

# Rubi 4.16.0.4 Integration Test Results

## on the problems in the test-suite directory "5 Inverse trig functions"

Test results for the 227 problems in "5.1.2 (d x)^m (a+b arcsin(c x))^n.m"

Problem 168: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{(a + b \operatorname{ArcSin}[c x])^3} dx$$

Optimal (type 4, 197 leaves, 16 steps):

$$\begin{aligned} & -\frac{x^2 \sqrt{1-c^2 x^2}}{2 b c (a + b \operatorname{ArcSin}[c x])^2} - \frac{x}{b^2 c^2 (a + b \operatorname{ArcSin}[c x])} + \frac{3 x^3}{2 b^2 (a + b \operatorname{ArcSin}[c x])} - \\ & \frac{\operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{8 b^3 c^3} + \frac{9 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3(a+b \operatorname{ArcSin}[c x])}{b}\right]}{8 b^3 c^3} - \\ & \frac{\operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{8 b^3 c^3} + \frac{9 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3(a+b \operatorname{ArcSin}[c x])}{b}\right]}{8 b^3 c^3} \end{aligned}$$

Result (type 4, 245 leaves, 16 steps):

$$\begin{aligned} & -\frac{x^2 \sqrt{1-c^2 x^2}}{2 b c (a + b \operatorname{ArcSin}[c x])^2} - \frac{x}{b^2 c^2 (a + b \operatorname{ArcSin}[c x])} + \frac{3 x^3}{2 b^2 (a + b \operatorname{ArcSin}[c x])} - \\ & \frac{9 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c x]\right]}{8 b^3 c^3} + \frac{9 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c x]\right]}{8 b^3 c^3} + \\ & \frac{\operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{b^3 c^3} - \frac{9 \operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c x]\right]}{8 b^3 c^3} + \\ & \frac{9 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c x]\right]}{8 b^3 c^3} + \frac{\operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{b^3 c^3} \end{aligned}$$

## Test results for the 703 problems in "5.1.4 (f x)^m (d+e x^2)^p (a+b arcsin(c x))^n.m"

Problem 45: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{x^4 (d - c^2 d x^2)^2} dx$$

Optimal (type 4, 259 leaves, 19 steps):

$$\begin{aligned} & -\frac{b c^3}{3 d^2 \sqrt{1 - c^2 x^2}} - \frac{b c}{6 d^2 x^2 \sqrt{1 - c^2 x^2}} - \frac{a + b \operatorname{ArcSin}[c x]}{3 d^2 x^3 (1 - c^2 x^2)} - \\ & \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])}{3 d^2 x (1 - c^2 x^2)} + \frac{5 c^4 x (a + b \operatorname{ArcSin}[c x])}{2 d^2 (1 - c^2 x^2)} - \\ & \frac{5 i c^3 (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d^2} - \frac{13 b c^3 \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{6 d^2} + \\ & \frac{5 i b c^3 \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{2 d^2} - \frac{5 i b c^3 \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{2 d^2} \end{aligned}$$

Result (type 4, 285 leaves, 19 steps):

$$\begin{aligned} & -\frac{5 b c^3}{6 d^2 \sqrt{1 - c^2 x^2}} + \frac{b c}{3 d^2 x^2 \sqrt{1 - c^2 x^2}} - \frac{b c \sqrt{1 - c^2 x^2}}{2 d^2 x^2} - \\ & \frac{a + b \operatorname{ArcSin}[c x]}{3 d^2 x^3 (1 - c^2 x^2)} - \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])}{3 d^2 x (1 - c^2 x^2)} + \frac{5 c^4 x (a + b \operatorname{ArcSin}[c x])}{2 d^2 (1 - c^2 x^2)} - \\ & \frac{5 i c^3 (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d^2} - \frac{13 b c^3 \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{6 d^2} + \\ & \frac{5 i b c^3 \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{2 d^2} - \frac{5 i b c^3 \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{2 d^2} \end{aligned}$$

Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{x^4 (d - c^2 d x^2)^3} dx$$

Optimal (type 4, 317 leaves, 23 steps):

$$\begin{aligned}
& \frac{b c^3}{12 d^3 (1 - c^2 x^2)^{3/2}} - \frac{b c}{6 d^3 x^2 (1 - c^2 x^2)^{3/2}} - \frac{29 b c^3}{24 d^3 \sqrt{1 - c^2 x^2}} - \frac{a + b \operatorname{ArcSin}[c x]}{3 d^3 x^3 (1 - c^2 x^2)^2} - \\
& \frac{7 c^2 (a + b \operatorname{ArcSin}[c x])}{3 d^3 x (1 - c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSin}[c x])}{12 d^3 (1 - c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSin}[c x])}{8 d^3 (1 - c^2 x^2)} - \\
& \frac{35 i c^3 (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{4 d^3} - \frac{19 b c^3 \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{6 d^3} + \\
& \frac{35 i b c^3 \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{8 d^3} - \frac{35 i b c^3 \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{8 d^3}
\end{aligned}$$

Result (type 4, 369 leaves, 23 steps):

$$\begin{aligned}
& -\frac{7 b c^3}{36 d^3 (1 - c^2 x^2)^{3/2}} + \frac{b c}{9 d^3 x^2 (1 - c^2 x^2)^{3/2}} - \frac{49 b c^3}{24 d^3 \sqrt{1 - c^2 x^2}} + \\
& \frac{5 b c}{9 d^3 x^2 \sqrt{1 - c^2 x^2}} - \frac{5 b c \sqrt{1 - c^2 x^2}}{6 d^3 x^2} - \frac{a + b \operatorname{ArcSin}[c x]}{3 d^3 x^3 (1 - c^2 x^2)^2} - \\
& \frac{7 c^2 (a + b \operatorname{ArcSin}[c x])}{3 d^3 x (1 - c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSin}[c x])}{12 d^3 (1 - c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSin}[c x])}{8 d^3 (1 - c^2 x^2)} - \\
& \frac{35 i c^3 (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{4 d^3} - \frac{19 b c^3 \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{6 d^3} + \\
& \frac{35 i b c^3 \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{8 d^3} - \frac{35 i b c^3 \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{8 d^3}
\end{aligned}$$

Problem 60: Result optimal but 2 more steps used.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{x^6} dx$$

Optimal (type 3, 187 leaves, 4 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{d - c^2 d x^2}}{20 x^4 \sqrt{1 - c^2 x^2}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{30 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{5 d x^5} - \\
& \frac{2 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{15 d x^3} - \frac{2 b c^5 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Result (type 3, 187 leaves, 6 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{d - c^2 d x^2}}{20 x^4 \sqrt{1 - c^2 x^2}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{30 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{5 d x^5} - \\
& \frac{2 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{15 d x^3} - \frac{2 b c^5 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

### Problem 61: Result optimal but 3 more steps used.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{x^8} dx$$

Optimal (type 3, 263 leaves, 4 steps):

$$\begin{aligned} & -\frac{b c \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{140 x^4 \sqrt{1 - c^2 x^2}} + \frac{2 b c^5 \sqrt{d - c^2 d x^2}}{105 x^2 \sqrt{1 - c^2 x^2}} - \\ & \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{7 d x^7} - \frac{4 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{35 d x^5} - \\ & \frac{8 c^4 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{105 d x^3} - \frac{8 b c^7 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{105 \sqrt{1 - c^2 x^2}} \end{aligned}$$

Result (type 3, 263 leaves, 7 steps):

$$\begin{aligned} & -\frac{b c \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{140 x^4 \sqrt{1 - c^2 x^2}} + \frac{2 b c^5 \sqrt{d - c^2 d x^2}}{105 x^2 \sqrt{1 - c^2 x^2}} - \\ & \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{7 d x^7} - \frac{4 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{35 d x^5} - \\ & \frac{8 c^4 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{105 d x^3} - \frac{8 b c^7 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{105 \sqrt{1 - c^2 x^2}} \end{aligned}$$

### Problem 62: Result optimal but 3 more steps used.

$$\int x^5 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 256 leaves, 3 steps):

$$\begin{aligned} & \frac{8 b x \sqrt{d - c^2 d x^2}}{105 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b x^3 \sqrt{d - c^2 d x^2}}{315 c^3 \sqrt{1 - c^2 x^2}} + \frac{b x^5 \sqrt{d - c^2 d x^2}}{175 c \sqrt{1 - c^2 x^2}} - \\ & \frac{b c x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{3 c^6 d} + \\ & \frac{2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^6 d^2} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^6 d^3} \end{aligned}$$

Result (type 3, 256 leaves, 6 steps):

$$\begin{aligned} & \frac{8 b x \sqrt{d - c^2 d x^2}}{105 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b x^3 \sqrt{d - c^2 d x^2}}{315 c^3 \sqrt{1 - c^2 x^2}} + \frac{b x^5 \sqrt{d - c^2 d x^2}}{175 c \sqrt{1 - c^2 x^2}} - \\ & \frac{b c x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{3 c^6 d} + \\ & \frac{2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^6 d^2} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^6 d^3} \end{aligned}$$

### Problem 63: Result optimal but 3 more steps used.

$$\int x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 183 leaves, 3 steps):

$$\frac{2 b x \sqrt{d - c^2 d x^2}}{15 c^3 \sqrt{1 - c^2 x^2}} + \frac{b x^3 \sqrt{d - c^2 d x^2}}{45 c \sqrt{1 - c^2 x^2}} - \frac{b c x^5 \sqrt{d - c^2 d x^2}}{25 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{3 c^4 d} + \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^4 d^2}$$

Result (type 3, 183 leaves, 6 steps):

$$\frac{2 b x \sqrt{d - c^2 d x^2}}{15 c^3 \sqrt{1 - c^2 x^2}} + \frac{b x^3 \sqrt{d - c^2 d x^2}}{45 c \sqrt{1 - c^2 x^2}} - \frac{b c x^5 \sqrt{d - c^2 d x^2}}{25 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{3 c^4 d} + \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^4 d^2}$$

### Problem 74: Result optimal but 2 more steps used.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{x^8} dx$$

Optimal (type 3, 231 leaves, 5 steps):

$$-\frac{b c d \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{1 - c^2 x^2}} + \frac{2 b c^3 d \sqrt{d - c^2 d x^2}}{35 x^4 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{70 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{7 d x^7} - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{35 d x^5} + \frac{2 b c^7 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{1 - c^2 x^2}}$$

Result (type 3, 231 leaves, 7 steps):

$$-\frac{b c d \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{1 - c^2 x^2}} + \frac{2 b c^3 d \sqrt{d - c^2 d x^2}}{35 x^4 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{70 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{7 d x^7} - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{35 d x^5} + \frac{2 b c^7 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{1 - c^2 x^2}}$$

### Problem 75: Result optimal but 3 more steps used.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{x^{10}} dx$$

Optimal (type 3, 308 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{72 x^8 \sqrt{1 - c^2 x^2}} + \frac{5 b c^3 d \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{420 x^4 \sqrt{1 - c^2 x^2}} - \frac{2 b c^7 d \sqrt{d - c^2 d x^2}}{315 x^2 \sqrt{1 - c^2 x^2}} \\
& - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{9 d x^9} - \frac{4 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{63 d x^7} \\
& - \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{315 d x^5} + \frac{8 b c^9 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{315 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Result (type 3, 308 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{72 x^8 \sqrt{1 - c^2 x^2}} + \frac{5 b c^3 d \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{420 x^4 \sqrt{1 - c^2 x^2}} - \frac{2 b c^7 d \sqrt{d - c^2 d x^2}}{315 x^2 \sqrt{1 - c^2 x^2}} \\
& - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{9 d x^9} - \frac{4 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{63 d x^7} \\
& - \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{315 d x^5} + \frac{8 b c^9 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{315 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Problem 76: Result optimal but 4 more steps used.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])}{x^{12}} dx$$

Optimal (type 3, 385 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{1 - c^2 x^2}} + \frac{b c^3 d \sqrt{d - c^2 d x^2}}{66 x^8 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{1386 x^6 \sqrt{1 - c^2 x^2}} \\
& - \frac{b c^7 d \sqrt{d - c^2 d x^2}}{770 x^4 \sqrt{1 - c^2 x^2}} - \frac{4 b c^9 d \sqrt{d - c^2 d x^2}}{1155 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{11 d x^{11}} \\
& - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{33 d x^9} - \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{231 d x^7} \\
& - \frac{16 c^6 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{1155 d x^5} + \frac{16 b c^{11} d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{1155 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Result (type 3, 385 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{1 - c^2 x^2}} + \frac{b c^3 d \sqrt{d - c^2 d x^2}}{66 x^8 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{1386 x^6 \sqrt{1 - c^2 x^2}} \\
& - \frac{b c^7 d \sqrt{d - c^2 d x^2}}{770 x^4 \sqrt{1 - c^2 x^2}} - \frac{4 b c^9 d \sqrt{d - c^2 d x^2}}{1155 x^2 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{11 d x^{11}} \\
& - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{33 d x^9} - \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{231 d x^7} \\
& - \frac{16 c^6 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{1155 d x^5} + \frac{16 b c^{11} d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{1155 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

### Problem 77: Result optimal but 3 more steps used.

$$\int x^7 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 375 leaves, 4 steps):

$$\begin{aligned} & \frac{16 b d x \sqrt{d - c^2 d x^2}}{1155 c^7 \sqrt{1 - c^2 x^2}} + \frac{8 b d x^3 \sqrt{d - c^2 d x^2}}{3465 c^5 \sqrt{1 - c^2 x^2}} + \frac{2 b d x^5 \sqrt{d - c^2 d x^2}}{1925 c^3 \sqrt{1 - c^2 x^2}} + \\ & \frac{b d x^7 \sqrt{d - c^2 d x^2}}{1617 c \sqrt{1 - c^2 x^2}} - \frac{4 b c d x^9 \sqrt{d - c^2 d x^2}}{297 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{1 - c^2 x^2}} - \\ & \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^8 d} + \frac{3 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^8 d^2} - \\ & \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{3 c^8 d^3} + \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcSin}[c x])}{11 c^8 d^4} \end{aligned}$$

Result (type 3, 375 leaves, 7 steps):

$$\begin{aligned} & \frac{16 b d x \sqrt{d - c^2 d x^2}}{1155 c^7 \sqrt{1 - c^2 x^2}} + \frac{8 b d x^3 \sqrt{d - c^2 d x^2}}{3465 c^5 \sqrt{1 - c^2 x^2}} + \frac{2 b d x^5 \sqrt{d - c^2 d x^2}}{1925 c^3 \sqrt{1 - c^2 x^2}} + \\ & \frac{b d x^7 \sqrt{d - c^2 d x^2}}{1617 c \sqrt{1 - c^2 x^2}} - \frac{4 b c d x^9 \sqrt{d - c^2 d x^2}}{297 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{1 - c^2 x^2}} - \\ & \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^8 d} + \frac{3 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^8 d^2} - \\ & \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{3 c^8 d^3} + \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcSin}[c x])}{11 c^8 d^4} \end{aligned}$$

### Problem 78: Result optimal but 3 more steps used.

$$\int x^5 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 301 leaves, 4 steps):

$$\begin{aligned} & \frac{8 b d x \sqrt{d - c^2 d x^2}}{315 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b d x^3 \sqrt{d - c^2 d x^2}}{945 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d x^5 \sqrt{d - c^2 d x^2}}{525 c \sqrt{1 - c^2 x^2}} - \\ & \frac{10 b c d x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{5 c^6 d} + \\ & \frac{2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^6 d^2} - \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{9 c^6 d^3} \end{aligned}$$

Result (type 3, 301 leaves, 7 steps):

$$\frac{8 b d x \sqrt{d - c^2 d x^2}}{315 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b d x^3 \sqrt{d - c^2 d x^2}}{945 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d x^5 \sqrt{d - c^2 d x^2}}{525 c \sqrt{1 - c^2 x^2}} -$$

$$\frac{10 b c d x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \text{ArcSin}[c x])}{5 c^6 d} +$$

$$\frac{2 (d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x])}{7 c^6 d^2} - \frac{(d - c^2 d x^2)^{9/2} (a + b \text{ArcSin}[c x])}{9 c^6 d^3}$$

**Problem 79: Result optimal but 3 more steps used.**

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b \text{ArcSin}[c x]) dx$$

Optimal (type 3, 227 leaves, 4 steps):

$$\frac{2 b d x \sqrt{d - c^2 d x^2}}{35 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d x^3 \sqrt{d - c^2 d x^2}}{105 c \sqrt{1 - c^2 x^2}} - \frac{8 b c d x^5 \sqrt{d - c^2 d x^2}}{175 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{1 - c^2 x^2}} -$$

$$\frac{(d - c^2 d x^2)^{5/2} (a + b \text{ArcSin}[c x])}{5 c^4 d} + \frac{(d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x])}{7 c^4 d^2}$$

Result (type 3, 227 leaves, 7 steps):

$$\frac{2 b d x \sqrt{d - c^2 d x^2}}{35 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d x^3 \sqrt{d - c^2 d x^2}}{105 c \sqrt{1 - c^2 x^2}} - \frac{8 b c d x^5 \sqrt{d - c^2 d x^2}}{175 \sqrt{1 - c^2 x^2}} + \frac{b c^3 d x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{1 - c^2 x^2}} -$$

$$\frac{(d - c^2 d x^2)^{5/2} (a + b \text{ArcSin}[c x])}{5 c^4 d} + \frac{(d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x])}{7 c^4 d^2}$$

**Problem 91: Result optimal but 2 more steps used.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \text{ArcSin}[c x])}{x^{10}} dx$$

Optimal (type 3, 282 leaves, 6 steps):

$$-\frac{b c^3 d^2 \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^5 d^2 \sqrt{d - c^2 d x^2}}{42 x^4 \sqrt{1 - c^2 x^2}} - \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{21 x^2 \sqrt{1 - c^2 x^2}} -$$

$$\frac{b c d^2 (1 - c^2 x^2)^{7/2} \sqrt{d - c^2 d x^2}}{72 x^8} - \frac{(d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x])}{9 d x^9} -$$

$$\frac{2 c^2 (d - c^2 d x^2)^{7/2} (a + b \text{ArcSin}[c x])}{63 d x^7} - \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2} \text{Log}[x]}{63 \sqrt{1 - c^2 x^2}}$$

Result (type 3, 282 leaves, 8 steps):



$$\begin{aligned}
& - \frac{b c^3 d^2 \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^5 d^2 \sqrt{d - c^2 d x^2}}{42 x^4 \sqrt{1 - c^2 x^2}} - \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{21 x^2 \sqrt{1 - c^2 x^2}} - \\
& \frac{b c d^2 (1 - c^2 x^2)^{7/2} \sqrt{d - c^2 d x^2}}{72 x^8} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{9 d x^9} - \\
& \frac{2 c^2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{63 d x^7} - \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{63 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Problem 92: Result optimal but 3 more steps used.

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])}{x^{12}} dx$$

Optimal (type 3, 361 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c d^2 \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{1 - c^2 x^2}} + \frac{23 b c^3 d^2 \sqrt{d - c^2 d x^2}}{792 x^8 \sqrt{1 - c^2 x^2}} - \\
& \frac{113 b c^5 d^2 \sqrt{d - c^2 d x^2}}{4158 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{924 x^4 \sqrt{1 - c^2 x^2}} + \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2}}{693 x^2 \sqrt{1 - c^2 x^2}} - \\
& \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{11 d x^{11}} - \frac{4 c^2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{99 d x^9} - \\
& \frac{8 c^4 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{693 d x^7} - \frac{8 b c^{11} d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{693 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Result (type 3, 361 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b c d^2 \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{1 - c^2 x^2}} + \frac{23 b c^3 d^2 \sqrt{d - c^2 d x^2}}{792 x^8 \sqrt{1 - c^2 x^2}} - \\
& \frac{113 b c^5 d^2 \sqrt{d - c^2 d x^2}}{4158 x^6 \sqrt{1 - c^2 x^2}} + \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{924 x^4 \sqrt{1 - c^2 x^2}} + \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2}}{693 x^2 \sqrt{1 - c^2 x^2}} - \\
& \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{11 d x^{11}} - \frac{4 c^2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{99 d x^9} - \\
& \frac{8 c^4 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{693 d x^7} - \frac{8 b c^{11} d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{693 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Problem 93: Result optimal but 3 more steps used.

$$\int x^5 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 354 leaves, 4 steps):

$$\frac{8 b d^2 x \sqrt{d - c^2 d x^2}}{693 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b d^2 x^3 \sqrt{d - c^2 d x^2}}{2079 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d^2 x^5 \sqrt{d - c^2 d x^2}}{1155 c \sqrt{1 - c^2 x^2}} - \frac{113 b c d^2 x^7 \sqrt{d - c^2 d x^2}}{4851 \sqrt{1 - c^2 x^2}} +$$

$$\frac{23 b c^3 d^2 x^9 \sqrt{d - c^2 d x^2}}{891 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d^2 x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^6 d} +$$

$$\frac{2 (d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{9 c^6 d^2} - \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcSin}[c x])}{11 c^6 d^3}$$

Result (type 3, 354 leaves, 7 steps):

$$\frac{8 b d^2 x \sqrt{d - c^2 d x^2}}{693 c^5 \sqrt{1 - c^2 x^2}} + \frac{4 b d^2 x^3 \sqrt{d - c^2 d x^2}}{2079 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d^2 x^5 \sqrt{d - c^2 d x^2}}{1155 c \sqrt{1 - c^2 x^2}} - \frac{113 b c d^2 x^7 \sqrt{d - c^2 d x^2}}{4851 \sqrt{1 - c^2 x^2}} +$$

$$\frac{23 b c^3 d^2 x^9 \sqrt{d - c^2 d x^2}}{891 \sqrt{1 - c^2 x^2}} - \frac{b c^5 d^2 x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^6 d} +$$

$$\frac{2 (d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{9 c^6 d^2} - \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcSin}[c x])}{11 c^6 d^3}$$

Problem 94: Result optimal but 3 more steps used.

$$\int x^3 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 278 leaves, 4 steps):

$$\frac{2 b d^2 x \sqrt{d - c^2 d x^2}}{63 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d^2 x^3 \sqrt{d - c^2 d x^2}}{189 c \sqrt{1 - c^2 x^2}} - \frac{b c d^2 x^5 \sqrt{d - c^2 d x^2}}{21 \sqrt{1 - c^2 x^2}} + \frac{19 b c^3 d^2 x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{1 - c^2 x^2}} -$$

$$\frac{b c^5 d^2 x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^4 d} + \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{9 c^4 d^2}$$

Result (type 3, 278 leaves, 7 steps):

$$\frac{2 b d^2 x \sqrt{d - c^2 d x^2}}{63 c^3 \sqrt{1 - c^2 x^2}} + \frac{b d^2 x^3 \sqrt{d - c^2 d x^2}}{189 c \sqrt{1 - c^2 x^2}} - \frac{b c d^2 x^5 \sqrt{d - c^2 d x^2}}{21 \sqrt{1 - c^2 x^2}} + \frac{19 b c^3 d^2 x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{1 - c^2 x^2}} -$$

$$\frac{b c^5 d^2 x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{1 - c^2 x^2}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcSin}[c x])}{7 c^4 d} + \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcSin}[c x])}{9 c^4 d^2}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi - c^2 \pi x^2} (a + b \operatorname{ArcSin}[c x]) dx$$

Optimal (type 3, 68 leaves, 3 steps):

$$-\frac{1}{4} b c \sqrt{\pi} x^2 + \frac{1}{2} x \sqrt{\pi - c^2 \pi x^2} (a + b \operatorname{ArcSin}[c x]) + \frac{\sqrt{\pi} (a + b \operatorname{ArcSin}[c x])^2}{4 b c}$$

Result (type 3, 116 leaves, 3 steps):

$$-\frac{b c x^2 \sqrt{1-c^2 x^2}}{4 \sqrt{1-c^2 x^2}} + \frac{1}{2} x \sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x]) + \frac{\sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 b c \sqrt{1-c^2 x^2}}$$

Problem 110: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a + b \operatorname{ArcSin}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 200 leaves, 5 steps):

$$\frac{3 b x^2 \sqrt{1-c^2 x^2}}{16 c^3 \sqrt{d - c^2 d x^2}} + \frac{b x^4 \sqrt{1-c^2 x^2}}{16 c \sqrt{d - c^2 d x^2}} - \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{8 c^4 d} - \frac{x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{4 c^2 d} + \frac{3 \sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{16 b c^5 \sqrt{d - c^2 d x^2}}$$

Result (type 3, 200 leaves, 6 steps):

$$\frac{3 b x^2 \sqrt{1-c^2 x^2}}{16 c^3 \sqrt{d - c^2 d x^2}} + \frac{b x^4 \sqrt{1-c^2 x^2}}{16 c \sqrt{d - c^2 d x^2}} - \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{8 c^4 d} - \frac{x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{4 c^2 d} + \frac{3 \sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{16 b c^5 \sqrt{d - c^2 d x^2}}$$

Problem 112: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSin}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 124 leaves, 3 steps):

$$\frac{b x^2 \sqrt{1-c^2 x^2}}{4 c \sqrt{d - c^2 d x^2}} - \frac{x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{2 c^2 d} + \frac{\sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 b c^3 \sqrt{d - c^2 d x^2}}$$

Result (type 3, 124 leaves, 4 steps):

$$\frac{b x^2 \sqrt{1-c^2 x^2}}{4 c \sqrt{d - c^2 d x^2}} - \frac{x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{2 c^2 d} + \frac{\sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 b c^3 \sqrt{d - c^2 d x^2}}$$

Problem 114: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 49 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 b c \sqrt{d - c^2 d x^2}}$$

Result (type 3, 49 leaves, 2 steps):

$$\frac{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{2 b c \sqrt{d-c^2 d x^2}}$$

Problem 115: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 145 leaves, 6 steps):

$$-\frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} +$$

$$\frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}}$$

Result (type 4, 145 leaves, 7 steps):

$$-\frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} +$$

$$\frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}}$$

Problem 117: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x^3 \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 229 leaves, 8 steps):

$$-\frac{b c \sqrt{1-c^2 x^2}}{2 x \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{2 d x^2} -$$

$$\frac{c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} +$$

$$\frac{i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 \sqrt{d-c^2 d x^2}} - \frac{i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 \sqrt{d-c^2 d x^2}}$$

Result (type 4, 229 leaves, 9 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{1-c^2 x^2}}{2 x \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{2 d x^2} - \\
& \frac{c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 \sqrt{d-c^2 d x^2}} - \frac{i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 119: Result valid but suboptimal antiderivative.**

$$\int \frac{x^5 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 221 leaves, 5 steps):

$$\begin{aligned}
& -\frac{5 b x \sqrt{d-c^2 d x^2}}{3 c^5 d^2 \sqrt{1-c^2 x^2}} - \frac{b x^3 \sqrt{d-c^2 d x^2}}{9 c^3 d^2 \sqrt{1-c^2 x^2}} + \frac{a+b \operatorname{ArcSin}[c x]}{c^6 d \sqrt{d-c^2 d x^2}} + \frac{2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{c^6 d^2} - \\
& \frac{(d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcSin}[c x])}{3 c^6 d^3} - \frac{b \sqrt{d-c^2 d x^2} \operatorname{ArcTanh}[c x]}{c^6 d^2 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 229 leaves, 8 steps):

$$\begin{aligned}
& -\frac{5 b x \sqrt{1-c^2 x^2}}{3 c^5 d \sqrt{d-c^2 d x^2}} - \frac{b x^3 \sqrt{1-c^2 x^2}}{9 c^3 d \sqrt{d-c^2 d x^2}} + \\
& \frac{x^4 (a+b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d-c^2 d x^2}} + \frac{8 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{3 c^6 d^2} + \\
& \frac{4 x^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{3 c^4 d^2} - \frac{b \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{c^6 d \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 120: Result optimal but 1 more steps used.**

$$\int \frac{x^4 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 214 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b x^2 \sqrt{1-c^2 x^2}}{4 c^3 d \sqrt{d-c^2 d x^2}} + \frac{x^3 (a+b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d-c^2 d x^2}} + \frac{3 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{2 c^4 d^2} - \\
& \frac{3 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{4 b c^5 d \sqrt{d-c^2 d x^2}} + \frac{b \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{2 c^5 d \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 214 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b x^2 \sqrt{1 - c^2 x^2}}{4 c^3 d \sqrt{d - c^2 d x^2}} + \frac{x^3 (a + b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} + \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{2 c^4 d^2} - \\
& \frac{3 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 b c^5 d \sqrt{d - c^2 d x^2}} + \frac{b \sqrt{1 - c^2 x^2} \operatorname{Log}[1 - c^2 x^2]}{2 c^5 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 121: Result valid but suboptimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{ArcSin}[c x])}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 142 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{d - c^2 d x^2}}{c^3 d^2 \sqrt{1 - c^2 x^2}} + \frac{a + b \operatorname{ArcSin}[c x]}{c^4 d \sqrt{d - c^2 d x^2}} + \\
& \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{c^4 d^2} - \frac{b \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}[c x]}{c^4 d^2 \sqrt{1 - c^2 x^2}}
\end{aligned}$$

Result (type 3, 146 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{1 - c^2 x^2}}{c^3 d \sqrt{d - c^2 d x^2}} + \frac{x^2 (a + b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} + \\
& \frac{2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{c^4 d^2} - \frac{b \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[c x]}{c^4 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 122: Result optimal but 1 more steps used.**

$$\int \frac{x^2 (a + b \operatorname{ArcSin}[c x])}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$\begin{aligned}
& \frac{x (a + b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 b c^3 d \sqrt{d - c^2 d x^2}} + \frac{b \sqrt{1 - c^2 x^2} \operatorname{Log}[1 - c^2 x^2]}{2 c^3 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 3, 135 leaves, 4 steps):

$$\begin{aligned}
& \frac{x (a + b \operatorname{ArcSin}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 b c^3 d \sqrt{d - c^2 d x^2}} + \frac{b \sqrt{1 - c^2 x^2} \operatorname{Log}[1 - c^2 x^2]}{2 c^3 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 125: Result optimal but 1 more steps used.**

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{x (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 220 leaves, 8 steps):

$$\frac{a + b \operatorname{ArcSin}[c x]}{d \sqrt{d - c^2 d x^2}} - \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} -$$

$$\frac{b \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[c x]}{d \sqrt{d - c^2 d x^2}} + \frac{i b \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} -$$

$$\frac{i b \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}}$$

Result (type 4, 220 leaves, 9 steps):

$$\frac{a + b \operatorname{ArcSin}[c x]}{d \sqrt{d - c^2 d x^2}} - \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} -$$

$$\frac{b \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[c x]}{d \sqrt{d - c^2 d x^2}} + \frac{i b \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}} -$$

$$\frac{i b \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d - c^2 d x^2}}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{x^2 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 150 leaves, 5 steps):

$$-\frac{a + b \operatorname{ArcSin}[c x]}{d x \sqrt{d - c^2 d x^2}} + \frac{2 c^2 x (a + b \operatorname{ArcSin}[c x])}{d \sqrt{d - c^2 d x^2}} +$$

$$\frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{d^2 \sqrt{1 - c^2 x^2}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{2 d^2 \sqrt{1 - c^2 x^2}}$$

Result (type 3, 150 leaves, 7 steps):

$$-\frac{a + b \operatorname{ArcSin}[c x]}{d x \sqrt{d - c^2 d x^2}} + \frac{2 c^2 x (a + b \operatorname{ArcSin}[c x])}{d \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{1 - c^2 x^2} \operatorname{Log}[x]}{d \sqrt{d - c^2 d x^2}} + \frac{b c \sqrt{1 - c^2 x^2} \operatorname{Log}[1 - c^2 x^2]}{2 d \sqrt{d - c^2 d x^2}}$$

Problem 127: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcSin}[c x]}{x^3 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{1-c^2 x^2}}{2 d x \sqrt{d-c^2 d x^2}} + \frac{3 c^2 (a+b \operatorname{ArcSin}[c x])}{2 d \sqrt{d-c^2 d x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{2 d x^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{3 c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d-c^2 d x^2}} - \frac{b c^2 \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{d \sqrt{d-c^2 d x^2}} + \\
& \frac{3 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 d \sqrt{d-c^2 d x^2}} - \frac{3 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 d \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 316 leaves, 12 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{1-c^2 x^2}}{2 d x \sqrt{d-c^2 d x^2}} + \frac{3 c^2 (a+b \operatorname{ArcSin}[c x])}{2 d \sqrt{d-c^2 d x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{2 d x^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{3 c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d \sqrt{d-c^2 d x^2}} - \frac{b c^2 \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{d \sqrt{d-c^2 d x^2}} + \\
& \frac{3 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 d \sqrt{d-c^2 d x^2}} - \frac{3 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 d \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 128: Result valid but suboptimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x^4 (d-c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 238 leaves, 5 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{d-c^2 d x^2}}{6 d^2 x^2 \sqrt{1-c^2 x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{3 d x^3 \sqrt{d-c^2 d x^2}} - \frac{4 c^2 (a+b \operatorname{ArcSin}[c x])}{3 d x \sqrt{d-c^2 d x^2}} + \\
& \frac{8 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d \sqrt{d-c^2 d x^2}} + \frac{5 b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{1-c^2 x^2}} + \frac{b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[1-c^2 x^2]}{2 d^2 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 238 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{1-c^2 x^2}}{6 d x^2 \sqrt{d-c^2 d x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{3 d x^3 \sqrt{d-c^2 d x^2}} - \frac{4 c^2 (a+b \operatorname{ArcSin}[c x])}{3 d x \sqrt{d-c^2 d x^2}} + \\
& \frac{8 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d \sqrt{d-c^2 d x^2}} + \frac{5 b c^3 \sqrt{1-c^2 x^2} \operatorname{Log}[x]}{3 d \sqrt{d-c^2 d x^2}} + \frac{b c^3 \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{2 d \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 129: Result optimal but 1 more steps used.**

$$\int \frac{x^6 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 293 leaves, 11 steps):



$$\begin{aligned}
& - \frac{b}{6 c^7 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{b x^2 \sqrt{1-c^2 x^2}}{4 c^5 d^2 \sqrt{d-c^2 d x^2}} + \frac{x^5 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{5 x^3 (a+b \operatorname{ArcSin}[c x])}{3 c^4 d^2 \sqrt{d-c^2 d x^2}} - \frac{5 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{2 c^6 d^3} + \\
& \frac{5 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{4 b c^7 d^2 \sqrt{d-c^2 d x^2}} - \frac{7 b \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{6 c^7 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 293 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b}{6 c^7 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{b x^2 \sqrt{1-c^2 x^2}}{4 c^5 d^2 \sqrt{d-c^2 d x^2}} + \frac{x^5 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{5 x^3 (a+b \operatorname{ArcSin}[c x])}{3 c^4 d^2 \sqrt{d-c^2 d x^2}} - \frac{5 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{2 c^6 d^3} + \\
& \frac{5 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{4 b c^7 d^2 \sqrt{d-c^2 d x^2}} - \frac{7 b \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{6 c^7 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 130: Result valid but suboptimal antiderivative.**

$$\int \frac{x^5 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 219 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{d-c^2 d x^2}}{6 c^5 d^3 (1-c^2 x^2)^{3/2}} + \frac{b x \sqrt{d-c^2 d x^2}}{c^5 d^3 \sqrt{1-c^2 x^2}} + \frac{a+b \operatorname{ArcSin}[c x]}{3 c^6 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{2 (a+b \operatorname{ArcSin}[c x])}{c^6 d^2 \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{c^6 d^3} + \frac{11 b \sqrt{d-c^2 d x^2} \operatorname{ArcTanh}[c x]}{6 c^6 d^3 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 234 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b x^3}{6 c^3 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{5 b x \sqrt{1-c^2 x^2}}{6 c^5 d^2 \sqrt{d-c^2 d x^2}} + \frac{x^4 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{4 x^2 (a+b \operatorname{ArcSin}[c x])}{3 c^4 d^2 \sqrt{d-c^2 d x^2}} - \frac{8 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])}{3 c^6 d^3} + \frac{11 b \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 c^6 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 131: Result optimal but 1 more steps used.**

$$\int \frac{x^4 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 212 leaves, 7 steps):

$$\begin{aligned}
& - \frac{b}{6 c^5 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{x^3 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{x (a+b \operatorname{ArcSin}[c x])}{c^4 d^2 \sqrt{d-c^2 d x^2}} + \frac{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{2 b c^5 d^2 \sqrt{d-c^2 d x^2}} - \frac{2 b \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 212 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b}{6 c^5 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{x^3 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{x (a+b \operatorname{ArcSin}[c x])}{c^4 d^2 \sqrt{d-c^2 d x^2}} + \frac{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2}{2 b c^5 d^2 \sqrt{d-c^2 d x^2}} - \frac{2 b \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{3 c^5 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 132: Result valid but suboptimal antiderivative.**

$$\int \frac{x^3 (a+b \operatorname{ArcSin}[c x])}{(d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 150 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{d-c^2 d x^2}}{6 c^3 d^3 (1-c^2 x^2)^{3/2}} + \frac{a+b \operatorname{ArcSin}[c x]}{3 c^4 d (d-c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSin}[c x]}{c^4 d^2 \sqrt{d-c^2 d x^2}} + \frac{5 b \sqrt{d-c^2 d x^2} \operatorname{ArcTanh}[c x]}{6 c^4 d^3 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 155 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b x}{6 c^3 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{x^2 (a+b \operatorname{ArcSin}[c x])}{3 c^2 d (d-c^2 d x^2)^{3/2}} - \\
& \frac{2 (a+b \operatorname{ArcSin}[c x])}{3 c^4 d^2 \sqrt{d-c^2 d x^2}} + \frac{5 b \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 c^4 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 136: Result optimal but 1 more steps used.**

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x (d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 291 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b c x}{6 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{a+b \operatorname{ArcSin}[c x]}{3 d (d-c^2 d x^2)^{3/2}} + \frac{a+b \operatorname{ArcSin}[c x]}{d^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{7 b \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 d^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 291 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b c x}{6 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} + \frac{a+b \operatorname{ArcSin}[c x]}{3 d (d-c^2 d x^2)^{3/2}} + \frac{a+b \operatorname{ArcSin}[c x]}{d^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{7 b \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 d^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{i b \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 137: Result valid but suboptimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x^2 (d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 224 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{d-c^2 d x^2}}{6 d^3 (1-c^2 x^2)^{3/2}} - \frac{a+b \operatorname{ArcSin}[c x]}{d x (d-c^2 d x^2)^{3/2}} + \frac{4 c^2 x (a+b \operatorname{ArcSin}[c x])}{3 d (d-c^2 d x^2)^{3/2}} + \\
& \frac{8 c^2 x (a+b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d-c^2 d x^2}} + \frac{b c \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{d^3 \sqrt{1-c^2 x^2}} + \frac{5 b c \sqrt{d-c^2 d x^2} \operatorname{Log}[1-c^2 x^2]}{6 d^3 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 224 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b c}{6 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{d x (d-c^2 d x^2)^{3/2}} + \frac{4 c^2 x (a+b \operatorname{ArcSin}[c x])}{3 d (d-c^2 d x^2)^{3/2}} + \\
& \frac{8 c^2 x (a+b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d-c^2 d x^2}} + \frac{b c \sqrt{1-c^2 x^2} \operatorname{Log}[x]}{d^2 \sqrt{d-c^2 d x^2}} + \frac{5 b c \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{6 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 138: Result optimal but 1 more steps used.**

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x^3 (d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 433 leaves, 15 steps):

$$\begin{aligned}
& \frac{b c}{4 d^2 x \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{5 b c^3 x}{12 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{3 b c \sqrt{1-c^2 x^2}}{4 d^2 x \sqrt{d-c^2 d x^2}} + \\
& \frac{5 c^2 (a+b \operatorname{ArcSin}[c x])}{6 d (d-c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSin}[c x]}{2 d x^2 (d-c^2 d x^2)^{3/2}} + \frac{5 c^2 (a+b \operatorname{ArcSin}[c x])}{2 d^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{5 c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{13 b c^2 \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 d^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{5 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 d^2 \sqrt{d-c^2 d x^2}} - \frac{5 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 433 leaves, 16 steps):

$$\begin{aligned}
& \frac{b c}{4 d^2 x \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{5 b c^3 x}{12 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{3 b c \sqrt{1-c^2 x^2}}{4 d^2 x \sqrt{d-c^2 d x^2}} + \\
& \frac{5 c^2 (a+b \operatorname{ArcSin}[c x])}{6 d (d-c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSin}[c x]}{2 d x^2 (d-c^2 d x^2)^{3/2}} + \frac{5 c^2 (a+b \operatorname{ArcSin}[c x])}{2 d^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{5 c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{d^2 \sqrt{d-c^2 d x^2}} - \frac{13 b c^2 \sqrt{1-c^2 x^2} \operatorname{ArcTanh}[c x]}{6 d^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{5 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{2 d^2 \sqrt{d-c^2 d x^2}} - \frac{5 i b c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{2 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 139: Result valid but suboptimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSin}[c x]}{x^4 (d-c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 310 leaves, 5 steps):

$$\begin{aligned}
& -\frac{b c^3 \sqrt{d-c^2 d x^2}}{6 d^3 (1-c^2 x^2)^{3/2}} - \frac{b c \sqrt{d-c^2 d x^2}}{6 d^3 x^2 \sqrt{1-c^2 x^2}} - \frac{a+b \operatorname{ArcSin}[c x]}{3 d x^3 (d-c^2 d x^2)^{3/2}} - \\
& \frac{2 c^2 (a+b \operatorname{ArcSin}[c x])}{d x (d-c^2 d x^2)^{3/2}} + \frac{8 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d (d-c^2 d x^2)^{3/2}} + \frac{16 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{8 b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{3 d^3 \sqrt{1-c^2 x^2}} + \frac{4 b c^3 \sqrt{d-c^2 d x^2} \operatorname{Log}[1-c^2 x^2]}{3 d^3 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Result (type 3, 310 leaves, 12 steps):

$$\begin{aligned}
& -\frac{b c^3}{6 d^2 \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}} - \frac{b c \sqrt{1-c^2 x^2}}{6 d^2 x^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{a+b \operatorname{ArcSin}[c x]}{3 d x^3 (d-c^2 d x^2)^{3/2}} - \frac{2 c^2 (a+b \operatorname{ArcSin}[c x])}{d x (d-c^2 d x^2)^{3/2}} + \frac{8 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d (d-c^2 d x^2)^{3/2}} + \\
& \frac{16 c^4 x (a+b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d-c^2 d x^2}} + \frac{8 b c^3 \sqrt{1-c^2 x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{d-c^2 d x^2}} + \frac{4 b c^3 \sqrt{1-c^2 x^2} \operatorname{Log}[1-c^2 x^2]}{3 d^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 142: Result optimal but 1 more steps used.**

$$\int \frac{(f x)^{3/2} (a+b \operatorname{ArcSin}[c x])}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 5, 137 leaves, 1 step):

$$\begin{aligned}
& \frac{1}{5 f \sqrt{d-c^2 d x^2}} 2 (f x)^{5/2} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right] - \\
& \left(4 b c (f x)^{7/2} \sqrt{1-c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]\right) / \left(35 f^2 \sqrt{d-c^2 d x^2}\right)
\end{aligned}$$

Result (type 5, 137 leaves, 2 steps):

$$\frac{1}{5 f \sqrt{d - c^2 d x^2}} 2 (f x)^{5/2} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right] - \left(4 b c (f x)^{7/2} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]\right) / \left(35 f^2 \sqrt{d - c^2 d x^2}\right)$$

Problem 152: Result optimal but 1 more steps used.

$$\int \frac{x^m (a + b \operatorname{ArcSin}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 5, 163 leaves, 1 step):

$$\left( x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \left( (1+m) \sqrt{d - c^2 d x^2} \right) - \left( b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \left( (2 + 3m + m^2) \sqrt{d - c^2 d x^2} \right)$$

Result (type 5, 163 leaves, 2 steps):

$$\left( x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \left( (1+m) \sqrt{d - c^2 d x^2} \right) - \left( b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \left( (2 + 3m + m^2) \sqrt{d - c^2 d x^2} \right)$$

Problem 153: Result optimal but 1 more steps used.

$$\int \frac{x^m (a + b \operatorname{ArcSin}[c x])}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 5, 272 leaves, 3 steps):

$$\begin{aligned}
& \frac{x^{1+m} (a + b \operatorname{ArcSin}[c x])}{d \sqrt{d - c^2 d x^2}} - \\
& \left( m x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \\
& \left( d (1+m) \sqrt{d - c^2 d x^2} \right) - \frac{b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{d (2+m) \sqrt{d - c^2 d x^2}} + \\
& \left( b c m x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( d (2 + 3 m + m^2) \sqrt{d - c^2 d x^2} \right)
\end{aligned}$$

Result (type 5, 272 leaves, 4 steps):

$$\begin{aligned}
& \frac{x^{1+m} (a + b \operatorname{ArcSin}[c x])}{d \sqrt{d - c^2 d x^2}} - \\
& \left( m x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \\
& \left( d (1+m) \sqrt{d - c^2 d x^2} \right) - \frac{b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{d (2+m) \sqrt{d - c^2 d x^2}} + \\
& \left( b c m x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( d (2 + 3 m + m^2) \sqrt{d - c^2 d x^2} \right)
\end{aligned}$$

**Problem 154: Result optimal but 1 more steps used.**

$$\int \frac{x^m (a + b \operatorname{ArcSin}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 5, 408 leaves, 5 steps):

$$\begin{aligned}
& \frac{x^{1+m} (a + b \operatorname{ArcSin}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{(2 - m) x^{1+m} (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \left( (2 - m) m x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \\
& \left( 3 d^2 (1+m) \sqrt{d - c^2 d x^2} \right) - \frac{b c (2 - m) x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 (2+m) \sqrt{d - c^2 d x^2}} - \\
& \frac{b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 (2+m) \sqrt{d - c^2 d x^2}} + \\
& \left( b c (2 - m) m x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( 3 d^2 (2 + 3 m + m^2) \sqrt{d - c^2 d x^2} \right)
\end{aligned}$$

Result (type 5, 408 leaves, 6 steps):

$$\begin{aligned} & \frac{x^{1+m} (a + b \operatorname{ArcSin}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{(2 - m) x^{1+m} (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} - \\ & \left( (2 - m) m x^{1+m} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / \\ & \left( 3 d^2 (1 + m) \sqrt{d - c^2 d x^2} \right) - \frac{b c (2 - m) x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d - c^2 d x^2}} - \\ & \frac{b c x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d - c^2 d x^2}} + \\ & \left( b c (2 - m) m x^{2+m} \sqrt{1 - c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\ & \left( 3 d^2 (2 + 3 m + m^2) \sqrt{d - c^2 d x^2} \right) \end{aligned}$$

Problem 235: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 337 leaves, 10 steps):

$$\begin{aligned} & \frac{15 b^2 x (1 - c^2 x^2)}{64 c^4 \sqrt{d - c^2 d x^2}} + \frac{b^2 x^3 (1 - c^2 x^2)}{32 c^2 \sqrt{d - c^2 d x^2}} - \\ & \frac{15 b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{64 c^5 \sqrt{d - c^2 d x^2}} + \frac{3 b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{8 c^3 \sqrt{d - c^2 d x^2}} + \\ & \frac{b x^4 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{8 c \sqrt{d - c^2 d x^2}} - \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{8 c^4 d} - \\ & \frac{x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 c^2 d} + \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{8 b c^5 \sqrt{d - c^2 d x^2}} \end{aligned}$$

Result (type 3, 337 leaves, 11 steps):

$$\begin{aligned} & \frac{15 b^2 x (1 - c^2 x^2)}{64 c^4 \sqrt{d - c^2 d x^2}} + \frac{b^2 x^3 (1 - c^2 x^2)}{32 c^2 \sqrt{d - c^2 d x^2}} - \\ & \frac{15 b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{64 c^5 \sqrt{d - c^2 d x^2}} + \frac{3 b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{8 c^3 \sqrt{d - c^2 d x^2}} + \\ & \frac{b x^4 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{8 c \sqrt{d - c^2 d x^2}} - \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{8 c^4 d} - \\ & \frac{x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{4 c^2 d} + \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{8 b c^5 \sqrt{d - c^2 d x^2}} \end{aligned}$$

### Problem 237: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 206 leaves, 5 steps):

$$\frac{b^2 x \sqrt{d - c^2 d x^2}}{4 c^2 d} - \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{4 c^3 \sqrt{d - c^2 d x^2}} + \frac{b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{2 c \sqrt{d - c^2 d x^2}} - \frac{x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 c^2 d} + \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{6 b c^3 \sqrt{d - c^2 d x^2}}$$

Result (type 3, 213 leaves, 6 steps):

$$\frac{b^2 x (1 - c^2 x^2)}{4 c^2 \sqrt{d - c^2 d x^2}} - \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{4 c^3 \sqrt{d - c^2 d x^2}} + \frac{b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{2 c \sqrt{d - c^2 d x^2}} - \frac{x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 c^2 d} + \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{6 b c^3 \sqrt{d - c^2 d x^2}}$$

### Problem 239: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 49 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c \sqrt{d - c^2 d x^2}}$$

Result (type 3, 49 leaves, 2 steps):

$$\frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c \sqrt{d - c^2 d x^2}}$$

### Problem 240: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{x \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 4, 257 leaves, 8 steps):



$$\begin{aligned}
& - \frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \frac{2 b^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 257 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \frac{2 b^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 242: Result optimal but 1 more steps used.

$$\int \frac{(a+b \operatorname{ArcSin}[c x])^2}{x^3 \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 402 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])}{x \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^2}{2 d x^2} - \\
& \frac{c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1-c^2 x^2}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{i b c^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \frac{b^2 c^2 \sqrt{1-c^2 x^2} \operatorname{PolyLog}\left[3, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 402 leaves, 14 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{x \sqrt{d - c^2 d x^2}} - \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 d x^2} - \\
& \frac{c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}\left[e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d - c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1 - c^2 x^2}\right]}{\sqrt{d - c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d - c^2 d x^2}} - \\
& \frac{i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}\left[2, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d - c^2 d x^2}} - \\
& \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[3, -e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d - c^2 d x^2}} + \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[3, e^{i \operatorname{ArcSin}[c x]}\right]}{\sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 245: Result optimal but 1 more steps used.**

$$\int \frac{x^4 (a + b \operatorname{ArcSin}[c x])^2}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 424 leaves, 14 steps):

$$\begin{aligned}
& - \frac{b^2 x (1 - c^2 x^2)}{4 c^4 d \sqrt{d - c^2 d x^2}} + \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{4 c^5 d \sqrt{d - c^2 d x^2}} - \frac{b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{2 c^3 d \sqrt{d - c^2 d x^2}} + \\
& \frac{x^3 (a + b \operatorname{ArcSin}[c x])^2}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{c^5 d \sqrt{d - c^2 d x^2}} + \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 c^4 d^2} - \\
& \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{2 b c^5 d \sqrt{d - c^2 d x^2}} + \frac{2 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSin}[c x]}\right]}{c^5 d \sqrt{d - c^2 d x^2}} - \\
& \frac{i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSin}[c x]}\right]}{c^5 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 424 leaves, 15 steps):

$$\begin{aligned}
& - \frac{b^2 x (1 - c^2 x^2)}{4 c^4 d \sqrt{d - c^2 d x^2}} + \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{4 c^5 d \sqrt{d - c^2 d x^2}} - \frac{b x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{2 c^3 d \sqrt{d - c^2 d x^2}} + \\
& \frac{x^3 (a + b \operatorname{ArcSin}[c x])^2}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{c^5 d \sqrt{d - c^2 d x^2}} + \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 c^4 d^2} - \\
& \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{2 b c^5 d \sqrt{d - c^2 d x^2}} + \frac{2 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSin}[c x]}\right]}{c^5 d \sqrt{d - c^2 d x^2}} - \\
& \frac{i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSin}[c x]}\right]}{c^5 d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

### Problem 247: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSin}[c x])^2}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 250 leaves, 7 steps):

$$\begin{aligned} & \frac{x (a + b \operatorname{ArcSin}[c x])^2}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{c^3 d \sqrt{d - c^2 d x^2}} - \\ & \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c^3 d \sqrt{d - c^2 d x^2}} + \frac{2 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}[1 + e^{2 i \operatorname{ArcSin}[c x]}]}{c^3 d \sqrt{d - c^2 d x^2}} - \\ & \frac{i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -e^{2 i \operatorname{ArcSin}[c x]}]}{c^3 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

Result (type 4, 250 leaves, 8 steps):

$$\begin{aligned} & \frac{x (a + b \operatorname{ArcSin}[c x])^2}{c^2 d \sqrt{d - c^2 d x^2}} - \frac{i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{c^3 d \sqrt{d - c^2 d x^2}} - \\ & \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c^3 d \sqrt{d - c^2 d x^2}} + \frac{2 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}[1 + e^{2 i \operatorname{ArcSin}[c x]}]}{c^3 d \sqrt{d - c^2 d x^2}} - \\ & \frac{i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -e^{2 i \operatorname{ArcSin}[c x]}]}{c^3 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

### Problem 250: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{x (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 467 leaves, 15 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{d \sqrt{d - c^2 d x^2}} + \frac{4 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 467 leaves, 16 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{d \sqrt{d - c^2 d x^2}} + \frac{4 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Problem 252: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{x^3 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 634 leaves, 26 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{d x \sqrt{d - c^2 d x^2}} + \frac{3 c^2 (a + b \operatorname{ArcSin}[c x])^2}{2 d \sqrt{d - c^2 d x^2}} - \\
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{2 d x^2 \sqrt{d - c^2 d x^2}} + \frac{4 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{3 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{3 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 634 leaves, 27 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])}{d x \sqrt{d - c^2 d x^2}} + \frac{3 c^2 (a + b \operatorname{ArcSin}[c x])^2}{2 d \sqrt{d - c^2 d x^2}} - \\
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{2 d x^2 \sqrt{d - c^2 d x^2}} + \frac{4 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{d \sqrt{d - c^2 d x^2}} + \\
& \frac{3 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{2 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} - \\
& \frac{3 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}} + \frac{3 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Problem 255: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a + b \operatorname{ArcSin}[c x])^2}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 421 leaves, 16 steps):

$$\begin{aligned}
& \frac{b^2 x}{3 c^4 d^2 \sqrt{d - c^2 d x^2}} - \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} - \frac{b x^2 (a + b \operatorname{ArcSin}[c x])}{3 c^3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \\
& \frac{x^3 (a + b \operatorname{ArcSin}[c x])^2}{3 c^2 d (d - c^2 d x^2)^{3/2}} - \frac{x (a + b \operatorname{ArcSin}[c x])^2}{c^4 d^2 \sqrt{d - c^2 d x^2}} + \frac{4 i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c^5 d^2 \sqrt{d - c^2 d x^2}} - \frac{8 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}[1 + e^{2 i \operatorname{ArcSin}[c x]}]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{4 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -e^{2 i \operatorname{ArcSin}[c x]}]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 421 leaves, 17 steps):

$$\begin{aligned}
& \frac{b^2 x}{3 c^4 d^2 \sqrt{d - c^2 d x^2}} - \frac{b^2 \sqrt{1 - c^2 x^2} \operatorname{ArcSin}[c x]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} - \frac{b x^2 (a + b \operatorname{ArcSin}[c x])}{3 c^3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \\
& \frac{x^3 (a + b \operatorname{ArcSin}[c x])^2}{3 c^2 d (d - c^2 d x^2)^{3/2}} - \frac{x (a + b \operatorname{ArcSin}[c x])^2}{c^4 d^2 \sqrt{d - c^2 d x^2}} + \frac{4 i \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^3}{3 b c^5 d^2 \sqrt{d - c^2 d x^2}} - \frac{8 b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{Log}[1 + e^{2 i \operatorname{ArcSin}[c x]}]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{4 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -e^{2 i \operatorname{ArcSin}[c x]}]}{3 c^5 d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 260: Result optimal but 1 more steps used.**

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{x (d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 577 leaves, 24 steps):

$$\begin{aligned}
& \frac{b^2}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{b c x (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \frac{(a + b \operatorname{ArcSin}[c x])^2}{3 d (d - c^2 d x^2)^{3/2}} + \\
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{d^2 \sqrt{d - c^2 d x^2}} + \frac{14 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{7 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{7 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 577 leaves, 25 steps):

$$\begin{aligned}
& \frac{b^2}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{b c x (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \frac{(a + b \operatorname{ArcSin}[c x])^2}{3 d (d - c^2 d x^2)^{3/2}} + \\
& \frac{(a + b \operatorname{ArcSin}[c x])^2}{d^2 \sqrt{d - c^2 d x^2}} + \frac{14 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{7 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{7 i b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 i b \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \frac{2 b^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$

**Problem 262: Result optimal but 1 more steps used.**

$$\int \frac{(a + b \operatorname{ArcSin}[c x])^2}{x^3 (d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 752 leaves, 38 steps):

$$\begin{aligned}
& \frac{b^2 c^2}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{b c (a + b \operatorname{ArcSin}[c x])}{d^2 x \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \frac{2 b c^3 x (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \\
& \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])^2}{6 d (d - c^2 d x^2)^{3/2}} - \frac{(a + b \operatorname{ArcSin}[c x])^2}{2 d x^2 (d - c^2 d x^2)^{3/2}} + \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])^2}{2 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{26 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{5 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{13 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{13 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \frac{5 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$

Result (type 4, 752 leaves, 39 steps):

$$\begin{aligned}
& \frac{b^2 c^2}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{b c (a + b \operatorname{ArcSin}[c x])}{d^2 x \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \frac{2 b c^3 x (a + b \operatorname{ArcSin}[c x])}{3 d^2 \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2}} + \\
& \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])^2}{6 d (d - c^2 d x^2)^{3/2}} - \frac{(a + b \operatorname{ArcSin}[c x])^2}{2 d x^2 (d - c^2 d x^2)^{3/2}} + \frac{5 c^2 (a + b \operatorname{ArcSin}[c x])^2}{2 d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{26 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{ArcTan}[e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^2 \operatorname{ArcTanh}[e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{ArcTanh}[\sqrt{1 - c^2 x^2}]}{d^2 \sqrt{d - c^2 d x^2}} + \\
& \frac{5 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{13 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{13 i b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSin}[c x]}]}{3 d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 i b c^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x]) \operatorname{PolyLog}[2, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{5 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, -e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}} + \frac{5 b^2 c^2 \sqrt{1 - c^2 x^2} \operatorname{PolyLog}[3, e^{i \operatorname{ArcSin}[c x]}]}{d^2 \sqrt{d - c^2 d x^2}}
\end{aligned}$$



Problem 272: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSin}[a x]^2}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{\sqrt{1 - a^2 x^2} \text{ArcSin}[a x]^3}{3 a \sqrt{c - a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{\sqrt{1 - a^2 x^2} \text{ArcSin}[a x]^3}{3 a \sqrt{c - a^2 c x^2}}$$

Problem 276: Unable to integrate problem.

$$\int x^m (d - c^2 d x^2)^3 (a + b \text{ArcSin}[c x])^2 dx$$

Optimal (type 5, 1312 leaves, 23 steps):

$$\begin{aligned}
& \frac{2 b^2 c^2 d^3 x^{3+m}}{(3+m)(7+m)^2} + \frac{30 b^2 c^2 d^3 x^{3+m}}{(3+m)^2 (5+m)(7+m)^2} + \frac{36 b^2 c^2 d^3 x^{3+m}}{(3+m)^2 (5+m)^2 (7+m)} + \\
& \frac{12 b^2 c^2 d^3 x^{3+m}}{(3+m)(5+m)^2 (7+m)} + \frac{48 b^2 c^2 d^3 x^{3+m}}{(3+m)^3 (5+m)(7+m)} + \frac{10 b^2 c^2 d^3 x^{3+m}}{(7+m)^2 (15+8m+m^2)} - \\
& \frac{10 b^2 c^4 d^3 x^{5+m}}{(5+m)^2 (7+m)^2} - \frac{4 b^2 c^4 d^3 x^{5+m}}{(5+m)(7+m)^2} - \frac{12 b^2 c^4 d^3 x^{5+m}}{(5+m)^3 (7+m)} + \frac{2 b^2 c^6 d^3 x^{7+m}}{(7+m)^3} - \\
& \frac{36 b c d^3 x^{2+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])}{(3+m)(5+m)^2 (7+m)} - \frac{48 b c d^3 x^{2+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])}{(3+m)^2 (5+m)(7+m)} - \\
& \frac{30 b c d^3 x^{2+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])}{(7+m)^2 (15+8m+m^2)} - \frac{10 b c d^3 x^{2+m} (1-c^2 x^2)^{3/2} (a+b \operatorname{ArcSin}[c x])}{(5+m)(7+m)^2} - \\
& \frac{12 b c d^3 x^{2+m} (1-c^2 x^2)^{3/2} (a+b \operatorname{ArcSin}[c x])}{(5+m)^2 (7+m)} - \frac{2 b c d^3 x^{2+m} (1-c^2 x^2)^{5/2} (a+b \operatorname{ArcSin}[c x])}{(7+m)^2} + \\
& \frac{48 d^3 x^{1+m} (a+b \operatorname{ArcSin}[c x])^2}{(5+m)(7+m)(3+4m+m^2)} + \frac{24 d^3 x^{1+m} (1-c^2 x^2) (a+b \operatorname{ArcSin}[c x])^2}{(7+m)(15+8m+m^2)} + \\
& \frac{6 d^3 x^{1+m} (1-c^2 x^2)^2 (a+b \operatorname{ArcSin}[c x])^2}{(5+m)(7+m)} + \frac{d^3 x^{1+m} (1-c^2 x^2)^3 (a+b \operatorname{ArcSin}[c x])^2}{7+m} - \\
& \left( 48 b c d^3 x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\
& \left( (2+m)(3+m)^2 (5+m)(7+m) \right) - \\
& \left( 30 b c d^3 x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\
& \left( (5+m)(7+m)^2 (6+5m+m^2) \right) - \\
& \left( 36 b c d^3 x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\
& \left( (5+m)^2 (7+m)(6+5m+m^2) \right) - \\
& \left( 96 b c d^3 x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\
& \left( (5+m)(7+m)(6+11m+6m^2+m^3) \right) + \\
& \left( 30 b^2 c^2 d^3 x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( (2+m)(3+m)^2 (5+m)(7+m)^2 \right) + \\
& \left( 36 b^2 c^2 d^3 x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( (2+m)(3+m)^2 (5+m)^2 (7+m) \right) + \\
& \left( 48 b^2 c^2 d^3 x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( (2+m)(3+m)^3 (5+m)(7+m) \right) + \\
& \left( 96 b^2 c^2 d^3 x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right] \right) / \\
& \left( (3+m)^2 (5+m)(7+m)(2+3m+m^2) \right)
\end{aligned}$$

Result (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m (d - c^2 d x^2)^3 (a + b \text{ArcSin}[c x])^2, x\right]$$

Problem 277: Unable to integrate problem.

$$\int x^m (d - c^2 d x^2)^2 (a + b \text{ArcSin}[c x])^2 dx$$

Optimal (type 5, 756 leaves, 13 steps):

$$\begin{aligned} & \frac{6 b^2 c^2 d^2 x^{3+m}}{(3+m)^2 (5+m)^2} + \frac{2 b^2 c^2 d^2 x^{3+m}}{(3+m) (5+m)^2} + \frac{8 b^2 c^2 d^2 x^{3+m}}{(3+m)^3 (5+m)} - \frac{2 b^2 c^4 d^2 x^{5+m}}{(5+m)^3} - \\ & \frac{6 b c d^2 x^{2+m} \sqrt{1 - c^2 x^2} (a + b \text{ArcSin}[c x])}{(3+m) (5+m)^2} - \frac{8 b c d^2 x^{2+m} \sqrt{1 - c^2 x^2} (a + b \text{ArcSin}[c x])}{(3+m)^2 (5+m)} - \\ & \frac{2 b c d^2 x^{2+m} (1 - c^2 x^2)^{3/2} (a + b \text{ArcSin}[c x])}{(5+m)^2} + \frac{8 d^2 x^{1+m} (a + b \text{ArcSin}[c x])^2}{(5+m) (3 + 4 m + m^2)} + \\ & \frac{4 d^2 x^{1+m} (1 - c^2 x^2) (a + b \text{ArcSin}[c x])^2}{15 + 8 m + m^2} + \frac{d^2 x^{1+m} (1 - c^2 x^2)^2 (a + b \text{ArcSin}[c x])^2}{5 + m} - \\ & \left( 8 b c d^2 x^{2+m} (a + b \text{ArcSin}[c x]) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\ & \left( (2+m) (3+m)^2 (5+m) \right) - \\ & \left( 6 b c d^2 x^{2+m} (a + b \text{ArcSin}[c x]) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\ & \left( (5+m)^2 (6 + 5 m + m^2) \right) - \\ & \left( 16 b c d^2 x^{2+m} (a + b \text{ArcSin}[c x]) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right] \right) / \\ & \left( (5+m) (6 + 11 m + 6 m^2 + m^3) \right) + \\ & \left( 6 b^2 c^2 d^2 x^{3+m} \text{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\ & \left( (2+m) (3+m)^2 (5+m)^2 \right) + \\ & \left( 8 b^2 c^2 d^2 x^{3+m} \text{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\ & \left( (2+m) (3+m)^3 (5+m) \right) + \\ & \left( 16 b^2 c^2 d^2 x^{3+m} \text{HypergeometricPFQ}\left[\left\{1, \frac{3}{2} + \frac{m}{2}, \frac{3}{2} + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}, \frac{5}{2} + \frac{m}{2}\right\}, c^2 x^2\right] \right) / \\ & \left( (3+m)^2 (5+m) (2 + 3 m + m^2) \right) \end{aligned}$$

Result (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m (d - c^2 d x^2)^2 (a + b \text{ArcSin}[c x])^2, x\right]$$

Problem 278: Unable to integrate problem.

$$\int x^m (d - c^2 d x^2) (a + b \text{ArcSin}[c x])^2 dx$$

Optimal (type 5, 371 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 b^2 c^2 d x^{3+m}}{(3+m)^3} - \frac{2 b c d x^{2+m} \sqrt{1-c^2 x^2} (a+b \operatorname{ArcSin}[c x])}{(3+m)^2} + \\
& \frac{2 d x^{1+m} (a+b \operatorname{ArcSin}[c x])^2}{3+4 m+m^2} + \frac{d x^{1+m} (1-c^2 x^2) (a+b \operatorname{ArcSin}[c x])^2}{3+m} - \\
& \frac{2 b c d x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{(2+m)(3+m)^2} - \\
& \frac{4 b c d x^{2+m} (a+b \operatorname{ArcSin}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 x^2\right]}{6+11 m+6 m^2+m^3} + \frac{1}{(2+m)(3+m)^3} \\
& 2 b^2 c^2 d x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right] + \\
& \left(4 b^2 c^2 d x^{3+m} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{3}{2}+\frac{m}{2}, \frac{3}{2}+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}, \frac{5}{2}+\frac{m}{2}\right\}, c^2 x^2\right]\right) / \\
& \left((3+m)^2 (2+3 m+m^2)\right)
\end{aligned}$$

Result (type 8, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^m (d-c^2 d x^2) (a+b \operatorname{ArcSin}[c x])^2, x\right]$$

**Problem 282: Result valid but suboptimal antiderivative.**

$$\int x^m (d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcSin}[c x])^2 dx$$

Optimal (type 8, 957 leaves, 12 steps):

$$\begin{aligned}
& \frac{10 b^2 c^2 d^2 x^{3+m} \sqrt{d - c^2 d x^2}}{(4+m)^3 (6+m)} + \frac{2 b^2 c^2 d^2 (52 + 15 m + m^2) x^{3+m} \sqrt{d - c^2 d x^2}}{(4+m)^2 (6+m)^3} - \\
& \frac{2 b^2 c^4 d^2 x^{5+m} \sqrt{d - c^2 d x^2}}{(6+m)^3} - \frac{30 b c d^2 x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(2+m)^2 (4+m) (6+m) \sqrt{1 - c^2 x^2}} - \\
& \frac{10 b c d^2 x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(6+m) (8 + 6 m + m^2) \sqrt{1 - c^2 x^2}} - \frac{2 b c d^2 x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(12 + 8 m + m^2) \sqrt{1 - c^2 x^2}} + \\
& \frac{10 b c^3 d^2 x^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(4+m)^2 (6+m) \sqrt{1 - c^2 x^2}} + \frac{4 b c^3 d^2 x^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(4+m) (6+m) \sqrt{1 - c^2 x^2}} - \\
& \frac{2 b c^5 d^2 x^{6+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(6+m)^2 \sqrt{1 - c^2 x^2}} + \frac{15 d^2 x^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{(6+m) (8 + 6 m + m^2)} + \\
& \frac{5 d x^{1+m} (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^2}{(4+m) (6+m)} + \frac{x^{1+m} (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^2}{6+m} + \\
& \frac{30 b^2 c^2 d^2 x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) (6+m) \sqrt{1 - c^2 x^2}} + \\
& \left( \frac{10 b^2 c^2 d^2 (10 + 3 m) x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{(2+m) (3+m) (4+m)^3 (6+m) \sqrt{1 - c^2 x^2}} + \right. \\
& \left. \frac{2 b^2 c^2 d^2 (264 + 130 m + 15 m^2) x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{(2+m) (3+m) (4+m)^2 (6+m)^3 \sqrt{1 - c^2 x^2}} + \right. \\
& \left. \frac{15 d^3 \operatorname{Unintegrable}\left[\frac{x^m (a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{(6+m) (8 + 6 m + m^2)} \right) /
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^m (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^2, x\right]$$

**Problem 283: Result valid but suboptimal antiderivative.**

$$\int x^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^2 dx$$

Optimal (type 8, 499 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 c^2 d x^{3+m} \sqrt{d - c^2 d x^2}}{(4 + m)^3} - \frac{6 b c d x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(2 + m)^2 (4 + m) \sqrt{1 - c^2 x^2}} - \\
& \frac{2 b c d x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(8 + 6 m + m^2) \sqrt{1 - c^2 x^2}} + \frac{2 b c^3 d x^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(4 + m)^2 \sqrt{1 - c^2 x^2}} + \\
& \frac{3 d x^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{8 + 6 m + m^2} + \frac{x^{1+m} (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^2}{4 + m} + \\
& \frac{6 b^2 c^2 d x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{(2 + m)^2 (3 + m) (4 + m) \sqrt{1 - c^2 x^2}} + \\
& \left( 2 b^2 c^2 d (10 + 3 m) x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right] \right) / \\
& \left( (2 + m) (3 + m) (4 + m)^3 \sqrt{1 - c^2 x^2} \right) + \frac{3 d^2 \operatorname{Unintegrateable}\left[\frac{x^m (a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{8 + 6 m + m^2}
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[x^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^2, x\right]$$

**Problem 284: Result valid but suboptimal antiderivative.**

$$\int x^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2 dx$$

Optimal (type 8, 203 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 b c x^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])}{(2 + m)^2 \sqrt{1 - c^2 x^2}} + \frac{x^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2}{2 + m} + \\
& \frac{2 b^2 c^2 x^{3+m} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{(2 + m)^2 (3 + m) \sqrt{1 - c^2 x^2}} + \\
& \frac{d \operatorname{Unintegrateable}\left[\frac{x^m (a + b \operatorname{ArcSin}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{2 + m}
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[x^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^2, x\right]$$

**Problem 298: Result optimal but 1 more steps used.**

$$\int \frac{\operatorname{ArcSin}[a x]^3}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{\sqrt{1-a^2 x^2} \operatorname{ArcSin}[a x]^4}{4 a \sqrt{c-a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{\sqrt{1-a^2 x^2} \operatorname{ArcSin}[a x]^4}{4 a \sqrt{c-a^2 c x^2}}$$

Problem 383: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcSin}[c x])^2} dx$$

Optimal (type 4, 150 leaves, 14 steps):

$$\begin{aligned} & -\frac{x(1-c^2 x^2)}{b c (a+b \operatorname{ArcSin}[c x])} + \frac{\operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{4 b^2 c^2} + \\ & \frac{3 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3(a+b \operatorname{ArcSin}[c x])}{b}\right]}{4 b^2 c^2} + \\ & \frac{\operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{4 b^2 c^2} + \frac{3 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3(a+b \operatorname{ArcSin}[c x])}{b}\right]}{4 b^2 c^2} \end{aligned}$$

Result (type 4, 198 leaves, 14 steps):

$$\begin{aligned} & -\frac{x(1-c^2 x^2)}{b c (a+b \operatorname{ArcSin}[c x])} - \frac{3 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c x]\right]}{4 b^2 c^2} + \\ & \frac{3 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c x]\right]}{4 b^2 c^2} + \\ & \frac{\operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{b^2 c^2} - \frac{3 \operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c x]\right]}{4 b^2 c^2} + \\ & \frac{3 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c x]\right]}{4 b^2 c^2} + \frac{\operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a+b \operatorname{ArcSin}[c x]}{b}\right]}{b^2 c^2} \end{aligned}$$

Problem 444: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\operatorname{ArcSin}[a x]}}{\sqrt{c-a^2 c x^2}} dx$$

Optimal (type 3, 44 leaves, 1 step):

$$\frac{2 \sqrt{1-a^2 x^2} \operatorname{ArcSin}[a x]^{3/2}}{3 a \sqrt{c-a^2 c x^2}}$$

Result (type 3, 44 leaves, 2 steps):

$$\frac{2 \sqrt{1 - a^2 x^2} \operatorname{ArcSin}[a x]^{3/2}}{3 a \sqrt{c - a^2 c x^2}}$$

Problem 449: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSin}[a x]^{3/2}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 44 leaves, 1 step):

$$\frac{2 \sqrt{1 - a^2 x^2} \operatorname{ArcSin}[a x]^{5/2}}{5 a \sqrt{c - a^2 c x^2}}$$

Result (type 3, 44 leaves, 2 steps):

$$\frac{2 \sqrt{1 - a^2 x^2} \operatorname{ArcSin}[a x]^{5/2}}{5 a \sqrt{c - a^2 c x^2}}$$

Problem 453: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSin}[a x]^{5/2}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 44 leaves, 1 step):

$$\frac{2 \sqrt{1 - a^2 x^2} \operatorname{ArcSin}[a x]^{7/2}}{7 a \sqrt{c - a^2 c x^2}}$$

Result (type 3, 44 leaves, 2 steps):

$$\frac{2 \sqrt{1 - a^2 x^2} \operatorname{ArcSin}[a x]^{7/2}}{7 a \sqrt{c - a^2 c x^2}}$$

Problem 457: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\operatorname{ArcSin}\left[\frac{x}{a}\right]}}{\sqrt{a^2 - x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 a \sqrt{1 - \frac{x^2}{a^2}} \operatorname{ArcSin}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2 - x^2}}$$

Result (type 3, 42 leaves, 2 steps):



$$\frac{2 a \sqrt{1 - \frac{x^2}{a^2}} \operatorname{ArcSin}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2 - x^2}}$$

Problem 462: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSin}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2 - x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 a \sqrt{1 - \frac{x^2}{a^2}} \operatorname{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2 - x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 a \sqrt{1 - \frac{x^2}{a^2}} \operatorname{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2 - x^2}}$$

Problem 465: Result optimal but 1 more steps used.

$$\int \frac{(c - a^2 c x^2)^{5/2}}{\sqrt{\operatorname{ArcSin}[a x]}} dx$$

Optimal (type 4, 244 leaves, 9 steps):

$$\begin{aligned} & \frac{5 c^2 \sqrt{c - a^2 c x^2} \sqrt{\operatorname{ArcSin}[a x]}}{8 a \sqrt{1 - a^2 x^2}} + \frac{3 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \operatorname{FresnelC}\left[2 \sqrt{\frac{2}{\pi}} \sqrt{\operatorname{ArcSin}[a x]}\right]}{16 a \sqrt{1 - a^2 x^2}} + \\ & \frac{c^2 \sqrt{\frac{\pi}{3}} \sqrt{c - a^2 c x^2} \operatorname{FresnelC}\left[2 \sqrt{\frac{3}{\pi}} \sqrt{\operatorname{ArcSin}[a x]}\right]}{32 a \sqrt{1 - a^2 x^2}} + \\ & \frac{15 c^2 \sqrt{\pi} \sqrt{c - a^2 c x^2} \operatorname{FresnelC}\left[\frac{2 \sqrt{\operatorname{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{32 a \sqrt{1 - a^2 x^2}} \end{aligned}$$

Result (type 4, 244 leaves, 10 steps):

$$\begin{aligned}
& \frac{5 c^2 \sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}{8 a \sqrt{1 - a^2 x^2}} + \frac{3 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[2 \sqrt{\frac{2}{\pi}} \sqrt{\text{ArcSin}[a x]}\right]}{16 a \sqrt{1 - a^2 x^2}} + \\
& \frac{c^2 \sqrt{\frac{\pi}{3}} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[2 \sqrt{\frac{3}{\pi}} \sqrt{\text{ArcSin}[a x]}\right]}{32 a \sqrt{1 - a^2 x^2}} + \\
& \frac{15 c^2 \sqrt{\pi} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[\frac{2 \sqrt{\text{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{32 a \sqrt{1 - a^2 x^2}}
\end{aligned}$$

Problem 466: Result optimal but 1 more steps used.

$$\int \frac{(c - a^2 c x^2)^{3/2}}{\sqrt{\text{ArcSin}[a x]}} dx$$

Optimal (type 4, 170 leaves, 7 steps):

$$\begin{aligned}
& \frac{3 c \sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}{4 a \sqrt{1 - a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[2 \sqrt{\frac{2}{\pi}} \sqrt{\text{ArcSin}[a x]}\right]}{8 a \sqrt{1 - a^2 x^2}} + \\
& \frac{c \sqrt{\pi} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[\frac{2 \sqrt{\text{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{2 a \sqrt{1 - a^2 x^2}}
\end{aligned}$$

Result (type 4, 170 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 c \sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}{4 a \sqrt{1 - a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[2 \sqrt{\frac{2}{\pi}} \sqrt{\text{ArcSin}[a x]}\right]}{8 a \sqrt{1 - a^2 x^2}} + \\
& \frac{c \sqrt{\pi} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[\frac{2 \sqrt{\text{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{2 a \sqrt{1 - a^2 x^2}}
\end{aligned}$$

Problem 467: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{c - a^2 c x^2}}{\sqrt{\text{ArcSin}[a x]}} dx$$

Optimal (type 4, 99 leaves, 5 steps):

$$\begin{aligned}
& \frac{\sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}{a \sqrt{1 - a^2 x^2}} + \frac{\sqrt{\pi} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[\frac{2 \sqrt{\text{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{2 a \sqrt{1 - a^2 x^2}}
\end{aligned}$$

Result (type 4, 99 leaves, 6 steps):

$$\frac{\sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}{a \sqrt{1 - a^2 x^2}} + \frac{\sqrt{\pi} \sqrt{c - a^2 c x^2} \text{FresnelC}\left[\frac{2 \sqrt{\text{ArcSin}[a x]}}{\sqrt{\pi}}\right]}{2 a \sqrt{1 - a^2 x^2}}$$

Problem 468: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1 - a^2 x^2} \sqrt{\text{ArcSin}[a x]}}{a \sqrt{c - a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1 - a^2 x^2} \sqrt{\text{ArcSin}[a x]}}{a \sqrt{c - a^2 c x^2}}$$

Problem 474: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c - a^2 c x^2} \text{ArcSin}[a x]^{3/2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$-\frac{2 \sqrt{1 - a^2 x^2}}{a \sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}$$

Result (type 3, 42 leaves, 2 steps):

$$-\frac{2 \sqrt{1 - a^2 x^2}}{a \sqrt{c - a^2 c x^2} \sqrt{\text{ArcSin}[a x]}}$$

Problem 479: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c - a^2 c x^2} \text{ArcSin}[a x]^{5/2}} dx$$

Optimal (type 3, 44 leaves, 1 step):

$$-\frac{2 \sqrt{1 - a^2 x^2}}{3 a \sqrt{c - a^2 c x^2} \text{ArcSin}[a x]^{3/2}}$$

Result (type 3, 44 leaves, 2 steps):

$$-\frac{2 \sqrt{1 - a^2 x^2}}{3 a \sqrt{c - a^2 c x^2} \text{ArcSin}[a x]^{3/2}}$$

### Problem 482: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$\frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{8 b c^3 (1+n) \sqrt{1 - c^2 x^2}} + \frac{1}{c^3 \sqrt{1 - c^2 x^2}}$$

$$i 2^{-2(3+n)} e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n}$$

$$\operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2}$$

$$(a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right]$$

Result (type 4, 259 leaves, 7 steps):

$$\frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{8 b c^3 (1+n) \sqrt{1 - c^2 x^2}} + \frac{1}{c^3 \sqrt{1 - c^2 x^2}}$$

$$i 2^{-2(3+n)} e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n}$$

$$\operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2}$$

$$(a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right]$$

### Problem 483: Result optimal but 1 more steps used.

$$\int x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 391 leaves, 9 steps):

$$\begin{aligned}
& - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} e^{-\frac{i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} e^{\frac{i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \left( \frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, \frac{i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} \\
& 3^{-1-n} e^{-\frac{3 i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{3 i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} 3^{-1-n} e^{\frac{3 i a}{b}} \sqrt{d-c^2 d x^2} \\
& (a+b \operatorname{ArcSin}[c x])^n \left( \frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3 i(a+b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 391 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} e^{-\frac{i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} e^{\frac{i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \left( \frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, \frac{i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} \\
& 3^{-1-n} e^{-\frac{3 i a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{3 i(a+b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{8 c^2 \sqrt{1-c^2 x^2}} 3^{-1-n} e^{\frac{3 i a}{b}} \sqrt{d-c^2 d x^2} \\
& (a+b \operatorname{ArcSin}[c x])^n \left( \frac{i(a+b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3 i(a+b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 484: Result optimal but 1 more steps used.**

$$\int \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$\begin{aligned}
& \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{2 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& i 2^{-3-n} e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-3-n} e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 259 leaves, 7 steps):

$$\begin{aligned}
& \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{2 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& i 2^{-3-n} e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-3-n} e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 485: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n}{x} dx$$

Optimal (type 8, 218 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{2 \sqrt{d - c^2 d x^2}} d e^{-\frac{i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{2 \sqrt{d - c^2 d x^2}} d e^{\frac{i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] + d \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n}{x}, x\right]$$

### Problem 486: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n}{x^2} dx$$

Optimal (type 8, 87 leaves, 3 steps):

$$-\frac{c d \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{b (1+n) \sqrt{d - c^2 d x^2}} + d \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x^2 \sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n}{x^2}, x\right]$$

### Problem 487: Result optimal but 1 more steps used.

$$\int x^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 684 leaves, 12 steps):

$$\begin{aligned} & \frac{d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{16 b c^3 (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\ & i 2^{-7-n} d e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\ & \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} d e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\ & (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-2n} d e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\ & \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-2n} d e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\ & (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\ & \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\ & \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] \end{aligned}$$

Result (type 4, 684 leaves, 13 steps):

$$\begin{aligned}
& \frac{d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{16 b c^3 (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\
& i 2^{-7-n} d e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} d e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-2n} d e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-2n} d e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 488: Result optimal but 1 more steps used.**

$$\int x (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 595 leaves, 12 steps):



$$\begin{aligned}
& - \frac{1}{16 c^2 \sqrt{1 - c^2 x^2}} d e^{-\frac{i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, - \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{16 c^2 \sqrt{1 - c^2 x^2}} d e^{\frac{i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 3^{-n} d e^{-\frac{3 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, - \frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 3^{-n} d e^{\frac{3 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} \\
& 5^{-1-n} d e^{-\frac{5 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, - \frac{5 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 5^{-1-n} d e^{\frac{5 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{5 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 595 leaves, 13 steps):

$$\begin{aligned}
& - \frac{1}{16 c^2 \sqrt{1 - c^2 x^2}} d e^{-\frac{i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, - \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{16 c^2 \sqrt{1 - c^2 x^2}} d e^{\frac{i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 3^{-n} d e^{-\frac{3 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, - \frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 3^{-n} d e^{\frac{3 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} \\
& 5^{-1-n} d e^{-\frac{5 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, - \frac{5 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 c^2 \sqrt{1 - c^2 x^2}} 5^{-1-n} d e^{\frac{5 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{5 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 489: Result optimal but 1 more steps used.**

$$\int (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{8 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& i 2^{-3-n} d e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-3-n} d e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} d e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} d e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 466 leaves, 10 steps):

$$\begin{aligned}
& \frac{3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{8 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& i 2^{-3-n} d e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-3-n} d e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} d e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-2(3+n)} d e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 490: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n}{x} dx$$

Optimal (type 8, 426 leaves, 15 steps):

$$\begin{aligned}
& \frac{1}{8 \sqrt{d - c^2 d x^2}} 5 d^2 e^{-\frac{i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{8 \sqrt{d - c^2 d x^2}} 5 d^2 e^{\frac{i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{8 \sqrt{d - c^2 d x^2}} 3^{-1-n} d^2 e^{-\frac{3 i a}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{8 \sqrt{d - c^2 d x^2}} 3^{-1-n} d^2 e^{\frac{3 i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{3 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + d^2 \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n}{x}, x\right]$$

**Problem 491: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n}{x^2} dx$$

Optimal (type 8, 297 leaves, 9 steps):

$$\begin{aligned}
& -\frac{3 c d^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{2 b (1 + n) \sqrt{d - c^2 d x^2}} + \frac{1}{\sqrt{d - c^2 d x^2}} i 2^{-3-n} c d^2 e^{-\frac{2 i a}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{\sqrt{d - c^2 d x^2}} i 2^{-3-n} c d^2 e^{\frac{2 i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + d^2 \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x^2 \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcSin}[c x])^n}{x^2}, x\right]$$

### Problem 492: Result optimal but 1 more steps used.

$$\int x^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 906 leaves, 15 steps):

$$\begin{aligned} & \frac{5 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{128 b c^3 (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\ & i 2^{-7-n} d^2 e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\ & \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} d^2 e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\ & (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(4+n)} d^2 e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\ & \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(4+n)} d^2 e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\ & (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\ & \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\ & \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\ & \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\ & i 2^{-11-3n} d^2 e^{-\frac{8 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\ & \operatorname{Gamma}\left[1+n, -\frac{8 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-11-3n} d^2 e^{\frac{8 i a}{b}} \sqrt{d - c^2 d x^2} \\ & (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{8 i (a + b \operatorname{ArcSin}[c x])}{b}\right] \end{aligned}$$

Result (type 4, 906 leaves, 16 steps):

$$\begin{aligned}
& \frac{5 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{128 b c^3 (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\
& i 2^{-7-n} d^2 e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} d^2 e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(4+n)} d^2 e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-2(4+n)} d^2 e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c^3 \sqrt{1 - c^2 x^2}} \\
& i 2^{-11-3 n} d^2 e^{-\frac{8 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{8 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{c^3 \sqrt{1 - c^2 x^2}} i 2^{-11-3 n} d^2 e^{\frac{8 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{8 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Problem 493: Result optimal but 1 more steps used.

$$\int x (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 815 leaves, 15 steps):

$$\begin{aligned}
& - \left( \left( 5 d^2 e^{-\frac{ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \right. \right. \\
& \quad \left. \left. \operatorname{Gamma}\left[1 + n, -\frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] \right) \right) / \left( 128 c^2 \sqrt{1 - c^2 x^2} \right) - \\
& \quad \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} 5 d^2 e^{\frac{ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 3^{1-n} d^2 e^{-\frac{3ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 3^{1-n} d^2 e^{\frac{3ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 5^{-n} d^2 e^{-\frac{5ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 5^{-n} d^2 e^{\frac{5ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 7^{-1-n} d^2 e^{-\frac{7ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{7i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} 7^{-1-n} d^2 e^{\frac{7ia}{b}} \sqrt{d - c^2 d x^2} \\
& \quad (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{7i(a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 815 leaves, 16 steps):

$$\begin{aligned}
& - \left( \left( 5 d^2 e^{-\frac{ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \right. \right. \\
& \quad \left. \left. \operatorname{Gamma}\left[1 + n, -\frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] \right) / \left( 128 c^2 \sqrt{1 - c^2 x^2} \right) \right) - \\
& \quad \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} 5 d^2 e^{\frac{ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 3^{1-n} d^2 e^{-\frac{3ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 3^{1-n} d^2 e^{\frac{3ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 5^{-n} d^2 e^{-\frac{5ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 5^{-n} d^2 e^{\frac{5ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, \frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} \\
& \quad 7^{-1-n} d^2 e^{-\frac{7ia}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \quad \operatorname{Gamma}\left[1 + n, -\frac{7i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{128 c^2 \sqrt{1 - c^2 x^2}} 7^{-1-n} d^2 e^{\frac{7ia}{b}} \sqrt{d - c^2 d x^2} \\
& \quad (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{7i(a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 494: Result optimal but 1 more steps used.**

$$\int (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n dx$$

Optimal (type 4, 698 leaves, 12 steps):



$$\begin{aligned}
& \frac{5 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{16 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& 15 i 2^{-7-n} d^2 e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} 15 i 2^{-7-n} d^2 e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} 3 i 2^{-7-2n} d^2 e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} 3 i 2^{-7-2n} d^2 e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

Result (type 4, 698 leaves, 13 steps):

$$\begin{aligned}
& \frac{5 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{16 b c (1+n) \sqrt{1 - c^2 x^2}} - \frac{1}{c \sqrt{1 - c^2 x^2}} \\
& 15 i 2^{-7-n} d^2 e^{-\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} 15 i 2^{-7-n} d^2 e^{\frac{2 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} 3 i 2^{-7-2n} d^2 e^{-\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{c \sqrt{1 - c^2 x^2}} 3 i 2^{-7-2n} d^2 e^{\frac{4 i a}{b}} \sqrt{d - c^2 d x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{c \sqrt{1 - c^2 x^2}} i 2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 i a}{b}} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6 i (a + b \operatorname{ArcSin}[c x])}{b}\right]
\end{aligned}$$

**Problem 495: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n}{x} dx$$

Optimal (type 8, 826 leaves, 27 steps):

$$\begin{aligned}
& \frac{1}{16 \sqrt{d - c^2 d x^2}} 11 d^3 e^{-\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{16 \sqrt{d - c^2 d x^2}} 11 d^3 e^{\frac{ia}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 \sqrt{d - c^2 d x^2}} 5 \times 3^{-1-n} d^3 e^{-\frac{3ia}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{8 \sqrt{d - c^2 d x^2}} 3^{-n} d^3 e^{-\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, -\frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{32 \sqrt{d - c^2 d x^2}} 5 \times 3^{-1-n} d^3 e^{\frac{3ia}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{8 \sqrt{d - c^2 d x^2}} 3^{-n} d^3 e^{\frac{3ia}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{3i(a + b \operatorname{ArcSin}[c x])}{b}\right] + \frac{1}{32 \sqrt{d - c^2 d x^2}} 5^{-1-n} d^3 e^{-\frac{5ia}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( -\frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{32 \sqrt{d - c^2 d x^2}} 5^{-1-n} d^3 e^{\frac{5ia}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i(a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1 + n, \frac{5i(a + b \operatorname{ArcSin}[c x])}{b}\right] + d^3 \operatorname{Unintegrateable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n}{x}, x\right]$$

**Problem 496: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n}{x^2} dx$$

Optimal (type 8, 501 leaves, 18 steps):

$$\begin{aligned}
& - \frac{15 c d^3 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^{1+n}}{8 b (1+n) \sqrt{d - c^2 d x^2}} + \frac{1}{\sqrt{d - c^2 d x^2}} \\
& i 2^{-2-n} c d^3 e^{-\frac{2 i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, -\frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{\sqrt{d - c^2 d x^2}} i 2^{-2-n} c d^3 e^{\frac{2 i a}{b}} \sqrt{1 - c^2 x^2} \\
& (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + \\
& \frac{1}{\sqrt{d - c^2 d x^2}} i 2^{-2(3+n)} c d^3 e^{-\frac{4 i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \\
& \left( - \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] - \frac{1}{\sqrt{d - c^2 d x^2}} \\
& i 2^{-2(3+n)} c d^3 e^{\frac{4 i a}{b}} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcSin}[c x])^n \left( \frac{i (a + b \operatorname{ArcSin}[c x])}{b} \right)^{-n} \\
& \operatorname{Gamma}\left[1+n, \frac{4 i (a + b \operatorname{ArcSin}[c x])}{b}\right] + d^3 \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSin}[c x])^n}{x^2 \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcSin}[c x])^n}{x^2}, x\right]$$

## Test results for the 474 problems in "5.1.5 Inverse sine functions.m"

Problem 229: Result valid but suboptimal antiderivative.

$$\int \frac{(c e + d e x)^2}{(a + b \operatorname{ArcSin}[c + d x])^3} dx$$

Optimal (type 4, 248 leaves, 18 steps):

$$\begin{aligned}
& - \frac{e^2 (c + d x)^2 \sqrt{1 - (c + d x)^2}}{2 b d (a + b \operatorname{ArcSin}[c + d x])^2} - \frac{e^2 (c + d x)}{b^2 d (a + b \operatorname{ArcSin}[c + d x])} + \frac{3 e^2 (c + d x)^3}{2 b^2 d (a + b \operatorname{ArcSin}[c + d x])} - \\
& \frac{e^2 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a + b \operatorname{ArcSin}[c + d x]}{b}\right]}{8 b^3 d} + \frac{9 e^2 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3 (a + b \operatorname{ArcSin}[c + d x])}{b}\right]}{8 b^3 d} - \\
& \frac{e^2 \operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a + b \operatorname{ArcSin}[c + d x]}{b}\right]}{8 b^3 d} + \frac{9 e^2 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3 (a + b \operatorname{ArcSin}[c + d x])}{b}\right]}{8 b^3 d}
\end{aligned}$$

Result (type 4, 306 leaves, 18 steps):

$$\begin{aligned}
& - \frac{e^2 (c + d x)^2 \sqrt{1 - (c + d x)^2}}{2 b d (a + b \operatorname{ArcSin}[c + d x])^2} - \frac{e^2 (c + d x)}{b^2 d (a + b \operatorname{ArcSin}[c + d x])} + \\
& \frac{3 e^2 (c + d x)^3}{2 b^2 d (a + b \operatorname{ArcSin}[c + d x])} - \frac{9 e^2 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c + d x]\right]}{8 b^3 d} + \\
& \frac{9 e^2 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{CosIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c + d x]\right]}{8 b^3 d} + \\
& \frac{e^2 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{CosIntegral}\left[\frac{a + b \operatorname{ArcSin}[c + d x]}{b}\right]}{b^3 d} - \frac{9 e^2 \operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a}{b} + \operatorname{ArcSin}[c + d x]\right]}{8 b^3 d} + \\
& \frac{9 e^2 \operatorname{Sin}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSin}[c + d x]\right]}{8 b^3 d} + \frac{e^2 \operatorname{Sin}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a + b \operatorname{ArcSin}[c + d x]}{b}\right]}{b^3 d}
\end{aligned}$$

Problem 338: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSin}[a + b x]}{\sqrt{c - c (a + b x)^2}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{1 - (a + b x)^2} \operatorname{ArcSin}[a + b x]^2}{2 b \sqrt{c - c (a + b x)^2}}$$

Result (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{1 - (a + b x)^2} \operatorname{ArcSin}[a + b x]^2}{2 b \sqrt{c - c (a + b x)^2}}$$

Problem 339: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSin}[a + b x]}{\sqrt{(1 - a^2) c - 2 a b c x - b^2 c x^2}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{1 - (a + b x)^2} \operatorname{ArcSin}[a + b x]^2}{2 b \sqrt{c - c (a + b x)^2}}$$

Result (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{1 - (a + b x)^2} \operatorname{ArcSin}[a + b x]^2}{2 b \sqrt{c - c (a + b x)^2}}$$

### Problem 470: Unable to integrate problem.

$$\int \frac{x}{\text{ArcSin}[\text{Sin}[x]]} dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\text{ArcSin}[\text{Sin}[x]] + \text{Log}[\text{ArcSin}[\text{Sin}[x]]] \left( -\text{ArcSin}[\text{Sin}[x]] + x \sqrt{\text{Cos}[x]^2} \text{Sec}[x] \right)$$

Result (type 8, 9 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{x}{\text{ArcSin}[\text{Sin}[x]]}, x\right]$$

### Problem 474: Unable to integrate problem.

$$\int \frac{\sqrt{1-x^2} + x \text{ArcSin}[x]}{\text{ArcSin}[x] - x^2 \text{ArcSin}[x]} dx$$

Optimal (type 3, 16 leaves, ? steps):

$$-\frac{1}{2} \text{Log}[1-x^2] + \text{Log}[\text{ArcSin}[x]]$$

Result (type 8, 32 leaves, 1 step):

$$\text{Unintegrable}\left[\frac{\sqrt{1-x^2} + x \text{ArcSin}[x]}{(1-x^2) \text{ArcSin}[x]}, x\right]$$

Test results for the 227 problems in "5.2.2 (d x)^m (a+b arccos(c x))^n.m"

### Problem 168: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{(a + b \text{ArcCos}[c x])^3} dx$$

Optimal (type 4, 197 leaves, 16 steps):

$$\begin{aligned} & \frac{x^2 \sqrt{1-c^2 x^2}}{2 b c (a + b \text{ArcCos}[c x])^2} - \frac{x}{b^2 c^2 (a + b \text{ArcCos}[c x])} + \frac{3 x^3}{2 b^2 (a + b \text{ArcCos}[c x])} - \\ & \frac{\text{CosIntegral}\left[\frac{a+b \text{ArcCos}[c x]}{b}\right] \text{Sin}\left[\frac{a}{b}\right]}{8 b^3 c^3} - \frac{9 \text{CosIntegral}\left[\frac{3(a+b \text{ArcCos}[c x])}{b}\right] \text{Sin}\left[\frac{3 a}{b}\right]}{8 b^3 c^3} + \\ & \frac{\text{Cos}\left[\frac{a}{b}\right] \text{SinIntegral}\left[\frac{a+b \text{ArcCos}[c x]}{b}\right]}{8 b^3 c^3} + \frac{9 \text{Cos}\left[\frac{3 a}{b}\right] \text{SinIntegral}\left[\frac{3(a+b \text{ArcCos}[c x])}{b}\right]}{8 b^3 c^3} \end{aligned}$$

Result (type 4, 246 leaves, 16 steps):

$$\begin{aligned}
& \frac{x^2 \sqrt{1 - c^2 x^2}}{2 b c (a + b \operatorname{ArcCos}[c x])^2} - \frac{x}{b^2 c^2 (a + b \operatorname{ArcCos}[c x])} + \frac{3 x^3}{2 b^2 (a + b \operatorname{ArcCos}[c x])} - \\
& \frac{9 \operatorname{CosIntegral}\left[\frac{a}{b} + \operatorname{ArcCos}[c x]\right] \operatorname{Sin}\left[\frac{a}{b}\right]}{8 b^3 c^3} + \frac{\operatorname{CosIntegral}\left[\frac{a + b \operatorname{ArcCos}[c x]}{b}\right] \operatorname{Sin}\left[\frac{a}{b}\right]}{b^3 c^3} - \\
& \frac{9 \operatorname{CosIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcCos}[c x]\right] \operatorname{Sin}\left[\frac{3 a}{b}\right]}{8 b^3 c^3} + \frac{9 \operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a}{b} + \operatorname{ArcCos}[c x]\right]}{8 b^3 c^3} + \\
& \frac{9 \operatorname{Cos}\left[\frac{3 a}{b}\right] \operatorname{SinIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcCos}[c x]\right]}{8 b^3 c^3} - \frac{\operatorname{Cos}\left[\frac{a}{b}\right] \operatorname{SinIntegral}\left[\frac{a + b \operatorname{ArcCos}[c x]}{b}\right]}{b^3 c^3}
\end{aligned}$$

---

Test results for the 33 problems in "5.2.4 (f x)^m (d+e x^2)^p (a+b arccos(c x))^n.m"

---

Test results for the 118 problems in "5.2.5 Inverse cosine functions.m"

---

Test results for the 166 problems in "5.3.2 (d x)^m (a+b arctan(c x^n))^p.m"

---

Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^7 (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 3, 124 leaves, 12 steps):

$$\begin{aligned}
& \frac{a b x^2}{4 c^3} + \frac{b^2 x^4}{24 c^2} + \frac{b^2 x^2 \operatorname{ArcTan}[c x^2]}{4 c^3} - \frac{b x^6 (a + b \operatorname{ArcTan}[c x^2])}{12 c} - \\
& \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{8 c^4} + \frac{1}{8} x^8 (a + b \operatorname{ArcTan}[c x^2])^2 - \frac{b^2 \operatorname{Log}[1 + c^2 x^4]}{6 c^4}
\end{aligned}$$

Result (type 4, 731 leaves, 62 steps):

$$\begin{aligned}
& \frac{a b x^2}{8 c^3} - \frac{23 i b^2 x^2}{192 c^3} + \frac{b^2 x^4}{128 c^2} - \frac{7 i b^2 x^6}{576 c} + \frac{b^2 x^8}{256} - \frac{3 b^2 (1 - i c x^2)^2}{32 c^4} + \frac{b^2 (1 - i c x^2)^3}{36 c^4} - \\
& \frac{b^2 (1 - i c x^2)^4}{256 c^4} - \frac{b^2 \operatorname{Log}[1 - i c x^2]}{24 c^4} - \frac{b^2 (1 - i c x^2) \operatorname{Log}[1 - i c x^2]}{16 c^4} - \frac{b^2 \operatorname{Log}[1 - i c x^2]^2}{32 c^4} - \\
& \frac{b x^4 (2 i a - b \operatorname{Log}[1 - i c x^2])}{32 c^2} + \frac{i b x^6 (2 i a - b \operatorname{Log}[1 - i c x^2])}{48 c} + \frac{1}{64} b x^8 (2 i a - b \operatorname{Log}[1 - i c x^2]) + \\
& \frac{1}{32} x^8 (2 a + i b \operatorname{Log}[1 - i c x^2])^2 + \frac{1}{192} i b (2 a + i b \operatorname{Log}[1 - i c x^2]) \\
& \left( \frac{48 (1 - i c x^2)}{c^4} - \frac{36 (1 - i c x^2)^2}{c^4} + \frac{16 (1 - i c x^2)^3}{c^4} - \frac{3 (1 - i c x^2)^4}{c^4} - \frac{12 \operatorname{Log}[1 - i c x^2]}{c^4} \right) + \\
& \frac{b (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right]}{16 c^4} + \frac{i b^2 x^6 \operatorname{Log}[1 + i c x^2]}{24 c} - \\
& \frac{b^2 (1 + i c x^2) \operatorname{Log}[1 + i c x^2]}{8 c^4} - \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^2)\right] \operatorname{Log}[1 + i c x^2]}{16 c^4} - \\
& \frac{1}{16} b x^8 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2] + \frac{b^2 \operatorname{Log}[1 + i c x^2]^2}{32 c^4} - \frac{1}{32} b^2 x^8 \operatorname{Log}[1 + i c x^2]^2 + \\
& \frac{5 b^2 \operatorname{Log}[1 + i c x^2]}{192 c^4} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right]}{16 c^4} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^2)\right]}{16 c^4}
\end{aligned}$$

**Problem 75: Result valid but suboptimal antiderivative.**

$$\int x^5 (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 4, 154 leaves, 10 steps):

$$\begin{aligned}
& \frac{b^2 x^2}{6 c^2} - \frac{b^2 \operatorname{ArcTan}[c x^2]}{6 c^3} - \frac{b x^4 (a + b \operatorname{ArcTan}[c x^2])}{6 c} - \frac{i (a + b \operatorname{ArcTan}[c x^2])^2}{6 c^3} + \\
& \frac{1}{6} x^6 (a + b \operatorname{ArcTan}[c x^2])^2 - \frac{b (a + b \operatorname{ArcTan}[c x^2]) \operatorname{Log}\left[\frac{2}{1 + i c x^2}\right]}{3 c^3} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i c x^2}\right]}{6 c^3}
\end{aligned}$$

Result (type 4, 647 leaves, 53 steps):



$$\begin{aligned}
& -\frac{i a b x^2}{6 c^2} + \frac{19 b^2 x^2}{72 c^2} - \frac{5 i b^2 x^4}{144 c} + \frac{b^2 x^6}{108} - \frac{i b^2 (1 - i c x^2)^2}{16 c^3} + \frac{i b^2 (1 - i c x^2)^3}{108 c^3} + \\
& \frac{i b^2 \operatorname{Log}[1 - i c x^2]}{12 c^3} + \frac{i b^2 (1 - i c x^2) \operatorname{Log}[1 - i c x^2]}{12 c^3} - \frac{i b^2 \operatorname{Log}[1 - i c x^2]^2}{24 c^3} + \\
& \frac{i b x^4 (2 i a - b \operatorname{Log}[1 - i c x^2])}{24 c} + \frac{1}{36} b x^6 (2 i a - b \operatorname{Log}[1 - i c x^2]) + \\
& \frac{1}{24} x^6 (2 a + i b \operatorname{Log}[1 - i c x^2])^2 + \frac{1}{72} i b (2 a + i b \operatorname{Log}[1 - i c x^2]) \\
& \left( \frac{18 i (1 - i c x^2)}{c^3} - \frac{9 i (1 - i c x^2)^2}{c^3} + \frac{2 i (1 - i c x^2)^3}{c^3} - \frac{6 i \operatorname{Log}[1 - i c x^2]}{c^3} \right) - \\
& \frac{i b (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right]}{12 c^3} + \frac{i b^2 x^4 \operatorname{Log}[1 + i c x^2]}{12 c} - \\
& \frac{i b^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^2)\right] \operatorname{Log}[1 + i c x^2]}{12 c^3} - \frac{1}{12} b x^6 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2] - \\
& \frac{i b^2 \operatorname{Log}[1 + i c x^2]^2}{24 c^3} - \frac{1}{24} b^2 x^6 \operatorname{Log}[1 + i c x^2]^2 - \frac{i b^2 \operatorname{Log}[i + c x^2]}{72 c^3} + \\
& \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right]}{12 c^3} - \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^2)\right]}{12 c^3}
\end{aligned}$$

**Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^3 (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 3, 90 leaves, 7 steps):

$$-\frac{a b x^2}{2 c} - \frac{b^2 x^2 \operatorname{ArcTan}[c x^2]}{2 c} + \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{4 c^2} + \frac{1}{4} x^4 (a + b \operatorname{ArcTan}[c x^2])^2 + \frac{b^2 \operatorname{Log}[1 + c^2 x^4]}{4 c^2}$$

Result (type 4, 612 leaves, 44 steps):

$$\begin{aligned}
& -\frac{3abx^2}{4c} + \frac{b^2x^4}{16} + \frac{b^2(1-icx^2)^2}{32c^2} + \frac{b^2(1+icx^2)^2}{32c^2} - \frac{b^2\operatorname{Log}[i-cx^2]}{16c^2} + \\
& \frac{3b^2(1-icx^2)\operatorname{Log}[1-icx^2]}{8c^2} + \frac{1}{16}bx^4(2ia-b\operatorname{Log}[1-icx^2]) + \\
& \frac{ib(1-icx^2)^2(2a+ib\operatorname{Log}[1-icx^2])}{16c^2} + \frac{(1-icx^2)(2a+ib\operatorname{Log}[1-icx^2])^2}{8c^2} - \\
& \frac{(1-icx^2)^2(2a+ib\operatorname{Log}[1-icx^2])^2}{16c^2} - \frac{b(2ia-b\operatorname{Log}[1-icx^2])\operatorname{Log}[\frac{1}{2}(1+icx^2)]}{8c^2} - \\
& \frac{1}{16}b^2x^4\operatorname{Log}[1+icx^2] + \frac{3b^2(1+icx^2)\operatorname{Log}[1+icx^2]}{8c^2} - \frac{b^2(1+icx^2)^2\operatorname{Log}[1+icx^2]}{16c^2} + \\
& \frac{b^2\operatorname{Log}[\frac{1}{2}(1-icx^2)]\operatorname{Log}[1+icx^2]}{8c^2} - \frac{1}{8}bx^4(2ia-b\operatorname{Log}[1-icx^2])\operatorname{Log}[1+icx^2] - \\
& \frac{b^2(1+icx^2)\operatorname{Log}[1+icx^2]^2}{8c^2} + \frac{b^2(1+icx^2)^2\operatorname{Log}[1+icx^2]^2}{16c^2} - \\
& \frac{b^2\operatorname{Log}[i+cx^2]}{16c^2} + \frac{b^2\operatorname{PolyLog}[2, \frac{1}{2}(1-icx^2)]}{8c^2} + \frac{b^2\operatorname{PolyLog}[2, \frac{1}{2}(1+icx^2)]}{8c^2}
\end{aligned}$$

**Problem 77: Result valid but suboptimal antiderivative.**

$$\int x (a + b \operatorname{ArcTan}[cx^2])^2 dx$$

Optimal (type 4, 101 leaves, 6 steps):

$$\begin{aligned}
& \frac{i(a + b \operatorname{ArcTan}[cx^2])^2}{2c} + \frac{1}{2}x^2(a + b \operatorname{ArcTan}[cx^2])^2 + \\
& \frac{b(a + b \operatorname{ArcTan}[cx^2])\operatorname{Log}[\frac{2}{1+icx^2}]}{c} + \frac{ib^2\operatorname{PolyLog}[2, 1 - \frac{2}{1+icx^2}]}{2c}
\end{aligned}$$

Result (type 4, 255 leaves, 28 steps):

$$\begin{aligned}
& \frac{i(1-icx^2)(2a+ib\operatorname{Log}[1-icx^2])^2}{8c} + \frac{ib(2ia-b\operatorname{Log}[1-icx^2])\operatorname{Log}[\frac{1}{2}(1+icx^2)]}{4c} + \\
& \frac{ib^2\operatorname{Log}[\frac{1}{2}(1-icx^2)]\operatorname{Log}[1+icx^2]}{4c} - \frac{1}{4}bx^2(2ia-b\operatorname{Log}[1-icx^2])\operatorname{Log}[1+icx^2] + \\
& \frac{ib^2(1+icx^2)\operatorname{Log}[1+icx^2]^2}{8c} - \frac{ib^2\operatorname{PolyLog}[2, \frac{1}{2}(1-icx^2)]}{4c} + \frac{ib^2\operatorname{PolyLog}[2, \frac{1}{2}(1+icx^2)]}{4c}
\end{aligned}$$

**Problem 79: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[cx^2])^2}{x^3} dx$$

Optimal (type 4, 97 leaves, 5 steps):

$$-\frac{1}{2} i c (a + b \operatorname{ArcTan}[c x^2])^2 - \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{2 x^2} +$$

$$b c (a + b \operatorname{ArcTan}[c x^2]) \operatorname{Log}\left[2 - \frac{2}{1 - i c x^2}\right] - \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - i c x^2}\right]$$

Result (type 4, 290 leaves, 24 steps):

$$2 a b c \operatorname{Log}[x] - \frac{(1 - i c x^2) (2 a + i b \operatorname{Log}[1 - i c x^2])^2}{8 x^2} +$$

$$\frac{1}{4} i b c (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right] + \frac{1}{4} i b^2 c \operatorname{Log}\left[\frac{1}{2} (1 - i c x^2)\right] \operatorname{Log}[1 + i c x^2] +$$

$$b (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2] + \frac{b^2 (1 + i c x^2) \operatorname{Log}[1 + i c x^2]^2}{8 x^2} +$$

$$\frac{1}{2} i b^2 c \operatorname{PolyLog}[2, -i c x^2] - \frac{1}{2} i b^2 c \operatorname{PolyLog}[2, i c x^2] -$$

$$\frac{1}{4} i b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right] + \frac{1}{4} i b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^2)\right]$$

**Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{x^5} dx$$

Optimal (type 3, 87 leaves, 9 steps):

$$-\frac{b c (a + b \operatorname{ArcTan}[c x^2])}{2 x^2} - \frac{1}{4} c^2 (a + b \operatorname{ArcTan}[c x^2])^2 -$$

$$\frac{(a + b \operatorname{ArcTan}[c x^2])^2}{4 x^4} + b^2 c^2 \operatorname{Log}[x] - \frac{1}{4} b^2 c^2 \operatorname{Log}[1 + c^2 x^4]$$

Result (type 4, 419 leaves, 46 steps):

$$b^2 c^2 \operatorname{Log}[x] - \frac{1}{4} b^2 c^2 \operatorname{Log}[i - c x^2] + \frac{i b c (2 i a - b \operatorname{Log}[1 - i c x^2])}{8 x^2} -$$

$$\frac{b c (1 - i c x^2) (2 a + i b \operatorname{Log}[1 - i c x^2])}{8 x^2} - \frac{1}{16} c^2 (2 a + i b \operatorname{Log}[1 - i c x^2])^2 -$$

$$\frac{(2 a + i b \operatorname{Log}[1 - i c x^2])^2}{16 x^4} + \frac{1}{8} b c^2 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right] +$$

$$\frac{i b^2 c \operatorname{Log}[1 + i c x^2]}{4 x^2} - \frac{1}{8} b^2 c^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^2)\right] \operatorname{Log}[1 + i c x^2] +$$

$$b (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2] + \frac{1}{16} b^2 c^2 \operatorname{Log}[1 + i c x^2]^2 + \frac{b^2 \operatorname{Log}[1 + i c x^2]^2}{16 x^4} -$$

$$\frac{1}{8} b^2 c^2 \operatorname{Log}[i + c x^2] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^2)\right]$$

### Problem 86: Result valid but suboptimal antiderivative.

$$\int x^3 (a + b \operatorname{ArcTan}[c x^2])^3 dx$$

Optimal (type 4, 149 leaves, 9 steps):

$$\begin{aligned} & -\frac{3 i b (a + b \operatorname{ArcTan}[c x^2])^2}{4 c^2} - \frac{3 b x^2 (a + b \operatorname{ArcTan}[c x^2])^2}{4 c} + \frac{(a + b \operatorname{ArcTan}[c x^2])^3}{4 c^2} + \\ & \frac{1}{4} x^4 (a + b \operatorname{ArcTan}[c x^2])^3 - \frac{3 b^2 (a + b \operatorname{ArcTan}[c x^2]) \operatorname{Log}\left[\frac{2}{1+i c x^2}\right]}{2 c^2} - \frac{3 i b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c x^2}\right]}{4 c^2} \end{aligned}$$

Result (type 4, 951 leaves, 155 steps):

$$\begin{aligned} & \frac{3 i b^2 (1 - i c x^2)^2 (2 i a - b \operatorname{Log}[1 - i c x^2])}{64 c^2} + \frac{3 i b (1 - i c x^2)^2 (2 i a - b \operatorname{Log}[1 - i c x^2])^2}{64 c^2} + \\ & \frac{3 b^2 (1 - i c x^2)^2 (2 a + i b \operatorname{Log}[1 - i c x^2])}{64 c^2} - \frac{3 i b (1 - i c x^2) (2 a + i b \operatorname{Log}[1 - i c x^2])^2}{16 c^2} + \\ & \frac{3 i b (1 - i c x^2)^2 (2 a + i b \operatorname{Log}[1 - i c x^2])^2}{64 c^2} + \frac{(1 - i c x^2) (2 a + i b \operatorname{Log}[1 - i c x^2])^3}{16 c^2} - \\ & \frac{(1 - i c x^2)^2 (2 a + i b \operatorname{Log}[1 - i c x^2])^3}{32 c^2} - \frac{3 i b^2 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right]}{8 c^2} + \\ & \frac{3 i b (2 i a - b \operatorname{Log}[1 - i c x^2])^2 \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right]}{32 c^2} + \\ & \frac{3 i b (2 a + i b \operatorname{Log}[1 - i c x^2])^2 \operatorname{Log}\left[\frac{1}{2} (1 + i c x^2)\right]}{32 c^2} - \\ & \frac{3 i b^3 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^2)\right] \operatorname{Log}[1 + i c x^2]}{8 c^2} + \frac{3 b^2 x^2 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2]}{8 c} + \\ & \frac{3}{32} i b x^4 (2 i a - b \operatorname{Log}[1 - i c x^2])^2 \operatorname{Log}[1 + i c x^2] - \\ & \frac{3 i b (2 a + i b \operatorname{Log}[1 - i c x^2])^2 \operatorname{Log}[1 + i c x^2]}{32 c^2} - \frac{3 i b^3 (1 + i c x^2) \operatorname{Log}[1 + i c x^2]^2}{16 c^2} + \\ & \frac{3}{32} i b^2 x^4 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2]^2 - \\ & \frac{3 b^2 (2 a + i b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2]^2}{32 c^2} + \frac{i b^3 (1 + i c x^2) \operatorname{Log}[1 + i c x^2]^3}{16 c^2} - \\ & \frac{i b^3 (1 + i c x^2)^2 \operatorname{Log}[1 + i c x^2]^3}{32 c^2} + \frac{3 i b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right]}{8 c^2} - \\ & \frac{3 i b^2 (2 i a - b \operatorname{Log}[1 - i c x^2]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right]}{16 c^2} - \\ & \frac{3 b^2 (2 a + i b \operatorname{Log}[1 - i c x^2]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^2)\right]}{16 c^2} - \frac{3 i b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^2)\right]}{8 c^2} \end{aligned}$$

### Problem 87: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \operatorname{ArcTan}[c x^2] \right)^3 dx$$

Optimal (type 4, 144 leaves, 6 steps):

$$\frac{i \left( a + b \operatorname{ArcTan}[c x^2] \right)^3}{2 c} + \frac{1}{2} x^2 \left( a + b \operatorname{ArcTan}[c x^2] \right)^3 + \frac{3 b \left( a + b \operatorname{ArcTan}[c x^2] \right)^2 \operatorname{Log}\left[\frac{2}{1+i c x^2}\right]}{2 c} +$$

$$\frac{3 i b^2 \left( a + b \operatorname{ArcTan}[c x^2] \right) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c x^2}\right]}{2 c} + \frac{3 b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+i c x^2}\right]}{4 c}$$

Result (type 4, 545 leaves, 82 steps):

$$\frac{3 b \left( 1 - i c x^2 \right) \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right)^2}{16 c} + \frac{3 b \left( 1 - i c x^2 \right) \left( 2 a + i b \operatorname{Log}\left[1 - i c x^2\right] \right)^2}{16 c} +$$

$$\frac{i \left( 1 - i c x^2 \right) \left( 2 a + i b \operatorname{Log}\left[1 - i c x^2\right] \right)^3}{16 c} + \frac{3 b \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right)^2 \operatorname{Log}\left[\frac{1}{2} \left( 1 + i c x^2 \right) \right]}{8 c} -$$

$$\frac{3 b \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right)^2 \operatorname{Log}\left[1 + i c x^2\right]}{16 c} + \frac{3}{16} i b x^2 \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right)^2 \operatorname{Log}\left[1 + i c x^2\right] +$$

$$\frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} \left( 1 - i c x^2 \right) \right] \operatorname{Log}\left[1 + i c x^2\right]^2}{8 c} + \frac{3 b^2 \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right) \operatorname{Log}\left[1 + i c x^2\right]^2}{16 c} +$$

$$\frac{3}{16} i b^2 x^2 \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right) \operatorname{Log}\left[1 + i c x^2\right]^2 + \frac{b^3 \left( 1 + i c x^2 \right) \operatorname{Log}\left[1 + i c x^2\right]^3}{16 c} -$$

$$\frac{3 b^2 \left( 2 i a - b \operatorname{Log}\left[1 - i c x^2\right] \right) \operatorname{PolyLog}\left[2, \frac{1}{2} \left( 1 - i c x^2 \right) \right]}{4 c} +$$

$$\frac{3 b^3 \operatorname{Log}\left[1 + i c x^2\right] \operatorname{PolyLog}\left[2, \frac{1}{2} \left( 1 + i c x^2 \right) \right]}{4 c} -$$

$$\frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} \left( 1 - i c x^2 \right) \right]}{4 c} - \frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} \left( 1 + i c x^2 \right) \right]}{4 c}$$

### Problem 89: Unable to integrate problem.

$$\int \frac{\left( a + b \operatorname{ArcTan}[c x^2] \right)^3}{x^3} dx$$

Optimal (type 4, 138 leaves, 6 steps):

$$-\frac{1}{2} i c \left( a + b \operatorname{ArcTan}[c x^2] \right)^3 - \frac{\left( a + b \operatorname{ArcTan}[c x^2] \right)^3}{2 x^2} + \frac{3}{2} b c \left( a + b \operatorname{ArcTan}[c x^2] \right)^2 \operatorname{Log}\left[2 - \frac{2}{1-i c x^2}\right] -$$

$$\frac{3}{2} i b^2 c \left( a + b \operatorname{ArcTan}[c x^2] \right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-i c x^2}\right] + \frac{3}{4} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-i c x^2}\right]$$

Result (type 8, 347 leaves, 16 steps):

$$\begin{aligned}
& \frac{3}{16} b c \operatorname{Log}\left[\frac{1}{i} c x^2\right] \left(2 a + \frac{1}{i} b \operatorname{Log}\left[1 - \frac{1}{i} c x^2\right]\right)^2 - \frac{\left(1 - \frac{1}{i} c x^2\right) \left(2 a + \frac{1}{i} b \operatorname{Log}\left[1 - \frac{1}{i} c x^2\right]\right)^3}{16 x^2} - \\
& \frac{3}{16} b^3 c \operatorname{Log}\left[-\frac{1}{i} c x^2\right] \operatorname{Log}\left[1 + \frac{1}{i} c x^2\right]^2 - \frac{\frac{1}{i} b^3 \left(1 + \frac{1}{i} c x^2\right) \operatorname{Log}\left[1 + \frac{1}{i} c x^2\right]^3}{16 x^2} + \\
& \frac{3}{8} \frac{1}{i} b^2 c \left(2 a + \frac{1}{i} b \operatorname{Log}\left[1 - \frac{1}{i} c x^2\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{1}{i} c x^2\right] - \\
& \frac{3}{8} b^3 c \operatorname{Log}\left[1 + \frac{1}{i} c x^2\right] \operatorname{PolyLog}\left[2, 1 + \frac{1}{i} c x^2\right] + \frac{3}{8} b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{1}{i} c x^2\right] + \\
& \frac{3}{8} b^3 c \operatorname{PolyLog}\left[3, 1 + \frac{1}{i} c x^2\right] + \frac{3}{8} \frac{1}{i} b \operatorname{Unintegrable}\left[\frac{\left(-2 \frac{1}{i} a + b \operatorname{Log}\left[1 - \frac{1}{i} c x^2\right]\right)^2 \operatorname{Log}\left[1 + \frac{1}{i} c x^2\right]}{x^3}, x\right] - \\
& \frac{3}{8} \frac{1}{i} b^2 \operatorname{Unintegrable}\left[\frac{\left(-2 \frac{1}{i} a + b \operatorname{Log}\left[1 - \frac{1}{i} c x^2\right]\right) \operatorname{Log}\left[1 + \frac{1}{i} c x^2\right]^2}{x^3}, x\right]
\end{aligned}$$

Problem 90: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTan}\left[c x^2\right]\right)^3}{x^5} dx$$

Optimal (type 4, 149 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3}{4} \frac{1}{i} b c^2 \left(a + b \operatorname{ArcTan}\left[c x^2\right]\right)^2 - \frac{3 b c \left(a + b \operatorname{ArcTan}\left[c x^2\right]\right)^2}{4 x^2} - \\
& \frac{1}{4} c^2 \left(a + b \operatorname{ArcTan}\left[c x^2\right]\right)^3 - \frac{\left(a + b \operatorname{ArcTan}\left[c x^2\right]\right)^3}{4 x^4} + \\
& \frac{3}{2} b^2 c^2 \left(a + b \operatorname{ArcTan}\left[c x^2\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 - \frac{1}{i} c x^2}\right] - \frac{3}{4} \frac{1}{i} b^3 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{1}{i} c x^2}\right]
\end{aligned}$$

Result (type 8, 533 leaves, 29 steps):

$$\begin{aligned}
& \frac{3}{4} a b^2 c^2 \operatorname{Log}[x] - \frac{3 b c (1 - i c x^2) (2 a + i b \operatorname{Log}[1 - i c x^2])^2}{32 x^2} + \\
& \frac{3}{32} i b c^2 \operatorname{Log}[i c x^2] (2 a + i b \operatorname{Log}[1 - i c x^2])^2 - \frac{1}{32} c^2 (2 a + i b \operatorname{Log}[1 - i c x^2])^3 - \\
& \frac{(2 a + i b \operatorname{Log}[1 - i c x^2])^3}{32 x^4} + \frac{3 b^3 c (1 + i c x^2) \operatorname{Log}[1 + i c x^2]^2}{32 x^2} + \\
& \frac{3}{32} i b^3 c^2 \operatorname{Log}[-i c x^2] \operatorname{Log}[1 + i c x^2]^2 - \frac{1}{32} i b^3 c^2 \operatorname{Log}[1 + i c x^2]^3 - \\
& \frac{i b^3 \operatorname{Log}[1 + i c x^2]^3}{32 x^4} + \frac{3}{16} i b^3 c^2 \operatorname{PolyLog}[2, -i c x^2] - \frac{3}{16} i b^3 c^2 \operatorname{PolyLog}[2, i c x^2] - \\
& \frac{3}{16} b^2 c^2 (2 a + i b \operatorname{Log}[1 - i c x^2]) \operatorname{PolyLog}[2, 1 - i c x^2] + \\
& \frac{3}{16} i b^3 c^2 \operatorname{Log}[1 + i c x^2] \operatorname{PolyLog}[2, 1 + i c x^2] + \\
& \frac{3}{16} i b^3 c^2 \operatorname{PolyLog}[3, 1 - i c x^2] - \frac{3}{16} i b^3 c^2 \operatorname{PolyLog}[3, 1 + i c x^2] + \\
& \frac{3}{8} i b \operatorname{Unintegrable}\left[\frac{(-2 i a + b \operatorname{Log}[1 - i c x^2])^2 \operatorname{Log}[1 + i c x^2]}{x^5}, x\right] - \\
& \frac{3}{8} i b^2 \operatorname{Unintegrable}\left[\frac{(-2 i a + b \operatorname{Log}[1 - i c x^2]) \operatorname{Log}[1 + i c x^2]^2}{x^5}, x\right]
\end{aligned}$$

**Problem 93: Result optimal but 1 more steps used.**

$$\int (d x)^m (a + b \operatorname{ArcTan}[c x^2]) dx$$

Optimal (type 5, 75 leaves, 2 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTan}[c x^2])}{d (1+m)} - \frac{2 b c (d x)^{3+m} \operatorname{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, -c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Result (type 5, 75 leaves, 3 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTan}[c x^2])}{d (1+m)} - \frac{2 b c (d x)^{3+m} \operatorname{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, -c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

**Problem 113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^{11} (a + b \operatorname{ArcTan}[c x^3])^2 dx$$

Optimal (type 3, 124 leaves, 12 steps):

$$\frac{a b x^3}{6 c^3} + \frac{b^2 x^6}{36 c^2} + \frac{b^2 x^3 \operatorname{ArcTan}[c x^3]}{6 c^3} - \frac{b x^9 (a + b \operatorname{ArcTan}[c x^3])}{18 c} - \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{12 c^4} + \frac{1}{12} x^{12} (a + b \operatorname{ArcTan}[c x^3])^2 - \frac{b^2 \operatorname{Log}[1 + c^2 x^6]}{9 c^4}$$

Result (type 4, 731 leaves, 62 steps):

$$\begin{aligned} & \frac{a b x^3}{12 c^3} - \frac{23 i b^2 x^3}{288 c^3} + \frac{b^2 x^6}{192 c^2} - \frac{7 i b^2 x^9}{864 c} + \frac{b^2 x^{12}}{384} - \\ & \frac{b^2 (1 - i c x^3)^2}{16 c^4} + \frac{b^2 (1 - i c x^3)^3}{54 c^4} - \frac{b^2 (1 - i c x^3)^4}{384 c^4} - \frac{b^2 \operatorname{Log}[i - c x^3]}{36 c^4} - \\ & \frac{b^2 (1 - i c x^3) \operatorname{Log}[1 - i c x^3]}{24 c^4} - \frac{b^2 \operatorname{Log}[1 - i c x^3]^2}{48 c^4} - \frac{b x^6 (2 i a - b \operatorname{Log}[1 - i c x^3])}{48 c^2} + \\ & \frac{i b x^9 (2 i a - b \operatorname{Log}[1 - i c x^3])}{72 c} + \frac{1}{96} b x^{12} (2 i a - b \operatorname{Log}[1 - i c x^3]) + \\ & \frac{1}{48} x^{12} (2 a + i b \operatorname{Log}[1 - i c x^3])^2 + \frac{1}{288} i b (2 a + i b \operatorname{Log}[1 - i c x^3]) \\ & \left( \frac{48 (1 - i c x^3)}{c^4} - \frac{36 (1 - i c x^3)^2}{c^4} + \frac{16 (1 - i c x^3)^3}{c^4} - \frac{3 (1 - i c x^3)^4}{c^4} - \frac{12 \operatorname{Log}[1 - i c x^3]}{c^4} \right) + \\ & \frac{b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right]}{24 c^4} + \frac{i b^2 x^9 \operatorname{Log}[1 + i c x^3]}{36 c} - \\ & \frac{b^2 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]}{12 c^4} - \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3]}{24 c^4} - \\ & \frac{1}{24} b x^{12} (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] + \frac{b^2 \operatorname{Log}[1 + i c x^3]^2}{48 c^4} - \frac{1}{48} b^2 x^{12} \operatorname{Log}[1 + i c x^3]^2 + \\ & \frac{5 b^2 \operatorname{Log}[i + c x^3]}{288 c^4} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right]}{24 c^4} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right]}{24 c^4} \end{aligned}$$

**Problem 114: Result valid but suboptimal antiderivative.**

$$\int x^8 (a + b \operatorname{ArcTan}[c x^3])^2 dx$$

Optimal (type 4, 154 leaves, 10 steps):

$$\begin{aligned} & \frac{b^2 x^3}{9 c^2} - \frac{b^2 \operatorname{ArcTan}[c x^3]}{9 c^3} - \frac{b x^6 (a + b \operatorname{ArcTan}[c x^3])}{9 c} - \frac{i (a + b \operatorname{ArcTan}[c x^3])^2}{9 c^3} + \\ & \frac{1}{9} x^9 (a + b \operatorname{ArcTan}[c x^3])^2 - \frac{2 b (a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}\left[\frac{2}{1 + i c x^3}\right]}{9 c^3} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i c x^3}\right]}{9 c^3} \end{aligned}$$

Result (type 4, 647 leaves, 53 steps):



$$\begin{aligned}
& -\frac{i a b x^3}{9 c^2} + \frac{19 b^2 x^3}{108 c^2} - \frac{5 i b^2 x^6}{216 c} + \frac{b^2 x^9}{162} - \frac{i b^2 (1 - i c x^3)^2}{24 c^3} + \frac{i b^2 (1 - i c x^3)^3}{162 c^3} + \\
& \frac{i b^2 \operatorname{Log}[1 - i c x^3]}{18 c^3} + \frac{i b^2 (1 - i c x^3) \operatorname{Log}[1 - i c x^3]}{18 c^3} - \frac{i b^2 \operatorname{Log}[1 - i c x^3]^2}{36 c^3} + \\
& \frac{i b x^6 (2 i a - b \operatorname{Log}[1 - i c x^3])}{36 c} + \frac{1}{54} b x^9 (2 i a - b \operatorname{Log}[1 - i c x^3]) + \\
& \frac{1}{36} x^9 (2 a + i b \operatorname{Log}[1 - i c x^3])^2 + \frac{1}{108} i b (2 a + i b \operatorname{Log}[1 - i c x^3]) \\
& \left( \frac{18 i (1 - i c x^3)}{c^3} - \frac{9 i (1 - i c x^3)^2}{c^3} + \frac{2 i (1 - i c x^3)^3}{c^3} - \frac{6 i \operatorname{Log}[1 - i c x^3]}{c^3} \right) - \\
& \frac{i b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right]}{18 c^3} + \frac{i b^2 x^6 \operatorname{Log}[1 + i c x^3]}{18 c} - \\
& \frac{i b^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3]}{18 c^3} - \frac{1}{18} b x^9 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] - \\
& \frac{i b^2 \operatorname{Log}[1 + i c x^3]^2}{36 c^3} - \frac{1}{36} b^2 x^9 \operatorname{Log}[1 + i c x^3]^2 - \frac{i b^2 \operatorname{Log}[i + c x^3]}{108 c^3} + \\
& \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right]}{18 c^3} - \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right]}{18 c^3}
\end{aligned}$$

**Problem 115:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^5 (a + b \operatorname{ArcTan}[c x^3])^2 dx$$

Optimal (type 3, 90 leaves, 7 steps):

$$-\frac{a b x^3}{3 c} - \frac{b^2 x^3 \operatorname{ArcTan}[c x^3]}{3 c} + \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{6 c^2} + \frac{1}{6} x^6 (a + b \operatorname{ArcTan}[c x^3])^2 + \frac{b^2 \operatorname{Log}[1 + c^2 x^6]}{6 c^2}$$

Result (type 4, 612 leaves, 44 steps):

$$\begin{aligned}
& -\frac{a b x^3}{2 c} + \frac{b^2 x^6}{24} + \frac{b^2 (1 - i c x^3)^2}{48 c^2} + \frac{b^2 (1 + i c x^3)^2}{48 c^2} - \frac{b^2 \operatorname{Log}[i - c x^3]}{24 c^2} + \\
& \frac{b^2 (1 - i c x^3) \operatorname{Log}[1 - i c x^3]}{4 c^2} + \frac{1}{24} b x^6 (2 i a - b \operatorname{Log}[1 - i c x^3]) + \\
& \frac{i b (1 - i c x^3)^2 (2 a + i b \operatorname{Log}[1 - i c x^3])}{24 c^2} + \frac{(1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{12 c^2} - \\
& \frac{(1 - i c x^3)^2 (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{24 c^2} - \frac{b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[\frac{1}{2} (1 + i c x^3)]}{12 c^2} - \\
& \frac{1}{24} b^2 x^6 \operatorname{Log}[1 + i c x^3] + \frac{b^2 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]}{4 c^2} - \frac{b^2 (1 + i c x^3)^2 \operatorname{Log}[1 + i c x^3]}{24 c^2} + \\
& \frac{b^2 \operatorname{Log}[\frac{1}{2} (1 - i c x^3)] \operatorname{Log}[1 + i c x^3]}{12 c^2} - \frac{1}{12} b x^6 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] - \\
& \frac{b^2 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^2}{12 c^2} + \frac{b^2 (1 + i c x^3)^2 \operatorname{Log}[1 + i c x^3]^2}{24 c^2} - \\
& \frac{b^2 \operatorname{Log}[i + c x^3]}{24 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - i c x^3)]}{12 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + i c x^3)]}{12 c^2}
\end{aligned}$$

**Problem 116: Result valid but suboptimal antiderivative.**

$$\int x^2 (a + b \operatorname{ArcTan}[c x^3])^2 dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$\begin{aligned}
& \frac{i (a + b \operatorname{ArcTan}[c x^3])^2}{3 c} + \frac{1}{3} x^3 (a + b \operatorname{ArcTan}[c x^3])^2 + \\
& \frac{2 b (a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}[\frac{2}{1 + i c x^3}]}{3 c} + \frac{i b^2 \operatorname{PolyLog}[2, 1 - \frac{2}{1 + i c x^3}]}{3 c}
\end{aligned}$$

Result (type 4, 255 leaves, 28 steps):

$$\begin{aligned}
& \frac{i (1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{12 c} + \frac{i b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[\frac{1}{2} (1 + i c x^3)]}{6 c} + \\
& \frac{i b^2 \operatorname{Log}[\frac{1}{2} (1 - i c x^3)] \operatorname{Log}[1 + i c x^3]}{6 c} - \frac{1}{6} b x^3 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] + \\
& \frac{i b^2 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^2}{12 c} - \frac{i b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - i c x^3)]}{6 c} + \frac{i b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + i c x^3)]}{6 c}
\end{aligned}$$

**Problem 118: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{x^4} dx$$

Optimal (type 4, 100 leaves, 5 steps):

$$-\frac{1}{3} i c (a + b \operatorname{ArcTan}[c x^3])^2 - \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{3 x^3} +$$

$$\frac{2}{3} b c (a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 - i c x^3}\right] - \frac{1}{3} i b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - i c x^3}\right]$$

Result (type 4, 290 leaves, 24 steps):

$$2 a b c \operatorname{Log}[x] - \frac{(1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{12 x^3} +$$

$$\frac{1}{6} i b c (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right] + \frac{1}{6} i b^2 c \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3] +$$

$$b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] + \frac{b^2 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^2}{12 x^3} +$$

$$\frac{1}{3} i b^2 c \operatorname{PolyLog}\left[2, -i c x^3\right] - \frac{1}{3} i b^2 c \operatorname{PolyLog}\left[2, i c x^3\right] -$$

$$\frac{1}{6} i b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right] + \frac{1}{6} i b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right]$$

**Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{x^7} dx$$

Optimal (type 3, 87 leaves, 9 steps):

$$-\frac{b c (a + b \operatorname{ArcTan}[c x^3])}{3 x^3} - \frac{1}{6} c^2 (a + b \operatorname{ArcTan}[c x^3])^2 -$$

$$\frac{(a + b \operatorname{ArcTan}[c x^3])^2}{6 x^6} + b^2 c^2 \operatorname{Log}[x] - \frac{1}{6} b^2 c^2 \operatorname{Log}[1 + c^2 x^6]$$

Result (type 4, 419 leaves, 46 steps):

$$b^2 c^2 \operatorname{Log}[x] - \frac{1}{6} b^2 c^2 \operatorname{Log}[i - c x^3] + \frac{i b c (2 i a - b \operatorname{Log}[1 - i c x^3])}{12 x^3} -$$

$$\frac{b c (1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])}{12 x^3} - \frac{1}{24} c^2 (2 a + i b \operatorname{Log}[1 - i c x^3])^2 -$$

$$\frac{(2 a + i b \operatorname{Log}[1 - i c x^3])^2}{24 x^6} + \frac{1}{12} b c^2 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right] +$$

$$\frac{i b^2 c \operatorname{Log}[1 + i c x^3]}{6 x^3} - \frac{1}{12} b^2 c^2 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3] +$$

$$b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3] + \frac{1}{24} b^2 c^2 \operatorname{Log}[1 + i c x^3]^2 + \frac{b^2 \operatorname{Log}[1 + i c x^3]^2}{24 x^6} -$$

$$\frac{1}{12} b^2 c^2 \operatorname{Log}[i + c x^3] - \frac{1}{12} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right] - \frac{1}{12} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right]$$

### Problem 120: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{x^{10}} dx$$

Optimal (type 4, 154 leaves, 9 steps):

$$\begin{aligned} & -\frac{b^2 c^2}{9 x^3} - \frac{1}{9} b^2 c^3 \operatorname{ArcTan}[c x^3] - \frac{b c (a + b \operatorname{ArcTan}[c x^3])}{9 x^6} + \\ & \frac{1}{9} i c^3 (a + b \operatorname{ArcTan}[c x^3])^2 - \frac{(a + b \operatorname{ArcTan}[c x^3])^2}{9 x^9} - \\ & \frac{2}{9} b c^3 (a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 - i c x^3}\right] + \frac{1}{9} i b^2 c^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - i c x^3}\right] \end{aligned}$$

Result (type 4, 536 leaves, 59 steps):

$$\begin{aligned} & -\frac{b^2 c^2}{9 x^3} - \frac{2}{3} a b c^3 \operatorname{Log}[x] + \frac{1}{18} i b^2 c^3 \operatorname{Log}[i - c x^3] + \frac{i b c (2 i a - b \operatorname{Log}[1 - i c x^3])}{36 x^6} + \\ & \frac{b c^2 (2 i a - b \operatorname{Log}[1 - i c x^3])}{18 x^3} - \frac{b c (2 a + i b \operatorname{Log}[1 - i c x^3])}{36 x^6} - \\ & \frac{i b c^2 (1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])}{18 x^3} - \frac{1}{36} i c^3 (2 a + i b \operatorname{Log}[1 - i c x^3])^2 - \\ & \frac{(2 a + i b \operatorname{Log}[1 - i c x^3])^2}{36 x^9} - \frac{1}{18} i b c^3 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right] + \\ & \frac{i b^2 c \operatorname{Log}[1 + i c x^3]}{18 x^6} - \frac{1}{18} i b^2 c^3 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3] + \\ & \frac{b (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]}{18 x^9} - \frac{1}{36} i b^2 c^3 \operatorname{Log}[1 + i c x^3]^2 + \\ & \frac{b^2 \operatorname{Log}[1 + i c x^3]^2}{36 x^9} - \frac{1}{9} i b^2 c^3 \operatorname{PolyLog}[2, -i c x^3] + \frac{1}{9} i b^2 c^3 \operatorname{PolyLog}[2, i c x^3] + \\ & \frac{1}{18} i b^2 c^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right] - \frac{1}{18} i b^2 c^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right] \end{aligned}$$

### Problem 121: Result valid but suboptimal antiderivative.

$$\int x^8 (a + b \operatorname{ArcTan}[c x^3])^3 dx$$

Optimal (type 4, 240 leaves, 13 steps):

$$\begin{aligned}
& \frac{a b^2 x^3}{3 c^2} + \frac{b^3 x^3 \operatorname{ArcTan}[c x^3]}{3 c^2} - \frac{b (a + b \operatorname{ArcTan}[c x^3])^2}{6 c^3} - \frac{b x^6 (a + b \operatorname{ArcTan}[c x^3])^2}{6 c} \\
& - \frac{i (a + b \operatorname{ArcTan}[c x^3])^3}{9 c^3} + \frac{1}{9} x^9 (a + b \operatorname{ArcTan}[c x^3])^3 - \frac{b (a + b \operatorname{ArcTan}[c x^3])^2 \operatorname{Log}\left[\frac{2}{1+i c x^3}\right]}{3 c^3} - \\
& \frac{b^3 \operatorname{Log}[1+c^2 x^6]}{6 c^3} - \frac{i b^2 (a + b \operatorname{ArcTan}[c x^3]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c x^3}\right]}{3 c^3} - \frac{b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+i c x^3}\right]}{6 c^3}
\end{aligned}$$

Result (type 4, 1867 leaves, 239 steps):

$$\begin{aligned}
& \frac{2 a b^2 x^3}{3 c^2} + \frac{7 i b^3 x^3}{216 c^2} - \frac{23 b^3 x^6}{432 c} + \frac{1}{324} i b^3 x^9 - \\
& \frac{b^3 (1-i c x^3)^2}{48 c^3} - \frac{b^3 (1+i c x^3)^2}{24 c^3} + \frac{b^3 (1+i c x^3)^3}{324 c^3} + \frac{7 b^3 \operatorname{Log}[i-c x^3]}{108 c^3} - \\
& \frac{b^3 (1-i c x^3) \operatorname{Log}[1-i c x^3]}{3 c^3} + \frac{b^3 \operatorname{Log}[1-i c x^3]^2}{72 c^3} - \frac{b^2 x^6 (2 i a - b \operatorname{Log}[1-i c x^3])}{24 c} - \\
& \frac{b^2 (1-i c x^3)^2 (2 i a - b \operatorname{Log}[1-i c x^3])}{48 c^3} - \frac{1}{72} i b x^9 (2 i a - b \operatorname{Log}[1-i c x^3])^2 - \\
& \frac{b (1-i c x^3)^2 (2 i a - b \operatorname{Log}[1-i c x^3])^2}{48 c^3} - \frac{i b^2 (1-i c x^3)^2 (2 a + i b \operatorname{Log}[1-i c x^3])}{16 c^3} + \\
& \frac{i b^2 (1-i c x^3)^3 (2 a + i b \operatorname{Log}[1-i c x^3])}{108 c^3} - \frac{b (1-i c x^3) (2 a + i b \operatorname{Log}[1-i c x^3])^2}{8 c^3} + \\
& \frac{b (1-i c x^3)^2 (2 a + i b \operatorname{Log}[1-i c x^3])^2}{16 c^3} - \frac{b (1-i c x^3)^3 (2 a + i b \operatorname{Log}[1-i c x^3])^2}{72 c^3} - \\
& \frac{i (1-i c x^3) (2 a + i b \operatorname{Log}[1-i c x^3])^3}{24 c^3} + \frac{i (1-i c x^3)^2 (2 a + i b \operatorname{Log}[1-i c x^3])^3}{24 c^3} - \\
& \frac{i (1-i c x^3)^3 (2 a + i b \operatorname{Log}[1-i c x^3])^3}{72 c^3} + \frac{1}{216} i b^2 (2 i a - b \operatorname{Log}[1-i c x^3]) \\
& \left( \frac{18 i (1-i c x^3)}{c^3} - \frac{9 i (1-i c x^3)^2}{c^3} + \frac{2 i (1-i c x^3)^3}{c^3} - \frac{6 i \operatorname{Log}[1-i c x^3]}{c^3} \right) + \\
& \frac{b^2 (2 i a - b \operatorname{Log}[1-i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1+i c x^3)\right]}{12 c^3} - \frac{b (2 i a - b \operatorname{Log}[1-i c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1+i c x^3)\right]}{24 c^3} + \\
& \frac{b (2 a + i b \operatorname{Log}[1-i c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1+i c x^3)\right]}{24 c^3} + \frac{b^3 x^6 \operatorname{Log}[1+i c x^3]}{18 c} - \\
& \frac{1}{108} i b^3 x^9 \operatorname{Log}[1+i c x^3] - \frac{11 b^3 (1+i c x^3) \operatorname{Log}[1+i c x^3]}{36 c^3} + \frac{b^3 (1+i c x^3)^2 \operatorname{Log}[1+i c x^3]}{12 c^3} - \\
& \frac{b^3 (1+i c x^3)^3 \operatorname{Log}[1+i c x^3]}{108 c^3} - \frac{b^3 \operatorname{Log}\left[\frac{1}{2} (1-i c x^3)\right] \operatorname{Log}[1+i c x^3]}{12 c^3} + \\
& \frac{b^2 x^6 (2 i a - b \operatorname{Log}[1-i c x^3]) \operatorname{Log}[1+i c x^3]}{12 c} + \frac{1}{24} i b x^9 (2 i a - b \operatorname{Log}[1-i c x^3])^2 \operatorname{Log}[1+i c x^3] -
\end{aligned}$$

$$\begin{aligned}
& \frac{b \left( 2a + i b \operatorname{Log} \left[ 1 - i c x^3 \right] \right)^2 \operatorname{Log} \left[ 1 + i c x^3 \right]}{24 c^3} - \frac{b^3 \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{72 c^3} + \frac{1}{72} i b^3 x^9 \operatorname{Log} \left[ 1 + i c x^3 \right]^2 + \\
& \frac{b^3 \left( 1 + i c x^3 \right) \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{8 c^3} - \frac{b^3 \left( 1 + i c x^3 \right)^2 \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{12 c^3} + \frac{b^3 \left( 1 + i c x^3 \right)^3 \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{72 c^3} - \\
& \frac{b^3 \operatorname{Log} \left[ \frac{1}{2} \left( 1 - i c x^3 \right) \right] \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{12 c^3} + \frac{1}{24} i b^2 x^9 \left( 2 i a - b \operatorname{Log} \left[ 1 - i c x^3 \right] \right) \operatorname{Log} \left[ 1 + i c x^3 \right]^2 - \\
& \frac{i b^2 \left( 2a + i b \operatorname{Log} \left[ 1 - i c x^3 \right] \right) \operatorname{Log} \left[ 1 + i c x^3 \right]^2}{24 c^3} - \frac{b^3 \left( 1 + i c x^3 \right) \operatorname{Log} \left[ 1 + i c x^3 \right]^3}{24 c^3} + \\
& \frac{b^3 \left( 1 + i c x^3 \right)^2 \operatorname{Log} \left[ 1 + i c x^3 \right]^3}{24 c^3} - \frac{b^3 \left( 1 + i c x^3 \right)^3 \operatorname{Log} \left[ 1 + i c x^3 \right]^3}{72 c^3} + \frac{b^3 \operatorname{Log} \left[ i + c x^3 \right]}{24 c^3} - \\
& \frac{b^3 \operatorname{PolyLog} \left[ 2, \frac{1}{2} \left( 1 - i c x^3 \right) \right]}{12 c^3} + \frac{b^2 \left( 2 i a - b \operatorname{Log} \left[ 1 - i c x^3 \right] \right) \operatorname{PolyLog} \left[ 2, \frac{1}{2} \left( 1 - i c x^3 \right) \right]}{12 c^3} + \\
& \frac{i b^2 \left( 2a + i b \operatorname{Log} \left[ 1 - i c x^3 \right] \right) \operatorname{PolyLog} \left[ 2, \frac{1}{2} \left( 1 - i c x^3 \right) \right]}{12 c^3} - \\
& \frac{b^3 \operatorname{PolyLog} \left[ 2, \frac{1}{2} \left( 1 + i c x^3 \right) \right]}{12 c^3} - \frac{b^3 \operatorname{Log} \left[ 1 + i c x^3 \right] \operatorname{PolyLog} \left[ 2, \frac{1}{2} \left( 1 + i c x^3 \right) \right]}{6 c^3} + \\
& \frac{b^3 \operatorname{PolyLog} \left[ 3, \frac{1}{2} \left( 1 - i c x^3 \right) \right]}{6 c^3} + \frac{b^3 \operatorname{PolyLog} \left[ 3, \frac{1}{2} \left( 1 + i c x^3 \right) \right]}{6 c^3}
\end{aligned}$$

Problem 122: Result valid but suboptimal antiderivative.

$$\int x^5 \left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right)^3 dx$$

Optimal (type 4, 147 leaves, 9 steps):

$$\begin{aligned}
& - \frac{i b \left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right)^2}{2 c^2} - \frac{b x^3 \left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right)^2}{2 c} + \frac{\left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right)^3}{6 c^2} + \\
& \frac{1}{6} x^6 \left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right)^3 - \frac{b^2 \left( a + b \operatorname{ArcTan} \left[ c x^3 \right] \right) \operatorname{Log} \left[ \frac{2}{1 + i c x^3} \right]}{c^2} - \frac{i b^3 \operatorname{PolyLog} \left[ 2, 1 - \frac{2}{1 + i c x^3} \right]}{2 c^2}
\end{aligned}$$

Result (type 4, 951 leaves, 155 steps):

$$\begin{aligned}
& \frac{i b^2 (1 - i c x^3)^2 (2 i a - b \operatorname{Log}[1 - i c x^3])}{32 c^2} + \frac{i b (1 - i c x^3)^2 (2 i a - b \operatorname{Log}[1 - i c x^3])^2}{32 c^2} + \\
& \frac{b^2 (1 - i c x^3)^2 (2 a + i b \operatorname{Log}[1 - i c x^3])}{32 c^2} - \frac{i b (1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{8 c^2} + \\
& \frac{i b (1 - i c x^3)^2 (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{32 c^2} + \frac{(1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^3}{24 c^2} - \\
& \frac{(1 - i c x^3)^2 (2 a + i b \operatorname{Log}[1 - i c x^3])^3}{48 c^2} - \frac{i b^2 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right]}{4 c^2} + \\
& \frac{i b (2 i a - b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right]}{16 c^2} + \\
& \frac{i b (2 a + i b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1 + i c x^3)\right]}{16 c^2} - \frac{i b^3 \operatorname{Log}\left[\frac{1}{2} (1 - i c x^3)\right] \operatorname{Log}[1 + i c x^3]}{4 c^2} + \\
& \frac{b^2 x^3 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]}{4 c} + \frac{1}{16} i b x^6 (2 i a - b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3] - \\
& \frac{i b (2 a + i b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3]}{16 c^2} - \frac{i b^3 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^2}{8 c^2} + \\
& \frac{1}{16} i b^2 x^6 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2 - \frac{b^2 (2 a + i b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2}{16 c^2} + \\
& \frac{i b^3 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^3}{24 c^2} - \frac{i b^3 (1 + i c x^3)^2 \operatorname{Log}[1 + i c x^3]^3}{48 c^2} + \\
& \frac{i b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right]}{4 c^2} - \frac{i b^2 (2 i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right]}{8 c^2} - \\
& \frac{b^2 (2 a + i b \operatorname{Log}[1 - i c x^3]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - i c x^3)\right]}{8 c^2} - \frac{i b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + i c x^3)\right]}{4 c^2}
\end{aligned}$$

**Problem 123: Result valid but suboptimal antiderivative.**

$$\int x^2 (a + b \operatorname{ArcTan}[c x^3])^3 dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\begin{aligned}
& \frac{i (a + b \operatorname{ArcTan}[c x^3])^3}{3 c} + \frac{1}{3} x^3 (a + b \operatorname{ArcTan}[c x^3])^3 + \frac{b (a + b \operatorname{ArcTan}[c x^3])^2 \operatorname{Log}\left[\frac{2}{1 + i c x^3}\right]}{c} + \\
& \frac{i b^2 (a + b \operatorname{ArcTan}[c x^3]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i c x^3}\right]}{c} + \frac{b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + i c x^3}\right]}{2 c}
\end{aligned}$$

Result (type 4, 545 leaves, 82 steps):

$$\begin{aligned}
& \frac{b(1 - i c x^3)(2i a - b \operatorname{Log}[1 - i c x^3])^2}{8c} + \frac{b(1 - i c x^3)(2a + i b \operatorname{Log}[1 - i c x^3])^2}{8c} + \\
& \frac{i(1 - i c x^3)(2a + i b \operatorname{Log}[1 - i c x^3])^3}{24c} + \frac{b(2i a - b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[\frac{1}{2}(1 + i c x^3)]}{4c} - \\
& \frac{b(2i a - b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3]}{8c} + \frac{1}{8} i b x^3 (2i a - b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3] + \\
& \frac{b^3 \operatorname{Log}[\frac{1}{2}(1 - i c x^3)] \operatorname{Log}[1 + i c x^3]^2}{4c} + \frac{b^2(2i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2}{8c} + \\
& \frac{\frac{1}{8} i b^2 x^3 (2i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2}{8} + \frac{b^3(1 + i c x^3) \operatorname{Log}[1 + i c x^3]^3}{24c} - \\
& \frac{b^2(2i a - b \operatorname{Log}[1 - i c x^3]) \operatorname{PolyLog}[2, \frac{1}{2}(1 - i c x^3)]}{2c} + \\
& \frac{b^3 \operatorname{Log}[1 + i c x^3] \operatorname{PolyLog}[2, \frac{1}{2}(1 + i c x^3)]}{2c} - \\
& \frac{b^3 \operatorname{PolyLog}[3, \frac{1}{2}(1 - i c x^3)]}{2c} - \frac{b^3 \operatorname{PolyLog}[3, \frac{1}{2}(1 + i c x^3)]}{2c}
\end{aligned}$$

### Problem 125: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTan}[c x^3])^3}{x^4} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\begin{aligned}
& -\frac{1}{3} i c (a + b \operatorname{ArcTan}[c x^3])^3 - \frac{(a + b \operatorname{ArcTan}[c x^3])^3}{3 x^3} + b c (a + b \operatorname{ArcTan}[c x^3])^2 \operatorname{Log}\left[2 - \frac{2}{1 - i c x^3}\right] - \\
& i b^2 c (a + b \operatorname{ArcTan}[c x^3]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - i c x^3}\right] + \frac{1}{2} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - i c x^3}\right]
\end{aligned}$$

Result (type 8, 347 leaves, 16 steps):

$$\begin{aligned}
& \frac{1}{8} b c \operatorname{Log}[i c x^3] (2a + i b \operatorname{Log}[1 - i c x^3])^2 - \frac{(1 - i c x^3)(2a + i b \operatorname{Log}[1 - i c x^3])^3}{24 x^3} - \\
& \frac{1}{8} b^3 c \operatorname{Log}[-i c x^3] \operatorname{Log}[1 + i c x^3]^2 - \frac{i b^3 (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^3}{24 x^3} + \\
& \frac{1}{4} i b^2 c (2a + i b \operatorname{Log}[1 - i c x^3]) \operatorname{PolyLog}[2, 1 - i c x^3] - \\
& \frac{1}{4} b^3 c \operatorname{Log}[1 + i c x^3] \operatorname{PolyLog}[2, 1 + i c x^3] + \frac{1}{4} b^3 c \operatorname{PolyLog}[3, 1 - i c x^3] + \\
& \frac{1}{4} b^3 c \operatorname{PolyLog}[3, 1 + i c x^3] + \frac{3}{8} i b \operatorname{Unintegrateable}\left[\frac{(-2i a + b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3]}{x^4}, x\right] - \\
& \frac{3}{8} i b^2 \operatorname{Unintegrateable}\left[\frac{(-2i a + b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2}{x^4}, x\right]
\end{aligned}$$



### Problem 126: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTan}[c x^3])^3}{x^7} dx$$

Optimal (type 4, 146 leaves, 8 steps):

$$\begin{aligned} & -\frac{1}{2} i b c^2 (a + b \operatorname{ArcTan}[c x^3])^2 - \frac{b c (a + b \operatorname{ArcTan}[c x^3])^2}{2 x^3} - \\ & \frac{1}{6} c^2 (a + b \operatorname{ArcTan}[c x^3])^3 - \frac{(a + b \operatorname{ArcTan}[c x^3])^3}{6 x^6} + \\ & b^2 c^2 (a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 - i c x^3}\right] - \frac{1}{2} i b^3 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - i c x^3}\right] \end{aligned}$$

Result (type 8, 533 leaves, 29 steps):

$$\begin{aligned} & \frac{3}{4} a b^2 c^2 \operatorname{Log}[x] - \frac{b c (1 - i c x^3) (2 a + i b \operatorname{Log}[1 - i c x^3])^2}{16 x^3} + \\ & \frac{1}{16} i b c^2 \operatorname{Log}[i c x^3] (2 a + i b \operatorname{Log}[1 - i c x^3])^2 - \frac{1}{48} c^2 (2 a + i b \operatorname{Log}[1 - i c x^3])^3 - \\ & \frac{(2 a + i b \operatorname{Log}[1 - i c x^3])^3}{48 x^6} + \frac{b^3 c (1 + i c x^3) \operatorname{Log}[1 + i c x^3]^2}{16 x^3} + \\ & \frac{1}{16} i b^3 c^2 \operatorname{Log}[-i c x^3] \operatorname{Log}[1 + i c x^3]^2 - \frac{1}{48} i b^3 c^2 \operatorname{Log}[1 + i c x^3]^3 - \\ & \frac{i b^3 \operatorname{Log}[1 + i c x^3]^3}{48 x^6} + \frac{1}{8} i b^3 c^2 \operatorname{PolyLog}[2, -i c x^3] - \frac{1}{8} i b^3 c^2 \operatorname{PolyLog}[2, i c x^3] - \\ & \frac{1}{8} b^2 c^2 (2 a + i b \operatorname{Log}[1 - i c x^3]) \operatorname{PolyLog}[2, 1 - i c x^3] + \\ & \frac{1}{8} i b^3 c^2 \operatorname{Log}[1 + i c x^3] \operatorname{PolyLog}[2, 1 + i c x^3] + \\ & \frac{1}{8} i b^3 c^2 \operatorname{PolyLog}[3, 1 - i c x^3] - \frac{1}{8} i b^3 c^2 \operatorname{PolyLog}[3, 1 + i c x^3] + \\ & \frac{3}{8} i b \operatorname{Unintegrable}\left[\frac{(-2 i a + b \operatorname{Log}[1 - i c x^3])^2 \operatorname{Log}[1 + i c x^3]}{x^7}, x\right] - \\ & \frac{3}{8} i b^2 \operatorname{Unintegrable}\left[\frac{(-2 i a + b \operatorname{Log}[1 - i c x^3]) \operatorname{Log}[1 + i c x^3]^2}{x^7}, x\right] \end{aligned}$$

### Problem 129: Result optimal but 1 more steps used.

$$\int (d x)^m (a + b \operatorname{ArcTan}[c x^3]) dx$$

Optimal (type 5, 75 leaves, 2 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTan}[c x^3])}{d (1+m)} - \frac{3 b c (d x)^{4+m} \operatorname{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, -c^2 x^6\right]}{d^4 (1+m) (4+m)}$$

Result (type 5, 75 leaves, 3 steps):

$$\frac{(dx)^{1+m} (a + b \operatorname{ArcTan}[cx^3])}{d(1+m)} - \frac{3bc(dx)^{4+m} \operatorname{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, -c^2x^6\right]}{d^4(1+m)(4+m)}$$

**Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^3 \left( a + b \operatorname{ArcTan}\left[\frac{c}{x}\right] \right)^2 dx$$

Optimal (type 3, 122 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{12} b^2 c^2 x^2 - \frac{1}{2} b c^3 x \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right) + \frac{1}{6} b c x^3 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right) - \\ & \frac{1}{4} c^4 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^2 + \frac{1}{4} x^4 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^2 - \frac{1}{3} b^2 c^4 \operatorname{Log}\left[1 + \frac{c^2}{x^2}\right] - \frac{2}{3} b^2 c^4 \operatorname{Log}[x] \end{aligned}$$

Result (type 4, 862 leaves, 88 steps):

$$\begin{aligned} & -\frac{1}{4} a b c^3 x - \frac{1}{8} i a b c^2 x^2 + \frac{1}{12} b^2 c^2 x^2 + \frac{1}{12} a b c x^3 - \frac{11}{48} b^2 c^4 \operatorname{Log}\left[i - \frac{c}{x}\right] - \\ & \frac{1}{8} i b^2 c^3 x \operatorname{Log}\left[1 - \frac{i c}{x}\right] + \frac{1}{16} b^2 c^2 x^2 \operatorname{Log}\left[1 - \frac{i c}{x}\right] + \frac{1}{24} i b^2 c x^3 \operatorname{Log}\left[1 - \frac{i c}{x}\right] - \\ & \frac{1}{8} b c^3 \left(1 - \frac{i c}{x}\right) x \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) + \frac{1}{16} i b c^2 x^2 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) + \\ & \frac{1}{24} b c x^3 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) - \frac{1}{16} c^4 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \\ & \frac{1}{16} x^4 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \frac{1}{4} i b^2 c^3 x \operatorname{Log}\left[1 + \frac{i c}{x}\right] - \frac{1}{12} i b^2 c x^3 \operatorname{Log}\left[1 + \frac{i c}{x}\right] - \\ & \frac{1}{4} i a b x^4 \operatorname{Log}\left[1 + \frac{i c}{x}\right] + \frac{1}{8} b^2 x^4 \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[1 + \frac{i c}{x}\right] + \frac{1}{16} b^2 c^4 \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 - \\ & \frac{1}{16} b^2 x^4 \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 - \frac{5}{48} b^2 c^4 \operatorname{Log}\left[i + \frac{c}{x}\right] + \frac{1}{4} i a b c^4 \operatorname{Log}[c - i x] - \frac{5}{48} b^2 c^4 \operatorname{Log}[c - i x] - \\ & \frac{1}{8} b^2 c^4 \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}[c - i x] - \frac{5}{48} b^2 c^4 \operatorname{Log}[c + i x] - \frac{1}{8} b^2 c^4 \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{Log}[c + i x] + \\ & \frac{1}{8} b^2 c^4 \operatorname{Log}\left[\frac{c - i x}{2 c}\right] \operatorname{Log}[c + i x] + \frac{1}{8} b^2 c^4 \operatorname{Log}[c - i x] \operatorname{Log}\left[\frac{c + i x}{2 c}\right] - \frac{1}{4} i a b c^4 \operatorname{Log}[x] - \\ & \frac{11}{24} b^2 c^4 \operatorname{Log}[x] - \frac{1}{8} b^2 c^4 \operatorname{Log}[c + i x] \operatorname{Log}\left[-\frac{i x}{c}\right] - \frac{1}{8} b^2 c^4 \operatorname{Log}[c - i x] \operatorname{Log}\left[\frac{i x}{c}\right] + \\ & \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{c - i x}{2 c}\right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{c + i x}{2 c}\right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, -\frac{i c}{x}\right] + \\ & \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, \frac{i c}{x}\right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, 1 - \frac{i x}{c}\right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog}\left[2, 1 + \frac{i x}{c}\right] \end{aligned}$$

### Problem 141: Result valid but suboptimal antiderivative.

$$\int x^2 \left( a + b \operatorname{ArcTan}\left[\frac{c}{x}\right] \right)^2 dx$$

Optimal (type 4, 152 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{3} b^2 c^2 x + \frac{1}{3} b^2 c^3 \operatorname{ArcCot}\left[\frac{x}{c}\right] + \frac{1}{3} b c x^2 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right) - \\ & \frac{1}{3} i c^3 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^2 + \frac{1}{3} x^3 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^2 + \\ & \frac{2}{3} b c^3 \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right) \operatorname{Log}\left[2 - \frac{2}{1 - \frac{ic}{x}}\right] - \frac{1}{3} i b^2 c^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{ic}{x}}\right] \end{aligned}$$

Result (type 4, 787 leaves, 73 steps):

$$\begin{aligned} & -\frac{1}{3} i a b c^2 x + \frac{1}{3} b^2 c^2 x + \frac{1}{6} a b c x^2 - \frac{1}{4} i b^2 c^3 \operatorname{Log}\left[i - \frac{c}{x}\right] + \frac{1}{6} b^2 c^2 x \operatorname{Log}\left[1 - \frac{ic}{x}\right] + \\ & \frac{1}{12} i b^2 c x^2 \operatorname{Log}\left[1 - \frac{ic}{x}\right] + \frac{1}{6} i b c^2 \left(1 - \frac{ic}{x}\right) x \left(2 a + i b \operatorname{Log}\left[1 - \frac{ic}{x}\right]\right) + \\ & \frac{1}{12} b c x^2 \left(2 a + i b \operatorname{Log}\left[1 - \frac{ic}{x}\right]\right) + \frac{1}{12} i c^3 \left(2 a + i b \operatorname{Log}\left[1 - \frac{ic}{x}\right]\right)^2 + \\ & \frac{1}{12} x^3 \left(2 a + i b \operatorname{Log}\left[1 - \frac{ic}{x}\right]\right)^2 - \frac{1}{6} i b^2 c x^2 \operatorname{Log}\left[1 + \frac{ic}{x}\right] - \frac{1}{3} i a b x^3 \operatorname{Log}\left[1 + \frac{ic}{x}\right] + \\ & \frac{1}{6} b^2 x^3 \operatorname{Log}\left[1 - \frac{ic}{x}\right] \operatorname{Log}\left[1 + \frac{ic}{x}\right] + \frac{1}{12} i b^2 c^3 \operatorname{Log}\left[1 + \frac{ic}{x}\right]^2 - \frac{1}{12} b^2 x^3 \operatorname{Log}\left[1 + \frac{ic}{x}\right]^2 + \\ & \frac{1}{12} i b^2 c^3 \operatorname{Log}\left[i + \frac{c}{x}\right] - \frac{1}{3} a b c^3 \operatorname{Log}[c - ix] + \frac{1}{12} i b^2 c^3 \operatorname{Log}[c - ix] - \\ & \frac{1}{6} i b^2 c^3 \operatorname{Log}\left[1 - \frac{ic}{x}\right] \operatorname{Log}[c - ix] - \frac{1}{12} i b^2 c^3 \operatorname{Log}[c + ix] + \frac{1}{6} i b^2 c^3 \operatorname{Log}\left[1 + \frac{ic}{x}\right] \operatorname{Log}[c + ix] - \\ & \frac{1}{6} i b^2 c^3 \operatorname{Log}\left[\frac{c - ix}{2c}\right] \operatorname{Log}[c + ix] + \frac{1}{6} i b^2 c^3 \operatorname{Log}[c - ix] \operatorname{Log}\left[\frac{c + ix}{2c}\right] - \\ & \frac{1}{3} a b c^3 \operatorname{Log}[x] + \frac{1}{6} i b^2 c^3 \operatorname{Log}[c + ix] \operatorname{Log}\left[-\frac{ix}{c}\right] - \frac{1}{6} i b^2 c^3 \operatorname{Log}[c - ix] \operatorname{Log}\left[\frac{ix}{c}\right] + \\ & \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c - ix}{2c}\right] - \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c + ix}{2c}\right] + \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, -\frac{ic}{x}\right] - \\ & \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, \frac{ic}{x}\right] - \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, 1 - \frac{ix}{c}\right] + \frac{1}{6} i b^2 c^3 \operatorname{PolyLog}\left[2, 1 + \frac{ix}{c}\right] \end{aligned}$$

### Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \left( a + b \operatorname{ArcTan}\left[\frac{c}{x}\right] \right)^2 dx$$

Optimal (type 3, 82 leaves, 9 steps):

$$b c x \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right) + \frac{1}{2} c^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 +$$

$$\frac{1}{2} x^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 + \frac{1}{2} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{c^2}{x^2} \right] + b^2 c^2 \operatorname{Log} [x]$$

Result (type 4, 663 leaves, 58 steps):

$$\begin{aligned} & \frac{1}{2} a b c x + \frac{1}{4} b^2 c^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] + \frac{1}{4} i b^2 c x \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] + \frac{1}{4} b c \left( 1 - \frac{i c}{x} \right) x \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right) + \\ & \frac{1}{8} c^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^2 + \frac{1}{8} x^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^2 - \frac{1}{2} i b^2 c x \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] - \\ & \frac{1}{2} i a b x^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] + \frac{1}{4} b^2 x^2 \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] - \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 - \\ & \frac{1}{8} b^2 x^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 - \frac{1}{2} i a b c^2 \operatorname{Log} [c - i x] + \frac{1}{4} b^2 c^2 \operatorname{Log} [c - i x] + \\ & \frac{1}{4} b^2 c^2 \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \operatorname{Log} [c - i x] + \frac{1}{4} b^2 c^2 \operatorname{Log} [c + i x] + \frac{1}{4} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] \operatorname{Log} [c + i x] - \\ & \frac{1}{4} b^2 c^2 \operatorname{Log} \left[ \frac{c - i x}{2 c} \right] \operatorname{Log} [c + i x] - \frac{1}{4} b^2 c^2 \operatorname{Log} [c - i x] \operatorname{Log} \left[ \frac{c + i x}{2 c} \right] + \frac{1}{2} i a b c^2 \operatorname{Log} [x] + \\ & \frac{1}{2} b^2 c^2 \operatorname{Log} [x] + \frac{1}{4} b^2 c^2 \operatorname{Log} [c + i x] \operatorname{Log} \left[ -\frac{i x}{c} \right] + \frac{1}{4} b^2 c^2 \operatorname{Log} [c - i x] \operatorname{Log} \left[ \frac{i x}{c} \right] - \\ & \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c - i x}{2 c} \right] - \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c + i x}{2 c} \right] - \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, -\frac{i c}{x} \right] - \\ & \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{i c}{x} \right] + \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{i x}{c} \right] + \frac{1}{4} b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 + \frac{i x}{c} \right] \end{aligned}$$

**Problem 143: Result valid but suboptimal antiderivative.**

$$\int \left( a + b \operatorname{ArcTan} \left[ \frac{c}{x} \right] \right)^2 dx$$

Optimal (type 4, 83 leaves, 6 steps):

$$i c \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 + x \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 -$$

$$2 b c \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ \frac{2 c}{c + i x} \right] + i b^2 c \operatorname{PolyLog} \left[ 2, 1 - \frac{2 c}{c + i x} \right]$$

Result (type 4, 478 leaves, 31 steps):

$$\begin{aligned}
& a^2 x + i a b x \operatorname{Log}\left[1 - \frac{i c}{x}\right] + \frac{1}{4} b^2 (i c - x) \operatorname{Log}\left[1 - \frac{i c}{x}\right]^2 - \\
& i a b x \operatorname{Log}\left[1 + \frac{i c}{x}\right] + \frac{1}{2} b^2 x \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[1 + \frac{i c}{x}\right] - \frac{1}{4} b^2 (i c + x) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 - \\
& \frac{1}{2} i b^2 c \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{Log}[-c - i x] + a b c \operatorname{Log}[c - i x] + \frac{1}{2} i b^2 c \operatorname{Log}[-c - i x] \operatorname{Log}\left[\frac{c - i x}{2 c}\right] + \\
& \frac{1}{2} i b^2 c \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}[-c + i x] + a b c \operatorname{Log}[c + i x] - \frac{1}{2} i b^2 c \operatorname{Log}[-c + i x] \operatorname{Log}\left[\frac{c + i x}{2 c}\right] - \\
& \frac{1}{2} i b^2 c \operatorname{Log}[-c - i x] \operatorname{Log}\left[-\frac{i x}{c}\right] + \frac{1}{2} i b^2 c \operatorname{Log}[-c + i x] \operatorname{Log}\left[\frac{i x}{c}\right] - \\
& \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, \frac{c - i x}{2 c}\right] + \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, \frac{c + i x}{2 c}\right] - \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, -\frac{i c}{x}\right] + \\
& \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, \frac{i c}{x}\right] + \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{i x}{c}\right] - \frac{1}{2} i b^2 c \operatorname{PolyLog}\left[2, 1 + \frac{i x}{c}\right]
\end{aligned}$$

Problem 145: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \operatorname{ArcTan}\left[\frac{c}{x}\right]\right)^2}{x^2} dx$$

Optimal (type 4, 96 leaves, 6 steps):

$$\begin{aligned}
& -\frac{i \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{c} - \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{x} - \\
& \frac{2 b \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right) \operatorname{Log}\left[\frac{2}{1 + \frac{i c}{x}}\right]}{c} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{i c}{x}}\right]}{c}
\end{aligned}$$

Result (type 4, 259 leaves, 28 steps):

$$\begin{aligned}
& -\frac{i \left(1 - \frac{i c}{x}\right) \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2}{4 c} + \frac{b \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{2 x} - \\
& \frac{i b^2 \left(1 + \frac{i c}{x}\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2}{4 c} - \frac{i b^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{Log}\left[-\frac{i c - x}{2 x}\right]}{2 c} - \\
& \frac{i b \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) \operatorname{Log}\left[\frac{i c + x}{2 x}\right]}{2 c} + \frac{i b^2 \operatorname{PolyLog}\left[2, -\frac{i c - x}{2 x}\right]}{2 c} - \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{i c + x}{2 x}\right]}{2 c}
\end{aligned}$$

Problem 146: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b \operatorname{ArcTan}\left[\frac{c}{x}\right]\right)^2}{x^3} dx$$

Optimal (type 3, 84 leaves, 7 steps):

$$\frac{a b}{c x} + \frac{b^2 \operatorname{ArcCot}\left[\frac{x}{c}\right]}{c x} - \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{2 c^2} - \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{2 x^2} - \frac{b^2 \operatorname{Log}\left[1 + \frac{c^2}{x^2}\right]}{2 c^2}$$

Result (type 4, 836 leaves, 66 steps):

$$\begin{aligned} & -\frac{b^2 \left(1 - \frac{i c}{x}\right)^2}{16 c^2} - \frac{b^2 \left(1 + \frac{i c}{x}\right)^2}{16 c^2} - \frac{i a b}{4 x^2} - \frac{b^2}{8 x^2} + \frac{3 a b}{2 c x} + \frac{i a b \operatorname{Log}\left[1 - \frac{c}{x}\right]}{2 c^2} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x}\right]}{8 c^2} - \\ & \frac{3 b^2 \left(1 - \frac{i c}{x}\right) \operatorname{Log}\left[1 - \frac{i c}{x}\right]}{4 c^2} + \frac{b^2 \operatorname{Log}\left[1 - \frac{i c}{x}\right]}{8 x^2} - \frac{i b \left(1 - \frac{i c}{x}\right)^2 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)}{8 c^2} - \\ & \frac{\left(1 - \frac{i c}{x}\right) \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2}{4 c^2} + \frac{\left(1 - \frac{i c}{x}\right)^2 \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2}{8 c^2} - \frac{3 b^2 \left(1 + \frac{i c}{x}\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{4 c^2} + \\ & \frac{b^2 \left(1 + \frac{i c}{x}\right)^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{8 c^2} + \frac{i a b \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{2 x^2} + \frac{b^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{8 x^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{4 x^2} + \\ & \frac{b^2 \left(1 + \frac{i c}{x}\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2}{4 c^2} - \frac{b^2 \left(1 + \frac{i c}{x}\right)^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2}{8 c^2} + \frac{b^2 \operatorname{Log}\left[1 + \frac{c}{x}\right]}{8 c^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[c - i x\right]}{4 c^2} - \\ & \frac{b^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{Log}\left[c + i x\right]}{4 c^2} + \frac{b^2 \operatorname{Log}\left[\frac{c - i x}{2 c}\right] \operatorname{Log}\left[c + i x\right]}{4 c^2} + \frac{b^2 \operatorname{Log}\left[c - i x\right] \operatorname{Log}\left[\frac{c + i x}{2 c}\right]}{4 c^2} - \\ & \frac{b^2 \operatorname{Log}\left[c + i x\right] \operatorname{Log}\left[-\frac{i x}{c}\right]}{4 c^2} - \frac{b^2 \operatorname{Log}\left[c - i x\right] \operatorname{Log}\left[\frac{i x}{c}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c - i x}{2 c}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c + i x}{2 c}\right]}{4 c^2} + \\ & \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{i c}{x}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{i c}{x}\right]}{4 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{i x}{c}\right]}{4 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 + \frac{i x}{c}\right]}{4 c^2} \end{aligned}$$

Problem 147: Unable to integrate problem.

$$\int x^3 \left(a + b \operatorname{ArcTan}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 214 leaves, 17 steps):

$$\begin{aligned} & \frac{1}{4} b^3 c^3 x + \frac{1}{4} b^3 c^4 \operatorname{ArcCot}\left[\frac{x}{c}\right] + \frac{1}{4} b^2 c^2 x^2 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right) - \\ & i b c^4 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2 - \frac{3}{4} b c^3 x \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2 + \\ & \frac{1}{4} b c x^3 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2 - \frac{1}{4} c^4 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3 + \frac{1}{4} x^4 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3 + \\ & 2 b^2 c^4 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 - \frac{i c}{x}}\right] - i b^3 c^4 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{i c}{x}}\right] \end{aligned}$$

Result (type 8, 1568 leaves, 139 steps):

$$\begin{aligned}
& -\frac{3}{8} a^2 b c^3 x - \frac{5}{16} i a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} i a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \\
& \frac{1}{8} a^2 b c x^3 + \frac{3}{8} i b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{i c}{x}\right]^2 \text{Log}\left[1 + \frac{i c}{x}\right], x\right] - \\
& \frac{3}{8} i b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}\left[1 + \frac{i c}{x}\right]^2, x\right] - \frac{11}{16} a b^2 c^4 \text{Log}\left[i - \frac{c}{x}\right] - \\
& \frac{1}{32} i b^3 c^4 \text{Log}\left[i - \frac{c}{x}\right] - \frac{3}{8} i a b^2 c^3 x \text{Log}\left[1 - \frac{i c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1 - \frac{i c}{x}\right] + \\
& \frac{1}{8} i a b^2 c x^3 \text{Log}\left[1 - \frac{i c}{x}\right] + \frac{5}{32} i b^2 c^3 \left(1 - \frac{i c}{x}\right) x \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right) + \\
& \frac{1}{32} b^2 c^2 x^2 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right) + \frac{5}{64} i b c^4 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 - \\
& \frac{3}{32} b c^3 \left(1 - \frac{i c}{x}\right) x \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \frac{3}{64} i b c^2 x^2 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \\
& \frac{1}{32} b c x^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 - \frac{1}{32} c^4 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^3 + \\
& \frac{1}{32} x^4 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^3 + \frac{3}{4} i a b^2 c^3 x \text{Log}\left[1 + \frac{i c}{x}\right] - \frac{5}{32} b^3 c^3 \left(1 + \frac{i c}{x}\right) x \text{Log}\left[1 + \frac{i c}{x}\right] - \\
& \frac{1}{32} i b^3 c^2 x^2 \text{Log}\left[1 + \frac{i c}{x}\right] - \frac{1}{4} i a b^2 c x^3 \text{Log}\left[1 + \frac{i c}{x}\right] - \frac{3}{8} i a^2 b x^4 \text{Log}\left[1 + \frac{i c}{x}\right] + \\
& \frac{3}{8} a b^2 x^4 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}\left[1 + \frac{i c}{x}\right] + \frac{3}{16} a b^2 c^4 \text{Log}\left[1 + \frac{i c}{x}\right]^2 + \frac{5}{64} i b^3 c^4 \text{Log}\left[1 + \frac{i c}{x}\right]^2 + \\
& \frac{3}{32} b^3 c^3 \left(1 + \frac{i c}{x}\right) x \text{Log}\left[1 + \frac{i c}{x}\right]^2 + \frac{3}{64} i b^3 c^2 x^2 \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \frac{1}{32} b^3 c x^3 \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \\
& \frac{3}{16} a b^2 x^4 \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \frac{1}{32} i b^3 c^4 \text{Log}\left[1 + \frac{i c}{x}\right]^3 + \frac{1}{32} i b^3 x^4 \text{Log}\left[1 + \frac{i c}{x}\right]^3 + \\
& \frac{1}{32} i b^3 c^4 \text{Log}\left[i + \frac{c}{x}\right] + \frac{3}{8} i a^2 b c^4 \text{Log}[c - i x] - \frac{5}{16} a b^2 c^4 \text{Log}[c - i x] - \\
& \frac{3}{8} a b^2 c^4 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}[c - i x] - \frac{5}{16} a b^2 c^4 \text{Log}[c + i x] - \frac{3}{8} a b^2 c^4 \text{Log}\left[1 + \frac{i c}{x}\right] \text{Log}[c + i x] + \\
& \frac{3}{8} a b^2 c^4 \text{Log}\left[\frac{c - i x}{2 c}\right] \text{Log}[c + i x] + \frac{3}{8} a b^2 c^4 \text{Log}[c - i x] \text{Log}\left[\frac{c + i x}{2 c}\right] + \\
& \frac{3}{32} i b^3 c^4 \text{Log}\left[1 + \frac{i c}{x}\right]^2 \text{Log}\left[-\frac{i c}{x}\right] + \frac{3}{32} i b c^4 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 \text{Log}\left[\frac{i c}{x}\right] - \\
& \frac{11}{8} a b^2 c^4 \text{Log}[x] - \frac{3}{8} a b^2 c^4 \text{Log}[c + i x] \text{Log}\left[-\frac{i x}{c}\right] - \frac{3}{8} a b^2 c^4 \text{Log}[c - i x] \text{Log}\left[\frac{i x}{c}\right] - \\
& \frac{3}{16} b^2 c^4 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right) \text{PolyLog}\left[2, 1 - \frac{i c}{x}\right] + \frac{3}{16} i b^3 c^4 \text{Log}\left[1 + \frac{i c}{x}\right] \text{PolyLog}\left[2, 1 + \frac{i c}{x}\right] + \\
& \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c - i x}{2 c}\right] + \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c + i x}{2 c}\right] + \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, -\frac{i c}{x}\right] + \\
& \frac{11}{32} i b^3 c^4 \text{PolyLog}\left[2, -\frac{i c}{x}\right] - \frac{11}{32} i b^3 c^4 \text{PolyLog}\left[2, \frac{i c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 - \frac{i x}{c}\right] - \\
& \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 + \frac{i x}{c}\right] + \frac{3}{16} i b^3 c^4 \text{PolyLog}\left[3, 1 - \frac{i c}{x}\right] - \frac{3}{16} i b^3 c^4 \text{PolyLog}\left[3, 1 + \frac{i c}{x}\right]
\end{aligned}$$

### Problem 148: Unable to integrate problem.

$$\int x^2 \left( a + b \operatorname{ArcTan} \left[ \frac{c}{x} \right] \right)^3 dx$$

Optimal (type 4, 229 leaves, 15 steps):

$$\begin{aligned} & b^2 c^2 x \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right) + \frac{1}{2} b c^3 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 + \\ & \frac{1}{2} b c x^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 - \frac{1}{3} i c^3 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^3 + \frac{1}{3} x^3 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^3 + \\ & b c^3 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 \operatorname{Log} \left[ 2 - \frac{2}{1 - \frac{i c}{x}} \right] + \frac{1}{2} b^3 c^3 \operatorname{Log} \left[ 1 + \frac{c^2}{x^2} \right] + b^3 c^3 \operatorname{Log} [x] - \\ & i b^2 c^3 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right) \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 - \frac{i c}{x}} \right] + \frac{1}{2} b^3 c^3 \operatorname{PolyLog} \left[ 3, -1 + \frac{2}{1 - \frac{i c}{x}} \right] \end{aligned}$$

Result (type 8, 1323 leaves, 103 steps):



$$\begin{aligned}
& -\frac{1}{2} i a^2 b c^2 x + \frac{3}{4} a b^2 c^2 x + \frac{1}{4} a^2 b c x^2 + \frac{3}{8} i b^3 \text{CannotIntegrate}\left[x^2 \text{Log}\left[1 - \frac{i c}{x}\right]^2 \text{Log}\left[1 + \frac{i c}{x}\right], x\right] - \\
& \frac{3}{8} i b^3 \text{CannotIntegrate}\left[x^2 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}\left[1 + \frac{i c}{x}\right]^2, x\right] - \frac{3}{4} i a b^2 c^3 \text{Log}\left[i - \frac{c}{x}\right] + \\
& \frac{1}{2} a b^2 c^2 x \text{Log}\left[1 - \frac{i c}{x}\right] + \frac{1}{4} i a b^2 c x^2 \text{Log}\left[1 - \frac{i c}{x}\right] + \frac{1}{8} b^2 c^2 \left(1 - \frac{i c}{x}\right) x \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right) + \\
& \frac{1}{16} b c^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \frac{1}{8} i b c^2 \left(1 - \frac{i c}{x}\right) x \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \\
& \frac{1}{16} b c x^2 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 + \frac{1}{24} i c^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^3 + \\
& \frac{1}{24} x^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^3 - \frac{1}{8} i b^3 c^2 \left(1 + \frac{i c}{x}\right) x \text{Log}\left[1 + \frac{i c}{x}\right] - \frac{1}{2} i a b^2 c x^2 \text{Log}\left[1 + \frac{i c}{x}\right] - \\
& \frac{1}{2} i a^2 b x^3 \text{Log}\left[1 + \frac{i c}{x}\right] + \frac{1}{2} a b^2 x^3 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}\left[1 + \frac{i c}{x}\right] + \frac{1}{4} i a b^2 c^3 \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \\
& \frac{1}{16} b^3 c^3 \text{Log}\left[1 + \frac{i c}{x}\right]^2 + \frac{1}{8} i b^3 c^2 \left(1 + \frac{i c}{x}\right) x \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \frac{1}{16} b^3 c x^2 \text{Log}\left[1 + \frac{i c}{x}\right]^2 - \\
& \frac{1}{4} a b^2 x^3 \text{Log}\left[1 + \frac{i c}{x}\right]^2 + \frac{1}{24} b^3 c^3 \text{Log}\left[1 + \frac{i c}{x}\right]^3 + \frac{1}{24} i b^3 x^3 \text{Log}\left[1 + \frac{i c}{x}\right]^3 - \\
& \frac{1}{2} a^2 b c^3 \text{Log}[c - i x] + \frac{1}{4} i a b^2 c^3 \text{Log}[c - i x] - \frac{1}{2} i a b^2 c^3 \text{Log}\left[1 - \frac{i c}{x}\right] \text{Log}[c - i x] - \\
& \frac{1}{4} i a b^2 c^3 \text{Log}[c + i x] + \frac{1}{2} i a b^2 c^3 \text{Log}\left[1 + \frac{i c}{x}\right] \text{Log}[c + i x] - \\
& \frac{1}{2} i a b^2 c^3 \text{Log}\left[\frac{c - i x}{2 c}\right] \text{Log}[c + i x] + \frac{1}{2} i a b^2 c^3 \text{Log}[c - i x] \text{Log}\left[\frac{c + i x}{2 c}\right] - \\
& \frac{1}{8} b^3 c^3 \text{Log}\left[1 + \frac{i c}{x}\right]^2 \text{Log}\left[-\frac{i c}{x}\right] + \frac{1}{8} b c^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right)^2 \text{Log}\left[\frac{i c}{x}\right] + \\
& \frac{1}{4} b^3 c^3 \text{Log}[x] + \frac{1}{2} i a b^2 c^3 \text{Log}[c + i x] \text{Log}\left[-\frac{i x}{c}\right] - \frac{1}{2} i a b^2 c^3 \text{Log}[c - i x] \text{Log}\left[\frac{i x}{c}\right] + \\
& \frac{1}{4} i b^2 c^3 \left(2 a + i b \text{Log}\left[1 - \frac{i c}{x}\right]\right) \text{PolyLog}\left[2, 1 - \frac{i c}{x}\right] - \frac{1}{4} b^3 c^3 \text{Log}\left[1 + \frac{i c}{x}\right] \text{PolyLog}\left[2, 1 + \frac{i c}{x}\right] + \\
& \frac{1}{2} i a b^2 c^3 \text{PolyLog}\left[2, \frac{c - i x}{2 c}\right] - \frac{1}{2} i a b^2 c^3 \text{PolyLog}\left[2, \frac{c + i x}{2 c}\right] + \frac{1}{2} i a b^2 c^3 \text{PolyLog}\left[2, -\frac{i c}{x}\right] - \\
& \frac{3}{8} b^3 c^3 \text{PolyLog}\left[2, -\frac{i c}{x}\right] - \frac{3}{8} b^3 c^3 \text{PolyLog}\left[2, \frac{i c}{x}\right] - \frac{1}{2} i a b^2 c^3 \text{PolyLog}\left[2, 1 - \frac{i x}{c}\right] + \\
& \frac{1}{2} i a b^2 c^3 \text{PolyLog}\left[2, 1 + \frac{i x}{c}\right] + \frac{1}{4} b^3 c^3 \text{PolyLog}\left[3, 1 - \frac{i c}{x}\right] + \frac{1}{4} b^3 c^3 \text{PolyLog}\left[3, 1 + \frac{i c}{x}\right]
\end{aligned}$$

**Problem 149: Unable to integrate problem.**

$$\int x \left( a + b \text{ArcTan}\left[\frac{c}{x}\right] \right)^3 dx$$

Optimal (type 4, 145 leaves, 8 steps):

$$\begin{aligned} & \frac{3}{2} i b c^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 + \frac{3}{2} b c x \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^2 + \\ & \frac{1}{2} c^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^3 + \frac{1}{2} x^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right)^3 - \\ & 3 b^2 c^2 \left( a + b \operatorname{ArcCot} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ 2 - \frac{2}{1 - \frac{i c}{x}} \right] + \frac{3}{2} i b^3 c^2 \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 - \frac{i c}{x}} \right] \end{aligned}$$

Result (type 8, 1058 leaves, 75 steps):

$$\begin{aligned} & \frac{3}{4} a^2 b c x + \frac{3}{8} i b^3 \operatorname{CannotIntegrate} \left[ x \operatorname{Log} \left[ 1 - \frac{i c}{x} \right]^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right], x \right] - \\ & \frac{3}{8} i b^3 \operatorname{CannotIntegrate} \left[ x \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2, x \right] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ i - \frac{c}{x} \right] + \\ & \frac{3}{4} i a b^2 c x \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] + \frac{3}{16} b c \left( 1 - \frac{i c}{x} \right) x \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^2 + \\ & \frac{1}{16} c^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^3 + \frac{1}{16} x^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^3 - \frac{3}{2} i a b^2 c x \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] - \\ & \frac{3}{4} i a^2 b x^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] + \frac{3}{4} a b^2 x^2 \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] - \frac{3}{8} a b^2 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 - \\ & \frac{3}{16} b^3 c \left( 1 + \frac{i c}{x} \right) x \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 - \frac{3}{8} a b^2 x^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 + \frac{1}{16} i b^3 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^3 + \\ & \frac{1}{16} i b^3 x^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^3 - \frac{3}{4} i a^2 b c^2 \operatorname{Log} [c - i x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - i x] + \\ & \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \operatorname{Log} [c - i x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c + i x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] \operatorname{Log} [c + i x] - \\ & \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ \frac{c - i x}{2 c} \right] \operatorname{Log} [c + i x] - \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - i x] \operatorname{Log} \left[ \frac{c + i x}{2 c} \right] - \\ & \frac{3}{16} i b^3 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right]^2 \operatorname{Log} \left[ -\frac{i c}{x} \right] - \frac{3}{16} i b c^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right)^2 \operatorname{Log} \left[ \frac{i c}{x} \right] + \\ & \frac{3}{2} a b^2 c^2 \operatorname{Log} [x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c + i x] \operatorname{Log} \left[ -\frac{i x}{c} \right] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - i x] \operatorname{Log} \left[ \frac{i x}{c} \right] + \\ & \frac{3}{8} b^2 c^2 \left( 2 a + i b \operatorname{Log} \left[ 1 - \frac{i c}{x} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{i c}{x} \right] - \frac{3}{8} i b^3 c^2 \operatorname{Log} \left[ 1 + \frac{i c}{x} \right] \operatorname{PolyLog} \left[ 2, 1 + \frac{i c}{x} \right] - \\ & \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c - i x}{2 c} \right] - \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c + i x}{2 c} \right] - \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, -\frac{i c}{x} \right] - \\ & \frac{3}{8} i b^3 c^2 \operatorname{PolyLog} \left[ 2, -\frac{i c}{x} \right] + \frac{3}{8} i b^3 c^2 \operatorname{PolyLog} \left[ 2, \frac{i c}{x} \right] + \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{i x}{c} \right] + \\ & \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 + \frac{i x}{c} \right] - \frac{3}{8} i b^3 c^2 \operatorname{PolyLog} \left[ 3, 1 - \frac{i c}{x} \right] + \frac{3}{8} i b^3 c^2 \operatorname{PolyLog} \left[ 3, 1 + \frac{i c}{x} \right] \end{aligned}$$

**Problem 150: Unable to integrate problem.**

$$\int \left( a + b \operatorname{ArcTan} \left[ \frac{c}{x} \right] \right)^3 dx$$

Optimal (type 4, 119 leaves, 6 steps):

$$\begin{aligned} & i c \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^3 + x \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^3 - 3 b c \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right)^2 \operatorname{Log}\left[\frac{2 c}{c + i x}\right] + \\ & 3 i b^2 c \left( a + b \operatorname{ArcCot}\left[\frac{x}{c}\right] \right) \operatorname{PolyLog}\left[2, 1 - \frac{2 c}{c + i x}\right] - \frac{3}{2} b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2 c}{c + i x}\right] \end{aligned}$$

Result (type 8, 805 leaves, 43 steps):

$$\begin{aligned} & a^3 x + \frac{3}{8} i b^3 \operatorname{CannotIntegrate}\left[\operatorname{Log}\left[1 - \frac{i c}{x}\right]^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right], x\right] - \\ & \frac{3}{8} i b^3 \operatorname{CannotIntegrate}\left[\operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2, x\right] + \frac{3}{2} i a^2 b x \operatorname{Log}\left[1 - \frac{i c}{x}\right] + \\ & \frac{3}{4} a b^2 (i c - x) \operatorname{Log}\left[1 - \frac{i c}{x}\right]^2 + \frac{1}{8} i b^3 (i c - x) \operatorname{Log}\left[1 - \frac{i c}{x}\right]^3 - \frac{3}{2} i a^2 b x \operatorname{Log}\left[1 + \frac{i c}{x}\right] + \\ & \frac{3}{2} a b^2 x \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}\left[1 + \frac{i c}{x}\right] - \frac{3}{4} a b^2 (i c + x) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 + \frac{1}{8} i b^3 (i c + x) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^3 - \\ & \frac{3}{2} i a b^2 c \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{Log}[-c - i x] + \frac{3}{2} a^2 b c \operatorname{Log}[c - i x] + \frac{3}{2} i a b^2 c \operatorname{Log}[-c - i x] \operatorname{Log}\left[\frac{c - i x}{2 c}\right] + \\ & \frac{3}{2} i a b^2 c \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{Log}[-c + i x] + \frac{3}{2} a^2 b c \operatorname{Log}[c + i x] - \frac{3}{2} i a b^2 c \operatorname{Log}[-c + i x] \operatorname{Log}\left[\frac{c + i x}{2 c}\right] + \\ & \frac{3}{8} b^3 c \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 \operatorname{Log}\left[-\frac{i c}{x}\right] + \frac{3}{8} b^3 c \operatorname{Log}\left[1 - \frac{i c}{x}\right]^2 \operatorname{Log}\left[\frac{i c}{x}\right] - \\ & \frac{3}{2} i a b^2 c \operatorname{Log}[-c - i x] \operatorname{Log}\left[-\frac{i x}{c}\right] + \frac{3}{2} i a b^2 c \operatorname{Log}[-c + i x] \operatorname{Log}\left[\frac{i x}{c}\right] + \\ & \frac{3}{4} b^3 c \operatorname{Log}\left[1 - \frac{i c}{x}\right] \operatorname{PolyLog}\left[2, 1 - \frac{i c}{x}\right] + \frac{3}{4} b^3 c \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{i c}{x}\right] - \\ & \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, \frac{c - i x}{2 c}\right] + \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, \frac{c + i x}{2 c}\right] - \\ & \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, -\frac{i c}{x}\right] + \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, \frac{i c}{x}\right] + \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{i x}{c}\right] - \\ & \frac{3}{2} i a b^2 c \operatorname{PolyLog}\left[2, 1 + \frac{i x}{c}\right] - \frac{3}{4} b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{i c}{x}\right] - \frac{3}{4} b^3 c \operatorname{PolyLog}\left[3, 1 + \frac{i c}{x}\right] \end{aligned}$$

**Problem 152: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{ArcTan}\left[\frac{c}{x}\right]\right)^3}{x^2} dx$$

Optimal (type 4, 136 leaves, 6 steps):

$$\begin{aligned} & -\frac{i \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3}{c} - \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3}{x} - \frac{3 b \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2 \operatorname{Log}\left[\frac{2}{1 + \frac{i c}{x}}\right]}{c} - \\ & \frac{3 i b^2 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{i c}{x}}\right]}{c} - \frac{3 b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + \frac{i c}{x}}\right]}{2 c} \end{aligned}$$

Result (type 4, 551 leaves, 82 steps):

$$\begin{aligned}
& - \frac{3 b \left(1 - \frac{i c}{x}\right) \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2}{8 c} - \frac{3 b \left(1 - \frac{i c}{x}\right) \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2}{8 c} - \\
& \frac{i \left(1 - \frac{i c}{x}\right) \left(2 a + i b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^3}{8 c} + \frac{3 b \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{8 c} - \\
& \frac{3 i b \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2 \operatorname{Log}\left[1 + \frac{i c}{x}\right]}{8 x} - \frac{3 b^2 \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2}{8 c} - \\
& \frac{3 i b^2 \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2}{8 x} - \frac{b^3 \left(1 + \frac{i c}{x}\right) \operatorname{Log}\left[1 + \frac{i c}{x}\right]^3}{8 c} - \\
& \frac{3 b^3 \operatorname{Log}\left[1 + \frac{i c}{x}\right]^2 \operatorname{Log}\left[-\frac{i c - x}{2 x}\right]}{4 c} - \frac{3 b \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right)^2 \operatorname{Log}\left[\frac{i c + x}{2 x}\right]}{4 c} + \\
& \frac{3 b^2 \left(2 i a - b \operatorname{Log}\left[1 - \frac{i c}{x}\right]\right) \operatorname{PolyLog}\left[2, -\frac{i c - x}{2 x}\right]}{2 c} - \\
& \frac{3 b^3 \operatorname{Log}\left[1 + \frac{i c}{x}\right] \operatorname{PolyLog}\left[2, \frac{i c + x}{2 x}\right]}{2 c} + \frac{3 b^3 \operatorname{PolyLog}\left[3, -\frac{i c - x}{2 x}\right]}{2 c} + \frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{i c + x}{2 x}\right]}{2 c}
\end{aligned}$$

**Problem 153: Unable to integrate problem.**

$$\int \frac{\left(a + b \operatorname{ArcTan}\left[\frac{c}{x}\right]\right)^3}{x^3} dx$$

Optimal (type 4, 147 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 i b \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{2 c^2} + \frac{3 b \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^2}{2 c x} - \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3}{2 c^2} - \\
& \frac{\left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right)^3}{2 x^2} + \frac{3 b^2 \left(a + b \operatorname{ArcCot}\left[\frac{x}{c}\right]\right) \operatorname{Log}\left[\frac{2}{1 + \frac{i c}{x}}\right]}{c^2} + \frac{3 i b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + \frac{i c}{x}}\right]}{2 c^2}
\end{aligned}$$

Result (type 8, 1316 leaves, 81 steps):

$$\begin{aligned}
& \frac{3 \, i \, b^3 \left(1 - \frac{i \, c}{x}\right)^2}{64 \, c^2} - \frac{3 \, a \, b^2 \left(1 + \frac{i \, c}{x}\right)^2}{16 \, c^2} - \frac{3 \, i \, b^3 \left(1 + \frac{i \, c}{x}\right)^2}{64 \, c^2} - \frac{3 \, i \, a^2 \, b}{8 \, x^2} - \frac{3 \, a \, b^2}{8 \, x^2} + \\
& \frac{3 \, a^2 \, b}{4 \, c \, x} - \frac{3 \, b^3}{2 \, c \, x} + \frac{3}{8} \, i \, b^3 \text{ CannotIntegrate} \left[ \frac{\text{Log} \left[1 - \frac{i \, c}{x}\right]^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]}{x^3}, x \right] - \\
& \frac{3}{8} \, i \, b^3 \text{ CannotIntegrate} \left[ \frac{\text{Log} \left[1 - \frac{i \, c}{x}\right] \text{Log} \left[1 + \frac{i \, c}{x}\right]^2}{x^3}, x \right] + \frac{3 \, i \, a^2 \, b \text{Log} \left[i - \frac{c}{x}\right]}{4 \, c^2} + \frac{3 \, a \, b^2 \text{Log} \left[i - \frac{c}{x}\right]}{8 \, c^2} - \\
& \frac{3 \, a \, b^2 \left(1 - \frac{i \, c}{x}\right) \text{Log} \left[1 - \frac{i \, c}{x}\right]}{4 \, c^2} + \frac{3 \, i \, b^3 \left(1 - \frac{i \, c}{x}\right) \text{Log} \left[1 - \frac{i \, c}{x}\right]}{4 \, c^2} + \frac{3 \, a \, b^2 \text{Log} \left[1 - \frac{i \, c}{x}\right]}{8 \, x^2} - \\
& \frac{3 \, b^2 \left(1 - \frac{i \, c}{x}\right)^2 \left(2 \, a + i \, b \text{Log} \left[1 - \frac{i \, c}{x}\right]\right)}{32 \, c^2} + \frac{3 \, i \, b \left(1 - \frac{i \, c}{x}\right) \left(2 \, a + i \, b \text{Log} \left[1 - \frac{i \, c}{x}\right]\right)^2}{8 \, c^2} - \\
& \frac{3 \, i \, b \left(1 - \frac{i \, c}{x}\right)^2 \left(2 \, a + i \, b \text{Log} \left[1 - \frac{i \, c}{x}\right]\right)^2}{32 \, c^2} - \frac{\left(1 - \frac{i \, c}{x}\right) \left(2 \, a + i \, b \text{Log} \left[1 - \frac{i \, c}{x}\right]\right)^3}{8 \, c^2} + \\
& \frac{\left(1 - \frac{i \, c}{x}\right)^2 \left(2 \, a + i \, b \text{Log} \left[1 - \frac{i \, c}{x}\right]\right)^3}{16 \, c^2} - \frac{9 \, a \, b^2 \left(1 + \frac{i \, c}{x}\right) \text{Log} \left[1 + \frac{i \, c}{x}\right]}{4 \, c^2} - \frac{3 \, i \, b^3 \left(1 + \frac{i \, c}{x}\right) \text{Log} \left[1 + \frac{i \, c}{x}\right]}{4 \, c^2} + \\
& \frac{3 \, a \, b^2 \left(1 + \frac{i \, c}{x}\right)^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]}{8 \, c^2} + \frac{3 \, i \, b^3 \left(1 + \frac{i \, c}{x}\right)^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]}{32 \, c^2} + \frac{3 \, i \, a^2 \, b \text{Log} \left[1 + \frac{i \, c}{x}\right]}{4 \, x^2} + \\
& \frac{3 \, a \, b^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]}{8 \, x^2} - \frac{3 \, a \, b^2 \text{Log} \left[1 - \frac{i \, c}{x}\right] \text{Log} \left[1 + \frac{i \, c}{x}\right]}{4 \, x^2} + \frac{3 \, a \, b^2 \left(1 + \frac{i \, c}{x}\right) \text{Log} \left[1 + \frac{i \, c}{x}\right]^2}{4 \, c^2} + \\
& \frac{3 \, i \, b^3 \left(1 + \frac{i \, c}{x}\right) \text{Log} \left[1 + \frac{i \, c}{x}\right]^2}{8 \, c^2} - \frac{3 \, a \, b^2 \left(1 + \frac{i \, c}{x}\right)^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]^2}{8 \, c^2} - \frac{3 \, i \, b^3 \left(1 + \frac{i \, c}{x}\right)^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]^2}{32 \, c^2} - \\
& \frac{i \, b^3 \left(1 + \frac{i \, c}{x}\right) \text{Log} \left[1 + \frac{i \, c}{x}\right]^3}{8 \, c^2} + \frac{i \, b^3 \left(1 + \frac{i \, c}{x}\right)^2 \text{Log} \left[1 + \frac{i \, c}{x}\right]^3}{16 \, c^2} + \frac{3 \, a \, b^2 \text{Log} \left[i + \frac{c}{x}\right]}{8 \, c^2} - \\
& \frac{3 \, a \, b^2 \text{Log} \left[1 - \frac{i \, c}{x}\right] \text{Log} [c - i \, x]}{4 \, c^2} - \frac{3 \, a \, b^2 \text{Log} \left[1 + \frac{i \, c}{x}\right] \text{Log} [c + i \, x]}{4 \, c^2} + \frac{3 \, a \, b^2 \text{Log} \left[\frac{c - i \, x}{2 \, c}\right] \text{Log} [c + i \, x]}{4 \, c^2} + \\
& \frac{3 \, a \, b^2 \text{Log} [c - i \, x] \text{Log} \left[\frac{c + i \, x}{2 \, c}\right]}{4 \, c^2} - \frac{3 \, a \, b^2 \text{Log} [c + i \, x] \text{Log} \left[-\frac{i \, x}{c}\right]}{4 \, c^2} - \frac{3 \, a \, b^2 \text{Log} [c - i \, x] \text{Log} \left[\frac{i \, x}{c}\right]}{4 \, c^2} + \\
& \frac{3 \, a \, b^2 \text{PolyLog} \left[2, \frac{c - i \, x}{2 \, c}\right]}{4 \, c^2} + \frac{3 \, a \, b^2 \text{PolyLog} \left[2, \frac{c + i \, x}{2 \, c}\right]}{4 \, c^2} + \frac{3 \, a \, b^2 \text{PolyLog} \left[2, -\frac{i \, c}{x}\right]}{4 \, c^2} + \\
& \frac{3 \, a \, b^2 \text{PolyLog} \left[2, \frac{i \, c}{x}\right]}{4 \, c^2} - \frac{3 \, a \, b^2 \text{PolyLog} \left[2, 1 - \frac{i \, x}{c}\right]}{4 \, c^2} - \frac{3 \, a \, b^2 \text{PolyLog} \left[2, 1 + \frac{i \, x}{c}\right]}{4 \, c^2}
\end{aligned}$$

## Test results for the 31 problems in "5.3.3 (d+e x)^m (a+b arctan(c x^n))^p.m"

Problem 21: Result optimal but 1 more steps used.

$$\int (d + e x)^2 (a + b \operatorname{ArcTan}[c x^2]) dx$$

Optimal (type 3, 250 leaves, 17 steps):

$$\begin{aligned} & -\frac{2 b e^2 x}{3 c} - \frac{b d^3 \operatorname{ArcTan}[c x^2]}{3 e} + \\ & \frac{(d + e x)^3 (a + b \operatorname{ArcTan}[c x^2])}{3 e} + \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c} x]}{3 \sqrt{2} c^{3/2}} - \\ & \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c} x]}{3 \sqrt{2} c^{3/2}} - \frac{b (3 c d^2 + e^2) \operatorname{Log}[1 - \sqrt{2} \sqrt{c} x + c x^2]}{6 \sqrt{2} c^{3/2}} + \\ & \frac{b (3 c d^2 + e^2) \operatorname{Log}[1 + \sqrt{2} \sqrt{c} x + c x^2]}{6 \sqrt{2} c^{3/2}} - \frac{b d e \operatorname{Log}[1 + c^2 x^4]}{2 c} \end{aligned}$$

Result (type 3, 250 leaves, 18 steps):

$$\begin{aligned} & -\frac{2 b e^2 x}{3 c} - \frac{b d^3 \operatorname{ArcTan}[c x^2]}{3 e} + \\ & \frac{(d + e x)^3 (a + b \operatorname{ArcTan}[c x^2])}{3 e} + \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c} x]}{3 \sqrt{2} c^{3/2}} - \\ & \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c} x]}{3 \sqrt{2} c^{3/2}} - \frac{b (3 c d^2 + e^2) \operatorname{Log}[1 - \sqrt{2} \sqrt{c} x + c x^2]}{6 \sqrt{2} c^{3/2}} + \\ & \frac{b (3 c d^2 + e^2) \operatorname{Log}[1 + \sqrt{2} \sqrt{c} x + c x^2]}{6 \sqrt{2} c^{3/2}} - \frac{b d e \operatorname{Log}[1 + c^2 x^4]}{2 c} \end{aligned}$$

Problem 23: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcTan}[c x^2]}{d + e x} dx$$

Optimal (type 4, 501 leaves, 19 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcTan}[c x^2]) \operatorname{Log}[d + e x]}{e} + \frac{b c \operatorname{Log}\left[\frac{e \left(1 - (-c^2)^{1/4} x\right)}{(-c^2)^{1/4} d + e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} + \\
& \frac{b c \operatorname{Log}\left[-\frac{e \left(1 + (-c^2)^{1/4} x\right)}{(-c^2)^{1/4} d - e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \frac{b c \operatorname{Log}\left[\frac{e \left(1 - \sqrt{-c^2} x\right)}{\sqrt{-c^2} d + e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{Log}\left[-\frac{e \left(1 + \sqrt{-c^2} x\right)}{\sqrt{-c^2} d - e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/4} (d + e x)}{(-c^2)^{1/4} d - e}\right]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{PolyLog}\left[2, \frac{\sqrt{-c^2} (d + e x)}{\sqrt{-c^2} d - e}\right]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/4} (d + e x)}{(-c^2)^{1/4} d + e}\right]}{2 \sqrt{-c^2} e} - \frac{b c \operatorname{PolyLog}\left[2, \frac{\sqrt{-c^2} (d + e x)}{\sqrt{-c^2} d + e}\right]}{2 \sqrt{-c^2} e}
\end{aligned}$$

Result (type 8, 30 leaves, 2 steps):

$$b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTan}[c x^2]}{d + e x}, x\right] + \frac{a \operatorname{Log}[d + e x]}{e}$$

**Problem 24: Result optimal but 1 more steps used.**

$$\int \frac{a + b \operatorname{ArcTan}[c x^2]}{(d + e x)^2} dx$$

Optimal (type 3, 328 leaves, 18 steps):

$$\begin{aligned}
& \frac{b c^2 d^3 \operatorname{ArcTan}[c x^2]}{e (c^2 d^4 + e^4)} - \frac{a + b \operatorname{ArcTan}[c x^2]}{e (d + e x)} + \\
& \frac{b \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c} x]}{\sqrt{2} (c^2 d^4 + e^4)} - \frac{b \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c} x]}{\sqrt{2} (c^2 d^4 + e^4)} - \\
& \frac{2 b c d e \operatorname{Log}[d + e x]}{c^2 d^4 + e^4} - \frac{b \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 - \sqrt{2} \sqrt{c} x + c x^2]}{2 \sqrt{2} (c^2 d^4 + e^4)} + \\
& \frac{b \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 + \sqrt{2} \sqrt{c} x + c x^2]}{2 \sqrt{2} (c^2 d^4 + e^4)} + \frac{b c d e \operatorname{Log}[1 + c^2 x^4]}{2 (c^2 d^4 + e^4)}
\end{aligned}$$

Result (type 3, 328 leaves, 19 steps):

$$\begin{aligned}
& \frac{b c^2 d^3 \operatorname{ArcTan}[c x^2]}{e (c^2 d^4 + e^4)} - \frac{a + b \operatorname{ArcTan}[c x^2]}{e (d + e x)} + \\
& \frac{b \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c} x]}{\sqrt{2} (c^2 d^4 + e^4)} - \frac{b \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c} x]}{\sqrt{2} (c^2 d^4 + e^4)} - \\
& \frac{2 b c d e \operatorname{Log}[d + e x]}{c^2 d^4 + e^4} - \frac{b \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 - \sqrt{2} \sqrt{c} x + c x^2]}{2 \sqrt{2} (c^2 d^4 + e^4)} + \\
& \frac{b \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 + \sqrt{2} \sqrt{c} x + c x^2]}{2 \sqrt{2} (c^2 d^4 + e^4)} + \frac{b c d e \operatorname{Log}[1 + c^2 x^4]}{2 (c^2 d^4 + e^4)}
\end{aligned}$$

**Problem 25: Result valid but suboptimal antiderivative.**

$$\int (d + e x) (a + b \operatorname{ArcTan}[c x^2])^2 dx$$

Optimal (type 4, 1325 leaves, 77 steps):

$$\begin{aligned}
& a^2 d x - \frac{2 (-1)^{3/4} a b d \operatorname{ArcTan}[(-1)^{3/4} \sqrt{c} x]}{\sqrt{c}} + \frac{(-1)^{3/4} b^2 d \operatorname{ArcTan}[(-1)^{3/4} \sqrt{c} x]^2}{\sqrt{c}} + \\
& \frac{i e (a + b \operatorname{ArcTan}[c x^2])^2}{2 c} + \frac{1}{2} e x^2 (a + b \operatorname{ArcTan}[c x^2])^2 + \frac{2 (-1)^{3/4} a b d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x]}{\sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x]^2}{\sqrt{c}} + \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTan}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTan}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{\sqrt{2} (-1)^{1/4} + \sqrt{c} x}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[-\frac{\sqrt{2} (-1)^{3/4} + \sqrt{c} x}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}[(-1)^{3/4} \sqrt{c} x] \operatorname{Log}\left[\frac{(1+i) (1 + (-1)^{1/4} \sqrt{c} x)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} +
\end{aligned}$$



$$\begin{aligned}
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& i a b d x \operatorname{Log}\left[1-i c x^2\right] + \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right]}{\sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1-i c x^2\right]}{\sqrt{c}} - \\
& \frac{1}{4} b^2 d x \operatorname{Log}\left[1-i c x^2\right]^2 + \frac{b e\left(a+b \operatorname{ArcTan}\left[c x^2\right]\right) \operatorname{Log}\left[\frac{2}{1+i c x^2}\right]}{c} - \\
& i a b d x \operatorname{Log}\left[1+i c x^2\right] - \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1+i c x^2\right]}{\sqrt{c}} + \frac{1}{2} b^2 d x \operatorname{Log}\left[1-i c x^2\right] \operatorname{Log}\left[1+i c x^2\right] - \\
& \frac{1}{4} b^2 d x \operatorname{Log}\left[1+i c x^2\right]^2 + \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} - \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{\sqrt{2}\left((-1)^{1/4}+\sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1+\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1+(-1)^{1/4} \sqrt{c} x\right)}{1+(-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \\
& \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1-i)\left(1+(-1)^{3/4} \sqrt{c} x\right)}{1+(-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}} + \frac{i b^2 e \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i c x^2}\right]}{2 c}
\end{aligned}$$

Result (type 4, 1554 leaves, 110 steps):

$$\begin{aligned}
& \frac{a^2(d+e x)^2}{2 e} + \frac{(-1)^{3/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{\sqrt{c}} + 2 a b d x \operatorname{ArcTan}\left[c x^2\right] + \\
& a b e x^2 \operatorname{ArcTan}\left[c x^2\right] + \frac{\sqrt{2} a b d \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{c} x\right]}{\sqrt{c}} - \frac{\sqrt{2} a b d \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{c} x\right]}{\sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right]^2}{\sqrt{c}} + \frac{2(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1-(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{2(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1+(-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{\sqrt{2} \left((-1)^{1/4} + \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{2 (-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[-\frac{\sqrt{2} \left((-1)^{3/4} + \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1+i) \left(1 + (-1)^{1/4} \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[\frac{(1-i) \left(1 + (-1)^{3/4} \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1 - i c x^2\right]}{\sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1 - i c x^2\right]}{\sqrt{c}} - \frac{1}{4} b^2 d x \operatorname{Log}\left[1 - i c x^2\right]^2 - \\
& \frac{i b^2 e \left(1 - i c x^2\right) \operatorname{Log}\left[1 - i c x^2\right]^2}{8 c} - \frac{i b^2 e \operatorname{Log}\left[1 - i c x^2\right] \operatorname{Log}\left[\frac{1}{2} \left(1 + i c x^2\right)\right]}{4 c} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTan}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1 + i c x^2\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{ArcTanh}\left[(-1)^{3/4} \sqrt{c} x\right] \operatorname{Log}\left[1 + i c x^2\right]}{\sqrt{c}} + \frac{i b^2 e \operatorname{Log}\left[\frac{1}{2} \left(1 - i c x^2\right)\right] \operatorname{Log}\left[1 + i c x^2\right]}{4 c} + \\
& \frac{1}{2} b^2 d x \operatorname{Log}\left[1 - i c x^2\right] \operatorname{Log}\left[1 + i c x^2\right] + \frac{1}{4} b^2 e x^2 \operatorname{Log}\left[1 - i c x^2\right] \operatorname{Log}\left[1 + i c x^2\right] - \\
& \frac{1}{4} b^2 d x \operatorname{Log}\left[1 + i c x^2\right]^2 + \frac{i b^2 e \left(1 + i c x^2\right) \operatorname{Log}\left[1 + i c x^2\right]^2}{8 c} - \\
& \frac{a b d \operatorname{Log}\left[1 - \sqrt{2} \sqrt{c} x + c x^2\right]}{\sqrt{2} \sqrt{c}} + \frac{a b d \operatorname{Log}\left[1 + \sqrt{2} \sqrt{c} x + c x^2\right]}{\sqrt{2} \sqrt{c}} - \frac{a b e \operatorname{Log}\left[1 + c^2 x^4\right]}{2 c} - \\
& \frac{i b^2 e \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 - i c x^2\right)\right]}{4 c} + \frac{i b^2 e \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 + i c x^2\right)\right]}{4 c} + \\
& \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} + \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{\sqrt{c}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{\sqrt{2} \left((-1)^{1/4} + \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}} + \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{\sqrt{c}} - \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1 + \frac{\sqrt{2} \left((-1)^{3/4} + \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \\
& \frac{(-1)^{1/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 + (-1)^{1/4} \sqrt{c} x\right)}{1 + (-1)^{3/4} \sqrt{c} x}\right]}{2 \sqrt{c}} - \frac{(-1)^{3/4} b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-1)^{3/4} \sqrt{c} x\right)}{1 + (-1)^{1/4} \sqrt{c} x}\right]}{2 \sqrt{c}}
\end{aligned}$$

**Problem 26: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{d + e x} dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTan}[c x^2])^2}{d + e x}, x\right]$$

Result (type 8, 56 leaves, 2 steps):

$$2 a b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTan}[c x^2]}{d + e x}, x\right] + b^2 \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTan}[c x^2]^2}{d + e x}, x\right] + \frac{a^2 \operatorname{Log}[d + e x]}{e}$$

**Problem 27: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{ArcTan}[c x^2])^2}{(d + e x)^2} dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTan}[c x^2])^2}{(d + e x)^2}, x\right]$$

Result (type 8, 363 leaves, 21 steps):

$$\begin{aligned}
& -\frac{a^2}{e(d+ex)} + \frac{2ab c^2 d^3 \operatorname{ArcTan}[c x^2]}{e(c^2 d^4 + e^4)} - \\
& \frac{2ab \operatorname{ArcTan}[c x^2]}{e(d+ex)} + \frac{\sqrt{2} ab \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{c} x]}{c^2 d^4 + e^4} - \\
& \frac{\sqrt{2} ab \sqrt{c} (c d^2 - e^2) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{c} x]}{c^2 d^4 + e^4} + b^2 \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTan}[c x^2]^2}{(d+ex)^2}, x\right] - \\
& \frac{4abcd e \operatorname{Log}[d+ex]}{c^2 d^4 + e^4} - \frac{ab \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 - \sqrt{2} \sqrt{c} x + c x^2]}{\sqrt{2} (c^2 d^4 + e^4)} + \\
& \frac{ab \sqrt{c} (c d^2 + e^2) \operatorname{Log}[1 + \sqrt{2} \sqrt{c} x + c x^2]}{\sqrt{2} (c^2 d^4 + e^4)} + \frac{abcd e \operatorname{Log}[1 + c^2 x^4]}{c^2 d^4 + e^4}
\end{aligned}$$

Problem 28: Result valid but suboptimal antiderivative.

$$\int (d+ex)^2 (a+b \operatorname{ArcTan}[c x^3]) dx$$

Optimal (type 3, 315 leaves, 24 steps):

$$\begin{aligned}
& -\frac{bde \operatorname{ArcTan}[c^{1/3} x]}{c^{2/3}} - \frac{bd^3 \operatorname{ArcTan}[c x^3]}{3e} + \frac{(d+ex)^3 (a+b \operatorname{ArcTan}[c x^3])}{3e} + \\
& \frac{bde \operatorname{ArcTan}[\sqrt{3} - 2c^{1/3} x]}{2c^{2/3}} - \frac{bde \operatorname{ArcTan}[\sqrt{3} + 2c^{1/3} x]}{2c^{2/3}} + \frac{\sqrt{3} b d^2 \operatorname{ArcTan}\left[\frac{1-2c^{2/3} x^2}{\sqrt{3}}\right]}{2c^{1/3}} + \\
& \frac{bd^2 \operatorname{Log}[1+c^{2/3} x^2]}{2c^{1/3}} - \frac{\sqrt{3} bde \operatorname{Log}[1-\sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4c^{2/3}} + \\
& \frac{\sqrt{3} bde \operatorname{Log}[1+\sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4c^{2/3}} - \frac{bd^2 \operatorname{Log}[1-c^{2/3} x^2 + c^{4/3} x^4]}{4c^{1/3}} - \frac{be^2 \operatorname{Log}[1+c^2 x^6]}{6c}
\end{aligned}$$

Result (type 3, 331 leaves, 25 steps):

$$\begin{aligned}
& \frac{a(d+ex)^3}{3e} - \frac{bde \operatorname{ArcTan}[c^{1/3} x]}{c^{2/3}} + bd^2 x \operatorname{ArcTan}[c x^3] + bde x^2 \operatorname{ArcTan}[c x^3] + \\
& \frac{1}{3} be^2 x^3 \operatorname{ArcTan}[c x^3] + \frac{bde \operatorname{ArcTan}[\sqrt{3} - 2c^{1/3} x]}{2c^{2/3}} - \frac{bde \operatorname{ArcTan}[\sqrt{3} + 2c^{1/3} x]}{2c^{2/3}} + \\
& \frac{\sqrt{3} b d^2 \operatorname{ArcTan}\left[\frac{1-2c^{2/3} x^2}{\sqrt{3}}\right]}{2c^{1/3}} + \frac{bd^2 \operatorname{Log}[1+c^{2/3} x^2]}{2c^{1/3}} - \frac{\sqrt{3} bde \operatorname{Log}[1-\sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4c^{2/3}} + \\
& \frac{\sqrt{3} bde \operatorname{Log}[1+\sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4c^{2/3}} - \frac{bd^2 \operatorname{Log}[1-c^{2/3} x^2 + c^{4/3} x^4]}{4c^{1/3}} - \frac{be^2 \operatorname{Log}[1+c^2 x^6]}{6c}
\end{aligned}$$

Problem 29: Result optimal but 1 more steps used.

$$\int (d+ex) (a+b \operatorname{ArcTan}[c x^3]) dx$$

Optimal (type 3, 285 leaves, 22 steps):

$$\begin{aligned}
 & - \frac{b e \operatorname{ArcTan}\left[c^{1/3} x\right]}{2 c^{2/3}} - \frac{b d^2 \operatorname{ArcTan}\left[c x^3\right]}{2 e} + \frac{(d+e x)^2 (a+b \operatorname{ArcTan}\left[c x^3\right])}{2 e} + \\
 & \frac{b e \operatorname{ArcTan}\left[\sqrt{3}-2 c^{1/3} x\right]}{4 c^{2/3}} - \frac{b e \operatorname{ArcTan}\left[\sqrt{3}+2 c^{1/3} x\right]}{4 c^{2/3}} + \frac{\sqrt{3} b d \operatorname{ArcTan}\left[\frac{1-2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} + \\
 & \frac{b d \operatorname{Log}\left[1+c^{2/3} x^2\right]}{2 c^{1/3}} - \frac{\sqrt{3} b e \operatorname{Log}\left[1-\sqrt{3} c^{1/3} x+c^{2/3} x^2\right]}{8 c^{2/3}} + \\
 & \frac{\sqrt{3} b e \operatorname{Log}\left[1+\sqrt{3} c^{1/3} x+c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b d \operatorname{Log}\left[1-c^{2/3} x^2+c^{4/3} x^4\right]}{4 c^{1/3}}
 \end{aligned}$$

Result (type 3, 285 leaves, 23 steps):

$$\begin{aligned}
 & - \frac{b e \operatorname{ArcTan}\left[c^{1/3} x\right]}{2 c^{2/3}} - \frac{b d^2 \operatorname{ArcTan}\left[c x^3\right]}{2 e} + \frac{(d+e x)^2 (a+b \operatorname{ArcTan}\left[c x^3\right])}{2 e} + \\
 & \frac{b e \operatorname{ArcTan}\left[\sqrt{3}-2 c^{1/3} x\right]}{4 c^{2/3}} - \frac{b e \operatorname{ArcTan}\left[\sqrt{3}+2 c^{1/3} x\right]}{4 c^{2/3}} + \frac{\sqrt{3} b d \operatorname{ArcTan}\left[\frac{1-2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} + \\
 & \frac{b d \operatorname{Log}\left[1+c^{2/3} x^2\right]}{2 c^{1/3}} - \frac{\sqrt{3} b e \operatorname{Log}\left[1-\sqrt{3} c^{1/3} x+c^{2/3} x^2\right]}{8 c^{2/3}} + \\
 & \frac{\sqrt{3} b e \operatorname{Log}\left[1+\sqrt{3} c^{1/3} x+c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b d \operatorname{Log}\left[1-c^{2/3} x^2+c^{4/3} x^4\right]}{4 c^{1/3}}
 \end{aligned}$$

**Problem 30: Unable to integrate problem.**

$$\int \frac{a+b \operatorname{ArcTan}\left[c x^3\right]}{d+e x} d x$$

Optimal (type 4, 739 leaves, 25 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcTan}[c x^3]) \operatorname{Log}[d + e x]}{e} + \frac{b c \operatorname{Log}\left[\frac{e(1 - (-c^2)^{1/6} x)}{(-c^2)^{1/6} d + e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{Log}\left[-\frac{e(1 + (-c^2)^{1/6} x)}{(-c^2)^{1/6} d - e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{Log}\left[-\frac{e((-1)^{1/3} + (-c^2)^{1/6} x)}{(-c^2)^{1/6} d - (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{Log}\left[-\frac{e((-1)^{2/3} + (-c^2)^{1/6} x)}{(-c^2)^{1/6} d - (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{Log}\left[\frac{(-1)^{2/3} e(1 + (-1)^{1/3} (-c^2)^{1/6} x)}{(-c^2)^{1/6} d + (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{Log}\left[\frac{(-1)^{1/3} e(1 + (-1)^{2/3} (-c^2)^{1/6} x)}{(-c^2)^{1/6} d + (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2 \sqrt{-c^2} e} - \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d - e}\right]}{2 \sqrt{-c^2} e} + \\
& \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d + e}\right]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d - (-1)^{1/3} e}\right]}{2 \sqrt{-c^2} e} - \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d + (-1)^{1/3} e}\right]}{2 \sqrt{-c^2} e} - \\
& \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d - (-1)^{2/3} e}\right]}{2 \sqrt{-c^2} e} + \frac{b c \operatorname{PolyLog}\left[2, \frac{(-c^2)^{1/6} (d + e x)}{(-c^2)^{1/6} d + (-1)^{2/3} e}\right]}{2 \sqrt{-c^2} e}
\end{aligned}$$

Result (type 8, 30 leaves, 2 steps):

$$b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTan}[c x^3]}{d + e x}, x\right] + \frac{a \operatorname{Log}[d + e x]}{e}$$

**Problem 31: Result optimal but 1 more steps used.**

$$\int \frac{a + b \operatorname{ArcTan}[c x^3]}{(d + e x)^2} dx$$

Optimal (type 3, 906 leaves, 34 steps):

$$\begin{aligned}
& - \frac{b c^{2/3} d e^3 \operatorname{ArcTan}[c^{1/3} x]}{c^2 d^6 + e^6} + \frac{b c^2 d^5 \operatorname{ArcTan}[c x^3]}{e (c^2 d^6 + e^6)} - \\
& \frac{a + b \operatorname{ArcTan}[c x^3]}{e (d + e x)} + \frac{b c^{2/3} d (\sqrt{3} c d^3 + e^3) \operatorname{ArcTan}[\sqrt{3} - 2 c^{1/3} x]}{2 (c^2 d^6 + e^6)} + \\
& \frac{b c^{2/3} d (\sqrt{3} c d^3 - e^3) \operatorname{ArcTan}[\sqrt{3} + 2 c^{1/3} x]}{2 (c^2 d^6 + e^6)} + \frac{\sqrt{3} b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{ArcTan}\left[\frac{1 + \frac{2 c^{2/3} x}{(-c^2)^{5/6}}}{\sqrt{3}}\right]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{\sqrt{3} b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{ArcTan}\left[\frac{c^{4/3} + 2 (-c^2)^{5/6} x}{\sqrt{3} c^{4/3}}\right]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} + \frac{b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{Log}[(-c^2)^{1/6} - c^{2/3} x]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{Log}[(-c^2)^{1/6} + c^{2/3} x]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} + \frac{3 b c d^2 e^2 \operatorname{Log}[d + e x]}{c^2 d^6 + e^6} + \\
& \frac{b c^{5/3} d^4 \operatorname{Log}[1 + c^{2/3} x^2]}{2 (c^2 d^6 + e^6)} - \frac{b c^{2/3} d (c d^3 - \sqrt{3} e^3) \operatorname{Log}[1 - \sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4 (c^2 d^6 + e^6)} - \\
& \frac{b c^{2/3} d (c d^3 + \sqrt{3} e^3) \operatorname{Log}[1 + \sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4 (c^2 d^6 + e^6)} + \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{Log}[(-c^2)^{1/3} - c^{2/3} (-c^2)^{1/6} x + c^{4/3} x^2]}{4 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{Log}[(-c^2)^{1/3} + c^{2/3} (-c^2)^{1/6} x + c^{4/3} x^2]}{4 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \frac{b c d^2 e^2 \operatorname{Log}[1 + c^2 x^6]}{2 (c^2 d^6 + e^6)}
\end{aligned}$$

Result (type 3, 906 leaves, 35 steps):

$$\begin{aligned}
& - \frac{b c^{2/3} d e^3 \operatorname{ArcTan}[c^{1/3} x]}{c^2 d^6 + e^6} + \frac{b c^2 d^5 \operatorname{ArcTan}[c x^3]}{e (c^2 d^6 + e^6)} - \\
& \frac{a + b \operatorname{ArcTan}[c x^3]}{e (d + e x)} + \frac{b c^{2/3} d (\sqrt{3} c d^3 + e^3) \operatorname{ArcTan}[\sqrt{3} - 2 c^{1/3} x]}{2 (c^2 d^6 + e^6)} + \\
& \frac{b c^{2/3} d (\sqrt{3} c d^3 - e^3) \operatorname{ArcTan}[\sqrt{3} + 2 c^{1/3} x]}{2 (c^2 d^6 + e^6)} + \frac{\sqrt{3} b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{ArcTan}\left[\frac{1 + \frac{2 c^{2/3} x}{(-c^2)^{5/6}}}{\sqrt{3}}\right]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{\sqrt{3} b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{ArcTan}\left[\frac{c^{4/3} + 2 (-c^2)^{5/6} x}{\sqrt{3} c^{4/3}}\right]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} + \frac{b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{Log}[(-c^2)^{1/6} - c^{2/3} x]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{Log}[(-c^2)^{1/6} + c^{2/3} x]}{2 (-c^2)^{2/3} (c^2 d^6 + e^6)} + \frac{3 b c d^2 e^2 \operatorname{Log}[d + e x]}{c^2 d^6 + e^6} + \\
& \frac{b c^{5/3} d^4 \operatorname{Log}[1 + c^{2/3} x^2]}{2 (c^2 d^6 + e^6)} - \frac{b c^{2/3} d (c d^3 - \sqrt{3} e^3) \operatorname{Log}[1 - \sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4 (c^2 d^6 + e^6)} - \\
& \frac{b c^{2/3} d (c d^3 + \sqrt{3} e^3) \operatorname{Log}[1 + \sqrt{3} c^{1/3} x + c^{2/3} x^2]}{4 (c^2 d^6 + e^6)} + \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 - e^3) \operatorname{Log}[(-c^2)^{1/3} - c^{2/3} (-c^2)^{1/6} x + c^{4/3} x^2]}{4 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \\
& \frac{b c^{5/3} e (\sqrt{-c^2} d^3 + e^3) \operatorname{Log}[(-c^2)^{1/3} + c^{2/3} (-c^2)^{1/6} x + c^{4/3} x^2]}{4 (-c^2)^{2/3} (c^2 d^6 + e^6)} - \frac{b c d^2 e^2 \operatorname{Log}[1 + c^2 x^6]}{2 (c^2 d^6 + e^6)}
\end{aligned}$$

Test results for the 1301 problems in "5.3.4 u (a+b arctan(c x))^p.m"

Problem 1137: Result valid but suboptimal antiderivative.

$$\int x^3 (d + e x^2)^3 (a + b \operatorname{ArcTan}[c x]) dx$$

Optimal (type 3, 240 leaves, ? steps):

$$\begin{aligned}
& \frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x}{40 c^9} - \\
& \frac{b (10 c^6 d^3 - 20 c^4 d^2 e + 15 c^2 d e^2 - 4 e^3) x^3}{120 c^7} - \frac{b e (20 c^4 d^2 - 15 c^2 d e + 4 e^2) x^5}{200 c^5} - \\
& \frac{b (15 c^2 d - 4 e) e^2 x^7}{280 c^3} - \frac{b e^3 x^9}{90 c} + \frac{b (c^2 d - e)^4 (c^2 d + 4 e) \operatorname{ArcTan}[c x]}{40 c^{10} e^2} - \\
& \frac{d (d + e x^2)^4 (a + b \operatorname{ArcTan}[c x])}{8 e^2} + \frac{(d + e x^2)^5 (a + b \operatorname{ArcTan}[c x])}{10 e^2}
\end{aligned}$$



Result (type 3, 285 leaves, 8 steps):

$$\begin{aligned} & \frac{b \left( 325 c^8 d^4 + 1815 c^6 d^3 e - 4977 c^4 d^2 e^2 + 4305 c^2 d e^3 - 1260 e^4 \right) x}{12600 c^9 e} + \\ & \frac{b \left( 5 c^6 d^3 + 750 c^4 d^2 e - 1071 c^2 d e^2 + 420 e^3 \right) x \left( d + e x^2 \right)}{12600 c^7 e} - \\ & \frac{b \left( 25 c^4 d^2 - 135 c^2 d e + 84 e^2 \right) x \left( d + e x^2 \right)^2}{4200 c^5 e} - \frac{b \left( 23 c^2 d - 36 e \right) x \left( d + e x^2 \right)^3}{2520 c^3 e} - \\ & \frac{b x \left( d + e x^2 \right)^4}{90 c e} + \frac{b \left( c^2 d - e \right)^4 \left( c^2 d + 4 e \right) \text{ArcTan}[c x]}{40 c^{10} e^2} - \\ & \frac{d \left( d + e x^2 \right)^4 \left( a + b \text{ArcTan}[c x] \right)}{8 e^2} + \frac{\left( d + e x^2 \right)^5 \left( a + b \text{ArcTan}[c x] \right)}{10 e^2} \end{aligned}$$

Problem 1292: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \text{ArcTan}[c x] \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{x^2} dx$$

Optimal (type 4, 100 leaves, 6 steps):

$$\begin{aligned} & \frac{c e \left( a + b \text{ArcTan}[c x] \right)^2}{b} - \frac{\left( a + b \text{ArcTan}[c x] \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{x} + \\ & \frac{1}{2} b c \left( d + e \text{Log}[1 + c^2 x^2] \right) \text{Log}\left[1 - \frac{1}{1 + c^2 x^2}\right] - \frac{1}{2} b c e \text{PolyLog}\left[2, \frac{1}{1 + c^2 x^2}\right] \end{aligned}$$

Result (type 4, 92 leaves, 8 steps):

$$\begin{aligned} & \frac{c e \left( a + b \text{ArcTan}[c x] \right)^2}{b} + b c d \text{Log}[x] - \frac{\left( a + b \text{ArcTan}[c x] \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{x} - \\ & \frac{b c \left( d + e \text{Log}[1 + c^2 x^2] \right)^2}{4 e} - \frac{1}{2} b c e \text{PolyLog}\left[2, -c^2 x^2\right] \end{aligned}$$

Problem 1294: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \text{ArcTan}[c x] \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{x^4} dx$$

Optimal (type 4, 189 leaves, 15 steps):

$$\begin{aligned} & - \frac{2 c^2 e \left( a + b \text{ArcTan}[c x] \right)}{3 x} - \frac{c^3 e \left( a + b \text{ArcTan}[c x] \right)^2}{3 b} + b c^3 e \text{Log}[x] - \frac{1}{3} b c^3 e \text{Log}[1 + c^2 x^2] - \\ & \frac{b c \left( 1 + c^2 x^2 \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{6 x^2} - \frac{\left( a + b \text{ArcTan}[c x] \right) \left( d + e \text{Log}[1 + c^2 x^2] \right)}{3 x^3} - \\ & \frac{1}{6} b c^3 \left( d + e \text{Log}[1 + c^2 x^2] \right) \text{Log}\left[1 - \frac{1}{1 + c^2 x^2}\right] + \frac{1}{6} b c^3 e \text{PolyLog}\left[2, \frac{1}{1 + c^2 x^2}\right] \end{aligned}$$

Result (type 4, 186 leaves, 17 steps):

$$\begin{aligned}
& - \frac{2 c^2 e (a + b \operatorname{ArcTan}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcTan}[c x])^2}{3 b} - \frac{1}{3} b c^3 d \operatorname{Log}[x] + \\
& b c^3 e \operatorname{Log}[x] - \frac{1}{3} b c^3 e \operatorname{Log}[1 + c^2 x^2] - \frac{b c (1 + c^2 x^2) (d + e \operatorname{Log}[1 + c^2 x^2])}{6 x^2} - \\
& \frac{(a + b \operatorname{ArcTan}[c x]) (d + e \operatorname{Log}[1 + c^2 x^2])}{3 x^3} + \frac{b c^3 (d + e \operatorname{Log}[1 + c^2 x^2])^2}{12 e} + \frac{1}{6} b c^3 e \operatorname{PolyLog}[2, -c^2 x^2]
\end{aligned}$$

Problem 1296: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTan}[c x]) (d + e \operatorname{Log}[1 + c^2 x^2])}{x^6} dx$$

Optimal (type 4, 248 leaves, 24 steps):

$$\begin{aligned}
& - \frac{7 b c^3 e}{60 x^2} - \frac{2 c^2 e (a + b \operatorname{ArcTan}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcTan}[c x])}{5 x} + \frac{c^5 e (a + b \operatorname{ArcTan}[c x])^2}{5 b} - \\
& \frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 + c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 + c^2 x^2])}{20 x^4} + \\
& \frac{b c^3 (1 + c^2 x^2) (d + e \operatorname{Log}[1 + c^2 x^2])}{10 x^2} - \frac{(a + b \operatorname{ArcTan}[c x]) (d + e \operatorname{Log}[1 + c^2 x^2])}{5 x^5} + \\
& \frac{1}{10} b c^5 (d + e \operatorname{Log}[1 + c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 + c^2 x^2}\right] - \frac{1}{10} b c^5 e \operatorname{PolyLog}\left[2, \frac{1}{1 + c^2 x^2}\right]
\end{aligned}$$

Result (type 4, 245 leaves, 26 steps):

$$\begin{aligned}
& - \frac{7 b c^3 e}{60 x^2} - \frac{2 c^2 e (a + b \operatorname{ArcTan}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcTan}[c x])}{5 x} + \frac{c^5 e (a + b \operatorname{ArcTan}[c x])^2}{5 b} + \\
& \frac{1}{5} b c^5 d \operatorname{Log}[x] - \frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 + c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 + c^2 x^2])}{20 x^4} + \\
& \frac{b c^3 (1 + c^2 x^2) (d + e \operatorname{Log}[1 + c^2 x^2])}{10 x^2} - \frac{(a + b \operatorname{ArcTan}[c x]) (d + e \operatorname{Log}[1 + c^2 x^2])}{5 x^5} - \\
& \frac{b c^5 (d + e \operatorname{Log}[1 + c^2 x^2])^2}{20 e} - \frac{1}{10} b c^5 e \operatorname{PolyLog}[2, -c^2 x^2]
\end{aligned}$$

---

Test results for the 70 problems in "5.3.5 u (a+b arctan(c+d x))^p.m"

---

Test results for the 385 problems in "5.3.6 Exponentials of inverse tangent.m"

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTan}[a x]}}{x (c + a^2 c x^2)} dx$$

Optimal (type 5, 65 leaves, 3 steps):

$$\frac{i e^{n \operatorname{ArcTan}[a x]}}{c n} - \frac{2 i e^{n \operatorname{ArcTan}[a x]} \operatorname{Hypergeometric2F1}\left[1, -\frac{i n}{2}, 1 - \frac{i n}{2}, e^{2 i \operatorname{ArcTan}[a x]}\right]}{c n}$$

Result (type 5, 132 leaves, 3 steps):

$$\frac{i (1 - i a x)^{\frac{i n}{2}} (1 + i a x)^{-\frac{i n}{2}}}{c n} - \frac{1}{c (2 + i n)}$$

$$2 (1 - i a x)^{1 + \frac{i n}{2}} (1 + i a x)^{-1 - \frac{i n}{2}} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i n}{2}, 2 + \frac{i n}{2}, \frac{1 - i a x}{1 + i a x}\right]$$

**Problem 345: Result valid but suboptimal antiderivative.**

$$\int \frac{e^{n \operatorname{ArcTan}[a x]}}{x^2 (c + a^2 c x^2)} dx$$

Optimal (type 5, 90 leaves, 5 steps):

$$\frac{i a e^{n \operatorname{ArcTan}[a x]} (i + n)}{c n} - \frac{e^{n \operatorname{ArcTan}[a x]}}{c x} -$$

$$\frac{2 i a e^{n \operatorname{ArcTan}[a x]} \operatorname{Hypergeometric2F1}\left[1, -\frac{i n}{2}, 1 - \frac{i n}{2}, -1 + \frac{2 i}{i + a x}\right]}{c}$$

Result (type 5, 180 leaves, 5 steps):

$$- \frac{a (1 - i n) (1 - i a x)^{\frac{i n}{2}} (1 + i a x)^{-\frac{i n}{2}}}{c n} - \frac{(1 - i a x)^{\frac{i n}{2}} (1 + i a x)^{-\frac{i n}{2}}}{c x} - \frac{1}{c (2 + i n)}$$

$$2 a n (1 - i a x)^{1 + \frac{i n}{2}} (1 + i a x)^{-1 - \frac{i n}{2}} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{i n}{2}, 2 + \frac{i n}{2}, \frac{1 - i a x}{1 + i a x}\right]$$

**Problem 346: Result valid but suboptimal antiderivative.**

$$\int \frac{e^{n \operatorname{ArcTan}[a x]}}{x^3 (c + a^2 c x^2)} dx$$

Optimal (type 5, 126 leaves, 6 steps):

$$\frac{i a^2 e^{n \operatorname{ArcTan}[a x]} (-2 + i n + n^2)}{2 c n} - \frac{e^{n \operatorname{ArcTan}[a x]}}{2 c x^2} - \frac{a e^{n \operatorname{ArcTan}[a x]} n}{2 c x} - \frac{1}{c n}$$

$$i a^2 e^{n \operatorname{ArcTan}[a x]} (-2 + n^2) \operatorname{Hypergeometric2F1}\left[1, -\frac{i n}{2}, 1 - \frac{i n}{2}, e^{2 i \operatorname{ArcTan}[a x]}\right]$$

Result (type 5, 242 leaves, 6 steps):

$$\begin{aligned}
& - \frac{a^2 (2i + n - in^2) (1 - ia x)^{\frac{in}{2}} (1 + ia x)^{-\frac{in}{2}}}{2cn} - \\
& \frac{(1 - ia x)^{\frac{in}{2}} (1 + ia x)^{-\frac{in}{2}}}{2cx^2} - \frac{an (1 - ia x)^{\frac{in}{2}} (1 + ia x)^{-\frac{in}{2}}}{2cx} + \frac{1}{c(2 + in)} \\
& a^2 (2 - n^2) (1 - ia x)^{1+\frac{in}{2}} (1 + ia x)^{-1-\frac{in}{2}} \text{Hypergeometric2F1}\left[1, 1 + \frac{in}{2}, 2 + \frac{in}{2}, \frac{1 - ia x}{1 + ia x}\right]
\end{aligned}$$

Test results for the 153 problems in "5.3.7 Inverse tangent functions.m"

Test results for the 234 problems in "5.4.1 Inverse cotangent functions.m"

Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCot}[a + bx]}{c + dx^2} dx$$

Optimal (type 4, 642 leaves, 15 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}-\sqrt{d}x)}{(b\sqrt{c}+(1-i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{ib(\sqrt{c}+i\sqrt{d}x)}{(b\sqrt{c}-(1+i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} - \\
& \frac{\text{Log}\left[-\frac{i-a-bx}{a+bx}\right] \text{Log}\left[\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+(1+i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{Log}\left[\frac{i+a+bx}{a+bx}\right] \text{Log}\left[-\frac{b(i\sqrt{c}+\sqrt{d}x)}{(b\sqrt{c}+i(i+a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} + \\
& \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}-ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}-(1+i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{(b\sqrt{c}+ia\sqrt{d})(i-a-bx)}{(b\sqrt{c}+(1+i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} - \\
& \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}-ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+(1-i)a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{(b\sqrt{c}+ia\sqrt{d})(i+a+bx)}{(b\sqrt{c}+i(i+a)\sqrt{d}}\right]}{4\sqrt{c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 655 leaves, 37 steps):

$$\begin{aligned}
& \frac{i \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \left( \operatorname{Log}\left[-\frac{i-a-bx}{a+bx}\right] + \operatorname{Log}[a+bx] - \operatorname{Log}[-i+a+bx] \right)}{2 \sqrt{c} \sqrt{d}} - \\
& \frac{i \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \left( \operatorname{Log}[a+bx] - \operatorname{Log}[i+a+bx] + \operatorname{Log}\left[\frac{i+a+bx}{a+bx}\right] \right)}{2 \sqrt{c} \sqrt{d}} + \\
& \frac{i \operatorname{Log}[-i+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{-c}-\sqrt{d} x)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} - \frac{i \operatorname{Log}[i+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{-c}-\sqrt{d} x)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} - \\
& \frac{i \operatorname{Log}[-i+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{-c}+\sqrt{d} x)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} + \frac{i \operatorname{Log}[i+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{-c}+\sqrt{d} x)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} + \\
& \frac{i \operatorname{PolyLog}\left[2, -\frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}-(i-a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} - \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{d}(i-a-bx)}{b\sqrt{-c}+(i-a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} + \\
& \frac{i \operatorname{PolyLog}\left[2, -\frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}-(i+a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}} - \frac{i \operatorname{PolyLog}\left[2, \frac{\sqrt{d}(i+a+bx)}{b\sqrt{-c}+(i+a)\sqrt{d}}\right]}{4 \sqrt{-c} \sqrt{d}}
\end{aligned}$$

---

Test results for the 12 problems in "5.4.2 Exponentials of inverse cotangent.m"

---

Test results for the 174 problems in "5.5.1 u (a+b arcsec(c x))^n.m"

---

Test results for the 50 problems in "5.5.2 Inverse secant functions.m"

---

Test results for the 178 problems in "5.6.1 u (a+b arccsc(c x))^n.m"

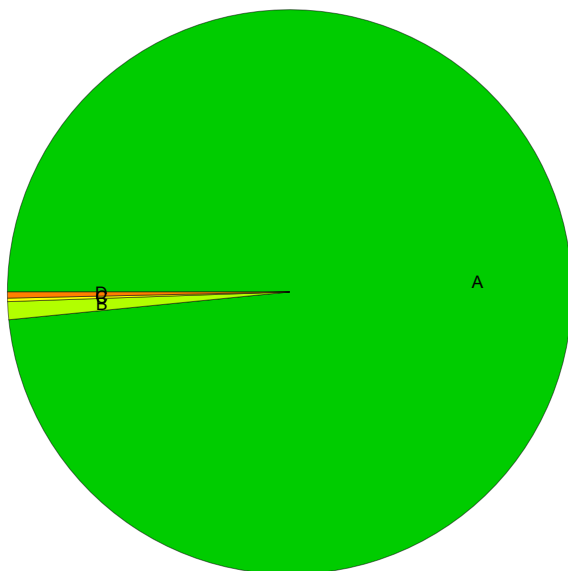
---

Test results for the 49 problems in "5.6.2 Inverse cosecant functions.m"

---

## Summary of Integration Test Results

4585 integration problems



A - 4513 optimal antiderivatives

B - 47 valid but suboptimal antiderivatives

C - 9 unnecessarily complex antiderivatives

D - 16 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives