# Mathematica 11.3 Integration Test Results

# Test results for the 294 problems in "4.7.2 trig^m (a trig+b trig)^n.m"

Problem 6: Result more than twice size of optimal antiderivative.

$$\int Csc[x]^{2} (a Cos[x] + b Sin[x]) dx$$

$$Optimal (type 3, 12 leaves, 5 steps): -b ArcTanh[Cos[x]] - a Csc[x]$$

$$Result (type 3, 25 leaves): -a Csc[x] - b Log[Cos[\frac{x}{2}]] + b Log[Sin[\frac{x}{2}]]$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[x]^3}{a\cos[x] + b\sin[x]} \, \mathrm{d}x$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{a^2 \ b \ x}{\left(a^2 + b^2\right)^2} + \frac{b \ x}{2 \ \left(a^2 + b^2\right)} - \frac{a^3 \ Log \left[a \ Cos \left[x\right] + b \ Sin \left[x\right]\right]}{\left(a^2 + b^2\right)^2} - \frac{b \ Cos \left[x\right] \ Sin \left[x\right]}{2 \ \left(a^2 + b^2\right)} - \frac{a \ Sin \left[x\right]^2}{2 \ \left(a^2 + b^2\right)}$$

Result (type 3, 94 leaves):

$$\begin{split} &\frac{1}{4\,\left(a^2+b^2\right)^2}\left(-4\,\dot{\mathbb{1}}\,\,a^3\,x+6\,a^2\,b\,\,x+2\,b^3\,x+4\,\dot{\mathbb{1}}\,\,a^3\,\text{ArcTan}\,[\,\text{Tan}\,[\,x\,]\,\,]\right.\\ &\left. a\,\left(a^2+b^2\right)\,\text{Cos}\,[\,2\,x\,]\,-2\,a^3\,\text{Log}\,[\,\left(a\,\text{Cos}\,[\,x\,]\,+b\,\text{Sin}\,[\,x\,]\,\right)^{\,2}\,\right] -a^2\,b\,\text{Sin}\,[\,2\,x\,]\,-b^3\,\text{Sin}\,[\,2\,x\,]\,\right) \end{split}$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int\!\frac{\text{Sin}[x]}{\text{a}\,\text{Cos}[x] + \text{b}\,\text{Sin}[x]}\,\mathrm{d}x$$

Optimal (type 3, 35 leaves, 2 steps):

$$\frac{b \, x}{a^2 + b^2} - \frac{a \, \text{Log} \, [\, a \, \text{Cos} \, [\, x \,] \, + b \, \text{Sin} \, [\, x \,] \, ]}{a^2 + b^2}$$

Result (type 3, 47 leaves):

$$\frac{1}{2\,\left(a^2+b^2\right)}\left(2\,\left(-\,\dot{\mathbb{1}}\,\,a+b\right)\,x+2\,\,\dot{\mathbb{1}}\,\,a\,\,\text{ArcTan}\,[\,\text{Tan}\,[\,x\,]\,\,]\,\,-\,a\,\,\text{Log}\,\left[\,\left(a\,\,\text{Cos}\,[\,x\,]\,+b\,\,\text{Sin}\,[\,x\,]\,\right)^{\,2}\,\right]\right)$$

# Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[x]^2}{\left(a\cos[x] + b\sin[x]\right)^2} dx$$

#### Optimal (type 3, 64 leaves, 4 steps):

$$- \, \frac{\left( \, a^2 \, - \, b^2 \, \right) \, \, x}{\left( \, a^2 \, + \, b^2 \, \right)^{\, 2}} \, + \, \frac{a}{\left( \, a^2 \, + \, b^2 \, \right) \, \, \left( \, b \, + \, a \, \, \text{Cot} \, [ \, x \, ] \, \, \right)} \, - \, \frac{2 \, a \, b \, \, \text{Log} \, [ \, a \, \, \text{Cos} \, [ \, x \, ] \, \, + \, b \, \, \text{Sin} \, [ \, x \, ] \, \, ]}{\left( \, a^2 \, + \, b^2 \, \right)^{\, 2}}$$

#### Result (type 3, 121 leaves):

# Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^3}{\left(\operatorname{a}\operatorname{Cos}[x] + \operatorname{b}\operatorname{Sin}[x]\right)^2} \, \mathrm{d}x$$

### Optimal (type 3, 118 leaves, 11 steps):

$$-\frac{\text{ArcTanh}\left[\text{Cos}\left[x\right]\right]}{2\,a^{2}}-\frac{2\,b^{2}\,\text{ArcTanh}\left[\text{Cos}\left[x\right]\right]}{a^{4}}-\frac{\left(a^{2}+b^{2}\right)\,\text{ArcTanh}\left[\text{Cos}\left[x\right]\right]}{a^{4}}+\\ \\ \frac{3\,b\,\sqrt{a^{2}+b^{2}}\,\,\text{ArcTanh}\left[\frac{b\,\text{Cos}\left[x\right]-a\,\text{Sin}\left[x\right]}{\sqrt{a^{2}+b^{2}}}\right]}{a^{4}}+\frac{2\,b\,\text{Csc}\left[x\right]}{a^{3}}-\frac{\text{Cot}\left[x\right]\,\text{Csc}\left[x\right]}{2\,a^{2}}+\frac{a^{2}+b^{2}}{a^{3}\,\left(a\,\text{Cos}\left[x\right]+b\,\text{Sin}\left[x\right]\right)}$$

#### Result (type 3, 270 leaves):

$$\frac{1}{8 \, a^4 \, \left(b + a \, \text{Cot} \, [x]\right)} \\ \left( -48 \, b \, \sqrt{a^2 + b^2} \, \operatorname{ArcTanh} \left[ \, \frac{-b + a \, \text{Tan} \left[ \frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right] \, \left(b + a \, \text{Cot} \, [x]\right) + 8 \, a^3 \, \text{Csc} \, [x] + 8 \, a \, b^2 \, \text{Csc} \, [x] - 12 \, a^2 \, b \, \text{Log} \left[ \text{Cos} \, \left[ \frac{x}{2} \right] \, \right] - 24 \, b^3 \, \text{Log} \left[ \text{Cos} \, \left[ \frac{x}{2} \right] \, \right] - 12 \, a^3 \, \text{Cot} \, [x] \, \text{Log} \left[ \text{Cos} \, \left[ \frac{x}{2} \right] \, \right] - 24 \, a \, b^2 \, \text{Cot} \, [x] \, \text{Log} \left[ \text{Sin} \, \left[ \frac{x}{2} \right] \, \right] + 24 \, b^3 \, \text{Log} \left[ \text{Sin} \, \left[ \frac{x}{2} \right] \, \right] + 24 \, b^3 \, \text{Log} \left[ \text{Sin} \, \left[ \frac{x}{2} \right] \, \right] + 12 \, a^2 \, b \, \text{Log} \left[ \text{Sin} \, \left[ \frac{x}{2} \right] \, \right] + 24 \, a \, b^2 \, \text{Cot} \, [x] \, \text{Log} \left[ \text{Sin} \, \left[ \frac{x}{2} \right] \, \right] + a^2 \, b \, \text{Sec} \, \left[ \frac{x}{2} \, \right]^2 + a^3 \, \text{Cot} \, [x] \, \text{Sec} \, \left[ \frac{x}{2} \, \right]^2 - a \, c \, \text{Cot} \, \left[ \frac{x}{2} \, \right]^2 \, \left( -4 \, a \, b \, \text{Cos} \, [x] + a^2 \, \text{Cot} \, [x] + b \, \left( a - 4 \, b \, \text{Sin} \, [x] \, \right) \right) + 8 \, a \, b^2 \, \text{Tan} \, \left[ \frac{x}{2} \, \right] + 8 \, a^2 \, b \, \text{Cot} \, [x] \, \text{Tan} \, \left[ \frac{x}{2} \, \right] \right)$$

# Problem 22: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[x]^3}{\left(a\cos[x] + b\sin[x]\right)^3} \, dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\begin{split} & - \frac{b \left( 3 \ a^2 - b^2 \right) \ x}{\left( a^2 + b^2 \right)^3} + \frac{a}{2 \left( a^2 + b^2 \right) \ \left( b + a \ \text{Cot}[x] \right)^2} + \\ & \frac{2 \ a \ b}{\left( a^2 + b^2 \right)^2 \left( b + a \ \text{Cot}[x] \right)} + \frac{a \left( a^2 - 3 \ b^2 \right) \ \text{Log}[a \ \text{Cos}[x] + b \ \text{Sin}[x]]}{\left( a^2 + b^2 \right)^3} \end{split}$$

Result (type 3, 114 leaves):

$$\begin{split} & \frac{b \, \left(-\, 3 \, \, a^2 \, + \, b^2\right) \, x}{\left(\, a^2 \, + \, b^2\,\right)^{\, 3}} \, + \, \frac{a \, \left(\, a^2 \, - \, 3 \, b^2\right) \, Log \left[\, a \, Cos \left[\, x\,\right] \, + \, b \, Sin \left[\, x\,\right] \, \right]}{\left(\, a^2 \, + \, b^2\,\right)^{\, 3}} \, + \\ & \frac{a^3}{2 \, \left(\, a \, - \, \dot{\mathbb{1}} \, b\,\right)^{\, 2} \, \left(\, a \, + \, \dot{\mathbb{1}} \, b\,\right)^{\, 2} \, \left(\, a \, Cos \left[\, x\,\right] \, + \, b \, Sin \left[\, x\,\right] \,\right)^{\, 2}} \, + \, \frac{3 \, a \, b \, Sin \left[\, x\,\right]}{\left(\, a^2 \, + \, b^2\,\right)^{\, 2} \, \left(\, a \, Cos \left[\, x\,\right] \, + \, b \, Sin \left[\, x\,\right] \,\right)} \end{split}$$

### Problem 24: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Sin}[x]}{\left(\text{a} \, \text{Cos}[x] + \text{b} \, \text{Sin}[x]\right)^3} \, \mathrm{d}x$$

Optimal (type 3, 15 leaves, 2 steps):

$$\frac{1}{2 a \left(b + a \cot \left[x\right]\right)^2}$$

Result (type 3, 47 leaves):

$$\frac{2 \, b^2 \, \text{Sin} \, [\, x \,]^{\, 2} \, + \, a \, \left(a \, + \, b \, \text{Sin} \, [\, 2 \, \, x \,] \,\right)}{2 \, a \, \left(a^2 \, + \, b^2\right) \, \left(a \, \text{Cos} \, [\, x \,] \, + \, b \, \text{Sin} \, [\, x \,] \,\right)^{\, 2}}$$

# Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a \cos \left[x\right] + b \sin \left[x\right]\right)^{3}} \, \mathrm{d}x$$

Optimal (type 3, 73 leaves, 3 steps):

$$-\frac{\text{ArcTanh}\left[\frac{b\,\text{Cos}\,[x]-a\,\text{Sin}\,[x]}{\sqrt{a^2+b^2}}\right]}{2\,\left(a^2+b^2\right)^{3/2}}-\frac{b\,\text{Cos}\,[x]-a\,\text{Sin}\,[x]}{2\,\left(a^2+b^2\right)\,\left(a\,\text{Cos}\,[x]+b\,\text{Sin}\,[x]\right)^2}$$

Result (type 3, 101 leaves):

$$\left( \left( a^2 + b^2 \right) \left( -b \operatorname{Cos}[x] + a \operatorname{Sin}[x] \right) + 2 \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[ \frac{-b + a \operatorname{Tan} \left[ \frac{x}{2} \right]}{\sqrt{a^2 + b^2}} \right] \left( a \operatorname{Cos}[x] + b \operatorname{Sin}[x] \right)^2 \right) / \left( 2 \left( a - i b \right)^2 \left( a \operatorname{Cos}[x] + b \operatorname{Sin}[x] \right)^2 \right)$$

Problem 29: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int Sin[c + dx]^{-n} (a Cos[c + dx] + i a Sin[c + dx])^n dx$$

Optimal (type 5, 66 leaves, 1 step):

#### Result (type 6, 2971 leaves):

$$\left\{ e^{-\ln (c + d \cdot x) + n \log [\cos (c + d \cdot x) + i \sin (c + d \cdot x)]} \left( \cos [c + d \cdot x] + i \sin [c + d \cdot x] \right) \frac{\sin [c + d \cdot x]}{\log [\cos (c + d \cdot x) + i \sin [c + d \cdot x])} \frac{\sin [c + d \cdot x] - 2n \tan \left[\frac{1}{2}(c + d \cdot x)\right]}{\log [\cos (c + d \cdot x) + i \sin [c + d \cdot x])} \right\}$$

$$\left( a \left( \cos [c + d \cdot x] + i \sin [c + d \cdot x] \right) \right)^n \sin [c + d \cdot x] - 2n \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \left( 1 + i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right)^{-2n} + \left( (-2 + n) \operatorname{AppellF1}\left[1 - n, -2 \, n, \, 1, \, 2 - n, \, -i \tan \left[\frac{1}{2}(c + d \cdot x)\right], \, i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \right) \right)$$

$$\left( \left( i + \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \left( i \left( -2 + n \right) \operatorname{AppellF1}\left[1 - n, -2 \, n, \, 1, \, 2 - n, \, -i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \right) \right) \right)$$

$$i \tan \left[\frac{1}{2}(c + d \cdot x)\right] + \left( 2n \operatorname{AppellF1}\left[2 - n, \, 1 - 2 \, n, \, 1, \, 3 - n, \, -i \right] \right)$$

$$- i \tan \left[\frac{1}{2}(c + d \cdot x)\right], \, i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \operatorname{Tan}\left[\frac{1}{2}(c + d \cdot x)\right] \right) \right)$$

$$\left( d \left( -1 + n \right) \left( \frac{1}{2(-1 + n)} \operatorname{Sec}\left[\frac{1}{2}(c + d \cdot x)\right]^2 \left( \cos [c + d \cdot x] + i \sin [c + d \cdot x]\right)^n \sin [c + d \cdot x]^{-n} \right)$$

$$\left( - \operatorname{Hypergeometric2F1}\left[1 - 2 \, n, \, 1 - n, \, 2 - n, \, -i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right)$$

$$\left( 1 + i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right)^{-2n} +$$

$$\left( (-2 + n) \operatorname{AppellF1}\left[1 - n, \, -2 \, n, \, 1, \, 2 - n, \, -i \tan \left[\frac{1}{2}(c + d \cdot x)\right], \, i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \right)$$

$$\left( \left( i + \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \left( i \left( -2 + n \right) \operatorname{AppellF1}\left[1 - n, \, -2 \, n, \, 1, \, 2 - n, \, -i \tan \left[\frac{1}{2}(c + d \cdot x)\right] \right) \right) \right)$$

$$3-n, -i \, {\sf Tan} \Big[ \frac{1}{2} \, \big( c + d \, x \big) \Big], \, i \, {\sf Tan} \Big[ \frac{1}{2} \, \big( c - d \, x \big) \Big] \Big] + {\sf AppellFI} \Big[ 2-n, -2n, 2, \\ 3-n, -i \, {\sf Tan} \Big[ \frac{1}{2} \, \big( c + d \, x \big) \Big], \, i \, {\sf Tan} \Big[ \frac{1}{2} \, \big( c + d \, x \big) \Big] \Big) \, {\sf Tan} \Big[ \frac{1}{2} \, \big( c + d \, x \big) \Big] \Big) \Big) - \frac{1}{-1 \cdot n} \, n \, {\sf Cos} \, \big( c + d \, x \big) \Big[ \big( c + d \, x \big) \Big] + i \, {\sf Sin} \, \big( c + d \, x \big) \Big] \\ \Big[ - {\sf Hypergeometric} \, 2 {\sf FI} \, \big[ 1-2n, 1-n, 2-n, -i \, {\sf Tan} \big[ \frac{1}{2} \, \big( c + d \, x \big) \big] \Big] \\ \Big[ \Big( -2+n) \, {\sf AppellFI} \, \big[ 1-n, -2n, 1, 2-n, -i \, {\sf Tan} \big[ \frac{1}{2} \, \big( c + d \, x \big) \big] \Big] \Big] \Big[ \Big( i + {\sf Tan} \big[ \frac{1}{2} \, \big( c + d \, x \big) \big] \Big] \Big[ i \, \big( -2+n) \, {\sf AppellFI} \, \big[ 1-n, -2n, 1, 2-n, -i \, {\sf Tan} \big[ \frac{1}{2} \, \big( c + d \, x \big) \big] \Big] \Big] \Big[ x \, \big( - d \, x \big) \Big] \Big] \Big] \\ -i \, {\sf Tan} \, \Big[ \frac{1}{2} \, \big( c + d \, x \big) \Big] \Big] \Big[ i \, \big( -2+n \big) \, {\sf AppellFI} \, \big[ 1-n, -2n, 1, 2-n, -i \, {\sf Tan} \big[ \frac{1}{2} \, \big( c + d \, x \big) \big] \Big] \Big] \Big] \Big[ x \, \big( - d \, x \big) \Big] \Big] \Big] \\ -i \, {\sf Tan} \, \big( i \, \big( c \, s \, \big) \, \big( -2+n \big) \, \big($$

$$\begin{split} &\left[1+i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right)^{-1-2n}\left)\left(1+i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right)^{-2n}-\\ &\left((-2+n)\,\text{AppellF1}\left[1-n,-2\,n,1,2-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right) \\ &\text{Sec}\left[\frac{1}{2}\left(c+d\,x\right)\right]^2\right)\bigg/\left(2\left(i+\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right)^2\left(i\,\left(-2+n\right)\,\text{AppellF1}\left[1-n,-2\,n,1,2-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right)\right) \\ &\left(2-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right]+\text{AppellF1}\left[2-n,1-2\,n,1,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right)\right) \\ &3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right) \\ &7-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right) \\ &7-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,-i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right],\,i\,\text{Tan}\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] \\ &8-2-n,\,2,\,3-n,\,3-n,\,3-n,\,3-n,\,3-n,\,3-n,\,3-n,$$

# Problem 37: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{2} (a Cos[c+dx] + b Sin[c+dx]) dx$$

Optimal (type 3, 24 leaves, 5 steps):

$$\frac{a\, ArcTanh \, [\, Sin \, [\, c \, + \, d\, x\, ]\,\,]}{d} \, + \, \frac{b\, Sec \, [\, c \, + \, d\, x\,]}{d}$$

Result (type 3, 81 leaves):

$$-\frac{a\, \text{Log} \left[\text{Cos} \left[\frac{c}{2} + \frac{d\,x}{2}\right] - \text{Sin} \left[\frac{c}{2} + \frac{d\,x}{2}\right]\right]}{d} + \frac{a\, \text{Log} \left[\text{Cos} \left[\frac{c}{2} + \frac{d\,x}{2}\right] + \text{Sin} \left[\frac{c}{2} + \frac{d\,x}{2}\right]\right]}{d} + \frac{b\, \text{Sec} \left[c + d\,x\right]}{d}$$

# Problem 41: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{6} (a Cos[c+dx] + b Sin[c+dx]) dx$$

Optimal (type 3, 74 leaves, 7 steps):

$$\frac{3 \text{ a ArcTanh}[Sin[c+dx]]}{8 \text{ d}} + \frac{b \text{ Sec}[c+dx]^5}{5 \text{ d}} + \frac{3 \text{ a Sec}[c+dx] \text{ Tan}[c+dx]}{8 \text{ d}} + \frac{a \text{ Sec}[c+dx]^3 \text{ Tan}[c+dx]}{4 \text{ d}}$$

Result (type 3, 207 leaves):

$$-\frac{3 \text{ a Log} \left[ \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] - \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right]}{8 \text{ d}} + \\ \frac{3 \text{ a Log} \left[ \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right]}{8 \text{ d}} + \frac{\text{b Sec} \left[ c + \text{d x} \right]^5}{5 \text{ d}} + \\ \frac{\text{a}}{16 \text{ d} \left( \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] - \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right)^4} + \frac{3 \text{ a}}{16 \text{ d} \left( \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] - \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right)^2} - \\ \frac{\text{a}}{16 \text{ d} \left( \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right)^4} - \frac{3 \text{ a}}{16 \text{ d} \left( \text{Cos} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( c + \text{d x} \right) \right] \right)^2}$$

# Problem 51: Result more than twice size of optimal antiderivative.

$$\int Sec[c + dx]^{3} (a Cos[c + dx] + b Sin[c + dx])^{2} dx$$

### Optimal (type 3, 67 leaves, 7 steps):

### Result (type 3, 181 leaves):

$$\begin{split} &\frac{1}{4\,d} \left[ 8\,a\,b + \left( -4\,a^2 + 2\,b^2 \right) \, \text{Log} \big[ \text{Cos} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, - \, \text{Sin} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, + \\ &4\,a^2 \, \text{Log} \big[ \text{Cos} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, + \, \text{Sin} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, \big] \, - \\ &2\,b^2 \, \text{Log} \big[ \text{Cos} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, + \, \text{Sin} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, \big] \, + \, \frac{b^2}{\left( \text{Cos} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, - \, \text{Sin} \, \big[ \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, \big)^2} \, + \\ &16\,a\,b\,\text{Sec} \, [\,c + d\,x\,] \, \, \text{Sin} \, \big[ \, \frac{1}{2} \, \left( c + d\,x \right) \, \big]^2 \, - \, \frac{b^2}{\left( \text{Cos} \, \big[ \, \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, + \, \text{Sin} \, \big[ \, \frac{1}{2} \, \left( c + d\,x \right) \, \big] \, \big)^2} \end{split}$$

# Problem 53: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,x\,\right]^{\,5} \,\left(\,a\,Cos\left[\,c\,+\,d\,x\,\right]\,+\,b\,Sin\left[\,c\,+\,d\,x\,\right]\,\right)^{\,2}\,\mathrm{d}x$$

### Optimal (type 3, 120 leaves, 9 steps):

$$\frac{a^{2} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{2\,d} - \frac{b^{2} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{8\,d} + \frac{2\,a\,b\,\operatorname{Sec}[c+d\,x]^{3}}{3\,d} + \\ \frac{a^{2} \operatorname{Sec}[c+d\,x]\,\operatorname{Tan}[c+d\,x]}{2\,d} - \frac{b^{2} \operatorname{Sec}[c+d\,x]\,\operatorname{Tan}[c+d\,x]}{8\,d} + \frac{b^{2} \operatorname{Sec}[c+d\,x]^{3}\,\operatorname{Tan}[c+d\,x]}{4\,d}$$

Result (type 3, 851 leaves):

$$\frac{a \, b \, Cos \, [c + d \, x]^2 \, \left(a + b \, Tan \, [c + d \, x]\right)^2}{3 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2} + \\ \left( \left( -4 \, a^2 + b^2 \right) \, Cos \, [c + d \, x]^2 \, Log \left[ Cos \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] - Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \left(a + b \, Tan \, [c + d \, x]\right)^2 \right) / \left(8 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) + \\ \left( \left( 4 \, a^2 - b^2 \right) \, Cos \, [c + d \, x]^2 \, Log \left[ Cos \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] + Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \left(a + b \, Tan \, [c + d \, x]\right)^2 \right) / \left(8 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) + \left(b^2 \, Cos \, [c + d \, x]^2 \, \left(a + b \, Tan \, [c + d \, x]\right)^2 \right) / \left(8 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) / \left(16 \, d \, \left(Cos \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] - Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right)^4 \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) + \left(12 \, a^2 + 8 \, a \, b - 3 \, b^2 \right) \, Cos \, [c + d \, x]^2 \, \left(a + b \, Tan \, [c + d \, x]\right)^2 \right) / \left(48 \, d \, \left(Cos \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] - Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right)^2 \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) + \left(a \, b \, Cos \, [c + d \, x]^2 \, Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \, \left(a + b \, Tan \, [c + d \, x]\right)^2 \right) / \left(3 \, d \, \left(Cos \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] - Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) + \left(b^2 \, Cos \, [c + d \, x]^2 \, Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) - \left(b^2 \, Cos \, [c + d \, x]^2 \, Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) - \left(a \, b \, Cos \, [c + d \, x]^2 \, Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) - \left(a \, b \, Cos \, [c + d \, x]^2 \, Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right) \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)^2 \right) - \left(a \, b \, Cos \, \left(c \, d \, x\right)^2 \, + Sin \, \left[ \frac{1}{2} \, \left(c + d \, x\right) \, \right] \right)^3 \, \left(a \, Cos \, [c + d \, x] + b$$

# Problem 55: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,x\,\right]^{\,7} \,\left(\,a\,Cos\left[\,c\,+\,d\,x\,\right]\,+\,b\,Sin\left[\,c\,+\,d\,x\,\right]\,\right)^{\,2}\,\mathbb{d}x$$

Optimal (type 3, 168 leaves, 11 steps):

$$\frac{3 \text{ a}^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{8 \text{ d}} - \frac{b^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{16 \text{ d}} + \frac{2 \text{ a} b \operatorname{Sec}[c+d\,x]^5}{5 \text{ d}} + \frac{3 \text{ a}^2 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{8 \text{ d}} - \frac{b^2 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{16 \text{ d}} + \frac{16 \text{ d}}{4 \text{ d}} + \frac{a^2 \operatorname{Sec}[c+d\,x]^3 \operatorname{Tan}[c+d\,x]}{24 \text{ d}} + \frac{b^2 \operatorname{Sec}[c+d\,x]^5 \operatorname{Tan}[c+d\,x]}{6 \text{ d}}$$

#### Result (type 3, 1175 leaves):

$$\frac{3 \text{ a b } \text{Cos} [c + d \, x]^2 \left( a + b \, \text{Tan} [c + d \, x] \right)^2}{20 \text{ d } \left( a \, \text{Cos} [c + d \, x] + b \, \text{Sin} [c + d \, x] \right)^2} + \\ \left( \left( -6 \, a^2 + b^2 \right) \, \text{Cos} [c + d \, x]^2 \, \text{Log} \left[ \text{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right] \left( a + b \, \text{Tan} [c + d \, x] \right)^2 \right) / \\ \left( \left( 6 \, a^2 + b^2 \right) \, \text{Cos} [c + d \, x]^2 \, \text{Log} \left[ \text{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right] \left( a + b \, \text{Tan} [c + d \, x] \right)^2 \right) / \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} [c + d \, x]^2 \, \text{Log} \left[ \text{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, \text{Tan} [c + d \, x] \right)^2 \right) / \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} [c + d \, x] \right)^2 \right) / \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} [c + d \, x] \right)^2 \right) / \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^6 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) / \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^4 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) + \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^4 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) + \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^3 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) + \\ \left( \left( 6 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^3 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) + \\ \left( \left( 3 \, a^2 + 12 \, a^2 - b^2 \right) \, \text{Cos} \left[ c + d \, x \right] \right) - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \left( a + b \, \text{Tan} \left[ c + d \, x \right] \right)^2 \right) / \\ \left( \left( a^2 \, b \, \text{Cos} \left[ c + d \, x \right) \right) - \text{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^3 \left( a \, \text{Cos} \left[ c + d \, x \right] + b \, \text{Sin} \left[ c + d \, x \right] \right)^2 \right) + \\ \left( \left( 3 \, a^2 \, b \, \text{Cos} \left[ c + d \, x \right] \right) - \text{Sin} \left[ \frac{1}{2} \left( c + d$$

$$\left( 10 \, d \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right)^5 \, \left( \mathsf{a} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] + \mathsf{b} \, \mathsf{Sin} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) + \\ \left( \left( -5 \, \mathsf{a}^2 + 4 \, \mathsf{a} \, \mathsf{b} \right) \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right]^2 \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Tan} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) \right) \\ \left( 80 \, \mathsf{d} \, \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right)^4 \, \left( \mathsf{a} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] + \mathsf{b} \, \mathsf{Sin} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) - \\ \left( 3 \, \mathsf{a} \, \mathsf{b} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right]^2 \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Tan} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) \right) \\ \left( 20 \, \mathsf{d} \, \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right)^3 \, \left( \mathsf{a} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] + \mathsf{b} \, \mathsf{Sin} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) + \\ \left( \left( -30 \, \mathsf{a}^2 + 12 \, \mathsf{a} \, \mathsf{b} + 5 \, \mathsf{b}^2 \right) \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right]^2 \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Tan} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) \right) \\ \left( 160 \, \mathsf{d} \, \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right)^2 \, \left( \mathsf{a} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] + \mathsf{b} \, \mathsf{Sin} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) - \\ \left( 3 \, \mathsf{a} \, \mathsf{b} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right]^2 \, \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right) \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Tan} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) \right) \right) \\ \left( 20 \, \mathsf{d} \, \left( \mathsf{Cos} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] + \mathsf{Sin} \left[ \frac{1}{2} \, \left( \mathsf{c} + \mathsf{d} \, \mathsf{x} \right) \right] \right) \, \left( \mathsf{a} \, \mathsf{Cos} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] + \mathsf{b} \, \mathsf{Sin} \left[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \right] \right)^2 \right) \right) \right)$$

### Problem 66: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,x\,\right]^{\,4} \,\left(\,a\,Cos\left[\,c\,+\,d\,x\,\right]\,+\,b\,Sin\left[\,c\,+\,d\,x\,\right]\,\right)^{\,3} \,\,\mathrm{d}x$$

### Optimal (type 3, 103 leaves, 9 steps):

$$\frac{a^{3} \, Arc Tanh \left[ Sin \left[ c + d \, x \right] \right]}{d} - \frac{3 \, a \, b^{2} \, Arc Tanh \left[ Sin \left[ c + d \, x \right] \right]}{2 \, d} + \frac{3 \, a^{2} \, b \, Sec \left[ c + d \, x \right]}{d} - \frac{b^{3} \, Sec \left[ c + d \, x \right]}{d} + \frac{3 \, a \, b^{2} \, Sec \left[ c + d \, x \right] \, Tan \left[ c + d \, x \right]}{2 \, d}$$

### Result (type 3, 293 leaves):

$$\begin{split} &\frac{1}{12\,d}\left(36\,a^2\,b - 10\,b^3 - 6\,a\,\left(2\,a^2 - 3\,b^2\right)\,Log\big[Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] - Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\big] + 12\,a^3 \\ &- Log\big[Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\big] - 18\,a\,b^2\,Log\big[Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\big] + \\ &\frac{9\,a\,b^2}{\left(Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] - Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\right)^2} + \\ &- 2\,b\,\left(18\,a^2 - b^2 + 2\,b^2\,Cos\,[c + d\,x] + \left(18\,a^2 - 5\,b^2\right)\,Cos\,\big[2\,\left(c + d\,x\right)\,\big]\right)\,Sec\,[c + d\,x]^3\,Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]^2 - \\ &\frac{9\,a\,b^2}{\left(Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\right)^2} + \\ &\frac{b^3}{\left(Cos\,\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big] + Sin\big[\frac{1}{2}\,\left(c + d\,x\right)\,\big]\right)^2} \end{split}$$

# Problem 67: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{5} \left(a Cos[c+dx] + b Sin[c+dx]\right)^{3} dx$$

Optimal (type 3, 30 leaves, 2 steps):

Result (type 3, 79 leaves):

$$\frac{1}{8\,d} Sec\,[\,c\,+\,d\,x\,]^{\,4} \\ \left(\,\left(\,6\,a^{2}\,b\,-\,2\,b^{3}\,\right)\,Cos\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\big]\,+\,a\,\left(\,6\,a\,b\,+\,2\,\left(\,a^{2}\,+\,b^{2}\,\right)\,Sin\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\big]\,+\,\left(\,a^{2}\,-\,b^{2}\,\right)\,Sin\,\big[\,4\,\left(\,c\,+\,d\,x\,\right)\,\big]\,\,\right)\,\right) \\ \left(\,6\,a^{2}\,b\,-\,2\,b^{3}\,\right)\,Cos\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\big]\,+\,a\,\left(\,6\,a\,b\,+\,2\,\left(\,a^{2}\,+\,b^{2}\,\right)\,Sin\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\big]\,+\,\left(\,a^{2}\,-\,b^{2}\,\right)\,Sin\,\big[\,4\,\left(\,c\,+\,d\,x\,\right)\,\big]\,\,\right)\,$$

# Problem 68: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,\,x\,\right]^{\,6} \, \left(\,a\,Cos \left[\,c\,+\,d\,\,x\,\right]\,+\,b\,Sin \left[\,c\,+\,d\,\,x\,\right]\,\right)^{\,3} \, \, \mathrm{d}x$$

Optimal (type 3, 158 leaves, 12 steps):

$$\frac{a^{3} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]\,]}{2\,d} - \frac{3\,a\,b^{2} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]\,]}{8\,d} + \frac{a^{2}\,b \operatorname{Sec}[c+d\,x]^{3}}{d} - \frac{b^{3} \operatorname{Sec}[c+d\,x]^{3}}{3\,d} + \frac{b^{3} \operatorname{Sec}[c+d\,x]^{5}}{5\,d} + \frac{a^{3} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{2\,d} - \frac{3\,a\,b^{2} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{8\,d} + \frac{3\,a\,b^{2} \operatorname{Sec}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{4\,d}$$

Result (type 3, 464 leaves):

$$\frac{1}{1920\,d} \, \operatorname{Sec} \, [\, c + d\, x \,]^{\,5} \, \left( 960\,a^2\,b + 64\,b^3 + 320\,\left( 3\,a^2\,b - b^3 \right) \, \operatorname{Cos} \left[ 2\,\left( c + d\, x \right) \right] \, - \\ 300\,a^3\,\operatorname{Cos} \left[ 3\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, - \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, + \\ 225\,a\,b^2\,\operatorname{Cos} \left[ 3\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, - \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, - \\ 60\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, - \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, + \, 45\,a\,b^2\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \\ \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, - \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, - \, 150\,a\,\left( 4\,a^2 - 3\,b^2 \right) \, \operatorname{Cos} \left[ c + d\, x \right] \\ \left( \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, - \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right) \, - \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, + \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, - \\ 300\,a^3\,\operatorname{Cos} \left[ 3\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, + \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, - \\ 225\,a\,b^2\,\operatorname{Cos} \left[ 3\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, + \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, - \\ 45\,a\,b^2\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \, + \, \operatorname{Sin} \left[ \frac{1}{2}\,\left( c + d\, x \right) \right] \right] \, + \, 240\,a^3\,\operatorname{Sin} \left[ 2\,\left( c + d\, x \right) \right] \, + \\ 540\,a\,b^2\,\operatorname{Sin} \left[ 2\,\left( c + d\, x \right) \right] \, + \, 120\,a^3\,\operatorname{Sin} \left[ 4\,\left( c + d\, x \right) \right] \, - \, 90\,a\,b^2\,\operatorname{Sin} \left[ 4\,\left( c + d\, x \right) \right] \right) \, - \\ 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 120\,a^3\,\operatorname{Sin} \left[ 4\,\left( c + d\, x \right) \right] \, - \, 90\,a\,b^2\,\operatorname{Sin} \left[ 4\,\left( c + d\, x \right) \right] \right) \, - \\ 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d\, x \right) \right] \, + \, 300\,a^3\,\operatorname{Cos} \left[ 5\,\left( c + d$$

### Problem 70: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{8} \left(a Cos[c+dx] + b Sin[c+dx]\right)^{3} dx$$

#### Optimal (type 3, 210 leaves, 14 steps):

$$\frac{3 \ a^{3} \ ArcTanh[Sin[c+d\,x]]}{8 \ d} - \frac{3 \ a \ b^{2} \ ArcTanh[Sin[c+d\,x]]}{16 \ d} + \frac{3 \ a^{2} \ b \ Sec[c+d\,x]^{5}}{5 \ d} - \frac{b^{3} \ Sec[c+d\,x]^{5}}{5 \ d} + \frac{b^{3} \ Sec[c+d\,x]^{7}}{7 \ d} + \frac{3 \ a^{3} \ Sec[c+d\,x] \ Tan[c+d\,x]}{8 \ d} - \frac{3 \ a \ b^{2} \ Sec[c+d\,x] \ Tan[c+d\,x]}{16 \ d} + \frac{16 \ d}{2 \ d} + \frac{a \ b^{2} \ Sec[c+d\,x]^{5} \ Tan[c+d\,x]^{5} \ Tan[c+d\,$$

Result (type 3, 637 leaves):

$$\frac{1}{35\,840\,d} \, \operatorname{Sec}\left[c + d\,x\right]^7 \left(10\,752\,a^2\,b + 1536\,b^3 + 3584\,\left(3\,a^2\,b - b^3\right)\,\operatorname{Cos}\left[2\,\left(c + d\,x\right)\right] - 4410\,a^3\,\operatorname{Cos}\left[3\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + \\ 2205\,a\,b^2\,\operatorname{Cos}\left[3\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 1470\,a^3\,\operatorname{Cos}\left[5\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + \\ 735\,a\,b^2\,\operatorname{Cos}\left[5\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 210\,a^3\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + 105\,a\,b^2\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right] \right] \\ \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - 3675\,a\,\left(2\,a^2 - b^2\right)\,\operatorname{Cos}\left[c + d\,x\right] \right] \\ \left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + \\ 4410\,a^3\,\operatorname{Cos}\left[3\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 2205\,a\,b^2\,\operatorname{Cos}\left[3\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + \\ 1470\,a^3\,\operatorname{Cos}\left[5\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] + \\ 210\,a^3\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 105\,a\,b^2\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 105\,a\,b^2\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right]\,\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\,\left(c + d\,x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\,\left(c + d\,x\right)\right]\right] - \\ 105\,a\,b^2\,\operatorname{Cos}\left[7\,\left(c + d\,x\right)\right] + 6790\,a\,b^2\,\operatorname{Sin}\left[2\,\left(c + d\,x\right)\right] + 2800\,a^3\,\operatorname{Sin}\left[4\,\left(c + d\,x\right)\right]\right) - \\ 1400\,a\,b^2\,\operatorname{Sin}\left[4\,\left(c + d\,x\right)\right] + 420\,a^3\,\operatorname{Sin}\left[6\,\left(c + d\,x\right)\right] - 210\,a\,b^2\,\operatorname{Sin}\left[6\,\left(c + d\,x\right)\right]\right)$$

# Problem 72: Result more than twice size of optimal antiderivative.

$$\int Sec[c + dx]^{10} (a Cos[c + dx] + b Sin[c + dx])^{3} dx$$

#### Optimal (type 3, 259 leaves, 16 steps):

$$\frac{5 \text{ a}^3 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{16 \text{ d}} - \frac{15 \text{ a} \text{ b}^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{128 \text{ d}} + \frac{3 \text{ a}^2 \text{ b} \operatorname{Sec}[c+d\,x]^7}{7 \text{ d}} - \frac{b^3 \operatorname{Sec}[c+d\,x]^7}{7 \text{ d}} + \frac{5 \text{ a}^3 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{16 \text{ d}} + \frac{5 \text{ a}^3 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{16 \text{ d}} + \frac{15 \text{ a} \text{ b}^2 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{128 \text{ d}} + \frac{5 \text{ a}^3 \operatorname{Sec}[c+d\,x]^3 \operatorname{Tan}[c+d\,x]}{24 \text{ d}} + \frac{5 \text{ a} \text{ b}^2 \operatorname{Sec}[c+d\,x]^3 \operatorname{Tan}[c+d\,x]}{64 \text{ d}} + \frac{3 \text{ a} \text{ b}^2 \operatorname{Sec}[c+d\,x]^7 \operatorname{Tan}[c+d\,x]}{8 \text{ d}} + \frac{3 \text{ a} \text{ b}^2 \operatorname{Sec}[c+d\,x]^7 \operatorname{Tan}[c+d\,x]}{8 \text{ d}}$$

#### Result (type 3, 1924 leaves):

$$\frac{5 b \left(-216 \, a^2 + 23 \, b^2\right) \cos \left[c + d \, x\right] \cdot 3 \left(a + b \, Tan \left[c + d \, x\right]\right)^3}{8064 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3} - \frac{1}{8064 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3}{\left[5 \left(8 \, a^3 - 3 \, a \, b^2\right) \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[5 \left(8 \, a^3 - 3 \, a \, b^2\right) \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128 \, d \left(a \cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right]^3\right] + \left[128$$

$$\left( 16128 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^2 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) + \\ \left( \cos \left[ c + d \, x \right]^3 \left( 144 \, a^2 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 13 \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \tan \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 1344 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^5 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) + \\ \left( \cos \left[ c + d \, x \right]^3 \left( 108 \, a^2 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \tan \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^5 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^5 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^3 \left( a \cos \left[ c + d \, x \right] \right) \right) \left( a + b \tan \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^3 \left( a \cos \left[ c + d \, x \right] \right) \right) \left( a + b \tan \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \right) \left( a + b \tan \left[ c + d \, x \right] \right)^3 \right) / \\ \left( 2016 \, d$$

Problem 77: Result unnecessarily involves imaginary or complex numbers.

$$\left[ \text{Cos}\left[c + dx\right]^{2} \left( a \, \text{Cos}\left[c + dx\right] + b \, \text{Sin}\left[c + dx\right] \right)^{4} \, dx \right]$$

Optimal (type 3, 301 leaves, 19 steps):

$$\frac{5 \, a^4 \, x}{16} + \frac{3}{8} \, a^2 \, b^2 \, x + \frac{b^4 \, x}{16} - \frac{2 \, a^3 \, b \, \mathsf{Cos} \, [c + d \, x]^{\, 6}}{3 \, d} + \frac{5 \, a^4 \, \mathsf{Cos} \, [c + d \, x] \, \mathsf{Sin} \, [c + d \, x]}{16 \, d} + \frac{3 \, a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x] \, \mathsf{Sin} \, [c + d \, x]}{8 \, d} + \frac{b^4 \, \mathsf{Cos} \, [c + d \, x] \, \mathsf{Sin} \, [c + d \, x]}{16 \, d} + \frac{5 \, a^4 \, \mathsf{Cos} \, [c + d \, x]^{\, 3} \, \mathsf{Sin} \, [c + d \, x]}{16 \, d} + \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 3} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} - \frac{a^2 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^{\, 5} \, \mathsf{Sin} \, [c + d \, x]^{\, 5} \, [c + d \, x]^{\, 5} \, \mathsf{Sin}$$

#### Result (type 3, 178 leaves):

$$\begin{split} &\frac{1}{192\,d} \left(12\,\left(a-\mathop{\dot{\mathbb{1}}} b\right)\,\left(a+\mathop{\dot{\mathbb{1}}} b\right)\,\left(5\,a^2+b^2\right)\,\left(c+d\,x\right) - 12\,a\,b\,\left(5\,a^2+3\,b^2\right)\,Cos\left[2\,\left(c+d\,x\right)\right] - \\ &24\,a^3\,b\,Cos\left[4\,\left(c+d\,x\right)\right] - 4\,a\,b\,\left(a^2-b^2\right)\,Cos\left[6\,\left(c+d\,x\right)\right] + 3\,\left(15\,a^4+6\,a^2\,b^2-b^4\right)\,Sin\left[2\,\left(c+d\,x\right)\right] + \\ &3\,\left(3\,a^4-6\,a^2\,b^2-b^4\right)\,Sin\left[4\,\left(c+d\,x\right)\right] + \left(a^4-6\,a^2\,b^2+b^4\right)\,Sin\left[6\,\left(c+d\,x\right)\right]\right) \end{split}$$

# Problem 84: Result more than twice size of optimal antiderivative.

$$\int Sec[c + dx]^{5} (a Cos[c + dx] + b Sin[c + dx])^{4} dx$$

#### Optimal (type 3, 168 leaves, 12 steps):

```
a^4 ArcTanh [Sin [c + dx]] 3 a^2 b<sup>2</sup> ArcTanh [Sin [c + dx]]
 \frac{3 a^2 b^2 Sec[c+dx] Tan[c+dx]}{d} - \frac{3 b^4 Sec[c+dx] Tan[c+dx]}{8 d} + \frac{b^4 Sec[c+dx] Tan[c+dx]^3}{4 d}
```

Result (type 3, 936 leaves):

$$\frac{2 \, a \, b \, \left(6 \, a^2 - 5 \, b^2\right) \, \cos \left[c + d \, x\right]^4 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^4}{3 \, d \, \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4} + \\ \left(\left(-8 \, a^4 + 24 \, a^2 \, b^2 - 3 \, b^4\right) \, \cos \left[c + d \, x\right]^4 \, \log \left[\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] - \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \\ \left(a + b \, Tan\left[c + d \, x\right]\right)^4\right) \bigg/ \left(8 \, d \, \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \left(8 \, a^4 - 24 \, a^2 \, b^2 + 3 \, b^4\right) \\ \left. \cos \left[c + d \, x\right]^4 \, \log \left[\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, \left(a + b \, Tan\left[c + d \, x\right]\right)^4\right) \bigg/ \\ \left(8 \, d \, \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \left(b^4 \, \cos \left[c + d \, x\right]^4 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^4\right) \bigg/ \\ \left(8 \, d \, \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \left(b^4 \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) \bigg/ \\ \left(16 \, d \, \left(\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] - \sin \left[\frac{1}{2} \, \left(c + d \, x\right]\right]\right)^4\right) \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \\ \left(16 \, d \, \left(\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] - \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^2 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \\ \left(2 \, a \, b^3 \, \cos \left[c + d \, x\right]^4 \, \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) - \\ \left(3 \, d \, \left(\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] - \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) - \\ \left(2 \, a \, b^3 \, \cos \left[c + d \, x\right]^4 \, \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) - \\ \left(2 \, a \, b^3 \, \cos \left[c + d \, x\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \\ \left(16 \, d \, \left(\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) - \\ \left(2 \, a \, b^3 \, \cos \left[c + d \, x\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \\ \left(16 \, d \, \left(\cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d \, x\right] + b \, \sin \left[c + d \, x\right]\right)^4\right) + \\ \left(2 \, a \, b^3 \, \cos \left[c + d \, x\right] + \sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \left(a \, \cos \left[c + d$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ \, c \, + \, d \, x \, \right]^{\, 6} \, \left( \, a \, Cos \left[ \, c \, + \, d \, x \, \right] \, + \, b \, Sin \left[ \, c \, + \, d \, x \, \right] \, \right)^{\, 4} \, \mathbb{d} \, x$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{\left(b + a \cot \left[c + d x\right]\right)^{5} Tan \left[c + d x\right]^{5}}{5 b d}$$

#### Result (type 3, 131 leaves):

$$\left( \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \, \left( 10 \, a \, b \, \left( a^2 - b^2 \right) \, Cos \left[ c + d \, x \right]^2 + \left( 5 \, a^4 - 10 \, a^2 \, b^2 + b^4 \right) \, Cos \left[ c + d \, x \right]^3 \, Sin \left[ c + d \, x \right] + b^2 \, \left( \left( 5 \, a^2 - b^2 \right) \, Sin \left[ 2 \, \left( c + d \, x \right) \, \right] + b \, \left( 5 \, a + b \, Tan \left[ c + d \, x \right] \, \right) \right) \right) \right) \right/ \left( 5 \, d \, \left( a \, Cos \left[ c + d \, x \right] + b \, Sin \left[ c + d \, x \right] \right)^4 \right)$$

### Problem 86: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{7} \left(a Cos[c+dx] + b Sin[c+dx]\right)^{4} dx$$

#### Optimal (type 3, 258 leaves, 16 steps):

$$\frac{a^{4} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{2\,d} - \frac{3\,a^{2}\,b^{2} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{4\,d} + \frac{4\,d}{5\,d} + \frac{b^{4} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{16\,d} + \frac{4\,a^{3}\,b \operatorname{Sec}[c+d\,x]^{3}}{3\,d} - \frac{4\,a\,b^{3} \operatorname{Sec}[c+d\,x]^{3}}{3\,d} + \frac{4\,a\,b^{3} \operatorname{Sec}[c+d\,x]^{5}}{5\,d} + \frac{a^{4} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{2\,d} + \frac{3\,a^{2}\,b^{2} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{4\,d} + \frac{b^{4} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{16\,d} + \frac{3\,a^{2}\,b^{2} \operatorname{Sec}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{2\,d} + \frac{b^{4} \operatorname{Sec}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Sec}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Tan}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Tan}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Tan}[c+d\,x]^{3} \operatorname{Tan}[c+d\,x]}{6\,d} + \frac{b^{4} \operatorname{Tan}[c+d$$

#### Result (type 3, 1342 leaves):

$$\frac{a \ b \ \left(20 \ a^2 - 11 \ b^2\right) \ Cos[c + d \ x]^4 \ \left(a + b \ Tan[c + d \ x]\right)^4}{30 \ d \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4} + \\ \left(\left(-8 \ a^4 + 12 \ a^2 \ b^2 - b^4\right) \ Cos[c + d \ x]^4 \ Log[Cos\left[\frac{1}{2} \left(c + d \ x\right)\right] - Sin\left[\frac{1}{2} \left(c + d \ x\right)\right]\right] \\ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/ \left(16 \ d \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4\right) + \\ \left(\left(8 \ a^4 - 12 \ a^2 \ b^2 + b^4\right) \ Cos[c + d \ x]^4 \ Log[Cos\left[\frac{1}{2} \left(c + d \ x\right)\right] + Sin\left[\frac{1}{2} \left(c + d \ x\right)\right]\right] \ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/ \\ \left(16 \ d \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4\right) + \left(b^4 \ Cos[c + d \ x]^4 \ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/ \\ \left(48 \ d \ \left(Cos\left[\frac{1}{2} \left(c + d \ x\right)\right] - Sin\left[\frac{1}{2} \left(c + d \ x\right)\right]\right)^6 \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4\right) + \\ \left(\left(30 \ a^2 \ b^2 + 8 \ a \ b^3 - 5 \ b^4\right) \ Cos[c + d \ x]^4 \ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/ \\ \left(30 \ a^2 \ b^2 + 8 \ a \ b^3 - 5 \ b^4\right) \ Cos[c + d \ x]\right) \bigg]^4 \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4\right) + \\ \left(\left(120 \ a^4 + 160 \ a^3 \ b - 180 \ a^2 \ b^2 - 88 \ a \ b^3 + 15 \ b^4\right) \ Cos[c + d \ x]^4 \ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/ \\ \left(480 \ d \ \left(Cos\left[\frac{1}{2} \left(c + d \ x\right)\right] - Sin\left[\frac{1}{2} \left(c + d \ x\right)\right]\right)^2 \ \left(a \ Cos[c + d \ x] + b \ Sin[c + d \ x]\right)^4\right) + \\ \left(a \ b^3 \ Cos[c + d \ x]^4 \ Sin\left[\frac{1}{2} \left(c + d \ x\right)\right] \ \left(a + b \ Tan[c + d \ x]\right)^4\right) \bigg/$$

$$\left( 5 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right)^5 \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) - \left( b^4 \cos \left[ c + d x \right]^4 \left( a + b \tan \left[ c + d x \right] \right)^4 \right) / \left( 48 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right)^6 \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) - \left( a b^3 \cos \left[ c + d x \right]^4 \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \left( a + b \tan \left[ c + d x \right] \right)^4 \right) / \left( 5 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right)^5 \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) + \left( \left( -30 a^2 b^2 + 8 a b^3 + 5 b^4 \right) \cos \left[ c + d x \right]^4 \left( a + b \tan \left[ c + d x \right] \right)^4 \right) / \left( 80 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right)^4 \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) + \left( \left( -120 a^4 + 160 a^3 b + 180 a^2 b^2 - 88 a b^3 - 15 b^4 \right) \cos \left[ c + d x \right]^4 \left( a + b \tan \left[ c + d x \right] \right)^4 \right) / \left( 480 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right)^2 \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) + \left( \cos \left[ c + d x \right]^4 \left( 20 a^3 b \sin \left[ \frac{1}{2} \left( c + d x \right) \right] - 11 a b^3 \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \left( a + b \tan \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] - \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \right) \left( a \cos \left[ c + d x \right] + b \sin \left[ c + d x \right] \right)^4 \right) / \left( 30 d \left( \cos \left[ \frac{1}{2} \left( c + d x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d x \right) \right] \right) \right) \left( a \cos \left[ c + d x \right] \right) \right) \left( a \cos \left[ c + d$$

# Problem 88: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx]^{9} (a Cos[c+dx] + b Sin[c+dx])^{4} dx$$

Optimal (type 3, 330 leaves, 19 steps):

$$\frac{3 \text{ a}^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{8 \text{ d}} - \frac{3 \text{ a}^2 \text{ b}^2 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{8 \text{ d}} + \frac{8 \text{ d}}{8 \text{ d}}$$

$$\frac{3 \text{ b}^4 \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{128 \text{ d}} + \frac{4 \text{ a}^3 \text{ b} \operatorname{Sec}[c+d\,x]^5}{5 \text{ d}} - \frac{4 \text{ a} \text{ b}^3 \operatorname{Sec}[c+d\,x]^5}{5 \text{ d}} + \frac{4 \text{ a} \text{ b}^3 \operatorname{Sec}[c+d\,x]^7}{7 \text{ d}} + \frac{3 \text{ a}^4 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{8 \text{ d}} + \frac{3 \text{ b}^4 \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{128 \text{ d}}$$

#### Result (type 3, 1732 leaves):

$$\frac{a \ b \ (42 \ a^2 - 17 \ b^2) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4}{140 \ d \ (a \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4} - \\ \left(3 \ (16 \ a^4 - 16 \ a^2 \ b^2 + b^4) \ Cos [c + d \ x]^4 \ Log [Cos [\frac{1}{2} \ (c + d \ x)] - Sin [\frac{1}{2} \ (c + d \ x)]] \right) \\ \left(a + b \ Tan [c + d \ x])^4\right) / \left(128 \ d \ (a \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) + \left(3 \ (16 \ a^4 - 16 \ a^2 \ b^2 + b^4) \right) \\ Cos [c + d \ x]^4 \ Log [Cos [\frac{1}{2} \ (c + d \ x)] + Sin [\frac{1}{2} \ (c + d \ x)]] \ (a + b \ Tan [c + d \ x])^4\right) / \\ \left(128 \ d \ (a \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left(128 \ d \ (a \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left(128 \ d \ (cos [\frac{1}{2} \ (c + d \ x)] - Sin [\frac{1}{2} \ (c + d \ x)]\right)^8 \ (a \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) + \\ \left((56 \ a^2 \ b^2 + 16 \ a \ b^3 - 7 \ b^4) \ Cos [c + d \ x]^4 \ (a + b \ Tan [c + d \ x])^4\right) / \\ \left((56 \ a^4 + 896 \ a^3 \ b - 256 \ a \ b^3 - 35 \ b^4) \ Cos [c + d \ x]^4 \ (a + b \ Tan [c + d \ x])^4\right) / \\ \left((560 \ a^4 + 896 \ a^3 \ b - 256 \ a \ b^3 - 35 \ b^4) \ Cos [c + d \ x]^4 \ (a + b \ Tan [c + d \ x])^4\right) / \\ \left((1680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^3 \ b - 1680 \ a^2 \ b^2 - 544 \ a \ b^3 + 105 \ b^4) \ Cos [c + d \ x] + b \ Sin [c + d \ x])^4\right) / \\ \left((3680 \ a^4 + 1344 \ a^4 + b \ Tan$$

$$\left( \left[ 14 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^7 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \left( -56 \, a^2 \, b^2 + 16 \, a \, b^3 + 7 \, b^4 \right) \cos \left[ c + d \, x \right]^4 \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) / \\ \left( 448 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^6 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \left( -560 \, a^4 + 896 \, a^3 \, b - 256 \, a \, b^3 + 35 \, b^4 \right) \cos \left[ c + d \, x \right]^4 \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) / \\ \left( 8960 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^4 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \left( -1680 \, a^4 + 1344 \, a^3 \, b + 1680 \, a^2 \, b^2 - 544 \, a \, b^3 - 105 \, b^4 \right) \cos \left[ c + d \, x \right]^4 \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( 8960 \, d \left( \cos \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^2 \left( a \cos \left[ c + d \, x \right] + b \sin \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( 42 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 17 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( 42 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 17 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( 7 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 17 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( 7 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 2 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( -7 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] - 2 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) + \\ \left( \cos \left[ c + d \, x \right]^4 \left( -42 \, a^3 \, b \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] + 17 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right) \left( a + b \, Tan \left[ c + d \, x \right] \right)^4 \right) \right) + \\ \left( \cos \left[ c + d \, x \right] + \sin \left[ \frac{1}{2} \left( c + d \, x \right) \right] + 17 \, a \, b^3 \sin \left[ \frac{1}{2} \left( c + d \, x \right] \right$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\left[ \mathsf{Cos}\left[ \mathsf{c} + \mathsf{d}\,\mathsf{x} \right]^{3} \, \left( \mathsf{a}\,\mathsf{Cos}\left[ \mathsf{c} + \mathsf{d}\,\mathsf{x} \right] + \mathsf{b}\,\mathsf{Sin}\left[ \mathsf{c} + \mathsf{d}\,\mathsf{x} \right] \right)^{5} \, \mathrm{d}\mathsf{x} \right]$$

Optimal (type 3, 426 leaves, 25 steps):

$$\frac{35 \, a^5 \, x}{128} + \frac{25}{64} \, a^3 \, b^2 \, x + \frac{15}{128} \, a \, b^4 \, x - \frac{5 \, a^2 \, b^3 \, \mathsf{Cos} \, [c + d \, x]^6}{3 \, d} - \frac{5 \, a^4 \, b \, \mathsf{Cos} \, [c + d \, x]^8}{8 \, d} + \frac{35 \, a^5 \, \mathsf{Cos} \, [c + d \, x] \, \mathsf{Sin} \, [c + d \, x]}{4 \, d} + \frac{25 \, a^3 \, b^2 \, \mathsf{Cos} \, [c + d \, x] \, \mathsf{Sin} \, [c + d \, x]}{64 \, d} + \frac{128 \, d}{64 \, d} + \frac{192 \, d}{64 \, d} + \frac{25 \, a^3 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^3 \, \mathsf{Sin} \, [c + d \, x]}{64 \, d} + \frac{192 \, d}{64 \, d} + \frac{192 \, d}{64 \, d} + \frac{7 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{64 \, d} + \frac{5 \, a^3 \, b^2 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{Sin} \, [c + d \, x]}{24 \, d} + \frac{5 \, a^5 \, \mathsf{Cos} \, [c + d \, x]^5 \, \mathsf{$$

#### Result (type 3, 259 leaves):

$$\begin{array}{l} \frac{1}{3072\;d}\;\left(120\;a\;\left(a-i\;b\right)\;\left(a+i\;b\right)\;\left(7\;a^2+3\;b^2\right)\;\left(c+d\;x\right) - \\ 24\;b\;\left(35\;a^4+30\;a^2\;b^2+3\;b^4\right)\;Cos\left[2\;\left(c+d\;x\right)\right] + 12\;b\;\left(-35\;a^4-10\;a^2\;b^2+b^4\right)\;Cos\left[4\;\left(c+d\;x\right)\right] + \\ 8\;b\;\left(-15\;a^4+10\;a^2\;b^2+b^4\right)\;Cos\left[6\;\left(c+d\;x\right)\right] - 3\;b\;\left(5\;a^4-10\;a^2\;b^2+b^4\right)\;Cos\left[8\;\left(c+d\;x\right)\right] + \\ 96\;a^3\;\left(7\;a^2+5\;b^2\right)\;Sin\left[2\;\left(c+d\;x\right)\right] + 24\;a\;\left(7\;a^4-10\;a^2\;b^2-5\;b^4\right)\;Sin\left[4\;\left(c+d\;x\right)\right] + \\ 32\;a^3\;\left(a^2-5\;b^2\right)\;Sin\left[6\;\left(c+d\;x\right)\right] + 3\;a\;\left(a^4-10\;a^2\;b^2+5\;b^4\right)\;Sin\left[8\;\left(c+d\;x\right)\right]\right) \end{array}$$

# Problem 98: Result more than twice size of optimal antiderivative.

$$\Big\lceil Sec\,[\,c\,+\,d\,x\,]\,\,\left(a\,Cos\,[\,c\,+\,d\,x\,]\,+\,b\,Sin\,[\,c\,+\,d\,x\,]\,\right)^{\,5}\,\text{d}x$$

### Optimal (type 3, 170 leaves, 8 steps):

$$\begin{split} &\frac{1}{8}\,a\,\left(3\,a^{4}+10\,a^{2}\,b^{2}+15\,b^{4}\right)\,x-\frac{b^{5}\,Log\,[Sin\,[\,c+d\,x\,]\,\,]}{d}+\frac{b^{5}\,Log\,[Tan\,[\,c+d\,x\,]\,\,]}{d}+\frac{\left(4\,b\,\left(5\,a^{4}-b^{4}\right)\,+5\,a\,\left(a^{2}-3\,b^{2}\right)\,\left(a^{2}+b^{2}\right)\,Cot\,[\,c+d\,x\,]\,\right)\,Sin\,[\,c+d\,x\,]^{\,2}}{8\,d}\\ &-\frac{8\,d}{\left(b\,\left(5\,a^{4}-10\,a^{2}\,b^{2}+b^{4}\right)\,+a\,\left(a^{4}-10\,a^{2}\,b^{2}+5\,b^{4}\right)\,Cot\,[\,c+d\,x\,]\,\right)\,Sin\,[\,c+d\,x\,]^{\,4}}{8\,d} \end{split}$$

Result (type 3, 408 leaves):

$$\frac{a \left(3 \, a^4 + 10 \, a^2 \, b^2 + 15 \, b^4\right) \, \left(c + d \, x\right) \, Cos \left[c + d \, x\right]^5 \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5}{8 \, d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5} - \\ \left(b \, \left(5 \, a^4 + 10 \, a^2 \, b^2 - 3 \, b^4\right) \, Cos \left[c + d \, x\right]^5 \, Cos \left[2 \, \left(c + d \, x\right)\right] \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5\right) / \\ \left(8 \, d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5\right) - \\ \left(b \, \left(5 \, a^4 - 10 \, a^2 \, b^2 + b^4\right) \, Cos \left[c + d \, x\right]^5 \, Cos \left[4 \, \left(c + d \, x\right)\right] \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5\right) / \\ \left(32 \, d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5\right) - \frac{b^5 \, Cos \left[c + d \, x\right]^5 \, Log \left[Cos \left[c + d \, x\right]\right] \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5}{d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5} + \\ \frac{a \, \left(a^4 - 5 \, b^4\right) \, Cos \left[c + d \, x\right]^5 \, Sin \left[2 \, \left(c + d \, x\right)\right] \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5}{4 \, d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5} + \\ \left(a \, \left(a^4 - 10 \, a^2 \, b^2 + 5 \, b^4\right) \, Cos \left[c + d \, x\right]^5 \, Sin \left[4 \, \left(c + d \, x\right)\right] \, \left(a + b \, Tan \left[c + d \, x\right]\right)^5\right) / \\ \left(32 \, d \, \left(a \, Cos \left[c + d \, x\right] + b \, Sin \left[c + d \, x\right]\right)^5\right)$$

# Problem 99: Result more than twice size of optimal antiderivative.

$$\int Sec \left[c + dx\right]^{2} \left(a Cos \left[c + dx\right] + b Sin \left[c + dx\right]\right)^{5} dx$$

#### Optimal (type 3, 205 leaves, 17 steps):

$$\frac{5 \, a \, b^4 \, ArcTanh [Sin[c+d\,x]]}{d} - \frac{10 \, a^2 \, b^3 \, Cos[c+d\,x]}{d} + \frac{2 \, b^5 \, Cos[c+d\,x]}{d} - \frac{5 \, a^4 \, b \, Cos[c+d\,x]^3}{3 \, d} + \frac{10 \, a^2 \, b^3 \, Cos[c+d\,x]}{d} + \frac{a^5 \, Sin[c+d\,x]}{d} - \frac{5 \, a^4 \, b \, Cos[c+d\,x]^3}{d} + \frac{10 \, a^3 \, b^3 \, Sin[c+d\,x]}{d} - \frac{a^5 \, Sin[c+d\,x]}{d} - \frac{a^5 \, Sin[c+d\,x]^3}{3 \, d} + \frac{10 \, a^3 \, b^2 \, Sin[c+d\,x]^3}{3 \, d} - \frac{5 \, a \, b^4 \, Sin[c+d\,x]^3}{3 \, d}$$

Result (type 3, 632 leaves):

$$\frac{b^{5} Cos[c+dx]^{5} \left(a+b Tan[c+dx]\right)^{5}}{d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}} - \frac{b \left(5 a^{4}+30 a^{2} b^{2}-7 b^{4}\right) Cos[c+dx]^{6} \left(a+b Tan[c+dx]\right)^{5}}{4 d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}} - \frac{b \left(5 a^{4}-10 a^{2} b^{2}+b^{4}\right) Cos[c+dx]^{5} Cos[3 \left(c+dx\right)] \left(a+b Tan[c+dx]\right)^{5}\right)}{4 d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}} - \frac{b \left(5 a^{4}-10 a^{2} b^{2}+b^{4}\right) Cos[c+dx]^{5} Cos[3 \left(c+dx\right)] \left(a+b Tan[c+dx]\right)^{5}\right)}{\left(12 d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} - \frac{b \left(5 a b^{4} Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)}{\left(d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} + \frac{b \left(5 a b^{4} Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)}{\left(d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} + \frac{b \left(5 a b^{4} Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)}{\left(d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} + \frac{b \left(5 Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)}{\left(d \left(Cos[\frac{1}{2} \left(c+dx\right)]-Sin[\frac{1}{2} \left(c+dx\right)]\right) \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} - \frac{b^{5} Cos[c+dx]+b Sin[\frac{1}{2} \left(c+dx\right)] \left(a+b Tan[c+dx]\right)^{5}\right)}{\left(a \left(Cos[\frac{1}{2} \left(c+dx\right)]+Sin[\frac{1}{2} \left(c+dx\right)]\right) \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} + \frac{b \left(a \left(3 a^{4}+10 a^{2} b^{2}-25 b^{4}\right) Cos[c+dx]^{5} Sin[c+dx] \left(a+b Tan[c+dx]\right)^{5}\right)}{\left(a \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)} + \frac{b \left(a \left(3 a^{4}+10 a^{2} b^{2}+5 b^{4}\right) Cos[c+dx]^{5} Sin[3 \left(c+dx\right)] \left(a+b Tan[c+dx]\right)^{5}\right)}{\left(12 d \left(a Cos[c+dx]+b Sin[c+dx]\right)^{5}\right)}$$

# Problem 100: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,x\,\right]^{\,3} \,\left(\,a\,Cos\left[\,c\,+\,d\,x\,\right]\,+\,b\,Sin\left[\,c\,+\,d\,x\,\right]\,\right)^{\,5} \,\,\mathrm{d}x$$

Optimal (type 3, 169 leaves, 7 steps):

$$\begin{split} &\frac{1}{2}\; a\; \left(a^4+10\; a^2\; b^2-15\; b^4\right)\; x-\frac{2\; b^3\; \left(5\; a^2-b^2\right)\; Log\left[Sin\left[c+d\; x\right]\right]}{d}\; +\frac{2\; b^3\; \left(5\; a^2-b^2\right)\; Log\left[Tan\left[c+d\; x\right]\right]}{d}\; +\frac{1}{2\; d}\left(b\; \left(5\; a^4-10\; a^2\; b^2+b^4\right)\; +a\; \left(a^4-10\; a^2\; b^2+5\; b^4\right)\; Cot\left[c+d\; x\right]\right)\; Sin\left[c+d\; x\right]^2\; +\frac{5\; a\; b^4\; Tan\left[c+d\; x\right]}{d}\; +\frac{b^5\; Tan\left[c+d\; x\right]^2}{2\; d}\; -\frac{b^5\; Tan\left[c+d\; x\right]^2}{2\; d}\; +\frac{b^5\; Tan\left[c+d\; x\right]^2}{2\; d}\; -\frac{b^5\; Tan\left[c+d\; x$$

Result (type 3, 382 leaves):

```
\frac{b^{5}\,Cos\,[\,c\,+\,d\,x\,]^{\,3}\,\left(a\,+\,b\,Tan\,[\,c\,+\,d\,x\,]\,\right)^{\,5}}{2\,d\,\left(a\,Cos\,[\,c\,+\,d\,x\,]\,+\,b\,Sin\,[\,c\,+\,d\,x\,]\,\right)^{\,5}}\,+
                 \frac{\text{a} \, \left(\text{a}^{4} + \text{10 a}^{2} \, \text{b}^{2} - \text{15 b}^{4}\right) \, \left(\text{c} + \text{d} \, \text{x}\right) \, \text{Cos} \, \left[\,\text{c} + \text{d} \, \text{x} \,\right]^{\,5} \, \left(\text{a} + \text{b} \, \text{Tan} \, \left[\,\text{c} + \text{d} \, \text{x} \,\right] \,\right)^{\,5}}{\text{2 d} \, \left(\text{a} \, \text{Cos} \, \left[\,\text{c} + \text{d} \, \text{x} \,\right] \, + \text{b} \, \text{Sin} \, \left[\,\text{c} + \text{d} \, \text{x} \,\right] \,\right)^{\,5}} - \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{d} \, \text{c} \,\right)^{\,5} + \text{cos} \, \left(\text{c} + \text{
                     \left(b\,\left(5\,a^{4}-10\,a^{2}\,b^{2}+b^{4}\right)\,Cos\,[\,c+d\,x\,]\,{}^{5}\,Cos\,\big[\,2\,\left(\,c+d\,x\,\right)\,\big]\,\left(\,a+b\,Tan\,[\,c+d\,x\,]\,\,\right)\,{}^{5}\right)\,\left/\,a^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}+b^{2}
                                    (4 d (a Cos [c + dx] + b Sin [c + dx])^5) -
                 \frac{2\,\left(5\,\,a^{2}\,b^{3}\,-\,b^{5}\right)\,\,Cos\,[\,c\,+\,d\,x\,]^{\,5}\,\,Log\,[\,Cos\,[\,c\,+\,d\,x\,]\,\,]\,\,\left(\,a\,+\,b\,\,Tan\,[\,c\,+\,d\,x\,]\,\,\right)^{\,5}}{d\,\left(\,a\,\,Cos\,[\,c\,+\,d\,x\,]\,\,+\,b\,\,Sin\,[\,c\,+\,d\,x\,]\,\,\right)^{\,5}}\,+
                 \frac{5\;a\;b^{4}\;Cos\,[\,c\;+\;d\;x\,]\,\,^{4}\;Sin\,[\,c\;+\;d\;x\,]\;\,\left(\,a\;+\;b\;Tan\,[\,c\;+\;d\;x\,]\;\,\right)^{\,5}}{d\;\left(\,a\;Cos\,[\,c\;+\;d\;x\,]\;+\;b\;Sin\,[\,c\;+\;d\;x\,]\;\,\right)^{\,5}}\;+
                     \left( a\, \left( a^{4}-10\, a^{2}\, b^{2}+5\, b^{4}\right)\, Cos\, [\, c+d\, x\, ]^{\, 5}\, Sin \left[\, 2\, \left(\, c+d\, x\right)\, \right]\, \left( a+b\, Tan\, [\, c+d\, x\, ]\, \right)^{\, 5}\right) \, \middle/\, a+b\, Tan\, [\, c+d\, x\, ]\, \left( a+b\, Tan\, [\, c+d\, x\, ]\, \right)^{\, 5}\right) \, \Big/\, a+b\, Tan\, [\, c+d\, x\, ]\, \left( a+b\, Tan\, [\, c+d\, x\, ]\, \right)^{\, 5}
                                      (4 d (a Cos [c + dx] + b Sin [c + dx])^5)
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# Problem 101: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ \, c \, + \, d \, x \, \right]^{\, 4} \, \left( \, a \, Cos \left[ \, c \, + \, d \, x \, \right] \, + \, b \, Sin \left[ \, c \, + \, d \, x \, \right] \, \right)^{\, 5} \, \mathrm{d}x$$

#### Optimal (type 3, 204 leaves, 17 steps):

$$\frac{10\,a^3\,b^2\,ArcTanh[Sin[c+d\,x]]}{d} - \frac{15\,a\,b^4\,ArcTanh[Sin[c+d\,x]]}{2\,d} - \frac{5\,a^4\,b\,Cos[c+d\,x]}{d} + \frac{10\,a^2\,b^3\,Cos[c+d\,x]}{d} + \frac{10\,a^2\,b^3\,Sec[c+d\,x]}{d} - \frac{2\,b^5\,Sec[c+d\,x]}{d} + \frac{b^5\,Sec[c+d\,x]^3}{3\,d} + \frac{a^5\,Sin[c+d\,x]}{d} - \frac{10\,a^3\,b^2\,Sin[c+d\,x]}{d} + \frac{15\,a\,b^4\,Sin[c+d\,x]}{2\,d} + \frac{5\,a\,b^4\,Sin[c+d\,x]}{2\,d} + \frac{2\,b^5\,Sec[c+d\,x]}{2\,d} + \frac{b^5\,Sec[c+d\,x]^3}{2\,d} + \frac{b^5\,Sec[c+d\,$$

Result (type 3, 892 leaves):

$$\frac{b^3 \left(-60 \, a^2 + 11 \, b^2\right) \, \cos \left[c + d \, x\right]^5 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5}{6 \, d \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5} - \frac{b \, \left(5 \, a^4 - 10 \, a^2 \, b^2 + b^4\right) \, \cos \left[c + d \, x\right]^6 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5}{d \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5} - \frac{b \, \left(5 \, a^4 - 10 \, a^2 \, b^2 + b^4\right) \, Cos\left[c + d \, x\right]^5 \, Log\left[Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] - Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(2 \, d \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(5 \, \left(4 \, a^3 \, b^2 - 3 \, a \, b^4\right) \, Cos\left[c + d \, x\right]^5 \, Log\left[Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] + Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(2 \, d \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(\left(15 \, a \, b^4 + b^5\right) \, Cos\left(c + d \, x\right)^5 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(2 \, d \, \left(Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] - Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^2 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) / \left(12 \, d \, \left(Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] - Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^2 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(b^5 \, Cos\left[c + d \, x\right]^5 \, Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right] \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(6 \, d \, \left(Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] + Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(\left(-15 \, a \, b^4 + b^5\right) \, Cos\left[c + d \, x\right]^5 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(12 \, d \, \left(Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] + Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^2 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(\left(-15 \, a \, b^4 + b^5\right) \, Cos\left[c + d \, x\right]^5 \, \left(a + b \, Tan\left[c + d \, x\right]\right)^5\right) / \left(12 \, d \, \left(Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right] + Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^2 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(\left(-15 \, a \, b^4 + b^5\right) \, Cos\left[c + d \, x\right]^5 \, Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \, \left(a \, Cos\left[c + d \, x\right] + b \, Sin\left[c + d \, x\right]\right)^5\right) + \left(\left(-15 \, a \, b^4 + b^5\right) \, Cos\left[c + d \, x\right]^5 \, Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right)^3 \, \left(a \, Cos\left[c + d \,$$

# Problem 102: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ c + dx \right]^5 \left( a Cos \left[ c + dx \right] + b Sin \left[ c + dx \right] \right)^5 dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$a \left( a^4 - 10 \ a^2 \ b^2 + 5 \ b^4 \right) \ x - \frac{b \left( 5 \ a^4 - 10 \ a^2 \ b^2 + b^4 \right) \ Log \left[ Cos \left[ c + d \ x \right] \ \right]}{d} + \frac{4 \ a \ b^2 \left( a^2 - b^2 \right) \ Tan \left[ c + d \ x \right]}{d} + \frac{b \left( 3 \ a^2 - b^2 \right) \left( a + b \ Tan \left[ c + d \ x \right] \right)^2}{2 \ d} + \frac{2 \ a \ b \left( a + b \ Tan \left[ c + d \ x \right] \right)^3}{3 \ d} + \frac{b \left( a + b \ Tan \left[ c + d \ x \right] \right)^4}{4 \ d}$$

#### Result (type 3, 369 leaves):

$$\frac{b^{5} \, Cos \, [c + d \, x] \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{4 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} - \frac{b^{3} \, \left(-5 \, a^{2} + b^{2} \right) \, Cos \, [c + d \, x]^{3} \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{a \, \left(a^{4} - 10 \, a^{2} \, b^{2} + 5 \, b^{4} \right) \, \left(c + d \, x \right) \, Cos \, [c + d \, x]^{5} \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{a \, b^{4} \, Cos \, [c + d \, x] \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{5 \, a \, b^{4} \, Cos \, [c + d \, x]^{2} \, Sin \, [c + d \, x] \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{3 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(10 \, Cos \, [c + d \, x]^{4} \, \left(3 \, a^{3} \, b^{2} \, Sin \, [c + d \, x] - 2 \, a \, b^{4} \, Sin \, [c + d \, x] \,\right) \, \left(a + b \, Tan \, [c + d \, x] \,\right)^{5}}{\left(3 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}\right)} + \frac{\left(3 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] \,\right)^{5}}{a \, d \, \left(a \, Cos \, [c + d \, x] \,\right)^{5}} + \frac{\left(a \, b \, Cos \, [c + d \, x] \,$$

### Problem 103: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ c + d \, x \right]^6 \, \left( a \, Cos \left[ c + d \, x \right] \, + b \, Sin \left[ c + d \, x \right] \right)^5 \, \mathrm{d}x$$

### Optimal (type 3, 224 leaves, 15 steps):

$$\frac{a^{5} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{d} - \frac{5 \, a^{3} \, b^{2} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{d} + \frac{15 \, a \, b^{4} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{8 \, d} + \frac{5 \, a^{4} \, b \operatorname{Sec}[c+d\,x]}{d} - \frac{10 \, a^{2} \, b^{3} \operatorname{Sec}[c+d\,x]}{d} + \frac{b^{5} \operatorname{Sec}[c+d\,x]}{d} + \frac{10 \, a^{2} \, b^{3} \operatorname{Sec}[c+d\,x]^{3}}{3 \, d} - \frac{2 \, b^{5} \operatorname{Sec}[c+d\,x]^{3}}{3 \, d} + \frac{b^{5} \operatorname{Sec}[c+d\,x]^{5}}{5 \, d} + \frac{5 \, a^{3} \, b^{2} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{d} - \frac{15 \, a \, b^{4} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]}{8 \, d} + \frac{5 \, a \, b^{4} \operatorname{Sec}[c+d\,x] \operatorname{Tan}[c+d\,x]^{3}}{4 \, d}$$

#### Result (type 3, 1219 leaves):

$$\frac{b \left(600 \ a^4 - 1000 \ a^2 \ b^2 + 89 \ b^4\right) \ Cos \left[c + d \ x\right]^5 \left(a + b \ Tan \left[c + d \ x\right]\right)^5}{120 \ d \left(a \ Cos \left[c + d \ x\right] + b \ Sin \left[c + d \ x\right]\right)^5} + \\ \left(\left(-8 \ a^5 + 40 \ a^3 \ b^2 - 15 \ a \ b^4\right) \ Cos \left[c + d \ x\right]^5 \ Log \left[Cos \left[\frac{1}{2} \left(c + d \ x\right)\right] - Sin \left[\frac{1}{2} \left(c + d \ x\right)\right]\right] \\ \left(a + b \ Tan \left[c + d \ x\right]\right)^5\right) \bigg/ \left(8 \ d \ \left(a \ Cos \left[c + d \ x\right] + b \ Sin \left[c + d \ x\right]\right)^5\right) + \\ \left(\left(8 \ a^5 - 40 \ a^3 \ b^2 + 15 \ a \ b^4\right) \ Cos \left[c + d \ x\right]^5 \ Log \left[Cos \left[\frac{1}{2} \left(c + d \ x\right)\right] + Sin \left[\frac{1}{2} \left(c + d \ x\right)\right]\right] \\ \left(a + b \ Tan \left[c + d \ x\right]\right)^5\right) \bigg/ \left(8 \ d \ \left(a \ Cos \left[c + d \ x\right] + b \ Sin \left[c + d \ x\right]\right)^5\right) + \\$$

$$\left( (25 \, a \, b^4 + 2 \, b^5) \, \cos(c + d \, x)^5 \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) / \\ \left( 80 \, d \, \left[ \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right] - \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right)^4 \, \left( a \, \cos(c + d \, x) + b \, \sin(c + d \, x) \right)^5 \right) + \\ \left( (600 \, a^3 \, b^2 + 200 \, a^2 \, b^3 - 375 \, a \, b^4 - 31 \, b^5) \, \cos(c + d \, x)^5 \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) / \\ \left( 240 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) - \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right)^2 \, \left( a \, \cos(c + d \, x) + b \, \sin(c + d \, x) \right)^5 \right) + \\ \left( b^5 \, \cos(c + d \, x)^5 \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) / \\ \left( 20 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) - \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right)^5 \, \left( a \, \cos(c + d \, x) + b \, \sin(c + d \, x) \right)^5 \right) - \\ \left( b^5 \, \cos(c + d \, x)^5 \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) / \\ \left( 20 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) + \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right)^5 \right) + \left( (-25 \, a \, b^4 + 2 \, b^5) \, \cos(c + d \, x)^5 \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) / \\ \left( 80 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) + \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right)^3 \, \left( a \, \cos(c + d \, x) + b \, \sin(c + d \, x) \right)^5 \right) + \\ \left( (-600 \, a^3 \, b^2 + 2000 \, a^2 \, b^3 + 375 \, a \, b^4 - 31 \, b^5 \right) \, \cos(c + d \, x)^5 \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) + \\ \left( (-600 \, a^3 \, b^2 + 2000 \, a^2 \, b^3 + 375 \, a \, b^4 - 31 \, b^5 \right) \, \cos(c + d \, x)^5 \, \left( a + b \, \tan(c + d \, x) \right)^5 \right) + \\ \left( 240 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) + \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right)^2 \, \left( a \, \cos(c + d \, x) + b \, \sin(c + d \, x) \right)^5 \right) + \\ \left( \cos(c + d \, x)^5 \, \left( -600 \, a^4 \, b \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right) + 10000 \, a^2 \, b^3 \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) - 89 \, b^5 \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right) \right) \right) \right) + \\ \left( 120 \, d \, \left( \cos\left(\frac{1}{2} \, \left( c + d \, x \right) \right) + \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right) \, \left( a \, \cos\left(c + d \, x \right) + b \, \sin\left(c + d \, x \right) \right)^5 \right) + \\ \left( \cos\left(c + d \, x\right)^5 \, \left( 200 \, a^2 \, b^3 \, \sin\left(\frac{1}{2} \, \left( c + d \, x \right) \right) \right) \, \left( a \, \cos\left(c + d \, x \right) + b \, \sin\left(c + d \, x \right) \right)^5 \right) + \\ \left( \cos\left(c + d \, x\right)^5 \, \left( 200 \, a^2 \, b^3 \, \sin\left(\frac{1}{2} \, \left( c$$

Problem 104: Result more than twice size of optimal antiderivative.

```
\int Sec \left[\,c + d\,x\,\right]^{\,7} \, \left(\,a\,Cos \left[\,c + d\,x\,\right] \, + \,b\,Sin \left[\,c + d\,x\,\right]\,\right)^{\,5} \, \mathrm{d}x
```

### Optimal (type 3, 30 leaves, 2 steps):

$$\frac{\left(b + a \cot \left[c + d x\right]\right)^{6} \tan \left[c + d x\right]^{6}}{6 b d}$$

#### Result (type 3, 370 leaves):

$$-\frac{b^3 \left(-5 \, a^2 + b^2\right) \, \text{Cos} \, [c + d \, x] \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5}{2 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5} + \\ \frac{b \, \left(5 \, a^4 - 10 \, a^2 \, b^2 + b^4\right) \, \text{Cos} \, [c + d \, x]^3 \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5}{2 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5} + \\ \frac{b^5 \, \text{Sec} \, [c + d \, x] \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5}{6 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5} + \\ \frac{a \, b^4 \, \text{Sin} \, [c + d \, x] \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5}{d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5} + \\ \left(2 \, \text{Cos} \, [c + d \, x]^2 \, \left(5 \, a^3 \, b^2 \, \text{Sin} \, [c + d \, x] - 3 \, a \, b^4 \, \text{Sin} \, [c + d \, x]\right) \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5\right) / \\ \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x] - 10 \, a^3 \, b^2 \, \text{Sin} \, [c + d \, x] + 3 \, a \, b^4 \, \text{Sin} \, [c + d \, x]\right) \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5\right) / \\ \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x] - 10 \, a^3 \, b^2 \, \text{Sin} \, [c + d \, x] + 3 \, a \, b^4 \, \text{Sin} \, [c + d \, x]\right) \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5\right) / \\ \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x] - 10 \, a^3 \, b^2 \, \text{Sin} \, [c + d \, x] + 3 \, a \, b^4 \, \text{Sin} \, [c + d \, x]\right) \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5\right) / \right) / \\ \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x] - 10 \, a^3 \, b^2 \, \text{Sin} \, [c + d \, x] + 3 \, a \, b^4 \, \text{Sin} \, [c + d \, x]\right) \, \left(a + b \, \text{Tan} \, [c + d \, x]\right)^5\right) / \right) / \\ \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \right) / \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \right) / \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \right) / / \left(3 \, d \, \left(a \, \text{Cos} \, [c + d \, x] + b \, \text{Sin} \, [c + d \, x]\right)^5\right) / \right) / \right) / / / / /$$

### Problem 105: Result more than twice size of optimal antiderivative.

$$\int Sec \left[ \, c \, + \, d \, x \, \right]^{\, 8} \, \left( a \, Cos \left[ \, c \, + \, d \, x \, \right] \, + \, b \, Sin \left[ \, c \, + \, d \, x \, \right] \, \right)^{\, 5} \, \mathrm{d}x$$

#### Optimal (type 3, 318 leaves, 19 steps):

$$\frac{a^{5} \operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{2\,d} = \frac{5\,a^{3}\,b^{2}\operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{4\,d} + \frac{4\,d}{5\,a\,b^{4}\operatorname{ArcTanh}[\operatorname{Sin}[c+d\,x]]}{16\,d} + \frac{5\,a^{4}\,b\,\operatorname{Sec}[c+d\,x]^{3}}{3\,d} = \frac{10\,a^{2}\,b^{3}\operatorname{Sec}[c+d\,x]^{3}}{3\,d} + \frac{b^{5}\operatorname{Sec}[c+d\,x]^{3}}{4\,d} + \frac{2\,a^{2}\,b^{3}\operatorname{Sec}[c+d\,x]^{5}}{4\,d} - \frac{2\,b^{5}\operatorname{Sec}[c+d\,x]^{5}}{5\,d} + \frac{b^{5}\operatorname{Sec}[c+d\,x]^{7}}{7\,d} + \frac{a^{5}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{2\,d} + \frac{5\,a^{3}\,b^{2}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{4\,d} + \frac{5\,a^{3}\,b^{2}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{2\,d} - \frac{5\,a^{3}\,b^{2}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{2\,d} - \frac{5\,a^{3}\,b^{2}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{6\,d} + \frac{5\,a^{3}\,b^{4}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{6\,d} - \frac{5\,a^{4}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{6\,d} + \frac{5\,a^{4}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{6\,d} - \frac{5\,a^{5}\,b^{4}\operatorname{Sec}[c+d\,x]\operatorname{Tan}[c+d\,x]}{6\,d} - \frac{5\,a^{5}\,b^{5}\,b^{5}\,b^{5}\,b^{5}}{6\,d} - \frac{5\,a^{5}\,b^$$

#### Result (type 3, 1677 leaves):

$$\frac{b \left(1400 \ a^4-1540 \ a^2 \ b^2+103 \ b^4\right) \ Cos \left[c+d \ x\right]^5 \ \left(a+b \ Tan \left[c+d \ x\right]\right)^5}{1680 \ d \ \left(a \ Cos \left[c+d \ x\right] + b \ Sin \left[c+d \ x\right]\right)^5} + \\ \left(\left(-8 \ a^5+20 \ a^3 \ b^2-5 \ a \ b^4\right) \ Cos \left[c+d \ x\right]^5 \ Log \left[Cos \left[\frac{1}{2} \left(c+d \ x\right)\right] - Sin \left[\frac{1}{2} \left(c+d \ x\right)\right]\right]$$

$$\left( a + b \operatorname{Tan} \left[ c + d \, x \right] \right)^{5} \right/ \left( 16 \, d \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 8 \, a^{5} - 20 \, a^{3} \, b^{2} + 5 \, a \, b^{4} \right) \operatorname{Cos} \left[ c + d \, x \right]^{5} \operatorname{Log} \left[ \operatorname{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] + \operatorname{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right] \\ \left( a + b \operatorname{Tan} \left[ c + d \, x \right] \right)^{5} \right/ \left( 16 \, d \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 35 \, a \, b^{4} + 3 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right] - \operatorname{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^{6} \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 350 \, a^{3} \, b^{2} + 140 \, a^{2} \, b^{3} - 175 \, a \, b^{4} - 18 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right]^{5} \left( a + b \operatorname{Tan} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 350 \, a^{3} \, b^{2} + 1400 \, a^{4} \, b^{2} - 175 \, a \, b^{4} - 18 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 380 \, a^{3} \, b^{2} + 1400 \, a^{4} \, b^{2} - 175 \, a \, b^{4} - 18 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 380 \, a^{3} \, b^{2} + 1400 \, a^{4} \, b^{2} - 1540 \, a^{2} \, b^{3} + 525 \, a \, b^{4} + 103 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right]^{5} \right) + \\ \left( \left( 380 \, a^{3} \, b^{2} + 1400 \, a^{4} \, b^{2} - 1540 \, a^{2} \, b^{3} + 525 \, a \, b^{4} + 103 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right]^{5} \right) + \\ \left( \left( 350 \, a^{3} \, b^{2} + 1400 \, a^{3} \, b^{2} - 1540 \, a^{2} \, b^{3} + 525 \, a \, b^{4} + 103 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right]^{5} \right) + \\ \left( \left( 56 \, d \left( \operatorname{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \operatorname{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^{7} \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( 56 \, d \left( \operatorname{Cos} \left[ \frac{1}{2} \left( c + d \, x \right) \right] - \operatorname{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^{7} \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( -35 \, a \, b^{4} + 3 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right] + \operatorname{Sin} \left[ \frac{1}{2} \left( c + d \, x \right) \right] \right)^{3} \left( a \operatorname{Cos} \left[ c + d \, x \right] + b \operatorname{Sin} \left[ c + d \, x \right] \right)^{5} \right) + \\ \left( \left( -35 \, a \, b^{4} + 3 \, b^{5} \right) \operatorname{Cos} \left[ c + d \, x \right] + \operatorname{Sin} \left$$

$$\left(a+b\,\mathsf{Tan}[c+d\,x]\right)^5 \right) / \\ \left(1680\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \, \left(a\,\mathsf{Cos}\left[c+d\,x\right] + b\,\mathsf{Sin}\left[c+d\,x\right]\right)^5 \right) + \\ \left(\mathsf{Cos}\left[c+d\,x\right]^5 \left(70\,a^2\,b^3\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 9\,b^5\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \, \left(a+b\,\mathsf{Tan}\left[c+d\,x\right]\right)^5 \right) / \\ \left(140\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^5 \, \left(a\,\mathsf{Cos}\left[c+d\,x\right] + b\,\mathsf{Sin}\left[c+d\,x\right]\right)^5 \right) + \\ \left(\mathsf{Cos}\left[c+d\,x\right]^5 \left(-70\,a^2\,b^3\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 9\,b^5\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \, \left(a+b\,\mathsf{Tan}\left[c+d\,x\right]\right)^5 \right) / \\ \left(140\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^5 \, \left(a\,\mathsf{Cos}\left[c+d\,x\right] + b\,\mathsf{Sin}\left[c+d\,x\right]\right)^5 \right) + \\ \left(\mathsf{Cos}\left[c+d\,x\right]^5 \left(1400\,a^4\,b\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 1540\,a^2\,b^3\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 103\,b^5\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] \right) \\ \left(a+b\,\mathsf{Tan}\left[c+d\,x\right]\right)^5 \right) / \\ \left(\mathsf{1680}\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 1540\,a^2\,b^3\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 103\,b^5\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] \right) \\ \left(a+b\,\mathsf{Tan}\left[c+d\,x\right]\right)^5 \right) / \\ \left(1680\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 1540\,a^2\,b^3\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 103\,b^5\,\mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] \right) \\ \left(a+b\,\mathsf{Tan}\left[c+d\,x\right]\right)^5 \right) / \\ \left(1680\,d\, \left(\mathsf{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \mathsf{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \mathsf{Sin}\left[c+d\,x\right]\right)^5 \right) \right)$$

# Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos [c + dx]^3}{a \cos [c + dx] + b \sin [c + dx]} dx$$

### Optimal (type 3, 119 leaves, 5 steps):

$$\begin{split} \frac{a\;b^2\;x}{\left(a^2+b^2\right)^2} + \frac{a\;x}{2\;\left(a^2+b^2\right)} + \frac{b\;Cos\,[\,c+d\,x\,]^{\,2}}{2\;\left(a^2+b^2\right)\;d} \;+ \\ \frac{b^3\;Log\,[\,a\;Cos\,[\,c+d\,x\,]\,\,+\,b\;Sin\,[\,c+d\,x\,]\,\,]}{\left(a^2+b^2\right)^2\;d} + \frac{a\;Cos\,[\,c+d\,x\,]\;Sin\,[\,c+d\,x\,]}{2\;\left(a^2+b^2\right)\;d} \end{split}$$

#### Result (type 3, 143 leaves):

$$\begin{split} &\frac{1}{4\,\left(a^2+b^2\right)^2\,d} \left(2\,a^3\,c+6\,a\,b^2\,c+4\,\dot{\mathbb{1}}\,b^3\,c+2\,a^3\,d\,x+6\,a\,b^2\,d\,x+4\,\dot{\mathbb{1}}\,b^3\,d\,x-4\,\dot{\mathbb{1}}\,b^3\,ArcTan\,[Tan\,[\,c+d\,x\,]\,\,]+b\,\left(a^2+b^2\right)\,Cos\,\big[\,2\,\left(c+d\,x\right)\,\big]+2\,b^3\,Log\,\big[\,\left(a\,Cos\,[\,c+d\,x\,]\,+b\,Sin\,[\,c+d\,x\,]\,\right)^{\,2}\big]+a^3\,Sin\,\big[\,2\,\left(c+d\,x\right)\,\big]+a\,b^2\,Sin\,\big[\,2\,\left(c+d\,x\right)\,\big]\,\big) \end{split}$$

# Problem 119: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + dx]^4}{\operatorname{a} \operatorname{Cos} [c + dx] + \operatorname{b} \operatorname{Sin} [c + dx]} dx$$

Optimal (type 3, 153 leaves, 7 steps):

$$\frac{ a \, \text{ArcTanh} [\text{Sin} [\, c + d \, x \,]\,\,] }{ 2 \, b^2 \, d } - \frac{ a \, \left( a^2 + b^2 \right) \, \text{ArcTanh} [\, \text{Sin} [\, c + d \, x \,]\,\,] }{ b^4 \, d } - \frac{ \left( a^2 + b^2 \right)^{3/2} \, \text{ArcTanh} \left[ \frac{ b \, \text{Cos} \, [c + d \, x] - a \, \text{Sin} \, [c + d \, x] \,}{ \sqrt{a^2 + b^2}} \right] }{ b^4 \, d } + \frac{ \left( a^2 + b^2 \right) \, \text{Sec} \, [\, c + d \, x \,] }{ b^3 \, d } + \frac{ \text{Sec} \, [\, c + d \, x]^{\, 3} }{ 3 \, b \, d } - \frac{ a \, \text{Sec} \, [\, c + d \, x \,] \, \, \text{Tan} \, [\, c + d \, x \,] }{ 2 \, b^2 \, d }$$

Result (type 3, 321 leaves):

$$\begin{split} &\frac{1}{24\,b^4\,d}\,\left(48\,\left(a^2+b^2\right)^{3/2}\,\mathsf{ArcTanh}\Big[\frac{-\,b+a\,\mathsf{Tan}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]}{\sqrt{a^2+b^2}}\Big]\,+\\ &\operatorname{Sec}\big[c+d\,x\big]^3\,\left(12\,a^2\,b+20\,b^3+12\,b\,\left(a^2+b^2\right)\,\mathsf{Cos}\Big[2\,\left(c+d\,x\right)\,\Big]\,+\\ &6\,a^3\,\mathsf{Cos}\Big[3\,\left(c+d\,x\right)\,\Big]\,\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,-\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,+\\ &9\,a\,b^2\,\mathsf{Cos}\Big[3\,\left(c+d\,x\right)\,\Big]\,\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,-\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,+\,9\,a\,\left(2\,a^2+3\,b^2\right)\,\mathsf{Cos}\big[c+d\,x\big]\,\Big]\,\\ &\left(\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,-\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,-\,\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,+\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,-\\ &6\,a^3\,\mathsf{Cos}\Big[3\,\left(c+d\,x\right)\,\Big]\,\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,+\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,-\\ &9\,a\,b^2\,\mathsf{Cos}\Big[3\,\left(c+d\,x\right)\,\Big]\,\mathsf{Log}\Big[\mathsf{Cos}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,+\,\mathsf{Sin}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,\Big]\,-\,6\,a\,b^2\,\mathsf{Sin}\Big[2\,\left(c+d\,x\right)\,\Big]\,\Big)\,\Big]\,\end{split}$$

# Problem 121: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + d x]^6}{\operatorname{a} \operatorname{Cos} [c + d x] + \operatorname{b} \operatorname{Sin} [c + d x]} dx$$

Optimal (type 3, 262 leaves, 11 steps):

$$-\frac{3 \, a \, ArcTanh[Sin[c+d\,x]]}{8 \, b^2 \, d} - \frac{a \, \left(a^2+b^2\right) \, ArcTanh[Sin[c+d\,x]]}{2 \, b^4 \, d} - \frac{2 \, b^4 \, d}{2 \, b^2 \, b^2 \, ArcTanh[Sin[c+d\,x]]} - \frac{a \, \left(a^2+b^2\right)^2 \, ArcTanh[Sin[c+d\,x]]}{b^6 \, d} - \frac{a \, \left(a^2+b^2\right)^2 \, ArcTanh[\frac{b \, Cos[c+d\,x] - a \, Sin[c+d\,x]}{\sqrt{a^2+b^2}}\right)}{b^6 \, d} + \frac{\left(a^2+b^2\right)^2 \, Sec[c+d\,x]}{3 \, b^3 \, d} + \frac{Sec[c+d\,x]^5}{5 \, b \, d} - \frac{3 \, a \, Sec[c+d\,x] \, Tan[c+d\,x]}{8 \, b^2 \, d} - \frac{a \, ArcTanh[\frac{b \, Cos[c+d\,x] - a \, Sin[c+d\,x]}{2 \, b^4 \, d}}{4 \, b^2 \, d}$$

Result (type 3, 1313 leaves): 
$$\left(\left(120 \, a^4 + 260 \, a^2 \, b^2 + 149 \, b^4\right) \, Sec[c+d\,x] \, \left(a \, Cos[c+d\,x] + b \, Sin[c+d\,x]\right)\right) / \left(120 \, b^5 \, d \, \left(a+b \, Tan[c+d\,x]\right)\right) + \left(2 \, \left(a-i\, b\right)^2 \, \left(a+i\, b\right)^2 \, \sqrt{a^2+b^2} \, ArcTanh\left[\frac{\sqrt{a^2+b^2} \, \left(-b \, Cos\left[\frac{1}{2} \, \left(c+d\,x\right)\right] + a \, Sin\left[\frac{1}{2} \, \left(c+d\,x\right)\right]\right)}{a^2 \, Cos\left[\frac{1}{2} \, \left(c+d\,x\right)\right] + b^2 \, Cos\left[\frac{1}{2} \, \left(c+d\,x\right)\right]}\right)$$

$$Sec[c+d\,x] \, \left(a \, Cos[c+d\,x] + b \, Sin[c+d\,x]\right) / \left(8 \, b^6 \, d \, \left(a+b \, Tan[c+d\,x]\right)\right) + \left(\left(-8 \, a^5 - 20 \, a^3 \, b^2 - 15 \, a \, b^4\right) \, Log\left[Cos\left[\frac{1}{2} \, \left(c+d\,x\right)\right] + Sin\left[\frac{1}{2} \, \left(c+d\,x\right)\right]\right] \, Sec[c+d\,x] \\ \left(a \, Cos[c+d\,x] + b \, Sin[c+d\,x]\right) / \left(8 \, b^6 \, d \, \left(a+b \, Tan[c+d\,x]\right)\right) + \left(-5 \, a + 2 \, b\right) \, Sec[c+d\,x] \, \left(a \, Cos[c+d\,x] + b \, Sin[c+d\,x]\right) + Sin[c+d\,x]\right)$$

$$\left(240\,b^4\,d\,\left(\text{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^2\,\left(a+b\,\text{Tan}[c+d\,x]\right)\right) + \\ \left(\text{Sec}\left[c+d\,x\right]\,\left(-20\,a^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 29\,b^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)\,\left(a\,\text{Cos}\left[c+d\,x\right] + b\,\text{Sin}\left[c+d\,x\right]\right)\right) / \\ \left(120\,b^3\,d\,\left(\text{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3\,\left(a+b\,\text{Tan}\left[c+d\,x\right]\right)\right) + \\ \left(\text{Sec}\left[c+d\,x\right]\,\left(20\,a^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 29\,b^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)\,\left(a\,\text{Cos}\left[c+d\,x\right] + b\,\text{Sin}\left[c+d\,x\right]\right)\right) / \\ \left(120\,b^3\,d\,\left(\text{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3\,\left(a+b\,\text{Tan}\left[c+d\,x\right]\right)\right) + \\ \left(\text{Sec}\left[c+d\,x\right]\,\left(-120\,a^4\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 260\,a^2\,b^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - 149\,b^4\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \\ \left(a\,\text{Cos}\left[c+d\,x\right] + b\,\text{Sin}\left[c+d\,x\right]\right)\right) / \\ \left(120\,b^5\,d\,\left(\text{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + \text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) + 260\,a^2\,b^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 149\,b^4\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \\ \left(a\,\text{Cos}\left[c+d\,x\right]\,\left(120\,a^4\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 260\,a^2\,b^2\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 149\,b^4\,\text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \\ \left(a\,\text{Cos}\left[c+d\,x\right] + b\,\text{Sin}\left[c+d\,x\right]\right) / \\ \left(120\,b^5\,d\,\left(\text{Cos}\left[\frac{1}{2}\,\left(c+d\,x\right)\right] - \text{Sin}\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right) \right) \left(a+b\,\text{Tan}\left[c+d\,x\right]\right)\right) \right) \\ \end{array}$$

Problem 124: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cos [c + dx]^2}{\left(a \cos [c + dx] + b \sin [c + dx]\right)^2} dx$$

Optimal (type 3, 82 leaves, 4 steps):

$$\frac{\left(a^{2}-b^{2}\right) \, x}{\left(a^{2}+b^{2}\right)^{2}} + \frac{2 \, a \, b \, \text{Log} \left[a \, \text{Cos} \left[c + d \, x\right] \, + b \, \text{Sin} \left[c + d \, x\right] \, \right]}{\left(a^{2}+b^{2}\right)^{2} \, d} - \frac{b}{\left(a^{2}+b^{2}\right) \, d \, \left(a + b \, \text{Tan} \left[c + d \, x\right] \right)}$$

Result (type 3, 192 leaves):

$$\left( a^2 \, \text{Cos} \, [\, c + d \, x \, ] \, \left( \, \left( \, a + \dot{\mathbb{1}} \, \, b \, \right)^2 \, \left( \, c + d \, x \, \right) \, + \, a \, b \, \text{Log} \left[ \, \left( \, a \, \text{Cos} \, [\, c + d \, x \, ] \, + \, b \, \text{Sin} \, [\, c + d \, x \, ] \, \right)^2 \, \right] \right) \, + \, \\ b \, \left( \, \left( \, a + \dot{\mathbb{1}} \, \, b \, \right) \, \left( - \dot{\mathbb{1}} \, b^2 + a \, b \, \left( \, 1 + \dot{\mathbb{1}} \, c + \dot{\mathbb{1}} \, d \, x \, \right) \, + \, a^2 \, \left( \, c + d \, x \, \right) \, \right) \, + \, \\ a^2 \, b \, \text{Log} \left[ \, \left( a \, \text{Cos} \, [\, c + d \, x \, ] \, + \, b \, \text{Sin} \, [\, c + d \, x \, ] \, \right)^2 \, \right] \right) \, \text{Sin} \left[ \, c + d \, x \, \right] \, - \, \\ 2 \, \dot{\mathbb{1}} \, a^2 \, b \, \text{ArcTan} \left[ \, \text{Tan} \, [\, c + d \, x \, ] \, \right] \, \left( a \, \text{Cos} \, [\, c + d \, x \, ] \, + \, b \, \text{Sin} \, [\, c + d \, x \, ] \, \right) \right) \, / \, \\ \left( a \, \left( \, a^2 + b^2 \right)^2 \, d \, \left( a \, \text{Cos} \, [\, c + d \, x \, ] \, + \, b \, \text{Sin} \, [\, c + d \, x \, ] \, \right) \right) \, .$$

Problem 129: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[c+dx]^3}{\left(\operatorname{a}\operatorname{Cos}[c+dx] + \operatorname{b}\operatorname{Sin}[c+dx]\right)^2} \, \mathrm{d}x$$

Optimal (type 3, 179 leaves, 11 steps):

$$\frac{2 \, a^2 \, ArcTanh \, [Sin \, [c + d \, x] \, ]}{b^4 \, d} + \frac{ArcTanh \, [Sin \, [c + d \, x] \, ]}{2 \, b^2 \, d} + \\ \frac{\left(a^2 + b^2\right) \, ArcTanh \, [Sin \, [c + d \, x] \, ]}{b^4 \, d} + \frac{3 \, a \, \sqrt{a^2 + b^2} \, ArcTanh \, \left[\frac{b \, Cos \, [c + d \, x] - a \, Sin \, [c + d \, x]}{\sqrt{a^2 + b^2}}\right]}{b^4 \, d} - \\ \frac{2 \, a \, Sec \, [c + d \, x]}{b^3 \, d} - \frac{a^2 + b^2}{b^3 \, d \, \left(a \, Cos \, [c + d \, x] + b \, Sin \, [c + d \, x]\right)} + \frac{Sec \, [c + d \, x] \, Tan \, [c + d \, x]}{2 \, b^2 \, d}$$

Result (type 3, 709 leaves):

$$-\frac{\left(a-i\,b\right)\,\left(a+i\,b\right)\,Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)}{b^{3}\,d\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} - \frac{2\,a\,Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} - \frac{2\,a\,Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} - \frac{6\,a\,\sqrt{a^{2}+b^{2}}\,ArcTanh\left[\frac{\sqrt{a^{2}+b^{2}}\,\left(-b\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+a\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)}{a^{2}\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+b^{2}\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}\right]} \right] \\ Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}\right) / \left(b^{4}\,d\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}\right) - \frac{\left(3\,\left(2\,a^{2}+b^{2}\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right]\,Sec\left[c+d\,x\right]^{2}}{\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}\right) / \left(2\,b^{4}\,d\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}\right) + \frac{\left(3\,\left(2\,a^{2}+b^{2}\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right]\,Sec\left[c+d\,x\right]^{2}}{\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}} - \frac{Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{\left(a\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^{2}\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} - \frac{Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^{2}\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} + \frac{2\,a\,Sec\left[c+d\,x\right]^{2}\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^{2}\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} + \frac{2\,a\,Sec\left[c+d\,x\right]^{2}\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^{2}\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}} + \frac{2\,a\,Sec\left[c+d\,x\right]^{2}\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\,\left(a\,Cos\left[c+d\,x\right]+b\,Sin\left[c+d\,x\right]\right)^{2}}{b^{3}\,d\,\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^{2}\,\left(a+b\,Tan\left[c+d\,x\right]\right)^{2}}$$

# Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int\! \frac{\text{Cos}\,[\,c\,+\,d\,x\,]^{\,3}}{\left(a\,\text{Cos}\,[\,c\,+\,d\,x\,]\,+\,b\,\text{Sin}\,[\,c\,+\,d\,x\,]\,\right)^{\,3}}\,\,\text{d}x$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{a \left(a^2 - 3 b^2\right) x}{\left(a^2 + b^2\right)^3} + \frac{b \left(3 a^2 - b^2\right) Log[a Cos[c + d x] + b Sin[c + d x]]}{\left(a^2 + b^2\right)^3 d} - \frac{b}{2 \left(a^2 + b^2\right) d \left(a + b Tan[c + d x]\right)^2} - \frac{2 a b}{\left(a^2 + b^2\right)^2 d \left(a + b Tan[c + d x]\right)}$$

Result (type 3, 154 leaves):

$$\frac{1}{2\,d} \left( \frac{2\,a\,\left(a^2-3\,b^2\right)\,\left(c+d\,x\right)}{\left(a^2+b^2\right)^3} - \frac{2\,b\,\left(-3\,a^2+b^2\right)\,\text{Log}\left[a\,\text{Cos}\left[c+d\,x\right]+b\,\text{Sin}\left[c+d\,x\right]\right]}{\left(a^2+b^2\right)^3} - \frac{b^3}{\left(a-i\,b\right)^2\,\left(a+i\,b\right)^2\,\left(a\,\text{Cos}\left[c+d\,x\right]+b\,\text{Sin}\left[c+d\,x\right]\right)^2} + \frac{6\,b^2\,\text{Sin}\left[c+d\,x\right]}{\left(a^2+b^2\right)^2\,\left(a\,\text{Cos}\left[c+d\,x\right]+b\,\text{Sin}\left[c+d\,x\right]\right)^2} \right) + \frac{1}{\left(a^2+b^2\right)^2\,\left(a\,\text{Cos}\left[c+d\,x\right]+b\,\text{Sin}\left[c+d\,x\right]\right)^2} \left(a^2+b^2\right)^2 \left(a^2+$$

### Problem 134: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Cos}\,[\,c + d\,x\,]}{\left(\,a\,\mathsf{Cos}\,[\,c + d\,x\,] \,+\, b\,\mathsf{Sin}\,[\,c + d\,x\,]\,\,\right)^{\,3}}\,\,\mathrm{d}x$$

Optimal (type 3, 22 leaves, 2 steps):

$$-\frac{1}{2 b d (a + b Tan [c + d x])^{2}}$$

Result (type 3, 57 leaves):

$$\frac{-\,b\,\text{Cos}\left[\,2\,\left(\,c\,+\,d\,x\,\right)\,\,\right]\,+\,a\,\text{Sin}\left[\,2\,\left(\,c\,+\,d\,x\,\right)\,\,\right]}{2\,\left(\,a^2\,+\,b^2\,\right)\,d\,\left(\,a\,\text{Cos}\left[\,c\,+\,d\,x\,\right]\,+\,b\,\text{Sin}\left[\,c\,+\,d\,x\,\right]\,\right)^{\,2}}$$

### Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a \cos \left[c + d x\right] + b \sin \left[c + d x\right]\right)^{3}} \, dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$-\frac{\mathsf{ArcTanh}\left[\frac{\mathsf{bCos}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]-\mathsf{aSin}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]}{\sqrt{\mathsf{a}^2+\mathsf{b}^2}}\right]}{2\left(\mathsf{a}^2+\mathsf{b}^2\right)^{3/2}\mathsf{d}}-\frac{\mathsf{bCos}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]-\mathsf{aSin}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]}{2\left(\mathsf{a}^2+\mathsf{b}^2\right)\mathsf{d}\left(\mathsf{aCos}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]+\mathsf{bSin}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]\right)^2}$$

Result (type 3, 132 leaves):

$$\left( \left( a^2 + b^2 \right) \; \left( - b \, \mathsf{Cos} \, [\, c + d \, x \,] \; + \, a \, \mathsf{Sin} \, [\, c + d \, x \,] \; \right) \; + \\ \\ 2 \, \sqrt{a^2 + b^2} \; \mathsf{ArcTanh} \left[ \; \frac{-b + a \, \mathsf{Tan} \left[ \frac{1}{2} \; \left( c + d \, x \right) \; \right]}{\sqrt{a^2 + b^2}} \; \right] \; \left( a \, \mathsf{Cos} \, [\, c + d \, x \,] \; + \, b \, \mathsf{Sin} \, [\, c + d \, x \,] \; \right)^2 \right) \\ \\ \left( 2 \; \left( a - \dot{\mathbb{1}} \; b \right)^2 \; \left( a + \dot{\mathbb{1}} \; b \right)^2 d \; \left( a \, \mathsf{Cos} \, [\, c + d \, x \,] \; + \, b \, \mathsf{Sin} \, [\, c + d \, x \,] \; \right)^2 \right)$$

# Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Sec} [c + d x]^{4}}{\left(a \operatorname{Cos} [c + d x] + b \operatorname{Sin} [c + d x]\right)^{3}} dx$$

Optimal (type 3, 383 leaves, 31 steps):

$$\frac{4 \, a^3 \, \text{ArcTanh}[\text{Sin}[c + d \, x]]}{b^6 \, d} = \frac{3 \, a \, \text{ArcTanh}[\text{Sin}[c + d \, x]]}{2 \, b^4 \, d} = \frac{6 \, a \, \left(a^2 + b^2\right) \, \text{ArcTanh}[\text{Sin}[c + d \, x]]}{b^6 \, d} = \frac{2 \, b^4 \, d}{2 \, b^4 \, d}$$

$$\frac{8 \, a^2 \, \sqrt{a^2 + b^2} \, \text{ArcTanh}\left[\frac{b \, \text{Cos}[c + d \, x] - a \, \text{Sin}[c + d \, x]}{\sqrt{a^2 + b^2}}\right]}{b^6 \, d} = \frac{\sqrt{a^2 + b^2} \, \text{ArcTanh}\left[\frac{b \, \text{Cos}[c + d \, x] - a \, \text{Sin}[c + d \, x]}{\sqrt{a^2 + b^2}}\right]}{b^6 \, d} + \frac{4 \, a^2 \, \text{Sec}[c + d \, x]}{b^5 \, d} + \frac{2 \, \left(a^2 + b^2\right) \, \text{Sec}[c + d \, x]}{b^5 \, d} + \frac{3 \, b^3 \, d}{3 \, b^3 \, d} = \frac{\left(a^2 + b^2\right) \, \left(b \, \text{Cos}[c + d \, x] - a \, \text{Sin}[c + d \, x]\right)}{2 \, b^4 \, d \, \left(a \, \text{Cos}[c + d \, x] + b \, \text{Sin}[c + d \, x]\right)^2} + \frac{4 \, a \, \left(a^2 + b^2\right)}{b^5 \, d} = \frac{3 \, a \, \text{Sec}[c + d \, x] \, \text{Tan}[c + d \, x]}{2 \, b^4 \, d}$$

Result (type 3, 688 leaves):

$$\frac{1}{12\,b^6\,d} \frac{1}{(a+b\,Tan[c+d\,x])^3} Sec[c+d\,x]^3 \left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right) }{\left(\frac{6\,b^2\,\left(a^2+b^2\right)^2\,Sin[c+d\,x]}{a} + \frac{6\,\left(a-i\,b\right)\,\left(a+i\,b\right)\,b\,\left(8\,a^2-b^2\right)\,\left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)}{a} + \frac{2\,b\,\left(36\,a^2+13\,b^2\right)\,\left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)^2 +}{\left(60\,\sqrt{a^2+b^2}\,\left(4\,a^2+b^2\right)\,ArcTanh\left[\frac{b}{2}\,\left(c+d\,x\right)\right]} \frac{-b+a\,Tan\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}{\sqrt{a^2+b^2}}\right] \left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)^2 +} \\ 30\,a\,\left(4\,a^2+3\,b^2\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] \left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)^2 -}{30\,a\,\left(4\,a^2+3\,b^2\right)\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] \left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)^2 +} \\ \frac{b^2\,\left(-9\,a+b\right)\,\left(a\,Cos[c+d\,x]+b\,Sin[c+d\,x]\right)^2}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^2} +} \\ \frac{2\,b^3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^2}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right)^3} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}{\left(Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}{\left(c+d\,x\right)} +} \\ \frac{2\,b\,3\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]}{\left(c+d$$

Problem 140: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\! \frac{Sec\,[\,c\,+\,d\,x\,]^{\,5}}{\left(a\,Cos\,[\,c\,+\,d\,x\,]\,\,+\,b\,Sin\,[\,c\,+\,d\,x\,]\,\,\right)^{\,3}}\,\,\mathrm{d}x$$

Optimal (type 3, 232 leaves, 3 steps):

$$-\frac{\left(a^{2}+b^{2}\right)^{3}}{2\;a^{2}\;b^{5}\;d\;\left(b+a\;Cot\left[c+d\;x\right]\right)^{2}}-\frac{\left(5\;a^{2}-b^{2}\right)\;\left(a^{2}+b^{2}\right)^{2}}{a^{2}\;b^{6}\;d\;\left(b+a\;Cot\left[c+d\;x\right]\right)}+\\ \\ \frac{3\;\left(a^{2}+b^{2}\right)\;\left(5\;a^{2}+b^{2}\right)\;Log\left[b+a\;Cot\left[c+d\;x\right]\right]}{b^{7}\;d}+\frac{3\;\left(a^{2}+b^{2}\right)\;\left(5\;a^{2}+b^{2}\right)\;Log\left[Tan\left[c+d\;x\right]\right]}{b^{7}\;d}-\\ \\ \frac{a\;\left(10\;a^{2}+9\;b^{2}\right)\;Tan\left[c+d\;x\right]}{b^{6}\;d}+\frac{3\;\left(2\;a^{2}+b^{2}\right)\;Tan\left[c+d\;x\right]^{2}}{2\;b^{5}\;d}-\frac{a\;Tan\left[c+d\;x\right]^{3}}{b^{4}\;d}+\frac{Tan\left[c+d\;x\right]^{4}}{4\;b^{3}\;d}$$

Result (type 3, 530 leaves):

$$-\frac{\left(a-i\,b\right)^2\,\left(a+i\,b\right)^2\,Sec\,[c+d\,x]^3\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)}{2\,b^5\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} - \\ \left(3\,\left(5\,a^4+6\,a^2\,b^2+b^4\right)\,Log\,[Cos\,[c+d\,x]]\,Sec\,[c+d\,x]^3\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3\right) \Big/ \\ \left(b^7\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3\right) + \left(3\,\left(5\,a^4+6\,a^2\,b^2+b^4\right)\,Log\,[a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3\right) \Big/ \\ Sec\,[c+d\,x]^3\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3\right) \Big/ \left(b^7\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3\right) + \\ \frac{\left(3\,a^2+b^2\right)\,Sec\,[c+d\,x]^5\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3}{b^5\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} + \\ \frac{Sec\,[c+d\,x]^7\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3}{4\,b^3\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} - \\ \frac{4\,b^3\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3}{4\,b^3\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} - \\ \left(2\,Sec\,[c+d\,x]^4\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3\,\left(5\,a^3\,Sin\,[c+d\,x]+4\,a\,b^2\,Sin\,[c+d\,x]\right)\right) \Big/ \\ \left(b^6\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3\right) - \left(5\,Sec\,[c+d\,x]^3\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)\right) \Big/ \left(b^6\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3\right) - \\ \frac{a\,Sec\,[c+d\,x]^5\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3\,Tan\,[c+d\,x]}{b^4\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} + \\ \frac{a\,Sec\,[c+d\,x]^5\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3}{b^4\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} + \\ \frac{a\,Sec\,[c+d\,x]^5\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3}{b^4\,d\,\left(a+b\,Tan\,[c+d\,x]\right)^3} + \\ \frac{a\,Sec\,[$$

Problem 141: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cos [c + d x]^4}{\left(a \cos [c + d x] + b \sin [c + d x]\right)^4} dx$$

Optimal (type 3, 165 leaves, 6 steps):

$$\frac{\left(a^4 - 6 \ a^2 \ b^2 + b^4\right) \ x}{\left(a^2 + b^2\right)^4} + \frac{4 \ a \ b \ \left(a^2 - b^2\right) \ Log \left[a \ Cos \left[c + d \ x\right] \ + b \ Sin \left[c + d \ x\right]\right]}{\left(a^2 + b^2\right)^4 \ d} - \frac{b}{3 \ \left(a^2 + b^2\right)^2 \ d \ \left(a + b \ Tan \left[c + d \ x\right]\right)^3} - \frac{b \ \left(3 \ a^2 - b^2\right)}{\left(a^2 + b^2\right)^2 \ d \ \left(a + b \ Tan \left[c + d \ x\right]\right)^2} - \frac{b \ \left(3 \ a^2 - b^2\right)}{\left(a^2 + b^2\right)^3 \ d \ \left(a + b \ Tan \left[c + d \ x\right]\right)}$$

Result (type 3, 419 leaves):

$$\frac{\left(a^2-2\,a\,b-b^2\right)\;\left(a^2+2\,a\,b-b^2\right)\;\left(c+d\,x\right)}{\left(a-\frac{i}{b}\,b\right)^4\;\left(a+\frac{i}{b}\,b\right)^4\;d} \\ + \left(4\;\left(\frac{i}{a}\,a^{10}\;b+a^9\,b^2+2\,\frac{i}{a}\,a^8\,b^3+2\,a^7\,b^4-2\,\frac{i}{a}\,a^4\,b^7-2\,a^3\,b^8-\frac{i}{a}\,a^2\,b^9-a\,b^{10}\right)\;\left(c+d\,x\right)\right) \Big/ \\ + \left(\left(a-\frac{i}{b}\,b\right)^8\,\left(a+\frac{i}{b}\,b\right)^7\,d\right) - \frac{4\,\frac{i}{b}\,\left(a^3\,b-a\,b^3\right)\;ArcTan[Tan[c+d\,x]\,]}{\left(a^2+b^2\right)^4\,d} \\ + \frac{2\,\left(a^3\,b-a\,b^3\right)\;Log\left[\left(a\,Cos\,[c+d\,x]+b\,Sin[c+d\,x]\right)^2\right]}{\left(a^2+b^2\right)^4\,d} \\ + \frac{b^4\,Sin\,[c+d\,x]}{3\,a\,\left(a-\frac{i}{b}\,b\right)^2\,\left(a+\frac{i}{b}\,b\right)^2\,d\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^3} - \\ \frac{b^3\,\left(6\,a^2+b^2\right)}{3\,a\,\left(a-\frac{i}{b}\,b\right)^3\,\left(a+\frac{i}{b}\,b\right)^3\,d\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)^2} + \\ \frac{2\,\left(9\,a^2\,b^2\,Sin\,[c+d\,x]-2\,b^4\,Sin\,[c+d\,x]\right)}{3\,a\,\left(a-\frac{i}{b}\,b\right)^3\,\left(a+\frac{i}{b}\,b\right)^3\,d\,\left(a\,Cos\,[c+d\,x]+b\,Sin\,[c+d\,x]\right)}$$

### Problem 143: Result more than twice size of optimal antiderivative.

$$\int\! \frac{\text{Cos}\,[\,c\,+\,d\,x\,]^{\,2}}{\left(\,a\,\text{Cos}\,[\,c\,+\,d\,x\,]\,\,+\,b\,\text{Sin}\,[\,c\,+\,d\,x\,]\,\,\right)^{\,4}}\,\,\text{d}x$$

Optimal (type 3, 30 leaves, 2 steps):

$$-\,\frac{\,\text{Cot}\,[\,c\,+\,d\,x\,]^{\,3}}{\,3\,b\,d\,\left(b\,+\,a\,\text{Cot}\,[\,c\,+\,d\,x\,]\,\right)^{\,3}}$$

Result (type 3, 124 leaves):

$$\begin{array}{l} \left(-\,6\,a\,b\,\left(a^2+b^2\right)\,Cos\,[\,c+d\,x\,]\,+\,\left(-\,6\,a^3\,b+2\,a\,b^3\right)\,Cos\,\big[\,3\,\left(\,c+d\,x\,\right)\,\big]\,+\,\\ 2\,\left(a^2-b^2\right)\,\left(3\,a^2+b^2+\left(3\,a^2-b^2\right)\,Cos\,\big[\,2\,\left(\,c+d\,x\,\right)\,\big]\,\right)\,Sin\,[\,c+d\,x\,]\,\right)\,\left/ \left(12\,a\,\left(a^2+b^2\right)^2\,d\,\left(a\,Cos\,[\,c+d\,x\,]\,+\,b\,Sin\,[\,c+d\,x\,]\,\right)^{\,3}\right) \end{array}$$

Problem 146: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[c+dx]}{\left(\operatorname{a}\operatorname{Cos}[c+dx]+\operatorname{b}\operatorname{Sin}[c+dx]\right)^{4}}\,\mathrm{d}x$$

Optimal (type 3, 231 leaves, 8 steps):

$$\frac{\mathsf{ArcTanh} \big[ \mathsf{Sin} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] \big)}{\mathsf{b}^{\mathsf{d}} \, \mathsf{d}} + \frac{\mathsf{a} \, \mathsf{ArcTanh} \Big[ \frac{\mathsf{bCos} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] + \mathsf{d} \, \mathsf{d}}{\mathsf{2} \, \mathsf{b}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{d}^{\mathsf{d}} \Big]} + \frac{\mathsf{a} \, \mathsf{ArcTanh} \Big[ \frac{\mathsf{bCos} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] + \mathsf{d} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \big]}{\mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{b}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \Big]} - \frac{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{cos} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{b}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \Big]}{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{cos} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{b} \, \mathsf{sin} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] \big)^{\mathsf{d}} - \frac{\mathsf{d} \, \mathsf{d} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} + \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^{\mathsf{d}} \big)}{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{cos} \big[ \mathsf{c} + \mathsf{d} \, \mathsf{x} \big] \, \mathsf{d}^{\mathsf{d}} \big)} + \frac{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d}^{\mathsf{d}} \big)}{\mathsf{d} \, \mathsf{d}^{\mathsf{d}} \, \mathsf{d}^$$

### Problem 169: Result more than twice size of optimal antiderivative.

$$\int \frac{Sec [c + d x]}{\left(a Cos [c + d x] + i a Sin [c + d x]\right)^{2}} dx$$

Optimal (type 3, 46 leaves, 8 steps):

$$-\frac{ArcTanh[Sin[c+dx]]}{a^{2}d} + \frac{2 i Cos[c+dx]}{a^{2}d} + \frac{2 Sin[c+dx]}{a^{2}d}$$

Result (type 3, 184 leaves):

$$\begin{split} &-\left(\left(\text{Sec}\left[\,c+d\,x\,\right)^{\,2}\right.\right.\right.\\ &\left.\left.\left(\text{Cos}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\left(2\,\,\dot{\mathbb{1}}\,+\,\text{Log}\left[\,\text{Cos}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,-\,\text{Sin}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right]\,-\,\text{Log}\left[\,\text{Cos}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right]\,+\,\\ &\left.\left.\text{Sin}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right]\right)\,+\,\left(2\,+\,\dot{\mathbb{1}}\,\,\text{Log}\left[\,\text{Cos}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,-\,\text{Sin}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right]\,-\,\\ &\left.\dot{\mathbb{1}}\,\,\text{Log}\left[\,\text{Cos}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,+\,\text{Sin}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right)\right)\,\,\text{Sin}\left[\,\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\right]\right)\\ &\left.\left(\text{Cos}\left[\,\frac{3}{2}\,\left(\,c+d\,x\,\right)\,\right]\,+\,\dot{\mathbb{1}}\,\,\text{Sin}\left[\,\frac{3}{2}\,\left(\,c+d\,x\,\right)\,\right]\,\right)\right)\right/\left(a^{2}\,d\,\left(\,-\,\dot{\mathbb{1}}\,+\,\text{Tan}\left[\,c+d\,x\,\right]\,\right)^{\,2}\right)\right) \end{split}$$

### Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + d x]^{3}}{(a \operatorname{Cos} [c + d x] + i a \operatorname{Sin} [c + d x])^{2}} dx$$

Optimal (type 3, 56 leaves, 8 steps):

$$\frac{3 \, ArcTanh \, [Sin \, [c+d \, x] \, ]}{2 \, a^2 \, d} - \frac{2 \, i \, Sec \, [c+d \, x]}{a^2 \, d} - \frac{Sec \, [c+d \, x] \, Tan \, [c+d \, x]}{2 \, a^2 \, d}$$

Result (type 3, 146 leaves):

$$-\frac{1}{4 a^2 d}$$

$$\operatorname{Sec}\left[c + d x\right]^2 \left(8 i \operatorname{Cos}\left[c + d x\right] + 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\left(c + d x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\left(c + d x\right)\right]\right] + 3 \operatorname{Cos}\left[2\left(c + d x\right)\right]\right)$$

$$\left(\operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\left(c + d x\right)\right] - \operatorname{Sin}\left[\frac{1}{2}\left(c + d x\right)\right]\right] - \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\left(c + d x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\left(c + d x\right)\right]\right]\right) - 3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2}\left(c + d x\right)\right] + \operatorname{Sin}\left[\frac{1}{2}\left(c + d x\right)\right]\right] + 2 \operatorname{Sin}\left[c + d x\right]\right)$$

### Problem 173: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + d x]^{5}}{\left(a \operatorname{Cos} [c + d x] + i a \operatorname{Sin} [c + d x]\right)^{2}} dx$$

Optimal (type 3, 84 leaves, 10 steps):

$$\frac{5\, Arc Tanh [Sin [c + d \, x]\,]}{8\, a^2\, d} - \frac{2\, i\, Sec [c + d \, x]^3}{3\, a^2\, d} + \frac{5\, Sec [c + d \, x]\, Tan [c + d \, x]}{8\, a^2\, d} - \frac{Sec [c + d \, x]^3\, Tan [c + d \, x]}{4\, a^2\, d}$$

Result (type 3, 215 leaves):

$$-\frac{1}{192\,a^{2}\,d}\,Sec\,[\,c+d\,x\,]^{\,4}\\ \left(128\,i\,Cos\,[\,c+d\,x\,]\,+45\,Log\,\big[Cos\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,-Sin\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,\big]\,+60\,Cos\,\big[\,2\,\left(\,c+d\,x\,\right)\,\big]}{\left(Log\,\big[Cos\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,-Sin\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,\right)\,-Log\,\big[Cos\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,+Sin\,\big[\frac{1}{2}\,\left(\,c+d\,x\,\right)\,\big]\,\big]\,+18\,Sin\,\big[\,c+d\,x\,\big]\,-30\,Sin\,\big[\,3\,\left(\,c+d\,x\,\right)\,\big]\,\big)}$$

### Problem 179: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos [c + dx]}{\left(a \cos [c + dx] + i a \sin [c + dx]\right)^{3}} dx$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{i \, \cot [c + d \, x]^2}{2 \, a^3 \, d \, (i + \cot [c + d \, x])^2}$$

Result (type 3, 77 leaves):

$$\frac{\text{i}\, \text{Cos}\left[2\, \left(c + d\, x\right)\,\right]}{4\, a^3\, d} \,+\, \frac{\text{i}\, \text{Cos}\left[4\, \left(c + d\, x\right)\,\right]}{8\, a^3\, d} \,+\, \frac{\text{Sin}\left[2\, \left(c + d\, x\right)\,\right]}{4\, a^3\, d} \,+\, \frac{\text{Sin}\left[4\, \left(c + d\, x\right)\,\right]}{8\, a^3\, d}$$

### Problem 185: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec} [c + d x]^{5}}{\left(a \operatorname{Cos} [c + d x] + i a \operatorname{Sin} [c + d x]\right)^{3}} dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{i \left(i - \mathsf{Cot}\left[c + \mathsf{d}x\right]\right)^{4} \mathsf{Tan}\left[c + \mathsf{d}x\right]^{4}}{4 \mathsf{a}^{3} \mathsf{d}}$$

Result (type 3, 90 leaves):

$$-\frac{1}{4 \, a^3 \, d} \, \dot{a} \, Sec \, [\, c \,] \, Sec \, [\, c \,+\, d \,x \,]^{\, 4} \, \left( 3 \, Cos \, [\, c \,] \,+\, 2 \, Cos \, [\, c \,+\, 2 \, d \,x \,] \,+\, 2 \, Cos \, [\, 3 \, c \,+\, 2 \, d \,x \,] \,-\, 4 \, a^3 \, d \, \right)$$

# Problem 188: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\operatorname{Sec}[x] + \operatorname{Tan}[x]} \, \mathrm{d}x$$

Optimal (type 3, 5 leaves, 3 steps):

Result (type 3, 16 leaves):

$$2 Log \left[ Cos \left[ \frac{x}{2} \right] + Sin \left[ \frac{x}{2} \right] \right]$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Tan}\,[x]}{\mathsf{Sec}\,[x] + \mathsf{Tan}\,[x]} \, \mathrm{d} x$$

Optimal (type 3, 11 leaves, 3 steps):

$$x + \frac{\mathsf{Cos}[x]}{1 + \mathsf{Sin}[x]}$$

Result (type 3, 25 leaves):

$$X - \frac{2 \, \text{Sin} \left[ \frac{x}{2} \right]}{\text{Cos} \left[ \frac{x}{2} \right] + \text{Sin} \left[ \frac{x}{2} \right]}$$

Problem 192: Result more than twice size of optimal antiderivative.

$$\int\!\frac{\text{Cot}\,[x]}{\text{Sec}\,[x]\,+\text{Tan}\,[x]}\,\mathrm{d}x$$

Optimal (type 3, 9 leaves, 4 steps):

Result (type 3, 20 leaves):

$$-x - Log\left[Cos\left[\frac{x}{2}\right]\right] + Log\left[Sin\left[\frac{x}{2}\right]\right]$$

Problem 193: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}[x]}{\mathsf{Sec}[x] + \mathsf{Tan}[x]} \, \mathrm{d}x$$

Optimal (type 3, 10 leaves, 2 steps):

$$-\frac{\mathsf{Cos}\,[\,x\,]}{\mathsf{1}+\mathsf{Sin}\,[\,x\,]}$$

Result (type 3, 23 leaves):

$$\frac{2\,\text{Sin}\!\left[\frac{x}{2}\right]}{\text{Cos}\!\left[\frac{x}{2}\right]+\text{Sin}\!\left[\frac{x}{2}\right]}$$

### Problem 199: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Cot}[x]}{\text{Sec}[x] - \text{Tan}[x]} \, dx$$

$$Optimal (type 3, 7 leaves, 4 steps): x - \text{ArcTanh}[\text{Cos}[x]]$$

$$Result (type 3, 18 leaves): x - \text{Log}[\text{Cos}[\frac{x}{2}]] + \text{Log}[\text{Sin}[\frac{x}{2}]]$$

# Problem 200: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Sec}[x]}{\operatorname{Sec}[x] - \operatorname{Tan}[x]} \, dx$$
Optimal (type 3, 11 leaves, 2 steps):
$$\frac{\operatorname{Cos}[x]}{1 - \operatorname{Sin}[x]}$$
Result (type 3, 25 leaves):
$$\frac{2 \operatorname{Sin}\left[\frac{x}{2}\right]}{\operatorname{Cos}\left[\frac{x}{2}\right] - \operatorname{Sin}\left[\frac{x}{2}\right]}$$

# Problem 203: Result more than twice size of optimal antiderivative.

$$\int \frac{\sin[x]}{\cot[x] + \csc[x]} dx$$
Optimal (type 3, 6 leaves, 3 steps):
$$x - \sin[x]$$
Result (type 3, 14 leaves):
$$2\left(\frac{x}{2} - \frac{\sin[x]}{2}\right)$$

# Problem 205: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Tan}[\mathsf{x}]}{\mathsf{Cot}[\mathsf{x}] + \mathsf{Csc}[\mathsf{x}]} \, d\mathsf{x}$$
Optimal (type 3, 7 leaves, 4 steps):
$$-\mathsf{x} + \mathsf{ArcTanh}[\mathsf{Sin}[\mathsf{x}]]$$
Result (type 3, 36 leaves):

$$-x - Log \Big[ Cos \Big[ \frac{x}{2} \Big] - Sin \Big[ \frac{x}{2} \Big] \Big] + Log \Big[ Cos \Big[ \frac{x}{2} \Big] + Sin \Big[ \frac{x}{2} \Big] \Big]$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int\!\frac{\text{Sin}[x]}{-\text{Cot}[x]+\text{Csc}[x]}\,\mathrm{d}x$$

Optimal (type 3, 4 leaves, 3 steps):

x + Sin[x]

Result (type 3, 14 leaves):

$$2\left(\frac{x}{2}+\frac{\sin[x]}{2}\right)$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int\!\frac{\mathsf{Tan}\,[\,x\,]}{-\,\mathsf{Cot}\,[\,x\,]\,+\,\mathsf{Csc}\,[\,x\,]}\,\,\mathrm{d}\,x$$

Optimal (type 3, 5 leaves, 4 steps):

x + ArcTanh[Sin[x]]

Result (type 3, 46 leaves):

$$2\left(\frac{x}{2} - \frac{1}{2} Log\left[Cos\left[\frac{x}{2}\right] - Sin\left[\frac{x}{2}\right]\right] + \frac{1}{2} Log\left[Cos\left[\frac{x}{2}\right] + Sin\left[\frac{x}{2}\right]\right]\right)$$

Problem 215: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{Csc \lceil c + dx \rceil + Sin \lceil c + dx \rceil} \, dx$$

Optimal (type 3, 23 leaves, 3 steps):

$$-\frac{\mathsf{ArcTanh}\Big[\frac{\mathsf{Cos}\,[\mathsf{c}+\mathsf{d}\,\mathsf{x}]}{\sqrt{2}}\Big]}{\sqrt{2}\;\mathsf{d}}$$

Result (type 3, 61 leaves):

$$-\frac{\text{ArcTanh}\Big[\frac{\text{Cos}[\textbf{c}]-(-\textbf{i}+\text{Sin}[\textbf{c}])}{\sqrt{2}}\frac{\text{Tan}\Big[\frac{\text{d}\textbf{x}}{2}\Big]}{\sqrt{2}}\Big]+\text{ArcTanh}\Big[\frac{\text{Cos}[\textbf{c}]-(\textbf{i}+\text{Sin}[\textbf{c}])}{\sqrt{2}}\frac{\text{Tan}\Big[\frac{\text{d}\textbf{x}}{2}\Big]}{\sqrt{2}}\Big]}{\sqrt{2}}$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Tan}\,[\,c + d\,x\,]}{\mathsf{Csc}\,[\,c + d\,x\,] + \mathsf{Sin}\,[\,c + d\,x\,]} \, \mathrm{d}x$$

#### Optimal (type 3, 29 leaves, 4 steps):

#### Result (type 3, 63 leaves):

$$-\frac{1}{2\,d} \left( \text{ArcTan} \left[ \text{Sin} \left[ \, c + d \, x \, \right] \, \right] \, + \\ - \, \text{Log} \left[ \text{Cos} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, - \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] - \, \text{Log} \left[ \text{Cos} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] \right) + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] \, \right] + \, \text{Sin} \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \right] \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2} \, \left( \, c + d \, x \, \right) \, \right] \, \left[ \, \frac{1}{2}$$

# Problem 225: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Tan}\,[\,c\,+\,d\,x\,]}{\mathsf{Csc}\,[\,c\,+\,d\,x\,]\,\,-\,\mathsf{Sin}\,[\,c\,+\,d\,x\,]}\,\,\mathrm{d}x$$

#### Optimal (type 3, 34 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}\left[\operatorname{Sin}\left[c+d\,x\right]\right]}{2\,d}+\frac{\operatorname{Sec}\left[c+d\,x\right]\,\operatorname{Tan}\left[c+d\,x\right]}{2\,d}$$

#### Result (type 3, 69 leaves):

# Problem 226: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Cot}\,[\,c + \mathsf{d}\,x\,]}{\mathsf{Csc}\,[\,c + \mathsf{d}\,x\,] - \mathsf{Sin}\,[\,c + \mathsf{d}\,x\,]} \,\mathrm{d}x$$

### Optimal (type 3, 11 leaves, 2 steps):

### Result (type 3, 68 leaves):

$$-\frac{\text{Log}\!\left[\text{Cos}\!\left[\frac{c}{2}+\frac{dx}{2}\right]-\text{Sin}\!\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{d}+\frac{\text{Log}\!\left[\text{Cos}\!\left[\frac{c}{2}+\frac{dx}{2}\right]+\text{Sin}\!\left[\frac{c}{2}+\frac{dx}{2}\right]\right]}{d}$$

# Problem 240: Result more than twice size of optimal antiderivative.

$$\int Sec[c+dx] \left(a Sin[c+dx] + b Tan[c+dx]\right)^2 dx$$

Optimal (type 3, 90 leaves, 7 steps):

$$-2 \ a \ b \ x + \frac{\left(2 \ a^2 - b^2\right) \ ArcTanh [Sin [c + d \ x]]}{2 \ d} - \frac{3 \ a^2 \ Sin [c + d \ x]}{2 \ d} + \frac{a \ b \ Tan [c + d \ x]}{d} + \frac{\left(b + a \ Cos [c + d \ x]\right)^2 \ Sec [c + d \ x] \ Tan [c + d \ x]}{2 \ d}$$

#### Result (type 3, 265 leaves):

$$\begin{split} &-\frac{1}{4\,d}\,\text{Sec}\,[\,c\,+\,d\,x\,]^{\,2}\,\left(4\,a\,b\,c\,+\,4\,a\,b\,d\,x\,+\,\right.\\ &-2\,a^{2}\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,]\,-\,b^{2}\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,-\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,]\,-\,2\,a^{2}\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,]\,+\,\\ &-2\,a^{2}\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,+\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,]\,+\,\\ &-\left.\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\left(\,4\,a\,b\,\left(\,c\,+\,d\,x\,\right)\,+\,\left(\,2\,a^{2}\,-\,b^{2}\,\right)\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,-\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,]\,+\,\\ &-\left.\left(\,-\,2\,a^{2}\,+\,b^{2}\,\right)\,\text{Log}\big[\,\text{Cos}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,+\,\text{Sin}\,\big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\big]\,+\,\\ &-\left.\left(\,a^{2}\,-\,2\,b^{2}\,\right)\,\text{Sin}\,[\,c\,+\,d\,x\,]\,\,-\,4\,a\,b\,\text{Sin}\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,+\,a^{2}\,\text{Sin}\,\big[\,3\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\big)\,\end{split}$$

### Problem 241: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c + d\,x\,\right]^{\,2} \, \left(a\,Sin \left[\,c + d\,x\,\right] \, + b\,Tan \left[\,c + d\,x\,\right]\,\right)^{\,2} \, \mathbb{d}x$$

#### Optimal (type 3, 99 leaves, 7 steps):

$$-a^{2} x - \frac{a b \operatorname{ArcTanh} [\operatorname{Sin} [c + d x]]}{d} + \frac{\left(2 a^{2} - b^{2}\right) \operatorname{Tan} [c + d x]}{3 d} + \frac{a b \operatorname{Sec} [c + d x] \operatorname{Tan} [c + d x]}{3 d} + \frac{\left(b + a \operatorname{Cos} [c + d x]\right)^{2} \operatorname{Sec} [c + d x]^{2} \operatorname{Tan} [c + d x]}{3 d}$$

#### Result (type 3, 201 leaves):

$$\begin{split} \frac{1}{12\,d}\,Sec\,[\,c\,+\,d\,x\,]^{\,3}\,\left(-\,9\,a\,Cos\,[\,c\,+\,d\,x\,]\,\,\left(a\,\left(\,c\,+\,d\,x\,\right)\,-\,\right.\right.\\ &\left.b\,Log\,\big[\,Cos\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,-\,Sin\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\big]\,+\,b\,Log\,\big[\,Cos\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,+\,Sin\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\big]\,-\,3\,a\,Cos\,\big[\,3\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\left(a\,\left(\,c\,+\,d\,x\,\right)\,\,-\,b\,Log\,\big[\,Cos\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,-\,Sin\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\big]\,+\,\\ &\left.b\,Log\,\big[\,Cos\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,+\,Sin\,\Big[\,\frac{1}{2}\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\big]\,\,+\,\\ &\left.2\,\left(\,3\,a^2\,+\,b^2\,+\,6\,a\,b\,Cos\,[\,c\,+\,d\,x\,]\,\,+\,\left(\,3\,a^2\,-\,b^2\,\right)\,Cos\,\big[\,2\,\left(\,c\,+\,d\,x\,\right)\,\,\big]\,\,\big)\,\,Sin\,[\,c\,+\,d\,x\,]\,\,\big)\,\end{split}$$

# Problem 242: Result more than twice size of optimal antiderivative.

$$\int Sec \left[\,c\,+\,d\,x\,\right]^{\,3}\,\left(\,a\,Sin\left[\,c\,+\,d\,x\,\right]\,+\,b\,Tan\left[\,c\,+\,d\,x\,\right]\,\right)^{\,2}\,\text{d}x$$

Optimal (type 3, 125 leaves, 9 steps):

$$-\frac{\left(4\,a^{2}+b^{2}\right)\,ArcTanh[Sin[c+d\,x]\,]}{8\,d} - \frac{2\,a\,b\,Tan[c+d\,x]}{3\,d} + \frac{\left(2\,a^{2}-b^{2}\right)\,Sec[c+d\,x]\,Tan[c+d\,x]}{8\,d} + \frac{a\,b\,Sec\,[c+d\,x]^{\,2}\,Tan\,[c+d\,x]}{6\,d} + \frac{\left(b+a\,Cos\,[c+d\,x]\right)^{\,2}\,Sec\,[c+d\,x]^{\,3}\,Tan\,[c+d\,x]}{4\,d} + \frac{\left(b+a\,Cos\,[c+d\,x$$

Result (type 3, 336 leaves):

Problem 264: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\cos[c+dx]^3}{\left(a\sin[c+dx]+b\tan[c+dx]\right)^3} dx$$

Optimal (type 3, 248 leaves, 6 steps):

$$\frac{b^{6}}{2\,a^{3}\,\left(a^{2}-b^{2}\right)^{2}\,d\,\left(b+a\,\text{Cos}\,\left[c+d\,x\right]\right)^{2}}-\frac{2\,b^{5}\,\left(3\,a^{2}-b^{2}\right)}{a^{3}\,\left(a^{2}-b^{2}\right)^{3}\,d\,\left(b+a\,\text{Cos}\,\left[c+d\,x\right]\right)}-\frac{\left(a\,\left(a^{2}+3\,b^{2}\right)-b\,\left(3\,a^{2}+b^{2}\right)\,\text{Cos}\,\left[c+d\,x\right]\right)}{2\,\left(a^{2}-b^{2}\right)^{3}\,d}-\frac{\left(2\,a+5\,b\right)\,\text{Log}\,\left[1-\text{Cos}\,\left[c+d\,x\right]\right]}{4\,\left(a+b\right)^{4}\,d}-\frac{\left(2\,a-5\,b\right)\,\text{Log}\,\left[1+\text{Cos}\,\left[c+d\,x\right]\right]}{4\,\left(a-b\right)^{4}\,d}-\frac{b^{4}\,\left(15\,a^{4}-4\,a^{2}\,b^{2}+b^{4}\right)\,\text{Log}\,\left[b+a\,\text{Cos}\,\left[c+d\,x\right]\right]}{a^{3}\,\left(a^{2}-b^{2}\right)^{4}\,d}$$

Result (type 3, 713 leaves):

$$\frac{b^6 \left(b + a \cos[c + d \, x]\right) \, Tan[c + d \, x]^3}{2 \, a^3 \left(-a + b\right)^2 \left(a + b\right)^2 \, d \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \frac{2 \, b^5 \left(-3 \, a^2 + b^2\right) \, \left(b + a \cos[c + d \, x]\right)^2 \, Tan[c + d \, x]^3}{a^3 \left(-a + b\right)^3 \, \left(a + b\right)^3 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \frac{2 \, i \, \left(a^5 - 4 \, a^3 \, b^2 - 9 \, a \, b^4\right) \, \left(c + d \, x\right) \, \left(b + a \cos[c + d \, x]\right)^3 \, Tan[c + d \, x]^3}{\left(a - b\right)^4 \, \left(a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \frac{i \, \left(-2 \, a - 5 \, b\right) \, Arc Tan[Tan[c + d \, x]] \, \left(b + a \cos[c + d \, x]\right)^3 \, Tan[c + d \, x]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \frac{i \, \left(-2 \, a + 5 \, b\right) \, Arc Tan[Tan[c + d \, x]] \, \left(b + a \cos[c + d \, x]\right)^3 \, Tan[c + d \, x]^3}{2 \, \left(-a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} + \frac{\left(b + a \cos[c + d \, x]\right)^3 \, Csc\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan[c + d \, x]^3}{8 \, \left(a + b\right)^3 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} + \frac{\left(-2 \, a + 5 \, b\right) \, \left(b + a \cos[c + d \, x]\right)^3 \, Log\left[\cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2\right] \, Tan[c + d \, x]^3\right) / \left(4 \, \left(-a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3\right) + \left(\left(-15 \, a^4 \, b^4 + 4 \, a^2 \, b^6 - b^8\right) \, \left(b + a \cos[c + d \, x]\right)^3 \, Log\left[b + a \cos[c + d \, x]\right] \, Tan[c + d \, x]^3\right) / \left(a^3 \, \left(-a^2 + b^2\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3\right) + \left(\left(-2 \, a - 5 \, b\right) \, \left(b + a \cos[c + d \, x]\right)^3 \, Log\left[\sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2\right] \, Tan[c + d \, x]^3\right) / \left(4 \, \left(a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3\right) + \left(\left(-2 \, a - 5 \, b\right) \, \left(b + a \cos[c + d \, x]\right)^3 \, Log\left[\sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2\right] \, Tan[c + d \, x]^3\right) / \left(4 \, \left(a + b\right)^4 \, d \, \left(a \sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3\right) + \left(b + a \cos[c + d \, x]\right)^3 \, Sec\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan[c + d \, x]$$

# Problem 265: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos [c + d x]^{2}}{(a \sin [c + d x] + b \tan [c + d x])^{3}} dx$$

Optimal (type 3, 232 leaves, 6 steps):

$$-\frac{b^{5}}{2\,a^{2}\,\left(a^{2}-b^{2}\right)^{2}\,d\,\left(b+a\,Cos\left[c+d\,x\right]\right)^{2}}+\frac{b^{4}\,\left(5\,a^{2}-b^{2}\right)}{a^{2}\,\left(a^{2}-b^{2}\right)^{3}\,d\,\left(b+a\,Cos\left[c+d\,x\right]\right)}+\\ \frac{\left(b\,\left(3\,a^{2}+b^{2}\right)-a\,\left(a^{2}+3\,b^{2}\right)\,Cos\left[c+d\,x\right]\right)}{2\,\left(a^{2}-b^{2}\right)^{3}\,d}-\frac{\left(a+4\,b\right)\,Log\left[1-Cos\left[c+d\,x\right]\right]}{4\,\left(a+b\right)^{4}\,d}+\\ \frac{\left(a-4\,b\right)\,Log\left[1+Cos\left[c+d\,x\right]\right]}{4\,\left(a-b\right)^{4}\,d}+\frac{2\,b^{3}\,\left(5\,a^{2}+b^{2}\right)\,Log\left[b+a\,Cos\left[c+d\,x\right]\right]}{\left(a^{2}-b^{2}\right)^{4}\,d}$$

Result (type 3, 477 leaves):

$$-\frac{b^5 \left(b + a \cos \left[c + d \, x\right]\right) \, Tan \left[c + d \, x\right]^3}{2 \, a^2 \, \left(-a + b\right)^2 \, \left(a + b\right)^2 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{b^4 \, \left(-5 \, a^2 + b^2\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^2 \, Tan \left[c + d \, x\right]^3}{a^2 \, \left(-a + b\right)^3 \, \left(a + b\right)^3 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{\left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Csc \left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan \left[c + d \, x\right]^3}{8 \, \left(a + b\right)^3 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]^3} + \frac{\left(a - 4 \, b\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[Cos \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, Tan \left[c + d \, x\right]^3}{2 \, \left(-a + b\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{\left(2 \, \left(5 \, a^2 \, b^3 + b^5\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[b + a \, Cos \left[c + d \, x\right]\right] \, Tan \left[c + d \, x\right]^3\right)}{\left(\left(-a^2 + b^2\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3\right) + \frac{\left(-a - 4 \, b\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[Sin \left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, Tan \left[c + d \, x\right]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{\left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Sec \left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan \left[c + d \, x\right]^3}{3 \, \left(a + b\right)^3 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3}$$

### Problem 266: Result more than twice size of optimal antiderivative.

$$\int \frac{\cos[c+dx]}{\left(a\sin[c+dx]+b\tan[c+dx]\right)^3} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$\frac{b^4}{2 \, a \, \left(a^2-b^2\right)^2 \, d \, \left(b+a \, \text{Cos} \, [\, c+d \, x\, ] \, \right)^2} - \frac{4 \, a \, b^3}{\left(a^2-b^2\right)^3 \, d \, \left(b+a \, \text{Cos} \, [\, c+d \, x\, ] \, \right)} - \frac{\left(a \, \left(a^2+3 \, b^2\right)-b \, \left(3 \, a^2+b^2\right) \, \text{Cos} \, [\, c+d \, x\, ] \, \right)}{2 \, \left(a^2-b^2\right)^3 \, d} - \frac{3 \, b \, \text{Log} \, [\, 1-\text{Cos} \, [\, c+d \, x\, ] \, ]}{4 \, \left(a+b\right)^4 \, d} + \frac{3 \, b \, \text{Log} \, [\, 1+\text{Cos} \, [\, c+d \, x\, ] \, ]}{4 \, \left(a-b\right)^4 \, d} - \frac{6 \, a \, b^2 \, \left(a^2+b^2\right) \, \text{Log} \, [\, b+a \, \text{Cos} \, [\, c+d \, x\, ] \, ]}{\left(a^2-b^2\right)^4 \, d} + \frac{1 \, a \, b^3}{4 \, \left(a-b\right)^4 \, d} - \frac{$$

Result (type 3, 458 leaves):

$$\frac{b^4 \left(b + a \cos[c + d \, x]\right) \, Tan[c + d \, x]^3}{2 \, a \, \left(-a + b\right)^2 \, \left(a + b\right)^2 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} + \\ \frac{4 \, a \, b^3 \, \left(b + a \, Cos[c + d \, x]\right)^2 \, Tan[c + d \, x]^3}{\left(-a + b\right)^3 \, \left(a + b\right)^3 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \\ \frac{\left(b + a \, Cos[c + d \, x]\right)^3 \, Csc\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan[c + d \, x]^3}{8 \, \left(a + b\right)^3 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} + \\ \frac{3 \, b \, \left(b + a \, Cos[c + d \, x]\right)^3 \, Log\left[Cos\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, Tan[c + d \, x]^3}{2 \, \left(-a + b\right)^4 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} - \\ \frac{\left(6 \, \left(a^3 \, b^2 + a \, b^4\right) \, \left(b + a \, Cos[c + d \, x]\right)^3 \, Log\left[b + a \, Cos[c + d \, x]\right] \, Tan[c + d \, x]^3\right) \, / \left(\left(-a^2 + b^2\right)^4 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3\right) - \\ \frac{3 \, b \, \left(b + a \, Cos[c + d \, x]\right)^3 \, Log\left[Sin\left[\frac{1}{2} \, \left(c + d \, x\right)\right]\right] \, Tan[c + d \, x]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3} + \\ \frac{\left(b + a \, Cos[c + d \, x]\right)^3 \, Sec\left[\frac{1}{2} \, \left(c + d \, x\right)\right]^2 \, Tan[c + d \, x]^3}{8 \, \left(-a + b\right)^3 \, d \, \left(a \, Sin[c + d \, x] + b \, Tan[c + d \, x]\right)^3}$$

Problem 267: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \! \frac{1}{ \left( a \, Sin \, [\, c \, + \, d \, \, x \,] \, + \, b \, Tan \, [\, c \, + \, d \, \, x \,] \, \right)^3} \, \, \text{d} x$$

Optimal (type 3, 229 leaves, 5 steps):

$$-\frac{b^{3}}{2\,\left(a^{2}-b^{2}\right)^{2}\,d\,\left(b+a\,Cos\,[\,c+d\,x\,]\,\right)^{2}} + \frac{b^{2}\,\left(3\,a^{2}+b^{2}\right)}{\left(a^{2}-b^{2}\right)^{3}\,d\,\left(b+a\,Cos\,[\,c+d\,x\,]\,\right)} + \\ \frac{\left(b\,\left(3\,a^{2}+b^{2}\right)-a\,\left(a^{2}+3\,b^{2}\right)\,Cos\,[\,c+d\,x\,]\,\right)\,Csc\,[\,c+d\,x\,]^{2}}{2\,\left(a^{2}-b^{2}\right)^{3}\,d} + \frac{\left(a-2\,b\right)\,Log\,[\,1-Cos\,[\,c+d\,x\,]\,]}{4\,\left(a+b\right)^{4}\,d} - \\ \frac{\left(a+2\,b\right)\,Log\,[\,1+Cos\,[\,c+d\,x\,]\,\,]}{4\,\left(a-b\right)^{4}\,d} + \frac{b\,\left(3\,a^{4}+8\,a^{2}\,b^{2}+b^{4}\right)\,Log\,[\,b+a\,Cos\,[\,c+d\,x\,]\,\,]}{\left(a^{2}-b^{2}\right)^{4}\,d}$$

Result (type 3, 696 leaves):

$$-\frac{b^3 \left(b + a \cos \left[c + d \, x\right]\right) \, Tan \left[c + d \, x\right]^3}{2 \, \left(-a + b\right)^2 \, \left(a + b\right)^2 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3}{\frac{b^2 \, \left(3 \, a^2 + b^2\right) \, \left(b + a \cos \left[c + d \, x\right]\right)^2 \, Tan \left[c + d \, x\right]^3}{\left(-a + b\right)^3 \, \left(a + b\right)^3 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{2 \, i \, \left(3 \, a^4 \, b + 8 \, a^2 \, b^3 + b^5\right) \, \left(c + d \, x\right) \, \left(b + a \cos \left[c + d \, x\right]\right)^3 \, Tan \left[c + d \, x\right]^3}{\left(a - b\right)^4 \, \left(a + b\right)^4 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{i \, \left(-a - 2 \, b\right) \, Arc Tan \left[Tan \left[c + d \, x\right]\right] \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Tan \left[c + d \, x\right]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{i \, \left(a - 2 \, b\right) \, Arc Tan \left[Tan \left[c + d \, x\right]\right] \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Tan \left[c + d \, x\right]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} - \frac{i \, \left(a - 2 \, b\right) \, Arc Tan \left[Tan \left[c + d \, x\right]\right] \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Tan \left[c + d \, x\right]^3}{2 \, \left(a + b\right)^4 \, d \, \left(a \sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{i \, \left(a - 2 \, b\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[\cos \left(\frac{1}{2} \, \left(c + d \, x\right)\right]^2\right] \, Tan \left[c + d \, x\right]^3}{4 \, \left(a - a \, b\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{i \, \left(3 \, a^4 \, b + 8 \, a^2 \, b^3 + b^5\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[b + a \, Cos \left[c + d \, x\right]\right]^3 \, Tan \left[c + d \, x\right]^3}{4 \, \left(a \, a \, b\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{i \, \left(3 \, a^4 \, b + 8 \, a^2 \, b^3 + b^5\right) \, \left(b + a \, Cos \left[c + d \, x\right]\right)^3 \, Log \left[b + a \, Cos \left[c + d \, x\right]\right]^3 \, Tan \left[c + d \, x\right]^3}{4 \, \left(a \, a \, b\right)^4 \, d \, \left(a \, Sin \left[c + d \, x\right] + b \, Tan \left[c + d \, x\right]\right)^3} + \frac{i \, \left(3 \, a^4 \, b + 8 \, a^2 \, b^3 + b^5\right) \, \left(b \, a^2 \, c \, c \, c^2 \, c^2$$

Problem 268: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Sec}\,[\,c + d\,x\,]}{\left(\mathsf{a}\,\mathsf{Sin}\,[\,c + d\,x\,] + \mathsf{b}\,\mathsf{Tan}\,[\,c + d\,x\,]\,\right)^3}\,\,\mathrm{d}x$$

Optimal (type 3, 231 leaves, 6 steps):

$$\frac{a \ b^2}{2 \ \left(a^2-b^2\right)^2 \ d \ \left(b+a \ \text{Cos} \left[c+d \ x\right]\right)^2} - \frac{2 \ a \ b \ \left(a^2+b^2\right)}{\left(a^2-b^2\right)^3 \ d \ \left(b+a \ \text{Cos} \left[c+d \ x\right]\right)} - \frac{\left(a \ \left(a^2+3 \ b^2\right)-b \ \left(3 \ a^2+b^2\right) \ \text{Cos} \left[c+d \ x\right]\right)}{2 \ \left(a^2-b^2\right)^3 \ d} + \frac{\left(2 \ a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{4 \ \left(a+b\right)^4 \ d} + \frac{\left(2 \ a+b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{4 \ \left(a-b\right)^4 \ d} + \frac{\left(2 \ a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(2 \ a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(2 \ a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b^2\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Cos} \left[c+d \ x\right]\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Log} \left[c+d \ x\right]\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Log} \left[c+d \ x\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Log} \left[c+d \ x\right]}{\left(a^2-b\right)^4 \ d} + \frac{\left(a-b\right) \ \text{Log} \left[1-\text{Log} \left[c+d \ x\right]}{\left(a^2-b\right)^4 \ d$$

Result (type 3, 703 leaves):

$$\frac{a\,b^2\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)\,\text{Tan}\,[c+d\,x]^3}{2\,\left(-a+b\right)^2\,\left(a+b\right)^2\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{2\,a\,b\,\left(-i\,a+b\right)\,\left(i\,a+b\right)\,\left(b+a\,\text{Cos}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3}{\left(-a+b\right)^3\,\left(a+b\right)^3\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{2\,i\,\left(a^5+8\,a^3\,b^2+3\,a\,b^4\right)\,\left(c+d\,x\right)\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Tan}\,[c+d\,x]^3}{\left(a-b\right)^4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} - \\ \frac{i\,\left(2\,a-b\right)\,\text{ArcTan}\,[\text{Tan}\,[c+d\,x]]\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Tan}\,[c+d\,x]^3}{2\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} - \\ \frac{i\,\left(2\,a+b\right)\,\text{ArcTan}\,[\text{Tan}\,[c+d\,x]]\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Tan}\,[c+d\,x]^3}{2\,\left(-a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Csc}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{2\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(2\,a+b\right)\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Log}\left[\text{Cos}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\right]\,\text{Tan}\,[c+d\,x]^3}{4\,\left(-a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left((-a^5-8\,a^3\,b^2-3\,a\,b^4\right)\,\left(b+a\,\text{Cos}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(2\,a-b\right)\,\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Log}\left[\text{Sin}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\right]\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^4\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^3\,d\,\left(a\,\text{Sin}\,[c+d\,x]+b\,\text{Tan}\,[c+d\,x]\right)^3} + \\ \frac{\left(b+a\,\text{Cos}\,[c+d\,x]\right)^3\,\text{Sec}\left(\frac{1}{2}\,\left(c+d\,x\right)\right)^2\,\text{Tan}\,[c+d\,x]^3}{4\,\left(a+b\right)^3\,d\,\left(a\,\text{Sin}\,[$$

# Problem 272: Result more than twice size of optimal antiderivative.

Optimal (type 5, 264 leaves, 8 steps):

$$\frac{\left(a^2-2\,b^2\right)\,\text{Cos}\,[\,c+d\,x\,]^{\,-1+m}\,\text{Sin}\,[\,c+d\,x\,]}{d\,m\,\left(2+m\right)} = \\ \frac{2\,a\,b\,\text{Cos}\,[\,c+d\,x\,]^{\,m}\,\text{Sin}\,[\,c+d\,x\,]}{d\,\left(2+3\,m+m^2\right)} = \\ \frac{Cos\,[\,c+d\,x\,]^{\,-1+m}\,\left(b+a\,\text{Cos}\,[\,c+d\,x\,]\,\right)^{\,2}\,\text{Sin}\,[\,c+d\,x\,]}{d\,\left(2+m\right)} = \\ \left(\left(a^2\,\left(1-m\right)-b^2\,\left(2+m\right)\right)\,\text{Cos}\,[\,c+d\,x\,]^{\,-1+m}\,\text{Hypergeometric}\,2F1\left[\frac{1}{2}\,,\,\frac{1}{2}\,\left(-1+m\right)\,,\,\frac{1+m}{2}\,,\,\text{Cos}\,[\,c+d\,x\,]^{\,2}\right]}{Sin\,[\,c+d\,x\,]} \right) \\ Sin\,[\,c+d\,x\,] \right) / \left(d\,\left(1-m\right)\,m\,\left(2+m\right)\,\sqrt{Sin\,[\,c+d\,x\,]^{\,2}}\right) = \\ \left(2\,a\,b\,\text{Cos}\,[\,c+d\,x\,]^{\,m}\,\text{Hypergeometric}\,2F1\left[\frac{1}{2}\,,\,\frac{m}{2}\,,\,\frac{2+m}{2}\,,\,\text{Cos}\,[\,c+d\,x\,]^{\,2}\right]\,\text{Sin}\,[\,c+d\,x\,]\right) / \\ \left(d\,m\,\left(1+m\right)\,\sqrt{\text{Sin}\,[\,c+d\,x\,]^{\,2}}\right)$$

Result (type 5, 890 leaves):

# Problem 276: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x] \sin[x]^2}{a \cos[x] + b \sin[x]} dx$$

$$-\frac{a\,b^2\,x}{\left(a^2+b^2\right)^2}+\frac{a\,x}{2\,\left(a^2+b^2\right)}+\frac{a^2\,b\,\text{Log}\,[\,a\,\text{Cos}\,[\,x\,]\,+\,b\,\text{Sin}\,[\,x\,]\,\,]}{\left(a^2+b^2\right)^2}-\frac{a\,\text{Cos}\,[\,x\,]\,\,\text{Sin}\,[\,x\,]}{2\,\left(a^2+b^2\right)}+\frac{b\,\text{Sin}\,[\,x\,]\,^2}{2\,\left(a^2+b^2\right)}$$

#### Result (type 3, 153 leaves):

$$-\frac{1}{8 \left(a^2+b^2\right)^2} \\ \left(-2 \, a^3 \, x - 6 \, \dot{\mathbb{1}} \, a^2 \, b \, x + 6 \, a \, b^2 \, x + 2 \, \dot{\mathbb{1}} \, b^3 \, x - 2 \, \dot{\mathbb{1}} \, b \, \left(-3 \, a^2+b^2\right) \, \mathsf{ArcTan} \left[\mathsf{Tan} \left[x\right]\right] + 2 \, b \, \left(a^2+b^2\right) \, \mathsf{Cos} \left[2 \, x\right] - 2 \, \left(a^2+b^2\right) \, \left(a \, x + b \, \mathsf{Log} \left[a \, \mathsf{Cos} \left[x\right] + b \, \mathsf{Sin} \left[x\right]\right]\right) - 3 \, a^2 \, b \, \mathsf{Log} \left[\left(a \, \mathsf{Cos} \left[x\right] + b \, \mathsf{Sin} \left[x\right]\right)^2\right] + b^3 \, \mathsf{Log} \left[\left(a \, \mathsf{Cos} \left[x\right] + b \, \mathsf{Sin} \left[x\right]\right)^2\right] + 2 \, a^3 \, \mathsf{Sin} \left[2 \, x\right] + 2 \, a \, b^2 \, \mathsf{Sin} \left[2 \, x\right]\right)$$

### Problem 278: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x]^2 \sin[x]}{a \cos[x] + b \sin[x]} dx$$

#### Optimal (type 3, 93 leaves, 7 steps):

$$-\frac{a^2\,b\,x}{\left(a^2+b^2\right)^2} + \frac{b\,x}{2\,\left(a^2+b^2\right)} - \frac{a\,b^2\,Log\,[\,a\,Cos\,[\,x\,]\,+b\,Sin\,[\,x\,]\,\,]}{\left(a^2+b^2\right)^2} + \frac{b\,Cos\,[\,x\,]\,Sin\,[\,x\,]}{2\,\left(a^2+b^2\right)} + \frac{a\,Sin\,[\,x\,]^{\,2}}{2\,\left(a^2+b^2\right)}$$

#### Result (type 3, 82 leaves):

$$\begin{split} &\frac{1}{4\,\left(a^2+b^2\right)^2} \left(4\,\dot{\mathbb{1}}\,a\,b^2\,\text{ArcTan}\,[\text{Tan}\,[\,x\,]\,\,]\,-a\,\left(a^2+b^2\right)\,\text{Cos}\,[\,2\,x\,]\,\,-\\ &2\,b\,\left(\,\left(a+\dot{\mathbb{1}}\,b\right)^2\,x\,+\,a\,b\,\text{Log}\,\big[\,\left(a\,\text{Cos}\,[\,x\,]\,+\,b\,\text{Sin}\,[\,x\,]\,\right)^{\,2}\,\big]\,\right)\,+\,b\,\left(a^2+b^2\right)\,\text{Sin}\,[\,2\,x\,]\,\right) \end{split}$$

### Problem 280: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{Cos}[x]^2 \, \mathsf{Sin}[x]^3}{\mathsf{a}\, \mathsf{Cos}[x] + \mathsf{b}\, \mathsf{Sin}[x]} \, \mathrm{d} x$$

#### Optimal (type 3, 176 leaves, 13 steps):

$$\frac{a^2 \, b^3 \, x}{\left(a^2 + b^2\right)^3} - \frac{a^2 \, b \, x}{2 \, \left(a^2 + b^2\right)^2} + \frac{b \, x}{8 \, \left(a^2 + b^2\right)} - \frac{a^3 \, b^2 \, \text{Log} \left[a \, \text{Cos} \left[x\right] + b \, \text{Sin} \left[x\right]\right]}{\left(a^2 + b^2\right)^3} + \frac{a^2 \, b \, \text{Cos} \left[x\right] \, \text{Sin} \left[x\right]}{2 \, \left(a^2 + b^2\right)^2} + \frac{b \, \text{Cos} \left[x\right] \, \text{Sin} \left[x\right]}{8 \, \left(a^2 + b^2\right)} - \frac{b \, \text{Cos} \left[x\right]^3 \, \text{Sin} \left[x\right]}{4 \, \left(a^2 + b^2\right)} - \frac{a \, b^2 \, \text{Sin} \left[x\right]^2}{2 \, \left(a^2 + b^2\right)^2} + \frac{a \, \text{Sin} \left[x\right]^4}{4 \, \left(a^2 + b^2\right)}$$

#### Result (type 3, 178 leaves):

$$\frac{1}{32 \left(a^2+b^2\right)^3} \\ \left(-12 \, a^4 \, b \, x - 32 \, \dot{\mathbb{1}} \, a^3 \, b^2 \, x + 24 \, a^2 \, b^3 \, x + 4 \, b^5 \, x + 32 \, \dot{\mathbb{1}} \, a^3 \, b^2 \, \text{ArcTan[Tan[x]]} - 4 \, a \, \left(a^4-b^4\right) \, \text{Cos[2\,x]} + a^5 \, \text{Cos[4\,x]} + 2 \, a^3 \, b^2 \, \text{Cos[4\,x]} + a \, b^4 \, \text{Cos[4\,x]} - 16 \, a^3 \, b^2 \, \text{Log[} \left(a \, \text{Cos[x]} + b \, \text{Sin[x]} \right)^2 \right] + 8 \, a^4 \, b \, \text{Sin[2\,x]} + 8 \, a^2 \, b^3 \, \text{Sin[2\,x]} - a^4 \, b \, \text{Sin[4\,x]} - 2 \, a^2 \, b^3 \, \text{Sin[4\,x]} - b^5 \, \text{Sin[4\,x]} \right)$$

Problem 282: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x]^3 \sin[x]^2}{a \cos[x] + b \sin[x]} dx$$

Optimal (type 3, 175 leaves, 13 steps):

$$\frac{a^{3}\,b^{2}\,x}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a\,b^{2}\,x}{2\,\left(a^{2}+b^{2}\right)^{2}}+\frac{a\,x}{8\,\left(a^{2}+b^{2}\right)}-\frac{b\,Cos\,[\,x\,]^{\,4}}{4\,\left(a^{2}+b^{2}\right)}+\frac{a^{2}\,b^{3}\,Log\,[\,a\,Cos\,[\,x\,]\,+\,b\,Sin\,[\,x\,]\,\,]}{\left(a^{2}+b^{2}\right)^{\,3}}-\frac{a\,b^{2}\,Cos\,[\,x\,]\,Sin\,[\,x\,]}{2\,\left(a^{2}+b^{2}\right)^{\,2}}+\frac{a\,Cos\,[\,x\,]\,Sin\,[\,x\,]}{8\,\left(a^{2}+b^{2}\right)}-\frac{a\,Cos\,[\,x\,]^{\,3}\,Sin\,[\,x\,]}{4\,\left(a^{2}+b^{2}\right)}-\frac{a^{2}\,b\,Sin\,[\,x\,]^{\,2}}{2\,\left(a^{2}+b^{2}\right)^{\,2}}$$

Result (type 3, 287 leaves):

$$-\frac{1}{32\,\left(a^2+b^2\right)^3}\,\left(-4\,a^5\,x+4\,\dot{\mathbb{1}}\,a^4\,b\,x-24\,a^3\,b^2\,x-24\,\dot{\mathbb{1}}\,a^2\,b^3\,x+12\,a\,b^4\,x+\right.\\ \left.4\,\dot{\mathbb{1}}\,b^5\,x-4\,\dot{\mathbb{1}}\,b\,\left(a^4-6\,a^2\,b^2+b^4\right)\,\text{ArcTan[Tan[x]]}+4\,b\,\left(-a^4+b^4\right)\,\text{Cos[2\,x]}+\\ \left.a^4\,b\,\text{Cos[4\,x]}+2\,a^2\,b^3\,\text{Cos[4\,x]}+b^5\,\text{Cos[4\,x]}-4\,a^4\,b\,\text{Log[a\,Cos[x]}+b\,\text{Sin[x]]}-8\,a^2\,b^3\,\text{Log[a\,Cos[x]}+b\,\text{Sin[x]]}-4\,b^5\,\text{Log[a\,Cos[x]}+b\,\text{Sin[x]]}+\\ \left.2\,a^4\,b\,\text{Log}\Big[\left(a\,\text{Cos[x]}+b\,\text{Sin[x]}\right)^2\Big]-12\,a^2\,b^3\,\text{Log}\Big[\left(a\,\text{Cos[x]}+b\,\text{Sin[x]}\right)^2\Big]+\\ \left.2\,b^5\,\text{Log}\Big[\left(a\,\text{Cos[x]}+b\,\text{Sin[x]}\right)^2\Big]+8\,a^3\,b^2\,\text{Sin[2\,x]}+\\ \left.8\,a\,b^4\,\text{Sin[2\,x]}+a^5\,\text{Sin[4\,x]}+2\,a^3\,b^2\,\text{Sin[4\,x]}+a\,b^4\,\text{Sin[4\,x]}\right)$$

Problem 284: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Cos}[x] \mathsf{Sin}[x]}{\left(\mathsf{a} \mathsf{Cos}[x] + \mathsf{b} \mathsf{Sin}[x]\right)^2} \, \mathrm{d}x$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{2 \, a \, b \, x}{\left(a^2 + b^2\right)^2} - \frac{\left(a^2 - b^2\right) \, Log \left[a \, Cos \left[x\right] \, + b \, Sin \left[x\right]\right]}{\left(a^2 + b^2\right)^2} - \frac{b \, Sin \left[x\right]}{\left(a^2 + b^2\right) \, \left(a \, Cos \left[x\right] \, + b \, Sin \left[x\right]\right)}$$

Result (type 3, 144 leaves):

$$\left( a \, \mathsf{Cos} \, [\, x \,] \, \left( -2 \, \dot{\mathbb{1}} \, \left( a + \dot{\mathbb{1}} \, b \right)^2 \, x + \left( -a^2 + b^2 \right) \, \mathsf{Log} \left[ \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right)^2 \, \right] \right) \, + \\ b \, \left( 2 \, \left( a + \dot{\mathbb{1}} \, b \right) \, \left( a \, \left( -1 - \dot{\mathbb{1}} \, x \right) + b \, \left( \dot{\mathbb{1}} + x \right) \right) \, + \left( -a^2 + b^2 \right) \, \mathsf{Log} \left[ \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right)^2 \, \right] \right) \, \mathsf{Sin} \, [\, x \,] \, + \\ 2 \, \dot{\mathbb{1}} \, \left( a^2 - b^2 \right) \, \mathsf{ArcTan} \, [\, \mathsf{Tan} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \right) \, \left/ \, \left( 2 \, \left( a^2 + b^2 \right)^2 \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \right) \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Sin} \, [\, x \,] \, \right) \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, + b \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Cos} \, [\, x \,] \, \right) \, \left( a \, \mathsf{Co$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x] \sin[x]^3}{(a \cos[x] + b \sin[x])^2} dx$$

Optimal (type 3, 129 leaves, 17 steps):

$$\begin{split} & \frac{b \, \left(3 \, a^3 - a \, b^2\right) \, x}{\left(a^2 + b^2\right)^3} - \frac{a^2 \, \left(a^2 - 3 \, b^2\right) \, \text{Log} \left[a \, \text{Cos} \left[x\right] \, + b \, \text{Sin} \left[x\right]\right]}{\left(a^2 + b^2\right)^3} - \\ & \frac{a \, b \, \text{Cos} \left[x\right] \, \text{Sin} \left[x\right]}{\left(a^2 + b^2\right)^2} - \frac{\left(a^2 - b^2\right) \, \text{Sin} \left[x\right]^2}{2 \, \left(a^2 + b^2\right)^2} - \frac{a^2 \, b \, \text{Sin} \left[x\right]}{\left(a^2 + b^2\right)^2 \, \left(a \, \text{Cos} \left[x\right] \, + b \, \text{Sin} \left[x\right]\right)} \end{split}$$

#### Result (type 3, 226 leaves):

$$\frac{1}{4 \left( a^2 + b^2 \right)^3 \left( a \, \mathsf{Cos} \, [x] + b \, \mathsf{Sin} \, [x] \right)} \\ \left( 4 \, \dot{\mathbb{1}} \, a^2 \, \left( a^2 - 3 \, b^2 \right) \, \mathsf{Arc} \mathsf{Tan} \, [\mathsf{Tan} \, [x] \, ] \, \left( a \, \mathsf{Cos} \, [x] + b \, \mathsf{Sin} \, [x] \right) + a \, \mathsf{Cos} \, [x] \, \left( \left( a^4 - b^4 \right) \, \mathsf{Cos} \, [2 \, x] + 2 \, a \, \left( 2 \, \left( \dot{\mathbb{1}} \, a - b \right)^3 \, x - a \, \left( a^2 - 3 \, b^2 \right) \, \mathsf{Log} \left[ \, \left( a \, \mathsf{Cos} \, [x] + b \, \mathsf{Sin} \, [x] \, \right)^2 \right] - b \, \left( a^2 + b^2 \right) \, \mathsf{Sin} \, [2 \, x] \, \right) \right) - b \, \mathsf{Sin} \, [x] \, \left( \left( -a^4 + b^4 \right) \, \mathsf{Cos} \, [2 \, x] + 2 \, a \, \left( 2 \, \left( a^3 \, \left( 1 + \dot{\mathbb{1}} \, x \right) + a \, b^2 \, \left( 1 - 3 \, \dot{\mathbb{1}} \, x \right) - 3 \, a^2 \, b \, x + b^3 \, x \right) + a \, \left( a^2 - 3 \, b^2 \right) \, \mathsf{Log} \left[ \, \left( a \, \mathsf{Cos} \, [x] + b \, \mathsf{Sin} \, [x] \, \right)^2 \right] + b \, \left( a^2 + b^2 \right) \, \mathsf{Sin} \, [2 \, x] \, \right) \right) \right)$$

### Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x]^3 \sin[x]}{\left(a \cos[x] + b \sin[x]\right)^2} dx$$

#### Optimal (type 3, 128 leaves, 17 steps):

$$-\frac{a b (a^{2}-3 b^{2}) x}{\left(a^{2}+b^{2}\right)^{3}} - \frac{b^{2} (3 a^{2}-b^{2}) Log[a Cos[x]+b Sin[x]]}{\left(a^{2}+b^{2}\right)^{3}} + \frac{a b Cos[x] Sin[x]}{\left(a^{2}+b^{2}\right)^{2}} + \frac{\left(a^{2}-b^{2}\right) Sin[x]^{2}}{2 \left(a^{2}+b^{2}\right)^{2}} + \frac{a b^{2} Cos[x]}{\left(a^{2}+b^{2}\right)^{2} \left(a Cos[x]+b Sin[x]\right)}$$

### Result (type 3, 221 leaves):

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\frac{1}{4\,\left(a^2+b^2\right)^3\,\left(a\,\text{Cos}\left[x\right]+b\,\text{Sin}\left[x\right]\right)}
             \left(-4\pm b^2\left(-3\,a^2+b^2\right)\,\text{ArcTan}\left[\text{Tan}\left[x\right]\right]\,\left(a\,\text{Cos}\left[x\right]+b\,\text{Sin}\left[x\right]\right)\\ -a\,\text{Cos}\left[x\right]\,\left(\left(a^4-b^4\right)\,\text{Cos}\left[2\,x\right]+b\,\text{Cos}\left[2\,x\right]\right)\right)
                                                                       2 \ b \ \left(2 \ \left(a + i \ b\right)^3 \ x - b \ \left(-3 \ a^2 + b^2\right) \ Log\left[ \ \left(a \ Cos\left[x\right] \ + b \ Sin\left[x\right] \ \right)^2 \right] - a \ \left(a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \right) \ + b \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ x\right] \ \left(-3 \ a^2 + b^2\right) \ Sin\left[2 \ a^2 + b^2\right] \ Sin\left[2 \ a^2 + 
                                  b \sin[x] \left( (-a^4 + b^4) \cos[2x] + 2b \left( -2 (a + i b) (a^2 x - b^2 (i + x) + a (b + 2 i b x)) + a (b + 2 i b x) \right) + a (b + 2 i b x) \right)
                                                                                                             (-3 a^2 b + b^3) Log[(a Cos[x] + b Sin[x])^2] + a (a^2 + b^2) Sin[2x])
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# Problem 292: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cos[x]^3 \sin[x]^3}{\left(a \cos[x] + b \sin[x]\right)^2} dx$$

Optimal (type 3, 210 leaves, 48 steps):

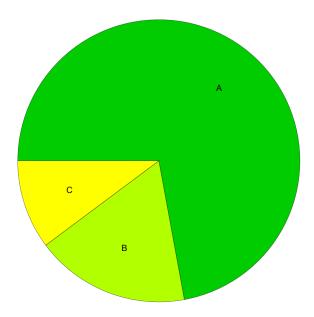
$$-\frac{3 \ a \ b \ \left(a^4-6 \ a^2 \ b^2+b^4\right) \ x}{4 \ \left(a^2+b^2\right)^4} - \frac{b^2 \cos \left[x\right]^4}{4 \ \left(a^2+b^2\right)^2} - \\ \frac{3 \ a^2 \ b^2 \ \left(a^2-b^2\right) \ Log \left[a \ Cos \left[x\right]+b \ Sin \left[x\right]\right]}{\left(a^2+b^2\right)^4} + \frac{a \ b \ \left(5 \ a^2-3 \ b^2\right) \ Cos \left[x\right] \ Sin \left[x\right]}{4 \ \left(a^2+b^2\right)^3} - \\ \frac{a \ b \ Cos \left[x\right]^3 \ Sin \left[x\right]}{2 \ \left(a^2+b^2\right)^2} - \frac{2 \ a^2 \ b^2 \ Sin \left[x\right]^2}{\left(a^2+b^2\right)^3} + \frac{a^2 \ Sin \left[x\right]^4}{4 \ \left(a^2+b^2\right)^2} - \frac{a^2 \ b^3 \ Sin \left[x\right]}{\left(a^2+b^2\right)^3 \ \left(a \ Cos \left[x\right]+b \ Sin \left[x\right]\right)}$$

#### Result (type 3, 409 leaves):

$$\frac{1}{32 \, \left(a^2 + b^2\right)^4} \left( -12 \, a \, b \, \left(a^2 - 3 \, b^2\right) \, \left(3 \, a^2 - b^2\right) \, x + 6 \, i \, \left(a^6 - 15 \, a^4 \, b^2 + 15 \, a^2 \, b^4 - b^6\right) \, x - 6 \, i \, \left(a^6 - 15 \, a^4 \, b^2 + 15 \, a^2 \, b^4 - b^6\right) \, ArcTan[Tan[x]] - 4 \, \left(a^2 + b^2\right) \, \left(a^4 - 6 \, a^2 \, b^2 + b^4\right) \, Cos[2 \, x] + \left(a^2 - b^2\right) \, \left(a^2 + b^2\right)^2 \, Cos[4 \, x] + 3 \, \left(a^6 - 15 \, a^4 \, b^2 + 15 \, a^2 \, b^4 - b^6\right) \, Log\left[\left(a \, Cos[x] + b \, Sin[x]\right)^2\right] + \frac{2 \, b \, \left(a^2 + b^2\right) \, \left(3 \, a^4 - 10 \, a^2 \, b^2 + 3 \, b^4\right) \, Sin[x]}{a \, Cos[x] + b \, Sin[x]} + \frac{2 \, b \, \left(a^2 + b^2\right)^2 \, \left(a \, Cos[x] \, \left(-2 \, i \, \left(a + i \, b\right)^2 \, x + \left(-a^2 + b^2\right) \, Log\left[\left(a \, Cos[x] + b \, Sin[x]\right)^2\right]\right) + b \, \left(2 \, \left(a + i \, b\right) \, \left(a \, \left(-1 - i \, x\right) + b \, \left(i + x\right)\right) + \left(-a^2 + b^2\right) \, Log\left[\left(a \, Cos[x] + b \, Sin[x]\right)^2\right]\right) \, Sin[x] + 2 \, i \, \left(a^2 - b^2\right) \, ArcTan[Tan[x]] \, \left(a \, Cos[x] + b \, Sin[x]\right)\right) \right) \right/ \\ \left(a \, Cos[x] + b \, Sin[x]\right) + 16 \, a \, b \, \left(a^4 - b^4\right) \, Sin[2 \, x] - 2 \, a \, b \, \left(a^2 + b^2\right)^2 \, Sin[4 \, x]\right)$$

# **Summary of Integration Test Results**

### 294 integration problems



- A 212 optimal antiderivatives
- B 52 more than twice size of optimal antiderivatives
- C 30 unnecessarily complex antiderivatives
- D 0 unable to integrate problems
- E 0 integration timeouts