\right)^2 \, n \, \left(d + e \, x\right)^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{2 \, e^4} - \frac{2 \, b \, g^2 \, \left(e \, f - d \, g\right) \, n \, \left(d + e \, x\right)^3 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{3 \, e^4} - \frac{b \, g^3 \, n \, \left(d + e \, x\right)^4 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{2 \, e^4} - \frac{b \, \left(e \, f - d \, g\right)^4 \, n \, \text{Log} \left[d + e \, x\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{3 \, e^4} + \frac{\left(f + g \, x\right)^4 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{4 \, g} - \frac{b \, \left(e \, f - d \, g\right)^4 \, n \, \text{Log} \left[d + e \, x\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{4 \, g} + \frac{\left(f + g \, x\right)^4 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{4 \, g} + \frac{b \, g \, a \, b \, b \, g \, a \, b \,$$

Result (type 3, 301 leaves, 6 steps):

$$\frac{2\,b^{2}\,\left(e\,f-d\,g\right)^{3}\,n^{2}\,x}{e^{3}}\,+\,\frac{3\,b^{2}\,g\,\left(e\,f-d\,g\right)^{2}\,n^{2}\,\left(d+e\,x\right)^{2}}{4\,e^{4}}\,+\,\frac{2\,b^{2}\,g^{2}\,\left(e\,f-d\,g\right)\,n^{2}\,\left(d+e\,x\right)^{3}}{9\,e^{4}}\,+\,\frac{b^{2}\,g^{3}\,n^{2}\,\left(d+e\,x\right)^{4}}{32\,e^{4}}\,+\,\frac{b^{2}\,\left(e\,f-d\,g\right)^{4}\,n^{2}\,Log\left[d+e\,x\right]^{2}}{4\,e^{4}\,g}\,-\,\frac{1}{24\,g}\,n^{2}\,\left(d+e\,x\right)^{2}\,\left(d+e\,x\right)^{2}\,\left(d+e\,x\right)^{2}\,\left(d+e\,x\right)^{2}\,+\,\frac{16\,g^{3}\,\left(e\,f-d\,g\right)\,\left(d+e\,x\right)^{3}}{e^{4}}\,+\,\frac{3\,g^{4}\,\left(d+e\,x\right)^{4}}{e^{4}}\,+\,\frac{12\,\left(e\,f-d\,g\right)^{4}\,Log\left[d+e\,x\right]}{e^{4}}\right)}{\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)^{2}}\,$$

#### Problem 45: Result valid but suboptimal antiderivative.

$$\int \left(f+g\,x\right)^{\,2}\,\left(a+b\,Log\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

Optimal (type 3, 287 leaves, 8 steps):

$$\frac{2\,b^{2}\,\left(e\,f-d\,g\right)^{2}\,n^{2}\,x}{e^{2}} + \frac{b^{2}\,g\,\left(e\,f-d\,g\right)\,n^{2}\,\left(d+e\,x\right)^{2}}{2\,e^{3}} + \frac{2\,b^{2}\,g^{2}\,n^{2}\,\left(d+e\,x\right)^{3}}{27\,e^{3}} + \frac{b^{2}\,\left(e\,f-d\,g\right)^{3}\,n^{2}\,Log\left[d+e\,x\right]^{2}}{3\,e^{3}\,g} - \frac{2\,b\,\left(e\,f-d\,g\right)^{2}\,n\,\left(d+e\,x\right)\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{e^{3}} - \frac{b\,g\,\left(e\,f-d\,g\right)\,n\,\left(d+e\,x\right)^{2}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{e^{3}} - \frac{2\,b\,\left(e\,f-d\,g\right)^{3}\,n\,Log\left[d+e\,x\right]^{2}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,e^{3}\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)^{2}}{3\,g} - \frac{2\,b\,\left(e\,f-d\,g\right)^{3}\,n\,Log\left[d+e\,x\right]^{2}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(f+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)^{2}}{3\,g} - \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)^{2}}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]\right)}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} + \frac{\left(g+g\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{n}\right]}{3\,g} +$$

Result (type 3, 243 leaves, 8 steps):

$$\frac{2\,b^{2}\,\left(e\,f-d\,g\right)^{2}\,n^{2}\,x}{e^{2}}\,+\,\frac{b^{2}\,g\,\left(e\,f-d\,g\right)\,n^{2}\,\left(d+e\,x\right)^{2}}{2\,e^{3}}\,+\,\frac{2\,b^{2}\,g^{2}\,n^{2}\,\left(d+e\,x\right)^{3}}{27\,e^{3}}\,+\,\frac{b^{2}\,\left(e\,f-d\,g\right)^{3}\,n^{2}\,Log\,[\,d+e\,x\,]\,^{2}}{3\,e^{3}\,g}\,-\\\\ \frac{b\,n\,\left(\frac{18\,g\,\left(e\,f-d\,g\right)^{2}\,\left(d+e\,x\right)}{e^{3}}\,+\,\frac{9\,g^{2}\,\left(e\,f-d\,g\right)\,\left(d+e\,x\right)^{2}}{e^{3}}\,+\,\frac{2\,g^{3}\,\left(d+e\,x\right)^{3}}{e^{3}}\,+\,\frac{6\,\left(e\,f-d\,g\right)^{3}\,Log\,[\,d+e\,x\,]}{e^{3}}\right)\,\left(a+b\,Log\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]\right)}{9\,g}\,+\,\frac{\left(f+g\,x\right)^{3}\,\left(a+b\,Log\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]\right)^{2}}{3\,g}$$

# Problem 50: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right)^{\, 2}}{\left(f+g\, x\right)^{\, 3}} \, \mathrm{d}x$$

Optimal (type 4, 202 leaves, 7 steps):

$$-\frac{b \, e \, n \, \left(d + e \, x\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)}{\left(e \, f - d \, g\right)^2 \, \left(f + g \, x\right)} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2}{2 \, g \, \left(f + g \, x\right)^2} + \\ \frac{b^2 \, e^2 \, n^2 \, Log\left[f + g \, x\right]}{g \, \left(e \, f - d \, g\right)^2} - \frac{b \, e^2 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, Log\left[1 + \frac{e \, f - d \, g}{g \, (d + e \, x)}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{b^2 \, e^2 \, n^2 \, PolyLog\left[2, \, -\frac{e \, f - d \, g}{g \, (d + e \, x)}\right]}{g \, \left(e \, f - d \, g\right)^2}$$

#### Result (type 4, 233 leaves, 9 steps):

$$-\frac{b\,e\,n\,\left(d+e\,x\right)\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)}{\left(e\,f-d\,g\right)^{\,2}\,\left(f+g\,x\right)} + \frac{e^{2}\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}{2\,g\,\left(e\,f-d\,g\right)^{\,2}} - \frac{\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)^{\,2}}{2\,g\,\left(f+g\,x\right)^{\,2}} + \\ \frac{b^{2}\,e^{2}\,n^{2}\,Log\left[f+g\,x\right]}{g\,\left(e\,f-d\,g\right)^{\,2}} - \frac{b\,e^{2}\,n\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^{\,n}\right]\right)\,Log\left[\frac{e\,(f+g\,x)}{e\,f-d\,g}\right]}{g\,\left(e\,f-d\,g\right)^{\,2}} - \frac{b^{2}\,e^{2}\,n^{2}\,PolyLog\left[2\,,\,-\frac{g\,(d+e\,x)}{e\,f-d\,g}\right]}{g\,\left(e\,f-d\,g\right)^{\,2}}$$

#### Problem 51: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,Log\left[\,c\,\left(d+e\,x\right)^{\,n}\,\right]\,\right)^{\,2}}{\left(\,f+g\,x\right)^{\,4}}\,\mathrm{d}x$$

#### Optimal (type 4, 317 leaves, 11 steps):

$$-\frac{b^{2} e^{2} n^{2}}{3 g (e f-d g)^{2} (f+g x)} - \frac{b^{2} e^{3} n^{2} Log[d+e x]}{3 g (e f-d g)^{3}} + \frac{b e n (a+b Log[c (d+e x)^{n}])}{3 g (e f-d g) (f+g x)^{2}} - \frac{2 b e^{2} n (d+e x) (a+b Log[c (d+e x)^{n}])}{3 (e f-d g)^{3} (f+g x)} - \frac{(a+b Log[c (d+e x)^{n}])^{2}}{3 (e f-d g)^{3}} + \frac{b^{2} e^{3} n^{2} Log[f+g x]}{g (e f-d g)^{3}} - \frac{2 b e^{3} n (a+b Log[c (d+e x)^{n}]) Log[1+\frac{e f-d g}{g (d+e x)}]}{3 g (e f-d g)^{3}} + \frac{2 b^{2} e^{3} n^{2} PolyLog[2, -\frac{e f-d g}{g (d+e x)}]}{3 g (e f-d g)^{3}}$$

#### Result (type 4, 347 leaves, 13 steps):

$$-\frac{b^{2} e^{2} n^{2}}{3 g (e f - d g)^{2} (f + g x)} - \frac{b^{2} e^{3} n^{2} Log [d + e x]}{3 g (e f - d g)^{3}} + \frac{b e n (a + b Log [c (d + e x)^{n}])}{3 g (e f - d g) (f + g x)^{2}} - \frac{2 b e^{2} n (d + e x) (a + b Log [c (d + e x)^{n}])}{3 (e f - d g)^{3} (f + g x)} + \frac{e^{3} (a + b Log [c (d + e x)^{n}])^{2}}{3 g (e f - d g)^{3}} - \frac{(a + b Log [c (d + e x)^{n}])^{2}}{3 g (f + g x)^{3}} + \frac{e^{3} (a + b Log [c (d + e x)^{n}])^{2}}{3 g (e f - d g)^{3}} - \frac{2 b e^{3} n (a + b Log [c (d + e x)^{n}]) Log [\frac{e (f + g x)}{e f - d g}]}{3 g (e f - d g)^{3}} - \frac{2 b^{2} e^{3} n^{2} PolyLog [2, -\frac{g (d + e x)}{e f - d g}]}{3 g (e f - d g)^{3}}$$

## Problem 58: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \left(d+e x\right)^{n}\right]\right)^{3}}{\left(f+g x\right)^{3}} \, dx$$

Optimal (type 4, 342 leaves, 9 steps):

$$-\frac{3 \, b \, e \, n \, \left(d + e \, x\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2}{2 \, \left(e \, f - d \, g\right)^2 \, \left(f + g \, x\right)} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^3}{2 \, g \, \left(f + g \, x\right)^2} + \frac{2 \, g \, \left(f + g \, x\right)^2}{2 \, g \, \left(f + g \, x\right)^n\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2 \, Log\left[1 + \frac{e \, f - d \, g}{g \, \left(d + e \, x\right)}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b \, e^2 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2 \, Log\left[1 + \frac{e \, f - d \, g}{g \, \left(d + e \, x\right)}\right]}{2 \, g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[2 \, d - e \, d \, g\right]}{2 \, g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[2 \, d - e \, d \, g\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[3 \, d - e \, d \, g\right]}{g \, \left(e \, f - d \, g\right)^2}$$

Result (type 4, 370 leaves, 12 steps):

$$\frac{3 \, b \, e \, n \, \left(d + e \, x\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2}{2 \, \left(e \, f - d \, g\right)^2 \, \left(f + g \, x\right)} + \frac{e^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^3}{2 \, g \, \left(e \, f - d \, g\right)^2} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^3}{2 \, g \, \left(f + g \, x\right)^2} + \frac{3 \, b^2 \, e^2 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, Log\left[\frac{e \, (f + g \, x)}{e \, f - d \, g}\right]}{e \, f - d \, g} - \frac{3 \, b \, e^2 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right)^2 \, Log\left[\frac{e \, (f + g \, x)}{e \, f - d \, g}\right]}{2 \, g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[2, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^2 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^2}$$

# Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \text{Log}\left[\, c \, \left(d+e \, x\right)^{\, n}\,\right]\,\right)^{\, 3}}{\left(\, f+g \, x\right)^{\, 4}} \, \, \mathrm{d} x$$

Optimal (type 4, 564 leaves, 16 steps):

$$\frac{b^{2} e^{2} n^{2} \left(d+ex\right) \left(a+b \, Log\left[c\, \left(d+ex\right)^{n}\right]\right)}{\left(e\, f-d\, g\right)^{3} \left(f+g\, x\right)} + \frac{b\, e\, n\, \left(a+b \, Log\left[c\, \left(d+ex\right)^{n}\right]\right)^{2}}{2\, g\, \left(e\, f-d\, g\right) \left(f+g\, x\right)^{2}} - \frac{b\, e^{2}\, n\, \left(d+e\, x\right) \left(a+b \, Log\left[c\, \left(d+e\, x\right)^{n}\right]\right)^{2}}{\left(e\, f-d\, g\right)^{3} \left(f+g\, x\right)} - \frac{\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{n}\right]\right)^{3}}{\left(g\, f+g\, x\right)^{3}} - \frac{b^{3} \, e^{3}\, n^{3} \, Log\left[f+g\, x\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{2} \, e^{3}\, n^{2} \, \left(a+b \, Log\left[c\, \left(d+e\, x\right)^{n}\right]\right) \, Log\left[\frac{e\, (f+g\, x)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{2} \, e^{3}\, n^{2} \, \left(a+b \, Log\left[c\, \left(d+e\, x\right)^{n}\right]\right) \, Log\left[\frac{e\, (f+g\, x)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{g\, \left(d+e\, x\right)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[3, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{g\, \left(d+e\, x\right)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{g\, \left(d+e\, x\right)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{g\, \left(d+e\, x\right)}{e\, f-d\, g}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}{g\, \left(d+e\, x\right)}\right]}{g\, \left(e\, f-d\, g\right)^{3}} + \frac{2\, b^{3} \, e^{3}\, n^{3} \, PolyLog\left[2, -\frac{e\, f-d\, g}$$

#### Result (type 4, 525 leaves, 21 steps):

$$\frac{b^2 \, e^2 \, n^2 \, \left(d + e \, x\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)}{\left(e \, f - d \, g\right)^3 \, \left(f + g \, x\right)} - \frac{b \, e^3 \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)^2}{2 \, g \, \left(e \, f - d \, g\right)^3} + \frac{b \, e \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)^2}{2 \, g \, \left(e \, f - d \, g\right)^3 \, \left(f + g \, x\right)^2} - \frac{b \, e^3 \, n \, \left(d + e \, x\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)^3}{3 \, g \, \left(e \, f - d \, g\right)^3 \, \left(f + g \, x\right)} - \frac{b \, e^3 \, n^3 \, \text{Log} \left[f + g \, x\right)^3}{3 \, g \, \left(f + g \, x\right)^3} - \frac{b \, e^3 \, n^3 \, \text{Log} \left[f + g \, x\right]}{g \, \left(e \, f - d \, g\right)^3} + \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right) \, \text{Log} \left[\frac{e \, (f + g \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^3} - \frac{b \, e^3 \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right)^2 \, \text{Log} \left[\frac{e \, (f + g \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^3} + \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x\right)^n\right]\right) \, \text{Log} \left[\frac{e \, (f + g \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^3} + \frac{2 \, b^3 \, e^3 \, n^3 \, \text{PolyLog} \left[2, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^3} + \frac{2 \, b^3 \, e^3 \, n^3 \, \text{PolyLog} \left[3, -\frac{g \, (d + e \, x)}{e \, f - d \, g}\right]}{g \, \left(e \, f - d \, g\right)^3}$$

#### Problem 85: Result valid but suboptimal antiderivative.

$$\int x^2 Log[c(a+bx)^n]^2 dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\frac{2\,a^{2}\,n^{2}\,x}{b^{2}} - \frac{a\,n^{2}\,\left(a+b\,x\right)^{2}}{2\,b^{3}} + \frac{2\,n^{2}\,\left(a+b\,x\right)^{3}}{27\,b^{3}} - \frac{a^{3}\,n^{2}\,Log\left[a+b\,x\right]^{2}}{3\,b^{3}} - \frac{2\,a^{2}\,n\,\left(a+b\,x\right)\,Log\left[c\,\left(a+b\,x\right)^{n}\right]}{b^{3}} + \frac{a\,n\,\left(a+b\,x\right)^{2}\,Log\left[c\,\left(a+b\,x\right)^{n}\right]}{5\,b^{3}} + \frac{2\,n^{2}\,\left(a+b\,x\right)^{3}\,Log\left[c\,\left(a+b\,x\right)^{n}\right]}{3\,b^{3}} + \frac{1}{3}\,x^{3}\,Log\left[c\,\left(a+b\,x\right)^{n}\right]^{2}$$

Result (type 3, 156 leaves, 7 steps):

$$\begin{split} &\frac{2\,\,a^{2}\,\,n^{2}\,\,x}{b^{2}}\,-\,\frac{a\,\,n^{2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}{2\,\,b^{3}}\,+\,\frac{2\,\,n^{2}\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{27\,\,b^{3}}\,-\,\frac{a^{3}\,\,n^{2}\,\,Log\,\left[\,a\,+\,b\,\,x\,\right]^{\,2}}{3\,\,b^{3}}\,-\,\\ &\frac{1}{9}\,n\,\left(\,\frac{18\,\,a^{2}\,\left(\,a\,+\,b\,\,x\,\right)}{b^{3}}\,-\,\frac{9\,\,a\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,2}}{b^{3}}\,+\,\frac{2\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,3}}{b^{3}}\,-\,\frac{6\,\,a^{3}\,\,Log\,\left[\,a\,+\,b\,\,x\,\right]}{b^{3}}\,\right)\,\,Log\,\left[\,c\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,n}\,\right]\,+\,\frac{1}{3}\,\,x^{3}\,\,Log\,\left[\,c\,\,\left(\,a\,+\,b\,\,x\,\right)^{\,n}\,\right]^{\,2} \end{split}$$

## Problem 86: Result valid but suboptimal antiderivative.

$$\int\!\frac{Log\!\left[\left.c\,\left(a+b\,x\right)^{\,n}\right]^{\,2}}{x^{4}}\,\mathrm{d}x$$

#### Optimal (type 4, 177 leaves, 11 steps):

$$-\frac{b^{2} n^{2}}{3 a^{2} x} - \frac{b^{3} n^{2} Log[x]}{a^{3}} + \frac{b^{3} n^{2} Log[a+bx]}{3 a^{3}} - \frac{b n Log[c (a+bx)^{n}]}{3 a x^{2}} + \frac{2 b^{2} n (a+bx) Log[c (a+bx)^{n}]}{3 a^{3} x} - \frac{b n Log[c (a+bx)^{n}]}{3 a^{3} x} - \frac{b n Log[c (a+bx)^{n}]}{3 a^{3} x} - \frac{2 b^{3} n^{2} PolyLog[2, \frac{a}{a+bx}]}{3 a^{3}} - \frac{b n Log[c (a+bx)^{n}]}{3 a^{3} a^{3}} - \frac{b n Log[c (a+bx)^{n}]}{3 a^{3}} - \frac{b n Log[c (a+bx)^{n}]}{3 a^{3}} - \frac{b n Log[c (a+b$$

#### Result (type 4, 193 leaves, 13 steps):

$$-\frac{b^{2} n^{2}}{3 a^{2} x} - \frac{b^{3} n^{2} Log[x]}{a^{3}} + \frac{b^{3} n^{2} Log[a+b x]}{3 a^{3}} - \frac{b n Log[c (a+b x)^{n}]}{3 a x^{2}} + \frac{2 b^{2} n (a+b x) Log[c (a+b x)^{n}]}{3 a^{3} x} + \frac{2 b^{3} n Log[c (a+b x)^{n}]}{3 a^{3} x} + \frac{2 b^{3} n Log[c (a+b x)^{n}]}{3 a^{3}} + \frac{2 b^{3} n^{2} PolyLog[2, 1 + \frac{b x}{a}]}{3 a^{3}}$$

# Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{(h+ix)^4 (a+b Log[c (e+fx)])}{de+dfx} dx$$

#### Optimal (type 3, 315 leaves, 8 steps):

$$-\frac{4 \, b \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} - \frac{3 \, b \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2}{2 \, d \, f^5} - \frac{4 \, b \, i^3 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^3}{9 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4}{16 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4}{16 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4}{16 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4}{16 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^2 \, \left(e + f \, x\right)^2 \, \left(a + b \, log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^5} + \frac{4 \, i \, \left(f \, h - e \, i\right)^3 \, \left(a + b \, log \left[c \, \left(e + f \, x\right)\right]\right)}{4 \, d \, f^5} + \frac{i^4 \, \left(e + f \, x\right)^4 \, \left(a + b \, log \left[c \, \left(e + f \, x\right)\right]\right)}{4 \, d \, f^5} + \frac{\left(f \, h - e \, i\right)^4 \, log \left[e + f \, x\right] \, \left(a + b \, log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^5} + \frac{10 \, d \, f^5}{4 \, d \, f^5} + \frac{10 \, d \, f^5}$$

Result (type 3, 260 leaves, 6 steps):

$$-\frac{4 \, b \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} - \frac{3 \, b \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2}{2 \, d \, f^5} - \frac{4 \, b \, i^3 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^3}{9 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4}{16 \, d \, f^5} - \frac{b \, \left(f \, h - e \, i\right)^4 \, Log \left[e + f \, x\right]^2}{2 \, d \, f^5} + \frac{12 \, d \, f^5}{12$$

#### Problem 176: Result valid but suboptimal antiderivative.

$$\int \frac{\left(h+ix\right)^{3} \left(a+b Log\left[c\left(e+fx\right)\right]\right)}{d e+d f x} dx$$

Optimal (type 3, 244 leaves, 8 steps):

$$-\frac{3 \, b \, i \, \left(f \, h - e \, i\right)^2 \, x}{d \, f^3} - \frac{3 \, b \, i^2 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^2}{4 \, d \, f^4} - \frac{b \, i^3 \, \left(e + f \, x\right)^3}{9 \, d \, f^4} - \frac{b \, \left(f \, h - e \, i\right)^3 \, Log \left[e + f \, x\right]^2}{2 \, d \, f^4} + \frac{3 \, i \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right) \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{3 \, i^3 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right) \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{3 \, d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{3 \, d \, f^4} + \frac{13 \, \left(f \, h - e \, i\right)^3 \, Log \left[e + f \, x\right] \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{3 \, d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, Log \left[e + f \, x\right] \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{3 \, d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, Log \left[e + f \, x\right] \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{3 \, d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, Log \left[e + f \, x\right] \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]}{d \, f^4} + \frac{13 \, \left(e + f \, x\right)^3 \,$$

#### Result (type 3, 204 leaves, 8 steps):

$$-\frac{3 \, b \, i \, \left( f \, h - e \, i \right)^2 \, x}{d \, f^3} - \frac{3 \, b \, i^2 \, \left( f \, h - e \, i \right) \, \left( e + f \, x \right)^2}{4 \, d \, f^4} - \frac{b \, i^3 \, \left( e + f \, x \right)^3}{9 \, d \, f^4} - \frac{b \, \left( f \, h - e \, i \right)^3 \, Log \left[ e + f \, x \right]^2}{2 \, d \, f^4} + \frac{\left( \frac{18 \, i \, \left( f \, h - e \, i \right)^2 \, \left( e + f \, x \right)^2}{f^3} + \frac{2 \, i^3 \, \left( e + f \, x \right)^3}{f^3} + \frac{6 \, \left( f \, h - e \, i \right)^3 \, Log \left[ e + f \, x \right]}{f^3} \right) \, \left( a + b \, Log \left[ c \, \left( e + f \, x \right) \, \right] \right)}{6 \, d \, f}$$

#### Problem 177: Result valid but suboptimal antiderivative.

$$\int \frac{(h+ix)^2 (a+b Log[c (e+fx)])}{de+dfx} dx$$

Optimal (type 3, 157 leaves, 7 steps):

$$-\frac{b \left(4 \, f \, h - 3 \, e \, i + f \, i \, x\right)^{2}}{4 \, d \, f^{3}} - \frac{b \left(f \, h - e \, i\right)^{2} \, Log \left[e + f \, x\right]^{2}}{2 \, d \, f^{3}} + \frac{2 \, i \left(f \, h - e \, i\right) \, \left(e + f \, x\right) \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^{3}} + \frac{i^{2} \, \left(e + f \, x\right)^{2} \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f^{3}} + \frac{\left(f \, h - e \, i\right)^{2} \, Log \left[e + f \, x\right] \, \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{a \, d \, f^{3}} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right] + \frac{1}{2} \left(a + b \, L$$

Result (type 3, 133 leaves, 7 steps):

$$-\frac{b \left(4 \, f \, h - 3 \, e \, i + f \, i \, x\right)^2}{4 \, d \, f^3} - \frac{b \left(f \, h - e \, i\right)^2 \, Log \left[e + f \, x\right]^2}{2 \, d \, f^3} + \frac{\left(\frac{4 \, i \, (f \, h - e \, i) \, (e + f \, x)}{f^2} + \frac{i^2 \, (e + f \, x)^2}{f^2} + \frac{2 \, (f \, h - e \, i)^2 \, Log \left[e + f \, x\right]}{f^2}\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right) \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)}{2 \, d \, f} + \frac{1}{2} \left(a + b \, Log \left[c \, \left(e + f \, x\right)\right]\right)$$

## Problem 180: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \, Log \left[ c \, \left( e + f \, x \right) \, \right]}{\left( d \, e + d \, f \, x \right) \, \left( h + i \, x \right)} \, dx$$

Optimal (type 4, 87 leaves, 4 steps):

$$-\frac{\left(\texttt{a}+\texttt{b}\,\texttt{Log}\!\left[\texttt{c}\,\left(\texttt{e}+\texttt{f}\,\texttt{x}\right)\,\right]\right)\,\texttt{Log}\!\left[\texttt{1}+\frac{\texttt{f}\,\texttt{h}-\texttt{e}\,\textbf{i}}{\texttt{i}\,\left(\texttt{e}+\texttt{f}\,\texttt{x}\right)}\right]}{\texttt{d}\,\left(\texttt{f}\,\texttt{h}-\texttt{e}\,\textbf{i}\right)}+\frac{\texttt{b}\,\texttt{PolyLog}\!\left[\texttt{2},\,-\frac{\texttt{f}\,\texttt{h}-\texttt{e}\,\textbf{i}}{\texttt{i}\,\left(\texttt{e}+\texttt{f}\,\texttt{x}\right)}\right]}{\texttt{d}\,\left(\texttt{f}\,\texttt{h}-\texttt{e}\,\textbf{i}\right)}$$

Result (type 4, 116 leaves, 6 steps):

$$\frac{\left(\texttt{a} + \texttt{b} \, \texttt{Log} \left[\texttt{c} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)\,\right]\right)^2}{\texttt{2} \, \texttt{b} \, \texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)} - \frac{\left(\texttt{a} + \texttt{b} \, \texttt{Log} \left[\texttt{c} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)\,\right]\right) \, \texttt{Log} \left[\frac{\texttt{f} \, \left(\texttt{h} + \texttt{i} \, \texttt{x}\right)}{\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}}\right]}{\texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)} - \frac{\texttt{b} \, \texttt{PolyLog} \left[\texttt{2} \, \texttt{,} \, - \frac{\texttt{i} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)}{\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}}\right]}{\texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)}$$

## Problem 181: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \, \text{Log} \left[\, c \, \left(\, e+f \, x\,\right)\,\,\right]}{\left(\, d \, e+d \, f \, x\,\right) \, \left(\, h+i \, x\,\right)^{\, 2}} \, \, \text{d} \, x$$

Optimal (type 4, 151 leaves, 7 steps):

$$-\frac{\text{i}\left(\text{e+fx}\right)\left(\text{a+b} \text{Log}\left[\text{c}\left(\text{e+fx}\right)\right]\right)}{\text{d}\left(\text{fh-ei}\right)^{2}\left(\text{h+ix}\right)}+\frac{\text{b} \text{f} \text{Log}\left[\text{h+ix}\right]}{\text{d}\left(\text{fh-ei}\right)^{2}}-\frac{\text{f}\left(\text{a+b} \text{Log}\left[\text{c}\left(\text{e+fx}\right)\right]\right) \text{Log}\left[\text{1}+\frac{\text{fh-ei}}{\text{i}\left(\text{e+fx}\right)}\right]}{\text{i}\left(\text{e+fx}\right)}+\frac{\text{b} \text{f} \text{PolyLog}\left[\text{2},-\frac{\text{fh-ei}}{\text{i}\left(\text{e+fx}\right)}\right]}{\text{d}\left(\text{fh-ei}\right)^{2}}$$

