# Rubi 4.16.0 Hyperbolic Integration Test Suite Results

Test results for the 502 problems in "6.1.1 (c+d x)^m (a+b sinh)^n.m"

Test results for the 102 problems in "6.1.3 (e x)^m (a+b sinh(c+d x^n))^p.m"

Test results for the 33 problems in "6.1.4 (d+e x)^m sinh(a+b x+c x^2)^n.m"

Test results for the 525 problems in "6.1.7 hyper^m (a+b sinh^n)^p.m"

Test results for the 369 problems in "6.1.5 Hyperbolic sine functions.m"

Test results for the 183 problems in "6.2.1 (c+d x)^m (a+b cosh)^n.m"

Test results for the 111 problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.m"

Test results for the 68 problems in "6.2.3 (e x)^m (a+b cosh(c+d x^n))^p.m"

Test results for the 33 problems in "6.2.4 (d+e x)^m cosh(a+b x+c x^2)^n.m"

Test results for the 85 problems in "6.2.7 hyper^m (a+b cosh^n)^p.m"

Test results for the 336 problems in "6.2.5 Hyperbolic cosine functions.m"

Test results for the 77 problems in "6.3.1 (c+d x)^m (a+b tanh)^n.m"

Problem 16: Unable to integrate problem.

$$\int \left(\,c\,+\,d\,\,x\,\right)\;\,\left(\,b\,\,Tanh\,[\,e\,+\,f\,\,x\,]\,\,\right)^{\,5/2}\,\,\mathrm{d}\,x$$

Optimal (type 4, 1392 leaves, 44 steps):

$$2 \, b^{5/2} \, d \, A \, \text{cran} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \left( c \cdot d \, x \right) \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \, d \, A \, \text{cranh} \left[ \frac{\sqrt{b \, \text{tamh} \, (c \cdot f \, x)}}}{\sqrt{b}} \right] = \left( -b \right)^{5/2} \,$$

Result (type 8, 137 leaves, 7 steps):

$$\frac{2 \, b^{5/2} \, d \, \text{ArcTan} \Big[ \, \frac{\sqrt{b \, \text{Tanh} \, [e+f \, x]}}{\sqrt{b}} \, \Big]}{3 \, f^2} \, + \, \frac{2 \, b^{5/2} \, d \, \text{ArcTanh} \Big[ \, \frac{\sqrt{b \, \text{Tanh} \, [e+f \, x]}}{\sqrt{b}} \, \Big]}{3 \, f^2} \, - \, \frac{4 \, b^2 \, d \, \sqrt{b \, \text{Tanh} \, [e+f \, x]}}{3 \, f^2} \, - \, \frac{2 \, b \, \left(c + d \, x\right) \, \left(b \, \text{Tanh} \, [e+f \, x] \, \right)^{3/2}}{3 \, f} \, + \, b^2 \, \text{Unintegrable} \Big[ \left(c + d \, x\right) \, \sqrt{b \, \text{Tanh} \, [e+f \, x]} \, , \, x \Big]$$

## Problem 17: Unable to integrate problem.

$$\int \left(c + dx\right) \left(b \, \mathsf{Tanh} \left[\, e + fx\,\right]\,\right)^{3/2} \, \mathrm{d}x$$

Optimal (type 4, 1363 leaves, 43 steps):

$$\frac{2 \, b^{3/2} \, d \, ArcTanl \left[ \sqrt{b \, Tanh [e-Fx]} \right]}{\sqrt{5}} \left[ (-b)^{3/2} \left\{ (c-d \, x) \, ArcTanh \left[ \sqrt{b \, Tanh [e-Fx]} \right]} {\sqrt{5}} \right] \left[ -b \right]^{3/2} \, d \, ArcTanh \left[ \sqrt{b \, Tanh [e-Fx]} \right]} {\sqrt{5}} \right]^2 \\ = \frac{2 \, b^{3/2} \, d \, ArcTanh \left[ \sqrt{b \, Tanh [e-Fx]} \right]}{\sqrt{5}} \left[ b^{3/2} \, d \, ArcTanh \left[ \sqrt{b \, Tanh [e-Fx]} \right]} {\sqrt{5}} \right]^2 \\ = \frac{2 \, d^2}{\sqrt{5}} \\ = \frac{2 \, d^2}{$$

Result (type 8, 108 leaves, 6 steps):

$$-\frac{2\,b^{3/2}\,d\,\text{ArcTan}\big[\frac{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{\sqrt{b}}\big]}{f^2}\,+\,\frac{2\,b^{3/2}\,d\,\text{ArcTanh}\big[\frac{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{\sqrt{b}}\big]}{f^2}\,-\,\frac{2\,b\,\left(c+d\,x\right)\,\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{f}\,+\,b^2\,\text{Unintegrable}\big[\frac{c+d\,x}{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}\text{, }x\big]$$

## Problem 18: Unable to integrate problem.

$$\int (c + dx) \sqrt{b \operatorname{Tanh}[e + fx]} dx$$

Optimal (type 4, 1280 leaves, 37 steps):

Result (type 8, 20 leaves, 0 steps):

Unintegrable  $[(c + dx) \sqrt{b Tanh [e + fx]}, x]$ 

## Problem 19: Unable to integrate problem.

$$\int \frac{c + d\,x}{\sqrt{b\, Tanh\, [\, e + f\, x\,]}} \, \, \text{d} x$$

Optimal (type 4, 1280 leaves, 37 steps):

$$\frac{\left(c+dx\right)\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{-\operatorname{b}}}\right]}{\sqrt{-\operatorname{b}}f} = \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{-\operatorname{b}}}\right]^2}{2\sqrt{-\operatorname{b}}f^2} + \frac{2\sqrt{-\operatorname{b}}f^2}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}}\right]^2}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{bTanh}(e+fx)}}\right]}{\sqrt{\operatorname{b}}\sqrt{\operatorname{bTanh}(e+fx)}} + \frac{d\operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{\operatorname{b$$

Result (type 8, 20 leaves, 0 steps):

Unintegrable 
$$\left[\frac{c + dx}{\sqrt{b \operatorname{Tanh}[e + fx]}}, x\right]$$

## Problem 20: Unable to integrate problem.

$$\int \frac{c + d\,x}{\left(b\, Tanh \left[\,e + f\,x\,\right]\,\right)^{\,3/2}}\, \mathrm{d}x$$

Optimal (type 4, 1365 leaves, 43 steps):

$$\frac{2 \, d \, A \, C \, Tan \left[ \frac{\sqrt{b \, Tanh \left( e + X \right)}}{\sqrt{b}} \right] \, \left( e + d \, x \right) \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + X \right)}}{\sqrt{b}} \right]}{\sqrt{b}} \, \left( 2 \, \left( - b \right)^{3/2} \, f^2 \right) \, \\ = 2 \, \left( - b \right)^{3/2} \, f^2 \, \left( - b \right)^{3/2} \, f^2 \, \right) \, \\ = \frac{\left[ (c + d \, x) \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}{\sqrt{b}} \right]}{\sqrt{b}} \, \left( 2 \, \left( - b \right)^{3/2} \, f^2 \right) \, \right) \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}{\sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b}}{\sqrt{b} \, \sqrt{b \, Tanh \left( e + f \, x \right)}} \right] \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b}}{\sqrt{b} \, \sqrt{b \, Tanh \left( e + f \, x \right)}} \right] \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}{\sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b}}{\sqrt{b} \, \sqrt{b \, Tanh \left( e + f \, x \right)}} \right] \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, \sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b} \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}} \right] \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}}{\sqrt{b} \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}} \right] \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}}{\sqrt{b} \, \sqrt{b}} \right]} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}}{\sqrt{b} \, \sqrt{b}} \right]} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}}{\sqrt{b} \, \sqrt{b}} \right]} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \right] \, Log \left[ \frac{2 \, \sqrt{b} \, \sqrt{b} \, Tanh \left( e + f \, x \right)}}{\sqrt{b} \, \sqrt{b}} \right]} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \, \left( - b \right)^{3/2} \, f^2} \right]} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \, \left( - b \right)^{3/2} \, f^2} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \, \left( - b \right)^{3/2} \, f^2} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e + f \, x \right)}}}{\sqrt{b} \, \sqrt{b}} \, \left( - b \right)^{3/2} \, f^2} \, d \, A \, C \, Tanh \left[ \frac{\sqrt{b \, Tanh \left( e +$$

Result (type 8, 110 leaves, 6 steps):

$$\frac{2\,\text{d}\,\text{ArcTan}\!\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]}{b^{3/2}\,f^2}\,+\,\frac{2\,\text{d}\,\text{ArcTanh}\!\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]}{b^{3/2}\,f^2}\,-\,\frac{2\,\left(\,c+d\,x\right)}{b\,f\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}\,+\,\frac{\text{Unintegrable}\left[\,\left(\,c+d\,x\right)\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\,\,,\,x\right]}{b^2}$$

## Problem 21: Result valid but suboptimal antiderivative.

$$\int (c + dx)^2 (b Tanh [e + fx])^{3/2} dx$$

Optimal (type 8, 1340 leaves, 38 steps):

