# Mathematica 11.3 Integration Test Results

Test results for the 286 problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int Sec \, [\,e + f\,x\,] \, \left(a + a\,Sec \, [\,e + f\,x\,] \,\right) \, \left(c - c\,Sec \, [\,e + f\,x\,] \,\right)^4 \, dx$$
 Optimal (type 3, 105 leaves, 12 steps): 
$$\frac{7\,a\,c^4\,ArcTanh \, [\,Sin \, [\,e + f\,x\,]\,]}{8\,f} - \frac{a\,c^4\,Sec \, [\,e + f\,x\,]\,\,Tan \, [\,e + f\,x\,]}{8\,f} - \frac{8\,f}{8\,f} + \frac{3\,a\,c^4\,Sec \, [\,e + f\,x\,]^{\,3}\,Tan \, [\,e + f\,x\,]}{4\,f} + \frac{4\,a\,c^4\,Tan \, [\,e + f\,x\,]^{\,3}}{3\,f} + \frac{a\,c^4\,Tan \, [\,e + f\,x\,]^{\,5}}{5\,f}$$

Result (type 3, 499 leaves):

$$-\frac{1}{3840\,f}\,a\,c^4\,Sec\,[e]\,Sec\,[e+f\,x]^5\,\Big(525\,Cos\,[2\,e+3\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)+\\ 525\,Cos\,[4\,e+3\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)+\\ 105\,Cos\,[4\,e+5\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)+\\ 105\,Cos\,[6\,e+5\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)+\\ 1050\,Cos\,[f\,x]\,\left(Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)+1050\,Cos\,[2\,e+f\,x]\\ \left(Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]-Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 525\,Cos\,[2\,e+3\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]+Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 525\,Cos\,[4\,e+3\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]+Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 105\,Cos\,[4\,e+5\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]+Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 105\,Cos\,[6\,e+5\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]+Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 105\,Cos\,[6\,e+5\,f\,x]\,Log\big[Cos\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]+Sin\big[\frac{1}{2}\,\big(e+f\,x\big)\,\big]\Big)-\\ 105\,Cos\,[6\,e+f\,x]+780\,Sin\,[e+2\,f\,x\big)+780\,Sin\,[3\,e+2\,f\,x\big)+640\,Sin\,[2\,e+3\,f\,x\big)-\\ 720\,Sin\,[4\,e+3\,f\,x\big]+30\,Sin\,[3\,e+4\,f\,x\big]+30\,Sin\,[5\,e+4\,f\,x\big]+272\,Sin\,[4\,e+5\,f\,x\big]\Big)$$

### Problem 2: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+aSec[e+fx]) (c-cSec[e+fx])^{3} dx$$

Optimal (type 3, 86 leaves, 9 steps):

$$\frac{5 \ a \ c^3 \ ArcTanh[Sin[e+fx]]}{8 \ f} - \frac{3 \ a \ c^3 \ Sec[e+fx] \ Tan[e+fx]}{8 \ f} - \frac{a \ c^3 \ Sec[e+fx]^3 \ Tan[e+fx]}{4 \ f} + \frac{2 \ a \ c^3 \ Tan[e+fx]^3}{3 \ f}$$

Result (type 3, 887 leaves):

#### Problem 3: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+aSec[e+fx]) (c-cSec[e+fx])^2 dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a\;c^2\;ArcTanh\,[\,Sin\,[\,e+f\,x\,]\,\,]}{2\,f}\;-\;\frac{a\;c^2\;Sec\,[\,e+f\,x\,]\;Tan\,[\,e+f\,x\,]}{2\,f}\;+\;\frac{a\;c^2\;Tan\,[\,e+f\,x\,]^{\,3}}{3\,f}$$

Result (type 3, 313 leaves):

$$-\frac{1}{48\,f}\,a\,c^{2}\,Sec\,[e]\,Sec\,[e+f\,x]^{\,3}\,\left(3\,Cos\,[2\,e+3\,f\,x]\,Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)+\\ 3\,Cos\,[4\,e+3\,f\,x]\,Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)+\\ 9\,Cos\,[f\,x]\,\left(Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)-\\ Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]+Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)+\\ \left(Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)-\\ \left(Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)-\\ 3\,Cos\,[2\,e+3\,f\,x]\,Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]+Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)-\\ 3\,Cos\,[4\,e+3\,f\,x]\,Log\,[Cos\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]+Sin\,\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)-\\ 12\,Sin\,[2\,e+f\,x]+6\,Sin\,[e+2\,f\,x]+6\,Sin\,[3\,e+2\,f\,x]+4\,Sin\,[2\,e+3\,f\,x]\right)$$

### Problem 14: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] \left(a+a\,Sec[e+fx]\right)^2 \left(c-c\,Sec[e+fx]\right) dx$$

Optimal (type 3, 61 leaves, 6 steps):

$$\frac{a^2 \, c \, ArcTanh \, [Sin \, [e+f \, x] \, ]}{2 \, f} \, - \, \frac{a^2 \, c \, Sec \, [e+f \, x] \, \, Tan \, [e+f \, x]}{2 \, f} \, - \, \frac{a^2 \, c \, Tan \, [e+f \, x]^3}{3 \, f}$$

Result (type 3, 124 leaves):

$$\frac{1}{12\,\text{f}}\mathsf{a}^2\,\mathsf{c}\,\left[-6\,\text{Log}\big[\text{Cos}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,-\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,\big]\,+\,6\,\text{Log}\big[\text{Cos}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,\big]\,-\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\big]\,+\,\text{Sin}\big[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{$$

$$\frac{3}{\left(\text{Cos}\left[\frac{1}{2}\left(e+fx\right)\right]-\text{Sin}\left[\frac{1}{2}\left(e+fx\right)\right]\right)^{2}}+\frac{3}{\left(\text{Cos}\left[\frac{1}{2}\left(e+fx\right)\right]+\text{Sin}\left[\frac{1}{2}\left(e+fx\right)\right]\right)^{2}}-4\,\text{Tan}\left[e+fx\right]^{3}\right)$$

### Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]\,\,\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{\,2}}{\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\mathrm{d} x$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{3 \, a^2 \, ArcTanh \, [Sin \, [e+f \, x] \, ]}{c \, f} \, -\frac{3 \, a^2 \, Tan \, [e+f \, x]}{c \, f} \, -\frac{2 \, \left(a^2+a^2 \, Sec \, [e+f \, x] \, \right) \, Tan \, [e+f \, x]}{f \, \left(c-c \, Sec \, [e+f \, x] \, \right)}$$

Result (type 3, 220 leaves):

$$\left(2\,\mathsf{a}^2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\mathsf{Sec}\left[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\,\mathsf{Sin}\left[\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(4\,\mathsf{Csc}\left[\,\frac{\mathsf{e}}{2}\,\right]\,\mathsf{Sec}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\mathsf{Sin}\left[\,\frac{\mathsf{f}\,\mathsf{x}}{2}\,\right]\,+\right. \\ \left.\left(-3\,\mathsf{Log}\left[\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,-\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right]\,+\,3\,\mathsf{Log}\left[\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,+\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right]\,+\,\\ \left.\mathsf{Sin}\left[\,\mathsf{f}\,\mathsf{x}\,\right]\,\left/\,\left(\left(\mathsf{Cos}\left[\,\frac{\mathsf{e}}{2}\,\right]\,-\,\mathsf{Sin}\left[\,\frac{\mathsf{e}}{2}\,\right]\right)\,\left(\mathsf{Cos}\left[\,\frac{\mathsf{e}}{2}\,\right]\,+\,\mathsf{Sin}\left[\,\frac{\mathsf{e}}{2}\,\right]\right)\,\left(\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,-\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\right) \\ \left.\left(\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,+\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right)\right)\right)\,\mathsf{Tan}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\right)\right/\,\left(\mathsf{f}\,\left(\,\mathsf{c}\,-\,\mathsf{c}\,\mathsf{Sec}\left[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)$$

#### Problem 21: Result more than twice size of optimal antiderivative.

$$\int Sec \left[e+fx\right] \; \left(a+a\,Sec \left[e+fx\right]\right)^3 \; \left(c-c\,Sec \left[e+fx\right]\right)^6 \, \text{d}x$$

Optimal (type 3, 227 leaves, 16 steps):

$$\frac{55 \, a^3 \, c^6 \, ArcTanh[Sin[e+fx]]}{128 \, f} - \frac{25 \, a^3 \, c^6 \, Sec[e+fx] \, Tan[e+fx]}{128 \, f} - \frac{128 \, f}{128 \, f} + \frac{128 \, f}{64 \, f} + \frac{5 \, a^3 \, c^6 \, Sec[e+fx] \, Tan[e+fx]^3}{24 \, f} + \frac{5 \, a^3 \, c^6 \, Sec[e+fx] \, Tan[e+fx]^3}{16 \, f} + \frac{a^3 \, c^6 \, Sec[e+fx] \, Tan[e+fx]^5}{6 \, f} - \frac{3 \, a^3 \, c^6 \, Sec[e+fx]^3 \, Tan[e+fx]^5}{8 \, f} + \frac{4 \, a^3 \, c^6 \, Tan[e+fx]^7}{7 \, f} + \frac{a^3 \, c^6 \, Tan[e+fx]^9}{9 \, f}$$

Result (type 3, 1686 leaves):

$$\begin{split} &\frac{1}{33\,554\,432\,f}\,9\,\text{Cos}\,[\,e+f\,x\,]^{\,9}\,\text{Csc}\,\big[\,\frac{e}{2}\,+\,\frac{f\,x}{2}\,\big]^{\,12}\,\text{Sec}\,\big[\,\frac{e}{2}\,+\,\frac{f\,x}{2}\,\big]^{\,6}\,\left(\,a+a\,\text{Sec}\,[\,e+f\,x\,]\,\,\right)^{\,3} \\ &\left(\,c-c\,\text{Sec}\,[\,e+f\,x\,]\,\,\right)^{\,6}\,\left(\,-\,1430\,\text{Log}\,\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\big]\,\,\right) \\ &\left. 1430\,\text{Log}\,\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\big]\,\,\right] - \frac{1}{32}\,\text{Sec}\,[\,e+f\,x\,]^{\,8} \\ &\left. \left(4601\,\text{Sin}\,[\,e+f\,x\,]\,+\,3589\,\text{Sin}\,\big[\,3\,\left(\,e+f\,x\,\right)\,\,\big]\,+\,5441\,\text{Sin}\,\big[\,5\,\left(\,e+f\,x\,\right)\,\,\big]\,-\,715\,\text{Sin}\,\big[\,7\,\left(\,e+f\,x\,\right)\,\,\big]\,\,\right) \right. \\ &\left. \frac{1}{16\,777\,216\,f}\,11\,\text{Cos}\,[\,e+f\,x\,]^{\,9}\,\text{Csc}\,\big[\,\frac{e}{2}\,+\,\frac{f\,x}{2}\,\big]^{\,12}\,\text{Sec}\,\big[\,\frac{e}{2}\,+\,\frac{f\,x}{2}\,\big]^{\,6}\,\left(\,a+a\,\text{Sec}\,[\,e+f\,x\,]\,\,\right)^{\,3} \end{split}$$

$$\begin{array}{l} (c-cSec[e+fx])^{6} \left( -210 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] - \sin \left[ \frac{1}{2} \left( e+fx \right) \right] \right] + \\ 210 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] + \sin \left[ \frac{1}{2} \left( e+fx \right) \right] + \frac{1}{32} Sec[e+fx]^{8} \\ (5053 \sin \left[ e+fx \right] + 2681 \sin \left[ 3 \left( e+fx \right) \right] + 805 \sin \left[ 5 \left( e+fx \right) \right] + 105 \sin \left[ 7 \left( e+fx \right) \right] \right) \right) + \\ \frac{1}{25165 824 f} 29 \cos \left[ e+fx \right]^{9} \csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^{22} Sec \left[ \frac{e}{2} + \frac{fx}{2} \right]^{6} \left( a+a Sec \left[ e+fx \right] \right)^{3} \\ \left( c-c Sec \left[ e+fx \right] \right)^{6} \left( -330 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] - \sin \left[ \frac{1}{2} \left( e+fx \right) \right] \right] + \\ 330 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] + \sin \left[ \frac{1}{2} \left( e+fx \right) \right] + 132 Sec \left[ e+fx \right] \right] \\ \left( -6103 \sin \left[ e+fx \right] + 4213 \sin \left[ 3 \left( e+fx \right) \right] + 1265 \sin \left[ 5 \left( e+fx \right) \right] + 165 \sin \left[ 7 \left( e+fx \right) \right] \right) \right) - \\ \frac{1}{838608 f} 5 \cos \left[ e+fx \right]^{9} \csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^{22} Sec \left[ \frac{e}{2} + \frac{fx}{2} \right]^{6} \left( a+a Sec \left[ e+fx \right] \right) \\ \left( c-c Sec \left[ e+fx \right] \right)^{6} \left( -858 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] - 5 in \left[ \frac{1}{2} \left( e+fx \right) \right] \right] + \\ 858 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] + Sin \left[ \frac{1}{2} \left( e+fx \right) \right] - 5 in \left[ \frac{1}{2} \left( e+fx \right) \right] \right) + \\ \frac{1}{33554432 f} \cos \left[ e+fx \right]^{9} \csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^{12} Sec \left[ \frac{e}{2} + \frac{fx}{2} \right]^{6} \left( a+a Sec \left[ e+fx \right] \right) \\ \left( c-c Sec \left[ e+fx \right]^{9} \right) - \left( 24310 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] - 3289 \sin \left[ 5 \left( e+fx \right) \right] + 249 \sin \left[ 7 \left( e+fx \right) \right] \right) \right) + \\ \frac{1}{3159} \cos \left[ e+fx \right]^{9} \cos \left[ e+fx \right]^{9} \cos \left[ \frac{e}{2} + \frac{fx}{2} \right]^{12} Sec \left[ \frac{e}{2} + \frac{fx}{2} \right]^{16} \left( a+a Sec \left[ e+fx \right] \right) \right] + \\ 24310 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] + 5 \sin \left[ \frac{1}{2} \left( e+fx \right) \right] - \frac{1}{32} Sec \left[ e+fx \right] \right] + \\ 24310 \log \left[ \cos \left[ \frac{1}{2} \left( e+fx \right) \right] + 5 \sin \left[ \frac{1}{2} \left( e+fx \right) \right] + 20613 \sin \left[ 7 \left( e+fx \right) \right] \right) \right) - \\ \frac{1}{8192} \cos \left[ e+fx \right]^{9} \cos \left[ e+fx \right]^{9} \cos \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] \right) \right] + \\ \frac{1}{192} \cos \left[ e+fx \right]^{9} \cos \left[ e+fx \right]^{9} \cos \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} Sec \left[ e+fx \right] + \frac{1}{2} S$$

$$\left( \frac{256 \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} - \frac{448 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 2} \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} + \frac{80 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 4} \, \mathsf{Tan} [\, e + f \, x \,]}{3 \, f} - \frac{40 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 6} \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} + \frac{\mathsf{Sec} [\, e + f \, x \,]^{\, 8} \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} \right) + \frac{1}{16384}$$

$$3 \, \mathsf{Cos} [\, e + f \, x \,]^{\, 9} \, \mathsf{Csc} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^{12} \, \mathsf{Sec} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^{\, 6} \, \left( a + a \, \mathsf{Sec} [\, e + f \, x \,] \right)^{\, 3} \, \left( c - c \, \mathsf{Sec} [\, e + f \, x \,] \right)^{\, 6}$$

$$\left( \frac{64 \, \mathsf{Tan} [\, e + f \, x \,]}{63 \, f} + \frac{32 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 2} \, \mathsf{Tan} [\, e + f \, x \,]}{63 \, f} + \frac{8 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 4} \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} \right) + \frac{1}{65 \, 536}$$

$$\mathsf{55} \, \mathsf{Cos} [\, e + f \, x \,]^{\, 9} \, \mathsf{Csc} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^{12} \, \mathsf{Sec} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^{\, 6} \, \left( a + a \, \mathsf{Sec} [\, e + f \, x \,] \right)^{\, 3} \, \left( c - c \, \mathsf{Sec} [\, e + f \, x \,] \right)^{\, 6}$$

$$\left( \frac{128 \, \mathsf{Tan} [\, e + f \, x \,]}{315 \, f} + \frac{64 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 2} \, \mathsf{Tan} [\, e + f \, x \,]}{315 \, f} + \frac{16 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 4} \, \mathsf{Tan} [\, e + f \, x \,]}{105 \, f} + \frac{8 \, \mathsf{Sec} [\, e + f \, x \,]^{\, 6} \, \mathsf{Tan} [\, e + f \, x \,]}{9 \, f} \right)$$

# Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,3}}{\mathsf{c}\,-\,\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 100 leaves, 6 steps):

$$-\frac{15 \, a^3 \, \text{ArcTanh} \, [\text{Sin} \, [\, e + f \, x \, ] \, ]}{2 \, c \, f} - \frac{10 \, a^3 \, \text{Tan} \, [\, e + f \, x \, ]}{c \, f} - \frac{5 \, a^3 \, \text{Sec} \, [\, e + f \, x \, ] \, \text{Tan} \, [\, e + f \, x \, ]}{2 \, c \, f} - \frac{2 \, a \, \left( a + a \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^2 \, \text{Tan} \, [\, e + f \, x \, ]}{f \, \left( c - c \, \text{Sec} \, [\, e + f \, x \, ] \, \right)}$$

Result (type 3, 287 leaves):

$$\begin{split} \frac{1}{16\,f\,\left(c-c\,Sec\,[\,e+f\,x\,]\,\right)}\,a^3\,Cos\,[\,e+f\,x\,]^{\,2}\,Sec\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]^4 \\ &\left(1+Sec\,[\,e+f\,x\,]\,\right)^3\,Tan\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\left(32\,Csc\,\left[\,\frac{e}{2}\,\right]\,Sec\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,Sin\,\left[\,\frac{f\,x}{2}\,\right]\,+ \\ &\left(-30\,Log\,\left[Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,-Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right]\,+30\,Log\,\left[Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,+Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right]\,+ \\ &\frac{1}{\left(Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,-Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\right)^2} - \frac{1}{\left(Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,+Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right)} \\ &\left(16\,Sin\,[\,f\,x\,]\,\right)\,\left/\,\left(\left(Cos\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\right]\right)\,\left(Cos\,\left[\,\frac{e}{2}\,\right]\,+Sin\,\left[\,\frac{e}{2}\,\right]\right)\,\left(Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right) - \\ &Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right)\,\left(Cos\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right) + Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\right) \right) \\ &\left(16\,Sin\,[\,f\,x\,]\,\right)\,\left(Cos\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\right]\,\right)\,\left(Cos\,\left[\,\frac{e}{2}\,\right]\,+Sin\,\left[\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right) + Sin\,\left[\,\frac{e}{2}\,\left(\,e+f\,x\,\right)\,\right] \right) \right) \\ &\left(16\,Sin\,\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\left(\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\left(\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\left(\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\left(\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{2}\,\right]\,-Sin\,\left[\,\frac{e}{2}\,\left(\,\frac{e}{2}\,\right]\,\right) \\ &\left(16\,Sin\,\left[\,\frac{e}{$$

### Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \, \left(a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3}{\left(c - c\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^2} \,\mathrm{d}x$$

Optimal (type 3, 119 leaves, 6 steps):

$$\begin{split} & \frac{5 \; a^3 \; ArcTanh \left[ Sin \left[ e + f \, x \right] \right]}{c^2 \; f} \; + \; \frac{5 \; a^3 \; Tan \left[ e + f \, x \right]}{c^2 \; f} \; - \\ & \frac{2 \; a \; \left( a + a \; Sec \left[ e + f \, x \right] \right)^2 \; Tan \left[ e + f \, x \right]}{3 \; f \; \left( c - c \; Sec \left[ e + f \, x \right] \right)^2} \; + \; \frac{10 \; \left( a^3 + a^3 \; Sec \left[ e + f \, x \right] \right) \; Tan \left[ e + f \, x \right]}{3 \; f \; \left( c^2 - c^2 \; Sec \left[ e + f \, x \right] \right)} \end{split}$$

Result (type 3, 671 leaves):

$$\left(2 \cos \left[e+fx\right] \csc \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^5 \left(a+a \sec \left[e+fx\right]\right)^3 \sin \left[\frac{fx}{2}\right] \tan \left[\frac{e}{2}+\frac{fx}{2}\right]\right) \right/ \\ \left(3 f \left(c-c \sec \left[e+fx\right]\right)^2\right) - \frac{2 \cos \left[e+fx\right] \cot \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^4 \left(a+a \sec \left[e+fx\right]\right)^3 \tan \left[\frac{e}{2}+\frac{fx}{2}\right]^2}{3 f \left(c-c \sec \left[e+fx\right]\right)^2} + \\ \left(10 \cos \left[e+fx\right] \csc \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^3 \left(a+a \sec \left[e+fx\right]\right)^3 \sin \left[\frac{fx}{2}\right] \tan \left[\frac{e}{2}+\frac{fx}{2}\right]^3\right) \right/ \\ \left(3 f \left(c-c \sec \left[e+fx\right]\right)^2\right) - \left(5 \cos \left[e+fx\right] \log \left[\cos \left[\frac{e}{2}+\frac{fx}{2}\right]-\sin \left[\frac{e}{2}+\frac{fx}{2}\right]\right]\right) \\ \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^2 \left(a+a \sec \left[e+fx\right]\right)^3 \tan \left[\frac{e}{2}+\frac{fx}{2}\right]^4\right) / \left(2 f \left(c-c \sec \left[e+fx\right]\right)^2\right) + \\ \left(5 \cos \left[e+fx\right] \log \left[\cos \left[\frac{e}{2}+\frac{fx}{2}\right]+\sin \left[\frac{e}{2}+\frac{fx}{2}\right]\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^2 \\ \left(a+a \sec \left[e+fx\right]\right)^3 \tan \left[\frac{e}{2}+\frac{fx}{2}\right]^4\right) / \left(2 f \left(c-c \sec \left[e+fx\right]\right)^2\right) + \\ \left(\cos \left[e+fx\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^2 \left(a+a \sec \left[e+fx\right]\right)^3 \sin \left[\frac{fx}{2}\right] \tan \left[\frac{e}{2}+\frac{fx}{2}\right]^4\right) / \\ \left(2 f \left(c-c \sec \left[e+fx\right]\right)^2 \left(\cos \left[\frac{e}{2}\right]-\sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2}+\frac{fx}{2}\right]-\sin \left[\frac{e}{2}+\frac{fx}{2}\right]\right)\right) + \\ \left(\cos \left[e+fx\right] \sec \left[\frac{e}{2}+\frac{fx}{2}\right]^2 \left(a+a \sec \left[e+fx\right]\right)^3 \sin \left[\frac{fx}{2}\right] \tan \left[\frac{e}{2}+\frac{fx}{2}\right]\right) \right) + \\ \left(2 f \left(c-c \sec \left[e+fx\right]\right)^2 \left(\cos \left[\frac{e}{2}\right]+\sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2}+\frac{fx}{2}\right]+\sin \left[\frac{e}{2}+\frac{fx}{2}\right]\right)\right) \right)$$

### Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + fx] \left(c - c \operatorname{Sec} [e + fx]\right)^{4}}{a + a \operatorname{Sec} [e + fx]} dx$$

Optimal (type 3, 121 leaves, 10 steps):

$$-\frac{35\,c^{4}\,ArcTanh\,[Sin\,[\,e+f\,x\,]\,\,]}{2\,a\,f} + \frac{28\,c^{4}\,Tan\,[\,e+f\,x\,]}{a\,f} - \\ \frac{21\,c^{4}\,Sec\,[\,e+f\,x\,]\,\,Tan\,[\,e+f\,x\,]}{2\,a\,f} + \frac{2\,c\,\left(\,c-c\,Sec\,[\,e+f\,x\,]\,\right)^{\,3}\,Tan\,[\,e+f\,x\,]}{f\,\left(\,a+a\,Sec\,[\,e+f\,x\,]\,\right)} + \frac{7\,c^{4}\,Tan\,[\,e+f\,x\,]^{\,3}}{3\,a\,f}$$

Result (type 3, 1036 leaves):

$$\left(35 \cos \left[e + f x\right]^{3} \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \csc \left[\frac{e}{2} + \frac{f x}{2}\right]^{6} \right. \\ \left. \log \left[\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right] \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4}\right) \left/ \left(16 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right)\right) - \left(35 \cos \left[e + f x\right]^{3} \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{6} \log \left[\cos \left[\frac{e}{2} + \frac{f x}{2}\right] + \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right] \right) \right. \\ \left. \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4}\right) \left/ \left(16 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right)\right) + \left(2 \cos \left[e + f x\right]\right)^{4}\right) \left/ \left(16 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right)\right) + \left(2 \cos \left[e + f x\right]\right)^{3} \cot \left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{7} \operatorname{Sec}\left[\frac{e}{2}\right] \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4} \sin \left[\frac{f x}{2}\right]\right) \right/ \left(f \left(a + a \operatorname{Sec}\left[e + f x\right]\right)\right) + \left(\cos \left[e + f x\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{f x}{2} + \frac{f x}{2}\right]\right) \left(\sin \left[\frac{f x}{2}\right]\right) \right) \left(48 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right)^{3}\right) + \left(\cos \left[e + f x\right]^{3} \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right] \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) \right/ \left(48 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right) \left(\cos \left[\frac{e}{2}\right] - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right/ \left(24 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right) \left(\cos \left[\frac{e}{2}\right] - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) + \left(\cos \left[e + f x\right]^{3} \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{6} \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4} \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right/ \left(48 \, f \left(a + a \operatorname{Sec}\left[e + f x\right]\right) \left(\cos \left[\frac{e}{2}\right] - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) + \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{6} \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4} \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right) + \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right] \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] + \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) \right) \right) + \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^{2} \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^{4} \operatorname{Csc}\left[\frac{$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec[e+fx] \left(c-cSec[e+fx]\right)^3}{a+aSec[e+fx]} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$-\frac{15\,c^{3}\,ArcTanh\,[Sin\,[\,e+f\,x\,]\,\,]}{2\,a\,f} + \frac{10\,c^{3}\,Tan\,[\,e+f\,x\,]}{a\,f} - \\ \frac{5\,c^{3}\,Sec\,[\,e+f\,x\,]\,\,Tan\,[\,e+f\,x\,]}{2\,a\,f} + \frac{2\,c\,\left(\,c-c\,Sec\,[\,e+f\,x\,]\,\right)^{\,2}\,Tan\,[\,e+f\,x\,]}{f\,\left(\,a+a\,Sec\,[\,e+f\,x\,]\,\right)}$$

#### Result (type 3, 287 leaves):

$$\begin{split} &\frac{1}{16\,a\,f\,\left(1+Sec\,[\,e+f\,x\,]\,\right)}\,Cos\,[\,e+f\,x\,]^{\,2}\,Cot\,\left[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,Csc\,\left[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]^{\,4} \\ &\left(\,c-c\,Sec\,[\,e+f\,x\,]\,\right)^{\,3}\,\left[-32\,Csc\,\left[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right]\,Sec\,\left[\frac{e}{2}\,\right]\,Sin\,\left[\frac{f\,x}{2}\,\right] + Cot\,\left[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\right] \\ &\left(-30\,Log\,\left[Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,-Sin\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,\right] + 30\,Log\,\left[Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,+Sin\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,\right] + \\ &\frac{1}{\left(Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,-Sin\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\right)^{\,2}} - \\ &\left(16\,Sin\,[\,f\,x\,]\,\right)\,\left(\,\left(Cos\,\left[\frac{e}{2}\,\right]\,-Sin\,\left[\frac{e}{2}\,\right]\right)\,\left(Cos\,\left[\frac{e}{2}\,\right]\,+Sin\,\left[\frac{e}{2}\,\right]\right) \\ &\left(Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,-Sin\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\right) \right) \\ &\left(Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\,-Sin\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\right) \\ &\left(Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right] \\ &\left(Cos\,\left[\frac{1}{2}\,\left(\,e+f\,x\right)\,\right]\right) \\ &\left(Cos\,\left[\frac{1}{2}\,$$

# Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} \left[\,e + f\,x\,\right] \, \left(\,c - c\,\operatorname{Sec} \left[\,e + f\,x\,\right]\,\right)^{\,2}}{a + a\,\operatorname{Sec} \left[\,e + f\,x\,\right]} \, \mathrm{d} x$$

Optimal (type 3, 74 leaves, 5 steps):

$$-\frac{3 c^{2} ArcTanh[Sin[e+fx]]}{af} + \frac{3 c^{2} Tan[e+fx]}{af} + \frac{2 (c^{2} - c^{2} Sec[e+fx]) Tan[e+fx]}{f (a+a Sec[e+fx])}$$

Result (type 3, 220 leaves):

$$\begin{split} &\left[2\,c^2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\mathsf{Sec}\left[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right] \\ &\quad \mathsf{Sin}\left[\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(4\,\mathsf{Csc}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\mathsf{Sec}\left[\,\frac{\mathsf{e}}{2}\,\right]\,\mathsf{Sin}\left[\,\frac{\mathsf{f}\,\mathsf{x}}{2}\,\right] + \mathsf{Cot}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right. \\ &\quad \left.\left(3\,\mathsf{Log}\left[\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,-\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right] - 3\,\mathsf{Log}\left[\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,+\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right] + \\ &\quad \left.\mathsf{Sin}\left[\,\mathsf{f}\,\mathsf{x}\,\right]\,\left/\,\left(\left(\mathsf{Cos}\left[\,\frac{\mathsf{e}}{2}\,\right]\,-\,\mathsf{Sin}\left[\,\frac{\mathsf{e}}{2}\,\right]\right)\,\left(\mathsf{Cos}\left[\,\frac{\mathsf{e}}{2}\,\right]\,+\,\mathsf{Sin}\left[\,\frac{\mathsf{e}}{2}\,\right]\right)\,\left(\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,-\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right) \\ &\quad \left.\left.\left(\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,+\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right)\right)\right)\right)\right/\,\left(\mathsf{a}\,\mathsf{f}\,\left(1+\,\mathsf{Sec}\left[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\,\right)\right) \end{split}$$

#### Problem 42: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec[e+fx] \left(c-c Sec[e+fx]\right)^5}{\left(a+a Sec[e+fx]\right)^2} dx$$

Optimal (type 3, 164 leaves, 11 steps):

$$\frac{105 \ c^{5} \ Arc Tanh [Sin [e+fx]]}{2 \ a^{2} \ f} - \frac{84 \ c^{5} \ Tan [e+fx]}{a^{2} \ f} + \frac{63 \ c^{5} \ Sec [e+fx] \ Tan [e+fx]}{2 \ a^{2} \ f} - \frac{6 \ c^{2} \ \left(c-c \ Sec [e+fx]\right)^{3} \ Tan [e+fx]}{5 \ \left(a^{2} + a^{2} \ Sec [e+fx]\right)} - \frac{7 \ c^{5} \ Tan [e+fx]^{3}}{a^{2} \ f} - \frac{7 \ c^{5} \ Tan [e+fx]^{3}}{a^{2} \$$

#### Result (type 3, 380 leaves):

$$\frac{1}{3072 \, a^2 \, f \, \left(1 + \operatorname{Sec}\left[e + f \, x\right]\right)^2} \\ \operatorname{Cot}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] \operatorname{Csc}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]^6 \, \left(c - c \operatorname{Sec}\left[e + f \, x\right]\right)^5 \, \left(20160 \operatorname{Cos}\left[e + f \, x\right]^3 \operatorname{Cot}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]^3 \right) \\ \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] - \operatorname{Sin}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] + \operatorname{Sin}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]\right]\right) + \\ \operatorname{Csc}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[e\right] \, \left(-1323 \operatorname{Sin}\left[\frac{f \, x}{2}\right] + 3247 \operatorname{Sin}\left[\frac{3 \, f \, x}{2}\right] - 2901 \operatorname{Sin}\left[e - \frac{f \, x}{2}\right] + \\ \operatorname{1197} \operatorname{Sin}\left[e + \frac{f \, x}{2}\right] - 3027 \operatorname{Sin}\left[2 \, e + \frac{f \, x}{2}\right] - 273 \operatorname{Sin}\left[e + \frac{3 \, f \, x}{2}\right] + 1827 \operatorname{Sin}\left[2 \, e + \frac{3 \, f \, x}{2}\right] - \\ \operatorname{1693} \operatorname{Sin}\left[3 \, e + \frac{3 \, f \, x}{2}\right] + 1995 \operatorname{Sin}\left[e + \frac{5 \, f \, x}{2}\right] - 117 \operatorname{Sin}\left[2 \, e + \frac{5 \, f \, x}{2}\right] + 1143 \operatorname{Sin}\left[3 \, e + \frac{5 \, f \, x}{2}\right] - \\ \operatorname{969} \operatorname{Sin}\left[4 \, e + \frac{5 \, f \, x}{2}\right] + 1173 \operatorname{Sin}\left[2 \, e + \frac{7 \, f \, x}{2}\right] + 1173 \operatorname{Sin}\left[3 \, e + \frac{7 \, f \, x}{2}\right] + 1173 \operatorname{Sin}\left[4 \, e + \frac{9 \, f \, x}{2}\right] + 352 \operatorname{Sin}\left[5 \, e + \frac{9 \, f \, x}{2}\right] \right) \right)$$

## Problem 43: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{c}\,-\,\mathsf{c}\,\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,4}}{\left(\,\mathsf{a}\,+\,\mathsf{a}\,\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,2}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 150 leaves, 7 steps):

$$\frac{35\,c^4\,ArcTanh\,[Sin\,[e+f\,x]\,]}{2\,a^2\,f} - \frac{70\,c^4\,Tan\,[e+f\,x]}{3\,a^2\,f} + \frac{35\,c^4\,Sec\,[e+f\,x]\,\,Tan\,[e+f\,x]}{6\,a^2\,f} + \frac{2\,c\,\left(c-c\,Sec\,[e+f\,x]\,\right)^3\,Tan\,[e+f\,x]}{3\,f\,\left(a+a\,Sec\,[e+f\,x]\right)^2} - \frac{14\,\left(c^2-c^2\,Sec\,[e+f\,x]\,\right)^2\,Tan\,[e+f\,x]}{3\,f\,\left(a^2+a^2\,Sec\,[e+f\,x]\,\right)}$$

Result (type 3, 349 leaves):

$$\begin{split} &\frac{1}{3 \, \mathsf{a}^2 \, \mathsf{f} \, \left( 1 + \mathsf{Sec} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right)^2} \, \mathsf{c}^4 \, \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \, \mathsf{Sec} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^2 \\ &- \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^3 \, \left[ -256 \, \mathsf{Cot} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Csc} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \, \mathsf{Sec} \left[ \frac{\mathsf{e}}{2} \right] \, \mathsf{Sin} \left[ \frac{\mathsf{f} \, \mathsf{x}}{2} \right] + 3 \, \mathsf{Cot} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^3 \\ &- 32 \, \mathsf{Csc} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^3 \, \mathsf{Sec} \left[ \frac{\mathsf{e}}{2} \right] \, \mathsf{Sin} \left[ \frac{\mathsf{f} \, \mathsf{x}}{2} \right] + 3 \, \mathsf{Cot} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^3 \\ &- \left[ -70 \, \mathsf{Log} \left[ \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] - \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right] + 70 \, \mathsf{Log} \left[ \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right] + \\ &- \frac{1}{\left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] - \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right)^2} - \frac{1}{\left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] - \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right) \left( \mathsf{Cos} \left[ \frac{\mathsf{e}}{2} \right] + \mathsf{Sin} \left[ \frac{\mathsf{e}}{2} \right] \right)} \\ &- \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] - \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right) \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \right) \right) \right) - \\ &- 32 \, \mathsf{Cot} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] \, \mathsf{Csc} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Tan} \left[ \frac{\mathsf{e}}{2} \right] \right) \right) \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right) \right] \right) \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] + \mathsf{Sin} \left[ \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{x} \right] \right] + \mathsf{e} \, \mathsf{f} \, \mathsf{f$$

### Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \, \left(c - c\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3}{\left(a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^2} \, \mathrm{d} x$$

Optimal (type 3, 119 leaves, 6 steps):

$$\begin{split} & \frac{5 \, c^3 \, ArcTanh \, [Sin \, [\, e+f \, x \, ] \, ]}{a^2 \, f} - \frac{5 \, c^3 \, Tan \, [\, e+f \, x \, ]}{a^2 \, f} + \\ & \frac{2 \, c \, \left(c-c \, Sec \, [\, e+f \, x \, ] \, \right)^2 \, Tan \, [\, e+f \, x \, ]}{3 \, f \, \left(a+a \, Sec \, [\, e+f \, x \, ] \, \right)^2} - \frac{10 \, \left(c^3-c^3 \, Sec \, [\, e+f \, x \, ] \, \right) \, Tan \, [\, e+f \, x \, ]}{3 \, f \, \left(a^2+a^2 \, Sec \, [\, e+f \, x \, ] \, \right)} \end{split}$$

Result (type 3, 671 leaves):

$$\left(5 \cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^4 \csc \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \right)$$

$$Log \left[\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right] \left(c - c \sec \left[e + f x\right]\right)^3 \right) \left/ \left(2 f \left(a + a \sec \left[e + f x\right]\right)^2\right) - \left(5 \cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]\right]^4 \csc \left[\frac{e}{2} + \frac{f x}{2}\right]^2 Log \left[\cos \left[\frac{e}{2} + \frac{f x}{2}\right] + \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right] \right)$$

$$\left(c - c \sec \left[e + f x\right]\right)^3 \right) \left/ \left(2 f \left(a + a \sec \left[e + f x\right]\right)^2\right) + \left(10 \cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^3 \csc \left[\frac{e}{2} + \frac{f x}{2}\right]^3 \operatorname{Sec}\left[\frac{e}{2}\right] \left(c - c \sec \left[e + f x\right]\right)^3 \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right/$$

$$\left(3 f \left(a + a \sec \left[e + f x\right]\right)^2\right) + \left(2 \cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \operatorname{Sec}\left[\frac{e}{2}\right] \left(c - c \sec \left[e + f x\right]\right)^3 \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right/$$

$$\left(3 f \left(a + a \sec \left[e + f x\right]\right)^2\right) + \left(\cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \left(c - c \sec \left[e + f x\right]\right)^3 \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right/$$

$$\left(2 f \left(a + a \sec \left[e + f x\right]\right)^2 \left(\cos \left[\frac{e}{2} - \sin \left[\frac{e}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] - \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) + \left(\cos \left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^4 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \left(c - c \sec \left[e + f x\right]\right)^3 \operatorname{Sin}\left[\frac{f x}{2}\right]\right) \right) + \left(2 f \left(a + a \sec \left[e + f x\right]\right)^2 \left(\cos \left[\frac{e}{2} - \frac{f x}{2}\right]\right) \left(\cos \left[\frac{e}{2} + \frac{f x}{2}\right] + \sin \left[\frac{e}{2} + \frac{f x}{2}\right]\right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^3 \operatorname{Tan}\left[\frac{e}{2}\right] \right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^3 \operatorname{Tan}\left[\frac{e}{2}\right] \right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^3 \operatorname{Tan}\left[\frac{e}{2}\right] \right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^3 \operatorname{Tan}\left[\frac{e}{2}\right] \right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} + \frac{f x}{2}\right]^2 \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \left(c - c \operatorname{Sec}\left[e + f x\right]\right)^3 \operatorname{Tan}\left[\frac{e}{2}\right] \right) \right) + 2 \left(2 \operatorname{Cos}\left[e + f x\right] \cot \left[\frac{e}{2} +$$

### Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^2\,\left(\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^2}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{\csc[e+fx]}{a^2 c^2 f} - \frac{\csc[e+fx]^3}{3 a^2 c^2 f}$$

Result (type 3, 100 leaves)

$$\frac{1}{a^2\,c^2} \Biggl( \frac{5\,\text{Cot}\left[\,\frac{1}{2}\,\left(\,e + f\,x\right)\,\,\right]}{12\,f} \,-\, \frac{\text{Cot}\left[\,\frac{1}{2}\,\left(\,e + f\,x\right)\,\,\right]\,\text{Csc}\left[\,\frac{1}{2}\,\left(\,e + f\,x\right)\,\,\right]^2}{24\,f} \,+\, \frac{1}{2} \left(\,e + f\,x\right) \left(\,e + f\,x\right) \left(\,e + f\,x\right)\,\left(\,e + f\,x\right)\,\left(\,e + f\,x\right)\,\,\right)}{24\,f} \,+\, \frac{1}{2} \left(\,e + f\,x\right) \left(\,e + f\,x\right) \left(\,e + f\,x\right) \left(\,e + f\,x\right)\,\left(\,e + f\,x\right)\,\left(\,e$$

$$\frac{5 \operatorname{Tan}\left[\frac{1}{2} \left(e+fx\right)\right]}{12 f} - \frac{\operatorname{Sec}\left[\frac{1}{2} \left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2} \left(e+fx\right)\right]}{24 f}$$

#### Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec[e+fx] \left(c-cSec[e+fx]\right)^4}{\left(a+aSec[e+fx]\right)^3} dx$$

#### Optimal (type 3, 164 leaves, 7 steps):

$$-\frac{7 c^{4} \operatorname{ArcTanh}[\operatorname{Sin}[e+fx]]}{a^{3} f} + \frac{7 c^{4} \operatorname{Tan}[e+fx]}{a^{3} f} + \frac{2 c \left(c-c \operatorname{Sec}[e+fx]\right)^{3} \operatorname{Tan}[e+fx]}{5 f \left(a+a \operatorname{Sec}[e+fx]\right)^{3}} - \frac{14 \left(c^{2}-c^{2} \operatorname{Sec}[e+fx]\right)^{2} \operatorname{Tan}[e+fx]}{15 a f \left(a+a \operatorname{Sec}[e+fx]\right)^{2}} + \frac{14 \left(c^{4}-c^{4} \operatorname{Sec}[e+fx]\right) \operatorname{Tan}[e+fx]}{3 f \left(a^{3}+a^{3} \operatorname{Sec}[e+fx]\right)}$$

#### Result (type 3, 826 leaves):

$$\left( 7 \cos \left[ e + f x \right] \cot \left[ \frac{e}{2} + \frac{f x}{2} \right]^{6} \csc \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \right.$$

$$\left. \log \left[ \cos \left[ \frac{e}{2} + \frac{f x}{2} \right] - \sin \left[ \frac{e}{2} + \frac{f x}{2} \right] \right] \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \right) / \left( 2 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \right) - \left( 7 \operatorname{Cos} \left[ e + f x \right] \cot \left[ \frac{e}{2} + \frac{f x}{2} \right]^{6} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \operatorname{Log} \left[ \cos \left[ \frac{e}{2} + \frac{f x}{2} \right] + \operatorname{Sin} \left[ \frac{e}{2} + \frac{f x}{2} \right] \right] \right)$$

$$\left( c - c \operatorname{Sec} \left[ e + f x \right] \cot \left[ \frac{e}{2} + \frac{f x}{2} \right]^{5} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{3} \operatorname{Sec} \left[ \frac{e}{2} \right] \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \operatorname{Sin} \left[ \frac{f x}{2} \right] \right) \right) /$$

$$\left( 15 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \right) +$$

$$\left( 8 \operatorname{Cos} \left[ e + f x \right] \cot \left[ \frac{e}{2} + \frac{f x}{2} \right]^{3} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{5} \operatorname{Sec} \left[ \frac{e}{2} \right] \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \operatorname{Sin} \left[ \frac{f x}{2} \right] \right) \right) /$$

$$\left( 15 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \right) +$$

$$\left( 2 \operatorname{Cos} \left[ e + f x \right] \operatorname{Cot} \left[ \frac{e}{2} + \frac{f x}{2} \right] \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{7} \operatorname{Sec} \left[ \frac{e}{2} \right] \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \operatorname{Sin} \left[ \frac{f x}{2} \right] \right) /$$

$$\left( 15 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \right) +$$

$$\left( 2 \operatorname{Cos} \left[ e + f x \right] \operatorname{Cot} \left[ \frac{e}{2} + \frac{f x}{2} \right] \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{7} \operatorname{Sec} \left[ \frac{e}{2} \right] \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \operatorname{Sin} \left[ \frac{f x}{2} \right] \right) /$$

$$\left( 5 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \right) +$$

$$\left( \operatorname{Cos} \left[ e + f x \right] \operatorname{Cot} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{6} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)^{4} \operatorname{Sin} \left[ \frac{f x}{2} \right] \right) /$$

$$\left( 2 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \left( \operatorname{Cos} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \left( \operatorname{Cos} \left[ \frac{e}{2} + \frac{f x}{2} \right] - \operatorname{Sin} \left[ \frac{e}{2} + \frac{f x}{2} \right] \right) \right) +$$

$$\left( \operatorname{Cos} \left[ e + f x \right] \operatorname{Cot} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{6} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \left( \operatorname{Cos} \left[ \frac{e}{2} + \frac{f x}{2} \right] + \operatorname{Sin} \left[ \frac{e}{2} + \frac{f x}{2} \right] \right) \right) +$$

$$\left( 2 f \left( a + a \operatorname{Sec} \left[ e + f x \right] \right)^{3} \left( \operatorname{Cos} \left[ \frac{e}{2} + \frac{f x}{2} \right)^{2} \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right] + \operatorname{Csc} \left[ \frac{$$

#### Problem 60: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)^3 \left(c-c\operatorname{Sec}[e+fx]\right)^3} \, dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$\frac{Csc\,[\,e+f\,x\,]}{a^3\,c^3\,f} - \frac{2\,Csc\,[\,e+f\,x\,]^{\,3}}{3\,a^3\,c^3\,f} + \frac{Csc\,[\,e+f\,x\,]^{\,5}}{5\,a^3\,c^3\,f}$$

$$-\frac{1}{a^{3} c^{3}} \left( -\frac{89 \, \text{Cot} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{240 \, f} + \frac{31 \, \text{Cot} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \, \text{Csc} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]^{2}}{480 \, f} - \frac{\text{Cot} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \, \text{Csc} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{160 \, f} - \frac{89 \, \text{Tan} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{240 \, f} + \frac{31 \, \text{Sec} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]^{2} \, \text{Tan} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{480 \, f} - \frac{\text{Sec} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{160 \, f} + \frac{160 \, f}{160 \, f}$$

#### Problem 61: Result more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^3\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^4}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 99 leaves, 7 step

$$-\frac{\text{Cot}\,[\,e+f\,x\,]^{\,7}}{7\,\,a^3\,\,c^4\,f} + \frac{\text{Csc}\,[\,e+f\,x\,]}{a^3\,\,c^4\,f} - \frac{\text{Csc}\,[\,e+f\,x\,]^{\,3}}{a^3\,\,c^4\,f} + \frac{3\,\text{Csc}\,[\,e+f\,x\,]^{\,5}}{5\,\,a^3\,\,c^4\,f} - \frac{\text{Csc}\,[\,e+f\,x\,]^{\,7}}{7\,\,a^3\,\,c^4\,f}$$

Result (type 3, 211 leaves):

$$\frac{1}{35\,840\,a^3\,c^4\,f}\,Csc\left[e\right]\,Csc\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]^2\,Csc\left[e+f\,x\right]^5 \\ \left(2912\,Sin\left[e\right]\,+\,416\,Sin\left[f\,x\right]\,-\,7620\,Sin\left[e+f\,x\right]\,+\,1905\,Sin\left[2\,\left(e+f\,x\right)\,\right]\,+\,\\ 3810\,Sin\left[3\,\left(e+f\,x\right)\,\right]\,-\,1524\,Sin\left[4\,\left(e+f\,x\right)\,\right]\,-\,762\,Sin\left[5\,\left(e+f\,x\right)\,\right]\,+\,\\ 381\,Sin\left[6\,\left(e+f\,x\right)\,\right]\,-\,2016\,Sin\left[2\,e+f\,x\right]\,+\,2080\,Sin\left[e+2\,f\,x\right]\,-\,1680\,Sin\left[3\,e+2\,f\,x\right]\,+\,\\ 240\,Sin\left[2\,e+3\,f\,x\right]\,+\,560\,Sin\left[4\,e+3\,f\,x\right]\,-\,880\,Sin\left[3\,e+4\,f\,x\right]\,+\,\\ 560\,Sin\left[5\,e+4\,f\,x\right]\,+\,400\,Sin\left[4\,e+5\,f\,x\right]\,-\,560\,Sin\left[6\,e+5\,f\,x\right]\,+\,80\,Sin\left[5\,e+6\,f\,x\right]\,\right)$$

### Problem 62: Result more than twice size of optimal antiderivative.

$$\int\! \frac{\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^3\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^5}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 120 leaves, 10 steps)

$$\frac{2\,Cot\,[\,e\,+\,f\,x\,]^{\,9}}{9\,\,a^{3}\,\,c^{5}\,\,f}\,+\,\frac{Csc\,[\,e\,+\,f\,x\,]}{a^{3}\,\,c^{5}\,\,f}\,-\,\frac{5\,Csc\,[\,e\,+\,f\,x\,]^{\,3}}{3\,\,a^{3}\,\,c^{5}\,\,f}\,+\,\frac{9\,Csc\,[\,e\,+\,f\,x\,]^{\,5}}{5\,\,a^{3}\,\,c^{5}\,\,f}\,-\,\frac{Csc\,[\,e\,+\,f\,x\,]^{\,7}}{a^{3}\,\,c^{5}\,\,f}\,+\,\frac{2\,Csc\,[\,e\,+\,f\,x\,]^{\,9}}{9\,\,a^{3}\,\,c^{5}\,\,f}$$

#### Result (type 3, 257 leaves):

```
\frac{-}{184\,320\,a^3\,c^5\,f\,\left(-1+Sec\,[\,e+f\,x\,]\,\right)^5\,\left(1+Sec\,[\,e+f\,x\,]\,\right)^3}
   Csc[e] Sec[e+fx]^7 (-33024 Sin[e] + 6144 Sin[fx] + 76455 Sin[e+fx] -
                         33 980 Sin[2(e+fx)] - 32 281 Sin[3(e+fx)] + 27 184 Sin[4(e+fx)] +
                        1699 \sin[5(e+fx)] - 6796 \sin[6(e+fx)] + 1699 \sin[7(e+fx)] + 22656 \sin[2e+fx] - 6796 \sin[6(e+fx)] + 6796 \sin[6(e+
                        17216 \sin[e + 2fx] + 4416 \sin[3e + 2fx] + 3200 \sin[2e + 3fx] - 15360 \sin[4e + 3fx] +
                        12160 Sin[3 e + 4 f x] - 1920 Sin[5 e + 4 f x] - 5120 Sin[4 e + 5 f x] + 5760 Sin[6 e + 5 f x] +
                        320 \sin[5 e + 6 f x] - 2880 \sin[7 e + 6 f x] + 640 \sin[6 e + 7 f x] Tan[e + f x]
```

Problem 68: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]\,\,\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)}{\sqrt{\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]}}\,\,\mathrm{d} x$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{2\sqrt{2} \text{ a ArcTan}\Big[\frac{\sqrt{c} \text{ Tan}[e+fx]}{\sqrt{2} \sqrt{c-c \, \text{Sec}[e+fx]}}\Big]}{\sqrt{c} \text{ f}} + \frac{2 \text{ a Tan}[e+fx]}{f\sqrt{c-c \, \text{Sec}[e+fx]}}$$

Result (type 3, 167 leaves):

$$-\left(\left(\text{i}\;\sqrt{2}\;\;\text{a}\;\left(-1+\text{e}^{\text{i}\;\left(e+f\,x\right)}\right)\right.\right.\\ \left.\left(\sqrt{2}\;\;\left(1+\text{e}^{\text{i}\;\left(e+f\,x\right)}\right)+2\,\sqrt{1+\text{e}^{2\,\text{i}\;\left(e+f\,x\right)}}\;\;\text{Log}\left[1-\text{e}^{\text{i}\;\left(e+f\,x\right)}\right]-2\,\sqrt{1+\text{e}^{2\,\text{i}\;\left(e+f\,x\right)}}\right]\right.\\ \left.\left.\left.\left(1+\text{e}^{\text{i}\;\left(e+f\,x\right)}\right)+\sqrt{2}\,\sqrt{1+\text{e}^{2\,\text{i}\;\left(e+f\,x\right)}}\;\right]\right)\right)\right/\left(\left(1+\text{e}^{2\,\text{i}\;\left(e+f\,x\right)}\right)\,f\,\sqrt{c-c\,\text{Sec}\left[e+f\,x\right]}\right)\right)$$

