Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions"

Test results for the 1917 problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b\,x^m}{2\,(a+b\,x)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x}}\right)dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x}}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^m\left(-\frac{b\,x}{a}\right)^{-m}\text{Hypergeometric2F1}\left[-\frac{1}{2},-m,\frac{1}{2},1+\frac{b\,x}{a}\right]}{\sqrt{a+b\,x}}-\frac{2\,m\,x^m\left(-\frac{b\,x}{a}\right)^{-m}\sqrt{a+b\,x}\;\text{Hypergeometric2F1}\left[\frac{1}{2},1-m,\frac{3}{2},1+\frac{b\,x}{a}\right]}{a}$$

Test results for the 3201 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 957: Result valid but suboptimal antiderivative.

$$\int (e\,x)^m\,(a-b\,x)^{2+n}\,(a+b\,x)^n\,dx$$

Optimal (type 5, 211 leaves, ? steps):

$$-\frac{(e\,x)^{1+m}\,(a-b\,x)^{1+n}\,(a+b\,x)^{1+n}}{e\,(3+m+2\,n)}+\left(2\,a^2\,(2+m+n)\,(e\,x)^{1+m}\,(a-b\,x)^n\,(a+b\,x)^n\left(1-\frac{b^2\,x^2}{a^2}\right)^{-n}\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2\,x^2}{a^2}\right]\right)\Big/\left(e\,(1+m)\,(3+m+2\,n)\right)-\frac{1}{e^2\,(2+m)}$$
$$2\,a\,b\,(e\,x)^{2+m}\,(a-b\,x)^n\,(a+b\,x)^n\left(1-\frac{b^2\,x^2}{a^2}\right)^{-n}\text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2\,x^2}{a^2}\right]$$

Result (type 5, 238 leaves, 11 steps):

$$\frac{1}{e\,(1+m)}a^2\,(e\,x)^{1+m}\,(a-b\,x)^n\,(a+b\,x)^n$$
$$\left(1-\frac{b^2\,x^2}{a^2}\right)^{-n}\text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2\,x^2}{a^2}\right]-\frac{1}{e^2\,(2+m)}$$
$$2\,a\,b\,(e\,x)^{2+m}\,(a-b\,x)^n\,(a+b\,x)^n\left(1-\frac{b^2\,x^2}{a^2}\right)^{-n}\text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2\,x^2}{a^2}\right]+$$
$$\frac{1}{e^3\,(3+m)}b^2\,(e\,x)^{3+m}\,(a-b\,x)^n\,(a+b\,x)^n\left(1-\frac{b^2\,x^2}{a^2}\right)^{-n}\text{Hypergeometric2F1}\left[\frac{3+m}{2},-n,\frac{5+m}{2},\frac{b^2\,x^2}{a^2}\right]$$

Problem 1001: Result unnecessarily involves higher level functions.

$$\int\frac{(1-a\,x)^{1-n}\,(1+a\,x)^{1+n}}{x^2}\,dx$$

Optimal (type 5, 106 leaves, 3 steps):

$$-\frac{1}{1-n}2\,a\,(1-a\,x)^{1-n}\,(1+a\,x)^{-1+n}\,\text{Hypergeometric2F1}\left[2,1-n,2-n,\frac{1-a\,x}{1+a\,x}\right]+$$
$$\frac{2^n\,a\,(1-a\,x)^{1-n}\,\text{Hypergeometric2F1}\left[1-n,-n,2-n,\frac{1}{2}\,(1-a\,x)\right]}{1-n}$$

Result (type 6, 48 leaves, 1 step):

$$\frac{2^{1-n}\,a\,(1+a\,x)^{2+n}\,\text{AppellF1}\left[2+n,-1+n,2,3+n,\frac{1}{2}\,(1+a\,x),1+a\,x\right]}{2+n}$$

Problem 1006: Result valid but suboptimal antiderivative.

$$\int\frac{(a-b\,x)^{-n}\,(a+b\,x)^{1+n}}{x}\,dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{(a-b\,x)^{1-n}\,(a+b\,x)^n}{2\,n}-\frac{a\,(a-b\,x)^{-n}\,(a+b\,x)^n\,\text{Hypergeometric2F1}\left[1,n,1+n,\frac{a+b\,x}{a-b\,x}\right]}{n}+\frac{1}{n\,(1+n)}$$
$$2^{-1-n}\,(1+2\,n)\,(a-b\,x)^{-n}\left(\frac{a-b\,x}{a}\right)^n\,(a+b\,x)^{1+n}\,\text{Hypergeometric2F1}\left[n,1+n,2+n,\frac{a+b\,x}{2\,a}\right]$$

Result (type 5, 173 leaves, 7 steps):

$$\frac{a\,(a-b\,x)^{-n}\,(a+b\,x)^{n}\,\text{Hypergeometric2F1}\left[1,\,-n,\,1-n,\,\frac{a-b\,x}{a+b\,x}\right]}{n}-\frac{1}{n}2^{n}\,a\,(a-b\,x)^{-n}\,(a+b\,x)^{n}\left(\frac{a+b\,x}{a}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,\frac{a-b\,x}{2\,a}\right]+\frac{1}{1+n}2^{-n}\,(a-b\,x)^{-n}\left(\frac{a-b\,x}{a}\right)^{n}(a+b\,x)^{1+n}\,\text{Hypergeometric2F1}\left[n,\,1+n,\,2+n,\,\frac{a+b\,x}{2\,a}\right]$$

Problem 1007: Result unnecessarily involves higher level functions.

$$\int\frac{(a-b\,x)^{-n}\,(a+b\,x)^{1+n}}{x^{2}}\,dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$-\frac{(a-b\,x)^{-n}\,(a+b\,x)^{1+n}}{x}+\frac{1}{n}b\,(1+2\,n)\,(a-b\,x)^{-n}\,(a+b\,x)^{n}\,\text{Hypergeometric2F1}\left[1,\,-n,\,1-n,\,\frac{a-b\,x}{a+b\,x}\right]-\frac{1}{n}2^{n}\,b\,(a-b\,x)^{-n}\,(a+b\,x)^{n}\left(\frac{a+b\,x}{a}\right)^{-n}\text{Hypergeometric2F1}\left[-n,\,-n,\,1-n,\,\frac{a-b\,x}{2\,a}\right]$$

Result (type 6, 76 leaves, 2 steps):

$$\frac{1}{a^{2}\,(2+n)}2^{-n}\,b\,(a-b\,x)^{-n}\left(\frac{a-b\,x}{a}\right)^{n}(a+b\,x)^{2+n}\,\text{AppellF1}\left[2+n,\,n,\,2,\,3+n,\,\frac{a+b\,x}{2\,a},\,\frac{a+b\,x}{a}\right]$$

Problem 2121: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(1-2\,x)^{3/2}\,(3+5\,x)^{2}}\,dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{6}{121\,\sqrt{1-2\,x}}-\frac{1}{11\,\sqrt{1-2\,x}\,(3+5\,x)}-\frac{6}{121}\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2\,x}\right]$$

Result (type 3, 70 leaves, 4 steps):

$$\frac{2}{11\,\sqrt{1-2\,x}\,(3+5\,x)}-\frac{15\,\sqrt{1-2\,x}}{121\,(3+5\,x)}-\frac{6}{121}\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2\,x}\right]$$

Problem 2144: Result valid but suboptimal antiderivative.

$$\int\frac{3+5\,x}{(1-2\,x)^{5/2}\,(2+3\,x)^{4}}\,dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$\frac{160}{3087\,(1-2x)^{3/2}}+\frac{160}{2401\sqrt{1-2x}}+\frac{1}{63\,(1-2x)^{3/2}\,(2+3x)^3}-\frac{16}{147\,(1-2x)^{3/2}\,(2+3x)^2}-\frac{16}{147\,(1-2x)^{3/2}\,(2+3x)}-\frac{160\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{2401}$$

Result (type 3, 130 leaves, 7 steps):

$$\frac{1}{63\,(1-2x)^{3/2}\,(2+3x)^3}+\frac{64}{441\,(1-2x)^{3/2}\,(2+3x)^2}+\frac{64}{147\sqrt{1-2x}\,(2+3x)^2}-\frac{80\sqrt{1-2x}}{343\,(2+3x)^2}-\frac{240\sqrt{1-2x}}{2401\,(2+3x)}-\frac{160\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{2401}$$

Problem 2145: Result valid but suboptimal antiderivative.

$$\int\frac{3+5x}{(1-2x)^{5/2}\,(2+3x)^5}\,dx$$

Optimal (type 3, 136 leaves, 8 steps):

$$\frac{215}{9604\,(1-2x)^{3/2}}+\frac{1935}{67228\sqrt{1-2x}}+\frac{1}{84\,(1-2x)^{3/2}\,(2+3x)^4}-\frac{43}{588\,(1-2x)^{3/2}\,(2+3x)^3}-\frac{129}{2744\,(1-2x)^{3/2}\,(2+3x)^2}-\frac{129}{2744\,(1-2x)^{3/2}\,(2+3x)}-\frac{1935\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{67228}$$

Result (type 3, 150 leaves, 8 steps):

$$\frac{1}{84\,(1-2x)^{3/2}\,(2+3x)^4}+\frac{43}{294\,(1-2x)^{3/2}\,(2+3x)^3}+\frac{387}{686\sqrt{1-2x}\,(2+3x)^3}-\frac{387\sqrt{1-2x}}{1372\,(2+3x)^3}-\frac{1935\sqrt{1-2x}}{19208\,(2+3x)^2}-\frac{5805\sqrt{1-2x}}{134456\,(2+3x)}-\frac{1935\sqrt{\frac{3}{7}}\,\text{ArcTanh}\left[\sqrt{\frac{3}{7}}\,\sqrt{1-2x}\right]}{67228}$$

Problem 2196: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(1-2x)^{5/2}\,(3+5x)^3}\,dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{35}{3993\,(1-2x)^{3/2}}+\frac{175}{14641\sqrt{1-2x}}-\frac{1}{22\,(1-2x)^{3/2}\,(3+5x)^2}-\frac{7}{242\,(1-2x)^{3/2}\,(3+5x)}-\frac{175\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2x}\right]}{14641}$$

Result (type 3, 110 leaves, 6 steps):

$$\frac{2}{33\,(1-2x)^{3/2}\,(3+5x)^2}+\frac{70}{363\sqrt{1-2x}\,(3+5x)^2}-\frac{875\sqrt{1-2x}}{7986\,(3+5x)^2}-\frac{875\sqrt{1-2x}}{29282\,(3+5x)}-\frac{175\sqrt{\frac{5}{11}}\,\text{ArcTanh}\left[\sqrt{\frac{5}{11}}\,\sqrt{1-2x}\right]}{14641}$$

Problem 3075: Result valid but suboptimal antiderivative.

$$\int\frac{(a+bx)^m\,(c+dx)^{-1-m}}{e+fx}\,dx$$

Optimal (type 5, 72 leaves, 1 step):

$$-\frac{(a+bx)^m\,(c+dx)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(be-af)\,(c+dx)}{(de-cf)\,(a+bx)}\right]}{(de-cf)\,m}$$

Result (type 5, 75 leaves, 1 step):

$$\left((a+bx)^{1+m}\,(c+dx)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(de-cf)\,(a+bx)}{(be-af)\,(c+dx)}\right]\right)\Big/\left((be-af)\,(1+m)\right)$$

Problem 3077: Result valid but suboptimal antiderivative.

$$\int\frac{(a+bx)^m\,(c+dx)^{-1-m}}{(e+fx)^3}\,dx$$

Optimal (type 5, 283 leaves, 4 steps):

$$-\frac{f\,(a+bx)^{1+m}\,(c+dx)^{-m}}{2\,(be-af)\,(de-cf)\,(e+fx)^2}-\frac{f\,(b\,(3de-cf\,(1-m))-adf\,(2+m))\,(a+bx)^{1+m}\,(c+dx)^{-m}}{2\,(be-af)^2\,(de-cf)^2\,(e+fx)}+$$
$$\Big(\left(2abdf\,(1+m)\,(2de+cfm)-b^2\,(2d^2e^2+4cdefm-c^2f^2\,(1-m)\,m)-a^2d^2f^2\,(2+3m+m^2)\right)$$
$$(a+bx)^m\,(c+dx)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(be-af)\,(c+dx)}{(de-cf)\,(a+bx)}\right]\Big)\Big/$$
$$\left(2\,(be-af)^2\,(de-cf)^3\,m\right)$$

Result (type 5, 300 leaves, 4 steps):

$$-\frac{f\,(a\,d\,f\,(2+m)-b\,(2\,d\,e+c\,f\,m))\,(a+b\,x)^{1+m}\,(c+d\,x)^{1-m}}{2\,(b\,c-a\,d)\,(b\,e-a\,f)\,(d\,e-c\,f)^2\,m\,(e+f\,x)^2}+\frac{d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{(b\,c-a\,d)\,(d\,e-c\,f)\,m\,(e+f\,x)^2}+\left(\left(2\,a\,b\,d\,f\,(1+m)\,(2\,d\,e+c\,f\,m)-b^2\,\left(2\,d^2\,e^2+4\,c\,d\,e\,f\,m-c^2\,f^2\,(1-m)\,m\right)-a^2\,d^2\,f^2\,\left(2+3\,m+m^2\right)\right)(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\,\text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]\right)\Big/\left(2\,(b\,e-a\,f)^3\,(d\,e-c\,f)^2\,m\,(1+m)\right)$$

Problem 3078: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{-1-m}}{(e+f\,x)^4}\,dx$$

Optimal (type 5, 498 leaves, 5 steps):

$$-\frac{f\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{3\,(b\,e-a\,f)\,(d\,e-c\,f)\,(e+f\,x)^3}-\frac{f\,(b\,(5\,d\,e-c\,f\,(2-m))-a\,d\,f\,(3+m))\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{6\,(b\,e-a\,f)^2\,(d\,e-c\,f)^2\,(e+f\,x)^2}-\left(f\,\left(a^2\,d^2\,f^2\,\left(6+5\,m+m^2\right)-a\,b\,d\,f\,\left(d\,e\,(15+8\,m)-c\,f\,\left(3-2\,m-2\,m^2\right)\right)+b^2\,\left(11\,d^2\,e^2-c\,d\,e\,f\,(7-8\,m)+c^2\,f^2\,\left(2-3\,m+m^2\right)\right)\right)(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\right)\Big/\left(6\,(b\,e-a\,f)^3\,(d\,e-c\,f)^3\,(e+f\,x)\right)+\left(\left(3\,a\,b^2\,d\,f\,(1+m)\,\left(6\,d^2\,e^2+6\,c\,d\,e\,f\,m-c^2\,f^2\,(1-m)\,m\right)-3\,a^2\,b\,d^2\,f^2\,(3\,d\,e+c\,f\,m)\,\left(2+3\,m+m^2\right)+a^3\,d^3\,f^3\,\left(6+11\,m+6\,m^2+m^3\right)-b^3\,\left(6\,d^3\,e^3+18\,c\,d^2\,e^2\,f\,m-9\,c^2\,d\,e\,f^2\,(1-m)\,m+c^3\,f^3\,m\,\left(2-3\,m+m^2\right)\right)\right)(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]\right)\Big/\left(6\,(b\,e-a\,f)^3\,(d\,e-c\,f)^4\,m\right)$$

Result (type 5, 520 leaves, 5 steps):

$$-\frac{f\,(a\,d\,f\,(3+m)-b\,(3\,d\,e+c\,f\,m))\,(a+b\,x)^{1+m}\,(c+d\,x)^{1-m}}{3\,(b\,c-a\,d)\,(b\,e-a\,f)\,(d\,e-c\,f)^2\,m\,(e+f\,x)^3}+\frac{d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{(b\,c-a\,d)\,(d\,e-c\,f)\,m\,(e+f\,x)^3}+\left(f\,\left(b^2\,\left(6\,d^2\,e^2+7\,c\,d\,e\,f\,m-c^2\,f^2\,(2-m)\,m\right)+a^2\,d^2\,f^2\,\left(6+5\,m+m^2\right)-a\,b\,d\,f\,(c\,f\,m\,(3+2\,m)+d\,e\,(12+7\,m))\right)(a+b\,x)^{1+m}\,(c+d\,x)^{1-m}\right)\Big/\left(6\,(b\,c-a\,d)\,(b\,e-a\,f)^2\,(d\,e-c\,f)^3\,m\,(e+f\,x)^2\right)+\left(\left(3\,a\,b^2\,d\,f\,(1+m)\,\left(6\,d^2\,e^2+6\,c\,d\,e\,f\,m-c^2\,f^2\,(1-m)\,m\right)-3\,a^2\,b\,d^2\,f^2\,(3\,d\,e+c\,f\,m)\,\left(2+3\,m+m^2\right)+a^3\,d^3\,f^3\,\left(6+11\,m+6\,m^2+m^3\right)-b^3\,\left(6\,d^3\,e^3+18\,c\,d^2\,e^2\,f\,m-9\,c^2\,d\,e\,f^2\,(1-m)\,m+c^3\,f^3\,m\,\left(2-3\,m+m^2\right)\right)\right)(a+b\,x)^{1+m}\,(c+d\,x)^{-1-m}\,\text{Hypergeometric2F1}\left[2,1+m,2+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]\right)\Big/\left(6\,(b\,e-a\,f)^4\,(d\,e-c\,f)^3\,m\,(1+m)\right)$$

Problem 3084: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{e+f x} \, dx$$

Optimal (type 5, 120 leaves, 2 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (d e-c f) (1+m)}+\frac{1}{(d e-c f)^2 m} f (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]$$

Result (type 5, 135 leaves, 2 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (d e-c f) (1+m)}-\left(f (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left((b e-a f) (d e-c f) (1+m)\right)$$

Problem 3085: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{(e+f x)^2} \, dx$$

Optimal (type 5, 233 leaves, 4 steps):

$$-\frac{d (a d f (2+m)-b (d e+c f (1+m))) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (b e-a f) (d e-c f)^2 (1+m)}-\frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{(b e-a f) (d e-c f) (e+f x)}-\left(f (a d f (2+m)-b (2 d e+c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]\right) \Big/ \left((b e-a f) (d e-c f)^3 m\right)$$

Result (type 5, 243 leaves, 4 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (d e-c f) (1+m) (e+f x)}+\frac{f (b d e+b c f (1+m)-a d f (2+m)) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) (b e-a f) (d e-c f)^2 (1+m) (e+f x)}+\left(f (a d f (2+m)-b (2 d e+c f m)) (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left((b e-a f)^2 (d e-c f)^2 (1+m)\right)$$

Problem 3086: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-2-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 432 leaves, 5 steps):

$$\Big(d \left(a^2 d^2 f^2 \left(6+5 m+m^2\right)+b^2 \left(2 d^2 e^2+5 c d e f (1+m)-c^2 f^2 \left(1-m^2\right)\right)-a b d f \left(d e (9+5 m)+c f \left(3+5 m+2 m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m}\Big) \Big/ \left(2 (b c-a d) (b e-a f)^2 (d e-c f)^3 (1+m)\right)-\frac{f (a+b x)^{1+m} (c+d x)^{-1-m}}{2 (b e-a f) (d e-c f) (e+f x)^2}-\frac{f (b (4 d e-c f (1-m))-a d f (3+m)) (a+b x)^{1+m} (c+d x)^{-1-m}}{2 (b e-a f)^2 (d e-c f)^2 (e+f x)}-\Big(f \left(2 a b d f (2+m) (3 d e+c f m)-b^2 \left(6 d^2 e^2+6 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(6+5 m+m^2\right)\right) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]\Big) \Big/ \left(2 (b e-a f)^2 (d e-c f)^4 m\right)$$

Result (type 5, 452 leaves, 5 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d) (d e-c f) (1+m) (e+f x)^2}+\frac{f (2 b d e+b c f (1+m)-a d f (3+m)) (a+b x)^{1+m} (c+d x)^{-m}}{2 (b c-a d) (b e-a f) (d e-c f)^2 (1+m) (e+f x)^2}+\Big(\left(f \left(a^2 d^2 f^2 \left(6+5 m+m^2\right)+b^2 \left(2 d^2 e^2+5 c d e f (1+m)-c^2 f^2 \left(1-m^2\right)\right)-a b d f \left(d e (9+5 m)+c f \left(3+5 m+2 m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-m}\right) \Big/ \left(2 (b c-a d) (b e-a f)^2 (d e-c f)^3 (1+m) (e+f x)\right)+\Big(f \left(2 a b d f (2+m) (3 d e+c f m)-b^2 \left(6 d^2 e^2+6 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(6+5 m+m^2\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\Big) \Big/ \left(2 (b e-a f)^3 (d e-c f)^3 (1+m)\right)$$

Problem 3093: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-3-m}}{e+f x} \, dx$$

Optimal (type 5, 196 leaves, 4 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c-a d) (d e-c f) (2+m)}+\frac{d (a d f (2+m)+b (d e-c f (3+m))) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d)^2 (d e-c f)^2 (1+m) (2+m)}-\frac{1}{(d e-c f)^3 m}f^2 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]$$

Result (type 5, 208 leaves, 4 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c-a d) (d e-c f) (2+m)}+\frac{d (b d e+a d f (2+m)-b c f (3+m)) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c-a d)^2 (d e-c f)^2 (1+m) (2+m)}+\left(f^2 (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right)\Big/\left((b e-a f) (d e-c f)^2 (1+m)\right)$$

Problem 3094: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-3-m}}{(e+f x)^2} \, dx$$

Optimal (type 5, 384 leaves, 5 steps):

$$-\frac{d (a d f (3+m)-b (d e+c f (2+m))) (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c-a d) (b e-a f) (d e-c f)^2 (2+m)}-\left(d \left(a^2 d^2 f^2 \left(6+5 m+m^2\right)-b^2 \left(d^2 e^2-c d e f (5+2 m)-c^2 f^2 \left(2+3 m+m^2\right)\right)-a b d f \left(d e (3+2 m)+c f \left(9+8 m+2 m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m}\right)\Big/\left((b c-a d)^2 (b e-a f) (d e-c f)^3 (1+m) (2+m)\right)-\frac{f (a+b x)^{1+m} (c+d x)^{-2-m}}{(b e-a f) (d e-c f) (e+f x)}+\left(f^2 (a d f (3+m)-b (3 d e+c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]\right)\Big/\left((b e-a f) (d e-c f)^4 m\right)$$

Result (type 5, 398 leaves, 5 steps):

$$-\left(\left(d\left(a^2 d^2 f^2\left(6+5 m+m^2\right)-b^2\left(d^2 e^2-c d e f(5+2 m)-c^2 f^2\left(2+3 m+m^2\right)\right)-a b d f\left(d e(3+2 m)+c f\left(9+8 m+2 m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-1-m}\right)\Big/\left((b c-a d)^2(b e-a f)(d e-c f)^3(1+m)(2+m)\right)\right)+\frac{d(a+b x)^{1+m}(c+d x)^{-2-m}}{(b c-a d)(d e-c f)(2+m)(e+f x)}+\frac{f(b d e+b c f(2+m)-a d f(3+m))(a+b x)^{1+m}(c+d x)^{-1-m}}{(b c-a d)(b e-a f)(d e-c f)^2(2+m)(e+f x)}-\left(f^2(a d f(3+m)-b(3 d e+c f m))(a+b x)^{1+m}(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right)\Big/\left((b e-a f)^2(d e-c f)^3(1+m)\right)$$

Problem 3101: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-4-m}}{e+f x}\,dx$$

Optimal (type 5, 330 leaves, 5 steps):

$$\frac{d(a+b x)^{1+m}(c+d x)^{-3-m}}{(b c-a d)(d e-c f)(3+m)}+\frac{d(a d f(3+m)+b(2 d e-c f(5+m)))(a+b x)^{1+m}(c+d x)^{-2-m}}{(b c-a d)^2(d e-c f)^2(2+m)(3+m)}+\left(d\left(a^2 d^2 f^2\left(6+5 m+m^2\right)+a b d f(3+m)(d e-c f(5+2 m))+b^2\left(2 d^2 e^2-c d e f(7+m)+c^2 f^2\left(11+6 m+m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-1-m}\right)\Big/\left((b c-a d)^3(d e-c f)^3(1+m)(2+m)(3+m)\right)+\frac{1}{(d e-c f)^4 m} f^3(a+b x)^m(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]$$

Result (type 5, 344 leaves, 5 steps):

$$\frac{d(a+b x)^{1+m}(c+d x)^{-3-m}}{(b c-a d)(d e-c f)(3+m)}+\frac{d(2 b d e+a d f(3+m)-b c f(5+m))(a+b x)^{1+m}(c+d x)^{-2-m}}{(b c-a d)^2(d e-c f)^2(2+m)(3+m)}+\left(d\left(a^2 d^2 f^2\left(6+5 m+m^2\right)+a b d f(3+m)(d e-c f(5+2 m))+b^2\left(2 d^2 e^2-c d e f(7+m)+c^2 f^2\left(11+6 m+m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-1-m}\right)\Big/\left((b c-a d)^3(d e-c f)^3(1+m)(2+m)(3+m)\right)-\left(f^3(a+b x)^{1+m}(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right)\Big/\left((b e-a f)(d e-c f)^3(1+m)\right)$$

Problem 3102: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-4-m}}{(e+f x)^2}\, dx$$

Optimal (type 5, 634 leaves, 6 steps):

$$-\frac{d\,(a d f\,(4+m)-b\,(d e+c f\,(3+m)))\,(a+b x)^{1+m}\,(c+d x)^{-3-m}}{(b c-a d)\,(b e-a f)\,(d e-c f)^2\,(3+m)}-\Big(d\,\big(a^2 d^2 f^2\,(12+7 m+m^2)-b^2\,(2 d^2 e^2-2 c d e f\,(4+m)-c^2 f^2\,(6+5 m+m^2))-2 a b d f\,(d e\,(2+m)+c f\,(10+6 m+m^2))\big)\,(a+b x)^{1+m}\,(c+d x)^{-2-m}\Big)\Big/\Big((b c-a d)^2\,(b e-a f)\,(d e-c f)^3\,(2+m)\,(3+m)\Big)-\Big(d\,\big(a^3 d^3 f^3\,(24+26 m+9 m^2+m^3)-a^2 b d^2 f^2\,(3+m)\,(d e\,(4+3 m)+c f\,(20+15 m+3 m^2))-b^3\,(2 d^3 e^3-2 c d^2 e^2 f\,(5+m)+c^2 d e f^2\,(26+17 m+3 m^2)+c^3 f^3\,(6+11 m+6 m^2+m^3))-a b^2 d f\,(2 d^2 e^2\,(2+m)-2 c d e f\,(16+15 m+3 m^2)-c^2 f^2\,(44+50 m+21 m^2+3 m^3))\big)\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\Big)\Big/\Big((b c-a d)^3\,(b e-a f)\,(d e-c f)^4\,(1+m)\,(2+m)\,(3+m)\Big)-\frac{f\,(a+b x)^{1+m}\,(c+d x)^{-3-m}}{(b e-a f)\,(d e-c f)\,(e+f x)}-\Big(f^3\,(a d f\,(4+m)-b\,(4 d e+c f m))\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\Big[1,-m,1-m,\frac{(b e-a f)\,(c+d x)}{(d e-c f)\,(a+b x)}\Big]\Big)\Big/\Big((b e-a f)\,(d e-c f)^5\,m\Big)$$

Result (type 5, 646 leaves, 6 steps):

$$-\Big(\Big(d\,\big(a^2 d^2 f^2\,(12+7 m+m^2)-b^2\,(2 d^2 e^2-2 c d e f\,(4+m)-c^2 f^2\,(6+5 m+m^2))-2 a b d f\,(d e\,(2+m)+c f\,(10+6 m+m^2))\big)\,(a+b x)^{1+m}\,(c+d x)^{-2-m}\Big)\Big/\Big((b c-a d)^2\,(b e-a f)\,(d e-c f)^3\,(2+m)\,(3+m)\Big)\Big)-\Big(d\,\big(a^3 d^3 f^3\,(24+26 m+9 m^2+m^3)-a^2 b d^2 f^2\,(3+m)\,(d e\,(4+3 m)+c f\,(20+15 m+3 m^2))-b^3\,(2 d^3 e^3-2 c d^2 e^2 f\,(5+m)+c^2 d e f^2\,(26+17 m+3 m^2)+c^3 f^3\,(6+11 m+6 m^2+m^3))-a b^2 d f\,(2 d^2 e^2\,(2+m)-2 c d e f\,(16+15 m+3 m^2)-c^2 f^2\,(44+50 m+21 m^2+3 m^3))\big)\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\Big)\Big/\Big((b c-a d)^3\,(b e-a f)\,(d e-c f)^4\,(1+m)\,(2+m)\,(3+m)\Big)+\frac{d\,(a+b x)^{1+m}\,(c+d x)^{-3-m}}{(b c-a d)\,(d e-c f)\,(3+m)\,(e+f x)}+\frac{f\,(b d e+b c f\,(3+m)-a d f\,(4+m))\,(a+b x)^{1+m}\,(c+d x)^{-2-m}}{(b c-a d)\,(b e-a f)\,(d e-c f)^2\,(3+m)\,(e+f x)}+\Big(f^3\,(a d f\,(4+m)-b\,(4 d e+c f m))\,(a+b x)^{1+m}\,(c+d x)^{-1-m}\,\text{Hypergeometric2F1}\Big[1,1+m,2+m,\frac{(d e-c f)\,(a+b x)}{(b e-a f)\,(c+d x)}\Big]\Big)\Big/\Big((b e-a f)^2\,(d e-c f)^4\,(1+m)\Big)$$

Problem 3110: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-5-m}}{e+f x} \, dx$$

Optimal (type 5, 557 leaves, 6 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-4-m}}{(b c-a d) (d e-c f) (4+m)}+\frac{d (a d f (4+m)+b (3 d e-c f (7+m))) (a+b x)^{1+m} (c+d x)^{-3-m}}{(b c-a d)^2 (d e-c f)^2 (3+m) (4+m)}+$$
$$\left(d \left(a^2 d^2 f^2 \left(12+7 m+m^2\right)+2 a b d f (4+m) (d e-c f (4+m))+b^2 \left(6 d^2 e^2-2 c d e f (10+m)+c^2 f^2 \left(26+9 m+m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-2-m}\right) \Big/ \left((b c-a d)^3 (d e-c f)^3 (2+m) (3+m) (4+m)\right)+$$
$$\left(d \left(a^3 d^3 f^3 \left(24+26 m+9 m^2+m^3\right)+a^2 b d^2 f^2 \left(12+7 m+m^2\right) (d e-c f (7+3 m))+a b^2 d f (4+m) \left(2 d^2 e^2-2 c d e f (5+m)+c^2 f^2 \left(26+17 m+3 m^2\right)\right)+b^3 \left(6 d^3 e^3-2 c d^2 e^2 f (13+m)+c^2 d e f^2 \left(46+11 m+m^2\right)-c^3 f^3 \left(50+35 m+10 m^2+m^3\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m}\right) \Big/ \left((b c-a d)^4 (d e-c f)^4 (1+m) (2+m) (3+m) (4+m)\right)-$$
$$\frac{1}{(d e-c f)^5 m} f^4 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]$$

Result (type 5, 569 leaves, 6 steps):

$$\frac{d (a+b x)^{1+m} (c+d x)^{-4-m}}{(b c-a d) (d e-c f) (4+m)}+\frac{d (3 b d e+a d f (4+m)-b c f (7+m)) (a+b x)^{1+m} (c+d x)^{-3-m}}{(b c-a d)^2 (d e-c f)^2 (3+m) (4+m)}+$$
$$\left(d \left(a^2 d^2 f^2 \left(12+7 m+m^2\right)+2 a b d f (4+m) (d e-c f (4+m))+b^2 \left(6 d^2 e^2-2 c d e f (10+m)+c^2 f^2 \left(26+9 m+m^2\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-2-m}\right) \Big/ \left((b c-a d)^3 (d e-c f)^3 (2+m) (3+m) (4+m)\right)+$$
$$\left(d \left(a^3 d^3 f^3 \left(24+26 m+9 m^2+m^3\right)+a^2 b d^2 f^2 \left(12+7 m+m^2\right) (d e-c f (7+3 m))+a b^2 d f (4+m) \left(2 d^2 e^2-2 c d e f (5+m)+c^2 f^2 \left(26+17 m+3 m^2\right)\right)+b^3 \left(6 d^3 e^3-2 c d^2 e^2 f (13+m)+c^2 d e f^2 \left(46+11 m+m^2\right)-c^3 f^3 \left(50+35 m+10 m^2+m^3\right)\right)\right) (a+b x)^{1+m} (c+d x)^{-1-m}\right) \Big/ \left((b c-a d)^4 (d e-c f)^4 (1+m) (2+m) (3+m) (4+m)\right)+$$
$$\left(f^4 (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1,1+m,2+m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left((b e-a f) (d e-c f)^4 (1+m)\right)$$

Problem 3116: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{1-m}}{e+f x} \, dx$$

Optimal (type 5, 230 leaves, 6 steps):

$$-\frac{d\,(d\,e-c\,f)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}}{(b\,c-a\,d)\,f^2\,m}-\frac{1}{f^2\,m}(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b\,e-a\,f)\,(c+d\,x)}{(d\,e-c\,f)\,(a+b\,x)}\right]+\left(d\,(b\,(d\,e-c\,f\,(1-m))-a\,d\,f\,m)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\right)\Big/\left(b\,(b\,c-a\,d)\,f^2\,m\,(1+m)\right)$$

Result (type 5, 220 leaves, 7 steps):

$$\frac{1}{f^2\,m}(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]-\frac{1}{f^2\,m}(d\,e-c\,f)\,(a+b\,x)^m\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]+\frac{1}{b\,f\,(1+m)}d\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]$$

Problem 3117: Result unnecessarily involves higher level functions.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{1-m}}{(e+f\,x)^2}\,dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$-\frac{(a+b\,x)^m\,(c+d\,x)^{1-m}}{f\,(e+f\,x)}+\frac{1}{f^2\,(b\,e-a\,f)\,m}(a\,d\,f\,(1-m)-b\,(d\,e-c\,f\,m))\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]+\frac{1}{f^2\,m}d\,(a+b\,x)^m\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]$$

Result (type 6, 108 leaves, 2 steps):

$$\left((b\,c-a\,d)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\text{AppellF1}\left[1+m,-1+m,2,2+m,-\frac{d\,(a+b\,x)}{b\,c-a\,d},-\frac{f\,(a+b\,x)}{b\,e-a\,f}\right]\right)\Big/\left((b\,e-a\,f)^2\,(1+m)\right)$$

Problem 3127: Result valid but suboptimal antiderivative.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{2-m}}{e+f\,x}\,dx$$

Optimal (type 5, 370 leaves, 6 steps):

$$-\left(\left(d\left(2 a b c d f^2 m-a^2 d^2 f^2 m-b^2\left(2 d^2 e^2-4 c d e f+c^2 f^2(2+m)\right)\right)(a+b x)^{1+m}(c+d x)^{-m}\right)\Big/\left(2 b^2(b c-a d) f^3 m\right)\right)+\frac{d^2(a+b x)^{2+m}(c+d x)^{-m}}{2 b^2 f}+\frac{1}{f^3 m}(d e-c f)^2(a+b x)^m(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]+\left(d\left(2 a b d f(d e-c f(2-m)) m+a^2 d^2 f^2(1-m) m-b^2\left(2 d^2 e^2-2 c d e f(2-m)+c^2 f^2\left(2-3 m+m^2\right)\right)\right)(a+b x)^{1+m}(c+d x)^{-m}\left(\frac{b(c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d(a+b x)}{b c-a d}\right]\right)\Big/\left(2 b^2(b c-a d) f^3 m(1+m)\right)$$

Result (type 5, 319 leaves, 10 steps):

$$-\frac{1}{f^3 m}(d e-c f)^2(a+b x)^m(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,m,1+m,\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]+\frac{1}{b^2 f(1+m)}d(b c-a d)(a+b x)^{1+m}(c+d x)^{-m}\left(\frac{b(c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,1+m,2+m,-\frac{d(a+b x)}{b c-a d}\right]+\frac{1}{f^3 m}(d e-c f)^2(a+b x)^m(c+d x)^{-m}\left(\frac{b(c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d(a+b x)}{b c-a d}\right]-\frac{1}{b f^2(1+m)}d(d e-c f)(a+b x)^{1+m}(c+d x)^{-m}\left(\frac{b(c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d(a+b x)}{b c-a d}\right]$$

Problem 3128: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(e+f x)^2}\,dx$$

Optimal (type 5, 316 leaves, 7 steps):

$$-\frac{2 d^2(d e-c f)(a+b x)^{1+m}(c+d x)^{-m}}{(b c-a d) f^3 m}+\frac{(d e-c f)^2(a+b x)^{1+m}(c+d x)^{-m}}{f^2(b e-a f)(e+f x)}+\frac{1}{f^3(b e-a f) m}(d e-c f)(a d f(2-m)-b(2 d e-c f m))(a+b x)^m(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]+\left(d^2(b(2 d e-c f(2-m))-a d f m)(a+b x)^{1+m}(c+d x)^{-m}\left(\frac{b(c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d(a+b x)}{b c-a d}\right]\right)\Big/\left(b(b c-a d) f^3 m(1+m)\right)$$

Result (type 6, 113 leaves, 2 steps):

$$\left((b c-a d)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-2+m,2,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]\right) \Big/ \left(b (b e-a f)^2 (1+m)\right)$$

Problem 3129: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(e+f x)^3} \, dx$$

Optimal (type 5, 362 leaves, 7 steps):

$$\frac{(b e-a f) (a+b x)^{-1+m} (c+d x)^{2-m}}{2 f^2 (e+f x)^2}+\frac{(a d f (2-m)-b (3 d e-c f (1+m))) (a+b x)^{-1+m} (c+d x)^{2-m}}{2 f^2 (d e-c f) (e+f x)}-\left(\left(2 a b d f (2-m) (d e-c f m)-b^2 \left(2 d^2 e^2-2 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(2-3 m+m^2\right)\right) (a+b x)^{-1+m} (c+d x)^{1-m} \text{Hypergeometric2F1}\left[1,-1+m,m,\frac{(d e-c f) (a+b x)}{(b e-a f) (c+d x)}\right]\right) \Big/ \left(2 f^3 (b e-a f) (d e-c f) (1-m)\right)-\frac{1}{f^3 (1-m)} d (b c-a d) (a+b x)^{-1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,-1+m,m,-\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 6, 110 leaves, 2 steps):

$$\left((b c-a d)^2 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m,-2+m,3,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]\right) \Big/ \left((b e-a f)^3 (1+m)\right)$$

Problem 3134: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{3-m}}{e+f x} \, dx$$

Optimal (type 5, 488 leaves, 7 steps):

$$\frac{b\,(b\,e-a\,f)^3\,(a+b\,x)^{-3+m}\,(c+d\,x)^{4-m}}{(b\,c-a\,d)\,f^4\,(3-m)}-$$
$$\frac{b\,(b\,(3\,d\,e-c\,f\,(1-m))-a\,d\,f\,(2+m))\,(a+b\,x)^{-2+m}\,(c+d\,x)^{4-m}}{6\,d^2\,f^2}+$$
$$\frac{b\,(a+b\,x)^{-1+m}\,(c+d\,x)^{4-m}}{3\,d\,f}-\frac{1}{f^4\,(3-m)}$$
$$(b\,e-a\,f)^3\,(a+b\,x)^{-3+m}\,(c+d\,x)^{3-m}\,\text{Hypergeometric2F1}\left[1,\,-3+m,\,-2+m,\,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]-$$
$$\frac{1}{6\,b^3\,d^2\,f^4\,(2-m)\,(3-m)}\,(b\,c-a\,d)^2\,(3\,a^2\,b\,d^2\,f^2\,(d\,e-c\,f\,(3-m))\,(1-m)\,m+$$
$$a^3\,d^3\,f^3\,m\,(2-3\,m+m^2)+3\,a\,b^2\,d\,f\,m\,(2\,d^2\,e^2-2\,c\,d\,e\,f\,(3-m)+c^2\,f^2\,(6-5\,m+m^2))-$$
$$b^3\,(6\,d^3\,e^3-6\,c\,d^2\,e^2\,f\,(3-m)+3\,c^2\,d\,e\,f^2\,(6-5\,m+m^2)-c^3\,f^3\,(6-11\,m+6\,m^2-m^3)))\,(a+b\,x)^{-2+m}$$
$$(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[-3+m,\,-2+m,\,-1+m,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]$$

Result (type 5, 417 leaves, 13 steps):

$$\frac{1}{f^4\,m}(d\,e-c\,f)^3\,(a+b\,x)^m\,(c+d\,x)^{-m}\,\text{Hypergeometric2F1}\left[1,\,m,\,1+m,\,\frac{(d\,e-c\,f)\,(a+b\,x)}{(b\,e-a\,f)\,(c+d\,x)}\right]+$$
$$\frac{1}{b^3\,f\,(1+m)}d\,(b\,c-a\,d)^2\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m$$
$$\text{Hypergeometric2F1}\left[-2+m,\,1+m,\,2+m,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]-$$
$$\frac{1}{b^2\,f^2\,(1+m)}d\,(b\,c-a\,d)\,(d\,e-c\,f)\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m$$
$$\text{Hypergeometric2F1}\left[-1+m,\,1+m,\,2+m,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]-\frac{1}{f^4\,m}(d\,e-c\,f)^3\,(a+b\,x)^m$$
$$(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[m,\,m,\,1+m,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]+\frac{1}{b\,f^3\,(1+m)}$$
$$d\,(d\,e-c\,f)^2\,(a+b\,x)^{1+m}\,(c+d\,x)^{-m}\left(\frac{b\,(c+d\,x)}{b\,c-a\,d}\right)^m\,\text{Hypergeometric2F1}\left[m,\,1+m,\,2+m,\,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]$$

Problem 3136: Result unnecessarily involves higher level functions.

$$\int\frac{(a+b\,x)^m\,(c+d\,x)^{3-m}}{(e+f\,x)^3}\,dx$$

Optimal (type 5, 453 leaves, 8 steps):

$$-\frac{3 d^3 (d e-c f) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) f^4 m}-\frac{(d e-c f)^3 (a+b x)^{1+m} (c+d x)^{-m}}{2 f^3 (b e-a f) (e+f x)^2}+$$
$$\left((d e-c f)^2 (b (5 d e+c f (1-m))-a d f (6-m)) (a+b x)^{1+m} (c+d x)^{-m}\right)\Big/$$
$$\left(2 f^3 (b e-a f)^2 (e+f x)\right)+\frac{1}{2 f^4 (b e-a f)^2 m}(d e-c f)$$
$$\left(2 a b d f (3-m) (2 d e-c f m)-b^2 \left(6 d^2 e^2-4 c d e f m-c^2 f^2 (1-m) m\right)-a^2 d^2 f^2 \left(6-5 m+m^2\right)\right)$$
$$(a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]+$$
$$\left(d^3 (b (3 d e-c f (3-m))-a d f m) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m\right.$$
$$\left.\text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]\right)\Big/\left(b (b c-a d) f^4 m (1+m)\right)$$

Result (type 6, 113 leaves, 2 steps):

$$\left((b c-a d)^3 (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m\right.$$
$$\left.\text{AppellF1}\left[1+m,-3+m,3,2+m,-\frac{d (a+b x)}{b c-a d},-\frac{f (a+b x)}{b e-a f}\right]\right)\Big/\left(b (b e-a f)^3 (1+m)\right)$$

Problem 3137: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{b c+a d+2 b d x}\, dx$$

Optimal (type 5, 245 leaves, 6 steps):

$$\frac{(b c-a d) (3-2 n) (a+b x)^{2-n} (c+d x)^{-1+n}}{8 b^3 (1-n)}+\frac{d (a+b x)^{3-n} (c+d x)^{-1+n}}{4 b^3}+\frac{1}{8 b^3 d (1-n)}$$
$$(b c-a d)^2 (a+b x)^{1-n} (c+d x)^{-1+n} \text{Hypergeometric2F1}\left[1,-1+n,n,-\frac{b (c+d x)}{d (a+b x)}\right]-$$
$$\frac{1}{8 b^2 d^2 (1-n) n}(b c-a d)^2 \left(1-2 n^2\right) (a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n$$
$$(c+d x)^n \text{Hypergeometric2F1}\left[-1+n,n,1+n,\frac{b (c+d x)}{b c-a d}\right]$$

Result (type 5, 319 leaves, 10 steps):

$$-\frac{1}{8 b^2 d^2 n}(b c-a d)^2 (a+b x)^{-n} (c+d x)^n \text{Hypergeometric2F1}\left[1,-n,1-n,-\frac{d (a+b x)}{b (c+d x)}\right]+\frac{1}{8 b^2 d^2 n}$$
$$(b c-a d)^2 (a+b x)^{-n} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{Hypergeometric2F1}\left[-n,-n,1-n,-\frac{d (a+b x)}{b c-a d}\right]-$$
$$\frac{1}{4 b d^2 (1+n)}(b c-a d) (a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{1+n}$$
$$\text{Hypergeometric2F1}\left[n,1+n,2+n,\frac{b (c+d x)}{b c-a d}\right]+\frac{1}{2 d^2 (2+n)}$$
$$(a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{2+n} \text{Hypergeometric2F1}\left[n,2+n,3+n,\frac{b (c+d x)}{b c-a d}\right]$$

Problem 3138: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{(b c+a d+2 b d x)^2} \, dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$-\frac{1}{4 b^3 d (1-n)}(b c-a d) (a+b x)^{1-n} (c+d x)^{-1+n} \text{Hypergeometric2F1}\left[2,1-n,2-n,-\frac{d (a+b x)}{b (c+d x)}\right]+$$
$$\frac{1}{4 b d^2 (1+n)}(a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^{1+n} \text{Hypergeometric2F1}\left[n,1+n,2+n,\frac{b (c+d x)}{b c-a d}\right]$$

Result (type 6, 113 leaves, 2 steps):

$$\left((a+b x)^{2-n} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n}\right.$$
$$\left.\text{AppellF1}\left[2-n,-1-n,2,3-n,-\frac{d (a+b x)}{b c-a d},-\frac{2 d (a+b x)}{b c-a d}\right]\right)\Big/\left(b^2 (b c-a d) (2-n)\right)$$

Problem 3139: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^{1-n} (c+d x)^{1+n}}{(b c+a d+2 b d x)^3} \, dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$-\frac{(b c-a d) (a+b x)^{1-n} (c+d x)^n}{8 b^2 d (b c+a d+2 b d x)^2}-\frac{(1+2 n) (a+b x)^{1-n} (c+d x)^n}{8 b^2 d (b c+a d+2 b d x)}-\frac{1}{8 b^2 d^2 n}$$
$$\left(1-2 n^2\right) (a+b x)^{-n} (c+d x)^n \text{Hypergeometric2F1}\left[1,n,1+n,-\frac{b (c+d x)}{d (a+b x)}\right]+\frac{1}{8 b^2 d^2 n}$$
$$(a+b x)^{-n} \left(-\frac{d (a+b x)}{b c-a d}\right)^n (c+d x)^n \text{Hypergeometric2F1}\left[n,n,1+n,\frac{b (c+d x)}{b c-a d}\right]$$

Result (type 6, 113 leaves, 2 steps):

$$\left((a+b x)^{2-n} (c+d x)^n \left(\frac{b (c+d x)}{b c-a d}\right)^{-n} \text{AppellF1}\left[2-n,-1-n,3,3-n,-\frac{d (a+b x)}{b c-a d},-\frac{2 d (a+b x)}{b c-a d}\right]\right) \Big/ \left(b^2 (b c-a d)^2 (2-n)\right)$$

Problem 3141: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{b c+a d+2 b d x} \, dx$$

Optimal (type 5, 231 leaves, 6 steps):

$$\frac{(b c-a d) (1+2 m) (a+b x)^{1+m} (c+d x)^{-m}}{8 b^3 m}+\frac{d (a+b x)^{2+m} (c+d x)^{-m}}{4 b^3}+\frac{1}{8 b^3 d m} (b c-a d)^2 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,-\frac{b (c+d x)}{d (a+b x)}\right]-\frac{1}{8 b^3 m (1+m)} (b c-a d) \left(1-4 m+2 m^2\right) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 5, 314 leaves, 10 steps):

$$-\frac{1}{8 b^3 d m} (b c-a d)^2 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,m,1+m,-\frac{d (a+b x)}{b (c+d x)}\right]+\frac{1}{2 b^3 (1+m)} (b c-a d) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]+\frac{1}{8 b^3 d m} (b c-a d)^2 (a+b x)^m (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,m,1+m,-\frac{d (a+b x)}{b c-a d}\right]+\frac{1}{4 b^3 (1+m)} (b c-a d) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]$$

Problem 3142: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(b c+a d+2 b d x)^2} \, dx$$

Optimal (type 5, 144 leaves, 4 steps):

$$-\frac{1}{4 b^3 d m}(b c-a d)(a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[2, m, 1+m, -\frac{d (a+b x)}{b (c+d x)}\right]+\frac{1}{4 b^3 d m}(b c-a d)(a+b x)^m (c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m, m, 1+m, -\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 6, 93 leaves, 2 steps):

$$\frac{1}{b^3 (1+m)}(a+b x)^{1+m} (c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m, -2+m, 2, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{2 d (a+b x)}{b c-a d}\right]$$

Problem 3143: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b x)^m (c+d x)^{2-m}}{(b c+a d+2 b d x)^3} \, dx$$

Optimal (type 5, 261 leaves, 7 steps):

$$\frac{(b c-a d)(a+b x)^{-1+m} (c+d x)^{2-m}}{8 b d^2 (b c+a d+2 b d x)^2}+\frac{(1-2 m)(a+b x)^{-1+m} (c+d x)^{2-m}}{8 b d^2 (b c+a d+2 b d x)}-\frac{1}{8 b^2 d^2 (1-m)}\left(1-4 m+2 m^2\right)(a+b x)^{-1+m} (c+d x)^{1-m} \text{Hypergeometric2F1}\left[1, -1+m, m, -\frac{d (a+b x)}{b (c+d x)}\right]-\frac{1}{8 b^3 d^2 (1-m)}(b c-a d)(a+b x)^{-1+m} (c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[-1+m, -1+m, m, -\frac{d (a+b x)}{b c-a d}\right]$$

Result (type 6, 103 leaves, 2 steps):

$$\left((a+b x)^{1+m} (c+d x)^{-m}\left(\frac{b (c+d x)}{b c-a d}\right)^m \text{AppellF1}\left[1+m, -2+m, 3, 2+m, -\frac{d (a+b x)}{b c-a d}, -\frac{2 d (a+b x)}{b c-a d}\right]\right) \Big/ \left(b^3 (b c-a d)(1+m)\right)$$

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Problem 111: Unable to integrate problem.

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x} \sqrt{g+h x}} \, dx$$

Optimal (type 4, 786 leaves, ? steps):

$$-\frac{2 d^3 \sqrt{a+b x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (d e-c f) (d g-c h) \sqrt{c+d x}}-\frac{2 b^3 \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}}{(b c-a d)^2 (b e-a f) (b g-a h) \sqrt{a+b x}}+$$

$$\left(2 b \left(a^2 d^2 f h-a b d^2 (f g+e h)+b^2 \left(2 d^2 e g+c^2 f h-c d (f g+e h)\right)\right) \sqrt{c+d x} \sqrt{e+f x} \sqrt{g+h x}\right) \Big/ \left((b c-a d)^2 (b e-a f) (d e-c f) (b g-a h) (d g-c h) \sqrt{a+b x}\right)-$$

$$\left(2 \sqrt{f g-e h} \left(a^2 d^2 f h-a b d^2 (f g+e h)+b^2 \left(2 d^2 e g+c^2 f h-c d (f g+e h)\right)\right) \sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right],-\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right]\right) \Big/$$

$$\left((b c-a d)^2 (b e-a f) (d e-c f) \sqrt{b g-a h} (d g-c h) \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x}\right)-$$

$$\left(4 b d \sqrt{\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}} \sqrt{g+h x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b g-a h} \sqrt{e+f x}}{\sqrt{f g-e h} \sqrt{a+b x}}\right],-\frac{(b c-a d) (f g-e h)}{(d e-c f) (b g-a h)}\right]\right) \Big/$$

$$\left((b c-a d)^2 \sqrt{b g-a h} \sqrt{f g-e h} \sqrt{c+d x} \sqrt{-\frac{(b e-a f) (g+h x)}{(f g-e h) (a+b x)}}\right)$$

Result (type 8, 39 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a+b x)^{3/2} (c+d x)^{3/2} \sqrt{e+f x} \sqrt{g+h x}},x\right]$$

Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (A+B x) (c+d x)^{-m}}{e+f x} \, dx$$

Optimal (type 5, 233 leaves, 5 steps):

$$-\frac{d (B e-A f) (a+b x)^{1+m} (c+d x)^{-m}}{(b c-a d) f^2 m}-\frac{1}{f^2 m}$$

$$(B e-A f) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1,-m,1-m,\frac{(b e-a f) (c+d x)}{(d e-c f) (a+b x)}\right]-$$

$$\left((a B d f m-b (B d e-A d f+B c f m)) (a+b x)^{1+m} (c+d x)^{-m} \left(\frac{b (c+d x)}{b c-a d}\right)^m \text{Hypergeometric2F1}\left[m,1+m,2+m,-\frac{d (a+b x)}{b c-a d}\right]\right) \Big/ \left(b (b c-a d) f^2 m (1+m)\right)$$

Result (type 5, 220 leaves, 7 steps):

$$\frac{1}{f^2 m}(B e - A f)\,(a+b x)^m\,(c+d x)^{-m}\,\text{Hypergeometric2F1}\left[1, m, 1+m, \frac{(d e - c f)\,(a+b x)}{(b e - a f)\,(c+d x)}\right] - \frac{1}{f^2 m}$$
$$(B e - A f)\,(a+b x)^m\,(c+d x)^{-m}\left(\frac{b\,(c+d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, m, 1+m, -\frac{d\,(a+b x)}{b c - a d}\right] +$$
$$\frac{1}{b f\,(1+m)} B\,(a+b x)^{1+m}\,(c+d x)^{-m}\left(\frac{b\,(c+d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1+m, 2+m, -\frac{d\,(a+b x)}{b c - a d}\right]$$

Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Test results for the 78 problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n (e+f x)^p.m"

Test results for the 35 problems in "1.1.1.7 P(x) (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Test results for the 1071 problems in "1.1.2.2 (c x)^m (a+b x^2)^p.m"

Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{a\,(2+m)\,x^{1+m}}{\sqrt{a+b x^2}} + \frac{b\,(3+m)\,x^{3+m}}{\sqrt{a+b x^2}}\right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m}\sqrt{a+b x^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{a\,x^{2+m}\sqrt{1+\frac{b x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{b x^2}{a}\right]}{\sqrt{a+b x^2}} +$$
$$\frac{b\,(3+m)\,x^{4+m}\sqrt{1+\frac{b x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, -\frac{b x^2}{a}\right]}{(4+m)\sqrt{a+b x^2}}$$

Problem 664: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,x^{1+m}}{\left(a+b\,x^2\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^2}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^m\sqrt{1+\frac{b\,x^2}{a}}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{2},\frac{2+m}{2},-\frac{b\,x^2}{a}\right]}{\sqrt{a+b\,x^2}}-\frac{b\,x^{2+m}\sqrt{1+\frac{b\,x^2}{a}}\ \text{Hypergeometric2F1}\left[\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{b\,x^2}{a}\right]}{a\,(2+m)\sqrt{a+b\,x^2}}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int(c\,x)^{13/3}\left(a+b\,x^2\right)^{1/3}dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$-\frac{5\,a^2\,c^3\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{108\,b^2}+\frac{a\,c\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{36\,b}+\frac{(c\,x)^{16/3}\left(a+b\,x^2\right)^{1/3}}{6\,c}-\frac{5\,a^3\,c^{13/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,(a+b\,x^2)^{1/3}}}{\sqrt{3}}\right]}{54\,\sqrt{3}\,b^{8/3}}-\frac{5\,a^3\,c^{13/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{108\,b^{8/3}}$$

Result (type 3, 275 leaves, 12 steps):

$$-\frac{5\,a^2\,c^3\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{108\,b^2}+\frac{a\,c\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{36\,b}+\frac{(c\,x)^{16/3}\left(a+b\,x^2\right)^{1/3}}{6\,c}-\frac{5\,a^3\,c^{13/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{54\,\sqrt{3}\,b^{8/3}}-\frac{5\,a^3\,c^{13/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{162\,b^{8/3}}+\frac{5\,a^3\,c^{13/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{(a+b\,x^2)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{(a+b\,x^2)^{1/3}}\right]}{324\,b^{8/3}}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int(c\,x)^{7/3}\left(a+b\,x^2\right)^{1/3}dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\frac{a\,c\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{12\,b}+\frac{(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{4\,c}+\frac{a^2\,c^{7/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{6\,\sqrt{3}\,b^{5/3}}+\frac{a^2\,c^{7/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,\left(a+b\,x^2\right)^{1/3}\right]}{12\,b^{5/3}}$$

Result (type 3, 244 leaves, 11 steps):

$$\frac{a\,c\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{12\,b}+\frac{(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{4\,c}+\frac{a^2\,c^{7/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{6\,\sqrt{3}\,b^{5/3}}+\frac{a^2\,c^{7/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{18\,b^{5/3}}-\frac{a^2\,c^{7/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{36\,b^{5/3}}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int (c\,x)^{1/3}\left(a+b\,x^2\right)^{1/3}\,dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$\frac{(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{2\,c}-\frac{a\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\,\sqrt{3}\,b^{2/3}}-\frac{a\,c^{1/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,\left(a+b\,x^2\right)^{1/3}\right]}{4\,b^{2/3}}$$

Result (type 3, 211 leaves, 10 steps):

$$\frac{(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{2\,c}-\frac{a\,c^{1/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{2\,\sqrt{3}\,b^{2/3}}-\frac{a\,c^{1/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{6\,b^{2/3}}+\frac{a\,c^{1/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{12\,b^{2/3}}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,x^2\right)^{1/3}}{(c\,x)^{5/3}}\,dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$-\frac{3\,\left(a+b\,x^2\right)^{1/3}}{2\,c\,(c\,x)^{2/3}}-\frac{\sqrt{3}\,b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\,c^{5/3}}-\frac{3\,b^{1/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,\left(a+b\,x^2\right)^{1/3}\right]}{4\,c^{5/3}}$$

Result (type 3, 208 leaves, 10 steps):

$$-\frac{3\left(a+b x^2\right)^{1/3}}{2 c (c x)^{2/3}}-\frac{\sqrt{3}\, b^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{2 c^{5/3}}-\frac{b^{1/3} \operatorname{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{2 c^{5/3}}+\frac{b^{1/3} \operatorname{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{4 c^{5/3}}$$

Problem 754: Result valid but suboptimal antiderivative.

$$\int (c x)^{13/3} \left(a+b x^2\right)^{4/3} dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{324 b^2}+\frac{a^2 c (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{108 b}+\frac{a (c x)^{16/3} \left(a+b x^2\right)^{1/3}}{18 c}+\frac{(c x)^{16/3} \left(a+b x^2\right)^{4/3}}{8 c}-\frac{5 a^4 c^{13/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{162 \sqrt{3}\, b^{8/3}}-\frac{5 a^4 c^{13/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{324 b^{8/3}}$$

Result (type 3, 303 leaves, 13 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{324 b^2}+\frac{a^2 c (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{108 b}+\frac{a (c x)^{16/3} \left(a+b x^2\right)^{1/3}}{18 c}+\frac{(c x)^{16/3} \left(a+b x^2\right)^{4/3}}{8 c}-\frac{5 a^4 c^{13/3} \operatorname{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3}\, c^{2/3}}\right]}{162 \sqrt{3}\, b^{8/3}}-\frac{5 a^4 c^{13/3} \operatorname{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{486 b^{8/3}}+\frac{5 a^4 c^{13/3} \operatorname{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{972 b^{8/3}}$$

Problem 755: Result valid but suboptimal antiderivative.

$$\int (c x)^{7/3} \left(a+b x^2\right)^{4/3} dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\frac{a^2 c (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{27 b}+\frac{a (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{9 c}+\frac{(c x)^{10/3} \left(a+b x^2\right)^{4/3}}{6 c}+\frac{2 a^3 c^{7/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{27 \sqrt{3}\, b^{5/3}}+\frac{a^3 c^{7/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{27 b^{5/3}}$$

Result (type 3, 272 leaves, 12 steps):

$$\frac{a^2 c (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{27 b}+\frac{a (c x)^{10/3} \left(a+b x^2\right)^{1/3}}{9 c}+\frac{(c x)^{10/3} \left(a+b x^2\right)^{4/3}}{6 c}+\frac{2 a^3 c^{7/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{27 \sqrt{3} b^{5/3}}+\frac{2 a^3 c^{7/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{81 b^{5/3}}-\frac{a^3 c^{7/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{81 b^{5/3}}$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int (c x)^{1/3} \left(a+b x^2\right)^{4/3} \, dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{a (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{3 c}+\frac{(c x)^{4/3} \left(a+b x^2\right)^{4/3}}{4 c}-\frac{a^2 c^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}}-\frac{a^2 c^{1/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 3, 243 leaves, 11 steps):

$$\frac{a (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{3 c}+\frac{(c x)^{4/3} \left(a+b x^2\right)^{4/3}}{4 c}-\frac{a^2 c^{1/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{3 \sqrt{3} b^{2/3}}-\frac{a^2 c^{1/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a^2 c^{1/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{\left(a+b x^2\right)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{\left(a+b x^2\right)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^2\right)^{4/3}}{(c x)^{5/3}} \, dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{2 b (c x)^{4/3} \left(a+b x^2\right)^{1/3}}{c^3}-\frac{3 \left(a+b x^2\right)^{4/3}}{2 c (c x)^{2/3}}-\frac{2 a b^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} \left(a+b x^2\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3}}-\frac{a b^{1/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} \left(a+b x^2\right)^{1/3}\right]}{c^{5/3}}$$

Result (type 3, 233 leaves, 11 steps):

$$\frac{2\,b\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{c^3}-\frac{3\left(a+b\,x^2\right)^{4/3}}{2\,c\,(c\,x)^{2/3}}-\frac{2\,a\,b^{1/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{\sqrt{3}\,c^{5/3}}-$$
$$\frac{2\,a\,b^{1/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{3\,c^{5/3}}+\frac{a\,b^{1/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{3\,c^{5/3}}$$

Problem 758: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,x^2\right)^{4/3}}{(c\,x)^{11/3}}\,dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$-\frac{3\,b\left(a+b\,x^2\right)^{1/3}}{2\,c^3\,(c\,x)^{2/3}}-\frac{3\left(a+b\,x^2\right)^{4/3}}{8\,c\,(c\,x)^{8/3}}-$$
$$\frac{\sqrt{3}\,b^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\,c^{11/3}}-\frac{3\,b^{4/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{4\,c^{11/3}}$$

Result (type 3, 234 leaves, 11 steps):

$$-\frac{3\,b\left(a+b\,x^2\right)^{1/3}}{2\,c^3\,(c\,x)^{2/3}}-\frac{3\left(a+b\,x^2\right)^{4/3}}{8\,c\,(c\,x)^{8/3}}-\frac{\sqrt{3}\,b^{4/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{2\,c^{11/3}}-$$
$$\frac{b^{4/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{2\,c^{11/3}}+\frac{b^{4/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{4\,c^{11/3}}$$

Problem 771: Result valid but suboptimal antiderivative.

$$\int \frac{(c\,x)^{19/3}}{\left(a+b\,x^2\right)^{2/3}}\,dx$$

Optimal (type 3, 198 leaves, 6 steps):

$$\frac{10\,a^2\,c^5\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{27\,b^3}-\frac{2\,a\,c^3\,(c\,x)^{10/3}\left(a+b\,x^2\right)^{1/3}}{9\,b^2}+\frac{c\,(c\,x)^{16/3}\left(a+b\,x^2\right)^{1/3}}{6\,b}+$$
$$\frac{20\,a^3\,c^{19/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{27\,\sqrt{3}\,b^{11/3}}+\frac{10\,a^3\,c^{19/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\left(a+b\,x^2\right)^{1/3}\right]}{27\,b^{11/3}}$$

Result (type 3, 278 leaves, 12 steps):

$$\frac{10 a^2 c^5 (c x)^{4/3} (a+b x^2)^{1/3}}{27 b^3}-\frac{2 a c^3 (c x)^{10/3} (a+b x^2)^{1/3}}{9 b^2}+\frac{c (c x)^{16/3} (a+b x^2)^{1/3}}{6 b}+\frac{20 a^3 c^{19/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{27 \sqrt{3} b^{11/3}}+\frac{20 a^3 c^{19/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{81 b^{11/3}}-\frac{10 a^3 c^{19/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{(a+b x^2)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{81 b^{11/3}}$$

Problem 772: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{13/3}}{(a+b x^2)^{2/3}} \, dx$$

Optimal (type 3, 167 leaves, 5 steps):

$$-\frac{5 a c^3 (c x)^{4/3} (a+b x^2)^{1/3}}{12 b^2}+\frac{c (c x)^{10/3} (a+b x^2)^{1/3}}{4 b}-\frac{5 a^2 c^{13/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a+b x^2)^{1/3}}}{\sqrt{3}}\right]}{6 \sqrt{3} b^{8/3}}-\frac{5 a^2 c^{13/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} (a+b x^2)^{1/3}\right]}{12 b^{8/3}}$$

Result (type 3, 247 leaves, 11 steps):

$$-\frac{5 a c^3 (c x)^{4/3} (a+b x^2)^{1/3}}{12 b^2}+\frac{c (c x)^{10/3} (a+b x^2)^{1/3}}{4 b}-\frac{5 a^2 c^{13/3} \text{ArcTan}\left[\frac{c^{2/3}+\frac{2 b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{6 \sqrt{3} b^{8/3}}-\frac{5 a^2 c^{13/3} \text{Log}\left[c^{2/3}-\frac{b^{1/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{18 b^{8/3}}+\frac{5 a^2 c^{13/3} \text{Log}\left[c^{4/3}+\frac{b^{2/3} (c x)^{4/3}}{(a+b x^2)^{2/3}}+\frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a+b x^2)^{1/3}}\right]}{36 b^{8/3}}$$

Problem 773: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{7/3}}{(a+b x^2)^{2/3}} \, dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{c (c x)^{4/3} (a+b x^2)^{1/3}}{2 b}+\frac{a c^{7/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a+b x^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3}}+\frac{a c^{7/3} \text{Log}\left[b^{1/3} (c x)^{2/3}-c^{2/3} (a+b x^2)^{1/3}\right]}{2 b^{5/3}}$$

Result (type 3, 209 leaves, 10 steps):

$$\frac{c\,(c\,x)^{4/3}\left(a+b\,x^2\right)^{1/3}}{2\,b}+\frac{a\,c^{7/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{\sqrt{3}\,b^{5/3}}+$$
$$\frac{a\,c^{7/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{3\,b^{5/3}}-\frac{a\,c^{7/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{6\,b^{5/3}}$$

Problem 774: Result valid but suboptimal antiderivative.

$$\int\frac{(c\,x)^{1/3}}{\left(a+b\,x^2\right)^{2/3}}\,dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$-\frac{\sqrt{3}\,c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{c^{2/3}\,\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\,b^{2/3}}-\frac{3\,c^{1/3}\,\text{Log}\left[b^{1/3}\,(c\,x)^{2/3}-c^{2/3}\,\left(a+b\,x^2\right)^{1/3}\right]}{4\,b^{2/3}}$$

Result (type 3, 183 leaves, 9 steps):

$$-\frac{\sqrt{3}\,c^{1/3}\,\text{ArcTan}\left[\frac{c^{2/3}+\frac{2\,b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}}{\sqrt{3}\,c^{2/3}}\right]}{2\,b^{2/3}}-$$
$$\frac{c^{1/3}\,\text{Log}\left[c^{2/3}-\frac{b^{1/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{2\,b^{2/3}}+\frac{c^{1/3}\,\text{Log}\left[c^{4/3}+\frac{b^{2/3}\,(c\,x)^{4/3}}{\left(a+b\,x^2\right)^{2/3}}+\frac{b^{1/3}\,c^{2/3}\,(c\,x)^{2/3}}{\left(a+b\,x^2\right)^{1/3}}\right]}{4\,b^{2/3}}$$

Test results for the 349 problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 301: Result unnecessarily involves higher level functions.

$$\int\frac{\left(1-2\,x^2\right)^m}{\sqrt{1-x^2}}\,dx$$

Optimal (type 5, 62 leaves, ? steps):

$$-\frac{2^{-2-m}\,\sqrt{x^2}\,\left(2-4\,x^2\right)^{1+m}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1+m}{2},\,\frac{3+m}{2},\,\left(1-2\,x^2\right)^2\right]}{(1+m)\,x}$$

Result (type 6, 23 leaves, 1 step):

$$x\,\text{AppellF1}\left[\frac{1}{2},\,-m,\,\frac{1}{2},\,\frac{3}{2},\,2\,x^2,\,x^2\right]$$

Test results for the 1156 problems in "1.1.2.4 (e x)^m (a+b x^2)^p

(c+d x^2)^q.m"

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 51 problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 174 problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 516: Result valid but suboptimal antiderivative.

$$\int x^4 \left(a+b x^3\right)^{1/3} dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{a x^2 \left(a+b x^3\right)^{1/3}}{18 b}+\frac{1}{6} x^5 \left(a+b x^3\right)^{1/3}+\frac{a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}}+\frac{a^2 \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{5/3}}$$

Result (type 3, 173 leaves, 9 steps):

$$\frac{a x^2 \left(a+b x^3\right)^{1/3}}{18 b}+\frac{1}{6} x^5 \left(a+b x^3\right)^{1/3}+\frac{a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{5/3}}+$$
$$\frac{a^2 \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 b^{5/3}}-\frac{a^2 \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 b^{5/3}}$$

Problem 517: Result valid but suboptimal antiderivative.

$$\int x \left(a+b x^3\right)^{1/3} dx$$

Optimal (type 3, 94 leaves, 2 steps):

$$\frac{1}{3} x^2 \left(a+b x^3\right)^{1/3}-\frac{a \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}-\frac{a \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 3, 145 leaves, 8 steps):

$$\frac{1}{3} x^2 \left(a+b x^3\right)^{1/3}-\frac{a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}}-\frac{a \operatorname{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}}+\frac{a \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 518: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^2} \, dx$$

Optimal (type 3, 88 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{x}-\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2} b^{1/3} \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]$$

Result (type 3, 138 leaves, 8 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{x}-\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$
$$\frac{1}{3} b^{1/3} \operatorname{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]+\frac{1}{6} b^{1/3} \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$-\frac{5 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2}+\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b}-\frac{5 a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{8/3}}-\frac{5 a^2 \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{8/3}}$$

Result (type 3, 176 leaves, 9 steps):

$$-\frac{5 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2}+\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b}-\frac{5 a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{8/3}}-$$
$$\frac{5 a^2 \operatorname{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 b^{8/3}}+\frac{5 a^2 \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 b^{8/3}}$$

Problem 568: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}+\frac{2\,a\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{a\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{5/3}}$$

Result (type 3, 148 leaves, 8 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}+\frac{2\,a\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{2\,a\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}-\frac{a\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}$$

Problem 569: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 72 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,b^{2/3}}$$

Problem 581: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 53 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{2}\,\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 87 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{6}\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-\frac{1}{3}\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]$$

Problem 2271: Result valid but suboptimal antiderivative.

$$\int\frac{x^3}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$-\frac{5\,a\,x\left(a+b\,x^{3/2}\right)^{1/3}}{9\,b^2}+\frac{x^{5/2}\left(a+b\,x^{3/2}\right)^{1/3}}{3\,b}-\frac{10\,a^2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\,b^{8/3}}-\frac{5\,a^2\,\text{Log}\left[b^{1/3}\sqrt{x}-\left(a+b\,x^{3/2}\right)^{1/3}\right]}{9\,b^{8/3}}$$

Result (type 3, 198 leaves, 10 steps):

$$-\frac{5\,a\,x\left(a+b\,x^{3/2}\right)^{1/3}}{9\,b^2}+\frac{x^{5/2}\left(a+b\,x^{3/2}\right)^{1/3}}{3\,b}-\frac{10\,a^2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\,b^{8/3}}-\frac{10\,a^2\,\text{Log}\left[1-\frac{b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}\right]}{27\,b^{8/3}}+\frac{5\,a^2\,\text{Log}\left[1+\frac{b^{2/3}\,x}{(a+b\,x^{3/2})^{2/3}}+\frac{b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}\right]}{27\,b^{8/3}}$$

Problem 2272: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{\text{Log}\left[b^{1/3}\sqrt{x}-\left(a+b\,x^{3/2}\right)^{1/3}\right]}{b^{2/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$-\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}-\frac{2\,\text{Log}\left[1-\frac{b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}\right]}{3\,b^{2/3}}+\frac{\text{Log}\left[1+\frac{b^{2/3}\,x}{(a+b\,x^{3/2})^{2/3}}+\frac{b^{1/3}\sqrt{x}}{(a+b\,x^{3/2})^{1/3}}\right]}{3\,b^{2/3}}$$

Problem 2686: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,n\,x^{-1+m+n}}{2\,\left(a+b\,x^n\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^n}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b\,x^n}{a}\right]}{\sqrt{a+b\,x^n}}-$$
$$\frac{b\,n\,x^{m+n}\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^n}{a}\right]}{2\,a\,(m+n)\,\sqrt{a+b\,x^n}}$$

Problem 2697: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{6\,a\,x^2}{b\,(4+m)\,\sqrt{a+b\,x^{-2+m}}}+\frac{x^m}{\sqrt{a+b\,x^{-2+m}}}\right)dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2\,x^3\,\sqrt{a+b\,x^{-2+m}}}{b\,(4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2\,a\,x^3\,\sqrt{1+\frac{b\,x^{-2+m}}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{3}{2-m},-\frac{1+m}{2-m},-\frac{b\,x^{-2+m}}{a}\right]}{b\,(4+m)\,\sqrt{a+b\,x^{-2+m}}}+$$
$$\frac{x^{1+m}\,\sqrt{1+\frac{b\,x^{-2+m}}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{1+m}{2-m},\frac{1-2\,m}{2-m},-\frac{b\,x^{-2+m}}{a}\right]}{(1+m)\,\sqrt{a+b\,x^{-2+m}}}$$

Problem 2699: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b\,n\,x^{-1+m+n}}{2\,\left(a+b\,x^n\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^n}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b\,x^n}{a}\right]}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n}\,\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^n}{a}\right]}{2\,a\,(m+n)\,\sqrt{a+b\,x^n}}$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{7/3}}{a-b\,x^3}\,dx$$

Optimal (type 5, 483 leaves, 22 steps):

$$-\frac{7}{5}\,a\,x\,\left(a+b\,x^3\right)^{1/3}-\frac{1}{5}\,x\,\left(a+b\,x^3\right)^{4/3}-\frac{4\times 2^{1/3}\,a^{5/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}}-\frac{2\times 2^{1/3}\,a^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}}-\frac{7\,a^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{5\,\left(a+b\,x^3\right)^{2/3}}-\frac{2\times 2^{1/3}\,a^{5/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}+\frac{2\times 2^{1/3}\,a^{5/3}\,\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}-\frac{4\times 2^{1/3}\,a^{5/3}\,\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}+\frac{2^{1/3}\,a^{5/3}\,\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}$$

Result (type 6, 56 leaves, 2 steps):

$$\frac{a\,x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},\,1,\,-\frac{7}{3},\,\frac{4}{3},\,\frac{b\,x^3}{a},\,-\frac{b\,x^3}{a}\right]}{\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 35: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{4/3}}{a-b\,x^3}\,dx$$

Optimal (type 5, 464 leaves, 21 steps):

$$-\frac{1}{2}\,x\,\left(a+b\,x^3\right)^{1/3}-\frac{2\times 2^{1/3}\,a^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}}-$$

$$\frac{2^{1/3}\,a^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{1/3}}-\frac{a\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{4}{3},\,-\frac{b\,x^3}{a}\right]}{2\,\left(a+b\,x^3\right)^{2/3}}-$$

$$\frac{2^{1/3}\,a^{2/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}+\frac{2^{1/3}\,a^{2/3}\,\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}-$$

$$\frac{2\times 2^{1/3}\,a^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{1/3}}+\frac{a^{2/3}\,\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\,b^{1/3}}$$

