Mathematica 11.3 Integration Test Results

Test results for the 136 problems in "8.4 Trig integral functions.m"

Problem 6: Unable to integrate problem.

Problem 39: Unable to integrate problem.

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\int \frac{\sin[b\,x] \, \sin[ntegral[b\,x]]}{x^3} \, dx
Optimal (type 4, 96 leaves, 14 steps):
b^2 \, \cos[ntegral[2\,b\,x]] - \frac{b \, \cos[b\,x] \, \sin[b\,x]}{2\,x} - \frac{\sin[b\,x]^2}{4\,x^2} - \frac{b \, \sin[2\,b\,x]}{4\,x} - \frac{\sin[b\,x]^2}{4\,x}
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$$\frac{b^2 \operatorname{CosIntegral}[2 \, b \, x] - \frac{2 \, x}{2 \, x} - \frac{4 \, x^2}{4 \, x^2} - \frac{4 \, x}{4 \, x}}{2 \, x} - \frac{b \operatorname{Cos}[b \, x] \operatorname{SinIntegral}[b \, x]}{2 \, x} - \frac{\operatorname{Sin}[b \, x] \operatorname{SinIntegral}[b \, x]}{2 \, x^2} - \frac{1}{4} \, b^2 \operatorname{SinIntegral}[b \, x]^2}$$

Result (type 8, 14 leaves):

Sin[bx] SinIntegral[bx]

Problem 41: Unable to integrate problem.

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\int \frac{\sin[b \, x] \, \sin[ntegral[b \, x]}{x} \, dx
Optimal (type 4, 10 leaves, 1 step):
\frac{1}{2} \sin[ntegral[b \, x]^{2}
Result (type 9, 26 leaves):
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Problem 47: Unable to integrate problem.

$$\int \frac{\mathsf{Cos}[b\,x]\,\mathsf{SinIntegral}[b\,x]}{\mathsf{x}^2}\,\mathrm{d} \mathsf{x}$$

Optimal (type 4, 44 leaves, 7 steps):

$$b \, \text{CosIntegral} \, [\, 2 \, b \, x \,] \, \, - \, \frac{ \, \text{Sin} \, [\, 2 \, b \, x \,] \,}{2 \, x} \, - \, \frac{ \, \text{Cos} \, [\, b \, x \,] \, \, \text{SinIntegral} \, [\, b \, x \,] \,}{x} \, - \, \frac{1}{2} \, b \, \, \text{SinIntegral} \, [\, b \, x \,] \,^2 \, \,$$

Result (type 8, 14 leaves):

$$\int \frac{\mathsf{Cos}\,[\,b\,\,x\,]\,\,\mathsf{SinIntegral}\,[\,b\,\,x\,]}{x^2}\,\,\mathrm{d}x$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int x \sin[a + bx] \sin[ntegral[c + dx]] dx$$

Optimal (type 4, 371 leaves, 24 steps):

$$\frac{\text{Cos}\left[\mathsf{a}-\mathsf{c}+\left(\mathsf{b}-\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}\,\left(\mathsf{b}-\mathsf{d}\right)} - \frac{\mathsf{Cos}\left[\mathsf{a}+\mathsf{c}+\left(\mathsf{b}+\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}\,\left(\mathsf{b}+\mathsf{d}\right)} - \frac{\mathsf{Cos}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \mathsf{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}-\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}-\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}^2} + \frac{\mathsf{Cos}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \mathsf{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}+\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}^2} + \frac{\mathsf{c}\,\mathsf{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}-\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}-\mathsf{d}\right)\mathsf{x}\right] \mathsf{Sin}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right]}{2\,\mathsf{b}\,\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}+\mathsf{d}\right)\mathsf{x}\right] \mathsf{Sin}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right]}{2\,\mathsf{b}\,\mathsf{d}} + \frac{\mathsf{Sin}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \mathsf{SinIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}-\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}-\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}^2} - \frac{\mathsf{c}\,\mathsf{Cos}\left[\mathsf{a}+\mathsf{b}\,\mathsf{x}\right] \mathsf{SinIntegral}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]}{\mathsf{b}} + \frac{\mathsf{Sin}\left[\mathsf{a}+\mathsf{b}\,\mathsf{x}\right] \mathsf{SinIntegral}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]}{\mathsf{b}^2} - \frac{\mathsf{c}\,\mathsf{Cos}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \mathsf{SinIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}+\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}\,\mathsf{d}} - \frac{\mathsf{Sin}\left[\mathsf{a}-\frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \mathsf{SinIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}}+\left(\mathsf{b}+\mathsf{d}\right)\mathsf{x}\right]}{2\,\mathsf{b}\,\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{d}}{\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{d}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}}{\mathsf{c}} - \frac{\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{c}\,\mathsf{$$