Problem 69: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)}{\left(c-c\operatorname{Sec}[e+fx]\right)^{3/2}} \, dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{\text{a ArcTan}\Big[\frac{\sqrt{c \ \text{Tan}\left[e+f\,x\right]}}{\sqrt{2} \ \sqrt{c-c \ \text{Sec}\left[e+f\,x\right]}}\Big]}{\sqrt{2} \ c^{3/2} \ f} - \frac{\text{a Tan}\left[e+f\,x\right]}{\text{f} \left(c-c \ \text{Sec}\left[e+f\,x\right]\right)^{3/2}}$$

Result (type 3, 298 leaves):

$$\begin{split} a &\left( \left[ 2\,e^{-\frac{1}{2}\,i\;\left(e+f\,x\right)}\,\sqrt{\frac{e^{i\;\left(e+f\,x\right)}}{1+e^{2\,i\;\left(e+f\,x\right)}}}\,\,\sqrt{1+e^{2\,i\;\left(e+f\,x\right)}} \right. \\ &\left. \left( Log\left[ 1-e^{i\;\left(e+f\,x\right)}\,\right] - Log\left[ 1+e^{i\;\left(e+f\,x\right)} + \sqrt{2}\,\,\sqrt{1+e^{2\,i\;\left(e+f\,x\right)}}\,\,\right] \right) \right. \\ &\left. Sec\left[ e+f\,x\right]^{3/2}\,Sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^3 \right/ \left( f\left( c-c\,Sec\left[ e+f\,x\right] \right)^{3/2} \right) + \\ &\left. \left( Sec\left[ e+f\,x\right]^2 \left( \frac{4\,Cos\left[ \frac{e}{2}\right]\,Cos\left[ \frac{f\,x}{2}\right]}{f} - \frac{2\,Cot\left[ \frac{e}{2}\right]\,Csc\left[ \frac{e}{2} + \frac{f\,x}{2}\right]}{f} + \frac{2\,Csc\left[ \frac{e}{2}\right]\,Csc\left[ \frac{e}{2} + \frac{f\,x}{2}\right]^2\,Sin\left[ \frac{f\,x}{2}\right]}{f} - \frac{4\,Sin\left[ \frac{e}{2}\right]\,Sin\left[ \frac{f\,x}{2}\right]}{f} \right) Sin\left[ \frac{e}{2} + \frac{f\,x}{2}\right]^3 \right/ \left( c-c\,Sec\left[ e+f\,x\right] \right)^{3/2} \right) \end{split}$$

Problem 70: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]\,\,\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)}{\left(\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{5/2}}\,\,\mathrm{d} x$$

Optimal (type 3, 113 leaves, 4 steps):

Result (type 3, 362 leaves):

$$\begin{split} a\left(\left[e^{-\frac{1}{2}\frac{i}{i}\cdot(e+fx)}\sqrt{\frac{e^{i\cdot(e+fx)}}{1+e^{2\,i\cdot(e+fx)}}}\right.\sqrt{1+e^{2\,i\cdot(e+fx)}}\right.\\ &\left.\left(-Log\left[1-e^{i\cdot(e+fx)}\right]+Log\left[1+e^{i\cdot(e+fx)}+\sqrt{2}\right.\sqrt{1+e^{2\,i\cdot(e+fx)}}\right]\right)\right)\\ Sec\left[e+fx\right]^{5/2}Sin\left[\frac{e}{2}+\frac{fx}{2}\right]^{5} \right/\left(2\,f\left(c-c\,Sec\left[e+fx\right]\right)^{5/2}\right)+\\ &\left.\left(Sec\left[e+fx\right]^{3}\left(-\frac{3\,Cos\left[\frac{e}{2}\right]\,Cos\left[\frac{fx}{2}\right]}{f}+\frac{7\,Cot\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]}{2\,f}-\frac{Cot\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{fx}{2}\right]^{3}}{f}-\frac{7\,Cosc\left[\frac{e}{2}\right]\,Cs$$

Problem 75: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \; \left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^2}{\sqrt{\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]}} \, \mathrm{d} x$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{4\sqrt{2} \text{ a}^2 \operatorname{ArcTan} \left[\frac{\sqrt{c} \text{ Tan} [e+fx]}{\sqrt{2} \sqrt{c-c} \operatorname{Sec} [e+fx]}\right]}{\sqrt{c} \text{ f}} + \frac{16 \text{ a}^2 \operatorname{Tan} [e+fx]}{3 \text{ f} \sqrt{c-c} \operatorname{Sec} [e+fx]} - \frac{2 \text{ a}^2 \sqrt{c-c} \operatorname{Sec} [e+fx]}{3 \text{ c} \text{ f}} \operatorname{Tan} [e+fx]$$

Result (type 3, 292 leaves):

Problem 76: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{2}}{\left(c-c\operatorname{Sec}[e+fx]\right)^{3/2}} dx$$

Optimal (type 3, 113 leaves, 4 steps):

$$\frac{3\,\sqrt{2}\,\,a^{2}\,\text{ArcTan}\big[\frac{\sqrt{c}\,\,\text{Tan}\,[e+f\,x]}{\sqrt{2}\,\,\sqrt{c-c}\,\,\text{Sec}\,[e+f\,x]}\big]}{c^{3/2}\,f} - \frac{2\,a^{2}\,\,\text{Tan}\,[\,e+f\,x\,]}{f\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{3/2}} - \frac{2\,a^{2}\,\,\text{Tan}\,[\,e+f\,x\,]}{c\,\,f\,\,\sqrt{c-c}\,\,\text{Sec}\,[\,e+f\,x\,]}$$

Result (type 3, 337 leaves):

$$\left(3 e^{-\frac{1}{2} i \left(e+fx\right)} \sqrt{\frac{e^{i \left(e+fx\right)}}{1+e^{2 i \left(e+fx\right)}}} \sqrt{1+e^{2 i \left(e+fx\right)}} \right. \\ \left(Log\left[1-e^{i \left(e+fx\right)}\right] - Log\left[1+e^{i \left(e+fx\right)} + \sqrt{2} \sqrt{1+e^{2 i \left(e+fx\right)}}\right]\right) Sec\left[\frac{e}{2} + \frac{fx}{2}\right] \\ \left(a+a Sec\left[e+fx\right]\right)^2 Tan\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right/ \left(f\sqrt{Sec\left[e+fx\right]} \left(c-c Sec\left[e+fx\right]\right)^{3/2}\right) + \\ \left(Sec\left[\frac{e}{2} + \frac{fx}{2}\right] \left(a+a Sec\left[e+fx\right]\right)^2 \left(\frac{4 Cos\left[\frac{e}{2}\right] Cos\left[\frac{fx}{2}\right]}{f} - \frac{Cot\left[\frac{e}{2}\right] Csc\left[\frac{e}{2} + \frac{fx}{2}\right]}{f} + \\ \frac{Csc\left[\frac{e}{2}\right] Csc\left[\frac{e}{2} + \frac{fx}{2}\right]^2 Sin\left[\frac{fx}{2}\right]}{f} - \frac{4 Sin\left[\frac{e}{2}\right] Sin\left[\frac{fx}{2}\right]}{f} \right) Tan\left[\frac{e}{2} + \frac{fx}{2}\right]^3 \right/ \left(c-c Sec\left[e+fx\right]\right)^{3/2}$$

Problem 77: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]\,\,\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{\,2}}{\left(\mathsf{c} - \mathsf{c}\,\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{\,5/2}}\,\,\mathrm{d}x$$

Optimal (type 3, 117 leaves, 4 steps):

$$-\frac{3 \ a^2 \, \text{ArcTan} \big[ \frac{\sqrt{c} \, \, \text{Tan} \, [e+f \, x]}{\sqrt{2} \, \, \sqrt{c-c} \, \text{Sec} \, [e+f \, x]} \, \big]}{4 \, \sqrt{2} \, \, c^{5/2} \, f} \, - \, \frac{a^2 \, \text{Tan} \, [\, e+f \, x\,]}{f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{5/2}} \, + \, \frac{5 \, a^2 \, \text{Tan} \, [\, e+f \, x\,]}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, f \, c\, \text{Sec} \, [\, e+f \, x\,] \, \right)^{3/2}} \, + \, \frac{1}{4 \, c \, f \, \left(c-c \, f \, c\, f \,$$

Result (type 3, 378 leaves):

$$\begin{split} &\frac{1}{4\,c^{2}\,f\left(-1+\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{\,2}\,\sqrt{c\,-\,c\,\mathsf{Sec}\,[\,e+f\,x\,]}} \\ &a^{2}\,\,e^{-\frac{1}{2}\,i\,\,(\,e+f\,x)}\,\,\mathsf{Csc}\,\Big[\frac{e}{2}\Big]\,\,\mathsf{Sec}\,\Big[\frac{1}{2}\,\,\big(\,e+f\,x\big)\,\Big]^{\,3}\,\,\sqrt{\,\mathsf{Sec}\,[\,e+f\,x\,]}\,\,\big(1+\mathsf{Sec}\,[\,e+f\,x\,]\,\big)^{\,2} \\ &\left(\left(e^{-\frac{3\,i\,e}{2}}\,\left(-1+e^{i\,e}\right)\,\left(\mathsf{Cos}\,\Big[\frac{f\,x}{2}\,\Big]+i\,\,\mathsf{Sin}\,\Big[\frac{f\,x}{2}\,\Big]\right)\,\left(-9\,i\,\,e^{i\,e}\,\,\big(1+e^{i\,e}\big)\,\,\mathsf{Cos}\,\Big[\frac{f\,x}{2}\,\Big]+i\,\,\big(1+e^{3\,i\,e}\big) \right. \\ &\left. \mathsf{Cos}\,\Big[\frac{3\,f\,x}{2}\,\Big]-9\,\,e^{i\,e}\,\mathsf{Sin}\,\Big[\frac{f\,x}{2}\,\Big]+9\,\,e^{2\,i\,e}\,\mathsf{Sin}\,\Big[\frac{f\,x}{2}\,\Big]+\mathsf{Sin}\,\Big[\frac{3\,f\,x}{2}\,\Big]-e^{3\,i\,e}\,\mathsf{Sin}\,\Big[\frac{3\,f\,x}{2}\,\Big]\right)\right)\right/ \\ &\left. \left(16\,\sqrt{\,\mathsf{Sec}\,[\,e+f\,x\,]\,}\,\right)+3\,\,\sqrt{\frac{e^{i\,\,(e+f\,x)}}{1+e^{2\,i\,\,(e+f\,x)}}}\,\,\sqrt{1+e^{2\,i\,\,(e+f\,x)}}\,\,\left(-\mathsf{Log}\,\big[\,1-e^{i\,\,(e+f\,x)}\,\big]\,+\mathsf{Log}\,\Big[\frac{1}{2}\,\,\big(\,e+f\,x\,\big)\,\,\Big]\right. \\ &\left. \mathsf{Log}\,\Big[1+e^{i\,\,(e+f\,x)}\,+\sqrt{2}\,\,\sqrt{1+e^{2\,i\,\,(e+f\,x)}}\,\,\Big]\right)\,\mathsf{Sin}\,\Big[\frac{e}{2}\,\Big]\,\mathsf{Sin}\,\Big[\frac{1}{2}\,\,\big(\,e+f\,x\,\big)\,\,\Big]^{\,4}\,\,\mathsf{Tan}\,\Big[\frac{1}{2}\,\,\big(\,e+f\,x\,\big)\,\,\Big] \end{aligned}$$

Problem 78: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{2}}{\left(c-c\operatorname{Sec}[e+fx]\right)^{7/2}} \, dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$-\frac{a^2\,\text{ArcTan}\!\left[\frac{\sqrt{c}\,\,\text{Tan}[e+f\,x]}{\sqrt{2}\,\,\sqrt{c-c\,\text{Sec}[e+f\,x]}}\right]}{16\,\,\sqrt{2}\,\,c^{7/2}\,f} - \frac{\left(a^2+a^2\,\text{Sec}[e+f\,x]\right)\,\,\text{Tan}[e+f\,x]}{3\,f\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{7/2}} + \\ \frac{a^2\,\,\text{Tan}[e+f\,x]}{4\,c\,f\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{5/2}} - \frac{a^2\,\,\text{Tan}[e+f\,x]}{16\,c^2\,f\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}}$$

Result (type 3, 486 leaves):

$$\left( e^{-\frac{1}{2}i \; (e+fx)} \sqrt{\frac{e^{i \; (e+fx)}}{1+e^{2i \; (e+fx)}}} \sqrt{1+e^{2i \; (e+fx)}} \right) \sqrt{1+e^{2i \; (e+fx)}}$$

$$\left( -Log \left[ 1 - e^{i \; (e+fx)} \right] + Log \left[ 1 + e^{i \; (e+fx)} + \sqrt{2} \; \sqrt{1+e^{2i \; (e+fx)}} \right] \right) Sec \left[ e + fx \right]^{3/2}$$

$$\left( a + a \, Sec \left[ e + fx \right] \right)^2 Sin \left[ \frac{e}{2} + \frac{fx}{2} \right]^3 Tan \left[ \frac{e}{2} + \frac{fx}{2} \right]^4 \right) / \left( 8 \; f \; \left( c - c \, Sec \left[ e + fx \right] \right)^{7/2} \right) +$$

$$\left[ Sec \left[ e + fx \right]^2 \left( a + a \, Sec \left[ e + fx \right] \right)^2 \left( \frac{7 \, Cos \left[ \frac{e}{2} \right] \, Cos \left[ \frac{e}{2} \right] \, Cos \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right] \right) +$$

$$\frac{17 \, Cot \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^3}{12 \; f} - \frac{Cot \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^5}{3 \; f} + \frac{43 \, Csc \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \, Sin \left[ \frac{fx}{2} \right] -$$

$$\frac{17 \, Csc \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^4 \, Sin \left[ \frac{fx}{2} \right]}{12 \; f} + \frac{Csc \left[ \frac{e}{2} \right] \, Csc \left[ \frac{e}{2} + \frac{fx}{2} \right]^6 \, Sin \left[ \frac{fx}{2} \right] -$$

$$\frac{7 \, Sin \left[ \frac{e}{2} \right] \, Sin \left[ \frac{fx}{2} \right]}{12 \; f} \right)$$

$$Sin \left[ \frac{e}{2} + \frac{fx}{2} \right]^3 \, Tan \left[ \frac{e}{2} + \frac{fx}{2} \right]^4 \right) / \left( c - c \, Sec \left[ e + fx \right] \right)^{7/2}$$

# Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{Sec \left[\,e + f\,x\,\right] \, \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,3}}{\sqrt{c - c\,Sec \left[\,e + f\,x\,\right]}} \, \mathrm{d}x$$

Optimal (type 3, 164 leaves, 5 steps):

$$-\frac{8\sqrt{2} \ a^{3} \operatorname{ArcTan} \left[ \frac{\sqrt{c} \ \operatorname{Tan} \left[ e+f \, x \right]}{\sqrt{2} \ \sqrt{c-c} \operatorname{Sec} \left[ e+f \, x \right]} \right]}{\sqrt{c} \ f} + \frac{8 \ a^{3} \ \operatorname{Tan} \left[ e+f \, x \right]}{f \sqrt{c-c} \operatorname{Sec} \left[ e+f \, x \right]} + \frac{2 \ a \ \left( a+a \operatorname{Sec} \left[ e+f \, x \right] \right)^{2} \operatorname{Tan} \left[ e+f \, x \right]}{f \sqrt{c-c} \operatorname{Sec} \left[ e+f \, x \right]} + \frac{4 \ \left( a^{3}+a^{3} \operatorname{Sec} \left[ e+f \, x \right] \right) \operatorname{Tan} \left[ e+f \, x \right]}{3 \ f \sqrt{c-c} \operatorname{Sec} \left[ e+f \, x \right]}$$

Result (type 3, 223 leaves):

$$\begin{split} -\left(\left(2 \text{ is } \text{ a}^{3} \text{ } \left(-1+\text{e}^{\text{i } (\text{e+f} \, x)}\right) \text{ } \left(73+105 \text{ e}^{\text{i } (\text{e+f} \, x)}+190 \text{ e}^{2 \text{ i } (\text{e+f} \, x)}+190 \text{ e}^{3 \text{ i } (\text{e+f} \, x)}+190 \text{ e}^{3 \text{ i } (\text{e+f} \, x)}\right) + \\ 105 \text{ e}^{4 \text{ i } (\text{e+f} \, x)}+73 \text{ e}^{5 \text{ i } (\text{e+f} \, x)}+60 \sqrt{2} \text{ } \left(1+\text{e}^{2 \text{ i } (\text{e+f} \, x)}\right)^{5/2} \text{ Log} \left[1-\text{e}^{\text{i } (\text{e+f} \, x)}\right] - \\ 60 \sqrt{2} \text{ } \left(1+\text{e}^{2 \text{ i } (\text{e+f} \, x)}\right)^{5/2} \text{ Log} \left[1+\text{e}^{\text{i } (\text{e+f} \, x)}+\sqrt{2} \text{ } \sqrt{1+\text{e}^{2 \text{ i } (\text{e+f} \, x)}}\right]\right) \bigg) \\ \left(15 \text{ } \left(1+\text{e}^{2 \text{ i } (\text{e+f} \, x)}\right)^{3} \text{ f } \sqrt{\text{c}-\text{c} \text{Sec} \left[\text{e+f} \, x\right]}\right)\right) \end{split}$$

Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] (a+a\operatorname{Sec}[e+fx])^3}{(c-c\operatorname{Sec}[e+fx])^{3/2}} dx$$

Optimal (type 3, 168 leaves, 5 steps):

$$\frac{10\,\sqrt{2}\,\,\mathsf{a}^3\,\mathsf{ArcTan}\Big[\frac{\sqrt{c\,\,\mathsf{Tan}[e+f\,x]}}{\sqrt{2}\,\,\sqrt{\mathsf{c-c}\,\mathsf{Sec}[e+f\,x]}}\Big]}{\mathsf{c}^{3/2}\,\mathsf{f}} - \frac{\mathsf{a}\,\,\big(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[e+f\,x]\,\big)^2\,\mathsf{Tan}\,[e+f\,x]}{\mathsf{f}\,\,\big(\mathsf{c}-\mathsf{c}\,\,\mathsf{Sec}\,[e+f\,x]\,\big)^{3/2}} - \frac{10\,\,\mathsf{a}^3\,\mathsf{Tan}\,[e+f\,x]}{\mathsf{c}\,\mathsf{f}\,\,\sqrt{\mathsf{c-c}\,\mathsf{Sec}\,[e+f\,x]}} - \frac{\mathsf{5}\,\,\big(\mathsf{a}^3+\mathsf{a}^3\,\mathsf{Sec}\,[e+f\,x]\,\big)\,\,\mathsf{Tan}\,[e+f\,x]}{3\,\,\mathsf{c}\,\mathsf{f}\,\,\sqrt{\mathsf{c-c}\,\mathsf{Sec}\,[e+f\,x]}} - \frac{\mathsf{5}\,\,\big(\mathsf{a}^3+\mathsf{a}^3\,\mathsf{Sec}\,[e+f\,x]\,\big)\,\,\mathsf{Tan}\,[e+f\,x]}{3\,\,\mathsf{c}\,\mathsf{f}\,\,\sqrt{\mathsf{c-c}\,\mathsf{Sec}\,[e+f\,x]}} - \frac{\mathsf{5}\,\,\mathsf{c}$$

Result (type 3, 377 leaves):

$$\left( 5 e^{-\frac{1}{2} i \left( e + f x \right)} \sqrt{\frac{e^{i \cdot \left( e + f x \right)}}{1 + e^{2 \cdot i \cdot \left( e + f x \right)}}} \sqrt{1 + e^{2 \cdot i \cdot \left( e + f x \right)}} \right) \sqrt{1 + e^{2 \cdot i \cdot \left( e + f x \right)}}$$

$$\left( Log \left[ 1 - e^{i \cdot \left( e + f x \right)} \right] - Log \left[ 1 + e^{i \cdot \left( e + f x \right)} + \sqrt{2} \sqrt{1 + e^{2 \cdot i \cdot \left( e + f x \right)}} \right] \right) Sec \left[ \frac{e}{2} + \frac{f x}{2} \right]^3$$

$$\left( a + a Sec \left[ e + f x \right] \right)^3 Tan \left[ \frac{e}{2} + \frac{f x}{2} \right]^3 \right) / \left( f Sec \left[ e + f x \right]^{3/2} \left( c - c Sec \left[ e + f x \right] \right)^{3/2} \right) +$$

$$\left( Cos \left[ e + f x \right] Sec \left[ \frac{e}{2} + \frac{f x}{2} \right]^3 \left( a + a Sec \left[ e + f x \right] \right)^3 \right)$$

$$\left( \frac{19 Cos \left[ \frac{e}{2} \right] Cos \left[ \frac{f x}{2} \right]}{3 \cdot f} - \frac{Cot \left[ \frac{e}{2} \right] Csc \left[ \frac{e}{2} + \frac{f x}{2} \right]}{f} + \frac{Cos \left[ \frac{e}{2} + \frac{f x}{2} \right] Sec \left[ e + f x \right]}{3 \cdot f} \right) +$$

$$\frac{Csc \left[ \frac{e}{2} \right] Csc \left[ \frac{e}{2} + \frac{f x}{2} \right]^2 Sin \left[ \frac{f x}{2} \right]}{f} - \frac{19 Sin \left[ \frac{e}{2} \right] Sin \left[ \frac{f x}{2} \right]}{3 \cdot f} \right) Tan \left[ \frac{e}{2} + \frac{f x}{2} \right]^3 / \left( c - c Sec \left[ e + f x \right] \right)^{3/2}$$

Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{3}}{\left(c-c\operatorname{Sec}[e+fx]\right)^{5/2}} dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$-\frac{15 \ a^{3} \ ArcTan \Big[\frac{\sqrt{c} \ Tan [e+fx]}{\sqrt{2} \ \sqrt{c-c} \ Sec [e+fx]}\Big]}{2 \ \sqrt{2} \ c^{5/2} \ f} - \frac{a \ \Big(a+a \ Sec [e+fx]\Big)^{2} \ Tan [e+fx]}{2 \ f \ \Big(c-c \ Sec [e+fx]\Big)^{5/2}} + \\ \frac{5 \ \Big(a^{3}+a^{3} \ Sec [e+fx]\Big) \ Tan [e+fx]}{4 \ c \ f \ \Big(c-c \ Sec [e+fx]\Big)^{3/2}} + \frac{15 \ a^{3} \ Tan [e+fx]}{4 \ c^{2} \ f \ \sqrt{c-c} \ Sec [e+fx]}$$

#### Result (type 3, 411 leaves):

$$\left(15\,e^{-\frac{1}{2}\,\mathrm{i}\,\left(e+f\,x\right)}\,\sqrt{\frac{e^{\mathrm{i}\,\left(e+f\,x\right)}}{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}}\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\right. \\ \left(Log\left[1-e^{\mathrm{i}\,\left(e+f\,x\right)}\,\right]-Log\left[1+e^{\mathrm{i}\,\left(e+f\,x\right)}+\sqrt{2}\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\right]\right)\,Sec\left[\frac{e}{2}+\frac{f\,x}{2}\right] \\ \left(a+a\,Sec\left[e+f\,x\right]\right)^{3}\,Tan\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{5} \right/\left(4\,f\,\sqrt{Sec\left[e+f\,x\right]}\,\left(c-c\,Sec\left[e+f\,x\right]\right)^{5/2}\right) + \\ \left[Sec\left[\frac{e}{2}+\frac{f\,x}{2}\right]\,\left(a+a\,Sec\left[e+f\,x\right]\right)^{3}\left(\frac{9\,Cos\left[\frac{e}{2}\right]\,Cos\left[\frac{f\,x}{2}\right]}{2\,f}-\frac{Cot\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{4\,f}-\frac{Cot\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{3}}{2\,f}+\frac{Csc\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{2}\,Sin\left[\frac{f\,x}{2}\right]}{4\,f}-\frac{9\,Sin\left[\frac{e}{2}\right]\,Sin\left[\frac{f\,x}{2}\right]}{2\,f}\right)\,Tan\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{5} \right/\left(c-c\,Sec\left[e+f\,x\right]\right)^{5/2} \\ \right.$$

Problem 90: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)\,\sqrt{\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}}\,\,\mathrm{d} \mathsf{x}$$

Optimal (type 3, 89 leaves, 3 steps):

$$-\frac{\text{ArcTan}\Big[\frac{\sqrt{c \text{ Tan}[e+fx]}}{\sqrt{2} \sqrt{c-c \text{ Sec}[e+fx]}}\Big]}{\sqrt{2} \text{ a } \sqrt{c} \text{ f}} + \frac{\text{Tan}[e+fx]}{\text{f} \left(\text{a}+\text{a } \text{Sec}[e+fx]\right) \sqrt{c-c \text{ Sec}[e+fx]}}$$

Result (type 3, 204 leaves):

$$-\left(\left(\text{i}\left(-1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\right)\,\left(\sqrt{2}\,\left(1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\right)\,+\,\left(1+\text{e}^{\text{i}\,\left(e+f\,x\right)}\right)\,\sqrt{1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}}\right.\text{Log}\left[1-\text{e}^{\text{i}\,\left(e+f\,x\right)}\right]\,-\,\left(1+\text{e}^{\text{i}\,\left(e+f\,x\right)}\right)\,\sqrt{1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}}\right.\text{Log}\left[1+\text{e}^{\text{i}\,\left(e+f\,x\right)}\right.+\,\sqrt{2}\,\sqrt{1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}}\right]\right)\right)\right/\\ \left(\sqrt{2}\,\,a\,\left(1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\right)^{2}\,f\,\left(1+\text{Sec}\left[e+f\,x\right]\right)\,\sqrt{c-c\,\text{Sec}\left[e+f\,x\right]}\right)\right)$$

### Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)\left(c-c\operatorname{Sec}[e+fx]\right)^{3/2}} \, dx$$

Optimal (type 3, 122 leaves, 4 steps):

$$-\frac{3\,\text{ArcTan}\Big[\frac{\sqrt{c}\,\,\text{Tan}[e+f\,x]}{\sqrt{2}\,\,\sqrt{c-c\,\,\text{Sec}\,[e+f\,x]}}\Big]}{4\,\sqrt{2}\,\,a\,\,c^{3/2}\,f} - \\ \frac{3\,\text{Tan}[e+f\,x]}{4\,a\,f\,\left(c-c\,\,\text{Sec}\,[e+f\,x]\right)^{3/2}} + \frac{\text{Tan}[e+f\,x]}{f\,\left(a+a\,\,\text{Sec}\,[e+f\,x]\right)\,\left(c-c\,\,\text{Sec}\,[e+f\,x]\right)^{3/2}}$$

Result (type 3, 220 leaves):

$$\begin{split} -\left(\left(e^{-2\,\text{i}\,\,(e+f\,x)}\,\,\text{Csc}\left[\,2\,\left(e+f\,x\right)\,\right]\right. \\ &\left(3-8\,\,e^{\text{i}\,\,(e+f\,x)}\,-4\,\,e^{3\,\text{i}\,\,(e+f\,x)}\,+e^{4\,\text{i}\,\,(e+f\,x)}\,-2\,\,e^{\frac{3}{2}\,\text{i}\,\,(e+f\,x)}\,\left(-4+3\,\sqrt{2}\,\,\sqrt{1+e^{2\,\text{i}\,\,(e+f\,x)}}\right. \\ &\left. -4+3\,\sqrt{2}\,\,\sqrt{1+e^{2\,\text{i}\,\,(e+f\,x)}}\right] \\ &\left. -4+3\,\sqrt{2}\,\,\sqrt{1+e^{2\,\text{i}\,\,(e+f\,x)}}\right$$

# Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{5/2}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 156 leaves, 5 steps):

$$\frac{15\,\text{ArcTan}\Big[\frac{\sqrt{c\,\,\text{Tan}[e+f\,x]}}{\sqrt{2}\,\,\sqrt{c-c\,\,\text{Sec}[e+f\,x]}}\Big]}{32\,\sqrt{2}\,\,a\,\,c^{5/2}\,f} - \frac{5\,\,\text{Tan}[\,e+f\,x\,]}{8\,\,a\,\,f\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{5/2}} + \\ \frac{\text{Tan}[\,e+f\,x\,]}{f\,\,\big(\,a+a\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{5/2}} - \frac{15\,\,\text{Tan}\,[\,e+f\,x\,]}{32\,\,a\,\,c\,\,f\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{3/2}}$$

Result (type 3, 441 leaves):

$$\begin{split} &\left[15\,e^{-\frac{1}{2}\,\mathrm{i}\,\left(e+f\,x\right)}\,\sqrt{\frac{e^{\,\mathrm{i}\,\left(e+f\,x\right)}}{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}}\,\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\,\,\mathsf{Cos}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2\right.\\ &\left.\left.\left(\mathsf{Log}\left[1-e^{\,\mathrm{i}\,\left(e+f\,x\right)}\right]-\mathsf{Log}\left[1+e^{\,\mathrm{i}\,\left(e+f\,x\right)}+\sqrt{2}\,\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\,\,\right]\right)\mathsf{Sec}\left[e+f\,x\right]^{7/2}\,\mathsf{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^5\right]\right/\\ &\left.\left(4\,f\,\left(a+a\,\mathsf{Sec}\left[e+f\,x\right]\right)\,\left(c-c\,\mathsf{Sec}\left[e+f\,x\right]\right)^{5/2}\right)+\left(\mathsf{Cos}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^2\mathsf{Sec}\left[e+f\,x\right]^4\right.\\ &\left.\left(-\frac{3\,\mathsf{Cos}\left[\frac{e}{2}\right]\,\mathsf{Cos}\left[\frac{f\,x}{2}\right]}{2\,f}+\frac{15\,\mathsf{Cot}\left[\frac{e}{2}\right]\,\mathsf{Csc}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{4\,f}-\frac{\mathsf{Cot}\left[\frac{e}{2}\right]\,\mathsf{Csc}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^3}{2\,f}-\frac{2\,\mathsf{Sec}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{f}-\frac{15\,\mathsf{Cot}\left[\frac{e}{2}\right]\,\mathsf{Csc}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^3}{2\,f}-\frac{3\,\mathsf{Sin}\left[\frac{e}{2}\right]\,\mathsf{Sin}\left[\frac{f\,x}{2}\right]}{2\,f}\right.\\ &\left.\mathsf{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^5\right/\left(\left(a+a\,\mathsf{Sec}\left[e+f\,x\right]\right)\,\left(c-c\,\mathsf{Sec}\left[e+f\,x\right]\right)^{5/2}\right) \end{split}$$

Problem 97: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^2 \sqrt{\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}} \,\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 138 leaves, 4 steps):

$$-\frac{\text{ArcTan}\big[\frac{\sqrt{c} \ \text{Tan}[e+fx]}{\sqrt{2} \ \sqrt{c-c} \ \text{Sec}[e+fx]}\big]}{2 \ \sqrt{2} \ a^2 \ \sqrt{c} \ f} + \frac{\text{Tan}[e+fx]}{3 \ f \ (a+a \ \text{Sec}[e+fx])^2 \ \sqrt{c-c} \ \text{Sec}[e+fx]}} + \frac{\text{Tan}[e+fx]}{2 \ f \ (a^2+a^2 \ \text{Sec}[e+fx]) \ \sqrt{c-c} \ \text{Sec}[e+fx]}}$$

Result (type 3, 296 leaves):

$$\left[ 2 \, e^{-\frac{1}{2} \, i \, \left(e + f \, x\right)} \, \mathsf{Cos} \left[ \, \frac{1}{2} \, \left(e + f \, x\right) \, \right] \right. \\ \left. \left( \sqrt{\frac{e^{i \, \left(e + f \, x\right)}}{1 + e^{2 \, i \, \left(e + f \, x\right)}}} \, \mathsf{Cos} \left[ \, \frac{1}{2} \, \left(e + f \, x\right) \, \right]^{3} \, \left( 5 \, \sqrt{2} \, \left( 1 + e^{i \, \left(e + f \, x\right)} \right) + 3 \, \sqrt{1 + e^{2 \, i \, \left(e + f \, x\right)}} \, \mathsf{Log} \left[ 1 - e^{i \, \left(e + f \, x\right)} \, \right] - 3 \, \sqrt{1 + e^{2 \, i \, \left(e + f \, x\right)}} \, \mathsf{Log} \left[ 1 + e^{i \, \left(e + f \, x\right)} + \sqrt{2} \, \sqrt{1 + e^{2 \, i \, \left(e + f \, x\right)}} \, \right] \right) + e^{\frac{1}{2} \, i \, \left(e + f \, x\right)} \, \sqrt{\mathsf{Sec} \left[e + f \, x\right]} \, - 7 \, e^{\frac{1}{2} \, i \, \left(e + f \, x\right)} \, \mathsf{Cos} \left[ \frac{1}{2} \, \left(e + f \, x\right) \, \right]^{2} \, \sqrt{\mathsf{Sec} \left[e + f \, x\right]} \, \mathsf{Sec} \left[e + f \, x\right] \, \mathsf{Sec} \left[e$$

Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\text{Sec}\,[\,e+f\,x\,]}{\left(\,a+a\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,2}\,\left(\,c-c\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,3/2}}\,\,\text{d}x$$

Optimal (type 3, 169 leaves, 5 steps):

$$-\frac{5\,\text{ArcTan}\big[\frac{\sqrt{c}\,\text{Tan}[e+f\,x]}{\sqrt{2}\,\sqrt{c-c\,\text{Sec}[e+f\,x]}}\big]}{8\,\sqrt{2}\,\,a^2\,c^{3/2}\,f} - \frac{5\,\text{Tan}[e+f\,x]}{8\,a^2\,f\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}} + \\ \frac{\text{Tan}[e+f\,x]}{3\,f\,\left(a+a\,\text{Sec}[e+f\,x]\right)^2\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}} + \frac{5\,\text{Tan}[e+f\,x]}{6\,f\,\left(a^2+a^2\,\text{Sec}[e+f\,x]\right)\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}}$$

Result (type 3, 395 leaves):

$$-\left(\left[5\ e^{-\frac{1}{2} i\ (e+f\,x)}\ \sqrt{\frac{e^{i}\ (e+f\,x)}{1+e^{2\,i}\ (e+f\,x)}}\ \sqrt{1+e^{2\,i}\ (e+f\,x)}\ \cos\left[\frac{e}{2}+\frac{f\,x}{2}\right]^4\right.$$
 
$$\left(Log\left[1-e^{i\ (e+f\,x)}\right]-Log\left[1+e^{i\ (e+f\,x)}+\sqrt{2}\ \sqrt{1+e^{2\,i}\ (e+f\,x)}\ \right]\right)Sec\left[e+f\,x\right]^{7/2}Sin\left[\frac{e}{2}+\frac{f\,x}{2}\right]^3\right)\bigg/\left(f\left(a+a\,Sec\left[e+f\,x\right]\right)^2\left(c-c\,Sec\left[e+f\,x\right]\right)^{3/2}\right)+$$
 
$$\left(Cos\left[\frac{e}{2}+\frac{f\,x}{2}\right]^4Sec\left[e+f\,x\right]^4\left[-\frac{26\,Cos\left[\frac{e}{2}\right]\,Cos\left[\frac{f\,x}{2}\right]}{3\,f}-\frac{Cot\left[\frac{e}{2}\right]\,Csc\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{f}+\frac{20\,Sec\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{3\,f}-\frac{2\,Sec\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{3\,f}\right]$$
 
$$\left(\frac{e}{2}+\frac{f\,x}{2}\right]^3\bigg)\bigg/\left(\left(a+a\,Sec\left[e+f\,x\right]\right)^2\left(c-c\,Sec\left[e+f\,x\right]\right)^{3/2}\right)$$

Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{Sec}\left[\,e + f\,x\,\right]}{\left(\,a + a\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,2}\,\left(\,c - c\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,5/2}}\,\text{d}x$$

Optimal (type 3, 203 leaves, 6 steps):

$$-\frac{35\,\text{ArcTan}\Big[\frac{\sqrt{c\,\,\text{Tan}[e+f\,x]}}{\sqrt{2}\,\,\sqrt{c-c\,\,\text{Sec}[e+f\,x]}}\Big]}{64\,\sqrt{2}\,\,a^2\,\,c^{5/2}\,\,f} - \frac{35\,\text{Tan}\,[e+f\,x]}{48\,\,a^2\,\,f\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{5/2}} + \\ \frac{\text{Tan}\,[\,e+f\,x\,]}{3\,\,f\,\,\big(\,a+a\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{\,2}\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{\,5/2}} + \\ \frac{7\,\text{Tan}\,[\,e+f\,x\,]}{6\,\,f\,\,\big(\,a^2+a^2\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{\,5/2}} - \frac{35\,\text{Tan}\,[\,e+f\,x\,]}{64\,\,a^2\,\,c\,\,f\,\,\big(\,c-c\,\,\text{Sec}\,[\,e+f\,x\,]\,\big)^{\,3/2}}$$

Result (type 3, 465 leaves):

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int\! \frac{\operatorname{Sec}\left[\,e + f\,x\,\right]}{\left(\,a + a\,\operatorname{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,3}\,\sqrt{c - c\,\operatorname{Sec}\left[\,e + f\,x\,\right]}}\,\,\mathrm{d}x$$

Optimal (type 3, 181 leaves, 5 steps):

$$-\frac{\text{ArcTan}\Big[\frac{\sqrt{c} \; \text{Tan}[e+fx]}{\sqrt{2} \; \sqrt{c-c} \, \text{Sec}[e+fx]}\Big]}{4 \; \sqrt{2} \; a^3 \; \sqrt{c} \; f} + \frac{\text{Tan}[e+fx]}{5 \; f \; \left(a+a \, \text{Sec}[e+fx]\right)^3 \; \sqrt{c-c} \, \text{Sec}[e+fx]}} + \frac{\text{Tan}[e+fx]}{6 \; a \; f \; \left(a+a \, \text{Sec}[e+fx]\right)^2 \; \sqrt{c-c} \, \text{Sec}[e+fx]}} + \frac{\text{Tan}[e+fx]}{4 \; f \; \left(a^3+a^3 \, \text{Sec}[e+fx]\right) \; \sqrt{c-c} \, \text{Sec}[e+fx]}}$$

Result (type 3, 334 leaves):

### Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\!\frac{\mathsf{Sec}\,[\,e+f\,x\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^3\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{3/2}}\,\,\mathrm{d}x$$

#### Optimal (type 3, 212 leaves, 6 steps):

$$-\frac{7\,\text{ArcTan}\!\left[\frac{\sqrt{c}\,\text{Tan}[e+f\,x]}{\sqrt{2}\,\sqrt{c-c\,\text{Sec}[e+f\,x]}}\right]}{16\,\sqrt{2}\,\,a^3\,c^{3/2}\,f} - \frac{7\,\text{Tan}[e+f\,x]}{16\,a^3\,f\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}} + \\ \frac{\text{Tan}[e+f\,x]}{5\,f\,\left(a+a\,\text{Sec}[e+f\,x]\right)^3\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}} + \frac{7\,\text{Tan}[e+f\,x]}{30\,a\,f\,\left(a+a\,\text{Sec}[e+f\,x]\right)^2\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}} + \\ \frac{7\,\text{Tan}[e+f\,x]}{12\,f\,\left(a^3+a^3\,\text{Sec}[e+f\,x]\right)\,\left(c-c\,\text{Sec}[e+f\,x]\right)^{3/2}}$$

Result (type 3, 417 leaves):

$$-\left(\left[7\,e^{-\frac{1}{2}\,\mathrm{i}\,\left(e+f\,x\right)}\,\sqrt{\frac{e^{\,\mathrm{i}\,\left(e+f\,x\right)}}{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}}\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\,\operatorname{Cos}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{6}\right.\right.$$

$$\left.\left(Log\left[1-e^{\,\mathrm{i}\,\left(e+f\,x\right)}\,\right]-Log\left[1+e^{\,\mathrm{i}\,\left(e+f\,x\right)}+\sqrt{2}\,\sqrt{1+e^{2\,\mathrm{i}\,\left(e+f\,x\right)}}\,\right]\right)\operatorname{Sec}\left[e+f\,x\right]^{9/2}\operatorname{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{3}\right/\left.\right.$$

$$\left(f\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{3}\left(c-c\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}\right)\right.+\left.\left.\left(\operatorname{Cos}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{6}\operatorname{Sec}\left[e+f\,x\right]^{5}\left(-\frac{278\operatorname{Cos}\left[\frac{e}{2}\right]\operatorname{Cos}\left[\frac{f\,x}{2}\right]}{15\,f}-\frac{\operatorname{Cot}\left[\frac{e}{2}\right]\operatorname{Csc}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{f}+\frac{242\operatorname{Sec}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{15\,f}-\frac{56\operatorname{Sec}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{3}}{15\,f}+\frac{2\operatorname{Sec}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{5}}{5\,f}+\frac{\operatorname{Csc}\left[\frac{e}{2}\right]\operatorname{Csc}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{2}\operatorname{Sin}\left[\frac{f\,x}{2}\right]}{f}+\frac{278\operatorname{Sin}\left[\frac{e}{2}\right]\operatorname{Sin}\left[\frac{f\,x}{2}\right]}{15\,f}\right)$$

$$\operatorname{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]^{3}\right/\left(\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{3}\left(c-c\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{3}\,\left(c-c\operatorname{Sec}\left[e+f\,x\right]\right)^{5/2}}\,\mathrm{d}x$$

Optimal (type 3, 246 leaves, 7 steps):

$$-\frac{63 \, \text{ArcTan} \Big[ \frac{\sqrt{c} \, \text{Tan} [\text{e+fx}]}{\sqrt{2} \, \sqrt{\text{c-c} \, \text{Sec} [\text{e+fx}]}} \Big]}{128 \, \sqrt{2} \, \text{a}^3 \, \text{c}^{5/2} \, \text{f}} - \frac{21 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{f} \, \left(\text{c-c} \, \text{Sec} [\, \text{e+fx}] \, \right)^{5/2}} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{f} \, \left(\text{c-c} \, \text{Sec} [\, \text{e+fx}] \, \right)^{5/2}} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{f} \, \left(\text{c-c} \, \text{Sec} [\, \text{e+fx}] \, \right)^{5/2}} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{a}^3 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+fx}]}{32 \, \text{Tan} [\, \text{e+fx}]} + \frac{3 \, \text{Tan} [\, \text{e+f$$

Result (type 3, 487 leaves):

### Problem 107: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ e + f \, x \right] \, \sqrt{a + a \, Sec \left[ e + f \, x \right]} \, \left( c - c \, Sec \left[ e + f \, x \right] \right)^{5/2} \, \mathrm{d}x$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{a\,\left(c-c\,Sec\left[\,e+f\,x\,\right]\,\right)^{\,5/2}\,Tan\left[\,e+f\,x\,\right]}{3\,f\,\sqrt{a+a\,Sec\left[\,e+f\,x\,\right]}}$$

Result (type 3, 87 leaves):

$$\begin{split} &\frac{1}{12\,\mathsf{f}}c^2\,\left(\mathsf{5}-\mathsf{6}\,\mathsf{Cos}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,+3\,\mathsf{Cos}\,\big[\,\mathsf{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\right)\,\mathsf{Csc}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,\right)\,\big]} \\ &-\mathsf{Sec}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]^{\,2}\,\sqrt{\mathsf{a}\,\left(\,\mathsf{1}+\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)}\,\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]} \end{split}$$

## Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec} \left[ \, e + f \, x \, \right] \, \sqrt{a + a} \, \operatorname{Sec} \left[ \, e + f \, x \, \right]}{\sqrt{c - c} \, \operatorname{Sec} \left[ \, e + f \, x \, \right]} \, \, \mathrm{d} x$$

Optimal (type 3, 51 leaves, 1 step):

$$\frac{a \log [1 - Sec [e + f x]] Tan [e + f x]}{f \sqrt{a + a Sec [e + f x]}} \sqrt{c - c Sec [e + f x]}$$

Result (type 3, 99 leaves):

$$-\left(\left(\mathrm{i}\left(-1+\mathrm{e}^{\mathrm{i}\;\left(e+f\,x\right)}\right)\;\left(2\,Log\left[1-\mathrm{e}^{\mathrm{i}\;\left(e+f\,x\right)}\;\right]-Log\left[1+\mathrm{e}^{2\;\mathrm{i}\;\left(e+f\,x\right)}\;\right]\right)\;\sqrt{a\;\left(1+Sec\left[e+f\,x\right]\right)}\;\right)\right/\left(\left(1+\mathrm{e}^{\mathrm{i}\;\left(e+f\,x\right)}\right)\;f\;\sqrt{c\;-c\;Sec\left[e+f\,x\right]\;}\right)\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{Sec [e + f x] \left(a + a Sec [e + f x]\right)^{3/2}}{\sqrt{c - c Sec [e + f x]}} dx$$

Optimal (type 3, 95 leaves, 2 steps):

$$\frac{2 \, a^2 \, \mathsf{Log} \, [\, 1 \, - \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ] \, ] \, \, \mathsf{Tan} \, [\, e \, + \, f \, x \, ]}{f \, \sqrt{a \, + \, a} \, \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ]} \, \, + \, \frac{a \, \sqrt{a \, + \, a} \, \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ]}{f \, \sqrt{c \, - \, c} \, \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ]} \, \, + \, \frac{a \, \sqrt{a \, + \, a} \, \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ]}{f \, \sqrt{c \, - \, c} \, \, \mathsf{Sec} \, [\, e \, + \, f \, x \, ]}$$

Result (type 3, 174 leaves):

$$\begin{split} &\left(\sqrt{2}\ \mathsf{a}\ \left(1 + \mathsf{Cos}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]\ \left(4\,\mathsf{Log}\left[\,\mathsf{1} - \,\mathsf{e}^{\,\mathrm{i}\,\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)}\,\,\right] - 2\,\mathsf{Log}\left[\,\mathsf{1} + \,\mathsf{e}^{2\,\,\mathrm{i}\,\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)}\,\,\right]\,\right)\,\,\mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]^{\,3/2} \\ &\sqrt{\mathsf{a}\,\left(1 + \mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]\,\,\right)}\,\,\left(\mathsf{Cos}\left[\,\frac{1}{2}\,\left(\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right)\,\,\right] + \,\mathrm{i}\,\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right)\,\,\right]\,\right)\,\mathsf{Sin}\left[\,\frac{1}{2}\,\left(\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right)\,\,\right]\right) \\ &\left(\left(1 + \,\mathsf{e}^{\,\mathrm{i}\,\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)}\,\right)\,\,\sqrt{\,\frac{\,\mathsf{e}^{\,\mathrm{i}\,\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)}}{\,1 + \,\mathsf{e}^{2\,\,\mathrm{i}\,\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)}}}\,\,\mathsf{f}\,\,\sqrt{\,\mathsf{c} - \mathsf{c}\,\,\mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]}\,\,\right) \end{split}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec} \left[ e + f \, x \right] \, \left( a + a \operatorname{Sec} \left[ e + f \, x \right] \right)^{3/2}}{\left( c - c \operatorname{Sec} \left[ e + f \, x \right] \right)^{3/2}} \, \mathrm{d} x$$

Optimal (type 3, 99 leaves, 2 steps):

$$-\frac{a\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}\,\,\,\text{Tan}\,[\,e+f\,x\,]}{f\,\left(c-c\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,3/2}}\,-\frac{a^2\,\text{Log}\,[\,1-\text{Sec}\,[\,e+f\,x\,]\,\,]\,\,\,\text{Tan}\,[\,e+f\,x\,]}{c\,f\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}\,\,\,\sqrt{c-c\,\text{Sec}\,[\,e+f\,x\,]}}$$

Result (type 3, 134 leaves):

$$-\left(\left(a\;\left(2-2\;Log\left[1-\mathrm{e}^{\mathrm{i}\;\left(e+f\,x\right)}\;\right]\right.\right.\right.\\ \left.\left.\left.\left(2\;Log\left[1-\mathrm{e}^{\mathrm{i}\;\left(e+f\,x\right)}\;\right]-Log\left[1+\mathrm{e}^{2\,\mathrm{i}\;\left(e+f\,x\right)}\;\right]\right)\right.\\ \left.\left.\left.\left(1+\mathsf{Sec}\left[e+f\,x\right]\right)\right.\right.\right.\\ \left.\left.\left(2\,\mathsf{c}\,\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right.\right.\\ \left.\left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right.\\ \left.\left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right.\\ \left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right.\\ \left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right)\right.\\ \left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right.\\ \left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right]\right)\right.\\ \left.\left(c\;\mathsf{f}\left(-1+\mathsf{Cos}\left[e+f\,x\right]\right)\right)\right]$$

Problem 126: Result more than twice size of optimal antiderivative.

$$\int Sec \left[e+fx\right] \ \left(a+a \, Sec \left[e+fx\right]\right)^{5/2} \, \sqrt{c-c \, Sec \left[e+fx\right]} \ dx$$

Optimal (type 3, 43 leaves, 1 step):

$$-\frac{c \left(a + a \operatorname{Sec}\left[e + f x\right]\right)^{5/2} \operatorname{Tan}\left[e + f x\right]}{3 f \sqrt{c - c \operatorname{Sec}\left[e + f x\right]}}$$

Result (type 3, 88 leaves):

$$\frac{1}{6\,\mathsf{f}}\mathsf{a}^2\,\mathsf{Cot}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(2+4\,\mathsf{Cos}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,+\,\mathsf{Cos}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]^{\,2}\,\mathsf{Sec}\left[\,\frac{1}{2}\,\left(\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,\right)\,\right]^{\,2}\right)\\ \mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]^{\,2}\,\sqrt{\mathsf{a}\,\left(1+\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)}\,\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}$$

Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{Sec [e + fx] (a + a Sec [e + fx])^{5/2}}{\sqrt{c - c Sec [e + fx]}} dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{4 \, a^3 \, \text{Log} \, [\, 1 - \text{Sec} \, [\, e + f \, x \, ] \, ] \, \, \text{Tan} \, [\, e + f \, x \, ]}{f \, \sqrt{a + a} \, \text{Sec} \, [\, e + f \, x \, ]} \, \sqrt{c - c \, \text{Sec} \, [\, e + f \, x \, ]}} \, + \\ \frac{2 \, a^2 \, \sqrt{a + a} \, \text{Sec} \, [\, e + f \, x \, ]}{f \, \sqrt{c - c} \, \text{Sec} \, [\, e + f \, x \, ]} \, + \frac{a \, \left(a + a \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^{3/2} \, \text{Tan} \, [\, e + f \, x \, ]}{2 \, f \, \sqrt{c - c} \, \text{Sec} \, [\, e + f \, x \, ]}}$$

Result (type 3, 328 leaves):

$$\left(4\,\sqrt{2}\,\,\mathrm{e}^{\frac{1}{2}\,\mathrm{i}\,\,(e+f\,x)}\,\,\sqrt{\,\,\frac{\left(1\,+\,\,\mathrm{e}^{\mathrm{i}\,\,(e+f\,x)}\,\right)^{\,2}}{1\,+\,\,\mathrm{e}^{2\,\mathrm{i}\,\,(e+f\,x)}}}\,\,\left(2\,\,Log\,\Big[1\,-\,\,\mathrm{e}^{\mathrm{i}\,\,(e+f\,x)}\,\,\Big]\,-\,Log\,\Big[1\,+\,\,\mathrm{e}^{2\,\mathrm{i}\,\,(e+f\,x)}\,\,\Big]\,\right)$$

$$\sqrt{\text{Sec}\left[e+f\,x\right]}\,\left(a\,\left(1+\text{Sec}\left[e+f\,x\right]\right)\right)^{5/2}\,\text{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]\Bigg|\Big/$$

$$\left(\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$^{\perp}$}}\stackrel{(e+f\,x)}{}\right)\,\sqrt{\frac{\mathop{\text{$\mathbb{E}$}}^{\text{$^{\perp}$}}\stackrel{(e+f\,x)}{}}{1+\mathop{\text{$\mathbb{E}$}}^{2\,\,\text{$^{\perp}$}}\stackrel{(e+f\,x)}{}}}\,\,\,f\,\left(1+\mathop{\text{Sec}}\left[\,e+f\,x\,\right]\,\right)^{5/2}\,\sqrt{c-c\,\mathop{\text{Sec}}\left[\,e+f\,x\,\right]}\,\right)+\left(\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}^{\text{$\perp$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{(e+f\,x)}{}\right)^{-1}\left(1+\mathop{\text{$\mathbb{E}$}}\stackrel{$$

$$\left[ \mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right] \,\,\sqrt{\,\left(\mathsf{1} + \mathsf{Cos}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]\,\right) \,\,\mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]} \,\,\left(\mathsf{a}\,\left(\mathsf{1} + \mathsf{Sec}\left[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,\right]\,\right)\,\right)^{\,5/2} \right]$$

$$\left(\frac{5\,\text{Sec}\left[\frac{e}{2}+\frac{f\,x}{2}\right]}{2\,f}+\frac{\text{Cos}\left[\frac{e}{2}+\frac{f\,x}{2}\right]\,\text{Sec}\left[e+f\,x\right]}{f}\right)\,\text{Sin}\left[\frac{e}{2}+\frac{f\,x}{2}\right]\right)\bigg/$$

$$\left(\left(1+\text{Sec}\left[e+f\,x\right]\right)^{5/2}\,\sqrt{c-c\,\text{Sec}\left[e+f\,x\right]}\right)$$

#### Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{Sec \left[e+fx\right] \, \left(a+a\, Sec \left[e+fx\right]\right)^{5/2}}{\left(c-c\, Sec \left[e+fx\right]\right)^{3/2}} \, \mathrm{d}x$$

Optimal (type 3, 145 leaves, 3 steps):

$$-\frac{a \left(a + a \operatorname{Sec}[e + f x]\right)^{3/2} \operatorname{Tan}[e + f x]}{f \left(c - c \operatorname{Sec}[e + f x]\right)^{3/2}} - \\ \frac{4 a^{3} \operatorname{Log}[1 - \operatorname{Sec}[e + f x]] \operatorname{Tan}[e + f x]}{c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{2 a^{2} \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{c f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 188 leaves):

$$\left( a^2 \left( 1 - 4 \, \text{Log} \left[ 1 - \text{e}^{\text{i} \, (\text{e+f} \, x)} \, \right] + \text{Cos} \left[ \, \text{e} + \, \text{f} \, x \, \right] \, \left( -5 + 8 \, \text{Log} \left[ 1 - \text{e}^{\text{i} \, (\text{e+f} \, x)} \, \right] - 4 \, \text{Log} \left[ 1 + \text{e}^{2 \, \text{i} \, (\text{e+f} \, x)} \, \right] \right) + 2 \, \text{Log} \left[ 1 + \text{e}^{2 \, \text{i} \, (\text{e+f} \, x)} \, \right] + \text{Cos} \left[ 2 \, \left( \text{e} + \, \text{f} \, x \right) \, \right] \left( -4 \, \text{Log} \left[ 1 - \text{e}^{\text{i} \, (\text{e+f} \, x)} \, \right] + 2 \, \text{Log} \left[ 1 + \text{e}^{2 \, \text{i} \, (\text{e+f} \, x)} \, \right] \right) \right)$$
 
$$\text{Sec} \left[ \text{e} + \, \text{f} \, x \right] \, \sqrt{a \, \left( 1 + \text{Sec} \left[ \text{e} + \, \text{f} \, x \right] \right)} \, \, \text{Tan} \left[ \frac{1}{2} \, \left( \text{e} + \, \text{f} \, x \right) \, \right] \right)$$
 
$$\left( \text{c} \, \text{f} \, \left( -1 + \text{Cos} \left[ \text{e} + \, \text{f} \, x \right] \right) \, \sqrt{c - c \, \text{Sec} \left[ \text{e} + \, \text{f} \, x \right]} \right)$$

### Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sec}\left[\,e + f\,x\,\right] \; \left(\,a + a\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,5/2}}{\left(\,c - c\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,5/2}} \; \text{d}x$$

Optimal (type 3, 145 leaves, 3 steps):

$$-\frac{a \left(a + a \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{3/2} \, \text{Tan} \, [\, e + f \, x \, ]}{2 \, f \left(c - c \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{5/2}} + \\ \frac{a^2 \, \sqrt{a + a \, \text{Sec} \, [\, e + f \, x \, ]} \, \, \text{Tan} \, [\, e + f \, x \, ]}{c \, f \left(c - c \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{3/2}} + \\ \frac{a^3 \, \text{Log} \, [\, 1 - \text{Sec} \, [\, e + f \, x \, ]\,) \, \, \text{Tan} \, [\, e + f \, x \, ]}{c^2 \, f \, \sqrt{a + a \, \text{Sec} \, [\, e + f \, x \, ]} \, \, \sqrt{c - c \, \text{Sec} \, [\, e + f \, x \, ]}}$$

Result (type 3, 182 leaves):

$$\begin{split} -\left(\left(a^{2}\,\left(4-6\,\text{Log}\left[1-\text{e}^{\text{i}\,\left(e+f\,x\right)}\,\right]+\text{Cos}\left[\,e+f\,x\,\right]\,\left(8\,\text{Log}\left[1-\text{e}^{\text{i}\,\left(e+f\,x\right)}\,\right]-4\,\text{Log}\left[1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\,\right]\right)+\\ &3\,\text{Log}\left[1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\,\right]+\text{Cos}\left[2\,\left(e+f\,x\right)\,\right]\,\left(-2\,\text{Log}\left[1-\text{e}^{\text{i}\,\left(e+f\,x\right)}\,\right]+\text{Log}\left[1+\text{e}^{2\,\text{i}\,\left(e+f\,x\right)}\,\right]\right)\right)\\ &\sqrt{a\,\left(1+\text{Sec}\left[\,e+f\,x\,\right]\,\right)}\,\,\text{Tan}\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right)\bigg/\left(2\,c^{2}\,f\,\left(-1+\text{Cos}\left[\,e+f\,x\,\right]\,\right)^{2}\,\sqrt{c-c\,\text{Sec}\left[\,e+f\,x\,\right]}\,\right)\right) \end{split}$$

## Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{c}\,-\,\mathsf{c}\,\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,5/2}}{\sqrt{\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{4 c^{3} Log [1 + Sec [e + f x]] Tan [e + f x]}{f \sqrt{a + a Sec [e + f x]} \sqrt{c - c Sec [e + f x]}} - \frac{2 c^{2} \sqrt{c - c Sec [e + f x]} Tan [e + f x]}{f \sqrt{a + a Sec [e + f x]}} - \frac{c (c - c Sec [e + f x])^{3/2} Tan [e + f x]}{2 f \sqrt{a + a Sec [e + f x]}}$$

Result (type 3, 141 leaves):

$$\begin{split} \left(c^2 \, \text{Cot} \left[\frac{1}{2} \, \left(e + f \, x\right)\,\right] \, \left(1 - 6 \, \text{Cos} \left[e + f \, x\right] \, + 8 \, \text{Log} \left[1 + \text{e}^{\text{i} \, \left(e + f \, x\right)}\,\right] \, + \\ & \quad \text{Cos} \left[2 \, \left(e + f \, x\right)\,\right] \, \left(8 \, \text{Log} \left[1 + \text{e}^{\text{i} \, \left(e + f \, x\right)}\,\right] - 4 \, \text{Log} \left[1 + \text{e}^{2 \, \text{i} \, \left(e + f \, x\right)}\,\right]\right) - 4 \, \text{Log} \left[1 + \text{e}^{2 \, \text{i} \, \left(e + f \, x\right)}\,\right]\right) \\ & \quad \text{Sec} \left[e + f \, x\right]^2 \, \sqrt{c - c \, \text{Sec} \left[e + f \, x\right]}\right) \bigg/ \left(2 \, f \, \sqrt{a \, \left(1 + \text{Sec} \left[e + f \, x\right]\,\right)}\right) \end{split}$$

#### Problem 134: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{c}\,-\,\mathsf{c}\,\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,3/\,2}}{\sqrt{\mathsf{a}\,+\,\mathsf{a}\,\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 94 leaves, 2 steps):

$$-\frac{2 c^2 \text{Log} [1+\text{Sec} [e+fx]] \text{Tan} [e+fx]}{f \sqrt{a+a} \text{Sec} [e+fx]} \sqrt{c-c} \frac{c \sqrt{c-c} \text{Sec} [e+fx]}{\sqrt{a+a} \text{Sec} [e+fx]} \frac{c \sqrt{c-c} \text{Sec} [e+fx]}{f \sqrt{a+a} \text{Sec} [e+fx]}$$

Result (type 3, 173 leaves):

$$\left(c \, \operatorname{e}^{-2 \, \mathrm{i} \, \left(e + f \, x\right)} \, \left(1 + \operatorname{e}^{2 \, \mathrm{i} \, \left(e + f \, x\right)}\right)^2 \, \mathsf{Cos}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] \, \mathsf{Cot}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] \\ \left(-1 + \mathsf{Cos}\left[e + f \, x\right] \, \left(4 \, \mathsf{Log}\left[1 + \operatorname{e}^{\mathrm{i} \, \left(e + f \, x\right)}\right] - 2 \, \mathsf{Log}\left[1 + \operatorname{e}^{2 \, \mathrm{i} \, \left(e + f \, x\right)}\right]\right)\right) \, \mathsf{Sec}\left[e + f \, x\right]^3 \, \sqrt{c - c \, \mathsf{Sec}\left[e + f \, x\right]} \\ \left(\mathsf{Cos}\left[\frac{1}{2} \, \left(e + f \, x\right)\right] + \mathrm{i} \, \mathsf{Sin}\left[\frac{1}{2} \, \left(e + f \, x\right)\right]\right)\right) \bigg/ \, \left(2 \, \left(1 + \operatorname{e}^{\mathrm{i} \, \left(e + f \, x\right)}\right) \, f \, \sqrt{\mathsf{a} \, \left(1 + \mathsf{Sec}\left[e + f \, x\right]\right)}\right) \right)$$

# Problem 135: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + f x] \sqrt{c - c \operatorname{Sec} [e + f x]}}{\sqrt{a + a \operatorname{Sec} [e + f x]}} dx$$

Optimal (type 3, 50 leaves, 1 step):

$$-\frac{c \log[1 + Sec[e + fx]] Tan[e + fx]}{f \sqrt{a + a Sec[e + fx]} \sqrt{c - c Sec[e + fx]}}$$

Result (type 3, 118 leaves):

$$\left( \begin{array}{c} \mathbb{i} \left( 1 + \mathbb{e}^{\mathbb{i} \; (e+f\,x)} \right) \sqrt{\frac{c \left( -1 + \mathbb{e}^{\mathbb{i} \; (e+f\,x)} \right)^2}{1 + \mathbb{e}^{2\,\mathbb{i} \; (e+f\,x)}}} \right. \left( 2 \, \text{Log} \left[ 1 + \mathbb{e}^{\mathbb{i} \; (e+f\,x)} \right] - \text{Log} \left[ 1 + \mathbb{e}^{2\,\mathbb{i} \; (e+f\,x)} \right] \right) \right) \\ \left( \left( -1 + \mathbb{e}^{\mathbb{i} \; (e+f\,x)} \right) \, f \sqrt{a \left( 1 + \text{Sec} \left[ e + f\,x \right] \right)} \right)$$

Problem 136: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + f x]}{\sqrt{a + a \operatorname{Sec} [e + f x]}} \sqrt{c - c \operatorname{Sec} [e + f x]} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$-\frac{\mathsf{ArcTanh}\left[\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right]\,\mathsf{Tan}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}{\mathsf{f}\,\sqrt{\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}}\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}$$

Result (type 3, 115 leaves):

$$-\left(\left(2\,\dot{\mathbb{1}}\,\left(-1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\right)\,\mathsf{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]^{2}\,\left(\mathsf{Log}\left[1-\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\,\right]-\mathsf{Log}\left[1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\,\right]\right)\,\mathsf{Sec}\left[\,e+f\,x\,\right]\right)\right/\left(\left(1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\right)\,f\,\sqrt{a\,\left(1+\mathsf{Sec}\left[\,e+f\,x\,\right]\,\right)}\,\,\sqrt{c-c\,\mathsf{Sec}\left[\,e+f\,x\,\right]}\,\right)\right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec} [e + f x]}{\sqrt{a + a \operatorname{Sec} [e + f x]}} \left( c - c \operatorname{Sec} [e + f x] \right)^{3/2} dx$$

Optimal (type 3, 95 leaves, 3 steps):

$$-\frac{\mathsf{Tan}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}{2\,\mathsf{f}\,\sqrt{\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}\,\left(\mathsf{c}\,-\,\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\right)^{3/2}} - \frac{\mathsf{Arc}\,\mathsf{Tanh}\,[\,\mathsf{Cos}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,]\,\,\mathsf{Tan}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}{2\,\mathsf{c}\,\mathsf{f}\,\sqrt{\,\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}\,\,\sqrt{\mathsf{c}\,-\,\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}}$$

Result (type 3, 129 leaves):

$$\left( \left( -1 - Log \left[ 1 - e^{i \cdot (e + f \cdot x)} \right. \right] + Cos \left[ e + f \cdot x \right] \cdot \left( Log \left[ 1 - e^{i \cdot (e + f \cdot x)} \right. \right) - Log \left[ 1 + e^{i \cdot (e + f \cdot x)} \right. \right) \right) \\ + Log \left[ 1 + e^{i \cdot (e + f \cdot x)} \right] \cdot \left( 2 \cdot c \cdot f \cdot \left( -1 + Cos \left[ e + f \cdot x \right] \right) \cdot \sqrt{a \cdot \left( 1 + Sec \left[ e + f \cdot x \right] \right)} \cdot \sqrt{c - c \cdot Sec \left[ e + f \cdot x \right]} \right) \right)$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int\!\frac{Sec\,[\,e+f\,x\,]}{\sqrt{a+a\,Sec\,[\,e+f\,x\,]}}\,\left(c-c\,Sec\,[\,e+f\,x\,]\,\right)^{5/2}\,\mathrm{d}x$$

Optimal (type 3, 140 leaves, 4 steps):

$$\frac{\mathsf{Tan}\,[\,e + f\,x\,]}{\mathsf{4}\,f\,\sqrt{\mathsf{a}\,+\mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\left(\mathsf{c}\,-\mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{5/2}} \\ \frac{\mathsf{Tan}\,[\,e + f\,x\,]}{\mathsf{4}\,\mathsf{c}\,f\,\sqrt{\mathsf{a}\,+\mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\left(\mathsf{c}\,-\mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{3/2}} \\ -\frac{\mathsf{ArcTanh}\,[\mathsf{Cos}\,[\,e + f\,x\,]\,\,\mathsf{Tan}\,[\,e + f\,x\,]}{\mathsf{4}\,\mathsf{c}^2\,f\,\sqrt{\mathsf{a}\,+\mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\sqrt{\mathsf{c}\,-\mathsf{c}\,\mathsf{Sec}\,[\,e + f\,x\,]}}$$

Result (type 3, 176 leaves):

$$\left( \left( 4+3 \, \text{Log} \left[ 1-\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] + \text{Cos} \left[ 2 \, \left( e+f \, x \right) \, \right] \, \left( \text{Log} \left[ 1-\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] - \text{Log} \left[ 1+\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] \right) - \\ 3 \, \text{Log} \left[ 1+\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] + \text{Cos} \left[ e+f \, x \right] \, \left( -6-4 \, \text{Log} \left[ 1-\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] + 4 \, \text{Log} \left[ 1+\text{e}^{\text{i} \, \left( e+f \, x \right)} \, \right] \right) \right) \\ \text{Tan} \left[ e+f \, x \right] \right) \left/ \, \left( 8 \, c^2 \, f \, \left( -1+\text{Cos} \left[ e+f \, x \right] \right)^2 \, \sqrt{a \, \left( 1+\text{Sec} \left[ e+f \, x \right] \right)} \, \sqrt{c-c \, \text{Sec} \left[ e+f \, x \right]} \right) \right) \right) \right) \right) \right\}$$

### Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}[e+fx] \left(c-c\operatorname{Sec}[e+fx]\right)^{5/2}}{\left(a+a\operatorname{Sec}[e+fx]\right)^{3/2}} \, dx$$

#### Optimal (type 3, 142 leaves, 3 steps):

$$\begin{split} & \frac{4\,c^3\,\text{Log}\,[\,1+\text{Sec}\,[\,e+f\,x\,]\,\,]\,\,\text{Tan}\,[\,e+f\,x\,]}{a\,f\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}\,\,\sqrt{c-c\,\text{Sec}\,[\,e+f\,x\,]}} \,\, + \\ & \frac{2\,c^2\,\sqrt{c-c\,\text{Sec}\,[\,e+f\,x\,]}\,\,\,\text{Tan}\,[\,e+f\,x\,]}{a\,f\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}} \,\, + \frac{c\,\left(\,c-c\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{3/2}\,\text{Tan}\,[\,e+f\,x\,]}{f\,\left(\,a+a\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{3/2}} \end{split}$$

#### Result (type 3, 183 leaves):

$$\begin{split} -\left(\left(c^{2} \, \text{Cot}\left[\frac{1}{2} \, \left(e + f \, x\right)\right.\right] \\ & \left.\left(-1 + 4 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] + \text{Cos}\left[e + f \, x\right] \, \left(-5 + 8 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] - 4 \, \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) + \\ & \left. \text{Cos}\left[2 \, \left(e + f \, x\right)\right.\right] \, \left(4 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] - 2 \, \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) - 2 \, \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) \\ & \left. \text{Sec}\left[e + f \, x\right] \, \sqrt{c - c \, \text{Sec}\left[e + f \, x\right]}\right) / \left(a \, f \, \left(1 + \text{Cos}\left[e + f \, x\right]\right) \, \sqrt{a \, \left(1 + \text{Sec}\left[e + f \, x\right]\right)}\right)\right) \end{split}$$

## Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]\,\left(c-c\,\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}}{\left(a+a\,\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}}\,\mathrm{d}x$$

### Optimal (type 3, 95 leaves, 2 steps):

$$\frac{c^2 \, \text{Log} \, [\, 1 + \text{Sec} \, [\, e + f \, x \, ] \, ] \, \, \text{Tan} \, [\, e + f \, x \, ]}{a \, f \, \sqrt{a + a} \, \text{Sec} \, [\, e + f \, x \, ]} \, \, \sqrt{c - c} \, \, \text{Sec} \, [\, e + f \, x \, ]} \, + \, \frac{c \, \, \sqrt{c - c} \, \, \text{Sec} \, [\, e + f \, x \, ]}{f \, \left( a + a \, \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^{3/2}}$$

### Result (type 3, 132 leaves):

$$\begin{split} -\left(\left(c\,\mathsf{Cot}\left[\frac{1}{2}\left(e+f\,x\right)\right]\,\left(-2+2\,\mathsf{Log}\left[1+e^{i\,\left(e+f\,x\right)}\right]\right.\right.\\ &\left.\left.\mathsf{Cos}\left[e+f\,x\right]\,\left(2\,\mathsf{Log}\left[1+e^{i\,\left(e+f\,x\right)}\right]-\mathsf{Log}\left[1+e^{2\,i\,\left(e+f\,x\right)}\right]\right)-\mathsf{Log}\left[1+e^{2\,i\,\left(e+f\,x\right)}\right]\right)\right.\\ &\left.\sqrt{c-c\,\mathsf{Sec}\left[e+f\,x\right]}\right)\middle/\left(a\,f\,\left(1+\mathsf{Cos}\left[e+f\,x\right]\right)\,\sqrt{a\,\left(1+\mathsf{Sec}\left[e+f\,x\right]\right)}\right)\right) \end{split}$$

## Problem 142: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}\sqrt{c-c\operatorname{Sec}\left[e+f\,x\right]}}\,\,\mathrm{d}x$$

Optimal (type 3, 95 leaves, 3 steps):

$$\frac{\text{Tan}\left[e+fx\right]}{2\,f\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\left[e+fx\right]\right)^{3/2}\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\left[e+fx\right]}}\,-\,\frac{\mathsf{ArcTanh}\left[\mathsf{Cos}\left[e+fx\right]\right]\,\mathsf{Tan}\left[e+fx\right]}{2\,\mathsf{a}\,f\sqrt{\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\left[e+fx\right]}\,\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\left[e+fx\right]}}$$

Result (type 3, 129 leaves):

$$\left( \left( -1 + \text{Log} \left[ 1 - \text{e}^{\text{i} \ (\text{e+f} \, x)} \right. \right] + \text{Cos} \left[ \text{e+f} \, x \right] \ \left( \text{Log} \left[ 1 - \text{e}^{\text{i} \ (\text{e+f} \, x)} \right. \right] - \text{Log} \left[ 1 + \text{e}^{\text{i} \ (\text{e+f} \, x)} \right. \right] \right) \\ - \text{Tan} \left[ \text{e+f} \, x \right] \right) \ \left/ \ \left( 2 \, \text{af} \left( 1 + \text{Cos} \left[ \text{e+f} \, x \right] \right) \ \sqrt{\text{a} \left( 1 + \text{Sec} \left[ \text{e+f} \, x \right] \right)} \ \sqrt{\text{c-c} \, \text{Sec} \left[ \text{e+f} \, x \right]} \right) \right. \right)$$

### Problem 143: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{3/2}\,\left(\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{3/2}}\, \mathtt{d}\mathsf{x}$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{\mathsf{Csc}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}{\mathsf{2}\,\mathsf{a}\,\mathsf{c}\,\mathsf{f}\,\sqrt{\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}}\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}\,-\,\frac{\mathsf{ArcTanh}\,[\mathsf{Cos}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]\,]\,\,\mathsf{Tan}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}{\mathsf{2}\,\mathsf{a}\,\mathsf{c}\,\mathsf{f}\,\sqrt{\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}}\,\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\mathsf{e}+\mathsf{f}\,\mathsf{x}]}$$

Result (type 3, 89 leaves):

$$\frac{\text{Csc}\left[\,e+f\,x\,\right] \,+\, \left(\text{Log}\left[\,1-\,\text{e}^{\,i\,\,\left(\,e+f\,x\,\right)}\,\,\right] \,-\, \text{Log}\left[\,1+\,\text{e}^{\,i\,\,\left(\,e+f\,x\,\right)}\,\,\right]\,\right)\,\,\text{Tan}\left[\,e+f\,x\,\right]}{2\,\,a\,\,c\,\,f\,\,\sqrt{\,a\,\,\left(\,1+\,\text{Sec}\left[\,e+f\,x\,\right]\,\right)}}\,\,\,\sqrt{\,c\,-\,c\,\,\text{Sec}\left[\,e+f\,x\,\right]}}$$

### Problem 144: Result unnecessarily involves imaginary or complex numbers.

$$\int\! \frac{\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{3/2}\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{5/2}}\, \mathbb{d}\mathsf{x}$$

Optimal (type 3, 146 leaves, 4 steps):

$$\frac{3\,\text{Csc}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}{8\,\text{a}\,\,\text{c}^{\,2}\,\text{f}\,\sqrt{\,\text{a}\,+\,\text{a}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}}\,\sqrt{\,\text{c}\,-\,\text{c}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}}\,-\,\frac{3\,\text{ArcTanh}\,[\,\text{Cos}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,]\,\,\text{Tan}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}{4\,\text{f}\,\,\big(\,\text{a}\,+\,\text{a}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\big)^{\,3/2}\,\,\big(\,\text{c}\,-\,\text{c}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\big)^{\,5/2}}\,-\,\frac{3\,\text{ArcTanh}\,[\,\text{Cos}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,]\,\,\text{Tan}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}{8\,\text{a}\,\,\text{c}^{\,2}\,\text{f}\,\sqrt{\,\text{a}\,+\,\text{a}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}}\,\,\sqrt{\,\text{c}\,-\,\text{c}\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]}$$

Result (type 3, 243 leaves):

$$\begin{array}{l} \left( \left( -2 + 6 \, \text{Log} \left[ 1 - \mathbb{e}^{i \, \, (e+f \, x)} \, \right] + 3 \, \text{Cos} \left[ 3 \, \left( e + f \, x \right) \, \right] \, \text{Log} \left[ 1 - \mathbb{e}^{i \, \, (e+f \, x)} \, \right] - \\ 2 \, \text{Cos} \left[ 2 \, \left( e + f \, x \right) \, \right] \, \left( 5 + 3 \, \text{Log} \left[ 1 - \mathbb{e}^{i \, \, (e+f \, x)} \, \right] - 3 \, \text{Log} \left[ 1 + \mathbb{e}^{i \, \, (e+f \, x)} \, \right] \right) - \\ 6 \, \text{Log} \left[ 1 + \mathbb{e}^{i \, \, (e+f \, x)} \, \right] - 3 \, \text{Cos} \left[ 3 \, \left( e + f \, x \right) \, \right] \, \text{Log} \left[ 1 + \mathbb{e}^{i \, \, (e+f \, x)} \, \right] \right) + \\ \text{Cos} \left[ e + f \, x \right] \, \left( 4 - 3 \, \text{Log} \left[ 1 - \mathbb{e}^{i \, \, (e+f \, x)} \, \right] + 3 \, \text{Log} \left[ 1 + \mathbb{e}^{i \, \, (e+f \, x)} \, \right] \right) \right) \, \text{Tan} \left[ e + f \, x \right] \right) \\ \left( 32 \, a \, c^2 \, f \, \left( -1 + \text{Cos} \left[ e + f \, x \right] \right)^2 \, \left( 1 + \text{Cos} \left[ e + f \, x \right] \right) \, \sqrt{a \, \left( 1 + \text{Sec} \left[ e + f \, x \right] \right)} \, \, \sqrt{c - c \, \text{Sec} \left[ e + f \, x \right]} \right) \right) \right) \\ \end{array}$$

### Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \,\,\left(c - c\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{5/2}}{\left(a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{5/2}}\,\,\mathrm{d}x$$

Optimal (type 3, 145 leaves, 3 steps):

$$-\frac{c^3 \, \text{Log} \, [\, 1 + \text{Sec} \, [\, e + f \, x \, ] \, ] \, \, \text{Tan} \, [\, e + f \, x \, ]}{a^2 \, f \, \sqrt{a + a} \, \text{Sec} \, [\, e + f \, x \, ]} \, \sqrt{c - c} \, \, \text{Sec} \, [\, e + f \, x \, ]} \, - \\ \frac{c^2 \, \sqrt{c - c} \, \, \text{Sec} \, [\, e + f \, x \, ]}{a \, f \, \left(a + a \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^{3/2}} \, + \frac{c \, \left(c - c \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^{3/2} \, \text{Tan} \, [\, e + f \, x \, ]}{2 \, f \, \left(a + a \, \text{Sec} \, [\, e + f \, x \, ] \, \right)^{5/2}}$$

Result (type 3, 178 leaves):

$$\begin{split} &\left(c^{2} \, \text{Cot}\left[\frac{1}{2} \, \left(e + f \, x\right)\right.\right] \\ &\left.\left(-4 + 6 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] + \text{Cos}\left[e + f \, x\right] \, \left(8 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] - 4 \, \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) + \\ &\left.\left.\text{Cos}\left[2 \, \left(e + f \, x\right)\right.\right] \, \left(2 \, \text{Log}\left[1 + e^{i \, \left(e + f \, x\right)}\right.\right] - \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) - 3 \, \text{Log}\left[1 + e^{2 \, i \, \left(e + f \, x\right)}\right.\right]\right) \\ &\left.\sqrt{c - c \, \text{Sec}\left[e + f \, x\right.\right]} \, \left/\left.\left(2 \, a^{2} \, f \, \left(1 + \text{Cos}\left[e + f \, x\right.\right]\right)^{2} \, \sqrt{a \, \left(1 + \text{Sec}\left[e + f \, x\right.\right]\right)}\right.\right) \end{split}$$

## Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)^{5/2}\sqrt{c-c\operatorname{Sec}[e+fx]}} \, dx$$

Optimal (type 3, 140 leaves, 4 steps):

Result (type 3, 176 leaves):

$$\begin{split} \left( \left( -4 + 3 \, \text{Log} \left[ 1 - \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] + \text{Cos} \left[ e + f \, x \right] \, \left( -6 + 4 \, \text{Log} \left[ 1 - \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] - 4 \, \text{Log} \left[ 1 + \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] \right) + \\ & \quad \text{Cos} \left[ 2 \, \left( e + f \, x \right) \, \right] \, \left( \text{Log} \left[ 1 - \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] - \text{Log} \left[ 1 + \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] \right) - 3 \, \text{Log} \left[ 1 + \mathbb{e}^{\text{i} \, \left( e + f \, x \right)} \, \right] \right) \, \text{Tan} \left[ e + f \, x \right] \right) \\ & \quad \left( 8 \, a^2 \, f \, \left( 1 + \text{Cos} \left[ e + f \, x \right] \right)^2 \, \sqrt{a \, \left( 1 + \text{Sec} \left[ e + f \, x \right] \right)} \, \sqrt{c - c \, \text{Sec} \left[ e + f \, x \right]} \right) \end{split}$$

## Problem 149: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{5/2}\,\left(c-c\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}}\,\mathrm{d}x$$

Optimal (type 3, 146 leaves, 4 steps):

#### Result (type 3, 242 leaves):

### Problem 150: Result unnecessarily involves imaginary or complex numbers.

$$\int\! \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{5/2} \left(\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{5/2}}\, \mathrm{d}\mathsf{x}$$

#### Optimal (type 3, 160 leaves, 4 steps):

$$\frac{3\,\text{Csc}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}{8\,\,\text{a}^2\,\,\text{c}^2\,\,\text{f}\,\,\sqrt{\,\text{a}\,+\,\text{a}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}\,\,\sqrt{\,\text{c}\,-\,\text{c}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}\,-\,\frac{3\,\,\text{ArcTanh}\,[\,\text{Cos}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]\,\,]\,\,\text{Tan}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}{4\,\,\text{a}^2\,\,\text{c}^2\,\,\text{f}\,\,\sqrt{\,\text{a}\,+\,\text{a}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}\,\,\sqrt{\,\text{c}\,-\,\text{c}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}\,-\,\frac{3\,\,\text{ArcTanh}\,[\,\text{Cos}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]\,\,]\,\,\text{Tan}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}{8\,\,\text{a}^2\,\,\text{c}^2\,\,\text{f}\,\,\sqrt{\,\text{a}\,+\,\text{a}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}\,\,\sqrt{\,\text{c}\,-\,\text{c}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\,\text{x}\,]}}$$

#### Result (type 3, 105 leaves):

$$\left( \left( 1 - 5 \, \mathsf{Cos} \left[ 2 \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \right) \, \mathsf{Csc} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^{\, 3} + 6 \, \left( \mathsf{Log} \left[ 1 - \mathsf{e}^{\mathtt{i} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right)} \, \right] - \mathsf{Log} \left[ 1 + \mathsf{e}^{\mathtt{i} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right)} \, \right] \right) \, \mathsf{Tan} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right] \right) \, \left( 16 \, \mathsf{a}^2 \, \mathsf{c}^2 \, \mathsf{f} \, \sqrt{\mathsf{a} \, \left( 1 + \mathsf{Sec} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right] \, \right)} \, \sqrt{\mathsf{c} - \mathsf{c} \, \mathsf{Sec} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]} \right) \right)$$

## Problem 151: Unable to integrate problem.

$$\int Sec \left[\,e + f\,x\,\right] \, \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,m} \, \left(\,c - c\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,n} \, \mathrm{d}x$$

### Optimal (type 5, 101 leaves, 3 steps):

$$-\frac{1}{f\left(1+2\,\text{m}\right)}2^{\frac{1}{2}+n}\,c\,\,\text{Hypergeometric}2\text{F1}\left[\,\frac{1}{2}+\text{m,}\,\,\frac{1}{2}-\text{n,}\,\,\frac{3}{2}+\text{m,}\,\,\frac{1}{2}\,\left(1+\text{Sec}\left[\,\text{e}+\text{f}\,\text{x}\,\right]\,\right)\,\right]\\ \left(1-\text{Sec}\left[\,\text{e}+\text{f}\,\text{x}\,\right]\,\right)^{\frac{1}{2}-n}\,\left(\text{a}+\text{a}\,\text{Sec}\left[\,\text{e}+\text{f}\,\text{x}\,\right]\,\right)^{\text{m}}\,\left(\text{c}-\text{c}\,\text{Sec}\left[\,\text{e}+\text{f}\,\text{x}\,\right]\,\right)^{-1+n}\,\text{Tan}\left[\,\text{e}+\text{f}\,\text{x}\,\right]$$

#### Result (type 8, 34 leaves):

$$\int Sec \left[e+fx\right] \, \left(a+a\, Sec \left[e+fx\right]\right)^m \, \left(c-c\, Sec \left[e+fx\right]\right)^n \, \mathrm{d}x$$

### Problem 154: Unable to integrate problem.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{m}}{c-c\operatorname{Sec}[e+fx]} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$-\left(\left(2^{\frac{1}{2}+m}\text{ a Hypergeometric2F1}\left[-\frac{1}{2},\,\frac{1}{2}-m,\,\frac{1}{2},\,\frac{1}{2}\left(1-\text{Sec}\left[e+f\,x\right]\right)\right]\right.\\ \left.\left(1+\text{Sec}\left[e+f\,x\right]\right)^{\frac{1}{2}-m}\,\left(a+a\,\text{Sec}\left[e+f\,x\right]\right)^{-1+m}\,\text{Tan}\left[e+f\,x\right]\right)\right/\,\left(f\left(c-c\,\text{Sec}\left[e+f\,x\right]\right)\right)\right)$$

Result (type 8, 34 leaves):

$$\int \frac{\mathsf{Sec}\, [\, e + f\, x\,] \, \left(\mathsf{a} + \mathsf{a}\, \mathsf{Sec}\, [\, e + f\, x\,]\,\right)^{\,\mathsf{m}}}{\mathsf{c} - \mathsf{c}\, \mathsf{Sec}\, [\, e + f\, x\,]} \, \mathrm{d} x$$

### Problem 155: Unable to integrate problem.

$$\int \frac{Sec[e+fx] \left(a+a\,Sec[e+fx]\right)^m}{\left(c-c\,Sec[e+fx]\right)^2} \, dx$$

Optimal (type 5, 92 leaves, 3 steps):

$$-\left(\left(2^{\frac{1}{2}+m} \text{ a Hypergeometric2F1}\left[-\frac{3}{2}\text{, } \frac{1}{2}-m\text{, } -\frac{1}{2}\text{, } \frac{1}{2}\left(1-\text{Sec}\left[e+f\,x\right]\right)\right]\right.\\ \left.\left(1+\text{Sec}\left[e+f\,x\right]\right)^{\frac{1}{2}-m}\left(a+a\,\text{Sec}\left[e+f\,x\right]\right)^{-1+m}\,\text{Tan}\left[e+f\,x\right]\right)\right/\left(3\,f\left(c-c\,\text{Sec}\left[e+f\,x\right]\right)^{2}\right)\right)$$

Result (type 8, 34 leaves)

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \; \left(\,a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{\,m}}{\left(\,c - c\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{\,2}} \; \mathrm{d}x$$

### Problem 156: Unable to integrate problem.

$$\int Sec \left[e + fx\right] \left(a + a Sec \left[e + fx\right]\right)^{m} \left(c - c Sec \left[e + fx\right]\right)^{5/2} dx$$

Optimal (type 3, 160 leaves, 3 steps):

$$-\frac{64 c^{3} (a + a Sec [e + fx])^{m} Tan [e + fx]}{f (5 + 2 m) (3 + 8 m + 4 m^{2}) \sqrt{c - c Sec [e + fx]}} - \frac{16 c^{2} (a + a Sec [e + fx])^{m} \sqrt{c - c Sec [e + fx]}}{f (15 + 16 m + 4 m^{2})}$$

$$\frac{2 c (a + a Sec [e + fx])^{m} (c - c Sec [e + fx])^{3/2} Tan [e + fx]}{f (5 + 2 m)}$$

Result (type 8, 36 leaves):

$$\int Sec \left[\,e + f\,x\,\right] \; \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,m} \; \left(\,c - c\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,5/2} \, \mathrm{d}x$$

## Problem 157: Unable to integrate problem.

$$\int Sec \left[e+f\,x\right] \, \left(a+a\,Sec \left[e+f\,x\right]\right)^m \, \left(c-c\,Sec \left[e+f\,x\right]\right)^{3/2} \, dx$$

Optimal (type 3, 100 leaves, 2 steps):

$$-\frac{8 \, c^2 \, \left(\mathsf{a} + \mathsf{a} \, \mathsf{Sec} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,]\,\right)^\mathsf{m} \, \mathsf{Tan} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,]}{\mathsf{f} \, \left(\mathsf{3} + \mathsf{8} \, \mathsf{m} + \mathsf{4} \, \mathsf{m}^2\right) \, \sqrt{\mathsf{c} - \mathsf{c} \, \mathsf{Sec} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,]}} \, - \frac{2 \, c \, \left(\mathsf{a} + \mathsf{a} \, \mathsf{Sec} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,]\,\right)^\mathsf{m} \, \sqrt{\mathsf{c} - \mathsf{c} \, \mathsf{Sec} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,]}}{\mathsf{f} \, \left(\mathsf{3} + \mathsf{2} \, \mathsf{m}\right)} \, \mathsf{Tan} \, [\,\mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \, \mathsf{m} \, \mathsf{$$

Result (type 8, 36 leaves):

$$\int Sec \left[e+fx\right] \, \left(a+a\, Sec \left[e+fx\right]\right)^m \, \left(c-c\, Sec \left[e+fx\right]\right)^{3/2} \, dx$$

### Problem 158: Unable to integrate problem.

$$\int Sec \left[\,e + f\,x\,\right] \, \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,m} \, \sqrt{\,c - c\,Sec \left[\,e + f\,x\,\right]} \ \, \mathrm{d}x$$

Optimal (type 3, 46 leaves, 1 step):

$$- \, \frac{2 \, c \, \left( a + a \, \text{Sec} \left[ \, e + f \, x \, \right] \, \right)^{\, m} \, \text{Tan} \left[ \, e + f \, x \, \right]}{f \, \left( 1 + 2 \, m \right) \, \sqrt{c - c \, \text{Sec} \left[ \, e + f \, x \, \right]}}$$

Result (type 8, 36 leaves):

$$\int Sec[e+fx] \left(a+a\,Sec[e+fx]\right)^m \sqrt{c-c\,Sec[e+fx]} \ dx$$

## Problem 159: Unable to integrate problem.

$$\int \frac{Sec[e+fx] \left(a+aSec[e+fx]\right)^{m}}{\sqrt{c-cSec[e+fx]}} dx$$

Optimal (type 5, 69 leaves, 2 steps):

$$-\left(\left(\mathsf{Hypergeometric2F1}\left[\mathbf{1},\,\frac{1}{2}+\mathsf{m},\,\frac{3}{2}+\mathsf{m},\,\frac{1}{2}\left(\mathbf{1}+\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)\right]\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)^{\mathsf{m}}\,\mathsf{Tan}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)\right/\left(\mathsf{f}\left(\mathbf{1}+\mathsf{2}\,\mathsf{m}\right)\,\sqrt{\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}\right)\right)$$

Result (type 8, 36 leaves):

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{m}}{\sqrt{c-c\operatorname{Sec}[e+fx]}} \, dx$$

### Problem 160: Unable to integrate problem.

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^{m}}{\left(c-c\operatorname{Sec}[e+fx]\right)^{3/2}} dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$-\left(\left(\mathsf{Hypergeometric2F1}\left[2\,,\,\frac{1}{2}\,+\,\mathsf{m}\,,\,\frac{3}{2}\,+\,\mathsf{m}\,,\,\frac{1}{2}\,\left(1\,+\,\mathsf{Sec}\left[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right]\,\right)\,\right]\,\left(\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\left[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right]\,\right)\right)\right)$$

$$\left(2\,\mathsf{c}\,\mathsf{f}\,\left(1\,+\,2\,\mathsf{m}\right)\,\sqrt{\mathsf{c}\,-\,\mathsf{c}\,\mathsf{Sec}\left[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right]\,}\right)\right)$$

Result (type 8, 36 leaves):

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,] \,\,\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{m}}}{\left(\,\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{3/2}}} \,\,\mathrm{d} x$$

### Problem 161: Unable to integrate problem.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,] \, \left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{m}}}{\left(\,\mathsf{c} - \mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{5/2}}} \, \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 74 leaves, 2 steps):

$$-\left(\left(\text{Hypergeometric2F1}\left[3,\,\frac{1}{2}+\text{m,}\,\frac{3}{2}+\text{m,}\,\frac{1}{2}\,\left(1+\text{Sec}\left[\,e+f\,x\,\right]\,\right)\,\right]\,\left(a+a\,\text{Sec}\left[\,e+f\,x\,\right]\,\right)^{\,\text{m}}\,\text{Tan}\left[\,e+f\,x\,\right]\,\right)\right/\left(4\,c^2\,f\,\left(1+2\,\text{m}\right)\,\sqrt{c-c\,\text{Sec}\left[\,e+f\,x\,\right]}\,\right)\right)$$

Result (type 8, 36 leaves):

$$\int \frac{\operatorname{Sec}[e+fx] \left(a+a\operatorname{Sec}[e+fx]\right)^m}{\left(c-c\operatorname{Sec}[e+fx]\right)^{5/2}} \, dx$$

## Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int Sec \left[e+fx\right] \ \left(a+a \, Sec \left[e+fx\right]\right)^m \ \left(c-c \, Sec \left[e+fx\right]\right)^{-3-m} \, \mathrm{d}x$$

Optimal (type 3, 169 leaves, 3 steps):

$$-\frac{\left(a + a \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{\,m} \, \left(c - c \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{-3 - m} \, \text{Tan} \, [\, e + f \, x\,]}{f \, \left(1 + 2 \, m\right)} \, + \\ \\ \frac{2 \, \left(a + a \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{\,1 + m} \, \left(c - c \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{\,-3 - m} \, \text{Tan} \, [\, e + f \, x\,]}{a \, f \, \left(3 + 8 \, m + 4 \, m^2\right)} \, - \\ \\ \frac{2 \, \left(a + a \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{\,2 + m} \, \left(c - c \, \text{Sec} \, [\, e + f \, x\,]\,\right)^{\,-3 - m} \, \text{Tan} \, [\, e + f \, x\,]}{a^2 \, f \, \left(1 + 2 \, m\right) \, \left(15 + 16 \, m + 4 \, m^2\right)}$$

Result (type 3, 321 leaves):

$$\begin{split} &-\frac{1}{\left(-1+\text{e}^{\frac{i}{2}}\left(\text{e}^{+\text{f}\,x}\right)\right)^{\,5}\,\text{f}\,\left(1+2\,\text{m}\right)\,\left(3+2\,\text{m}\right)}\,\left(5+2\,\text{m}\right)} \\ &\pm\,2^{3+\text{m}}\,\left(-\,\dot{\mathbb{1}}\,\,\text{e}^{-\frac{1}{2}\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(-1+\text{e}^{\frac{i}{2}}\,\,(\text{e}^{+\text{f}\,x})\,\,\right)\right)^{-2\,\text{m}}\,\left(1+\text{e}^{\frac{i}{2}}\,\,(\text{e}^{+\text{f}\,x})\,\,\right)\,\left(\frac{\text{e}^{\frac{i}{2}}\,\,(\text{e}^{+\text{f}\,x})}}{1+\text{e}^{2\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}}\right)^{-\text{m}} \\ &\left(\frac{\left(1+\text{e}^{\frac{i}{2}}\,\,(\text{e}^{+\text{f}\,x})\,\,\right)^{\,2}}{1+\text{e}^{2\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}}\right)^{\,\text{m}}\,\left(7+12\,\text{m}+4\,\text{m}^{2}-4\,\text{e}^{\frac{i}{2}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)-4\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+\text{e}^{2\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+4\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\,\left(3+2\,\text{m}\right)+2\,\text{e}^{3\,\dot{\mathbb{1}}\,\,(\text{e}^{+\text{f}\,x})}\,\left$$

Problem 163: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right] \, \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,m} \, \left(\,c - c\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,-2 - m} \, \mathrm{d}x$$

Optimal (type 3, 104 leaves, 2 steps):

$$-\frac{\left(a + a \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{\,m} \, \left(c - c \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{\,-2 - m} \, \text{Tan} \, [\, e + f \, x \, ]}{f \, \left(1 + 2 \, m\right)} + \\ \frac{\left(a + a \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{\,1 + m} \, \left(c - c \, \text{Sec} \, [\, e + f \, x \, ]\,\right)^{\,-2 - m} \, \text{Tan} \, [\, e + f \, x \, ]}{a \, f \, \left(3 + 8 \, m + 4 \, m^2\right)}$$

Result (type 3, 250 leaves):

Problem 164: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right] \,\,\left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,m} \,\,\left(\,c - c\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,-1 - m} \,\,\mathrm{d}x$$

Optimal (type 3, 47 leaves, 1 step):

$$-\frac{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{m}}\,\left(\mathsf{c}-\mathsf{c}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,\mathsf{-1-m}}\,\mathsf{Tan}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\,\mathsf{f}\,\left(\mathsf{1}+\mathsf{2}\,\mathsf{m}\right)}$$

Result (type 3, 208 leaves):

$$\begin{split} &-\frac{1}{f+2\,f\,m}2^{1+m}\,e^{-\frac{1}{2}\,\mathrm{i}\,\,(e+f\,x)}\,\left(-\,\mathrm{i}\,\,e^{-\frac{1}{2}\,\mathrm{i}\,\,(e+f\,x)}\,\left(-1+e^{\mathrm{i}\,\,(e+f\,x)}\,\right)\right)^{-1-2\,m}\,\left(1+e^{\mathrm{i}\,\,(e+f\,x)}\,\right)\\ &\left(\frac{e^{\mathrm{i}\,\,(e+f\,x)}}{1+e^{2\,\mathrm{i}\,\,(e+f\,x)}}\right)^{-m}\left(\frac{\left(1+e^{\mathrm{i}\,\,(e+f\,x)}\,\right)^2}{1+e^{2\,\mathrm{i}\,\,(e+f\,x)}}\right)^m\,Sec\,[\,e+f\,x\,]^{\,1+m}\,\left(1+Sec\,[\,e+f\,x\,]\,\right)^{-m}\\ &\left(a\,\left(1+Sec\,[\,e+f\,x\,]\,\right)\right)^m\,\left(c-c\,Sec\,[\,e+f\,x\,]\,\right)^{-1-m}\,Sin\,\!\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]^{2\,\,(1+m)} \end{split}$$

### Problem 165: Unable to integrate problem.

$$\int Sec[e+fx] \left(a+aSec[e+fx]\right)^{m} \left(c-cSec[e+fx]\right)^{-m} dx$$

Optimal (type 5, 101 leaves, 3 steps):

$$-\frac{1}{f\left(1+2\,\text{m}\right)}2^{\frac{1}{2}-\text{m}}\,c\,\,\text{Hypergeometric}2F1\Big[\,\frac{1}{2}\,+\,\text{m}\,,\,\,\frac{1}{2}\,+\,\text{m}\,,\,\,\frac{3}{2}\,+\,\text{m}\,,\,\,\frac{1}{2}\,\left(\,1\,+\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\,\right)\,\Big]\\ \left(\,1\,-\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\,\right)^{\frac{1}{2}+\text{m}}\,\left(\,\text{a}\,+\,\text{a}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\,\right)^{\,\text{m}}\,\left(\,\text{c}\,-\,\text{c}\,\,\text{Sec}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]\,\,\right)^{-1-\text{m}}\,\,\text{Tan}\,[\,\text{e}\,+\,\text{f}\,\text{x}\,]$$

Result (type 8, 36 leaves):

$$\int Sec \left[e+fx\right] \, \left(a+a\,Sec \left[e+fx\right]\right)^m \, \left(c-c\,Sec \left[e+fx\right]\right)^{-m} \, \mathrm{d}x$$

### Problem 166: Unable to integrate problem.

$$\left\lceil \text{Sec}\left[\,e + f\,x\,\right] \, \left(\,a + a\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,\text{m}} \, \left(\,c - c\,\text{Sec}\left[\,e + f\,x\,\right]\,\right)^{\,\text{1-m}} \, \mathbb{d}\,x \right\rceil$$

Optimal (type 5, 99 leaves, 3 steps):

$$-\frac{1}{f\left(1+2\,\text{m}\right)}2^{\frac{3}{2}-\text{m}}\,c\,\,\text{Hypergeometric}2\text{F1}\!\left[-\frac{1}{2}+\text{m,}\,\,\frac{1}{2}+\text{m,}\,\,\frac{3}{2}+\text{m,}\,\,\frac{1}{2}\left(1+\text{Sec}\left[e+f\,x\right]\right)\right]\\ \left(1-\text{Sec}\left[e+f\,x\right]\right)^{-\frac{1}{2}+\text{m}}\,\left(a+a\,\text{Sec}\left[e+f\,x\right]\right)^{\text{m}}\,\left(c-c\,\text{Sec}\left[e+f\,x\right]\right)^{-\text{m}}\,\text{Tan}\left[e+f\,x\right]$$

Result (type 8, 38 leaves):

$$\int Sec \left[\,e\,+\,f\,x\,\right] \; \left(\,a\,+\,a\,Sec \left[\,e\,+\,f\,x\,\right]\,\right)^{\,m} \; \left(\,c\,-\,c\,Sec \left[\,e\,+\,f\,x\,\right]\,\right)^{\,1-m} \; \text{d}\,x$$

### Problem 167: Unable to integrate problem.

$$\int Sec \, [\, e \, + \, f \, x \, ] \, \, \left( \, a \, + \, a \, Sec \, [\, e \, + \, f \, x \, ] \, \, \right)^{\, m} \, \, \left( \, c \, - \, c \, Sec \, [\, e \, + \, f \, x \, ] \, \, \right)^{\, 2 - m} \, \, \mathrm{d} \, x$$

Optimal (type 5, 101 leaves, 3 steps):

$$-\frac{1}{f\left(1+2\,\text{m}\right)}2^{\frac{5}{2}-\text{m}}\,c^2\,\text{Hypergeometric}2\text{F1}\!\left[-\frac{3}{2}+\text{m,}\,\frac{1}{2}+\text{m,}\,\frac{3}{2}+\text{m,}\,\frac{1}{2}\left(1+\text{Sec}\left[e+f\,x\right]\right)\right]\\ \left(1-\text{Sec}\left[e+f\,x\right]\right)^{-\frac{1}{2}+\text{m}}\,\left(a+a\,\text{Sec}\left[e+f\,x\right]\right)^{\text{m}}\left(c-c\,\text{Sec}\left[e+f\,x\right]\right)^{-\text{m}}\,\text{Tan}\left[e+f\,x\right]$$

Result (type 8, 38 leaves):

$$\int Sec[e+fx] \left(a+aSec[e+fx]\right)^{m} \left(c-cSec[e+fx]\right)^{2-m} dx$$

### Problem 168: Result more than twice size of optimal antiderivative.

$$\int Sec \left[e+fx\right]^2 \left(a+a \, Sec \left[e+fx\right]\right)^3 \, \left(c-c \, Sec \left[e+fx\right]\right) \, \text{d}x$$

#### Optimal (type 3, 105 leaves, 10 steps):

$$\frac{a^{3} \ c \ ArcTanh \ [Sin \ [e+fx] \ ]}{4 \ f} + \frac{a^{3} \ c \ Sec \ [e+fx] \ Tan \ [e+fx]}{4 \ f} - \frac{4 \ f}{2 \ f} - \frac{a^{3} \ c \ Sec \ [e+fx]^{3} \ Tan \ [e+fx]^{3}}{3 \ f} - \frac{a^{3} \ c \ Tan \ [e+fx]^{5}}{5 \ f}$$

#### Result (type 3, 276 leaves):

$$-\frac{a^{3} c Log \left[Cos \left[\frac{1}{2} \left(e+fx\right)\right]-Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right]}{4 f}+\frac{a^{3} c Log \left[Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right]}{4 f}-\frac{a^{3} c}{8 f \left(Cos \left[\frac{1}{2} \left(e+fx\right)\right]-Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{4}}+\frac{a^{3} c Log \left[Cos \left[\frac{1}{2} \left(e+fx\right)\right]-Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)}{8 f \left(Cos \left[\frac{1}{2} \left(e+fx\right)\right]-Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}+\frac{a^{3} c}{8 f \left(Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}+\frac{a^{3} c Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}{8 f \left(Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}+\frac{a^{3} c Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}{15 f }+\frac{a^{3} c Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]\right)^{2}}{15 f }+\frac{a^{3} c Cos \left[\frac{1}{2} \left(e+fx\right)\right]+Sin \left[\frac{1}{2} \left(e+fx\right)\right]}{15 f }+\frac{a^{3} c Cos \left[\frac{1}{2} \left(e+fx\right)\right]}{15 f }+\frac{a^{3} c Co$$

# Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} \left[ e + f x \right]^{2} \left( c - c \operatorname{Sec} \left[ e + f x \right] \right)}{a + a \operatorname{Sec} \left[ e + f x \right]} \, dx$$

### Optimal (type 3, 56 leaves, 5 steps):

$$\frac{2\,c\, ArcTanh \left[Sin\left[e+f\,x\right]\,\right]}{a\,f}\,-\,\frac{c\, Tan\left[e+f\,x\right]}{a\,f}\,-\,\frac{2\,c\, Tan\left[e+f\,x\right]}{f\, \left(a+a\, Sec\left[e+f\,x\right]\,\right)}$$

