Rules for integrands of the form $(d + e\,x)^m \, \mathrm{Tanh}\left[a + b\,x + c\,x^2\right]^n$

1: $\int \mathrm{Tanh}\left[a + b\,x + c\,x^2\right]^n \, dx$

- **Rule:**

$$\int \mathrm{Tanh}\left[a + b\,x + c\,x^2\right]^n \, dx \;\longrightarrow\; \int \mathrm{Tanh}\left[a + b\,x + c\,x^2\right]^n \, dx$$

- **Program code:**

```
Int[Tanh[a_.+b_.*x_+c_.*x_^2]^n_.,x_Symbol] :=
  Integral[Tanh[a+b*x+c*x^2]^n,x] /;
FreeQ[{a,b,c,n},x]
```

```
Int[Coth[a_.+b_.*x_+c_.*x_^2]^n_.,x_Symbol] :=
  Integral[Coth[a+b*x+c*x^2]^n,x] /;
FreeQ[{a,b,c,n},x]
```

2. $\int (d + e\,x)^m \, \mathrm{Tanh}\left[a + b\,x + c\,x^2\right]^n \, dx$

1: $\int (d + e\,x) \, \mathrm{Tanh}\left[a + b\,x + c\,x^2\right] \, dx$

- **Rule:**

$$\int (d + e\,x) \, \mathrm{Tanh}\left[a + b\,x + c\,x^2\right] \, dx \;\longrightarrow\; \frac{e \, \mathrm{Log}\left[\mathrm{Cosh}\left[a + b\,x + c\,x^2\right]\right]}{2\,c} + \frac{2\,c\,d - b\,e}{2\,c} \int \mathrm{Tanh}\left[a + b\,x + c\,x^2\right] \, dx$$

- **Program code:**

```
Int[(d_.+e_.*x_)*Tanh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Log[Cosh[a+b*x+c*x^2]]/(2*c) +
  (2*c*d-b*e)/(2*c)*Int[Tanh[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x]
```

```
Int[(d_.+e_.*x_)*Coth[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  e*Log[Sinh[a+b*x+c*x^2]]/(2*c) +
  (2*c*d-b*e)/(2*c)*Int[Coth[a+b*x+c*x^2],x] /;
FreeQ[{a,b,c,d,e},x]
```

x: $\int (d + e x)^m \, \text{Tanh}\left[a + b x + c x^2\right] dx$ when $m > 1$

- Note: **This rule is valid, but to be useful need a rule for reducing integrands of the form** $x^m \, \text{Log}\left[\text{Cosh}\left[a + b x + c x^2\right]\right].$

- Rule: **If $m > 1$, then**

$$\int x^m \, \text{Tanh}\left[a + b x + c x^2\right] dx \longrightarrow$$

$$\frac{x^{m-1} \, \text{Log}\left[\text{Cosh}\left[a + b x + c x^2\right]\right]}{2 c} - \frac{b}{2 c} \int x^{m-1} \, \text{Tanh}\left[a + b x + c x^2\right] dx - \frac{m - 1}{2 c} \int x^{m-2} \, \text{Log}\left[\text{Cosh}\left[a + b x + c x^2\right]\right] dx$$

- Program code:

```
(* Int[x_^m_*Tanh[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  x^(m-1)*Log[Cosh[a+b*x+c*x^2]]/(2*c) -
  b/(2*c)*Int[x^(m-1)*Tanh[a+b*x+c*x^2],x] -
  (m-1)/(2*c)*Int[x^(m-2)*Log[Cosh[a+b*x+c*x^2]],x] /;
FreeQ[{a,b,c},x] && GtQ[m,1] *)
```

```
(* Int[x_^m_*Coth[a_.+b_.*x_+c_.*x_^2],x_Symbol] :=
  x^(m-1)*Log[Sinh[a+b*x+c*x^2]]/(2*c) -
  b/(2*c)*Int[x^(m-1)*Coth[a+b*x+c*x^2],x] -
  (m-1)/(2*c)*Int[x^(m-2)*Log[Sinh[a+b*x+c*x^2]],x] /;
FreeQ[{a,b,c},x] && GtQ[m,1] *)
```

2: $\int (d + e x)^m \, \text{Tanh}\left[a + b x + c x^2\right]^n dx$

- Rule:

$$\int (d + e x)^m \, \text{Tanh}\left[a + b x + c x^2\right]^n dx \longrightarrow \int (d + e x)^m \, \text{Tanh}\left[a + b x + c x^2\right]^n dx$$

- Program code:

```
Int[(d_.+e_.*x_)^m_.*Tanh[a_.+b_.*x_+c_.*x_^2]^n_.,x_Symbol] :=
  Integral[(d+e*x)^m*Tanh[a+b*x+c*x^2]^n,x] /;
FreeQ[{a,b,c,d,e,m,n},x]
```

```
Int[(d_.+e_.*x_)^m_.*Coth[a_.+b_.*x_+c_.*x_^2]^n_.,x_Symbol] :=
  Integral[(d+e*x)^m*Coth[a+b*x+c*x^2]^n,x] /;
FreeQ[{a,b,c,d,e,m,n},x]
```