Result (type 4, 345 leaves):

$$\begin{split} &\frac{1}{4\,b^2\,d} e^{-i\,\,(a+c)} \\ &\left(b\,d\left(-\frac{e^{-i\,\,(b+d)\,\,x}}{b+d} + \frac{e^{i\,\,(2\,a+b\,x-d\,x)}}{b-d}\right) - i\,\,\left(b\,c - i\,d\right)\,e^{\frac{i\,\left(-b\,c+\left(2\,a+c\right)\,d\right)}{d}} \, \text{ExpIntegralEi}\left[\,\frac{i\,\,\left(b-d\right)\,\,\left(c+d\,x\right)}{d}\,\right] + \\ &\left(-i\,b\,c + d\right)\,e^{\frac{i\,c\,\,(b+d)}{d}} \, \text{ExpIntegralEi}\left[-\frac{i\,\,\left(b+d\right)\,\,\left(c+d\,x\right)}{d}\,\right]\right) + \frac{1}{4\,b^2\,d} \\ &e^{-i\,\,(a-c)}\,\left(b\,d\,\left(\frac{e^{-i\,\,(b-d)\,\,x}}{b-d} - \frac{e^{i\,\,(2\,a+(b+d)\,\,x)}}{b+d}\right) + i\,\,\left(b\,c + i\,d\right)\,e^{\frac{i\,c\,\,(b-d)}{d}} \, \text{ExpIntegralEi}\left[-\frac{i\,\,\left(b-d\right)\,\,\left(c+d\,x\right)}{d}\right] + \\ &\left(i\,b\,c + d\right)\,e^{-\frac{i\,\,(b\,c-2\,a\,d+c\,d)}{d}} \, \text{ExpIntegralEi}\left[\frac{i\,\,\left(b+d\right)\,\,\left(c+d\,x\right)}{d}\right]\right) - \\ &\frac{\left(b\,x\,Cos\,[\,a+b\,x\,]\, - Sin\,[\,a+b\,x\,]\,\right) \, SinIntegral\,[\,c+d\,x\,]}{b^2} \end{split}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int Sin[a+bx] SinIntegral[c+dx] dx$$

Optimal (type 4, 154 leaves, 9 steps):

$$-\frac{\text{CosIntegral}\left[\frac{c \cdot (b-d)}{d} + \left(b-d\right) \cdot x\right] \cdot \text{Sin}\left[a - \frac{b \cdot c}{d}\right]}{2 \cdot b} + \\ \frac{\text{CosIntegral}\left[\frac{c \cdot (b+d)}{d} + \left(b+d\right) \cdot x\right] \cdot \text{Sin}\left[a - \frac{b \cdot c}{d}\right]}{2 \cdot b} - \frac{\text{Cos}\left[a - \frac{b \cdot c}{d}\right] \cdot \text{SinIntegral}\left[\frac{c \cdot (b-d)}{d} + \left(b-d\right) \cdot x\right]}{2 \cdot b} - \\ \frac{\text{Cos}\left[a + b \cdot x\right] \cdot \text{SinIntegral}\left[c + d \cdot x\right]}{b} + \frac{\text{Cos}\left[a - \frac{b \cdot c}{d}\right] \cdot \text{SinIntegral}\left[\frac{c \cdot (b+d)}{d} + \left(b+d\right) \cdot x\right]}{2 \cdot b} - \\ \frac{2 \cdot b}{b} - \frac{2$$

Result (type 4, 168 leaves):

$$\begin{split} &\frac{1}{4\,b} \\ &\dot{\mathbb{I}} \,\, \text{$e^{-\frac{i\,\left(b\,c+a\,d\right)}{d}}$} \, \left(-\,\text{$e^{\frac{2\,i\,b\,c}{d}}$ } \, \text{ExpIntegralEi} \left[-\,\frac{\dot{\mathbb{I}}\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\,\right] \, + \, \text{$e^{2\,i\,a}$ } \, \text{ExpIntegralEi} \left[\,\frac{\dot{\mathbb{I}}\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\,\right] \, + \\ &\, e^{\frac{2\,i\,b\,c}{d}} \, \, \text{ExpIntegralEi} \left[-\,\frac{\dot{\mathbb{I}}\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\,\right] \, - \, e^{2\,i\,a} \, \, \text{ExpIntegralEi} \left[\,\frac{\dot{\mathbb{I}}\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\,\right] \, + \\ &\, 4\,\dot{\mathbb{I}}\,\,e^{\frac{i\,\left(b\,c+a\,d\right)}{d}} \, \, \text{Cos}\left[a+b\,x\right] \, \, \text{SinIntegral}\left[c+d\,x\right] \, \right) \end{split}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int x \cos[a + bx] \sin[ntegral[c + dx]] dx$$

Optimal (type 4, 370 leaves, 24 steps):

$$\frac{c \cos\left[a-\frac{b\,c}{d}\right] \operatorname{CosIntegral}\left[\frac{c\,(b-d)}{d}+\left(b-d\right)\,x\right]}{2\,b\,d} - \frac{c \cos\left[a-\frac{b\,c}{d}\right] \operatorname{CosIntegral}\left[\frac{c\,(b+d)}{d}+\left(b+d\right)\,x\right]}{2\,b\,d} + \frac{\operatorname{CosIntegral}\left[\frac{c\,(b-d)}{d}+\left(b-d\right)\,x\right] \operatorname{Sin}\left[a-\frac{b\,c}{d}\right]}{2\,b^2} - \frac{\operatorname{Sin}\left[a-c+\left(b-d\right)\,x\right]}{2\,b\,\left(b-d\right)} + \frac{\operatorname{Sin}\left[a+c+\left(b+d\right)\,x\right]}{2\,b\,\left(b+d\right)} + \frac{\operatorname{CosIntegral}\left[\frac{c\,(b+d)}{d}+\left(b-d\right)\,x\right]}{2\,b\,\left(b-d\right)} + \frac{\operatorname{Sin}\left[a+c+\left(b+d\right)\,x\right]}{2\,b\,\left(b+d\right)} + \frac{\operatorname{Cos}\left[a-\frac{b\,c}{d}\right] \operatorname{SinIntegral}\left[\frac{c\,(b-d)}{d}+\left(b-d\right)\,x\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right] \operatorname{SinIntegral}\left[\frac{c\,(b-d)}{d}+\left(b-d\right)\,x\right]}{b^2} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right] \operatorname{SinIntegral}\left[\frac{c\,(b+d)}{d}+\left(b+d\right)\,x\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-\frac{b\,c}{d}\right]}{2\,b\,d} + \frac{c\,\operatorname{Sin}\left[a-$$

Result (type 4, 343 leaves):

$$\begin{split} &-\frac{1}{4\,b^2\,d}e^{-i\;(a+c)}\,\left(-\,\dot{\mathbb{I}}\,b\,d\,\left(\frac{e^{-i\;(b+d)\;x}}{b+d}\,+\,\frac{e^{i\;(2\,a+(b-d)\;x)}}{b-d}\right)\,+\\ &-\left(-\,b\,c\,+\,\dot{\mathbb{I}}\,d\right)\,e^{\frac{i\,(-b\,c+(2\,a+c)\;d)}{d}}\,\text{ExpIntegralEi}\!\left[\frac{\dot{\mathbb{I}}\,\left(b-d\right)\,\left(c\,+\,d\,x\right)}{d}\right]\,+\\ &-\left(b\,c\,+\,\dot{\mathbb{I}}\,d\right)\,e^{\frac{i\,c\,(b+d)}{d}}\,\text{ExpIntegralEi}\!\left[-\,\frac{\dot{\mathbb{I}}\,\left(b+d\right)\,\left(c\,+\,d\,x\right)}{d}\right]\right)\,+\,\frac{1}{4\,b^2\,d}e^{-i\;(a-c)}\\ &-\left(-\,\dot{\mathbb{I}}\,b\,d\,\left(\frac{e^{-i\;(b-d)\;x}}{b-d}\,+\,\frac{e^{i\;(2\,a+(b+d)\;x)}}{b+d}\right)\,+\,\left(b\,c\,+\,\dot{\mathbb{I}}\,d\right)\,e^{\frac{i\,c\,(b-d)}{d}}\,\text{ExpIntegralEi}\!\left[-\,\frac{\dot{\mathbb{I}}\,\left(b-d\right)\,\left(c\,+\,d\,x\right)}{d}\right]\,+\\ &-\left(-\,b\,c\,+\,\dot{\mathbb{I}}\,d\right)\,e^{2\,\dot{\mathbb{I}}\,a-\frac{i\,c\,(b+d)}{d}}\,\text{ExpIntegralEi}\!\left[\,\frac{\dot{\mathbb{I}}\,\left(b+d\right)\,\left(c\,+\,d\,x\right)}{d}\,\right]\right)\,+\,\frac{\left(\text{Cos}\,[\,a\,+\,b\,x\,]\,\,+\,b\,x\,\text{Sin}\,[\,a\,+\,b\,x\,]\,\right)\,\text{SinIntegral}\,[\,c\,+\,d\,x\,]}{b^2} \end{split}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \cos[a+bx] \sin Integral[c+dx] dx$$

Optimal (type 4, 153 leaves, 9 steps):

$$-\frac{\text{Cos}\left[a-\frac{b\,c}{d}\right]\text{ CosIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)\,x\right]}{2\,b}+\\ \frac{\text{Cos}\left[a-\frac{b\,c}{d}\right]\text{ CosIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)\,x\right]}{2\,b}+\frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)\,x\right]}{2\,b}+\\ \frac{\text{Sin}\left[a+b\,x\right]\text{ SinIntegral}\left[c+d\,x\right]}{b}-\frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)\,x\right]}{2\,b}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}{2\,b}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}{2\,b}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}{2\,b}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}{2\,b}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}+\frac{c\,\left(b+d\right)}{a}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}+\frac{c\,\left(b+d\right)}{a}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right]\text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)}+\frac{c\,\left(b+d\right)}{a}-\\ \frac{\text{Sin}\left[a-\frac{b\,c}$$

Result (type 4, 164 leaves):

$$\begin{split} &\frac{1}{4\,b} \text{e}^{-\frac{i\,\left(b\,c+a\,d\right)}{d}} \left(-\,\text{e}^{\frac{2\,i\,b\,c}{d}}\,\,\text{ExpIntegralEi}\left[-\frac{i\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\right] - \\ &\text{e}^{2\,i\,a}\,\,\text{ExpIntegralEi}\left[\,\frac{i\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\right] + \text{e}^{\frac{2\,i\,b\,c}{d}}\,\,\text{ExpIntegralEi}\left[-\frac{i\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\right] + \\ &\text{e}^{2\,i\,a}\,\,\text{ExpIntegralEi}\left[\,\frac{i\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\right] + 4\,\,\text{e}^{\frac{i\,\left(b\,c+a\,d\right)}{d}}\,\,\text{Sin[a+b\,x]}\,\,\text{SinIntegral[c+d\,x]} \right] \end{split}$$

Problem 108: Unable to integrate problem.

$$\int \frac{\text{CosIntegral}[b\,x]\,\,\text{Sin}[b\,x]}{x^2}\,\text{d}x$$

Optimal (type 4, 44 leaves, 7 steps):

$$\frac{1}{2} \ b \ CosIntegral \ [b \ x]^2 + b \ CosIntegral \ [2 \ b \ x] \ - \ \frac{CosIntegral \ [b \ x] \ Sin \ [b \ x]}{x} \ - \ \frac{Sin \ [2 \ b \ x]}{2 \ x}$$

Result (type 8, 14 leaves):

$$\int \frac{\mathsf{CosIntegral}[b\,x]\,\mathsf{Sin}[b\,x]}{x^2}\,\mathrm{d}x$$

Problem 114: Unable to integrate problem.

$$\int \frac{\mathsf{Cos}\,[\,b\,x\,]\,\,\mathsf{CosIntegral}\,[\,b\,x\,]}{x^3}\,\,\mathrm{d}x$$

Optimal (type 4, 97 leaves, 14 steps):

$$-\frac{\text{Cos} \left[b \, x\right]^2}{4 \, x^2} - \frac{\text{Cos} \left[b \, x\right] \, \text{CosIntegral} \left[b \, x\right]}{2 \, x^2} - \frac{1}{4} \, b^2 \, \text{CosIntegral} \left[b \, x\right]^2 - \\ b^2 \, \text{CosIntegral} \left[2 \, b \, x\right] + \frac{b \, \text{Cos} \left[b \, x\right] \, \text{Sin} \left[b \, x\right]}{2 \, x} + \frac{b \, \text{CosIntegral} \left[b \, x\right] \, \text{Sin} \left[b \, x\right]}{2 \, x} + \frac{b \, \text{Sin} \left[2 \, b \, x\right]}{4 \, x}$$

Result (type 8, 14 leaves):

$$\int \frac{\mathsf{Cos}[b\,x]\,\,\mathsf{CosIntegral}[b\,x]}{\mathsf{x}^3}\,\,\mathrm{d}\,\mathsf{x}$$

Problem 131: Result unnecessarily involves imaginary or complex numbers.

$$x$$
 CosIntegral [c + dx] Sin[a + bx] dx

Optimal (type 4, 371 leaves, 24 steps):

$$\frac{c \, \text{Cos}\left[a - \frac{b \, c}{d}\right] \, \text{CosIntegral}\left[\frac{c \, (b - d)}{d} + \left(b - d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Cos}\left[a - \frac{b \, c}{d}\right] \, \text{CosIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{b \, b} = \frac{c \, \text{Cos}\left[a - \frac{b \, c}{d}\right] \, \text{CosIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{2 \, b \, d}{2 \, b^2} + \frac{c \, \text{CosIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right] \, \text{Sin}\left[a - \frac{b \, c}{d}\right]}{2 \, b^2} + \frac{c \, \text{Sin}\left[a - c + \left(b - d\right) \, x\right]}{2 \, b \, \left(b - d\right)} + \frac{s \, \text{Sin}\left[a + c + \left(b + d\right) \, x\right]}{2 \, b \, \left(b + d\right)} = \frac{c \, \text{Cos}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b - d)}{d} + \left(b - d\right) \, x\right]}{2 \, b \, d} + \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b - d)}{d} + \left(b - d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b - d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d} + \left(b + d\right) \, x\right]}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d}\right] + \left(b + d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d}\right] + \left(b + d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d}\right] + \left(b + d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d}\right] + \left(b + d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b + d)}{d}\right] + \left(b - d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, \text{SinIntegral}\left[\frac{c \, (b \, d)}{d}\right] + \left(b - d\right) \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, x}{2 \, b \, d} = \frac{c \, \text{Sin}\left[a - \frac{b \, c}{d}\right] \, x}{2 \, b \, d} = \frac{c \,$$

