# Mathematica 11.3 Integration Test Results

## Test results for the 62 problems in "4.2.1.1 (a+b cos)^n.m"

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(3+5\,\mathsf{Cos}\,[\,\mathsf{c}+\mathsf{d}\,\mathsf{x}\,]\,\right)^4}\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 140 leaves, 6 steps):

$$\frac{279 \, \text{Log} \left[\,2 \, \text{Cos} \left[\,\frac{1}{2} \, \left(\,c + d \,x\,\right)\,\,\right] \, - \, \text{Sin} \left[\,\frac{1}{2} \, \left(\,c + d \,x\,\right)\,\,\right]\,\,}{32 \, 768 \, d} \, - \, \frac{279 \, \text{Log} \left[\,2 \, \text{Cos} \left[\,\frac{1}{2} \, \left(\,c + d \,x\,\right)\,\,\right] \, + \, \text{Sin} \left[\,\frac{1}{2} \, \left(\,c + d \,x\,\right)\,\,\right]\,\,}{32 \, 768 \, d} \, + \, \frac{5 \, \text{Sin} \left[\,c + d \,x\,\right]}{48 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)^{\,2}} \, - \, \frac{25 \, \text{Sin} \left[\,c + d \,x\,\right]}{512 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)^{\,2}} \, + \, \frac{995 \, \text{Sin} \left[\,c + d \,x\,\right]}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \, 576 \, d \, \left(\,3 + 5 \, \text{Cos} \left[\,c + d \,x\,\right]\,\,\right)} \, + \, \frac{32 \, 768 \, d}{24 \,$$

Result (type 3, 296 leaves):

$$\frac{1}{393\,216\,d\,\left(3+5\,Cos\left[c+d\,x\right]\right)^3} \left(467\,046\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 765\,855\,Cos\left[c+d\,x\right] \left(Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + 376\,650\,Cos\left[2\,\left(c+d\,x\right)\right] \\ \left(Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 467\,046\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 226\,140\,Sin\left[c+d\,x\right] + 190\,800\,Sin\left[2\,\left(c+d\,x\right)\right] + 99\,500\,Sin\left[3\,\left(c+d\,x\right)\right] \right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(3-5\,Cos\,[\,c+d\,x\,]\,\right)^4}\,\mathrm{d}x$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{279 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right] - 2 \, \text{Sin} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right]\right]}{32 \, 768 \, \text{d}} + \frac{279 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right] + 2 \, \text{Sin} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right]\right]}{32 \, 768 \, \text{d}} - \frac{5 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{48 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)^{3}} + \frac{25 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{512 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)^{2}} - \frac{995 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{24 \, 576 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)}$$

#### Result (type 3, 288 leaves):

$$\frac{1}{393\,216\,d\,\left(-3+5\,Cos\left[c+d\,x\right]\right)^3} \left(467\,046\,Log\left[Cos\left[\frac{1}{2}\left(c+d\,x\right)\right]-2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] - 2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] - 2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right] - 2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right] - 2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right]\right] - 2\,Sin\left[\frac{1}{2}\left(c+d\,x\right)\right] - 2\,Sin\left[\frac{1}{2}\left(c+$$

## Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-3+5\cos\left[c+d\,x\right]\right)^4}\,\mathrm{d}x$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{279 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right] - 2 \, \text{Sin} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right]\right]}{32768 \, \text{d}} + \frac{279 \, \text{Log} \left[\text{Cos} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right] + 2 \, \text{Sin} \left[\frac{1}{2} \, \left(\text{c} + \text{d} \, \text{x}\right)\,\right]\right]}{32768 \, \text{d}} - \frac{5 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{48 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)^{3}} + \frac{25 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{512 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)^{2}} - \frac{995 \, \text{Sin} \left[\text{c} + \text{d} \, \text{x}\right]}{24576 \, \text{d} \, \left(3 - 5 \, \text{Cos} \left[\text{c} + \text{d} \, \text{x}\right]\right)}$$

Result (type 3, 288 leaves):

$$\frac{1}{393\,216\,d\,\left(-3+5\,Cos\left[c+d\,x\right]\right)^3} \left(467\,046\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 765\,855\,Cos\left[c+d\,x\right] \left(Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + 376\,650\,Cos\left[2\,\left(c+d\,x\right)\right] \right) - \\ \left(Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] \right) - \\ 467\,046\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + 2\,Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 226\,140\,Sin\left[c+d\,x\right] - 190\,800\,Sin\left[2\,\left(c+d\,x\right)\right] + 99\,500\,Sin\left[3\,\left(c+d\,x\right)\right] \right)$$

### Problem 49: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(-3-5\cos\left[c+d\,x\right]\right)^4}\,\mathrm{d}x$$

#### Optimal (type 3, 140 leaves, 6 steps):

$$\frac{279 \, \text{Log} \left[2 \, \text{Cos} \left[\frac{1}{2} \, \left(c + d \, x\right)\,\right] - \text{Sin} \left[\frac{1}{2} \, \left(c + d \, x\right)\,\right]\right]}{32 \, 768 \, d} - \frac{279 \, \text{Log} \left[2 \, \text{Cos} \left[\frac{1}{2} \, \left(c + d \, x\right)\,\right] + \text{Sin} \left[\frac{1}{2} \, \left(c + d \, x\right)\,\right]\right]}{32 \, 768 \, d} + \frac{5 \, \text{Sin} \left[c + d \, x\right]}{48 \, d \, \left(3 + 5 \, \text{Cos} \left[c + d \, x\right]\right)^3} - \frac{25 \, \text{Sin} \left[c + d \, x\right]}{512 \, d \, \left(3 + 5 \, \text{Cos} \left[c + d \, x\right]\right)^2} + \frac{995 \, \text{Sin} \left[c + d \, x\right]}{24 \, 576 \, d \, \left(3 + 5 \, \text{Cos} \left[c + d \, x\right]\right)}$$

#### Result (type 3, 296 leaves):

$$\frac{1}{393\,216\,d\,\left(3+5\,Cos\left[c+d\,x\right]\right)^3} \left(467\,046\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 765\,855\,Cos\left[c+d\,x\right] \left(Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]+Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + 376\,650\,Cos\left[2\,\left(c+d\,x\right)\right] \\ \left(Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right]-Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 467\,046\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] - \\ 104\,625\,Cos\left[3\,\left(c+d\,x\right)\right]\,Log\left[2\,Cos\left[\frac{1}{2}\,\left(c+d\,x\right)\right] + Sin\left[\frac{1}{2}\,\left(c+d\,x\right)\right]\right] + \\ 226\,140\,Sin\left[c+d\,x\right] + 190\,800\,Sin\left[2\,\left(c+d\,x\right)\right] + 99\,500\,Sin\left[3\,\left(c+d\,x\right)\right] \right)$$

### Problem 56: Result more than twice size of optimal antiderivative.

$$\int (a + b \cos [c + dx])^{4/3} dx$$

Optimal (type 6, 108 leaves, 3 steps):

$$\left( \sqrt{2} \ \left( a + b \right) \ \mathsf{AppellF1} \left[ \frac{1}{2} \text{, } \frac{1}{2} \text{, } -\frac{4}{3} \text{, } \frac{3}{2} \text{, } \frac{1}{2} \left( 1 - \mathsf{Cos} \left[ c + \mathsf{d} \, x \right] \right) \text{, } \frac{b \left( 1 - \mathsf{Cos} \left[ c + \mathsf{d} \, x \right] \right)}{a + b} \right]$$
 
$$\left( a + b \, \mathsf{Cos} \left[ c + \mathsf{d} \, x \right] \right)^{1/3} \, \mathsf{Sin} \left[ c + \mathsf{d} \, x \right] \right) \bigg/ \left( \mathsf{d} \, \sqrt{1 + \mathsf{Cos} \left[ c + \mathsf{d} \, x \right]} \ \left( \frac{a + b \, \mathsf{Cos} \left[ c + \mathsf{d} \, x \right]}{a + b} \right)^{1/3} \right)$$

Result (type 6, 246 leaves):

Result (type 6, 246 leaves). 
$$-\frac{1}{16 \, b \, d} 3 \, \left( a + b \, \mathsf{Cos} \, [c + d \, x] \right)^{1/3} \, \mathsf{Csc} \, [c + d \, x]$$
 
$$\left( 4 \, \left( -a^2 + b^2 \right) \, \mathsf{AppellF1} \left[ \frac{1}{3}, \, \frac{1}{2}, \, \frac{1}{2}, \, \frac{4}{3}, \, \frac{a + b \, \mathsf{Cos} \, [c + d \, x]}{a - b}, \, \frac{a + b \, \mathsf{Cos} \, [c + d \, x]}{a + b} \right]$$
 
$$\sqrt{-\frac{b \, \left( -1 + \mathsf{Cos} \, [c + d \, x] \right)}{a + b}} \, \sqrt{-\frac{b \, \left( 1 + \mathsf{Cos} \, [c + d \, x] \right)}{a - b}} +$$
 
$$5 \, \mathsf{a} \, \mathsf{AppellF1} \left[ \frac{4}{3}, \, \frac{1}{2}, \, \frac{1}{2}, \, \frac{7}{3}, \, \frac{a + b \, \mathsf{Cos} \, [c + d \, x]}{a - b}, \, \frac{a + b \, \mathsf{Cos} \, [c + d \, x]}{a + b} \right] \sqrt{-\frac{b \, \left( -1 + \mathsf{Cos} \, [c + d \, x] \right)}{a + b}}$$
 
$$\sqrt{-\frac{b \, \left( 1 + \mathsf{Cos} \, [c + d \, x] \right)}{a - b}} \, \left( a + b \, \mathsf{Cos} \, [c + d \, x] \right) - 4 \, b^2 \, \mathsf{Sin} \, [c + d \, x]^2}$$

## Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b \cos \left[c+d x\right]\right)^{4/3}} \, \mathrm{d}x$$

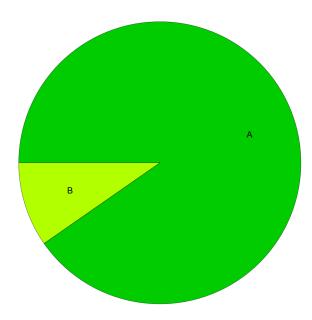
Optimal (type 6, 110 leaves, 3 steps):

$$\frac{\sqrt{2} \ \mathsf{AppellF1} \left[ \frac{1}{2} \text{, } \frac{1}{2} \text{, } \frac{4}{3} \text{, } \frac{3}{2} \text{, } \frac{1}{2} \left( 1 - \mathsf{Cos} \left[ c + d \, x \right] \right) \text{, } \frac{b \, (1 - \mathsf{Cos} \left[ c + d \, x \right])}{a + b} \right] \, \left( \frac{a + b \, \mathsf{Cos} \left[ c + d \, x \right]}{a + b} \right)^{1/3} \, \mathsf{Sin} \left[ c + d \, x \right]} \, \left( a + b \right) \, d \, \sqrt{1 + \mathsf{Cos} \left[ c + d \, x \right]} \, \left( a + b \, \mathsf{Cos} \left[ c + d \, x \right] \right)^{1/3} \, \mathsf{Sin} \left[ c + d \, x \right] \, \left( a + b \, \mathsf{Cos} \left[ c + d \, x \right] \right)^{1/3} \, \mathsf{Sin} \left[ c + d \, x \right] \, \left( a + b \, \mathsf{Cos} \left[ c + d \, x \right] \right)^{1/3} \, \mathsf{Sin} \left[ c + d \, x \right] \, \left( a + b \, \mathsf{Cos} \left[ c + d \, x \right] \right)^{1/3} \, \mathsf{Sin} \left[ c + d \, x \right] \, \mathsf{Cos} \left$$

Result (type 6, 268 leaves):

# **Summary of Integration Test Results**

## 62 integration problems



- A 56 optimal antiderivatives
- B 6 more than twice size of optimal antiderivatives
- C 0 unnecessarily complex antiderivatives
- D 0 unable to integrate problems
- E 0 integration timeouts