#### Result (type 3, 154 leaves):

$$-\frac{1}{a}c\left(\frac{2 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right] - \text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]}{f}\right) - \frac{2 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right] + \text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]}{f} + \frac{\text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]}{f \left(\text{Cos} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right] - \text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]} + \frac{\text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]}{f \left(\text{Cos} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right] + \text{Sin} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]} + \frac{2 \, \text{Tan} \left[\frac{1}{2} \, \left(e+fx\right)\right.\right]}{f}$$

Problem 172: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + f x]^{2} (c - c \operatorname{Sec} [e + f x])}{(a + a \operatorname{Sec} [e + f x])^{2}} dx$$

Optimal (type 3, 70 leaves, 4 steps)

$$-\frac{\text{c ArcTanh}\left[\text{Sin}\left[\text{e}+\text{f x}\right]\right]}{\text{a}^{2}\,\text{f}}+\frac{7\,\text{c Tan}\left[\text{e}+\text{f x}\right]}{3\,\text{a}^{2}\,\text{f}\left(1+\text{Sec}\left[\text{e}+\text{f x}\right]\right)}-\frac{2\,\text{c Tan}\left[\text{e}+\text{f x}\right]}{3\,\text{f}\left(\text{a}+\text{a Sec}\left[\text{e}+\text{f x}\right]\right)^{2}}$$

Result (type 3, 335 leaves):

$$\begin{split} &\frac{1}{6\,a^2\,f\,\left(1+Sec\,[e+f\,x]\,\right)^2}\,c\,Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\,Sec\left[\frac{e}{2}\right] \\ &Sec\,[e+f\,x]^2\,\left(3\,Cos\,\left[e+\frac{3\,f\,x}{2}\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] + \\ &3\,Cos\,\left[2\,e+\frac{3\,f\,x}{2}\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] + \\ &9\,Cos\left[\frac{f\,x}{2}\right]\,\left(Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] - \\ &Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]+Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] + 9\,Cos\left[e+\frac{f\,x}{2}\right] \\ &\left(Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] - Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right] + Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] - \\ &3\,Cos\left[e+\frac{3\,f\,x}{2}\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right] + Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] - \\ &3\,Cos\left[2\,e+\frac{3\,f\,x}{2}\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right] + Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]\right] + \\ &24\,Sin\left[\frac{f\,x}{2}\right] - 6\,Sin\left[e+\frac{f\,x}{2}\right] + 10\,Sin\left[e+\frac{3\,f\,x}{2}\right] \end{split}$$

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(g\, \mathsf{Sec}\, [\,e + f\,x\,]\,\right)^{\,p} \, \left(\mathsf{a} + \mathsf{a}\, \mathsf{Sec}\, [\,e + f\,x\,]\,\right)^{\,2} \, \left(\mathsf{c} - \mathsf{c}\, \mathsf{Sec}\, [\,e + f\,x\,]\,\right) \, \mathrm{d} x$$

Optimal (type 5, 140 leaves, 5 steps):

$$\begin{split} &-\frac{1}{3\,\mathsf{f}}\mathsf{a}^2\,\mathsf{c}\,\left(\mathsf{Cos}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,2}\right)^{\frac{3+p}{2}}\,\mathsf{Hypergeometric}2\mathsf{F1}\big[\,\frac{3}{2}\,,\,\,\frac{3+p}{2}\,,\,\,\frac{5}{2}\,,\,\,\mathsf{Sin}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,2}\big] \\ &\quad \left(\mathsf{g}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\right)^{\,\mathsf{p}}\,\mathsf{Tan}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,3}\,-\,\,\frac{1}{3\,\mathsf{f}\,\mathsf{g}}\mathsf{a}^{2}\,\mathsf{c}\,\left(\mathsf{Cos}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,2}\right)^{\frac{4+p}{2}} \\ &\quad \mathsf{Hypergeometric}2\mathsf{F1}\big[\,\frac{3}{2}\,,\,\,\frac{4+p}{2}\,,\,\,\frac{5}{2}\,,\,\,\mathsf{Sin}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,2}\big]\,\left(\mathsf{g}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\right)^{\,1+p}\,\mathsf{Tan}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^{\,3} \end{split}$$

Result (type 6, 13496 leaves):

$$\frac{1}{32 f} \text{Cos} [e + fx]^4 \left(\text{Cos} [e + fx]^2\right)^{\frac{1}{2} (-1+p)} \text{Csc} \left[\frac{e}{2} + \frac{fx}{2}\right]^2$$

$$\begin{split} & \text{Hypergeometric2F1} \Big[ \frac{1}{2}, \frac{3 \cdot p}{2}, \frac{3}{2}, \text{Sin}[e + f \, x]^2 \Big] \text{Sec} \Big[ \frac{e}{2} + \frac{f \, x}{2} \Big]^4 \\ & \left[ g \, \text{Sec}[e + f \, x] \right]^p \left( a + a \, \text{Sec}[e + f \, x] \right)^2 \left( c - c \, \text{Sec}[e + f \, x] \right) \text{Sin}[e + f \, x] + \frac{1}{16 \, f} \\ & \text{Cos} \left[ e + f \, x \right]^3 \left( \text{Cos}[e + f \, x]^2 \right)^{p/2} \text{Csc} \Big[ \frac{e}{2} + \frac{f \, x}{2} \Big]^2 \text{Hypergeometric2F1} \Big[ \frac{1}{2}, \frac{4 + p}{2}, \frac{3}{2}, \frac{3}{2}, \frac{5 \text{In}[e + f \, x]^2} \Big] \\ & \text{Sec} \Big[ \frac{e}{2} + \frac{f \, x}{2} \Big]^3 \left( g \, \text{Sec}[e + f \, x] \right)^p \left( a + a \, \text{Sec}[e + f \, x] \right)^2 \left( c - c \, \text{Sec}[e + f \, x] \right) \text{Sin}[e + f \, x] - \frac{3}{2} \\ & \text{Sec} \Big[ \frac{e}{2} + \frac{f \, x}{2} \Big]^2 \text{Sec} \Big[ \frac{e}{2} + \frac{f \, x}{2} \Big]^4 \text{Sec}[e + f \, x]^{-2 + p} \left( g \, \text{Sec}[e + f \, x] \right)^p \left( a + a \, \text{Sec}[e + f \, x] \right)^2 \right) \\ & \left( c - c \, \text{Sec}[e + f \, x] \right) \left( -5 \text{Ce}[e + f \, x]^{-2 + p} + 2 \, \text{Cos} \left[ 2 \left( e + f \, x \right) \right] \text{Sec}[e + f \, x]^{2 + p} \right) \text{Tan} \Big[ \frac{1}{2} \left( e - f \, x \right) \right] \\ & \left( c - c \, \text{Sec}[e + f \, x] \right)^2 \right)^p \left( \left[ 4 \, \text{AppellF1} \Big[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \right]^2, -\text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \right) \\ & \left( -1 + \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \right)^p \left( \left[ 4 \, \text{AppellF1} \Big[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \right) - \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) + 2 \left[ \left( -1 + f \, x \right) \Big]^2 \right] \\ & \left( -1 + \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \right)^p \left( \left[ 4 \, \text{AppellF1} \Big[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) - \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) + 2 \left[ \left( -1 + f \, x \right) \Big]^2 \right] \\ & \left( -1 + \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right)^p \left( \left[ 4 \, \text{AppellF1} \Big[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) - \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) + 2 \left[ \left( - f \, x \right) \Big]^2 \right] + 2 \left[ \left( - f \, x \right) \Big]^2 \right] \\ & \left( -1 + \text{Tan} \Big[ \frac{1}{2}, \left( e + f \, x \right) \Big]^2 \right) \left( \left[ 1 + \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) - \text{Tan} \Big[ \frac{1}{2} \left( e + f \, x \right) \Big]^2 \right) + 2 \left[ \left( -1 + f \, x \right) \Big]^2 \right) \right] \\ & \left( -1 + \text{Tan} \Big[ \frac$$

$$\left[ -\frac{1}{\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)^{3}} 12 \operatorname{Sec}\left[\frac{1}{2}\left(e + f x\right)\right]^{2} \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2} \left(\frac{1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}}{1 - Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}}\right)^{2} \right]$$

$$\left(\left[4 \operatorname{AppellF1}\left[\frac{1}{2}, p, 1 - p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \right)$$

$$\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)^{2} \right) / \left(\left[1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \left[3 \operatorname{AppellF1}\left[\frac{1}{2}, p, 1 - p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right], -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + p \operatorname{AppellF1}\left[\frac{3}{2}, p, 2 - p, \frac{5}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + p \operatorname{AppellF1}\left[\frac{3}{2}, 1 + p, 2 - p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + p \operatorname{AppellF1}\left[\frac{3}{2}, 1 + p, 2 - p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right]$$

$$\left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 1 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \right) \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right]$$

$$\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 1 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 1 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{5}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(-1 + Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \left(2 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) \right) \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(2 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) \right)$$

$$\left(2 \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{3}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(2 \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{5}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) \right)$$

$$\left(2 \operatorname{AppellF1}\left[\frac{3}{2}, 3 + p, -p, \frac{5}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)$$

$$\left(2 \operatorname{AppellF1}\left[\frac{3}{2}, 3 + p, -p, \frac{5}{2}, Tan\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) \right)$$

$$\left(\left[1 \operatorname{AppellF1}\left[\frac{3}{2}, 1 + p, \frac{3}{2}, T$$

$$1-p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, 1 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, 1 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) / \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, 1 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + \left( 1 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, 2 + p, -p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) - \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right] + \left( 2 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, 2 + p, 1 - p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) + \left( 2 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, 3 + p, -p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) + \left( 2 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, 3 + p, -p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) + \left( \frac{\operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + \left( \frac{\operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + \left( \frac{\operatorname{AppellF1} \left[ \frac{1}{2}, e + f x \right) \right)^2 \right) / \left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) - \left( \left( 4 \operatorname{AppellF1} \left[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) - \left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \left( \left( 1 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) - \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$\left( 3 \, \mathsf{AppellFI} \left[ \frac{1}{2}, \, 1 + \mathsf{p}, \, \, \mathsf{p}, \, \frac{3}{2}, \, \mathsf{Tan} \right[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right] + \\ 2 \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 1 + \mathsf{p}, \, 1 - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \right[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ (1+\mathsf{p}) \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 2 + \mathsf{p}, \, -\mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) - \\ -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) - \\ \left( 2 \, \mathsf{AppellFI} \left[ \frac{1}{2}, \, 2 + \mathsf{p}, \, -\mathsf{p}, \, \frac{3}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right] \right) + \\ \frac{2}{3} \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 2 + \mathsf{p}, \, -\mathsf{p}, \, \frac{3}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \frac{2}{3} \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 2 + \mathsf{p}, \, -\mathsf{p}, \, \frac{3}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \frac{2}{3} \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 3 + \mathsf{p}, \, -\mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \frac{2}{3} \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 3 + \mathsf{p}, \, -\mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right) + \\ \frac{1}{3} \left( -\mathsf{I} \, \mathsf{Im} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \frac{1}{3} \left( -\mathsf{Im} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right)^2 \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \frac{1}{3} \left( -\mathsf{Im} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right) \\ \frac{1}{3} \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{$$

$$\left( \text{AppellF1} \left[ \frac{1}{2}, 2 + p, -p, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] + \\ \frac{2}{3} \left( p \text{AppellF1} \left[ \frac{3}{2}, 2 + p, 1 - p, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] + \\ (2 + p) \text{ AppellF1} \left[ \frac{3}{2}, 3 + p, -p, \frac{5}{2}, \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right]$$

$$\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] - \left[ 4 \text{ AppellF1} \left[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \right]$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] - \left[ 1 + \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right]$$

$$\left( 2 \left( \left( -1 + p \right) \text{ AppellF1} \left[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \text{ Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{ Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{ Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$-\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{ Sec} \left[ \frac{1}{2} \left( e + f x \right)$$

$$(2+p) \operatorname{AppellF1}\left[\frac{3}{2}, 3+p, -p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] \right)$$
 
$$\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] + \frac{2}{3}\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \right]$$
 
$$\left(p\left[-\frac{3}{3}\left(1-p\right) \operatorname{AppellF1}\left[\frac{5}{2}, 2+p, 2-p, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]$$
 
$$-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]$$
 
$$\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] + \left(2+p\right) \left(\frac{3}{5}\operatorname{p AppellF1}\left[\frac{5}{2}, 3+p, -p, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]$$
 
$$\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2 \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{AppellF1}\left[\frac{1}{2}, 2+p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \right]$$
 
$$\left(\operatorname{AppellF1}\left[\frac{3}{2}, 2+p, -p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] \operatorname{Cas}\left[\frac{e+fx}{2}\right]^2 \operatorname{Sec}\left[\frac{e+fx}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]$$
 
$$\left(\operatorname{Cas}\left[2\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] - \operatorname{Tan}\left[\frac{1}{2}\left(e+$$

$$\begin{split} &-\text{Tan}\big[\frac{1}{2}\left(e+fx)\big]^2\big]\right)\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\big)+\\ &\left(12\text{AppelIFI}\big[\frac{1}{2},2+p,-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2,-\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\big]\\ &\left(1+\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)\right)\bigg/\\ &\left(3\text{AppelIFI}\big[\frac{1}{2},2+p,-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2,-\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right]+\\ &2\left(p\text{AppelIFI}\big[\frac{3}{2},2+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2,-\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right]+\\ &(2+p)\text{AppelIFI}\big[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)+\\ &\left(4\text{AppelIFI}\big[\frac{1}{2},3+p,-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(\text{AppelIFI}\big[\frac{1}{2},3+p,-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(\text{AppelIFI}\big[\frac{1}{2},3+p,-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,1-p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,p,p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\text{AppelIFI}\big[\frac{3}{2},3+p,p,p,\frac{3}{2},\text{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\right)-\\ &\left(3+p\right)\frac{1}{2}\left(e+fx\right)\right)^2\right)^2 -\frac{1}{2}\left(2+fx\right)^2\right)^2 -\frac{1}{2}\left(2+fx\right)^2\right)^2$$

$$\left(12 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \right) \\ = \left(-1 + \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) \right) / \\ \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + \\ 2 \left(p \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, 1 - p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + \\ \left(2 + p\right) \operatorname{AppellF1}\left[\frac{3}{2}, 3 + p, -p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, \\ -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2} + \\ \left(4 \operatorname{AppellF1}\left[\frac{1}{2}, 3 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] \right) / \\ \left(\operatorname{AppellF1}\left[\frac{1}{2}, 3 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right] + \\ \left(3 + p\right) \operatorname{AppellF1}\left[\frac{3}{2}, 3 + p, 1 - p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) + \\ \left(3 + p\right) \operatorname{AppellF1}\left[\frac{3}{2}, 4 + p, -p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}, -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \\ \frac{1}{\left(1 - \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)^{2}} \operatorname{Sec}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\left(\frac{1 + \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}}{1 - \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}}\right) - \\ -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) / \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, 1 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \\ -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)^{2} / \left(3 \operatorname{AppellF1}\left[\frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \\ -\operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right)^{2} / \left(3 \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) + \\ \left(1 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) + \\ \left(1 \operatorname{AppellF1}\left[\frac{1}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) + \\ \left(1 \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) - \operatorname{Tan}\left[\frac{1}{2}\left(e + f x\right)\right]^{2}\right) + \\ \left(1 \operatorname{AppellF1}\left[\frac{3}{2}, 2 + p, -p, \frac{3}{2}, \operatorname{Tan}\left$$

$$- \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big) \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big) + \\ \Big[ 4 \text{AppellFI} \Big[ \frac{1}{2}, 3 + p, -p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2, - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \Big] \Big/ \\ \Big[ \text{AppellFI} \Big[ \frac{1}{2}, 3 + p, -p, \frac{3}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2, - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + \\ \Big[ \frac{2}{3} \left( p \text{AppellFI} \Big[ \frac{3}{2}, 3 + p, 1 - p, \frac{5}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2, - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + \\ \Big[ \text{(3 + p) AppellFI} \Big[ \frac{3}{2}, 4 + p, -p, \frac{5}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2, \\ - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \Big( \frac{1 + \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^{-1 + p} \\ \\ \frac{1}{2} \frac{\text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^{-1 + p} \\ \\ \frac{1}{2} \frac{\text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^{-1 + p} \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^{-1} \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2 \Big( \frac{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2}{1 - \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2} \Big)^2 \Big)^2$$

$$\begin{split} \left( \mathsf{AppellF1} \Big[ \frac{1}{2}, \, 3+p, \, -p, \, \frac{3}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \Big] + \\ \frac{2}{3} \left( p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 3+p, \, 1-p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \Big] + \\ (3+p) \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 4+p, \, -p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \\ -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \Big] \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \Big) - \\ \frac{1}{\left( 1 - \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right)^3} \, 2 \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big] \, \left( \frac{1 + \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2}{1 - \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2} \right)^p \\ \left( -\left( \left( 6 \, \mathsf{AppellF1} \Big[ \frac{1}{2}, \, 1+p, \, -p, \, \frac{3}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right) \\ \left( -\left( \left( 6 \, \mathsf{AppellF1} \Big[ \frac{1}{2}, \, 1+p, \, -p, \, \frac{3}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right) \\ \left( 3 \, \mathsf{AppellF1} \Big[ \frac{1}{2}, \, 1+p, \, -p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) + \\ \left( 1 + p \right) \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 2+p, \, -p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \\ - \left( 3 \, \left( \frac{1}{3} \, p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 1+p, \, 1-p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right] \\ - \left( 3 \, \left( \frac{1}{3} \, p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 2+p, \, -p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right] \\ - \left( 3 \, \left( \frac{1}{3} \, p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 2+p, \, -p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right) \\ - \left( 3 \, \left( \frac{1}{3} \, p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 2+p, \, -p, \, \frac{3}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right) \right) \\ - \left( 3 \, \left( \frac{1}{3} \, p \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 1+p, \, 1-p, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2, \, -\mathsf{Tan} \Big[ \frac{1}{2} \left( e + fx \right) \Big]^2 \right) \right) \\$$

$$2 \left( \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 2 + \mathsf{p}, \, 1 - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \right[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right] + \\ (2 + \mathsf{p}) \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 3 + \mathsf{p}, \, - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \right[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \\ -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) + \\ \left[ 12 \left( \frac{1}{3} \, \mathsf{p} \, \mathsf{AppellFI} \left[ \frac{3}{2}, \, 2 + \mathsf{p}, \, 1 - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right] \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right] + \frac{1}{3} \left( 2 + \mathsf{p} \right) \right. \\ \mathsf{AppellFI} \left[ \frac{3}{2}, \, 3 + \mathsf{p}, \, - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \right] \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right]^2 \right) \\ \mathsf{Sec} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \right$$

$$\begin{split} & \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}] \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, \\ & \frac{1}{3}\left(1+p\right) \operatorname{AppellF1}[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}] \\ & \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)] + 2\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, \\ & \left(p\left(-\frac{3}{5}\left(1-p\right) \operatorname{AppellF1}[\frac{5}{2},1+p,2-p,\frac{7}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, \\ & -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, \\ & -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}] \\ & \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)] + \left(1+p\right)\left(\frac{3}{5}\operatorname{pAppellF1}[\frac{5}{2},2+p, \\ & 1-p,\frac{7}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}] \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, \\ & -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2} \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}) \right) \\ & \left(\beta\operatorname{AppellF1}[\frac{1}{2},1+p,-p,\frac{3}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}) + \left(1+p\right)\operatorname{AppellF1}[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}) \right] \\ & \left(1+p\right)\operatorname{AppellF1}[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}\right) \\ & \left(2\operatorname{AppellF1}[\frac{1}{2},2+p,-p,\frac{3}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}\right) \\ & \left(2\operatorname{AppellF1}[\frac{1}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}\right) \\ & \left(2\operatorname{AppellF1}[\frac{3}{2},2+p,1-p,\frac{5}{2},\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}\right) \\ & \operatorname{Sec}[\frac{1}{2}\left(e+fx\right)]^{2}\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}, -\operatorname{Tan}[\frac{1}{2}\left(e+fx\right)]^{2}\right) \\ & \operatorname{Sec}[\frac{1}{2}\left(e$$

$$-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]\text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}, -\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]$$

$$AppellF1\left[\frac{5}{2},3+p,1-p,\frac{7}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}, -\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]$$

$$\text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}, -\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]$$

$$\text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}, -\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]$$

$$\frac{1}{2}\left(e+fx\right)\right]^{2}, -\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right]$$

$$-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \text{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(p\text{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{3}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(p\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(2+p\right)\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(2+p\right)\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(3+p\right)\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,1-p,\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + 3$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,1-p,\frac{5}{2},\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + 3$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2},3+p,1-p,\frac{5}{2},\frac{5}{2},\text{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + 3$$

$$\left(\frac{3}{2}p\text{AppellF1}\left[\frac{3}{2$$

$$-\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]\right)\right)\right)\Big/$$

$$\left(\mathsf{AppellF1}\left[\frac{1}{2},\,3+\mathsf{p,-p,\frac{3}{2}},\,\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2,\,-\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right]+\frac{2}{3}$$

$$\left(\mathsf{p}\,\mathsf{AppellF1}\left[\frac{3}{2},\,3+\mathsf{p,\,1-p,\,\frac{5}{2}},\,\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2,\,-\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right]+$$

$$\left(3+\mathsf{p}\right)\,\mathsf{AppellF1}\left[\frac{3}{2},\,4+\mathsf{p,-p,\,\frac{5}{2}},\,\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right],$$

$$-\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right)\,\operatorname{Tan}\left[\frac{1}{2}\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\right]^2\right)$$

# Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\left\lceil \left( g \, \mathsf{Sec} \, [\, e + f \, x \, ] \, \right)^{\, p} \, \left( \mathsf{a} + \mathsf{a} \, \mathsf{Sec} \, [\, e + f \, x \, ] \, \right) \, \left( \mathsf{c} - \mathsf{c} \, \mathsf{Sec} \, [\, e + f \, x \, ] \, \right) \, \mathbb{d} \, x \right.$$

Optimal (type 5, 65 leaves, 2 steps):

$$-\frac{1}{3\,f}$$

$$a\,c\,\left(\text{Cos}\,[\,e+f\,x\,]^{\,2}\right)^{\frac{3+p}{2}}\,\text{Hypergeometric}2\text{F1}\!\left[\,\frac{3}{2}\,,\,\,\frac{3+p}{2}\,,\,\,\frac{5}{2}\,,\,\,\text{Sin}\,[\,e+f\,x\,]^{\,2}\,\right]\,\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,p}\,\text{Tan}\,[\,e+f\,x\,]^{\,3}$$

Result (type 6, 6864 leaves):

-ac 
$$\left( -\left( \left( \cos \left[ 2 \left( e + f x \right) \right] \csc \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \sec \left[ \frac{1}{2} \left( e + f x \right) \right]^{3} \left( -1 + \sec \left[ e + f x \right] \right) \left( g \sec \left[ e + f x \right] \right)^{p} \left( 1 + \left( e + f x \right) \right)^{2} \right) \right) \left( \left( 6 \operatorname{AppellF1} \left[ \frac{1}{2}, \, p, \, 1 - p, \, \frac{3}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) \right) \left( \left( 1 + \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) \left( \left( 1 + \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \, p, \, 1 - p, \, \frac{3}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) \left( \left( 1 + \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) \right)$$

$$\left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \, p, \, 1 - p, \, \frac{3}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2}, \, -\tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) + 2 \right)$$

$$\left( \left( -1 + p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, \, 1 + p, \, 1 - p, \, \frac{5}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2}, \, -\tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right)$$

$$\operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) - \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \, 1 + p, \, -p, \, \frac{3}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right)$$

$$\left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, \, 1 + p, \, -p, \, \frac{3}{2}, \, \tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2}, \, -\tan \left[ \frac{1}{2} \left( e + f x \right) \right]^{2} \right) \right) +$$

$$2 \left( \mathsf{p} \mathsf{AppellF1} \left[ \frac{3}{2}, 1 + \mathsf{p}, 1 - \mathsf{p}, \frac{5}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right] + \\ (1 + \mathsf{p}) \, \mathsf{AppellF1} \left[ \frac{3}{2}, 2 - \mathsf{p}, -\mathsf{p}, \frac{5}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, \\ -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right] \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right) - \\ \left( 2 \, \mathsf{AppellF1} \left[ \frac{1}{2}, 2 + \mathsf{p}, -\mathsf{p}, \frac{3}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right] \right) / \\ \left( \mathsf{AppellF1} \left[ \frac{1}{2}, 2 + \mathsf{p}, -\mathsf{p}, \frac{3}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right] \right) + \\ \left( 2 + \mathsf{p}) \, \mathsf{AppellF1} \left[ \frac{3}{2}, 2 + \mathsf{p}, 1 - \mathsf{p}, \frac{5}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right) + \\ \left( 2 + \mathsf{p}) \, \mathsf{AppellF1} \left[ \frac{3}{2}, 3 + \mathsf{p}, -\mathsf{p}, \frac{5}{2}, \mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2, -\mathsf{Tan} \left[ \frac{1}{2} \left( \mathsf{e} + \mathsf{f} \mathsf{x} \right) \right]^2 \right) / \\ \left( 2 + \mathsf{f} \, \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{e} \, \mathsf{e} \, \mathsf{e} \, \mathsf{f} \, \mathsf{e} \, \mathsf{e$$

$$\begin{split} &\left[\frac{\sec\left[\frac{1}{2}\left(e+fx\right)\right]^{2} \tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}}{1-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}} + \left[\sec\left[\frac{1}{2}\left(e+fx\right)\right]^{2} \tan\left[\frac{1}{2}\left(e+fx\right)\right] \right] \\ &\left[\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)\right] \middle/ \left(1-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)^{2} \\ &\left[\left(6 \operatorname{AppellF1}\left[\frac{1}{2},p,1-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \\ &\left[-1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \middle/ \left(\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) \\ &\left[3 \operatorname{AppellF1}\left[\frac{1}{2},p,1-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] \\ &-2\left(\left(-1+p\right)\operatorname{AppellF1}\left[\frac{3}{2},p,2-p,\frac{5}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) \\ &-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + p\operatorname{AppellF1}\left[\frac{3}{2},1+p,1-p,\frac{5}{2},\\ &-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] - \tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) \tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) - \\ &\left(3 \operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) - \left(3 \operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)\right) - \\ &\left(3 \operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) - \left(3 \operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)\right) - \\ &-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + 2\left(p\operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) - \\ &-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right] + \left(1+p\operatorname{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{5}{2},\\ &-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)\right] + \left(2\operatorname{AppellF1}\left[\frac{1}{2},2+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right)\right] + \\ &\left(2+p\operatorname{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{3}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2},-\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(2+p\operatorname{AppellF1}\left[\frac{3}{2},2+p,1-p,\frac{5}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(2+p\operatorname{AppellF1}\left[\frac{3}{2},2+p,1-p,\frac{5}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(2+p\operatorname{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(2+p\operatorname{AppellF1}\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) \tan\left[\frac{1}{2}\left(e+fx\right)\right] + \\ &\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \frac{1}{2}\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \frac{1}{2}\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \frac{1}{2}\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \frac{1}{2}\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \\ &\left(1+\tan\left[\frac{1}{2}\left(e+fx\right)\right]^{2}\right) + \frac{1}{2}\left$$

$$\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)^2 \left( 3 \text{AppellFI} \left[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \left( \left( -1 + p \right) \text{AppellFI} \left[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right] \right) \right)^2 + 2 \left( \left( -1 + p \right) \text{AppellFI} \left[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right] \right) \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)^2 - \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + p \text{AppellFI} \left[ \frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \left( 3 \text{AppellFI} \left[ \frac{1}{2}, p, 1 - p, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \left( \left( -1 + p \right) \text{AppellFI} \left[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right] \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) - \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + p \text{AppellFI} \left[ \frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right)$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + 2 \left( \left( -1 + p \right) \text{AppellFI} \left[ \frac{3}{2}, 1 + p \right) - p, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) + \frac{3}{2}, \text{T$$

$$\begin{split} &\operatorname{Tan}\big[\frac{1}{2}\left(e+fx\right)\big]^2\big) - \left(3\left(\frac{1}{3}\operatorname{pAppellF1}\left[\frac{3}{2},1+p,1-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right), \\ &-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] + \frac{1}{3}\left(1+p\right) \\ &\operatorname{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] \\ &\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]\right)\left(-1+\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right)\right) / \\ &\left(3\operatorname{AppellF1}\left[\frac{1}{2},1+p,-p,\frac{3}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] + \\ &2\left(\operatorname{pAppellF1}\left[\frac{3}{2},1+p,1-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] + \\ &\left(1+p\right)\operatorname{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right] + \\ &\left(1+p\right)\operatorname{AppellF1}\left[\frac{3}{2},2+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] + \\ &\frac{1}{3}\left(2+p\right)\operatorname{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right]\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right] \right) \right) / \\ &\left(\operatorname{AppellF1}\left[\frac{3}{2},2+p,1-p,\frac{3}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) - \left(\operatorname{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) - \operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) + \\ &\left(2+p\right)\operatorname{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) - \left(\operatorname{AppellF1}\left[\frac{3}{2},3+p,-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) + \\ &\operatorname{AppellF1}\left[\frac{3}{2},p,2-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) + \\ &\operatorname{AppellF1}\left[\frac{3}{2},1+p,1-p,\frac{5}{2},\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &\operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2,-\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \operatorname{Sec}\left[\frac{1}{2}\left(e+fx\right)\right]^2\right) \\ &\operatorname{Tan}\left[\frac{1}{2}\left(e+fx\right)\right]^2$$

$$\begin{split} \frac{7}{2}, & \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2, & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, p, 2 - p, \frac{5}{2}, \right) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \frac{5}{2}, \right) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \text{AppellF1} \Big[ \frac{3}{2}, 1 + p, 1 - p, \frac{5}{2}, \text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big) \\ & -\text{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] \text{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^2 \Big] + 2 \left( \left( -1 + p \right) \left( -1 + p \right) \Big[ \frac{1}{2} \left( -1 + p \right) \Big] + 2 \left( -1 + p \right) \Big[ \frac{1}{2} \left( -1 + p \right) \Big] + 2 \left( -1 + p \right) \Big[ \frac{1}{2} \left( -1 + p \right) \Big] + 2 \left( -1 + p \right) \Big[ \frac{1}{2} \left( -1 + p \right) \Big] + 2 \left( -1 + p \right) \Big[ \frac{1}{2} \left( -1 +$$

$$(1+p) \left( \frac{3}{5} \, \mathsf{p} \, \mathsf{AppellFI} [\frac{5}{2}, \, 2+\mathsf{p}, \, 1-\mathsf{p}, \, \frac{7}{2}, \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \\ -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] \, \mathsf{Sec} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})] + \\ \frac{3}{5} \, (2+p) \, \mathsf{AppellFI} [\frac{5}{2}, \, 3+\mathsf{p}, \, -\mathsf{p}, \, \frac{7}{2}, \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \\ -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] \, \mathsf{Sec} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \\ -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] \, \mathsf{Sec} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \, -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] + \\ 2 \, \left( \mathsf{p} \, \mathsf{AppellFI} [\frac{3}{2}, \, 1+\mathsf{p}, \, 1-\mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \, -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] + \\ (1+p) \, \mathsf{AppellFI} [\frac{3}{2}, \, 2+\mathsf{p}, \, -\mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2, \, -\mathsf{Tan} [\frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x})]^2 \right] \\ -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right] \\ -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right] \\ -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right] \\ -\mathsf{Sec} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right] \\ -\mathsf{Sec} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \\ -\mathsf{Sec} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \\ -\mathsf{Sec} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \\ -\mathsf{Sec} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \, \mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2, \, -\mathsf{Tan} \left[ \frac{1}{2} \, (\mathsf{e}+\mathsf{f}\,\mathsf{x}) \right]^2 \right) \\ -\mathsf{Tan} \left[ \frac$$

$$- \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \text{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) \right) / \\ \left( \text{AppellF1} \left[ \frac{1}{2}, 2 + p, -p, \frac{3}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] + \\ \frac{2}{3} \left( p \text{AppellF1} \left[ \frac{3}{2}, 2 + p, 1 - p, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] + \\ \left( 2 + p \right) \text{AppellF1} \left[ \frac{3}{2}, 3 + p, -p, \frac{5}{2}, \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2, \\ -\text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right] \text{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \right) \right) \right) + \\ \frac{1}{8 f} \left( \text{Cos} \left[ e + f x \right]^2 \right)^{1 + \frac{10}{2}} \text{Csc} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \text{Hypergeometric2F1} \right[ \\ \frac{1}{2}, \\ \frac{3 + p}{2}, \\ \frac{3}{2}, \\ \text{Sin} \left[ e + f x \right]^2 \right] \left( -1 + \text{Sec} \left[ e + f x \right] \right) \\ \left( g \\ \text{Sec} \left[ e + f x \right] \right)^p \left( 1 + \text{Sec} \left[ e + f x \right] \right) \\ \text{Sec} \left[ e + f x \right] \right) \text{Tan} \left[ e + f x \right] \right)$$

Problem 176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(g \operatorname{Sec}[e+fx]\right)^{p} \left(c-c \operatorname{Sec}[e+fx]\right)}{a+a \operatorname{Sec}[e+fx]} dx$$

Optimal (type 5, 180 leaves, 6 steps):

$$-\left(\left(c\,g\,\left(1-2\,p\right)\,\mathsf{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1-p}{2},\,\frac{3-p}{2},\,\mathsf{Cos}\,[e+f\,x]^{\,2}\right]\right.\\ \left.\left(g\,\mathsf{Sec}\,[e+f\,x]\right)^{-1+p}\,\mathsf{Sin}\,[e+f\,x]\right)\bigg/\left(a\,f\,\left(1-p\right)\,\sqrt{\mathsf{Sin}\,[e+f\,x]^{\,2}}\right)\right)+\\ \left(2\,c\,\mathsf{Hypergeometric2F1}\left[\frac{1}{2},\,-\frac{p}{2},\,\frac{2-p}{2},\,\mathsf{Cos}\,[e+f\,x]^{\,2}\right]\,\left(g\,\mathsf{Sec}\,[e+f\,x]\right)^{p}\,\mathsf{Sin}\,[e+f\,x]\right)\bigg/\left(a\,f\,\sqrt{\mathsf{Sin}\,[e+f\,x]^{\,2}}\right)-\frac{2\,c\,\left(g\,\mathsf{Sec}\,[e+f\,x]\right)^{p}\,\mathsf{Tan}\,[e+f\,x]}{f\,\left(a+a\,\mathsf{Sec}\,[e+f\,x]\right)}$$

Result (type 6, 3396 leaves):

$$- \left( \left[ 6 \operatorname{c} \operatorname{Sec}[e+fx]^p \left( g \operatorname{Sec}[e+fx] \right)^p \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^3 \right. \\ \left. \left( - \left[ \left( \left[ \left( \operatorname{AppellF1} \left[ \frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] \operatorname{Cos} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] \right/ \\ \left. \left( 3 \operatorname{AppellF1} \left[ \frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] + \\ 2 \left( \left( -1+p \right) \operatorname{AppellF1} \left[ \frac{3}{2}, p, 2-p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] + \\ \operatorname{AppellF1} \left[ \frac{3}{2}, 1+p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) - \\ \operatorname{AppellF1} \left[ \frac{1}{2}, p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) \right) + \\ \operatorname{AppellF1} \left[ \frac{1}{2}, p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] + \\ \operatorname{AppellF1} \left[ \frac{1}{2}, p, -p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right] + \\ \operatorname{AppellF1} \left[ \frac{3}{2}, p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) + \\ \operatorname{AppellF1} \left[ \frac{3}{2}, p, 1-p, \frac{5}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2, -\operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) \right) \right) \right) \right)$$

$$\left( \operatorname{af} \left( 3 \operatorname{Sec} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \operatorname{Sec} \left[ e+fx \right]^p \left( - \left( \left( \operatorname{AppellF1} \left[ \frac{1}{2}, p, 1-p, \frac{3}{2}, \operatorname{Tan} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

$$\left( \operatorname{af} \left( 3 \operatorname{Sec} \left[ \frac{1}{2} \left( e+fx \right) \right]^2 \operatorname{Sec} \left[ e+fx \right]^2 \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \right) \right) \right) \right) \right) \right)$$

$$\left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \operatorname{Sec} \left[ e+fx \right]^2 \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \right) \right) \right) \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right)^2 \right) \right) \right) \right) \right) \right) \right)$$

$$\left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right) \right) \left( \operatorname{af} \left( \frac{1}{2} \left( e+fx \right) \right) \right) \right) \right) \right) \right) \right) \left( \operatorname$$

$$\begin{split} & \mathsf{p} \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 1 + \mathsf{p}, \, 1 - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2, \, - \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \Big] \\ & \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big] + 3 \, \bigg( -\frac{1}{3} \, \big( 1 - \mathsf{p} \big) \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, \mathsf{p}, \, 2 - \mathsf{p}, \, \frac{5}{2}, \, \\ & \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2, \, - \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \Big] \, \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \\ & = \frac{1}{3} \, \mathsf{p} \, \mathsf{AppellF1} \Big[ \frac{3}{2}, \, 1 + \mathsf{p}, \, 1 - \mathsf{p}, \, \frac{5}{2}, \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2, \, - \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big] + 2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \Big] \\ & = \mathsf{Sec} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big[ \frac{1}{2} \, \big( \mathsf{e} + \mathsf{f} \, \mathsf{x} \big) \Big]^2 \, \mathsf{Tan} \Big$$

# Problem 177: Unable to integrate problem.

$$\int \frac{\left(g\, \text{Sec}\, [\, e + f\, x\, ]\,\right)^{\,p}\, \left(c - c\, \text{Sec}\, [\, e + f\, x\, ]\,\right)}{\left(a + a\, \text{Sec}\, [\, e + f\, x\, ]\,\right)^{\,2}}\, \, \text{d} x}$$

Optimal (type 5, 226 leaves, 7 steps):

$$-\left(\left(c\,g\,\left(3-4\,p\right)\,\text{Hypergeometric}2F1\Big[\frac{1}{2},\,\frac{1-p}{2},\,\frac{3-p}{2},\,\cos{[\,e+f\,x\,]^{\,2}}\Big]\right.\\ \left.\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{-1+p}\,\text{Sin}\,[\,e+f\,x\,]\,\right)\bigg/\left(3\,a^{2}\,f\,\sqrt{\,\text{Sin}\,[\,e+f\,x\,]^{\,2}}\,\right)\right)+\\ \left(c\,\left(5-4\,p\right)\,\text{Hypergeometric}2F1\Big[\frac{1}{2},\,-\frac{p}{2},\,\frac{2-p}{2},\,\cos{[\,e+f\,x\,]^{\,2}}\Big]\,\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{p}\,\text{Sin}\,[\,e+f\,x\,]\,\right)\bigg/\\ \left(3\,a^{2}\,f\,\sqrt{\,\text{Sin}\,[\,e+f\,x\,]^{\,2}}\,\right)-\\ \frac{c\,\left(5-4\,p\right)\,\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{p}\,\text{Tan}\,[\,e+f\,x\,]}{3\,a^{2}\,f\,\left(1+\text{Sec}\,[\,e+f\,x\,]\,\right)}-\\ \frac{2\,c\,\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{p}\,\text{Tan}\,[\,e+f\,x\,]}{3\,f\,\left(a+a\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,2}}$$

Result (type 8, 36 leaves):

$$\int \frac{\left(g\, \text{Sec}\, [\, e + f\, x\, ]\,\right)^p\, \left(c - c\, \text{Sec}\, [\, e + f\, x\, ]\,\right)}{\left(a + a\, \text{Sec}\, [\, e + f\, x\, ]\,\right)^2}\, \, \text{d} x}$$

## Problem 180: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]^{5/2}}{\sqrt{a+a\operatorname{Sec}[e+fx]}} dx$$

Optimal (type 3, 140 leaves, 8 steps):

$$-\frac{2\, \text{ArcSinh} \big[\frac{\sqrt{a}\, \text{Tan}[\text{e+f}\, x]}{\sqrt{\text{a+a}\, \text{Sec}[\text{e+f}\, x]}}\big]}{\sqrt{a}\, \text{cf}} + \frac{\frac{\text{ArcTanh} \big[\frac{\sqrt{a}\, \sqrt{\text{Sec}[\text{e+f}\, x]}\, \text{Sin}[\text{e+f}\, x]}{\sqrt{2}\, \sqrt{\text{a+a}\, \text{Sec}[\text{e+f}\, x]}}\big]}{\sqrt{2}\, \sqrt{a}\, \text{cf}} + \frac{\text{Csc}\, [\text{e+f}\, x]\, \sqrt{\text{a+a}\, \text{Sec}[\text{e+f}\, x]}}{\text{acf}\, \sqrt{\text{Sec}[\text{e+f}\, x]}}$$

Result (type 3, 724 leaves):

$$\begin{cases} Sec \left[ e + fx \right]^{3/2} \sqrt{\left( 1 + Cos \left[ e + fx \right] \right) Sec \left[ e + fx \right]} & \sqrt{1 + Sec \left[ e + fx \right]} \end{cases} \\ \left( -\frac{2 \cot \left[ e \right]}{f} + \frac{Csc \left[ \frac{e}{2} \right] Csc \left[ \frac{e}{2} + \frac{fx}{2} \right] Sin \left[ \frac{fx}{2} \right]}{f} + \frac{Sec \left[ \frac{e}{2} \right] Sec \left[ \frac{e}{2} + \frac{fx}{2} \right] Sin \left[ \frac{fx}{2} \right]}{f} \right) Sin \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \right) / \\ \left( \sqrt{a} \left( 1 + Sec \left[ e + fx \right] \right) \left( c - c Sec \left[ e + fx \right] \right) \right) + \\ \left( Cos \left[ e + fx \right] \left( Log \left[ 1 - 2 Sec \left[ e + fx \right] - 3 Sec \left[ e + fx \right]^2 - 2 \sqrt{2} \sqrt{Sec \left[ e + fx \right]} \sqrt{1 + Sec \left[ e + fx \right]} \sqrt{-1 + Sec \left[ e + fx \right]^2} \right) - Log \left[ 1 - 2 Sec \left[ e + fx \right] - 3 Sec \left[ e + fx \right]^2 + 2 \sqrt{2} \sqrt{Sec \left[ e + fx \right]} \sqrt{1 + Sec \left[ e + fx \right]} \sqrt{-1 + Sec \left[ e + fx \right]^2} \right] \right) \\ \left( 1 + Sec \left[ e + fx \right] \right)^{3/2} \sqrt{-1 + Sec \left[ e + fx \right]^2} Sin \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 Sin \left[ e + fx \right] \right) / \\ \left( 2 f \left( 1 + Cos \left[ e + fx \right] \right) \sqrt{2 - 2 Cos \left[ e + fx \right]^2} \sqrt{1 - Cos \left[ e + fx \right]^2} \right) \\ \sqrt{a \left( 1 + Sec \left[ e + fx \right] \right)} \left( c - c Sec \left[ e + fx \right] \right) \right) + \\ \left( Cos \left[ e + fx \right] \left( -8 Log \left[ 1 + Sec \left[ e + fx \right] \right] \right) + 8 Log \left[ \sqrt{Sec \left[ e + fx \right]} \right] + Sec \left[ e + fx \right]^{3/2} + 2 \sqrt{2} \sqrt{Sec \left[ e + fx \right]} \sqrt{1 + Sec \left[ e + fx \right]^2} \right) + Log \left[ 1 - 2 Sec \left[ e + fx \right] - 3 Sec \left[$$

## Problem 181: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(g\operatorname{Sec}\left[e+fx\right]\right)^{3/2}}{\sqrt{a+a\operatorname{Sec}\left[e+fx\right]}\,\left(c-c\operatorname{Sec}\left[e+fx\right]\right)}\,\mathrm{d}x$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{g^{3/2}\operatorname{ArcTanh}\big[\frac{\sqrt{a}\sqrt{g}\operatorname{Tan[e+fx]}}{\sqrt{2}\sqrt{g\operatorname{Sec}[e+fx]}}\sqrt{a+a\operatorname{Sec}[e+fx]}}{\sqrt{2}\sqrt{a}\operatorname{c}f}+\frac{g\operatorname{Cot}[e+fx]\sqrt{g\operatorname{Sec}[e+fx]}\sqrt{a+a\operatorname{Sec}[e+fx]}}{a\operatorname{c}f}$$

Result (type 3, 431 leaves):

$$\left( g \operatorname{Sec} \left[ e + f x \right] \right)^{3/2} \sqrt{\left( 1 + \operatorname{Cos} \left[ e + f x \right] \right) \operatorname{Sec} \left[ e + f x \right]} \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]}$$

$$\left( -\frac{2 \operatorname{Cot} \left[ e \right]}{f} + \frac{\operatorname{Csc} \left[ \frac{e}{2} \right] \operatorname{Csc} \left[ \frac{e}{2} + \frac{f x}{2} \right] \operatorname{Sin} \left[ \frac{f x}{2} \right]}{f} + \frac{\operatorname{Sec} \left[ \frac{e}{2} \right] \operatorname{Sec} \left[ \frac{e}{2} + \frac{f x}{2} \right] \operatorname{Sin} \left[ \frac{f x}{2} \right]}{f} \right) \operatorname{Sin} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \right) / \left( \sqrt{a \left( 1 + \operatorname{Sec} \left[ e + f x \right] \right)} \left( c - c \operatorname{Sec} \left[ e + f x \right] \right) \right) + \left( \left( \operatorname{Log} \left[ 1 - 2 \operatorname{Sec} \left[ e + f x \right] - 3 \operatorname{Sec} \left[ e + f x \right] \right) \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]} \right) - \operatorname{Log} \left[ 1 - 2 \operatorname{Sec} \left[ e + f x \right] - 3 \operatorname{Sec} \left[ e + f x \right]^{2} + 2 \sqrt{2} \sqrt{\operatorname{Sec} \left[ e + f x \right]} \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]} \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]^{2}} \right) / \left( g \operatorname{Sec} \left[ e + f x \right] \right) / \left( 1 + \operatorname{Sec} \left[ e + f x \right] \right) \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]^{2}} \operatorname{Sin} \left[ \frac{e}{2} + \frac{f x}{2} \right]^{2} \operatorname{Sin} \left[ e + f x \right] \right) / \left( 2 f \left( 1 + \operatorname{Cos} \left[ e + f x \right] \right) \sqrt{2 - 2 \operatorname{Cos} \left[ e + f x \right]^{2}} \sqrt{1 - \operatorname{Cos} \left[ e + f x \right]^{2}} \right)$$

$$\operatorname{Sec} \left[ e + f x \right]^{5/2} \sqrt{a \left( 1 + \operatorname{Sec} \left[ e + f x \right] \right)} \left( c - c \operatorname{Sec} \left[ e + f x \right] \right) \right)$$

Problem 183: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\text{Sec}\,[\,e+f\,x\,]^{\,2}}{\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}}\,\sqrt{c-c\,\text{Sec}\,[\,e+f\,x\,]}\,\,\text{d}x$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\text{Log}\left[\text{Tan}\left[\,e + f\,x\,\right]\,\right]\,\,\text{Tan}\left[\,e + f\,x\,\right]}{f\,\sqrt{a + a\,\text{Sec}\left[\,e + f\,x\,\right]}}\,\,\sqrt{c - c\,\text{Sec}\left[\,e + f\,x\,\right]}$$

Result (type 3, 129 leaves):

$$-\left(\left(2\,\dot{\mathbb{1}}\,\left(-1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\right)\,\mathsf{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\,\right]^{2}\,\left(\mathsf{Log}\left[1-\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\,\right]+\mathsf{Log}\left[1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\,\right]-\mathsf{Log}\left[1+\mathbb{e}^{2\,\dot{\mathbb{1}}\,\left(e+f\,x\right)}\,\right]\right)\right)$$

$$\mathsf{Sec}\left[e+f\,x\right]\right)\bigg/\left(\left(1+\mathbb{e}^{\dot{\mathbb{1}}\,\left(e+f\,x\right)}\right)\,f\,\sqrt{a\,\left(1+\mathsf{Sec}\left[e+f\,x\right]\right)}\,\,\sqrt{c-c\,\mathsf{Sec}\left[e+f\,x\right]}\,\right)\bigg)$$

Problem 186: Result more than twice size of optimal antiderivative.

$$\int Sec \left[e + f x\right] \left(a + a Sec \left[e + f x\right]\right) \left(c + d Sec \left[e + f x\right]\right)^{3} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\frac{a\;\left(8\;c^{3}+12\;c^{2}\;d+12\;c\;d^{2}+3\;d^{3}\right)\;ArcTanh\left[Sin\left[e+f\,x\right]\right]}{8\;f}+\frac{a\;\left(3\;c^{3}+16\;c^{2}\;d+12\;c\;d^{2}+4\;d^{3}\right)\;Tan\left[e+f\,x\right]}{6\;f}+\frac{a\;d\;\left(6\;c^{2}+20\;c\;d+9\;d^{2}\right)\;Sec\left[e+f\,x\right]\;Tan\left[e+f\,x\right]}{24\;f}+\frac{a\;\left(3\;c+4\;d\right)\;\left(c+d\;Sec\left[e+f\,x\right]\right)^{2}\;Tan\left[e+f\,x\right]}{12\;f}+\frac{a\;\left(c+d\;Sec\left[e+f\,x\right]\right)^{3}\;Tan\left[e+f\,x\right]}{4\;f}$$

Result (type 3, 1107 leaves):

$$a \left( \left( (-8\,c^3 - 12\,c^2\,d - 12\,c\,d^2 - 3\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] \, \text{Log} \left[ \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \, - \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right] \right) \\ \ \, \text{Sec} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]^2 \left( 1 + \, \text{Sec} \left[ e + f \, x \right] \right) \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^3 \right) / \left( 16\,f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^3 \right) + \\ \ \, \left( \left( 8\,c^3 + 12\,c^2\,d + 12\,c\,d^2 + 3\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] \, ^4 \, \text{Log} \left[ \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \, + \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right) \\ \ \, \text{Sec} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]^2 \left( 1 + \, \text{Sec} \left[ e + f \, x \right] \right) \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^3 \right) / \\ \ \, \left( d^3 \, \text{Cos} \left[ e + f \, x \right] \, \right)^3 \, \left( \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \, + \, \left( 36\,c^2\,d + 48\,c\,d^2 + 13\,d^3 \right) \, \text{Cos} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] - \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right)^4 \right) + \\ \ \, \left( (c + d \, \text{Sec} \left[ e + f \, x \right] \, \right)^3 \right) / \left( 96\,f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \, \right)^3 \, \left( \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right) - \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right)^3 \right) / \\ \ \, \left( 32\,f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \, \right)^3 \, \left( \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right) \right) + \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right) \right) / \\ \ \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^3 \right) / \left( 96\,f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right) \, \left( \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right)^3 \right) / \\ \ \, \left( 32\,f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^3 \, \left( \cos \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right] \right) \right) \right) / \\ \ \, \left( (-36\,c^2\,d - 48\,c\,d^2 - 13\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] + \, \text{Sin} \left[ \frac{1}{2} \, \left( e + f \, x \right) \right] \right) \right) \right) / \\ \ \, \left( (-36\,c^2\,d - 48\,c\,d^2 - 13\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] + \, \text{Sec} \left[ \frac{1}{2} \, \left( e + f \, x \right) \right] \right) \right) \right) / \\ \ \, \left( (-36\,c^2\,d - 48\,c\,d^2 - 13\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] \right) \right) \left( - \, \text{Cos} \left[ e + f \, x \right] \right) \right) \right) / \\ \ \, \left( (-36\,c^2\,d - 48\,c\,d^2 - 13\,d^3 \right) \, \text{Cos} \left[ e + f \, x \right] \right) \right) \left( - \, \text{Cos} \left[ e + f \, x \right] \right) \right) \right) / \left( e + \, d \, \text{Sec} \left[ \frac{1}{2} \, \left( e$$

$$\left( \text{Cos} \left[ e + f \, x \right]^4 \, \text{Sec} \left[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \, \left( 1 + \text{Sec} \left[ e + f \, x \right] \right) \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^3 \, \left( 3 \, c^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] + 2 \, d^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) \right) + 2 \, d^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) \right)$$

$$\left( 6 \, f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^3 \, \left( \text{Cos} \left[ \frac{1}{2} \left( e + f \, x \right) \right] - \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) + 2 \, d^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) \right)$$

$$\left( \text{Cos} \left[ e + f \, x \right]^4 \, \text{Sec} \left[ \frac{1}{2} \left( e + f \, x \right) \right]^2 \, \left( 1 + \text{Sec} \left[ e + f \, x \right] \right) \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^3 \, \left( 3 \, c^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] + 2 \, d^3 \, \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) \right)$$

$$\left( 6 \, f \, \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^3 \, \left( \text{Cos} \left[ \frac{1}{2} \left( e + f \, x \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( e + f \, x \right) \right] \right) \right) \right)$$

### Problem 187: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right] \, \left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right) \, \left(\,c + d\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,2} \, \mathrm{d}x$$

Optimal (type 3, 108 leaves, 6 steps):

$$\frac{a\;\left(2\;c^{2}+2\;c\;d+d^{2}\right)\;ArcTanh\left[Sin\left[e+f\,x\right]\right]}{2\;f}\;+\;\frac{2\;a\;\left(c^{2}+3\;c\;d+d^{2}\right)\;Tan\left[e+f\,x\right]}{3\;f}\;+\\ \frac{a\;d\;\left(2\;c+3\;d\right)\;Sec\left[e+f\,x\right]\;Tan\left[e+f\,x\right]}{6\;f}\;+\;\frac{a\;\left(c+d\;Sec\left[e+f\,x\right]\right)^{2}\;Tan\left[e+f\,x\right]}{3\;f}$$

#### Result (type 3, 240 leaves):

$$\frac{1}{24\,f\left(-1+Tan\left[\frac{1}{2}\left(e+f\,x\right)\right]^2\right)^3} \\ a\,Sec\,\Big[\frac{1}{2}\left(e+f\,x\right)\Big]^6\,\left(9\,\left(2\,c^2+2\,c\,d+d^2\right)\,Cos\,[e+f\,x]\,\left(Log\left[Cos\left[\frac{1}{2}\left(e+f\,x\right)\right]-Sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right]-Log\left[Cos\left[\frac{1}{2}\left(e+f\,x\right)\right]+Sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right]\right) \\ +3\,\left(2\,c^2+2\,c\,d+d^2\right)\,Cos\,\Big[3\,\left(e+f\,x\right)\Big] \\ \left(Log\left[Cos\left[\frac{1}{2}\left(e+f\,x\right)\right]-Sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right]-Log\left[Cos\left[\frac{1}{2}\left(e+f\,x\right)\right]+Sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right]\right) \\ -4\,\left(3\,c^2+6\,c\,d+4\,d^2+3\,d\,\left(2\,c+d\right)\,Cos\,[e+f\,x]+\left(3\,c^2+6\,c\,d+2\,d^2\right)\,Cos\,\Big[2\left(e+f\,x\right)\right]\right)\,Sin\,[e+f\,x]\right) \\ \end{array}$$

## Problem 188: Result more than twice size of optimal antiderivative.

Optimal (type 3, 56 leaves, 5 steps):

$$\frac{a \, \left(2 \, c + d\right) \, ArcTanh \left[Sin \left[e + f \, x\right]\,\right]}{2 \, f} + \frac{a \, \left(c + d\right) \, Tan \left[e + f \, x\right]}{f} + \frac{a \, d \, Sec \left[e + f \, x\right] \, Tan \left[e + f \, x\right]}{2 \, f}$$

Result (type 3, 154 leaves):

$$\begin{split} \frac{1}{4\,\mathsf{f}}\mathsf{a} &\left[-2\,\left(2\,\mathsf{c} + \mathsf{d}\right)\,\mathsf{Log}\big[\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] - \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\,\right] + \\ &4\,\mathsf{c}\,\mathsf{Log}\big[\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\,\big] + \\ &2\,\mathsf{d}\,\mathsf{Log}\big[\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\,\big] + \frac{\mathsf{d}}{\left(\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] - \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\right)^2} - \\ &\frac{\mathsf{d}}{\left(\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\right)^2} + 4\,\left(\mathsf{c} + \mathsf{d}\right)\,\mathsf{Tan}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,] \end{split}$$

## Problem 195: Result more than twice size of optimal antiderivative.

$$\int Sec \left[e+fx\right] \ \left(a+a \, Sec \left[e+fx\right]\right)^2 \ \left(c+d \, Sec \left[e+fx\right]\right)^2 \, dx$$

Optimal (type 3, 176 leaves, 8 steps):

$$\frac{a^2 \left(12 \, c^2 + 16 \, c \, d + 7 \, d^2\right) \, ArcTanh \left[Sin\left[e + f \, x\right]\right]}{8 \, f} - \\ \frac{a^2 \left(c^3 - 8 \, c^2 \, d - 20 \, c \, d^2 - 8 \, d^3\right) \, Tan\left[e + f \, x\right]}{6 \, d \, f} - \frac{a^2 \left(2 \, c \, \left(c - 8 \, d\right) - 21 \, d^2\right) \, Sec\left[e + f \, x\right] \, Tan\left[e + f \, x\right]}{24 \, f} - \\ \frac{a^2 \left(c - 8 \, d\right) \, \left(c + d \, Sec\left[e + f \, x\right]\right)^2 \, Tan\left[e + f \, x\right]}{12 \, d \, f} + \frac{a^2 \, \left(c + d \, Sec\left[e + f \, x\right]\right)^3 \, Tan\left[e + f \, x\right]}{4 \, d \, f}$$

Result (type 3, 479 leaves):

$$\begin{split} &-\frac{1}{192\,f}\,a^2\,\text{Sec}\,[\,e\,+\,f\,x\,]^{\,4}\,\left(108\,c^2\,\text{Log}\big[\,\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,+\,\\ &-144\,c\,d\,\text{Log}\big[\,\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,]\,+\,\\ &-63\,d^2\,\text{Log}\big[\,\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,]\,+\,12\,\left(12\,c^2\,+\,16\,c\,d\,+\,7\,d^2\right)\,\text{Cos}\,\Big[\,2\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\\ &-\left(\text{Log}\big[\,\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,]\,-\,\text{Log}\big[\,\text{Cos}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big]\,+\,\\ &-3\,\left(12\,c^2\,+\,16\,c\,d\,+\,7\,d^2\right)\,\text{Cos}\,\Big[\,4\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,\text{Log}\big[\,\text{Cos}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,+\,\text{Sin}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big]\,-\,\\ &-108\,c^2\,\text{Log}\big[\,\text{Cos}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big]\,-\,\\ &-144\,c\,d\,\text{Log}\big[\,\text{Cos}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big]\,-\,\\ &-144\,c\,d\,\text{Log}\,\Big[\,\text{Cos}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\Big[\,\frac{1}{2}\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big]\,-\,24\,c^2\,\text{Sin}\,[\,e\,+\,f\,x\,]\,\,-\,96\,c\,d\,\text{Sin}\,[\,e\,+\,f\,x\,)\,\,\big]\,-\,\\ &-90\,d^2\,\text{Sin}\,[\,e\,+\,f\,x\,)\,\,-\,96\,c^2\,\text{Sin}\,[\,2\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,224\,c\,d\,\text{Sin}\,[\,2\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,128\,d^2\,\text{Sin}\,[\,2\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,\\ &-24\,c^2\,\text{Sin}\,[\,3\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,96\,c\,d\,\text{Sin}\,[\,3\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,-\,32\,d^2\,\text{Sin}\,[\,3\,\left(\,e\,+\,f\,x\,\right)\,\,\big]\,\,\big)\,\,$$

## Problem 196: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+aSec[e+fx])^2 (c+dSec[e+fx]) dx$$

Optimal (type 3, 103 leaves, 6 steps):

$$\frac{a^2 \, \left(3 \, c + 2 \, d\right) \, ArcTanh \left[Sin \left[e + f \, x\right]\right.\right]}{2 \, f} + \frac{2 \, a^2 \, \left(3 \, c + 2 \, d\right) \, Tan \left[e + f \, x\right]}{3 \, f} + \\ \frac{a^2 \, \left(3 \, c + 2 \, d\right) \, Sec \left[e + f \, x\right] \, Tan \left[e + f \, x\right]}{6 \, f} + \frac{d \, \left(a + a \, Sec \left[e + f \, x\right]\right)^2 \, Tan \left[e + f \, x\right]}{3 \, f}$$

Result (type 3, 993 leaves):

$$\left( \left( -3\,c - 2\,d \right) \cos\left[ e + f\,x \right]^{3} \log\left[ \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] - \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right] \sec\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^{4} \\ \left( a + a \sec\left[ e + f\,x \right] \right)^{2} \left( c + d \sec\left[ e + f\,x \right] \right) \right) / \left( 8\,f \left( d + c \cos\left[ e + f\,x \right] \right) \right) + \\ \left( \left( 3\,c + 2\,d \right) \cos\left[ e + f\,x \right]^{3} \log\left[ \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right] \sec\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^{4} \\ \left( a + a \sec\left[ e + f\,x \right] \right)^{2} \left( c + d \sec\left[ e + f\,x \right] \right) \right) / \left( 8\,f \left( d + c \cos\left[ e + f\,x \right] \right) \right) + \\ \left( d \cos\left[ e + f\,x \right]^{3} \sec\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^{4} \left( a + a \sec\left[ e + f\,x \right] \right)^{2} \left( c + d \sec\left[ e + f\,x \right] \right) \sin\left[ \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] - \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] - \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) \right) + \\ \left( \cos\left[ e + f\,x \right]^{3} \sec\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^{4} \left( a + a \sec\left[ e + f\,x \right] \right)^{2} \left( c + d \sec\left[ e + f\,x \right] \right) \\ \left( 3\,c \cos\left[ \frac{e}{2} \right] + 7\,d \cos\left[ \frac{e}{2} \right] - 3\,c \sin\left[ \frac{e}{2} \right] - 5\,d \sin\left[ \frac{e}{2} \right] \right) \right) / \\ \left( 48\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] - \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] - \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) + \\ \left( \cos\left[ e + f\,x \right]^{3} \sec\left[ \frac{e}{2} + \frac{f\,x}{2} \right]^{4} \left( a + a \sec\left[ e + f\,x \right] \right)^{2} \\ \left( c + d \sec\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] - \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] - \sin\left[ \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] - \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] + \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] + \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] + \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[ e + f\,x \right] \right) \left( \cos\left[ \frac{e}{2} \right] + \sin\left[ \frac{e}{2} \right] \right) \left( \cos\left[ \frac{e}{2} + \frac{f\,x}{2} \right] + \sin\left[ \frac{e}{2} + \frac{f\,x}{2} \right] \right) \right) / \\ \left( 24\,f \left( d + c \cos\left[$$

Problem 197: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] (a+a\operatorname{Sec}[e+fx])^2}{c+d\operatorname{Sec}[e+fx]} dx$$

Optimal (type 3, 95 leaves, 8 steps):

$$-\frac{a^{2} \, \left(c-2 \, d\right) \, ArcTanh \left[Sin \left[e+f \, x\right]\,\right]}{d^{2} \, f} + \frac{2 \, a^{2} \, \left(c-d\right)^{3/2} \, ArcTanh \left[\frac{\sqrt{c-d} \, Tan \left[\frac{1}{2} \, \left(e+f \, x\right)\,\right]}{\sqrt{c+d}}\right]}{d^{2} \, \sqrt{c+d}} + \frac{a^{2} \, Tan \left[e+f \, x\right]}{d \, f}$$

Result (type 3, 329 leaves):

$$\begin{split} &\frac{1}{4\,d^2\,f\left(c+d\,Sec\left[e+f\,x\right]\right)}\,a^2\,Cos\left[e+f\,x\right]\,\left(d+c\,Cos\left[e+f\,x\right]\right)\,Sec\left[\frac{1}{2}\,\left(e+f\,x\right)\right]^4\\ &\left(1+Sec\left[e+f\,x\right]\right)^2\left(\left(c-2\,d\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]-\\ &\left(c-2\,d\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]-\\ &\left(2\,i\,\left(c-d\right)^2\,ArcTan\left[\frac{\left(i\,Cos\left[e\right]+Sin\left[e\right]\right)\left(c\,Sin\left[e\right]+\left(-d+c\,Cos\left[e\right]\right)\,Tan\left[\frac{f\,x}{2}\right]\right)}{\sqrt{c^2-d^2}}\sqrt{\left(Cos\left[e\right]-i\,Sin\left[e\right]\right)^2}\right]\\ &\left(Cos\left[e\right]-i\,Sin\left[e\right]\right)\right/\left(\sqrt{c^2-d^2}\,\sqrt{\left(Cos\left[e\right]-i\,Sin\left[e\right]\right)^2}\right)+\\ &\frac{d\,Sin\left[\frac{f\,x}{2}\right]}{\left(Cos\left[\frac{e}{2}\right]-Sin\left[\frac{e}{2}\right]\right)\left(Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right)}\\ &\frac{d\,Sin\left[\frac{f\,x}{2}\right]}{\left(Cos\left[\frac{e}{2}\right]+Sin\left[\frac{e}{2}\right]\right)\left(Cos\left[\frac{1}{2}\,\left(e+f\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right)} \end{split}$$

Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{a}\,+\,\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\right)^{\,2}}{\left(\,\mathsf{c}\,+\,\mathsf{d}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\right)^{\,2}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 117 leaves, 8 steps):

$$\begin{split} \frac{a^2 \, \text{ArcTanh} \left[ \text{Sin} \left[ e + f \, x \right] \, \right]}{d^2 \, f} \, - \\ \frac{2 \, a^2 \, \sqrt{c - d} \, \left( c + 2 \, d \right) \, \text{ArcTanh} \left[ \frac{\sqrt{c - d} \, \left[ \text{Tan} \left[ \frac{1}{2} \, \left( e + f \, x \right) \, \right]}{\sqrt{c + d}} \, \right]}{d^2 \, \left( c + d \right)^{3/2} \, f} \, - \, \frac{a^2 \, \left( c - d \right) \, \text{Tan} \left[ e + f \, x \right]}{d \, \left( c + d \right) \, f \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)} \end{split}$$

Result (type 3, 312 leaves):

$$\begin{split} &\frac{1}{4\,d^2\,f\left(c+d\,\text{Sec}\left[e+f\,x\right]\right)^2}\,a^2\,\left(d+c\,\text{Cos}\left[e+f\,x\right]\right)\,\text{Sec}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]^4\\ &\left(1+\text{Sec}\left[e+f\,x\right]\right)^2\left[-\left(d+c\,\text{Cos}\left[e+f\,x\right]\right)\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]-\text{Sin}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]+\\ &\left(d+c\,\text{Cos}\left[e+f\,x\right]\right)\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]+\\ &\left(2\,\left(c^2+c\,d-2\,d^2\right)\,\text{ArcTan}\left[\frac{\left(i\,\text{Cos}\left[e\right]+\text{Sin}\left[e\right]\right)\left(c\,\text{Sin}\left[e\right]+\left(-d+c\,\text{Cos}\left[e\right]\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)}{\sqrt{c^2-d^2}\,\sqrt{\left(\text{Cos}\left[e\right]-i\,\text{Sin}\left[e\right]\right)^2}}\right]\\ &\left(d+c\,\text{Cos}\left[e+f\,x\right]\right)\,\left(i\,\text{Cos}\left[e\right]+\text{Sin}\left[e\right]\right)\right/\left(\left(c+d\right)\,\sqrt{c^2-d^2}\,\sqrt{\left(\text{Cos}\left[e\right]-i\,\text{Sin}\left[e\right]\right)^2}\right)+\\ &\frac{\left(c-d\right)\,d\,\left(d\,\text{Sin}\left[e\right]-c\,\text{Sin}\left[f\,x\right]\right)}{c\,\left(c+d\right)\,\left(\text{Cos}\left[\frac{e}{2}\right]-\text{Sin}\left[\frac{e}{2}\right]\right)\,\left(\text{Cos}\left[\frac{e}{2}\right]+\text{Sin}\left[\frac{e}{2}\right]\right)}\right) \end{split}$$

### Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,] \, \left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,2}}{\left(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)^{\,3}} \, \, \mathrm{d} \mathsf{x}$$

Optimal (type 3, 130 leaves, 5 steps):

$$\frac{3 \, a^2 \, ArcTanh \Big[ \frac{\sqrt{c-d} \, Tan \Big[ \frac{1}{2} \, (e+f \, x) \, \Big]}{\sqrt{c-d}} \Big]}{\sqrt{c-d} \, \left(c+d\right)^{5/2} \, f} \, + \, \frac{\left(a^2 + a^2 \, Sec \, [\, e+f \, x\,] \, \right) \, Tan \, [\, e+f \, x\,]}{2 \, \left(c+d\right) \, f \, \left(c+d \, Sec \, [\, e+f \, x\,] \, \right)^2} \, + \, \frac{3 \, a^2 \, Tan \, [\, e+f \, x\,]}{2 \, \left(c+d\right)^2 \, f \, \left(c+d \, Sec \, [\, e+f \, x\,] \, \right)}$$

Result (type 3, 249 leaves):

$$\left( a^2 \left( d + c \, \mathsf{Cos} \, [e + f \, x] \right) \, \mathsf{Sec} \left[ \frac{1}{2} \left( e + f \, x \right) \right]^4 \, \mathsf{Sec} \, [e + f \, x] \, \left( 1 + \mathsf{Sec} \, [e + f \, x] \right)^2 \right. \\ \left. \left( - \left( \left[ 6 \, \mathring{\mathtt{i}} \, \mathsf{ArcTan} \left[ \, \left( \left[ \mathring{\mathtt{i}} \, \mathsf{Cos} \, [e] + \mathsf{Sin} \, [e] \right) \right) \, \left( \mathsf{c} \, \mathsf{Sin} \, [e] + \left( - \mathsf{d} + \mathsf{c} \, \mathsf{Cos} \, [e] \right) \, \mathsf{Tan} \left[ \frac{f \, x}{2} \right] \right) \right) \right/ \\ \left. \left( \sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{ \left( \mathsf{Cos} \, [e] - \mathring{\mathtt{i}} \, \mathsf{Sin} \, [e] \right)^2} \right) \right] \, \left( \mathsf{d} + \mathsf{c} \, \mathsf{Cos} \, [e + f \, x] \right)^2 \, \left( \mathsf{Cos} \, [e] - \mathring{\mathtt{i}} \, \mathsf{Sin} \, [e] \right) \right) \right/ \\ \left. \left( \sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{ \left( \mathsf{Cos} \, [e] - \mathring{\mathtt{i}} \, \mathsf{Sin} \, [e] \right)^2} \right) \right) + \frac{\left( \mathsf{c} - \mathsf{d} \right) \, \left( \mathsf{c} + \mathsf{d} \right) \, \mathsf{Sec} \, [e] \, \left( - \mathsf{d} \, \mathsf{Sin} \, [e] + \mathsf{c} \, \mathsf{Sin} \, [e] + \mathsf{c} \, \mathsf{Sin} \, [e] \right) \right) \right/ \\ \left. \left( \mathsf{d} + \mathsf{c} \, \mathsf{Cos} \, [e + f \, x] \right) \, \mathsf{Sec} \, [e] \, \left( \left( \mathsf{c}^2 - \mathsf{d} \, \mathsf{c} \, \mathsf{d} - \mathsf{2} \, \mathsf{d}^2 \right) \, \mathsf{Sin} \, [e] + \mathsf{c} \, \left( \mathsf{d} \, \mathsf{c} + \mathsf{d} \right) \, \mathsf{Sin} \, [f \, x] \right) \right) \right) \right/ \\ \left( \mathsf{8} \, \left( \mathsf{c} + \mathsf{d} \right)^2 \, \mathsf{f} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{Sec} \, [e + f \, x] \right)^3 \right)$$

# Problem 204: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+aSec[e+fx])^3 (c+dSec[e+fx]) dx$$

Optimal (type 3, 125 leaves, 10 steps):

$$\frac{5 \, a^3 \, \left(4 \, c + 3 \, d\right) \, ArcTanh[Sin[e + f \, x]]}{8 \, f} + \\ \frac{a^3 \, \left(4 \, c + 3 \, d\right) \, Tan[e + f \, x]}{f} + \frac{3 \, a^3 \, \left(4 \, c + 3 \, d\right) \, Sec[e + f \, x] \, Tan[e + f \, x]}{8 \, f} + \\ \frac{d \, \left(a + a \, Sec[e + f \, x]\right)^3 \, Tan[e + f \, x]}{4 \, f} + \frac{a^3 \, \left(4 \, c + 3 \, d\right) \, Tan[e + f \, x]^3}{12 \, f}$$

Result (type 3, 273 leaves):

$$-\frac{1}{1536\,f}\,a^3\,\left(1+\text{Cos}\,[\,e+f\,x\,]\,\right)^3\,\text{Sec}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\Big]^6\,\text{Sec}\,[\,e+f\,x\,]^4\,\left(120\,\left(4\,c+3\,d\right)\,\text{Cos}\,[\,e+f\,x\,]^4\right)\\ \left(\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\Big]\,-\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\Big]\,\Big]\,-\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\Big]\,+\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\Big]\,\Big]\right)\\ -\text{Sec}\,[\,e\,]\,\left(-24\,\left(11\,c+9\,d\right)\,\text{Sin}\,[\,e\,]\,+\,\left(36\,c+69\,d\right)\,\text{Sin}\,[\,f\,x\,]\,+\,36\,c\,\text{Sin}\,[\,2\,e+f\,x\,]\,+\,\\ 69\,d\,\text{Sin}\,[\,2\,e+f\,x\,]\,+\,280\,c\,\text{Sin}\,[\,e+2\,f\,x\,]\,+\,264\,d\,\text{Sin}\,[\,e+2\,f\,x\,]\,-\,72\,c\,\text{Sin}\,[\,3\,e+2\,f\,x\,]\,-\,\\ 24\,d\,\text{Sin}\,[\,3\,e+2\,f\,x\,]\,+\,36\,c\,\text{Sin}\,[\,2\,e+3\,f\,x\,]\,+\,45\,d\,\text{Sin}\,[\,2\,e+3\,f\,x\,]\,+\,36\,c\,\text{Sin}\,[\,4\,e+3\,f\,x\,]\,+\,\\ 45\,d\,\text{Sin}\,[\,4\,e+3\,f\,x\,]\,+\,88\,c\,\text{Sin}\,[\,3\,e+4\,f\,x\,]\,+\,72\,d\,\text{Sin}\,[\,3\,e+4\,f\,x\,]\,\right)$$

Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + f x] (a + a \operatorname{Sec} [e + f x])^{3}}{c + d \operatorname{Sec} [e + f x]} dx$$

Optimal (type 3, 153 leaves, 9 steps):

$$\frac{a^{3} \, \text{ArcTanh} [\text{Sin}[\,e+f\,x\,]\,\,]}{2 \, \text{df}} + \frac{a^{3} \, \left(c^{2} - 3 \, c \, d + 3 \, d^{2}\right) \, \text{ArcTanh} [\,\text{Sin}[\,e+f\,x\,]\,\,]}{d^{3} \, f} - \frac{2 \, a^{3} \, \left(c - d\right)^{5/2} \, \text{ArcTanh} \left[\frac{\sqrt{c-d} \, \, \text{Tan}\left[\frac{1}{2} \, \left(e+f\,x\right)\,\right]}{\sqrt{c+d}}\right]}{d^{3} \, \sqrt{c+d} \, f} - \frac{a^{3} \, \left(c - 3 \, d\right) \, \text{Tan}\left[e+f\,x\right]}{d^{2} \, f} + \frac{a^{3} \, \text{Sec}\left[e+f\,x\right] \, \text{Tan}\left[e+f\,x\right]}{2 \, d \, f}$$

Result (type 3, 419 leaves):

$$\begin{split} &\frac{1}{32\,d^3\,f\left(c+d\,Sec\,[e+f\,x]\right)}\,a^3\,Cos\,[e+f\,x]^2\,\left(d+c\,Cos\,[e+f\,x]\right)\,Sec\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\Big]^6\\ &\left(1+Sec\,[e+f\,x]\right)^3\,\Bigg[-2\,\left(2\,c^2-6\,c\,d+7\,d^2\right)\,Log\,\Big[Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]-Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big] +\\ &2\,\left(2\,c^2-6\,c\,d+7\,d^2\right)\,Log\,\Big[Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big]+\Bigg[8\,\left(c-d\right)^3\\ &ArcTan\,\Big[\frac{\left(i\,Cos\,[e]+Sin\,[e]\right)\,\left(c\,Sin\,[e]+\left(-d+c\,Cos\,[e]\right)\,Tan\,\Big[\frac{f\,x}{2}\,\Big]\right)}{\sqrt{c^2-d^2}\,\sqrt{\left(Cos\,[e]-i\,Sin\,[e]\right)^2}}\Bigg]\,\left(i\,Cos\,[e]+Sin\,[e]\right)\Bigg)\Bigg/\\ &\left(\sqrt{c^2-d^2}\,\sqrt{\left(Cos\,[e]-i\,Sin\,[e]\right)^2}\right)+\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]-Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}-\\ &\frac{4\,\left(c-3\,d\right)\,d\,Sin\,\Big[\frac{f\,x}{2}\,\Big]}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}-\\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}\Bigg(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\right)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)} \\ &\frac{d^2}{\left(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+Sin\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)}\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big(Cos\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big)\Big($$

Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \; \left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3}{\left(\,\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^2} \; \mathrm{d} x$$

Optimal (type 3, 161 leaves, 9 steps):

$$-\frac{a^{3} \, \left(2 \, c - 3 \, d\right) \, ArcTanh \left[Sin \left[e + f \, x\right]\right]}{d^{3} \, f} + \frac{2 \, a^{3} \, \left(c - d\right)^{3/2} \, \left(2 \, c + 3 \, d\right) \, ArcTanh \left[\frac{\sqrt{c - d} \, Tan \left[\frac{1}{2} \, \left(e + f \, x\right)\right]}{\sqrt{c + d}}\right]}{d^{3} \, \left(c + d\right)^{3/2} \, f} + \frac{2 \, a^{3} \, c \, Tan \left[e + f \, x\right]}{d^{2} \, \left(c + d\right) \, f} - \frac{\left(c - d\right) \, \left(a^{3} + a^{3} \, Sec \left[e + f \, x\right]\right) \, Tan \left[e + f \, x\right]}{d \, \left(c + d\right) \, f \, \left(c + d\right) \, f \, \left(c + d\right) \, f \, \left(c + d\right) \, f}$$

Result (type 3, 979 leaves):

$$\left( (2\,c - 3\,d) \, Cos \, [e + f \, x] \, \left( d + c \, Cos \, (e + f \, x) \right)^2 \, Log \left[ Cos \left[ \frac{e}{2} + \frac{f \, x}{2} \right] - Sin \left[ \frac{e}{2} + \frac{f \, x}{2} \right] \right]$$
 
$$Sec \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \, \left( a + a \, Sec \, [e + f \, x] \right)^3 \right) / \left( B \, d^3 \, f \, \left( c + d \, Sec \, [e + f \, x] \right)^2 \right) +$$
 
$$\left( \left( -2\,c + 3\,d \right) \, Cos \, [e + f \, x] \, \left( d + c \, Cos \, [e + f \, x] \right)^2 \, Log \left[ Cos \left[ \frac{e}{2} + \frac{f \, x}{2} \right] + Sin \left[ \frac{e}{2} + \frac{f \, x}{2} \right] \right]$$
 
$$Sec \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \, \left( a + a \, Sec \, [e + f \, x] \right)^3 \right) / \left( B \, d^3 \, f \, \left( c + d \, Sec \, [e + f \, x] \right)^2 \right) +$$
 
$$\left( \left( -c + d \right)^2 \, \left( 2\,c + 3\,d \right) \, Cos \, [e + f \, x] \, \left( d + c \, Cos \, [e + f \, x] \right)^2 \, Sec \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \, \left( a + a \, Sec \, [e + f \, x] \right)^3 \right)$$
 
$$\left( -\left[ \left( \left[ a \, Arc \, Tan \left[ Sec \, \left[ \frac{f \, x}{2} \right] \right] \, \left( \frac{Cos \, [e]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \, [2 \, e] - i \, Sin \, [2 \, e]}} \right) - \frac{i \, Sin \, [e + \frac{f \, x}{2}]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \, [2 \, e] - i \, Sin \, [2 \, e]}} \right) \right) -$$
 
$$\left( Arc \, Tan \left[ Sec \, \left[ \frac{f \, x}{2} \right] \, \left( \frac{Cos \, [e]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \, [2 \, e] - i \, Sin \, [2 \, e]}} \right) \right) - \frac{i \, Sin \, [e]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \, [2 \, e] - i \, Sin \, [2 \, e]}} \right) \right)$$
 
$$\left( -i \, d \, Sin \, \left[ \frac{f \, x}{2} \right] + i \, c \, Sin \, \left[ e + \frac{f \, x}{2} \right] \right) \right] Sin \, [e] \right) /$$
 
$$\left( -i \, d \, Sin \, \left[ \frac{f \, x}{2} \right] + i \, c \, Sin \, \left[ e + \frac{f \, x}{2} \right] \right) \right] Sin \, [e] \right) /$$
 
$$\left( -4 \, d^3 \, \sqrt{c^2 - d^2} \, \sqrt{Cos \, [2 \, e] - i \, Sin \, [2 \, e]} \right) \right) \right) / \left( \left( c + d \right) \, \left( c + d \, Sec \, [e + f \, x] \right)^2 \right) +$$
 
$$\left( Cos \, [e + f \, x] \, \left( d + c \, Cos \, [e + f \, x] \right) Sec \, \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \, \left( a + a \, Sec \, [e + f \, x] \right)^3 \right)$$
 
$$\left( -c^2 \, d \, Sin \, [e] + 2 \, c \, d^2 \, Sin \, [e] - d^3 \, Sin \, [e] + c^3 \, Sin \, [f \, x] - 2 \, c^2 \, d \, Sin \, [f \, x] + c \, d^2 \, Sin \, [f \, x] \right) \right) /$$
 
$$\left( 8 \, c^3 \, c \, \left( c + d \, \right) \, f \, \left( c + d \, Sec \, [e + f \, x] \right)^2 \, \left( Cos \, \left[ \frac{e}{2} \right] - Sin \, \left[ \frac{e}{2} \right] \right) \left( Cos \, \left[ \frac{e}{2} \right] + Sin \, \left[ \frac{e}{2} \right] \right) \right)$$
 
$$\left( -c^2 \, d \, Sin \, [e] + f \, x_1 \right)^2 \, \left( Cos \, \left[ \frac{e}{2}$$

Problem 207: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{Sec \left[e+fx\right] \ \left(a+a \, Sec \left[e+fx\right]\right)^3}{\left(c+d \, Sec \left[e+fx\right]\right)^3} \, dx$$

Optimal (type 3, 188 leaves, 9 steps):

$$\frac{a^{3} \, \text{ArcTanh} \left[\text{Sin} \left[e + f \, x\right]\right]}{d^{3} \, f} - \frac{a^{3} \, \sqrt{c - d} \, \left(2 \, c^{2} + 6 \, c \, d + 7 \, d^{2}\right) \, \text{ArcTanh} \left[\frac{\sqrt{c - d} \, \text{Tan} \left[\frac{1}{2} \, \left(e + f \, x\right)\right]}{\sqrt{c + d}}\right]}{d^{3} \, \left(c + d\right)^{5/2} \, f} - \frac{\left(c - d\right) \, \left(a^{3} + a^{3} \, \text{Sec} \left[e + f \, x\right]\right) \, \text{Tan} \left[e + f \, x\right]}{2 \, d \, \left(c + d\right) \, \left(2 \, c + 5 \, d\right) \, \text{Tan} \left[e + f \, x\right]} - \frac{a^{3} \, \left(c - d\right) \, \left(2 \, c + 5 \, d\right) \, \text{Tan} \left[e + f \, x\right]}{2 \, d^{2} \, \left(c + d\right)^{2} \, f \, \left(c + d \, \text{Sec} \left[e + f \, x\right]\right)}$$

Result (type 3, 393 leaves):

$$\begin{split} \frac{1}{32\,d^3\,f\left(c+d\,\text{Sec}\,[\,e+f\,x\,]\,\right)^3}\,a^3\,\left(d+c\,\text{Cos}\,[\,e+f\,x\,]\,\right)\,\text{Sec}\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]^6 \\ &\left(1+\text{Sec}\,[\,e+f\,x\,]\,\right)^3\left[-4\,\left(d+c\,\text{Cos}\,[\,e+f\,x\,]\,\right)^2\,\text{Log}\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]-\text{Sin}\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\,\right] + \\ &4\,\left(d+c\,\text{Cos}\,[\,e+f\,x\,]\,\right)^2\,\text{Log}\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]+\text{Sin}\Big[\frac{1}{2}\,\left(e+f\,x\right)\,\Big]\Big] + \left[4\,\left(2\,c^3+4\,c^2\,d+c\,d^2-7\,d^3\right)\right] \\ &+ \text{ArcTan}\Big[\frac{\left(\text{i}\,\text{Cos}\,[\,e\,]\,+\,\text{Sin}\,[\,e\,]\,\right)\,\left(c\,\text{Sin}\,[\,e\,]\,+\,\left(-d+c\,\text{Cos}\,[\,e\,]\,\right)\,\text{Tan}\Big[\frac{f\,x}{2}\Big]\right)}{\sqrt{c^2-d^2}\,\sqrt{\left(\text{Cos}\,[\,e\,]\,-\,\text{i}\,\text{Sin}\,[\,e\,]\,\right)^2}}\,\left(d+c\,\text{Cos}\,[\,e+f\,x\,]\,\right)^2 \\ &+ \left(\text{i}\,\text{Cos}\,[\,e\,]\,+\,\text{Sin}\,[\,e\,]\,\right) \left/\left(\left(c+d\right)^2\,\sqrt{c^2-d^2}\,\sqrt{\left(\text{Cos}\,[\,e\,]\,-\,\text{i}\,\text{Sin}\,[\,e\,]\,\right)^2}\right) + \frac{1}{c^2\,\left(c+d\right)^2} \\ &+ \left(c-d\right)\,d\,\text{Sec}\,[\,e\,]\,\left(\left(2\,c^4+6\,c^3\,d+5\,c^2\,d^2+12\,c\,d^3+2\,d^4\right)\,\text{Sin}\,[\,e\,]\,-\,c\,\left(d\,\left(7\,c^2+18\,c\,d+2\,d^2\right)\right) \\ &+ \text{Sin}\,[\,f\,x\,]\,-\,d\,\left(c^2+6\,c\,d+2\,d^2\right)\,\text{Sin}\,[\,2\,e+f\,x\,]\,+\,c\,\left(2\,c^2+6\,c\,d+d^2\right)\,\text{Sin}\,[\,e+2\,f\,x\,]\,\right)\,\right) \end{split}$$

Problem 208: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \; \left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3}{\left(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^4} \; \mathrm{d} x$$

Optimal (type 3, 178 leaves, 6 steps):

Result (type 3, 398 leaves):

$$\frac{1}{192 \left(c+d\right)^3 f \left(c+d \operatorname{Sec}\left[e+fx\right]\right)^4}$$

$$a^3 \left(d+c \operatorname{Cos}\left[e+fx\right]\right) \operatorname{Sec}\left[\frac{1}{2} \left(e+fx\right)\right]^6 \operatorname{Sec}\left[e+fx\right] \left(1+\operatorname{Sec}\left[e+fx\right]\right)^3$$

$$\left(-\left(\left(120 \operatorname{i} \operatorname{ArcTan}\left[\left(\operatorname{i} \operatorname{Cos}\left[e\right]+\operatorname{Sin}\left[e\right]\right) \left(c \operatorname{Sin}\left[e\right]+\left(-d+c \operatorname{Cos}\left[e\right]\right) \operatorname{Tan}\left[\frac{fx}{2}\right]\right)\right)\right) / \left(\sqrt{c^2-d^2} \sqrt{\left(\operatorname{Cos}\left[e\right]-\operatorname{i} \operatorname{Sin}\left[e\right]\right)^2}\right)\right] \left(d+c \operatorname{Cos}\left[e+fx\right]\right)^3$$

$$\left(\operatorname{Cos}\left[e\right]-\operatorname{i} \operatorname{Sin}\left[e\right]\right)\right) / \left(\sqrt{c^2-d^2} \sqrt{\left(\operatorname{Cos}\left[e\right]-\operatorname{i} \operatorname{Sin}\left[e\right]\right)^2}\right)\right) +$$

$$\frac{1}{c^3} \left(c \operatorname{Sec}\left[e\right] \left(6 \left(8 c^4+6 c^3 d+30 c^2 d^2+9 c d^3+2 d^4\right) \operatorname{Sin}\left[fx\right] -$$

$$3 \left(6 c^4-3 c^3 d+30 c^2 d^2+18 c d^3+4 d^4\right) \operatorname{Sin}\left[2 e+fx\right]+c \left(3 \left(3 c^3+38 c^2 d+12 c d^2+2 d^3\right) \right)$$

$$\operatorname{Sin}\left[e+2 fx\right]+3 \left(3 c^3-6 c^2 d-6 c d^2-2 d^3\right) \operatorname{Sin}\left[3 e+2 fx\right]+c \left(22 c^2+9 c d+2 d^2\right)$$

$$\operatorname{Sin}\left[2 e+3 fx\right]\right)\right)-2 d \left(66 c^4+27 c^3 d+50 c^2 d^2+18 c d^3+4 d^4\right) \operatorname{Tan}\left[e\right]\right)\right)$$

### Problem 210: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} \left[ e + f x \right] \left( c + d \operatorname{Sec} \left[ e + f x \right] \right)^{4}}{a + a \operatorname{Sec} \left[ e + f x \right]} \, dx$$

Optimal (type 3, 183 leaves, 7 steps):

$$\frac{d \left(8 \, c^3 - 12 \, c^2 \, d + 12 \, c \, d^2 - 3 \, d^3\right) \, ArcTanh \left[Sin\left[e + f \, x\right]\right]}{2 \, a \, f} - \frac{\left(3 \, c - 4 \, d\right) \, d \, \left(c + d \, Sec\left[e + f \, x\right]\right)^2 \, Tan\left[e + f \, x\right]}{3 \, a \, f} + \frac{\left(c - d\right) \, \left(c + d \, Sec\left[e + f \, x\right]\right)^3 \, Tan\left[e + f \, x\right]}{f \left(a + a \, Sec\left[e + f \, x\right]\right)} - \frac{1}{6 \, a \, f} d \, \left(4 \, \left(3 \, c^3 - 16 \, c^2 \, d + 12 \, c \, d^2 - 4 \, d^3\right) + d \, \left(6 \, c^2 - 20 \, c \, d + 9 \, d^2\right) \, Sec\left[e + f \, x\right]\right) \, Tan\left[e + f \, x\right]$$

Result (type 3, 1243 leaves):

## Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e+fx] \left(c+d \operatorname{Sec} [e+fx]\right)^{3}}{a+a \operatorname{Sec} [e+fx]} \, dx$$

Optimal (type 3, 117 leaves, 6 steps):

$$\frac{3 \ d \ \left(2 \ c^{2}-2 \ c \ d+d^{2}\right) \ ArcTanh \left[Sin \left[e+f x\right]\right]}{2 \ a \ f} + \frac{\left(c-d\right) \ \left(c+d \ Sec \left[e+f x\right]\right)^{2} \ Tan \left[e+f x\right]}{f \ \left(a+a \ Sec \left[e+f x\right]\right)} - \frac{d \ \left(4 \ \left(c^{2}-3 \ c \ d+d^{2}\right)+\left(2 \ c-3 \ d\right) \ d \ Sec \left[e+f x\right]\right) \ Tan \left[e+f x\right]}{2 \ a \ f}$$

Result (type 3, 275 leaves):

$$\begin{split} &\frac{1}{\text{a}\,f\,\left(1+\text{Cos}\,[\,e+f\,x\,]\,\right)}\,\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\big]^6\,\,\text{Sec}\,[\,e+f\,x\,]^{\,2} \\ &\left(16\,d^3\,\text{Csc}\,[\,e+f\,x\,]^{\,3}\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\big]^4\,+\,\left(-1+\text{Tan}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]^{\,2}\right)\,\left(3\,d\,\left(2\,c^2-2\,c\,d+d^2\right)\right) \\ &\left(\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,\Big]\,-\,\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,+\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,\Big]\,-\,\\ &2\,\left(\,c^3-3\,c^2\,d+9\,c\,d^2-3\,d^3\right)\,\,\text{Tan}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,-\,3\,d\,\left(\,2\,c^2-2\,c\,d+d^2\right) \\ &\left(\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,-\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,\Big]\,-\,\text{Log}\,\Big[\text{Cos}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,+\,\text{Sin}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]\,\Big]\,\Big) \\ &\left(\text{Tan}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]^2\,+\,2\,\left(\,c-d\,\right)^3\,\,\text{Tan}\,\Big[\frac{1}{2}\,\left(\,e+f\,x\,\right)\,\,\Big]^3\,\Big)\,\Big) \end{split}$$

## Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + fx] (c + d \operatorname{Sec} [e + fx])^{2}}{a + a \operatorname{Sec} [e + fx]} dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{\left(2\,c-d\right)\,d\,ArcTanh\left[Sin\left[e+f\,x\right]\,\right]}{a\,f}\,+\,\frac{d^2\,Tan\left[e+f\,x\right]}{a\,f}\,+\,\frac{\left(c-d\right)^2\,Tan\left[e+f\,x\right]}{f\left(a+a\,Sec\left[e+f\,x\right]\right)}$$

Result (type 3, 237 leaves):

$$\begin{split} &\left(2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right)\,\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)^2 \\ &\left(\left(\mathsf{c}-\mathsf{d}\right)^2\,\mathsf{Sec}\left[\frac{\mathsf{e}}{2}\,\right]\,\mathsf{Sin}\left[\frac{\mathsf{f}\,\mathsf{x}}{2}\right]+\mathsf{d}\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(-\left(2\,\mathsf{c}-\mathsf{d}\right)\right) \\ &\left(\mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]-\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)-\mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]+\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\right)+\\ &\left(\mathsf{d}\,\mathsf{Sin}\left[\mathsf{f}\,\mathsf{x}\right]\right)\bigg/\left(\left(\mathsf{Cos}\left[\frac{\mathsf{e}}{2}\right]-\mathsf{Sin}\left[\frac{\mathsf{e}}{2}\right]\right)\left(\mathsf{Cos}\left[\frac{\mathsf{e}}{2}\right]+\mathsf{Sin}\left[\frac{\mathsf{e}}{2}\right]\right) \\ &\left(\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]-\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\left(\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]+\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\right)\right)\bigg)\bigg)\bigg/\\ &\left(\mathsf{a}\,\mathsf{f}\,\left(\mathsf{d}+\mathsf{c}\,\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)^2\left(\mathsf{1}+\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)\right) \end{split}$$

# Problem 213: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,] \,\,\left(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)}{\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]} \,\,\mathrm{d} \mathsf{x}$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{d \operatorname{ArcTanh} \left[\operatorname{Sin} \left[e+f x\right]\right]}{a f} + \frac{\left(c-d\right) \operatorname{Tan} \left[e+f x\right]}{f\left(a+a \operatorname{Sec} \left[e+f x\right]\right)}$$

Result (type 3. 109 leaves):

$$\begin{split} \left(2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(\mathsf{d}\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right.\\ &\left.\left.\left(-\mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]-\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right] + \mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right] + \mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\right)\right) \\ &\left.\left(\mathsf{c}-\mathsf{d}\right)\,\mathsf{Sec}\left[\frac{\mathsf{e}}{2}\right]\,\mathsf{Sin}\left[\frac{\mathsf{f}\,\mathsf{x}}{2}\right]\right)\right)\bigg/\,\left(\mathsf{a}\,\mathsf{f}\,\left(\mathsf{1}+\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)\right) \end{split}$$

# Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)\,\left(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,\mathsf{e} + \mathsf{f}\,\mathsf{x}\,]\,\right)}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 83 leaves, 4 steps):

$$-\frac{2\,d\,\text{ArcTanh}\Big[\,\frac{\sqrt{c-d}\,\,\text{Tan}\Big[\frac{1}{2}\,\,(e+f\,x)\,\Big]\,}{\sqrt{c+d}}\,\Big]}{a\,\,\Big(c-d\Big)^{\,3/2}\,\sqrt{c+d}\,\,\,f}\,+\,\,\frac{\text{Tan}\,[\,e+f\,x\,]}{\Big(\,c-d\Big)\,\,f\,\,\Big(a+a\,\text{Sec}\,[\,e+f\,x\,]\,\Big)}$$

Result (type 3, 160 leaves):

$$\left( 2 \, \mathsf{Cos} \left[ \frac{1}{2} \, \left( e + \mathsf{f} \, \mathsf{x} \right) \right] \right. \\ \left. \left( \left[ 2 \, \mathsf{d} \, \mathsf{ArcTan} \left[ \frac{\left( \mathbb{i} \, \mathsf{Cos} \left[ e \right] + \mathsf{Sin} \left[ e \right] \right) \, \left( \mathsf{c} \, \mathsf{Sin} \left[ e \right] + \left( -\mathsf{d} + \mathsf{c} \, \mathsf{Cos} \left[ e \right] \right) \, \mathsf{Tan} \left[ \frac{\mathsf{f} \, \mathsf{x}}{2} \right] \right)}{\sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{\left( \mathsf{Cos} \left[ e \right] - \mathbb{i} \, \mathsf{Sin} \left[ e \right] \right)^2}} \right] \, \mathsf{Cos} \left[ \frac{1}{2} \, \left( e + \mathsf{f} \, \mathsf{x} \right) \right] \\ \left( \mathbb{i} \, \mathsf{Cos} \left[ e \right] + \mathsf{Sin} \left[ e \right] \right) \right) \bigg/ \left( \sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{\left( \mathsf{Cos} \left[ e \right] - \mathbb{i} \, \mathsf{Sin} \left[ e \right] \right)^2} \right) + \\ \mathsf{Sec} \left[ \frac{e}{2} \right] \, \mathsf{Sin} \left[ \frac{\mathsf{f} \, \mathsf{x}}{2} \right] \right) \bigg) \bigg/ \left( \mathsf{a} \, \left( \mathsf{c} - \mathsf{d} \right) \, \mathsf{f} \, \left( \mathsf{1} + \mathsf{Cos} \left[ e + \mathsf{f} \, \mathsf{x} \right] \right) \right)$$

Problem 215: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)\left(c+d\operatorname{Sec}[e+fx]\right)^{2}} dx$$

Optimal (type 3, 145 leaves, 6 steps):

$$-\frac{2 \, d \, \left(2 \, c + d\right) \, \text{ArcTanh} \left[\frac{\sqrt{c - d} \, \, \text{Tan} \left[\frac{1}{2} \, \left(e + f \, x\right)\right]}{\sqrt{c + d}}\right]}{a \, \left(c - d\right)^{5/2} \, \left(c + d\right)^{3/2} \, f} \\ \\ \frac{\left(c + 2 \, d\right) \, \text{Tan} \left[e + f \, x\right]}{\left(c - d\right)^2 \, \left(c + d\right) \, f \, \left(a + a \, \text{Sec} \left[e + f \, x\right]\right)} - \frac{d \, \, \text{Tan} \left[e + f \, x\right]}{\left(c^2 - d^2\right) \, f \, \left(a + a \, \text{Sec} \left[e + f \, x\right]\right) \, \left(c + d \, \text{Sec} \left[e + f \, x\right]\right)}$$

Result (type 3, 286 leaves):

$$\left( 2 \operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right] \left( d + c \operatorname{Cos} \left[ e + f x \right] \right) \operatorname{Sec} \left[ e + f x \right]^{3} \right. \\ \left( \left( 2 \operatorname{d} \left( 2 \operatorname{c} + d \right) \operatorname{ArcTan} \left[ \frac{\left( i \operatorname{Cos} \left[ e \right] + \operatorname{Sin} \left[ e \right] \right) \left( \operatorname{c} \operatorname{Sin} \left[ e \right] + \left( - \operatorname{d} + \operatorname{c} \operatorname{Cos} \left[ e \right] \right) \operatorname{Tan} \left[ \frac{f x}{2} \right] \right)}{\sqrt{c^{2} - d^{2}}} \right) \right. \\ \left. \left. \operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right] \left( d + c \operatorname{Cos} \left[ e + f x \right] \right) \left( i \operatorname{Cos} \left[ e \right] + \operatorname{Sin} \left[ e \right] \right) \right) \right. \\ \left. \left( \left( c + d \right) \sqrt{c^{2} - d^{2}} \sqrt{\left( \operatorname{Cos} \left[ e \right] - i \operatorname{Sin} \left[ e \right] \right)^{2}} \right) + \left( \operatorname{d} + \operatorname{c} \operatorname{Cos} \left[ e + f x \right] \right) \operatorname{Sec} \left[ \frac{e}{2} \right] \operatorname{Sin} \left[ \frac{f x}{2} \right] + \\ \left. \frac{d^{2} \operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right] \left( - \operatorname{d} \operatorname{Sin} \left[ e \right] + \operatorname{c} \operatorname{Sin} \left[ f x \right] \right)}{\operatorname{c} \left( \left( c + d \right) \left( \operatorname{Cos} \left[ \frac{e}{2} \right] - \operatorname{Sin} \left[ \frac{e}{2} \right] \right) \left( \operatorname{Cos} \left[ \frac{e}{2} \right] + \operatorname{Sin} \left[ \frac{e}{2} \right] \right)} \right) \right) \right/ \\ \left( a \left( c - d \right)^{2} \operatorname{f} \left( 1 + \operatorname{Sec} \left[ e + f x \right] \right) \left( c + \operatorname{d} \operatorname{Sec} \left[ e + f x \right] \right)^{2} \right)$$

Problem 216: Result unnecessarily involves complex numbers and more than

#### twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)\,\left(c+d\operatorname{Sec}[e+fx]\right)^3}\,\mathrm{d}x$$

Optimal (type 3, 207 leaves, 7 steps):

$$-\frac{3 \text{ d } \left(2 \text{ c}^2+2 \text{ c } \text{ d}+\text{ d}^2\right) \text{ ArcTanh} \left[\frac{\sqrt{c-d} \text{ Tan} \left[\frac{1}{2} \left(e+f x\right)\right]}{\sqrt{c+d}}\right]}{\text{ a } \left(c-d\right)^{7/2} \left(c+d\right)^{5/2} \text{ f}} + \frac{\text{ d } \left(2 \text{ c}+3 \text{ d}\right) \text{ Tan} \left[e+f x\right]}{2 \text{ a } \left(c-d\right)^2 \left(c+d\right) \text{ f } \left(c+d \text{ Sec} \left[e+f x\right]\right)^2} + \\ \frac{\text{ Tan} \left[e+f x\right]}{\left(c-d\right) \text{ f } \left(a+a \text{ Sec} \left[e+f x\right]\right) \left(c+d \text{ Sec} \left[e+f x\right]\right)^2} + \frac{\text{ d } \left(2 \text{ c}+d\right) \left(c+4 \text{ d}\right) \text{ Tan} \left[e+f x\right]}{2 \text{ a } \left(c-d\right)^3 \left(c+d\right)^2 \text{ f } \left(c+d \text{ Sec} \left[e+f x\right]\right)}$$

#### Result (type 3, 1422 leaves):

$$\left[ \left( 2\,c^2 + 2\,c\,d + d^2 \right) \, \text{Cos} \left[ \frac{e}{2} + \frac{f\,x}{2} \right]^2 \, \left( d + c\, \text{Cos} \left[ e + f\,x \right] \right)^3 \right. \\ \left. \text{Sec} \left[ e + f\,x \right]^4 \left[ - \left( \left[ 6\,i\,d\, \text{ArcTan} \left[ \text{Sec} \left[ \frac{f\,x}{2} \right] \, \left( \frac{\text{Cos} \left[ e \right]}{\sqrt{c^2 - d^2} \, \sqrt{\text{Cos} \left[ 2\,e \right] - i\,\text{Sin} \left[ 2\,e \right]}} \right. - \frac{i\,\text{Sin} \left[ e \right]}{\sqrt{c^2 - d^2} \, \sqrt{\text{Cos} \left[ 2\,e \right] - i\,\text{Sin} \left[ 2\,e \right]}} \right) \left[ - i\,d\, \text{Sin} \left[ \frac{f\,x}{2} \right] + i\,c\, \text{Sin} \left[ e + \frac{f\,x}{2} \right] \right) \right] \, \text{Cos} \left[ e \right] \right] \\ \left. \left( \sqrt{c^2 - d^2} \, \sqrt{\text{Cos} \left[ 2\,e \right] - i\,\text{Sin} \left[ 2\,e \right]}} \right) - \frac{i\,\text{Sin} \left[ e \right]}{\sqrt{c^2 - d^2} \, \sqrt{\text{Cos} \left[ 2\,e \right] - i\,\text{Sin} \left[ 2\,e \right]}} \right) \right] \\ \left. \left( - i\,d\, \text{Sin} \left[ \frac{f\,x}{2} \right] + i\,c\, \text{Sin} \left[ e + \frac{f\,x}{2} \right] \right) \right] \, \text{Sin} \left[ e \right] \right) \right/ \\ \left( \sqrt{c^2 - d^2} \, \sqrt{\text{Cos} \left[ 2\,e \right] - i\,\text{Sin} \left[ 2\,e \right]} \right) \right) \right] / \\ \left( \left( - c\,+ d \right)^3 \, \left( c\,+ d \right)^2 \, \left( a + a\,\text{Sec} \left[ e + f\,x \right] \right) \, \left( c\,+ d\,\text{Sec} \left[ e + f\,x \right] \right)^3 \right) + \frac{1}{8\,c^2 \, \left( - c\,+ d \right)^3 \, \left( c\,+ d \right)^2 \, f \, \left( a + a\,\text{Sec} \left[ e + f\,x \right] \right) \, \left( c\,+ d\,\text{Sec} \left[ e + f\,x \right] \right)^3} \right] \\ \text{Cos} \left[ \frac{e}{2} + \frac{f\,x}{2} \right] \\ \left( d + c\,\text{Cos} \left[ e + f\,x \right] \right) \, \text{Sec} \left[ \frac{e}{2} \right] \\ \text{Sec} \left[ e \right] \\ \text{Sec} \left[ e \right] \\ \text{Sec} \left[ e \right] \right. \\ \left. \left( 8\,c^5\,d\,\text{Sin} \left[ \frac{f\,x}{2} \right] + 10\,c^4\,d^2\,\text{Sin} \left[ \frac{f\,x}{2} \right] - 11\,c^3\,d^3\,\text{Sin} \left[ \frac{f\,x}{2} \right] - 17\,c^2\,d^4\,\text{Sin} \left[ \frac{f\,x}{2} \right] - 22\,c^4\,d^2\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 22\,c^4\,d^2\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 22\,c^4\,d^2\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 27\,c^3\,d^3\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 22\,c^4\,d^3\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 22\,c^4\,d^3\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 27\,c^3\,d^3\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] - 5\,c^2\,d^4\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] + 2\,c\,d^5\,\text{Sin} \left[ \frac{3\,f\,x}{2} \right] + 2\,c\,d^5\,\text{Si$$

$$\begin{split} &4\,c^6\,Sin\Big[e-\frac{f\,x}{2}\Big] + 8\,c^5\,d\,Sin\Big[e-\frac{f\,x}{2}\Big] + 18\,c^4\,d^2\,Sin\Big[e-\frac{f\,x}{2}\Big] + \\ &35\,c^3\,d^3\,Sin\Big[e-\frac{f\,x}{2}\Big] + 25\,c^2\,d^4\,Sin\Big[e-\frac{f\,x}{2}\Big] + 2\,c\,d^5\,Sin\Big[e-\frac{f\,x}{2}\Big] - \\ &2\,d^6\,Sin\Big[e-\frac{f\,x}{2}\Big] - 4\,c^6\,Sin\Big[e+\frac{f\,x}{2}\Big] - 8\,c^5\,d\,Sin\Big[e+\frac{f\,x}{2}\Big] - 6\,c^4\,d^2\,Sin\Big[e+\frac{f\,x}{2}\Big] - 7\,c^3\,d^3\,Sin\Big[e+\frac{f\,x}{2}\Big] + 5\,c^2\,d^4\,Sin\Big[e+\frac{f\,x}{2}\Big] + 2\,c\,d^5\,Sin\Big[e+\frac{f\,x}{2}\Big] - 2\,d^6\,Sin\Big[e+\frac{f\,x}{2}\Big] + \\ &8\,c^5\,d\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] + 22\,c^4\,d^2\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] + 17\,c^3\,d^3\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] + \\ &13\,c^2\,d^4\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] + 2\,c\,d^5\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] - 2\,d^6\,Sin\Big[2\,e+\frac{f\,x}{2}\Big] + 2\,c^6\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] + \\ &4\,c^5\,d\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] - 4\,c^4\,d^2\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] - 19\,c^3\,d^3\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] - \\ &5\,c^2\,d^4\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] + 2\,c\,d^5\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] - 8\,c^5\,d\,Sin\Big[2\,e+\frac{3\,f\,x}{2}\Big] - \\ &5\,c^2\,d^4\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] - c^3\,d^3\,Sin\Big[e+\frac{3\,f\,x}{2}\Big] + 2\,c^2\,d^4\,Sin\Big[2\,e+\frac{3\,f\,x}{2}\Big] - \\ &2\,c\,d^5\,Sin\Big[2\,e+\frac{3\,f\,x}{2}\Big] + 2\,c^6\,Sin\Big[3\,e+\frac{3\,f\,x}{2}\Big] + 2\,c^2\,d^4\,Sin\Big[3\,e+\frac{3\,f\,x}{2}\Big] - \\ &2\,c\,d^5\,Sin\Big[3\,e+\frac{3\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[e+\frac{5\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[e+\frac{5\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[e+\frac{5\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[e+\frac{5\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[e+\frac{5\,f\,x}{2}\Big] - 2\,c^6\,Sin\Big[2\,e+\frac{5\,f\,x}{2}\Big] - 2\,$$

# Problem 217: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec \left[e+fx\right] \ \left(c+d \, Sec \left[e+fx\right]\right)^5}{\left(a+a \, Sec \left[e+fx\right]\right)^2} \ dx$$

Optimal (type 3, 258 leaves, 8 steps):

$$\frac{5 \; \left(2 \; c-d\right) \; d^2 \; \left(2 \; c^2-3 \; c \; d+2 \; d^2\right) \; ArcTanh [Sin [e+fx]]}{2 \; a^2 \; f} \; - \\ \frac{d \; \left(c^2+10 \; c \; d-12 \; d^2\right) \; \left(c+d \; Sec \left[e+fx\right]\right)^2 \; Tan \left[e+fx\right]}{3 \; a^2 \; f} \; + \\ \frac{\left(c-d\right) \; \left(c+10 \; d\right) \; \left(c+d \; Sec \left[e+fx\right]\right)^3 \; Tan \left[e+fx\right]}{3 \; f \; \left(a^2+a^2 \; Sec \left[e+fx\right]\right)} \; + \; \frac{\left(c-d\right) \; \left(c+d \; Sec \left[e+fx\right]\right)^4 \; Tan \left[e+fx\right]}{3 \; f \; \left(a+a \; Sec \left[e+fx\right]\right)^2} \; - \\ \frac{1}{6 \; a^2 \; f} d \; \left(4 \; \left(c^4+10 \; c^3 \; d-44 \; c^2 \; d^2+40 \; c \; d^3-12 \; d^4\right) \; + \; d \; \left(2 \; c^3+20 \; c^2 \; d-57 \; c \; d^2+30 \; d^3\right) \; Sec \left[e+fx\right]\right)} \; Tan \left[e+fx\right]$$

#### Result (type 3, 743 leaves):

$$\left(10 \left(-4 \, c^3 \, d^2 + 8 \, c^2 \, d^3 - 7 \, c \, d^4 + 2 \, d^5\right) \, \cos\left[\frac{e}{2} + \frac{f \, x}{2}\right]^4 \\ - \cos\left[e + f \, x\right]^3 \, \log\left[\cos\left[\frac{1}{2} \left(e + f \, x\right)\right] - \sin\left[\frac{1}{2} \left(e + f \, x\right)\right]\right] \, \left(c + d \, \sec\left[e + f \, x\right]\right)^5\right) \Big/ \\ \left(f \left(d + c \, \cos\left[e + f \, x\right]\right)^5 \, \left(a + a \, \sec\left[e + f \, x\right]\right)^2\right) - \left(10 \, \left(-4 \, c^3 \, d^2 + 8 \, c^2 \, d^3 - 7 \, c \, d^4 + 2 \, d^5\right) \, \cos\left[\frac{e}{2} + \frac{f \, x}{2}\right]^4 \, \cos\left[e + f \, x\right]^3 \\ - \log\left[\cos\left[\frac{1}{2} \left(e + f \, x\right)\right] + \sin\left[\frac{1}{2} \left(e + f \, x\right)\right]\right] \, \left(c + d \, \sec\left[e + f \, x\right]\right)^5\right) \Big/ \\ \left(f \left(d + c \, \cos\left[e + f \, x\right]\right)^5 \, \left(a + a \, \sec\left[e + f \, x\right]\right)^2\right) + \frac{1}{24 \, f \, \left(d + c \, \cos\left[e + f \, x\right]\right)^5} \left(a + a \, \sec\left[e + f \, x\right]\right)^2 \right) + \frac{1}{24 \, f \, \left(d + c \, \cos\left[e + f \, x\right]\right)^5} \left(a + a \, \sec\left[e + f \, x\right]\right)^2$$

$$\cos\left[\frac{e}{2} + \frac{f \, x}{2}\right]^4 \, \sec\left[\frac{1}{2} \left(e + f \, x\right)\right]^3 \, \left(c + d \, \sec\left[e + f \, x\right]\right)^5 \right) + 60 \, c^3 \, d^2 \, \sin\left[\frac{1}{2} \left(e + f \, x\right)\right] - 15 \, c \, d^4 \, \sin\left[\frac{1}{2} \left(e + f \, x\right)\right] + 18 \, d^5 \, \sin\left[\frac{1}{2} \left(e + f \, x\right)\right] - 2 \, c^5 \, \sin\left[\frac{3}{2} \left(e + f \, x\right)\right] + 18 \, d^5 \, \sin\left[\frac{3}{2} \left(e + f \, x\right)\right] - 2 \, c^5 \, \sin\left[\frac{3}{2} \left(e + f \, x\right)\right] + 10 \, d^3 \, d^3 \, \sin\left[\frac{3}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{3}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{5}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{7}{2} \left(e + f \, x\right)\right] - 105 \, c \, d^4 \, \sin\left[\frac{5}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{7}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{7}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{7}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{7}{2} \left(e + f \, x\right)\right] + 100 \, c^3 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] + 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] + 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, \sin\left[\frac{9}{2} \left(e + f \, x\right)\right] - 100 \, c^4 \, d^3 \, d^3$$

### Problem 219: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec[e+fx] \left(c+dSec[e+fx]\right)^3}{\left(a+aSec[e+fx]\right)^2} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{ \left( \text{3 c} - \text{2 d} \right) \, d^2 \, \text{ArcTanh} \left[ \text{Sin} \left[ e + f \, x \right] \, \right] }{ a^2 \, f} + \frac{ \left( \text{c} - \text{d} \right) \, \left( \text{c} + \text{d Sec} \left[ e + f \, x \right] \right)^2 \, \text{Tan} \left[ e + f \, x \right] }{ 3 \, f \, \left( \text{a} + \text{a Sec} \left[ e + f \, x \right] \right)^2 } + \frac{ \left( \text{c}^3 + 4 \, \text{c}^2 \, \text{d} - 12 \, \text{c} \, \text{d}^2 + 10 \, \text{d}^3 - \left( \text{c} - 4 \, \text{d} \right) \, \text{d}^2 \, \text{Sec} \left[ e + f \, x \right] \right) \, \text{Tan} \left[ e + f \, x \right] }{ 3 \, f \, \left( \text{a}^2 + \text{a}^2 \, \text{Sec} \left[ e + f \, x \right] \right) }$$

Result (type 3, 294 leaves):

$$\begin{split} &\frac{1}{3\,\mathsf{a}^2\,\mathsf{f}\,\big(1+\mathsf{Cos}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\big)\,\big)^2}\,2\,\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]^6\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,\,\left(\,\mathsf{6}\,\mathsf{d}^2\,\left(\,-\,\mathsf{3}\,\mathsf{c}\,+\,2\,\mathsf{d}\,\right)\,\right) \\ &\left(\,\mathsf{Log}\,\big[\,\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,-\,\mathsf{Sin}\,\big[\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\big]\,-\,\mathsf{Log}\,\big[\,\mathsf{Cos}\,\big[\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,+\,\mathsf{Sin}\,\big[\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\big]\,-\,\mathsf{8}\,\left(\,\mathsf{c}\,-\,\mathsf{d}\,\right)^3\,\mathsf{Csc}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,^3\,\mathsf{Sin}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]^4\,+\,32\,\left(\,\mathsf{c}\,-\,\mathsf{d}\,\right)^3\,\mathsf{Csc}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]^5\,\mathsf{Sin}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]^8\,+\,\\ &2\,\left(\,2\,\mathsf{c}^3\,+\,3\,\mathsf{c}^2\,\mathsf{d}\,-\,12\,\mathsf{c}\,\mathsf{d}^2\,+\,13\,\mathsf{d}^3\right)\,\mathsf{Tan}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,+\,6\,\left(\,3\,\mathsf{c}\,-\,2\,\mathsf{d}\,\right)\,\mathsf{d}^2\,\\ &\left(\,\mathsf{Log}\,\big[\,\mathsf{Cos}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,-\,\mathsf{Sin}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\big]\,-\,\mathsf{Log}\,\big[\,\mathsf{Cos}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,+\,\mathsf{Sin}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]\,\big]\,\big)\,\\ &\mathsf{Tan}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]^2\,-\,2\,\left(\,\mathsf{c}\,-\,\mathsf{d}\,\right)^2\,\left(\,2\,\mathsf{c}\,+\,\mathsf{7}\,\mathsf{d}\,\right)\,\mathsf{Tan}\,\big[\,\frac{1}{2}\,\left(\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,\right)\,\big]^3\,\big)\,\end{split}$$

## Problem 220: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \ \left(\,c + d\,\mathsf{Sec}\,[\,e + f\,x\,]\,\,\right)^{\,2}}{\left(\,a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\,\right)^{\,2}} \,\,\mathrm{d}x$$

Optimal (type 3, 89 leaves, 6 steps):

$$\frac{d^2 \, Arc Tanh \, [\, Sin \, [\, e + f \, x \,] \,\,]}{a^2 \, f} \, + \, \frac{\left(\, c \, - \, d\,\right)^{\, 2} \, Tan \, [\, e \, + \, f \, x \,]}{3 \, f \, \left(\, a \, + \, a \, Sec \, [\, e \, + \, f \, x \,] \,\,\right)^{\, 2}} \, + \, \frac{\left(\, c \, - \, d\,\right) \, \, \left(\, c \, + \, 5 \, d\,\right) \, \, Tan \, [\, e \, + \, f \, x \,]}{3 \, f \, \left(\, a^2 \, + \, a^2 \, Sec \, [\, e \, + \, f \, x \,] \,\,\right)}$$

Result (type 3, 181 leaves):

$$-\left(\left(2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\,\left(\mathsf{6}\,\mathsf{d}^2\,\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]^3\right.\right.\\ \left.\left.\left(\mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]-\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)-\mathsf{Log}\left[\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]+\mathsf{Sin}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\right)\right)\\ \left.\left(\mathsf{c}-\mathsf{d}\right)^2\mathsf{Sec}\left[\frac{\mathsf{e}}{2}\right]\mathsf{Sin}\left[\frac{\mathsf{f}\,\mathsf{x}}{2}\right]-\mathsf{4}\,\left(\mathsf{c}^2+\mathsf{c}\,\mathsf{d}-\mathsf{2}\,\mathsf{d}^2\right)\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]^2\mathsf{Sec}\left[\frac{\mathsf{e}}{2}\right]\mathsf{Sin}\left[\frac{\mathsf{f}\,\mathsf{x}}{2}\right]+\\ \left.\left(\mathsf{c}-\mathsf{d}\right)^2\mathsf{Cos}\left[\frac{1}{2}\,\left(\mathsf{e}+\mathsf{f}\,\mathsf{x}\right)\,\right]\mathsf{Tan}\left[\frac{\mathsf{e}}{2}\right]\right)\right)\right/\left(\mathsf{3}\,\mathsf{a}^2\,\mathsf{f}\,\left(\mathsf{1}+\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right)^2\right)\right)$$

### Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+a\operatorname{Sec}[e+fx]\right)^2\left(c+d\operatorname{Sec}[e+fx]\right)} \, dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$\frac{2\,d^{2}\,\text{ArcTanh}\Big[\frac{\sqrt{c-d}\,\,\text{Tan}\Big[\frac{1}{2}\,(e+f\,x)\Big]}{\sqrt{c+d}}\Big]}{a^{2}\,\left(c-d\right)^{5/2}\,\sqrt{c+d}\,\,f} + \frac{\text{Tan}\,[\,e+f\,x\,]}{3\,\left(c-d\right)\,f\,\left(a+a\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{2}} + \frac{\left(c-4\,d\right)\,\text{Tan}\,[\,e+f\,x\,]}{3\,\left(c-d\right)^{2}\,f\,\left(a^{2}+a^{2}\,\text{Sec}\,[\,e+f\,x\,]\,\right)}$$

Result (type 3, 209 leaves):

$$\begin{split} &\left(\text{Cos}\left[\frac{1}{2}\left(e+fx\right)\right]\right. \\ &\left.\left(-\left(\left[24\,\dot{\text{i}}\,d^2\,\text{ArcTan}\left[\left(\left(\dot{\text{i}}\,\text{Cos}\left[e\right]+\text{Sin}\left[e\right]\right)\left(c\,\text{Sin}\left[e\right]+\left(-d+c\,\text{Cos}\left[e\right]\right)\,\text{Tan}\left[\frac{f\,x}{2}\right]\right)\right)\right/ \\ &\left.\left(\sqrt{c^2-d^2}\,\,\sqrt{\left(\text{Cos}\left[e\right]-\dot{\text{i}}\,\text{Sin}\left[e\right]\right)^2}\,\right)\right]\,\text{Cos}\left[\frac{1}{2}\left(e+f\,x\right)\right]^3 \\ &\left.\left(\text{Cos}\left[e\right]-\dot{\text{i}}\,\text{Sin}\left[e\right]\right)\right)\right/\left(\sqrt{c^2-d^2}\,\,\sqrt{\left(\text{Cos}\left[e\right]-\dot{\text{i}}\,\text{Sin}\left[e\right]\right)^2}\,\right)\right) + \\ &\left.\text{Sec}\left[\frac{e}{2}\right]\,\left(3\,\left(c-3\,d\right)\,\text{Sin}\left[\frac{f\,x}{2}\right]-3\,\left(c-2\,d\right)\,\text{Sin}\left[e+\frac{f\,x}{2}\right]+\left(2\,c-5\,d\right)\,\text{Sin}\left[e+\frac{3\,f\,x}{2}\right]\right)\right)\right)\right/ \\ &\left.\left(3\,a^2\,\left(c-d\right)^2\,f\,\left(1+\text{Cos}\left[e+f\,x\right]\right)^2\right) \end{split}$$

# Problem 223: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{2}\,\left(c+d\operatorname{Sec}\left[e+f\,x\right]\right)^{2}}\,\mathrm{d}x$$

Optimal (type 3, 211 leaves, 7 steps):

$$\begin{split} &\frac{2\,d^{2}\,\left(3\,c+2\,d\right)\,ArcTanh\left[\frac{\sqrt{c-d}\,Tan\left[\frac{1}{2}\,\left(e+f\,x\right)\right]}{\sqrt{c+d}}\right]}{a^{2}\,\left(c-d\right)^{7/2}\,\left(c+d\right)^{3/2}\,f} + \frac{d\,\left(c^{2}-6\,c\,d-10\,d^{2}\right)\,Tan\left[e+f\,x\right]}{3\,a^{2}\,\left(c-d\right)^{3}\,\left(c+d\right)\,f\,\left(c+d\,Sec\left[e+f\,x\right]\right)} + \\ &\frac{\left(c-6\,d\right)\,Tan\left[e+f\,x\right]}{3\,a^{2}\,\left(c-d\right)^{2}\,f\,\left(1+Sec\left[e+f\,x\right]\right)\,\left(c+d\,Sec\left[e+f\,x\right]\right)} + \\ &\frac{Tan\left[e+f\,x\right]}{3\,\left(c-d\right)\,f\,\left(a+a\,Sec\left[e+f\,x\right]\right)^{2}\,\left(c+d\,Sec\left[e+f\,x\right]\right)} \end{split}$$

Result (type 3, 764 leaves):

$$\begin{cases} \left(3 \, c + 2 \, d\right) \, Cos \left[\frac{e}{2} + \frac{f \, x}{2}\right]^4 \, \left(d + c \, Cos \left[e + f \, x\right]\right)^2 \\ Sec \left[e + f \, x\right]^4 \, \left[\left[8 \, i \, d^2 \, ArcTan \left[Sec \left[\frac{f \, x}{2}\right] \, \left(\frac{Cos \left[e\right]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} \right. \right. \\ \left. \frac{i \, Sin \left[e\right]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} \right) \left(-i \, d \, Sin \left[\frac{f \, x}{2}\right] + i \, c \, Sin \left[e + \frac{f \, x}{2}\right]\right) \right] \, Cos \left[e\right] \\ \left(\sqrt{c^2 - d^2} \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} \right) + \left\{8 \, d^2 \, ArcTan \left[Sec \left[\frac{f \, x}{2}\right]\right] \right. \\ \left(\frac{Cos \left[e\right]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} - \frac{i \, Sin \left[e\right]}{\sqrt{c^2 - d^2} \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} \right) \\ \left(-i \, d \, Sin \left[\frac{f \, x}{2}\right] + i \, c \, Sin \left[e + \frac{f \, x}{2}\right]\right) \right] \\ Sin \left[e\right] \right) / \left(\sqrt{c^2 - d^2} \, f \, \sqrt{Cos \left[2 \, e\right] - i \, Sin \left[2 \, e\right]}} \right) \right) / \\ \left(\left(-c + d\right)^3 \, \left(c + d\right) \, \left(a + a \, Sec \left[e + f \, x\right]\right)^2 \, \left(c + d \, Sec \left[e + f \, x\right]\right)^2\right) - \\ 2 \, Cos \left[\frac{e}{2} + \frac{f \, x}{2}\right] \, \left(d + c \, Cos \left[e + f \, x\right]\right)^2 \, Sec \left[\frac{e}{2}\right] \, Sec \left[e + f \, x\right]^4 \, \left(d + a \, Sec \left[e + f \, x\right]\right)^2 \, \left(c + d \, Sec \left[e + f \, x\right]\right)^2 \right. \\ \left\{8 \right. \\ \left. \, Cos \left[\frac{e}{2} + \frac{f \, x}{2}\right]^3 \, \left(d + c \, Cos \left[e + f \, x\right]\right)^2 \, \left(c + d \, Sec \left[e + f \, x\right]\right)^2 \right. \\ \left. \left(3 \, \left(-c + d\right)^3 \, f \, \left(a + a \, Sec \left[e + f \, x\right]\right)^2 \, \left(c + d \, Sec \left[e + f \, x\right]\right)^2 \right) - \\ \left\{4 \, Cos \left[\frac{e}{2} + \frac{f \, x}{2}\right]^4 \, \left(d + c \, Cos \left[e + f \, x\right]\right) \, Sec \left[e + f \, x\right]^4 \, \\ \left. \left(d^4 \, Sin \left[e\right] - c \, d^3 \, Sin \left[f \, x\right]\right) \right) / \\ \left[c \, \left(-c \, d\right)^3 \, \left(c \, d\right) \, f \, \left(a + a \, Sec \left[e + f \, x\right]\right)^2 \, \left(c + d \, Sec \left[e + f \, x\right]\right)^2 \right. \\ \left. \left(Cos \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right) \, \left(Cos \left[\frac{e}{2} + Sin \left[\frac{e}{2}\right]\right) \right) - \\ \left. \left(Cos \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right) \, \left(c \, c \, \left[\frac{e}{2} + Sin \left[\frac{e}{2}\right]\right) \right) - \\ \left. \left(Cos \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right) \, \left(c \, c \, \left[\frac{e}{2} + Sin \left[\frac{e}{2}\right]\right) \right) - \\ \left. \left(2 \, Cos \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right) \, \left(c \, c \, \left[\frac{e}{2} + Sin \left[\frac{e}{2}\right]\right) \right) - \\ \left. \left(c \, c \, c \, \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right] \, \left(c \, c \, \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right]\right] \, \left[\frac{e}{2} - Sin \left[\frac{e}{2}\right] \right] \right$$

Problem 224: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]}{\left(\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,2}\,\left(\mathsf{c}+\mathsf{d}\,\mathsf{Sec}\,[\,\mathsf{e}+\mathsf{f}\,\mathsf{x}\,]\,\right)^{\,3}}\,\,\mathrm{d}\,\mathsf{x}$$

#### Optimal (type 3, 284 leaves, 8 steps):

$$\frac{d^2 \left(12 \, c^2 + 16 \, c \, d + 7 \, d^2\right) \, ArcTanh \left[\frac{\sqrt{c-d} \, Tan \left[\frac{1}{2} \, (e+f\,x)\right]}{\sqrt{c+d}}\right]}{\sqrt{c+d}} + \\ \frac{d \left(2 \, c^2 - 16 \, c \, d - 21 \, d^2\right) \, Tan \left[e+f\,x\right]}{6 \, a^2 \, \left(c-d\right)^3 \, \left(c+d\right) \, f \, \left(c+d \, Sec \left[e+f\,x\right]\right)^2} + \\ \frac{\left(c-8 \, d\right) \, Tan \left[e+f\,x\right]}{3 \, a^2 \, \left(c-d\right)^2 \, f \, \left(1+Sec \left[e+f\,x\right]\right) \, \left(c+d \, Sec \left[e+f\,x\right]\right)^2} + \\ \frac{Tan \left[e+f\,x\right]}{3 \, \left(c-d\right) \, f \, \left(a+a \, Sec \left[e+f\,x\right]\right)^2 \, \left(c+d \, Sec \left[e+f\,x\right]\right)^2} + \\ \frac{d \, \left(2 \, c^3 - 16 \, c^2 \, d - 59 \, c \, d^2 - 32 \, d^3\right) \, Tan \left[e+f\,x\right]}{6 \, a^2 \, \left(c-d\right)^4 \, \left(c+d\right)^2 \, f \, \left(c+d \, Sec \left[e+f\,x\right]\right)}$$

#### Result (type 3, 2220 leaves):

$$\begin{cases} \left(12\,c^2 + 16\,c\,d + 7\,d^2\right)\, Cos\left[\frac{e}{2} + \frac{f\,x}{2}\right]^4 \, \left(d + c\, Cos\left[e + f\,x\right]\right)^3 \\ Sec\left[e + f\,x\right]^5 \left(-\left(\left(4\,i\,d^2\,ArcTan\left[Sec\left[\frac{f\,x}{2}\right]\right] \left(\frac{Cos\left[e\right]}{\sqrt{c^2 - d^2}\,\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}}\right) - \frac{i\,Sin\left[e\right]}{\sqrt{c^2 - d^2}\,\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}} \right) \left(-i\,d\,Sin\left[\frac{f\,x}{2}\right] + i\,c\,Sin\left[e + \frac{f\,x}{2}\right]\right)\right) \, Cos\left[e\right] \right) / \\ \left(\sqrt{c^2 - d^2}\,\,f\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}\right) - \left(4\,d^2\,ArcTan\left[Sec\left[\frac{f\,x}{2}\right] \right] \right) \\ \left(\sqrt{c^2 - d^2}\,\,f\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}\right) - \frac{i\,Sin\left[e\right]}{\sqrt{c^2 - d^2}\,\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}}\right) \\ \left(-i\,d\,Sin\left[\frac{f\,x}{2}\right] + i\,c\,Sin\left[e + \frac{f\,x}{2}\right]\right)\right) \, Sin\left[e\right] \right) / \\ \left(\sqrt{c^2 - d^2}\,\,f\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}\right) \right) \right) / \\ \left(\sqrt{c^2 - d^2}\,\,f\,\sqrt{Cos\left[2\,e\right] - i\,Sin\left[2\,e\right]}\right) \right) \right) / \\ \left(\left(-c\,c\,d\right)^4\,\left(c\,+ d\right)^2\,\left(a + a\,Sec\left[e + f\,x\right]\right)^2\,\left(c\,+ d\,Sec\left[e + f\,x\right]\right)^3 \right) + \frac{1}{48\,c^2\,\left(-c\,+ d\right)^4\,\left(c\,+ d\right)^2\,f\,\left(a + a\,Sec\left[e + f\,x\right]\right)^2\,\left(c\,+ d\,Sec\left[e\,+ f\,x\right]\right)^3} \right) \\ Cos\left[e\right] \\ Sec\left[e\right] \\ Sec\left$$

$$\begin{aligned} &6\operatorname{cd}^6\operatorname{Sin}[\frac{3}{2}] + 6\operatorname{d}^7\operatorname{Sin}[\frac{3}{2}] - 12\operatorname{c}^7\operatorname{Sin}[e - \frac{f}{2}] + 2\operatorname{0}\operatorname{c}^6\operatorname{d}\operatorname{Sin}[e - \frac{f}{2}] + \\ &23\operatorname{6}\operatorname{c}^5\operatorname{d}^2\operatorname{Sin}[e - \frac{f}{2}] + 62\operatorname{8}\operatorname{d}^4\operatorname{d}^3\operatorname{Sin}[e - \frac{f}{2}] + 77\operatorname{8}\operatorname{d}^3\operatorname{d}^4\operatorname{Sin}[e - \frac{f}{2}] + \\ &42\operatorname{0}\operatorname{c}^2\operatorname{d}^5\operatorname{Sin}[e - \frac{f}{2}] + 4\operatorname{8}\operatorname{c}\operatorname{d}^6\operatorname{Sin}[e - \frac{f}{2}] - 1\operatorname{8}\operatorname{d}^7\operatorname{Sin}[e - \frac{f}{2}] + \\ &12\operatorname{c}^7\operatorname{Sin}[e + \frac{f}{2}] - 2\operatorname{0}\operatorname{c}^6\operatorname{d}\operatorname{Sin}[e + \frac{f}{2}] - 2\operatorname{36}\operatorname{c}^5\operatorname{d}^2\operatorname{Sin}[e + \frac{f}{2}] + \\ &4\operatorname{60}\operatorname{c}^4\operatorname{d}^3\operatorname{Sin}[e + \frac{f}{2}] - 3\operatorname{10}\operatorname{c}^3\operatorname{d}^4\operatorname{Sin}[e + \frac{f}{2}] + 3\operatorname{9}\operatorname{c}^2\operatorname{d}^5\operatorname{Sin}[e + \frac{f}{2}] + \\ &4\operatorname{8}\operatorname{c}\operatorname{d}^6\operatorname{Sin}[e + \frac{f}{2}] - 3\operatorname{10}\operatorname{c}^3\operatorname{d}^4\operatorname{Sin}[e + \frac{f}{2}] + 3\operatorname{9}\operatorname{c}^2\operatorname{d}^5\operatorname{Sin}[e + \frac{f}{2}] + \\ &4\operatorname{8}\operatorname{c}\operatorname{d}^6\operatorname{Sin}[e + \frac{f}{2}] - 1\operatorname{8}\operatorname{d}^7\operatorname{Sin}[e + \frac{f}{2}] - 1\operatorname{6}\operatorname{c}^7\operatorname{Sin}[2e + \frac{f}{2}] + 1\operatorname{4}\operatorname{c}^6\operatorname{d}\operatorname{Sin}[2e + \frac{f}{2}] + \\ &22\operatorname{0}\operatorname{c}^5\operatorname{d}^2\operatorname{Sin}[2e + \frac{f}{2}] + 5\operatorname{92}\operatorname{c}^4\operatorname{d}^3\operatorname{Sin}[2e + \frac{f}{2}] + 5\operatorname{92}\operatorname{c}^3\operatorname{d}^4\operatorname{Sin}[2e + \frac{f}{2}] + \\ &22\operatorname{0}\operatorname{c}^5\operatorname{d}^7\operatorname{Sin}[2e + \frac{f}{2}] + 3\operatorname{6}\operatorname{c}^6\operatorname{d}\operatorname{Sin}[2e + \frac{f}{2}] + 12\operatorname{6}\operatorname{c}^5\operatorname{d}^7\operatorname{Sin}[2e + \frac{f}{2}] - \\ &6\operatorname{c}^7\operatorname{Sin}[e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[e + \frac{3fx}{2}] + 12\operatorname{6}\operatorname{c}^5\operatorname{d}^7\operatorname{Sin}[e + \frac{3fx}{2}] + \\ &114\operatorname{c}^4\operatorname{d}^3\operatorname{Sin}[e + \frac{3fx}{2}] - 15\operatorname{9}\operatorname{c}^3\operatorname{d}^4\operatorname{Sin}[e + \frac{3fx}{2}] - 14\operatorname{4}\operatorname{c}^2\operatorname{d}^5\operatorname{Sin}[e + \frac{3fx}{2}] - \\ &6\operatorname{c}^4\operatorname{Sin}[e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[e + \frac{3fx}{2}] + 12\operatorname{c}^4\operatorname{d}^3\operatorname{Sin}[2e + \frac{3fx}{2}] - 16\operatorname{c}^6\operatorname{d}\operatorname{Sin}[2e + \frac{3fx}{2}] - \\ &22\operatorname{6}\operatorname{c}^5\operatorname{d}^2\operatorname{Sin}[2e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[3e + \frac{3fx}{2}] + 12\operatorname{c}^6\operatorname{d}^3\operatorname{Sin}[2e + \frac{3fx}{2}] - 16\operatorname{c}^6\operatorname{d}\operatorname{Sin}[2e + \frac{3fx}{2}] - \\ &22\operatorname{6}\operatorname{c}^5\operatorname{d}^3\operatorname{Sin}[3e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[3e + \frac{3fx}{2}] + 12\operatorname{c}^6\operatorname{d}^3\operatorname{Sin}[2e + \frac{3fx}{2}] - 14\operatorname{c}^6\operatorname{d}\operatorname{Sin}[2e + \frac{3fx}{2}] - \\ &22\operatorname{6}\operatorname{c}^5\operatorname{d}^3\operatorname{Sin}[3e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[3e + \frac{3fx}{2}] + 12\operatorname{c}^6\operatorname{d}^3\operatorname{Sin}[3e + \frac{3fx}{2}] - \\ &22\operatorname{6}\operatorname{c}^5\operatorname{d}^3\operatorname{Sin}[3e + \frac{3fx}{2}] + 6\operatorname{c}^6\operatorname{d}\operatorname{Sin}[3e + \frac{3fx}{2}] + 12\operatorname{c}^6\operatorname{d}^3\operatorname{Sin}[3e + \frac{3fx}{2}] - \\$$

$$42 c^{5} d^{2} Sin \left[4 e + \frac{5 f x}{2}\right] + 24 c^{4} d^{3} Sin \left[4 e + \frac{5 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{5 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{5 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{5 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{5 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{5 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] + 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{3} d^{4} Sin \left[4 e + \frac{7 f x}{2}\right] - 27 c^{4} d^{3} Sin \left[4$$

### Problem 225: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec \left[\,e + f\,x\,\right] \; \left(\,c + d\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,6}}{\left(\,a + a\,Sec \left[\,e + f\,x\,\right]\,\right)^{\,3}} \; \mathrm{d}x$$

#### Optimal (type 3, 363 leaves, 9 steps):

Optimal (type 3, 363 leaves, 9 steps): 
$$\frac{d^3 \left(40 \, c^3 - 90 \, c^2 \, d + 78 \, c \, d^2 - 23 \, d^3\right) \, ArcTanh[Sin[e+fx]]}{2 \, a^3 \, f} - \frac{1}{15 \, a^3 \, f}$$

$$2 \, d \left(2 \, c^5 + 18 \, c^4 \, d + 107 \, c^3 \, d^2 - 472 \, c^2 \, d^3 + 456 \, c \, d^4 - 136 \, d^5\right) \, Tan[e+fx] - \frac{1}{30 \, a^3 \, f}$$

$$d^2 \left(4 \, c^4 + 36 \, c^3 \, d + 216 \, c^2 \, d^2 - 626 \, c \, d^3 + 345 \, d^4\right) \, Sec[e+fx] \, Tan[e+fx] - \frac{d \left(2 \, c^3 + 18 \, c^2 \, d + 111 \, c \, d^2 - 136 \, d^3\right) \, \left(c + d \, Sec[e+fx]\right)^2 \, Tan[e+fx]}{15 \, a^3 \, f} + \frac{\left(c-d\right) \, \left(2 \, c^2 + 18 \, c \, d + 115 \, d^2\right) \, \left(c + d \, Sec[e+fx]\right)^3 \, Tan[e+fx]}{15 \, f \left(a^3 + a^3 \, Sec[e+fx]\right)} + \frac{\left(c-d\right) \, \left(c + d \, Sec[e+fx]\right)^5 \, Tan[e+fx]}{15 \, a \, f \left(a + a \, Sec[e+fx]\right)^2} + \frac{\left(c-d\right) \, \left(c + d \, Sec[e+fx]\right)^5 \, Tan[e+fx]}{5 \, f \left(a + a \, Sec[e+fx]\right)^3}$$

#### Result (type 3, 1338 leaves):

$$\left( 4 \left( -40 \, c^3 \, d^3 + 90 \, c^2 \, d^4 - 78 \, c \, d^5 + 23 \, d^6 \right) \, \text{Cos} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \\ \text{Cos} \left[ e + f \, x \right]^3 \, \text{Log} \left[ \text{Cos} \left[ \frac{e}{2} + \frac{f \, x}{2} \right] - \text{Sin} \left[ \frac{e}{2} + \frac{f \, x}{2} \right] \right] \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^6 \right) / \\ \left( f \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^6 \, \left( a + a \, \text{Sec} \left[ e + f \, x \right] \right)^3 \right) - \left( 4 \, \left( -40 \, c^3 \, d^3 + 90 \, c^2 \, d^4 - 78 \, c \, d^5 + 23 \, d^6 \right) \right) \\ \text{Cos} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^6 \, \text{Cos} \left[ e + f \, x \right]^3 \, \text{Log} \left[ \text{Cos} \left[ \frac{e}{2} + \frac{f \, x}{2} \right] + \text{Sin} \left[ \frac{e}{2} + \frac{f \, x}{2} \right] \right] \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^6 \right) / \\ \left( f \left( d + c \, \text{Cos} \left[ e + f \, x \right] \right)^6 \, \left( a + a \, \text{Sec} \left[ e + f \, x \right] \right)^3 \right) + \\ \left( 2 \, \text{Cos} \left[ \frac{e}{2} + \frac{f \, x}{2} \right]^2 \, \text{Cos} \left[ e + f \, x \right]^3 \, \text{Sec} \left[ \frac{e}{2} \right] \, \left( c + d \, \text{Sec} \left[ e + f \, x \right] \right)^6 \, \left( c^6 \, \text{Sin} \left[ \frac{e}{2} \right] - 6 \, c^5 \, d \, \text{Sin} \left[ \frac{e}{2} \right] \right) + \\ 15 \, c^4 \, d^2 \, \text{Sin} \left[ \frac{e}{2} \right] - 20 \, c^3 \, d^3 \, \text{Sin} \left[ \frac{e}{2} \right] + 15 \, c^2 \, d^4 \, \text{Sin} \left[ \frac{e}{2} \right] - 6 \, c \, d^5 \, \text{Sin} \left[ \frac{e}{2} \right] + d^6 \, \text{Sin} \left[ \frac{e}{2} \right] \right) \right) / \\ \right)$$

$$\left(5 f \left(d + c \cos[e + f x)\right)^{6} \left(a + a \operatorname{Sec}[e + f x]\right)^{3}\right) + \\ \left(8 \cos\left[\frac{e}{2} + \frac{f x}{2}\right]^{4} \cos[e + f x]^{3} \sec\left[\frac{e}{2}\right] \left(c + d \operatorname{Sec}[e + f x]\right)^{6} \left(-4 c^{6} \sin\left[\frac{e}{2}\right] + 9 c^{5} d \sin\left[\frac{e}{2}\right] + \\ 15 c^{4} d^{2} \sin\left[\frac{e}{2}\right] - 70 c^{3} d^{3} \sin\left[\frac{e}{2}\right] + 90 c^{2} d^{4} \sin\left[\frac{e}{2}\right] - 51 c d^{5} \sin\left[\frac{e}{2}\right] + 11 d^{6} \sin\left[\frac{e}{2}\right]\right) \right) \right/ \\ \left(15 f \left(d + c \cos[e + f x]\right)^{6} \left(a + a \operatorname{Sec}[e + f x]\right)^{3}\right) + \\ \left(2 \cos\left[\frac{e}{2} + \frac{f x}{2}\right] \cos[e + f x]^{3} \operatorname{Sec}\left[\frac{e}{2}\right] \left(c + d \operatorname{Sec}[e + f x]\right)^{6} \left(c^{6} \sin\left[\frac{f x}{2}\right] + 11 d^{6} \sin\left[\frac{f x}{2}\right]\right) \right) \right/ \\ \left(15 f \left(d + c \cos[e + f x]\right)^{6} \left(a + a \operatorname{Sec}[e + f x]\right)^{6} \left(c^{6} \sin\left[\frac{f x}{2}\right] - 6 c d^{5} \sin\left[\frac{f x}{2}\right] + d^{6} \sin\left[\frac{f x}{2}\right]\right) \right) \right/ \\ \left(15 f \left(d + c \cos[e + f x]\right)^{6} \left(a + a \operatorname{Sec}[e + f x]\right)^{3}\right) + \\ \left(15 f \left(d + c \cos[e + f x]\right)^{6} \left(a + a \operatorname{Sec}[e + f x]\right)^{3}\right) + \\ \left(15 f \left(d + c \cos[e + f x]\right)^{6} \left(a + a \operatorname{Sec}\left[e + f x]\right)^{3}\right) + \\ \left(15 f \left(d + c \cos\left[\frac{f x}{2}\right] + 9c^{5} d \sin\left[\frac{f x}{2}\right] + 15c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 51c d^{5} \sin\left[\frac{f x}{2}\right] + 11 d^{6} \sin\left[\frac{f x}{2}\right] + 15c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 51c d^{5} \sin\left[\frac{f x}{2}\right] + 11 d^{6} \sin\left[\frac{f x}{2}\right] + 15c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 51c d^{5} \sin\left[\frac{f x}{2}\right] + 11d^{6} \sin\left[\frac{f x}{2}\right] + 15c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 51c d^{5} \sin\left[\frac{f x}{2}\right] + 11d^{6} \sin\left[\frac{f x}{2}\right] + 15c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 51c d^{5} \sin\left[\frac{f x}{2}\right] + 11d^{6} \sin\left[\frac{f x}{2}\right] + 13c^{4} d^{2} \sin\left[\frac{f x}{2}\right] - 70c^{3} d^{3} \sin\left[\frac{f x}{2}\right] + 90c^{2} d^{4} \sin\left[\frac{f x}{2}\right] - 61c d^{5} \sin\left[\frac{f x}{2}\right] + 12c^{4} d^{5} \sin\left[\frac{f x}{$$

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,] \ \left(\,c + d\,\mathsf{Sec}\,[\,e + f\,x\,]\,\,\right)^{\,3}}{\left(\,a + a\,\mathsf{Sec}\,[\,e + f\,x\,]\,\,\right)^{\,3}} \,\,\mathrm{d}x$$

Optimal (type 3, 133 leaves, 6 steps):

$$\frac{ \, d^3 \, Arc Tanh \, [Sin \, [e+f \, x] \, ] }{ \, a^3 \, f } \, + \, \frac{ \left( \, c-d \right) \, \left( \, c+d \, Sec \, [e+f \, x] \, \right)^2 \, Tan \, [e+f \, x] }{ \, 5 \, f \, \left( \, a+a \, Sec \, [e+f \, x] \, \right)^3 } \, + \\ \left( \, \left( \, c-d \right) \, \left( \, 2 \, \left( \, 2 \, c^2 + 8 \, c \, d + 11 \, d^2 \right) \, + \, \left( \, 2 \, c^2 + 11 \, c \, d + 29 \, d^2 \right) \, Sec \, [e+f \, x] \, \right) \, Tan \, [e+f \, x] \, \right) \, \left( \, 15 \, a\, f \, \left( \, a+a \, Sec \, [e+f \, x] \, \right)^2 \right)$$

Result (type 3, 295 leaves):

$$\begin{split} &\frac{1}{30\,a^3\,f\left(1+\text{Cos}\left[e+f\,x\right]\right)^3}\left(-\,240\,d^3\,\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]^6\right.\\ &\left.\left(\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]-\text{Sin}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]-\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\right]\right)+\\ &\left.\left(c-d\right)\,\text{Cos}\left[\frac{1}{2}\,\left(e+f\,x\right)\right]\,\text{Sec}\left[\frac{e}{2}\right]\right.\\ &\left.\left(5\,\left(8\,c^2+17\,c\,d+29\,d^2\right)\,\text{Sin}\left[\frac{f\,x}{2}\right]-15\,\left(2\,c^2+5\,c\,d+5\,d^2\right)\,\text{Sin}\left[e+\frac{f\,x}{2}\right]+20\,c^2\,\text{Sin}\left[e+\frac{3\,f\,x}{2}\right]+\\ &\left.65\,c\,d\,\text{Sin}\left[e+\frac{3\,f\,x}{2}\right]+95\,d^2\,\text{Sin}\left[e+\frac{3\,f\,x}{2}\right]-15\,c^2\,\text{Sin}\left[2\,e+\frac{3\,f\,x}{2}\right]-15\,c\,d\,\text{Sin}\left[2\,e+\frac{3\,f\,x}{2}\right]-\\ &\left.15\,d^2\,\text{Sin}\left[2\,e+\frac{3\,f\,x}{2}\right]+7\,c^2\,\text{Sin}\left[2\,e+\frac{5\,f\,x}{2}\right]+16\,c\,d\,\text{Sin}\left[2\,e+\frac{5\,f\,x}{2}\right]+22\,d^2\,\text{Sin}\left[2\,e+\frac{5\,f\,x}{2}\right]\right)\right) \end{split}$$

## Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec} [e+fx]}{\left(a+a\operatorname{Sec} [e+fx]\right)^3 \left(c+d\operatorname{Sec} [e+fx]\right)} \, dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$-\frac{2\,d^{3}\,ArcTanh\left[\frac{\sqrt{c-d}\,Tan\left[\frac{1}{2}\,(e+f\,x)\right]}{\sqrt{c+d}}\right]}{a^{3}\,\left(c-d\right)^{7/2}\,\sqrt{c+d}\,f}+\frac{Tan\left[\,e+f\,x\,\right]}{5\,\left(\,c-d\right)\,f\,\left(\,a+a\,Sec\left[\,e+f\,x\,\right]\,\right)^{3}}+\\ \frac{\left(\,2\,c-7\,d\right)\,Tan\left[\,e+f\,x\,\right]}{15\,a\,\left(\,c-d\right)^{\,2}\,f\,\left(\,a+a\,Sec\left[\,e+f\,x\,\right]\,\right)^{\,2}}+\frac{\left(\,2\,c^{2}-9\,c\,d+22\,d^{2}\right)\,Tan\left[\,e+f\,x\,\right]}{15\,\left(\,c-d\right)^{\,3}\,f\,\left(\,a^{3}+a^{3}\,Sec\left[\,e+f\,x\,\right]\,\right)}$$

Result (type 3, 345 leaves):

$$\frac{1}{30 \, \mathsf{a}^3 \, \left(\mathsf{c} - \mathsf{d}\right)^3 \, \mathsf{f} \, \left(1 + \mathsf{Cos}\left[\mathsf{e} + \mathsf{f} \, \mathsf{x}\right]\right)^3 } \\ \mathsf{Cos}\left[\frac{1}{2} \, \left(\mathsf{e} + \mathsf{f} \, \mathsf{x}\right)\,\right] \left(\left[480 \, \mathsf{d}^3 \, \mathsf{ArcTan}\left[\frac{\left(\mathrm{i} \, \mathsf{Cos}\left[\mathsf{e}\right] + \mathsf{Sin}\left[\mathsf{e}\right]\right) \, \left(\mathsf{c} \, \mathsf{Sin}\left[\mathsf{e}\right] + \left(-\mathsf{d} + \mathsf{c} \, \mathsf{Cos}\left[\mathsf{e}\right]\right) \, \mathsf{Tan}\left[\frac{\mathsf{f} \, \mathsf{x}}{2}\right]\right)}{\sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{\left(\mathsf{Cos}\left[\mathsf{e}\right] - \mathrm{i} \, \mathsf{Sin}\left[\mathsf{e}\right]\right)^2}}\right] \\ \mathsf{Cos}\left[\frac{1}{2} \, \left(\mathsf{e} + \mathsf{f} \, \mathsf{x}\right)\right]^5 \, \left(\mathrm{i} \, \mathsf{Cos}\left[\mathsf{e}\right] + \mathsf{Sin}\left[\mathsf{e}\right]\right) \right) \bigg/ \left(\sqrt{\mathsf{c}^2 - \mathsf{d}^2} \, \sqrt{\left(\mathsf{Cos}\left[\mathsf{e}\right] - \mathrm{i} \, \mathsf{Sin}\left[\mathsf{e}\right]\right)^2}\right) + \mathsf{Sec}\left[\frac{\mathsf{e}}{2}\right] \\ \left(5 \, \left(8 \, \mathsf{c}^2 - \mathsf{27} \, \mathsf{c} \, \mathsf{d} + \mathsf{37} \, \mathsf{d}^2\right) \, \mathsf{Sin}\left[\frac{\mathsf{f} \, \mathsf{x}}{2}\right] - \mathsf{15} \, \left(2 \, \mathsf{c}^2 - \mathsf{7} \, \mathsf{c} \, \mathsf{d} + \mathsf{9} \, \mathsf{d}^2\right) \, \mathsf{Sin}\left[\mathsf{e} + \frac{\mathsf{f} \, \mathsf{x}}{2}\right] + 20 \, \mathsf{c}^2 \, \mathsf{Sin}\left[\mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] - \\ \mathsf{75} \, \mathsf{c} \, \mathsf{d} \, \mathsf{Sin}\left[\mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] + \mathsf{115} \, \mathsf{d}^2 \, \mathsf{Sin}\left[\mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] - \mathsf{15} \, \mathsf{c}^2 \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] + \mathsf{45} \, \mathsf{c} \, \mathsf{d} \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] - \\ \mathsf{45} \, \mathsf{d}^2 \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{3 \, \mathsf{f} \, \mathsf{x}}{2}\right] + \mathsf{7} \, \mathsf{c}^2 \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{5 \, \mathsf{f} \, \mathsf{x}}{2}\right] - 2\mathsf{4} \, \mathsf{c} \, \mathsf{d} \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{5 \, \mathsf{f} \, \mathsf{x}}{2}\right] + \mathsf{32} \, \mathsf{d}^2 \, \mathsf{Sin}\left[2 \, \mathsf{e} + \frac{5 \, \mathsf{f} \, \mathsf{x}}{2}\right]\right) \right)$$

Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\left(a+a\operatorname{Sec}\left[e+f\,x\right]\right)^{3}\,\left(c+d\operatorname{Sec}\left[e+f\,x\right]\right)^{2}}\,\mathrm{d}x$$

Optimal (type 3, 288 leaves, 8 steps)

$$-\frac{2\,d^3\,\left(4\,c+3\,d\right)\,\text{ArcTanh}\Big[\frac{\sqrt{c-d}\,\,\text{Tan}\Big[\frac{1}{2}\,\left(e+f\,x\right)\Big]}{\sqrt{c+d}}\Big]}{\sigma^3\,\left(c-d\right)^{9/2}\,\left(c+d\right)^{3/2}\,f} + \frac{d\,\left(2\,c^3-12\,c^2\,d+43\,c\,d^2+72\,d^3\right)\,\text{Tan}\,[\,e+f\,x\,]}{15\,a^3\,\left(c-d\right)^4\,\left(c+d\right)\,f\,\left(c+d\,\text{Sec}\,[\,e+f\,x\,]\,\right)} + \frac{Tan\,[\,e+f\,x\,]}{5\,\left(c-d\right)\,f\,\left(a+a\,\text{Sec}\,[\,e+f\,x\,]\,\right)^3\,\left(c+d\,\text{Sec}\,[\,e+f\,x\,]\,\right)} + \frac{\left(2\,c-9\,d\right)\,\text{Tan}\,[\,e+f\,x\,]}{\left(2\,c-9\,d\right)\,\text{Tan}\,[\,e+f\,x\,]} + \frac{\left(2\,c^2-12\,c\,d+45\,d^2\right)\,\text{Tan}\,[\,e+f\,x\,]}{\left(2\,c^2-12\,c\,d+45\,d^2\right)\,\text{Tan}\,[\,e+f\,x\,]} + \frac{\left(2\,c^2-12\,c\,d+45\,d^2\right)\,\text{Tan}\,[\,e+f\,x\,]}{\left(2\,c^2-12\,c\,d+45\,d^2\right)\,\text{Tan}\,[\,$$

