# Rubi 4.16.0.4 Integration Test Results

# on the problems in the test-suite directory "4 Trig functions"

Test results for the 538 problems in "4.1.0 (a sin)^m (b trg)^n.m"

Test results for the 348 problems in "4.1.10 (c+d x)^m (a+b sin)^n.m"

Test results for the 72 problems in "4.1.1.1 (a+b sin)^n.m"

Test results for the 113 problems in "4.1.11 (e x)^m (a+b x^n)^p sin.m"

Test results for the 357 problems in "4.1.12 (e x)^m (a+b sin(c+d x^n))^p.m"

Test results for the 653 problems in "4.1.1.2 (g cos)^p (a+b sin)^m.m"

Problem 648: Result valid but suboptimal antiderivative.

$$\int \left(e\, Cos\, [\, c\, +d\, x\, ]\,\right)^{\,-3-m}\, \left(a\, +\, b\, Sin\, [\, c\, +\, d\, x\, ]\,\right)^{\,m}\, \mathrm{d}x$$

Optimal (type 5, 311 leaves, ? steps):

$$\frac{\left(e \cos \left[c + d \, x\right]\right)^{-m} \, \text{Sec}\left[c + d \, x\right]^{4} \, \left(-1 + \sin \left[c + d \, x\right]\right) \, \left(1 + \sin \left[c + d \, x\right]\right) \, \left(a + b \sin \left[c + d \, x\right]\right)^{1+m}}{\left(a - b\right) \, d \, e^{3} \, \left(2 + m\right)} + \frac{1}{\left(a - b\right)^{2} \, d \, e^{3} \, m \, \left(2 + m\right)} \\ \left(-2 \, b + a \, \left(2 + m\right)\right) \, \left(e \cos \left[c + d \, x\right]\right)^{-m} \, \text{Sec}\left[c + d \, x\right]^{4} \, \left(-1 + \sin \left[c + d \, x\right]\right) \, \left(1 + \sin \left[c + d \, x\right]\right)^{2} \, \left(a + b \sin \left[c + d \, x\right]\right)^{1+m} - \\ \frac{1}{\left(a - b\right)^{3} \, d \, e^{3} \, m \, \left(1 + m\right)} \left(-b^{2} + a^{2} \, \left(1 + m\right)\right) \, \left(e \cos \left[c + d \, x\right]\right)^{-m} \, \text{Hypergeometric} \\ \text{Sec}\left[c + d \, x\right]^{4} \, \left(1 + \sin \left[c + d \, x\right]\right)^{3} \, \left(\frac{\left(a + b\right) \, \left(1 + \sin \left[c + d \, x\right]\right)}{\left(a - b\right) \, \left(-1 + \sin \left[c + d \, x\right]\right)} \right)^{\frac{1}{2} \, \left(-2 + m\right)} \, \left(a + b \sin \left[c + d \, x\right]\right)^{1+m}$$

Result (type 5, 420 leaves, 5 steps):

$$\frac{\left(e \cos \left[c + d \, x\right]\right)^{-2-m} \, \left(a + b \, \sin \left[c + d \, x\right]\right)^{1+m}}{\left(a - b\right) \, d \, e \, \left(2 + m\right)} - \\ \left(b \, \left(e \cos \left[c + d \, x\right]\right)^{-2-m} \, \text{Hypergeometric} \\ 2F1 \left[1 + m, \, \frac{2+m}{2}, \, 2+m, \, \frac{2 \, \left(a + b \, \sin \left[c + d \, x\right]\right)}{\left(a + b\right) \, \left(1 + \sin \left[c + d \, x\right]\right)}\right] \, \left(1 - \sin \left[c + d \, x\right]\right) \, \left(-\frac{\left(a - b\right) \, \left(1 - \sin \left[c + d \, x\right]\right)}{\left(a + b\right) \, \left(1 + \sin \left[c + d \, x\right]\right)} \right)^{m/2} \\ \left(a + b \, \sin \left[c + d \, x\right]\right)^{1+m} \right) \bigg/ \, \left(\left(a^2 - b^2\right) \, d \, e \, \left(1 + m\right) \, \left(2 + m\right)\right) + \frac{a \, \left(e \, \cos \left[c + d \, x\right]\right)^{-2-m} \, \left(1 + \sin \left[c + d \, x\right]\right) \, \left(a + b \, \sin \left[c + d \, x\right]\right)^{1+m}}{\left(a^2 - b^2\right) \, d \, e \, \left(2 + m\right)} \\ \left(2^{-m/2} \, a \, \left(a + b + a \, m\right) \, \left(e \, \cos \left[c + d \, x\right]\right)^{-2-m} \, \text{Hypergeometric} \\ 2F1 \left[-\frac{m}{2}, \, \frac{2+m}{2}, \, \frac{2-m}{2}, \, \frac{\left(a - b\right) \, \left(1 - \sin \left[c + d \, x\right]\right)}{2 \, \left(a + b \, \sin \left[c + d \, x\right]\right)}\right] \\ \left(1 - \sin \left[c + d \, x\right]\right) \, \left(\frac{\left(a + b\right) \, \left(1 + \sin \left[c + d \, x\right]\right)}{a + b \, \sin \left[c + d \, x\right]}\right)^{\frac{2-m}{2}} \, \left(a + b \, \sin \left[c + d \, x\right]\right)^{1+m} \right) / \, \left(\left(a - b\right) \, \left(a + b\right)^2 \, d \, e \, m \, \left(2 + m\right)\right)$$

Test results for the 36 problems in "4.1.13 (d+e x)^m sin(a+b x+c x^2)^n.m"

Test results for the 208 problems in "4.1.1.3 (g tan)^p (a+b sin)^m.m"

Test results for the 837 problems in "4.1.2.1 (a+b sin)^m (c+d sin)^n.m"

Test results for the 1563 problems in "4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n.m"

Problem 1479: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} \left[ e + f \, x \right]^2 \, \left( a + b \, \operatorname{Sin} \left[ e + f \, x \right] \right)^{3/2}}{\sqrt{d \, \operatorname{Sin} \left[ e + f \, x \right]}} \, \mathrm{d} x$$

Optimal (type 4, 312 leaves, ? steps):

$$\frac{\operatorname{Sec}\left[e+fx\right]\left(b+a\operatorname{Sin}\left[e+fx\right]\right)\sqrt{a+b\operatorname{Sin}\left[e+fx\right]}}{f\sqrt{d\operatorname{Sin}\left[e+fx\right]}} - \frac{f\sqrt{d\operatorname{Sin}\left[e+fx\right]}}{\left(a+b\right)^{3/2}\sqrt{-\frac{a\left(-1+\operatorname{Csc}\left[e+fx\right]\right)}{a+b}}}\sqrt{\frac{a\left(1+\operatorname{Csc}\left[e+fx\right]\right)}{a-b}}}{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d}\sqrt{a+b\operatorname{Sin}\left[e+fx\right]}}{\sqrt{a+b}\sqrt{d\operatorname{Sin}\left[e+fx\right]}}\right], -\frac{a+b}{a-b}\right]\operatorname{Tan}\left[e+fx\right]}{\sqrt{d}f} - \frac{\sqrt{d}f}{\left(a+b\right)\sqrt{-\frac{a\left(-1+\operatorname{Csc}\left[e+fx\right]\right)}{a+b}}\sqrt{\frac{b+a\operatorname{Csc}\left[e+fx\right]}{-a+b}}}}{\left(b\left(a+b\right)\sqrt{-\frac{a\left(-1+\operatorname{Csc}\left[e+fx\right]\right)}{a+b}}\sqrt{\frac{b+a\operatorname{Csc}\left[e+fx\right]}{-a+b}}}\right]\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a\operatorname{Csc}\left[e+fx\right]}{a-b}}\right], \frac{-a+b}{a+b}\right]\left(1+\operatorname{Sin}\left[e+fx\right]\right)\operatorname{Tan}\left[e+fx\right]\right)}{\sqrt{d\operatorname{Sin}\left[e+fx\right]}}$$

$$\left(f\sqrt{\frac{a\left(1+\operatorname{Csc}\left[e+fx\right]\right)}{a-b}}\sqrt{d\operatorname{Sin}\left[e+fx\right]}\sqrt{a+b\operatorname{Sin}\left[e+fx\right]}\right)$$

Result (type 8, 37 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\operatorname{Sec}[e+fx]^{2}(a+b\operatorname{Sin}[e+fx])^{3/2}}{\sqrt{d\operatorname{Sin}[e+fx]}},x\right]$$

## Problem 1480: Unable to integrate problem.

$$\int \frac{Sec \left[e+fx\right]^4 \, \left(a+b \, Sin \left[e+fx\right]\right)^{5/2}}{\sqrt{d \, Sin \left[e+fx\right]}} \, \mathrm{d}x$$

Optimal (type 4, 366 leaves, ? steps):

Result (type 8, 87 leaves, 1 step):

$$\frac{\text{Sec}\left[\text{e}+\text{f}\,\text{x}\right]^{3}\,\sqrt{\text{d}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]}\,\left(\text{a}+\text{b}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]\right)^{5/2}}{3\,\text{d}\,\text{f}}+\frac{5}{6}\,\text{a}\,\text{Unintegrable}\left[\frac{\text{Sec}\left[\text{e}+\text{f}\,\text{x}\right]^{2}\,\left(\text{a}+\text{b}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]\right)^{3/2}}{\sqrt{\text{d}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]}},\,\text{x}\right]$$

## Problem 1515: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} [e + f x]^{6} (a + b \operatorname{Sin} [e + f x])^{9/2}}{\sqrt{d \operatorname{Sin} [e + f x]}} dx$$

Optimal (type 4, 502 leaves, ? steps):

$$\frac{3 \, a \, b \, \left(-2 \, a^2 + b^2\right) \, Cos\left[e + f \, x\right] \, \sqrt{a + b \, Sin\left[e + f \, x\right]}}{5 \, f \, \sqrt{d \, Sin\left[e + f \, x\right]}} + \frac{5 \, f \, \sqrt{d \, Sin\left[e + f \, x\right]}}{5 \, d \, f} + \frac{1}{20 \, d \, f$$

Result (type 8, 87 leaves, 1 step):

$$\frac{\text{Sec}\left[e+fx\right]^{5}\sqrt{\text{d}\,\text{Sin}\left[e+fx\right]}}{5\,\text{d}\,f} + \frac{9}{10}\,\text{a}\,\text{Unintegrable}\left[\frac{\text{Sec}\left[e+fx\right]^{4}\,\left(a+b\,\text{Sin}\left[e+fx\right]\right)^{7/2}}{\sqrt{\text{d}\,\text{Sin}\left[e+fx\right]}}\text{, }x\right]$$

Test results for the 51 problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

Test results for the 358 problems in "4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin).m"

Test results for the 19 problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

Test results for the 34 problems in "4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2).m"

Test results for the 594 problems in "4.1.7 (d trig)^m (a+b (c sin)^n)^p.m"

Problem 391: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} [c + d x]^{2}}{a + b \operatorname{Sin} [c + d x]^{3}} dx$$

Optimal (type 3, 299 leaves, ? steps):

$$\frac{2 \, \left(-1\right)^{2/3} \, b^{2/3} \, \text{ArcTan} \left[ \, \frac{(-1)^{1/3} \, b^{1/3} - a^{1/3} \, \text{Tan} \left[ \frac{1}{2} \, \left( c + d \, x \right) \, \right]}{\sqrt{a^{2/3} - \left(-1\right)^{2/3} \, b^{2/3}}} \, \right]}{3 \, a^{2/3} \, \left(a^{2/3} - \left(-1\right)^{2/3} \, b^{2/3}\right)^{3/2} \, d} \, - \, \frac{2 \, b^{2/3} \, \text{ArcTan} \left[ \, \frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[ \frac{1}{2} \, \left( c + d \, x \right) \, \right]}{\sqrt{a^{2/3} - b^{2/3}}} \, \right]}{3 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a^{2/3} \, \left(a^{2/3} - b^{2/3}\right)^{3/2} \, d} \, + \, \frac{1}{2 \, a$$

$$\frac{2 \left(-1\right)^{1/3} b^{2/3} \operatorname{ArcTan} \left[\frac{\left(-1\right)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan} \left[\frac{1}{2} \left(c + d \, x\right)\right]}{\sqrt{a^{2/3} + \left(-1\right)^{1/3} b^{2/3}}}\right]}{3 \, a^{2/3} \left(a^{2/3} + \left(-1\right)^{1/3} b^{2/3}\right)^{3/2} d} + \frac{\operatorname{Sec} \left[c + d \, x\right] \, \left(b - a \, \operatorname{Sin} \left[c + d \, x\right]\right)}{\left(-a^2 + b^2\right) \, d}$$

Result (type 8, 25 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\operatorname{Sec}[c+dx]^2}{a+b\operatorname{Sin}[c+dx]^3}, x\right]$$

Problem 392: Unable to integrate problem.

$$\int \frac{\operatorname{Sec} [c + dx]^4}{a + b \operatorname{Sin} [c + dx]^3} dx$$

Optimal (type 3, 1093 leaves, ? steps):

$$-\frac{2 \left(-1\right)^{2/3} \, a^{2/3} \, b^{8/3} \, \text{ArcTan} \left[\frac{(-1)^{3/5} b^{1/3} a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}} \right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d} \\ -\frac{2 \, b^2 \, \left(2 \, a^2 + b^2\right) \, \text{ArcTan} \left[\frac{(-1)^{3/5} b^{3/3} - a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \, a^{2/3} \, \sqrt{a^{2/3} - b^{2/3}}} + \frac{2 \, b^2 \, \left(2 \, a^2 + b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{\sqrt{a^{2/3} - b^{2/3}}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - b^{2/3}}} \right]}{3 \, a^{2/3} \, \sqrt{a^{2/3} - b^{2/3}}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}} \right]}{3 \, \sqrt{a^{2/3} - b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}} \right]}{3 \, \sqrt{a^{2/3} - b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} \, b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} \, b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} \, b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} \, b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d}} \right]}{3 \, \sqrt{(-1)^{1/3} \, a^{2/3} + b^{2/3} \, \left(a^2 - b^2\right)^2 \, d}} + \frac{2 \, b^{4/3} \, \left(a^2 + 2 \, b^2\right) \, \text{ArcTan} \left[\frac{b^{1/3} + a^{1/3} \, \text{Tan} \left[\frac{1}{2} \, (\text{c} + \text{d} \, x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} \, b^{2/3} \, b^{2/3}} \, \left(a^2 - b^2\right)^2 \, d}} \right]} + \frac{2 \, b^{4/3$$

Result (type 8, 25 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\operatorname{Sec}[c+dx]^4}{a+b\operatorname{Sin}[c+dx]^3}, x\right]$$

## Problem 593: Unable to integrate problem.

$$\left\lceil \sqrt{\,a\,+\,\left(c\,Cos\,[\,e\,+\,f\,x\,]\,+\,b\,Sin\,[\,e\,+\,f\,x\,]\,\right)^{\,2}}\,\,\text{d}x\right.$$

Optimal (type 4, 79 leaves, 3 steps):

$$\frac{\text{EllipticE}\left[e+fx+\text{ArcTan}\left[b,c\right],-\frac{b^2+c^2}{a}\right]\sqrt{a+\left(c\,\text{Cos}\left[e+f\,x\right]+b\,\text{Sin}\left[e+f\,x\right]\right)^2}}{f\sqrt{1+\frac{\left(c\,\text{Cos}\left[e+f\,x\right]+b\,\text{Sin}\left[e+f\,x\right]\right)^2}{a}}}$$

Result (type 8, 115 leaves, 3 steps):

$$\begin{split} &\frac{1}{2} \; \text{$\dot{\text{i}}$ CannotIntegrate} \Big[ \frac{\text{Sec} \, [\, e + f \, x \, ] \, ^2 \, \sqrt{a + \text{Cos} \, [\, e + f \, x \, ] \, ^2 \, \left(c + b \, \text{Tan} \, [\, e + f \, x \, ] \, \right)^2}}{ & \text{$\dot{\text{i}} - \text{Tan} \, [\, e + f \, x \, ] \, }} \text{, } x \, \Big] + \\ &\frac{1}{2} \, \, \text{$\dot{\text{i}} \text{ CannotIntegrate}} \Big[ \frac{\text{Sec} \, [\, e + f \, x \, ] \, ^2 \, \sqrt{a + \text{Cos} \, [\, e + f \, x \, ] \, ^2 \, \left(c + b \, \text{Tan} \, [\, e + f \, x \, ] \, \right)^2}}{ & \text{$\dot{\text{i}} + \text{Tan} \, [\, e + f \, x \, ]}} \text{, } x \Big] \end{split}$$

## Problem 594: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a + (c \cos[e + fx] + b \sin[e + fx])^2}} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$\frac{\text{EllipticF}\big[\text{e+fx+ArcTan[b,c],}-\frac{b^2+c^2}{a}\big]\sqrt{1+\frac{(\text{cCos[e+fx]+bSin[e+fx]})^2}{a}}}{\text{f}\sqrt{a+\left(\text{cCos[e+fx]}+\text{bSin[e+fx]}\right)^2}}$$

Result (type 8, 115 leaves, 3 steps):

$$\frac{1}{2} \, \, \dot{\mathbb{I}} \, \, \mathsf{CannotIntegrate} \Big[ \frac{\mathsf{Sec} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,]^{\, 2}}{\left( \dot{\mathbb{I}} - \mathsf{Tan} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right) \, \sqrt{\mathsf{a} + \mathsf{Cos} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,]^{\, 2} \, \left( \mathsf{c} + \mathsf{b} \, \mathsf{Tan} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right)^{\, 2}} \,, \, \, \mathsf{x} \, \Big] + \\ \frac{1}{2} \, \, \dot{\mathbb{I}} \, \, \mathsf{CannotIntegrate} \Big[ \frac{\mathsf{Sec} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,]^{\, 2} \, \left( \mathsf{c} + \mathsf{b} \, \mathsf{Tan} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right)^{\, 2}}{\left( \dot{\mathbb{I}} + \mathsf{Tan} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right) \, \sqrt{\mathsf{a} + \mathsf{Cos} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,]^{\, 2} \, \left( \mathsf{c} + \mathsf{b} \, \mathsf{Tan} \, [\, \mathsf{e} + \mathsf{f} \, \mathsf{x} \,] \right)^{\, 2}} \,, \, \, \mathsf{x} \, \Big]$$

Test results for the 9 problems in "4.1.8 (a+b sin)^m (c+d trig)^n.m"

Test results for the 19 problems in "4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p.m"

Test results for the 294 problems in "4.2.0 (a cos)^m (b trg)^n.m"

Test results for the 189 problems in "4.2.10 (c+d x)^m (a+b cos)^n.m"

Test results for the 62 problems in "4.2.1.1 (a+b cos)^n.m"

Test results for the 99 problems in "4.2.12 (e x)^m (a+b cos(c+d x^n))^p.m"

Test results for the 88 problems in "4.2.1.2 (g sin)^p (a+b cos)^m.m"

Test results for the 34 problems in "4.2.13 (d+e x)^m cos(a+b x+c x^2)^n.m"

Test results for the 22 problems in "4.2.1.3 (g tan)^p (a+b cos)^m.m"

Test results for the 932 problems in "4.2.2.1 (a+b cos)^m (c+d cos)^n.m"

Test results for the 4 problems in "4.2.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n.m"

Test results for the 1 problems in "4.2.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n.m"

Test results for the 644 problems in "4.2.3.1 (a+b cos)^m (c+d cos)^n (A+B cos).m"

Test results for the 393 problems in "4.2.4.1 (a+b cos)^m (A+B cos+C cos^2).m"

Test results for the 1541 problems in "4.2.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2).m"

Test results for the 98 problems in "4.2.7 (d trig)^m (a+b (c cos)^n)^p.m"

Test results for the 21 problems in "4.2.8 (a+b cos)^m (c+d trig)^n.m"

Test results for the 20 problems in "4.2.9 trig^m (a+b cos^n+c cos^(2 n))^p.m"

Test results for the 387 problems in "4.3.0 (a trg)^m (b tan)^n.m"

Test results for the 63 problems in "4.3.10 (c+d x)^m (a+b tan)^n.m"

Problem 17: Unable to integrate problem.

$$\int \left( \frac{x^2}{\sqrt{\text{Tan} \left[ a + b \ x^2 \right]}} + \frac{\sqrt{\text{Tan} \left[ a + b \ x^2 \right]}}{b} + x^2 \, \text{Tan} \left[ a + b \ x^2 \right]^{3/2} \right) \, \text{d}x$$

Optimal (type 3, 17 leaves, ? steps):

$$\frac{x\,\sqrt{\,\text{Tan}\,\big[\,a\,+\,b\,\,x^2\,\big]}}{b}$$

Result (type 8, 55 leaves, 1 step):

$$\text{Unintegrable} \Big[ \frac{x^2}{\sqrt{\text{Tan} \big[ a + b \ x^2 \big]}} \text{, } x \Big] + \frac{\text{Unintegrable} \Big[ \sqrt{\text{Tan} \big[ a + b \ x^2 \big]} \text{ , } x \Big]}{b} + \text{Unintegrable} \Big[ x^2 \, \text{Tan} \big[ a + b \ x^2 \big]^{3/2} \text{, } x \Big]$$

Test results for the 66 problems in "4.3.11 (e x)^m (a+b tan(c+d x^n))^p.m"

Test results for the 700 problems in "4.3.1.2 (d sec)^m (a+b tan)^n.m"

Test results for the 91 problems in "4.3.1.3 (d sin)^m (a+b tan)^n.m"

Test results for the 1328 problems in "4.3.2.1 (a+b tan)^m (c+d tan)^n.m"

Test results for the 855 problems in "4.3.3.1 (a+b tan)^m (c+d tan)^n (A+B tan).m"

Test results for the 171 problems in "4.3.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2).m"

Test results for the 499 problems in "4.3.7 (d trig)^m (a+b (c tan)^n)^p.m"

Test results for the 51 problems in "4.3.9 trig^m (a+b tan^n+c tan^(2 n))^p.m"

Test results for the 52 problems in "4.4.0 (a trg)^m (b cot)^n.m"

Test results for the 61 problems in "4.4.10 (c+d x)^m (a+b cot)^n.m"

Test results for the 23 problems in "4.4.1.2 (d csc)^m (a+b cot)^n.m"

Test results for the 19 problems in "4.4.1.3 (d cos)^m (a+b cot)^n.m"

Test results for the 106 problems in "4.4.2.1 (a+b cot)^m (c+d cot)^n.m"

Test results for the 64 problems in "4.4.7 (d trig)^m (a+b (c cot)^n)^p.m"

Test results for the 32 problems in "4.4.9 trig^m (a+b cot^n+c cot^(2 n))^p.m"

Test results for the 299 problems in "4.5.0 (a sec)^m (b trg)^n.m"

Test results for the 46 problems in "4.5.10 (c+d x)^m (a+b sec)^n.m"

Test results for the 83 problems in "4.5.11 (e x)^m (a+b sec(c+d x^n))^p.m"

Test results for the 879 problems in "4.5.1.2 (d sec)^n (a+b sec)^m.m"

Problem 286: Result unnecessarily involves higher level functions.

$$\int Sec [c + dx]^{5/3} (a + a Sec [c + dx])^{2/3} dx$$

Optimal (type 5, 327 leaves, ? steps):

$$= \frac{3 \text{ a Sec} \left[ \text{c} + \text{d x} \right]^{5/3} \text{ Sin} \left[ \text{c} + \text{d x} \right]}{2 \text{ d } \left( \text{a } \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right) \right)^{2/3}} + \frac{9 \text{ Sec} \left[ \text{c} + \text{d x} \right]^{2/3} \left( \text{a } \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right) \right)^{2/3} \text{ Sin} \left[ \text{c} + \text{d x} \right]}{4 \text{ d}} - \frac{9 \left( \text{a } \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right) \right)^{2/3} \text{ Tan} \left[ \text{c} + \text{d x} \right]}{4 \text{ d} \left( \frac{1}{1 + \text{Cos} \left[ \text{c} + \text{d x} \right]} \right)^{1/3} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{1/3}} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{1/3} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{1/3} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{1/3} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{1/3} \right) - \left[ 5 \text{ Hypergeometric} 2F1 \left[ \frac{1}{3}, \frac{3}{4}, \frac{7}{4}, \text{ Tan} \left[ \frac{1}{2} \left( \text{c} + \text{d x} \right) \right]^{4} \right] \left( \text{Cos} \left[ \text{c} + \text{d x} \right] \text{ Sec} \left[ \frac{1}{2} \left( \text{c} + \text{d x} \right) \right]^{4} \right)^{1/3} \left( \text{a } \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right) \right)^{2/3} \text{ Tan} \left[ \text{c} + \text{d x} \right]^{3} \right) \right/ \left( 8 \text{ d} \left( \frac{1}{1 + \text{Cos} \left[ \text{c} + \text{d x} \right]} \right)^{1/3} \left( 1 + \text{Sec} \left[ \text{c} + \text{d x} \right] \right)^{10/3} \right) \right)$$

Result (type 6, 79 leaves, 3 steps):

$$\frac{1}{\text{d} \left(1 + \text{Sec}\left[c + \text{d}\,x\right]\right)^{7/6}} 2 \times 2^{1/6} \, \text{AppellF1}\left[\frac{1}{2}, -\frac{2}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \text{Sec}\left[c + \text{d}\,x\right], \frac{1}{2} \left(1 - \text{Sec}\left[c + \text{d}\,x\right]\right)\right] \left(\text{a} + \text{a}\,\text{Sec}\left[c + \text{d}\,x\right]\right)^{2/3} \, \text{Tan}\left[c + \text{d}\,x\right]$$

# Test results for the 306 problems in "4.5.1.3 (d sin)^n (a+b sec)^m.m"

#### Problem 271: Result optimal but 2 more steps used.

Optimal (type 5, 115 leaves, 4 steps):

$$\frac{\text{Hypergeometric2F1}\left[\texttt{1, 1+n, 2+n, } \frac{\texttt{a+b Sec}\left[\texttt{c+d x}\right]}{\texttt{a-b}} \left(\texttt{a+b Sec}\left[\texttt{c+d x}\right]\right)^{\texttt{1+n}}}{\texttt{2} \left(\texttt{a-b}\right) \texttt{d} \left(\texttt{1+n}\right)} - \frac{\text{Hypergeometric2F1}\left[\texttt{1, 1+n, 2+n, } \frac{\texttt{a+b Sec}\left[\texttt{c+d x}\right]}{\texttt{a+b}}\right] \left(\texttt{a+b Sec}\left[\texttt{c+d x}\right]\right)^{\texttt{1+n}}}{\texttt{2} \left(\texttt{a+b}\right) \texttt{d} \left(\texttt{1+n}\right)}$$

Result (type 5, 115 leaves, 6 steps):

$$\frac{\text{Hypergeometric2F1}\left[\textbf{1,1+n,2+n,} \frac{a+b\,\text{Sec}\left[c+d\,x\right]}{a-b}\right]\,\left(a+b\,\text{Sec}\left[c+d\,x\right]\right)^{1+n}}{2\,\left(a-b\right)\,d\,\left(1+n\right)} - \frac{\text{Hypergeometric2F1}\left[\textbf{1,1+n,2+n,} \frac{a+b\,\text{Sec}\left[c+d\,x\right]}{a+b}\right]\,\left(a+b\,\text{Sec}\left[c+d\,x\right]\right)^{1+n}}{2\,\left(a+b\right)\,d\,\left(1+n\right)}$$

## Problem 276: Unable to integrate problem.

Optimal (type 6, 424 leaves, ? steps):

$$-\frac{1}{2\sqrt{2}\,d}$$

$$3 \text{ AppellF1} \Big[ -\frac{1}{2}, \frac{5}{2}, -n, \frac{1}{2}, \frac{1}{2}\, \left( 1 - \text{Sec}[c + d\,x] \right), \frac{b\left( 1 - \text{Sec}[c + d\,x] \right)}{a + b} \Big] \, \text{Cot}[c + d\,x] \, \sqrt{1 + \text{Sec}[c + d\,x]} \, \left( a + b \, \text{Sec}[c + d\,x] \right)^n \left( \frac{a + b \, \text{Sec}[c + d\,x]}{a + b} \right)^{-n} - \frac{1}{6\sqrt{2}} \, \frac{1}{d} \, \text{AppellF1} \Big[ -\frac{3}{2}, \frac{5}{2}, -n, -\frac{1}{2}, \frac{1}{2}\, \left( 1 - \text{Sec}[c + d\,x] \right), \frac{b\left( 1 - \text{Sec}[c + d\,x] \right)}{a + b} \Big] \, \text{Cot}[c + d\,x]^3$$

$$(1 + \text{Sec}[c + d\,x])^{3/2} \, \left( a + b \, \text{Sec}[c + d\,x] \right)^n \left( \frac{a + b \, \text{Sec}[c + d\,x]}{a + b} \right)^{-n} + \frac{1}{\sqrt{2} \, d\sqrt{1 + \text{Sec}[c + d\,x]}}$$

$$\text{AppellF1} \Big[ \frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2}\, \left( 1 - \text{Sec}[c + d\,x] \right), \frac{b\left( 1 - \text{Sec}[c + d\,x] \right)}{a + b} \Big] \, \left( a + b \, \text{Sec}[c + d\,x] \right)^n \left( \frac{a + b \, \text{Sec}[c + d\,x]}{a + b} \right)^{-n} \, \text{Tan}[c + d\,x] + \frac{1}{2\sqrt{2} \, d\sqrt{1 + \text{Sec}[c + d\,x]}}$$

$$\text{AppellF1} \Big[ \frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2}\, \left( 1 - \text{Sec}[c + d\,x] \right), \frac{b\left( 1 - \text{Sec}[c + d\,x] \right)}{a + b} \Big] \, \left( a + b \, \text{Sec}[c + d\,x] \right)^n \left( \frac{a + b \, \text{Sec}[c + d\,x]}{a + b} \right)^{-n} \, \text{Tan}[c + d\,x]$$

$$\text{Result (type 8, 23 leaves, 0 steps):}$$

Unintegrable  $\left[\operatorname{Csc}\left[c+d\,x\right]^{4}\left(a+b\operatorname{Sec}\left[c+d\,x\right]\right)^{n},\,x\right]$ 

# Test results for the 365 problems in "4.5.1.4 (d tan)^n (a+b sec)^m.m"

Problem 207: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Tan}[e+fx]^2}{\left(a+a\,\mathsf{Sec}[e+fx]\right)^{9/2}}\,\mathrm{d}x$$

Optimal (type 3, 177 leaves, ? steps):

$$-\frac{2\, \text{ArcTan} \Big[\frac{\sqrt{a}\, \text{Tan}[e+f\,x]}{\sqrt{a+a}\, \text{Sec}[e+f\,x]}\Big]}{a^{9/2}\, f} + \frac{91\, \text{ArcTan} \Big[\frac{\sqrt{a}\, \text{Tan}[e+f\,x]}{\sqrt{2}\, \sqrt{a+a}\, \text{Sec}[e+f\,x]}\Big]}{32\, \sqrt{2}\, a^{9/2}\, f} + \frac{32\, \sqrt{2}\, a^{9/2}\, f}{11\, \text{Tan}[e+f\,x]} + \frac{27\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{27\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{27\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac{11\, \text{Tan}[e+f\,x]}{24\, a^2\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{5/2}} + \frac{11\, \text{Tan}[e+f\,x]}{32\, a^3\, f\, \left(a+a\, \text{Sec}[e+f\,x]\right)^{3/2}} + \frac$$

Result (type 3, 227 leaves, 7 steps):

$$-\frac{2\, \text{ArcTan} \Big[ \frac{\sqrt{a} \, \text{Tan} [e+f\,x]}{\sqrt{a+a} \, \text{Sec} [e+f\,x]} \Big]}{a^{9/2} \, f} + \frac{91\, \text{ArcTan} \Big[ \frac{\sqrt{a} \, \text{Tan} [e+f\,x]}{\sqrt{2} \, \sqrt{a+a} \, \text{Sec} [e+f\,x]} \Big]}{32 \, \sqrt{2} \, a^{9/2} \, f} + \frac{27 \, \text{Sec} \Big[ \frac{1}{2} \, \Big( e+f\,x \Big) \, \Big]^2 \, \text{Sin} [e+f\,x]}{64 \, a^4 \, f \, \sqrt{a+a} \, \text{Sec} [e+f\,x]}} + \frac{11 \, \text{Cos} [e+f\,x] \, \text{Sec} \Big[ \frac{1}{2} \, \Big( e+f\,x \Big) \, \Big]^4 \, \text{Sin} [e+f\,x]}{96 \, a^4 \, f \, \sqrt{a+a} \, \text{Sec} [e+f\,x]} + \frac{\text{Cos} [e+f\,x]^2 \, \text{Sec} \Big[ \frac{1}{2} \, \Big( e+f\,x \Big) \, \Big]^6 \, \text{Sin} [e+f\,x]}{24 \, a^4 \, f \, \sqrt{a+a} \, \text{Sec} [e+f\,x]}}$$

## Problem 347: Unable to integrate problem.

$$\int \frac{\left(d \, \mathsf{Tan} \, [\, e + f \, x\, ]\,\right)^n}{\mathsf{a} + \mathsf{b} \, \mathsf{Sec} \, [\, e + f \, x\, ]} \, \mathbb{d} x$$

Optimal (type 6, 266 leaves, ? steps):

$$\frac{1}{a\,f\,\left(1-n\right)}d\,\mathsf{AppellF1}\!\left[1-n,\,\frac{1-n}{2},\,\frac{1-n}{2},\,2-n,\,\frac{a+b}{a+b\,\mathsf{Sec}\,[e+f\,x]},\,\frac{a-b}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right]\left(-\frac{b\,\left(1-\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b\,\mathsf{Sec}\,[e+f\,x]}\right)^{\frac{1-n}{2}}\left(\frac{b\,\left(1+\mathsf{Sec}\,[e+f\,x]\right)}{a+b$$

Result (type 8, 25 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(d \operatorname{Tan}[e+fx]\right)^{n}}{a+b \operatorname{Sec}[e+fx]}, x\right]$$

# Test results for the 241 problems in "4.5.2.1 (a+b sec)^m (c+d sec)^n.m"

## Problem 217: Unable to integrate problem.

$$\int \frac{\left(c + d \operatorname{Sec}\left[e + f x\right]\right)^{3/2}}{\sqrt{a + b \operatorname{Sec}\left[e + f x\right]}} \, dx$$

Optimal (type 4, 652 leaves, ? steps):

$$-\left[\left(2\,c\,\left(c+d\right)\,\text{Cot}[e+fx]\,\text{EllipticPi}\left[\frac{a\,\left(c+d\right)}{\left(a+b\right)\,c},\,\text{ArcSin}\left[\sqrt{\frac{\left(a+b\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\right],\,\frac{\left(a-b\right)\,\left(c+d\right)}{\left(a+b\right)\,\left(c-d\right)}\right]\sqrt{\frac{\left(b\,c-a\,d\right)\,\left(1+\text{Sec}[e+fx]\right)}{\left(c-d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\right]$$

$$\left(a+b\,\text{Sec}\left[e+fx]\right)^{3/2}\sqrt{\frac{\left(a+b\right)\,\left(b\,c-a\,d\right)\,\left(-1+\text{Sec}[e+fx]\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\right]/\left(a+b\right)\,f\sqrt{c+d\,\text{Sec}[e+fx]}\right)}\right]} / \left(a+b\right)\,f\sqrt{c+d\,\text{Sec}[e+fx]}\right)$$

$$\left(a+b\,\text{Sec}\left[e+fx\right]\right)^{3/2}\sqrt{\frac{\left(a+b\right)\,\left(c+d\right)}{\left(a+b\right)\,d},\,\text{ArcSin}\left[\sqrt{\frac{\left(a+b\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\right],\,\frac{\left(a-b\right)\,\left(c+d\right)}{\left(a+b\right)\,\left(c-d\right)}\right]\sqrt{\frac{\left(b\,c-a\,d\right)\,\left(1+\text{Sec}[e+fx]\right)}{\left(c-d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\right)}$$

$$\left(a+b\,\text{Sec}\left[e+fx\right]\right)^{3/2}\sqrt{-\frac{\left(a+b\right)\,\left(-b\,c+a\,d\right)\,\left(-1+\text{Sec}[e+fx]\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,^2\left(a+b\,\text{Sec}[e+fx]\right)}}} / \left(b\,\left(a+b\right)\,f\sqrt{c+d\,\text{Sec}[e+fx]}\right) + \frac{1}{a\,b\,f\sqrt{\frac{\left(a+b\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}}}}{2\,\left(b\,c-a\,d\right)\,\text{Cot}\left[e+fx\right]\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(a+b\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}}\right],\,\frac{\left(a-b\right)\,\left(c+d\right)}{\left(a+b\right)\,\left(c-d\right)}\right]$$

$$\sqrt{\frac{\left(b\,c-a\,d\right)\,\left(-1+\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}}\sqrt{\frac{\left(a+b\right)\,\left(c+d\,\text{Sec}[e+fx]\right)}{\left(c+d\right)\,\left(a+b\,\text{Sec}[e+fx]\right)}}\sqrt{\sqrt{c+d\,\text{Sec}[e+fx]}}$$

Result (type 8, 31 leaves, 0 steps):

Unintegrable 
$$\left[\frac{\left(c+d\,Sec\,[\,e+f\,x\,]\right)^{3/2}}{\sqrt{a+b\,Sec\,[\,e+f\,x\,]}},\,x\right]$$

Test results for the 286 problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n.m"

Test results for the 634 problems in "4.5.3.1 (a+b sec)^m (d sec)^n (A+B sec).m"

Test results for the 70 problems in "4.5.4.1 (a+b sec)^m (A+B sec+C sec^2).m"

Test results for the 1373 problems in "4.5.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2).m"

Test results for the 470 problems in "4.5.7 (d trig)^m (a+b (c sec)^n)^p.m"

Problem 132: Unable to integrate problem.

$$\int \left( a + b \, \text{Sec} \left[ \, e + f \, x \, \right]^{\, 2} \right)^{\, p} \, \left( d \, \text{Sin} \left[ \, e + f \, x \, \right] \, \right)^{\, m} \, \mathrm{d} x$$

Optimal (type 6, 123 leaves, ? steps):

$$\begin{split} &\frac{1}{f\left(1+m\right)} AppellF1\Big[\,\frac{1+m}{2}\,,\,\,\frac{1}{2}+p\,,\,-p\,,\,\,\frac{3+m}{2}\,,\,\,Sin[\,e+f\,x\,]^{\,2}\,,\,\,\frac{a\,Sin[\,e+f\,x\,]^{\,2}}{a+b}\Big] \\ &\left(Cos\,[\,e+f\,x\,]^{\,2}\right)^{\frac{1}{2}+p}\,\left(a+b\,Sec\,[\,e+f\,x\,]^{\,2}\right)^{p}\,\left(d\,Sin[\,e+f\,x\,]\right)^{m}\,\left(\frac{a+b-a\,Sin[\,e+f\,x\,]^{\,2}}{a+b}\right)^{-p}\,Tan[\,e+f\,x\,] \end{split}$$

Result (type 8, 27 leaves, 0 steps):

Unintegrable 
$$\left[\left(a+b\,Sec\left[e+f\,x\right]^{2}\right)^{p}\,\left(d\,Sin\left[e+f\,x\right]\right)^{m}$$
,  $x\right]$ 

Problem 228: Result valid but suboptimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right]^{\,5}\,\sqrt{\,a + b\,Sec \left[\,e + f\,x\,\right]^{\,2}}\,\,\mathrm{d}x$$

Optimal (type 4, 372 leaves, 11 steps):

$$-\frac{\left(2\,a^{2}-3\,a\,b-8\,b^{2}\right)\,\text{Sin}[e+f\,x]\,\,\sqrt{\text{Sec}[e+f\,x]^{2}\,\left(a+b-a\,\text{Sin}[e+f\,x]^{2}\right)}}{15\,b^{2}\,f}+\frac{1}{15\,b^{2}\,f}\\ +\frac{1}{15\,b^{2}\,f\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^{2}}{a+b}}}\\ +\frac{1}{15\,b^{2}\,f\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^{2}}}}\\ +\frac{1}{15\,b^{2}\,f\,\sqrt{1-\frac{a\,\text{Si$$

#### Result (type 4, 471 leaves, 11 steps):

$$-\frac{\left(2\,a^{2}-3\,a\,b-8\,b^{2}\right)\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,Sin[\,e+f\,x]\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}}{15\,b^{2}\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}}\,+\\ \left(\left(2\,a^{2}-3\,a\,b-8\,b^{2}\right)\sqrt{Cos\,[e+f\,x]^{2}}\,EllipticE\left[ArcSin[Sin[\,e+f\,x]\,]\right]\,,\,\,\frac{a}{a+b}\right]\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}\,\right)\Big/\\ \left[15\,b^{2}\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\sqrt{1-\frac{a\,Sin[\,e+f\,x]^{2}}{a+b}}\right]-\\ \left((a-8\,b)\,\left(a+b\right)\sqrt{Cos\,[e+f\,x]^{2}}\,EllipticF\left[ArcSin[Sin[\,e+f\,x]\,]\right]\,,\,\,\frac{a}{a+b}\right]\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\sqrt{1-\frac{a\,Sin[\,e+f\,x]^{2}}{a+b}}\Big/\\ \left[15\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}\right)+\frac{\left(a+4\,b\right)\,Sec\,[e+f\,x]\,\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}\,Tan[\,e+f\,x]}{15\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}\,Tan[\,e+f\,x]}+\\ \frac{Sec\,[e+f\,x]^{3}\,\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\sqrt{a+b-a\,Sin[\,e+f\,x]^{2}}\,Tan[\,e+f\,x]}{5\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}}\,Tan[\,e+f\,x]}$$

## Problem 229: Result valid but suboptimal antiderivative.

$$\int Sec [e + f x]^3 \sqrt{a + b Sec [e + f x]^2} dx$$

Optimal (type 4, 288 leaves, 10 steps):

$$\frac{\left(a+2\,b\right)\,\text{Sin}\left[e+f\,x\right]\,\sqrt{\text{Sec}\left[e+f\,x\right]^{\,2}\,\left(a+b-a\,\text{Sin}\left[e+f\,x\right]^{\,2}\right)}}{3\,b\,f} - \frac{\left(a+2\,b\right)\,\sqrt{\text{Cos}\left[e+f\,x\right]^{\,2}}\,\,\text{EllipticE}\!\left[\text{ArcSin}\left[\text{Sin}\left[e+f\,x\right]\right]\,,\,\,\frac{a}{a+b}\right]\,\sqrt{\text{Sec}\left[e+f\,x\right]^{\,2}\,\left(a+b-a\,\text{Sin}\left[e+f\,x\right]^{\,2}\right)}}{3\,b\,f\,\sqrt{1-\frac{a\,\text{Sin}\left[e+f\,x\right]^{\,2}}{a+b}}}$$

$$\left[ 2 \left( \mathsf{a} + \mathsf{b} \right) \sqrt{\mathsf{Cos}\left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^2} \; \mathsf{EllipticF}\left[ \mathsf{ArcSin}\left[ \mathsf{Sin}\left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right] \right] , \; \frac{\mathsf{a}}{\mathsf{a} + \mathsf{b}} \right] \sqrt{\mathsf{Sec}\left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^2 \left( \mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin}\left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^2 \right)} \; \sqrt{1 - \frac{\mathsf{a} \, \mathsf{Sin}\left[ \mathsf{e} + \mathsf{f} \, \mathsf{x} \right]^2}{\mathsf{a} + \mathsf{b}}} \right] \right)$$

$$\left( 3\,f\, \left( a+b-a\,Sin\, [\,e+f\,x\,]^{\,2} \right) \right) \,+\, \frac{Sec\, [\,e+f\,x\,]^{\,2}\, \left( s+b-a\,Sin\, [\,e+f\,x\,]^{\,2} \right)}{3\,f} \,\, Tan\, [\,e+f\,x\,]^{\,2} \, \left( s+b-a\,Sin\, [\,e+f\,x\,]^{\,2} \right) \, Tan\, [\,e+f\,x\,]^{\,2} \, Tan\, [\,e+f\,x\,$$

Result (type 4, 364 leaves, 10 steps):

$$\frac{\left(a+2\,b\right)\,\sqrt{a+b\,Sec\,[e+f\,x]^{\,2}}\,\,Sin\,[e+f\,x]\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{\,2}}}{3\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{\,2}}} - \\ \left(\left(a+2\,b\right)\,\sqrt{Cos\,[e+f\,x]^{\,2}}\,\,EllipticE\,\big[ArcSin\,[Sin\,[e+f\,x]\,]\,\,,\,\,\frac{a}{a+b}\big]\,\,\sqrt{a+b\,Sec\,[e+f\,x]^{\,2}}}\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{\,2}}\,\,\right) / \\ \left(3\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{\,2}}\,\,\sqrt{1-\frac{a\,Sin\,[e+f\,x]^{\,2}}{a+b}}\right) + \\ \left(3\,b\,f\,\sqrt{b+a$$

$$\frac{2\left(\mathsf{a}+\mathsf{b}\right)\sqrt{\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}\;\mathsf{EllipticF}\left[\mathsf{ArcSin}\left[\mathsf{Sin}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right],\,\frac{\mathsf{a}}{\mathsf{a}+\mathsf{b}}\right]\sqrt{\mathsf{a}+\mathsf{b}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}\,\sqrt{1-\frac{\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a}+\mathsf{b}}}}\\ \frac{3\,\mathsf{f}\sqrt{\mathsf{b}+\mathsf{a}\,\mathsf{Cos}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}\,\sqrt{\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}}{\sqrt{\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}\;\mathsf{Tan}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}\\ \frac{\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\,\sqrt{\mathsf{a}+\mathsf{b}\,\mathsf{Sec}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}{\sqrt{\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}\;\mathsf{Tan}\left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]}$$

#### Problem 230: Result valid but suboptimal antiderivative.

$$\int Sec [e + fx] \sqrt{a + b Sec [e + fx]^2} dx$$

Optimal (type 4, 218 leaves, 10 steps):

$$\frac{\operatorname{Sin}[e+fx] \sqrt{\operatorname{Sec}[e+fx]^2 (a+b-a \operatorname{Sin}[e+fx]^2)}}{f}$$

$$\frac{\sqrt{\text{Cos}\left[e+fx\right]^2} \ \text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right], \frac{a}{a+b}\right] \sqrt{\text{Sec}\left[e+fx\right]^2 \left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)}}{f\sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}} + \frac{1}{f\left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)}$$

$$\left(a+b\right) \sqrt{\text{Cos}\left[e+fx\right]^2} \text{ EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right]\right], \ \frac{a}{a+b}\right] \sqrt{\text{Sec}\left[e+fx\right]^2 \left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)} \sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}$$

Result (type 4, 271 leaves, 10 steps):

$$\frac{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,} \,\, \text{Sin}\,[e+f\,x]\,\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}}{f\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}} - \frac{\sqrt{\cos\,[e+f\,x]^2\,\,} \,\, \sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}}{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,} \,\, \sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}}{\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,} \,\, \sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}}{\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,} \,\, \sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}}{\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2\,\,}} + \frac{\sqrt{a+b\,\text{Sec}$$

#### Problem 231: Result valid but suboptimal antiderivative.

$$\int cos[e+fx] \sqrt{a+b Sec[e+fx]^2} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{\sqrt{\text{Cos}\,[e+f\,x]^{\,2}}}{\int \sqrt{\text{Los}\,[e+f\,x]^{\,2}}} \, \left[\text{Sin}\,[e+f\,x]^{\,2}\right] \sqrt{\text{Sec}\,[e+f\,x]^{\,2}} \left(a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}\right)} + \int \sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^{\,2}}{a+b}} \, dx$$

Result (type 4, 103 leaves, 5 steps):

$$\frac{\sqrt{\text{Cos}\left[e+fx\right]^2} \; \text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right], \; \frac{a}{a+b}\right] \sqrt{a+b\,\text{Sec}\left[e+fx\right]^2}}{\sqrt{b+a\,\text{Cos}\left[e+fx\right]^2}} \sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}$$

# Problem 232: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx]^3 \sqrt{a+b} \, Sec[e+fx]^2 \, dx$$

Optimal (type 4, 246 leaves, 9 steps):

$$\frac{\text{Cos}\left[e+fx\right]^{2} \, \text{Sin}\left[e+fx\right] \, \sqrt{\text{Sec}\left[e+fx\right]^{2} \, \left(a+b-a \, \text{Sin}\left[e+fx\right]^{2}\right)}}{3 \, f} + \frac{\left(2 \, a+b\right) \, \sqrt{\text{Cos}\left[e+fx\right]^{2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right], \, \frac{a}{a+b}\right] \, \sqrt{\text{Sec}\left[e+fx\right]^{2} \, \left(a+b-a \, \text{Sin}\left[e+fx\right]^{2}\right)}}{3 \, a \, f \, \sqrt{1-\frac{a \, \text{Sin}\left[e+fx\right]^{2}}{a+b}}} - \frac{3 \, a \, f \, \sqrt{1-\frac{a \, \text{Sin}\left[e+fx\right]^{2}}{a+b}}}{3 \, a \, f \, \sqrt{1-\frac{a \, \text{Sin}\left[e+fx\right]^{2}}{a+b}}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a \, \text{Sin}\left[e+fx\right]^{2}}{3 \, a \, b}} - \frac{1 \, a$$

Result (type 4, 299 leaves, 9 steps):

 $(3 a f (a + b - a Sin[e + fx]^2))$ 

$$\frac{\text{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2\sqrt{\mathsf{a} + \mathsf{b}\,\text{Sec}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}}{\mathsf{3}\,\mathsf{f}\,\sqrt{\mathsf{b} + \mathsf{a}\,\text{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}}} + \frac{\mathsf{3}\,\mathsf{f}\,\sqrt{\mathsf{b} + \mathsf{a}\,\text{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}}}{\left(\left(2\,\mathsf{a} + \mathsf{b}\right)\sqrt{\mathsf{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}\right) \text{EllipticE}\left[\mathsf{ArcSin}\left[\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]\right]\right], \frac{\mathsf{a}}{\mathsf{a} + \mathsf{b}}\right]\sqrt{\mathsf{a} + \mathsf{b}\,\mathsf{Sec}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}}\sqrt{\mathsf{a} + \mathsf{b} - \mathsf{a}\,\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}\right) / \frac{\mathsf{b}\,\left(\mathsf{a} + \mathsf{b}\right)\sqrt{\mathsf{cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}}\sqrt{\mathsf{1} - \frac{\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a} + \mathsf{b}}}} - \frac{\mathsf{b}\,\left(\mathsf{a} + \mathsf{b}\right)\sqrt{\mathsf{cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}} \\ \mathsf{3}\,\mathsf{a}\,\mathsf{f}\,\sqrt{\mathsf{b} + \mathsf{a}\,\mathsf{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2} \;\; \mathsf{EllipticF}\left[\mathsf{ArcSin}\left[\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]\right], \frac{\mathsf{a}}{\mathsf{a} + \mathsf{b}}\right]\sqrt{\mathsf{a} + \mathsf{b}\,\mathsf{Sec}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}} \sqrt{\mathsf{1} - \frac{\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a} + \mathsf{b}}}$$

#### Problem 233: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx]^5 \sqrt{a+b} \, Sec[e+fx]^2 \, dx$$

Optimal (type 4, 338 leaves, 10 steps):

Optimal (type 4, 338 leaves, TO steps): 
$$\frac{2 \, \left( 2\, a - b \right) \, \mathsf{Cos} \left[ e + f \, x \right]^2 \, \mathsf{Sin} \left[ e + f \, x \right] \, \sqrt{\mathsf{Sec} \left[ e + f \, x \right]^2 \, \left( a + b - a \, \mathsf{Sin} \left[ e + f \, x \right]^2 \right)} }{15\, a \, f} + \frac{15\, a \, f}{15\, a^2 \, f \, \sqrt{1 - \frac{a \, \mathsf{Sin} \left[ e + f \, x \right]^2}{a + b}}} + \frac{1}{15\, a^2 \, f \, \sqrt{1 - \frac{a \, \mathsf{Sin} \left[ e + f \, x \right]^2}{a + b}}}$$

$$\left( 8\, a^2 + 3\, a \, b - 2\, b^2 \right) \, \sqrt{\mathsf{Cos} \left[ e + f \, x \right]^2} \, \, \mathsf{EllipticE} \left[ \mathsf{ArcSin} \left[ \mathsf{Sin} \left[ e + f \, x \right] \right] \, , \, \frac{a}{a + b} \right] \, \sqrt{\mathsf{Sec} \left[ e + f \, x \right]^2 \, \left( a + b - a \, \mathsf{Sin} \left[ e + f \, x \right]^2 \right)} \, - \frac{a \, \mathsf{Sin} \left[ e + f \, x \right]^2}{a + b} \right]$$

$$\left( 15\, a^2 \, f \, \left( a + b - a \, \mathsf{Sin} \left[ e + f \, x \right]^2 \right) \, \right)$$

Result (type 4, 400 leaves, 10 steps):

$$\frac{2 \left(2 \, a - b\right) \, Cos \left[e + f \, x\right]^2 \, \sqrt{a + b \, Sec \left[e + f \, x\right]^2} \, Sin \left[e + f \, x\right] \, \sqrt{a + b - a \, Sin \left[e + f \, x\right]^2}}{15 \, a \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2}} + \\ \frac{Cos \left[e + f \, x\right]^2 \, \sqrt{a + b \, Sec \left[e + f \, x\right]^2} \, Sin \left[e + f \, x\right] \, \left(a + b - a \, Sin \left[e + f \, x\right]^2\right)^{3/2}}{5 \, a \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2}} + \\ \frac{5 \, a \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2} \, Sin \left[e + f \, x\right]^2}{5 \, a \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2}} \, EllipticE \left[ArcSin \left[Sin \left[e + f \, x\right]\right], \, \frac{a}{a + b}\right] \, \sqrt{a + b \, Sec \left[e + f \, x\right]^2} \, \sqrt{a + b - a \, Sin \left[e + f \, x\right]^2}\right) / \\ \left[15 \, a^2 \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2} \, \sqrt{1 - \frac{a \, Sin \left[e + f \, x\right]^2}{a + b}}\right] - \\ \left[2 \, \left(2 \, a - b\right) \, b \, \left(a + b\right) \, \sqrt{Cos \left[e + f \, x\right]^2} \, EllipticF \left[ArcSin \left[Sin \left[e + f \, x\right]\right], \, \frac{a}{a + b}\right] \, \sqrt{a + b \, Sec \left[e + f \, x\right]^2} \, \sqrt{1 - \frac{a \, Sin \left[e + f \, x\right]^2}{a + b}}\right) / \\ \left[15 \, a^2 \, f \, \sqrt{b + a \, Cos \left[e + f \, x\right]^2} \, \sqrt{a + b - a \, Sin \left[e + f \, x\right]^2}\right)$$

#### Problem 241: Result valid but suboptimal antiderivative.

$$\int Sec[e+fx]^5 (a+b Sec[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 450 leaves, 12 steps):

Result (type 4, 572 leaves, 12 steps):

$$-\frac{2 \left(a + 2 b\right) \left(a^{2} - 4 a b - 4 b^{2}\right) \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \operatorname{Sin}[e + f x] \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}}{35 b^{2} f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}}} + \frac{35 b^{2} f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}}}{\left(2 \left(a + 2 b\right) \left(a^{2} - 4 a b - 4 b^{2}\right) \sqrt{\operatorname{Cos}[e + f x]^{2}} \operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}] \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}}\right)}{\left(35 b^{2} f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^{2}}{a + b}}\right) - \left((a + b) \left(a^{2} - 16 a b - 16 b^{2}\right) \sqrt{\operatorname{Cos}[e + f x]^{2}} \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[e + f x]], \frac{a}{a + b}] \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \sqrt{1 - \frac{a \operatorname{Sin}[e + f x]^{2}}{a + b}}\right)}\right) + \frac{\left(a^{2} + 11 a b + 8 b^{2}\right) \operatorname{Sec}[e + f x] \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}} \operatorname{Tan}[e + f x]}{35 b f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}} \operatorname{Tan}[e + f x]} + \frac{2 \left(4 a + 3 b\right) \operatorname{Sec}[e + f x]^{3} \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}} \operatorname{Tan}[e + f x]}{35 f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}} \operatorname{Tan}[e + f x]} + \frac{b \operatorname{Sec}[e + f x]^{3} \sqrt{a + b \operatorname{Sec}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}}{7 f \sqrt{b + a \operatorname{Cos}[e + f x]^{2}} \sqrt{a + b - a \operatorname{Sin}[e + f x]^{2}}} \operatorname{Tan}[e + f x]}$$

# Problem 242: Result valid but suboptimal antiderivative.

$$\int Sec[e+fx]^3 (a+b Sec[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 371 leaves, 11 steps):

$$\frac{\left(3\,a^2+13\,a\,b+8\,b^2\right)\,\text{Sin}[e+f\,x]\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}{15\,b\,f} - \frac{1}{15\,b\,f\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^2}{a+b}}} \\ - \frac{1}{15\,b\,f\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^2}{a+b}}} \\ \left(3\,a^2+13\,a\,b+8\,b^2\right)\,\,\sqrt{\text{Cos}[e+f\,x]^2\,\,\,\text{EllipticE}\big[\text{ArcSin}[\text{Sin}[e+f\,x]]\,\,,\,\,\frac{a}{a+b}\big]\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}} + \\ \left((a+b)\,\,\big(9\,a+8\,b\big)\,\,\sqrt{\text{Cos}[e+f\,x]^2\,\,\,\,\text{EllipticF}\big[\text{ArcSin}[\text{Sin}[e+f\,x]]\,\,,\,\,\frac{a}{a+b}\big]\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}\,\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^2}{a+b}} \right) \\ \left(15\,f\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)\big) + \frac{2\,\,\big(3\,a+2\,b\big)\,\,\text{Sec}[e+f\,x]\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}\,\,\,\text{Tan}[e+f\,x]}{15\,f} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}\,\,\,\text{Tan}[e+f\,x]}{5\,f} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}{6\,a+b+a\,\text{Sin}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}{6\,a+b+a\,\text{Sin}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}{6\,a+b+a\,\text{Sin}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}{6\,a+b+a\,\text{Sin}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}} + \\ \frac{b\,\text{Sec}[e+f\,x]^3\,\,\sqrt{\text{Sec}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)}}}{6\,a+b+a\,\text{Sin}[e+f\,x]^2\,\,\big(a+b-a\,\text{Sin}[e+f\,x]^2\big)} + \\ \frac{b\,\text{S$$

Result (type 4, 470 leaves, 11 steps):

$$\frac{\left(3\,a^{2}+13\,a\,b+8\,b^{2}\right)\sqrt{a+b\,Sec\,[e+f\,x]^{2}\,\,Sin\,[e+f\,x]\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{2}}}{15\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}}-\frac{15\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\,Sin\,[e+f\,x]^{2}}{\left(\left(3\,a^{2}+13\,a\,b+8\,b^{2}\right)\sqrt{Cos\,[e+f\,x]^{2}}\,\,EllipticE\left[ArcSin\,[Sin\,[e+f\,x]]\right],\,\,\frac{a}{a+b}\right]\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{2}}}\right)}{\left[15\,b\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\,\sqrt{1-\frac{a\,Sin\,[e+f\,x]^{2}}{a+b}}\right]}+\frac{2\,\left(3\,a+2\,b\right)\,Sec\,[e+f\,x]\,\,\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{2}}\,\,Tan\,[e+f\,x]}}{15\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{2}}\,\,Tan\,[e+f\,x]}}+\frac{b\,Sec\,[e+f\,x]^{3}\,\sqrt{a+b\,Sec\,[e+f\,x]^{2}}\,\,\sqrt{a+b-a\,Sin\,[e+f\,x]^{2}}\,\,Tan\,[e+f\,x]}}{5\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^{2}}}\,\,Tan\,[e+f\,x]}$$

#### Problem 243: Result valid but suboptimal antiderivative.

$$\int Sec \left[e + f x\right] \left(a + b Sec \left[e + f x\right]^{2}\right)^{3/2} dx$$

$$\frac{2 \left(2 \, a + b\right) \, \text{Sin}\left[e + f \, x\right] \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}}{3 \, f} - \frac{1}{3 \, f \sqrt{1 - \frac{a \, \text{Sin}\left[e + f \, x\right]^2}{a + b}}}$$

$$2 \left(2 \, a + b\right) \, \sqrt{\text{Cos}\left[e + f \, x\right]^2} \, \text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e + f \, x\right]\right], \, \frac{a}{a + b}\right] \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}} + \left((a + b) \, \left(3 \, a + 2 \, b\right) \, \sqrt{\text{Cos}\left[e + f \, x\right]^2} \, \text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e + f \, x\right]\right], \, \frac{a}{a + b}\right] \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}} \, \sqrt{1 - \frac{a \, \text{Sin}\left[e + f \, x\right]^2}{a + b}} \right) \right)$$

$$\left(3 \, f \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)\right) + \frac{b \, \text{Sec}\left[e + f \, x\right] \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}}{3 \, f} \, \text{Tan}\left[e + f \, x\right] \right)$$

Result (type 4, 366 leaves, 10 steps):

$$\frac{2\left(2\,a+b\right)\,\sqrt{a+b\,Sec\,[e+f\,x]^2}\,\,Sin[e+f\,x]\,\,\sqrt{a+b-a\,Sin[e+f\,x]^2}}{3\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^2}} = \frac{2\left(2\,a+b\right)\,\sqrt{cos\,[e+f\,x]^2}\,\,BllipticE\left[ArcSin[Sin[e+f\,x]]\,,\,\,\frac{a}{a+b}\right]\,\sqrt{a+b\,Sec\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,Sin[e+f\,x]^2}\,\right)}{\left(3\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^2}\,\,\sqrt{1-\frac{a\,Sin[e+f\,x]^2}{a+b}}\right)} + \frac{\left(a+b\right)\,\left(3\,a+2\,b\right)\,\sqrt{Cos\,[e+f\,x]^2}\,\,BllipticF\left[ArcSin[Sin[e+f\,x]]\,,\,\,\frac{a}{a+b}\right]\,\sqrt{a+b\,Sec\,[e+f\,x]^2}\,\,\sqrt{1-\frac{a\,Sin[e+f\,x]^2}{a+b}}\right)}{\left(3\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,Sin[e+f\,x]^2}\,\,Dhamber} + \frac{b\,Sec\,[e+f\,x]\,\,\sqrt{a+b\,Sec\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,Sin[e+f\,x]^2}\,\,Bhamber}{3\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^2}}\,\sqrt{a+b-a\,Sin[e+f\,x]^2}\,\,Bhamber} + \frac{b\,Sec\,[e+f\,x]\,\,\sqrt{a+b\,Sec\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,Sin[e+f\,x]^2}}{3\,f\,\sqrt{b+a\,Cos\,[e+f\,x]^2}}$$

## Problem 244: Result valid but suboptimal antiderivative.

$$\left\lceil \text{Cos}\left[\,e + f\,x\,\right] \, \left(a + b\,\text{Sec}\left[\,e + f\,x\,\right]^{\,2}\right)^{\,3/\,2} \, \text{d}x \right.$$

Optimal (type 4, 224 leaves, 9 steps):

$$\frac{b\, Sin\, [\, e + f\, x\, ]\,\,\, \sqrt{Sec\, [\, e + f\, x\, ]^{\, 2}\,\, \left(\, a + b - a\, Sin\, [\, e + f\, x\, ]^{\, 2}\,\right)}}{f} \,\,\, +$$

$$\frac{\left(\text{a-b}\right)\sqrt{\text{Cos}\left[\text{e+fx}\right]^2}}{\text{f}\sqrt{1-\frac{\text{a}\,\text{Sin}\left[\text{e+fx}\right]^2}{\text{a+b}}}}\frac{\left\{\text{Sec}\left[\text{e+fx}\right]^2\left(\text{a+b-a}\,\text{Sin}\left[\text{e+fx}\right]^2\right)\right\}}{\left\{\text{f}\left(\text{a+b-a}\,\text{Sin}\left[\text{e+fx}\right]^2\right)\right\}} + \frac{1}{\text{f}\left(\text{a+b-a}\,\text{Sin}\left[\text{e+fx}\right]^2\right)}$$

$$b \left(a+b\right) \sqrt{\text{Cos}\left[e+fx\right]^2} \ \text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right]\right], \ \frac{a}{a+b}\right] \sqrt{\text{Sec}\left[e+fx\right]^2 \left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)} \ \sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}{a+b} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}{a+b} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}} = \frac{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{$$

Result (type 4, 277 leaves, 9 steps):

$$\frac{b\,\sqrt{\,a+b\,Sec\,[\,e+f\,x\,]^{\,2}}\,\,Sin\,[\,e+f\,x\,]\,\,\sqrt{\,a+b-a\,Sin\,[\,e+f\,x\,]^{\,2}}}{f\,\sqrt{\,b+a\,Cos\,[\,e+f\,x\,]^{\,2}}}\,+\\ \left(\,\left(\,a-b\right)\,\sqrt{\,Cos\,[\,e+f\,x\,]^{\,2}}\,\,EllipticE\big[ArcSin\,[\,Sin\,[\,e+f\,x\,]\,\,]\,\,,\,\,\frac{a}{a+b}\,\big]\,\,\sqrt{\,a+b\,Sec\,[\,e+f\,x\,]^{\,2}}}\,\,\sqrt{\,a+b-a\,Sin\,[\,e+f\,x\,]^{\,2}}\,\,\right)\Big/\\ \left(\,f\,\sqrt{\,b+a\,Cos\,[\,e+f\,x\,]^{\,2}}\,\,\sqrt{\,1-\frac{a\,Sin\,[\,e+f\,x\,]^{\,2}}{a+b}}\,\,\right)\,+$$

$$\frac{b \left(\mathsf{a} + \mathsf{b}\right) \sqrt{\mathsf{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2} \; \mathsf{EllipticF}\left[\mathsf{ArcSin}\left[\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]\right], \; \frac{\mathsf{a}}{\mathsf{a} + \mathsf{b}}\right] \sqrt{\mathsf{a} + \mathsf{b}\,\mathsf{Sec}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2} \; \sqrt{1 - \frac{\mathsf{a}\,\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a} + \mathsf{b}}} \\ \mathsf{f}\,\sqrt{\mathsf{b} + \mathsf{a}\,\mathsf{Cos}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2} \; \sqrt{\mathsf{a} + \mathsf{b} - \mathsf{a}\,\mathsf{Sin}\left[\mathsf{e} + \mathsf{f}\,\mathsf{x}\right]^2}$$

# Problem 245: Result valid but suboptimal antiderivative.

$$\left\lceil \text{Cos}\left[\,e + f\,x\,\right]^{\,3} \,\left(\,a + b\,\text{Sec}\left[\,e + f\,x\,\right]^{\,2}\right)^{\,3/2}\,\text{d}x\right.$$

Optimal (type 4, 241 leaves, 9 steps):

$$\frac{a \cos \left[e+fx\right]^2 \sin \left[e+fx\right] \sqrt{Sec\left[e+fx\right]^2 \left(a+b-a \sin \left[e+fx\right]^2\right)}}{3 f} + \frac{1}{3 f \sqrt{1-\frac{a \sin \left[e+fx\right]^2}{a+b}}}$$

$$2 \left(a+2b\right) \sqrt{\cos \left[e+fx\right]^2} \text{ EllipticE} \left[ArcSin\left[Sin\left[e+fx\right]\right], \frac{a}{a+b}\right] \sqrt{Sec\left[e+fx\right]^2 \left(a+b-a \sin \left[e+fx\right]^2\right)} - \left(b \left(a+b\right) \sqrt{\cos \left[e+fx\right]^2} \text{ EllipticF} \left[ArcSin\left[Sin\left[e+fx\right]\right], \frac{a}{a+b}\right] \sqrt{Sec\left[e+fx\right]^2 \left(a+b-a \sin \left[e+fx\right]^2\right)} \sqrt{1-\frac{a \sin \left[e+fx\right]^2}{a+b}}\right) / \left(3 f \left(a+b-a \sin \left[e+fx\right]^2\right)\right)$$

Result (type 4, 294 leaves, 9 steps):

$$\frac{a \cos \left[e+fx\right]^2 \sqrt{a+b \sec \left[e+fx\right]^2} \cdot \sin \left[e+fx\right] \sqrt{a+b-a \sin \left[e+fx\right]^2}}{3 f \sqrt{b+a \cos \left[e+fx\right]^2}} + \\ \frac{3 f \sqrt{b+a \cos \left[e+fx\right]^2}}{3 f \sqrt{b+a \cos \left[e+fx\right]^2}} \cdot \left[2 \left(a+2b\right) \sqrt{\cos \left[e+fx\right]^2} \cdot \left[\text{EllipticE}\left[\text{ArcSin}\left[\sin \left[e+fx\right]\right]\right], \frac{a}{a+b}\right] \sqrt{a+b \sec \left[e+fx\right]^2}} \sqrt{a+b-a \sin \left[e+fx\right]^2}\right) \right/ \\ \frac{3 f \sqrt{b+a \cos \left[e+fx\right]^2}}{a+b} - \\ \frac{b \left(a+b\right) \sqrt{\cos \left[e+fx\right]^2}}{3 f \sqrt{b+a \cos \left[e+fx\right]^2}} \cdot \left[\text{EllipticF}\left[\text{ArcSin}\left[\sin \left[e+fx\right]\right], \frac{a}{a+b}\right] \sqrt{a+b \sec \left[e+fx\right]^2}} \sqrt{1-\frac{a \sin \left[e+fx\right]^2}{a+b}}} \\ \frac{3 f \sqrt{b+a \cos \left[e+fx\right]^2}}{3 f \sqrt{b+a \cos \left[e+fx\right]^2}} \sqrt{a+b-a \sin \left[e+fx\right]^2}} \right) \sqrt{a+b \sec \left[e+fx\right]^2}$$

#### Problem 246: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx]^5 (a+b Sec[e+fx]^2)^{3/2} dx$$

Optimal (type 4, 319 leaves, 10 steps):

$$-\frac{2 \left(a-3 \left(a+b\right)\right) \cos \left[e+fx\right]^{2} \sin \left[e+fx\right] \sqrt{\sec \left[e+fx\right]^{2} \left(a+b-a \sin \left[e+fx\right]^{2}\right)}}{15 \, f} + \frac{a \cos \left[e+fx\right]^{4} \sin \left[e+fx\right] \sqrt{\sec \left[e+fx\right]^{2} \left(a+b-a \sin \left[e+fx\right]^{2}\right)}}{5 \, f} + \frac{1}{15 \, a \, f \, \sqrt{1-\frac{a \sin \left[e+fx\right]^{2}}{a+b}}} \\ \left(8 \, a^{2}+13 \, a \, b+3 \, b^{2}\right) \sqrt{\cos \left[e+fx\right]^{2}} \, EllipticE\left[ArcSin\left[Sin\left[e+fx\right]\right], \, \frac{a}{a+b}\right] \sqrt{\sec \left[e+fx\right]^{2} \left(a+b-a \sin \left[e+fx\right]^{2}\right)}} - \\ \left(b \, \left(a+b\right) \, \left(4 \, a+3 \, b\right) \sqrt{\cos \left[e+fx\right]^{2}} \, EllipticF\left[ArcSin\left[Sin\left[e+fx\right]\right], \, \frac{a}{a+b}\right] \sqrt{\sec \left[e+fx\right]^{2} \left(a+b-a \sin \left[e+fx\right]^{2}\right)}} \sqrt{1-\frac{a \sin \left[e+fx\right]^{2}}{a+b}} \right) / \\ \left(15 \, a \, f \, \left(a+b-a \sin \left[e+fx\right]^{2}\right)\right)$$

#### Result (type 4, 395 leaves, 10 steps):

$$\frac{2 \left( a - 3 \left( a + b \right) \right) Cos \left[ e + f x \right]^2 \sqrt{a + b Sec} \left[ e + f x \right]^2}{15 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} + \frac{15 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2}{5 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} + \frac{a \, Cos \left[ e + f x \right]^4 \sqrt{a + b} \, Sec \left[ e + f x \right]^2}{5 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} + \frac{5 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2}{5 \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} + \frac{a}{a + b} \left[ \left( 8 \, a^2 + 13 \, a \, b + 3 \, b^2 \right) \sqrt{Cos \left[ e + f x \right]^2} \, EllipticE \left[ ArcSin \left[ Sin \left[ e + f x \right] \right] , \frac{a}{a + b} \right] \sqrt{a + b} \, Sec \left[ e + f x \right]^2} \, \sqrt{a + b - a} \, Sin \left[ e + f x \right]^2} \right) / \left[ 15 \, a \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} \, EllipticF \left[ ArcSin \left[ Sin \left[ e + f x \right] \right] , \frac{a}{a + b} \right] \sqrt{a + b} \, Sec \left[ e + f x \right]^2} \, \sqrt{1 - \frac{a \, Sin \left[ e + f x \right]^2}{a + b}} \right) / \left[ 15 \, a \, f \sqrt{b + a} \, Cos \left[ e + f x \right]^2} \, \sqrt{a + b - a} \, Sin \left[ e + f x \right]^2} \right] \right)$$

## Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]^5}{\sqrt{a+b\operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 4, 330 leaves, 10 steps):

$$\frac{2 \left(\mathsf{a}-\mathsf{b}\right) \, \mathsf{EllipticE} \left[\mathsf{ArcSin} \left[\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right], \, \frac{\mathsf{a}}{\mathsf{a}+\mathsf{b}}\right] \left(\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2\right)}{3 \, \mathsf{b}^2 \, \mathsf{f} \, \sqrt{\mathsf{Cos} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2} \, \sqrt{\mathsf{Sec} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2 \left(\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2\right)} \, \sqrt{1-\frac{\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a}+\mathsf{b}}}} - \frac{\left(\mathsf{a}-2\,\mathsf{b}\right) \, \mathsf{EllipticF} \left[\mathsf{ArcSin} \left[\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]\right], \, \frac{\mathsf{a}}{\mathsf{a}+\mathsf{b}}\right] \, \sqrt{1-\frac{\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}{\mathsf{a}+\mathsf{b}}}} \right]}{3 \, \mathsf{b}^2 \, \mathsf{f} \, \sqrt{\mathsf{Cos} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2 \, \left(\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2\right)}} + \frac{\mathsf{Sec} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^3 \, \left(\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2\right) \, \sqrt{\mathsf{In} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2}}{3 \, \mathsf{b}\,\mathsf{f} \, \sqrt{\mathsf{Sec} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2 \, \left(\mathsf{a}+\mathsf{b}-\mathsf{a}\,\mathsf{Sin} \left[\mathsf{e}+\mathsf{f}\,\mathsf{x}\right]^2\right)}}$$

Result (type 4, 380 leaves, 10 steps):

$$\frac{2\left(a-b\right)\sqrt{b+a}\cos\left[e+fx\right]^{2}}{3b^{2}f\sqrt{\cos\left[e+fx\right]^{2}}} \frac{\text{EllipticE}\left[\text{ArcSin}\left[\sin\left[e+fx\right]\right], \frac{a}{a+b}\right]\sqrt{a+b-a}\sin\left[e+fx\right]^{2}}}{\sqrt{1-\frac{a\sin\left[e+fx\right]^{2}}{a+b}}} - \frac{\left(a-2b\right)\sqrt{b+a}\cos\left[e+fx\right]^{2}}{3bf\sqrt{\cos\left[e+fx\right]^{2}}} \frac{\text{EllipticF}\left[\text{ArcSin}\left[\sin\left[e+fx\right]\right], \frac{a}{a+b}\right]\sqrt{1-\frac{a\sin\left[e+fx\right]^{2}}{a+b}}}{3bf\sqrt{\cos\left[e+fx\right]^{2}}\sqrt{a+b}\sec\left[e+fx\right]^{2}}\sqrt{a+b-a}\sin\left[e+fx\right]^{2}} - \frac{2\left(a-b\right)\sqrt{b+a}\cos\left[e+fx\right]^{2}}{3b^{2}f\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]^{2}}{3b^{2}f\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{b+a}\cos\left[e+fx\right]^{2}}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{b+a}\cos\left[e+fx\right]^{2}}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\sin\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}} \frac{\sqrt{a+b-a}\cos\left[e+fx\right]}{3bf\sqrt{a+b}\sec\left[e+fx\right]^{2}}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]^3}{\sqrt{a+b\operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 4, 170 leaves, 7 steps):

$$-\frac{\sqrt{a}\sqrt{a+b}}{b\,f\,\sqrt{\text{Cos}\,[e+f\,x]^{\,2}}}\sqrt{\text{Sec}\,[e+f\,x]^{\,2}\left(a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}\right)}}{\sqrt{\text{Sec}\,[e+f\,x]^{\,2}}}\sqrt{\frac{a+b}{a}}\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^{\,2}}{a+b}}} + \frac{\text{Sec}\,[e+f\,x]\left(a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}\right)\,\text{Tan}\,[e+f\,x]^{\,2}}}{b\,f\,\sqrt{\text{Sec}\,[e+f\,x]^{\,2}\left(a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}\right)}}$$

Result (type 4, 202 leaves, 7 steps):

$$-\frac{\sqrt{a}\ \sqrt{a+b}\ \sqrt{b+a}\cos\left[e+fx\right]^2\ EllipticE\left[ArcSin\left[\frac{\sqrt{a}\ Sin\left[e+fx\right]}{\sqrt{a+b}}\right],\ \frac{a+b}{a}\right]\sqrt{1-\frac{a\,Sin\left[e+fx\right]^2}{a+b}}}{b\,f\,\sqrt{\cos\left[e+fx\right]^2}\ \sqrt{a+b}Sec\left[e+fx\right]^2} \sqrt{a+b-a\,Sin\left[e+fx\right]^2}} + \frac{\sqrt{b+a\,Cos\left[e+fx\right]^2}\ Sec\left[e+fx\right]\sqrt{a+b-a\,Sin\left[e+fx\right]^2}}{b\,f\,\sqrt{a+b\,Sec\left[e+fx\right]^2}}$$

#### Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]}{\sqrt{a+b\operatorname{Sec}[e+fx]^2}} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right],\ \frac{a}{a+b}\right]\sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}}{f\,\sqrt{\text{Cos}\left[e+fx\right]^2}\,\,\sqrt{\text{Sec}\left[e+fx\right]^2\,\left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)}}$$

Result (type 4, 103 leaves, 5 steps):

$$\frac{\sqrt{b+a\cos\left[e+f\,x\right]^{\,2}} \; EllipticF\left[ArcSin\left[Sin\left[e+f\,x\right]\right], \; \frac{a}{a+b}\right] \, \sqrt{1-\frac{a\,Sin\left[e+f\,x\right]^{\,2}}{a+b}}}{f\,\sqrt{\cos\left[e+f\,x\right]^{\,2}} \; \sqrt{a+b\,Sec\left[e+f\,x\right]^{\,2}} \; \sqrt{a+b-a\,Sin\left[e+f\,x\right]^{\,2}}}$$

## Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]}{\sqrt{a+b\,\text{Sec}[e+fx]^2}}\,dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$\frac{\sqrt{\mathsf{a}+\mathsf{b}} \; \mathsf{EllipticE} \left[\mathsf{ArcSin} \left[ \frac{\sqrt{\mathsf{a} \; \mathsf{Sin} \left[\mathsf{e}+\mathsf{f} \, \mathsf{x} \right]}}{\sqrt{\mathsf{a}+\mathsf{b}}} \right], \; \frac{\mathsf{a}+\mathsf{b}}{\mathsf{a}} \right] \sqrt{1 - \frac{\mathsf{a} \, \mathsf{Sin} \left[\mathsf{e}+\mathsf{f} \, \mathsf{x} \right]^2}{\mathsf{a}+\mathsf{b}}}}{\sqrt{\mathsf{a}} \; \mathsf{f} \; \sqrt{\mathsf{Cos} \left[\mathsf{e}+\mathsf{f} \, \mathsf{x} \right]^2} \; \sqrt{\mathsf{Sec} \left[\mathsf{e}+\mathsf{f} \, \mathsf{x} \right]^2 \left(\mathsf{a}+\mathsf{b}-\mathsf{a} \, \mathsf{Sin} \left[\mathsf{e}+\mathsf{f} \, \mathsf{x} \right]^2 \right)}}$$

Result (type 4, 128 leaves, 5 steps):

$$\frac{\sqrt{a+b} \ \sqrt{b+a \, \text{Cos} \, [e+f\,x]^2} \ \text{EllipticE} \big[ \text{ArcSin} \big[ \frac{\sqrt{a} \ \text{Sin} [e+f\,x]}{\sqrt{a+b}} \big] \text{, } \frac{a+b}{a} \big] \ \sqrt{1-\frac{a \, \text{Sin} [e+f\,x]^2}{a+b}} }{\sqrt{a} \ f \ \sqrt{\text{Cos} \, [e+f\,x]^2} \ \sqrt{a+b \, \text{Sec} \, [e+f\,x]^2}} \ \sqrt{a+b-a \, \text{Sin} \, [e+f\,x]^2}}$$

## Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]^3}{\sqrt{a+b\,\text{Sec}[e+fx]^2}}\,\mathrm{d}x$$

Optimal (type 4, 255 leaves, 9 steps):

$$\frac{\text{Sin}[\text{e}+\text{fx}] \left(\text{a}+\text{b}-\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2\right)}{3\,\text{a}\,\text{f}\,\sqrt{\text{Sec}[\text{e}+\text{fx}]^2\left(\text{a}+\text{b}-\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2\right)}} + \frac{2\,\left(\text{a}-\text{b}\right)\,\text{EllipticE}\big[\text{ArcSin}[\text{Sin}[\text{e}+\text{fx}]],\,\frac{\text{a}}{\text{a}+\text{b}}\big]\,\left(\text{a}+\text{b}-\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2\right)}{3\,\text{a}^2\,\text{f}\,\sqrt{\text{Cos}[\text{e}+\text{fx}]^2}\,\,\sqrt{\text{Sec}[\text{e}+\text{fx}]^2\left(\text{a}+\text{b}-\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2\right)}}\,\sqrt{1-\frac{\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2}{\text{a}+\text{b}}}} = \left(\text{a}-\text{2}\,\text{b}\right)\,\text{b}\,\text{EllipticF}\big[\text{ArcSin}[\text{Sin}[\text{e}+\text{fx}]],\,\frac{\text{a}}{\text{a}+\text{b}}\big]}\,\sqrt{1-\frac{\text{a}\,\text{Sin}[\text{e}+\text{fx}]^2}{\text{a}+\text{b}}}}\right]$$

$$\frac{\left(\text{a-2b}\right)\text{ b}\text{ EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[\text{e+fx}\right]\right], \frac{\text{a}}{\text{a+b}}\right]\sqrt{1-\frac{\text{a}\text{Sin}\left[\text{e+fx}\right]^2}{\text{a+b}}}}{3\text{ a}^2\text{ f}\sqrt{\text{Cos}\left[\text{e+fx}\right]^2}\sqrt{\text{Sec}\left[\text{e+fx}\right]^2\left(\text{a+b-a}\text{Sin}\left[\text{e+fx}\right]^2\right)}}$$

#### Result (type 4, 296 leaves, 9 steps):

$$\frac{\sqrt{b+a} \cos [e+fx]^2 \sin [e+fx] \sqrt{a+b-a} \sin [e+fx]^2}{3 a f \sqrt{a+b} \sec [e+fx]^2} + \\ \frac{2 \left(a-b\right) \sqrt{b+a} \cos [e+fx]^2 }{3 a^2 f \sqrt{\cos [e+fx]^2} \sqrt{a+b} \sec [e+fx]^2 \sqrt{1-\frac{a \sin [e+fx]^2}{a+b}}} - \\ \frac{\left(a-2b\right) b \sqrt{b+a} \cos [e+fx]^2 }{3 a^2 f \sqrt{\cos [e+fx]^2}} \left[ \text{EllipticF} \left[ \text{ArcSin} \left[ \text{Sin} \left[e+fx\right] \right], \frac{a}{a+b} \right] \sqrt{1-\frac{a \sin [e+fx]^2}{a+b}} \right]} \\ \frac{\left(a-2b\right) b \sqrt{b+a} \cos \left[e+fx\right]^2 }{3 a^2 f \sqrt{\cos [e+fx]^2}} \sqrt{a+b} \sec \left[e+fx\right]^2 \sqrt{a+b-a} \sin \left[e+fx\right]^2}$$

## Problem 262: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]^5}{\sqrt{a+b\operatorname{Sec}[e+fx]^2}} dx$$

$$\frac{4 \; \left( a - b \right) \; Sin[e + fx] \; \left( a + b - a \, Sin[e + fx]^2 \right)}{15 \; a^2 \; f \; \sqrt{Sec} \; [e + fx]^2 \; \left( a + b - a \, Sin[e + fx]^2 \right)} \; + \; \frac{Cos[e + fx]^2 \; Sin[e + fx] \; \left( a + b - a \, Sin[e + fx]^2 \right)}{5 \; a \; f \; \sqrt{Sec} \; [e + fx]^2 \; \left( a + b - a \, Sin[e + fx]^2 \right)} \; + \; \frac{\left( 8 \; a^2 - 7 \; a \; b + 8 \; b^2 \right) \; EllipticE \left[ ArcSin[Sin[e + fx]] \; , \; \frac{a}{a + b} \right] \; \left( a + b - a \, Sin[e + fx]^2 \right)}{15 \; a^3 \; f \; \sqrt{Cos[e + fx]^2} \; \sqrt{Sec[e + fx]^2 \; \left( a + b - a \, Sin[e + fx]^2 \right)}} \; - \; \frac{b \; \left( 4 \; a^2 - 3 \; a \; b + 8 \; b^2 \right) \; EllipticF \left[ ArcSin[Sin[e + fx]] \; , \; \frac{a}{a + b} \right] \; \sqrt{1 - \frac{a \, Sin[e + fx]^2}{a + b}}} \; \\ \frac{b \; \left( 4 \; a^2 - 3 \; a \; b + 8 \; b^2 \right) \; EllipticF \left[ ArcSin[Sin[e + fx]] \; , \; \frac{a}{a + b} \right] \; \sqrt{1 - \frac{a \, Sin[e + fx]^2}{a + b}}} \; }{15 \; a^3 \; f \; \sqrt{Cos[e + fx]^2} \; \sqrt{Sec[e + fx]^2 \; \left( a + b - a \, Sin[e + fx]^2 \right)}} \; }$$

