Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions"

Test results for the 1917 problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\,\frac{b\,x^m}{2\,\left(\,a\,+\,b\,x\right)^{\,3/\,2}}\,+\,\frac{m\,x^{-1+m}}{\sqrt{\,a\,+\,b\,x}}\,\right)\,\text{d}x$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b} x}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^{m}\left(-\frac{b\,x}{a}\right)^{-m}\,\text{Hypergeometric2F1}\left[-\frac{1}{2}\text{, -m, }\frac{1}{2}\text{, }1+\frac{b\,x}{a}\right]}{\sqrt{a+b\,x}}-\\ \frac{2\,m\,x^{m}\left(-\frac{b\,x}{a}\right)^{-m}\,\sqrt{a+b\,x}\,\,\text{Hypergeometric2F1}\left[\frac{1}{2}\text{, }1-\text{m, }\frac{3}{2}\text{, }1+\frac{b\,x}{a}\right]}$$

a

Test results for the 3201 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 957: Result valid but suboptimal antiderivative.

$$\int \left(\,e\;x\,\right)^{\,m}\;\left(\,a\,-\,b\;x\,\right)^{\,2+n}\;\left(\,a\,+\,b\;x\,\right)^{\,n}\,\mathrm{d}x$$

Optimal (type 5, 211 leaves, ? steps):

$$-\frac{\left(\text{e x}\right)^{\text{1+m }}\left(\text{a - b x}\right)^{\text{1+n }}\left(\text{a + b x}\right)^{\text{1+n }}}{\text{e }\left(\text{3 + m + 2 n}\right)} + \left(\text{2 a}^{2}\left(\text{2 + m + n}\right)\left(\text{e x}\right)^{\text{1+m }}\left(\text{a - b x}\right)^{\text{n}}\left(\text{a + b x}\right)^{\text{n}}\left(\text{1 - }\frac{\text{b}^{2}\,\text{x}^{2}}{\text{a}^{2}}\right)^{-\text{n}}} \right.$$

$$+ \left(\text{Hypergeometric2F1}\left[\frac{1+\text{m}}{2}, -\text{n, }\frac{3+\text{m}}{2}, \frac{\text{b}^{2}\,\text{x}^{2}}{\text{a}^{2}}\right]\right) \left/\left(\text{e }\left(\text{1 + m}\right)\left(\text{3 + m + 2 n}\right)\right) - \frac{1}{\text{e}^{2}\left(\text{2 + m}\right)} \right.$$

$$+ \left(\text{2 a b }\left(\text{e x}\right)^{\text{2+m }}\left(\text{a - b x}\right)^{\text{n}}\left(\text{a + b x}\right)^{\text{n}}\left(\text{1 - }\frac{\text{b}^{2}\,\text{x}^{2}}{\text{a}^{2}}\right)^{-\text{n}}\right) + \left(\text{Hypergeometric2F1}\left[\frac{2+\text{m}}{2}, -\text{n, }\frac{4+\text{m}}{2}, \frac{\text{b}^{2}\,\text{x}^{2}}{\text{a}^{2}}\right]\right) \right.$$

Result (type 5, 238 leaves, 11 steps):

$$\begin{split} &\frac{1}{e\left(1+m\right)}a^{2}\;\left(e\,x\right)^{\,1+m}\;\left(a-b\,x\right)^{\,n}\;\left(a+b\,x\right)^{\,n}\\ &\left(1-\frac{b^{2}\,x^{2}}{a^{2}}\right)^{-n}\;\text{Hypergeometric}2\text{F1}\left[\,\frac{1+m}{2}\,\text{, -n, }\,\frac{3+m}{2}\,\text{, }\,\frac{b^{2}\,x^{2}}{a^{2}}\,\right]\,-\,\frac{1}{e^{2}\;\left(2+m\right)}\\ &2\,a\,b\,\left(e\,x\right)^{\,2+m}\;\left(a-b\,x\right)^{\,n}\;\left(a+b\,x\right)^{\,n}\;\left(1-\frac{b^{2}\,x^{2}}{a^{2}}\right)^{-n}\;\text{Hypergeometric}2\text{F1}\left[\,\frac{2+m}{2}\,\text{, -n, }\,\frac{4+m}{2}\,\text{, }\,\frac{b^{2}\,x^{2}}{a^{2}}\,\right]\,+\,\\ &\frac{1}{e^{3}\;\left(3+m\right)}b^{2}\;\left(e\,x\right)^{\,3+m}\;\left(a-b\,x\right)^{\,n}\;\left(a+b\,x\right)^{\,n}\;\left(1-\frac{b^{2}\,x^{2}}{a^{2}}\right)^{-n}\;\text{Hypergeometric}2\text{F1}\left[\,\frac{3+m}{2}\,\text{, -n, }\,\frac{5+m}{2}\,\text{, }\,\frac{b^{2}\,x^{2}}{a^{2}}\,\right] \end{split}$$

Test results for the 159 problems in "1.1.1.4 (a+b x) n (c+d x) n (e+f x) p (g+h x) q .m"

Problem 111: Unable to integrate problem.

$$\int \frac{1}{\left(a + b \, x\right)^{3/2} \, \left(c + d \, x\right)^{3/2} \, \sqrt{e + f \, x} \, \sqrt{g + h \, x}} \, \, \mathrm{d} \, x$$

Optimal (type 4, 786 leaves, ? steps):

$$- \frac{2\,d^3\,\sqrt{a + b\,x}\,\,\sqrt{e + f\,x}\,\,\sqrt{g + h\,x}}{\left(b\,c - a\,d\right)^2\,\left(d\,e - c\,f\right)\,\left(d\,g - c\,h\right)\,\,\sqrt{c + d\,x}} - \frac{2\,b^3\,\sqrt{c + d\,x}\,\,\sqrt{e + f\,x}\,\,\sqrt{g + h\,x}}{\left(b\,c - a\,d\right)^2\,\left(b\,e - a\,f\right)\,\left(b\,g - a\,h\right)\,\,\sqrt{a + b\,x}} + \left(2\,b\,\left(a^2\,d^2\,f\,h - a\,b\,d^2\,\left(f\,g + e\,h\right) + b^2\,\left(2\,d^2\,e\,g + c^2\,f\,h - c\,d\,\left(f\,g + e\,h\right)\right)\right)\,\,\sqrt{c + d\,x}\,\,\sqrt{e + f\,x}\,\,\sqrt{g + h\,x}}\right) / \left(\left(b\,c - a\,d\right)^2\,\left(b\,e - a\,f\right)\,\left(d\,e - c\,f\right)\,\left(b\,g - a\,h\right)\,\left(d\,g - c\,h\right)\,\sqrt{a + b\,x}}\right) - \left(2\,\sqrt{f\,g - e\,h}\,\,\left(a^2\,d^2\,f\,h - a\,b\,d^2\,\left(f\,g + e\,h\right) + b^2\,\left(2\,d^2\,e\,g + c^2\,f\,h - c\,d\,\left(f\,g + e\,h\right)\right)\right)\,\,\sqrt{c + d\,x}}\right) / \left(\frac{b\,e - a\,f\right)\,\left(g + h\,x\right)}{\left(f\,g - e\,h\right)\,\left(a + b\,x\right)}\,\, EllipticE\left[ArcSin\left[\frac{\sqrt{b\,g - a\,h}\,\,\sqrt{e + f\,x}}{\sqrt{f\,g - e\,h}\,\,\sqrt{a + b\,x}}\right], - \frac{\left(b\,c - a\,d\right)\,\left(f\,g - e\,h\right)}{\left(d\,e - c\,f\right)\,\left(b\,g - a\,h\right)}\right]\right] / / \\ \left(b\,c - a\,d\right)^2\,\left(b\,e - a\,f\right)\,\left(d\,e - c\,f\right)\,\sqrt{b\,g - a\,h}\,\,\left(d\,g - c\,h\right)\,\sqrt{\frac{\left(b\,e - a\,f\right)\,\left(c + d\,x\right)}{\left(d\,e - c\,f\right)\,\left(a + b\,x\right)}}\,\sqrt{g + h\,x}}\right) - \frac{\left(b\,c - a\,d\right)\,\left(f\,g - e\,h\right)}{\left(d\,e - c\,f\right)\,\left(a + b\,x\right)}\,\sqrt{g + h\,x}}$$

$$EllipticF\left[ArcSin\left[\frac{\sqrt{b\,g - a\,h}\,\,\sqrt{e + f\,x}}{\sqrt{f\,g - e\,h}\,\,\sqrt{a + b\,x}}\right], - \frac{\left(b\,c - a\,d\right)\,\left(f\,g - e\,h\right)}{\left(d\,e - c\,f\right)\,\left(b\,g - a\,h\right)}\right] / / \\ \left(b\,c - a\,d\right)^2\,\sqrt{b\,g - a\,h}\,\,\sqrt{f\,g - e\,h}\,\,\sqrt{c + d\,x}\,\,\sqrt{\frac{\left(b\,e - a\,f\right)\,\left(g + h\,x\right)}{\left(f\,g - e\,h\right)\,\left(a + b\,x\right)}}\right)$$

$$Result(type\,8,\,39\,leaves,\,0\,steps):$$

$$CannotIntegrate\left[\frac{1}{\left(a + b\,x\right)^{3/2}\left(c + d\,x\right)^{3/2}\,\sqrt{e + f\,x}\,\,\sqrt{g + h\,x}}\,,x\right]$$

Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Test results for the 78 problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n $(e+f x)^p.m''$

Test results for the 35 problems in "1.1.1.7 P(x) $(a+b x)^m (c+d x)^n$ $(e+f x)^p (g+h x)^q.m$

Test results for the 1071 problems in "1.1.2.2 (c x) m (a+b x 2) p .m"

$$\int \! \left(\frac{a \, \left(2 + m\right) \, x^{1+m}}{\sqrt{a + b \, x^2}} + \frac{b \, \left(3 + m\right) \, x^{3+m}}{\sqrt{a + b \, x^2}} \right) \, \mathrm{d} x$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m} \sqrt{a + b x^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{\text{a } x^{2+\text{m}} \sqrt{1+\frac{\text{b } x^2}{\text{a}}} \text{ Hypergeometric2F1} \left[\frac{1}{2}, \frac{2+\text{m}}{2}, \frac{4+\text{m}}{2}, -\frac{\text{b } x^2}{\text{a}}\right]}{\sqrt{\text{a } + \text{b } x^2}} + \\ \text{b } \left(3+\text{m}\right) x^{4+\text{m}} \sqrt{1+\frac{\text{b } x^2}{\text{a}}} \text{ Hypergeometric2F1} \left[\frac{1}{2}, \frac{4+\text{m}}{2}, \frac{6+\text{m}}{2}, -\frac{\text{b } x^2}{\text{a}}\right]$$

Problem 664: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\, \frac{b \; x^{1+m}}{\left(\, a \, + \, b \; x^2 \,\right)^{\, 3/2}} \, + \, \frac{m \; x^{-1+m}}{\sqrt{\, a \, + \, b \; x^2}} \, \right) \, \mathrm{d} x$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b \ x^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^{m}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\,\text{Hypergeometric2F1}\!\left[\frac{1}{2}\text{,}\,\frac{m}{2}\text{,}\,\frac{\frac{2+m}{2}\text{,}\,-\frac{b\,x^{2}}{a}\right]}{\sqrt{a+b\,x^{2}}}\,-$$

$$\frac{b\;x^{2+m}\;\sqrt{1+\frac{b\;x^2}{a}}\;\;\text{Hypergeometric2F1}\left[\,\frac{3}{2}\text{, }\frac{2+m}{2}\text{, }\frac{4+m}{2}\text{, }-\frac{b\;x^2}{a}\,\right]}{a\;\left(\,2+m\right)\;\sqrt{a+b\;x^2}}$$

Test results for the 349 problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-2\;x^2\right)^m}{\sqrt{1-x^2}}\; \text{d}\,x$$

Optimal (type 5, 62 leaves, ? steps):

$$-\frac{2^{-2-m}\,\sqrt{x^2}\,\left(2-4\,x^2\right)^{1+m}\,\text{Hypergeometric}2\text{F1}\!\left[\frac{1}{2}\,,\,\frac{\frac{1+m}{2}},\,\frac{\frac{3+m}{2}},\,\left(1-2\,x^2\right)^2\right]}{\left(1+m\right)\,x}$$

Result (type 6, 23 leaves, 1 step):

x AppellF1
$$\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right]$$

Test results for the 1156 problems in "1.1.2.4 (e x)^m (a+b x^2)^p $(c+d x^2)^q.m''$

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q $(e+f x^2)^r.m''$

Test results for the 51 problems in "1.1.2.6 (g x) $^{\text{m}}$ (a+b x $^{\text{2}}$) $^{\text{p}}$ (c+d x^2)^a (e+f x^2)^r.m"

Test results for the 174 problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 2686: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\!\left(-\,\frac{b\;n\;x^{-1+m+n}}{2\;\left(\,a\,+\,b\;x^{n}\,\right)^{\,3/2}}\,+\,\frac{m\;x^{-1+m}}{\sqrt{\,a\,+\,b\;x^{n}}}\,\right)\,\mathrm{d}\!\!/\,x$$

Optimal (type 3, 15 leaves, ? steps):

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^{m}\sqrt{1+\frac{b\,x^{n}}{a}} \text{ Hypergeometric2F1}\Big[\frac{1}{2},\frac{m}{n},\frac{\frac{m+n}{n},-\frac{b\,x^{n}}{a}\Big]}{\sqrt{a+b\,x^{n}}} - \frac{b\,n\,x^{m+n}\sqrt{1+\frac{b\,x^{n}}{a}} \text{ Hypergeometric2F1}\Big[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^{n}}{a}\Big]}{2\,2\,(m+n)\,\sqrt{2+b\,x^{n}}}$$

Problem 2697: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \! \left(\frac{6 \ a \ x^2}{b \ (4+m) \ \sqrt{a+b \ x^{-2+m}}} + \frac{x^m}{\sqrt{a+b \ x^{-2+m}}} \right) \, \mathrm{d}x$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 \, x^3 \, \sqrt{a + b \, x^{-2+m}}}{b \, (4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2 \text{ a } x^3 \sqrt{1 + \frac{b \, x^{-2+m}}{a}} \text{ Hypergeometric2F1} \Big[\frac{1}{2}\text{, } -\frac{3}{2-m}\text{, } -\frac{1+m}{2-m}\text{, } -\frac{b \, x^{-2+m}}{a} \Big]}{b \, (4+m) \, \sqrt{a+b \, x^{-2+m}}} + \\ \frac{x^{1+m} \, \sqrt{1 + \frac{b \, x^{-2+m}}{a}} \text{ Hypergeometric2F1} \Big[\frac{1}{2}\text{, } -\frac{1+m}{2-m}\text{, } \frac{1-2\, m}{2-m}\text{, } -\frac{b \, x^{-2+m}}{a} \Big]}{\Big(1+m \Big) \, \sqrt{a+b \, x^{-2+m}}}$$

Problem 2699: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(- \, \frac{b \; n \; x^{-1+m+n}}{2 \; \left(\, a \, + \, b \; x^n \, \right)^{\, 3/2}} \, + \, \frac{m \; x^{-1+m}}{\sqrt{a \, + \, b \; x^n}} \, \right) \; \text{d} \, x$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\;x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \, \sqrt{1 + \frac{b \, x^n}{a}} \, \, \text{Hypergeometric2F1} \Big[\, \frac{1}{2} \, , \, \frac{m}{n} \, , \, \frac{m+n}{n} \, , \, - \frac{b \, x^n}{a} \, \Big]}{\sqrt{a + b \, x^n}} \, - \\ \\ \frac{b \, n \, x^{m+n} \, \sqrt{1 + \frac{b \, x^n}{a}} \, \, \text{Hypergeometric2F1} \Big[\, \frac{3}{2} \, , \, \frac{m+n}{n} \, , \, 2 + \frac{m}{n} \, , \, - \frac{b \, x^n}{a} \, \Big]}{2 \, a \, (m+n) \, \sqrt{a + b \, x^n}}$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Test results for the 1081 problems in "1.1.3.4 (e x) m (a+b x n) p $(c+d x^n)^q.m$

Problem 455: Result unnecessarily involves higher level functions.

$$\int\!\frac{x^6}{\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,\text{d}x$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2\,x\,\left(4\,c\,+\,d\,x^3\right)}{81\,c\,d^2\,\left(8\,c\,-\,d\,x^3\right)\,\sqrt{c\,+\,d\,x^3}} \,-\, \left[2\,\sqrt{2\,+\,\sqrt{3}}\,\,\left(c^{1/3}\,+\,d^{1/3}\,x\right)\right. \\ \left.\sqrt{\frac{c^{2/3}\,-\,c^{1/3}\,d^{1/3}\,x\,+\,d^{2/3}\,x^2}{\left(\left(1\,+\,\sqrt{3}\,\right)\,c^{1/3}\,+\,d^{1/3}\,x\right)^2}}\,\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1\,-\,\sqrt{3}\,\right)\,c^{1/3}\,+\,d^{1/3}\,x}{\left(1\,+\,\sqrt{3}\,\right)\,c^{1/3}\,+\,d^{1/3}\,x}\right]\,\text{, } -7\,-\,4\,\sqrt{3}\,\right]\right] \right/ \\ \left.\left(81\,\times\,3^{1/4}\,c\,d^{7/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}\,+\,d^{1/3}\,x\right)}{\left(\left(1\,+\,\sqrt{3}\,\right)\,c^{1/3}\,+\,d^{1/3}\,x\right)^2}}\,\,\sqrt{c\,+\,d\,x^3}\right] \right.$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^{7}\sqrt{1+\frac{d\,x^{3}}{c}}}{448\,c^{3}\,\sqrt{c+d\,x^{3}}} \, AppellF1\left[\frac{7}{3},\,2,\,\frac{3}{2},\,\frac{10}{3},\,\frac{d\,x^{3}}{8\,c},\,-\frac{d\,x^{3}}{c}\right]$$

Test results for the 46 problems in "1.1.3.6 (g x) m (a+b x n) p (c+d $x^n)^q (e+f x^n)^r.m''$

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 problems in "1.1.4.2 (c x) m (a x j +b x^n)^p.m"

Test results for the 298 problems in "1.1.4.3 (e x) $^{\text{h}}$ m (a x $^{\text{i}}$ +b x $^{\text{k}}$) $^{\text{p}}$ $(c+d x^n)^q.m''$

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b $x+c x^2)^p.m''$

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b $x+c x^2)^p.m''$

Problem 833: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2} \, \mathrm{d}x$$

Optimal (type 3, 91 leaves, ? steps):

$$-\operatorname{ArcCosh}[x] + \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}}\right] + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)} \operatorname{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10} \left(-1 + \sqrt{5} \right)} \ \sqrt{-1 + x} \ \sqrt{1 + x} \ \operatorname{ArcTan} \left[\frac{2 - \left(1 - \sqrt{5} \right) x}{\sqrt{2 \left(-1 + \sqrt{5} \right)} \ \sqrt{-1 + x^2}} \right]}{\sqrt{-1 + x^2}} - \frac{\sqrt{\frac{1}{10} \left(1 + \sqrt{5} \right)} \ \sqrt{-1 + x} \ \sqrt{1 + x} \ \operatorname{ArcTanh} \left[\frac{2 - \left(1 + \sqrt{5} \right) x}{\sqrt{2 \left(1 + \sqrt{5} \right)} \ \sqrt{-1 + x^2}} \right]}{\sqrt{-1 + x^2}} - \frac{\sqrt{\frac{1}{10} \left(1 + \sqrt{5} \right)} \ \sqrt{-1 + x} \ \sqrt{1 + x} \ \operatorname{ArcTanh} \left[\frac{2 - \left(1 + \sqrt{5} \right) x}{\sqrt{2 \left(1 + \sqrt{5} \right)} \ \sqrt{-1 + x^2}} \right]}{\sqrt{-1 + x^2}}$$

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c $x^2)^p (d+e x+f x^2)^q.m''$

Test results for the 400 problems in "1.2.1.9 P(x) $(d+e x)^m (a+b x+c)$ x^2)^p.m"

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 174: Unable to integrate problem.

$$\int \frac{\sqrt{a+b\,x^2}}{\sqrt{1-x^4}}\,\text{d}x$$

Optimal (type 4, 112 leaves, ? steps):

$$\frac{\text{a}\,\sqrt{1-x^2}\,\,\sqrt{\frac{\text{a}\,\left(1+x^2\right)}{\text{a}+\text{b}\,x^2}}\,\,\text{EllipticPi}\left[\,\frac{\text{b}}{\text{a}+\text{b}}\,\text{, }\text{ArcSin}\left[\,\frac{\sqrt{\text{a}+\text{b}}\,\,x}{\sqrt{\text{a}+\text{b}\,x^2}}\,\right]\,\text{, }-\frac{\text{a}-\text{b}}{\text{a}+\text{b}}\,\right]}{\sqrt{\text{a}+\text{b}}\,\,\sqrt{1+x^2}\,\,\sqrt{\frac{\text{a}\,\left(1-x^2\right)}{\text{a}+\text{b}\,x^2}}}$$

Result (type 8, 25 leaves, 0 steps):

Unintegrable
$$\left[\frac{\sqrt{a+b\;x^2}}{\sqrt{1-x^4}},\;x\right]$$

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b $x^2+c x^4)^p.m''$

Problem 374: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\,d\,+\,e\,\,x^2\,\right)^{\,3/2}}{x^2\,\left(\,a\,+\,b\,\,x^2\,+\,c\,\,x^4\,\right)}\;\mathrm{d} x$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\,\sqrt{d+e\,x^2}}{a\,x}\,-\,\frac{\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,e\right)^{\,3/2}\,ArcTan\,\left[\,\frac{\sqrt{\,2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,e\,\,\,x}}{\sqrt{\,b-\sqrt{b^2-4\,a\,c}\,\,\,\,}\sqrt{d+e\,x^2}}\,\right]}{\sqrt{\,b^2-4\,a\,c}\,\,\left(b-\sqrt{b^2-4\,a\,c}\,\,\right)^{\,3/2}}\,+\,\frac{\left(2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,e\,\,x}{\sqrt{b^2-4\,a\,c}\,\,\left(b-\sqrt{b^2-4\,a\,c}\,\right)^{\,3/2}}\,+\,\frac{1}{2}\,\left(b-\sqrt{b^2-4\,a\,c}\,\,\left(b-\sqrt{b^2-4\,a\,c}\,\right)^{\,3/2}}\right)^{\,3/2}}$$

$$\frac{\left(2\,c\;d - \left(b + \sqrt{b^2 - 4\,a\;c}\;\right)\,e\right)^{\,3/2}\,\text{ArcTan}\,\big[\,\frac{\sqrt{2\,c\,d - \left(b + \sqrt{b^2 - 4\,a\;c}\;\right)\,e}\,\,x}{\sqrt{b + \sqrt{b^2 - 4\,a\;c}}\,\,\sqrt{d + e\;x^2}}\,\big]}{\sqrt{b^2 - 4\,a\;c}\,\,\left(b + \sqrt{b^2 - 4\,a\;c}\;\right)^{\,3/2}}$$

Result (type 3, 432 leaves, 16 steps):

$$\frac{d\sqrt{d+e\,x^2}}{a\,x} = \frac{d\sqrt{d+e\,x^2}}{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)}} = \frac{d\sqrt{e}\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}} = \frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)}\,e}{\sqrt{b-\sqrt{b^2-4\,a\,c}}} = \frac{x}{\sqrt{d+e\,x^2}} = \frac{\sqrt{2\,c\,d-\left(b-\sqrt{b^2-4\,a\,c}\right)}\,e}{\sqrt{b-\sqrt{b^2-4\,a\,c}}} = \frac{x}{\sqrt{d+e\,x^2}} = \frac{\sqrt{2\,c\,d-\left(b+\sqrt{b^2-4\,a\,c}\right)}\,e}{\sqrt{b+\sqrt{b^2-4\,a\,c}}} = \frac{x}{\sqrt{d+e\,x^2}} = \frac{\sqrt{e}\,\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\,ArcTanh\left[\frac{\sqrt{e}\,x}{\sqrt{d+e\,x^2}}\right]}{a} = \frac{\sqrt{e}\,\left(d-\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\,ArcTanh\left[\frac{\sqrt{e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{d+e\,x^2}} = \frac{\sqrt{e}\,\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\,ArcTanh\left[\frac{\sqrt{e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{d+e\,x^2}} = \frac{\sqrt{e}\,\left(d+\frac{b\,d-2\,a\,e}{\sqrt{b^2-4\,a\,c}}\right)\,ArcTanh\left[\frac{\sqrt{e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{e}\,\left(d+\frac{e}\,x}{\sqrt{e}\,x}\right)} = \frac{\sqrt{e}\,\left(d+\frac{e}\,x}{\sqrt{e}\,x}\right)}{\sqrt{e}\,\left(d+\frac{e}\,x}{\sqrt{e}\,x}\right)} = \frac{\sqrt{e}\,\left(d+\frac{e}\,x}{\sqrt{e}\,x}\right)}{\sqrt{e}\,\left(d+\frac{e}\,x}{\sqrt{e}\,x}\right)}$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Test results for the 145 problems in "1.2.2.6 P(x) (d x) m (a+b x 2 +c x^4)^p.m"

Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b $x^2+c x^4)^p.m''$

Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Test results for the 664 problems in "1.2.3.2 (d x) m (a+b x n +c x n (2 n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 \left(a^2 + 2 a b x^3 + b^2 x^6\right)^{3/2} dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^{2} \left(a + b \ x^{3}\right)^{3} \sqrt{a^{2} + 2 \ a \ b \ x^{3} + b^{2} \ x^{6}}}{12 \ b^{3}} - \\ \frac{2 \ a \left(a + b \ x^{3}\right)^{4} \sqrt{a^{2} + 2 \ a \ b \ x^{3} + b^{2} \ x^{6}}}{15 \ b^{3}} + \frac{\left(a + b \ x^{3}\right)^{5} \sqrt{a^{2} + 2 \ a \ b \ x^{3} + b^{2} \ x^{6}}}{18 \ b^{3}}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 \ x^9 \ \sqrt{a^2 + 2 \ a \ b \ x^3 + b^2 \ x^6}}{9 \ \left(a + b \ x^3\right)} + \frac{a^2 \ b \ x^{12} \ \sqrt{a^2 + 2 \ a \ b \ x^3 + b^2 \ x^6}}{4 \ \left(a + b \ x^3\right)} + \\ \frac{a \ b^2 \ x^{15} \ \sqrt{a^2 + 2 \ a \ b \ x^3 + b^2 \ x^6}}{5 \ \left(a + b \ x^3\right)} + \frac{b^3 \ x^{18} \ \sqrt{a^2 + 2 \ a \ b \ x^3 + b^2 \ x^6}}{18 \ \left(a + b \ x^3\right)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \! \left(\frac{\left(a^2 + 2 \ a \ b \ x^{1/3} + b^2 \ x^{2/3} \right)^p}{x^2} - \frac{2 \ b^3 \ \left(1 - 2 \ p \right) \ \left(1 - p \right) \ p \ \left(a^2 + 2 \ a \ b \ x^{1/3} + b^2 \ x^{2/3} \right)^p}{3 \ a^3 \ x} \right) \ \text{d}x$$

Optimal (type 3, 146 leaves, ? steps):

$$-\frac{\left(a+b\ x^{1/3}\right)\ \left(a^2+2\ a\ b\ x^{1/3}+b^2\ x^{2/3}\right)^p}{a\ x}+\frac{b\ \left(1-p\right)\ \left(a+b\ x^{1/3}\right)\ \left(a^2+2\ a\ b\ x^{1/3}+b^2\ x^{2/3}\right)^p}{a^2\ x^{2/3}}-\frac{b^2\ \left(1-2\ p\right)\ \left(1-p\right)\ \left(a+b\ x^{1/3}\right)\ \left(a^2+2\ a\ b\ x^{1/3}+b^2\ x^{2/3}\right)^p}{a^3\ x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{1}{a^3 \, \left(1+2 \, p\right)} 2 \, b^3 \, \left(1-2 \, p\right) \, \left(1-p\right) \, p \, \left(1+\frac{b \, x^{1/3}}{a}\right) \, \left(a^2+2 \, a \, b \, x^{1/3}+b^2 \, x^{2/3}\right)^p$$

Hypergeometric2F1[1, 1 + 2 p, 2 (1 + p), 1 +
$$\frac{b x^{1/3}}{a}$$
] + $\frac{1}{a^3 (1 + 2 p)}$

$$3\;b^{3}\;\left(1\;+\;\frac{b\;x^{1/3}}{a}\right)\;\left(a^{2}\;+\;2\;a\;b\;x^{1/3}\;+\;b^{2}\;x^{2/3}\right)^{p}\;\\ \text{Hypergeometric2F1}\left[\;4\;,\;\;1\;+\;2\;p\;,\;\;2\;\left(1\;+\;p\right)\;,\;\;1\;+\;\frac{b\;x^{1/3}}{a}\;\right]$$

Test results for the 96 problems in "1.2.3.3 ($d+e \times n$)^q ($a+b \times n+c$ $x^{(2 n)}p.m$ "

Test results for the 156 problems in "1.2.3.4 (f x) n (d+e x n) q (a+b $x^n+c x^(2 n))^p.m''$

Test results for the 17 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c $x^{(2 n)}p.m$ "

Test results for the 140 problems in "1.2.4.2 (d x) n (a x q +b x n +c $x^{(2 n-q)}^{p.m}$

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\left[\, \left(\, b \, \, x^{1+p} \, \, \left(\, b \, \, x \, + \, c \, \, x^{3} \, \right)^{\, p} \, + \, 2 \, \, c \, \, x^{3+p} \, \, \left(\, b \, \, x \, + \, c \, \, x^{3} \, \right)^{\, p} \right) \, \, \mathbb{d} \, x \right.$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p} \ \left(b \ x + c \ x^{3}\right)^{1+p}}{2 \ \left(1 + p\right)}$$

Result (type 5, 116 leaves, 7 steps):

$$\begin{split} &\frac{1}{2\left(1+p\right)}b\;x^{2+p}\;\left(1+\frac{c\;x^{2}}{b}\right)^{-p}\;\left(b\;x+c\;x^{3}\right)^{p}\;\text{Hypergeometric2F1}\!\left[-p\text{, 1}+p\text{, 2}+p\text{, }-\frac{c\;x^{2}}{b}\right]\;+\\ &\frac{1}{2+p}c\;x^{4+p}\left(1+\frac{c\;x^{2}}{b}\right)^{-p}\;\left(b\;x+c\;x^{3}\right)^{p}\;\text{Hypergeometric2F1}\!\left[-p\text{, 2}+p\text{, 3}+p\text{, }-\frac{c\;x^{2}}{b}\right] \end{split}$$

Problem 221: Result valid but suboptimal antiderivative.

$$\left(1+2x\right)\left(x+x^{2}\right)^{3}\left(-18+7\left(x+x^{2}\right)^{3}\right)^{2}dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 \ x^4 \ \left(1+x\right)^4 - 36 \ x^7 \ \left(1+x\right)^7 + \frac{49}{10} \ x^{10} \ \left(1+x\right)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81\,x^{4} + 324\,x^{5} + 486\,x^{6} + 288\,x^{7} - 171\,x^{8} - 756\,x^{9} - \frac{12\,551\,x^{10}}{10} - 1211\,x^{11} - \frac{1071\,x^{12}}{2} + 336\,x^{13} + 993\,x^{14} + \frac{6174\,x^{15}}{5} + 1029\,x^{16} + 588\,x^{17} + \frac{441\,x^{18}}{2} + 49\,x^{19} + \frac{49\,x^{20}}{10}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int x^3 \ \left(1+x\right)^3 \ \left(1+2 \ x\right) \ \left(-18+7 \ x^3 \ \left(1+x\right)^3\right)^2 \ \mathrm{d} \, x$$

Optimal (type 1, 33 leaves, ? steps):

$$81 \ x^4 \ \left(1+x\right)^4 - 36 \ x^7 \ \left(1+x\right)^7 + \frac{49}{10} \ x^{10} \ \left(1+x\right)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81 \, x^4 + 324 \, x^5 + 486 \, x^6 + 288 \, x^7 - 171 \, x^8 - 756 \, x^9 - \frac{12551 \, x^{10}}{10} - 1211 \, x^{11} - \frac{1071 \, x^{12}}{2} + 336 \, x^{13} + 993 \, x^{14} + \frac{6174 \, x^{15}}{5} + 1029 \, x^{16} + 588 \, x^{17} + \frac{441 \, x^{18}}{2} + 49 \, x^{19} + \frac{49 \, x^{20}}{10}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20 \times + 4 \times^2}{9 - 10 \times^2 + x^4} \, dx$$

Optimal (type 3, 31 leaves, ? steps):

$$Log[1-x] - \frac{1}{2}Log[3-x] + \frac{3}{2}Log[1+x] - 2Log[3+x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2}\operatorname{ArcTanh}\left[\frac{x}{3}\right]+\frac{\operatorname{ArcTanh}\left[x\right]}{2}+\frac{5}{4}\operatorname{Log}\left[1-x^{2}\right]-\frac{5}{4}\operatorname{Log}\left[9-x^{2}\right]$$

Problem 393: Unable to integrate problem.

$$\int\!\frac{\left(1+x^2\right)^2}{a\,x^6+b\,\left(1+x^2\right)^3}\,\text{d}x$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\Big[\frac{\sqrt{a^{1/3}+b^{1/3}}}{b^{1/6}} \Big]}{3\sqrt{a^{1/3}+b^{1/3}}} \, b^{5/6} \, + \, \frac{\text{ArcTan}\Big[\frac{\sqrt{-(-1)^{1/3}}\,a^{1/3}+b^{1/3}}}{3\sqrt{-\left(-1\right)^{1/3}}\,a^{1/3}+b^{1/3}} \, b^{5/6}} + \, \frac{\text{ArcTan}\Big[\frac{\sqrt{(-1)^{2/3}}\,a^{1/3}+b^{1/3}}}{b^{1/6}} \, x\Big]}{3\sqrt{\left(-1\right)^{2/3}}\,a^{1/3}+b^{1/3}} \, b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

CannotIntegrate
$$\left[\frac{1}{a x^6 + b (1 + x^2)^3}, x\right]$$
 +

$$\text{2 CannotIntegrate} \left[\, \frac{ \, \, x^2 \, }{ \, \text{a} \, \, x^6 \, + \, b \, \left(1 + x^2 \right)^3 } \, \text{, } \, x \, \right] \, + \, \text{CannotIntegrate} \left[\, \frac{ \, \, x^4 \, }{ \, \text{a} \, \, x^6 \, + \, b \, \left(1 + x^2 \right)^3 } \, \text{, } \, x \, \right]$$

Problem 493: Unable to integrate problem.

$$\int \left(\frac{3 \, \left(-47 + 228 \, x + 120 \, x^2 + 19 \, x^3 \right)}{\left(3 + x + x^4 \right)^4} + \frac{42 - 320 \, x - 75 \, x^2 - 8 \, x^3}{\left(3 + x + x^4 \right)^3} + \frac{30 \, x}{\left(3 + x + x^4 \right)^2} \right) \, \mathrm{d}x$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3 \, x+5 \, x^2+x^4-5 \, x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$-\frac{19}{4 \left(3 + x + x^{4}\right)^{3}} + \frac{1}{\left(3 + x + x^{4}\right)^{2}} - \frac{621}{4} CannotIntegrate \left[\frac{1}{\left(3 + x + x^{4}\right)^{4}}, x\right] + \frac{1}{\left(3 + x + x^{4}\right)^{3}} + \frac{1}{\left(3 + x + x^{4}\right)^{4}} + \frac{1}{\left(3 + x + x^{4}\right)^{4}}$$

684 CannotIntegrate
$$\left[\frac{x}{\left(3+x+x^4\right)^4}, x\right] + 360$$
 CannotIntegrate $\left[\frac{x^2}{\left(3+x+x^4\right)^4}, x\right] + 360$

44 CannotIntegrate
$$\left[\frac{1}{\left(3+x+x^4\right)^3}$$
, $x\right]$ – 320 CannotIntegrate $\left[\frac{x}{\left(3+x+x^4\right)^3}$, $x\right]$ –

75 CannotIntegrate
$$\left[\frac{x^2}{\left(3+x+x^4\right)^3}$$
, $x\right]$ + 30 CannotIntegrate $\left[\frac{x}{\left(3+x+x^4\right)^2}$, $x\right]$

Problem 494: Unable to integrate problem.

$$\int \left(\frac{-\,3\,+\,10\,\,x\,+\,4\,\,x^{3}\,-\,30\,\,x^{5}}{\left(\,3\,+\,x\,+\,x^{4}\,\right)^{\,3}}\,-\,\frac{3\,\,\left(\,1\,+\,4\,\,x^{3}\,\right)\,\,\left(\,2\,-\,3\,\,x\,+\,5\,\,x^{2}\,+\,x^{4}\,-\,5\,\,x^{6}\,\right)}{\left(\,3\,+\,x\,+\,x^{4}\,\right)^{\,4}} \right) \,\,\mathrm{d}x$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3 x+5 x^2+x^4-5 x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\begin{split} &\frac{7}{2\left(3+x+x^4\right)^3} - \frac{63\,x}{22\left(3+x+x^4\right)^3} - \frac{12\,x^2}{\left(3+x+x^4\right)^3} - \frac{5\,x^3}{\left(3+x+x^4\right)^3} + \frac{3\,x^4}{2\left(3+x+x^4\right)^3} - \\ &\frac{10\,x^6}{\left(3+x+x^4\right)^3} - \frac{1}{2\left(3+x+x^4\right)^2} + \frac{5\,x^2}{\left(3+x+x^4\right)^2} + \frac{144}{11}\,\text{CannotIntegrate}\Big[\,\frac{1}{\left(3+x+x^4\right)^4},\,x\Big] + \\ &\frac{828}{11}\,\text{CannotIntegrate}\Big[\,\frac{x}{\left(3+x+x^4\right)^4},\,x\Big] + 18\,\text{CannotIntegrate}\Big[\,\frac{x^2}{\left(3+x+x^4\right)^4},\,x\Big] - \\ &4\,\text{CannotIntegrate}\Big[\,\frac{1}{\left(3+x+x^4\right)^3},\,x\Big] - 20\,\text{CannotIntegrate}\Big[\,\frac{x}{\left(3+x+x^4\right)^3},\,x\Big] \end{split}$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 197: Unable to integrate problem.

$$\int \frac{\left(d^3 + e^3 x^3\right)^p}{d + e x} \, dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\begin{split} &\frac{1}{e\,p} \left(d^3 + e^3\,x^3 \right)^p \left(1 + \frac{2\,\left(d + e\,x \right)}{\left(-3 + \dot{\mathbb{1}}\,\sqrt{3}\,\right)\,d} \right)^{-p} \left(1 - \frac{2\,\left(d + e\,x \right)}{\left(3 + \dot{\mathbb{1}}\,\sqrt{3}\,\right)\,d} \right)^{-p} \\ &\text{AppellF1} \Big[\, p \text{, } -p \text{, } -p \text{, } 1 + p \text{, } -\frac{2\,\left(d + e\,x \right)}{\left(-3 + \dot{\mathbb{1}}\,\sqrt{3}\,\right)\,d} \,, \, \frac{2\,\left(d + e\,x \right)}{\left(3 + \dot{\mathbb{1}}\,\sqrt{3}\,\right)\,d} \, \Big] \end{split}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate
$$\left[\frac{\left(d^3+e^3\,x^3\right)^p}{d+e\,x},\,x\right]$$

Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\!\left(\frac{\sqrt{a\,x^{2\,n}}}{\sqrt{1+x^{n}}}\,+\,\frac{2\,x^{-n}\,\sqrt{a\,x^{2\,n}}}{\left(2+n\right)\,\sqrt{1+x^{n}}}\right)\,\text{d}x$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2 \, x^{1-n} \, \sqrt{a \, x^{2 \, n}} \, \sqrt{1 + x^n}}{2 + n}$$

Result (type 5, 80 leaves, 5 steps):

$$\frac{x\,\sqrt{a\,x^{2\,n}}\,\,\text{Hypergeometric2F1}\Big[\frac{1}{2},\,1+\frac{1}{n},\,2+\frac{1}{n},\,-x^n\Big]}{1+n}+\frac{2\,x^{1-n}\,\sqrt{a\,x^{2\,n}}\,\,\text{Hypergeometric2F1}\Big[\frac{1}{2},\,\frac{1}{n},\,1+\frac{1}{n},\,-x^n\Big]}{2+n}$$

Problem 616: Unable to integrate problem.

$$\begin{split} \int \frac{1}{x^2} \left(a + b \, x + c \, x^2 \right)^m \, \left(d + e \, x + f \, x^2 + g \, x^3 \right)^n \\ \left(- a \, d + \left(b \, d \, m + a \, e \, n \right) \, x + \left(c \, d + b \, e + a \, f + 2 \, c \, d \, m + b \, e \, m + b \, e \, n + 2 \, a \, f \, n \right) \, x^2 + \\ \left(2 \, c \, e + 2 \, b \, f + 2 \, a \, g + 2 \, c \, e \, m + b \, f \, m + c \, e \, n + 2 \, b \, f \, n + 3 \, a \, g \, n \right) \, x^3 + \\ \left(3 \, c \, f + 3 \, b \, g + 2 \, c \, f \, m + b \, g \, m + 2 \, c \, f \, n + 3 \, b \, g \, n \right) \, x^4 + c \, g \, \left(4 + 2 \, m + 3 \, n \right) \, x^5 \right) \, \mathrm{d} x \end{split}$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(a+b\;x+c\;x^{2}\right)^{1+m}\;\left(d+e\;x+f\;x^{2}+g\;x^{3}\right)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$\begin{array}{l} \left(c\,\left(d+2\,d\,m\right)+b\,e\,\left(1+m+n\right)+a\,f\,\left(1+2\,n\right)\right) \\ \text{CannotIntegrate}\left[\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right] -\\ a\,d\,\text{CannotIntegrate}\left[\,\frac{\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^2},\,x\right] +\\ \left(b\,d\,m+a\,e\,n\right)\,\text{CannotIntegrate}\left[\,\frac{\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x},\,x\right] +\\ \left(c\,e\,\left(2+2\,m+n\right)+b\,f\,\left(2+m+2\,n\right)+a\,g\,\left(2+3\,n\right)\right) \\ \text{CannotIntegrate}\left[\,x\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right] +\\ \left(c\,f\,\left(3+2\,m+2\,n\right)+b\,g\,\left(3+m+3\,n\right)\right)\,\text{CannotIntegrate}\left[\,x^2\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right] +\\ c\,g\,\left(4+2\,m+3\,n\right)\,\text{CannotIntegrate}\left[\,x^3\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\right] \end{array}$$

Problem 617: Unable to integrate problem.

$$\int \frac{1}{x^3} \left(a + b \, x + c \, x^2 \right)^m \, \left(d + e \, x + f \, x^2 + g \, x^3 \right)^n \, \left(-2 \, a \, d + \left(-b \, d - a \, e + b \, d \, m + a \, e \, n \right) \, x + \left(2 \, c \, d \, m + b \, e \, m + b \, e \, n + 2 \, a \, f \, n \right) \, x^2 + \left(c \, e + b \, f + a \, g + 2 \, c \, e \, m + b \, f \, m + c \, e \, n + 2 \, b \, f \, n + 3 \, a \, g \, n \right) \, x^3 + \left(2 \, c \, f + 2 \, b \, g + 2 \, c \, f \, m + b \, g \, m + 2 \, c \, f \, n + 3 \, b \, g \, n \right) \, x^4 + c \, g \, \left(3 + 2 \, m + 3 \, n \right) \, x^5 \right) \, \mathrm{d} x$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{\left(\,a\,+\,b\,\,x\,+\,c\,\,x^2\,\right)^{\,1+m}\,\,\left(\,d\,+\,e\,\,x\,+\,f\,\,x^2\,+\,g\,\,x^3\,\right)^{\,1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$\begin{array}{l} \left(\text{ce}\,\left(1+2\,\text{m}+n\right)+\text{b}\,f\left(1+\text{m}+2\,n\right)+\text{a}\,g\left(1+3\,n\right)\right) \\ \text{CannotIntegrate}\left[\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\,\right] - \\ 2\,\text{a}\,d\,\text{CannotIntegrate}\left[\,\frac{\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^3},\,x\,\right] - \\ \left(\text{b}\,d\,\left(1-\text{m}\right)+\text{a}\,e\,\left(1-\text{n}\right)\right)\,\text{CannotIntegrate}\left[\,\frac{\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x^2},\,x\,\right] + \\ \left(2\,\text{c}\,d\,m+2\,\text{a}\,f\,n+b\,e\,\left(m+n\right)\right)\,\text{CannotIntegrate}\left[\,\frac{\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n}{x},\,x\,\right] + \\ \left(2\,\text{c}\,f\,\left(1+m+n\right)+\text{b}\,g\,\left(2+m+3\,n\right)\right)\,\text{CannotIntegrate}\left[\,x\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\,\right] + \\ \left(2\,\text{c}\,g\,\left(3+2\,m+3\,n\right)\,\text{CannotIntegrate}\left[\,x^2\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\,\right] + \\ \left(3\,\text{c}\,g\,\left(3+2\,m+3\,n\right)\,\text{CannotIntegrate}\left[\,x^2\,\left(a+b\,x+c\,x^2\right)^m\,\left(d+e\,x+f\,x^2+g\,x^3\right)^n,\,x\,\right] + \\ \left(3\,\text{c}\,g\,\left(3+2\,m+3\,n\right)\,\text{C$$

Problem 941: Result unnecessarily involves higher level functions.

$$\int\!\left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right)\,\mathrm{d}x$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{\left(1-x^{6}\right)^{2/3}}{5\,x^{5}}+\frac{1}{5}\,x\,\left(1-x^{6}\right)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^{6}\right]}{5x^{5}}+x \text{ Hypergeometric2F1}\left[-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^{6}\right]$$

Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x\,\sqrt{-1+x^2}} \ dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} \left(3 \; x + \sqrt{-1 + x^2} \; \right) \; \sqrt{1 - x^2 + x \; \sqrt{-1 + x^2}} \; + \; \frac{3 \; \text{ArcSin} \left[\, x - \sqrt{-1 + x^2} \; \, \right]}{4 \; \sqrt{2}}$$

Result (type 8, 24 leaves, 0 steps):

CannotIntegrate
$$\left[\sqrt{1-x^2+x\,\sqrt{-1+x^2}}\right]$$
 , x

Problem 996: Unable to integrate problem.

$$\int \frac{\sqrt{-x + \sqrt{x} \sqrt{1 + x}}}{\sqrt{1 + x}} dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left(\sqrt{x} + 3\sqrt{1+x} \right) \sqrt{-x + \sqrt{x}} \sqrt{1+x} - \frac{3 ArcSin \left[\sqrt{x} - \sqrt{1+x} \right]}{2\sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

CannotIntegrate
$$\left[\frac{\sqrt{-x+\sqrt{x}}\sqrt{1+x}}{\sqrt{1+x}}\right]$$
, x

Problem 997: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\,\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,\mathrm{d}x$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2\left(1+\sqrt{5}\right)} \ \operatorname{ArcTan}\left[\sqrt{-2+\sqrt{5}} \ \left(x+\sqrt{1+x^2}\right)\right] + \sqrt{2\left(-1+\sqrt{5}\right)} \ \operatorname{ArcTanh}\left[\sqrt{2+\sqrt{5}} \ \left(x+\sqrt{1+x^2}\right)\right]$$

Result (type 3, 319 leaves, 25 steps):

$$-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}} \ \operatorname{ArcTan} \Big[\sqrt{\frac{2}{1+\sqrt{5}}} \ x\Big] - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)} \ \operatorname{ArcTan} \Big[\sqrt{\frac{2}{1+\sqrt{5}}} \ x\Big] - \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}} \ \operatorname{ArcTan} \Big[\sqrt{\frac{2}{-1+\sqrt{5}}} \ \sqrt{1+x^2}\ \Big] - \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)} \ \operatorname{ArcTan} \Big[\sqrt{\frac{2}{-1+\sqrt{5}}} \ \sqrt{1+x^2}\ \Big] - \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)} \ \operatorname{ArcTanh} \Big[\sqrt{\frac{2}{-1+\sqrt{5}}} \ x\Big] - \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)} \ \operatorname{ArcTanh} \Big[\sqrt{\frac{2}{-1+\sqrt{5}}} \ x\Big] - \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)} \ \operatorname{ArcTanh} \Big[\sqrt{\frac{2}{1+\sqrt{5}}} \ \sqrt{1+x^2}\ \Big] + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)} \ \operatorname{ArcTanh} \Big[\sqrt{\frac{2}{1+\sqrt{5}}} \ \sqrt{1+x^2}\ \Big]$$

Problem 1017: Result valid but suboptimal antiderivative.

$$\int \frac{1-x^2}{\left(1-x+x^2\right) \; \left(1-x^3\right)^{2/3}} \, \mathrm{d}x$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \ \text{ArcTan} \Big[\frac{1 - \frac{2 \cdot 2^{3/3} \, (1-x)}{(1-x^3)^{3/3}} \Big]}{2^{2/3}} - \frac{\text{Log} \Big[1 + 2 \, \left(1 - x \right)^3 - x^3 \Big]}{2 \times 2^{2/3}} + \frac{3 \, \text{Log} \Big[2^{1/3} \, \left(1 - x \right) + \left(1 - x^3 \right)^{1/3} \Big]}{2 \times 2^{2/3}}$$

Result (type 3, 425 leaves, 42 steps):

$$\frac{2^{1/3} \, \text{ArcTan} \left[\frac{1 - \frac{2 \cdot 2^{1/3} \, (1 - x)}{\sqrt{3}} \right]}{\sqrt{3}} + \frac{\text{ArcTan} \left[\frac{1 + \frac{2^{1/3} \, (1 - x)}{\sqrt{3}}}{\sqrt{3}} \right]}{2^{2/3} \, \sqrt{3}} - \frac{\text{ArcTan} \left[\frac{1 - \frac{2 \cdot 2^{1/3} \, x}{(1 - x^3)^{1/3}} \right]}{2^{2/3} \, \sqrt{3}} + \frac{\text{ArcTan} \left[\frac{1 + 2^{2/3} \, (1 - x)}{\sqrt{3}} \right]}{2^{2/3} \, \sqrt{3}} + \frac{\text{ArcTan} \left[\frac{1 + 2^{2/3} \, (1 - x^3)^{1/3}}{\sqrt{3}} \right]}{2^{2/3} \, \sqrt{3}} + \frac{\text{ArcTan} \left[\frac{1 + 2^{2/3} \, (1 - x)}{\sqrt{3}} \right]}{3 \times 2^{2/3}} - \frac{\text{Log} \left[1 + \frac{2^{2/3} \, (1 - x)^2}{(1 - x^3)^{2/3}} - \frac{2^{1/3} \, (1 - x)}{(1 - x^3)^{1/3}} \right]}{3 \times 2^{2/3}} + \frac{1}{3} \times 2^{1/3} \, \text{Log} \left[1 + \frac{2^{1/3} \, \left(1 - x \right)}{\left(1 - x^3 \right)^{1/3}} \right] - \frac{\text{Log} \left[2 \times 2^{1/3} + \frac{(1 - x)^2}{(1 - x^3)^{2/3}} + \frac{2^{2/3} \, (1 - x)}{(1 - x^3)^{1/3}} \right]}{6 \times 2^{2/3}} - \frac{\text{Log} \left[2^{1/3} - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[- 2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)^{1/3} \right]}{2 \times 2^{2/3}} - \frac{\text{Log} \left[-2^{1/3} \, x - \left(1 - x^3 \right)$$

Problem 1018: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{-1+x^4} \ \left(1+x^4\right)} \ \text{d} \, x$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4}\operatorname{ArcTan}\Big[\frac{1+x^2}{x\sqrt{-1+x^4}}\Big]-\frac{1}{4}\operatorname{ArcTanh}\Big[\frac{1-x^2}{x\sqrt{-1+x^4}}\Big]$$

Result (type 3, 47 leaves, 9 steps):

$$\left(-\frac{1}{8}-\frac{\dot{\mathbb{I}}}{8}\right) \, \mathsf{ArcTan} \, \Big[\, \frac{\left(1+\dot{\mathbb{I}}\,\right) \, x}{\sqrt{-1+x^4}} \, \Big] \, + \, \left(\frac{1}{8}+\frac{\dot{\mathbb{I}}}{8}\right) \, \mathsf{ArcTanh} \, \Big[\, \frac{\left(1+\dot{\mathbb{I}}\,\right) \, x}{\sqrt{-1+x^4}} \, \Big]$$

Problem 1023: Unable to integrate problem.

$$\int \left(1 + x + x^2 + x^3\right)^{-n} \left(1 - x^4\right)^n dx$$

Optimal (type 3, 34 leaves, ? steps):

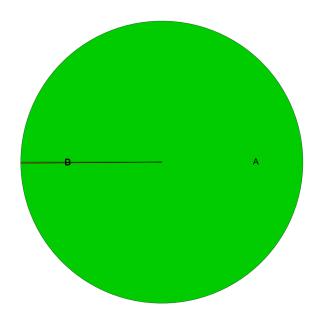
$$- \ \frac{\left(1-x\right) \ \left(1+x+x^2+x^3\right)^{-n} \ \left(1-x^4\right)^n}{1+n}$$

Result (type 8, 25 leaves, 0 steps):

CannotIntegrate
$$\left[\left(1 + x + x^2 + x^3 \right)^{-n} \left(1 - x^4 \right)^n, x \right]$$

Summary of Integration Test Results

26125 integration problems



- A 26092 optimal antiderivatives
- B 9 valid but suboptimal antiderivatives
- C 13 unnecessarily complex antiderivatives
- D 11 unable to integrate problems
- E 0 integration timeouts
- F 0 invalid antiderivatives