

# Mathematica 11.3 Integration Test Results

## Test results for the 950 problems in "4.7.7 Trig functions.m"

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\begin{aligned} & -\frac{\text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + b x\right] \sin\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right]}{2\sqrt{-c}\sqrt{d}} + \frac{\text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - b x\right] \sin\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right]}{2\sqrt{-c}\sqrt{d}} \\ & -\frac{\cos\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - b x\right]}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + b x\right]}{2\sqrt{-c}\sqrt{d}} \end{aligned}$$

Result (type 4, 172 leaves):

$$\begin{aligned} & \frac{1}{2\sqrt{c}\sqrt{d}} \\ & i \left( \text{CosIntegral}\left[b \left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] \sin\left[a - \frac{i b \sqrt{c}}{\sqrt{d}}\right] - \text{CosIntegral}\left[b \left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] \sin\left[a + \frac{i b \sqrt{c}}{\sqrt{d}}\right] + \right. \\ & \quad \left. \cos\left[a - \frac{i b \sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[b \left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] + \cos\left[a + \frac{i b \sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{i b \sqrt{c}}{\sqrt{d}} - b x\right] \right) \end{aligned}$$

Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b - \frac{a}{x^2}} \sin[x]}{\sqrt{a - b x^2}} dx$$

Optimal (type 4, 28 leaves, 3 steps):

$$\frac{\sqrt{b - \frac{a}{x^2}} x \text{SinIntegral}[x]}{\sqrt{a - b x^2}}$$

Result (type 4, 46 leaves):

$$\frac{i \sqrt{b - \frac{a}{x^2}} x \left( \text{ExpIntegralEi}[-i x] - \text{ExpIntegralEi}[i x] \right)}{2 \sqrt{a - b x^2}}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x (1 + \sin[\log[x]])} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$-\frac{\cos[\log[x]]}{1 + \sin[\log[x]]}$$

Result (type 3, 26 leaves):

$$\frac{2 \sin\left[\frac{\log[x]}{2}\right]}{\cos\left[\frac{\log[x]}{2}\right] + \sin\left[\frac{\log[x]}{2}\right]}$$

Problem 36: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sin\left[\frac{a + b x}{c + d x}\right] dx$$

Optimal (type 4, 100 leaves, 5 steps):

$$\frac{(b c - a d) \cos\left[\frac{b}{d}\right] \text{CosIntegral}\left[\frac{b c - a d}{d (c + d x)}\right]}{d^2} + \frac{(c + d x) \sin\left[\frac{a + b x}{c + d x}\right]}{d} + \frac{(b c - a d) \sin\left[\frac{b}{d}\right] \text{SinIntegral}\left[\frac{b c - a d}{d (c + d x)}\right]}{d^2}$$

Result (type 4, 918 leaves):

$$\begin{aligned}
& \frac{1}{2d} (b c^2 - a c d) \\
& \left( \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} - \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} \right) - \\
& \frac{1}{2d} (-b c^2 + a c d) \left( \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} - \right. \\
& \left. \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} \right) - \frac{1}{2d} i (b c^2 - a c d) \\
& \left( \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} - \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} \right) - \\
& \frac{1}{2d} i (-b c^2 + a c d) \\
& \left( \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} - \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(b c - a d)} \right) + \\
& x \cos\left[\frac{-b c + a d}{d(c+dx)}\right] \sin\left[\frac{b}{d}\right] + x \cos\left[\frac{b}{d}\right] \sin\left[\frac{-b c + a d}{d(c+dx)}\right] - \frac{1}{d^2} \\
& (-b c + a d) \left( \cos\left[\frac{b}{d}\right] \text{CosIntegral}\left[\frac{-b c + a d}{d(c+dx)}\right] - \sin\left[\frac{b}{d}\right] \text{SinIntegral}\left[\frac{-b c + a d}{d(c+dx)}\right] \right)
\end{aligned}$$

**Problem 37: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sin\left[\frac{a+bx}{c+dx}\right]^2 dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$\begin{aligned}
& \frac{(b c - a d) \text{CosIntegral}\left[\frac{2(b c - a d)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right]}{d^2} + \\
& \frac{(c+dx) \sin\left[\frac{a+bx}{c+dx}\right]^2}{d} - \frac{(b c - a d) \cos\left[\frac{2b}{d}\right] \text{SinIntegral}\left[\frac{2(b c - a d)}{d(c+dx)}\right]}{d^2}
\end{aligned}$$

Result (type 4, 401 leaves):

$$\begin{aligned}
 & -\frac{1}{d}(-b c^2 + a c d) \\
 & \left( \frac{e^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{4ib}{d}}\right) \left(-e^{\frac{4ia}{c+dx}} + e^{\frac{4ibc}{d(c+dx)}}\right)}{8(b c - a d)} - \frac{e^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{4ib}{d}}\right) \left(e^{\frac{4ia}{c+dx}} + e^{\frac{4ibc}{d(c+dx)}}\right)}{8(b c - a d)} \right) - \\
 & \frac{1}{2} x \cos\left[\frac{2b}{d}\right] \cos\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + \frac{1}{2} x \sin\left[\frac{2b}{d}\right] \sin\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + \frac{1}{2d^2} \\
 & \left( d^2 x + 2bc \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right] - 2ad \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right] + \right. \\
 & \left. 2bc \cos\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] - 2ad \cos\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \right)
 \end{aligned}$$

Problem 38: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sin\left[\frac{a+bx}{c+dx}\right]^3 dx$$

Optimal (type 4, 194 leaves, 9 steps):

$$\begin{aligned}
 & \frac{3(b c - a d) \cos\left[\frac{b}{d}\right] \operatorname{CosIntegral}\left[\frac{b c - a d}{d(c+dx)}\right]}{4 d^2} - \\
 & \frac{3(b c - a d) \cos\left[\frac{3b}{d}\right] \operatorname{CosIntegral}\left[\frac{3(b c - a d)}{d(c+dx)}\right]}{4 d^2} + \frac{(c+dx) \sin\left[\frac{a+bx}{c+dx}\right]^3}{d} + \\
 & \frac{3(b c - a d) \sin\left[\frac{b}{d}\right] \operatorname{SinIntegral}\left[\frac{b c - a d}{d(c+dx)}\right]}{4 d^2} - \frac{3(b c - a d) \sin\left[\frac{3b}{d}\right] \operatorname{SinIntegral}\left[\frac{3(b c - a d)}{d(c+dx)}\right]}{4 d^2}
 \end{aligned}$$

Result (type 4, 657 leaves):

$$\begin{aligned}
& -\frac{1}{4d} 3(-bc^2 + acd) \\
& \left( \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(bc-ad)} - \frac{i e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(bc-ad)} \right) + \\
& \frac{1}{4d} 3(-bc^2 + acd) \left( \frac{i e^{-\frac{3i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{6ib}{d}}\right) \left(-e^{\frac{6ia}{c+dx}} + e^{\frac{6ibc}{d(c+dx)}}\right)}{12(bc-ad)} - \right. \\
& \left. \frac{i e^{-\frac{3i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{6ib}{d}}\right) \left(e^{\frac{6ia}{c+dx}} + e^{\frac{6ibc}{d(c+dx)}}\right)}{12(bc-ad)} \right) + \frac{3}{4} x \cos\left[\frac{-bc+ad}{d(c+dx)}\right] \sin\left[\frac{b}{d}\right] - \\
& \frac{1}{4} x \cos\left[\frac{3(-bc+ad)}{d(c+dx)}\right] \sin\left[\frac{3b}{d}\right] + \frac{3}{4} x \cos\left[\frac{b}{d}\right] \sin\left[\frac{-bc+ad}{d(c+dx)}\right] - \\
& \frac{1}{4} x \cos\left[\frac{3b}{d}\right] \sin\left[\frac{3(-bc+ad)}{d(c+dx)}\right] + \frac{1}{4d^2} \\
& 3(-bc+ad) \left( -\cos\left[\frac{b}{d}\right] \text{CosIntegral}\left[\frac{-bc+ad}{d(c+dx)}\right] + \cos\left[\frac{3b}{d}\right] \text{CosIntegral}\left[\frac{3(-bc+ad)}{d(c+dx)}\right] + \right. \\
& \left. \sin\left[\frac{b}{d}\right] \text{SinIntegral}\left[\frac{-bc+ad}{d(c+dx)}\right] - \sin\left[\frac{3b}{d}\right] \text{SinIntegral}\left[\frac{3(-bc+ad)}{d(c+dx)}\right] \right)
\end{aligned}$$

**Problem 46: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[a+bx]}{c+dx^2} dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$\begin{aligned}
& \frac{\cos\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right]}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{CosIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right]}{2\sqrt{-c}\sqrt{d}} + \\
& \frac{\sin\left[a + \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right]}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left[a - \frac{b\sqrt{-c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right]}{2\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 172 leaves):

$$\begin{aligned}
& -\frac{1}{2\sqrt{c}\sqrt{d}} \\
& i \left( \cos\left[a + \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{CosIntegral}\left[b\left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] - \cos\left[a - \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{CosIntegral}\left[b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] + \right. \\
& \quad \left. \sin\left[a - \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right] + \sin\left[a + \frac{ib\sqrt{c}}{\sqrt{d}}\right] \text{SinIntegral}\left[\frac{ib\sqrt{c}}{\sqrt{d}} - bx\right] \right)
\end{aligned}$$

**Problem 51: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos\left[\frac{a+bx}{c+dx}\right] dx$$

Optimal (type 4, 101 leaves, 5 steps):

$$\begin{aligned}
& \frac{(c+dx) \cos\left[\frac{a+bx}{c+dx}\right]}{d} - \frac{(bc-ad) \text{CosIntegral}\left[\frac{bc-ad}{d(c+dx)}\right] \sin\left[\frac{b}{d}\right]}{d^2} + \\
& \frac{(bc-ad) \cos\left[\frac{b}{d}\right] \text{SinIntegral}\left[\frac{bc-ad}{d(c+dx)}\right]}{d^2}
\end{aligned}$$

Result (type 4, 317 leaves):

$$\begin{aligned}
& \frac{1}{d} (-bc^2 + acd) \\
& \left( \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{2ib}{d}}\right) \left(-e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(bc-ad)} - \frac{e^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{2ib}{d}}\right) \left(e^{\frac{2ia}{c+dx}} + e^{\frac{2ibc}{d(c+dx)}}\right)}{4(bc-ad)} \right) + \\
& x \cos\left[\frac{b}{d}\right] \cos\left[\frac{-bc+ad}{d(c+dx)}\right] - x \sin\left[\frac{b}{d}\right] \sin\left[\frac{-bc+ad}{d(c+dx)}\right] + \frac{1}{d^2} \\
& (-bc+ad) \left( \text{CosIntegral}\left[\frac{-bc+ad}{d(c+dx)}\right] \sin\left[\frac{b}{d}\right] + \cos\left[\frac{b}{d}\right] \text{SinIntegral}\left[\frac{-bc+ad}{d(c+dx)}\right] \right)
\end{aligned}$$

**Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos\left[\frac{a+bx}{c+dx}\right]^2 dx$$

Optimal (type 4, 107 leaves, 6 steps):

$$\frac{(c+dx) \cos\left[\frac{a+bx}{c+dx}\right]^2}{d} - \frac{(bc-ad) \operatorname{CosIntegral}\left[\frac{2(bc-ad)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right]}{d^2} +$$

$$\frac{(bc-ad) \cos\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(bc-ad)}{d(c+dx)}\right]}{d^2}$$

Result (type 4, 400 leaves):

$$\frac{1}{d}(-bc^2 + acd)$$

$$\left( \frac{e^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}} \left(-1 + e^{\frac{4ib}{d}}\right) \left(-e^{\frac{4ia}{c+dx}} + e^{\frac{4ibc}{d(c+dx)}}\right)}{8(bc-ad)} - \frac{e^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}} \left(1 + e^{\frac{4ib}{d}}\right) \left(e^{\frac{4ia}{c+dx}} + e^{\frac{4ibc}{d(c+dx)}}\right)}{8(bc-ad)} \right) +$$

$$\frac{1}{2} x \cos\left[\frac{2b}{d}\right] \cos\left[\frac{2(-bc+ad)}{d(c+dx)}\right] - \frac{1}{2} x \sin\left[\frac{2b}{d}\right] \sin\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + \frac{1}{2d^2}$$

$$\left( d^2 x - 2bc \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right] + 2ad \operatorname{CosIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \sin\left[\frac{2b}{d}\right] - \right.$$

$$\left. 2bc \cos\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] + 2ad \cos\left[\frac{2b}{d}\right] \operatorname{SinIntegral}\left[\frac{2(-bc+ad)}{d(c+dx)}\right] \right)$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \operatorname{Sec}[c+dx]}}{1+\cos[c+dx]} dx$$

Optimal (type 4, 92 leaves, 2 steps):

$$\frac{\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Tan}[c+dx]}{1+\operatorname{Sec}[c+dx]}\right], \frac{a-b}{a+b}\right] \sqrt{\frac{1}{1+\operatorname{Sec}[c+dx]}} \sqrt{a+b \operatorname{Sec}[c+dx]}}{d \sqrt{\frac{a+b \operatorname{Sec}[c+dx]}{(a+b)(1+\operatorname{Sec}[c+dx])}}}$$

Result (type 4, 1979 leaves):

$$\left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sqrt{a+b \operatorname{Sec}[c+dx]} \left( -2 \sin[c+dx] + 2 \tan\left[\frac{1}{2}(c+dx)\right] \right) \right) / (d(1+\cos[c+dx])) +$$

$$\left( \cos\left[\frac{c}{2} + \frac{dx}{2}\right]^2 \sec\left[\frac{1}{2}(c+dx)\right]^5 \right.$$

$$\left( \frac{b}{\sqrt{b+a \cos[c+dx]} \sqrt{\operatorname{Sec}[c+dx]}} + \frac{a \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{b+a \cos[c+dx]}} + \frac{b \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{b+a \cos[c+dx]}} + \right.$$

$$\left. \frac{a \cos[2(c+dx)] \sqrt{\operatorname{Sec}[c+dx]}}{\sqrt{b+a \cos[c+dx]}} \right) \sqrt{1+\operatorname{Sec}[c+dx]} \sqrt{a+b \operatorname{Sec}[c+dx]}$$

$$\begin{aligned}
& \left( 2 \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad \left. \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \left( -\sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right) / \\
& \left( 4 d \sqrt{\frac{1}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \sqrt{\sec [c + d x]} \right. \\
& \quad \left. - \left( \left( a \sec \left[ \frac{1}{2} (c + d x) \right] \right)^5 \sqrt{1 + \sec [c + d x]} \sin [c + d x] \right. \right. \\
& \quad \left. \left( 2 \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \right. \\
& \quad \left. \left. \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \left( -\sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right) \right) / \\
& \quad \left( 8 \left( \frac{1}{1 + \cos [c + d x]} \right)^{3/2} \sqrt{b + a \cos [c + d x]} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \right) - \\
& \quad \left( 3 \sqrt{b + a \cos [c + d x]} \sec \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{1 + \sec [c + d x]} \sin [c + d x] \right. \\
& \quad \left( 2 \cos \left[ \frac{1}{2} (c + d x) \right] \sqrt{\frac{\cos [c + d x]}{1 + \cos [c + d x]}} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \tan \left[ \frac{1}{2} (c + d x) \right] \right], \frac{a - b}{a + b} \right] + \right. \\
& \quad \left. \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \left( -\sin \left[ \frac{1}{2} (c + d x) \right] + \sin \left[ \frac{3}{2} (c + d x) \right] \right) \right) / \\
& \quad \left( 8 \sqrt{\frac{1}{1 + \cos [c + d x]}} \sqrt{\frac{b + a \cos [c + d x]}{(a + b) (1 + \cos [c + d x])}} \right) - \\
& \quad \left( \sqrt{b + a \cos [c + d x]} \sec \left[ \frac{1}{2} (c + d x) \right]^5 \sqrt{1 + \sec [c + d x]} \right.
\end{aligned}$$



$$\begin{aligned}
& \left( -\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a \cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) \\
& \left( 2 \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \left. \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \left( -\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) \Bigg) / \\
& \left( 8 \left( \frac{1}{1+\cos[c+dx]} \right)^{3/2} \left( \frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])} \right)^{3/2} \right) + \\
& \left( 5 \sqrt{b+a \cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{1+\operatorname{Sec}[c+dx]} \right. \\
& \left. \left( 2 \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \right. \\
& \left. \left. \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \left( -\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) \right. \\
& \left. \tan\left[\frac{1}{2}(c+dx)\right] \right) / \left( 8 \left( \frac{1}{1+\cos[c+dx]} \right)^{3/2} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right) + \\
& \frac{1}{4 \left( \frac{1}{1+\cos[c+dx]} \right)^{3/2} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}}} \sqrt{b+a \cos[c+dx]} \\
& \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \sqrt{1+\operatorname{Sec}[c+dx]} \\
& \left( \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \left( -\frac{1}{2} \cos\left[\frac{1}{2}(c+dx)\right] + \frac{3}{2} \cos\left[\frac{3}{2}(c+dx)\right] \right) - \right. \\
& \left. \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \sin\left[\frac{1}{2}(c+dx)\right] + \right. \\
& \left. \frac{1}{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}}} \cos\left[\frac{1}{2}(c+dx)\right] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] \right. \\
& \left. \left( \frac{\cos[c+dx] \sin[c+dx]}{(1+\cos[c+dx])^2} - \frac{\sin[c+dx]}{1+\cos[c+dx]} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( \left( -\frac{a \sin[c+dx]}{(a+b)(1+\cos[c+dx])} + \frac{(b+a \cos[c+dx]) \sin[c+dx]}{(a+b)(1+\cos[c+dx])^2} \right) \right. \\
& \quad \left. \left( -\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right) \right) / \left( 2 \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \right) + \\
& \quad \frac{\sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right] \sqrt{1 - \frac{(a-b) \tan\left[\frac{1}{2}(c+dx)\right]^2}{a+b}}}{\sqrt{1 - \tan\left[\frac{1}{2}(c+dx)\right]^2}} + \\
& \quad \left( \sqrt{b+a \cos[c+dx]} \operatorname{Sec}\left[\frac{1}{2}(c+dx)\right]^5 \operatorname{Sec}[c+dx] \right. \\
& \quad \left( 2 \cos\left[\frac{1}{2}(c+dx)\right] \sqrt{\frac{\cos[c+dx]}{1+\cos[c+dx]}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(c+dx)\right]\right], \frac{a-b}{a+b}\right] + \right. \\
& \quad \left. \left. \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \left( -\sin\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{3}{2}(c+dx)\right] \right) \tan[c+dx] \right) / \right. \\
& \quad \left. \left( 8 \left( \frac{1}{1+\cos[c+dx]} \right)^{3/2} \sqrt{\frac{b+a \cos[c+dx]}{(a+b)(1+\cos[c+dx])}} \sqrt{1+\operatorname{Sec}[c+dx]} \right) \right) \right)
\end{aligned}$$

**Problem 64: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[a+bx] \sec[2a+2bx] dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[a+bx]]}{b} + \frac{\sqrt{2} \operatorname{ArcTanh}[\sqrt{2} \sin[a+bx]]}{b}$$

Result (type 3, 331 leaves):

$$\frac{1}{4b} \left( \frac{(2+2i) \left( (-1-i) + \sqrt{2} \right) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a+bx) \right] - (-1+\sqrt{2}) \sin \left[ \frac{1}{2} (a+bx) \right]}{(1+\sqrt{2}) \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right]} \right)}{(-1+i) + \sqrt{2}} - \right.$$

$$2i\sqrt{2} \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a+bx) \right] - (1+\sqrt{2}) \sin \left[ \frac{1}{2} (a+bx) \right]}{(-1+\sqrt{2}) \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right]} \right] +$$

$$4 \log \left[ \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right] \right] - 4 \log \left[ \cos \left[ \frac{1}{2} (a+bx) \right] + \sin \left[ \frac{1}{2} (a+bx) \right] \right] +$$

$$2\sqrt{2} \log \left[ \sqrt{2} + 2 \sin[a+bx] \right] - \sqrt{2} \log \left[ 2 - \sqrt{2} \cos[a+bx] - \sqrt{2} \sin[a+bx] \right] +$$

$$\left. \frac{1}{(-1+i) + \sqrt{2}} (1-i) \left( (-1-i) + \sqrt{2} \right) \log \left[ 2 + \sqrt{2} \cos[a+bx] - \sqrt{2} \sin[a+bx] \right] \right)$$

**Problem 65: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[a+bx] \sec[2(a+bx)] dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[\sin[a+bx]]}{b} + \frac{\sqrt{2} \operatorname{ArcTanh}[\sqrt{2} \sin[a+bx]]}{b}$$

Result (type 3, 331 leaves):

$$\frac{1}{4b} \left( \frac{(2+2i) \left( (-1-i) + \sqrt{2} \right) \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a+bx) \right] - (-1+\sqrt{2}) \sin \left[ \frac{1}{2} (a+bx) \right]}{(1+\sqrt{2}) \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right]} \right)}{(-1+i) + \sqrt{2}} - \right.$$

$$2i\sqrt{2} \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{1}{2} (a+bx) \right] - (1+\sqrt{2}) \sin \left[ \frac{1}{2} (a+bx) \right]}{(-1+\sqrt{2}) \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right]} \right] +$$

$$4 \log \left[ \cos \left[ \frac{1}{2} (a+bx) \right] - \sin \left[ \frac{1}{2} (a+bx) \right] \right] - 4 \log \left[ \cos \left[ \frac{1}{2} (a+bx) \right] + \sin \left[ \frac{1}{2} (a+bx) \right] \right] +$$

$$2\sqrt{2} \log \left[ \sqrt{2} + 2 \sin[a+bx] \right] - \sqrt{2} \log \left[ 2 - \sqrt{2} \cos[a+bx] - \sqrt{2} \sin[a+bx] \right] +$$

$$\left. \frac{1}{(-1+i) + \sqrt{2}} (1-i) \left( (-1-i) + \sqrt{2} \right) \log \left[ 2 + \sqrt{2} \cos[a+bx] - \sqrt{2} \sin[a+bx] \right] \right)$$

**Problem 74: Result unnecessarily involves complex numbers and more than**

twice size of optimal antiderivative.

$$\int \sin[x] \tan[2x] \, dx$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2} \sin[x]\right]}{\sqrt{2}} - \sin[x]$$

Result (type 3, 179 leaves):

$$\begin{aligned} & -\frac{1}{4\sqrt{2}} \left( 2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + \right. \\ & \quad 2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] - 2 \log[\sqrt{2} + 2 \sin[x]] + \\ & \quad \left. \log[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \log[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + 4\sqrt{2} \sin[x] \right) \end{aligned}$$

Problem 76: Result is not expressed in closed-form.

$$\int \sin[x] \tan[4x] \, dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$\frac{1}{4} \sqrt{2 - \sqrt{2}} \text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2 - \sqrt{2}}}\right] + \frac{1}{4} \sqrt{2 + \sqrt{2}} \text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2 + \sqrt{2}}}\right] - \sin[x]$$

Result (type 7, 96 leaves):

$$\begin{aligned} & \frac{1}{16} \text{RootSum}\left[1 + \#1^8 \&, \frac{1}{\#1^7} \left( 2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] - i \log[1 - 2 \cos[x] \#1 + \#1^2] + \right. \right. \\ & \quad \left. \left. 2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^6 - i \log[1 - 2 \cos[x] \#1 + \#1^2] \#1^6 \right) \& \right] - \sin[x] \end{aligned}$$

Problem 77: Result is not expressed in closed-form.

$$\int \sin[x] \tan[5x] \, dx$$

Optimal (type 3, 112 leaves, 10 steps):

$$\begin{aligned} & \frac{1}{5} \text{ArcTanh}[\sin[x]] - \frac{1}{20} (1 - \sqrt{5}) \log[1 - \sqrt{5} - 4 \sin[x]] - \frac{1}{20} (1 + \sqrt{5}) \log[1 + \sqrt{5} - 4 \sin[x]] + \\ & \quad \frac{1}{20} (1 - \sqrt{5}) \log[1 - \sqrt{5} + 4 \sin[x]] + \frac{1}{20} (1 + \sqrt{5}) \log[1 + \sqrt{5} + 4 \sin[x]] - \sin[x] \end{aligned}$$

Result (type 7, 248 leaves):

$$\frac{1}{20} \left( \text{RootSum} \left[ 1 - \#1^2 + \#1^4 - \#1^6 + \#1^8 \&, \right. \right. \\ \left. \left( 6 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] - 3 i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] - 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^2 + \right. \right. \\ \left. i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \#1^2 - 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^4 + \right. \\ \left. i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \#1^4 + 6 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^6 - \right. \\ \left. \left. 3 i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \#1^6 \right) / \left( -\#1 + 2 \#1^3 - 3 \#1^5 + 4 \#1^7 \right) \& \right] - \\ 4 \left( \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] - \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] + 5 \sin[x] \right) \right)$$

Problem 78: Result is not expressed in closed-form.

$$\int \sin[x] \tan[6x] \, dx$$

Optimal (type 3, 89 leaves, 10 steps):

$$\frac{\operatorname{ArcTanh} \left[ \sqrt{2} \sin[x] \right]}{3 \sqrt{2}} + \frac{1}{6} \sqrt{2 - \sqrt{3}} \operatorname{ArcTanh} \left[ \frac{2 \sin[x]}{\sqrt{2 - \sqrt{3}}} \right] + \\ \frac{1}{6} \sqrt{2 + \sqrt{3}} \operatorname{ArcTanh} \left[ \frac{2 \sin[x]}{\sqrt{2 + \sqrt{3}}} \right] - \sin[x]$$

Result (type 7, 366 leaves):

$$\frac{1}{24} \left( \text{RootSum} \left[ 1 - \#1^4 + \#1^8 \&, \frac{1}{-\#1^3 + 2 \#1^7} \right. \right. \\ \left. \left( 4 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] - 2 i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] - 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^2 + \right. \right. \\ \left. i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \#1^2 - 2 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^4 + i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \right. \\ \left. \#1^4 + 4 \operatorname{ArcTan} \left[ \frac{\sin[x]}{\cos[x] - \#1} \right] \#1^6 - 2 i \operatorname{Log} \left[ 1 - 2 \cos[x] \#1 + \#1^2 \right] \#1^6 \right) \& \right] - \\ \sqrt{2} \left( 2 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] + 2 i \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] - \right. \\ \left. 2 \operatorname{Log} \left[ \sqrt{2} + 2 \sin[x] \right] + \operatorname{Log} \left[ 2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] + \right. \\ \left. \left. \operatorname{Log} \left[ 2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right] + 12 \sqrt{2} \sin[x] \right) \right)$$

**Problem 80: Result more than twice size of optimal antiderivative.**

$$\int \cot[2x] \sin[x] \, dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\sin[x]] + \sin[x]$$

Result (type 3, 41 leaves):

$$\frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \sin[x]$$

**Problem 82: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cot[4x] \sin[x] \, dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\sin[x]] - \frac{\operatorname{ArcTanh}[\sqrt{2} \sin[x]]}{2\sqrt{2}} + \sin[x]$$

Result (type 3, 223 leaves):

$$\begin{aligned} & \frac{1}{8\sqrt{2}} \left( 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + \right. \\ & 2\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - 2\sqrt{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \operatorname{Log}[\sqrt{2} + 2 \sin[x]] + \\ & \left. \operatorname{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \operatorname{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + 8\sqrt{2} \sin[x] \right) \end{aligned}$$

**Problem 83: Result more than twice size of optimal antiderivative.**

$$\int \cot[5x] \sin[x] \, dx$$

Optimal (type 3, 82 leaves, 6 steps):

$$\begin{aligned} & -\frac{1}{5} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \operatorname{ArcTanh}\left[2 \sqrt{\frac{2}{5 + \sqrt{5}}} \sin[x]\right] - \\ & \frac{1}{5} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{5}} (5 + \sqrt{5}) \sin[x]\right] + \sin[x] \end{aligned}$$

Result (type 3, 201 leaves):

$$\frac{(-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(-3 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right]}{\sqrt{50 - 10\sqrt{5}}} - \frac{(-1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(5 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{10 - 2\sqrt{5}}}\right]}{\sqrt{50 - 10\sqrt{5}}} +$$

$$\frac{(1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(-5 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{2(5 + \sqrt{5})}}\right]}{\sqrt{10(5 + \sqrt{5})}} - \frac{(1 + \sqrt{5}) \operatorname{ArcTanh}\left[\frac{(3 + \sqrt{5}) \tan\left[\frac{x}{2}\right]}{\sqrt{2(5 + \sqrt{5})}}\right]}{\sqrt{10(5 + \sqrt{5})}} + \sin[x]$$

**Problem 84: Result more than twice size of optimal antiderivative.**

$$\int \cot[6x] \sin[x] \, dx$$

Optimal (type 3, 38 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh}[\sin[x]] - \frac{1}{6} \operatorname{ArcTanh}[2 \sin[x]] - \frac{\operatorname{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{3}}\right]}{2\sqrt{3}} + \sin[x]$$

Result (type 3, 99 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \operatorname{ArcTanh}\left[\frac{\tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2\sqrt{3} \operatorname{ArcTanh}\left[\sqrt{3} \tan\left[\frac{x}{2}\right]\right] + 2 \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \right.$$

$$\left. 2 \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \log[1 - 2 \sin[x]] - \log[1 + 2 \sin[x]] + 12 \sin[x] \right)$$

**Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[2x] \sin[x] \, dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sqrt{2} \cos[x]]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}} \left( 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] - 2i \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + \right.$$

$$\left. 4 \operatorname{ArcTanh}\left[\sqrt{2} + \tan\left[\frac{x}{2}\right]\right] - \log[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] + \log[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] \right)$$

**Problem 87: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[4x] \sin[x] \, dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} + \frac{\operatorname{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}}$$

Result (type 3, 5090 leaves):

$$\left( \left( -2(-1)^{3/8}(1+\sqrt{2})x - \left( 2(-1)^{1/4}(-2-(1-i))(-1)^{5/8} + (-1)^{5/8}\sqrt{2} \right) \right. \right. \\ \left. \left. \operatorname{ArcTan}\left[ \frac{-\cos[x] + (1+\sqrt{2})\sin[x]}{2(-1)^{3/8} + \cos[x] - \sqrt{2}\cos[x] + \sin[x]} \right] \right) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) - \right. \\ \left( 2(1-i)^{3/2}2^{1/4} \left( (-3-i) + 2(-1)^{5/8} + (2+i)\sqrt{2} - (2+2i)(-1)^{3/8}\sqrt{2} + 2(-1)^{5/8}\sqrt{2} \right) \right. \\ \left. \operatorname{ArcTan}\left[ \frac{(1+i) + i\sqrt{2} + ((-1+i) + 2(-1)^{3/8} + \sqrt{2})\tan\left[\frac{x}{2}\right]}{\sqrt{1-i}2^{3/4}} \right] \right) / \\ \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) + 2(-1)^{3/8}\log\left[\sec\left[\frac{x}{2}\right]^2\right] + \\ \left( (-1)^{3/4}(-2-(1-i))(-1)^{5/8} + (-1)^{5/8}\sqrt{2} \right) \\ \log\left[-\sec\left[\frac{x}{2}\right]^4(-2+(1-i)\sqrt{2} + 2(-1)^{3/8}(-1+\sqrt{2}))\cos[x] + \sqrt{2}\cos[2x] - \right. \\ \left. 2(-1)^{3/8}\sin[x] + \sqrt{2}\sin[2x] \right] \Big) / \left( (-1+i) + 2(-1)^{3/8} + \sqrt{2} \right) \\ \left( -\left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( ((-1+i) + \sqrt{1-i}\sqrt{1+i}) \left( -(-1-i)^{3/2}(1-i)^{1/4}(1+i)^{1/4} - (1+i)\cos[x] + \right. \right. \right. \right. \\ \left. \left. \left. i\sqrt{1-i}\sqrt{1+i}\cos[x] + (1-i)\sin[x] + \sqrt{1-i}\sqrt{1+i}\sin[x] \right) \right) \right) - \\ \sin[x] / \left( \sqrt{1-i}(1-i)^{1/4}(1+i)^{1/4} \left( (-1+i) + \sqrt{1-i}\sqrt{1+i} \right) \right. \\ \left( -(-1-i)^{3/2}(1-i)^{1/4}(1+i)^{1/4} - (1+i)\cos[x] + i\sqrt{1-i}\sqrt{1+i}\cos[x] + \right. \\ \left. (1-i)\sin[x] + \sqrt{1-i}\sqrt{1+i}\sin[x] \right) \Big) - \left( i\sqrt{1-i}(1-i)^{1/4}(1+i)^{1/4}\sin[x] \right) / \\ \left( 2 \left( (-1+i) + \sqrt{1-i}\sqrt{1+i} \right) \left( -(-1-i)^{3/2}(1-i)^{1/4}(1+i)^{1/4} - (1+i)\cos[x] + \right. \right. \\ \left. \left. i\sqrt{1-i}\sqrt{1+i}\cos[x] + (1-i)\sin[x] + \sqrt{1-i}\sqrt{1+i}\sin[x] \right) \right) \Big) /$$



$$\begin{aligned}
& \left( -2 (-1)^{3/8} (1 + \sqrt{2}) - \left( 2 (-1)^{1/4} (-2 - (1 - i)) (-1)^{5/8} + (-1)^{5/8} \sqrt{2} \right) \right. \\
& \quad \left( \frac{(1 + \sqrt{2}) \cos[x] + \sin[x]}{2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} - \left( (\cos[x] - \sin[x] + \sqrt{2} \sin[x]) \right. \right. \\
& \quad \left. \left. (-\cos[x] + (1 + \sqrt{2}) \sin[x]) \right) \right) / \left( 2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x] \right)^2 \Bigg) / \\
& \quad \left( \left( (-1 + i) + 2 (-1)^{3/8} + \sqrt{2} \right) \left( 1 + \frac{(-\cos[x] + (1 + \sqrt{2}) \sin[x])^2}{(2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x])^2} \right) \right) + 2 \\
& \quad (-1)^{3/8} \tan\left[\frac{x}{2}\right] - \\
& \quad \left( (-1)^{3/4} (-2 - (1 - i)) (-1)^{5/8} + (-1)^{5/8} \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \\
& \quad \left( -\sec\left[\frac{x}{2}\right]^4 (-2 (-1)^{3/8} \cos[x] + 2 \sqrt{2} \cos[2x] - 2 (-1)^{3/8} (-1 + \sqrt{2}) \sin[x] - \right. \\
& \quad \left. 2 \sqrt{2} \sin[2x]) - 2 \sec\left[\frac{x}{2}\right]^4 (-2 + (1 - i) \sqrt{2} + 2 (-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \right. \\
& \quad \left. \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x]) \tan\left[\frac{x}{2}\right] \right) \Bigg) / \\
& \quad \left( \left( (-1 + i) + 2 (-1)^{3/8} + \sqrt{2} \right) (-2 + (1 - i) \sqrt{2} + 2 (-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \right. \\
& \quad \left. \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x]) \right) - \\
& \quad \left( (1 - i) \left( (-3 - i) + 2 (-1)^{5/8} + (2 + i) \sqrt{2} - (2 + 2i) (-1)^{3/8} \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \right) / \\
& \quad \left( \sqrt{2} \left( 1 + \frac{1}{\sqrt{2}} \left( \frac{1}{4} + \frac{i}{4} \right) \left( (1 + i) + i \sqrt{2} + ((-1 + i) + 2 (-1)^{3/8} + \sqrt{2}) \tan\left[\frac{x}{2}\right] \right)^2 \right) \right) \Bigg) + \\
& \quad \left( \left( (-2 - 2i) \left( (1 - i) + \sqrt{2} + (-1)^{7/8} \sqrt{2} \right) x + \right. \right. \\
& \quad \left. \left( 2 + 2i \right) (-1)^{3/8} \left( 2 - (1 - i) (-1)^{5/8} + (1 - i) (-1)^{7/8} - \sqrt{2} \right) \right. \\
& \quad \left. \operatorname{ArcTan}\left[ \frac{\sin[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right] + \right. \\
& \quad \left( 4 - 4i \right) \left( (1 + 3i) + (1 - i) (-1)^{1/8} + (1 + 2i) (-1)^{3/8} - \right. \\
& \quad \left. (2 + 2i) (-1)^{5/8} + (2 + i) (-1)^{7/8} - (1 + 2i) \sqrt{2} \right) \\
& \quad \left. \operatorname{ArcTan}\left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right) \right] \right) + \\
& \quad 2 (-1)^{7/8} \sqrt{2} (-1 + (-1)^{1/4}) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \log\left[\sec\left[\frac{x}{2}\right]^2\right] + \\
& \quad (-1)^{7/8} \left( (-2 - 2i) + 2 (-1)^{5/8} - 2 (-1)^{7/8} + (1 + i) \sqrt{2} \right) \\
& \quad \log\left[\sec\left[\frac{x}{2}\right]^4 \left( 1 - 3 (-1)^{1/4} + 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -i + (-1)^{1/4} \right) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \Big) \Big) \\
& \left( i \Big/ \left( \sqrt{1-i} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{-1-i} (1-i)^{3/4} (1+i)^{1/4} + \right. \right. \right. \\
& \quad \left. \left. \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) + \\
& \quad 1 \Big/ \left( \sqrt{1+i} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{-1-i} (1-i)^{3/4} (1+i)^{1/4} + \right. \right. \\
& \quad \left. \left. \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) - (2 \sin[x]) \Big/ \\
& \quad \left( \sqrt{-1-i} (1-i)^{1/4} (1+i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right)^2 \left( \sqrt{-1-i} (1-i)^{3/4} (1+i)^{1/4} + \right. \right. \\
& \quad \left. \left. \sqrt{1-i} \cos[x] - \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) \Big) \Big) \Big/ \\
& \left( (-2-2i) \left( (1-i) + \sqrt{2} + (-1)^{7/8} \sqrt{2} \right) + \left( (2+2i) (-1)^{3/8} \left( 2 - (1-i) (-1)^{5/8} + \right. \right. \right. \\
& \quad \left. \left. (1-i) (-1)^{7/8} - \sqrt{2} \right) \left( - \left( \sin[x] \left( (-1)^{3/4} \cos[x] - \sin[x] + (-1)^{1/4} \sin[x] \right) \right) \Big/ \right. \right. \\
& \quad \left. \left( - (-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2 \right) + \\
& \quad \left. \frac{\cos[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right) \Big) \Big/ \\
& \left( 1 + \frac{\sin[x]^2}{\left( -(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2} \right) + \\
& 2(-1)^{7/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right] + \\
& \left( (-1)^{7/8} \left( (-2-2i) + 2(-1)^{5/8} - 2(-1)^{7/8} + (1+i) \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \quad \left( \sec\left[\frac{x}{2}\right]^4 \left( 2(-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2(-1)^{3/4} \cos[2x] - \right. \right. \\
& \quad \left. \left. 2(-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \sin[x] + 2(-i + (-1)^{1/4}) \sin[2x] \right) + \right. \\
& \quad \left. 2 \sec\left[\frac{x}{2}\right]^4 \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] - (-i + (-1)^{1/4}) \right. \right. \\
& \quad \left. \left. \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \Big) \Big/ \\
& \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] - (-i + (-1)^{1/4}) \cos[2x] + \right. \\
& \quad \left. 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) + \\
& \left( 2(-1)^{5/8} \left( (1+3i) + (1-i) (-1)^{1/8} + (1+2i) (-1)^{3/8} - (2+2i) (-1)^{5/8} + \right. \right. \\
& \quad \left. \left. (2+i) (-1)^{7/8} - (1+2i) \sqrt{2} \right) \left( -1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \right) \Big/
\end{aligned}$$

$$\begin{aligned}
& \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right)^2 \right) \Bigg) + \\
& \left( 2 (-1)^{1/8} (1 + (-1)^{1/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) x - \right. \\
& \quad 2 (-1)^{3/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \\
& \quad \text{ArcTan}\left[\frac{\sin[x]}{(-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \sin[x])}\right] - 4 \left( (3 - i) - \right. \\
& \quad \left. 2 (-1)^{1/8} + 2 (-1)^{3/8} - (2 - i) \sqrt{2} + (-1)^{1/8} \sqrt{2} - (2 + i) (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2} \right) \\
& \quad \left. \text{ArcTan}\left[\left(\frac{1}{2} + \frac{i}{2}\right) (-1)^{5/8} \left( i + (-1)^{3/4} + (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right) \right] \right) - \\
& \quad 2 (-1)^{7/8} (1 + (-1)^{3/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \log\left[\sec\left[\frac{x}{2}\right]^2\right] + \\
& \quad (-1)^{7/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \\
& \quad \log\left[-\sec\left[\frac{x}{2}\right]^4 (-1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] + \right. \\
& \quad \left. (-i + (-1)^{1/4}) \cos[2x] + 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \Bigg) \\
& \left( 1 / \left( \sqrt{1 - i} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 (-\sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} + \right. \right. \\
& \quad \left. \left. \sqrt{1 - i} \cos[x] - \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) - \\
& \quad i / \left( \sqrt{1 + i} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 (-\sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} + \right. \\
& \quad \left. \sqrt{1 - i} \cos[x] - \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \Bigg) + (2 \sin[x]) / \\
& \quad \left( \sqrt{-1 + i} (1 - i)^{3/4} (1 + i)^{1/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 (-\sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} + \right. \\
& \quad \left. \sqrt{1 - i} \cos[x] - \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \Bigg) \Bigg) / \\
& \left( 2 (-1)^{1/8} (1 + (-1)^{1/4}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) - \right. \\
& \quad \left( 2 (-1)^{3/8} (2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}) \right. \\
& \quad \left. \left( - \left( (\sin[x] ((-1)^{3/4} \cos[x] - (-1 + (-1)^{1/4}) \sin[x])) \right) / \right. \right. \\
& \quad \left. \left. \left( (-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \sin[x]) \right)^2 \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \frac{\cos[x]}{\left(-1 + (-1)^{1/4}\right) \cos[x] + (-1)^{5/8} \left(\sqrt{2} + (-1)^{1/8} \sin[x]\right)} \right) \right) \right) / \\
& \left( 1 + \frac{\sin[x]^2}{\left(\left(-1 + (-1)^{1/4}\right) \cos[x] + (-1)^{5/8} \left(\sqrt{2} + (-1)^{1/8} \sin[x]\right)\right)^2} \right) - \\
& 2 (-1)^{7/8} \left(1 + (-1)^{3/4}\right) \left(1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}\right) \tan\left[\frac{x}{2}\right] - \\
& \left( (-1)^{7/8} \left(2 - \sqrt{2} - (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2}\right) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \quad \left( -\sec\left[\frac{x}{2}\right]^4 \left(2 (-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2 (-1)^{3/4} \cos[2x] + \right. \right. \\
& \quad \quad \left. 2 (-1)^{5/8} \sqrt{2} \left(-1 + (-1)^{1/4}\right) \sin[x] - 2 \left(-i + (-1)^{1/4}\right) \sin[2x]\right) - \\
& \quad \left. 2 \sec\left[\frac{x}{2}\right]^4 \left(-1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} \left(-1 + (-1)^{1/4}\right) \cos[x] + \left(-i + (-1)^{1/4}\right) \right. \right. \\
& \quad \quad \left. \cos[2x] + 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x]\right) \tan\left[\frac{x}{2}\right] \left. \right) \right) / \\
& \left( -1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} \left(-1 + (-1)^{1/4}\right) \cos[x] + \left(-i + (-1)^{1/4}\right) \cos[2x] + \right. \\
& \quad \left. 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) - \\
& \left( (1 + i) (-1)^{5/8} \left(1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}\right) \left( (3 - i) - 2 (-1)^{1/8} + 2 (-1)^{3/8} - \right. \right. \\
& \quad \left. \left. (2 - i) \sqrt{2} + (-1)^{1/8} \sqrt{2} - (2 + i) (-1)^{3/8} \sqrt{2} + (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \right) / \\
& \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + \left(1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}\right) \tan\left[\frac{x}{2}\right] \right)^2 \right) \right) + \\
& \left( \left( -4 \sqrt{-1 - i} \left(-1 + \sqrt{2}\right) \operatorname{ArcTanh}\left[ \frac{-i \left( (1 + i) + \sqrt{2} \right) + \left( (1 + i) + 2 (-1)^{5/8} - \sqrt{2} \right) \tan\left[\frac{x}{2}\right]}{\sqrt{-1 - i} 2^{3/4}} \right] + \right. \right. \\
& \quad (-1)^{1/8} 2^{1/4} \left( 2 \operatorname{ArcTan}\left[ \frac{\cos[x] + (1 + \sqrt{2}) \sin[x]}{2 (-1)^{5/8} + (-1 + \sqrt{2}) \cos[x] + \sin[x]} \right] - \right. \\
& \quad i \left( 2 (1 + \sqrt{2}) x + 2 \log\left[\sec\left[\frac{x}{2}\right]^2\right] - \log\left[\sec\left[\frac{x}{2}\right]^4 \left(2 - (1 + i) \sqrt{2} + 2 (-1)^{5/8} \right. \right. \right. \\
& \quad \quad \left. \left. \left. (-1 + \sqrt{2}) \cos[x] - \sqrt{2} \cos[2x] + 2 (-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right] \right) \right) \right) \right) + \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right) \left( -\frac{2 (1 - i)^{1/4} (1 + i)^{1/4}}{\sqrt{-1 + i}} - (1 + i) \cos[x] + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 - i} \sqrt{1 + i} \cos[x] + (1 - i) \sin[x] - i \sqrt{1 - i} \sqrt{1 + i} \sin[x] \right) \right) \right) - \\
& (2 \sin[x]) / \left( (-1 + i)^{5/2} (1 - i)^{1/4} (1 + i)^{1/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{2(1-i)^{1/4}(1+i)^{1/4}}{\sqrt{-1+i}} - (1+i)\cos[x] + \sqrt{1-i}\sqrt{1+i}\cos[x] + \right. \\
& \quad \left. (1-i)\sin[x] - i\sqrt{1-i}\sqrt{1+i}\sin[x] \right) + \left( i(1-i)^{1/4}(1+i)^{1/4}\sin[x] \right) / \\
& \left( (-1+i)^{3/2} \left( (-1-i) + \sqrt{1-i}\sqrt{1+i} \right) \left( -\frac{2(1-i)^{1/4}(1+i)^{1/4}}{\sqrt{-1+i}} - (1+i)\cos[x] + \right. \right. \\
& \quad \left. \left. \sqrt{1-i}\sqrt{1+i}\cos[x] + (1-i)\sin[x] - i\sqrt{1-i}\sqrt{1+i}\sin[x] \right) \right) / \\
& \left( -\frac{2^{1/4} \left( (1+i) + 2(-1)^{5/8} - \sqrt{2} \right) (-1+\sqrt{2}) \operatorname{Sec}\left[\frac{x}{2}\right]^2}{1 + \frac{\left(\frac{1-i}{4}\right) \left(-i((1+i)+\sqrt{2}) + ((1+i)+2(-1)^{5/8}-\sqrt{2})\tan\left[\frac{x}{2}\right]\right)^2}{\sqrt{2}}} + (-1)^{1/8} 2^{1/4} \right. \\
& \quad \left( \left( 2 \left( \frac{(1+\sqrt{2})\cos[x] - \sin[x]}{2(-1)^{5/8} + (-1+\sqrt{2})\cos[x] + \sin[x]} - \left( \cos[x] - (-1+\sqrt{2})\sin[x] \right) \right. \right. \right. \\
& \quad \left. \left. \left( \cos[x] + (1+\sqrt{2})\sin[x] \right) \right) / \left( 2(-1)^{5/8} + (-1+\sqrt{2})\cos[x] + \sin[x] \right)^2 \right) / \\
& \quad \left( 1 + \frac{\left( \cos[x] + (1+\sqrt{2})\sin[x] \right)^2}{\left( 2(-1)^{5/8} + (-1+\sqrt{2})\cos[x] + \sin[x] \right)^2} \right) - i \left( 2(1+\sqrt{2}) + 2\tan\left[\frac{x}{2}\right] - \right. \\
& \quad \left( \cos\left[\frac{x}{2}\right]^4 \left( \operatorname{Sec}\left[\frac{x}{2}\right]^4 \left( 2(-1)^{5/8}\cos[x] + 2\sqrt{2}\cos[2x] - 2(-1)^{5/8}(-1+\sqrt{2})\sin[x] + \right. \right. \right. \\
& \quad \left. \left. 2\sqrt{2}\sin[2x] \right) + 2\operatorname{Sec}\left[\frac{x}{2}\right]^4 \left( 2 - (1+i)\sqrt{2} + 2(-1)^{5/8}(-1+\sqrt{2})\cos[x] - \right. \right. \\
& \quad \left. \left. \sqrt{2}\cos[2x] + 2(-1)^{5/8}\sin[x] + \sqrt{2}\sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) / \left( 2 - (1+i)\sqrt{2} + \right. \\
& \quad \left. \left. 2(-1)^{5/8}(-1+\sqrt{2})\cos[x] - \sqrt{2}\cos[2x] + 2(-1)^{5/8}\sin[x] + \sqrt{2}\sin[2x] \right) \right) / \left. \right)
\end{aligned}$$

**Problem 89: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sec[6x] \sin[x] \, dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh}\left[\sqrt{2}\cos[x]\right]}{3\sqrt{2}} + \frac{\operatorname{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} + \frac{\operatorname{ArcTanh}\left[\frac{2\cos[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 3, 678 leaves):

$$\begin{aligned}
& \left( \frac{1}{6} + \frac{i}{6} \right) (-1)^{1/4} \text{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \text{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right) \right] - \\
& \left( \frac{1}{6} + \frac{i}{6} \right) (-1)^{3/4} \text{ArcTanh} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \text{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right) \right] + \\
& \frac{1}{12 (2 + \sqrt{2})} \left( 1 + \sqrt{2} \right) \left( x + 2 \sqrt{3} \text{ArcTanh} \left[ \frac{2 + (2 + \sqrt{2}) \tan \left[ \frac{x}{2} \right]}{\sqrt{6}} \right] - \right. \\
& \quad \left. \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \text{Log} \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 \left( \sqrt{2} - 2 \cos [x] + 2 \sin [x] \right) \right] \right) - \\
& \frac{1}{12 \sqrt{2}} \left( x - 2 \sqrt{3} \text{ArcTanh} \left[ \frac{\sqrt{2} + (-1 + \sqrt{2}) \tan \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \right. \\
& \quad \left. \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \left( 1 + \sqrt{2} \cos [x] - \sqrt{2} \sin [x] \right) \right] \right) + \\
& \left( \left( 2 (\sqrt{2} + \sqrt{3}) \text{ArcTanh} \left[ \frac{2 + (2 + \sqrt{6}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] + \right. \right. \\
& \quad \left. \left. (3 + \sqrt{6}) \left( x - \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \text{Log} \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 \left( \sqrt{6} - 2 \cos [x] + 2 \sin [x] \right) \right] \right) \right) \right) \\
& \quad \left( 1 + \sqrt{6} \sin [x] \right) \left( 3 + \sqrt{6} - (2 + \sqrt{6}) \cos [x] + (2 + \sqrt{6}) \sin [x] \right) \Bigg) / \\
& \left( 12 \left( (12 + 5 \sqrt{6}) \cos [2x] + 2 \cos [x] (5 + 2 \sqrt{6} + 5 \sqrt{6} \sin [x]) - \right. \right. \\
& \quad \left. \left. 2 (12 + 5 \sqrt{6} + 4 (5 + 2 \sqrt{6}) \sin [x] - 6 \sin [2x]) \right) \right) + \\
& \left( (-2 (-2 + \sqrt{6}) \text{ArcTanh} [\sqrt{2} + (\sqrt{2} - \sqrt{3}) \tan \left[ \frac{x}{2} \right]] + \right. \\
& \quad \left. (3 \sqrt{2} - 2 \sqrt{3}) \left( x - \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \text{Log} \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 (\sqrt{3} + \sqrt{2} \cos [x] - \sqrt{2} \sin [x]) \right] \right) \right) \\
& \quad \left( \sqrt{2} - 2 \sqrt{3} \sin [x] \right) (-3 + \sqrt{6} - (-2 + \sqrt{6}) \cos [x] + (-2 + \sqrt{6}) \sin [x]) \Bigg) / \\
& \left( 24 \left( (-12 + 5 \sqrt{6}) \cos [2x] + 2 \cos [x] (-5 + 2 \sqrt{6} + 5 \sqrt{6} \sin [x]) - \right. \right. \\
& \quad \left. \left. 2 (-12 + 5 \sqrt{6} + 4 (-5 + 2 \sqrt{6}) \sin [x] + 6 \sin [2x]) \right) \right)
\end{aligned}$$

**Problem 90: Result more than twice size of optimal antiderivative.**

$$\int \csc [2x] \sin [x] \, dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\frac{1}{2} \text{ArcTanh}[\sin[x]]$$

Result (type 3, 37 leaves):

$$\frac{1}{2} \left( -\text{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right)$$

**Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \csc[4x] \sin[x] \, dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \text{ArcTanh}[\sin[x]] + \frac{\text{ArcTanh}[\sqrt{2} \sin[x]]}{2\sqrt{2}}$$

Result (type 3, 218 leaves):

$$\begin{aligned} & \frac{1}{8\sqrt{2}} \left( -2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] - 2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + \right. \\ & 2\sqrt{2} \text{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - 2\sqrt{2} \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + 2 \text{Log}[\sqrt{2} + 2 \sin[x]] - \\ & \left. \text{Log}[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] - \text{Log}[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]] \right) \end{aligned}$$

**Problem 94: Result more than twice size of optimal antiderivative.**

$$\int \csc[6x] \sin[x] \, dx$$

Optimal (type 3, 36 leaves, 7 steps):

$$\frac{1}{6} \text{ArcTanh}[\sin[x]] + \frac{1}{6} \text{ArcTanh}[2 \sin[x]] - \frac{\text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{3}}\right]}{2\sqrt{3}}$$

Result (type 3, 95 leaves):

$$\begin{aligned} & \frac{1}{12} \left( -2\sqrt{3} \text{ArcTanh}\left[\frac{\tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2\sqrt{3} \text{ArcTanh}\left[\sqrt{3} \tan\left[\frac{x}{2}\right]\right] - \right. \\ & \left. 2 \text{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 2 \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \text{Log}[1 - 2 \sin[x]] + \text{Log}[1 + 2 \sin[x]] \right) \end{aligned}$$

**Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [x] \tan [2 x] \, dx$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \cos [x]}{\sqrt{2}}\right]}{\sqrt{2}} - \cos [x]$$

Result (type 3, 183 leaves):

$$\begin{aligned} & \frac{1}{4 \sqrt{2}} \left( 2 i \text{ArcTan}\left[\frac{\cos \left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin \left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos \left[\frac{x}{2}\right] - \sin \left[\frac{x}{2}\right]}\right] - \right. \\ & \quad \left. 2 i \text{ArcTan}\left[\frac{\cos \left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin \left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos \left[\frac{x}{2}\right] - \sin \left[\frac{x}{2}\right]}\right] + 4 \text{ArcTanh}\left[\sqrt{2} + \tan \left[\frac{x}{2}\right]\right] - \right. \\ & \quad \left. 4 \sqrt{2} \cos [x] - \text{Log}\left[2 - \sqrt{2} \cos [x] - \sqrt{2} \sin [x]\right] + \text{Log}\left[2 + \sqrt{2} \cos [x] - \sqrt{2} \sin [x]\right] \right) \end{aligned}$$

**Problem 106: Result more than twice size of optimal antiderivative.**

$$\int \cos [x] \tan [3 x] \, dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 \cos [x]}{\sqrt{3}}\right]}{\sqrt{3}} - \cos [x]$$

Result (type 3, 48 leaves):

$$-\frac{\text{ArcTanh}\left[\frac{-2 + \tan \left[\frac{x}{2}\right]}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\text{ArcTanh}\left[\frac{2 + \tan \left[\frac{x}{2}\right]}{\sqrt{3}}\right]}{\sqrt{3}} - \cos [x]$$

**Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [x] \tan [4 x] \, dx$$

Optimal (type 3, 71 leaves, 6 steps):

$$\frac{1}{4} \sqrt{2 - \sqrt{2}} \text{ArcTanh}\left[\frac{2 \cos [x]}{\sqrt{2 - \sqrt{2}}}\right] + \frac{1}{4} \sqrt{2 + \sqrt{2}} \text{ArcTanh}\left[\frac{2 \cos [x]}{\sqrt{2 + \sqrt{2}}}\right] - \cos [x]$$