Result (type 3, 1772 leaves):

$$\begin{split} &\left(4\,c+3\,d\right)\,\text{Cos}\,\big[\frac{e}{2}+\frac{f\,x}{2}\big]^6\,\left(d+c\,\text{Cos}\,[e+f\,x]\,\right)^2 \\ &\text{Sec}\,[e+f\,x]^5\,\left(\left(16\,\dot{\mathbb{1}}\,d^3\,\text{ArcTan}\,\big[\text{Sec}\,\big[\frac{f\,x}{2}\big]\,\left(\frac{\text{Cos}\,[e]}{\sqrt{c^2-d^2}\,\,\sqrt{\text{Cos}\,[2\,e]-\dot{\mathbb{1}}\,\text{Sin}\,[2\,e]}}\right. - \right. \\ &\left. -\frac{\dot{\mathbb{1}}\,\text{Sin}\,[e]}{\sqrt{c^2-d^2}\,\,\sqrt{\text{Cos}\,[2\,e]-\dot{\mathbb{1}}\,\text{Sin}\,[2\,e]}}\right)\left(-\dot{\mathbb{1}}\,d\,\text{Sin}\,\big[\frac{f\,x}{2}\big] + \dot{\mathbb{1}}\,c\,\text{Sin}\,\big[e+\frac{f\,x}{2}\big]\right)\right]\,\text{Cos}\,[e]\right) \bigg/ \\ &\left(\sqrt{c^2-d^2}\,\,f\,\sqrt{\text{Cos}\,[2\,e]-\dot{\mathbb{1}}\,\text{Sin}\,[2\,e]}}\right) + \left(16\,d^3\,\text{ArcTan}\,\big[\text{Sec}\,\big[\frac{f\,x}{2}\big]\right] \end{split}$$

$$\left(\frac{\cos[e]}{\sqrt{c^2-d^2}} \cdot \sqrt{\cos[2e] - i \sin[2e]} - \frac{i \sin[e]}{\sqrt{c^2-d^2}} \cdot \sqrt{\cos[2e] - i \sin[2e]} \right)$$

$$\left(-i d \sin\left[\frac{fx}{2}\right] + i c \sin\left[e + \frac{fx}{2}\right]\right) \right]$$

$$\sin[e] \left/ \left/ \left(\sqrt{c^2-d^2} \cdot f \sqrt{\cos[2e] - i \sin[2e]} \right) \right) \right) \right/$$

$$\left(\left(-c + d\right)^4 \cdot \left(c + d\right) \cdot \left(a + a \sec[e + fx]\right)^3 \cdot \left(c + d \sec[e + fx]\right)^2\right) + \frac{1}{1200 \cdot \left(-c + d\right)^4 \cdot \left(c + d\right) \cdot f \cdot \left(a + a \sec[e + fx]\right)^3 \cdot \left(c + d \sec[e + fx]\right)^2\right) + \frac{1}{1200 \cdot \left(-c + d\right)^4 \cdot \left(c + d\right) \cdot f \cdot \left(a + a \sec[e + fx]\right)^3 \cdot \left(c + d \sec[e + fx]\right)^2}$$

$$\cos\left[\frac{e}{2} + \frac{fx}{2}\right]$$

$$\left(d + c \cos[e + fx]\right) \cdot \sec\left[\frac{e}{2}\right] \cdot \sec[e]$$

$$\sec[e + fx]^5$$

$$\left(-55 \cdot c^3 \sin\left[\frac{fx}{2}\right] + 135 \cdot c^4 \cdot d \sin\left[\frac{fx}{2}\right] - 20 \cdot c^3 \cdot d^2 \cdot \sin\left[\frac{fx}{2}\right] - 810 \cdot c^2 \cdot d^3 \cdot \sin\left[\frac{fx}{2}\right] - 450 \cdot c \cdot d^4 \cdot \sin\left[\frac{fx}{2}\right] + 150 \cdot d^5 \cdot \sin\left[\frac{fx}{2}\right] + 47 \cdot c^5 \cdot \sin\left[\frac{3fx}{2}\right] - 137 \cdot c^4 \cdot d \cdot \sin\left[\frac{3fx}{2}\right] + 88 \cdot c^3 \cdot d^3 \cdot \sin\left[\frac{3fx}{2}\right] + 812 \cdot c^2 \cdot d^3 \cdot \sin\left[\frac{3fx}{2}\right] + 690 \cdot c^4 \cdot \sin\left[\frac{3fx}{2}\right] + 765 \cdot \sin\left[\frac{3fx}{2}\right] - 990 \cdot c^4 \cdot \sin\left[e - \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e - \frac{fx}{2}\right] + 1030 \cdot c^2 \cdot d^3 \cdot \sin\left[e - \frac{fx}{2}\right] - 990 \cdot c^4 \cdot \sin\left[e - \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e - \frac{fx}{2}\right] + 1030 \cdot c^2 \cdot d^3 \cdot \sin\left[e + \frac{fx}{2}\right] + 765 \cdot c \cdot d^4 \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^2 \cdot d^3 \cdot \sin\left[e + \frac{fx}{2}\right] - 675 \cdot c \cdot d^4 \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^2 \cdot d^3 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^2 \cdot d^3 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^4 \cdot d \cdot \sin\left[e + \frac{fx}{2}\right] - 150 \cdot d^5 \cdot \sin\left[e + \frac{fx}{2}\right] - 810 \cdot c^$$

$$20 c^{5} Sin \left[e + \frac{5 f x}{2}\right] - 76 c^{4} d Sin \left[e + \frac{5 f x}{2}\right] + 106 c^{3} d^{2} Sin \left[e + \frac{5 f x}{2}\right] + \\ 346 c^{2} d^{3} Sin \left[e + \frac{5 f x}{2}\right] + 219 c d^{4} Sin \left[e + \frac{5 f x}{2}\right] + 15 d^{5} Sin \left[e + \frac{5 f x}{2}\right] - \\ 15 c^{5} Sin \left[2 e + \frac{5 f x}{2}\right] + 45 c^{4} d Sin \left[2 e + \frac{5 f x}{2}\right] - 30 c^{3} d^{2} Sin \left[2 e + \frac{5 f x}{2}\right] - \\ 90 c^{2} d^{3} Sin \left[2 e + \frac{5 f x}{2}\right] + 75 c d^{4} Sin \left[2 e + \frac{5 f x}{2}\right] + 15 d^{5} Sin \left[2 e + \frac{5 f x}{2}\right] + \\ 20 c^{5} Sin \left[3 e + \frac{5 f x}{2}\right] - 76 c^{4} d Sin \left[3 e + \frac{5 f x}{2}\right] + 106 c^{3} d^{2} Sin \left[3 e + \frac{5 f x}{2}\right] + \\ 346 c^{2} d^{3} Sin \left[3 e + \frac{5 f x}{2}\right] + 144 c d^{4} Sin \left[3 e + \frac{5 f x}{2}\right] - 15 d^{5} Sin \left[3 e + \frac{5 f x}{2}\right] - \\ 15 c^{5} Sin \left[4 e + \frac{5 f x}{2}\right] + 45 c^{4} d Sin \left[4 e + \frac{5 f x}{2}\right] - 30 c^{3} d^{2} Sin \left[4 e + \frac{5 f x}{2}\right] - \\ 90 c^{2} d^{3} Sin \left[4 e + \frac{5 f x}{2}\right] - 15 d^{5} Sin \left[4 e + \frac{5 f x}{2}\right] + 7 c^{5} Sin \left[2 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[2 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[3 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] - \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] + \\ 27 c^{4} d Sin \left[4 e + \frac{7 f x}{2}\right] + 38 c^{3} d^{2} Sin \left[4 e + \frac{7 f x}{2}\right] + 7 c^{5} Sin \left[4 e + \frac{7 f x}{2}\right] + \\ 27 c^$$

Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]}{\left(\mathsf{a} + \mathsf{a}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3\,\left(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^3}\,\,\mathrm{d}x$$

Optimal (type 3, 368 leaves, 9 steps):

$$\frac{d^{3}\left(20\,c^{2}+30\,c\,d+13\,d^{2}\right)\,ArcTanh\left[\frac{\sqrt{c-d}\,Tan\left[\frac{1}{2}\,\left(e+f\,x\right)\right]}{\sqrt{c+d}}\right]}{a^{3}\left(c-d\right)^{11/2}\left(c+d\right)^{5/2}\,f}+\\ \frac{d\,\left(4\,c^{3}-30\,c^{2}\,d+146\,c\,d^{2}+195\,d^{3}\right)\,Tan\left[e+f\,x\right]}{30\,a^{3}\left(c-d\right)^{4}\left(c+d\right)\,f\left(c+d\,Sec\left[e+f\,x\right]\right)^{2}}+\\ \frac{Tan\left[e+f\,x\right]}{5\left(c-d\right)\,f\left(a+a\,Sec\left[e+f\,x\right]\right)^{3}\left(c+d\,Sec\left[e+f\,x\right]\right)^{2}}+\\ \frac{\left(2\,c-11\,d\right)\,Tan\left[e+f\,x\right]}{15\,a\,\left(c-d\right)^{2}\,f\left(a+a\,Sec\left[e+f\,x\right]\right)^{2}\left(c+d\,Sec\left[e+f\,x\right]\right)^{2}}+\\ \frac{\left(2\,c^{2}-15\,c\,d+76\,d^{2}\right)\,Tan\left[e+f\,x\right]}{15\left(c-d\right)^{3}\,f\left(a^{3}+a^{3}\,Sec\left[e+f\,x\right]\right)\left(c+d\,Sec\left[e+f\,x\right]\right)^{2}}+\\ \frac{d\,\left(4\,c^{4}-30\,c^{3}\,d+142\,c^{2}\,d^{2}+525\,c\,d^{3}+304\,d^{4}\right)\,Tan\left[e+f\,x\right]}{30\,a^{3}\,\left(c-d\right)^{5}\left(c+d\right)^{2}\,f\left(c+d\,Sec\left[e+f\,x\right]\right)}$$

Result (type 3, 1096 leaves):

$$\left( 4 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^4 \left( d + c \cos \left[ e + f x \right] \right)^3 \sec \left[ \frac{e}{2} \right] \sec \left[ e + f x \right]^6 \left( -8 c \sin \left[ \frac{e}{2} \right] + 23 d \sin \left[ \frac{e}{2} \right] \right) \right) \right/ \\ \left( 15 \left( -c + d \right)^4 f \left( a + a \sec \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) + \\ \left( 20 c^2 + 30 c d + 13 d^2 \right) \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^6 \left[ d + c \cos \left[ e + f x \right] \right]^3 \sec \left[ e + f x \right]^6 \\ \left( - \left( \left[ 8 \text{ i d}^3 \text{ ArcTan} \left[ \sec \left[ \frac{f x}{2} \right] \left( \frac{\cos \left[ e \right]}{\sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]}} \right) - \left[ d \sin \left[ \frac{f x}{2} \right] + i c \sin \left[ e + \frac{f x}{2} \right] \right) \right] \cos \left[ e \right] \right) \right/ \\ \left( \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) - \left[ 8 d^3 \text{ ArcTan} \left[ \sec \left[ \frac{f x}{2} \right] \right] \right] \cos \left[ e \right] \right) \right/ \\ \left( \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) - \left[ 8 d^3 \text{ ArcTan} \left[ \sec \left[ \frac{f x}{2} \right] \right] \right] \cos \left[ e \right] \right) \right/ \\ \left( \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) - \left[ 8 d^3 \text{ ArcTan} \left[ \sec \left[ \frac{f x}{2} \right] \right] \right] \cos \left[ e \right] \right) \right/ \\ \left( \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) - \left[ 8 d^3 \text{ ArcTan} \left[ \sec \left[ \frac{f x}{2} \right] \right] \right) \right/ \\ \left( \left( -c + d \right)^3 \left( c + d \sec \left[ e + f x \right] \right) \right) \sin \left[ e \right] \right) \left/ \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) \right/ \\ \left( \left( -c + d \right)^3 \left( c + d \sec \left[ e + f x \right] \right) \right) \sin \left[ e \right] \right) \left/ \sqrt{c^2 - d^2} \sqrt{\cos \left[ 2 e \right] - i \sin \left[ 2 e \right]} \right) \right) \right/ \\ \left( \left( -c + d \right)^3 \left( c + d \sec \left[ e + f x \right] \right) \right) \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 2 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right] \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 4 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right] \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 8 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^5 \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 8 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^5 \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 8 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^6 \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 8 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^6 \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right] \right)^3 \right) - \\ \left( 6 \cos \left[ \frac{e}{2} + \frac{f x}{2} \right]^6 \left( d + c \cos \left[ e + f x \right] \right)^3 \left( c + d \sec \left[ e + f x \right$$

Problem 239: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(g \operatorname{Sec}\left[e + f x\right]\right)^{3/2} \sqrt{a + a \operatorname{Sec}\left[e + f x\right]}}{c + d \operatorname{Sec}\left[e + f x\right]} \, dx$$

Optimal (type 3, 149 leaves, 5 steps):

$$\frac{2\,\sqrt{a}\,\,g^{3/2}\,\text{ArcTanh}\Big[\,\frac{\sqrt{a}\,\,\sqrt{g}\,\,\text{Tan}[\,e+f\,x\,]}{\sqrt{g\,\text{Sec}\,[\,e+f\,x\,]}\,\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}}\,\Big]}{d\,\,f}\\\\ 2\,\sqrt{a}\,\,\sqrt{c}\,\,g^{3/2}\,\,\text{ArcTanh}\Big[\,\frac{\sqrt{a}\,\,\sqrt{c}\,\,\sqrt{g}\,\,\text{Tan}[\,e+f\,x\,]}{\sqrt{c+d}\,\,\sqrt{g\,\text{Sec}\,[\,e+f\,x\,]}}\,\sqrt{a+a\,\text{Sec}\,[\,e+f\,x\,]}}\,\Big]}\\ d\,\sqrt{c+d}\,\,f$$

Result (type 3, 427 leaves):

$$\begin{split} \frac{1}{4\left(\text{i}+\sqrt{2}\right)\,\text{d}\,\sqrt{\text{c}+\text{d}}\,\,\text{f}\,\sqrt{\text{g}\,\text{Sec}\,[\text{e}+\text{f}\,\text{x})}} \\ &\left(-2\,\text{i}+\sqrt{2}\right)\,\text{g}^2\left(2\,\sqrt{\text{c}+\text{d}}\,\,\text{ArcTan}\Big[\frac{\text{Cos}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]-\left(-1+\sqrt{2}\right)\,\text{Sin}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\left(1+\sqrt{2}\right)\,\text{Cos}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]-\text{Sin}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right]+\\ &2\,\sqrt{\text{c}+\text{d}}\,\,\text{ArcTan}\Big[\frac{\text{Cos}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]-\left(1+\sqrt{2}\right)\,\text{Sin}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\left(-1+\sqrt{2}\right)\,\text{Cos}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]-\text{Sin}\left[\frac{1}{4}\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right]+\\ &i\left(2\,\sqrt{\text{c}+\text{d}}\,\,\text{Log}\Big[\sqrt{2}\,+2\,\text{Sin}\Big[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]\right]-\\ &\sqrt{\text{c}+\text{d}}\,\,\text{Log}\Big[2-\sqrt{2}\,\,\text{Cos}\left[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]\right]-\\ &\sqrt{\text{c}+\text{d}}\,\,\text{Log}\Big[2+\sqrt{2}\,\,\text{Cos}\left[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]-\sqrt{2}\,\,\text{Sin}\Big[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]\Big]+2\,\sqrt{\text{c}}\\ &\text{Log}\Big[\sqrt{2}\,\,\sqrt{\text{c}+\text{d}}\,-2\,\sqrt{\text{c}}\,\,\text{Sin}\Big[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]\Big]-\\ &2\,\sqrt{\text{c}}\,\,\text{Log}\Big[\sqrt{2}\,\,\sqrt{\text{c}+\text{d}}\,+2\,\sqrt{\text{c}}\,\,\text{Sin}\Big[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\right]\Big]\Big)\right)\,\text{Sec}\Big[\frac{1}{2}\left(\text{e}+\text{f}\,\text{x}\right)\Big]\,\sqrt{\text{a}\,\,\big(1+\text{Sec}\,[\text{e}+\text{f}\,\text{x}]\big)}} \end{split}$$

Problem 243: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\left(g\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{5/2}}{\sqrt{\mathsf{a}+\mathsf{a}\,\mathsf{Sec}\,[\,e+f\,x\,]\,}\,\left(\,c+d\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)}\,\,\mathrm{d}x$$

Optimal (type 3, 231 leaves, 8 steps):

$$\frac{2\,g^{5/2}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{g}\,\,\text{Tan}[e+f\,x]}{\sqrt{g\,\,\text{Sec}\,[e+f\,x]}\,\,\sqrt{a+a\,\,\text{Sec}\,[e+f\,x]}}\Big]}{\sqrt{a}\,\,d\,\,f} + \frac{\sqrt{2}\,\,g^{5/2}\,\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\,\sqrt{g}\,\,\text{Tan}\,[e+f\,x]}{\sqrt{2}\,\,\sqrt{g\,\,\text{Sec}\,[e+f\,x]}\,\,\sqrt{a+a\,\,\text{Sec}\,[e+f\,x]}}\Big]}{\sqrt{a}\,\,\left(\,c-d\right)\,\,f} - \frac{2\,c^{3/2}\,g^{5/2}\,\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\,\sqrt{c}\,\,\sqrt{g}\,\,\text{Tan}\,[e+f\,x]}{\sqrt{c+d}\,\,\sqrt{g\,\,\text{Sec}\,[e+f\,x]}\,\,\sqrt{a+a\,\,\text{Sec}\,[e+f\,x]}}\Big]}{\sqrt{a}\,\,\left(\,c-d\right)\,\,d\,\,\sqrt{c+d}\,\,\,f}$$

Result (type 3, 1097 leaves):

$$\frac{1}{2\left(i+\sqrt{2}\right)d\left(-c+d\right)\sqrt{c+d}} \frac{1}{\sqrt{a}\left(1+Sec\left[e+fx\right]\right)} \frac{1}{2}e^{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]}{\left(-2\left(-2\,i+\sqrt{2}\right)\left(c-d\right)\sqrt{c+d}} \frac{1}{Arctan}\left[\frac{\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\left(-1+\sqrt{2}\right)\sin\left[\frac{1}{4}\left(e+fx\right)\right]}{\left(1+\sqrt{2}\right)\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\sin\left[\frac{1}{4}\left(e+fx\right)\right]}\right] - \\ 2\left(-2\,i+\sqrt{2}\right)\left(c-d\right)\sqrt{c+d} \frac{1}{Arctan}\left[\frac{\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\left(1+\sqrt{2}\right)\sin\left[\frac{1}{4}\left(e+fx\right)\right]}{\left(-1+\sqrt{2}\right)\sin\left[\frac{1}{4}\left(e+fx\right)\right]}\right] + \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\sin\left[\frac{1}{4}\left(e+fx\right)\right]\right] + \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\sin\left[\frac{1}{4}\left(e+fx\right)\right]\right] - \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\cos\left[\frac{1}{4}\left(e+fx\right)\right]-\sin\left[\frac{1}{4}\left(e+fx\right)\right]\right] - \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\cos\left[\frac{1}{4}\left(e+fx\right)\right]+\sin\left[\frac{1}{4}\left(e+fx\right)\right]\right] - \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\sqrt{2}+2\sin\left[\frac{1}{2}\left(e+fx\right)\right]+\sin\left[\frac{1}{4}\left(e+fx\right)\right]\right] - \\ 4\,\frac{1}{2}d\sqrt{c+d} \log\left[\sqrt{2}+2\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - 2\,\frac{1}{2}\sqrt{2}\sqrt{c+d} \log\left[\sqrt{2}+2\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2-\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2-\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2-\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{2}\sqrt{c+d} \log\left[2+\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]-\sqrt{2}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]-2\,\frac{1}{2}\sqrt{2}\cos\left[\frac{1}{2}\left(e+fx\right)\right]\right] - \\ 2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right]-2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right]-2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right]-2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right] + \\ 2\,\frac{1}{2}\sqrt{2}\cos\left[\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left[\frac{1}{2}\left(e+fx\right)\right]\right]$$

## Problem 245: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+bSec[e+fx]) (c+dSec[e+fx])^{3} dx$$

#### Optimal (type 3, 180 leaves, 7 steps):

$$\frac{\left(8 \text{ a } \text{c}^3 + 12 \text{ b } \text{c}^2 \text{ d} + 12 \text{ a } \text{c } \text{d}^2 + 3 \text{ b } \text{d}^3\right) \text{ ArcTanh} [\text{Sin} [\text{e} + \text{f} \text{x}]]}{8 \text{ f}} + \frac{\left(4 \text{ a } \text{d} \left(4 \text{ c}^2 + \text{d}^2\right) + 3 \text{ b } \left(\text{c}^3 + 4 \text{ c } \text{d}^2\right)\right) \text{ Tan} [\text{e} + \text{f} \text{x}]}{6 \text{ f}} + \frac{\text{d} \left(6 \text{ b } \text{c}^2 + 20 \text{ a } \text{c } \text{d} + 9 \text{ b } \text{d}^2\right) \text{ Sec} [\text{e} + \text{f} \text{x}] \text{ Tan} [\text{e} + \text{f} \text{x}]}{24 \text{ f}} + \frac{\left(3 \text{ b } \text{c} + 4 \text{ a } \text{d}\right) \left(\text{c} + \text{d Sec} [\text{e} + \text{f} \text{x}]\right)^2 \text{ Tan} [\text{e} + \text{f} \text{x}]}{12 \text{ f}} + \frac{\text{b} \left(\text{c} + \text{d Sec} [\text{e} + \text{f} \text{x}]\right)^3 \text{ Tan} [\text{e} + \text{f} \text{x}]}{4 \text{ f}}$$

#### Result (type 3, 1179 leaves):

$$\left( \left( -8\,a\,c^3 - 12\,b\,c^2\,d - 12\,a\,c\,d^2 - 3\,b\,d^3 \right) \, Cos\left[ e + f\,x \right]^4 \, Log\left[ Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] - Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right] \right) \\ \left( \left( 8\,a\,c^3 + 12\,b\,c^2\,d + 12\,a\,c\,d^2 + 3\,b\,d^3 \right) \, Cos\left[ e + f\,x \right]^4 \, Log\left[ Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \, \left( d + c\,Cos\left[ e + f\,x \right] \, \right)^3 \right) + \left( \left( 8\,a\,c^3 + 12\,b\,c^2\,d + 12\,a\,c\,d^2 + 3\,b\,d^3 \right) \, Cos\left[ e + f\,x \right]^4 \, Log\left[ Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] + Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right] \right) \\ \left( \left( 8\,a\,c^3 + 12\,b\,c^2\,d + 12\,a\,c\,d^2 + 3\,b\,d^3 \right) \, Cos\left[ e + f\,x \right]^4 \, Log\left[ Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] + Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right) \right) \\ \left( \left( 3\,a\,b\,c^3 + 12\,b\,c^2\,d + 12\,a\,c\,d^2 + 3\,b\,d^3 \right) \, Cos\left[ e + f\,x \right] \right)^3 \right) \left( \left( 4\,a\,b\,Sec\left[ e + f\,x \right] \right) \, \left( 4\,b\,Sec\left[ e + f\,x \right] \right)^3 \right) \right) \\ \left( \left( 4\,b\,d^3\,Cos\left[ e + f\,x \right]^4 \, \left( a + b\,Sec\left[ e + f\,x \right] \right) \, \left( c + d\,Sec\left[ e + f\,x \right] \right)^3 \, Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] - Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right)^4 \right) + \\ \left( \left( 36\,b\,c^2\,d + 36\,a\,c\,d^2 + 12\,b\,c\,d^2 + 4\,a\,d^3 + 9\,b\,d^3 \right) \, Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] - Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right)^2 \right) - \\ \left( b\,d^3\,Cos\left[ e + f\,x \right] \, \left( d + c\,Cos\left[ e + f\,x \right] \right) \, \left( c + d\,Sec\left[ e + f\,x \right] \right)^3 \right) \right) \\ \left( \left( 4\,b\,d^3\,Cos\left[ e + f\,x \right] \, \right) \, \left( d + c\,Cos\left[ e + f\,x \right] \right)^3 \, \left( Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] + Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right)^2 \right) - \\ \left( b\,d^3\,Cos\left[ e + f\,x \right] \, \left( d + c\,Cos\left[ e + f\,x \right] \right) \, \left( c + d\,Sec\left[ e + f\,x \right] \right)^3 \right) \right) \\ \left( \left( 16\,f \, \left( b + a\,Cos\left[ e + f\,x \right] \right) \, \left( d + c\,Cos\left[ e + f\,x \right] \right)^3 \, \left( Cos\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] + Sin\left[ \frac{1}{2} \, \left( e + f\,x \right) \, \right] \right)^4 \right) + \\ \left( \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3 \right) \\ \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3 \right) \\ \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3 \right) \\ \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3 \right) \\ \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3 \right) \\ \left( -36\,b\,c^2\,d - 36\,a\,c\,d^2 - 12\,b\,c\,d^2 - 4\,a\,d^3 - 9\,b\,d^3$$

$$\left( \text{6f} \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] - \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right)^{3} \right) + \\ \left( \cos \left[ e + f x \right]^{4} \left( a + b \sec \left[ e + f x \right] \right) \left( c + d \sec \left[ e + f x \right] \right)^{3} \right) \\ \left( 3 b c d^{2} \sin \left[ \frac{1}{2} \left( e + f x \right) \right] + a d^{3} \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) / \\ \left( 6 f \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right)^{3} \right) + \\ \left( \cos \left[ e + f x \right]^{4} \left( a + b \sec \left[ e + f x \right] \right) \left( c + d \sec \left[ e + f x \right] \right)^{3} \left( 3 b c^{3} \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) + \\ \left( 3 f \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] - \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) / \\ \left( 3 f \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] - \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) / \\ \left( 3 f \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] + 2 a d^{3} \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) / \\ \left( 3 f \left( b + a \cos \left[ e + f x \right] \right) \left( d + c \cos \left[ e + f x \right] \right)^{3} \left( \cos \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) \right)$$

## Problem 246: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ e + f \, x \right] \, \left( a + b \, Sec \left[ e + f \, x \right] \right) \, \left( c + d \, Sec \left[ e + f \, x \right] \right)^2 \, \text{d} x$$

#### Optimal (type 3, 115 leaves, 6 steps):

$$\frac{\left(2\,b\,c\,d + a\,\left(2\,c^2 + d^2\right)\right)\,\mathsf{ArcTanh}\,[\mathsf{Sin}\,[e + f\,x]\,]}{2\,f} + \frac{2\,\left(3\,a\,c\,d + b\,\left(c^2 + d^2\right)\right)\,\mathsf{Tan}\,[e + f\,x]}{3\,f} + \frac{d\,\left(2\,b\,c + 3\,a\,d\right)\,\mathsf{Sec}\,[e + f\,x]\,\,\mathsf{Tan}\,[e + f\,x]}{6\,f} + \frac{b\,\left(c + d\,\mathsf{Sec}\,[e + f\,x]\right)^2\,\mathsf{Tan}\,[e + f\,x]}{3\,f}$$

#### Result (type 3, 239 leaves):

$$\begin{split} \frac{1}{6\,f}\,Sec\,[\,e+f\,x\,]^{\,3}\,\left(-\frac{9}{4}\,\left(2\,b\,c\,d+a\,\left(2\,c^{2}+d^{2}\right)\right)\,Cos\,[\,e+f\,x\,] \right. \\ \left. \left. \left(Log\big[Cos\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]-Sin\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]\,\right] - Log\big[Cos\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]\,\right) - \frac{3}{4}\,\left(2\,b\,c\,d+a\,\left(2\,c^{2}+d^{2}\right)\right)\,Cos\big[3\,\left(e+f\,x\right)\,\big] \\ \left. \left(Log\big[Cos\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]-Sin\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]\,\right] - Log\big[Cos\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(e+f\,x\right)\,\big]\,\right) + \left(3\,b\,c^{2}+6\,a\,c\,d+4\,b\,d^{2}+3\,d\,\left(2\,b\,c+a\,d\right)\,Cos\,[\,e+f\,x\,] + \left(3\,b\,c^{2}+6\,a\,c\,d+2\,b\,d^{2}\right)\,Cos\big[\,2\,\left(e+f\,x\right)\,\big]\,\right) \\ Sin\,[\,e+f\,x\,]\,\right) \end{split}$$

## Problem 247: Result more than twice size of optimal antiderivative.

$$\int Sec[e+fx] (a+bSec[e+fx]) (c+dSec[e+fx]) dx$$

Optimal (type 3, 61 leaves, 5 steps):

$$\frac{\left(2\,a\,c+b\,d\right)\,\mathsf{ArcTanh}\,[\,\mathsf{Sin}\,[\,e+f\,x\,]\,\,]}{2\,f}\,+\,\frac{\left(b\,c+a\,d\right)\,\mathsf{Tan}\,[\,e+f\,x\,]}{f}\,+\,\frac{b\,d\,\mathsf{Sec}\,[\,e+f\,x\,]\,\,\mathsf{Tan}\,[\,e+f\,x\,]}{2\,f}$$

Result (type 3, 164 leaves):

$$\begin{split} &\frac{1}{4\,\mathsf{f}} \left[ -2\,\left(2\,\mathsf{a}\,\mathsf{c} + \mathsf{b}\,\mathsf{d}\right)\,\mathsf{Log}\big[\mathsf{Cos}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] - \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] \right] + \\ &4\,\mathsf{a}\,\mathsf{c}\,\mathsf{Log}\big[\mathsf{Cos}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] \right] + \\ &2\,\mathsf{b}\,\mathsf{d}\,\mathsf{Log}\big[\mathsf{Cos}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] \right] + \frac{\mathsf{b}\,\mathsf{d}}{\left(\mathsf{Cos}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] - \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\right)^2} - \\ &\frac{\mathsf{b}\,\mathsf{d}}{\left(\mathsf{Cos}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big] + \mathsf{Sin}\big[\frac{1}{2}\,\left(\mathsf{e} + \mathsf{f}\,\mathsf{x}\right)\,\big]\right)^2} + 4\,\left(\mathsf{b}\,\mathsf{c} + \mathsf{a}\,\mathsf{d}\right)\,\mathsf{Tan}\,[\mathsf{e} + \mathsf{f}\,\mathsf{x}] \end{split}$$

## Problem 252: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(c+d\operatorname{Sec}[e+fx]\right)^{4}}{a+b\operatorname{Sec}[e+fx]} dx$$

Optimal (type 3, 247 leaves, 12 steps):

$$\frac{d^{3} \left(4 \ b \ c - a \ d\right) \ ArcTanh[Sin[e + f \ x]]}{2 \ b^{2} \ f} + \frac{d \left(2 \ b \ c - a \ d\right) \left(2 \ b^{2} \ c^{2} - 2 \ a \ b \ c \ d + a^{2} \ d^{2}\right) \ ArcTanh[Sin[e + f \ x]]}{b^{4} \ f} + \frac{d^{4} \ Tan[e + f \ x]}{b \ f} + \frac{d^{4} \ Tan[e + f \ x]}{b \ f} + \frac{d^{4} \ Tan[e + f \ x]}{b \ f} + \frac{d^{4} \ Tan[e + f \ x]}{3 \ b \ f} + \frac{d^{4} \ Tan[e + f \ x]}{3 \ b \ f}$$

Result (type 3, 1150 leaves):

$$-\left[\left(2\left(b\,c-a\,d\right)^4 Arc Tanh\left[\frac{(-a+b)\,Tan\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{a^2-b^2}}\right] \cos\left[e+f\,x\right]^3 \left(b+a\cos\left[e+f\,x\right]\right)\right] \\ + \left(\left(c+d\,\sec\left[e+f\,x\right]\right)^4\right] \bigg/ \left(b^4\sqrt{a^2-b^2}\,f\left(d+c\cos\left[e+f\,x\right]\right)^4 \left(a+b\,\sec\left[e+f\,x\right]\right)\right)\right) + \\ + \left(\left(-8\,b^3\,c^3\,d+12\,a\,b^2\,c^2\,d^2-8\,a^2\,b\,c\,d^3-4\,b^3\,c\,d^3+2\,a^3\,d^4+a\,b^2\,d^4\right) \cos\left[e+f\,x\right]^3 \\ + \left(b+a\cos\left[e+f\,x\right]\right) \log\left[\cos\left[\frac{1}{2}\left(e+f\,x\right)\right]-\sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right] \left(c+d\,\sec\left[e+f\,x\right]\right)^4\right] \bigg/ \\ + \left(2\,b^4\,f\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right)\right) + \\ + \left(\left(8\,b^3\,c^3\,d-12\,a\,b^2\,c^2\,d^2+8\,a^2\,b\,c\,d^3+4\,b^3\,c\,d^3-2\,a^3\,d^4-a\,b^2\,d^4\right) \cos\left[e+f\,x\right]^3 \\ + \left(b+a\cos\left[e+f\,x\right]\right) \log\left[\cos\left[\frac{1}{2}\left(e+f\,x\right]\right]+\sin\left[\frac{1}{2}\left(e+f\,x\right]\right]\right] \left(c+d\,\sec\left[e+f\,x\right]\right)^4\right] \bigg/ \\ + \left(2\,b^4\,f\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right) + \\ + \left(\left(12\,b\,c\,d^3-3\,a\,d^4+b\,d^4\right)\cos\left[e+f\,x\right]^3\left(b+a\cos\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right)^4\right) \bigg/ \\ + \left(12\,b^2\,f\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right)\left(\cos\left[\frac{1}{2}\left(e+f\,x\right]\right]-\sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)^2\right) + \\ + \left(d^4\cos\left[e+f\,x\right]^3\left(b+a\cos\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right)^4\right) \bigg/ \\ + \left(b^4\cos\left[e+f\,x\right]^3\left(b+a\cos\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right)^4\right) \bigg/ \\ + \left(b^4\cos\left[e+f\,x\right]^3\left(b+a\cos\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right)\left(a+b\,\sec\left[e+f\,x\right]\right)\right) \bigg/ \\ + \left(c^2b^2\,\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right)\left(\cos\left[\frac{1}{2}\left(e+f\,x\right)\right]+\sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right)^2\right) + \\ + \left((-12\,b\,c\,d^3+3\,a\,d^4-b\,d^4\right)\cos\left[e+f\,x\right]^3\left(b+a\cos\left[e+f\,x\right]\right)\left(\cos\left[\frac{1}{2}\left(e+f\,x\right)\right]+\sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right) \bigg/ \\ + \left(c^2b^2\,\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right) \right) \bigg/ \\ + \left(c^2b^2\,\left(d+c\cos\left[e+f\,x\right]\right)^4\left(a+b\,\sec\left[e+f\,x\right]\right)\left(c+d\,\sec\left[e+f\,x\right]\right) \left(a+b\,\sec\left[e+f\,x\right]\right) \bigg/ \\ + \left(a^2b^2\,\left(a+b^2\right)^2\left(a+b^2\right)^2\left(a+b^2\right)^2\right) \bigg/ \\ + \left(a^2b^2\,\left(a+b^2\right)^2\left(a+b^2\right)^2\left(a+b^2\right)^2\right) \bigg/ \\ + \left(a^2b^2\,\left(a+b^2\right)^2\left(a+b^2\right)^2\left(a+b^2\right)^2\right) \bigg/ \right) \bigg/ \\ + \left(a^2b^2\,\left(a+b^2\right)^2\left(a+b^2\right)^2\left(a+b^2\right)^2\left(a+b^2\right$$

## Problem 253: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(c+d\operatorname{Sec}[e+fx]\right)^{3}}{a+b\operatorname{Sec}[e+fx]} dx$$

Optimal (type 3, 170 leaves, 10 steps):

$$\frac{d^{3} \, ArcTanh \, [Sin \, [e+f \, x] \, ]}{2 \, b \, f} + \frac{d \, \left(3 \, b^{2} \, c^{2} - 3 \, a \, b \, c \, d + a^{2} \, d^{2}\right) \, ArcTanh \, [Sin \, [e+f \, x] \, ]}{b^{3} \, f} + \frac{2 \, \left(b \, c - a \, d\right)^{3} \, ArcTanh \left[\frac{\sqrt{a-b} \, Tan \left[\frac{1}{2} \, (e+f \, x) \, \right]}{\sqrt{a+b}} \, \right]}{\sqrt{a-b} \, b^{3} \, \sqrt{a+b} \, f} + \frac{d^{2} \, \left(3 \, b \, c - a \, d\right) \, Tan \, [e+f \, x]}{b^{2} \, f} + \frac{d^{3} \, Sec \, [e+f \, x] \, Tan \, [e+f \, x]}{2 \, b \, f}$$

#### Result (type 3, 389 leaves):

$$\frac{1}{4\,b^{3}\,f\,\left(\text{d}+\text{c}\,\text{Cos}\,[\,\text{e}+\text{f}\,\text{x}\,]\,\right)^{\,3}\,\left(\text{a}+\text{b}\,\text{Sec}\,[\,\text{e}+\text{f}\,\text{x}\,]\,\right)}}{\left(\text{cos}\,[\,\text{e}+\text{f}\,\text{x}\,]^{\,2}\,\left(\text{b}+\text{a}\,\text{Cos}\,[\,\text{e}+\text{f}\,\text{x}\,]\,\right)\,\left(\text{c}+\text{d}\,\text{Sec}\,[\,\text{e}+\text{f}\,\text{x}\,]\,\right)^{\,3}} \left[\frac{8\,\left(-\,\text{b}\,\text{c}+\text{a}\,\text{d}\right)^{\,3}\,\text{ArcTanh}\left[\frac{\left(-\,\text{a}+\text{b}\right)\,\text{Tan}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\sqrt{a^{2}-b^{2}}}\right]}{\sqrt{a^{2}-b^{2}}}\right]}{\sqrt{a^{2}-b^{2}}} - \\ 2\,d\,\left(-\,6\,\text{a}\,\text{b}\,\text{c}\,\text{d}+2\,\text{a}^{2}\,d^{2}+b^{2}\,\left(6\,\text{c}^{2}+d^{2}\right)\right)\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]-\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]\right]} + \\ 2\,d\,\left(-\,6\,\text{a}\,\text{b}\,\text{c}\,\text{d}+2\,\text{a}^{2}\,d^{2}+b^{2}\,\left(6\,\text{c}^{2}+d^{2}\right)\right)\,\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]\right]} + \\ \frac{b^{2}\,d^{3}}{\left(\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]-\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}}{\left(\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right)} - \\ \frac{b^{2}\,d^{3}}{\left(\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right)} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}} - \\ \frac{b^{2}\,d^{3}}{\left(\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right)} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}} - \\ \frac{b^{2}\,d^{3}}{\left(\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]+\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}\right)} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}} + \frac{4\,\text{b}\,d^{2}\,\left(3\,\text{b}\,\text{c}-\text{a}\,\text{d}\right)\,\text{Sin}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]}{\text{Cos}\left[\frac{1}{2}\,\left(\text{e}+\text{f}\,\text{x}\right)\right]} + \frac{4\,\text{b}\,d^{2}\,\left($$

# Problem 258: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \left(c+d\operatorname{Sec}[e+fx]\right)^{5}}{\left(a+b\operatorname{Sec}[e+fx]\right)^{2}} dx$$

Optimal (type 3, 379 leaves, 16 steps):

$$\frac{d^4 \left(5 \ b \ c-2 \ a \ d\right) \ ArcTanh[Sin[e+fx]]}{2 \ b^3 \ f} + \frac{d^2 \left(10 \ b^3 \ c^3-20 \ a \ b^2 \ c^2 \ d+15 \ a^2 \ b \ c \ d^2-4 \ a^3 \ d^3\right) \ ArcTanh[Sin[e+fx]]}{b^5 \ f} + \frac{2 \left(b \ c-a \ d\right)^5 \ ArcTanh\left[\frac{\sqrt{a-b} \ Tan\left[\frac{1}{2} \ (e+fx)\right]}{\sqrt{a+b}}\right]}{a \ (a-b)^{3/2} \ b^3 \ (a+b)^{3/2} \ f} + \frac{2 \left(b \ c-a \ d\right)^4 \ \left(b \ c+4 \ a \ d\right) \ ArcTanh\left[\frac{\sqrt{a-b} \ Tan\left[\frac{1}{2} \ (e+fx)\right]}{\sqrt{a+b}}\right]}{a \ \sqrt{a-b} \ b^5 \ \sqrt{a+b} \ f} - \frac{\left(b \ c-a \ d\right)^5 \ Sin[e+fx]}{b^4 \ \left(a^2-b^2\right) \ f \ \left(b+a \ Cos[e+fx]\right)} + \frac{d^5 \ Tan[e+fx]}{b^2 \ f} + \frac{d^3 \ \left(10 \ b^2 \ c^2-10 \ a \ b \ c \ d+3 \ a^2 \ d^2\right) \ Tan[e+fx]}{b^4 \ f} + \frac{d^4 \ \left(5 \ b \ c-2 \ a \ d\right) \ Sec[e+fx] \ Tan[e+fx]}{3 \ b^2 \ f} + \frac{d^5 \ Tan[e+fx]^3}{3 \ b^2 \ f}$$

Result (type 3, 1137 leaves):

$$-\left(\left[2\left(b\,c-a\,d\right)^4\left(-a\,b\,c-4\,a^2\,d+5\,b^2\,d\right)\,ArcTanh\left[\frac{\left(-a+b\right)\,Tan\left[\frac{1}{2}\left(e+f\,x\right)\right]}{\sqrt{a^2-b^2}}\right]\right]$$

$$Cos\left[e+f\,x\right]^3\left(b+a\,Cos\left[e+f\,x\right]\right)^2\left(c+d\,Sec\left[e+f\,x\right]\right)^5\right/$$

$$\left(b^5\sqrt{a^2-b^2}\left(-a^2+b^2\right)f\left(d+c\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)\right)+$$

$$\left(\left(-20\,b^3\,c^3\,d^2+40\,a\,b^2\,c^2\,d^3-30\,a^2\,b\,c\,d^4-5\,b^3\,c\,d^4+8\,a^3\,d^5+2\,a\,b^2\,d^5\right)\,Cos\left[e+f\,x\right]^3$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^2\,Log\left[Cos\left[\frac{1}{2}\left(e+f\,x\right)\right]-Sin\left[\frac{1}{2}\left(e+f\,x\right)\right]\right]\left(c+d\,Sec\left[e+f\,x\right]\right)^5\right)\right/$$

$$\left(2\,b^5\,f\left(d+c\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)+$$

$$\left(\left(20\,b^3\,c^3\,d^2-40\,a\,b^2\,c^2\,d^3+30\,a^2\,b\,c\,d^4+5\,b^3\,c\,d^4-8\,a^3\,d^5-2\,a\,b^2\,d^5\right)\,Cos\left[e+f\,x\right]^3$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)+$$

$$\frac{1}{24\,b^4\left(-a^2+b^2\right)\,f\left(d+c\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)}{\left(b+a\,Cos\left[e+f\,x\right]\right)}\left(c+d\,Sec\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^5\left(a+b\,Sec\left[e+f\,x\right]\right)^2\right)$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^3\left(a+b\,Cos\left[e+f\,x\right]\right)^2\right)$$

$$\left(b+a\,Cos\left[e+f\,x\right]\right)^3\left(a+b\,Cos\left[e$$

# Problem 264: Unable to integrate problem.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]\,\,\,\sqrt{\,a + b\,\mathsf{Sec}\,[\,e + f\,x\,]\,\,}}{c + d\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\mathrm{d} x$$

Optimal (type 4, 213 leaves, 3 steps):

$$\begin{split} &\frac{1}{d\,f}2\,\sqrt{a+b}\,\,\mathsf{Cot}\,[\,e+f\,x\,]\,\,\mathsf{EllipticF}\,\big[\mathsf{ArcSin}\,\big[\,\frac{\sqrt{\,a+b}\,\mathsf{Sec}\,[\,e+f\,x\,]\,}{\sqrt{\,a+b}}\,\big]\,,\,\,\frac{a+b}{a-b}\,\big]\\ &\sqrt{\frac{b\,\,\big(1-\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a+b}}\,\,\sqrt{-\frac{b\,\,\big(1+\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a-b}}\,\,-\\ &\left[2\,\,\big(b\,c-a\,d\big)\,\,\mathsf{EllipticPi}\,\big[\,\frac{2\,d}{c+d}\,,\,\,\mathsf{ArcSin}\,\big[\,\frac{\sqrt{1-\mathsf{Sec}\,[\,e+f\,x\,]\,}}{\sqrt{2}}\,\big]\,,\,\,\frac{2\,b}{a+b}\,\big]\,\,\sqrt{\frac{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}{a+b}}\,\,\\ &\mathrm{Tan}\,[\,e+f\,x\,]\,\,\Bigg|\,\,\left(d\,\,\big(\,c+d\big)\,\,f\,\sqrt{\,a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}\,\,\sqrt{\,-\,\mathsf{Tan}\,[\,e+f\,x\,]^{\,2}}\,\right) \end{split}$$

#### Result (type 8, 35 leaves):

$$\int \frac{\operatorname{Sec}[e+fx] \sqrt{a+b\operatorname{Sec}[e+fx]}}{c+d\operatorname{Sec}[e+fx]} \, dx$$

## Problem 265: Unable to integrate problem.

$$\int \frac{\operatorname{Sec}\left[\,e + f\,x\,\right]\,\,\sqrt{\,a + b\,\operatorname{Sec}\left[\,e + f\,x\,\right]\,}}{\sqrt{\,c + d\,\operatorname{Sec}\left[\,e + f\,x\,\right]}}\,\,\mathrm{d}x$$

Optimal (type 4, 196 leaves, 1 step):

$$\frac{1}{d\,\sqrt{\frac{a+b}{c+d}}}\,\,f$$

$$2\,\text{Cot}\,[\,e+f\,x\,]\,\,\text{EllipticPi}\,\big[\,\frac{b\,\left(\,c+d\right)}{\left(\,a+b\right)\,d}\,,\,\,\text{ArcSin}\,\big[\,\frac{\sqrt{\frac{a+b}{c+d}}\,\,\sqrt{\,c+d\,\text{Sec}\,[\,e+f\,x\,]}}{\sqrt{\,a+b\,\text{Sec}\,[\,e+f\,x\,]}}\,\big]\,,\,\,\frac{\left(\,a-b\right)\,\left(\,c+d\right)}{\left(\,a+b\right)\,\left(\,c-d\right)}\,\big]\\ \sqrt{\frac{\left(\,b\,c-a\,d\right)\,\left(\,1-\text{Sec}\,[\,e+f\,x\,]\,\right)}{\left(\,c+d\right)\,\left(\,a+b\,\text{Sec}\,[\,e+f\,x\,]\,\right)}}\,\,\sqrt{\frac{\left(\,b\,c-a\,d\right)\,\left(\,1+\text{Sec}\,[\,e+f\,x\,]\,\right)}{\left(\,c-d\right)\,\left(\,a+b\,\text{Sec}\,[\,e+f\,x\,]\,\right)}}\,\,\left(\,a+b\,\text{Sec}\,[\,e+f\,x\,]\,\right)}$$

$$\int \frac{\operatorname{Sec}[e+fx] \sqrt{a+b\operatorname{Sec}[e+fx]}}{\sqrt{c+d\operatorname{Sec}[e+fx]}} \, dx$$

# Problem 269: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} [e + f x]^{2}}{\sqrt{a + b \operatorname{Sec} [e + f x]}} dx$$

Optimal (type 4, 396 leaves, 3 steps):

$$\frac{1}{b \ d \sqrt{\frac{a+b}{c+d}}} \ f$$

$$2 \, \text{Cot} \, [\text{e} + \text{f} \, \text{x}] \, \text{EllipticPi} \, \Big[ \frac{b \, \left( \text{c} + \text{d} \right)}{\left( \text{a} + \text{b} \right) \, d}, \, \text{ArcSin} \, \Big[ \frac{\sqrt{\frac{a+b}{c+d}} \, \sqrt{c+d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}]}{\sqrt{a+b} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}]} \, \Big], \, \frac{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \right)}{\left( \text{a} + \text{b} \right) \, \left( \text{c} - \text{d} \right)} \, \Big[ \frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{c} - \text{d} \right) \, \left( \text{a} + \text{b} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{c} - \text{d} \right) \, \left( \text{a} + \text{b} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)} \, \sqrt{\frac{\left( \text{a} + \text{b} \right) \, \left( \text{c} - \text{d} \right)}{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{a} + \text{b} \right) \, \left( \text{c} - \text{d} \right)}{\left( \text{a} + \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} + \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} + \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{a} \, \text{d} \right) \, \left( 1 + \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{d} \, \text{d} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}{\left( \text{a} - \text{b} \right) \, \left( \text{c} + \text{d} \, \text{Sec} \, [\text{e} + \text{f} \, \text{x}] \right)}} \, \sqrt{\frac{\left( \text{b} \, \text{c} - \text{d} \, \text{d} \,$$

Result (type 8, 39 leaves)

$$\int \frac{\operatorname{Sec} [e + f x]^{2}}{\sqrt{a + b \operatorname{Sec} [e + f x]}} \, dx$$

Problem 270: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(g\operatorname{Sec}\left[e+f\,x\right]\right)^{3/2}\,\sqrt{c+d\operatorname{Sec}\left[e+f\,x\right]}}{a+b\operatorname{Sec}\left[e+f\,x\right]}\,\mathrm{d}x$$

Optimal (type 4, 170 leaves, 7 steps):

$$\frac{2\,d\,g\,\sqrt{\frac{d+c\,Cos\,[e+f\,x]}{c+d}}}{b\,f\,\sqrt{c+d\,Sec\,[e+f\,x]}}} \,\,EllipticPi\,\Big[\,2,\,\frac{1}{2}\,\left(e+f\,x\right),\,\frac{2\,c}{c+d}\,\Big]\,\,\sqrt{g\,Sec\,[e+f\,x]}}{} \\ + \\ \left[2\,\left(b\,c-a\,d\right)\,g\,\sqrt{\frac{d+c\,Cos\,[e+f\,x]}{c+d}}\,\,EllipticPi\,\Big[\,\frac{2\,a}{a+b},\,\frac{1}{2}\,\left(e+f\,x\right),\,\frac{2\,c}{c+d}\,\Big]\,\,\sqrt{g\,Sec\,[e+f\,x]}}\right] \\ \left(b\,\left(a+b\right)\,f\,\sqrt{c+d\,Sec\,[e+f\,x]}\right)$$

Result (type 4, 223 leaves):

$$-\left(\left[2\,\text{i}\,g\,\sqrt{-\frac{c\,\left(-1+\text{Cos}\,[e+f\,x]\right)}{c+d}}\,\sqrt{\frac{c\,\left(1+\text{Cos}\,[e+f\,x]\right)}{c-d}}\,\,\text{Cot}\,[e+f\,x]\right]}\right) \left[\text{EllipticPi}\left[1-\frac{c}{d}\,,\,\,\text{i}\,\,\text{ArcSinh}\left[\sqrt{\frac{1}{c-d}}\,\,\sqrt{d+c\,\text{Cos}\,[e+f\,x]}\,\right]\,,\,\,\frac{-c+d}{c+d}\right]\right] - \\ \text{EllipticPi}\left[\frac{a\,\left(-c+d\right)}{-b\,c+a\,d}\,,\,\,\,\text{i}\,\,\text{ArcSinh}\left[\sqrt{\frac{1}{c-d}}\,\,\sqrt{d+c\,\text{Cos}\,[e+f\,x]}\,\right]\,,\,\,\frac{-c+d}{c+d}\right]\right) \\ \sqrt{g\,\text{Sec}\,[e+f\,x]}\,\,\sqrt{c+d\,\text{Sec}\,[e+f\,x]}\,\, \left|\sqrt{\left[b\,\sqrt{\frac{1}{c-d}}\,\,f\,\sqrt{d+c\,\text{Cos}\,[e+f\,x]}\,\right]}\right| \right) \left[\frac{d}{d+c\,\text{Cos}\,[e+f\,x]}\right] \left[\frac{d}{d+c\,\text{Cos}\,[e+f\,x]}\right] \left[\frac{d}{d+c\,\text{Cos}\,[e+f\,x]}\right] \left[\frac{d}{d+c\,\text{Cos}\,[e+f\,x]}\right] \left[\frac{d}{d+c\,\text{Cos}\,[e+f\,x]}\right] \right]$$

## Problem 272: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{g \operatorname{Sec}[e+fx]} \sqrt{c+d \operatorname{Sec}[e+fx]}}{a+b \operatorname{Cos}[e+fx]} dx$$

Optimal (type 4, 168 leaves, 8 steps):

$$\frac{2\,d\,\sqrt{\frac{d+c\,\mathsf{Cos}\,[e+f\,x]}{c+d}}}{\mathsf{a}\,f\,\sqrt{c+d\,\mathsf{Sec}\,[e+f\,x]}}} \,\, + \,\, \\ \frac{\mathsf{a}\,f\,\sqrt{c+d\,\mathsf{Sec}\,[e+f\,x]}}{\mathsf{c}\,+d} \,\, + \,\, \\ \left[2\,\left(\mathsf{a}\,\mathsf{c}\,-\mathsf{b}\,\mathsf{d}\right)\,\sqrt{\frac{d+c\,\mathsf{Cos}\,[e+f\,x]}{c+d}}\,\, \mathsf{EllipticPi}\left[\frac{2\,\mathsf{b}}{\mathsf{a}+\mathsf{b}},\,\frac{1}{2}\,\left(\mathsf{e}\,+\mathsf{f}\,\mathsf{x}\right),\,\frac{2\,\mathsf{c}}{\mathsf{c}+\mathsf{d}}\right]\,\sqrt{\mathsf{g}\,\mathsf{Sec}\,[e+f\,x]}}\right]}{\left(\mathsf{a}\,\left(\mathsf{a}\,+\mathsf{b}\right)\,\mathsf{f}\,\sqrt{\mathsf{c}\,+\mathsf{d}\,\mathsf{Sec}\,[e+f\,x]}\right)}$$

#### Result (type 4, 222 leaves):

$$-\left(\left[2\,\dot{\mathbb{I}}\,\sqrt{-\frac{c\,\left(-1+\mathsf{Cos}\,[e+f\,x]\,\right)}{c+d}}\,\sqrt{\frac{c\,\left(1+\mathsf{Cos}\,[e+f\,x]\right)}{c-d}}\,\,\mathsf{Cot}\,[e+f\,x]\right]}\,\,\mathsf{Cot}\,[e+f\,x]\right)\\ \left(\mathsf{EllipticPi}\,\Big[1-\frac{c}{d}\,,\,\dot{\mathbb{I}}\,\mathsf{ArcSinh}\,\Big[\sqrt{\frac{1}{c-d}}\,\,\sqrt{d+c\,\mathsf{Cos}\,[e+f\,x]}\,\,\Big]\,,\,\frac{-c+d}{c+d}\,\Big] - \mathsf{EllipticPi}\,\Big[\frac{b\,\left(-c+d\right)}{-a\,c+b\,d}\,,\,\dot{\mathbb{I}}\,\mathsf{ArcSinh}\,\Big[\sqrt{\frac{1}{c-d}}\,\,\sqrt{d+c\,\mathsf{Cos}\,[e+f\,x]}\,\,\Big]\,,\,\frac{-c+d}{c+d}\,\Big]\right)\\ \sqrt{g\,\mathsf{Sec}\,[e+f\,x]}\,\,\sqrt{c+d\,\mathsf{Sec}\,[e+f\,x]}\,\,\Bigg)\bigg/\left(a\,\sqrt{\frac{1}{c-d}}\,\,f\,\sqrt{d+c\,\mathsf{Cos}\,[e+f\,x]}\,\,\Big)\bigg)$$

## Problem 273: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \sqrt{a+b\operatorname{Sec}[e+fx]}}{c+c\operatorname{Sec}[e+fx]} dx$$

Optimal (type 4, 95 leaves, 1 step):

$$\frac{\text{EllipticE}\left[\text{ArcSin}\left[\frac{\text{Tan}\left[e+f\,x\right]}{1+\text{Sec}\left[e+f\,x\right]}\right],\,\,\frac{a-b}{a+b}\right]\,\sqrt{\frac{1}{1+\text{Sec}\left[e+f\,x\right]}}}\,\,\sqrt{a+b\,\text{Sec}\left[e+f\,x\right]}}{\text{c}\,\sqrt{\frac{a+b\,\text{Sec}\left[e+f\,x\right]}{(a+b)\,\,(1+\text{Sec}\left[e+f\,x\right])}}}$$

#### Result (type 4, 1999 leaves):

Result(type 4, 1999 leaves): 
$$\left( \cos \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \operatorname{Sec}\left[ e + fx \right] \sqrt{a + b \operatorname{Sec}\left[ e + fx \right]} \right. \left( - 2 \operatorname{Sin}\left[ e + fx \right] + 2 \operatorname{Tan}\left[ \frac{1}{2} \left( e + fx \right) \right] \right) \right) / \left( f \left( c + c \operatorname{Sec}\left[ e + fx \right] \right) \right) + \\ \left( \cos \left[ \frac{e}{2} + \frac{fx}{2} \right]^2 \operatorname{Sec}\left[ \frac{1}{2} \left( e + fx \right) \right]^5 \left( \frac{b}{\sqrt{b + a \operatorname{Cos}\left[ e + fx \right]}} \sqrt{\operatorname{Sec}\left[ e + fx \right]} \right. + \frac{a \sqrt{\operatorname{Sec}\left[ e + fx \right]}}{\sqrt{b + a \operatorname{Cos}\left[ e + fx \right]}} \right. + \\ \left. \frac{b \sqrt{\operatorname{Sec}\left[ e + fx \right]}}{\sqrt{b + a \operatorname{Cos}\left[ e + fx \right]}} + \frac{a \operatorname{Cos}\left[ 2 \left( e + fx \right) \right] \sqrt{\operatorname{Sec}\left[ e + fx \right]}}{\sqrt{b + a \operatorname{Cos}\left[ e + fx \right]}} \right) \\ \sqrt{\operatorname{Sec}\left[ e + fx \right]} \sqrt{1 + \operatorname{Sec}\left[ e + fx \right]} \sqrt{a + b \operatorname{Sec}\left[ e + fx \right]} \\ \left( 2 \operatorname{Cos}\left[ \frac{1}{2} \left( e + fx \right) \right] \sqrt{\frac{\operatorname{Cos}\left[ e + fx \right]}{1 + \operatorname{Cos}\left[ e + fx \right]}} \right. \operatorname{EllipticE}\left[ \operatorname{ArcSin}\left[ \operatorname{Tan}\left[ \frac{1}{2} \left( e + fx \right) \right] \right], \frac{a - b}{a + b} \right] + \\ \sqrt{\frac{b + a \operatorname{Cos}\left[ e + fx \right]}{\left( a + b \right) \left( 1 + \operatorname{Cos}\left[ e + fx \right]} \right)}} \left( - \operatorname{Sin}\left[ \frac{1}{2} \left( e + fx \right) \right] + \operatorname{Sin}\left[ \frac{3}{2} \left( e + fx \right) \right] \right) \right) \right) / \\ 4 \left. f \left( \frac{1}{1 + \operatorname{Cos}\left[ e + fx \right]} \right)^{3/2} \sqrt{\frac{b + a \operatorname{Cos}\left[ e + fx \right]}{\left( a + b \right) \left( 1 + \operatorname{Cos}\left[ e + fx \right] \right)}} \right. \left( c + c \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right) \right) / \left( - \operatorname{Sec}\left[ e + fx \right] \right) \right)$$

$$- \left( \left( a \operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^{5} \sqrt{1 + \operatorname{Sec} \left[ e + f x \right]} \operatorname{Sin} \left[ e + f x \right] \right) \right)$$

$$\left[ 2 \, \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \, \sqrt{\frac{\mathsf{Cos} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]}{1 + \mathsf{Cos} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]}} \right. \\ \left. \mathsf{EllipticE} \left[ \mathsf{ArcSin} \left[ \mathsf{Tan} \left[ \, \frac{1}{2} \, \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right] \, \right] , \, \, \frac{\mathsf{a} - \mathsf{b}}{\mathsf{a} + \mathsf{b}} \right] + \mathsf{cos} \left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right] \right] \right] \right]$$

$$\sqrt{\frac{b + a \cos [e + f x]}{(a + b)}} \left( -\sin \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{3}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right)^{3/2} \sqrt{b + a \cos [e + f x]} \sqrt{\frac{b + a \cos [e + f x]}{(a + b)} \left( 1 + \cos [e + f x] \right)} \right) - \left( 3 \sqrt{b + a \cos [e + f x]} \right) \sin [e + f x]$$
 
$$\left( 3 \sqrt{b + a \cos [e + f x]} \right) \left[ -\sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right]^5 \sqrt{1 + \sec [e + f x]} \right] \sin [e + f x]$$
 
$$\left( 2 \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \right) \left[ -\sin \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{3}{2} \left( e + f x \right) \right] \right) \right] /$$
 
$$\left( 8 \sqrt{\frac{1}{1 + \cos [e + f x]}} \sqrt{\frac{b + a \cos [e + f x]}{(a + b)} \left( 1 + \cos [e + f x]} \right)} \right) - \left( \sqrt{b + a \cos [e + f x]} \sqrt{\frac{b + a \cos [e + f x]}{(a + b)} \left( 1 + \cos [e + f x]} \right)} \right) -$$
 
$$\left( \sqrt{b + a \cos [e + f x]} \sqrt{\frac{b + a \cos [e + f x]}{(a + b)} \left( 1 + \cos [e + f x]} \right)} \right) - \left( 2 \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right) \left[ -\sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right] + \frac{a - b}{a + b} \right] +$$
 
$$\sqrt{\frac{b + a \cos [e + f x]}{(a + b)} \left( 1 + \cos [e + f x]} \right)} \left( -\sin \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{3}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \sin \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) - \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) \right) /$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) - \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right)$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) - \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right)$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) - \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right)$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right) - \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \right)$$
 
$$\left( 8 \left( \frac{1}{1 + \cos [e + f x]} \right) \cos \left[ \frac{1}{2} \left( \frac{1}$$

$$\begin{split} & \operatorname{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \bigg) \bigg/ \left[ 8 \left( \frac{1}{1 + \cos \left[ e + f x \right]} \right)^{3/2} \sqrt{\frac{b + a \cos \left[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)}} \right] + \\ & \frac{1}{4 \left( \frac{1}{1 + \cos \left[ e + f x \right]} \right)^{3/2} \sqrt{\frac{b + a \cos \left[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)}} \sqrt{b + a \cos \left[ e + f x \right]} \\ & \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^{3/2} \sqrt{\frac{b + a \cos \left[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)}} \left( - \frac{1}{2} \cos \left[ \frac{1}{2} \left( e + f x \right) \right] + \frac{3}{2} \cos \left[ \frac{3}{2} \left( e + f x \right) \right] \right) - \\ & \sqrt{\frac{b + a \cos \left[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)}} \operatorname{EllipticE} \Big[ \operatorname{ArcSin} \Big[ \operatorname{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \Big], \frac{a - b}{a + b} \Big] \operatorname{Sin} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \\ & \frac{1}{\sqrt{\frac{\cos \left[ e + f x \right]}{1 + \cos \left[ e + f x \right]}}} \operatorname{Cos} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \operatorname{EllipticE} \Big[ \operatorname{ArcSin} \Big[ \operatorname{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \Big], \frac{a - b}{a + b} \Big] \\ & \left( \frac{\cos \left[ e + f x \right] \operatorname{Sin} \Big[ e + f x \right]}{\left( 1 + \cos \left[ e + f x \right] \right)^{2}} - \frac{\operatorname{Sin} \Big[ e + f x \right]}{1 + \cos \left[ e + f x \right]} \right) + \\ & \left( \left[ - \frac{a \sin \left[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)} + \frac{\left( b + a \cos \left[ e + f x \right] \right) \operatorname{Sin} \Big[ e + f x \right]}{\left( a + b \right) \left( 1 + \cos \left[ e + f x \right] \right)} \right) + \\ & \sqrt{-\sin \left[ \frac{1}{2} \left( e + f x \right) \right] + \sin \left[ \frac{3}{2} \left( e + f x \right) \right]} \right) \sqrt{1 - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^{2}} \\ & \sqrt{b + a \cos \left[ e + f x \right]} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^{3} \operatorname{Sec} \Big[ e + f x \Big] \\ & \left[ 2 \cos \left[ \frac{1}{2} \left( e + f x \right) \right] \sqrt{\frac{\cos \left[ e + f x \right]}{1 + \cos \left[ e + f x \right]}}} \operatorname{EllipticE} \Big[ \operatorname{ArcSin} \Big[ \operatorname{Tan} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \Big], \frac{a - b}{a + b} \right] + \\ & \left[ \sqrt{b + a \cos \left[ e + f x \right]} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big]^{3} \operatorname{Sec} \Big[ e + f x \Big] \right] \right] + \frac{1}{a + b} \right] + \\ & \left[ \sqrt{b + a \cos \left[ e + f x \right]} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \right] \right] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] \right] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left( e + f x \right) \Big] + \frac{1}{2} \operatorname{Sec} \Big[ \frac{1}{2} \left$$

$$\sqrt{\frac{b+a\cos\left[e+fx\right]}{\left(a+b\right)\,\left(1+\cos\left[e+fx\right]\right)}}\,\left(-\sin\left[\frac{1}{2}\,\left(e+fx\right)\,\right]+\sin\left[\frac{3}{2}\,\left(e+fx\right)\,\right]\right)\right)\, Tan\left[e+fx\right]} / \\ \left(8\,\left(\frac{1}{1+\cos\left[e+fx\right]}\right)^{3/2}\,\sqrt{\frac{b+a\cos\left[e+fx\right]}{\left(a+b\right)\,\left(1+\cos\left[e+fx\right]\right)}}\,\sqrt{1+\sec\left[e+fx\right]}\right)\right)$$

## Problem 274: Unable to integrate problem.

$$\int \frac{\left(g\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{3/2}\,\sqrt{\,a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}}{c+c\,\mathsf{Sec}\,[\,e+f\,x\,]}\,\,\mathrm{d}x$$

Optimal (type 4, 295 leaves, 11 steps):

$$\frac{g\; \left(b+a\, \mathsf{Cos}\, [e+f\, x]\right) \; \mathsf{EllipticE}\left[\frac{1}{2}\; \left(e+f\, x\right), \, \frac{2\, \mathsf{a}}{\mathsf{a}+\mathsf{b}}\right] \; \sqrt{g\, \mathsf{Sec}\, [e+f\, x]}}{c\, f\, \sqrt{\frac{b+a\, \mathsf{Cos}\, [e+f\, x]}{\mathsf{a}+\mathsf{b}}} \; \sqrt{\mathsf{a}+\mathsf{b}\, \mathsf{Sec}\, [e+f\, x]}} \\ + \frac{\left(\mathsf{a}-\mathsf{b}\right) \; \mathsf{g}\, \sqrt{\frac{b+a\, \mathsf{Cos}\, [e+f\, x]}{\mathsf{a}+\mathsf{b}}} \; \; \mathsf{EllipticF}\left[\frac{1}{2}\; \left(e+f\, x\right), \, \frac{2\, \mathsf{a}}{\mathsf{a}+\mathsf{b}}\right] \; \sqrt{g\, \mathsf{Sec}\, [e+f\, x]}}{c\, f\, \sqrt{\mathsf{a}+\mathsf{b}\, \mathsf{Sec}\, [e+f\, x]}} \\ + \frac{2\, \mathsf{b}\, \mathsf{g}\, \sqrt{\frac{b+a\, \mathsf{Cos}\, [e+f\, x]}{\mathsf{a}+\mathsf{b}}} \; \; \mathsf{EllipticPi}\left[2, \, \frac{1}{2}\; \left(e+f\, x\right), \, \frac{2\, \mathsf{a}}{\mathsf{a}+\mathsf{b}}\right] \; \sqrt{g\, \mathsf{Sec}\, [e+f\, x]}}{c\, f\, \sqrt{\mathsf{a}+\mathsf{b}\, \mathsf{Sec}\, [e+f\, x]}} \\ - \frac{c\, f\, \sqrt{\mathsf{a}+\mathsf{b}\, \mathsf{Sec}\, [e+f\, x]}}{\mathsf{g}\, \left(\mathsf{b}+\mathsf{a}\, \mathsf{Cos}\, [e+f\, x]\right) \; \sqrt{g\, \mathsf{Sec}\, [e+f\, x]}} \; \mathsf{Sin}\, [e+f\, x]}{f\, \left(\mathsf{c}+\mathsf{c}\, \mathsf{Cos}\, [e+f\, x]\right) \; \sqrt{\mathsf{a}+\mathsf{b}\, \mathsf{Sec}\, [e+f\, x]}}$$

Result (type 8, 41 leaves):

$$\int \frac{\left(g\, \mathsf{Sec}\, [\, e + f\, x\, ]\,\right)^{3/2}\, \sqrt{\, a + b\, \mathsf{Sec}\, [\, e + f\, x\, ]\,}}{c + c\, \mathsf{Sec}\, [\, e + f\, x\, ]}\, \, \mathrm{d} x$$

Problem 275: Result more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]}{\sqrt{\,\mathsf{a}\,+\,\mathsf{b}\,\mathsf{Sec}\,[\,\mathsf{e}\,+\,\mathsf{f}\,\mathsf{x}\,]\,}}\,\,\mathrm{d}\mathsf{x}$$

Optimal (type 4, 209 leaves, 3 steps):

$$-\frac{1}{\left(a-b\right)\,c\,f}2\,\sqrt{a+b}\,\,\mathsf{Cot}\,[\,e+f\,x]\,\,\mathsf{EllipticF}\,\big[\mathsf{ArcSin}\,\big[\,\frac{\sqrt{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]}}{\sqrt{a+b}}\,\big]\,,\,\,\frac{a+b}{a-b}\big]$$
 
$$\sqrt{\frac{b\,\,\big(1-\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a+b}}\,\,\sqrt{-\frac{b\,\,\big(1+\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a-b}}\,\,+$$
 
$$\left(\mathsf{EllipticE}\big[\mathsf{ArcSin}\,\big[\,\frac{\mathsf{Tan}\,[\,e+f\,x\,]}{1+\mathsf{Sec}\,[\,e+f\,x\,]}\,\big]\,,\,\,\frac{a-b}{a+b}\,\big]\,\,\sqrt{\frac{1}{1+\mathsf{Sec}\,[\,e+f\,x\,]}}\,\,\sqrt{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]}\right)/$$
 
$$\left((a-b)\,\,c\,f\,\,\sqrt{\frac{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]}{\left(a+b\right)\,\,\left(1+\mathsf{Sec}\,[\,e+f\,x\,]\,\right)}}\right)$$

#### Result (type 4, 2173 leaves):

$$\begin{split} &\left(\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \left(b + a\cos\left[e + fx\right]\right) \operatorname{Sec}\left[e + fx\right]^2 \left(\frac{2\sin\left[e + fx\right]}{-a + b} - \frac{2\operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]}{-a + b}\right)\right)\right/ \\ &\left(f\sqrt{a + b\sec\left[e + fx\right]} \left(c + c\sec\left[e + fx\right]\right)\right) - \\ &\left(2\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \left(-\frac{b}{\left(-a + b\right)\sqrt{b + a\cos\left[e + fx\right]}} - \frac{a\sqrt{\operatorname{Sec}\left[e + fx\right]}}{\left(-a + b\right)\sqrt{b + a\cos\left[e + fx\right]}} - \frac{a\sqrt{\operatorname{Sec}\left[e + fx\right]}}{\left(-a + b\right)\sqrt{b + a\cos\left[e + fx\right]}} + \frac{b\sqrt{\operatorname{Sec}\left[e + fx\right]}}{\left(-a + b\right)\sqrt{b + a\cos\left[e + fx\right]}} - \frac{a\cos\left[2\left(e + fx\right)\right]\sqrt{\operatorname{Sec}\left[e + fx\right]}}{\left(-a + b\right)\sqrt{b + a\cos\left[e + fx\right]}} \right) \operatorname{Sec}\left[e + fx\right]^{3/2} \\ &\sqrt{\cos\left[\frac{1}{2}\left(e + fx\right)\right]^2\operatorname{Sec}\left[e + fx\right]} \left(a - b\right)\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right], \\ &\frac{a + b}{a - b}\right]\sqrt{\frac{\left(b + a\cos\left[e + fx\right]\right)\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^2}{a + b}} + \sqrt{2}\sqrt{\frac{a - b}{a + b}}\sqrt{\frac{\cos\left[e + fx\right]}{1 + \cos\left[e + fx\right]}} \\ &\left(b + a\cos\left[e + fx\right]\right)\operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right] - \left(-1 + \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]^2\right) \right/ \\ &\left(\frac{a - b}{a + b}\right)^{3/2}\left(a + b\right)f\sqrt{\operatorname{Cos}\left[e + fx\right]\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^4}\sqrt{a + b\operatorname{Sec}\left[e + fx\right]} \end{aligned}$$

$$\left(c + c \operatorname{Sec}[e + fx]\right) \left[ -\left(\left[2\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^{2} \sqrt{\operatorname{Cos}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\operatorname{Sec}\left[e + fx\right]} \right] \right. \\ \left. \left. \left(a - b\right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right], \frac{a + b}{a - b}\right] \right. \\ \left. \sqrt{\frac{\left(b + a\operatorname{Cos}\left[e + fx\right]\right)\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}}{a + b} + \\ \sqrt{2} \sqrt{\frac{a - b}{a + b}} \sqrt{\frac{\operatorname{Cos}\left[e + fx\right]}{1 + \operatorname{Cos}\left[e + fx\right]}} \left(b + a\operatorname{Cos}\left[e + fx\right]\right) \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right] \right] \right] \right. \\ \left. \left(\frac{a - b}{a + b}\right)^{3/2} \left(a + b\right) \sqrt{b + a\operatorname{Cos}\left[e + fx\right]} \sqrt{\operatorname{Cos}\left[e + fx\right]\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^{4}} \right) - \\ \left[a \sqrt{\operatorname{Cos}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\operatorname{Sec}\left[e + fx\right]} \operatorname{Sin}\left[e + fx\right] \left(\left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}}\right] + \sqrt{2} \sqrt{\frac{a - b}{a + b}} \right. \right. \\ \left. \sqrt{\frac{\operatorname{Cos}\left[e + fx\right]}{1 + \operatorname{Cos}\left[e + fx\right]}} \left(b + a\operatorname{Cos}\left[e + fx\right]\right) \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right] \left. \left(-1 + \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\right) \right] \right. \\ \left. \left(\frac{a - b}{a + b}\right)^{3/2} \left(a + b\right) \left(b + a\operatorname{Cos}\left[e + fx\right]\right)^{3/2} \sqrt{\operatorname{Cos}\left[e + fx\right]\operatorname{Sec}\left[\frac{1}{2}\left(e + fx\right)\right]^{4}} + \\ \left. \sqrt{\operatorname{Cos}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\operatorname{Sec}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right], \right. \\ \left. \left. \sqrt{\operatorname{Cos}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\operatorname{Sec}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right], \right. \\ \left. \sqrt{\operatorname{Cos}\left[\frac{1}{2}\left(e + fx\right)\right]^{2}\operatorname{Sec}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right], \right. \right. \\ \left. \sqrt{\operatorname{Cos}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right] \right. \right. \\ \left. \sqrt{\operatorname{Cos}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left[\frac{1}{2}\left(e + fx\right)\right]\right] \right] \right. \right. \\ \left. \sqrt{\operatorname{Cos}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Tan}\left(\frac{1}{2}\left(e + fx\right)\right]\right] \right] \right. \right. \\ \left. \sqrt{\operatorname{Cos}\left[e + fx\right]} \left. \left(a - b\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a - b}{a + b}} \operatorname{Elliptic$$

$$\left( \left( a - b \right) \sqrt{\frac{a - b}{a + b}} \operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \sqrt{\frac{\left( b + a \operatorname{Cos} \left[ e + f x \right) \right) \operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2}{a + b}} \right.$$

$$\sqrt{1 - \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2} \right) / \left[ 2 \sqrt{1 - \frac{\left( a - b \right) \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2}{a + b}} \right]$$

$$\left( \left( a - b \right) \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{a - b}{a + b}} \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right] \right], \frac{a + b}{a - b} \right] \right.$$

$$\sqrt{\frac{\left( b + a \operatorname{Cos} \left[ e + f x \right] \right) \operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^2}{a + b}} +$$

$$\sqrt{2} \sqrt{\frac{a - b}{a + b}} \sqrt{\frac{\operatorname{Cos} \left[ e + f x \right]}{1 + \operatorname{Cos} \left[ e + f x \right]}} \left( b + a \operatorname{Cos} \left[ e + f x \right] \right) \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right] \right.$$

$$\left. \left( -1 + \operatorname{Tan} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \right) \left( -\operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right] \operatorname{Sec} \left[ e + f x \right] \operatorname{Sin} \left[ \frac{1}{2} \left( e + f x \right) \right] +$$

$$\left. \operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \operatorname{Sec} \left[ e + f x \right] \operatorname{Tan} \left[ e + f x \right] \right.$$

$$\sqrt{\operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \operatorname{Sec} \left[ e + f x \right]} \sqrt{\operatorname{Cos} \left[ e + f x \right] \operatorname{Sec} \left[ \frac{1}{2} \left( e + f x \right) \right]^4}$$

$$\sqrt{\operatorname{Cos} \left[ \frac{1}{2} \left( e + f x \right) \right]^2 \operatorname{Sec} \left[ e + f x \right]}$$

Problem 276: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Sec}\,[\,e\,+\,f\,x\,]^{\,2}}{\sqrt{\mathsf{a}\,+\,\mathsf{b}\,\mathsf{Sec}\,[\,e\,+\,f\,x\,]\,}}\,\,\mathrm{d}x$$

Optimal (type 4, 214 leaves, 3 steps):

$$\frac{1}{\left(a-b\right)\,b\,c\,f}2\,a\,\sqrt{a+b}\,\,\text{Cot}\,[\,e+f\,x\,]\,\,\text{EllipticF}\,\big[\,\text{ArcSin}\,\big[\,\frac{\sqrt{a+b\,\text{Sec}\,[\,e+f\,x\,]\,}}{\sqrt{a+b}}\,\big]\,,\,\,\frac{a+b}{a-b}\,\big]$$
 
$$\sqrt{\frac{b\,\left(1-\text{Sec}\,[\,e+f\,x\,]\,\right)}{a+b}}\,\,\sqrt{-\frac{b\,\left(1+\text{Sec}\,[\,e+f\,x\,]\,\right)}{a-b}}\,\,-$$
 
$$\left[\text{EllipticE}\,\big[\,\text{ArcSin}\,\big[\,\frac{\text{Tan}\,[\,e+f\,x\,]\,}{1+\text{Sec}\,[\,e+f\,x\,]}\,\big]\,,\,\,\frac{a-b}{a+b}\,\big]\,\,\sqrt{\frac{1}{1+\text{Sec}\,[\,e+f\,x\,]}}\,\,\sqrt{a+b\,\text{Sec}\,[\,e+f\,x\,]}\,\,\right]/$$
 
$$\left(\left(a-b\right)\,c\,f\,\,\sqrt{\frac{a+b\,\text{Sec}\,[\,e+f\,x\,]\,}{\left(a+b\right)\,\left(1+\text{Sec}\,[\,e+f\,x\,]\,\right)}}\,\right)$$

#### Result (type 4, 1482 leaves):

Result(type+, Figure 1 and Eaves). 
$$8 \text{ a } \text{Cos} \left[ \frac{e}{2} + \frac{f \times}{2} \right]^2 \text{ Cos} \left[ \frac{1}{2} \left( e + f \times \right) \right]^2 \sqrt{\frac{b + a \text{ Cos} \left[ e + f \times \right]}{\left( a + b \right) \left( 1 + \text{ Cos} \left[ e + f \times \right] \right)}}$$
 
$$EllipticPi \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} \left( e + f \times \right) \right] \right], \frac{a - b}{a + b} \right] \sqrt{\text{Cos} \left[ e + f \times \right] \text{Sec} \left[ \frac{1}{2} \left( e + f \times \right) \right]^4}$$
 
$$\text{Sec} \left[ e + f \times \right]^{3/2} \sqrt{1 + \text{Sec} \left[ e + f \times \right]} / \left( \left( -a + b \right) f \sqrt{a + b \text{ Sec} \left[ e + f \times \right]} \right) \left( c + c \text{ Sec} \left[ e + f \times \right] \right) \right) -$$
 
$$\left[ 4 \text{ b } \text{Cos} \left[ \frac{e}{2} + \frac{f \times}{2} \right]^2 \text{Cos} \left[ \frac{1}{2} \left( e + f \times \right) \right]^2 \sqrt{\frac{b + a \text{ Cos} \left[ e + f \times \right]}{\left( a + b \right) \left( 1 + \text{ Cos} \left[ e + f \times \right] \right)}} \right) } \sqrt{\text{Cos} \left[ e + f \times \right] \text{Sec} \left[ \frac{1}{2} \left( e + f \times \right) \right]^4} \right]$$
 
$$\text{EllipticPi} \left[ -1, -\text{ArcSin} \left[ \text{Tan} \left[ \frac{1}{2} \left( e + f \times \right) \right] \right], \frac{a - b}{a + b} \right] \sqrt{\text{Cos} \left[ e + f \times \right] \text{Sec} \left[ \frac{1}{2} \left( e + f \times \right) \right]^4} \right]$$
 
$$\text{Sec} \left[ e + f \times \right]^{3/2} \sqrt{1 + \text{Sec} \left[ e + f \times \right]} \right) / \left( \left( -a + b \right) f \sqrt{a + b \text{ Sec} \left[ e + f \times \right]} \right) \left( c + c \text{ Sec} \left[ e + f \times \right] \right) \right) +$$
 
$$\left( \text{Cos} \left[ \frac{e}{2} + \frac{f \times}{2} \right]^2 \left( b + a \text{ Cos} \left[ e + f \times \right] \right) \text{Sec} \left[ e + f \times \right]^2 - \frac{2 \text{ Sin} \left[ e + f \times \right]}{-a + b} + \frac{2 \text{ Tan} \left[ \frac{1}{2} \left( e + f \times \right) \right]}{-a + b} \right) \right) /$$
 
$$\left( f \sqrt{a + b \text{ Sec} \left[ e + f \times \right]} \right) \left( c + c \text{ Sec} \left[ e + f \times \right] \right) \right) -$$
 
$$\left( 2 \text{ Cos} \left[ \frac{e}{2} + \frac{f \times}{2} \right]^2 \sqrt{b + a \text{ Cos} \left[ e + f \times \right]} \text{ Sec} \left[ e + f \times \right]^{3/2} \sqrt{\frac{1 - \text{Tan} \left[ \frac{1}{2} \left( e + f \times \right) \right]^2}{-a + b}} \right) / \right) / \right)$$

$$\begin{split} b \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \sqrt{1 - \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2} + a \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^3 \\ \sqrt{1 - \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2} - b \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^3 \sqrt{1 - \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2} + \\ 4 \stackrel{!}{=} a \, \text{EllipticPi} \big[ -\frac{a+b}{a-b}, \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ \sqrt{\frac{a+b-a \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2 + b \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2} - \\ 2 \stackrel{!}{=} b \, \text{EllipticPi} \big[ -\frac{a+b}{a-b}, \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ \sqrt{\frac{a+b-a \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2 + b \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big]^2} + \\ 4 \stackrel{!}{=} a \, \text{EllipticPi} \big[ -\frac{a+b}{a-b}, \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} b \, \text{EllipticPi} \big[ -\frac{a+b}{a-b}, \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} b \, \text{EllipticPi} \big[ -\frac{a+b}{a-b}, \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} b \, \text{EllipticE} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticE} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticF} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticF} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticF} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticF} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{Tan} \big[ \frac{1}{2} \left( e + f x \right) \big] \big], \frac{a+b}{a-b} \big] \\ -2 \stackrel{!}{=} (a-b) \, \text{EllipticF} \big[ \text{ i ArcSinh} \big[ \sqrt{\frac{-a+b}{a+b}} & \, \text{T$$

$$\left( \left( -\mathsf{a} + \mathsf{b} \right) \sqrt{\frac{-\mathsf{a} + \mathsf{b}}{\mathsf{a} + \mathsf{b}}} \ \mathsf{f} \sqrt{\mathsf{a} + \mathsf{b} \, \mathsf{Sec} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,]} \ \left( \mathsf{c} + \mathsf{c} \, \mathsf{Sec} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right) \ \left( \mathsf{1} + \mathsf{Tan} \left[ \frac{\mathsf{1}}{\mathsf{2}} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right]^2 \right)^{3/2} \right)$$
 
$$\sqrt{\frac{\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Tan} \left[ \frac{\mathsf{1}}{\mathsf{2}} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right]^2 + \mathsf{b} \, \mathsf{Tan} \left[ \frac{\mathsf{1}}{\mathsf{2}} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right]^2}{\mathsf{1} + \mathsf{Tan} \left[ \frac{\mathsf{1}}{\mathsf{2}} \left( \mathsf{e} + \mathsf{f} \, \mathsf{x} \right) \, \right]^2} \right) }$$

Problem 277: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\!\frac{\left(g\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{\,3/2}}{\sqrt{\,a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}\,\left(\,c+c\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)}\,\,\mathrm{d}x$$

Optimal (type 4, 229 leaves, 7 steps):

$$\frac{g\,\left(b+a\,Cos\,[\,e+f\,x\,]\,\right)\,\,EllipticE\left[\,\frac{1}{2}\,\left(\,e+f\,x\,\right)\,,\,\,\frac{2\,a}{a+b}\,\right]\,\,\sqrt{g\,Sec\,[\,e+f\,x\,]}}{\left(\,a-b\right)\,\,c\,f\,\sqrt{\frac{b+a\,Cos\,[\,e+f\,x\,]}{a+b}}}\,\,\sqrt{\,a+b\,Sec\,[\,e+f\,x\,]}}$$

$$\frac{g\sqrt{\frac{b+a\cos[e+fx]}{a+b}} \quad EllipticF\left[\frac{1}{2}\left(e+fx\right), \frac{2a}{a+b}\right]\sqrt{gSec[e+fx]}}{cf\sqrt{a+bSec[e+fx]}}$$

$$\frac{g\,\left(b+a\,Cos\left[e+f\,x\right]\right)\,\sqrt{g\,Sec\left[e+f\,x\right]}\,\,Sin\left[e+f\,x\right]}{\left(a-b\right)\,f\left(c+c\,Cos\left[e+f\,x\right]\right)\,\sqrt{a+b\,Sec\left[e+f\,x\right]}}$$

Result (type 6, 1019 leaves):

$$\left( \frac{2 \operatorname{Csc}[e]}{2} + \frac{\operatorname{f} x}{2} \right)^2 \left( b + a \operatorname{Cos}[e + \operatorname{f} x] \right) \left( \operatorname{gSec}[e + \operatorname{f} x] \right)^{3/2}$$

$$\left( \frac{2 \operatorname{Csc}[e]}{\left( -a + b \right) \operatorname{f}} + \frac{2 \operatorname{Sec}\left[\frac{e}{2}\right] \operatorname{Sec}\left[\frac{e}{2} + \frac{\operatorname{f} x}{2}\right] \operatorname{Sin}\left[\frac{\operatorname{f} x}{2}\right]}{\left( -a + b \right) \operatorname{f}} \right) \right) / \left( \sqrt{a + b \operatorname{Sec}[e + \operatorname{f} x]} \left( c + c \operatorname{Sec}[e + \operatorname{f} x] \right) \right) +$$

$$\frac{1}{\left( -a + b \right) \operatorname{f} \sqrt{1 + \operatorname{Cot}[e]^2} \sqrt{a + b \operatorname{Sec}[e + \operatorname{f} x]} \left( c + c \operatorname{Sec}[e + \operatorname{f} x] \right)}$$

$$\operatorname{AppellF1}\left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\operatorname{Csc}[e] \left( b - a \sqrt{1 + \operatorname{Cot}[e]^2} \operatorname{Sin}[e] \operatorname{Sin}[\operatorname{f} x - \operatorname{ArcTan}[\operatorname{Cot}[e]]] \right)}{a \sqrt{1 + \operatorname{Cot}[e]^2}} \right)$$

$$\frac{Csc[e] \left[b-a\sqrt{1+Cot[e]^2} \; Sin[e] \; Sin[e] \; Sin[e] \; ArcTan[Cot[e]]]\right]}{a\sqrt{1+Cot[e]^2}} \Big] \; Cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2} \\ \sqrt{b+aCos[e+fx]} \; Csc\left[\frac{e}{2}\right] \; Sec\left[\frac{e}{2}\right] \; \left(g \; Sec[e+fx]\right)^{3/2} \; Sec[fx - ArcTan[Cot[e]]] \\ \sqrt{\frac{a\sqrt{1+Cot[e]^2} - a\sqrt{1+Cot[e]^2} \; Sin[fx - ArcTan[Cot[e]]]}{a\sqrt{1+Cot[e]^2} + a\sqrt{1+Cot[e]^2} \; b \; Csc[e]} \\ \sqrt{\frac{a\sqrt{1+Cot[e]^2} + a\sqrt{1+Cot[e]^2} \; b \; Csc[e]}{\sqrt{b-a\sqrt{1+Cot[e]^2} \; Sin[e] \; Sin[fx - ArcTan[Cot[e]]]}} + \\ \sqrt{\frac{a\cos\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \sqrt{b+aCos[e+fx]} \; Csc\left[\frac{e}{2}\right] \; Sec\left[\frac{e}{2}\right] \; \left(g \; Sec[e+fx]\right)^{3/2}} \\ \left(\left[AppellF1\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\left[\left(Sec[e] \left[b+aCos[e] \; Cos[fx+ArcTan[Tan[e]]\right]\right) \right] \right. \\ \sqrt{1+Tan[e]^2}\right]\right) / \left[a\sqrt{1+Tan[e]^2} \left(1 - \frac{b \; Sec[e]}{a\sqrt{1+Tan[e]^2}}\right)\right]\right), \\ -\left[\left(Sec[e] \left[b+aCos[e] \; Cos[fx+ArcTan[Tan[e]]]\right] \sqrt{1+Tan[e]^2}\right)\right/ \\ \left[a\sqrt{1+Tan[e]^2} \sqrt{\left(\left[a\sqrt{1+Tan[e]^2} - aCos[fx+ArcTan[Tan[e]]]\right] \sqrt{1+Tan[e]^2}\right)}\right/ \\ \left(b\; Sec[e] + a\sqrt{1+Tan[e]^2}\right) \sqrt{\left(\left[a\sqrt{1+Tan[e]^2} - aCos[fx+ArcTan[Tan[e]]]\right] \sqrt{1+Tan[e]^2}\right)} \\ \sqrt{b+aCos[e] \; Cos[fx+ArcTan[Tan[e]]] \; \sqrt{1+Tan[e]^2}} + \\ \left(\frac{Sin[fx+ArcTan[Tan[e]]] \; Tan[e]}{\sqrt{1+Tan[e]^2}} + \left(2\; a\; Cos[e] \; \left[b+a\; Cos[e] \; Cos[e] \;$$

#### Problem 278: Unable to integrate problem.

$$\int\!\frac{\left(g\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)^{5/2}}{\sqrt{\mathsf{a}+\mathsf{b}\,\mathsf{Sec}\,[\,e+f\,x\,]}\,\left(\mathsf{c}+\mathsf{c}\,\mathsf{Sec}\,[\,e+f\,x\,]\,\right)}\,\,\mathrm{d} x$$

Optimal (type 4, 312 leaves, 11 steps):

$$-\frac{g^2\left(b+a\,Cos\left[e+f\,x\right]\right)\,EllipticE\left[\frac{1}{2}\,\left(e+f\,x\right),\,\frac{2\,a}{a+b}\right]\,\sqrt{g\,Sec\left[e+f\,x\right]}}{\left(a-b\right)\,c\,f\,\sqrt{\frac{b+a\,Cos\left[e+f\,x\right]}{a+b}}}\,\,\sqrt{a+b\,Sec\left[e+f\,x\right]}}$$

$$\frac{g^2 \sqrt{\frac{b + a \, Cos[e + f \, x]}{a + b}} \quad \text{EllipticF}\left[\frac{1}{2} \left(e + f \, x\right), \, \frac{2 \, a}{a + b}\right] \sqrt{g \, Sec[e + f \, x]}}{c \, f \, \sqrt{a + b \, Sec[e + f \, x]}} +$$

$$\frac{2 g^2 \sqrt{\frac{b+a \cos [e+fx]}{a+b}}}{\text{c f } \sqrt{a+b \sec [e+fx]}} \text{ EllipticPi} \left[2, \frac{1}{2} \left(e+fx\right), \frac{2a}{a+b}\right] \sqrt{g \sec [e+fx]}}{+\frac{1}{2} \left(e+fx\right)}$$

$$\frac{g^2 \left(b + a \cos[e + fx]\right) \sqrt{g \sec[e + fx]} \sin[e + fx]}{\left(a - b\right) f\left(c + c \cos[e + fx]\right) \sqrt{a + b \sec[e + fx]}}$$

#### Result (type 8, 41 leaves

$$\int\!\frac{\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{\,5/2}}{\sqrt{\,a+b\,\text{Sec}\,[\,e+f\,x\,]\,}\,\left(\,c+c\,\text{Sec}\,[\,e+f\,x\,]\,\right)}\,\,\text{d}x$$

# Problem 279: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} [e + f x] \sqrt{a + b \operatorname{Sec} [e + f x]}}{c + d \operatorname{Sec} [e + f x]} dx$$

Optimal (type 4, 213 leaves, 3 steps):

$$\begin{split} &\frac{1}{d\,f}2\,\sqrt{a+b}\,\,\mathsf{Cot}\,[\,e+f\,x\,]\,\,\mathsf{EllipticF}\,\big[\mathsf{ArcSin}\,\big[\,\frac{\sqrt{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}}{\sqrt{a+b}}\,\big]\,,\,\,\frac{a+b}{a-b}\,\big]\\ &\sqrt{\frac{b\,\,\big(1-\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a+b}}\,\,\sqrt{-\frac{b\,\,\big(1+\mathsf{Sec}\,[\,e+f\,x\,]\,\big)}{a-b}}\,\,-\\ &\left[2\,\,\big(b\,c-a\,d\big)\,\,\mathsf{EllipticPi}\,\big[\,\frac{2\,d}{c+d}\,,\,\mathsf{ArcSin}\,\big[\,\frac{\sqrt{1-\mathsf{Sec}\,[\,e+f\,x\,]\,}}{\sqrt{2}}\,\big]\,,\,\,\frac{2\,b}{a+b}\,\big]\,\,\sqrt{\frac{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}{a+b}}\,\,\\ &\mathrm{Tan}\,[\,e+f\,x\,]\,\,\Bigg|\,\,\left(d\,\,\big(\,c+d\big)\,\,f\,\sqrt{a+b\,\mathsf{Sec}\,[\,e+f\,x\,]\,}\,\,\sqrt{-\mathsf{Tan}\,[\,e+f\,x\,]^{\,2}}\,\right) \end{split}$$

Result (type 8, 35 leaves):

$$\int \frac{\operatorname{Sec} [e+fx] \sqrt{a+b \operatorname{Sec} [e+fx]}}{c+d \operatorname{Sec} [e+fx]} \, dx$$

Problem 280: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(g\,\mathsf{Sec}\,[\,e + f\,x\,]\,\right)^{3/2}\,\sqrt{\,\mathsf{a} + \mathsf{b}\,\mathsf{Sec}\,[\,e + f\,x\,]\,}}{\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,e + f\,x\,]}\,\,\mathrm{d} x$$

Optimal (type 4, 170 leaves, 7 steps):

Result (type 4, 223 leaves):

$$-\left(\left(2 \text{ i g } \sqrt{-\frac{a \left(-1+\text{Cos}\left[e+fx\right]\right)}{a+b}} \sqrt{\frac{a \left(1+\text{Cos}\left[e+fx\right]\right)}{a-b}} \right. \text{Cot}\left[e+fx\right]\right)$$

$$\left(\text{EllipticPi}\left[1-\frac{a}{b}, \text{ i ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \text{Cos}\left[e+fx\right]}\right], \frac{-a+b}{a+b}\right] - \left(\frac{a-b}{b-b}, \text{ i ArcSinh}\left[\sqrt{\frac{1}{a-b}} \sqrt{b+a \text{Cos}\left[e+fx\right]}\right], \frac{-a+b}{a+b}\right]\right)$$

$$\sqrt{g \text{Sec}\left[e+fx\right]} \sqrt{a+b \text{Sec}\left[e+fx\right]} / \left(\sqrt{\frac{1}{a-b}} d f \sqrt{b+a \text{Cos}\left[e+fx\right]}\right)$$

## Problem 281: Unable to integrate problem.

$$\int \frac{\mathsf{Sec}\,[\,e + f\,x\,]}{\sqrt{\mathsf{a} + \mathsf{b}\,\mathsf{Sec}\,[\,e + f\,x\,]}} \,\, \big(\mathsf{c} + \mathsf{d}\,\mathsf{Sec}\,[\,e + f\,x\,]\,\big)} \,\, \mathrm{d} x$$

Optimal (type 4, 102 leaves, 1 step):

$$\left( 2 \, \text{EllipticPi} \left[ \frac{2 \, d}{c + d}, \, \text{ArcSin} \left[ \frac{\sqrt{1 - \text{Sec} \left[ e + f \, x \right]}}{\sqrt{2}} \right], \, \frac{2 \, b}{a + b} \right] \, \sqrt{\frac{a + b \, \text{Sec} \left[ e + f \, x \right]}{a + b}} \, \, \text{Tan} \left[ e + f \, x \right] \right) \right)$$

Result (type 8, 35 leaves)

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]}{\sqrt{a+b\,\operatorname{Sec}\left[e+f\,x\right]}}\,\left(c+d\,\operatorname{Sec}\left[e+f\,x\right]\right)}\,\mathrm{d}x$$

# Problem 282: Unable to integrate problem.

$$\int \frac{\operatorname{Sec}\left[e+f\,x\right]^{\,2}}{\sqrt{a+b\,\operatorname{Sec}\left[e+f\,x\right]}\,\left(c+d\,\operatorname{Sec}\left[e+f\,x\right]\right)}\,\mathrm{d}x$$

Optimal (type 4, 209 leaves, 3 steps):

$$\frac{1}{b \, d \, f} 2 \, \sqrt{a + b} \, \, \mathsf{Cot}[\, e + f \, x] \, \, \mathsf{EllipticF}[\, \mathsf{ArcSin}[\, \frac{\sqrt{a + b \, \mathsf{Sec}\, [\, e + f \, x \,]}}{\sqrt{a + b}} \, \big] \, , \, \, \frac{a + b}{a - b} \big]$$
 
$$\sqrt{\frac{b \, \left(1 - \mathsf{Sec}\, [\, e + f \, x \,]\,\right)}{a + b}} \, \sqrt{-\frac{b \, \left(1 + \mathsf{Sec}\, [\, e + f \, x \,]\,\right)}{a - b}} \, -$$
 
$$\left(2 \, c \, \, \mathsf{EllipticPi}[\, \frac{2 \, d}{c + d} \, , \, \, \mathsf{ArcSin}[\, \frac{\sqrt{1 - \mathsf{Sec}\, [\, e + f \, x \,]}}{\sqrt{2}} \, \big] \, , \, \, \frac{2 \, b}{a + b} \big] \, \sqrt{\frac{a + b \, \mathsf{Sec}\, [\, e + f \, x \,]}{a + b}} \, \, \mathsf{Tan}[\, e + f \, x \,] \right)$$
 
$$\left(d \, \left(c + d\right) \, f \, \sqrt{a + b \, \mathsf{Sec}\, [\, e + f \, x \,]} \, \sqrt{-\mathsf{Tan}\, [\, e + f \, x \,]^{\, 2}} \right)$$

Result (type 8, 37 leaves):

$$\int \frac{\operatorname{Sec} [e + f x]^{2}}{\sqrt{a + b \operatorname{Sec} [e + f x]}} dx$$

## Problem 284: Result unnecessarily involves imaginary or complex numbers.

$$\int\!\frac{\left(g\,\text{Sec}\,[\,e+f\,x\,]\,\right)^{5/2}}{\sqrt{a+b\,\text{Sec}\,[\,e+f\,x\,]}\,\left(\,c+d\,\text{Sec}\,[\,e+f\,x\,]\,\right)}\,\,\text{d}x$$

Optimal (type 4, 166 leaves, 7 steps):

$$\frac{2\,g^2\,\sqrt{\frac{b+a\,\text{Cos}\,[e+f\,x]}{a+b}}\,\,\,\text{EllipticPi}\big[\,2\,,\,\,\frac{1}{2}\,\,\big(\,e+f\,x\big)\,\,,\,\,\frac{2\,a}{a+b}\,\big]\,\,\sqrt{g\,\text{Sec}\,[\,e+f\,x\,]}}{d\,f\,\sqrt{a+b\,\text{Sec}\,[\,e+f\,x\,]}}\,-\,\\ \\ \left(2\,c\,g^2\,\sqrt{\frac{b+a\,\text{Cos}\,[\,e+f\,x\,]}{a+b}}\,\,\,\text{EllipticPi}\big[\,\frac{2\,c}{c+d}\,,\,\,\frac{1}{2}\,\,\big(\,e+f\,x\big)\,\,,\,\,\frac{2\,a}{a+b}\,\big]\,\,\sqrt{g\,\text{Sec}\,[\,e+f\,x\,]}}\right) \\ \left(d\,\,\big(\,c+d\big)\,\,f\,\sqrt{a+b\,\text{Sec}\,[\,e+f\,x\,]}\,\,\big)$$

Result (type 4, 246 leaves):

$$-\left(\left[2\ \text{i}\ \text{g}\ \sqrt{-\frac{a\left(-1+\text{Cos}\left[e+fx\right]\right)}{a+b}}\ \sqrt{\frac{a\left(1+\text{Cos}\left[e+fx\right]\right)}{a-b}}\ \sqrt{b+a\,\text{Cos}\left[e+fx\right]}\ \text{Cot}\left[e+fx\right]\right)\right]$$

$$\left(\left[-b\ c+a\ d\right]\ \text{EllipticPi}\left[1-\frac{a}{b}\ ,\ \text{i}\ \text{ArcSinh}\left[\sqrt{\frac{1}{a-b}}\ \sqrt{b+a\,\text{Cos}\left[e+fx\right]}\ \right],\ \frac{-a+b}{a+b}\right]+b\ c\ \text{EllipticPi}\left[\frac{\left(a-b\right)\ c}{-b\ c+a\ d}\ ,\ \text{i}\ \text{ArcSinh}\left[\sqrt{\frac{1}{a-b}}\ \sqrt{b+a\,\text{Cos}\left[e+fx\right]}\ \right],\ \frac{-a+b}{a+b}\right]\right)$$

$$\left(\text{g}\ \text{Sec}\left[e+fx\right]\right)^{3/2}\left/\sqrt{\frac{1}{a-b}}\ b\ d\ \left(-b\ c+a\ d\right)\ f\ \sqrt{a+b\,\text{Sec}\left[e+fx\right]}\ \right)\right)$$

## Problem 285: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx] \operatorname{Tan}[e+fx]^{4}}{\left(c-c \operatorname{Sec}[e+fx]\right)^{7}} dx$$

Optimal (type 3, 67 leaves, 4 steps):

$$\frac{\text{Cot}\left[\frac{1}{2}\left(e+f\,x\right)\right]^{5}}{20\,c^{7}\,f}-\frac{\text{Cot}\left[\frac{1}{2}\left(e+f\,x\right)\right]^{7}}{14\,c^{7}\,f}+\frac{\text{Cot}\left[\frac{1}{2}\left(e+f\,x\right)\right]^{9}}{36\,c^{7}\,f}$$

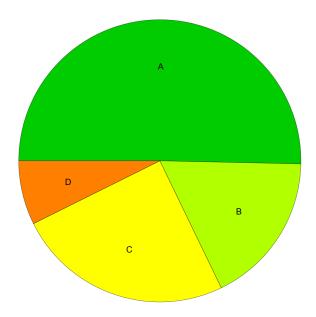
Result (type 3, 151 leaves):

$$\frac{1}{23\,063\,040\,c^{7}\,f}$$

$$Csc\left[\frac{e}{2}\right] Csc\left[\frac{1}{2}\left(e+fx\right)\right]^{9} \left(-971\,082\,Sin\left[\frac{fx}{2}\right] - 718\,830\,Sin\left[e+\frac{fx}{2}\right] + 467\,208\,Sin\left[e+\frac{3\,fx}{2}\right] + 659\,400\,Sin\left[2\,e+\frac{3\,fx}{2}\right] - 303\,192\,Sin\left[2\,e+\frac{5\,fx}{2}\right] - 179\,640\,Sin\left[3\,e+\frac{5\,fx}{2}\right] + 659\,Sin\left[3\,e+\frac{7\,fx}{2}\right] +$$

# **Summary of Integration Test Results**

#### 286 integration problems



- A 144 optimal antiderivatives
- B 50 more than twice size of optimal antiderivatives
- C 71 unnecessarily complex antiderivatives
- D 21 unable to integrate problems
- E 0 integration timeouts