# Mathematica 11.3 Integration Test Results

# Test results for the 311 problems in "8.1 Error functions.m"

Problem 26: Unable to integrate problem.

$$\int \frac{\mathsf{Erf}[\,b\,x\,]^{\,2}}{x^{3}}\,\mathrm{d}x$$

Optimal (type 4, 67 leaves, 5 steps):

$$-\frac{2\,b\,\,\mathrm{e}^{-b^2\,x^2}\,\mathsf{Erf}[\,b\,\,x\,]}{\sqrt{\pi}\,\,x}\,-\,b^2\,\,\mathsf{Erf}[\,b\,\,x\,]^{\,2}\,-\,\frac{\mathsf{Erf}[\,b\,\,x\,]^{\,2}}{2\,x^2}\,+\,\frac{2\,b^2\,\,\mathsf{ExpIntegralEi}\big[\,-\,2\,b^2\,\,x^2\,\big]}{\pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\mathsf{Erf}[b\,x]^2}{x^3}\,\mathrm{d}x$$

Problem 27: Unable to integrate problem.

$$\int\!\frac{\text{Erf}[\,b\,x\,]^{\,2}}{x^{5}}\,\text{d}x$$

Optimal (type 4, 125 leaves, 8 steps):

$$-\frac{b^2 e^{-2 b^2 x^2}}{3 \pi x^2} - \frac{b e^{-b^2 x^2} Erf[b x]}{3 \sqrt{\pi} x^3} + \frac{2 b^3 e^{-b^2 x^2} Erf[b x]}{3 \sqrt{\pi} x} + \frac{1}{3 b^4} Erf[b x]^2 - \frac{Erf[b x]^2}{4 x^4} - \frac{4 b^4 ExpIntegralEi[-2 b^2 x^2]}{3 \pi}$$

Result (type 8, 12 leaves):

$$\int\!\frac{\text{Erf}[\,b\,x\,]^{\,2}}{x^{5}}\,\text{d}x$$

Problem 28: Unable to integrate problem.

$$\int\!\frac{\text{Erf}\,[\,b\,\,x\,]^{\,2}}{x^{7}}\,\text{d}\,x$$

Optimal (type 4, 177 leaves, 12 steps):

$$-\frac{b^2 \, \mathrm{e}^{-2 \, b^2 \, x^2}}{15 \, \pi \, x^4} + \frac{2 \, b^4 \, \mathrm{e}^{-2 \, b^2 \, x^2}}{9 \, \pi \, x^2} - \frac{2 \, b \, \mathrm{e}^{-b^2 \, x^2} \, \mathrm{Erf} [\, b \, x\,]}{15 \, \sqrt{\pi} \, x^5} + \frac{4 \, b^3 \, \mathrm{e}^{-b^2 \, x^2} \, \mathrm{Erf} [\, b \, x\,]}{45 \, \sqrt{\pi} \, x^3} - \frac{8 \, b^5 \, \mathrm{e}^{-b^2 \, x^2} \, \mathrm{Erf} [\, b \, x\,]}{45 \, \sqrt{\pi} \, x} - \frac{4}{45} \, b^6 \, \mathrm{Erf} [\, b \, x\,]^2 - \frac{\mathrm{Erf} [\, b \, x\,]^2}{6 \, x^6} + \frac{28 \, b^6 \, \mathrm{ExpIntegralEi} \big[\, -2 \, b^2 \, x^2 \big]}{45 \, \pi}$$

Result (type 8, 12 leaves):

$$\int\!\frac{\text{Erf}\,[\,b\,x\,]^{\,2}}{x^{7}}\,\text{d}\,x$$

#### Problem 72: Unable to integrate problem.

$$\int e^{c+b^2 x^2} \operatorname{Erf}[b x] dx$$

Optimal (type 5, 29 leaves, 1 step):

b e c x 2 Hypergeometric  
PFQ [ {1, 1}, {
$$\frac{3}{2}$$
, 2}, b 2 x 2 ]  $\sqrt{\pi}$ 

Result (type 8, 18 leaves):

$$\int e^{c+b^2 x^2} \operatorname{Erf}[b x] dx$$

#### Problem 98: Unable to integrate problem.

$$\left[ \mathsf{Cos} \left[ \mathsf{c} + \dot{\mathtt{i}} \; \mathsf{b}^2 \; \mathsf{x}^2 \right] \; \mathsf{Erf} \left[ \; \mathsf{b} \; \mathsf{x} \; \right] \; \mathrm{d} \mathsf{x} \right]$$

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{\text{e}^{\text{ic}\,\sqrt{\pi}\,\,\text{Erf}\,[\,b\,\,x\,]^{\,2}}}{\text{8}\,b}\,+\,\frac{\text{b}\,\,\text{e}^{-\text{ic}\,\,x^{\,2}\,\,\text{HypergeometricPFQ}\left[\,\{\,1,\,\,1\,\}\,,\,\,\left\{\,\frac{3}{2}\,,\,\,2\,\right\}\,,\,\,b^{\,2}\,\,x^{\,2}\,\right]}{2\,\sqrt{\pi}}$$

Result (type 8, 20 leaves):

$$\int Cos[c + ib^2 x^2] Erf[bx] dx$$

## Problem 99: Unable to integrate problem.

$$\int Cos[c - i b^2 x^2] Erf[b x] dx$$

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{\text{e}^{-\text{ic}\,\sqrt{\pi}\,\,\text{Erf}\,[\,b\,\,x\,]^{\,2}}}{8\,\,b}\,+\,\frac{b\,\,\text{e}^{\,\text{ic}}\,\,x^{\,2}\,\text{HypergeometricPFQ}\big[\,\{\,\textbf{1,\,1}\,\}\,,\,\,\big\{\frac{\,3\,}{\,2}\,,\,\,2\,\big\}\,,\,\,b^{\,2}\,\,x^{\,2}\,\big]}{2\,\,\sqrt{\pi}}$$

Result (type 8, 20 leaves):

$$\int Cos \left[ c - i b^2 x^2 \right] Erf[b x] dx$$

# Problem 129: Unable to integrate problem.

$$\int\!\frac{\text{Erfc}\,[\,b\;x\,]^{\,2}}{x^{3}}\,\text{d}x$$

Optimal (type 4, 67 leaves, 5 steps):

$$\frac{2\,b\,\,\mathrm{e}^{-b^2\,x^2}\,\mathsf{Erfc}\,[\,b\,\,x\,]}{\sqrt{\pi}\,\,x}\,-\,b^2\,\,\mathsf{Erfc}\,[\,b\,\,x\,]^{\,2}\,-\,\frac{\,\mathsf{Erfc}\,[\,b\,\,x\,]^{\,2}}{2\,x^2}\,+\,\frac{2\,b^2\,\,\mathsf{ExpIntegralEi}\,\big[\,-\,2\,\,b^2\,\,x^2\,\big]}{\pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\text{Erfc}\,[\,b\,\,x\,]^{\,2}}{x^3}\,\text{d}x$$

## Problem 130: Unable to integrate problem.

$$\int \frac{\text{Erfc}\,[\,b\,\,x\,]^{\,2}}{x^5}\,\text{d}x$$

Optimal (type 4, 125 leaves, 8 steps):

$$-\frac{b^2 e^{-2 b^2 x^2}}{3 \pi x^2} + \frac{b e^{-b^2 x^2} \operatorname{Erfc} [b x]}{3 \sqrt{\pi} x^3} - \frac{2 b^3 e^{-b^2 x^2} \operatorname{Erfc} [b x]}{3 \sqrt{\pi} x} + \frac{1}{3} b^4 \operatorname{Erfc} [b x]^2 - \frac{\operatorname{Erfc} [b x]^2}{4 x^4} - \frac{4 b^4 \operatorname{ExpIntegralEi} [-2 b^2 x^2]}{3 \pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\mathrm{Erfc}\,[\,b\,\,x\,]^{\,2}}{x^{5}}\,\mathrm{d}x$$

## Problem 131: Unable to integrate problem.

$$\int \frac{\mathsf{Erfc}\,[\,b\,\,x\,]^{\,2}}{\mathsf{x}^{7}}\,\mathrm{d}x$$

Optimal (type 4, 177 leaves, 12 steps):

$$-\frac{b^{2} e^{-2 b^{2} x^{2}}}{15 \pi x^{4}}+\frac{2 b^{4} e^{-2 b^{2} x^{2}}}{9 \pi x^{2}}+\frac{2 b e^{-b^{2} x^{2}} Erfc[b x]}{15 \sqrt{\pi} x^{5}}-\frac{4 b^{3} e^{-b^{2} x^{2}} Erfc[b x]}{45 \sqrt{\pi} x^{3}}+\frac{8 b^{5} e^{-b^{2} x^{2}} Erfc[b x]}{45 \sqrt{\pi} x}-\frac{4}{45} b^{6} Erfc[b x]^{2}-\frac{Erfc[b x]^{2}}{6 x^{6}}+\frac{28 b^{6} ExpIntegralEi[-2 b^{2} x^{2}]}{45 \pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\mathsf{Erfc}\,[\,b\,\,x\,]^{\,2}}{x^7}\,\mathrm{d}x$$

## Problem 138: Unable to integrate problem.

$$\int (c + dx)^2 \operatorname{Erfc}[a + bx]^2 dx$$

Optimal (type 4, 375 leaves, 16 steps):

$$\frac{d \left(b \, c - a \, d\right) \, e^{-2 \, (a + b \, x)^2}}{b^3 \, \pi} + \frac{d^2 \, e^{-2 \, (a + b \, x)^2} \, \left(a + b \, x\right)}{3 \, b^3 \, \pi} - \frac{\left(b \, c - a \, d\right)^2 \, \sqrt{\frac{2}{\pi}} \, \operatorname{Erf}\left[\sqrt{2} \, \left(a + b \, x\right)\right]}{b^3} - \frac{5 \, d^2 \, \operatorname{Erf}\left[\sqrt{2} \, \left(a + b \, x\right)\right]}{6 \, b^3 \, \sqrt{2 \, \pi}} - \frac{2 \, d^2 \, e^{-(a + b \, x)^2} \, \operatorname{Erfc}\left[a + b \, x\right]}{3 \, b^3 \, \sqrt{\pi}} - \frac{2 \, \left(b \, c - a \, d\right)^2 \, e^{-(a + b \, x)^2} \, \operatorname{Erfc}\left[a + b \, x\right]}{b^3 \, \sqrt{\pi}} - \frac{2 \, d^2 \, e^{-(a + b \, x)^2} \, \left(a + b \, x\right)^2 \, \operatorname{Erfc}\left[a + b \, x\right]}{3 \, b^3 \, \sqrt{\pi}} - \frac{2 \, d^2 \, e^{-(a + b \, x)^2} \, \left(a + b \, x\right)^2 \, \operatorname{Erfc}\left[a + b \, x\right]}{3 \, b^3 \, \sqrt{\pi}} - \frac{d \, \left(b \, c - a \, d\right) \, \operatorname{Erfc}\left[a + b \, x\right]^2}{2 \, b^3} + \frac{\left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right) \, \operatorname{Erfc}\left[a + b \, x\right]^2}{b^3} + \frac{d^2 \, \left(a + b \, x\right)^3 \, \operatorname{Erfc}\left[a + b \, x\right]^2}{3 \, b^3}$$

Result (type 8, 18 leaves):

$$\int (c + dx)^2 \operatorname{Erfc}[a + bx]^2 dx$$

## Problem 175: Unable to integrate problem.

$$\int e^{c+b^2 x^2} \operatorname{Erfc}[b x] dx$$

Optimal (type 5, 50 leaves, 3 steps):

$$\frac{\text{e}^{\text{c}}\,\sqrt{\pi}\,\,\text{Erfi}\,[\,\text{b}\,\,x\,]}{2\,\text{b}}\,\,-\,\,\frac{\text{b}\,\,\text{e}^{\text{c}}\,\,x^2\,\,\text{HypergeometricPFQ}\big[\,\{\text{1, 1}\}\,,\,\,\big\{\frac{3}{2}\,,\,\,2\big\}\,,\,\,\text{b}^2\,\,x^2\,\big]}{\sqrt{\pi}}$$

Result (type 8, 18 leaves):

$$\int e^{c+b^2 x^2} \operatorname{Erfc}[b x] dx$$

## Problem 201: Unable to integrate problem.

$$\int Cos \left[c + i b^2 x^2\right] Erfc[b x] dx$$

Optimal (type 5, 85 leaves, 6 steps):

$$-\frac{\mathrm{e}^{\mathrm{i}\,\mathrm{c}\,\sqrt{\pi}\,\,\mathrm{Erfc}\,[\,\mathrm{b}\,\mathrm{x}\,]^{\,2}}{8\,\mathrm{b}} + \frac{\mathrm{e}^{-\mathrm{i}\,\mathrm{c}\,\sqrt{\pi}\,\,\mathrm{Erfi}\,[\,\mathrm{b}\,\mathrm{x}\,]}}{4\,\mathrm{b}} - \frac{\mathrm{b}\,\mathrm{e}^{-\mathrm{i}\,\mathrm{c}\,\,\mathrm{x}^{\,2}\,\mathrm{HypergeometricPFQ}\big[\,\{\mathrm{1,\,1}\}\,,\,\big\{\frac{3}{2}\,,\,2\big\}\,,\,\,\mathrm{b}^{2}\,\mathrm{x}^{2}\big]}{2\,\sqrt{\pi}}$$

#### Result (type 8, 20 leaves):

$$\int Cos[c + i b^2 x^2] Erfc[b x] dx$$

## Problem 202: Unable to integrate problem.

$$\int Cos \left[c - i b^2 x^2\right] \, Erfc \left[b x\right] \, dx$$

#### Optimal (type 5, 85 leaves, 6 steps):

$$-\frac{\mathrm{e}^{-\mathrm{i}\,c}\,\sqrt{\pi}\,\,\mathrm{Erfc}\,[\,b\,x\,]^{\,2}}{8\,b}+\frac{\mathrm{e}^{\mathrm{i}\,c}\,\sqrt{\pi}\,\,\mathrm{Erfi}\,[\,b\,x\,]}{4\,b}-\frac{b\,\mathrm{e}^{\mathrm{i}\,c}\,x^{\,2}\,\mathrm{HypergeometricPFQ}\big[\,\{1,\,1\}\,,\,\big\{\frac{3}{2},\,2\big\}\,,\,b^{\,2}\,x^{\,2}\big]}{2\,\sqrt{\pi}}$$

#### Result (type 8, 20 leaves):

$$\int Cos[c - i b^2 x^2] Erfc[b x] dx$$

## Problem 228: Unable to integrate problem.

$$\int x^5 \operatorname{Erfi}[b x]^2 dx$$

#### Optimal (type 4, 175 leaves, 12 steps):

$$\begin{split} &\frac{11 \, e^{2 \, b^2 \, x^2}}{12 \, b^6 \, \pi} - \frac{7 \, e^{2 \, b^2 \, x^2} \, x^2}{12 \, b^4 \, \pi} + \frac{e^{2 \, b^2 \, x^2} \, x^4}{6 \, b^2 \, \pi} - \frac{5 \, e^{b^2 \, x^2} \, x \, \text{Erfi} \, [\, b \, x\,]}{4 \, b^5 \, \sqrt{\pi}} + \\ &\frac{5 \, e^{b^2 \, x^2} \, x^3 \, \text{Erfi} \, [\, b \, x\,]}{6 \, b^3 \, \sqrt{\pi}} - \frac{e^{b^2 \, x^2} \, x^5 \, \text{Erfi} \, [\, b \, x\,]}{3 \, b \, \sqrt{\pi}} + \frac{5 \, \text{Erfi} \, [\, b \, x\,]^2}{16 \, b^6} + \frac{1}{6} \, x^6 \, \text{Erfi} \, [\, b \, x\,]^2 \end{split}$$

#### Result (type 8, 12 leaves):

$$\int x^5 \operatorname{Erfi}[b x]^2 dx$$

## Problem 229: Unable to integrate problem.

$$\int x^3 \operatorname{Erfi}[b x]^2 dx$$

#### Optimal (type 4, 124 leaves, 8 steps):

$$-\frac{\mathrm{e}^{2\,b^{2}\,x^{2}}}{2\,b^{4}\,\pi}+\frac{\mathrm{e}^{2\,b^{2}\,x^{2}}\,x^{2}}{4\,b^{2}\,\pi}+\frac{3\,\mathrm{e}^{b^{2}\,x^{2}}\,x\,\mathrm{Erfi}\,[\,b\,x\,]}{4\,b^{3}\,\sqrt{\pi}}-\frac{\mathrm{e}^{b^{2}\,x^{2}}\,x^{3}\,\mathrm{Erfi}\,[\,b\,x\,]}{2\,b\,\sqrt{\pi}}-\frac{3\,\mathrm{Erfi}\,[\,b\,x\,]^{\,2}}{16\,b^{4}}+\frac{1}{4}\,x^{4}\,\mathrm{Erfi}\,[\,b\,x\,]^{\,2}$$

#### Result (type 8, 12 leaves):

$$\int x^3 \operatorname{Erfi}[b \, x]^2 \, dx$$

#### Problem 230: Unable to integrate problem.

$$\int x \operatorname{Erfi}[b x]^2 dx$$

Optimal (type 4, 71 leaves, 5 steps):

$$\frac{e^{2\,b^2\,x^2}}{2\,b^2\,\pi} = \frac{e^{b^2\,x^2}\,x\,\text{Erfi}\,[\,b\,\,x\,]}{b\,\sqrt{\pi}} + \frac{\text{Erfi}\,[\,b\,\,x\,]^{\,2}}{4\,b^2} + \frac{1}{2}\,x^2\,\text{Erfi}\,[\,b\,\,x\,]^{\,2}$$

Result (type 8, 10 leaves):

$$\int x \operatorname{Erfi}[bx]^2 dx$$

## Problem 232: Unable to integrate problem.

$$\int\!\frac{\text{Erfi}\,[\,b\,\,x\,]^{\,2}}{x^{3}}\,\text{d}x$$

Optimal (type 4, 65 leaves, 5 steps):

$$-\frac{2\,b\,e^{b^2\,x^2}\,\text{Erfi}\,[\,b\,x\,]}{\sqrt{\pi}\,\,x}\,+\,b^2\,\text{Erfi}\,[\,b\,x\,]^{\,2}\,-\,\frac{\text{Erfi}\,[\,b\,x\,]^{\,2}}{2\,x^2}\,+\,\frac{2\,b^2\,\text{ExpIntegralEi}\,\big[\,2\,b^2\,x^2\,\big]}{\pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\text{Erfi}\,[\,b\;x\,]^{\,2}}{x^{3}}\,\text{d}x$$

## Problem 233: Unable to integrate problem.

$$\int \frac{\text{Erfi}[b\,x]^2}{x^5}\,\mathrm{d}x$$

Optimal (type 4, 123 leaves, 8 steps):

$$-\frac{b^{2} e^{2 b^{2} x^{2}}}{3 \pi x^{2}} - \frac{b e^{b^{2} x^{2}} Erfi[b x]}{3 \sqrt{\pi} x^{3}} - \frac{2 b^{3} e^{b^{2} x^{2}} Erfi[b x]}{3 \sqrt{\pi} x} + \frac{1}{3} b^{4} Erfi[b x]^{2} - \frac{Erfi[b x]^{2}}{4 x^{4}} + \frac{4 b^{4} ExpIntegralEi[2 b^{2} x^{2}]}{3 \pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\mathrm{Erfi}[b\,x]^2}{x^5} \,\mathrm{d}x$$

# Problem 234: Unable to integrate problem.

$$\int\!\frac{\text{Erfi}\,[\,b\;x\,]^{\,2}}{x^{7}}\,\text{d}x$$

Optimal (type 4, 174 leaves, 12 steps):

$$-\frac{b^{2} e^{2 b^{2} x^{2}}}{15 \pi x^{4}} - \frac{2 b^{4} e^{2 b^{2} x^{2}}}{9 \pi x^{2}} - \frac{2 b e^{b^{2} x^{2}} Erfi[b x]}{15 \sqrt{\pi} x^{5}} - \frac{4 b^{3} e^{b^{2} x^{2}} Erfi[b x]}{45 \sqrt{\pi} x^{3}} - \frac{8 b^{5} e^{b^{2} x^{2}} Erfi[b x]}{45 \sqrt{\pi} x} + \frac{4}{45} b^{6} Erfi[b x]^{2} - \frac{Erfi[b x]^{2}}{6 x^{6}} + \frac{28 b^{6} ExpIntegralEi[2 b^{2} x^{2}]}{45 \pi}$$

Result (type 8, 12 leaves):

$$\int \frac{\text{Erfi}\,[\,b\,\,x\,]^{\,2}}{x^7}\,\text{d}x$$

#### Problem 241: Unable to integrate problem.

$$\int (c + dx)^2 \operatorname{Erfi}[a + bx]^2 dx$$

Optimal (type 4, 366 leaves, 16 steps):

$$\frac{d \left(b \, c - a \, d\right) \, e^{2 \, (a + b \, x)^2}}{b^3 \, \pi} + \frac{d^2 \, e^{2 \, (a + b \, x)^2} \, \left(a + b \, x\right)}{3 \, b^3 \, \pi} + \frac{2 \, d^2 \, e^{(a + b \, x)^2} \, Erfi \left[a + b \, x\right]}{3 \, b^3 \, \sqrt{\pi}} - \frac{2 \, \left(b \, c - a \, d\right)^2 \, e^{(a + b \, x)^2} \, Erfi \left[a + b \, x\right]}{b^3 \, \sqrt{\pi}} - \frac{2 \, d \, \left(b \, c - a \, d\right) \, e^{(a + b \, x)^2} \, \left(a + b \, x\right) \, Erfi \left[a + b \, x\right]}{b^3 \, \sqrt{\pi}} - \frac{2 \, d^2 \, e^{(a + b \, x)^2} \, \left(a + b \, x\right)^2 \, Erfi \left[a + b \, x\right]}{3 \, b^3 \, \sqrt{\pi}} + \frac{d \, \left(b \, c - a \, d\right) \, Erfi \left[a + b \, x\right]^2}{2 \, b^3} + \frac{d \, \left(b \, c - a \, d\right) \, \left(a + b \, x\right)^2 \, Erfi \left[a + b \, x\right]^2}{b^3} + \frac{d \, \left(b \, c - a \, d\right) \, \left(a + b \, x\right)^2 \, Erfi \left[a + b \, x\right]^2}{b^3} + \frac{d \, \left(b \, c - a \, d\right) \, \left(a + b \, x\right)^2 \, Erfi \left[a + b \, x\right]^2}{b^3} - \frac{5 \, d^2 \, Erfi \left[\sqrt{2} \, \left(a + b \, x\right)\right]}{6 \, b^3 \, \sqrt{2 \, \pi}} + \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(b \, c - a \, d\right)^2 \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(a + b \, x\right)^2}{b^3} - \frac{d \, \left(a + b$$

Result (type 8, 18 leaves):

$$\int (c + dx)^2 \operatorname{Erfi}[a + bx]^2 dx$$

## Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int (c + dx) \operatorname{Erfi}[a + bx]^2 dx$$

Optimal (type 4, 184 leaves, 10 steps):

$$\frac{d \, e^{2 \, (a+b \, x)^{\, 2}}}{2 \, b^{2} \, \pi} - \frac{2 \, \left(b \, c - a \, d\right) \, e^{\, (a+b \, x)^{\, 2}} \, \text{Erfi} \, [\, a + b \, x\,]}{b^{2} \, \sqrt{\pi}} - \frac{d \, e^{\, (a+b \, x)^{\, 2}} \, \left(a + b \, x\right) \, \text{Erfi} \, [\, a + b \, x\,]}{b^{2} \, \sqrt{\pi}} + \frac{d \, \text{Erfi} \, [\, a + b \, x\,]^{\, 2}}{4 \, b^{2}} + \frac{\left(b \, c - a \, d\right) \, \left(a + b \, x\right) \, \text{Erfi} \, [\, a + b \, x\,]^{\, 2}}{b^{2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2} \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}} + \frac{d \, \left(a + b \, x\right)^{\, 2}}{b^{\, 2}}$$

Result (type 4, 189 leaves):

$$\begin{split} \frac{1}{4\,b^2\,\pi} \\ \left( \left( 4\,a\,b\,c + d - 2\,a^2\,d \right) \,\pi\,\text{Erfc} \left[ -\,\dot{\mathbb{1}}\, \left( a + b\,x \right) \,\right] \,\text{Erfc} \left[ \,\dot{\mathbb{1}}\, \left( a + b\,x \right) \,\right] + 2\, \left( d\,e^{2\,\left( a + b\,x \right)^{\,2}} + 4\,a\,b\,c\,\pi + d\,\pi - 2\,a^2\,d\,\pi + 2\,\dot{\mathbb{1}}\,b\,c\,\sqrt{2\,\pi} \, - 2\,\dot{\mathbb{1}}\,a\,d\,\sqrt{2\,\pi} \, - 2\,e^{\,\left( a + b\,x \right)^{\,2}}\,\sqrt{\pi}\, \left( 2\,b\,c - a\,d + b\,d\,x \right) \,\text{Erfi} \left[ a + b\,x \right] + b^2\,\pi\,x\, \left( 2\,c + d\,x \right) \,\text{Erfi} \left[ a + b\,x \right]^{\,2} + 2\, \left( b\,c - a\,d \right)\,\sqrt{2\,\pi}\,\,\text{Erfi} \left[ \sqrt{2}\, \left( a + b\,x \right) \,\right] \right) \end{split}$$

#### Problem 280: Unable to integrate problem.

$$\int \frac{e^{-b^2 \, x^2} \, \text{Erfi} \, [\, b \, x\,]}{x^2} \, \text{d} x$$

Optimal (type 5, 60 leaves, 3 steps):

$$-\frac{\text{e}^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,x\,]}{x}\,-\,\frac{2\,b^3\,x^2\,\text{HypergeometricPFQ}\big[\,\{1,\,1\}\,,\,\big\{\frac{3}{2}\,,\,2\big\}\,,\,-b^2\,x^2\,\big]}{\sqrt{\pi}}\,+\,\frac{2\,b\,\text{Log}\,[\,x\,]}{\sqrt{\pi}}$$

Result (type 9, 26 leaves):

$$-\frac{1}{2}\,b\,\text{MeijerG}\!\left[\,\{\,\{\,0\,\}\,,\,\,\{\,1\,\}\,\,\}\,,\,\,\left\{\,\{\,0\,,\,\,0\,\}\,,\,\,\left\{\,-\,\frac{1}{2}\,\right\}\,\right\},\,\,b^2\,\,x^2\,\right]$$

## Problem 281: Unable to integrate problem.

$$\int\!\frac{\text{e}^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,x\,]}{x^4}\,\text{d}x$$

Optimal (type 5, 105 leaves, 5 steps)

$$-\frac{b}{3\,\sqrt{\pi}\,\,x^{2}}-\frac{e^{-b^{2}\,x^{2}}\,\text{Erfi}\,[\,b\,x\,]}{3\,x^{3}}+\frac{2\,b^{2}\,e^{-b^{2}\,x^{2}}\,\text{Erfi}\,[\,b\,x\,]}{3\,x}+\\ \frac{4\,b^{5}\,x^{2}\,\text{HypergeometricPFQ}\big[\,\{1,\,1\}\,,\,\big\{\frac{3}{2}\,,\,2\big\}\,,\,-b^{2}\,x^{2}\big]}{3\,\sqrt{\pi}}-\frac{4\,b^{3}\,\text{Log}\,[\,x\,]}{3\,\sqrt{\pi}}$$

Result (type 9, 29 leaves):

$$-\frac{\text{bMeijerG}\left[\left\{\{\emptyset\},\,\{2\}\right\},\,\left\{\{\emptyset,\,1\},\,\left\{-\frac{1}{2}\right\}\right\},\,b^2\,x^2\right]}{2\,x^2}$$

#### Problem 282: Unable to integrate problem.

$$\int\!\frac{\text{e}^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,x\,]}{x^6}\,\text{d}x$$

Optimal (type 5, 144 leaves, 7 steps):

$$-\frac{b}{10\,\sqrt{\pi}\,\,x^4} + \frac{2\,b^3}{15\,\sqrt{\pi}\,\,x^2} - \frac{e^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,\,x\,]}{5\,x^5} + \frac{2\,b^2\,e^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,\,x\,]}{15\,x^3} - \\ \frac{4\,b^4\,e^{-b^2\,x^2}\,\text{Erfi}\,[\,b\,\,x\,]}{15\,x} - \frac{8\,b^7\,x^2\,\text{HypergeometricPFQ}\big[\,\{1,\,1\}\,,\,\big\{\frac{3}{2}\,,\,2\big\}\,,\,-b^2\,x^2\big]}{15\,\sqrt{\pi}} + \frac{8\,b^5\,\text{Log}\,[\,x\,]}{15\,\sqrt{\pi}}$$

Result (type 9, 29 leaves):

$$-\frac{b \, \text{MeijerG}\left[\left\{\{\{\emptyset\}, \{3\}\}, \left\{\{\emptyset, 2\}, \left\{-\frac{1}{2}\right\}\right\}, b^2 \, x^2\right]}{2 \, x^4}\right]}{2 \, x^4}$$

## Problem 304: Unable to integrate problem.

$$\int \text{Erfi}[b \, x] \, \text{Sin} \Big[ c + i b^2 \, x^2 \Big] \, dx$$

Optimal (type 5, 67 leaves, 4 steps):

$$\frac{\text{i} \ \text{e}^{-\text{i} \ \text{c}} \ \sqrt{\pi} \ \text{Erfi} \ [\text{b} \ \text{x}]^{\, 2}}{\text{8} \ \text{b}} \ - \ \frac{\text{i} \ \text{b} \ \text{e}^{\text{i} \ \text{c}} \ \text{x}^{\, 2} \ \text{HypergeometricPFQ} \left[ \ \{\text{1, 1}\} \ , \ \left\{\frac{3}{2} \ , \ 2\right\} \ , \ -\text{b}^{\, 2} \ \text{x}^{\, 2} \right]}{2 \sqrt{\pi}}$$

Result (type 8, 20 leaves):

$$\int \text{Erfi}[b \, x] \, \text{Sin} \Big[c + i \, b^2 \, x^2\Big] \, dx$$

## Problem 305: Unable to integrate problem.

Optimal (type 5, 67 leaves, 4 steps):

$$-\frac{\,\,\mathrm{i}\,\,\mathrm{e}^{\mathrm{i}\,\mathrm{c}}\,\sqrt{\pi}\,\,\mathrm{Erfi}\,[\,b\,\,x\,]^{\,2}}{8\,b}\,+\,\,\frac{\,\,\mathrm{i}\,\,b\,\,\mathrm{e}^{-\mathrm{i}\,\mathrm{c}}\,\,x^{\,2}\,\,\mathrm{HypergeometricPFQ}\big[\,\{1,\,1\}\,\text{,}\,\,\big\{\frac{3}{2}\,\text{,}\,\,2\big\}\,\text{,}\,\,-\,b^{\,2}\,\,x^{\,2}\,\big]}{2\,\sqrt{\pi}}$$

Result (type 8, 20 leaves):

## Problem 306: Unable to integrate problem.

$$\left\lceil \mathsf{Cos}\left[\,\mathsf{c}\,+\,\mathtt{i}\,\,\mathsf{b}^2\,\mathsf{x}^2\,\right]\,\mathsf{Erfi}\left[\,\mathsf{b}\,\mathsf{x}\,\right]\,\,\mathrm{d}\,\mathsf{x}\right.$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{\text{e}^{-\text{ic}\,\sqrt{\pi}\,\,\text{Erfi}\,[\,b\,\,x\,]^{\,2}}}{8\,\,b}\,+\,\frac{b\,\,\text{e}^{\,\text{ic}}\,\,x^{\,2}\,\,\text{HypergeometricPFQ}\,\big[\,\{\,\mathbf{1},\,\,\mathbf{1}\,\}\,\,,\,\,\big\{\,\frac{3}{\,2}\,,\,\,2\,\big\}\,,\,\,-\,b^{\,2}\,\,x^{\,2}\,\big]}{2\,\sqrt{\pi}}$$

Result (type 8, 20 leaves):

$$\int Cos[c + i b^2 x^2] Erfi[b x] dx$$

# Problem 307: Unable to integrate problem.

$$\int Cos \left[ c - i b^2 x^2 \right] Erfi[b x] dx$$

Optimal (type 5, 63 leaves, 4 steps):

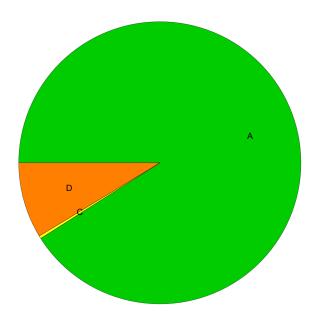
$$\frac{\text{e}^{\text{ic}\,\sqrt{\pi}\,\,\text{Erfi}\,[\,b\,\,x\,]^{\,2}}}{8\,b}\,+\,\frac{b\,\,\text{e}^{-\text{ic}\,\,x^{\,2}\,\,\text{HypergeometricPFQ}\left[\,\,\{\,1,\,\,1\,\}\,\,,\,\,\left\{\,\frac{3}{2}\,,\,\,2\,\right\}\,,\,\,\,-\,\,b^{\,2}\,\,x^{\,2}\,\right]}}{2\,\,\sqrt{\pi}}$$

Result (type 8, 20 leaves):

$$\int Cos[c - i b^2 x^2] Erfi[b x] dx$$

# **Summary of Integration Test Results**

#### 311 integration problems



- A 283 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 1 unnecessarily complex antiderivatives
- D 27 unable to integrate problems
- E 0 integration timeouts