Result (type 4, 332 leaves):

$$-\frac{1}{4\,b^2\,d}e^{-i\;(a+c)}$$

$$\left(-i\,b\,d\left(\frac{e^{-i\;(b+d)\,x}}{b+d}+\frac{e^{i\;(2\,c-b\,x+d\,x)}}{b-d}\right)+\left(b\,c+i\,d\right)\,e^{\frac{i\,c\;(b+d)}{d}}\,\text{ExpIntegralEi}\!\left[-\frac{i\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\right]+\right.$$

$$\left(b\,c+i\,d\right)\,e^{\frac{i\,c\;(b+d)}{d}}\,\text{ExpIntegralEi}\!\left[-\frac{i\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\right]\right)-\frac{1}{4\,b^2\,d}$$

$$e^{i\;(a-c)}\left(i\,b\,d\left(\frac{e^{i\;(b-d)\,x}}{b-d}+\frac{e^{i\;(2\,c+(b+d)\,x)}}{b+d}\right)+\left(b\,c-i\,d\right)\,e^{-\frac{i\,c\;(b-d)}{d}}\,\text{ExpIntegralEi}\!\left[\frac{i\,\left(b-d\right)\,\left(c+d\,x\right)}{d}\right]+\right.$$

$$\left(b\,c-i\,d\right)\,e^{-\frac{i\,c\;(b-d)}{d}}\,\text{ExpIntegralEi}\!\left[\frac{i\,\left(b+d\right)\,\left(c+d\,x\right)}{d}\right]\right)-$$

$$\frac{\text{CosIntegral}\,[\,c+d\,x\,]\,\left(b\,x\,\text{Cos}\,[\,a+b\,x\,]\,-\,\text{Sin}\,[\,a+b\,x\,]\,\right)}{b^2}$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

Optimal (type 4, 154 leaves, 9 steps):

$$\frac{\text{Cos}\left[\mathsf{a} - \frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \, \text{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}-\mathsf{d}\right)}{\mathsf{d}} + \left(\mathsf{b}-\mathsf{d}\right)\,\mathsf{x}\right]}{2\,\mathsf{b}} - \frac{2\,\mathsf{b}}{\mathsf{cos}\left[\mathsf{a}+\mathsf{b}\,\mathsf{x}\right] \, \text{CosIntegral}\left[\mathsf{c}+\mathsf{d}\,\mathsf{x}\right]}{\mathsf{b}} + \frac{\mathsf{Cos}\left[\mathsf{a} - \frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \, \mathsf{CosIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}} + \left(\mathsf{b}+\mathsf{d}\right)\,\mathsf{x}\right]}{2\,\mathsf{b}} - \frac{2\,\mathsf{b}}{\mathsf{cos}\left[\mathsf{a} - \frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \, \mathsf{SinIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}} + \left(\mathsf{b}+\mathsf{d}\right)\,\mathsf{x}\right]}{2\,\mathsf{b}} - \frac{\mathsf{Sin}\left[\mathsf{a} - \frac{\mathsf{b}\,\mathsf{c}}{\mathsf{d}}\right] \, \mathsf{SinIntegral}\left[\frac{\mathsf{c}\,\left(\mathsf{b}+\mathsf{d}\right)}{\mathsf{d}} + \left(\mathsf{b}+\mathsf{d}\right)\,\mathsf{x}\right]}{2\,\mathsf{b}}$$

Result (type 4, 144 leaves):

$$\begin{split} &\frac{1}{4\,b} \left(-4\,\text{Cos}\,[\,a + b\,x\,]\,\,\text{CosIntegral}\,[\,c + d\,x\,] \,\, + \\ &\left(\text{ExpIntegralEi}\left[-\frac{\dot{\mathbb{I}}\,\left(b - d \right)\,\left(c + d\,x \right)}{d} \,\right] \,\, + \,\text{ExpIntegralEi}\left[-\frac{\dot{\mathbb{I}}\,\left(b + d \right)\,\left(c + d\,x \right)}{d} \,\right] \right) \\ &\left(\text{Cos}\left[a - \frac{b\,c}{d} \,\right] \,\, - \,\dot{\mathbb{I}}\,\,\text{Sin}\left[a - \frac{b\,c}{d} \,\right] \right) \,\, + \\ &\left(\text{ExpIntegralEi}\left[\,\frac{\dot{\mathbb{I}}\,\left(b - d \right)\,\left(c + d\,x \right)}{d} \,\right] \,\, + \,\, \text{ExpIntegralEi}\left[\,\frac{\dot{\mathbb{I}}\,\left(b + d \right)\,\left(c + d\,x \right)}{d} \,\right] \right) \\ &\left(\text{Cos}\left[a - \frac{b\,c}{d} \,\right] \,\, + \,\,\dot{\mathbb{I}}\,\,\text{Sin}\left[a - \frac{b\,c}{d} \,\right] \right) \right) \end{split}$$

Problem 134: Result unnecessarily involves imaginary or complex numbers.

$$\int x \cos[a + b x] \cos[ntegral[c + d x]] dx$$

Optimal (type 4, 370 leaves, 24 steps):

$$\frac{\text{Cos}\left[a-c+\left(b-d\right)x\right]}{2 \text{ b} \left(b-d\right)} + \frac{\text{Cos}\left[a+c+\left(b+d\right)x\right]}{2 \text{ b} \left(b+d\right)} - \\ \frac{\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ CosIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Cos}\left[a+b\,x\right] \text{ CosIntegral}\left[c+d\,x\right]}{b^2} - \\ \frac{\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ CosIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{c\,\text{CosIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)x\right] \text{ Sin}\left[a-\frac{b\,c}{d}\right]}{2 \text{ b} d} + \\ \frac{c\,\text{CosIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right] \text{ Sin}\left[a-\frac{b\,c}{d}\right]}{2 \text{ b} d} + \frac{x\,\text{CosIntegral}\left[c+d\,x\right] \text{ Sin}\left[a+b\,x\right]}{b} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)x\right]}{2 \text{ b} d} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b-d\right)}{d}+\left(b-d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b} d} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Sin}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \\ \frac{c\,\text{Cos}\left[a-\frac{b\,c}{d}\right] \text{ SinIntegral}\left[\frac{c\,\left(b+d\right)}{d}+\left(b+d\right)x\right]}{2 \text{ b}^2} + \frac{\text{Cos}\left[a-\frac{b\,c}{d}\right]}{2 \text{ b}^2} + \frac{\text{Cos}\left[a-\frac{b\,c}{d}\right]}{2 \text{ b}^2} + \frac{\text{Cos}$$

Result (type 4, 347 leaves):

$$\begin{split} &\frac{1}{4\,b^2\,d}\,\dot{\mathbb{I}}\,\,e^{-i\,\,(a+c)}\\ &\left(-\,\dot{\mathbb{I}}\,b\,d\,\left(\frac{e^{-i\,\,(b+d)\,\,x}}{b+d}\,+\,\frac{e^{i\,\,(2\,a+\,(b-d)\,\,x)}}{b-d}\right)\,+\,\left(-\,b\,\,c\,+\,\dot{\mathbb{I}}\,d\right)\,\,e^{\frac{i\,\left(-b\,c+\,(2\,a+c)\,d\right)}{d}}\,\,\text{ExpIntegralEi}\left[\,\frac{\dot{\mathbb{I}}\,\,\left(b-d\right)\,\,\left(c\,+\,d\,\,x\right)}{d}\,\right]\,+\,\left(b\,\,c\,+\,\dot{\mathbb{I}}\,d\right)\,\,e^{\frac{i\,c\,\,(b-d)}{d}}\,\,\text{ExpIntegralEi}\left[\,-\,\frac{\dot{\mathbb{I}}\,\,\left(b+d\right)\,\,\left(c\,+\,d\,\,x\right)}{d}\,\right]\,\right)\,+\,\frac{1}{4\,b^2\,d}\,\dot{\mathbb{I}}\,\,e^{-i\,\,(a-c)}\\ &\left(-\,\dot{\mathbb{I}}\,b\,d\,\left(\frac{e^{-i\,\,(b-d)\,\,x}}{b-d}\,+\,\frac{e^{i\,\,(2\,a+\,(b+d)\,\,x)}}{b+d}\right)\,+\,\left(b\,\,c\,+\,\dot{\mathbb{I}}\,d\right)\,\,e^{\frac{i\,c\,\,(b-d)}{d}}\,\,\text{ExpIntegralEi}\left[\,-\,\frac{\dot{\mathbb{I}}\,\,\left(b-d\right)\,\,\left(c\,+\,d\,\,x\right)}{d}\,\right]\,+\,\\ &\left(-\,b\,\,c\,+\,\dot{\mathbb{I}}\,d\right)\,\,e^{2\,i\,\,a-\frac{i\,c\,\,(b+d)}{d}}\,\,\text{ExpIntegralEi}\left[\,\frac{\dot{\mathbb{I}}\,\,\left(b+d\right)\,\,\left(c\,+\,d\,\,x\right)}{d}\,\right]\,\right)\,+\,\\ &\frac{CosIntegral\,[\,c\,+\,d\,\,x\,]\,\,\left(Cos\,[\,a\,+\,b\,\,x\,]\,\,+\,b\,\,x\,\,Sin\,[\,a\,+\,b\,\,x\,]\,\right)}{b^2} \end{split}$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \cos[a + bx] \cos[ntegral[c + dx]] dx$$

Optimal (type 4, 153 leaves, 9 steps):

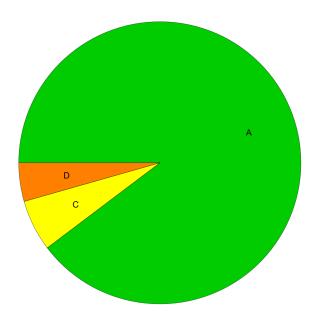
$$-\frac{\text{CosIntegral}\left[\frac{c\ (b-d)}{d} + \left(b-d\right)\ x\right] \, \text{Sin}\left[a - \frac{b\,c}{d}\right]}{2\,b} - \\ \frac{\text{CosIntegral}\left[\frac{c\ (b+d)}{d} + \left(b+d\right)\ x\right] \, \text{Sin}\left[a - \frac{b\,c}{d}\right]}{2\,b} + \frac{\text{CosIntegral}\left[c+d\ x\right] \, \text{Sin}\left[a+b\ x\right]}{b} - \\ \frac{\text{Cos}\left[a - \frac{b\,c}{d}\right] \, \text{SinIntegral}\left[\frac{c\ (b-d)}{d} + \left(b-d\right)\ x\right]}{2\,b} - \frac{\text{Cos}\left[a - \frac{b\,c}{d}\right] \, \text{SinIntegral}\left[\frac{c\ (b+d)}{d} + \left(b+d\right)\ x\right]}{2\,b}$$

Result (type 4, 153 leaves):

$$\begin{split} &\frac{1}{4\,b} \\ &\left(\text{i} \,\, \text{e}^{-\frac{i\,\left(b\,c+a\,d\right)}{d}} \left(-\,\text{e}^{\frac{2\,i\,b\,c}{d}} \, \text{ExpIntegralEi} \left[-\,\frac{\text{i}\,\left(b-d\right)\,\left(c+d\,x\right)}{d} \,\right] + \text{e}^{2\,i\,a} \, \text{ExpIntegralEi} \left[\,\frac{\text{i}\,\left(b-d\right)\,\left(c+d\,x\right)}{d} \,\right] - \\ &\left. \text{e}^{\frac{2\,i\,b\,c}{d}} \, \text{ExpIntegralEi} \left[-\,\frac{\text{i}\,\left(b+d\right)\,\left(c+d\,x\right)}{d} \,\right] + \text{e}^{2\,i\,a} \, \text{ExpIntegralEi} \left[\,\frac{\text{i}\,\left(b+d\right)\,\left(c+d\,x\right)}{d} \,\right] \right) + \\ & 4 \, \text{CosIntegral} \left[c+d\,x \right] \, \text{Sin} \left[a+b\,x \right] \right) \end{split}$$

Summary of Integration Test Results

136 integration problems



- A 122 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 8 unnecessarily complex antiderivatives
- D 6 unable to integrate problems
- E 0 integration timeouts