Result (type 3, 5854 leaves):

$$-\cos [x] + \left( \left( (2 - 2 i) (-1)^{3/8} x + \left( 2 \sqrt{2} \left( (2 + 2 i) - (1 + 3 i) (-1)^{3/8} - (1 + i) \sqrt{2} + (1 + 2 i) (-1)^{3/8} \sqrt{2} \right) \right. \right. \right.$$



$$\begin{aligned}
& \left. \operatorname{ArcTan} \left[ \frac{\cos [x] + (1 + \sqrt{2}) \sin [x]}{2 (-1)^{5/8} + (-1 + \sqrt{2}) \cos [x] + \sin [x]} \right] \right) / \\
& \left( (-1 - i) - 2 (-1)^{5/8} + \sqrt{2} \right) + \left( 4 \times 2^{3/4} \left( (-1 + i) + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \right. \\
& \left. \operatorname{ArcTanh} \left[ \frac{-i \left( (1 + i) + \sqrt{2} \right) + \left( (1 + i) + 2 (-1)^{5/8} - \sqrt{2} \right) \tan \left[ \frac{x}{2} \right]}{\sqrt{-1 - i} 2^{3/4}} \right] \right) / \\
& \left( \sqrt{-1 - i} \left( (1 + i) + 2 (-1)^{5/8} - \sqrt{2} \right) \right) - (1 - i) (-1)^{3/8} \sqrt{2} (-2 + \sqrt{2}) \log \left[ \sec \left[ \frac{x}{2} \right]^2 \right] + \\
& \left( i \sqrt{2} \left( (2 + 2i) - (1 + 3i) (-1)^{3/8} - (1 + i) \sqrt{2} + (1 + 2i) (-1)^{3/8} \sqrt{2} \right) \right. \\
& \left. \log \left[ \sec \left[ \frac{x}{2} \right]^4 \left( 2 - (1 + i) \sqrt{2} + 2 (-1)^{5/8} (-1 + \sqrt{2}) \cos [x] - \sqrt{2} \cos [2x] + \right. \right. \right. \\
& \left. \left. \left. 2 (-1)^{5/8} \sin [x] + \sqrt{2} \sin [2x] \right) \right] \right) / \left( (-1 - i) - 2 (-1)^{5/8} + \sqrt{2} \right) \Bigg) \\
& \left( \left( \frac{1}{2} + \frac{i}{2} \right) / \left( \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right) \left( -\frac{2 (1 - i)^{1/4} (1 + i)^{1/4}}{\sqrt{-1 + i}} - (1 + i) \cos [x] + \right. \right. \right. \\
& \left. \left. \left. \sqrt{1 - i} \sqrt{1 + i} \cos [x] + (1 - i) \sin [x] - i \sqrt{1 - i} \sqrt{1 + i} \sin [x] \right) \right) \right) + (2 \sin [x]) / \\
& \left( (-1 + i)^{3/2} (1 - i)^{3/4} (1 + i)^{3/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right) \left( -\frac{2 (1 - i)^{1/4} (1 + i)^{1/4}}{\sqrt{-1 + i}} - \right. \right. \\
& \left. \left. (1 + i) \cos [x] + \sqrt{1 - i} \sqrt{1 + i} \cos [x] + (1 - i) \sin [x] - i \sqrt{1 - i} \sqrt{1 + i} \sin [x] \right) \right) \Bigg) + \\
& (2 i \sin [x]) / \left( (-1 + i)^{3/2} (1 - i)^{5/4} (1 + i)^{1/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right) \right. \\
& \left( -\frac{2 (1 - i)^{1/4} (1 + i)^{1/4}}{\sqrt{-1 + i}} - (1 + i) \cos [x] + \sqrt{1 - i} \sqrt{1 + i} \cos [x] + \right. \\
& \left. \left. (1 - i) \sin [x] - i \sqrt{1 - i} \sqrt{1 + i} \sin [x] \right) \right) \Bigg) \Bigg) / \\
& \left( (2 - 2i) (-1)^{3/8} + \left( 2 \sqrt{2} \left( (2 + 2i) - (1 + 3i) (-1)^{3/8} - (1 + i) \sqrt{2} + (1 + 2i) (-1)^{3/8} \sqrt{2} \right) \right. \right. \\
& \left. \left. \frac{(1 + \sqrt{2}) \cos [x] - \sin [x]}{2 (-1)^{5/8} + (-1 + \sqrt{2}) \cos [x] + \sin [x]} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left( \cos[x] - (-1 + \sqrt{2}) \sin[x] \right) \left( \cos[x] + (1 + \sqrt{2}) \sin[x] \right)}{\left( 2(-1)^{5/8} + (-1 + \sqrt{2}) \cos[x] + \sin[x] \right)^2} \right) \Bigg/ \\
& \left( \left( (-1 - i) - 2(-1)^{5/8} + \sqrt{2} \right) \left( 1 + \frac{\left( \cos[x] + (1 + \sqrt{2}) \sin[x] \right)^2}{\left( 2(-1)^{5/8} + (-1 + \sqrt{2}) \cos[x] + \sin[x] \right)^2} \right) \right) - \\
& (1 - i) (-1)^{3/8} \sqrt{2} (-2 + \sqrt{2}) \tan\left[\frac{x}{2}\right] + \\
& \left( i \sqrt{2} \left( (2 + 2i) - (1 + 3i) (-1)^{3/8} - (1 + i) \sqrt{2} + (1 + 2i) (-1)^{3/8} \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \left( \sec\left[\frac{x}{2}\right]^4 \left( 2(-1)^{5/8} \cos[x] + 2\sqrt{2} \cos[2x] - 2(-1)^{5/8} (-1 + \sqrt{2}) \sin[x] + \right. \right. \\
& \left. \left. 2\sqrt{2} \sin[2x] \right) + 2 \sec\left[\frac{x}{2}\right]^4 \left( 2 - (1 + i) \sqrt{2} + 2(-1)^{5/8} (-1 + \sqrt{2}) \cos[x] - \right. \right. \\
& \left. \left. \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \Bigg/ \\
& \left( \left( (-1 - i) - 2(-1)^{5/8} + \sqrt{2} \right) \left( 2 - (1 + i) \sqrt{2} + 2(-1)^{5/8} (-1 + \sqrt{2}) \cos[x] - \right. \right. \\
& \left. \left. \sqrt{2} \cos[2x] + 2(-1)^{5/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right) - \\
& \left. \frac{(1 - i) \left( (-1 + i) + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2}{1 + \frac{\left( \frac{1-i}{4} \right) \left( -i \left( (1+i) + \sqrt{2} \right) + (1+i) + 2(-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right]^2}{\sqrt{2}}} \right) + \\
& \left( \left( (-2 + 2i) (-1)^{5/8} x - \left( 2\sqrt{2} \left( (-2 - 2i) - (3 + i) (-1)^{5/8} + (1 + i) \sqrt{2} + (2 + i) (-1)^{5/8} \sqrt{2} \right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcTan}\left[ \frac{-\cos[x] + (1 + \sqrt{2}) \sin[x]}{2(-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} \right] \right) \right) \Bigg/ \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) + \\
& \left( 2(1 - i)^{3/2} 2^{3/4} \left( (1 + i) - \sqrt{2} + (1 + i) (-1)^{3/8} \sqrt{2} \right) \right. \\
& \left. \operatorname{ArcTan}\left[ \frac{(1 + i) + i \sqrt{2} + \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) \tan\left[\frac{x}{2}\right]}{\sqrt{1 - i} 2^{3/4}} \right] \right) \Bigg/ \\
& \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) - (1 - i) (-1)^{5/8} \sqrt{2} (-2 + \sqrt{2}) \log\left[\sec\left[\frac{x}{2}\right]^2\right] + \\
& \left( i \sqrt{2} \left( (-2 - 2i) - (3 + i) (-1)^{5/8} + (1 + i) \sqrt{2} + (2 + i) (-1)^{5/8} \sqrt{2} \right) \right. \\
& \left. \log\left[-\sec\left[\frac{x}{2}\right]^4 \left( -2 + (1 - i) \sqrt{2} + 2(-1)^{3/8} (-1 + \sqrt{2}) \cos[x] + \sqrt{2} \cos[2x] - \right. \right. \right. \\
& \left. \left. \left. 2(-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right] \right) \Bigg/ \left( (-1 + i) + 2(-1)^{3/8} + \sqrt{2} \right) \\
& \left( - \left( \left( \frac{1}{2} + \frac{i}{2} \right) \right) \Bigg/ \left( \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right) \left( -(-1 - i)^{3/2} (1 - i)^{1/4} (1 + i)^{1/4} - (1 + i) \cos[x] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x]}{\left( \sqrt{-1-i} \sin[x] \right) / \left( (1-i)^{3/4} (1+i)^{3/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \right.} \right. \\
& \quad \left. \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \cos[x] + i \sqrt{1-i} \sqrt{1+i} \cos[x] + \right. \right. \\
& \quad \left. \left. (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) - \left( i \sqrt{-1-i} (1-i)^{3/4} \sin[x] \right) / \\
& \quad \left( 2 (1+i)^{1/4} \left( (-1+i) + \sqrt{1-i} \sqrt{1+i} \right) \left( -(-1-i)^{3/2} (1-i)^{1/4} (1+i)^{1/4} - (1+i) \right. \right. \\
& \quad \left. \left. \cos[x] + i \sqrt{1-i} \sqrt{1+i} \cos[x] + (1-i) \sin[x] + \sqrt{1-i} \sqrt{1+i} \sin[x] \right) \right) \Bigg) / \\
& \left( (-2+2i) (-1)^{5/8} - \left( 2\sqrt{2} \left( (-2-2i) - (3+i) (-1)^{5/8} + (1+i) \sqrt{2} + (2+i) (-1)^{5/8} \sqrt{2} \right) \right. \right. \\
& \quad \left. \left( \frac{(1+\sqrt{2}) \cos[x] + \sin[x]}{2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x]} - \left( (\cos[x] - \sin[x] + \sqrt{2} \sin[x]) \right. \right. \right. \\
& \quad \left. \left. (-\cos[x] + (1+\sqrt{2}) \sin[x]) \right) \right) / \left( 2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x] \right)^2 \Bigg) / \\
& \left( \left( (-1+i) + 2 (-1)^{3/8} + \sqrt{2} \right) \left( 1 + \frac{(-\cos[x] + (1+\sqrt{2}) \sin[x])^2}{\left( 2 (-1)^{3/8} + \cos[x] - \sqrt{2} \cos[x] + \sin[x] \right)^2} \right) \right) - \\
& (1-i) (-1)^{5/8} \sqrt{2} (-2+\sqrt{2}) \tan\left[\frac{x}{2}\right] - \\
& \left( i \sqrt{2} \left( (-2-2i) - (3+i) (-1)^{5/8} + (1+i) \sqrt{2} + (2+i) (-1)^{5/8} \sqrt{2} \right) \cos\left[\frac{x}{2}\right]^4 \right. \\
& \quad \left( -\sec\left[\frac{x}{2}\right]^4 \left( -2 (-1)^{3/8} \cos[x] + 2\sqrt{2} \cos[2x] - 2 (-1)^{3/8} (-1+\sqrt{2}) \sin[x] - \right. \right. \\
& \quad \left. \left. 2\sqrt{2} \sin[2x] \right) - 2 \sec\left[\frac{x}{2}\right]^4 \left( -2 + (1-i) \sqrt{2} + 2 (-1)^{3/8} (-1+\sqrt{2}) \cos[x] + \right. \right. \\
& \quad \left. \left. \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \Bigg) / \\
& \left( \left( (-1+i) + 2 (-1)^{3/8} + \sqrt{2} \right) \left( -2 + (1-i) \sqrt{2} + 2 (-1)^{3/8} (-1+\sqrt{2}) \cos[x] + \right. \right. \\
& \quad \left. \left. \sqrt{2} \cos[2x] - 2 (-1)^{3/8} \sin[x] + \sqrt{2} \sin[2x] \right) \right) + \\
& \left. \frac{(1-i) \left( (1+i) - \sqrt{2} + (1+i) (-1)^{3/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2}{1 + \frac{\left( \frac{1+i}{4} \right) \left( (1+i) + i \sqrt{2} + (-1+i) + 2 (-1)^{3/8} + \sqrt{2} \right) \tan\left[\frac{x}{2}\right]^2}{\sqrt{2}}} \right) + \\
& \left( \left( -2 (-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) x - \right. \right. \\
& \quad \left. \left. 2 (-1)^{3/8} \left( (4+4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3+3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{ArcTan}\left[ \frac{\sin[x]}{-(-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right] - (4+4i) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \\
& \text{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan \left[ \frac{x}{2} \right] \right) \right] + \\
& 2 (-1)^{7/8} \left( (-3 - i) + (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \\
& (-1)^{7/8} \left( (4 + 4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3 + 3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \\
& \text{Log} \left[ \text{Sec} \left[ \frac{x}{2} \right]^4 \left( 1 - 3 (-1)^{1/4} + 2 (-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - \right. \right. \\
& \quad \left. \left. (-i + (-1)^{1/4}) \cos[2x] + 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \right] \Bigg) \\
& \left( - \left( i / \left( \sqrt{1 - i} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) \right) - \\
& 1 / \left( \sqrt{1 + i} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \right. \right. \\
& \quad \left. \left. \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) - \\
& (2 \sin[x]) / \left( \sqrt{-1 - i} (1 - i)^{1/4} (1 + i)^{3/4} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \right. \\
& \quad \left. \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - \right. \right. \\
& \quad \left. \left. i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) - (2i (1 + i)^{3/4} \sin[x]) / \\
& \left( \sqrt{-1 - i} (1 - i)^{3/4} \left( (-1 + i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( -\sqrt{-1 - i} (1 - i)^{3/4} (1 + i)^{1/4} - \right. \right. \\
& \quad \left. \left. \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] - i \sqrt{1 - i} \sin[x] - i \sqrt{1 + i} \sin[x] \right) \right) \Bigg) / \\
& \left( -2 (-1)^{3/8} (1 + (-1)^{1/4}) (-2 + \sqrt{2}) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) - \right. \\
& \left( 2 (-1)^{3/8} \left( (4 + 4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3 + 3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right. \\
& \quad \left. \left( - \left( (\sin[x] \left( (-1)^{3/4} \cos[x] - \sin[x] + (-1)^{1/4} \sin[x] \right)) / \right. \right. \right. \\
& \quad \left. \left. \left( - (-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2 \right) + \right. \\
& \quad \left. \left. \frac{\cos[x]}{- (-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x]} \right) \right) / \\
& \left( 1 + \frac{\sin[x]^2}{\left( - (-1)^{5/8} \sqrt{2} + \cos[x] - (-1)^{1/4} \cos[x] + (-1)^{3/4} \sin[x] \right)^2} \right) + \\
& 2 (-1)^{7/8} \left( (-3 - i) + (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan \left[ \frac{x}{2} \right] + \\
& \left( (-1)^{7/8} \left( (4 + 4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3 + 3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \cos\left[\frac{x}{2}\right]^4 \left( \sec\left[\frac{x}{2}\right]^4 \left( 2(-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2(-1)^{3/4} \cos[2x] - \right. \right. \\
& \quad \left. \left. 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \sin[x] + 2(-i + (-1)^{1/4}) \sin[2x] \right) + \right. \\
& \quad \left. 2 \sec\left[\frac{x}{2}\right]^4 \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - (-i + (-1)^{1/4}) \right. \right. \\
& \quad \left. \left. \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \right) \Bigg) \Bigg/ \\
& \left( 1 - 3(-1)^{1/4} + 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] - (-i + (-1)^{1/4}) \cos[2x] + \right. \\
& \quad \left. 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) + \\
& \left( 2(-1)^{1/8} (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + \right. \right. \\
& \quad \left. \left. (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \right) \Bigg/ \\
& \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + (-1 + (-1)^{1/4} - (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right]^2 \right) \right) \Bigg) + \\
& \left( \left( -2(-1)^{3/8} (1 + (-1)^{1/4}) (-2 + \sqrt{2}) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) x + \right. \right. \\
& \quad \left. \left. 2(-1)^{3/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{ArcTan}\left[ \frac{\sin[x]}{(-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} (\sqrt{2} + (-1)^{1/8} \sin[x])} \right] + (4 + 4i) \right. \right. \\
& \quad \left. \left. \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + (1 + 2i) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \right. \right. \\
& \quad \left. \left. \operatorname{ArcTan}\left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{5/8} \left( i + (-1)^{3/4} + (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \tan\left[\frac{x}{2}\right] \right) \right] \right) + \right. \\
& \quad \left. \left. 2(-1)^{7/8} \left( (3 + i) - (2 + i) \sqrt{2} \right) (1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2}) \log\left[\sec\left[\frac{x}{2}\right]^2\right] - \right. \right. \\
& \quad \left. \left. (-1)^{7/8} \left( (4 + 4i) - 2(-1)^{5/8} + 2(-1)^{7/8} - (3 + 3i) \sqrt{2} + 2(-1)^{5/8} \sqrt{2} - 2(-1)^{7/8} \sqrt{2} \right) \right. \right. \\
& \quad \left. \left. \log\left[-\sec\left[\frac{x}{2}\right]^4 (-1 + 3(-1)^{1/4} - 2(-1)^{5/8} \sqrt{2} (-1 + (-1)^{1/4}) \cos[x] + \right. \right. \right. \\
& \quad \left. \left. \left. (-i + (-1)^{1/4}) \cos[2x] + 2(-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \right] \right) \right) \Bigg) \\
& \left( - \left( 1 \Bigg/ \left( \sqrt{1 - i} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( \sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] + i \sqrt{1 - i} \sin[x] + i \sqrt{1 + i} \sin[x] \right) \right) \right) \Bigg) + \right. \\
& \quad \left. i \Bigg/ \left( \sqrt{1 + i} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( \sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} - \sqrt{1 - i} \cos[x] + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{1 + i} \cos[x] + i \sqrt{1 - i} \sin[x] + i \sqrt{1 + i} \sin[x] \right) \right) \right) - \left( 2i (1 - i)^{3/4} \sin[x] \right) \Bigg/ \\
& \left( \sqrt{-1 + i} (1 + i)^{3/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( \sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} - \right. \right. \\
& \quad \left. \left. \sqrt{1 - i} \cos[x] + \sqrt{1 + i} \cos[x] + i \sqrt{1 - i} \sin[x] + i \sqrt{1 + i} \sin[x] \right) \right) + \left( 2 \sin[x] \right) \Bigg/ \\
& \left( \sqrt{-1 + i} (1 - i)^{3/4} (1 + i)^{1/4} \left( (-1 - i) + \sqrt{1 - i} \sqrt{1 + i} \right)^2 \left( \sqrt{-1 + i} (1 - i)^{1/4} (1 + i)^{3/4} - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left( \sqrt{1-i} \cos[x] + \sqrt{1+i} \cos[x] + i \sqrt{1-i} \sin[x] + i \sqrt{1+i} \sin[x] \right) \right) \Bigg/ \\
& \left( -2 (-1)^{3/8} \left( 1 + (-1)^{1/4} \right) \left( -2 + \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) + \right. \\
& \left( 2 (-1)^{3/8} \left( (4 + 4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3 + 3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right. \\
& \left. \left( - \left( \sin[x] \left( (-1)^{3/4} \cos[x] - (-1 + (-1)^{1/4}) \sin[x] \right) \right) / \right. \right. \\
& \left. \left( (-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} \left( \sqrt{2} + (-1)^{1/8} \sin[x] \right) \right)^2 \right) + \\
& \left. \frac{\cos[x]}{(-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} \left( \sqrt{2} + (-1)^{1/8} \sin[x] \right)} \right) \Bigg/ \\
& \left( 1 + \frac{\sin[x]^2}{\left( (-1 + (-1)^{1/4}) \cos[x] + (-1)^{5/8} \left( \sqrt{2} + (-1)^{1/8} \sin[x] \right) \right)^2} \right) + \\
& 2 (-1)^{7/8} \left( (3 + i) - (2 + i) \sqrt{2} \right) \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right] + \\
& \left( (-1)^{7/8} \left( (4 + 4i) - 2 (-1)^{5/8} + 2 (-1)^{7/8} - (3 + 3i) \sqrt{2} + 2 (-1)^{5/8} \sqrt{2} - 2 (-1)^{7/8} \sqrt{2} \right) \right. \\
& \cos\left[\frac{x}{2}\right]^4 \left( -\sec\left[\frac{x}{2}\right]^4 \left( 2 (-1)^{3/8} \sqrt{2} \cos[x] + 2 \cos[2x] + 2 (-1)^{3/4} \cos[2x] + \right. \right. \\
& \left. 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \sin[x] - 2 \left( -i + (-1)^{1/4} \right) \sin[2x] \right) - \\
& \left. 2 \sec\left[\frac{x}{2}\right]^4 \left( -1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] + \left( -i + (-1)^{1/4} \right) \right. \right. \\
& \left. \cos[2x] + 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) \tan\left[\frac{x}{2}\right] \Bigg) \Bigg/ \\
& \left( -1 + 3 (-1)^{1/4} - 2 (-1)^{5/8} \sqrt{2} \left( -1 + (-1)^{1/4} \right) \cos[x] + \left( -i + (-1)^{1/4} \right) \cos[2x] + \right. \\
& \left. 2 (-1)^{3/8} \sqrt{2} \sin[x] + \sin[2x] + (-1)^{3/4} \sin[2x] \right) - \\
& \left( 2 (-1)^{1/8} \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \left( (-1 + i) + (1 + i) (-1)^{1/8} - (2 + i) (-1)^{3/8} + \right. \right. \\
& \left. \left( 1 + 2i \right) (-1)^{7/8} + \sqrt{2} + (1 - i) (-1)^{5/8} \sqrt{2} \right) \sec\left[\frac{x}{2}\right]^2 \Bigg) \Bigg/ \\
& \left( 1 - \frac{1}{2} (-1)^{3/4} \left( i + (-1)^{3/4} + \left( 1 - (-1)^{1/4} + (-1)^{5/8} \sqrt{2} \right) \tan\left[\frac{x}{2}\right] \right)^2 \right) \Bigg)
\end{aligned}$$

**Problem 108: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \tan[5x] \, dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$\frac{1}{5} \sqrt{\frac{1}{2} (5 + \sqrt{5})} \operatorname{ArcTanh} \left[ 2 \sqrt{\frac{2}{5 + \sqrt{5}}} \cos [x] \right] +$$

$$\frac{1}{5} \sqrt{\frac{1}{2} (5 - \sqrt{5})} \operatorname{ArcTanh} \left[ \sqrt{\frac{2}{5} (5 + \sqrt{5})} \cos [x] \right] - \cos [x]$$

Result (type 3, 215 leaves):

$$\frac{(1 + \sqrt{5}) \operatorname{ArcTanh} \left[ \frac{4 - (-1 + \sqrt{5}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2} (5 + \sqrt{5})} \right]}{\sqrt{10} (5 + \sqrt{5})} + \frac{(1 + \sqrt{5}) \operatorname{ArcTanh} \left[ \frac{4 + (-1 + \sqrt{5}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2} (5 + \sqrt{5})} \right]}{\sqrt{10} (5 + \sqrt{5})} +$$

$$\frac{(-1 + \sqrt{5}) \operatorname{ArcTanh} \left[ \frac{4 - (1 + \sqrt{5}) \tan \left[ \frac{x}{2} \right]}{\sqrt{10 - 2 \sqrt{5}}} \right]}{\sqrt{50 - 10 \sqrt{5}}} + \frac{(-1 + \sqrt{5}) \operatorname{ArcTanh} \left[ \frac{4 + (1 + \sqrt{5}) \tan \left[ \frac{x}{2} \right]}{\sqrt{10 - 2 \sqrt{5}}} \right]}{\sqrt{50 - 10 \sqrt{5}}} - \cos [x]$$

Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos [x] \tan [6 x] \, dx$$

Optimal (type 3, 89 leaves, 10 steps):

$$\frac{\operatorname{ArcTanh} \left[ \sqrt{2} \cos [x] \right]}{3 \sqrt{2}} + \frac{1}{6} \sqrt{2 - \sqrt{3}} \operatorname{ArcTanh} \left[ \frac{2 \cos [x]}{\sqrt{2 - \sqrt{3}}} \right] +$$

$$\frac{1}{6} \sqrt{2 + \sqrt{3}} \operatorname{ArcTanh} \left[ \frac{2 \cos [x]}{\sqrt{2 + \sqrt{3}}} \right] - \cos [x]$$

Result (type 3, 776 leaves):

$$\begin{aligned}
& \left( -\frac{1}{6} - \frac{i}{6} \right) (-1)^{1/4} \text{ArcTan} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{1/4} \text{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right) \right] + \\
& \left( \frac{1}{6} + \frac{i}{6} \right) (-1)^{3/4} \text{ArcTanh} \left[ \left( \frac{1}{2} + \frac{i}{2} \right) (-1)^{3/4} \text{Sec} \left[ \frac{x}{2} \right] \left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right) \right] - \\
& \cos[x] + \frac{1}{12\sqrt{2}} \left( x + 2\sqrt{3} \text{ArcTanh} \left[ \frac{\sqrt{2} + (-1 + \sqrt{2}) \tan \left[ \frac{x}{2} \right]}{\sqrt{3}} \right] - \right. \\
& \left. \log \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \log \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \left( 1 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right) \right] \right) + \\
& \left( (1 + \sqrt{2}) \left( x - 2\sqrt{3} \text{ArcTanh} \left[ \frac{2 + (2 + \sqrt{2}) \tan \left[ \frac{x}{2} \right]}{\sqrt{6}} \right] - \log \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \right. \right. \\
& \left. \left. \log \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 \left( \sqrt{2} - 2 \cos[x] + 2 \sin[x] \right) \right] \right) \right) \\
& \left( 2 + \sqrt{2} \sin[x] \right) \left( 1 + \sqrt{2} - (2 + \sqrt{2}) \cos[x] + (2 + \sqrt{2}) \sin[x] \right) \Bigg) / \\
& \left( 12 \left( -12 - 9\sqrt{2} + 4(3 + 2\sqrt{2}) \cos[x] + (4 + 3\sqrt{2}) \cos[2x] - 18 \sin[x] - 12\sqrt{2} \sin[x] + \right. \right. \\
& \left. \left. 4 \sin[2x] + 3\sqrt{2} \sin[2x] \right) \right) - \left( \left( 2(-2 + \sqrt{6}) \text{ArcTanh} \left[ \sqrt{2} + (\sqrt{2} - \sqrt{3}) \tan \left[ \frac{x}{2} \right] \right] + \right. \right. \\
& \left. \left( 3\sqrt{2} - 2\sqrt{3} \right) \left( x - \log \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \log \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 \left( \sqrt{3} + \sqrt{2} \cos[x] - \sqrt{2} \sin[x] \right) \right] \right) \right) \right) \\
& \left( \sqrt{2} - \sqrt{3} \sin[x] \right) \left( -3 + \sqrt{6} - (-2 + \sqrt{6}) \cos[x] + (-2 + \sqrt{6}) \sin[x] \right) \Bigg) / \\
& \left( 12 \left( -36 + 15\sqrt{6} + (20 - 8\sqrt{6}) \cos[x] + (12 - 5\sqrt{6}) \cos[2x] - \right. \right. \\
& \left. \left. 50 \sin[x] + 20\sqrt{6} \sin[x] + 12 \sin[2x] - 5\sqrt{6} \sin[2x] \right) \right) + \\
& \left( \left( -2(\sqrt{2} + \sqrt{3}) \text{ArcTanh} \left[ \frac{2 + (2 + \sqrt{6}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] + \right. \right. \\
& \left. \left( 3 + \sqrt{6} \right) \left( x - \log \left[ \text{Sec} \left[ \frac{x}{2} \right]^2 \right] + \log \left[ -\text{Sec} \left[ \frac{x}{2} \right]^2 \left( \sqrt{6} - 2 \cos[x] + 2 \sin[x] \right) \right] \right) \right) \right) \\
& \left( 2 + \sqrt{6} \sin[x] \right) \left( 3 + \sqrt{6} - (2 + \sqrt{6}) \cos[x] + (2 + \sqrt{6}) \sin[x] \right) \Bigg) / \\
& \left( 12 \left( -36 - 15\sqrt{6} + 4(5 + 2\sqrt{6}) \cos[x] + (12 + 5\sqrt{6}) \cos[2x] - \right. \right. \\
& \left. \left. 50 \sin[x] - 20\sqrt{6} \sin[x] + 12 \sin[2x] + 5\sqrt{6} \sin[2x] \right) \right)
\end{aligned}$$

**Problem 110: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \cot[2x] \, dx$$



Optimal (type 3, 10 leaves, 4 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cos [x]] + \cos [x]$$

Result (type 3, 25 leaves):

$$\cos [x] - \frac{1}{2} \operatorname{Log}\left[\cos \left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\sin \left[\frac{x}{2}\right]\right]$$

**Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos [x] \cot [4 x] \, dx$$

Optimal (type 3, 28 leaves, 6 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\cos [x]] - \frac{\operatorname{ArcTanh}\left[\sqrt{2} \cos [x]\right]}{2 \sqrt{2}} + \cos [x]$$

Result (type 3, 73 leaves):

$$\frac{1}{4} \left( (-1 - i) (-1)^{3/4} \operatorname{ArcTanh}\left[\frac{-1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] - \right. \\ \left. (1 - i) (-1)^{1/4} \operatorname{ArcTanh}\left[\frac{1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + 4 \cos [x] - \operatorname{Log}\left[\cos \left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\sin \left[\frac{x}{2}\right]\right] \right)$$

**Problem 114: Result more than twice size of optimal antiderivative.**

$$\int \cos [x] \cot [6 x] \, dx$$

Optimal (type 3, 38 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh}[\cos [x]] - \frac{1}{6} \operatorname{ArcTanh}[2 \cos [x]] - \frac{\operatorname{ArcTanh}\left[\frac{2 \cos [x]}{\sqrt{3}}\right]}{2 \sqrt{3}} + \cos [x]$$

Result (type 3, 87 leaves):

$$\frac{1}{12} \left( 2 \sqrt{3} \operatorname{ArcTanh}\left[\frac{-2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2 \sqrt{3} \operatorname{ArcTanh}\left[\frac{2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] + 12 \cos [x] - \right. \\ \left. 2 \operatorname{Log}\left[\cos \left[\frac{x}{2}\right]\right] + \operatorname{Log}[1 - 2 \cos [x]] - \operatorname{Log}[1 + 2 \cos [x]] + 2 \operatorname{Log}\left[\sin \left[\frac{x}{2}\right]\right] \right)$$

**Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \sec[2x] dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2} \sin[x]\right]}{\sqrt{2}}$$

Result (type 3, 174 leaves):

$$\frac{1}{4\sqrt{2}} \left( -2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (-1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] - 2i \text{ArcTan}\left[\frac{\cos\left[\frac{x}{2}\right] - (1 + \sqrt{2}) \sin\left[\frac{x}{2}\right]}{(-1 + \sqrt{2}) \cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}\right] + \right. \\ \left. 2 \log\left[\sqrt{2} + 2 \sin[x]\right] - \log\left[2 - \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right] - \log\left[2 + \sqrt{2} \cos[x] - \sqrt{2} \sin[x]\right] \right)$$

**Problem 118: Result is not expressed in closed-form.**

$$\int \cos[x] \sec[4x] dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2(2-\sqrt{2})}} - \frac{\text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2(2+\sqrt{2})}}$$

Result (type 7, 91 leaves):

$$\frac{1}{16} \text{RootSum}\left[1 + \#1^8 \&, \frac{1}{\#1^5} \left( 2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] - i \log\left[1 - 2 \cos[x] \#1 + \#1^2\right] + \right. \right. \\ \left. \left. 2 \text{ArcTan}\left[\frac{\sin[x]}{\cos[x] - \#1}\right] \#1^2 - i \log\left[1 - 2 \cos[x] \#1 + \#1^2\right] \#1^2 \right) \&\right]$$

**Problem 120: Result is not expressed in closed-form.**

$$\int \cos[x] \sec[6x] dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$-\frac{\text{ArcTanh}\left[\sqrt{2} \sin[x]\right]}{3\sqrt{2}} + \frac{\text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2-\sqrt{3}}}\right]}{6\sqrt{2-\sqrt{3}}} + \frac{\text{ArcTanh}\left[\frac{2 \sin[x]}{\sqrt{2+\sqrt{3}}}\right]}{6\sqrt{2+\sqrt{3}}}$$

Result (type 7, 356 leaves):

$$\frac{1}{24}$$

$$\left( \sqrt{2} \left( 2 \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] + 2 \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{2} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{2} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right]} \right] - 2 \operatorname{Log} \left[ \sqrt{2} + 2 \sin [x] \right] + \operatorname{Log} \left[ 2 - \sqrt{2} \cos [x] - \sqrt{2} \sin [x] \right] + \operatorname{Log} \left[ 2 + \sqrt{2} \cos [x] - \sqrt{2} \sin [x] \right] \right) + \operatorname{RootSum} \left[ 1 - \#1^4 + \#1^8 \&, \frac{1}{-\#1^3 + 2 \#1^7} \left( 2 \operatorname{ArcTan} \left[ \frac{\sin [x]}{\cos [x] - \#1} \right] - \operatorname{Log} \left[ 1 - 2 \cos [x] \#1 + \#1^2 \right] + 2 \operatorname{ArcTan} \left[ \frac{\sin [x]}{\cos [x] - \#1} \right] \#1^2 - \operatorname{Log} \left[ 1 - 2 \cos [x] \#1 + \#1^2 \right] \#1^2 + 2 \operatorname{ArcTan} \left[ \frac{\sin [x]}{\cos [x] - \#1} \right] \#1^4 - \operatorname{Log} \left[ 1 - 2 \cos [x] \#1 + \#1^2 \right] \#1^4 + 2 \operatorname{ArcTan} \left[ \frac{\sin [x]}{\cos [x] - \#1} \right] \#1^6 - \operatorname{Log} \left[ 1 - 2 \cos [x] \#1 + \#1^2 \right] \#1^6 \right) \& \right] \right)$$

**Problem 121: Result more than twice size of optimal antiderivative.**

$$\int \cos [2 x] \sec [x] \, dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$-\operatorname{ArcTanh}[\sin [x]] + 2 \sin [x]$$

Result (type 3, 37 leaves):

$$\operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] - \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] + 2 \sin [x]$$

**Problem 123: Result more than twice size of optimal antiderivative.**

$$\int \cos [x] \csc [2 x] \, dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cos [x]]$$

Result (type 3, 21 leaves):

$$\frac{1}{2} \left( -\operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] \right] + \operatorname{Log} \left[ \sin \left[ \frac{x}{2} \right] \right] \right)$$

**Problem 125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \cos[x] \csc[4x] \, dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$-\frac{1}{4} \operatorname{ArcTanh}[\cos[x]] + \frac{\operatorname{ArcTanh}\left[\sqrt{2} \cos[x]\right]}{2\sqrt{2}}$$

Result (type 3, 66 leaves):

$$\frac{1}{4} \left( (1+i) (-1)^{3/4} \operatorname{ArcTanh}\left[\frac{-1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] + \sqrt{2} \operatorname{ArcTanh}\left[\frac{1 + \tan\left[\frac{x}{2}\right]}{\sqrt{2}}\right] - \log\left[\cos\left[\frac{x}{2}\right]\right] + \log\left[\sin\left[\frac{x}{2}\right]\right] \right)$$

**Problem 127: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \csc[6x] \, dx$$

Optimal (type 3, 36 leaves, 7 steps):

$$-\frac{1}{6} \operatorname{ArcTanh}[\cos[x]] - \frac{1}{6} \operatorname{ArcTanh}[2 \cos[x]] + \frac{\operatorname{ArcTanh}\left[\frac{2 \cos[x]}{\sqrt{3}}\right]}{2\sqrt{3}}$$

Result (type 3, 83 leaves):

$$\frac{1}{12} \left( -2\sqrt{3} \operatorname{ArcTanh}\left[\frac{-2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] + 2\sqrt{3} \operatorname{ArcTanh}\left[\frac{2 + \tan\left[\frac{x}{2}\right]}{\sqrt{3}}\right] - 2 \log\left[\cos\left[\frac{x}{2}\right]\right] + \log[1 - 2 \cos[x]] - \log[1 + 2 \cos[x]] + 2 \log\left[\sin\left[\frac{x}{2}\right]\right] \right)$$

**Problem 174: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{a - a \sin[e + f x]} (c + c \sin[e + f x])^{3/2}}{x} \, dx$$

Optimal (type 4, 186 leaves, 11 steps):

$$\begin{aligned} & c \cos[e] \operatorname{CosIntegral}[f x] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} + \\ & \frac{1}{2} c \operatorname{CosIntegral}[2 f x] \operatorname{Sec}[e + f x] \sin[2 e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\ & c \operatorname{Sec}[e + f x] \sin[e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[f x] + \\ & \frac{1}{2} c \cos[2 e] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[2 f x] \end{aligned}$$

Result (type 4, 150 leaves):

$$\left( c e^{-i(e-fx)} \sqrt{-i c e^{-i(e+fx)} (i + e^{i(e+fx)})^2} (2 e^{ie} \operatorname{ExpIntegralEi}[-i f x] + 2 e^{3ie} \operatorname{ExpIntegralEi}[i f x] + i (\operatorname{ExpIntegralEi}[-2 i f x] - e^{4ie} \operatorname{ExpIntegralEi}[2 i f x])) \sqrt{a - a \sin[e + f x]} \right) / \left( 2 \sqrt{2} (1 + e^{2ie(e+fx)}) \right)$$

Problem 175: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a - a \sin[e + f x]} (c + c \sin[e + f x])^{3/2}}{x^2} dx$$

Optimal (type 4, 273 leaves, 13 steps):

$$\begin{aligned} & - \frac{c \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}{x} + \\ & c f \cos[2 e] \operatorname{CosIntegral}[2 f x] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\ & c f \operatorname{CosIntegral}[f x] \operatorname{Sec}[e + f x] \sin[e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\ & \frac{c \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \sin[2 e + 2 f x]}{2 x} - \\ & c f \cos[e] \operatorname{Sec}[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[f x] - \\ & c f \operatorname{Sec}[e + f x] \sin[2 e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[2 f x] \end{aligned}$$

Result (type 4, 231 leaves):

$$\left( c e^{-i(e+fx)} \sqrt{-i c e^{-i(e+fx)} (i + e^{i(e+fx)})^2} \left( (-i - 2 e^{ie(e+fx)} - 2 e^{3ie(e+fx)} + i e^{4ie(e+fx)} - 2 i e^{ie(e+2fx)} f x \operatorname{ExpIntegralEi}[-i f x] + 2 i e^{3ie+2ifx} f x \operatorname{ExpIntegralEi}[i f x] + 2 e^{2ifx} f x \operatorname{ExpIntegralEi}[-2 i f x] + 2 e^{2ie(2e+fx)} f x \operatorname{ExpIntegralEi}[2 i f x]) \sqrt{a - a \sin[e + f x]} \right) \right) / \left( 2 \sqrt{2} (1 + e^{2ie(e+fx)}) x \right)$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a - a \sin[e + f x]} (c + c \sin[e + f x])^{3/2}}{x^3} dx$$

Optimal (type 4, 385 leaves, 15 steps):

$$\begin{aligned}
& - \frac{c \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}{2 x^2} - \frac{1}{2 x} \\
& c f \cos[2 e + 2 f x] \sec[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\
& \frac{1}{2} c f^2 \cos[e] \operatorname{CosIntegral}[f x] \sec[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\
& c f^2 \operatorname{CosIntegral}[2 f x] \sec[e + f x] \sin[2 e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} - \\
& \frac{c \sec[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \sin[2 e + 2 f x]}{4 x^2} + \\
& \frac{1}{2} c f^2 \sec[e + f x] \sin[e] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[f x] - \\
& c f^2 \cos[2 e] \sec[e + f x] \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \operatorname{SinIntegral}[2 f x] + \\
& \frac{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]} \tan[e + f x]}{2 x}
\end{aligned}$$

Result (type 4, 317 leaves):

$$\begin{aligned}
& \frac{1}{4 \sqrt{2} \left( -i + e^{i(e+fx)} \right) \sqrt{-i c e^{-i(e+fx)} \left( i + e^{i(e+fx)} \right)^2 x^2}} \\
& c^2 e^{-2 i(e+fx)} \left( i + e^{i(e+fx)} \right) \left( -1 + 2 i e^{i(e+fx)} + 2 i e^{3 i(e+fx)} + e^{4 i(e+fx)} + 2 i f x + 2 e^{i(e+fx)} f x - \right. \\
& \quad 2 e^{3 i(e+fx)} f x + 2 i e^{4 i(e+fx)} f x + 2 i e^{i(e+2fx)} f^2 x^2 \operatorname{ExpIntegralEi}[-i f x] + \\
& \quad 2 i e^{3 i e + 2 i f x} f^2 x^2 \operatorname{ExpIntegralEi}[i f x] - 4 e^{2 i f x} f^2 x^2 \operatorname{ExpIntegralEi}[-2 i f x] + \\
& \quad \left. 4 e^{2 i(2e+fx)} f^2 x^2 \operatorname{ExpIntegralEi}[2 i f x] \right) \sqrt{a - a \sin[e + f x]}
\end{aligned}$$

**Problem 182: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 \sqrt{a - a \sin[e + f x]}}{(c + c \sin[e + f x])^{3/2}} dx$$

Optimal (type 3, 280 leaves, 34 steps):

$$\begin{aligned}
& - \frac{2 a x}{c f^2 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{2 a \operatorname{ArcTanh}[\sin[e + f x]] \cos[e + f x]}{c f^3 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \\
& \frac{2 a \cos[e + f x] \log[\cos[e + f x]]}{c f^3 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} - \frac{a x^2 \sec[e + f x]}{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \\
& \frac{2 a x \sin[e + f x]}{c f^2 \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}} + \frac{a x^2 \tan[e + f x]}{c f \sqrt{a - a \sin[e + f x]} \sqrt{c + c \sin[e + f x]}}
\end{aligned}$$

Result (type 3, 178 leaves):

$$\begin{aligned}
& - \left( \left( \cos\left[\frac{1}{2}(e + f x)\right] + \sin\left[\frac{1}{2}(e + f x)\right] \right) \sqrt{a - a \sin[e + f x]} \right. \\
& \quad \left( 2 i f x + f^2 x^2 + 2 f x \cos[e + f x] - 2 \log[1 + e^{2 i(e+fx)}] + 2 i f x \sin[e + f x] - \right. \\
& \quad \left. \left. 2 \log[1 + e^{2 i(e+fx)}] \sin[e + f x] + 4 i \operatorname{ArcTan}[e^{i(e+fx)}] (1 + \sin[e + f x]) \right) \right) / \\
& \quad \left( f^3 \left( \cos\left[\frac{1}{2}(e + f x)\right] - \sin\left[\frac{1}{2}(e + f x)\right] \right) (c (1 + \sin[e + f x]))^{3/2} \right)
\end{aligned}$$

**Problem 185: Result more than twice size of optimal antiderivative.**

$$\int (a + a \cos[x]) (A + B \sec[x]) dx$$

Optimal (type 3, 18 leaves, 5 steps):

$$a (A + B) x + a B \operatorname{ArcTanh}[\sin[x]] + a A \sin[x]$$

Result (type 3, 51 leaves):

$$a A x + a B x - a B \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + a B \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + a A \sin[x]$$

**Problem 189: Result more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[x]}{a + a \cos[x]} dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$\frac{B \operatorname{ArcTanh}[\sin[x]]}{a} + \frac{(A - B) \sin[x]}{a + a \cos[x]}$$

Result (type 3, 71 leaves):

$$-\frac{1}{a(1 + \cos[x])} - \frac{2 \cos\left[\frac{x}{2}\right] \left( B \cos\left[\frac{x}{2}\right] \left( \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) + (-A + B) \sin\left[\frac{x}{2}\right] \right)}{a(1 + \cos[x])}$$

**Problem 193: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos[x])^{5/2} (A + B \sec[x]) dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$2 a^{5/2} B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a + a \cos[x]}}\right] + \frac{2 a^3 (32 A + 35 B) \sin[x]}{15 \sqrt{a + a \cos[x]}} + \frac{2}{15} a^2 (8 A + 5 B) \sqrt{a + a \cos[x]} \sin[x] + \frac{2}{5} a A (a + a \cos[x])^{3/2} \sin[x]$$

Result (type 3, 283 leaves):

$$\begin{aligned} & \frac{1}{60} a^2 \sqrt{a (1 + \cos [x])} \sec \left[ \frac{x}{2} \right] \left( -30 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] - \right. \\ & 30 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] + 30 \sqrt{2} B \log \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] - \\ & 15 \sqrt{2} B \log \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] - 15 \sqrt{2} B \log \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] + \\ & \left. 300 A \sin \left[ \frac{x}{2} \right] + 300 B \sin \left[ \frac{x}{2} \right] + 50 A \sin \left[ \frac{3x}{2} \right] + 20 B \sin \left[ \frac{3x}{2} \right] + 6 A \sin \left[ \frac{5x}{2} \right] \right) \end{aligned}$$

**Problem 194: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + a \cos [x])^{3/2} (A + B \sec [x]) dx$$

Optimal (type 3, 72 leaves, 5 steps):

$$2 a^{3/2} B \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [x]}{\sqrt{a + a \cos [x]}} \right] + \frac{2 a^2 (4 A + 3 B) \sin [x]}{3 \sqrt{a + a \cos [x]}} + \frac{2}{3} a A \sqrt{a + a \cos [x]} \sin [x]$$

Result (type 3, 263 leaves):

$$\begin{aligned} & \frac{1}{12} a \sqrt{a (1 + \cos [x])} \sec \left[ \frac{x}{2} \right] \left( -6 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (-1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] - \right. \\ & 6 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - (1 + \sqrt{2}) \sin \left[ \frac{x}{4} \right]}{(-1 + \sqrt{2}) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] + \\ & 6 \sqrt{2} B \log \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] - 3 \sqrt{2} B \log \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] - \\ & \left. 3 \sqrt{2} B \log \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] + 36 A \sin \left[ \frac{x}{2} \right] + 24 B \sin \left[ \frac{x}{2} \right] + 4 A \sin \left[ \frac{3x}{2} \right] \right) \end{aligned}$$

**Problem 195: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + a \cos [x]} (A + B \sec [x]) dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$2 \sqrt{a} B \operatorname{ArcTanh} \left[ \frac{\sqrt{a} \sin [x]}{\sqrt{a + a \cos [x]}} \right] + \frac{2 a A \sin [x]}{\sqrt{a + a \cos [x]}}$$

Result (type 3, 244 leaves):



$$\begin{aligned} & \frac{1}{4} \sqrt{a(1+\cos[x])} \sec\left[\frac{x}{2}\right] \left( -2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - \right. \\ & \quad 2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] + \\ & \quad 2\sqrt{2} B \log\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \log\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - \\ & \quad \left. \sqrt{2} B \log\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] + 8A \sin\left[\frac{x}{2}\right] \right) \end{aligned}$$

**Problem 196: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[x]}{\sqrt{a + a \cos[x]}} dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{2B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{a+a \cos[x]}}\right]}{\sqrt{a}} + \frac{\sqrt{2}(A-B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin[x]}{\sqrt{2}\sqrt{a+a \cos[x]}}\right]}{\sqrt{a}}$$

Result (type 3, 307 leaves):

$$\begin{aligned} & \frac{1}{2\sqrt{a(1+\cos[x])}} \cos\left[\frac{x}{2}\right] \left( -2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (-1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - \right. \\ & \quad 2i\sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos\left[\frac{x}{4}\right] - (1+\sqrt{2})\sin\left[\frac{x}{4}\right]}{(-1+\sqrt{2})\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]}\right] - 4A \log\left[\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]\right] + \\ & \quad 4B \log\left[\cos\left[\frac{x}{4}\right] - \sin\left[\frac{x}{4}\right]\right] + 4A \log\left[\cos\left[\frac{x}{4}\right] + \sin\left[\frac{x}{4}\right]\right] - \\ & \quad 4B \log\left[\cos\left[\frac{x}{4}\right] + \sin\left[\frac{x}{4}\right]\right] + 2\sqrt{2} B \log\left[\sqrt{2} + 2\sin\left[\frac{x}{2}\right]\right] - \\ & \quad \left. \sqrt{2} B \log\left[2 - \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] - \sqrt{2} B \log\left[2 + \sqrt{2}\cos\left[\frac{x}{2}\right] - \sqrt{2}\sin\left[\frac{x}{2}\right]\right] \right) \end{aligned}$$

**Problem 197: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \sec[x]}{(a + a \cos[x])^{3/2}} dx$$

Optimal (type 3, 92 leaves, 7 steps):

$$\frac{2 B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [x]}{\sqrt{a+a \cos [x]}}\right]}{a^{3 / 2}}+\frac{(A-5 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [x]}{\sqrt{2} \sqrt{a+a \cos [x]}}\right]}{2 \sqrt{2} a^{3 / 2}}+\frac{(A-B) \sin [x]}{2(a+a \cos [x])^{3 / 2}}$$

Result (type 3, 524 leaves):

$$\begin{aligned} & \frac{1}{4 a \sqrt{a(1+\cos [x])}} \operatorname{Sec}\left[\frac{x}{2}\right]\left(-4 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos \left[\frac{x}{4}\right]-\left(-1+\sqrt{2}\right) \sin \left[\frac{x}{4}\right]}{\left(1+\sqrt{2}\right) \cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]}\right] \cos \left[\frac{x}{2}\right]^2-\right. \\ & 4 i \sqrt{2} B \operatorname{ArcTan}\left[\frac{\cos \left[\frac{x}{4}\right]-\left(1+\sqrt{2}\right) \sin \left[\frac{x}{4}\right]}{\left(-1+\sqrt{2}\right) \cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]}\right] \cos \left[\frac{x}{2}\right]^2-A \log \left[\cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]\right]+ \\ & 5 B \log \left[\cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]\right]-A \cos [x] \log \left[\cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]\right]+ \\ & 5 B \cos [x] \log \left[\cos \left[\frac{x}{4}\right]-\sin \left[\frac{x}{4}\right]\right]+A \log \left[\cos \left[\frac{x}{4}\right]+\sin \left[\frac{x}{4}\right]\right]- \\ & 5 B \log \left[\cos \left[\frac{x}{4}\right]+\sin \left[\frac{x}{4}\right]\right]+A \cos [x] \log \left[\cos \left[\frac{x}{4}\right]+\sin \left[\frac{x}{4}\right]\right]- \\ & 5 B \cos [x] \log \left[\cos \left[\frac{x}{4}\right]+\sin \left[\frac{x}{4}\right]\right]+2 \sqrt{2} B \log \left[\sqrt{2}+2 \sin \left[\frac{x}{2}\right]\right]+ \\ & 2 \sqrt{2} B \cos [x] \log \left[\sqrt{2}+2 \sin \left[\frac{x}{2}\right]\right]-\sqrt{2} B \log \left[2-\sqrt{2} \cos \left[\frac{x}{2}\right]-\sqrt{2} \sin \left[\frac{x}{2}\right]\right]- \\ & \sqrt{2} B \cos [x] \log \left[2-\sqrt{2} \cos \left[\frac{x}{2}\right]-\sqrt{2} \sin \left[\frac{x}{2}\right]\right]-\sqrt{2} B \log \left[2+\sqrt{2} \cos \left[\frac{x}{2}\right]-\sqrt{2} \sin \left[\frac{x}{2}\right]\right]- \\ & \left.\sqrt{2} B \cos [x] \log \left[2+\sqrt{2} \cos \left[\frac{x}{2}\right]-\sqrt{2} \sin \left[\frac{x}{2}\right]\right]+2 A \sin \left[\frac{x}{2}\right]-2 B \sin \left[\frac{x}{2}\right]\right) \end{aligned}$$

**Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{A+B \operatorname{Sec}[x]}{(a+a \cos [x])^{5 / 2}} d x$$

Optimal (type 3, 120 leaves, 8 steps):

$$\frac{2 B \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [x]}{\sqrt{a+a \cos [x]}}\right]}{a^{5 / 2}}+\frac{(3 A-43 B) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sin [x]}{\sqrt{2} \sqrt{a+a \cos [x]}}\right]}{16 \sqrt{2} a^{5 / 2}}+\frac{(A-B) \sin [x]}{4(a+a \cos [x])^{5 / 2}}+\frac{(3 A-11 B) \sin [x]}{16 a(a+a \cos [x])^{3 / 2}}$$

Result (type 3, 393 leaves):

$$\frac{1}{8 \left( a \left( 1 + \cos [x] \right) \right)^{5/2} \left( B + A \cos [x] \right)}$$

$$\cos \left[ \frac{x}{2} \right]^5 \cos [x] \left( A + B \sec [x] \right) \left( -32 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - \left( -1 + \sqrt{2} \right) \sin \left[ \frac{x}{4} \right]}{\left( 1 + \sqrt{2} \right) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] - \right.$$

$$32 i \sqrt{2} B \operatorname{ArcTan} \left[ \frac{\cos \left[ \frac{x}{4} \right] - \left( 1 + \sqrt{2} \right) \sin \left[ \frac{x}{4} \right]}{\left( -1 + \sqrt{2} \right) \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right]} \right] + 2 \left( -3 A + 43 B \right) \log \left[ \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right] +$$

$$2 \left( 3 A - 43 B \right) \log \left[ \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right] + 32 \sqrt{2} B \log \left[ \sqrt{2} + 2 \sin \left[ \frac{x}{2} \right] \right] -$$

$$16 \sqrt{2} B \log \left[ 2 - \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] - 16 \sqrt{2} B \log \left[ 2 + \sqrt{2} \cos \left[ \frac{x}{2} \right] - \sqrt{2} \sin \left[ \frac{x}{2} \right] \right] +$$

$$\left. \frac{A - B}{\left( \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right)^4} + \frac{3 A - 11 B}{\left( \cos \left[ \frac{x}{4} \right] - \sin \left[ \frac{x}{4} \right] \right)^2} + \frac{-A + B}{\left( \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right)^4} + \frac{-3 A + 11 B}{\left( \cos \left[ \frac{x}{4} \right] + \sin \left[ \frac{x}{4} \right] \right)^2} \right)$$

**Problem 228: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\left( a \cos [c + d x] + b \sin [c + d x] \right)^3} dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh} \left[ \frac{b \cos [c + d x] - a \sin [c + d x]}{\sqrt{a^2 + b^2}} \right]}{2 \left( a^2 + b^2 \right)^{3/2} d} - \frac{b \cos [c + d x] - a \sin [c + d x]}{2 \left( a^2 + b^2 \right) d \left( a \cos [c + d x] + b \sin [c + d x] \right)^2}$$

Result (type 3, 132 leaves):

$$\left( \left( a^2 + b^2 \right) \left( -b \cos [c + d x] + a \sin [c + d x] \right) + \right.$$

$$2 \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{-b + a \tan \left[ \frac{1}{2} (c + d x) \right]}{\sqrt{a^2 + b^2}} \right] \left( a \cos [c + d x] + b \sin [c + d x] \right)^2 \Bigg) /$$

$$\left( 2 \left( a - i b \right)^2 \left( a + i b \right)^2 d \left( a \cos [c + d x] + b \sin [c + d x] \right)^2 \right)$$

**Problem 232: Result unnecessarily involves higher level functions.**

$$\int \left( a \cos [c + d x] + b \sin [c + d x] \right)^{7/2} dx$$

Optimal (type 4, 186 leaves, 4 steps):

$$\begin{aligned}
& -\frac{1}{21d} 10 (a^2 + b^2) (b \cos[c + dx] - a \sin[c + dx]) \sqrt{a \cos[c + dx] + b \sin[c + dx]} - \\
& \frac{2 (b \cos[c + dx] - a \sin[c + dx]) (a \cos[c + dx] + b \sin[c + dx])^{5/2}}{7d} + \\
& \left( 10 (a^2 + b^2)^2 \operatorname{EllipticF}\left[\frac{1}{2} (c + dx - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos[c + dx] + b \sin[c + dx]}{\sqrt{a^2 + b^2}}} \right) / \\
& (21d \sqrt{a \cos[c + dx] + b \sin[c + dx]})
\end{aligned}$$