Result (type 4, 181 leaves, 9 steps):

$$-\frac{\mathrm{i} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \right)}{\mathsf{d} \, \left( \mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i} \right)^2 \, \left( \mathsf{h} + \mathsf{i} \, \mathsf{x} \right)} + \frac{\mathsf{f} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \right)^2}{2 \, \mathsf{b} \, \mathsf{d} \, \left( \mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i} \right)^2} + \frac{\mathsf{b} \, \mathsf{f} \, \mathsf{Log} \left[ \mathsf{h} + \mathsf{i} \, \mathsf{x} \right]}{\mathsf{d} \, \left( \mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i} \right)^2} - \frac{\mathsf{f} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \right) \, \mathsf{Log} \left[ \frac{\mathsf{f} \, \left( \mathsf{h} + \mathsf{i} \, \mathsf{x} \right)}{\mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i}} \right]}{\mathsf{d} \, \left( \mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i} \right)^2} - \frac{\mathsf{b} \, \mathsf{f} \, \mathsf{PolyLog} \left[ \mathsf{2} \, \mathsf{,} \, - \frac{\mathsf{i} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right)}{\mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i}} \right]}{\mathsf{d} \, \left( \mathsf{f} \, \mathsf{h} - \mathsf{e} \, \mathsf{i} \right)^2}$$

## Problem 182: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \log[c(e + fx)]}{(de + dfx)(h + ix)^3} dx$$

Optimal (type 4, 250 leaves, 11 steps):

$$-\frac{b\,f}{2\,d\,\left(f\,h-e\,i\right)^{\,2}\,\left(h+i\,x\right)} - \frac{b\,f^{\,2}\,Log\,[\,e+f\,x\,]}{2\,d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]}{2\,d\,\left(f\,h-e\,i\right)\,\left(h+i\,x\right)^{\,2}} - \frac{f\,i\,\left(e+f\,x\right)\,\left(a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]\right)}{d\,\left(f\,h-e\,i\right)^{\,3}\,\left(h+i\,x\right)} + \frac{3\,b\,f^{\,2}\,Log\,[\,h+i\,x\,]}{2\,d\,\left(f\,h-e\,i\right)^{\,3}} - \frac{f^{\,2}\,\left(a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]\right)\,Log\,[\,1+\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\,\,]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{b\,f^{\,2}\,PolyLog\,[\,2,\,-\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\,\,]}{d\,\left(f\,h-e\,i\right)^{\,3}}$$

Result (type 4, 282 leaves, 13 steps):

$$-\frac{b\,f}{2\,d\,\left(f\,h-e\,i\right)^{2}\,\left(h+i\,x\right)} - \frac{b\,f^{2}\,Log\,[\,e+f\,x\,]}{2\,d\,\left(f\,h-e\,i\right)^{3}} + \frac{a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]}{2\,d\,\left(f\,h-e\,i\right)\,\left(h+i\,x\right)^{2}} - \frac{f\,i\,\left(e+f\,x\right)\,\left(a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]\right)}{d\,\left(f\,h-e\,i\right)^{3}\,\left(h+i\,x\right)} + \frac{f^{2}\,\left(a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]\right)\,Log\,\left[\frac{f\,(h+i\,x)}{f\,h-e\,i}\,\,]}{2\,b\,d\,\left(f\,h-e\,i\right)^{3}} + \frac{3\,b\,f^{2}\,Log\,[\,h+i\,x\,]}{2\,d\,\left(f\,h-e\,i\right)^{3}} - \frac{f^{2}\,\left(a+b\,Log\,[\,c\,\left(e+f\,x\right)\,\,]\right)\,Log\,\left[\frac{f\,(h+i\,x)}{f\,h-e\,i}\,\,]}{d\,\left(f\,h-e\,i\right)^{3}} - \frac{b\,f^{2}\,PolyLog\,[\,2\,,\,-\frac{i\,(e+f\,x)}{f\,h-e\,i}\,\,]}{d\,\left(f\,h-e\,i\right)^{3}}$$

#### Problem 183: Result valid but suboptimal antiderivative.

$$\int \frac{\left(h + i x\right)^4 \left(a + b Log\left[c \left(e + f x\right)\right]\right)^2}{d e + d f x} dx$$

Optimal (type 3, 579 leaves, 32 steps):

$$-\frac{4 \, a \, b \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} + \frac{8 \, b^2 \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} + \frac{3 \, b^2 \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2}{2 \, d \, f^5} + \frac{8 \, b^2 \, i^3 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^3}{27 \, d \, f^5} + \frac{b^2 \, i^4 \, \left(e + f \, x\right)^4}{32 \, d \, f^5} + \frac{7 \, b^2 \, \left(f \, h - e \, i\right)^4 \, Log\left[e + f \, x\right]}{d \, f^5} - \frac{4 \, b \, i \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right) \, Log\left[c \, \left(e + f \, x\right)\right]}{d \, f^5} - \frac{4 \, b \, i \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right) \, d \, f^5}{d \, f^5} - \frac{3 \, b \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2 \, \left(a + b \, Log\left[c \, \left(e + f \, x\right)\right]\right)}{d \, f^5} - \frac{8 \, b \, i^3 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^3 \, \left(a + b \, Log\left[c \, \left(e + f \, x\right)\right]\right)}{9 \, d \, f^5} - \frac{9 \, d \, f^5}{6 \, d \, f^5} - \frac{b \, i^4 \, \left(e + f \, x\right)^4 \, \left(a + b \, Log\left[c \, \left(e + f \, x\right)\right]\right)}{6 \, d \, f^5} - \frac{7 \, b \, \left(f \, h - e \, i\right)^4 \, Log\left[e + f \, x\right] \, \left(a + b \, Log\left[c \, \left(e + f \, x\right)\right]\right)}{6 \, d \, f^5} + \frac{2 \, i \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right) \, \left(a + b \, Log\left[c \, \left(e + f \, x\right)\right]\right)^2}{2 \, d \, f^5} + \frac{2 \, d \, f^5}{2 \, d \, f^5} - \frac{2 \, d \, f^5}{2 \, d \, f^5} - \frac{2 \, d \, f^5}{2 \, d \, f^5} + \frac{2 \, d \, f^5}{2 \, d \, f^5} + \frac{2 \, d \, f^5}{2 \, d \, f^5} - \frac{2 \, d \, f^5}{2 \, d \, f^5} + \frac{2 \, d \, f^5}{2 \, d \, f^5} - \frac{2 \, d \, f^5}{2 \, d \, f^5} + \frac$$

Result (type 3, 672 leaves, 30 steps):

$$-\frac{4 \, a \, b \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} + \frac{8 \, b^2 \, i \, \left(f \, h - e \, i\right)^3 \, x}{d \, f^4} + \frac{3 \, b^2 \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2}{2 \, d \, f^5} + \frac{8 \, b^2 \, i^3 \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right)^4}{32 \, d \, f^5} + \frac{7 \, b^2 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]^2}{12 \, d \, f^5} - \frac{4 \, b^2 \, i \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right) \, Log[c \, \left(e + f \, x\right)]}{d \, f^5} - \frac{b \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2 \, \left(a + b \, Log[c \, \left(e + f \, x\right)]\right)}{2 \, d \, f^5} - \frac{1}{9 \, d \, f^3}$$

$$b \, \left(f \, h - e \, i\right) \, \left(\frac{18 \, i \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)}{f^2} + \frac{9 \, i^2 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^2}{f^2} + \frac{2 \, i^3 \, \left(e + f \, x\right)^3}{f^2} + \frac{6 \, \left(f \, h - e \, i\right)^3 \, Log[e + f \, x]}{f^2} \right) \, \left(a + b \, Log[c \, \left(e + f \, x\right)]\right) - \frac{1}{24 \, d \, f^2} \, \left(\frac{48 \, i \, \left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right)}{f^3} + \frac{36 \, i^2 \, \left(f \, h - e \, i\right)^2 \, \left(e + f \, x\right)^2}{f^3} + \frac{16 \, i^3 \, \left(f \, h - e \, i\right) \, \left(e + f \, x\right)^3}{f^3} + \frac{3 \, i^4 \, \left(e + f \, x\right)^4}{f^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]}{f^3} \right) + \frac{\left(f \, h - e \, i\right)^3 \, \left(e + f \, x\right) \, \left(e + f \, x\right) \, \left(e + f \, x\right) \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^4}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^4}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^4}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, Log[e + f \, x]}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right)^4 \, \left(e + f \, x\right)^3}{g^3} + \frac{12 \, \left(f \, h - e \, i\right$$

### Problem 188: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(e + f x\right)\right]\right)^{2}}{\left(d e + d f x\right) \left(h + i x\right)} dx$$

Optimal (type 4, 142 leaves, 5 steps):

$$-\frac{\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\!\left[\mathsf{c}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right.\right)^{2}\,\mathsf{Log}\!\left[\mathsf{1}+\frac{\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}}{\mathsf{i}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)}\right.\right)}{\mathsf{d}\,\left(\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}\right)}+\frac{2\,\mathsf{b}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\!\left[\mathsf{c}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right.\right]\right)\,\mathsf{PolyLog}\!\left[\mathsf{2},\,-\frac{\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}}{\mathsf{i}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)}\right.\right]}{\mathsf{d}\,\left(\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}\right)}+\frac{2\,\mathsf{b}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\!\left[\mathsf{c}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right.\right]\right)\,\mathsf{PolyLog}\!\left[\mathsf{2},\,-\frac{\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}}{\mathsf{i}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)}\right.\right]}{\mathsf{d}\,\left(\mathsf{f}\,\mathsf{h}-\mathsf{e}\,\mathbf{i}\right)}$$

Result (type 4, 168 leaves, 8 steps):

$$\frac{\left(\texttt{a} + \texttt{b} \, \texttt{Log} \left[\texttt{c} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)\,\right]\right)^3}{\texttt{3} \, \texttt{b} \, \texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)} - \frac{\left(\texttt{a} + \texttt{b} \, \texttt{Log} \left[\texttt{c} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)\,\right]\right)^2 \, \texttt{Log} \left[\frac{\texttt{f} \, (\texttt{h} + \texttt{i} \, \texttt{x})}{\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}}\right]}{\texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)} - \frac{2 \, \texttt{b} \, \left(\texttt{a} + \texttt{b} \, \texttt{Log} \left[\texttt{c} \, \left(\texttt{e} + \texttt{f} \, \texttt{x}\right)\,\right]\right) \, \texttt{PolyLog} \left[\texttt{2}, \, -\frac{\texttt{i} \, (\texttt{e} + \texttt{f} \, \texttt{x})}{\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}}\right]}{\texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)} + \frac{2 \, \texttt{b}^2 \, \texttt{PolyLog} \left[\texttt{3}, \, -\frac{\texttt{i} \, (\texttt{e} + \texttt{f} \, \texttt{x})}{\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}}\right]}{\texttt{d} \, \left(\texttt{f} \, \texttt{h} - \texttt{e} \, \texttt{i}\right)}$$

## Problem 189: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[c\, \left(e+f\, x\right)\,\right]\right)^{2}}{\left(d\, e+d\, f\, x\right)\, \left(h+i\, x\right)^{2}}\, \mathrm{d}x$$

Optimal (type 4, 273 leaves, 9 steps):

$$-\frac{i\left(e+fx\right)\left(a+b\log\left[c\left(e+fx\right)\right]\right)^{2}}{d\left(fh-e\,i\right)^{2}\left(h+i\,x\right)}+\frac{2\,b\,f\left(a+b\log\left[c\left(e+fx\right)\right]\right)\log\left[\frac{f\left(h+i\,x\right)}{fh-e\,i}\right]}{d\left(fh-e\,i\right)^{2}}-\frac{f\left(a+b\log\left[c\left(e+f\,x\right)\right]\right)^{2}\log\left[1+\frac{fh-e\,i}{i\left(e+f\,x\right)}\right]}{d\left(fh-e\,i\right)^{2}}+\frac{2\,b^{2}\,f\,PolyLog\left[2,-\frac{i\left(e+f\,x\right)}{fh-e\,i}\right]}{d\left(fh-e\,i\right)^{2}}+\frac{2\,b^{2}\,f\,PolyLog\left[2,-\frac{i\left(e+f\,x\right)}{fh-e\,i}\right]}{d\left(fh-e\,i\right)^{2}}+\frac{2\,b^{2}\,f\,PolyLog\left[3,-\frac{fh-e\,i}{i\left(e+f\,x\right)}\right]}{d\left(fh-e\,i\right)^{2}}$$

### Result (type 4, 300 leaves, 12 steps):

$$-\frac{i \left(e+fx\right) \left(a+b \, \text{Log} \left[c \, \left(e+fx\right)\right]\right)^{2}}{d \left(fh-e \, i\right)^{2} \left(h+i \, x\right)} + \frac{f \left(a+b \, \text{Log} \left[c \, \left(e+fx\right)\right]\right)^{3}}{3 \, b \, d \, \left(fh-e \, i\right)^{2}} + \\ \frac{2 \, b \, f \left(a+b \, \text{Log} \left[c \, \left(e+fx\right)\right]\right) \, \text{Log} \left[\frac{f \, (h+i \, x)}{f \, h-e \, i}\right]}{f \, h-e \, i} - \frac{f \left(a+b \, \text{Log} \left[c \, \left(e+fx\right)\right]\right)^{2} \, \text{Log} \left[\frac{f \, (h+i \, x)}{f \, h-e \, i}\right]}{d \, \left(fh-e \, i\right)^{2}} + \\ \frac{2 \, b^{2} \, f \, \text{PolyLog} \left[2, -\frac{i \, \left(e+fx\right)}{f \, h-e \, i}\right]}{f \, h-e \, i} - \frac{2 \, b \, f \, \left(a+b \, \text{Log} \left[c \, \left(e+fx\right)\right]\right) \, \text{PolyLog} \left[2, -\frac{i \, \left(e+fx\right)}{f \, h-e \, i}\right]}{d \, \left(fh-e \, i\right)^{2}} + \frac{2 \, b^{2} \, f \, \text{PolyLog} \left[3, -\frac{i \, \left(e+fx\right)}{f \, h-e \, i}\right]}{d \, \left(fh-e \, i\right)^{2}}$$

## Problem 190: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log \left[c \left(e + f x\right)\right]\right)^{2}}{\left(d e + d f x\right) \left(h + i x\right)^{3}} dx$$

Optimal (type 4, 485 leaves, 16 steps):

$$\frac{b\,fi\,\left(e+f\,x\right)\,\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)}{d\,\left(f\,h-e\,i\right)^{\,3}\,\left(h+i\,x\right)} + \frac{\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)^{\,2}}{2\,d\,\left(f\,h-e\,i\right)\,\left(h+i\,x\right)^{\,2}} - \frac{f\,i\,\left(e+f\,x\right)\,\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)^{\,2}}{d\,\left(f\,h-e\,i\right)^{\,3}\,\left(h+i\,x\right)} - \frac{b^{\,2}\,f^{\,2}\,Log\left[h+i\,x\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{b\,f^{\,2}\,\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)\,Log\left[\frac{f\,(h+i\,x)}{f\,h-e\,i}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{b\,f^{\,2}\,\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)\,Log\left[1+\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} - \frac{f^{\,2}\,\left(a+b\,Log\left[c\,\left(e+f\,x\right)\,\right]\right)^{\,2}\,Log\left[1+\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} - \frac{b^{\,2}\,f^{\,2}\,PolyLog\left[2\,,\,-\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{2\,b^{\,2}\,f^{\,2}\,PolyLog\left[2\,,\,-\frac{f\,h-e\,i}{f\,h-e\,i}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{2\,b^{\,2}\,f^{\,2}\,PolyLog\left[3\,,\,-\frac{f\,h-e\,i}{i\,\left(e+f\,x\right)}\right]}{d\,\left(f\,h-e\,i\right)^{\,3}} + \frac{2\,b^{\,2}\,f^{\,2}\,PolyLog\left$$

Result (type 4, 453 leaves, 21 steps):

### Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + e x\right)^{n}\right]\right)^{2}}{x^{3} \left(f + g x^{2}\right)} dx$$

Optimal (type 4, 551 leaves, 23 steps):

$$\frac{b^{2} e^{2} \, n^{2} \, \text{Log}\left[x\right]}{d^{2} \, f} = \frac{b \, \text{e} \, n \, \left(d + e \, x\right) \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right)}{d^{2} \, f} = \frac{\left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2}}{d^{2} \, f} + \frac{g \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, \text{Log}\left[\frac{e \, \left(\sqrt{-f} \, + \sqrt{g} \, x\right)}{e \, \sqrt{-f} \, + d \, \sqrt{g}}\right]}{2 \, f^{2}} + \frac{g \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, \text{Log}\left[\frac{e \, \left(\sqrt{-f} \, + \sqrt{g} \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{2 \, f^{2}} - \frac{b \, g \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, \text{Log}\left[\frac{e \, \left(\sqrt{-f} \, + \sqrt{g} \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{2 \, f^{2}} - \frac{b \, g \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right) \, \text{PolyLog}\left[2, \, -\frac{\sqrt{g} \, \left(d + e \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{d^{2} \, f} + \frac{b \, g \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right) \, \text{PolyLog}\left[2, \, -\frac{\sqrt{g} \, \left(d + e \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{f^{2}} + \frac{b \, g \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + e \, x\right)^{n}\right]\right) \, \text{PolyLog}\left[2, \, -\frac{\sqrt{g} \, \left(d + e \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{f^{2}} - \frac{b^{2} \, g \, n^{2} \, \text{PolyLog}\left[3, \, -\frac{\sqrt{g} \, \left(d + e \, x\right)}{e \, \sqrt{-f} \, + d \, \sqrt{g}}\right]}{f^{2}} + \frac{2 \, b^{2} \, g \, n^{2} \, \text{PolyLog}\left[3, \, 1 + \frac{e \, x}{d}\right]}{f^{2}}$$

Result (type 4, 575 leaves, 25 steps):

$$\frac{b^{2} \, e^{2} \, n^{2} \, Log\left[x\right]}{d^{2} \, f} - \frac{b \, e \, n \, \left(d + e \, x\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)}{d^{2} \, f} - \frac{b \, e^{2} \, n \, Log\left[-\frac{e \, x}{d}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)}{d^{2} \, f} + \frac{e^{2} \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2}}{2 \, d^{2} \, f} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2}}{f^{2}} + \frac{g \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, Log\left[\frac{e \, \left(\sqrt{-f} \, -\sqrt{g} \, x\right)}{e \, \sqrt{-f} \, + d \, \sqrt{g}}\right]}{2 \, f^{2}} + \frac{g \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, Log\left[\frac{e \, \left(\sqrt{-f} \, -\sqrt{g} \, x\right)}{e \, \sqrt{-f} \, + d \, \sqrt{g}}\right]}{2 \, f^{2}} + \frac{g \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right)^{2} \, Log\left[\frac{e \, \left(\sqrt{-f} \, -\sqrt{g} \, x\right)}{e \, \sqrt{-f} \, + d \, \sqrt{g}}\right]}{f^{2}} + \frac{g \, g \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^{n}\right]\right) \, PolyLog\left[2, \, -\frac{\sqrt{g} \, \left(d + e \, x\right)}{e \, \sqrt{-f} \, - d \, \sqrt{g}}\right]}{f^{2}} + \frac{g \, g \, n^{2} \, PolyLog\left[2, \, \frac{\sqrt{g} \, \left(d + e \, x\right)}{d}\right]}{f^{2}} - \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{\sqrt{g} \, \left(d + e \, x\right)}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} - \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}{f^{2}} + \frac{g \, b^{2} \, g \, n^{2} \, PolyLog\left[3, \, \frac{1 + \frac{e \, x}{d}}{d}\right]}$$

## Problem 319: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[\, c \, \left(d+e \, x\right)^{\, n}\,\right]\,\right)^{\, 2}}{x^4 \, \left(f+g \, x^2\right)} \, \mathrm{d} x$$

Optimal (type 4, 694 leaves, 26 steps):

$$\frac{b^{2}e^{2}n^{2}}{3d^{2}fx} - \frac{b^{2}e^{3}n^{2}Log[x]}{d^{3}f} + \frac{b^{2}e^{3}n^{2}Log[d+ex]}{3d^{3}f} - \frac{ben(a+bLog[c(d+ex)^{n}])}{3dfx^{2}} + \frac{2begnLog[-\frac{ex}{d}](a+bLog[c(d+ex)^{n}])}{3d^{3}fx} - \frac{2begnLog[-\frac{ex}{d}](a+bLog[c(d+ex)^{n}])}{df^{2}} - \frac{2begnLog[-\frac{ex}{d}](a+bLog[c(d+ex)^{n}])}{df^{2}} - \frac{(a+bLog[c(d+ex)^{n}])^{2}}{3fx^{3}} + \frac{g(d+ex)(a+bLog[c(d+ex)^{n}])^{2}}{df^{2}x} + \frac{g^{3/2}(a+bLog[c(d+ex)^{n}])^{2}Log[\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}]}{2(-f)^{5/2}} - \frac{2be^{3}n(a+bLog[c(d+ex)^{n}])Log[1-\frac{d}{d+ex}]}{3d^{3}f} - \frac{2b^{2}e^{3}n^{2}PolyLog[2,\frac{d}{d+ex}]}{3d^{3}f} - \frac{2b^{2}e^{3}n^{2}PolyLog[2,\frac{d}{d+ex}]}{3d^{3}f} - \frac{2b^{2}e^{3}n^{2}PolyLog[2,\frac{d}{d+ex}]}{(-f)^{5/2}} - \frac{2b^{2}e^{3}n^{2}PolyLog[2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}]}{(-f)^{5/2}} - \frac{2b^{2}e^{3}n^{2}PolyLog[2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}]}{(-f)^{5/2}} - \frac{b^{2}g^{3/2}n^{2}PolyLog[3,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}]}{(-f)^{5/2}} - \frac{b^{2}g^{3/2}n^{2}PolyLog[3$$

### Result (type 4, 717 leaves, 28 steps):

$$\frac{b^2 \, e^2 \, n^2}{3 \, d^2 \, f \, x} - \frac{b^2 \, e^3 \, n^2 \, \text{Log}[x]}{d^3 \, f} + \frac{b^2 \, e^3 \, n^2 \, \text{Log}[d + e \, x]}{3 \, d^3 \, f} - \frac{b \, e \, n \, \left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)}{3 \, d \, f \, x^2} + \frac{2 \, b \, e^3 \, n \, \text{Log}[-\frac{e \, x}{d}] \, \left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)}{3 \, d^3 \, f \, x} + \frac{2 \, b \, e \, g \, n \, \text{Log}[-\frac{e \, x}{d}] \, \left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)}{3 \, d^3 \, f} - \frac{2 \, b \, e \, g \, n \, \text{Log}[-\frac{e \, x}{d}] \, \left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)}{3 \, d^3 \, f} - \frac{e^3 \, \left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} + \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left(d + e \, x\right)^n]\right)^2}{3 \, d^3 \, f} - \frac{\left(a + b \, \text{Log}[c \, \left$$

# Problem 324: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \left(d+e x\right)^{n}\right]\right)^{2}}{x^{3} \left(f+g x^{2}\right)^{2}} dx$$

Optimal (type 4, 970 leaves, 36 steps):

$$\frac{b^{2}e^{2}n^{2} \log[x]}{d^{2}f^{2}} - \frac{b \, en \, \left(d + ex\right) \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}]\right)}{d^{2}f^{2}} + \frac{e^{2}g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}]\right)^{2}}{2 \, f^{2} \, \left(e^{2}f + d^{2}g\right)} - \frac{\left(a + b \, \log[c \, \left(d + ex\right)^{n}]\right)^{2}}{2 \, f^{2}x^{2}} - \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}]\right)^{2}}{2 \, f^{2} \, \left(f + gx^{2}\right)} - \frac{2 \, g \, \log\left[-\frac{ex}{d}\right] \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right]\right)^{2}}{f^{3}} - \frac{b \, e\left(e \, f + d \, \sqrt{-f} \, \sqrt{g}\right) \, g \, n \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}]\right) \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}{2 \, f^{3} \, \left(e^{2} \, f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \log\left[\frac{e\left(\sqrt{-f} \, \sqrt{g}\, x\right)}{e\sqrt{-f} \, d \, \sqrt{g}}\right]}}{2 \, f^{3} \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \left(e^{2}f + d^{2}g\right)}{2 \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \left(e^{2}f + d^{2}g\right)}{2 \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d + ex\right)^{n}\right)\right)^{2} \, \left(e^{2}f + d^{2}g\right)}{2 \, \left(e^{2}f + d^{2}g\right)} + \frac{g \, \left(a + b \, \log\left[c \, \left(d$$

Result (type 4, 994 leaves, 38 steps):

$$\frac{b^{2}e^{2} n^{2} Log[x]}{d^{2}f^{2}} = \frac{b \, e \, n \, (d + e \, x) \, (a + b \, Log[c \, (d + e \, x)^{n}])}{d^{2}f^{2}} - \frac{b \, e^{2} \, n \, Log[c \, (d + e \, x)^{n}])}{d^{2}f^{2}} + \frac{e^{2} \, (a + b \, Log[c \, (d + e \, x)^{n}])^{2}}{2 \, d^{2}f^{2}} + \frac{e^{2} \, g \, (a + b \, Log[c \, (d + e \, x)^{n}])^{2}}{2 \, f^{2} \, (e^{2} \, f + d^{2} \, g)} - \frac{(a + b \, Log[c \, (d + e \, x)^{n}])^{2}}{2 \, f^{2}x^{2}} - \frac{g \, (a + b \, Log[c \, (d + e \, x)^{n}])^{2}}{2 \, f^{2} \, (f + g \, x^{2})} - \frac{2 \, f^{2} \, (f + g \, x^{2})}{2 \, f^{2} \, (e^{2} \, f + d^{2} \, g)} - \frac{2 \, f^{2} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)}{2 \, f^{3} \, (e^{2} \, f + d^{2} \, g)} + \frac{2 \, f^{3} \, ($$

# Problem 363: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}[fx^m] \left(a + b \text{Log}\left[c \left(d + e x\right)^n\right]\right)}{x^2} dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{b \, e \, m \, n \, Log\left[x\right]}{d} \, - \, \frac{b \, e \, n \, Log\left[1 + \frac{d}{e \, x}\right] \, Log\left[f \, x^m\right]}{d} \, - \, \frac{b \, e \, m \, n \, Log\left[d + e \, x\right]}{d} \, - \, \left(\frac{m}{x} + \frac{Log\left[f \, x^m\right]}{x}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, + \, \frac{b \, e \, m \, n \, PolyLog\left[2, -\frac{d}{e \, x}\right]}{d} \, - \, \left(\frac{m}{x} + \frac{m}{x} + \frac{m}{x}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, + \, \frac{m \, n \, PolyLog\left[2, -\frac{d}{e \, x}\right]}{d} \, - \, \left(\frac{m}{x} + \frac{m}{x} + \frac{m}{x}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, + \, \frac{m \, n \, PolyLog\left[2, -\frac{d}{e \, x}\right]}{d} \, - \, \left(\frac{m}{x} + \frac{m}{x} + \frac{m}{x} + \frac{m}{x}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x\right)^n\right]\right) \, + \, \frac{m \, n \, PolyLog\left[2, -\frac{d}{e \, x}\right]}{d} \, - \, \left(\frac{m}{x} + \frac{m}{x} + \frac$$