Result (type 6, 55 leaves, 2 steps):

$$\frac{x\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},\,1,\,-\frac{4}{3},\,\frac{4}{3},\,\frac{b\,x^3}{a},\,-\frac{b\,x^3}{a}\right]}{\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{a-b\,x^3}\,dx$$

Optimal (type 3, 398 leaves, 14 steps):

$$-\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^{1/3}\,b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\,a^{1/3}\,b^{1/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,a^{1/3}\,b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,a^{1/3}\,b^{1/3}}-$$

$$\frac{2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,a^{1/3}\,b^{1/3}}+\frac{\text{Log}\left[2\times2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times2^{2/3}\,a^{1/3}\,b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{1}{3},1,-\frac{1}{3},\frac{4}{3},\frac{b\,x^3}{a},-\frac{b\,x^3}{a}\right]}{a\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^3\right)\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 452 leaves, 17 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\,a^{4/3}\,b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{2/3}\sqrt{3}\,a^{4/3}\,b^{1/3}}+$$

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,a\left(a+b\,x^3\right)^{2/3}}-$$

$$\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times2^{2/3}\,a^{4/3}\,b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times2^{2/3}\,a^{4/3}\,b^{1/3}}-$$

$$\frac{\text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times2^{2/3}\,a^{4/3}\,b^{1/3}}+\frac{\text{Log}\left[2\times2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{12\times2^{2/3}\,a^{4/3}\,b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},1,\frac{2}{3},\frac{4}{3},\frac{b\,x^3}{a},-\frac{b\,x^3}{a}\right]}{a\left(a+b\,x^3\right)^{2/3}}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^3\right)\left(a+b\,x^3\right)^{5/3}}\,dx$$

Optimal (type 5, 473 leaves, 21 steps):

$$\frac{x}{4 a^2 \left(a+b x^3\right)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}\, a^{7/3} b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{2/3} \sqrt{3}\, a^{7/3} b^{1/3}}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 a^2 \left(a+b x^3\right)^{2/3}}-\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{7/3} b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{7/3} b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{7/3} b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{7/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},1,\frac{5}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a^2 \left(a+b x^3\right)^{2/3}}$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^3\right) \left(a+b x^3\right)^{8/3}} \, dx$$

Optimal (type 5, 492 leaves, 22 steps):

$$\frac{x}{10 a^2 \left(a+b x^3\right)^{5/3}}+\frac{13 x}{40 a^3 \left(a+b x^3\right)^{2/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{2/3} \sqrt{3}\, a^{10/3} b^{1/3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{8\times 2^{2/3} \sqrt{3}\, a^{10/3} b^{1/3}}+\frac{9 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{20 a^3 \left(a+b x^3\right)^{2/3}}-\frac{\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{10/3} b^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{24\times 2^{2/3} a^{10/3} b^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{12\times 2^{2/3} a^{10/3} b^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{48\times 2^{2/3} a^{10/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x\left(1+\frac{b x^3}{a}\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},1,\frac{8}{3},\frac{4}{3},\frac{b x^3}{a},-\frac{b x^3}{a}\right]}{a^3\left(a+b x^3\right)^{2/3}}$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b x^3\right)^{8/3}}{c+d x^3}\,dx$$

Optimal (type 3, 331 leaves, 5 steps):

$$-\frac{b\left(6 b c-11 a d\right) x\left(a+b x^3\right)^{2/3}}{18 d^2}+\frac{b x\left(a+b x^3\right)^{5/3}}{6 d}+\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\, d^3}-\frac{\left(b c-a d\right)^{8/3}\text{ArcTan}\left[\frac{1+\frac{2\left(b c-a d\right)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d^3}-\frac{\left(b c-a d\right)^{8/3}\text{Log}\left[c+d x^3\right]}{6 c^{2/3} d^3}+\frac{\left(b c-a d\right)^{8/3}\text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d^3}-\frac{b^{2/3}\left(9 b^2 c^2-24 a b c d+20 a^2 d^2\right)\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x\left(a+b x^3\right)^{2/3}\text{AppellF1}\left[\frac{1}{3},-\frac{8}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b x^3\right)^{5/3}}{c+d x^3}\,dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{b x\left(a+b x^3\right)^{2/3}}{3 d}-\frac{b^{2/3}\left(3 b c-5 a d\right)\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\, d^2}+\frac{\left(b c-a d\right)^{5/3}\text{ArcTan}\left[\frac{1+\frac{2\left(b c-a d\right)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} d^2}+\frac{\left(b c-a d\right)^{5/3}\text{Log}\left[c+d x^3\right]}{6 c^{2/3} d^2}-\frac{\left(b c-a d\right)^{5/3}\text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} d^2}+\frac{b^{2/3}\left(3 b c-5 a d\right)\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 d^2}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a\,x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{5}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Problem 88: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{2/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d}-\frac{\left(b\,c-a\,d\right)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,d}-\frac{\left(b\,c-a\,d\right)^{2/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,d}+\frac{\left(b\,c-a\,d\right)^{2/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,d}-\frac{b^{2/3}\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{2\,d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{c\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b\,x^3\right)^{1/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}+\frac{\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}-\frac{\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{\sqrt{3}\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}-\frac{\text{Log}\left[c^{1/3}-\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}+\frac{\text{Log}\left[c^{2/3}+\frac{\left(b\,c-a\,d\right)^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{c^{1/3}\,\left(b\,c-a\,d\right)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,c^{2/3}\,\left(b\,c-a\,d\right)^{1/3}}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b\,x^3\right)^{4/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{b\,x}{a\,(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}}-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,(b\,c-a\,d)^{4/3}}-\frac{d\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,(b\,c-a\,d)^{4/3}}+\frac{d\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps) :

$$\frac{b\,x}{a\,(b\,c-a\,d)\,\left(a+b\,x^3\right)^{1/3}}-\frac{d\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{\sqrt{3}\,c^{2/3}\,(b\,c-a\,d)^{4/3}}+\frac{d\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,c^{2/3}\,(b\,c-a\,d)^{4/3}}-\frac{d\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,c^{2/3}\,(b\,c-a\,d)^{4/3}}$$

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{\left(a+b\,x^3\right)^{7/3}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 226 leaves, 4 steps) :

$$\frac{b\,x}{4\,a\,(b\,c-a\,d)\,\left(a+b\,x^3\right)^{4/3}}+\frac{b\,(3\,b\,c-7\,a\,d)\,x}{4\,a^2\,(b\,c-a\,d)^2\,\left(a+b\,x^3\right)^{1/3}}+\frac{d^2\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,(b\,c-a\,d)^{7/3}}+\frac{d^2\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,(b\,c-a\,d)^{7/3}}-\frac{d^2\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{2/3}\,(b\,c-a\,d)^{7/3}}$$

Result (type 5, 621 leaves, 2 steps) :

$$-\frac{1}{40\,c^4\,(b\,c-a\,d)^2\,x^5\,(a+b\,x^3)^{10/3}}\Bigg(70\,c^4\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^2+105\,c^3\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^2+45\,c^2\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^2+$$
$$280\,c^5\,(a+b\,x^3)^3+420\,c^4\,d\,x^3\,(a+b\,x^3)^3+180\,c^3\,d^2\,x^6\,(a+b\,x^3)^3-$$
$$280\,c^5\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$420\,c^4\,d\,x^3\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$180\,c^3\,d^2\,x^6\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$33\,c^2\,(b\,c-a\,d)^3\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$60\,c\,d\,(b\,c-a\,d)^3\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$27\,d^2\,(b\,c-a\,d)^3\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{10}{3},\frac{13}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$9\,c^2\,(b\,c-a\,d)^3\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$18\,c\,d\,(b\,c-a\,d)^3\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$9\,d^2\,(b\,c-a\,d)^3\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 92: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,x^3)^{10/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 280 leaves, 5 steps):

$$\frac{b\,x}{7\,a\,(b\,c-a\,d)\,(a+b\,x^3)^{7/3}}+\frac{b\,(6\,b\,c-13\,a\,d)\,x}{28\,a^2\,(b\,c-a\,d)^2\,(a+b\,x^3)^{4/3}}+\frac{b\,(18\,b^2\,c^2-57\,a\,b\,c\,d+67\,a^2\,d^2)\,x}{28\,a^3\,(b\,c-a\,d)^3\,(a+b\,x^3)^{1/3}}-$$
$$\frac{d^3\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{2/3}\,(b\,c-a\,d)^{10/3}}-\frac{d^3\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{2/3}\,(b\,c-a\,d)^{10/3}}+\frac{d^3\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{2\,c^{2/3}\,(b\,c-a\,d)^{10/3}}$$

Result (type 5, 1172 leaves, 2 steps):

$$-\frac{1}{5096\,c^5\,(b\,c-a\,d)^3\,x^8\,(a+b\,x^3)^{13/3}}$$

$$\Big(7280\,c^5\,(b\,c-a\,d)^2\,x^6\,(a+b\,x^3)^2+16\,380\,c^4\,d\,(b\,c-a\,d)^2\,x^9\,(a+b\,x^3)^2+$$
$$14\,040\,c^3\,d^2\,(b\,c-a\,d)^2\,x^{12}\,(a+b\,x^3)^2+4212\,c^2\,d^3\,(b\,c-a\,d)^2\,x^{15}\,(a+b\,x^3)^2+$$
$$12\,740\,c^6\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^3+28\,665\,c^5\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^3+$$
$$24\,570\,c^4\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^3+7371\,c^3\,d^3\,(b\,c-a\,d)\,x^{12}\,(a+b\,x^3)^3+$$
$$50\,960\,c^7\,(a+b\,x^3)^4+114\,660\,c^6\,d\,x^3\,(a+b\,x^3)^4+98\,280\,c^5\,d^2\,x^6\,(a+b\,x^3)^4+$$
$$29\,484\,c^4\,d^3\,x^9\,(a+b\,x^3)^4-50\,960\,c^7\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$114\,660\,c^6\,d\,x^3\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$98\,280\,c^5\,d^2\,x^6\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$29\,484\,c^4\,d^3\,x^9\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$5796\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{Hypergeometric2F1}\Big[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$15\,246\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{Hypergeometric2F1}\Big[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$13\,608\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{Hypergeometric2F1}\Big[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$4158\,d^3\,(b\,c-a\,d)^4\,x^{21}\,\text{Hypergeometric2F1}\Big[2,\frac{13}{3},\frac{16}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$2646\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{13}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$7560\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{13}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$7182\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{13}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$2268\,d^3\,(b\,c-a\,d)^4\,x^{21}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{13}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$378\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{13}{3}\Big\},\Big\{1,1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$1134\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{13}{3}\Big\},\Big\{1,1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$1134\,c\,d^2\,(b\,c-a\,d)^4\,x^{18}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{13}{3}\Big\},\Big\{1,1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$378\,d^3\,(b\,c-a\,d)^4\,x^{21}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{13}{3}\Big\},\Big\{1,1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]\Big)$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{8/3}}{\left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 351 leaves, 5 steps):

$$\frac{b\,(2 b c-a d)\, x \left(a+b x^3\right)^{2/3}}{3 c d^2}-\frac{(b c-a d)\, x \left(a+b x^3\right)^{5/3}}{3 c d \left(c+d x^3\right)}-\frac{2 b^{5/3} (3 b c-4 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, d^3}+\frac{2 (b c-a d)^{5/3} (3 b c+a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c^{5/3} d^3}+\frac{(b c-a d)^{5/3} (3 b c+a d) \operatorname{Log}\left[c+d x^3\right]}{9 c^{5/3} d^3}-\frac{(b c-a d)^{5/3} (3 b c+a d) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{3 c^{5/3} d^3}+\frac{b^{5/3} (3 b c-4 a d) \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{3 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x \left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{8}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 99: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{\left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 301 leaves, 4 steps):

$$-\frac{(b c-a d)\, x \left(a+b x^3\right)^{2/3}}{3 c d \left(c+d x^3\right)}+\frac{b^{5/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d^2}-\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c^{5/3} d^2}-\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{Log}\left[c+d x^3\right]}{18 c^{5/3} d^2}+\frac{(b c-a d)^{2/3} (3 b c+2 a d) \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{6 c^{5/3} d^2}-\frac{b^{5/3} \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d^2}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a x \left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{5}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,x^3\right)^{2/3}}{\left(c+d\,x^3\right)^2}\,dx$$

Optimal (type 3, 182 leaves, 2 steps):

$$\frac{x\left(a+b\,x^3\right)^{2/3}}{3\,c\left(c+d\,x^3\right)}+\frac{2\,a\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{1/3}}+\frac{a\,\text{Log}\left[c+d\,x^3\right]}{9\,c^{5/3}\,(b\,c-a\,d)^{1/3}}-\frac{a\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{3\,c^{5/3}\,(b\,c-a\,d)^{1/3}}$$

Result (type 3, 241 leaves, 8 steps):

$$\frac{x\left(a+b\,x^3\right)^{2/3}}{3\,c\left(c+d\,x^3\right)}+\frac{2\,a\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{3\,\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{1/3}}-$$

$$\frac{2\,a\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{9\,c^{5/3}\,(b\,c-a\,d)^{1/3}}+\frac{a\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(a+b\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{9\,c^{5/3}\,(b\,c-a\,d)^{1/3}}$$

Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)^2}\,dx$$

Optimal (type 3, 217 leaves, 2 steps):

$$-\frac{d\,x\left(a+b\,x^3\right)^{2/3}}{3\,c\,(b\,c-a\,d)\left(c+d\,x^3\right)}+\frac{(3\,b\,c-2\,a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{4/3}}+$$

$$\frac{(3\,b\,c-2\,a\,d)\,\text{Log}\left[c+d\,x^3\right]}{18\,c^{5/3}\,(b\,c-a\,d)^{4/3}}-\frac{(3\,b\,c-2\,a\,d)\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{6\,c^{5/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 3, 276 leaves, 8 steps):

$$-\frac{d\,x\left(a+b\,x^3\right)^{2/3}}{3\,c\,(b\,c-a\,d)\left(c+d\,x^3\right)}+\frac{(3\,b\,c-2\,a\,d)\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{3\,\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{4/3}}-$$

$$\frac{(3\,b\,c-2\,a\,d)\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{9\,c^{5/3}\,(b\,c-a\,d)^{4/3}}+\frac{(3\,b\,c-2\,a\,d)\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(a+b\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{18\,c^{5/3}\,(b\,c-a\,d)^{4/3}}$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)^2} \, dx$$

Optimal (type 3, 261 leaves, 4 steps):

$$\frac{b\,(3 b c+a d)\,x}{3 a c\,(b c-a d)^2 \left(a+b x^3\right)^{1/3}}-\frac{d\,x}{3 c\,(b c-a d)\left(a+b x^3\right)^{1/3}\left(c+d x^3\right)}-\frac{2 d\,(3 b c-a d)\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3}\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, c^{5/3} (b c-a d)^{7/3}}-\frac{d\,(3 b c-a d)\,\text{Log}\left[c+d x^3\right]}{9 c^{5/3} (b c-a d)^{7/3}}+\frac{d\,(3 b c-a d)\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{3 c^{5/3} (b c-a d)^{7/3}}$$

Result (type 5, 625 leaves, 2 steps):

$$-\frac{1}{420\,(b\,c-a\,d)^2\,x^5\,(c+d\,x^3)}c\,(a+b\,x^3)^{2/3}\Bigg(6860+\frac{13\,720\,d\,x^3}{c}+\frac{6300\,d^2\,x^6}{c^2}-\frac{525\,(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}-\frac{1890\,d\,(b\,c-a\,d)\,x^6}{c^2\,(a+b\,x^3)}-$$
$$\frac{945\,d^2\,(b\,c-a\,d)\,x^9}{c^3\,(a+b\,x^3)}-6860\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$\frac{13\,720\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]}{c}-$$
$$\frac{6300\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]}{c^2}+$$
$$\frac{2240\,(b\,c-a\,d)\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]}{c\,(a+b\,x^3)}+$$
$$\frac{5320\,d\,(b\,c-a\,d)\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]}{c^2\,(a+b\,x^3)}+$$
$$\frac{2520\,d^2\,(b\,c-a\,d)\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]}{c^3\,(a+b\,x^3)}-\frac{1}{c^3\,(a+b\,x^3)^3}$$
$$54\,(b\,c-a\,d)^3\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-\frac{1}{c^4\,(a+b\,x^3)^3}$$
$$108\,d\,(b\,c-a\,d)^3\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$\frac{1}{c^5\,(a+b\,x^3)^3}54\,d^2\,(b\,c-a\,d)^3\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{(a+b\,x^3)^{7/3}\,(c+d\,x^3)^2}\,dx$$

Optimal (type 3, 324 leaves, 5 steps):

$$\frac{b\,(3\,b\,c+4\,a\,d)\,x}{12\,a\,c\,(b\,c-a\,d)^2\,(a+b\,x^3)^{4/3}}+\frac{b\,(9\,b^2\,c^2-33\,a\,b\,c\,d-4\,a^2\,d^2)\,x}{12\,a^2\,c\,(b\,c-a\,d)^3\,(a+b\,x^3)^{1/3}}-$$

$$\frac{d\,x}{3\,c\,(b\,c-a\,d)\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)}+\frac{d^2\,(9\,b\,c-2\,a\,d)\,\text{ArcTan}\Big[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\Big]}{3\,\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{10/3}}+$$

$$\frac{d^2\,(9\,b\,c-2\,a\,d)\,\text{Log}[c+d\,x^3]}{18\,c^{5/3}\,(b\,c-a\,d)^{10/3}}-\frac{d^2\,(9\,b\,c-2\,a\,d)\,\text{Log}\Big[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\Big]}{6\,c^{5/3}\,(b\,c-a\,d)^{10/3}}$$

Result (type 5, 1214 leaves, 2 steps):

$$\frac{1}{21840\,c^5\,(b\,c-a\,d)^3\,x^8\,(a+b\,x^3)^{10/3}\,(c+d\,x^3)}$$

$$\Big(26130\,c^5\,(b\,c-a\,d)^2\,x^6\,(a+b\,x^3)^2+89505\,c^4\,d\,(b\,c-a\,d)^2\,x^9\,(a+b\,x^3)^2+$$
$$84240\,c^3\,d^2\,(b\,c-a\,d)^2\,x^{12}\,(a+b\,x^3)^2+26325\,c^2\,d^3\,(b\,c-a\,d)^2\,x^{15}\,(a+b\,x^3)^2+$$
$$748020\,c^6\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^3+2113020\,c^5\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^3+$$
$$1916460\,c^4\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^3+589680\,c^3\,d^3\,(b\,c-a\,d)\,x^{12}\,(a+b\,x^3)^3-$$
$$2002000\,c^7\,(a+b\,x^3)^4-5460000\,c^6\,d\,x^3\,(a+b\,x^3)^4-$$
$$4914000\,c^5\,d^2\,x^6\,(a+b\,x^3)^4-1506960\,c^4\,d^3\,x^9\,(a+b\,x^3)^4-$$
$$1248520\,c^6\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$3478020\,c^5\,d\,(b\,c-a\,d)\,x^6\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$3144960\,c^4\,d^2\,(b\,c-a\,d)\,x^9\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$966420\,c^3\,d^3\,(b\,c-a\,d)\,x^{12}\,(a+b\,x^3)^3\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$2002000\,c^7\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$5460000\,c^6\,d\,x^3\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$4914000\,c^5\,d^2\,x^6\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$1506960\,c^4\,d^3\,x^9\,(a+b\,x^3)^4\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$7938\,c^3\,(b\,c-a\,d)^4\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{10}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$22680\,c^2\,d\,(b\,c-a\,d)^4\,x^{15}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{10}{3}\Big\},\Big\{1,\frac{16}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$

$$21546\, c\, d^2\, (b\, c - a\, d)^4\, x^{18}\, \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right] +$$

$$6804\, d^3\, (b\, c - a\, d)^4\, x^{21}\, \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right] +$$

$$1134\, c^3\, (b\, c - a\, d)^4\, x^{12}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right] +$$

$$3402\, c^2\, d\, (b\, c - a\, d)^4\, x^{15}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right] +$$

$$3402\, c\, d^2\, (b\, c - a\, d)^4\, x^{18}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right] +$$

$$1134\, d^3\, (b\, c - a\, d)^4\, x^{21}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b\, c - a\, d)\, x^3}{c\, (a + b\, x^3)}\right]\Bigg)$$

Problem 109: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b\, x^3)^{14/3}}{(c + d\, x^3)^3}\, dx$$

Optimal (type 3, 541 leaves, 7 steps):

$$-\frac{b\, (2\, b\, c - a\, d)\, (18\, b^2\, c^2 - 18\, a\, b\, c\, d - 5\, a^2\, d^2)\, x\, (a + b\, x^3)^{2/3}}{18\, c^2\, d^4} + \frac{b\, (18\, b^2\, c^2 - 10\, a\, b\, c\, d - 5\, a^2\, d^2)\, x\, (a + b\, x^3)^{5/3}}{18\, c^2\, d^3} - \frac{(b\, c - a\, d)\, x\, (a + b\, x^3)^{11/3}}{6\, c\, d\, (c + d\, x^3)^2} -$$

$$\frac{(b\, c - a\, d)\, (12\, b\, c + 5\, a\, d)\, x\, (a + b\, x^3)^{8/3}}{18\, c^2\, d^2\, (c + d\, x^3)} + \frac{b^{8/3}\, (54\, b^2\, c^2 - 126\, a\, b\, c\, d + 77\, a^2\, d^2)\, \text{ArcTan}\left[\frac{1 + \frac{2\, b^{1/3}\, x}{(a + b\, x^3)^{1/3}}}{\sqrt{3}}\right]}{9\, \sqrt{3}\, d^5} -$$

$$\frac{(b\, c - a\, d)^{8/3}\, (54\, b^2\, c^2 + 18\, a\, b\, c\, d + 5\, a^2\, d^2)\, \text{ArcTan}\left[\frac{1 + \frac{2\, (b\, c - a\, d)^{1/3}\, x}{c^{1/3}\, (a + b\, x^3)^{1/3}}}{\sqrt{3}}\right]}{9\, \sqrt{3}\, c^{8/3}\, d^5} -$$

$$\frac{(b\, c - a\, d)^{8/3}\, (54\, b^2\, c^2 + 18\, a\, b\, c\, d + 5\, a^2\, d^2)\, \text{Log}\left[c + d\, x^3\right]}{54\, c^{8/3}\, d^5} + \frac{1}{18\, c^{8/3}\, d^5}$$

$$(b\, c - a\, d)^{8/3}\, (54\, b^2\, c^2 + 18\, a\, b\, c\, d + 5\, a^2\, d^2)\, \text{Log}\left[\frac{(b\, c - a\, d)^{1/3}\, x}{c^{1/3}} - (a + b\, x^3)^{1/3}\right] -$$

$$\frac{b^{8/3}\, (54\, b^2\, c^2 - 126\, a\, b\, c\, d + 77\, a^2\, d^2)\, \text{Log}\left[-b^{1/3}\, x + (a + b\, x^3)^{1/3}\right]}{18\, d^5}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^4\,x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},\,-\frac{14}{3},\,3,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{c^3\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Problem 110: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{11/3}}{\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 458 leaves, 6 steps):

$$\frac{b\,\left(18\,b^2\,c^2-7\,a\,b\,c\,d-5\,a^2\,d^2\right)\,x\,\left(a+b\,x^3\right)^{2/3}}{18\,c^2\,d^3}-\frac{\left(b\,c-a\,d\right)\,x\,\left(a+b\,x^3\right)^{8/3}}{6\,c\,d\,\left(c+d\,x^3\right)^2}-$$

$$\frac{\left(b\,c-a\,d\right)\,\left(9\,b\,c+5\,a\,d\right)\,x\,\left(a+b\,x^3\right)^{5/3}}{18\,c^2\,d^2\,\left(c+d\,x^3\right)}-\frac{b^{8/3}\,\left(9\,b\,c-11\,a\,d\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,d^4}+$$

$$\frac{\left(b\,c-a\,d\right)^{5/3}\,\left(27\,b^2\,c^2+12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,d^4}+$$

$$\frac{\left(b\,c-a\,d\right)^{5/3}\,\left(27\,b^2\,c^2+12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{Log}\left[c+d\,x^3\right]}{54\,c^{8/3}\,d^4}-\frac{1}{18\,c^{8/3}\,d^4}$$

$$\left(b\,c-a\,d\right)^{5/3}\,\left(27\,b^2\,c^2+12\,a\,b\,c\,d+5\,a^2\,d^2\right)\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]+$$

$$\frac{b^{8/3}\,\left(9\,b\,c-11\,a\,d\right)\,\text{Log}\left[-b^{1/3}\,x+\left(a+b\,x^3\right)^{1/3}\right]}{6\,d^4}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^3\,x\,\left(a+b\,x^3\right)^{2/3}\,\text{AppellF1}\left[\frac{1}{3},\,-\frac{11}{3},\,3,\,\frac{4}{3},\,-\frac{b\,x^3}{a},\,-\frac{d\,x^3}{c}\right]}{c^3\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}}$$

Problem 111: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{8/3}}{\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 391 leaves, 5 steps):

$$-\frac{(b c-a d) x \left(a+b x^3\right)^{5/3}}{6 c d \left(c+d x^3\right)^2}-\frac{(b c-a d) (6 b c+5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 d^2 \left(c+d x^3\right)}+$$

$$\frac{b^{8/3} \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-\frac{(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^3}-$$

$$\frac{(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} d^3}+\frac{1}{18 c^{8/3} d^3}$$

$$(b c-a d)^{2/3} \left(9 b^2 c^2+6 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]-$$

$$\frac{b^{8/3} \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 d^3}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},-\frac{8}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{5/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 217 leaves, 3 steps):

$$\frac{x \left(a+b x^3\right)^{5/3}}{6 c \left(c+d x^3\right)^2}+\frac{5 a x \left(a+b x^3\right)^{2/3}}{18 c^2 \left(c+d x^3\right)}+\frac{5 a^2 \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{1/3}}+$$

$$\frac{5 a^2 \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{1/3}}-\frac{5 a^2 \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{1/3}}$$

Result (type 3, 276 leaves, 9 steps):

$$\frac{x \left(a+b x^3\right)^{5/3}}{6 c \left(c+d x^3\right)^2}+\frac{5 a x \left(a+b x^3\right)^{2/3}}{18 c^2 \left(c+d x^3\right)}+\frac{5 a^2 \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{1/3}}-$$

$$\frac{5 a^2 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 c^{8/3} (b c-a d)^{1/3}}+\frac{5 a^2 \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 c^{8/3} (b c-a d)^{1/3}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 267 leaves, 3 steps):

$$-\frac{d x \left(a+b x^3\right)^{5/3}}{6 c (b c-a d) \left(c+d x^3\right)^2}+\frac{(6 b c-5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 (b c-a d) \left(c+d x^3\right)}+\frac{a (6 b c-5 a d) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{4/3}}+\frac{a (6 b c-5 a d) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{4/3}}-\frac{a (6 b c-5 a d) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{4/3}}$$

Result (type 3, 326 leaves, 9 steps):

$$-\frac{d x \left(a+b x^3\right)^{5/3}}{6 c (b c-a d) \left(c+d x^3\right)^2}+\frac{(6 b c-5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 (b c-a d) \left(c+d x^3\right)}+\frac{a (6 b c-5 a d) \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{4/3}}-\frac{a (6 b c-5 a d) \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{27 c^{8/3} (b c-a d)^{4/3}}+\frac{a (6 b c-5 a d) \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{54 c^{8/3} (b c-a d)^{4/3}}$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 307 leaves, 4 steps):

$$-\frac{d x \left(a+b x^3\right)^{2/3}}{6 c (b c-a d) \left(c+d x^3\right)^2}-\frac{d (9 b c-5 a d) x \left(a+b x^3\right)^{2/3}}{18 c^2 (b c-a d)^2 \left(c+d x^3\right)}+\frac{\left(9 b^2 c^2-12 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{7/3}}+\frac{\left(9 b^2 c^2-12 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{7/3}}-\frac{\left(9 b^2 c^2-12 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{7/3}}$$

Result (type 5, 167 leaves, 2 steps):

$$-\left(\left(x\left(c\,d\left(3\,b^2\,c\,x^3\left(4\,c+3\,d\,x^3\right)-a^2\,d\left(8\,c+5\,d\,x^3\right)+a\,b\left(12\,c^2+c\,d\,x^3-5\,d^2\,x^6\right)\right)-2\left(9\,b^2\,c^2-12\,a\,b\,c\,d+5\,a^2\,d^2\right)\left(c+d\,x^3\right)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\left(a+b\,x^3\right)}\right]\right)\right)\Big/\left(18\,c^3\,(b\,c-a\,d)^2\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)^2\right)\right)$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^3\right)^{4/3}\left(c+d\,x^3\right)^3}\,dx$$

Optimal (type 3, 377 leaves, 5 steps):

$$-\frac{d\,x}{6\,c\,(b\,c-a\,d)\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)^2}+\frac{b\,(6\,b\,c+a\,d)\,x}{6\,a\,c\,(b\,c-a\,d)^2\left(a+b\,x^3\right)^{1/3}\left(c+d\,x^3\right)}+\frac{d\left(18\,b^2\,c^2+15\,a\,b\,c\,d-5\,a^2\,d^2\right)x\left(a+b\,x^3\right)^{2/3}}{18\,a\,c^2\,(b\,c-a\,d)^3\left(c+d\,x^3\right)}-\frac{d\left(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2\right)\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,c^{8/3}\,(b\,c-a\,d)^{10/3}}-\frac{d\left(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2\right)\text{Log}\left[c+d\,x^3\right]}{54\,c^{8/3}\,(b\,c-a\,d)^{10/3}}+\frac{d\left(27\,b^2\,c^2-18\,a\,b\,c\,d+5\,a^2\,d^2\right)\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{18\,c^{8/3}\,(b\,c-a\,d)^{10/3}}$$

Result (type 5, 428 leaves, 2 steps):

$$-\frac{1}{16380\,c^5\,(b\,c-a\,d)^3\,x^8\left(a+b\,x^3\right)^{7/3}\left(c+d\,x^3\right)^2}\left(65\,c^2\left(a+b\,x^3\right)^2\left(14000\,a^2\,c^5+21896\,a\,b\,c^5\,x^3+48104\,a^2\,c^4\,d\,x^3+8391\,b^2\,c^5\,x^6+70802\,a\,b\,c^4\,d\,x^6+60807\,a^2\,c^3\,d^2\,x^6+24417\,b^2\,c^4\,d\,x^9+81534\,a\,b\,c^3\,d^2\,x^9+33657\,a^2\,c^2\,d^3\,x^9+23409\,b^2\,c^3\,d^2\,x^{12}+38652\,a\,b\,c^2\,d^3\,x^{12}+7155\,a^2\,c\,d^4\,x^{12}+7425\,b^2\,c^2\,d^3\,x^{15}+5940\,a\,b\,c\,d^4\,x^{15}+243\,a^2\,d^5\,x^{15}-28\left(c+d\,x^3\right)^2\left(27\,b^2\,c^2\,x^6\left(7\,c+6\,d\,x^3\right)+9\,a\,b\,c\,x^3\left(73\,c^2+104\,c\,d\,x^3+33\,d^2\,x^6\right)+a^2\left(500\,c^3+843\,c^2\,d\,x^3+375\,c\,d^2\,x^6+27\,d^3\,x^9\right)\right)\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\left(a+b\,x^3\right)}\right]\right)-486\,(b\,c-a\,d)^4\,x^{12}\left(c+d\,x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\left(a+b\,x^3\right)}\right]\right)$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{7/3} \left(c+d x^3\right)^3} \, dx$$

Optimal (type 3, 463 leaves, 6 steps):

$$-\frac{d x}{6 c (b c-a d) \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)^2}+\frac{b (3 b c+2 a d) x}{12 a c (b c-a d)^2 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}+\frac{b \left(9 b^2 c^2-42 a b c d-2 a^2 d^2\right) x}{12 a^2 c (b c-a d)^3 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)}+$$
$$\frac{d \left(27 b^3 c^3-135 a b^2 c^2 d-42 a^2 b c d^2+10 a^3 d^3\right) x \left(a+b x^3\right)^{2/3}}{36 a^2 c^2 (b c-a d)^4 \left(c+d x^3\right)}+$$
$$\frac{d^2 \left(54 b^2 c^2-24 a b c d+5 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c-a d)^{13/3}}+$$
$$\frac{d^2 \left(54 b^2 c^2-24 a b c d+5 a^2 d^2\right) \text{Log}\left[c+d x^3\right]}{54 c^{8/3} (b c-a d)^{13/3}}-$$
$$\frac{d^2 \left(54 b^2 c^2-24 a b c d+5 a^2 d^2\right) \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{18 c^{8/3} (b c-a d)^{13/3}}$$

Result (type 5, 1990 leaves, 2 steps):

$$-\frac{1}{524160 c^6 (b c-a d)^4 x^{11} \left(a+b x^3\right)^{10/3} \left(c+d x^3\right)^2}$$
$$\Bigg(522756 c^6 (b c-a d)^3 x^9 \left(a+b x^3\right)^2+1516320 c^5 d (b c-a d)^3 x^{12} \left(a+b x^3\right)^2+$$
$$2198664 c^4 d^2 (b c-a d)^3 x^{15} \left(a+b x^3\right)^2+1415232 c^3 d^3 (b c-a d)^3 x^{18} \left(a+b x^3\right)^2+$$
$$341172 c^2 d^4 (b c-a d)^3 x^{21} \left(a+b x^3\right)^2+28042560 c^7 (b c-a d)^2 x^6 \left(a+b x^3\right)^3+$$
$$107602560 c^6 d (b c-a d)^2 x^9 \left(a+b x^3\right)^3+157697280 c^5 d^2 (b c-a d)^2 x^{12} \left(a+b x^3\right)^3+$$
$$101088000 c^4 d^3 (b c-a d)^2 x^{15} \left(a+b x^3\right)^3+24261120 c^3 d^4 (b c-a d)^2 x^{18} \left(a+b x^3\right)^3-$$
$$265470660 c^8 (b c-a d) x^3 \left(a+b x^3\right)^4-1019636800 c^7 d (b c-a d) x^6 \left(a+b x^3\right)^4-$$
$$1466086440 c^6 d^2 (b c-a d) x^9 \left(a+b x^3\right)^4-930252960 c^5 d^3 (b c-a d) x^{12} \left(a+b x^3\right)^4-$$
$$221899860 c^4 d^4 (b c-a d) x^{15} \left(a+b x^3\right)^4+335877360 c^9 \left(a+b x^3\right)^5+$$
$$1279532800 c^8 d x^3 \left(a+b x^3\right)^5+1823334240 c^7 d^2 x^6 \left(a+b x^3\right)^5+$$
$$1151579520 c^6 d^3 x^9 \left(a+b x^3\right)^5+273939120 c^5 d^4 x^{12} \left(a+b x^3\right)^5-$$
$$67420080 c^7 (b c-a d)^2 x^6 \left(a+b x^3\right)^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-$$
$$259692160 c^6 d (b c-a d)^2 x^9 \left(a+b x^3\right)^3 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-$$

$$377\,700\,960\, c^5 d^2 (b c - a d)^2 x^{12} (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$241\,113\,600\, c^4 d^3 (b c - a d)^2 x^{15} (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$57\,723\,120\, c^3 d^4 (b c - a d)^2 x^{18} (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$349\,440\,000\, c^8 (b c - a d) x^3 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$1\,339\,520\,000\, c^7 d (b c - a d) x^6 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$1\,921\,920\,000\, c^6 d^2 (b c - a d) x^9 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$1\,218\,147\,840\, c^5 d^3 (b c - a d) x^{12} (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$290\,384\,640\, c^4 d^4 (b c - a d) x^{15} (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$335\,877\,360\, c^9 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$1\,279\,532\,800\, c^8 d x^3 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$1\,823\,334\,240\, c^7 d^2 x^6 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$1\,151\,579\,520\, c^6 d^3 x^9 (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$273\,939\,120\, c^5 d^4 x^{12} (a + b x^3)^5 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$57\,834\, c^4 (b c - a d)^5 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$224\,532\, c^3 d (b c - a d)^5 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$326\,592\, c^2 d^2 (b c - a d)^5 x^{21} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$210\,924\, c d^3 (b c - a d)^5 x^{24} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$51\,030\, d^4 (b c - a d)^5 x^{27} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$5103\, c^4 \left(b c - a d\right)^5 x^{15}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{\left(b c - a d\right) x^3}{c \left(a + b x^3\right)}\right] - 20412\, c^3 d \left(b c - a d\right)^5 x^{18}$$
$$\text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{\left(b c - a d\right) x^3}{c \left(a + b x^3\right)}\right] - 30618\, c^2 d^2$$
$$\left(b c - a d\right)^5 x^{21}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{\left(b c - a d\right) x^3}{c \left(a + b x^3\right)}\right] - 20412$$
$$c\, d^3 \left(b c - a d\right)^5 x^{24}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{\left(b c - a d\right) x^3}{c \left(a + b x^3\right)}\right] -$$
$$\left. 5103\, d^4 \left(b c - a d\right)^5 x^{27}\, \text{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{\left(b c - a d\right) x^3}{c \left(a + b x^3\right)}\right]\right)$$

Test results for the 1081 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(8 c - d x^3\right)^2 \left(c + d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2 x \left(4 c + d x^3\right)}{81 c d^2 \left(8 c - d x^3\right) \sqrt{c + d x^3}} - \left(2 \sqrt{2 + \sqrt{3}} \left(c^{1/3} + d^{1/3} x\right) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(81 \times 3^{1/4} c\, d^{7/3} \sqrt{\frac{c^{1/3} \left(c^{1/3} + d^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3}\right)$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \sqrt{1 + \frac{d x^3}{c}}\, \text{AppellF1}\left[\frac{7}{3}, 2, \frac{3}{2}, \frac{10}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{448 c^3 \sqrt{c + d x^3}}$$

Problem 574: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 268 leaves, 6 steps):

$$-\frac{7 a x^2 \left(a+b x^3\right)^{1/3}}{18 b^2 d}-\frac{x^5 \left(a+b x^3\right)^{1/3}}{6 b d}+\frac{11 a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3} d}-\frac{2^{1/3} a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{8/3} d}+\frac{a^2 \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{8/3} d}+\frac{11 a^2 \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{8/3} d}-\frac{a^2 \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{8/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{8}{3},-\frac{1}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{8 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 575: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$-\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 b d}+\frac{4 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d}-\frac{2^{1/3} a \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3} d}+\frac{a \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{2/3} b^{5/3} d}+\frac{2 a \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{3 b^{5/3} d}-\frac{a \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{2/3} b^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5 \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3},-\frac{1}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 576: Result unnecessarily involves higher level functions.

$$\int \frac{x \left(a+b x^3\right)^{1/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;b^{2/3}\,d}-\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;b^{2/3}\,d}+\frac{\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times 2^{2/3}\,b^{2/3}\,d}+$$
$$\frac{\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}\,d}-\frac{\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,b^{2/3}\,d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]}{2\,a\,d\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 577: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 156 leaves, 3 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{a\,d\,x}-\frac{2^{1/3}\,b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;a\,d}+$$
$$\frac{b^{1/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times 2^{2/3}\,a\,d}-\frac{b^{1/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a\,d}$$

Result (type 5, 77 leaves, 2 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}\,\left(1-\frac{b\,x^3}{a}\right)^{1/3}\,\text{Hypergeometric2F1}\left[-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{2\,b\,x^3}{a-b\,x^3}\right]}{a\,d\,x\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 578: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b\,x^3\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{4\,a\,d\,x^4}-\frac{5\,b\,\left(a+b\,x^3\right)^{1/3}}{4\,a^2\,d\,x}-\frac{2^{1/3}\,b^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\;a^2\,d}+$$
$$\frac{b^{4/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times 2^{2/3}\,a^2\,d}-\frac{b^{4/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^2\,d}$$

Result (type 5, 117 leaves, 2 steps):

$$-\left(\left(\left(a^2+4\,a\,b\,x^3+3\,b^2\,x^6-b\,x^3\,\left(a+3\,b\,x^3\right)\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,1,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]+\right.\right.\right.$$
$$\left.\left.\left.3\,b\,x^3\,\left(a-b\,x^3\right)\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,2,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)\right/\left(4\,a^2\,d\,x^4\,\left(a+b\,x^3\right)^{2/3}\right)\right)$$

Problem 579: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^8\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{7\,a\,d\,x^7}-\frac{2\,b\,\left(a+b\,x^3\right)^{1/3}}{7\,a^2\,d\,x^4}-\frac{8\,b^2\,\left(a+b\,x^3\right)^{1/3}}{7\,a^3\,d\,x}-$$
$$\frac{2^{1/3}\,b^{7/3}\,\text{ArcTan}\left[\frac{1+\frac{2\cdot2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^3\,d}+\frac{b^{7/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times2^{2/3}\,a^3\,d}-\frac{b^{7/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^3\,d}$$

Result (type 5, 244 leaves, 2 steps):

$$-\frac{1}{28\,a^3\,d\,x^7\,\left(a+b\,x^3\right)^{2/3}}\left(4\,a^3+10\,a^2\,b\,x^3+24\,a\,b^2\,x^6+18\,b^3\,x^9-\right.$$
$$2\,b\,x^3\,\left(2\,a^2+3\,a\,b\,x^3+9\,b^2\,x^6\right)\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,1,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$15\,a^2\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,2,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]+12\,a\,b^2\,x^6$$
$$\text{Hypergeometric2F1}\left[\frac{2}{3},\,2,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]-27\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},\,2,\,\frac{5}{3},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$\left.9\,b\,x^3\,\left(a-b\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},\,2,\,2\right\},\,\left\{1,\,\frac{5}{3}\right\},\,\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 580: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{10\,a\,d\,x^{10}}-\frac{11\,b\,\left(a+b\,x^3\right)^{1/3}}{70\,a^2\,d\,x^7}-\frac{37\,b^2\,\left(a+b\,x^3\right)^{1/3}}{140\,a^3\,d\,x^4}-\frac{169\,b^3\,\left(a+b\,x^3\right)^{1/3}}{140\,a^4\,d\,x}-$$
$$\frac{2^{1/3}\,b^{10/3}\,\text{ArcTan}\left[\frac{1+\frac{2\cdot2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^4\,d}+\frac{b^{10/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times2^{2/3}\,a^4\,d}-\frac{b^{10/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{2/3}\,a^4\,d}$$

Result (type 5, 423 leaves, 2 steps):

$$-\frac{1}{280\,a^4\,d\,x^{10}\,\left(a+b\,x^3\right)^{2/3}}\left(28\,a^4+64\,a^3\,b\,x^3+90\,a^2\,b^2\,x^6+\right.$$
$$216\,a\,b^3\,x^9+162\,b^4\,x^{12}-28\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$36\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-54\,a\,b^3\,x^9$$
$$\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-162\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$117\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$99\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+81\,a\,b^3\,x^9$$
$$\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-297\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-$$
$$54\,b\,x^3\,\left(a-b\,x^3\right)^2\,\left(2\,a+3\,b\,x^3\right)\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]+$$
$$\left.27\,b\,x^3\,\left(a-b\,x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{5}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 581: Result unnecessarily involves higher level functions.

$$\int\frac{x^6\,\left(a+b\,x^3\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Optimal (type 5, 521 leaves, 22 steps):

$$-\frac{3\,a\,x\,\left(a+b\,x^3\right)^{1/3}}{5\,b^2\,d}-\frac{x^4\,\left(a+b\,x^3\right)^{1/3}}{5\,b\,d}-\frac{2^{1/3}\,a^{5/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{7/3}\,d}-$$
$$\frac{a^{5/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}\,b^{7/3}\,d}-\frac{2\,a^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{5\,b^2\,d\,\left(a+b\,x^3\right)^{2/3}}-$$
$$\frac{a^{5/3}\,\text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\,b^{7/3}\,d}+\frac{a^{5/3}\,\text{Log}\left[1+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}-\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\times 2^{2/3}\,b^{7/3}\,d}-$$
$$\frac{2^{1/3}\,a^{5/3}\,\text{Log}\left[1+\frac{2^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{7/3}\,d}+\frac{a^{5/3}\,\text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3}\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{2^{2/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\times 2^{2/3}\,b^{7/3}\,d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{b\,x^3}{a},\frac{b\,x^3}{a}\right]}{7\,a\,d\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 582: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a + b x^3\right)^{1/3}}{a d - b d x^3} \, dx$$

Optimal (type 5, 494 leaves, 21 steps):

$$-\frac{x \left(a + b x^3\right)^{1/3}}{2 b d} - \frac{2^{1/3} a^{2/3} \text{ArcTan}\left[\frac{1 - \frac{2\, 2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{4/3} d} - \frac{a^{2/3} \text{ArcTan}\left[\frac{1 + \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, b^{4/3} d} - \frac{a x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 b d \left(a + b x^3\right)^{2/3}} - \frac{a^{2/3} \text{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{\left(a + b x^3\right)^{1/3}}\right]}{3 \times 2^{2/3} b^{4/3} d} + \frac{a^{2/3} \text{Log}\left[1 + \frac{2^{2/3} \left(a^{1/3} + b^{1/3} x\right)^2}{\left(a + b x^3\right)^{2/3}} - \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{3 \times 2^{2/3} b^{4/3} d} - \frac{2^{1/3} a^{2/3} \text{Log}\left[1 + \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{3 b^{4/3} d} + \frac{a^{2/3} \text{Log}\left[2 \times 2^{1/3} + \frac{\left(a^{1/3} + b^{1/3} x\right)^2}{\left(a + b x^3\right)^{2/3}} + \frac{2^{2/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{6 \times 2^{2/3} b^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4 \left(a + b x^3\right)^{1/3} \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{4 a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 583: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a + b x^3\right)^{1/3}}{a d - b d x^3} \, dx$$

Optimal (type 3, 416 leaves, 14 steps):

$$-\frac{2^{1/3} \text{ArcTan}\left[\frac{1 - \frac{2\, 2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{1/3} b^{1/3} d} - \frac{\text{ArcTan}\left[\frac{1 + \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{1/3} b^{1/3} d} - \frac{\text{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{\left(a + b x^3\right)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3} d} + \frac{\text{Log}\left[1 + \frac{2^{2/3} \left(a^{1/3} + b^{1/3} x\right)^2}{\left(a + b x^3\right)^{2/3}} - \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3} d} - \frac{2^{1/3} \text{Log}\left[1 + \frac{2^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{3 a^{1/3} b^{1/3} d} + \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{\left(a^{1/3} + b^{1/3} x\right)^2}{\left(a + b x^3\right)^{2/3}} + \frac{2^{2/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(a + b x^3\right)^{1/3}}\right]}{6 \times 2^{2/3} a^{1/3} b^{1/3} d}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 584: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^3\left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 496 leaves, 21 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{2 a d x^2}-\frac{2^{1/3} b^{2/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{4/3} d}-\frac{b^{2/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}\, a^{4/3} d}+\frac{b x\left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{2 a d\left(a+b x^3\right)^{2/3}}-\frac{b^{2/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{4/3} d}+\frac{b^{2/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{4/3} d}-\frac{2^{1/3} b^{2/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{4/3} d}+\frac{b^{2/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{2 a d x^2\left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 585: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^6\left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 523 leaves, 22 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{5 a d x^5}-\frac{3 b \left(a+b x^3\right)^{1/3}}{5 a^2 d x^2}-\frac{2^{1/3} b^{5/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{7/3} d}-$$

$$\frac{b^{5/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}\, a^{7/3} d}+\frac{2 b^2 x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{5 a^2 d \left(a+b x^3\right)^{2/3}}-$$

$$\frac{b^{5/3} \text{Log}\left[2^{2/3}-\frac{a^{1/3}+b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{7/3} d}+\frac{b^{5/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{2/3} a^{7/3} d}-$$

$$\frac{2^{1/3} b^{5/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{7/3} d}+\frac{b^{5/3} \text{Log}\left[2\times 2^{1/3}+\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{6\times 2^{2/3} a^{7/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{1/3} \text{AppellF1}\left[-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d x^5 \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 593: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 264 leaves, 5 steps):

$$-\frac{4 a x \left(a+b x^3\right)^{2/3}}{9 b^2 d}-\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 b d}-\frac{14 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3}\, b^{7/3} d}+\frac{2^{2/3} a^2 \text{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{7/3} d}+$$

$$\frac{a^2 \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{7/3} d}-\frac{a^2 \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{7/3} d}+\frac{7 a^2 \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{9 b^{7/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},-\frac{2}{3},1,\frac{10}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{7 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 594: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{a d-b d x^3}\, dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$-\frac{x\left(a+b x^3\right)^{2/3}}{3 b d}-\frac{5 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{4/3} d}+\frac{2^{2/3} a \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d}+$$
$$\frac{a \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{4/3} d}-\frac{a \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{4/3} d}+\frac{5 a \operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{4/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{4}{3},-\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{4 a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 595: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 3, 200 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} d}+\frac{2^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} d}+\frac{\operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} b^{1/3} d}-$$
$$\frac{\operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} b^{1/3} d}+\frac{\operatorname{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 b^{1/3} d}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 596: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^3\left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 157 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a d x^2}+\frac{2^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2\, 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a d}+$$
$$\frac{b^{2/3} \operatorname{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a d}-\frac{b^{2/3} \operatorname{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a d}$$

Result (type 5, 79 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \left(1-\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{2 b x^3}{a-b x^3}\right]}{2 a d x^2 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 597: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^6 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 182 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 a d x^5}-\frac{7 b \left(a+b x^3\right)^{2/3}}{10 a^2 d x^2}+\frac{2^{2/3} b^{5/3} \text{ArcTan}\left[\frac{1+\frac{2\ 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^2 d}+\frac{b^{5/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a^2 d}-\frac{b^{5/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^2 d}$$

Result (type 5, 121 leaves, 2 steps):

$$-\left(\left(2 a^2+5 a b x^3+3 b^2 x^6-4 b x^3 \left(2 a+3 b x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]+12 b x^3 \left(a-b x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2 b x^3}{a+b x^3}\right]\right)\Big/\left(10 a^2 d x^5 \left(a+b x^3\right)^{1/3}\right)\right)$$

Problem 598: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^9 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 3, 209 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 a d x^8}-\frac{b \left(a+b x^3\right)^{2/3}}{4 a^2 d x^5}-\frac{5 b^2 \left(a+b x^3\right)^{2/3}}{8 a^3 d x^2}+\frac{2^{2/3} b^{8/3} \text{ArcTan}\left[\frac{1+\frac{2\ 2^{1/3} b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^3 d}+\frac{b^{8/3} \text{Log}\left[a d-b d x^3\right]}{3\times 2^{1/3} a^3 d}-\frac{b^{8/3} \text{Log}\left[2^{1/3} b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2^{1/3} a^3 d}$$

Result (type 5, 244 leaves, 2 steps):

$$-\frac{1}{40\,a^3\,d\,x^8\,\left(a+b\,x^3\right)^{1/3}}\left(5\,a^3+11\,a^2\,b\,x^3+15\,a\,b^2\,x^6+9\,b^3\,x^9-4\,b\,x^3\,\left(5\,a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+42\,a^2\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+12\,a\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-54\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-18\,b\,x^3\,\left(a-b\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 599: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{x^{12}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{\left(a+b\,x^3\right)^{2/3}}{11\,a\,d\,x^{11}}-\frac{13\,b\,\left(a+b\,x^3\right)^{2/3}}{88\,a^2\,d\,x^8}-\frac{49\,b^2\,\left(a+b\,x^3\right)^{2/3}}{220\,a^3\,d\,x^5}-\frac{293\,b^3\,\left(a+b\,x^3\right)^{2/3}}{440\,a^4\,d\,x^2}+\frac{2^{2/3}\,b^{11/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a^4\,d}+\frac{b^{11/3}\,\text{Log}\left[a\,d-b\,d\,x^3\right]}{3\times2^{1/3}\,a^4\,d}-\frac{b^{11/3}\,\text{Log}\left[2^{1/3}\,b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2^{1/3}\,a^4\,d}$$

Result (type 5, 391 leaves, 2 steps):

$$-\frac{1}{440\,a^4\,d\,x^{11}\,\left(a+b\,x^3\right)^{1/3}}\left(40\,a^4+85\,a^3\,b\,x^3+99\,a^2\,b^2\,x^6+135\,a\,b^3\,x^9+81\,b^4\,x^{12}-160\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-180\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-216\,a\,b^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-324\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+396\,a^3\,b\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]+198\,a^2\,b^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-594\,b^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{2\,b\,x^3}{a+b\,x^3}\right]-54\,b\,x^3\,\left(a-b\,x^3\right)^2\,\left(5\,a+6\,b\,x^3\right)\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]+54\,b\,x^3\,\left(a-b\,x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2,2\right\},\left\{1,1,\frac{4}{3}\right\},\frac{2\,b\,x^3}{a+b\,x^3}\right]\right)$$

Problem 600: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$-\frac{9 a x^2 \left(a+b x^3\right)^{2/3}}{28 b^2 d}-\frac{x^5 \left(a+b x^3\right)^{2/3}}{7 b d}+\frac{2^{2/3} a^{7/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{8/3} d}+$$

$$\frac{a^{7/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, b^{8/3} d}-\frac{19 a^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{28 b^2 d \left(a+b x^3\right)^{1/3}}+$$

$$\frac{a^{7/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{8/3} d}+\frac{a^{7/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{8/3} d}-$$

$$\frac{2^{2/3} a^{7/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{8/3} d}-\frac{a^{7/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{8/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{8 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 601: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{2/3}}{a d-b d x^3} \, dx$$

Optimal (type 5, 485 leaves, 13 steps):

$$-\frac{x^2\left(a+b x^3\right)^{2/3}}{4 b d}+\frac{2^{2/3} a^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{5/3} d}+$$
$$\frac{a^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, b^{5/3} d}-\frac{3 a x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 b d\left(a+b x^3\right)^{1/3}}+$$
$$\frac{a^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{5/3} d}+\frac{a^{4/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{5/3} d}-$$
$$\frac{2^{2/3} a^{4/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{5/3} d}-\frac{a^{4/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{5 a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 602: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(a+b x^3\right)^{2/3}}{a d-b d x^3}\, dx$$

Optimal (type 5, 457 leaves, 11 steps):

$$\frac{2^{2/3} a^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{a^{1/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}\, b^{2/3} d}-$$
$$\frac{x^2\left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 d\left(a+b x^3\right)^{1/3}}+$$
$$\frac{a^{1/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2\left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} b^{2/3} d}+\frac{a^{1/3} \text{Log}\left[1+\frac{2^{2/3}\left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} b^{2/3} d}-$$
$$\frac{2^{2/3} a^{1/3} \text{Log}\left[1+\frac{2^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{2/3} d}-\frac{a^{1/3} \text{Log}\left[\frac{b^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}\left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} b^{2/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{2 a d\left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 603: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^2 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 483 leaves, 13 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{a d x}+\frac{2^{2/3} b^{1/3} \operatorname{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, a^{2/3} d}+$$
$$\frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}\, a^{2/3} d}+\frac{b x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{2 a d \left(a+b x^3\right)^{1/3}}+$$
$$\frac{b^{1/3} \operatorname{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6 \times 2^{1/3} a^{2/3} d}+\frac{b^{1/3} \operatorname{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 \times 2^{1/3} a^{2/3} d}-$$
$$\frac{2^{2/3} b^{1/3} \operatorname{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{2/3} d}-\frac{b^{1/3} \operatorname{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} a^{2/3} d}$$

Result (type 6, 64 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \operatorname{AppellF1}\left[-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{a d x \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 604: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^5 \left(a d-b d x^3\right)} \, dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{4 a d x^4}-\frac{3 b \left(a+b x^3\right)^{2/3}}{2 a^2 d x}+\frac{2^{2/3} b^{4/3} \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^{5/3} d}+$$

$$\frac{b^{4/3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} a^{5/3} d}+\frac{3 b^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{b x^3}{a}\right]}{4 a^2 d \left(a+b x^3\right)^{1/3}}+$$

$$\frac{b^{4/3} \text{Log}\left[\frac{\left(a^{1/3}-b^{1/3} x\right)^2 \left(a^{1/3}+b^{1/3} x\right)}{a}\right]}{6\times 2^{1/3} a^{5/3} d}+\frac{b^{4/3} \text{Log}\left[1+\frac{2^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2}{\left(a+b x^3\right)^{2/3}}-\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3\times 2^{1/3} a^{5/3} d}-$$

$$\frac{2^{2/3} b^{4/3} \text{Log}\left[1+\frac{2^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(a+b x^3\right)^{1/3}}\right]}{3 a^{5/3} d}-\frac{b^{4/3} \text{Log}\left[\frac{b^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{a^{1/3}}-\frac{2^{2/3} b^{1/3} \left(a+b x^3\right)^{1/3}}{a^{1/3}}\right]}{2\times 2^{1/3} a^{5/3} d}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{\left(a+b x^3\right)^{2/3} \text{AppellF1}\left[-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{b x^3}{a},\frac{b x^3}{a}\right]}{4 a d x^4 \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 612: Result valid but suboptimal antiderivative.

$$\int \frac{x^6}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$-\frac{1}{3} x \left(1-x^3\right)^{2/3}+\frac{2 \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-$$

$$\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}-\frac{1}{3} \text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 226 leaves, 15 steps):

$$-\frac{1}{3} x \left(1-x^3\right)^{2/3}+\frac{2 \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{1}{9} \text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-$$

$$\frac{2}{9} \text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 613: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\operatorname{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\operatorname{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}+\frac{1}{2}\operatorname{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{1}{6}\operatorname{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{1}{3}\operatorname{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{\operatorname{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\operatorname{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 614: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\operatorname{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\operatorname{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\operatorname{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\operatorname{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 615: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 105 leaves, 3 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{2 x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 139 leaves, 8 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{2 x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 616: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}+\frac{\left(1-x^3\right)^{2/3}}{5 x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{5/3}}{5 x^5}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 617: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{8 x^8}+\frac{\left(1-x^3\right)^{2/3}}{20 x^5}-\frac{17 \left(1-x^3\right)^{2/3}}{40 x^2}+$$
$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 175 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{2 x^2}-\frac{\left(1-x^3\right)^{5/3}}{5 x^5}-\frac{\left(1-x^3\right)^{8/3}}{8 x^8}+$$
$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 618: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 271 leaves, 12 steps):

$$-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{8}x^8\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},1,\frac{11}{3},x^3,-x^3\right]$$

Problem 619: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 254 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5}x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]$$

Problem 620: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[(1-x)\ (1+x)^2\right]}{12\times 2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\ (1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\ (1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}\ x^2\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 621: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\ (1-x^3)^{1/3}\ (1+x^3)}\,dx$$

Optimal (type 5, 270 leaves, 12 steps):

$$-\frac{(1-x^3)^{2/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-$$
$$\frac{1}{2}\ x^2\ \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)\ (1+x)^2\right]}{12\times 2^{1/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\ (1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\ (1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},x^3,-x^3\right]}{x}$$

Problem 622: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^5\ (1-x^3)^{1/3}\ (1+x^3)}\,dx$$

Optimal (type 5, 289 leaves, 14 steps):

$$-\frac{(1-x^3)^{2/3}}{4\ x^4}+\frac{(1-x^3)^{2/3}}{2\ x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+$$
$$\frac{1}{4}\ x^2\ \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)\ (1+x)^2\right]}{12\times 2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\ (1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\ (1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\ (1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},x^3,-x^3\right]}{4\,x^4}$$

Problem 629: Result valid but suboptimal antiderivative.

$$\int\frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$-\frac{1}{3}x^2\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+$$
$$\frac{\text{Log}\left[1+x^3\right]}{6\times2^{2/3}}+\frac{1}{6}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times2^{2/3}}$$

Result (type 3, 228 leaves, 14 steps):

$$-\frac{1}{3}x^2\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{1}{18}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]+$$
$$\frac{1}{9}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times2^{2/3}}$$

Problem 630: Result valid but suboptimal antiderivative.

$$\int\frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-$$
$$\frac{\text{Log}\left[1+x^3\right]}{6\times2^{2/3}}-\frac{1}{2}\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times2^{2/3}}$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{1}{6}\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-$$
$$\frac{1}{3}\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 631: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3} x-(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 632: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$$

Optimal (type 3, 103 leaves, 2 steps):

$$-\frac{(1-x^3)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[-2^{1/3} x-(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 137 leaves, 8 steps):

$$-\frac{(1-x^3)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 633: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{4 x^4}+\frac{\left(1-x^3\right)^{1/3}}{4 x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{4/3}}{4 x^4}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Problem 634: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 291 leaves, 15 steps):

$$-\frac{1}{2} x \left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7} x^7 \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},1,\frac{10}{3},x^3,-x^3\right]$$

Problem 635: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 294 leaves, 18 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3} \sqrt{3}}+$$
$$\frac{1}{2} x \,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{4} x^4 \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right]$$

Problem 636: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 293 leaves, 16 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}}+\frac{1}{2}\,x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 21 leaves, 1 step):

$$x\,\text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]$$

Problem 637: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 294 leaves, 16 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{2\,x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},x^3,-x^3\right]}{2\,x^2}$$

Problem 645: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$\frac{x^4}{2\left(1-x^3\right)^{1/3}}+\frac{5}{6}x\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+$$
$$\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{6}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{10}x^{10}\,\text{AppellF1}\left[\frac{10}{3},\frac{4}{3},1,\frac{13}{3},x^3,-x^3\right]$$

Problem 646: Result unnecessarily involves higher level functions.

$$\int\frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-$$
$$\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7}x^{7}\,\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]$$

Problem 647: Result unnecessarily involves higher level functions.

$$\int\frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 38 leaves, 1 step):

$$\frac{x^4\,\text{Hypergeometric2F1}\left[\frac{4}{3},\frac{4}{3},\frac{7}{3},\frac{2x^3}{1+x^3}\right]}{4\left(1+x^3\right)^{4/3}}$$

Problem 648: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$\frac{x}{2\left(1-x^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Problem 649: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$\frac{1}{2x^2\left(1-x^3\right)^{1/3}}-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 5, 204 leaves, 1 step):

$$-\frac{1}{14x^5\left(1-x^3\right)^{7/3}}\Bigg(14+56x^3-91x^6-42x^9+63x^{12}-7\left(1-x^3\right)^2\left(2+12x^3+9x^6\right)$$
$$\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]-30x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]-$$
$$84x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]-54x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]-$$
$$18x^6\left(1+x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},-\frac{2x^3}{1-x^3}\right]\Bigg)$$

Problem 650: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{1}{2x^5\left(1-x^3\right)^{1/3}}-\frac{7\left(1-x^3\right)^{2/3}}{10x^5}-\frac{4\left(1-x^3\right)^{2/3}}{5x^2}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{12\times2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times2^{1/3}}$$

Result (type 5, 397 leaves, 1 step):

$$-\frac{1}{70x^8\left(1-x^3\right)^{7/3}}\left(28-182x^3-476x^6+819x^9+378x^{12}-567x^{15}-28\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]+182x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]+476x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]-819x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]-378x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]+567x^{15}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},-\frac{2x^3}{1-x^3}\right]-36x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]+342x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]+972x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]+594x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},-\frac{2x^3}{1-x^3}\right]+54x^6\left(1+x^3\right)^2\left(1+6x^3\right)\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},-\frac{2x^3}{1-x^3}\right]+54x^6\left(1+x^3\right)^3\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},-\frac{2x^3}{1-x^3}\right]\right)$$

Problem 651: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{1}{2x^8\left(1-x^3\right)^{1/3}}-\frac{5\left(1-x^3\right)^{2/3}}{8x^8}-\frac{13\left(1-x^3\right)^{2/3}}{20x^5}-\frac{49\left(1-x^3\right)^{2/3}}{40x^2}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{12\times2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{4\times2^{1/3}}$$

Result (type 5, 612 leaves, 1 step):

$$-\frac{1}{280\,x^{11}\left(1-x^3\right)^{4/3}\left(-1+x^3\right)}\Bigg(-70+308\,x^3-1162\,x^6-2856\,x^9+$$
$$4914\,x^{12}+2268\,x^{15}-3402\,x^{18}+70\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]-$$
$$308\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]+1162\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$2856\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]-$$
$$4914\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]-2268\,x^{15}$$
$$\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]+3402\,x^{18}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$66\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2\,x^3}{-1+x^3}\right]-312\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$2268\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$6696\,x^{15}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$4050\,x^{18}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{2\,x^3}{-1+x^3}\right]+$$
$$27\,x^6\left(1+x^3\right)^2\left(-7+18\,x^3+105\,x^6\right)\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$54\,x^6\left(1+x^3\right)^3\left(-1+15\,x^3\right)\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$81\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$324\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$486\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$324\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]+$$
$$81\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{2\,x^3}{-1+x^3}\right]\Bigg)$$

Problem 652: Result unnecessarily involves higher level functions.

$$\int\frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{x^5}{2\left(1-x^3\right)^{1/3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-$$
$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times 2^{1/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{11}x^{11}\,\text{AppellF1}\left[\frac{11}{3},\frac{4}{3},1,\frac{14}{3},x^3,-x^3\right]$$

Problem 653: Result unnecessarily involves higher level functions.

$$\int\frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\frac{x^2}{2\left(1-x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\sqrt{3}}-$$
$$\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times 2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{8}x^8\,\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]$$

Problem 654: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\frac{x^2}{2\,(1-x^3)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\,\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\,\sqrt{3}}-\frac{1}{4}\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5}\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]$$

Problem 655: Result unnecessarily involves higher level functions.

$$\int\frac{x}{(1-x^3)^{4/3}\,(1+x^3)}\,dx$$

Optimal (type 5, 274 leaves, 11 steps):

$$\frac{x^2}{2\,(1-x^3)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{4\times 2^{1/3}\,\sqrt{3}}-\frac{1}{4}\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{24\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{8\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{4}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 656: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\,(1-x^3)^{4/3}\,(1+x^3)}\,dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\frac{1}{2x\left(1-x^3\right)^{1/3}}-\frac{3\left(1-x^3\right)^{2/3}}{2x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{1/3}\sqrt{3}}-$$

$$\frac{3}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times2^{1/3}}-$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}+\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times2^{1/3}}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},x^3,-x^3\right]}{x}$$

Problem 657: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 308 leaves, 14 steps):

$$\frac{1}{2x^4\left(1-x^3\right)^{1/3}}-\frac{3\left(1-x^3\right)^{2/3}}{4x^4}-\frac{\left(1-x^3\right)^{2/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}+$$

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{4\times2^{1/3}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24\times2^{1/3}}+$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{8\times2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},x^3,-x^3\right]}{4x^4}$$

Problem 665: Result unnecessarily involves higher level functions.

$$\int\frac{x^7\left(a+b\,x^3\right)^{1/3}}{c+d\,x^3}\,dx$$

Optimal (type 3, 336 leaves, 6 steps):

$$-\frac{(6bc-ad)\,x^2\,(a+bx^3)^{1/3}}{18\,b\,d^2}+\frac{x^5\,(a+bx^3)^{1/3}}{6\,d}-\frac{(9b^2c^2-3abcd-a^2d^2)\,\text{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}\,b^{5/3}\,d^3}+\frac{c^{5/3}\,(bc-ad)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^3}-\frac{c^{5/3}\,(bc-ad)^{1/3}\,\text{Log}[c+dx^3]}{6\,d^3}-\frac{(9b^2c^2-3abcd-a^2d^2)\,\text{Log}\left[b^{1/3}x-(a+bx^3)^{1/3}\right]}{18\,b^{5/3}\,d^3}+\frac{c^{5/3}\,(bc-ad)^{1/3}\,\text{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}}-(a+bx^3)^{1/3}\right]}{2\,d^3}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^8\,(a+bx^3)^{1/3}\,\text{AppellF1}\left[\frac{8}{3},-\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]}{8\,c\left(1+\frac{bx^3}{a}\right)^{1/3}}$$

Problem 666: Result unnecessarily involves higher level functions.

$$\int\frac{x^4\,(a+bx^3)^{1/3}}{c+dx^3}\,dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{x^2\,(a+bx^3)^{1/3}}{3\,d}+\frac{(3bc-ad)\,\text{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\,b^{2/3}\,d^2}-\frac{c^{2/3}\,(bc-ad)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2}+\frac{c^{2/3}\,(bc-ad)^{1/3}\,\text{Log}[c+dx^3]}{6\,d^2}+\frac{(3bc-ad)\,\text{Log}\left[b^{1/3}x-(a+bx^3)^{1/3}\right]}{6\,b^{2/3}\,d^2}-\frac{c^{2/3}\,(bc-ad)^{1/3}\,\text{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}}-(a+bx^3)^{1/3}\right]}{2\,d^2}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^5\,(a+bx^3)^{1/3}\,\text{AppellF1}\left[\frac{5}{3},-\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right]}{5\,c\left(1+\frac{bx^3}{a}\right)^{1/3}}$$

Problem 667: Result unnecessarily involves higher level functions.

$$\int\frac{x\,(a+bx^3)^{1/3}}{c+dx^3}\,dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d}+\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{1/3}\,d}-\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{1/3}\,d}-$$
$$\frac{b^{1/3}\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,d}+\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{1/3}\,d}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^2\,\left(a+b\,x^3\right)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,c\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}$$

Problem 668: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^2\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 168 leaves, 3 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{c\,x}-\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{4/3}}+$$
$$\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{4/3}}-\frac{\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{4/3}}$$

Result (type 5, 87 leaves, 2 steps):

$$-\frac{1}{c\,x\,\left(1+\frac{b\,x^3}{a}\right)^{1/3}}\left(a+b\,x^3\right)^{1/3}\,\left(1+\frac{d\,x^3}{c}\right)^{1/3}\,\text{Hypergeometric2F1}\left[-\frac{1}{3},-\frac{1}{3},\frac{2}{3},-\frac{c\,\left(\frac{b\,x^3}{a}-\frac{d\,x^3}{c}\right)}{c+d\,x^3}\right]$$

Problem 669: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^5\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 204 leaves, 4 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{4\,c\,x^4}-\frac{\left(b\,c-4\,a\,d\right)\,\left(a+b\,x^3\right)^{1/3}}{4\,a\,c^2\,x}+\frac{d\,\left(b\,c-a\,d\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{7/3}}-$$
$$\frac{d\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{7/3}}+\frac{d\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{7/3}}$$

Result (type 5, 145 leaves, 2 steps):

$$-\left(\left(2\,c\,\left(a+b\,x^3\right)\,\left(c-3\,d\,x^3\right)-\left(b\,c-a\,d\right)\,x^3\,\left(c-3\,d\,x^3\right)\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+3\,\left(b\,c-a\,d\right)\,x^3\,\left(c+d\,x^3\right)\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)\Big/\left(8\,c^3\,x^4\,\left(a+b\,x^3\right)^{2/3}\right)\right)$$

Problem 670: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^3\right)^{1/3}}{x^8\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$-\frac{\left(a+b\,x^3\right)^{1/3}}{7\,c\,x^7}-\frac{\left(b\,c-7\,a\,d\right)\,\left(a+b\,x^3\right)^{1/3}}{28\,a\,c^2\,x^4}+\frac{\left(3\,b^2\,c^2+7\,a\,b\,c\,d-28\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{1/3}}{28\,a^2\,c^3\,x}-\frac{d^2\,\left(b\,c-a\,d\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{10/3}}+\frac{d^2\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{10/3}}-\frac{d^2\,\left(b\,c-a\,d\right)^{1/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{10/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$-\frac{1}{56 c^4 x^7 \left(a+b x^3\right)^{2/3}}\left(8 a c^3+8 b c^3 x^3-12 a c^2 d x^3-12 b c^2 d x^6+36 a c d^2 x^6+36 b c d^2 x^9-\right.$$
$$2 \left(b c-a d\right) x^3 \left(2 c^2-3 c d x^3+9 d^2 x^6\right) \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$
$$15 b c^3 x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$15 a c^2 d x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$12 b c^2 d x^6 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$
$$12 a c d^2 x^6 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$27 b c d^2 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$
$$27 a d^3 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$\left.9 \left(b c-a d\right) x^3 \left(c+d x^3\right)^2 \text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]\right)$$

Problem 671: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{x^{11} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{10 c x^{10}}-\frac{\left(b c-10 a d\right) \left(a+b x^3\right)^{1/3}}{70 a c^2 x^7}+\frac{\left(3 b^2 c^2+5 a b c d-35 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{140 a^2 c^3 x^4}-$$
$$\frac{\left(9 b^3 c^3+15 a b^2 c^2 d+35 a^2 b c d^2-140 a^3 d^3\right) \left(a+b x^3\right)^{1/3}}{140 a^3 c^4 x}+\frac{d^3 \left(b c-a d\right)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{13/3}}-$$
$$\frac{d^3 \left(b c-a d\right)^{1/3} \text{Log}\left[c+d x^3\right]}{6 c^{13/3}}+\frac{d^3 \left(b c-a d\right)^{1/3} \text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{13/3}}$$

Result (type 5, 905 leaves, 2 steps):

$$-\frac{1}{560 c^5 x^{10} \left(a+b x^3\right)^{2/3}}\left(56 a c^4+56 b c^4 x^3-72 a c^3 d x^3-72 b c^3 d x^6+108 a c^2 d^2 x^6+108 b c^2 d^2 x^9-\right.$$
$$324 a c d^3 x^9-324 b c d^3 x^{12}-28 b c^4 x^3 \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$

$$28\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$36\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$36\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$54\,b\,c^2\,d^2\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$54\,a\,c\,d^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$162\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$162\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$117\,b\,c^4\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$117\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$99\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$99\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$81\,b\,c^2\,d^2\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$81\,a\,c\,d^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$297\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$297\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},2,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-54\,(b\,c-a\,d)\,x^3$$
$$(2\,c-3\,d\,x^3)\,(c+d\,x^3)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{5}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$27\,(b\,c-a\,d)\,x^3\,(c+d\,x^3)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{5}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 684: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 334 leaves, 5 steps):

$$-\frac{(3 b c-a d) x \left(a+b x^3\right)^{2/3}}{9 b d^2}+\frac{x^4 \left(a+b x^3\right)^{2/3}}{6 d}+\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{4/3} d^3}-\frac{c^{4/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^3}+\frac{c^{4/3} (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^3}-\frac{\left(9 b^2 c^2-6 a b c d-a^2 d^2\right) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{18 b^{4/3} d^3}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^7 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},-\frac{2}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 685: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 \left(a+b x^3\right)^{2/3}}{c+d x^3} \, dx$$

Optimal (type 3, 272 leaves, 4 steps):

$$\frac{x \left(a+b x^3\right)^{2/3}}{3 d}-\frac{(3 b c-2 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{1/3} d^2}+\frac{c^{1/3} (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+\frac{c^{1/3} (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 d^2}-\frac{c^{1/3} (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}+\frac{(3 b c-2 a d) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{1/3} d^2}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^4 \left(a+b x^3\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},-\frac{2}{3},1,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 686: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{c+d x^3}\, dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3}\,\mathrm{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\; d}-\frac{\left(b c-a d\right)^{2/3}\,\mathrm{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\; c^{2/3}\, d}-\frac{\left(b c-a d\right)^{2/3}\,\mathrm{Log}\left[c+d x^3\right]}{6\, c^{2/3}\, d}+\frac{\left(b c-a d\right)^{2/3}\,\mathrm{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2\, c^{2/3}\, d}-\frac{b^{2/3}\,\mathrm{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2\, d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^{2/3}\,\mathrm{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 687: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^3 \left(c+d x^3\right)}\, dx$$

Optimal (type 3, 169 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2\, c\, x^2}+\frac{\left(b c-a d\right)^{2/3}\,\mathrm{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\; c^{5/3}}+\frac{\left(b c-a d\right)^{2/3}\,\mathrm{Log}\left[c+d x^3\right]}{6\, c^{5/3}}-\frac{\left(b c-a d\right)^{2/3}\,\mathrm{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2\, c^{5/3}}$$

Result (type 5, 89 leaves, 2 steps):

$$-\left(\left(\left(a+b x^3\right)^{2/3} \left(1+\frac{d x^3}{c}\right)^{2/3}\,\mathrm{Hypergeometric2F1}\left[-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]\right)\Big/\left(2\, c\, x^2 \left(1+\frac{b x^3}{a}\right)^{2/3}\right)\right)$$

Problem 688: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^6 \left(c+d x^3\right)}\, dx$$

Optimal (type 3, 206 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 c x^5}-\frac{(2 b c-5 a d)\left(a+b x^3\right)^{2/3}}{10 a c^2 x^2}-\frac{d (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{8/3}}-$$
$$\frac{d (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{8/3}}+\frac{d (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{8/3}}$$

Result (type 5, 148 leaves, 2 steps):

$$-\Bigg(\Bigg(c\left(a+b x^3\right)\left(2 c-3 d x^3\right)-$$
$$2 (b c-a d) x^3 \left(2 c-3 d x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]+6 (b c-a d)$$
$$x^3 \left(c+d x^3\right) \text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]\Bigg)\Bigg/\left(10 c^3 x^5 \left(a+b x^3\right)^{1/3}\right)\Bigg)$$

Problem 689: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{2/3}}{x^9 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 257 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 c x^8}-\frac{(b c-4 a d)\left(a+b x^3\right)^{2/3}}{20 a c^2 x^5}+$$
$$\frac{\left(3 b^2 c^2+8 a b c d-20 a^2 d^2\right)\left(a+b x^3\right)^{2/3}}{40 a^2 c^3 x^2}+\frac{d^2 (b c-a d)^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{11/3}}+$$
$$\frac{d^2 (b c-a d)^{2/3} \text{Log}\left[c+d x^3\right]}{6 c^{11/3}}-\frac{d^2 (b c-a d)^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{11/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$-\frac{1}{40\,c^4\,x^8\,\left(a+b\,x^3\right)^{1/3}}\Big(5\,a\,c^3+5\,b\,c^3\,x^3-6\,a\,c^2\,d\,x^3-6\,b\,c^2\,d\,x^6+9\,a\,c\,d^2\,x^6+9\,b\,c\,d^2\,x^9-$$
$$2\,\left(b\,c-a\,d\right)\,x^3\,\left(5\,c^2-6\,c\,d\,x^3+9\,d^2\,x^6\right)\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,1,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$
$$21\,b\,c^3\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$
$$21\,a\,c^2\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$
$$6\,b\,c^2\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$
$$6\,a\,c\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$
$$27\,b\,c\,d^2\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$
$$27\,a\,d^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,2,\,\frac{4}{3},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$
$$9\,\left(b\,c-a\,d\right)\,x^3\,\left(c+d\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},\,2,\,2\right\},\,\left\{1,\,\frac{4}{3}\right\},\,\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\Big)$$

Problem 690: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,x^3\right)^{2/3}}{x^{12}\,\left(c+d\,x^3\right)}\,dx$$

Optimal (type 3, 320 leaves, 6 steps):

$$-\frac{\left(a+b\,x^3\right)^{2/3}}{11\,c\,x^{11}}-\frac{\left(2\,b\,c-11\,a\,d\right)\,\left(a+b\,x^3\right)^{2/3}}{88\,a\,c^2\,x^8}+\frac{\left(6\,b^2\,c^2+11\,a\,b\,c\,d-44\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{2/3}}{220\,a^2\,c^3\,x^5}-$$
$$\frac{\left(18\,b^3\,c^3+33\,a\,b^2\,c^2\,d+88\,a^2\,b\,c\,d^2-220\,a^3\,d^3\right)\,\left(a+b\,x^3\right)^{2/3}}{440\,a^3\,c^4\,x^2}-\frac{d^3\,\left(b\,c-a\,d\right)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{14/3}}-$$
$$\frac{d^3\,\left(b\,c-a\,d\right)^{2/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{14/3}}+\frac{d^3\,\left(b\,c-a\,d\right)^{2/3}\,\text{Log}\left[\frac{\left(b\,c-a\,d\right)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{14/3}}$$

Result (type 5, 819 leaves, 2 steps):

$$-\frac{1}{440\,c^5\,x^{11}\,\left(a+b\,x^3\right)^{1/3}}\left(40\,a\,c^4+40\,b\,c^4\,x^3-45\,a\,c^3\,d\,x^3-45\,b\,c^3\,d\,x^6+54\,a\,c^2\,d^2\,x^6+54\,b\,c^2\,d^2\,x^9-\right.$$

$$81\,a\,c\,d^3\,x^9-81\,b\,c\,d^3\,x^{12}-80\,b\,c^4\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$80\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$90\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$90\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$108\,b\,c^2\,d^2\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$108\,a\,c\,d^3\,x^9\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$162\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$162\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$198\,b\,c^4\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$198\,a\,c^3\,d\,x^3\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$99\,b\,c^3\,d\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$99\,a\,c^2\,d^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$297\,b\,c\,d^3\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-$$

$$297\,a\,d^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{1}{3},2,\frac{4}{3},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]-27\,\left(b\,c-a\,d\right)\,x^3$$

$$\left(5\,c-6\,d\,x^3\right)\,\left(c+d\,x^3\right)^2\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2\right\},\left\{1,\frac{4}{3}\right\},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]+$$

$$\left.27\,\left(b\,c-a\,d\right)\,x^3\,\left(c+d\,x^3\right)^3\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{3},2,2,2\right\},\left\{1,1,\frac{4}{3}\right\},\frac{\left(b\,c-a\,d\right)\,x^3}{c\,\left(a+b\,x^3\right)}\right]\right)$$

Problem 702: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

Optimal (type 3, 334 leaves, 6 steps):

$$-\frac{(6 b c-7 a d) x^2 \left(a+b x^3\right)^{1/3}}{18 d^2}+\frac{b x^5 \left(a+b x^3\right)^{1/3}}{6 d}-\frac{\left(9 b^2 c^2-12 a b c d+2 a^2 d^2\right) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{2/3} d^3}+\frac{c^{2/3} (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3}-\frac{c^{2/3} (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 d^3}-\frac{\left(9 b^2 c^2-12 a b c d+2 a^2 d^2\right) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{18 b^{2/3} d^3}+\frac{c^{2/3} (b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^3}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^5 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{5}{3},-\frac{4}{3},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{5 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 703: Result unnecessarily involves higher level functions.

$$\int \frac{x \left(a+b x^3\right)^{4/3}}{c+d x^3} \, dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\frac{b x^2 \left(a+b x^3\right)^{1/3}}{3 d}+\frac{b^{1/3} (3 b c-4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^2}-\frac{(b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d^2}+\frac{(b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{1/3} d^2}+\frac{b^{1/3} (3 b c-4 a d) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 d^2}-\frac{(b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} d^2}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^2 \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},-\frac{4}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 704: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^2 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 254 leaves, 5 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{c x}-\frac{b^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d}+\frac{(b c-a d)^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{4/3} d}-\frac{(b c-a d)^{4/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{4/3} d}-\frac{b^{4/3} \operatorname{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d}+\frac{(b c-a d)^{4/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} d}$$

Result (type 6, 63 leaves, 2 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3} \operatorname{AppellF1}\left[-\frac{1}{3},-\frac{4}{3},1,\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c x \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 705: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^5 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 201 leaves, 4 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{4 c x^4}-\frac{(5 b c-4 a d) \left(a+b x^3\right)^{1/3}}{4 c^2 x}-\frac{(b c-a d)^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{7/3}}+\frac{(b c-a d)^{4/3} \operatorname{Log}\left[c+d x^3\right]}{6 c^{7/3}}-\frac{(b c-a d)^{4/3} \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 90 leaves, 2 steps):

$$-\left(\left(a \left(a+b x^3\right)^{1/3} \left(1+\frac{d x^3}{c}\right)^{4/3} \operatorname{Hypergeometric2F1}\left[-\frac{4}{3},-\frac{4}{3},-\frac{1}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]\right) \Big/ \left(4 c x^4 \left(1+\frac{b x^3}{a}\right)^{1/3}\right)\right)$$

Problem 706: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^8 \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{7 c x^7}-\frac{(8 b c-7 a d) \left(a+b x^3\right)^{1/3}}{28 c^2 x^4}-\frac{\left(4 b^2 c^2-35 a b c d+28 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{28 a c^3 x}+\frac{d (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}}-\frac{d (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{10/3}}+\frac{d (b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 169 leaves, 2 steps):

$$\left(12 c (b c-a d) x^3 \left(a+b x^3\right) \left(c+d x^3\right) \text{Hypergeometric2F1}\left[-\frac{1}{3},2,\frac{2}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]-\left(4 c-3 d x^3\right) \left(c \left(a+b x^3\right) \left(5 b c x^3+a \left(c-4 d x^3\right)\right)-2 (b c-a d)^2 x^6 \text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b c-a d) x^3}{c \left(a+b x^3\right)}\right]\right)\right)\Big/\left(28 c^4 x^7 \left(a+b x^3\right)^{2/3}\right)$$

Problem 707: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^3\right)^{4/3}}{x^{11} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{a \left(a+b x^3\right)^{1/3}}{10 c x^{10}}-\frac{(11 b c-10 a d) \left(a+b x^3\right)^{1/3}}{70 c^2 x^7}-\frac{\left(2 b^2 c^2-40 a b c d+35 a^2 d^2\right) \left(a+b x^3\right)^{1/3}}{140 a c^3 x^4}+\frac{\left(6 b^3 c^3+20 a b^2 c^2 d-175 a^2 b c d^2+140 a^3 d^3\right) \left(a+b x^3\right)^{1/3}}{140 a^2 c^4 x}-\frac{d^2 (b c-a d)^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{13/3}}+\frac{d^2 (b c-a d)^{4/3} \text{Log}\left[c+d x^3\right]}{6 c^{13/3}}-\frac{d^2 (b c-a d)^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{13/3}}$$

Result (type 5, 260 leaves, 2 steps):

$$\frac{1}{140\,c^5\,x^{10}\,(a+b\,x^3)^{2/3}}\left(6\,c\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)\,(11\,c^2+2\,c\,d\,x^3-9\,d^2\,x^6)\,\text{Hypergeometric2F1}\left[-\frac{1}{3},2,\frac{2}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-(14\,c^2-12\,c\,d\,x^3+9\,d^2\,x^6)\left(c\,(a+b\,x^3)\,(5\,b\,c\,x^3+a\,(c-4\,d\,x^3))-2\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\right)-18\,c\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)\,(c+d\,x^3)^2\,\text{HypergeometricPFQ}\left[\left\{-\frac{1}{3},2,2\right\},\left\{\frac{2}{3},1\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\right)$$

Problem 708: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,x^3)^{4/3}}{x^{14}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 392 leaves, 7 steps):

$$-\frac{a\,(a+b\,x^3)^{1/3}}{13\,c\,x^{13}}-\frac{(14\,b\,c-13\,a\,d)\,(a+b\,x^3)^{1/3}}{130\,c^2\,x^{10}}-\frac{(4\,b^2\,c^2-143\,a\,b\,c\,d+130\,a^2\,d^2)\,(a+b\,x^3)^{1/3}}{910\,a\,c^3\,x^7}+\frac{(12\,b^3\,c^3+26\,a\,b^2\,c^2\,d-520\,a^2\,b\,c\,d^2+455\,a^3\,d^3)\,(a+b\,x^3)^{1/3}}{1820\,a^2\,c^4\,x^4}-\frac{1}{1820\,a^3\,c^5\,x}(36\,b^4\,c^4+78\,a\,b^3\,c^3\,d+260\,a^2\,b^2\,c^2\,d^2-2275\,a^3\,b\,c\,d^3+1820\,a^4\,d^4)\,(a+b\,x^3)^{1/3}+\frac{d^3\,(b\,c-a\,d)^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{16/3}}-\frac{d^3\,(b\,c-a\,d)^{4/3}\,\text{Log}[c+d\,x^3]}{6\,c^{16/3}}+\frac{d^3\,(b\,c-a\,d)^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{2\,c^{16/3}}$$

Result (type 5, 1446 leaves, 2 steps):

$$-\frac{1}{1820\,c^6\,x^{13}\,(a+b\,x^3)^{2/3}}\left(140\,a^2\,c^5+840\,a\,b\,c^5\,x^3-686\,a^2\,c^4\,d\,x^3+700\,b^2\,c^5\,x^6-1316\,a\,b\,c^4\,d\,x^6+612\,a^2\,c^3\,d^2\,x^6-630\,b^2\,c^4\,d\,x^9+1152\,a\,b\,c^3\,d^2\,x^9-513\,a^2\,c^2\,d^3\,x^9+540\,b^2\,c^3\,d^2\,x^{12}-918\,a\,b\,c^2\,d^3\,x^{12}+324\,a^2\,c\,d^4\,x^{12}-405\,b^2\,c^2\,d^3\,x^{15}+324\,a\,b\,c\,d^4\,x^{15}-828\,a\,b\,c^5\,x^3\,\text{Hypergeometric2F1}\left[-\frac{1}{3},2,\frac{2}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+828\,a^2\,c^4\,d\,x^3\,\text{Hypergeometric2F1}\left[-\frac{1}{3},2,\frac{2}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-\right.$$

$828\, b^2 c^5 x^6 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$918\, a\, b\, c^4 d\, x^6 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$90\, a^2 c^3 d^2 x^6 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$90\, b^2 c^4 d\, x^9 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$234\, a\, b\, c^3 d^2 x^9 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$324\, a^2 c^2 d^3 x^9 \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$324\, b^2 c^3 d^2 x^{12} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$918\, a\, b\, c^2 d^3 x^{12} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$594\, a^2 c\, d^4 x^{12} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$594\, b^2 c^2 d^3 x^{15} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$594\, a\, b\, c\, d^4 x^{15} \text{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$280\, b^2 c^5 x^6 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$560\, a\, b\, c^4 d\, x^6 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$280\, a^2 c^3 d^2 x^6 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$252\, b^2 c^4 d\, x^9 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$504\, a\, b\, c^3 d^2 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$252\, a^2 c^2 d^3 x^9 \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] -$

$216\, b^2 c^3 d^2 x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d)\, x^3}{c\, (a + b x^3)}\right] +$

$$432\,a\,b\,c^2\,d^3\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$216\,a^2\,c\,d^4\,x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$162\,b^2\,c^2\,d^3\,x^{15}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$324\,a\,b\,c\,d^4\,x^{15}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$162\,a^2\,d^5\,x^{15}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$54\,c\,(b\,c-a\,d)\,x^3\,(a+b\,x^3)\,(7\,c-6\,d\,x^3)\,(c+d\,x^3)^2$$
$$\text{HypergeometricPFQ}\left[\left\{-\frac{1}{3},2,2\right\},\left\{\frac{2}{3},1\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-54\,c\,(b\,c-a\,d)\,x^3$$
$$(a+b\,x^3)\,(c+d\,x^3)^3\,\text{HypergeometricPFQ}\left[\left\{-\frac{1}{3},2,2,2\right\},\left\{\frac{2}{3},1,1\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 721: Result valid but suboptimal antiderivative.

$$\int\frac{x^6}{(a+b\,x^3)^{1/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\frac{x\,(a+b\,x^3)^{2/3}}{3\,b\,d}-\frac{(3\,b\,c+a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{4/3}\,d^2}+\frac{c^{4/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\,(b\,c-a\,d)^{1/3}}+\frac{c^{4/3}\,\text{Log}\left[c+d\,x^3\right]}{6\,d^2\,(b\,c-a\,d)^{1/3}}-$$
$$\frac{c^{4/3}\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{2\,d^2\,(b\,c-a\,d)^{1/3}}+\frac{(3\,b\,c+a\,d)\,\text{Log}\left[-b^{1/3}\,x+(a+b\,x^3)^{1/3}\right]}{6\,b^{4/3}\,d^2}$$

Result (type 3, 394 leaves, 15 steps):

$$\frac{x\,(a+b\,x^3)^{2/3}}{3\,b\,d}-\frac{(3\,b\,c+a\,d)\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{4/3}\,d^2}+\frac{c^{4/3}\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}}{\sqrt{3}\,c^{1/3}}\right]}{\sqrt{3}\,d^2\,(b\,c-a\,d)^{1/3}}+$$
$$\frac{(3\,b\,c+a\,d)\,\text{Log}\left[1-\frac{b^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{9\,b^{4/3}\,d^2}-\frac{(3\,b\,c+a\,d)\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{(a+b\,x^3)^{2/3}}+\frac{b^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{18\,b^{4/3}\,d^2}-$$
$$\frac{c^{4/3}\,\text{Log}\left[c^{1/3}-\frac{(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{3\,d^2\,(b\,c-a\,d)^{1/3}}+\frac{c^{4/3}\,\text{Log}\left[c^{2/3}+\frac{(b\,c-a\,d)^{2/3}\,x^2}{(a+b\,x^3)^{2/3}}+\frac{c^{1/3}\,(b\,c-a\,d)^{1/3}\,x}{(a+b\,x^3)^{1/3}}\right]}{6\,d^2\,(b\,c-a\,d)^{1/3}}$$

Problem 722: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}-\frac{c^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{1/3}}-\frac{c^{1/3} \text{Log}\left[c+d x^3\right]}{6 d\, (b c-a d)^{1/3}}+$$
$$\frac{c^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d\, (b c-a d)^{1/3}}-\frac{\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 b^{1/3} d}$$

Result (type 3, 346 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{1/3} d}-\frac{c^{1/3} \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{1/3}}-\frac{\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{1/3} d}+\frac{\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 b^{1/3} d}+$$
$$\frac{c^{1/3} \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 d\, (b c-a d)^{1/3}}-\frac{c^{1/3} \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 d\, (b c-a d)^{1/3}}$$

Problem 723: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{1/3}}-\frac{\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{1/3}}-\frac{\text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{1/3}}+\frac{\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{1/3}}$$

Problem 724: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a c x^2}-\frac{d \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{5/3} (b c-a d)^{1/3}}-\frac{d \operatorname{Log}\left[c+d x^3\right]}{6 c^{5/3} (b c-a d)^{1/3}}+\frac{d \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{5/3} (b c-a d)^{1/3}}$$

Result (type 3, 235 leaves, 8 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{2 a c x^2}-\frac{d \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{5/3} (b c-a d)^{1/3}}+$$
$$\frac{d \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{5/3} (b c-a d)^{1/3}}-\frac{d \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{5/3} (b c-a d)^{1/3}}$$

Problem 725: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 214 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{5 a c x^5}+\frac{(3 b c+5 a d) \left(a+b x^3\right)^{2/3}}{10 a^2 c^2 x^2}+$$
$$\frac{d^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{8/3} (b c-a d)^{1/3}}+\frac{d^2 \operatorname{Log}\left[c+d x^3\right]}{6 c^{8/3} (b c-a d)^{1/3}}-\frac{d^2 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{8/3} (b c-a d)^{1/3}}$$

Result (type 3, 271 leaves, 9 steps):

$$\frac{(b c+a d) \left(a+b x^3\right)^{2/3}}{2 a^2 c^2 x^2}-\frac{\left(a+b x^3\right)^{5/3}}{5 a^2 c x^5}+\frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{8/3} (b c-a d)^{1/3}}-$$
$$\frac{d^2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{8/3} (b c-a d)^{1/3}}+\frac{d^2 \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{8/3} (b c-a d)^{1/3}}$$

Problem 726: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 \left(a+b x^3\right)^{1/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 262 leaves, 5 steps):

$$-\frac{\left(a+b x^3\right)^{2/3}}{8 a c x^8}+\frac{(3 b c+4 a d) \left(a+b x^3\right)^{2/3}}{20 a^2 c^2 x^5}-\frac{\left(9 b^2 c^2+12 a b c d+20 a^2 d^2\right) \left(a+b x^3\right)^{2/3}}{40 a^3 c^3 x^2}-\frac{d^3 \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{11/3} (b c-a d)^{1/3}}-\frac{d^3 \text{Log}\left[c+d x^3\right]}{6 c^{11/3} (b c-a d)^{1/3}}+\frac{d^3 \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{11/3} (b c-a d)^{1/3}}$$

Result (type 3, 317 leaves, 9 steps):

$$-\frac{\left(b^2 c^2+a b c d+a^2 d^2\right) \left(a+b x^3\right)^{2/3}}{2 a^3 c^3 x^2}+\frac{(2 b c+a d) \left(a+b x^3\right)^{5/3}}{5 a^3 c^2 x^5}-\frac{\left(a+b x^3\right)^{8/3}}{8 a^3 c x^8}-\frac{d^3 \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{11/3} (b c-a d)^{1/3}}+\frac{d^3 \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{11/3} (b c-a d)^{1/3}}-\frac{d^3 \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{11/3} (b c-a d)^{1/3}}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 279 leaves, 5 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 b d}+\frac{(3 b c+2 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d^2}-\frac{c^{5/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c-a d)^{2/3}}+\frac{c^{5/3} \text{Log}\left[c+d x^3\right]}{6 d^2 (b c-a d)^{2/3}}+\frac{(3 b c+2 a d) \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{6 b^{5/3} d^2}-\frac{c^{5/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2 (b c-a d)^{2/3}}$$

Result (type 3, 400 leaves, 16 steps):

$$\frac{x^2 \left(a+b x^3\right)^{1/3}}{3 b d}+\frac{(3 b c+2 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d^2}-\frac{c^{5/3} \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} d^2 (b c-a d)^{2/3}}+\frac{(3 b c+2 a d) \text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{5/3} d^2}-\frac{(3 b c+2 a d) \text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{5/3} d^2}-\frac{c^{5/3} \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 d^2 (b c-a d)^{2/3}}+\frac{c^{5/3} \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 d^2 (b c-a d)^{2/3}}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{c^{2/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{2/3}}-\frac{c^{2/3} \text{Log}\left[c+d x^3\right]}{6 d\, (b c-a d)^{2/3}}-$$
$$\frac{\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3} d}+\frac{c^{2/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d\, (b c-a d)^{2/3}}$$

Result (type 3, 346 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3} d}+\frac{c^{2/3} \text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, d\, (b c-a d)^{2/3}}-$$
$$\frac{\text{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 b^{2/3} d}+\frac{\text{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 b^{2/3} d}+$$
$$\frac{c^{2/3} \text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 d\, (b c-a d)^{2/3}}-\frac{c^{2/3} \text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 d\, (b c-a d)^{2/3}}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 149 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{1/3} (b c-a d)^{2/3}}+\frac{\text{Log}\left[c+d x^3\right]}{6 c^{1/3} (b c-a d)^{2/3}}-\frac{\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{1/3} (b c-a d)^{2/3}}$$

Result (type 3, 208 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{1/3} (b c-a d)^{2/3}}-\frac{\text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 c^{1/3} (b c-a d)^{2/3}}+\frac{\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 c^{1/3} (b c-a d)^{2/3}}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 173 leaves, 3 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a c x}+\frac{d \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{4/3} (b c-a d)^{2/3}}-\frac{d \operatorname{Log}\left[c+d x^3\right]}{6 c^{4/3} (b c-a d)^{2/3}}+\frac{d \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{4/3} (b c-a d)^{2/3}}$$

Result (type 3, 232 leaves, 8 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{a c x}+\frac{d \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{4/3} (b c-a d)^{2/3}}+$$

$$\frac{d \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{4/3} (b c-a d)^{2/3}}-\frac{d \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{4/3} (b c-a d)^{2/3}}$$

Problem 742: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^3\right)^{2/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 215 leaves, 4 steps):

$$-\frac{\left(a+b x^3\right)^{1/3}}{4 a c x^4}+\frac{(3 b c+4 a d) \left(a+b x^3\right)^{1/3}}{4 a^2 c^2 x}-$$

$$\frac{d^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{7/3} (b c-a d)^{2/3}}+\frac{d^2 \operatorname{Log}\left[c+d x^3\right]}{6 c^{7/3} (b c-a d)^{2/3}}-\frac{d^2 \operatorname{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{7/3} (b c-a d)^{2/3}}$$

Result (type 3, 269 leaves, 9 steps):

$$\frac{(b c+a d) \left(a+b x^3\right)^{1/3}}{a^2 c^2 x}-\frac{\left(a+b x^3\right)^{4/3}}{4 a^2 c x^4}-\frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{7/3} (b c-a d)^{2/3}}-$$

$$\frac{d^2 \operatorname{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{7/3} (b c-a d)^{2/3}}+\frac{d^2 \operatorname{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{7/3} (b c-a d)^{2/3}}$$

Problem 754: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 322 leaves, 5 steps):

$$\frac{a x^4}{b (b c-a d) \left(a+b x^3\right)^{1/3}}+\frac{(b c-4 a d) x \left(a+b x^3\right)^{2/3}}{3 b^2 d (b c-a d)}-\frac{(3 b c+4 a d) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{7/3} d^2}+\frac{c^{7/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c-a d)^{4/3}}+\frac{c^{7/3} \text{Log}\left[c+d x^3\right]}{6 d^2 (b c-a d)^{4/3}}-\frac{c^{7/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2 (b c-a d)^{4/3}}+\frac{(3 b c+4 a d) \text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{6 b^{7/3} d^2}$$

Result (type 6, 67 leaves, 2 steps):

$$\frac{x^{10} \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{10}{3},\frac{4}{3},1,\frac{13}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{10 a c \left(a+b x^3\right)^{1/3}}$$

Problem 755: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 260 leaves, 4 steps):

$$\frac{a x}{b (b c-a d) \left(a+b x^3\right)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d}-\frac{c^{4/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d (b c-a d)^{4/3}}-\frac{c^{4/3} \text{Log}\left[c+d x^3\right]}{6 d (b c-a d)^{4/3}}+\frac{c^{4/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 d (b c-a d)^{4/3}}-\frac{\text{Log}\left[-b^{1/3} x+\left(a+b x^3\right)^{1/3}\right]}{2 b^{4/3} d}$$

Result (type 6, 67 leaves, 2 steps):

$$\frac{x^7 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{7 a c \left(a+b x^3\right)^{1/3}}$$

Problem 756: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 172 leaves, 3 steps):

$$-\frac{x}{(b c-a d)\left(a+b x^3\right)^{1/3}}+\frac{c^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(b c-a d)^{4/3}}+\frac{c^{1/3}\,\text{Log}\left[c+d x^3\right]}{6 (b c-a d)^{4/3}}-\frac{c^{1/3}\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 (b c-a d)^{4/3}}$$

Result (type 5, 92 leaves, 2 steps):

$$\frac{x^4 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{4}{3},\frac{4}{3},\frac{7}{3},-\frac{c \left(\frac{b x^3}{a}-\frac{d x^3}{c}\right)}{c+d x^3}\right]}{4 a c \left(a+b x^3\right)^{1/3} \left(1+\frac{d x^3}{c}\right)^{4/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{b x}{a (b c-a d)\left(a+b x^3\right)^{1/3}}-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{4/3}}-\frac{d\,\text{Log}\left[c+d x^3\right]}{6 c^{2/3} (b c-a d)^{4/3}}+\frac{d\,\text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{2/3} (b c-a d)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps):

$$\frac{b x}{a (b c-a d)\left(a+b x^3\right)^{1/3}}-\frac{d\,\text{ArcTan}\left[\frac{c^{1/3}+\frac{2 (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}\, c^{1/3}}\right]}{\sqrt{3}\, c^{2/3} (b c-a d)^{4/3}}+\frac{d\,\text{Log}\left[c^{1/3}-\frac{(b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 c^{2/3} (b c-a d)^{4/3}}-\frac{d\,\text{Log}\left[c^{2/3}+\frac{(b c-a d)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{c^{1/3} (b c-a d)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 c^{2/3} (b c-a d)^{4/3}}$$

Problem 758: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)}\, dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$\frac{b}{a\,(b\,c-a\,d)\,x^2\,(a+b\,x^3)^{1/3}}-\frac{(3\,b\,c-a\,d)\,(a+b\,x^3)^{2/3}}{2\,a^2\,c\,(b\,c-a\,d)\,x^2}+$$

$$\frac{d^2\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{5/3}\,(b\,c-a\,d)^{4/3}}+\frac{d^2\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{5/3}\,(b\,c-a\,d)^{4/3}}-\frac{d^2\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-(a+b\,x^3)^{1/3}\right]}{2\,c^{5/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 542 leaves, 2 steps):

$$\frac{1}{14\,c^4\,(b\,c-a\,d)\,x^5\,(a+b\,x^3)^{7/3}}\Bigg(28\,c^4\,(a+b\,x^3)^2+168\,c^3\,d\,x^3\,(a+b\,x^3)^2+$$
$$126\,c^2\,d^2\,x^6\,(a+b\,x^3)^2-28\,c^4\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$168\,c^3\,d\,x^3\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$126\,c^2\,d^2\,x^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$15\,c^2\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$42\,c\,d\,(b\,c-a\,d)^2\,x^9\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$27\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$9\,c^2\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$18\,c\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$9\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Problem 759: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^6\,(a+b\,x^3)^{4/3}\,(c+d\,x^3)}\,dx$$

Optimal (type 3, 287 leaves, 5 steps):

$$\frac{b}{a\,(b\,c-a\,d)\,x^5\,\left(a+b\,x^3\right)^{1/3}}-\frac{(6\,b\,c-a\,d)\,\left(a+b\,x^3\right)^{2/3}}{5\,a^2\,c\,(b\,c-a\,d)\,x^5}+\frac{\left(18\,b^2\,c^2-3\,a\,b\,c\,d-5\,a^2\,d^2\right)\,\left(a+b\,x^3\right)^{2/3}}{10\,a^3\,c^2\,(b\,c-a\,d)\,x^2}-$$

$$\frac{d^3\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c-a\,d)^{1/3}\,x}{c^{1/3}\,\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,c^{8/3}\,(b\,c-a\,d)^{4/3}}-\frac{d^3\,\text{Log}\left[c+d\,x^3\right]}{6\,c^{8/3}\,(b\,c-a\,d)^{4/3}}+\frac{d^3\,\text{Log}\left[\frac{(b\,c-a\,d)^{1/3}\,x}{c^{1/3}}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,c^{8/3}\,(b\,c-a\,d)^{4/3}}$$

Result (type 5, 950 leaves, 2 steps) :

$$\frac{1}{70\,c^5\,(b\,c-a\,d)\,x^8\,(a+b\,x^3)^{7/3}}$$
$$\Bigg(56\,c^5\,(a+b\,x^3)^2-252\,c^4\,d\,x^3\,(a+b\,x^3)^2-1512\,c^3\,d^2\,x^6\,(a+b\,x^3)^2-1134\,c^2\,d^3\,x^9\,(a+b\,x^3)^2-$$
$$56\,c^5\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$252\,c^4\,d\,x^3\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$1512\,c^3\,d^2\,x^6\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$1134\,c^2\,d^3\,x^9\,(a+b\,x^3)^2\,\text{Hypergeometric2F1}\Big[\frac{1}{3},1,\frac{4}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]-$$
$$18\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$171\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$486\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$297\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{Hypergeometric2F1}\Big[2,\frac{7}{3},\frac{10}{3},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$27\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$216\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$351\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$162\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,\frac{7}{3}\Big\},\Big\{1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$27\,c^3\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{7}{3}\Big\},\Big\{1,1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$81\,c^2\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{7}{3}\Big\},\Big\{1,1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$81\,c\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{7}{3}\Big\},\Big\{1,1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]+$$
$$27\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\Big[\Big\{2,2,2,\frac{7}{3}\Big\},\Big\{1,1,\frac{10}{3}\Big\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\Big]\Bigg)$$

Problem 760: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{1}{x^9 \left(a+b x^3\right)^{4/3} \left(c+d x^3\right)} \, dx$$

Optimal (type 3, 351 leaves, 6 steps):

$$\frac{b}{a \left(b c-a d\right) x^8 \left(a+b x^3\right)^{1/3}}-\frac{\left(9 b c-a d\right) \left(a+b x^3\right)^{2/3}}{8 a^2 c \left(b c-a d\right) x^8}+\frac{\left(9 b c-4 a d\right) \left(3 b c+a d\right) \left(a+b x^3\right)^{2/3}}{20 a^3 c^2 \left(b c-a d\right) x^5}-\frac{\left(81 b^3 c^3-9 a b^2 c^2 d-12 a^2 b c d^2-20 a^3 d^3\right) \left(a+b x^3\right)^{2/3}}{40 a^4 c^3 \left(b c-a d\right) x^2}+\frac{d^4 \text{ArcTan}\left[\frac{1+\frac{2 \left(b c-a d\right)^{1/3} x}{c^{1/3} \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{11/3} \left(b c-a d\right)^{4/3}}+\frac{d^4 \text{Log}\left[c+d x^3\right]}{6 c^{11/3} \left(b c-a d\right)^{4/3}}-\frac{d^4 \text{Log}\left[\frac{\left(b c-a d\right)^{1/3} x}{c^{1/3}}-\left(a+b x^3\right)^{1/3}\right]}{2 c^{11/3} \left(b c-a d\right)^{4/3}}$$

Result (type 5, 1486 leaves, 2 steps):

$$\frac{1}{560 c^6 \left(b c-a d\right) x^{11} \left(a+b x^3\right)^{7/3}}$$
$$\Bigg(280 c^6 \left(a+b x^3\right)^2-672 c^5 d x^3 \left(a+b x^3\right)^2+3024 c^4 d^2 x^6 \left(a+b x^3\right)^2+18144 c^3 d^3 x^9 \left(a+b x^3\right)^2+$$
$$13608 c^2 d^4 x^{12} \left(a+b x^3\right)^2-280 c^6 \left(a+b x^3\right)^2 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$
$$672 c^5 d x^3 \left(a+b x^3\right)^2 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$3024 c^4 d^2 x^6 \left(a+b x^3\right)^2 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$18144 c^3 d^3 x^9 \left(a+b x^3\right)^2 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$13608 c^2 d^4 x^{12} \left(a+b x^3\right)^2 \text{Hypergeometric2F1}\left[\frac{1}{3},1,\frac{4}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$66 c^4 \left(b c-a d\right)^2 x^6 \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$
$$312 c^3 d \left(b c-a d\right)^2 x^9 \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$2268 c^2 d^2 \left(b c-a d\right)^2 x^{12} \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$6696 c d^3 \left(b c-a d\right)^2 x^{15} \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]-$$
$$4050 d^4 \left(b c-a d\right)^2 x^{18} \text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{10}{3},\frac{\left(b c-a d\right) x^3}{c \left(a+b x^3\right)}\right]+$$

$$189\,c^4\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$108\,c^3\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$3618\,c^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$6156\,c\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$2835\,d^4\,(b\,c-a\,d)^2\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]+$$
$$54\,c^4\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$648\,c^3\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$2268\,c^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$2376\,c\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$810\,d^4\,(b\,c-a\,d)^2\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$81\,c^4\,(b\,c-a\,d)^2\,x^6\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$324\,c^3\,d\,(b\,c-a\,d)^2\,x^9\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$486\,c^2\,d^2\,(b\,c-a\,d)^2\,x^{12}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$324\,c\,d^3\,(b\,c-a\,d)^2\,x^{15}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]-$$
$$81\,d^4\,(b\,c-a\,d)^2\,x^{18}\,\text{HypergeometricPFQ}\left[\left\{2,2,2,2,\frac{7}{3}\right\},\left\{1,1,1,\frac{10}{3}\right\},\frac{(b\,c-a\,d)\,x^3}{c\,(a+b\,x^3)}\right]\Bigg)$$

Test results for the 46 problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Problem 1412: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b x+c x^2\right)^{4/3}}{(b d+2 c d x)^{11/3}}\,dx$$

Optimal (type 3, 320 leaves, 8 steps):

$$-\frac{3 \left(d (b+2 c x)\right)^{4/3} \left(a+b x+c x^2\right)^{1/3}}{16 c^2 \left(b^2-4 a c\right) d^5}+\frac{9 \left(d (b+2 c x)\right)^{4/3} \left(a+b x+c x^2\right)^{4/3}}{16 c \left(b^2-4 a c\right)^2 d^5}+\frac{3 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right) d (b d+2 c d x)^{8/3}}-\frac{9 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right)^2 d^3 (b d+2 c d x)^{2/3}}-\frac{\sqrt{3}\, \text{ArcTan}\left[\frac{1+\frac{2^{1/3} \left(d (b+2 c x)\right)^{2/3}}{c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}}}{\sqrt{3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}-\frac{3 \text{Log}\left[\left(d (b+2 c x)\right)^{2/3}-2^{2/3} c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}\right]}{32\times 2^{2/3} c^{7/3} d^{11/3}}$$

Result (type 3, 468 leaves, 14 steps):

$$-\frac{3 \left(d (b+2 c x)\right)^{4/3} \left(a+b x+c x^2\right)^{1/3}}{16 c^2 \left(b^2-4 a c\right) d^5}+\frac{9 \left(d (b+2 c x)\right)^{4/3} \left(a+b x+c x^2\right)^{4/3}}{16 c \left(b^2-4 a c\right)^2 d^5}+\frac{3 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right) d (b d+2 c d x)^{8/3}}-\frac{9 \left(a+b x+c x^2\right)^{7/3}}{4 \left(b^2-4 a c\right)^2 d^3 (b d+2 c d x)^{2/3}}-\frac{\sqrt{3}\, \text{ArcTan}\left[\frac{c^{1/3} d^{2/3}+\frac{2^{1/3} \left(d (b+2 c x)\right)^{2/3}}{\left(a+b x+c x^2\right)^{1/3}}}{\sqrt{3}\, c^{1/3} d^{2/3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}-\frac{\text{Log}\left[-\frac{2^{1/3} \left(d (b+2 c x)\right)^{2/3}-2 c^{1/3} d^{2/3} \left(a+b x+c x^2\right)^{1/3}}{\left(a+b x+c x^2\right)^{1/3}}\right]}{16\times 2^{2/3} c^{7/3} d^{11/3}}+\frac{1}{32\times 2^{2/3} c^{7/3} d^{11/3}} \text{Log}\left[\left(\left(d (b+2 c x)\right)^{4/3}+2^{2/3} c^{1/3} d^{2/3} \left(d (b+2 c x)\right)^{2/3} \left(a+b x+c x^2\right)^{1/3}+2\times 2^{1/3} c^{2/3} d^{4/3} \left(a+b x+c x^2\right)^{2/3}\right) \Big/ \left(a+b x+c x^2\right)^{2/3}\right]$$

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

Problem 833: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{-1+x}\,\sqrt{1+x}}{1+x-x^2}\,dx$$

