

## Rubi 4.16.0 Hyperbolic Integration Test Results

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Test results for the 502 problems in "6.1.1  $(c+dx)^m (a+b \sinh)^n$ .m"

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Test results for the 102 problems in "6.1.3  $(ex)^m (a+b \sinh(c+dx^n))^p$ .m"

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Test results for the 33 problems in "6.1.4  $(d+ex)^m \sinh(a+bx+cx^2)^n$ .m"

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Test results for the 525 problems in "6.1.7  $\text{hyper}^m (a+b \sinh^n)^p$ .m"

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Test results for the 369 problems in "6.1.5 Hyperbolic sine functions.m"

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Test results for the 183 problems in "6.2.1  $(c+dx)^m (a+b \cosh)^n$ .m"

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Test results for the 111 problems in "6.2.2  $(ex)^m (a+b x^n)^p \cosh$ .m"

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Test results for the 68 problems in "6.2.3  $(ex)^m (a+b \cosh(c+dx^n))^p$ .m"

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Test results for the 33 problems in "6.2.4  $(d+ex)^m \cosh(a+bx+cx^2)^n$ .m"

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Test results for the 85 problems in "6.2.7  $\text{hyper}^m (a+b \cosh^n)^p$ .m"

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Test results for the 336 problems in "6.2.5 Hyperbolic cosine functions.m"

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Test results for the 77 problems in "6.3.1  $(c+dx)^m (a+b \tanh)^n$ .m"

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Problem 16: Unable to integrate problem.

$$\int (c + d x) (b \tanh[e + f x])^{5/2} dx$$

Optimal (type 4, 1392 leaves, 44 steps):

$$\begin{aligned}
& \frac{2 b^{5/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} - \frac{(-b)^{5/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{f} - \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2 f^2} + \frac{2 b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} + \\
& \frac{b^{5/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f} + \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 f^2} - \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} + \\
& \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} - \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} - \\
& \frac{b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} + \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{(-b)^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} - \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} + \\
& \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \frac{b^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \\
& \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} - \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} + \\
& \frac{(-b)^{5/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{4 b^2 d \sqrt{b \operatorname{Tanh}[e+f x]}}{3 f^2} - \frac{2 b (c+d x) (b \operatorname{Tanh}[e+f x])^{3/2}}{3 f}
\end{aligned}$$

Result (type 8, 137 leaves, 7 steps):

$$\frac{2 b^{5/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} + \frac{2 b^{5/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{3 f^2} - \frac{4 b^2 d \sqrt{b \operatorname{Tanh}[e+f x]}}{3 f^2} - \frac{2 b (c+d x) (b \operatorname{Tanh}[e+f x])^{3/2}}{3 f} + b^2 \operatorname{Unintegrable}\left[(c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}, x\right]$$

Problem 17: Unable to integrate problem.

$$\int (c+d x) (b \operatorname{Tanh}[e+f x])^{3/2} dx$$

Optimal (type 4, 1363 leaves, 43 steps):

$$\begin{aligned}
& - \frac{2 b^{3/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} - \frac{(-b)^{3/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{f} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2 f^2} + \\
& \frac{2 b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} + \frac{b^{3/2} (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f} + \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 f^2} - \\
& \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} + \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^2} - \\
& \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} - \frac{b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 f^2} + \\
& \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{(-b)^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} - \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 f^2} + \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \\
& \frac{b^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 f^2} + \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} - \\
& \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} + \frac{(-b)^{3/2} d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \frac{2 b(c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}}{f}
\end{aligned}$$

Result (type 8, 108 leaves, 6 steps):

$$- \frac{2 b^{3/2} d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} + \frac{2 b^{3/2} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} - \frac{2 b (c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}}{f} + b^2 \operatorname{Unintegrable}\left[\frac{c+d x}{\sqrt{b \operatorname{Tanh}[e+f x]}}, x\right]$$

Problem 18: Unable to integrate problem.

$$\int (c+d x) \sqrt{b \operatorname{Tanh}[e+f x]} dx$$

Optimal (type 4, 1280 leaves, 37 steps):

$$\begin{aligned}
& - \frac{\sqrt{-b} (c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right]}{f} - \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right]^2}{2 f^2} + \\
& \frac{\sqrt{b} (c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right]}{f} + \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right]^2}{2 f^2} - \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \tanh[e + f x]}}\right]}{f^2} + \\
& \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \tanh[e + f x]}}\right]}{f^2} - \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \tanh[e + f x]})}\right]}{2 f^2} - \\
& \frac{\sqrt{b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \tanh[e + f x]})}\right]}{2 f^2} + \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}}\right]}{f^2} - \\
& \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 (\sqrt{b} - \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 (\sqrt{b} + \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right)}\right]}{2 f^2} - \\
& \frac{\sqrt{-b} d \operatorname{ArcTanh}\left[\frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 + \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}}\right]}{f^2} - \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \tanh[e + f x]}}\right]}{2 f^2} - \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \tanh[e + f x]}}\right]}{2 f^2} + \\
& \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \tanh[e + f x]})}\right]}{4 f^2} + \frac{\sqrt{b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \tanh[e + f x]})}\right]}{4 f^2} + \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}}\right]}{2 f^2} - \\
& \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2 (\sqrt{b} - \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} - \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 + \frac{2 (\sqrt{b} + \sqrt{b \tanh[e + f x]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}\right)}\right]}{4 f^2} + \frac{\sqrt{-b} d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{\sqrt{b \tanh[e + f x]}}{\sqrt{-b}}}\right]}{2 f^2}
\end{aligned}$$

Result (type 8, 20 leaves, 0 steps):

$\operatorname{Unintegrable}\left[(c + d x) \sqrt{b \tanh[e + f x]}, x\right]$

### Problem 19: Unable to integrate problem.

$$\int \frac{c + d x}{\sqrt{b \tanh [e + f x]}} dx$$

Optimal (type 4, 1280 leaves, 37 steps):



$$\begin{aligned}
& - \frac{(c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right]}{\sqrt{-b} f} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right]^2}{2 \sqrt{-b} f^2} + \\
& \frac{(c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right]}{\sqrt{b} f} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right]^2}{2 \sqrt{b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}[e + f x]}}\right]}{\sqrt{b} f^2} + \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]}}\right]}{\sqrt{b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}\right]}{2 \sqrt{b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}\right]}{2 \sqrt{b} f^2} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}}\right]}{\sqrt{-b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 (\sqrt{b} - \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right)}\right]}{2 \sqrt{-b} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right)}\right]}{2 \sqrt{-b} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 + \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}}\right]}{\sqrt{-b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}[e + f x]}}\right]}{2 \sqrt{b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]}}\right]}{2 \sqrt{b} f^2} + \\
& \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} - \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} - \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b} (\sqrt{-b} + \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}\right]}{4 \sqrt{b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}}\right]}{2 \sqrt{-b} f^2} - \\
& \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2 (\sqrt{b} - \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right)}\right]}{4 \sqrt{-b} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1 + \frac{2 (\sqrt{b} + \sqrt{b \operatorname{Tanh}[e + f x]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}\right)}\right]}{4 \sqrt{-b} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{\sqrt{b \operatorname{Tanh}[e + f x]}}{\sqrt{-b}}}\right]}{2 \sqrt{-b} f^2}
\end{aligned}$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{c + d x}{\sqrt{b \operatorname{Tanh}[e + f x]}}, x\right]$$

## Problem 20: Unable to integrate problem.

$$\int \frac{c + d x}{(b \tanh[e + f x])^{3/2}} dx$$

Optimal (type 4, 1365 leaves, 43 steps):

$$\begin{aligned}
& \frac{2 d \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} - \frac{(c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{(-b)^{3/2} f} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{2(-b)^{3/2} f^2} + \frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} + \\
& \frac{(c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{2 b^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^2} + \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 b^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{2 b^{3/2} f^2} + \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2(-b)^{3/2} f^2} - \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{2(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 b^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{2 b^{3/2} f^2} + \\
& \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 b^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{4 b^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2(-b)^{3/2} f^2} - \\
& \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4(-b)^{3/2} f^2} - \frac{d \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{4(-b)^{3/2} f^2} + \frac{d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{2(-b)^{3/2} f^2} - \frac{2(c+d x)}{b f \sqrt{b \operatorname{Tanh}[e+f x]}}
\end{aligned}$$

Result (type 8, 110 leaves, 6 steps):

$$\frac{2 \operatorname{d} \operatorname{ArcTan}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} + \frac{2 \operatorname{d} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} - \frac{2 (c+d x)}{b f \sqrt{b \operatorname{Tanh}[e+f x]}} + \frac{\operatorname{Unintegrable}\left[(c+d x) \sqrt{b \operatorname{Tanh}[e+f x]}, x\right]}{b^2}$$

Problem 21: Result valid but suboptimal antiderivative.

$$\int (c+d x)^2 (b \operatorname{Tanh}[e+f x])^{3/2} dx$$

Optimal (type 8, 1340 leaves, 38 steps):

$$\begin{aligned} & \frac{4 (-b)^{3/2} d (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{f^2} + \frac{2 (-b)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{f^3} + \\ & \frac{4 b^{3/2} d (c+d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{f^2} + \frac{2 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{f^3} - \frac{4 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^3} + \\ & \frac{4 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^3} - \frac{2 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b}) (\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{f^3} - \\ & \frac{2 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b} (\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b}) (\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{f^3} - \frac{4 (-b)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^3} + \\ & \frac{2 (-b)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 (\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b}) \left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{f^3} + \frac{2 (-b)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 (\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b}) \left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{f^3} + \\ & \frac{4 (-b)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{f^3} - \frac{2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^3} - \\ & \frac{2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{f^3} + \frac{b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b} (\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b}) (\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{f^3} + \end{aligned}$$

$$\begin{aligned}
& \frac{b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{b}(\sqrt{-b} + \sqrt{b \tanh[e+fx]})}{(\sqrt{-b} + \sqrt{b}) (\sqrt{b} + \sqrt{b \tanh[e+fx]})}\right]}{f^3} - \frac{2(-b)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{b \tanh[e+fx]}}{\sqrt{-b}}}\right]}{f^3} + \\
& \frac{(-b)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2(\sqrt{b} - \sqrt{b \tanh[e+fx]})}{(\sqrt{-b} + \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e+fx]}}{\sqrt{-b}}\right)}\right]}{f^3} + \frac{(-b)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 + \frac{2(\sqrt{b} + \sqrt{b \tanh[e+fx]})}{(\sqrt{-b} - \sqrt{b}) \left(1 - \frac{\sqrt{b \tanh[e+fx]}}{\sqrt{-b}}\right)}\right]}{f^3} - \\
& \frac{2(-b)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{\sqrt{b \tanh[e+fx]}}{\sqrt{-b}}}\right]}{f^3} - \frac{2b(c+dx)^2 \sqrt{b \tanh[e+fx]}}{f} + b^2 \operatorname{Unintegrable}\left[\frac{(c+dx)^2}{\sqrt{b \tanh[e+fx]}}, x\right]
\end{aligned}$$

Result (type 8, 79 leaves, 1 step):

$$-\frac{2b(c+dx)^2 \sqrt{b \tanh[e+fx]}}{f} + b^2 \operatorname{Unintegrable}\left[\frac{(c+dx)^2}{\sqrt{b \tanh[e+fx]}}, x\right] + \frac{4bd \operatorname{Unintegrable}\left[(c+dx) \sqrt{b \tanh[e+fx]}, x\right]}{f}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{(c+dx)^2}{(b \tanh[e+fx])^{3/2}} dx$$

Optimal (type 8, 1342 leaves, 38 steps):

$$\begin{aligned}
& \frac{4 d (c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]}{(-b)^{3/2} f^2} + \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right]^2}{(-b)^{3/2} f^3} + \frac{4 d (c + d x) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]}{b^{3/2} f^2} + \\
& \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right]^2}{b^{3/2} f^3} - \frac{4 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^3} + \frac{4 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^3} - \\
& \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{b^{3/2} f^3} - \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{b^{3/2} f^3} - \\
& \frac{4 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^3} + \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{(-b)^{3/2} f^3} + \\
& \frac{2 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{(-b)^{3/2} f^3} + \frac{4 d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^3} - \frac{2 d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^3} - \\
& \frac{2 d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}}{\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]}}\right]}{b^{3/2} f^3} + \frac{d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{b^{3/2} f^3} + \frac{d^2 \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{b}(\sqrt{-b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}\right]}{b^{3/2} f^3} - \\
& \frac{2 d^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^3} + \frac{d^2 \operatorname{PolyLog}\left[2, 1-\frac{2(\sqrt{b}-\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}+\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{(-b)^{3/2} f^3} + \frac{d^2 \operatorname{PolyLog}\left[2, 1+\frac{2(\sqrt{b}+\sqrt{b \operatorname{Tanh}[e+f x]})}{(\sqrt{-b}-\sqrt{b})\left(1-\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}\right)}\right]}{(-b)^{3/2} f^3} - \\
& \frac{2 d^2 \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\frac{\sqrt{b \operatorname{Tanh}[e+f x]}}{\sqrt{-b}}}\right]}{(-b)^{3/2} f^3} - \frac{2 (c + d x)^2}{b f \sqrt{b \operatorname{Tanh}[e+f x]}} + \frac{\operatorname{Unintegrable}\left[(c + d x)^2 \sqrt{b \operatorname{Tanh}[e+f x]}, x\right]}{b^2}
\end{aligned}$$

Result (type 8, 83 leaves, 1 step):

$$- \frac{2 (c + d x)^2}{b f \sqrt{b \operatorname{Tanh}[e+f x]}} + \frac{4 d \operatorname{Unintegrable}\left[\frac{c+d x}{\sqrt{b \operatorname{Tanh}[e+f x]}}, x\right]}{b f} + \frac{\operatorname{Unintegrable}\left[(c + d x)^2 \sqrt{b \operatorname{Tanh}[e+f x]}, x\right]}{b^2}$$

## Test results for the 263 problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.m"

## Test results for the 247 problems in "6.3.2 Hyperbolic tangent functions.m"

Problem 146: Unable to integrate problem.

$$\int x^3 \tanh[a + 2 \log[x]] \, dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} e^{-2a} \log[1 + e^{2a} x^4]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[x^3 Tanh[a + 2 Log[x]], x]

Problem 147: Unable to integrate problem.

$$\int x^2 \tanh[a + 2 \log[x]] \, dx$$

Optimal (type 3, 151 leaves, 11 steps):

$$\frac{x^3}{3} + \frac{e^{-3a/2} \operatorname{ArcTan}\left[\frac{1 - \sqrt{2} e^{a/2} x}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{e^{-3a/2} \operatorname{ArcTan}\left[\frac{1 + \sqrt{2} e^{a/2} x}{\sqrt{2}}\right]}{\sqrt{2}} - \frac{e^{-3a/2} \log\left[\frac{1 - \sqrt{2} e^{a/2} x + e^a x^2}{2\sqrt{2}}\right]}{2\sqrt{2}} + \frac{e^{-3a/2} \log\left[\frac{1 + \sqrt{2} e^{a/2} x + e^a x^2}{2\sqrt{2}}\right]}{2\sqrt{2}}$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[x^2 Tanh[a + 2 Log[x]], x]

Problem 148: Unable to integrate problem.

$$\int x \tanh[a + 2 \log[x]] \, dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTan}[e^a x^2]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[x Tanh[a + 2 Log[x]], x]

### Problem 149: Unable to integrate problem.

$$\int \text{Tanh}[a + 2 \text{Log}[x]] \, dx$$

Optimal (type 3, 145 leaves, 11 steps):

$$x + \frac{e^{-a/2} \text{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{\sqrt{2}} - \frac{e^{-a/2} \text{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{-a/2} \text{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}} - \frac{e^{-a/2} \text{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}}$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate[Tanh[a + 2 Log[x]], x]

### Problem 151: Unable to integrate problem.

$$\int \frac{\text{Tanh}[a + 2 \text{Log}[x]]}{x^2} \, dx$$

Optimal (type 3, 147 leaves, 11 steps):

$$\frac{1}{x} - \frac{e^{a/2} \text{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{a/2} \text{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{\sqrt{2}} + \frac{e^{a/2} \text{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}} - \frac{e^{a/2} \text{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{2\sqrt{2}}$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[ $\frac{\text{Tanh}[a + 2 \text{Log}[x]]}{x^2}$ , x]

### Problem 152: Unable to integrate problem.

$$\int \frac{\text{Tanh}[a + 2 \text{Log}[x]]}{x^3} \, dx$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{1}{2x^2} + e^a \text{ArcTan}[e^a x^2]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[ $\frac{\text{Tanh}[a + 2 \text{Log}[x]]}{x^3}$ , x]



### Problem 153: Unable to integrate problem.

$$\int x^3 \tanh[a + 2 \log[x]]^2 dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{e^{-2a}}{1 + e^{2a} x^4} - e^{-2a} \log[1 + e^{2a} x^4]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate[x<sup>3</sup> Tanh[a + 2 Log[x]]<sup>2</sup>, x]

### Problem 154: Unable to integrate problem.

$$\int x^2 \tanh[a + 2 \log[x]]^2 dx$$

Optimal (type 3, 173 leaves, 12 steps):

$$\begin{aligned} & \frac{x^3}{3} + \frac{x^3}{1 + e^{2a} x^4} + \frac{3 e^{-3a/2} \operatorname{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2 \sqrt{2}} - \frac{3 e^{-3a/2} \operatorname{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2 \sqrt{2}} - \\ & \frac{3 e^{-3a/2} \log[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4 \sqrt{2}} + \frac{3 e^{-3a/2} \log[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4 \sqrt{2}} \end{aligned}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate[x<sup>2</sup> Tanh[a + 2 Log[x]]<sup>2</sup>, x]

### Problem 155: Unable to integrate problem.

$$\int x \tanh[a + 2 \log[x]]^2 dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 + e^{2a} x^4} - e^{-a} \operatorname{ArcTan}[e^a x^2]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[x Tanh[a + 2 Log[x]]<sup>2</sup>, x]

## Problem 156: Unable to integrate problem.

$$\int \text{Tanh}[a + 2 \text{Log}[x]]^2 dx$$

Optimal (type 3, 165 leaves, 13 steps):

$$x + \frac{x}{1 + e^{2a} x^4} + \frac{e^{-a/2} \text{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{e^{-a/2} \text{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2\sqrt{2}} + \frac{e^{-a/2} \text{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}} - \frac{e^{-a/2} \text{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}}$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[Tanh[a + 2 Log[x]]^2, x]

## Problem 158: Unable to integrate problem.

$$\int \frac{\text{Tanh}[a + 2 \text{Log}[x]]^2}{x^2} dx$$

Optimal (type 3, 190 leaves, 12 steps):

$$-\frac{1}{x(1 + e^{2a} x^4)} - \frac{2e^{2a} x^3}{1 + e^{2a} x^4} + \frac{e^{a/2} \text{ArcTan}[1 - \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{e^{a/2} \text{ArcTan}[1 + \sqrt{2} e^{a/2} x]}{2\sqrt{2}} - \frac{e^{a/2} \text{Log}[1 - \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}} + \frac{e^{a/2} \text{Log}[1 + \sqrt{2} e^{a/2} x + e^a x^2]}{4\sqrt{2}}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate[\frac{\text{Tanh}[a + 2 \text{Log}[x]]^2}{x^2}, x]

## Problem 159: Unable to integrate problem.

$$\int \frac{\text{Tanh}[a + 2 \text{Log}[x]]^2}{x^3} dx$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\frac{1}{2x^2(1 + e^{2a} x^4)} - \frac{3e^{2a} x^2}{2(1 + e^{2a} x^4)} - e^a \text{ArcTan}[e^a x^2]$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Tanh}[a + 2 \text{Log}[x]]^2}{x^3}, x\right]$$

### Problem 160: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}[a + 2 \text{Log}[x]] dx$$

Optimal (type 5, 60 leaves, 3 steps):

$$\frac{(e x)^{1+m}}{e (1+m)} - \frac{2 (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, -e^{2a} x^4\right]}{e (1+m)}$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \text{Tanh}[a + 2 \text{Log}[x]], x\right]$$

### Problem 161: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}[a + 2 \text{Log}[x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{(e x)^{1+m}}{e (1+m)} + \frac{(e x)^{1+m}}{e (1+e^{2a} x^4)} - \frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, -e^{2a} x^4\right]}{e}$$

Result (type 8, 17 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \text{Tanh}[a + 2 \text{Log}[x]]^2, x\right]$$

### Problem 162: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}[a + 2 \text{Log}[x]]^3 dx$$

Optimal (type 5, 176 leaves, 5 steps):

$$\frac{(3+m)(5+m)(e x)^{1+m}}{8 e (1+m)} - \frac{(e x)^{1+m} (1 - e^{2a} x^4)^2}{4 e (1 + e^{2a} x^4)^2} - \frac{e^{-2a} (e x)^{1+m} (e^{2a} (3-m) + e^{4a} (5+m) x^4)}{8 e (1 + e^{2a} x^4)} - \frac{(9 + 2m + m^2) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, -e^{2a} x^4\right]}{4 e (1+m)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate[(e x)<sup>m</sup> Tanh[a + 2 Log[x]]<sup>3</sup>, x]

### Problem 163: Unable to integrate problem.

$$\int \text{Tanh}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x (1 - e^{2a} x^{2b})^{-p} (-1 + e^{2a} x^{2b})^p \text{AppellF1}\left[\frac{1}{2b}, -p, p, \frac{1}{2} \left(2 + \frac{1}{b}\right), e^{2a} x^{2b}, -e^{2a} x^{2b}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[Tanh[a + b Log[x]]<sup>p</sup>, x]

### Problem 164: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{(e x)^{1+m} (1 - e^{2a} x^{2b})^{-p} (-1 + e^{2a} x^{2b})^p \text{AppellF1}\left[\frac{1+m}{2b}, -p, p, 1 + \frac{1+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right]}{e (1+m)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate[(e x)<sup>m</sup> Tanh[a + b Log[x]]<sup>p</sup>, x]

### Problem 165: Unable to integrate problem.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{2}\right]^p dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2^{-p} e^{-2a} (-1 + e^{2a} x)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 - e^{2a} x)\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate[Tanh[ $\frac{1}{2} (2a + \text{Log}[x])$ ]<sup>p</sup>, x]

### Problem 166: Unable to integrate problem.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{4}\right]^p dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$e^{-4a} \left(-1 + e^{2a} \sqrt{x}\right)^{1+p} \left(1 + e^{2a} \sqrt{x}\right)^{1-p} - \frac{2^{1-p} e^{-4a} p \left(-1 + e^{2a} \sqrt{x}\right)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} \left(1 - e^{2a} \sqrt{x}\right)\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

$$\text{CannotIntegrate}\left[\text{Tanh}\left[\frac{1}{4} (4a + \text{Log}[x])\right]^p, x\right]$$

### Problem 167: Unable to integrate problem.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{6}\right]^p dx$$

Optimal (type 5, 158 leaves, 5 steps):

$$\frac{-e^{-6a} p \left(-1 + e^{2a} x^{1/3}\right)^{1+p} \left(1 + e^{2a} x^{1/3}\right)^{1-p} + e^{-4a} \left(-1 + e^{2a} x^{1/3}\right)^{1+p} \left(1 + e^{2a} x^{1/3}\right)^{1-p} x^{1/3} + 2^{-p} e^{-6a} (1 + 2p^2) \left(-1 + e^{2a} x^{1/3}\right)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} \left(1 - e^{2a} x^{1/3}\right)\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

$$\text{CannotIntegrate}\left[\text{Tanh}\left[\frac{1}{6} (6a + \text{Log}[x])\right]^p, x\right]$$

### Problem 168: Unable to integrate problem.

$$\int \text{Tanh}\left[a + \frac{\text{Log}[x]}{8}\right]^p dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\frac{\frac{1}{3} e^{-12a} \left(-1 + e^{2a} x^{1/4}\right)^{1+p} \left(1 + e^{2a} x^{1/4}\right)^{1-p} \left(e^{4a} (3 + 2p^2) - 2e^{6a} p x^{1/4}\right) + e^{-4a} \left(-1 + e^{2a} x^{1/4}\right)^{1+p} \left(1 + e^{2a} x^{1/4}\right)^{1-p} \sqrt{x} - 2^{2-p} e^{-8a} p (2 + p^2) \left(-1 + e^{2a} x^{1/4}\right)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} \left(1 - e^{2a} x^{1/4}\right)\right]}{3 (1+p)}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate[ $\text{Tanh}\left[\frac{1}{8}(8a + \text{Log}[x])\right]^p, x]$

### Problem 169: Unable to integrate problem.

$$\int \text{Tanh}[a + \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^2)^{-p} (-1 + e^{2a} x^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate[ $\text{Tanh}[a + \text{Log}[x]]^p, x]$

### Problem 170: Unable to integrate problem.

$$\int \text{Tanh}[a + 2 \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^4)^{-p} (-1 + e^{2a} x^4)^p \text{AppellF1}\left[\frac{1}{4}, -p, p, \frac{5}{4}, e^{2a} x^4, -e^{2a} x^4\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[ $\text{Tanh}[a + 2 \text{Log}[x]]^p, x]$

### Problem 171: Unable to integrate problem.

$$\int \text{Tanh}[a + 3 \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x (1 - e^{2a} x^6)^{-p} (-1 + e^{2a} x^6)^p \text{AppellF1}\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^6, -e^{2a} x^6\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[ $\text{Tanh}[a + 3 \text{Log}[x]]^p, x]$

### Problem 172: Unable to integrate problem.

$$\int x^3 \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 59 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4 \operatorname{Hypergeometric2F1}\left[1, \frac{2}{b d n}, 1 + \frac{2}{b d n}, -e^{2 a d} \left(c x^n\right)^{2 b d}\right]$$

Result (type 8, 19 leaves, 0 steps):

`CannotIntegrate` $\left[x^3 \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 173: Unable to integrate problem.

$$\int x^2 \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3} x^3 \operatorname{Hypergeometric2F1}\left[1, \frac{3}{2 b d n}, 1 + \frac{3}{2 b d n}, -e^{2 a d} \left(c x^n\right)^{2 b d}\right]$$

Result (type 8, 19 leaves, 0 steps):

`CannotIntegrate` $\left[x^2 \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 174: Unable to integrate problem.

$$\int x \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$\frac{x^2}{2} - x^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, -e^{2 a d} \left(c x^n\right)^{2 b d}\right]$$

Result (type 8, 17 leaves, 0 steps):

`CannotIntegrate` $\left[x \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 175: Unable to integrate problem.

$$\int \operatorname{Tanh}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 53 leaves, 4 steps):

$$x - 2 \times \text{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right], x\right]$$

Problem 177: Unable to integrate problem.

$$\int \frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^2} dx$$

Optimal (type 5, 59 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \text{Hypergeometric2F1}\left[1, -\frac{1}{2 b d n}, 1 - \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^2}, x\right]$$

Problem 178: Unable to integrate problem.

$$\int \frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^3} dx$$

Optimal (type 5, 56 leaves, 4 steps):

$$-\frac{1}{2 x^2} + \frac{\text{Hypergeometric2F1}\left[1, -\frac{1}{b d n}, 1 - \frac{1}{b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{x^2}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]}{x^3}, x\right]$$

Problem 179: Unable to integrate problem.

$$\int x^3 \text{Tanh}\left[d (a + b \text{Log}[c x^n])\right]^2 dx$$

Optimal (type 5, 133 leaves, 5 steps):



$$\frac{1}{4} \left( 1 + \frac{4}{b d n} \right) x^4 + \frac{x^4 \left( 1 - e^{2 a d} (c x^n)^{2 b d} \right)}{b d n \left( 1 + e^{2 a d} (c x^n)^{2 b d} \right)} - \frac{2 x^4 \text{Hypergeometric2F1} \left[ 1, \frac{2}{b d n}, 1 + \frac{2}{b d n}, -e^{2 a d} (c x^n)^{2 b d} \right]}{b d n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $\left[ x^3 \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2, x \right]$

**Problem 180: Unable to integrate problem.**

$$\int x^2 \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2 dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{1}{3} \left( 1 + \frac{3}{b d n} \right) x^3 + \frac{x^3 \left( 1 - e^{2 a d} (c x^n)^{2 b d} \right)}{b d n \left( 1 + e^{2 a d} (c x^n)^{2 b d} \right)} - \frac{2 x^3 \text{Hypergeometric2F1} \left[ 1, \frac{3}{2 b d n}, 1 + \frac{3}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d} \right]}{b d n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $\left[ x^2 \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2, x \right]$

**Problem 181: Unable to integrate problem.**

$$\int x \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2 dx$$

Optimal (type 5, 131 leaves, 5 steps):

$$\frac{1}{2} \left( 1 + \frac{2}{b d n} \right) x^2 + \frac{x^2 \left( 1 - e^{2 a d} (c x^n)^{2 b d} \right)}{b d n \left( 1 + e^{2 a d} (c x^n)^{2 b d} \right)} - \frac{2 x^2 \text{Hypergeometric2F1} \left[ 1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, -e^{2 a d} (c x^n)^{2 b d} \right]}{b d n}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $\left[ x \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2, x \right]$

**Problem 182: Unable to integrate problem.**

$$\int \tanh \left[ d \left( a + b \log [c x^n] \right) \right]^2 dx$$

Optimal (type 5, 127 leaves, 5 steps):

$$\left(1 + \frac{1}{b d n}\right) x + \frac{x \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)}{b d n \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 x \text{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{b d n}$$

Result (type 8, 17 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right]^2, x\right]$$

Problem 184: Unable to integrate problem.

$$\int \frac{\text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right]^2}{x^2} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$-\frac{1 - \frac{1}{b d n}}{x} + \frac{1 - e^{2 a d} (c x^n)^{2 b d}}{b d n x \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 \text{Hypergeometric2F1}\left[1, -\frac{1}{2 b d n}, 1 - \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{b d n x}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right]^2}{x^2}, x\right]$$

Problem 185: Unable to integrate problem.

$$\int \frac{\text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right]^2}{x^3} dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{2 - b d n}{2 b d n x^2} + \frac{1 - e^{2 a d} (c x^n)^{2 b d}}{b d n x^2 \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 \text{Hypergeometric2F1}\left[1, -\frac{1}{b d n}, 1 - \frac{1}{b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{b d n x^2}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right]^2}{x^3}, x\right]$$

Problem 189: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}\left[d \left(a + b \text{Log}[c x^n]\right)\right] dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{(e x)^{1+m} - 2 (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2 b d n}, 1 + \frac{1+m}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{e (1+m)}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \tanh\left[d (a + b \log[c x^n])\right], x\right]$$

### Problem 190: Unable to integrate problem.

$$\int (e x)^m \tanh\left[d (a + b \log[c x^n])\right]^2 dx$$

Optimal (type 5, 169 leaves, 5 steps):

$$\frac{(1+m+b d n) (e x)^{1+m}}{b d e (1+m) n} + \frac{(e x)^{1+m} (1 - e^{2 a d} (c x^n)^{2 b d})}{b d e n (1 + e^{2 a d} (c x^n)^{2 b d})} - \frac{2 (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2 b d n}, 1 + \frac{1+m}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{b d e n}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \tanh\left[d (a + b \log[c x^n])\right]^2, x\right]$$

### Problem 191: Unable to integrate problem.

$$\int (e x)^m \tanh\left[d (a + b \log[c x^n])\right]^3 dx$$

Optimal (type 5, 307 leaves, 6 steps):

$$\frac{(1+m+b d n) (1+m+2 b d n) (e x)^{1+m}}{2 b^2 d^2 e (1+m) n^2} - \frac{(e x)^{1+m} (1 - e^{2 a d} (c x^n)^{2 b d})^2}{2 b d e n (1 + e^{2 a d} (c x^n)^{2 b d})^2} + \frac{e^{-2 a d} (e x)^{1+m} \left( \frac{e^{2 a d} (1+m-2 b d n)}{n} - \frac{e^{4 a d} (1+m+2 b d n) (c x^n)^{2 b d}}{n} \right)}{2 b^2 d^2 e n (1 + e^{2 a d} (c x^n)^{2 b d})} - \frac{(1+2 m+m^2+2 b^2 d^2 n^2) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2 b d n}, 1 + \frac{1+m}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]}{b^2 d^2 e (1+m) n^2}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \tanh\left[d (a + b \log[c x^n])\right]^3, x\right]$$

### Problem 192: Unable to integrate problem.

$$\int \text{Tanh}\left[d \left(a + b \text{Log}\left[c x^n\right]\right)\right]^p dx$$

Optimal (type 6, 115 leaves, 4 steps):

$$x \left(1 - e^{2ad} (c x^n)^{2bd}\right)^{-p} \left(-1 + e^{2ad} (c x^n)^{2bd}\right)^p \text{AppellF1}\left[\frac{1}{2bdn}, -p, p, 1 + \frac{1}{2bdn}, e^{2ad} (c x^n)^{2bd}, -e^{2ad} (c x^n)^{2bd}\right]$$

Result (type 8, 17 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\text{Tanh}\left[d \left(a + b \text{Log}\left[c x^n\right]\right)\right]^p, x\right]$$

### Problem 193: Unable to integrate problem.

$$\int (e x)^m \text{Tanh}\left[d \left(a + b \text{Log}\left[c x^n\right]\right)\right]^p dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e(1+m)} (e x)^{1+m} \left(1 - e^{2ad} (c x^n)^{2bd}\right)^{-p} \left(-1 + e^{2ad} (c x^n)^{2bd}\right)^p \text{AppellF1}\left[\frac{1+m}{2bdn}, -p, p, 1 + \frac{1+m}{2bdn}, e^{2ad} (c x^n)^{2bd}, -e^{2ad} (c x^n)^{2bd}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \text{Tanh}\left[d \left(a + b \text{Log}\left[c x^n\right]\right)\right]^p, x\right]$$

Test results for the 61 problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

Test results for the 53 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Test results for the 224 problems in "6.4.2 Hyperbolic cotangent functions.m"

### Problem 151: Unable to integrate problem.

$$\int x^3 \text{Coth}\left[a + 2 \text{Log}\left[x\right]\right] dx$$

Optimal (type 3, 30 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{1}{2} e^{-2a} \operatorname{Log}[1 - e^{2a} x^4]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[ $x^3 \operatorname{Coth}[a + 2 \operatorname{Log}[x]]$ ], x]

### Problem 152: Unable to integrate problem.

$$\int x^2 \operatorname{Coth}[a + 2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{x^3}{3} + e^{-3a/2} \operatorname{ArcTan}[e^{a/2} x] - e^{-3a/2} \operatorname{ArcTanh}[e^{a/2} x]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[ $x^2 \operatorname{Coth}[a + 2 \operatorname{Log}[x]]$ ], x]

### Problem 153: Unable to integrate problem.

$$\int x \operatorname{Coth}[a + 2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTanh}[e^a x^2]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[ $x \operatorname{Coth}[a + 2 \operatorname{Log}[x]]$ ], x]

### Problem 154: Unable to integrate problem.

$$\int \operatorname{Coth}[a + 2 \operatorname{Log}[x]] \, dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$x - e^{-a/2} \operatorname{ArcTan}[e^{a/2} x] - e^{-a/2} \operatorname{ArcTanh}[e^{a/2} x]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate[ $\operatorname{Coth}[a + 2 \operatorname{Log}[x]]$ ], x]

## Problem 156: Unable to integrate problem.

$$\int \frac{\text{Coth}[a + 2 \text{Log}[x]]}{x^2} dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{1}{x} + e^{a/2} \text{ArcTan}[e^{a/2} x] - e^{a/2} \text{ArcTanh}[e^{a/2} x]$$

Result (type 8, 13 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}[a + 2 \text{Log}[x]]}{x^2}, x\right]$$

## Problem 157: Unable to integrate problem.

$$\int \frac{\text{Coth}[a + 2 \text{Log}[x]]}{x^3} dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\frac{1}{2x^2} - e^a \text{ArcTanh}[e^a x^2]$$

Result (type 8, 13 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}[a + 2 \text{Log}[x]]}{x^3}, x\right]$$

## Problem 158: Unable to integrate problem.

$$\int x^3 \text{Coth}[a + 2 \text{Log}[x]]^2 dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{e^{-2a}}{1 - e^{2a} x^4} + e^{-2a} \text{Log}[1 - e^{2a} x^4]$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[x^3 \text{Coth}[a + 2 \text{Log}[x]]^2, x\right]$$

### Problem 159: Unable to integrate problem.

$$\int x^2 \operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 - e^{2a} x^4} + \frac{3}{2} e^{-3a/2} \operatorname{ArcTan}[e^{a/2} x] - \frac{3}{2} e^{-3a/2} \operatorname{ArcTanh}[e^{a/2} x]$$

Result (type 8, 15 leaves, 0 steps):

`CannotIntegrate[x^2 Coth[a + 2 Log[x]]^2, x]`

### Problem 160: Unable to integrate problem.

$$\int x \operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 - e^{2a} x^4} - e^{-a} \operatorname{ArcTanh}[e^a x^2]$$

Result (type 8, 13 leaves, 0 steps):

`CannotIntegrate[x Coth[a + 2 Log[x]]^2, x]`

### Problem 161: Unable to integrate problem.

$$\int \operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$x + \frac{x}{1 - e^{2a} x^4} - \frac{1}{2} e^{-a/2} \operatorname{ArcTan}[e^{a/2} x] - \frac{1}{2} e^{-a/2} \operatorname{ArcTanh}[e^{a/2} x]$$

Result (type 8, 11 leaves, 0 steps):

`CannotIntegrate[Coth[a + 2 Log[x]]^2, x]`

### Problem 163: Unable to integrate problem.

$$\int \frac{\operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2}{x^2} dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$-\frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4} - \frac{1}{2}e^{a/2}\text{ArcTan}[e^{a/2}x] + \frac{1}{2}e^{a/2}\text{ArcTanh}[e^{a/2}x]$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}[a+2\text{Log}[x]]^2}{x^2}, x\right]$$

Problem 164: Unable to integrate problem.

$$\int \frac{\text{Coth}[a+2\text{Log}[x]]^2}{x^3} dx$$

Optimal (type 3, 60 leaves, 5 steps):

$$-\frac{1}{2x^2(1-e^{2a}x^4)} + \frac{3e^{2a}x^2}{2(1-e^{2a}x^4)} + e^a\text{ArcTanh}[e^ax^2]$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}[a+2\text{Log}[x]]^2}{x^3}, x\right]$$

Problem 165: Unable to integrate problem.

$$\int (ex)^m \text{Coth}[a+2\text{Log}[x]] dx$$

Optimal (type 5, 59 leaves, 3 steps):

$$\frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, e^{2a}x^4\right]}{e(1+m)}$$

Result (type 8, 15 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(ex)^m \text{Coth}[a+2\text{Log}[x]], x\right]$$

Problem 166: Unable to integrate problem.

$$\int (ex)^m \text{Coth}[a+2\text{Log}[x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):



$$\frac{(e x)^{1+m}}{e (1+m)} + \frac{(e x)^{1+m}}{e (1 - e^{2a} x^4)} - \frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, e^{2a} x^4\right]}{e}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate[(e x)<sup>m</sup> Coth[a + 2 Log[x]]<sup>2</sup>, x]

### Problem 167: Unable to integrate problem.

$$\int (e x)^m \text{Coth}[a + 2 \text{Log}[x]]^3 dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{(3+m)(5+m)(e x)^{1+m}}{8 e (1+m)} - \frac{(e x)^{1+m} (1 + e^{2a} x^4)^2}{4 e (1 - e^{2a} x^4)^2} - \frac{e^{-2a} (e x)^{1+m} (e^{2a} (3-m) - e^{4a} (5+m) x^4)}{8 e (1 - e^{2a} x^4)} - \frac{(9 + 2m + m^2) (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{4}, \frac{5+m}{4}, e^{2a} x^4\right]}{4 e (1+m)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate[(e x)<sup>m</sup> Coth[a + 2 Log[x]]<sup>3</sup>, x]

### Problem 168: Unable to integrate problem.

$$\int \text{Coth}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x (-1 - e^{2a} x^{2b})^p (1 + e^{2a} x^{2b})^{-p} \text{AppellF1}\left[\frac{1}{2b}, p, -p, \frac{1}{2} \left(2 + \frac{1}{b}\right), e^{2a} x^{2b}, -e^{2a} x^{2b}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[Coth[a + b Log[x]]<sup>p</sup>, x]

### Problem 169: Unable to integrate problem.

$$\int (e x)^m \text{Coth}[a + b \text{Log}[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{(e x)^{1+m} (-1 - e^{2a} x^{2b})^p (1 + e^{2a} x^{2b})^{-p} \text{AppellF1}\left[\frac{1+m}{2b}, p, -p, 1 + \frac{1+m}{2b}, e^{2a} x^{2b}, -e^{2a} x^{2b}\right]}{e (1+m)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate[(e x)^m Coth[a + b Log[x]]^p, x]

Problem 170: Unable to integrate problem.

$$\int \text{Coth}\left[a + \frac{\text{Log}[x]}{2}\right]^p dx$$

Optimal (type 5, 52 leaves, 2 steps):

$$-\frac{2^{-p} e^{-2a} (-1 - e^{2a} x)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 + e^{2a} x)\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate[Coth[1/2 (2a + Log[x])]^p, x]

Problem 171: Unable to integrate problem.

$$\int \text{Coth}\left[a + \frac{\text{Log}[x]}{4}\right]^p dx$$

Optimal (type 5, 108 leaves, 4 steps):

$$e^{-4a} (-1 - e^{2a} \sqrt{x})^{1+p} (1 - e^{2a} \sqrt{x})^{1-p} - \frac{2^{1-p} e^{-4a} p (-1 - e^{2a} \sqrt{x})^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 + e^{2a} \sqrt{x})\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate[Coth[1/4 (4a + Log[x])]^p, x]

Problem 172: Unable to integrate problem.

$$\int \text{Coth}\left[a + \frac{\text{Log}[x]}{6}\right]^p dx$$

Optimal (type 5, 162 leaves, 5 steps):

$$\frac{e^{-6a} p \left(-1 - e^{2a} x^{1/3}\right)^{1+p} \left(1 - e^{2a} x^{1/3}\right)^{1-p} + e^{-4a} \left(-1 - e^{2a} x^{1/3}\right)^{1+p} \left(1 - e^{2a} x^{1/3}\right)^{1-p} x^{1/3} - 2^{-p} e^{-6a} (1 + 2p^2) \left(-1 - e^{2a} x^{1/3}\right)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 + e^{2a} x^{1/3})\right]}{1+p}$$

Result (type 8, 15 leaves, 1 step):

$$\text{CannotIntegrate}\left[\text{Coth}\left[\frac{1}{6} (6a + \text{Log}[x])\right]\right]^p, x]$$

Problem 173: Unable to integrate problem.

$$\int \text{Coth}\left[a + \frac{\text{Log}[x]}{8}\right]^p dx$$

Optimal (type 5, 194 leaves, 5 steps):

$$\frac{\frac{1}{3} e^{-12a} \left(-1 - e^{2a} x^{1/4}\right)^{1+p} \left(1 - e^{2a} x^{1/4}\right)^{1-p} \left(e^{4a} (3 + 2p^2) + 2 e^{6a} p x^{1/4}\right) + e^{-4a} \left(-1 - e^{2a} x^{1/4}\right)^{1+p} \left(1 - e^{2a} x^{1/4}\right)^{1-p} \sqrt{x} - 2^{2-p} e^{-8a} p (2 + p^2) \left(-1 - e^{2a} x^{1/4}\right)^{1+p} \text{Hypergeometric2F1}\left[p, 1+p, 2+p, \frac{1}{2} (1 + e^{2a} x^{1/4})\right]}{3 (1+p)}$$

Result (type 8, 15 leaves, 1 step):

$$\text{CannotIntegrate}\left[\text{Coth}\left[\frac{1}{8} (8a + \text{Log}[x])\right]\right]^p, x]$$

Problem 174: Unable to integrate problem.

$$\int \text{Coth}[a + \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(-1 - e^{2a} x^2\right)^p \left(1 + e^{2a} x^2\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, p, -p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right]$$

Result (type 8, 9 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\text{Coth}[a + \text{Log}[x]]^p, x\right]$$

Problem 175: Unable to integrate problem.

$$\int \text{Coth}[a + 2 \text{Log}[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left( -1 - e^{2a} x^4 \right)^p \left( 1 + e^{2a} x^4 \right)^{-p} \text{AppellF1} \left[ \frac{1}{4}, p, -p, \frac{5}{4}, e^{2a} x^4, -e^{2a} x^4 \right]$$

Result (type 8, 11 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \text{Coth} [a + 2 \text{Log} [x]]^p, x \right]$$

### Problem 176: Unable to integrate problem.

$$\int \text{Coth} [a + 3 \text{Log} [x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left( -1 - e^{2a} x^6 \right)^p \left( 1 + e^{2a} x^6 \right)^{-p} \text{AppellF1} \left[ \frac{1}{6}, p, -p, \frac{7}{6}, e^{2a} x^6, -e^{2a} x^6 \right]$$

Result (type 8, 11 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \text{Coth} [a + 3 \text{Log} [x]]^p, x \right]$$

### Problem 177: Unable to integrate problem.

$$\int x^3 \text{Coth} [d (a + b \text{Log} [c x^n])] dx$$

Optimal (type 5, 58 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4 \text{Hypergeometric2F1} \left[ 1, \frac{2}{b d n}, 1 + \frac{2}{b d n}, e^{2 a d} (c x^n)^{2 b d} \right]$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ x^3 \text{Coth} [d (a + b \text{Log} [c x^n])] , x \right]$$

### Problem 178: Unable to integrate problem.

$$\int x^2 \text{Coth} [d (a + b \text{Log} [c x^n])] dx$$

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3} x^3 \text{Hypergeometric2F1} \left[ 1, \frac{3}{2 b d n}, 1 + \frac{3}{2 b d n}, e^{2 a d} (c x^n)^{2 b d} \right]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate $\left[x^2 \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 179: Unable to integrate problem.

$$\int x \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{x^2}{2} - x^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate $\left[x \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 180: Unable to integrate problem.

$$\int \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 52 leaves, 4 steps):

$$x - 2 x \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate $\left[\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right], x\right]$

### Problem 182: Unable to integrate problem.

$$\int \frac{\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^2} dx$$

Optimal (type 5, 58 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2 b d n}, 1 - \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate $\left[\frac{\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^2}, x\right]$

## Problem 183: Unable to integrate problem.

$$\int \frac{\text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]}{x^3} dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{1}{2x^2} + \frac{\text{Hypergeometric2F1}\left[1, -\frac{1}{bdn}, 1 - \frac{1}{bdn}, e^{2ad}(cx^n)^{2bd}\right]}{x^2}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]}{x^3}, x\right]$$

## Problem 184: Unable to integrate problem.

$$\int x^3 \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 132 leaves, 5 steps):

$$\frac{1}{4} \left(1 + \frac{4}{bdn}\right) x^4 + \frac{x^4 \left(1 + e^{2ad}(cx^n)^{2bd}\right)}{bdn \left(1 - e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^4 \text{Hypergeometric2F1}\left[1, \frac{2}{bdn}, 1 + \frac{2}{bdn}, e^{2ad}(cx^n)^{2bd}\right]}{bdn}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[x^3 \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2, x\right]$$

## Problem 185: Unable to integrate problem.

$$\int x^2 \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{1}{3} \left(1 + \frac{3}{bdn}\right) x^3 + \frac{x^3 \left(1 + e^{2ad}(cx^n)^{2bd}\right)}{bdn \left(1 - e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^3 \text{Hypergeometric2F1}\left[1, \frac{3}{2bdn}, 1 + \frac{3}{2bdn}, e^{2ad}(cx^n)^{2bd}\right]}{bdn}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[x^2 \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2, x\right]$$

### Problem 186: Unable to integrate problem.

$$\int x \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{1}{2} \left(1 + \frac{2}{b d n}\right) x^2 + \frac{x^2 \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)}{b d n \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 x^2 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{b d n}$$

Result (type 8, 19 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[x \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2, x\right]$$

### Problem 187: Unable to integrate problem.

$$\int \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 126 leaves, 5 steps):

$$\left(1 + \frac{1}{b d n}\right) x + \frac{x \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)}{b d n \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 x \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{b d n}$$

Result (type 8, 17 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2, x\right]$$

### Problem 189: Unable to integrate problem.

$$\int \frac{\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2}{x^2} dx$$

Optimal (type 5, 134 leaves, 5 steps):

$$-\frac{1 - \frac{1}{b d n}}{x} + \frac{1 + e^{2 a d} (c x^n)^{2 b d}}{b d n x \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2 b d n}, 1 - \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{b d n x}$$

Result (type 8, 21 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^2}{x^2}, x\right]$$

## Problem 190: Unable to integrate problem.

$$\int \frac{\text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2}{x^3} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$\frac{2 - b d n}{2 b d n x^2} + \frac{1 + e^{2 a d} (c x^n)^{2 b d}}{b d n x^2 \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 \text{Hypergeometric2F1}\left[1, -\frac{1}{b d n}, 1 - \frac{1}{b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{b d n x^2}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2}{x^3}, x\right]$$

## Problem 194: Unable to integrate problem.

$$\int (e x)^m \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 87 leaves, 4 steps):

$$\frac{(e x)^{1+m}}{e (1+m)} - \frac{2 (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2 b d n}, 1 + \frac{1+m}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{e (1+m)}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right], x\right]$$

## Problem 195: Unable to integrate problem.

$$\int (e x)^m \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 168 leaves, 5 steps):

$$\frac{(1+m+b d n) (e x)^{1+m}}{b d e (1+m) n} + \frac{(e x)^{1+m} \left(1 + e^{2 a d} (c x^n)^{2 b d}\right)}{b d e n \left(1 - e^{2 a d} (c x^n)^{2 b d}\right)} - \frac{2 (e x)^{1+m} \text{Hypergeometric2F1}\left[1, \frac{1+m}{2 b d n}, 1 + \frac{1+m}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]}{b d e n}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(e x)^m \text{Coth}\left[d\left(a + b \log\left[c x^n\right]\right)\right]^2, x\right]$$



### Problem 196: Unable to integrate problem.

$$\int (e x)^m \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^3 dx$$

Optimal (type 5, 306 leaves, 6 steps):

$$\frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2} + \frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n} + \frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2bdn}, 1+\frac{1+m}{2bdn}, e^{2ad}(cx^n)^{2bd}\right]}{b^2d^2e(1+m)n^2}$$

Result (type 8, 23 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[(ex)^m \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^3, x\right]$$

### Problem 197: Unable to integrate problem.

$$\int \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^p dx$$

Optimal (type 6, 115 leaves, 4 steps):

$$x \left(-1 - e^{2ad}(cx^n)^{2bd}\right)^p \left(1 + e^{2ad}(cx^n)^{2bd}\right)^{-p} \operatorname{AppellF1}\left[\frac{1}{2bdn}, p, -p, 1 + \frac{1}{2bdn}, e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right]$$

Result (type 8, 17 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^p, x\right]$$

### Problem 198: Unable to integrate problem.

$$\int (e x)^m \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^p dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e(1+m)} (ex)^{1+m} \left(-1 - e^{2ad}(cx^n)^{2bd}\right)^p \left(1 + e^{2ad}(cx^n)^{2bd}\right)^{-p} \operatorname{AppellF1}\left[\frac{1+m}{2bdn}, p, -p, 1 + \frac{1+m}{2bdn}, e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[(ex)^m \operatorname{Coth}\left[d \left(a + b \operatorname{Log}\left[c x^n\right]\right)\right]^p, x\right]$$

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Test results for the 16 problems in "6.5.1 (c+d x)^m (a+b sech)^n.m"

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Test results for the 84 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

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Test results for the 220 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

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Test results for the 201 problems in "6.5.3 Hyperbolic secant functions.m"

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Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( (1 - b^2 n^2) \operatorname{Sech}[a + b \operatorname{Log}[c x^n]] + 2 b^2 n^2 \operatorname{Sech}[a + b \operatorname{Log}[c x^n]]^3 \right) dx$$

Optimal (type 3, 40 leaves, ? steps):

$$x \operatorname{Sech}[a + b \operatorname{Log}[c x^n]] + b n x \operatorname{Sech}[a + b \operatorname{Log}[c x^n]] \operatorname{Tanh}[a + b \operatorname{Log}[c x^n]]$$

Result (type 5, 139 leaves, 9 steps):

$$2 e^a (1 - b n) x (c x^n)^b \operatorname{Hypergeometric2F1}\left[1, \frac{b + \frac{1}{n}}{2 b}, \frac{1}{2} \left(3 + \frac{1}{b n}\right), -e^{2 a} (c x^n)^{2 b}\right] +$$

$$\frac{16 b^2 e^{3 a} n^2 x (c x^n)^{3 b} \operatorname{Hypergeometric2F1}\left[3, \frac{3 b + \frac{1}{n}}{2 b}, \frac{1}{2} \left(5 + \frac{1}{b n}\right), -e^{2 a} (c x^n)^{2 b}\right]}{1 + 3 b n}$$


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Test results for the 29 problems in "6.6.1 (c+d x)^m (a+b csch)^n.m"

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Test results for the 83 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

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Test results for the 27 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

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Test results for the 175 problems in "6.6.3 Hyperbolic cosecant functions.m"

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Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int \left( - (1 - b^2 n^2) \operatorname{Csch}[a + b \operatorname{Log}[c x^n]] + 2 b^2 n^2 \operatorname{Csch}[a + b \operatorname{Log}[c x^n]]^3 \right) dx$$

Optimal (type 3, 42 leaves, ? steps):

$$-x \operatorname{Csch}[a + b \operatorname{Log}[c x^n]] - b n x \operatorname{Coth}[a + b \operatorname{Log}[c x^n]] \operatorname{Csch}[a + b \operatorname{Log}[c x^n]]$$

Result (type 5, 137 leaves, 9 steps):

$$\begin{aligned} & 2 e^a (1 - b n) x (c x^n)^b \operatorname{Hypergeometric2F1}\left[1, \frac{b + \frac{1}{n}}{2b}, \frac{1}{2} \left(3 + \frac{1}{bn}\right), e^{2a} (c x^n)^{2b}\right] - \\ & \frac{16 b^2 e^{3a} n^2 x (c x^n)^{3b} \operatorname{Hypergeometric2F1}\left[3, \frac{3b + \frac{1}{n}}{2b}, \frac{1}{2} \left(5 + \frac{1}{bn}\right), e^{2a} (c x^n)^{2b}\right]}{1 + 3 b n} \end{aligned}$$

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Test results for the 1059 problems in "6.7.1 Hyperbolic functions.m"