Result (type 4, 120 leaves, 8 steps):

$$\begin{split} &\frac{b\,e\,m\,n\,Log\,[\,x\,]}{d}\,+\,\frac{b\,e\,n\,Log\,[\,f\,x^m\,]^{\,2}}{2\,d\,m}\,-\,\frac{b\,e\,m\,n\,Log\,[\,d\,+\,e\,\,x\,]}{d}\,-\\ &\left(\frac{m}{x}\,+\,\frac{Log\,[\,f\,x^m\,]}{x}\,\right)\,\left(a\,+\,b\,Log\,\big[\,c\,\left(d\,+\,e\,\,x\,\right)^{\,n}\,\big]\,\right)\,-\,\frac{b\,e\,n\,Log\,[\,f\,x^m\,]\,\,Log\,\big[\,1\,+\,\frac{e\,x}{d}\,\big]}{d}\,-\,\frac{b\,e\,m\,n\,PolyLog\,\big[\,2\,,\,-\,\frac{e\,x}{d}\,\big]}{d}\,\end{split}$$

## Problem 364: Result valid but suboptimal antiderivative.

$$\int \frac{Log[fx^m] \left(a+b Log[c \left(d+e x\right)^n]\right)}{x^3} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$-\frac{3 \, b \, e \, m \, n}{4 \, d \, x} - \frac{b \, e^2 \, m \, n \, Log \left[x\right]}{4 \, d^2} - \frac{b \, e \, n \, Log \left[f \, x^m\right]}{2 \, d \, x} + \frac{b \, e^2 \, n \, Log \left[1 + \frac{d}{e \, x}\right] \, Log \left[f \, x^m\right]}{2 \, d^2} + \\ \frac{b \, e^2 \, m \, n \, Log \left[d + e \, x\right]}{4 \, d^2} - \frac{1}{4} \left(\frac{m}{x^2} + \frac{2 \, Log \left[f \, x^m\right]}{x^2}\right) \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) - \frac{b \, e^2 \, m \, n \, PolyLog \left[2 \, , \, -\frac{d}{e \, x}\right]}{2 \, d^2}$$

### Result (type 4, 175 leaves, 9 steps):

$$-\frac{3 \, b \, e \, m \, n}{4 \, d \, x} \, - \frac{b \, e^2 \, m \, n \, Log \, [x]}{4 \, d^2} \, - \frac{b \, e \, n \, Log \, [f \, x^m]}{2 \, d \, x} \, - \frac{b \, e^2 \, n \, Log \, [f \, x^m]^2}{4 \, d^2 m} \, + \frac{b \, e^2 \, m \, n \, Log \, [d + e \, x]}{4 \, d^2} \, - \frac{1}{4 \, d^2 m} \, - \frac{1}{4 \, d^2 m}$$

## Problem 365: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}\,[\,f\,x^m\,]\,\,\left(\,a\,+\,b\,\,\text{Log}\,\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]\,\right)}{x^4}\,\,\text{d}\,x$$

Optimal (type 4, 193 leaves, 9 steps):

$$-\frac{5 \text{ bem n}}{36 \text{ d } x^2} + \frac{4 \text{ b } e^2 \text{ m n}}{9 \text{ d}^2 \text{ x}} + \frac{b \text{ e}^3 \text{ m n Log}[x]}{9 \text{ d}^3} - \frac{b \text{ e n Log}[f \text{ x}^m]}{6 \text{ d } x^2} + \frac{b \text{ e}^2 \text{ n Log}[f \text{ x}^m]}{3 \text{ d}^2 \text{ x}} - \frac{b \text{ e}^3 \text{ n Log}[1 + \frac{d}{e \text{ x}}] \text{ Log}[f \text{ x}^m]}{3 \text{ d}^3} - \frac{b \text{ e}^3 \text{ m n Log}[d + e \text{ x}]}{9 \text{ d}^3} - \frac{1}{9} \left(\frac{m}{x^3} + \frac{3 \text{ Log}[f \text{ x}^m]}{x^3}\right) \left(a + b \text{ Log}[c \left(d + e \text{ x}\right)^n]\right) + \frac{b \text{ e}^3 \text{ m n PolyLog}[2, -\frac{d}{e \text{ x}}]}{3 \text{ d}^3}$$

Result (type 4, 212 leaves, 10 steps):

$$-\frac{5 \text{ bem n}}{36 \text{ d } x^2} + \frac{4 \text{ b } e^2 \text{ m n}}{9 \text{ d}^2 \text{ x}} + \frac{b \text{ e}^3 \text{ m n Log}[x]}{9 \text{ d}^3} - \frac{b \text{ e n Log}[f \text{ x}^m]}{6 \text{ d } x^2} + \frac{b \text{ e}^2 \text{ n Log}[f \text{ x}^m]}{3 \text{ d}^2 \text{ x}} + \frac{b \text{ e}^3 \text{ n Log}[f \text{ x}^m]^2}{6 \text{ d}^3 \text{ m}} - \frac{b \text{ e}^3 \text{ m n Log}[d + e \text{ x}]}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3} + \frac{1}{9 \text{ d}^3 \text{ m n Log}[f \text{ x}^m]}{3 \text{ d}^3} + \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]} + \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]} + \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m n log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m log}[f \text{ x}^m]^2}{9 \text{ d}^3} - \frac{1}{9 \text{ d}^3 \text{ m log}[f \text{ x}^m]^2}{9$$

## Problem 366: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}[fx^m] \left(a + b \text{Log}\left[c \left(d + e x\right)^n\right]\right)}{x^5} dx$$

Optimal (type 4, 230 leaves, 11 steps):

$$-\frac{7 \text{ b e m n}}{144 \text{ d } x^3} + \frac{3 \text{ b } e^2 \text{ m n}}{32 \text{ d}^2 \text{ } x^2} - \frac{5 \text{ b } e^3 \text{ m n}}{16 \text{ d}^3 \text{ x}} - \frac{\text{ b } e^4 \text{ m n Log}[x]}{16 \text{ d}^4} - \frac{\text{ b e n Log}[f \text{ } x^m]}{12 \text{ d } x^3} + \frac{\text{ b } e^2 \text{ n Log}[f \text{ } x^m]}{8 \text{ d}^2 \text{ } x^2} - \frac{\text{ b } e^3 \text{ n Log}[f \text{ } x^m]}{4 \text{ d}^3 \text{ x}} + \frac{\text{ b } e^4 \text{ m n Log}[d + e \text{ x}]}{16 \text{ d}^4} - \frac{1}{16} \left(\frac{\text{m}}{x^4} + \frac{4 \text{ Log}[f \text{ } x^m]}{x^4}\right) \left(\text{a + b Log}[\text{c } \left(\text{d + e x}\right)^n]\right) - \frac{\text{b } e^4 \text{ m n PolyLog}[2, -\frac{d}{ex}]}{4 \text{ d}^4}$$

### Result (type 4, 249 leaves, 11 steps):

$$-\frac{7 \text{ b e m n}}{144 \text{ d } x^3} + \frac{3 \text{ b } e^2 \text{ m n}}{32 \text{ d}^2 \text{ } x^2} - \frac{5 \text{ b } e^3 \text{ m n}}{16 \text{ d}^3 \text{ x}} - \frac{\text{ b } e^4 \text{ m n Log}[\textbf{x}]}{16 \text{ d}^4} - \frac{\text{ b } e \text{ n Log}[\textbf{f} \textbf{x}^m]}{12 \text{ d } x^3} + \frac{\text{ b } e^2 \text{ n Log}[\textbf{f} \textbf{x}^m]}{8 \text{ d}^2 \text{ } x^2} - \frac{\text{ b } e^3 \text{ n Log}[\textbf{f} \textbf{x}^m]}{4 \text{ d}^3 \text{ x}} - \frac{\text{ b } e^4 \text{ n Log}[\textbf{f} \textbf{x}^m]^2}{8 \text{ d}^4 \text{ m}} + \frac{\text{ b } e^4 \text{ m n Log}[\textbf{d} + e \text{ x}]}{16 \text{ d}^4} - \frac{1}{16} \left(\frac{\text{m}}{\text{x}^4} + \frac{4 \text{ Log}[\textbf{f} \textbf{x}^m]}{\text{x}^4}\right) \left(\text{a + b Log}[\text{c} \left(\text{d} + e \text{ x}\right)^n]\right) + \frac{\text{b } e^4 \text{ n Log}[\textbf{f} \textbf{x}^m] \text{ Log}[\textbf{f} \textbf{x}^m] \text{ Log}[\textbf{f} + \frac{e \text{ x}}{\text{d}}]}{4 \text{ d}^4} + \frac{\text{b } e^4 \text{ m n PolyLog}[\textbf{2}, -\frac{e \text{ x}}{\text{d}}]}{4 \text{ d}^4}$$

# Problem 367: Result valid but suboptimal antiderivative.

$$\left\lceil x^2 \, \mathsf{Log} \big[ \, \mathsf{f} \, x^m \big] \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \big[ \, \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, x \right)^n \big] \, \right)^2 \, \mathrm{d} x$$

Optimal (type 4, 705 leaves, 52 steps):

$$\frac{2 \, a \, b \, d^2 \, m \, n \, x}{9 \, e^2} - \frac{71 \, b^2 \, d^2 \, m \, n}{54 \, e^2} + \frac{b \, d^2 \, m \, n}{9 \, e^2} + \frac{19 \, b^2 \, d \, m \, n}{54 \, e} - \frac{2}{27} \, b^2 \, m \, n^2 \, x^3 - \frac{2 \, a \, b \, d^2 \, n \, x \, Log [f \, x^m]}{3 \, e^2} + \frac{11 \, b^2 \, d^2 \, n^2 \, x \, Log [f \, x^m]}{9 \, e^2} - \frac{5 \, b^2 \, d \, m \, n^2 \, Log [f \, x^m]}{18 \, e} + \frac{22}{27} \, b^2 \, n^2 \, x^3 \, Log [f \, x^m] + \frac{23 \, b^2 \, d^3 \, m \, n^2 \, Log [d \, + e \, x]}{54 \, e^3} + \frac{5 \, b^2 \, d^3 \, m \, n^2 \, Log [-\frac{e \, x}{d}] \, Log [d \, + e \, x]}{9 \, e^3} - \frac{5 \, b^2 \, d^3 \, m \, n^2 \, Log [f \, x^m] \, Log [$$

Result (type 4, 902 leaves, 50 steps):

$$\frac{2 \, a \, b \, d^3 \, m \, n \, x}{3 \, e^2} - \frac{151 \, b^2 \, d^2 \, m \, n^2 \, x}{54 \, e^2} - \frac{a \, b \, d \, m \, n^2 \, x^2}{6 \, e} + \frac{7 \, b^2 \, d \, m \, n^2 \, x^2}{27 \, e} + \frac{2}{27} \, a \, b \, m \, n \, x^3 - \frac{4}{81} \, b^2 \, m \, n^2 \, x^3 + \frac{b^2 \, d \, m \, n^2 \, (d + e \, x)^3}{6 \, e^3} - \frac{2 \, b^2 \, m \, n^2 \, (d + e \, x)^3}{81 \, e^3} + \frac{11 \, a \, b \, d^3 \, m \, n \, \log[x]}{9 \, e^3} + \frac{23 \, b^2 \, d^3 \, m \, n^2 \, \log[x]}{54 \, e^3} + \frac{2 \, b^2 \, d^2 \, n^2 \, x \, \log[f \, x^m]}{e^2} - \frac{b^2 \, d \, n^2 \, (d + e \, x)^2 \, \log[f \, x^m]}{2 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n^2 \, \log[f \, x^m]}{27 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n^2 \, \log[d + e \, x)^2}{3 \, e^3} + \frac{2 \, a \, b \, d^3 \, m \, n \, \log[-\frac{e \, x}{d}] \, \log[d + e \, x)}{3 \, e^3} + \frac{b^2 \, d^3 \, m \, n^2 \, \log[d + e \, x)^2}{9 \, e^3} + \frac{b^2 \, d^3 \, m \, n^2 \, \log[x] \, \log[x] \, \log[d + e \, x)^2}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n^2 \, \log[x] \, \log[x] \, \log[x] \, \log[x] \, (d + e \, x)^n]}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n \, \log[x] \, \log[x] \, (d + e \, x)^n}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n \, \log[x] \, \log[x] \, (d + e \, x)^n]}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n \, \log[x] \, \log[x] \, (d + e \, x)^n]}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, n \, \log[x] \, \log[x] \, \log[x] \, (d + e \, x)^n]}{3 \, e^3} + \frac{2 \, b^2 \, d^3 \, m \, \log[x] \, \log[x]$$

## Problem 370: Unable to integrate problem.

$$\int \frac{\text{Log}[fx^m] \left(a + b \text{Log}\left[c \left(d + e x\right)^n\right]\right)^2}{x} dx$$

Optimal (type 4, 823 leaves, ? steps):

Result (type 8, 72 leaves, 1 step):

$$\frac{\text{Log}\left[\text{f}\,x^{\text{m}}\right]^{2}\,\left(\text{a}+\text{b}\,\text{Log}\left[\text{c}\,\left(\text{d}+\text{e}\,x\right)^{\text{n}}\right]\right)^{2}}{2\,\,\text{m}}\,-\,\frac{\text{b}\,\text{e}\,\text{n}\,\text{Unintegrable}\left[\frac{\text{Log}\left[\text{f}\,x^{\text{m}}\right]^{2}\,\left(\text{a}+\text{b}\,\text{Log}\left[\text{c}\,\left(\text{d}+\text{e}\,x\right)^{\text{n}}\right]\right)}{\text{d}+\text{e}\,x},\,x\right]}{\text{m}}$$

## Problem 374: Unable to integrate problem.

$$\int \frac{\text{Log}[x] \text{Log}[a+bx]^2}{x} dx$$

Optimal (type 4, 519 leaves, ? steps):

$$\frac{1}{12} \left( \log \left[ -\frac{bx}{a} \right]^4 + 6 \log \left[ -\frac{bx}{a} \right]^2 \log \left[ -\frac{bx}{a+bx} \right]^2 - 4 \left( \log \left[ -\frac{bx}{a} \right] + \log \left[ \frac{a}{a+bx} \right] \right) \log \left[ -\frac{bx}{a+bx} \right]^3 + \\ \log \left[ -\frac{bx}{a+bx} \right]^4 + 6 \log \left[ x \right]^2 \log \left[ a+bx \right]^2 + 4 \left( 2 \log \left[ -\frac{bx}{a} \right]^3 - 3 \log \left[ x \right]^2 \log \left[ a+bx \right] \right) \log \left[ 1 + \frac{bx}{a} \right] + \\ 6 \left( \log \left[ x \right] - \log \left[ -\frac{bx}{a} \right] \right) \left( \log \left[ x \right] + 3 \log \left[ -\frac{bx}{a} \right] \right) \log \left[ 1 + \frac{bx}{a} \right]^2 - 4 \log \left[ -\frac{bx}{a} \right]^2 \log \left[ -\frac{bx}{a+bx} \right] \left( \log \left[ -\frac{bx}{a} \right] + 3 \log \left[ 1 + \frac{bx}{a} \right] \right) + \\ 12 \left( \log \left[ -\frac{bx}{a} \right]^2 - 2 \log \left[ -\frac{bx}{a} \right] \left( \log \left[ -\frac{bx}{a+bx} \right] + \log \left[ 1 + \frac{bx}{a} \right] \right) + 2 \log \left[ x \right] \left( -\log \left[ a+bx \right] + \log \left[ 1 + \frac{bx}{a} \right] \right) \right) \operatorname{PolyLog} \left[ 2, -\frac{bx}{a} \right] - \\ 12 \log \left[ -\frac{bx}{a+bx} \right]^2 \operatorname{PolyLog} \left[ 2, \frac{bx}{a+bx} \right] + 12 \left( \log \left[ -\frac{bx}{a} \right] - \log \left[ -\frac{bx}{a+bx} \right] \right)^2 \operatorname{PolyLog} \left[ 2, 1 + \frac{bx}{a} \right] + \\ 24 \left( \log \left[ x \right] - \log \left[ -\frac{bx}{a} \right] \right) \log \left[ 1 + \frac{bx}{a} \right] \operatorname{PolyLog} \left[ 2, 1 + \frac{bx}{a} \right] + 24 \left( \log \left[ -\frac{bx}{a+bx} \right] + \log \left[ a+bx \right] \right) \operatorname{PolyLog} \left[ 3, -\frac{bx}{a} \right] + \\ 24 \log \left[ -\frac{bx}{a+bx} \right] \operatorname{PolyLog} \left[ 3, \frac{bx}{a+bx} \right] + 24 \left( -\log \left[ x \right] + \log \left[ -\frac{bx}{a+bx} \right] \right) \operatorname{PolyLog} \left[ 3, 1 + \frac{bx}{a} \right] - \\ 24 \left( \operatorname{PolyLog} \left[ 4, -\frac{bx}{a} \right] + \operatorname{PolyLog} \left[ 4, \frac{bx}{a+bx} \right] - \operatorname{PolyLog} \left[ 4, 1 + \frac{bx}{a} \right] \right) \right)$$

Result (type 8, 40 leaves, 1 step):

$$\frac{1}{2} \operatorname{Log}[x]^{2} \operatorname{Log}[a+bx]^{2} - b \operatorname{Unintegrable}\left[\frac{\operatorname{Log}[x]^{2} \operatorname{Log}[a+bx]}{a+bx}, x\right]$$

### Problem 379: Result valid but suboptimal antiderivative.

$$\left\lceil x^2 \, \left( a + b \, \text{Log} \left[ \, c \, \left( \, d + e \, x \, \right)^{\, n} \, \right] \, \right) \, \left( \, f + g \, \text{Log} \left[ \, c \, \left( \, d + e \, x \, \right)^{\, n} \, \right] \, \right) \, \, \text{d} \, x \right.$$

Optimal (type 3, 258 leaves, 7 steps):

$$\frac{2 \, b \, d^2 \, g \, n^2 \, x}{e^2} - \frac{b \, d \, g \, n^2 \, \left(d + e \, x\right)^2}{2 \, e^3} + \frac{2 \, b \, g \, n^2 \, \left(d + e \, x\right)^3}{27 \, e^3} - \frac{b \, d^3 \, g \, n^2 \, Log \left[d + e \, x\right]^2}{3 \, e^3} + \frac{1}{3} \, x^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) \, \left(f + g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) - \frac{d^2 \, n \, \left(d + e \, x\right) \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{2 \, e^3} - \frac{d \, n \, \left(d + e \, x\right)^2 \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{2 \, e^3} - \frac{d^3 \, n \, Log \left[d + e \, x\right] \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{3 \, e^3}$$

Result (type 3, 258 leaves, 13 steps):

$$\begin{split} &\frac{2 \, b \, d^2 \, g \, n^2 \, x}{e^2} - \frac{b \, d \, g \, n^2 \, \left(d + e \, x\right)^2}{2 \, e^3} + \frac{2 \, b \, g \, n^2 \, \left(d + e \, x\right)^3}{27 \, e^3} - \frac{b \, d^3 \, g \, n^2 \, Log \left[d + e \, x\right]^2}{3 \, e^3} - \frac{1}{8 \, g \, n} \left(\frac{18 \, d^2 \, \left(d + e \, x\right)}{e^3} - \frac{9 \, d \, \left(d + e \, x\right)^2}{e^3} + \frac{2 \, \left(d + e \, x\right)^3}{e^3} - \frac{6 \, d^3 \, Log \left[d + e \, x\right]}{e^3}\right) \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) - \frac{1}{18} \, b \, n \, \left(\frac{18 \, d^2 \, \left(d + e \, x\right)}{e^3} - \frac{9 \, d \, \left(d + e \, x\right)^2}{e^3} + \frac{2 \, \left(d + e \, x\right)^3}{e^3} - \frac{6 \, d^3 \, Log \left[d + e \, x\right]}{e^3}\right) \, \left(f + g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) + \frac{1}{3} \, x^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) \, \left(f + g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) \end{split}$$

### Problem 380: Result valid but suboptimal antiderivative.

$$\left\lceil x \; \left( \mathsf{a} + \mathsf{b} \; \mathsf{Log} \left[ \; \mathsf{c} \; \left( \mathsf{d} + \mathsf{e} \; x \right)^{\, \mathsf{n}} \right] \; \right) \; \left( \mathsf{f} + \mathsf{g} \; \mathsf{Log} \left[ \; \mathsf{c} \; \left( \mathsf{d} + \mathsf{e} \; x \right)^{\, \mathsf{n}} \right] \right) \; \mathrm{d} x \right.$$

Optimal (type 3, 196 leaves, 7 steps):

$$-\frac{2 \, b \, d \, g \, n^2 \, x}{e} + \frac{b \, g \, n^2 \, \left(d + e \, x\right)^2}{4 \, e^2} + \frac{b \, d^2 \, g \, n^2 \, Log \left[d + e \, x\right]^2}{2 \, e^2} + \\ \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) \, \left(f + g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right) + \frac{d \, n \, \left(d + e \, x\right) \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{e^2} - \\ \frac{n \, \left(d + e \, x\right)^2 \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{4 \, e^2} - \frac{d^2 \, n \, Log \left[d + e \, x\right] \, \left(b \, f + a \, g + 2 \, b \, g \, Log \left[c \, \left(d + e \, x\right)^n\right]\right)}{2 \, e^2}$$

Result (type 3, 206 leaves, 13 steps):

$$-\frac{2 \ b \ d \ g \ n^2 \ x}{e} + \frac{b \ g \ n^2 \ \left(d + e \ x\right)^2}{4 \ e^2} + \frac{b \ d^2 \ g \ n^2 \ Log \left[d + e \ x\right]^2}{2 \ e^2} + \frac{1}{4} \ g \ n \left(\frac{4 \ d \ \left(d + e \ x\right)}{e^2} - \frac{\left(d + e \ x\right)^2}{e^2} - \frac{2 \ d^2 \ Log \left[d + e \ x\right]}{e^2}\right) \ \left(a + b \ Log \left[c \ \left(d + e \ x\right)^n\right]\right) + \frac{1}{4} \ b \ n \left(\frac{4 \ d \ \left(d + e \ x\right)}{e^2} - \frac{2 \ d^2 \ Log \left[d + e \ x\right]}{e^2}\right) \ \left(f + g \ Log \left[c \ \left(d + e \ x\right)^n\right]\right) + \frac{1}{2} \ x^2 \ \left(a + b \ Log \left[c \ \left(d + e \ x\right)^n\right]\right) \ \left(f + g \ Log \left[c \ \left(d + e \ x\right)^n\right]\right)$$

### Problem 381: Result valid but suboptimal antiderivative.

$$\left\lceil \left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^{\mathsf{n}} \right] \right) \, \left( \mathsf{f} + \mathsf{g} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^{\mathsf{n}} \right] \right) \, \mathbb{d} \mathsf{x} \right] \right\rangle$$

Optimal (type 3, 110 leaves, 6 steps):

$$\begin{split} & - \left( b \, f + a \, g \right) \, n \, x + 2 \, b \, g \, n^2 \, x \, - \, \frac{2 \, b \, g \, n \, \left( d + e \, x \right) \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right] \,}{e} \, + \\ & \times \, \left( a + b \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right] \right) \, \left( f + g \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right] \right) \, + \, \frac{d \, \left( b \, f + a \, g + 2 \, b \, g \, Log \left[ c \, \left( d + e \, x \right)^{\, n} \right] \right)^{\, 2}}{4 \, b \, e \, g} \end{split}$$

Result (type 3, 130 leaves, 11 steps):

$$-\,b\,f\,n\,x\,-\,a\,g\,n\,x\,+\,2\,b\,g\,n^{2}\,x\,-\,\frac{2\,b\,g\,n\,\left(\,d\,+\,e\,\,x\,\right)\,\,Log\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]}{e}\,\,+\,\\ \frac{\,d\,g\,\left(\,a\,+\,b\,Log\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]\,\right)^{\,2}}{2\,b\,e}\,\,+\,x\,\,\left(\,a\,+\,b\,Log\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]\,\right)\,\,\left(\,f\,+\,g\,Log\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]\,\right)\,\,+\,\frac{\,b\,d\,\,\left(\,f\,+\,g\,Log\left[\,c\,\,\left(\,d\,+\,e\,\,x\,\right)^{\,n}\,\right]\,\right)^{\,2}}{2\,e\,g}$$

### Problem 382: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right) \, \left(f+g \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right)}{x} \, \mathrm{d}x$$

Optimal (type 4, 158 leaves, 6 steps):

$$\begin{split} & \text{Log} \left[ x \right] \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^\mathsf{n} \right] \right) \, \left( \mathsf{f} + \mathsf{g} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^\mathsf{n} \right] \right) - \frac{\mathsf{Log} \left[ \mathsf{x} \right] \, \left( \mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^\mathsf{n} \right] \right)^2}{4 \, \mathsf{b} \, \mathsf{g}} \, + \\ & \frac{\mathsf{Log} \left[ - \frac{\mathsf{e} \, \mathsf{x}}{\mathsf{d}} \right] \, \left( \mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^\mathsf{n} \right] \right)^2}{4 \, \mathsf{b} \, \mathsf{g}} + \mathsf{n} \, \left( \mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[ \mathsf{c} \, \left( \mathsf{d} + \mathsf{e} \, \mathsf{x} \right)^\mathsf{n} \right] \right) \, \mathsf{PolyLog} \left[ 2 \, , \, 1 + \frac{\mathsf{e} \, \mathsf{x}}{\mathsf{d}} \right] - 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{PolyLog} \left[ 3 \, , \, 1 + \frac{\mathsf{e} \, \mathsf{x}}{\mathsf{d}} \right] \, \mathsf{exp} \right] \, \mathsf{PolyLog} \left[ \mathsf{g} \, \mathsf{g} \,$$

Result (type 4, 219 leaves, 11 steps):

$$-\frac{g \, \text{Log} \big[ x \big] \, \left( a + b \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \big)^2}{2 \, b} + \frac{g \, \text{Log} \big[ - \frac{e \, x}{d} \big] \, \left( a + b \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right)^2}{2 \, b} + \\ \\ \text{Log} \big[ x \big] \, \left( a + b \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \big) \, \left( f + g \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right) - \frac{b \, \text{Log} \big[ x \big] \, \left( f + g \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right)^2}{2 \, g} + \frac{b \, \text{Log} \big[ - \frac{e \, x}{d} \big] \, \left( f + g \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right)^2}{2 \, g} + \\ \\ g \, n \, \left( a + b \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right) \, \text{PolyLog} \big[ 2 \text{, } 1 + \frac{e \, x}{d} \big] + b \, n \, \left( f + g \, \text{Log} \big[ c \, \left( d + e \, x \right)^n \big] \right) \, \text{PolyLog} \big[ 2 \text{, } 1 + \frac{e \, x}{d} \big] - 2 \, b \, g \, n^2 \, \text{PolyLog} \big[ 3 \text{, } 1 + \frac{e \, x}{d} \big]$$

## Problem 383: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right) \, \left(f+g \, Log\left[c\, \left(d+e\, x\right)^{\, n}\right]\right)}{x^2} \, \mathrm{d}x$$

Optimal (type 4, 96 leaves, 4 steps):

$$-\frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[\mathsf{c} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{\,\mathsf{n}}\right]\right) \, \left(\mathsf{f} + \mathsf{g} \, \mathsf{Log} \left[\mathsf{c} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{\,\mathsf{n}}\right]\right)}{\mathsf{x}} + \frac{\mathsf{e} \, \mathsf{n} \, \left(\mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[\mathsf{c} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{\,\mathsf{n}}\right]\right) \, \mathsf{Log} \left[\mathsf{1} - \frac{\mathsf{d}}{\mathsf{d} + \mathsf{e} \, \mathsf{x}}\right]}{\mathsf{d}} - \frac{2 \, \mathsf{b} \, \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{PolyLog} \left[\mathsf{2} \, \mathsf{,} \, \frac{\mathsf{d}}{\mathsf{d} + \mathsf{e} \, \mathsf{x}}\right]}{\mathsf{d}} + \frac{\mathsf{e} \, \mathsf{n} \, \left(\mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[\mathsf{c} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{\,\mathsf{n}}\right]\right) \, \mathsf{Log} \left[\mathsf{1} - \frac{\mathsf{d}}{\mathsf{d} + \mathsf{e} \, \mathsf{x}}\right]}{\mathsf{d}} + \frac{\mathsf{e} \, \mathsf{n} \, \left(\mathsf{b} \, \mathsf{f} + \mathsf{a} \, \mathsf{g} + 2 \, \mathsf{b} \, \mathsf{g} \, \mathsf{Log} \left[\mathsf{c} \, \left(\mathsf{d} + \mathsf{e} \, \mathsf{x}\right)^{\,\mathsf{n}}\right]\right) \, \mathsf{Log} \left[\mathsf{n} \, \mathsf{d} + \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{PolyLog} \left[\mathsf{n} \, \mathsf{d} + \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{PolyLog} \left[\mathsf{n} \, \mathsf{d} + \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{polyLog} \left[\mathsf{n} \, \mathsf{d} + \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{polyLog} \left[\mathsf{n} \, \mathsf{d} + \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{polyLog} \left[\mathsf{n} \, \mathsf{d} \, \mathsf{e} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{polyLog} \left[\mathsf{n} \, \mathsf{g} \, \mathsf{n}^2 \, \mathsf{g} \, \mathsf{polyLog} \left[\mathsf{n} \, \mathsf{g} \, \mathsf$$

Result (type 4, 169 leaves, 11 steps):

$$\frac{e\,g\,n\,Log\left[-\frac{e\,x}{d}\right]\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^n\right]\right)}{d}\,-\,\frac{e\,g\,\left(a+b\,Log\left[c\,\left(d+e\,x\right)^n\right]\right)^2}{2\,b\,d}\,+\,\frac{b\,e\,n\,Log\left[-\frac{e\,x}{d}\right]\,\left(f+g\,Log\left[c\,\left(d+e\,x\right)^n\right]\right)}{d}\,-\,\frac{2\,b\,d}{\left(a+b\,Log\left[c\,\left(d+e\,x\right)^n\right]\right)^2}\,-\,\frac{b\,e\,\left(f+g\,Log\left[c\,\left(d+e\,x\right)^n\right]\right)^2}{2\,d\,g}\,+\,\frac{2\,b\,e\,g\,n^2\,PolyLog\left[2\,,\,1+\frac{e\,x}{d}\right]}{d}\,$$

## Problem 384: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \left(d+e x\right)^{n}\right]\right) \left(f+g \log \left[c \left(d+e x\right)^{n}\right]\right)}{x^{3}} dx$$

Optimal (type 4, 156 leaves, 7 steps):

Result (type 4, 265 leaves, 17 steps):

$$\frac{b \, e^2 \, g \, n^2 \, Log[x]}{d^2} - \frac{e \, g \, n \, \left(d + e \, x\right) \, \left(a + b \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)}{2 \, d^2 \, x} - \frac{e^2 \, g \, n \, Log\big[-\frac{e \, x}{d}\big] \, \left(a + b \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)}{2 \, d^2} + \frac{e^2 \, g \, \left(a + b \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)^2}{4 \, b \, d^2} - \frac{b \, e \, n \, \left(d + e \, x\right) \, \left(f + g \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)}{2 \, d^2} - \frac{b \, e^2 \, n \, Log\big[-\frac{e \, x}{d}\big] \, \left(f + g \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)}{2 \, d^2} - \frac{b \, e^2 \, n \, Log\big[c \, \left(d + e \, x\right)^n\big]\right)}{2 \, d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}{d}\big]}{d^2} - \frac{b \, e^2 \, g \, n^2 \, PolyLog\big[2, \, 1 + \frac{e \, x}$$

# Problem 385: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \log \left[c \left(d+e x\right)^{n}\right]\right) \left(f+g \log \left[c \left(d+e x\right)^{n}\right]\right)}{x^{4}} dx$$

Optimal (type 4, 234 leaves, 11 steps):

$$-\frac{b \, e^2 \, g \, n^2}{3 \, d^2 \, x} - \frac{b \, e^3 \, g \, n^2 \, Log \, [x]}{d^3} + \frac{b \, e^3 \, g \, n^2 \, Log \, [d + e \, x]}{3 \, d^3} - \frac{\left(a + b \, Log \, [c \, \left(d + e \, x\right)^n]\right) \, \left(f + g \, Log \, [c \, \left(d + e \, x\right)^n]\right)}{3 \, x^3} - \frac{e \, n \, \left(b \, f + a \, g + 2 \, b \, g \, Log \, [c \, \left(d + e \, x\right)^n]\right)}{6 \, d \, x^2} + \frac{e^3 \, n \, \left(b \, f + a \, g + 2 \, b \, g \, Log \, [c \, \left(d + e \, x\right)^n]\right)}{3 \, d^3 \, x} + \frac{e^3 \, n \, \left(b \, f + a \, g + 2 \, b \, g \, Log \, [c \, \left(d + e \, x\right)^n]\right) \, Log \, \left[1 - \frac{d}{d + e \, x}\right]}{3 \, d^3} - \frac{2 \, b \, e^3 \, g \, n^2 \, PolyLog \, \left[2, \, \frac{d}{d + e \, x}\right]}{3 \, d^3}$$

Result (type 4, 365 leaves, 25 steps):

$$-\frac{b\,e^2\,g\,n^2}{3\,d^2\,x} - \frac{b\,e^3\,g\,n^2\,Log\,[x]}{d^3} + \frac{b\,e^3\,g\,n^2\,Log\,[d+e\,x]}{3\,d^3} - \frac{e\,g\,n\,\left(a+b\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{6\,d\,x^2} + \frac{e^2\,g\,n\,\left(d+e\,x\right)\,\left(a+b\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{3\,d^3\,x} + \frac{e^3\,g\,n\,Log\,\left[-\frac{e\,x}{d}\right]\,\left(a+b\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{3\,d^3} - \frac{e^3\,g\,\left(a+b\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{6\,b\,d^3} - \frac{b\,e\,n\,\left(f+g\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{6\,d\,x^2} + \frac{b\,e^2\,n\,\left(d+e\,x\right)\,\left(f+g\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{3\,d^3\,x} + \frac{b\,e^3\,n\,Log\,\left[-\frac{e\,x}{d}\right]\,\left(f+g\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{3\,d^3} - \frac{e\,a\,n\,Log\,\left[-\frac{e\,x}{d}\right]\,\left(f+g\,Log\,\left[c\,\left(d+e\,x\right)^n\right]\right)}{3\,d^3} - \frac{e\,a\,n\,Log\,\left[-\frac{e\,x}{d}\right]\,\left(f+g\,Log\,\left[c\,\left(d+e\,x\right)^n\right]}{3\,d^3} -$$

## Problem 428: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)^{\,3}\,\left(a+b\,\text{Log}\!\left[\,c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\,\right]\,\right)^{\,2}\,\text{d}x$$

Optimal (type 3, 409 leaves, 9 steps):

$$\frac{2\,b^{2}\,\left(f\,g-e\,h\right)^{3}\,p^{2}\,q^{2}\,x}{f^{3}} + \frac{3\,b^{2}\,h\,\left(f\,g-e\,h\right)^{2}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{2}}{4\,f^{4}} + \\ \frac{2\,b^{2}\,h^{2}\,\left(f\,g-e\,h\right)\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{3}}{9\,f^{4}} + \frac{b^{2}\,h^{3}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{4}}{32\,f^{4}} + \frac{b^{2}\,\left(f\,g-e\,h\right)^{4}\,p^{2}\,q^{2}\,Log\left[e+f\,x\right]^{2}}{4\,f^{4}\,h} - \\ \frac{2\,b\,\left(f\,g-e\,h\right)^{3}\,p\,q\,\left(e+f\,x\right)\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{f^{4}} - \frac{3\,b\,h\,\left(f\,g-e\,h\right)^{2}\,p\,q\,\left(e+f\,x\right)^{2}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{2\,f^{4}} - \frac{2\,b\,h^{3}\,p\,q\,\left(e+f\,x\right)^{4}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{8\,f^{4}} - \frac{b\,\left(f\,g-e\,h\right)^{4}\,p\,q\,Log\left[e+f\,x\right]\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{2\,f^{4}\,h} + \frac{\left(g+h\,x\right)^{4}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{4\,h} - \frac{b\,\left(f\,g-e\,h\right)^{4}\,p\,q\,Log\left[e+f\,x\right]\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{4\,h} - \frac{\left(g+h\,x\right)^{4}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{4\,h} - \frac{\left(g+h\,x\right)^{4}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]}{4\,h} - \frac{\left(g+h\,x\right)^{4}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right)}{4\,h} - \frac{\left(g+h\,x\right)$$

Result (type 3, 325 leaves, 7 steps):

$$\begin{split} &\frac{2\,b^{2}\,\left(f\,g-e\,h\right)^{3}\,p^{2}\,q^{2}\,x}{f^{3}}\,+\,\frac{3\,b^{2}\,h\,\left(f\,g-e\,h\right)^{2}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{2}}{4\,f^{4}}\,+\,\\ &\frac{2\,b^{2}\,h^{2}\,\left(f\,g-e\,h\right)\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{3}}{9\,f^{4}}\,+\,\frac{b^{2}\,h^{3}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{4}}{32\,f^{4}}\,+\,\frac{b^{2}\,\left(f\,g-e\,h\right)^{4}\,p^{2}\,q^{2}\,Log\left[e+f\,x\right]^{2}}{4\,f^{4}\,h}\,-\,\frac{1}{24\,h}\,\\ &b\,p\,q\,\left(\frac{48\,h\,\left(f\,g-e\,h\right)^{3}\,\left(e+f\,x\right)}{f^{4}}\,+\,\frac{36\,h^{2}\,\left(f\,g-e\,h\right)^{2}\,\left(e+f\,x\right)^{2}}{f^{4}}\,+\,\frac{16\,h^{3}\,\left(f\,g-e\,h\right)\,\left(e+f\,x\right)^{3}}{f^{4}}\,+\,\frac{3\,h^{4}\,\left(e+f\,x\right)^{4}}{f^{4}}\,+\,\frac{12\,\left(f\,g-e\,h\right)^{4}\,Log\left[e+f\,x\right]}{f^{4}}\,\right)^{2}\,d^{2}\,h^{2}\,d^{2}\,h^{2}\,d^{2}\,h^{2}\,h^{2}\,d^{2}\,h^$$

### Problem 429: Result valid but suboptimal antiderivative.

$$\int \left(g+h\,x\right)^{\,2}\,\left(a+b\,\text{Log}\left[\,c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\,\right]\,\right)^{\,2}\,\text{d}x$$

Optimal (type 3, 323 leaves, 9 steps):

$$\frac{2\,b^{2}\,\left(f\,g-e\,h\right)^{2}\,p^{2}\,q^{2}\,x}{f^{2}} + \frac{b^{2}\,h\,\left(f\,g-e\,h\right)\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{2}}{2\,f^{3}} + \frac{2\,b^{2}\,h^{2}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{3}}{27\,f^{3}} + \frac{b^{2}\,\left(f\,g-e\,h\right)^{3}\,p^{2}\,q^{2}\,Log\,[e+f\,x]^{2}}{3\,f^{3}\,h} - \frac{2\,b\,\left(f\,g-e\,h\right)^{2}\,p\,q\,\left(e+f\,x\right)\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{f^{3}} - \frac{b\,h\,\left(f\,g-e\,h\right)\,p\,q\,\left(e+f\,x\right)^{2}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{f^{3}} - \frac{2\,b\,h^{2}\,p\,q\,\left(e+f\,x\right)^{3}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{9\,f^{3}} - \frac{2\,b\,\left(f\,g-e\,h\right)^{3}\,p\,q\,Log\,[e+f\,x]\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,f^{3}\,h} + \frac{\left(g+h\,x\right)^{3}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,\left(e+f\,x\right)^{3}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,\left(e+f\,x\right)^{3}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,Log\,[e+f\,x]\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,Log\,[e+f\,x]\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,\left(e+f\,x\right)^{3}\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,Log\,[e+f\,x]\,\left(a+b\,Log\,[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{3\,h} + \frac{2\,b\,h^{2}\,p\,q\,Log\,[e+f\,x]\,\left(a+b\,Log\,[e+f\,x]\,h^{2}\,h^$$

Result (type 3, 264 leaves, 9 steps):

$$\frac{2\,b^{2}\,\left(f\,g-e\,h\right)^{2}\,p^{2}\,q^{2}\,x}{f^{2}} + \frac{b^{2}\,h\,\left(f\,g-e\,h\right)\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{2}}{2\,f^{3}} + \frac{2\,b^{2}\,h^{2}\,p^{2}\,q^{2}\,\left(e+f\,x\right)^{3}}{27\,f^{3}} + \frac{b^{2}\,\left(f\,g-e\,h\right)^{3}\,p^{2}\,q^{2}\,Log\left[e+f\,x\right]^{2}}{3\,f^{3}\,h} \\ \frac{b\,p\,q\,\left(\frac{18\,h\,\left(f\,g-e\,h\right)^{2}\,\left(e+f\,x\right)}{f^{3}} + \frac{9\,h^{2}\,\left(f\,g-e\,h\right)\,\left(e+f\,x\right)^{2}}{f^{3}} + \frac{2\,h^{3}\,\left(e+f\,x\right)^{3}}{f^{3}} + \frac{6\,\left(f\,g-e\,h\right)^{3}\,Log\left[e+f\,x\right]}{f^{3}}\right)\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)}{9\,h} + \frac{\left(g+h\,x\right)^{3}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{p}\right)^{q}\right]\right)^{2}}{3\,h} \\ \\ + \frac{2\,h^{2}\,h^{2$$

# Problem 434: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,Log\left[\,c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\,\right]\,\right)^{\,2}}{\left(g+h\,x\right)^{\,3}}\,\mathrm{d}x$$

Optimal (type 4, 222 leaves, 8 steps):

$$-\frac{b\,f\,p\,q\,\left(e+f\,x\right)\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)}{\left(f\,g-e\,h\right)^{\,2}\,\left(g+h\,x\right)}-\frac{\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)^{\,2}}{2\,h\,\left(g+h\,x\right)^{\,2}}+\\ \\ \frac{b^{2}\,f^{2}\,p^{2}\,q^{2}\,Log\left[g+h\,x\right]}{h\,\left(f\,g-e\,h\right)^{\,2}}-\frac{b\,f^{2}\,p\,q\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)\,Log\left[1+\frac{f\,g-e\,h}{h\,\left(e+f\,x\right)}\right]}{h\,\left(e+f\,x\right)}+\frac{b^{2}\,f^{2}\,p^{2}\,q^{2}\,PolyLog\left[2,-\frac{f\,g-e\,h}{h\,\left(e+f\,x\right)}\right]}{h\,\left(f\,g-e\,h\right)^{\,2}}$$

### Result (type 4, 257 leaves, 10 steps):

$$-\frac{b\,f\,p\,q\,\left(e+f\,x\right)\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)}{\left(f\,g-e\,h\right)^{\,2}\,\left(g+h\,x\right)} + \frac{f^{\,2}\,\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)^{\,2}}{2\,h\,\left(f\,g-e\,h\right)^{\,2}} - \frac{\left(a+b\,Log\left[c\,\left(d\,\left(e+f\,x\right)^{\,p}\right)^{\,q}\right]\right)^{\,2}}{2\,h\,\left(g+h\,x\right)^{\,2}} + \frac{b^{\,2}\,f^{\,2}\,p^{\,2}\,q^{\,2}\,Log\left[g+h\,x\right]}{h\,\left(f\,g-e\,h\right)^{\,2}} - \frac{b^{\,2}\,f^{\,2}\,p^{\,2}\,q^{\,2}\,PolyLog\left[2\,,\,-\frac{h\,\left(e+f\,x\right)}{f\,g-e\,h}\right]}{h\,\left(f\,g-e\,h\right)^{\,2}} + \frac{b^{\,2}\,f^{\,2}\,p^{\,2}\,q^{\,2}\,PolyLog\left[2\,,\,-\frac{h\,\left(e+f\,x\right)}{f\,g-e\,h}\right]}{h\,\left(e\,h\,g-e\,h\right)^{\,2}} + \frac{b^{\,2}\,f^{\,2}\,p^{\,2}\,q^{\,2}\,p^{$$

# Problem 440: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \text{Log}\left[\, c \, \left(d \, \left(e+f \, x\right)^{\, p}\right)^{\, q}\,\right]\,\right)^{\, 3}}{\left(g+h \, x\right)^{\, 3}} \, \mathrm{d} x$$

### Optimal (type 4, 376 leaves, 10 steps):

$$\frac{3 \, b \, f \, p \, q \, \left(e + f \, x\right) \, \left(a + b \, Log \left[c \, \left(d \, \left(e + f \, x\right)^{\, p}\right)^{\, q}\right]\right)^{\, 2}}{2 \, \left(f \, g - e \, h\right)^{\, 2} \, \left(g + h \, x\right)} - \frac{\left(a + b \, Log \left[c \, \left(d \, \left(e + f \, x\right)^{\, p}\right)^{\, q}\right]\right)^{\, 3}}{2 \, h \, \left(g + h \, x\right)^{\, 2}} + \frac{2 \, h \, \left(g + h \, x\right)^{\, 2}}{2 \, h \, \left(g + h \, x\right)^{\, 2}} - \frac{3 \, b \, f^{\, 2} \, p^{\, 2} \, q^{\, 2} \, \left(a + b \, Log \left[c \, \left(d \, \left(e + f \, x\right)^{\, p}\right)^{\, q}\right]\right) \, Log \left[\frac{f \, (g + h \, x)}{f \, g - e \, h}\right]}{f \, g - e \, h}} - \frac{3 \, b \, f^{\, 2} \, p \, q \, \left(a + b \, Log \left[c \, \left(d \, \left(e + f \, x\right)^{\, p}\right)^{\, q}\right]\right)^{\, 2} \, Log \left[1 + \frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{2 \, h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[2 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[2 \, , \, -\frac{h \, \left(e + f \, x\right)}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[2 \, , \, -\frac{h \, \left(e + f \, x\right)}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, p^{\, 3} \, q^{\, 3} \, PolyLog \left[3 \, , \, -\frac{f \, g - e \, h}{h \, \left(e + f \, x\right)}\right]}{h \, \left(f \, g - e \, h\right)^{\, 2}} + \frac{3 \, b^{\, 3} \, f^{\, 2} \, g^{\, 3} \,$$

### Result (type 4, 408 leaves, 13 steps):

$$-\frac{3 \, b \, f \, p \, q \, \left(e + f \, x\right) \, \left(a + b \, Log\left[c \, \left(d \, \left(e + f \, x\right)^{p}\right)^{q}\right]\right)^{2}}{2 \, \left(f \, g - e \, h\right)^{2} \, \left(g + h \, x\right)} + \frac{f^{2} \, \left(a + b \, Log\left[c \, \left(d \, \left(e + f \, x\right)^{p}\right)^{q}\right]\right)^{3}}{2 \, h \, \left(f \, g - e \, h\right)^{2}} - \frac{\left(a + b \, Log\left[c \, \left(d \, \left(e + f \, x\right)^{p}\right)^{q}\right]\right)^{3}}{2 \, h \, \left(g + h \, x\right)^{2}} + \frac{3 \, b^{2} \, f^{2} \, p^{2} \, q^{2} \, \left(a + b \, Log\left[c \, \left(d \, \left(e + f \, x\right)^{p}\right)^{q}\right]\right) \, Log\left[\frac{f \, (g + h \, x)}{f \, g - e \, h}\right]}{2 \, h \, \left(f \, g - e \, h\right)^{2}} - \frac{3 \, b \, f^{2} \, p \, q \, \left(a + b \, Log\left[c \, \left(d \, \left(e + f \, x\right)^{p}\right)^{q}\right]\right)^{2} \, Log\left[\frac{f \, (g + h \, x)}{f \, g - e \, h}\right]}{2 \, h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[2, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{2} \, p^{3} \, q^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{3} \, p^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h \, \left(f \, g - e \, h\right)^{2}} + \frac{3 \, b^{3} \, f^{3} \, p^{3} \, PolyLog\left[3, \, -\frac{h \, (e + f \, x)}{f \, g - e \, h}\right]}{h$$

# Test results for the 641 problems in "3.4 u (a+b log(c (d+e x^m)^n))^p.m"

## Problem 77: Result valid but suboptimal antiderivative.

$$\int x^5 Log \left[ c \left( a + b x^2 \right)^p \right]^2 dx$$

Optimal (type 3, 215 leaves, 8 steps):

$$\frac{a^{2} p^{2} x^{2}}{b^{2}} - \frac{a p^{2} \left(a + b x^{2}\right)^{2}}{4 b^{3}} + \frac{p^{2} \left(a + b x^{2}\right)^{3}}{27 b^{3}} - \frac{a^{3} p^{2} Log \left[a + b x^{2}\right]^{2}}{6 b^{3}} - \frac{a^{2} p \left(a + b x^{2}\right) Log \left[c \left(a + b x^{2}\right)^{p}\right]}{b^{3}} + \frac{a^{3} p Log \left[c \left(a + b x^{2}\right)^{p}\right]}{2 b^{3}} + \frac{a^{3} p Log \left[c \left(a + b x^{2}\right)^{p}\right]}{3 b^{3}} + \frac{1}{6} x^{6} Log \left[c \left(a + b x^{2}\right)^{p}\right]^{2}$$

Result (type 3, 175 leaves, 8 steps):

$$\begin{split} &\frac{a^2 \ p^2 \ x^2}{b^2} - \frac{a \ p^2 \ \left(a + b \ x^2\right)^2}{4 \ b^3} + \frac{p^2 \ \left(a + b \ x^2\right)^3}{27 \ b^3} - \frac{a^3 \ p^2 \ Log \left[a + b \ x^2\right]^2}{6 \ b^3} - \\ &\frac{1}{18} \ p \left(\frac{18 \ a^2 \ \left(a + b \ x^2\right)}{b^3} - \frac{9 \ a \ \left(a + b \ x^2\right)^2}{b^3} + \frac{2 \ \left(a + b \ x^2\right)^3}{b^3} - \frac{6 \ a^3 \ Log \left[a + b \ x^2\right]}{b^3}\right) \ Log \left[c \ \left(a + b \ x^2\right)^p\right] + \frac{1}{6} \ x^6 \ Log \left[c \ \left(a + b \ x^2\right)^p\right]^2 \end{split}$$

### Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[c\left(a+b\,x^2\right)^p\right]^2}{v^5}\,\mathrm{d}x$$

Optimal (type 4, 129 leaves, 8 steps):

$$\frac{b^2 \, p^2 \, Log \, [\, x\,]}{a^2} \, - \, \frac{b \, p \, \left(a + b \, x^2\right) \, Log \, \left[\, c \, \left(a + b \, x^2\right)^{\, p}\,\right]}{2 \, a^2 \, x^2} \, - \, \frac{Log \, \left[\, c \, \left(a + b \, x^2\right)^{\, p}\,\right]^{\, 2}}{4 \, x^4} \, - \, \frac{b^2 \, p \, Log \, \left[\, c \, \left(a + b \, x^2\right)^{\, p}\,\right] \, Log \, \left[\, 1 - \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, + \, \frac{b^2 \, p^2 \, PolyLog \, \left[\, 2 \, , \, \frac{a}{a + b \, x^2}\,\right]}{2 \, a^2} \, +$$

Result (type 4, 147 leaves, 10 steps):

$$\frac{b^2 \, p^2 \, Log\left[x\right]}{a^2} - \frac{b \, p \, \left(a + b \, x^2\right) \, Log\left[c \, \left(a + b \, x^2\right)^p\right]}{2 \, a^2 \, x^2} - \frac{b^2 \, p \, Log\left[-\frac{b \, x^2}{a}\right] \, Log\left[c \, \left(a + b \, x^2\right)^p\right]}{2 \, a^2} + \frac{b^2 \, Log\left[c \, \left(a + b \, x^2\right)^p\right]^2}{4 \, a^2} - \frac{Log\left[c \, \left(a + b \, x^2\right)^p\right]^2}{4 \, x^4} - \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2, \, 1 + \frac{b \, x^2}{a}\right]}{2 \, a^2} + \frac{b^2 \, p^2 \, PolyLog\left[2,$$

## Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{Log \left[ c \left( a + b x^2 \right)^p \right]^2}{x^7} \, \mathrm{d}x$$

Optimal (type 4, 193 leaves, 12 steps):

$$-\frac{b^{2} p^{2}}{6 a^{2} x^{2}} - \frac{b^{3} p^{2} Log[x]}{a^{3}} + \frac{b^{3} p^{2} Log[a + b x^{2}]}{6 a^{3}} - \frac{b p Log[c (a + b x^{2})^{p}]}{6 a x^{4}} + \frac{b^{2} p (a + b x^{2}) Log[c (a + b x^{2})^{p}]}{6 a^{3}} - \frac{Log[c (a + b x^{2})^{p}]^{2}}{6 a^{6}} + \frac{b^{3} p Log[c (a + b x^{2})^{p}] Log[1 - \frac{a}{a + b x^{2}}]}{3 a^{3}} - \frac{b^{3} p^{2} PolyLog[2, \frac{a}{a + b x^{2}}]}{3 a^{3}}$$

Result (type 4, 211 leaves, 14 steps):

$$-\frac{b^{2} \, p^{2}}{6 \, a^{2} \, x^{2}} - \frac{b^{3} \, p^{2} \, Log \, [\, x\,]}{a^{3}} + \frac{b^{3} \, p^{2} \, Log \, [\, a + b \, x^{2}\,]}{6 \, a^{3}} - \frac{b \, p \, Log \, [\, c \, \left(\, a + b \, x^{2}\,\right)^{\, p}\,]}{6 \, a \, x^{4}} + \frac{b^{2} \, p \, \left(\, a + b \, x^{2}\,\right) \, Log \, [\, c \, \left(\, a + b \, x^{2}\,\right)^{\, p}\,]}{3 \, a^{3} \, x^{2}} + \frac{b^{3} \, p \, Log \, [\, c \, \left(\, a + b \, x^{2}\,\right)^{\, p}\,]}{6 \, a^{3}} - \frac{b^{3} \, Log \, [\, c \, \left(\, a + b \, x^{2}\,\right)^{\, p}\,]^{\, 2}}{6 \, a^{3}} - \frac{Log \, [\, c \, \left(\, a + b \, x^{2}\,\right)^{\, p}\,]^{\, 2}}{6 \, x^{6}} + \frac{b^{3} \, p^{2} \, PolyLog \, [\, 2 \, , \, 1 + \frac{b \, x^{2}}{a}\,]}{3 \, a^{3}}$$

### Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[c\,\left(a+b\,x^2\right)^p\right]^3}{x^5}\,\mathrm{d}x$$

Optimal (type 4, 219 leaves, 10 steps):

$$\frac{3 \, b^{2} \, p^{2} \, Log\left[-\frac{b \, x^{2}}{a}\right] \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]}{2 \, a^{2}} - \frac{3 \, b \, p \, \left(a + b \, x^{2}\right) \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{4 \, a^{2} \, x^{2}} - \frac{Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{3}}{4 \, x^{4}} - \frac{3 \, b^{2} \, p \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2} \, Log\left[1 - \frac{a}{a + b \, x^{2}}\right]}{4 \, a^{2}} + \frac{3 \, b^{2} \, p^{2} \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2} \, PolyLog\left[2, \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{2}} + \frac{3 \, b^{2} \, p^{3} \, PolyLog\left[3, \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{2}} + \frac{3 \, b^{2} \, p^{3} \, PolyLog\left[3, \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{2}}$$

Result (type 4, 236 leaves, 13 steps):

$$\frac{3 \, b^{2} \, p^{2} \, Log\left[\left(-\frac{b \, x^{2}}{a}\right] \, Log\left[\left(-\frac{b \, x^{2}}{a}\right] \, Log\left[\left(-\frac{b \, x^{2}}{a}\right) \, Log\left(\left(-\frac{b \, x^{2}}{a}\right) \, Log\left(\left$$

# Problem 97: Result valid but suboptimal antiderivative.

$$\int \frac{Log\left[c\left(a+bx^2\right)^p\right]^3}{x^7} \, dx$$

Optimal (type 4, 352 leaves, 17 steps):

$$\frac{b^{3} \, p^{3} \, Log\left[x\right]}{a^{3}} - \frac{b^{2} \, p^{2} \, \left(a + b \, x^{2}\right) \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]}{2 \, a^{3} \, x^{2}} - \frac{b^{3} \, p^{2} \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]}{a^{3}} - \frac{b^{3} \, p^{2} \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{4 \, a \, x^{4}} + \frac{b^{2} \, p \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{2 \, a^{3} \, x^{2}} - \frac{Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{3}}{6 \, x^{6}} - \frac{b^{3} \, p^{2} \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right] \, Log\left[1 - \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{3}} + \frac{b^{3} \, p \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2} \, Log\left[1 - \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{3}} + \frac{b^{3} \, p^{3} \, PolyLog\left[2, \, \frac{a}{a + b \, x^{2}}\right]}{2 \, a^{3}} - \frac{b^{3} \, p^{2} \, Log\left[c \, \left(a + b \, x^{2}\right)^{p}\right] \, PolyLog\left[2, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[2, \, 1 + \frac{b \, x^{2}}{a}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} + \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, PolyLog\left[3, \, \frac{a}{a + b \, x^{2}}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, Po$$

Result (type 4, 331 leaves, 22 steps):

$$\frac{b^{3} \, p^{3} \, \text{Log}\left[x\right]}{a^{3}} - \frac{b^{2} \, p^{2} \, \left(a + b \, x^{2}\right) \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]}{2 \, a^{3} \, x^{2}} - \frac{3 \, b^{3} \, p^{2} \, \text{Log}\left[-\frac{b \, x^{2}}{a}\right] \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]}{2 \, a^{3}} + \frac{b^{3} \, p \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{4 \, a^{3}} - \frac{b \, p \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{4 \, a^{3}} + \frac{b^{2} \, p \, \left(a + b \, x^{2}\right) \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{2 \, a^{3} \, x^{2}} + \frac{b^{3} \, p \, \text{Log}\left[-\frac{b \, x^{2}}{a}\right] \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{2}}{2 \, a^{3}} - \frac{b^{3} \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right]^{3}}{6 \, a^{3}} - \frac{b^{3} \, p^{3} \, \text{PolyLog}\left[2, \, 1 + \frac{b \, x^{2}}{a}\right]}{2 \, a^{3}} + \frac{b^{3} \, p^{2} \, \text{Log}\left[c \, \left(a + b \, x^{2}\right)^{p}\right] \, \text{PolyLog}\left[2, \, 1 + \frac{b \, x^{2}}{a}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, \text{PolyLog}\left[3, \, 1 + \frac{b \, x^{2}}{a}\right]}{a^{3}} - \frac{b^{3} \, p^{3} \, p^{3}$$

## Problem 163: Result valid but suboptimal antiderivative.

$$\int (fx)^{-1+3n} Log[c (d + ex^n)^p]^2 dx$$

Optimal (type 3, 372 leaves, 9 steps):

$$\frac{2\,d^{2}\,p^{2}\,x^{1-2\,n}\,\left(f\,x\right)^{-1+3\,n}}{e^{2}\,n} - \frac{d\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{2}}{2\,e^{3}\,n} + \frac{2\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{3}}{27\,e^{3}\,n} - \frac{d^{3}\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(g\,x\right)^{-1+3\,n}\,\log\left[d+e\,x^{n}\right]^{2}}{3\,e^{3}\,n} - \frac{2\,d^{2}\,p\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{2}\,\log\left[c\,\left(d+e\,x^{n}\right)^{p}\right]}{3\,e^{3}\,n} - \frac{2\,d^{2}\,p\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{2}\,\log\left[c\,\left(d+e\,x^{n}\right)^{p}\right]}{e^{3}\,n} - \frac{2\,p\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{2}\,\log\left[c\,\left(d+e\,x^{n}\right)^{p}\right]}{e^{3}\,n} - \frac{2\,d^{3}\,p\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{p}\,\left($$

Result (type 3, 278 leaves, 9 steps):

$$\frac{2\,d^{2}\,p^{2}\,x^{1-2\,n}\,\left(f\,x\right)^{-1+3\,n}}{e^{2}\,n}-\frac{d\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{2}}{2\,e^{3}\,n}+\frac{2\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(d+e\,x^{n}\right)^{3}}{27\,e^{3}\,n}-\frac{d^{3}\,p^{2}\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,Log\left[d+e\,x^{n}\right]^{2}}{3\,e^{3}\,n}-\frac{p\,x^{1-3\,n}\,\left(f\,x\right)^{-1+3\,n}\,\left(\frac{18\,d^{2}\,\left(d+e\,x^{n}\right)}{e^{3}}-\frac{9\,d\,\left(d+e\,x^{n}\right)^{3}}{e^{3}}-\frac{6\,d^{3}\,Log\left[d+e\,x^{n}\right]}{e^{3}}\right)\,Log\left[c\,\left(d+e\,x^{n}\right)^{p}\right]}{9\,n}+\frac{x\,\left(f\,x\right)^{-1+3\,n}\,Log\left[c\,\left(d+e\,x^{n}\right)^{p}\right]^{2}}{3\,n}$$

# Problem 168: Result valid but suboptimal antiderivative.

$$\int (fx)^{-1-2n} Log[c(d+ex^n)^p]^2 dx$$

Optimal (type 4, 200 leaves, 9 steps):

$$\frac{e^{2}\,p^{2}\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,Log\left[x\right]}{d^{2}}\,-\,\frac{e\,p\,x^{1+n}\,\left(f\,x\right)^{-1-2\,n}\,\left(d+e\,x^{n}\right)\,Log\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]}{d^{2}\,n}\,-\,\frac{x\,\left(f\,x\right)^{-1-2\,n}\,Log\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,Log\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{-1-2\,n}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,n}\,-\,\frac{e^{2}\,p\,x^{1+2\,n}\,\left(f\,x\right)^{\,2}\,PolyLog\left[c\,\left(d+e\,x^{n}\right)^{\,p}\right]^{\,2}}{2\,$$

Result (type 4, 238 leaves, 11 steps):

$$\frac{e^2 \ p^2 \ x^{1+2 \ n} \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \text{Log} \left[x\right]}{d^2} - \frac{e \ p \ x^{1+n} \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \left(\text{d} + e \ x^n\right) \ \text{Log} \left[c \ \left(\text{d} + e \ x^n\right)^p\right]}{d^2 \ n} - \frac{e^2 \ p \ x^{1+2 \ n} \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \text{Log} \left[c \ \left(\text{d} + e \ x^n\right)^p\right]}{d^2 \ n} - \frac{x \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \text{Log} \left[c \ \left(\text{d} + e \ x^n\right)^p\right]}{d^2 \ n} - \frac{e^2 \ p^2 \ x^{1+2 \ n} \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \text{Log} \left[c \ \left(\text{d} + e \ x^n\right)^p\right]}{d^2 \ n} - \frac{e^2 \ p^2 \ x^{1+2 \ n} \ \left(\text{f} \ x\right)^{-1-2 \ n} \ \text{PolyLog} \left[2, \ 1 + \frac{e \ x^n}{d}\right]}{d^2 \ n}$$

## Problem 408: Result valid but suboptimal antiderivative.

Optimal (type 3, 480 leaves, 8 steps):

$$\frac{5 \ b^{2} \ d^{4} \ n^{2} \ \left(d+e \sqrt{x}\right)^{2}}{2 \ e^{6}} - \frac{40 \ b^{2} \ d^{3} \ n^{2} \ \left(d+e \sqrt{x}\right)^{3}}{27 \ e^{6}} + \frac{5 \ b^{2} \ d^{2} \ n^{2} \ \left(d+e \sqrt{x}\right)^{4}}{8 \ e^{6}} - \frac{4 \ b^{2} \ d \ n^{2} \ \left(d+e \sqrt{x}\right)^{5}}{25 \ e^{6}} + \frac{b^{2} \ n^{2} \ \left(d+e \sqrt{x}\right)^{6}}{54 \ e^{6}} - \frac{4 \ b^{2} \ d^{5} \ n^{2} \ \sqrt{x}}{e^{5}} + \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+e \sqrt{x}\right]^{2}}{3 \ e^{6}} + \frac{4 \ b \ d^{5} \ n \ \left(d+e \sqrt{x}\right) \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{e^{6}} - \frac{5 \ b \ d^{4} \ n \ \left(d+e \sqrt{x}\right)^{2} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{e^{6}} - \frac{40 \ b \ d^{3} \ n \ \left(d+e \sqrt{x}\right)^{3} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{9 \ e^{6}} - \frac{5 \ b \ d^{2} \ n \ \left(d+e \sqrt{x}\right)^{4} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{2 \ e^{6}} - \frac{4 \ b \ d \ n \ \left(d+e \sqrt{x}\right)^{5} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{5 \ e^{6}} - \frac{b \ n \ \left(d+e \sqrt{x}\right)^{6} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)}{3 \ e^{6}} + \frac{1}{3} \ x^{3} \ \left(a+b \ Log \left[c \ \left(d+e \sqrt{x}\right)^{n}\right]\right)^{2}$$

### Result (type 3, 355 leaves, 8 steps):

$$\frac{5 \, b^2 \, d^4 \, n^2 \, \left(d + e \, \sqrt{x} \,\right)^2}{2 \, e^6} - \frac{40 \, b^2 \, d^3 \, n^2 \, \left(d + e \, \sqrt{x} \,\right)^3}{27 \, e^6} + \frac{5 \, b^2 \, d^2 \, n^2 \, \left(d + e \, \sqrt{x} \,\right)^4}{8 \, e^6} - \frac{4 \, b^2 \, d \, n^2 \, \left(d + e \, \sqrt{x} \,\right)^5}{54 \, e^6} - \frac{4 \, b^2 \, d^5 \, n^2 \, \sqrt{x}}{e^5} + \frac{b^2 \, d^6 \, n^2 \, \text{Log} \left[d + e \, \sqrt{x} \,\right]^2}{3 \, e^6} + \frac{1}{90} \, b \, n$$
 
$$\left( \frac{360 \, d^5 \, \left(d + e \, \sqrt{x} \,\right)}{e^6} - \frac{450 \, d^4 \, \left(d + e \, \sqrt{x} \,\right)^2}{e^6} + \frac{400 \, d^3 \, \left(d + e \, \sqrt{x} \,\right)^3}{e^6} - \frac{225 \, d^2 \, \left(d + e \, \sqrt{x} \,\right)^4}{e^6} + \frac{72 \, d \, \left(d + e \, \sqrt{x} \,\right)^5}{e^6} - \frac{10 \, \left(d + e \, \sqrt{x} \,\right)^6}{e^6} - \frac{60 \, d^6 \, \text{Log} \left[d + e \, \sqrt{x} \,\right]}{e^6} \right) \right)$$
 
$$\left( a + b \, \text{Log} \left[ c \, \left(d + e \, \sqrt{x} \,\right)^n \right] \right) + \frac{1}{3} \, x^3 \, \left( a + b \, \text{Log} \left[ c \, \left(d + e \, \sqrt{x} \,\right)^n \right] \right)^2$$

# Problem 409: Result valid but suboptimal antiderivative.

$$\int x \left(a + b \operatorname{Log}\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2} dx$$

Optimal (type 3, 342 leaves, 8 steps):

$$\frac{3 \, b^2 \, d^2 \, n^2 \, \left(d + e \, \sqrt{x}\,\right)^2}{2 \, e^4} - \frac{4 \, b^2 \, d \, n^2 \, \left(d + e \, \sqrt{x}\,\right)^3}{9 \, e^4} + \frac{b^2 \, n^2 \, \left(d + e \, \sqrt{x}\,\right)^4}{16 \, e^4} - \frac{4 \, b^2 \, d^3 \, n^2 \, \sqrt{x}}{e^3} + \frac{b^2 \, d^4 \, n^2 \, \text{Log} \left[d + e \, \sqrt{x}\,\right]^2}{2 \, e^4} + \frac{4 \, b \, d \, n \, \left(d + e \, \sqrt{x}\,\right)^3 \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{e^4} - \frac{3 \, b \, d^2 \, n \, \left(d + e \, \sqrt{x}\,\right)^2 \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{e^4} + \frac{4 \, b \, d \, n \, \left(d + e \, \sqrt{x}\,\right)^3 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{3 \, e^4} - \frac{b \, n \, \left(d + e \, \sqrt{x}\,\right)^3 \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{e^4} + \frac{1}{2} \, x^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)^2$$

Result (type 3, 263 leaves, 8 steps):

$$\frac{3 \ b^{2} \ d^{2} \ n^{2} \left(d+e \ \sqrt{x} \ \right)^{2}}{2 \ e^{4}} - \frac{4 \ b^{2} \ d \ n^{2} \left(d+e \ \sqrt{x} \ \right)^{3}}{9 \ e^{4}} + \frac{b^{2} \ n^{2} \left(d+e \ \sqrt{x} \ \right)^{4}}{16 \ e^{4}} - \frac{4 \ b^{2} \ d^{3} \ n^{2} \ \sqrt{x}}{e^{3}} + \frac{b^{2} \ d^{4} \ n^{2} \ Log \left[d+e \ \sqrt{x} \ \right]^{2}}{2 \ e^{4}} + \frac{16 \ d \left(d+e \ \sqrt{x} \ \right)^{3}}{e^{4}} - \frac{36 \ d^{2} \left(d+e \ \sqrt{x} \ \right)^{2}}{e^{4}} + \frac{16 \ d \left(d+e \ \sqrt{x} \ \right)^{3}}{e^{4}} - \frac{3 \ \left(d+e \ \sqrt{x} \ \right)^{4}}{e^{4}} - \frac{12 \ d^{4} \ Log \left[d+e \ \sqrt{x} \ \right]}{e^{4}} \left(a+b \ Log \left[c \ \left(d+e \ \sqrt{x} \ \right)^{n}\right] \right) + \frac{1}{2} x^{2} \left(a+b \ Log \left[c \ \left(d+e \ \sqrt{x} \ \right)^{n}\right] \right)^{2}$$

### Problem 412: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2}}{x^{2}} dx$$

Optimal (type 4, 155 leaves, 8 steps):

$$-\frac{2 \, b \, e \, n \, \left(d + e \, \sqrt{x}\,\right) \, \left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{d^2 \, \sqrt{x}} - \frac{2 \, b \, e^2 \, n \, Log\left[1 - \frac{d}{d + e \, \sqrt{x}}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{d^2} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\,\right)^n\right]\right)}{d^2} + \frac{b^2 \, e^2 \, n^2 \, Log\left[x\right]}{d^2} + \frac{2 \, b^2 \, e^2 \, n^2 \, PolyLog\left[2, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^2}$$

Result (type 4, 176 leaves, 10 steps):

$$-\frac{2 \text{ b e n } \left(d+e \sqrt{x}\right) \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^{n}\right]\right)}{d^{2} \sqrt{x}}+\frac{e^{2} \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^{n}\right]\right)^{2}}{d^{2}}-\frac{\left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^{n}\right]\right)^{2}}{x}-\frac{2 \text{ b } e^{2} \text{ n } \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^{n}\right]\right) \text{ Log}\left[-\frac{e \sqrt{x}}{d}\right]}{x}+\frac{b^{2} e^{2} \text{ n}^{2} \text{ Log}\left[x\right]}{d^{2}}-\frac{2 \text{ b}^{2} e^{2} \text{ n}^{2} \text{ PolyLog}\left[2, 1+\frac{e \sqrt{x}}{d}\right]}{d^{2}}$$

# Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2}}{x^{3}} dx$$

Optimal (type 4, 293 leaves, 16 steps):

$$-\frac{b^{2} e^{2} n^{2}}{6 d^{2} x} + \frac{5 b^{2} e^{3} n^{2}}{6 d^{3} \sqrt{x}} - \frac{5 b^{2} e^{4} n^{2} Log[d + e \sqrt{x}]}{6 d^{4}} - \frac{b e n \left(a + b Log[c \left(d + e \sqrt{x}\right)^{n}]\right)}{3 d x^{3/2}} + \frac{b e^{2} n \left(a + b Log[c \left(d + e \sqrt{x}\right)^{n}]\right)}{2 d^{2} x} - \frac{b e^{3} n \left(d + e \sqrt{x}\right) \left(a + b Log[c \left(d + e \sqrt{x}\right)^{n}]\right)}{d^{4} \sqrt{x}} - \frac{b e^{4} n Log[1 - \frac{d}{d + e \sqrt{x}}] \left(a + b Log[c \left(d + e \sqrt{x}\right)^{n}]\right)}{d^{4}} - \frac{\left(a + b Log[c \left(d + e \sqrt{x}\right)^{n}]\right)^{2}}{2 x^{2}} + \frac{11 b^{2} e^{4} n^{2} Log[x]}{12 d^{4}} + \frac{b^{2} e^{4} n^{2} PolyLog[2, \frac{d}{d + e \sqrt{x}}]}{d^{4}}$$

Result (type 4, 318 leaves, 18 steps):

$$\frac{b^2 \, e^2 \, n^2}{6 \, d^2 \, x} + \frac{5 \, b^2 \, e^3 \, n^2}{6 \, d^3 \, \sqrt{x}} - \frac{5 \, b^2 \, e^4 \, n^2 \, \text{Log} \big[ \, d + e \, \sqrt{x} \, \big]}{6 \, d^4} - \frac{b \, e \, n \, \big( a + b \, \text{Log} \big[ \, c \, \big( d + e \, \sqrt{x} \, \big)^n \big] \big)}{3 \, d \, x^{3/2}} + \frac{b \, e^2 \, n \, \big( a + b \, \text{Log} \big[ \, c \, \big( d + e \, \sqrt{x} \, \big)^n \big] \big)}{2 \, d^2 \, x} - \frac{b \, e^3 \, n \, \big( d + e \, \sqrt{x} \, \big) \, \big( a + b \, \text{Log} \big[ \, c \, \big( d + e \, \sqrt{x} \, \big)^n \big] \big)}{2 \, d^4} + \frac{e^4 \, \left( a + b \, \text{Log} \big[ \, c \, \big( d + e \, \sqrt{x} \, \big)^n \big] \big)^2}{2 \, d^4} - \frac{\left( a + b \, \text{Log} \big[ \, c \, \big( d + e \, \sqrt{x} \, \big)^n \big] \big)^2}{2 \, x^2} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{2 \, x^2} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyLog \big[ \, 2 \, , \, 1 + \frac{e \, \sqrt{x}}{d} \big]}{d^4} - \frac{b^2 \, e^4 \, n^2 \, PolyL$$

## Problem 414: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2}}{x^{4}} dx$$

Optimal (type 4, 408 leaves, 24 steps):

$$-\frac{b^{2} e^{2} n^{2}}{30 d^{2} x^{2}} + \frac{b^{2} e^{3} n^{2}}{10 d^{3} x^{3/2}} - \frac{47 b^{2} e^{4} n^{2}}{180 d^{4} x} + \frac{77 b^{2} e^{5} n^{2}}{90 d^{5} \sqrt{x}} - \frac{77 b^{2} e^{6} n^{2} Log \left[d + e \sqrt{x}\right]}{90 d^{6}} - \frac{2 b e^{n} \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{15 d x^{5/2}} + \frac{b e^{2} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{6 d^{2} x^{2}} - \frac{2 b e^{n} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{3 d^{4} x} - \frac{2 b e^{5} n \left(d + e \sqrt{x}\right) \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{3 d^{6} \sqrt{x}} - \frac{2 b e^{5} n \left(d + e \sqrt{x}\right) \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{3 d^{6} \sqrt{x}} - \frac{2 b e^{6} n Log \left[1 - \frac{d}{d + e \sqrt{x}}\right] \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{3 d^{6} \sqrt{x}} - \frac{2 b e^{6} n^{2} Log \left[x\right]}{3 x^{3}} + \frac{137 b^{2} e^{6} n^{2} Log \left[x\right]}{180 d^{6}} + \frac{2 b^{2} e^{6} n^{2} PolyLog \left[2 + \frac{d}{d + e \sqrt{x}}\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 x^{3}} + \frac{1 a^{2} b^{2} e^{6} n^{2} Log \left[x\right]}{180 d^{6}} + \frac{2 b^{2} e^{6} n^{2} PolyLog \left[2 + \frac{d}{d + e \sqrt{x}}\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{2 b^{2} e^{6} n^{2} PolyLog \left[2 + \frac{d}{d + e \sqrt{x}}\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{2 b^{2} e^{6} n^{2} PolyLog \left[x\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} - \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{2 b^{2} e^{6} n^{2} Log \left[x\right]}{3 d^{6}} - \frac{2 b^{2} Log \left[x\right]}{3 d^{6}} - \frac{2 b^{2}$$

#### Result (type 4, 432 leaves, 26 steps):

$$-\frac{b^{2} e^{2} n^{2}}{30 d^{2} x^{2}} + \frac{b^{2} e^{3} n^{2}}{10 d^{3} x^{3/2}} - \frac{47 b^{2} e^{4} n^{2}}{180 d^{4} x} + \frac{77 b^{2} e^{5} n^{2}}{90 d^{5} \sqrt{x}} - \frac{77 b^{2} e^{6} n^{2} Log \left[d + e \sqrt{x}\right]}{90 d^{6}} - \frac{2 b e n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{15 d x^{5/2}} + \frac{b e^{2} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{6 d^{2} x^{2}} - \frac{2 b e^{3} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{9 d^{3} x^{3/2}} + \frac{b e^{4} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)}{3 d^{4} x} - \frac{2 b e^{5} n \left(d + e \sqrt{x}\right)^{n}\right] + \frac{e^{6} \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2}}{3 d^{6}} - \frac{\left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{2}}{3 x^{3}} - \frac{2 b e^{6} n \left(a + b Log \left[c \left(d + e \sqrt{x}\right)^{n}\right]\right) - \frac{2 b e^{6} n^{2} Log \left[x\right]}{3 d^{6}} + \frac{137 b^{2} e^{6} n^{2} Log \left[x\right]}{180 d^{6}} - \frac{2 b^{2} e^{6} n^{2} PolyLog \left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{3 d^{6}}$$

### Problem 419: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{3}}{x^{2}} dx$$

Optimal (type 4, 263 leaves, 10 steps):

$$\frac{3 \, b \, e \, n \, \left(d + e \, \sqrt{x}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\right)^n\right]\right)^2}{d^2 \, \sqrt{x}} \qquad \qquad \qquad d^2 \\ \frac{\left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\right)^n\right]\right)^3}{x} + \frac{6 \, b^2 \, e^2 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, \sqrt{x}\right)^n\right]\right) \, Log\left[-\frac{e \, \sqrt{x}}{d}\right]}{d^2} + \frac{6 \, b^3 \, e^2 \, n^3 \, PolyLog\left[2, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^2} + \frac{6 \, b^3 \, e^2 \, n^3 \, PolyLog\left[2, \, \frac{1 + \frac{e \, \sqrt{x}}{d}}{d}\right]}{d^2} + \frac{6 \, b^3 \, e^2 \, n^3 \, PolyLog\left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^2}$$

Result (type 4, 283 leaves, 13 steps):

$$-\frac{3 \text{ be n } \left(d+e \sqrt{x}\right) \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right)^2}{d^2 \sqrt{x}} + \frac{e^2 \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right)^3}{d^2} - \frac{\left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right)^3}{x} + \frac{6 b^2 e^2 n^2 \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right) \text{ Log}\left[-\frac{e \sqrt{x}}{d}\right]}{d^2} - \frac{3 b e^2 n \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right)^2 \text{ Log}\left[-\frac{e \sqrt{x}}{d}\right]}{d^2} + \frac{6 b^3 e^2 n^3 \text{ PolyLog}\left[2, 1+\frac{e \sqrt{x}}{d}\right]}{d^2} - \frac{6 b^2 e^2 n^2 \left(a+b \text{ Log}\left[c \left(d+e \sqrt{x}\right)^n\right]\right) \text{ PolyLog}\left[2, 1+\frac{e \sqrt{x}}{d}\right]}{d^2} + \frac{6 b^3 e^2 n^3 \text{ PolyLog}\left[3, 1+\frac{e \sqrt{x}}{d}\right]}{d^2}$$

### Problem 420: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e \sqrt{x}\right)^{n}\right]\right)^{3}}{x^{3}} dx$$

Optimal (type 4, 573 leaves, 28 steps):

$$\frac{b^{3} \, e^{3} \, n^{3}}{2 \, d^{3} \, \sqrt{x}} + \frac{b^{3} \, e^{4} \, n^{3} \, \text{Log} \left[d + e \, \sqrt{x} \,\right]}{2 \, d^{4}} - \frac{b^{2} \, e^{2} \, n^{2} \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)}{2 \, d^{2} \, x} + \frac{5 \, b^{2} \, e^{3} \, n^{2} \, \left(d + e \, \sqrt{x} \,\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)}{2 \, d^{4} \, \sqrt{x}} + \frac{5 \, b^{2} \, e^{4} \, n^{2} \, \text{Log} \left[1 - \frac{d}{d + e \, \sqrt{x}}\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)}{2 \, d^{4} \, 2 \, x} - \frac{b \, e \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)^{2}}{2 \, d^{3} \, 2} + \frac{3 \, b \, e^{2} \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)^{2}}{4 \, d^{2} \, x} - \frac{3 \, b \, e^{4} \, n \, \text{Log} \left[1 - \frac{d}{d + e \, \sqrt{x}}\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right)^{2}}{2 \, d^{4}} - \frac{3 \, b^{2} \, e^{4} \, n^{2} \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, \sqrt{x} \,\right)^{n}\right]\right) \, \text{Log} \left[-\frac{e \, \sqrt{x}}{d}\right]}{2 \, d^{4}} - \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[2, \, \frac{d}{d + e \, \sqrt{x}}\right]}{2 \, d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[2, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog} \left[3, \, \frac{d}{d + e \, \sqrt{x}}\right]}{d^{4}} + \frac{3 \, b^{3} \, e^{4} \, n^{3} \, \text{PolyLog$$

Result (type 4, 550 leaves, 35 steps):

$$\frac{b^{3} e^{3} n^{3}}{2 d^{3} \sqrt{x}} + \frac{b^{3} e^{4} n^{3} \log\left[d + e\sqrt{x}\right]}{2 d^{4}} - \frac{b^{2} e^{2} n^{2} \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)}{2 d^{2} x} + \frac{5 b^{2} e^{3} n^{2} \left(d + e\sqrt{x}\right) \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)}{2 d^{4} \sqrt{x}} - \frac{5 b e^{4} n \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)^{2}}{2 d^{3} \sqrt{x}} + \frac{3 b e^{2} n \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)^{2}}{4 d^{2} x} - \frac{3 b e^{3} n \left(d + e\sqrt{x}\right) \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)^{2}}{2 d^{4} \sqrt{x}} + \frac{e^{4} \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)^{3}}{2 d^{4}} - \frac{\left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right)^{3}}{2 x^{2}} + \frac{11 b^{2} e^{4} n^{2} \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right) \log\left[-\frac{e\sqrt{x}}{d}\right]}{2 d^{4}} - \frac{3 b^{3} e^{4} n^{3} \log\left[x\right]}{2 d^{4}} + \frac{11 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[2, 1 + \frac{e\sqrt{x}}{d}\right]}{2 d^{4}} - \frac{3 b^{2} e^{4} n^{2} \left(a + b \log\left[c \left(d + e\sqrt{x}\right)^{n}\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{3 b^{3} e^{4} n^{3} \operatorname{PolyLog}\left[3, 1 + \frac{e\sqrt{x}}{d}\right]}{d^{4}} + \frac{$$

## Problem 429: Result valid but suboptimal antiderivative.

$$\int x^2 \left( a + b \, \text{Log} \left[ c \, \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 \, dx$$

Optimal (type 4, 404 leaves, 24 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ \sqrt{x}}{90 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x}{180 \ d^{4}} - \frac{b^{2} \ e^{3} \ n^{2} \ x^{3/2}}{10 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{2}}{30 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{\sqrt{x}}\right]}{90 \ d^{6}} + \frac{2 \ b \ e^{5} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d^{6}} - \frac{b \ e^{4} \ n \ x \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d^{4}} + \frac{2 \ b \ e^{3} \ n \ x^{3/2} \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{9 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{2} \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{15 \ d} + \frac{2 \ b \ e^{6} \ n \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{3 \ d^{6}} - \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d$$

Result (type 4, 428 leaves, 26 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ \sqrt{x}}{90 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x}{180 \ d^{4}} - \frac{b^{2} \ e^{3} \ n^{2} \ x^{3/2}}{10 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{2}}{30 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{\sqrt{x}}\right]}{90 \ d^{6}} + \frac{2 \ b \ e^{5} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d^{6}} - \frac{b \ e^{4} \ n \ x \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d^{4}} + \frac{2 \ b \ e^{3} \ n \ x^{3/2} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{9 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{2} \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{15 \ d} - \frac{e^{6} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{3 \ d^{6}} + \frac{1}{3} \ d^{6} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{180 \ d^{6}} + \frac{2 \ b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{3 \ d^{6}}$$

# Problem 430: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \, Log \left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 288 leaves, 16 steps):

$$-\frac{5 \ b^{2} \ e^{3} \ n^{2} \ \sqrt{x}}{6 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x}{6 \ d^{2}} + \frac{5 \ b^{2} \ e^{4} \ n^{2} \ Log \left[d + \frac{e}{\sqrt{x}}\right]}{6 \ d^{4}} + \frac{b \ e^{3} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{4}} - \frac{b \ e^{3} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{4}} + \frac{b \ e^{3} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{4}} + \frac{b \ e^{3} \ n \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]}{3 \ d} + \frac{b \ e^{4} \ n \ Log \left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{4}} + \frac{11 \ b^{2} \ e^{4} \ n^{2} \ Log \left[x\right]}{12 \ d^{4}} - \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^{4}}$$

Result (type 4, 311 leaves, 18 steps):

$$-\frac{5 \ b^{2} \ e^{3} \ n^{2} \sqrt{x}}{6 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x}{6 \ d^{2}} + \frac{5 \ b^{2} \ e^{4} \ n^{2} \ Log \left[d + \frac{e}{\sqrt{x}}\right]}{6 \ d^{4}} + \frac{b \ e^{3} \ n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \ \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{4}} - \frac{b \ e^{2} \ n \ x \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ d^{2}} + \frac{b \ e \ n \ x^{3/2} \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ d} - \frac{e^{4} \left(a + b \ Log \left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{2 \ d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \sqrt{x}}\right]}{d^{4}} + \frac{b^{2} \ e^{4} \ n^{2}$$

## Problem 431: Result valid but suboptimal antiderivative.

$$\int \left( a + b \, \text{Log} \left[ c \, \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 152 leaves, 9 steps):

$$\frac{2 \text{ be n } \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{2}} + \frac{2 \text{ be}^{2} \text{ n Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{2}} + \frac{2 \text{ be}^{2} \text{ n Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{d^{2}} + \frac{2 \text{ be}^{2} \text{ n Log}\left[x\right]}{d^{2}} - \frac{2 \text{ be}^{2} \text{ e}^{2} \text{ n PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^{2}}$$

Result (type 4, 174 leaves, 11 steps):

$$\frac{2 \text{ be n } \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^2} - \frac{e^2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} + x \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + 2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + 2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{$$

## Problem 434: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \, Log\left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{x^3} \, dx$$

Optimal (type 3, 341 leaves, 8 steps):

$$-\frac{3 \ b^{2} \ d^{2} \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{2}}{2 \ e^{4}} + \frac{4 \ b^{2} \ d \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{3}}{9 \ e^{4}} - \frac{b^{2} \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{4}}{16 \ e^{4}} + \frac{4 \ b^{2} \ d^{3} \ n^{2}}{e^{3} \sqrt{x}} - \frac{b^{2} \ d^{4} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{2 \ e^{4}} - \frac{4 \ b \ d^{3} \ n \left(d+\frac{e}{\sqrt{x}}\right) \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{e^{4}} + \frac{3 \ b \ d^{2} \ n \left(d+\frac{e}{\sqrt{x}}\right)^{2} \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{e^{4}} - \frac{4 \ b \ d \ n \left(d+\frac{e}{\sqrt{x}}\right)^{3} \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{3 \ e^{4}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{e^{4}} - \frac{\left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{e^{4}} - \frac{\left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[d+\frac{e}{\sqrt{x}}\right] \left(a+b \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{2 \ x^{2}} + \frac{b \ d^{4} \ n \ Log \left[c \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{2 \ x^{2}} + \frac{b \$$

Result (type 3, 263 leaves, 8 steps):

$$-\frac{3 \ b^{2} \ d^{2} \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{2}}{2 \ e^{4}} + \frac{4 \ b^{2} \ d \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{3}}{9 \ e^{4}} - \frac{b^{2} \ n^{2} \left(d+\frac{e}{\sqrt{x}}\right)^{4}}{16 \ e^{4}} + \frac{4 \ b^{2} \ d^{3} \ n^{2}}{e^{3} \sqrt{x}} - \frac{b^{2} \ d^{4} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{2 \ e^{4}} - \frac{1}{2 \ d^{4} \ Log \left[d+\frac{e}{\sqrt{x}}\right]}{e^{4}} - \frac{1}{2 \ e^{4}} - \frac{1$$

## Problem 435: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{x^4} \, dx$$

Optimal (type 3, 480 leaves, 8 steps):

$$-\frac{5 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{2}}{2 \ e^{6}} + \frac{40 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{3}}{27 \ e^{6}} - \frac{5 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{4}}{8 \ e^{6}} + \frac{4 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{5}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{54 \ e^{6}} + \frac{4 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \sqrt{x}} - \frac{b^{2} \ d^{6} \ n^{2} \ \log\left[d+\frac{e}{\sqrt{x}}\right]^{2}}{8 \ e^{6}} - \frac{4 \ b \ d^{5} \ n \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{8 \ e^{6}} + \frac{4 \ b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} + \frac{4 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \sqrt{x}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{8 \ e^{6}} + \frac{4 \ b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{n}\right]}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2}$$

Result (type 3, 355 leaves, 8 steps):

$$-\frac{5 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{2}}{2 \ e^{6}} + \frac{40 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{3}}{27 \ e^{6}} - \frac{5 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{4}}{8 \ e^{6}} + \frac{4 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{5}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{54 \ e^{6}} + \frac{4 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ \sqrt{x}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{3 \ e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{\sqrt{x}}\right]^{2}}{e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} - \frac{60 \ d^{6} \ Log \left[d+\frac{e}{\sqrt{x}}\right]}{e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)^{6}}{e^{6}} - \frac{10 \ \left(d+\frac{e}{\sqrt{x}}\right)$$

### Problem 436: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \, \text{Log} \left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^3 \, dx$$

Optimal (type 4, 569 leaves, 28 steps):

$$\frac{b^{3} \, e^{3} \, n^{3} \, \sqrt{x}}{2 \, d^{3}} - \frac{b^{3} \, e^{4} \, n^{3} \, \text{Log} \left[d + \frac{e}{\sqrt{x}}\right]}{2 \, d^{4}} - \frac{5 \, b^{2} \, e^{3} \, n^{2} \, \left(d + \frac{e}{\sqrt{x}}\right) \, \sqrt{x} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \, d^{4}} + \frac{b^{2} \, e^{2} \, n^{2} \, x \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \, d^{2}} - \frac{5 \, b^{2} \, e^{4} \, n^{2} \, \text{Log} \left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)}{2 \, d^{4}} + \frac{3 \, b \, e^{3} \, n \, \left(d + \frac{e}{\sqrt{x}}\right) \, \sqrt{x} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{2 \, d^{4}} - \frac{3 \, b \, e^{4} \, n \, \text{Log} \left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{2 \, d^{4}} + \frac{3 \, b \, e^{4} \, n \, \text{Log} \left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{2 \, d^{4}} + \frac{1}{2 \, d^{4}} + \frac{1}{$$

Result (type 4, 546 leaves, 35 steps):

$$\frac{b^{3} \, e^{3} \, n^{3} \, \sqrt{x}}{2 \, d^{3}} - \frac{b^{3} \, e^{4} \, n^{3} \, \text{Log} \left[\, d + \frac{e}{\sqrt{x}} \,\right]}{2 \, d^{4}} - \frac{5 \, b^{2} \, e^{3} \, n^{2} \, \left(\, d + \frac{e}{\sqrt{x}} \,\right) \, \sqrt{x} \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right] \,\right)}{2 \, d^{4}} + \frac{2 \, d^{2}}{2 \, d^{2}} + \frac{5 \, b^{2} \, e^{3} \, n^{2} \, \left(\, d + \frac{e}{\sqrt{x}} \,\right) \, \sqrt{x} \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right] \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b \, e^{3} \, n \, \left(\, d + \frac{e}{\sqrt{x}} \,\right) \, \sqrt{x} \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right] \,\right)^{\, 2}}{2 \, d^{4}} - \frac{3 \, b \, e^{2} \, n \, x \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right] \,\right)^{\, 2}}{4 \, d^{2}} + \frac{b \, e^{3} \, n \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} - \frac{3 \, b \, e^{3} \, n \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{b \, e^{3} \, n \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 3}}{2 \, d^{4}} + \frac{1}{2} \, x^{2} \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 3} \, - \frac{3 \, b \, e^{3} \, n \, x \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right) \,\right)^{\, 2}}{2 \, d^{4}} + \frac{11 \, b^{3} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right) \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{2} \, \left(\, a + b \, \text{Log} \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right) \,\right)^{\, 2} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog \left[\, c \, \left(\, d + \frac{e}{\sqrt{x}} \,\right)^{\, n} \,\right)^{\, 2}}{2 \, d^{4}} + \frac{3 \, b^{2} \, e^{4} \, n^{3} \, PolyLog$$

## Problem 437: Result valid but suboptimal antiderivative.

$$\int \left( a + b \log \left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^3 dx$$

Optimal (type 4, 260 leaves, 11 steps):

$$\frac{3 \text{ be n } \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{d^{2}} + \frac{3 \text{ be}^{2} \text{ n Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right)^{2}}{d^{2}} + \frac{x \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right) - \frac{6 \text{ b}^{2} \text{ e}^{2} \text{ n}^{2} \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right]\right) \text{ Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^{2}} - \frac{6 \text{ b}^{3} \text{ e}^{2} \text{ n}^{3} \text{ PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^{2}} - \frac{6 \text{ b}^{3} \text{ e}^{2} \text{ n}^{3} \text{ PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^{2}} - \frac{6 \text{ b}^{3} \text{ e}^{2} \text{ n}^{3} \text{ PolyLog}\left[3, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^{2}}$$

Result (type 4, 281 leaves, 14 steps):

$$\frac{3 \text{ be n } \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} - \frac{e^2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3}{d^2} + x \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 - \frac{6 \text{ b}^2 e^2 n^2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \text{ Log}\left[-\frac{e}{d\sqrt{x}}\right]}{d^2} + \frac{3 \text{ b} e^2 n \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 \text{ Log}\left[-\frac{e}{d\sqrt{x}}\right]}{d^2} - \frac{6 \text{ b}^3 e^2 n^3 \text{ PolyLog}\left[2, 1 + \frac{e}{d\sqrt{x}}\right]}{d^2} + \frac{6 \text{ b}^2 e^2 n^2 \left(a + b \text{ Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \text{ PolyLog}\left[2, 1 + \frac{e}{d\sqrt{x}}\right]}{d^2} - \frac{6 \text{ b}^3 e^2 n^3 \text{ PolyLog}\left[3, 1 + \frac{e}{d\sqrt{x}}\right]}{d^2}$$

## Problem 450: Result valid but suboptimal antiderivative.

$$\int \! x^2 \, \left( a + b \, \text{Log} \left[ \, c \, \left( d + e \, x^{1/3} \right)^n \, \right] \, \right)^2 \, \mathrm{d} x$$

Optimal (type 3, 680 leaves, 8 steps):

$$\begin{array}{c} -\frac{6 \, b^2 \, d^7 \, n^2 \, \left(d + e \, x^{1/3}\right)^2}{e^9} + \frac{56 \, b^2 \, d^6 \, n^2 \, \left(d + e \, x^{1/3}\right)^3}{9 \, e^9} - \frac{21 \, b^2 \, d^5 \, n^2 \, \left(d + e \, x^{1/3}\right)^4}{4 \, e^9} + \frac{84 \, b^2 \, d^4 \, n^2 \, \left(d + e \, x^{1/3}\right)^5}{25 \, e^9} \\ -\frac{14 \, b^2 \, d^3 \, n^2 \, \left(d + e \, x^{1/3}\right)^6}{9 \, e^9} + \frac{24 \, b^2 \, d^2 \, n^2 \, \left(d + e \, x^{1/3}\right)^7}{49 \, e^9} - \frac{3 \, b^2 \, d \, n^2 \, \left(d + e \, x^{1/3}\right)^8}{32 \, e^9} + \frac{2 \, b^2 \, n^2 \, \left(d + e \, x^{1/3}\right)^9}{243 \, e^9} + \frac{6 \, b^2 \, d^8 \, n^2 \, x^{1/3}}{e^8} - \frac{6 \, b \, d^8 \, n \, \left(d + e \, x^{1/3}\right)}{32 \, e^9} - \frac{6 \, b \, d^8 \, n \, \left(d + e \, x^{1/3}\right)}{32 \, e^9} + \frac{12 \, b \, d^7 \, n \, \left(d + e \, x^{1/3}\right)^9}{423 \, e^9} + \frac{6 \, b^2 \, d^8 \, n^2 \, x^{1/3}}{e^8} - \frac{6 \, b \, d^8 \, n \, \left(d + e \, x^{1/3}\right)}{32 \, e^9} - \frac{6 \, b \, d^8 \, n \, \left(d + e \, x^{1/3}\right)}{32 \, e^9} + \frac{12 \, b \, d^7 \, n \, \left(d + e \, x^{1/3}\right)^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{e^9} - \frac{56 \, b \, d^6 \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} + \frac{21 \, b \, d^5 \, n \, \left(d + e \, x^{1/3}\right)^4 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{e^9} - \frac{24 \, b \, d^4 \, n \, \left(d + e \, x^{1/3}\right)^5 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} + \frac{28 \, b \, d^3 \, n \, \left(d + e \, x^{1/3}\right)^6 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{24 \, b \, d^2 \, n \, \left(d + e \, x^{1/3}\right)^7 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^7 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^7 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{32 \, e^9} - \frac{2b \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Lo$$

Result (type 3, 491 leaves, 8 steps):

$$-\frac{6\ b^{2}\ d^{7}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{2}}{e^{9}}+\frac{56\ b^{2}\ d^{6}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{3}}{9\ e^{9}}-\frac{21\ b^{2}\ d^{5}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{4}}{4\ e^{9}}+\frac{84\ b^{2}\ d^{4}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{5}}{25\ e^{9}}-\frac{14\ b^{2}\ d^{3}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{6}}{9\ e^{9}}+\frac{24\ b^{2}\ d^{2}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{9}}{25\ e^{9}}+\frac{2b^{2}\ d^{9}\ n^{2}\ Log\left[d+e\ x^{1/3}\right]^{2}}{243\ e^{9}}+\frac{2b^{2}\ n^{2}\ \left(d+e\ x^{1/3}\right)^{9}}{243\ e^{9}}+\frac{6\ b^{2}\ d^{8}\ n^{2}\ x^{1/3}}{e^{8}}-\frac{b^{2}\ d^{9}\ n^{2}\ Log\left[d+e\ x^{1/3}\right]^{2}}{3\ e^{9}}-\frac{1}{3780}$$

$$b\ n\ \left(\frac{22\ 680\ d^{8}\ \left(d+e\ x^{1/3}\right)}{e^{9}}-\frac{45\ 360\ d^{7}\ \left(d+e\ x^{1/3}\right)^{2}}{e^{9}}+\frac{70\ 560\ d^{6}\ \left(d+e\ x^{1/3}\right)^{3}}{e^{9}}-\frac{79\ 380\ d^{5}\ \left(d+e\ x^{1/3}\right)^{4}}{e^{9}}+\frac{63\ 504\ d^{4}\ \left(d+e\ x^{1/3}\right)^{5}}{e^{9}}-\frac{2520\ d^{9}\ Log\left[d+e\ x^{1/3}\right]}{e^{9}}-\frac{3780}{e^{9}}$$

$$\left(a+b\ Log\left[c\ \left(d+e\ x^{1/3}\right)^{n}\right]\right)+\frac{1}{3}\ x^{3}\ \left(a+b\ Log\left[c\ \left(d+e\ x^{1/3}\right)^{n}\right]\right)^{2}$$

## Problem 451: Result valid but suboptimal antiderivative.

$$\int x \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2} dx$$

### Optimal (type 3, 480 leaves, 8 steps):

$$\frac{15 \, b^2 \, d^4 \, n^2 \, \left(d + e \, x^{1/3}\right)^2}{4 \, e^6} = \frac{20 \, b^2 \, d^3 \, n^2 \, \left(d + e \, x^{1/3}\right)^3}{9 \, e^6} + \frac{15 \, b^2 \, d^2 \, n^2 \, \left(d + e \, x^{1/3}\right)^4}{16 \, e^6} = \frac{6 \, b^2 \, d \, n^2 \, \left(d + e \, x^{1/3}\right)^5}{25 \, e^6} + \frac{b^2 \, n^2 \, \left(d + e \, x^{1/3}\right)^6}{36 \, e^6} - \frac{6 \, b^2 \, d^5 \, n^2 \, x^{1/3}}{e^5} + \frac{b^2 \, d^6 \, n^2 \, Log \left[d + e \, x^{1/3}\right]^2}{2 \, e^6} + \frac{6 \, b \, d^5 \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{e^6} - \frac{15 \, b \, d^4 \, n \, \left(d + e \, x^{1/3}\right)^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{2 \, e^6} + \frac{20 \, b \, d^3 \, n \, \left(d + e \, x^{1/3}\right)^3 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{3 \, e^6} - \frac{15 \, b \, d^2 \, n \, \left(d + e \, x^{1/3}\right)^4 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{4 \, e^6} + \frac{6 \, b \, d \, n \, \left(d + e \, x^{1/3}\right)^5 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{5 \, e^6} - \frac{b \, n \, \left(d + e \, x^{1/3}\right)^6 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{6 \, e^6} - \frac{b \, d^6 \, n \, Log \left[d + e \, x^{1/3}\right] \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} - \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{e^6} + \frac{1}{2} \, x^2 \, \left(a + b \, Log \left[c$$

Result (type 3, 355 leaves, 8 steps):

$$\frac{15 \ b^2 \ d^4 \ n^2 \ \left(d + e \ x^{1/3}\right)^2}{4 \ e^6} - \frac{20 \ b^2 \ d^3 \ n^2 \ \left(d + e \ x^{1/3}\right)^3}{9 \ e^6} + \frac{15 \ b^2 \ d^2 \ n^2 \ \left(d + e \ x^{1/3}\right)^4}{16 \ e^6} - \frac{6 \ b^2 \ d \ n^2 \ \left(d + e \ x^{1/3}\right)^6}{25 \ e^6} + \frac{b^2 \ n^2 \ \left(d + e \ x^{1/3}\right)^6}{36 \ e^6} - \frac{6 \ b^2 \ d^5 \ n^2 \ x^{1/3}}{e^5} + \frac{b^2 \ d^6 \ n^2 \ Log \left[d + e \ x^{1/3}\right]^2}{2 \ e^6} + \frac{1}{60} \ b \ n \\ \left(\frac{360 \ d^5 \ \left(d + e \ x^{1/3}\right)}{e^6} - \frac{450 \ d^4 \ \left(d + e \ x^{1/3}\right)^2}{e^6} + \frac{400 \ d^3 \ \left(d + e \ x^{1/3}\right)^3}{e^6} - \frac{225 \ d^2 \ \left(d + e \ x^{1/3}\right)^4}{e^6} + \frac{72 \ d \ \left(d + e \ x^{1/3}\right)^5}{e^6} - \frac{10 \ \left(d + e \ x^{1/3}\right)^6}{e^6} - \frac{60 \ d^6 \ Log \left[d + e \ x^{1/3}\right]}{e^6} \right) \\ \left(a + b \ Log \left[c \ \left(d + e \ x^{1/3}\right)^n\right]\right) + \frac{1}{2} \ x^2 \ \left(a + b \ Log \left[c \ \left(d + e \ x^{1/3}\right)^n\right]\right)^2$$

### Problem 452: Result valid but suboptimal antiderivative.

$$\int (a + b \log [c (d + e x^{1/3})^n])^2 dx$$

#### Optimal (type 3, 267 leaves, 8 steps):

$$-\frac{3 \ b^{2} \ d \ n^{2} \ \left(d+e \ x^{1/3}\right)^{2}}{2 \ e^{3}} + \frac{2 \ b^{2} \ n^{2} \ \left(d+e \ x^{1/3}\right)^{3}}{9 \ e^{3}} + \frac{6 \ b^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{2}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{6 \ b^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{6 \ b^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+e \ x^{1/$$

#### Result (type 3, 210 leaves, 8 steps):

$$-\frac{3 \ b^{2} \ d \ n^{2} \ \left(d+e \ x^{1/3}\right)^{2}}{2 \ e^{3}} + \frac{2 \ b^{2} \ n^{2} \ \left(d+e \ x^{1/3}\right)^{3}}{9 \ e^{3}} + \frac{6 \ b^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{2}} - \frac{b^{2} \ d^{3} \ n^{2} \ Log\left[d+e \ x^{1/3}\right]^{2}}{e^{3}} - \frac{1}{8 \ b^{2} \ \left(d+e \ x^{1/3}\right)^{2}}{e^{3}} - \frac{9 \ d \ \left(d+e \ x^{1/3}\right)^{2}}{e^{3}} + \frac{2 \ \left(d+e \ x^{1/3}\right)^{3}}{e^{3}} - \frac{6 \ d^{3} \ Log\left[d+e \ x^{1/3}\right]}{e^{3}} \right) \ \left(a+b \ Log\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right) + x \ \left(a+b \ Log\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right)^{2} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} - \frac{1}{8 \ d^{2} \ Log\left[d+e \ x^{1/3}\right]}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} - \frac{1}{8 \ d^{2} \ Log\left[d+e \ x^{1/3}\right]}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ d^{2} \ n^{2} \ x^{1/3}}{e^{3}} + \frac{1}{8 \ d^{2} \ n^{2} \ n^{$$

# Problem 454: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[\, c \, \left(d+e \, x^{1/3}\right)^{\, n}\,\right]\,\right)^{\, 2}}{x^2} \, \mathrm{d}x$$

Optimal (type 4, 231 leaves, 12 steps):

$$-\frac{b^{2} \, e^{2} \, n^{2}}{d^{2} \, x^{1/3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[d + e \, x^{1/3}\right]}{d^{3}} - \frac{b \, e \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{d \, x^{2/3}} + \frac{2 \, b \, e^{3} \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{d^{3} \, x^{1/3}} + \frac{2 \, b \, e^{3} \, n \, Log\left[1 - \frac{d}{d + e \, x^{1/3}}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{d^{3}} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)^{2}}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{2 \, b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{1/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{1/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, Log\left[x\right$$

#### Result (type 4, 253 leaves, 14 steps):

$$-\frac{b^{2} e^{2} n^{2}}{d^{2} x^{1/3}} + \frac{b^{2} e^{3} n^{2} Log \left[d + e x^{1/3}\right]}{d^{3}} - \frac{b e n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{d x^{2/3}} + \\ \frac{2 b e^{2} n \left(d + e x^{1/3}\right) \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{d^{3} x^{1/3}} - \frac{e^{3} \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2}}{d^{3}} - \frac{\left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2}}{x} + \\ \frac{2 b e^{3} n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right) Log \left[-\frac{e x^{1/3}}{d}\right]}{d^{3}} - \frac{b^{2} e^{3} n^{2} Log \left[x\right]}{d^{3}} + \frac{2 b^{2} e^{3} n^{2} PolyLog \left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^{3}}$$

### Problem 455: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, Log\left[c\, \left(d+e\, x^{1/3}\right)^n\right]\right)^2}{x^3} \, dx$$

#### Optimal (type 4, 405 leaves, 24 steps):

$$-\frac{b^{2} \, e^{2} \, n^{2}}{20 \, d^{2} \, x^{4/3}} + \frac{3 \, b^{2} \, e^{3} \, n^{2}}{20 \, d^{3} \, x} - \frac{47 \, b^{2} \, e^{4} \, n^{2}}{120 \, d^{4} \, x^{2/3}} + \frac{77 \, b^{2} \, e^{5} \, n^{2}}{60 \, d^{5} \, x^{1/3}} - \frac{77 \, b^{2} \, e^{6} \, n^{2} \, \text{Log} \left[d + e \, x^{1/3}\right]}{60 \, d^{6}} - \frac{b \, e^{n} \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{5 \, d \, x^{5/3}} + \frac{b \, e^{2} \, n \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{4 \, d^{2} \, x^{4/3}} - \frac{b \, e^{5} \, n \, \left(d + e \, x^{1/3}\right)^{n}\right]}{2 \, d^{4} \, x^{2/3}} - \frac{b \, e^{5} \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{d^{6} \, x^{1/3}} - \frac{b \, e^{5} \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^{n}\right]\right)}{d^{6} \, x^{1/3}} - \frac{b^{2} \, e^{6} \, n^{2} \, \text{Log} \left[x\right]}{2 \, x^{2}} + \frac{137 \, b^{2} \, e^{6} \, n^{2} \, \text{Log} \left[x\right]}{180 \, d^{6}} + \frac{b^{2} \, e^{6} \, n^{2} \, \text{PolyLog} \left[2, \, \frac{d}{d + e \, x^{1/3}}\right]}{d^{6}}$$

#### Result (type 4, 430 leaves, 26 steps):

$$-\frac{b^{2} e^{2} n^{2}}{20 d^{2} x^{4/3}} + \frac{3 b^{2} e^{3} n^{2}}{20 d^{3} x} - \frac{47 b^{2} e^{4} n^{2}}{120 d^{4} x^{2/3}} + \frac{77 b^{2} e^{5} n^{2}}{60 d^{5} x^{1/3}} - \frac{77 b^{2} e^{6} n^{2} Log \left[d + e x^{1/3}\right]}{60 d^{6}} - \frac{b e n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{5 d x^{5/3}} + \frac{b e^{2} n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{4 d^{2} x^{4/3}} - \frac{b e^{3} n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{3 d^{3} x} + \frac{b e^{4} n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{2 d^{4} x^{2/3}} - \frac{b e^{5} n \left(d + e x^{1/3}\right)^{n}\right] \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)}{d^{6} x^{1/3}} + \frac{e^{6} \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2}}{2 d^{6}} - \frac{\left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2}}{2 x^{2}} - \frac{b e^{6} n \left(a + b Log \left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{2}}{2 x^{2}} - \frac{b^{2} e^{6} n^{2} PolyLog \left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^{6}}$$

### Problem 461: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e x^{1/3}\right)^n\right]\right)^3}{x^2} \, dx$$

Optimal (type 4, 439 leaves, 17 steps):

$$-\frac{3 \ b^{2} \ e^{2} \ n^{2} \ \left(d+e \ x^{1/3}\right) \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right)}{d^{3} \ x^{1/3}} - \frac{3 \ b^{2} \ e^{3} \ n^{2} \ \text{Log}\left[1-\frac{d}{d+e \ x^{1/3}}\right] \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right)}{d^{3}} - \frac{3 \ b \ e^{n} \ \left(d+e \ x^{1/3}\right)^{n}\right)^{2}}{2 \ d \ x^{2/3}} + \frac{3 \ b \ e^{2} \ n \ \left(d+e \ x^{1/3}\right) \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right)^{2}}{d^{3} \ x^{1/3}} + \frac{3 \ b \ e^{3} \ n \ \text{Log}\left[1-\frac{d}{d+e \ x^{1/3}}\right] \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right)^{2}}{d^{3}} - \frac{6 \ b^{2} \ e^{3} \ n^{2} \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right) \ \text{Log}\left[-\frac{e \ x^{1/3}}{d}\right]}{d^{3}} + \frac{b^{3} \ e^{3} \ n^{3} \ \text{Log}\left[x\right]}{d^{3}} + \frac{3 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[2, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{2} \ e^{3} \ n^{2} \ \left(a+b \ \text{Log}\left[c \ \left(d+e \ x^{1/3}\right)^{n}\right]\right) \ \text{PolyLog}\left[2, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[2, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{d^{3}} - \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+e \ x^{1/3}}\right]}{$$

Result (type 4, 414 leaves, 22 steps):

$$-\frac{3 \, b^2 \, e^2 \, n^2 \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{d^3 \, x^{1/3}} + \frac{3 \, b \, e^3 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3} - \frac{3 \, b \, e n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d \, x^{2/3}} + \frac{3 \, b \, e^2 \, n \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{d^3 \, x^{1/3}} - \frac{e^3 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^3}{d^3 \, x^{1/3}} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^3}{x} - \frac{9 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^3}{d^3} + \frac{3 \, b \, e^3 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2 \, Log\left[-\frac{e \, x^{1/3}}{d}\right]}{x} + \frac{b^3 \, e^3 \, n^3 \, Log\left[x\right]}{d^3} - \frac{9 \, b^3 \, e^3 \, n^3 \, PolyLog\left[2, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} + \frac{6 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right) \, PolyLog\left[2, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3}}{d}\right]}{d^3} - \frac{6 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{1/3$$

### Problem 462: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \log\left[c \left(d + e x^{1/3}\right)^{n}\right]\right)^{3}}{x^{3}} dx$$

Optimal (type 4, 765 leaves, 62 steps):

$$\frac{b^3 \, e^3 \, n^3}{20 \, d^3 \, x} + \frac{3 \, b^3 \, e^4 \, n^3}{10 \, d^4 \, x^{2/3}} - \frac{71 \, b^3 \, e^5 \, n^3}{40 \, d^5 \, x^{1/3}} + \frac{71 \, b^3 \, e^6 \, n^3 \, \log \left[d + e \, x^{1/3}\right]}{40 \, d^6} - \frac{3 \, b^2 \, e^2 \, n^2 \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{20 \, d^2 \, x^{4/3}} + \frac{9 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{40 \, d^4 \, x^{2/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{20 \, d^6 \, x^{1/3}} + \frac{3 \, b \, e^4 \, n \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{8 \, d^2 \, x^{4/3}} - \frac{b \, e^3 \, n \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^4 \, n \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{8 \, d^2 \, x^{4/3}} - \frac{b \, e^3 \, n \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^5 \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^5 \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^5 \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^5 \, n \, \left(d + e \, x^{1/3}\right) \, \left(a + b \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^3 \, x} + \frac{3 \, b \, e^6 \, n \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^6 \, x^{1/3}} + \frac{3 \, b^2 \, e^6 \, n^2 \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)^2}{2 \, d^6 \, x^{1/3}} + \frac{3 \, b^2 \, e^6 \, n^3 \, \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right) \log \left[-\frac{e \, x^{1/3}}{d}\right]}{2 \, d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, \log \left[x\right]}{2 \, d^6} + \frac{77 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]}{d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]}{d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]}{d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3}\right)^n\right]}{d^6} + \frac{3 \, b^3 \, e^6 \, n^3 \, poly \log \left[c \, \left(d + e \, x^{1/3$$

#### Result (type 4, 742 leaves, 73 steps):

$$\frac{b^3 \, e^3 \, n^3}{20 \, d^3 \, x} + \frac{3 \, b^3 \, e^4 \, n^3}{10 \, d^4 \, x^{2/3}} - \frac{71 \, b^3 \, e^5 \, n^3}{40 \, d^5 \, x^{1/3}} + \frac{71 \, b^3 \, e^6 \, n^3 \, \text{Log} \left[d + e \, x^{1/3}\right]}{40 \, d^6} - \frac{3 \, b^2 \, e^2 \, n^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{20 \, d^2 \, x^{4/3}} + \frac{9 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + e \, x^{1/3}\right)^n\right]\right)}{40 \, d^4 \, x^{2/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{77 \, b^2 \, e^5 \, n^2 \, \left(d + e \, x^{1/3}\right)}{20 \, d^6 \, x^{1/3}} + \frac{9 \, b^2 \, n^3 \, \left(a + b \, b \, b \, b \, b \, \left(d + e \, x^{1/3}\right)^n\right)}{20 \, d^6 \, x^{1/3}} + \frac{9 \, b^2 \, n^3 \, n^3 \, \left(a + b \, b \, b \, b \, c^3 \, n^3 \, \left(a + b \, b \, b \, c^3 \, n^3 \, \left(a + b \, b \, b \, c^3 \, n^3 \, \left(a + b \, b \, b \, c^3 \,$$

# Problem 471: Result valid but suboptimal antiderivative.

$$\int x^3 \left(a + b \operatorname{Log}\left[c \left(d + e x^{2/3}\right)^n\right]\right)^2 dx$$

Optimal (type 3, 482 leaves, 8 steps):

$$\frac{15 \, b^2 \, d^4 \, n^2 \, \left(d + e \, x^{2/3}\right)^2}{8 \, e^6} - \frac{10 \, b^2 \, d^3 \, n^2 \, \left(d + e \, x^{2/3}\right)^3}{9 \, e^6} + \frac{15 \, b^2 \, d^2 \, n^2 \, \left(d + e \, x^{2/3}\right)^4}{32 \, e^6} - \frac{3 \, b^2 \, d \, n^2 \, \left(d + e \, x^{2/3}\right)^5}{e^5} + \frac{b^2 \, d^6 \, n^2 \, x^{2/3}}{4 \, e^6} + \frac{3 \, b \, d^5 \, n \, \left(d + e \, x^{2/3}\right)^3 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{e^6} - \frac{15 \, b \, d^4 \, n \, \left(d + e \, x^{2/3}\right)^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{4 \, e^6} + \frac{10 \, b \, d^3 \, n \, \left(d + e \, x^{2/3}\right)^3 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{3 \, e^6} - \frac{15 \, b \, d^2 \, n \, \left(d + e \, x^{2/3}\right)^4 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{8 \, e^6} + \frac{3 \, b \, d \, n \, \left(d + e \, x^{2/3}\right)^3 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{5 \, e^6} - \frac{b \, n \, \left(d + e \, x^{2/3}\right)^6 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{12 \, e^6} - \frac{b \, d^6 \, n \, Log\left[d + e \, x^{2/3}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{2 \, e^6} + \frac{1}{4} \, x^4 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{2 \, e^6}$$

#### Result (type 3, 355 leaves, 8 steps):

$$\frac{15 \ b^2 \ d^4 \ n^2 \ \left(d + e \ x^{2/3}\right)^2}{8 \ e^6} - \frac{10 \ b^2 \ d^3 \ n^2 \ \left(d + e \ x^{2/3}\right)^3}{9 \ e^6} + \frac{15 \ b^2 \ d^2 \ n^2 \ \left(d + e \ x^{2/3}\right)^4}{32 \ e^6} - \frac{3 \ b^2 \ d \ n^2 \ \left(d + e \ x^{2/3}\right)^6}{25 \ e^6} + \frac{b^2 \ n^2 \ \left(d + e \ x^{2/3}\right)^6}{72 \ e^6} - \frac{3 \ b^2 \ d^5 \ n^2 \ x^{2/3}}{e^5} + \frac{b^2 \ d^6 \ n^2 \ Log \left[d + e \ x^{2/3}\right]^2}{4 \ e^6} + \frac{1}{120} \ b \ n \\ \left(\frac{360 \ d^5 \ \left(d + e \ x^{2/3}\right)}{e^6} - \frac{450 \ d^4 \ \left(d + e \ x^{2/3}\right)^2}{e^6} + \frac{400 \ d^3 \ \left(d + e \ x^{2/3}\right)^3}{e^6} - \frac{225 \ d^2 \ \left(d + e \ x^{2/3}\right)^4}{e^6} + \frac{72 \ d \ \left(d + e \ x^{2/3}\right)^5}{e^6} - \frac{10 \ \left(d + e \ x^{2/3}\right)^6}{e^6} - \frac{60 \ d^6 \ Log \left[d + e \ x^{2/3}\right]}{e^6} \right)^2 \left(a + b \ Log \left[c \ \left(d + e \ x^{2/3}\right)^n\right]\right)^2$$

### Problem 472: Result valid but suboptimal antiderivative.

$$\left\lceil x \, \left( a + b \, \text{Log} \left[ \, c \, \left( d + e \, x^{2/3} \right)^{\, n} \, \right] \, \right)^{\, 2} \, \text{d} x \right.$$

Optimal (type 3, 275 leaves, 8 steps):

$$-\frac{3 \frac{b^2 d n^2 \left(d+e x^{2/3}\right)^2}{4 e^3}+\frac{b^2 n^2 \left(d+e x^{2/3}\right)^3}{9 e^3}+\frac{3 \frac{b^2 d^2 n^2 x^{2/3}}{e^2}-\frac{b^2 d^3 n^2 Log \left[d+e x^{2/3}\right]^2}{2 e^3}-\frac{3 b d^2 n \left(d+e x^{2/3}\right) \left(a+b Log \left[c \left(d+e x^{2/3}\right)^n\right]\right)}{e^3}+\frac{3 b d n \left(d+e x^{2/3}\right)^2 \left(a+b Log \left[c \left(d+e x^{2/3}\right)^n\right]\right)}{2 e^3}-\frac{b n \left(d+e x^{2/3}\right)^3 \left(a+b Log \left[c \left(d+e x^{2/3}\right)^n\right]\right)}{3 e^3}+\frac{b d^3 n Log \left[d+e x^{2/3}\right] \left(a+b Log \left[c \left(d+e x^{2/3}\right)^n\right]\right)}{e^3}+\frac{1}{2} x^2 \left(a+b Log \left[c \left(d+e x^{2/3}\right)^n\right]\right)^2}{e^3}$$

Result (type 3, 217 leaves, 8 steps):

$$-\frac{3 \ b^2 \ d \ n^2 \ \left(d+e \ x^{2/3}\right)^2}{4 \ e^3} + \frac{b^2 \ n^2 \ \left(d+e \ x^{2/3}\right)^3}{9 \ e^3} + \frac{3 \ b^2 \ d^2 \ n^2 \ x^{2/3}}{e^2} - \frac{b^2 \ d^3 \ n^2 \ Log \left[d+e \ x^{2/3}\right]^2}{2 \ e^3} - \frac{1}{6} \ b \ n \left(\frac{18 \ d^2 \ \left(d+e \ x^{2/3}\right)}{e^3} - \frac{9 \ d \ \left(d+e \ x^{2/3}\right)^2}{e^3} + \frac{2 \ \left(d+e \ x^{2/3}\right)^3}{e^3} - \frac{6 \ d^3 \ Log \left[d+e \ x^{2/3}\right]}{e^3}\right) \left(a+b \ Log \left[c \ \left(d+e \ x^{2/3}\right)^n\right]\right) + \frac{1}{2} \ x^2 \ \left(a+b \ Log \left[c \ \left(d+e \ x^{2/3}\right)^n\right]\right)^2$$

## Problem 474: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \ Log\left[c \ \left(d+e \ x^{2/3}\right)^n\right]\right)^2}{x^3} \, \mathrm{d}x$$

#### Optimal (type 4, 238 leaves, 12 steps):

$$-\frac{b^{2} \, e^{2} \, n^{2}}{2 \, d^{2} \, x^{2/3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[d + e \, x^{2/3}\right]}{2 \, d^{3}} - \frac{b \, e \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^{n}\right]\right)}{2 \, d \, x^{4/3}} + \frac{b \, e^{2} \, n \, \left(d + e \, x^{2/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^{n}\right]\right)}{d^{3} \, x^{2/3}} + \frac{b \, e^{3} \, n \, Log\left[1 - \frac{d}{d + e \, x^{2/3}}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^{n}\right]\right)}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, Log\left[x\right]}{d^{3}} - \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d + e \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d \, x^{2/3}}\right]}{d^{3}} + \frac{b^{2} \, e^{3} \, n^{2} \, PolyLog\left[2, \, \frac{d}{d \, x^{2/3}}\right]}{$$

#### Result (type 4, 261 leaves, 14 steps):

$$-\frac{b^{2} e^{2} n^{2}}{2 d^{2} x^{2/3}} + \frac{b^{2} e^{3} n^{2} Log \left[d + e x^{2/3}\right]}{2 d^{3}} - \frac{b e n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{2 d x^{4/3}} + \\ \frac{b e^{2} n \left(d + e x^{2/3}\right) \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{d^{3} x^{2/3}} - \frac{e^{3} \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)^{2}}{2 d^{3}} - \frac{\left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)^{2}}{2 x^{2}} + \\ \frac{b e^{3} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right) Log \left[-\frac{e x^{2/3}}{d}\right]}{d^{3}} - \frac{b^{2} e^{3} n^{2} Log \left[x\right]}{d^{3}} + \frac{b^{2} e^{3} n^{2} PolyLog \left[2, 1 + \frac{e x^{2/3}}{d}\right]}{d^{3}}$$

# Problem 475: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \text{Log}\left[c\, \left(d+e\, x^{2/3}\right)^n\right]\right)^2}{x^5} \, dx$$

Optimal (type 4, 412 leaves, 24 steps):

$$-\frac{b^{2} e^{2} n^{2}}{40 d^{2} x^{8/3}} + \frac{3 b^{2} e^{3} n^{2}}{40 d^{3} x^{2}} - \frac{47 b^{2} e^{4} n^{2}}{240 d^{4} x^{4/3}} + \frac{77 b^{2} e^{5} n^{2}}{120 d^{5} x^{2/3}} - \frac{77 b^{2} e^{6} n^{2} Log \left[d + e x^{2/3}\right]}{120 d^{6}} - \frac{b e n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{10 d x^{10/3}} + \frac{b e^{2} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{8 d^{2} x^{8/3}} - \frac{b e^{3} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{4 d^{4} x^{4/3}} - \frac{b e^{5} n \left(d + e x^{2/3}\right)\left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{2 d^{6} x^{2/3}} - \frac{b e^{5} n \left(d + e x^{2/3}\right)\left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{2 d^{6} x^{2/3}} - \frac{b e^{6} n^{2} Log \left[x\right]}{4 x^{4}} + \frac{137 b^{2} e^{6} n^{2} Log \left[x\right]}{180 d^{6}} + \frac{b^{2} e^{6} n^{2} PolyLog \left[2, \frac{d}{d + e x^{2/3}}\right]}{2 d^{6}}$$

#### Result (type 4, 436 leaves, 26 steps):

$$-\frac{b^{2} e^{2} n^{2}}{40 d^{2} x^{8/3}} + \frac{3 b^{2} e^{3} n^{2}}{40 d^{3} x^{2}} - \frac{47 b^{2} e^{4} n^{2}}{240 d^{4} x^{4/3}} + \frac{77 b^{2} e^{5} n^{2}}{120 d^{5} x^{2/3}} - \frac{77 b^{2} e^{6} n^{2} Log \left[d + e x^{2/3}\right]}{120 d^{6}} - \frac{b e n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{10 d x^{10/3}} + \frac{b e^{2} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{8 d^{2} x^{8/3}} - \frac{b e^{3} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{6 d^{3} x^{2}} + \frac{b e^{4} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{4 d^{4} x^{4/3}} - \frac{b e^{5} n \left(d + e x^{2/3}\right) \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)}{2 d^{6} x^{2/3}} + \frac{e^{6} \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)^{2}}{4 d^{6}} - \frac{\left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)^{2}}{4 x^{4}} - \frac{b e^{6} n \left(a + b Log \left[c \left(d + e x^{2/3}\right)^{n}\right]\right)^{2}}{4 d^{6}} - \frac{b^{2} e^{6} n^{2} PolyLog \left[2, 1 + \frac{e x^{2/3}}{d}\right]}{2 d^{6}}$$

### Problem 484: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \ Log\left[c \ \left(d+e \ x^{2/3}\right)^n\right]\right)^3}{v^3} \ \mathrm{d}x$$

Optimal (type 4, 451 leaves, 17 steps):

$$\frac{3 \, b^2 \, e^2 \, n^2 \, \left(d + e \, x^{2/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{2 \, d^3 \, x^{2/3}} - \frac{3 \, b^2 \, e^3 \, n^2 \, Log\left[1 - \frac{d}{d + e \, x^{2/3}}\right] \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{2 \, d^3 \, x^{2/3}} - \frac{3 \, b \, e^3 \, n \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{2 \, d^3 \, x^{2/3}} + \frac{3 \, b \, e^3 \, n \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{2 \, d^3} - \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right) \, Log\left[-\frac{e \, x^{2/3}}{d}\right]}{2 \, d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, Log\left[x\right]}{2 \, d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[2, \frac{d}{d + e \, x^{2/3}}\right]}{2 \, d^3} - \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right) \, PolyLog\left[2, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[2, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d + e \, x^{2/3}}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \frac{d}{d \, a \, b^3}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, Pol$$

Result (type 4, 428 leaves, 22 steps):

$$\frac{3 \, b^2 \, e^2 \, n^2 \, \left(d + e \, x^{2/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)}{2 \, d^3 \, x^{2/3}} + \frac{3 \, b \, e^3 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{4 \, d^3} - \frac{3 \, b \, e \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{4 \, d \, x^{4/3}} + \frac{3 \, b \, e^2 \, n \, \left(d + e \, x^{2/3}\right) \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2}{2 \, d^3 \, x^{2/3}} - \frac{e^3 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^3}{2 \, d^3} - \frac{\left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^3}{2 \, x^2} - \frac{9 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^3}{2 \, d^3} + \frac{3 \, b \, e^3 \, n \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right)^2 \, Log\left[-\frac{e \, x^{2/3}}{d}\right]}{2 \, d^3} + \frac{b^3 \, e^3 \, n^3 \, Log\left[x\right]}{d^3} - \frac{9 \, b^3 \, e^3 \, n^3 \, PolyLog\left[2, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, Log\left[c \, \left(d + e \, x^{2/3}\right)^n\right]\right) \, PolyLog\left[2, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, PolyLog\left[3, \, 1 + \frac{e \, x^{2/3}}{d}\right]}{$$

### Problem 497: Result valid but suboptimal antiderivative.

$$\int \! x^2 \, \left( a + b \, \text{Log} \left[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \, \right] \, \right)^2 \, \text{d}x$$

#### Optimal (type 4, 572 leaves, 36 steps):

$$\frac{481 \, b^2 \, e^8 \, n^2 \, x^{1/3}}{420 \, d^8} - \frac{341 \, b^2 \, e^7 \, n^2 \, x^{2/3}}{840 \, d^7} + \frac{743 \, b^2 \, e^6 \, n^2 \, x}{3780 \, d^6} - \frac{533 \, b^2 \, e^5 \, n^2 \, x^{4/3}}{5040 \, d^5} + \frac{73 \, b^2 \, e^4 \, n^2 \, x^{5/3}}{1260 \, d^4} - \frac{5 \, b^2 \, e^3 \, n^2 \, x^2}{168 \, d^3} + \frac{b^2 \, e^2 \, n^2 \, x^{7/3}}{84 \, d^2} - \frac{481 \, b^2 \, e^9 \, n^2 \, \text{Log} \left[d + \frac{e}{x^{1/3}}\right]}{420 \, d^9} - \frac{2 \, b \, e^8 \, n \, \left(d + \frac{e}{x^{1/3}}\right) \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 \, d^9} + \frac{b \, e^7 \, n \, x^{2/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 \, d^7} - \frac{2 \, b \, e^6 \, n \, x \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 \, d^9} + \frac{b \, e^5 \, n \, x^{4/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{6 \, d^5} + \frac{2 \, b \, e^4 \, n \, x^{5/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{15 \, d^4} + \frac{b \, e^n \, n \, x^2 \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{21 \, d^2} + \frac{b \, e^n \, n \, x^{8/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b \, e^n \, n \, x^{8/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} + \frac{b \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} + \frac{b \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 \, d} - \frac{2 \, b^2 \, e^n \, n \, x^{1/3} \, \left(a + b \, \text{Log} \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]$$

Result (type 4, 596 leaves, 38 steps):

$$\frac{481 \, b^2 \, e^8 \, n^2 \, x^{1/3}}{420 \, d^8} - \frac{341 \, b^2 \, e^7 \, n^2 \, x^{2/3}}{840 \, d^7} + \frac{743 \, b^2 \, e^6 \, n^2 \, x}{3780 \, d^6} - \frac{533 \, b^2 \, e^5 \, n^2 \, x^{4/3}}{5040 \, d^5} + \frac{73 \, b^2 \, e^4 \, n^2 \, x^{5/3}}{1260 \, d^4} - \frac{5 \, b^2 \, e^3 \, n^2 \, x^2}{168 \, d^3} + \frac{b^2 \, e^2 \, n^2 \, x^{7/3}}{84 \, d^2} - \frac{481 \, b^2 \, e^9 \, n^2 \, Log \left[d + \frac{e}{x^{1/3}}\right]}{420 \, d^9} + \frac{2 \, b \, e^8 \, n \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]}{3 \, d^9} + \frac{b \, e^7 \, n \, x^{2/3} \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} - \frac{2 \, b \, e^6 \, n \, x \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{9 \, d^3} - \frac{2 \, b \, e^4 \, n \, x^{5/3} \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{15 \, d^4} + \frac{b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{9 \, d^3} - \frac{2 \, b \, e^4 \, n \, x^{5/3} \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{15 \, d^4} + \frac{b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{9 \, d^3} - \frac{2 \, b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} - \frac{2 \, b \, e^9 \, n \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{12 \, d^2} + \frac{b \, e^3 \, n \, x^2 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{12 \, e^9 \, n^2 \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]} + \frac{e^9 \, e^9 \, n^2 \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, \left(a + b \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{3 \, d^9} + \frac{e^9 \, e^9 \, n^2 \, Log \left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n \right]\right)}{$$

### Problem 498: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \, \text{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 \, dx$$

Optimal (type 4, 400 leaves, 24 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ x^{1/3}}{600 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x^{2/3}}{120 \ d^{4}} - \frac{3 \ b^{2} \ e^{3} \ n^{2} \ x}{20 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{4/3}}{20 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{x^{1/3}}\right]}{600 \ d^{6}} + \frac{b \ e^{5} \ n \left(d + \frac{e}{x^{1/3}}\right) \ x^{1/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{d^{6}} - \frac{b \ e^{4} \ n \ x^{2/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{3 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{4/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{4 \ d^{2}} + \frac{b \ e \ n \ x^{5/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{5 \ d} + \frac{b \ e^{6} \ n \ Log \left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{d^{6}} + \frac{1}{2} \ x^{2} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2} + \frac{137 \ b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{180 \ d^{6}} - \frac{b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^{6}}$$

Result (type 4, 423 leaves, 26 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ x^{1/3}}{60 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x^{2/3}}{120 \ d^{4}} - \frac{3 \ b^{2} \ e^{3} \ n^{2} \ x}{20 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{4/3}}{20 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{x^{1/3}}\right]}{60 \ d^{6}} + \frac{b \ e^{5} \ n \left(d + \frac{e}{x^{1/3}}\right) \ x^{1/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{d^{6}} - \frac{b \ e^{4} \ n \ x^{2/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{2 \ d^{4}} + \frac{b \ e^{3} \ n \ x \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{3 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{4/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{5 \ d} - \frac{e^{6} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{6}} + \frac{1}{2} x^{2} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2} + \frac{b \ e^{6} \ n \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right) \ Log \left[-\frac{e}{d \ x^{1/3}}\right]}{180 \ d^{6}} + \frac{137 \ b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \ x^{1/3}}\right]}{d^{6}}$$

# Problem 499: Result valid but suboptimal antiderivative.

$$\int \left(a + b \, \text{Log} \, \Big[ \, c \, \left(d + \frac{e}{x^{1/3}} \right)^n \, \Big] \, \right)^2 \, \text{d}x$$

Optimal (type 4, 227 leaves, 13 steps):

$$\frac{b^2 \, e^2 \, n^2 \, x^{1/3}}{d^2} - \frac{b^2 \, e^3 \, n^2 \, \text{Log} \Big[ d + \frac{e}{x^{1/3}} \Big]}{d^3} - \frac{2 \, b \, e^2 \, n \, \left( d + \frac{e}{x^{1/3}} \right) \, x^{1/3} \, \left( a + b \, \text{Log} \Big[ c \, \left( d + \frac{e}{x^{1/3}} \right)^n \Big] \right)}{d^3} + \frac{b \, e \, n \, x^{2/3} \, \left( a + b \, \text{Log} \Big[ c \, \left( d + \frac{e}{x^{1/3}} \right)^n \Big] \right)}{d} - \frac{2 \, b \, e^3 \, n \, \text{Log} \Big[ 1 - \frac{d}{d + \frac{e}{x^{1/3}}} \Big] \, \left( a + b \, \text{Log} \Big[ c \, \left( d + \frac{e}{x^{1/3}} \right)^n \Big] \right)}{d^3} + x \, \left( a + b \, \text{Log} \Big[ c \, \left( d + \frac{e}{x^{1/3}} \right)^n \Big] \right)^2 - \frac{b^2 \, e^3 \, n^2 \, \text{Log} \, [x]}{d^3} + \frac{2 \, b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[ 2 \text{, } \frac{d}{d + \frac{e}{x^{1/3}}} \Big] }{d^3} + \frac{d^3}{d^3} + \frac{2 \, b^2 \, e^3 \, n^2 \, \text{Log} \, [x]}{d^3} + \frac{2 \, b^2 \, e^3 \, n^2 \, \text{PolyLog} \, \left[ 2 \text{, } \frac{d}{d + \frac{e}{x^{1/3}}} \right] }{d^3} + \frac{d^3}{d^3} + \frac{d^3}{d$$

#### Result (type 4, 248 leaves, 15 steps):

$$\frac{b^2 \, e^2 \, n^2 \, x^{1/3}}{d^2} - \frac{b^2 \, e^3 \, n^2 \, \text{Log} \big[ \, d + \frac{e}{x^{1/3}} \big]}{d^3} - \frac{2 \, b \, e^2 \, n \, \left( d + \frac{e}{x^{1/3}} \right) \, x^{1/3} \, \left( a + b \, \text{Log} \big[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \big] \, \right)}{d^3} + \frac{b \, e \, n \, x^{2/3} \, \left( a + b \, \text{Log} \big[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \big] \, \right)}{d} + \frac{e^3 \, \left( a + b \, \text{Log} \big[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \big] \, \right)^2}{d^3} + x \, \left( a + b \, \text{Log} \big[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \big] \, \right)^2 - \frac{e^3 \, \left( a + b \, \text{Log} \big[ \, c \, \left( d + \frac{e}{x^{1/3}} \right)^n \big] \, \right)}{d^3} - \frac{e^3 \, e^3 \, n^2 \, \text{Log} \big[ \, x \big]}{d^3} - \frac{2 \, b^2 \, e^3 \, n^2 \, \text{PolyLog} \big[ \, 2 \, , \, 1 + \frac{e}{d \, x^{1/3}} \big]}{d^3}$$

# Problem 501: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{x^{2}} dx$$

#### Optimal (type 3, 269 leaves, 8 steps):

$$\frac{3 \ b^2 \ d \ n^2 \ \left(d + \frac{e}{x^{1/3}}\right)^2}{2 \ e^3} - \frac{2 \ b^2 \ n^2 \ \left(d + \frac{e}{x^{1/3}}\right)^3}{9 \ e^3} - \frac{6 \ b^2 \ d^2 \ n^2}{e^2 \ x^{1/3}} + \frac{b^2 \ d^3 \ n^2 \ Log \left[d + \frac{e}{x^{1/3}}\right]^2}{e^3} + \frac{6 \ b \ d^2 \ n \ \left(d + \frac{e}{x^{1/3}}\right) \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \frac{3 \ b \ d \ n \ \left(d + \frac{e}{x^{1/3}}\right)^2 \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} + \frac{2 \ b \ n \ \left(d + \frac{e}{x^{1/3}}\right)^3 \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \frac{2 \ b \ d^3 \ n \ Log \left[d + \frac{e}{x^{1/3}}\right] \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]}{e^3} - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]}{e^3} - \frac{\left(a + b \ L$$

#### Result (type 3, 212 leaves, 8 steps):

$$\begin{split} &\frac{3 \ b^2 \ d \ n^2 \ \left(d + \frac{e}{x^{1/3}}\right)^2}{2 \ e^3} - \frac{2 \ b^2 \ n^2 \ \left(d + \frac{e}{x^{1/3}}\right)^3}{9 \ e^3} - \frac{6 \ b^2 \ d^2 \ n^2}{e^2 \ x^{1/3}} + \frac{b^2 \ d^3 \ n^2 \ Log \left[d + \frac{e}{x^{1/3}}\right]^2}{e^3} + \\ &\frac{1}{3} \ b \ n \left(\frac{18 \ d^2 \ \left(d + \frac{e}{x^{1/3}}\right)}{e^3} - \frac{9 \ d \ \left(d + \frac{e}{x^{1/3}}\right)^2}{e^3} + \frac{2 \ \left(d + \frac{e}{x^{1/3}}\right)^3}{e^3} - \frac{6 \ d^3 \ Log \left[d + \frac{e}{x^{1/3}}\right]}{e^3} \right) \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right] \right) - \frac{\left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{x} \end{split}$$

### Problem 502: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \, Log\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{x^3} \, dx$$

#### Optimal (type 3, 479 leaves, 8 steps):

$$-\frac{15 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{2}}{4 \ e^{6}} + \frac{20 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{3}}{9 \ e^{6}} - \frac{15 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{4}}{16 \ e^{6}} + \frac{6 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{5}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{36 \ e^{6}} + \frac{6 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{1/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{1/3}}\right]^{2}}{2 \ e^{6}} - \frac{6 \ b \ d^{5} \ n \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{6 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{1/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{2 \ e^{6} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{n} \right]}{2 \ e^{6}} + \frac{b^{2} \ d^{6} \ n^{2} \ n^{2$$

Result (type 3, 355 leaves, 8 steps):

$$-\frac{15 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{2}}{4 \ e^{6}} + \frac{20 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{3}}{9 \ e^{6}} - \frac{15 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{4}}{16 \ e^{6}} + \frac{6 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{5}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{36 \ e^{6}} + \frac{6 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{1/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{1/3}}\right]^{2}}{2 \ e^{6}} - \frac{16 \ b^{2} \ d^{6} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{25 \ e^{6}} + \frac{6 \ b^{2} \ d^{5} \ n^{2}}{6^{5} \ x^{1/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{1/3}}\right]^{2}}{2 \ e^{6}} - \frac{10 \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{e^{6}} - \frac{b^{2} \ d^{6} \ n^{2} \ log \left[d+\frac{e}{x^{1/3}}\right]^{6}}{2 \ e^{6}} - \frac{60 \ d^{6} \ Log \left[d+\frac{e}{x^{1/3}}\right]}{2 \ e^{6}} - \frac{10 \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{e^{6}} - \frac{60 \ d^{6} \ Log \left[d+\frac{e}{x^{1/3}}\right]}{e^{6}} - \frac{10 \ \left(d+\frac{e}{x^{1/3}}\right)^{6}}{e^{6}} - \frac{10 \ \left(d+$$

### Problem 503: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \log \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^3 dx$$

Optimal (type 4, 759 leaves, 62 steps):

$$\frac{71 \ b^{3} \ e^{5} \ n^{3} \ x^{1/3}}{40 \ d^{5}} - \frac{3 \ b^{3} \ e^{4} \ n^{3} \ x^{2/3}}{10 \ d^{4}} + \frac{b^{3} \ e^{3} \ n^{3} \ x}{20 \ d^{3}} - \frac{71 \ b^{3} \ e^{6} \ n^{3} \ Log \left[d + \frac{e}{x^{1/3}}\right]}{40 \ d^{6}} - \frac{77 \ b^{2} \ e^{5} \ n^{2} \left(d + \frac{e}{x^{1/3}}\right)^{x} \left(x^{1/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20 \ d^{6}} + \frac{40 \ d^{6}}{20 \ d^{3}} - \frac{9 \ b^{2} \ e^{3} \ n^{2} \ x \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20 \ d^{3}} + \frac{3 \ b^{2} \ e^{2} \ n^{2} \ x^{4/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20 \ d^{6}} - \frac{9 \ b^{2} \ e^{3} \ n^{2} \ x \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20 \ d^{6}} + \frac{3 \ b \ e^{5} \ n \left(d + \frac{e}{x^{1/3}}\right) \ x^{1/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{20 \ d^{6}} - \frac{3 \ b \ e^{4} \ n \ x^{2/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{4 \ d^{4}} + \frac{b \ e^{3} \ n \ x \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{3}} - \frac{3 \ b \ e^{5} \ n \ x^{4/3} \left(a + b \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8 \ d^{2}} + \frac{3 \ b \ e^{6} \ n \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{3}} + \frac{3 \ b \ e^{6} \ n \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{3}} + \frac{3 \ b \ e^{6} \ n \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8 \ d^{2}} + \frac{3 \ b \ e^{6} \ n \ Log \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8 \ d^{2}} + \frac{3 \ b^{2} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8 \ d^{2}} + \frac{3 \ b^{2} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8 \ d^{6}} + \frac{3 \ b^{2} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{6}} + \frac{3 \ b^{2} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{3}}{6 \ d^{6}} + \frac{3 \ b^{3} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{3}}{6 \ d^{6}} + \frac{3 \ b^{3} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]}{6 \ d^{6}} + \frac{3 \ b^{3} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d + \frac{e}{x^{1/3}}\right)^{n}\right]}{6 \ d^{6}} + \frac{3 \ b^{3} \ e^{6} \ n^{3} \ PolyLog \left[c \left(d +$$

Result (type 4, 736 leaves, 73 steps):

$$\frac{71\,b^{3}\,e^{5}\,n^{3}\,x^{1/3}}{40\,d^{5}} - \frac{3\,b^{3}\,e^{4}\,n^{3}\,x^{2/3}}{10\,d^{4}} + \frac{b^{3}\,e^{3}\,n^{3}\,x}{20\,d^{3}} - \frac{71\,b^{3}\,e^{6}\,n^{3}\,\text{Log}\left[d + \frac{e}{x^{1/3}}\right]}{40\,d^{6}} - \frac{77\,b^{2}\,e^{5}\,n^{2}\,\left(d + \frac{e}{x^{1/3}}\right)\,x^{1/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20\,d^{6}} + \frac{47\,b^{2}\,e^{4}\,n^{2}\,x^{2/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{40\,d^{4}} - \frac{9\,b^{2}\,e^{3}\,n^{2}\,x\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20\,d^{3}} + \frac{3\,b^{2}\,e^{2}\,n^{2}\,x^{4/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)}{20\,d^{2}} + \frac{77\,b\,e^{6}\,n\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{40\,d^{6}} + \frac{3\,b\,e^{5}\,n\,\left(d + \frac{e}{x^{1/3}}\right)\,x^{1/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2\,d^{6}} - \frac{3\,b\,e^{4}\,n\,x^{2/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{4\,d^{4}} + \frac{b\,e^{3}\,n\,x\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2\,d^{3}} - \frac{3\,b\,e^{2}\,n\,x^{4/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{8\,d^{2}} + \frac{3\,b\,e\,n\,x^{5/3}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{10\,d} - \frac{e^{6}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{3}}{2\,d^{6}} + \frac{1}{2}\,x^{2}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{3} - \frac{137\,b^{3}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{2}\,\left(a + b\,\text{Log}\left[c\,\left(d + \frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} - \frac{1}{2}\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} - \frac{3\,b^{3}\,e^{6}\,n^{3}\,\text{PolyLog}\left[3,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{d^{6}} - \frac{3\,b^{3}\,e^{6}\,n^{3}\,\text{PolyLog}\left[3,\,1 + \frac{e}{d\,x^{1/3}}\right]}{20\,d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/3}}\right]}{d^{6}} + \frac{3\,b^{2}\,e^{6}\,n^{3}\,\text{PolyLog}\left[2,\,1 + \frac{e}{d\,x^{1/$$

## Problem 504: Result valid but suboptimal antiderivative.

$$\int \left(a + b \log \left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 dx$$

Optimal (type 4, 436 leaves, 18 steps):

$$\frac{3 \ b^{2} \ e^{2} \ n^{2} \ \left(d+\frac{e}{x^{1/3}}\right) \ x^{1/3} \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right)}{d^{3}} + \frac{3 \ b^{2} \ e^{3} \ n^{2} \ \text{Log}\left[1-\frac{d}{d+\frac{e}{x^{1/3}}}\right] \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right)}{d^{3}} - \frac{3 \ b \ e^{2} \ n \ \left(d+\frac{e}{x^{1/3}}\right) \ x^{1/3} \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{d^{3}} + \frac{3 \ b \ e \ n \ x^{2/3} \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{2 \ d} - \frac{3 \ b \ e^{3} \ n \ \text{Log}\left[1-\frac{d}{d+\frac{e}{x^{1/3}}}\right] \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right)^{2}}{d^{3}} + x \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right) + \frac{6 \ b^{2} \ e^{3} \ n^{2} \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right) \ \text{Log}\left[-\frac{e}{d \ x^{1/3}}\right]}{d^{3}} + \frac{b^{3} \ e^{3} \ n^{3} \ \text{Log}\left[x\right]}{d^{3}} - \frac{3 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[2, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{2} \ e^{3} \ n^{2} \ \left(a+b \ \text{Log}\left[c \ \left(d+\frac{e}{x^{1/3}}\right)^{n}\right]\right) \ \text{PolyLog}\left[2, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[2, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3} \ n^{3} \ \text{PolyLog}\left[3, \frac{d}{d+\frac{e}{x^{1/3}}}\right]}{d^{3}} + \frac{6 \ b^{3} \ e^{3$$

Result (type 4, 410 leaves, 23 steps):

$$\frac{3 \, b^2 \, e^2 \, n^2 \, \left(d + \frac{e}{x^{1/3}}\right) \, x^{1/3} \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} - \frac{3 \, b \, e^3 \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 \, d^3} - \frac{3 \, b \, e^2 \, n \, \left(d + \frac{e}{x^{1/3}}\right) \, x^{1/3} \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{d^3} + \frac{3 \, b \, e^3 \, n \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3}{d^3} + x \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 + \frac{e^3 \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3}{d^3} + x \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 + \frac{e^3 \, e^3 \, n^2 \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 + \frac{e^3 \, e^3 \, n^3 \, \text{Log}\left[x\right]}{d^3} + \frac{e^3 \, e^3 \, n^3 \, \text{Log}\left[x\right]}{d^3} + \frac{e^3 \, e^3 \, n^3 \, \text{PolyLog}\left[2, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} - \frac{6 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \, \text{PolyLog}\left[2, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{6 \, b^3 \, e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, n^3 \, \text{PolyLog}\left[3, \, 1 + \frac{e}{d \, x^{1/3}}\right]}{d^3} + \frac{e^3 \, n^3 \, n^3 \, n^3 \, n^3 \,$$

### Problem 516: Result valid but suboptimal antiderivative.

$$\int x^3 \left( a + b \, \text{Log} \left[ c \, \left( d + \frac{e}{x^{2/3}} \right)^n \right] \right)^2 \, dx$$

Optimal (type 4, 412 leaves, 24 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ x^{2/3}}{120 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x^{4/3}}{240 \ d^{4}} - \frac{3 \ b^{2} \ e^{3} \ n^{2} \ x^{2}}{40 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{8/3}}{40 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{x^{2/3}}\right]}{120 \ d^{6}} + \frac{b \ e^{5} \ n \left(d + \frac{e}{x^{2/3}}\right) \ x^{2/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ d^{6}} - \frac{b \ e^{4} \ n \ x^{4/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ d^{4}} + \frac{b \ e^{3} \ n \ x^{2} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{6 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{8/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{8 \ d^{2}} + \frac{b \ e \ n \ x^{10/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{10 \ d} + \frac{b \ e \ n \ x^{10/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ d^{6}} + \frac{b \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d} - \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d} - \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{2 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{2 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{2 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{2 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ Log \left[x\right]}{10$$

Result (type 4, 436 leaves, 26 steps):

$$-\frac{77 \ b^{2} \ e^{5} \ n^{2} \ x^{2/3}}{120 \ d^{5}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x^{4/3}}{240 \ d^{4}} - \frac{3 \ b^{2} \ e^{3} \ n^{2} \ x^{2}}{40 \ d^{3}} + \frac{b^{2} \ e^{2} \ n^{2} \ x^{8/3}}{40 \ d^{2}} + \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[d + \frac{e}{x^{2/3}}\right]}{120 \ d^{6}} + \frac{b \ e^{5} \ n \left(d + \frac{e}{x^{2/3}}\right)^{n} \left[ x^{2/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right) - \frac{b \ e^{4} \ n \ x^{4/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ d^{4}} + \frac{b \ e^{3} \ n \ x^{2} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{6 \ d^{3}} - \frac{b \ e^{2} \ n \ x^{8/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{10 \ d} - \frac{e^{6} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{4 \ d^{6}} + \frac{1}{4} x^{4} \left[a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2} + \frac{b \ e^{6} \ n \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right) \ Log \left[-\frac{e}{d \ x^{2/3}}\right]}{180 \ d^{6}} + \frac{1}{180 \ d^{6}} + \frac{b^{2} \ e^{6} \ n^{2} \ PolyLog \left[2, \ 1 + \frac{e}{d \ x^{2/3}}\right]}{2 \ d^{6}}$$

# Problem 517: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \log \left[ c \left( d + \frac{e}{x^{2/3}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 239 leaves, 12 steps):

$$\frac{b^2 \, e^2 \, n^2 \, x^{2/3}}{2 \, d^2} - \frac{b^2 \, e^3 \, n^2 \, \text{Log} \Big[d + \frac{e}{x^{2/3}}\Big]}{2 \, d^3} - \frac{b \, e^2 \, n \, \Big(d + \frac{e}{x^{2/3}}\Big) \, x^{2/3} \, \Big(a + b \, \text{Log} \Big[c \, \Big(d + \frac{e}{x^{2/3}}\Big)^n\Big]\Big)}{d^3} + \frac{b \, e \, n \, x^{4/3} \, \Big(a + b \, \text{Log} \Big[c \, \Big(d + \frac{e}{x^{2/3}}\Big)^n\Big]\Big)}{2 \, d} - \frac{b \, e^3 \, n \, \text{Log} \Big[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\Big] \, \Big(a + b \, \text{Log} \Big[c \, \Big(d + \frac{e}{x^{2/3}}\Big)^n\Big]\Big)}{d^3} + \frac{1}{2} \, x^2 \, \Big(a + b \, \text{Log} \Big[c \, \Big(d + \frac{e}{x^{2/3}}\Big)^n\Big]\Big)^2 - \frac{b^2 \, e^3 \, n^2 \, \text{Log} \big[x\big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{Log} \big[x\big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{Log} \big[x\big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \, n^2 \, \text{PolyLog} \Big[2 \, , \, \frac{d}{d + \frac{e}{x^{2/3}}}\Big]}{d^3} + \frac{b^2 \, e^3 \,$$

Result (type 4, 264 leaves, 14 steps):

$$\frac{b^2\,e^2\,n^2\,x^{2/3}}{2\,d^2} - \frac{b^2\,e^3\,n^2\,Log\left[d + \frac{e}{x^{2/3}}\right]}{2\,d^3} - \frac{b\,e^2\,n\,\left(d + \frac{e}{x^{2/3}}\right)\,x^{2/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{d^3} + \\ \frac{b\,e\,n\,x^{4/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2\,d} + \frac{e^3\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2\,d^3} + \frac{1}{2}\,x^2\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 - \\ \frac{b\,e^3\,n\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)\,Log\left[-\frac{e}{d\,x^{2/3}}\right]}{d^3} - \frac{b^2\,e^3\,n^2\,Log\left[x\right]}{d^3} - \frac{b^2\,e^3\,n^2\,PolyLog\left[2\,,\,1 + \frac{e}{d\,x^{2/3}}\right]}{d^3}$$

# Problem 519: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b \, \text{Log}\left[c \, \left(d+\frac{e}{x^{2/3}}\right)^n\right]\right)^2}{x^3} \, dx$$

Optimal (type 3, 276 leaves, 8 steps):

$$\frac{3 \ b^{2} \ d \ n^{2} \left(d+\frac{e}{x^{2/3}}\right)^{2}}{4 \ e^{3}} - \frac{b^{2} \ n^{2} \left(d+\frac{e}{x^{2/3}}\right)^{3}}{9 \ e^{3}} - \frac{3 \ b^{2} \ d^{2} \ n^{2}}{e^{2} \ x^{2/3}} + \frac{b^{2} \ d^{3} \ n^{2} \ Log \left[d+\frac{e}{x^{2/3}}\right]^{2}}{2 \ e^{3}} + \frac{3 \ b \ d^{2} \ n \left(d+\frac{e}{x^{2/3}}\right) \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{e^{3}} - \frac{3 \ b \ d \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{2} \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ e^{3}} + \frac{b \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{3} \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ a^{3}} - \frac{b \ d^{3} \ n \ Log \left[d+\frac{e}{x^{2/3}}\right] \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ x^{2}} - \frac{\left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{2 \ x^{2}}$$

Result (type 3, 217 leaves, 8 steps):

$$\begin{split} &\frac{3\;b^2\;d\;n^2\;\left(d+\frac{e}{x^{2/3}}\right)^2}{4\;e^3} - \frac{b^2\;n^2\;\left(d+\frac{e}{x^{2/3}}\right)^3}{9\;e^3} - \frac{3\;b^2\;d^2\;n^2}{e^2\;x^{2/3}} + \frac{b^2\;d^3\;n^2\;Log\left[d+\frac{e}{x^{2/3}}\right]^2}{2\;e^3} + \\ &\frac{1}{6}\;b\;n\left[\frac{18\;d^2\;\left(d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{9\;d\;\left(d+\frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2\;\left(d+\frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6\;d^3\;Log\left[d+\frac{e}{x^{2/3}}\right]}{e^3}\right] \left(a+b\;Log\left[c\;\left(d+\frac{e}{x^{2/3}}\right)^n\right]\right) - \frac{\left(a+b\;Log\left[c\;\left(d+\frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2\;x^2} + \frac{2\;d^3\;n^2\;Log\left[d+\frac{e}{x^{2/3}}\right]}{e^3} - \frac{6\;d^3\;Log\left[d+\frac{e}{x^{2/3}}\right]}{e^3} + \frac{2\;d^3\;n^2\;Log\left[d+\frac{e}{x^{2/3}}\right]}{2\;x^2} + \frac{2\;d^3\;n^2\;Log\left[d+\frac{e}{x^{2/3}}\right]^2}{2\;x^2} + \frac{2\;d^3\;n^2\;Log\left[d+\frac{e}{x^{2/3}}$$

## Problem 520: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{x^5} \, dx$$

#### Optimal (type 3, 482 leaves, 8 steps):

$$-\frac{15 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{2}}{8 \ e^{6}} + \frac{10 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{3}}{9 \ e^{6}} - \frac{15 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{4}}{32 \ e^{6}} + \frac{3 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{5}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{72 \ e^{6}} + \frac{3 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{2/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{2/3}}\right]^{2}}{4 \ e^{6}} - \frac{3 \ b \ d^{5} \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{n} \left(d+\frac{e}{x^{2/3}}\right)^{n} \right]}{e^{6}} + \frac{15 \ b \ d^{2} \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{n} \left(d+\frac{e}{x^{2/3}}\right)^{2} \ \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} - \frac{10 \ b \ d^{3} \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{3} \ \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{8 \ e^{6}} + \frac{3 \ b \ d \ n \ \left(d+\frac{e}{x^{2/3}}\right)^{n} \left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{5 \ e^{6}} + \frac{b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{8 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{5 \ e^{6}} + \frac{b \ d^{6} \ n \ Log \left[d+\frac{e}{x^{2/3}}\right] \ \left(a+b \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]}{4 \ e^{6}} + \frac{2 \ b \ d^{6} \ n \ Log \left[c \ \left(d+\frac{e}{x^{2/3}}\right)^$$

### Result (type 3, 355 leaves, 8 steps):

$$-\frac{15 \ b^{2} \ d^{4} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{2}}{8 \ e^{6}} + \frac{10 \ b^{2} \ d^{3} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{3}}{9 \ e^{6}} - \frac{15 \ b^{2} \ d^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{4}}{32 \ e^{6}} + \frac{3 \ b^{2} \ d \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{25 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{72 \ e^{6}} + \frac{3 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{2/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{2/3}}\right]^{2}}{4 \ e^{6}} - \frac{10 \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{4 \ e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{e^{6}} + \frac{3 \ b^{2} \ d^{5} \ n^{2}}{e^{5} \ x^{2/3}} - \frac{b^{2} \ d^{6} \ n^{2} \ Log \left[d+\frac{e}{x^{2/3}}\right]^{2}}{4 \ e^{6}} - \frac{10 \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{e^{6}} - \frac{b^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right)^{6}}{e^{6}} - \frac{b^{2} \ n^{2} \ n^{2}$$

### Problem 524: Result valid but suboptimal antiderivative.

$$\int x^3 \, \left( a + b \, \text{Log} \left[ \, c \, \left( d + \frac{e}{x^{2/3}} \right)^n \, \right] \, \right)^3 \, \mathrm{d}x$$

Optimal (type 4, 773 leaves, 62 steps):

$$\frac{71 \ b^{3} \ e^{5} \ n^{3} \ x^{2/3}}{80 \ d^{5}} - \frac{3 \ b^{5} \ e^{4} \ n^{3} \ x^{4/3}}{20 \ d^{4}} + \frac{b^{3} \ e^{3} \ n^{3} \ x^{2}}{40 \ d^{3}} - \frac{71 \ b^{3} \ e^{6} \ n^{3} \ Log \left[d + \frac{e}{x^{2/3}}\right]}{80 \ d^{6}} - \frac{77 \ b^{2} \ e^{5} \ n^{2} \ \left(d + \frac{e}{x^{2/3}}\right)^{n} \left(2 + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40 \ d^{6}} + \frac{47 \ b^{2} \ e^{4} \ n^{2} \ x^{4/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40 \ d^{2}} + \frac{3 \ b^{2} \ e^{2} \ n^{2} \ x^{8/3} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40 \ d^{2}} - \frac{77 \ b^{2} \ e^{6} \ n^{2} \ Log \left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40 \ d^{6}} + \frac{3 \ b^{2} \ e^{3} \ n^{2} \ x^{2} \ \left(a + b \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40 \ d^{6}} - \frac{3 \ b^{2} \ e^{6} \ n^{2} \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40 \ d^{6}} - \frac{3 \ b^{2} \ e^{6} \ n^{2} \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40 \ d^{6}} + \frac{3 \ b^{2} \ e^{6} \ n^{2} \ Log \left[c \ \left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40 \ d^{6}} + \frac{3 \ b^{2} \ e^{6} \ n^{2} \ h^{2} \ h^$$

Result (type 4, 746 leaves, 73 steps):

$$\frac{71\,b^{3}\,e^{5}\,n^{3}\,x^{2/3}}{80\,d^{5}} - \frac{3\,b^{3}\,e^{4}\,n^{3}\,x^{4/3}}{20\,d^{4}} + \frac{b^{3}\,e^{3}\,n^{3}\,x^{2}}{40\,d^{3}} - \frac{71\,b^{3}\,e^{6}\,n^{3}\,Log\left[d + \frac{e}{x^{2/3}}\right]}{80\,d^{6}} - \frac{77\,b^{2}\,e^{5}\,n^{2}\,\left(d + \frac{e}{x^{2/3}}\right)\,x^{2/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40\,d^{6}} + \frac{47\,b^{2}\,e^{4}\,n^{2}\,x^{4/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{80\,d^{4}} - \frac{9\,b^{2}\,e^{3}\,n^{2}\,x^{2}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40\,d^{3}} + \frac{3\,b^{2}\,e^{2}\,n^{2}\,x^{8/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40\,d^{2}} + \frac{77\,b^{2}\,e^{5}\,n^{2}\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]}{80\,d^{6}} + \frac{40\,d^{6}}{40\,d^{6}} + \frac{9\,b^{2}\,e^{3}\,n^{2}\,x^{2}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40\,d^{3}} + \frac{3\,b^{2}\,e^{2}\,n^{2}\,x^{8/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)}{40\,d^{2}} + \frac{3\,b\,e^{5}\,n\,\left(d + \frac{e}{x^{2/3}}\right)\,x^{2/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40\,d^{6}} + \frac{3\,b\,e^{6}\,n\,x^{4/3}\,\left(a + b\,Log\left[c\,\left(d + \frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{40\,d^{6}} + \frac{3\,b\,e^{6}\,n\,x^{4/3}\,e^{6}\,n\,x^{4/3}\,e^{6}\,n\,x^{4/3}\,e^{6}\,n\,x^{4/3}\,e^{6}\,n\,x^{4/3}\,e^{6}\,n\,x^{4/3}\,e^{6}$$

## Problem 525: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \log \left[ c \left( d + \frac{e}{x^{2/3}} \right)^n \right] \right)^3 dx$$

Optimal (type 4, 451 leaves, 17 steps):

$$\frac{3 \, b^2 \, e^2 \, n^2 \, \left(d + \frac{e}{x^{2/3}}\right) \, x^{2/3} \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 \, d^3} + \frac{3 \, b^2 \, e^3 \, n^2 \, \text{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 \, d^3} - \frac{3 \, b \, e^2 \, n \, \left(d + \frac{e}{x^{2/3}}\right) \, x^{2/3} \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 \, d^3} + \frac{3 \, b \, e \, n \, x^{4/3} \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 \, d} - \frac{3 \, b \, e^3 \, n \, \text{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 \, d^3} + \frac{1}{2} \, x^2 \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) + \frac{3 \, b^2 \, e^3 \, n^2 \, \left(a + b \, \text{Log}\left[c \, \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \, \text{Log}\left[-\frac{e}{d \, x^{2/3}}\right]}{d^3} + \frac{b^3 \, e^3 \, n^3 \, \text{Log}\left[x\right]}{d^3} - \frac{3 \, b^3 \, e^3 \, n^3 \, \text{PolyLog}\left[2, \, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{2 \, d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, \text{PolyLog}\left[2, \, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, \text{PolyLog}\left[3, \, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3} + \frac{3 \, b^3 \, e^3 \, n^3 \, \text{PolyLog}\left[3, \, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3} + \frac{d^3}{d^3} + \frac{d$$

Result (type 4, 428 leaves, 22 steps):

$$\frac{3 \ b^{2} \ e^{2} \ n^{2} \ \left(d+\frac{e}{x^{2/3}}\right) \ x^{2/3} \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)}{2 \ d^{3}} - \frac{3 \ b \ e^{3} \ n \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{4 \ d^{3}} - \frac{3 \ b \ e^{2} \ n \ \left(d+\frac{e}{x^{2/3}}\right) \ x^{2/3} \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{3}} + \frac{3 \ b \ e \ n \ x^{4/3} \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2}}{2 \ d^{3}} + \frac{e^{3} \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3}}{2 \ d^{3}} + \frac{1}{2} \ x^{2} \ \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{2} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2} \left(a+b \ Log\left[c \ \left(d+\frac{e}{x^{2/3}}\right)^{n}\right]\right)^{3} + \frac{1}{2} \ x^{2$$

Test results for the 314 problems in "3.5 Logarithm functions.m"