Rubi 4.16.0 Independent Integration Test Results

Test results for the 175 problems in "Apostol Problems.m"

Test results for the 113 problems in "Moses Problems.m"

Test results for the 705 problems in "Timofeev Problems.m"

Problem 69: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSin}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2-x^2}}\, dx$$

Optimal (type 3, 42 leaves, 1 step) :

$$\frac{2\, a\, \sqrt{1-\frac{x^2}{a^2}}\ \text{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5\, \sqrt{a^2-x^2}}$$

Result (type 3, 42 leaves, 2 steps) :

$$\frac{2\, a\, \sqrt{1-\frac{x^2}{a^2}}\ \text{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5\, \sqrt{a^2-x^2}}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-x}\ x\ (1+x)^{2/3}}{-(1-x)^{5/6}\ (1+x)^{1/3}+(1-x)^{2/3}\ \sqrt{1+x}}\, dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}(1+x)^{1/3}+\frac{1}{4}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}(1-x)^{1/3}(1+x)^{2/3}(1+3x)+\frac{1}{12}(1-x)^{1/6}(1+x)^{5/6}(2+3x)-\frac{1}{12}(1-x)^{5/6}(1+x)^{1/6}(10+3x)+\frac{1}{6}\text{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right]-\frac{4\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{5}{6}\text{ArcTan}\left[\frac{(1-x)^{1/3}-(1+x)^{1/3}}{(1-x)^{1/6}(1+x)^{1/6}}\right]+\frac{\text{ArcTanh}\left[\frac{\sqrt{3}(1-x)^{1/6}(1+x)^{1/6}}{(1-x)^{1/3}+(1+x)^{1/3}}\right]}{6\sqrt{3}}$$

Result (type 3, 522 leaves, 46 steps):

$$\frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}(1+x)^{1/6}+\frac{1}{6}(1-x)^{2/3}(1+x)^{1/3}-\frac{1}{4}(1-x)^{5/3}(1+x)^{1/3}+\frac{1}{3}(1-x)^{1/3}(1+x)^{2/3}-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3}+\frac{5}{12}(1-x)^{1/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{7/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{5/6}(1+x)^{7/6}+\frac{1}{4}x\sqrt{1-x^2}+\frac{\text{ArcSin}[x]}{4}-\frac{2}{3}\text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right]+\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{3\sqrt{3}}+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{9}\text{Log}[1-x]+\frac{1}{9}\text{Log}[1+x]+\frac{1}{3}\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}-\frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}+\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}+\frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}-\frac{1}{3}\text{Log}\left[1+\frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right]$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left((-1+x)^2(1+x)\right)^{1/3}}\,dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(3-3x)^{1/3}}\right]}{3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[-\frac{8}{3}(-1+x)\right]}{2\times 3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[1+\frac{3^{1/3}(1+x)^{1/3}}{(3-3x)^{1/3}}\right]}{2\left(1-x-x^2+x^3\right)^{1/3}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int \frac{\left((-1+x)^2 (1+x)\right)^{1/3}}{x^2}\,dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{\left((-1+x)^2 (1+x)\right)^{1/3}}{x}-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\sqrt{3}\operatorname{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]+\frac{\operatorname{Log}[x]}{6}-\frac{2}{3}\operatorname{Log}[1+x]-\frac{3}{2}\operatorname{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]-\frac{1}{2}\operatorname{Log}\left[1+\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 3, 404 leaves, 6 steps):

$$-\frac{\left(1-x-x^2+x^3\right)^{1/3}}{x}-\frac{3\times 3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\right]}{(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\right]}{(3-3x)^{2/3}(1+x)^{1/3}}+\frac{\left(1-x-x^2+x^3\right)^{1/3}\operatorname{Log}[x]}{2\times 3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\operatorname{Log}\left[\frac{4(1+x)}{3}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3\times 3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\operatorname{Log}\left[1+\frac{(3-3x)^{1/3}}{3^{1/3}(1+x)^{1/3}}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\operatorname{Log}\left[\left(\frac{2}{3}\right)^{2/3}(3-3x)^{1/3}-\frac{2^{2/3}(1+x)^{1/3}}{3^{1/3}}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3}\operatorname{ArcTan}\left[\frac{1+\frac{2(-3+x)}{\left(9+3x-5x^2+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\operatorname{Log}[1+x]-\frac{3}{2}\operatorname{Log}\left[1-\frac{-3+x}{\left(9+3x-5x^2+x^3\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(9-3x)^{2/3}(1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(9-3x)^{1/3}}\right]}{3^{1/6}\left(9+3x-5x^2+x^3\right)^{1/3}}-\frac{(9-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[-\frac{32}{3}(-3+x)\right]}{2\times 3^{2/3}\left(9+3x-5x^2+x^3\right)^{1/3}}-\frac{3^{1/3}(9-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[1+\frac{3^{1/3}(1+x)^{1/3}}{(9-3x)^{1/3}}\right]}{2\left(9+3x-5x^2+x^3\right)^{1/3}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x+\sqrt{1+x+x^2}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-x+\sqrt{1+x+x^2}-\frac{3}{2}\,\text{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]$$

Result (type 3, 59 leaves, 3 steps):

$$\frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)}+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]-\frac{3}{2}\,\text{Log}\left[1+2\left(x+\sqrt{1+x+x^2}\right)\right]$$

Problem 306: Result valid but suboptimal antiderivative.

$$\int\left(x\left(1-x^2\right)\right)^{1/3}dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{1}{2}x\left(x\left(1-x^2\right)\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{2x-\left(x\left(1-x^2\right)\right)^{1/3}}{\sqrt{3}\left(x\left(1-x^2\right)\right)^{1/3}}\right]}{2\sqrt{3}}+\frac{\text{Log}[x]}{12}-\frac{1}{4}\,\text{Log}\left[x+\left(x\left(1-x^2\right)\right)^{1/3}\right]$$

Result (type 3, 200 leaves, 12 steps):

$$\frac{1}{2}x\left(x-x^3\right)^{1/3}-\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{ArcTan}\left[\frac{1-\frac{2x^{2/3}}{\left(1-x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}\left(x-x^3\right)^{2/3}}+\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{Log}\left[1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{12\left(x-x^3\right)^{2/3}}-\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{Log}\left[1+\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{6\left(x-x^3\right)^{2/3}}$$

Problem 314: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(-1+x^3\right)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 78 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}x}{(2+x^3)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[-1+x^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}x-\left(2+x^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 107 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2x}{3^{1/6}(2+x^3)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}x}{(2+x^3)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}x^2}{(2+x^3)^{2/3}}+\frac{3^{1/3}x}{(2+x^3)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 319: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(3x+3x^2+x^3\right)\left(3+3x+3x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}(1+x)}{(2+(1+x)^3)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[1-(1+x)^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}(1+x)-\left(2+(1+x)^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(1+x)}{3^{1/6}(2+(1+x)^3)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}(1+x)}{(2+(1+x)^3)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}(1+x)^2}{(2+(1+x)^3)^{2/3}}+\frac{3^{1/3}(1+x)}{(2+(1+x)^3)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Tan}[x]}{\left(-1+\sqrt{\text{Tan}[x]}\right)^2}\,dx$$

Optimal (type 3, 84 leaves, ? steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[\frac{1-\text{Tan}[x]}{\sqrt{2}\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{1+\text{Tan}[x]}{\sqrt{2}\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{1}{2}\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Result (type 3, 133 leaves, 19 steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}+\frac{1}{2}\text{Log}[\text{Cos}[x]]+$$
$$\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]-\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Problem 416: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Cos}[2x]-\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2}\,\text{Log}\left[\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sec}[x]\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\right]-$$
$$\frac{\text{ArcSin}[\text{Cos}[x]-\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{ArcTanh}[\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{Sin}[2x]}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}$$

Result (type 3, 234 leaves, 27 steps):

$$-2\,\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-\sqrt{2}\,\text{ArcSinh}[\text{Tan}[x]]\,\text{Cot}[x]\,\left(\text{Sec}[x]^2\right)^{3/2}\sqrt{\text{Cos}[x]\,\text{Sin}[x]}\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-$$
$$\frac{\sqrt{2}\,\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}+\frac{\sqrt{2}\,\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}-$$
$$\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]^2\left(-\text{Cos}[2x]+2\,\text{Tan}[x]^2\right)}{\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}\,dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2\,\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]-\frac{11\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]}{4\sqrt{2}}+\frac{\text{Tan}[x]}{2\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{2\,\text{Tan}[x]^3}{3\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{3\,\text{Tan}[x]}{4\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}$$

Result (type 3, 208 leaves, 21 steps):

$$\frac{3\,\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Cot}[x]\left(1-\text{Tan}[x]^2\right)}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Tan}[x]\left(1-\text{Tan}[x]^2\right)}{3\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}-\frac{11\,\text{ArcTan}\left[\sqrt{-1+\text{Tan}[x]^2}\right]\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}+\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-1+\text{Tan}[x]^2}}{\sqrt{2}}\right]\text{Tan}[x]}{\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}$$

Problem 455: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Sin}[x]^6\,\text{Tan}[x]}{\text{Cos}[2x]^{3/4}}\,dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{1-\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{1+\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+\frac{1}{36}\text{Cos}[2x]^{9/4}$$

Result (type 3, 154 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+$$
$$\frac{1}{36}\text{Cos}[2x]^{9/4}+\frac{\text{Log}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int e^{x/2}\,x^2\,\text{Cos}[x]^3\,dx$$

Optimal (type 3, 187 leaves, ? steps):

$$-\frac{132}{125}e^{x/2}\text{Cos}[x]+\frac{18}{25}e^{x/2}x\,\text{Cos}[x]+\frac{48}{185}e^{x/2}x^2\,\text{Cos}[x]+\frac{2}{37}e^{x/2}x^2\,\text{Cos}[x]^3-\frac{428\,e^{x/2}\,\text{Cos}[3x]}{50653}+\frac{70\,e^{x/2}\,x\,\text{Cos}[3x]}{1369}-$$
$$\frac{24}{125}e^{x/2}\text{Sin}[x]-\frac{24}{25}e^{x/2}x\,\text{Sin}[x]+\frac{96}{185}e^{x/2}x^2\,\text{Sin}[x]+\frac{12}{37}e^{x/2}x^2\,\text{Cos}[x]^2\,\text{Sin}[x]-\frac{792\,e^{x/2}\,\text{Sin}[3x]}{50653}-\frac{24\,e^{x/2}\,x\,\text{Sin}[3x]}{1369}$$

Result (type 3, 253 leaves, 31 steps):

$$-\frac{6687696\, e^{x/2}\, \text{Cos}[x]}{6331625}+\frac{24792\, e^{x/2}\, x\, \text{Cos}[x]}{34225}+\frac{48}{185}\, e^{x/2}\, x^2\, \text{Cos}[x]+\frac{16\, e^{x/2}\, \text{Cos}[x]^3}{50653}-\frac{8\, e^{x/2}\, x\, \text{Cos}[x]^3}{1369}+\frac{2}{37}\, e^{x/2}\, x^2\, \text{Cos}[x]^3-\frac{432\, e^{x/2}\, \text{Cos}[3x]}{50653}+\frac{72\, e^{x/2}\, x\, \text{Cos}[3x]}{1369}-\frac{1218672\, e^{x/2}\, \text{Sin}[x]}{6331625}-\frac{32556\, e^{x/2}\, x\, \text{Sin}[x]}{34225}+\frac{96}{185}\, e^{x/2}\, x^2\, \text{Sin}[x]+\frac{96\, e^{x/2}\, \text{Cos}[x]^2\, \text{Sin}[x]}{50653}-\frac{48\, e^{x/2}\, x\, \text{Cos}[x]^2\, \text{Sin}[x]}{1369}+\frac{12}{37}\, e^{x/2}\, x^2\, \text{Cos}[x]^2\, \text{Sin}[x]-\frac{816\, e^{x/2}\, \text{Sin}[3x]}{50653}-\frac{12\, e^{x/2}\, x\, \text{Sin}[3x]}{1369}$$

Problem 614: Result valid but suboptimal antiderivative.

$$\int \left(1+x^4\right)\left(1-2\,\text{Log}[x]+\text{Log}[x]^3\right)\,dx$$

Optimal (type 3, 60 leaves, 13 steps):

$$-3x+\frac{169\,x^5}{625}+4\,x\,\text{Log}[x]-\frac{44}{125}\,x^5\,\text{Log}[x]-3\,x\,\text{Log}[x]^2-\frac{3}{25}\,x^5\,\text{Log}[x]^2+x\,\text{Log}[x]^3+\frac{1}{5}\,x^5\,\text{Log}[x]^3$$

Result (type 3, 73 leaves, 13 steps):

$$-3x+\frac{169\,x^5}{625}+6\,x\,\text{Log}[x]+\frac{6}{125}\,x^5\,\text{Log}[x]-\frac{2}{5}\left(5x+x^5\right)\text{Log}[x]-3\,x\,\text{Log}[x]^2-\frac{3}{25}\,x^5\,\text{Log}[x]^2+x\,\text{Log}[x]^3+\frac{1}{5}\,x^5\,\text{Log}[x]^3$$

Problem 695: Result valid but suboptimal antiderivative.

$$\int \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]dx$$

Optimal (type 3, 55 leaves, ? steps):

$$-\frac{\sqrt{2}\,a\,\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}}+(a+x)\,\text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

Result (type 3, 118 leaves, 8 steps):

$$-\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\sqrt{-\frac{a-x}{a+x}}\,(a+x)+x\,\text{ArcSin}\left[\sqrt{-\frac{a-x}{a+x}}\right]-\frac{a\,\sqrt{\frac{a}{a+x}}\,\text{ArcTanh}\left[\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\,\sqrt{-\frac{a}{a+x}}}\right]}{\sqrt{-\frac{a}{a+x}}}$$

Test results for the 50 problems in "Charlwood Problems.m"

Problem 3: Unable to integrate problem.

$$\int -\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]\,dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{4\sqrt{2}}-\left(\frac{3}{8}+x\right)\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]$$

Result (type 8, 60 leaves, 3 steps):

$$-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]+\frac{\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}},x\right]}{2\sqrt{2}}$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[1+x\sqrt{1+x^2}\right]\,dx$$

Optimal (type 3, 97 leaves, ? steps):

$$-2x+\sqrt{2\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]-\sqrt{2\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Result (type 3, 332 leaves, 32 steps):

$$-2x-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{1+x^2}\right]+$$
$$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{1+x^2}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]+$$
$$\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{1+x^2}\right]-\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{1+x^2}\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3}+\frac{1}{3}\text{ArcTan}\left[\frac{\text{Cos}[x]\,\left(1+\text{Cos}[x]^2\right)\,\text{Sin}[x]}{1+\text{Cos}[x]^2\,\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\right]$$

Result (type 4, 289 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{\text{Tan}[x]}{\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}\right]\,\text{Cos}[x]^2\,\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}{2\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}-$$

$$\frac{\left(1+\sqrt{3}\right)\,\text{Cos}[x]^2\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\,\left(\sqrt{3}+\text{Tan}[x]^2\right)\,\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}}{4\times 3^{1/4}\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}+$$

$$\left(\left(2+\sqrt{3}\right)\,\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{1}{6}\left(3-2\,\sqrt{3}\right),2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\,\left(\sqrt{3}+\text{Tan}[x]^2\right)\,\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/$$
$$\left(4\times 3^{1/4}\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcTan}\left[x+\sqrt{1-x^2}\right]\,dx$$

Optimal (type 3, 141 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$
$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2\,x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\text{ArcTanh}\left[x\,\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 269 leaves, 40 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}}+\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}-\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 152 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 286 leaves, 32 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{2\sqrt{3}}-\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{2\sqrt{3}}+\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]}{\sqrt{-1+\text{Sec}[x]^4}}\,dx$$

Optimal (type 3, 28 leaves, ? steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]\,\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Sin}[x]}{\sqrt{2\,\text{Sin}[x]^2-\text{Sin}[x]^4}}\right]\sqrt{1-\text{Cos}[x]^4}\,\text{Sec}[x]^2}{\sqrt{2}\,\sqrt{-1+\text{Sec}[x]^4}}$$

Test results for the 376 problems in "Stewart Problems.m"

Test results for the 284 problems in "Hearn Problems.m"

Problem 169: Unable to integrate problem.

$$\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Result (type 8, 69 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2}x+2x^2}\,x}{\left(-1+e^{x^2}x\right)^2},x\right]+2\,\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2}x+2x^2}\,x^3}{\left(-1+e^{x^2}x\right)^2},x\right]$$

Problem 278: Unable to integrate problem.

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2\left(-1+2x^2\right)}-\text{ArcTanh}\left[\frac{x(2+x)\left(7-x+27x^2+33x^3\right)}{\left(2+37x^2+31x^3\right)\sqrt{1+2x^2+4x^3+x^4}}\right]$$

Result (type 8, 354 leaves, 10 steps):

$$\frac{9}{4}\,\text{CannotIntegrate}\left[\frac{1}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{4}\,\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}-2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]+\text{CannotIntegrate}\left[\frac{x}{\sqrt{1+2x^2+4x^3+x^4}},x\right]+\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{4}\,\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}+2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{8}\,\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{1}{8}\left(15+\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{8}\,\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{1}{8}\left(15-\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{17}{2}\,\text{CannotIntegrate}\left[\frac{x}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]$$

Problem 279: Unable to integrate problem.

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4}\,\text{ArcTanh}\left[\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right]-\frac{1}{2}\,\text{ArcTanh}\left[\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right]+\frac{9}{4}\,\text{ArcTanh}\left[\frac{(11+7y)\sqrt{1-5y-5y^2}}{3(7+5y)\sqrt{1-y-y^2}}\right]$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}},y\right]+\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}},y\right]-\frac{3}{2}\,\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}},y\right]$$

Problem 281: Unable to integrate problem.

$$\int\left(\sqrt{9-4\sqrt{2}}\;x-\sqrt{2}\;\sqrt{1+4x+2x^2+x^4}\right)dx$$

Optimal (type 4, 4030 leaves, ? steps):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\;x^2-\sqrt{2}\left(-\frac{1}{3}\sqrt{1+4x+2x^2+x^4}+\frac{1}{3}(1+x)\sqrt{1+4x+2x^2+x^4}+\right.$$
$$\frac{4\,i\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{1+4x+2x^2+x^4}}{4\times2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x}-$$
$$\left(8\times2^{2/3}\sqrt{\frac{3}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}\right.$$
$$\sqrt{\left(\left(i\left(-19899+3445\sqrt{33}+\left(-26+6\sqrt{33}\right)^{2/3}\left(-2574+466\sqrt{33}\right)+\left(-26+6\sqrt{33}\right)^{1/3}\left(-19899+3445\sqrt{33}\right)+\left(59697-10335\sqrt{33}\right)x\right)\right)\right/}$$
$$\left(\left(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\right.$$
$$\left.\left.\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\right)$$
$$\sqrt{1+4x+2x^2+x^4}\;\text{EllipticE}\left[\text{ArcSin}\left[\left(\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.}\right.\right.\right.$$
$$\left.4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\Big/\left(\sqrt{\frac{39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{39-13\,i\sqrt{3}+9\,i\sqrt{11}-9\sqrt{33}+4\left(3+i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}}\right.$$
$$\left.\left.\sqrt{\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}\right)\right],$$

$$\frac{4\,(21+7\,\mathrm{i}\,\sqrt{3}-3\,\mathrm{i}\,\sqrt{11}-3\,\sqrt{33})+(3-\mathrm{i}\,\sqrt{3}-3\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}}{4\,(21-7\,\mathrm{i}\,\sqrt{3}+3\,\mathrm{i}\,\sqrt{11}-3\,\sqrt{33})+(3+\mathrm{i}\,\sqrt{3}+3\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}}]\Bigg)\Big/$$

$$\Big(\Big(4\times 2^{2/3}-(-13+3\,\sqrt{33})^{1/3}-2^{1/3}\,(-13+3\,\sqrt{33})^{2/3}+3\,(-13+3\,\sqrt{33})^{1/3}\,x\Big)$$

$$\sqrt{\Big((\mathrm{i}\,(1+x))\Big/\Big(\Big(104-24\,\sqrt{33}+(-13-13\,\mathrm{i}\,\sqrt{3}+9\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+4\,\mathrm{i}\,(\mathrm{i}+\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}\Big)}$$

$$\Big(26-6\,\sqrt{33}+(-13+13\,\mathrm{i}\,\sqrt{3}-9\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+(-4-4\,\mathrm{i}\,\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}+6\,(-13+3\,\sqrt{33})\,x\Big)\Big)\Big)$$

$$\sqrt{\Big(26-6\,\sqrt{33}+(-13+13\,\mathrm{i}\,\sqrt{3}-9\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+(-4-4\,\mathrm{i}\,\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}+6\,(-13+3\,\sqrt{33})\,x\Big)}$$

$$\sqrt{\Big(26-6\,\sqrt{33}+(-13-13\,\mathrm{i}\,\sqrt{3}+9\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+4\,\mathrm{i}\,(\mathrm{i}+\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}+6\,(-13+3\,\sqrt{33})\,x\Big)}\Big)+$$

$$\Bigg(\Big(2^{1/3}\,(13-13\,\mathrm{i}\,\sqrt{3}+9\,\mathrm{i}\,\sqrt{11}-3\,\sqrt{33})+4\times 2^{2/3}\,(1+\mathrm{i}\,\sqrt{3})\,(-13+3\,\sqrt{33})^{1/3}+20\,(-13+3\,\sqrt{33})^{2/3}\Big)$$

$$\Big(4\times 2^{2/3}\,(\mathrm{i}+\sqrt{3})+8\,\mathrm{i}\,(-13+3\,\sqrt{33})^{1/3}+2^{1/3}\,(-\mathrm{i}+\sqrt{3})\,(-13+3\,\sqrt{33})^{2/3}\Big)\sqrt{\frac{52-12\,\sqrt{33}-2^{1/3}\,(-13+3\,\sqrt{33})^{4/3}+4\,(-26+6\,\sqrt{33})^{2/3}}{-13+3\,\sqrt{33}+4\,(-26+6\,\sqrt{33})^{1/3}}}$$

$$\sqrt{\Big(\frac{1}{1+x}\Big(-8\,\mathrm{i}\,(-13+3\,\sqrt{33})+(-43\,\mathrm{i}-13\,\sqrt{3}+9\,\sqrt{11}+5\,\mathrm{i}\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+(2\,\mathrm{i}+4\,\sqrt{3}-2\,\mathrm{i}\,\sqrt{33})\,(-26+6\,\sqrt{33})^{2/3}+}$$

$$\Big(8\,\mathrm{i}\,(-13+3\,\sqrt{33})+(13\,\mathrm{i}-13\,\sqrt{3}+9\,\sqrt{11}-3\,\mathrm{i}\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+4\,(\mathrm{i}+\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}\Big)\,x\Big)\Big)$$

$$\sqrt{1+4\,x+2\,x^2+x^4}\;\text{EllipticF}\Big[\text{ArcSin}\Big[\Big(\sqrt{52-12\,\sqrt{33}-2^{1/3}\,(-13+3\,\sqrt{33})^{4/3}+4\,(-26+6\,\sqrt{33})^{2/3}}$$

$$\sqrt{\Big(26-6\,\sqrt{33}+(-13-13\,\mathrm{i}\,\sqrt{3}+9\,\mathrm{i}\,\sqrt{11}+3\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}+4\,\mathrm{i}\,(\mathrm{i}+\sqrt{3})\,(-26+6\,\sqrt{33})^{2/3}+6\,(-13+3\,\sqrt{33})\,x\Big)}\Big)\Big/$$

$$\Big(2^{1/6}\,\sqrt{3}\,(-13+3\,\sqrt{33})^{2/3}\,\sqrt{39+13\,\mathrm{i}\,\sqrt{3}-9\,\mathrm{i}\,\sqrt{11}-9\,\sqrt{33}+4\,(3-\mathrm{i}\,\sqrt{3})\,(-26+6\,\sqrt{33})^{1/3}}\,\sqrt{1+x}\Big)\Big],$$

$$\frac{4\,(21\,\mathrm{i}-7\,\sqrt{3}+3\,\sqrt{11}-3\,\mathrm{i}\,\sqrt{33})+(3\,\mathrm{i}+\sqrt{3}+3\,\sqrt{11}+3\,\mathrm{i}\,\sqrt{33})\,(-26+6\,\sqrt{33})^{1/3}}{-56\,\sqrt{3}+24\,\sqrt{11}+2\,(\sqrt{3}+3\,\sqrt{11})\,(-26+6\,\sqrt{33})^{1/3}}]\Bigg)\Big/$$

$$\Big(3\times 2^{2/3}\times 3^{3/4}\,(-13+3\sqrt{33})^{1/3}\sqrt{39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}-9\sqrt{33}+4\,(3-\mathrm{i}\sqrt{3})\,(-26+6\sqrt{33})^{1/3}}\,\sqrt{1+x}$$

$$(4\times 2^{2/3}\,(-\mathrm{i}+\sqrt{3})-2\,\mathrm{i}\,(-13+3\sqrt{33})^{1/3}+2^{1/3}\,(\mathrm{i}+\sqrt{3})\,(-13+3\sqrt{33})^{2/3}+6\,\mathrm{i}\,(-13+3\sqrt{33})^{1/3}\,x)$$

$$\sqrt{(26-6\sqrt{33}+(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33})\,(-26+6\sqrt{33})^{1/3}+4\,\mathrm{i}\,(\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{2/3}+6\,(-13+3\sqrt{33})\,x)}$$

$$\sqrt{\Big(\Big(8\,(-13+3\sqrt{33})-(5-3\,\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+\sqrt{33})\,(-26+6\sqrt{33})^{2/3}+(-26+6\sqrt{33})^{1/3}\,(-41+15\,\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+7\sqrt{33})+}$$

$$(104-24\sqrt{33}+(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33})\,(-26+6\sqrt{33})^{1/3}+4\,\mathrm{i}\,(\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{2/3})\,x\Big)\Big/$$

$$\Big((-39-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+9\sqrt{33}+4\,\mathrm{i}\,(3\,\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{1/3})\,(1+x)\Big)\Big)\Big)+$$

$$\Big((4\times 2^{2/3}+2\,(-13+3\sqrt{33})^{1/3}-2^{1/3}\,(-13+3\sqrt{33})^{2/3})\,(4\times 2^{2/3}\,(\mathrm{i}+\sqrt{3})-4\,\mathrm{i}\,(-13+3\sqrt{33})^{1/3}+2^{1/3}\,(-\mathrm{i}+\sqrt{3})\,(-13+3\sqrt{33})^{2/3})$$

$$(4\times 2^{2/3}\,(-\mathrm{i}+\sqrt{3})+4\,\mathrm{i}\,(-13+3\sqrt{33})^{1/3}+2^{1/3}\,(\mathrm{i}+\sqrt{3})\,(-13+3\sqrt{33})^{2/3})$$

$$\sqrt{\Big((-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\,(-3\,\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{1/3})\Big/}$$

$$(104-24\sqrt{33}+(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33})\,(-26+6\sqrt{33})^{1/3}+(-4-4\,\mathrm{i}\sqrt{3})\,(-26+6\sqrt{33})^{2/3})\Big)\,\sqrt{1+x}$$

$$\sqrt{\Big(\Big(104-24\sqrt{33}+2\,(1+14\,\mathrm{i}\sqrt{3}-6\,\mathrm{i}\sqrt{11}+\sqrt{33})\,(-26+6\sqrt{33})^{1/3}+(-7-\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+\sqrt{33})\,(-26+6\sqrt{33})^{2/3}+}$$

$$2\,(-52+12\sqrt{33}+2^{1/3}\,(-13+3\sqrt{33})^{4/3}-4\,(-26+6\sqrt{33})^{2/3})\,x\Big)\Big/$$

$$\Big((-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\,(-3\,\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{1/3})\,(1+x)\Big)\Big)$$

$$\sqrt{\Big(\Big(104-24\sqrt{33}+2\,(1-14\,\mathrm{i}\sqrt{3}+6\,\mathrm{i}\sqrt{11}+\sqrt{33})\,(-26+6\sqrt{33})^{1/3}+(-7+\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+\sqrt{33})\,(-26+6\sqrt{33})^{2/3}+}$$

$$2\,(-52+12\sqrt{33}+2^{1/3}\,(-13+3\sqrt{33})^{4/3}-4\,(-26+6\sqrt{33})^{2/3})\,x\Big)\Big/$$

$$\Big((-39-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+9\sqrt{33}+4\,\mathrm{i}\,(3\,\mathrm{i}+\sqrt{3})\,(-26+6\sqrt{33})^{1/3})\,(1+x)\Big)\Big)\,\sqrt{1+4\,x+2\,x^2+x^4}$$

$$\mathrm{EllipticPi}\Big[\frac{2^{1/3}\,(4\times 2^{1/3}\,(-3\,\mathrm{i}+\sqrt{3})+(3\,\mathrm{i}+\sqrt{3})\,(-13+3\sqrt{33})^{2/3})}{4\times 2^{2/3}\,(-\mathrm{i}+\sqrt{3})-8\,\mathrm{i}\,(-13+3\sqrt{33})^{1/3}+2^{1/3}\,(\mathrm{i}+\sqrt{3})\,(-13+3\sqrt{33})^{2/3}},$$

$$\text{ArcSin}\Big[\Big(\sqrt{13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x}\Big)\Big/$$
$$\Big(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{\Big(\Big(-39+13\,i\sqrt{3}-9\,i\sqrt{11}+9\sqrt{33}-4\,i\left(-3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/}$$
$$\Big(104-24\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big)\sqrt{1+x}\Big)\Big],$$
$$\frac{4\left(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{4\left(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\Big]\Big)\Big/$$
$$\Big(2^{1/6}\sqrt{3}\left(4\times2^{2/3}\left(i+\sqrt{3}\right)+2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}-6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$
$$\left(4\times2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$
$$\sqrt{13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x}\Big)\Big)$$

Result (type 8, 47 leaves, 1 step):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\,x^2-\sqrt{2}\,\text{CannotIntegrate}\left[\sqrt{1+4x+2x^2+x^4},x\right]$$

Problem 284: Unable to integrate problem.

$$\int\frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Optimal (type 3, 71 leaves, ? steps):

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\text{Log}\left[1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right]-\left(-1+\sqrt{2}\right)\text{Log}\left[-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right]\right)$$

Result (type 8, 248 leaves, 5 steps):

$$2\,\text{CannotIntegrate}\left[\frac{1}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+4\,\text{CannotIntegrate}\left[\frac{x}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{x^2}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+12\,\text{CannotIntegrate}\left[\frac{x^7}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$10\,\text{CannotIntegrate}\left[\frac{x^8}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+\frac{1}{2}\text{Log}\left[1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right]$$

Test results for the 9 problems in "Jeffrey Problems.m"

Problem 2: Result valid but suboptimal antiderivative.

$$\int\frac{1+\text{Cos}[x]+2\,\text{Sin}[x]}{3+\text{Cos}[x]^2+2\,\text{Sin}[x]-2\,\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 38 leaves, 43 steps) :

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]+\text{Cot}\left[\frac{x}{2}\right]-\frac{\text{Sin}[x]}{1-\text{Cos}[x]}$$

Problem 3: Result valid but suboptimal antiderivative.

$$\int\frac{2+\text{Cos}[x]+5\,\text{Sin}[x]}{4\,\text{Cos}[x]-2\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[x]-2\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1-3\,\text{Cos}[x]+\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 42 leaves, 25 steps) :

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[1+\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int\frac{3+7\,\text{Cos}[x]+2\,\text{Sin}[x]}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-5\,\text{Sin}[x]-\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1+\text{Cos}[x]-2\,\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 31 leaves, 32 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 5: Unable to integrate problem.

$$\int \frac{-1+4\,\text{Cos}[x]+5\,\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 43 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{3+\text{Cos}[x]}\right]-2\,\text{ArcTan}\left[\frac{3\,\text{Sin}[x]+7\,\text{Cos}[x]\,\text{Sin}[x]}{1+2\,\text{Cos}[x]+5\,\text{Cos}[x]^2}\right]$$

Result (type 8, 79 leaves, 2 steps):

$$\text{CannotIntegrate}\left[\frac{1}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-4\,\text{Cos}[x]^3},x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+5\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]$$

Problem 6: Unable to integrate problem.

$$\int \frac{-5+2\,\text{Cos}[x]+7\,\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{2\,\text{Cos}[x]\,\text{Sin}[x]}{1-\text{Cos}[x]+2\,\text{Cos}[x]^2}\right]$$

Result (type 8, 81 leaves, 2 steps):

$$-5\,\text{CannotIntegrate}\left[\frac{1}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+7\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]$$

Test results for the 7 problems in "Hebisch Problems.m"

Problem 2: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}} \left(2-x^2\right)}{2 x+x^3} \, dx$$

Optimal (type 4, 10 leaves, ? steps):

$$\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 76 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}-x}, x\right]+\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}+x}, x\right]$$

Problem 3: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}} \left(2+2 x+3 x^2-x^3+2 x^4\right)}{2 x+x^3} \, dx$$

Optimal (type 4, 28 leaves, ? steps):

$$e^{\frac{x}{2+x^2}} \left(2+x^2\right)+\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 131 leaves, 5 steps):

$$-\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}}, x\right]+\left(1+i \sqrt{2}\right) \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}-x}, x\right]+$$
$$\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right]+2\,\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}} x, x\right]-\left(1-i \sqrt{2}\right) \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}+x}, x\right]$$

Problem 5: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{-1+x^2}} \left(1-3 x-x^2+x^3\right)}{1-x-x^2+x^3} \, dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\frac{1}{-1+x^2}}\,(1+x)$$

Result (type 8, 75 leaves, 6 steps):

$$\text{CannotIntegrate}\left[e^{\frac{1}{-1+x^2}},x\right]+\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1-x},x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{(-1+x)^2},x\right]+\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1+x},x\right]$$

Problem 7: Unable to integrate problem.

$$\int\frac{e^{x+\frac{1}{\text{Log}[x]}}\left(-1+(1+x)\,\text{Log}[x]^2\right)}{\text{Log}[x]^2}\,dx$$

Optimal (type 3, 10 leaves, ? steps):

$$e^{x+\frac{1}{\text{Log}[x]}}\,x$$

Result (type 8, 40 leaves, 2 steps):

$$\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}},x\right]+\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}\,x,x\right]-\text{CannotIntegrate}\left[\frac{e^{x+\frac{1}{\text{Log}[x]}}}{\text{Log}[x]^2},x\right]$$

Test results for the 8 problems in "Wester Problems.m"

Test results for the 116 problems in "Welz Problems.m"

Problem 9: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-$$
$$\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 3, 365 leaves, 18 steps) :

$$\frac{2\,(1-2\,x)\,\sqrt{x}}{5\,(1+x-x^2)}-\frac{2\,(1-2\,x)\,\sqrt{-1+x^2}}{5\,(1+x-x^2)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\,\sqrt{5}\right)}\,\text{ArcTan}\Big[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\Big]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\text{ArcTan}\Big[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\Big]-\frac{2}{5}\sqrt{\frac{1}{5}\left(-2+5\,\sqrt{5}\right)}\,\text{ArcTan}\Big[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\Big]-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\,\sqrt{5}\right)}\,\text{ArcTanh}\Big[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\Big]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\text{ArcTanh}\Big[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\Big]-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\,\sqrt{5}\right)}\,\text{ArcTanh}\Big[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\Big]$$

Problem 10: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4\,x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\,\sqrt{5}}\,\text{ArcTan}\Big[\frac{1}{2}\sqrt{2+2\,\sqrt{5}}\,\sqrt{x}\Big]-\frac{1}{50}\sqrt{-110+50\,\sqrt{5}}\,\text{ArcTan}\Big[\frac{\sqrt{-2+2\,\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\Big]-\frac{1}{25}\sqrt{110+50\,\sqrt{5}}\,\text{ArcTanh}\Big[\frac{1}{2}\sqrt{-2+2\,\sqrt{5}}\,\sqrt{x}\Big]-\frac{1}{50}\sqrt{110+50\,\sqrt{5}}\,\text{ArcTanh}\Big[\frac{\sqrt{2+2\,\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\Big]$$

Result (type 3, 541 leaves, 25 steps) :

$$\frac{2\left(1-2x\right)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{\left(1-2x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{\left(3-x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{\left(2+x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+$$

$$\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]-\frac{1}{5}\sqrt{\frac{1}{10}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]-\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 29: Result valid but suboptimal antiderivative.

$$\int x^3\,\text{Log}[2+x]^3\,\text{Log}[3+x]\,dx$$

Optimal (type 4, 606 leaves, 359 steps):

$$-\frac{302177\,x}{1152}+\frac{8029\,x^2}{2304}-\frac{763\,x^3}{3456}+\frac{3\,x^4}{256}+\frac{377}{64}(2+x)^2-\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\mathrm{Log}[2+x]-\frac{187}{64}x^2\,\mathrm{Log}[2+x]+\frac{83}{288}x^3\,\mathrm{Log}[2+x]-\frac{3}{128}x^4\,\mathrm{Log}[2+x]+\frac{6733}{32}(2+x)\,\mathrm{Log}[2+x]-\frac{377}{32}(2+x)^2\,\mathrm{Log}[2+x]+\frac{71}{72}(2+x)^3\,\mathrm{Log}[2+x]-\frac{3}{64}(2+x)^4\,\mathrm{Log}[2+x]-\frac{43}{12}\mathrm{Log}[2+x]^2-\frac{17}{48}x^3\,\mathrm{Log}[2+x]^2+\frac{3}{64}x^4\,\mathrm{Log}[2+x]^2-\frac{1251}{16}(2+x)\,\mathrm{Log}[2+x]^2+\frac{273}{32}(2+x)^2\,\mathrm{Log}[2+x]^2-\frac{3}{4}(2+x)^3\,\mathrm{Log}[2+x]^2+\frac{3}{64}(2+x)^4\,\mathrm{Log}[2+x]^2+\frac{65}{4}(2+x)\,\mathrm{Log}[2+x]^3-\frac{33}{8}(2+x)^2\,\mathrm{Log}[2+x]^3+\frac{3}{4}(2+x)^3\,\mathrm{Log}[2+x]^3-\frac{1}{16}(2+x)^4\,\mathrm{Log}[2+x]^3+\frac{3891}{128}\mathrm{Log}[3+x]-\frac{115}{48}x^2\,\mathrm{Log}[3+x]+\frac{37}{144}x^3\,\mathrm{Log}[3+x]-\frac{3}{128}x^4\,\mathrm{Log}[3+x]+\frac{415}{12}(3+x)\,\mathrm{Log}[3+x]-\frac{4083}{32}\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]-25\,x\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{13}{4}x^2\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]-\frac{7}{12}x^3\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{3}{32}x^4\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{963}{16}\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]+6\,x\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{3}{2}x^2\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]+\frac{1}{2}x^3\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{3}{16}x^4\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{81}{4}\mathrm{Log}[2+x]^3\,\mathrm{Log}[3+x]+\frac{1}{4}x^4\,\mathrm{Log}[2+x]^3\,\mathrm{Log}[3+x]-\frac{5609}{96}\mathrm{PolyLog}[2,-2-x]+\frac{563}{8}\mathrm{Log}[2+x]\,\mathrm{PolyLog}[2,-2-x]-\frac{195}{4}\mathrm{Log}[2+x]^2\,\mathrm{PolyLog}[2,-2-x]-\frac{563}{8}\mathrm{PolyLog}[3,-2-x]+\frac{195}{2}\mathrm{Log}[2+x]\,\mathrm{PolyLog}[3,-2-x]-\frac{195}{2}\mathrm{PolyLog}[4,-2-x]$$

Result (type 4, 679 leaves, 359 steps):

$$-\frac{302177\,x}{1152}+\frac{8029\,x^2}{2304}-\frac{763\,x^3}{3456}+\frac{3\,x^4}{256}+\frac{377}{64}(2+x)^2-\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\mathrm{Log}[2+x]-\frac{187}{64}x^2\,\mathrm{Log}[2+x]+\frac{83}{288}x^3\,\mathrm{Log}[2+x]-\frac{3}{128}x^4\,\mathrm{Log}[2+x]+\frac{6365}{32}(2+x)\,\mathrm{Log}[2+x]-\frac{273}{32}(2+x)^2\,\mathrm{Log}[2+x]+\frac{1}{2}(2+x)^3\,\mathrm{Log}[2+x]-\frac{3}{128}(2+x)^4\,\mathrm{Log}[2+x]+\frac{1}{128}\left(384(2+x)-144(2+x)^2+32(2+x)^3-3(2+x)^4-192\,\mathrm{Log}[2+x]\right)\mathrm{Log}[2+x]+\frac{17}{72}\left(36(2+x)-9(2+x)^2+(2+x)^3-24\,\mathrm{Log}[2+x]\right)\mathrm{Log}[2+x]+\frac{43}{12}\mathrm{Log}[2+x]^2-\frac{17}{48}x^3\,\mathrm{Log}[2+x]^2+\frac{3}{64}x^4\,\mathrm{Log}[2+x]^2-\frac{1251}{16}(2+x)\,\mathrm{Log}[2+x]^2+\frac{273}{32}(2+x)^2\,\mathrm{Log}[2+x]^2-\frac{3}{4}(2+x)^3\,\mathrm{Log}[2+x]^2+\frac{3}{64}(2+x)^4\,\mathrm{Log}[2+x]^2+\frac{65}{4}(2+x)\,\mathrm{Log}[2+x]^3-\frac{33}{8}(2+x)^2\,\mathrm{Log}[2+x]^3+\frac{3}{4}(2+x)^3\,\mathrm{Log}[2+x]^3-\frac{1}{16}(2+x)^4\,\mathrm{Log}[2+x]^3+\frac{3891}{128}\mathrm{Log}[3+x]-\frac{115}{48}x^2\,\mathrm{Log}[3+x]+\frac{37}{144}x^3\,\mathrm{Log}[3+x]-\frac{3}{128}x^4\,\mathrm{Log}[3+x]+\frac{415}{12}(3+x)\,\mathrm{Log}[3+x]-\frac{4083}{32}\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]-25\,x\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{13}{4}x^2\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]-\frac{7}{12}x^3\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{3}{32}x^4\,\mathrm{Log}[2+x]\,\mathrm{Log}[3+x]+\frac{963}{16}\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]+6\,x\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{3}{2}x^2\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]+\frac{1}{2}x^3\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{3}{16}x^4\,\mathrm{Log}[2+x]^2\,\mathrm{Log}[3+x]-\frac{81}{4}\mathrm{Log}[2+x]^3\,\mathrm{Log}[3+x]+\frac{1}{4}x^4\,\mathrm{Log}[2+x]^3\,\mathrm{Log}[3+x]-\frac{5609}{96}\mathrm{PolyLog}[2,-2-x]+\frac{563}{8}\mathrm{Log}[2+x]\,\mathrm{PolyLog}[2,-2-x]-\frac{195}{4}\mathrm{Log}[2+x]^2\,\mathrm{PolyLog}[2,-2-x]-\frac{563}{8}\mathrm{PolyLog}[3,-2-x]+\frac{195}{2}\mathrm{Log}[2+x]\,\mathrm{PolyLog}[3,-2-x]-\frac{195}{2}\mathrm{PolyLog}[4,-2-x]$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(2-3x+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-x)}{\sqrt{3}\left(2-3x+x^2\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\left(2-3x+x^2\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 176 leaves, 2 steps):

$$-\frac{\sqrt{3}\,(-2+x)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{1/3}(-2+x)^{2/3}}{\sqrt{3}\,(-1+x)^{1/3}}\right]}{2\times 2^{1/3}\left(2-3x+x^2\right)^{1/3}}+\frac{3\,(-2+x)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}}-2^{1/3}(-1+x)^{1/3}\right]}{4\times 2^{1/3}\left(2-3x+x^2\right)^{1/3}}-\frac{(-2+x)^{1/3}(-1+x)^{1/3}\,\text{Log}[x]}{2\times 2^{1/3}\left(2-3x+x^2\right)^{1/3}}$$

Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(-5+7x-3x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\left(-5+7x-3x^2+x^3\right)^{1/3}}\right]+\frac{1}{4}\text{Log}[1-x]-\frac{3}{4}\text{Log}\left[1-x+\left(-5+7x-3x^2+x^3\right)^{1/3}\right]$$

Result (type 3, 131 leaves, 5 steps):

$$\frac{\sqrt{3}\left(4+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{\left(4+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(4(-1+x)+(-1+x)^3\right)^{1/3}}-\frac{3\left(4+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\left(4+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]}{4\left(4(-1+x)+(-1+x)^3\right)^{1/3}}$$

Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(x\left(-q+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\left(x\left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\text{Log}\left[-x+\left(x\left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 117 leaves, 5 steps) :

$$\frac{\sqrt{3}\,x^{1/3}\left(-q+x^2\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,x^{2/3}}{\left(-q+x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-q\,x+x^3\right)^{1/3}}-\frac{3\,x^{1/3}\left(-q+x^2\right)^{1/3}\,\text{Log}\left[x^{2/3}-\left(-q+x^2\right)^{1/3}\right]}{4\left(-q\,x+x^3\right)^{1/3}}$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(\left(-1+x\right)\left(q-2\,x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(-1+x\right)}{\sqrt{3}\left(\left(-1+x\right)\left(q-2\,x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}\left[1-x\right]-\frac{3}{4}\,\text{Log}\left[1-x+\left(\left(-1+x\right)\left(q-2\,x+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 145 leaves, 5 steps) :

$$\frac{\sqrt{3}\left(-1+q+\left(-1+x\right)^2\right)^{1/3}\left(-1+x\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\left(-1+x\right)^{2/3}}{\left(-1+q+\left(-1+x\right)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-\left(1-q\right)\left(-1+x\right)+\left(-1+x\right)^3\right)^{1/3}}-\frac{3\left(-1+q+\left(-1+x\right)^2\right)^{1/3}\left(-1+x\right)^{1/3}\,\text{Log}\left[-\left(-1+q+\left(-1+x\right)^2\right)^{1/3}+\left(-1+x\right)^{2/3}\right]}{4\left(-\left(1-q\right)\left(-1+x\right)+\left(-1+x\right)^3\right)^{1/3}}$$

Problem 43: Unable to integrate problem.

$$\int\frac{1}{x\left(\left(-1+x\right)\left(q-2\,q\,x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,q^{1/3}\left(-1+x\right)}{\sqrt{3}\left(\left(-1+x\right)\left(q-2\,q\,x+x^2\right)\right)^{1/3}}\right]}{2\,q^{1/3}}+\frac{\text{Log}\left[1-x\right]}{4\,q^{1/3}}+\frac{\text{Log}\left[x\right]}{2\,q^{1/3}}-\frac{3\,\text{Log}\left[-q^{1/3}\left(-1+x\right)+\left(\left(-1+x\right)\left(q-2\,q\,x+x^2\right)\right)^{1/3}\right]}{4\,q^{1/3}}$$

Result (type 8, 677 leaves, 2 steps) :

$$\frac{1}{3\left(-q+3 q x+(-1-2 q) x^2+x^3\right)^{1/3}}\left(-1-2 q-\frac{1-5 q+4 q^2+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{-(-1+q)^3 q}\right)^{2/3}}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{-(-1+q)^3 q}\right)^{1/3}}+3 x\right)^{1/3}$$

$$\left(-1+5 q-4 q^2+\frac{(1-4 q)^2 (1-q)^2}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}}+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}+\right.$$

$$\left.\frac{3\left(1-5 q+4 q^2+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}\right)\left(\frac{1}{3}(-1-2 q)+x\right)}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{1/3}}+9\left(\frac{1}{3}(-1-2 q)+x\right)^2\right)^{1/3}$$

$$\text{Unintegrable}\left[3\Big/\left(x\left(-1-2 q-\frac{1-5 q+4 q^2+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{-(-1+q)^3 q}\right)^{2/3}}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{-(-1+q)^3 q}\right)^{1/3}}+3 x\right)^{1/3}\right.\right.$$

$$\left(-1+5 q-4 q^2+\frac{(1-4 q)^2 (1-q)^2}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}}+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}+\right.$$

$$\left.\left.\left.9\left(\frac{1}{3}(-1-2 q)+x\right)^2+\frac{\left(1-5 q+4 q^2+\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{2/3}\right)(-1-2 q+3 x)}{\left(1+6 q-15 q^2+8 q^3+3 \sqrt{3} \sqrt{(1-q)^3 q}\right)^{1/3}}\right)^{1/3}\right),x\right]$$

Problem 44: Unable to integrate problem.

$$\int \frac{2-(1+k) x}{\left((1-x) x (1-k x)\right)^{1/3} (1-(1+k) x)}\, dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2 k^{1/3} x}{((1-x) x (1-k x))^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2 k^{1/3}}+\frac{\text{Log}[1-(1+k) x]}{2 k^{1/3}}-\frac{3\,\text{Log}\left[-k^{1/3} x+((1-x) x (1-k x))^{1/3}\right]}{2 k^{1/3}}$$

Result (type 8, 139 leaves, 3 steps):

$$\frac{3 (1-x)^{1/3} x (1-k x)^{1/3} \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},x,k x\right]}{2 \left((1-x) x (1-k x)\right)^{1/3}}+\frac{(1-x)^{1/3} x^{1/3} (1-k x)^{1/3} \text{CannotIntegrate}\left[\frac{1}{(1-x)^{1/3} x^{1/3} (1+(-1-k) x) (1-k x)^{1/3}},x\right]}{\left((1-x) x (1-k x)\right)^{1/3}}$$

Problem 45: Unable to integrate problem.

$$\int \frac{1-k x}{(1+(-2+k) x) \left((1-x) x (1-k x)\right)^{2/3}} \, dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-k x)}{(1-k)^{1/3} \left((1-x) x (1-k x)\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} (1-k)^{1/3}}+\frac{\text{Log}\left[1-(2-k) x\right]}{2^{2/3} (1-k)^{1/3}}+\frac{\text{Log}\left[1-k x\right]}{2\times 2^{2/3} (1-k)^{1/3}}-\frac{3 \text{Log}\left[-1+k x+2^{2/3} (1-k)^{1/3} \left((1-x) x (1-k x)\right)^{1/3}\right]}{2\times 2^{2/3} (1-k)^{1/3}}$$

Result (type 8, 78 leaves, 1 step):

$$\frac{(1-x)^{2/3} x^{2/3} (1-k x)^{2/3} \text{CannotIntegrate}\left[\frac{(1-k x)^{1/3}}{(1-x)^{2/3} x^{2/3} (1+(-2+k) x)},x\right]}{\left((1-x) x (1-k x)\right)^{2/3}}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x+c x^2}{\left(1-x+x^2\right) \left(1-x^3\right)^{1/3}} \, dx$$

Optimal (type 3, 493 leaves, 19 steps):

$$\frac{(a+b) \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{(a+b) \text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}-\frac{c \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{(a-c) \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{(b+c) \text{ArcTan}\left[\frac{1+2^{2/3} \left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+$$
$$\frac{(a+b) \text{Log}\left[(1-x) (1+x)^2\right]}{12\times 2^{1/3}}-\frac{(a-c) \text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{(b+c) \text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{(a+b) \text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{(a+b) \text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+$$
$$\frac{(b+c) \text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}+\frac{(a-c) \text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}+\frac{1}{2} c \text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{(a+b) \text{Log}\left[-1+x+2^{2/3} \left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 576 leaves, 7 steps):

$$-\frac{c\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\left(2a+b-i\sqrt{3}\,b-\left(1+i\sqrt{3}\right)c\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(2a+b+i\sqrt{3}\,b-c+i\sqrt{3}\,c\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+\frac{\left(3\,i\,b-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{12\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(3\,i\,b+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{12\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\frac{1}{2}c\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{\left(3\,i\,b-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-$$

$$\frac{\left(3\,i\,b+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right]$$

Result (type 4, 204 leaves, 9 steps):

$$\frac{2\,i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2}{1-i\,a}\right]}{\sqrt{-(a-x)\left(1+x^2\right)}}+\frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticPi}\left[\frac{2}{1-i\left(a-\sqrt{1+a^2}\right)},\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2}{1-i\,a}\right]}{\left(1-i\left(a-\sqrt{1+a^2}\right)\right)\sqrt{-(a-x)\left(1+x^2\right)}}$$

Problem 56: Result valid but suboptimal antiderivative.

$$\int x \left(1-x^3\right)^{1/3} \, dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\frac{1}{3} x^2 \left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}-\frac{1}{6} \text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 3, 107 leaves, 8 steps):

$$\frac{1}{3} x^2 \left(1-x^3\right)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}}+\frac{1}{18} \text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{1}{9} \text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]$$

Problem 58: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{1+x} \, dx$$

Optimal (type 3, 482 leaves, 25 steps):

$$\left(1-x^3\right)^{1/3}+\frac{2^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{1/3} \text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$
$$\frac{2^{1/3} \text{ArcTan}\left[\frac{1+2^{2/3} \left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{3}\times 2^{1/3} \text{Log}\left[1+x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3} \text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]-$$
$$\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}-\frac{1}{2} \text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3} x-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{1+x},x\right]$$

Problem 59: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Optimal (type 3, 280 leaves, 19 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[-3\,(-1+x)\,(1-x+x^2)\right]}{2\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[-2^{1/3}\,(-1+x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}x+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 79 leaves, 2 steps):

$$\frac{2\,i\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{1+i\sqrt{3}-2x},x\right]}{\sqrt{3}}+\frac{2\,i\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{-1+i\sqrt{3}+2x},x\right]}{\sqrt{3}}$$

Problem 60: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{2+x}\,dx$$

Optimal (type 6, 232 leaves, 12 steps):

$$\left(1-x^3\right)^{1/3}+\frac{1}{2}x\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right]-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+3^{1/6}\,\text{ArcTan}\left[\frac{1-\frac{3^{2/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]-3^{1/6}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(1-x^3\right)^{1/3}}{3\times 3^{1/6}}\right]-\frac{\text{Log}\left[8+x^3\right]}{3^{1/3}}+\frac{1}{2}\times 3^{2/3}\,\text{Log}\left[3^{2/3}-\left(1-x^3\right)^{1/3}\right]-\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{1}{2}\times 3^{2/3}\,\text{Log}\left[-\frac{1}{2}\times 3^{2/3}x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{2+x},x\right]$$

Problem 61: Unable to integrate problem.

$$\int \frac{2+x}{\left(1+x+x^2\right)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 168 leaves, 9 steps) :

$$-\frac{x^2\,\text{AppellF1}\left[\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right]}{2\times 2^{1/3}}+\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}\,x}{\left(2+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}+\frac{\text{ArcTan}\left[\frac{3^{1/3}+2\,\left(2+x^3\right)^{1/3}}{3^{5/6}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-x^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}-\left(2+x^3\right)^{1/3}\right]}{2\times 3^{1/3}}-\frac{\text{Log}\left[3^{1/3}\,x-\left(2+x^3\right)^{1/3}\right]}{3^{1/3}}$$

Result (type 8, 81 leaves, 2 steps) :

$$\left(1-i\sqrt{3}\right)\,\text{Unintegrable}\left[\frac{1}{\left(1-i\sqrt{3}+2\,x\right)\,\left(2+x^3\right)^{1/3}},x\right]+\left(1+i\sqrt{3}\right)\,\text{Unintegrable}\left[\frac{1}{\left(1+i\sqrt{3}+2\,x\right)\,\left(2+x^3\right)^{1/3}},x\right]$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-2+a+x}{\left(-a+x\right)\,\sqrt{\left(2-a\right)\,a\,x+\left(-1-2\,a+a^2\right)\,x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

0

Result (type 4, 529 leaves, 5 steps) :

$$\frac{2\,(1-a)\,\sqrt{x}\,\sqrt{(2-a)\,a-\left(1+2\,a-a^2\right)\,x+x^2}\;\text{ArcTan}\Big[\frac{\sqrt{-1+2\,a-a^2}\,\sqrt{x}}{\sqrt{(2-a)\,a-\left(1+2\,a-a^2\right)\,x+x^2}}\Big]}{a\,\sqrt{-1+2\,a-a^2}\,\sqrt{(2-a)\,a\,x-\left(1+2\,a-a^2\right)\,x^2+x^3}}+$$

$$\left(((2-a)\,a)^{3/4}\,\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-\left(1+2\,a-a^2\right)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\;\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\Big],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\Big]\right)\Big/$$

$$\left(a\,\sqrt{(2-a)\,a\,x-\left(1+2\,a-a^2\right)\,x^2+x^3}\right)+\left((2-a)\,\left(1-\sqrt{(2-a)\,a}\right)\,\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-\left(1+2\,a-a^2\right)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\right.$$

$$\left.\text{EllipticPi}\Big[\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\,\sqrt{(2-a)\,a}},\,2\,\text{ArcTan}\Big[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\Big],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\Big]\right)\Big/\left(((2-a)\,a)^{3/4}\,\sqrt{(2-a)\,a\,x-\left(1+2\,a-a^2\right)\,x^2+x^3}\right)$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-a+(-1+2\,a)\,x}{(-a+x)\,\sqrt{a^2\,x-\left(-1+2\,a+a^2\right)\,x^2+(-1+2\,a)\,x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\Big[\frac{-a^2+2\,a\,x+x^2-2\,\left(x+\sqrt{(1-x)\,x\,\left(a^2+x-2\,a\,x\right)}\right)}{(a-x)^2}\Big]$$

Result (type 4, 180 leaves, 7 steps):

$$-\frac{2\,(1-2\,a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticF}\Big[\text{ArcSin}\big[\sqrt{x}\big],\,-\frac{1-2\,a}{a^2}\Big]}{\sqrt{a^2\,x+\left(1-2\,a-a^2\right)\,x^2-(1-2\,a)\,x^3}}+\frac{4\,(1-a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticPi}\Big[\frac{1}{a},\,\text{ArcSin}\big[\sqrt{x}\big],\,-\frac{1-2\,a}{a^2}\Big]}{\sqrt{a^2\,x+\left(1-2\,a-a^2\right)\,x^2-(1-2\,a)\,x^3}}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-x^3\right) \left(a+b x^3\right)^{1/3}} \, dx$$

Optimal (type 3, 98 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 (a+b)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a+b)^{1/3}}+\frac{\text{Log}\left[1-x^3\right]}{6 (a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 (a+b)^{1/3}}$$

Result (type 3, 135 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 (a+b)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a+b)^{1/3}}-\frac{\text{Log}\left[1-\frac{(a+b)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{3 (a+b)^{1/3}}+\frac{\text{Log}\left[1+\frac{(a+b)^{2/3} x^2}{\left(a+b x^3\right)^{2/3}}+\frac{(a+b)^{1/3} x}{\left(a+b x^3\right)^{1/3}}\right]}{6 (a+b)^{1/3}}$$

Problem 95: Unable to integrate problem.

$$\int \frac{1+x}{\left(1+x+x^2\right) \left(a+b x^3\right)^{1/3}} \, dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 (a+b)^{1/3} x}{\left(a+b x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a+b)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2 \left(a+b x^3\right)^{1/3}}{(a+b)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a+b)^{1/3}}+\frac{\text{Log}\left[(a+b)^{1/3}-\left(a+b x^3\right)^{1/3}\right]}{2 (a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3} x-\left(a+b x^3\right)^{1/3}\right]}{2 (a+b)^{1/3}}$$

Result (type 8, 91 leaves, 2 steps):

$$\frac{1}{3} \left(3-i \sqrt{3}\right) \text{Unintegrable}\left[\frac{1}{\left(1-i \sqrt{3}+2 x\right) \left(a+b x^3\right)^{1/3}},x\right]+\frac{1}{3} \left(3+i \sqrt{3}\right) \text{Unintegrable}\left[\frac{1}{\left(1+i \sqrt{3}+2 x\right) \left(a+b x^3\right)^{1/3}},x\right]$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}\,x-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 3, 122 leaves, 7 steps) :

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,x}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\,x}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\left(1-x^3\right)^{1/3}\,\left(1+x^3\right)}\,dx$$

Optimal (type 3, 233 leaves, 8 steps) :

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\,\sqrt{3}}+\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\,\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step) :

$$\frac{1}{2}\,x^2\,\text{AppellF1}\left[\frac{2}{3},\,\frac{1}{3},\,1,\,\frac{5}{3},\,x^3,\,-x^3\right]$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+x}{\left(1-x+x^2\right)\,\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 4 steps) :

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$
$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$
$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(1+x)^2}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 5 steps):

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$
$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$
$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1-x}{\left(1+x+x^2\right)\left(1+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 119 leaves, ? steps):

$$-\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}\ (1+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\ (1+x)^2}{(1+x^3)^{2/3}}-\frac{2^{1/3}\ (1+x)}{(1+x^3)^{1/3}}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\ (1+x)}{(1+x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 399 leaves, 4 steps):

$$\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}-2x\right)}{(1+x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}-2x\right)}{(1+x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$
$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}-2x\right)\left(1-i\sqrt{3}+2x\right)^2\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{\left(i+\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}-2x\right)\left(1+i\sqrt{3}+2x\right)^2\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$
$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}-2x+2\times 2^{2/3}\left(1+x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}-2x+2\times 2^{2/3}\left(1+x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 103: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2}\,dx$$

Optimal (type 5, 43 leaves, 5 steps):

$$\frac{1}{\left(1-x^3\right)^{1/3}}+\frac{x}{\left(1-x^3\right)^{1/3}}-x^2\ \text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right]$$

Result (type 8, 151 leaves, 2 steps):

$$-\frac{4}{3}\ \text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4\,i\ \text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}-2x},x\right]}{3\sqrt{3}}-$$
$$\frac{4}{3}\ \text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4\,i\ \text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+i\sqrt{3}+2x},x\right]}{3\sqrt{3}}$$

Problem 104: Unable to integrate problem.

$$\int \frac{1-x}{\left(1+x+x^2\right)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 5, 43 leaves, 5 steps):

$$\frac{1}{\left(1-x^3\right)^{1/3}}+\frac{x}{\left(1-x^3\right)^{1/3}}-x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right]$$

Result (type 8, 87 leaves, 2 steps):

$$-\left(1+i\sqrt{3}\right)\,\text{Unintegrable}\left[\frac{1}{\left(1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{1/3}},x\right]-\left(1-i\sqrt{3}\right)\,\text{Unintegrable}\left[\frac{1}{\left(1+i\sqrt{3}+2x\right)\left(1-x^3\right)^{1/3}},x\right]$$

Problem 108: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{a+b\,x}\,dx$$

Optimal (type 6, 384 leaves, 13 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{2\,b}-\frac{\left(a^3+b^3\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right]}{2\,a^2\,b^2}+\frac{a^2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}-\frac{\left(a^3+b^3\right)^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\left(a^3+b^3\right)^{1/3}x}{a\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+$$

$$\frac{\left(a^3+b^3\right)^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2b\left(1-x^3\right)^{1/3}}{\left(a^3+b^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+\frac{a\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]}{2\,b^2}-\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[a^3+b^3x^3\right]}{3\,b^3}+$$

$$\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[-\frac{\left(a^3+b^3\right)^{1/3}x}{a}-\left(1-x^3\right)^{1/3}\right]}{2\,b^3}-\frac{a^2\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]}{2\,b^3}+\frac{\left(a^3+b^3\right)^{2/3}\,\text{Log}\left[\left(a^3+b^3\right)^{1/3}-b\left(1-x^3\right)^{1/3}\right]}{2\,b^3}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{a+b\,x},x\right]$$

Problem 109: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Optimal (type 5, 234 leaves, 13 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{x\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{2x^2\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+$$
$$\frac{1}{3}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{3\times 2^{1/3}}$$

Result (type 8, 151 leaves, 2 steps):

$$-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+i\sqrt{3}-2x},x\right]}{3\sqrt{3}}-$$
$$\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}+2x},x\right]}{3\sqrt{3}}$$

Problem 110: Unable to integrate problem.

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Optimal (type 3, 199 leaves, 14 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{1-x+x^2}-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$
$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}+\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 159 leaves, 6 steps):

$$-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4}{3}\left(1+i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4}{3}\left(1-i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]$$

Problem 111: Unable to integrate problem.

$$\int\frac{(1-x^3)^{2/3}}{1+x}\,dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{1}{2}(1-x^3)^{2/3}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{3\,\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{1+x},x\right]$$

Problem 112: Unable to integrate problem.

$$\int\frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\frac{1}{2}(1-x^3)^{2/3}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{3\,\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 19 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+x},\ x\right]$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 21 leaves, 1 step):

$$x\,\text{AppellF1}\left[\frac{1}{3},\,-\frac{2}{3},\,1,\,\frac{4}{3},\,x^3,\,-x^3\right]$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 250 leaves, 10 steps):

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\,\frac{2}{3},\,\frac{5}{3},\,x^3\right]+$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}x^2\,\text{AppellF1}\left[\frac{2}{3},\,-\frac{2}{3},\,1,\,\frac{5}{3},\,x^3,\,-x^3\right]$$

Problem 115: Unable to integrate problem.

$$\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 383 leaves, ? steps):

$$-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$

$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+$$

$$\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]+\frac{\text{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 101 leaves, 2 steps):

$$-\frac{2}{3}\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1-x},x\right]-\frac{1}{3}\left(1+(-1)^{2/3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1+(-1)^{1/3}x},x\right]-\frac{1}{3}\left(1-(-1)^{1/3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}x},x\right]$$

Problem 116: Result unnecessarily involves higher level functions.

$$\int\frac{(1-x^3)^{1/3}}{1+x^3}\,dx$$

Optimal (type 3, 272 leaves, 14 steps):

$$\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}-$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}$$

Result (type 6, 21 leaves, 1 step):

$$x\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]$$

Test results for the 14 problems in "Bronstein Problems.m"

Problem 12: Unable to integrate problem.

$$\int \frac{x^2+2\,x\,\text{Log}[x]+\text{Log}[x]^2+(1+x)\,\sqrt{x+\text{Log}[x]}}{x^3+2\,x^2\,\text{Log}[x]+x\,\text{Log}[x]^2}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\text{Log}[x]-\frac{2}{\sqrt{x+\text{Log}[x]}}$$

Result (type 8, 65 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(x+\text{Log}[x])^{3/2}},x\right]-\text{CannotIntegrate}\left[\frac{1}{\text{Log}[x]\,(x+\text{Log}[x])^{3/2}},x\right]-\text{CannotIntegrate}\left[\frac{1}{\text{Log}[x]^2\,\sqrt{x+\text{Log}[x]}},x\right]+\text{CannotIntegrate}\left[\frac{\sqrt{x+\text{Log}[x]}}{x\,\text{Log}[x]^2},x\right]+\text{Log}[x]$$

Test results for the 35 problems in "Bondarenko Problems.m"

Problem 21: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(\text{Cos}[x]+\text{Cos}[3\,x])^5}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\frac{523}{256}\,\text{ArcTanh}[\text{Sin}[x]]+\frac{1483\,\text{ArcTanh}\left[\sqrt{2}\,\text{Sin}[x]\right]}{512\,\sqrt{2}}+\frac{\text{Sin}[x]}{32\,(1-2\,\text{Sin}[x]^2)^4}-\frac{17\,\text{Sin}[x]}{192\,(1-2\,\text{Sin}[x]^2)^3}+\frac{203\,\text{Sin}[x]}{768\,(1-2\,\text{Sin}[x]^2)^2}-\frac{437\,\text{Sin}[x]}{512\,(1-2\,\text{Sin}[x]^2)}-\frac{43}{256}\,\text{Sec}[x]\,\text{Tan}[x]-\frac{1}{128}\,\text{Sec}[x]^3\,\text{Tan}[x]$$

Result (type 3, 786 leaves, 45 steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]-\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-\frac{1}{128\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^4}+\frac{1}{64\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^3}-\frac{47}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^2}+\frac{45}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}+\frac{1}{128\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^4}-\frac{1}{64\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^3}+\frac{47}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^2}-\frac{45}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}-\frac{7-17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{119\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{11\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{1-43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}-\frac{65\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{451\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-\frac{89+15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}+\frac{7+17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{11\left(1-3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{119\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}+\frac{65\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{1+43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{89-15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-\frac{451\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Tanh}[x]}{\sqrt{e^x+e^{2x}}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2\,e^{-x}\sqrt{e^x+e^{2x}}-\frac{\text{ArcTan}\left[\frac{i-(1-2i)\,e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1+i}}+\frac{\text{ArcTan}\left[\frac{i+(1+2i)\,e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1-i}}$$

Result (type 3, 147 leaves, 11 steps):

$$\frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}}-\frac{(1-i)^{3/2}\sqrt{e^x}\,\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1-i}\,\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}-\frac{(1+i)^{3/2}\sqrt{e^x}\,\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1+i}\,\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[x^2+\sqrt{1-x^2}\right] dx$$

Optimal (type 3, 185 leaves, ? steps):

$$-2 x-\text{ArcSin}[x]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+x\,\text{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Result (type 3, 349 leaves, 31 steps):

$$-2 x-\text{ArcSin}[x]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]-\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\ x}{\sqrt{1-x^2}}\right]+x\,\text{Log}\left[x^2+\sqrt{1-x^2}\right]$$