Optimal (type 3, 91 leaves, ? steps):

$$-\text{ArcCosh}[x]+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\,\sqrt{-1+x}}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\,\sqrt{-1+x}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 400 problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 174: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\,x^2}}{\sqrt{1-x^4}}\,dx$$

Optimal (type 4, 112 leaves, ? steps):

$$\frac{a\,\sqrt{1-x^2}\,\sqrt{\frac{a\,(1+x^2)}{a+b\,x^2}}\,\text{EllipticPi}\left[\frac{b}{a+b},\,\text{ArcSin}\left[\frac{\sqrt{a+b}\,x}{\sqrt{a+b\,x^2}}\right],\,-\frac{a-b}{a+b}\right]}{\sqrt{a+b}\,\sqrt{1+x^2}\,\sqrt{\frac{a\,(1-x^2)}{a+b\,x^2}}}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{a+b\,x^2}}{\sqrt{1-x^4}},\,x\right]$$

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 374: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^{3/2}}{x^2\,\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\sqrt{d+e x^2}}{a x}-\frac{\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\ \left(b-\sqrt{b^2-4 a c}\right)^{3/2}}+$$

$$\frac{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\ \left(b+\sqrt{b^2-4 a c}\right)^{3/2}}$$

Result (type 3, 432 leaves, 16 steps):

$$-\frac{d\sqrt{d+e x^2}}{a x}-$$

$$\left(\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ \left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b-\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]\right) \Big/$$

$$\left(2 a \sqrt{b-\sqrt{b^2-4 a c}}\right)-$$

$$\left(\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ \left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\ x}{\sqrt{b+\sqrt{b^2-4 a c}}\ \sqrt{d+e x^2}}\right]\right) \Big/$$

$$\left(2 a \sqrt{b+\sqrt{b^2-4 a c}}\right)+\frac{d\sqrt{e}\ \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{a}-$$

$$\frac{\sqrt{e}\ \left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{2 a}-\frac{\sqrt{e}\ \left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\ x}{\sqrt{d+e x^2}}\right]}{2 a}$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Test results for the 145 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Test results for the 664 problems in "1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} \, dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^2 \left(a+b x^3\right)^3 \sqrt{a^2+2 a b x^3+b^2 x^6}}{12 b^3}-\frac{2 a \left(a+b x^3\right)^4 \sqrt{a^2+2 a b x^3+b^2 x^6}}{15 b^3}+\frac{\left(a+b x^3\right)^5 \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 b^3}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 x^9 \sqrt{a^2+2 a b x^3+b^2 x^6}}{9 \left(a+b x^3\right)}+\frac{a^2 b x^{12} \sqrt{a^2+2 a b x^3+b^2 x^6}}{4 \left(a+b x^3\right)}+\frac{a b^2 x^{15} \sqrt{a^2+2 a b x^3+b^2 x^6}}{5 \left(a+b x^3\right)}+\frac{b^3 x^{18} \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 \left(a+b x^3\right)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \left(\frac{\left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{x^2}-\frac{2 b^3 (1-2 p) (1-p) p \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{3 a^3 x}\right) dx$$

Optimal (type 3, 146 leaves, ? steps):

$$-\frac{\left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a x}+\frac{b (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^2 x^{2/3}}-\frac{b^2 (1-2 p) (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^3 x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{1}{a^3\,(1+2\,p)}2\,b^3\,(1-2\,p)\,(1-p)\,p\left(1+\frac{b\,x^{1/3}}{a}\right)\left(a^2+2\,a\,b\,x^{1/3}+b^2\,x^{2/3}\right)^p$$
$$\text{Hypergeometric2F1}\left[1,\,1+2\,p,\,2\,(1+p),\,1+\frac{b\,x^{1/3}}{a}\right]+\frac{1}{a^3\,(1+2\,p)}$$
$$3\,b^3\left(1+\frac{b\,x^{1/3}}{a}\right)\left(a^2+2\,a\,b\,x^{1/3}+b^2\,x^{2/3}\right)^p\text{Hypergeometric2F1}\left[4,\,1+2\,p,\,2\,(1+p),\,1+\frac{b\,x^{1/3}}{a}\right]$$

Test results for the 96 problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 156 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 17 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.m"

Test results for the 140 problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(b\,x^{1+p}\,\left(b\,x+c\,x^3\right)^p+2\,c\,x^{3+p}\,\left(b\,x+c\,x^3\right)^p\right)\,dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p}\,\left(b\,x+c\,x^3\right)^{1+p}}{2\,(1+p)}$$

Result (type 5, 116 leaves, 7 steps):

$$\frac{1}{2\,(1+p)}b\,x^{2+p}\left(1+\frac{c\,x^2}{b}\right)^{-p}\left(b\,x+c\,x^3\right)^p\text{Hypergeometric2F1}\left[-p,\,1+p,\,2+p,\,-\frac{c\,x^2}{b}\right]+$$
$$\frac{1}{2+p}c\,x^{4+p}\left(1+\frac{c\,x^2}{b}\right)^{-p}\left(b\,x+c\,x^3\right)^p\text{Hypergeometric2F1}\left[-p,\,2+p,\,3+p,\,-\frac{c\,x^2}{b}\right]$$

Problem 221: Result valid but suboptimal antiderivative.

$$\int (1+2x)\,(x+x^2)^3\,\left(-18+7\,(x+x^2)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81\,x^4\,(1+x)^4-36\,x^7\,(1+x)^7+\frac{49}{10}\,x^{10}\,(1+x)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81\,x^4+324\,x^5+486\,x^6+288\,x^7-171\,x^8-756\,x^9-\frac{12551\,x^{10}}{10}-1211\,x^{11}-\frac{1071\,x^{12}}{2}+336\,x^{13}+993\,x^{14}+\frac{6174\,x^{15}}{5}+1029\,x^{16}+588\,x^{17}+\frac{441\,x^{18}}{2}+49\,x^{19}+\frac{49\,x^{20}}{10}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int x^3\,(1+x)^3\,(1+2x)\,\left(-18+7\,x^3\,(1+x)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81\,x^4\,(1+x)^4-36\,x^7\,(1+x)^7+\frac{49}{10}\,x^{10}\,(1+x)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81\,x^4+324\,x^5+486\,x^6+288\,x^7-171\,x^8-756\,x^9-\frac{12551\,x^{10}}{10}-1211\,x^{11}-\frac{1071\,x^{12}}{2}+336\,x^{13}+993\,x^{14}+\frac{6174\,x^{15}}{5}+1029\,x^{16}+588\,x^{17}+\frac{441\,x^{18}}{2}+49\,x^{19}+\frac{49\,x^{20}}{10}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20\,x+4\,x^2}{9-10\,x^2+x^4}\,dx$$

Optimal (type 3, 31 leaves, ? steps):

$$\text{Log}[1-x]-\frac{1}{2}\,\text{Log}[3-x]+\frac{3}{2}\,\text{Log}[1+x]-2\,\text{Log}[3+x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2}\,\text{ArcTanh}\left[\frac{x}{3}\right]+\frac{\text{ArcTanh}[x]}{2}+\frac{5}{4}\,\text{Log}\left[1-x^2\right]-\frac{5}{4}\,\text{Log}\left[9-x^2\right]$$

Problem 393: Unable to integrate problem.

$$\int \frac{\left(1+x^2\right)^2}{a x^6+b \left(1+x^2\right)^3}\,dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{a^{1/3}+b^{1/3}}\;b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3}a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{-(-1)^{1/3}a^{1/3}+b^{1/3}}\;b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3}a^{1/3}+b^{1/3}}\;x}{b^{1/6}}\right]}{3\sqrt{(-1)^{2/3}a^{1/3}+b^{1/3}}\;b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{1}{a x^6+b \left(1+x^2\right)^3},x\right]+2\,\text{CannotIntegrate}\left[\frac{x^2}{a x^6+b \left(1+x^2\right)^3},x\right]+\text{CannotIntegrate}\left[\frac{x^4}{a x^6+b \left(1+x^2\right)^3},x\right]$$

Problem 493: Unable to integrate problem.

$$\int\left(\frac{3\left(-47+228 x+120 x^2+19 x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320 x-75 x^2-8 x^3}{\left(3+x+x^4\right)^3}+\frac{30 x}{\left(3+x+x^4\right)^2}\right)dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3 x+5 x^2+x^4-5 x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$-\frac{19}{4\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}-\frac{621}{4}\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},x\right]+684\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},x\right]+360\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},x\right]+44\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},x\right]-320\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},x\right]-75\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^3},x\right]+30\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^2},x\right]$$

Problem 494: Unable to integrate problem.

$$\int\left(\frac{-3+10 x+4 x^3-30 x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4 x^3\right)\left(2-3 x+5 x^2+x^4-5 x^6\right)}{\left(3+x+x^4\right)^4}\right)dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3x+5x^2+x^4-5x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\frac{7}{2\left(3+x+x^4\right)^3}-\frac{63x}{22\left(3+x+x^4\right)^3}-\frac{12x^2}{\left(3+x+x^4\right)^3}-\frac{5x^3}{\left(3+x+x^4\right)^3}+\frac{3x^4}{2\left(3+x+x^4\right)^3}-\frac{10x^6}{\left(3+x+x^4\right)^3}-\frac{1}{2\left(3+x+x^4\right)^2}+\frac{5x^2}{\left(3+x+x^4\right)^2}+\frac{144}{11}\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},x\right]+\frac{828}{11}\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},x\right]+18\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},x\right]-4\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},x\right]-20\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},x\right]$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 20: Unable to integrate problem.

$$\int\frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 187 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2dx}{\left(2c^3+d^3x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}\,c^2d}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(2c+dx)}{\left(2c^3+d^3x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2c^2d}-\frac{\text{Log}[c+dx]}{2c^2d}-\frac{\text{Log}\left[dx-\left(2c^3+d^3x^3\right)^{1/3}\right]}{4c^2d}+\frac{3\,\text{Log}\left[d(2c+dx)-d\left(2c^3+d^3x^3\right)^{1/3}\right]}{4c^2d}$$

Result (type 8, 27 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}},x\right]$$

Problem 21: Unable to integrate problem.

$$\int\frac{1}{\left(1+2^{1/3}x\right)\left(1+x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 147 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2x}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(2^{2/3}+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-$$
$$\frac{\text{Log}\left[1+2^{1/3}x\right]}{2^{2/3}}-\frac{\text{Log}\left[x-(1+x^3)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[2+2^{1/3}x-2^{1/3}(1+x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(1+2^{1/3}x)(1+x^3)^{2/3}},x\right]$$

Problem 22: Unable to integrate problem.

$$\int\frac{1}{(1-2^{1/3}x)(1-x^3)^{2/3}}\,dx$$

Optimal (type 3, 159 leaves, 1 step):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{2/3}-2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1-2^{1/3}x\right]}{2^{2/3}}+$$
$$\frac{\text{Log}\left[-x-(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}-\frac{3\,\text{Log}\left[-2+2^{1/3}x+2^{1/3}(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 26 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(1-2^{1/3}x)(1-x^3)^{2/3}},x\right]$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int(c+d\,x)^4\,(a+b\,x^3)^{1/3}\,dx$$

Optimal (type 5, 387 leaves, 11 steps):

$$\frac{3\,a\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{2\,b}+\frac{a\,d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,b}+$$

$$\frac{1}{30}\left(a+b\,x^3\right)^{1/3}\left(15\,c^4\,x+40\,c^3\,d\,x^2+45\,c^2\,d^2\,x^3+24\,c\,d^3\,x^4+5\,d^4\,x^5\right)-\frac{4\,a\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}}+$$

$$\frac{a^2\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{5/3}}+\frac{a\,c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,\left(a+b\,x^3\right)^{2/3}}+$$

$$\frac{a\,c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{5\,\left(a+b\,x^3\right)^{2/3}}-$$

$$\frac{2\,a\,c^3\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{2/3}}+\frac{a^2\,d^4\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{18\,b^{5/3}}$$

Result (type 5, 498 leaves, 23 steps):

$$\frac{3\,a\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{2\,b}+\frac{a\,d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{18\,b}+$$

$$\frac{1}{30}\left(a+b\,x^3\right)^{1/3}\left(15\,c^4\,x+40\,c^3\,d\,x^2+45\,c^2\,d^2\,x^3+24\,c\,d^3\,x^4+5\,d^4\,x^5\right)-\frac{4\,a\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}}+$$

$$\frac{a^2\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{9\,\sqrt{3}\,b^{5/3}}+\frac{a\,c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,\left(a+b\,x^3\right)^{2/3}}+$$

$$\frac{a\,c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{5\,\left(a+b\,x^3\right)^{2/3}}-\frac{4\,a\,c^3\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{2/3}}+$$

$$\frac{a^2\,d^4\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{27\,b^{5/3}}+\frac{2\,a\,c^3\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{2/3}}-\frac{a^2\,d^4\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{54\,b^{5/3}}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int (c+d\,x)^3\,\left(a+b\,x^3\right)^{1/3}\,dx$$

Optimal (type 5, 242 leaves, 9 steps):

$$\frac{3\,a\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{4\,b}+\frac{a\,d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{10\,b}+$$

$$\frac{1}{20}\,\left(a+b\,x^3\right)^{1/3}\,\left(10\,c^3\,x+20\,c^2\,d\,x^2+15\,c\,d^2\,x^3+4\,d^3\,x^4\right)-\frac{a\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+$$

$$\frac{a\,\left(5\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{10\,b\,\left(a+b\,x^3\right)^{2/3}}-\frac{a\,c^2\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 5, 297 leaves, 15 steps):

$$\frac{3\,a\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{4\,b}+\frac{a\,d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{10\,b}+\frac{1}{20}\,\left(a+b\,x^3\right)^{1/3}\,\left(10\,c^3\,x+20\,c^2\,d\,x^2+15\,c\,d^2\,x^3+4\,d^3\,x^4\right)-$$

$$\frac{a\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{a\,\left(5\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{10\,b\,\left(a+b\,x^3\right)^{2/3}}-$$

$$\frac{a\,c^2\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{a\,c^2\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,b^{2/3}}$$

Problem 25: Result valid but suboptimal antiderivative.

$$\int (c+d\,x)^2\,\left(a+b\,x^3\right)^{1/3}\,dx$$

Optimal (type 5, 192 leaves, 8 steps):

$$\frac{a\,d^2\,\left(a+b\,x^3\right)^{1/3}}{4\,b}+\frac{1}{12}\,\left(a+b\,x^3\right)^{1/3}\,\left(6\,c^2\,x+8\,c\,d\,x^2+3\,d^2\,x^3\right)-\frac{2\,a\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}}+$$

$$\frac{a\,c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,\left(a+b\,x^3\right)^{2/3}}-\frac{a\,c\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{2/3}}$$

Result (type 5, 245 leaves, 14 steps):

$$\frac{a\,d^2\,\left(a+b\,x^3\right)^{1/3}}{4\,b}+\frac{1}{12}\,\left(a+b\,x^3\right)^{1/3}\,\left(6\,c^2\,x+8\,c\,d\,x^2+3\,d^2\,x^3\right)-$$

$$\frac{2\,a\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{2/3}}+\frac{a\,c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,\left(a+b\,x^3\right)^{2/3}}-$$

$$\frac{2\,a\,c\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{2/3}}+\frac{a\,c\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{2/3}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int (c + d x) \left(a + b x^3\right)^{1/3} dx$$

Optimal (type 5, 155 leaves, 6 steps):

$$\frac{1}{6} \left(3 c x + 2 d x^2\right) \left(a + b x^3\right)^{1/3} - \frac{a d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}} + \frac{a c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 \left(a + b x^3\right)^{2/3}} - \frac{a d \operatorname{Log}\left[b^{1/3} x - \left(a + b x^3\right)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 5, 207 leaves, 12 steps):

$$\frac{1}{6} \left(3 c x + 2 d x^2\right) \left(a + b x^3\right)^{1/3} - \frac{a d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}\, b^{2/3}} + \frac{a c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 \left(a + b x^3\right)^{2/3}} - \frac{a d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{9 b^{2/3}} + \frac{a d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}} + \frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 27: Unable to integrate problem.

$$\int \frac{\left(a + b x^3\right)^{1/3}}{c + d x} dx$$

Optimal (type 6, 435 leaves, 13 steps):

$$\frac{\left(a+b x^3\right)^{1/3}}{d}+\frac{x \left(a+b x^3\right)^{1/3} \text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c \left(1+\frac{b x^3}{a}\right)^{1/3}}+$$

$$\frac{b^{1/3} c \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}-\frac{\left(b c^3-a d^3\right)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+$$

$$\frac{\left(b c^3-a d^3\right)^{1/3} \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+\frac{\left(b c^3-a d^3\right)^{1/3} \text{Log}\left[c^3+d^3 x^3\right]}{3 d^2}+$$

$$\frac{b^{1/3} c \text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}-\frac{\left(b c^3-a d^3\right)^{1/3} \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}-$$

$$\frac{\left(b c^3-a d^3\right)^{1/3} \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{2 d^2}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(a+b x^3\right)^{1/3}}{c+d x},x\right]$$

Problem 28: Unable to integrate problem.

$$\int \frac{\left(a+b x^3\right)^{1/3}}{(c+d x)^2} \, dx$$

Optimal (type 6, 818 leaves, 20 steps):

$$-\frac{c^2\left(a+b x^3\right)^{1/3}}{d\left(c^3+d^3 x^3\right)}-\frac{d x^2\left(a+b x^3\right)^{1/3}}{c^3+d^3 x^3}+\frac{x\left(a+b x^3\right)^{1/3}\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^2\left(1+\frac{b x^3}{a}\right)^{1/3}}-$$

$$\frac{d^3 x^4\left(a+b x^3\right)^{1/3}\text{AppellF1}\left[\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2 c^5\left(1+\frac{b x^3}{a}\right)^{1/3}}-\frac{b^{1/3}\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2}+$$

$$\frac{2 a d\,\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3} x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\,c\left(b c^3-a d^3\right)^{2/3}}+\frac{\left(3 b c^3-2 a d^3\right)\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3} x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}\,c\,d^2\left(b c^3-a d^3\right)^{2/3}}-\frac{b c^2\,\text{ArcTan}\left[\frac{1-\frac{2 d\left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,d^2\left(b c^3-a d^3\right)^{2/3}}-$$

$$\frac{b c^2\,\text{Log}\left[c^3+d^3 x^3\right]}{6 d^2\left(b c^3-a d^3\right)^{2/3}}-\frac{a d\,\text{Log}\left[c^3+d^3 x^3\right]}{9 c\left(b c^3-a d^3\right)^{2/3}}-\frac{\left(3 b c^3-2 a d^3\right)\text{Log}\left[c^3+d^3 x^3\right]}{18 c d^2\left(b c^3-a d^3\right)^{2/3}}-$$

$$\frac{b^{1/3}\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 d^2}+\frac{a d\,\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c\left(b c^3-a d^3\right)^{2/3}}+$$

$$\frac{\left(3 b c^3-2 a d^3\right)\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{6 c d^2\left(b c^3-a d^3\right)^{2/3}}+\frac{b c^2\,\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d\left(a+b x^3\right)^{1/3}\right]}{2 d^2\left(b c^3-a d^3\right)^{2/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(a+b x^3\right)^{1/3}}{(c+d x)^2},x\right]$$

Problem 33: Unable to integrate problem.

$$\int\frac{1}{(c+d x)\left(a+b x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 333 leaves, 10 steps):

$$-\frac{d x^2\left(1+\frac{b x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2 c^2\left(a+b x^3\right)^{1/3}}+$$

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\left(b c^3-a d^3\right)^{1/3} x}{c\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(b c^3-a d^3\right)^{1/3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 d\left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(b c^3-a d^3\right)^{1/3}}+\frac{\text{Log}\left[c^3+d^3 x^3\right]}{3\left(b c^3-a d^3\right)^{1/3}}-$$

$$\frac{\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2\left(b c^3-a d^3\right)^{1/3}}-\frac{\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d\left(a+b x^3\right)^{1/3}\right]}{2\left(b c^3-a d^3\right)^{1/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c+d x)\left(a+b x^3\right)^{1/3}},x\right]$$

Problem 34: Unable to integrate problem.

$$\int \frac{1}{(c+d\,x)^2\,(a+b\,x^3)^{1/3}}\,dx$$

Optimal (type 6, 761 leaves, 17 steps):

$$\frac{c^2\,d^2\,(a+b\,x^3)^{2/3}}{(b\,c^3-a\,d^3)\,(c^3+d^3\,x^3)}-\frac{c\,d^3\,x\,(a+b\,x^3)^{2/3}}{(b\,c^3-a\,d^3)\,(c^3+d^3\,x^3)}-\frac{d\,x^2\left(1+\frac{b\,x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{c^3\,(a+b\,x^3)^{1/3}}+$$

$$\frac{d^4\,x^5\left(1+\frac{b\,x^3}{a}\right)^{1/3}\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{d^3\,x^3}{c^3}\right]}{5\,c^6\,(a+b\,x^3)^{1/3}}+\frac{2\,a\,d^3\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c^3-a\,d^3)^{1/3}\,x}{c\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c\,(b\,c^3-a\,d^3)^{4/3}}+$$

$$\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{ArcTan}\left[\frac{1+\frac{2\,(b\,c^3-a\,d^3)^{1/3}\,x}{c\,(a+b\,x^3)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,c\,(b\,c^3-a\,d^3)^{4/3}}-\frac{b\,c^2\,\text{ArcTan}\left[\frac{1-\frac{2\,d\,(a+b\,x^3)^{1/3}}{(b\,c^3-a\,d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(b\,c^3-a\,d^3)^{4/3}}+\frac{b\,c^2\,\text{Log}\left[c^3+d^3\,x^3\right]}{6\,(b\,c^3-a\,d^3)^{4/3}}+$$

$$\frac{a\,d^3\,\text{Log}\left[c^3+d^3\,x^3\right]}{9\,c\,(b\,c^3-a\,d^3)^{4/3}}+\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{Log}\left[c^3+d^3\,x^3\right]}{18\,c\,(b\,c^3-a\,d^3)^{4/3}}-\frac{a\,d^3\,\text{Log}\left[\frac{(b\,c^3-a\,d^3)^{1/3}\,x}{c}-(a+b\,x^3)^{1/3}\right]}{3\,c\,(b\,c^3-a\,d^3)^{4/3}}-$$

$$\frac{(3\,b\,c^3-2\,a\,d^3)\,\text{Log}\left[\frac{(b\,c^3-a\,d^3)^{1/3}\,x}{c}-(a+b\,x^3)^{1/3}\right]}{6\,c\,(b\,c^3-a\,d^3)^{4/3}}-\frac{b\,c^2\,\text{Log}\left[(b\,c^3-a\,d^3)^{1/3}+d\,(a+b\,x^3)^{1/3}\right]}{2\,(b\,c^3-a\,d^3)^{4/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d\,x)^2\,(a+b\,x^3)^{1/3}},x\right]$$

Problem 35: Unable to integrate problem.

$$\int \frac{1}{(c+d\,x)^3\,(a+b\,x^3)^{1/3}}\,dx$$

Optimal (type 6, 1513 leaves, 32 steps):

$$\frac{3 c^4 d^2 (a+b x^3)^{2/3}}{2 (b c^3-a d^3) (c^3+d^3 x^3)^2}-\frac{3 c^3 d^3 x (a+b x^3)^{2/3}}{2 (b c^3-a d^3) (c^3+d^3 x^3)^2}+$$
$$\frac{4 b c^4 d^2 (a+b x^3)^{2/3}}{3 (b c^3-a d^3)^2 (c^3+d^3 x^3)}-\frac{c d^2 (b c^3-3 a d^3) (a+b x^3)^{2/3}}{3 (b c^3-a d^3)^2 (c^3+d^3 x^3)}+$$
$$\frac{d^3 (3 b c^3-7 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)}-\frac{d^3 (9 b c^3-5 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)}-$$
$$\frac{7 d^3 (3 b c^3+a d^3) x (a+b x^3)^{2/3}}{18 (b c^3-a d^3)^2 (c^3+d^3 x^3)}-\frac{3 d x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},3,\frac{5}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2 c^4 (a+b x^3)^{1/3}}+$$
$$\frac{6 d^4 x^5 \left(1+\frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},3,\frac{8}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{5 c^7 (a+b x^3)^{1/3}}+\frac{2 a^2 d^6 \text{ArcTan}\left[\frac{1+\frac{2 (b c^3-a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}}+$$
$$\frac{7 a d^3 (3 b c^3-a d^3) \text{ArcTan}\left[\frac{1+\frac{2 (b c^3-a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}}+\frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \text{ArcTan}\left[\frac{1+\frac{2 (b c^3-a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3-a d^3)^{7/3}}-$$
$$\frac{4 b^2 c^4 \text{ArcTan}\left[\frac{1-\frac{2 d (a+b x^3)^{1/3}}{(b c^3-a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3-a d^3)^{7/3}}+\frac{b c (b c^3-3 a d^3) \text{ArcTan}\left[\frac{1-\frac{2 d (a+b x^3)^{1/3}}{(b c^3-a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3-a d^3)^{7/3}}+$$
$$\frac{2 b^2 c^4 \text{Log}[c^3+d^3 x^3]}{9 (b c^3-a d^3)^{7/3}}+\frac{a^2 d^6 \text{Log}[c^3+d^3 x^3]}{27 c^2 (b c^3-a d^3)^{7/3}}-\frac{b c (b c^3-3 a d^3) \text{Log}[c^3+d^3 x^3]}{18 (b c^3-a d^3)^{7/3}}+$$
$$\frac{7 a d^3 (3 b c^3-a d^3) \text{Log}[c^3+d^3 x^3]}{54 c^2 (b c^3-a d^3)^{7/3}}+\frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \text{Log}[c^3+d^3 x^3]}{54 c^2 (b c^3-a d^3)^{7/3}}-$$
$$\frac{a^2 d^6 \text{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{9 c^2 (b c^3-a d^3)^{7/3}}-\frac{7 a d^3 (3 b c^3-a d^3) \text{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{18 c^2 (b c^3-a d^3)^{7/3}}-$$
$$\frac{(9 b^2 c^6-12 a b c^3 d^3+5 a^2 d^6) \text{Log}\left[\frac{(b c^3-a d^3)^{1/3} x}{c}-(a+b x^3)^{1/3}\right]}{18 c^2 (b c^3-a d^3)^{7/3}}-$$
$$\frac{2 b^2 c^4 \text{Log}\left[(b c^3-a d^3)^{1/3}+d (a+b x^3)^{1/3}\right]}{3 (b c^3-a d^3)^{7/3}}+\frac{b c (b c^3-3 a d^3) \text{Log}\left[(b c^3-a d^3)^{1/3}+d (a+b x^3)^{1/3}\right]}{6 (b c^3-a d^3)^{7/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^3 (a+b x^3)^{1/3}},x\right]$$

Problem 36: Result valid but suboptimal antiderivative.

$$\int \frac{(c+d\,x)^4}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 306 leaves, 10 steps):

$$\frac{6\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}-\frac{4\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+$$
$$\frac{2\,a\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}+$$
$$\frac{c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-$$
$$\frac{2\,c^3\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{b^{2/3}}+\frac{a\,d^4\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{3\,b^{5/3}}$$

Result (type 5, 416 leaves, 22 steps):

$$\frac{6\,c^2\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^4\,x^2\,\left(a+b\,x^3\right)^{1/3}}{3\,b}-\frac{4\,c^3\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+$$
$$\frac{2\,a\,d^4\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\,\sqrt{3}\,b^{5/3}}+\frac{c^4\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}+$$
$$\frac{c\,d^3\,x^4\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{4\,c^3\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+$$
$$\frac{2\,a\,d^4\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}+\frac{2\,c^3\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}-\frac{a\,d^4\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{9\,b^{5/3}}$$

Problem 37: Result valid but suboptimal antiderivative.

$$\int \frac{(c+d\,x)^3}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 187 leaves, 8 steps):

$$\frac{3\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{2\,b}-\frac{\sqrt{3}\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}}+\frac{\left(2\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,b\,\left(a+b\,x^3\right)^{2/3}}-\frac{3\,c^2\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,b^{2/3}}$$

Result (type 5, 239 leaves, 14 steps):

$$\frac{3\,c\,d^2\,\left(a+b\,x^3\right)^{1/3}}{b}+\frac{d^3\,x\,\left(a+b\,x^3\right)^{1/3}}{2\,b}-\frac{\sqrt{3}\,c^2\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}}+\frac{\left(2\,b\,c^3-a\,d^3\right)\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{2\,b\,\left(a+b\,x^3\right)^{2/3}}-\frac{c^2\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{b^{2/3}}+\frac{c^2\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{2\,b^{2/3}}$$

Problem 38: Result valid but suboptimal antiderivative.

$$\int\frac{\left(c+d\,x\right)^2}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 141 leaves, 7 steps):

$$\frac{d^2\,\left(a+b\,x^3\right)^{1/3}}{b}-\frac{2\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{c\,d\,\text{Log}\left[b^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{b^{2/3}}$$

Result (type 5, 195 leaves, 13 steps):

$$\frac{d^2\,\left(a+b\,x^3\right)^{1/3}}{b}-\frac{2\,c\,d\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^{2/3}}+\frac{c^2\,x\,\left(1+\frac{b\,x^3}{a}\right)^{2/3}\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\,x^3}{a}\right]}{\left(a+b\,x^3\right)^{2/3}}-\frac{2\,c\,d\,\text{Log}\left[1-\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}+\frac{c\,d\,\text{Log}\left[1+\frac{b^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{b^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,b^{2/3}}$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int\frac{c+d\,x}{\left(a+b\,x^3\right)^{2/3}}\,dx$$

Optimal (type 5, 121 leaves, 5 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}+\frac{c\, x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{\left(a+b x^3\right)^{2/3}}-\frac{d\,\text{Log}\left[b^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 172 leaves, 11 steps):

$$-\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b^{2/3}}+\frac{c\, x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b x^3}{a}\right]}{\left(a+b x^3\right)^{2/3}}-\frac{d\,\text{Log}\left[1-\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 b^{2/3}}+\frac{d\,\text{Log}\left[1+\frac{b^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{b^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 b^{2/3}}$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(c+d x)\left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 6, 332 leaves, 10 steps):

$$\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c \left(a+b x^3\right)^{2/3}}+\frac{d\,\text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{2/3}}-\frac{d\,\text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{2/3}}-\frac{d\,\text{Log}\left[c^3+d^3 x^3\right]}{3 \left(b c^3-a d^3\right)^{2/3}}+\frac{d\,\text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{2 \left(b c^3-a d^3\right)^{2/3}}+\frac{d\,\text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{2 \left(b c^3-a d^3\right)^{2/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)\left(a+b x^3\right)^{2/3}},x\right]$$

Problem 41: Unable to integrate problem.

$$\int \frac{1}{(c+d x)^2\left(a+b x^3\right)^{2/3}}\, dx$$

Optimal (type 6, 760 leaves, 18 steps):

$$\frac{c^2 d^2 \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)}+\frac{d^4 x^2 \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^2 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{2 c^5 \left(a+b x^3\right)^{2/3}}+\frac{2 a d^4 \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c \left(b c^3-a d^3\right)^{5/3}}+$$

$$\frac{2 d \left(3 b c^3-a d^3\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c \left(b c^3-a d^3\right)^{5/3}}-\frac{2 b c^2 d \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} \left(b c^3-a d^3\right)^{5/3}}-\frac{b c^2 d \text{Log}\left[c^3+d^3 x^3\right]}{3 \left(b c^3-a d^3\right)^{5/3}}-$$

$$\frac{a d^4 \text{Log}\left[c^3+d^3 x^3\right]}{9 c \left(b c^3-a d^3\right)^{5/3}}-\frac{d \left(3 b c^3-a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{9 c \left(b c^3-a d^3\right)^{5/3}}+\frac{a d^4 \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c \left(b c^3-a d^3\right)^{5/3}}+$$

$$\frac{d \left(3 b c^3-a d^3\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c \left(b c^3-a d^3\right)^{5/3}}+\frac{b c^2 d \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{\left(b c^3-a d^3\right)^{5/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^2 \left(a+b x^3\right)^{2/3}},x\right]$$

Problem 42: Unable to integrate problem.

$$\int \frac{1}{(c+d x)^3 \left(a+b x^3\right)^{2/3}} \, dx$$

Optimal (type 6, 1357 leaves, 30 steps):

$$\frac{3 c^4 d^2 \left(a+b x^3\right)^{1/3}}{2 \left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)^2}+\frac{3 c^2 d^4 x^2 \left(a+b x^3\right)^{1/3}}{2 \left(b c^3-a d^3\right) \left(c^3+d^3 x^3\right)^2}+\frac{5 b c^4 d^2 \left(a+b x^3\right)^{1/3}}{3 \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}-$$

$$\frac{c d^2 \left(b c^3-6 a d^3\right) \left(a+b x^3\right)^{1/3}}{6 \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+\frac{d^4 \left(9 b c^3-4 a d^3\right) x^2 \left(a+b x^3\right)^{1/3}}{6 c \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+$$

$$\frac{d^4 \left(3 b c^3+2 a d^3\right) x^2 \left(a+b x^3\right)^{1/3}}{3 c \left(b c^3-a d^3\right)^2 \left(c^3+d^3 x^3\right)}+\frac{x \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{c^3 \left(a+b x^3\right)^{2/3}}-$$

$$\frac{7 d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{4 c^6 \left(a+b x^3\right)^{2/3}}+$$

$$\frac{d^6 x^7 \left(1+\frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{b x^3}{a},-\frac{d^3 x^3}{c^3}\right]}{7 c^9 \left(a+b x^3\right)^{2/3}}+\frac{2 a d^4 \left(6 b c^3-a d^3\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{ArcTan}\left[\frac{1+\frac{2 \left(b c^3-a d^3\right)^{1/3} x}{c \left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 \left(b c^3-a d^3\right)^{8/3}}-\frac{10 b^2 c^4 d \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{b c d \left(b c^3-6 a d^3\right) \text{ArcTan}\left[\frac{1-\frac{2 d \left(a+b x^3\right)^{1/3}}{\left(b c^3-a d^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} \left(b c^3-a d^3\right)^{8/3}}-\frac{5 b^2 c^4 d \text{Log}\left[c^3+d^3 x^3\right]}{9 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{b c d \left(b c^3-6 a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{18 \left(b c^3-a d^3\right)^{8/3}}-\frac{a d^4 \left(6 b c^3-a d^3\right) \text{Log}\left[c^3+d^3 x^3\right]}{9 c^2 \left(b c^3-a d^3\right)^{8/3}}-$$

$$\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{Log}\left[c^3+d^3 x^3\right]}{18 c^2 \left(b c^3-a d^3\right)^{8/3}}+\frac{a d^4 \left(6 b c^3-a d^3\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{3 c^2 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{d \left(9 b^2 c^6-6 a b c^3 d^3+2 a^2 d^6\right) \text{Log}\left[\frac{\left(b c^3-a d^3\right)^{1/3} x}{c}-\left(a+b x^3\right)^{1/3}\right]}{6 c^2 \left(b c^3-a d^3\right)^{8/3}}+$$

$$\frac{5 b^2 c^4 d \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{3 \left(b c^3-a d^3\right)^{8/3}}-$$

$$\frac{b c d \left(b c^3-6 a d^3\right) \text{Log}\left[\left(b c^3-a d^3\right)^{1/3}+d \left(a+b x^3\right)^{1/3}\right]}{6 \left(b c^3-a d^3\right)^{8/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^3 \left(a+b x^3\right)^{2/3}},x\right]$$

Problem 197: Unable to integrate problem.

$$\int \frac{\left(d^3+e^3 x^3\right)^p}{d+e x}\,dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{1}{e\,p}\left(d^3+e^3 x^3\right)^p \left(1+\frac{2\,(d+e x)}{\left(-3+i\sqrt{3}\right) d}\right)^{-p} \left(1-\frac{2\,(d+e x)}{\left(3+i\sqrt{3}\right) d}\right)^{-p}$$
$$\text{AppellF1}\left[p,\,-p,\,-p,\,1+p,\,-\frac{2\,(d+e x)}{\left(-3+i\sqrt{3}\right) d},\,\frac{2\,(d+e x)}{\left(3+i\sqrt{3}\right) d}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(d^3+e^3 x^3\right)^p}{d+e x},\,x\right]$$

Problem 227: Result valid but suboptimal antiderivative.

$$\int x^m \left(c\,\left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 3, 161 leaves, 3 steps):

$$\frac{a^3 c\, x^{1+m}\sqrt{c\,\left(a+b x^2\right)^2}}{(1+m)\,\left(a+b x^2\right)}+\frac{3 a^2 b\, c\, x^{3+m}\sqrt{c\,\left(a+b x^2\right)^2}}{(3+m)\,\left(a+b x^2\right)}+$$
$$\frac{3 a b^2 c\, x^{5+m}\sqrt{c\,\left(a+b x^2\right)^2}}{(5+m)\,\left(a+b x^2\right)}+\frac{b^3 c\, x^{7+m}\sqrt{c\,\left(a+b x^2\right)^2}}{(7+m)\,\left(a+b x^2\right)}$$

Result (type 3, 205 leaves, 4 steps):

$$\frac{a^3 c\, x^{1+m}\sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(1+m)\,\left(a+b x^2\right)}+\frac{3 a^2 b\, c\, x^{3+m}\sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(3+m)\,\left(a+b x^2\right)}+$$
$$\frac{3 a b^2 c\, x^{5+m}\sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(5+m)\,\left(a+b x^2\right)}+\frac{b^3 c\, x^{7+m}\sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{(7+m)\,\left(a+b x^2\right)}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int x^5 \left(c\,\left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 4 steps):

$$\frac{a^3 c x^6 \sqrt{c \left(a+b x^2\right)^2}}{6 \left(a+b x^2\right)}+\frac{3 a^2 b c x^8 \sqrt{c \left(a+b x^2\right)^2}}{8 \left(a+b x^2\right)}+\frac{3 a b^2 c x^{10} \sqrt{c \left(a+b x^2\right)^2}}{10 \left(a+b x^2\right)}+\frac{b^3 c x^{12} \sqrt{c \left(a+b x^2\right)^2}}{12 \left(a+b x^2\right)}$$

Result (type 2, 134 leaves, 4 steps):

$$\frac{a^2 c \left(a+b x^2\right)^3 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{8 b^3}-\frac{a c \left(a+b x^2\right)^4 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 b^3}+\frac{c \left(a+b x^2\right)^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{12 b^3}$$

Problem 229: Result valid but suboptimal antiderivative.

$$\int x^4 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)}+\frac{b^3 c x^{11} \sqrt{c \left(a+b x^2\right)^2}}{11 \left(a+b x^2\right)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 \left(a+b x^2\right)}+\frac{3 a^2 b c x^7 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{7 \left(a+b x^2\right)}+\frac{a b^2 c x^9 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{3 \left(a+b x^2\right)}+\frac{b^3 c x^{11} \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{11 \left(a+b x^2\right)}$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{a c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^2}}{8 b^2}+\frac{c \left(a+b x^2\right)^4 \sqrt{c \left(a+b x^2\right)^2}}{10 b^2}$$

Result (type 2, 78 leaves, 4 steps):

$$-\frac{a \left(a+b x^2\right) \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{8 b^2}+\frac{\left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{5/2}}{10 b^2 c}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^3 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)}+\frac{3 a^2 b c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{3 a b^2 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}+\frac{b^3 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{9 \left(a+b x^2\right)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^3 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{3 \left(a+b x^2\right)}+\frac{3 a^2 b c x^5 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{5 \left(a+b x^2\right)}+$$
$$\frac{3 a b^2 c x^7 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{7 \left(a+b x^2\right)}+\frac{b^3 c x^9 \sqrt{a^2 c+2 a b c x^2+b^2 c x^4}}{9 \left(a+b x^2\right)}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int \left(c \left(a+b x^2\right)^2\right)^{3/2} dx$$

Optimal (type 2, 135 leaves, 3 steps):

$$\frac{a^3 c x \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{a^2 b c x^3 \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}+\frac{3 a b^2 c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)}+\frac{b^3 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)}$$

Result (type 2, 175 leaves, 4 steps):

$$\frac{a^3 x \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}+\frac{a^2 b x^3 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{\left(a+b x^2\right)^3}+$$
$$\frac{3 a b^2 x^5 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{5 \left(a+b x^2\right)^3}+\frac{b^3 x^7 \left(a^2 c+2 a b c x^2+b^2 c x^4\right)^{3/2}}{7 \left(a+b x^2\right)^3}$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^2\right)^{3/2}}{x} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{3 a^2 b c x^2 \sqrt{c \left(a+b x^2\right)^2}}{2 \left(a+b x^2\right)}+\frac{3 a b^2 c x^4 \sqrt{c \left(a+b x^2\right)^2}}{4 \left(a+b x^2\right)}+$$
$$\frac{b^3 c x^6 \sqrt{c \left(a+b x^2\right)^2}}{6 \left(a+b x^2\right)}+\frac{a^3 c \sqrt{c \left(a+b x^2\right)^2} \operatorname{Log}[x]}{a+b x^2}$$

Result (type 3, 183 leaves, 5 steps):

$$\frac{3\,a^2\,b\,c\,x^2\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{2\,\left(a+b\,x^2\right)}+\frac{3\,a\,b^2\,c\,x^4\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{4\,\left(a+b\,x^2\right)}+\frac{b^3\,c\,x^6\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{6\,\left(a+b\,x^2\right)}+\frac{a^3\,c\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}\,\mathrm{Log}[x]}{a+b\,x^2}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int\frac{\left(c\,\left(a+b\,x^2\right)^2\right)^{3/2}}{x^2}\,dx$$

Optimal (type 2, 134 leaves, 3 steps):

$$-\frac{a^3\,c\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{x\,\left(a+b\,x^2\right)}+\frac{3\,a^2\,b\,c\,x\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{a+b\,x^2}+\frac{a\,b^2\,c\,x^3\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{a+b\,x^2}+\frac{b^3\,c\,x^5\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{5\,\left(a+b\,x^2\right)}$$

Result (type 2, 178 leaves, 4 steps):

$$-\frac{a^3\,c\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{x\,\left(a+b\,x^2\right)}+\frac{3\,a^2\,b\,c\,x\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{a+b\,x^2}+\frac{a\,b^2\,c\,x^3\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{a+b\,x^2}+\frac{b^3\,c\,x^5\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{5\,\left(a+b\,x^2\right)}$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int\frac{\left(c\,\left(a+b\,x^2\right)^2\right)^{3/2}}{x^3}\,dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{a^3\,c\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{2\,x^2\,\left(a+b\,x^2\right)}+\frac{3\,a\,b^2\,c\,x^2\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{2\,\left(a+b\,x^2\right)}+\frac{b^3\,c\,x^4\,\sqrt{c\,\left(a+b\,x^2\right)^2}}{4\,\left(a+b\,x^2\right)}+\frac{3\,a^2\,b\,c\,\sqrt{c\,\left(a+b\,x^2\right)^2}\,\mathrm{Log}[x]}{a+b\,x^2}$$

Result (type 3, 184 leaves, 5 steps):

$$-\frac{a^3\,c\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{2\,x^2\,\left(a+b\,x^2\right)}+\frac{3\,a\,b^2\,c\,x^2\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{2\,\left(a+b\,x^2\right)}+\frac{b^3\,c\,x^4\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}}{4\,\left(a+b\,x^2\right)}+\frac{3\,a^2\,b\,c\,\sqrt{a^2\,c+2\,a\,b\,c\,x^2+b^2\,c\,x^4}\,\mathrm{Log}[x]}{a+b\,x^2}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} dx$$

Optimal (type 3, 253 leaves, 8 steps):

$$\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{21}{320} a^2 c x^3 \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{3}{40} a c x^3 \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{12} c x^3 \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{21 a^{9/2} c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcSinh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{1024 b^{3/2} \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 254 leaves, 9 steps):

$$\frac{7}{128} a^3 c x^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{21 a^5 c x \sqrt{c \left(a+b x^2\right)^3}}{1024 b \left(a+b x^2\right)}+\frac{21 a^4 c x^3 \sqrt{c \left(a+b x^2\right)^3}}{512 \left(a+b x^2\right)}+\frac{21}{320} a^2 c x^3 \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{3}{40} a c x^3 \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{12} c x^3 \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{21 a^6 c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a+b x^2}}\right]}{1024 b^{3/2} \left(a+b x^2\right)^{3/2}}$$

Problem 239: Result valid but suboptimal antiderivative.

$$\int \left(c \left(a+b x^2\right)^3\right)^{3/2} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$\frac{21}{128} a^3 c x \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^4 c x \sqrt{c \left(a+b x^2\right)^3}}{256 \left(a+b x^2\right)}+\frac{21}{160} a^2 c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{80} a c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{10} c x \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^{7/2} c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcSinh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{256 \sqrt{b} \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 208 leaves, 8 steps):

$$\frac{21}{128} a^3 c x \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^4 c x \sqrt{c \left(a+b x^2\right)^3}}{256 \left(a+b x^2\right)}+$$
$$\frac{21}{160} a^2 c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{80} a c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+$$
$$\frac{1}{10} c x \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}+\frac{63 a^5 c \sqrt{c \left(a+b x^2\right)^3}\ \text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a+b x^2}}\right]}{256 \sqrt{b} \left(a+b x^2\right)^{3/2}}$$

Problem 240: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x}\, dx$$

Optimal (type 3, 192 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c \left(a+b x^2\right)^3}+\frac{a^4 c \sqrt{c \left(a+b x^2\right)^3}}{a+b x^2}+$$
$$\frac{1}{5} a^2 c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{7} a c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+$$
$$\frac{1}{9} c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{a^3 c \sqrt{c \left(a+b x^2\right)^3}\ \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 194 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c \left(a+b x^2\right)^3}+\frac{a^4 c \sqrt{c \left(a+b x^2\right)^3}}{a+b x^2}+$$
$$\frac{1}{5} a^2 c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{1}{7} a c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}+$$
$$\frac{1}{9} c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}-\frac{a^{9/2} c \sqrt{c \left(a+b x^2\right)^3}\ \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{\left(a+b x^2\right)^{3/2}}$$

Problem 241: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x^2}\, dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c \left(a+b x^2\right)^3}+\frac{315 a^3 b c x \sqrt{c \left(a+b x^2\right)^3}}{128 \left(a+b x^2\right)}+$$
$$\frac{21}{16} a b c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{8} b c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-$$
$$\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{x}+\frac{315 a^{5/2} \sqrt{b}\, c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcSinh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{128 \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 209 leaves, 8 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c \left(a+b x^2\right)^3}+\frac{315 a^3 b c x \sqrt{c \left(a+b x^2\right)^3}}{128 \left(a+b x^2\right)}+$$
$$\frac{21}{16} a b c x \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{8} b c x \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-$$
$$\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{x}+\frac{315 a^4 \sqrt{b}\, c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a+b x^2}}\right]}{128 \left(a+b x^2\right)^{3/2}}$$

Problem 242: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \left(a+b x^2\right)^3\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 202 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c \left(a+b x^2\right)^3}+\frac{9 a^3 b c \sqrt{c \left(a+b x^2\right)^3}}{2 \left(a+b x^2\right)}+$$
$$\frac{9}{10} a b c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{14} b c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-$$
$$\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{2 x^2}-\frac{9 a^2 b c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{2 \left(1+\frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 204 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c \left(a+b x^2\right)^3}+\frac{9 a^3 b c \sqrt{c \left(a+b x^2\right)^3}}{2 \left(a+b x^2\right)}+\frac{9}{10} a b c \left(a+b x^2\right) \sqrt{c \left(a+b x^2\right)^3}+\frac{9}{14} b c \left(a+b x^2\right)^2 \sqrt{c \left(a+b x^2\right)^3}-\frac{c \left(a+b x^2\right)^3 \sqrt{c \left(a+b x^2\right)^3}}{2 x^2}-\frac{9 a^{7/2} b c \sqrt{c \left(a+b x^2\right)^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{2 \left(a+b x^2\right)^{3/2}}$$

Problem 243: Result valid but suboptimal antiderivative.

$$\int x^2 \left(\frac{c}{a+b x^2}\right)^{3/2} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{c x \sqrt{\frac{c}{a+b x^2}}}{b}+\frac{\sqrt{a}\, c \sqrt{\frac{c}{a+b x^2}} \sqrt{1+\frac{b x^2}{a}}\, \text{ArcSinh}\left[\frac{\sqrt{b}\, x}{\sqrt{a}}\right]}{b^{3/2}}$$

Result (type 3, 75 leaves, 4 steps):

$$-\frac{c x \sqrt{\frac{c}{a+b x^2}}}{b}+\frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{a+b x^2}\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x}{\sqrt{a+b x^2}}\right]}{b^{3/2}}$$

Problem 246: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a+b x^2}\right)^{3/2}}{x} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a+b x^2}}}{a}-\frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{1+\frac{b x^2}{a}}\, \text{ArcTanh}\left[\sqrt{1+\frac{b x^2}{a}}\right]}{a}$$

Result (type 3, 73 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a+b x^2}}}{a}-\frac{c \sqrt{\frac{c}{a+b x^2}} \sqrt{a+b x^2}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^2}}{\sqrt{a}}\right]}{a^{3/2}}$$

Problem 248: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a+b x^2}\right)^{3/2}}{x^3} dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$-\frac{3\,b\,c\,\sqrt{\frac{c}{a+b\,x^2}}}{2\,a^2}-\frac{c\,\sqrt{\frac{c}{a+b\,x^2}}}{2\,a\,x^2}+\frac{3\,b\,c\,\sqrt{\frac{c}{a+b\,x^2}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\text{ArcTanh}\left[\sqrt{1+\frac{b\,x^2}{a}}\right]}{2\,a^2}$$

Result (type 3, 112 leaves, 6 steps):

$$\frac{c\,\sqrt{\frac{c}{a+b\,x^2}}}{a\,x^2}-\frac{3\,c\,\sqrt{\frac{c}{a+b\,x^2}}\,\left(a+b\,x^2\right)}{2\,a^2\,x^2}+\frac{3\,b\,c\,\sqrt{\frac{c}{a+b\,x^2}}\,\sqrt{a+b\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^2}}{\sqrt{a}}\right]}{2\,a^{5/2}}$$

Problem 249: Result valid but suboptimal antiderivative.

$$\int x^7\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,dx$$

Optimal (type 2, 138 leaves, 4 steps):

$$-\frac{2\,a^3\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)}{7\,b^4}+\frac{6\,a^2\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)^2}{11\,b^4}-\frac{2\,a\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)^3}{5\,b^4}+\frac{2\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)^4}{19\,b^4}$$

Result (type 2, 152 leaves, 4 steps):

$$-\frac{2\,a^3\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{3/2}}{7\,b^4}+\frac{6\,a^2\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{5/2}}{11\,b^4}-\frac{2\,a\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{7/2}}{5\,b^4}+\frac{2\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{9/2}}{19\,b^4}$$

Problem 250: Result valid but suboptimal antiderivative.

$$\int x^5\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,dx$$

Optimal (type 2, 102 leaves, 4 steps):

$$\frac{2\,a^2\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)}{7\,b^3}-\frac{4\,a\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)^2}{11\,b^3}+\frac{2\,\left(c\,\sqrt{a+b\,x^2}\right)^{3/2}\,\left(a+b\,x^2\right)^3}{15\,b^3}$$

Result (type 2, 113 leaves, 4 steps):

$$\frac{2\,a^2\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{3/2}}{7\,b^3}-\frac{4\,a\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{5/2}}{11\,b^3}+\frac{2\,c\,\sqrt{c\,\sqrt{a+b\,x^2}}\,\left(a+b\,x^2\right)^{7/2}}{15\,b^3}$$

Problem 251: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c \sqrt{a+b x^2}\right)^{3/2} \, dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{2 a \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)}{7 b^2}+\frac{2 \left(c \sqrt{a+b x^2}\right)^{3/2} \left(a+b x^2\right)^2}{11 b^2}$$

Result (type 2, 74 leaves, 4 steps):

$$-\frac{2 a c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{3/2}}{7 b^2}+\frac{2 c \sqrt{c \sqrt{a+b x^2}} \left(a+b x^2\right)^{5/2}}{11 b^2}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x} \, dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{2}{3} \left(c \sqrt{a+b x^2}\right)^{3/2}+\frac{\left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTan}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/4}}-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTanh}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 3, 141 leaves, 7 steps):

$$\frac{2}{3} c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}+$$
$$\frac{a^{3/4} c \sqrt{c \sqrt{a+b x^2}} \text{ArcTan}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{\left(a+b x^2\right)^{1/4}}-\frac{a^{3/4} c \sqrt{c \sqrt{a+b x^2}} \text{ArcTanh}\left[\frac{\left(a+b x^2\right)^{1/4}}{a^{1/4}}\right]}{\left(a+b x^2\right)^{1/4}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x^3} \, dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{2 x^2}+\frac{3 b \left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTan}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1+\frac{b x^2}{a}\right)^{3/4}}-$$
$$\frac{3 b \left(c \sqrt{a+b x^2}\right)^{3/2} \text{ArcTanh}\left[\left(1+\frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 3, 151 leaves, 7 steps):

$$-\frac{c\sqrt{c\sqrt{a+b x^2}}\ \sqrt{a+b x^2}}{2 x^2}+\frac{3 b c\sqrt{c\sqrt{a+b x^2}}\ \text{ArcTan}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right]}{4 a^{1/4}\left(a+b x^2\right)^{1/4}}-\frac{3 b c\sqrt{c\sqrt{a+b x^2}}\ \text{ArcTanh}\left[\frac{(a+b x^2)^{1/4}}{a^{1/4}}\right]}{4 a^{1/4}\left(a+b x^2\right)^{1/4}}$$

Problem 255: Result valid but suboptimal antiderivative.

$$\int x^2\left(c\sqrt{a+b x^2}\right)^{3/2}dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{2 a x\left(c\sqrt{a+b x^2}\right)^{3/2}}{15 b}+\frac{2}{9}x^3\left(c\sqrt{a+b x^2}\right)^{3/2}-\frac{4 a^2 x\left(c\sqrt{a+b x^2}\right)^{3/2}}{15 b\left(a+b x^2\right)}+\frac{4 a^{3/2}\left(c\sqrt{a+b x^2}\right)^{3/2}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\ x}{\sqrt{a}}\right],2\right]}{15 b^{3/2}\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 191 leaves, 6 steps):

$$-\frac{4 a^2 c x\sqrt{c\sqrt{a+b x^2}}}{15 b\sqrt{a+b x^2}}+\frac{2 a c x\sqrt{c\sqrt{a+b x^2}}\ \sqrt{a+b x^2}}{15 b}+\frac{2}{9}c x^3\sqrt{c\sqrt{a+b x^2}}\ \sqrt{a+b x^2}+\frac{4 a^{5/2} c\sqrt{c\sqrt{a+b x^2}}\left(1+\frac{b x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\ x}{\sqrt{a}}\right],2\right]}{15 b^{3/2}\sqrt{a+b x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int\left(c\sqrt{a+b x^2}\right)^{3/2}dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$\frac{2}{5}x\left(c\sqrt{a+b x^2}\right)^{3/2}+\frac{6 a x\left(c\sqrt{a+b x^2}\right)^{3/2}}{5\left(a+b x^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+b x^2}\right)^{3/2}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\ x}{\sqrt{a}}\right],2\right]}{5\sqrt{b}\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 146 leaves, 5 steps):

$$\frac{6 a c x\sqrt{c\sqrt{a+b x^2}}}{5\sqrt{a+b x^2}}+\frac{2}{5}c x\sqrt{c\sqrt{a+b x^2}}\ \sqrt{a+b x^2}-\frac{6 a^{3/2} c\sqrt{c\sqrt{a+b x^2}}\left(1+\frac{b x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\ x}{\sqrt{a}}\right],2\right]}{5\sqrt{b}\ \sqrt{a+b x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\sqrt{a+b x^2}\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$-\frac{\left(c\sqrt{a+b x^2}\right)^{3/2}}{x}+\frac{3 b x\left(c\sqrt{a+b x^2}\right)^{3/2}}{a+b x^2}-\frac{3\sqrt{b}\left(c\sqrt{a+b x^2}\right)^{3/2}\,\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{\sqrt{a}\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 142 leaves, 5 steps):

$$\frac{3 b c x\sqrt{c\sqrt{a+b x^2}}}{\sqrt{a+b x^2}}-\frac{c\sqrt{c\sqrt{a+b x^2}}\,\sqrt{a+b x^2}}{x}-\frac{1}{\sqrt{a+b x^2}}$$
$$3\sqrt{a}\,\sqrt{b}\,c\sqrt{c\sqrt{a+b x^2}}\left(1+\frac{b x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c\sqrt{a+b x^2}\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$-\frac{\left(c\sqrt{a+b x^2}\right)^{3/2}}{3 x^3}-\frac{b\left(c\sqrt{a+b x^2}\right)^{3/2}}{2 a x}+$$
$$\frac{b^2 x\left(c\sqrt{a+b x^2}\right)^{3/2}}{2 a\left(a+b x^2\right)}-\frac{b^{3/2}\left(c\sqrt{a+b x^2}\right)^{3/2}\,\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2 a^{3/2}\left(1+\frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 193 leaves, 6 steps):

$$\frac{b^2 c x\sqrt{c\sqrt{a+b x^2}}}{2 a\sqrt{a+b x^2}}-\frac{c\sqrt{c\sqrt{a+b x^2}}\,\sqrt{a+b x^2}}{3 x^3}-\frac{b c\sqrt{c\sqrt{a+b x^2}}\,\sqrt{a+b x^2}}{2 a x}-$$
$$\frac{b^{3/2} c\sqrt{c\sqrt{a+b x^2}}\left(1+\frac{b x^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a}}\right],2\right]}{2\sqrt{a}\,\sqrt{a+b x^2}}$$

Problem 318: Result valid but suboptimal antiderivative.

$$\int x^5\sqrt{a+\frac{b}{c+d x^2}}\,dx$$

Optimal (type 3, 216 leaves, 7 steps):

$$-\frac{\left(b^2+4abc-8a^2c^2\right)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3}-\frac{(b+4ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3}+\frac{\left(c+dx^2\right)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3}+\frac{b\left(b^2+4abc+8a^2c^2\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{16a^{5/2}d^3}$$

Result (type 3, 259 leaves, 9 steps):

$$\frac{\left(b^2+4abc+8a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3}-\frac{(3b+8ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{24a^2d^3}+\frac{x^2\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{6ad^2}+\frac{b\left(b^2+4abc+8a^2c^2\right)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right]}{16a^{5/2}d^3\sqrt{b+a\left(c+dx^2\right)}}$$

Problem 319: Result valid but suboptimal antiderivative.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2}-\frac{b(b+4ac)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{8a^{3/2}d^2}$$

Result (type 3, 181 leaves, 8 steps):

$$-\frac{(b+4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a\left(c+dx^2\right)\right)}{4ad^2}-\frac{b(b+4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right]}{8a^{3/2}d^2\sqrt{b+a\left(c+dx^2\right)}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x} \, dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]-\frac{\sqrt{b+a c} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{\sqrt{c}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{a} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{\sqrt{b+a \left(c+d x^2\right)}}-\frac{\sqrt{b+a c} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^3} \, dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{2 c x^2}+\frac{b d \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{2 c^{3/2} \sqrt{b+a c}}$$

Result (type 3, 140 leaves, 6 steps):

$$-\frac{\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}{2 c x^2}+\frac{b d \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{2 c^{3/2} \sqrt{b+a c} \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^2}}}{x^5}\,dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{(5\,b+4\,a\,c)\,d\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,c^2\,(b+a\,c)\,x^2}-\frac{\left(c+d\,x^2\right)^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,c^2\,x^4}-\frac{b\,(3\,b+4\,a\,c)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{8\,c^{5/2}\,(b+a\,c)^{3/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(3\,b+4\,a\,c)\,d\,\left(c+d\,x^2\right)\,\sqrt{a+\frac{b}{c+d\,x^2}}}{8\,c^2\,(b+a\,c)\,x^2}-\frac{\left(c+d\,x^2\right)\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\left(b+a\,\left(c+d\,x^2\right)\right)}{4\,c\,(b+a\,c)\,x^4}-\frac{b\,(3\,b+4\,a\,c)\,d^2\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}\right]}{8\,c^{5/2}\,(b+a\,c)^{3/2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}$$

Problem 324: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d\,x^2}}}{x^7}\,dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$-\frac{\left(11\,b^2+20\,a\,b\,c+8\,a^2\,c^2\right)\,d^2\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{16\,c^3\,(b+a\,c)^2\,x^2}+\frac{(3\,b+4\,a\,c)\,d\,\left(c+d\,x^2\right)^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,c^3\,(b+a\,c)\,x^4}-\frac{\left(c+d\,x^2\right)^3\,\left(\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}\right)^{3/2}}{6\,c^2\,(b+a\,c)\,x^6}+\frac{b\,\left(5\,b^2+12\,a\,b\,c+8\,a^2\,c^2\right)\,d^3\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{16\,c^{7/2}\,(b+a\,c)^{5/2}}$$

Result (type 3, 271 leaves, 9 steps):

$$-\frac{\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}{6 c x^6}+\frac{(5 b+4 a c) d \left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}{24 c^2 (b+a c) x^4}-$$

$$\frac{(5 b+2 a c)(3 b+4 a c) d^2 \left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}{48 c^3 (b+a c)^2 x^2}+$$

$$\left(b \left(5 b^2+12 a b c+8 a^2 c^2\right) d^3 \sqrt{c+d x^2}\,\sqrt{a+\frac{b}{c+d x^2}}\ \text{ArcTanh}\left[\frac{\sqrt{b+a c}\,\sqrt{c+d x^2}}{\sqrt{c}\,\sqrt{b+a\left(c+d x^2\right)}}\right]\right)\Big/$$

$$\left(16 c^{7/2} (b+a c)^{5/2}\sqrt{b+a\left(c+d x^2\right)}\right)$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int x^4 \sqrt{a+\frac{b}{c+d x^2}}\, dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$-\frac{\left(2 b^2+7 a b c-3 a^2 c^2\right) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a^2 d^2}+$$

$$\frac{(b-3 a c) x \left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a d^2}+\frac{x^3 \left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d}+$$

$$\left(\sqrt{c}\left(2 b^2+7 a b c-3 a^2 c^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(15 a^2 d^{5/2}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\right)-\frac{c^{3/2}(b-3 a c)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{15 a d^{5/2}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 478 leaves, 8 steps):

$$-\frac{\left(2 b^2+7 a b c-3 a^2 c^2\right) x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 a^2 d^2 \sqrt{b+a\left(c+d x^2\right)}}+$$

$$\frac{(b-3 a c) x\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 a d^2 \sqrt{b+a\left(c+d x^2\right)}}+\frac{x^3\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 d \sqrt{b+a\left(c+d x^2\right)}}+$$

$$\left(\sqrt{c}\left(2 b^2+7 a b c-3 a^2 c^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) /$$

$$\left(15 a^2 d^{5/2} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{b+a\left(c+d x^2\right)}\right)-$$

$$\left(c^{3/2}(b-3 a c) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) /$$

$$\left(15 a d^{5/2} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{b+a\left(c+d x^2\right)}\right)$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int x^2 \sqrt{a+\frac{b}{c+d x^2}}\, dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\frac{(b-a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 a d}+\frac{x\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 d}-$$

$$\frac{\sqrt{c}(b-a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a d^{3/2} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-$$

$$\frac{c^{3/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 d^{3/2} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 370 leaves, 7 steps):

$$\frac{(b-a c)\, x \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}}{3 a d \sqrt{b+a \left(c+d x^2\right)}}+\frac{x \left(c+d x^2\right) \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}}{3 d \sqrt{b+a \left(c+d x^2\right)}}-$$

$$\left(\sqrt{c}\, (b-a c) \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(3 a d^{3/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\, \sqrt{b+a \left(c+d x^2\right)}\right)-$$

$$\frac{c^{3/2} \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 d^{3/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\, \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int \sqrt{a+\frac{b}{c+d x^2}}\, dx$$

Optimal (type 4, 213 leaves, 6 steps):

$$x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}-\frac{\sqrt{c}\, \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{d}\, \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}+$$

$$\frac{\sqrt{c}\, \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{d}\, \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 279 leaves, 6 steps):

$$\frac{x \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}}{\sqrt{b+a \left(c+d x^2\right)}}-\frac{\sqrt{c}\, \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{d}\, \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\, \sqrt{b+a \left(c+d x^2\right)}}+$$

$$\frac{\sqrt{c}\, \sqrt{b+a c+a d x^2}\, \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{d}\, \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\, \sqrt{b+a \left(c+d x^2\right)}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^2}\,dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\frac{d x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c}-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x}-\frac{\sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{c} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}+\frac{a \sqrt{c} \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 353 leaves, 8 steps):

$$\frac{d x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c \sqrt{b+a \left(c+d x^2\right)}}-\frac{\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c x \sqrt{b+a \left(c+d x^2\right)}}-\frac{\sqrt{d} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{\sqrt{c} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}}+\left(a \sqrt{c} \sqrt{d} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}\, \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right) \Bigg/ \left((b+a c) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}\right)$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+d x^2}}}{x^4}\,dx$$

Optimal (type 4, 362 leaves, 8 steps):

$$-\frac{(2b+ac)\,d^2\,x\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2\,(b+ac)}-\frac{(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3}+\frac{(2b+ac)\,d\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2\,(b+ac)\,x}+$$

$$\frac{(2b+ac)\,d^{3/2}\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3c^{3/2}\,(b+ac)\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}-$$

$$\frac{a\,d^{3/2}\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3\sqrt{c}\,(b+ac)\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

Result (type 4, 472 leaves, 8 steps):

$$-\frac{(2b+ac)\,d^2\,x\,\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^2\,(b+ac)\,\sqrt{b+a\,(c+dx^2)}}-$$

$$\frac{(c+dx^2)\,\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\,\sqrt{b+a\,(c+dx^2)}}+\frac{(2b+ac)\,d\,(c+dx^2)\,\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^2\,(b+ac)\,x\,\sqrt{b+a\,(c+dx^2)}}+$$

$$\left((2b+ac)\,d^{3/2}\,\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]\right)\Big/$$

$$\left(3c^{3/2}\,(b+ac)\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\,\sqrt{b+a\,(c+dx^2)}\right)-$$

$$\frac{a\,d^{3/2}\,\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3\sqrt{c}\,(b+ac)\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\,\sqrt{b+a\,(c+dx^2)}}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2}-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c x^5}+$$

$$\frac{(4 b+3 a c) d \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^2 (b+a c) x^3}-\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^2 \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2 x}-$$

$$\left(\left(8 b^2+13 a b c+3 a^2 c^2\right) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Bigg/$$

$$\left(15 c^{5/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\right)+$$

$$\frac{a (4 b+3 a c) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{3/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 598 leaves, 9 steps) :

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^3 x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^3 (b+a c)^2 \sqrt{b+a \left(c+d x^2\right)}}-$$

$$\frac{\left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c x^5 \sqrt{b+a \left(c+d x^2\right)}}+\frac{(4 b+3 a c) d \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^2 (b+a c) x^3 \sqrt{b+a \left(c+d x^2\right)}}-$$

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) d^2 \left(c+d x^2\right) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{15 c^3 (b+a c)^2 x \sqrt{b+a \left(c+d x^2\right)}}-\Bigg(\left(8 b^2+13 a b c+3 a^2 c^2\right)$$

$$d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\Bigg) \Bigg/$$

$$\left(15 c^{5/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}\right)+$$

$$\left(a (4 b+3 a c) d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Bigg/$$

$$\left(15 c^{3/2} (b+a c)^2 \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{b+a \left(c+d x^2\right)}\right)$$

Problem 331: Result valid but suboptimal antiderivative.

$$\int x^5 \left(a+\frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 3, 249 leaves, 8 steps):

$$-\frac{b c^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d^3}-\frac{\left(5 b^2+60 a b c-24 a^2 c^2\right) \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{48 a d^3}-$$

$$\frac{(b+12 a c) \left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 d^3}+\frac{\left(c+d x^2\right)^3 \left(\frac{b+a c+a d x^2}{c+d x^2}\right)^{5/2}}{6 a d^3}-$$

$$\frac{b \left(b^2+12 a b c-24 a^2 c^2\right) \text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{3/2} d^3}$$

Result (type 3, 311 leaves, 10 steps):

$$-\frac{\left(b^2+12\,a\,b\,c-24\,a^2\,c^2\right)\left(c+d\,x^2\right)\sqrt{a+\frac{b}{c+d\,x^2}}}{16\,a\,d^3}-\frac{\left(b^2+12\,a\,b\,c-24\,a^2\,c^2\right)\left(c+d\,x^2\right)\sqrt{a+\frac{b}{c+d\,x^2}}\left(b+a\left(c+d\,x^2\right)\right)}{24\,a\,b\,d^3}-\frac{c^2\sqrt{a+\frac{b}{c+d\,x^2}}\left(b+a\left(c+d\,x^2\right)\right)^2}{b\,d^3}+\frac{\left(c+d\,x^2\right)\sqrt{a+\frac{b}{c+d\,x^2}}\left(b+a\left(c+d\,x^2\right)\right)^2}{6\,a\,d^3}-\left(b\left(b^2+12\,a\,b\,c-24\,a^2\,c^2\right)\sqrt{c+d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\left(c+d\,x^2\right)}}\right]\right)\Big/\left(16\,a^{3/2}\,d^3\sqrt{b+a\left(c+d\,x^2\right)}\right)$$

Problem 332: Result valid but suboptimal antiderivative.

$$\int x^3\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\frac{b\,c\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{d^2}+\frac{\left(5\,b-4\,a\,c\right)\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,d^2}+\frac{a\left(c+d\,x^2\right)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,d^2}+\frac{3\,b\left(b-4\,a\,c\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{a}}\right]}{8\,\sqrt{a}\,d^2}$$

Result (type 3, 222 leaves, 9 steps):

$$\frac{3\left(b-4\,a\,c\right)\left(c+d\,x^2\right)\sqrt{a+\frac{b}{c+d\,x^2}}}{8\,d^2}+\frac{\left(b-4\,a\,c\right)\left(c+d\,x^2\right)\sqrt{a+\frac{b}{c+d\,x^2}}\left(b+a\left(c+d\,x^2\right)\right)}{4\,b\,d^2}+\frac{c\sqrt{a+\frac{b}{c+d\,x^2}}\left(b+a\left(c+d\,x^2\right)\right)^2}{b\,d^2}+\frac{3\,b\left(b-4\,a\,c\right)\sqrt{c+d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\left(c+d\,x^2\right)}}\right]}{8\,\sqrt{a}\,d^2\sqrt{b+a\left(c+d\,x^2\right)}}$$

Problem 334: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x}\,dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$\frac{b\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c}+a^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{a}}\right]-\frac{(b+a\,c)^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{c^{3/2}}$$

Result (type 3, 206 leaves, 10 steps):

$$\frac{b\sqrt{a+\frac{b}{c+d\,x^2}}}{c}+\frac{a^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\,(c+d\,x^2)}}\right]}{\sqrt{b+a\,(c+d\,x^2)}}-$$

$$\frac{(b+a\,c)^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{c^{3/2}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^3}\,dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{3\,b\,d\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{2\,c^2}-\frac{(c+d\,x^2)\left(\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}\right)^{3/2}}{2\,c\,x^2}+\frac{3\,b\,\sqrt{b+a\,c}\,d\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{2\,c^{5/2}}$$

Result (type 3, 170 leaves, 7 steps):

$$-\frac{3\,b\,d\sqrt{a+\frac{b}{c+d\,x^2}}}{2\,c^2}-\frac{\sqrt{a+\frac{b}{c+d\,x^2}}\,\left(b+a\,(c+d\,x^2)\right)}{2\,c\,x^2}+$$

$$\frac{3\,b\,\sqrt{b+a\,c}\,d\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{2\,c^{5/2}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 336: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^5}\,dx$$

Optimal (type 3, 205 leaves, 7 steps):

$$\frac{b\,d^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^3}+\frac{(9\,b+4\,a\,c)\,d\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{8\,c^3\,x^2}-\frac{(b+a\,c)\,(c+d\,x^2)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{4\,c^3\,x^4}-\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{8\,c^{7/2}\,\sqrt{b+a\,c}}$$

Result (type 3, 260 leaves, 8 steps):

$$\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\sqrt{a+\frac{b}{c+d\,x^2}}}{8\,c^3\,(b+a\,c)}+\frac{(5\,b+4\,a\,c)\,d\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\left(b+a\,(c+d\,x^2)\right)}{8\,c^2\,(b+a\,c)\,x^2}-\frac{\sqrt{a+\frac{b}{c+d\,x^2}}\,\left(b+a\,(c+d\,x^2)\right)^2}{4\,c\,(b+a\,c)\,x^4}-\frac{3\,b\,(5\,b+4\,a\,c)\,d^2\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{8\,c^{7/2}\,\sqrt{b+a\,c}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 337: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^7}\,dx$$

Optimal (type 3, 292 leaves, 8 steps):

$$-\frac{b\,d^3\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^4}-\frac{(79\,b^2+108\,a\,b\,c+24\,a^2\,c^2)\,d^2\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{48\,c^4\,(b+a\,c)\,x^2}+\frac{(11\,b+12\,a\,c)\,d\,(c+d\,x^2)^2\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{24\,c^4\,x^4}-\frac{(c+d\,x^2)^3\left(\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}\right)^{5/2}}{6\,c^2\,(b+a\,c)\,x^6}+\frac{b\,(35\,b^2+60\,a\,b\,c+24\,a^2\,c^2)\,d^3\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{16\,c^{9/2}\,(b+a\,c)^{3/2}}$$

Result (type 3, 287 leaves, 10 steps):

$$-\frac{\left(105 b^2+110 a b c+8 a^2 c^2\right) d^3 \sqrt{a+\frac{b}{c+d x^2}}}{48 c^4 (b+a c)}-$$

$$\frac{(b+a c) \sqrt{a+\frac{b}{c+d x^2}}}{6 c x^6}+\frac{7 b d \sqrt{a+\frac{b}{c+d x^2}}}{24 c^2 x^4}-\frac{b (35 b+32 a c) d^2 \sqrt{a+\frac{b}{c+d x^2}}}{48 c^3 (b+a c) x^2}+$$

$$\left(b \left(35 b^2+60 a b c+24 a^2 c^2\right) d^3 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]\right) \Big/ \left(16 c^{9/2} (b+a c)^{3/2} \sqrt{b+a \left(c+d x^2\right)}\right)$$

Problem 338: Result valid but suboptimal antiderivative.

$$\int x^4 \left(a+\frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$\frac{\left(b^2-14 a b c+a^2 c^2\right) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 a d^2}+\frac{(7 b-a c) x \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d^2}+$$

$$\frac{6 a x^3 \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d}-\frac{x^3 \left(b+a c+a d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d}-$$

$$\left(\sqrt{c} \left(b^2-14 a b c+a^2 c^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(5 a d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}\right)-\frac{c^{3/2} (7 b-a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 d^{5/2} \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}}}$$

Result (type 4, 526 leaves, 9 steps):

$$\frac{\left(b^2-14\,a\,b\,c+a^2\,c^2\right)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,a\,d^2\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+$$

$$\frac{\left(7\,b-a\,c\right)\,x\,\left(c+d\,x^2\right)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,d^2\,\sqrt{b+a\,\left(c+d\,x^2\right)}}+$$

$$\frac{6\,a\,x^3\,\left(c+d\,x^2\right)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{5\,d\,\sqrt{b+a\,\left(c+d\,x^2\right)}}-\frac{x^3\,\left(b+a\,c+a\,d\,x^2\right)^{3/2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{d\,\sqrt{b+a\,\left(c+d\,x^2\right)}}-$$

$$\left(\sqrt{c}\,\left(b^2-14\,a\,b\,c+a^2\,c^2\right)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(5\,a\,d^{5/2}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\right)-$$

$$\left(c^{3/2}\,\left(7\,b-a\,c\right)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(5\,d^{5/2}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{\left(b+a\,c\right)\,\left(c+d\,x^2\right)}}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\right)$$

Problem 339: Result valid but suboptimal antiderivative.

$$\int x^2\,\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}\,dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\frac{(7b-ac)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x\left(b+ac+adx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}-$$

$$\frac{\sqrt{c}\,(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3d^{3/2}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}}+$$

$$\frac{\sqrt{c}\,(3b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{3d^{3/2}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}}$$