Result (type 5, 205 leaves):

$$\begin{aligned}
& \frac{1}{42d} \\
& \left( \sqrt{a \cos[c + dx] + b \sin[c + dx]} (-23b(a^2 + b^2) \cos[c + dx] + (-9a^2b + 3b^3) \cos[3(c + dx)]) + 2 \right. \\
& \quad a(13a^2 + 7b^2 + 3(a^2 - 3b^2) \cos[2(c + dx)]) \sin[c + dx] + \\
& \quad \left( 20(a^2 + b^2)^2 \sqrt{\cos[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \right. \right. \\
& \quad \left. \left. \sin[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]]^2 \tan[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]] \right) \right) / \\
& \quad \left( \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin[c + dx + \operatorname{ArcTan}\left[\frac{a}{b}\right]]} \right)
\end{aligned}$$

**Problem 233: Result unnecessarily involves higher level functions.**

$$\int (a \cos[c + dx] + b \sin[c + dx])^{5/2} dx$$

Optimal (type 4, 131 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{5d} 2 (b \cos[c + dx] - a \sin[c + dx]) (a \cos[c + dx] + b \sin[c + dx])^{3/2} + \\
& \left( 6(a^2 + b^2) \operatorname{EllipticE}\left[\frac{1}{2} (c + dx - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos[c + dx] + b \sin[c + dx]} \right) / \\
& \left( 5d \sqrt{\frac{a \cos[c + dx] + b \sin[c + dx]}{\sqrt{a^2 + b^2}}} \right)
\end{aligned}$$

Result (type 5, 256 leaves):

$$\frac{1}{5 b d} \left( \sqrt{a \cos [c+d x]+b \sin [c+d x]} \right. \\
\left. \left( 6 a \left( a^2+b^2 \right)-2 a b^2 \cos \left[ 2 \left( c+d x \right) \right]+b \left( a^2-b^2 \right) \sin \left[ 2 \left( c+d x \right) \right] \right)-\right. \\
\left. \left( 3 \left( a^2+b^2 \right)^2 \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \left( b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\right.\right.\right. \right. \\
\left. \left. \left. \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2 \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]+\sqrt{\sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2} \right. \right. \right. \\
\left. \left. \left. \left( 2 a \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]-b \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \right) \right) \right) \right) / \\
\left. \left( \left( a \sqrt{1+\frac{b^2}{a^2}} \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right] \right)^{3 / 2} \sqrt{\sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]\right]^2} \right) \right)$$

**Problem 234: Result unnecessarily involves higher level functions.**

$$\int (a \cos [c+d x]+b \sin [c+d x])^{3 / 2} d x$$

Optimal (type 4, 131 leaves, 3 steps):

$$-\frac{2 \left( b \cos [c+d x]-a \sin [c+d x] \right) \sqrt{a \cos [c+d x]+b \sin [c+d x]}}{3 d}+ \\
\left( 2 \left( a^2+b^2 \right) \operatorname{EllipticF}\left[\frac{1}{2} \left( c+d x-\operatorname{ArcTan}[a, b] \right), 2\right] \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}} \right) / \\
\left( 3 d \sqrt{a \cos [c+d x]+b \sin [c+d x]} \right)$$

Result (type 5, 143 leaves):

$$\frac{1}{3d} 2 \left( (-b \cos[c+dx] + a \sin[c+dx]) \sqrt{a \cos[c+dx] + b \sin[c+dx]} + \right. \\ \left. (a^2 + b^2) \sqrt{\cos[c+dx + \text{ArcTan}[\frac{a}{b}]]^2} \right. \\ \left. \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[c+dx + \text{ArcTan}[\frac{a}{b}]]^2\right] \tan[c+dx + \text{ArcTan}[\frac{a}{b}]]\right) / \\ \left( \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin[c+dx + \text{ArcTan}[\frac{a}{b}]]} \right)$$

**Problem 235: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a \cos[c+dx] + b \sin[c+dx]} dx$$

Optimal (type 4, 75 leaves, 2 steps):

$$\left( 2 \text{EllipticE}\left[\frac{1}{2}(c+dx - \text{ArcTan}[a, b]), 2\right] \sqrt{a \cos[c+dx] + b \sin[c+dx]} \right) / \\ \left( d \sqrt{\frac{a \cos[c+dx] + b \sin[c+dx]}{\sqrt{a^2 + b^2}}} \right)$$

Result (type 5, 268 leaves):

$$\begin{aligned}
& \left( \cos \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right] \right. \\
& \quad \left. - b (a^2 + b^2) \operatorname{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right]^2 \right] \right. \\
& \quad \left. \sin \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right] + \sqrt{\sin \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right]^2} \left( -2 a (a^2 + b^2) \right. \right. \\
& \quad \left. \left. \cos \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right] + 2 a^2 \sqrt{1 + \frac{b^2}{a^2}} \sqrt{a \sqrt{1 + \frac{b^2}{a^2}} \cos \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right]} \right. \right. \\
& \quad \left. \left. \sqrt{a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right]} + b (a^2 + b^2) \sin \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right] \right) \right) \Bigg/ \\
& \quad \left( b d \left( a \sqrt{1 + \frac{b^2}{a^2}} \cos \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right] \right)^{3/2} \sqrt{\sin \left[ c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right] \right]^2} \right)
\end{aligned}$$

**Problem 236: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a \cos [c + d x] + b \sin [c + d x]}} dx$$

Optimal (type 4, 75 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF} \left[ \frac{1}{2} \left( c + d x - \operatorname{ArcTan} [a, b] \right), 2 \right] \sqrt{\frac{a \cos [c + d x] + b \sin [c + d x]}{\sqrt{a^2 + b^2}}}}{d \sqrt{a \cos [c + d x] + b \sin [c + d x]}}$$

Result (type 5, 92 leaves):

$$\begin{aligned}
& \left( 2 \sqrt{\cos \left[ c + d x + \operatorname{ArcTan} \left[ \frac{a}{b} \right] \right]^2} \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin \left[ c + d x + \operatorname{ArcTan} \left[ \frac{a}{b} \right] \right]^2 \right] \right. \\
& \quad \left. \tan \left[ c + d x + \operatorname{ArcTan} \left[ \frac{a}{b} \right] \right] \right) \Bigg/ \left( d \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin \left[ c + d x + \operatorname{ArcTan} \left[ \frac{a}{b} \right] \right]} \right)
\end{aligned}$$

**Problem 237: Result unnecessarily involves higher level functions and more**

than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a \cos [c+d x]+b \sin [c+d x]\right)^{3 / 2}} d x$$

Optimal (type 4, 138 leaves, 3 steps):

$$\begin{aligned} & -\frac{2\left(b \cos [c+d x]-a \sin [c+d x]\right)}{\left(a^2+b^2\right) d \sqrt{a \cos [c+d x]+b \sin [c+d x]}}- \\ & \left(2 \operatorname{EllipticE}\left[\frac{1}{2}(c+d x-\operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos [c+d x]+b \sin [c+d x]}\right) / \\ & \left(\left(a^2+b^2\right) d \sqrt{\frac{a \cos [c+d x]+b \sin [c+d x]}{\sqrt{a^2+b^2}}}\right) \end{aligned}$$

Result (type 5, 322 leaves):

$$\begin{aligned} & \frac{\sqrt{a \cos [c+d x]+b \sin [c+d x]}\left(-\frac{2}{a b}+\frac{2 \sin [c+d x]}{a(a \cos [c+d x]+b \sin [c+d x])}\right)}{d}-\frac{1}{b d} \\ & \left(-\left(b \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]^2\right]\right.\right. \\ & \left.\left.\sin [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]\right) / \left(a \sqrt{1+\frac{b^2}{a^2}} \sqrt{1-\cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]}\right.\right. \\ & \left.\left.\sqrt{a \sqrt{\frac{a^2+b^2}{a^2}} \cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]] \sqrt{1+\cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]}\right)}\right)- \\ & \left.\frac{2 a^2 \sqrt{1+\frac{b^2}{a^2}} \cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]}{a^2+b^2}-\frac{b \sin [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]}{a \sqrt{1+\frac{b^2}{a^2}}}\right) \\ & \left.\sqrt{a \sqrt{1+\frac{b^2}{a^2}} \cos [c+d x-\operatorname{ArcTan}\left[\frac{b}{a}\right]]}\right) \end{aligned}$$



### Problem 238: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a \cos[c + d x] + b \sin[c + d x])^{5/2}} dx$$

Optimal (type 4, 142 leaves, 3 steps):

$$-\frac{2(b \cos[c + d x] - a \sin[c + d x])}{3(a^2 + b^2) d (a \cos[c + d x] + b \sin[c + d x])^{3/2}} + \frac{2 \operatorname{EllipticF}\left[\frac{1}{2}(c + d x - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{\frac{a \cos[c + d x] + b \sin[c + d x]}{\sqrt{a^2 + b^2}}}}{3(a^2 + b^2) d \sqrt{a \cos[c + d x] + b \sin[c + d x]}}$$

Result (type 5, 145 leaves):

$$\frac{1}{3(a^2 + b^2) d} \left( \frac{-b \cos[c + d x] + a \sin[c + d x]}{(a \cos[c + d x] + b \sin[c + d x])^{3/2}} + \left( \sqrt{\cos[c + d x + \operatorname{ArcTan}\left[\frac{a}{b}\right]]^2} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[c + d x + \operatorname{ArcTan}\left[\frac{a}{b}\right]]^2\right] \right. \right. \\ \left. \left. \tan[c + d x + \operatorname{ArcTan}\left[\frac{a}{b}\right]] \right) / \left( \sqrt{\sqrt{1 + \frac{a^2}{b^2}} b \sin[c + d x + \operatorname{ArcTan}\left[\frac{a}{b}\right]]} \right) \right)$$

### Problem 239: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a \cos[c + d x] + b \sin[c + d x])^{7/2}} dx$$

Optimal (type 4, 197 leaves, 4 steps):

$$-\frac{2(b \cos[c + d x] - a \sin[c + d x])}{5(a^2 + b^2) d (a \cos[c + d x] + b \sin[c + d x])^{5/2}} - \frac{6(b \cos[c + d x] - a \sin[c + d x])}{5(a^2 + b^2)^2 d \sqrt{a \cos[c + d x] + b \sin[c + d x]}} - \frac{\left( 6 \operatorname{EllipticE}\left[\frac{1}{2}(c + d x - \operatorname{ArcTan}[a, b]), 2\right] \sqrt{a \cos[c + d x] + b \sin[c + d x]} \right)}{\left( 5(a^2 + b^2)^2 d \sqrt{\frac{a \cos[c + d x] + b \sin[c + d x]}{\sqrt{a^2 + b^2}}} \right)}$$

Result (type 5, 277 leaves):

$$\begin{aligned}
& \frac{1}{5 b (a^2 + b^2) d} \\
& \left( - \left( \left( 2 \left( 3 a^2 \cos [c + d x]^3 - a b \sin [c + d x] + 6 a b \cos [c + d x]^2 \sin [c + d x] + b^2 \cos [c + d x] \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( 1 + 3 \sin [c + d x]^2 \right) \right) \right) / \left( a \cos [c + d x] + b \sin [c + d x] \right)^{5/2} \right) + \\
& \left( \cos [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]] \left( 3 b \operatorname{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]]^2 \right] \right. \right. \\
& \quad \left. \left. \sin [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]] - 3 \sqrt{\sin [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]]^2} \right. \right. \\
& \quad \left. \left. \left( -2 a \cos [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]] + b \sin [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]] \right) \right) \right) / \\
& \left( \left( a \sqrt{1 + \frac{b^2}{a^2}} \cos [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]] \right)^{3/2} \sqrt{\sin [c + d x - \operatorname{ArcTan} \left[ \frac{b}{a} \right]]^2} \right) \right)
\end{aligned}$$

**Problem 240: Result unnecessarily involves higher level functions.**

$$\int (2 \cos [c + d x] + 3 \sin [c + d x])^{7/2} dx$$

Optimal (type 4, 120 leaves, 4 steps):

$$\begin{aligned}
& \frac{130 \times 13^{3/4} \operatorname{EllipticF} \left[ \frac{1}{2} \left( c + d x - \operatorname{ArcTan} \left[ \frac{3}{2} \right] \right), 2 \right]}{21 d} - \\
& \frac{130 \left( 3 \cos [c + d x] - 2 \sin [c + d x] \right) \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]}}{21 d} - \\
& \frac{2 \left( 3 \cos [c + d x] - 2 \sin [c + d x] \right) \left( 2 \cos [c + d x] + 3 \sin [c + d x] \right)^{5/2}}{7 d}
\end{aligned}$$

Result (type 5, 153 leaves):

$$\begin{aligned}
& \frac{1}{42 d} \left( - \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]} \right. \\
& \quad \left( 897 \cos [c + d x] + 27 \cos [3 (c + d x)] - 598 \sin [c + d x] + 138 \sin [3 (c + d x)] \right) + 260 \times 13^{3/4} \\
& \quad \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin [c + d x + \operatorname{ArcTan} \left[ \frac{2}{3} \right]]^2 \right] \sec [c + d x + \operatorname{ArcTan} \left[ \frac{2}{3} \right]] \\
& \quad \sqrt{- \left( -1 + \sin [c + d x + \operatorname{ArcTan} \left[ \frac{2}{3} \right]] \right) \sin [c + d x + \operatorname{ArcTan} \left[ \frac{2}{3} \right]]} \sqrt{1 + \sin [c + d x + \operatorname{ArcTan} \left[ \frac{2}{3} \right]]} \right)
\end{aligned}$$

**Problem 241: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (2 \cos [c + d x] + 3 \sin [c + d x])^{5/2} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{78 \times 13^{1/4} \operatorname{EllipticE}\left[\frac{1}{2} \left(c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{5 d} - \frac{2 \left(3 \cos [c + d x] - 2 \sin [c + d x]\right) \left(2 \cos [c + d x] + 3 \sin [c + d x]\right)^{3/2}}{5 d}$$

Result (type 5, 199 leaves):

$$\frac{1}{5 d} \left( \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]} \left( 52 - 12 \cos [2 (c + d x)] - 5 \sin [2 (c + d x)] \right) - \frac{13 \times 13^{1/4} \left( 4 \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]] - 3 \sin [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]] \right)}{\sqrt{\cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]}} - \left( 39 \times 13^{1/4} \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]^2\right] \sin [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]] \right) / \left( \sqrt{-\left(-1 + \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]\right) \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]} \sqrt{1 + \cos [c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]]} \right) \right)$$

**Problem 242: Result unnecessarily involves higher level functions.**

$$\int (2 \cos [c + d x] + 3 \sin [c + d x])^{3/2} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{2 \times 13^{3/4} \operatorname{EllipticF}\left[\frac{1}{2} \left(c + d x - \operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{3 d} - \frac{2 \left(3 \cos [c + d x] - 2 \sin [c + d x]\right) \sqrt{2 \cos [c + d x] + 3 \sin [c + d x]}}{3 d}$$

Result (type 5, 133 leaves):

$$\frac{1}{3d} \left( 2 \left( -3 \cos [c+dx] + 2 \sin [c+dx] \right) \sqrt{2 \cos [c+dx] + 3 \sin [c+dx]} + \right. \\ \left. 2 \times 13^{3/4} \text{HypergeometricPFQ} \left[ \left\{ \frac{1}{4}, \frac{1}{2} \right\}, \left\{ \frac{5}{4} \right\}, \sin \left[ c+dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right]^2 \right] \sec \left[ c+dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right] \right. \\ \left. \sqrt{-\left( -1 + \sin \left[ c+dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right] \right) \sin \left[ c+dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right]} \sqrt{1 + \sin \left[ c+dx + \text{ArcTan} \left[ \frac{2}{3} \right] \right]} \right)$$

**Problem 243: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{2 \cos [c+dx] + 3 \sin [c+dx]} \, dx$$

Optimal (type 4, 27 leaves, 2 steps):

$$\frac{2 \times 13^{1/4} \text{EllipticE} \left[ \frac{1}{2} \left( c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right), 2 \right]}{d}$$

Result (type 5, 184 leaves):

$$\frac{1}{3d} \left( -4 \times 13^{1/4} \sqrt{\cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]} + \right. \\ \left. 4 \sqrt{2 \cos [c+dx] + 3 \sin [c+dx]} + \frac{3 \times 13^{1/4} \sin \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]}{\sqrt{\cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right}}} - \right. \\ \left. \left( 3 \times 13^{1/4} \text{HypergeometricPFQ} \left[ \left\{ -\frac{1}{2}, -\frac{1}{4} \right\}, \left\{ \frac{3}{4} \right\}, \cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]^2 \right] \right. \right. \\ \left. \left. \sin \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right] \right) / \left( \sqrt{-\left( -1 + \cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right] \right) \cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]} \right. \right. \\ \left. \left. \sqrt{1 + \cos \left[ c+dx - \text{ArcTan} \left[ \frac{3}{2} \right] \right]} \right) \right)$$

Problem 244: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 \cos [c+d x]+3 \sin [c+d x]}} d x$$

Optimal (type 4, 27 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2}\left(c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{13^{1/4} d}$$

Result (type 5, 88 leaves):

$$\frac{1}{13^{1/4} d} {}_2\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\},\left\{\frac{5}{4}\right\}, \sin \left[c+d x+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]^2\right] \sec \left[c+d x+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right] \\ \sqrt{-\left(-1+\sin \left[c+d x+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right)\right) \sin \left[c+d x+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]} \sqrt{1+\sin \left[c+d x+\operatorname{ArcTan}\left[\frac{2}{3}\right]\right]}$$

Problem 245: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(2 \cos [c+d x]+3 \sin [c+d x]\right)^{3/2}} d x$$

Optimal (type 4, 73 leaves, 3 steps):

$$-\frac{2 \operatorname{EllipticE}\left[\frac{1}{2}\left(c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{13^{3/4} d}-\frac{2\left(3 \cos [c+d x]-2 \sin [c+d x]\right)}{13 d \sqrt{2 \cos [c+d x]+3 \sin [c+d x]}}$$

Result (type 5, 190 leaves):

$$\begin{aligned}
& \frac{1}{3d} \left( \frac{4 \sqrt{\cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]}}{13^{3/4}} - \right. \\
& \frac{2 \cos[c + dx]}{\sqrt{2 \cos[c + dx] + 3 \sin[c + dx]}} - \frac{3 \sin\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]}{13^{3/4} \sqrt{\cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]}} + \\
& \left. \left( 3 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]^2\right] \sin\left[c + dx - \arctan\left[\frac{3}{2}\right]\right] \right) / \right. \\
& \left. \left( 13^{3/4} \sqrt{-\left(-1 + \cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]\right) \cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]} \right. \right. \\
& \left. \left. \sqrt{1 + \cos\left[c + dx - \arctan\left[\frac{3}{2}\right]\right]} \right) \right)
\end{aligned}$$

**Problem 246: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 \cos[c + dx] + 3 \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 75 leaves, 3 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2} \left(c + dx - \arctan\left[\frac{3}{2}\right]\right), 2\right]}{39 \times 13^{1/4} d} - \frac{2 (3 \cos[c + dx] - 2 \sin[c + dx])}{39 d (2 \cos[c + dx] + 3 \sin[c + dx])^{3/2}}$$

Result (type 5, 157 leaves):

$$\begin{aligned}
& \left( -78 \cos[c + dx] + 52 \sin[c + dx] + \right. \\
& \sqrt{2} 13^{3/4} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin\left[c + dx + \arctan\left[\frac{2}{3}\right]\right]^2\right] \\
& \sec\left[c + dx + \arctan\left[\frac{2}{3}\right]\right] (2 \cos[c + dx] + 3 \sin[c + dx])^{3/2} \sqrt{1 + \sin\left[c + dx + \arctan\left[\frac{2}{3}\right]\right]} \\
& \left. \sqrt{-1 + \cos\left[2 \left(c + dx + \arctan\left[\frac{2}{3}\right]\right)\right] + 2 \sin\left[c + dx + \arctan\left[\frac{2}{3}\right]\right]} \right) / \\
& (507 d (2 \cos[c + dx] + 3 \sin[c + dx])^{3/2})
\end{aligned}$$

### Problem 247: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(2 \cos [c+d x]+3 \sin [c+d x])^{7 / 2}} d x$$

Optimal (type 4, 120 leaves, 4 steps):

$$-\frac{6 \operatorname{EllipticE}\left[\frac{1}{2}\left(c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right), 2\right]}{65 \times 13^{3 / 4} d}-\frac{2\left(3 \cos [c+d x]-2 \sin [c+d x]\right)}{65 d\left(2 \cos [c+d x]+3 \sin [c+d x]\right)^{5 / 2}}-\frac{6\left(3 \cos [c+d x]-2 \sin [c+d x]\right)}{845 d \sqrt{2 \cos [c+d x]+3 \sin [c+d x]}}$$

Result (type 5, 224 leaves):

$$\begin{aligned} & \frac{1}{65 d} \left( \frac{4 \sqrt{\cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}}{13^{3 / 4}} + \right. \\ & \quad \left( -33 \cos [c+d x]+5 \cos \left[ 3\left( c+d x\right)\right]-4\left(\sin [c+d x]+3 \sin \left[ 3\left( c+d x\right)\right]\right) \right) / \\ & \quad \left( 2\left( 2 \cos [c+d x]+3 \sin [c+d x]\right)^{5 / 2}\right)-\frac{3 \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}{13^{3 / 4} \sqrt{\cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]}}+ \\ & \quad \left( 3 \operatorname{HypergeometricPFQ}\left[\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\}, \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]^2\right] \sin \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]\right) / \\ & \quad \left( 13^{3 / 4} \sqrt{-\left(-1+\cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right)\right) \cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \right. \\ & \quad \left. \sqrt{1+\cos \left[ c+d x-\operatorname{ArcTan}\left[\frac{3}{2}\right]\right]} \right) \end{aligned}$$

### Problem 267: Result more than twice size of optimal antiderivative.

$$\int (a \sec [x]+b \tan [x]) d x$$

Optimal (type 3, 12 leaves, 3 steps):

$$a \operatorname{ArcTanh}[\sin [x]]-b \operatorname{Log}[\cos [x]]$$

Result (type 3, 42 leaves):

$$-b \operatorname{Log}[\cos [x]]-a \operatorname{Log}\left[\cos \left[\frac{x}{2}\right]-\sin \left[\frac{x}{2}\right]\right]+a \operatorname{Log}\left[\cos \left[\frac{x}{2}\right]+\sin \left[\frac{x}{2}\right]\right]$$

**Problem 274: Result more than twice size of optimal antiderivative.**

$$\int (\sec [x] + \tan [x])^4 dx$$

Optimal (type 3, 30 leaves, 5 steps):

$$x + \frac{2 \cos [x]^3}{3 (1 - \sin [x])^3} - \frac{2 \cos [x]}{1 - \sin [x]}$$

Result (type 3, 64 leaves):

$$- \left( \left( -3 (8 + 3 x) \cos \left[ \frac{x}{2} \right] + (16 + 3 x) \cos \left[ \frac{3 x}{2} \right] + 6 (4 + 2 x + x \cos [x]) \sin \left[ \frac{x}{2} \right] \right) / \right. \\ \left. \left( 6 \left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right)^3 \right) \right)$$

**Problem 275: Result more than twice size of optimal antiderivative.**

$$\int (\sec [x] + \tan [x])^3 dx$$

Optimal (type 3, 18 leaves, 4 steps):

$$\log [1 - \sin [x]] + \frac{2}{1 - \sin [x]}$$

Result (type 3, 38 leaves):

$$2 \log \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + \frac{2}{\left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right)^2}$$

**Problem 277: Result more than twice size of optimal antiderivative.**

$$\int (\sec [x] + \tan [x]) dx$$

Optimal (type 3, 13 leaves, 3 steps):

$$-2 \log \left[ \cos \left[ \frac{1}{4} (\pi + 2 x) \right] \right]$$

Result (type 3, 38 leaves):

$$- \log [\cos [x]] - \log \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + \log \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right]$$

**Problem 278: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sec [x] + \tan [x]} dx$$

Optimal (type 3, 5 leaves, 3 steps):



$$\text{Log}[1 + \text{Sin}[x]]$$

Result (type 3, 16 leaves):

$$2 \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]$$

**Problem 280: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(\sec[x] + \tan[x])^3} dx$$

Optimal (type 3, 16 leaves, 4 steps):

$$-\text{Log}[1 + \text{Sin}[x]] - \frac{2}{1 + \text{Sin}[x]}$$

Result (type 3, 34 leaves):

$$-2 \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \frac{2}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^2}$$

**Problem 281: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(\sec[x] + \tan[x])^4} dx$$

Optimal (type 3, 26 leaves, 4 steps):

$$x - \frac{2 \cos[x]^3}{3 (1 + \text{Sin}[x])^3} + \frac{2 \cos[x]}{1 + \text{Sin}[x]}$$

Result (type 3, 62 leaves):

$$\left( 3 (-8 + 3x) \cos\left[\frac{x}{2}\right] + (16 - 3x) \cos\left[\frac{3x}{2}\right] + 6 (-4 + 2x + x \cos[x]) \sin\left[\frac{x}{2}\right] \right) / \left( 6 \left( \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right)^3 \right)$$

**Problem 287: Result more than twice size of optimal antiderivative.**

$$\int (a \cot[x] + b \csc[x]) dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-b \text{ArcTanh}[\cos[x]] + a \text{Log}[\sin[x]]$$

Result (type 3, 25 leaves):

$$-b \text{Log}\left[\cos\left[\frac{x}{2}\right]\right] + b \text{Log}\left[\sin\left[\frac{x}{2}\right]\right] + a \text{Log}[\sin[x]]$$

**Problem 297: Result more than twice size of optimal antiderivative.**

$$\int (\cot [x] + \csc [x]) \, dx$$

Optimal (type 3, 9 leaves, 3 steps):

$$-\text{ArcTanh}[\cos [x]] + \text{Log}[\sin [x]]$$

Result (type 3, 20 leaves):

$$-\text{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \text{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \text{Log}[\sin [x]]$$

**Problem 306: Result more than twice size of optimal antiderivative.**

$$\int (\csc [x] - \sin [x]) \, dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcTanh}[\cos [x]] + \cos [x]$$

Result (type 3, 19 leaves):

$$\cos [x] - \text{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \text{Log}\left[\sin\left[\frac{x}{2}\right]\right]$$

**Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(\csc [x] - \sin [x])^4} \, dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$\frac{\tan [x]^5}{5} + \frac{\tan [x]^7}{7}$$

Result (type 3, 37 leaves):

$$\frac{2 \tan [x]}{35} + \frac{1}{35} \sec [x]^2 \tan [x] - \frac{8}{35} \sec [x]^4 \tan [x] + \frac{1}{7} \sec [x]^6 \tan [x]$$

**Problem 312: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(\csc [x] - \sin [x])^6} \, dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\frac{\tan [x]^7}{7} + \frac{2 \tan [x]^9}{9} + \frac{\tan [x]^{11}}{11}$$

Result (type 3, 57 leaves):

$$-\frac{8 \tan [x]}{693}-\frac{4}{693} \sec [x]^2 \tan [x]-\frac{1}{231} \sec [x]^4 \tan [x]+$$

$$\frac{113}{693} \sec [x]^6 \tan [x]-\frac{23}{99} \sec [x]^8 \tan [x]+\frac{1}{11} \sec [x]^{10} \tan [x]$$

**Problem 318: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{\csc [x]-\sin [x]}} d x$$

Optimal (type 3, 60 leaves, 8 steps):

$$\frac{\frac{\operatorname{ArcTan}\left[\sqrt{-\sin [x]}\right] \cos [x]}{\sqrt{\cos [x] \cot [x] \sqrt{-\sin [x]}}}-\frac{\operatorname{ArcTanh}\left[\sqrt{-\sin [x]}\right] \cos [x]}{\sqrt{\cos [x] \cot [x] \sqrt{-\sin [x]}}}}$$

Result (type 5, 37 leaves):

$$2 \sqrt{\cos [x] \cot [x]} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sec [x]^2\right] \sec [x](-\tan [x]^2)^{1 / 4}$$

**Problem 319: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(\csc [x]-\sin [x]\right)^{3 / 2}} d x$$

Optimal (type 3, 80 leaves, 9 steps):

$$\frac{\frac{\sec [x]}{2 \sqrt{\cos [x] \cot [x]}}+\frac{\frac{\operatorname{ArcTan}\left[\sqrt{-\sin [x]}\right] \cot [x] \sqrt{-\sin [x]}}{4 \sqrt{\cos [x] \cot [x]}}}{\frac{\operatorname{ArcTanh}\left[\sqrt{-\sin [x]}\right] \cot [x] \sqrt{-\sin [x]}}{4 \sqrt{\cos [x] \cot [x]}}}+$$

Result (type 5, 42 leaves):

$$\frac{\sec [x]\left(3+\frac{\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sec [x]^2\right]}{(-\tan [x]^2)^{1 / 4}}\right)}{6 \sqrt{\cos [x] \cot [x]}}$$

**Problem 320: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\left(\csc [x]-\sin [x]\right)^{5 / 2}} d x$$

Optimal (type 3, 99 leaves, 10 steps):

$$\begin{aligned}
& - \frac{3 \operatorname{ArcTan}[\sqrt{-\sin[x]}] \cos[x]}{32 \sqrt{\cos[x] \cot[x]} \sqrt{-\sin[x]}} + \\
& \frac{3 \operatorname{ArcTanh}[\sqrt{-\sin[x]}] \cos[x]}{32 \sqrt{\cos[x] \cot[x]} \sqrt{-\sin[x]}} - \frac{3 \tan[x]}{16 \sqrt{\cos[x] \cot[x]}} + \frac{\sec[x]^2 \tan[x]}{4 \sqrt{\cos[x] \cot[x]}}
\end{aligned}$$

Result (type 5, 57 leaves):

$$\begin{aligned}
& \left( (5 - 3 \cos[2x]) \sec[x]^2 \tan[x] - \right. \\
& \left. 6 \cot[x] \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \sec[x]^2\right] (-\tan[x]^2)^{1/4} \right) / \left( 32 \sqrt{\cos[x] \cot[x]} \right)
\end{aligned}$$

**Problem 321: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(\csc[x] - \sin[x])^{7/2}} dx$$

Optimal (type 3, 118 leaves, 11 steps):

$$\begin{aligned}
& \frac{5 \sec[x]}{192 \sqrt{\cos[x] \cot[x]}} - \frac{5 \sec[x]^3}{48 \sqrt{\cos[x] \cot[x]}} - \frac{5 \operatorname{ArcTan}[\sqrt{-\sin[x]}] \cot[x] \sqrt{-\sin[x]}}{128 \sqrt{\cos[x] \cot[x]}} - \\
& \frac{5 \operatorname{ArcTanh}[\sqrt{-\sin[x]}] \cot[x] \sqrt{-\sin[x]}}{128 \sqrt{\cos[x] \cot[x]}} + \frac{\sec[x]^3 \tan[x]^2}{6 \sqrt{\cos[x] \cot[x]}}
\end{aligned}$$

Result (type 5, 63 leaves):

$$\begin{aligned}
& \frac{1}{192} \sqrt{\cos[x] \cot[x]} \csc[x] \sec[x] \left( -5 + 57 \sec[x]^2 - 84 \sec[x]^4 + \right. \\
& \left. 32 \sec[x]^6 + 5 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \sec[x]^2\right] (-\tan[x]^2)^{3/4} \right)
\end{aligned}$$

**Problem 323: Result more than twice size of optimal antiderivative.**

$$\int (-\cos[x] + \sec[x])^3 dx$$

Optimal (type 3, 34 leaves, 6 steps):

$$-\frac{5}{2} \operatorname{ArcTanh}[\sin[x]] + \frac{5 \sin[x]}{2} + \frac{5 \sin[x]^3}{6} + \frac{1}{2} \sin[x]^3 \tan[x]^2$$

Result (type 3, 85 leaves):

$$\begin{aligned}
& \frac{1}{12} \left( 30 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] - 30 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] + \right. \\
& \left. \frac{3}{\left(\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right)^2} - \frac{3}{\left(\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right)^2} + 27 \sin[x] - \sin[3x] \right)
\end{aligned}$$

### Problem 325: Result more than twice size of optimal antiderivative.

$$\int (-\cos[x] + \sec[x]) \, dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$\text{ArcTanh}[\sin[x]] - \sin[x]$$

Result (type 3, 37 leaves):

$$-\text{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \text{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - \sin[x]$$

### Problem 328: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-\cos[x] + \sec[x])^3} \, dx$$

Optimal (type 3, 17 leaves, 4 steps):

$$\frac{\csc[x]^3}{3} - \frac{\csc[x]^5}{5}$$

Result (type 3, 93 leaves):

$$\begin{aligned} & \frac{11}{240} \cot\left[\frac{x}{2}\right] + \frac{11}{480} \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^2 - \frac{1}{160} \cot\left[\frac{x}{2}\right] \csc\left[\frac{x}{2}\right]^4 + \\ & \frac{11}{240} \tan\left[\frac{x}{2}\right] + \frac{11}{480} \sec\left[\frac{x}{2}\right]^2 \tan\left[\frac{x}{2}\right] - \frac{1}{160} \sec\left[\frac{x}{2}\right]^4 \tan\left[\frac{x}{2}\right] \end{aligned}$$

### Problem 329: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-\cos[x] + \sec[x])^4} \, dx$$

Optimal (type 3, 17 leaves, 2 steps):

$$-\frac{1}{5} \cot[x]^5 - \frac{\cot[x]^7}{7}$$

Result (type 3, 37 leaves):

$$-\frac{2 \cot[x]}{35} - \frac{1}{35} \cot[x] \csc[x]^2 + \frac{8}{35} \cot[x] \csc[x]^4 - \frac{1}{7} \cot[x] \csc[x]^6$$

### Problem 330: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(-\cos[x] + \sec[x])^5} \, dx$$

Optimal (type 3, 25 leaves, 4 steps):

$$-\frac{1}{5} \operatorname{Csc}[x]^5 + \frac{2 \operatorname{Csc}[x]^7}{7} - \frac{\operatorname{Csc}[x]^9}{9}$$

Result (type 3, 165 leaves):

$$\begin{aligned} & -\frac{649 \operatorname{Cot}\left[\frac{x}{2}\right]}{80640} - \frac{649 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^2}{161280} - \frac{31 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^4}{53760} + \frac{37 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^6}{32256} - \frac{\operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^8}{4608} \\ & -\frac{649 \operatorname{Tan}\left[\frac{x}{2}\right]}{80640} - \frac{649 \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right]}{161280} - \frac{31 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \operatorname{Tan}\left[\frac{x}{2}\right]}{53760} + \frac{37 \operatorname{Sec}\left[\frac{x}{2}\right]^6 \operatorname{Tan}\left[\frac{x}{2}\right]}{32256} - \frac{\operatorname{Sec}\left[\frac{x}{2}\right]^8 \operatorname{Tan}\left[\frac{x}{2}\right]}{4608} \end{aligned}$$

**Problem 331: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^6} dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$-\frac{1}{7} \operatorname{Cot}[x]^7 - \frac{2 \operatorname{Cot}[x]^9}{9} - \frac{\operatorname{Cot}[x]^{11}}{11}$$

Result (type 3, 57 leaves):

$$\begin{aligned} & \frac{8 \operatorname{Cot}[x]}{693} + \frac{4}{693} \operatorname{Cot}[x] \operatorname{Csc}[x]^2 + \frac{1}{231} \operatorname{Cot}[x] \operatorname{Csc}[x]^4 - \\ & \frac{113}{693} \operatorname{Cot}[x] \operatorname{Csc}[x]^6 + \frac{23}{99} \operatorname{Cot}[x] \operatorname{Csc}[x]^8 - \frac{1}{11} \operatorname{Cot}[x] \operatorname{Csc}[x]^{10} \end{aligned}$$

**Problem 332: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-\cos[x] + \sec[x])^7} dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\operatorname{Csc}[x]^7}{7} - \frac{\operatorname{Csc}[x]^9}{3} + \frac{3 \operatorname{Csc}[x]^{11}}{11} - \frac{\operatorname{Csc}[x]^{13}}{13}$$

Result (type 3, 237 leaves):

$$\begin{aligned} & \frac{10027 \operatorname{Cot}\left[\frac{x}{2}\right]}{6150144} + \frac{10027 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^2}{12300288} + \frac{755 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^4}{4100096} - \\ & \frac{101 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^6}{768768} - \frac{101 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^8}{878592} + \frac{79 \operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^{10}}{1171456} - \\ & \frac{\operatorname{Cot}\left[\frac{x}{2}\right] \operatorname{Csc}\left[\frac{x}{2}\right]^{12}}{106496} + \frac{10027 \operatorname{Tan}\left[\frac{x}{2}\right]}{6150144} + \frac{10027 \operatorname{Sec}\left[\frac{x}{2}\right]^2 \operatorname{Tan}\left[\frac{x}{2}\right]}{12300288} + \frac{755 \operatorname{Sec}\left[\frac{x}{2}\right]^4 \operatorname{Tan}\left[\frac{x}{2}\right]}{4100096} - \\ & \frac{101 \operatorname{Sec}\left[\frac{x}{2}\right]^6 \operatorname{Tan}\left[\frac{x}{2}\right]}{768768} - \frac{101 \operatorname{Sec}\left[\frac{x}{2}\right]^8 \operatorname{Tan}\left[\frac{x}{2}\right]}{878592} + \frac{79 \operatorname{Sec}\left[\frac{x}{2}\right]^{10} \operatorname{Tan}\left[\frac{x}{2}\right]}{1171456} - \frac{\operatorname{Sec}\left[\frac{x}{2}\right]^{12} \operatorname{Tan}\left[\frac{x}{2}\right]}{106496} \end{aligned}$$

### Problem 337: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{-\cos[x] + \sec[x]}} dx$$

Optimal (type 3, 52 leaves, 8 steps):

$$\frac{\frac{\text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} - \frac{\text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}}{1}$$

Result (type 5, 37 leaves):

$$-2 \left( -\cot[x]^2 \right)^{1/4} \csc[x] \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[x]^2\right] \sqrt{\sin[x] \tan[x]}$$

### Problem 338: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(-\cos[x] + \sec[x])^{3/2}} dx$$

Optimal (type 3, 72 leaves, 9 steps):

$$-\frac{\csc[x]}{2 \sqrt{\sin[x] \tan[x]}} + \frac{\frac{\text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}}{4 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} + \frac{\frac{\text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}}{4 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}$$

Result (type 5, 49 leaves):

$$\frac{\frac{1}{6} \csc[x] \left( -3 \cot[x]^2 + (-\cot[x]^2)^{3/4} \text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[x]^2\right] \right)}{\sec[x] \sqrt{\sin[x] \tan[x]}}$$

### Problem 339: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(-\cos[x] + \sec[x])^{5/2}} dx$$

Optimal (type 3, 91 leaves, 10 steps):

$$\frac{\frac{3 \cot[x]}{16 \sqrt{\sin[x] \tan[x]}} - \frac{\cot[x] \csc[x]^2}{4 \sqrt{\sin[x] \tan[x]}}}{1} + \frac{\frac{3 \text{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{32 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} + \frac{3 \text{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{32 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}}{1}$$

Result (type 5, 53 leaves):

$$\frac{\csc[x] \left( -5 - 3 \cos[2x] + \frac{6 \cos[x]^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \csc[x]^2\right]}{(-\cot[x]^2)^{7/4}} \right)}{32 (\sin[x] \tan[x])^{3/2}}$$

### Problem 340: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(-\cos[x] + \sec[x])^{7/2}} dx$$

Optimal (type 3, 110 leaves, 11 steps):

$$-\frac{5 \csc[x]}{192 \sqrt{\sin[x] \tan[x]}} + \frac{5 \csc[x]^3}{48 \sqrt{\sin[x] \tan[x]}} - \frac{\cot[x]^2 \csc[x]^3}{6 \sqrt{\sin[x] \tan[x]}} - \frac{5 \operatorname{ArcTan}[\sqrt{\cos[x]}] \sin[x]}{128 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}} - \frac{5 \operatorname{ArcTanh}[\sqrt{\cos[x]}] \sin[x]}{128 \sqrt{\cos[x]} \sqrt{\sin[x] \tan[x]}}$$

Result (type 5, 63 leaves):

$$-\frac{1}{192} \csc[x] \left( -5 + 57 \csc[x]^2 - 84 \csc[x]^4 + 32 \csc[x]^6 + 5 (-\cot[x]^2)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \csc[x]^2\right] \right) \sec[x] \sqrt{\sin[x] \tan[x]}$$

### Problem 341: Result more than twice size of optimal antiderivative.

$$\int (\sin[x] + \tan[x])^4 dx$$

Optimal (type 3, 55 leaves, 18 steps):

$$-\frac{61x}{8} - 2 \operatorname{ArcTanh}[\sin[x]] + \frac{19}{8} \cos[x] \sin[x] + \frac{1}{4} \cos[x]^3 \sin[x] - \frac{4 \sin[x]^3}{3} + 5 \tan[x] + 2 \sec[x] \tan[x] + \frac{\tan[x]^3}{3}$$

Result (type 3, 129 leaves):

$$\frac{1}{768} \sec[x]^3 \left( -72 \cos[x] \left( 61x - 16 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 16 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) - 24 \cos[3x] \left( 61x - 16 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 16 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] \right) + 1395 \sin[x] + 672 \sin[2x] + 1265 \sin[3x] + 129 \sin[5x] + 32 \sin[6x] + 3 \sin[7x] \right)$$

### Problem 343: Result more than twice size of optimal antiderivative.

$$\int (\sin[x] + \tan[x])^2 dx$$

Optimal (type 3, 25 leaves, 9 steps):

$$-\frac{x}{2} + 2 \operatorname{ArcTanh}[\sin[x]] - 2 \sin[x] - \frac{1}{2} \cos[x] \sin[x] + \tan[x]$$

Result (type 3, 60 leaves):



$$-\frac{x}{2} - 2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + 2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right] - 2 \sin[x] - \frac{1}{8} \sec[x] \sin[3x] + \frac{7 \tan[x]}{8}$$

**Problem 351: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + C \sin[x]}{(b \cos[x] + c \sin[x])^3} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{A \operatorname{ArcTanh}\left[\frac{c \cos[x] - b \sin[x]}{\sqrt{b^2 + c^2}}\right]}{2 (b^2 + c^2)^{3/2}} + \frac{b C - A c \cos[x] + A b \sin[x]}{2 (b^2 + c^2) (b \cos[x] + c \sin[x])^2} - \frac{c^2 C \cos[x] - b c C \sin[x]}{(b^2 + c^2)^2 (b \cos[x] + c \sin[x])}$$

Result (type 3, 132 leaves):

$$\left( 2 A b \sqrt{b^2 + c^2} \operatorname{ArcTanh}\left[\frac{-c + b \tan\left[\frac{x}{2}\right]}{\sqrt{b^2 + c^2}}\right] (b \cos[x] + c \sin[x])^2 + \right. \\ \left. (b^2 + c^2) (-A b c \cos[x] + A b^2 \sin[x] + 2 c^2 C \sin[x]^2 + b C (b + c \sin[2x])) \right) / \\ (2 b (b - i c)^2 (b + i c)^2 (b \cos[x] + c \sin[x])^2)$$

**Problem 354: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{A + B \cos[x]}{(b \cos[x] + c \sin[x])^3} dx$$

Optimal (type 3, 116 leaves, 4 steps):

$$-\frac{A \operatorname{ArcTanh}\left[\frac{c \cos[x] - b \sin[x]}{\sqrt{b^2 + c^2}}\right]}{2 (b^2 + c^2)^{3/2}} - \frac{B c + A c \cos[x] - A b \sin[x]}{2 (b^2 + c^2) (b \cos[x] + c \sin[x])^2} - \frac{b B c \cos[x] - b^2 B \sin[x]}{(b^2 + c^2)^2 (b \cos[x] + c \sin[x])}$$

Result (type 3, 118 leaves):

$$\left( 2 A \sqrt{b^2 + c^2} \operatorname{ArcTanh}\left[\frac{-c + b \tan\left[\frac{x}{2}\right]}{\sqrt{b^2 + c^2}}\right] (b \cos[x] + c \sin[x])^2 + \right. \\ \left. (b^2 + c^2) (-A c \cos[x] - B c \cos[2x] + b (A + 2 B \cos[x]) \sin[x]) \right) / \\ (2 (b - i c)^2 (b + i c)^2 (b \cos[x] + c \sin[x])^2)$$

**Problem 355: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x] \right)^4 dx$$

Optimal (type 3, 246 leaves, 6 steps):

$$\begin{aligned} & \frac{35}{8} (b^2 + c^2)^2 x - \frac{35 c (b^2 + c^2)^{3/2} \cos [d + e x]}{8 e} + \frac{35 b (b^2 + c^2)^{3/2} \sin [d + e x]}{8 e} - \frac{1}{24 e} \\ & 35 (b^2 + c^2) (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right) - \\ & \frac{1}{12 e} 7 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^2 - \\ & \frac{1}{4 e} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^3 \end{aligned}$$

Result (type 3, 238 leaves):

$$\begin{aligned} & \frac{1}{96 e} \left( 420 (b^2 + c^2)^2 (d + e x) - \right. \\ & 672 (b - i c) (b + i c) c \sqrt{b^2 + c^2} \cos [d + e x] - 336 b c (b^2 + c^2) \cos [2 (d + e x)] + \\ & 32 c (-3 b^2 + c^2) \sqrt{b^2 + c^2} \cos [3 (d + e x)] - 12 b c (b^2 - c^2) \cos [4 (d + e x)] + \\ & 672 b (b - i c) (b + i c) \sqrt{b^2 + c^2} \sin [d + e x] + 168 (b^4 - c^4) \sin [2 (d + e x)] + \\ & \left. 32 b (b^2 - 3 c^2) \sqrt{b^2 + c^2} \sin [3 (d + e x)] + 3 (b^4 - 6 b^2 c^2 + c^4) \sin [4 (d + e x)] \right) \end{aligned}$$

**Problem 356: Result unnecessarily involves imaginary or complex numbers.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^3 dx$$

Optimal (type 3, 178 leaves, 5 steps):

$$\begin{aligned} & \frac{5}{2} (b^2 + c^2)^{3/2} x - \frac{5 c (b^2 + c^2) \cos [d + e x]}{2 e} + \frac{5 b (b^2 + c^2) \sin [d + e x]}{2 e} - \frac{1}{6 e} \\ & 5 \sqrt{b^2 + c^2} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right) - \\ & \frac{1}{3 e} (c \cos [d + e x] - b \sin [d + e x]) \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^2 \end{aligned}$$

Result (type 3, 163 leaves):

$$\begin{aligned} & \frac{1}{12 e} \left( 30 (b - i c) (b + i c) \sqrt{b^2 + c^2} (d + e x) - 45 c (b^2 + c^2) \cos [d + e x] - \right. \\ & 18 b c \sqrt{b^2 + c^2} \cos [2 (d + e x)] + c (-3 b^2 + c^2) \cos [3 (d + e x)] + 45 b (b^2 + c^2) \sin [d + e x] + \\ & \left. 9 (b^2 - c^2) \sqrt{b^2 + c^2} \sin [2 (d + e x)] + b (b^2 - 3 c^2) \sin [3 (d + e x)] \right) \end{aligned}$$

**Problem 361: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^3} dx$$

Optimal (type 3, 191 leaves, 3 steps):

$$\begin{aligned}
& - \frac{c \cos [d+e x]-b \sin [d+e x]}{5 \sqrt{b^2+c^2} e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^3} - \\
& \frac{2(c \cos [d+e x]-b \sin [d+e x])}{15\left(b^2+c^2\right) e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^2} - \\
& \frac{2\left(c-\sqrt{b^2+c^2} \sin [d+e x]\right)}{15 c\left(b^2+c^2\right) e(c \cos [d+e x]-b \sin [d+e x])}
\end{aligned}$$

Result (type 3, 420 leaves):

$$\begin{aligned}
& \frac{1}{120 c\left(b^2+c^2\right) e(c \cos [d+e x]-b \sin [d+e x])^5} \\
& \left(-76 b^4 c-152 b^2 c^3-76 c^5+90 b c\left(b^2+c^2\right)^{3 / 2} \cos [d+e x]+20 c\left(-b^4+c^4\right) \cos [2(d+e x)]+\right. \\
& 10 b^3 c \sqrt{b^2+c^2} \cos [3(d+e x)]+10 b c^3 \sqrt{b^2+c^2} \cos [3(d+e x)]- \\
& 4 b^3 c \sqrt{b^2+c^2} \cos [5(d+e x)]+4 b c^3 \sqrt{b^2+c^2} \cos [5(d+e x)]+10 b^4 \sqrt{b^2+c^2} \sin [d+e x]+ \\
& 110 b^2 c^2 \sqrt{b^2+c^2} \sin [d+e x]+100 c^4 \sqrt{b^2+c^2} \sin [d+e x]-40 b^3 c^2 \sin [2(d+e x)]- \\
& 40 b c^4 \sin [2(d+e x)]-5 b^4 \sqrt{b^2+c^2} \sin [3(d+e x)]+5 c^4 \sqrt{b^2+c^2} \sin [3(d+e x)]+ \\
& \left. b^4 \sqrt{b^2+c^2} \sin [5(d+e x)]-6 b^2 c^2 \sqrt{b^2+c^2} \sin [5(d+e x)]+c^4 \sqrt{b^2+c^2} \sin [5(d+e x)]\right)
\end{aligned}$$

**Problem 362: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^4} d x$$

Optimal (type 3, 259 leaves, 4 steps):

$$\begin{aligned}
& - \frac{c \cos [d+e x]-b \sin [d+e x]}{7 \sqrt{b^2+c^2} e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^4} - \\
& \frac{3(c \cos [d+e x]-b \sin [d+e x])}{35\left(b^2+c^2\right) e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^3} - \\
& \frac{2(c \cos [d+e x]-b \sin [d+e x])}{35\left(b^2+c^2\right)^{3 / 2} e\left(\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]\right)^2} - \\
& \frac{2\left(c-\sqrt{b^2+c^2} \sin [d+e x]\right)}{35 c\left(b^2+c^2\right)^{3 / 2} e(c \cos [d+e x]-b \sin [d+e x])}
\end{aligned}$$

Result (type 3, 533 leaves):

$$\frac{1}{1120 c (b^2 + c^2) e (-c \cos [d + e x] + b \sin [d + e x])^7} \\ \left( 832 b^4 c \sqrt{b^2 + c^2} + 1664 b^2 c^3 \sqrt{b^2 + c^2} + 832 c^5 \sqrt{b^2 + c^2} - 1190 b c (b^2 + c^2)^2 \cos [d + e x] + \right. \\ 448 c \sqrt{b^2 + c^2} (b^4 - c^4) \cos [2 (d + e x)] - 112 b^5 c \cos [3 (d + e x)] + 56 b^3 c^3 \cos [3 (d + e x)] + \\ 168 b c^5 \cos [3 (d + e x)] + 28 b^5 c \cos [5 (d + e x)] - 28 b c^5 \cos [5 (d + e x)] - \\ 6 b^5 c \cos [7 (d + e x)] + 20 b^3 c^3 \cos [7 (d + e x)] - 6 b c^5 \cos [7 (d + e x)] - \\ 35 b^6 \sin [d + e x] - 1295 b^4 c^2 \sin [d + e x] - 2485 b^2 c^4 \sin [d + e x] - 1225 c^6 \sin [d + e x] + \\ 896 b^3 c^2 \sqrt{b^2 + c^2} \sin [2 (d + e x)] + 896 b c^4 \sqrt{b^2 + c^2} \sin [2 (d + e x)] + 21 b^6 \sin [3 (d + e x)] - \\ 189 b^4 c^2 \sin [3 (d + e x)] - 161 b^2 c^4 \sin [3 (d + e x)] + 49 c^6 \sin [3 (d + e x)] - \\ 7 b^6 \sin [5 (d + e x)] + 35 b^4 c^2 \sin [5 (d + e x)] + 35 b^2 c^4 \sin [5 (d + e x)] - 7 c^6 \sin [5 (d + e x)] + \\ \left. b^6 \sin [7 (d + e x)] - 15 b^4 c^2 \sin [7 (d + e x)] + 15 b^2 c^4 \sin [7 (d + e x)] - c^6 \sin [7 (d + e x)] \right)$$

**Problem 366: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{2 a + 2 a \cos [d + e x] + 2 c \sin [d + e x]} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{\log \left[ a + c \tan \left[ \frac{1}{2} (d + e x) \right] \right]}{2 c e}$$

Result (type 3, 57 leaves):

$$\frac{1}{2} \left( -\frac{\log \left[ \cos \left[ \frac{1}{2} (d + e x) \right] \right]}{c e} + \frac{\log \left[ a \cos \left[ \frac{1}{2} (d + e x) \right] + c \sin \left[ \frac{1}{2} (d + e x) \right] \right]}{c e} \right)$$

**Problem 369: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 a + 2 a \cos [d + e x] + 2 c \sin [d + e x])^4} dx$$

Optimal (type 3, 207 leaves, 5 steps):

$$-\frac{a (5 a^2 + 3 c^2) \log \left[ a + c \tan \left[ \frac{1}{2} (d + e x) \right] \right]}{32 c^7 e} - \\ \frac{c \cos [d + e x] - a \sin [d + e x]}{48 c^2 e (a + a \cos [d + e x] + c \sin [d + e x])^3} + \frac{5 (a c \cos [d + e x] - a^2 \sin [d + e x])}{96 c^4 e (a + a \cos [d + e x] + c \sin [d + e x])^2} - \\ \frac{c (15 a^2 + 4 c^2) \cos [d + e x] - a (15 a^2 + 4 c^2) \sin [d + e x]}{96 c^6 e (a + a \cos [d + e x] + c \sin [d + e x])}$$

Result (type 3, 492 leaves):

$$\begin{aligned}
& \frac{1}{384 c^7 e \left( a + a \cos [d + e x] + c \sin [d + e x] \right)^4} \\
& \cos \left[ \frac{1}{2} (d + e x) \right] \left( a \cos \left[ \frac{1}{2} (d + e x) \right] + c \sin \left[ \frac{1}{2} (d + e x) \right] \right) \\
& \left( 192 (5 a^3 + 3 a c^2) \cos \left[ \frac{1}{2} (d + e x) \right]^3 \log \left[ \cos \left[ \frac{1}{2} (d + e x) \right] \right] \right. \\
& \quad \left. \left( a \cos \left[ \frac{1}{2} (d + e x) \right] + c \sin \left[ \frac{1}{2} (d + e x) \right] \right)^3 - 192 (5 a^3 + 3 a c^2) \cos \left[ \frac{1}{2} (d + e x) \right]^3 \right. \\
& \quad \left. \log \left[ a \cos \left[ \frac{1}{2} (d + e x) \right] + c \sin \left[ \frac{1}{2} (d + e x) \right] \right] \left( a \cos \left[ \frac{1}{2} (d + e x) \right] + c \sin \left[ \frac{1}{2} (d + e x) \right] \right)^3 + \right. \\
& \quad \frac{1}{a} c \left( 150 a^5 c + 130 a^3 c^3 + 24 a c^5 + 3 a c (25 a^4 + 25 a^2 c^2 - 4 c^4) \cos [d + e x] - \right. \\
& \quad 6 (25 a^5 c + 15 a^3 c^3 + 4 a c^5) \cos [2 (d + e x)] - 75 a^5 c \cos [3 (d + e x)] - \\
& \quad 35 a^3 c^3 \cos [3 (d + e x)] - 4 a c^5 \cos [3 (d + e x)] + 150 a^6 \sin [d + e x] + \\
& \quad 255 a^4 c^2 \sin [d + e x] + 129 a^2 c^4 \sin [d + e x] + 12 c^6 \sin [d + e x] + 120 a^6 \sin [2 (d + e x)] + \\
& \quad 72 a^4 c^2 \sin [2 (d + e x)] + 36 a^2 c^4 \sin [2 (d + e x)] + 30 a^6 \sin [3 (d + e x)] - \\
& \quad \left. \left. 37 a^4 c^2 \sin [3 (d + e x)] - 27 a^2 c^4 \sin [3 (d + e x)] - 4 c^6 \sin [3 (d + e x)] \right) \right)
\end{aligned}$$

**Problem 370: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{2 a + 2 a \cos [d + e x] + 2 a \sin [d + e x]} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{\log \left[ 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right]}{2 a e}$$

Result (type 3, 50 leaves):

$$\frac{-\frac{\log \left[ \cos \left[ \frac{1}{2} (d + e x) \right] \right]}{e} + \frac{\log \left[ \cos \left[ \frac{1}{2} (d + e x) \right] + \sin \left[ \frac{1}{2} (d + e x) \right] \right]}{e}}{2 a}$$

**Problem 378: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2 a - 2 a \cos [d + e x] + 2 c \sin [d + e x])^2} dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$\frac{a \log \left[ a + c \cot \left[ \frac{1}{2} (d + e x) \right] \right]}{4 c^3 e} - \frac{c \cos [d + e x] + a \sin [d + e x]}{4 c^2 e (a - a \cos [d + e x] + c \sin [d + e x])}$$

Result (type 3, 229 leaves):

$$\begin{aligned}
& - \frac{1}{4 c^3 e \left( a - a \cos [d + e x] + c \sin [d + e x] \right)^2} \\
& \sin \left[ \frac{1}{2} (d + e x) \right] \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right) \left( \cos [d + e x] \right. \\
& \quad \left. \left( a^2 + 2 c^2 - 2 a^2 \log \left[ \sin \left[ \frac{1}{2} (d + e x) \right] \right] + 2 a^2 \log \left[ c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right] \right) + \right. \\
& \quad \left. a \left( a \left( -1 + 2 \log \left[ \sin \left[ \frac{1}{2} (d + e x) \right] \right] - 2 \log \left[ c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right] \right) + \right. \\
& \quad \left. c \left( 1 + 2 \log \left[ \sin \left[ \frac{1}{2} (d + e x) \right] \right] - 2 \log \left[ c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right] \right) \right. \\
& \quad \left. \sin [d + e x] \right) \Big)
\end{aligned}$$

**Problem 379: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a - 2a \cos [d + e x] + 2c \sin [d + e x])^3} dx$$

Optimal (type 3, 134 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(3a^2 + c^2) \log \left[ a + c \cot \left[ \frac{1}{2} (d + e x) \right] \right]}{16 c^5 e} - \\
& \frac{c \cos [d + e x] + a \sin [d + e x]}{16 c^2 e (a - a \cos [d + e x] + c \sin [d + e x])^2} + \frac{3 (a c \cos [d + e x] + a^2 \sin [d + e x])}{16 c^4 e (a - a \cos [d + e x] + c \sin [d + e x])}
\end{aligned}$$

Result (type 3, 350 leaves):

$$\begin{aligned}
& \frac{1}{8 c^5 e (a - a \cos [d + e x] + c \sin [d + e x])^3} \sin \left[ \frac{1}{2} (d + e x) \right] \\
& \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right) \left( c^2 (-i a + c) (i a + c) \sin \left[ \frac{1}{2} (d + e x) \right]^2 - \right. \\
& \quad 6 a (a^2 + c^2) \sin \left[ \frac{1}{2} (d + e x) \right]^3 \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right) - \\
& \quad c^2 \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right)^2 + 4 (3 a^2 + c^2) \log \left[ \sin \left[ \frac{1}{2} (d + e x) \right] \right] \\
& \quad \sin \left[ \frac{1}{2} (d + e x) \right]^2 \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right)^2 - \\
& \quad 4 (3 a^2 + c^2) \log \left[ c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right] \\
& \quad \sin \left[ \frac{1}{2} (d + e x) \right]^2 \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right)^2 + \\
& \quad 3 a c \left( c \cos \left[ \frac{1}{2} (d + e x) \right] + a \sin \left[ \frac{1}{2} (d + e x) \right] \right)^2 \sin [d + e x] \Big)
\end{aligned}$$

### Problem 380: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(2a - 2a \cos[d + ex] + 2c \sin[d + ex])^4} dx$$

Optimal (type 3, 207 leaves, 5 steps):

$$\frac{a(5a^2 + 3c^2) \operatorname{Log}\left[a + c \cot\left[\frac{1}{2}(d + ex)\right]\right]}{32c^7 e} - \frac{c \cos[d + ex] + a \sin[d + ex]}{48c^2 e (a - a \cos[d + ex] + c \sin[d + ex])^3} + \frac{5(a c \cos[d + ex] + a^2 \sin[d + ex])}{96c^4 e (a - a \cos[d + ex] + c \sin[d + ex])^2} - \frac{c(15a^2 + 4c^2) \cos[d + ex] + a(15a^2 + 4c^2) \sin[d + ex]}{96c^6 e (a - a \cos[d + ex] + c \sin[d + ex])}$$

Result (type 3, 494 leaves):

$$\frac{1}{384c^7 e (a - a \cos[d + ex] + c \sin[d + ex])^4} \sin\left[\frac{1}{2}(d + ex)\right] \left( c \cos\left[\frac{1}{2}(d + ex)\right] + a \sin\left[\frac{1}{2}(d + ex)\right] \right) \left( 150a^6 + 130a^4 c^2 + 24a^2 c^4 - 225a^6 \cos[d + ex] - 255a^4 c^2 \cos[d + ex] - 42a^2 c^4 \cos[d + ex] - 24c^6 \cos[d + ex] + 90a^6 \cos[2(d + ex)] + 174a^4 c^2 \cos[2(d + ex)] - 15a^6 \cos[3(d + ex)] - 49a^4 c^2 \cos[3(d + ex)] + 18a^2 c^4 \cos[3(d + ex)] + 8c^6 \cos[3(d + ex)] - 192(5a^3 + 3ac^2) \operatorname{Log}\left[\sin\left[\frac{1}{2}(d + ex)\right]\right] \sin\left[\frac{1}{2}(d + ex)\right]^3 \left( c \cos\left[\frac{1}{2}(d + ex)\right] + a \sin\left[\frac{1}{2}(d + ex)\right] \right)^3 + 192(5a^3 + 3ac^2) \operatorname{Log}\left[ c \cos\left[\frac{1}{2}(d + ex)\right] + a \sin\left[\frac{1}{2}(d + ex)\right] \right] \sin\left[\frac{1}{2}(d + ex)\right]^3 \left( c \cos\left[\frac{1}{2}(d + ex)\right] + a \sin\left[\frac{1}{2}(d + ex)\right] \right)^3 + 75a^5 c \sin[d + ex] + 75a^3 c^3 \sin[d + ex] - 12a c^5 \sin[d + ex] - 60a^5 c \sin[2(d + ex)] - 156a^3 c^3 \sin[2(d + ex)] - 12a c^5 \sin[2(d + ex)] + 15a^5 c \sin[3(d + ex)] + 79a^3 c^3 \sin[3(d + ex)] + 20a c^5 \sin[3(d + ex)] \right)$$

### Problem 384: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{2a + 2b \cos[d + ex] + 2a \sin[d + ex]} dx$$

Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\operatorname{Log}\left[a + b \cot\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{2be}$$

Result (type 3, 93 leaves):

$$\frac{1}{2} \left( \frac{\text{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right] + \sin\left[\frac{1}{2}(d+ex)\right]\right]}{be} - \frac{1}{be} \right. \\ \left. \text{Log}\left[a \cos\left[\frac{1}{2}(d+ex)\right] + b \cos\left[\frac{1}{2}(d+ex)\right] + a \sin\left[\frac{1}{2}(d+ex)\right] - b \sin\left[\frac{1}{2}(d+ex)\right]\right] \right)$$

**Problem 387: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a + 2b \cos[d+ex] + 2a \sin[d+ex])^4} dx$$

Optimal (type 3, 215 leaves, 5 steps):

$$\frac{a(5a^2 + 3b^2) \text{Log}\left[a + b \cot\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{32b^7e} - \\ \frac{a \cos[d+ex] - b \sin[d+ex]}{48b^2e(a + b \cos[d+ex] + a \sin[d+ex])^3} + \frac{5(a^2 \cos[d+ex] - ab \sin[d+ex])}{96b^4e(a + b \cos[d+ex] + a \sin[d+ex])^2} - \\ \frac{a(15a^2 + 4b^2) \cos[d+ex] - b(15a^2 + 4b^2) \sin[d+ex]}{96b^6e(a + b \cos[d+ex] + a \sin[d+ex])}$$

Result (type 3, 632 leaves):

$$\frac{1}{384b^7e} \left( -12a(5a^2 + 3b^2) \text{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right] + \sin\left[\frac{1}{2}(d+ex)\right]\right] + \right. \\ 12a(5a^2 + 3b^2) \text{Log}\left[(a+b) \cos\left[\frac{1}{2}(d+ex)\right] + (a-b) \sin\left[\frac{1}{2}(d+ex)\right]\right] + \\ (b(150a^6 + 130a^4b^2 + 24a^2b^4 - 3a^2(25a^4 - 50a^3b + 5a^2b^2 - 30ab^3 + 4b^4) \cos[d+ex] - \\ 6a^2(15a^4 + 20a^3b + 9a^2b^2 + 2ab^3 - 2b^4) \cos[2(d+ex)] + 15a^6 \cos[3(d+ex)] - \\ 30a^5b \cos[3(d+ex)] - 41a^4b^2 \cos[3(d+ex)] - 38a^3b^3 \cos[3(d+ex)] - \\ 12a^2b^4 \cos[3(d+ex)] - 8ab^5 \cos[3(d+ex)] + 225a^6 \sin[d+ex] + \\ 75a^5b \sin[d+ex] + 180a^4b^2 \sin[d+ex] + 15a^3b^3 \sin[d+ex] + \\ 27a^2b^4 \sin[d+ex] + 12ab^5 \sin[d+ex] + 12b^6 \sin[d+ex] - 60a^6 \sin[2(d+ex)] + \\ 120a^5b \sin[2(d+ex)] + 54a^4b^2 \sin[2(d+ex)] + 102a^3b^3 \sin[2(d+ex)] + \\ 6a^2b^4 \sin[2(d+ex)] + 6ab^5 \sin[2(d+ex)] - 15a^6 \sin[3(d+ex)] - \\ 45a^5b \sin[3(d+ex)] - 4a^4b^2 \sin[3(d+ex)] + 3a^3b^3 \sin[3(d+ex)] + \\ 15a^2b^4 \sin[3(d+ex)] + 4ab^5 \sin[3(d+ex)] + 4b^6 \sin[3(d+ex)])) \Big/ \\ \left( (a+b) \left( \cos\left[\frac{1}{2}(d+ex)\right] + \sin\left[\frac{1}{2}(d+ex)\right] \right)^3 \right. \\ \left. \left( (a+b) \cos\left[\frac{1}{2}(d+ex)\right] + (a-b) \sin\left[\frac{1}{2}(d+ex)\right] \right)^3 \right)$$

**Problem 391: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{2a + 2b \cos[d+ex] - 2a \sin[d+ex]} dx$$



Optimal (type 3, 33 leaves, 2 steps):

$$\frac{\operatorname{Log}\left[a + b \operatorname{Tan}\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{2 b e}$$

Result (type 3, 96 leaves):

$$-\frac{\operatorname{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right] - \sin\left[\frac{1}{2}(d+ex)\right]\right]}{2 b e} + \frac{1}{2 b e} \operatorname{Log}\left[a \cos\left[\frac{1}{2}(d+ex)\right] + b \cos\left[\frac{1}{2}(d+ex)\right] - a \sin\left[\frac{1}{2}(d+ex)\right] + b \sin\left[\frac{1}{2}(d+ex)\right]\right]$$

**Problem 394: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(2a + 2b \cos[d+ex] - 2a \sin[d+ex])^4} dx$$

Optimal (type 3, 215 leaves, 5 steps):

$$-\frac{a(5a^2 + 3b^2) \operatorname{Log}\left[a + b \operatorname{Tan}\left[\frac{d}{2} + \frac{\pi}{4} + \frac{ex}{2}\right]\right]}{32 b^7 e} + \frac{a \cos[d+ex] + b \sin[d+ex]}{48 b^2 e (a + b \cos[d+ex] - a \sin[d+ex])^3} - \frac{5(a^2 \cos[d+ex] + a b \sin[d+ex])}{96 b^4 e (a + b \cos[d+ex] - a \sin[d+ex])^2} + \frac{a(15a^2 + 4b^2) \cos[d+ex] + b(15a^2 + 4b^2) \sin[d+ex]}{96 b^6 e (a + b \cos[d+ex] - a \sin[d+ex])}$$

Result (type 3, 636 leaves):

$$\begin{aligned} & \frac{1}{384 b^7 e} \left( 12a(5a^2 + 3b^2) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d+ex)\right] - \sin\left[\frac{1}{2}(d+ex)\right]\right] - \right. \\ & 12a(5a^2 + 3b^2) \operatorname{Log}\left[(a+b) \cos\left[\frac{1}{2}(d+ex)\right] + (-a+b) \sin\left[\frac{1}{2}(d+ex)\right]\right] + \\ & (b(-150a^6 - 130a^4b^2 - 24a^2b^4 + 3a^2(25a^4 - 50a^3b + 5a^2b^2 - 30ab^3 + 4b^4) \cos[d+ex] + \\ & 6a^2(15a^4 + 20a^3b + 9a^2b^2 + 2ab^3 - 2b^4) \cos[2(d+ex)] - 15a^6 \cos[3(d+ex)] + \\ & 30a^5b \cos[3(d+ex)] + 41a^4b^2 \cos[3(d+ex)] + 38a^3b^3 \cos[3(d+ex)] + \\ & 12a^2b^4 \cos[3(d+ex)] + 8ab^5 \cos[3(d+ex)] + 225a^6 \sin[d+ex] + \\ & 75a^5b \sin[d+ex] + 180a^4b^2 \sin[d+ex] + 15a^3b^3 \sin[d+ex] + \\ & 27a^2b^4 \sin[d+ex] + 12ab^5 \sin[d+ex] + 12b^6 \sin[d+ex] - 60a^6 \sin[2(d+ex)] + \\ & 120a^5b \sin[2(d+ex)] + 54a^4b^2 \sin[2(d+ex)] + 102a^3b^3 \sin[2(d+ex)] + \\ & 6a^2b^4 \sin[2(d+ex)] + 6ab^5 \sin[2(d+ex)] - 15a^6 \sin[3(d+ex)] - \\ & 45a^5b \sin[3(d+ex)] - 4a^4b^2 \sin[3(d+ex)] + 3a^3b^3 \sin[3(d+ex)] + \\ & 15a^2b^4 \sin[3(d+ex)] + 4ab^5 \sin[3(d+ex)] + 4b^6 \sin[3(d+ex)]) \Big) / \\ & \left( (a+b) \left( \cos\left[\frac{1}{2}(d+ex)\right] - \sin\left[\frac{1}{2}(d+ex)\right] \right)^3 \right. \\ & \left. \left( (a+b) \cos\left[\frac{1}{2}(d+ex)\right] + (-a+b) \sin\left[\frac{1}{2}(d+ex)\right] \right)^3 \right) \end{aligned}$$

Problem 402: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[d + ex] + c \sin[d + ex])^4} dx$$

Optimal (type 3, 292 leaves, 6 steps):

$$\frac{a (2 a^2 + 3 (b^2 + c^2)) \operatorname{ArcTan}\left[\frac{c + (a-b) \tan\left[\frac{1}{2}(d+ex)\right]}{\sqrt{a^2 - b^2 - c^2}}\right]}{(a^2 - b^2 - c^2)^{7/2} e} +$$

$$\frac{c \cos[d + ex] - b \sin[d + ex]}{3 (a^2 - b^2 - c^2) e (a + b \cos[d + ex] + c \sin[d + ex])^3} +$$

$$\frac{5 (a c \cos[d + ex] - a b \sin[d + ex])}{6 (a^2 - b^2 - c^2)^2 e (a + b \cos[d + ex] + c \sin[d + ex])^2} +$$

$$\frac{(c (11 a^2 + 4 (b^2 + c^2)) \cos[d + ex] - b (11 a^2 + 4 (b^2 + c^2)) \sin[d + ex])}{(6 (a^2 - b^2 - c^2)^3 e (a + b \cos[d + ex] + c \sin[d + ex]))}$$

Result (type 3, 606 leaves):

$$\frac{1}{24 e} \left( \frac{24 a (2 a^2 + 3 (b^2 + c^2)) \operatorname{ArcTan}\left[\frac{c + (a-b) \tan\left[\frac{1}{2}(d+ex)\right]}{\sqrt{-a^2 + b^2 + c^2}}\right]}{(-a^2 + b^2 + c^2)^{7/2}} + \right.$$

$$\frac{1}{b (-a^2 + b^2 + c^2)^3 (a + b \cos[d + ex] + c \sin[d + ex])^3} (44 a^5 c + 82 a^3 b^2 c +$$

$$24 a b^4 c + 82 a^3 c^3 + 48 a b^2 c^3 + 24 a c^5 + 30 a^2 b c (2 a^2 + 3 (b^2 + c^2)) \cos[d + ex] -$$

$$6 a c (-2 b^4 + 2 b^2 c^2 + 4 c^4 + a^2 (7 b^2 + 11 c^2)) \cos[2 (d + ex)] - 22 a^2 b^3 c \cos[3 (d + ex)] -$$

$$8 b^5 c \cos[3 (d + ex)] - 22 a^2 b c^3 \cos[3 (d + ex)] - 16 b^3 c^3 \cos[3 (d + ex)] -$$

$$8 b c^5 \cos[3 (d + ex)] + 72 a^4 b^2 \sin[d + ex] - 9 a^2 b^4 \sin[d + ex] + 12 b^6 \sin[d + ex] +$$

$$132 a^4 c^2 \sin[d + ex] + 72 a^2 b^2 c^2 \sin[d + ex] + 36 b^4 c^2 \sin[d + ex] + 81 a^2 c^4 \sin[d + ex] +$$

$$36 b^2 c^4 \sin[d + ex] + 12 c^6 \sin[d + ex] + 54 a^3 b^3 \sin[2 (d + ex)] + 6 a b^5 \sin[2 (d + ex)] +$$

$$78 a^3 b c^2 \sin[2 (d + ex)] + 48 a b^3 c^2 \sin[2 (d + ex)] + 42 a b c^4 \sin[2 (d + ex)] +$$

$$11 a^2 b^4 \sin[3 (d + ex)] + 4 b^6 \sin[3 (d + ex)] + 4 b^4 c^2 \sin[3 (d + ex)] -$$

$$11 a^2 c^4 \sin[3 (d + ex)] - 4 b^2 c^4 \sin[3 (d + ex)] - 4 c^6 \sin[3 (d + ex)]) \left. \right)$$

Problem 403: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (2 + 3 \cos[d + ex] + 5 \sin[d + ex])^{5/2} dx$$

Optimal (type 4, 185 leaves, 7 steps):

$$\begin{aligned}
 & \frac{796 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{15 e} + \\
 & \frac{64 \operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{\sqrt{2 + \sqrt{34}} e} - \frac{1}{15 e} \\
 & \frac{32 \left(5 \cos[d + e x] - 3 \sin[d + e x]\right) \sqrt{2 + 3 \cos[d + e x] + 5 \sin[d + e x]}}{5 e} - \\
 & \frac{2 \left(5 \cos[d + e x] - 3 \sin[d + e x]\right) \left(2 + 3 \cos[d + e x] + 5 \sin[d + e x]\right)^{3/2}}{5 e}
 \end{aligned}$$

Result (type 6, 536 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{2 + 3 \cos[d + ex] + 5 \sin[d + ex]} \\
& \left( \frac{796}{25} - \frac{44}{3} \cos[d + ex] - 6 \cos[2(d + ex)] + \frac{44}{5} \sin[d + ex] - \frac{16}{5} \sin[2(d + ex)] \right) + \\
& \frac{1}{15e} \sqrt{\frac{34}{17 + \sqrt{34}}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}\right], \\
& -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \operatorname{Sec}\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \sqrt{1 - \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} \\
& \sqrt{-\frac{1 + \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} + \frac{1}{75e} \\
& 13532 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}\right], \right. \right. \right. \\
& \left. \left. - \frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) / \right. \\
& \left( 17 \sqrt{1 - \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \sqrt{-\frac{1 + \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{-17 + \sqrt{34}}} \right. \\
& \left. \left. \sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \right) \right) - \\
& \left. \frac{\frac{3}{17} \left( 2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) - \frac{5 \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}}}{\sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}} \right)
\end{aligned}$$

Problem 404: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \left( 2 + 3 \cos [d + e x] + 5 \sin [d + e x] \right)^{3/2} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{16 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE} \left[ \frac{1}{2} \left( d + e x - \operatorname{ArcTan} \left[ \frac{5}{3} \right] \right), \frac{2}{15} \left( 17 - \sqrt{34} \right) \right]}{3 e} +$$

$$\frac{20 \operatorname{EllipticF} \left[ \frac{1}{2} \left( d + e x - \operatorname{ArcTan} \left[ \frac{5}{3} \right] \right), \frac{2}{15} \left( 17 - \sqrt{34} \right) \right]}{\sqrt{2 + \sqrt{34}} e} -$$

$$\frac{2 \left( 5 \cos [d + e x] - 3 \sin [d + e x] \right) \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]}}{3 e}$$

Result (type 6, 512 leaves):

$$\begin{aligned}
& \frac{1}{e} \left( \frac{16}{5} - \frac{10}{3} \cos[d + ex] + 2 \sin[d + ex] \right) \sqrt{2 + 3 \cos[d + ex] + 5 \sin[d + ex]} + \\
& \frac{1}{3e} 46 \sqrt{\frac{34}{17 + \sqrt{34}}} \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, \right. \\
& \left. -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \right] \sec[d + ex + \operatorname{ArcTan}[\frac{3}{5}]] \sqrt{1 - \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} \\
& \sqrt{-\frac{1 + \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} + \frac{1}{15e} \\
& 272 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \right] \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) / \right. \\
& \left. \left( 17 \sqrt{1 - \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \sqrt{-\frac{1 + \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{-17 + \sqrt{34}}} \right. \right. \\
& \left. \left. \sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \right) \right) - \\
& \left. \frac{\frac{3}{17} \left( 2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) - \frac{5 \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}}}{\sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}} \right)
\end{aligned}$$

Problem 405: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{2 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{e}$$

Result (type 6, 326 leaves):

$$\begin{aligned} & \frac{1}{15 e \sqrt{2 + \sqrt{34}} \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] \sqrt{\sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]^2}} \\ & \left( -15 \sqrt{30} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{34} + 17 \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{-17 + \sqrt{34}}\right], \right. \\ & \quad \left. \frac{\sqrt{34} + 17 \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{17 + \sqrt{34}} \right] \sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] + \\ & \quad \left( -75 \cos [d + e x] + 45 \sin [d + e x] + 2 \sqrt{30} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \right. \right. \\ & \quad \left. \left. \frac{\sqrt{34} + 17 \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{-17 + \sqrt{34}}, \frac{\sqrt{34} + 17 \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{17 + \sqrt{34}} \right] \right. \\ & \quad \left. \sqrt{\cos \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]^2} \sqrt{2 + \sqrt{34}} \cos \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right] \sec \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \right. \\ & \quad \left. \left. \sqrt{2 + \sqrt{34}} \sin \left[d + e x + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \right) \sqrt{\sin \left[d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right]^2} \right) \end{aligned}$$

Problem 406: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{2 \operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{\sqrt{2 + \sqrt{34}} e}$$

Result (type 6, 128 leaves):

$$\frac{1}{e} \sqrt{\frac{2}{15}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{\sqrt{34} + 17 \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{-17 + \sqrt{34}}, \frac{\sqrt{34} + 17 \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{17 + \sqrt{34}}\right] \sqrt{\cos\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]^2 \sec\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]} \sqrt{2 + \sqrt{34} \sin\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}$$

Problem 407: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(2 + 3 \cos[d + ex] + 5 \sin[d + ex])^{3/2}} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$-\frac{\sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + ex - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{15 e} - \frac{5 \cos[d + ex] - 3 \sin[d + ex]}{15 e \sqrt{2 + 3 \cos[d + ex] + 5 \sin[d + ex]}}$$

Result (type 6, 528 leaves):



$$\begin{aligned}
& \frac{\sqrt{2+3\cos[dx]+5\sin[dx]}\left(-\frac{34}{225}+\frac{2(5+17\sin[dx])}{45(2+3\cos[dx]+5\sin[dx])}\right)}{e} \\
& - \frac{1}{15e}\sqrt{\frac{34}{17+\sqrt{34}}}\operatorname{AppellF1}\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},-\frac{2+\sqrt{34}\sin\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{\sqrt{34}\left(1-\sqrt{\frac{2}{17}}\right)}\right], \\
& - \frac{2+\sqrt{34}\sin\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{\sqrt{34}\left(-1-\sqrt{\frac{2}{17}}\right)}\operatorname{Sec}\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]\sqrt{1-\sin\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]} \\
& \sqrt{-\frac{1+\sin\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}{-17+\sqrt{34}}}\sqrt{2+\sqrt{34}\sin\left[d+ex+\operatorname{ArcTan}\left[\frac{3}{5}\right]\right]}-\frac{1}{75e} \\
& 17\left(-\left(5\sqrt{\frac{1}{34}(17+\sqrt{34})}\operatorname{AppellF1}\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{\sqrt{34}\left(1-\sqrt{\frac{2}{17}}\right)}\right],\right.\right. \\
& \left.-\frac{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{\sqrt{34}\left(-1-\sqrt{\frac{2}{17}}\right)}\sin\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]\right)/ \\
& \left(17\sqrt{1-\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}\sqrt{-\frac{1+\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{-17+\sqrt{34}}}\right. \\
& \left.\left.\sqrt{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}\right)\right)- \\
& \frac{\frac{3}{17}\left(2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right)\right)-\frac{5\sin\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}{\sqrt{34}}}{\sqrt{2+\sqrt{34}\cos\left[d+ex-\operatorname{ArcTan}\left[\frac{5}{3}\right]\right]}}
\end{aligned}$$

Problem 408: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(2 + 3 \cos[d + e x] + 5 \sin[d + e x])^{5/2}} dx$$

Optimal (type 4, 187 leaves, 7 steps):

$$\begin{aligned} & \frac{4 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{675 e} + \\ & \frac{\operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{45 \sqrt{2 + \sqrt{34}} e} - \\ & \frac{5 \cos[d + e x] - 3 \sin[d + e x]}{45 e (2 + 3 \cos[d + e x] + 5 \sin[d + e x])^{3/2}} + \frac{4 (5 \cos[d + e x] - 3 \sin[d + e x])}{675 e \sqrt{2 + 3 \cos[d + e x] + 5 \sin[d + e x]}} \end{aligned}$$

Result (type 6, 564 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{2 + 3 \cos[d + ex] + 5 \sin[d + ex]} \\
& \left( \frac{136}{10125} + \frac{-115 - 136 \sin[d + ex]}{2025 (2 + 3 \cos[d + ex] + 5 \sin[d + ex])} + \frac{2 (5 + 17 \sin[d + ex])}{135 (2 + 3 \cos[d + ex] + 5 \sin[d + ex])^2} \right) + \\
& \frac{1}{675 e} \sqrt{\frac{17}{2 (17 + \sqrt{34})}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, \right. \\
& \left. -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)}\right] \sec[d + ex + \operatorname{ArcTan}[\frac{3}{5}]] \sqrt{1 - \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} \\
& \sqrt{-\frac{1 + \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} + \frac{1}{3375 e} \\
& 68 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}, \right. \right. \right. \right. \\
& \left. \left. \left. -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)}\right] \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) / \right. \\
& \left( 17 \sqrt{1 - \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \sqrt{-\frac{1 + \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{-17 + \sqrt{34}}} \right. \\
& \left. \left. \left. \sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \right) \right) - \right. \\
& \left. \frac{\frac{3}{17} (2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] - \frac{5 \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}})}{\sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}} \right)
\end{aligned}$$

Problem 409: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{1}{(2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{7/2}} dx$$

Optimal (type 4, 233 leaves, 8 steps):

$$\begin{aligned} & - \frac{199 \sqrt{2 + \sqrt{34}} \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{101250 e} - \\ & \frac{8 \operatorname{EllipticF}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{5}{3}\right]\right), \frac{2}{15} \left(17 - \sqrt{34}\right)\right]}{3375 \sqrt{2 + \sqrt{34}} e} - \\ & \frac{5 \cos [d + e x] - 3 \sin [d + e x]}{75 e (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{5/2}} + \frac{8 (5 \cos [d + e x] - 3 \sin [d + e x])}{3375 e (2 + 3 \cos [d + e x] + 5 \sin [d + e x])^{3/2}} - \\ & \frac{199 (5 \cos [d + e x] - 3 \sin [d + e x])}{101250 e \sqrt{2 + 3 \cos [d + e x] + 5 \sin [d + e x]}} \end{aligned}$$

Result (type 6, 598 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{2 + 3 \cos[d + ex] + 5 \sin[d + ex]} \left( -\frac{3383}{759375} + \frac{-305 - 272 \sin[d + ex]}{10125 (2 + 3 \cos[d + ex] + 5 \sin[d + ex])^2} + \right. \\
& \quad \left. \frac{2 (5 + 17 \sin[d + ex])}{225 (2 + 3 \cos[d + ex] + 5 \sin[d + ex])^3} + \frac{1595 + 3383 \sin[d + ex]}{151875 (2 + 3 \cos[d + ex] + 5 \sin[d + ex])} \right) - \\
& \quad \frac{1}{50625 e} \sqrt{\frac{17}{2 (17 + \sqrt{34})}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}\right], \\
& \quad - \frac{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \operatorname{Sec}\left[d + ex + \operatorname{ArcTan}\left[\frac{3}{5}\right]\right] \sqrt{1 - \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} \\
& \quad \sqrt{-\frac{1 + \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]}{-17 + \sqrt{34}}} \sqrt{2 + \sqrt{34} \sin[d + ex + \operatorname{ArcTan}[\frac{3}{5}]]} - \frac{1}{506250 e} \\
& \quad 3383 \left( - \left( \left( 5 \sqrt{\frac{1}{34} (17 + \sqrt{34})} \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(1 - \sqrt{\frac{2}{17}}\right)}\right], \right. \right. \right. \\
& \quad \left. \left. - \frac{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34} \left(-1 - \sqrt{\frac{2}{17}}\right)} \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] \right) / \right. \\
& \quad \left( 17 \sqrt{1 - \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \sqrt{-\frac{1 + \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{-17 + \sqrt{34}}} \right. \\
& \quad \left. \left. \sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]} \right) \right) - \\
& \quad \left. \frac{\frac{3}{17} (2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]] - \frac{5 \sin[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}{\sqrt{34}})}{\sqrt{2 + \sqrt{34} \cos[d + ex - \operatorname{ArcTan}[\frac{5}{3}]]}} \right)
\end{aligned}$$

**Problem 410: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \cos[d + ex] + c \sin[d + ex])^{5/2} dx$$

Optimal (type 4, 347 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{15e} 16 (a c \cos[d + ex] - a b \sin[d + ex]) \sqrt{a + b \cos[d + ex] + c \sin[d + ex]} - \\ & \frac{1}{5e} 2 (c \cos[d + ex] - b \sin[d + ex]) (a + b \cos[d + ex] + c \sin[d + ex])^{3/2} + \\ & \left( 2 (23 a^2 + 9 (b^2 + c^2)) \operatorname{EllipticE}\left[\frac{1}{2} (d + ex - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \right. \\ & \quad \left. \sqrt{a + b \cos[d + ex] + c \sin[d + ex]} \right) / \left( 15 e \sqrt{\frac{a + b \cos[d + ex] + c \sin[d + ex]}{a + \sqrt{b^2 + c^2}}} \right) - \\ & \left( 16 a (a^2 - b^2 - c^2) \operatorname{EllipticF}\left[\frac{1}{2} (d + ex - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \right. \\ & \quad \left. \sqrt{\frac{a + b \cos[d + ex] + c \sin[d + ex]}{a + \sqrt{b^2 + c^2}}} \right) / \left( 15 e \sqrt{a + b \cos[d + ex] + c \sin[d + ex]} \right) \end{aligned}$$

Result (type 6, 3767 leaves):

$$\begin{aligned} & \frac{1}{e} \sqrt{a + b \cos[d + ex] + c \sin[d + ex]} \left( \frac{2 b (23 a^2 + 9 b^2 + 9 c^2)}{15 c} - \frac{22}{15} a c \cos[d + ex] - \right. \\ & \quad \left. \frac{2}{5} b c \cos[2 (d + ex)] + \frac{22}{15} a b \sin[d + ex] + \frac{1}{5} (b^2 - c^2) \sin[2 (d + ex)] \right) + \\ & \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c e} 2 a^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right], \\ & - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \operatorname{Sec}[d + ex + \operatorname{ArcTan}[\frac{b}{c}]] \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} c e} \\
& 34 a b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} e} \\
& 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right.
\end{aligned}$$

$$\begin{aligned}
 & - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
 & \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
 & \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{15 c e} \\
 & 23 a^2 b^2 \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \right. \\
 & \left. \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \right. \\
 & \left. \left. \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right)
 \end{aligned}$$



$$\begin{aligned}
& \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{5 c e} \\
& 3 b^4 \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right. \right. \\
& \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \left. \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{15 e} \\
 & 23 a^2 c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right. \right. \\
 & \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
 \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{5 e} \\
& 6 b^2 c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right. \right. \\
& \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \left. \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{5 e} \\
 & 3 c^3 \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \right) \right. \right. \\
 & \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
 \end{aligned}$$

$$\left( \frac{\sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}$$

**Problem 411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b \cos [d + e x] + c \sin [d + e x])^{3/2} dx$$

Optimal (type 4, 283 leaves, 6 steps):

$$\begin{aligned} & -\frac{1}{3e} 2 (c \cos [d + e x] - b \sin [d + e x]) \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} + \\ & \left( 8 a \text{EllipticE} \left[ \frac{1}{2} (d + e x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \right) / \\ & \left( 3 e \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}} \right) - \\ & \left( 2 (a^2 - b^2 - c^2) \text{EllipticF} \left[ \frac{1}{2} (d + e x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \right. \\ & \left. \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}} \right) / (3 e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}) \end{aligned}$$

Result (type 6, 2190 leaves):

$$\frac{1}{e} \left( \frac{8 a b}{3 c} - \frac{2}{3} c \cos [d + e x] + \frac{2}{3} b \sin [d + e x] \right) \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} +$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 + \frac{b^2}{c^2} c e}} 2 a^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2} c} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2} c}}\right) c}\right], \\
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2} c} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2} c}}\right) c} \sec\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2} c e}} \\
& 2 b^2 \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2} c} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2} c}}\right) c}\right], \\
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2} c} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2} c} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2} c}}\right) c} \sec\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}} e} \\
 & 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
 & \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \sec\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{3 c e} \\
 & 4 a b^2 \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
 & \left. \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
 & \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{3 e} \\
 & 4 a c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right) \right)
 \end{aligned}$$



$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \\
& \left. - \frac{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \\
& \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}}
\end{aligned}$$

**Problem 412:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \, dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\left( 2 \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos[d + e x] + c \sin[d + e x]} \right) /$$

$$\left( e \sqrt{\frac{a + b \cos[d + e x] + c \sin[d + e x]}{a + \sqrt{b^2 + c^2}}} \right)$$

Result (type 6, 1408 leaves):

$$\frac{2 b \sqrt{a + b \cos[d + e x] + c \sin[d + e x]}}{c e} + \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c e}$$

$$2 a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right.$$

$$\left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \sec[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + e x + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} + \frac{1}{c e}}$$

$$b^2 \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos[d + e x - \operatorname{ArcTan}[\frac{c}{b}]]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right) \right) \right)$$

$$\begin{aligned}
 & - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. \left. - \frac{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \right) - \\
 & \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{e} \\
 & c \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \\
& \left. - \frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - a + b \sqrt{\frac{b^2 + c^2}{b^2}}}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \\
& \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}}
\end{aligned}$$

**Problem 413:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b \cos [d + e x] + c \sin [d + e x]}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\left( 2 \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}} \right) \Bigg/ \left( e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \right)$$

Result (type 6, 285 leaves):

$$\frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c e} {}_2F_1\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right]$$

$$\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a - \sqrt{1 + \frac{b^2}{c^2}} c}, \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + \sqrt{1 + \frac{b^2}{c^2}} c}$$

$$\sec\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \sqrt{-\frac{\sqrt{1 + \frac{b^2}{c^2}} c \left(-1 + \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]\right)}{a + \sqrt{1 + \frac{b^2}{c^2}} c}}$$

$$\sqrt{\frac{\sqrt{1 + \frac{b^2}{c^2}} c \left(1 + \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]\right)}{-a + \sqrt{1 + \frac{b^2}{c^2}} c}} \sqrt{\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + \sqrt{1 + \frac{b^2}{c^2}} c}}$$

**Problem 414: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\left(a + b \cos[d + e x] + c \sin[d + e x]\right)^{3/2}} dx$$

Optimal (type 4, 186 leaves, 3 steps):

$$\frac{2 \left(c \cos[d + e x] - b \sin[d + e x]\right)}{\left(a^2 - b^2 - c^2\right) e \sqrt{a + b \cos[d + e x] + c \sin[d + e x]}} +$$

$$\left(2 \operatorname{EllipticE}\left[\frac{1}{2} \left(d + e x - \operatorname{ArcTan}\left[\frac{b}{c}\right]\right), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}}\right] \sqrt{a + b \cos[d + e x] + c \sin[d + e x]}\right) /$$

$$\left(\left(a^2 - b^2 - c^2\right) e \sqrt{\frac{a + b \cos[d + e x] + c \sin[d + e x]}{a + \sqrt{b^2 + c^2}}}\right)$$

Result (type 6, 1540 leaves):

$$\frac{1}{e} \sqrt{a + b \cos[d + e x] + c \sin[d + e x]}$$

$$\left(-\frac{2 \left(b^2 + c^2\right)}{b c \left(-a^2 + b^2 + c^2\right)} + \frac{2 \left(a c + b^2 \sin[d + e x] + c^2 \sin[d + e x]\right)}{b \left(-a^2 + b^2 + c^2\right) \left(a + b \cos[d + e x] + c \sin[d + e x]\right)}\right) -$$

$$\begin{aligned}
& \left( 2 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \right) / \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2) e \right) - \\
& \quad \left( b^2 \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}, \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right] \sin\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \Bigg) / (c (-a^2 + b^2 + c^2) e) - \\
& \frac{1}{(-a^2 + b^2 + c^2) e} c \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \\
& \left. \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
\end{aligned}$$

**Problem 415:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos [d + e x] + c \sin [d + e x])^{5/2}} dx$$

Optimal (type 4, 382 leaves, 7 steps):



$$\begin{aligned}
& \frac{2 (c \cos [d+e x]-b \sin [d+e x])}{3\left(a^2-b^2-c^2\right) e\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^{3 / 2}}+ \\
& \frac{8(a c \cos [d+e x]-a b \sin [d+e x])}{3\left(a^2-b^2-c^2\right)^2 e \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}}+ \\
& \left(8 a \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}\right) / \\
& \left(3\left(a^2-b^2-c^2\right)^2 e \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}\right)- \\
& \left(2 \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[b, c]), \frac{2 \sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b \cos [d+e x]+c \sin [d+e x]}{a+\sqrt{b^2+c^2}}}\right) / \\
& \left(3\left(a^2-b^2-c^2\right) e \sqrt{a+b \cos [d+e x]+c \sin [d+e x]}\right)
\end{aligned}$$

Result(type 6, 2408 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{a+b \cos [d+e x]+c \sin [d+e x]} \\
& \left(\frac{8 a\left(b^2+c^2\right)}{3 b c\left(a^2-b^2-c^2\right)^2}+\frac{2\left(a c+b^2 \sin [d+e x]+c^2 \sin [d+e x]\right)}{3 b\left(-a^2+b^2+c^2\right)\left(a+b \cos [d+e x]+c \sin [d+e x]\right)^2}-\right. \\
& \left.\frac{2\left(3 a^2 c+b^2 c+c^3+4 a b^2 \sin [d+e x]+4 a c^2 \sin [d+e x]\right)}{3 b\left(-a^2+b^2+c^2\right)^2\left(a+b \cos [d+e x]+c \sin [d+e x]\right)}\right)+ \\
& \left(2 a^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2},-\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c},\right.\right. \\
& \left.-\frac{a+\sqrt{1+\frac{b^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1+\frac{b^2}{c^2}}\left(-1-\frac{a}{\sqrt{1+\frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}}-c \sqrt{\frac{b^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a+c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a+c \sqrt{\frac{b^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Bigg/ \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 e \right) + \\
& \left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \right. \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \Bigg/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 e \right) + \\
& \left( 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \sec \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 e \right) + \right. \\
& \left( 4 a b^2 \right) - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \right. \\
& \left. \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
 & \left( \frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) - \\
 & \frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \left/ \left( 3 c \left( -a^2 + b^2 + c^2 \right)^2 e \right) + \right. \\
 & \left( 4 a c \left( - \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right/ \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \left. \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
 \end{aligned}$$

$$\left( \frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}} \right) - \frac{2 b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \Bigg) / \left( 3 (-a^2 + b^2 + c^2)^2 e \right)$$

Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos [d + e x] + c \sin [d + e x])^{7/2}} dx$$

Optimal (type 4, 490 leaves, 8 steps):

$$\begin{aligned} & \frac{2 (c \cos [d + e x] - b \sin [d + e x])}{5 (a^2 - b^2 - c^2) e (a + b \cos [d + e x] + c \sin [d + e x])^{5/2}} + \\ & \frac{16 (a c \cos [d + e x] - a b \sin [d + e x])}{15 (a^2 - b^2 - c^2)^2 e (a + b \cos [d + e x] + c \sin [d + e x])^{3/2}} + \left( 2 (23 a^2 + 9 (b^2 + c^2)) \right. \\ & \quad \left. \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \right) / \\ & \left( 15 (a^2 - b^2 - c^2)^3 e \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}} \right) - \\ & \left( 16 a \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos [d + e x] + c \sin [d + e x]}{a + \sqrt{b^2 + c^2}}} \right) / \\ & \left( 15 (a^2 - b^2 - c^2)^2 e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \right) + \\ & \left( 2 (c (23 a^2 + 9 (b^2 + c^2)) \cos [d + e x] - b (23 a^2 + 9 (b^2 + c^2)) \sin [d + e x]) \right) / \\ & \left( 15 (a^2 - b^2 - c^2)^3 e \sqrt{a + b \cos [d + e x] + c \sin [d + e x]} \right) \end{aligned}$$

Result (type 6, 4116 leaves):

$$\frac{1}{e} \sqrt{a + b \cos [d + e x] + c \sin [d + e x]}$$

$$\begin{aligned}
& \left( -\frac{2(b^2 + c^2)(23a^2 + 9b^2 + 9c^2)}{15bc(-a^2 + b^2 + c^2)^3} + \frac{2(ac + b^2 \sin[d + ex] + c^2 \sin[d + ex])}{5b(-a^2 + b^2 + c^2)(a + b \cos[d + ex] + c \sin[d + ex])^3} - \right. \\
& \frac{2(5a^2c + 3b^2c + 3c^3 + 8ab^2 \sin[d + ex] + 8ac^2 \sin[d + ex])}{15b(-a^2 + b^2 + c^2)^2(a + b \cos[d + ex] + c \sin[d + ex])^2} + \\
& \left. \left( 2(15a^3c + 17ab^2c + 17ac^3 + 23a^2b^2 \sin[d + ex] + 9b^4 \sin[d + ex] + \right. \right. \\
& \left. \left. 23a^2c^2 \sin[d + ex] + 18b^2c^2 \sin[d + ex] + 9c^4 \sin[d + ex]) \right) / \right. \\
& \left. \left( 15b(-a^2 + b^2 + c^2)^3(a + b \cos[d + ex] + c \sin[d + ex]) \right) \right) - \\
& \left( 2a^3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \left. \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec}\left[d + ex + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]} \\
& \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \right) / \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^3 e \right) - \\
& \left( 34a^2b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \left/ \left( 15 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^3 e \right) - \right. \\
& \left. \left( 34 a c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \right. \right. \\
& \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
& \left. \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \right) / \left( 15 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^3 e \right) - \\
 & \left( 23 a^2 b^2 - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right], \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) / \right. \\
 & \left. \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \right. \\
 & \left. \left. \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \right. \\
 & \left. \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \right) -
 \end{aligned}$$



$$\begin{aligned}
& \left( \frac{2b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) / \left( 15c \left( -a^2+b^2+c^2 \right)^3 e \right) - \\
& \left( 3b^4 - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \\
& \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
& \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} -
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2b \left( \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2+c^2} - \frac{c \sin \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) / \left( 5c \left( -a^2+b^2+c^2 \right)^3 e \right) - \\
 & \left( 23 a^2 c \left( - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \\
 & \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \\
 & \left( \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+e x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) -
 \end{aligned}$$

$$\begin{aligned}
& \left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) \Bigg/ \left( 15 \left( -a^2+b^2+c^2 \right)^3 e \right) - \\
& \frac{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}{\left( 6b^2c - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \Bigg/ \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \left. \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right. \\
& \left. \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) -
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2b \left( \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{b^2+c^2} - \frac{c \sin\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right)}{\sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}} \right) / \left( 5 \left( -a^2+b^2+c^2 \right)^3 e \right) - \\
 & \left( 3c^3 - \left( c \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)} \right. \right. \right. \\
 & \left. \left. - \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{b\sqrt{1+\frac{c^2}{b^2}} \left(-1-\frac{a}{b\sqrt{1+\frac{c^2}{b^2}}}\right)} \right] \sin\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right] \right) / \\
 & \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]} \right. \\
 & \left. \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos\left[d+ex-\text{ArcTan}\left[\frac{c}{b}\right]\right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right) -
 \end{aligned}$$

$$\left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) \bigg/ \left( 5 (-a^2+b^2+c^2)^3 e \right)$$

$$\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ d+ex - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}$$

**Problem 420: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{5+4 \cos [d+e x]+3 \sin [d+e x]}} d x$$

Optimal (type 3, 48 leaves, 3 steps):

$$\frac{\sqrt{\frac{2}{5}} \text{ArcTanh} \left[ \frac{\sin \left[ d+ex - \text{ArcTan} \left[ \frac{3}{4} \right] \right]}{\sqrt{2} \sqrt{1+\cos \left[ d+ex - \text{ArcTan} \left[ \frac{3}{4} \right] \right]}} \right]}{e}$$

Result (type 3, 101 leaves):

$$-\left( \left( \left( \frac{2}{5} + \frac{6i}{5} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \text{ArcTan} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left( -1 + 3 \tan \left[ \frac{1}{4} (d+ex) \right] \right) \right] \right) \right. \\ \left. \left( 3 \cos \left[ \frac{1}{2} (d+ex) \right] + \sin \left[ \frac{1}{2} (d+ex) \right] \right) \right) \bigg/ \left( e \sqrt{5+4 \cos [d+ex]+3 \sin [d+ex]} \right)$$

**Problem 421: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(5+4 \cos [d+e x]+3 \sin [d+e x])^{3/2}} d x$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{\text{ArcTanh} \left[ \frac{\sin \left[ d+ex - \text{ArcTan} \left[ \frac{3}{4} \right] \right]}{\sqrt{2} \sqrt{1+\cos \left[ d+ex - \text{ArcTan} \left[ \frac{3}{4} \right] \right]}} \right]}{10 \sqrt{10} e} - \frac{3 \cos [d+ex] - 4 \sin [d+ex]}{10 e (5+4 \cos [d+ex]+3 \sin [d+ex])^{3/2}}$$

Result (type 3, 154 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{250} - \frac{i}{125} \right) \left( 3 \cos \left[ \frac{1}{2} (d + e x) \right] + \sin \left[ \frac{1}{2} (d + e x) \right] \right) \right. \right. \\
& \quad \left( (5 + 10i) \left( \cos \left[ \frac{1}{2} (d + e x) \right] - 3 \sin \left[ \frac{1}{2} (d + e x) \right] \right) - \right. \\
& \quad \left. (1 - i) \sqrt{20 + 15i} \operatorname{ArcTan} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left( -1 + 3 \tan \left[ \frac{1}{4} (d + e x) \right] \right) \right] \right. \\
& \quad \left. \left. \left. \left( 3 \cos \left[ \frac{1}{2} (d + e x) \right] + \sin \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \right) \right) / \left( e (5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{3/2} \right)
\end{aligned}$$

**Problem 422: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\begin{aligned}
& \frac{3 \operatorname{ArcTanh} \left[ \frac{\sin [d + e x - \operatorname{ArcTan} [\frac{3}{4}]]}{\sqrt{2} \sqrt{1 + \cos [d + e x - \operatorname{ArcTan} [\frac{3}{4}]]}} \right]}{400 \sqrt{10} e} - \\
& \frac{3 \cos [d + e x] - 4 \sin [d + e x]}{20 e (5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{5/2}} - \frac{3 (3 \cos [d + e x] - 4 \sin [d + e x])}{400 e (5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{3/2}}
\end{aligned}$$

Result (type 3, 180 leaves):

$$\begin{aligned}
& - \left( \left( \left( \frac{1}{20000} - \frac{i}{10000} \right) \left( 3 \cos \left[ \frac{1}{2} (d + e x) \right] + \sin \left[ \frac{1}{2} (d + e x) \right] \right) \right. \right. \\
& \quad \left( (-6 + 6i) \sqrt{20 + 15i} \operatorname{ArcTan} \left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{\frac{4}{5} + \frac{3i}{5}} \left( -1 + 3 \tan \left[ \frac{1}{4} (d + e x) \right] \right) \right] \right) \\
& \quad \left( 3 \cos \left[ \frac{1}{2} (d + e x) \right] + \sin \left[ \frac{1}{2} (d + e x) \right] \right)^4 + \\
& \quad (5 + 10i) \left( 55 \cos \left[ \frac{1}{2} (d + e x) \right] + 39 \cos \left[ \frac{3}{2} (d + e x) \right] - 165 \sin \left[ \frac{1}{2} (d + e x) \right] - \right. \\
& \quad \left. \left. 27 \sin \left[ \frac{3}{2} (d + e x) \right] \right) \right) \right) / \left( e (5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{5/2} \right)
\end{aligned}$$

**Problem 427: Result unnecessarily involves complex numbers and more than**

twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-5 + 4 \cos [d + e x] + 3 \sin [d + e x]}} dx$$

Optimal (type 3, 49 leaves, 3 steps):

$$\frac{\sqrt{\frac{2}{5}} \operatorname{ArcTan}\left[\frac{\sin\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2} \sqrt{-1+\cos\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{e}$$

Result (type 3, 99 leaves):

$$\left( \left( \frac{2}{5} + \frac{6i}{5} \right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \operatorname{ArcTanh}\left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left( 3 + \tan\left[ \frac{1}{4} (d + e x) \right] \right) \right] \right. \\ \left. \left( \cos\left[ \frac{1}{2} (d + e x) \right] - 3 \sin\left[ \frac{1}{2} (d + e x) \right] \right) \right) / \left( e \sqrt{-5 + 4 \cos [d + e x] + 3 \sin [d + e x]} \right)$$

Problem 428: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(-5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sin\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2} \sqrt{-1+\cos\left[d+e x-\operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{10 \sqrt{10} e} + \frac{3 \cos [d + e x] - 4 \sin [d + e x]}{10 e (-5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{3/2}}$$

Result (type 3, 152 leaves):

$$\left( \left( \frac{1}{250} - \frac{i}{125} \right) \left( \cos\left[ \frac{1}{2} (d + e x) \right] - 3 \sin\left[ \frac{1}{2} (d + e x) \right] \right) \right. \\ \left( (-1 + i) \sqrt{-20 - 15i} \operatorname{ArcTanh}\left[ \left( \frac{1}{10} + \frac{3i}{10} \right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left( 3 + \tan\left[ \frac{1}{4} (d + e x) \right] \right) \right] \right) \\ \left( \cos\left[ \frac{1}{2} (d + e x) \right] - 3 \sin\left[ \frac{1}{2} (d + e x) \right] \right)^2 + \\ \left. \left( 5 + 10i \right) \left( 3 \cos\left[ \frac{1}{2} (d + e x) \right] + \sin\left[ \frac{1}{2} (d + e x) \right] \right) \right) / \left( e (-5 + 4 \cos [d + e x] + 3 \sin [d + e x])^{3/2} \right)$$

**Problem 429: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(-5 + 4 \cos[d + e x] + 3 \sin[d + e x])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTan}\left[\frac{\sin\left[d + e x - \operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}{\sqrt{2} \sqrt{-1 + \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{3}{4}\right]\right]}}\right]}{400 \sqrt{10} e} + \frac{3 \cos[d + e x] - 4 \sin[d + e x]}{20 e (-5 + 4 \cos[d + e x] + 3 \sin[d + e x])^{5/2}} - \frac{3 (3 \cos[d + e x] - 4 \sin[d + e x])}{400 e (-5 + 4 \cos[d + e x] + 3 \sin[d + e x])^{3/2}}$$

Result (type 3, 178 leaves):

$$\left( \left( \frac{1}{10000} + \frac{i}{20000} \right) \left( \cos\left[\frac{1}{2}(d + e x)\right] - 3 \sin\left[\frac{1}{2}(d + e x)\right] \right) \right. \\ \left( (6 + 6i) \sqrt{-20 - 15i} \operatorname{ArcTanh}\left[\left(\frac{1}{10} + \frac{3i}{10}\right) \sqrt{-\frac{4}{5} - \frac{3i}{5}} \left(3 + \tan\left[\frac{1}{4}(d + e x)\right]\right)\right] \right. \\ \left. \left( \cos\left[\frac{1}{2}(d + e x)\right] - 3 \sin\left[\frac{1}{2}(d + e x)\right] \right)^4 + (10 - 5i) \right. \\ \left. \left( 165 \cos\left[\frac{1}{2}(d + e x)\right] - 27 \cos\left[\frac{3}{2}(d + e x)\right] + 55 \sin\left[\frac{1}{2}(d + e x)\right] - 39 \sin\left[\frac{3}{2}(d + e x)\right] \right) \right) \right) / \\ (e (-5 + 4 \cos[d + e x] + 3 \sin[d + e x])^{5/2})$$

**Problem 430: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x] \right)^{7/2} dx$$

Optimal (type 3, 258 leaves, 4 steps):



$$\begin{aligned}
& - \frac{256 (b^2 + c^2)^{3/2} (c \cos[d + ex] - b \sin[d + ex])}{35 e \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} - \frac{1}{35 e} \\
& 64 (b^2 + c^2) (c \cos[d + ex] - b \sin[d + ex]) \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} - \frac{1}{35 e} \\
& 24 \sqrt{b^2 + c^2} (c \cos[d + ex] - b \sin[d + ex]) \left( \sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{3/2} - \\
& \frac{1}{7 e} 2 (c \cos[d + ex] - b \sin[d + ex]) \left( \sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{5/2}
\end{aligned}$$

Result (type 4, 11888 leaves):

$$\begin{aligned}
& \frac{1}{e} \sqrt{b^2 + c^2} \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \\
& \left( \frac{24 b (b^2 + c^2)}{5 c} - \frac{2}{5} c \sqrt{b^2 + c^2} \cos[d + ex] - \frac{6}{5} b c \cos[2 (d + ex)] + \frac{2}{5} b \sqrt{b^2 + c^2} \sin[d + ex] + \right. \\
& \quad \left. \frac{3}{5} (b^2 - c^2) \sin[2 (d + ex)] \right) + \frac{1}{e} \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \\
& \left( \frac{88 b (b^2 + c^2)^{3/2}}{35 c} - \frac{173}{70} c (b^2 + c^2) \cos[d + ex] - \frac{2}{35} b c \sqrt{b^2 + c^2} \cos[2 (d + ex)] - \right. \\
& \quad \left. \frac{1}{14} c (3 b^2 - c^2) \cos[3 (d + ex)] + \frac{173}{70} b (b^2 + c^2) \sin[d + ex] + \right. \\
& \quad \left. \frac{1}{35} (b^2 - c^2) \sqrt{b^2 + c^2} \sin[2 (d + ex)] + \frac{1}{14} b (b^2 - 3 c^2) \sin[3 (d + ex)] \right) - \left( 1024 b (-i b + c) \right. \\
& \quad \left. (b^2 + c^2)^2 \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + ex)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + ex)])}} \right], 1 \right] - \right. \right. \\
& \quad \left. \left. 2 \text{EllipticPi}[-1, \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + ex)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + ex)])}} \right], 1 \right] \right) \right. \\
& \quad \left. \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( -i + \tan\left[\frac{1}{2} (d + ex)\right] \right) \right. \\
& \quad \left. \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + ex)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + ex)])}} \left( c + (-b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2} (d + ex)\right] \right) \right) \Bigg/ \\
& \left( 35 (b + i c - \sqrt{b^2 + c^2})^2 (b + i c + \sqrt{b^2 + c^2}) e (1 + \cos[d + ex]) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \\
& \left(b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) \Bigg) + \\
& \frac{1}{35 c e (1 + \cos[d + ex])} \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} 256 (b^2 + c^2)^{5/2} \\
& \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( \left( -b + c \tan\left[\frac{1}{2}(d + ex)\right] \right) \right. \\
& \sqrt{\left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2\right)} \\
& \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2\right) \left(b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \right. \\
& \left. \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) \Bigg) \Bigg/ \left((b^2 + c^2) \left(1 + \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} \\
& \sqrt{\left(b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) \Bigg) - \\
& \left( \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2\right) \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \right. \right. \\
& \left. \left. \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2\right)} \\
& \left(b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) \Bigg) \\
& \left( \left( 2 c^2 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1\right] + 2 i \operatorname{EllipticPi}\left[ \right. \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \Bigg/ \left(\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right. \\
& \left.\sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2\right) \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right)} + \left(8b^3 \left(-b + i c + \sqrt{b^2 + c^2}\right)\right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}}\right], 1\right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}}\right], 1\right] \right) \\
& \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left. \left(-b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left.\sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2\right) \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + ex)^2 \right] \Bigg) + \left( 4b^5 \left( -b + ic + \sqrt{b^2 + c^2} \right) \right. \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{(ib + c - i\sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] - 2 \\
& ic \text{EllipticPi} \left[ \frac{(b + ic - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - ic - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \left. \text{ArcSin} \left[ \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{(ib + c - i\sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] \right) \\
& \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{(ib + c - i\sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \Bigg) / \left( c^2 (b - ic - \sqrt{b^2 + c^2}) \right) \\
& \left( -b - ic + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan \left[ \frac{1}{2} (d + ex) \right] + \right. \\
& \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + ex)^2 \right] \right) \Bigg) + \left( 4bc^2 \left( -b + ic + \sqrt{b^2 + c^2} \right) \right. \\
& \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2}) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{(ib + c - i\sqrt{b^2 + c^2}) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] - \\
& 2ic \text{EllipticPi} \left[ \frac{(b + ic - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - ic - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2\right.}\right. \\
& \left.\left.\left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)\right) - \\
& \left(8 b^2 \sqrt{b^2 + c^2} \left(-b + i c + \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1\right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{i}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[ \frac{1}{2} (d + ex) \right] + (-b + \sqrt{b^2 + c^2}) \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right) \Bigg) - \\
& \left( 8b^4 \sqrt{b^2 + c^2} \left( (-b + ic + \sqrt{b^2 + c^2}) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{(b + ic - \sqrt{b^2 + c^2})(i + \frac{c}{b - \sqrt{b^2 + c^2}})}{(b - ic - \sqrt{b^2 + c^2})(-i + \frac{c}{b - \sqrt{b^2 + c^2}})} \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] \right) \right. \right. \\
& \left. \left. (-i + \tan[\frac{1}{2}(d + ex)]) \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right. \right. \\
& \left. \left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[ \frac{1}{2} (d + ex) \right] \right) \right) \Bigg) / \left( c^2 (b - ic - \sqrt{b^2 + c^2}) (-b - ic + \sqrt{b^2 + c^2}) \right) \\
& \left( \frac{i}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[ \frac{1}{2} (d + ex) \right] + (-b + \sqrt{b^2 + c^2}) \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right) \Bigg) + \\
& \left( 4b(b^2 + c^2) \left( (-b + ic + \sqrt{b^2 + c^2}) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] - 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)^2\right.}\right. \\
& \left.\left.\left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)\right) + \\
& \left(4 b^3 (b^2 + c^2) \left(\left(-b + i c + \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}}, 1\right] - 2\right.\right. \\
& \left.\left.\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}\right) - 2\right. \\
& \left.\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)},\right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right]\right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( c^2 \left( b - ic - \sqrt{b^2 + c^2} \right) \right. \\
& \left( -b - ic + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \left( 8b^3 \left( -b + ic - \sqrt{b^2 + c^2} \right) \right. \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] - 2 \\
& ic \text{EllipticPi}\left[\frac{\left( b + ic + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - ic + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( -b - ic - \sqrt{b^2 + c^2} \right) \right. \\
& \left( b - ic + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \left( 4b^5 \left( -b + ic - \sqrt{b^2 + c^2} \right) \right.
\end{aligned}$$



$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2 \\
& i c \text{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \\
& \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(c^2 \left(-b - i c - \sqrt{b^2 + c^2}\right)\right) \\
& \left(b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \\
& \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2\right) \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \right. \\
& \left. \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \Bigg) + \left(4 b c^2 \left(-b + i c - \sqrt{b^2 + c^2}\right)\right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - \right. \\
& \left. 2 i c \text{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left(-b-ic-\sqrt{b^2+c^2}\right)\left(b-ic+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \\
& \left. \left. \left( b+\sqrt{b^2+c^2} + 2c\tan\left[\frac{1}{2}(d+ex)\right] + \left(-b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) - \\
& \left( 4b(b^2+c^2) \left( \left(-b+ic-\sqrt{b^2+c^2}\right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. ic \text{EllipticPi}\left[ \frac{\left(b+ic+\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(b-ic+\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSin}\left[ \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] \right) \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right. \\
& \left. \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left(-b-ic-\sqrt{b^2+c^2}\right)\left(b-ic+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \\
& \left. \left. \left( b+\sqrt{b^2+c^2} + 2c\tan\left[\frac{1}{2}(d+ex)\right] + \left(-b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \left( c^2 (-b - i c - \sqrt{b^2 + c^2}) \right. \\
& \quad \left. (b - i c + \sqrt{b^2 + c^2}) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. (-b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) \right) + \left( 2 b^3 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \Bigg/ \\
& \left( c \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) + \\
& \left( 2bc \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1\right] \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right. \\
& \quad \left. \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right] \Bigg/ \\
& \left( \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) - \\
& \left( 2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1\right] \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right. \\
& \quad \left. \sqrt{\frac{\left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right] \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{i}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(\frac{i}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \\
& \left( c \left(-\frac{i}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{i}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) - \\
& \left( b c \left( 2i \left( -\frac{1}{2}i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right) \right) \right) \right. \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right) \right) \right], 1 \right) - \\
& \quad \left( i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF}\left[ \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[ \sqrt{\left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right)} \right] \right. \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right) \right] \right), 1 \right] \Bigg/ \\
& \quad \left( 2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) + \left( 2c \text{EllipticPi}\left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[ \sqrt{\left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right)} \right] \right. \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right) \right] \right), 1 \right] \Bigg/ \\
& \quad \left( \left( b - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \Bigg) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \\
& \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + \right. \right.}
\end{aligned}$$



$$\left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] - \right. \right. \right. \right. \\ \left. \left. \left. \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right)} \right) / \\ \left( (b^2+c^2) \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \sqrt{\left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] - \right. \right. \right. \\ \left. \left. \left. b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} - \right. \\ \left. \sqrt{\left( b + 2c \tan\left[\frac{1}{2}(d+ex)\right] - b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right)$$

**Problem 431: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex] \right)^{5/2} dx$$

Optimal (type 3, 190 leaves, 3 steps):

$$-\frac{64(b^2+c^2)(c \cos[d+ex] - b \sin[d+ex])}{15e \sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]} - \frac{1}{15e} \\ 16 \sqrt{b^2+c^2} (c \cos[d+ex] - b \sin[d+ex]) \sqrt{\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]} - \\ \frac{1}{5e} 2 (c \cos[d+ex] - b \sin[d+ex]) \left( \sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex] \right)^{3/2}$$

Result (type 4, 11771 leaves):

$$\frac{1}{e} \sqrt{b^2+c^2} \left( \frac{4b \sqrt{b^2+c^2}}{3c} - \frac{4}{3} c \cos[d+ex] + \frac{4}{3} b \sin[d+ex] \right) \\ \sqrt{\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]} + \frac{1}{e} \sqrt{\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]} \\ \left( \frac{44b(b^2+c^2)}{15c} - \frac{2}{15} c \sqrt{b^2+c^2} \cos[d+ex] - \frac{2}{5} b c \cos[2(d+ex)] + \right. \\ \left. \frac{2}{15} b \sqrt{b^2+c^2} \sin[d+ex] + \frac{1}{5} (b^2-c^2) \sin[2(d+ex)] \right) - \left( 256b(-ib+c)(b^2+c^2)^{3/2} \right)$$

$$\begin{aligned}
& \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1 \right] - \right. \\
& \quad \left. 2 \text{EllipticPi} \left[ -1, \text{ArcSin} \left[ \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \quad \sqrt{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]} \left( -i + \tan\left[\frac{1}{2} (d + e x)\right] \right) \\
& \quad \sqrt{-\frac{(-b - i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(-b + i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \left( c + (-b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2} (d + e x)\right] \right) \Bigg/ \\
& \quad \left( 15 (b + i c - \sqrt{b^2 + c^2})^2 (b + i c + \sqrt{b^2 + c^2}) e (1 + \cos[d + e x]) \right. \\
& \quad \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]}{(1 + \cos[d + e x])^2}} \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2} (d + e x)\right] \right)^2 \right)} \\
& \quad \left. \left( b + 2 c \tan\left[\frac{1}{2} (d + e x)\right] - b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \right) \right) \Bigg) + \\
& \quad \frac{1}{15 c e (1 + \cos[d + e x])} \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]}{(1 + \cos[d + e x])^2}} \\
& \quad 64 (b^2 + c^2)^2 \sqrt{\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]} \\
& \quad \left( \left( \left( -b + c \tan\left[\frac{1}{2} (d + e x)\right] \right) \sqrt{\left( b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] - \right. \right. \right. \\
& \quad \left. \left. b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2} (d + e x)\right] \right)^2 \right)} \right. \right. \\
& \quad \left. \left( b + 2 c \tan\left[\frac{1}{2} (d + e x)\right] - b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \right) \right) \Bigg) \Bigg/ \\
& \quad \left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \sqrt{\left( b + 2 c \tan\left[\frac{1}{2} (d + e x)\right] - b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \right. \right. \right.
\end{aligned}$$



$$\begin{aligned}
& \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg) - \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + \right. \right. \right. \\
& \left. \left. \left. 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF} \left[ \right. \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] + 2i \operatorname{EllipticPi} \left[ \right. \right. \right. \right. \\
& \left. \left. \left. \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \Bigg) \Bigg/ \left( \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) + \left( 8b^3 \left( \left( -b + ic + \sqrt{b^2 + c^2} \right) \right. \right. \right. \\
& \left. \left. \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(-b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right)\right. \\
& \left.\sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right) \left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \right.\right.\right. \\
& \left.\left.\left(-b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)} + \left(4 b^5 \left(-b + i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] - 2\right. \\
& \left.\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( c^2 \left( b - ic - \sqrt{b^2 + c^2} \right) \right. \\
& \left( -b - ic + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + 4bc^2 \left( -b + ic + \sqrt{b^2 + c^2} \right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] - \\
& 2ic \operatorname{EllipticPi}\left[\frac{\left( b + ic - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - ic - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( b - ic - \sqrt{b^2 + c^2} \right) \left( -b - ic + \sqrt{b^2 + c^2} \right) \right. \\
& \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} - \\
& \left( 8b^2 \sqrt{b^2 + c^2} \left( -b + ic + \sqrt{b^2 + c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1] - 2 \\
& i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1\right] \\
& \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right) \\
& \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2\right)} \\
& \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \Bigg) - \\
& \left(8 b^4 \sqrt{b^2 + c^2} \left(-b + i c + \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1\right] - 2\right. \right. \\
& i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( c^2 \left( b - i c - \sqrt{b^2 + c^2} \right) \left( -b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left. \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right)} \right. \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \Bigg) + \\
& \left( 4 b (b^2 + c^2) \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. i c \text{EllipticPi}\left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)} \right], 1 \right] \right. \right. \\
& \left. \left. \text{ArcSin}\left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1 \right] \right) \Bigg) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left( b - i c - \sqrt{b^2 + c^2} \right) \left( -b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left. \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right)} \right. \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \right. \\
& \quad i c \text{EllipticPi} \left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \left. \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right) \\
& \quad \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \left( c^2 \left( b - i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left( -b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \quad \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \Bigg) + \left( 8 b^3 \left( \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \right. \\
& \quad \left. \left. i c \text{EllipticPi} \left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left.\left(b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2\right) \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \right. \right. \\
& \left. \left. \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)} \Bigg) + 4 b^5 \left(\left(-b + i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi} \left[ \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)} \right], \right. \\
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \right. \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(c^2 \left(-b - i c - \sqrt{b^2 + c^2}\right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left. \right) + \left( 4bc^2 \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - \right. \\
& \quad \left. 2ic \text{EllipticPi}\left[\frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \left/ \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \right. \right. \\
& \quad \left. \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \right.} \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) - \\
& \quad \left( 4b(b^2 + c^2) \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)^2\right.}\right. \\
& \left.\left.\left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)\right) - \\
& \left(4 b^3 (b^2 + c^2) \left(\left(-b + i c - \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}}, 1\right] - 2\right.\right. \\
& \left.\left.\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}\right) - 2\right. \\
& \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \right. \\
& \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \left( 2 b^3 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF}\left[ \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right. \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right] \Bigg/ \\
& \left( c \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right. \right. \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right)} + \right. \\
& \left( 2 b c \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF}\left[ \operatorname{ArcSin}\left[ \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}\Bigg/ \\
& \left(\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)}\right. \\
& \quad \left.\left(b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]+\left(-b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)\Bigg)- \\
& \left(2b^2\sqrt{b^2+c^2}\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\right.\right.\right. \\
& \quad \left.\left.\left.\sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\right],1\right]\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right. \\
& \quad \left.\sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)\Bigg/ \\
& \left(c\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)}\right. \\
& \quad \left.\left(b+\sqrt{b^2+c^2}+2c\tan\left[\frac{1}{2}(d+ex)\right]+\left(-b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)\Bigg)- \\
& \left(bc\left(2i\left(-\frac{1}{2}i\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\text{EllipticE}\left[\text{ArcSin}\left[\right.\right.\right.\right.\right. \\
& \quad \left.\left.\left.\sqrt{\left(\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)}\right)\right)\right.\right. \\
& \quad \left.\left.\left(\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)\right)\right],1\right)- \\
& \quad \left(i\left(i\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)+i\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\right)\text{EllipticF}\left[\right.\right. \\
& \quad \left.\left.\text{ArcSin}\left[\sqrt{\left(\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)}\right]\right)\right)\Bigg/
\end{aligned}$$



$$\begin{aligned}
& \left( 2 \left( -\frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) + \left( 2 c \operatorname{EllipticPi} \left[ \frac{\frac{c}{b - \sqrt{b^2 + c^2}}}{-\frac{c}{b - \sqrt{b^2 + c^2}}}, \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \sqrt{\left( \left( -\frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2} (d + e x) \right) \right)} \right] \right. \right. \\
& \quad \left. \left. \left( \left( \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{1}{2} (d + e x) \right) \right) \right] \right], 1 \right) \Bigg/ \\
& \left( \left( b - \sqrt{b^2 + c^2} \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \left( -\frac{1}{2} (d + e x) \right) \\
& \sqrt{\frac{\left( -\frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2} (d + e x) \right)}{\left( \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{1}{2} (d + e x) \right)}} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right. \\
& \quad \left. \left. e x) \right] \right) + \left( \frac{1}{2} (d + e x) \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \Bigg) \Bigg/ \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \\
& \quad \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \Bigg/ \\
& \left( (b^2 + c^2) \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \sqrt{\left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] - \right. \\
& \quad \left. b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \\
& \left. \sqrt{\left( b + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] - b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right)
\end{aligned}$$

**Problem 432: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( \sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x] \right)^{3/2} dx$$

Optimal (type 3, 126 leaves, 2 steps):

$$-\frac{8\sqrt{b^2+c^2}\left(c\cos[d+ex]-b\sin[d+ex]\right)}{3e\sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}}-\frac{1}{3e}$$

$$2\left(c\cos[d+ex]-b\sin[d+ex]\right)\sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}$$

Result (type 4, 11679 leaves):

$$\frac{2b\sqrt{b^2+c^2}\sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}}{ce}+\frac{1}{e}$$

$$\left(\frac{2b\sqrt{b^2+c^2}}{3c}-\frac{2}{3}c\cos[d+ex]+\frac{2}{3}b\sin[d+ex]\right)\sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}-$$

$$\left(32b(-ib+c)(b^2+c^2)\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{(-b-ic+\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}}{(-b+ic+\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}}\right],1\right)-2\operatorname{EllipticPi}\left[-1,\operatorname{ArcSin}\left[\sqrt{-\frac{(-b-ic+\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}}{(-b+ic+\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}}\right],1\right]\right)$$

$$\sqrt{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)$$

$$\sqrt{-\frac{(-b-ic+\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(-b+ic+\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}}\left(c+\left(-b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]\right)}\Bigg/$$

$$\left(3\left(b+ic-\sqrt{b^2+c^2}\right)^2\left(b+ic+\sqrt{b^2+c^2}\right)e(1+\cos[d+ex])\right.$$

$$\sqrt{\frac{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}{(1+\cos[d+ex])^2}}\sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right.$$

$$\left.\left.\left(b+2c\tan\left[\frac{1}{2}(d+ex)\right]-b\tan\left[\frac{1}{2}(d+ex)\right]^2+\sqrt{b^2+c^2}\left(1+\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)\right)\right)}\Bigg)+$$

$$\frac{1}{3ce(1+\cos[d+ex])\sqrt{\frac{\sqrt{b^2+c^2}+b\cos[d+ex]+c\sin[d+ex]}{(1+\cos[d+ex])^2}}}\frac{8(b^2+c^2)^{3/2}}{}$$

$$\begin{aligned}
& \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( \left( -b + c \tan\left[\frac{1}{2}(d + ex)\right] \right) \right. \\
& \sqrt{\left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2 \right)} \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \right. \right. \\
& \quad \left. \left. \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) / \left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} \\
& \sqrt{\left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} \Bigg) - \\
& \left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \right. \right. \right. \right. \\
& \quad \left. \left. \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \quad \left. \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right)} \Bigg) \\
& \left( \left( 2c^2 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)} \right], 1 \right] + 2i \operatorname{EllipticPi}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \right. \\
& \quad \left. \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right) \right. \\
& \quad \left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) / \left( \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \right. \right. \\
& \quad \left. \left. \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) + \left( 8b^3 \left( -b + ic + \sqrt{b^2+c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+ic\sqrt{b^2+c^2} \right) \left( i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-ic\sqrt{b^2+c^2} \right) \left( -i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] - 2 \right. \\
& \quad \left. ic \text{EllipticPi}\left[\frac{\left( b+ic-\sqrt{b^2+c^2} \right) \left( i+\frac{c}{b-\sqrt{b^2+c^2}} \right)}{\left( b-ic-\sqrt{b^2+c^2} \right) \left( -i+\frac{c}{b-\sqrt{b^2+c^2}} \right)}, \right. \\
& \quad \left. \text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+ic\sqrt{b^2+c^2} \right) \left( i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-ic\sqrt{b^2+c^2} \right) \left( -i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i+\tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -ib+c+ic\sqrt{b^2+c^2} \right) \left( i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-ic\sqrt{b^2+c^2} \right) \left( -i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}} \\
& \quad \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left( b-ic-\sqrt{b^2+c^2} \right) \right. \\
& \quad \left( -b-ic+\sqrt{b^2+c^2} \right) \left( i-\frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \\
& \quad \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \right. \right. \\
& \quad \left. \left. \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) + \left( 4b^5 \left( -b + ic + \sqrt{b^2+c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib+c+ic\sqrt{b^2+c^2} \right) \left( i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}{\left( ib+c-ic\sqrt{b^2+c^2} \right) \left( -i+\tan\left[\frac{1}{2}(d+ex)\right] \right)}} \right], 1 \right] - 2 \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(c^2 \left(b - i c - \sqrt{b^2 + c^2}\right)\right. \\
& \left.(-b - i c + \sqrt{b^2 + c^2}) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right)\right. \\
& \left.\sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right) \left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \right.\right.\right. \\
& \left.\left.\left.(-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)\right)} + \left(4 b c^2 \left(-b + i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] - \right. \\
& \left.2 i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( (b - ic - \sqrt{b^2 + c^2}) (-b - ic + \sqrt{b^2 + c^2}) \right) \\
& \left( ic - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) - \\
& \left( 8b^2 \sqrt{b^2 + c^2} \left( (-b + ic + \sqrt{b^2 + c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(-icb + c + i\sqrt{b^2 + c^2})(ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(icb + c - i\sqrt{b^2 + c^2})(-ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{(b + ic - \sqrt{b^2 + c^2}) \left( ic + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - ic - \sqrt{b^2 + c^2}) \left( -ic + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-icb + c + i\sqrt{b^2 + c^2})(ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(icb + c - i\sqrt{b^2 + c^2})(-ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1 \right] \right) \right) \\
& (-ic + \operatorname{Tan}[\frac{1}{2}(d + ex)]) \sqrt{\frac{(-icb + c + i\sqrt{b^2 + c^2})(ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(icb + c - i\sqrt{b^2 + c^2})(-ic + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( (b - ic - \sqrt{b^2 + c^2}) (-b - ic + \sqrt{b^2 + c^2}) \right) \\
& \left( ic - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( 8 b^4 \sqrt{b^2 + c^2} \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \operatorname{EllipticPi} \left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \left( c^2 \left( b - i c - \sqrt{b^2 + c^2} \right) \left( -b - i c + \sqrt{b^2 + c^2} \right) \right) \\
& \quad \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right.} \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \Bigg) + \\
& \quad \left( 4 b (b^2 + c^2) \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \operatorname{EllipticPi} \left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2\right)} \right. \\
& \left.\left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right) \Bigg) + \\
& \left(4 b^3 (b^2 + c^2) \left(-b + i c + \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1\right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(c^2 \left(b - i c - \sqrt{b^2 + c^2}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \left( -b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg) + \left( 8b^3 \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \text{EllipticPi}\left[\frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \Bigg) / \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \quad \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg) + \left( 4b^5 \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -ib + c + i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c - i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(c^2 \left(-b - i c - \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right)\right. \\
& \left.\sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right) \left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \right.\right.\right. \\
& \left.\left.\left(-b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)} + \left(4 b c^2 \left(-b + i c - \sqrt{b^2 + c^2}\right)\right. \\
& \left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] - \right. \\
& \left.2 i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left.\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) - \\
& \left( 4 b (b^2 + c^2) \left( \left( -b + i c - \sqrt{b^2 + c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \right. \\
& \left. \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right. \\
& \left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right) \Bigg/ \\
& \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \text{EllipticPi} \left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin} \left[ \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right) \\
& \quad \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left/ \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \right. \right. \\
& \quad \left. \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \right. \\
& \quad \left. \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right)} + \left( 2 b^3 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( c \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right.} \right. \\
& \quad \left. \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \Bigg) + \\
& \left( 2bc \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{\left(-i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1 \right] \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \right. \\
& \quad \left. \sqrt{\frac{\left(-i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(i + \frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)}} \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \right] \Bigg) \Bigg/ \\
& \left( \left( -i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( i + \frac{c}{b-\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right.} \right. \\
& \quad \left. \left( b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left( -b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \Bigg) - \\
& \left( 2b^2\sqrt{b^2+c^2} \left( -i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}}], 1] \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \\
& \sqrt{\frac{\left(-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \\
& \left(c \left(-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right)\right) - \\
& \left(b c \left(2i \left(-\frac{1}{2}i \left(\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \text{EllipticE}\left[\text{ArcSin}\left[\right.\right.\right.\right. \right. \\
& \quad \sqrt{\left(\left(\left(-\frac{i}{b}b + c + i\sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)} \Bigg/ \\
& \quad \left.\left.\left(\left(\frac{i}{b}b + c - i\sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)\right)\right], 1] - \right. \\
& \quad \left.\left(i \left(i \left(-\frac{i}{b} - \frac{c}{b - \sqrt{b^2 + c^2}}\right) + i \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \text{EllipticF}\left[\right.\right.\right. \right. \\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{i}{b}b + c + i\sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)} \Bigg/ \right. \\
& \quad \left.\left.\left(\left(\frac{i}{b}b + c - i\sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)\right)\right], 1] \right) \Bigg/ \\
& \quad \left(2 \left(-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) + \left(2c \text{EllipticPi}\left[\frac{\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}}{-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}}, \right.\right. \\
& \quad \text{ArcSin}\left[\sqrt{\left(\left(\left(-\frac{i}{b}b + c + i\sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)} \Bigg/ \right. \\
& \quad \left.\left.\left(\left(\frac{i}{b}b + c - i\sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)\right)\right)\right], 1] \right) \Bigg/ \\
& \quad \left.\left.\left(\left(b - \sqrt{b^2 + c^2}\right) \left(-\frac{i}{b} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right)\right)\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \\
& + \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2 \Bigg) \Bigg) / \\
& \left(\sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \right.\right.\right. \\
& \left.\left.\left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right)} + \right. \\
& \left(c \sqrt{b^2 + c^2} \left(2 \frac{1}{2} \left(-\frac{1}{2} \frac{1}{b - \sqrt{b^2 + c^2}} \left(\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right) \text{EllipticE}\left[\text{ArcSin}\left[\right.\right.\right.\right. \right. \\
& \left.\left.\left.\left.\left(\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}\right)\right], 1\right] - \right.\right. \\
& \left.\left(\frac{1}{2} \left(\frac{1}{2} \left(-\frac{1}{2} - \frac{c}{b - \sqrt{b^2 + c^2}}\right) + \frac{1}{2} \left(\frac{1}{2} - \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \text{EllipticF}\left[\right.\right.\right. \\
& \left.\left.\left.\text{ArcSin}\left[\sqrt{\left(\left(\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}\right)\right]\right], 1\right]\right) \right. \\
& \left.\left(2 \left(-\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) + 2 c \text{EllipticPi}\left[\frac{\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}}{-\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}}, \right.\right. \\
& \left.\left.\text{ArcSin}\left[\sqrt{\left(\left(\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}\right)\right]\right], 1\right]\right) \Bigg) / \\
& \left(\left(b - \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)\right) \Bigg) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \\
& \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right)
\end{aligned}$$



$$\begin{aligned}
& \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \\
& \sqrt{-\frac{(-b - ic + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(-b + ic + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \left( c + (-b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( (b + ic - \sqrt{b^2 + c^2})^2 (b + ic + \sqrt{b^2 + c^2}) e (1 + \cos[d + ex]) \right. \\
& \left. \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} \sqrt{\left( (1 + \tan\left[\frac{1}{2}(d + ex)\right])^2 \right. \right.} \\
& \left. \left. \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) + \\
& \frac{1}{c e (1 + \cos[d + ex]) \sqrt{\frac{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}}} 2 (b^2 + c^2) \\
& \sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( \left( -b + c \tan\left[\frac{1}{2}(d + ex)\right] \right) \right. \\
& \left. \sqrt{\left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2 \right)} \right. \\
& \left. \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \Bigg/ \left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} \right. \\
& \left. \sqrt{\left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) -} \\
& \left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right.} \\
& \left. \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + 2c \tan\left[\frac{1}{2}(d + ex)\right] - b \tan\left[\frac{1}{2}(d + ex)\right]^2 + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) \Bigg/ \left( (b^2 + c^2) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 2 c^2 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] + 2 i \operatorname{EllipticPi} \left[ \right. \right. \\
& \quad \left. \left. \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \left( \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left. \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) + 8 b^3 \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \operatorname{EllipticPi} \left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( (b - ic - \sqrt{b^2 + c^2}) \right. \\
& \left. (-b - ic + \sqrt{b^2 + c^2}) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) + 4b^5 \left( (-b + ic + \sqrt{b^2 + c^2}) \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1\right] - 2 \right. \\
& \left. ic \operatorname{EllipticPi}\left[\frac{(b + ic - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - ic - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1\right] \right) \right. \\
& \left. (-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]) \sqrt{\frac{(-ib + c + i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c - i\sqrt{b^2 + c^2})(-i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right) \right. \\
& \left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right) \Bigg/ \left( c^2 (b - ic - \sqrt{b^2 + c^2}) \right. \\
& \left. (-b - ic + \sqrt{b^2 + c^2}) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (-b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) + 4bc^2 \left( (-b + ic + \sqrt{b^2 + c^2}) \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - \\
& 2 i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \right) \\
& \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right) \\
& \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2\right)} \\
& \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \Bigg) - \\
& \left(8 b^2 \sqrt{b^2 + c^2} \left(-b + i c + \sqrt{b^2 + c^2}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2 \right. \\
& \left. i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left(b-ic-\sqrt{b^2+c^2}\right)\left(-b-ic+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \\
& \left. \left. \left( b+\sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(-b+\sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) - \\
& \left( 8b^4\sqrt{b^2+c^2} \left( \left(-b+ic+\sqrt{b^2+c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{\left(b+ic-\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(b-ic-\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b-\sqrt{b^2+c^2}}\right)}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] \right) \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c+i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c-i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right. \\
& \left. \left( -\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( c^2 \left(b-ic-\sqrt{b^2+c^2}\right)\left(-b-ic+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left( i - \frac{c}{b-\sqrt{b^2+c^2}} \right) \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \\
& \left. \left. \left( b+\sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(-b+\sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}}]{}, 1] - 2 \right. \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}}]{}, 1] \right] \right) \right. \right. \\
& \quad \left. \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right. \\
& \quad \left. \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \Bigg/ \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \right) \\
& \quad \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right.} \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + (-b + \sqrt{b^2 + c^2}) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \Bigg) + \\
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c + \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(-i b + c + i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c - i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}}]{}, 1] - 2 \right. \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{(b - i c - \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(c^2 \left(b - i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left. \left(-b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left. \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \right.\right. \right. \\
& \left. \left. \left. \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)\right)} + 8 b^3 \left(-b + i c - \sqrt{b^2 + c^2}\right) \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1\right] - 2 \\
& i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left. \right) + \left( 4b^5 \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \text{EllipticPi}\left[\frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \quad \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \left. \right) / \\
& \quad \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \quad \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \right.} \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} + 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \left. \right) + \\
& \quad \left( 4b c^2 \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1] - 2 \\
& i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1\right] \\
& \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \\
& \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right) \left(i - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \right. \\
& \left.\left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2\right)} \right. \\
& \left.\left(b + \sqrt{b^2 + c^2} + 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right) \Bigg) - \\
& \left(4 b \left(b^2 + c^2\right) \left(-b + i c - \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c + i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c - i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}], 1\right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}\right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}}}, 1 \right] \right) \\
& \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}}} \\
& \left(-\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \\
& \left( \left(-b - \frac{1}{2} c - \sqrt{b^2 + c^2}\right) \left(b - \frac{1}{2} c + \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} - \frac{c}{b - \sqrt{b^2 + c^2}}\right) \right. \\
& \left. \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \sqrt{\left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)^2\right)} \right. \\
& \left. \left(b + \sqrt{b^2 + c^2} + 2 c \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right] + \left(-b + \sqrt{b^2 + c^2}\right) \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]^2\right) \right) \Bigg) - \\
& \left(4 b^3 (b^2 + c^2) \left( \left(-b + \frac{1}{2} c - \sqrt{b^2 + c^2}\right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}}}, 1 \right] - 2 \right. \right. \right. \\
& \left. \frac{1}{2} c \operatorname{EllipticPi} \left[ \frac{\left(b + \frac{1}{2} c + \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}{\left(b - \frac{1}{2} c + \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \frac{c}{b - \sqrt{b^2 + c^2}}\right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}}}, 1 \right] \right) \right. \\
& \left. \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-\frac{1}{2} b + c + \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(\frac{1}{2} b + c - \frac{1}{2} \sqrt{b^2 + c^2}\right) \left(-\frac{1}{2} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \right. \\
& \left( b - i c + \sqrt{b^2 + c^2} \right) \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \\
& \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \\
& \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \left( 2b^3 \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right. \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)} \right], 1 \right] \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( c \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right. \right. \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} + 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( -b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right)} + \right. \\
& \left. \left( 2bc \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)}} \left[1, 1\right] \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right) \\
& \sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)}} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right) \Bigg/ \\
& \left( \left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d+ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(-b + \sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2\right) \right) - \\
& \left( 2b^2\sqrt{b^2+c^2} \left(-\frac{c}{b-\sqrt{b^2+c^2}}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{c}{b-\sqrt{b^2+c^2}}\right], 1\right] \right. \\
& \quad \left. \sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)}} \left[1, 1\right] \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right) \right. \\
& \quad \left. \sqrt{\frac{\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)}{\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)}} \left(-\frac{c}{b-\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right]\right) \right) \Bigg/ \\
& \left( c \left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d+ex)\right]\right)^2} \right. \\
& \quad \left. \left(b + \sqrt{b^2+c^2} + 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(-b + \sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2\right) \right) - \\
& \left( bc \left( 2 \left(-\frac{c}{b-\sqrt{b^2+c^2}}\right) \left(\frac{c}{b-\sqrt{b^2+c^2}}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{c}{b-\sqrt{b^2+c^2}}\right], 1\right] \right. \right. \\
& \quad \left. \left. \sqrt{\left(\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\right)} \right) \right) \Bigg/ \\
& \quad \left( \left(\frac{c}{b-\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b-\sqrt{b^2+c^2}}\right) \right) \left[1, 1\right] -
\end{aligned}$$



$$\begin{aligned}
& \left( i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] \right) / \\
& \left( 2 \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) + \left( 2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b - \sqrt{b^2 + c^2}}}{-i + \frac{c}{b - \sqrt{b^2 + c^2}}}, \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] \right) / \\
& \left( \left( b - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \\
& \sqrt{\frac{\left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right.} \right. \\
& \quad \left. \left. e x) \right] \right) + \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b - \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \left. \right) \right) / \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( b + \sqrt{b^2 + c^2} + 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \\
& \quad \left. \left. \left( -b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) + \\
& \left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b - \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \right. \right. \right. \right. \right. \right. \\
& \quad \sqrt{\left( \left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c - i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] - \right. \\
& \quad \left. \left( i \left( i \left( -i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b - \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \right. \right. \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c + i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \right. \right. \right.
\end{aligned}$$



$$\int \frac{1}{\sqrt{\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{(b^2 + c^2)^{1/4} \sin[d + ex - \operatorname{ArcTan}[b, c]]}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos[d + ex - \operatorname{ArcTan}[b, c]]}}\right]}{(b^2 + c^2)^{1/4} e}$$

Result (type 4, 63 264 leaves): Display of huge result suppressed!

**Problem 435: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]\right)^{3/2}} dx$$

Optimal (type 3, 160 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{(b^2 + c^2)^{1/4} \sin[d + ex - \operatorname{ArcTan}[b, c]]}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos[d + ex - \operatorname{ArcTan}[b, c]]}}\right]}{2 \sqrt{2} (b^2 + c^2)^{3/4} e} - \frac{c \cos[d + ex] - b \sin[d + ex]}{2 \sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 436: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\left(\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]\right)^{5/2}} dx$$

Optimal (type 3, 226 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTanh}\left[\frac{(b^2 + c^2)^{1/4} \sin[d + ex - \operatorname{ArcTan}[b, c]]}{\sqrt{2} \sqrt{\sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} \cos[d + ex - \operatorname{ArcTan}[b, c]]}}\right]}{16 \sqrt{2} (b^2 + c^2)^{5/4} e} - \frac{c \cos[d + ex] - b \sin[d + ex]}{4 \sqrt{b^2 + c^2} e \left(\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]\right)^{5/2}} - \frac{3 (c \cos[d + ex] - b \sin[d + ex])}{16 (b^2 + c^2) e \left(\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 437: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{5/2} dx$$

Optimal (type 3, 196 leaves, 3 steps):

$$\begin{aligned} & -\frac{64 (b^2 + c^2) (c \cos[d + ex] - b \sin[d + ex])}{15 e \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} + \frac{1}{15 e} \\ & 16 \sqrt{b^2 + c^2} (c \cos[d + ex] - b \sin[d + ex]) \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} - \\ & \frac{1}{5 e} 2 (c \cos[d + ex] - b \sin[d + ex]) \left( -\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{3/2} \end{aligned}$$

Result (type 4, 11602 leaves):

$$\begin{aligned} & \frac{1}{e} \sqrt{b^2 + c^2} \left( \frac{4 b \sqrt{b^2 + c^2}}{3 c} + \frac{4}{3} c \cos[d + ex] - \frac{4}{3} b \sin[d + ex] \right) \\ & \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} + \frac{1}{e} \\ & \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( \frac{44 b (b^2 + c^2)}{15 c} + \frac{2}{15} c \sqrt{b^2 + c^2} \cos[d + ex] - \right. \\ & \quad \left. \frac{2}{5} b c \cos[2(d + ex)] - \frac{2}{15} b \sqrt{b^2 + c^2} \sin[d + ex] + \frac{1}{5} (b^2 - c^2) \sin[2(d + ex)] \right) - \\ & \left( 256 b c (b^2 + c^2)^{5/2} \left( \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] - \right. \right. \\ & \quad \left. \left. 2 \text{EllipticPi}[-1, \text{ArcSin} \left[ \sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] \right) \\ & \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \\ & \left( -\frac{(b + i c + \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2}(d + ex)])} \right)^{3/2} \\ & \left( -c + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right] \right) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg/ \end{aligned}$$

$$\begin{aligned}
& \left( 15 \left( b + i c + \sqrt{b^2 + c^2} \right)^3 \left( b^2 + c^2 - b \sqrt{b^2 + c^2} \right) e \left( 1 + \cos [d + e x] \right) \right. \\
& \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}{(1 + \cos [d + e x])^2}} \\
& \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \sqrt{\left( - \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right.} \\
& \left. \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) \left. \right) + \\
& \frac{1}{15 c e \left( 1 + \cos [d + e x] \right) \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]}{(1 + \cos [d + e x])^2}}} \\
& 64 \left( b^2 + c^2 \right)^2 \sqrt{-\sqrt{b^2 + c^2} + b \cos [d + e x] + c \sin [d + e x]} \\
& \left( \left( \left( -b + c \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \right.} \right. \right. \\
& \left. \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \sqrt{\left( - \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right.} \\
& \left. \left. b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) \right) / \\
& \left( \left( b^2 + c^2 \right) \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \sqrt{\left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) + \right.} \right. \\
& \left. \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) + \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - \right. \right.} \\
& \left. \left. 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \right) \\
& \sqrt{\left( - \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) + \right.} \right.}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right]^2 \right) \left( 2c^2 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i \operatorname{EllipticF} \left[ \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] + 2i \operatorname{EllipticPi} \left[ \right. \right. \right. \\
& \left. \left. \left. \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin} \left[ \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] \right) \right) \\
& \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \left/ \left( \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right. \right. \\
& \left. \left. \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2c \tan \left[ \frac{1}{2} (d + ex) \right] + \right. \right. \right. \right. \\
& \left. \left. \left. (b + \sqrt{b^2 + c^2}) \tan \left[ \frac{1}{2} (d + ex) \right]^2 \right) \right) \right) + 8b^3 \left( (-b + ic + \sqrt{b^2 + c^2}) \right. \\
& \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi} \left[ \frac{(b + ic - \sqrt{b^2 + c^2})(i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - ic - \sqrt{b^2 + c^2})(-i + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], \right. \\
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}} \right], 1 \right] \right) \\
& \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) + \left( 4 b^5 \left( (-b + i c + \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi}\left[\frac{(b + i c - \sqrt{b^2 + c^2}) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{(b - i c - \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}} \right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right])}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( c^2 (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) + \left( 4 b c^2 \left( (-b + i c + \sqrt{b^2 + c^2}) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2 \\
& i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \\
& \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right) \\
& \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2\right)} \\
& \left(-b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2} (d + e x)\right] + \left(b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \Bigg) - \\
& \left(4 b (b^2 + c^2) \left(-b + i c + \sqrt{b^2 + c^2}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2 \right. \\
& \left. i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] \right)
\end{aligned}$$



$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left(b-i c-\sqrt{b^2+c^2}\right)\left(-b-i c+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right)\left(i-\frac{c}{b+\sqrt{b^2+c^2}}\right) \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)} \right. \\
& \left. \left(-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+\left(b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right) \Bigg) - \\
& \left( 4b^3(b^2+c^2) \left( \left(-b+ic+\sqrt{b^2+c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{\left(b+ic-\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{\left(b-ic-\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \\
& \left( c^2\left(b-ic-\sqrt{b^2+c^2}\right)\left(-b-ic+\sqrt{b^2+c^2}\right)\left(-\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c}\right) \right. \\
& \left. \left(i-\frac{c}{b+\sqrt{b^2+c^2}}\right) \sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)\left(-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+\right)} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( (b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2} (d + e x)^2\right] \right) \Bigg) + \left( 8 b^3 \left( (-b + i c - \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}}\right], 1\right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi}\left[\frac{(b + i c + \sqrt{b^2 + c^2}) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{(b - i c + \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}}\right], 1\right] \right) \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] \right) \Bigg) \Bigg/ \\
& \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \right. \right. \right. \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)^2\right] \right) \right) \Bigg) + \left( 4 b^5 \left( (-b + i c - \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right])}}\right], 1\right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi}\left[\frac{(b + i c + \sqrt{b^2 + c^2}) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{(b - i c + \sqrt{b^2 + c^2}) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \\
& \left(c^2 \left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left.\left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \right.\right. \right. \\
& \left.\left.\left(b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)} \right] + \left(4 b c^2 \left(-b + i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1\right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \\
& \left( 8b^2 \sqrt{b^2 + c^2} \left( \left( -b + ic - \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. ic \text{EllipticPi}\left[ \frac{\left( b + ic + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - ic + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \right. \right. \\
& \left. \left. \left. \text{ArcSin}\left[ \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \right. \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right. \right. \\
& \left. \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right) \sqrt{\left( \left( -b - ic - \sqrt{b^2 + c^2} \right) \left( b - ic + \sqrt{b^2 + c^2} \right) \right.} \\
& \left. \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right)} + \\
& \left( 8b^4 \sqrt{b^2 + c^2} \left( \left( -b + ic - \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(c^2 \left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)^2\right.}\right. \\
& \left.\left.\left(-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)\right) + \\
& \left(4 b \left(b^2 + c^2\right) \left(-b + i c - \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] - 2\right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)},\right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right]\right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( -b - ic - \sqrt{b^2 + c^2} \right) \left( b - ic + \sqrt{b^2 + c^2} \right) \right. \\
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( ic - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) + \\
& \left( 4b^3(b^2 + c^2) \left( \left( -b + ic - \sqrt{b^2 + c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{\left( b + ic + \sqrt{b^2 + c^2} \right) \left( ic + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - ic + \sqrt{b^2 + c^2} \right) \left( -ic + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \right. \\
& \left. \left( -ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -ic + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right. \\
& \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \right) \Bigg/ \\
& \left( c^2 \left( -b - ic - \sqrt{b^2 + c^2} \right) \left( b - ic + \sqrt{b^2 + c^2} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( ic - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x)^2 \right] \Bigg) + \left( 2 b^3 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \right. \right. \\
& \text{ArcSin} \left[ \frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \\
& \left. \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right] \Bigg/ \\
& \left( c \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right.} \right. \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \Bigg) + \\
& \left( 2 b c \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \right. \right. \right. \\
& \left. \left. \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \left. \sqrt{\frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right] \right] \Bigg/ \\
& \left( \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right.} \right. \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \Bigg) +
\end{aligned}$$





$$\begin{aligned}
& \left( \left( \left( \frac{b + c + i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right) \right), 1 \right) / \\
& \left( \left( \left( \frac{b + \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \\
& \sqrt{\frac{\left( \frac{-i b + c - i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{\left( \frac{i b + c + i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right.} \\
& \left. \left. ex) \right] \right) + \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2} \right) / \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + ex) \right] + \right. \right.} \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2} \right) - \\
& \left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \right. \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left( \frac{-i b + c - i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right] \right) \right] \right), 1 \right) - \right. \\
& \left. \left( i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \text{ArcSin} \left[ \sqrt{\left( \left( \frac{-i b + c - i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right]} \right) \right] \right) \right] \right), 1 \right) \right) / \\
& \left( \left( \frac{i b + c + i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right) \right), 1 \right) / \\
& \left( 2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) + \left( 2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \right. \right. \\
& \left. \left. \text{ArcSin} \left[ \sqrt{\left( \left( \frac{-i b + c - i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right]} \right) \right] \right) \right) / \\
& \left( \left( \frac{i b + c + i \sqrt{b^2 + c^2}}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \right) \right), 1 \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( \left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \\
& \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + ex) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right. \\
& \left. \left. ex) \right] \right) + \left( i + \tan \left[ \frac{1}{2} (d + ex) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \Bigg) \Bigg) / \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \tan \left[ \frac{1}{2} (d + ex) \right] + \right. \right. \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + ex) \right]^2 \right) \right)} \Bigg) \Bigg) / \\
& \left( (b^2 + c^2) \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) \sqrt{\left( -b + \sqrt{b^2 + c^2} - 2c \tan \left[ \frac{1}{2} (d + ex) \right] + \right. \\
& \left. b \tan \left[ \frac{1}{2} (d + ex) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + ex) \right]^2 \right)} \\
& \sqrt{\left( -2c \tan \left[ \frac{1}{2} (d + ex) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) +} \\
& \left. \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + ex) \right] \right)^2 \right) \Bigg) \Bigg)
\end{aligned}$$

**Problem 438: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( -\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex] \right)^{3/2} dx$$

Optimal (type 3, 130 leaves, 2 steps):

$$\begin{aligned}
& \frac{8 \sqrt{b^2 + c^2} (c \cos[d + ex] - b \sin[d + ex])}{3 e \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}} - \frac{1}{3 e} \\
& 2 (c \cos[d + ex] - b \sin[d + ex]) \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}
\end{aligned}$$

Result (type 4, 11512 leaves):

$$\begin{aligned}
 & -\frac{2 b \sqrt{b^2+c^2} \sqrt{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}}{c e}+\frac{1}{e} \\
 & \left(-\frac{2 b \sqrt{b^2+c^2}}{3 c}-\frac{2}{3} c \cos [d+e x]+\frac{2}{3} b \sin [d+e x]\right) \sqrt{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}+ \\
 & \left(32 b c\left(b^2+c^2\right)^2\left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{\left(b+i c+\sqrt{b^2+c^2}\right)\left(i+\tan \left[\frac{1}{2}(d+e x)\right]}\right)}{\left(b-i c+\sqrt{b^2+c^2}\right)\left(-i+\tan \left[\frac{1}{2}(d+e x)\right]}\right)}\right], 1\right)-\right. \\
 & \left.2 \operatorname{EllipticPi}\left[-1, \operatorname{ArcSin}\left[\sqrt{-\frac{\left(b+i c+\sqrt{b^2+c^2}\right)\left(i+\tan \left[\frac{1}{2}(d+e x)\right]}\right)}{\left(b-i c+\sqrt{b^2+c^2}\right)\left(-i+\tan \left[\frac{1}{2}(d+e x)\right]}\right)}\right], 1\right]\right) \\
 & \sqrt{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}\left(-i+\tan \left[\frac{1}{2}(d+e x)\right]\right) \\
 & \left(-\frac{\left(b+i c+\sqrt{b^2+c^2}\right)\left(i+\tan \left[\frac{1}{2}(d+e x)\right]\right)}{\left(b-i c+\sqrt{b^2+c^2}\right)\left(-i+\tan \left[\frac{1}{2}(d+e x)\right]\right)}\right)^{3 / 2} \\
 & \left(-c+\left(b+\sqrt{b^2+c^2}\right) \tan \left[\frac{1}{2}(d+e x)\right]\right)\left(1+\tan \left[\frac{1}{2}(d+e x)\right]^2\right)\left.\right) / \\
 & \left(3\left(b+i c+\sqrt{b^2+c^2}\right)^3\left(b^2+c^2-b \sqrt{b^2+c^2}\right) e\left(1+\cos [d+e x]\right)\right. \\
 & \left.\sqrt{\frac{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}{\left(1+\cos [d+e x]\right)^2}}\right. \\
 & \left.\left(i+\tan \left[\frac{1}{2}(d+e x)\right]\right)^2 \sqrt{\left(-\left(1+\tan \left[\frac{1}{2}(d+e x)\right]^2\right)\right.}\right. \\
 & \left.\left.\left(-2 c \tan \left[\frac{1}{2}(d+e x)\right]+b\left(-1+\tan \left[\frac{1}{2}(d+e x)\right]^2\right)+\sqrt{b^2+c^2}\left(1+\tan \left[\frac{1}{2}(d+e x)\right]^2\right)\right)\right)\right)-} \\
 & \frac{1}{3 c e\left(1+\cos [d+e x]\right)} \sqrt{\frac{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}{\left(1+\cos [d+e x]\right)^2}} \\
 & 8\left(b^2+c^2\right)^{3 / 2} \sqrt{-\sqrt{b^2+c^2}+b \cos [d+e x]+c \sin [d+e x]}
 \end{aligned}$$

$$\begin{aligned}
& \left( \left( -b + c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \right.} \right. \\
& \quad \sqrt{b^2 + c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \sqrt{\left( -\left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( -2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \right.} \right. \\
& \quad \left. \left. b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) \right) / \\
& \quad \left( (b^2 + c^2) \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \sqrt{\left( -2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) + \right.} \right. \\
& \quad \left. \left. \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) + \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - \right.} \right. \\
& \quad \left. \left. 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2 + c^2} \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \sqrt{\left( -\left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( -2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) + \right.} \right. \\
& \quad \left. \left. \sqrt{b^2 + c^2} \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) \left( \left( 2c^2 \left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -\operatorname{EllipticF}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2}(d+ex) \right)}{\left( \frac{1}{2}(d+ex) \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right]}, 1 \right] + 2 \operatorname{EllipticPi}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\frac{1}{2}(d+ex)}{b + \sqrt{b^2 + c^2}}, \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2}(d+ex) \right)}{\left( \frac{1}{2}(d+ex) \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right]}, 1 \right] \right) \right) \\
& \quad \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2}(d+ex) \right)}{\left( \frac{1}{2}(d+ex) \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right)}} \\
& \quad \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\left( \left( \frac{1}{2}(d+ex) \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \frac{1}{2}(d+ex) \right) \right)} \\
& \quad \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d+ex)\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( (b + \sqrt{b^2 + c^2}) \operatorname{Tan} \left[ \frac{1}{2} (d + e x)^2 \right] \right) + \left( 8 b^3 \left( (-b + i c + \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c - \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], 1 \right] \right. \\
& \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] \right) \\
& \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \left( (b - i c - \sqrt{b^2 + c^2}) (-b - i c + \sqrt{b^2 + c^2}) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} [\frac{1}{2} (d + e x)]) + \right. \\
& \left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x)^2 \right] \right) + \left( 4 b^5 \left( (-b + i c + \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \operatorname{Tan} [\frac{1}{2} (d + e x)])}} \right], 1 \right] - 2 \right. \right. \\
& \left. \left. i c \operatorname{EllipticPi} \left[ \frac{(b + i c - \sqrt{b^2 + c^2}) (i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - i c - \sqrt{b^2 + c^2}) (-i + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], 1 \right] \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \\
& \left(c^2 \left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left.\left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2}(d + e x)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2}(d + e x)\right] + \right.\right. \right. \\
& \left.\left.\left(b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + e x)\right]^2\right)\right)} + \left(4 b c^2 \left(-b + i c + \sqrt{b^2 + c^2}\right) \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1\right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + e x)\right]\right) \Bigg/ \left(\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[ \frac{1}{2} (d + ex) \right] + (b + \sqrt{b^2 + c^2}) \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right) \Bigg) - \\
& \left( 4b(b^2 + c^2) \left( \left( -b + ic + \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[ \text{ArcSin}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] - 2 \right. \right. \\
& \quad \left. \left. ic \text{EllipticPi}\left[ \frac{\left( b + ic - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - ic - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)} \right. \right. \right. \\
& \quad \left. \left. \left. \text{ArcSin}\left[ \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] \right) \right. \right. \\
& \quad \left. \left( -i + \tan\left[ \frac{1}{2} (d + ex) \right] \right) \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}} \right. \right. \\
& \quad \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[ \frac{1}{2} (d + ex) \right] \right) \right) \Bigg/ \left( \left( b - ic - \sqrt{b^2 + c^2} \right) \left( -b - ic + \sqrt{b^2 + c^2} \right) \right) \\
& \quad \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right.} \\
& \quad \left. \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[ \frac{1}{2} (d + ex) \right] + (b + \sqrt{b^2 + c^2}) \tan\left[ \frac{1}{2} (d + ex) \right]^2 \right) \right) \Bigg) - \\
& \left( 4b^3(b^2 + c^2) \left( \left( -b + ic + \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[ \text{ArcSin}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \tan\left[ \frac{1}{2} (d + ex) \right] \right)}} \right], 1 \right] - 2 \right. \right.
\end{aligned}$$

$$\text{I c EllipticPi} \left[ \frac{\left( b + \text{I c} - \sqrt{b^2 + c^2} \right) \left( \text{I} + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - \text{I c} - \sqrt{b^2 + c^2} \right) \left( -\text{I} + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right.$$

$$\left. \text{ArcSin} \left[ \sqrt{\frac{\left( -\text{I b} + c - \text{I} \sqrt{b^2 + c^2} \right) \left( \text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \text{I b} + c + \text{I} \sqrt{b^2 + c^2} \right) \left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right]$$

$$\left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -\text{I b} + c - \text{I} \sqrt{b^2 + c^2} \right) \left( \text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \text{I b} + c + \text{I} \sqrt{b^2 + c^2} \right) \left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}$$

$$\left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/$$

$$\left( c^2 \left( b - \text{I c} - \sqrt{b^2 + c^2} \right) \left( -b - \text{I c} + \sqrt{b^2 + c^2} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right.$$

$$\left( \text{I} - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \text{Tan} \left[ \frac{1}{2} (d + e x) \right] + \right.$$

$$\left. \left( b + \sqrt{b^2 + c^2} \right) \text{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \Bigg) + \left( 8 b^3 \left( -b + \text{I c} - \sqrt{b^2 + c^2} \right) \right.$$

$$\left. \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{\left( -\text{I b} + c - \text{I} \sqrt{b^2 + c^2} \right) \left( \text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \text{I b} + c + \text{I} \sqrt{b^2 + c^2} \right) \left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 \right.$$

$$\text{I c EllipticPi} \left[ \frac{\left( b + \text{I c} + \sqrt{b^2 + c^2} \right) \left( \text{I} + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - \text{I c} + \sqrt{b^2 + c^2} \right) \left( -\text{I} + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right.$$

$$\left. \text{ArcSin} \left[ \sqrt{\frac{\left( -\text{I b} + c - \text{I} \sqrt{b^2 + c^2} \right) \left( \text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \text{I b} + c + \text{I} \sqrt{b^2 + c^2} \right) \left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right]$$

$$\left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -\text{I b} + c - \text{I} \sqrt{b^2 + c^2} \right) \left( \text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \text{I b} + c + \text{I} \sqrt{b^2 + c^2} \right) \left( -\text{I} + \text{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}}$$



$$\begin{aligned}
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( (-b - ic - \sqrt{b^2 + c^2}) (b - ic + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( ic - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) + \left( 4b^5 \left( (-b + ic - \sqrt{b^2 + c^2}) \right. \right. \\
& \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-ib + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[\frac{(b + ic + \sqrt{b^2 + c^2})(i + \frac{c}{b + \sqrt{b^2 + c^2}})}{(b - ic + \sqrt{b^2 + c^2})(-ib + \frac{c}{b + \sqrt{b^2 + c^2}})} \right], 1\right] \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-ib + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right], 1\right] \right) \right. \\
& \left. (-ib + \operatorname{Tan}[\frac{1}{2}(d + ex)]) \sqrt{\frac{(-ib + c - i\sqrt{b^2 + c^2})(i + \operatorname{Tan}[\frac{1}{2}(d + ex)])}{(ib + c + i\sqrt{b^2 + c^2})(-ib + \operatorname{Tan}[\frac{1}{2}(d + ex)])}} \right) \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \\
& \left( c^2 (-b - ic - \sqrt{b^2 + c^2}) (b - ic + \sqrt{b^2 + c^2}) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( ic - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) (-b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right. \\
& \left. \left. \left. (b + \sqrt{b^2 + c^2}) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) + \left( 4bc^2 \left( (-b + ic - \sqrt{b^2 + c^2}) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{1}{2}b+c-\frac{1}{2}\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{1}{2}b+c+\frac{1}{2}\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right]-2 \\
& \frac{1}{2}c \text{EllipticPi}\left[\frac{\left(b+\frac{1}{2}c+\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{\left(b-\frac{1}{2}c+\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-\frac{1}{2}b+c-\frac{1}{2}\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{1}{2}b+c+\frac{1}{2}\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right] \\
& \left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)\sqrt{\frac{\left(-\frac{1}{2}b+c-\frac{1}{2}\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{1}{2}b+c+\frac{1}{2}\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left(-\frac{c}{b+\sqrt{b^2+c^2}}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)\left/\left(\left(-b-\frac{1}{2}c-\sqrt{b^2+c^2}\right)\left(b-\frac{1}{2}c+\sqrt{b^2+c^2}\right)\right.\right. \\
& \left.\left.\left(\frac{-b-\sqrt{b^2+c^2}}{c}-\frac{-b+\sqrt{b^2+c^2}}{c}\right)\left(\frac{1}{2}-\frac{c}{b+\sqrt{b^2+c^2}}\right)\sqrt{\left(\left(1+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)^2\right.\right.\right. \\
& \left.\left.\left.\left(-b+\sqrt{b^2+c^2}-2c\text{Tan}\left[\frac{1}{2}(d+ex)\right]+\left(b+\sqrt{b^2+c^2}\right)\text{Tan}\left[\frac{1}{2}(d+ex)\right]^2\right)\right)\right)\right)+ \\
& \left(8b^2\sqrt{b^2+c^2}\left(\left(-b+\frac{1}{2}c-\sqrt{b^2+c^2}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{1}{2}b+c-\frac{1}{2}\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{1}{2}b+c+\frac{1}{2}\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right]-2\right.\right. \\
& \left.\frac{1}{2}c \text{EllipticPi}\left[\frac{\left(b+\frac{1}{2}c+\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{\left(b-\frac{1}{2}c+\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(-\frac{1}{2}b+c-\frac{1}{2}\sqrt{b^2+c^2}\right)\left(\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{1}{2}b+c+\frac{1}{2}\sqrt{b^2+c^2}\right)\left(-\frac{1}{2}+\text{Tan}\left[\frac{1}{2}(d+ex)\right]\right)}}\right], 1\right]
\end{aligned}$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \\
& \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( \left(-b-i c-\sqrt{b^2+c^2}\right)\left(b-i c+\sqrt{b^2+c^2}\right) \right. \\
& \left. \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \\
& \left. \left. \left( -b+\sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(b+\sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) + \\
& \left( 8b^4\sqrt{b^2+c^2} \left( \left(-b+ic-\sqrt{b^2+c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] - 2 \right. \right. \\
& \left. \left. ic \operatorname{EllipticPi}\left[ \frac{\left(b+ic+\sqrt{b^2+c^2}\right)\left(i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}{\left(b-ic+\sqrt{b^2+c^2}\right)\left(-i+\frac{c}{b+\sqrt{b^2+c^2}}\right)}, \right. \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right], 1\right] \right) \right. \\
& \left. \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{\left(-ib+c-i\sqrt{b^2+c^2}\right)\left(i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(ib+c+i\sqrt{b^2+c^2}\right)\left(-i+\tan\left[\frac{1}{2}(d+ex)\right]\right)}} \right. \\
& \left. \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( c^2 \left(-b-i c-\sqrt{b^2+c^2}\right)\left(b-i c+\sqrt{b^2+c^2}\right) \right. \right. \\
& \left. \left. \left( \frac{-b-\sqrt{b^2+c^2}}{c} - \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \sqrt{\left( \left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right.} \right. \right. \\
& \left. \left. \left. \left( -b+\sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + \left(b+\sqrt{b^2+c^2}\right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}}]{}, 1] - 2 \right. \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right], 1] \right] \right. \\
& \quad \left. (-i + \tan[\frac{1}{2} (d + e x)]) \sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}} \right. \\
& \quad \left. \left. \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan[\frac{1}{2} (d + e x)] \right) \right) \right] \left/ \left( (-b - i c - \sqrt{b^2 + c^2}) (b - i c + \sqrt{b^2 + c^2}) \right. \right. \\
& \quad \left. \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan[\frac{1}{2} (d + e x)] \right)^2 \right. \right. \\
& \quad \left. \left. \left( -b + \sqrt{b^2 + c^2} - 2 c \tan[\frac{1}{2} (d + e x)] + (b + \sqrt{b^2 + c^2}) \tan[\frac{1}{2} (d + e x)]^2 \right) \right) \right) \right] + \\
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{(-i b + c - i \sqrt{b^2 + c^2}) (i + \tan[\frac{1}{2} (d + e x)])}{(i b + c + i \sqrt{b^2 + c^2}) (-i + \tan[\frac{1}{2} (d + e x)])}}]{}, 1] - 2 \right. \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{(b + i c + \sqrt{b^2 + c^2}) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{(b - i c + \sqrt{b^2 + c^2}) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -\mathfrak{i} b + c - \mathfrak{i} \sqrt{b^2 + c^2} \right) \left( \mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \mathfrak{i} b + c + \mathfrak{i} \sqrt{b^2 + c^2} \right) \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \\
& \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -\mathfrak{i} b + c - \mathfrak{i} \sqrt{b^2 + c^2} \right) \left( \mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \mathfrak{i} b + c + \mathfrak{i} \sqrt{b^2 + c^2} \right) \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \left( c^2 \left( -b - \mathfrak{i} c - \sqrt{b^2 + c^2} \right) \left( b - \mathfrak{i} c + \sqrt{b^2 + c^2} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left. \left( \mathfrak{i} - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) + \left( 2 b^3 \left( -\mathfrak{i} - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left( -\mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right. \\
& \left. \sqrt{\frac{\left( -\mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( \mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -\mathfrak{i} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right] \Bigg/ \\
& \left( c \left( -\mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( \mathfrak{i} + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right)} \right) + \left( 2 b c \left( -\mathfrak{i} - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}], 1\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right) \\
& \sqrt{\frac{\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}\Bigg/ \\
& \left(\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)}\right. \\
& \quad \left.\left(-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+\left(b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)+ \\
& \left(2b^2\sqrt{b^2+c^2}\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{c}{b+\sqrt{b^2+c^2}}\right], 1\right]\right) \\
& \sqrt{\frac{\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}], 1\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right) \\
& \sqrt{\frac{\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}{\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}}\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)}\Bigg/ \\
& \left(c\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}\right)\sqrt{\left(\left(1+\tan\left[\frac{1}{2}(d+ex)\right]\right)^2\right)}\right. \\
& \quad \left.\left(-b+\sqrt{b^2+c^2}-2c\tan\left[\frac{1}{2}(d+ex)\right]+\left(b+\sqrt{b^2+c^2}\right)\tan\left[\frac{1}{2}(d+ex)\right]^2\right)\right)- \\
& \left(bc\left(2\frac{c}{b+\sqrt{b^2+c^2}}\left(-\frac{c}{b+\sqrt{b^2+c^2}}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{c}{b+\sqrt{b^2+c^2}}\right], 1\right]\right)\right. \\
& \quad \left.\sqrt{\left(\left(-b+\sqrt{b^2+c^2}\right)\left(\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)}\right) \\
& \quad \left.\left(\left(b+\sqrt{b^2+c^2}\right)\left(-\frac{c}{b+\sqrt{b^2+c^2}}+\tan\left[\frac{1}{2}(d+ex)\right]\right)\right)\right), 1\right)-
\end{aligned}$$

$$\begin{aligned}
& \left( i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] \right) / \\
& \left( 2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) + \left( 2 c \text{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] \right) / \\
& \left( \left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \\
& \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right. \\
& \quad \left. \left. e x) \right] \right) + \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) / \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \\
& \quad \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \right) - \\
& \left( c \sqrt{b^2 + c^2} \left( 2 i \left( -\frac{1}{2} i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticE} \left[ \text{ArcSin} \left[ \right. \right. \right. \right. \right. \right. \\
& \quad \sqrt{\left( \left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \\
& \quad \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right] \right], 1 \right] - \right. \\
& \quad \left. \left( i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF} \left[ \right. \right. \right. \right. \\
& \quad \text{ArcSin} \left[ \sqrt{\left( \left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) /} \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \Bigg] , 1 \Bigg) / \\
& \left( 2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) + \left( 2 c \operatorname{EllipticPi} \left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \right. \right. \\
& \operatorname{ArcSin} \left[ \sqrt{\left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) / \right. \\
& \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \right) \right) \right] , 1 \right] \Bigg) / \\
& \left( \left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \Bigg) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \\
& \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + \right. \right.} \right. \\
& \left. \left. e x) \right] \right) + \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \Bigg) \Bigg) / \\
& \left( \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \right. \\
& \left. \left. \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \right) \Bigg) \Bigg) / \\
& \left( (b^2 + c^2) \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \sqrt{\left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right.} \\
& \left. b \tan \left[ \frac{1}{2} (d + e x) \right]^2 + \sqrt{b^2 + c^2} \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right) \\
& \sqrt{\left( -2 c \tan \left[ \frac{1}{2} (d + e x) \right] + b \left( -1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) +} \\
& \left. \sqrt{b^2 + c^2} \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 439: Result unnecessarily involves higher level functions and more



than twice size of optimal antiderivative.

$$\int \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \, dx$$

Optimal (type 3, 57 leaves, 1 step):

$$-\frac{2(c \cos[d + ex] - b \sin[d + ex])}{e \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}$$

Result (type 4, 11415 leaves):

$$\begin{aligned} & \frac{2 b \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}}{c e} - \\ & \left( 8 b c (b^2 + c^2)^{3/2} \left( \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] - \right. \\ & \quad \left. 2 \text{EllipticPi}\left[-1, \text{ArcSin}\left[\sqrt{-\frac{(b + i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])}}\right]}, 1\right] \right) \\ & \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]} \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \\ & \left( -\frac{(b + i c + \sqrt{b^2 + c^2})(i + \tan[\frac{1}{2}(d + ex)])}{(b - i c + \sqrt{b^2 + c^2})(-i + \tan[\frac{1}{2}(d + ex)])} \right)^{3/2} \\ & \left( -c + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right] \right) \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg/ \\ & \left( (b + i c + \sqrt{b^2 + c^2})^3 (b^2 + c^2 - b \sqrt{b^2 + c^2}) e (1 + \cos[d + ex]) \right. \\ & \quad \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos[d + ex] + c \sin[d + ex]}{(1 + \cos[d + ex])^2}} \\ & \quad \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)^2 \sqrt{\left( -\left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\ & \quad \left. \left. \left( -2 c \tan\left[\frac{1}{2}(d + ex)\right] + b \left( -1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) + \sqrt{b^2 + c^2} \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \right) \right) \Bigg) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{c e \left(1 + \cos[d + e x]\right) \sqrt{\frac{-\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]}{(1 + \cos[d + e x])^2}}} \\
& 2 (b^2 + c^2) \sqrt{-\sqrt{b^2 + c^2} + b \cos[d + e x] + c \sin[d + e x]} \\
& \left( \left( \left( -b + c \tan\left[\frac{1}{2} (d + e x)\right] \right) \sqrt{\left( -b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2} (d + e x)\right] + b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \right.} \right. \right. \\
& \quad \left. \sqrt{b^2 + c^2} \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \sqrt{\left( -\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \left(-2 c \tan\left[\frac{1}{2} (d + e x)\right] + \right. \right.} \\
& \quad \left. \left. b \left(-1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) + \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right)} \right) \right) / \\
& \left( (b^2 + c^2) \left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \sqrt{\left( -2 c \tan\left[\frac{1}{2} (d + e x)\right] + b \left(-1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) + \right.} \right. \\
& \quad \left. \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right) \right) + \sqrt{\left( \left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \left(-b + \sqrt{b^2 + c^2} - \right. \right.} \\
& \quad \left. \left. 2 c \tan\left[\frac{1}{2} (d + e x)\right] + b \tan\left[\frac{1}{2} (d + e x)\right]^2 + \sqrt{b^2 + c^2} \tan\left[\frac{1}{2} (d + e x)\right]^2 \right) \right) \sqrt{\left( -\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) \left(-2 c \tan\left[\frac{1}{2} (d + e x)\right] + \right. \right.} \\
& \quad \left. \left. b \left(-1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right) + \sqrt{b^2 + c^2} \left(1 + \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right) \right) \right) \left( \left( 2 c^2 \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i \operatorname{EllipticF}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)} \right], 1\right] + 2 i \operatorname{EllipticPi}\left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \operatorname{ArcSin}\left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)} \right], 1\right] \right) \right) \\
& \left( -i + \tan\left[\frac{1}{2} (d + e x)\right] \right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right)$$

$$\sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right.$$

$$\left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg) + \left( 8b^3 \left( \left( -b + ic + \sqrt{b^2 + c^2} \right) \right. \right.$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] - 2$$

$$ic \operatorname{EllipticPi}\left[\frac{\left( b + ic - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - ic - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right.$$

$$\left. \operatorname{ArcSin}\left[\sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] \right)$$

$$\left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -ib + c - i\sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( ib + c + i\sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}}$$

$$\left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/$$

$$\left( \left( b - ic - \sqrt{b^2 + c^2} \right) \left( -b - ic + \sqrt{b^2 + c^2} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right.$$

$$\left. \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \right. \right. \right.$$

$$\left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \Bigg) + \left( 4b^5 \left( \left( -b + ic + \sqrt{b^2 + c^2} \right) \right. \right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2$$

$$i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)},\right.$$

$$\left.\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right]$$

$$\left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}$$

$$\left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/$$

$$\left(c^2 \left(b - i c - \sqrt{b^2 + c^2}\right) \left(-b - i c + \sqrt{b^2 + c^2}\right) \left(-\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c}\right)\right.$$

$$\left.\left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \tan\left[\frac{1}{2} (d + e x)\right]\right)^2 \left(-b + \sqrt{b^2 + c^2} - 2 c \tan\left[\frac{1}{2} (d + e x)\right] +\right.\right.}\right.$$

$$\left.\left.\left(b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2} (d + e x)\right]^2\right)\right) + \left(4 b c^2 \left(-b + i c + \sqrt{b^2 + c^2}\right)\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right] - 2\right.$$

$$i c \text{EllipticPi}\left[\frac{\left(b + i c - \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c - \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)},\right.$$

$$\left.\text{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2} (d + e x)\right]\right)}}\right], 1\right]$$

$$\begin{aligned}
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \\
& \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \right. \\
& \left. \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right. \right.} \\
& \left. \left. \left( -b+\sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) - \\
& \left( 4b(b^2+c^2) \left( (-b+ic+\sqrt{b^2+c^2}) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[ \frac{(b+ic-\sqrt{b^2+c^2})(i+\frac{c}{b+\sqrt{b^2+c^2}})}{(b-ic-\sqrt{b^2+c^2})(-i+\frac{c}{b+\sqrt{b^2+c^2}})}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \right], 1 \right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d+ex)\right] \right) \sqrt{\frac{(-ib+c-i\sqrt{b^2+c^2})(i+\tan[\frac{1}{2}(d+ex)])}{(ib+c+i\sqrt{b^2+c^2})(-i+\tan[\frac{1}{2}(d+ex)])}} \\
& \left( -\frac{c}{b+\sqrt{b^2+c^2}} + \tan\left[\frac{1}{2}(d+ex)\right] \right) \Bigg/ \left( (b-ic-\sqrt{b^2+c^2})(-b-ic+\sqrt{b^2+c^2}) \right. \\
& \left. \left( -\frac{-b-\sqrt{b^2+c^2}}{c} + \frac{-b+\sqrt{b^2+c^2}}{c} \right) \left( i - \frac{c}{b+\sqrt{b^2+c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right] \right)^2 \right. \right.} \\
& \left. \left. \left( -b+\sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + (b+\sqrt{b^2+c^2}) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( \left( -b + i c + \sqrt{b^2 + c^2} \right) \text{EllipticF} \left[ \text{ArcSin} \left[ \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \\
& \quad \left. i c \text{EllipticPi} \left[ \frac{\left( b + i c - \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - i c - \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin} \left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] \right) \\
& \quad \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \quad \left( c^2 \left( b - i c - \sqrt{b^2 + c^2} \right) \left( -b - i c + \sqrt{b^2 + c^2} \right) \left( -\frac{-b - \sqrt{b^2 + c^2}}{c} + \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \quad \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \tan \left[ \frac{1}{2} (d + e x) \right] + \right. \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} \right) \tan \left[ \frac{1}{2} (d + e x) \right]^2 \right)} \Bigg) + \left( 8 b^3 \left( \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan \left[ \frac{1}{2} (d + e x) \right] \right)}} \right], 1 \right] - 2 \right. \right. \\
& \quad \left. \left. i c \text{EllipticPi} \left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}} \right], 1 \right] \\
& \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \\
& \left( \left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right) \left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \right. \\
& \left. \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]^2\right) \left(-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right] + \right.\right. \right. \\
& \left. \left.\left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]^2\right)\right)} \right] + \left(4 b^5 \left(-b + i c - \sqrt{b^2 + c^2}\right) \right. \\
& \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi} \left[ \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin} \left[ \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}} \right], 1 \right] \right. \\
& \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[\frac{1}{2} (d + e x)\right]\right) \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] \right) + \right.} \\
& \left. \left( b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \left. \right) + \left( 4 b c^2 \left( -b + i c - \sqrt{b^2 + c^2} \right) \right. \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] - 2 \\
& i c \text{EllipticPi}\left[\frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] \right) \\
& \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right] \right) \left. \right) / \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \tan\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \left. \right) + \\
& \left( 8 b^2 \sqrt{b^2 + c^2} \left( -b + i c - \sqrt{b^2 + c^2} \right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1\right] - 2 \right.
\end{aligned}$$



$$\begin{aligned}
& \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \\
& \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}} \\
& \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \Bigg/ \left(\left(-b - i c - \sqrt{b^2 + c^2}\right) \left(b - i c + \sqrt{b^2 + c^2}\right)\right. \\
& \left.\left(\frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c}\right) \left(i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(\left(1 + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)^2\right.}\right. \\
& \left.\left.\left(-b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right] + \left(b + \sqrt{b^2 + c^2}\right) \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]^2\right)\right)\right) + \\
& \left(8 b^4 \sqrt{b^2 + c^2} \left(-b + i c - \sqrt{b^2 + c^2}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] - 2\right. \right. \\
& \left. \frac{\left(b + i c + \sqrt{b^2 + c^2}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}{\left(b - i c + \sqrt{b^2 + c^2}\right) \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right)}, \right. \\
& \left. \operatorname{ArcSin}\left[\sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}, 1\right] \right) \\
& \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right) \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \operatorname{Tan}\left[\frac{1}{2} (d + e x)\right]\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) + \\
& \left( 4 b (b^2 + c^2) \left( -b + i c - \sqrt{b^2 + c^2} \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[ \right. \right. \right. \\
& \left. \left. \left. \frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)} \right], 1 \right] - 2 \right. \\
& \left. i c \operatorname{EllipticPi}\left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[ \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \right], 1 \right] \right) \\
& \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)}} \\
& \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right) \Bigg/ \left( \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \right. \\
& \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] \right)^2 \right.} \\
& \left. \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right] + \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan}\left[\frac{1}{2}(d + ex)\right]^2 \right) \right) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& \left( 4 b^3 (b^2 + c^2) \left( (-b + i c - \sqrt{b^2 + c^2}) \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] - 2 \right. \right. \\
& \quad i c \operatorname{EllipticPi} \left[ \frac{\left( b + i c + \sqrt{b^2 + c^2} \right) \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}{\left( b - i c + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right)}, \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \right) \\
& \quad \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \sqrt{\frac{\left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}} \\
& \quad \left( -\frac{c}{b + \sqrt{b^2 + c^2}} + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \Bigg/ \\
& \quad \left( c^2 \left( -b - i c - \sqrt{b^2 + c^2} \right) \left( b - i c + \sqrt{b^2 + c^2} \right) \left( \frac{-b - \sqrt{b^2 + c^2}}{c} - \frac{-b + \sqrt{b^2 + c^2}}{c} \right) \right. \\
& \quad \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \sqrt{\left( \left( 1 + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)^2 \right) \left( -b + \sqrt{b^2 + c^2} - 2 c \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) +} \\
& \quad \left. \left( b + \sqrt{b^2 + c^2} \right) \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right]^2 \right) \Bigg) + \left( 2 b^3 \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \operatorname{EllipticF} \left[ \right. \right. \\
& \quad \left. \left. \operatorname{ArcSin} \left[ \frac{\left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)}{\left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right)} \right], 1 \right] \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (d + e x) \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \Bigg/ \\
& \left( c \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(-b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) + \\
& \left( 2bc \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1\right] \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right. \\
& \quad \left. \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right] \Bigg/ \\
& \left( \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(-b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + (b + \sqrt{b^2 + c^2}) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) + \\
& \left( 2b^2 \sqrt{b^2 + c^2} \left(-i - \frac{c}{b + \sqrt{b^2 + c^2}}\right) \text{EllipticF}\left[\text{ArcSin}\left[ \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \right], 1\right] \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right. \\
& \quad \left. \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}} \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right) \right] \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \Bigg/ \\
& \left( c \left(-i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \left(i + \frac{c}{b + \sqrt{b^2 + c^2}}\right) \sqrt{\left(1 + \tan\left[\frac{1}{2}(d + ex)\right]\right)^2} \right. \\
& \quad \left. \left(-b + \sqrt{b^2 + c^2} - 2c \tan\left[\frac{1}{2}(d + ex)\right] + \left(b + \sqrt{b^2 + c^2}\right) \tan\left[\frac{1}{2}(d + ex)\right]^2\right) \right) - \\
& \left( b c \left( 2i \left( -\frac{1}{2}i \left( i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \text{EllipticE}\left[\text{ArcSin}\left[ \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right)} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right), 1 \right] - \right. \right. \\
& \quad \left. \left. \left( i \left( i \left( -i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) + i \left( i - \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \text{EllipticF}\left[ \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \text{ArcSin}\left[ \sqrt{\left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right)} \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right), 1 \right] \right) \right) \Bigg/ \\
& \left( 2 \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) + \left( 2c \text{EllipticPi}\left[ \frac{i + \frac{c}{b + \sqrt{b^2 + c^2}}}{-i + \frac{c}{b + \sqrt{b^2 + c^2}}}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[ \sqrt{\left( \left( -i b + c - i \sqrt{b^2 + c^2} \right) \left( i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right)} \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left( \left( i b + c + i \sqrt{b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \right) \right), 1 \right] \right) \Bigg/ \\
& \left( \left( b + \sqrt{b^2 + c^2} \right) \left( -i + \frac{c}{b + \sqrt{b^2 + c^2}} \right) \right) \Bigg) \left( -i + \tan\left[\frac{1}{2}(d + ex)\right] \right) \\
& \sqrt{\frac{\left(-i b + c - i \sqrt{b^2 + c^2}\right) \left(i + \tan\left[\frac{1}{2}(d + ex)\right]\right)}{\left(i b + c + i \sqrt{b^2 + c^2}\right) \left(-i + \tan\left[\frac{1}{2}(d + ex)\right]\right)} \left(-\frac{c}{b + \sqrt{b^2 + c^2}} + \tan\left[\frac{1}{2}(d + \right. \right.}
\end{aligned}$$



$$\left( \sqrt{\left( \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \left( -b + \sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + \right. \right. \right. \right. \\ \left. \left. \left( b + \sqrt{b^2+c^2} \right) \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right)} \right) / \\ \left( (b^2+c^2) \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \sqrt{\left( -b + \sqrt{b^2+c^2} - 2c \tan\left[\frac{1}{2}(d+ex)\right] + \right. \right. \right. \\ \left. \left. b \tan\left[\frac{1}{2}(d+ex)\right]^2 + \sqrt{b^2+c^2} \tan\left[\frac{1}{2}(d+ex)\right]^2 \right)} \right) \\ \sqrt{\left( -2c \tan\left[\frac{1}{2}(d+ex)\right] + b \left( -1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) + \right.} \\ \left. \left. \sqrt{b^2+c^2} \left( 1 + \tan\left[\frac{1}{2}(d+ex)\right]^2 \right) \right) \right)} \right)$$

**Problem 440:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]}} dx$$

Optimal (type 3, 91 leaves, 3 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{(b^2+c^2)^{1/4} \sin[d+ex - \operatorname{ArcTan}[b,c]]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2} + \sqrt{b^2+c^2} \cos[d+ex - \operatorname{ArcTan}[b,c]]}}\right]}{(b^2+c^2)^{1/4} e}$$

Result (type 4, 61 904 leaves): Display of huge result suppressed!

**Problem 441:** Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left( -\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex] \right)^{3/2}} dx$$

Optimal (type 3, 164 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{(b^2+c^2)^{1/4} \sin[d+ex-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}} \cos[d+ex-\text{ArcTan}[b,c]]}\right]}{2\sqrt{2} (b^2+c^2)^{3/4} e} + \frac{c \cos[d+ex] - b \sin[d+ex]}{2\sqrt{b^2+c^2} e \left(-\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 442: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\left(-\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]\right)^{5/2}} dx$$

Optimal (type 3, 232 leaves, 5 steps):

$$3 \text{ArcTan}\left[\frac{(b^2+c^2)^{1/4} \sin[d+ex-\text{ArcTan}[b,c]]}{\sqrt{2} \sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}} \cos[d+ex-\text{ArcTan}[b,c]]}\right] - \frac{16\sqrt{2} (b^2+c^2)^{5/4} e}{c \cos[d+ex] - b \sin[d+ex]} - \frac{4\sqrt{b^2+c^2} e \left(-\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]\right)^{5/2}}{3 (c \cos[d+ex] - b \sin[d+ex])} - \frac{16 (b^2+c^2) e \left(-\sqrt{b^2+c^2} + b \cos[d+ex] + c \sin[d+ex]\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

**Problem 448: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \sec[d+ex] + c \tan[d+ex])^{3/2}}{\sec[d+ex]^{3/2}} dx$$

Optimal (type 4, 371 leaves, 7 steps):



$$\begin{aligned}
& - \left( \left( 2 \left( c \cos[d + ex] - a \sin[d + ex] \right) \left( a + b \sec[d + ex] + c \tan[d + ex] \right)^{3/2} \right) / \right. \\
& \quad \left. \left( 3 e \sec[d + ex]^{3/2} \left( b + a \cos[d + ex] + c \sin[d + ex] \right) \right) \right) + \\
& \left( 8 b \operatorname{EllipticE} \left[ \frac{1}{2} (d + ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \left( a + b \sec[d + ex] + c \tan[d + ex] \right)^{3/2} \right) / \\
& \left( 3 e \sec[d + ex]^{3/2} \left( b + a \cos[d + ex] + c \sin[d + ex] \right) \sqrt{\frac{b + a \cos[d + ex] + c \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} \right) + \\
& \left( 2 (a^2 - b^2 + c^2) \operatorname{EllipticF} \left[ \frac{1}{2} (d + ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right. \\
& \quad \left. \sqrt{\frac{b + a \cos[d + ex] + c \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} \left( a + b \sec[d + ex] + c \tan[d + ex] \right)^{3/2} \right) / \\
& \quad \left( 3 e \sec[d + ex]^{3/2} \left( b + a \cos[d + ex] + c \sin[d + ex] \right)^2 \right)
\end{aligned}$$

Result (type 6, 2490 leaves):

$$\begin{aligned}
& \left( \left( \frac{8 a b}{3 c} - \frac{2}{3} c \cos[d + ex] + \frac{2}{3} a \sin[d + ex] \right) \left( a + b \sec[d + ex] + c \tan[d + ex] \right)^{3/2} \right) / \\
& \quad \left( e \sec[d + ex]^{3/2} \left( b + a \cos[d + ex] + c \sin[d + ex] \right) \right) + \\
& \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \right. \right. \\
& \quad \left. \left. -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin[d + ex + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \sec[d + ex + \operatorname{ArcTan}[\frac{a}{c}]] \right. \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{a}{c}]]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \sin[d + ex + \operatorname{ArcTan}[\frac{a}{c}]]} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}} (a + b \sec[d + e x] + c \tan[d + e x])^{3/2}} \right) / \\
 & \left( 3 \sqrt{1 + \frac{a^2}{c^2}} c e \sec[d + e x]^{3/2} (b + a \cos[d + e x] + c \sin[d + e x])^{3/2} \right) + \\
 & \left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, \right. \right. \\
 & \quad \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \sec\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right) \\
 & \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \\
 & \left( \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}} (a + b \sec[d + e x] + c \tan[d + e x])^{3/2}} \right) / \\
 & \left( \sqrt{1 + \frac{a^2}{c^2}} c e \sec[d + e x]^{3/2} (b + a \cos[d + e x] + c \sin[d + e x])^{3/2} \right) +
 \end{aligned}$$

$$\begin{aligned}
& \left( 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, \right. \right. \\
& \quad \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right. \\
& \quad \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}} (a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x])^{3/2}} \right) / \\
& \quad \left( 3 \sqrt{1 + \frac{a^2}{c^2}} e \operatorname{Sec}[d + e x]^{3/2} (b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x])^{3/2} \right) + \\
& \quad \left( 4 a^2 b \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \right) \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right/ \\
 & \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
 & \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
 & \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \\
 & \left. \frac{2 a \left( \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right}}} \left( a + b \sec \left[ d + e x \right] + c \tan \left[ d + e x \right] \right)^{3/2} \right/ \\
 & \left( 3 c e \sec \left[ d + e x \right]^{3/2} \left( b + a \cos \left[ d + e x \right] + c \sin \left[ d + e x \right] \right)^{3/2} \right) + \\
 & \left( 4 b c \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& \left. \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right/ \\
& \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
& \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \\
& \left. \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right) (a + b \sec [d + e x] + c \tan [d + e x])^{3/2} / \\
& (3 e \sec [d + e x]^{3/2} (b + a \cos [d + e x] + c \sin [d + e x])^{3/2})
\end{aligned}$$

**Problem 449:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b \sec [d + e x] + c \tan [d + e x]}}{\sqrt{\sec [d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]} \right) /$$

$$\left( e \sqrt{\operatorname{Sec}[d + e x]} \sqrt{\frac{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right)$$

Result (type 6, 1580 leaves):

$$\frac{2 a \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]}}{c e \sqrt{\operatorname{Sec}[d + e x]}} +$$

$$\left( 2 b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \right. \right.$$

$$\left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \operatorname{Sec}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]$$

$$\sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}$$

$$\sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Sin}[d + e x + \operatorname{ArcTan}[\frac{a}{c}]]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}} \sqrt{a + b \operatorname{Sec}[d + e x] + c \operatorname{Tan}[d + e x]} \right) /$$

$$\left( \sqrt{1 + \frac{a^2}{c^2}} c e \sqrt{\operatorname{Sec}[d + e x]} \sqrt{b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x]} \right) +$$

$$\left( a^2 \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right. \right.$$

$$\left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) /$$

$$\left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right.$$

$$\left. \sqrt{\frac{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} - \frac{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}} \right) -$$

$$\left( \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right) \sqrt{a + b \sec \left[ d + e x \right] + c \tan \left[ d + e x \right]} /$$

$$\left( c e \sqrt{\sec \left[ d + e x \right]} \sqrt{b + a \cos \left[ d + e x \right] + c \sin \left[ d + e x \right]} \right) +$$

$$\begin{aligned}
 & \left( c - \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right. \right. \right. \\
 & \quad \left. \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) \right) / \\
 & \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
 & \quad \left. \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \right. \\
 & \quad \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \\
 & \left( \frac{2 a \left( \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}} \right) \sqrt{a + b \sec \left[ d + e x \right] + c \tan \left[ d + e x \right]} / \\
 & \left( e \sqrt{\sec \left[ d + e x \right]} \sqrt{b + a \cos \left[ d + e x \right] + c \sin \left[ d + e x \right]} \right)
 \end{aligned}$$

Problem 450: Result unnecessarily involves higher level functions and more



than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\sec[d+ex]}}{\sqrt{a+b\sec[d+ex]+c\tan[d+ex]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[a, c]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\sec[d+ex]} \right. \\ \left. \sqrt{\frac{b+a\cos[d+ex]+c\sin[d+ex]}{b+\sqrt{a^2+c^2}}} \right) / \left( e \sqrt{a+b\sec[d+ex]+c\tan[d+ex]} \right)$$

Result (type 6, 339 leaves):

$$\left( 2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]}{b-\sqrt{1+\frac{a^2}{c^2}}c}\right], \right. \\ \left. \frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]}{b+\sqrt{1+\frac{a^2}{c^2}}c} \right] \sqrt{\sec[d+ex]} \sec[d+ex+\operatorname{ArcTan}[\frac{a}{c}]] \\ \sqrt{b+a\cos[d+ex]+c\sin[d+ex]} \sqrt{-\frac{\sqrt{1+\frac{a^2}{c^2}}c(-1+\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]])}{b+\sqrt{1+\frac{a^2}{c^2}}c}} \\ \sqrt{\frac{\sqrt{1+\frac{a^2}{c^2}}c(1+\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]])}{-b+\sqrt{1+\frac{a^2}{c^2}}c}} \sqrt{\frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin[d+ex+\operatorname{ArcTan}[\frac{a}{c}]]}{b+\sqrt{1+\frac{a^2}{c^2}}c}}} \right) / \\ \left( \sqrt{1+\frac{a^2}{c^2}}c e \sqrt{a+b\sec[d+ex]+c\tan[d+ex]} \right)$$

**Problem 451: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\sec [d+e x]^{3 / 2}}{\left(a+b \sec [d+e x]+c \tan [d+e x]\right)^{3 / 2}} d x$$

Optimal (type 4, 240 leaves, 4 steps):

$$\begin{aligned} & -\left(\left(2 \sec [d+e x]^{3 / 2}\left(c \cos [d+e x]-a \sin [d+e x]\right)\left(b+a \cos [d+e x]+c \sin [d+e x]\right)\right) / \right. \\ & \quad \left.\left(\left(a^2-b^2+c^2\right) e\left(a+b \sec [d+e x]+c \tan [d+e x]\right)^{3 / 2}\right)\right)- \\ & \quad \left(2 \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sec [d+e x]^{3 / 2}\right. \\ & \quad \left.\left(b+a \cos [d+e x]+c \sin [d+e x]\right)^2\right) / \\ & \quad \left(\left(a^2-b^2+c^2\right) e \sqrt{\frac{b+a \cos [d+e x]+c \sin [d+e x]}{b+\sqrt{a^2+c^2}}}\left(a+b \sec [d+e x]+c \tan [d+e x]\right)^{3 / 2}\right) \end{aligned}$$

Result (type 6, 1732 leaves):

$$\begin{aligned} & \left(\sec [d+e x]^{3 / 2}\left(b+a \cos [d+e x]+c \sin [d+e x]\right)^2\right. \\ & \quad \left.\left(-\frac{2\left(a^2+c^2\right)}{a c\left(a^2-b^2+c^2\right)}+\frac{2\left(b c+a^2 \sin [d+e x]+c^2 \sin [d+e x]\right)}{a\left(a^2-b^2+c^2\right)\left(b+a \cos [d+e x]+c \sin [d+e x]\right)}\right) / \\ & \quad \left(e\left(a+b \sec [d+e x]+c \tan [d+e x]\right)^{3 / 2}\right)- \\ & \quad \left(2 b \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2},-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c},\right.\right. \\ & \quad \left.\left.-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(-1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c}\right] \sec [d+e x]^{3 / 2}\right. \\ & \quad \left.\sec \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]\left(b+a \cos [d+e x]+c \sin [d+e x]\right)^{3 / 2}\right) \end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \\
 & \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}} \right) / \\
 & \left( \sqrt{1 + \frac{a^2}{c^2}} c (a^2 - b^2 + c^2) e (a + b \sec[d + e x] + c \tan[d + e x])^{3/2} \right) - \\
 & \left( a^2 \sec[d + e x]^{3/2} (b + a \cos[d + e x] + c \sin[d + e x])^{3/2} \right. \\
 & \left. - \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right], \right. \right. \\
 & \left. \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \right] \sin\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \right. \\
& \sqrt{b + a \sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \left. \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \right) - \\
& \frac{2 a \left( \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)}{\sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}} \Bigg/ \\
& \left( c \left( a^2 - b^2 + c^2 \right) e \left( a + b \sec \left[ d + e x \right] + c \tan \left[ d + e x \right] \right)^{3/2} \right) - \\
& \left( c \sec \left[ d + e x \right]^{3/2} \left( b + a \cos \left[ d + e x \right] + c \sin \left[ d + e x \right] \right)^{3/2} \right. \\
& \left. \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right/ \\
& \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
& \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) - \\
& \left. \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}}}{a^2 + c^2} \right/ \\
& \sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \left( (a^2 - b^2 + c^2) e (a + b \operatorname{Sec} [d + e x] + c \operatorname{Tan} [d + e x])^{3/2} \right)
\end{aligned}$$

**Problem 452:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [d + e x]^{5/2}}{(a + b \operatorname{Sec} [d + e x] + c \operatorname{Tan} [d + e x])^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned}
& - \left( \left( 2 \operatorname{Sec}[d+ex]^{5/2} (c \cos[d+ex] - a \sin[d+ex]) (b + a \cos[d+ex] + c \sin[d+ex]) \right) / \right. \\
& \quad \left. \left( 3 (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d+ex] + c \tan[d+ex])^{5/2} \right) \right) + \\
& \left( 8 \operatorname{Sec}[d+ex]^{5/2} (b c \cos[d+ex] - a b \sin[d+ex]) (b + a \cos[d+ex] + c \sin[d+ex])^2 \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2)^2 e (a + b \operatorname{Sec}[d+ex] + c \tan[d+ex])^{5/2} \right) + \\
& \left( 8 b \operatorname{EllipticE} \left[ \frac{1}{2} (d+ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right. \\
& \quad \left. \operatorname{Sec}[d+ex]^{5/2} (b + a \cos[d+ex] + c \sin[d+ex])^3 \right) / \\
& \left( 3 (a^2 - b^2 + c^2)^2 e \sqrt{\frac{b + a \cos[d+ex] + c \sin[d+ex]}{b + \sqrt{a^2 + c^2}}} (a + b \operatorname{Sec}[d+ex] + c \tan[d+ex])^{5/2} \right) + \\
& \left( 2 \operatorname{EllipticF} \left[ \frac{1}{2} (d+ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \operatorname{Sec}[d+ex]^{5/2} \right. \\
& \quad \left. (b + a \cos[d+ex] + c \sin[d+ex])^2 \sqrt{\frac{b + a \cos[d+ex] + c \sin[d+ex]}{b + \sqrt{a^2 + c^2}}} \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2) e (a + b \operatorname{Sec}[d+ex] + c \tan[d+ex])^{5/2} \right)
\end{aligned}$$

Result (type 6, 2708 leaves):

$$\begin{aligned}
& \left( \operatorname{Sec}[d+ex]^{5/2} (b + a \cos[d+ex] + c \sin[d+ex])^3 \right. \\
& \quad \left( \frac{8 b (a^2 + c^2)}{3 a c (-a^2 + b^2 - c^2)^2} + \frac{2 (b c + a^2 \sin[d+ex] + c^2 \sin[d+ex])}{3 a (a^2 - b^2 + c^2) (b + a \cos[d+ex] + c \sin[d+ex])^2} - \right. \\
& \quad \left. \frac{2 (a^2 c + 3 b^2 c + c^3 + 4 a^2 b \sin[d+ex] + 4 b c^2 \sin[d+ex])}{3 a (a^2 - b^2 + c^2)^2 (b + a \cos[d+ex] + c \sin[d+ex])} \right) \Bigg) / \\
& \quad \left( e (a + b \operatorname{Sec}[d+ex] + c \tan[d+ex])^{5/2} \right) + \left( 2 a^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin[d+ex + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin[d+ex + \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \right)
\end{aligned}$$

$$\sec [d+e x]^{5 / 2} \sec \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]\left(b+a \cos [d+e x]+c \sin [d+e x]\right)^{5 / 2}$$

$$\sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}}-c \sqrt{\frac{a^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b+c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b+c \sqrt{\frac{a^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}}+c \sqrt{\frac{a^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b+c \sqrt{\frac{a^2+c^2}{c^2}}}} \quad /$$

$$\left(3 \sqrt{1+\frac{a^2}{c^2}} c\left(a^2-b^2+c^2\right)^2 e\left(a+b \sec [d+e x]+c \tan [d+e x]\right)^{5 / 2}+\right.$$

$$\left.2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2},-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c},\right.\right.$$

$$\left.-\frac{b+\sqrt{1+\frac{a^2}{c^2}} c \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1+\frac{a^2}{c^2}}\left(-1-\frac{b}{\sqrt{1+\frac{a^2}{c^2}} c}\right) c}\right] \sec [d+e x]^{5 / 2}$$

$$\sec \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]\left(b+a \cos [d+e x]+c \sin [d+e x]\right)^{5 / 2}$$

$$\sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}}-c \sqrt{\frac{a^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b+c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b+c \sqrt{\frac{a^2+c^2}{c^2}} \sin \left[d+e x+\operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}$$

$$\begin{aligned}
 & \left( \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) / \\
 & \left( \sqrt{1 + \frac{a^2}{c^2}} c (a^2 - b^2 + c^2)^2 e (a + b \sec[d + e x] + c \tan[d + e x])^{5/2} \right) + \\
 & \left( 2 c \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, \right. \right. \\
 & \quad \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \sec[d + e x]^{5/2} \right. \\
 & \quad \left. \sec\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] (b + a \cos[d + e x] + c \sin[d + e x])^{5/2} \right. \\
 & \quad \left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \right. \\
 & \quad \left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) / \\
 & \left( 3 \sqrt{1 + \frac{a^2}{c^2}} (a^2 - b^2 + c^2)^2 e (a + b \sec[d + e x] + c \tan[d + e x])^{5/2} \right) +
 \end{aligned}$$



$$\left( 4 a^2 b \operatorname{Sec}[d + e x]^{5/2} \left( b + a \operatorname{Cos}[d + e x] + c \operatorname{Sin}[d + e x] \right)^{5/2} \right.$$

$$\left. - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right], \right. \right. \right.$$

$$\left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \right] \operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \right) /$$

$$\left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right.$$

$$\sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}$$

$$\left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) -$$

$$\begin{aligned}
 & \left( \frac{2 a \left( b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{a^2 + c^2} - \frac{c \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}}} \right) \sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \Bigg/ \\
 & \left( 3 c \left( a^2 - b^2 + c^2 \right)^2 e \left( a + b \operatorname{Sec} \left[ d + e x \right] + c \operatorname{Tan} \left[ d + e x \right] \right)^{5/2} + \right. \\
 & \left. 4 b c \operatorname{Sec} \left[ d + e x \right]^{5/2} \left( b + a \cos \left[ d + e x \right] + c \sin \left[ d + e x \right] \right)^{5/2} \right. \\
 & \left. - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right) \Bigg/ \right. \\
 & \left. \left( a \sqrt{1 + \frac{c^2}{a^2}} \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \right. \\
 & \left. \left. \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \right) \right.
 \end{aligned}$$

$$\left( \frac{\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}}}{\frac{2a \left( \frac{b+a \sqrt{1+\frac{c^2}{a^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a^2+c^2} - \frac{c \sin\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1+\frac{c^2}{a^2}}}\right)}{\sqrt{b+a \sqrt{1+\frac{c^2}{a^2}} \cos\left[d+ex - \operatorname{ArcTan}\left[\frac{c}{a}\right]\right}}}\right) /$$

$$\left(3(a^2 - b^2 + c^2)^2 e (a + b \sec[d+ex] + c \tan[d+ex])^{5/2}\right)$$

**Problem 453: Attempted integration timed out after 120 seconds.**

$$\int \cos[d+ex]^{3/2} (a + b \sec[d+ex] + c \tan[d+ex])^{3/2} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned} & - \left( \left( 2 \cos[d+ex]^{3/2} (c \cos[d+ex] - a \sin[d+ex]) (a + b \sec[d+ex] + c \tan[d+ex])^{3/2} \right) / \right. \\ & \quad \left. (3 e (b + a \cos[d+ex] + c \sin[d+ex])) \right) + \\ & \left( 8 b \cos[d+ex]^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} (d+ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b + \sqrt{a^2+c^2}}\right] \right. \\ & \quad \left. (a + b \sec[d+ex] + c \tan[d+ex])^{3/2} \right) / \\ & \left( 3 e (b + a \cos[d+ex] + c \sin[d+ex]) \sqrt{\frac{b + a \cos[d+ex] + c \sin[d+ex]}{b + \sqrt{a^2+c^2}}} \right) + \\ & \left( 2 (a^2 - b^2 + c^2) \cos[d+ex]^{3/2} \operatorname{EllipticF}\left[\frac{1}{2} (d+ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b + \sqrt{a^2+c^2}}\right] \right. \\ & \quad \left. \sqrt{\frac{b + a \cos[d+ex] + c \sin[d+ex]}{b + \sqrt{a^2+c^2}}} (a + b \sec[d+ex] + c \tan[d+ex])^{3/2} \right) / \\ & \quad (3 e (b + a \cos[d+ex] + c \sin[d+ex])^2) \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 454: Attempted integration timed out after 120 seconds.

$$\int \sqrt{\cos [d+e x]} \sqrt{a+b \sec [d+e x]+c \tan [d+e x]} d x$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \sqrt{\cos [d+e x]} \operatorname{EllipticE}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{a+b \sec [d+e x]+c \tan [d+e x]} \right) / \left( e \sqrt{\frac{b+a \cos [d+e x]+c \sin [d+e x]}{b+\sqrt{a^2+c^2}}} \right)$$

Result (type 1, 1 leaves):

???

Problem 455: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{\cos [d+e x]} \sqrt{a+b \sec [d+e x]+c \tan [d+e x]}} d x$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(d+e x-\operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\frac{b+a \cos [d+e x]+c \sin [d+e x]}{b+\sqrt{a^2+c^2}}} \right) / \left( e \sqrt{\cos [d+e x]} \sqrt{a+b \sec [d+e x]+c \tan [d+e x]} \right)$$

Result (type 4, 506 leaves):

$$\begin{aligned}
& \left( 4 \left( i a - i b + c + \sqrt{a^2 - b^2 + c^2} \right) \right. \\
& \quad \text{EllipticF} \left[ \text{ArcSin} \left[ \sqrt{\left( \left( -i a + i b + c + \sqrt{a^2 - b^2 + c^2} \right) (-\cos[d + e x] + i \sin[d + e x]) \right)} \right] \right. \\
& \quad \quad \left. \left( i a - i b + c + \sqrt{a^2 - b^2 + c^2} \right) \right], \frac{b + i \sqrt{a^2 - b^2 + c^2}}{b - i \sqrt{a^2 - b^2 + c^2}} \right] \\
& \quad \sqrt{\frac{\left( -i a + i b + c + \sqrt{a^2 - b^2 + c^2} \right) (-\cos[d + e x] + i \sin[d + e x])}{i a - i b + c + \sqrt{a^2 - b^2 + c^2}}} \\
& \quad (\cos[d + e x] + i \sin[d + e x]) \sqrt{-\frac{i \left( -c + \sqrt{a^2 - b^2 + c^2} + (a - b) \tan\left[\frac{1}{2}(d + e x)\right] \right)}{\left( -i a + i b - c + \sqrt{a^2 - b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + e x)\right] \right)}}} \\
& \quad \sqrt{-\frac{i \left( c + \sqrt{a^2 - b^2 + c^2} + (-a + b) \tan\left[\frac{1}{2}(d + e x)\right] \right)}{\left( i a - i b + c + \sqrt{a^2 - b^2 + c^2} \right) \left( -i + \tan\left[\frac{1}{2}(d + e x)\right] \right)}}} \right) / \\
& \quad \left( \left( a + i \left( i b + c + \sqrt{a^2 - b^2 + c^2} \right) \right) e^{\sqrt{\cos[d + e x]} \sqrt{a + b \sec[d + e x] + c \tan[d + e x]}} \right)
\end{aligned}$$

**Problem 456: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{\cos[d + e x]^{3/2} (a + b \sec[d + e x] + c \tan[d + e x])^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\begin{aligned}
& - \left( \left( 2 (c \cos[d + e x] - a \sin[d + e x]) (b + a \cos[d + e x] + c \sin[d + e x]) \right) / \right. \\
& \quad \left. \left( (a^2 - b^2 + c^2) e \cos[d + e x]^{3/2} (a + b \sec[d + e x] + c \tan[d + e x])^{3/2} \right) \right) - \\
& \quad \left( 2 \text{EllipticE} \left[ \frac{1}{2} (d + e x - \text{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] (b + a \cos[d + e x] + c \sin[d + e x])^2 \right) / \\
& \quad \left( (a^2 - b^2 + c^2) e \cos[d + e x]^{3/2} \right. \\
& \quad \quad \left. \sqrt{\frac{b + a \cos[d + e x] + c \sin[d + e x]}{b + \sqrt{a^2 + c^2}}} (a + b \sec[d + e x] + c \tan[d + e x])^{3/2} \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

### Problem 457: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\cos[d + ex]^{5/2} (a + b \sec[d + ex] + c \tan[d + ex])^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned} & - \left( \left( 2 (c \cos[d + ex] - a \sin[d + ex]) (b + a \cos[d + ex] + c \sin[d + ex]) \right) / \right. \\ & \quad \left( 3 (a^2 - b^2 + c^2) e \cos[d + ex]^{5/2} (a + b \sec[d + ex] + c \tan[d + ex])^{5/2} \right) + \\ & \quad \left( 8 (b c \cos[d + ex] - a b \sin[d + ex]) (b + a \cos[d + ex] + c \sin[d + ex])^2 \right) / \\ & \quad \left( 3 (a^2 - b^2 + c^2)^2 e \cos[d + ex]^{5/2} (a + b \sec[d + ex] + c \tan[d + ex])^{5/2} \right) + \\ & \quad \left( 8 b \operatorname{EllipticE} \left[ \frac{1}{2} (d + ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] (b + a \cos[d + ex] + c \sin[d + ex])^3 \right) / \\ & \quad \left( 3 (a^2 - b^2 + c^2)^2 e \cos[d + ex]^{5/2} \right. \\ & \quad \left. \sqrt{\frac{b + a \cos[d + ex] + c \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} (a + b \sec[d + ex] + c \tan[d + ex])^{5/2} \right) + \\ & \quad \left( 2 \operatorname{EllipticF} \left[ \frac{1}{2} (d + ex - \operatorname{ArcTan}[a, c]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right. \\ & \quad \left. (b + a \cos[d + ex] + c \sin[d + ex])^2 \sqrt{\frac{b + a \cos[d + ex] + c \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} \right) / \\ & \quad \left( 3 (a^2 - b^2 + c^2) e \cos[d + ex]^{5/2} (a + b \sec[d + ex] + c \tan[d + ex])^{5/2} \right) \end{aligned}$$

Result (type 1, 1 leaves):

???

### Problem 461: Result more than twice size of optimal antiderivative.

$$\int \frac{\csc[x]}{2 + 2 \cot[x] + 3 \csc[x]} dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$x + 2 \operatorname{ArcTan} \left[ \frac{\cos[x] - \sin[x]}{2 + \cos[x] + \sin[x]} \right]$$

Result (type 3, 51 leaves):

$$-\operatorname{ArcTan} \left[ \frac{\cos\left[\frac{x}{2}\right]}{2 \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]} \right] + \operatorname{ArcTan} \left[ \sec\left[\frac{x}{2}\right] \left( 2 \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right) \right]$$

**Problem 462:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + c \cot[d + ex] + b \csc[d + ex])^{3/2}}{\csc[d + ex]^{3/2}} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned} & \left( 8 b (a + c \cot[d + ex] + b \csc[d + ex])^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} (d + ex - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \right) / \\ & \left( 3 e \csc[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex]) \sqrt{\frac{b + c \cos[d + ex] + a \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} \right) + \\ & \left( 2 (a^2 - b^2 + c^2) (a + c \cot[d + ex] + b \csc[d + ex])^{3/2} \right. \\ & \quad \left. \operatorname{EllipticF}\left[\frac{1}{2} (d + ex - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \sqrt{\frac{b + c \cos[d + ex] + a \sin[d + ex]}{b + \sqrt{a^2 + c^2}}} \right) / \\ & \left( 3 e \csc[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex])^2 \right) - \\ & \left( 2 (a + c \cot[d + ex] + b \csc[d + ex])^{3/2} (a \cos[d + ex] - c \sin[d + ex]) \right) / \\ & \left( 3 e \csc[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex]) \right) \end{aligned}$$

Result (type 6, 2490 leaves):

$$\begin{aligned} & \left( (a + c \cot[d + ex] + b \csc[d + ex])^{3/2} \left( \frac{8 b c}{3 a} - \frac{2}{3} a \cos[d + ex] + \frac{2}{3} c \sin[d + ex] \right) \right) / \\ & (e \csc[d + ex]^{3/2} (b + c \cos[d + ex] + a \sin[d + ex])) + \\ & \left( 4 a b (a + c \cot[d + ex] + b \csc[d + ex])^{3/2} \right. \\ & \quad \left. \left( a \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos[d + ex - \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \right) \right) \end{aligned}$$

$$\begin{aligned}
 & \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right/ \\
 & \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right. \\
 & \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \\
 & \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \\
 & \left( \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \\
 & \left. \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right) / \\
 & \left( 3 e \csc [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])^{3/2} + \right. \\
 & \left. 4 b c^2 (a + c \cot [d + e x] + b \csc [d + e x])^{3/2} \right)
 \end{aligned}$$



$$\begin{aligned}
& \left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) / \right. \\
& \quad \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right. \\
& \quad \left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right. \\
& \quad \left. \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \right. \\
& \quad \left. \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) / \right. \\
& \quad \left. \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right) / \\
& \left( 3 a e \csc \left[ d + e x \right]^{3/2} \left( b + c \cos \left[ d + e x \right] + a \sin \left[ d + e x \right] \right)^{3/2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 2 a \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a \sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1+\frac{c^2}{a^2}}\left(1-\frac{b}{a \sqrt{1+\frac{c^2}{a^2}}}\right)}, \right. \right. \\
& \quad \left. \left. -\frac{b+a \sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1+\frac{c^2}{a^2}}\left(-1-\frac{b}{a \sqrt{1+\frac{c^2}{a^2}}}\right)}\right] \right. \\
& \quad (a+c \cot [d+e x]+b \csc [d+e x])^{3 / 2} \sec \left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \\
& \quad \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}}-a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b+a \sqrt{\frac{a^2+c^2}{a^2}}}} \\
& \quad \sqrt{b+a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]} \\
& \quad \left. \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}}+a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b+a \sqrt{\frac{a^2+c^2}{a^2}}}} \right) / \\
& \quad \left( 3 \sqrt{1+\frac{c^2}{a^2}} e \csc [d+e x]^{3 / 2} (b+c \cos [d+e x]+a \sin [d+e x])^{3 / 2} \right) + \\
& \quad \left( 2 b^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a \sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin}\left[d+e x+\operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1+\frac{c^2}{a^2}}\left(1-\frac{b}{a \sqrt{1+\frac{c^2}{a^2}}}\right)}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \\
& (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2} \operatorname{Sec}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right] \\
& \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \\
& \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]} \\
& \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \Bigg/ \\
& \left( a \sqrt{1 + \frac{c^2}{a^2}} e \operatorname{Csc}[d + e x]^{3/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^{3/2} \right) + \\
& \left( 2 c^2 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)}\right], \right. \\
& \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin}\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left(-1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}}\right)} \right] (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2}
\end{aligned}$$

$$\begin{aligned} & \sec \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} - a \sqrt{\frac{a^2+c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \\ & \sqrt{b + a \sqrt{\frac{a^2+c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\ & \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \Bigg/ \\ & \left( 3 a \sqrt{1 + \frac{c^2}{a^2}} e \csc [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])^{3/2} \right) \end{aligned}$$

**Problem 463:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + c \cot [d + e x] + b \csc [d + e x]}}{\sqrt{\csc [d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\begin{aligned} & \left( 2 \sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan} [c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right) / \\ & \left( e \sqrt{\csc [d + e x]} \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}} \right) \end{aligned}$$

Result (type 6, 1580 leaves):

$$\frac{2 c \sqrt{a + c \cot [d + e x] + b \csc [d + e x]}}{a e \sqrt{\csc [d + e x]}} + \left( a \sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \right)$$

$$\begin{aligned}
& \left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) \right) / \right. \\
& \quad \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right. \\
& \quad \left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right. \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \\
& \quad \left( \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) / \\
& \quad \left( \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right) / \\
& \quad \left( e \sqrt{\csc \left[ d + e x \right]} \sqrt{b + c \cos \left[ d + e x \right]} + a \sin \left[ d + e x \right] \right) +
\end{aligned}$$

$$\left( c^2 \sqrt{a + c \cot[d + ex] + b \csc[d + ex]} \right.$$

$$\left. - \left( \left( a \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, \right. \right. \right. \right.$$

$$\left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \sin\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right) \right) /$$

$$\left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right.$$

$$\left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \right.$$

$$\left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos\left[d + ex - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) -$$

$$\left( \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) /$$

$$\sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}$$

$$\left( a e \sqrt{c \sec \left[ d + e x \right]} \sqrt{b + c \cos \left[ d + e x \right] + a \sin \left[ d + e x \right]} \right) +$$

$$\left( 2 b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \right. \right.$$

$$- \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)},$$

$$- \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \Big]$$

$$\sqrt{a + c \cot \left[ d + e x \right] + b \csc \left[ d + e x \right]}$$

$$\sec \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]$$

$$\sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}}$$

$$\sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}$$

$$\sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}} + a \sqrt{\frac{a^2+c^2}{a^2}} \sin\left[d + e x + \operatorname{ArcTan}\left[\frac{c}{a}\right]\right]}{-b + a \sqrt{\frac{a^2+c^2}{a^2}}}} \Bigg/$$

$$\left( a \sqrt{1 + \frac{c^2}{a^2}} e^{\sqrt{\operatorname{Csc}[d + e x]} \sqrt{b + c \cos[d + e x] + a \sin[d + e x]}} \right)$$

**Problem 464:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\operatorname{Csc}[d + e x]}}{\sqrt{a + c \cot[d + e x] + b \operatorname{Csc}[d + e x]}} dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \sqrt{\operatorname{Csc}[d + e x]} \operatorname{EllipticF}\left[\frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \right.$$

$$\left. \sqrt{\frac{b + c \cos[d + e x] + a \sin[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) \Bigg/ \left( e \sqrt{a + c \cot[d + e x] + b \operatorname{Csc}[d + e x]} \right)$$

Result (type 6, 339 leaves):



$$\begin{aligned}
& \left( 2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b - a \sqrt{1 + \frac{c^2}{a^2}}}, \right. \right. \\
& \quad \left. \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{1 + \frac{c^2}{a^2}}} \right] \sqrt{\operatorname{Csc} [d + e x]} \operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right. \\
& \quad \left. \sqrt{b + c \operatorname{Cos} [d + e x] + a \operatorname{Sin} [d + e x]} \sqrt{-\frac{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 + \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{b + a \sqrt{1 + \frac{c^2}{a^2}}}} \right. \right. \\
& \quad \left. \sqrt{\frac{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 + \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right)}{-b + a \sqrt{1 + \frac{c^2}{a^2}}}} \sqrt{b + a \sqrt{1 + \frac{c^2}{a^2}} \operatorname{Sin} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \right) / \\
& \quad \left( a \sqrt{1 + \frac{c^2}{a^2}} e^{\sqrt{a + c \operatorname{Cot} [d + e x] + b \operatorname{Csc} [d + e x]}} \right)
\end{aligned}$$

**Problem 465:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc} [d + e x]^{3/2}}{\left( a + c \operatorname{Cot} [d + e x] + b \operatorname{Csc} [d + e x] \right)^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\begin{aligned}
& - \left( 2 \operatorname{Csc}[d + e x]^{3/2} \right. \\
& \quad \left. \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2 \right) / \\
& \quad \left( (a^2 - b^2 + c^2) e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2} \sqrt{\frac{b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) - \\
& \quad (2 \operatorname{Csc}[d + e x]^{3/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]) (a \operatorname{Cos}[d + e x] - c \operatorname{Sin}[d + e x])) / \\
& \quad ((a^2 - b^2 + c^2) e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2})
\end{aligned}$$

Result (type 6, 1732 leaves):

$$\begin{aligned}
& \left( \operatorname{Csc}[d + e x]^{3/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2 \right. \\
& \quad \left( -\frac{2(a^2 + c^2)}{a c (a^2 - b^2 + c^2)} + \frac{2(a b + a^2 \operatorname{Sin}[d + e x] + c^2 \operatorname{Sin}[d + e x])}{c (a^2 - b^2 + c^2) (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])} \right) \Big) / \\
& \quad (e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{3/2}) - \\
& \quad \left( a \operatorname{Csc}[d + e x]^{3/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^{3/2} \right. \\
& \quad \left( - \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}[d + e x - \operatorname{ArcTan}[\frac{a}{c}]]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \operatorname{Sin}[d + e x - \operatorname{ArcTan}[\frac{a}{c}]] \right) \right) /
\end{aligned}$$

$$\left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} - c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right.$$

$$\sqrt{b + c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}$$

$$\left. \sqrt{\frac{c \sqrt{\frac{a^2+c^2}{c^2}} + c \sqrt{\frac{a^2+c^2}{c^2}} \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2+c^2}{c^2}}}} \right) -$$

$$\frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c}$$

$$\left. \frac{\sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}}{\sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}} \right) /$$

$$\left( (a^2 - b^2 + c^2) e (a + c \cot [d + e x] + b \csc [d + e x])^{3/2} \right) -$$

$$\left( c^2 \csc [d + e x]^{3/2} (b + c \cos [d + e x] + a \sin [d + e x])^{3/2} \right)$$

$$\left( \left( \left( a \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \text{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \right) \right) \right)$$

$$\begin{aligned}
 & \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right/ \\
 & \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right. \\
 & \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \\
 & \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \\
 & \left. \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \right/ \\
 & \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \\
 & \left( a \left( a^2 - b^2 + c^2 \right) e \left( a + c \cot \left[ d + e x \right] + b \csc \left[ d + e x \right] \right)^{3/2} - \right. \\
 & \left. 2 b \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right], \right.
 \end{aligned}$$

$$\begin{aligned}
& - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \\
& \operatorname{Csc} [d + e x]^{3/2} \operatorname{Sec} \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \\
& (b + c \cos [d + e x] + a \sin [d + e x])^{3/2} \\
& \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \\
& \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \Bigg/ \\
& \left( a (a^2 - b^2 + c^2) \sqrt{1 + \frac{c^2}{a^2}} e (a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{3/2} \right)
\end{aligned}$$

**Problem 466:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc} [d + e x]^{5/2}}{(a + c \cot [d + e x] + b \operatorname{Csc} [d + e x])^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned}
& \left( 8 b \operatorname{Csc}[d + e x]^{5/2} \right. \\
& \quad \left. \operatorname{EllipticE}\left[\frac{1}{2}(d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^3 \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2)^2 e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{5/2} \sqrt{\frac{b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) + \\
& \quad \left( 2 \operatorname{Csc}[d + e x]^{5/2} \operatorname{EllipticF}\left[\frac{1}{2}(d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}}\right] \right. \\
& \quad \left. (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2 \sqrt{\frac{b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2) e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{5/2} \right) - \\
& \quad \left( 2 \operatorname{Csc}[d + e x]^{5/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]) (a \operatorname{Cos}[d + e x] - c \operatorname{Sin}[d + e x]) \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2) e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{5/2} \right) + \\
& \quad \left( 8 \operatorname{Csc}[d + e x]^{5/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2 (a b \operatorname{Cos}[d + e x] - b c \operatorname{Sin}[d + e x]) \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2)^2 e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{5/2} \right)
\end{aligned}$$

Result (type 6, 2708 leaves):

$$\begin{aligned}
& \left( \operatorname{Csc}[d + e x]^{5/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^3 \right. \\
& \quad \left( \frac{8 b (a^2 + c^2)}{3 a c (-a^2 + b^2 - c^2)^2} + \frac{2 (a b + a^2 \operatorname{Sin}[d + e x] + c^2 \operatorname{Sin}[d + e x])}{3 c (a^2 - b^2 + c^2) (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2} - \right. \\
& \quad \left. \frac{2 (a^3 + 3 a b^2 + a c^2 + 4 a^2 b \operatorname{Sin}[d + e x] + 4 b c^2 \operatorname{Sin}[d + e x])}{3 c (a^2 - b^2 + c^2)^2 (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])} \right) \Bigg) / \\
& \quad \left( e (a + c \operatorname{Cot}[d + e x] + b \operatorname{Csc}[d + e x])^{5/2} \right) + \\
& \quad \left( 4 a b \operatorname{Csc}[d + e x]^{5/2} (b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^{5/2} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( \left( a \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( 1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} \left( -1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) c} \right] \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right) / \right. \\
& \quad \left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right. \\
& \quad \left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right. \\
& \quad \left. \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \right. \\
& \quad \left. \frac{2 c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) / \right. \\
& \quad \left. \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + e x - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \right) / \\
& \left( 3 \left( a^2 - b^2 + c^2 \right)^2 e \left( a + c \cot \left[ d + e x \right] + b \csc \left[ d + e x \right] \right)^{5/2} \right) +
\end{aligned}$$

$$\left( 4 b c^2 \operatorname{Csc}[d + e x]^{5/2} \left( b + c \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x] \right)^{5/2} \right.$$

$$\left. \left( - \left( a \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c}, \right. \right. \right. \right.$$

$$\left. \left. \left. - \frac{b + \sqrt{1 + \frac{a^2}{c^2}} c \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{\sqrt{1 + \frac{a^2}{c^2}} \left(-1 - \frac{b}{\sqrt{1 + \frac{a^2}{c^2}} c}\right) c} \right] \operatorname{Sin}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right] \right) \right) /$$

$$\left( \sqrt{1 + \frac{a^2}{c^2}} c \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} - c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right.$$

$$\left. \sqrt{b + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]} \right.$$

$$\left. \left. \sqrt{\frac{c \sqrt{\frac{a^2 + c^2}{c^2}} + c \sqrt{\frac{a^2 + c^2}{c^2}} \operatorname{Cos}\left[d + e x - \operatorname{ArcTan}\left[\frac{a}{c}\right]\right]}{-b + c \sqrt{\frac{a^2 + c^2}{c^2}}}} \right) - \right.$$



$$\begin{aligned}
& \left( \frac{2c \left( b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + ex - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right] \right)}{a^2 + c^2} - \frac{a \sin \left[ d + ex - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]}{\sqrt{1 + \frac{a^2}{c^2}} c} \right) \bigg/ \\
& \sqrt{b + \sqrt{1 + \frac{a^2}{c^2}} c \cos \left[ d + ex - \operatorname{ArcTan} \left[ \frac{a}{c} \right] \right]} \bigg) \\
& \left( 3a \left( a^2 - b^2 + c^2 \right)^2 e \left( a + c \cot \left[ d + ex \right] + b \csc \left[ d + ex \right] \right)^{5/2} + \right. \\
& \left. 2a \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right. \right. \right. \\
& \left. \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \right. \\
& \left. \csc \left[ d + ex \right]^{5/2} \sec \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \right. \\
& \left. \left( b + c \cos \left[ d + ex \right] + a \sin \left[ d + ex \right] \right)^{5/2} \right. \\
& \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \\
& \sqrt{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + ex + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) \bigg/
\end{aligned}$$

$$\begin{aligned}
& \left( 3 \left( a^2 - b^2 + c^2 \right)^2 \sqrt{1 + \frac{c^2}{a^2}} e \left( a + c \cot [d + e x] + b \csc [d + e x] \right)^{5/2} \right) + \\
& \left( 2 b^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( 1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right. \right. \\
& \quad \left. \left. - \frac{b + a \sqrt{1 + \frac{c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1 + \frac{c^2}{a^2}} \left( -1 - \frac{b}{a \sqrt{1 + \frac{c^2}{a^2}}} \right)} \right] \csc [d + e x]^{5/2} \right. \\
& \quad \left. \sec \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] \left( b + c \cos [d + e x] + a \sin [d + e x] \right)^{5/2} \right. \\
& \quad \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} - a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right. \\
& \quad \left. \sqrt{\frac{b + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a^2}} \right. \\
& \quad \left. \sqrt{\frac{a \sqrt{\frac{a^2 + c^2}{a^2}} + a \sqrt{\frac{a^2 + c^2}{a^2}} \sin \left[ d + e x + \operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b + a \sqrt{\frac{a^2 + c^2}{a^2}}}} \right) / \\
& \left( a \left( a^2 - b^2 + c^2 \right)^2 \sqrt{1 + \frac{c^2}{a^2}} e \left( a + c \cot [d + e x] + b \csc [d + e x] \right)^{5/2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( 2 c^2 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{b+a \sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1+\frac{c^2}{a^2}} \left( 1-\frac{b}{a \sqrt{1+\frac{c^2}{a^2}}} \right)} \right], \right. \\
& \quad \left. -\frac{b+a \sqrt{1+\frac{c^2}{a^2}} \operatorname{Sin} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{a \sqrt{1+\frac{c^2}{a^2}} \left( -1-\frac{b}{a \sqrt{1+\frac{c^2}{a^2}}} \right)} \right] \operatorname{Csc} [d+e x]^{5/2} \\
& \quad \operatorname{Sec} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right] (b+c \operatorname{Cos} [d+e x]+a \operatorname{Sin} [d+e x])^{5/2} \\
& \quad \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}}-a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{b+a \sqrt{\frac{a^2+c^2}{a^2}}}} \\
& \quad \sqrt{b+a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]} \\
& \quad \left. \sqrt{\frac{a \sqrt{\frac{a^2+c^2}{a^2}}+a \sqrt{\frac{a^2+c^2}{a^2}} \operatorname{Sin} \left[ d+e x+\operatorname{ArcTan} \left[ \frac{c}{a} \right] \right]}{-b+a \sqrt{\frac{a^2+c^2}{a^2}}}} \right] / \\
& \quad \left( 3 a \left( a^2-b^2+c^2 \right)^2 \sqrt{1+\frac{c^2}{a^2}} e \left( a+c \operatorname{Cot} [d+e x]+b \operatorname{Csc} [d+e x] \right)^{5/2} \right)
\end{aligned}$$

**Problem 467:** Attempted integration timed out after 120 seconds.

$$\int (a+c \operatorname{Cot} [d+e x]+b \operatorname{Csc} [d+e x])^{3/2} \operatorname{Sin} [d+e x]^{3/2} dx$$

Optimal (type 4, 371 leaves, 7 steps):

$$\begin{aligned}
& \left( 8 b \left( a + c \cot [d + e x] + b \csc [d + e x] \right)^{3/2} \right. \\
& \quad \left. \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \sin [d + e x]^{3/2} \right) / \\
& \quad \left( 3 e \left( b + c \cos [d + e x] + a \sin [d + e x] \right) \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}} \right) + \\
& \quad \left( 2 \left( a^2 - b^2 + c^2 \right) \left( a + c \cot [d + e x] + b \csc [d + e x] \right)^{3/2} \right. \\
& \quad \left. \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \sin [d + e x]^{3/2} \right. \\
& \quad \left. \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}} \right) / \left( 3 e \left( b + c \cos [d + e x] + a \sin [d + e x] \right)^2 \right) - \\
& \quad \left( 2 \left( a + c \cot [d + e x] + b \csc [d + e x] \right)^{3/2} \sin [d + e x]^{3/2} \left( a \cos [d + e x] - c \sin [d + e x] \right) \right) / \\
& \quad \left( 3 e \left( b + c \cos [d + e x] + a \sin [d + e x] \right) \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

### Problem 468: Unable to integrate problem.

$$\int \sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \sqrt{\sin [d + e x]} \, dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\begin{aligned}
& \left( 2 \sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right. \\
& \quad \left. \sqrt{\sin [d + e x]} \right) / \left( e \sqrt{\frac{b + c \cos [d + e x] + a \sin [d + e x]}{b + \sqrt{a^2 + c^2}}} \right)
\end{aligned}$$

Result (type 8, 35 leaves):

$$\int \sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \sqrt{\sin [d + e x]} \, dx$$

### Problem 469: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + c \cot [d + e x] + b \csc [d + e x]} \sqrt{\sin [d + e x]}} \, dx$$

Optimal (type 4, 118 leaves, 3 steps):

$$\left( 2 \operatorname{EllipticF}\left[\frac{1}{2}(d+ex - \operatorname{ArcTan}[c, a]), \frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right] \sqrt{\frac{b+c\cos[d+ex] + a\sin[d+ex]}{b+\sqrt{a^2+c^2}}} \right) / \\ \left( e\sqrt{a+c\cot[d+ex] + b\csc[d+ex]} \sqrt{\sin[d+ex]} \right)$$

Result (type 4, 719 leaves):

$$\left( 4 \left( i a + b - c - i \sqrt{a^2 - b^2 + c^2} \right) (1 + \cos[d+ex]) \sqrt{\csc[d+ex]} \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{(-a - i b + i c + \sqrt{a^2 - b^2 + c^2})(i + \tan[\frac{1}{2}(d+ex)])}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2})(-i + \tan[\frac{1}{2}(d+ex)])}}\right], \right. \\ \left. \frac{i b + \sqrt{a^2 - b^2 + c^2}}{i b - \sqrt{a^2 - b^2 + c^2}} \sqrt{\frac{b+c\cos[d+ex] + a\sin[d+ex]}{(1+\cos[d+ex])^2}} \left(-i + \tan\left[\frac{1}{2}(d+ex)\right]\right)^2 \right. \\ \left. \sqrt{\frac{(-a - i b + i c + \sqrt{a^2 - b^2 + c^2})(i + \tan[\frac{1}{2}(d+ex)])}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2})(-i + \tan[\frac{1}{2}(d+ex)])}} \sqrt{\cot\left[\frac{1}{2}(d+ex)\right] + \tan\left[\frac{1}{2}(d+ex)\right]} \right. \\ \left. \sqrt{\frac{i(a - \sqrt{a^2 - b^2 + c^2} + b \tan[\frac{1}{2}(d+ex)] - c \tan[\frac{1}{2}(d+ex)])}{(-a + i b - i c + \sqrt{a^2 - b^2 + c^2})(-i + \tan[\frac{1}{2}(d+ex)])}} \right. \\ \left. \sqrt{-\frac{i(a + \sqrt{a^2 - b^2 + c^2} + b \tan[\frac{1}{2}(d+ex)] - c \tan[\frac{1}{2}(d+ex)])}{(a - i b + i c + \sqrt{a^2 - b^2 + c^2})(-i + \tan[\frac{1}{2}(d+ex)])}} \sqrt{\frac{\tan[\frac{1}{2}(d+ex)]}{1 + \tan[\frac{1}{2}(d+ex)]^2}} \right) / \\ \left( (a + i b - i c - \sqrt{a^2 - b^2 + c^2}) e \sqrt{a+c\cot[d+ex] + b\csc[d+ex]} \sqrt{\left(1 + \tan\left[\frac{1}{2}(d+ex)\right]\right)^2} \right. \\ \left. \left(b + c + 2a \tan\left[\frac{1}{2}(d+ex)\right] + b \tan\left[\frac{1}{2}(d+ex)\right]^2 - c \tan\left[\frac{1}{2}(d+ex)\right]^2\right) \right)$$

**Problem 470: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(a+c\cot[d+ex] + b\csc[d+ex])^{3/2} \sin[d+ex]^{3/2}} dx$$

Optimal (type 4, 240 leaves, 4 steps):

$$\begin{aligned}
& - \left( \left( 2 \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] (b + c \cos[d + e x] + a \sin[d + e x])^2 \right) / \right. \\
& \quad \left( (a^2 - b^2 + c^2) e (a + c \cot[d + e x] + b \csc[d + e x])^{3/2} \right. \\
& \quad \left. \sin[d + e x]^{3/2} \sqrt{\frac{b + c \cos[d + e x] + a \sin[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) \Bigg) - \\
& \quad \frac{2 (b + c \cos[d + e x] + a \sin[d + e x]) (a \cos[d + e x] - c \sin[d + e x])}{(a^2 - b^2 + c^2) e (a + c \cot[d + e x] + b \csc[d + e x])^{3/2} \sin[d + e x]^{3/2}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 471: Attempted integration timed out after 120 seconds.**

$$\int \frac{1}{(a + c \cot[d + e x] + b \csc[d + e x])^{5/2} \sin[d + e x]^{5/2}} dx$$

Optimal (type 4, 492 leaves, 8 steps):

$$\begin{aligned}
& \left( 8 b \operatorname{EllipticE} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] (b + c \cos[d + e x] + a \sin[d + e x])^3 \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2)^2 e (a + c \cot[d + e x] + b \csc[d + e x])^{5/2} \right. \\
& \quad \left. \sin[d + e x]^{5/2} \sqrt{\frac{b + c \cos[d + e x] + a \sin[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) + \\
& \quad \left( 2 \operatorname{EllipticF} \left[ \frac{1}{2} (d + e x - \operatorname{ArcTan}[c, a]), \frac{2 \sqrt{a^2 + c^2}}{b + \sqrt{a^2 + c^2}} \right] \right. \\
& \quad \left. (b + c \cos[d + e x] + a \sin[d + e x])^2 \sqrt{\frac{b + c \cos[d + e x] + a \sin[d + e x]}{b + \sqrt{a^2 + c^2}}} \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2) e (a + c \cot[d + e x] + b \csc[d + e x])^{5/2} \sin[d + e x]^{5/2} \right) - \\
& \quad \left( 2 (b + c \cos[d + e x] + a \sin[d + e x]) (a \cos[d + e x] - c \sin[d + e x]) \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2) e (a + c \cot[d + e x] + b \csc[d + e x])^{5/2} \sin[d + e x]^{5/2} \right) + \\
& \quad \left( 8 (b + c \cos[d + e x] + a \sin[d + e x])^2 (a b \cos[d + e x] - b c \sin[d + e x]) \right) / \\
& \quad \left( 3 (a^2 - b^2 + c^2)^2 e (a + c \cot[d + e x] + b \csc[d + e x])^{5/2} \sin[d + e x]^{5/2} \right)
\end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 475: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\cos^2[x] - \sin^2[x]} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTanh}[2 \cos[x] \sin[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2} \operatorname{Log}[\cos[x] - \sin[x]] + \frac{1}{2} \operatorname{Log}[\cos[x] + \sin[x]]$$

**Problem 503: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{d + e \sin[x]}{a + b \sin[x] + c \sin^2[x]} dx$$

Optimal (type 3, 242 leaves, 7 steps):

$$\frac{\sqrt{2} \left( e + \frac{2 c d - b e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTan} \left[ \frac{2 c + (b - \sqrt{b^2 - 4 a c}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}}} \right]}{\sqrt{b^2 - 2 c (a + c) - b \sqrt{b^2 - 4 a c}}} +$$

$$\frac{\sqrt{2} \left( e - \frac{2 c d - b e}{\sqrt{b^2 - 4 a c}} \right) \operatorname{ArcTan} \left[ \frac{2 c + (b + \sqrt{b^2 - 4 a c}) \tan \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}}} \right]}{\sqrt{b^2 - 2 c (a + c) + b \sqrt{b^2 - 4 a c}}}$$

Result (type 3, 286 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-\frac{b^2}{2} + 2ac}} \\
& \left( \left( -2 \, i \, c \, d + \left( i \, b + \sqrt{-b^2 + 4ac} \right) e \right) \operatorname{ArcTan} \left[ \frac{2c + \left( b - i \sqrt{-b^2 + 4ac} \right) \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) - i b \sqrt{-b^2 + 4ac}}} \right] \right) / \\
& \left( \sqrt{b^2 - 2c(a+c) - i b \sqrt{-b^2 + 4ac}} \right) + \\
& \left( \left( 2 \, i \, c \, d + \left( -i \, b + \sqrt{-b^2 + 4ac} \right) e \right) \operatorname{ArcTan} \left[ \frac{2c + \left( b + i \sqrt{-b^2 + 4ac} \right) \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2} \sqrt{b^2 - 2c(a+c) + i b \sqrt{-b^2 + 4ac}}} \right] \right) / \\
& \left( \sqrt{b^2 - 2c(a+c) + i b \sqrt{-b^2 + 4ac}} \right) \Bigg)
\end{aligned}$$

**Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{Tan}[d + ex]) (b^2 + 2ab \operatorname{Tan}[d + ex] + a^2 \operatorname{Tan}[d + ex]^2)^2 dx$$

Optimal (type 3, 144 leaves, 7 steps):

$$\begin{aligned}
& a(a^2 - 3b^2)(a^2 + b^2)x + \frac{b(3a^2 - b^2)(a^2 + b^2) \operatorname{Log}[\operatorname{Cos}[d + ex]]}{e} - \frac{a(a^4 - b^4) \operatorname{Tan}[d + ex]}{e} + \\
& \frac{b(a^2 + b^2)(b + a \operatorname{Tan}[d + ex])^2}{2e} + \frac{(a^2 + b^2)(b + a \operatorname{Tan}[d + ex])^3}{3e} + \frac{b(b + a \operatorname{Tan}[d + ex])^4}{4e}
\end{aligned}$$

Result (type 3, 578 leaves):



$$\frac{a^4 b \cos[d + ex] (b + a \tan[d + ex])^4 (a + b \tan[d + ex])}{4 e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex])} +$$

$$\frac{a^2 b (a^2 + 3 b^2) \cos[d + ex]^3 (b + a \tan[d + ex])^4 (a + b \tan[d + ex])}{e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex])} -$$

$$\left( a (-i a + b) (i a + b) (-a^2 + 3 b^2) (d + ex) \cos[d + ex]^5 (b + a \tan[d + ex])^4 (a + b \tan[d + ex]) \right) /$$

$$\left( e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex]) \right) +$$

$$\left( (3 a^4 b + 2 a^2 b^3 - b^5) \cos[d + ex]^5 \log[\cos[d + ex]] (b + a \tan[d + ex])^4 (a + b \tan[d + ex]) \right) /$$

$$\left( e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex]) \right) +$$

$$\left( \cos[d + ex]^2 (a^5 \sin[d + ex] + 4 a^3 b^2 \sin[d + ex]) (b + a \tan[d + ex])^4 (a + b \tan[d + ex]) \right) /$$

$$\left( 3 e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex]) \right) +$$

$$\left( 2 \cos[d + ex]^4 (-2 a^5 \sin[d + ex] + a^3 b^2 \sin[d + ex] + 6 a b^4 \sin[d + ex]) (b + a \tan[d + ex])^4 \right.$$

$$\left. (a + b \tan[d + ex]) \right) / \left( 3 e (b \cos[d + ex] + a \sin[d + ex])^4 (a \cos[d + ex] + b \sin[d + ex]) \right)$$

**Problem 512: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \tan[d + ex]}{b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2} dx$$

Optimal (type 3, 101 leaves, 4 steps):

$$-\frac{a(a^2 - 3b^2)x}{(a^2 + b^2)^2} + \frac{b(3a^2 - b^2) \log[b \cos[d + ex] + a \sin[d + ex]]}{(a^2 + b^2)^2 e} - \frac{a^2 - b^2}{(a^2 + b^2) e (b + a \tan[d + ex])}$$

Result (type 3, 219 leaves):

$$\frac{1}{2 b (a^2 + b^2)^2 e (b + a \tan[d + ex])}$$

$$\left( b^2 (-2 (a - i b)^3 (d + ex) - b (-3 a^2 + b^2) \log[(b \cos[d + ex] + a \sin[d + ex])^2]) + \right.$$

$$a (2 (a - i b) (a^3 - a^2 b (-i + d + ex) + b^3 (-i + d + ex) + i a b^2 (i + 2 d + 2 ex)) -$$

$$b^2 (-3 a^2 + b^2) \log[(b \cos[d + ex] + a \sin[d + ex])^2]) \tan[d + ex] +$$

$$\left. 2 i b^2 (-3 a^2 + b^2) \operatorname{ArcTan}[\tan[d + ex]] (b + a \tan[d + ex]) \right)$$

**Problem 513: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \tan[d + ex]}{(b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2)^2} dx$$

Optimal (type 3, 197 leaves, 6 steps):

$$\frac{a (a^4 - 10 a^2 b^2 + 5 b^4) x}{(a^2 + b^2)^4} - \frac{b (5 a^4 - 10 a^2 b^2 + b^4) \operatorname{Log}[b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]]}{(a^2 + b^2)^4 e} - \frac{a^2 - b^2}{3 (a^2 + b^2) e (b + a \operatorname{Tan}[d + e x])^3} - \frac{b (3 a^2 - b^2)}{2 (a^2 + b^2)^2 e (b + a \operatorname{Tan}[d + e x])^2} + \frac{a^4 - 6 a^2 b^2 + b^4}{(a^2 + b^2)^3 e (b + a \operatorname{Tan}[d + e x])}$$

Result (type 3, 1098 leaves):

$$\begin{aligned} & \left( (-5 i a^{11} b + 5 a^{10} b^2 - 5 i a^9 b^3 + 5 a^8 b^4 + 14 i a^7 b^5 - 14 a^6 b^6 + 22 i a^5 b^7 - 22 a^4 b^8 + 7 i a^3 b^9 - 7 a^2 b^{10} - \right. \\ & \quad \left. i a b^{11} + b^{12}) (d + e x) \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( (a - i b)^3 (a + i b)^4 (-i a + b)^4 (i a + b)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) \right. \\ & \quad \left. (b + a \operatorname{Tan}[d + e x])^4 \right) - \\ & \left( i (-5 a^4 b + 10 a^2 b^3 - b^5) \operatorname{ArcTan}[\operatorname{Tan}[d + e x]] \operatorname{Sec}[d + e x]^3 \right. \\ & \quad \left. (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( (a^2 + b^2)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) + \\ & \left( (-5 a^4 b + 10 a^2 b^3 - b^5) \operatorname{Log}[(b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^2] \right. \\ & \quad \left. \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x])^4 (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( 2 (a^2 + b^2)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) + \\ & \left( \operatorname{Sec}[d + e x]^3 (b \operatorname{Cos}[d + e x] + a \operatorname{Sin}[d + e x]) \right. \\ & \quad (-12 a^8 b \operatorname{Cos}[d + e x] + 24 a^6 b^3 \operatorname{Cos}[d + e x] + 36 a^4 b^5 \operatorname{Cos}[d + e x] + 9 a^7 b^2 (d + e x) \operatorname{Cos}[d + e x] - \\ & \quad 81 a^5 b^4 (d + e x) \operatorname{Cos}[d + e x] - 45 a^3 b^6 (d + e x) \operatorname{Cos}[d + e x] + 45 a b^8 (d + e x) \operatorname{Cos}[d + e x] + \\ & \quad 8 a^8 b \operatorname{Cos}[3 (d + e x)] - 54 a^6 b^3 \operatorname{Cos}[3 (d + e x)] - 44 a^4 b^5 \operatorname{Cos}[3 (d + e x)] + \\ & \quad 18 a^2 b^7 \operatorname{Cos}[3 (d + e x)] - 9 a^7 b^2 (d + e x) \operatorname{Cos}[3 (d + e x)] + 93 a^5 b^4 (d + e x) \operatorname{Cos}[3 (d + e x)] - \\ & \quad 75 a^3 b^6 (d + e x) \operatorname{Cos}[3 (d + e x)] + 15 a b^8 (d + e x) \operatorname{Cos}[3 (d + e x)] - \\ & \quad 12 a^9 \operatorname{Sin}[d + e x] + 51 a^7 b^2 \operatorname{Sin}[d + e x] + 81 a^5 b^4 \operatorname{Sin}[d + e x] + 9 a^3 b^6 \operatorname{Sin}[d + e x] - \\ & \quad 9 a b^8 \operatorname{Sin}[d + e x] + 9 a^8 b (d + e x) \operatorname{Sin}[d + e x] - 81 a^6 b^3 (d + e x) \operatorname{Sin}[d + e x] - \\ & \quad 45 a^4 b^5 (d + e x) \operatorname{Sin}[d + e x] + 45 a^2 b^7 (d + e x) \operatorname{Sin}[d + e x] + 4 a^9 \operatorname{Sin}[3 (d + e x)] - \\ & \quad 31 a^7 b^2 \operatorname{Sin}[3 (d + e x)] + 5 a^5 b^4 \operatorname{Sin}[3 (d + e x)] + 31 a^3 b^6 \operatorname{Sin}[3 (d + e x)] - \\ & \quad 9 a b^8 \operatorname{Sin}[3 (d + e x)] - 3 a^8 b (d + e x) \operatorname{Sin}[3 (d + e x)] + 39 a^6 b^3 (d + e x) \operatorname{Sin}[3 (d + e x)] - \\ & \quad \left. 105 a^4 b^5 (d + e x) \operatorname{Sin}[3 (d + e x)] + 45 a^2 b^7 (d + e x) \operatorname{Sin}[3 (d + e x)]) (a + b \operatorname{Tan}[d + e x]) \right) / \\ & \left( 12 b (-i a + b)^4 (i a + b)^4 e (a \operatorname{Cos}[d + e x] + b \operatorname{Sin}[d + e x]) (b + a \operatorname{Tan}[d + e x])^4 \right) \end{aligned}$$

**Problem 517: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{Tan}[d + e x]}{(b^2 + 2 a b \operatorname{Tan}[d + e x] + a^2 \operatorname{Tan}[d + e x]^2)^{3/2}} dx$$

Optimal (type 3, 316 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(a^2 - b^2) (b + a \tan[d + ex])}{2 (a^2 + b^2) e (b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2)^{3/2}} - \\
& \left( (a^4 - 6 a^2 b^2 + b^4) \log[b \cos[d + ex] + a \sin[d + ex]] (b + a \tan[d + ex])^3 \right) / \\
& \left( (a^2 + b^2)^3 e (b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2)^{3/2} \right) - \\
& \frac{4 b (a^2 - b^2) x (a b + a^2 \tan[d + ex])^3}{a^2 (a^2 + b^2)^3 (b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2)^{3/2}} - \left( b (3 a^2 - b^2) (a b + a^2 \tan[d + ex])^3 \right) / \\
& \left( (a^2 + b^2)^2 e (a^3 b + a^4 \tan[d + ex]) (b^2 + 2 a b \tan[d + ex] + a^2 \tan[d + ex]^2)^{3/2} \right)
\end{aligned}$$

Result(type 3, 293 leaves):

$$\begin{aligned}
& \frac{1}{2 (a^2 + b^2)^3 e (b + a \tan[d + ex]) \sqrt{(b + a \tan[d + ex])^2}} \left( (-a^6 + a^2 b^4) \sec[d + ex]^2 + \right. \\
& 2 i (a^4 - 6 a^2 b^2 + b^4) \operatorname{ArcTan}[\tan[d + ex]] (b + a \tan[d + ex])^2 + (b + a \tan[d + ex]) \\
& \left. \left( b (-2 i (a - i b)^4 (d + ex) - (a^4 - 6 a^2 b^2 + b^4) \log[(b \cos[d + ex] + a \sin[d + ex])^2]) \right) + \right. \\
& a \left( 2 (a - i b) (a^2 b (4 i - 3 d - 3 ex) + b^3 (-2 i + d + ex) - i a^3 (4 i + d + ex) + i a b^2 (2 i + 3 d + \right. \\
& \left. 3 ex)) - (a^4 - 6 a^2 b^2 + b^4) \log[(b \cos[d + ex] + a \sin[d + ex])^2] \right) \tan[d + ex] \left. \right)
\end{aligned}$$

**Problem 518: Result more than twice size of optimal antiderivative.**

$$\int (a + b \sec[d + ex]) (b^2 + 2 a b \sec[d + ex] + a^2 \sec[d + ex]^2)^2 dx$$

Optimal (type 3, 184 leaves, 8 steps):

$$\begin{aligned}
& a b^4 x + \frac{b (19 a^4 + 56 a^2 b^2 + 8 b^4) \operatorname{ArcTanh}[\sin[d + ex]]}{8 e} + \\
& \frac{a (4 a^4 + 50 a^2 b^2 + 19 b^4) \tan[d + ex]}{6 e} + \frac{a^2 b (41 a^2 + 26 b^2) \sec[d + ex] \tan[d + ex]}{24 e} + \\
& \frac{(4 a^2 + 7 b^2) (a b + a^2 \sec[d + ex])^2 \tan[d + ex]}{12 a e} + \frac{b (a b + a^2 \sec[d + ex])^3 \tan[d + ex]}{4 a^2 e}
\end{aligned}$$

Result(type 3, 590 leaves):

$$\begin{aligned}
& \frac{a b^4 (d + e x)}{e} + \frac{(-19 a^4 b - 56 a^2 b^3 - 8 b^5) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right]}{8 e} + \\
& \frac{(19 a^4 b + 56 a^2 b^3 + 8 b^5) \operatorname{Log}\left[\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right]}{8 e} + \\
& \frac{a^4 b}{16 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^4} + \frac{4 a^5 + 57 a^4 b + 16 a^3 b^2 + 72 a^2 b^3}{48 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^2} - \\
& \frac{a^4 b}{16 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^4} + \frac{-4 a^5 - 57 a^4 b - 16 a^3 b^2 - 72 a^2 b^3}{48 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^2} + \\
& \frac{a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 4 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right]}{6 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)^3} + \frac{a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 4 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right]}{6 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)^3} + \\
& \left(2 \left(a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 13 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right] + 6 a b^4 \sin\left[\frac{1}{2}(d + e x)\right]\right)\right) / \\
& \left(3 e \left(\cos\left[\frac{1}{2}(d + e x)\right] - \sin\left[\frac{1}{2}(d + e x)\right]\right)\right) + \\
& \left(2 \left(a^5 \sin\left[\frac{1}{2}(d + e x)\right] + 13 a^3 b^2 \sin\left[\frac{1}{2}(d + e x)\right] + 6 a b^4 \sin\left[\frac{1}{2}(d + e x)\right]\right)\right) / \\
& \left(3 e \left(\cos\left[\frac{1}{2}(d + e x)\right] + \sin\left[\frac{1}{2}(d + e x)\right]\right)\right)
\end{aligned}$$

**Problem 548: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos [x] + C \sin [x]}{a + b \cos [x] + i b \sin [x]} dx$$

Optimal (type 3, 92 leaves, 1 step):

$$\begin{aligned}
& -\frac{b (B + i C) x}{2 a^2} - \frac{(i b^2 (B + i C) + a^2 (i B + C)) \operatorname{Log}[a + b \cos [x] + i b \sin [x]]}{2 a^2 b} + \\
& \frac{(i B - C) (\cos [x] - i \sin [x])}{2 a}
\end{aligned}$$

Result (type 3, 195 leaves):

$$\begin{aligned}
& \frac{(a^2 B - b^2 B - i a^2 C - i b^2 C) x}{4 a^2 b} - \frac{(a^2 B + b^2 B - i a^2 C + i b^2 C) \operatorname{ArcTan}\left[\frac{(a+b) \cos \left[\frac{x}{2}\right]}{-a \sin \left[\frac{x}{2}\right] + b \sin \left[\frac{x}{2}\right]}\right]}{2 a^2 b} + \\
& \frac{i (B + i C) \cos [x]}{2 a} - \frac{(a^2 B + b^2 B - i a^2 C + i b^2 C) \operatorname{Log}\left[a^2 + b^2 + 2 a b \cos [x]\right]}{4 a^2 b} + \frac{(B + i C) \sin [x]}{2 a}
\end{aligned}$$

**Problem 549: Result more than twice size of optimal antiderivative.**

$$\int \frac{B \cos [x] + C \sin [x]}{a + b \cos [x] - i b \sin [x]} dx$$

Optimal (type 3, 90 leaves, 1 step):

$$-\frac{b(B-iC)x}{2a^2} + \frac{(ia^2(B+iC) + b^2(iB+C)) \operatorname{Log}[a+b\cos[x] - ib\sin[x]]}{2a^2b} - \frac{(iB+C)(\cos[x] + i\sin[x])}{2a}$$

Result (type 3, 195 leaves):

$$\frac{(a^2B - b^2B + ia^2C + ib^2C)x}{4a^2b} + \frac{(a^2B + b^2B + ia^2C - ib^2C) \operatorname{ArcTan}\left[\frac{(a+b)\cos\left[\frac{x}{2}\right]}{a\sin\left[\frac{x}{2}\right] - b\sin\left[\frac{x}{2}\right]}\right]}{2a^2b} - \frac{i(B-iC)\cos[x]}{2a} + \frac{i(a^2B + b^2B + ia^2C - ib^2C) \operatorname{Log}[a^2 + b^2 + 2ab\cos[x]]}{4a^2b} + \frac{(B-iC)\sin[x]}{2a}$$

**Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+b\cos[x] + c\sin[x])^{5/2} (d+be\cos[x] + ce\sin[x]) dx$$

Optimal (type 4, 390 leaves, 8 steps):

$$\left( 2(161a^2d + 63(b^2+c^2)d + 15a^3e + 145a(b^2+c^2)e) \right. \\ \left. \operatorname{EllipticE}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{a+b\cos[x] + c\sin[x]} \right) / \\ \left( 105 \sqrt{\frac{a+b\cos[x] + c\sin[x]}{a+\sqrt{b^2+c^2}}} \right) - \left( 2(a^2-b^2-c^2)(56ad + 15a^2e + 25(b^2+c^2)e) \right. \\ \left. \operatorname{EllipticF}\left[\frac{1}{2}(x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right] \sqrt{\frac{a+b\cos[x] + c\sin[x]}{a+\sqrt{b^2+c^2}}} \right) / \\ \left( 105 \sqrt{a+b\cos[x] + c\sin[x]} \right) - \frac{2}{7}(a+b\cos[x] + c\sin[x])^{5/2}(ce\cos[x] - be\sin[x]) - \\ \frac{2}{35}(a+b\cos[x] + c\sin[x])^{3/2}(c(7d+5ae)\cos[x] - b(7d+5ae)\sin[x]) - \\ \frac{2}{105} \sqrt{a+b\cos[x] + c\sin[x]} \\ (c(56ad + 15a^2e + 25(b^2+c^2)e)\cos[x] - b(56ad + 15a^2e + 25(b^2+c^2)e)\sin[x])$$

Result (type 6, 7823 leaves):

$$\sqrt{a+b\cos[x] + c\sin[x]} \left( \frac{2b(161a^2d + 63b^2d + 63c^2d + 15a^3e + 145ab^2e + 145ac^2e)}{105c} - \frac{1}{210}c(308ad + 180a^2e + 115b^2e + 115c^2e)\cos[x] - \frac{2}{35}bc(7d + 15ae)\cos[2x] - \right.$$

$$\begin{aligned}
& \frac{1}{14} c (3 b^2 - c^2) e \cos [3 x] + \frac{1}{210} b (308 a d + 180 a^2 e + 115 b^2 e + 115 c^2 e) \sin [x] + \\
& \frac{1}{35} (b^2 - c^2) (7 d + 15 a e) \sin [2 x] + \frac{1}{14} b (b^2 - 3 c^2) e \sin [3 x] \Big) + \\
& \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c} 2 a^3 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \\
& \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \sec \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}} c} \\
& 34 a b^2 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \\
& \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \sec \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]
\end{aligned}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{15 \sqrt{1 + \frac{b^2}{c^2}}}$$

$$34 a c d \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c},\right.$$

$$\left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} + \frac{1}{7 \sqrt{1 + \frac{b^2}{c^2}} c}$$

$$18 a^2 b^2 e \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c},\right.$$

$$\begin{aligned}
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}} c} \\
& 10 b^4 e \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \\
& \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \right. \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{7 \sqrt{1 + \frac{b^2}{c^2}}}
\end{aligned}$$



$$\begin{aligned}
& 18 a^2 c \, e \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}}} \\
& 20 b^2 c \, e \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{21 \sqrt{1 + \frac{b^2}{c^2}}} \\
 & 10 c^3 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
 & \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{15 c} \\
 & 23 a^2 b^2 d \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
& \left. \left. - \frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \right. \\
& \left. \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right. \\
& \left. \frac{1}{5 c} \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) + \frac{1}{5 c} \\
& 3 b^4 d \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \Bigg/
\end{aligned}$$

$$\begin{aligned}
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. \sqrt{\frac{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}{b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
 & \left. \sqrt{\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}} \right) + \frac{23}{15} a^2 c d \\
 & \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)
 \end{aligned}$$

$$\begin{aligned}
& \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
& \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} + \frac{6}{5} b^2 c d \\
& \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right) \right. \\
& \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) \\
& \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) -
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{3}{5} c^3 d \\
 & \left( \frac{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \\
 & \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \\
 & \left( \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{1}{7c} a^3 b^2 e \\
 & \left( \frac{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
 \end{aligned}$$

$$\begin{aligned}
& \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) / \\
& \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \quad \left. \sqrt{\frac{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} - a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) - \\
& \quad \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) \\
& \quad \frac{1}{21c} 29 a b^4 e \\
& \quad \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) + \frac{1}{21c} 29 a b^4 e \\
& \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
& \left. \left. - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} + \frac{1}{7} a^3 c e \right. \\
& \left. - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right. \right. \right. \\
& \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \Bigg/ \right.
\end{aligned}$$



$$\begin{aligned}
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
& \left. \sqrt{\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2}} \right) + \frac{58}{21} a b^2 c e \\
& \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right. \right. \right. \\
& \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)
\end{aligned}$$

$$\begin{aligned}
 & \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
 & \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} + \frac{29}{21} a c^3 e \\
 & \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right) \right) \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) \\
 & \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) -
 \end{aligned}$$

$$\frac{\frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}$$

**Problem 557: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \left( a + b \cos [x] + c \sin [x] \right)^{3/2} \left( d + b e \cos [x] + c e \sin [x] \right) dx$$

Optimal (type 4, 294 leaves, 7 steps):

$$\begin{aligned} & \left( 2 \left( 20 a d + 3 a^2 e + 9 (b^2 + c^2) e \right) \text{EllipticE} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \right. \\ & \quad \left. \sqrt{a + b \cos [x] + c \sin [x]} \right) / \left( 15 \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) - \\ & \left( 2 (a^2 - b^2 - c^2) (5 d + 3 a e) \text{EllipticF} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \right. \\ & \quad \left. \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) / \left( 15 \sqrt{a + b \cos [x] + c \sin [x]} \right) - \\ & \frac{2}{5} (a + b \cos [x] + c \sin [x])^{3/2} (c e \cos [x] - b e \sin [x]) - \\ & \frac{2}{15} \sqrt{a + b \cos [x] + c \sin [x]} (c (5 d + 3 a e) \cos [x] - b (5 d + 3 a e) \sin [x]) \end{aligned}$$

Result (type 6, 5218 leaves):

$$\begin{aligned} & \sqrt{a + b \cos [x] + c \sin [x]} \left( \frac{2 b (20 a d + 3 a^2 e + 9 b^2 e + 9 c^2 e)}{15 c} - \frac{2}{15} c (5 d + 6 a e) \cos [x] - \right. \\ & \quad \left. \frac{2}{5} b c e \cos [2 x] + \frac{2}{15} b (5 d + 6 a e) \sin [x] + \frac{1}{5} (b^2 - c^2) e \sin [2 x] \right) + \\ & \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c} 2 a^2 d \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \text{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right], \end{aligned}$$

$$\begin{aligned}
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}} c} \\
& 2 b^2 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \\
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}}}
\end{aligned}$$

$$\begin{aligned}
& 2 c d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{5 \sqrt{1 + \frac{b^2}{c^2}} c} \\
& 8 a b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}
\end{aligned}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{5 \sqrt{1 + \frac{b^2}{c^2}}}$$

$$8 a c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c},\right.$$

$$\left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}$$

$$\sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{3 c}$$

$$4 a b^2 d \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right), \right.$$

$$\begin{aligned}
 & \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. - \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) \\
 & \left. \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) + \frac{4}{3} a c d \\
 & \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. \sqrt{\frac{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}{b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
 & \left. \sqrt{\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}} \right) + \frac{1}{5 c} a^2 b^2 e \\
 & \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)
 \end{aligned}$$



$$\begin{aligned}
& \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
& \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} + \frac{1}{5 c} \\
& 3 b^4 e \left( - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right) \right. \right. \\
& \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) \\
& \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) -
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{1}{5} a^2 c e \\
 & \left( \frac{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{\left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right) \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \right. \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \\
 & \left( \sqrt{a+b} \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{6}{5} b^2 c e \\
 & \left( \frac{\sqrt{a+b} \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{\left( \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{6}{5} b^2 c e} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \quad \left. \left. \left. - \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \right) / \\
 & \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \quad \left. \sqrt{\frac{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} - \frac{a+b\sqrt{\frac{b^2+c^2}{b^2}}}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) \\
 & \quad \left. \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{3}{5} c^3 e \\
 & \quad \left. \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \\
 & \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
& \left. \left. - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \right. \\
& \left. \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)
\end{aligned}$$

**Problem 558: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \sqrt{a + b \cos [x] + c \sin [x]} \left( d + b e \cos [x] + c e \sin [x] \right) dx$$

Optimal (type 4, 229 leaves, 6 steps):

$$\begin{aligned}
& \left( 2 (3d + ae) \operatorname{EllipticE} \left[ \frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos[x] + c \sin[x]} \right) / \\
& \left( 3 \sqrt{\frac{a + b \cos[x] + c \sin[x]}{a + \sqrt{b^2 + c^2}}} \right) - \\
& \left( 2 (a^2 - b^2 - c^2) e \operatorname{EllipticF} \left[ \frac{1}{2} (x - \operatorname{ArcTan}[b, c]), \frac{2\sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos[x] + c \sin[x]}{a + \sqrt{b^2 + c^2}}} \right) / \\
& \left( 3 \sqrt{a + b \cos[x] + c \sin[x]} \right) - \frac{2}{3} \sqrt{a + b \cos[x] + c \sin[x]} (c e \cos[x] - b e \sin[x])
\end{aligned}$$

Result (type 6, 3006 leaves):

$$\begin{aligned}
& \sqrt{a + b \cos[x] + c \sin[x]} \left( \frac{2b(3d + ae)}{3c} - \frac{2}{3} c e \cos[x] + \frac{2}{3} b e \sin[x] \right) + \\
& \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c} {}_2F_1 \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right], \\
& - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin[x + \operatorname{ArcTan}[\frac{b}{c}]]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}} c}
\end{aligned}$$

$$\begin{aligned}
& 2 b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} + \frac{1}{3 \sqrt{1 + \frac{b^2}{c^2}}} \\
& 2 c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \\
& \quad \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}} + \frac{1}{c}} \\
& b^2 d \left( - \left( \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)}, \right. \right. \right. \\
& \quad \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}}\right)} \sin\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right) / \\
& \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \quad \left. \sqrt{\frac{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
& \quad \left( \frac{2 b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right)}{b^2+c^2} - \frac{c \sin\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) + \\
& \quad \left( \sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \right)
\end{aligned}$$

$$\begin{aligned}
 & c d \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \quad \left. \left. - \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \quad \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \quad \left. \sqrt{\frac{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} - \frac{1}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}} \right) - \\
 & \quad \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) + \frac{1}{3c} \\
 & \quad \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) + \frac{1}{3c} \\
 & a b^2 e \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
& \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \left. \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
& \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) + \frac{1}{3} a c e \\
& \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \left. \right) + \frac{1}{3} a c e \\
& \left( \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right/ \right. \right. \right. \right.
\end{aligned}$$

$$\left( \frac{b \sqrt{1 + \frac{c^2}{b^2}}}{\sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\ \left. \frac{\sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\ \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\ \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \right)$$

Problem 559: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{\sqrt{a + b \cos [x] + c \sin [x]}} dx$$

Optimal (type 4, 180 leaves, 5 steps):

$$\left( 2 e \text{EllipticE} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos [x] + c \sin [x]} \right) / \\ \left( \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) + \\ \left( 2 (d - a e) \text{EllipticF} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) / \\ (\sqrt{a + b \cos [x] + c \sin [x]})$$

Result (type 6, 1319 leaves):

$$\frac{2 b e \sqrt{a + b \cos [x] + c \sin [x]}}{c} + \frac{1}{\sqrt{1 + \frac{b^2}{c^2}} c}$$

$$\begin{aligned}
& 2 \, d \, \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} \, c \, \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} \, c}\right) c}, \right. \\
& \quad \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} \, c \, \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} \, c}\right) c} \right] \text{Sec}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right] \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \, \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \, \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \, \text{Sin}\left[x + \text{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} + \frac{1}{c}} \\
& \quad b^2 e \left( - \left( \left( c \, \text{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \, \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right. \right. \right. \\
& \quad \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \, \text{Cos}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \right] \text{Sin}\left[x - \text{ArcTan}\left[\frac{c}{b}\right]\right] \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
 & \left. \sqrt{\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2}} \right) + \\
 & c e \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)
 \end{aligned}$$

$$\left( \frac{\sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] - a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) - \frac{\frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}$$

**Problem 560: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{(a + b \cos [x] + c \sin [x])^{3/2}} dx$$

Optimal (type 4, 250 leaves, 6 steps):

$$\left( 2 (d - a e) \text{EllipticE} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos [x] + c \sin [x]} \right) / \left( (a^2 - b^2 - c^2) \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) + \frac{2 e \text{EllipticF} \left[ \frac{1}{2} (x - \text{ArcTan}[b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}}}{\sqrt{a + b \cos [x] + c \sin [x]}} + \frac{2 (c (d - a e) \cos [x] - b (d - a e) \sin [x])}{(a^2 - b^2 - c^2) \sqrt{a + b \cos [x] + c \sin [x]}}$$

Result (type 6, 3176 leaves):

$$\sqrt{a + b \cos [x] + c \sin [x]} \left( \frac{2 (b^2 + c^2) (-d + a e)}{b c (-a^2 + b^2 + c^2)} - \frac{(2 (-a c d + a^2 c e - b^2 d \sin [x] - c^2 d \sin [x] + a b^2 e \sin [x] + a c^2 e \sin [x]))}{(b (-a^2 + b^2 + c^2) (a + b \cos [x] + c \sin [x]))} \right) -$$

$$\begin{aligned}
& \left( 2 a d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
& \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \right) / \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2) \right) + \\
& \left( 2 b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2) \right) + \right. \\
& \left( 2 c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
& \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right) \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2) \right) - \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{1}{c(-a^2 + b^2 + c^2)} b^2 d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right], \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \right. \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \left. \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} - a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \right) - \\
 & \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
 & \left. \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} - \frac{1}{-a^2 + b^2 + c^2} \right) \\
 & c d \left( \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right], \right. \right. \right.
 \end{aligned}$$



$$\begin{aligned}
& - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
& \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
& \left. - \frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}} \right) \\
& \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \Bigg/ \\
& \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} + \frac{1}{c \left( -a^2 + b^2 + c^2 \right)} \\
& a b^2 e \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \Bigg/
\end{aligned}$$

$$\begin{aligned}
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. \sqrt{\frac{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}{b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \left( \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \\
 & \left. \sqrt{\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}} \right) + \frac{1}{-a^2 + b^2 + c^2} \\
 & a c e \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)
 \end{aligned}$$

$$\left( \frac{\sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}{\sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right) \sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}$$

**Problem 561: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{d + b e \cos [x] + c e \sin [x]}{(a + b \cos [x] + c \sin [x])^{5/2}} dx$$

Optimal (type 4, 378 leaves, 7 steps):

$$\left( 2 (4 a d - a^2 e - 3 (b^2 + c^2) e) \text{EllipticE} \left[ \frac{1}{2} (x - \text{ArcTan} [b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{a + b \cos [x] + c \sin [x]} \right) / \left( 3 (a^2 - b^2 - c^2)^2 \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) - \left( 2 (d - a e) \text{EllipticF} \left[ \frac{1}{2} (x - \text{ArcTan} [b, c]), \frac{2 \sqrt{b^2 + c^2}}{a + \sqrt{b^2 + c^2}} \right] \sqrt{\frac{a + b \cos [x] + c \sin [x]}{a + \sqrt{b^2 + c^2}}} \right) / \left( 3 (a^2 - b^2 - c^2) \sqrt{a + b \cos [x] + c \sin [x]} \right) + \frac{2 (c (d - a e) \cos [x] - b (d - a e) \sin [x])}{3 (a^2 - b^2 - c^2) (a + b \cos [x] + c \sin [x])^{3/2}} + \frac{2 (c (4 a d - a^2 e - 3 (b^2 + c^2) e) \cos [x] - b (4 a d - a^2 e - 3 (b^2 + c^2) e) \sin [x])}{3 (a^2 - b^2 - c^2)^2 \sqrt{a + b \cos [x] + c \sin [x]}}$$

Result (type 6, 5554 leaves):

$$\sqrt{a + b \cos [x] + c \sin [x]} \left( - \frac{2 (b^2 + c^2) (-4 a d + a^2 e + 3 b^2 e + 3 c^2 e)}{3 b c (-a^2 + b^2 + c^2)^2} - \frac{2 (-a c d + a^2 c e - b^2 d \sin [x] - c^2 d \sin [x] + a b^2 e \sin [x] + a c^2 e \sin [x])}{3 b (-a^2 + b^2 + c^2) (a + b \cos [x] + c \sin [x])^2} + \frac{2 (-3 a^2 c d - b^2 c d - c^3 d + 4 a b^2 c e + 4 a c^3 e - 4 a b^2 d \sin [x] - 4 a c^2 d \sin [x] +$$

$$\begin{aligned}
 & \left( a^2 b^2 e^{\sin[x]} + 3 b^4 e^{\sin[x]} + a^2 c^2 e^{\sin[x]} + 6 b^2 c^2 e^{\sin[x]} + 3 c^4 e^{\sin[x]} \right) / \\
 & \left( 3 b \left( -a^2 + b^2 + c^2 \right)^2 \left( a + b \cos[x] + c \sin[x] \right) \right) + \\
 & \left( 2 a^2 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \right. \\
 & \quad \left. \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \right. \\
 & \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]} \right. \\
 & \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \right) / \left( \sqrt{1 + \frac{b^2}{c^2}} c \left( -a^2 + b^2 + c^2 \right)^2 \right) + \\
 & \left( 2 b^2 d \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( 1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c}, \right. \right. \\
 & \quad \left. \left. -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right]}{\sqrt{1 + \frac{b^2}{c^2}} \left( -1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c} \right) c} \right] \operatorname{Sec} \left[ x + \operatorname{ArcTan} \left[ \frac{b}{c} \right] \right] \right.
 \end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c (-a^2 + b^2 + c^2)^2 \right) + \right. \\
 & \left. \left( 2 c d \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \right. \right. \\
 & \left. \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c} \right] \sec\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right) \right. \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
 & \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 \right) - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( 8 a b^2 e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
 & \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right. \\
 & \quad \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} - c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \sqrt{a + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
 & \quad \left. \sqrt{\frac{c \sqrt{\frac{b^2 + c^2}{c^2}} + c \sqrt{\frac{b^2 + c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2 + c^2}{c^2}}} \right) / \left( 3 \sqrt{1 + \frac{b^2}{c^2}} c \left(-a^2 + b^2 + c^2\right)^2 \right) - \\
 & \left( 8 a c e \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}, \right. \right. \\
 & \quad \left. \left. - \frac{a + \sqrt{1 + \frac{b^2}{c^2}} c \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{\sqrt{1 + \frac{b^2}{c^2}} \left(-1 - \frac{a}{\sqrt{1 + \frac{b^2}{c^2}} c}\right) c}\right] \operatorname{Sec}\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right] \right)
 \end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} - c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \sqrt{a + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]} \\
& \sqrt{\frac{c \sqrt{\frac{b^2+c^2}{c^2}} + c \sqrt{\frac{b^2+c^2}{c^2}} \sin\left[x + \operatorname{ArcTan}\left[\frac{b}{c}\right]\right]}{-a + c \sqrt{\frac{b^2+c^2}{c^2}}}} \left/ \left( 3 \sqrt{1 + \frac{b^2}{c^2}} (-a^2 + b^2 + c^2)^2 \right) + \right. \\
& \left( 4 a b^2 d \left( - \left( c \operatorname{AppellF1}\left[-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)}\right], \right. \right. \right. \\
& \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left(-1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}}\right)} \sin\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right] \right) \right/ \\
& \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \left. \sqrt{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos\left[x - \operatorname{ArcTan}\left[\frac{c}{b}\right]\right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) -
\end{aligned}$$

$$\begin{aligned}
 & \left. \frac{\frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) \Bigg/ \left( 3c \left( -a^2 + b^2 + c^2 \right)^2 \right) + \\
 & \frac{1}{3 \left( -a^2 + b^2 + c^2 \right)^2} 4acd - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \\
 & \left. \left. \left. - \frac{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1+\frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \Bigg/ \\
 & \left( b \sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) \\
 & \left( \sqrt{a+b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \\
 & \frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}} \right) \\
 & \left. \frac{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right) -
 \end{aligned}$$



$$\begin{aligned}
& \left( a^2 b^2 e \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. - \frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( -1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) / \right. \\
& \quad \left( b\sqrt{1+\frac{c^2}{b^2}} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} - b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right. \\
& \quad \left. \left. \left( \sqrt{a+b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b\sqrt{\frac{b^2+c^2}{b^2}} + b\sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a+b\sqrt{\frac{b^2+c^2}{b^2}}}} \right) - \right. \right. \\
& \quad \left. \left. \frac{2b \left( a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}}} \right) \right) / \left( 3c \left( -a^2 + b^2 + c^2 \right)^2 \right) - \right. \\
& \quad \left. \left. \sqrt{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]} \right) \right) \\
& \frac{1}{c \left( -a^2 + b^2 + c^2 \right)^2} b^4 e \left( - \left( \left( c \operatorname{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a+b\sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \operatorname{ArcTan} \left[ \frac{c}{b} \right] \right]}{b\sqrt{1+\frac{c^2}{b^2}} \left( 1 - \frac{a}{b\sqrt{1+\frac{c^2}{b^2}}} \right)} \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \Bigg/ \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right. \\
 & \left. \sqrt{\frac{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \right. \\
 & \left. - \frac{1}{3 \left( -a^2 + b^2 + c^2 \right)^2} \right) \\
 & \frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \\
 & \frac{1}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} - \frac{1}{3 \left( -a^2 + b^2 + c^2 \right)^2} \\
 & a^2 c e \left( - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) \right) \Bigg/
 \end{aligned}$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right.$$

$$\left. \sqrt{\frac{a + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} + b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right) -$$

$$\frac{2 b \left( \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} - \frac{1}{(-a^2 + b^2 + c^2)^2}$$

$$2 b^2 c e \left( - \left( \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right) \right. \right.$$

$$\left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right) \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) /$$

$$\left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2+c^2}{b^2}} - b \sqrt{\frac{b^2+c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2+c^2}{b^2}}}} \right)$$

$$\begin{aligned}
 & \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) - \\
 & \frac{\frac{2 b \left( a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2 + c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}}}}{\sqrt{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}} - \frac{1}{(-a^2 + b^2 + c^2)^2} \\
 & c^3 e \left( - \left( c \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( 1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right. \right. \right. \\
 & \left. \left. - \frac{a + b \sqrt{1 + \frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1 + \frac{c^2}{b^2}} \left( -1 - \frac{a}{b \sqrt{1 + \frac{c^2}{b^2}}} \right)} \right] \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right) / \\
 & \left( b \sqrt{1 + \frac{c^2}{b^2}} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} - b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) \\
 & \left( \sqrt{a + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]} \sqrt{\frac{b \sqrt{\frac{b^2 + c^2}{b^2}} + b \sqrt{\frac{b^2 + c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{-a + b \sqrt{\frac{b^2 + c^2}{b^2}}}} \right) -
 \end{aligned}$$

$$\frac{\frac{2b \left( a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right] \right)}{b^2+c^2} - \frac{c \sin \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}{b \sqrt{1+\frac{c^2}{b^2}}}}{\sqrt{a+b \sqrt{1+\frac{c^2}{b^2}} \cos \left[ x - \text{ArcTan} \left[ \frac{c}{b} \right] \right]}}$$

**Problem 581: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{a+b \cos [x] \sin [x]} dx$$

Optimal (type 4, 225 leaves, 9 steps):

$$-\frac{i x \operatorname{Log}\left[1-\frac{i b e^{2 i x}}{2 a-\sqrt{4 a^2-b^2}}\right]}{\sqrt{4 a^2-b^2}}+\frac{i x \operatorname{Log}\left[1-\frac{i b e^{2 i x}}{2 a+\sqrt{4 a^2-b^2}}\right]}{\sqrt{4 a^2-b^2}}-\frac{\operatorname{PolyLog}\left[2,\frac{i b e^{2 i x}}{2 a-\sqrt{4 a^2-b^2}}\right]}{2 \sqrt{4 a^2-b^2}}+\frac{\operatorname{PolyLog}\left[2,\frac{i b e^{2 i x}}{2 a+\sqrt{4 a^2-b^2}}\right]}{2 \sqrt{4 a^2-b^2}}$$

Result (type 4, 789 leaves):

$$\begin{aligned}
& \frac{1}{2} \left( \frac{\pi \operatorname{ArcTan}\left[\frac{b+2a \tan[x]}{\sqrt{4a^2-b^2}}\right]}{\sqrt{4a^2-b^2}} + \frac{1}{\sqrt{-4a^2+b^2}} \right. \\
& \left( 2 \operatorname{ArcCos}\left[-\frac{2a}{b}\right] \operatorname{ArcTanh}\left[\frac{(2a-b) \cot\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] + (\pi-4x) \operatorname{ArcTanh}\left[\frac{(2a+b) \tan\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] - \right. \\
& \left. \left( \operatorname{ArcCos}\left[-\frac{2a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{(2a-b) \cot\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] \right) \right. \\
& \left. \operatorname{Log}\left[\frac{(2a+b) \left(-2a+b-i\sqrt{-4a^2+b^2}\right) \left(1+i \cot\left[\frac{\pi}{4}+x\right]\right)}{b \left(2a+b+\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}\right] - \right. \\
& \left( \operatorname{ArcCos}\left[-\frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a-b) \cot\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] \right) \\
& \left. \operatorname{Log}\left[\frac{(2a+b) \left(2ia-ib+\sqrt{-4a^2+b^2}\right) \left(i+\cot\left[\frac{\pi}{4}+x\right]\right)}{b \left(2a+b+\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}\right] + \right. \\
& \left( \operatorname{ArcCos}\left[-\frac{2a}{b}\right] + 2i \left( \operatorname{ArcTanh}\left[\frac{(2a-b) \cot\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] + \operatorname{ArcTanh}\left[\frac{(2a+b) \tan\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] \right) \right) \\
& \left. \operatorname{Log}\left[\frac{(-1)^{1/4} \sqrt{-4a^2+b^2} e^{-ix}}{2\sqrt{b} \sqrt{a+b \cos[x]} \sin[x]}\right] + \right. \\
& \left( \operatorname{ArcCos}\left[-\frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a-b) \cot\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] - 2i \operatorname{ArcTanh}\left[\frac{(2a+b) \tan\left[\frac{\pi}{4}+x\right]}{\sqrt{-4a^2+b^2}}\right] \right) \\
& \left. \operatorname{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right) \sqrt{-4a^2+b^2} e^{ix}}{\sqrt{b} \sqrt{2a+b \sin[2x]}}\right] + \right. \\
& i \left( \operatorname{PolyLog}\left[2, \frac{(2a-i\sqrt{-4a^2+b^2}) \left(2a+b-\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}{b \left(2a+b+\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}\right] - \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(2a+i\sqrt{-4a^2+b^2}) \left(2a+b-\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}{b \left(2a+b+\sqrt{-4a^2+b^2} \cot\left[\frac{\pi}{4}+x\right]\right)}\right] \right) \Bigg)
\end{aligned}$$

**Problem 588: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin[ax]^3}{x (ax \cos[ax] - \sin[ax])^2} dx$$

Optimal (type 4, 56 leaves, 4 steps):

$$\frac{\cos[ax]}{ax} + \frac{\sin[ax]}{a^2 x^2} + \frac{\sin[ax]^2}{a^2 x^2 (ax \cos[ax] - \sin[ax])} + \text{SinIntegral}[ax]$$

Result (type 4, 242 leaves):

$$\frac{1}{2ax \cos[ax] - 2\sin[ax]} \left( (1 + \cos[2ax] + i a e^{ix} \cos[ax] \text{ExpIntegralEi}[-1 - i ax] - i a e^{ix} \cos[ax] \text{ExpIntegralEi}[-1 + i ax] - i e^{\cos[ax]} \cos[ax] (ax \cos[ax] - \sin[ax]) + i e^{\cos[ax]} \cos[ax] (ax \cos[ax] - \sin[ax]) - i e^{\cos[ax]} \cos[ax] \sin[ax] + i e^{\cos[ax]} \cos[ax] \sin[ax] + 2ax \cos[ax] \sin[ax] - 2\sin[ax] \sin[ax] + a e^{ix} \cos[ax] \sin[ax] - e^{\sin[ax]} \sin[ax] - a e^{ix} \cos[ax] \sin[ax] + e^{\sin[ax]} \sin[ax]) \right)$$

**Problem 597: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[ax]^3}{x (\cos[ax] + ax \sin[ax])^2} dx$$

Optimal (type 4, 56 leaves, 4 steps):

$$\frac{\cos[ax]}{a^2 x^2} + \text{CosIntegral}[ax] - \frac{\sin[ax]}{ax} - \frac{\cos[ax]^2}{a^2 x^2 (\cos[ax] + ax \sin[ax])}$$

Result (type 4, 237 leaves):

$$\frac{1}{2 (\cos[ax] + ax \sin[ax])} \left( (-1 + \cos[2ax] - e^{\cos[ax]} \cos[ax] \text{CosIntegral}[i + ax] + e^{\cos[ax]} \cos[ax] \text{ExpIntegralEi}[-1 - i ax] + e^{\cos[ax]} \cos[ax] \text{ExpIntegralEi}[-1 + i ax] - a e^{ix} \cos[ax] \sin[ax] + a e^{ix} \exp[ax] \sin[ax] + 2 \cos[ax] (\cos[ax] + ax \sin[ax]) - e^{\cos[ax]} \cos[ax] (\cos[ax] + ax \sin[ax]) - i e^{\cos[ax]} \cos[ax] \sin[ax] - i a e^{ix} \sin[ax] \sin[ax] - i e^{\cos[ax]} \cos[ax] \sin[ax] - i a e^{ix} \sin[ax] \sin[ax]) \right)$$

**Problem 623: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{c \tan[a + bx]} \tan[2(a + bx)]} dx$$

Optimal (type 3, 100 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c} \tan[2a + 2bx]}{\sqrt{-c + c \sec[2a + 2bx]}}\right]}{b \sqrt{c}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{c} \tan[2a + 2bx]}{\sqrt{2} \sqrt{-c + c \sec[2a + 2bx]}}\right]}{\sqrt{2} b \sqrt{c}}$$

Result (type 6, 170 leaves):

$$\left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] \sin[a+bx]^2 \tan[a+bx] \right) /$$

$$\left( b \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] + \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] - 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \right.$$

$$\left. \cot[a+bx]^2, -\cot[a+bx]^2\right] \tan[a+bx]^2 \right) \sqrt{c \tan[a+bx] \tan[2(a+bx)]}$$

**Problem 624: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[2(a+bx)]}{\sqrt{c \tan[a+bx] \tan[2(a+bx)]}} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{2b\sqrt{c}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{2}\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{\sqrt{2}b\sqrt{c}} + \frac{\sin[2a+2bx]}{2b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}$$

Result (type 6, 226 leaves):

$$\frac{1}{4bc} \left( \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] \cos[2(a+bx)] \tan[a+bx] \right) / \right.$$

$$\left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] + \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] - \right.$$

$$\left. 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2\right] \tan[a+bx]^2 \right) +$$

$$\cot[a+bx] \left( 2 \cos[2(a+bx)] + \operatorname{ArcTan}\left[\sqrt{-1+\tan[a+bx]^2}\right] \sqrt{-1+\tan[a+bx]^2} \right)$$

$$\sqrt{c \tan[a+bx] \tan[2(a+bx)]}$$

**Problem 625: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[2(a+bx)]^2}{\sqrt{c \tan[a+bx] \tan[2(a+bx)]}} dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{8b\sqrt{c}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{2}\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}\right]}{\sqrt{2}b\sqrt{c}} +$$

$$\frac{\sin[2a+2bx]}{8b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}} + \frac{\cos[2a+2bx] \sin[2a+2bx]}{4b\sqrt{-c+c \operatorname{Sec}[2a+2bx]}}$$

Result (type 6, 235 leaves):



$$\left( \left( 42 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] \sin[a+bx]^2 \tan[a+bx] \right) / \right. \\ \left( 2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] + \right. \\ \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] - \right. \\ \left. 3 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] \tan[a+bx]^2 \right) + \\ \left( 2 (1 + \cos[2(a+bx)]) + \cos[4(a+bx)] \right) + \operatorname{ArcTan} \left[ \sqrt{-1 + \tan[a+bx]^2} \right] \sqrt{-1 + \tan[a+bx]^2} \\ \tan[2(a+bx)] \Big) / \left( 16 b \sqrt{c \tan[a+bx] \tan[2(a+bx)]} \right)$$

**Problem 630: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c \tan[a+bx] \tan[2(a+bx)])^{3/2}} dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$-\frac{\operatorname{ArcTanh} \left[ \frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{-c+c \sec[2a+2bx]}} \right]}{b c^{3/2}} + \frac{5 \operatorname{ArcTanh} \left[ \frac{\sqrt{c} \tan[2a+2bx]}{\sqrt{2} \sqrt{-c+c \sec[2a+2bx]}} \right]}{4 \sqrt{2} b c^{3/2}} - \frac{\tan[2a+2bx]}{4 b (-c+c \sec[2a+2bx])^{3/2}}$$

Result (type 6, 226 leaves):

$$\frac{1}{8 b c^2} \left( - \left( \left( 12 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] \cos[2(a+bx)] \tan[a+bx] \right) / \right. \right. \\ \left( 2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] + \right. \\ \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] - \right. \\ \left. 3 \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a+bx]^2, -\cot[a+bx]^2 \right] \tan[a+bx]^2 \right) \Big) - \\ \cot[a+bx] \left( -2 + \csc[a+bx]^2 + \operatorname{ArcTan} \left[ \sqrt{-1 + \tan[a+bx]^2} \right] \sqrt{-1 + \tan[a+bx]^2} \right) \\ \sqrt{c \tan[a+bx] \tan[2(a+bx)]}$$

**Problem 631: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[2(a+bx)]}{(c \tan[a+bx] \tan[2(a+bx)])^{3/2}} dx$$

Optimal (type 3, 178 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2 a+2 b x]}{\sqrt{-c+c \sec [2 a+2 b x]}}\right]}{2 b c^{3/2}} + \frac{9 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2 a+2 b x]}{\sqrt{2} \sqrt{-c+c \sec [2 a+2 b x]}}\right]}{4 \sqrt{2} b c^{3/2}} - \\
& \frac{\sin [2 a+2 b x]}{4 b (-c+c \sec [2 a+2 b x])^{3/2}} - \frac{3 \sin [2 a+2 b x]}{4 b c \sqrt{-c+c \sec [2 a+2 b x]}}
\end{aligned}$$

Result (type 6, 249 leaves):

$$\begin{aligned}
& \frac{1}{8 b c^2} \left( -2 \cot [a+b x] - \cot [a+b x] \csc [a+b x]^2 + 4 \sin [2 (a+b x)] - \right. \\
& 3 \operatorname{ArcTan}\left[\sqrt{-1+\tan [a+b x]^2}\right] \cot [a+b x] \sqrt{-1+\tan [a+b x]^2} - \\
& \left( 18 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot [a+b x]^2, -\cot [a+b x]^2\right] \cos [2 (a+b x)] \tan [a+b x] \right) / \\
& \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot [a+b x]^2, -\cot [a+b x]^2\right] + \right. \\
& \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot [a+b x]^2, -\cot [a+b x]^2\right] - 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \right. \right. \\
& \left. \left. \cot [a+b x]^2, -\cot [a+b x]^2\right] \tan [a+b x]^2 \right) \left. \right) \sqrt{c \tan [a+b x] \tan [2 (a+b x)]}
\end{aligned}$$

**Problem 632: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos [2 (a+b x)]^2}{(c \tan [a+b x] \tan [2 (a+b x)])^{3/2}} dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$\begin{aligned}
& - \frac{19 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2 a+2 b x]}{\sqrt{-c+c \sec [2 a+2 b x]}}\right]}{8 b c^{3/2}} + \frac{13 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \tan[2 a+2 b x]}{\sqrt{2} \sqrt{-c+c \sec [2 a+2 b x]}}\right]}{4 \sqrt{2} b c^{3/2}} - \\
& \frac{\cos [2 a+2 b x] \sin [2 a+2 b x]}{4 b (-c+c \sec [2 a+2 b x])^{3/2}} - \frac{7 \sin [2 a+2 b x]}{8 b c \sqrt{-c+c \sec [2 a+2 b x]}} - \frac{\cos [2 a+2 b x] \sin [2 a+2 b x]}{2 b c \sqrt{-c+c \sec [2 a+2 b x]}}
\end{aligned}$$

Result (type 6, 251 leaves):

$$\begin{aligned} & \left( (-9 \cos[a + b x] + 4 \cos[3(a + b x)] + \cos[5(a + b x)]) \csc[a + b x] - \right. \\ & \left( 114 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a + b x]^2, -\cot[a + b x]^2\right] \sin[a + b x]^2 \tan[a + b x] \right) / \\ & \left( 2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \cot[a + b x]^2, -\cot[a + b x]^2\right] + \right. \\ & \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \cot[a + b x]^2, -\cot[a + b x]^2\right] - \right. \\ & \left. 3 \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \cot[a + b x]^2, -\cot[a + b x]^2\right] \tan[a + b x]^2 \right) - \\ & \left. 7 \operatorname{ArcTan}\left[\sqrt{-1 + \tan[a + b x]^2}\right] \sqrt{-1 + \tan[a + b x]^2} \tan[2(a + b x)] \right) / \\ & \left( 16 b c \sqrt{c \tan[a + b x] \tan[2(a + b x)]} \right) \end{aligned}$$

**Problem 634: Result unnecessarily involves higher level functions.**

$$\int \frac{\csc[x]^2 \sec[x]}{\sqrt{\sin[2x]} (-2 + \tan[x])} dx$$

Optimal (type 3, 69 leaves, 6 steps):

$$\frac{\cos[x]}{2 \sqrt{\sin[2x]}} + \frac{\cos[x] \cot[x]}{3 \sqrt{\sin[2x]}} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{\tan[x]}}{\sqrt{2}}\right] \sin[x]}{2 \sqrt{2} \sqrt{\sin[2x]} \sqrt{\tan[x]}}$$

Result (type 4, 119 leaves):

$$\begin{aligned} & \frac{1}{4} \sqrt{\sin[2x]} \left( \left( 1 + \frac{2 \cot[x]}{3} \right) \csc[x] + 5 \sqrt{\frac{\cos[x]}{-2 + 2 \cos[x]}} \right. \\ & \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[-\frac{2}{-1 + \sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \right. \\ & \left. \left. \operatorname{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \right) \sec[x] \sqrt{\tan\left[\frac{x}{2}\right]} \right) \end{aligned}$$

**Problem 635: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[x]^2 \sin[x]}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}} dx$$

Optimal (type 3, 79 leaves, 6 steps):

$$\frac{\cos[x]^4 \sin[x]}{3 \sin[2x]^{5/2}} + \frac{\cos[x]^3 \sin[x]^2}{2 \sin[2x]^{5/2}} - \frac{5 \operatorname{ArcTanh}\left[\frac{\sqrt{\tan[x]}}{\sqrt{2}}\right] \sin[x]^5}{2 \sqrt{2} \sin[2x]^{5/2} \tan[x]^{5/2}}$$

Result (type 4, 139 leaves):

$$\begin{aligned} & - \left( \left( \csc[x] (2 \cos[x] - \sin[x]) \sqrt{\sin[2x]} \right. \right. \\ & \quad \left. \left( -\frac{1}{3} (3 + 2 \cot[x]) \csc[x] - 5 \sqrt{\frac{\cos[x]}{-2 + 2 \cos[x]}} \left( \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \right. \right. \right. \right. \\ & \quad \left. \left. \operatorname{EllipticPi}\left[-\frac{2}{-1 + \sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] + \operatorname{EllipticPi}\left[\frac{1}{2} (-1 + \sqrt{5}), \right. \right. \right. \\ & \quad \left. \left. \left. -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan\left[\frac{x}{2}\right]}}\right], -1\right] \right) \sec[x] \sqrt{\tan\left[\frac{x}{2}\right]} \right) \right) / (16 (-1 + 2 \cot[x])) \end{aligned}$$

**Problem 636: Result unnecessarily involves higher level functions.**

$$\int \frac{\cos[x]^3 \cos[2x]}{(\sin[x]^2 - \sin[2x]) \sin[2x]^{5/2}} dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$\frac{\cos[x]^5}{5 \sin[2x]^{5/2}} + \frac{\cos[x]^4 \sin[x]}{6 \sin[2x]^{5/2}} - \frac{3 \cos[x]^3 \sin[x]^2}{4 \sin[2x]^{5/2}} + \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{\tan[x]}}{\sqrt{2}}\right] \sin[x]^5}{4 \sqrt{2} \sin[2x]^{5/2} \tan[x]^{5/2}}$$

Result (type 4, 188 leaves):

$$\frac{1}{960} \sec [x] \sqrt{\sin [2 x]} \left( -114 \cot [x] + 20 \cot [x]^2 + 24 \cot [x] \csc [x]^2 - \right.$$

$$45 \sqrt{2} \sqrt{\frac{\cos [x]}{-1 + \cos [x]}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan \left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan \left[\frac{x}{2}\right]} -$$

$$45 \sqrt{2} \sqrt{\frac{\cos [x]}{-1 + \cos [x]}} \operatorname{EllipticPi}\left[-\frac{2}{-1 + \sqrt{5}}, -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan \left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan \left[\frac{x}{2}\right]} -$$

$$45 \sqrt{2} \sqrt{\frac{\cos [x]}{-1 + \cos [x]}} \operatorname{EllipticPi}\left[\frac{1}{2}(-1 + \sqrt{5}), -\operatorname{ArcSin}\left[\frac{1}{\sqrt{\tan \left[\frac{x}{2}\right]}}\right], -1\right] \sqrt{\tan \left[\frac{x}{2}\right]} \left. \right)$$

**Problem 638: Result more than twice size of optimal antiderivative.**

$$\int (b \sec [c + d x] + a \sin [c + d x])^3 (a \cos [c + d x] + b \sec [c + d x] \tan [c + d x]) dx$$

Optimal (type 3, 26 leaves, 1 step):

$$\frac{(b \sec [c + d x] + a \sin [c + d x])^4}{4 d}$$

Result (type 3, 938 leaves):

$$\begin{aligned}
& \left( 8 b^4 \cos[c + d x] (b \sec[c + d x] + a \sin[c + d x])^3 (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) + \\
& \left( a^4 \cos[4 c] \cos[4 d x] \cos[c + d x]^5 (b \sec[c + d x] + a \sin[c + d x])^3 \right. \\
& \quad \left. (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) + \\
& \left( 16 a b^2 \cos[c + d x]^3 \sec[c] (3 a \cos[c] + 2 b \sin[c]) \right. \\
& \quad \left. (b \sec[c + d x] + a \sin[c + d x])^3 (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) - \\
& \left( 4 a^3 \cos[2 d x] \cos[c + d x]^5 (a \cos[2 c] + 4 b \sin[2 c]) \right. \\
& \quad \left. (b \sec[c + d x] + a \sin[c + d x])^3 (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) + \\
& \left( 32 a b^3 \cos[c + d x]^2 \sec[c] \sin[d x] (b \sec[c + d x] + a \sin[c + d x])^3 \right. \\
& \quad \left. (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) + \\
& \left( 32 a^3 b \cos[c + d x]^4 \sec[c] \sin[d x] (b \sec[c + d x] + a \sin[c + d x])^3 \right. \\
& \quad \left. (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) + \\
& \left( 4 a^3 \cos[c + d x]^5 (-4 b \cos[2 c] + a \sin[2 c]) \sin[2 d x] \right. \\
& \quad \left. (b \sec[c + d x] + a \sin[c + d x])^3 (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right) - \\
& \left( a^4 \cos[c + d x]^5 \sin[4 c] \sin[4 d x] (b \sec[c + d x] + a \sin[c + d x])^3 \right. \\
& \quad \left. (a \cos[c + d x] + b \sec[c + d x] \tan[c + d x]) \right) / \\
& \left( d (3 a \cos[c + d x] + a \cos[3 c + 3 d x] + 4 b \sin[c + d x]) (2 b + a \sin[2 c + 2 d x])^3 \right)
\end{aligned}$$

**Problem 654: Result more than twice size of optimal antiderivative.**

$$\int \cos[x]^3 (a + b \cos[x]^2)^3 \sin[x] \, dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$\frac{a (a + b \cos[x]^2)^4}{8 b^2} - \frac{(a + b \cos[x]^2)^5}{10 b^2}$$

Result (type 3, 137 leaves):

$$\frac{1}{32} \left( -12 a^2 b \cos[x]^4 - 8 a b^2 \cos[x]^6 - 2 b^3 \cos[x]^8 - 4 a^3 \cos[2x] - 4 a^2 b \cos[x]^3 \cos[3x] - \right. \\ \left. a^3 \cos[4x] - \frac{1}{32} a b^2 (48 \cos[2x] + 36 \cos[4x] + 16 \cos[6x] + 3 \cos[8x]) - \right. \\ \left. \frac{1}{320} b^3 (140 \cos[2x] + 100 \cos[4x] + 50 \cos[6x] + 15 \cos[8x] + 2 \cos[10x]) \right)$$

**Problem 657: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]^2 \sin[x]}{\sqrt{1 - \cos[x]^6}} dx$$

Optimal (type 3, 9 leaves, 3 steps):

$$-\frac{1}{3} \text{ArcSin}[\cos[x]^3]$$

Result (type 4, 162 leaves):

$$-\left( \left( i \cos[x]^2 \text{EllipticPi}\left[\frac{3}{2} + \frac{i\sqrt{3}}{2}, i \text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{3}}} \tan[x]\right], \frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right] \sin[x] \right. \right. \\ \left. \left. \sqrt{1 - \frac{2i \tan[x]^2}{-3i+\sqrt{3}}} \sqrt{1 + \frac{2i \tan[x]^2}{3i+\sqrt{3}}} \right) / \left( \sqrt{2} \sqrt{-\frac{i}{-3i+\sqrt{3}}} \sqrt{1 - \cos[x]^6} \right) \right)$$

**Problem 670: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \cos[x] \sqrt{1 + \csc[x]} dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\text{ArcTanh}[\sqrt{1 + \csc[x]}] + \sqrt{1 + \csc[x]} \sin[x]$$

Result (type 6, 5067 leaves):

$$\sin[x] \sqrt{\csc[x] (1 + \sin[x])} - \\ \left( 4 \cos\left[\frac{x}{4}\right]^3 \sqrt{1 + \csc[x]} \sin\left[\frac{x}{4}\right] \left( \frac{\cos\left[\frac{x}{2}\right] \sqrt{1 + \csc[x]}}{2 (\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right])} - \frac{\sqrt{1 + \csc[x]} \sin\left[\frac{x}{2}\right]}{2 (\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right])} \right) \right. \\ \left( - \left( \left( 75 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) / \right. \right. \\ \left( 5 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + 2 \left( -2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, \right. \right. \right. \\ \left. \left. \left. -\tan\left[\frac{x}{4}\right]^2\right] + \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right) \right) \right) +$$





[illegible]

$$\begin{aligned}
& -\tan\left[\frac{x}{4}\right]^2 + \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \tan\left[\frac{x}{4}\right]^2 \Big) \tan\left[\frac{x}{4}\right]^2 \Big) + \\
& \left( 27 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \tan\left[\frac{x}{4}\right] \right) / \\
& \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& 2 \left( -2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right) \right) \Big) - \\
& \frac{1}{15 \left( \cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right] \right)} 4 \cos\left[\frac{x}{4}\right]^3 \sqrt{1 + \csc[x]} \sin\left[\frac{x}{4}\right] \\
& \left( - \left( \left( 75 \left( -\frac{1}{10} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \right. \right. \right. \right. \\
& \left. \left. \left. \frac{1}{20} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \right) \right) / \right. \\
& \left( 5 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + 2 \right. \\
& \left( -2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right) \right) + \\
& \frac{1}{4} \sec\left[\frac{x}{4}\right]^2 \left( \left( 70 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) / \left( 7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + 2 \left( -2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right) + \right. \\
& \left. \left( 27 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \tan\left[\frac{x}{4}\right] \right) / \right. \\
& \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \\
& 2 \left( -2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right) \Big) + \left( 75 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \left( \left( -2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + 5 \right. \right. \right. \\
& \left( -\frac{1}{10} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \right. \\
& \left. \left. \frac{1}{20} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \right) + 2 \right. \\
& \left. \tan\left[\frac{x}{4}\right]^2 \left( -\frac{5}{18} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \right. \right.
\end{aligned}$$

[illegible]

$$\begin{aligned}
& 2 \tan\left[\frac{x}{4}\right]^2 \left( -\frac{7}{22} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 2, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \right. \\
& \quad \tan\left[\frac{x}{4}\right] + \frac{21}{44} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{5}{2}, 1, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \\
& \quad \tan\left[\frac{x}{4}\right] - 2 \left( -\frac{7}{11} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 3, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right. \\
& \quad \quad \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{7}{44} \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 2, \frac{15}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \\
& \quad \quad \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \left. \right) \Bigg/ \left( 7 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \tan\left[\frac{x}{4}\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{x}{4}\right]^2\right] + 2 \left( -2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \right. \\
& \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right)^2 - \right. \\
& \quad \left( 27 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \tan\left[\frac{x}{4}\right] \right. \\
& \quad \left( \left( -2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \right. \\
& \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \right. \\
& \quad 9 \left( -\frac{5}{18} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \right. \\
& \quad \quad \left. \frac{5}{36} \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) + \\
& \quad 2 \tan\left[\frac{x}{4}\right]^2 \left( -\frac{9}{26} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{3}{2}, 2, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \right. \\
& \quad \tan\left[\frac{x}{4}\right] + \frac{27}{52} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{5}{2}, 1, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \sec\left[\frac{x}{4}\right]^2 \\
& \quad \tan\left[\frac{x}{4}\right] - 2 \left( -\frac{9}{13} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{1}{2}, 3, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right. \\
& \quad \quad \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] + \frac{9}{52} \operatorname{AppellF1}\left[\frac{13}{4}, \frac{3}{2}, 2, \frac{17}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \\
& \quad \quad \sec\left[\frac{x}{4}\right]^2 \tan\left[\frac{x}{4}\right] \right) \left. \right) \Bigg/ \left( 9 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \tan\left[\frac{x}{4}\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{x}{4}\right]^2\right] + 2 \left( -2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] + \right. \right. \\
& \quad \quad \left. \left. \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \tan\left[\frac{x}{4}\right]^2, -\tan\left[\frac{x}{4}\right]^2\right] \right) \tan\left[\frac{x}{4}\right]^2 \right)^2 \right) \Bigg) \Bigg)
\end{aligned}$$

**Problem 673: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]}{\sqrt{2 \sin[x] + \sin[x]^2}} dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$2 \operatorname{ArcTanh}\left[\frac{\sin[x]}{\sqrt{2 \sin[x] + \sin[x]^2}}\right]$$

Result (type 3, 40 leaves):

$$\frac{2 \operatorname{ArcSinh}\left[\frac{\sqrt{\sin[x]}}{\sqrt{2}}\right] \sqrt{\sin[x]} \sqrt{2 + \sin[x]}}{\sqrt{\sin[x] (2 + \sin[x])}}$$

**Problem 676: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \sec[\sin[x]] dx$$

Optimal (type 3, 4 leaves, 2 steps):

$$\operatorname{ArcTanh}[\sin[\sin[x]]]$$

Result (type 3, 37 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{\sin[x]}{2}\right] - \sin\left[\frac{\sin[x]}{2}\right]\right] + \operatorname{Log}\left[\cos\left[\frac{\sin[x]}{2}\right] + \sin\left[\frac{\sin[x]}{2}\right]\right]$$

**Problem 677: Result more than twice size of optimal antiderivative.**

$$\int \cos[x] \sin[x]^3 (a + b \sin[x]^2)^3 dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{a (a + b \sin[x]^2)^4}{8 b^2} + \frac{(a + b \sin[x]^2)^5}{10 b^2}$$

Result (type 3, 128 leaves):

$$\frac{1}{10240} \left( -20 (64 a^3 + 24 a b^2 + 7 b^3) \cos[2x] + 20 (16 a^3 + 18 a b^2 + 5 b^3) \cos[4x] + \right. \\ \left. b (-10 b (16 a + 5 b) \cos[6x] + 15 b (2 a + b) \cos[8x] - 2 b^2 \cos[10x] + \right. \\ \left. 3840 a^2 \sin[x]^4 + 2560 a b \sin[x]^6 + 640 b^2 \sin[x]^8 - 1280 a^2 \sin[x]^3 \sin[3x] \right)$$

**Problem 691: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^2}{1 - \tan[x]^2} dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTanh}[2 \cos[x] \sin[x]]$$

Result (type 3, 23 leaves):

$$-\frac{1}{2} \operatorname{Log}[\cos[x] - \sin[x]] + \frac{1}{2} \operatorname{Log}[\cos[x] + \sin[x]]$$

**Problem 705: Result more than twice size of optimal antiderivative.**

$$\int \sec[x]^2 \tan[x]^6 (1 + \tan[x]^2)^3 dx$$

Optimal (type 3, 33 leaves, 4 steps):

$$\frac{\tan[x]^7}{7} + \frac{\tan[x]^9}{3} + \frac{3 \tan[x]^{11}}{11} + \frac{\tan[x]^{13}}{13}$$

Result (type 3, 67 leaves):

$$\begin{aligned} & -\frac{16 \tan[x]}{3003} - \frac{8 \sec[x]^2 \tan[x]}{3003} - \frac{2 \sec[x]^4 \tan[x]}{1001} - \frac{5 \sec[x]^6 \tan[x]}{3003} + \\ & \frac{53}{429} \sec[x]^8 \tan[x] - \frac{27}{143} \sec[x]^{10} \tan[x] + \frac{1}{13} \sec[x]^{12} \tan[x] \end{aligned}$$

**Problem 709: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^2}{\sqrt{4 - \sec[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\operatorname{ArcSin}\left[\frac{\tan[x]}{\sqrt{3}}\right]$$

Result (type 3, 43 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{\sin[x]}{\sqrt{1+2\cos[2x]}}\right] \sqrt{1+2\cos[2x]} \sec[x]}{\sqrt{4 - \sec[x]^2}}$$

**Problem 710: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sec[x]^2}{\sqrt{1 - 4 \tan[x]^2}} dx$$

Optimal (type 3, 9 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcSin}[2 \tan[x]]$$

Result (type 3, 52 leaves):

$$\frac{\operatorname{ArcTan}\left[\frac{2\sqrt{2}\sin[x]}{\sqrt{-3+5\cos[2x]}}\right] \sqrt{-3+5\cos[2x]} \sec[x]}{2\sqrt{2-8\tan[x]^2}}$$

Problem 711: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec [x]^2}{\sqrt{-4+\tan [x]^2}} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\text{ArcTanh}\left[\frac{\tan [x]}{\sqrt{-4+\tan [x]^2}}\right]$$

Result (type 3, 51 leaves):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2} \sin [x]}{\sqrt{3+5 \cos [2 x]}}\right] \sqrt{3+5 \cos [2 x]} \sec [x]}{\sqrt{2} \sqrt{-4+\tan [x]^2}}$$

Problem 712: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1-\cot [x]^2} \sec [x]^2 dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$\text{ArcSin}[\cot [x]] + \sqrt{1-\cot [x]^2} \tan [x]$$

Result (type 3, 52 leaves):

$$\left(-\text{ArcTan}\left[\frac{\cos [x]}{\sqrt{-\cos [2 x]}}\right] \cos [x] \sqrt{-\cos [2 x]} + \cos [2 x]\right) \sqrt{1-\cot [x]^2} \sec [2 x] \tan [x]$$

Problem 728: Result more than twice size of optimal antiderivative.

$$\int \frac{\sec [x] \tan [x]}{\sqrt{4+\sec [x]^2}} dx$$

Optimal (type 3, 5 leaves, 3 steps):

$$\text{ArcCsch}[2 \cos [x]]$$

Result (type 3, 38 leaves):

$$\frac{\text{ArcTanh}\left[\sqrt{3+2 \cos [2 x]}\right] \sqrt{3+2 \cos [2 x]} \sec [x]}{\sqrt{4+\sec [x]^2}}$$

Problem 738: Result more than twice size of optimal antiderivative.

$$\int \frac{\cot [6 x] \csc [6 x]}{(5-11 \csc [6 x]^2)^2} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$-\frac{\text{ArcTanh}\left[\sqrt{\frac{5}{11}} \sin[6x]\right]}{60\sqrt{55}} + \frac{\sin[6x]}{60(11 - 5\sin[6x]^2)}$$

Result (type 3, 97 leaves):

$$\frac{\left(17\sqrt{55}\left(\log[\sqrt{55} - 5\sin[6x]] - \log[\sqrt{55} + 5\sin[6x]]\right) + 5\sqrt{55}\cos[12x]\right.}{\left.\left(\log[\sqrt{55} - 5\sin[6x]] - \log[\sqrt{55} + 5\sin[6x]]\right) + 220\sin[6x]\right)} \bigg/ (6600(17 + 5\cos[12x]))$$

**Problem 759: Result more than twice size of optimal antiderivative.**

$$\int (\cos[x]^{12} \sin[x]^{10} - \cos[x]^{10} \sin[x]^{12}) dx$$

Optimal (type 3, 12 leaves, ? steps):

$$\frac{1}{11} \cos[x]^{11} \sin[x]^{11}$$

Result (type 3, 49 leaves):

$$\frac{21 \sin[2x]}{1048576} - \frac{15 \sin[6x]}{1048576} + \frac{15 \sin[10x]}{2097152} - \frac{5 \sin[14x]}{2097152} + \frac{\sin[18x]}{2097152} - \frac{\sin[22x]}{23068672}$$

**Problem 779: Result more than twice size of optimal antiderivative.**

$$\int 3x^2 \cos[7 + x^3] dx$$

Optimal (type 3, 6 leaves, 3 steps):

$$\sin[7 + x^3]$$

Result (type 3, 23 leaves):

$$3 \left( \frac{1}{3} \cos[x^3] \sin[7] + \frac{1}{3} \cos[7] \sin[x^3] \right)$$

**Problem 781: Result more than twice size of optimal antiderivative.**

$$\int x \sin[1 + x^2] dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{1}{2} \cos[1 + x^2]$$

Result (type 3, 21 leaves):

$$-\frac{1}{2} \cos[1] \cos[x^2] + \frac{1}{2} \sin[1] \sin[x^2]$$



**Problem 782: Result more than twice size of optimal antiderivative.**

$$\int x \cos[1 + x^2] \, dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$\frac{1}{2} \sin[1 + x^2]$$

Result (type 3, 21 leaves):

$$\frac{1}{2} \cos[x^2] \sin[1] + \frac{1}{2} \cos[1] \sin[x^2]$$

**Problem 784: Result more than twice size of optimal antiderivative.**

$$\int x^2 \sin[1 + x^3] \, dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{1}{3} \cos[1 + x^3]$$

Result (type 3, 21 leaves):

$$-\frac{1}{3} \cos[1] \cos[x^3] + \frac{1}{3} \sin[1] \sin[x^3]$$

**Problem 802: Result more than twice size of optimal antiderivative.**

$$\int \sec[x] (1 - \sin[x]) \, dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\log[1 + \sin[x]]$$

Result (type 3, 36 leaves):

$$\log[\cos[x]] - \log\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \log\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]$$

**Problem 803: Result more than twice size of optimal antiderivative.**

$$\int (1 + \cos[x]) \csc[x] \, dx$$

Optimal (type 3, 7 leaves, 2 steps):

$$\log[1 - \cos[x]]$$

Result (type 3, 20 leaves):

$$-\operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] + \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}[\sin[x]]$$

**Problem 805: Result more than twice size of optimal antiderivative.**

$$\int \csc[2x] (\cos[x] + \sin[x]) \, dx$$

Optimal (type 3, 15 leaves, 6 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\cos[x]] + \frac{1}{2} \operatorname{ArcTanh}[\sin[x]]$$

Result (type 3, 61 leaves):

$$-\frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right]\right] - \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\sin\left[\frac{x}{2}\right]\right] + \frac{1}{2} \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]\right]$$

**Problem 806: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x] (-3 + 2 \sin[x])}{2 - 3 \sin[x] + \sin[x]^2} \, dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\operatorname{Log}[2 - 3 \sin[x] + \sin[x]^2]$$

Result (type 3, 26 leaves):

$$2 \operatorname{Log}\left[\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]\right] + \operatorname{Log}[2 - \sin[x]]$$

**Problem 807: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[x]^2 \sin[x]}{5 + \cos[x]^2} \, dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$\sqrt{5} \operatorname{ArcTan}\left[\frac{\cos[x]}{\sqrt{5}}\right] - \cos[x]$$

Result (type 3, 82 leaves):

$$\frac{1}{20} \left( -\sqrt{5} \operatorname{ArcTan}\left[\frac{\cos[x]}{\sqrt{5}}\right] + 21 \sqrt{5} \operatorname{ArcTan}\left[\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}} \tan\left[\frac{x}{2}\right]\right] + 21 \sqrt{5} \operatorname{ArcTan}\left[\frac{1}{\sqrt{5}} + \sqrt{\frac{6}{5}} \tan\left[\frac{x}{2}\right]\right] - 20 \cos[x] \right)$$

### Problem 825: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{Sec}[5 - x^2] \, dx$$

Optimal (type 3, 13 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{ArcTanh}[\operatorname{Sin}[5 - x^2]]$$

Result (type 3, 63 leaves):

$$\frac{1}{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{5}{2} - \frac{x^2}{2}\right] - \operatorname{Sin}\left[\frac{5}{2} - \frac{x^2}{2}\right]\right] - \frac{1}{2} \operatorname{Log}\left[\operatorname{Cos}\left[\frac{5}{2} - \frac{x^2}{2}\right] + \operatorname{Sin}\left[\frac{5}{2} - \frac{x^2}{2}\right]\right]$$

### Problem 826: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}\left[\frac{1}{x}\right]}{x^2} \, dx$$

Optimal (type 3, 5 leaves, 2 steps):

$$\operatorname{ArcTanh}\left[\operatorname{Cos}\left[\frac{1}{x}\right]\right]$$

Result (type 3, 21 leaves):

$$\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2x}\right]\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\frac{1}{2x}\right]\right]$$

### Problem 834: Result more than twice size of optimal antiderivative.

$$\int 35 \operatorname{Cos}[x]^3 \operatorname{Sin}[x]^4 \, dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$7 \operatorname{Sin}[x]^5 - 5 \operatorname{Sin}[x]^7$$

Result (type 3, 33 leaves):

$$35 \left( \frac{3 \operatorname{Sin}[x]}{64} - \frac{1}{64} \operatorname{Sin}[3x] - \frac{1}{320} \operatorname{Sin}[5x] + \frac{1}{448} \operatorname{Sin}[7x] \right)$$

### Problem 850: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Tan}[c + dx]}{\sqrt{a \operatorname{Sin}[c + dx]^2}} \, dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a \sin[c+dx]^2}}{\sqrt{a}}\right]}{\sqrt{a} d}$$

Result (type 3, 73 leaves):

$$\left( \left( -\text{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] - \sin\left[\frac{1}{2}(c+dx)\right]\right] + \text{Log}\left[\cos\left[\frac{1}{2}(c+dx)\right] + \sin\left[\frac{1}{2}(c+dx)\right]\right] \right) \sin[c+dx] \right) / \left( d \sqrt{a \sin[c+dx]^2} \right)$$

**Problem 861: Result more than twice size of optimal antiderivative.**

$$\int \sec[x] \sqrt{\sec[x] + \tan[x]} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$2 \sqrt{\sec[x] (1 + \sin[x])}$$

Result (type 3, 37 leaves):

$$2 \sqrt{\frac{\cos\left[\frac{x}{2}\right] + \sin\left[\frac{x}{2}\right]}{\cos\left[\frac{x}{2}\right] - \sin\left[\frac{x}{2}\right]}}$$

**Problem 885: Result more than twice size of optimal antiderivative.**

$$\int (-\cos[x] + \sin[x]) (\cos[x] + \sin[x])^5 dx$$

Optimal (type 3, 11 leaves, 1 step):

$$-\frac{1}{6} (\cos[x] + \sin[x])^6$$

Result (type 3, 25 leaves):

$$\frac{1}{4} \cos[4x] - \frac{5}{8} \sin[2x] + \frac{1}{24} \sin[6x]$$

**Problem 894: Result more than twice size of optimal antiderivative.**

$$\int \sin[x] \tan[x]^5 dx$$

Optimal (type 3, 34 leaves, 5 steps):

$$\frac{15}{8} \text{ArcTanh}[\sin[x]] - \frac{15 \sin[x]}{8} - \frac{5}{8} \sin[x] \tan[x]^2 + \frac{1}{4} \sin[x] \tan[x]^4$$

Result (type 3, 113 leaves):

$$\frac{1}{16} \left( -30 \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right] + 30 \operatorname{Log} \left[ \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right] + \frac{1}{\left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right)^4} - \frac{9}{\left( \cos \left[ \frac{x}{2} \right] - \sin \left[ \frac{x}{2} \right] \right)^2} - \frac{1}{\left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right)^4} + \frac{9}{\left( \cos \left[ \frac{x}{2} \right] + \sin \left[ \frac{x}{2} \right] \right)^2} - 16 \sin [x] \right)$$

**Problem 904: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{Sec} [1+x] \operatorname{Tan} [1+x] dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$-\operatorname{ArcTanh} [\sin [1+x]] + x \operatorname{Sec} [1+x]$$

Result (type 3, 47 leaves):

$$\operatorname{Log} \left[ \cos \left[ \frac{1+x}{2} \right] - \sin \left[ \frac{1+x}{2} \right] \right] - \operatorname{Log} \left[ \cos \left[ \frac{1+x}{2} \right] + \sin \left[ \frac{1+x}{2} \right] \right] + x \operatorname{Sec} [1+x]$$

**Problem 906: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sin [2x]}{\sqrt{9 - \cos [x]^4}} dx$$

Optimal (type 3, 11 leaves, 5 steps):

$$-\operatorname{ArcSin} \left[ \frac{\cos [x]^2}{3} \right]$$

Result (type 3, 26 leaves):

$$\frac{i}{2} \operatorname{Log} \left[ i \cos [x]^2 + \sqrt{9 - \cos [x]^4} \right]$$

**Problem 910: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{-1 + \operatorname{Sec} [x]}{1 - \operatorname{Tan} [x]} dx$$

Optimal (type 3, 37 leaves, 6 steps):

$$-\frac{x}{2} + \frac{\operatorname{ArcTanh} \left[ \frac{\cos [x] (1 + \operatorname{Tan} [x])}{\sqrt{2}} \right]}{\sqrt{2}} + \frac{1}{2} \operatorname{Log} [\cos [x] - \sin [x]]$$

Result (type 3, 40 leaves):

$$\frac{1}{2} \left( -x + (2 - 2i) (-1)^{1/4} \operatorname{ArcTanh} \left[ \frac{1 + \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{2}} \right] + \operatorname{Log} [\cos [x] - \sin [x]] \right)$$

### Problem 912: Result unnecessarily involves higher level functions.

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x]}} dx$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

Result (type 5, 68 leaves):

$$-\frac{1}{3 (\sin[x]^2)^{3/4}} 2 \sqrt{\cos[x]} \sqrt{\sin[x]} \left( 3 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \cos[x]^2\right] \sin[x] + \right. \\ \left. \cos[x] \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \cos[x]^2\right] \sqrt{\sin[x]^2} \right)$$

### Problem 927: Result more than twice size of optimal antiderivative.

$$\int x^5 \sec[a + b x^3]^7 \tan[a + b x^3] dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$-\frac{5 \operatorname{ArcTanh}[\sin[a + b x^3]]}{336 b^2} + \frac{x^3 \sec[a + b x^3]^7}{21 b} - \frac{5 \sec[a + b x^3] \tan[a + b x^3]}{336 b^2} - \\ \frac{5 \sec[a + b x^3]^3 \tan[a + b x^3]}{504 b^2} - \frac{\sec[a + b x^3]^5 \tan[a + b x^3]}{126 b^2}$$

Result (type 3, 352 leaves):

$$\frac{1}{64512 b^2} \sec[a + b x^3]^7 \left( 3072 b x^3 + 105 \cos[5(a + b x^3)] \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] - \sin\left[\frac{1}{2}(a + b x^3)\right]\right] + \right. \\ 15 \cos[7(a + b x^3)] \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] - \sin\left[\frac{1}{2}(a + b x^3)\right]\right] + \\ 525 \cos[a + b x^3] \left( \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] - \sin\left[\frac{1}{2}(a + b x^3)\right]\right] - \right. \\ \left. \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] + \sin\left[\frac{1}{2}(a + b x^3)\right]\right] \right) + 315 \cos[3(a + b x^3)] \\ \left( \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] - \sin\left[\frac{1}{2}(a + b x^3)\right]\right] - \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] + \sin\left[\frac{1}{2}(a + b x^3)\right]\right] \right) - \\ 105 \cos[5(a + b x^3)] \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] + \sin\left[\frac{1}{2}(a + b x^3)\right]\right] - \\ 15 \cos[7(a + b x^3)] \log\left[\cos\left[\frac{1}{2}(a + b x^3)\right] + \sin\left[\frac{1}{2}(a + b x^3)\right]\right] - \\ \left. 566 \sin[2(a + b x^3)] - 200 \sin[4(a + b x^3)] - 30 \sin[6(a + b x^3)] \right)$$

**Problem 943: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\cos[a+bx]^4 - \sin[a+bx]^4}{\cos[a+bx]^4 + \sin[a+bx]^4} dx$$

Optimal (type 3, 72 leaves, 4 steps):

$$-\frac{\operatorname{Log}\left[1 - \sqrt{2} \tan[a+bx] + \tan[a+bx]^2\right]}{2\sqrt{2}b} + \frac{\operatorname{Log}\left[1 + \sqrt{2} \tan[a+bx] + \tan[a+bx]^2\right]}{2\sqrt{2}b}$$

Result (type 3, 102 leaves):

$$-\frac{1}{4\left(5i + \sqrt{2}\right)b} \\ i\left(-2i + 5\sqrt{2}\right)\left(\operatorname{Log}\left[-1 - 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}\right] - \operatorname{Log}\left[-1 + 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}\right]\right)$$

**Problem 945: Result more than twice size of optimal antiderivative.**

$$\int \frac{\cos[a+bx]^2 - \sin[a+bx]^2}{\cos[a+bx]^2 + \sin[a+bx]^2} dx$$

Optimal (type 3, 16 leaves, 6 steps):

$$\frac{\cos[a+bx] \sin[a+bx]}{b}$$

Result (type 3, 33 leaves):

$$\frac{\cos[2bx] \sin[2a]}{2b} + \frac{\cos[2a] \sin[2bx]}{2b}$$

**Problem 950: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{-\csc[a+bx]^4 + \sec[a+bx]^4}{\csc[a+bx]^4 + \sec[a+bx]^4} dx$$

Optimal (type 3, 72 leaves, 4 steps):

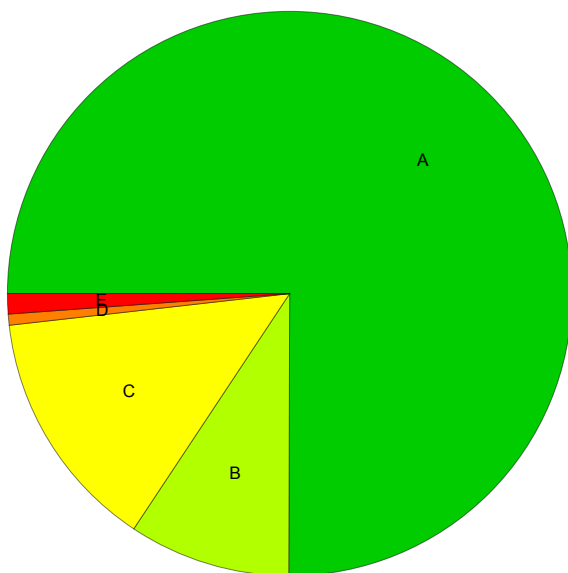
$$\frac{\operatorname{Log}\left[1 - \sqrt{2} \tan[a+bx] + \tan[a+bx]^2\right]}{2\sqrt{2}b} - \frac{\operatorname{Log}\left[1 + \sqrt{2} \tan[a+bx] + \tan[a+bx]^2\right]}{2\sqrt{2}b}$$

Result (type 3, 102 leaves):

$$\frac{1}{4\left(5i + \sqrt{2}\right)b} \\ i\left(-2i + 5\sqrt{2}\right)\left(\operatorname{Log}\left[-1 - 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}\right] - \operatorname{Log}\left[-1 + 2i\sqrt{2}e^{2i(a+bx)} + e^{4i(a+bx)}\right]\right)$$

## Summary of Integration Test Results

950 integration problems



A - 713 optimal antiderivatives

B - 88 more than twice size of optimal antiderivatives

C - 132 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 11 integration timeouts