Result (type 4, 395 leaves, 10 steps):

$$\frac{4 \left(a-b\right) \sqrt{b+a \cos \left[e+fx\right]^2 \sin \left[e+fx\right] \sqrt{a+b-a \sin \left[e+fx\right]^2}}{15 \, a^2 \, f \sqrt{a+b \sec \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{b+a \cos \left[e+fx\right]^2 \sin \left[e+fx\right] \sqrt{a+b-a \sin \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \sec \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{b+a \cos \left[e+fx\right]^2 \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \sec \left[e+fx\right]^2}} + \frac{\left(8 \, a^2-7 \, a \, b+8 \, b^2\right) \sqrt{b+a \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{\cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b-a \sin \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \sec \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b-a \sin \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b-a \sin \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b-a \sin \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a \, f \sqrt{a+b \cos \left[e+fx\right]^2}} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{\cos \left[e+fx\right]^2 \sqrt{a+b \cos \left[e+fx\right]^2}}{5 \, a+b \cos \left[e+fx\right]^2} + \frac{$$

Problem 270: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec} [e + f x]^{5}}{(a + b \operatorname{Sec} [e + f x]^{2})^{3/2}} dx$$

Optimal (type 4, 289 leaves, 10 steps):

$$\frac{a \left(2 \, a + b\right) \, \text{Sin}\left[e + f \, x\right]}{b^2 \, \left(a + b\right) \, f \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}} - \frac{\left(2 \, a + b\right) \, \text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e + f \, x\right]\right], \, \frac{a}{a + b}\right] \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}{b^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}\left[e + f \, x\right]^2} \, \sqrt{\text{Sec}\left[e + f \, x\right]^2 \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)}} \, \sqrt{1 - \frac{a \, \text{Sin}\left[e + f \, x\right]^2}{a + b}} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, \text{Sin}\left[e + f \, x\right]^2\right)} + \frac{1}{a + b} \, \left(a + b - a \, x\right)} + \frac{1}{a + b} \, \left(a + b - a \, x\right)} + \frac{1}{a + b} \, \left(a$$

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e+fx\right]\right],\frac{a}{a+b}\right]\sqrt{1-\frac{a\,\text{Sin}\left[e+fx\right]^2}{a+b}}}{b\,\text{f}\,\sqrt{\text{Cos}\left[e+fx\right]^2}\,\sqrt{\text{Sec}\left[e+fx\right]^2\left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)}}+\frac{\text{Sec}\left[e+fx\right]\,\text{Tan}\left[e+fx\right]}{b\,\text{f}\,\sqrt{\text{Sec}\left[e+fx\right]^2\left(a+b-a\,\text{Sin}\left[e+fx\right]^2\right)}}$$

Result (type 4, 367 leaves, 10 steps):

$$\frac{a \left(2\,a+b\right) \sqrt{b+a \, \text{Cos}\, [e+f\,x]^2 \, \text{Sin}\, [e+f\,x]}}{b^2 \left(a+b\right) f \sqrt{a+b \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}} - \\ \frac{\left(2\,a+b\right) \sqrt{b+a \, \text{Cos}\, [e+f\,x]^2 \, \text{EllipticE}\left[\text{ArcSin}\, [\text{Sin}\, [e+f\,x]\,]\,,\,\, \frac{a}{a+b}\right] \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}}{b^2 \left(a+b\right) f \sqrt{\text{Cos}\, [e+f\,x]^2 \, \sqrt{a+b \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{1-\frac{a \, \text{Sin}\, [e+f\,x]^2}{a+b}}}}} + \\ \frac{\sqrt{b+a \, \text{Cos}\, [e+f\,x]^2 \, \text{EllipticF}\left[\text{ArcSin}\, [\text{Sin}\, [e+f\,x]\,]\,,\,\, \frac{a}{a+b}\right] \sqrt{1-\frac{a \, \text{Sin}\, [e+f\,x]^2}{a+b}}}}{b \, f \sqrt{\text{Cos}\, [e+f\,x]^2 \, \sqrt{a+b \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}}} + \\ \frac{\sqrt{b+a \, \text{Cos}\, [e+f\,x]^2 \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}}}{b \, f \sqrt{a+b \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}} + \frac{\sqrt{b+a \, \text{Cos}\, [e+f\,x]^2 \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}}}{b \, f \sqrt{a+b \, \text{Sec}\, [e+f\,x]^2 \, \sqrt{a+b-a \, \text{Sin}\, [e+f\,x]^2}}}}$$

## Problem 271: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]^{3}}{(a+b\operatorname{Sec}[e+fx]^{2})^{3/2}} dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$-\frac{a \, \text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]}{b \, \left(\text{a} + b\right) \, \text{f} \, \sqrt{\text{Sec}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2} \, \left(\text{a} + b - a \, \text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}\right)}} + \frac{\text{EllipticE}\big[\text{ArcSin}[\,\text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}\,] \, \left(\text{a} + b - a \, \text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}\right)}{b \, \left(\text{a} + b\right) \, \text{f} \, \sqrt{\text{Cos}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}} \, \sqrt{\text{Sec}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2} \, \left(\text{a} + b - a \, \text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}\right)}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,\text{e} + \text{f}\,\text{x}\,]^{\,2}}{a + b}}}$$

Result (type 4, 182 leaves, 7 steps):

$$-\frac{a\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^{\,2}}\,\,\text{Sin}\,[e+f\,x]}{b\,\left(a+b\right)\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^{\,2}}}\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}}}{\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}}} + \frac{\sqrt{b+a\,\text{Cos}\,[e+f\,x]^{\,2}}\,\,\text{EllipticE}\big[\text{ArcSin}\,[\text{Sin}\,[e+f\,x]\,]\,\,,\,\,\frac{a}{a+b}\big]\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^{\,2}}}}{b\,\left(a+b\right)\,f\,\sqrt{\text{Cos}\,[e+f\,x]^{\,2}}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^{\,2}}}\,\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^{\,2}}{a+b}}}$$

## Problem 272: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+b\operatorname{Sec}[e+fx]^{2}\right)^{3/2}} dx$$

#### Optimal (type 4, 229 leaves, 9 steps):

$$\frac{Sin[e+fx]}{(a+b) f \sqrt{Sec[e+fx]^2 (a+b-a Sin[e+fx]^2)}}$$

$$\frac{\text{EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[e+f\,x\right]\right],\frac{a}{a+b}\right]\left(a+b-a\,\text{Sin}\left[e+f\,x\right]^2\right)}{a\left(a+b\right)\,f\,\sqrt{\text{Cos}\left[e+f\,x\right]^2}\,\sqrt{\text{Sec}\left[e+f\,x\right]^2\left(a+b-a\,\text{Sin}\left[e+f\,x\right]^2\right)}}\,\sqrt{1-\frac{a\,\text{Sin}\left[e+f\,x\right]^2}{a+b}}}\,+\frac{\text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[e+f\,x\right]\right],\frac{a}{a+b}\right]\,\sqrt{1-\frac{a\,\text{Sin}\left[e+f\,x\right]^2}{a+b}}}}{a\,f\,\sqrt{\text{Cos}\left[e+f\,x\right]^2}\,\sqrt{\text{Sec}\left[e+f\,x\right]^2\,\left(a+b-a\,\text{Sin}\left[e+f\,x\right]^2\right)}}$$

#### Result (type 4, 284 leaves, 9 steps):

$$\frac{\sqrt{b+a\cos\left[e+fx\right]^2}\,\sin\left[e+fx\right]}{\left(a+b\right)\,f\,\sqrt{a+b\sec\left[e+fx\right]^2}\,\sqrt{a+b-a\sin\left[e+fx\right]^2}} - \frac{\sqrt{b+a\cos\left[e+fx\right]^2}\,\left[\text{EllipticE}\left[\text{ArcSin}\left[\sin\left[e+fx\right]\right],\,\frac{a}{a+b}\right]\,\sqrt{a+b-a\sin\left[e+fx\right]^2}}}{a\,\left(a+b\right)\,f\,\sqrt{\cos\left[e+fx\right]^2}\,\sqrt{a+b\sec\left[e+fx\right]^2}\,\sqrt{1-\frac{a\sin\left[e+fx\right]^2}{a+b}}} + \frac{\sqrt{b+a\cos\left[e+fx\right]^2}\,\left[\text{EllipticE}\left[\text{ArcSin}\left[\sin\left[e+fx\right]\right],\,\frac{a}{a+b}\right]\,\sqrt{a+b-a\sin\left[e+fx\right]^2}}}{a\,\left(a+b\right)\,f\,\sqrt{\cos\left[e+fx\right]^2}\,\sqrt{a+b\sec\left[e+fx\right]^2}}$$

$$\frac{\sqrt{b+a\cos\left[e+fx\right]^2} \; EllipticF\left[ArcSin\left[Sin\left[e+fx\right]\right], \; \frac{a}{a+b}\right] \sqrt{1-\frac{aSin\left[e+fx\right]^2}{a+b}}}{a\,f\,\sqrt{\cos\left[e+fx\right]^2} \; \sqrt{a+bSec\left[e+fx\right]^2} \; \sqrt{a+b-aSin\left[e+fx\right]^2}}$$

## Problem 273: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Cos}\,[\,e + f\,x\,]}{\left(\,a + b\,\mathsf{Sec}\,[\,e + f\,x\,]^{\,2}\,\right)^{\,3/2}}\,\mathrm{d}x$$

#### Optimal (type 4, 240 leaves, 9 steps):

$$-\frac{b \sin[e+fx]}{a (a+b) f \sqrt{Sec[e+fx]^2 (a+b-a \sin[e+fx]^2)}}$$

$$\frac{\left(\text{a}+2\text{ b}\right)\text{ EllipticE}\left[\text{ArcSin}\left[\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]\right],\,\frac{\text{a}}{\text{a}+\text{b}}\right]\left(\text{a}+\text{b}-\text{a}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]^2\right)}{\left(\text{a}+\text{b}\right)\text{f}\sqrt{\text{Cos}}\left[\text{e}+\text{f}\,\text{x}\right]^2}\,\sqrt{\text{Sec}\left[\text{e}+\text{f}\,\text{x}\right]^2\left(\text{a}+\text{b}-\text{a}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]^2\right)}}\,\sqrt{1-\frac{\text{a}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]^2}{\text{a}+\text{b}}}}-\frac{2\text{ b}\,\text{EllipticF}\left[\text{ArcSin}\left[\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]\right],\,\frac{\text{a}}{\text{a}+\text{b}}\right]\sqrt{1-\frac{\text{a}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]^2}{\text{a}+\text{b}}}}\right]}{\text{a}^2\text{ f}\sqrt{\text{Cos}}\left[\text{e}+\text{f}\,\text{x}\right]^2}\,\sqrt{\text{Sec}\left[\text{e}+\text{f}\,\text{x}\right]^2\left(\text{a}+\text{b}-\text{a}\,\text{Sin}\left[\text{e}+\text{f}\,\text{x}\right]^2\right)}}$$

$$\frac{b\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{Sin}\,[e+f\,x]}{a\,\left(a+b\right)\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}} + \\ \frac{\left(a+2\,b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{EllipticE}\big[\text{ArcSin}\,[\text{Sin}\,[e+f\,x]\,]\,\,,\,\,\frac{a}{a+b}\big]\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}}{a^2\,\left(a+b\right)\,f\,\sqrt{\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}} \\ \frac{2\,b\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{EllipticF}\big[\text{ArcSin}\,[\text{Sin}\,[e+f\,x]\,]\,\,,\,\,\frac{a}{a+b}\big]\,\sqrt{1-\frac{a\,\text{Sin}\,[e+f\,x]^2}{a+b}}} \\ \frac{a^2\,f\,\sqrt{\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}} \\ \sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}\,[e+f\,x]^2}}$$

#### Problem 274: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]^3}{(a+b \operatorname{Sec}[e+fx]^2)^{3/2}} dx$$

Optimal (type 4, 335 leaves, 10 steps):

$$\frac{b \cos [e + fx]^2 \sin [e + fx]}{a (a + b) f \sqrt{Sec [e + fx]^2 (a + b - a Sin [e + fx]^2)}} + \frac{(a + 4b) \sin [e + fx] (a + b - a Sin [e + fx]^2)}{3 a^2 (a + b) f \sqrt{Sec [e + fx]^2 (a + b - a Sin [e + fx]^2)}} + \frac{(2 a^2 - 3 a b - 8 b^2) EllipticE [ArcSin [Sin [e + fx]], \frac{a}{a + b}] (a + b - a Sin [e + fx]^2)}{3 a^3 (a + b) f \sqrt{Cos [e + fx]^2} \sqrt{Sec [e + fx]^2 (a + b - a Sin [e + fx]^2)}} - \frac{(a - 8b) b EllipticF [ArcSin [Sin [e + fx]], \frac{a}{a + b}] \sqrt{1 - \frac{a Sin [e + fx]^2}{a + b}}}}{3 a^3 f \sqrt{Cos [e + fx]^2} \sqrt{Sec [e + fx]^2 (a + b - a Sin [e + fx]^2)}}$$

Result (type 4, 399 leaves, 10 steps):

$$\frac{b \operatorname{Cos}[e+fx]^2 \sqrt{b+a \operatorname{Cos}[e+fx]^2} \operatorname{Sin}[e+fx]}{a \left(a+b\right) f \sqrt{a+b \operatorname{Sec}[e+fx]^2} \sqrt{a+b-a \operatorname{Sin}[e+fx]^2}} + \frac{\left(a+4b\right) \sqrt{b+a \operatorname{Cos}[e+fx]^2} \operatorname{Sin}[e+fx] \sqrt{a+b-a \operatorname{Sin}[e+fx]^2}}{3 a^2 \left(a+b\right) f \sqrt{a+b \operatorname{Sec}[e+fx]^2}} + \frac{\left(2 a^2-3 a b-8 b^2\right) \sqrt{b+a \operatorname{Cos}[e+fx]^2} \sqrt{a+b-a \operatorname{Sin}[e+fx]^2}}{3 a^3 \left(a+b\right) f \sqrt{\operatorname{Cos}[e+fx]^2} \operatorname{EllipticE} \left[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]^2 \sqrt{1-\frac{a \operatorname{Sin}[e+fx]^2}{a+b}}\right]} - \frac{\left(a-8b\right) b \sqrt{b+a \operatorname{Cos}[e+fx]^2} \operatorname{EllipticF} \left[\operatorname{ArcSin}[\operatorname{Sin}[e+fx]^2 \sqrt{1-\frac{a \operatorname{Sin}[e+fx]^2}{a+b}}\right]}{3 a^3 f \sqrt{\operatorname{Cos}[e+fx]^2} \sqrt{a+b \operatorname{Sec}[e+fx]^2} \sqrt{a+b-a \operatorname{Sin}[e+fx]^2}}$$

#### Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [e + f x]^5}{(a + b \sec [e + f x]^2)^{3/2}} dx$$

Optimal (type 4, 436 leaves, 11 steps):

$$\frac{b \, \mathsf{Cos} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^4 \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]}{a \, (\mathsf{a} + \mathsf{b}) \, \mathsf{f} \, \sqrt{\mathsf{Sec} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, (\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2)}} + \frac{(\mathsf{a} \, \mathsf{a}^2 - \mathsf{5} \, \mathsf{a} \, \mathsf{b} - 24 \, \mathsf{b}^2) \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}] \, \left(\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}{15 \, \mathsf{a}^3 \, \left(\mathsf{a} + \mathsf{b}\right) \, \mathsf{f} \, \sqrt{\mathsf{Sec} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, \left(\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}} + \frac{\left(\mathsf{a} + \mathsf{6} \, \mathsf{b}\right) \, \mathsf{Cos} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}] \, \left(\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}{5 \, \mathsf{a}^2 \, \left(\mathsf{a} + \mathsf{b}\right) \, \mathsf{f} \, \sqrt{\mathsf{Sec} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, \left(\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}} + \frac{\left(\mathsf{a} + \mathsf{6} \, \mathsf{b}\right) \, \mathsf{Cos} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, \mathsf{Sec} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}{5 \, \mathsf{a}^2 \, \left(\mathsf{a} + \mathsf{b}\right) \, \mathsf{f} \, \sqrt{\mathsf{Sec} \, [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2 \, \left(\mathsf{a} + \mathsf{b} - \mathsf{a} \, \mathsf{Sin} [\mathsf{e} + \mathsf{f} \, \mathsf{x}]^2\right)}} + \frac{\mathsf{a} \, \mathsf{a} \, \mathsf{b} \, \mathsf{b} \, \mathsf{f} \, \mathsf{d} \, \mathsf{b} \, \mathsf{f} \, \mathsf{d} \, \mathsf{b} \, \mathsf{d} \, \mathsf{d} \, \mathsf{b} \, \mathsf{d} \, \mathsf{d} \, \mathsf{b} \, \mathsf{d} \, \mathsf{d}$$

Result (type 4, 509 leaves, 11 steps):

$$\frac{b \cos [e+fx]^4 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{a (a+b) f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx] \sqrt{a+b-a \sin [e+fx]^2}}{15 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx] \sqrt{a+b-a \sin [e+fx]^2}}{15 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a^3 \, b^3 \, (a+b) f \sqrt{a+b \sec [e+fx]^2}} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \, a^2 - 5 \, a \, b - 24 \, b^2\right) \sqrt{b+a \cos [e+fx]^2}}{5 \, a+b \cos [e+fx]^2} + \frac{\left(4 \,$$

#### Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]^5}{\left(a+b\operatorname{Sec}[e+fx]^2\right)^{5/2}} \, dx$$

Optimal (type 4, 321 leaves, 10 steps):

$$-\frac{2 \text{ a } (\text{a} + 2 \text{ b}) \text{ Sin}[\text{e} + \text{f} \text{x}]}{3 \text{ b}^2 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Sec}[\text{e} + \text{f} \text{x}]^2 \left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)}} - \frac{\text{a } \text{Sin}[\text{e} + \text{f} \text{x}]}{3 \text{ b } \left(\text{a} + \text{b}\right) \text{ f } \left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)} \sqrt{\text{Sec}[\text{e} + \text{f} \text{x}]^2 \left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)}} + \frac{2 \left(\text{a} + 2 \text{ b}\right) \text{ EllipticE}\left[\text{ArcSin}[\text{Sin}[\text{e} + \text{f} \text{x}]^2, \frac{\text{a}}{\text{a} + \text{b}}\right] \left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)}{\left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)}} - \frac{2 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b} - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2\right)}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \left(\text{a} + \text{b}\right) - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \left(\text{a} + \text{b}\right) - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \left(\text{a} + \text{b}\right) - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \left(\text{a} + \text{b}\right) - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{Cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \left(\text{a} + \text{b}\right) - \text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}}{\sqrt{1 - \frac{\text{a} \text{Sin}[\text{e} + \text{f} \text{x}]^2}{\text{a} + \text{b}}}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{f} \text{x}]^2}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{f} \text{x}]^2}}{\text{a} + \text{b}}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{f} \text{x}]^2} \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{f} \text{x}]^2}} - \frac{1 \left(\text{a} + \text{b}\right)^2 \text{ f } \sqrt{\text{cos}[\text{e} + \text{$$

Result (type 4, 383 leaves, 10 steps):

$$\frac{\sqrt{b+a\cos\left[e+fx\right]^{2}}}{3b\left(a+b\right)f\sqrt{\cos\left[e+fx\right]^{2}}}\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sin\left[e+fx\right]\right],\frac{a}{a+b}\right]\sqrt{1-\frac{a\sin\left[e+fx\right]^{2}}{a+b}}}{3b\left(a+b\right)f\sqrt{\cos\left[e+fx\right]^{2}}}\sqrt{a+b\sec\left[e+fx\right]^{2}}\sqrt{a+b-a\sin\left[e+fx\right]^{2}}$$

#### Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec} [e+fx]^3}{\left(a+b\operatorname{Sec} [e+fx]^2\right)^{5/2}} \, \mathrm{d}x$$

#### Optimal (type 4, 319 leaves, 10 steps):

$$=\frac{\left(a-b\right)\operatorname{Sin}[e+fx]}{3\,b\,\left(a+b\right)^2\,f\,\sqrt{\operatorname{Sec}\left[e+fx\right]^2\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}}+\frac{\operatorname{Sin}[e+fx]}{3\,\left(a+b\right)\,f\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)\,\sqrt{\operatorname{Sec}\left[e+fx\right]^2\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}}+\frac{\left(a-b\right)\,\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\operatorname{Sin}\left[e+fx\right]\right],\,\frac{a}{a+b}\right]\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}{\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}+\frac{3\,a\,b\,\left(a+b\right)^2\,f\,\sqrt{\operatorname{Cos}\left[e+fx\right]^2\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}}{\sqrt{1-\frac{a\operatorname{Sin}\left[e+fx\right]^2}{a+b}}}+\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\operatorname{Sin}\left[e+fx\right]\right],\,\frac{a}{a+b}\right]\,\sqrt{1-\frac{a\operatorname{Sin}\left[e+fx\right]^2}{a+b}}}{3\,a\,\left(a+b\right)\,f\,\sqrt{\operatorname{Cos}\left[e+fx\right]^2\,\left(a+b-a\operatorname{Sin}\left[e+fx\right]^2\right)}}$$

Result (type 4, 381 leaves, 10 steps):

$$\frac{\sqrt{b+a\cos[e+fx]^2} \; Sin[e+fx]}{3 \; (a+b) \; f \sqrt{a+b \, Sec[e+fx]^2} \; \left(a+b-a \, Sin[e+fx]^2\right)^{3/2}} - \frac{\left(a-b\right) \sqrt{b+a\cos[e+fx]^2} \; Sin[e+fx]}{3 \; b \; \left(a+b\right)^2 \; f \sqrt{a+b \, Sec[e+fx]^2} \; \sqrt{a+b-a \, Sin[e+fx]^2}} \\ \frac{\left(a-b\right) \sqrt{b+a\cos[e+fx]^2} \; EllipticE\left[ArcSin[Sin[e+fx]], \frac{a}{a+b}\right] \sqrt{a+b-a \, Sin[e+fx]^2}}{3 \; a \; b \; \left(a+b\right)^2 \; f \sqrt{\cos[e+fx]^2} \; \sqrt{a+b \, Sec[e+fx]^2} \; \sqrt{1-\frac{a \, Sin[e+fx]^2}{a+b}}} \\ \frac{\sqrt{b+a\cos[e+fx]^2} \; EllipticF\left[ArcSin[Sin[e+fx]], \frac{a}{a+b}\right] \sqrt{1-\frac{a \, Sin[e+fx]^2}{a+b}}}{3 \; a \; \left(a+b\right) \; f \sqrt{\cos[e+fx]^2} \; \sqrt{a+b \, Sec[e+fx]^2} \; \sqrt{a+b-a \, Sin[e+fx]^2}} \\ \frac{\sqrt{b+a\cos[e+fx]^2} \; \sqrt{a+b \, Sec[e+fx]^2} \; \sqrt{a+b \, Sec[e+fx]^2} \; \sqrt{a+b-a \, Sin[e+fx]^2}}{\sqrt{a+b-a \, Sin[e+fx]^2}}$$

#### Problem 285: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Sec}[e+fx]}{\left(a+b\operatorname{Sec}[e+fx]^{2}\right)^{5/2}} dx$$

Optimal (type 4, 327 leaves, 10 steps):

$$\frac{2 \left(2 \, a + b\right) \, \text{Sin}[e + f \, x]}{3 \, a \, \left(a + b\right)^2 \, f \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} - \frac{b \, \text{Sin}[e + f \, x]}{3 \, a \, \left(a + b\right) \, f \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right) \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} - \frac{2 \, \left(2 \, a + b\right) \, \text{EllipticE}\left[\text{ArcSin}[\text{Sin}[e + f \, x]], \, \frac{a}{a + b}\right] \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}{3 \, a^2 \, \left(a + b\right)^2 \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, \text{EllipticF}\left[\text{ArcSin}[\text{Sin}[e + f \, x]], \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, \text{Sin}[e + f \, x]^2}{a + b}}} + \frac{3 \, a^2 \, \left(a + b\right) \, \text{EllipticF}\left[\text{ArcSin}[\text{Sin}[e + f \, x]], \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, \text{Sin}[e + f \, x]^2}{a + b}}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[e + f \, x]^2\right)}} + \frac{3 \, a^2 \, \left(a + b\right) \, f \, \sqrt{\text{Cos}[e + f \, x]^2} \, \sqrt{\text{Sec}[e + f \, x]^2} \, \sqrt{\text{Sec}[e$$

Result (type 4, 389 leaves, 10 steps):

$$\frac{b\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{Sin}[e+f\,x]}{3\,a\,\left(a+b\right)\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\left(a+b-a\,\text{Sin}[e+f\,x]^2\right)^{3/2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{Sin}[e+f\,x]}{3\,a\,\left(a+b\right)^2\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}[e+f\,x]^2}} {3\,a\,\left(a+b\right)^2\,f\,\sqrt{\cos\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^2}{a+b}}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}[e+f\,x]^2}\,\,\sqrt{a+b-a\,\text{Sin}[e+f\,x]^2}} {3\,a^2\,\left(a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\text{EllipticF}\left[\text{ArcSin}\,[\text{Sin}[e+f\,x]^2\,\,\sqrt{1-\frac{a\,\text{Sin}[e+f\,x]^2}{a+b}}\right]} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a^2\,\left(a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a^2\,\left(a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a\,\left(a+b\right)^2\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a\,\left(a+b\right)^2\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a\,\left(a+b\right)^2\,f\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Sec}\,[e+f\,x]^2}} {3\,a\,\left(a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}\,\,\sqrt{a+b\,\text{Cos}\,[e+f\,x]^2}} {3\,a\,\left(a+b\right)\,\sqrt{b+a\,\text{Cos}\,[e+f\,x]^2}} + \frac{2\,\left(2\,a+b\right)\,\sqrt{b+a\,\text{Cos}$$

#### Problem 286: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]}{\left(a+b\operatorname{Sec}[e+fx]^{2}\right)^{5/2}} dx$$

Optimal (type 4, 349 leaves, 10 steps):

$$\frac{2 \, b \, \left(3 \, a + 2 \, b\right) \, \text{Sin}[\,e + f \, x]}{3 \, a^2 \, \left(a + b\right)^2 \, f \, \sqrt{\text{Sec}[\,e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right)}} - \frac{b \, \text{Cos}[\,e + f \, x]^2 \, \text{Sin}[\,e + f \, x]}{3 \, a \, \left(a + b\right) \, f \, \left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right) \, \sqrt{\text{Sec}[\,e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right)}} + \frac{\left(3 \, a^2 + 13 \, a \, b + 8 \, b^2\right) \, \text{EllipticE}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}\right] \, \left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right)}{\left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right)} - \frac{3 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{\text{Cos}[\,e + f \, x]^2 \, \left(a + b - a \, \text{Sin}[\,e + f \, x]^2\right)}}{\sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,\text{Sin}[\,e + f \, x]^{\,}\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,b + f \, x]^2\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,b + f \, x]^2\right], \, \frac{a}{a + b}} \, \sqrt{1 - \frac{a \, \text{Sin}[\,e + f \, x]^2}{a + b}}} - \frac{b}{a \, b}} + \frac{b \, \left(9 \, a + 8 \, b\right) \, \text{EllipticF}\left[\text{ArcSin}[\,b + f \, x]^2\right], \, \frac{a}{a \, b}} + \frac{b}{a \, b}} + \frac{b}{a \, b}} + \frac{b}{a \, b}} + \frac{b}{a \, b}} + \frac{b$$

Result (type 4, 411 leaves, 10 steps):

$$-\frac{b \cos [e+fx]^2 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{3 a (a+b) f \sqrt{a+b \sec [e+fx]^2} (a+b-a \sin [e+fx]^2)^{3/2}} - \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+2 b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (3 a+b) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{3 a^2 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} = \frac{2 b (a+b) \sqrt{b+a \cos [e+fx]^2} \sin$$

#### Problem 287: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [e + f x]^3}{\left(a + b \operatorname{Sec} [e + f x]^2\right)^{5/2}} dx$$

Optimal (type 4, 441 leaves, 11 steps):

$$\frac{2 \, b \, (4 \, a + 3 \, b) \, Cos [\, e + f \, x\,]^2 \, (a + b - a \, Sin [\, e + f \, x\,]^2)}{3 \, a^2 \, (a + b)^2 \, f \, \sqrt{Sec} [\, e + f \, x\,]^2 \, (a + b - a \, Sin [\, e + f \, x\,]^2)} + \frac{b \, Cos [\, e + f \, x\,]^4 \, Sin [\, e + f \, x\,]}{3 \, a \, (a + b) \, f \, (a + b - a \, Sin [\, e + f \, x\,]^2) \, \sqrt{Sec} [\, e + f \, x\,]^2 \, (a + b - a \, Sin [\, e + f \, x\,]^2)} + \frac{(a^2 + 11 \, a \, b + 8 \, b^2) \, Sin [\, e + f \, x\,] \, (a + b - a \, Sin [\, e + f \, x\,]^2)}{3 \, a^3 \, (a + b)^2 \, f \, \sqrt{Sec} [\, e + f \, x\,]^2 \, (a + b - a \, Sin [\, e + f \, x\,]^2)} + \frac{2 \, (a + 2 \, b) \, (a^2 - 4 \, a \, b - 4 \, b^2) \, EllipticE \left[ArcSin [Sin [\, e + f \, x\,]^2 \, , \, \frac{a}{a + b}\right] \, (a + b - a \, Sin [\, e + f \, x\,]^2)}{3 \, a^4 \, (a + b)^2 \, f \, \sqrt{Cos} \, [\, e + f \, x\,]^2} \, \sqrt{Sec} \, [\, e + f \, x\,]^2 \, (a + b - a \, Sin [\, e + f \, x\,]^2)} - \frac{b \, (a^2 - 16 \, a \, b - 16 \, b^2) \, EllipticF \left[ArcSin [Sin [\, e + f \, x\,]^2 \, , \, \frac{a}{a + b}\right] \, \sqrt{1 - \frac{a \, Sin [\, e + f \, x\,]^2}{a + b}}} \,$$

Result (type 4, 512 leaves, 11 steps):

$$-\frac{b \cos [e+fx]^4 \sqrt{b+a \cos [e+fx]^2}}{3 a (a+b) f \sqrt{a+b \sec [e+fx]^2}} \frac{\sin [e+fx]}{(a+b-a \sin [e+fx]^2)^{3/2}} - \frac{2 b (4 a+3 b) \cos [e+fx]^2 \sqrt{b+a \cos [e+fx]^2}}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} \frac{\sin [e+fx]}{\sqrt{a+b-a \sin [e+fx]^2}} + \frac{(a^2+11 a b+8 b^2) \sqrt{b+a \cos [e+fx]^2}}{3 a^3 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} \frac{(a^2+11 a b+8 b^2) \sqrt{b+a \cos [e+fx]^2}}{3 a^3 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} + \frac{(a^2+11 a b+8 b^2) \sqrt{b+a \cos [e+fx]^2}}{3 a^3 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} \frac{(a+b)^2 f \sqrt{a+b \sec [e+fx]^2}}{a+b} + \frac{a}{a+b} \sqrt{a+b-a \sin [e+fx]^2}$$

$$\left(2 (a+2b) (a^2-4ab-4b^2) \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b \sec [e+fx]^2} \sqrt{1-\frac{a \sin [e+fx]^2}{a+b}} - \frac{b (a^2-16ab-16b^2) \sqrt{b+a \cos [e+fx]^2}}{\sqrt{a+b \cos [e+fx]^2}} \frac{1-\frac{a \sin [e+fx]^2}{a+b}}{a+b} - \frac{a}{a+b} \sqrt{1-\frac{a \sin [e+fx]^2}{a+b}}} \right)$$

$$3 a^4 (a+b) f \sqrt{\cos [e+fx]^2} \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}$$

#### Problem 288: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[e+fx]^5}{(a+b\operatorname{Sec}[e+fx]^2)^{5/2}} dx$$

#### Optimal (type 4, 559 leaves, 12 steps):

$$-\frac{2 \, b \, \left(5 \, a + 4 \, b\right) \, Cos\left[e + f \, x\right]^4 \, Sin\left[e + f \, x\right]}{3 \, a^2 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} - \frac{b \, Cos\left[e + f \, x\right]^6 \, Sin\left[e + f \, x\right]}{3 \, a \, \left(a + b\right) \, f \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right) \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{2 \, \left(2 \, a^3 - 3 \, a^2 \, b - 42 \, a \, b^2 - 32 \, b^3\right) \, Sin\left[e + f \, x\right] \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^4 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, Sin\left[e + f \, x\right] \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, \left(3 \, a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, Cos\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}{15 \, a^3 \, \left(a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, \left(3 \, a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]^2\right)}} + \frac{\left(3 \, a^2 + 61 \, a \, b + 48 \, b^2\right) \, \left(3 \, a + b\right)^2 \, f \, \sqrt{Sec\left[e + f \, x\right]^2 \, \left(a + b - a \, Sin\left[e + f \, x\right]$$

$$\frac{b \cos [e+fx]^6 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{3 a (a+b) f \sqrt{a+b \sec [e+fx]^2} (a+b-a \sin [e+fx]^2)^{3/2}} - \frac{2 b (5 a+4 b) \cos [e+fx]^4 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]}{3 a^2 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 (2 a^3-3 a^2 b-42 a b^2-32 b^3) \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{15 a^4 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} + \frac{2 (3 a^2+61 a b+48 b^2) \cos [e+fx]^2 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{15 a^3 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} + \frac{(3 a^2+61 a b+48 b^2) \cos [e+fx]^2 \sqrt{b+a \cos [e+fx]^2} \sin [e+fx]^2}{15 a^3 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}} + \frac{(3 a^4-11 a^3 b+27 a^2 b^2+184 a b^3+128 b^4) \sqrt{b+a \cos [e+fx]^2}}{15 a^5 (a+b)^2 f \sqrt{\cos [e+fx]^2} \sqrt{a+b \sec [e+fx]^2}} = \frac{2 b (5 a+4 b) \cos [e+fx]^4 \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{15 a^4 (a+b)^2 f \sqrt{a+b \sec [e+fx]^2}}{15 a^5 (a+b)^2 f \sqrt{\cos [e+fx]^2} \sqrt{a+b \sec [e+fx]^2}} = \frac{2 b (5 a+4 b) \cos [e+fx]^4 \sqrt{b+a \cos [e+fx]^2} \sqrt{a+b-a \sin [e+fx]^2}}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5 a+4 b) \cos [e+fx]^2}{15 a^5 (a+b)^2 f \sqrt{a+b-a \sin [e+fx]^2}} + \frac{2 b (5$$

#### Problem 298: Unable to integrate problem.

$$\int \left( d \, \mathsf{Sec} \, [\, e + f \, x \, ] \, \right)^{m} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Sec} \, [\, e + f \, x \, ]^{\, 2} \right)^{p} \, \mathrm{d} x$$

Optimal (type 6, 111 leaves, ? steps):

$$\begin{split} &\frac{1}{\text{fm}} \text{AppellF1} \Big[ \frac{\text{m}}{2}, \, \frac{1}{2}, \, -\text{p}, \, \frac{2+\text{m}}{2}, \, \text{Sec} \, [\, \text{e} + \text{f} \, \text{x} \, ]^{\, 2}, \, -\frac{\text{b} \, \text{Sec} \, [\, \text{e} + \text{f} \, \text{x} \, ]^{\, 2}}{\text{a}} \, \Big] \\ &\quad \text{Cot} \, [\, \text{e} + \text{f} \, \text{x} \, ] \, \left( \text{d} \, \text{Sec} \, [\, \text{e} + \text{f} \, \text{x} \, ] \, \right)^{\, \text{m}} \, \left( \text{a} + \text{b} \, \text{Sec} \, [\, \text{e} + \text{f} \, \text{x} \, ]^{\, 2} \right)^{\, \text{p}} \, \left( 1 + \frac{\text{b} \, \text{Sec} \, [\, \text{e} + \text{f} \, \text{x} \, ]^{\, 2}}{\text{a}} \right)^{-\text{p}} \, \sqrt{-\text{Tan} \, [\, \text{e} + \text{f} \, \text{x} \, ]^{\, 2}} \end{split}$$

Result (type 8, 27 leaves, 0 steps):

$$Unintegrable\left[\,\left(d\,Sec\,[\,e+f\,x\,]\,\right)^{\,m}\,\left(a+b\,Sec\,[\,e+f\,x\,]^{\,2}\right)^{\,p}\text{, }x\,\right]$$

#### Problem 299: Result valid but suboptimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right]^{\,3} \, \left(a + b\,Sec \left[\,e + f\,x\,\right]^{\,2}\right)^{\,p} \,\mathrm{d}x$$

Optimal (type 6, 103 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, \ 2+p, \ -p, \ \frac{3}{2}, \ Sin[e+fx]^2, \ \frac{a \, Sin[e+fx]^2}{a+b} \Big] \\ & \left( Cos[e+fx]^2 \right)^p \, Sin[e+fx] \, \left( Sec[e+fx]^2 \left( a+b-a \, Sin[e+fx]^2 \right) \right)^p \, \left( 1 - \frac{a \, Sin[e+fx]^2}{a+b} \right)^{-p} \end{split}$$

Result (type 6, 124 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, \ 2+p, \ -p, \ \frac{3}{2}, \ Sin[e+fx]^2, \ \frac{a \, Sin[e+fx]^2}{a+b} \Big] \ \left( Cos[e+fx]^2 \right)^p \\ & \left( b+a \, Cos[e+fx]^2 \right)^{-p} \ \left( a+b \, Sec[e+fx]^2 \right)^p \, Sin[e+fx] \ \left( a+b-a \, Sin[e+fx]^2 \right)^p \ \left( 1-\frac{a \, Sin[e+fx]^2}{a+b} \right)^{-p} \end{split}$$

#### Problem 300: Result valid but suboptimal antiderivative.

$$\int Sec \left[\,e + f\,x\,\right] \, \left(\,a + b\,Sec \left[\,e + f\,x\,\right]^{\,2}\right)^{\,p} \, \mathrm{d}x$$

Optimal (type 6, 103 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1\Big[\frac{1}{2},\ 1+p,\ -p,\ \frac{3}{2},\ Sin[e+fx]^2,\ \frac{a\,Sin[e+fx]^2}{a+b}\Big] \\ &\left(Cos\,[e+f\,x]^2\right)^p\,Sin[e+f\,x]\,\left(Sec\,[e+f\,x]^2\,\left(a+b-a\,Sin[e+f\,x]^2\right)\right)^p\,\left(1-\frac{a\,Sin[e+f\,x]^2}{a+b}\right)^{-p} \end{split}$$

Result (type 6, 124 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, \ 1+p, \ -p, \ \frac{3}{2}, \ Sin[e+fx]^2, \ \frac{a \, Sin[e+fx]^2}{a+b} \Big] \ \left( Cos[e+fx]^2 \right)^p \\ &\left( b+a \, Cos[e+fx]^2 \right)^{-p} \ \left( a+b \, Sec[e+fx]^2 \right)^p \, Sin[e+fx] \ \left( a+b-a \, Sin[e+fx]^2 \right)^p \ \left( 1-\frac{a \, Sin[e+fx]^2}{a+b} \right)^{-p} \end{split}$$

# Problem 301: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx] (a+bSec[e+fx]^2)^p dx$$

Optimal (type 6, 101 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, \, p, \, -p, \, \frac{3}{2}, \, Sin \, [\, e + f \, x \, ]^{\, 2}, \, \, \frac{a \, Sin \, [\, e + f \, x \, ]^{\, 2}}{a + b} \Big] \\ &\quad \left( Cos \, [\, e + f \, x \, ]^{\, 2} \right)^{p} \, Sin \, [\, e + f \, x \, ] \, \left( Sec \, [\, e + f \, x \, ]^{\, 2} \, \left( a + b - a \, Sin \, [\, e + f \, x \, ]^{\, 2} \right) \right)^{p} \, \left( 1 - \frac{a \, Sin \, [\, e + f \, x \, ]^{\, 2}}{a + b} \right)^{-p} \end{split}$$

Result (type 6, 122 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, \ p, \ -p, \ \frac{3}{2}, \ Sin[e+fx]^2, \ \frac{a \, Sin[e+fx]^2}{a+b} \Big] \ \left( Cos[e+fx]^2 \right)^p \\ & \left( b+a \, Cos[e+fx]^2 \right)^{-p} \ \left( a+b \, Sec[e+fx]^2 \right)^p \, Sin[e+fx] \ \left( a+b-a \, Sin[e+fx]^2 \right)^p \ \left( 1-\frac{a \, Sin[e+fx]^2}{a+b} \right)^{-p} \end{split}$$

#### Problem 302: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx]^3 (a+b Sec[e+fx]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1\Big[\frac{1}{2}, -1+p, -p, \frac{3}{2}, Sin[e+fx]^2, \frac{a Sin[e+fx]^2}{a+b}\Big] \\ &\left(Cos[e+fx]^2\right)^p Sin[e+fx] \left(Sec[e+fx]^2 \left(a+b-a Sin[e+fx]^2\right)\right)^p \left(1-\frac{a Sin[e+fx]^2}{a+b}\right)^{-p} \end{split}$$

Result (type 6, 124 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, -1 + p, -p, \frac{3}{2}, Sin[e+fx]^2, \frac{a Sin[e+fx]^2}{a+b} \Big] \left( Cos[e+fx]^2 \right)^p \\ &\left( b + a Cos[e+fx]^2 \right)^{-p} \left( a + b Sec[e+fx]^2 \right)^p Sin[e+fx] \left( a + b - a Sin[e+fx]^2 \right)^p \left( 1 - \frac{a Sin[e+fx]^2}{a+b} \right)^{-p} \end{split}$$

#### Problem 303: Result valid but suboptimal antiderivative.

$$\int Cos[e+fx]^5 (a+b Sec[e+fx]^2)^p dx$$

Optimal (type 6, 103 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} AppellF1 \Big[ \frac{1}{2}, -2 + p, -p, \frac{3}{2}, \, Sin \, [e + f \, x]^2, \, \frac{a \, Sin \, [e + f \, x]^2}{a + b} \Big] \\ &\quad \left( Cos \, [e + f \, x]^2 \right)^p \, Sin \, [e + f \, x] \, \left( Sec \, [e + f \, x]^2 \left( a + b - a \, Sin \, [e + f \, x]^2 \right) \right)^p \, \left( 1 - \frac{a \, Sin \, [e + f \, x]^2}{a + b} \right)^{-p} \end{split}$$

Result (type 6, 124 leaves, 5 steps):

$$\begin{split} &\frac{1}{f} \text{AppellF1} \Big[ \frac{1}{2}, -2 + p, -p, \frac{3}{2}, \, \text{Sin} \, [\, e + f \, x \, ]^{\, 2}, \, \frac{a \, \text{Sin} \, [\, e + f \, x \, ]^{\, 2}}{a + b} \Big] \, \left( \text{Cos} \, [\, e + f \, x \, ]^{\, 2} \right)^{p} \\ &\left( b + a \, \text{Cos} \, [\, e + f \, x \, ]^{\, 2} \right)^{-p} \, \left( a + b \, \text{Sec} \, [\, e + f \, x \, ]^{\, 2} \right)^{p} \, \text{Sin} \, [\, e + f \, x \, ]^{\, 2} \right)^{p} \, \left( 1 - \frac{a \, \text{Sin} \, [\, e + f \, x \, ]^{\, 2}}{a + b} \right)^{-p} \end{split}$$

Test results for the 70 problems in "4.6.0 (a csc)^m (b trg)^n.m"

Test results for the 84 problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.m"

Test results for the 59 problems in "4.6.1.2 (d csc)^n (a+b csc)^m.m"

Test results for the 16 problems in "4.6.1.3 (d cos)^n (a+b csc)^m.m"

Test results for the 23 problems in "4.6.1.4 (d cot)^n (a+b csc)^m.m"

Test results for the 24 problems in "4.6.3.1 (a+b csc)^m (d csc)^n (A+B csc).m"

Test results for the 1 problems in "4.6.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2).m"

Test results for the 27 problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.m"

Test results for the 254 problems in "4.7.1 (c trig)^m (d trig)^n.m"

Test results for the 294 problems in "4.7.2 trig^m (a trig+b trig)^n.m"

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^3}{\left(a\cos[x] + b\sin[x]\right)^2} \, dx$$

Optimal (type 3, 107 leaves, ? steps):

$$\frac{6 \ a^2 \ b \ ArcTanh \left[ \frac{-b+a \ Tan \left[ \frac{x}{2} \right]}{\sqrt{a^2+b^2}} \right]}{\left( a^2+b^2 \right)^{5/2}} + \frac{3 \ a \ \left( a^2-b^2 \right) + a \ \left( a^2+b^2 \right) \ Cos \left[ 2 \ x \right] - b \ \left( a^2+b^2 \right) \ Sin \left[ 2 \ x \right]}{2 \ \left( a^2+b^2 \right)^2 \ \left( a \ Cos \left[ x \right] + b \ Sin \left[ x \right] \right)}$$

Result (type 3, 283 leaves, 19 steps):

$$-\frac{3 \text{ a}^2 \text{ ArcTanh} \Big[\frac{b \text{ Cos}[x] - a \text{ Sin}[x]}{\sqrt{a^2 + b^2}}\Big]}{b \left(a^2 + b^2\right)^{3/2}} - \frac{2 \text{ a}^2 \text{ b} \text{ ArcTanh} \Big[\frac{b - a \text{ Tan} \Big[\frac{x}{2}\Big]}{\sqrt{a^2 + b^2}}\Big]}{\left(a^2 + b^2\right)^{5/2}} + \frac{2 \text{ a}^2 \left(3 \text{ a}^2 + b^2\right) \text{ ArcTanh} \Big[\frac{b - a \text{ Tan} \Big[\frac{x}{2}\Big]}{\sqrt{a^2 + b^2}}\Big]}{b \left(a^2 + b^2\right)^{5/2}} - \frac{\text{Cos}[x]}{b^2} + \frac{3 \text{ a}^3 \text{ Sin}[x]}{b^3 \left(a^2 + b^2\right)} - \frac{2 \text{ a}^3 \text{ Cos} \Big[\frac{x}{2}\Big]^2 \left(2 \text{ a} \text{ b} + \left(a^2 - b^2\right) \text{ Tan} \Big[\frac{x}{2}\Big]\right)}{b^3 \left(a^2 + b^2\right)^2} + \frac{2 \text{ a}^2 \left(a + b \text{ Tan} \Big[\frac{x}{2}\Big]\right)}{\left(a^2 + b^2\right)^2 \left(a + 2 \text{ b} \text{ Tan} \Big[\frac{x}{2}\Big]^2\right)}$$

#### Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^2}{\left(a\cos[x] + b\sin[x]\right)^3} dx$$

Optimal (type 3, 92 leaves, ? steps):

$$-\frac{\left(a^{2}-2\;b^{2}\right)\;ArcTanh\left[\;\frac{-b+a\;Tan\left[\frac{x}{2}\right]}{\sqrt{a^{2}+b^{2}}}\;\right]}{\left(a^{2}+b^{2}\right)^{5/2}}+\frac{a\;\left(3\;a\;b\;Cos\left[x\right]\;+\;\left(a^{2}+4\;b^{2}\right)\;Sin\left[x\right]\;\right)}{2\;\left(a^{2}+b^{2}\right)^{2}\;\left(a\;Cos\left[x\right]\;+\;b\;Sin\left[x\right]\right)^{2}}$$

Result (type 3, 300 leaves, 13 steps):

$$\frac{2 \, \mathsf{a}^2 \, \mathsf{ArcTanh} \left[ \frac{\mathsf{b} \, \mathsf{Cos} \, [x] - \mathsf{a} \, \mathsf{Sin} \, [x]}{\sqrt{\mathsf{a}^2 + \mathsf{b}^2}} \right]}{\mathsf{b}^2 \, \left( \mathsf{a}^2 + \mathsf{b}^2 \right)^{3/2}} - \frac{\mathsf{ArcTanh} \left[ \frac{\mathsf{b} \, \mathsf{Cos} \, [x] - \mathsf{a} \, \mathsf{Sin} \, [x]}{\sqrt{\mathsf{a}^2 + \mathsf{b}^2}} \right]}{\mathsf{b}^2 \, \sqrt{\mathsf{a}^2 + \mathsf{b}^2}} - \frac{\mathsf{a}^2 \, \left( 2 \, \mathsf{a}^2 - \mathsf{b}^2 \right) \, \mathsf{ArcTanh} \left[ \frac{\mathsf{b} - \mathsf{a} \, \mathsf{Tanh} \left[ \frac{\mathsf{x}}{2} \right]}{\sqrt{\mathsf{a}^2 + \mathsf{b}^2}} \right]}{\mathsf{b}^2 \, \left( \mathsf{a}^2 + \mathsf{b}^2 \right)^{3/2}} + \frac{\mathsf{2} \, \left( \mathsf{a} \, \mathsf{b} + \left( \mathsf{a}^2 + \mathsf{2} \, \mathsf{b}^2 \right) \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right] \right)}{\mathsf{b} \, \left( \mathsf{a}^2 + \mathsf{b}^2 \right) \, \left( \mathsf{a} \, \mathsf{Cos} \, [x] + \mathsf{b} \, \mathsf{Sin} \, [x] \right)} + \frac{\mathsf{2} \, \left( \mathsf{a} \, \mathsf{b} + \left( \mathsf{a}^2 + \mathsf{2} \, \mathsf{b}^2 \right) \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right] \right)}{\mathsf{a} \, \left( \mathsf{a}^2 + \mathsf{b}^2 \right) \, \left( \mathsf{a} \, \mathsf{Cos} \, [x] + \mathsf{b} \, \mathsf{Sin} \, [x] \right)} + \frac{\mathsf{2} \, \left( \mathsf{a} \, \mathsf{b} + \left( \mathsf{a}^2 + \mathsf{2} \, \mathsf{b} \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right] \right)}{\mathsf{a} \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right]^2} - \frac{\mathsf{4} \, \mathsf{a}^4 + \mathsf{3} \, \mathsf{a}^2 \, \mathsf{b}^2 + \mathsf{2} \, \mathsf{b}^4 + \mathsf{a} \, \mathsf{b} \, \left( \mathsf{5} \, \mathsf{a}^2 + \mathsf{2} \, \mathsf{b}^2 \right) \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right]}{\mathsf{a} \, \mathsf{b} \, \left( \mathsf{a}^2 + \mathsf{b}^2 \right)^2 \, \left( \mathsf{a} + \mathsf{2} \, \mathsf{b} \, \mathsf{Tan} \left[ \frac{\mathsf{x}}{2} \right]^2 \right)}$$

#### Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[c+dx]^3}{\left(a\cos[c+dx]+b\sin[c+dx]\right)^2} dx$$

Optimal (type 3, 138 leaves, ? steps):

$$-\frac{3 \ a \ b^{2} \ ArcTanh\Big[\frac{b \ Cos \ [c+d \ x] - a \ Sin \ [c+d \ x]}{\sqrt{a^{2}+b^{2}}}\Big]}{\left(a^{2}+b^{2}\right)^{5/2} \ d} + \frac{2 \ a \ b \ Cos \ [c+d \ x]}{\left(a^{2}+b^{2}\right)^{2} \ d} + \frac{\left(a^{2}-b^{2}\right) \ Sin \ [c+d \ x]}{\left(a^{2}+b^{2}\right)^{2} \ d} - \frac{b^{3}}{\left(a^{2}+b^{2}\right)^{2} \ d \ \left(a \ Cos \ [c+d \ x] + b \ Sin \ [c+d \ x]\right)}$$

Result (type 3, 231 leaves, 11 steps):

$$\begin{split} &\frac{2\;b^4\,\text{ArcTanh}\Big[\frac{b^{-a\,\text{Tan}\Big[\frac{1}{2}\;(c+d\,x)\,\Big]}{\sqrt{a^2+b^2}}\,\Big]}{a\;\left(a^2+b^2\right)^{5/2}\,d} - \frac{2\;b^2\;\left(3\;a^2+b^2\right)\;\text{ArcTanh}\Big[\frac{b^{-a\,\text{Tan}\Big[\frac{1}{2}\;(c+d\,x)\,\Big]}{\sqrt{a^2+b^2}}\,\Big]}{a\;\left(a^2+b^2\right)^{5/2}\,d} + \\ &\frac{2\;\left(2\;a\;b+\left(a^2-b^2\right)\;\text{Tan}\Big[\frac{1}{2}\;\left(c+d\,x\right)\,\Big]\right)}{\left(a^2+b^2\right)^2\,d\;\left(1+\,\text{Tan}\Big[\frac{1}{2}\;\left(c+d\,x\right)\,\Big]^2\right)} - \frac{2\;b^3\;\left(a+b\,\text{Tan}\Big[\frac{1}{2}\;\left(c+d\,x\right)\,\Big]\right)}{a\;\left(a^2+b^2\right)^2\,d\;\left(a+2\;b\,\text{Tan}\Big[\frac{1}{2}\;\left(c+d\,x\right)\,\Big] - a\,\text{Tan}\Big[\frac{1}{2}\;\left(c+d\,x\right)\,\Big]^2\right)} \end{split}$$

#### Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[c+dx]^4}{\left(a\cos[c+dx]+b\sin[c+dx]\right)^3} dx$$

Optimal (type 3, 216 leaves, ? steps):

$$-\frac{3 \ b^{2} \left(4 \ a^{2}-b^{2}\right) \ ArcTanh \Big[\frac{b-a \ Tan \Big[\frac{1}{2} \ (c+d \ x)\Big]}{\sqrt{a^{2}+b^{2}}}\Big]}{\left(a^{2}+b^{2}\right)^{7/2} \ d} + \frac{b \left(3 \ a^{2}-b^{2}\right) \ Cos \ [c+d \ x]}{\left(a^{2}+b^{2}\right)^{3} \ d} + \frac{a \left(a^{2}-3 \ b^{2}\right) \ Sin \ [c+d \ x]}{\left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{4} \ Sin \ [c+d \ x]}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(8 \ a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(8 \ a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(8 \ a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(8 \ a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(8 \ a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)^{3} \ d} + \frac{b^{3} \left(a^{2}+b^{2}\right)}{2 \ a \left(a^{2}+b^{2}\right)} + \frac{b^{3} \left(a^{2$$

Result (type 3, 492 leaves, 15 steps):

$$-\frac{3 \ b^{4} \ \left(a^{2}+2 \ b^{2}\right) \ ArcTanh\left[\frac{b-a \ Tan\left[\frac{1}{2} \ (c+d \ x)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{a^{2} \ \left(a^{2}+b^{2}\right)^{7/2} \ d} + \frac{4 \ b^{4} \ \left(3 \ a^{2}+2 \ b^{2}\right) \ ArcTanh\left[\frac{b-a \ Tan\left[\frac{1}{2} \ (c+d \ x)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{a^{2} \ \left(a^{2}+b^{2}\right)^{7/2} \ d} - \frac{2 \ b^{2} \ \left(6 \ a^{4}+3 \ a^{2} \ b^{2}+b^{4}\right) \ ArcTanh\left[\frac{b-a \ Tan\left[\frac{1}{2} \ (c+d \ x)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{a^{2} \ \left(a^{2}+b^{2}\right)^{7/2} \ d} + \frac{2 \ b^{4} \ \left(a^{2}+b^{2}\right)^{7/2} \ d}{a^{2} \ \left(a^{2}+b^{2}\right)^{7/2} \ d} - \frac{2 \ b^{4} \ \left(a \ b+\left(a^{2}+2 \ b^{2}\right) \ Tan\left[\frac{1}{2} \ (c+d \ x)\right]\right)}{a^{3} \ \left(a^{2}+b^{2}\right)^{3} \ d \ \left(1+Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]^{2}\right)} + \frac{2 \ b^{4} \ \left(a \ b+\left(a^{2}+2 \ b^{2}\right) \ Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]\right)}{a^{3} \ \left(a^{2}+b^{2}\right)^{3} \ d \ \left(a+2 \ b \ Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]^{2}\right)} - \frac{4 \ b^{3} \ \left(2 \ a^{4}-b^{4}+a \ b \ \left(3 \ a^{2}+2 \ b^{2}\right) \ Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]\right)}{a^{3} \ \left(a^{2}+b^{2}\right)^{3} \ d \ \left(a+2 \ b \ Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]\right)} - a \ Tan\left[\frac{1}{2} \ \left(c+d \ x\right)\right]^{2}\right)}$$

#### Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[c+dx]^2}{\left(a\cos[c+dx]+b\sin[c+dx]\right)^3} dx$$

Optimal (type 3, 119 leaves, ? steps):

$$\frac{\left(2\;a^{2}-b^{2}\right)\;ArcTanh\left[\frac{-b+a\;Tan\left[\frac{1}{2}\;(c+d\;x)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{\left(a^{2}+b^{2}\right)^{5/2}\;d}-\frac{b\;\left(\left(4\;a^{2}+b^{2}\right)\;Cos\left[c+d\;x\right]+3\;a\;b\;Sin\left[c+d\;x\right]\right)}{2\;\left(a^{2}+b^{2}\right)^{2}\;d\;\left(a\;Cos\left[c+d\;x\right]+b\;Sin\left[c+d\;x\right]\right)^{2}}$$

Result (type 3, 225 leaves, 6 steps):

$$-\frac{\left(2\,a^{2}-b^{2}\right)\,\text{ArcTanh}\Big[\,\frac{b-a\,\text{Tan}\Big[\frac{1}{2}\,\left(c+d\,x\right)\,\Big]}{\sqrt{a^{2}+b^{2}}}\,\Big]}{\left(a^{2}+b^{2}\right)^{5/2}\,d}\,+\,\frac{2\,b^{2}\,\left(a\,b+\left(a^{2}+2\,b^{2}\right)\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\right)}{a^{3}\,\left(a^{2}+b^{2}\right)\,d\,\left(a+2\,b\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,-\,a\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]^{2}\right)^{2}}\,-\,\frac{b\,\left(4\,a^{4}+3\,a^{2}\,b^{2}+2\,b^{4}+a\,b\,\left(5\,a^{2}+2\,b^{2}\right)\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\right)}{a^{3}\,\left(a^{2}+b^{2}\right)^{2}\,d\,\left(a+2\,b\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]\,-\,a\,\text{Tan}\Big[\,\frac{1}{2}\,\left(c+d\,x\right)\,\Big]^{2}\right)}$$

# Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[c+dx]^3}{\left(a\cos[c+dx]+b\sin[c+dx]\right)^4} dx$$

Optimal (type 3, 157 leaves, ? steps):

$$\frac{a\;\left(2\;a^{2}-3\;b^{2}\right)\;ArcTanh\left[\frac{-b+a\,Tan\left[\frac{1}{2}\;\left(c+d\;x\right)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{\left(a^{2}+b^{2}\right)^{7/2}\;d}\;+\;\frac{-3\;\left(3\;a^{4}\;b-a^{2}\;b^{3}+b^{5}\right)\;Cos\left[2\;\left(c+d\;x\right)\right]+\frac{1}{2}\;b\;\left(-9\;a^{2}+b^{2}\right)\;\left(2\;\left(a^{2}+b^{2}\right)+3\;a\;b\;Sin\left[2\;\left(c+d\;x\right)\right]\right)}{6\;\left(a^{2}+b^{2}\right)^{3}\;d\;\left(a\;Cos\left[c+d\;x\right]+b\;Sin\left[c+d\;x\right]\right)^{3}}$$

Result (type 3, 362 leaves, 7 steps):

$$-\frac{a\;\left(2\;a^{2}-3\;b^{2}\right)\;\text{ArcTanh}\left[\frac{b-a\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]}{\sqrt{a^{2}+b^{2}}}\right]}{\left(a^{2}+b^{2}\right)^{7/2}d} - \frac{8\;b^{3}\;\left(a\;\left(a^{2}+2\;b^{2}\right)+b\;\left(3\;a^{2}+4\;b^{2}\right)\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]\right)}{3\;a^{5}\;\left(a^{2}+b^{2}\right)\;d\;\left(a+2\;b\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]-a\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]^{2}\right)^{3}} + \\ \frac{2\;b^{2}\;\left(b\;\left(15\;a^{4}+18\;a^{2}\;b^{2}+8\;b^{4}\right)+a\;\left(9\;a^{4}+30\;a^{2}\;b^{2}+16\;b^{4}\right)\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]\right)}{3\;a^{5}\;\left(a^{2}+b^{2}\right)^{2}\;d\;\left(a+2\;b\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]-a\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]^{2}\right)} - \\ \frac{b\;\left(6\;a^{6}+9\;a^{4}\;b^{2}+12\;a^{2}\;b^{4}+4\;b^{6}+a\;b\;\left(9\;a^{4}+6\;a^{2}\;b^{2}+2\;b^{4}\right)\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]\right)}{a^{4}\;\left(a^{2}+b^{2}\right)^{3}\;d\;\left(a+2\;b\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]-a\;\text{Tan}\left[\frac{1}{2}\;\left(c+d\;x\right)\right]^{2}\right)}$$

Test results for the 397 problems in "4.7.3 (c+d x)^m trig^n trig^p.m"

Test results for the 9 problems in "4.7.4 x^m (a+b trig^n)^p.m"

Test results for the 330 problems in "4.7.5 x^m trig(a+b log(c x^n))^p.m"

Problem 135: Unable to integrate problem.

$$\int x^3 \, \mathsf{Tan} \, [\, \mathsf{a} + \mathrm{i} \, \mathsf{Log} \, [\, \mathsf{x} \, ] \, ] \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 47 leaves, 5 steps):

$$-i e^{2ia} x^2 + \frac{i x^4}{4} + i e^{4ia} Log[e^{2ia} + x^2]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^3 Tan [a + i Log [x]], x]$ 

Problem 136: Unable to integrate problem.

$$\int x^2 \, \mathsf{Tan} \, [\, \mathsf{a} + \mathrm{i} \, \mathsf{Log} \, [\, \mathsf{x} \, ] \,\,] \,\, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 43 leaves, 5 steps):

$$-2 \,\, \dot{\mathbb{1}} \,\, e^{2 \,\dot{\mathbb{1}} \,\, a} \,\, x \,+\, \frac{\,\dot{\mathbb{1}} \,\, x^3}{3} \,+\, 2 \,\, \dot{\mathbb{1}} \,\, e^{3 \,\dot{\mathbb{1}} \,\, a} \,\, \text{ArcTan} \, \Big[ \, e^{-\dot{\mathbb{1}} \,\, a} \,\, x \,\Big]$$

Result (type 8, 15 leaves, 0 steps): CannotIntegrate  $[x^2 Tan [a + i Log [x]], x]$ 

# Problem 137: Unable to integrate problem.

$$\int x \, \mathsf{Tan} \, [\, \mathsf{a} + \dot{\mathsf{n}} \, \mathsf{Log} \, [\, \mathsf{x} \,] \,] \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 33 leaves, 5 steps):

$$\frac{\mathbb{i} x^2}{2} - \mathbb{i} e^{2 \mathbb{i} a} Log \left[ e^{2 \mathbb{i} a} + x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[x Tan[a + i Log[x]], x]

# Problem 138: Unable to integrate problem.

$$\int \mathsf{Tan}\left[\mathsf{a} + i \mathsf{Log}\left[\mathsf{x}\right]\right] \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 27 leaves, 4 steps):

$$\mathbb{i} \ x - 2 \ \mathbb{i} \ \mathbb{e}^{\mathbb{i} \ \mathsf{a}} \ \mathsf{ArcTan} \left[ \ \mathbb{e}^{-\mathbb{i} \ \mathsf{a}} \ x \, \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[Tan[a + i Log[x]], x]

# Problem 140: Unable to integrate problem.

$$\int \frac{\mathsf{Tan}[\mathsf{a} + \mathsf{i} \mathsf{Log}[\mathsf{x}]]}{\mathsf{x}^2} \, d\mathsf{x}$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{1}{x} + 2 i e^{-i a} ArcTan \left[ e^{-i a} x \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\mathsf{Tan}\left[\mathsf{a} + i \; \mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{x}^2}, \; \mathsf{x}\right]$$

#### Problem 141: Unable to integrate problem.

$$\int \frac{\mathsf{Tan}\left[\mathsf{a} + i \; \mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 35 leaves, 4 steps):

$$\frac{1}{2 x^2} - 1 e^{-2 i a} Log \left[ 1 + \frac{e^{2 i a}}{x^2} \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $\left[\frac{\mathsf{Tan}\left[\mathsf{a} + i \mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{v}^3}, \mathsf{x}\right]$ 

#### Problem 142: Unable to integrate problem.

$$\int \frac{\mathsf{Tan}\left[\mathsf{a} + i \; \mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{x}^4} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{\dot{\mathbb{1}}}{3 \, x^3} - \frac{2 \, \dot{\mathbb{1}} \, \, \mathrm{e}^{-2 \, \dot{\mathbb{1}} \, \, a}}{x} - 2 \, \dot{\mathbb{1}} \, \, \mathrm{e}^{-3 \, \dot{\mathbb{1}} \, \, a} \, \, \mathsf{ArcTan} \left[ \, \mathrm{e}^{-\dot{\mathbb{1}} \, \, a} \, \, x \, \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan[a+i Log[x]]}{x^4}, x\right]$$

# Problem 143: Unable to integrate problem.

$$\int x^3 \operatorname{Tan} \left[ a + i \operatorname{Log} \left[ x \right] \right]^2 dx$$

Optimal (type 3, 63 leaves, 5 steps):

$$2 e^{2 i a} x^{2} - \frac{x^{4}}{4} - \frac{2 e^{6 i a}}{e^{2 i a} + x^{2}} - 4 e^{4 i a} Log \left[ e^{2 i a} + x^{2} \right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $| x^3 \text{ Tan} [a + i \text{ Log} [x]]^2$ , x |

# Problem 144: Unable to integrate problem.

$$\int x^2 \, \mathsf{Tan} \, [\, \mathsf{a} + \mathrm{i} \, \mathsf{Log} \, [\, \mathsf{x} \, ] \, ]^{\, 2} \, \, \mathrm{d} x$$

Optimal (type 3, 62 leaves, 6 steps):

$$6 e^{2 i a} x - \frac{x^3}{3} - \frac{2 e^{2 i a} x^3}{e^{2 i a} + x^2} - 6 e^{3 i a} ArcTan[e^{-i a} x]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x^2 Tan [a + i Log [x]]^2, x]$ 

#### Problem 145: Unable to integrate problem.

$$\int x \operatorname{Tan}[a + i \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 51 leaves, 5 steps):

$$-\frac{x^{2}}{2} + \frac{2 e^{4 i a}}{e^{2 i a} + x^{2}} + 2 e^{2 i a} Log \left[ e^{2 i a} + x^{2} \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $\left[ x \operatorname{Tan} \left[ a + i \operatorname{Log} \left[ x \right] \right]^{2}, x \right]$ 

# Problem 146: Unable to integrate problem.

$$\int \mathsf{Tan} \left[ \mathsf{a} + i \mathsf{Log} \left[ \mathsf{x} \right] \right]^2 d\mathsf{x}$$

Optimal (type 3, 46 leaves, 6 steps):

$$-x - \frac{2 e^{2 i a} x}{e^{2 i a} + x^{2}} + 2 e^{i a} ArcTan[e^{-i a} x]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Tan} \left[ a + i \text{Log} \left[ x \right] \right]^2, x \right]$ 

#### Problem 148: Unable to integrate problem.

$$\int \frac{\mathsf{Tan}\left[\mathsf{a} + i \mathsf{Log}\left[\mathsf{x}\right]\right]^2}{\mathsf{x}^2} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 60 leaves, 5 steps):

$$\frac{\text{e}^{2\,\text{i}\,\text{a}}}{\text{x}\,\left(\text{e}^{2\,\text{i}\,\text{a}}+\text{x}^2\right)}+\frac{3\,\text{x}}{\text{e}^{2\,\text{i}\,\text{a}}+\text{x}^2}+2\,\text{e}^{-\text{i}\,\text{a}}\,\text{ArcTan}\!\left[\,\text{e}^{-\text{i}\,\text{a}}\,\text{x}\,\right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Tan}[a + i \text{Log}[x]]^2}{x^2}, x\right]$$

#### Problem 149: Unable to integrate problem.

$$\int \frac{\mathsf{Tan}\left[\mathsf{a} + i \; \mathsf{Log}\left[\mathsf{x}\right]\right]^2}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 55 leaves, 4 steps):

$$-\,\frac{2\,\,{\text {e}}^{-2\,\,{\text {i}}\,\,a}}{1+\,\frac{{\text {e}}^{2\,\,{\text {i}}\,\,a}}{x^2}}+\,\frac{1}{2\,x^2}-2\,\,{\text {e}}^{-2\,\,{\text {i}}\,\,a}\,\,\text{Log}\,\Big[\,1+\,\frac{{\text {e}}^{2\,\,{\text {i}}\,\,a}}{x^2}\,\Big]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan[a+i Log[x]]^2}{x^3}, x\right]$$

# Problem 150: Unable to integrate problem.

$$\int (e x)^m Tan[a + i Log[x]] dx$$

Optimal (type 5, 71 leaves, 4 steps):

$$-\frac{\text{i}\left(\text{e}\,\text{X}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1}+\text{m}\right)} + \frac{2\,\,\text{i}\,\,\left(\text{e}\,\text{X}\right)^{\,\text{1+m}}\,\text{Hypergeometric2F1}\!\left[\text{1,}\,\,\frac{1}{2}\,\left(-\text{1}-\text{m}\right),\,\,\frac{\text{1-m}}{2},\,\,-\frac{\text{e}^{2\,\text{i}\,\text{a}}}{\text{x}^2}\right]}{\text{e}\,\left(\text{1}+\text{m}\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate 
$$[(ex)^m Tan[a + i Log[x]], x]$$

#### Problem 151: Unable to integrate problem.

$$\int (e x)^m Tan[a + i Log[x]]^2 dx$$

Optimal (type 5, 77 leaves, 5 steps):

$$-\frac{x\;\left(e\;x\right)^{\;m}}{1+m}+\frac{2\;x\;\left(e\;x\right)^{\;m}}{1+\frac{e^{2\;i\;a}}{v^{2}}}-2\;x\;\left(e\;x\right)^{\;m}\; \text{Hypergeometric2F1}\Big[1\text{, }\frac{1}{2}\;\left(-1-m\right)\text{, }\frac{1-m}{2}\text{, }-\frac{e^{2\;i\;a}}{x^{2}}\Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan[a + i Log[x]]^2, x]$ 

# Problem 152: Unable to integrate problem.

$$\int (e x)^m \operatorname{Tan}[a + i \operatorname{Log}[x]]^3 dx$$

Optimal (type 5, 184 leaves, 6 steps):

$$-\frac{\frac{\text{i} \left(1-\text{m}\right) \text{ m x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1+\text{m}\right)} + \frac{\frac{\text{i} \left(1-\frac{\text{e}^{2 \text{i a}}}{\text{x}^{2}}\right)^{2} \text{ x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1+\frac{\text{e}^{2 \text{i a}}}{\text{x}^{2}}\right)^{2}} + \frac{\frac{\text{i} \left(\text{e}^{-2 \text{i a}} \left(\text{e}^{2 \text{i a}} \left(3+\text{m}\right)+\frac{\text{e}^{4 \text{i a}} \left(1-\text{m}\right)}{\text{x}^{2}}\right) \text{ x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1+\frac{\text{e}^{2 \text{i a}}}{\text{x}^{2}}\right)} - \frac{\text{i} \left(3+2 \text{ m}+\text{m}^{2}\right) \text{ x } \left(\text{e x}\right)^{\text{m}} \text{ Hypergeometric2F1}\left[1,\frac{1}{2} \left(-1-\text{m}\right),\frac{1-\text{m}}{2},-\frac{\text{e}^{2 \text{i a}}}{\text{x}^{2}}\right]}{2}\right]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan [a + i Log [x]]^3, x]$ 

#### Problem 153: Unable to integrate problem.

$$\int \mathsf{Tan} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 142 leaves, 4 steps):

$$x \, \left(1 - e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right)^{\,-p} \, \left(\frac{\,\mathrm{i}\, \left(1 - e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right)}{\,1 + e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}}\right)^p \, \left(1 + e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right)^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -e^{2\,\mathrm{i}\,a} \, x^{2\,\mathrm{i}\,b}\right]^p \, \mathsf{AppellF1} \left[-\,\frac{\,\mathrm{i}}{2\,b}, \, -p, \, p, \, 1 - \frac{\,\mathrm{i}}{2\,b}, \, -p, \, 1 - \frac{\,\mathrm{i}}{$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate [Tan[a + b Log[x]]<sup>p</sup>, x]

# Problem 154: Unable to integrate problem.

$$\int (e x)^m \operatorname{Tan}[a + b \operatorname{Log}[x]]^p dx$$

Optimal (type 6, 162 leaves, 4 steps):

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan [a + b Log[x]]^p, x]$ 

# Problem 155: Unable to integrate problem.

$$\int \mathsf{Tan} \left[ \mathsf{a} + \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, d\mathsf{x}$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\,i\,a}\,x^{2\,i}\right)^{-p}\,\left(\frac{\,\dot{\mathbb{1}}\,\left(1-e^{2\,i\,a}\,x^{2\,i}\right)}{\,1+e^{2\,i\,a}\,x^{2\,i}}\right)^{p}\,\left(1+e^{2\,i\,a}\,x^{2\,i}\right)^{p}\,x\,\\ \text{AppellF1}\left[-\frac{\,\dot{\mathbb{1}}}{2}\,\text{, -p, p, }1-\frac{\,\dot{\mathbb{1}}}{2}\,\text{, }e^{2\,i\,a}\,x^{2\,i}\,\text{, -e}^{2\,i\,a}\,x^{2\,i}\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate [Tan[a + Log[x]]<sup>p</sup>, x]

#### Problem 156: Unable to integrate problem.

$$\int \mathsf{Tan} \left[ \mathsf{a} + \mathsf{2} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\,i\,a}\,x^{4\,i}\right)^{-p}\,\left(\frac{\,\dot{\mathbb{I}}\,\left(1-e^{2\,i\,a}\,x^{4\,i}\right)}{\,1+e^{2\,i\,a}\,x^{4\,i}}\right)^{p}\,\left(1+e^{2\,i\,a}\,x^{4\,i}\right)^{p}\,x\,\\ \text{AppellF1}\left[-\frac{\,\dot{\mathbb{I}}\,}{4}\,,\,-p\,,\,p\,,\,1-\frac{\,\dot{\mathbb{I}}\,}{4}\,,\,\,e^{2\,i\,a}\,x^{4\,i}\,,\,\,-e^{2\,i\,a}\,x^{4\,i}\right]^{p}\,\left(1+e^{2\,i\,a}\,x^{4\,i}\right)^{p}\,x\,\\ \text{AppellF1}\left[-\frac{\,\dot{\mathbb{I}}\,}{4}\,,\,\,-p\,,\,\,p\,,\,\,1-\frac{\,\dot{\mathbb{I}}\,}{4}\,,\,\,e^{2\,i\,a}\,x^{4\,i}\,,\,\,-e^{2\,i\,a}\,x^{4\,i}\,\right]^{p}\,x^$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate Tan [a + 2 Log[x]]<sup>p</sup>, x

# Problem 157: Unable to integrate problem.

$$\int \operatorname{Tan}[a + 3 \operatorname{Log}[x]]^{p} dx$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}\right)^{-p} \left(\frac{{\rm i}\, \left(1-e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}\right)}{1+e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}}\right)^{p} \left(1+e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}\right)^{p} \\ x \, {\rm AppellF1}\left[-\frac{{\rm i}}{6}, \, -p, \, p, \, 1-\frac{{\rm i}}{6}, \, e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}, \, -e^{2\, {\rm i}\, a}\, x^{6\, {\rm i}}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tan[a + 3 Log[x]]^p, x]$ 

# Problem 158: Unable to integrate problem.

$$\int x^3 \, \mathsf{Tan} \big[ \, \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \big[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \big] \, \right) \, \big] \, \, \mathrm{d} x$$

Optimal (type 5, 71 leaves, 4 steps):

$$-\frac{\dot{\mathbb{1}} \ x^{4}}{4} + \frac{1}{2} \,\dot{\mathbb{1}} \ x^{4} \ \text{Hypergeometric2F1} \Big[ 1 \text{, } -\frac{2\,\dot{\mathbb{1}}}{b\,d\,n} \text{, } 1 - \frac{2\,\dot{\mathbb{1}}}{b\,d\,n} \text{, } - \text{e}^{2\,\dot{\mathbb{1}}\,a\,d} \, \left( c\, x^{n} \right)^{\,2\,\dot{\mathbb{1}}\,b\,d} \Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 Tan [d (a + b Log [c x^n])], x]$ 

#### Problem 159: Unable to integrate problem.

$$\int x^2 \, \mathsf{Tan} \big[ \, \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \big[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \big] \, \right) \, \big] \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 5, 75 leaves, 4 steps):

$$-\frac{\dot{\mathbb{1}} \ x^{3}}{3} + \frac{2}{3} \ \dot{\mathbb{1}} \ x^{3} \ \text{Hypergeometric2F1} \Big[ 1, -\frac{3 \ \dot{\mathbb{1}}}{2 \ b \ d \ n}, \ 1 - \frac{3 \ \dot{\mathbb{1}}}{2 \ b \ d \ n}, - e^{2 \ \dot{\mathbb{1}} \ a \ d} \ \left( c \ x^{n} \right)^{2 \ \dot{\mathbb{1}} \ b \ d} \Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^2 Tan [d (a + b Log [c x^n])], x]$ 

#### Problem 160: Unable to integrate problem.

$$\left\lceil x \, \mathsf{Tan} \left[ \, \mathsf{d} \, \left( \, \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \right] \, \right) \, \right] \, \mathbb{d} \, \mathsf{x} \right.$$

Optimal (type 5, 69 leaves, 4 steps):

$$-\,\frac{\mathop{\dot{\mathbb{L}}} \,\, x^2}{2} + \mathop{\dot{\mathbb{L}}} \,\, x^2\,\, \text{Hypergeometric} \\ 2\text{F1} \Big[\,\textbf{1}\,,\,\, -\,\frac{\mathop{\dot{\mathbb{L}}}}{b\,\,d\,\,n}\,,\,\, \textbf{1} \,-\,\frac{\mathop{\dot{\mathbb{L}}}}{b\,\,d\,\,n}\,,\,\, -\,\mathbb{e}^{2\,\mathop{\dot{\mathbb{L}}} \,a\,d}\,\, \left(\,c\,\,x^n\,\right)^{\,2\,\mathop{\dot{\mathbb{L}}} \,b\,d}\,\Big]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x Tan [d (a + b Log [c x^n])], x]$ 

# Problem 161: Unable to integrate problem.

$$\left\lceil \text{Tan} \left[ \text{d} \, \left( \text{a} + \text{b} \, \text{Log} \left[ \text{c} \, x^{\text{n}} \right] \right) \, \right] \, \text{d} x \right.$$

Optimal (type 5, 67 leaves, 4 steps):

$$-\,\dot{\mathbb{1}}\,\,x\,+\,2\,\,\dot{\mathbb{1}}\,\,x\,\,\text{Hypergeometric}\\ 2F1\Big[\,\textbf{1,}\,\,-\,\frac{\dot{\mathbb{1}}}{2\,b\,d\,n}\,,\,\,1\,-\,\frac{\dot{\mathbb{1}}}{2\,b\,d\,n}\,,\,\,-\,\boldsymbol{\mathbb{e}}^{2\,\dot{\mathbb{1}}\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,\dot{\mathbb{1}}\,b\,d}\,\Big]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[Tan[d(a+bLog[cx^n])], x]$ 

#### Problem 163: Unable to integrate problem.

$$\int\! \frac{Tan\big[\,d\,\left(\,a\,+\,b\,Log\,[\,c\,\,x^{n}\,]\,\right)\,\big]}{x^{2}}\,\,\mathrm{d}x$$

Optimal (type 5, 71 leaves, 4 steps):

$$\frac{1}{x} - \frac{2 i \text{ Hypergeometric2F1} \left[1, \frac{i}{2 \text{ bdn}}, 1 + \frac{i}{2 \text{ bdn}}, -e^{2 i \text{ ad}} \left(c x^n\right)^{2 i \text{ bd}}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan\left[d\left(a+b\,Log\left[c\,x^{n}\right]\right)\right]}{x^{2}}$$
,  $x\right]$ 

#### Problem 164: Unable to integrate problem.

$$\int \frac{\mathsf{Tan} \left[ d \left( a + b \mathsf{Log} \left[ c x^{n} \right] \right) \right]}{x^{3}} \, dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$\frac{\frac{1}{2}x^{2}}{2x^{2}} = \frac{\frac{1}{2} \text{ Hypergeometric 2F1}\left[1, \frac{\frac{1}{b \text{ dn}}, 1 + \frac{1}{b \text{ dn}}, -e^{2 \text{ i ad}} \left(c x^{n}\right)^{2 \text{ i bd}}\right]}{x^{2}}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]}{x^{3}}$$
,  $x\right]$ 

# Problem 165: Unable to integrate problem.

$$\int x^3 \, Tan \left[ d \left( a + b \, Log \left[ c \, x^n \right] \right) \right]^2 \, dx$$

Optimal (type 5, 159 leaves, 5 steps):

$$\frac{\left(4\,\,\dot{\mathbb{1}}\,-\,b\,\,d\,\,n\right)\,\,x^{4}}{4\,\,b\,\,d\,\,n}\,\,+\,\,\frac{\,\dot{\mathbb{1}}\,\,x^{4}\,\,\left(1\,-\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n\,\,\left(1\,+\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,x^{4}\,\,Hypergeometric 2F1}{b\,\,d\,\,n}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}}{b\,\,d\,\,n}\,\,,\,\,\,1\,-\,\,\frac{2\,\,\dot{\mathbb{1}}}{b\,\,d\,\,n}\,\,,\,\,\,-\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[x^3 Tan [d (a + b Log [c x^n])]^2, x]$ 

#### Problem 166: Unable to integrate problem.

$$\left\lceil x^2\, \mathsf{Tan}\left[\,\mathsf{d}\, \left(\,\mathsf{a} + \mathsf{b}\, \mathsf{Log}\left[\,\mathsf{c}\,\, x^n\,\right]\,\right)\,\right]^{\,2}\, \mathrm{d} x\right.$$

Optimal (type 5, 163 leaves, 5 steps):

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[x^2 Tan [d (a + b Log [c x^n])]^2, x]$ 

#### Problem 167: Unable to integrate problem.

Optimal (type 5, 159 leaves, 5 steps):

$$\frac{\left(2\,\,\dot{\mathbb{1}}\,-\,b\,\,d\,\,n\right)\,\,x^{2}}{2\,\,b\,\,d\,\,n}\,\,+\,\,\frac{\,\dot{\mathbb{1}}\,\,x^{2}\,\left(1\,-\,\,\mathbb{e}^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n\,\,\left(1\,+\,\,\mathbb{e}^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,x^{2}\,\,Hypergeometric2F1\!\left[1,\,\,-\,\,\frac{\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,1\,-\,\,\frac{\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,-\,\,\mathbb{e}^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right]}{b\,\,d\,\,n}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x Tan [d (a + b Log [c x^n])]^2, x]$ 

#### Problem 168: Unable to integrate problem.

Optimal (type 5, 154 leaves, 5 steps):

$$\frac{\left(\frac{\text{i}-\text{bdn}}{\text{bdn}}\right)\text{x}}{\text{bdn}} + \frac{\frac{\text{i}\text{x}\left(1-\text{e}^{2\text{iad}}\left(\text{c}\text{x}^{\text{n}}\right)^{2\text{ibd}}\right)}{\text{bdn}\left(1+\text{e}^{2\text{iad}}\left(\text{c}\text{x}^{\text{n}}\right)^{2\text{ibd}}\right)}}{\text{bdn}\left(\text{c}\text{x}^{\text{n}}\right)^{2\text{ibd}}\right)} - \frac{2\text{i}\text{x}\text{Hypergeometric2F1}\left[1,-\frac{\text{i}}{2\text{bdn}},1-\frac{\text{i}}{2\text{bdn}},1-\frac{\text{e}^{2\text{iad}}\left(\text{c}\text{x}^{\text{n}}\right)^{2\text{ibd}}\right]}{\text{bdn}}\right]}{\text{bdn}}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[Tan[d(a+bLog[cx^n])]^2, x]$ 

#### Problem 170: Unable to integrate problem.

$$\int \frac{\mathsf{Tan} \left[ d \left( a + b \, \mathsf{Log} \left[ c \, x^n \right] \right) \right]^2}{x^2} \, \mathrm{d} x$$

Optimal (type 5, 157 leaves, 5 steps):

$$\frac{1+\frac{\mathrm{i}}{b\,d\,n}}{x}+\frac{\mathrm{i}\,\left(1-\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b\,d}\right)}{b\,d\,n\,x\,\left(1+\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b\,d}\right)}-\frac{2\,\mathrm{i}\,\mathsf{Hypergeometric}2\mathsf{F1}\!\left[1,\,\frac{\mathrm{i}}{2\,b\,d\,n},\,1+\frac{\mathrm{i}}{2\,b\,d\,n},\,-\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b\,d}\right]}{b\,d\,n\,x}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{2}},x\right]$$

#### Problem 171: Unable to integrate problem.

$$\int \frac{\mathsf{Tan} \left[ d \left( a + b \mathsf{Log} \left[ c x^{n} \right] \right) \right]^{2}}{x^{3}} \, \mathrm{d}x$$

Optimal (type 5, 156 leaves, 5 steps):

$$\frac{1+\frac{2\,\mathrm{i}}{b\,\mathrm{d}\,n}}{2\,x^2}+\frac{\mathrm{i}\,\left(1-\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,d}\right)}{b\,\mathrm{d}\,n\,x^2\,\left(1+\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,d}\right)}-\frac{2\,\mathrm{i}\,\mathsf{Hypergeometric2F1}\!\left[1,\frac{\mathrm{i}}{b\,\mathrm{d}\,n},\,1+\frac{\mathrm{i}}{b\,\mathrm{d}\,n},\,-\mathrm{e}^{2\,\mathrm{i}\,a\,d}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,d}\right]}{b\,\mathrm{d}\,n\,x^2}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tan\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{3}},x\right]$$

# Problem 175: Unable to integrate problem.

$$\label{eq:continuous} \left[ \, \left( \, e \, \, x \, \right) \, ^m \, \mathsf{Tan} \left[ \, d \, \, \left( \, a \, + \, b \, \, \mathsf{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right] \, \, \mathrm{d} \, x \, \right.$$

Optimal (type 5, 101 leaves, 4 steps):

$$-\frac{\dot{\mathbb{1}} \ (e \ x)^{\, \mathbf{1} + m}}{e \ (\mathbf{1} + m)} + \frac{2 \ \dot{\mathbb{1}} \ (e \ x)^{\, \mathbf{1} + m} \ Hypergeometric 2F1}{\left[ \mathbf{1} \text{, } -\frac{\dot{\mathbb{1}} \ (\mathbf{1} + m)}{2 \, b \, d \, n} \text{, } \mathbf{1} - \frac{\dot{\mathbb{1}} \ (\mathbf{1} + m)}{2 \, b \, d \, n} \text{, } - \mathbb{e}^{2 \, \dot{\mathbb{1}} \, a \, d} \ (c \ x^n)^{\, 2 \, \dot{\mathbb{1}} \, b \, d} \right]}{e \ (\mathbf{1} + m)}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan [d(a+b Log[cx^n])], x]$ 

#### Problem 176: Unable to integrate problem.

$$\int (e x)^m Tan \left[d \left(a + b Log \left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 196 leaves, 5 steps):

$$\frac{\left(\frac{i}{\mathbb{I}}\left(\mathbf{1}+\mathbf{m}\right)-b\,d\,n\right)\,\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}}{b\,d\,e\,\left(\mathbf{1}+\mathbf{m}\right)\,n}+\frac{\frac{i}{\mathbb{I}}\,\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}\left(\mathbf{1}-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}{b\,d\,e\,n\,\left(\mathbf{1}+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}-\frac{2\,i\,\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}\,Hypergeometric2F1\left[\mathbf{1},\,-\frac{i\,\left(\mathbf{1}+\mathbf{m}\right)}{2\,b\,d\,n},\,\mathbf{1}-\frac{i\,\left(\mathbf{1}+\mathbf{m}\right)}{2\,b\,d\,n},\,-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right]}{b\,d\,e\,n}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan [d(a+bLog[cx^n])]^2, x]$ 

#### Problem 177: Unable to integrate problem.

$$\int \left( e \, x \right)^m \mathsf{Tan} \left[ \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, x^n \right] \right) \right]^3 \, \mathrm{d} x$$

Optimal (type 5, 351 leaves, 6 steps):

#### Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tan[d(a+bLog[cx^n])]^3$ , x]

#### Problem 178: Unable to integrate problem.

$$\int\! Tan \! \left[ \! \begin{array}{c} d \end{array} \! \left( a + b \ Log \! \left[ c \ x^n \right] \right) \, \right]^p \, \mathrm{d}\!\! \left[ x \right]$$

Optimal (type 6, 190 leaves, 5 steps):

$$x \, \left( 1 - \mathbb{e}^{2\, \text{iad}} \, \left( c \, \, x^n \right)^{\, 2\, \text{ibd}} \right)^{\, - p} \, \left( \frac{\, \text{i} \, \left( 1 - \mathbb{e}^{2\, \text{iad}} \, \left( c \, \, x^n \right)^{\, 2\, \text{ibd}} \right) \,}{\, 1 + \mathbb{e}^{2\, \text{iad}} \, \left( c \, \, x^n \right)^{\, 2\, \text{ibd}}} \right)^p \, \left( 1 + \mathbb{e}^{2\, \text{iad}} \, \left( c \, \, x^n \right)^{\, 2\, \text{ibd}} \right)^p$$

AppellF1
$$\left[-\frac{i}{2 \, b \, d \, n}, -p, p, 1 - \frac{i}{2 \, b \, d \, n}, e^{2 \, i \, a \, d} \, \left(c \, x^n\right)^{2 \, i \, b \, d}, -e^{2 \, i \, a \, d} \, \left(c \, x^n\right)^{2 \, i \, b \, d}\right]$$

#### Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[ Tan \left[ d \left( a + b Log \left[ c x^n \right] \right) \right]^p, x \right]$ 

# Problem 179: Unable to integrate problem.

$$\left\lceil \left( e\,x \right)^{\,m}\, \mathsf{Tan} \left[ \,d\, \left( \mathsf{a} + \mathsf{b}\, \mathsf{Log} \left[ \,c\,\, \mathsf{x}^{\mathsf{n}} \,\right] \,\right) \,\right]^{\mathsf{p}} \, \mathrm{d} x \right.$$

Optimal (type 6, 210 leaves, 5 steps):

$$\begin{split} &\frac{1}{e\,\left(1+m\right)}\,\left(e\,x\right)^{\,1+m}\,\left(1-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)^{-p}\,\left(\frac{\,\mathrm{i}\,\left(1-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}{1+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}}\right)^{p}\\ &\left(1+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)^{p}\,\mathsf{AppellF1}\!\left[-\frac{\,\mathrm{i}\,\left(1+m\right)}{2\,b\,d\,n}\,,\,-p\,,\,p\,,\,1-\frac{\,\mathrm{i}\,\left(1+m\right)}{2\,b\,d\,n}\,,\,\,e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\,,\,-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right] \end{split}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $\left[ (e x)^m Tan \left[ d \left( a + b Log \left[ c x^n \right] \right) \right]^p$ , x

# Problem 186: Unable to integrate problem.

$$\int x^3 \, \text{Cot} \, [\, a + \text{i} \, \, \text{Log} \, [\, x \, ] \,\,] \,\, \text{d} \, x$$

Optimal (type 3, 49 leaves, 5 steps):

$$-i e^{2ia} x^2 - \frac{i x^4}{4} - i e^{4ia} Log[e^{2ia} - x^2]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^3 \cot [a + i \log [x]], x]$ 

# Problem 187: Unable to integrate problem.

$$\int x^2 \cot [a + i \log [x]] dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$-2 i e^{2 i a} x - \frac{i x^3}{3} + 2 i e^{3 i a} ArcTanh [e^{-i a} x]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Cot} [a + i \text{ Log} [x]], x]$ 

# Problem 188: Unable to integrate problem.

$$\label{eq:cot_angle} \boxed{x \, \text{Cot} \, [\, a \, + \, \dot{\mathbb{1}} \, \, \text{Log} \, [\, x \, ] \, ] \, \, \mathbb{d} \, x}$$

Optimal (type 3, 35 leaves, 5 steps):

$$-\frac{i x^{2}}{2} - i e^{2 i a} Log [e^{2 i a} - x^{2}]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate[x Cot[a + i Log[x]], x]

#### Problem 189: Unable to integrate problem.

$$-i x + 2i e^{i a} ArcTanh [e^{-i a} x]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate [Cot[a + i Log[x]], x]

# Problem 191: Unable to integrate problem.

$$\int \frac{\mathsf{Cot}[\mathsf{a} + \mathsf{i} \mathsf{Log}[\mathsf{x}]]}{\mathsf{x}^2} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 29 leaves, 4 steps):

$$-\frac{\dot{\mathbb{I}}}{x} + 2 \,\dot{\mathbb{I}} \,\, e^{-i\,a} \,\, \text{ArcTanh} \left[ \,e^{-i\,a} \,\, x \,\right]$$

Result (type 8, 15 leaves, 0 steps):

$$\label{eq:cannotIntegrate} \begin{split} & \mathsf{CannotIntegrate} \big[ \, \frac{\mathsf{Cot} \, [\, a \, + \, \dot{\mathtt{i}} \, \, \mathsf{Log} \, [\, x \, ] \, \,]}{x^2} \, \text{, } x \, \big] \end{split}$$

# Problem 192: Unable to integrate problem.

$$\int \frac{\mathsf{Cot}[\mathsf{a} + i \mathsf{Log}[\mathsf{x}]]}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{1}{2 x^2} - 1 e^{-2 i a} Log \left[1 - \frac{e^{2 i a}}{x^2}\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}\left[a+i\text{Log}\left[x\right]\right]}{x^{3}}, x\right]$$

#### Problem 193: Unable to integrate problem.

$$\int \frac{\text{Cot}\left[\,\mathsf{a}\,+\,\dot{\mathbb{1}}\,\,\mathsf{Log}\left[\,\mathsf{x}\,\right]\,\right]}{\mathsf{x}^4}\,\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 45 leaves, 5 steps):

$$-\frac{\text{i}}{3 \, \text{x}^3} - \frac{2 \, \text{i} \, \text{e}^{-2 \, \text{i} \, \text{a}}}{\text{x}} + 2 \, \text{i} \, \text{e}^{-3 \, \text{i} \, \text{a}} \, \text{ArcTanh} \left[ \, \text{e}^{-\text{i} \, \text{a}} \, \text{x} \, \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}[a + i \text{Log}[x]]}{x^4}, x\right]$$

# Problem 194: Unable to integrate problem.

$$\int x^3 \cot [a + i \log [x]]^2 dx$$

Optimal (type 3, 67 leaves, 5 steps):

$$-2\; {\text {e}}^{2\; {\text {i}}\; a}\; x^2 - \frac{x^4}{4} - \frac{2\; {\text {e}}^{6\; {\text {i}}\; a}}{{\text {e}}^{2\; {\text {i}}\; a} - x^2} - 4\; {\text {e}}^{4\; {\text {i}}\; a}\; \text{Log} \left[\; {\text {e}}^{2\; {\text {i}}\; a} - x^2\; \right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x^3 \cot [a + i \log [x]]^2, x]$ 

# Problem 195: Unable to integrate problem.

$$\int x^2 \cot [a + i \log [x]]^2 dx$$

Optimal (type 3, 64 leaves, 6 steps):

$$-6 \, e^{2 \, i \, a} \, x - \frac{x^3}{3} - \frac{2 \, e^{2 \, i \, a} \, x^3}{e^{2 \, i \, a} - x^2} + 6 \, e^{3 \, i \, a} \, \text{ArcTanh} \left[ \, e^{-i \, a} \, x \, \right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x^2 \cot [a + i \log [x]]^2, x]$ 

# Problem 196: Unable to integrate problem.

$$\int x \cot [a + i \log [x]]^2 dx$$

Optimal (type 3, 55 leaves, 5 steps):

$$-\frac{x^{2}}{2}-\frac{2\,e^{4\,i\,a}}{e^{2\,i\,a}-x^{2}}-2\,e^{2\,i\,a}\,Log\left[\,e^{2\,i\,a}-x^{2}\,\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $| x \cot [a + i \log [x]]^2$ , x |

# Problem 197: Unable to integrate problem.

Optimal (type 3, 48 leaves, 6 steps):

$$-\,x\,-\,\frac{2\,\,{\rm e}^{2\,\,i\,\,a}\,\,x}{\,{\rm e}^{2\,\,i\,\,a}\,-\,x^2}\,+\,2\,\,{\rm e}^{\,i\,\,a}\,\,{\rm ArcTanh}\,\Big[\,\,{\rm e}^{-\,i\,\,a}\,\,x\,\Big]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Cot} \left[ a + i \text{Log} \left[ x \right] \right]^2, x \right]$ 

#### Problem 199: Unable to integrate problem.

$$\int \frac{\mathsf{Cot}\,[\,\mathsf{a}\,+\,\dot{\mathtt{i}}\,\,\mathsf{Log}\,[\,\mathsf{x}\,]\,]^{\,2}}{\mathsf{x}^{2}}\,\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 64 leaves, 5 steps):

$$\frac{\,\,\mathrm{e}^{2\,\mathrm{i}\,a}\,\,}{x\,\,\left(\,\mathrm{e}^{2\,\mathrm{i}\,a}\,-\,x^2\right)}\,-\,\frac{3\,x}{\,\,\mathrm{e}^{2\,\mathrm{i}\,a}\,-\,x^2}\,-\,2\,\,\mathrm{e}^{-\mathrm{i}\,a}\,\,\text{ArcTanh}\,\big[\,\mathrm{e}^{-\mathrm{i}\,a}\,x\,\big]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[\frac{\cot[a+i \log[x]]^2}{x^2}, x\right]$ 

# Problem 200: Unable to integrate problem.

$$\int \frac{\mathsf{Cot}\left[\mathsf{a} + i \; \mathsf{Log}\left[\mathsf{x}\right]\right]^2}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 57 leaves, 4 steps):

$$\frac{2 e^{-2 i a}}{1 - \frac{e^{2 i a}}{v^2}} + \frac{1}{2 x^2} + 2 e^{-2 i a} Log \left[1 - \frac{e^{2 i a}}{x^2}\right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}[a + i \text{Log}[x]]^2}{x^3}, x\right]$$

# Problem 201: Unable to integrate problem.

$$\int (e x)^m \cot[a + i \log[x]] dx$$

Optimal (type 5, 70 leaves, 4 steps):

$$\frac{\dot{\mathbb{I}} \ \left(e \ x\right)^{1+m}}{e \ \left(1+m\right)} - \frac{2 \ \dot{\mathbb{I}} \ \left(e \ x\right)^{1+m} \ Hypergeometric 2F1 \left[1, \ \frac{1}{2} \ \left(-1-m\right), \ \frac{1-m}{2}, \ \frac{e^{2 \ \dot{\mathbb{I}} \ a}}{x^2}\right]}{e \ \left(1+m\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m \cot[a + i \log[x]], x]$ 

#### Problem 202: Unable to integrate problem.

$$\int (e x)^m \cot [a + i \log [x]]^2 dx$$

Optimal (type 5, 77 leaves, 5 steps):

$$-\frac{x \; (\text{e x})^{\,\text{m}}}{1+\text{m}} + \frac{2 \; x \; (\text{e x})^{\,\text{m}}}{1-\frac{\text{e}^{2 \; i \; a}}{\text{x}^2}} - 2 \; x \; (\text{e x})^{\,\text{m}} \; \text{Hypergeometric2F1} \Big[ 1 \text{, } \frac{1}{2} \; \Big( -1-\text{m} \Big) \; \text{, } \frac{1-\text{m}}{2} \; \text{, } \frac{\text{e}^{2 \; i \; a}}{\text{x}^2} \Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Cot[a + i Log[x]]^2, x]$ 

# Problem 203: Unable to integrate problem.

$$\int (e x)^m \cot [a + i \log [x]]^3 dx$$

Optimal (type 5, 169 leaves, 6 steps):

$$\frac{\text{i} \left(1-\text{m}\right) \text{ m x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1+\text{m}\right)} - \frac{\text{i} \left(1+\frac{e^{2 \text{i a}}}{x^{2}}\right)^{2} \text{x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1-\frac{e^{2 \text{i a}}}{x^{2}}\right)^{2}} - \frac{\text{i} \left(3+\text{m}-\frac{e^{2 \text{i a}} \left(1-\text{m}\right)}{x^{2}}\right) \text{x } \left(\text{e x}\right)^{\text{m}}}{2 \left(1-\frac{e^{2 \text{i a}}}{x^{2}}\right)} + \frac{\text{i} \left(3+2\text{m}+\text{m}^{2}\right) \text{x } \left(\text{e x}\right)^{\text{m}} \text{ Hypergeometric2F1}\left[1,\frac{1}{2} \left(-1-\text{m}\right),\frac{1-\text{m}}{2},\frac{e^{2 \text{i a}}}{x^{2}}\right]}{1+\text{m}}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[(ex)^m \cot[a + i \log[x]]^3, x]$ 

#### Problem 204: Unable to integrate problem.

Optimal (type 6, 142 leaves, 4 steps):

$$X \left( 1 - e^{2 i a} X^{2 i b} \right)^{p} \left( 1 + e^{2 i a} X^{2 i b} \right)^{-p} \left( - \frac{i \left( 1 + e^{2 i a} X^{2 i b} \right)}{1 - e^{2 i a} X^{2 i b}} \right)^{p} AppellF1 \left[ - \frac{i}{2 b}, p, -p, 1 - \frac{i}{2 b}, e^{2 i a} X^{2 i b}, -e^{2 i a} X^{2 i b} \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate [Cot[a + b Log[x]]<sup>p</sup>, x]

#### Problem 205: Unable to integrate problem.

$$\int (e x)^m \cot [a + b \log [x]]^p dx$$

Optimal (type 6, 162 leaves, 4 steps):

$$\frac{1}{e\,\left(1+m\right)}\left(e\,x\right)^{\,1+m}\,\left(1-e^{2\,i\,a}\,x^{2\,i\,b}\right)^{\,p}\,\left(1+e^{2\,i\,a}\,x^{2\,i\,b}\right)^{\,-p}\,\left(-\,\frac{i\,\left(1+e^{2\,i\,a}\,x^{2\,i\,b}\right)}{1-e^{2\,i\,a}\,x^{2\,i\,b}}\right)^{\,p}\,AppellF1\left[\,-\,\frac{i\,\left(1+m\right)}{2\,b},\,p,\,-p,\,1-\,\frac{i\,\left(1+m\right)}{2\,b},\,e^{2\,i\,a}\,x^{2\,i\,b},\,-e^{2\,i\,a}\,x^{2\,i\,b}\right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Cot[a + b Log[x]]^p, x]$ 

# Problem 206: Unable to integrate problem.

$$\int \mathsf{Cot} [\mathsf{a} + \mathsf{Log}[\mathsf{x}]]^{\mathsf{p}} \, d\mathsf{x}$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\,i\,a}\,x^{2\,i}\right)^{\,p}\,\left(1+e^{2\,i\,a}\,x^{2\,i}\right)^{\,-p}\,\left(-\,\frac{\,\dot{\mathbb{I}}\,\left(1+e^{2\,i\,a}\,x^{2\,i}\right)}{1-e^{2\,i\,a}\,x^{2\,i}}\right)^{\,p}\,x\,\, \text{AppellF1}\left[\,-\,\frac{\,\dot{\mathbb{I}}}{2}\,,\,\,p\,,\,\,-\,p\,,\,\,1\,-\,\frac{\,\dot{\mathbb{I}}}{2}\,,\,\,e^{2\,i\,a}\,x^{2\,i}\,,\,\,-\,e^{2\,i\,a}\,x^{2\,i}\,\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate [Cot[a + Log[x]]<sup>p</sup>, x]

# Problem 207: Unable to integrate problem.

$$\int Cot[a + 2 Log[x]]^p dx$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\,i\,a}\,x^{4\,i}\right)^{\,p}\,\left(1+e^{2\,i\,a}\,x^{4\,i}\right)^{\,-p}\,\left(-\,\frac{\dot{\mathbb{1}}\,\left(1+e^{2\,i\,a}\,x^{4\,i}\right)}{1-e^{2\,i\,a}\,x^{4\,i}}\right)^{\,p}\,x\,\\ \text{AppellF1}\left[\,-\,\frac{\dot{\mathbb{1}}}{4}\,,\,\,p\,,\,\,-p\,,\,\,1-\frac{\dot{\mathbb{1}}}{4}\,,\,\,e^{2\,i\,a}\,x^{4\,i}\,,\,\,-e^{2\,i\,a}\,x^{4\,i}\,\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate Cot[a + 2 Log[x]]<sup>p</sup>, x

# Problem 208: Unable to integrate problem.

$$\int \cot [a + 3 \log [x]]^p dx$$

Optimal (type 6, 120 leaves, 4 steps):

$$\left(1-e^{2\,i\,a}\,x^{6\,i}\right)^{\,p}\,\left(1+e^{2\,i\,a}\,x^{6\,i}\right)^{\,-p}\,\left(-\,\frac{\,^{\dot{1}}\,\left(1+e^{2\,i\,a}\,x^{6\,i}\right)}{1-e^{2\,i\,a}\,x^{6\,i}}\right)^{\,p}\,x\,\, \text{AppellF1}\left[\,-\,\frac{\,^{\dot{1}}}{6}\,,\,\,p\,,\,\,-p\,,\,\,1-\frac{\,^{\dot{1}}}{6}\,,\,\,e^{2\,i\,a}\,x^{6\,i}\,,\,\,-e^{2\,i\,a}\,x^{6\,i}\,\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Cot} \left[ a + 3 \text{ Log} \left[ x \right] \right]^p, x \right]$ 

# Problem 209: Unable to integrate problem.

$$\left\lceil x^3 \, \text{Cot} \left[ \, d \, \left( \, a + b \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right] \, \mathrm{d} x \right.$$

Optimal (type 5, 70 leaves, 4 steps):

$$\frac{\dot{\mathbb{I}} \ x^4}{4} - \frac{1}{2} \,\dot{\mathbb{I}} \ x^4 \ \text{Hypergeometric2F1} \Big[ 1, -\frac{2\,\dot{\mathbb{I}}}{b\,d\,n}, \ 1 - \frac{2\,\dot{\mathbb{I}}}{b\,d\,n}, \ e^{2\,\dot{\mathbb{I}}\,a\,d} \, \left( c\, x^n \right)^{2\,\dot{\mathbb{I}}\,b\,d} \Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 \cot[d(a+b \log[cx^n])], x]$ 

# Problem 210: Unable to integrate problem.

$$\left\lceil x^2 \, \text{Cot} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right] \, \mathrm{d} x \right.$$

Optimal (type 5, 74 leaves, 4 steps):

$$\frac{\text{i} \ x^{3}}{3} - \frac{2}{3} \text{i} \ x^{3} \ \text{Hypergeometric2F1} \Big[ 1 \text{,} -\frac{3 \, \text{i}}{2 \, \text{b} \, \text{d} \, \text{n}} \text{,} \ 1 - \frac{3 \, \text{i}}{2 \, \text{b} \, \text{d} \, \text{n}} \text{,} \ \text{e}^{2 \, \text{i} \, \text{a} \, \text{d}} \, \left( c \, x^{n} \right)^{2 \, \text{i} \, \text{b} \, \text{d}} \Big]$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $\begin{bmatrix} x^2 \text{ Cot} [d (a + b \text{ Log} [c x^n])], x \end{bmatrix}$ 

# Problem 211: Unable to integrate problem.

Optimal (type 5, 68 leaves, 4 steps):

$$\frac{\dot{\mathbb{I}} \ x^2}{2} - \dot{\mathbb{I}} \ x^2 \ \text{Hypergeometric2F1} \Big[ 1, -\frac{\dot{\mathbb{I}}}{b \ d \ n}, \ 1 - \frac{\dot{\mathbb{I}}}{b \ d \ n}, \ e^{2 \ \dot{\mathbb{I}} \ a \ d} \ \Big( c \ x^n \Big)^{2 \ \dot{\mathbb{I}} \ b \ d} \Big]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\begin{bmatrix} x \cot [d (a + b \log [c x^n])], x \end{bmatrix}$ 

# Problem 212: Unable to integrate problem.

Optimal (type 5, 66 leaves, 4 steps):

$$i \times -2 i \times Hypergeometric 2F1 \left[1, -\frac{i}{2 h d n}, 1 - \frac{i}{2 h d n}, e^{2 i a d} (c x^n)^{2 i b d}\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Cot} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right], x \right]$ 

#### Problem 214: Unable to integrate problem.

$$\int \frac{\mathsf{Cot}\left[\mathsf{d}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\right]\right)\right]}{\mathsf{x}^{\mathsf{2}}}\,\mathsf{d}\mathsf{x}$$

Optimal (type 5, 70 leaves, 4 steps):

$$-\frac{\dot{\mathbb{I}}}{x} + \frac{2 i \text{ Hypergeometric2F1} \left[1, \frac{i}{2 \text{ bdn}}, 1 + \frac{i}{2 \text{ bdn}}, e^{2 i \text{ ad}} \left(c x^{n}\right)^{2 i \text{ bd}}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\cot\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]}{x^{2}},x\right]$$

# Problem 215: Unable to integrate problem.

$$\int\! \frac{\text{Cot} \left[\text{d} \left(\text{a} + \text{b} \, \text{Log} \left[\text{c} \, \, \text{x}^{\text{n}} \, \right] \right) \, \right]}{\text{x}^{3}} \, \text{d} \, \text{x}$$

Optimal (type 5, 68 leaves, 4 steps):

$$-\frac{i}{2 x^{2}} + \frac{i \text{ Hypergeometric2F1} \left[1, \frac{i}{b \text{ dn}}, 1 + \frac{i}{b \text{ dn}}, e^{2 i \text{ ad}} \left(c x^{n}\right)^{2 i \text{ bd}}\right]}{x^{2}}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}\left[d\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)\right]}{x^{3}},\,x\right]$$

#### Problem 216: Unable to integrate problem.

$$\left\lceil x^3 \, \text{Cot} \left[\, d \, \left(\, a \, + \, b \, \, \text{Log} \left[\, c \, \, x^n \, \right] \, \right) \, \right]^{\, 2} \, \mathrm{d} x \right.$$

Optimal (type 5, 158 leaves, 5 steps):

$$\frac{\left(4\,\,\dot{\mathbb{1}}\,-\,b\,\,d\,\,n\right)\,\,x^{4}}{4\,b\,d\,n}\,+\,\,\frac{\dot{\mathbb{1}}\,\,x^{4}\,\left(1\,+\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n\,\,\left(1\,-\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,x^{4}\,\,Hypergeometric 2F1\left[\,1\,,\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,1\,-\,\,\frac{2\,\,\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right]}{b\,\,d\,\,n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$[x^3 \text{ Cot} [d (a + b \text{ Log} [c x^n])]^2, x]$$

# Problem 217: Unable to integrate problem.

$$\left\lceil x^2 \, \text{Cot} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^2 \, \text{d} \, x \right.$$

Optimal (type 5, 162 leaves, 5 steps):

$$\frac{\left(3\,\,\dot{\mathbb{1}}\,-\,b\,\,d\,\,n\right)\,\,x^{3}}{3\,\,b\,\,d\,\,n}\,\,+\,\,\frac{\,\dot{\mathbb{1}}\,\,x^{3}\,\,\left(1\,+\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n\,\,\left(1\,-\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}\,\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,x^{3}\,\,Hypergeometric 2F1\left[\,\mathbf{1}\,,\,\,-\,\,\frac{3\,\,\dot{\mathbb{1}}}{2\,\,b\,\,d\,\,n}\,,\,\,\mathbf{1}\,-\,\,\frac{3\,\,\dot{\mathbb{1}}}{2\,\,b\,\,d\,\,n}\,,\,\,\mathbf{e}^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right]}{b\,\,d\,\,n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Cot} [d (a + b \text{ Log} [c x^n])]^2, x]$ 

#### Problem 218: Unable to integrate problem.

$$\left\lceil x \, \text{Cot} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^2 \, \mathrm{d} x \right.$$

Optimal (type 5, 158 leaves, 5 steps):

$$\frac{\left(2\,\,\dot{\mathbb{1}}\,-\,b\,\,d\,\,n\right)\,\,x^{2}}{2\,\,b\,\,d\,\,n}\,+\,\,\frac{\,\dot{\mathbb{1}}\,\,x^{2}\,\,\left(\mathbf{1}\,+\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}{b\,\,d\,\,n\,\,\left(\mathbf{1}\,-\,\,e^{2\,\,\dot{\mathbb{1}}\,\,a\,\,d}\,\,\left(c\,\,x^{n}\right)^{\,2\,\,\dot{\mathbb{1}}\,\,b\,\,d}\right)}\,-\,\,\frac{2\,\,\dot{\mathbb{1}}\,\,x^{2}\,\,Hypergeometric 2F1\left[\,\mathbf{1}\,,\,\,-\,\,\frac{\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,\mathbf{1}\,-\,\,\frac{\dot{\mathbb{1}}}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,b\,\,d\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,d\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,n}\,,\,\,\,\mathbf{0}\,-\,\,\frac{\dot{\mathbb{1}}\,\,a\,\,n}{b\,\,n}\,,\,\,\,\mathbf{0}\,\,\,\mathbf{0}\,\,\,\mathbf{0}\,\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,\,\mathbf{0}\,$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x \cot [d (a + b \log [c x^n])]^2, x]$ 

#### Problem 219: Unable to integrate problem.

Optimal (type 5, 153 leaves, 5 steps):

$$\frac{\left( \left\| - b \, d \, n \right) \, x}{b \, d \, n} + \frac{\left\| x \, \left( 1 + e^{2 \, i \, a \, d} \, \left( c \, x^n \right)^{2 \, i \, b \, d} \right)}{b \, d \, n \, \left( 1 - e^{2 \, i \, a \, d} \, \left( c \, x^n \right)^{2 \, i \, b \, d} \right)} - \frac{2 \, \left\| x \, \text{Hypergeometric2F1} \left[ 1, \, - \frac{i}{2 \, b \, d \, n}, \, 1 - \frac{i}{2 \, b \, d \, n}, \, e^{2 \, i \, a \, d} \, \left( c \, x^n \right)^{2 \, i \, b \, d} \right]}{b \, d \, n}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Cot} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right]^2, x \right]$ 

#### Problem 221: Unable to integrate problem.

$$\int\! \frac{\text{Cot} \left[\text{d} \, \left(\text{a} + \text{b} \, \text{Log} \, [\text{c} \, \, x^n \, ] \, \right) \, \right]^2}{x^2} \, \text{d} x$$

Optimal (type 5, 156 leaves, 5 steps)

$$\frac{1+\frac{\mathrm{i}}{b\,\mathsf{d}\,\mathsf{n}}}{\mathsf{X}} + \frac{\mathrm{i}\,\left(1+\mathrm{e}^{2\,\mathrm{i}\,\mathsf{a}\,\mathsf{d}}\,\left(c\,\,\mathsf{x}^\mathsf{n}\right)^{\,2\,\mathrm{i}\,b\,\mathsf{d}}\right)}{b\,\mathsf{d}\,\mathsf{n}\,\mathsf{x}\,\left(1-\mathrm{e}^{2\,\mathrm{i}\,\mathsf{a}\,\mathsf{d}}\,\left(c\,\,\mathsf{x}^\mathsf{n}\right)^{\,2\,\mathrm{i}\,b\,\mathsf{d}}\right)} - \frac{2\,\mathrm{i}\,\mathsf{Hypergeometric}2\mathsf{F1}\!\left[1,\,\frac{\mathrm{i}}{2\,\mathsf{b}\,\mathsf{d}\,\mathsf{n}},\,1+\frac{\mathrm{i}}{2\,\mathsf{b}\,\mathsf{d}\,\mathsf{n}},\,\,\mathrm{e}^{2\,\mathrm{i}\,\mathsf{a}\,\mathsf{d}}\,\left(c\,\,\mathsf{x}^\mathsf{n}\right)^{\,2\,\mathrm{i}\,b\,\mathsf{d}}\right]}{b\,\mathsf{d}\,\mathsf{n}\,\mathsf{x}}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}\left[d\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)\right]^{2}}{x^{2}},\,x\right]$$

# Problem 222: Unable to integrate problem.

$$\int \frac{\mathsf{Cot} \left[ \mathsf{d} \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^{\mathsf{n}} \right] \right) \right]^{2}}{\mathsf{x}^{3}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 155 leaves, 5 steps):

$$\frac{1+\frac{2\,\mathrm{i}}{b\,\mathrm{d}\,n}}{2\,x^2}+\frac{\mathrm{i}\,\left(1+\mathrm{e}^{2\,\mathrm{i}\,a\,\mathrm{d}}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,\mathrm{d}}\right)}{b\,\mathrm{d}\,n\,x^2\,\left(1-\mathrm{e}^{2\,\mathrm{i}\,a\,\mathrm{d}}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,\mathrm{d}}\right)}-\frac{2\,\mathrm{i}\,\mathsf{Hypergeometric2F1}\!\left[1,\,\frac{\mathrm{i}}{b\,\mathrm{d}\,n},\,1+\frac{\mathrm{i}}{b\,\mathrm{d}\,n},\,\,e^{2\,\mathrm{i}\,a\,\mathrm{d}}\,\left(c\,x^n\right)^{\,2\,\mathrm{i}\,b\,\mathrm{d}}\right]}{b\,\mathrm{d}\,n\,x^2}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Cot}\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]^{2}}{x^{3}},x\right]$$

#### Problem 226: Unable to integrate problem.

$$\[ \left( e \, x \right)^m \, \text{Cot} \left[ d \, \left( a + b \, \text{Log} \left[ c \, x^n \right] \right) \right] \, d x \]$$

Optimal (type 5, 100 leaves, 4 steps):

$$\frac{\dot{\mathbb{1}} \; \left(e \; x\right)^{\; 1+m}}{e \; \left(1+m\right)} \; - \; \frac{2 \; \dot{\mathbb{1}} \; \left(e \; x\right)^{\; 1+m} \; Hypergeometric 2F1}{\left[1, -\frac{\dot{\mathbb{1}} \; \left(1+m\right)}{2 \; b \; d \; n}, \; 1-\frac{\dot{\mathbb{1}} \; \left(1+m\right)}{2 \; b \; d \; n}, \; e^{2 \; \dot{\mathbb{1}} \; a \; d} \; \left(c \; x^n\right)^{\; 2 \; \dot{\mathbb{1}} \; b \; d}\right]}{e \; \left(1+m\right)}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[(ex)^m \cot[d(a+b Log[cx^n])]$ , x]

#### Problem 227: Unable to integrate problem.

$$\int (e x)^m \cot \left[d \left(a + b \log \left[c x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 195 leaves, 5 steps):

$$\frac{\left(\frac{i}{\mathbb{I}}\left(\mathbf{1}+\mathbf{m}\right)-b\,d\,n\right)\,\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}}{b\,d\,e\,\left(\mathbf{1}+\mathbf{m}\right)\,n}+\frac{\frac{i}{\mathbb{I}}\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}\left(\mathbf{1}+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}{b\,d\,e\,n\,\left(\mathbf{1}-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}-\frac{2\,i\,\left(e\,x\right)^{\,\mathbf{1}+\mathbf{m}}\,\mathsf{Hypergeometric2F1}\left[\mathbf{1},\,-\frac{i\,\left(\mathbf{1}+\mathbf{m}\right)}{2\,b\,d\,n},\,\mathbf{1}-\frac{i\,\left(\mathbf{1}+\mathbf{m}\right)}{2\,b\,d\,n},\,\mathbf{e}^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right]}{b\,d\,e\,n}$$

Result (type 8, 23 leaves, 0 steps):

# Problem 228: Unable to integrate problem.

$$\label{eq:cot_alpha} \left[\,\left(\,e\;x\,\right)^{\,m}\;\text{Cot}\left[\,d\;\left(\,a\;+\;b\;\,\text{Log}\left[\,c\;\,x^{n}\,\right]\,\right)\;\right]^{\,3}\;\text{d}\,x\,\right.$$

Optimal (type 5, 350 leaves, 6 steps):

$$\frac{\left( \dot{\mathbb{1}} \, \left( 1+m \right) - b \, d \, n \right) \, \left( 1+m+2 \, \dot{\mathbb{1}} \, b \, d \, n \right) \, \left( e \, x \right)^{1+m}}{2 \, b^2 \, d^2 \, e \, \left( 1+m \right) \, n^2} + \frac{\left( e \, x \right)^{1+m} \, \left( 1+e^{2 \, \dot{\mathbb{1}} \, a \, d} \, \left( c \, x^n \right)^{2 \, \dot{\mathbb{1}} \, b \, d} \right)^2}{2 \, b \, d \, e \, n \, \left( 1-e^{2 \, \dot{\mathbb{1}} \, a \, d} \, \left( c \, x^n \right)^{2 \, \dot{\mathbb{1}} \, b \, d} \right)^2} + \frac{\dot{\mathbb{1}} \, e^{-2 \, \dot{\mathbb{1}} \, a \, d} \, \left( e \, x \right)^{1+m} \, \left( \frac{e^{2 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d} \, \left( 1+m-2 \, \dot{\mathbb{1}} \, b \, d \, n \right)}{n} + \frac{e^{4 \, \dot{\mathbb{1}} \, a \, d$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $\left[ (e x)^m \text{Cot} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right]^3$ ,  $x \right]$ 

# Problem 229: Unable to integrate problem.

Optimal (type 6, 190 leaves, 5 steps):

$$x \left( 1 - e^{2iad} \left( c \, x^n \right)^{2ibd} \right)^p \left( 1 + e^{2iad} \left( c \, x^n \right)^{2ibd} \right)^{-p} \left( - \frac{i \left( 1 + e^{2iad} \left( c \, x^n \right)^{2ibd} \right)}{1 - e^{2iad} \left( c \, x^n \right)^{2ibd}} \right)^p$$

$$AppellF1 \left[ -\frac{i}{2bdn}, p, -p, 1 - \frac{i}{2bdn}, e^{2iad} \left( c \, x^n \right)^{2ibd}, -e^{2iad} \left( c \, x^n \right)^{2ibd} \right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Cot} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right]^p, x \right]$ 

# Problem 230: Unable to integrate problem.

$$\label{eq:cot_alpha} \left[\,\left(\,e\;x\,\right)^{\,m}\;\text{Cot}\left[\,d\;\left(\,a\,+\,b\;\text{Log}\left[\,c\;x^{n}\,\right]\,\right)\,\right]^{\,p}\;\mathrm{d}\!\!\mid\! x$$

Optimal (type 6, 210 leaves, 5 steps):

$$\begin{split} &\frac{1}{e\,\left(1+m\right)}\,\left(e\,x\right)^{\,1+m}\,\left(1-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)^{\,p}\,\left(1+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)^{\,-p}\\ &\left(-\,\frac{i\,\left(1+e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right)}{1-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}}\right)^{\,p}\,AppellF1\!\left[-\,\frac{i\,\left(1+m\right)}{2\,b\,d\,n}\text{, p, -p, }1-\frac{i\,\left(1+m\right)}{2\,b\,d\,n}\text{, }e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\text{, }-e^{2\,i\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,i\,b\,d}\right] \end{split}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m \cot[d(a+b \log[cx^n])]^p$ , x

# Problem 259: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( - \left( 1 + b^2 \, n^2 \right) \, \mathsf{Sec} \left[ \, \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \right] \, \right] \, + \, 2 \, b^2 \, n^2 \, \mathsf{Sec} \left[ \, \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \right] \, \right]^3 \right) \, \mathrm{d} x$$

Optimal (type 3, 41 leaves, ? steps):

$$-\,x\,\mathsf{Sec}\left[\,\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\,\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\,\right]\,\right]\,+\,\mathsf{b}\,\,\mathsf{n}\,\,\mathsf{x}\,\mathsf{Sec}\left[\,\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\,\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\,\right]\,\right]\,\mathsf{Tan}\left[\,\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\,\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\,\right]\,\right]$$

Result (type 5, 175 leaves, 7 steps):

$$-2\,e^{\mathrm{i}\,a}\,\left(1-\mathrm{i}\,b\,n\right)\,x\,\left(c\,x^{n}\right)^{\,\mathrm{i}\,b}\, \\ \text{Hypergeometric} \\ 2\text{F1}\left[1,\,\frac{1}{2}\left(1-\frac{\mathrm{i}}{b\,n}\right),\,\frac{1}{2}\left(3-\frac{\mathrm{i}}{b\,n}\right),\,-e^{2\,\mathrm{i}\,a}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b}\right] \\ + \frac{16\,b^{2}\,e^{3\,\mathrm{i}\,a}\,n^{2}\,x\,\left(c\,x^{n}\right)^{\,3\,\mathrm{i}\,b}\, \\ \text{Hypergeometric} \\ 2\text{F1}\left[3,\,\frac{1}{2}\left(3-\frac{\mathrm{i}}{b\,n}\right),\,\frac{1}{2}\left(5-\frac{\mathrm{i}}{b\,n}\right),\,-e^{2\,\mathrm{i}\,a}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b}\right]}{1+3\,\mathrm{i}\,b\,n}$$

# Problem 260: Result unnecessarily involves higher level functions.

$$\int \! x^m \, \text{Sec} \left[ \, a + 2 \, \text{Log} \left[ \, c \, \, x^{\frac{1}{2} \, \sqrt{- \, \left( \, 1 + m \, \right)^{\, 2}}} \, \, \right] \, \right]^{\, 3} \, \, \mathrm{d} x$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m}\,\text{Sec}\left[\,a+2\,\text{Log}\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]}{2\,\,\left(\,1+m\right)}\,+\,\,\frac{x^{1+m}\,\text{Sec}\left[\,a+2\,\text{Log}\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]\,\,\text{Tan}\left[\,a+2\,\text{Log}\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]}{2\,\,\sqrt{-\,\left(\,1+m\right)^{\,2}}}$$

Result (type 5, 146 leaves, 3 steps):

Problem 301: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\left\lceil \left( - \left( 1 + b^2 \ n^2 \right) \ \mathsf{Csc} \left[ \, \mathsf{a} + \mathsf{b} \ \mathsf{Log} \left[ \, \mathsf{c} \ x^n \, \right] \, \right] + 2 \ b^2 \ n^2 \ \mathsf{Csc} \left[ \, \mathsf{a} + \mathsf{b} \ \mathsf{Log} \left[ \, \mathsf{c} \ x^n \, \right] \, \right]^3 \right) \ \mathrm{d} x$$

Optimal (type 3, 42 leaves, ? steps):

$$- x \, \mathsf{Csc} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^\mathsf{n} \, \right] \, \right] \\ - \mathsf{b} \, \mathsf{n} \, \mathsf{x} \, \mathsf{Cot} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^\mathsf{n} \, \right] \, \right] \\ \mathsf{Csc} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^\mathsf{n} \, \right] \, \right] \\$$

Result (type 5, 172 leaves, 7 steps):

$$2\,e^{\mathrm{i}\,a}\,\left(\mathrm{i}\,+\,b\,n\right)\,x\,\left(c\,x^{n}\right)^{\,\mathrm{i}\,b}\, \\ \text{Hypergeometric} \\ 2\mathrm{F1}\!\left[1,\,\frac{1}{2}\left(1-\frac{\mathrm{i}}{b\,n}\right),\,\frac{1}{2}\left(3-\frac{\mathrm{i}}{b\,n}\right),\,e^{2\,\mathrm{i}\,a}\,\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b}\right] \\ -\frac{16\,b^{2}\,e^{3\,\mathrm{i}\,a}\,n^{2}\,x\,\left(c\,x^{n}\right)^{\,3\,\mathrm{i}\,b}\, \\ \text{Hypergeometric} \\ 2\mathrm{F1}\!\left[3,\,\frac{1}{2}\left(3-\frac{\mathrm{i}}{b\,n}\right),\,\frac{1}{2}\left(5-\frac{\mathrm{i}}{b\,n}\right),\,e^{2\,\mathrm{i}\,a}\left(c\,x^{n}\right)^{\,2\,\mathrm{i}\,b}\right]}{\mathrm{i}\,-3\,b\,n}$$

Problem 302: Result unnecessarily involves higher level functions.

$$\left\lceil x^{m}\, \text{Csc}\left[\, a + 2\, \text{Log}\left[\, c\,\, x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]^{\,3}\, \text{d}\, x\right.$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m}\,Csc\left[\,a+2\,Log\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\,\right]}{2\,\,\left(\,1+m\right)}\,-\,\,\frac{x^{1+m}\,Cot\left[\,a+2\,Log\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]\,Csc\left[\,a+2\,Log\left[\,c\,\,x^{\frac{1}{2}\,\sqrt{-\,(1+m)^{\,2}}}\,\,\right]\,\right]}{2\,\,\sqrt{-\,\left(\,1+m\right)^{\,2}}}$$

Result (type 5, 142 leaves, 3 steps):

$$-\frac{1}{\frac{1}{1+\frac{1}{2}m-3\sqrt{-\left(1+m\right)^{2}}}}8\,\,e^{3\,\frac{1}{2}\,a}\,x^{1+m}\,\left(c\,\,x^{\frac{1}{2}\sqrt{-\left(1+m\right)^{2}}}\right)^{6\,\frac{1}{2}}\,\\ \text{Hypergeometric2F1}\left[\,3\,,\,\,\frac{1}{2}\left(3\,-\,\,\frac{\frac{1}{2}\left(1+m\right)}{\sqrt{-\left(1+m\right)^{2}}}\right)\,,\,\,\frac{1}{2}\left(5\,-\,\,\frac{\frac{1}{2}\left(1+m\right)}{\sqrt{-\left(1+m\right)^{2}}}\right)\,,\,\,e^{2\,\frac{1}{2}\,a}\,\left(c\,\,x^{\frac{1}{2}\sqrt{-\left(1+m\right)^{2}}}\right)^{4\,\frac{1}{2}}\left(1+m\right)^{2}\left(1+m$$

# Test results for the 142 problems in "4.7.6 $f^{(a+b)} x+c x^2$ " trig(d+e x+f x^2)^n.m"

# Problem 28: Unable to integrate problem.

```
\int_{a}^{b} F^{c(a+bx)} (fx)^{m} Sin[d+ex] dx
Optimal (type 4, 139 leaves, ? steps):
    e^{-i\,d}\,\,F^{a\,c}\,\left(\text{f}\,x\right)^{\text{m}}\,\text{Gamma}\left[\,\mathbf{1}\,+\,\text{m}\,,\,\,x\,\,\left(\,i\,\,e\,-\,b\,\,c\,\,\text{Log}\,[\,F\,]\,\,\right)\,\,\right]\,\,\left(\,x\,\,\left(\,i\,\,e\,-\,b\,\,c\,\,\text{Log}\,[\,F\,]\,\,\right)\,\right)^{\,-\,\text{m}}
                                               2 (e + i b c Log[F])
   e^{id} F^{ac} (fx)^m Gamma [1+m, -x (ie+bcLog[F])] (-x (ie+bcLog[F]))^{-m}
                                                2 (e – i b c Log[F])
Result (type 8, 24 leaves, 1 step):
CannotIntegrate [F^{ac+bcx}(fx)^m Sin[d+ex], x]
```

#### Problem 32: Unable to integrate problem.

```
\left\lceil f\,F^{c\,\,(a+b\,x)}\,\,\left(f\,x\right)^{\,m}\,\left(e\,x\,Cos\,[\,d+e\,x\,]\,\,+\,\,\left(1+m+b\,c\,x\,Log\,[\,F\,]\,\right)\,Sin\,[\,d+e\,x\,]\,\right)\,\,\mathrm{d}x
Optimal (type 3, 23 leaves, ? steps):
fF^{c(a+bx)} x (fx)^m Sin[d+ex]
Result (type 8, 89 leaves, 6 steps):
e CannotIntegrate \left\lceil \, F^{a\,\,c\,+\,b\,\,c\,\,x} \,\, \left( \, f\,\,x \, \right)^{\,1+m}\, Cos\, \left[ \, d\,+\,e\,\,x \, \right] , x \, \right\rceil \,+\,
   f(1+m) CannotIntegrate F^{ac+bcx}(fx)^m Sin [d+ex], x + bc CannotIntegrate F^{ac+bcx}(fx)^{1+m} Sin [d+ex]
```

# Test results for the 950 problems in "4.7.7 Trig functions.m"

# Problem 759: Result valid but suboptimal antiderivative.

```
\left( \left( \cos [x]^{12} \sin [x]^{10} - \cos [x]^{10} \sin [x]^{12} \right) dx \right)
```

Optimal (type 3, 12 leaves, ? steps):

$$\frac{1}{11}$$
 Cos [x] 11 Sin [x] 11

Result (type 3, 129 leaves, 25 steps):

$$\frac{3 \cos \left[x\right]^{11} \sin \left[x\right]}{5632} - \frac{3 \cos \left[x\right]^{13} \sin \left[x\right]}{5632} + \frac{1}{512} \cos \left[x\right]^{11} \sin \left[x\right]^{3} - \frac{7 \cos \left[x\right]^{13} \sin \left[x\right]^{3}}{2816} + \frac{7 \cos \left[x\right]^{11} \sin \left[x\right]^{5}}{1280} - \frac{7}{880} \cos \left[x\right]^{13} \sin \left[x\right]^{5} + \frac{1}{80} \cos \left[x\right]^{11} \sin \left[x\right]^{7} - \frac{9}{440} \cos \left[x\right]^{13} \sin \left[x\right]^{7} + \frac{1}{40} \cos \left[x\right]^{11} \sin \left[x\right]^{9} - \frac{1}{22} \cos \left[x\right]^{13} \sin \left[x\right]^{9} + \frac{1}{22} \cos \left[x\right]^{11} \sin \left[x\right]^$$

#### Problem 796: Unable to integrate problem.

$$\int e^{Sin[x]} Sec[x]^{2} (x Cos[x]^{3} - Sin[x]) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{Sin[x]} \left(-1 + x Cos[x]\right) Sec[x]$$

Result (type 8, 24 leaves, 2 steps):

CannotIntegrate  $\left[e^{Sin[x]} \times Cos[x], x\right]$  - CannotIntegrate  $\left[e^{Sin[x]} Sec[x] Tan[x], x\right]$ 

# Problem 858: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\cos\left[x\right]^{3/2} \sqrt{3\cos\left[x\right] + \sin\left[x\right]}} \, \mathrm{d}x$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{2\sqrt{3}\cos[x] + \sin[x]}{\sqrt{\cos[x]}}$$

Result (type 3, 88 leaves, 5 steps):

$$\frac{2 \, \mathsf{Cos}\left[\frac{x}{2}\right]^2 \, \left(3 + 2 \, \mathsf{Tan}\left[\frac{x}{2}\right] - 3 \, \mathsf{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{\left.\mathsf{Cos}\left[\frac{x}{2}\right]^2 \, \left(3 + 2 \, \mathsf{Tan}\left[\frac{x}{2}\right] - 3 \, \mathsf{Tan}\left[\frac{x}{2}\right]^2\right)}} \, \sqrt{\left.\mathsf{Cos}\left[\frac{x}{2}\right]^2 \, \left(1 - \mathsf{Tan}\left[\frac{x}{2}\right]^2\right)}}$$

# Problem 859: Unable to integrate problem.

$$\int \frac{\operatorname{Csc}[x] \sqrt{\operatorname{Cos}[x] + \operatorname{Sin}[x]}}{\operatorname{Cos}[x]^{3/2}} dx$$

Optimal (type 3, 44 leaves, ? steps):

$$- \, \text{Log} \, [\, \text{Sin} \, [\, x \,] \,] \, + \, 2 \, \, \text{Log} \, \Big[ \, - \, \sqrt{\text{Cos} \, [\, x \,]} \, + \, \sqrt{\text{Cos} \, [\, x \,] \, + \, \text{Sin} \, [\, x \,]} \, \Big] \, + \, \frac{2 \, \sqrt{\text{Cos} \, [\, x \,] \, + \, \text{Sin} \, [\, x \,]}}{\sqrt{\text{Cos} \, [\, x \,]}}$$

Result (type 8, 20 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\operatorname{Csc}[x]\sqrt{\operatorname{Cos}[x]+\operatorname{Sin}[x]}}{\operatorname{Cos}[x]^{3/2}},x\right]$$

#### Problem 860: Result valid but suboptimal antiderivative.

$$\int\!\frac{\text{Cos}[x] + \text{Sin}[x]}{\sqrt{1 + \text{Sin}[2\,x]}}\,\text{d}x$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{x\sqrt{1+Sin[2x]}}{Cos[x]+Sin[x]}$$

Result (type 3, 72 leaves, 17 steps):

$$\frac{2\,\mathsf{ArcTan}\!\left[\mathsf{Tan}\!\left[\frac{\mathsf{x}}{2}\right]\right]\,\mathsf{Cos}\!\left[\frac{\mathsf{x}}{2}\right]^2\,\left(1+2\,\mathsf{Tan}\!\left[\frac{\mathsf{x}}{2}\right]-\mathsf{Tan}\!\left[\frac{\mathsf{x}}{2}\right]^2\right)}{\sqrt{\mathsf{Cos}\!\left[\frac{\mathsf{x}}{2}\right]^4\,\left(1+2\,\mathsf{Tan}\!\left[\frac{\mathsf{x}}{2}\right]-\mathsf{Tan}\!\left[\frac{\mathsf{x}}{2}\right]^2\right)^2}}$$

#### Problem 912: Result valid but suboptimal antiderivative.

$$\int \frac{\mathsf{Cos}[x] + \mathsf{Sin}[x]}{\sqrt{\mathsf{Cos}[x]}} \, \mathsf{d}x$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2}\ \text{ArcTan} \Big[ 1 - \frac{\sqrt{2}\ \sqrt{\text{Sin}\left[x\right]}}{\sqrt{\text{Cos}\left[x\right]}} \Big] + \sqrt{2}\ \text{ArcTan} \Big[ 1 + \frac{\sqrt{2}\ \sqrt{\text{Sin}\left[x\right]}}{\sqrt{\text{Cos}\left[x\right]}} \Big]$$

Result (type 3, 243 leaves, 22 steps):

# Problem 914: Unable to integrate problem.

$$\left\lceil \left( \textbf{10} \ \textbf{x}^{\textbf{9}} \ \textbf{Cos} \left[ \textbf{x}^{\textbf{5}} \ \textbf{Log} \left[ \textbf{x} \right] \right. \right] - \textbf{x}^{\textbf{10}} \ \left( \textbf{x}^{\textbf{4}} + \textbf{5} \ \textbf{x}^{\textbf{4}} \ \textbf{Log} \left[ \textbf{x} \right] \right) \ \textbf{Sin} \left[ \textbf{x}^{\textbf{5}} \ \textbf{Log} \left[ \textbf{x} \right] \right. \right) \ \text{d}\textbf{x} \right.$$

Optimal (type 3, 11 leaves, ? steps):

$$x^{10} Cos [x^5 Log[x]]$$

Result (type 8, 48 leaves, 4 steps):

10 CannotIntegrate  $\begin{bmatrix} x^9 \cos x^5 \log x \end{bmatrix}$ , x = -1 CannotIntegrate  $\begin{bmatrix} x^{14} \sin x^5 \log x \end{bmatrix}$ , x = -5 CannotIntegrate  $\begin{bmatrix} x^{14} \cos x \end{bmatrix}$ , x = -5 CannotIntegrate  $\begin{bmatrix} x^{14} \cos x \end{bmatrix}$ 

# Problem 915: Unable to integrate problem.

$$\int Cos\left[\frac{x}{2}\right]^2 Tan\left[\frac{\pi}{4} + \frac{x}{2}\right] dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x}{2} - \frac{\cos[x]}{2} - \log\left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $\left[\cos\left[\frac{x}{2}\right]^2 Tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x\right]$ 

# Problem 931: Unable to integrate problem.

$$\int \left( \frac{x^4}{b \, \sqrt{x^3 + 3 \, \text{Sin} \, [\, a + b \, x \, ]}} + \frac{x^2 \, \text{Cos} \, [\, a + b \, x \, ]}{\sqrt{x^3 + 3 \, \text{Sin} \, [\, a + b \, x \, ]}} + \frac{4 \, x \, \sqrt{x^3 + 3 \, \text{Sin} \, [\, a + b \, x \, ]}}{3 \, b} \right) \, \text{d}x$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^2 \sqrt{x^3 + 3 \sin{[a + b x]}}}{3 b}$$

Result (type 8, 82 leaves, 1 step):

$$\frac{\text{CannotIntegrate}\left[\frac{x^4}{\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}\text{, x}\right]}{b} + \text{CannotIntegrate}\left[\frac{x^2\,\text{Cos}\,[a+b\,x]}{\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}\text{, x}\right] + \frac{4\,\text{CannotIntegrate}\big[x\,\sqrt{x^3+3\,\text{Sin}[a+b\,x]}\,\,\text{, x}\big]}{3\,b}$$

# Problem 933: Unable to integrate problem.

$$\int \frac{\text{Cos}[x] + \text{Sin}[x]}{\mathbb{e}^{-x} + \text{Sin}[x]} \, dx$$

Optimal (type 3, 9 leaves, ? steps):

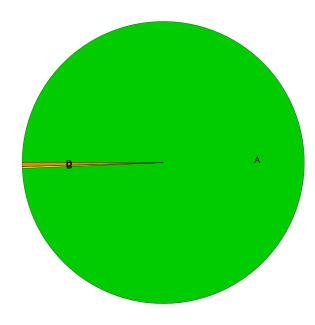
$$Log[1 + e^{x} Sin[x]]$$

Result (type 8, 36 leaves, 5 steps):

$$x - {\sf CannotIntegrate} \Big[ \frac{1}{1 + {\sf e}^{\sf X} {\sf Sin}[{\sf X}]} \text{, } {\sf X} \Big] - {\sf CannotIntegrate} \Big[ \frac{{\sf Cot}[{\sf X}]}{1 + {\sf e}^{\sf X} {\sf Sin}[{\sf X}]} \text{, } {\sf X} \Big] + {\sf Log}[{\sf Sin}[{\sf X}]]$$

# **Summary of Integration Test Results**

# 22551 integration problems



- A 22402 optimal antiderivatives
- B 47 valid but suboptimal antiderivatives
- C 5 unnecessarily complex antiderivatives
- D 97 unable to integrate problems
- E 0 integration timeouts
- F 0 invalid antiderivatives