Optimid (type 6, 1340 leaves, 30 steply). 
$$\frac{4 \left(-b\right)^{3/2} d \left(c + dx\right) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right]}{\sqrt{-b}} + 2 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right]}{\sqrt{-b}} + \frac{2 \left(b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right]}{\sqrt{-b}} + \frac{2 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{b}}\right]}{\sqrt{b}} + \frac{4 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{-b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}}\right]}{\sqrt{b}} + \frac{4 b^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2 \sqrt{b}}{\sqrt{-b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}}\right]}{\sqrt{b}} + \frac{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}}\right]}{\sqrt{-b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}} + \frac{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}}\right]}{\sqrt{-b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}} + \frac{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}\right)}{\sqrt{-b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}\right]}{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}\right)}{\sqrt{-b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}\right]}{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + fx\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + fx\right)}\right)}{\sqrt{-b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}\right]}$$

$$= 2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh}\left(e + fx\right)}}\right]}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}$$

$$= 2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}\right]}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}$$

$$= 2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}\right]}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}$$

$$= 2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}\right]}{\sqrt{b} + \sqrt{b} \operatorname{Tanh}\left(e + fx\right)}}$$

$$\frac{b^{3/2} \, d^2 \, \text{PolyLog} \Big[ 2 \text{, } 1 - \frac{2 \, \sqrt{b} \, \left( \sqrt{-b} \, + \sqrt{b \, \text{Tanh} [e+f \, x]} \, \right)}{\left( \sqrt{-b} \, + \sqrt{b} \, \right) \, \left( \sqrt{b} \, + \sqrt{b \, \text{Tanh} [e+f \, x]} \, \right)}}{f^3} - \frac{2 \, \left( -b \right)^{3/2} \, d^2 \, \text{PolyLog} \Big[ 2 \text{, } 1 - \frac{2}{1 - \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{-b}}} \, + \frac{1}{1 - \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}} \right]}{f^3} + \frac{\left( -b \right)^{3/2} \, d^2 \, \text{PolyLog} \Big[ 2 \text{, } 1 + \frac{2 \, \left( \sqrt{b} \, + \sqrt{b \, \text{Tanh} [e+f \, x]} \, \right)}{\left( \sqrt{-b} \, + \sqrt{b} \, \right) \, \left( 1 - \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}}{\sqrt{-b}} \right)}}{f^3} + \frac{\left( -b \right)^{3/2} \, d^2 \, \text{PolyLog} \Big[ 2 \text{, } 1 + \frac{2 \, \left( \sqrt{b} \, + \sqrt{b \, \text{Tanh} [e+f \, x]} \, \right)}{\left( \sqrt{-b} \, - \sqrt{b} \, \right) \, \left( 1 - \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}}{\sqrt{-b}} \right)}}{f^3} - \frac{2 \, b \, \left( c + d \, x \right)^2 \, \sqrt{b \, \text{Tanh} [e+f \, x]}}}{f} + b^2 \, \text{Unintegrable} \Big[ \frac{\left( c + d \, x \right)^2}{\sqrt{b \, \text{Tanh} [e+f \, x]}} \text{, } x \Big]}$$

Result (type 8, 79 leaves, 1 step):

$$-\frac{2\,b\,\left(c+d\,x\right)^{2}\,\sqrt{b\,Tanh\,[\,e+f\,x\,]}}{f}+b^{2}\,Unintegrable\,\left[\frac{\left(c+d\,x\right)^{\,2}}{\sqrt{b\,Tanh\,[\,e+f\,x\,]}}\,\text{, }x\,\right]+\frac{4\,b\,d\,Unintegrable\,\left[\,\left(c+d\,x\right)\,\sqrt{b\,Tanh\,[\,e+f\,x\,]}\,\,\text{, }x\,\right]}{f}$$

## Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c+d\,x\right)^2}{\left(b\,Tanh\left[e+f\,x\right]\right)^{3/2}}\,\mathrm{d}x$$

Optimal (type 8, 1342 leaves, 38 steps):

$$\frac{4 \text{d} \left(\text{c} \mid \text{d} \right) \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\sqrt{D}} = 2 \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]^2}{\left(-b\right)^{3/2} \notin 2} + 4 \frac{d \left(\text{c} \mid \text{d} \right) \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\sqrt{D}} = 2 \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}}{\sqrt{D}}\right]}{\sqrt{D}} + 4 \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right]}{\sqrt{D}} + 4 \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right]}{\sqrt{D}} + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right]}{\sqrt{D}} + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{\text{b Tranh}\left(\text{c} + \text{f} \right)}}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\sqrt{D}} + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right] \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\sqrt{D}} + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]} \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]} \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right]}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}} \log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right]}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}}\right]}}{\log \left[\frac{2\sqrt{D}}{\sqrt{D}}\right] + \frac{d^2 \text{AncTanh} \left[\frac{\sqrt{D} \text{Tranh}\left(\text{c} + \text{f} \right)}}{\sqrt{D}$$

Result (type 8, 83 leaves, 1 step):

Result (type 8, 83 leaves, 1 step):
$$-\frac{2\left(c+d\,x\right)^{2}}{b\,f\,\sqrt{b\,Tanh\left[e+f\,x\right]}} + \frac{4\,d\,Unintegrable\left[\frac{c+d\,x}{\sqrt{b\,Tanh\left[e+f\,x\right]}},\,x\right]}{b\,f} + \frac{Unintegrable\left[\left(c+d\,x\right)^{2}\,\sqrt{b\,Tanh\left[e+f\,x\right]},\,x\right]}{b^{2}}$$

## Test results for the 263 problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.m"

## Test results for the 247 problems in "6.3.2 Hyperbolic tangent functions.m"

## Problem 146: Unable to integrate problem.

```
x^3 Tanh [a + 2 Log [x]] dx
```

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} e^{-2a} Log [1 + e^{2a} x^4]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [a + 2 Log [x]], x]$ 

## Problem 147: Unable to integrate problem.

$$\int x^2 Tanh[a + 2 Log[x]] dx$$

Optimal (type 3, 151 leaves, 11 steps):

$$\frac{x^3}{3} + \frac{e^{-3\text{ a}/2} \, \text{ArcTan} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} - \frac{e^{-3\text{ a}/2} \, \text{ArcTan} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} - \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x + e^{a} \, x^2 \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^{a} \, x + e^{a} \, x + e^{a} \, x + e^{a} \, x \Big]}{2 \, \sqrt{2}} + \frac{e^{-3\text{ a}/2} \, e^{a/2} \, x + e^{a} \,$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [a + 2 Log [x]], x]$ 

#### Problem 148: Unable to integrate problem.

$$x \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTan} \left[ e^a x^2 \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[x Tanh[a + 2 Log[x]], x]

## Problem 149: Unable to integrate problem.

$$\int \mathsf{Tanh}\left[\mathsf{a} + \mathsf{2}\,\mathsf{Log}\left[\mathsf{x}\right]\right]\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 145 leaves, 11 steps):

$$X + \frac{ e^{-a/2} \, \text{ArcTan} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X \Big] }{\sqrt{2}} - \frac{ e^{-a/2} \, \text{ArcTan} \Big[ 1 + \sqrt{2} \, e^{a/2} \, X \Big] }{\sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} - \frac{ e^{-a/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X + e^a \, X^2 \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, X + e^a \, X + e^a \, X \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^a \, X + e^a \, X + e^a \, X \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^a \, X + e^a \, X + e^a \, X \Big] }{2 \, \sqrt{2}} + \frac{ e^{-a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^a \, X + e^a \, X + e^a \, X$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate [Tanh [a + 2 Log[x]], x]

## Problem 151: Unable to integrate problem.

$$\int \frac{Tanh[a+2 Log[x]]}{x^2} dx$$

Optimal (type 3, 147 leaves, 11 steps):

$$\frac{1}{x} - \frac{e^{a/2} \, \text{ArcTan} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} + \frac{e^{a/2} \, \text{ArcTan} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} + \frac{e^{a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x + e^a \, x^2 \Big]}{2 \, \sqrt{2}} - \frac{e^{a/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^a \, x^2 \Big]}{2 \, \sqrt{2}}$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2Log[x]]}{x^2}, x\right]$$

## Problem 152: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\,[\,\mathsf{a}\,+\,2\,\mathsf{Log}\,[\,\mathsf{x}\,]\,\,]}{\mathsf{x}^3}\,\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{1}{2 x^2} + e^a \operatorname{ArcTan} \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2Log[x]]}{x^3}, x\right]$$

## Problem 153: Unable to integrate problem.

$$\int x^3 \, Tanh \, [\, a + 2 \, Log \, [\, x \, ] \, \,]^{\, 2} \, \, d \, x$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{e^{-2a}}{1 + e^{2a}x^4} - e^{-2a} Log \left[1 + e^{2a}x^4\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [a + 2 Log [x]]^2, x]$ 

## Problem 154: Unable to integrate problem.

$$\int x^2 \operatorname{Tanh} [a + 2 \operatorname{Log} [x]]^2 dx$$

Optimal (type 3, 173 leaves, 12 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 + e^{2\,a}\,x^4} + \frac{3\,e^{-3\,a/2}\,\text{ArcTan}\big[1 - \sqrt{2}\,\,e^{a/2}\,x\big]}{2\,\sqrt{2}} - \frac{3\,e^{-3\,a/2}\,\text{ArcTan}\big[1 + \sqrt{2}\,\,e^{a/2}\,x\big]}{2\,\sqrt{2}} - \frac{3\,e^{-3\,a/2}\,\text{Log}\big[1 - \sqrt{2}\,\,e^{a/2}\,x + e^{a}\,x^2\big]}{4\,\sqrt{2}} + \frac{3\,e^{-3\,a/2}\,\text{Log}\big[1 + \sqrt{2}\,\,e^{a/2}\,x + e^{a}\,x^2\big]}{4\,\sqrt{2}}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [a + 2 Log [x]]^2, x]$ 

## Problem 155: Unable to integrate problem.

$$\int x Tanh[a + 2 Log[x]]^2 dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 + e^{2a} x^4} - e^{-a} ArcTan [e^a x^2]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x Tanh [a + 2 Log [x]]^2, x]$ 

## Problem 156: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + 2 \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^2 \, \mathrm{d} \mathsf{x}$$

Optimal (type 3, 165 leaves, 13 steps):

$$X + \frac{X}{1 + e^{2\,a}\,X^4} + \frac{e^{-a/2}\,\text{ArcTan} \left[1 - \sqrt{2}\,\,e^{a/2}\,X\right]}{2\,\sqrt{2}} - \frac{e^{-a/2}\,\text{ArcTan} \left[1 + \sqrt{2}\,\,e^{a/2}\,X\right]}{2\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} - \frac{e^{-a/2}\,\text{Log} \left[1 + \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} - \frac{e^{-a/2}\,\text{Log} \left[1 + \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} - \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} - \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} - \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X + e^{a}\,X^2\right]}{4\,\sqrt{2}} + \frac{e^{-a/2}\,\text{Log} \left[1 - \sqrt{2}\,\,e^{a/2}\,X + e^{a}\,X + e$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + 2 Log[x]]^2, x]$ 

## Problem 158: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\,[\,\mathsf{a}\,+\,2\,\mathsf{Log}\,[\,\mathsf{x}\,]\,\,]^{\,2}}{\mathsf{x}^2}\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 190 leaves, 12 steps):

$$\begin{split} & -\frac{1}{x\,\left(1+\,e^{2\,a}\,x^4\right)} - \frac{2\,e^{2\,a}\,x^3}{1+\,e^{2\,a}\,x^4} + \frac{\,e^{a/2}\,\text{ArcTan}\!\left[1-\sqrt{2}\,\,e^{a/2}\,x\right]}{2\,\sqrt{2}} - \\ & \frac{\,e^{a/2}\,\text{ArcTan}\!\left[1+\sqrt{2}\,\,e^{a/2}\,x\right]}{2\,\sqrt{2}} - \frac{\,e^{a/2}\,\text{Log}\!\left[1-\sqrt{2}\,\,e^{a/2}\,x+e^{a}\,x^2\right]}{4\,\sqrt{2}} + \frac{\,e^{a/2}\,\text{Log}\!\left[1+\sqrt{2}\,\,e^{a/2}\,x+e^{a}\,x^2\right]}{4\,\sqrt{2}} \end{split}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2Log[x]]^2}{x^2}, x\right]$$

## Problem 159: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{a} + 2 \mathsf{Log}\left[\mathsf{x}\right]\right]^2}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\,\frac{1}{2\,{x^{2}}\,\left(1+{{e}^{2\,a}}\,{x^{4}}\right)}\,-\,\frac{3\,{{e}^{2\,a}}\,{x^{2}}}{2\,\left(1+{{e}^{2\,a}}\,{x^{4}}\right)}\,-\,{{e}^{a}}\,\text{ArcTan}\!\left[\,{{e}^{a}}\,{x^{2}}\right]$$

Result (type 8, 15 leaves, 0 steps):

$$\label{eq:cannotIntegrate} CannotIntegrate \Big[ \frac{Tanh \left[ a + 2 \, Log \left[ x \right] \, \right]^2}{x^3} \text{, } x \Big]$$

## Problem 160: Unable to integrate problem.

$$\int (e x)^m Tanh[a + 2 Log[x]] dx$$

Optimal (type 5, 60 leaves, 3 steps):

$$\frac{\left(\left(e\;x\right)^{\;1+m}}{e\;\left(1+m\right)}\;-\;\frac{2\;\left(\left(e\;x\right)^{\;1+m}\;Hypergeometric2F1\left[\;1,\;\frac{1+m}{4}\;,\;\frac{5+m}{4}\;,\;-\,\odot^{2\;a}\;x^{4}\right]}{e\;\left(1+m\right)}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + 2 Log [x]], x]$ 

## Problem 161: Unable to integrate problem.

$$\int (e x)^m Tanh [a + 2 Log [x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} + \frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+}\mathbb{e}^{\text{2-a}}\,x^{\text{4}}\right)} - \frac{\left(\text{e x}\right)^{\text{1+m}}\,\text{Hypergeometric2F1}\left[\text{1,}\,\frac{\text{1+m}}{4}\text{,}\,\frac{\text{5+m}}{4}\text{,}\,-\mathbb{e}^{\text{2-a}}\,x^{\text{4}}\right]}{\text{e}}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + 2 Log [x]]^2, x]$ 

#### Problem 162: Unable to integrate problem.

$$\int (e x)^m Tanh [a + 2 Log[x]]^3 dx$$

Optimal (type 5, 176 leaves, 5 steps):

$$\frac{ \left(3+\text{m}\right) \; \left(5+\text{m}\right) \; \left(e\,x\right)^{\,1+\text{m}}}{8\; e\; \left(1+\text{m}\right)} - \frac{\left(e\,x\right)^{\,1+\text{m}} \; \left(1-e^{2\,a}\,x^4\right)^{\,2}}{4\; e\; \left(1+e^{2\,a}\,x^4\right)^{\,2}} - \\ \\ \frac{e^{-2\,a} \; \left(e\,x\right)^{\,1+\text{m}} \; \left(e^{2\,a} \; \left(3-\text{m}\right) \; + \; e^{4\,a} \; \left(5+\text{m}\right) \; x^4\right)}{8\; e\; \left(1+e^{2\,a}\,x^4\right)} - \frac{\left(9+2\,\text{m}+\text{m}^2\right) \; \left(e\,x\right)^{\,1+\text{m}} \; \text{Hypergeometric} 2\text{F1} \left[1,\; \frac{1+\text{m}}{4},\; \frac{5+\text{m}}{4},\; -e^{2\,a}\,x^4\right]}{4\; e\; \left(1+\text{m}\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + 2 Log [x]]^3, x]$ 

## Problem 163: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left( 1 - e^{2a} x^{2b} \right)^{-p} \left( -1 + e^{2a} x^{2b} \right)^{p} AppellF1 \left[ \frac{1}{2b}, -p, p, \frac{1}{2} \left( 2 + \frac{1}{b} \right), e^{2a} x^{2b}, -e^{2a} x^{2b} \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate [Tanh[a + b Log[x]]<sup>p</sup>, x]

## Problem 164: Unable to integrate problem.

$$\int (e x)^m Tanh [a + b Log[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}} \left(\text{1}-\text{e}^{\text{2 a }} \text{x}^{\text{2 b}}\right)^{\text{-p}} \left(-\text{1}+\text{e}^{\text{2 a }} \text{x}^{\text{2 b}}\right)^{\text{p}} \text{AppellF1} \left[\frac{\text{1+m}}{\text{2 b}},\text{-p,p,1}+\frac{\text{1+m}}{\text{2 b}},\text{e}^{\text{2 a }} \text{x}^{\text{2 b}},\text{-e}^{\text{2 a }} \text{x}^{\text{2 b}}\right]}{\text{e }\left(\text{1}+\text{m}\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh[a+bLog[x]]^p, x]$ 

## Problem 165: Unable to integrate problem.

$$\int\! Tanh \left[\, a\, +\, \frac{Log\left[\, x\,\right]}{2}\,\right]^p\, \mathrm{d} x$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2^{-p}\,\,\mathrm{e}^{-2\,a}\,\,\left(-\,1\,+\,\,\mathrm{e}^{2\,a}\,\,x\right)^{\,1\,+\,p}\,\,\text{Hypergeometric} 2F1\!\left[\,p\,,\,\,1\,+\,p\,,\,\,2\,+\,p\,,\,\,\frac{1}{2}\,\,\left(\,1\,-\,\,\mathrm{e}^{2\,a}\,\,x\,\right)\,\,\right]}{1\,+\,p}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ Tanh \left[ \frac{1}{2} \left( 2 a + Log[x] \right) \right]^p, x \right]$$

## Problem 166: Unable to integrate problem.

$$\int Tanh \left[ a + \frac{Log[x]}{4} \right]^p dx$$

Optimal (type 5, 106 leaves, 4 steps):

$$e^{-4\,a}\,\left(-\,1\,+\,e^{2\,a}\,\sqrt{x}\,\right)^{\,1+p}\,\left(1\,+\,e^{2\,a}\,\sqrt{x}\,\right)^{\,1-p}\,-\,\frac{2^{1-p}\,e^{-4\,a}\,p\,\left(-\,1\,+\,e^{2\,a}\,\sqrt{x}\,\right)^{\,1+p}\,\text{Hypergeometric2F1}\!\left[\,p,\,\,1\,+\,p,\,\,2\,+\,p,\,\,\frac{1}{2}\,\left(1\,-\,e^{2\,a}\,\sqrt{x}\,\right)\,\right]}{1\,+\,p}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ Tanh \left[ \frac{1}{4} \left( 4a + Log[x] \right) \right]^p, x \right]$ 

## Problem 167: Unable to integrate problem.

$$\int Tanh \left[ a + \frac{Log[x]}{6} \right]^p dx$$

Optimal (type 5, 158 leaves, 5 steps):

$$-\,e^{-6\,a}\,p\,\left(-\,1\,+\,e^{2\,a}\,x^{1/3}\right)^{\,1+p}\,\left(1\,+\,e^{2\,a}\,x^{1/3}\right)^{\,1-p}\,+\,e^{-4\,a}\,\left(-\,1\,+\,e^{2\,a}\,x^{1/3}\right)^{\,1+p}\,\left(1\,+\,e^{2\,a}\,x^{1/3}\right)^{\,1-p}\,x^{1/3}\,+\\ \frac{2^{-p}\,e^{-6\,a}\,\left(1\,+\,2\,p^2\right)\,\left(-\,1\,+\,e^{2\,a}\,x^{1/3}\right)^{\,1+p}\,\text{Hypergeometric}2\text{F1}\!\left[\,\text{p, 1}\,+\,\text{p, 2}\,+\,\text{p, }\frac{1}{2}\,\left(1\,-\,e^{2\,a}\,x^{1/3}\right)\,\right]}{1\,+\,p}\,$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ Tanh \left[ \frac{1}{6} \left( 6 a + Log[x] \right) \right]^p, x \right]$ 

## Problem 168: Unable to integrate problem.

$$\int Tanh \left[ a + \frac{Log[x]}{8} \right]^p dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\frac{1}{3}\,e^{-12\,a}\,\left(-1+e^{2\,a}\,x^{1/4}\right)^{1+p}\,\left(1+e^{2\,a}\,x^{1/4}\right)^{1-p}\,\left(e^{4\,a}\,\left(3+2\,p^2\right)-2\,e^{6\,a}\,p\,x^{1/4}\right)+e^{-4\,a}\,\left(-1+e^{2\,a}\,x^{1/4}\right)^{1+p}\,\left(1+e^{2\,a}\,x^{1/4}\right)^{1-p}\,\sqrt{x}-2\,e^{6\,a}\,p\,\left(2+p^2\right)\,\left(-1+e^{2\,a}\,x^{1/4}\right)^{1+p}\,\text{Hypergeometric}\\ \frac{2^{2-p}\,e^{-8\,a}\,p\,\left(2+p^2\right)\,\left(-1+e^{2\,a}\,x^{1/4}\right)^{1+p}\,\text{Hypergeometric}\\ 3\,\left(1+p\right)}{3\,\left(1+p\right)}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ Tanh \left[ \frac{1}{8} \left( 8 a + Log[x] \right) \right]^p, x \right]$$

## Problem 169: Unable to integrate problem.

$$\int Tanh [a + Log[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(1-e^{2a}x^{2}\right)^{-p} \left(-1+e^{2a}x^{2}\right)^{p} AppellF1\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a}x^{2}, -e^{2a}x^{2}\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + Log[x]]^p, x]$ 

## Problem 170: Unable to integrate problem.

$$\int Tanh [a + 2 Log [x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \, \left( 1 - \mathrm{e}^{2\,\mathsf{a}} \, \, x^4 \right)^{\,-p} \, \left( -\,1 \,+\, \mathrm{e}^{2\,\mathsf{a}} \, \, x^4 \right)^{\,p} \, \mathsf{AppellF1} \big[ \, \frac{1}{4} \,\text{, -p, p, } \, \frac{5}{4} \,\text{, } \, \mathrm{e}^{2\,\mathsf{a}} \, \, x^4 \,\text{, -e}^{2\,\mathsf{a}} \, \, x^4 \, \big]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate Tanh a + 2 Log [x] ] p, x

## Problem 171: Unable to integrate problem.

Tanh [a + 3 Log [x]] 
$$^p$$
 dx

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(1-e^{2a} x^{6}\right)^{-p} \left(-1+e^{2a} x^{6}\right)^{p} AppellF1\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^{6}, -e^{2a} x^{6}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh[a+3Log[x]]^p, x]$ 

## Problem 172: Unable to integrate problem.

$$\left\lceil x^3 \, \mathsf{Tanh} \left[ d \, \left( a + b \, \mathsf{Log} \left[ c \, \, x^n \, \right] \right) \, \right] \, \mathrm{d} x \right.$$

Optimal (type 5, 59 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4$$
 Hypergeometric2F1[1,  $\frac{2}{b d n}$ ,  $1 + \frac{2}{b d n}$ ,  $-e^{2ad} (c x^n)^{2bd}$ ]

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [d (a + b Log [c x^n])], x]$ 

## Problem 173: Unable to integrate problem.

$$\int x^2 \, \mathsf{Tanh} \big[ \, \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \big[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \big] \, \right) \, \big] \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3}x^3$$
 Hypergeometric2F1[1,  $\frac{3}{2 \, b \, d \, n}$ ,  $1 + \frac{3}{2 \, b \, d \, n}$ ,  $-e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}$ ]

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [d (a + b Log [c x^n])], x]$ 

## Problem 174: Unable to integrate problem.

$$\left\lceil x \, \mathsf{Tanh} \left[ \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \, \mathsf{x}^{\mathsf{n}} \right] \right) \, \right] \, \mathrm{d} x \right.$$

Optimal (type 5, 55 leaves, 4 steps):

$$\frac{x^2}{2}$$
 -  $x^2$  Hypergeometric2F1  $\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, -e^{2ad} (c x^n)^{2bd}\right]$ 

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x Tanh [d (a + b Log [c x^n])], x]$ 

## Problem 175: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{d} \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^{\mathsf{n}} \right] \right) \right] \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 53 leaves, 4 steps):

$$x - 2 x Hypergeometric 2F1 [1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, -e^{2 a d} (c x^n)^{2 b d}]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[Tanh[d(a+bLog[cx^n])], x]$ 

## Problem 177: Unable to integrate problem.

$$\int\! \frac{\mathsf{Tanh} \left[ \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \, [\, \mathsf{c} \, \, \mathsf{x}^n \,] \, \right) \, \right]}{\mathsf{x}^2} \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 59 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \text{ Hypergeometric} 2F1 \left[1, -\frac{1}{2 \text{ bdn}}, 1 - \frac{1}{2 \text{ bdn}}, -e^{2 \text{ ad}} \left(c x^n\right)^{2 \text{ bd}}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Tanh}\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]}{x^{2}},x\right]$$

## Problem 178: Unable to integrate problem.

$$\int \frac{Tanh \left[d \left(a + b Log \left[c x^{n}\right]\right)\right]}{x^{3}} dx$$

Optimal (type 5, 56 leaves, 4 steps):

$$-\frac{1}{2 x^2} + \frac{\text{Hypergeometric2F1} \left[1, -\frac{1}{b d n}, 1 - \frac{1}{b d n}, -e^{2 a d} \left(c x^n\right)^{2 b d}\right]}{x^2}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]}{x^{3}},x\right]$$

## Problem 179: Unable to integrate problem.

$$\left\lceil x^3 \, \mathsf{Tanh} \left[ \, \mathsf{d} \, \left( \, \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, x^n \, \right] \, \right) \, \right]^2 \, \mathrm{d} x \right.$$

Optimal (type 5, 133 leaves, 5 steps):

$$\frac{1}{4} \left( 1 + \frac{4}{b \, d \, n} \right) \, x^4 + \frac{x^4 \, \left( 1 - \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)}{b \, d \, n \, \left( 1 + \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)} - \frac{2 \, x^4 \, \text{Hypergeometric2F1} \left[ 1, \, \frac{2}{b \, d \, n}, \, 1 + \frac{2}{b \, d \, n}, \, 1 - \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right]}{b \, d \, n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $\begin{bmatrix} x^3 & Tanh \\ d & (a + b Log \\ c & x^n \end{bmatrix}) \end{bmatrix}^2$ ,  $x \end{bmatrix}$ 

## Problem 180: Unable to integrate problem.

$$\left\lceil x^2 \, \mathsf{Tanh} \left[ \, \mathsf{d} \, \left( \, \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, x^n \, \right] \, \right) \, \right]^2 \, \mathrm{d} x \right.$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{1}{3} \left(1 + \frac{3}{b \, d \, n}\right) \, x^3 \, + \, \frac{x^3 \, \left(1 - \text{e}^{2 \, \text{a} \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right)}{b \, d \, n \, \left(1 + \text{e}^{2 \, \text{a} \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right)} \, - \, \frac{2 \, x^3 \, \text{Hypergeometric2F1} \left[1, \, \frac{3}{2 \, b \, d \, n}, \, 1 + \frac{3}{2 \, b \, d \, n}, \, - \, \text{e}^{2 \, \text{a} \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right]}{b \, d \, n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [d (a + b Log [c x^n])]^2, x]$ 

## Problem 181: Unable to integrate problem.

$$\int x Tanh \left[d \left(a + b Log \left[c x^{n}\right]\right)\right]^{2} dx$$

Optimal (type 5, 131 leaves, 5 steps):

$$\frac{1}{2} \left( 1 + \frac{2}{b \, d \, n} \right) \, x^2 \, + \, \frac{x^2 \, \left( 1 - e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)}{b \, d \, n \, \left( 1 + e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)} \, - \, \frac{2 \, x^2 \, \text{Hypergeometric2F1} \left[ 1, \, \frac{1}{b \, d \, n}, \, 1 + \frac{1}{b \, d \, n}, \, 1 + \frac{1}{b \, d \, n}, \, -e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right]}{b \, d \, n}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $\left[x Tanh \left[d \left(a + b Log \left[c x^{n}\right]\right)\right]^{2}, x\right]$ 

## Problem 182: Unable to integrate problem.

$$\bigg\lceil \text{Tanh} \left[ \text{d} \, \left( \text{a} + \text{b} \, \text{Log} \left[ \text{c} \, \, x^{\text{n}} \, \right] \right) \, \right]^2 \, \text{d} \, x$$

Optimal (type 5, 127 leaves, 5 steps):

$$\left(1+\frac{1}{b\,d\,n}\right)\,x+\frac{x\,\left(1-{\,\mathrm{e}^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}\right)}{b\,d\,n\,\left(1+{\,\mathrm{e}^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}\right)}-\frac{2\,x\,\text{Hypergeometric} 2\text{F1}\!\left[1,\,\frac{1}{2\,b\,d\,n},\,1+\frac{1}{2\,b\,d\,n},\,-{\,\mathrm{e}^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}\right]}{b\,d\,n}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[Tanh [d (a + b Log [c x^n])]^2, x]$ 

## Problem 184: Unable to integrate problem.

$$\int \frac{Tanh \left[d \left(a + b Log \left[c x^{n}\right]\right)\right]^{2}}{x^{2}} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$-\frac{1-\frac{1}{b\,d\,n}}{x}+\frac{1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}{b\,d\,n\,x\,\left(1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,Hypergeometric2F1\!\left[1,\,-\frac{1}{2\,b\,d\,n},\,1-\frac{1}{2\,b\,d\,n},\,1-\frac{e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n\,x}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{2}},x\right]$$

#### Problem 185: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{d}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\mathsf{x}^{\mathsf{n}}\right]\right)\right]^{2}}{\mathsf{x}^{\mathsf{3}}}\,\mathrm{d}\mathsf{x}$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{2 - b \, d \, n}{2 \, b \, d \, n \, x^{2}} + \frac{1 - e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}}{b \, d \, n \, x^{2} \, \left(1 + e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right)} - \frac{2 \, \text{Hypergeometric2F1} \left[1, \, -\frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, -e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right]}{b \, d \, n \, x^{2}}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{3}},x\right]$$

## Problem 189: Unable to integrate problem.

$$\int (e x)^m Tanh [d (a + b Log[c x^n])] dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} - \frac{2 \left(\text{e x}\right)^{\text{1+m}} \text{Hypergeometric2F1}\left[\text{1, } \frac{\text{1+m}}{\text{2bdn}}, \text{1} + \frac{\text{1+m}}{\text{2bdn}}, \text{-} \text{e}^{\text{2ad}} \left(\text{c } \text{x}^{\text{n}}\right)^{\text{2bd}}\right]}{\text{e }\left(\text{1+m}\right)}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [d(a+bLog[cx^n])], x]$ 

## Problem 190: Unable to integrate problem.

$$\int \left(e\,x\right)^m \, \mathsf{Tanh}\left[d\,\left(a+b\,\mathsf{Log}\left[c\,\,x^n\,\right]\right)\,\right]^2 \, \mathrm{d}x$$

Optimal (type 5, 169 leaves, 5 steps):

$$\frac{\left(1+\textit{m}+\textit{b}\,\textit{d}\,\textit{n}\right)\,\,\left(\textit{e}\,\textit{x}\right)^{\,1+\textit{m}}}{\textit{b}\,\textit{d}\,\textit{e}\,\left(1+\textit{m}\right)\,\textit{n}}+\\ \\ \frac{\left(\textit{e}\,\textit{x}\right)^{\,1+\textit{m}}\,\left(1-\textit{e}^{\,2\,\textit{a}\,\textit{d}}\,\left(\textit{c}\,\textit{x}^{\textit{n}}\right)^{\,2\,\textit{b}\,\textit{d}}\right)}{\textit{b}\,\textit{d}\,\textit{e}\,\textit{n}\,\left(1+\textit{e}^{\,2\,\textit{a}\,\textit{d}}\,\left(\textit{c}\,\textit{x}^{\textit{n}}\right)^{\,2\,\textit{b}\,\textit{d}}\right)}-\\ \\ \frac{2\,\,\left(\textit{e}\,\textit{x}\right)^{\,1+\textit{m}}\,\textit{Hypergeometric}2\textit{F1}\left[\,\textbf{1},\,\,\frac{1+\textit{m}}{2\,\textit{b}\,\textit{d}\,\textit{n}}\,,\,\,1+\frac{1+\textit{m}}{2\,\textit{b}\,\textit{d}\,\textit{n}}\,,\,\,-\textit{e}^{\,2\,\textit{a}\,\textit{d}}\,\left(\textit{c}\,\textit{x}^{\textit{n}}\right)^{\,2\,\textit{b}\,\textit{d}}\right)}{\textit{b}\,\textit{d}\,\textit{e}\,\textit{n}} \\ \\ \\ \textit{b}\,\textit{d}\,\textit{e}\,\textit{n}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh[d(a+bLog[cx^n])]^2, x]$ 

#### Problem 191: Unable to integrate problem.

$$\left\lceil \left(\,e\,x\,\right)^{\,m}\,\mathsf{Tanh}\left[\,d\,\left(\,a\,+\,b\,\mathsf{Log}\left[\,c\,\,x^{n}\,\right]\,\right)\,\right]^{\,3}\,\,\mathrm{d}x\right.$$

Optimal (type 5, 307 leaves, 6 steps):

$$\frac{\left(1+\text{m}+\text{b}\,\text{d}\,\text{n}\right)\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}}{2\,\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}}-\frac{\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{2\,\text{b}\,\text{d}}\right)^{2}}{2\,\text{b}\,\text{d}\,\text{e}\,\text{n}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{2\,\text{b}\,\text{d}}\right)^{2}}+\frac{\text{e}^{-2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(1+\text{m}-2\,\text{b}\,\text{d}\,\text{n}\right)}{\text{n}}-\frac{\text{e}^{4\,\text{a}\,\text{d}}\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{2\,\text{b}\,\text{d}}}{\text{n}}\right)}{2\,\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\text{n}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{2\,\text{b}\,\text{d}}\right)}-\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\text{e}\,\text{e}\,\text{e}^{2\,\text{e}\,\text{d}}\,\left(\text{e}\,\text{e}\,\text{x}\right)^{1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{e}\,\text{d}}\,\left(\text{e}\,\text{e}\,\text{e}\,\text{e}^{2\,\text{e}\,\text{d}}\,\left(\text{e}\,\text{e}\,\text{e}\,\text{e}^{2\,\text{e}\,\text{e}\,\text{d}}\,\left(\text{e}\,\text{e}\,\text{e}\,\text{e}^{2\,\text{e}\,\text{e}\,\text{e}\,\text{e}\,\text{e}^{2}\,\text{e}\,\text{e}\,\text{e}^{2}\,\text{e}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}\,\text{e}^{2}\,\text{$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh[d(a+bLog[cx^n])]^3$ , x]

## Problem 192: Unable to integrate problem.

$$\bigg\lceil \text{Tanh} \left[ \text{d} \, \left( \text{a} + \text{b} \, \text{Log} \left[ \, \text{c} \, \, \text{x}^{\text{n}} \, \right] \, \right) \, \right]^{\text{p}} \, \text{d} \, \text{x}$$

Optimal (type 6, 115 leaves, 4 steps):

$$x \left(1 - e^{2ad} (c x^n)^{2bd}\right)^{-p} \left(-1 + e^{2ad} (c x^n)^{2bd}\right)^{p} AppellF1 \left[\frac{1}{2bdn}, -p, p, 1 + \frac{1}{2bdn}, e^{2ad} (c x^n)^{2bd}, -e^{2ad} (c x^n)^{2bd}\right]$$

Result (type 8, 17 leaves, 0 steps):

 $CannotIntegrate \big[ Tanh \big[ d \left( a + b Log \left[ c \ x^n \right] \right) \big]^p \text{, } x \big]$ 

## Problem 193: Unable to integrate problem.

$$\left[\left.\left(\,e\,x\right)\,^{m}\,\mathsf{Tanh}\left[\,d\,\left(\,a\,+\,b\,\mathsf{Log}\left[\,c\,\,x^{n}\,\right]\,\right)\,\right]^{\,p}\,\mathrm{d}x\right.$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e\,\left(1+m\right)}\,(e\,x)^{\,1+m}\,\left(1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)^{-p}\,\left(-1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)^{p}\,AppellF1\Big[\,\frac{1+m}{2\,b\,d\,n},\,-p,\,p,\,1+\frac{1+m}{2\,b\,d\,n},\,\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $\left[ \ (e\ x)^{\ m}\ Tanh \left[ \ d\ \left( a\ +\ b\ Log \left[ \ c\ x^n\ \right] \ \right)\ \right]^p$ ,  $x \right]$ 

## Test results for the 61 problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

Test results for the 53 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Test results for the 224 problems in "6.4.2 Hyperbolic cotangent functions.m"

## Problem 151: Unable to integrate problem.

$$\int x^3 \operatorname{Coth}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 30 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{1}{2} e^{-2a} Log \left[1 - e^{2a} x^4\right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [a + 2 \text{ Log } [x]], x]$ 

## Problem 152: Unable to integrate problem.

$$\int x^2 \, \text{Coth} \, [\, a + 2 \, \text{Log} \, [\, x \, ] \, ] \, \, \text{d} \, x$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{x^3}{3} + e^{-3 a/2} \operatorname{ArcTan} \left[ e^{a/2} x \right] - e^{-3 a/2} \operatorname{ArcTanh} \left[ e^{a/2} x \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Coth } [a + 2 \text{ Log } [x]], x]$ 

## Problem 153: Unable to integrate problem.

$$\int x \operatorname{Coth}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTanh} \left[ e^a x^2 \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[x Coth[a + 2 Log[x]], x]

## Problem 154: Unable to integrate problem.

$$\int Coth[a + 2 Log[x]] dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$x-\operatorname{e}^{-a/2}\operatorname{ArcTan}\left[\operatorname{e}^{a/2}x\right]-\operatorname{e}^{-a/2}\operatorname{ArcTanh}\left[\operatorname{e}^{a/2}x\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate[Coth[a + 2 Log[x]], x]

## Problem 156: Unable to integrate problem.

$$\int \frac{\text{Coth}\,[\,a+2\,\text{Log}\,[\,x\,]\,\,]}{x^2}\,\,\text{d}\,x$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{1}{x} + e^{a/2} \operatorname{ArcTan} \left[ \, e^{a/2} \, \, x \, \right] \, - \, e^{a/2} \operatorname{ArcTanh} \left[ \, e^{a/2} \, \, x \, \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}[a+2 Log[x]]}{x^2}, x\right]$$

## Problem 157: Unable to integrate problem.

$$\int \frac{\text{Coth}[a+2 \, \text{Log}[x]]}{x^3} \, dx$$

Optimal (type 3, 21 leaves, 4 steps):

$$\frac{1}{2 x^2} - e^a \operatorname{ArcTanh} \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

$$\label{eq:cannotIntegrate} CannotIntegrate \Big[ \, \frac{Coth \, [\, a \, + \, 2 \, Log \, [\, x \, ] \, ]}{x^3} \, , \, \, x \, \Big]$$

## Problem 158: Unable to integrate problem.

$$\int x^3 \, \mathsf{Coth} \, [\, \mathsf{a} + \mathsf{2} \, \mathsf{Log} \, [\, \mathsf{x} \,] \,]^{\, \mathsf{2}} \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{e^{-2a}}{1 - e^{2a}x^4} + e^{-2a} Log [1 - e^{2a}x^4]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$[x^3 \text{ Coth } [a + 2 \text{ Log } [x]]^2, x]$$

## Problem 159: Unable to integrate problem.

$$\int x^2 \, \text{Coth} \, [\, a + 2 \, \text{Log} \, [\, x \,] \,\,]^{\,2} \, \, \text{d} \, x$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 - e^{2\,a}\,x^4} + \frac{3}{2}\,e^{-3\,a/2}\,\text{ArcTan}\!\left[\,e^{a/2}\,x\,\right] - \frac{3}{2}\,e^{-3\,a/2}\,\text{ArcTanh}\!\left[\,e^{a/2}\,x\,\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Coth } [a + 2 \text{ Log } [x]]^2, x]$ 

## Problem 160: Unable to integrate problem.

$$\int x \operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 - e^{2a} x^4} - e^{-a} \operatorname{ArcTanh} \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate [x Coth[a + 2 Log[x]]<sup>2</sup>, x]

## Problem 161: Unable to integrate problem.

Optimal (type 3, 60 leaves, 7 steps):

$$\mathsf{X} + \frac{\mathsf{X}}{1 - \mathsf{e}^{2\,\mathsf{a}}\,\mathsf{X}^4} - \frac{1}{2}\,\,\mathsf{e}^{-\mathsf{a}/2}\,\mathsf{ArcTan}\!\left[\,\mathsf{e}^{\mathsf{a}/2}\,\mathsf{X}\,\right] - \frac{1}{2}\,\,\mathsf{e}^{-\mathsf{a}/2}\,\mathsf{ArcTanh}\!\left[\,\mathsf{e}^{\mathsf{a}/2}\,\mathsf{X}\,\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate [Coth[a + 2 Log[x]]<sup>2</sup>, x]

## Problem 163: Unable to integrate problem.

$$\int \frac{\operatorname{Coth}\left[\mathsf{a} + 2\operatorname{Log}\left[\mathsf{x}\right]\right]^{2}}{\mathsf{x}^{2}} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 86 leaves, 6 steps):

$$-\frac{1}{x\,\left(1-{\,{\rm e}^{2\,a}\,x^4}\right)}\,+\,\frac{2\,{\,{\rm e}^{2\,a}\,x^3}}{1-{\,{\rm e}^{2\,a}\,x^4}}\,-\,\frac{1}{2}\,{\,{\rm e}^{a/2}\,{\rm ArcTan}}\Big[\,{\,{\rm e}^{a/2}\,x}\,\Big]\,+\,\frac{1}{2}\,{\,{\rm e}^{a/2}\,{\rm ArcTanh}}\Big[\,{\,{\rm e}^{a/2}\,x}\,\Big]$$

Result (type 8, 15 leaves, 0 steps):

$$\label{eq:cannotIntegrate} CannotIntegrate \Big[ \frac{Coth \left[ a + 2 \, Log \left[ x \right] \, \right]^2}{x^2} \text{, } x \Big]$$

## Problem 164: Unable to integrate problem.

$$\int \frac{\text{Coth}\left[\,a\,+\,2\,\,\text{Log}\left[\,x\,\right]\,\,\right]^{\,2}}{x^{3}}\,\,\text{d}\,x$$

Optimal (type 3, 60 leaves, 5 steps):

$$-\,\frac{1}{2\,{{x}^{2}\,\left( 1-{{e}^{2\,a}\,{{x}^{4}}} \right)}}\,+\,\frac{3\,{{e}^{2\,a}\,{{x}^{2}}}}{2\,\left( 1-{{e}^{2\,a}\,{{x}^{4}}} \right)}\,+\,{{e}^{a}}\,\text{ArcTanh}\left[ \,{{e}^{a}\,{{x}^{2}}} \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}[a+2\log[x]]^2}{x^3}, x\right]$$

## Problem 165: Unable to integrate problem.

$$\int (ex)^m Coth[a+2 Log[x]] dx$$

Optimal (type 5, 59 leaves, 3 steps):

$$\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1+m}\right)}\,-\,\frac{2\,\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}\,\text{Hypergeometric}2\text{F1}\!\left[\text{1,}\,\,\frac{1+\text{m}}{4}\,,\,\,\frac{5+\text{m}}{4}\,,\,\,\text{e}^{2\,\text{a}}\,\text{x}^4\right]}{\text{e}\,\left(\text{1+m}\right)}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$[(ex)^m Coth[a + 2 Log[x]], x]$$

## Problem 166: Unable to integrate problem.

$$\int (e x)^m Coth[a + 2 Log[x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} + \frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1}-\text{e}^{\text{2a}}\,\text{x}^{\text{4}}\right)} - \frac{\left(\text{e x}\right)^{\text{1+m}}\,\text{Hypergeometric2F1}\left[\text{1,}\,\frac{\frac{1+m}{4}}{\text{,}},\,\frac{\frac{5+m}{4}}{\text{,}},\,\text{e}^{\text{2a}}\,\text{x}^{\text{4}}\right]}{\text{e}}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth [a + 2 Log [x]]^2, x]$ 

## Problem 167: Unable to integrate problem.

$$\int (e x)^m Coth[a + 2 Log[x]]^3 dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{ \left(3+m\right) \; \left(5+m\right) \; \left(e\,x\right)^{\,1+m}}{8 \, e \; \left(1+m\right)} - \frac{\left(e\,x\right)^{\,1+m} \; \left(1+e^{2\,a}\,x^4\right)^{\,2}}{4 \, e \; \left(1-e^{2\,a}\,x^4\right)^{\,2}} - \\ \frac{e^{-2\,a} \; \left(e\,x\right)^{\,1+m} \; \left(e^{2\,a} \; \left(3-m\right) - e^{4\,a} \; \left(5+m\right) \; x^4\right)}{8 \, e \; \left(1-e^{2\,a}\,x^4\right)} - \frac{\left(9+2\,m+m^2\right) \; \left(e\,x\right)^{\,1+m} \; \text{Hypergeometric2F1} \left[1, \; \frac{1+m}{4}, \; \frac{5+m}{4}, \; e^{2\,a} \; x^4\right]}{4 \, e \; \left(1+m\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth[a+2 Log[x]]^3, x]$ 

## Problem 168: Unable to integrate problem.

$$\int Coth [a + b Log[x]]^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left(-1 - e^{2a} x^{2b}\right)^{p} \left(1 + e^{2a} x^{2b}\right)^{-p} AppellF1\left[\frac{1}{2b}, p, -p, \frac{1}{2}\left(2 + \frac{1}{b}\right), e^{2a} x^{2b}, -e^{2a} x^{2b}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + b \text{Log} \left[ x \right] \right]^p, x \right]$ 

## Problem 169: Unable to integrate problem.

$$(e x)^m Coth[a + b Log[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}\left(-\text{1}-\text{e}^{\text{2 a }}\text{x}^{\text{2 b}}\right)^{\text{p}}\left(\text{1}+\text{e}^{\text{2 a }}\text{x}^{\text{2 b}}\right)^{-\text{p}}\text{AppellF1}\left[\frac{\text{1+m}}{\text{2 b}},\text{ p, -p, 1}+\frac{\text{1+m}}{\text{2 b}},\text{ e}^{\text{2 a }}\text{x}^{\text{2 b}},\text{ -e}^{\text{2 a }}\text{x}^{\text{2 b}}\right]}{\text{e }\left(\text{1}+\text{m}\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth[a+b Log[x]]^p, x]$ 

## Problem 170: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{2}\right]^p dx$$

Optimal (type 5, 52 leaves, 2 steps):

$$-\frac{2^{-p}\,\,\mathbb{e}^{-2\,\mathsf{a}}\,\left(-\,\mathsf{1}\,-\,\,\mathbb{e}^{2\,\mathsf{a}}\,\,\mathsf{x}\right)^{\,\mathsf{1}\,+\,\mathsf{p}}\,\,\mathsf{Hypergeometric}\,\mathsf{2F1}\left[\,\mathsf{p,\,\,1}\,+\,\mathsf{p,\,\,2}\,+\,\mathsf{p,\,\,}\frac{\,\mathsf{1}}{\,\mathsf{2}}\,\left(\,\mathsf{1}\,+\,\,\mathbb{e}^{2\,\mathsf{a}}\,\,\mathsf{x}\,\right)\,\,\right]}{\,\mathsf{1}\,+\,\mathsf{p}}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ \text{Coth} \left[ \frac{1}{2} \left( 2 \text{ a} + \text{Log} \left[ x \right] \right) \right]^p, x \right]$ 

## Problem 171: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{4}\right]^p dx$$

Optimal (type 5, 108 leaves, 4 steps):

$$e^{-4\,a} \, \left( -\,1 \,-\, e^{2\,a} \, \sqrt{x} \, \right)^{1+p} \, \left( 1 \,-\, e^{2\,a} \, \sqrt{x} \, \right)^{1-p} \,-\, \frac{2^{1-p} \, e^{-4\,a} \, p \, \left( -\,1 \,-\, e^{2\,a} \, \sqrt{x} \, \right)^{1+p} \, \text{Hypergeometric2F1} \big[ \, p, \, 1 \,+\, p, \, 2 \,+\, p, \, \frac{1}{2} \, \left( 1 \,+\, e^{2\,a} \, \sqrt{x} \, \right) \, \big] }{1 \,+\, p} \, \left( 1 \,+\, e^{2\,a} \, \sqrt{x} \, \right) \, \left( 1 \,+\, e^$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ \text{Coth} \left[ \frac{1}{4} (4 \text{ a} + \text{Log}[x]) \right]^p, x \right]$ 

## Problem 172: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{6}\right]^p dx$$

Optimal (type 5, 162 leaves, 5 steps):

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ \text{Coth} \left[ \frac{1}{c} \left( 6 \text{ a} + \text{Log} [x] \right) \right]^p, x \right]$ 

## Problem 173: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{8}\right]^p dx$$

Optimal (type 5, 194 leaves, 5 steps):

$$\frac{1}{3}\,\,\mathrm{e}^{-12\,\mathsf{a}}\,\left(-\,1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)^{\,1+\mathsf{p}}\,\left(1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)^{\,1-\mathsf{p}}\,\left(\mathrm{e}^{4\,\mathsf{a}}\,\left(3\,+\,2\,\,\mathsf{p}^2\right)\,+\,2\,\,\mathrm{e}^{6\,\mathsf{a}}\,\,\mathsf{p}\,\,x^{1/4}\right)\,+\,\mathrm{e}^{-4\,\mathsf{a}}\,\left(-\,1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)^{\,1+\mathsf{p}}\,\left(1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)^{\,1-\mathsf{p}}\,\sqrt{x}\,\,-\,2^{\,\mathsf{p}}\,\,\mathrm{e}^{-8\,\mathsf{a}}\,\,\mathsf{p}\,\left(2\,+\,\mathsf{p}^2\right)\,\left(-\,1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)^{\,1+\mathsf{p}}\,\,\mathsf{Hypergeometric} 2\mathsf{F1}\!\left[\,\mathsf{p}\,,\,1\,+\,\mathsf{p}\,,\,2\,+\,\mathsf{p}\,,\,\,\frac{1}{2}\,\left(1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,x^{1/4}\right)\,\right]}{3\,\left(1\,+\,\mathsf{p}\right)}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ \text{Coth} \left[ \frac{1}{8} \left( 8 \text{ a} + \text{Log} \left[ x \right] \right) \right]^{p}, x \right]$ 

#### Problem 174: Unable to integrate problem.

$$\int Coth [a + Log[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \, \left( -1 - \operatorname{e}^{2\,a} \, x^2 \right)^p \, \left( 1 + \operatorname{e}^{2\,a} \, x^2 \right)^{-p} \, \text{AppellF1} \left[ \, \frac{1}{2} \text{, p, -p, } \, \frac{3}{2} \text{, } \operatorname{e}^{2\,a} \, x^2 \text{, -e}^{2\,a} \, x^2 \right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + \text{Log} \left[ x \right] \right]^p, x \right]$ 

## Problem 175: Unable to integrate problem.

$$\int Coth[a + 2 Log[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \, \left( -1 - \operatorname{e}^{2\,a} \, x^4 \right)^p \, \left( 1 + \operatorname{e}^{2\,a} \, x^4 \right)^{-p} \, \text{AppellF1} \left[ \, \frac{1}{4} \text{, p, -p, } \, \frac{5}{4} \text{, } \operatorname{e}^{2\,a} \, x^4 \, \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + 2 \text{Log} \left[ x \right] \right]^p, x \right]$ 

## Problem 176: Unable to integrate problem.

$$\int Coth [a + 3 Log [x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(-1 - e^{2a} x^{6}\right)^{p} \left(1 + e^{2a} x^{6}\right)^{-p} AppellF1\left[\frac{1}{6}, p, -p, \frac{7}{6}, e^{2a} x^{6}, -e^{2a} x^{6}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + 3 \log \left[ x \right] \right]^p, x \right]$ 

## Problem 177: Unable to integrate problem.

Optimal (type 5, 58 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4$$
 Hypergeometric2F1  $\left[1, \frac{2}{b \, d \, n}, 1 + \frac{2}{b \, d \, n}, e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right]$ 

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [d (a + b \text{ Log } [c x^n])], x]$ 

## Problem 178: Unable to integrate problem.

$$\int x^2 \, \mathsf{Coth} \big[ \, d \, \left( \, \mathsf{a} \, + \, \mathsf{b} \, \mathsf{Log} \big[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \big] \, \right) \, \big] \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3} x^3$$
 Hypergeometric2F1[1,  $\frac{3}{2 b d n}$ ,  $1 + \frac{3}{2 b d n}$ ,  $e^{2 a d} (c x^n)^{2 b d}$ ]

Result (type 8, 19 leaves, 0 steps):

## Problem 179: Unable to integrate problem.

$$\left\lceil x \, \mathsf{Coth} \left[ \, d \, \left( a + b \, \mathsf{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right] \, \mathbb{d} \, x \right.$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{x^2}{2}$$
 -  $x^2$  Hypergeometric2F1[1,  $\frac{1}{b d n}$ ,  $1 + \frac{1}{b d n}$ ,  $e^{2 a d} (c x^n)^{2 b d}$ ]

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x Coth[d(a+b Log[cx^n])], x]$ 

## Problem 180: Unable to integrate problem.

Optimal (type 5, 52 leaves, 4 steps):

$$x - 2x$$
 Hypergeometric2F1  $\left[1, \frac{1}{2 b d n}, 1 + \frac{1}{2 b d n}, e^{2 a d} (c x^n)^{2 b d}\right]$ 

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right] \right], x \right]$ 

## Problem 182: Unable to integrate problem.

$$\int \frac{\mathsf{Coth} \left[ d \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{c} \, \mathsf{x}^{\mathsf{n}} \right] \right) \right]}{\mathsf{x}^{\mathsf{2}}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 58 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \text{ Hypergeometric} 2F1 \left[1, -\frac{1}{2 \text{ bdn}}, 1 - \frac{1}{2 \text{ bdn}}, e^{2 \text{ ad}} \left(c x^{n}\right)^{2 \text{ bd}}\right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

$$CannotIntegrate \Big[ \, \frac{Coth \big[ d \, \big( a + b \, Log \, [ \, c \, \, x^n \, ] \, \big) \, \Big]}{x^2} \, \text{, } x \, \Big]$$

## Problem 183: Unable to integrate problem.

$$\int \frac{\mathsf{Coth} \left[ d \left( a + b \mathsf{Log} \left[ c x^n \right] \right) \right]}{x^3} \, dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{1}{2 x^2} + \frac{\text{Hypergeometric2F1} \left[1, -\frac{1}{b d n}, 1 - \frac{1}{b d n}, e^{2 a d} \left(c x^n\right)^{2 b d}\right]}{x^2}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]}{x^{3}},x\right]$$

## Problem 184: Unable to integrate problem.

$$\left\lceil x^3 \, \text{Coth} \left[ \, d \, \left( \, a + b \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^2 \, \mathrm{d} x \right.$$

Optimal (type 5, 132 leaves, 5 steps):

$$\frac{1}{4} \left(1 + \frac{4}{b \, d \, n}\right) \, x^4 + \frac{x^4 \, \left(1 + e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right)}{b \, d \, n \, \left(1 - e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right)} - \frac{2 \, x^4 \, \text{Hypergeometric2F1} \left[1, \, \frac{2}{b \, d \, n}, \, 1 + \frac{2}{b \, d \, n}, \, e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right]}{b \, d \, n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [d (a + b \text{ Log } [c x^n])]^2, x]$ 

#### Problem 185: Unable to integrate problem.

$$\int \! x^2 \, \text{Coth} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^2 \, \mathrm{d} x$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{1}{3} \left( 1 + \frac{3}{b \, d \, n} \right) \, x^3 \, + \, \frac{x^3 \, \left( 1 + e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)}{b \, d \, n \, \left( 1 - e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)} - \frac{2 \, x^3 \, \text{Hypergeometric2F1} \left[ 1, \, \frac{3}{2 \, b \, d \, n}, \, 1 + \frac{3}{2 \, b \, d \, n}, \, e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right]}{b \, d \, n}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[ x^{2} \operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c \ x^{n} \right] \right) \right]^{2}$$
,  $x \right]$ 

## Problem 186: Unable to integrate problem.

$$\left\lceil x \, \text{Coth} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^{\, 2} \, \mathrm{d} x \right.$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{1}{2}\left(1+\frac{2}{b\,d\,n}\right)\,x^2+\frac{x^2\,\left(1+e^{2\,a\,d}\,\left(c\,x^n\right)^{2\,b\,d}\right)}{b\,d\,n\,\left(1-e^{2\,a\,d}\,\left(c\,x^n\right)^{2\,b\,d}\right)}-\frac{2\,x^2\,\text{Hypergeometric2F1}\!\left[1,\,\frac{1}{b\,d\,n},\,1+\frac{1}{b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^n\right)^{2\,b\,d}\right]}{b\,d\,n}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x Coth [d (a + b Log [c x^n])]^2$ , x]

## Problem 187: Unable to integrate problem.

Optimal (type 5, 126 leaves, 5 steps):

$$\left(1+\frac{1}{b\,d\,n}\right)\,x+\frac{x\,\left(1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}{b\,d\,n\,\left(1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,x\,\text{Hypergeometric2F1}\!\left[1,\,\frac{1}{\,2\,b\,d\,n},\,1+\frac{1}{\,2\,b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right]^2, x \right]$ 

#### Problem 189: Unable to integrate problem.

$$\int \frac{\operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c x^{n} \right] \right) \right]^{2}}{x^{2}} dx$$

Optimal (type 5, 134 leaves, 5 steps):

$$-\frac{1-\frac{1}{b\,d\,n}}{x}+\frac{1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}{b\,d\,n\,x\,\left(1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,\text{Hypergeometric2F1}\!\left[1,-\frac{1}{2\,b\,d\,n},\,1-\frac{1}{2\,b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n\,x}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]^{2}}{x^{2}}, x\right]$$

## Problem 190: Unable to integrate problem.

$$\int \frac{\text{Coth} \left[d \left(a + b \text{Log} \left[c x^{n}\right]\right)\right]^{2}}{x^{3}} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$\frac{2 - b \, d \, n}{2 \, b \, d \, n \, x^{2}} + \frac{1 + e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}}{b \, d \, n \, x^{2} \, \left(1 - e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right)} - \frac{2 \, \text{Hypergeometric2F1} \left[1, \, -\frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right]}{b \, d \, n \, x^{2}}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\log\left[cx^{n}\right]\right)\right]^{2}}{x^{3}},x\right]$$

## Problem 194: Unable to integrate problem.

$$\left\lceil \left( e \; x \right)^{\; m} \; \mathsf{Coth} \left[ \; d \; \left( \mathsf{a} \; + \; \mathsf{b} \; \mathsf{Log} \left[ \; c \; \; x^n \; \right] \; \right) \; \right] \; \mathrm{d} \! \mid \! x \right.$$

Optimal (type 5, 87 leaves, 4 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} = \frac{2 \ \left(\text{e x}\right)^{\text{1+m}} \, \text{Hypergeometric2F1}\left[\text{1, } \frac{\text{1+m}}{\text{2 b d n}}, \, \text{1} + \frac{\text{1+m}}{\text{2 b d n}}, \, \text{e}^{\text{2 a d }\left(\text{c } \, \text{x}^{\text{n}}\right)^{\text{2 b d }}}\right]}{\text{e }\left(\text{1+m}\right)}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$[(ex)^m Coth[d(a+b Log[cx^n])], x]$$

## Problem 195: Unable to integrate problem.

$$\left\lceil \left(e\,x\right)^{\,m}\, \text{Coth} \left[\,d\,\left(a+b\, \text{Log}\left[\,c\,\,x^{n}\,\right]\,\right)\,\right]^{\,2}\, \text{d}x\right.$$

Optimal (type 5, 168 leaves, 5 steps):

$$\frac{\left(1+\text{m}+\text{bdn}\right) \; \left(\text{ex}\right)^{1+\text{m}}}{\text{bde} \left(1+\text{m}\right) \; \text{n}} + \\ \frac{\left(\text{ex}\right)^{1+\text{m}} \left(1+\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right)}{\text{bden} \left(1-\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right)} - \\ \frac{2 \; \left(\text{ex}\right)^{1+\text{m}} \; \text{Hypergeometric} \\ \text{cx}^{\text{n}} \left(1+\frac{1+\text{m}}{2\,\text{bdn}}\right) \; \text{for } \\ \text{bden} \left(1-\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right)} - \\ \frac{2 \; \left(\text{ex}\right)^{1+\text{m}} \; \text{Hypergeometric} \\ \text{bden} \left(1+\frac{1+\text{m}}{2\,\text{bdn}}\right) \; \text{for } \\ \text{for } \\ \text{bden} \left(1+\frac{1+\text{m}}{2\,\text{bdn}}\right) \; \text{for } \\ \text$$

Result (type 8, 23 leaves, 0 steps):

$$CannotIntegrate \left[ \; (e \; x) \, ^m \, Coth \left[ \, d \, \left( a \, + \, b \, Log \left[ \, c \, \, x^n \, \right] \, \right) \, \right]^2 \text{, } x \, \right]$$

## Problem 196: Unable to integrate problem.

$$\left\lceil \left(\,e\,x\,\right)^{\,m}\,\text{Coth}\left[\,d\,\left(\,a\,+\,b\,\,\text{Log}\left[\,c\,\,x^{n}\,\right]\,\right)\,\right]^{\,3}\,\,\text{d}x\right.$$

Optimal (type 5, 306 leaves, 6 steps):

$$\frac{\left(1+\text{m}+\text{b}\,\text{d}\,\text{n}\right)\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(e\,x\right)^{\,1+\text{m}}}{2\,\,\text{b}^{2}\,\,\text{d}^{2}\,\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} - \frac{\left(e\,x\right)^{\,1+\text{m}}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}}{2\,\,\text{b}\,\text{d}\,\text{e}\,\text{n}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}} + \frac{\text{e}^{-2\,\text{a}\,\text{d}}\,\left(e\,x\right)^{\,1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(1+\text{m}-2\,\text{b}\,\text{d}\,\text{n}\right)}{\text{n}} + \frac{\text{e}^{4\,\text{a}\,\text{d}}\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{n}}}{2\,\,\text{b}\,\text{d}\,\text{e}\,\text{n}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)} \\ - \frac{\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(e\,x\right)^{\,1+\text{m}}\,\text{Hypergeometric}2\text{F1}\!\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,\frac{e^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}\right)} \\ - \frac{\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(e\,x\right)^{\,1+\text{m}}\,\text{Hypergeometric}2\text{F1}\!\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,\frac{e^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{b}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}\right)} \\ - \frac{\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(e\,x\right)^{\,1+\text{m}}\,\text{Hypergeometric}2\text{F1}\!\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,\frac{e^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{b}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}\right)} \\ - \frac{\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(e\,x\right)^{\,1+\text{m}}\,\text{Hypergeometric}2\text{F1}\!\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,\frac{e^{2\,\text{a}\,\text{d}}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{b}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\left(c\,x^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}\right)} \\ - \frac{\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\,\text{d}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{d}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^{2}\,\text{e}^$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth [d(a+b Log[cx^n])]^3$ , x]

## Problem 197: Unable to integrate problem.

Optimal (type 6, 115 leaves, 4 steps):

$$x \left( -1 - e^{2\,a\,d} \, \left( c \, \, x^n \right)^{\,2\,b\,d} \right)^p \, \left( 1 + e^{2\,a\,d} \, \left( c \, \, x^n \right)^{\,2\,b\,d} \right)^{\,-p} \\ AppellF1 \left[ \, \frac{1}{2\,b\,d\,n}, \, \, p_{\bullet} - p_{\bullet} \, \, 1 + \frac{1}{2\,b\,d\,n}, \, \, e^{2\,a\,d} \, \left( c \, \, x^n \right)^{\,2\,b\,d}, \, \, -e^{2\,a\,d} \, \left( c \, \, x^n \right)^{\,2\,b\,d} \right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[Coth[d(a+bLog[cx^n])]^p, x]$ 

## Problem 198: Unable to integrate problem.

$$\label{eq:continuous} \left[\,\left(\,e\;x\,\right)^{\,m}\;\text{Coth}\left[\,d\;\left(\,a\,+\,b\;\text{Log}\left[\,c\;x^{n}\,\right]\,\right)\,\,\right]^{\,p}\;\text{d}x$$

Optimal (type 6, 135 leaves, 4 steps):

$$\frac{1}{e\,\left(1+m\right)}\left(e\,x\right)^{\,1+m}\,\left(-\,1\,-\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)^{p}\,\left(1\,+\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)^{\,-p}\,AppellF1\left[\,\frac{1\,+\,m}{2\,b\,d\,n},\,p,\,-p,\,1\,+\,\frac{1\,+\,m}{2\,b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth [d(a+b Log[cx^n])]^p, x]$ 

Test results for the 16 problems in "6.5.1 (c+d x)^m (a+b sech)^n.m"

Test results for the 84 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

Test results for the 220 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

Test results for the 201 problems in "6.5.3 Hyperbolic secant functions.m"

Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\begin{split} &\int \left(\left(1-b^2\,n^2\right)\,\text{Sech}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]\,+2\,b^2\,n^2\,\text{Sech}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]^3\right)\,\mathrm{d}x\\ &\text{Optimal (type 3, }40\,\text{leaves, }?\text{ steps):}\\ &x\,\text{Sech}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]\,+b\,n\,x\,\text{Sech}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]\,\text{Tanh}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]\\ &\text{Result (type 5, }139\,\text{leaves, }9\text{ steps):}\\ &2\,e^a\,\left(1-b\,n\right)\,x\,\left(c\,\,x^n\right)^b\,\text{Hypergeometric2F1}\left[\,1,\,\,\frac{b+\frac{1}{n}}{2\,b}\,,\,\,\frac{1}{2}\,\left(3+\frac{1}{b\,n}\right)\,,\,\,-e^{2\,a}\,\left(c\,\,x^n\right)^{2\,b}\,\right]\,+\\ &\frac{16\,b^2\,e^{3\,a}\,n^2\,x\,\left(c\,\,x^n\right)^{3\,b}\,\text{Hypergeometric2F1}\left[\,3,\,\,\frac{3\,b+\frac{1}{n}}{2\,b}\,,\,\,\frac{1}{2}\,\left(5+\frac{1}{b\,n}\right)\,,\,\,-e^{2\,a}\,\left(c\,\,x^n\right)^{2\,b}\,\right]}{1+3\,b\,n} \end{split}$$

Test results for the 29 problems in "6.6.1 (c+d x)^m (a+b csch)^n.m"

Test results for the 83 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

Test results for the 27 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

Test results for the 175 problems in "6.6.3 Hyperbolic cosecant functions.m"

Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal

## antiderivative.

$$\int \left(-\left(1-b^2\,n^2\right)\,\operatorname{Csch}\left[a+b\,\operatorname{Log}\left[c\,x^n\right]\right]+2\,b^2\,n^2\,\operatorname{Csch}\left[a+b\,\operatorname{Log}\left[c\,x^n\right]\right]^3\right)\,\mathrm{d}x$$
 Optimal (type 3, 42 leaves, ? steps): 
$$-x\,\operatorname{Csch}\left[a+b\,\operatorname{Log}\left[c\,x^n\right]\right]-b\,n\,x\,\operatorname{Coth}\left[a+b\,\operatorname{Log}\left[c\,x^n\right]\right]\,\operatorname{Csch}\left[a+b\,\operatorname{Log}\left[c\,x^n\right]\right]$$
 Result (type 5, 137 leaves, 9 steps): 
$$2\,\mathrm{e}^a\,\left(1-b\,n\right)\,x\,\left(c\,x^n\right)^b\,\operatorname{Hypergeometric}2F1\left[1,\,\frac{b+\frac{1}{n}}{2\,b},\,\frac{1}{2}\left(3+\frac{1}{b\,n}\right),\,\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}\right] - \frac{16\,b^2\,\mathrm{e}^{3\,a}\,n^2\,x\,\left(c\,x^n\right)^{3\,b}\,\operatorname{Hypergeometric}2F1\left[3,\,\frac{3\,b+\frac{1}{n}}{2\,b},\,\frac{1}{2}\left(5+\frac{1}{b\,n}\right),\,\mathrm{e}^{2\,a}\,\left(c\,x^n\right)^{2\,b}\right] }{1+3\,b\,n}$$

Test results for the 1059 problems in "6.7.1 Hyperbolic functions.m"