Result (type 4, 430 leaves, 8 steps):

$$\frac{(7b-ac)\,x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+$$

$$\frac{4ax\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x\left(b+ac+adx^2\right)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}-$$

$$\left(\sqrt{c}\,(7b-ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]\right)\Big/$$

$$\left(3d^{3/2}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}\right)+$$

$$\left(\sqrt{c}\,(3b-ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]\right)\Big/$$

$$\left(3d^{3/2}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}\sqrt{b+a\left(c+dx^2\right)}\right)$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\frac{b\,x\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c}-\frac{(b-a\,c)\,x\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c}+\frac{(b-a\,c)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}+$$

$$\frac{a\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{d}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}$$

Result (type 4, 348 leaves, 7 steps):

$$\frac{b\,x\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c\sqrt{b+a\,(c+d\,x^2)}}-\frac{(b-a\,c)\,x\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}{c\sqrt{b+a\,(c+d\,x^2)}}+$$

$$\left((b-a\,c)\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\;\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{b+a\,(c+d\,x^2)}\right)+$$

$$\frac{a\sqrt{c}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}\;\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{d}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{b+a\,(c+d\,x^2)}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^2}\,dx$$

Optimal (type 4, 312 leaves, 8 steps):

$$\frac{b\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c\,x}+\frac{(2\,b+a\,c)\,d\,x\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^2}-\frac{(2\,b+a\,c)\left(c+d\,x^2\right)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c^2\,x}-$$

$$\frac{(2\,b+a\,c)\sqrt{d}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{c^{3/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}}+$$

$$\frac{a\sqrt{d}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}}$$

Result (type 4, 422 leaves, 8 steps):

$$\frac{b\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{c\,x\sqrt{b+a\left(c+d\,x^2\right)}}+\frac{(2\,b+a\,c)\,d\,x\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{c^2\sqrt{b+a\left(c+d\,x^2\right)}}-$$

$$\frac{(2\,b+a\,c)\left(c+d\,x^2\right)\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{c^2\,x\sqrt{b+a\left(c+d\,x^2\right)}}-$$

$$\left((2\,b+a\,c)\sqrt{d}\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(c^{3/2}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}\sqrt{b+a\left(c+d\,x^2\right)}\right)+$$

$$\frac{a\sqrt{d}\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{\sqrt{c}\sqrt{\frac{c\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\left(c+d\,x^2\right)}}\sqrt{b+a\left(c+d\,x^2\right)}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 388 leaves, 9 steps):

$$\frac{b\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{c\,x^3}-\frac{(8\,b+a\,c)\,d^2\,x\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{3\,c^3}-\frac{(4\,b+a\,c)\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{3\,c^2\,x^3}+\frac{(8\,b+a\,c)\,d\,(c+d\,x^2)\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{3\,c^3\,x}+\frac{(8\,b+a\,c)\,d^{3/2}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,c^{5/2}\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}-\frac{a\,(4\,b+a\,c)\,d^{3/2}\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{3\,c^{3/2}\,(b+a\,c)\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}$$

Result (type 4, 520 leaves, 9 steps):

$$\frac{b\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{c\,x^3\sqrt{b+a\,(c+d\,x^2)}}-\frac{(8\,b+a\,c)\,d^2\,x\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{3\,c^3\sqrt{b+a\,(c+d\,x^2)}}-\frac{(4\,b+a\,c)\,(c+d\,x^2)\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{3\,c^2\,x^3\sqrt{b+a\,(c+d\,x^2)}}+\frac{(8\,b+a\,c)\,d\,(c+d\,x^2)\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}}{3\,c^3\,x\sqrt{b+a\,(c+d\,x^2)}}+\left((8\,b+a\,c)\,d^{3/2}\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/\left(3\,c^{5/2}\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\sqrt{b+a\,(c+d\,x^2)}\right)-\left(a\,(4\,b+a\,c)\,d^{3/2}\sqrt{b+a\,c+a\,d\,x^2}\sqrt{a+\frac{b}{c+d\,x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/\left(3\,c^{3/2}\,(b+a\,c)\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\sqrt{b+a\,(c+d\,x^2)}\right)$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 494 leaves, 10 steps):

$$\frac{b\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^5}+\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c)}-\frac{(6 b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^2 x^5}+\frac{(8 b+a c) d \left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^3 x^3}-\frac{\left(16 b^2+16 a b c+a^2 c^2\right) d^2 \left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c) x}-\left(\left(16 b^2+16 a b c+a^2 c^2\right) d^{5/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Bigg/\left(5 c^{7/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\right)+\frac{a (8 b+a c) d^{5/2}\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{5 c^{5/2} (b+a c)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 648 leaves, 10 steps):

$$\frac{b\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}}{c\,x^5\sqrt{b+a\,(c+d\,x^2)}}+\frac{(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,d^3\,x\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}}{5\,c^4\,(b+a\,c)\sqrt{b+a\,(c+d\,x^2)}}-$$

$$\frac{(6\,b+a\,c)\,(c+d\,x^2)\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}}{5\,c^2\,x^5\sqrt{b+a\,(c+d\,x^2)}}+$$

$$\frac{(8\,b+a\,c)\,d\,(c+d\,x^2)\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}}{5\,c^3\,x^3\sqrt{b+a\,(c+d\,x^2)}}-$$

$$\frac{(16\,b^2+16\,a\,b\,c+a^2\,c^2)\,d^2\,(c+d\,x^2)\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}}{5\,c^4\,(b+a\,c)\,x\sqrt{b+a\,(c+d\,x^2)}}-$$

$$\left((16\,b^2+16\,a\,b\,c+a^2\,c^2)\,d^{5/2}\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(5\,c^{7/2}\,(b+a\,c)\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\ \sqrt{b+a\,(c+d\,x^2)}\right)+$$

$$\left(a\,(8\,b+a\,c)\,d^{5/2}\sqrt{b+a\,c+a\,d\,x^2}\ \sqrt{a+\frac{b}{c+d\,x^2}}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(5\,c^{5/2}\,(b+a\,c)\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\ \sqrt{b+a\,(c+d\,x^2)}\right)$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int\frac{x^5}{\sqrt{a+\frac{b}{c+d\,x^2}}}\,dx$$

Optimal (type 3, 225 leaves, 7 steps):

$$\frac{\left(5 b^2+12 a b c+8 a^2 c^2\right)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{16 a^3 d^3}-\frac{\left(5 b+8 a c\right)\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 a^2 d^3}+$$

$$\frac{x^2\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{6 a d^2}-\frac{b\left(5 b^2+12 a b c+8 a^2 c^2\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{7/2} d^3}$$

Result (type 3, 267 leaves, 9 steps):

$$\frac{\left(5 b^2+12 a b c+8 a^2 c^2\right)\left(b+a\left(c+d x^2\right)\right)}{16 a^3 d^3\sqrt{a+\frac{b}{c+d x^2}}}-\frac{\left(5 b+8 a c\right)\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{24 a^2 d^3\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{x^2\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{6 a d^2\sqrt{a+\frac{b}{c+d x^2}}}-\frac{b\left(5 b^2+12 a b c+8 a^2 c^2\right)\sqrt{b+a\left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+d x^2}}{\sqrt{b+a\left(c+d x^2\right)}}\right]}{16 a^{7/2} d^3\sqrt{c+d x^2}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int\frac{x^3}{\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$-\frac{\left(3 b+4 a c\right)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 a^2 d^2}+\frac{\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 a d^2}+\frac{b\left(3 b+4 a c\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 a^{5/2} d^2}$$

Result (type 3, 189 leaves, 8 steps):

$$-\frac{\left(3 b+4 a c\right)\left(b+a\left(c+d x^2\right)\right)}{8 a^2 d^2\sqrt{a+\frac{b}{c+d x^2}}}+\frac{\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{4 a d^2\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{b\left(3 b+4 a c\right)\sqrt{b+a\left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+d x^2}}{\sqrt{b+a\left(c+d x^2\right)}}\right]}{8 a^{5/2} d^2\sqrt{c+d x^2}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \sqrt{a+\frac{b}{c+d x^2}}} \, dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}-\frac{\sqrt{c}\ \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{\sqrt{b+a c}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{b+a \left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a \left(c+d x^2\right)}}\right]}{\sqrt{a} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}-\frac{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{\sqrt{b+a c} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 \sqrt{a+\frac{b}{c+d x^2}}} \, dx$$

Optimal (type 3, 108 leaves, 5 steps):

$$-\frac{\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{2 \left(b+a c\right) x^2}-\frac{b\, d\ \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{2 \sqrt{c} \left(b+a c\right)^{3/2}}$$

Result (type 3, 148 leaves, 6 steps):

$$-\frac{b+a \left(c+d x^2\right)}{2 \left(b+a c\right) x^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{b\, d \sqrt{b+a \left(c+d x^2\right)}\ \text{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{2 \sqrt{c} \left(b+a c\right)^{3/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \sqrt{a+\frac{b}{c+d x^2}}} \, dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{(b+4ac)\,d\,(c+d x^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c\,(b+ac)^2x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c\,(b+ac)\,x^4}+\frac{b\,(b+4ac)\,d^2\,\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{8c^{3/2}\,(b+ac)^{5/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(b+4ac)\,d\,(b+a(c+dx^2))}{8c\,(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(c+dx^2)\,(b+a(c+dx^2))}{4c\,(b+ac)\,x^4\sqrt{a+\frac{b}{c+dx^2}}}+$$

$$\frac{b\,(b+4ac)\,d^2\sqrt{b+a(c+dx^2)}\;\text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{8c^{3/2}\,(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

Optimal (type 4, 443 leaves, 8 steps):

$$-\frac{(4b+3ac)\,x\,(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{x^3\,(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(8b^2+13abc+3a^2c^2)\,x\,(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-$$

$$\left(\sqrt{c}\,(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]\right)\Big/$$

$$\left(15a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}\right)+$$

$$\frac{c^{3/2}\,(4b+3ac)\,(b+ac+adx^2)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+ac}\right]}{15a^2d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

Result (type 4, 498 leaves, 8 steps):

$$-\frac{(4 b+3 a c)\, x \sqrt{b+a c+a d x^2}\, \sqrt{b+a\left(c+d x^2\right)}}{15 a^2 d^2 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{x^3 \sqrt{b+a c+a d x^2}\, \sqrt{b+a\left(c+d x^2\right)}}{5 a d \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\left(8 b^2+13 a b c+3 a^2 c^2\right) x \sqrt{b+a c+a d x^2}\, \sqrt{b+a\left(c+d x^2\right)}}{15 a^3 d^2\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\left(\sqrt{c}\left(8 b^2+13 a b c+3 a^2 c^2\right) \sqrt{b+a c+a d x^2}\right.$$

$$\left.\sqrt{b+a\left(c+d x^2\right)}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(15 a^3 d^{5/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\, \sqrt{a+\frac{b}{c+d x^2}}\right)+$$

$$\left(c^{3/2}(4 b+3 a c) \sqrt{b+a c+a d x^2}\, \sqrt{b+a\left(c+d x^2\right)}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(15 a^2 d^{5/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\, \sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+d x^2}}}\, dx$$

Optimal (type 4, 354 leaves, 7 steps):

$$\frac{x\left(b+a c+a d x^2\right)}{3 a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(2 b+a c)\, x\left(b+a c+a d x^2\right)}{3 a^2 d\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+$$

$$\frac{\sqrt{c}\,(2 b+a c)\left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a^2 d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-$$

$$\frac{c^{3/2}\left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{3 a d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\, \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 398 leaves, 7 steps):

$$\frac{x\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3 a d\sqrt{a+\frac{b}{c+d x^2}}}-\frac{(2 b+a c)\,x\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3 a^2 d\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\left(\sqrt{c}\,(2 b+a c)\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(3 a^2 d^{3/2}\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}\right)-$$

$$\left(c^{3/2}\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(3 a d^{3/2}\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 4, 286 leaves, 6 steps):

$$\frac{x\left(b+a c+a d x^2\right)}{a\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{\sqrt{c}\left(b+a c+a d x^2\right)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{a\sqrt{d}\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+$$

$$\frac{c^{3/2}\left(b+a c+a d x^2\right)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\sqrt{d}\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 319 leaves, 6 steps):

$$\frac{x\sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}}{a\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\left(\sqrt{c}\ \sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}\ \texttt{EllipticE}\left[\texttt{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(a\sqrt{d}\ \left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\ \sqrt{a+\frac{b}{c+d x^2}}\right)+$$

$$\left(c^{3/2}\sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}\ \texttt{EllipticF}\left[\texttt{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left((b+a c)\sqrt{d}\ \left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\ \sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 353: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^2\sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 4, 343 leaves, 8 steps):

$$-\frac{b+a c+a d x^2}{(b+a c)\,x\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{d x\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{\sqrt{c}\ \sqrt{d}\ \left(b+a c+a d x^2\right)\texttt{EllipticE}\left[\texttt{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}+$$

$$\frac{\sqrt{c}\ \sqrt{d}\ \left(b+a c+a d x^2\right)\texttt{EllipticF}\left[\texttt{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{(b+a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 387 leaves, 8 steps):

$$-\frac{\sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}}{(b+a c)\ x\ \sqrt{a+\frac{b}{c+d x^2}}}+\frac{d\ x\ \sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}}{(b+a c)\ \left(c+d x^2\right)\ \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\left(\sqrt{c}\ \sqrt{d}\ \sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a c}\right]\right)\Big/$$

$$\left((b+a c)\ \left(c+d x^2\right)\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\ \left(c+d x^2\right)}}\ \sqrt{a+\frac{b}{c+d x^2}}\right)+$$

$$\left(\sqrt{c}\ \sqrt{d}\ \sqrt{b+a c+a d x^2}\ \sqrt{b+a\left(c+d x^2\right)}\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a c}\right]\right)\Big/$$

$$\left((b+a c)\ \left(c+d x^2\right)\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\ \left(c+d x^2\right)}}\ \sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 354: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^4\ \sqrt{a+\frac{b}{c+d x^2}}}\,dx$$

Optimal (type 4, 431 leaves, 8 steps):

$$-\frac{b+a c+a d x^2}{3\ (b+a c)\ x^3\ \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(b-a c)\ d\ \left(b+a c+a d x^2\right)}{3\ c\ (b+a c)^2\ x\ \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(b-a c)\ d^2\ x\ \left(b+a c+a d x^2\right)}{3\ c\ (b+a c)^2\ \left(c+d x^2\right)\ \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\frac{(b-a c)\ d^{3/2}\ \left(b+a c+a d x^2\right)\ \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a c}\right]}{3\ \sqrt{c}\ (b+a c)^2\ \left(c+d x^2\right)\ \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\ \left(c+d x^2\right)}}}-$$

$$\frac{a\ \sqrt{c}\ d^{3/2}\ \left(b+a c+a d x^2\right)\ \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\ x}{\sqrt{c}}\right],\ \frac{b}{b+a c}\right]}{3\ (b+a c)^2\ \left(c+d x^2\right)\ \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\ \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\ \left(c+d x^2\right)}}}$$

Result (type 4, 486 leaves, 8 steps):

$$-\frac{\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3\left(b+a c\right) x^3\sqrt{a+\frac{b}{c+d x^2}}}-\frac{\left(b-a c\right) d\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3 c\left(b+a c\right)^2 x\sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\left(b-a c\right) d^2 x\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}}{3 c\left(b+a c\right)^2\left(c+d x^2\right)\sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\left(\left(b-a c\right) d^{3/2}\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(3\sqrt{c}\left(b+a c\right)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{\left(b+a c\right)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}\right)-$$

$$\left(a\sqrt{c}\,d^{3/2}\sqrt{b+a c+a d x^2}\,\sqrt{b+a\left(c+d x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$

$$\left(3\left(b+a c\right)^2\left(c+d x^2\right)\sqrt{\frac{c\left(b+a c+a d x^2\right)}{\left(b+a c\right)\left(c+d x^2\right)}}\,\sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 355: Result valid but suboptimal antiderivative.

$$\int\frac{x^5}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 310 leaves, 8 steps):

$$-\frac{\left(b+a c\right)^2\left(c+d x^2\right)^3}{a b^2 d^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{16 a^4 d^3}-$$

$$\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 a^3 b d^3}+$$

$$\frac{\left(7 b^2+12 a b c+6 a^2 c^2\right)\left(c+d x^2\right)^3\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{6 a^2 b^2 d^3}-\frac{b\left(35 b^2+60 a b c+24 a^2 c^2\right)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{9/2} d^3}$$

Result (type 3, 323 leaves, 10 steps):

$$\frac{(b+a c)^2\left(c+d x^2\right)^2}{a^2 b d^3 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(b+a\left(c+d x^2\right)\right)}{16 a^4 d^3 \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{\left(35 b^2+60 a b c+24 a^2 c^2\right)\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{24 a^3 b d^3 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{\left(c+d x^2\right)^2\left(b+a\left(c+d x^2\right)\right)}{6 a^2 d^3 \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{b\left(35 b^2+60 a b c+24 a^2 c^2\right)\sqrt{b+a\left(c+d x^2\right)}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+d x^2}}{\sqrt{b+a\left(c+d x^2\right)}}\right]}{16 a^{9/2} d^3 \sqrt{c+d x^2}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{b(b+a c)}{a^3 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(7 b+4 a c)\left(c+d x^2\right)\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 a^3 d^2}+$$

$$\frac{\left(c+d x^2\right)^2\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 a^2 d^2}+\frac{3 b(5 b+4 a c)\,\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 a^{7/2} d^2}$$

Result (type 3, 242 leaves, 9 steps):

$$-\frac{(b+a c)\left(c+d x^2\right)^2}{a b d^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{3(5 b+4 a c)\left(b+a\left(c+d x^2\right)\right)}{8 a^3 d^2 \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{(5 b+4 a c)\left(c+d x^2\right)\left(b+a\left(c+d x^2\right)\right)}{4 a^2 b d^2 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{3 b(5 b+4 a c)\sqrt{b+a\left(c+d x^2\right)}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+d x^2}}{\sqrt{b+a\left(c+d x^2\right)}}\right]}{8 a^{7/2} d^2 \sqrt{c+d x^2}\sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{b}{a\,(b+a\,c)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{a}}\right]}{a^{3/2}}-\frac{c^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{(b+a\,c)^{3/2}}$$

Result (type 3, 214 leaves, 10 steps):

$$-\frac{b}{a\,(b+a\,c)\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{\sqrt{b+a\,(c+d\,x^2)}\,\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{c+d\,x^2}}{\sqrt{b+a\,(c+d\,x^2)}}\right]}{a^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\frac{c^{3/2}\,\sqrt{b+a\,(c+d\,x^2)}\,\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{(b+a\,c)^{3/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 3, 146 leaves, 6 steps):

$$\frac{3\,b\,d}{2\,(b+a\,c)^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{c+d\,x^2}{2\,(b+a\,c)\,x^2\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{3\,b\,\sqrt{c}\,\,d\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}{\sqrt{b+a\,c}}\right]}{2\,(b+a\,c)^{5/2}}$$

Result (type 3, 174 leaves, 7 steps):

$$\frac{3\,b\,d}{2\,(b+a\,c)^2\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{c+d\,x^2}{2\,(b+a\,c)\,x^2\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\frac{3\,b\,\sqrt{c}\,\,d\,\sqrt{b+a\,(c+d\,x^2)}\,\,\text{ArcTanh}\left[\frac{\sqrt{b+a\,c}\,\sqrt{c+d\,x^2}}{\sqrt{c}\,\sqrt{b+a\,(c+d\,x^2)}}\right]}{2\,(b+a\,c)^{5/2}\,\sqrt{c+d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+d x^2}\right)^{3/2}} \, dx$$

Optimal (type 3, 212 leaves, 7 steps):

$$-\frac{a b d^2}{(b+a c)^3 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(3 b-4 a c) d \left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 (b+a c)^3 x^2}-\frac{\left(c+d x^2\right)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 (b+a c)^2 x^4}-\frac{3 b (b-4 a c) d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{8 \sqrt{c} (b+a c)^{7/2}}$$

Result (type 3, 246 leaves, 8 steps):

$$\frac{3 b (b-4 a c) d^2}{8 c (b+a c)^3 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{(b-4 a c) d \left(c+d x^2\right)}{8 c (b+a c)^2 x^2 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{\left(c+d x^2\right)^2}{4 c (b+a c) x^4 \sqrt{a+\frac{b}{c+d x^2}}}-\frac{3 b (b-4 a c) d^2 \sqrt{b+a \left(c+d x^2\right)} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a \left(c+d x^2\right)}}\right]}{8 \sqrt{c} (b+a c)^{7/2} \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 361: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}} \, dx$$

Optimal (type 4, 482 leaves, 9 steps):

$$-\frac{x^3\left(c+d x^2\right)}{a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(8 b+a c)\, x\left(b+a c+a d x^2\right)}{5 a^3 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+$$

$$\frac{6 x^3\left(b+a c+a d x^2\right)}{5 a^2 d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{\left(16 b^2+16 a b c+a^2 c^2\right) x\left(b+a c+a d x^2\right)}{5 a^4 d^2\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-$$

$$\left(\sqrt{c}\left(16 b^2+16 a b c+a^2 c^2\right)\left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(5 a^4 d^{5/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}\right)+$$

$$\frac{c^{3/2}(8 b+a c)\left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\, x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a^3 d^{5/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 559 leaves, 9 steps) :

$$-\frac{x^3 \left(c+d x^2\right) \sqrt{b+a \left(c+d x^2\right)}}{a d \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{(8 b+a c) x \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)}}{5 a^3 d^2 \sqrt{a+\frac{b}{c+d x^2}}}+\frac{6 x^3 \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)}}{5 a^2 d \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\frac{\left(16 b^2+16 a b c+a^2 c^2\right) x \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)}}{5 a^4 d^2 \left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\left(\sqrt{c} \left(16 b^2+16 a b c+a^2 c^2\right) \sqrt{b+a c+a d x^2}\right.$$

$$\left.\sqrt{b+a \left(c+d x^2\right)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(5 a^4 d^{5/2} \left(c+d x^2\right) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}\right)+$$

$$\left(c^{3/2} (8 b+a c) \sqrt{b+a c+a d x^2} \sqrt{b+a \left(c+d x^2\right)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]\right) \Big/$$

$$\left(5 a^3 d^{5/2} \left(c+d x^2\right) \sqrt{\frac{c \left(b+a c+a d x^2\right)}{(b+a c) \left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 362: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}} \, dx$$

Optimal (type 4, 409 leaves, 8 steps):

$$-\frac{x\left(c+d x^2\right)}{a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{4 x\left(b+a c+a d x^2\right)}{3 a^2 d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}-\frac{(8 b+a c) x\left(b+a c+a d x^2\right)}{3 a^3 d\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+$$

$$\frac{\sqrt{c}\,(8 b+a c)\left(b+a c+a d x^2\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a^3 d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}-$$

$$\frac{c^{3/2}(4 b+a c)\left(b+a c+a d x^2\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]}{3 a^2 (b+a c) d^{3/2}\left(c+d x^2\right) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}}}$$

Result (type 4, 475 leaves, 8 steps):

$$-\frac{x\left(c+d x^2\right) \sqrt{b+a\left(c+d x^2\right)}}{a d \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}+\frac{4 x \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}}{3 a^2 d \sqrt{a+\frac{b}{c+d x^2}}}-$$

$$\frac{(8 b+a c) x \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)}}{3 a^3 d\left(c+d x^2\right) \sqrt{a+\frac{b}{c+d x^2}}}+$$

$$\left(\sqrt{c}\,(8 b+a c) \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$
$$\left(3 a^3 d^{3/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}\right)-$$

$$\left(c^{3/2}(4 b+a c) \sqrt{b+a c+a d x^2} \sqrt{b+a\left(c+d x^2\right)} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a c}\right]\right)\Big/$$
$$\left(3 a^2 (b+a c) d^{3/2}\left(c+d x^2\right) \sqrt{\frac{c\left(b+a c+a d x^2\right)}{(b+a c)\left(c+d x^2\right)}} \sqrt{a+\frac{b}{c+d x^2}}\right)$$

Problem 363: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$-\frac{b\,x}{a\,(b+a\,c)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(2\,b+a\,c)\,x\,\left(b+a\,c+a\,d\,x^2\right)}{a^2\,(b+a\,c)\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-$$

$$\frac{\sqrt{c}\,(2\,b+a\,c)\,\left(b+a\,c+a\,d\,x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a^2\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}+$$

$$\frac{c^{3/2}\,\left(b+a\,c+a\,d\,x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}$$

Result (type 4, 411 leaves, 7 steps):

$$-\frac{b\,x\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{a\,(b+a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{(2\,b+a\,c)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{a^2\,(b+a\,c)\,\left(c+d\,x^2\right)\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\left(\sqrt{c}\,(2\,b+a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(a^2\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)+$$

$$\left(c^{3/2}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(a\,(b+a\,c)\,\sqrt{d}\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 410 leaves, 8 steps):

$$-\frac{b}{a\,(b+a\,c)\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(b-a\,c)\,(b+a\,c+a\,d\,x^2)}{a\,(b+a\,c)^2\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-\frac{(b-a\,c)\,d\,x\,(b+a\,c+a\,d\,x^2)}{a\,(b+a\,c)^2\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+$$

$$\frac{\sqrt{c}\,(b-a\,c)\,\sqrt{d}\,(b+a\,c+a\,d\,x^2)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{a\,(b+a\,c)^2\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}+$$

$$\frac{c^{3/2}\,\sqrt{d}\,(b+a\,c+a\,d\,x^2)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]}{(b+a\,c)^2\,(c+d\,x^2)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}}$$

Result (type 4, 476 leaves, 8 steps):

$$-\frac{b\,\sqrt{b+a\,(c+d\,x^2)}}{a\,(b+a\,c)\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+$$

$$\frac{(b-a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}}{a\,(b+a\,c)^2\,x\,\sqrt{a+\frac{b}{c+d\,x^2}}}-\frac{(b-a\,c)\,d\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}}{a\,(b+a\,c)^2\,(c+d\,x^2)\,\sqrt{a+\frac{b}{c+d\,x^2}}}+$$

$$\left(\sqrt{c}\,(b-a\,c)\,\sqrt{d}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left(a\,(b+a\,c)^2\,(c+d\,x^2)\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)+$$

$$\left(c^{3/2}\,\sqrt{d}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,(c+d\,x^2)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\frac{b}{b+a\,c}\right]\right)\Big/$$

$$\left((b+a\,c)^2\,(c+d\,x^2)\,\sqrt{\frac{c\,(b+a\,c+a\,d\,x^2)}{(b+a\,c)\,(c+d\,x^2)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^4\left(a+\frac{b}{c+d\,x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 490 leaves, 9 steps):

$$-\frac{b}{a\,(b+a\,c)\,x^3\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(3\,b-a\,c)\,\left(b+a\,c+a\,d\,x^2\right)}{3\,a\,(b+a\,c)^2\,x^3\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-$$

$$\frac{(7\,b-a\,c)\,d\,\left(b+a\,c+a\,d\,x^2\right)}{3\,(b+a\,c)^3\,x\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}+\frac{(7\,b-a\,c)\,d^2\,x\,\left(b+a\,c+a\,d\,x^2\right)}{3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}}-$$

$$\frac{\sqrt{c}\,(7\,b-a\,c)\,d^{3/2}\,\left(b+a\,c+a\,d\,x^2\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]}{3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}+$$

$$\frac{\sqrt{c}\,(3\,b-a\,c)\,d^{3/2}\,\left(b+a\,c+a\,d\,x^2\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]}{3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{\frac{b+a\,c+a\,d\,x^2}{c+d\,x^2}}\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}}$$

Result (type 4, 567 leaves, 9 steps):

$$-\frac{b\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{a\,(b+a\,c)\,x^3\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{(3\,b-a\,c)\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{3\,a\,(b+a\,c)^2\,x^3\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\frac{(7\,b-a\,c)\,d\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{3\,(b+a\,c)^3\,x\,\sqrt{a+\frac{b}{c+d\,x^2}}}+\frac{(7\,b-a\,c)\,d^2\,x\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}}{3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{a+\frac{b}{c+d\,x^2}}}-$$

$$\left(\sqrt{c}\,(7\,b-a\,c)\,d^{3/2}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]\right)\Bigg/$$

$$\left(3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)+$$

$$\left(\sqrt{c}\,(3\,b-a\,c)\,d^{3/2}\,\sqrt{b+a\,c+a\,d\,x^2}\,\sqrt{b+a\,\left(c+d\,x^2\right)}\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right],\,\frac{b}{b+a\,c}\right]\right)\Bigg/$$

$$\left(3\,(b+a\,c)^3\,\left(c+d\,x^2\right)\,\sqrt{\frac{c\,\left(b+a\,c+a\,d\,x^2\right)}{(b+a\,c)\,\left(c+d\,x^2\right)}}\,\sqrt{a+\frac{b}{c+d\,x^2}}\right)$$

Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{\sqrt{a\,x^{2\,n}}}{\sqrt{1+x^n}}+\frac{2\,x^{-n}\,\sqrt{a\,x^{2\,n}}}{(2+n)\,\sqrt{1+x^n}}\right)\,dx$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2\,x^{1-n}\,\sqrt{a\,x^{2\,n}}\,\sqrt{1+x^n}}{2+n}$$

Result (type 5, 80 leaves, 5 steps):

$$\frac{x\,\sqrt{a\,x^{2\,n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,1+\frac{1}{n},\,2+\frac{1}{n},\,-x^n\right]}{1+n}+\frac{2\,x^{1-n}\,\sqrt{a\,x^{2\,n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1}{n},\,1+\frac{1}{n},\,-x^n\right]}{2+n}$$

Problem 616: Unable to integrate problem.

$$\int\frac{1}{x^2}\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n\left(-a\,d+\left(b\,d\,m+a\,e\,n\right)\,x+\left(c\,d+b\,e+a\,f+2\,c\,d\,m+b\,e\,m+b\,e\,n+2\,a\,f\,n\right)\,x^2+\left(2\,c\,e+2\,b\,f+2\,a\,g+2\,c\,e\,m+b\,f\,m+c\,e\,n+2\,b\,f\,n+3\,a\,g\,n\right)\,x^3+\left(3\,c\,f+3\,b\,g+2\,c\,f\,m+b\,g\,m+2\,c\,f\,n+3\,b\,g\,n\right)\,x^4+c\,g\,\left(4+2\,m+3\,n\right)\,x^5\right)\,dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(a+b\,x+c\,x^2\right)^{1+m}\left(d+e\,x+f\,x^2+g\,x^3\right)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$\left(c\,\left(d+2\,d\,m\right)+b\,e\,\left(1+m+n\right)+a\,f\,\left(1+2\,n\right)\right)\,\text{CannotIntegrate}\left[\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right]-a\,d\,\text{CannotIntegrate}\left[\frac{\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^2},\,x\right]+\left(b\,d\,m+a\,e\,n\right)\,\text{CannotIntegrate}\left[\frac{\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x},\,x\right]+\left(c\,e\,\left(2+2\,m+n\right)+b\,f\,\left(2+m+2\,n\right)+a\,g\,\left(2+3\,n\right)\right)\,\text{CannotIntegrate}\left[x\,\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right]+\left(c\,f\,\left(3+2\,m+2\,n\right)+b\,g\,\left(3+m+3\,n\right)\right)\,\text{CannotIntegrate}\left[x^2\,\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right]+c\,g\,\left(4+2\,m+3\,n\right)\,\text{CannotIntegrate}\left[x^3\,\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right]$$

Problem 617: Unable to integrate problem.

$$\int\frac{1}{x^3}\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n\left(-2\,a\,d+\left(-b\,d-a\,e+b\,d\,m+a\,e\,n\right)\,x+\left(2\,c\,d\,m+b\,e\,m+b\,e\,n+2\,a\,f\,n\right)\,x^2+\left(c\,e+b\,f+a\,g+2\,c\,e\,m+b\,f\,m+c\,e\,n+2\,b\,f\,n+3\,a\,g\,n\right)\,x^3+\left(2\,c\,f+2\,b\,g+2\,c\,f\,m+b\,g\,m+2\,c\,f\,n+3\,b\,g\,n\right)\,x^4+c\,g\,\left(3+2\,m+3\,n\right)\,x^5\right)\,dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(a+b\,x+c\,x^2\right)^{1+m}\left(d+e\,x+f\,x^2+g\,x^3\right)^{1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$(c\,e\,(1+2\,m+n)+b\,f\,(1+m+2\,n)+a\,g\,(1+3\,n))\ \text{CannotIntegrate}\left[\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\ x\right]-2\,a\,d\,\text{CannotIntegrate}\left[\frac{\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^3},\ x\right]-(b\,d\,(1-m)+a\,e\,(1-n))\ \text{CannotIntegrate}\left[\frac{\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^2},\ x\right]+(2\,c\,d\,m+2\,a\,f\,n+b\,e\,(m+n))\ \text{CannotIntegrate}\left[\frac{\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x},\ x\right]+(2\,c\,f\,(1+m+n)+b\,g\,(2+m+3\,n))\ \text{CannotIntegrate}\left[x\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\ x\right]+c\,g\,(3+2\,m+3\,n)\ \text{CannotIntegrate}\left[x^2\left(a+b\,x+c\,x^2\right)^m\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\ x\right]$$

Problem 852: Result valid but suboptimal antiderivative.

$$\int\sqrt{\frac{1}{-1+x^2}}\,dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\sqrt{1-x^2}\ \sqrt{\frac{1}{-1+x^2}}\ \text{ArcSin}[x]$$

Result (type 3, 33 leaves, 3 steps):

$$\sqrt{\frac{1}{-1+x^2}}\ \sqrt{-1+x^2}\ \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 853: Result valid but suboptimal antiderivative.

$$\int\sqrt{\frac{1+x^2}{-1+x^4}}\,dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\sqrt{1-x^2}\ \sqrt{\frac{1}{-1+x^2}}\ \text{ArcSin}[x]$$

Result (type 3, 33 leaves, 4 steps):

$$\sqrt{\frac{1}{-1+x^2}}\ \sqrt{-1+x^2}\ \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 941: Result unnecessarily involves higher level functions.

$$\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right) dx$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{\left(1-x^6\right)^{2/3}}{5 x^5}+\frac{1}{5} x \left(1-x^6\right)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^6\right]}{5 x^5}+x\,\text{Hypergeometric2F1}\left[-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^6\right]$$

Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x \sqrt{-1+x^2}}\, dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} \left(3 x+\sqrt{-1+x^2}\right) \sqrt{1-x^2+x \sqrt{-1+x^2}}+\frac{3 \text{ArcSin}\left[x-\sqrt{-1+x^2}\right]}{4 \sqrt{2}}$$

Result (type 8, 24 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\sqrt{1-x^2+x \sqrt{-1+x^2}},x\right]$$

Problem 996: Unable to integrate problem.

$$\int \frac{\sqrt{-x+\sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}}\, dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left(\sqrt{x}+3 \sqrt{1+x}\right) \sqrt{-x+\sqrt{x} \sqrt{1+x}}-\frac{3 \text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]}{2 \sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}},x\right]$$

Problem 997: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\ \left(x+\sqrt{1+x^2}\right)\right]+\sqrt{2\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\ \left(x+\sqrt{1+x^2}\right)\right]$$

Result (type 3, 319 leaves, 25 steps):

$$-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]$$

Problem 1016: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(3+3x+x^2\right)\left(3+3x+3x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\cdot 3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[1-(1+x)^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}(1+x)-\left(2+(1+x)^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(1+x)}{3^{1/6}\left(2+(1+x)^3\right)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{3^{1/3}(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 1017: Unable to integrate problem.

$$\int \frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-\frac{\text{Log}\left[1+2\,(1-x)^3-x^3\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[2^{1/3}\,(1-x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 103 leaves, 5 steps) :

$$-\left(1+i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{1}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},x\right]-$$
$$\left(1-i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{1}{\left(-1+i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},x\right]-$$
$$x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]$$

Problem 1018: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{-1+x^4}\left(1+x^4\right)}\,dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4}\text{ArcTan}\left[\frac{1+x^2}{x\sqrt{-1+x^4}}\right]-\frac{1}{4}\text{ArcTanh}\left[\frac{1-x^2}{x\sqrt{-1+x^4}}\right]$$

Result (type 3, 47 leaves, 9 steps) :

$$\left(-\frac{1}{8}-\frac{i}{8}\right)\text{ArcTan}\left[\frac{(1+i)\,x}{\sqrt{-1+x^4}}\right]+\left(\frac{1}{8}+\frac{i}{8}\right)\text{ArcTanh}\left[\frac{(1+i)\,x}{\sqrt{-1+x^4}}\right]$$

Problem 1023: Unable to integrate problem.

$$\int \left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n\,dx$$

Optimal (type 3, 34 leaves, ? steps):

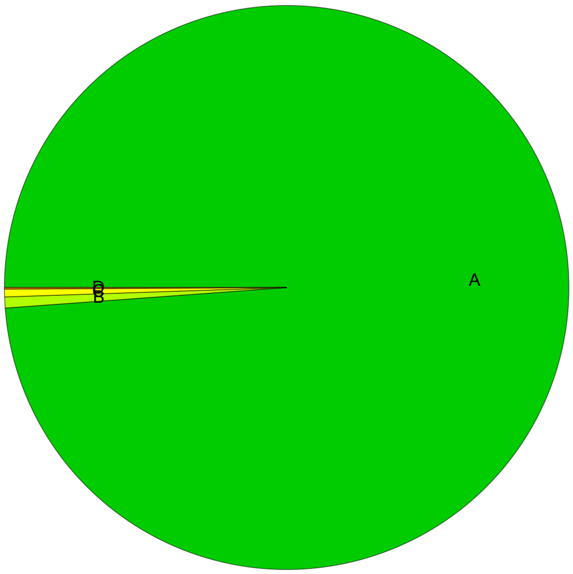
$$-\frac{(1-x)\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n}{1+n}$$

Result (type 8, 25 leaves, 0 steps) :

$$\text{CannotIntegrate}\left[\left(1+x+x^2+x^3\right)^{-n}\left(1-x^4\right)^n,x\right]$$

Summary of Integration Test Results

26125 integration problems



A - 25822 optimal antiderivatives

B - 164 valid but suboptimal antiderivatives

C - 116 unnecessarily complex antiderivatives

D - 23 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives