Mathematica 11.3 Integration Test Results

Test results for the 58 problems in "7.1.4b (f x) n m (d+e x 2) p (a+b arcsinh(c x)) n m"

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{d + e x^2} dx$$

Optimal (type 4, 485 leaves, 18 steps):

$$\frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]\right) \, \mathsf{Log} \left[1 - \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}} - \frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]\right) \, \mathsf{Log} \left[1 + \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, - \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}} + \frac{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}} \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, + \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} - \frac{\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]\right) \, \mathsf{Log} \left[1 + \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, + \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}} + \frac{\mathsf{b} \, \mathsf{PolyLog} \left[2, \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, - \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}}} + \frac{\mathsf{b} \, \mathsf{PolyLog} \left[2, \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, - \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}}} + \frac{\mathsf{b} \, \mathsf{PolyLog} \left[2, \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, + \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}}} + \frac{\mathsf{b} \, \mathsf{PolyLog} \left[2, \frac{\sqrt{\mathsf{e} \, } \, \mathsf{e}^{\mathsf{ArcSinh} \left[\mathsf{c} \, \mathsf{x}\right]}}{\mathsf{c} \, \sqrt{-\mathsf{d}} \, + \sqrt{-\mathsf{c}^2 \, \mathsf{d} + \mathsf{e}}}} \right]}{2 \, \sqrt{-\mathsf{d}} \, \sqrt{\mathsf{e}}}}$$

Result (type 4, 775 leaves):

$$\frac{1}{16\sqrt{d}\sqrt{e}}\left[16 \text{ a ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \right]$$

$$4\,b\,\left[8\,\,\dot{\mathbb{1}}\,\operatorname{ArcSin}\Big[\,\frac{\sqrt{1+\frac{c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\operatorname{ArcTan}\Big[\,\frac{\left(c\,\sqrt{d}\,-\sqrt{e}\,\right)\,\operatorname{Cot}\left[\,\frac{1}{4}\,\left(\pi+2\,\,\dot{\mathbb{1}}\,\operatorname{ArcSinh}\left[\,c\,\,x\,\right]\,\right)\,\Big]}{\sqrt{c^2\,d-e}}\,\Big]\,-\frac{1}{2}\,\left[\frac{\left(c\,\sqrt{d}\,-\sqrt{e}\,\right)\,\operatorname{Cot}\left[\,\frac{1}{4}\,\left(\pi+2\,\,\dot{\mathbb{1}}\,\operatorname{ArcSinh}\left[\,c\,\,x\,\right]\,\right)\,\right]}{\sqrt{c^2\,d-e}}\,\right]$$

$$\left[\pi + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \right] \operatorname{Log} \left[1 - \frac{\operatorname{i} \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] =$$

$$\left[\pi + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 - \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \right] \operatorname{Log} \left[1 + \frac{\operatorname{i} \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] -$$

$$\left[\pi - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 - \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \right] \operatorname{Log} \left[1 - \frac{\operatorname{i} \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] +$$

$$\left[\pi - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 + \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \right] \operatorname{Log} \left[1 + \frac{\operatorname{i} \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] +$$

$$\left[\pi - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \operatorname{Log} \left[c \left(\sqrt{d} - \operatorname{i} \, \sqrt{e} \, x \right) \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \operatorname{Log} \left[c \left(\sqrt{d} - \operatorname{i} \, \sqrt{e} \, x \right) \right] -$$

$$\left[\pi - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \operatorname{Log} \left[c \left(\sqrt{d} - \operatorname{i} \, \sqrt{e} \, x \right) \right] - 2 \operatorname{i} \operatorname{ArcSinh}[c \, x] \operatorname{Log} \left[c \left(\sqrt{d} - \operatorname{i} \, \sqrt{e} \, x \right) \right] -$$

$$2 \operatorname{i} \left[\operatorname{PolyLog} \left[2, - \frac{\operatorname{i} \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] \right] +$$

$$2 \operatorname{i} \left[\operatorname{PolyLog} \left[2, - \frac{\operatorname{i} \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] \right] +$$

$$\operatorname{PolyLog} \left[2, - \frac{\operatorname{i} \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \, e^{\operatorname{ArcSinh}[c \, x)}}{\sqrt{e}} \right] \right]$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \, ArcSinh \, [\, c \, \, x \,]}{\left(d + e \, x^2\right)^2} \, \, \mathrm{d} x$$

Optimal (type 4, 707 leaves, 26 steps):

Result (type 4, 1129 leaves):

$$\frac{\text{a x}}{\text{2 d } \left(\text{d} + \text{e } \text{x}^2\right)} + \frac{\text{a ArcTan}\left[\frac{\sqrt{\text{e}} \text{ x}}{\sqrt{\text{d}}}\right]}{\text{2 d}^{3/2} \sqrt{\text{e}}} + \\$$

$$b = -\frac{\frac{ArcSinh[c\,x]}{i\,\sqrt{d}\,+\sqrt{e}\,\,x}}{-\frac{ArcSinh[c\,x]}{i\,\sqrt{d}\,+\sqrt{e}\,\,x}} - \frac{\frac{c\,Log\left[\frac{2\,e\left(\sqrt{e}\,-i\,c^2\,\sqrt{d}\,\,x+\sqrt{-c^2\,d+e}\,\,\sqrt{1+c^2\,x^2}\,\right)}{c\,\sqrt{-c^2\,d+e}\,\,\left(i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)}\right]}{\sqrt{-c^2\,d+e}} + \frac{\frac{ArcSinh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}}{+\frac{ArcSinh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}} + \frac{c\,Log\left[-\frac{2\,e\left(\sqrt{e}\,+i\,c^2\,\sqrt{d}\,\,x+\sqrt{-c^2\,d+e}\,\,\sqrt{1+c^2\,x^2}\,\right)}{c\,\sqrt{-c^2\,d+e}\,\,\left(-i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)}\right]}{\sqrt{-c^2\,d+e}} + \frac{ArcSinh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}}{+\frac{ArcSinh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}} + \frac{ArcSinh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}} + \frac{ArcSinh[c\,x]}{-i\,\sqrt$$

$$\frac{1}{32\,d^{3/2}\,\sqrt{e}}\left[-\,\mathrm{i}\,\left(\pi-2\,\,\mathrm{i}\,\,\mathsf{ArcSinh}\,[\,c\,\,x\,]\,\right)^2+32\,\,\mathrm{i}\,\,\mathsf{ArcSin}\,\Big[\,\frac{\sqrt{1+\frac{c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\right]$$

$$\operatorname{ArcTan}\Big[\frac{\left(\operatorname{c}\sqrt{\operatorname{d}}-\sqrt{\operatorname{e}}\right)\operatorname{Cot}\left[\frac{1}{4}\left(\pi+2\operatorname{i}\operatorname{ArcSinh}\left[\operatorname{c}x\right]\right)\right]}{\sqrt{\operatorname{c}^{2}\operatorname{d}-\operatorname{e}}}\Big]+4\left(\pi+4\operatorname{ArcSin}\Big[\frac{\sqrt{1+\frac{\operatorname{c}\sqrt{\operatorname{d}}}{\sqrt{\operatorname{e}}}}}{\sqrt{2}}\Big]-\operatorname{ArcTan}\Big[\frac{\operatorname{cot}\left(\operatorname{c}x\right)}{\sqrt{2}}\right]+\operatorname{ArcSin}\left(\operatorname{c}x\right)$$

$$2 \, \, \mathtt{i} \, \, \mathsf{ArcSinh} \, [\, c \, \, x \,] \, \left[\, \mathsf{Log} \, \Big[\, \mathsf{1} - \frac{\, \mathtt{i} \, \, \left(- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, \, \mathbb{e}^{\mathsf{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \right] \, + \, \mathsf{4} \, \right]$$

$$\left(\pi - 4 \, \text{ArcSin} \Big[\, \frac{\sqrt{1 + \frac{c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, \right) \, \text{Log} \Big[\, 1 \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, \right) \, + \, \left(\frac{1}{2} \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, \right) \, + \, \left(\frac{1}{2} \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, \right) \, + \, \left(\frac{1}{2} \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, - \, 2 \, \, \text{ii} \, \, \text{ArcSinh} \, [\, c \, \, x \,] \, + \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \, \sqrt{d} \, + \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, - \, \frac{\text{ii} \, \, \left(c \,$$

$$8 \ \ \dot{\mathbb{I}} \ \left[\text{PolyLog} \left[2 \text{, } \frac{\dot{\mathbb{I}} \ \left(- c \ \sqrt{d} \ + \sqrt{c^2 \ d - e} \ \right) \ e^{\text{ArcSinh} \left[c \ x \right]}}{\sqrt{e}} \right] \ + \right.$$

$$\mathsf{PolyLog}\Big[2\text{, }-\frac{\mathrm{i}\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d-e}\,\right)\,\,\mathrm{e}^{\mathsf{ArcSinh}\,[c\,x]}}{\sqrt{e}}\,\Big]\Bigg)\Bigg]+$$

$$\frac{1}{32\,\text{d}^{3/2}\,\sqrt{e}}\,\left[\,\dot{\mathbb{1}}\,\left(\pi-2\,\,\dot{\mathbb{1}}\,\,\text{ArcSinh}\,[\,\text{c}\,\,\text{x}\,]\,\right)^{\,2}-32\,\,\dot{\mathbb{1}}\,\,\text{ArcSin}\,\left[\,\frac{\sqrt{\,1-\frac{\text{c}\,\sqrt{d}\,\,}{\sqrt{e}}\,\,}}{\sqrt{2}}\,\right]\right.$$

$$\operatorname{ArcTan} \Big[\, \frac{ \left(c \, \sqrt{d} \, + \sqrt{e} \, \right) \, \operatorname{Cot} \left[\, \frac{1}{4} \, \left(\pi + 2 \, \, \underline{\mathrm{i}} \, \operatorname{ArcSinh} \left[\, c \, \, x \, \right] \, \right) \, \right] }{ \sqrt{c^2 \, d - e}} \, \Big] \, - \\$$

$$4\left[\pi + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] - 2 \operatorname{i} \operatorname{ArcSinh}\left[c x\right]\right]$$

$$Log \left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{ArcSinh[c x]}}{\sqrt{e}}\right] - 4$$

$$\left[\pi - 4 \, \text{ArcSin} \Big[\, \frac{\sqrt{1 - \frac{c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, - 2 \, \text{$\mathbb{1}$ ArcSinh} \, [\, c \, x \,] \, \right] \, \text{Log} \Big[1 \, - \, \frac{ \, \text{$\mathbb{1}$} \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}}{\sqrt{e}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}} \, \Big] \, + \, \frac{1 \, \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, e^{\text{ArcSinh} \, [\, c \, x \,]}} \, \Big] \, + \, \frac$$

$$8 \ i \ \left[PolyLog \left[2, -\frac{i \left(-c \sqrt{d} + \sqrt{c^2 d - e} \right) e^{ArcSinh[c x]}}{\sqrt{e}} \right] + \right]$$

PolyLog[2,
$$\frac{i\left(c\sqrt{d} + \sqrt{c^2 d - e}\right) e^{ArcSinh[cx]}}{\sqrt{e}}$$
]

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b \operatorname{ArcSinh}[c x]\right)^{2}}{d + e x^{2}} dx$$

Optimal (type 4, 739 leaves, 22 steps):

Result (type 4, 3196 leaves):

$$\frac{1}{8\sqrt{d}\sqrt{e}}\left[8\,a^2\,\mathsf{ArcTan}\Big[\,\frac{\sqrt{e}\,x}{\sqrt{d}}\,\Big]\,+\right.$$

$$4 \text{ a b} \left[8 \text{ i } \operatorname{ArcSin} \left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{ArcTan} \left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\operatorname{Cot} \left[\frac{1}{4}\left(\pi+2 \text{ i } \operatorname{ArcSinh} \left[c \times\right]\right)\right]}{\sqrt{c^2 \, d-e}}\right] - \frac{\sqrt{c^2 \, d-e}}{\sqrt{c^2 \, d-e}}\right] = \frac{\sqrt{c^2 \, d-e}}{\sqrt{c^2 \, d-e}}$$

$$8 \ \verb"iArcSin" \Big[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \ \mathsf{ArcTan} \Big[\frac{\left(c \sqrt{d} + \sqrt{e}\right) \ \mathsf{Cot} \Big[\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \right) + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big] + \frac{1}{2} \left(\frac{1}{4} \left(\pi + 2 \ \verb"iArcSinh" [c \ x] \right) \Big]}{\sqrt{c^2 \ d - e}} \Big]$$

$$\left[\pi + 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] - 2 \operatorname{id} \operatorname{ArcSinh}\left[c \times 1\right]\right] \operatorname{Log}\left[1 - \frac{\operatorname{id}\left(-c\sqrt{d} + \sqrt{c^2 d - e}\right) \operatorname{e}^{\operatorname{ArcSinh}\left[c \times 1\right)}}{\sqrt{e}}\right] - 2 \operatorname{id} \operatorname{ArcSinh}\left[c \times 1\right]$$

$$\left[\pi + 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh} \left[c \, x \right] \right] \operatorname{Log} \left[1 + \frac{\operatorname{i} \left(- c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, \operatorname{e}^{\operatorname{ArcSinh} \left[c \, x \right]}}{\sqrt{e}} \right] - 2 \operatorname{i} \operatorname{ArcSinh} \left[c \, x \right] \right] \operatorname{Log} \left[1 + \frac{\operatorname{i} \left(- c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, \operatorname{e}^{\operatorname{ArcSinh} \left[c \, x \right]}}{\sqrt{e}} \right] - 2 \operatorname{i} \operatorname{ArcSinh} \left[c \, x \right]$$

$$\left(\pi - 4 \operatorname{ArcSin} \left[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] - 2 \operatorname{i} \operatorname{ArcSinh} \left[c \, x \right] \right) \operatorname{Log} \left[1 - \frac{\operatorname{i} \left(c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, \operatorname{e}^{\operatorname{ArcSinh} \left[c \, x \right]}}{\sqrt{e}} \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \, x \right] \right] + \operatorname{Ind} \left[\operatorname{ArcSinh} \left[c \,$$

$$\left(\pi - 4 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] - 2 \operatorname{i} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{Log}\left[1 + \frac{\operatorname{i}\left(c \sqrt{d} + \sqrt{c^2 d - e}\right) e^{\operatorname{ArcSinh}\left[c \times\right]}}{\sqrt{e}}\right] + \left(\frac{1}{2} \operatorname{ArcSinh}\left[c \times\right]\right) \operatorname{ArcSinh}\left[c \times\right]$$

$$\left(\pi - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x]\right) \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} - \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] + 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} - \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - \left(\pi - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x]\right) \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right] - 2 \stackrel{.}{\text{i}} \operatorname{ArcSinh}[\operatorname{c} x] \operatorname{Log}\left[\operatorname{c}\left(\sqrt{d} + \stackrel{.}{\text{i}} \sqrt{e} \ x\right)\right]$$

$$\mathsf{PolyLog} \Big[2, - \frac{i \left(c \sqrt{d} + \sqrt{c^2 d - e} \right) e^{\mathsf{ArcSinh}[c x]}}{\sqrt{e}} \Big] + \frac{1}{\sqrt{e}}$$

$$2 \ \ \ \, i \ \left[PolyLog \left[2 \text{, } -\frac{\text{i} \ \left(-c \ \sqrt{d} \ + \sqrt{c^2 \ d - e} \ \right) \ \text{e}^{ArcSinh\left[c \ x\right]}}{\sqrt{e}} \right] \ + \right.$$

$$\text{PolyLog} \Big[2 \text{, } \frac{ \text{i} \left(c \sqrt{d} + \sqrt{c^2 d - e} \right) \, \text{e}^{\text{ArcSinh}[c \, x]}}{\sqrt{e}} \, \Big] \right) \Bigg] + 4 \, b^2$$

$$8 \ \ \text{i ArcSin} \Big[\frac{\sqrt{1 + \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \ \text{ArcSinh} [c \ x] \ \text{ArcTan} \Big[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \Big] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ - \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ + \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ + \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{\left(c \ \sqrt{d} \ - \sqrt{e} \ \right) \ \text{Cot} \left[\frac{1}{4} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ \right]}{\sqrt{c^2 \ d - e}} \right] \ + \frac{1}{\sqrt{c^2 \ d - e}} \left[\frac{1}{\sqrt{e}} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ + \frac{1}{\sqrt{e}} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right]}{\sqrt{c^2 \ d - e}} \right] \ + \frac{1}{\sqrt{e}} \left[\frac{1}{\sqrt{e}} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right) \ + \frac{1}{\sqrt{e}} \left[\frac{1}{\sqrt{e}} \left(\pi + 2 \ \ \text{i ArcSinh} [c \ x] \ \right]}{\sqrt{e}} \right]$$

$$8 \text{ i ArcSin} \left[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \text{ ArcSinh} \left[c \text{ x} \right]$$

$$\operatorname{ArcTan}\Big[\, \frac{\left(c \, \sqrt{d} \, + \sqrt{e} \, \right) \, \operatorname{Cot}\left[\, \frac{1}{4} \, \left(\pi + 2 \, \operatorname{i} \, \operatorname{ArcSinh}\left[\, c \, \, x \, \right] \, \right) \, \right]}{\sqrt{c^2 \, d - e}} \, \Big] \, - \, 8 \, \operatorname{i} \, \operatorname{ArcSin}\Big[\, \frac{\sqrt{1 + \frac{c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big]$$

$$\begin{split} & \text{ArcSinh} \, [\, c \, \, x \,] \, \, \, \text{ArcTan} \, \Big[\, \left(\, c \, \sqrt{d} \, - \sqrt{e} \, \right) \, \left(\text{Cosh} \, \big[\, \frac{1}{2} \, \text{ArcSinh} \, [\, c \, \, x \,] \, \, \right) \, - \, \text{i} \, \, \text{Sinh} \, \big[\, \frac{1}{2} \, \text{ArcSinh} \, [\, c \, \, x \,] \, \, \big] \, \right) \Big) \Big/ \\ & \left(\sqrt{c^2 \, d - e} \, \left(\text{Cosh} \, \big[\, \frac{1}{2} \, \text{ArcSinh} \, [\, c \, \, x \,] \, \, \right) \, + \, \text{i} \, \, \text{Sinh} \, \big[\, \frac{1}{2} \, \text{ArcSinh} \, [\, c \, \, x \,] \, \, \big] \, \right) \Big) \, \Big] \, + \, \end{split}$$

$$8 i ArcSin \left[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] ArcSinh[c x] ArcTan \left[\frac{1}{\sqrt{2}} \right]$$

$$\begin{split} &\left(\left(c\;\sqrt{d}\;+\sqrt{e}\;\right)\;\left(\text{Cosh}\left[\frac{1}{2}\;\text{ArcSinh}\left[c\;x\right]\;\right]-\text{i}\;\text{Sinh}\left[\frac{1}{2}\;\text{ArcSinh}\left[c\;x\right]\;\right]\right)\right)\bigg/\\ &\left(\sqrt{c^2\;d-e}\;\left(\text{Cosh}\left[\frac{1}{2}\;\text{ArcSinh}\left[c\;x\right]\;\right]+\text{i}\;\text{Sinh}\left[\frac{1}{2}\;\text{ArcSinh}\left[c\;x\right]\;\right]\right)\right)\right]\;+\end{split}$$

$$\pi \, \operatorname{ArcSinh} \left[c \, \, x \, \right] \, \operatorname{Log} \left[1 - \frac{ \, \dot{\mathbb{1}} \, \left(- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right) \, \, e^{\operatorname{ArcSinh} \left[\, c \, \, x \, \right]}}{\sqrt{e}} \, \right] \, + \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right] \, \left[- \, c \, \sqrt{d} \, + \sqrt{c^2 \, d - e} \, \right]$$

$$\label{eq:log_loss} \dot{\mathbb{1}} \; \text{ArcSinh} \, [\, c \; x \,] \,^2 \; \text{Log} \, \Big[\, 1 \, - \, \frac{\dot{\mathbb{1}} \; \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \; \mathbb{e}^{\text{ArcSinh} \, [\, c \; x \,]}}{\sqrt{e}} \, \Big] \; - \, \frac{\partial}{\partial x} \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \; \right) \, \left(- \, c \; \sqrt{d} \; + \sqrt{c^2 \; d - e} \;$$

$$\begin{split} & 4 \text{ArcSin} \big[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \big] \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(- c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & i \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(- c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & \pi \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & 4 \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & \pi \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & \pi \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] - \\ & 4 \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & i \, \text{ArcSinh} \big[c \, x \big]^2 \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right)}{\sqrt{e}} \, e^{\text{ArcSinh} (c \, x)}} \Big] + \\ & i \, \text{ArcSinh} \big[c \, x \big]^2 \, \text{Log} \Big[1 + \frac{\sqrt{e} \, e^{\text{ArcSinh} (c \, x)}}{i \, c \, \sqrt{d} - \sqrt{-c^2 \, d + e}} \Big] - i \, \text{ArcSinh} \big[c \, x \big]^2 \, \text{Log} \Big[1 + \frac{\sqrt{e} \, e^{\text{ArcSinh} (c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \Big] - \\ & \text{Log} \Big[1 - \frac{\sqrt{e} \, e^{\text{ArcSinh} (c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \Big] + i \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \Big] - \\ & \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} - \sqrt{c^2 \, d + e}} \Big] + i \, \text{ArcSinh} \big[c \, x \big] \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} - \sqrt{c^2 \, d + e}} \Big] + i \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} - \sqrt{c^2 \, d + e}} \Big] + i \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} - \sqrt{c^2 \, d + e}} \Big] + i \, \text{ArcSinh} \big[c \, x \big] \, \text{Log} \Big[1 + \frac{i \left(c \, \sqrt{d} - \sqrt{c^2 \, d + e} \right)}{i \, c \, \sqrt{d} - \sqrt$$

$$\begin{split} & 4 \text{ArcSin} \big[\frac{\sqrt{1 - \frac{c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \big] \text{ ArcSinh} [c \, x] \, \text{Log} \big[1 + \frac{i \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] - \\ & i \, \text{ArcSinh} [c \, x]^2 \, \text{Log} \big[1 + \frac{i \left(-c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] + \\ & \pi \, \text{ArcSinh} [c \, x] \, \text{Log} \big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] - \\ & 4 \, \text{ArcSinh} [c \, x] \, \text{Log} \big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] - \\ & \pi \, \text{ArcSinh} [c \, x]^2 \, \text{Log} \big[1 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] - \\ & \pi \, \text{ArcSinh} [c \, x] \, \text{Log} \big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] + \\ & 4 \, \text{ArcSinh} [c \, x] \, \text{Log} \big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] + \\ & 4 \, \text{ArcSinh} [c \, x]^2 \, \text{Log} \big[1 + \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] + \\ & 2 \, i \, \text{ArcSinh} [c \, x] \, \text{PolyLog} \big[2 - \frac{i \left(c \, \sqrt{d} + \sqrt{c^2 \, d - e} \right) \left(c \, x + \sqrt{1 + c^2 \, x^2} \right)}{\sqrt{e}} \big] + \\ & 2 \, i \, \text{ArcSinh} [c \, x] \, \text{PolyLog} \big[2 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \big] + \\ & 2 \, i \, \text{ArcSinh} [c \, x] \, \text{PolyLog} \big[2 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \big] + \\ & 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \big] - 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}} \big] - \\ & 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}}} \big] + 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}}} \big] - \\ & 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}}} \big] + 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSinh} [c \, x)}}{i \, c \, \sqrt{d} + \sqrt{-c^2 \, d + e}}} \big] + \\ & 2 \, i \, \text{PolyLog} \big[3 - \frac{\sqrt{e} \, e^{\text{ArcSi$$

Problem 44: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int\!\frac{a+b\,\text{ArcSinh}\,[\,c\,\,x\,]}{\left(\,d+e\,\,x^2\right)^{\,3/2}}\,\,\text{d}\,x$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{x\,\left(\texttt{a}+\texttt{b}\,\texttt{ArcSinh}\,[\,\texttt{c}\,\,\texttt{x}\,]\,\right)}{\mathsf{d}\,\sqrt{\mathsf{d}+\mathsf{e}\,\,\texttt{x}^2}}-\frac{\mathsf{b}\,\mathsf{ArcTanh}\,\Big[\frac{\sqrt{\mathsf{e}}\,\,\sqrt{1+\mathsf{c}^2\,\,\texttt{x}^2}}{\mathsf{c}\,\sqrt{\mathsf{d}+\mathsf{e}\,\,\texttt{x}^2}}\,\Big]}{\mathsf{d}\,\sqrt{\mathsf{e}}}$$

Result (type 6, 166 leaves):

$$\begin{split} \frac{1}{\sqrt{d+e\,x^2}} x &\left(\left[2\,b\,c\,x\,\text{AppellF1} \left[1\,,\,\frac{1}{2}\,,\,\frac{1}{2}\,,\,2\,,\,-c^2\,x^2\,,\,-\frac{e\,x^2}{d} \right] \right) \right/ \\ &\left(\sqrt{1+c^2\,x^2}\, \left(-4\,d\,\text{AppellF1} \left[1\,,\,\frac{1}{2}\,,\,\frac{1}{2}\,,\,2\,,\,-c^2\,x^2\,,\,-\frac{e\,x^2}{d} \right] + x^2 \left(e\,\text{AppellF1} \left[2\,,\,\frac{1}{2}\,,\,\frac{3}{2}\,,\,3\,,\,-c^2\,x^2\,,\,-\frac{e\,x^2}{d} \right] \right) \right) \right) + \frac{a+b\,\text{ArcSinh} \left[c\,x \right]}{d} \right) \end{split}$$

Problem 45: Result unnecessarily involves higher level functions.

$$\int \frac{a+b \, ArcSinh \, [\, c \, \, x \,]}{\left(\, d+e \, \, x^2 \right)^{5/2}} \, \mathrm{d}x$$

Optimal (type 3, 146 leaves, 7 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{3\,d\,\left(c^2\,d-e\right)\,\sqrt{d+e\,x^2}} + \frac{x\,\left(a+b\,\text{ArcSinh}\,[\,c\,x\,]\,\right)}{3\,d\,\left(d+e\,x^2\right)^{3/2}} + \\ \\ \frac{2\,x\,\left(a+b\,\text{ArcSinh}\,[\,c\,x\,]\,\right)}{3\,d^2\,\sqrt{d+e\,x^2}} - \frac{2\,b\,\text{ArcTanh}\,\Big[\,\frac{\sqrt{e}\,\sqrt{1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\,\Big]}{3\,d^2\,\sqrt{e}}$$

Result (type 6, 235 leaves):

$$\begin{split} \frac{1}{3\,d^2\,\left(d+e\,x^2\right)^{3/2}} \left(-\frac{b\,c\,d\,\sqrt{1+c^2\,x^2}}{c^2\,d-e} + a\,x\,\left(3\,d+2\,e\,x^2\right) \,+ \\ \left(4\,b\,c\,d\,x^2\,\left(d+e\,x^2\right)\,\text{AppellF1} \left[\,\mathbf{1},\,\,\frac{1}{2}\,,\,\,\frac{1}{2}\,,\,\,2\,,\,\,-c^2\,x^2\,,\,\,-\frac{e\,x^2}{d}\,\right] \,\right) \left/\,\,\left(\sqrt{1+c^2\,x^2}\,\right. \\ \left. \left(-4\,d\,\text{AppellF1} \left[\,\mathbf{1},\,\,\frac{1}{2}\,,\,\,\frac{1}{2}\,,\,\,2\,,\,\,-c^2\,x^2\,,\,\,-\frac{e\,x^2}{d}\,\right] \,+\,x^2\,\left(e\,\text{AppellF1} \left[\,2\,,\,\,\frac{1}{2}\,,\,\,\frac{3}{2}\,,\,\,3\,,\,\,-c^2\,x^2\,,\,\,-\frac{e\,x^2}{d}\,\right] \,+\,x^2\,\left(e\,\text{AppellF1} \left[\,2\,,\,\,\frac{3}{2}\,,\,\,\frac{3}{2}\,,\,\,3\,,\,\,-c^2\,x^2\,,\,\,-\frac{e\,x^2}{d}\,\right] \,\right) \right) \,+\,b\,x\,\left(3\,d+2\,e\,x^2\right)\,\text{ArcSinh}\left[\,c\,x\,\right] \end{split}$$

Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{(d + e x^2)^{7/2}} dx$$

Optimal (type 3, 227 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{15\,d\,\left(c^2\,d-e\right)\,\left(d+e\,x^2\right)^{3/2}}\,-\,\frac{2\,b\,c\,\left(3\,c^2\,d-2\,e\right)\,\sqrt{1+c^2\,x^2}}{15\,d^2\,\left(c^2\,d-e\right)^2\,\sqrt{d+e\,x^2}}\,+\,\frac{x\,\left(a+b\,ArcSinh\,[\,c\,x\,]\,\right)}{5\,d\,\left(d+e\,x^2\right)^{5/2}}\,+\,\frac{3\,b\,ArcTanh\,\left[\,\frac{\sqrt{e}\,\sqrt{1+c^2\,x^2}\,}{\sqrt{1+c^2\,x^2}}\,\right]}{2\,b\,ArcTanh\,\left[\,\frac{\sqrt{e}\,\sqrt{1+c^2\,x^2}\,}{\sqrt{1+c^2\,x^2}}\,\right]}$$

$$\frac{4 \, x \, \left(\text{a} + \text{b} \, \text{ArcSinh} \left[\text{c} \, x \, \right]\right)}{15 \, d^2 \, \left(\text{d} + \text{e} \, x^2\right)^{3/2}} + \frac{8 \, x \, \left(\text{a} + \text{b} \, \text{ArcSinh} \left[\text{c} \, x \, \right]\right)}{15 \, d^3 \, \sqrt{\text{d} + \text{e} \, x^2}} - \frac{8 \, \text{b} \, \text{ArcTanh} \left[\frac{\sqrt{\text{e}} \, \sqrt{1 + \text{c}^2 \, x^2}}{\text{c} \, \sqrt{\text{d} + \text{e} \, x^2}}\right]}{15 \, d^3 \, \sqrt{\text{e}}}$$

Result (type 6, 308 leaves):

$$\left(-\frac{b c d^2 \sqrt{1 + c^2 x^2} \left(d + e x^2\right)}{c^2 d - e} - \frac{2 b c d \left(3 c^2 d - 2 e\right) \sqrt{1 + c^2 x^2} \left(d + e x^2\right)^2}{\left(-c^2 d + e\right)^2} + a x \left(15 d^2 + 20 d e x^2 + 8 e^2 x^4\right) + \left(16 b c d x^2 \left(d + e x^2\right)^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e x^2}{d}\right]\right) \middle/ \left(\sqrt{1 + c^2 x^2}\right)$$

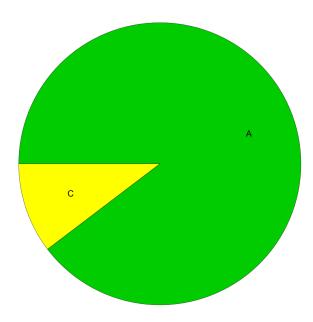
$$\left(-4 d \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left(e \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right]\right) \right) +$$

$$c^2 d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right] \right) \middle) +$$

$$b x \left(15 d^2 + 20 d e x^2 + 8 e^2 x^4\right) \text{ArcSinh}\left[c x\right] \middle/ \left(15 d^3 \left(d + e x^2\right)^{5/2}\right)$$

Summary of Integration Test Results

58 integration problems



- A 52 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 6 unnecessarily complex antiderivatives
- D 0 unable to integrate problems
- E 0 integration timeouts