Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "0 Independent test suites"

Test results for the 175 problems in "Apostol Problems.m"

Test results for the 35 problems in "Bondarenko Problems.m"

Problem 7: Unable to integrate problem.

$$\int \frac{\text{Log}[1+x]}{x\sqrt{1+\sqrt{1+x}}}\,dx$$

Optimal (type 4, 291 leaves, ? steps):

$$-8\,\text{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]-\frac{2\,\text{Log}[1+x]}{\sqrt{1+\sqrt{1+x}}}-\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\text{Log}[1+x]+$$
$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+$$
$$\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-$$
$$\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Log}[1+x]}{x\sqrt{1+\sqrt{1+x}}},x\right]$$

Problem 8: Unable to integrate problem.

$$\int \frac{\sqrt{1+\sqrt{1+x}}\,\text{Log}[1+x]}{x}\,dx$$

Optimal (type 4, 308 leaves, ? steps):

$$-16\sqrt{1+\sqrt{1+x}}+16\,\text{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]+4\sqrt{1+\sqrt{1+x}}\,\text{Log}[1+x]-2\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\text{Log}[1+x]+4\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-4\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-2\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-2\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+2\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1+\sqrt{1+x}}\,\text{Log}[1+x]}{x},x\right]$$

Problem 21: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(\text{Cos}[x]+\text{Cos}[3x])^5}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]+\frac{1483\,\text{ArcTanh}\left[\sqrt{2}\,\text{Sin}[x]\right]}{512\sqrt{2}}+\frac{\text{Sin}[x]}{32\left(1-2\,\text{Sin}[x]^2\right)^4}-\frac{17\,\text{Sin}[x]}{192\left(1-2\,\text{Sin}[x]^2\right)^3}+\frac{203\,\text{Sin}[x]}{768\left(1-2\,\text{Sin}[x]^2\right)^2}-\frac{437\,\text{Sin}[x]}{512\left(1-2\,\text{Sin}[x]^2\right)}-\frac{43}{256}\text{Sec}[x]\,\text{Tan}[x]-\frac{1}{128}\text{Sec}[x]^3\,\text{Tan}[x]$$

Result (type 3, 786 leaves, 45 steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]-\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-$$

$$\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$

$$\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$

$$\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-\frac{1}{128\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^4}+$$

$$\frac{1}{64\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^3}-\frac{47}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^2}+\frac{45}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}+\frac{1}{128\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^4}-$$

$$\frac{1}{64\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^3}+\frac{47}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^2}-\frac{45}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}-\frac{7-17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+$$

$$\frac{119\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{11\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{1-43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}-$$

$$\frac{65\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{451\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-$$

$$\frac{89+15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}+\frac{7+17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{11\left(1-3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-$$

$$\frac{119\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}+\frac{65\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+$$

$$\frac{1+43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{89-15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-\frac{451\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Tanh}[x]}{\sqrt{e^x+e^{2x}}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2\,e^{-x}\sqrt{e^x+e^{2x}}-\frac{\text{ArcTan}\left[\frac{i-(1-2\,i)\,e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1+i}}+\frac{\text{ArcTan}\left[\frac{i+(1+2\,i)\,e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1-i}}$$

Result (type 3, 147 leaves, 11 steps):

$$\frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}}-\frac{(1-i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1-i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}-\frac{(1+i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1+i}\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[x^2+\sqrt{1-x^2}\right]dx$$

Optimal (type 3, 185 leaves, ? steps):

$$-2x-\text{ArcSin}[x]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+x\,\text{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Result (type 3, 349 leaves, 31 steps):

$$-2\,x-\mathrm{ArcSin}[x]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\;\mathrm{ArcTan}\Big[\sqrt{\frac{2}{1+\sqrt{5}}}\;x\Big]+$$

$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\;\mathrm{ArcTan}\Big[\sqrt{\frac{2}{1+\sqrt{5}}}\;x\Big]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\;\mathrm{ArcTan}\Big[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+$$

$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\;\mathrm{ArcTan}\Big[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\;\mathrm{ArcTanh}\Big[\sqrt{\frac{2}{-1+\sqrt{5}}}\;x\Big]+$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\;\mathrm{ArcTanh}\Big[\sqrt{\frac{2}{-1+\sqrt{5}}}\;x\Big]-2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\;\mathrm{ArcTanh}\Big[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]-$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\;\mathrm{ArcTanh}\Big[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+x\,\mathrm{Log}\Big[x^2+\sqrt{1-x^2}\Big]$$

Test results for the 14 problems in "Bronstein Problems.m"

Problem 12: Unable to integrate problem.

$$\int\frac{x^2+2\,x\,\mathrm{Log}[x]+\mathrm{Log}[x]^2+(1+x)\sqrt{x+\mathrm{Log}[x]}}{x^3+2\,x^2\,\mathrm{Log}[x]+x\,\mathrm{Log}[x]^2}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\mathrm{Log}[x]-\frac{2}{\sqrt{x+\mathrm{Log}[x]}}$$

Result (type 8, 65 leaves, 3 steps):

$$\mathrm{CannotIntegrate}\Big[\frac{1}{(x+\mathrm{Log}[x])^{3/2}},\,x\Big]-\mathrm{CannotIntegrate}\Big[\frac{1}{\mathrm{Log}[x]\;(x+\mathrm{Log}[x])^{3/2}},\,x\Big]-$$

$$\mathrm{CannotIntegrate}\Big[\frac{1}{\mathrm{Log}[x]^2\sqrt{x+\mathrm{Log}[x]}},\,x\Big]+\mathrm{CannotIntegrate}\Big[\frac{\sqrt{x+\mathrm{Log}[x]}}{x\,\mathrm{Log}[x]^2},\,x\Big]+\mathrm{Log}[x]$$

Test results for the 50 problems in "Charlwood Problems.m"

Problem 3: Unable to integrate problem.

$$\int -\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]\,dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{4\sqrt{2}}-\left(\frac{3}{8}+x\right)\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]$$

Result (type 8, 60 leaves, 3 steps):

$$-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]+\frac{\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}},x\right]}{2\sqrt{2}}$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[1+x\sqrt{1+x^2}\right]\,dx$$

Optimal (type 3, 97 leaves, ? steps):

$$-2x+\sqrt{2\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]-\sqrt{2\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Result (type 3, 332 leaves, 32 steps):

$$-2x-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+$$
$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \mathrm{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+$$
$$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \mathrm{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+$$
$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \mathrm{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-$$
$$\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \mathrm{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+x\,\mathrm{Log}\left[1+x\sqrt{1+x^2}\right]$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Cos}[x]^2}{\sqrt{1+\mathrm{Cos}[x]^2+\mathrm{Cos}[x]^4}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3}+\frac{1}{3}\mathrm{ArcTan}\left[\frac{\mathrm{Cos}[x]\ \left(1+\mathrm{Cos}[x]^2\right)\ \mathrm{Sin}[x]}{1+\mathrm{Cos}[x]^2\sqrt{1+\mathrm{Cos}[x]^2+\mathrm{Cos}[x]^4}}\right]$$

Result (type 4, 289 leaves, 5 steps) :

$$\frac{\text{ArcTan}\left[\frac{\text{Tan}[x]}{\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}\right]\,\text{Cos}[x]^2\,\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}{2\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}-$$

$$\left(\left(1+\sqrt{3}\right)\,\text{Cos}[x]^2\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\,\frac{1}{4}\left(2-\sqrt{3}\right)\right]\,\left(\sqrt{3}+\text{Tan}[x]^2\right)\right.$$

$$\left.\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/\left(4\times 3^{1/4}\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)+$$

$$\left(\left(2+\sqrt{3}\right)\,\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{1}{6}\left(3-2\,\sqrt{3}\right),\,2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\,\frac{1}{4}\left(2-\sqrt{3}\right)\right]\right.$$

$$\left.\left(\sqrt{3}+\text{Tan}[x]^2\right)\,\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/\left(4\times 3^{1/4}\,\sqrt{\text{Cos}[x]^4\,\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcTan}\left[x+\sqrt{1-x^2}\right]\,dx$$

Optimal (type 3, 141 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2\,x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\,\text{ArcTanh}\left[x\,\sqrt{1-x^2}\right]-\frac{1}{8}\,\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 269 leaves, 40 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}}+$$

$$\frac{1}{12}\left(3i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}-$$

$$\frac{1}{12}\left(3i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 152 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-$$

$$\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 286 leaves, 32 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{2\sqrt{3}}-$$

$$\frac{1}{12}\left(3i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{2\sqrt{3}}+$$

$$\frac{1}{12}\left(3i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]}{\sqrt{-1+\text{Sec}[x]^4}}\,dx$$

Optimal (type 3, 28 leaves, ? steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]\,\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Sin}[x]}{\sqrt{2\,\text{Sin}[x]^2-\text{Sin}[x]^4}}\right]\sqrt{1-\text{Cos}[x]^4}\,\text{Sec}[x]^2}{\sqrt{2}\,\sqrt{-1+\text{Sec}[x]^4}}$$

Problem 45: Unable to integrate problem.

$$\int \sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\,dx$$

Optimal (type 3, 337 leaves, ? steps):

$$\sqrt{2}\left(\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]-\right.$$
$$\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]-$$
$$\sqrt{1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]+$$
$$\left.\sqrt{-1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]\right)$$
$$\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]}\,\sqrt{1+\text{Sec}[x]}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}},\,x\right]$$

Test results for the 284 problems in "Hearn Problems.m"

Problem 169: Unable to integrate problem.

$$\int \frac{e^{1-e^{x^2} x+2 x^2} \left(x+2 x^3\right)}{\left(1-e^{x^2} x\right)^2} \, dx$$

Optimal (type 3, 25 leaves, ? steps):

$$-\frac{e^{1-e^{x^2} x}}{-1+e^{x^2} x}$$

Result (type 8, 69 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x}{\left(-1+e^{x^2} x\right)^2}, x\right]+2\, \text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x^3}{\left(-1+e^{x^2} x\right)^2}, x\right]$$

Problem 278: Unable to integrate problem.

$$\int \frac{-8-8 x-x^2-3 x^3+7 x^4+4 x^5+2 x^6}{\left(-1+2 x^2\right)^2 \sqrt{1+2 x^2+4 x^3+x^4}} \, dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1+2 x) \sqrt{1+2 x^2+4 x^3+x^4}}{2 \left(-1+2 x^2\right)}-\text{ArcTanh}\left[\frac{x (2+x) \left(7-x+27 x^2+33 x^3\right)}{\left(2+37 x^2+31 x^3\right) \sqrt{1+2 x^2+4 x^3+x^4}}\right]$$

Result (type 8, 354 leaves, 10 steps):

$$\frac{9}{4}\,\text{CannotIntegrate}\Big[\frac{1}{\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{13}{4}\,\text{CannotIntegrate}\Big[\frac{1}{\left(\sqrt{2}-2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\Big]+$$
$$\text{CannotIntegrate}\Big[\frac{x}{\sqrt{1+2x^2+4x^3+x^4}},x\Big]+\frac{1}{2}\,\text{CannotIntegrate}\Big[\frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{13}{4}\,\text{CannotIntegrate}\Big[\frac{1}{\left(\sqrt{2}+2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{13}{8}\,\text{CannotIntegrate}\Big[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{1}{8}\left(15+\sqrt{2}\right)\text{CannotIntegrate}\Big[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{13}{8}\,\text{CannotIntegrate}\Big[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{1}{8}\left(15-\sqrt{2}\right)\text{CannotIntegrate}\Big[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\Big]-$$
$$\frac{17}{2}\,\text{CannotIntegrate}\Big[\frac{x}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\Big]$$

Problem 279: Unable to integrate problem.

$$\int\frac{(1+2y)\sqrt{1-5y-5y^2}}{y\,(1+y)\,(2+y)\sqrt{1-y-y^2}}\,dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4}\,\text{ArcTanh}\Big[\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\Big]-$$
$$\frac{1}{2}\,\text{ArcTanh}\Big[\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\Big]+\frac{9}{4}\,\text{ArcTanh}\Big[\frac{(11+7y)\sqrt{1-5y-5y^2}}{3\,(7+5y)\sqrt{1-y-y^2}}\Big]$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{1}{2}\,\text{CannotIntegrate}\Big[\frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}},y\Big]+$$
$$\text{CannotIntegrate}\Big[\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}},y\Big]-\frac{3}{2}\,\text{CannotIntegrate}\Big[\frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}},y\Big]$$

Problem 281: Unable to integrate problem.

$$\int \left(\sqrt{9-4\sqrt{2}}\ x-\sqrt{2}\ \sqrt{1+4x+2x^2+x^4}\right) dx$$

Optimal (type 4, 4030 leaves, ? steps):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\ x^2-\sqrt{2}\left(-\frac{1}{3}\sqrt{1+4x+2x^2+x^4}\ +\right.$$

$$\frac{1}{3}(1+x)\sqrt{1+4x+2x^2+x^4}+\left(4\,i\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{1+4x+2x^2+x^4}\right)\Big/$$

$$\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+\right.$$

$$\left.6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)-\left(8\times 2^{2/3}\sqrt{\frac{3}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}\right.$$

$$\sqrt{\left(\left(i\left(-19899+3445\sqrt{33}+\left(-26+6\sqrt{33}\right)^{2/3}\left(-2574+466\sqrt{33}\right)+\right.\right.\right.}$$

$$\left.\left.\left(-26+6\sqrt{33}\right)^{1/3}\left(-19899+3445\sqrt{33}\right)+\left(59697-10335\sqrt{33}\right)x\right)\right)\Big/$$

$$\left(\left(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\right.$$

$$\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.$$

$$\left.\left.\left.\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\right)\sqrt{1+4x+2x^2+x^4}$$

$$\text{EllipticE}\Big[\text{ArcSin}\Big[\left(\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\right.}\right.$$

$$\left.\left.\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\Big/$$

$$\left(\sqrt{\left(\left(39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big/\right.}\right.$$

$$\left.\left.\left(39-13\,i\sqrt{3}+9\,i\sqrt{11}-9\sqrt{33}+4\left(3+i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\right)\right)$$

$$\sqrt{\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.}$$

$$\left.\left.\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\Big],$$

$$\left(4\left(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big/$$

$$\left(4\left(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\right)+\right.$$

$$\left.\left.\left(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big]\right)\Big/$$

$$\Bigg(\Big(4\times 2^{2/3}-\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}+3\left(-13+3\sqrt{33}\right)^{1/3}x\Big)$$

$$\sqrt{\Big(\left(\mathrm{i}\,(1+x)\right)\Big/\Big(\Big(104-24\sqrt{33}+\left(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big(26-6\sqrt{33}+\left(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)\Big)\Big)}$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)}$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)}\;+$$

$$\Bigg(\Big(2^{1/3}\left(13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}-3\sqrt{33}\right)+4\times 2^{2/3}\left(1+\mathrm{i}\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{1/3}+20\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\Big(4\times 2^{2/3}\left(\mathrm{i}+\sqrt{3}\right)+8\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\sqrt{\frac{52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}$$

$$\sqrt{\Big(\frac{1}{1+x}\Big(-8\,\mathrm{i}\left(-13+3\sqrt{33}\right)+\left(-43\,\mathrm{i}-13\sqrt{3}+9\sqrt{11}+5\,\mathrm{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(2\,\mathrm{i}+4\sqrt{3}-2\,\mathrm{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+\Big(8\,\mathrm{i}\left(-13+3\sqrt{33}\right)+\left(13\,\mathrm{i}-13\sqrt{3}+9\sqrt{11}-3\,\mathrm{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)x\Big)\Big)}\;\sqrt{1+4x+2x^2+x^4}$$

$$\mathrm{EllipticF}\Big[\mathrm{ArcSin}\Big[\Big(\sqrt{\Big(52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}\Big)}\sqrt{\Big(26-6\sqrt{33}+\left(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)}\Big)\Big/\Big(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{\Big(39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}-9\sqrt{33}+4\left(3-\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)}\sqrt{1+x}\Big)\Big],$$

$$\Big(4\left(21\,\mathrm{i}-7\sqrt{3}+3\sqrt{11}-3\,\mathrm{i}\sqrt{33}\right)+\left(3\,\mathrm{i}+\sqrt{3}+3\sqrt{11}+3\,\mathrm{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/\Big(-56\sqrt{3}+24\sqrt{11}+2\left(\sqrt{3}+3\sqrt{11}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big]\Bigg)\Bigg/\Big(3\times 2^{2/3}\times 3^{3/4}$$

$$\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{\left(39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}-9\sqrt{33}+4\left(3-\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)}$$

$$\sqrt{1+x}\,\Big(4\times2^{2/3}\left(-\mathrm{i}+\sqrt{3}\right)-2\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+$$

$$2^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}x\Big)$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$4\,\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)$$

$$\sqrt{\Big(\Big(8\left(-13+3\sqrt{33}\right)-\left(5-3\,\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+\left(-26+6\sqrt{33}\right)^{1/3}}$$

$$\left(-41+15\,\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+7\sqrt{33}\right)+\Big(104-24\sqrt{33}+\Big(-13-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+$$

$$3\sqrt{33}\Big)\left(-26+6\sqrt{33}\right)^{1/3}+4\,\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)x\Big)\Big/$$

$$\Big(\Big(-39-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+9\sqrt{33}+4\,\mathrm{i}\left(3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)(1+x)\Big)\Big)\Big)+$$

$$\Big(\Big(4\times2^{2/3}+2\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\Big(4\times2^{2/3}\left(\mathrm{i}+\sqrt{3}\right)-4\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\Big(4\times2^{2/3}\left(-\mathrm{i}+\sqrt{3}\right)+4\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\sqrt{\Big(\Big(-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/}$$

$$\Big(104-24\sqrt{33}+\left(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+$$

$$\left(-4-4\,\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big)\sqrt{1+x}$$

$$\sqrt{\Big(\Big(104-24\sqrt{33}+2\left(1+14\,\mathrm{i}\sqrt{3}-6\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$\left(-7-\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+$$

$$2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\Big)\Big/$$

$$\Big(\Big(-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)(1+x)\Big)\Big)$$

$$\sqrt{\Big(\Big(104-24\sqrt{33}+2\left(1-14\,\mathrm{i}\sqrt{3}+6\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$\left(-7+\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+$$

$$2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\Big)\Big/\Big(\Big(-39-13\,\mathrm{i}\sqrt{3}+$$

$$9\,\mathrm{i}\sqrt{11}+9\sqrt{33}+4\,\mathrm{i}\left(3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)(1+x)\Big)\Big)\sqrt{1+4x+2x^2+x^4}$$

$$\mathtt{EllipticPi}\Big[\Big(2^{1/3}\Big(4\times2^{1/3}\left(-3\,\mathrm{i}+\sqrt{3}\right)+\left(3\,\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big)\Big)\Big/$$

$$\Big(4\times2^{2/3}\left(-\mathrm{i}+\sqrt{3}\right)-8\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big),$$

$$\text{ArcSin}\Big[\Big(\sqrt{\left(13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x\right)}\Big)\Big/$$

$$\Big(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{\Big(\Big(-39+13\,i\sqrt{3}-9\,i\sqrt{11}+9\sqrt{33}-4\,i\left(-3\,i+\sqrt{3}\right)}$$

$$\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/\Big(104-24\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)$$

$$\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big)\sqrt{1+x}\Big)\Big],$$

$$\Big(4\left(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/$$

$$\Big(4\left(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\right)+$$

$$\left(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big]\Big)\Big/$$

$$\Big(2^{1/6}\sqrt{3}\Big(4\times2^{2/3}\left(i+\sqrt{3}\right)+2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}-$$

$$6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\Big)\Big(4\times2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+$$

$$2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\Big)$$

$$\sqrt{\left(13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x\right)}\Big)$$

Result (type 8, 47 leaves, 1 step):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\;x^2-\sqrt{2}\;\text{CannotIntegrate}\left[\sqrt{1+4x+2x^2+x^4},\,x\right]$$

Problem 284: Unable to integrate problem.

$$\int\frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Optimal (type 3, 71 leaves, ? steps):

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\text{Log}\left[1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right]-\left(-1+\sqrt{2}\right)\text{Log}\left[-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right]\right)$$

Result (type 8, 248 leaves, 5 steps):

$$2\,\text{CannotIntegrate}\left[\frac{1}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{x}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{x^2}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$12\,\text{CannotIntegrate}\left[\frac{x^7}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$10\,\text{CannotIntegrate}\left[\frac{x^8}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$\frac{1}{2}\,\text{Log}\left[1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right]$$

Test results for the 7 problems in "Hebisch Problems.m"

Problem 2: Unable to integrate problem.

$$\int\frac{e^{\frac{x}{2+x^2}}\left(2-x^2\right)}{2x+x^3}\,dx$$

Optimal (type 4, 10 leaves, ? steps):

$$\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 76 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}-x},x\right]+\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x},x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}+x},x\right]$$

Problem 3: Unable to integrate problem.

$$\int\frac{e^{\frac{x}{2+x^2}}\left(2+2x+3x^2-x^3+2x^4\right)}{2x+x^3}\,dx$$

Optimal (type 4, 28 leaves, ? steps):

$$e^{\frac{x}{2+x^2}}\left(2+x^2\right)+\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 131 leaves, 5 steps):

$$-\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}}, x\right] + \left(1+i\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}-x}, x\right] + \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right] + 2\,\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}}\,x, x\right] - \left(1-i\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}+x}, x\right]$$

Problem 5: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{-1+x^2}}\left(1-3x-x^2+x^3\right)}{1-x-x^2+x^3}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\frac{1}{-1+x^2}}(1+x)$$

Result (type 8, 75 leaves, 6 steps):

$$\text{CannotIntegrate}\left[e^{\frac{1}{-1+x^2}}, x\right] + \frac{1}{2}\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1-x}, x\right] - \text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{(-1+x)^2}, x\right] + \frac{1}{2}\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1+x}, x\right]$$

Problem 7: Unable to integrate problem.

$$\int \frac{e^{x+\frac{1}{\text{Log}[x]}}\left(-1+(1+x)\,\text{Log}[x]^2\right)}{\text{Log}[x]^2}\,dx$$

Optimal (type 3, 10 leaves, ? steps):

$$e^{x+\frac{1}{\text{Log}[x]}}\,x$$

Result (type 8, 40 leaves, 2 steps):

$$\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}, x\right] + \text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}\,x, x\right] - \text{CannotIntegrate}\left[\frac{e^{x+\frac{1}{\text{Log}[x]}}}{\text{Log}[x]^2}, x\right]$$

Test results for the 9 problems in "Jeffrey Problems.m"

Problem 2: Result valid but suboptimal antiderivative.

$$\int \frac{1+\text{Cos}[x]+2\,\text{Sin}[x]}{3+\text{Cos}[x]^2+2\,\text{Sin}[x]-2\,\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 38 leaves, 43 steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]+\text{Cot}\left[\frac{x}{2}\right]-\frac{\text{Sin}[x]}{1-\text{Cos}[x]}$$

Problem 3: Result valid but suboptimal antiderivative.

$$\int\frac{2+\text{Cos}[x]+5\,\text{Sin}[x]}{4\,\text{Cos}[x]-2\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[x]-2\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1-3\,\text{Cos}[x]+\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 42 leaves, 25 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[1+\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int\frac{3+7\,\text{Cos}[x]+2\,\text{Sin}[x]}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-5\,\text{Sin}[x]-\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1+\text{Cos}[x]-2\,\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 31 leaves, 32 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 5: Unable to integrate problem.

$$\int\frac{-1+4\,\text{Cos}[x]+5\,\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 43 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{3+\text{Cos}[x]}\right]-2\,\text{ArcTan}\left[\frac{3\,\text{Sin}[x]+7\,\text{Cos}[x]\,\text{Sin}[x]}{1+2\,\text{Cos}[x]+5\,\text{Cos}[x]^2}\right]$$

Result (type 8, 79 leaves, 2 steps):

$$\text{CannotIntegrate}\left[\frac{1}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-4\,\text{Cos}[x]^3},x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+$$
$$5\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]$$

Problem 6: Unable to integrate problem.

$$\int \frac{-5+2\,\mathrm{Cos}[x]+7\,\mathrm{Cos}[x]^2}{-1+2\,\mathrm{Cos}[x]-9\,\mathrm{Cos}[x]^2+4\,\mathrm{Cos}[x]^3}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$x-2\,\mathrm{ArcTan}\left[\frac{2\,\mathrm{Cos}[x]\,\mathrm{Sin}[x]}{1-\mathrm{Cos}[x]+2\,\mathrm{Cos}[x]^2}\right]$$

Result (type 8, 81 leaves, 2 steps):

$$-5\,\mathrm{CannotIntegrate}\left[\frac{1}{-1+2\,\mathrm{Cos}[x]-9\,\mathrm{Cos}[x]^2+4\,\mathrm{Cos}[x]^3},x\right]+$$
$$2\,\mathrm{CannotIntegrate}\left[\frac{\mathrm{Cos}[x]}{-1+2\,\mathrm{Cos}[x]-9\,\mathrm{Cos}[x]^2+4\,\mathrm{Cos}[x]^3},x\right]+$$
$$7\,\mathrm{CannotIntegrate}\left[\frac{\mathrm{Cos}[x]^2}{-1+2\,\mathrm{Cos}[x]-9\,\mathrm{Cos}[x]^2+4\,\mathrm{Cos}[x]^3},x\right]$$

Test results for the 113 problems in "Moses Problems.m"

Test results for the 376 problems in "Stewart Problems.m"

Test results for the 705 problems in "Timofeev Problems.m"

Problem 69: Result optimal but 1 more steps used.

$$\int \frac{\mathrm{ArcSin}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2-x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2\,a\,\sqrt{1-\frac{x^2}{a^2}}\,\mathrm{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2-x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2\,a\,\sqrt{1-\frac{x^2}{a^2}}\,\mathrm{ArcSin}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2-x^2}}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-x}\; x\; (1+x)^{2/3}}{-(1-x)^{5/6}\,(1+x)^{1/3}+(1-x)^{2/3}\,\sqrt{1+x}}\,dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}(1+x)^{1/3}+\frac{1}{4}\sqrt{1-x}\;x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}(1-x)^{1/3}(1+x)^{2/3}(1+3x)+\frac{1}{12}(1-x)^{1/6}(1+x)^{5/6}(2+3x)-\frac{1}{12}(1-x)^{5/6}(1+x)^{1/6}(10+3x)+\frac{1}{6}\text{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right]-\frac{4\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-2(1+x)^{1/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{5}{6}\text{ArcTan}\left[\frac{(1-x)^{1/3}-(1+x)^{1/3}}{(1-x)^{1/6}(1+x)^{1/6}}\right]+\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,(1-x)^{1/6}(1+x)^{1/6}}{(1-x)^{1/3}+(1+x)^{1/3}}\right]}{6\sqrt{3}}$$

Result (type 3, 522 leaves, 46 steps):

$$\frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}(1+x)^{1/6}+\frac{1}{6}(1-x)^{2/3}(1+x)^{1/3}-\frac{1}{4}(1-x)^{5/3}(1+x)^{1/3}+\frac{1}{3}(1-x)^{1/3}(1+x)^{2/3}-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3}+\frac{5}{12}(1-x)^{1/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{7/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{5/6}(1+x)^{7/6}+\frac{1}{4}x\sqrt{1-x^2}+\frac{\text{ArcSin}[x]}{4}-\frac{2}{3}\text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right]+\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1-x)^{1/3}}{\sqrt{3}\,(1+x)^{1/3}}\right]}{3\sqrt{3}}+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{9}\text{Log}[1-x]+\frac{1}{9}\text{Log}[1+x]+\frac{1}{3}\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}-\frac{\sqrt{3}\,(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}+\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}+\frac{\sqrt{3}\,(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}-\frac{1}{3}\text{Log}\left[1+\frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right]$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left((-1+x)^2\,(1+x)\right)^{1/3}}\,dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\Big[\frac{1+\frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\Big]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\Big[1-\frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\Big]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{ArcTan}\Big[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(3-3x)^{1/3}}\Big]}{3^{1/6}(1-x-x^2+x^3)^{1/3}}-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\Big[-\frac{8}{3}(-1+x)\Big]}{2\times 3^{2/3}(1-x-x^2+x^3)^{1/3}}-\frac{3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\Big[1+\frac{3^{1/3}(1+x)^{1/3}}{(3-3x)^{1/3}}\Big]}{2(1-x-x^2+x^3)^{1/3}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int\frac{((-1+x)^2(1+x))^{1/3}}{x^2}\,dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{((-1+x)^2(1+x))^{1/3}}{x}-\frac{\text{ArcTan}\Big[\frac{1-\frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\Big]}{\sqrt{3}}-\sqrt{3}\,\text{ArcTan}\Big[\frac{1+\frac{2(-1+x)}{((-1+x)^2(1+x))^{1/3}}}{\sqrt{3}}\Big]+\frac{\text{Log}[x]}{6}-\frac{2}{3}\text{Log}[1+x]-\frac{3}{2}\text{Log}\Big[1-\frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\Big]-\frac{1}{2}\text{Log}\Big[1+\frac{-1+x}{((-1+x)^2(1+x))^{1/3}}\Big]$$

Result (type 3, 404 leaves, 6 steps):

$$-\frac{(1-x-x^2+x^3)^{1/3}}{x}-\frac{3\times 3^{1/6}(1-x-x^2+x^3)^{1/3}\,\text{ArcTan}\Big[\frac{1}{\sqrt{3}}-\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\Big]}{(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{1/6}(1-x-x^2+x^3)^{1/3}\,\text{ArcTan}\Big[\frac{1}{\sqrt{3}}+\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\Big]}{(3-3x)^{2/3}(1+x)^{1/3}}+\frac{(1-x-x^2+x^3)^{1/3}\,\text{Log}[x]}{2\times 3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}(1-x-x^2+x^3)^{1/3}\,\text{Log}\Big[\frac{4(1+x)}{3}\Big]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3\times 3^{2/3}(1-x-x^2+x^3)^{1/3}\,\text{Log}\Big[1+\frac{(3-3x)^{1/3}}{3^{1/3}(1+x)^{1/3}}\Big]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}(1-x-x^2+x^3)^{1/3}\,\text{Log}\Big[\left(\frac{2}{3}\right)^{2/3}(3-3x)^{1/3}-\frac{2^{2/3}(1+x)^{1/3}}{3^{1/3}}\Big]}{2(3-3x)^{2/3}(1+x)^{1/3}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(9+3x-5x^2+x^3)^{1/3}}\,dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2\,(-3+x)}{\left(9+3\,x-5\,x^2+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\,\text{Log}[1+x]-\frac{3}{2}\,\text{Log}\left[1-\frac{-3+x}{\left(9+3\,x-5\,x^2+x^3\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(9-3\,x)^{2/3}\,(1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\,(1+x)^{1/3}}{3^{1/6}\,(9-3\,x)^{1/3}}\right]}{3^{1/6}\,\left(9+3\,x-5\,x^2+x^3\right)^{1/3}}-$$
$$\frac{(9-3\,x)^{2/3}\,(1+x)^{1/3}\,\text{Log}\left[-\frac{32}{3}\,(-3+x)\right]}{2\times 3^{2/3}\,\left(9+3\,x-5\,x^2+x^3\right)^{1/3}}-\frac{3^{1/3}\,(9-3\,x)^{2/3}\,(1+x)^{1/3}\,\text{Log}\left[1+\frac{3^{1/3}\,(1+x)^{1/3}}{(9-3\,x)^{1/3}}\right]}{2\,\left(9+3\,x-5\,x^2+x^3\right)^{1/3}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x+\sqrt{1+x+x^2}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-x+\sqrt{1+x+x^2}-\frac{3}{2}\,\text{ArcSinh}\left[\frac{1+2\,x}{\sqrt{3}}\right]+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]$$

Result (type 3, 59 leaves, 3 steps):

$$\frac{3}{2\,\left(1+2\,\left(x+\sqrt{1+x+x^2}\right)\right)}+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]-\frac{3}{2}\,\text{Log}\left[1+2\,\left(x+\sqrt{1+x+x^2}\right)\right]$$

Problem 306: Result valid but suboptimal antiderivative.

$$\int\left(x\,\left(1-x^2\right)\right)^{1/3}\,dx$$

Optimal (type 3, 93 leaves, 6 steps):

$$\frac{1}{2}\,x\,\left(x\,\left(1-x^2\right)\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{2\,x-\left(x\,\left(1-x^2\right)\right)^{1/3}}{\sqrt{3}\,\left(x\,\left(1-x^2\right)\right)^{1/3}}\right]}{2\,\sqrt{3}}+\frac{\text{Log}[x]}{12}-\frac{1}{4}\,\text{Log}\left[x+\left(x\,\left(1-x^2\right)\right)^{1/3}\right]$$

Result (type 3, 200 leaves, 12 steps):

$$\frac{1}{2}\,x\,\left(x-x^3\right)^{1/3}-\frac{x^{2/3}\,\left(1-x^2\right)^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,x^{2/3}}{\left(1-x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\,\sqrt{3}\,\left(x-x^3\right)^{2/3}}+$$
$$\frac{x^{2/3}\,\left(1-x^2\right)^{2/3}\,\text{Log}\left[1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{12\,\left(x-x^3\right)^{2/3}}-\frac{x^{2/3}\,\left(1-x^2\right)^{2/3}\,\text{Log}\left[1+\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{6\,\left(x-x^3\right)^{2/3}}$$

Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(-1+x^3\right)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 78 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}\,x}{\left(2+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[-1+x^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}\,x-\left(2+x^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 107 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,x}{3^{1/6}\left(2+x^3\right)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}\,x}{\left(2+x^3\right)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}\,x^2}{\left(2+x^3\right)^{2/3}}+\frac{3^{1/3}\,x}{\left(2+x^3\right)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 319: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(3\,x+3\,x^2+x^3\right)\left(3+3\,x+3\,x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2\,3^{1/3}\,(1+x)}{\left(2+(1+x)^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}-\frac{\text{Log}\left[1-(1+x)^3\right]}{6\times 3^{1/3}}+\frac{\text{Log}\left[3^{1/3}\,(1+x)-\left(2+(1+x)^3\right)^{1/3}\right]}{2\times 3^{1/3}}$$

Result (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(1+x)}{3^{1/6}\left(2+(1+x)^3\right)^{1/3}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-\frac{3^{1/3}\,(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{3\times 3^{1/3}}-\frac{\text{Log}\left[1+\frac{3^{2/3}\,(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{3^{1/3}\,(1+x)}{\left(2+(1+x)^3\right)^{1/3}}\right]}{6\times 3^{1/3}}$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Tan}[x]}{\left(-1+\sqrt{\text{Tan}[x]}\right)^2}\,dx$$

Optimal (type 3, 84 leaves, ? steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[\frac{1-\text{Tan}[x]}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{1+\text{Tan}[x]}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{1}{2}\,\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Result (type 3, 133 leaves, 19 steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}+\frac{1}{2}\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]-\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Problem 416: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Cos}[2\,x]-\sqrt{\text{Sin}[2\,x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2}\,\text{Log}\left[\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sec}[x]\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\right]-\frac{\text{ArcSin}[\text{Cos}[x]-\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2\,x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{ArcTanh}[\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2\,x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{Sin}[2\,x]}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}$$

Result (type 3, 234 leaves, 27 steps):

$$-2\,\text{Sec}[x]^2\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-\sqrt{2}\,\text{ArcSinh}[\text{Tan}[x]]\,\text{Cot}[x]\,\left(\text{Sec}[x]^2\right)^{3/2}\,\sqrt{\text{Cos}[x]\,\text{Sin}[x]}\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-\frac{\sqrt{2}\,\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\,\text{Sec}[x]^2\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}+\frac{\sqrt{2}\,\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\,\text{Sec}[x]^2\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}-\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\,\text{Sec}[x]^2\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\,\text{Sec}[x]^2\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Sec}[x]^2\,\left(-\text{Cos}[2\,x]+2\,\text{Tan}[x]^2\right)}{\left(\text{Tan}[x]\,\text{Tan}[2\,x]\right)^{3/2}}\,dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2\,\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x]\ \text{Tan}[2\,x]}}\right]-\frac{11\,\text{ArcTanh}\left[\frac{\sqrt{2}\ \text{Tan}[x]}{\sqrt{\text{Tan}[x]\ \text{Tan}[2\,x]}}\right]}{4\,\sqrt{2}}+\frac{\text{Tan}[x]}{2\,\left(\text{Tan}[x]\ \text{Tan}[2\,x]\right)^{3/2}}+\frac{2\,\text{Tan}[x]^3}{3\,\left(\text{Tan}[x]\ \text{Tan}[2\,x]\right)^{3/2}}+\frac{3\,\text{Tan}[x]}{4\,\sqrt{\text{Tan}[x]\ \text{Tan}[2\,x]}}$$

Result (type 3, 208 leaves, 21 steps):

$$\frac{3\,\text{Tan}[x]}{4\,\sqrt{2}\,\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Cot}[x]\ \left(1-\text{Tan}[x]^2\right)}{4\,\sqrt{2}\,\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Tan}[x]\ \left(1-\text{Tan}[x]^2\right)}{3\,\sqrt{2}\,\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}-\frac{11\,\text{ArcTan}\left[\sqrt{-1+\text{Tan}[x]^2}\right]\ \text{Tan}[x]}{4\,\sqrt{2}\,\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\,\sqrt{-1+\text{Tan}[x]^2}}+\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-1+\text{Tan}[x]^2}}{\sqrt{2}}\right]\ \text{Tan}[x]}{\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\,\sqrt{-1+\text{Tan}[x]^2}}$$

Problem 455: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Sin}[x]^6\ \text{Tan}[x]}{\text{Cos}[2\,x]^{3/4}}\,dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{1-\sqrt{\text{Cos}[2\,x]}}{\sqrt{2}\ \text{Cos}[2\,x]^{1/4}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{1+\sqrt{\text{Cos}[2\,x]}}{\sqrt{2}\ \text{Cos}[2\,x]^{1/4}}\right]}{\sqrt{2}}+\frac{7}{4}\,\text{Cos}[2\,x]^{1/4}-\frac{1}{5}\,\text{Cos}[2\,x]^{5/4}+\frac{1}{36}\,\text{Cos}[2\,x]^{9/4}$$

Result (type 3, 154 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[1-\sqrt{2}\ \text{Cos}[2\,x]^{1/4}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\ \text{Cos}[2\,x]^{1/4}\right]}{\sqrt{2}}+\frac{7}{4}\,\text{Cos}[2\,x]^{1/4}-\frac{1}{5}\,\text{Cos}[2\,x]^{5/4}+\frac{1}{36}\,\text{Cos}[2\,x]^{9/4}+\frac{\text{Log}\left[1-\sqrt{2}\ \text{Cos}[2\,x]^{1/4}+\sqrt{\text{Cos}[2\,x]}\right]}{2\,\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\ \text{Cos}[2\,x]^{1/4}+\sqrt{\text{Cos}[2\,x]}\right]}{2\,\sqrt{2}}$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int e^{x/2}\,x^2\,\text{Cos}[x]^3\,dx$$

Optimal (type 3, 187 leaves, ? steps):

$$-\frac{132}{125}\,e^{x/2}\,\text{Cos}[x]+\frac{18}{25}\,e^{x/2}\,x\,\text{Cos}[x]+\frac{48}{185}\,e^{x/2}\,x^2\,\text{Cos}[x]+\frac{2}{37}\,e^{x/2}\,x^2\,\text{Cos}[x]^3-\frac{428\,e^{x/2}\,\text{Cos}[3\,x]}{50653}+\frac{70\,e^{x/2}\,x\,\text{Cos}[3\,x]}{1369}-\frac{24}{125}\,e^{x/2}\,\text{Sin}[x]-\frac{24}{25}\,e^{x/2}\,x\,\text{Sin}[x]+\frac{96}{185}\,e^{x/2}\,x^2\,\text{Sin}[x]+\frac{12}{37}\,e^{x/2}\,x^2\,\text{Cos}[x]^2\,\text{Sin}[x]-\frac{792\,e^{x/2}\,\text{Sin}[3\,x]}{50653}-\frac{24\,e^{x/2}\,x\,\text{Sin}[3\,x]}{1369}$$

Result (type 3, 253 leaves, 31 steps):

$$-\frac{6687696\, e^{x/2}\,\text{Cos}[x]}{6331625}+\frac{24792\, e^{x/2}\, x\,\text{Cos}[x]}{34225}+\frac{48}{185}\, e^{x/2}\, x^2\,\text{Cos}[x]+\frac{16\, e^{x/2}\,\text{Cos}[x]^3}{50653}-\frac{8\, e^{x/2}\, x\,\text{Cos}[x]^3}{1369}+\frac{2}{37}\, e^{x/2}\, x^2\,\text{Cos}[x]^3-\frac{432\, e^{x/2}\,\text{Cos}[3\,x]}{50653}+\frac{72\, e^{x/2}\, x\,\text{Cos}[3\,x]}{1369}-\frac{1218672\, e^{x/2}\,\text{Sin}[x]}{6331625}-\frac{32556\, e^{x/2}\, x\,\text{Sin}[x]}{34225}+\frac{96}{185}\, e^{x/2}\, x^2\,\text{Sin}[x]+\frac{96\, e^{x/2}\,\text{Cos}[x]^2\,\text{Sin}[x]}{50653}-\frac{48\, e^{x/2}\, x\,\text{Cos}[x]^2\,\text{Sin}[x]}{1369}+\frac{12}{37}\, e^{x/2}\, x^2\,\text{Cos}[x]^2\,\text{Sin}[x]-\frac{816\, e^{x/2}\,\text{Sin}[3\,x]}{50653}-\frac{12\, e^{x/2}\, x\,\text{Sin}[3\,x]}{1369}$$

Problem 614: Result valid but suboptimal antiderivative.

$$\int \left(1+x^4\right)\left(1-2\,\text{Log}[x]+\text{Log}[x]^3\right)\, dx$$

Optimal (type 3, 60 leaves, 13 steps):

$$-3\,x+\frac{169\,x^5}{625}+4\,x\,\text{Log}[x]-\frac{44}{125}\,x^5\,\text{Log}[x]-3\,x\,\text{Log}[x]^2-\frac{3}{25}\,x^5\,\text{Log}[x]^2+x\,\text{Log}[x]^3+\frac{1}{5}\,x^5\,\text{Log}[x]^3$$

Result (type 3, 73 leaves, 13 steps):

$$-3\,x+\frac{169\,x^5}{625}+6\,x\,\text{Log}[x]+\frac{6}{125}\,x^5\,\text{Log}[x]-\frac{2}{5}\left(5\,x+x^5\right)\text{Log}[x]-3\,x\,\text{Log}[x]^2-\frac{3}{25}\,x^5\,\text{Log}[x]^2+x\,\text{Log}[x]^3+\frac{1}{5}\,x^5\,\text{Log}[x]^3$$

Problem 695: Result valid but suboptimal antiderivative.

$$\int \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right] dx$$

Optimal (type 3, 55 leaves, ? steps):

$$-\frac{\sqrt{2}\, a\,\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}}+(a+x)\,\text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

Result (type 3, 118 leaves, 8 steps):

$$-\sqrt{2}\,\sqrt{\frac{a}{a+x}}\,\sqrt{-\frac{a-x}{a+x}}\,(a+x)+x\,\text{ArcSin}\left[\sqrt{-\frac{a-x}{a+x}}\right]-\frac{a\,\sqrt{\frac{a}{a+x}}\,\text{ArcTanh}\left[\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\,\sqrt{-\frac{a}{a+x}}}\right]}{\sqrt{-\frac{a}{a+x}}}$$

Test results for the 116 problems in "Welz Problems.m"

Problem 9: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\mathrm{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-$$
$$\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\mathrm{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-$$
$$\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\mathrm{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-$$
$$\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\mathrm{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 3, 365 leaves, 18 steps):

$$\frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]+$$
$$\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\mathrm{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{2}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\mathrm{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 10: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2 \sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4 x}{5 \left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25} \sqrt{-110+50 \sqrt{5}}\, \text{ArcTan}\left[\frac{1}{2} \sqrt{2+2 \sqrt{5}}\, \sqrt{x}\right]-$$
$$\frac{1}{50} \sqrt{-110+50 \sqrt{5}}\, \text{ArcTan}\left[\frac{\sqrt{-2+2 \sqrt{5}}\, \sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right) x}\right]-$$
$$\frac{1}{25} \sqrt{110+50 \sqrt{5}}\, \text{ArcTanh}\left[\frac{1}{2} \sqrt{-2+2 \sqrt{5}}\, \sqrt{x}\right]-$$
$$\frac{1}{50} \sqrt{110+50 \sqrt{5}}\, \text{ArcTanh}\left[\frac{\sqrt{2+2 \sqrt{5}}\, \sqrt{-1+x^2}}{2-x-\sqrt{5}\, x}\right]$$

Result (type 3, 541 leaves, 25 steps):

$$\frac{2\left(1-2x\right)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{\left(1-2x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{\left(3-x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+$$

$$\frac{\left(2+x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{10}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+$$

$$\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 29: Result valid but suboptimal antiderivative.

$$\int x^3\,\text{Log}[2+x]^3\,\text{Log}[3+x]\,dx$$

Optimal (type 4, 606 leaves, 359 steps) :

$$-\frac{302177\,x}{1152}+\frac{8029\,x^2}{2304}-\frac{763\,x^3}{3456}+\frac{3\,x^4}{256}+\frac{377}{64}\,(2+x)^2-\frac{71}{216}\,(2+x)^3+\frac{3}{256}\,(2+x)^4+\frac{2069}{144}\,\text{Log}[2+x]-\frac{187}{64}\,x^2\,\text{Log}[2+x]+\frac{83}{288}\,x^3\,\text{Log}[2+x]-\frac{3}{128}\,x^4\,\text{Log}[2+x]+\frac{6733}{32}\,(2+x)\,\text{Log}[2+x]-\frac{377}{32}\,(2+x)^2\,\text{Log}[2+x]+\frac{71}{72}\,(2+x)^3\,\text{Log}[2+x]-\frac{3}{64}\,(2+x)^4\,\text{Log}[2+x]-\frac{43}{12}\,\text{Log}[2+x]^2-\frac{17}{48}\,x^3\,\text{Log}[2+x]^2+\frac{3}{64}\,x^4\,\text{Log}[2+x]^2-\frac{1251}{16}\,(2+x)\,\text{Log}[2+x]^2+\frac{273}{32}\,(2+x)^2\,\text{Log}[2+x]^2-\frac{3}{4}\,(2+x)^3\,\text{Log}[2+x]^2+\frac{3}{64}\,(2+x)^4\,\text{Log}[2+x]^2+\frac{65}{4}\,(2+x)\,\text{Log}[2+x]^3-\frac{33}{8}\,(2+x)^2\,\text{Log}[2+x]^3+\frac{3}{4}\,(2+x)^3\,\text{Log}[2+x]^3-\frac{1}{16}\,(2+x)^4\,\text{Log}[2+x]^3+\frac{3891}{128}\,\text{Log}[3+x]-\frac{115}{48}\,x^2\,\text{Log}[3+x]+\frac{37}{144}\,x^3\,\text{Log}[3+x]-\frac{3}{128}\,x^4\,\text{Log}[3+x]+\frac{415}{12}\,(3+x)\,\text{Log}[3+x]-\frac{4083}{32}\,\text{Log}[2+x]\,\text{Log}[3+x]-25\,x\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{13}{4}\,x^2\,\text{Log}[2+x]\,\text{Log}[3+x]-\frac{7}{12}\,x^3\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{3}{32}\,x^4\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{963}{16}\,\text{Log}[2+x]^2\,\text{Log}[3+x]+6\,x\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{3}{2}\,x^2\,\text{Log}[2+x]^2\,\text{Log}[3+x]+\frac{1}{2}\,x^3\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{3}{16}\,x^4\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{81}{4}\,\text{Log}[2+x]^3\,\text{Log}[3+x]+\frac{1}{4}\,x^4\,\text{Log}[2+x]^3\,\text{Log}[3+x]-\frac{5609}{96}\,\text{PolyLog}[2,-2-x]+\frac{563}{8}\,\text{Log}[2+x]\,\text{PolyLog}[2,-2-x]-\frac{195}{4}\,\text{Log}[2+x]^2\,\text{PolyLog}[2,-2-x]-\frac{563}{8}\,\text{PolyLog}[3,-2-x]+\frac{195}{2}\,\text{Log}[2+x]\,\text{PolyLog}[3,-2-x]-\frac{195}{2}\,\text{PolyLog}[4,-2-x]$$

Result (type 4, 679 leaves, 359 steps) :

$$-\frac{302177\,x}{1152}+\frac{8029\,x^2}{2304}-\frac{763\,x^3}{3456}+\frac{3\,x^4}{256}+\frac{377}{64}(2+x)^2-\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\text{Log}[2+x]-\frac{187}{64}x^2\,\text{Log}[2+x]+\frac{83}{288}x^3\,\text{Log}[2+x]-\frac{3}{128}x^4\,\text{Log}[2+x]+\frac{6365}{32}(2+x)\,\text{Log}[2+x]-\frac{273}{32}(2+x)^2\,\text{Log}[2+x]+\frac{1}{2}(2+x)^3\,\text{Log}[2+x]-\frac{3}{128}(2+x)^4\,\text{Log}[2+x]+\frac{1}{128}\left(384(2+x)-144(2+x)^2+32(2+x)^3-3(2+x)^4-192\,\text{Log}[2+x]\right)\text{Log}[2+x]+\frac{17}{72}\left(36(2+x)-9(2+x)^2+(2+x)^3-24\,\text{Log}[2+x]\right)\text{Log}[2+x]+\frac{43}{12}\text{Log}[2+x]^2-\frac{17}{48}x^3\,\text{Log}[2+x]^2+\frac{3}{64}x^4\,\text{Log}[2+x]^2-\frac{1251}{16}(2+x)\,\text{Log}[2+x]^2+\frac{273}{32}(2+x)^2\,\text{Log}[2+x]^2-\frac{3}{4}(2+x)^3\,\text{Log}[2+x]^2+\frac{3}{64}(2+x)^4\,\text{Log}[2+x]^2+\frac{65}{4}(2+x)\,\text{Log}[2+x]^3-\frac{33}{8}(2+x)^2\,\text{Log}[2+x]^3+\frac{3}{4}(2+x)^3\,\text{Log}[2+x]^3-\frac{1}{16}(2+x)^4\,\text{Log}[2+x]^3+\frac{3891}{128}\text{Log}[3+x]-\frac{115}{48}x^2\,\text{Log}[3+x]+\frac{37}{144}x^3\,\text{Log}[3+x]-\frac{3}{128}x^4\,\text{Log}[3+x]+\frac{415}{12}(3+x)\,\text{Log}[3+x]-\frac{4083}{32}\text{Log}[2+x]\,\text{Log}[3+x]-25\,x\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{13}{4}x^2\,\text{Log}[2+x]\,\text{Log}[3+x]-\frac{7}{12}x^3\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{3}{32}x^4\,\text{Log}[2+x]\,\text{Log}[3+x]+\frac{963}{16}\text{Log}[2+x]^2\,\text{Log}[3+x]+6\,x\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{3}{2}x^2\,\text{Log}[2+x]^2\,\text{Log}[3+x]+\frac{1}{2}x^3\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{3}{16}x^4\,\text{Log}[2+x]^2\,\text{Log}[3+x]-\frac{81}{4}\text{Log}[2+x]^3\,\text{Log}[3+x]+\frac{1}{4}x^4\,\text{Log}[2+x]^3\,\text{Log}[3+x]-\frac{5609}{96}\text{PolyLog}[2,-2-x]+\frac{563}{8}\text{Log}[2+x]\,\text{PolyLog}[2,-2-x]-\frac{195}{4}\text{Log}[2+x]^2\,\text{PolyLog}[2,-2-x]-\frac{563}{8}\text{PolyLog}[3,-2-x]+\frac{195}{2}\text{Log}[2+x]\,\text{PolyLog}[3,-2-x]-\frac{195}{2}\text{PolyLog}[4,-2-x]$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x\left(2-3x+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-x)}{\sqrt{3}\left(2-3x+x^2\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\left(2-3x+x^2\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 176 leaves, 2 steps):

$$-\frac{\sqrt{3}\,\left(-2+x\right)^{1/3}\left(-1+x\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{1/3}\,(-2+x)^{2/3}}{\sqrt{3}\,(-1+x)^{1/3}}\right]}{2\times 2^{1/3}\left(2-3\,x+x^2\right)^{1/3}}+$$

$$\frac{3\left(-2+x\right)^{1/3}\left(-1+x\right)^{1/3}\text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}}-2^{1/3}\left(-1+x\right)^{1/3}\right]}{4\times 2^{1/3}\left(2-3\,x+x^2\right)^{1/3}}-\frac{\left(-2+x\right)^{1/3}\left(-1+x\right)^{1/3}\text{Log}[x]}{2\times 2^{1/3}\left(2-3\,x+x^2\right)^{1/3}}$$

Problem 40: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(-1+x\right)}{\sqrt{3}\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}}\right]+$$

$$\frac{1}{4}\text{Log}[1-x]-\frac{3}{4}\text{Log}\left[1-x+\left(-5+7\,x-3\,x^2+x^3\right)^{1/3}\right]$$

Result (type 3, 131 leaves, 5 steps):

$$\frac{\sqrt{3}\left(4+\left(-1+x\right)^2\right)^{1/3}\left(-1+x\right)^{1/3}\text{ArcTan}\left[\frac{1+\frac{2\,(-1+x)^{2/3}}{\left(4+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(4\left(-1+x\right)+\left(-1+x\right)^3\right)^{1/3}}-$$

$$\left(3\left(4+\left(-1+x\right)^2\right)^{1/3}\left(-1+x\right)^{1/3}\text{Log}\left[-\left(4+\left(-1+x\right)^2\right)^{1/3}+\left(-1+x\right)^{2/3}\right]\right)\Big/$$
$$\left(4\left(4\left(-1+x\right)+\left(-1+x\right)^3\right)^{1/3}\right)$$

Problem 41: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(x\left(-q+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,x}{\sqrt{3}\left(x\left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\text{Log}\left[-x+\left(x\left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 117 leaves, 5 steps):

$$\frac{\sqrt{3}\,x^{1/3}\left(-q+x^2\right)^{1/3}\text{ArcTan}\left[\frac{1+\frac{2\,x^{2/3}}{\left(-q+x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-q\,x+x^3\right)^{1/3}}-\frac{3\,x^{1/3}\left(-q+x^2\right)^{1/3}\text{Log}\left[x^{2/3}-\left(-q+x^2\right)^{1/3}\right]}{4\left(-q\,x+x^3\right)^{1/3}}$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\text{Log}[1-x]-\frac{3}{4}\text{Log}\left[1-x+\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 145 leaves, 5 steps):

$$\frac{\sqrt{3}\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{\left(-1+q+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}}-\left(3\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\left(-1+q+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]\right)\Big/\left(4\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}\right)$$

Problem 43: Unable to integrate problem.

$$\int \frac{1}{x\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2q^{1/3}(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\right]}{2q^{1/3}}+\frac{\text{Log}[1-x]}{4q^{1/3}}+\frac{\text{Log}[x]}{2q^{1/3}}-\frac{3\,\text{Log}\left[-q^{1/3}(-1+x)+\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}\right]}{4q^{1/3}}$$

Result (type 8, 677 leaves, 2 steps):

$$\frac{1}{3\left(-q+3qx+(-1-2q)x^2+x^3\right)^{1/3}}$$

$$\left(-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3 q}\right)^{2/3}}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3 q}\right)^{1/3}}+3x\right)^{1/3}$$

$$\left(-1+5q-4q^2+\frac{(1-4q)^2(1-q)^2}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}}+\right.$$
$$\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}+$$
$$\left(3\left(1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}\right)\left(\frac{1}{3}(-1-2q)+x\right)\right)\Big/$$
$$\left.\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{1/3}+9\left(\frac{1}{3}(-1-2q)+x\right)^2\right)^{1/3}$$

$$\text{Unintegrable}\left[3\Big/\left(x\left(-1-2q-\left(1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3 q}\right)^{2/3}\right)\Big/\right.\right.\right.$$
$$\left.\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3 q}\right)^{1/3}+3x\right)^{1/3}$$
$$\left(-1+5q-4q^2+\frac{(1-4q)^2(1-q)^2}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}}+\right.$$
$$\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}+9\left(\frac{1}{3}(-1-2q)+x\right)^2+$$
$$\left(\left(1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{2/3}\right)(-1-2q+3x)\right)\Big/$$
$$\left.\left.\left.\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3 q}\right)^{1/3}\right)^{1/3}\right),\ x\right]$$

Problem 44: Unable to integrate problem.

$$\int \frac{2-(1+k)x}{\left((1-x)x(1-kx)\right)^{1/3}\left(1-(1+k)x\right)}\,dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,k^{1/3}\,x}{((1-x)\,x\,(1-k\,x))^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2\,k^{1/3}}+\frac{\text{Log}[1-(1+k)\,x]}{2\,k^{1/3}}-\frac{3\,\text{Log}\left[-k^{1/3}\,x+((1-x)\,x\,(1-k\,x))^{1/3}\right]}{2\,k^{1/3}}$$

Result (type 8, 139 leaves, 3 steps):

$$\frac{3\,(1-x)^{1/3}\,x\,(1-k\,x)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},x,k\,x\right]}{2\,((1-x)\,x\,(1-k\,x))^{1/3}}+$$

$$\left((1-x)^{1/3}\,x^{1/3}\,(1-k\,x)^{1/3}\,\text{CannotIntegrate}\left[\frac{1}{(1-x)^{1/3}\,x^{1/3}\,(1+(-1-k)\,x)\,(1-k\,x)^{1/3}},x\right]\right)\Big/$$
$$((1-x)\,x\,(1-k\,x))^{1/3}$$

Problem 45: Unable to integrate problem.

$$\int\frac{1-k\,x}{(1+(-2+k)\,x)\,((1-x)\,x\,(1-k\,x))^{2/3}}\,dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-k\,x)}{(1-k)^{1/3}\,((1-x)\,x\,(1-k\,x))^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}[1-(2-k)\,x]}{2^{2/3}\,(1-k)^{1/3}}+$$
$$\frac{\text{Log}[1-k\,x]}{2\times 2^{2/3}\,(1-k)^{1/3}}-\frac{3\,\text{Log}\left[-1+k\,x+2^{2/3}\,(1-k)^{1/3}\,((1-x)\,x\,(1-k\,x))^{1/3}\right]}{2\times 2^{2/3}\,(1-k)^{1/3}}$$

Result (type 8, 78 leaves, 1 step):

$$\frac{(1-x)^{2/3}\,x^{2/3}\,(1-k\,x)^{2/3}\,\text{CannotIntegrate}\left[\frac{(1-k\,x)^{1/3}}{(1-x)^{2/3}\,x^{2/3}\,(1+(-2+k)\,x)},x\right]}{((1-x)\,x\,(1-k\,x))^{2/3}}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x+c\,x^2}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 493 leaves, 19 steps):

$$\frac{(a+b)\,\mathrm{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{(a+b)\,\mathrm{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-\frac{c\,\mathrm{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$

$$\frac{(a-c)\,\mathrm{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{(b+c)\,\mathrm{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{(a+b)\,\mathrm{Log}\left[(1-x)\,(1+x)^2\right]}{12\times 2^{1/3}}-$$

$$\frac{(a-c)\,\mathrm{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{(b+c)\,\mathrm{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{(a+b)\,\mathrm{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6\times 2^{1/3}}-$$

$$\frac{(a+b)\,\mathrm{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{(b+c)\,\mathrm{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{(a-c)\,\mathrm{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+$$

$$\frac{1}{2}c\,\mathrm{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{(a+b)\,\mathrm{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 576 leaves, 7 steps):

$$-\frac{c\,\mathrm{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\left(2a+b-i\sqrt{3}\,b-\left(1+i\sqrt{3}\right)c\right)\mathrm{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(2a+b+i\sqrt{3}\,b-c+i\sqrt{3}\,c\right)\mathrm{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\left(\left(3\,i\,b-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\mathrm{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]\right)\Big/$$
$$\left(12\times 2^{1/3}\left(i+\sqrt{3}\right)\right)+$$
$$\left(\left(3\,i\,b+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\mathrm{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]\right)\Big/$$
$$\left(12\times 2^{1/3}\left(i-\sqrt{3}\right)\right)+\frac{1}{2}c\,\mathrm{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{1}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}$$
$$\left(3\,i\,b-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\mathrm{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}(1-x^3)^{1/3}\right]-$$
$$\frac{1}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}\left(3\,i\,b+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\mathrm{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}(1-x^3)^{1/3}\right]$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\,(1+x^2)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\ \text{ArcTan}\left[\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\ (-a+x)}{\sqrt{(-a+x)\ (1+x^2)}}\right]$$

Result (type 4, 204 leaves, 9 steps):

$$\frac{2\,\text{i}\sqrt{\frac{a-x}{\text{i}+a}}\ \sqrt{1+x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\text{i}\,x}}{\sqrt{2}}\right],\frac{2}{1-\text{i}\,a}\right]}{\sqrt{-(a-x)\ (1+x^2)}}+$$
$$\left(4\sqrt{1+a^2}\ \sqrt{\frac{a-x}{\text{i}+a}}\ \sqrt{1+x^2}\ \text{EllipticPi}\left[\frac{2}{1-\text{i}\left(a-\sqrt{1+a^2}\right)},\text{ArcSin}\left[\frac{\sqrt{1-\text{i}\,x}}{\sqrt{2}}\right],\frac{2}{1-\text{i}\,a}\right]\right)\Big/$$
$$\left(\left(1-\text{i}\left(a-\sqrt{1+a^2}\right)\right)\sqrt{-(a-x)\ (1+x^2)}\right)$$

Problem 56: Result valid but suboptimal antiderivative.

$$\int x\ (1-x^3)^{1/3}\ dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\frac{1}{3}x^2\ (1-x^3)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{1}{6}\text{Log}\left[-x-(1-x^3)^{1/3}\right]$$

Result (type 3, 107 leaves, 8 steps):

$$\frac{1}{3}x^2\ (1-x^3)^{1/3}-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{1}{18}\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-\frac{1}{9}\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]$$

Problem 58: Unable to integrate problem.

$$\int\frac{(1-x^3)^{1/3}}{1+x}\ dx$$

Optimal (type 3, 482 leaves, 25 steps):

$$\left(1-x^3\right)^{1/3}+\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$

$$\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\,\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}+$$

$$\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}-\frac{1}{2}\,\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[-2^{1/3}\,x-\left(1-x^3\right)^{1/3}\right]}{2^{2/3}}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{1+x},\,x\right]$$

Problem 59: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Optimal (type 3, 280 leaves, 19 steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,(-1+x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}}-$$

$$\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\,\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}}-\frac{\text{Log}\left[-3\,(-1+x)\,\left(1-x+x^2\right)\right]}{2\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+$$

$$\frac{3\,\text{Log}\left[-2^{1/3}\,(-1+x)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}+\frac{1}{2}\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}\,x+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 79 leaves, 2 steps):

$$\frac{2\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{1+i\,\sqrt{3}-2\,x},\,x\right]}{\sqrt{3}}+\frac{2\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{-1+i\,\sqrt{3}+2\,x},\,x\right]}{\sqrt{3}}$$

Problem 60: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{1/3}}{2+x}\,dx$$

Optimal (type 6, 232 leaves, 12 steps):

$$\left(1-x^3\right)^{1/3}+\frac{1}{2}\,x\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-\frac{x^3}{8}\right]-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+$$
$$3^{1/6}\,\text{ArcTan}\left[\frac{1-\frac{3^{2/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]-3^{1/6}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\left(1-x^3\right)^{1/3}}{3\times3^{1/6}}\right]-\frac{\text{Log}\left[8+x^3\right]}{3^{1/3}}+$$
$$\frac{1}{2}\times3^{2/3}\,\text{Log}\left[3^{2/3}-\left(1-x^3\right)^{1/3}\right]-\text{Log}\left[-x-\left(1-x^3\right)^{1/3}\right]+\frac{1}{2}\times3^{2/3}\,\text{Log}\left[-\frac{1}{2}\times3^{2/3}\,x-\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{1/3}}{2+x},x\right]$$

Problem 61: Unable to integrate problem.

$$\int\frac{2+x}{\left(1+x+x^2\right)\left(2+x^3\right)^{1/3}}\,dx$$

Optimal (type 6, 168 leaves, 9 steps):

$$-\frac{x^2\,\text{AppellF1}\left[\frac{2}{3},1,\frac{1}{3},\frac{5}{3},x^3,-\frac{x^3}{2}\right]}{2\times2^{1/3}}+\frac{2\,\text{ArcTan}\left[\frac{1+\frac{2\times3^{1/3}x}{\left(2+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}}+$$
$$\frac{\text{ArcTan}\left[\frac{3^{1/3}+2\left(2+x^3\right)^{1/3}}{3^{5/6}}\right]}{3^{5/6}}+\frac{\text{Log}\left[1-x^3\right]}{6\times3^{1/3}}+\frac{\text{Log}\left[3^{1/3}-\left(2+x^3\right)^{1/3}\right]}{2\times3^{1/3}}-\frac{\text{Log}\left[3^{1/3}x-\left(2+x^3\right)^{1/3}\right]}{3^{1/3}}$$

Result (type 8, 81 leaves, 2 steps):

$$\left(1-i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1-i\sqrt{3}+2x\right)\left(2+x^3\right)^{1/3}},x\right]+$$
$$\left(1+i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1+i\sqrt{3}+2x\right)\left(2+x^3\right)^{1/3}},x\right]$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-2+a+x}{\left(-a+x\right)\sqrt{\left(2-a\right)a\,x+\left(-1-2a+a^2\right)x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

0

Result (type 4, 529 leaves, 5 steps):

$$\left(2\,(1-a)\,\sqrt{x}\,\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}\,\text{ArcTan}\left[\frac{\sqrt{-1+2\,a-a^2}\,\sqrt{x}}{\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}}\right]\right)\Big/$$

$$\left(a\,\sqrt{-1+2\,a-a^2}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)+$$

$$\left(((2-a)\,a)^{3/4}\,\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\,\text{EllipticF}\left[\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/\left(a\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)+$$

$$\left((2-a)\,\left(1-\sqrt{(2-a)\,a}\right)\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\,\sqrt{(2-a)\,a}},\,2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/$$

$$\left(((2-a)\,a)^{3/4}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a+(-1+2\,a)\,x}{(-a+x)\,\sqrt{a^2\,x-(-1+2\,a+a^2)\,x^2+(-1+2\,a)\,x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\left[\frac{-a^2+2\,a\,x+x^2-2\left(x+\sqrt{(1-x)\,x\,(a^2+x-2\,a\,x)}\right)}{(a-x)^2}\right]$$

Result (type 4, 180 leaves, 7 steps):

$$-\left(\left(2\,(1-2\,a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{x}\right],-\frac{1-2\,a}{a^2}\right]\right)\Big/\left(\sqrt{a^2\,x+\left(1-2\,a-a^2\right)\,x^2-(1-2\,a)\,x^3}\right)\right)+$$

$$\left(4\,(1-a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticPi}\left[\frac{1}{a},\text{ArcSin}\left[\sqrt{x}\right],-\frac{1-2\,a}{a^2}\right]\right)\Big/\left(\sqrt{a^2\,x+\left(1-2\,a-a^2\right)\,x^2-(1-2\,a)\,x^3}\right)$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(1-x^3\right)\,\left(a+b\,x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 98 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,(a+b)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+\frac{\text{Log}\left[1-x^3\right]}{6\,(a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,(a+b)^{1/3}}$$

Result (type 3, 135 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,(a+b)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}-\frac{\text{Log}\left[1-\frac{(a+b)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{3\,(a+b)^{1/3}}+\frac{\text{Log}\left[1+\frac{(a+b)^{2/3}\,x^2}{\left(a+b\,x^3\right)^{2/3}}+\frac{(a+b)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}\right]}{6\,(a+b)^{1/3}}$$

Problem 95: Unable to integrate problem.

$$\int\frac{1+x}{\left(1+x+x^2\right)\,\left(a+b\,x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,(a+b)^{1/3}\,x}{\left(a+b\,x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2\,\left(a+b\,x^3\right)^{1/3}}{(a+b)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,(a+b)^{1/3}}+$$

$$\frac{\text{Log}\left[(a+b)^{1/3}-\left(a+b\,x^3\right)^{1/3}\right]}{2\,(a+b)^{1/3}}-\frac{\text{Log}\left[(a+b)^{1/3}\,x-\left(a+b\,x^3\right)^{1/3}\right]}{2\,(a+b)^{1/3}}$$

Result (type 8, 91 leaves, 2 steps):

$$\frac{1}{3}\left(3-i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1-i\sqrt{3}+2x\right)\left(a+bx^3\right)^{1/3}},x\right]+$$
$$\frac{1}{3}\left(3+i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1+i\sqrt{3}+2x\right)\left(a+bx^3\right)^{1/3}},x\right]$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times2^{1/3}}$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12\times2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+x}{\left(1-x+x^2\right)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 4 steps):

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$

$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-$$

$$\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(1+x)^2}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 5 steps):

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$

$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-$$

$$\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1-x}{\left(1+x+x^2\right)\left(1+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 119 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1+x)^2}{\left(1+x^3\right)^{2/3}}-\frac{2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}(1+x)}{\left(1+x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 399 leaves, 4 steps) :

$$\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}-2x\right)}{\left(1+x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}-2x\right)}{\left(1+x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$

$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[\left(1-i\sqrt{3}-2x\right)\left(1-i\sqrt{3}+2x\right)^2\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-$$

$$\frac{\left(i+\sqrt{3}\right)\text{Log}\left[\left(1+i\sqrt{3}-2x\right)\left(1+i\sqrt{3}+2x\right)^2\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}-2x+2\times 2^{2/3}\left(1+x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}-2x+2\times 2^{2/3}\left(1+x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 103: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+x+x^2\right)^2}\,dx$$

Optimal (type 5, 43 leaves, 5 steps):

$$\frac{1}{\left(1-x^3\right)^{1/3}}+\frac{x}{\left(1-x^3\right)^{1/3}}-x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right]$$

Result (type 8, 151 leaves, 2 steps):

$$-\frac{4}{3}\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}-2x},x\right]}{3\sqrt{3}}-$$

$$\frac{4}{3}\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+i\sqrt{3}+2x},x\right]}{3\sqrt{3}}$$

Problem 104: Unable to integrate problem.

$$\int \frac{1-x}{\left(1+x+x^2\right)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 5, 43 leaves, 5 steps):

$$\frac{1}{\left(1-x^3\right)^{1/3}}+\frac{x}{\left(1-x^3\right)^{1/3}}-x^2\,\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{4}{3},\frac{5}{3},x^3\right]$$

Result (type 8, 87 leaves, 2 steps):

$$-\left(1+i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{1/3}},x\right]-\left(1-i\sqrt{3}\right)\text{Unintegrable}\left[\frac{1}{\left(1+i\sqrt{3}+2x\right)\left(1-x^3\right)^{1/3}},x\right]$$

Problem 108: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{2/3}}{a+bx}\,dx$$

Optimal (type 6, 384 leaves, 13 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{2b}-\frac{\left(a^3+b^3\right)x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-\frac{b^3x^3}{a^3}\right]}{2a^2b^2}+\frac{a^2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}-\frac{\left(a^3+b^3\right)^{2/3}\text{ArcTan}\left[\frac{1-\frac{2\left(a^3+b^3\right)^{1/3}x}{a\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+\frac{\left(a^3+b^3\right)^{2/3}\text{ArcTan}\left[\frac{1+\frac{2b\left(1-x^3\right)^{1/3}}{\left(a^3+b^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b^3}+\frac{a\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]}{2b^2}-\frac{\left(a^3+b^3\right)^{2/3}\text{Log}\left[a^3+b^3x^3\right]}{3b^3}+\frac{\left(a^3+b^3\right)^{2/3}\text{Log}\left[-\frac{\left(a^3+b^3\right)^{1/3}x}{a}-\left(1-x^3\right)^{1/3}\right]}{2b^3}-\frac{a^2\,\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]}{2b^3}+\frac{\left(a^3+b^3\right)^{2/3}\text{Log}\left[\left(a^3+b^3\right)^{1/3}-b\left(1-x^3\right)^{1/3}\right]}{2b^3}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{a+bx},x\right]$$

Problem 109: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Optimal (type 5, 234 leaves, 13 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{x\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}+\frac{2x^2\left(1-x^3\right)^{2/3}}{3\left(1+x^3\right)}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{1}{3}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{3\times2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{3\times2^{1/3}}$$

Result (type 8, 151 leaves, 2 steps):

$$-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+i\sqrt{3}-2x},x\right]}{3\sqrt{3}}-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4\,i\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{-1+i\sqrt{3}+2x},x\right]}{3\sqrt{3}}$$

Problem 110: Unable to integrate problem.

$$\int\frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$$

Optimal (type 3, 199 leaves, 14 steps):

$$\frac{\left(1-x^3\right)^{2/3}}{1-x+x^2}-\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}+\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 8, 159 leaves, 6 steps):

$$-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]+\frac{4}{3}\left(1+i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2},x\right]-\frac{4}{3}\,\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]+\frac{4}{3}\left(1-i\sqrt{3}\right)\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2},x\right]$$

Problem 111: Unable to integrate problem.

$$\int\frac{\left(1-x^3\right)^{2/3}}{1+x}\,dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 19 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+x},\ x\right]$$

Problem 112: Unable to integrate problem.

$$\int \frac{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 177 leaves, 6 steps):

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\ \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{2\times 2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]+\frac{3\ \text{Log}\left[-1+x+2^{2/3}\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 19 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\left(1-x^3\right)^{2/3}}{1+x},\ x\right]$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{2^{2/3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-\left(1-x^3\right)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+\left(1-x^3\right)^{1/3}\right]$$

Result (type 6, 21 leaves, 1 step):

$$x\ \text{AppellF1}\left[\frac{1}{3},-\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right]$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 250 leaves, 10 steps):

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-$$

$$\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{6\times 2^{1/3}}+$$

$$\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]-\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2}\,x^2\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]$$

Problem 115: Unable to integrate problem.

$$\int\frac{(1-x)\,(1-x^3)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 383 leaves, ? steps):

$$-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}\,x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{6\times 2^{1/3}}-$$

$$\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]+$$

$$\frac{\text{Log}\left[-2^{1/3}\,x-(1-x^3)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\,\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 8, 101 leaves, 2 steps):

$$-\frac{2}{3}\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1-x},x\right]-\frac{1}{3}\left(1+(-1)^{2/3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1+(-1)^{1/3}\,x},x\right]-$$

$$\frac{1}{3}\left(1-(-1)^{1/3}\right)\text{CannotIntegrate}\left[\frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}\,x},x\right]$$

Problem 116: Result unnecessarily involves higher level functions.

$$\int\frac{(1-x^3)^{1/3}}{1+x^3}\,dx$$

Optimal (type 3, 272 leaves, 14 steps):

$$\frac{2^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,\sqrt{3}}+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}+\frac{1}{3}\times 2^{1/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}\,(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}$$

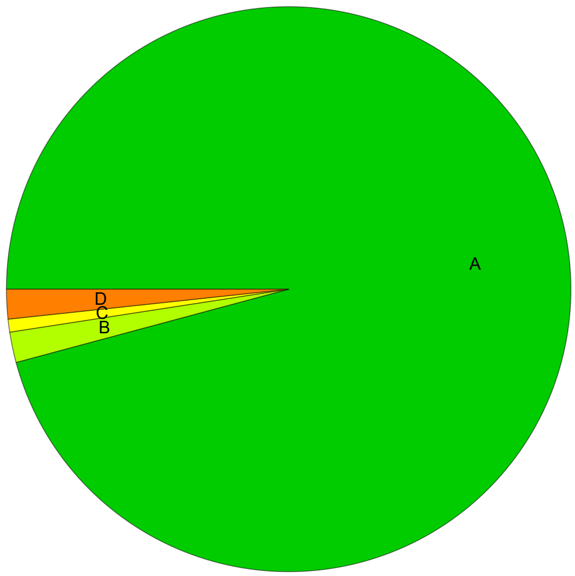
Result (type 6, 21 leaves, 1 step):

$$x\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right]$$

Test results for the 8 problems in "Wester Problems.m"

Summary of Integration Test Results

1892 integration problems



A - 1813 optimal antiderivatives

B - 33 valid but suboptimal antiderivatives

C - 14 unnecessarily complex antiderivatives

D - 32 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives