Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions"

Test results for the 156 problems in "7.1.2 (d x)^m (a+b arcsinh(c x))^n.m"

Test results for the 663 problems in "7.1.4 (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n.m"

Problem 45: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\,\left(d+c^2\,d\,x^2\right)^2}\,dx$$

Optimal (type 4, 239 leaves, 19 steps):

$$-\frac{b\,c^3}{3\,d^2\,\sqrt{1+c^2\,x^2}}-\frac{b\,c}{6\,d^2\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^2\,x^3\,\left(1+c^2\,x^2\right)}+\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,x\,\left(1+c^2\,x^2\right)}+\frac{5\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d^2\,\left(1+c^2\,x^2\right)}+\frac{5\,c^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d^2}+\frac{13\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^2}-\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}+\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}$$

Result (type 4, 264 leaves, 19 steps):

$$\frac{5\,b\,c^3}{6\,d^2\,\sqrt{1+c^2\,x^2}}+\frac{b\,c}{3\,d^2\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,d^2\,x^2}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^2\,x^3\,\left(1+c^2\,x^2\right)}+\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,x\,\left(1+c^2\,x^2\right)}+\frac{5\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d^2\,\left(1+c^2\,x^2\right)}+\frac{5\,c^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d^2}+\frac{13\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^2}-\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}+\frac{5\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d^2}$$

Problem 54: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\,\left(d+c^2\,d\,x^2\right)^3}\,dx$$

Optimal (type 4, 295 leaves, 23 steps):

$$-\frac{b\,c^3}{12\,d^3\,\left(1+c^2\,x^2\right)^{3/2}}-\frac{b\,c}{6\,d^3\,x^2\,\left(1+c^2\,x^2\right)^{3/2}}+\frac{29\,b\,c^3}{24\,d^3\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^3\,x^3\,\left(1+c^2\,x^2\right)^2}+\frac{7\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^3\,x\,\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{12\,d^3\,\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,d^3\,\left(1+c^2\,x^2\right)}+\frac{35\,c^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{4\,d^3}+\frac{19\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^3}-\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}+\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}$$

Result (type 4, 345 leaves, 23 steps):

$$\frac{7\,b\,c^3}{36\,d^3\,\left(1+c^2\,x^2\right)^{3/2}}+\frac{b\,c}{9\,d^3\,x^2\,\left(1+c^2\,x^2\right)^{3/2}}+\frac{49\,b\,c^3}{24\,d^3\,\sqrt{1+c^2\,x^2}}+\frac{5\,b\,c}{9\,d^3\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{5\,b\,c\,\sqrt{1+c^2\,x^2}}{6\,d^3\,x^2}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d^3\,x^3\,\left(1+c^2\,x^2\right)^2}+\frac{7\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^3\,x\,\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{12\,d^3\,\left(1+c^2\,x^2\right)^2}+\frac{35\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,d^3\,\left(1+c^2\,x^2\right)}+\frac{35\,c^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{4\,d^3}+\frac{19\,b\,c^3\,\text{ArcTanh}\left[\sqrt{1+c^2\,x^2}\right]}{6\,d^3}-\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}+\frac{35\,i\,b\,c^3\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{8\,d^3}$$

Problem 56: Result valid but suboptimal antiderivative.

$$\int x^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$-\frac{b\sqrt{\pi}\ x^2}{16\ c}-\frac{1}{16}\ b\ c\ \sqrt{\pi}\ x^4+\frac{\sqrt{\pi}\ x\ \sqrt{1+c^2\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)}{8\ c^2}+\frac{1}{4}\ x^3\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)-\frac{\sqrt{\pi}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)^2}{16\ b\ c^3}$$

Result (type 3, 181 leaves, 5 steps):

$$-\frac{b\ x^2\ \sqrt{\pi+c^2\ \pi\ x^2}}{16\ c\ \sqrt{1+c^2\ x^2}}-\frac{b\ c\ x^4\ \sqrt{\pi+c^2\ \pi\ x^2}}{16\ \sqrt{1+c^2\ x^2}}+\frac{x\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)}{8\ c^2}+\frac{1}{4}\ x^3\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)-\frac{\sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)^2}{16\ b\ c^3\ \sqrt{1+c^2\ x^2}}$$

Problem 57: Result valid but suboptimal antiderivative.

$$\int x\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)\ dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{b\ \sqrt{\pi}\ x}{3\ c}-\frac{1}{9}\ b\ c\ \sqrt{\pi}\ x^3+\frac{\left(\pi+c^2\ \pi\ x^2\right)^{3/2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)}{3\ c^2\ \pi}$$

Result (type 3, 105 leaves, 2 steps):

$$-\frac{b\ x\ \sqrt{\pi+c^2\ \pi\ x^2}}{3\ c\ \sqrt{1+c^2\ x^2}}-\frac{b\ c\ x^3\ \sqrt{\pi+c^2\ \pi\ x^2}}{9\ \sqrt{1+c^2\ x^2}}+\frac{\left(\pi+c^2\ \pi\ x^2\right)^{3/2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)}{3\ c^2\ \pi}$$

Problem 58: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)\ dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$-\frac{1}{4}\ b\ c\ \sqrt{\pi}\ x^2+\frac{1}{2}\ x\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)+\frac{\sqrt{\pi}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)^2}{4\ b\ c}$$

Result (type 3, 111 leaves, 3 steps):

$$-\frac{b\ c\ x^2\ \sqrt{\pi+c^2\ \pi\ x^2}}{4\ \sqrt{1+c^2\ x^2}}+\frac{1}{2}\ x\ \sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)+\frac{\sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)^2}{4\ b\ c\ \sqrt{1+c^2\ x^2}}$$

Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi+c^2\ \pi\ x^2}\ \left(a+b\ \text{ArcSinh}[c\ x]\right)}{x}\ dx$$

Optimal (type 4, 89 leaves, 8 steps):

$$-b\,c\,\sqrt{\pi}\;x+\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)-2\,\sqrt{\pi}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-b\,\sqrt{\pi}\;\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]+b\,\sqrt{\pi}\;\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 177 leaves, 8 steps):

$$-\frac{b\,c\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{\sqrt{1+c^2\,x^2}}+\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{2\,\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{b\,\sqrt{\pi+c^2\,\pi\,x^2}\;\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}+\frac{b\,\sqrt{\pi+c^2\,\pi\,x^2}\;\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}$$

Problem 60: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^2}\,dx$$

Optimal (type 3, 61 leaves, 3 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+\frac{c\,\sqrt{\pi}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b}+b\,c\,\sqrt{\pi}\;\text{Log}[x]$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+\frac{c\,\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,\sqrt{1+c^2\,x^2}}+\frac{b\,c\,\sqrt{\pi+c^2\,\pi\,x^2}\;\text{Log}[x]}{\sqrt{1+c^2\,x^2}}$$

Problem 61: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^3}\,dx$$

Optimal (type 4, 113 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{\pi}}{2\,x}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-c^2\,\sqrt{\pi}\;\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-\frac{1}{2}\,b\,c^2\,\sqrt{\pi}\;\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]+\frac{1}{2}\,b\,c^2\,\sqrt{\pi}\;\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 201 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}{2\,x\,\sqrt{1+c^2\,x^2}}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-\frac{c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{b\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{1+c^2\,x^2}}+\frac{b\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{1+c^2\,x^2}}$$

Problem 62: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^4}\,dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{b\,c\,\sqrt{\pi}}{6\,x^2}-\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{1}{3}\,b\,c^3\,\sqrt{\pi}\,\text{Log}[x]$$

Result (type 3, 106 leaves, 3 steps):

$$-\frac{b\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}{6\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{b\,c^3\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{Log}[x]}{3\,\sqrt{1+c^2\,x^2}}$$

Problem 64: Result valid but suboptimal antiderivative.

$$\int x^2\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{b\,\pi^{3/2}\,x^2}{32\,c}-\frac{7}{96}\,b\,c\,\pi^{3/2}\,x^4-\frac{1}{36}\,b\,c^3\,\pi^{3/2}\,x^6+\frac{\pi^{3/2}\,x\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{16\,c^2}+\frac{1}{8}\,\pi\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{6}\,x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{\pi^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{32\,b\,c^3}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{b\,\pi\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}{32\,c\,\sqrt{1+c^2\,x^2}}-\frac{7\,b\,c\,\pi\,x^4\,\sqrt{\pi+c^2\,\pi\,x^2}}{96\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x^6\,\sqrt{\pi+c^2\,\pi\,x^2}}{36\,\sqrt{1+c^2\,x^2}}+\frac{\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{16\,c^2}+\frac{1}{8}\,\pi\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{6}\,x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{32\,b\,c^3\,\sqrt{1+c^2\,x^2}}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right) \, dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{b\,\pi^{3/2}\,x}{5\,c} - \frac{2}{15}\,b\,c\,\pi^{3/2}\,x^3 - \frac{1}{25}\,b\,c^3\,\pi^{3/2}\,x^5 + \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Result (type 3, 146 leaves, 3 steps):

$$-\frac{b\,\pi\,x\,\sqrt{\pi + c^2 \pi x^2}}{5\,c\,\sqrt{1 + c^2 x^2}} - \frac{2\,b\,c\,\pi\,x^3\,\sqrt{\pi + c^2 \pi x^2}}{15\,\sqrt{1 + c^2 x^2}} - \frac{b\,c^3\,\pi\,x^5\,\sqrt{\pi + c^2 \pi x^2}}{25\,\sqrt{1 + c^2 x^2}} + \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Problem 66: Result valid but suboptimal antiderivative.

$$\int \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right) \, dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$-\frac{5}{16}\,b\,c\,\pi^{3/2}\,x^2 - \frac{1}{16}\,b\,c^3\,\pi^{3/2}\,x^4 + \frac{3}{8}\,\pi\,x\,\sqrt{\pi + c^2 \pi x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right) + \frac{1}{4}\,x\,\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right) + \frac{3\,\pi^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c}$$

Result (type 3, 180 leaves, 6 steps):

$$-\frac{5\,b\,c\,\pi\,x^2\,\sqrt{\pi + c^2 \pi x^2}}{16\,\sqrt{1 + c^2 x^2}} - \frac{b\,c^3\,\pi\,x^4\,\sqrt{\pi + c^2 \pi x^2}}{16\,\sqrt{1 + c^2 x^2}} + \frac{3}{8}\,\pi\,x\,\sqrt{\pi + c^2 \pi x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right) + \frac{1}{4}\,x\,\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right) + \frac{3\,\pi\,\sqrt{\pi + c^2 \pi x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c\,\sqrt{1 + c^2 x^2}}$$

Problem 67: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right)}{x} \, dx$$

Optimal (type 4, 134 leaves, 10 steps):

$$-\frac{4}{3}\,b\,c\,\pi^{3/2}\,x - \frac{1}{9}\,b\,c^3\,\pi^{3/2}\,x^3 + \pi\,\sqrt{\pi + c^2 \pi x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right) + \frac{1}{3}\,\left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\,\text{ArcSinh}[c\,x]\right) - 2\,\pi^{3/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right] - b\,\pi^{3/2}\,\text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\,x]}\right] + b\,\pi^{3/2}\,\text{PolyLog}\left[2, e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 249 leaves, 10 steps):

$$-\frac{4\,b\,c\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{3\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}{9\,\sqrt{1+c^2\,x^2}}+$$
$$\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{3}\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}+\frac{b\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}$$

Problem 68: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^2}\,dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{1}{4}\,b\,c^3\,\pi^{3/2}\,x^2+\frac{3}{2}\,c^2\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+\frac{3\,c\,\pi^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b}+b\,c\,\pi^{3/2}\,\text{Log}[x]$$

Result (type 3, 177 leaves, 6 steps):

$$-\frac{b\,c^3\,\pi\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}{4\,\sqrt{1+c^2\,x^2}}+\frac{3}{2}\,c^2\,\pi\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}+$$
$$\frac{3\,c\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,\sqrt{1+c^2\,x^2}}+\frac{b\,c\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{Log}[x]}{\sqrt{1+c^2\,x^2}}$$

Problem 69: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^3}\,dx$$

Optimal (type 4, 155 leaves, 11 steps):

$$-\frac{b\,c\,\pi^{3/2}}{2\,x}-b\,c^3\,\pi^{3/2}\,x+\frac{3}{2}\,c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-3\,c^2\,\pi^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-$$
$$\frac{3}{2}\,b\,c^2\,\pi^{3/2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]+\frac{3}{2}\,b\,c^2\,\pi^{3/2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 270 leaves, 11 steps):

$$-\frac{b\,c\,\pi\sqrt{\pi+c^2\,\pi\,x^2}}{2\,x\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\pi\,x\sqrt{\pi+c^2\,\pi\,x^2}}{\sqrt{1+c^2\,x^2}}+\frac{3}{2}\,c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,x^2}-\frac{3\,c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-\frac{3\,b\,c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{1+c^2\,x^2}}+\frac{3\,b\,c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\sqrt{1+c^2\,x^2}}$$

Problem 70: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^4}\,dx$$

Optimal (type 3, 115 leaves, 6 steps):

$$-\frac{b\,c\,\pi^{3/2}}{6\,x^2}-\frac{c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}-\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,x^3}+\frac{c^3\,\pi^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b}+\frac{4}{3}\,b\,c^3\,\pi^{3/2}\,\text{Log}[x]$$

Result (type 3, 184 leaves, 6 steps):

$$-\frac{b\,c\,\pi\sqrt{\pi+c^2\,\pi\,x^2}}{6\,x^2\sqrt{1+c^2\,x^2}}-\frac{c^2\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}-\frac{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,x^3}+\frac{c^3\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\sqrt{1+c^2\,x^2}}+\frac{4\,b\,c^3\,\pi\sqrt{\pi+c^2\,\pi\,x^2}\,\text{Log}[x]}{3\sqrt{1+c^2\,x^2}}$$

Problem 72: Result valid but suboptimal antiderivative.

$$\int x^2\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)dx$$

Optimal (type 3, 213 leaves, 12 steps):

$$-\frac{5\,b\,\pi^{5/2}\,x^2}{256\,c}-\frac{59}{768}\,b\,c\,\pi^{5/2}\,x^4-\frac{17}{288}\,b\,c^3\,\pi^{5/2}\,x^6-\frac{1}{64}\,b\,c^5\,\pi^{5/2}\,x^8+\frac{5\,\pi^{5/2}\,x\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{128\,c^2}+\frac{5}{64}\,\pi^2\,x^3\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5}{48}\,\pi\,x^3\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{8}\,x^3\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{5\,\pi^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{256\,b\,c^3}$$

Result (type 3, 337 leaves, 12 steps):

$$-\frac{5\,b\,\pi^2\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}{256\,c\,\sqrt{1+c^2\,x^2}}-\frac{59\,b\,c\,\pi^2\,x^4\,\sqrt{\pi+c^2\,\pi\,x^2}}{768\,\sqrt{1+c^2\,x^2}}-\frac{17\,b\,c^3\,\pi^2\,x^6\,\sqrt{\pi+c^2\,\pi\,x^2}}{288\,\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,c^5\,\pi^2\,x^8\,\sqrt{\pi+c^2\,\pi\,x^2}}{64\,\sqrt{1+c^2\,x^2}}+\frac{5\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{128\,c^2}+$$
$$\frac{5}{64}\,\pi^2\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5}{48}\,\pi\,x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{8}\,x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-\frac{5\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{256\,b\,c^3\,\sqrt{1+c^2\,x^2}}$$

Problem 73: Result valid but suboptimal antiderivative.

$$\int x\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$-\frac{b\,\pi^{5/2}\,x}{7\,c}-\frac{1}{7}\,b\,c\,\pi^{5/2}\,x^3-\frac{3}{35}\,b\,c^3\,\pi^{5/2}\,x^5-\frac{1}{49}\,b\,c^5\,\pi^{5/2}\,x^7+\frac{\left(\pi+c^2\,\pi\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^2\,\pi}$$

Result (type 3, 193 leaves, 3 steps):

$$-\frac{b\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{7\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,\pi^2\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}{7\,\sqrt{1+c^2\,x^2}}-\frac{3\,b\,c^3\,\pi^2\,x^5\,\sqrt{\pi+c^2\,\pi\,x^2}}{35\,\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,c^5\,\pi^2\,x^7\,\sqrt{\pi+c^2\,\pi\,x^2}}{49\,\sqrt{1+c^2\,x^2}}+\frac{\left(\pi+c^2\,\pi\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^2\,\pi}$$

Problem 74: Result valid but suboptimal antiderivative.

$$\int \left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{25}{96}\,b\,c\,\pi^{5/2}\,x^2-\frac{5}{96}\,b\,c^3\,\pi^{5/2}\,x^4-\frac{b\,\pi^{5/2}\,\left(1+c^2\,x^2\right)^3}{36\,c}+$$
$$\frac{5}{16}\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5}{24}\,\pi\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{6}\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5\,\pi^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{32\,b\,c}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{25\,b\,c\,\pi^2\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}{96\,\sqrt{1+c^2\,x^2}}-\frac{5\,b\,c^3\,\pi^2\,x^4\,\sqrt{\pi+c^2\,\pi\,x^2}}{96\,\sqrt{1+c^2\,x^2}}-\frac{b\,\pi^2\,\left(1+c^2\,x^2\right)^{5/2}\,\sqrt{\pi+c^2\,\pi\,x^2}}{36\,c}+$$
$$\frac{5}{16}\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5}{24}\,\pi\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{6}\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{5\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{32\,b\,c\,\sqrt{1+c^2\,x^2}}$$

Problem 75: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x}\,dx$$

Optimal (type 4, 179 leaves, 13 steps):

$$-\frac{23}{15}\,b\,c\,\pi^{5/2}\,x-\frac{11}{45}\,b\,c^3\,\pi^{5/2}\,x^3-\frac{1}{25}\,b\,c^5\,\pi^{5/2}\,x^5+$$
$$\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{3}\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{5}\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-2\,\pi^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]-$$
$$b\,\pi^{5/2}\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]+b\,\pi^{5/2}\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]$$

Result (type 4, 329 leaves, 13 steps):

$$-\frac{23\,b\,c\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}{15\,\sqrt{1+c^2\,x^2}}-\frac{11\,b\,c^3\,\pi^2\,x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}{45\,\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,c^5\,\pi^2\,x^5\,\sqrt{\pi+c^2\,\pi\,x^2}}{25\,\sqrt{1+c^2\,x^2}}+\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+$$
$$\frac{1}{3}\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)+\frac{1}{5}\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{2\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}-$$
$$\frac{b\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}+\frac{b\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{1+c^2\,x^2}}$$

Problem 76: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x^2}\,dx$$

Optimal (type 3, 157 leaves, 10 steps):

$$-\frac{9}{16} b c^3 \pi^{5/2} x^2 - \frac{1}{16} b c^5 \pi^{5/2} x^4 + \frac{15}{8} c^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right) + \frac{5}{4} c^2 \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \text{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \text{ArcSinh}[c x]\right)}{x} + \frac{15 c \pi^{5/2} \left(a + b \text{ArcSinh}[c x]\right)^2}{16 b} + b c \pi^{5/2} \text{Log}[x]$$

Result (type 3, 257 leaves, 10 steps):

$$-\frac{9 b c^3 \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} + \frac{15}{8} c^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right) + \frac{5}{4} c^2 \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \text{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \text{ArcSinh}[c x]\right)}{x} + \frac{15 c \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right)^2}{16 b \sqrt{1 + c^2 x^2}} + \frac{b c \pi^2 \sqrt{\pi + c^2 \pi x^2} \text{Log}[x]}{\sqrt{1 + c^2 x^2}}$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \text{ArcSinh}[c x]\right)}{x^3} \, dx$$

Optimal (type 4, 205 leaves, 13 steps):

$$-\frac{b c \pi^{5/2}}{2 x} - \frac{7}{3} b c^3 \pi^{5/2} x - \frac{1}{9} b c^5 \pi^{5/2} x^3 + \frac{5}{2} c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right) + \frac{5}{6} c^2 \pi \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \text{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \text{ArcSinh}[c x]\right)}{2 x^2} - 5 c^2 \pi^{5/2} \left(a + b \text{ArcSinh}[c x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right] - \frac{5}{2} b c^2 \pi^{5/2} \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c x]}\right] + \frac{5}{2} b c^2 \pi^{5/2} \text{PolyLog}\left[2, e^{\text{ArcSinh}[c x]}\right]$$

Result (type 4, 355 leaves, 13 steps):

$$-\frac{b c \pi^2 \sqrt{\pi + c^2 \pi x^2}}{2 x \sqrt{1 + c^2 x^2}} - \frac{7 b c^3 \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{3 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2}}{9 \sqrt{1 + c^2 x^2}} + \frac{5}{2} c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right) + \frac{5}{6} c^2 \pi \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b \text{ArcSinh}[c x]\right) - \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b \text{ArcSinh}[c x]\right)}{2 x^2} - \frac{5 c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b \text{ArcSinh}[c x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{\sqrt{1 + c^2 x^2}} - \frac{5 b c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}} + \frac{5 b c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \text{PolyLog}\left[2, e^{\text{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b\,\text{ArcSinh}[c\,x]\right)}{x^4} \, dx$$

Optimal (type 3, 166 leaves, 10 steps):

$$-\frac{b\,c\,\pi^{5/2}}{6\,x^2} - \frac{1}{4}\,b\,c^5\,\pi^{5/2}\,x^2 + \frac{5}{2}\,c^4\,\pi^2\,x\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right) - \frac{5\,c^2\,\pi\,\left(\pi + c^2\,\pi\,x^2\right)^{3/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{3\,x} - \frac{\left(\pi + c^2\,\pi\,x^2\right)^{5/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{3\,x^3} + \frac{5\,c^3\,\pi^{5/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b} + \frac{7}{3}\,b\,c^3\,\pi^{5/2}\,\text{Log}[x]$$

Result (type 3, 266 leaves, 10 steps):

$$-\frac{b\,c\,\pi^2\,\sqrt{\pi + c^2\,\pi\,x^2}}{6\,x^2\,\sqrt{1 + c^2\,x^2}} - \frac{b\,c^5\,\pi^2\,x^2\,\sqrt{\pi + c^2\,\pi\,x^2}}{4\,\sqrt{1 + c^2\,x^2}} + \frac{5}{2}\,c^4\,\pi^2\,x\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right) - \frac{5\,c^2\,\pi\,\left(\pi + c^2\,\pi\,x^2\right)^{3/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{3\,x} - \frac{\left(\pi + c^2\,\pi\,x^2\right)^{5/2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{3\,x^3} + \frac{5\,c^3\,\pi^2\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,\sqrt{1 + c^2\,x^2}} + \frac{7\,b\,c^3\,\pi^2\,\sqrt{\pi + c^2\,\pi\,x^2}\,\text{Log}[x]}{3\,\sqrt{1 + c^2\,x^2}}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{\pi + c^2\,\pi\,x^2}} \, dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$-\frac{8\,b\,x}{15\,c^5\,\sqrt{\pi}} + \frac{4\,b\,x^3}{45\,c^3\,\sqrt{\pi}} - \frac{b\,x^5}{25\,c\,\sqrt{\pi}} + \frac{8\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{15\,c^6\,\pi} - \frac{4\,x^2\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{15\,c^4\,\pi} + \frac{x^4\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Result (type 3, 215 leaves, 6 steps):

$$-\frac{8\,b\,x\,\sqrt{1 + c^2\,x^2}}{15\,c^5\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{4\,b\,x^3\,\sqrt{1 + c^2\,x^2}}{45\,c^3\,\sqrt{\pi + c^2\,\pi\,x^2}} - \frac{b\,x^5\,\sqrt{1 + c^2\,x^2}}{25\,c\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{8\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{15\,c^6\,\pi} - \frac{4\,x^2\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{15\,c^4\,\pi} + \frac{x^4\,\sqrt{\pi + c^2\,\pi\,x^2}\,\left(a + b\,\text{ArcSinh}[c\,x]\right)}{5\,c^2\,\pi}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \left(a+b \operatorname{ArcSinh}[c x]\right)}{\sqrt{\pi+c^2 \pi x^2}} \, dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{3 b x^2}{16 c^3 \sqrt{\pi}}-\frac{b x^4}{16 c \sqrt{\pi}}-\frac{3 x \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{8 c^4 \pi}+\frac{x^3 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{4 c^2 \pi}+\frac{3 \left(a+b \operatorname{ArcSinh}[c x]\right)^2}{16 b c^5 \sqrt{\pi}}$$

Result (type 3, 170 leaves, 5 steps):

$$\frac{3 b x^2 \sqrt{1+c^2 x^2}}{16 c^3 \sqrt{\pi+c^2 \pi x^2}}-\frac{b x^4 \sqrt{1+c^2 x^2}}{16 c \sqrt{\pi+c^2 \pi x^2}}-\frac{3 x \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{8 c^4 \pi}+\frac{x^3 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{4 c^2 \pi}+\frac{3 \left(a+b \operatorname{ArcSinh}[c x]\right)^2}{16 b c^5 \sqrt{\pi}}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a+b \operatorname{ArcSinh}[c x]\right)}{\sqrt{\pi+c^2 \pi x^2}} \, dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{2 b x}{3 c^3 \sqrt{\pi}}-\frac{b x^3}{9 c \sqrt{\pi}}-\frac{2 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^4 \pi}+\frac{x^2 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^2 \pi}$$

Result (type 3, 142 leaves, 4 steps):

$$\frac{2 b x \sqrt{1+c^2 x^2}}{3 c^3 \sqrt{\pi+c^2 \pi x^2}}-\frac{b x^3 \sqrt{1+c^2 x^2}}{9 c \sqrt{\pi+c^2 \pi x^2}}-\frac{2 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^4 \pi}+\frac{x^2 \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^2 \pi}$$

Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a+b \operatorname{ArcSinh}[c x]\right)}{\sqrt{\pi+c^2 \pi x^2}} \, dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$-\frac{b x^2}{4 c \sqrt{\pi}}+\frac{x \sqrt{\pi+c^2 \pi x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)}{2 c^2 \pi}-\frac{\left(a+b \operatorname{ArcSinh}[c x]\right)^2}{4 b c^3 \sqrt{\pi}}$$

Result (type 3, 97 leaves, 3 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,\pi}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{\pi}}$$

Problem 84: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{b\,x}{c\,\sqrt{\pi}}+\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi}$$

Result (type 3, 64 leaves, 2 steps):

$$-\frac{b\,x\,\sqrt{1+c^2\,x^2}}{c\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi}$$

Problem 87: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 41 leaves, 2 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,x}+\frac{b\,c\,\text{Log}[x]}{\sqrt{\pi}}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\pi\,x}+\frac{b\,c\,\sqrt{1+c^2\,x^2}\,\text{Log}[x]}{\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 88: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\,\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 4, 115 leaves, 8 steps):

$$-\frac{b\,c}{2\,\sqrt{\pi}\,x}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi\,x^2}+\frac{c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{\pi}}+$$
$$\frac{b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{\pi}}-\frac{b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{\pi}}$$

Result (type 4, 137 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi\,x^2}+\frac{c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{\pi}}+\frac{b\,c^2\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{\pi}}-\frac{b\,c^2\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{\pi}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\,\sqrt{\pi+c^2\,\pi\,x^2}}\,dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{b\,c}{6\,\sqrt{\pi}\,x^2}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{2\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x}-\frac{2\,b\,c^3\,\text{Log}[x]}{3\,\sqrt{\pi}}$$

Result (type 3, 141 leaves, 4 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{6\,x^2\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x^3}+\frac{2\,c^2\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,\pi\,x}-\frac{2\,b\,c^3\,\sqrt{1+c^2\,x^2}\,\text{Log}[x]}{3\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 91: Result valid but suboptimal antiderivative.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b\,x^2}{4\,c^3\,\pi^{3/2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{3\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^4\,\pi^2}-\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^5\,\pi^{3/2}}-\frac{b\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c^5\,\pi^{3/2}}$$

Result (type 3, 181 leaves, 7 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^3\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{3\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^4\,\pi^2}-\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^5\,\pi^{3/2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,c^5\,\pi\,\sqrt{\pi+c^2\,\pi\,x^2}}$$

Problem 93: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\left(\pi + c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$-\frac{x \left(a + b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^3\,\pi^{3/2}} + \frac{b\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c^3\,\pi^{3/2}}$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{x \left(a + b\,\text{ArcSinh}[c\,x]\right)}{c^2\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{\left(a + b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c^3\,\pi^{3/2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c^3\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\left(\pi + c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$-\frac{a + b\,\text{ArcSinh}[c\,x]}{c^2\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{b\,\text{ArcTan}[c\,x]}{c^2\,\pi^{3/2}}$$

Result (type 3, 70 leaves, 2 steps):

$$-\frac{a + b\,\text{ArcSinh}[c\,x]}{c^2\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} + \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{ArcTan}[c\,x]}{c^2\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int \frac{a + b\,\text{ArcSinh}[c\,x]}{\left(\pi + c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{x \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} - \frac{b\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c\,\pi^{3/2}}$$

Result (type 3, 76 leaves, 2 steps):

$$\frac{x \left(a + b\,\text{ArcSinh}[c\,x]\right)}{\pi\,\sqrt{\pi + c^2\,\pi\,x^2}} - \frac{b\,\sqrt{1 + c^2\,x^2}\,\text{Log}\left[1 + c^2\,x^2\right]}{2\,c\,\pi\,\sqrt{\pi + c^2\,\pi\,x^2}}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{a + b\,\text{ArcSinh}[c\,x]}{x \left(\pi + c^2\,\pi\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 94 leaves, 8 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\,\text{ArcTan}[c\,x]}{\pi^{3/2}}-\frac{2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{b\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}$$

Result (type 4, 119 leaves, 8 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{b\sqrt{1+c^2x^2}\,\text{ArcTan}[c\,x]}{\pi\sqrt{\pi+c^2\pi x^2}}-\frac{2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{b\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\left(\pi+c^2\pi x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 162 leaves, 11 steps):

$$-\frac{b\,c}{2\pi^{3/2}x}-\frac{3c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\,c^2\,\text{ArcTan}[c\,x]}{\pi^{3/2}}+\frac{3c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{3\,b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\pi^{3/2}}-\frac{3\,b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\pi^{3/2}}$$

Result (type 4, 212 leaves, 11 steps):

$$-\frac{b\,c\sqrt{1+c^2x^2}}{2\pi x\sqrt{\pi+c^2\pi x^2}}-\frac{3c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\pi\sqrt{\pi+c^2\pi x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\pi x^2\sqrt{\pi+c^2\pi x^2}}+\frac{b\,c^2\sqrt{1+c^2x^2}\,\text{ArcTan}[c\,x]}{\pi\sqrt{\pi+c^2\pi x^2}}+\frac{3c^2\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{3/2}}+\frac{3\,b\,c^2\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\pi^{3/2}}-\frac{3\,b\,c^2\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\pi^{3/2}}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int\frac{x^6\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(\pi+c^2\pi x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 192 leaves, 11 steps):

$$-\frac{b x^2}{4 c^5 \pi^{5/2}}-\frac{b}{6 c^7 \pi^{5/2} \left(1+c^2 x^2\right)}-\frac{x^5 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{5 x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{5 x \sqrt{\pi+c^2 \pi x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c^6 \pi^3}-\frac{5 \left(a+b \text{ArcSinh}[c x]\right)^2}{4 b c^7 \pi^{5/2}}-\frac{7 b \text{Log}\left[1+c^2 x^2\right]}{6 c^7 \pi^{5/2}}$$

Result (type 3, 256 leaves, 11 steps):

$$-\frac{b}{6 c^7 \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^5 \pi^2 \sqrt{\pi+c^2 \pi x^2}}-\frac{x^5 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{5 x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{5 x \sqrt{\pi+c^2 \pi x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c^6 \pi^3}-\frac{5 \left(a+b \text{ArcSinh}[c x]\right)^2}{4 b c^7 \pi^{5/2}}-\frac{7 b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{6 c^7 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 102: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \left(a+b \text{ArcSinh}[c x]\right)}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^{5/2} \left(1+c^2 x^2\right)}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^5 \pi^{5/2}}+\frac{2 b \text{Log}\left[1+c^2 x^2\right]}{3 c^5 \pi^{5/2}}$$

Result (type 3, 178 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^4 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^5 \pi^{5/2}}+\frac{2 b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{3 c^5 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a+b \text{ArcSinh}[c x]\right)}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^{5/2} \left(1+c^2 x^2\right)}+\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{b \text{Log}\left[1+c^2 x^2\right]}{6 c^3 \pi^{5/2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}+\frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}-\frac{b \sqrt{1+c^2 x^2} \operatorname{Log}\left[1+c^2 x^2\right]}{6 c^3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 105: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \operatorname{ArcSinh}[c x])}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$\frac{b x}{6 c \pi^{5/2} \left(1+c^2 x^2\right)}-\frac{a+b \operatorname{ArcSinh}[c x]}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{b \operatorname{ArcTan}[c x]}{6 c^2 \pi^{5/2}}$$

Result (type 3, 114 leaves, 3 steps):

$$\frac{b x}{6 c \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}-\frac{a+b \operatorname{ArcSinh}[c x]}{3 c^2 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 c^2 \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 106: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{\left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{b}{6 c \pi^{5/2} \left(1+c^2 x^2\right)}+\frac{x (a+b \operatorname{ArcSinh}[c x])}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{2 x (a+b \operatorname{ArcSinh}[c x])}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}-\frac{b \operatorname{Log}\left[1+c^2 x^2\right]}{3 c \pi^{5/2}}$$

Result (type 3, 147 leaves, 4 steps):

$$\frac{b}{6 c \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}+\frac{x (a+b \operatorname{ArcSinh}[c x])}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+$$
$$\frac{2 x (a+b \operatorname{ArcSinh}[c x])}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}-\frac{b \sqrt{1+c^2 x^2} \operatorname{Log}\left[1+c^2 x^2\right]}{3 c \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x \left(\pi+c^2 \pi x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 148 leaves, 11 steps):

$$-\frac{b\,c\,x}{6\,\pi^{5/2}\,\left(1+c^2\,x^2\right)}+\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}+\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{7\,b\,\text{ArcTan}[c\,x]}{6\,\pi^{5/2}}-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}-\frac{b\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{b\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}$$

Result (type 4, 187 leaves, 11 steps):

$$-\frac{b\,c\,x}{6\,\pi^2\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}+\frac{a+b\,\text{ArcSinh}[c\,x]}{\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{7\,b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}-\frac{b\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{b\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}$$

Problem 109: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 247 leaves, 15 steps):

$$-\frac{3\,b\,c}{4\,\pi^{5/2}\,x}+\frac{b\,c}{4\,\pi^{5/2}\,x\,\left(1+c^2\,x^2\right)}+\frac{5\,b\,c^3\,x}{12\,\pi^{5/2}\,\left(1+c^2\,x^2\right)}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{6\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{13\,b\,c^2\,\text{ArcTan}[c\,x]}{6\,\pi^{5/2}}+\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}-\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}$$

Result (type 4, 325 leaves, 15 steps):

$$\frac{b\,c}{4\,\pi^2\,x\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{5\,b\,c^3\,x}{12\,\pi^2\,\sqrt{1+c^2\,x^2}\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{3\,b\,c\,\sqrt{1+c^2\,x^2}}{4\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{6\,\pi\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,\pi\,x^2\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}}-\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{13\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,\pi^2\,\sqrt{\pi+c^2\,\pi\,x^2}}+\frac{5\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\pi^{5/2}}+\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}-\frac{5\,b\,c^2\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\pi^{5/2}}$$

Problem 120: Result optimal but 3 more steps used.

$$\int x^3 \sqrt{d+c^2 d x^2}\, (a+b\,\text{ArcSinh}[c\,x])\, dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$\frac{2 b x \sqrt{d+c^2 d x^2}}{15 c^3 \sqrt{1+c^2 x^2}}-\frac{b x^3 \sqrt{d+c^2 d x^2}}{45 c \sqrt{1+c^2 x^2}}-\frac{b c x^5 \sqrt{d+c^2 d x^2}}{25 \sqrt{1+c^2 x^2}}-\frac{\left(d+c^2 d x^2\right)^{3/2} (a+b\,\text{ArcSinh}[c\,x])}{3 c^4 d}+\frac{\left(d+c^2 d x^2\right)^{5/2} (a+b\,\text{ArcSinh}[c\,x])}{5 c^4 d^2}$$

Result (type 3, 175 leaves, 6 steps):

$$\frac{2 b x \sqrt{d+c^2 d x^2}}{15 c^3 \sqrt{1+c^2 x^2}}-\frac{b x^3 \sqrt{d+c^2 d x^2}}{45 c \sqrt{1+c^2 x^2}}-\frac{b c x^5 \sqrt{d+c^2 d x^2}}{25 \sqrt{1+c^2 x^2}}-\frac{\left(d+c^2 d x^2\right)^{3/2} (a+b\,\text{ArcSinh}[c\,x])}{3 c^4 d}+\frac{\left(d+c^2 d x^2\right)^{5/2} (a+b\,\text{ArcSinh}[c\,x])}{5 c^4 d^2}$$

Problem 128: Result optimal but 3 more steps used.

$$\int x^3 \left(d+c^2 d x^2\right)^{3/2} (a+b\,\text{ArcSinh}[c\,x])\, dx$$

Optimal (type 3, 217 leaves, 4 steps):

$$\frac{2 b d x \sqrt{d+c^2 d x^2}}{35 c^3 \sqrt{1+c^2 x^2}}-\frac{b d x^3 \sqrt{d+c^2 d x^2}}{105 c \sqrt{1+c^2 x^2}}-\frac{8 b c d x^5 \sqrt{d+c^2 d x^2}}{175 \sqrt{1+c^2 x^2}}-\frac{b c^3 d x^7 \sqrt{d+c^2 d x^2}}{49 \sqrt{1+c^2 x^2}}-\frac{\left(d+c^2 d x^2\right)^{5/2} (a+b\,\text{ArcSinh}[c\,x])}{5 c^4 d}+\frac{\left(d+c^2 d x^2\right)^{7/2} (a+b\,\text{ArcSinh}[c\,x])}{7 c^4 d^2}$$

Result (type 3, 217 leaves, 7 steps):

$$\frac{2 b d x \sqrt{d+c^2 d x^2}}{35 c^3 \sqrt{1+c^2 x^2}}-\frac{b d x^3 \sqrt{d+c^2 d x^2}}{105 c \sqrt{1+c^2 x^2}}-\frac{8 b c d x^5 \sqrt{d+c^2 d x^2}}{175 \sqrt{1+c^2 x^2}}-\frac{b c^3 d x^7 \sqrt{d+c^2 d x^2}}{49 \sqrt{1+c^2 x^2}}-\frac{\left(d+c^2 d x^2\right)^{5/2} (a+b\,\text{ArcSinh}[c\,x])}{5 c^4 d}+\frac{\left(d+c^2 d x^2\right)^{7/2} (a+b\,\text{ArcSinh}[c\,x])}{7 c^4 d^2}$$

Problem 136: Result optimal but 3 more steps used.

$$\int x^3 \left(d+c^2 d x^2\right)^{5/2} (a+b\,\text{ArcSinh}[c\,x])\, dx$$

Optimal (type 3, 266 leaves, 4 steps):

$$\frac{2\,b\,d^2\,x\,\sqrt{d+c^2\,d\,x^2}}{63\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d^2\,x^3\,\sqrt{d+c^2\,d\,x^2}}{189\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,d^2\,x^5\,\sqrt{d+c^2\,d\,x^2}}{21\,\sqrt{1+c^2\,x^2}}-\frac{19\,b\,c^3\,d^2\,x^7\,\sqrt{d+c^2\,d\,x^2}}{441\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^5\,d^2\,x^9\,\sqrt{d+c^2\,d\,x^2}}{81\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{9\,c^4\,d^2}$$

Result (type 3, 266 leaves, 7 steps):

$$\frac{2\,b\,d^2\,x\,\sqrt{d+c^2\,d\,x^2}}{63\,c^3\,\sqrt{1+c^2\,x^2}}-\frac{b\,d^2\,x^3\,\sqrt{d+c^2\,d\,x^2}}{189\,c\,\sqrt{1+c^2\,x^2}}-\frac{b\,c\,d^2\,x^5\,\sqrt{d+c^2\,d\,x^2}}{21\,\sqrt{1+c^2\,x^2}}-\frac{19\,b\,c^3\,d^2\,x^7\,\sqrt{d+c^2\,d\,x^2}}{441\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^5\,d^2\,x^9\,\sqrt{d+c^2\,d\,x^2}}{81\,\sqrt{1+c^2\,x^2}}-\frac{\left(d+c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{7\,c^4\,d}+\frac{\left(d+c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{9\,c^4\,d^2}$$

Problem 146: Result optimal but 1 more steps used.

$$\int \frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 192 leaves, 5 steps):

$$\frac{3\,b\,x^2\,\sqrt{1+c^2\,x^2}}{16\,c^3\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{1+c^2\,x^2}}{16\,c\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^4\,d}+\frac{x^3\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{4\,c^2\,d}+\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 192 leaves, 6 steps):

$$\frac{3\,b\,x^2\,\sqrt{1+c^2\,x^2}}{16\,c^3\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{1+c^2\,x^2}}{16\,c\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{8\,c^4\,d}+\frac{x^3\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{4\,c^2\,d}+\frac{3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{16\,b\,c^5\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 148: Result optimal but 1 more steps used.

$$\int \frac{x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{d+c^2\,d\,x^2}}+\frac{x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,d}-\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c\,\sqrt{d+c^2\,d\,x^2}}+\frac{x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^2\,d}-\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 150: Result optimal but 1 more steps used.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{\sqrt{d + c^2 d\, x^2}}\, dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1 + c^2 x^2}\, (a + b\, \text{ArcSinh}[c\, x])^2}{2\, b\, c\, \sqrt{d + c^2 d\, x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1 + c^2 x^2}\, (a + b\, \text{ArcSinh}[c\, x])^2}{2\, b\, c\, \sqrt{d + c^2 d\, x^2}}$$

Problem 151: Result optimal but 1 more steps used.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{x \sqrt{d + c^2 d\, x^2}}\, dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{2 \sqrt{1 + c^2 x^2}\, (a + b\, \text{ArcSinh}[c\, x])\, \text{ArcTanh}\left[e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}} - \frac{b \sqrt{1 + c^2 x^2}\, \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}} + \frac{b \sqrt{1 + c^2 x^2}\, \text{PolyLog}\left[2, e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}}$$

Result (type 4, 122 leaves, 7 steps):

$$-\frac{2 \sqrt{1 + c^2 x^2}\, (a + b\, \text{ArcSinh}[c\, x])\, \text{ArcTanh}\left[e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}} - \frac{b \sqrt{1 + c^2 x^2}\, \text{PolyLog}\left[2, -e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}} + \frac{b \sqrt{1 + c^2 x^2}\, \text{PolyLog}\left[2, e^{\text{ArcSinh}[c\, x]}\right]}{\sqrt{d + c^2 d\, x^2}}$$

Problem 153: Result optimal but 1 more steps used.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{x^3 \sqrt{d + c^2 d\, x^2}}\, dx$$

Optimal (type 4, 203 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,x^2}+\frac{c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 203 leaves, 9 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,x^2}+\frac{c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$\frac{5\,b\,x\,\sqrt{d+c^2\,d\,x^2}}{3\,c^5\,d^2\,\sqrt{1+c^2\,x^2}}-\frac{b\,x^3\,\sqrt{d+c^2\,d\,x^2}}{9\,c^3\,d^2\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{c^6\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^6\,d^2}+\frac{\left(d+c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^6\,d^3}+\frac{b\,\sqrt{d+c^2\,d\,x^2}\,\text{ArcTan}[c\,x]}{c^6\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 220 leaves, 8 steps):

$$\frac{5\,b\,x\,\sqrt{1+c^2\,x^2}}{3\,c^5\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{1+c^2\,x^2}}{9\,c^3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{8\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^6\,d^2}+\frac{4\,x^2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2}+\frac{b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{c^6\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 156: Result optimal but 1 more steps used.

$$\int \frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^3 d \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{3 x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c^4 d^2}-\frac{3 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{4 b c^5 d \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{2 c^5 d \sqrt{d+c^2 d x^2}}$$

Result (type 3, 206 leaves, 8 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^3 d \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{3 x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)}{2 c^4 d^2}-\frac{3 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{4 b c^5 d \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{2 c^5 d \sqrt{d+c^2 d x^2}}$$

Problem 157: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a+b \text{ArcSinh}[c x]\right)}{\left(d+c^2 d x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 136 leaves, 4 steps):

$$-\frac{b x \sqrt{d+c^2 d x^2}}{c^3 d^2 \sqrt{1+c^2 x^2}}+\frac{a+b \text{ArcSinh}[c x]}{c^4 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)}{c^4 d^2}-\frac{b \sqrt{d+c^2 d x^2} \text{ArcTan}[c x]}{c^4 d^2 \sqrt{1+c^2 x^2}}$$

Result (type 3, 141 leaves, 5 steps):

$$-\frac{b x \sqrt{1+c^2 x^2}}{c^3 d \sqrt{d+c^2 d x^2}}-\frac{x^2 \left(a+b \text{ArcSinh}[c x]\right)}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{2 \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)}{c^4 d^2}-\frac{b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{c^4 d \sqrt{d+c^2 d x^2}}$$

Problem 158: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a+b \text{ArcSinh}[c x]\right)}{\left(d+c^2 d x^2\right)^{3/2}} \, dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^3 d \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{2 c^3 d \sqrt{d+c^2 d x^2}}$$

Result (type 3, 130 leaves, 4 steps):

$$-\frac{x \left(a+b \text{ArcSinh}[c x]\right)}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{2 b c^3 d \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{2 c^3 d \sqrt{d+c^2 d x^2}}$$

Problem 161: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,\sqrt{1+c^2\,x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 194 leaves, 9 steps):

$$\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,\sqrt{1+c^2\,x^2}\,\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,\sqrt{1+c^2\,x^2}\,\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 162: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^2\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 143 leaves, 5 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,c^2\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}\,\,\text{Log}[x]}{d^2\,\sqrt{1+c^2\,x^2}}+\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}\,\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 143 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{2\,c^2\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{1+c^2\,x^2}\,\,\text{Log}[x]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{1+c^2\,x^2}\,\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 163: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^3\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 287 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,d\,x^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 287 leaves, 12 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{2\,d\,x\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{2\,d\,x^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,c^2\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2\,d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2\,x^2}\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 164: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{x^4\,\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 228 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d+c^2\,d\,x^2}}{6\,d^2\,x^2\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d\,x^3\,\sqrt{d+c^2\,d\,x^2}}+\frac{4\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,x\,\sqrt{d+c^2\,d\,x^2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{5\,b\,c^3\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}[x]}{3\,d^2\,\sqrt{1+c^2\,x^2}}-\frac{b\,c^3\,\sqrt{d+c^2\,d\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d^2\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 228 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{6\,d\,x^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d\,x^3\,\sqrt{d+c^2\,d\,x^2}}+\frac{4\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,x\,\sqrt{d+c^2\,d\,x^2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{5\,b\,c^3\,\sqrt{1+c^2\,x^2}\,\text{Log}[x]}{3\,d\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c^3\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{2\,d\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 165: Result optimal but 1 more steps used.

$$\int \frac{x^6 \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 281 leaves, 11 steps):

$$-\frac{b}{6\,c^7\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{5\,x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{5\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^6\,d^3}-\frac{5\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^7\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{7\,b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{6\,c^7\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 281 leaves, 12 steps):

$$-\frac{b}{6\,c^7\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2\,x^2}}{4\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^5\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{5\,x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{5\,x\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^6\,d^3}-\frac{5\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{4\,b\,c^7\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{7\,b\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{6\,c^7\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 166: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 \left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\frac{b\,x\,\sqrt{d+c^2\,d\,x^2}}{6\,c^5\,d^3\,\left(1+c^2\,x^2\right)^{3/2}}-\frac{b\,x\,\sqrt{d+c^2\,d\,x^2}}{c^5\,d^3\,\sqrt{1+c^2\,x^2}}-\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,c^6\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}+\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^6\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{c^6\,d^3}-\frac{11\,b\,\sqrt{d+c^2\,d\,x^2}\,\text{ArcTan}[c\,x]}{6\,c^6\,d^3\,\sqrt{1+c^2\,x^2}}$$

Result (type 3, 225 leaves, 9 steps):

$$-\frac{b\,x^3}{6\,c^3\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{5\,b\,x\,\sqrt{1+c^2\,x^2}}{6\,c^5\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^2\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}-\frac{4\,x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d+c^2\,d\,x^2}}+\frac{8\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,c^6\,d^3}-\frac{11\,b\,\sqrt{1+c^2\,x^2}\,\text{ArcTan}[c\,x]}{6\,c^6\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 167: Result optimal but 1 more steps used.

$$\int \frac{x^4 \left(a+b \operatorname{ArcSinh}[c x]\right)}{\left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 203 leaves, 7 steps):

$$\frac{b}{6 c^5 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{x \left(a+b \operatorname{ArcSinh}[c x]\right)}{c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^2}{2 b c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2} \operatorname{Log}\left[1+c^2 x^2\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 203 leaves, 8 steps):

$$\frac{b}{6 c^5 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{x \left(a+b \operatorname{ArcSinh}[c x]\right)}{c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right)^2}{2 b c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2} \operatorname{Log}\left[1+c^2 x^2\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 168: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a+b \operatorname{ArcSinh}[c x]\right)}{\left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$-\frac{b x \sqrt{d+c^2 d x^2}}{6 c^3 d^3 \left(1+c^2 x^2\right)^{3/2}}+\frac{a+b \operatorname{ArcSinh}[c x]}{3 c^4 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{a+b \operatorname{ArcSinh}[c x]}{c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b \sqrt{d+c^2 d x^2} \operatorname{ArcTan}[c x]}{6 c^4 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 149 leaves, 5 steps):

$$-\frac{b x}{6 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{x^2 \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{2 \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 c^4 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 172: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 262 leaves, 11 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{a+b \text{ArcSinh}[c x]}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{a+b \text{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{7 b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 262 leaves, 12 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{a+b \text{ArcSinh}[c x]}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{a+b \text{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{7 b \sqrt{1+c^2 x^2} \text{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}-\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{b \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 173: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^2 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 214 leaves, 5 steps):

$$-\frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 \left(1+c^2 x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{d x \left(d+c^2 d x^2\right)^{3/2}}-\frac{4 c^2 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{8 c^2 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{b c \sqrt{d+c^2 d x^2} \text{Log}[x]}{d^3 \sqrt{1+c^2 x^2}}+\frac{5 b c \sqrt{d+c^2 d x^2} \text{Log}\left[1+c^2 x^2\right]}{6 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 214 leaves, 8 steps):

$$-\frac{b c}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{a+b \text{ArcSinh}[c x]}{d x \left(d+c^2 d x^2\right)^{3/2}}-\frac{4 c^2 x (a+b \text{ArcSinh}[c x])}{3 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{8 c^2 x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{b c \sqrt{1+c^2 x^2} \text{Log}[x]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c \sqrt{1+c^2 x^2} \text{Log}\left[1+c^2 x^2\right]}{6 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 174: Result optimal but 1 more steps used.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^3 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 4, 400 leaves, 15 steps):

$$\frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 b c^2 \sqrt{1+c^2 x^2} \text{ ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{ PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{ PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 400 leaves, 16 steps):

$$\frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{a+b \text{ArcSinh}[c x]}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)}{2 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 b c^2 \sqrt{1+c^2 x^2} \text{ ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}}+\frac{5 c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{ PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}-\frac{5 b c^2 \sqrt{1+c^2 x^2} \text{ PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{2 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \text{ArcSinh}[c x]}{x^4 \left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 3, 297 leaves, 5 steps):

$$\frac{b c^3 \sqrt{d+c^2 d x^2}}{6 d^3 \left(1+c^2 x^2\right)^{3/2}}-\frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 x^2 \sqrt{1+c^2 x^2}}-\frac{a+b \text{ArcSinh}[c x]}{3 d x^3 \left(d+c^2 d x^2\right)^{3/2}}+\frac{2 c^2 \left(a+b \text{ArcSinh}[c x]\right)}{d x \left(d+c^2 d x^2\right)^{3/2}}+\frac{8 c^4 x \left(a+b \text{ArcSinh}[c x]\right)}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{16 c^4 x \left(a+b \text{ArcSinh}[c x]\right)}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{8 b c^3 \sqrt{d+c^2 d x^2} \text{ Log}[x]}{3 d^3 \sqrt{1+c^2 x^2}}-\frac{4 b c^3 \sqrt{d+c^2 d x^2} \text{ Log}\left[1+c^2 x^2\right]}{3 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 297 leaves, 12 steps):

$$\frac{b\,c^3}{6\,d^2\,\sqrt{1+c^2\,x^2}\,\sqrt{d+c^2\,d\,x^2}}-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{6\,d^2\,x^2\,\sqrt{d+c^2\,d\,x^2}}-$$
$$\frac{a+b\,\text{ArcSinh}[c\,x]}{3\,d\,x^3\,\left(d+c^2\,d\,x^2\right)^{3/2}}+\frac{2\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,x\,\left(d+c^2\,d\,x^2\right)^{3/2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\left(d+c^2\,d\,x^2\right)^{3/2}}+$$
$$\frac{16\,c^4\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{8\,b\,c^3\,\sqrt{1+c^2\,x^2}\,\text{Log}[x]}{3\,d^2\,\sqrt{d+c^2\,d\,x^2}}-\frac{4\,b\,c^3\,\sqrt{1+c^2\,x^2}\,\text{Log}\left[1+c^2\,x^2\right]}{3\,d^2\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 194: Result optimal but 1 more steps used.

$$\int\frac{x^m\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 5, 161 leaves, 1 step):

$$\left(x^{1+m}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1+m}{2},\,\frac{3+m}{2},\,-c^2\,x^2\right]\right)\Big/$$
$$\left(\left(1+m\right)\,\sqrt{d+c^2\,d\,x^2}\right)-$$
$$\left(b\,c\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,\,1+\frac{m}{2},\,1+\frac{m}{2}\right\},\,\left\{\frac{3}{2}+\frac{m}{2},\,2+\frac{m}{2}\right\},\,-c^2\,x^2\right]\right)\Big/$$
$$\left(\left(2+3\,m+m^2\right)\,\sqrt{d+c^2\,d\,x^2}\right)$$

Result (type 5, 161 leaves, 2 steps):

$$\left(x^{1+m}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1+m}{2},\,\frac{3+m}{2},\,-c^2\,x^2\right]\right)\Big/$$
$$\left(\left(1+m\right)\,\sqrt{d+c^2\,d\,x^2}\right)-$$
$$\left(b\,c\,x^{2+m}\,\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,\,1+\frac{m}{2},\,1+\frac{m}{2}\right\},\,\left\{\frac{3}{2}+\frac{m}{2},\,2+\frac{m}{2}\right\},\,-c^2\,x^2\right]\right)\Big/$$
$$\left(\left(2+3\,m+m^2\right)\,\sqrt{d+c^2\,d\,x^2}\right)$$

Problem 195: Result optimal but 1 more steps used.

$$\int\frac{x^m\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{\left(d+c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 268 leaves, 3 steps):

$$\frac{x^{1+m} \left(a+b \operatorname{ArcSinh}[c x]\right)}{d \sqrt{d+c^2 d x^2}}-\left(m x^{1+m} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right) \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]\right)\Big/\left(d (1+m) \sqrt{d+c^2 d x^2}\right)-\frac{b c x^{2+m} \sqrt{1+c^2 x^2} \operatorname{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2 x^2\right]}{d (2+m) \sqrt{d+c^2 d x^2}}+\left(b c m x^{2+m} \sqrt{1+c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]\right)\Big/\left(d \left(2+3 m+m^2\right) \sqrt{d+c^2 d x^2}\right)$$

Result (type 5, 268 leaves, 4 steps):

$$\frac{x^{1+m} \left(a+b \operatorname{ArcSinh}[c x]\right)}{d \sqrt{d+c^2 d x^2}}-\left(m x^{1+m} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right) \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]\right)\Big/\left(d (1+m) \sqrt{d+c^2 d x^2}\right)-\frac{b c x^{2+m} \sqrt{1+c^2 x^2} \operatorname{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2 x^2\right]}{d (2+m) \sqrt{d+c^2 d x^2}}+\left(b c m x^{2+m} \sqrt{1+c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]\right)\Big/\left(d \left(2+3 m+m^2\right) \sqrt{d+c^2 d x^2}\right)$$

Problem 196: Result optimal but 1 more steps used.

$$\int \frac{x^m \left(a+b \operatorname{ArcSinh}[c x]\right)}{\left(d+c^2 d x^2\right)^{5/2}} \, dx$$

Optimal (type 5, 402 leaves, 5 steps):

$$\frac{x^{1+m} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 d \left(d+c^2 d x^2\right)^{3/2}}+\frac{(2-m) x^{1+m} \left(a+b \operatorname{ArcSinh}[c x]\right)}{3 d^2 \sqrt{d+c^2 d x^2}}-\left((2-m) m x^{1+m} \sqrt{1+c^2 x^2} \left(a+b \operatorname{ArcSinh}[c x]\right) \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2 x^2\right]\right)\Big/\left(3 d^2 (1+m) \sqrt{d+c^2 d x^2}\right)-\frac{b c (2-m) x^{2+m} \sqrt{1+c^2 x^2} \operatorname{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2 x^2\right]}{3 d^2 (2+m) \sqrt{d+c^2 d x^2}}-\frac{b c x^{2+m} \sqrt{1+c^2 x^2} \operatorname{Hypergeometric2F1}\left[2,\frac{2+m}{2},\frac{4+m}{2},-c^2 x^2\right]}{3 d^2 (2+m) \sqrt{d+c^2 d x^2}}+\left(b c (2-m) m x^{2+m} \sqrt{1+c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2 x^2\right]\right)\Big/\left(3 d^2 \left(2+3 m+m^2\right) \sqrt{d+c^2 d x^2}\right)$$

Result (type 5, 402 leaves, 6 steps):

$$\frac{x^{1+m}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\left(d+c^2\,d\,x^2\right)^{3/2}}+\frac{(2-m)\,x^{1+m}\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\sqrt{d+c^2\,d\,x^2}}-$$
$$\left((2-m)\,m\,x^{1+m}\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2\,x^2\right]\right)\Big/$$
$$\left(3\,d^2\,(1+m)\,\sqrt{d+c^2\,d\,x^2}\right)-\frac{b\,c\,(2-m)\,x^{2+m}\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}-$$
$$\frac{b\,c\,x^{2+m}\sqrt{1+c^2\,x^2}\,\text{Hypergeometric2F1}\left[2,\frac{2+m}{2},\frac{4+m}{2},-c^2\,x^2\right]}{3\,d^2\,(2+m)\,\sqrt{d+c^2\,d\,x^2}}+$$
$$\left(b\,c\,(2-m)\,m\,x^{2+m}\sqrt{1+c^2\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},-c^2\,x^2\right]\right)\Big/$$
$$\left(3\,d^2\,\left(2+3\,m+m^2\right)\sqrt{d+c^2\,d\,x^2}\right)$$

Problem 252: Result valid but suboptimal antiderivative.

$$\int\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,dx$$

Optimal (type 3, 300 leaves, 16 steps):

$$\frac{245\,b^2\,\pi^{5/2}\,x\,\sqrt{1+c^2\,x^2}}{1152}+\frac{65\,b^2\,\pi^{5/2}\,x\,\left(1+c^2\,x^2\right)^{3/2}}{1728}+\frac{1}{108}\,b^2\,\pi^{5/2}\,x\,\left(1+c^2\,x^2\right)^{5/2}-$$
$$\frac{115\,b^2\,\pi^{5/2}\,\text{ArcSinh}[c\,x]}{1152\,c}-\frac{5}{16}\,b\,c\,\pi^{5/2}\,x^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)-$$
$$\frac{5\,b\,\pi^{5/2}\left(1+c^2\,x^2\right)^2\left(a+b\,\text{ArcSinh}[c\,x]\right)}{48\,c}-\frac{b\,\pi^{5/2}\left(1+c^2\,x^2\right)^3\left(a+b\,\text{ArcSinh}[c\,x]\right)}{18\,c}+$$
$$\frac{5}{16}\,\pi^2\,x\,\sqrt{\pi+c^2\,\pi\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2+\frac{5}{24}\,\pi\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2+$$
$$\frac{1}{6}\,x\,\left(\pi+c^2\,\pi\,x^2\right)^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^2+\frac{5\,\pi^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{48\,b\,c}$$

Result (type 3, 420 leaves, 16 steps):

$$\frac{245\, b^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{1152} + \frac{65\, b^2 \pi^2 x \left(1 + c^2 x^2\right) \sqrt{\pi + c^2 \pi x^2}}{1728} + \frac{1}{108}\, b^2 \pi^2 x \left(1 + c^2 x^2\right)^2 \sqrt{\pi + c^2 \pi x^2} - \frac{115\, b^2 \pi^2 \sqrt{\pi + c^2 \pi x^2}\, \text{ArcSinh}[c\, x]}{1152\, c \sqrt{1 + c^2 x^2}} - \frac{5\, b\, c\, \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{16 \sqrt{1 + c^2 x^2}} - \frac{5\, b\, \pi^2 \left(1 + c^2 x^2\right)^{3/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{48\, c} - \frac{b\, \pi^2 \left(1 + c^2 x^2\right)^{5/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{18\, c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{5}{24} \pi x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{1}{6} x \left(\pi + c^2 \pi x^2\right)^{5/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{5\, \pi^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^3}{48\, b\, c \sqrt{1 + c^2 x^2}}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 dx$$

Optimal (type 3, 210 leaves, 10 steps):

$$\frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 + c^2 x^2} + \frac{1}{32} b^2 \pi^{3/2} x \left(1 + c^2 x^2\right)^{3/2} - \frac{9\, b^2 \pi^{3/2} \text{ArcSinh}[c\, x]}{64\, c} - \frac{3}{8} b\, c\, \pi^{3/2} x^2 \left(a + b\, \text{ArcSinh}[c\, x]\right) - \frac{b\, \pi^{3/2} \left(1 + c^2 x^2\right)^2 \left(a + b\, \text{ArcSinh}[c\, x]\right)}{8\, c} + \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{1}{4} x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{\pi^{3/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^3}{8\, b\, c}$$

Result (type 3, 294 leaves, 10 steps):

$$\frac{15}{64} b^2 \pi x \sqrt{\pi + c^2 \pi x^2} + \frac{1}{32} b^2 \pi x \left(1 + c^2 x^2\right) \sqrt{\pi + c^2 \pi x^2} - \frac{9\, b^2 \pi \sqrt{\pi + c^2 \pi x^2}\, \text{ArcSinh}[c\, x]}{64\, c \sqrt{1 + c^2 x^2}} - \frac{3\, b\, c\, \pi x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{8 \sqrt{1 + c^2 x^2}} - \frac{b\, \pi \left(1 + c^2 x^2\right)^{3/2} \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{8\, c} + \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{1}{4} x \left(\pi + c^2 \pi x^2\right)^{3/2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{\pi \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^3}{8\, b\, c \sqrt{1 + c^2 x^2}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 dx$$

Optimal (type 3, 122 leaves, 5 steps):

$$\frac{1}{4} b^2 \sqrt{\pi}\, x \sqrt{1+c^2 x^2} - \frac{b^2 \sqrt{\pi}\, \text{ArcSinh}[c\, x]}{4\, c} - \frac{1}{2} b\, c \sqrt{\pi}\, x^2 \left(a + b\, \text{ArcSinh}[c\, x]\right) + \frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{\sqrt{\pi} \left(a + b\, \text{ArcSinh}[c\, x]\right)^3}{6\, b\, c}$$

Result (type 3, 184 leaves, 5 steps):

$$\frac{1}{4} b^2 x \sqrt{\pi + c^2 \pi x^2} - \frac{b^2 \sqrt{\pi + c^2 \pi x^2}\, \text{ArcSinh}[c\, x]}{4\, c \sqrt{1+c^2 x^2}} - \frac{b\, c\, x^2 \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)}{2 \sqrt{1+c^2 x^2}} + \frac{1}{2} x \sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2 + \frac{\sqrt{\pi + c^2 \pi x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^3}{6\, b\, c \sqrt{1+c^2 x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{\left(\pi + c^2 \pi x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$\frac{\left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{c\, \pi^{3/2}} + \frac{x \left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{2\, b \left(a + b\, \text{ArcSinh}[c\, x]\right) \text{Log}\left[1 + e^{2\, \text{ArcSinh}[c\, x]}\right]}{c\, \pi^{3/2}} - \frac{b^2\, \text{PolyLog}\left[2, -e^{2\, \text{ArcSinh}[c\, x]}\right]}{c\, \pi^{3/2}}$$

Result (type 4, 179 leaves, 6 steps):

$$\frac{x \left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{\pi \sqrt{\pi + c^2 \pi x^2}} + \frac{\sqrt{1+c^2 x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{c\, \pi \sqrt{\pi + c^2 \pi x^2}} - \frac{2\, b \sqrt{1+c^2 x^2} \left(a + b\, \text{ArcSinh}[c\, x]\right) \text{Log}\left[1 + e^{2\, \text{ArcSinh}[c\, x]}\right]}{c\, \pi \sqrt{\pi + c^2 \pi x^2}} - \frac{b^2 \sqrt{1+c^2 x^2}\, \text{PolyLog}\left[2, -e^{2\, \text{ArcSinh}[c\, x]}\right]}{c\, \pi \sqrt{\pi + c^2 \pi x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b\, \text{ArcSinh}[c\, x]\right)^2}{\left(\pi + c^2 \pi x^2\right)^{5/2}}\, dx$$

Optimal (type 4, 204 leaves, 9 steps):

$$-\frac{b^2 x}{3 \pi^{5/2} \sqrt{1+c^2 x^2}}+\frac{b \left(a+b \text{ArcSinh}[c x]\right)}{3 c \pi^{5/2} \left(1+c^2 x^2\right)}+$$
$$\frac{2 \left(a+b \text{ArcSinh}[c x]\right)^2}{3 c \pi^{5/2}}+\frac{x \left(a+b \text{ArcSinh}[c x]\right)^2}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+\frac{2 x \left(a+b \text{ArcSinh}[c x]\right)^2}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}-$$
$$\frac{4 b \left(a+b \text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2 \text{ArcSinh}[c x]}\right]}{3 c \pi^{5/2}}-\frac{2 b^2 \text{PolyLog}\left[2,-e^{2 \text{ArcSinh}[c x]}\right]}{3 c \pi^{5/2}}$$

Result (type 4, 292 leaves, 9 steps):

$$-\frac{b^2 x}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{b \left(a+b \text{ArcSinh}[c x]\right)}{3 c \pi^2 \sqrt{1+c^2 x^2} \sqrt{\pi+c^2 \pi x^2}}+\frac{x \left(a+b \text{ArcSinh}[c x]\right)^2}{3 \pi \left(\pi+c^2 \pi x^2\right)^{3/2}}+$$
$$\frac{2 x \left(a+b \text{ArcSinh}[c x]\right)^2}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}}+\frac{2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{3 c \pi^2 \sqrt{\pi+c^2 \pi x^2}}-$$
$$\frac{4 b \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2 \text{ArcSinh}[c x]}\right]}{3 c \pi^2 \sqrt{\pi+c^2 \pi x^2}}-$$
$$\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-e^{2 \text{ArcSinh}[c x]}\right]}{3 c \pi^2 \sqrt{\pi+c^2 \pi x^2}}$$

Problem 291: Result optimal but 1 more steps used.

$$\int \frac{x^4 \left(a+b \text{ArcSinh}[c x]\right)^2}{\sqrt{d+c^2 d x^2}} \, dx$$

Optimal (type 3, 323 leaves, 10 steps):

$$-\frac{15 b^2 x \left(1+c^2 x^2\right)}{64 c^4 \sqrt{d+c^2 d x^2}}+\frac{b^2 x^3 \left(1+c^2 x^2\right)}{32 c^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{15 b^2 \sqrt{1+c^2 x^2} \text{ArcSinh}[c x]}{64 c^5 \sqrt{d+c^2 d x^2}}+\frac{3 b x^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{8 c^3 \sqrt{d+c^2 d x^2}}-$$
$$\frac{b x^4 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)}{8 c \sqrt{d+c^2 d x^2}}-\frac{3 x \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{8 c^4 d}+$$
$$\frac{x^3 \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^2}{4 c^2 d}+\frac{\sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^3}{8 b c^5 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 323 leaves, 11 steps):

$$-\frac{15 b^2 x \left(1+c^2 x^2\right)}{64 c^4 \sqrt{d+c^2 d x^2}}+\frac{b^2 x^3 \left(1+c^2 x^2\right)}{32 c^2 \sqrt{d+c^2 d x^2}}+\frac{15 b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{64 c^5 \sqrt{d+c^2 d x^2}}+\frac{3 b x^2 \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])}{8 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^4 \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])}{8 c \sqrt{d+c^2 d x^2}}-\frac{3 x \sqrt{d+c^2 d x^2}\ (a+b\,\text{ArcSinh}[c x])^2}{8 c^4 d}+\frac{x^3 \sqrt{d+c^2 d x^2}\ (a+b\,\text{ArcSinh}[c x])^2}{4 c^2 d}+\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])^3}{8 b c^5 \sqrt{d+c^2 d x^2}}$$

Problem 293: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a+b\,\text{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}\,dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{b^2 x \left(1+c^2 x^2\right)}{4 c^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{4 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])}{2 c \sqrt{d+c^2 d x^2}}+\frac{x \sqrt{d+c^2 d x^2}\ (a+b\,\text{ArcSinh}[c x])^2}{2 c^2 d}-\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])^3}{6 b c^3 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 204 leaves, 6 steps):

$$\frac{b^2 x \left(1+c^2 x^2\right)}{4 c^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{4 c^3 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])}{2 c \sqrt{d+c^2 d x^2}}+\frac{x \sqrt{d+c^2 d x^2}\ (a+b\,\text{ArcSinh}[c x])^2}{2 c^2 d}-\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])^3}{6 b c^3 \sqrt{d+c^2 d x^2}}$$

Problem 295: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}\,dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])^3}{3 b c \sqrt{d+c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c x])^3}{3 b c \sqrt{d+c^2 d x^2}}$$

Problem 296: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x\,\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 223 leaves, 8 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}-\frac{2\,b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}$$

Result (type 4, 223 leaves, 9 steps):

$$-\frac{2\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b\,\sqrt{1+c^2 x^2}\,(a+b\,\text{ArcSinh}[c\,x])\,\text{PolyLog}\left[2,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}+\frac{2\,b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,\,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}-\frac{2\,b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,\,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2\,d\,x^2}}$$

Problem 298: Result optimal but 1 more steps used.

$$\int \frac{(a+b\,\text{ArcSinh}[c\,x])^2}{x^3\,\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 360 leaves, 13 steps):

$$-\frac{b\,c\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x\,\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2}+\frac{c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 360 leaves, 14 steps):

$$-\frac{b\,c\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{x\,\sqrt{d+c^2 d\,x^2}}-\frac{\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2}+\frac{c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b\,c^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}+\frac{b^2\,c^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{\sqrt{d+c^2 d\,x^2}}$$

Problem 301: Result optimal but 1 more steps used.

$$\int \frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 400 leaves, 14 steps):

$$\frac{b^2\,x\,\left(1+c^2 x^2\right)}{4\,c^4\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{ArcSinh}[c\,x]}{4\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b\,x^2\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{2\,c^3\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{x^3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^2\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}+\frac{3\,x\,\sqrt{d+c^2 d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,c^4\,d^2}-\frac{\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^3}{2\,b\,c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{2\,b\,\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}-\frac{b^2\,\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c\,x]}\right]}{c^5\,d\,\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 400 leaves, 15 steps):

$$\frac{b^2 x \left(1+c^2 x^2\right)}{4 c^4 d \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\, \text{ArcSinh}[c x]}{4 c^5 d \sqrt{d+c^2 d x^2}}-\frac{b x^2 \sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)}{2 c^3 d \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^2 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^5 d \sqrt{d+c^2 d x^2}}+\frac{3 x \sqrt{d+c^2 d x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^2}{2 c^4 d^2}-\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^3}{2 b c^5 d \sqrt{d+c^2 d x^2}}-\frac{2 b \sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2 \text{ArcSinh}[c x]}\right]}{c^5 d \sqrt{d+c^2 d x^2}}-\frac{b^2 \sqrt{1+c^2 x^2}\, \text{PolyLog}\left[2,-e^{2 \text{ArcSinh}[c x]}\right]}{c^5 d \sqrt{d+c^2 d x^2}}$$

Problem 303: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a+b\, \text{ArcSinh}[c x]\right)^2}{\left(d+c^2 d x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 233 leaves, 7 steps):

$$-\frac{x \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^2 d \sqrt{d+c^2 d x^2}}-\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^3 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^3}{3 b c^3 d \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2 \text{ArcSinh}[c x]}\right]}{c^3 d \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2}\, \text{PolyLog}\left[2,-e^{2 \text{ArcSinh}[c x]}\right]}{c^3 d \sqrt{d+c^2 d x^2}}$$

Result (type 4, 233 leaves, 8 steps):

$$-\frac{x \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^2 d \sqrt{d+c^2 d x^2}}-\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^2}{c^3 d \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right)^3}{3 b c^3 d \sqrt{d+c^2 d x^2}}+\frac{2 b \sqrt{1+c^2 x^2} \left(a+b\, \text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2 \text{ArcSinh}[c x]}\right]}{c^3 d \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2}\, \text{PolyLog}\left[2,-e^{2 \text{ArcSinh}[c x]}\right]}{c^3 d \sqrt{d+c^2 d x^2}}$$

Problem 306: Result optimal but 1 more steps used.

$$\int \frac{\left(a+b\, \text{ArcSinh}[c x]\right)^2}{x \left(d+c^2 d x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 412 leaves, 15 steps):

$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{d\sqrt{d+c^2 d\,x^2}}-\frac{4\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-$$
$$\frac{2\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-$$
$$\frac{2\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,i\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-\frac{2\,i\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-\frac{2\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}$$

Result (type 4, 412 leaves, 16 steps):

$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{d\sqrt{d+c^2 d\,x^2}}-\frac{4\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-$$
$$\frac{2\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-$$
$$\frac{2\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,i\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-\frac{2\,i\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,b\sqrt{1+c^2 x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}+$$
$$\frac{2\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}-\frac{2\,b^2\sqrt{1+c^2 x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\sqrt{d+c^2 d\,x^2}}$$

Problem 308: Result optimal but 1 more steps used.

$$\int\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{x^3\,\left(d+c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 573 leaves, 26 steps):

$$-\frac{b\,c\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,x\,\sqrt{d+c^2d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,\sqrt{d+c^2d\,x^2}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2\,\sqrt{d+c^2d\,x^2}}+\frac{4\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}$$

Result (type 4, 573 leaves, 27 steps):

$$-\frac{b\,c\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,x\,\sqrt{d+c^2d\,x^2}}-\frac{3\,c^2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,\sqrt{d+c^2d\,x^2}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,d\,x^2\,\sqrt{d+c^2d\,x^2}}+\frac{4\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2\,\text{ArcTanh}\left[e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{ArcTanh}\left[\sqrt{1+c^2x^2}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{2\,i\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b\,c^2\,\sqrt{1+c^2x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{PolyLog}\left[2,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}-\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,-e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}+\frac{3\,b^2\,c^2\,\sqrt{1+c^2x^2}\,\text{PolyLog}\left[3,e^{\text{ArcSinh}[c\,x]}\right]}{d\,\sqrt{d+c^2d\,x^2}}$$

Problem 311: Result optimal but 1 more steps used.

$$\int\frac{x^4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{\left(d+c^2d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 398 leaves, 16 steps):

$$-\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \left(a+b\,\text{ArcSinh}[c x]\right)}{3 c^3 d^2 \sqrt{1+c^2 x^2}\ \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{x \left(a+b\,\text{ArcSinh}[c x]\right)^2}{c^4 d^2 \sqrt{d+c^2 d x^2}}-\frac{4 \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^3}{3 b c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{8 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{4 b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 398 leaves, 17 steps):

$$-\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}}+\frac{b^2 \sqrt{1+c^2 x^2}\ \text{ArcSinh}[c x]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}-\frac{b x^2 \left(a+b\,\text{ArcSinh}[c x]\right)}{3 c^3 d^2 \sqrt{1+c^2 x^2}\ \sqrt{d+c^2 d x^2}}-\frac{x^3 \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^2 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{x \left(a+b\,\text{ArcSinh}[c x]\right)^2}{c^4 d^2 \sqrt{d+c^2 d x^2}}-\frac{4 \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^2}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^3}{3 b c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{8 b \sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right) \text{Log}\left[1+e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}+\frac{4 b^2 \sqrt{1+c^2 x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 316: Result optimal but 1 more steps used.

$$\int \frac{\left(a+b\,\text{ArcSinh}[c x]\right)^2}{x \left(d+c^2 d x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 518 leaves, 24 steps):

$$-\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{(a+b \text{ArcSinh}[c x])^2}{3 d (d+c^2 d x^2)^{3/2}}+$$
$$\frac{(a+b \text{ArcSinh}[c x])^2}{d^2 \sqrt{d+c^2 d x^2}}-\frac{14 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 518 leaves, 25 steps):

$$-\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c x (a+b \text{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}+\frac{(a+b \text{ArcSinh}[c x])^2}{3 d (d+c^2 d x^2)^{3/2}}+$$
$$\frac{(a+b \text{ArcSinh}[c x])^2}{d^2 \sqrt{d+c^2 d x^2}}-\frac{14 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x])^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-$$
$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{7 i b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \text{ArcSinh}[c x]) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+$$
$$\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{2 b^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 318: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcSinh}[c x])^2}{x^3 (d+c^2 d x^2)^{5/2}} \, dx$$

Optimal (type 4, 687 leaves, 38 steps):

$$\begin{aligned}
&\frac{b^2 c^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c \left(a+b \text{ArcSinh}[c x]\right)}{d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{2 b c^3 x \left(a+b \text{ArcSinh}[c x]\right)}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\\
&\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{2 d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{26 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{5 c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 c^2 \sqrt{1+c^2 x^2} \text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Result (type 4, 687 leaves, 39 steps):

$$\begin{aligned}
&\frac{b^2 c^2}{3 d^2 \sqrt{d+c^2 d x^2}}-\frac{b c \left(a+b \text{ArcSinh}[c x]\right)}{d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\frac{2 b c^3 x \left(a+b \text{ArcSinh}[c x]\right)}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}}-\\
&\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{6 d \left(d+c^2 d x^2\right)^{3/2}}-\frac{\left(a+b \text{ArcSinh}[c x]\right)^2}{2 d x^2 \left(d+c^2 d x^2\right)^{3/2}}-\frac{5 c^2 \left(a+b \text{ArcSinh}[c x]\right)^2}{2 d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{26 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{ArcTan}\left[e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{5 c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^2 \text{ArcTanh}\left[e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\frac{b^2 c^2 \sqrt{1+c^2 x^2} \text{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\\
&\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,-i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}+\frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[2,i e^{\text{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{5 b c^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right) \text{PolyLog}\left[2,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}-\\
&\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,-e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}+\frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \text{PolyLog}\left[3,e^{\text{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int x^m \left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcSinh}[c x])^2 \, dx$$

Optimal (type 8, 935 leaves, 12 steps):

$$\frac{10 b^2 c^2 d^2 x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^3 (6+m)}+\frac{2 b^2 c^2 d^2 \left(52+15 m+m^2\right) x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^2 (6+m)^3}+$$
$$\frac{2 b^2 c^4 d^2 x^{5+m} \sqrt{d+c^2 d x^2}}{(6+m)^3}-\frac{30 b c d^2 x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(2+m)^2 (4+m) (6+m) \sqrt{1+c^2 x^2}}-$$
$$\frac{10 b c d^2 x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(6+m) \left(8+6 m+m^2\right) \sqrt{1+c^2 x^2}}-\frac{2 b c d^2 x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{\left(12+8 m+m^2\right) \sqrt{1+c^2 x^2}}-$$
$$\frac{10 b c^3 d^2 x^{4+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(4+m)^2 (6+m) \sqrt{1+c^2 x^2}}-\frac{4 b c^3 d^2 x^{4+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(4+m) (6+m) \sqrt{1+c^2 x^2}}-$$
$$\frac{2 b c^5 d^2 x^{6+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(6+m)^2 \sqrt{1+c^2 x^2}}+\frac{15 d^2 x^{1+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{(6+m) \left(8+6 m+m^2\right)}+$$
$$\frac{5 d x^{1+m} \left(d+c^2 d x^2\right)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2}{(4+m) (6+m)}+\frac{x^{1+m} \left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcSinh}[c x])^2}{6+m}+$$
$$\frac{30 b^2 c^2 d^2 x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) (6+m) \sqrt{1+c^2 x^2}}+$$
$$\left(10 b^2 c^2 d^2 (10+3 m) x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2 x^2\right]\right) \Big/$$
$$\left((2+m) (3+m) (4+m)^3 (6+m) \sqrt{1+c^2 x^2}\right)+$$
$$\left(2 b^2 c^2 d^2 \left(264+130 m+15 m^2\right) x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2 x^2\right]\right) \Big/$$
$$\left((2+m) (3+m) (4+m)^2 (6+m)^3 \sqrt{1+c^2 x^2}\right)+\frac{15 d^3 \operatorname{Unintegrable}\left[\frac{x^m (a+b \operatorname{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}},x\right]}{(6+m) \left(8+6 m+m^2\right)}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^m \left(d+c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcSinh}[c x])^2,x\right]$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int x^m \left(d+c^2 d x^2\right)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2 \, dx$$

Optimal (type 8, 487 leaves, 7 steps):

$$\frac{2 b^2 c^2 d x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^3} - \frac{6 b c d x^{2+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])}{(2+m)^2 (4+m) \sqrt{1+c^2 x^2}} - \frac{2 b c d x^{2+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])}{(8+6 m+m^2) \sqrt{1+c^2 x^2}} - \frac{2 b c^3 d x^{4+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])}{(4+m)^2 \sqrt{1+c^2 x^2}} + \frac{3 d x^{1+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^2}{8+6 m+m^2} + \frac{x^{1+m} (d+c^2 d x^2)^{3/2} (a+b \text{ArcSinh}[c x])^2}{4+m} + \frac{6 b^2 c^2 d x^{3+m} \sqrt{d+c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) \sqrt{1+c^2 x^2}} + \left(2 b^2 c^2 d (10+3 m) x^{3+m} \sqrt{d+c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]\right) \Big/ \left((2+m) (3+m) (4+m)^3 \sqrt{1+c^2 x^2}\right) + \frac{3 d^2 \text{Unintegrable}\left[\frac{x^m (a+b \text{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}, x\right]}{8+6 m+m^2}$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m (d+c^2 d x^2)^{3/2} (a+b \text{ArcSinh}[c x])^2, x\right]$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int x^m \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^2 \, dx$$

Optimal (type 8, 198 leaves, 3 steps):

$$-\frac{2 b c x^{2+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])}{(2+m)^2 \sqrt{1+c^2 x^2}} + \frac{x^{1+m} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^2}{2+m} + \frac{2 b^2 c^2 x^{3+m} \sqrt{d+c^2 d x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) \sqrt{1+c^2 x^2}} + \frac{d \text{Unintegrable}\left[\frac{x^m (a+b \text{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}, x\right]}{2+m}$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[x^m \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^2, x\right]$$

Problem 337: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a x]^3}{\sqrt{c+a^2 c x^2}} \, dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a x]^4}{4 a \sqrt{c+a^2 c x^2}}$$

Result (type 3, 40 leaves, 2 steps):

$$\frac{\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a x]^4}{4 a \sqrt{c+a^2 c x^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1+c^2 x^2}}{(a+b\,\text{ArcSinh}[c x])^2}\, dx$$

Optimal (type 4, 149 leaves, 14 steps):

$$-\frac{x\left(1+c^2 x^2\right)}{b c\,(a+b\,\text{ArcSinh}[c x])}+\frac{\text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c x]}{b}\right]}{4 b^2 c^2}+\frac{3\,\text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{4 b^2 c^2}-\frac{\text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c x]}{b}\right]}{4 b^2 c^2}-\frac{3\,\text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b\,\text{ArcSinh}[c x])}{b}\right]}{4 b^2 c^2}$$

Result (type 4, 198 leaves, 14 steps):

$$-\frac{x\left(1+c^2 x^2\right)}{b c\,(a+b\,\text{ArcSinh}[c x])}-\frac{3\,\text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c x]\right]}{4 b^2 c^2}+\frac{3\,\text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 a}{b}+3\,\text{ArcSinh}[c x]\right]}{4 b^2 c^2}+\frac{\text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b\,\text{ArcSinh}[c x]}{b}\right]}{b^2 c^2}+\frac{3\,\text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c x]\right]}{4 b^2 c^2}-\frac{3\,\text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 a}{b}+3\,\text{ArcSinh}[c x]\right]}{4 b^2 c^2}-\frac{\text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b\,\text{ArcSinh}[c x]}{b}\right]}{b^2 c^2}$$

Problem 474: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcSinh}[a x]}}{\sqrt{c+a^2 c x^2}}\, dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1+a^2 x^2}\ \text{ArcSinh}[a x]^{3/2}}{3 a \sqrt{c+a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{3/2}}{3\,a\sqrt{c+a^2 c\,x^2}}$$

Problem 479: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a\,x]^{3/2}}{\sqrt{c+a^2 c\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{5/2}}{5\,a\sqrt{c+a^2 c\,x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{5/2}}{5\,a\sqrt{c+a^2 c\,x^2}}$$

Problem 483: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcSinh}[a\,x]^{5/2}}{\sqrt{c+a^2 c\,x^2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{7/2}}{7\,a\sqrt{c+a^2 c\,x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2\sqrt{1+a^2 x^2}\ \text{ArcSinh}[a\,x]^{7/2}}{7\,a\sqrt{c+a^2 c\,x^2}}$$

Problem 487: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcSinh}\left[\frac{x}{a}\right]}}{\sqrt{a^2+x^2}}\,dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2\,a\sqrt{1+\frac{x^2}{a^2}}\ \text{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3\sqrt{a^2+x^2}}$$

Result (type 3, 39 leaves, 2 steps):

$$\frac{2\,a\,\sqrt{1+\frac{x^2}{a^2}}\;\mathrm{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3\,\sqrt{a^2+x^2}}$$

Problem 492: Result optimal but 1 more steps used.

$$\int \frac{\mathrm{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2+x^2}}\,dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2\,a\,\sqrt{1+\frac{x^2}{a^2}}\;\mathrm{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2+x^2}}$$

Result (type 3, 39 leaves, 2 steps):

$$\frac{2\,a\,\sqrt{1+\frac{x^2}{a^2}}\;\mathrm{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2+x^2}}$$

Problem 495: Result optimal but 1 more steps used.

$$\int \frac{\left(c+a^2\,c\,x^2\right)^{5/2}}{\sqrt{\mathrm{ArcSinh}[a\,x]}}\,dx$$

Optimal (type 4, 396 leaves, 18 steps):

$$\frac{5\,c^2\,\sqrt{c+a^2\,c\,x^2}\;\sqrt{\mathrm{ArcSinh}[a\,x]}}{8\,a\,\sqrt{1+a^2\,x^2}}+\frac{3\,c^2\,\sqrt{\pi}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erf}\left[2\,\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{15\,c^2\,\sqrt{\frac{\pi}{2}}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erf}\left[\sqrt{2}\;\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c^2\,\sqrt{\frac{\pi}{6}}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erf}\left[\sqrt{6}\;\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+\frac{3\,c^2\,\sqrt{\pi}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erfi}\left[2\,\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{15\,c^2\,\sqrt{\frac{\pi}{2}}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erfi}\left[\sqrt{2}\;\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+\frac{c^2\,\sqrt{\frac{\pi}{6}}\;\sqrt{c+a^2\,c\,x^2}\;\mathrm{Erfi}\left[\sqrt{6}\;\sqrt{\mathrm{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}$$

Result (type 4, 396 leaves, 19 steps):

$$\frac{5\,c^2\,\sqrt{c+a^2\,c\,x^2}\,\sqrt{\text{ArcSinh}[a\,x]}}{8\,a\,\sqrt{1+a^2\,x^2}}+\frac{3\,c^2\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{15\,c^2\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c^2\,\sqrt{\frac{\pi}{6}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{6}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+\frac{3\,c^2\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{15\,c^2\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}+\frac{c^2\,\sqrt{\frac{\pi}{6}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{6}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{64\,a\,\sqrt{1+a^2\,x^2}}$$

Problem 496: Result optimal but 1 more steps used.

$$\int\frac{\left(c+a^2\,c\,x^2\right)^{3/2}}{\sqrt{\text{ArcSinh}[a\,x]}}\,dx$$

Optimal (type 4, 264 leaves, 13 steps):

$$\frac{3\,c\,\sqrt{c+a^2\,c\,x^2}\,\sqrt{\text{ArcSinh}[a\,x]}}{4\,a\,\sqrt{1+a^2\,x^2}}+\frac{c\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\,\sqrt{1+a^2\,x^2}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\,\sqrt{1+a^2\,x^2}}$$

Result (type 4, 264 leaves, 14 steps):

$$\frac{3\,c\,\sqrt{c+a^2\,c\,x^2}\,\sqrt{\text{ArcSinh}[a\,x]}}{4\,a\,\sqrt{1+a^2\,x^2}}+\frac{c\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\,\sqrt{1+a^2\,x^2}}+$$
$$\frac{c\,\sqrt{\pi}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{32\,a\,\sqrt{1+a^2\,x^2}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a\,x]}\right]}{4\,a\,\sqrt{1+a^2\,x^2}}$$

Problem 497: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{c+a^2\,c\,x^2}}{\sqrt{\text{ArcSinh}[a\,x]}}\,dx$$

Optimal (type 4, 156 leaves, 8 steps):

$$\frac{\sqrt{c+a^2 c x^2}\,\sqrt{\text{ArcSinh}[a x]}}{a\sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a x]}\right]}{4 a\sqrt{1+a^2 x^2}}+$$
$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a x]}\right]}{4 a\sqrt{1+a^2 x^2}}$$

Result (type 4, 156 leaves, 9 steps):

$$\frac{\sqrt{c+a^2 c x^2}\,\sqrt{\text{ArcSinh}[a x]}}{a\sqrt{1+a^2 x^2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a x]}\right]}{4 a\sqrt{1+a^2 x^2}}+$$
$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c+a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcSinh}[a x]}\right]}{4 a\sqrt{1+a^2 x^2}}$$

Problem 498: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\,\sqrt{\text{ArcSinh}[a x]}}\,dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{2\sqrt{1+a^2 x^2}\,\sqrt{\text{ArcSinh}[a x]}}{a\sqrt{c+a^2 c x^2}}$$

Result (type 3, 40 leaves, 2 steps):

$$\frac{2\sqrt{1+a^2 x^2}\,\sqrt{\text{ArcSinh}[a x]}}{a\sqrt{c+a^2 c x^2}}$$

Problem 504: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\,\text{ArcSinh}[a x]^{3/2}}\,dx$$

Optimal (type 3, 40 leaves, 1 step):

$$-\frac{2\sqrt{1+a^2 x^2}}{a\sqrt{c+a^2 c x^2}\,\sqrt{\text{ArcSinh}[a x]}}$$

Result (type 3, 40 leaves, 2 steps):

$$-\frac{2\sqrt{1+a^2 x^2}}{a\sqrt{c+a^2 c x^2}\,\sqrt{\text{ArcSinh}[a x]}}$$

Problem 509: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c+a^2 c x^2}\,\text{ArcSinh}[a x]^{5/2}}\,dx$$

Optimal (type 3, 42 leaves, 1 step):

$$-\frac{2\sqrt{1+a^2 x^2}}{3 a \sqrt{c+a^2 c x^2}\ \text{ArcSinh}[a x]^{3/2}}$$

Result (type 3, 42 leaves, 2 steps):

$$-\frac{2\sqrt{1+a^2 x^2}}{3 a \sqrt{c+a^2 c x^2}\ \text{ArcSinh}[a x]^{3/2}}$$

Problem 512: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$-\frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^{1+n}}{8 b c^3 (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 \left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 \left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]$$

Result (type 4, 235 leaves, 7 steps):

$$-\frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^{1+n}}{8 b c^3 (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 \left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 \left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]$$

Problem 513: Result optimal but 1 more steps used.

$$\int x \sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c x]\right)^n dx$$

Optimal (type 4, 355 leaves, 9 steps):

$$\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}3^{-1-n}\,e^{-\frac{3a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}e^{-\frac{a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}e^{a/b}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}3^{-1-n}\,e^{\frac{3a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Result (type 4, 355 leaves, 10 steps):

$$\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}3^{-1-n}\,e^{-\frac{3a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}e^{-\frac{a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}e^{a/b}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{8\,c^2\,\sqrt{1+c^2\,x^2}}3^{-1-n}\,e^{\frac{3a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{3\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Problem 514: Result optimal but 1 more steps used.

$$\int\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$\frac{\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{1+c^2\,x^2}}+\frac{1}{c\,\sqrt{1+c^2\,x^2}}2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{c\,\sqrt{1+c^2\,x^2}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Result (type 4, 235 leaves, 7 steps):

$$\frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{1+c^2 x^2}}+\frac{1}{c\,\sqrt{1+c^2 x^2}}$$
$$2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\,-\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]-\frac{1}{c\,\sqrt{1+c^2 x^2}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{2\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right]$$

Problem 515: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x}\,dx$$

Optimal (type 8, 198 leaves, 6 steps):

$$\frac{1}{2\,\sqrt{d+c^2 d x^2}}d\,e^{-\frac{a}{b}}\,\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n$$
$$\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{2\,\sqrt{d+c^2 d x^2}}$$
$$d\,e^{a/b}\,\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+$$
$$d\,\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x\,\sqrt{d+c^2 d x^2}},\,x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x},\,x\right]$$

Problem 516: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x^2}\,dx$$

Optimal (type 8, 83 leaves, 3 steps):

$$\frac{c\,d\,\sqrt{1+c^2 x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{b\,(1+n)\,\sqrt{d+c^2 d x^2}}+d\,\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x^2\,\sqrt{d+c^2 d x^2}},\,x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{d+c^2 d x^2}\ \left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x^2},\,x\right]$$

Problem 517: Result optimal but 1 more steps used.

$$\int x^2 \left(d+c^2 d x^2\right)^{3/2} (a+b \text{ArcSinh}[c x])^n \, dx$$

Optimal (type 4, 616 leaves, 12 steps):

$$-\frac{d \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^{1+n}}{16 b c^3 (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\times 3^{-1-n} d e^{-\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{6 (a+b \text{ArcSinh}[c x])}{b}\right]+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-2 n} d e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 (a+b \text{ArcSinh}[c x])}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n} d e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 (a+b \text{ArcSinh}[c x])}{b}\right]+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n} d e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 (a+b \text{ArcSinh}[c x])}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-2 n} d e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 (a+b \text{ArcSinh}[c x])}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\times 3^{-1-n} d e^{\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \text{ArcSinh}[c x])^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{6 (a+b \text{ArcSinh}[c x])}{b}\right]$$

Result (type 4, 616 leaves, 13 steps):

$$-\frac{d\sqrt{d+c^2 d x^2}\left(a+b\,\text{ArcSinh}[c x]\right)^{1+n}}{16\,b\,c^3\,(1+n)\,\sqrt{1+c^2 x^2}}+\frac{1}{c^3\,\sqrt{1+c^2 x^2}}$$

$$2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\,\sqrt{d+c^2 d x^2}\,\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,-\frac{6\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{c^3\,\sqrt{1+c^2 x^2}}2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\,\sqrt{d+c^2 d x^2}$$

$$\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,-\frac{4\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]-$$

$$\frac{1}{c^3\,\sqrt{1+c^2 x^2}}2^{-7-n}\,d\,e^{-\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}\,\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,-\frac{2\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{c^3\,\sqrt{1+c^2 x^2}}2^{-7-n}\,d\,e^{\frac{2a}{b}}\,\sqrt{d+c^2 d x^2}$$

$$\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{2\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]-$$

$$\frac{1}{c^3\,\sqrt{1+c^2 x^2}}2^{-7-2n}\,d\,e^{\frac{4a}{b}}\,\sqrt{d+c^2 d x^2}\,\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,\frac{4\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1+c^2 x^2}}2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\,\sqrt{d+c^2 d x^2}$$

$$\left(a+b\,\text{ArcSinh}[c x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\,\frac{6\left(a+b\,\text{ArcSinh}[c x]\right)}{b}\right]$$

Problem 518: Result optimal but 1 more steps used.

$$\int x\left(d+c^2 d x^2\right)^{3/2}\left(a+b\,\text{ArcSinh}[c x]\right)^n\,dx$$

Optimal (type 4, 542 leaves, 12 steps):

$$\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 5^{-1-n}\, d\, e^{-\frac{5 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{5\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 3^{-n}\, d\, e^{-\frac{3 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{3\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{16\, c^2 \sqrt{1+c^2 x^2}} d\, e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right]+\frac{1}{16\, c^2 \sqrt{1+c^2 x^2}} d\, e^{a/b} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 3^{-n}\, d\, e^{\frac{3 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{3\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 5^{-1-n}\, d\, e^{\frac{5 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{5\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]$$

Result (type 4, 542 leaves, 13 steps):

$$\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 5^{-1-n}\, d\, e^{-\frac{5 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{5\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 3^{-n}\, d\, e^{-\frac{3 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{3\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{16\, c^2 \sqrt{1+c^2 x^2}} d\, e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, -\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right]+\frac{1}{16\, c^2 \sqrt{1+c^2 x^2}} d\, e^{a/b} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 3^{-n}\, d\, e^{\frac{3 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{3\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{32\, c^2 \sqrt{1+c^2 x^2}} 5^{-1-n}\, d\, e^{\frac{5 a}{b}} \sqrt{d+c^2 d x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\, \frac{5\left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]$$

Problem 519: Result optimal but 1 more steps used.

$$\int \left(d+c^2 d x^2\right)^{3/2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \, dx$$

Optimal (type 4, 420 leaves, 9 steps):

$$\frac{3 d \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^{1+n}}{8 b c (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} d e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-3-n} d e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-3-n} d e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} d e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]$$

Result (type 4, 420 leaves, 10 steps):

$$\frac{3 d \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^{1+n}}{8 b c (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} d e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-3-n} d e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-3-n} d e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-2 (3+n)} d e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \,\text{ArcSinh}[c x]\right)^n \left(\frac{a+b \,\text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 \left(a+b \,\text{ArcSinh}[c x]\right)}{b}\right]$$

Problem 520: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+c^2 d x^2\right)^{3/2} \left(a+b \,\text{ArcSinh}[c x]\right)^n}{x} \, dx$$

Optimal (type 8, 389 leaves, 15 steps):

$$\frac{1}{8\sqrt{d+c^2 d x^2}} 3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{8\sqrt{d+c^2 d x^2}} 5 d^2 e^{-\frac{a}{b}} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcSinh}[c x]}{b}\right]+\frac{1}{8\sqrt{d+c^2 d x^2}} 5 d^2 e^{a/b} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcSinh}[c x]}{b}\right]+\frac{1}{8\sqrt{d+c^2 d x^2}} 3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]+d^2 \text{Unintegrable}\left[\frac{\left(a+b \text{ArcSinh}[c x]\right)^n}{x \sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(d+c^2 d x^2\right)^{3/2} \left(a+b \text{ArcSinh}[c x]\right)^n}{x},x\right]$$

Problem 521: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+c^2 d x^2\right)^{3/2} \left(a+b \text{ArcSinh}[c x]\right)^n}{x^2} \, dx$$

Optimal (type 8, 272 leaves, 9 steps):

$$\frac{3 c d^2 \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^{1+n}}{2 b (1+n) \sqrt{d+c^2 d x^2}}+\frac{1}{\sqrt{d+c^2 d x^2}} 2^{-3-n} c d^2 e^{-\frac{2a}{b}} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{\sqrt{d+c^2 d x^2}} 2^{-3-n} c d^2 e^{\frac{2a}{b}} \sqrt{1+c^2 x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]+d^2 \text{Unintegrable}\left[\frac{\left(a+b \text{ArcSinh}[c x]\right)^n}{x^2 \sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(d+c^2 d x^2\right)^{3/2} \left(a+b \text{ArcSinh}[c x]\right)^n}{x^2},x\right]$$

Problem 522: Result optimal but 1 more steps used.

$$\int x^2 \left(d+c^2 d x^2\right)^{5/2} \left(a+b \text{ArcSinh}[c x]\right)^n \, dx$$

Optimal (type 4, 816 leaves, 15 steps):

$$-\frac{5\, d^2 \sqrt{d+c^2 d\, x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^{1+n}}{128\, b\, c^3\, (1+n)\, \sqrt{1+c^2 x^2}}+\frac{1}{c^3 \sqrt{1+c^2 x^2}}$$

$$2^{-11-3 n}\, d^2\, e^{-\frac{8 a}{b}} \sqrt{d+c^2 d\, x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,-\frac{8 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\times 3^{-1-n}\, d^2\, e^{-\frac{6 a}{b}} \sqrt{d+c^2 d\, x^2}$$

$$\left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{6 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+$$

$$\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (4+n)}\, d^2\, e^{-\frac{4 a}{b}} \sqrt{d+c^2 d\, x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,-\frac{4 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\, d^2\, e^{-\frac{2 a}{b}} \sqrt{d+c^2 d\, x^2}$$

$$\left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]+$$

$$\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\, d^2\, e^{\frac{2 a}{b}} \sqrt{d+c^2 d\, x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\frac{2 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-2 (4+n)}\, d^2\, e^{\frac{4 a}{b}} \sqrt{d+c^2 d\, x^2}$$

$$\left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{4 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]-$$

$$\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-7-n}\times 3^{-1-n}\, d^2\, e^{\frac{6 a}{b}} \sqrt{d+c^2 d\, x^2}\, \left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\frac{6 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]-\frac{1}{c^3 \sqrt{1+c^2 x^2}} 2^{-11-3 n}\, d^2\, e^{\frac{8 a}{b}} \sqrt{d+c^2 d\, x^2}$$

$$\left(a+b\, \text{ArcSinh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcSinh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{8 \left(a+b\, \text{ArcSinh}[c\, x]\right)}{b}\right]$$

Result (type 4, 816 leaves, 16 steps):

$$-\frac{5\,d^2\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{128\,b\,c^3\,(1+n)\,\sqrt{1+c^2\,x^2}}+\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}$$

$$2^{-11-3\,n}\,d^2\,e^{-\frac{8\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,-\frac{8\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}$$

$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{6\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$

$$\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-2\,(4+n)}\,d^2\,e^{-\frac{4\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,-\frac{4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-7-n}\,d^2\,e^{-\frac{2\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}$$

$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$

$$\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-7-n}\,d^2\,e^{\frac{2\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-2\,(4+n)}\,d^2\,e^{\frac{4\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}$$

$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-$$

$$\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\,\frac{6\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1+c^2\,x^2}}2^{-11-3\,n}\,d^2\,e^{\frac{8\,a}{b}}\,\sqrt{d+c^2\,d\,x^2}$$

$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{8\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Problem 523: Result optimal but 1 more steps used.

$$\int x\,\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 745 leaves, 15 steps):

$$\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d+c^2 d x^2} \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{7\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d+c^2 d x^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d+c^2 d x^2} \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{3\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 5\, d^2 e^{-\frac{a}{b}} \sqrt{d+c^2 d x^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 5\, d^2 e^{a/b} \sqrt{d+c^2 d x^2} \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d+c^2 d x^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d+c^2 d x^2} \left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{5\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2 x^2}} 7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d+c^2 d x^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{7\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Result (type 4, 745 leaves, 16 steps) :

$$\frac{1}{128\,c^2\sqrt{1+c^2x^2}}7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d+c^2dx^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{7\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2x^2}}5^{-n}d^2e^{-\frac{5a}{b}}\sqrt{d+c^2dx^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{5\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2x^2}}3^{1-n}d^2e^{-\frac{3a}{b}}\sqrt{d+c^2dx^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{3\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2x^2}}5\,d^2e^{-\frac{a}{b}}\sqrt{d+c^2dx^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2x^2}}5\,d^2e^{a/b}\sqrt{d+c^2dx^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2x^2}}3^{1-n}d^2e^{\frac{3a}{b}}\sqrt{d+c^2dx^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{3\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+$$
$$\frac{1}{128\,c^2\sqrt{1+c^2x^2}}5^{-n}d^2e^{\frac{5a}{b}}\sqrt{d+c^2dx^2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{5\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{128\,c^2\sqrt{1+c^2x^2}}7^{-1-n}d^2e^{\frac{7a}{b}}\sqrt{d+c^2dx^2}$$
$$\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{7\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]$$

Problem 524: Result optimal but 1 more steps used.

$$\int\left(d+c^2dx^2\right)^{5/2}\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 632 leaves, 12 steps):

$$\frac{5 d^2 \sqrt{d + c^2 d x^2} \left(a + b \text{ArcSinh}[c x]\right)^{1+n}}{16 b c (1 + n) \sqrt{1 + c^2 x^2}} + \frac{1}{c \sqrt{1 + c^2 x^2}}$$

$$2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d + c^2 d x^2} \left(a + b \text{ArcSinh}[c x]\right)^n \left(-\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1 + n, -\frac{6 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right] + \frac{1}{c \sqrt{1 + c^2 x^2}} 3 \times 2^{-7-2 n} d^2 e^{-\frac{4 a}{b}} \sqrt{d + c^2 d x^2}$$

$$\left(a + b \text{ArcSinh}[c x]\right)^n \left(-\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{4 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right] +$$

$$\frac{1}{c \sqrt{1 + c^2 x^2}} 15 \times 2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d + c^2 d x^2} \left(a + b \text{ArcSinh}[c x]\right)^n \left(-\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1 + n, -\frac{2 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right] - \frac{1}{c \sqrt{1 + c^2 x^2}} 15 \times 2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d + c^2 d x^2}$$

$$\left(a + b \text{ArcSinh}[c x]\right)^n \left(\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{2 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right] -$$

$$\frac{1}{c \sqrt{1 + c^2 x^2}} 3 \times 2^{-7-2 n} d^2 e^{\frac{4 a}{b}} \sqrt{d + c^2 d x^2} \left(a + b \text{ArcSinh}[c x]\right)^n \left(\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1 + n, \frac{4 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right] - \frac{1}{c \sqrt{1 + c^2 x^2}} 2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d + c^2 d x^2}$$

$$\left(a + b \text{ArcSinh}[c x]\right)^n \left(\frac{a + b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{6 \left(a + b \text{ArcSinh}[c x]\right)}{b}\right]$$

Result (type 4, 632 leaves, 13 steps) :

$$\frac{5 d^2 \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^{1+n}}{16 b c (1+n) \sqrt{1+c^2 x^2}}+\frac{1}{c \sqrt{1+c^2 x^2}}$$

$$2^{-7-n}\times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,-\frac{6 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]+\frac{1}{c \sqrt{1+c^2 x^2}} 3\times 2^{-7-2 n} d^2 e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2}$$

$$\left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{4 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]+$$

$$\frac{1}{c \sqrt{1+c^2 x^2}} 15\times 2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(-\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,-\frac{2 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 15\times 2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2}$$

$$\left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]-$$

$$\frac{1}{c \sqrt{1+c^2 x^2}} 3\times 2^{-7-2 n} d^2 e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} \left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n}$$

$$\text{Gamma}\left[1+n,\frac{4 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]-\frac{1}{c \sqrt{1+c^2 x^2}} 2^{-7-n}\times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d+c^2 d x^2}$$

$$\left(a+b \text{ArcSinh}[c x]\right)^n \left(\frac{a+b \text{ArcSinh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{6 \left(a+b \text{ArcSinh}[c x]\right)}{b}\right]$$

Problem 525: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+c^2 d x^2\right)^{5/2} \left(a+b \text{ArcSinh}[c x]\right)^n}{x} \, dx$$

Optimal (type 8, 755 leaves, 27 steps):

$$\frac{1}{32\sqrt{d+c^2 d x^2}} 5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] - \frac{1}{32\sqrt{d+c^2 d x^2}}$$
$$5\times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] + \frac{1}{8\sqrt{d+c^2 d x^2}} 3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2}$$
$$(a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] +$$
$$\frac{1}{16\sqrt{d+c^2 d x^2}} 11\, d^3 e^{-\frac{a}{b}} \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^n \left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right] + \frac{1}{16\sqrt{d+c^2 d x^2}} 11\, d^3 e^{a/b} \sqrt{1+c^2 x^2}$$
$$(a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right] -$$
$$\frac{1}{32\sqrt{d+c^2 d x^2}} 5\times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] + \frac{1}{8\sqrt{d+c^2 d x^2}} 3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2}$$
$$(a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] +$$
$$\frac{1}{32\sqrt{d+c^2 d x^2}} 5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{1+c^2 x^2}\ (a+b\,\text{ArcSinh}[c\,x])^n \left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcSinh}[c\,x])}{b}\right] + d^3\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcSinh}[c\,x])^n}{x\sqrt{d+c^2 d x^2}},x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(d+c^2 d x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x},x\right]$$

Problem 526: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^{5/2}\,(a+b\,\text{ArcSinh}[c\,x])^n}{x^2}\,dx$$

Optimal (type 8, 454 leaves, 18 steps):

$$\frac{15\,c\,d^3\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^{1+n}}{8\,b\,(1+n)\,\sqrt{d+c^2\,d\,x^2}}+\frac{1}{\sqrt{d+c^2\,d\,x^2}}2^{-2\,(3+n)}\,c\,d^3\,e^{-\frac{4\,a}{b}}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+\frac{1}{\sqrt{d+c^2\,d\,x^2}}2^{-2-n}\,c\,d^3\,e^{-\frac{2\,a}{b}}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,-\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{\sqrt{d+c^2\,d\,x^2}}2^{-2-n}\,c\,d^3\,e^{\frac{2\,a}{b}}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{2\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]-\frac{1}{\sqrt{d+c^2\,d\,x^2}}2^{-2\,(3+n)}\,c\,d^3\,e^{\frac{4\,a}{b}}\,\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcSinh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\,\frac{4\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{b}\right]+d^3\,\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x^2\,\sqrt{d+c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(d+c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^n}{x^2},\,x\right]$$

Test results for the 371 problems in "7.1.5 Inverse hyperbolic sine functions.m"

Problem 50: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{\sqrt{d+c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c\,\sqrt{d+c^2\,d\,x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2\,x^2}\,\left(a+b\,\text{ArcSinh}[c\,x]\right)^2}{2\,b\,c\,\sqrt{d+c^2\,d\,x^2}}$$

Problem 170: Result valid but suboptimal antiderivative.

$$\int\frac{(c\,e+d\,e\,x)^2}{\left(a+b\,\text{ArcSinh}[c+d\,x]\right)^3}\,dx$$

Optimal (type 4, 246 leaves, 18 steps):

$$-\frac{e^2 (c+d x)^2 \sqrt{1+(c+d x)^2}}{2 b d (a+b \text{ArcSinh}[c+d x])^2}-\frac{e^2 (c+d x)}{b^2 d (a+b \text{ArcSinh}[c+d x])}-\frac{3 e^2 (c+d x)^3}{2 b^2 d (a+b \text{ArcSinh}[c+d x])}-\frac{e^2 \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b \text{ArcSinh}[c+d x]}{b}\right]}{8 b^3 d}+\frac{9 e^2 \text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 (a+b \text{ArcSinh}[c+d x])}{b}\right]}{8 b^3 d}+\frac{e^2 \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b \text{ArcSinh}[c+d x]}{b}\right]}{8 b^3 d}-\frac{9 e^2 \text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b \text{ArcSinh}[c+d x])}{b}\right]}{8 b^3 d}$$

Result (type 4, 305 leaves, 18 steps):

$$-\frac{e^2 (c+d x)^2 \sqrt{1+(c+d x)^2}}{2 b d (a+b \text{ArcSinh}[c+d x])^2}-\frac{e^2 (c+d x)}{b^2 d (a+b \text{ArcSinh}[c+d x])}-\frac{3 e^2 (c+d x)^3}{2 b^2 d (a+b \text{ArcSinh}[c+d x])}-\frac{9 e^2 \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c+d x]\right]}{8 b^3 d}+\frac{9 e^2 \text{Cosh}\left[\frac{3 a}{b}\right] \text{CoshIntegral}\left[\frac{3 a}{b}+3 \text{ArcSinh}[c+d x]\right]}{8 b^3 d}+\frac{e^2 \text{Cosh}\left[\frac{a}{b}\right] \text{CoshIntegral}\left[\frac{a+b \text{ArcSinh}[c+d x]}{b}\right]}{b^3 d}+\frac{9 e^2 \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcSinh}[c+d x]\right]}{8 b^3 d}-\frac{9 e^2 \text{Sinh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 a}{b}+3 \text{ArcSinh}[c+d x]\right]}{8 b^3 d}-\frac{e^2 \text{Sinh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b \text{ArcSinh}[c+d x]}{b}\right]}{b^3 d}$$

Problem 369: Unable to integrate problem.

$$\int \frac{x}{\text{ArcSinh}[\text{Sinh}[x]]} \, dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\text{ArcSinh}[\text{Sinh}[x]]+\text{Log}[\text{ArcSinh}[\text{Sinh}[x]]] \left(-\text{ArcSinh}[\text{Sinh}[x]]+x \sqrt{\text{Cosh}[x]^2} \text{ Sech}[x]\right)$$

Result (type 8, 9 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{x}{\text{ArcSinh}[\text{Sinh}[x]]}, x\right]$$

Test results for the 166 problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.m"

Problem 163: Result valid but suboptimal antiderivative.

$$\int \sqrt{f x} (a+b \text{ArcCosh}[c x])^2 \, dx$$

Optimal (type 5, 128 leaves, 2 steps):

$$\frac{2\,(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{3\,f}-\frac{1}{15\,f^2\,\sqrt{-1+c\,x}}$$
$$8\,b\,c\,(f\,x)^{5/2}\,\sqrt{1-c\,x}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]-$$
$$\frac{16\,b^2\,c^2\,(f\,x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{105\,f^3}$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{2\,(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{3\,f}-$$
$$\left(8\,b\,c\,(f\,x)^{5/2}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]\right)\Big/$$
$$\left(15\,f^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-\frac{16\,b^2\,c^2\,(f\,x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{105\,f^3}$$

Test results for the 569 problems in "7.2.4 (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Problem 58: Result optimal but 1 more steps used.

$$\int x^4\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,dx$$

Optimal (type 3, 278 leaves, 7 steps):

$$\frac{b\,x^2\,\sqrt{d-c^2\,d\,x^2}}{32\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^4\,\sqrt{d-c^2\,d\,x^2}}{96\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^6\,\sqrt{d-c^2\,d\,x^2}}{36\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{16\,c^4}-\frac{x^3\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{24\,c^2}+$$
$$\frac{1}{6}\,x^5\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{32\,b\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 278 leaves, 8 steps):

$$\frac{b\,x^2\,\sqrt{d-c^2\,d\,x^2}}{32\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^4\,\sqrt{d-c^2\,d\,x^2}}{96\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^6\,\sqrt{d-c^2\,d\,x^2}}{36\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{16\,c^4}-\frac{x^3\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{24\,c^2}+$$
$$\frac{1}{6}\,x^5\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])-\frac{\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{32\,b\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 59: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 201 leaves, 5 steps):

$$\frac{b x^2 \sqrt{d - c^2 d x^2}}{16 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2}}{16 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) - \frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{16 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 201 leaves, 6 steps):

$$\frac{b x^2 \sqrt{d - c^2 d x^2}}{16 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2}}{16 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) - \frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{16 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 60: Result optimal but 1 more steps used.

$$\int \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 124 leaves, 3 steps):

$$-\frac{b c x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{1}{2} x \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) - \frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{4 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 124 leaves, 4 steps):

$$-\frac{b c x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{1}{2} x \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x]) - \frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{4 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 61: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])}{x^2} \, dx$$

Optimal (type 3, 118 leaves, 3 steps):

$$-\frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])}{x} + \frac{c \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{2 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 118 leaves, 4 steps):

$$-\frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])}{x} + \frac{c \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^2}{2 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{x^6}\,dx$$

Optimal (type 3, 199 leaves, 4 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{20 x^4 \sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{30 x^2 \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{5 d x^5}-\frac{2 c^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{15 d x^3}-\frac{2 b c^5 \sqrt{d-c^2 d x^2}\,\text{Log}[x]}{15 \sqrt{-1+c x}\,\sqrt{1+c x}}$$

Result (type 3, 226 leaves, 5 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{20 x^4 \sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{30 x^2 \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{5 x^5}+\frac{c^2 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{15 x^3}+\frac{2 c^4 \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{15 x}-\frac{2 b c^5 \sqrt{d-c^2 d x^2}\,\text{Log}[x]}{15 \sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 64: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])}{x^8}\,dx$$

Optimal (type 3, 279 leaves, 4 steps):

$$-\frac{b c \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{b c^3 \sqrt{d-c^2 d x^2}}{140 x^4 \sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{2 b c^5 \sqrt{d-c^2 d x^2}}{105 x^2 \sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{7 d x^7}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{35 d x^5}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{105 d x^3}-\frac{8 b c^7 \sqrt{d-c^2 d x^2}\,\text{Log}[x]}{105 \sqrt{-1+c x}\,\sqrt{1+c x}}$$

Result (type 3, 303 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{42\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}}{140\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}{105\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,x^7}+\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{35\,x^5}+\frac{4\,c^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{105\,x^3}+\frac{8\,c^6\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{105\,x}-\frac{8\,b\,c^7\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{105\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 3, 272 leaves, 3 steps):

$$\frac{8\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{105\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{315\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^5\,\sqrt{d-c^2\,d\,x^2}}{175\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^7\,\sqrt{d-c^2\,d\,x^2}}{49\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^6\,d}+\frac{2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^6\,d^2}-\frac{\left(d-c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^6\,d^3}$$

Result (type 3, 302 leaves, 4 steps):

$$\frac{8\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{105\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{315\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^5\,\sqrt{d-c^2\,d\,x^2}}{175\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^7\,\sqrt{d-c^2\,d\,x^2}}{49\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{8\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{105\,c^6}-\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{35\,c^4}-\frac{x^4\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^2}$$

Problem 66: Result valid but suboptimal antiderivative.

$$\int x^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$\frac{2\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,d}+\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^4\,d^2}$$

Result (type 3, 214 leaves, 4 steps):

$$\frac{2\,b\,x\,\sqrt{d-c^2\,d\,x^2}}{15\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,x^3\,\sqrt{d-c^2\,d\,x^2}}{45\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c\,x^5\,\sqrt{d-c^2\,d\,x^2}}{25\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{15\,c^4}-\frac{x^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^2}$$

Problem 68: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}\,dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{b\,c\,x\,\sqrt{d-c^2\,d\,x^2}}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 213 leaves, 9 steps):

$$-\frac{b\,c\,x\,\sqrt{d-c^2\,d\,x^2}}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 69: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^3}\,dx$$

Optimal (type 4, 235 leaves, 8 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{2\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,x^2}+\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 235 leaves, 9 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{2\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,x^2}+\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 70: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^5}\,dx$$

Optimal (type 4, 315 leaves, 10 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{12\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}}{8\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,x^4}+\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{8\,x^2}+\frac{c^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,c^4\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,c^4\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 315 leaves, 11 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{12\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}}{8\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,x^4}+\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{8\,x^2}+\frac{c^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,c^4\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,c^4\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \text{ArcCosh}[c x]\right)}{x^8} \, dx$$

Optimal (type 3, 247 leaves, 5 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^3 d \sqrt{d-c^2 d x^2}}{35 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{70 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{7 d x^7}-\frac{2 c^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{35 d x^5}+\frac{2 b c^7 d \sqrt{d-c^2 d x^2} \, \text{Log}[x]}{35 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 322 leaves, 6 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{42 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^3 d \sqrt{d-c^2 d x^2}}{35 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{70 x^2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{3 c^2 d \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{35 x^5}-\frac{c^4 d \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{35 x^3}-\frac{2 c^6 d \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{35 x}-\frac{d (1-c x) (1+c x) \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{7 x^7}+\frac{2 b c^7 d \sqrt{d-c^2 d x^2} \, \text{Log}[x]}{35 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \text{ArcCosh}[c x]\right)}{x^{10}} \, dx$$

Optimal (type 3, 328 leaves, 5 steps):

$$-\frac{b c d \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d \sqrt{d-c^2 d x^2}}{420 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b c^7 d \sqrt{d-c^2 d x^2}}{315 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{9 d x^9}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{63 d x^7}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{315 d x^5}+\frac{8 b c^9 d \sqrt{d-c^2 d x^2} \, \text{Log}[x]}{315 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 401 leaves, 6 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{72\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5\,b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{189\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{420\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{315\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c^2\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{21\,x^7}-\frac{c^4\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{105\,x^5}-\frac{4\,c^6\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,x^3}-\frac{8\,c^8\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,x}-\frac{d\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,x^9}+\frac{8\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{315\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 79: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^{12}}\,dx$$

Optimal (type 3, 409 leaves, 5 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{110\,x^{10}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{66\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{1386\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{770\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}}{1155\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,d\,x^{11}}-\frac{2\,c^2\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{33\,d\,x^9}-\frac{8\,c^4\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{231\,d\,x^7}-\frac{16\,c^6\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,d\,x^5}+\frac{16\,b\,c^{11}\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{1155\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 480 leaves, 6 steps):

$$-\frac{b\,c\,d\,\sqrt{d-c^2\,d\,x^2}}{110\,x^{10}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,\sqrt{d-c^2\,d\,x^2}}{66\,x^8\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d\,\sqrt{d-c^2\,d\,x^2}}{1386\,x^6\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^7\,d\,\sqrt{d-c^2\,d\,x^2}}{770\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{4\,b\,c^9\,d\,\sqrt{d-c^2\,d\,x^2}}{1155\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c^2\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{33\,x^9}-\frac{c^4\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{231\,x^7}-\frac{2\,c^6\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{385\,x^5}-\frac{8\,c^8\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,x^3}-\frac{16\,c^{10}\,d\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{1155\,x}-\frac{d\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{11\,x^{11}}+\frac{16\,b\,c^{11}\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{1155\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int x^7 \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])\, dx$$

Optimal (type 3, 399 leaves, 4 steps):

$$\frac{16 b d x \sqrt{d-c^2 d x^2}}{1155 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d x^3 \sqrt{d-c^2 d x^2}}{3465 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d x^5 \sqrt{d-c^2 d x^2}}{1925 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d x^7 \sqrt{d-c^2 d x^2}}{1617 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{4 b c d x^9 \sqrt{d-c^2 d x^2}}{297 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 d x^{11} \sqrt{d-c^2 d x^2}}{121 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b\,\text{ArcCosh}[c x])}{5 c^8 d}+\frac{3 \left(d-c^2 d x^2\right)^{7/2} (a+b\,\text{ArcCosh}[c x])}{7 c^8 d^2}-\frac{\left(d-c^2 d x^2\right)^{9/2} (a+b\,\text{ArcCosh}[c x])}{3 c^8 d^3}+\frac{\left(d-c^2 d x^2\right)^{11/2} (a+b\,\text{ArcCosh}[c x])}{11 c^8 d^4}$$

Result (type 3, 460 leaves, 5 steps):

$$\frac{16 b d x \sqrt{d-c^2 d x^2}}{1155 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d x^3 \sqrt{d-c^2 d x^2}}{3465 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d x^5 \sqrt{d-c^2 d x^2}}{1925 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d x^7 \sqrt{d-c^2 d x^2}}{1617 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{4 b c d x^9 \sqrt{d-c^2 d x^2}}{297 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^3 d x^{11} \sqrt{d-c^2 d x^2}}{121 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{16 d (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{1155 c^8}-\frac{8 d x^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{231 c^6}-\frac{2 d x^4 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{33 c^4}-\frac{d x^6 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{11 c^2}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int x^5 \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])\, dx$$

Optimal (type 3, 321 leaves, 4 steps):

$$\frac{8\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{315\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{945\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{525\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{10\,b\,c\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{441\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{81\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^6\,d}+\frac{2\,\left(d-c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^6\,d^2}-\frac{\left(d-c^2\,d\,x^2\right)^{9/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,c^6\,d^3}$$

Result (type 3, 366 leaves, 5 steps):

$$\frac{8\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{315\,c^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{945\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{525\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{10\,b\,c\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{441\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^9\,\sqrt{d-c^2\,d\,x^2}}{81\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{8\,d\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{315\,c^6}-\frac{4\,d\,x^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{63\,c^4}-\frac{d\,x^4\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{9\,c^2}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int x^3\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,dx$$

Optimal (type 3, 243 leaves, 4 steps):

$$\frac{2\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{35\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{105\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{8\,b\,c\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{175\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{49\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{5\,c^4\,d}+\frac{\left(d-c^2\,d\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^4\,d^2}$$

Result (type 3, 272 leaves, 5 steps):

$$\frac{2\,b\,d\,x\,\sqrt{d-c^2\,d\,x^2}}{35\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}{105\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{8\,b\,c\,d\,x^5\,\sqrt{d-c^2\,d\,x^2}}{175\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,d\,x^7\,\sqrt{d-c^2\,d\,x^2}}{49\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{2\,d\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{35\,c^4}-\frac{d\,x^2\,(1-c\,x)^2\,(1+c\,x)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{7\,c^2}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x]) \, dx$$

Optimal (type 3, 293 leaves, 10 steps):

$$-\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b d^2 \left(1-c^2 x^2\right)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+\frac{5}{24} d x \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])+$$
$$\frac{1}{6} x \left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])-\frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 324 leaves, 9 steps):

$$-\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{b d^2 \left(1-c^2 x^2\right)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+$$
$$\frac{5}{24} d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])+$$
$$\frac{1}{6} d^2 x (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-\frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x^2} \, dx$$

Optimal (type 3, 284 leaves, 12 steps):

$$\frac{9 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^4 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])-$$
$$\frac{5}{4} c^2 d x \left(d-c^2 d x^2\right)^{3/2} (a+b \text{ArcCosh}[c x])-\frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x}+$$
$$\frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{16 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 315 leaves, 11 steps):

$$\frac{9\,b\,c^3\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{16\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d^2\,x^4\,\sqrt{d-c^2\,d\,x^2}}{16\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{15}{8}\,c^2\,d^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{5}{4}\,c^2\,d^2\,x\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)-\frac{d^2\,\left(1-c\,x\right)^2\,\left(1+c\,x\right)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x}+\frac{15\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{16\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 91: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^4}\,dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$-\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}}{6\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5}{2}\,c^4\,d^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)+\frac{5\,c^2\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x}-\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3}-\frac{5\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{7\,b\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 324 leaves, 11 steps):

$$-\frac{b\,c\,d^2\,\sqrt{d-c^2\,d\,x^2}}{6\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5}{2}\,c^4\,d^2\,x\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)+\frac{5\,c^2\,d^2\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x}-\frac{d^2\,\left(1-c\,x\right)^2\,\left(1+c\,x\right)^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3}-\frac{5\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{4\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{7\,b\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 92: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{20 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{11 b c^3 d^2 \sqrt{d-c^2 d x^2}}{30 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{c^4 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{x}+\frac{c^2 d \left(d-c^2 d x^2\right)^{3/2} \left(a+b \text{ArcCosh}[c x]\right)}{3 x^3}-\frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{5 x^5}+\frac{c^5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^2}{2 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^5 d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{15 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 324 leaves, 11 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{20 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{11 b c^3 d^2 \sqrt{d-c^2 d x^2}}{30 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{c^4 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{x}+\frac{c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{3 x^3}-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)}{5 x^5}+\frac{c^5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^2}{2 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^5 d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{15 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \text{ArcCosh}[c x]\right)}{x^{10}} \, dx$$

Optimal (type 3, 314 leaves, 6 steps):

$$-\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 \left(1-c^2 x^2\right)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} \left(a+b \text{ArcCosh}[c x]\right)}{9 d x^9}-\frac{2 c^2 \left(d-c^2 d x^2\right)^{7/2} \left(a+b \text{ArcCosh}[c x]\right)}{63 d x^7}-\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 448 leaves, 7 steps):

$$-\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c d^2 \left(1-c^2 x^2\right)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{c^4 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{21 x^5}+\frac{c^6 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x^3}+\frac{2 c^8 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x}+\frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{63 x^7}-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{9 x^9}-\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \text{ArcCosh}[c x])}{x^{12}} \, dx$$

Optimal (type 3, 385 leaves, 5 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{110 x^{10} \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 \sqrt{d-c^2 d x^2}}{792 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c^5 d^2 \sqrt{d-c^2 d x^2}}{4158 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{924 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2}}{693 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{11 d x^{11}}-\frac{4 c^2 \left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{99 d x^9}-\frac{8 c^4 \left(d-c^2 d x^2\right)^{7/2} (a+b \text{ArcCosh}[c x])}{693 d x^7}-\frac{8 b c^{11} d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{693 \sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 3, 519 leaves, 6 steps):

$$-\frac{b c d^2 \sqrt{d-c^2 d x^2}}{110 x^{10} \sqrt{-1+c x} \sqrt{1+c x}}+\frac{23 b c^3 d^2 \sqrt{d-c^2 d x^2}}{792 x^8 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{113 b c^5 d^2 \sqrt{d-c^2 d x^2}}{4158 x^6 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{924 x^4 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2}}{693 x^2 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{5 c^4 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{231 x^7}+\frac{c^6 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{231 x^5}+\frac{4 c^8 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{693 x^3}+\frac{8 c^{10} d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{693 x}+\frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{99 x^9}-\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{11 x^{11}}-\frac{8 b c^{11} d^2 \sqrt{d-c^2 d x^2} \text{Log}[x]}{693 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int x^7 \left(d-c^2 d x^2\right)^{5/2} (a+b\,\text{ArcCosh}[c x])\,dx$$

Optimal (type 3, 458 leaves, 4 steps):

$$\frac{16 b d^2 x \sqrt{d-c^2 d x^2}}{3003 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d^2 x^3 \sqrt{d-c^2 d x^2}}{9009 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d^2 x^5 \sqrt{d-c^2 d x^2}}{5005 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b d^2 x^7 \sqrt{d-c^2 d x^2}}{21021 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{53 b c d^2 x^9 \sqrt{d-c^2 d x^2}}{3861 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{27 b c^3 d^2 x^{11} \sqrt{d-c^2 d x^2}}{1573 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{13} \sqrt{d-c^2 d x^2}}{169 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{7/2} (a+b\,\text{ArcCosh}[c x])}{7 c^8 d}+\frac{\left(d-c^2 d x^2\right)^{9/2} (a+b\,\text{ArcCosh}[c x])}{3 c^8 d^2}-\frac{3 \left(d-c^2 d x^2\right)^{11/2} (a+b\,\text{ArcCosh}[c x])}{11 c^8 d^3}+\frac{\left(d-c^2 d x^2\right)^{13/2} (a+b\,\text{ArcCosh}[c x])}{13 c^8 d^4}$$

Result (type 3, 527 leaves, 5 steps):

$$\frac{16 b d^2 x \sqrt{d-c^2 d x^2}}{3003 c^7 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{8 b d^2 x^3 \sqrt{d-c^2 d x^2}}{9009 c^5 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b d^2 x^5 \sqrt{d-c^2 d x^2}}{5005 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{5 b d^2 x^7 \sqrt{d-c^2 d x^2}}{21021 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{53 b c d^2 x^9 \sqrt{d-c^2 d x^2}}{3861 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{27 b c^3 d^2 x^{11} \sqrt{d-c^2 d x^2}}{1573 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c^5 d^2 x^{13} \sqrt{d-c^2 d x^2}}{169 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{16 d^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{3003 c^8}-\frac{8 d^2 x^2 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{429 c^6}-\frac{6 d^2 x^4 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{143 c^4}-\frac{d^2 x^6 (1-c x)^3 (1+c x)^3 \sqrt{d-c^2 d x^2} (a+b\,\text{ArcCosh}[c x])}{13 c^2}$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int x^5 \left(d-c^2 d x^2\right)^{5/2} (a+b\,\text{ArcCosh}[c x])\,dx$$

Optimal (type 3, 378 leaves, 4 steps):

$$\frac{8\, b\, d^2\, x\, \sqrt{d-c^2\, d\, x^2}}{693\, c^5\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{4\, b\, d^2\, x^3\, \sqrt{d-c^2\, d\, x^2}}{2079\, c^3\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{b\, d^2\, x^5\, \sqrt{d-c^2\, d\, x^2}}{1155\, c\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{113\, b\, c\, d^2\, x^7\, \sqrt{d-c^2\, d\, x^2}}{4851\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{23\, b\, c^3\, d^2\, x^9\, \sqrt{d-c^2\, d\, x^2}}{891\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{b\, c^5\, d^2\, x^{11}\, \sqrt{d-c^2\, d\, x^2}}{121\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{\left(d-c^2\, d\, x^2\right)^{7/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{7\, c^6\, d}+\frac{2\, \left(d-c^2\, d\, x^2\right)^{9/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{9\, c^6\, d^2}-\frac{\left(d-c^2\, d\, x^2\right)^{11/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{11\, c^6\, d^3}$$

Result (type 3, 429 leaves, 5 steps):

$$\frac{8\, b\, d^2\, x\, \sqrt{d-c^2\, d\, x^2}}{693\, c^5\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{4\, b\, d^2\, x^3\, \sqrt{d-c^2\, d\, x^2}}{2079\, c^3\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{b\, d^2\, x^5\, \sqrt{d-c^2\, d\, x^2}}{1155\, c\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{113\, b\, c\, d^2\, x^7\, \sqrt{d-c^2\, d\, x^2}}{4851\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{23\, b\, c^3\, d^2\, x^9\, \sqrt{d-c^2\, d\, x^2}}{891\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{b\, c^5\, d^2\, x^{11}\, \sqrt{d-c^2\, d\, x^2}}{121\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{8\, d^2\, (1-c\, x)^3\, (1+c\, x)^3\, \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{693\, c^6}-\frac{4\, d^2\, x^2\, (1-c\, x)^3\, (1+c\, x)^3\, \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{99\, c^4}-\frac{d^2\, x^4\, (1-c\, x)^3\, (1+c\, x)^3\, \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{11\, c^2}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int x^3\, \left(d-c^2\, d\, x^2\right)^{5/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)\, dx$$

Optimal (type 3, 298 leaves, 4 steps):

$$\frac{2\, b\, d^2\, x\, \sqrt{d-c^2\, d\, x^2}}{63\, c^3\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{b\, d^2\, x^3\, \sqrt{d-c^2\, d\, x^2}}{189\, c\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{b\, c\, d^2\, x^5\, \sqrt{d-c^2\, d\, x^2}}{21\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+\frac{19\, b\, c^3\, d^2\, x^7\, \sqrt{d-c^2\, d\, x^2}}{441\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{b\, c^5\, d^2\, x^9\, \sqrt{d-c^2\, d\, x^2}}{81\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}-\frac{\left(d-c^2\, d\, x^2\right)^{7/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{7\, c^4\, d}+\frac{\left(d-c^2\, d\, x^2\right)^{9/2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{9\, c^4\, d^2}$$

Result (type 3, 331 leaves, 5 steps):

$$\frac{2\,b\,d^2\,x\,\sqrt{d-c^2\,d\,x^2}}{63\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d^2\,x^3\,\sqrt{d-c^2\,d\,x^2}}{189\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{b\,c\,d^2\,x^5\,\sqrt{d-c^2\,d\,x^2}}{21\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{19\,b\,c^3\,d^2\,x^7\,\sqrt{d-c^2\,d\,x^2}}{441\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^5\,d^2\,x^9\,\sqrt{d-c^2\,d\,x^2}}{81\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{2\,d^2\,(1-c\,x)^3\,(1+c\,x)^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{63\,c^4}-$$
$$\frac{d^2\,x^2\,(1-c\,x)^3\,(1+c\,x)^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{9\,c^2}$$

Problem 103: Result valid but suboptimal antiderivative.

$$\int\sqrt{1-x^2}\,\mathrm{ArcCosh}[x]\,dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$-\frac{\sqrt{1-x}\,x^2}{4\,\sqrt{-1+x}}+\frac{1}{2}\,x\,\sqrt{1-x^2}\,\mathrm{ArcCosh}[x]-\frac{\sqrt{1-x}\,\mathrm{ArcCosh}[x]^2}{4\,\sqrt{-1+x}}$$

Result (type 3, 84 leaves, 4 steps):

$$-\frac{x^2\,\sqrt{1-x^2}}{4\,\sqrt{-1+x}\,\sqrt{1+x}}+\frac{1}{2}\,x\,\sqrt{1-x^2}\,\mathrm{ArcCosh}[x]-\frac{\sqrt{1-x^2}\,\mathrm{ArcCosh}[x]^2}{4\,\sqrt{-1+x}\,\sqrt{1+x}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int\frac{x^5\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{8\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{15\,c^5\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{45\,c^3\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{b\,x^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{25\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{15\,c^6\,d}-$$
$$\frac{4\,x^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{15\,c^4\,d}-\frac{x^4\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}{5\,c^2\,d}$$

Result (type 3, 260 leaves, 7 steps):

$$-\frac{8\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{15\,c^5\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{45\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^5\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{25\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{15\,c^6\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{15\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^4\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{5\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 105: Result valid but suboptimal antiderivative.

$$\int\frac{x^4\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$-\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^4\,d}-\frac{x^3\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{4\,c^2\,d}+\frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^2}{16\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 228 leaves, 6 steps):

$$-\frac{3\,b\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{16\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^3\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{4\,c^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^2}{16\,b\,c^5\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 106: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 156 leaves, 4 steps):

$$-\frac{2\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{3\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^4\,d}-\frac{x^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^2\,d}$$

Result (type 3, 172 leaves, 5 steps):

$$-\frac{2\,b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{3\,c^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{9\,c\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^4\,\sqrt{d-c^2\,d\,x^2}}-\frac{x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])}{3\,c^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b \, \text{ArcCosh}[c\, x]\right)}{\sqrt{d - c^2 d\, x^2}} \, dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$-\frac{b\, x^2 \sqrt{-1 + c\, x} \sqrt{1 + c\, x}}{4\, c \sqrt{d - c^2 d\, x^2}} - \frac{x \sqrt{d - c^2 d\, x^2} \left(a + b \, \text{ArcCosh}[c\, x]\right)}{2\, c^2\, d} + \frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b \, \text{ArcCosh}[c\, x]\right)^2}{4\, b\, c^3 \sqrt{d - c^2 d\, x^2}}$$

Result (type 3, 140 leaves, 4 steps):

$$-\frac{b\, x^2 \sqrt{-1 + c\, x} \sqrt{1 + c\, x}}{4\, c \sqrt{d - c^2 d\, x^2}} - \frac{x\, (1 - c\, x)\, (1 + c\, x) \left(a + b \, \text{ArcCosh}[c\, x]\right)}{2\, c^2 \sqrt{d - c^2 d\, x^2}} + \frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b \, \text{ArcCosh}[c\, x]\right)^2}{4\, b\, c^3 \sqrt{d - c^2 d\, x^2}}$$

Problem 108: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a + b \, \text{ArcCosh}[c\, x]\right)}{\sqrt{d - c^2 d\, x^2}} \, dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$-\frac{b\, x \sqrt{-1 + c\, x} \sqrt{1 + c\, x}}{c \sqrt{d - c^2 d\, x^2}} - \frac{\sqrt{d - c^2 d\, x^2} \left(a + b \, \text{ArcCosh}[c\, x]\right)}{c^2\, d}$$

Result (type 3, 80 leaves, 3 steps):

$$-\frac{b\, x \sqrt{-1 + c\, x} \sqrt{1 + c\, x}}{c \sqrt{d - c^2 d\, x^2}} - \frac{(1 - c\, x)\, (1 + c\, x) \left(a + b \, \text{ArcCosh}[c\, x]\right)}{c^2 \sqrt{d - c^2 d\, x^2}}$$

Problem 109: Result optimal but 1 more steps used.

$$\int \frac{a + b \, \text{ArcCosh}[c\, x]}{\sqrt{d - c^2 d\, x^2}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b \, \text{ArcCosh}[c\, x]\right)^2}{2\, b\, c \sqrt{d - c^2 d\, x^2}}$$

Result (type 3, 53 leaves, 2 steps):

$$\frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b \, \text{ArcCosh}[c\, x]\right)^2}{2\, b\, c \sqrt{d - c^2 d\, x^2}}$$

Problem 110: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x \sqrt{d - c^2 d x^2}} \, dx$$

Optimal (type 4, 151 leaves, 6 steps):

$$\frac{2 \sqrt{-1 + c x} \sqrt{1 + c x} \left(a + b \operatorname{ArcCosh}[c x]\right) \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}} - \frac{i b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}} + \frac{i b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}}$$

Result (type 4, 151 leaves, 7 steps):

$$\frac{2 \sqrt{-1 + c x} \sqrt{1 + c x} \left(a + b \operatorname{ArcCosh}[c x]\right) \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}} - \frac{i b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}} + \frac{i b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d - c^2 d x^2}}$$

Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 \sqrt{d - c^2 d x^2}} \, dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$-\frac{\sqrt{d - c^2 d x^2} \left(a + b \operatorname{ArcCosh}[c x]\right)}{d x} - \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[x]}{\sqrt{d - c^2 d x^2}}$$

Result (type 3, 79 leaves, 3 steps):

$$-\frac{(1 - c x) (1 + c x) \left(a + b \operatorname{ArcCosh}[c x]\right)}{x \sqrt{d - c^2 d x^2}} - \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[x]}{\sqrt{d - c^2 d x^2}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^3 \sqrt{d - c^2 d x^2}} \, dx$$

Optimal (type 4, 238 leaves, 8 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,x\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,d\,x^2}+\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 246 leaves, 9 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{2\,x\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}+\frac{i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,x^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,x^3}-\frac{2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,x}-\frac{2\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 171 leaves, 5 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,x^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,c^2\,(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 114: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{\left(d - c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$-\frac{5\,b\,x\,\sqrt{d - c^2 d\,x^2}}{3\,c^5 d^2 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}} - \frac{b\,x^3 \sqrt{d - c^2 d\,x^2}}{9\,c^3 d^2 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}} + \frac{a + b \operatorname{ArcCosh}[c\,x]}{c^6 d \sqrt{d - c^2 d\,x^2}} + \frac{2\sqrt{d - c^2 d\,x^2}\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)}{c^6 d^2} - \frac{\left(d - c^2 d\,x^2\right)^{3/2} \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{3\,c^6 d^3} - \frac{b\sqrt{d - c^2 d\,x^2}\,\operatorname{ArcTanh}[c\,x]}{c^6 d^2 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}}$$

Result (type 3, 262 leaves, 5 steps):

$$\frac{5\,b\,x\,\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}}{3\,c^5 d \sqrt{d - c^2 d\,x^2}} + \frac{b\,x^3 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}}{9\,c^3 d \sqrt{d - c^2 d\,x^2}} + \frac{x^4 \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{c^2 d \sqrt{d - c^2 d\,x^2}} + \frac{8\,(1 - c\,x)\,(1 + c\,x)\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)}{3\,c^6 d \sqrt{d - c^2 d\,x^2}} + \frac{4\,x^2 (1 - c\,x)\,(1 + c\,x)\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)}{3\,c^4 d \sqrt{d - c^2 d\,x^2}} + \frac{b\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\operatorname{ArcTanh}[c\,x]}{c^6 d \sqrt{d - c^2 d\,x^2}}$$

Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{\left(d - c^2 d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 226 leaves, 7 steps):

$$\frac{b\,x^2 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}}{4\,c^3 d \sqrt{d - c^2 d\,x^2}} + \frac{x^3 \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{c^2 d \sqrt{d - c^2 d\,x^2}} + \frac{3\,x\,\sqrt{d - c^2 d\,x^2}\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)}{2\,c^4 d^2} - \frac{3\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)^2}{4\,b\,c^5 d \sqrt{d - c^2 d\,x^2}} - \frac{b\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\operatorname{Log}\left[1 - c^2 x^2\right]}{2\,c^5 d \sqrt{d - c^2 d\,x^2}}$$

Result (type 3, 237 leaves, 8 steps):

$$\frac{b\,x^2 \sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}}{4\,c^3 d \sqrt{d - c^2 d\,x^2}} + \frac{x^3 \left(a + b \operatorname{ArcCosh}[c\,x]\right)}{c^2 d \sqrt{d - c^2 d\,x^2}} + \frac{3\,x\,(1 - c\,x)\,(1 + c\,x)\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)}{2\,c^4 d \sqrt{d - c^2 d\,x^2}} - \frac{3\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\left(a + b \operatorname{ArcCosh}[c\,x]\right)^2}{4\,b\,c^5 d \sqrt{d - c^2 d\,x^2}} - \frac{b\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\operatorname{Log}\left[1 - c^2 x^2\right]}{2\,c^5 d \sqrt{d - c^2 d\,x^2}}$$

Problem 119: Result optimal but 1 more steps used.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 84 leaves, 2 steps):

$$\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 84 leaves, 3 steps):

$$\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,c\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{6\,d^2\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,x\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\sqrt{d-c^2\,d\,x^2}}+\frac{5\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,d^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,d^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 250 leaves, 6 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d\,x^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d\,x^3\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{4\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,x\,\sqrt{d-c^2\,d\,x^2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{5\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,d\,\sqrt{d-c^2\,d\,x^2}}-\frac{b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{2\,d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{x^5\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 243 leaves, 5 steps):

$$\frac{b\,x\,\sqrt{d-c^2\,d\,x^2}}{c^5\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,x\,\sqrt{d-c^2\,d\,x^2}}{6\,c^5\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)}+\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,c^6\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}-\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{c^6\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{c^6\,d^3}+\frac{11\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}[c\,x]}{6\,c^6\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 280 leaves, 6 steps):

$$-\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c^5\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^5\,d^2\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^2\,d^2\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^6\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{11\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{6\,c^6\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 158 leaves, 4 steps):

$$\frac{b\,x\,\sqrt{d-c^2\,d\,x^2}}{6\,c^3\,d^3\,\left(-1+c\,x\right)^{3/2}\,\left(1+c\,x\right)^{3/2}}+\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,c^4\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{5\,b\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}[c\,x]}{6\,c^4\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 243 leaves, 5 steps):

$$\frac{b\,x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^3\,d^2\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{c^4\,d^2\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}+\frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c\,d^2\,\left(1-c\,x\right)\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}+\frac{\left(1-c\,x\right)^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,c^4\,d^2\,\left(1+c\,x\right)\,\sqrt{d-c^2\,d\,x^2}}-\frac{5\,b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTanh}[c\,x]}{6\,c^4\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 127: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,c^3\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{6\,c^3\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 160 leaves, 5 steps):

$$\frac{b\sqrt{-1+c x}\,\sqrt{1+c x}}{6 c^3 d^2\left(1-c^2 x^2\right)\sqrt{d-c^2 d x^2}}+\frac{x^3\left(a+b\,\text{ArcCosh}[c x]\right)}{3 d^2\left(1-c x\right)\left(1+c x\right)\sqrt{d-c^2 d x^2}}+\frac{b\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Log}\left[1-c^2 x^2\right]}{6 c^3 d^2\sqrt{d-c^2 d x^2}}$$

Problem 128: Result valid but suboptimal antiderivative.

$$\int\frac{x\left(a+b\,\text{ArcCosh}[c x]\right)}{\left(d-c^2 d x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\frac{b x\sqrt{-1+c x}\,\sqrt{1+c x}}{6 c d\left(d-c^2 d x^2\right)^{3/2}}+\frac{a+b\,\text{ArcCosh}[c x]}{3 c^2 d\left(d-c^2 d x^2\right)^{3/2}}+\frac{b\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{ArcTanh}[c x]}{6 c^2 d^2\sqrt{d-c^2 d x^2}}$$

Result (type 3, 154 leaves, 4 steps):

$$\frac{b x\sqrt{-1+c x}\,\sqrt{1+c x}}{6 c d^2\left(1-c^2 x^2\right)\sqrt{d-c^2 d x^2}}+\frac{a+b\,\text{ArcCosh}[c x]}{3 c^2 d^2\left(1-c x\right)\left(1+c x\right)\sqrt{d-c^2 d x^2}}+\frac{b\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{ArcTanh}[c x]}{6 c^2 d^2\sqrt{d-c^2 d x^2}}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int\frac{a+b\,\text{ArcCosh}[c x]}{x^2\left(d-c^2 d x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 248 leaves, 5 steps):

$$-\frac{b c\sqrt{d-c^2 d x^2}}{6 d^3\sqrt{-1+c x}\,\sqrt{1+c x}\left(1-c^2 x^2\right)}-\frac{a+b\,\text{ArcCosh}[c x]}{d x\left(d-c^2 d x^2\right)^{3/2}}+\frac{4 c^2 x\left(a+b\,\text{ArcCosh}[c x]\right)}{3 d\left(d-c^2 d x^2\right)^{3/2}}+\frac{8 c^2 x\left(a+b\,\text{ArcCosh}[c x]\right)}{3 d^2\sqrt{d-c^2 d x^2}}+\frac{b c\sqrt{d-c^2 d x^2}\,\text{Log}[x]}{d^3\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{5 b c\sqrt{d-c^2 d x^2}\,\text{Log}\left[1-c^2 x^2\right]}{6 d^3\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Result (type 3, 279 leaves, 6 steps):

$$\frac{b c\sqrt{-1+c x}\,\sqrt{1+c x}}{6 d^2\left(1-c^2 x^2\right)\sqrt{d-c^2 d x^2}}+\frac{8 c^2 x\left(a+b\,\text{ArcCosh}[c x]\right)}{3 d^2\sqrt{d-c^2 d x^2}}-\frac{a+b\,\text{ArcCosh}[c x]}{d^2 x\left(1-c x\right)\left(1+c x\right)\sqrt{d-c^2 d x^2}}+\frac{4 c^2 x\left(a+b\,\text{ArcCosh}[c x]\right)}{3 d^2\left(1-c x\right)\left(1+c x\right)\sqrt{d-c^2 d x^2}}-\frac{b c\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Log}[x]}{d^2\sqrt{d-c^2 d x^2}}-\frac{5 b c\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Log}\left[1-c^2 x^2\right]}{6 d^2\sqrt{d-c^2 d x^2}}$$

Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\,\text{ArcCosh}[c\,x]}{x^4\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 338 leaves, 5 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}}{6\,d^3\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^3\,\sqrt{d-c^2\,d\,x^2}}{6\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)}-\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d\,x^3\,\left(d-c^2\,d\,x^2\right)^{3/2}}-\frac{2\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d\,x\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d\,\left(d-c^2\,d\,x^2\right)^{3/2}}+\frac{16\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{8\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}[x]}{3\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{4\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{Log}\left[1-c^2\,x^2\right]}{3\,d^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 3, 383 leaves, 6 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d^2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{6\,d^2\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}}+\frac{16\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{a+b\,\text{ArcCosh}[c\,x]}{3\,d^2\,x^3\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}-\frac{2\,c^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{d^2\,x\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}+\frac{8\,c^4\,x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,d^2\,(1-c\,x)\,(1+c\,x)\,\sqrt{d-c^2\,d\,x^2}}-\frac{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}[x]}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}-\frac{4\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Log}\left[1-c^2\,x^2\right]}{3\,d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 134: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]}{\left(c-a^2\,c\,x^2\right)^{7/2}}\,dx$$

Optimal (type 3, 246 leaves, 8 steps):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{20\,a\,c^3\,\left(1-a^2\,x^2\right)^2\,\sqrt{c-a^2\,c\,x^2}}+\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{15\,a\,c^3\,\left(1-a^2\,x^2\right)\,\sqrt{c-a^2\,c\,x^2}}+\frac{x\,\text{ArcCosh}[a\,x]}{5\,c\,\left(c-a^2\,c\,x^2\right)^{5/2}}+\frac{4\,x\,\text{ArcCosh}[a\,x]}{15\,c^2\,\left(c-a^2\,c\,x^2\right)^{3/2}}+\frac{8\,x\,\text{ArcCosh}[a\,x]}{15\,c^3\,\sqrt{c-a^2\,c\,x^2}}-\frac{4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Log}\left[1-a^2\,x^2\right]}{15\,a\,c^3\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 276 leaves, 7 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}}{20 a c^3 \left(1-a^2 x^2\right)^2 \sqrt{c-a^2 c x^2}}+\frac{2 \sqrt{-1+a x}\ \sqrt{1+a x}}{15 a c^3 \left(1-a^2 x^2\right) \sqrt{c-a^2 c x^2}}+$$
$$\frac{8 x \operatorname{ArcCosh}[a x]}{15 c^3 \sqrt{c-a^2 c x^2}}+\frac{x \operatorname{ArcCosh}[a x]}{5 c^3 (1-a x)^2 (1+a x)^2 \sqrt{c-a^2 c x^2}}+$$
$$\frac{4 x \operatorname{ArcCosh}[a x]}{15 c^3 (1-a x) (1+a x) \sqrt{c-a^2 c x^2}}-\frac{4 \sqrt{-1+a x}\ \sqrt{1+a x}\ \operatorname{Log}\left[1-a^2 x^2\right]}{15 a c^3 \sqrt{c-a^2 c x^2}}$$

Problem 135: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}}\, dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$-\frac{3 x^2 \sqrt{-1+a x}}{16 a^3 \sqrt{1-a x}}-\frac{x^4 \sqrt{-1+a x}}{16 a \sqrt{1-a x}}-\frac{3 x \sqrt{1-a^2 x^2}\ \operatorname{ArcCosh}[a x]}{8 a^4}-$$
$$\frac{x^3 \sqrt{1-a^2 x^2}\ \operatorname{ArcCosh}[a x]}{4 a^2}+\frac{3 \sqrt{-1+a x}\ \operatorname{ArcCosh}[a x]^2}{16 a^5 \sqrt{1-a x}}$$

Result (type 3, 206 leaves, 6 steps):

$$-\frac{3 x^2 \sqrt{-1+a x}\ \sqrt{1+a x}}{16 a^3 \sqrt{1-a^2 x^2}}-\frac{x^4 \sqrt{-1+a x}\ \sqrt{1+a x}}{16 a \sqrt{1-a^2 x^2}}-\frac{3 x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{8 a^4 \sqrt{1-a^2 x^2}}-$$
$$\frac{x^3 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{4 a^2 \sqrt{1-a^2 x^2}}+\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \operatorname{ArcCosh}[a x]^2}{16 a^5 \sqrt{1-a^2 x^2}}$$

Problem 136: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}}\, dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{2 x \sqrt{-1+a x}}{3 a^3 \sqrt{1-a x}}-\frac{x^3 \sqrt{-1+a x}}{9 a \sqrt{1-a x}}-\frac{2 \sqrt{1-a^2 x^2}\ \operatorname{ArcCosh}[a x]}{3 a^4}-\frac{x^2 \sqrt{1-a^2 x^2}\ \operatorname{ArcCosh}[a x]}{3 a^2}$$

Result (type 3, 158 leaves, 5 steps):

$$-\frac{2 x \sqrt{-1+a x}\ \sqrt{1+a x}}{3 a^3 \sqrt{1-a^2 x^2}}-\frac{x^3 \sqrt{-1+a x}\ \sqrt{1+a x}}{9 a \sqrt{1-a^2 x^2}}-$$
$$\frac{2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{3 a^4 \sqrt{1-a^2 x^2}}-\frac{x^2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{3 a^2 \sqrt{1-a^2 x^2}}$$

Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$-\frac{x^2 \sqrt{-1+a x}}{4 a \sqrt{1-a x}}-\frac{x \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]}{2 a^2}+\frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{4 a^3 \sqrt{1-a x}}$$

Result (type 3, 125 leaves, 4 steps):

$$-\frac{x^2 \sqrt{-1+a x} \sqrt{1+a x}}{4 a \sqrt{1-a^2 x^2}}-\frac{x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{2 a^2 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{4 a^3 \sqrt{1-a^2 x^2}}$$

Problem 138: Result valid but suboptimal antiderivative.

$$\int \frac{x \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$-\frac{x \sqrt{-1+a x}}{a \sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]}{a^2}$$

Result (type 3, 73 leaves, 3 steps):

$$-\frac{x \sqrt{-1+a x} \sqrt{1+a x}}{a \sqrt{1-a^2 x^2}}-\frac{(1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{a^2 \sqrt{1-a^2 x^2}}$$

Problem 139: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{2 a \sqrt{1-a x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{2 a \sqrt{1-a^2 x^2}}$$

Problem 140: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]}{x \sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 4, 103 leaves, 6 steps):

$$\frac{2\sqrt{-1+a\,x}\;\text{ArcCosh}[a\,x]\;\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{i\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{i\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}$$

Result (type 4, 142 leaves, 7 steps):

$$\frac{2\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{ArcCosh}[a\,x]\;\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{i\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+\frac{i\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 141: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a\,x]}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\frac{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}{x}-\frac{a\sqrt{-1+a\,x}\;\text{Log}[x]}{\sqrt{1-a\,x}}$$

Result (type 3, 72 leaves, 3 steps):

$$-\frac{(1-a\,x)\;(1+a\,x)\;\text{ArcCosh}[a\,x]}{x\sqrt{1-a^2\,x^2}}-\frac{a\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Log}[x]}{\sqrt{1-a^2\,x^2}}$$

Problem 142: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a\,x]}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 167 leaves, 8 steps):

$$\frac{a\sqrt{-1+a\,x}}{2\,x\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}{2\,x^2}+\frac{a^2\sqrt{-1+a\,x}\;\text{ArcCosh}[a\,x]\;\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{i\,a^2\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\sqrt{1-a\,x}}+\frac{i\,a^2\sqrt{-1+a\,x}\;\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\sqrt{1-a\,x}}$$

Result (type 4, 230 leaves, 9 steps):

$$\frac{a\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{2\,x\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{2\,x^2\,\sqrt{1-a^2\,x^2}}+\frac{a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-\frac{i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}+\frac{i\,a^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}$$

Problem 143: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 5, 98 leaves, 1 step):

$$\frac{2\,(f\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f}+\frac{1}{35\,f^2\,\sqrt{1-c\,x}}4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]$$

Result (type 5, 111 leaves, 2 steps):

$$\frac{2\,(f\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{5\,f}+\frac{1}{35\,f^2\,\sqrt{1-c^2\,x^2}}4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]$$

Problem 144: Result optimal but 1 more steps used.

$$\int\frac{(f\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 5, 141 leaves, 1 step):

$$\frac{1}{5\,f\,\sqrt{d-c^2\,d\,x^2}}2\,(f\,x)^{5/2}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]+\left(4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]\right)\Big/\left(35\,f^2\,\sqrt{d-c^2\,d\,x^2}\right)$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{1}{5\,f\,\sqrt{d-c^2\,d\,x^2}}2\,(f\,x)^{5/2}\,\sqrt{1-c^2\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]+\left(4\,b\,c\,(f\,x)^{7/2}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]\right)\Big/\left(35\,f^2\,\sqrt{d-c^2\,d\,x^2}\right)$$

Problem 153: Result valid but suboptimal antiderivative.

$$\int (f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])\,dx$$

Optimal (type 5, 278 leaves, 3 steps):

$$-\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}}{f^2\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{(f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])}{f\,(2+m)}+\left((f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]\right)\Big/\left(f\,(2+3\,m+m^2)\,\sqrt{1-c\,x}\,\sqrt{1+c\,x}\right)-\left(b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 5, 288 leaves, 4 steps):

$$-\frac{b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}}{f^2\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{(f\,x)^{1+m}\,\sqrt{d-c^2\,d\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])}{f\,(2+m)}+\left((f\,x)^{1+m}\,\sqrt{1-c^2\,x^2}\,\sqrt{d-c^2\,d\,x^2}\,\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2\,x^2\right]\right)\Big/\left(f\,(2+3\,m+m^2)\,(1-c\,x)\,(1+c\,x)\right)-\left(b\,c\,(f\,x)^{2+m}\,\sqrt{d-c^2\,d\,x^2}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2\,x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 154: Result optimal but 1 more steps used.

$$\int \frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 5, 176 leaves, 1 step):

$$\left((f x)^{1+m}\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2 x^2\right]\right)\Big/\left(f\,(1+m)\,\sqrt{d-c^2 d x^2}\right)+\left(b\,c\,(f x)^{2+m}\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2 x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)\,\sqrt{d-c^2 d x^2}\right)$$

Result (type 5, 176 leaves, 2 steps):

$$\left((f x)^{1+m}\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2 x^2\right]\right)\Big/\left(f\,(1+m)\,\sqrt{d-c^2 d x^2}\right)+\left(b\,c\,(f x)^{2+m}\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2 x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)\,\sqrt{d-c^2 d x^2}\right)$$

Problem 160: Result optimal but 1 more steps used.

$$\int \frac{(f x)^m\,(a+b\,\text{ArcCosh}[c x])}{\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}}\,dx$$

Optimal (type 5, 188 leaves, 1 step):

$$\left((f x)^{1+m}\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2 x^2\right]\right)\Big/\left(f\,(1+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}\right)+\left(b\,c\,(f x)^{2+m}\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2 x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}\right)$$

Result (type 5, 188 leaves, 2 steps):

$$\left((f x)^{1+m}\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2 x^2\right]\right)\Big/\left(f\,(1+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}\right)+\left(b\,c\,(f x)^{2+m}\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{HypergeometricPFQ}\left[\left\{1,1+\frac{m}{2},1+\frac{m}{2}\right\},\left\{\frac{3}{2}+\frac{m}{2},2+\frac{m}{2}\right\},c^2 x^2\right]\right)\Big/\left(f^2\,(1+m)\,(2+m)\,\sqrt{d1+c\,d1\,x}\,\sqrt{d2-c\,d2\,x}\right)$$

Problem 163: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m\,\text{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 5, 128 leaves, 1 step):

$$\frac{(f x)^{1+m} \text{ArcCosh}[a x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{f (1+m)}+$$
$$\left(a (f x)^{2+m} \sqrt{-1+a x} \text{ HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, a^2 x^2\right]\right) \Big/$$
$$\left(f^2 (1+m) (2+m) \sqrt{1-a x}\right)$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{(f x)^{1+m} \text{ArcCosh}[a x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{f (1+m)}+$$
$$\left(a (f x)^{2+m} \sqrt{-1+a x} \sqrt{1+a x} \text{ HypergeometricPFQ}\left[\left\{1, 1+\frac{m}{2}, 1+\frac{m}{2}\right\}, \left\{\frac{3}{2}+\frac{m}{2}, 2+\frac{m}{2}\right\}, a^2 x^2\right]\right) \Big/$$
$$\left(f^2 (1+m) (2+m) \sqrt{1-a^2 x^2}\right)$$

Problem 170: Result optimal but 1 more steps used.

$$\int x^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2 \, dx$$

Optimal (type 3, 371 leaves, 16 steps):

$$-\frac{856 b^2 \sqrt{d-c^2 d x^2}}{3375 c^4}+\frac{22 b^2 x^2 \sqrt{d-c^2 d x^2}}{3375 c^2}+\frac{2}{125} b^2 x^4 \sqrt{d-c^2 d x^2}+\frac{4 a b x \sqrt{d-c^2 d x^2}}{15 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{4 b^2 x \sqrt{d-c^2 d x^2} \text{ ArcCosh}[c x]}{15 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b x^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{45 c \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{2 b c x^5 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{25 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^4}-$$
$$\frac{x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^2}+\frac{1}{5} x^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2$$

Result (type 3, 371 leaves, 17 steps):

$$-\frac{856 b^2 \sqrt{d-c^2 d x^2}}{3375 c^4}+\frac{22 b^2 x^2 \sqrt{d-c^2 d x^2}}{3375 c^2}+\frac{2}{125} b^2 x^4 \sqrt{d-c^2 d x^2}+\frac{4 a b x \sqrt{d-c^2 d x^2}}{15 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{4 b^2 x \sqrt{d-c^2 d x^2} \text{ ArcCosh}[c x]}{15 c^3 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 b x^3 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{45 c \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{2 b c x^5 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{25 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^4}-$$
$$\frac{x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^2}+\frac{1}{5} x^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2$$

Problem 171: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2 \, dx$$

Optimal (type 3, 319 leaves, 11 steps):

$$-\frac{b^2 x \sqrt{d - c^2 d x^2}}{64 c^2} + \frac{1}{32} b^2 x^3 \sqrt{d - c^2 d x^2} - \frac{b^2 \sqrt{d - c^2 d x^2} \,\text{ArcCosh}[c x]}{64 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^2 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)}{8 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2 - \frac{\sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^3}{24 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 319 leaves, 12 steps):

$$-\frac{b^2 x \sqrt{d - c^2 d x^2}}{64 c^2} + \frac{1}{32} b^2 x^3 \sqrt{d - c^2 d x^2} - \frac{b^2 \sqrt{d - c^2 d x^2} \,\text{ArcCosh}[c x]}{64 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^2 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)}{8 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^4 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{x \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2}{8 c^2} + \frac{1}{4} x^3 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2 - \frac{\sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^3}{24 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 173: Result optimal but 1 more steps used.

$$\int \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2 \, dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{1}{4} b^2 x \sqrt{d - c^2 d x^2} + \frac{b^2 \sqrt{d - c^2 d x^2} \,\text{ArcCosh}[c x]}{4 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^2 \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{1}{2} x \sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^2 - \frac{\sqrt{d - c^2 d x^2} \left(a + b \,\text{ArcCosh}[c x]\right)^3}{6 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 204 leaves, 6 steps):

$$\frac{1}{4} b^2 x \sqrt{d-c^2 d x^2}+\frac{b^2 \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{4 c \sqrt{-1+c x} \sqrt{1+c x}}-\frac{b c x^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{2 \sqrt{-1+c x} \sqrt{1+c x}}+\frac{1}{2} x \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^3}{6 b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 174: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{x}\, dx$$

Optimal (type 4, 402 leaves, 12 steps):

$$2 b^2 \sqrt{d-c^2 d x^2}-\frac{2 a b c x \sqrt{d-c^2 d x^2}}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b^2 c x \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{\sqrt{-1+c x} \sqrt{1+c x}}+\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2-\frac{2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b^2 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b^2 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Result (type 4, 402 leaves, 13 steps):

$$2 b^2 \sqrt{d-c^2 d x^2}-\frac{2 a b c x \sqrt{d-c^2 d x^2}}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 b^2 c x \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{\sqrt{-1+c x} \sqrt{1+c x}}+\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2-\frac{2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\,\text{PolyLog}\left[2,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}-\frac{2 i b^2 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}+\frac{2 i b^2 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[3,i e^{\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{x^2}\, dx$$

Optimal (type 4, 234 leaves, 7 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x}+\frac{c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b^2 c\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 234 leaves, 8 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x}-\frac{c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^3}{3\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{2\,b\,c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b^2 c\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 176: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{x^3}\,dx$$

Optimal (type 4, 427 leaves, 12 steps):

$$-\frac{b\,c\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,x^2}+\frac{c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b^2 c^2\sqrt{d-c^2 d x^2}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b\,c^2\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b^2 c^2\sqrt{d-c^2 d x^2}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 427 leaves, 13 steps):

$$-\frac{b\,c\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,x^2}+$$

$$\frac{c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b^2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{i\,b\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{i\,b^2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{i\,b^2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 177: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^4}\,dx$$

Optimal (type 4, 336 leaves, 11 steps):

$$\frac{b^2\,c^2\,\sqrt{d-c^2\,d\,x^2}}{3\,x}-\frac{b^2\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{b\,c\,\left(1-c^2\,x^2\right)\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{c^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{3\,d\,x^3}-$$

$$\frac{2\,b\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b^2\,c^3\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 4, 344 leaves, 11 steps):

$$\frac{b^2 c^2 \sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3 \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{3 \sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{b c \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x}\ \sqrt{1+c x}}+\frac{c^3 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{(1-c x) (1+c x) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{3 x^3}-$$
$$\frac{2 b c^3 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{b^2 c^3 \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 183: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{x^2}\,dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$-\frac{1}{4} b^2 c^2 d x \sqrt{d-c^2 d x^2}-\frac{5 b^2 c d \sqrt{d-c^2 d x^2}\ \text{ArcCosh}[c x]}{4 \sqrt{-1+c x}\ \sqrt{1+c x}}+$$
$$\frac{3 b c^3 d x^2 \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{2 \sqrt{-1+c x}\ \sqrt{1+c x}}+\frac{b c d \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])}{\sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{3}{2} c^2 d x \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2+\frac{c d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^2}{\sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])^2}{x}+\frac{c d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])^3}{2 b \sqrt{-1+c x}\ \sqrt{1+c x}}+$$
$$\frac{2 b c d \sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c x])\ \text{Log}\left[1+e^{-2\,\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}-$$
$$\frac{b^2 c d \sqrt{d-c^2 d x^2}\ \text{PolyLog}\left[2,-e^{-2\,\text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Result (type 4, 465 leaves, 15 steps):

$$-\frac{1}{4} b^2 c^2 d x \sqrt{d-c^2 d x^2}-\frac{5 b^2 c d \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{4 \sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{3 b c^3 d x^2 \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{2 \sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{b c d \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{\sqrt{-1+c x}\, \sqrt{1+c x}}-\frac{3}{2} c^2 d x \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2-\frac{c d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{\sqrt{-1+c x}\, \sqrt{1+c x}}-\frac{d (1-c x) (1+c x) \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{x}+\frac{c d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{2 b \sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{2 b c d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])\, \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{b^2 c d \sqrt{d-c^2 d x^2}\, \text{PolyLog}\left[2,-e^{2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x}\, \sqrt{1+c x}}$$

Problem 185: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\, \text{ArcCosh}[c x])^2}{x^4}\, dx$$

Optimal (type 4, 426 leaves, 18 steps):

$$\frac{b^2 c^2 d \sqrt{d-c^2 d x^2}}{3 x}-\frac{b^2 c^3 d \sqrt{d-c^2 d x^2}\, \text{ArcCosh}[c x]}{3 \sqrt{-1+c x}\, \sqrt{1+c x}}-\frac{b c d \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{c^2 d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{x}-\frac{4 c^3 d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x}\, \sqrt{1+c x}}-\frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\, \text{ArcCosh}[c x])^2}{3 x^3}-\frac{c^3 d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^3}{3 b \sqrt{-1+c x}\, \sqrt{1+c x}}-\frac{8 b c^3 d \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])\, \text{Log}\left[1+e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\, \sqrt{1+c x}}+\frac{4 b^2 c^3 d \sqrt{d-c^2 d x^2}\, \text{PolyLog}\left[2,-e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x}\, \sqrt{1+c x}}$$

Result (type 4, 438 leaves, 18 steps):

$$\frac{b^2 c^2 d \sqrt{d - c^2 d x^2}}{3 x} - \frac{b^2 c^3 d \sqrt{d - c^2 d x^2}\ \text{ArcCosh}[c x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{b c d \left(1 - c^2 x^2\right) \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} +$$
$$\frac{c^2 d \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{x} + \frac{4 c^3 d \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{3 x^3} - \frac{c^3 d \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^3}{3 b \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{8 b c^3 d \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) \text{Log}\left[1 + e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{4 b^2 c^3 d \sqrt{d - c^2 d x^2}\ \text{PolyLog}\left[2, -e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 191: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d - c^2 d x^2\right)^{5/2} \left(a + b \text{ArcCosh}[c x]\right)^2}{x^2} \, dx$$

Optimal (type 4, 607 leaves, 25 steps):

$$-\frac{31}{64} b^2 c^2 d^2 x \sqrt{d - c^2 d x^2} - \frac{1}{32} b^2 c^2 d^2 x (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} -$$
$$\frac{89 b^2 c d^2 \sqrt{d - c^2 d x^2}\ \text{ArcCosh}[c x]}{64 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{15 b c^3 d^2 x^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} +$$
$$\frac{b c d^2 \left(1 - c^2 x^2\right) \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{\sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{b c d^2 \left(1 - c^2 x^2\right)^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2 +$$
$$\frac{c d^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^2}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5}{4} c^2 d x \left(d - c^2 d x^2\right)^{3/2} \left(a + b \text{ArcCosh}[c x]\right)^2 -$$
$$\frac{\left(d - c^2 d x^2\right)^{5/2} \left(a + b \text{ArcCosh}[c x]\right)^2}{x} + \frac{5 c d^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right)^3}{8 b \sqrt{-1 + c x} \sqrt{1 + c x}} +$$
$$\frac{2 b c d^2 \sqrt{d - c^2 d x^2} \left(a + b \text{ArcCosh}[c x]\right) \text{Log}\left[1 + e^{-2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{b^2 c d^2 \sqrt{d - c^2 d x^2}\ \text{PolyLog}\left[2, -e^{-2 \text{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 4, 638 leaves, 24 steps):

$$-\frac{31}{64} b^2 c^2 d^2 x \sqrt{d-c^2 d x^2} - \frac{1}{32} b^2 c^2 d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} -$$
$$\frac{89 b^2 c d^2 \sqrt{d-c^2 d x^2} \, \text{ArcCosh}[c x]}{64 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{15 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}} +$$
$$\frac{b c d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])}{\sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{b c d^2 (1-c^2 x^2)^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])}{8 \sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])^2 - \frac{c d^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])^2}{\sqrt{-1+c x} \sqrt{1+c x}} -$$
$$\frac{5}{4} c^2 d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])^2 -$$
$$\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])^2}{x} +$$
$$\frac{5 c d^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x])^3}{8 b \sqrt{-1+c x} \sqrt{1+c x}} +$$
$$\frac{2 b c d^2 \sqrt{d-c^2 d x^2} (a+b \, \text{ArcCosh}[c x]) \, \text{Log}\left[1+e^{2 \, \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} +$$
$$\frac{b^2 c d^2 \sqrt{d-c^2 d x^2} \, \text{PolyLog}\left[2, -e^{2 \, \text{ArcCosh}[c x]}\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \, \text{ArcCosh}[c x])^2}{x^4} \, dx$$

Optimal (type 4, 638 leaves, 30 steps):

$$\frac{7}{12} b^2 c^4 d^2 x \sqrt{d - c^2 d x^2} + \frac{b^2 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}}{3 x} +$$
$$\frac{23 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \text{ArcCosh}[c x]}{12 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5 b c^5 d^2 x^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{7 b c^3 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{b c d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])^2 -$$
$$\frac{7 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 c^2 d (d - c^2 d x^2)^{3/2} (a + b \text{ArcCosh}[c x])^2}{3 x} -$$
$$\frac{(d - c^2 d x^2)^{5/2} (a + b \text{ArcCosh}[c x])^2}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x])^3}{6 b \sqrt{-1 + c x} \sqrt{1 + c x}} -$$
$$\frac{14 b c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \text{ArcCosh}[c x]) \text{Log}\left[1 + e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} +$$
$$\frac{7 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \text{PolyLog}\left[2, -e^{-2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 4, 669 leaves, 29 steps):

$$\frac{7}{12} b^2 c^4 d^2 x \sqrt{d-c^2 d x^2}+\frac{b^2 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2}}{3 x}+$$
$$\frac{23 b^2 c^3 d^2 \sqrt{d-c^2 d x^2} \text{ArcCosh}[c x]}{12 \sqrt{-1+c x} \sqrt{1+c x}}-\frac{5 b c^5 d^2 x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{2 \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{7 b c^3 d^2 (1-c^2 x^2) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{3 \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{b c d^2 (1-c^2 x^2)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])}{3 x^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{5}{2} c^4 d^2 x \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2+\frac{7 c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 \sqrt{-1+c x} \sqrt{1+c x}}+$$
$$\frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 x}-$$
$$\frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{3 x^3}-$$
$$\frac{5 c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^3}{6 b \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{14 b c^3 d^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}-$$
$$\frac{7 b^2 c^3 d^2 \sqrt{d-c^2 d x^2} \text{PolyLog}\left[2,-e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 421 leaves, 16 steps):

$$-\frac{16 a b x \sqrt{-1+c x} \sqrt{1+c x}}{15 c^5 \sqrt{d-c^2 d x^2}}-\frac{4144 b^2 (1-c x) (1+c x)}{3375 c^6 \sqrt{d-c^2 d x^2}}-$$
$$\frac{272 b^2 x^2 (1-c x) (1+c x)}{3375 c^4 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x^4 (1-c x) (1+c x)}{125 c^2 \sqrt{d-c^2 d x^2}}-$$
$$\frac{16 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{15 c^5 \sqrt{d-c^2 d x^2}}-\frac{8 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{45 c^3 \sqrt{d-c^2 d x^2}}-$$
$$\frac{2 b x^5 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{25 c \sqrt{d-c^2 d x^2}}-\frac{8 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^6 d}-$$
$$\frac{4 x^2 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{15 c^4 d}-\frac{x^4 \sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{5 c^2 d}$$

Result (type 3, 445 leaves, 17 steps):

$$-\frac{16\,a\,b\,x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{15\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{4144\,b^2\,(1-c\,x)\,(1+c\,x)}{3375\,c^6\sqrt{d-c^2\,d\,x^2}}-\frac{272\,b^2\,x^2\,(1-c\,x)\,(1+c\,x)}{3375\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b^2\,x^4\,(1-c\,x)\,(1+c\,x)}{125\,c^2\sqrt{d-c^2\,d\,x^2}}-\frac{16\,b^2\,x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{15\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{8\,b\,x^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{45\,c^3\sqrt{d-c^2\,d\,x^2}}-\frac{2\,b\,x^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{25\,c\sqrt{d-c^2\,d\,x^2}}-\frac{8\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{15\,c^6\sqrt{d-c^2\,d\,x^2}}-\frac{4\,x^2\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{15\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{x^4\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{5\,c^2\sqrt{d-c^2\,d\,x^2}}$$

Problem 195: Result valid but suboptimal antiderivative.

$$\int\frac{x^4\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 3, 355 leaves, 11 steps):

$$-\frac{15\,b^2\,x\,(1-c\,x)\,(1+c\,x)}{64\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,x^3\,(1-c\,x)\,(1+c\,x)}{32\,c^2\sqrt{d-c^2\,d\,x^2}}+\frac{15\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{64\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{3\,b\,x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^3\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{8\,c^4\,d}-\frac{x^3\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,c^2\,d}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{8\,b\,c^5\sqrt{d-c^2\,d\,x^2}}$$

Result (type 3, 371 leaves, 12 steps):

$$-\frac{15\,b^2\,x\,(1-c\,x)\,(1+c\,x)}{64\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{b^2\,x^3\,(1-c\,x)\,(1+c\,x)}{32\,c^2\sqrt{d-c^2\,d\,x^2}}+\frac{15\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcCosh}[c\,x]}{64\,c^5\sqrt{d-c^2\,d\,x^2}}-\frac{3\,b\,x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c^3\sqrt{d-c^2\,d\,x^2}}-\frac{b\,x^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}{8\,c\sqrt{d-c^2\,d\,x^2}}-\frac{3\,x\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{8\,c^4\sqrt{d-c^2\,d\,x^2}}-\frac{x^3\,(1-c\,x)\,(1+c\,x)\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,c^2\sqrt{d-c^2\,d\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])^3}{8\,b\,c^5\sqrt{d-c^2\,d\,x^2}}$$

Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 292 leaves, 9 steps):

$$-\frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}}-\frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}{9 c \sqrt{d-c^2 d x^2}}-\frac{2 \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{3 c^4 d}-\frac{x^2 \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{3 c^2 d}$$

Result (type 3, 308 leaves, 10 steps):

$$-\frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}}-\frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}}-\frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}{9 c \sqrt{d-c^2 d x^2}}-\frac{2 (1-c x) (1+c x) \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{3 c^4 \sqrt{d-c^2 d x^2}}-\frac{x^2 (1-c x) (1+c x) \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{3 c^2 \sqrt{d-c^2 d x^2}}$$

Problem 197: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 226 leaves, 5 steps):

$$-\frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}}+\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}}-\frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}{2 c \sqrt{d-c^2 d x^2}}-\frac{x \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^2}{2 c^2 d}+\frac{\sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^3}{6 b c^3 \sqrt{d-c^2 d x^2}}$$

Result (type 3, 234 leaves, 6 steps):

$$-\frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}}+\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}}-\frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{2 c \sqrt{d-c^2 d x^2}}-\frac{x (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{2 c^2 \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^3}{6 b c^3 \sqrt{d-c^2 d x^2}}$$

Problem 198: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\frac{2 a b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}}-\frac{2 b^2 (1-c x) (1+c x)}{c^2 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{c \sqrt{d-c^2 d x^2}}-\frac{\sqrt{d-c^2 d x^2} (a+b \text{ArcCosh}[c x])^2}{c^2 d}$$

Result (type 3, 163 leaves, 5 steps):

$$-\frac{2 a b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}}-\frac{2 b^2 (1-c x) (1+c x)}{c^2 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \text{ArcCosh}[c x]}{c \sqrt{d-c^2 d x^2}}-\frac{(1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{c^2 \sqrt{d-c^2 d x^2}}$$

Problem 199: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^3}{3 b c \sqrt{d-c^2 d x^2}}$$

Result (type 3, 53 leaves, 2 steps):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^3}{3 b c \sqrt{d-c^2 d x^2}}$$

Problem 200: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcCosh}[c x])^2}{x \sqrt{d-c^2 d x^2}} \, dx$$

Optimal (type 4, 273 leaves, 8 steps):

$$\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$
$$2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{1}{\sqrt{d-c^2\,d\,x^2}}2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 273 leaves, 9 steps):

$$\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$
$$2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{1}{\sqrt{d-c^2\,d\,x^2}}2\,i\,b\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2\,i\,b^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 201: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 186 leaves, 6 steps):

$$-\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{d\,x}-$$
$$\frac{2\,b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{b^2\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 194 leaves, 7 steps):

$$\frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2\,b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{b^2\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 202: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^3\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 430 leaves, 12 steps):

$$\frac{b\,c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x\sqrt{d-c^2\,d\,x^2}}-\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,d\,x^2}+$$
$$\frac{c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{b^2\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$
$$i\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{1}{\sqrt{d-c^2\,d\,x^2}}i\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{i\,b^2\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{i\,b^2\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 438 leaves, 13 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{x\,\sqrt{d-c^2\,d\,x^2}}-\frac{(1-c\,x)\,(1+c\,x)\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{2\,x^2\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2\,\text{ArcTan}\left[e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{ArcTan}\left[\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right]}{\sqrt{d-c^2\,d\,x^2}}-\frac{1}{\sqrt{d-c^2\,d\,x^2}}$$
$$i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{1}{\sqrt{d-c^2\,d\,x^2}}i\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[c\,x]}\right]+$$
$$\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{i\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[c\,x]}\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 203: Result valid but suboptimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{x^4\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 328 leaves, 9 steps):

$$\frac{b^2\,c^2\,(1-c\,x)\,(1+c\,x)}{3\,x\,\sqrt{d-c^2\,d\,x^2}}+\frac{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{3\,x^2\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{3\,\sqrt{d-c^2\,d\,x^2}}-$$
$$\frac{\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{3\,d\,x^3}-\frac{2\,c^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{3\,d\,x}-$$
$$\frac{4\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1+e^{-2\,\text{ArcCosh}[c\,x]}\right]}{3\,\sqrt{d-c^2\,d\,x^2}}+$$
$$\frac{2\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{PolyLog}\left[2,\,-e^{-2\,\text{ArcCosh}[c\,x]}\right]}{3\,\sqrt{d-c^2\,d\,x^2}}$$

Result (type 4, 344 leaves, 10 steps):

$$\frac{b^2 c^2 (1-c x) (1+c x)}{3 x \sqrt{d-c^2 d x^2}}+\frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])}{3 x^2 \sqrt{d-c^2 d x^2}}+\frac{2 c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{3 \sqrt{d-c^2 d x^2}}-\frac{(1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 x^3 \sqrt{d-c^2 d x^2}}-\frac{2 c^2 (1-c x) (1+c x) (a+b \text{ArcCosh}[c x])^2}{3 x \sqrt{d-c^2 d x^2}}-\frac{4 b c^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1+e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}-\frac{2 b^2 c^3 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,-e^{2 \text{ArcCosh}[c x]}\right]}{3 \sqrt{d-c^2 d x^2}}$$

Problem 209: Result optimal but 1 more steps used.

$$\int \frac{(a+b \text{ArcCosh}[c x])^2}{\left(d-c^2 d x^2\right)^{3/2}} \, dx$$

Optimal (type 4, 198 leaves, 6 steps):

$$\frac{x (a+b \text{ArcCosh}[c x])^2}{d \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{c d \sqrt{d-c^2 d x^2}}-\frac{2 b \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}-\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}$$

Result (type 4, 198 leaves, 7 steps):

$$\frac{x (a+b \text{ArcCosh}[c x])^2}{d \sqrt{d-c^2 d x^2}}+\frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^2}{c d \sqrt{d-c^2 d x^2}}-\frac{2 b \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x]) \text{Log}\left[1-e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}-\frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \text{PolyLog}\left[2,e^{2 \text{ArcCosh}[c x]}\right]}{c d \sqrt{d-c^2 d x^2}}$$

Problem 225: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \text{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 243 leaves, 11 steps):

$$-\frac{15\,x\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{64\,a^4}-\frac{x^3\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{32\,a^2}+\frac{15\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{64\,a^5\,\sqrt{1-a\,x}}-\frac{3\,x^2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{8\,a^3\,\sqrt{1-a\,x}}-\frac{x^4\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{8\,a\,\sqrt{1-a\,x}}-\frac{3\,x\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{8\,a^4}-\frac{x^3\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{4\,a^2}+\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^3}{8\,a^5\,\sqrt{1-a\,x}}$$

Result (type 3, 329 leaves, 12 steps):

$$-\frac{15\,x\,(1-a\,x)\,(1+a\,x)}{64\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^3\,(1-a\,x)\,(1+a\,x)}{32\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{15\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{64\,a^5\,\sqrt{1-a^2\,x^2}}-\frac{3\,x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{8\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{x^4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{8\,a\,\sqrt{1-a^2\,x^2}}-\frac{3\,x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{8\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^3\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{4\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^3}{8\,a^5\,\sqrt{1-a^2\,x^2}}$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{x^3\,\text{ArcCosh}[a\,x]^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 177 leaves, 8 steps):

$$-\frac{40\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{27\,a^4}-\frac{2\,x^2\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}}{27\,a^2}-\frac{4\,x\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{3\,a^3\,\sqrt{1-a\,x}}-\frac{2\,x^3\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]}{9\,a\,\sqrt{1-a\,x}}-\frac{2\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{3\,a^4}-\frac{x^2\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^2}{3\,a^2}$$

Result (type 3, 237 leaves, 9 steps):

$$-\frac{40\,(1-a\,x)\,(1+a\,x)}{27\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^2\,(1-a\,x)\,(1+a\,x)}{27\,a^2\,\sqrt{1-a^2\,x^2}}-\frac{4\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{3\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{9\,a\,\sqrt{1-a^2\,x^2}}-\frac{2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{3\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^2}{3\,a^2\,\sqrt{1-a^2\,x^2}}$$

Problem 227: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\,\text{ArcCosh}[a\,x]^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$-\frac{x\sqrt{1-a x}\ \sqrt{1+a x}}{4 a^2}+\frac{\sqrt{-1+a x}\ \text{ArcCosh}[a x]}{4 a^3\sqrt{1-a x}}-\frac{x^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]}{2 a\sqrt{1-a x}}-\frac{x\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^2}{2 a^2}+\frac{\sqrt{-1+a x}\ \text{ArcCosh}[a x]^3}{6 a^3\sqrt{1-a x}}$$

Result (type 3, 207 leaves, 6 steps):

$$-\frac{x(1-a x)(1+a x)}{4 a^2\sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]}{4 a^3\sqrt{1-a^2 x^2}}-\frac{x^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]}{2 a\sqrt{1-a^2 x^2}}-\frac{x(1-a x)(1+a x)\ \text{ArcCosh}[a x]^2}{2 a^2\sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3}{6 a^3\sqrt{1-a^2 x^2}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int\frac{x\ \text{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 79 leaves, 3 steps):

$$-\frac{2\sqrt{1-a x}\ \sqrt{1+a x}}{a^2}-\frac{2 x\sqrt{-1+a x}\ \text{ArcCosh}[a x]}{a\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^2}{a^2}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{2(1-a x)(1+a x)}{a^2\sqrt{1-a^2 x^2}}-\frac{2 x\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]}{a\sqrt{1-a^2 x^2}}-\frac{(1-a x)(1+a x)\ \text{ArcCosh}[a x]^2}{a^2\sqrt{1-a^2 x^2}}$$

Problem 229: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a x}\ \text{ArcCosh}[a x]^3}{3 a\sqrt{1-a x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3}{3 a\sqrt{1-a^2 x^2}}$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a x]^2}{x\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\frac{2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{2 i \sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{2 i \sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{2 i \sqrt{-1+a x}\ \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{2 i \sqrt{-1+a x}\ \text{PolyLog}\left[3,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 248 leaves, 9 steps):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{2 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{2 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+\frac{2 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,-i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-\frac{2 i \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^2}{x^2\sqrt{1-a^2 x^2}}\, dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{a\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2}{\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^2}{x}-\frac{2 a\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{a\sqrt{-1+a x}\ \text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 174 leaves, 7 steps):

$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]^2}{x\sqrt{1-a^2 x^2}}-$$
$$\frac{2 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[2,\,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int\frac{\text{ArcCosh}[a x]^2}{x^3\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 296 leaves, 12 steps):

$$\frac{a\sqrt{-1+a x}\ \text{ArcCosh}[a x]}{x\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]^2}{2 x^2}+$$
$$\frac{a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{a^2\sqrt{-1+a x}\ \text{ArcTan}\left[\sqrt{-1+a x}\ \sqrt{1+a x}\right]}{\sqrt{1-a x}}-$$
$$\frac{i\,a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+$$
$$\frac{i\,a^2\sqrt{-1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+$$
$$\frac{i\,a^2\sqrt{-1+a x}\ \text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-\frac{i\,a^2\sqrt{-1+a x}\ \text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}$$

Result (type 4, 398 leaves, 13 steps):

$$\frac{a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]}{x\sqrt{1-a^2 x^2}}-\frac{(1-a x)\ (1+a x)\ \text{ArcCosh}[a x]^2}{2 x^2\sqrt{1-a^2 x^2}}+$$
$$\frac{a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2\ \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcTan}\left[\sqrt{-1+a x}\ \sqrt{1+a x}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{i\, a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,-i\, e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$
$$\frac{i\, a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\ \text{PolyLog}\left[2,i\, e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$
$$\frac{i\, a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,-i\, e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$
$$\frac{i\, a^2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,i\, e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int (f x)^m\ \left(d-c^2 d x^2\right)^{5/2}\ (a+b\ \text{ArcCosh}[c x])^2\, dx$$

Optimal (type 8, 1153 leaves, 22 steps) :

$$-\frac{10\, b^2 c^2 d^2 (f x)^{3+m} \sqrt{d-c^2 d x^2}}{f^3 (4+m)^3 (6+m)}-\frac{2\, b^2 c^2 d^2 \left(52+15 m+m^2\right) (f x)^{3+m} \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}}{f^3 (4+m)^2 (6+m)^3 (1-c x) (1+c x)}+$$

$$\frac{2\, b^2 c^4 d^2 (f x)^{5+m} \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}}{f^5 (6+m)^3 (1-c x) (1+c x)}-\frac{2\, b\, c\, d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^2 (2+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{30\, b\, c\, d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{10\, b\, c\, d^2 (f x)^{2+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^2 (2+m) (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{10\, b\, c^3 d^2 (f x)^{4+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^4 (4+m)^2 (6+m) \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{4\, b\, c^3 d^2 (f x)^{4+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^4 (4+m) (6+m) \sqrt{-1+c x} \sqrt{1+c x}}-$$

$$\frac{2\, b\, c^5 d^2 (f x)^{6+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])}{f^6 (6+m)^2 \sqrt{-1+c x} \sqrt{1+c x}}+$$

$$\frac{15\, d^2 (f x)^{1+m} \sqrt{d-c^2 d x^2}\, (a+b\, \text{ArcCosh}[c x])^2}{f (6+m) \left(8+6 m+m^2\right)}+$$

$$\frac{5\, d\, (f x)^{1+m} \left(d-c^2 d x^2\right)^{3/2} (a+b\, \text{ArcCosh}[c x])^2}{f (4+m) (6+m)}+\frac{(f x)^{1+m} \left(d-c^2 d x^2\right)^{5/2} (a+b\, \text{ArcCosh}[c x])^2}{f (6+m)}-$$

$$\left(30\, b^2 c^2 d^2 (f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]\right)\Big/$$

$$\left(f^3 (2+m)^2 (3+m) (4+m) (6+m) (1-c x) (1+c x)\right)-\left(10\, b^2 c^2 d^2 (10+3 m)\right.$$

$$\left.(f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]\right)\Big/$$

$$\left(f^3 (2+m) (3+m) (4+m)^3 (6+m) (1-c x) (1+c x)\right)-\left(2\, b^2 c^2 d^2 \left(264+130 m+15 m^2\right)\right.$$

$$\left.(f x)^{3+m} \sqrt{1-c^2 x^2} \sqrt{d-c^2 d x^2}\, \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2 x^2\right]\right)\Big/$$

$$\left(f^3 (2+m) (3+m) (4+m)^2 (6+m)^3 (1-c x) (1+c x)\right)+$$

$$\frac{15\, d^3\, \text{Unintegrable}\left[\frac{(f x)^m (a+b\, \text{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}},x\right]}{(6+m) \left(8+6 m+m^2\right)}$$

Result (type 8, 73 leaves, 1 step):

$$\left(d^2 \sqrt{d-c^2 d x^2}\, \text{Unintegrable}\left[(f x)^m (-1+c x)^{5/2} (1+c x)^{5/2} (a+b\, \text{ArcCosh}[c x])^2,x\right]\right)\Big/$$

$$\left(\sqrt{-1+c x} \sqrt{1+c x}\right)$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int (f x)^m \left(d - c^2 d x^2\right)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2 \, dx$$

Optimal (type 8, 583 leaves, 13 steps):

$$-\frac{2 b^2 c^2 d (f x)^{3+m} \sqrt{d - c^2 d x^2}}{f^3 (4+m)^3} - \frac{6 b c d (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 b c d (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m) (4+m) \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^3 d (f x)^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^4 (4+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{3 d (f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{f \left(8 + 6 m + m^2\right)} + \frac{(f x)^{1+m} \left(d - c^2 d x^2\right)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{f (4+m)} - \left(6 b^2 c^2 d (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]\right) \Big/ \left(f^3 (2+m)^2 (3+m) (4+m) (1 - c x) (1 + c x)\right) - \left(2 b^2 c^2 d (10 + 3 m) (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]\right) \Big/ \left(f^3 (2+m) (3+m) (4+m)^3 (1 - c x) (1 + c x)\right) + \frac{3 d^2 \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{8 + 6 m + m^2}$$

Result (type 8, 72 leaves, 1 step):

$$-\left(\left(d \sqrt{d - c^2 d x^2} \operatorname{Unintegrable}\left[(f x)^m (-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2, x\right]\right) \Big/ \left(\sqrt{-1 + c x} \sqrt{1 + c x}\right)\right)$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int (f x)^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 \, dx$$

Optimal (type 8, 239 leaves, 5 steps):

$$-\frac{2 b c (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{f (2+m)} - \left(2 b^2 c^2 (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]\right) \Big/ \left(f^3 (2+m)^2 (3+m) (1 - c x) (1 + c x)\right) + \frac{d \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{2 + m}$$

Result (type 8, 70 leaves, 1 step):

$$\left(\sqrt{d-c^2\,d\,x^2}\;\text{Unintegrable}\left[(f\,x)^m\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2,\,x\right]\right)\Big/\left(\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\sqrt{d-c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^2}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\left(d-c^2\,d\,x^2\right)^{3/2}},\,x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^2}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 238: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}},\,x\right]$$

Result (type 8, 73 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^2}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}},\,x\right]}{d^2\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 239: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\text{ArcCosh}[c\,x]^2}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\text{ArcCosh}[c\,x]^2}{\sqrt{1-c^2\,x^2}},\,x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{(f\,x)^m\,\text{ArcCosh}[c\,x]^2}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 248: Result optimal but 1 more steps used.

$$\int\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^3\,dx$$

Optimal (type 3, 231 leaves, 6 steps):

$$-\frac{3\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3}{4}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]+\frac{3\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^2}{8\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{3\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^2}{4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^3-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^4}{8\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 3, 231 leaves, 7 steps):

$$-\frac{3\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{3}{4}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]+\frac{3\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^2}{8\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{3\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^2}{4\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^3-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^4}{8\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 249: Result optimal but 1 more steps used.

$$\int\frac{\text{ArcCosh}[a\,x]^3}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 46 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^4}{4\,a\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^4}{4 a \sqrt{c-a^2 c x^2}}$$

Problem 250: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^3}{\left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 241 leaves, 7 steps):

$$\frac{x\,\text{ArcCosh}[a x]^3}{c \sqrt{c-a^2 c x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\,\text{PolyLog}\left[2,\ e^{2 \text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}+\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,\ e^{2 \text{ArcCosh}[a x]}\right]}{2 a c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 241 leaves, 8 steps):

$$\frac{x\,\text{ArcCosh}[a x]^3}{c \sqrt{c-a^2 c x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^3}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^2 \,\text{Log}\left[1-e^{2 \text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}-\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]\,\text{PolyLog}\left[2,\ e^{2 \text{ArcCosh}[a x]}\right]}{a c \sqrt{c-a^2 c x^2}}+\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{PolyLog}\left[3,\ e^{2 \text{ArcCosh}[a x]}\right]}{2 a c \sqrt{c-a^2 c x^2}}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{x^4\,\text{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 3, 315 leaves, 13 steps):

$$-\frac{45 x^2 \sqrt{-1+a x}}{128 a^3 \sqrt{1-a x}}-\frac{3 x^4 \sqrt{-1+a x}}{128 a \sqrt{1-a x}}-\frac{45 x \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{64 a^4}-\frac{3 x^3 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{32 a^2}+\frac{45 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{128 a^5 \sqrt{1-a x}}-\frac{9 x^2 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{16 a^3 \sqrt{1-a x}}-\frac{3 x^4 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{16 a \sqrt{1-a x}}-\frac{3 x \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{8 a^4}-\frac{x^3 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{4 a^2}+\frac{3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^4}{32 a^5 \sqrt{1-a x}}$$

Result (type 3, 427 leaves, 14 steps):

$$-\frac{45 x^2 \sqrt{-1+a x} \sqrt{1+a x}}{128 a^3 \sqrt{1-a^2 x^2}}-\frac{3 x^4 \sqrt{-1+a x} \sqrt{1+a x}}{128 a \sqrt{1-a^2 x^2}}-\frac{45 x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{64 a^4 \sqrt{1-a^2 x^2}}-\frac{3 x^3 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{32 a^2 \sqrt{1-a^2 x^2}}+\frac{45 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{128 a^5 \sqrt{1-a^2 x^2}}-\frac{9 x^2 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{16 a^3 \sqrt{1-a^2 x^2}}-\frac{3 x^4 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{16 a \sqrt{1-a^2 x^2}}-\frac{3 x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{8 a^4 \sqrt{1-a^2 x^2}}-\frac{x^3 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{4 a^2 \sqrt{1-a^2 x^2}}+\frac{3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^4}{32 a^5 \sqrt{1-a^2 x^2}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \operatorname{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}} \, dx$$

Optimal (type 3, 243 leaves, 10 steps):

$$-\frac{40 x \sqrt{-1+a x}}{9 a^3 \sqrt{1-a x}}-\frac{2 x^3 \sqrt{-1+a x}}{27 a \sqrt{1-a x}}-\frac{40 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a^4}-\frac{2 x^2 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a^2}-\frac{2 x \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{a^3 \sqrt{1-a x}}-\frac{x^3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{3 a \sqrt{1-a x}}-\frac{2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{3 a^4}-\frac{x^2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{3 a^2}$$

Result (type 3, 329 leaves, 11 steps):

$$-\frac{40\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{9\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{27\,a\,\sqrt{1-a^2\,x^2}}-\frac{40\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{9\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{2\,x^2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{9\,a^2\,\sqrt{1-a^2\,x^2}}-\frac{2\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{a^3\,\sqrt{1-a^2\,x^2}}-\frac{x^3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{3\,a\,\sqrt{1-a^2\,x^2}}-\frac{2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{3\,a^4\,\sqrt{1-a^2\,x^2}}-\frac{x^2\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{3\,a^2\,\sqrt{1-a^2\,x^2}}$$

Problem 255: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\,\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$-\frac{3\,x^2\,\sqrt{-1+a\,x}}{8\,a\,\sqrt{1-a\,x}}-\frac{3\,x\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{4\,a^2}+\frac{3\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{8\,a^3\,\sqrt{1-a\,x}}-\frac{3\,x^2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{4\,a\,\sqrt{1-a\,x}}-\frac{x\,\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{2\,a^2}+\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^4}{8\,a^3\,\sqrt{1-a\,x}}$$

Result (type 3, 257 leaves, 7 steps):

$$-\frac{3\,x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{8\,a\,\sqrt{1-a^2\,x^2}}-\frac{3\,x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{4\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{3\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{8\,a^3\,\sqrt{1-a^2\,x^2}}-\frac{3\,x^2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{4\,a\,\sqrt{1-a^2\,x^2}}-\frac{x\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{2\,a^2\,\sqrt{1-a^2\,x^2}}+\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^4}{8\,a^3\,\sqrt{1-a^2\,x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{6\,x\,\sqrt{-1+a\,x}}{a\,\sqrt{1-a\,x}}-\frac{6\,\sqrt{1-a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]}{a^2}-\frac{3\,x\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2}{a\,\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\,\text{ArcCosh}[a\,x]^3}{a^2}$$

Result (type 3, 153 leaves, 5 steps):

$$-\frac{6\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}{a\,\sqrt{1-a^2\,x^2}}-\frac{6\,(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]}{a^2\,\sqrt{1-a^2\,x^2}}-\frac{3\,x\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^2}{a\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,(1+a\,x)\,\text{ArcCosh}[a\,x]^3}{a^2\,\sqrt{1-a^2\,x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^4}{4\,a\,\sqrt{1-a\,x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{ArcCosh}[a\,x]^4}{4\,a\,\sqrt{1-a^2\,x^2}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 265 leaves, 10 steps):

$$\frac{2\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{3\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{3\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{6\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{6\,i\,\sqrt{-1+a\,x}\,\text{ArcCosh}[a\,x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-\frac{6\,i\,\sqrt{-1+a\,x}\,\text{PolyLog}\left[4,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{6\,i\,\sqrt{-1+a\,x}\,\text{PolyLog}\left[4,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}$$

Result (type 4, 356 leaves, 11 steps):

$$\frac{2\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$

$$\frac{3\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]^2\,\text{PolyLog}\left[2,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$

$$\frac{3\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]^2\,\text{PolyLog}\left[2,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$

$$\frac{6\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[3,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$

$$\frac{6\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[3,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$

$$\frac{6\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{PolyLog}\left[4,-i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$

$$\frac{6\,i\sqrt{-1+a x}\sqrt{1+a x}\,\text{PolyLog}\left[4,i\,e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}$$

Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^3}{x^2\sqrt{1-a^2 x^2}}\,dx$$

Optimal (type 4, 166 leaves, 7 steps):

$$\frac{a\sqrt{-1+a x}\,\text{ArcCosh}[a x]^3}{\sqrt{1-a x}}-\frac{\sqrt{1-a^2 x^2}\,\text{ArcCosh}[a x]^3}{x}-$$

$$\frac{3 a\sqrt{-1+a x}\,\text{ArcCosh}[a x]^2\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}-$$

$$\frac{3 a\sqrt{-1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}}+\frac{3 a\sqrt{-1+a x}\,\text{PolyLog}\left[3,-e^{2\,\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a x}}$$

Result (type 4, 229 leaves, 8 steps):

$$\frac{a\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}}-\frac{(1-a x)(1+a x)\,\text{ArcCosh}[a x]^3}{x\sqrt{1-a^2 x^2}}-$$

$$\frac{3 a\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]^2\,\text{Log}\left[1+e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}-$$

$$\frac{3 a\sqrt{-1+a x}\sqrt{1+a x}\,\text{ArcCosh}[a x]\,\text{PolyLog}\left[2,-e^{2\,\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}}+$$

$$\frac{3 a\sqrt{-1+a x}\sqrt{1+a x}\,\text{PolyLog}\left[3,-e^{2\,\text{ArcCosh}[a x]}\right]}{2\sqrt{1-a^2 x^2}}$$

Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a\,x]^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Optimal (type 4, 460 leaves, 18 steps):

$$\frac{3\,a\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]^2}{2\,x\,\sqrt{1-a\,x}}-\frac{\sqrt{1-a^2\,x^2}\,\,\text{ArcCosh}[a\,x]^3}{2\,x^2}-$$
$$\frac{6\,a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+$$
$$\frac{a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a\,x}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a\,x}}+$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}-$$
$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{PolyLog}\left[4,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}+\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\text{PolyLog}\left[4,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a\,x}}$$

Result (type 4, 614 leaves, 19 steps):

$$\frac{3\,a\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]^2}{2\,x\,\sqrt{1-a^2\,x^2}}-\frac{(1-a\,x)\,\,(1+a\,x)\,\,\text{ArcCosh}[a\,x]^3}{2\,x^2\,\sqrt{1-a^2\,x^2}}-$$

$$\frac{6\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+$$

$$\frac{a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]^3\,\text{ArcTan}\left[e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}-$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]^2\,\text{PolyLog}\left[2,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{2\,\sqrt{1-a^2\,x^2}}+$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{PolyLog}\left[3,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{ArcCosh}[a\,x]\,\,\text{PolyLog}\left[3,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}-$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{PolyLog}\left[4,\,-i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}+$$

$$\frac{3\,i\,a^2\,\sqrt{-1+a\,x}\,\,\sqrt{1+a\,x}\,\,\text{PolyLog}\left[4,\,i\,e^{\text{ArcCosh}[a\,x]}\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 261: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{1-c^2\,x^2}},\,x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\text{Unintegrable}\left[\frac{(f\,x)^m\,\,(a+b\,\text{ArcCosh}[c\,x])^3}{\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 267: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^5\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^5\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^5\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^5\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 268: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^4\sqrt{-1+c\,x}}$$

Result (type 4, 371 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\,\text{Sinh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 269: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\,\text{Sinh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 270: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1-c^2 x^2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 245 leaves, 10 steps):

$$-\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{3\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{3\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 271: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{a+b\,\mathrm{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1-c\,x}\ \mathrm{Cosh}\left[\frac{2\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{2\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \mathrm{Sinh}\left[\frac{2\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{2\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2\,a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{2\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{2\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2\,a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 272: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2 x^2}}{x\,(a+b\,\mathrm{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 116 leaves, 6 steps):

$$-\frac{\sqrt{-1+c\,x}\ \mathrm{Cosh}\left[\frac{a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{b\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\ \mathrm{Sinh}\left[\frac{a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{b\,\sqrt{1-c\,x}}+\mathrm{Unintegrable}\left[\frac{1}{x\,\sqrt{1-c^2 x^2}\ (a+b\,\mathrm{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 176 leaves, 7 steps):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 273: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^2\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 65 leaves, 3 steps):

$$-\frac{c\,\sqrt{-1+c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{1-c^2 x^2}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 115 leaves, 4 steps):

$$\frac{c\,\sqrt{1-c^2 x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 274: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4\,(a+b\,\text{ArcCosh}[c\,x])},\ x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{x^4\,(a+b\,\text{ArcCosh}[c\,x])},\ x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{x^3\,(1-c^2 x^2)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{3\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5a}{b}\right]\text{CoshIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{7a}{b}\right]\text{CoshIntegral}\left[\frac{7(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{5a}{b}\right]\text{SinhIntegral}\left[\frac{5(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{7a}{b}\right]\text{SinhIntegral}\left[\frac{7(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^4\sqrt{-1+c\,x}}$$

Result (type 4, 497 leaves, 16 steps):

$$-\frac{3\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{3\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{3a}{b}\right]\ \text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{5a}{b}\right]\ \text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2x^2}\ \text{Cosh}\left[\frac{7a}{b}\right]\ \text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{3\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{3\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{3a}{b}\right]\ \text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{5a}{b}\right]\ \text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2x^2}\ \text{Sinh}\left[\frac{7a}{b}\right]\ \text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^4\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{6\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{6\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\;\text{Cosh}\left[\frac{6\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{4\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\;\text{Sinh}\left[\frac{6\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 278: Result valid but suboptimal antiderivative.

$$\int \frac{x\,\left(1-c^2\,x^2\right)^{3/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$-\frac{\sqrt{1-c\,x}\,\operatorname{Cosh}\left[\frac{a}{b}\right]\operatorname{CoshIntegral}\left[\frac{a+b\operatorname{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{3\,\sqrt{1-c\,x}\,\operatorname{Cosh}\left[\frac{3\,a}{b}\right]\operatorname{CoshIntegral}\left[\frac{3\,(a+b\operatorname{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\operatorname{Cosh}\left[\frac{5\,a}{b}\right]\operatorname{CoshIntegral}\left[\frac{5\,(a+b\operatorname{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\operatorname{Sinh}\left[\frac{a}{b}\right]\operatorname{SinhIntegral}\left[\frac{a+b\operatorname{ArcCosh}[c\,x]}{b}\right]}{8\,b\,c^2\,\sqrt{-1+c\,x}}-\frac{3\,\sqrt{1-c\,x}\,\operatorname{Sinh}\left[\frac{3\,a}{b}\right]\operatorname{SinhIntegral}\left[\frac{3\,(a+b\operatorname{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\operatorname{Sinh}\left[\frac{5\,a}{b}\right]\operatorname{SinhIntegral}\left[\frac{5\,(a+b\operatorname{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 371 leaves, 13 steps):

$$-\frac{\sqrt{1-c^2\,x^2}\,\operatorname{Cosh}\left[\frac{a}{b}\right]\operatorname{CoshIntegral}\left[\frac{a}{b}+\operatorname{ArcCosh}[c\,x]\right]}{8\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\,\sqrt{1-c^2\,x^2}\,\operatorname{Cosh}\left[\frac{3\,a}{b}\right]\operatorname{CoshIntegral}\left[\frac{3\,a}{b}+3\operatorname{ArcCosh}[c\,x]\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\,\operatorname{Cosh}\left[\frac{5\,a}{b}\right]\operatorname{CoshIntegral}\left[\frac{5\,a}{b}+5\operatorname{ArcCosh}[c\,x]\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\,\operatorname{Sinh}\left[\frac{a}{b}\right]\operatorname{SinhIntegral}\left[\frac{a}{b}+\operatorname{ArcCosh}[c\,x]\right]}{8\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\,\sqrt{1-c^2\,x^2}\,\operatorname{Sinh}\left[\frac{3\,a}{b}\right]\operatorname{SinhIntegral}\left[\frac{3\,a}{b}+3\operatorname{ArcCosh}[c\,x]\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\,\operatorname{Sinh}\left[\frac{5\,a}{b}\right]\operatorname{SinhIntegral}\left[\frac{5\,a}{b}+5\operatorname{ArcCosh}[c\,x]\right]}{16\,b\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 279: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2\,x^2\right)^{3/2}}{a+b\operatorname{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{2\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{4\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c\,\sqrt{-1+c\,x}}-\frac{3\,\sqrt{1-c\,x}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{2\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{4\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 304 leaves, 10 steps):

$$\frac{\sqrt{1-c^2\,x^2}\ \text{Cosh}\left[\frac{2\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\ \text{Cosh}\left[\frac{4\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{3\,\sqrt{1-c^2\,x^2}\ \text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{8\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\ \text{Sinh}\left[\frac{2\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\ \text{Sinh}\left[\frac{4\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b\,c\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 280: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2\,x^2\right)^{3/2}}{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 215 leaves, 15 steps):

$$-\frac{5\,\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\frac{5\,\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x\,\sqrt{1-c^2\,x^2}\ \left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 301 leaves, 16 steps):

$$\frac{5\sqrt{1-c^2x^2}\ \mathrm{Cosh}\left[\frac{a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \mathrm{Cosh}\left[\frac{3\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2x^2}\ \mathrm{Sinh}\left[\frac{a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\ \mathrm{Sinh}\left[\frac{3\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \mathrm{Unintegrable}\left[\frac{1}{x\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\mathrm{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 281: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{3/2}}{x^2\,\left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 163 leaves, 9 steps):

$$\frac{c\sqrt{-1+c\,x}\ \mathrm{Cosh}\left[\frac{2\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,\sqrt{1-c\,x}}-\frac{3\,c\,\sqrt{-1+c\,x}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{2\,b\,\sqrt{1-c\,x}}-\frac{c\,\sqrt{-1+c\,x}\ \mathrm{Sinh}\left[\frac{2\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,\sqrt{1-c\,x}}+\mathrm{Unintegrable}\left[\frac{1}{x^2\sqrt{1-c^2x^2}\ \left(a+b\,\mathrm{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 240 leaves, 10 steps):

$$-\frac{c\,\sqrt{1-c^2x^2}\ \mathrm{Cosh}\left[\frac{2\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2\,a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{2\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{3\,c\,\sqrt{1-c^2x^2}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{2\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{c\,\sqrt{1-c^2x^2}\ \mathrm{Sinh}\left[\frac{2\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2\,a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{2\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\ \mathrm{Unintegrable}\left[\frac{1}{x^2\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\mathrm{ArcCosh}[c\,x])},\,x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 282: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^4 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(1-c^2 x^2\right)^{5/2}}{a+b \operatorname{ArcCosh}[c x]} \, dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{3\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{128\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^4\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{7\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{256\,b\,c^4\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{9\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{9\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{256\,b\,c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{128\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{7\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{256\,b\,c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{9\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{9\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{256\,b\,c^4\sqrt{-1+c\,x}}$$

Result (type 4, 497 leaves, 16 steps) :

$$-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{128\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\,\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{7a}{b}\right]\,\text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{9a}{b}\right]\,\text{CoshIntegral}\left[\frac{9a}{b}+9\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{128\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\,\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{7a}{b}\right]\,\text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{9a}{b}\right]\,\text{SinhIntegral}\left[\frac{9a}{b}+9\,\text{ArcCosh}[c\,x]\right]}{256\,b\,c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 285: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 439 leaves, 15 steps):

$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{8\,a}{b}\right]\,\text{CoshIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{128\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{5\,\sqrt{1-c\,x}\,\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{128\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c^3\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Sinh}\left[\frac{8\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{128\,b\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 556 leaves, 16 steps) :

$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{2a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{4a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{4a}{b}+4\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{6a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{6a}{b}+6\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Cosh}\left[\frac{8a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{8a}{b}+8\,\mathrm{ArcCosh}[c\,x]\right]}{128\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{5\sqrt{1-c^2 x^2}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{128\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{2a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2a}{b}+2\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{4a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{4a}{b}+4\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{6a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{6a}{b}+6\,\mathrm{ArcCosh}[c\,x]\right]}{32\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2 x^2}\ \mathrm{Sinh}\left[\frac{8a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{8a}{b}+8\,\mathrm{ArcCosh}[c\,x]\right]}{128\,b\,c^3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 286: Result valid but suboptimal antiderivative.

$$\int \frac{x\left(1-c^2 x^2\right)^{5/2}}{a+b\,\mathrm{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$-\frac{5\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{9\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{5\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\ \text{Cosh}\left[\frac{7\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{5\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-\frac{9\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}+\frac{5\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{5\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\ \text{Sinh}\left[\frac{7\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b\,c^2\sqrt{-1+c\,x}}$$

Result (type 4, 497 leaves, 16 steps):

$$-\frac{5\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{9\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3a}{b}\right]\text{CoshIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{5\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{5a}{b}\right]\text{CoshIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{7a}{b}\right]\text{CoshIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{5\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{9\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{3a}{b}\right]\text{SinhIntegral}\left[\frac{3a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{5\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{5a}{b}\right]\text{SinhIntegral}\left[\frac{5a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{\sqrt{1-c^2x^2}\,\text{Sinh}\left[\frac{7a}{b}\right]\text{SinhIntegral}\left[\frac{7a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b\,c^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 287: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{5/2}}{a+b\,\text{ArcCosh}[c\,x]}\,dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\frac{15\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\,\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\,\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c\,\sqrt{-1+c\,x}}-\frac{15\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\,\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,c\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\;\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{32\,b\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 430 leaves, 13 steps):

$$\frac{15\sqrt{1-c^2x^2}\;\text{Cosh}\left[\frac{2a}{b}\right]\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\;\text{Cosh}\left[\frac{4a}{b}\right]\text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2x^2}\;\text{Cosh}\left[\frac{6a}{b}\right]\text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2x^2}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{16\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{15\sqrt{1-c^2x^2}\;\text{Sinh}\left[\frac{2a}{b}\right]\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\;\text{Sinh}\left[\frac{4a}{b}\right]\text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{16\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\;\text{Sinh}\left[\frac{6a}{b}\right]\text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{32\,b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2x^2\right)^{5/2}}{x\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 309 leaves, 27 steps):

$$-\frac{11\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,\sqrt{1-c\,x}}+\frac{7\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\ \text{Cosh}\left[\frac{5\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+\frac{11\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b\,\sqrt{1-c\,x}}-\frac{7\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\ \text{Sinh}\left[\frac{5\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b\,\sqrt{1-c\,x}}+\text{Unintegrable}\left[\frac{1}{x\,\sqrt{1-c^2\,x^2}\ (a+b\,\text{ArcCosh}[c\,x])},\ x\right]$$

Result (type 8, 421 leaves, 28 steps):

$$\frac{11\sqrt{1-c^2\,x^2}\ \text{Cosh}\left[\frac{a}{b}\right]\ \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{7\sqrt{1-c^2\,x^2}\ \text{Cosh}\left[\frac{3\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\ \text{Cosh}\left[\frac{5\,a}{b}\right]\ \text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{11\sqrt{1-c^2\,x^2}\ \text{Sinh}\left[\frac{a}{b}\right]\ \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}+\frac{7\sqrt{1-c^2\,x^2}\ \text{Sinh}\left[\frac{3\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\ \text{Sinh}\left[\frac{5\,a}{b}\right]\ \text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\ \text{Unintegrable}\left[\frac{1}{x\,\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])},\ x\right]}{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}}$$

Problem 289: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^2 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 254 leaves, 18 steps):

$$\frac{c \sqrt{-1+c x} \operatorname{Cosh}\left[\frac{2 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{b \sqrt{1-c x}}-\frac{c \sqrt{-1+c x} \operatorname{Cosh}\left[\frac{4 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 b \sqrt{1-c x}}-\frac{15 c \sqrt{-1+c x} \operatorname{Log}[a+b \operatorname{ArcCosh}[c x]]}{8 b \sqrt{1-c x}}-\frac{c \sqrt{-1+c x} \operatorname{Sinh}\left[\frac{2 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{b \sqrt{1-c x}}+\frac{c \sqrt{-1+c x} \operatorname{Sinh}\left[\frac{4 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4 (a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 b \sqrt{1-c x}}+\operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{1-c^2 x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 357 leaves, 19 steps):

$$-\frac{c \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{2 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2 a}{b}+2 \operatorname{ArcCosh}[c x]\right]}{b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{c \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{4 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4 a}{b}+4 \operatorname{ArcCosh}[c x]\right]}{8 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{15 c \sqrt{1-c^2 x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[c x]]}{8 b \sqrt{-1+c x} \sqrt{1+c x}}+\frac{c \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{2 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2 a}{b}+2 \operatorname{ArcCosh}[c x]\right]}{b \sqrt{-1+c x} \sqrt{1+c x}}-\frac{c \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{4 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4 a}{b}+4 \operatorname{ArcCosh}[c x]\right]}{8 b \sqrt{-1+c x} \sqrt{1+c x}}-\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 290: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{5/2}}{x^3 \left(a+b \,\text{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{5/2} (1+c x)^{5/2}}{x^3 (a+b \,\text{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 291: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{5/2}}{x^4 \left(a+b \,\text{ArcCosh}[c x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{5/2}}{x^4 \left(a+b \,\text{ArcCosh}[c x]\right)}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{5/2} (1+c x)^{5/2}}{x^4 (a+b \,\text{ArcCosh}[c x])}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 292: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]} \, dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{\sqrt{-1+a x}\ \text{CoshIntegral}[2 \,\text{ArcCosh}[a x]]}{2 a^5 \sqrt{1-a x}}+\frac{\sqrt{-1+a x}\ \text{CoshIntegral}[4 \,\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a x}}+\frac{3 \sqrt{-1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a x}}$$

Result (type 4, 137 leaves, 6 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[2 \,\text{ArcCosh}[a x]]}{2 a^5 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[4 \,\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a^2 x^2}}+\frac{3 \sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1-a^2 x^2}}$$

Problem 293: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]} \, dx$$

Optimal (type 4, 65 leaves, 5 steps):

$$\frac{3\sqrt{-1+a x}\ \text{CoshIntegral}[\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1-a x}}+\frac{\sqrt{-1+a x}\ \text{CoshIntegral}[3\,\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1-a x}}$$

Result (type 4, 91 leaves, 6 steps):

$$\frac{3\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[3\,\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1-a^2 x^2}}$$

Problem 294: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 4, 65 leaves, 4 steps):

$$\frac{\sqrt{-1+a x}\ \text{CoshIntegral}[2\,\text{ArcCosh}[a x]]}{2 a^3 \sqrt{1-a x}}+\frac{\sqrt{-1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{2 a^3 \sqrt{1-a x}}$$

Result (type 4, 91 leaves, 5 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[2\,\text{ArcCosh}[a x]]}{2 a^3 \sqrt{1-a^2 x^2}}+\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Log}[\text{ArcCosh}[a x]]}{2 a^3 \sqrt{1-a^2 x^2}}$$

Problem 295: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 4, 28 leaves, 2 steps):

$$\frac{\sqrt{-1+a x}\ \text{CoshIntegral}[\text{ArcCosh}[a x]]}{a^2 \sqrt{1-a x}}$$

Result (type 4, 41 leaves, 3 steps):

$$\frac{\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{CoshIntegral}[\text{ArcCosh}[a x]]}{a^2 \sqrt{1-a^2 x^2}}$$

Problem 296: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-a^2 x^2}\ \text{ArcCosh}[a x]}\,dx$$

Optimal (type 3, 28 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\;\text{Log}[\text{ArcCosh}[a\,x]]}{a\,\sqrt{1-a\,x}}$$

Result (type 3, 41 leaves, 2 steps):

$$\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Log}[\text{ArcCosh}[a\,x]]}{a\,\sqrt{1-a^2\,x^2}}$$

Problem 297: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\,\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]},\,x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Unintegrable}\left[\frac{1}{x\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{ArcCosh}[a\,x]},\,x\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 298: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]}\,dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{1-a^2\,x^2}\;\text{ArcCosh}[a\,x]},\,x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{Unintegrable}\left[\frac{1}{x^2\,\sqrt{-1+a\,x}\;\sqrt{1+a\,x}\;\text{ArcCosh}[a\,x]},\,x\right]}{\sqrt{1-a^2\,x^2}}$$

Problem 299: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2\,x^2}\;(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$\frac{3\sqrt{-1+c\,x}\ \mathrm{Cosh}\left[\frac{a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^4\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\ \mathrm{Cosh}\left[\frac{3\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{3\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^4\,\sqrt{1-c\,x}}-\frac{3\sqrt{-1+c\,x}\ \mathrm{Sinh}\left[\frac{a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{a+b\,\mathrm{ArcCosh}[c\,x]}{b}\right]}{4\,b\,c^4\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\ \mathrm{Sinh}\left[\frac{3\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{3\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{4\,b\,c^4\,\sqrt{1-c\,x}}$$

Result (type 4, 245 leaves, 10 steps):

$$\frac{3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \mathrm{Cosh}\left[\frac{a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^4\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \mathrm{Cosh}\left[\frac{3\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^4\,\sqrt{1-c^2\,x^2}}-\frac{3\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \mathrm{Sinh}\left[\frac{a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{a}{b}+\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^4\,\sqrt{1-c^2\,x^2}}-\frac{\sqrt{-1+c\,x}\ \sqrt{1+c\,x}\ \mathrm{Sinh}\left[\frac{3\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{3\,a}{b}+3\,\mathrm{ArcCosh}[c\,x]\right]}{4\,b\,c^4\,\sqrt{1-c^2\,x^2}}$$

Problem 300: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{\sqrt{1-c^2\,x^2}\ \left(a+b\,\mathrm{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{-1+c\,x}\ \mathrm{Cosh}\left[\frac{2\,a}{b}\right]\ \mathrm{CoshIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c^3\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\ \mathrm{Log}[a+b\,\mathrm{ArcCosh}[c\,x]]}{2\,b\,c^3\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\ \mathrm{Sinh}\left[\frac{2\,a}{b}\right]\ \mathrm{SinhIntegral}\left[\frac{2\,(a+b\,\mathrm{ArcCosh}[c\,x])}{b}\right]}{2\,b\,c^3\,\sqrt{1-c\,x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\;\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c^3\,\sqrt{1-c^2\,x^2}}+$$

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{2\,b\,c^3\,\sqrt{1-c^2\,x^2}}-$$

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Sinh}\left[\frac{2\,a}{b}\right]\;\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{2\,b\,c^3\,\sqrt{1-c^2\,x^2}}$$

Problem 301: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\sqrt{1-c^2\,x^2}\;(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{\sqrt{-1+c\,x}\;\text{Cosh}\left[\frac{a}{b}\right]\;\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b\,c^2\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\;\text{Sinh}\left[\frac{a}{b}\right]\;\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b\,c^2\,\sqrt{1-c\,x}}$$

Result (type 4, 114 leaves, 5 steps):

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Cosh}\left[\frac{a}{b}\right]\;\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,c^2\,\sqrt{1-c^2\,x^2}}-$$

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Sinh}\left[\frac{a}{b}\right]\;\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{b\,c^2\,\sqrt{1-c^2\,x^2}}$$

Problem 302: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\sqrt{1-c^2\,x^2}\;(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 3, 35 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Log}[a+b\,\text{ArcCosh}[c\,x]]}{b\,c\,\sqrt{1-c^2\,x^2}}$$

Problem 303: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x\,\sqrt{1-c^2\,x^2}\;(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 304: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 305: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^2}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 306: Result valid but suboptimal antiderivative.

$$\int\frac{x}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{(1-c^2x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},x\right]$$

Result (type 8, 66 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{x}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 307: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 27 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 65 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{1}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 308: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2\,x^2}}\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x\,\left(-1+c\,x\right)^{3/2}\,\left(1+c\,x\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Problem 309: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2\,x^2}}\sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x^2\,\left(-1+c\,x\right)^{3/2}\,\left(1+c\,x\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]$$

Problem 310: Result valid but suboptimal antiderivative.

$$\int \frac{x^m \left(1-c^2 x^2\right)^{3/2}}{a+b \operatorname{ArcCosh}[c x]} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m \left(1-c^2 x^2\right)^{3/2}}{a+b \operatorname{ArcCosh}[c x]}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{x^m (-1+c x)^{3/2} (1+c x)^{3/2}}{a+b \operatorname{ArcCosh}[c x]}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 311: Result valid but suboptimal antiderivative.

$$\int \frac{x^m \sqrt{1-c^2 x^2}}{a+b \operatorname{ArcCosh}[c x]} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m \sqrt{1-c^2 x^2}}{a+b \operatorname{ArcCosh}[c x]}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{x^m \sqrt{-1+c x}\ \sqrt{1+c x}}{a+b \operatorname{ArcCosh}[c x]}, x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 312: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\sqrt{1-c^2 x^2}\ (a+b \operatorname{ArcCosh}[c x])} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\sqrt{1-c^2 x^2}\ (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \text{Unintegrable}\left[\frac{x^m}{\sqrt{-1+c x}\ \sqrt{1+c x}\ (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 313: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\, \text{ArcCosh}[c\, x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\, \text{ArcCosh}[c\, x]\right)}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \text{Unintegrable}\left[\frac{x^m}{(-1+c\, x)^{3/2}\, (1+c\, x)^{3/2}\, (a+b\, \text{ArcCosh}[c\, x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\, \text{ArcCosh}[c\, x]\right)} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\left(1-c^2 x^2\right)^{5/2} \left(a+b\, \text{ArcCosh}[c\, x]\right)}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \text{Unintegrable}\left[\frac{x^m}{(-1+c\, x)^{5/2}\, (1+c\, x)^{5/2}\, (a+b\, \text{ArcCosh}[c\, x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 320: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \sqrt{1-c^2 x^2}}{\left(a+b\, \text{ArcCosh}[c\, x]\right)^2} \, dx$$

Optimal (type 4, 350 leaves, 22 steps):

$$-\frac{x^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\sqrt{1-c^2x^2}}{b\,c\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2c^4\sqrt{-1+c\,x}}-\frac{3\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b^2c^4\sqrt{-1+c\,x}}+\frac{3\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}}+\frac{5\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}}$$

Result (type 4, 429 leaves, 23 steps):

$$\frac{x^3\,(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2x^2}}{b\,c\,\sqrt{-1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{3\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b^2c^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int\frac{x^2\sqrt{1-c^2x^2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 4, 154 leaves, 16 steps):

$$-\frac{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^3\,\sqrt{-1+c\,x}}+\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b^2\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 185 leaves, 17 steps):

$$\frac{x^2\,(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{2\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\sqrt{1-c^2\,x^2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 248 leaves, 14 steps):

$$-\frac{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}}-\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}}+\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 418 leaves, 15 steps):

$$\frac{x\,(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{3\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2\,x^2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 177 leaves, 8 steps):

$$\frac{(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{1-c^2\,x^2}}{x^2\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 97 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}\ \sqrt{1+c x}\ \sqrt{1-c^2 x^2}}{b c x^2\ (a+b\,\text{ArcCosh}[c x])}+\frac{2\sqrt{1-c x}\ \text{Unintegrable}\left[\frac{1}{x^3\ (a+b\,\text{ArcCosh}[c x])},x\right]}{b c \sqrt{-1+c x}}$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{(1-c x)\ \sqrt{1+c x}\ \sqrt{1-c^2 x^2}}{b c x^2 \sqrt{-1+c x}\ (a+b\,\text{ArcCosh}[c x])}+\frac{2\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{1}{x^3\ (a+b\,\text{ArcCosh}[c x])},x\right]}{b c \sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^3\ (a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^3\ (a+b\,\text{ArcCosh}[c x])^2},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c x}\ \sqrt{1+c x}}{x^3\ (a+b\,\text{ArcCosh}[c x])^2},x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^4\ (a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4\ (a+b\,\text{ArcCosh}[c x])^2},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{\sqrt{-1+c x}\ \sqrt{1+c x}}{x^4\ (a+b\,\text{ArcCosh}[c x])^2},x\right]}{\sqrt{-1+c x}\ \sqrt{1+c x}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int \frac{x^2\ (1-c^2 x^2)^{3/2}}{(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 4, 354 leaves, 21 steps):

$$-\frac{x^2\sqrt{-1+c x}\,\sqrt{1+c x}\,\left(1-c^2x^2\right)^{3/2}}{b c\,(a+b\,\text{ArcCosh}[c x])}-\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\,\text{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}}-\frac{\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\,\text{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c^3\sqrt{-1+c x}}+\frac{3\sqrt{1-c x}\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\,\text{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}}+\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{2a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}}+\frac{\sqrt{1-c x}\,\text{Cosh}\left[\frac{4a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{4 b^2 c^3\sqrt{-1+c x}}-\frac{3\sqrt{1-c x}\,\text{Cosh}\left[\frac{6a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c x])}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}}$$

Result (type 4, 439 leaves, 20 steps):

$$\frac{x^2\,(1-c x)^2\,(1+c x)^{3/2}\sqrt{1-c^2x^2}}{b c\sqrt{-1+c x}\,(a+b\,\text{ArcCosh}[c x])}-\frac{\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c x]\right]\,\text{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c x]\right]\,\text{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{3\sqrt{1-c^2x^2}\,\text{CoshIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c x]\right]\,\text{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{2a}{b}\right]\,\text{SinhIntegral}\left[\frac{2a}{b}+2\,\text{ArcCosh}[c x]\right]}{16 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}+\frac{\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{4a}{b}\right]\,\text{SinhIntegral}\left[\frac{4a}{b}+4\,\text{ArcCosh}[c x]\right]}{4 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}-\frac{3\sqrt{1-c^2x^2}\,\text{Cosh}\left[\frac{6a}{b}\right]\,\text{SinhIntegral}\left[\frac{6a}{b}+6\,\text{ArcCosh}[c x]\right]}{16 b^2 c^3\sqrt{-1+c x}\,\sqrt{1+c x}}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\left(1-c^2x^2\right)^{3/2}}{(a+b\,\text{ArcCosh}[c x])^2}\,dx$$

Optimal (type 4, 348 leaves, 24 steps):

$$-\frac{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{3/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^2\,\sqrt{-1+c\,x}}-\frac{9\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}}+\frac{5\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}}-\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b^2\,c^2\,\sqrt{-1+c\,x}}+\frac{9\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}}-\frac{5\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 429 leaves, 23 steps):

$$\frac{x\,(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{9\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{5\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{9\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{5\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2\,x^2\right)^{3/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 246 leaves, 11 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{3/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b^2\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 305 leaves, 11 steps):

$$\frac{(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{2\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 332: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2\,x^2\right)^{3/2}}{x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 156 leaves, 3 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{3/2}}{b\,c\,x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{2\,\sqrt{1-c\,x}\,\text{Unintegrable}\left[\frac{-1+c^2\,x^2}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{2\,c\,\sqrt{1-c\,x}\,\text{Unintegrable}\left[\frac{-1+c^2\,x^2}{x\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{-1+c\,x}}$$

Result (type 8, 189 leaves, 2 steps):

$$\frac{(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,x^2\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{2\,\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{-1+c^2\,x^2}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{2\,c\,\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{-1+c^2\,x^2}{x\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 333: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2 x^2\right)^{3/2}}{x^3 \left(a+b \operatorname{ArcCosh}[c x]\right)^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{(-1+c x)^{3/2} (1+c x)^{3/2}}{x^3 (a+b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 334: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{\sqrt{-1+c x} \sqrt{1+c x} \left(1-c^2 x^2\right)^{3/2}}{b c x^4 \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{4 \sqrt{1-c x}\ \text{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x}}$$

Result (type 8, 126 leaves, 2 steps):

$$\frac{(1-c x)^2 (1+c x)^{3/2} \sqrt{1-c^2 x^2}}{b c x^4 \sqrt{-1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)}-\frac{4 \sqrt{1-c^2 x^2}\ \text{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(1-c^2 x^2\right)^{5/2}}{\left(a+b \operatorname{ArcCosh}[c x]\right)^2} \, dx$$

Optimal (type 4, 454 leaves, 30 steps):

$$-\frac{x^2 \sqrt{-1+c\,x}\;\sqrt{1+c\,x}\;\left(1-c^2\,x^2\right)^{5/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c\,x}\;\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}-$$

$$\frac{\sqrt{1-c\,x}\;\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{3\,\sqrt{1-c\,x}\;\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}-$$

$$\frac{\sqrt{1-c\,x}\;\text{CoshIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{8\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}}-$$

$$\frac{3\,\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}+$$

$$\frac{\sqrt{1-c\,x}\;\text{Cosh}\left[\frac{8\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{8\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}}$$

Result (type 4, 565 leaves, 29 steps) :

$$\frac{x^2\,(1-c\,x)^3\,(1+c\,x)^{5/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{8\,a}{b}+8\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{8\,a}{b}\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{8\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{8\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{8\,a}{b}+8\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 336: Result valid but suboptimal antiderivative.

$$\int \frac{x\,\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 448 leaves, 30 steps) :

$$-\frac{x\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{5/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{5\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}-$$

$$\frac{27\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}+$$

$$\frac{25\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}-$$

$$\frac{7\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{7\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}-$$

$$\frac{5\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}+$$

$$\frac{27\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}-$$

$$\frac{25\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}+$$

$$\frac{7\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{7\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{7\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}}$$

Result (type 4, 555 leaves, 29 steps):

$$\frac{x\,(1-c\,x)^3\,(1+c\,x)^{5/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{5\,\sqrt{1-c^2\,x^2}\,\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}-$$

$$\frac{27\,\sqrt{1-c^2\,x^2}\,\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}+$$

$$\frac{25\,\sqrt{1-c^2\,x^2}\,\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{5\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}-$$

$$\frac{7\,\sqrt{1-c^2\,x^2}\,\,\text{CoshIntegral}\left[\frac{7\,a}{b}+7\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{7\,a}{b}\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}-$$

$$\frac{5\,\sqrt{1-c^2\,x^2}\,\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}+$$

$$\frac{27\,\sqrt{1-c^2\,x^2}\,\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}-$$

$$\frac{25\,\sqrt{1-c^2\,x^2}\,\,\text{Cosh}\left[\frac{5\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}+$$

$$\frac{7\,\sqrt{1-c^2\,x^2}\,\,\text{Cosh}\left[\frac{7\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{7\,a}{b}+7\,\text{ArcCosh}[c\,x]\right]}{64\,b^2\,c^2\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}$$

Problem 337: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 4, 351 leaves, 14 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{5/2}}{b\,c\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{15\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{CoshIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{15\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c\,x}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}}$$

Result (type 4, 436 leaves, 14 steps):

$$\frac{(1-c\,x)^3\,(1+c\,x)^{5/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{15\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{CoshIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{6\,a}{b}\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{15\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{4\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{3\,\sqrt{1-c^2\,x^2}\,\text{Cosh}\left[\frac{6\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{6\,a}{b}+6\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 339: Result valid but suboptimal antiderivative.

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 160 leaves, 3 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(1-c^2\,x^2\right)^{5/2}}{b\,c\,x^2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\,\sqrt{1-c\,x}\,\text{Unintegrable}\left[\frac{\left(-1+c^2\,x^2\right)^2}{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,c\,\sqrt{-1+c\,x}}+$$
$$\frac{4\,c\,\sqrt{1-c\,x}\,\text{Unintegrable}\left[\frac{\left(-1+c^2\,x^2\right)^2}{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,\sqrt{-1+c\,x}}$$

Result (type 8, 193 leaves, 2 steps):

$$\frac{\left(1-c\,x\right)^3\,\left(1+c\,x\right)^{5/2}\,\sqrt{1-c^2\,x^2}}{b\,c\,x^2\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\,\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{\left(-1+c^2\,x^2\right)^2}{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{4\,c\,\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{\left(-1+c^2\,x^2\right)^2}{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)},\,x\right]}{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2\,x^2\right)^{5/2}}{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2\,x^2\right)^{5/2}}{x^3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}}{x^3\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int\frac{\left(1-c^2\,x^2\right)^{5/2}}{x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(1-c^2\,x^2\right)^{5/2}}{x^4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2\,x^2}\,\text{Unintegrable}\left[\frac{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}}{x^4\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 337 leaves, 13 steps):

$$-\frac{x^5\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{5\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^6\sqrt{1-c\,x}}-\frac{15\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^6\sqrt{1-c\,x}}-\frac{5\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^6\sqrt{1-c\,x}}+\frac{5\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{8\,b^2\,c^6\sqrt{1-c\,x}}+\frac{15\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^6\sqrt{1-c\,x}}+\frac{5\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{16\,b^2\,c^6\sqrt{1-c\,x}}$$

Result (type 4, 424 leaves, 14 steps):

$$-\frac{x^5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^2\,c^6\sqrt{1-c^2 x^2}}-\frac{15\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{3\,a}{b}\right]}{16\,b^2\,c^6\sqrt{1-c^2 x^2}}-\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]\text{Sinh}\left[\frac{5\,a}{b}\right]}{16\,b^2\,c^6\sqrt{1-c^2 x^2}}+\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c\,x]\right]}{8\,b^2\,c^6\sqrt{1-c^2 x^2}}+\frac{15\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{3\,a}{b}\right]\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^6\sqrt{1-c^2 x^2}}+\frac{5\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{5\,a}{b}\right]\text{SinhIntegral}\left[\frac{5\,a}{b}+5\,\text{ArcCosh}[c\,x]\right]}{16\,b^2\,c^6\sqrt{1-c^2 x^2}}$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 236 leaves, 10 steps):

$$-\frac{x^4\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^5\,\sqrt{1-c\,x}}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{b^2\,c^5\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c\,x}}$$

Result (type 4, 301 leaves, 11 steps):

$$-\frac{x^4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^5\,\sqrt{1-c^2\,x^2}}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{4\,a}{b}\right]}{2\,b^2\,c^5\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c^5\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{4\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{4\,a}{b}+4\,\text{ArcCosh}[c\,x]\right]}{2\,b^2\,c^5\,\sqrt{1-c^2\,x^2}}$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 237 leaves, 10 steps):

$$-\frac{x^3 \sqrt{-1+c x}}{b c \sqrt{1-c x}\,(a+b \text{ArcCosh}[c x])}-\frac{3 \sqrt{-1+c x}\, \text{CoshIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right] \text{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^4 \sqrt{1-c x}}-$$
$$\frac{3 \sqrt{-1+c x}\, \text{CoshIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{3 a}{b}\right]}{4 b^2 c^4 \sqrt{1-c x}}+$$
$$\frac{3 \sqrt{-1+c x}\, \text{Cosh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a+b \text{ArcCosh}[c x]}{b}\right]}{4 b^2 c^4 \sqrt{1-c x}}+$$
$$\frac{3 \sqrt{-1+c x}\, \text{Cosh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 (a+b \text{ArcCosh}[c x])}{b}\right]}{4 b^2 c^4 \sqrt{1-c x}}$$

Result (type 4, 298 leaves, 11 steps):

$$-\frac{x^3 \sqrt{-1+c x}\, \sqrt{1+c x}}{b c \sqrt{1-c^2 x^2}\,(a+b \text{ArcCosh}[c x])}-$$
$$\frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^4 \sqrt{1-c^2 x^2}}-$$
$$\frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{CoshIntegral}\left[\frac{3 a}{b}+3 \text{ArcCosh}[c x]\right] \text{Sinh}\left[\frac{3 a}{b}\right]}{4 b^2 c^4 \sqrt{1-c^2 x^2}}+$$
$$\frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Cosh}\left[\frac{a}{b}\right] \text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c x]\right]}{4 b^2 c^4 \sqrt{1-c^2 x^2}}+$$
$$\frac{3 \sqrt{-1+c x}\, \sqrt{1+c x}\, \text{Cosh}\left[\frac{3 a}{b}\right] \text{SinhIntegral}\left[\frac{3 a}{b}+3 \text{ArcCosh}[c x]\right]}{4 b^2 c^4 \sqrt{1-c^2 x^2}}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-c^2 x^2}\,(a+b \text{ArcCosh}[c x])^2}\, dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{x^2 \sqrt{-1+c x}}{b c \sqrt{1-c x}\,(a+b \text{ArcCosh}[c x])}-\frac{\sqrt{-1+c x}\, \text{CoshIntegral}\left[\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right] \text{Sinh}\left[\frac{2 a}{b}\right]}{b^2 c^3 \sqrt{1-c x}}+$$
$$\frac{\sqrt{-1+c x}\, \text{Cosh}\left[\frac{2 a}{b}\right] \text{SinhIntegral}\left[\frac{2 (a+b \text{ArcCosh}[c x])}{b}\right]}{b^2 c^3 \sqrt{1-c x}}$$

Result (type 4, 175 leaves, 8 steps):

$$-\frac{x^2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]\,\text{Sinh}\left[\frac{2\,a}{b}\right]}{b^2\,c^3\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{2\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{2\,a}{b}+2\,\text{ArcCosh}[c\,x]\right]}{b^2\,c^3\,\sqrt{1-c^2\,x^2}}$$

Problem 346: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{x\,\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^2\,c^2\,\sqrt{1-c\,x}}+\frac{\sqrt{-1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b^2\,c^2\,\sqrt{1-c\,x}}$$

Result (type 4, 169 leaves, 6 steps):

$$-\frac{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^2\,c^2\,\sqrt{1-c^2\,x^2}}+\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]}{b^2\,c^2\,\sqrt{1-c^2\,x^2}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 3, 37 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}$$

Result (type 3, 50 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,x\sqrt{1-c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{-1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x^2\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\sqrt{1-c\,x}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,x\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x^2\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\sqrt{1-c^2 x^2}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,x^2\sqrt{1-c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{2\sqrt{-1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\sqrt{1-c\,x}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,x^2\sqrt{1-c^2 x^2}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{1}{x^3\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\sqrt{1-c^2 x^2}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(1-c^2 x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^3}{(1-c^2 x^2)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2 x^2}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\left[\frac{x^3}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2} \, dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{x^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2 x^2\right)^2 \left(a+b\,\text{ArcCosh}[c\,x]\right)},x\right]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 8, 127 leaves, 2 steps):

$$-\frac{x^2 \sqrt{-1+c\,x}}{b\,c\,(1-c\,x)\sqrt{1+c\,x}\,\sqrt{1-c^2 x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2 x^2\right)^2 \left(a+b\,\text{ArcCosh}[c\,x]\right)},x\right]}{b\,c\,\sqrt{1-c^2 x^2}}$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2} \, dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2},x\right]$$

Result (type 8, 66 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2 x^2}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2},x\right]$$

Problem 353: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)^2} \, dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2 x^2\right)^{3/2} \left(a+b\,\text{ArcCosh}[c\,x]\right)}+\frac{2\,c\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2 x^2\right)^2 \left(a+b\,\text{ArcCosh}[c\,x]\right)},x\right]}{b\,\sqrt{1-c\,x}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,(1-c\,x)\,\sqrt{1+c\,x}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{2\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^2\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,\sqrt{1-c^2\,x^2}}$$

Problem 354: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,\left(1-c^2\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\,\left(1-c^2\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2\,x^2}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Problem 355: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\,\left(1-c^2\,x^2\right)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2\,x^2}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x^2\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 108 leaves, 2 steps):

$$-\frac{x^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{4\,\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x^3}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 8, 129 leaves, 2 steps):

$$-\frac{x^4\sqrt{-1+c\,x}}{b\,c\,(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}-\frac{4\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^3}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},x\right]}{b\,c\,\sqrt{1-c^2\,x^2}}$$

Problem 357: Result valid but suboptimal antiderivative.

$$\int\frac{x^3}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^3}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^3}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int\frac{x^2}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x^2}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2}\,dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{\left(1-c^2\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^2},x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])}-\frac{4\,c\,\sqrt{-1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{1-c\,x}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1+c\,x}}{b\,c\,(1-c\,x)^2\,(1+c\,x)^{3/2}\,\sqrt{1-c^2\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}-$$
$$\frac{4\,c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+c^2\,x^2\right)^3\,(a+b\,\text{ArcCosh}[c\,x])},\,x\right]}{b\,\sqrt{1-c^2\,x^2}}$$

Problem 361: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{1}{\sqrt{1-c^2\,x^2}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\text{Unintegrable}\left[\frac{1}{x\,(-1+c\,x)^{5/2}\,(1+c\,x)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]$$

Problem 362: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2\,\left(1-c^2\,x^2\right)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{1}{\sqrt{1-c^2x^2}}\sqrt{-1+cx}\,\sqrt{1+cx}\;\text{Unintegrable}\Big[\frac{1}{x^2\,(-1+cx)^{5/2}\,(1+cx)^{5/2}\,(a+b\,\text{ArcCosh}[c\,x])^2},\,x\Big]$$

Problem 363: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,(1-c^2x^2)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\Big[\frac{(f\,x)^m\,(1-c^2x^2)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2},\,x\Big]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{1-c^2x^2}\;\text{Unintegrable}\Big[\frac{(f\,x)^m\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}}{(a+b\,\text{ArcCosh}[c\,x])^2},\,x\Big]}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m\,\sqrt{1-c^2x^2}}{(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\Big[\frac{(f\,x)^m\,\sqrt{1-c^2x^2}}{(a+b\,\text{ArcCosh}[c\,x])^2},\,x\Big]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{1-c^2x^2}\;\text{Unintegrable}\Big[\frac{(f\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{(a+b\,\text{ArcCosh}[c\,x])^2},\,x\Big]}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int\frac{(f\,x)^m}{\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}\,dx$$

Optimal (type 8, 91 leaves, 1 step):

$$-\frac{(f\,x)^m\,\sqrt{-1+c\,x}}{b\,c\,\sqrt{1-c\,x}\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{f\,m\,\sqrt{-1+c\,x}\;\text{Unintegrable}\Big[\frac{(f\,x)^{-1+m}}{a+b\,\text{ArcCosh}[c\,x]},\,x\Big]}{b\,c\,\sqrt{1-c\,x}}$$

Result (type 8, 117 leaves, 2 steps):

$$-\frac{(f\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{b\,c\,\sqrt{1-c^2x^2}\,(a+b\,\text{ArcCosh}[c\,x])}+\frac{f\,m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\;\text{Unintegrable}\Big[\frac{(f\,x)^{-1+m}}{a+b\,\text{ArcCosh}[c\,x]},\,x\Big]}{b\,c\,\sqrt{1-c^2x^2}}$$

Problem 366: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m}{\left(1-c^2 x^2\right)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{1}{\sqrt{1-c^2 x^2}} \sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{(f x)^m}{(-1+c x)^{3/2} (1+c x)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Problem 367: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{\left(1-c^2 x^2\right)^{5/2} (a+b \operatorname{ArcCosh}[c x])^2} \, dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m}{\left(1-c^2 x^2\right)^{5/2} (a+b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{ Unintegrable}\left[\frac{(f x)^m}{(-1+c x)^{5/2} (1+c x)^{5/2} (a+b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 368: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3} \, dx$$

Optimal (type 3, 32 leaves, 1 step):

$$-\frac{\sqrt{-1+a x}}{2 a \sqrt{1-a x} \operatorname{ArcCosh}[a x]^2}$$

Result (type 3, 45 leaves, 2 steps):

$$-\frac{\sqrt{-1+a x} \sqrt{1+a x}}{2 a \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}$$

Problem 380: Result optimal but 1 more steps used.

$$\int \sqrt{c-a^2 c x^2} \sqrt{\operatorname{ArcCosh}[a x]} \, dx$$

Optimal (type 4, 205 leaves, 10 steps):

$$\frac{1}{2} x \sqrt{c-a^2 c x^2} \sqrt{\text{ArcCosh}[a x]}-\frac{\sqrt{c-a^2 c x^2} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{\sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{\sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Result (type 4, 205 leaves, 11 steps):

$$\frac{1}{2} x \sqrt{c-a^2 c x^2} \sqrt{\text{ArcCosh}[a x]}-\frac{\sqrt{c-a^2 c x^2} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{\sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{\sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{16 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Problem 381: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}[a x]}}{\sqrt{c-a^2 c x^2}} \, dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{c-a^2 c x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^{3/2}}{3 a \sqrt{c-a^2 c x^2}}$$

Problem 382: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}[a x]}}{\left(c-a^2 c x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x \sqrt{\text{ArcCosh}[a x]}}{c \sqrt{c-a^2 c x^2}}+\frac{a \sqrt{-1+a x} \sqrt{1+a x} \text{Unintegrable}\left[\frac{x}{\left(1-a^2 x^2\right) \sqrt{\text{ArcCosh}[a x]}}, x\right]}{2 c \sqrt{c-a^2 c x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x \sqrt{\text{ArcCosh}[a x]}}{c \sqrt{c-a^2 c x^2}}+\frac{a \sqrt{-1+a x} \sqrt{1+a x} \text{Unintegrable}\left[\frac{x}{\left(1-a^2 x^2\right) \sqrt{\text{ArcCosh}[a x]}}, x\right]}{2 c \sqrt{c-a^2 c x^2}}$$

Problem 385: Result optimal but 1 more steps used.

$$\int \sqrt{c-a^2 c x^2}\ \text{ArcCosh}[a x]^{3/2}\, dx$$

Optimal (type 4, 302 leaves, 11 steps):

$$\frac{3\sqrt{c-a^2 c x^2}\ \sqrt{\text{ArcCosh}[a x]}}{16 a\sqrt{-1+a x}\ \sqrt{1+a x}}-\frac{3 a x^2\sqrt{c-a^2 c x^2}\ \sqrt{\text{ArcCosh}[a x]}}{8\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{1}{2} x\sqrt{c-a^2 c x^2}\ \text{ArcCosh}[a x]^{3/2}-\frac{\sqrt{c-a^2 c x^2}\ \text{ArcCosh}[a x]^{5/2}}{5 a\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{3\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2 c x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a x]}\right]}{64 a\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{3\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2 c x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a x]}\right]}{64 a\sqrt{-1+a x}\ \sqrt{1+a x}}$$

Result (type 4, 302 leaves, 12 steps):

$$\frac{3\sqrt{c-a^2 c x^2}\ \sqrt{\text{ArcCosh}[a x]}}{16 a\sqrt{-1+a x}\ \sqrt{1+a x}}-\frac{3 a x^2\sqrt{c-a^2 c x^2}\ \sqrt{\text{ArcCosh}[a x]}}{8\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{1}{2} x\sqrt{c-a^2 c x^2}\ \text{ArcCosh}[a x]^{3/2}-\frac{\sqrt{c-a^2 c x^2}\ \text{ArcCosh}[a x]^{5/2}}{5 a\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{3\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2 c x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a x]}\right]}{64 a\sqrt{-1+a x}\ \sqrt{1+a x}}+\frac{3\sqrt{\frac{\pi}{2}}\ \sqrt{c-a^2 c x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}[a x]}\right]}{64 a\sqrt{-1+a x}\ \sqrt{1+a x}}$$

Problem 386: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^{3/2}}{\sqrt{c-a^2 c x^2}}\, dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{5/2}}{5 a\sqrt{c-a^2 c x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{5/2}}{5 a\sqrt{c-a^2 c x^2}}$$

Problem 387: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^{3/2}}{\left(c-a^2 c x^2\right)^{3/2}}\, dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x\,\text{ArcCosh}[a\,x]^{3/2}}{c\,\sqrt{c-a^2\,c\,x^2}}+\frac{3\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}[a\,x]}}{1-a^2\,x^2},x\right]}{2\,c\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x\,\text{ArcCosh}[a\,x]^{3/2}}{c\,\sqrt{c-a^2\,c\,x^2}}+\frac{3\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}[a\,x]}}{1-a^2\,x^2},x\right]}{2\,c\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 389: Result optimal but 1 more steps used.

$$\int\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}\,dx$$

Optimal (type 4, 330 leaves, 13 steps):

$$\frac{15}{32}\,x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}+\frac{5\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{16\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{5\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{7/2}}{7\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 330 leaves, 14 steps):

$$\frac{15}{32}\,x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}+\frac{5\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{16\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{5\,a\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{3/2}}{8\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{1}{2}\,x\,\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{5/2}-\frac{\sqrt{c-a^2\,c\,x^2}\,\text{ArcCosh}[a\,x]^{7/2}}{7\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{15\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{256\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 390: Result optimal but 1 more steps used.

$$\int\frac{\text{ArcCosh}[a\,x]^{5/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Optimal (type 3, 48 leaves, 1 step):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{7/2}}{7 a\sqrt{c-a^2 c x^2}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{2\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{ArcCosh}[a x]^{7/2}}{7 a\sqrt{c-a^2 c x^2}}$$

Problem 391: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}[a x]^{5/2}}{\left(c-a^2 c x^2\right)^{3/2}}\, dx$$

Optimal (type 8, 94 leaves, 1 step):

$$\frac{x\,\text{ArcCosh}[a x]^{5/2}}{c\sqrt{c-a^2 c x^2}}+\frac{5 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Unintegrable}\left[\frac{x\,\text{ArcCosh}[a x]^{3/2}}{1-a^2 x^2},x\right]}{2 c\sqrt{c-a^2 c x^2}}$$

Result (type 8, 94 leaves, 2 steps):

$$\frac{x\,\text{ArcCosh}[a x]^{5/2}}{c\sqrt{c-a^2 c x^2}}+\frac{5 a\sqrt{-1+a x}\ \sqrt{1+a x}\ \text{Unintegrable}\left[\frac{x\,\text{ArcCosh}[a x]^{3/2}}{1-a^2 x^2},x\right]}{2 c\sqrt{c-a^2 c x^2}}$$

Problem 393: Result optimal but 1 more steps used.

$$\int \sqrt{a^2-x^2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\, dx$$

Optimal (type 4, 211 leaves, 10 steps):

$$\frac{1}{2} x\sqrt{a^2-x^2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}-\frac{a\sqrt{a^2-x^2}\ \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}+$$

$$\frac{a\sqrt{\frac{\pi}{2}}\ \sqrt{a^2-x^2}\ \text{Erf}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\ \right]}{16\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}-\frac{a\sqrt{\frac{\pi}{2}}\ \sqrt{a^2-x^2}\ \text{Erfi}\left[\sqrt{2}\ \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\ \right]}{16\sqrt{-1+\frac{x}{a}}\ \sqrt{1+\frac{x}{a}}}$$

Result (type 4, 211 leaves, 11 steps):

$$\frac{1}{2} x \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}-\frac{a \sqrt{a^2-x^2} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+$$

$$\frac{a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}-\frac{a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}$$

Problem 394: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{\sqrt{a^2-x^2}} \, dx$$

Optimal (type 3, 50 leaves, 1 step):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2-x^2}}$$

Result (type 3, 50 leaves, 2 steps):

$$\frac{2 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2-x^2}}$$

Problem 395: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{\left(a^2-x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 97 leaves, 1 step):

$$\frac{x \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{a^2 \sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{Unintegrable}\left[\frac{x}{\left(1-\frac{x^2}{a^2}\right) \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}, x\right]}{2 a^3 \sqrt{a^2-x^2}}$$

Result (type 8, 97 leaves, 2 steps):

$$\frac{x \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{a^2 \sqrt{a^2-x^2}}+\frac{\sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}} \text{Unintegrable}\left[\frac{x}{\left(1-\frac{x^2}{a^2}\right) \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}, x\right]}{2 a^3 \sqrt{a^2-x^2}}$$

Problem 398: Result optimal but 1 more steps used.

$$\int \sqrt{a^2-x^2}\, \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2} dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$\frac{3 a \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}-\frac{3 x^2 \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{8 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{1}{2} x \sqrt{a^2-x^2}\, \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}-\frac{a \sqrt{a^2-x^2}\, \text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{3 a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2}\, \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{3 a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2}\, \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}$$

Result (type 4, 316 leaves, 12 steps):

$$\frac{3 a \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{16 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}-\frac{3 x^2 \sqrt{a^2-x^2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{8 a \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{1}{2} x \sqrt{a^2-x^2}\, \text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}-\frac{a \sqrt{a^2-x^2}\, \text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{3 a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2}\, \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}+\frac{3 a \sqrt{\frac{\pi}{2}} \sqrt{a^2-x^2}\, \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}\right]}{64 \sqrt{-1+\frac{x}{a}} \sqrt{1+\frac{x}{a}}}$$

Problem 399: Result optimal but 1 more steps used.

$$\int \frac{\text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2-x^2}} dx$$

Optimal (type 3, 50 leaves, 1 step):

$$\frac{2\,a\,\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2-x^2}}$$

Result (type 3, 50 leaves, 2 steps):

$$\frac{2\,a\,\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\text{ArcCosh}\left[\frac{x}{a}\right]^{5/2}}{5\,\sqrt{a^2-x^2}}$$

Problem 400: Result optimal but 1 more steps used.

$$\int\frac{\text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 97 leaves, 1 step):

$$\frac{x\,\text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{a^2\,\sqrt{a^2-x^2}}+\frac{3\,\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{1-\frac{x^2}{a^2}},x\right]}{2\,a^3\,\sqrt{a^2-x^2}}$$

Result (type 8, 97 leaves, 2 steps):

$$\frac{x\,\text{ArcCosh}\left[\frac{x}{a}\right]^{3/2}}{a^2\,\sqrt{a^2-x^2}}+\frac{3\,\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\text{Unintegrable}\left[\frac{x\,\sqrt{\text{ArcCosh}\left[\frac{x}{a}\right]}}{1-\frac{x^2}{a^2}},x\right]}{2\,a^3\,\sqrt{a^2-x^2}}$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int\frac{x}{\sqrt{1-x^2}\,\sqrt{\text{ArcCosh}[x]}}\,dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$\frac{\sqrt{\pi}\,\sqrt{-1+x}\,\text{Erf}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\,\sqrt{1-x}}+\frac{\sqrt{\pi}\,\sqrt{-1+x}\,\text{Erfi}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\,\sqrt{1-x}}$$

Result (type 4, 83 leaves, 7 steps):

$$\frac{\sqrt{\pi}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{Erf}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\,\sqrt{1-x^2}}+\frac{\sqrt{\pi}\,\sqrt{-1+x}\,\sqrt{1+x}\,\text{Erfi}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2\,\sqrt{1-x^2}}$$

Problem 402: Result optimal but 1 more steps used.

$$\int\frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 4, 438 leaves, 18 steps):

$$-\frac{5 c^2 \sqrt{c-a^2 c x^2} \sqrt{\text{ArcCosh}[a x]}}{8 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 c^2 \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{6} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 c^2 \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{6} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Result (type 4, 438 leaves, 19 steps):

$$-\frac{5 c^2 \sqrt{c-a^2 c x^2} \sqrt{\text{ArcCosh}[a x]}}{8 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 c^2 \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{6} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{3 c^2 \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{6} \sqrt{\text{ArcCosh}[a x]}\right]}{64 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Problem 403: Result optimal but 1 more steps used.

$$\int \frac{\left(c-a^2 c x^2\right)^{3/2}}{\sqrt{\text{ArcCosh}[a x]}} \, dx$$

Optimal (type 4, 294 leaves, 13 steps):

$$-\frac{3 c \sqrt{c-a^2 c x^2} \sqrt{\text{ArcCosh}[a x]}}{4 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{c \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{32 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{4 a \sqrt{-1+a x} \sqrt{1+a x}}-\frac{c \sqrt{\pi} \sqrt{c-a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcCosh}[a x]}\right]}{32 a \sqrt{-1+a x} \sqrt{1+a x}}+\frac{c \sqrt{\frac{\pi}{2}} \sqrt{c-a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{4 a \sqrt{-1+a x} \sqrt{1+a x}}$$

Result (type 4, 294 leaves, 14 steps):

$$-\frac{3\,c\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}-\frac{c\,\sqrt{\pi}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[2\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{32\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{c\,\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 404: Result optimal but 1 more steps used.

$$\int\frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 4, 175 leaves, 8 steps):

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Result (type 4, 175 leaves, 9 steps):

$$-\frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a\,x]}\right]}{4\,a\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}}$$

Problem 405: Result optimal but 1 more steps used.

$$\int\frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\text{ArcCosh}[a\,x]}}\,dx$$

Optimal (type 3, 46 leaves, 1 step):

$$\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{c-a^2\,c\,x^2}}$$

Result (type 3, 46 leaves, 2 steps):

$$\frac{2\,\sqrt{-1+a\,x}\,\sqrt{1+a\,x}\,\sqrt{\text{ArcCosh}[a\,x]}}{a\,\sqrt{c-a^2\,c\,x^2}}$$

Problem 406: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c - a^2 c x^2\right)^{3/2} \sqrt{\text{ArcCosh}[a x]}} \, dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(c - a^2 c x^2\right)^{3/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$-\frac{\sqrt{-1 + a x} \sqrt{1 + a x} \, \text{Unintegrable}\left[\frac{1}{(-1+a x)^{3/2} (1+a x)^{3/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]}{c \sqrt{c - a^2 c x^2}}$$

Problem 407: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(c - a^2 c x^2\right)^{5/2} \sqrt{\text{ArcCosh}[a x]}} \, dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(c - a^2 c x^2\right)^{5/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]$$

Result (type 8, 66 leaves, 1 step):

$$\frac{\sqrt{-1 + a x} \sqrt{1 + a x} \, \text{Unintegrable}\left[\frac{1}{(-1+a x)^{5/2} (1+a x)^{5/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]}{c^2 \sqrt{c - a^2 c x^2}}$$

Problem 410: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{c - a^2 c x^2}}{\text{ArcCosh}[a x]^{3/2}} \, dx$$

Optimal (type 4, 170 leaves, 9 steps):

$$-\frac{2 \sqrt{-1 + a x} \sqrt{1 + a x} \sqrt{c - a^2 c x^2}}{a \sqrt{\text{ArcCosh}[a x]}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \, \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{a \sqrt{-1 + a x} \sqrt{1 + a x}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c - a^2 c x^2} \, \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcCosh}[a x]}\right]}{a \sqrt{-1 + a x} \sqrt{1 + a x}}$$

Result (type 4, 176 leaves, 10 steps):

$$\frac{2\left(1-a x\right)\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}}{a\sqrt{-1+a x}\,\sqrt{\text{ArcCosh}[a x]}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{c-a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{a\sqrt{-1+a x}\,\sqrt{1+a x}}$$

Problem 411: Result optimal but 1 more steps used.

$$\int\frac{1}{\sqrt{c-a^2 c x^2}\,\text{ArcCosh}[a x]^{3/2}}\,dx$$

Optimal (type 3, 46 leaves, 1 step):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}$$

Result (type 3, 46 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}$$

Problem 412: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(c-a^2 c x^2\right)^{3/2}\,\text{ArcCosh}[a x]^{3/2}}\,dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\left(c-a^2 c x^2\right)^{3/2}\sqrt{\text{ArcCosh}[a x]}}+\frac{4 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^2\sqrt{\text{ArcCosh}[a x]}},x\right]}{c\sqrt{c-a^2 c x^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}}{a c\left(1-a x\right)\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}+\frac{4 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^2\sqrt{\text{ArcCosh}[a x]}},x\right]}{c\sqrt{c-a^2 c x^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(c-a^2 c x^2\right)^{5/2}\,\text{ArcCosh}[a x]^{3/2}}\,dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{a\left(c-a^2 c x^2\right)^{5/2}\sqrt{\text{ArcCosh}[a x]}}-\frac{8 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^3\sqrt{\text{ArcCosh}[a x]}},x\right]}{c^2\sqrt{c-a^2 c x^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}}{a c^2 (1-a x)^2 (1+a x)^{3/2}\sqrt{c-a^2 c x^2}\,\sqrt{\text{ArcCosh}[a x]}}-\frac{8 a\sqrt{-1+a x}\,\sqrt{1+a x}\,\text{Unintegrable}\left[\frac{x}{\left(-1+a^2 x^2\right)^3\sqrt{\text{ArcCosh}[a x]}},x\right]}{c^2\sqrt{c-a^2 c x^2}}$$

Problem 415: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{c-a^2 c x^2}}{\text{ArcCosh}[a x]^{5/2}}\,dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}}{3 a\,\text{ArcCosh}[a x]^{3/2}}-\frac{8 x\sqrt{c-a^2 c x^2}}{3\sqrt{\text{ArcCosh}[a x]}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{3 a\sqrt{-1+a x}\,\sqrt{1+a x}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{3 a\sqrt{-1+a x}\,\sqrt{1+a x}}$$

Result (type 4, 207 leaves, 8 steps):

$$\frac{2 (1-a x)\sqrt{1+a x}\,\sqrt{c-a^2 c x^2}}{3 a\sqrt{-1+a x}\,\text{ArcCosh}[a x]^{3/2}}-\frac{8 x\sqrt{c-a^2 c x^2}}{3\sqrt{\text{ArcCosh}[a x]}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2 c x^2}\,\text{Erf}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{3 a\sqrt{-1+a x}\,\sqrt{1+a x}}+\frac{2\sqrt{2\pi}\,\sqrt{c-a^2 c x^2}\,\text{Erfi}\left[\sqrt{2}\,\sqrt{\text{ArcCosh}[a x]}\right]}{3 a\sqrt{-1+a x}\,\sqrt{1+a x}}$$

Problem 416: Result optimal but 1 more steps used.

$$\int\frac{1}{\sqrt{c-a^2 c x^2}\,\text{ArcCosh}[a x]^{5/2}}\,dx$$

Optimal (type 3, 48 leaves, 1 step):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{3 a\sqrt{c-a^2 c x^2}\,\text{ArcCosh}[a x]^{3/2}}$$

Result (type 3, 48 leaves, 2 steps):

$$-\frac{2\sqrt{-1+a x}\,\sqrt{1+a x}}{3 a\sqrt{c-a^2 c x^2}\,\text{ArcCosh}[a x]^{3/2}}$$

Problem 419: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \, dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^{1+n}}{8 b c^3 (1 + n) \sqrt{-1 + c x} \sqrt{1 + c x}} + \left(2^{-2 (3+n)} e^{-\frac{4 a}{b}} \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \left(-\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{4 (a + b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(c^3 \sqrt{-1 + c x} \sqrt{1 + c x}\right) - \left(2^{-2 (3+n)} e^{\frac{4 a}{b}} \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \left(\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{4 (a + b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(c^3 \sqrt{-1 + c x} \sqrt{1 + c x}\right)$$

Result (type 4, 253 leaves, 7 steps):

$$-\frac{\sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^{1+n}}{8 b c^3 (1 + n) \sqrt{-1 + c x} \sqrt{1 + c x}} + \left(2^{-2 (3+n)} e^{-\frac{4 a}{b}} \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \left(-\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{4 (a + b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(c^3 \sqrt{-1 + c x} \sqrt{1 + c x}\right) - \left(2^{-2 (3+n)} e^{\frac{4 a}{b}} \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \left(\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{4 (a + b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(c^3 \sqrt{-1 + c x} \sqrt{1 + c x}\right)$$

Problem 420: Result optimal but 1 more steps used.

$$\int x \sqrt{d - c^2 d x^2} \, (a + b \operatorname{ArcCosh}[c x])^n \, dx$$

Optimal (type 4, 379 leaves, 9 steps):

$$\left(3^{-1-n}\, e^{-\frac{3a}{b}} \sqrt{d - c^2 d x^2}\, (a + b \,\text{ArcCosh}[c x])^n \left(-\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{3 (a + b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) - \left(e^{-\frac{a}{b}} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(-\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{a + b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) + \left(e^{a/b} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{a + b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) - \left(3^{-1-n}\, e^{\frac{3a}{b}} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{3 (a + b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right)$$

Result (type 4, 379 leaves, 10 steps):

$$\left(3^{-1-n}\, e^{-\frac{3a}{b}} \sqrt{d - c^2 d x^2}\, (a + b \,\text{ArcCosh}[c x])^n \left(-\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{3 (a + b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) - \left(e^{-\frac{a}{b}} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(-\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{a + b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) + \left(e^{a/b} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{a + b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right) - \left(3^{-1-n}\, e^{\frac{3a}{b}} \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \left(\frac{a + b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{3 (a + b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(8 c^2 \sqrt{-1 + c x}\, \sqrt{1 + c x}\right)$$

Problem 421: Result optimal but 1 more steps used.

$$\int \sqrt{d - c^2 d x^2}\, (a + b\,\text{ArcCosh}[c x])^n \, dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\left(2^{-3-n}\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$
$$\left(2^{-3-n}\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 4, 253 leaves, 7 steps):

$$-\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^{1+n}}{2\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\left(2^{-3-n}\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$
$$\left(2^{-3-n}\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 422: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{d-c^2 d x^2}\ (a+b\,\text{ArcCosh}[c\,x])^n}{x}\,dx$$

Optimal (type 8, 211 leaves, 6 steps):

$$-\frac{1}{2\,\sqrt{d-c^2 d x^2}}d\,e^{-\frac{a}{b}}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])^n$$
$$\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]+\frac{1}{2\,\sqrt{d-c^2 d x^2}}$$
$$d\,e^{a/b}\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\ (a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]+d\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 245 leaves, 7 steps):

$$\left(e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(2 \sqrt{-1+c x} \sqrt{1+c x}\right) - \left(e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(2 \sqrt{-1+c x} \sqrt{1+c x}\right) - \frac{\sqrt{d-c^2 d x^2} \text{ Unintegrable}\left[\frac{(a+b \text{ArcCosh}[c x])^n}{x \sqrt{-1+c x} \sqrt{1+c x}},x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 423: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n}{x^2} \, dx$$

Optimal (type 8, 91 leaves, 3 steps):

$$-\frac{c d \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \text{ArcCosh}[c x]\right)^{1+n}}{b (1+n) \sqrt{d-c^2 d x^2}} + d \text{ Unintegrable}\left[\frac{(a+b \text{ArcCosh}[c x])^n}{x^2 \sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 125 leaves, 4 steps):

$$\frac{c \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^{1+n}}{b (1+n) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{\sqrt{d-c^2 d x^2} \text{ Unintegrable}\left[\frac{(a+b \text{ArcCosh}[c x])^n}{x^2 \sqrt{-1+c x} \sqrt{1+c x}},x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 424: Result optimal but 1 more steps used.

$$\int x^2 \left(d-c^2 d x^2\right)^{3/2} \left(a+b \text{ArcCosh}[c x]\right)^n \, dx$$

Optimal (type 4, 658 leaves, 12 steps):

$$-\frac{d\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{16\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 4, 658 leaves, 13 steps):

$$-\frac{d\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^{1+n}}{16\,b\,c^3\,(1+n)\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d\,e^{-\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-2n}\,d\,e^{-\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\,d\,e^{-\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\,d\,e^{\frac{2a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-2n}\,d\,e^{\frac{4a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d\,e^{\frac{6a}{b}}\sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c\,x])^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{6\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 425: Result optimal but 1 more steps used.

$$\int x\left(d-c^2 d x^2\right)^{3/2}(a+b\,\text{ArcCosh}[c\,x])^n\,dx$$

Optimal (type 4, 578 leaves, 12 steps):

$$-\left(\left(5^{-1-n}\, d\, e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.\right.$$
$$\left.\left.\text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(32\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)+$$
$$\left(3^{-n}\, d\, e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(32\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$
$$\left(d\, e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\right)\Big/\left(16\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$
$$\left(d\, e^{a/b} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]\right)\Big/\left(16\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$
$$\left(3^{-n}\, d\, e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(32\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$
$$\left(5^{-1-n}\, d\, e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c\,x])^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]\right)\Big/\left(32\,c^2 \sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 4, 578 leaves, 13 steps):

$$-\left(\left(5^{-1-n} d\, e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(32 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)\right)+$$

$$\left(3^{-n} d\, e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(32 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)-$$

$$\left(d\, e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(16 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$

$$\left(d\, e^{a/b} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(16 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)-$$

$$\left(3^{-n} d\, e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(32 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$

$$\left(5^{-1-n} d\, e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2}\, (a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5\,(a+b\,\text{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(32 c^2 \sqrt{-1+c x}\,\sqrt{1+c x}\right)$$

Problem 426: Result optimal but 1 more steps used.

$$\int \left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])^n \, dx$$

Optimal (type 4, 450 leaves, 9 steps):

$$-\frac{3 d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^{1+n}}{8 b c (1+n) \sqrt{-1+c x}\,\sqrt{1+c x}}-$$
$$\left(2^{-2(3+n)} d\, e^{-\frac{4a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$
$$\left(2^{-3-n} d\, e^{-\frac{2a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)-$$
$$\left(2^{-3-n} d\, e^{\frac{2a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$
$$\left(2^{-2(3+n)} d\, e^{\frac{4a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)$$

Result (type 4, 450 leaves, 10 steps):

$$-\frac{3 d \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^{1+n}}{8 b c (1+n) \sqrt{-1+c x}\,\sqrt{1+c x}}-$$
$$\left(2^{-2(3+n)} d\, e^{-\frac{4a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$
$$\left(2^{-3-n} d\, e^{-\frac{2a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)-$$
$$\left(2^{-3-n} d\, e^{\frac{2a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)+$$
$$\left(2^{-2(3+n)} d\, e^{\frac{4a}{b}} \sqrt{d-c^2 d x^2}\,(a+b\,\text{ArcCosh}[c x])^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4(a+b\,\text{ArcCosh}[c x])}{b}\right]\right)\Big/\left(c \sqrt{-1+c x}\,\sqrt{1+c x}\right)$$

Problem 427: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \,\text{ArcCosh}[c x]\right)^n}{x} \, dx$$

Optimal (type 8, 414 leaves, 15 steps):

$$\frac{1}{8 \sqrt{d-c^2 d x^2}} 3^{-1-n} d^2 e^{-\frac{3 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 \left(a+b \,\text{ArcCosh}[c x]\right)}{b}\right]-\frac{1}{8 \sqrt{d-c^2 d x^2}} 5 d^2 e^{-\frac{a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right]+\frac{1}{8 \sqrt{d-c^2 d x^2}} 5 d^2 e^{a/b} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \,\text{ArcCosh}[c x]}{b}\right]-\frac{1}{8 \sqrt{d-c^2 d x^2}} 3^{-1-n} d^2 e^{\frac{3 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 \left(a+b \,\text{ArcCosh}[c x]\right)}{b}\right]+d^2 \,\text{Unintegrable}\left[\frac{\left(a+b \,\text{ArcCosh}[c x]\right)^n}{x \sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 441 leaves, 16 steps):

$$-\left(\left(3^{-1-n} d\, e^{-\frac{3 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3 \left(a+b \,\text{ArcCosh}[c x]\right)}{b}\right]\right)\Big/\left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)\right)+\left(5 d\, e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \,\text{ArcCosh}[c x]}{b}\right]\right)\Big/\left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)-\left(5 d\, e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \,\text{ArcCosh}[c x]}{b}\right]\right)\Big/\left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)+\left(3^{-1-n} d\, e^{\frac{3 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b \,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3 \left(a+b \,\text{ArcCosh}[c x]\right)}{b}\right]\right)\Big/\left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)-\frac{d \sqrt{d-c^2 d x^2}\, \text{Unintegrable}\left[\frac{\left(a+b \,\text{ArcCosh}[c x]\right)^n}{x \sqrt{-1+c x} \sqrt{1+c x}},x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 428: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} \left(a+b \operatorname{ArcCosh}[c x]\right)^n}{x^2} \, dx$$

Optimal (type 8, 291 leaves, 9 steps):

$$-\frac{3 c d^2 \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^{1+n}}{2 b (1+n) \sqrt{d-c^2 d x^2}}+\frac{1}{\sqrt{d-c^2 d x^2}} 2^{-3-n} c d^2 e^{-\frac{2 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{2 \left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]-\frac{1}{\sqrt{d-c^2 d x^2}} 2^{-3-n} c d^2 e^{\frac{2 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{2 \left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]+d^2 \operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{ArcCosh}[c x]\right)^n}{x^2 \sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 320 leaves, 10 steps):

$$\frac{3 c d \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^{1+n}}{2 b (1+n) \sqrt{-1+c x} \sqrt{1+c x}}-\left(2^{-3-n} c d e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{2 \left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(\sqrt{-1+c x} \sqrt{1+c x}\right)+\left(2^{-3-n} c d e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{2 \left(a+b \operatorname{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(\sqrt{-1+c x} \sqrt{1+c x}\right)-\frac{d \sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{ArcCosh}[c x]\right)^n}{x^2 \sqrt{-1+c x} \sqrt{1+c x}},x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 429: Result optimal but 1 more steps used.

$$\int x^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \operatorname{ArcCosh}[c x]\right)^n \, dx$$

Optimal (type 4, 870 leaves, 15 steps):

$$-\frac{5\,d^2\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{128\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\left(2^{-11-3\,n}\,d^2\,e^{-\frac{8\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{8\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-2\,(4+n)}\,d^2\,e^{-\frac{4\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\,d^2\,e^{-\frac{2\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\,d^2\,e^{\frac{2\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-2\,(4+n)}\,d^2\,e^{\frac{4\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-11-3\,n}\,d^2\,e^{\frac{8\,a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{8\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 4, 870 leaves, 16 steps):

$$-\frac{5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{128\,b\,c^3\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\left(2^{-11-3\,n}\,d^2\,e^{-\frac{8\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{8\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-2\,(4+n)}\,d^2\,e^{-\frac{4\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\,d^2\,e^{-\frac{2\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\,d^2\,e^{\frac{2\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-2\,(4+n)}\,d^2\,e^{\frac{4\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-11-3\,n}\,d^2\,e^{\frac{8\,a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{8\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 430: Result optimal but 1 more steps used.

$$\int x\,\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 4, 793 leaves, 15 steps):

$$\left(7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{7 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{5 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{3 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(5 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(5 d^2 e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{3 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{5 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d-c^2 d x^2} \left(a+b \text{ArcCosh}[c x]\right)^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{7 \left(a+b \text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right)$$

Result (type 4, 793 leaves, 16 steps) :

$$\left(7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{7 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{5 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{3 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(5 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(5 d^2 e^{a/b} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{3 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) +$$
$$\left(5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{5 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right) -$$
$$\left(7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n,\frac{7 (a+b \operatorname{ArcCosh}[c x])}{b}\right]\right) \Big/ \left(128 c^2 \sqrt{-1+c x} \sqrt{1+c x}\right)$$

Problem 431: Result optimal but 1 more steps used.

$$\int \left(d-c^2 d x^2\right)^{5/2} (a+b \operatorname{ArcCosh}[c x])^n \, dx$$

Optimal (type 4, 674 leaves, 12 steps):

$$-\frac{5\,d^2\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{16\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(3\times 2^{-7-2n}\,d^2\,e^{-\frac{4a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(15\times 2^{-7-n}\,d^2\,e^{-\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(15\times 2^{-7-n}\,d^2\,e^{\frac{2a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(3\times 2^{-7-2n}\,d^2\,e^{\frac{4a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6a}{b}}\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 4, 674 leaves, 13 steps):

$$-\frac{5\,d^2\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{16\,b\,c\,(1+n)\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{-\frac{6a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,-\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(3\times 2^{-7-2n}\,d^2\,e^{-\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,-\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(15\times 2^{-7-n}\,d^2\,e^{-\frac{2a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,-\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(15\times 2^{-7-n}\,d^2\,e^{\frac{2a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,\frac{2\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(3\times 2^{-7-2n}\,d^2\,e^{\frac{4a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,\frac{4\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(2^{-7-n}\times 3^{-1-n}\,d^2\,e^{\frac{6a}{b}}\,\sqrt{d-c^2\,d\,x^2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\,\frac{6\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]\right)\Big/\left(c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 432: Result valid but suboptimal antiderivative.

$$\int\frac{\left(d-c^2\,d\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{x}\,dx$$

Optimal (type 8, 804 leaves, 27 steps) :

$$-\frac{1}{32\sqrt{d-c^2 d x^2}}5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5(a+b \text{ArcCosh}[c x])}{b}\right]-$$
$$\frac{1}{32\sqrt{d-c^2 d x^2}}5\times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3(a+b \text{ArcCosh}[c x])}{b}\right]+\frac{1}{8\sqrt{d-c^2 d x^2}}$$
$$3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{3(a+b \text{ArcCosh}[c x])}{b}\right]-\frac{1}{16\sqrt{d-c^2 d x^2}}11 d^3 e^{-\frac{a}{b}} \sqrt{-1+c x} \sqrt{1+c x}$$
$$(a+b \text{ArcCosh}[c x])^n \left(-\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b \text{ArcCosh}[c x]}{b}\right]+$$
$$\frac{1}{16\sqrt{d-c^2 d x^2}}11 d^3 e^{a/b} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n$$
$$\left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b \text{ArcCosh}[c x]}{b}\right]+\frac{1}{32\sqrt{d-c^2 d x^2}}$$
$$5\times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{3(a+b \text{ArcCosh}[c x])}{b}\right]-\frac{1}{8\sqrt{d-c^2 d x^2}}3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+c x} \sqrt{1+c x}$$
$$(a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3(a+b \text{ArcCosh}[c x])}{b}\right]+$$
$$\frac{1}{32\sqrt{d-c^2 d x^2}}5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \text{ArcCosh}[c x])^n \left(\frac{a+b \text{ArcCosh}[c x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{5(a+b \text{ArcCosh}[c x])}{b}\right]+d^3 \text{Unintegrable}\left[\frac{(a+b \text{ArcCosh}[c x])^n}{x\sqrt{d-c^2 d x^2}},x\right]$$

Result (type 8, 841 leaves, 28 steps):

$$\left(5^{-1-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{5\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(32 \sqrt{-1+c x} \sqrt{1+c x}\right)+$$
$$\left(5\times 3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(32 \sqrt{-1+c x} \sqrt{1+c x}\right)-$$
$$\left(3^{-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{3\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)+$$
$$\left(11 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(16 \sqrt{-1+c x} \sqrt{1+c x}\right)-$$
$$\left(11 d^2 e^{a/b} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c x]}{b}\right]\right) \Big/ \left(16 \sqrt{-1+c x} \sqrt{1+c x}\right)-$$
$$\left(5\times 3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(32 \sqrt{-1+c x} \sqrt{1+c x}\right)+$$
$$\left(3^{-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{3\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(8 \sqrt{-1+c x} \sqrt{1+c x}\right)-$$
$$\left(5^{-1-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} \left(a+b\,\text{ArcCosh}[c x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{5\left(a+b\,\text{ArcCosh}[c x]\right)}{b}\right]\right) \Big/ \left(32 \sqrt{-1+c x} \sqrt{1+c x}\right)-$$
$$\frac{d^2 \sqrt{d-c^2 d x^2}\ \text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcCosh}[c x]\right)^n}{x \sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 433: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b\,\text{ArcCosh}[c x]\right)^n}{x^2}\, dx$$

Optimal (type 8, 485 leaves, 18 steps) :

$$-\frac{15\, c\, d^3 \sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^{1+n}}{8\, b\, (1+n)\, \sqrt{d-c^2\, d\, x^2}}-\frac{1}{\sqrt{d-c^2\, d\, x^2}}2^{-2\,(3+n)}\, c\, d^3\, e^{-\frac{4\,a}{b}} \sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,-\frac{4\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]+\frac{1}{\sqrt{d-c^2\, d\, x^2}}2^{-2-n}\, c\, d^3\, e^{-\frac{2\,a}{b}} \sqrt{-1+c\, x}\, \sqrt{1+c\, x}$$
$$\left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{2\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]-$$
$$\frac{1}{\sqrt{d-c^2\, d\, x^2}}2^{-2-n}\, c\, d^3\, e^{\frac{2\,a}{b}} \sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n$$
$$\left(\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1+n,\frac{2\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]+\frac{1}{\sqrt{d-c^2\, d\, x^2}}$$
$$2^{-2\,(3+n)}\, c\, d^3\, e^{\frac{4\,a}{b}} \sqrt{-1+c\, x}\, \sqrt{1+c\, x}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}$$
$$\text{Gamma}\left[1+n,\frac{4\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]+d^3\, \text{Unintegrable}\left[\frac{\left(a+b\, \text{ArcCosh}[c\, x]\right)^n}{x^2 \sqrt{d-c^2\, d\, x^2}},x\right]$$

Result (type 8, 522 leaves, 19 steps):

$$\frac{15\, c\, d^2 \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^{1+n}}{8\, b\, (1+n)\, \sqrt{-1+c\, x}\, \sqrt{1+c\, x}}+$$
$$\left(2^{-2\,(3+n)}\, c\, d^2\, e^{-\frac{4\,a}{b}} \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{4\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]\right)\Big/\left(\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\right)-$$
$$\left(2^{-2-n}\, c\, d^2\, e^{-\frac{2\,a}{b}} \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,-\frac{2\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]\right)\Big/\left(\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\right)+$$
$$\left(2^{-2-n}\, c\, d^2\, e^{\frac{2\,a}{b}} \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{2\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]\right)\Big/\left(\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\right)-$$
$$\left(2^{-2\,(3+n)}\, c\, d^2\, e^{\frac{4\,a}{b}} \sqrt{d-c^2\, d\, x^2}\, \left(a+b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a+b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n}\right.$$
$$\left.\text{Gamma}\left[1+n,\frac{4\, \left(a+b\, \text{ArcCosh}[c\, x]\right)}{b}\right]\right)\Big/\left(\sqrt{-1+c\, x}\, \sqrt{1+c\, x}\right)-$$
$$\frac{d^2 \sqrt{d-c^2\, d\, x^2}\, \text{Unintegrable}\left[\frac{\left(a+b\, \text{ArcCosh}[c\, x]\right)^n}{x^2 \sqrt{-1+c\, x}\, \sqrt{1+c\, x}},x\right]}{\sqrt{-1+c\, x}\, \sqrt{1+c\, x}}$$

Problem 434: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \left(a + b\, \text{ArcCosh}[c\, x]\right)^n}{\sqrt{1 - c^2 x^2}} \, dx$$

Optimal (type 4, 323 leaves, 9 steps):

$$\frac{1}{8 c^4 \sqrt{1 - c\, x}} 3^{-1-n} e^{-\frac{3 a}{b}} \sqrt{-1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{3 \left(a + b\, \text{ArcCosh}[c\, x]\right)}{b}\right] + \frac{1}{8 c^4 \sqrt{1 - c\, x}} 3\, e^{-\frac{a}{b}} \sqrt{-1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right] - \frac{1}{8 c^4 \sqrt{1 - c\, x}} 3\, e^{a/b} \sqrt{-1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right] - \frac{1}{8 c^4 \sqrt{1 - c\, x}} 3^{-1-n} e^{\frac{3 a}{b}} \sqrt{-1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{3 \left(a + b\, \text{ArcCosh}[c\, x]\right)}{b}\right]$$

Result (type 4, 375 leaves, 10 steps):

$$\frac{1}{8 c^4 \sqrt{1 - c^2 x^2}} 3^{-1-n} e^{-\frac{3 a}{b}} \sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{3 \left(a + b\, \text{ArcCosh}[c\, x]\right)}{b}\right] + \frac{1}{8 c^4 \sqrt{1 - c^2 x^2}} 3\, e^{-\frac{a}{b}} \sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(-\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, -\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right] - \frac{1}{8 c^4 \sqrt{1 - c^2 x^2}} 3\, e^{a/b} \sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right] - \frac{1}{8 c^4 \sqrt{1 - c^2 x^2}} 3^{-1-n} e^{\frac{3 a}{b}} \sqrt{-1 + c\, x} \sqrt{1 + c\, x} \left(a + b\, \text{ArcCosh}[c\, x]\right)^n \left(\frac{a + b\, \text{ArcCosh}[c\, x]}{b}\right)^{-n} \text{Gamma}\left[1 + n, \frac{3 \left(a + b\, \text{ArcCosh}[c\, x]\right)}{b}\right]$$

Problem 435: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b\, \text{ArcCosh}[c\, x]\right)^n}{\sqrt{1 - c^2 x^2}} \, dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$\frac{\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{1-c\,x}}+\frac{1}{c^3\,\sqrt{1-c\,x}}2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1-c\,x}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]$$

Result (type 4, 250 leaves, 7 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{1-c^2\,x^2}}+\frac{1}{c^3\,\sqrt{1-c^2\,x^2}}2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]-\frac{1}{c^3\,\sqrt{1-c^2\,x^2}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]$$

Problem 436: Result valid but suboptimal antiderivative.

$$\int\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Optimal (type 4, 154 leaves, 4 steps):

$$\frac{1}{2\,c^2\,\sqrt{1-c\,x}}e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{2\,c^2\,\sqrt{1-c\,x}}e^{a/b}\,\sqrt{-1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]$$

Result (type 4, 180 leaves, 5 steps):

$$\frac{1}{2\,c^2\,\sqrt{1-c^2\,x^2}}e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{2\,c^2\,\sqrt{1-c^2\,x^2}}e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]$$

Problem 437: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \, \text{ArcCosh}[c \, x])^n}{\sqrt{1 - c^2 x^2}} \, dx$$

Optimal (type 3, 43 leaves, 1 step):

$$\frac{\sqrt{-1 + c \, x} \, (a + b \, \text{ArcCosh}[c \, x])^{1+n}}{b \, c \, (1 + n) \, \sqrt{1 - c \, x}}$$

Result (type 3, 56 leaves, 2 steps):

$$\frac{\sqrt{-1 + c \, x} \, \sqrt{1 + c \, x} \, (a + b \, \text{ArcCosh}[c \, x])^{1+n}}{b \, c \, (1 + n) \, \sqrt{1 - c^2 x^2}}$$

Problem 438: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \, \text{ArcCosh}[c \, x])^n}{x \, \sqrt{1 - c^2 x^2}} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \, \text{ArcCosh}[c \, x])^n}{x \, \sqrt{1 - c^2 x^2}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1 + c \, x} \, \sqrt{1 + c \, x} \, \text{Unintegrable}\left[\frac{(a+b \, \text{ArcCosh}[c \, x])^n}{x \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}}, x\right]}{\sqrt{1 - c^2 x^2}}$$

Problem 439: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \, \text{ArcCosh}[c \, x])^n}{x^2 \, \sqrt{1 - c^2 x^2}} \, dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \, \text{ArcCosh}[c \, x])^n}{x^2 \, \sqrt{1 - c^2 x^2}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1 + c \, x} \, \sqrt{1 + c \, x} \, \text{Unintegrable}\left[\frac{(a+b \, \text{ArcCosh}[c \, x])^n}{x^2 \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}}, x\right]}{\sqrt{1 - c^2 x^2}}$$

Problem 440: Result optimal but 1 more steps used.

$$\int \frac{x^3 \left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 379 leaves, 9 steps):

$$\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3^{-1-n}\,e^{-\frac{3\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]+\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}} 3\,e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3\,e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x} \left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]- \frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3^{-1-n}\,e^{\frac{3\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]$$

Result (type 4, 379 leaves, 10 steps):

$$\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3^{-1-n}\,e^{-\frac{3\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,-\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]+\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}} 3\,e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n} \text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3\,e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x} \left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]- \frac{1}{8\,c^4\,\sqrt{d-c^2\,d\,x^2}}3^{-1-n}\,e^{\frac{3\,a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n \left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\,\text{Gamma}\left[1+n,\frac{3\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{b}\right]$$

Problem 441: Result optimal but 1 more steps used.

$$\int \frac{x^2 \left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}+\frac{1}{c^3\,\sqrt{d-c^2\,d\,x^2}}2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]-\frac{1}{c^3\,\sqrt{d-c^2\,d\,x^2}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]$$

Result (type 4, 253 leaves, 7 steps):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^{1+n}}{2\,b\,c^3\,(1+n)\,\sqrt{d-c^2\,d\,x^2}}+\frac{1}{c^3\,\sqrt{d-c^2\,d\,x^2}}2^{-3-n}\,e^{-\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]-\frac{1}{c^3\,\sqrt{d-c^2\,d\,x^2}}2^{-3-n}\,e^{\frac{2a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{2\,(a+b\,\text{ArcCosh}[c\,x])}{b}\right]$$

Problem 442: Result optimal but 1 more steps used.

$$\int\frac{x\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 4, 182 leaves, 4 steps):

$$\frac{1}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]$$

Result (type 4, 182 leaves, 5 steps):

$$\frac{1}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}e^{-\frac{a}{b}}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,-\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]-\frac{1}{2\,c^2\,\sqrt{d-c^2\,d\,x^2}}e^{a/b}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\left(\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right)^{-n}\text{Gamma}\left[1+n,\frac{a+b\,\text{ArcCosh}[c\,x]}{b}\right]$$

Problem 443: Result optimal but 1 more steps used.

$$\int \frac{(a + b\,\text{ArcCosh}[c\,x])^n}{\sqrt{d - c^2 d\,x^2}}\,dx$$

Optimal (type 3, 57 leaves, 1 step):

$$\frac{\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,(a + b\,\text{ArcCosh}[c\,x])^{1+n}}{b\,c\,(1 + n)\,\sqrt{d - c^2 d\,x^2}}$$

Result (type 3, 57 leaves, 2 steps):

$$\frac{\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,(a + b\,\text{ArcCosh}[c\,x])^{1+n}}{b\,c\,(1 + n)\,\sqrt{d - c^2 d\,x^2}}$$

Problem 444: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d - c^2 d\,x^2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{d - c^2 d\,x^2}},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{d - c^2 d\,x^2}}$$

Problem 445: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d - c^2 d\,x^2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{d - c^2 d\,x^2}},\,x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1 + c\,x}\,\sqrt{1 + c\,x}\,\text{Unintegrable}\left[\frac{(a+b\,\text{ArcCosh}[c\,x])^n}{x^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{d - c^2 d\,x^2}}$$

Problem 446: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2 \left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}}, x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \,\text{Unintegrable}\left[\frac{x^2 (a+b \,\text{ArcCosh}[c\, x])^n}{(-1+c\, x)^{3/2} (1+c\, x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d\, x^2}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int \frac{x \left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x \left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \,\text{Unintegrable}\left[\frac{x (a+b \,\text{ArcCosh}[c\, x])^n}{(-1+c\, x)^{3/2} (1+c\, x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d\, x^2}}$$

Problem 448: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}} \, dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b \,\text{ArcCosh}[c\, x]\right)^n}{\left(d - c^2 d\, x^2\right)^{3/2}}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$-\frac{\sqrt{-1 + c\, x} \sqrt{1 + c\, x} \,\text{Unintegrable}\left[\frac{(a+b \,\text{ArcCosh}[c\, x])^n}{(-1+c\, x)^{3/2} (1+c\, x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d\, x^2}}$$

Problem 449: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b\,\mathrm{ArcCosh}[c\,x])^n}{x\,(d - c^2\,d\,x^2)^{3/2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\mathrm{Unintegrable}\left[\frac{(a + b\,\mathrm{ArcCosh}[c\,x])^n}{x\,(d - c^2\,d\,x^2)^{3/2}},\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\mathrm{Unintegrable}\left[\frac{(a+b\,\mathrm{ArcCosh}[c\,x])^n}{x\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 450: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b\,\mathrm{ArcCosh}[c\,x])^n}{x^2\,(d - c^2\,d\,x^2)^{3/2}}\,dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\mathrm{Unintegrable}\left[\frac{(a + b\,\mathrm{ArcCosh}[c\,x])^n}{x^2\,(d - c^2\,d\,x^2)^{3/2}},\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\mathrm{Unintegrable}\left[\frac{(a+b\,\mathrm{ArcCosh}[c\,x])^n}{x^2\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}},\,x\right]}{d\,\sqrt{d-c^2\,d\,x^2}}$$

Problem 451: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m\,(a + b\,\mathrm{ArcCosh}[c\,x])^n}{\sqrt{1 - c^2\,x^2}}\,dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\mathrm{Unintegrable}\left[\frac{(f\,x)^m\,(a + b\,\mathrm{ArcCosh}[c\,x])^n}{\sqrt{1 - c^2\,x^2}},\,x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\mathrm{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\mathrm{ArcCosh}[c\,x])^n}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{1-c^2\,x^2}}$$

Problem 457: Result valid but suboptimal antiderivative.

$$\int (f\,x)^m\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[(f\,x)^m\,\left(d-c^2\,d\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n,\,x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\left(\left(d\,\sqrt{d-c^2\,d\,x^2}\,\,\text{Unintegrable}\left[(f\,x)^m\,(-1+c\,x)^{3/2}\,(1+c\,x)^{3/2}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n,\,x\right]\right)\Big/\left(\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)\right)$$

Problem 458: Result valid but suboptimal antiderivative.

$$\int (f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[(f\,x)^m\,\sqrt{d-c^2\,d\,x^2}\,\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n,\,x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\left(\sqrt{d-c^2\,d\,x^2}\,\,\text{Unintegrable}\left[(f\,x)^m\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n,\,x\right]\right)\Big/\left(\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 459: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\sqrt{d-c^2\,d\,x^2}},\,x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,\,\text{Unintegrable}\left[\frac{(f\,x)^m\,(a+b\,\text{ArcCosh}[c\,x])^n}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}},\,x\right]}{\sqrt{d-c^2\,d\,x^2}}$$

Problem 460: Result valid but suboptimal antiderivative.

$$\int \frac{(f\,x)^m\,\left(a+b\,\text{ArcCosh}[c\,x]\right)^n}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a+b\,\text{ArcCosh}[c x])^n}{\left(d-c^2 d x^2\right)^{3/2}}, x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1+c x}\,\sqrt{1+c x}\,\text{Unintegrable}\left[\frac{(f x)^m (a+b\,\text{ArcCosh}[c x])^n}{(-1+c x)^{3/2} (1+c x)^{3/2}}, x\right]}{d\sqrt{d-c^2 d x^2}}$$

Test results for the 296 problems in "7.2.5 Inverse hyperbolic cosine functions.m"

Problem 61: Unable to integrate problem.

$$\int \frac{\left(d-c^2 d x^2\right)^{3/2} (a+b\,\text{ArcCosh}[c x])}{f+g x}\,dx$$

Optimal (type 4, 1270 leaves, ? steps):

$$-\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}}{g^3}+\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{a\,d\,(2+3\,c\,x-2\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{6\,g}+\frac{b\,c\,d\,x\,(-12-9\,c\,x+4\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{36\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}-\frac{a\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{2\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{b\,d\,(2+3\,c\,x-2\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{6\,g}-\frac{b\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]^2}{4\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$

$$\frac{d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}-$$

$$\frac{2\,a\,d\,(c\,f-g)^{3/2}\,(c\,f+g)^{3/2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c\,f+g}\,\sqrt{1+c\,x}}{\sqrt{c\,f-g}\,\sqrt{-1+c\,x}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$

$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$

$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$

$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$

$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Result (type 8, 1150 leaves, 28 steps):

$$\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$

$$\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{g^3\,(1-c\,x)\,(1+c\,x)}-\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}+$$

$$\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$

$$\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$

$$\frac{d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$

$$\left(a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\left[\frac{g+c^2\,f\,x}{\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}}\right]\right)\Big/$$
$$\left(g^4\,(1-c\,x)\,(1+c\,x)\right)-$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$
$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$
$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$
$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)+$$

$$\left(b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]\right)\Big/$$
$$\left(g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)-$$

$$\left(c\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\left[(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x]),\,x\right]\right)\Big/$$
$$\left(g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\right)$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int \frac{(c\, e + d\, e\, x)^2}{(a + b\, \text{ArcCosh}[c + d\, x])^3} \, dx$$

Optimal (type 4, 252 leaves, 18 steps):

$$-\frac{e^2 \sqrt{-1+c+d\,x}\,(c+d\,x)^2 \sqrt{1+c+d\,x}}{2\,b\,d\,(a+b\,\text{ArcCosh}[c+d\,x])^2}+\frac{e^2\,(c+d\,x)}{b^2\,d\,(a+b\,\text{ArcCosh}[c+d\,x])}-\frac{3\,e^2\,(c+d\,x)^3}{2\,b^2\,d\,(a+b\,\text{ArcCosh}[c+d\,x])}-\frac{e^2\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^3\,d}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c+d\,x])}{b}\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{8\,b^3\,d}+\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,(a+b\,\text{ArcCosh}[c+d\,x])}{b}\right]}{8\,b^3\,d}$$

Result (type 4, 311 leaves, 18 steps):

$$-\frac{e^2 \sqrt{-1+c+d\,x}\,(c+d\,x)^2 \sqrt{1+c+d\,x}}{2\,b\,d\,(a+b\,\text{ArcCosh}[c+d\,x])^2}+\frac{e^2\,(c+d\,x)}{b^2\,d\,(a+b\,\text{ArcCosh}[c+d\,x])}-\frac{3\,e^2\,(c+d\,x)^3}{2\,b^2\,d\,(a+b\,\text{ArcCosh}[c+d\,x])}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c+d\,x]\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{8\,b^3\,d}+\frac{e^2\,\text{CoshIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]\,\text{Sinh}\left[\frac{a}{b}\right]}{b^3\,d}-\frac{9\,e^2\,\text{CoshIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c+d\,x]\right]\,\text{Sinh}\left[\frac{3\,a}{b}\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a}{b}+\text{ArcCosh}[c+d\,x]\right]}{8\,b^3\,d}+\frac{9\,e^2\,\text{Cosh}\left[\frac{3\,a}{b}\right]\,\text{SinhIntegral}\left[\frac{3\,a}{b}+3\,\text{ArcCosh}[c+d\,x]\right]}{8\,b^3\,d}-\frac{e^2\,\text{Cosh}\left[\frac{a}{b}\right]\,\text{SinhIntegral}\left[\frac{a+b\,\text{ArcCosh}[c+d\,x]}{b}\right]}{b^3\,d}$$

Test results for the 243 problems in "7.3.2 (d x)^m (a+b arctanh(c x^n))^p.m"

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^7\,(a+b\,\text{ArcTanh}[c\,x^2])^2\,dx$$

Optimal (type 3, 125 leaves, 12 steps):

$$\frac{a b x^2}{4 c^3}+\frac{b^2 x^4}{24 c^2}+\frac{b^2 x^2 \text{ArcTanh}\left[c x^2\right]}{4 c^3}+\frac{b x^6\left(a+b \text{ArcTanh}\left[c x^2\right]\right)}{12 c}-\frac{\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2}{8 c^4}+\frac{1}{8} x^8\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2+\frac{b^2 \text{Log}\left[1-c^2 x^4\right]}{6 c^4}$$

Result (type 4, 636 leaves, 62 steps):

$$\frac{a b x^2}{8 c^3}+\frac{23 b^2 x^2}{192 c^3}+\frac{b^2 x^4}{128 c^2}-\frac{7 b^2 x^6}{576 c}-\frac{b^2 x^8}{256}+\frac{3 b^2\left(1-c x^2\right)^2}{32 c^4}-\frac{b^2\left(1-c x^2\right)^3}{36 c^4}+\frac{b^2\left(1-c x^2\right)^4}{256 c^4}-\frac{5 b^2 \text{Log}\left[1-c x^2\right]}{192 c^4}+\frac{b^2\left(1-c x^2\right) \text{Log}\left[1-c x^2\right]}{16 c^4}+\frac{b^2 \text{Log}\left[1-c x^2\right]^2}{32 c^4}-\frac{b x^4\left(2 a-b \text{Log}\left[1-c x^2\right]\right)}{32 c^2}+\frac{b x^6\left(2 a-b \text{Log}\left[1-c x^2\right]\right)}{48 c}-\frac{1}{64} b x^8\left(2 a-b \text{Log}\left[1-c x^2\right]\right)+\frac{1}{32} x^8\left(2 a-b \text{Log}\left[1-c x^2\right]\right)^2-\frac{1}{192} b\left(2 a-b \text{Log}\left[1-c x^2\right]\right)\left(\frac{48\left(1-c x^2\right)}{c^4}-\frac{36\left(1-c x^2\right)^2}{c^4}+\frac{16\left(1-c x^2\right)^3}{c^4}-\frac{3\left(1-c x^2\right)^4}{c^4}-\frac{12 \text{Log}\left[1-c x^2\right]}{c^4}\right)-\frac{b\left(2 a-b \text{Log}\left[1-c x^2\right]\right) \text{Log}\left[\frac{1}{2}\left(1+c x^2\right)\right]}{16 c^4}+\frac{b^2 \text{Log}\left[1+c x^2\right]}{24 c^4}+\frac{b^2 x^6 \text{Log}\left[1+c x^2\right]}{24 c}+\frac{b^2\left(1+c x^2\right) \text{Log}\left[1+c x^2\right]}{8 c^4}+\frac{b^2 \text{Log}\left[\frac{1}{2}\left(1-c x^2\right)\right] \text{Log}\left[1+c x^2\right]}{16 c^4}+\frac{1}{16} b x^8\left(2 a-b \text{Log}\left[1-c x^2\right]\right) \text{Log}\left[1+c x^2\right]-\frac{b^2 \text{Log}\left[1+c x^2\right]^2}{32 c^4}+\frac{1}{32} b^2 x^8 \text{Log}\left[1+c x^2\right]^2+\frac{b^2 \text{PolyLog}\left[2, \frac{1}{2}\left(1-c x^2\right)\right]}{16 c^4}+\frac{b^2 \text{PolyLog}\left[2, \frac{1}{2}\left(1+c x^2\right)\right]}{16 c^4}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2 dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\frac{b^2 x^2}{6 c^2}-\frac{b^2 \text{ArcTanh}\left[c x^2\right]}{6 c^3}+\frac{b x^4\left(a+b \text{ArcTanh}\left[c x^2\right]\right)}{6 c}+\frac{\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2}{6 c^3}+\frac{1}{6} x^6\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2-\frac{b\left(a+b \text{ArcTanh}\left[c x^2\right]\right) \text{Log}\left[\frac{2}{1-c x^2}\right]}{3 c^3}-\frac{b^2 \text{PolyLog}\left[2, 1-\frac{2}{1-c x^2}\right]}{6 c^3}$$

Result (type 4, 536 leaves, 53 steps):

$$-\frac{a\,b\,x^2}{6\,c^2}+\frac{19\,b^2\,x^2}{72\,c^2}-\frac{5\,b^2\,x^4}{144\,c}-\frac{b^2\,x^6}{108}+\frac{b^2\,\left(1-c\,x^2\right)^2}{16\,c^3}-\frac{b^2\,\left(1-c\,x^2\right)^3}{108\,c^3}+$$
$$\frac{b^2\,\text{Log}\left[1-c\,x^2\right]}{72\,c^3}-\frac{b^2\,\left(1-c\,x^2\right)\,\text{Log}\left[1-c\,x^2\right]}{12\,c^3}+\frac{b^2\,\text{Log}\left[1-c\,x^2\right]^2}{24\,c^3}+$$
$$\frac{b\,x^4\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{24\,c}-\frac{1}{36}\,b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)+\frac{1}{24}\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2-$$
$$\frac{1}{72}\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\left(\frac{18\,\left(1-c\,x^2\right)}{c^3}-\frac{9\,\left(1-c\,x^2\right)^2}{c^3}+\frac{2\,\left(1-c\,x^2\right)^3}{c^3}-\frac{6\,\text{Log}\left[1-c\,x^2\right]}{c^3}\right)+$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{12\,c^3}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]}{12\,c^3}+\frac{b^2\,x^4\,\text{Log}\left[1+c\,x^2\right]}{12\,c}+$$
$$\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]}{12\,c^3}+\frac{1}{12}\,b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]+\frac{b^2\,\text{Log}\left[1+c\,x^2\right]^2}{24\,c^3}+$$
$$\frac{1}{24}\,b^2\,x^6\,\text{Log}\left[1+c\,x^2\right]^2-\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{12\,c^3}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{12\,c^3}$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a\,b\,x^2}{2\,c}+\frac{b^2\,x^2\,\text{ArcTanh}\left[c\,x^2\right]}{2\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{4\,c^2}+\frac{1}{4}\,x^4\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^4\right]}{4\,c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\frac{3\,a\,b\,x^2}{4\,c}-\frac{b^2\,x^4}{16}+\frac{b^2\,\left(1-c\,x^2\right)^2}{32\,c^2}+\frac{b^2\,\left(1+c\,x^2\right)^2}{32\,c^2}-\frac{b^2\,\text{Log}\left[1-c\,x^2\right]}{16\,c^2}+\frac{3\,b^2\,\left(1-c\,x^2\right)\,\text{Log}\left[1-c\,x^2\right]}{8\,c^2}-$$
$$\frac{1}{16}\,b\,x^4\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)+\frac{b\,\left(1-c\,x^2\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{16\,c^2}-$$
$$\frac{\left(1-c\,x^2\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{8\,c^2}+\frac{\left(1-c\,x^2\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{16\,c^2}-$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{8\,c^2}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]}{16\,c^2}+\frac{1}{16}\,b^2\,x^4\,\text{Log}\left[1+c\,x^2\right]+$$
$$\frac{3\,b^2\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]}{8\,c^2}-\frac{b^2\,\left(1+c\,x^2\right)^2\,\text{Log}\left[1+c\,x^2\right]}{16\,c^2}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]}{8\,c^2}+$$
$$\frac{1}{8}\,b\,x^4\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]-\frac{b^2\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^2}{8\,c^2}+$$
$$\frac{b^2\,\left(1+c\,x^2\right)^2\,\text{Log}\left[1+c\,x^2\right]^2}{16\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{8\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{8\,c^2}$$

Problem 67: Result valid but suboptimal antiderivative.

$$\int x\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 4, 94 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{2\,c}+\frac{1}{2}\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2-\frac{b\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{Log}\left[\frac{2}{1-c\,x^2}\right]}{c}-\frac{b^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^2}\right]}{2\,c}$$

Result (type 4, 207 leaves, 28 steps):

$$-\frac{\left(1-c\,x^2\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{8\,c}+\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{4\,c}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]}{4\,c}+\frac{1}{4}\,b\,x^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]+\frac{b^2\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^2}{8\,c}-\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]}{4\,c}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]}{4\,c}$$

Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{x^3}\,dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{1}{2}\,c\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{2\,x^2}+b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{Log}\left[2-\frac{2}{1+c\,x^2}\right]-\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,x^2}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$2\,a\,b\,c\,\text{Log}[x]-\frac{\left(1-c\,x^2\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{8\,x^2}-\frac{1}{4}\,b\,c\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^2\right)\right]-\frac{1}{4}\,b^2\,c\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^2\right)\right]\,\text{Log}\left[1+c\,x^2\right]-\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)\,\text{Log}\left[1+c\,x^2\right]}{4\,x^2}-\frac{b^2\,\left(1+c\,x^2\right)\,\text{Log}\left[1+c\,x^2\right]^2}{8\,x^2}-\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,-c\,x^2\right]+\frac{1}{2}\,b^2\,c\,\text{PolyLog}\left[2,\,c\,x^2\right]+\frac{1}{4}\,b^2\,c\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^2\right)\right]-\frac{1}{4}\,b^2\,c\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^2\right)\right]$$

Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b\,\text{ArcTanh}[c\,x^2])^2}{x^5}\,dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$-\frac{b\,c\,(a + b\,\text{ArcTanh}[c\,x^2])}{2\,x^2} + \frac{1}{4}\,c^2\,(a + b\,\text{ArcTanh}[c\,x^2])^2 - \frac{(a + b\,\text{ArcTanh}[c\,x^2])^2}{4\,x^4} + b^2\,c^2\,\text{Log}[x] - \frac{1}{4}\,b^2\,c^2\,\text{Log}[1 - c^2\,x^4]$$

Result (type 4, 360 leaves, 46 steps):

$$b^2\,c^2\,\text{Log}[x] - \frac{1}{8}\,b^2\,c^2\,\text{Log}[1 - c\,x^2] - \frac{b\,c\,(2\,a - b\,\text{Log}[1 - c\,x^2])}{8\,x^2} - \frac{b\,c\,(1 - c\,x^2)\,(2\,a - b\,\text{Log}[1 - c\,x^2])}{8\,x^2} + \frac{1}{16}\,c^2\,(2\,a - b\,\text{Log}[1 - c\,x^2])^2 - \frac{(2\,a - b\,\text{Log}[1 - c\,x^2])^2}{16\,x^4} + \frac{1}{8}\,b\,c^2\,(2\,a - b\,\text{Log}[1 - c\,x^2])\,\text{Log}\left[\frac{1}{2}\,(1 + c\,x^2)\right] - \frac{1}{4}\,b^2\,c^2\,\text{Log}[1 + c\,x^2] - \frac{b^2\,c\,\text{Log}[1 + c\,x^2]}{4\,x^2} - \frac{1}{8}\,b^2\,c^2\,\text{Log}\left[\frac{1}{2}\,(1 - c\,x^2)\right]\,\text{Log}[1 + c\,x^2] - \frac{b\,(2\,a - b\,\text{Log}[1 - c\,x^2])\,\text{Log}[1 + c\,x^2]}{8\,x^4} + \frac{1}{16}\,b^2\,c^2\,\text{Log}[1 + c\,x^2]^2 - \frac{b^2\,\text{Log}[1 + c\,x^2]^2}{16\,x^4} - \frac{1}{8}\,b^2\,c^2\,\text{PolyLog}\left[2, \frac{1}{2}\,(1 - c\,x^2)\right] - \frac{1}{8}\,b^2\,c^2\,\text{PolyLog}\left[2, \frac{1}{2}\,(1 + c\,x^2)\right]$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int x^3\,(a + b\,\text{ArcTanh}[c\,x^2])^3\,dx$$

Optimal (type 4, 141 leaves, 9 steps):

$$\frac{3\,b\,(a + b\,\text{ArcTanh}[c\,x^2])^2}{4\,c^2} + \frac{3\,b\,x^2\,(a + b\,\text{ArcTanh}[c\,x^2])^2}{4\,c} - \frac{(a + b\,\text{ArcTanh}[c\,x^2])^3}{4\,c^2} + \frac{1}{4}\,x^4\,(a + b\,\text{ArcTanh}[c\,x^2])^3 - \frac{3\,b^2\,(a + b\,\text{ArcTanh}[c\,x^2])\,\text{Log}\left[\frac{2}{1-c\,x^2}\right]}{2\,c^2} - \frac{3\,b^3\,\text{PolyLog}\left[2, 1 - \frac{2}{1-c\,x^2}\right]}{4\,c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$-\frac{3 b \left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2}{16 c^2}-\frac{\left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{16 c^2}+\frac{\left(1-c x^2\right)^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{32 c^2}+\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[\frac{1}{2} \left(1+c x^2\right)\right]}{8 c^2}+\frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} \left(1-c x^2\right)\right] \operatorname{Log}\left[1+c x^2\right]}{8 c^2}+\frac{3 b^2 x^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]}{8 c}-\frac{3 b \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]}{32 c^2}+\frac{3}{32} b x^4 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]+\frac{3 b^3 \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^2}{16 c^2}-\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2}{32 c^2}+\frac{3}{32} b^2 x^4 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2-\frac{b^3 \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^3}{16 c^2}+\frac{b^3 \left(1+c x^2\right)^2 \operatorname{Log}\left[1+c x^2\right]^3}{32 c^2}-\frac{3 b^3 \operatorname{PolyLog}\left[2,\frac{1}{2} \left(1-c x^2\right)\right]}{8 c^2}+\frac{3 b^3 \operatorname{PolyLog}\left[2,\frac{1}{2} \left(1+c x^2\right)\right]}{8 c^2}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int x \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3 dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\frac{\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3}{2 c}+\frac{1}{2} x^2 \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^3-\frac{3 b \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2 \operatorname{Log}\left[\frac{2}{1-c x^2}\right]}{2 c}-\frac{3 b^2 \left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right) \operatorname{PolyLog}\left[2,1-\frac{2}{1-c x^2}\right]}{2 c}+\frac{3 b^3 \operatorname{PolyLog}\left[3,1-\frac{2}{1-c x^2}\right]}{4 c}$$

Result (type 4, 390 leaves, 82 steps):

$$-\frac{\left(1-c x^2\right) \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^3}{16 c}+\frac{3 b \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[\frac{1}{2} \left(1+c x^2\right)\right]}{8 c}-\frac{3 b \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]}{16 c}+\frac{3}{16} b x^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 \operatorname{Log}\left[1+c x^2\right]+\frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} \left(1-c x^2\right)\right] \operatorname{Log}\left[1+c x^2\right]^2}{8 c}+\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2}{16 c}+\frac{3}{16} b^2 x^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{Log}\left[1+c x^2\right]^2+\frac{b^3 \left(1+c x^2\right) \operatorname{Log}\left[1+c x^2\right]^3}{16 c}-\frac{3 b^2 \left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) \operatorname{PolyLog}\left[2,\frac{1}{2} \left(1-c x^2\right)\right]}{4 c}+\frac{3 b^3 \operatorname{Log}\left[1+c x^2\right] \operatorname{PolyLog}\left[2,\frac{1}{2} \left(1+c x^2\right)\right]}{4 c}-\frac{3 b^3 \operatorname{PolyLog}\left[3,\frac{1}{2} \left(1-c x^2\right)\right]}{4 c}-\frac{3 b^3 \operatorname{PolyLog}\left[3,\frac{1}{2} \left(1+c x^2\right)\right]}{4 c}$$

Problem 80: Unable to integrate problem.

$$\int \frac{(a + b \,\text{ArcTanh}[c\, x^2])^3}{x^3} \, dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{1}{2} c\, (a + b\, \text{ArcTanh}[c\, x^2])^3 - \frac{(a + b\, \text{ArcTanh}[c\, x^2])^3}{2\, x^2} + \frac{3}{2} b\, c\, (a + b\, \text{ArcTanh}[c\, x^2])^2 \,\text{Log}\left[2 - \frac{2}{1 + c\, x^2}\right] - \frac{3}{2} b^2 c\, (a + b\, \text{ArcTanh}[c\, x^2])\, \text{PolyLog}\left[2, -1 + \frac{2}{1 + c\, x^2}\right] - \frac{3}{4} b^3 c\, \text{PolyLog}\left[3, -1 + \frac{2}{1 + c\, x^2}\right]$$

Result (type 8, 284 leaves, 16 steps):

$$\frac{3}{16} b\, c\, \text{Log}[c\, x^2]\, (2\, a - b\, \text{Log}[1 - c\, x^2])^2 - \frac{(1 - c\, x^2)\, (2\, a - b\, \text{Log}[1 - c\, x^2])^3}{16\, x^2} + \frac{3}{16} b^3 c\, \text{Log}[-c\, x^2]\, \text{Log}[1 + c\, x^2]^2 - \frac{b^3 (1 + c\, x^2)\, \text{Log}[1 + c\, x^2]^3}{16\, x^2} - \frac{3}{8} b^2 c\, (2\, a - b\, \text{Log}[1 - c\, x^2])\, \text{PolyLog}[2, 1 - c\, x^2] + \frac{3}{8} b^3 c\, \text{Log}[1 + c\, x^2]\, \text{PolyLog}[2, 1 + c\, x^2] - \frac{3}{8} b^3 c\, \text{PolyLog}[3, 1 - c\, x^2] - \frac{3}{8} b^3 c\, \text{PolyLog}[3, 1 + c\, x^2] + \frac{3}{8} b\, \text{Unintegrable}\left[\frac{(-2\, a + b\, \text{Log}[1 - c\, x^2])^2\, \text{Log}[1 + c\, x^2]}{x^3}, x\right] - \frac{3}{8} b^2\, \text{Unintegrable}\left[\frac{(-2\, a + b\, \text{Log}[1 - c\, x^2])\, \text{Log}[1 + c\, x^2]^2}{x^3}, x\right]$$

Problem 81: Unable to integrate problem.

$$\int \frac{(a + b \,\text{ArcTanh}[c\, x^2])^3}{x^5} \, dx$$

Optimal (type 4, 139 leaves, 8 steps):

$$\frac{3}{4} b\, c^2 (a + b\, \text{ArcTanh}[c\, x^2])^2 - \frac{3\, b\, c\, (a + b\, \text{ArcTanh}[c\, x^2])^2}{4\, x^2} + \frac{1}{4} c^2 (a + b\, \text{ArcTanh}[c\, x^2])^3 - \frac{(a + b\, \text{ArcTanh}[c\, x^2])^3}{4\, x^4} + \frac{3}{2} b^2 c^2 (a + b\, \text{ArcTanh}[c\, x^2])\, \text{Log}\left[2 - \frac{2}{1 + c\, x^2}\right] - \frac{3}{4} b^3 c^2\, \text{PolyLog}\left[2, -1 + \frac{2}{1 + c\, x^2}\right]$$

Result (type 8, 437 leaves, 29 steps):

$$\frac{3}{4} a b^2 c^2 \text{Log}[x] - \frac{3 b c \left(1 - c x^2\right) \left(2 a - b \text{Log}\left[1 - c x^2\right]\right)^2}{32 x^2} +$$
$$\frac{3}{32} b c^2 \text{Log}\left[c x^2\right] \left(2 a - b \text{Log}\left[1 - c x^2\right]\right)^2 + \frac{1}{32} c^2 \left(2 a - b \text{Log}\left[1 - c x^2\right]\right)^3 -$$
$$\frac{\left(2 a - b \text{Log}\left[1 - c x^2\right]\right)^3}{32 x^4} - \frac{3 b^3 c \left(1 + c x^2\right) \text{Log}\left[1 + c x^2\right]^2}{32 x^2} - \frac{3}{32} b^3 c^2 \text{Log}\left[-c x^2\right] \text{Log}\left[1 + c x^2\right]^2 +$$
$$\frac{1}{32} b^3 c^2 \text{Log}\left[1 + c x^2\right]^3 - \frac{b^3 \text{Log}\left[1 + c x^2\right]^3}{32 x^4} - \frac{3}{16} b^3 c^2 \text{PolyLog}\left[2, -c x^2\right] +$$
$$\frac{3}{16} b^3 c^2 \text{PolyLog}\left[2, c x^2\right] - \frac{3}{16} b^2 c^2 \left(2 a - b \text{Log}\left[1 - c x^2\right]\right) \text{PolyLog}\left[2, 1 - c x^2\right] -$$
$$\frac{3}{16} b^3 c^2 \text{Log}\left[1 + c x^2\right] \text{PolyLog}\left[2, 1 + c x^2\right] - \frac{3}{16} b^3 c^2 \text{PolyLog}\left[3, 1 - c x^2\right] +$$
$$\frac{3}{16} b^3 c^2 \text{PolyLog}\left[3, 1 + c x^2\right] + \frac{3}{8} b \,\text{Unintegrable}\left[\frac{\left(-2 a + b \text{Log}\left[1 - c x^2\right]\right)^2 \text{Log}\left[1 + c x^2\right]}{x^5}, x\right] -$$
$$\frac{3}{8} b^2 \text{Unintegrable}\left[\frac{\left(-2 a + b \text{Log}\left[1 - c x^2\right]\right) \text{Log}\left[1 + c x^2\right]^2}{x^5}, x\right]$$

Problem 82: Result optimal but 1 more steps used.

$$\int (d x)^{5/2} \left(a + b \text{ArcTanh}\left[c x^2\right]\right) dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{8 b d (d x)^{3/2}}{21 c} + \frac{2 b d^{5/2} \text{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{\sqrt{2} b d^{5/2} \text{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} -$$
$$\frac{\sqrt{2} b d^{5/2} \text{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{2 (d x)^{7/2} \left(a + b \text{ArcTanh}\left[c x^2\right]\right)}{7 d} - \frac{2 b d^{5/2} \text{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} -$$
$$\frac{b d^{5/2} \text{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}} + \frac{b d^{5/2} \text{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}$$

Result (type 3, 317 leaves, 17 steps):

$$\frac{8 b d (d x)^{3/2}}{21 c} + \frac{2 b d^{5/2} \text{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{\sqrt{2} b d^{5/2} \text{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} -$$
$$\frac{\sqrt{2} b d^{5/2} \text{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{2 (d x)^{7/2} \left(a + b \text{ArcTanh}\left[c x^2\right]\right)}{7 d} - \frac{2 b d^{5/2} \text{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} -$$
$$\frac{b d^{5/2} \text{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}} + \frac{b d^{5/2} \text{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{7 \sqrt{2} c^{7/4}}$$

Problem 83: Result optimal but 1 more steps used.

$$\int (d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{8\,b\,d\,\sqrt{d\,x}}{5\,c}-\frac{2\,b\,d^{3/2}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}-\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{2\,(d\,x)^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d}-\frac{2\,b\,d^{3/2}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}-\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}$$

Result (type 3, 317 leaves, 17 steps):

$$\frac{8\,b\,d\,\sqrt{d\,x}}{5\,c}-\frac{2\,b\,d^{3/2}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}-\frac{\sqrt{2}\,b\,d^{3/2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{2\,(d\,x)^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d}-\frac{2\,b\,d^{3/2}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,c^{5/4}}+\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}-\frac{b\,d^{3/2}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,c^{5/4}}$$

Problem 84: Result optimal but 1 more steps used.

$$\int \sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\frac{2\,b\,\sqrt{d}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}-\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{2\,(d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d}-\frac{2\,b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}-\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}$$

Result (type 3, 301 leaves, 16 steps):

$$\frac{2\,b\,\sqrt{d}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}-\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{\sqrt{2}\,b\,\sqrt{d}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{2\,(d\,x)^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,d}-\frac{2\,b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,c^{3/4}}+\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}-\frac{b\,\sqrt{d}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\,\sqrt{2}\,c^{3/4}}$$

Problem 85: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{\sqrt{d\,x}}\,dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$-\frac{2\,b\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{2\,\sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d}-\frac{2\,b\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}+\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}$$

Result (type 3, 285 leaves, 16 steps):

$$-\frac{2\,b\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{\sqrt{2}\,b\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}+\frac{2\,\sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{d}-\frac{2\,b\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{c^{1/4}\sqrt{d}}-\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}+\frac{b\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,c^{1/4}\sqrt{d}}$$

Problem 86: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{3/2}}\,dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$-\frac{2\,b\,c^{1/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{2\,(a+b\,\text{ArcTanh}[c\,x^2])}{d\sqrt{d\,x}}+\frac{2\,b\,c^{1/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}-\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}$$

Result (type 3, 285 leaves, 16 steps):

$$-\frac{2\,b\,c^{1/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{\sqrt{2}\,b\,c^{1/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}-\frac{2\,(a+b\,\text{ArcTanh}[c\,x^2])}{d\sqrt{d\,x}}+\frac{2\,b\,c^{1/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{d^{3/2}}+\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}-\frac{b\,c^{1/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{\sqrt{2}\,d^{3/2}}$$

Problem 87: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}[c\,x^2]}{(d\,x)^{5/2}}\,dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\frac{2\,b\,c^{3/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}+\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{2\,(a+b\,\text{ArcTanh}[c\,x^2])}{3\,d\,(d\,x)^{3/2}}+\frac{2\,b\,c^{3/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}+\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}$$

Result (type 3, 301 leaves, 16 steps):

$$\frac{2\,b\,c^{3/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}+\frac{\sqrt{2}\,b\,c^{3/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{2\,(a+b\,\text{ArcTanh}[c\,x^2])}{3\,d\,(d\,x)^{3/2}}+\frac{2\,b\,c^{3/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{3\,d^{5/2}}-\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}+\frac{b\,c^{3/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{3\sqrt{2}\,d^{5/2}}$$

Problem 88: Result optimal but 1 more steps used.

$$\int \frac{a + b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{7/2}}\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$-\frac{8\,b\,c}{5\,d^3\,\sqrt{d\,x}} - \frac{2\,b\,c^{5/4}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} + \frac{\sqrt{2}\,b\,c^{5/4}\,\text{ArcTan}\left[1 - \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{\sqrt{2}\,b\,c^{5/4}\,\text{ArcTan}\left[1 + \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{2\,\left(a + b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d\,(d\,x)^{5/2}} + \frac{2\,b\,c^{5/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{b\,c^{5/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x - \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,d^{7/2}} + \frac{b\,c^{5/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x + \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,d^{7/2}}$$

Result (type 3, 317 leaves, 17 steps):

$$-\frac{8\,b\,c}{5\,d^3\,\sqrt{d\,x}} - \frac{2\,b\,c^{5/4}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} + \frac{\sqrt{2}\,b\,c^{5/4}\,\text{ArcTan}\left[1 - \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{\sqrt{2}\,b\,c^{5/4}\,\text{ArcTan}\left[1 + \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{2\,\left(a + b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{5\,d\,(d\,x)^{5/2}} + \frac{2\,b\,c^{5/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{5\,d^{7/2}} - \frac{b\,c^{5/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x - \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,d^{7/2}} + \frac{b\,c^{5/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x + \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{5\,\sqrt{2}\,d^{7/2}}$$

Problem 89: Result optimal but 1 more steps used.

$$\int \frac{a + b\,\text{ArcTanh}\left[c\,x^2\right]}{(d\,x)^{9/2}}\,dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$-\frac{8\,b\,c}{21\,d^3\,(d\,x)^{3/2}} + \frac{2\,b\,c^{7/4}\,\text{ArcTan}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}} + \frac{\sqrt{2}\,b\,c^{7/4}\,\text{ArcTan}\left[1 - \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}} - \frac{\sqrt{2}\,b\,c^{7/4}\,\text{ArcTan}\left[1 + \frac{\sqrt{2}\,c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}} - \frac{2\,\left(a + b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{7\,d\,(d\,x)^{7/2}} + \frac{2\,b\,c^{7/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\,\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}} + \frac{b\,c^{7/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x - \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{7\,\sqrt{2}\,d^{9/2}} - \frac{b\,c^{7/4}\,\text{Log}\left[\sqrt{d} + \sqrt{c}\,\sqrt{d}\,x + \sqrt{2}\,c^{1/4}\,\sqrt{d\,x}\right]}{7\,\sqrt{2}\,d^{9/2}}$$

Result (type 3, 317 leaves, 17 steps):

$$-\frac{8\,b\,c}{21\,d^3\,(d\,x)^{3/2}}+\frac{2\,b\,c^{7/4}\,\text{ArcTan}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}}+\frac{\sqrt{2}\,b\,c^{7/4}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}}-$$
$$\frac{\sqrt{2}\,b\,c^{7/4}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}}-\frac{2\,(a+b\,\text{ArcTanh}[c\,x^2])}{7\,d\,(d\,x)^{7/2}}+\frac{2\,b\,c^{7/4}\,\text{ArcTanh}\left[\frac{c^{1/4}\sqrt{d\,x}}{\sqrt{d}}\right]}{7\,d^{9/2}}+$$
$$\frac{b\,c^{7/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x-\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{7\sqrt{2}\,d^{9/2}}-\frac{b\,c^{7/4}\,\text{Log}\left[\sqrt{d}+\sqrt{c}\,\sqrt{d}\,x+\sqrt{2}\,c^{1/4}\sqrt{d\,x}\right]}{7\sqrt{2}\,d^{9/2}}$$

Problem 90: Unable to integrate problem.

$$\int\sqrt{d\,x}\,\left(a+b\,\text{ArcTanh}[c\,x^2]\right)^2\,dx$$

Optimal (type 4, 6327 leaves, 238 steps):

$$-\frac{8}{9}\,a\,b\,x\sqrt{d\,x}-\frac{2\sqrt{2}\,a\,b\sqrt{d\,x}\,\text{ArcTan}\left[1-\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{3\,c^{3/4}\sqrt{x}}+$$
$$\frac{2\sqrt{2}\,a\,b\sqrt{d\,x}\,\text{ArcTan}\left[1+\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{3\,c^{3/4}\sqrt{x}}-\frac{2\,i\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]^2}{3\,(-c)^{3/4}\sqrt{x}}-$$
$$\frac{2\,i\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]^2}{3\,c^{3/4}\sqrt{x}}-\frac{2\,b^2\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]^2}{3\,(-c)^{3/4}\sqrt{x}}-$$
$$\frac{2\,b^2\sqrt{d\,x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]^2}{3\,c^{3/4}\sqrt{x}}+\frac{4\,b^2\sqrt{d\,x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$
$$\frac{4\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$
$$\frac{2\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$
$$\frac{2\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$
$$\frac{2\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$
$$\frac{4\,b^2\sqrt{d\,x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{4\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\;-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\;+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\;-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\;+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{4\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{4\,b^2\,\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,c^{1/4}\sqrt{x}}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\,\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}\;-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{4\,b^2\sqrt{d\,x}\;\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,c^{1/4}\sqrt{x}}\right]}{3\,c^{3/4}\sqrt{x}}-\frac{4\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+c^{1/4}\sqrt{x}}\right]}{3\,c^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}+$$

$$\frac{2\,b^2\sqrt{d\,x}\;\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\sqrt{x}\right)}\right]}{3\,c^{3/4}\sqrt{x}}-$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}+\frac{\sqrt{2}\, a b \sqrt{d x}\, \text{Log}\left[1-\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{\sqrt{2}\, a b \sqrt{d x}\, \text{Log}\left[1+\sqrt{2}\, c^{1/4} \sqrt{x}+\sqrt{c}\, x\right]}{3 c^{3/4} \sqrt{x}}+\frac{4}{9} b^2 x \sqrt{d x}\, \text{Log}\left[1-c x^2\right]+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{3 (-c)^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1-c x^2\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{4}{9} b x \sqrt{d x} \left(2 a-b\, \text{Log}\left[1-c x^2\right]\right)+\frac{2 b \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \left(2 a-b\, \text{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{2 b \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \left(2 a-b\, \text{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}}+$$

$$\frac{1}{6} x \sqrt{d x} \left(2 a-b\, \text{Log}\left[1-c x^2\right]\right)^2+\frac{2}{3} a b x \sqrt{d x}\, \text{Log}\left[1+c x^2\right]-$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{3 (-c)^{3/4} \sqrt{x}}+\frac{2 b^2 \sqrt{d x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}}+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{3 (-c)^{3/4} \sqrt{x}}-\frac{2 b^2 \sqrt{d x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \text{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}}-$$

$$\frac{1}{3} b^2 x \sqrt{d x}\, \text{Log}\left[1-c x^2\right] \text{Log}\left[1+c x^2\right]+\frac{1}{6} b^2 x \sqrt{d x}\, \text{Log}\left[1+c x^2\right]^2+$$

$$\frac{2 b^2 \sqrt{d x}\, \text{PolyLog}\left[2,1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}}-\frac{2 i\, b^2 \sqrt{d x}\, \text{PolyLog}\left[2,1-\frac{2}{1-i\, (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d x}\, \text{PolyLog}\left[2,1+\frac{2 (-c)^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d x}\, \text{PolyLog}\left[2,1-\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3 (-c)^{3/4} \sqrt{x}}-$$

$$\frac{\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{(1+\mathrm{i})\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}-\frac{2\,\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1+\mathrm{i}\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{2\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}+$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{(1-\mathrm{i})\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}}\right]}{3\,(-c)^{3/4}\sqrt{x}}+\frac{2\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\, c^{1/4}\right)\left(1-\mathrm{i}\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,(-c)^{3/4}\sqrt{x}}-\frac{2\,\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2}{1-\mathrm{i}\, c^{1/4}\sqrt{x}}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1+\frac{2\, c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{\mathrm{i}\, b^2 \sqrt{d\, x}\, \text{PolyLog}\left[2, 1-\frac{2\, c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-\mathrm{i}\, c^{1/4}\sqrt{x}\right)}\right]}{3\, c^{3/4}\sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}-c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}+c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}-\frac{i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{3\, c^{3/4} \sqrt{x}}-$$

$$\frac{2\, i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2}{1+i\, c^{1/4} \sqrt{x}}\right]}{3\, c^{3/4} \sqrt{x}}+\frac{2\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3\, c^{3/4} \sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}-\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}+$$

$$\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}+\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}-\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, c^{3/4} \sqrt{x}}+$$

$$\frac{i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3\, (-c)^{3/4} \sqrt{x}}-$$

$$\frac{b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{2\, (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3\, (-c)^{3/4} \sqrt{x}}-\frac{i\, b^2 \sqrt{d\, x}\; \text{PolyLog}\left[2,\, 1-\frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{3\, c^{3/4} \sqrt{x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\sqrt{d\, x}\, \left(a+b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2,\, x\right]$$

Problem 91: Unable to integrate problem.

$$\int \frac{\left(a+b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{\sqrt{d\, x}}\, dx$$

Optimal (type 4, 6177 leaves, 241 steps):

$$\frac{2\, a^2\, x}{\sqrt{d\, x}}-\frac{2\, \sqrt{2}\, a\, b\, \sqrt{x}\; \text{ArcTan}\left[1-\sqrt{2}\, c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d\, x}}+\frac{2\, \sqrt{2}\, a\, b\, \sqrt{x}\; \text{ArcTan}\left[1+\sqrt{2}\, c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d\, x}}+$$

$$\frac{2\,i\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]^2}{(-c)^{1/4}\sqrt{d\,x}}-\frac{4\,a\,b\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]}{c^{1/4}\sqrt{d\,x}}+\frac{2\,i\,b^2\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]^2}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]^2}{(-c)^{1/4}\sqrt{d\,x}}-\frac{4\,a\,b\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]}{c^{1/4}\sqrt{d\,x}}-$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]^2}{c^{1/4}\sqrt{d\,x}}+\frac{4\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-(-c)^{1/4}\sqrt{x}}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{4\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{4\,b^2\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{4\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}-$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-\mathrm{i})\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-\mathrm{i}\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{4\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\,c^{1/4}\right)\,\left(1-\mathrm{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}-c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}+c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1+\mathrm{i})\,\left(1-c^{1/4}\,\sqrt{x}\right)}{1-\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{4\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\,-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\,+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\,-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\,+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\,\left(1+c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{\sqrt{2}\,a\,b\,\sqrt{x}\,\text{Log}\left[1-\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{\sqrt{2}\,a\,b\,\sqrt{x}\,\text{Log}\left[1+\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{2\,a\,b\,x\,\text{Log}\left[1-c\,x^2\right]}{\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{b^2\,x\,\text{Log}\left[1-c\,x^2\right]^2}{2\,\sqrt{d\,x}}+\frac{2\,a\,b\,x\,\text{Log}\left[1+c\,x^2\right]}{\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{b^2\,x\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[1+c\,x^2\right]}{\sqrt{d\,x}}+\frac{b^2\,x\,\text{Log}\left[1+c\,x^2\right]^2}{2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{(1+i)\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{i\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{(1-i)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-\mathrm{i}\,c^{1/4}\right)\,\left(1-\mathrm{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}+\frac{2\,\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}-c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}+c^{1/4}\right)\,\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{(1+\mathrm{i})\,\left(1-c^{1/4}\,\sqrt{x}\right)}{1-\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{2\,\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{2\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1+c^{1/4}\,\sqrt{x}}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\,\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}+\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\,\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,c^{1/4}\,\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-\frac{b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,c^{1/4}\,\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\,\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{c^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{\mathrm{i}\,b^2\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\,\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+\mathrm{i}\,c^{1/4}\right)\,\left(1-\mathrm{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{(-c)^{1/4}\,\sqrt{d\,x}}-$$

$$\frac{b^2\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{(-c)^{1/4}\sqrt{d\,x}}+\frac{i\,b^2\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{(1-i)\left(1+c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{c^{1/4}\sqrt{d\,x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{\sqrt{d\,x}},x\right]$$

Problem 92: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2}{(d\,x)^{3/2}}\,dx$$

Optimal (type 4, 6334 leaves, 197 steps):

$$-\frac{2\sqrt{2}\,a\,b\,c^{1/4}\sqrt{x}\,\text{ArcTan}\left[1-\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{d\sqrt{d\,x}}+\frac{2\sqrt{2}\,a\,b\,c^{1/4}\sqrt{x}\,\text{ArcTan}\left[1+\sqrt{2}\,c^{1/4}\sqrt{x}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+\frac{2\,i\,b^2c^{1/4}\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+$$

$$\frac{2\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}+\frac{2\,b^2c^{1/4}\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]^2}{d\sqrt{d\,x}}-$$

$$\frac{4\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2}{1-(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-$$

$$\frac{4\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+\frac{1}{d\sqrt{d\,x}}$$

$$2\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]+$$

$$\frac{1}{d\sqrt{d\,x}}2\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+$$

$$\frac{4\,b^2(-c)^{1/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+\frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]+$$

$$\frac{1}{d\,\sqrt{d\,x}}2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{1}{d\,\sqrt{d\,x}}2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{1}{d\,\sqrt{d\,x}}2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+\frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\mathrm{i}\,\sqrt{-\sqrt{-c}}\,+c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}-c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left(\mathrm{i}\,(-c)^{1/4}+c^{1/4}\right)\left(1-\mathrm{i}\,c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{(1+\mathrm{i})\left(1-c^{1/4}\,\sqrt{x}\right)}{1-\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2}{1+\mathrm{i}\,c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+\frac{4\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2}{1+c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\,-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}\,+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\,-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{c}}\;\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}\,+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\;\mathrm{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\;\mathrm{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+\frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+\text{i}\,c^{1/4}\right)\left(1-\text{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-\text{i})\left(1+c^{1/4}\,\sqrt{x}\right)}{1-\text{i}\,c^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{\sqrt{2}\,a\,b\,c^{1/4}\,\sqrt{x}\,\text{Log}\left[1-\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{d\,\sqrt{d\,x}}-\frac{\sqrt{2}\,a\,b\,c^{1/4}\,\sqrt{x}\,\text{Log}\left[1+\sqrt{2}\,c^{1/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1-c\,x^2\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{d\,\sqrt{d\,x}}-\frac{\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{2\,d\,\sqrt{d\,x}}-$$

$$\frac{2\,a\,b\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}+\frac{b^2\,\text{Log}\left[1-c\,x^2\right]\,\text{Log}\left[1+c\,x^2\right]}{d\,\sqrt{d\,x}}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]^2}{2\,d\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}+\frac{2\,\text{i}\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\text{i}\,(-c)^{1/4}\,\sqrt{x}}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{\text{i}\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,\,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\text{i}\,\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-\text{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}-$$

$$\frac{\text{i}\,b^2\,(-c)^{1/4}\,\sqrt{x}\,\text{PolyLog}\left[2,\,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\text{i}\,\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-\text{i}\,(-c)^{1/4}\,\sqrt{x}\right)}\right]}{d\,\sqrt{d\,x}}+$$

$$\frac{i\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{i\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{(1-i)\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{2\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-$$

$$\frac{i\,b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{b^2\,(-c)^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}+$$

$$\frac{2\,i\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-i\,c^{1/4}\sqrt{x}}\right]}{d\sqrt{d\,x}}-\frac{i\,b^2\,c^{1/4}\sqrt{x}\,\text{PolyLog}\left[2,1+\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{d\sqrt{d\,x}}-$$

$$\frac{i\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}-$$

$$\frac{i\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}-c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}-$$

$$\frac{i\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}+c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}+$$

$$\frac{i\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{d \sqrt{d\, x}}+\frac{2\, i\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2}{1+i\, c^{1/4} \sqrt{x}}\right]}{d \sqrt{d\, x}}-$$

$$\frac{2\, b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d\, x}}+\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}-$$

$$\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}-$$

$$\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{c}}\ \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}-$$

$$\frac{i\, b^2\, (-c)^{1/4} \sqrt{x}\ \text{PolyLog}\left[2,\ 1-\frac{2\, (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d\, x}}+$$

$$\frac{b^2 (-c)^{1/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i\, b^2 c^{1/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{(d\, x)^{3/2}}, x\right]$$

Problem 93: Unable to integrate problem.

$$\int \frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{(d\, x)^{5/2}} \, dx$$

Optimal (type 4, 6520 leaves, 197 steps):

$$-\frac{2 \sqrt{2}\, a\, b\, c^{3/4} \sqrt{x}\, \text{ArcTan}\left[1 - \sqrt{2}\, c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} + \frac{2 \sqrt{2}\, a\, b\, c^{3/4} \sqrt{x}\, \text{ArcTan}\left[1 + \sqrt{2}\, c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} - \frac{2 i\, b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{2 i\, b^2 c^{3/4} \sqrt{x}\, \text{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x}\, \text{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{4 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{4 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{1}{3 d^2 \sqrt{d x}} 2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}} - (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right] - \frac{1}{3 d^2 \sqrt{d x}} 2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}} + (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right] + \frac{2 b^2 (-c)^{3/4} \sqrt{x}\, \text{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \text{Log}\left[\frac{(1+i) \left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{4\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+(-c)^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}+\frac{1}{3\,d^2\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]+$$

$$\frac{1}{3\,d^2\,\sqrt{d\,x}}2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{1}{3\,d^2\,\sqrt{d\,x}}2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{1}{3\,d^2\,\sqrt{d\,x}}2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]+$$

$$\frac{2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{1-i\,(-c)^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{1}{3\,d^2\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\,\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2\,(-c)^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1-c^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\,\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left(i\,(-c)^{1/4}-c^{1/4}\right)\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left(i\,(-c)^{1/4}+c^{1/4}\right)\left(1-i\,c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{(1+i)\left(1-c^{1/4}\,\sqrt{x}\right)}{1-i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+i\,c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}+\frac{4\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2}{1+c^{1/4}\,\sqrt{x}}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[-\frac{2\,c^{1/4}\left(1-(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\text{Log}\left[\frac{2\,c^{1/4}\left(1+(-c)^{1/4}\,\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4}\,\sqrt{x}\right)}\right]}{3\,d^2\,\sqrt{d\,x}}-\frac{1}{3\,d^2\,\sqrt{d\,x}}$$

$$2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]-$$

$$\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{2\,(-c)^{1/4}\left(1+c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\text{Log}\left[\frac{(1-i)\left(1+c^{1/4}\sqrt{x}\right)}{1-i\,c^{1/4}\sqrt{x}}\right]}{3\,d^2\sqrt{d\,x}}-$$

$$\frac{\sqrt{2}\,a\,b\,c^{3/4}\sqrt{x}\,\text{Log}\left[1-\sqrt{2}\,c^{1/4}\sqrt{x}+\sqrt{c}\,x\right]}{3\,d^2\sqrt{d\,x}}+\frac{\sqrt{2}\,a\,b\,c^{3/4}\sqrt{x}\,\text{Log}\left[1+\sqrt{2}\,c^{1/4}\sqrt{x}+\sqrt{c}\,x\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[1-c\,x^2\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[1-c\,x^2\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b\,c^{3/4}\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b\,c^{3/4}\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)}{3\,d^2\sqrt{d\,x}}-\frac{\left(2\,a-b\,\text{Log}\left[1-c\,x^2\right]\right)^2}{6\,d^2\,x\sqrt{d\,x}}-$$

$$\frac{2\,a\,b\,\text{Log}\left[1+c\,x^2\right]}{3\,d^2\,x\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTan}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[1+c\,x^2\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\sqrt{x}\,\text{ArcTan}\left[c^{1/4}\sqrt{x}\right]\text{Log}\left[1+c\,x^2\right]}{3\,d^2\sqrt{d\,x}}-\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{ArcTanh}\left[(-c)^{1/4}\sqrt{x}\right]\text{Log}\left[1+c\,x^2\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{2\,b^2\,c^{3/4}\sqrt{x}\,\text{ArcTanh}\left[c^{1/4}\sqrt{x}\right]\text{Log}\left[1+c\,x^2\right]}{3\,d^2\sqrt{d\,x}}+\frac{b^2\,\text{Log}\left[1-c\,x^2\right]\text{Log}\left[1+c\,x^2\right]}{3\,d^2\,x\sqrt{d\,x}}-\frac{b^2\,\text{Log}\left[1+c\,x^2\right]^2}{6\,d^2\,x\sqrt{d\,x}}-$$

$$\frac{2\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-(-c)^{1/4}\sqrt{x}}\right]}{3\,d^2\sqrt{d\,x}}-\frac{2\,i\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{i\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{PolyLog}\left[2,1+\frac{2\,(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,d^2\sqrt{d\,x}}+$$

$$\frac{i\,b^2\,(-c)^{3/4}\sqrt{x}\,\text{PolyLog}\left[2,1-\frac{2\,(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3\,d^2\sqrt{d\,x}}-$$

$$\frac{i\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{(1+i)\left(1-(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{2\, i\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2}{1+i\,(-c)^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2}{1+(-c)^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} +$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} +$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{c}}\,\sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{i\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{(1-i)\left(1+(-c)^{1/4}\sqrt{x}\right)}{1-i\,(-c)^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} +$$

$$\frac{i\, b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-i\,c^{1/4}\right)\left(1-i\,(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} +$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4}\left(1-c^{1/4}\sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} -$$

$$\frac{2\, i\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 - \frac{2}{1-i\,c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{i\, b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1 + \frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}}\,\sqrt{x}\right)}{\left(i\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i\,c^{1/4}\sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} +$$

$$\frac{i\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{i\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}-c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{i\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i\, (-c)^{1/4}+c^{1/4}\right) \left(1-i\, c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{i\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i\, c^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-\frac{2\, i\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1+i\, c^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{2\, b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3\, d^2 \sqrt{d\, x}}+\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{-c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}-$$

$$\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+\sqrt{-\sqrt{c}}\, \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1+\frac{2\, c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2\, c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{i\, b^2\, (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2,\, 1-\frac{2\, (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i\, c^{1/4}\right) \left(1-i\, (-c)^{1/4} \sqrt{x}\right)}\right]}{3\, d^2 \sqrt{d\, x}}+$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1-\frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}}-$$

$$\frac{i b^2 c^{3/4} \sqrt{x}\, \text{PolyLog}\left[2, 1-\frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b \text{ArcTanh}\left[c x^2\right]\right)^2}{(d x)^{5/2}}, x\right]$$

Problem 96: Result optimal but 1 more steps used.

$$\int (d x)^m \left(a+b \text{ArcTanh}\left[c x^2\right]\right) dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{(d x)^{1+m} \left(a+b \text{ArcTanh}\left[c x^2\right]\right)}{d (1+m)}-\frac{2 b c (d x)^{3+m} \text{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Result (type 5, 74 leaves, 3 steps):

$$\frac{(d x)^{1+m} \left(a+b \text{ArcTanh}\left[c x^2\right]\right)}{d (1+m)}-\frac{2 b c (d x)^{3+m} \text{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^{11} \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2 dx$$

Optimal (type 3, 125 leaves, 12 steps):

$$\frac{a b x^3}{6 c^3}+\frac{b^2 x^6}{36 c^2}+\frac{b^2 x^3 \text{ArcTanh}\left[c x^3\right]}{6 c^3}+\frac{b x^9 \left(a+b \text{ArcTanh}\left[c x^3\right]\right)}{18 c}-$$

$$\frac{\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2}{12 c^4}+\frac{1}{12} x^{12} \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2+\frac{b^2 \text{Log}\left[1-c^2 x^6\right]}{9 c^4}$$

Result (type 4, 636 leaves, 62 steps):

$$\frac{a\,b\,x^3}{12\,c^3}+\frac{23\,b^2\,x^3}{288\,c^3}+\frac{b^2\,x^6}{192\,c^2}-\frac{7\,b^2\,x^9}{864\,c}-\frac{b^2\,x^{12}}{384}+\frac{b^2\,\left(1-c\,x^3\right)^2}{16\,c^4}-$$
$$\frac{b^2\,\left(1-c\,x^3\right)^3}{54\,c^4}+\frac{b^2\,\left(1-c\,x^3\right)^4}{384\,c^4}-\frac{5\,b^2\,\text{Log}\left[1-c\,x^3\right]}{288\,c^4}+\frac{b^2\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{24\,c^4}+$$
$$\frac{b^2\,\text{Log}\left[1-c\,x^3\right]^2}{48\,c^4}-\frac{b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{48\,c^2}+\frac{b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{72\,c}-$$
$$\frac{1}{96}\,b\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+\frac{1}{48}\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-\frac{1}{288}\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)$$
$$\left(\frac{48\,\left(1-c\,x^3\right)}{c^4}-\frac{36\,\left(1-c\,x^3\right)^2}{c^4}+\frac{16\,\left(1-c\,x^3\right)^3}{c^4}-\frac{3\,\left(1-c\,x^3\right)^4}{c^4}-\frac{12\,\text{Log}\left[1-c\,x^3\right]}{c^4}\right)-$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{24\,c^4}+\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{36\,c^4}+\frac{b^2\,x^9\,\text{Log}\left[1+c\,x^3\right]}{36\,c}+$$
$$\frac{b^2\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]}{12\,c^4}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{24\,c^4}+$$
$$\frac{1}{24}\,b\,x^{12}\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]^2}{48\,c^4}+$$
$$\frac{1}{48}\,b^2\,x^{12}\,\text{Log}\left[1+c\,x^3\right]^2+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{24\,c^4}+\frac{b^2\,\text{PolyLog}\left[2,\,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{24\,c^4}$$

Problem 117: Result valid but suboptimal antiderivative.

$$\int x^8\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2\,dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\frac{b^2\,x^3}{9\,c^2}-\frac{b^2\,\text{ArcTanh}\left[c\,x^3\right]}{9\,c^3}+\frac{b\,x^6\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)}{9\,c}+\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{9\,c^3}+$$
$$\frac{1}{9}\,x^9\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{2\,b\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\,\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{9\,c^3}-\frac{b^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,x^3}\right]}{9\,c^3}$$

Result (type 4, 536 leaves, 53 steps):

$$-\frac{a\,b\,x^3}{9\,c^2}+\frac{19\,b^2\,x^3}{108\,c^2}-\frac{5\,b^2\,x^6}{216\,c}-\frac{b^2\,x^9}{162}+\frac{b^2\,\left(1-c\,x^3\right)^2}{24\,c^3}-\frac{b^2\,\left(1-c\,x^3\right)^3}{162\,c^3}+$$
$$\frac{b^2\,\text{Log}\left[1-c\,x^3\right]}{108\,c^3}-\frac{b^2\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{18\,c^3}+\frac{b^2\,\text{Log}\left[1-c\,x^3\right]^2}{36\,c^3}+$$
$$\frac{b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{36\,c}-\frac{1}{54}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+\frac{1}{36}\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2-$$
$$\frac{1}{108}\,b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\left(\frac{18\,\left(1-c\,x^3\right)}{c^3}-\frac{9\,\left(1-c\,x^3\right)^2}{c^3}+\frac{2\,\left(1-c\,x^3\right)^3}{c^3}-\frac{6\,\text{Log}\left[1-c\,x^3\right]}{c^3}\right)+$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{18\,c^3}-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{18\,c^3}+\frac{b^2\,x^6\,\text{Log}\left[1+c\,x^3\right]}{18\,c}+$$
$$\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{18\,c^3}+\frac{1}{18}\,b\,x^9\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]+\frac{b^2\,\text{Log}\left[1+c\,x^3\right]^2}{36\,c^3}+$$
$$\frac{1}{36}\,b^2\,x^9\,\text{Log}\left[1+c\,x^3\right]^2-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{18\,c^3}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{18\,c^3}$$

Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^5\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2\,dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a\,b\,x^3}{3\,c}+\frac{b^2\,x^3\,\text{ArcTanh}\left[c\,x^3\right]}{3\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{6\,c^2}+\frac{1}{6}\,x^6\,\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^6\right]}{6\,c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\frac{a\,b\,x^3}{2\,c}-\frac{b^2\,x^6}{24}+\frac{b^2\,\left(1-c\,x^3\right)^2}{48\,c^2}+\frac{b^2\,\left(1+c\,x^3\right)^2}{48\,c^2}-\frac{b^2\,\text{Log}\left[1-c\,x^3\right]}{24\,c^2}+\frac{b^2\,\left(1-c\,x^3\right)\,\text{Log}\left[1-c\,x^3\right]}{4\,c^2}-$$
$$\frac{1}{24}\,b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)+\frac{b\,\left(1-c\,x^3\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)}{24\,c^2}-$$
$$\frac{\left(1-c\,x^3\right)\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,c^2}+\frac{\left(1-c\,x^3\right)^2\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{24\,c^2}-$$
$$\frac{b\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{12\,c^2}-\frac{b^2\,\text{Log}\left[1+c\,x^3\right]}{24\,c^2}+\frac{1}{24}\,b^2\,x^6\,\text{Log}\left[1+c\,x^3\right]+$$
$$\frac{b^2\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]}{4\,c^2}-\frac{b^2\,\left(1+c\,x^3\right)^2\,\text{Log}\left[1+c\,x^3\right]}{24\,c^2}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\,\left(1-c\,x^3\right)\right]\,\text{Log}\left[1+c\,x^3\right]}{12\,c^2}+$$
$$\frac{1}{12}\,b\,x^6\,\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\,\text{Log}\left[1+c\,x^3\right]-\frac{b^2\,\left(1+c\,x^3\right)\,\text{Log}\left[1+c\,x^3\right]^2}{12\,c^2}+$$
$$\frac{b^2\,\left(1+c\,x^3\right)^2\,\text{Log}\left[1+c\,x^3\right]^2}{24\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1-c\,x^3\right)\right]}{12\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\,\left(1+c\,x^3\right)\right]}{12\,c^2}$$

Problem 119: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2 dx$$

Optimal (type 4, 96 leaves, 6 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{3\,c}+\frac{1}{3}x^3\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{2\,b\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{Log}\left[\frac{2}{1-c\,x^3}\right]}{3\,c}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-c\,x^3}\right]}{3\,c}$$

Result (type 4, 207 leaves, 28 steps):

$$-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,c}+\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]}{6\,c}+\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]}{6\,c}+\frac{1}{6}b\,x^3\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]+\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{12\,c}-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]}{6\,c}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]}{6\,c}$$

Problem 121: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{x^4}\,dx$$

Optimal (type 4, 90 leaves, 5 steps):

$$\frac{1}{3}c\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)^2}{3\,x^3}+\frac{2}{3}b\,c\left(a+b\,\text{ArcTanh}\left[c\,x^3\right]\right)\text{Log}\left[2-\frac{2}{1+c\,x^3}\right]-\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^3}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$2\,a\,b\,c\,\text{Log}[x]-\frac{\left(1-c\,x^3\right)\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)^2}{12\,x^3}-\frac{1}{6}b\,c\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[\frac{1}{2}\left(1+c\,x^3\right)\right]-\frac{1}{6}b^2\,c\,\text{Log}\left[\frac{1}{2}\left(1-c\,x^3\right)\right]\text{Log}\left[1+c\,x^3\right]-\frac{b\left(2\,a-b\,\text{Log}\left[1-c\,x^3\right]\right)\text{Log}\left[1+c\,x^3\right]}{6\,x^3}-\frac{b^2\left(1+c\,x^3\right)\text{Log}\left[1+c\,x^3\right]^2}{12\,x^3}-\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,-c\,x^3\right]+\frac{1}{3}b^2\,c\,\text{PolyLog}\left[2,c\,x^3\right]+\frac{1}{6}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-c\,x^3\right)\right]-\frac{1}{6}b^2\,c\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+c\,x^3\right)\right]$$

Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b\,\mathrm{ArcTanh}[c\,x^3])^2}{x^7}\,dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$-\frac{b\,c\,(a + b\,\mathrm{ArcTanh}[c\,x^3])}{3\,x^3} + \frac{1}{6}\,c^2\,(a + b\,\mathrm{ArcTanh}[c\,x^3])^2 - \frac{(a + b\,\mathrm{ArcTanh}[c\,x^3])^2}{6\,x^6} + b^2\,c^2\,\mathrm{Log}[x] - \frac{1}{6}\,b^2\,c^2\,\mathrm{Log}[1 - c^2\,x^6]$$

Result (type 4, 360 leaves, 46 steps):

$$b^2\,c^2\,\mathrm{Log}[x] - \frac{1}{12}\,b^2\,c^2\,\mathrm{Log}[1 - c\,x^3] - \frac{b\,c\,(2\,a - b\,\mathrm{Log}[1 - c\,x^3])}{12\,x^3} - \frac{b\,c\,(1 - c\,x^3)\,(2\,a - b\,\mathrm{Log}[1 - c\,x^3])}{12\,x^3} + \frac{1}{24}\,c^2\,(2\,a - b\,\mathrm{Log}[1 - c\,x^3])^2 - \frac{(2\,a - b\,\mathrm{Log}[1 - c\,x^3])^2}{24\,x^6} + \frac{1}{12}\,b\,c^2\,(2\,a - b\,\mathrm{Log}[1 - c\,x^3])\,\mathrm{Log}\left[\frac{1}{2}\,(1 + c\,x^3)\right] - \frac{1}{6}\,b^2\,c^2\,\mathrm{Log}[1 + c\,x^3] - \frac{b^2\,c\,\mathrm{Log}[1 + c\,x^3]}{6\,x^3} - \frac{1}{12}\,b^2\,c^2\,\mathrm{Log}\left[\frac{1}{2}\,(1 - c\,x^3)\right]\,\mathrm{Log}[1 + c\,x^3] - \frac{b\,(2\,a - b\,\mathrm{Log}[1 - c\,x^3])\,\mathrm{Log}[1 + c\,x^3]}{12\,x^6} + \frac{1}{24}\,b^2\,c^2\,\mathrm{Log}[1 + c\,x^3]^2 - \frac{b^2\,\mathrm{Log}[1 + c\,x^3]^2}{24\,x^6} - \frac{1}{12}\,b^2\,c^2\,\mathrm{PolyLog}\left[2, \frac{1}{2}\,(1 - c\,x^3)\right] - \frac{1}{12}\,b^2\,c^2\,\mathrm{PolyLog}\left[2, \frac{1}{2}\,(1 + c\,x^3)\right]$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b\,\mathrm{ArcTanh}[c\,x^3])^2}{x^{10}}\,dx$$

Optimal (type 4, 144 leaves, 9 steps):

$$-\frac{b^2\,c^2}{9\,x^3} + \frac{1}{9}\,b^2\,c^3\,\mathrm{ArcTanh}[c\,x^3] - \frac{b\,c\,(a + b\,\mathrm{ArcTanh}[c\,x^3])}{9\,x^6} + \frac{1}{9}\,c^3\,(a + b\,\mathrm{ArcTanh}[c\,x^3])^2 - \frac{(a + b\,\mathrm{ArcTanh}[c\,x^3])^2}{9\,x^9} + \frac{2}{9}\,b\,c^3\,(a + b\,\mathrm{ArcTanh}[c\,x^3])\,\mathrm{Log}\left[2 - \frac{2}{1 + c\,x^3}\right] - \frac{1}{9}\,b^2\,c^3\,\mathrm{PolyLog}\left[2, -1 + \frac{2}{1 + c\,x^3}\right]$$

Result (type 4, 420 leaves, 59 steps):

$$-\frac{b^2 c^2}{9 x^3}+\frac{2}{3} a b c^3 \text{Log}[x]-\frac{b c \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{18 x^6}+\frac{b c^2 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{18 x^3}-\frac{b c^2 \left(1-c x^3\right) \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{18 x^3}+\frac{1}{36} c^3 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2-\frac{\left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2}{36 x^9}-\frac{1}{18} b c^3 \left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{Log}\left[\frac{1}{2} \left(1+c x^3\right)\right]+\frac{1}{18} b^2 c^3 \text{Log}\left[1+c x^3\right]-\frac{b^2 c \text{Log}\left[1+c x^3\right]}{18 x^6}-\frac{1}{18} b^2 c^3 \text{Log}\left[\frac{1}{2} \left(1-c x^3\right)\right] \text{Log}\left[1+c x^3\right]-\frac{b \left(2 a-b \text{Log}\left[1-c x^3\right]\right) \text{Log}\left[1+c x^3\right]}{18 x^9}-\frac{1}{36} b^2 c^3 \text{Log}\left[1+c x^3\right]^2-\frac{b^2 \text{Log}\left[1+c x^3\right]^2}{36 x^9}-\frac{1}{9} b^2 c^3 \text{PolyLog}\left[2,-c x^3\right]+\frac{1}{9} b^2 c^3 \text{PolyLog}\left[2,c x^3\right]+\frac{1}{18} b^2 c^3 \text{PolyLog}\left[2,\frac{1}{2} \left(1-c x^3\right)\right]-\frac{1}{18} b^2 c^3 \text{PolyLog}\left[2,\frac{1}{2} \left(1+c x^3\right)\right]$$

Problem 124: Result valid but suboptimal antiderivative.

$$\int x^8 \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3 dx$$

Optimal (type 4, 231 leaves, 13 steps):

$$\frac{a b^2 x^3}{3 c^2}+\frac{b^3 x^3 \text{ArcTanh}\left[c x^3\right]}{3 c^2}-\frac{b \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2}{6 c^3}+\frac{b x^6 \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2}{6 c}+\frac{\left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3}{9 c^3}+\frac{1}{9} x^9 \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^3-\frac{b \left(a+b \text{ArcTanh}\left[c x^3\right]\right)^2 \text{Log}\left[\frac{2}{1-c x^3}\right]}{3 c^3}+\frac{b^3 \text{Log}\left[1-c^2 x^6\right]}{6 c^3}-\frac{b^2 \left(a+b \text{ArcTanh}\left[c x^3\right]\right) \text{PolyLog}\left[2,1-\frac{2}{1-c x^3}\right]}{3 c^3}+\frac{b^3 \text{PolyLog}\left[3,1-\frac{2}{1-c x^3}\right]}{6 c^3}$$

Result (type 4, 1421 leaves, 239 steps):

$$\frac{2 a b^2 x^3}{3 c^2}-\frac{7 b^3 x^3}{216 c^2}-\frac{23 b^3 x^6}{432 c}+\frac{b^3 x^9}{324}+\frac{b^3 \left(1-c x^3\right)^2}{48 c^3}+\frac{b^3 \left(1+c x^3\right)^2}{24 c^3}-\frac{b^3 \left(1+c x^3\right)^3}{324 c^3}-\frac{b^3 \text{Log}\left[1-c x^3\right]}{24 c^3}+\frac{b^3 \left(1-c x^3\right) \text{Log}\left[1-c x^3\right]}{3 c^3}-\frac{b^3 \text{Log}\left[1-c x^3\right]^2}{72 c^3}-\frac{b^2 x^6 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{24 c}+\frac{b^2 \left(1-c x^3\right)^2 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{12 c^3}-\frac{b^2 \left(1-c x^3\right)^3 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)}{108 c^3}-\frac{1}{72} b x^9 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2-\frac{b \left(1-c x^3\right) \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2}{8 c^3}+\frac{b \left(1-c x^3\right)^2 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2}{12 c^3}-\frac{b \left(1-c x^3\right)^3 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^2}{72 c^3}-\frac{\left(1-c x^3\right) \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^3}{24 c^3}+\frac{\left(1-c x^3\right)^2 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^3}{24 c^3}-\frac{\left(1-c x^3\right)^3 \left(2 a-b \text{Log}\left[1-c x^3\right]\right)^3}{72 c^3}+$$

$$\frac{1}{216} b^2 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \left(\frac{18 \left(1 - c x^3\right)}{c^3} - \frac{9 \left(1 - c x^3\right)^2}{c^3} + \frac{2 \left(1 - c x^3\right)^3}{c^3} - \frac{6 \,\text{Log}\left[1 - c x^3\right]}{c^3}\right) -$$
$$\frac{b^2 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \text{Log}\left[\frac{1}{2} \left(1 + c x^3\right)\right]}{12 c^3} + \frac{b \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right)^2 \text{Log}\left[\frac{1}{2} \left(1 + c x^3\right)\right]}{12 c^3} -$$
$$\frac{7 b^3 \,\text{Log}\left[1 + c x^3\right]}{108 c^3} + \frac{b^3 x^6 \,\text{Log}\left[1 + c x^3\right]}{18 c} - \frac{1}{108} b^3 x^9 \,\text{Log}\left[1 + c x^3\right] +$$
$$\frac{11 b^3 \left(1 + c x^3\right) \text{Log}\left[1 + c x^3\right]}{36 c^3} - \frac{b^3 \left(1 + c x^3\right)^2 \text{Log}\left[1 + c x^3\right]}{12 c^3} + \frac{b^3 \left(1 + c x^3\right)^3 \text{Log}\left[1 + c x^3\right]}{108 c^3} +$$
$$\frac{b^3 \,\text{Log}\left[\frac{1}{2} \left(1 - c x^3\right)\right] \text{Log}\left[1 + c x^3\right]}{12 c^3} + \frac{b^2 x^6 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \text{Log}\left[1 + c x^3\right]}{12 c} -$$
$$\frac{b \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right)^2 \text{Log}\left[1 + c x^3\right]}{24 c^3} + \frac{1}{24} b x^9 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right)^2 \text{Log}\left[1 + c x^3\right] +$$
$$\frac{b^3 \,\text{Log}\left[1 + c x^3\right]^2}{72 c^3} + \frac{1}{72} b^3 x^9 \,\text{Log}\left[1 + c x^3\right]^2 - \frac{b^3 \left(1 + c x^3\right) \text{Log}\left[1 + c x^3\right]^2}{8 c^3} +$$
$$\frac{b^3 \left(1 + c x^3\right)^2 \text{Log}\left[1 + c x^3\right]^2}{12 c^3} - \frac{b^3 \left(1 + c x^3\right)^3 \text{Log}\left[1 + c x^3\right]^2}{72 c^3} + \frac{b^3 \,\text{Log}\left[\frac{1}{2} \left(1 - c x^3\right)\right] \text{Log}\left[1 + c x^3\right]^2}{12 c^3} +$$
$$\frac{b^2 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \text{Log}\left[1 + c x^3\right]^2}{24 c^3} + \frac{1}{24} b^2 x^9 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \text{Log}\left[1 + c x^3\right]^2 +$$
$$\frac{b^3 \left(1 + c x^3\right) \text{Log}\left[1 + c x^3\right]^3}{24 c^3} - \frac{b^3 \left(1 + c x^3\right)^2 \text{Log}\left[1 + c x^3\right]^3}{24 c^3} + \frac{b^3 \left(1 + c x^3\right)^3 \text{Log}\left[1 + c x^3\right]^3}{72 c^3} +$$
$$\frac{b^3 \,\text{PolyLog}\left[2, \frac{1}{2} \left(1 - c x^3\right)\right]}{12 c^3} - \frac{b^2 \left(2 a - b \,\text{Log}\left[1 - c x^3\right]\right) \text{PolyLog}\left[2, \frac{1}{2} \left(1 - c x^3\right)\right]}{6 c^3} +$$
$$\frac{b^3 \,\text{PolyLog}\left[2, \frac{1}{2} \left(1 + c x^3\right)\right]}{12 c^3} + \frac{b^3 \,\text{Log}\left[1 + c x^3\right] \text{PolyLog}\left[2, \frac{1}{2} \left(1 + c x^3\right)\right]}{6 c^3} -$$
$$\frac{b^3 \,\text{PolyLog}\left[3, \frac{1}{2} \left(1 - c x^3\right)\right]}{6 c^3} - \frac{b^3 \,\text{PolyLog}\left[3, \frac{1}{2} \left(1 + c x^3\right)\right]}{6 c^3}$$

Problem 125: Result valid but suboptimal antiderivative.

$$\int x^5 \left(a + b \,\text{ArcTanh}\left[c x^3\right]\right)^3 dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$\frac{b \left(a + b \,\text{ArcTanh}\left[c x^3\right]\right)^2}{2 c^2} + \frac{b x^3 \left(a + b \,\text{ArcTanh}\left[c x^3\right]\right)^2}{2 c} - \frac{\left(a + b \,\text{ArcTanh}\left[c x^3\right]\right)^3}{6 c^2} +$$
$$\frac{1}{6} x^6 \left(a + b \,\text{ArcTanh}\left[c x^3\right]\right)^3 - \frac{b^2 \left(a + b \,\text{ArcTanh}\left[c x^3\right]\right) \text{Log}\left[\frac{2}{1 - c x^3}\right]}{c^2} - \frac{b^3 \,\text{PolyLog}\left[2, 1 - \frac{2}{1 - c x^3}\right]}{2 c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$-\frac{b\left(1-c x^3\right)\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2}{8 c^2}-\frac{\left(1-c x^3\right)\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^3}{24 c^2}+\frac{\left(1-c x^3\right)^2\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^3}{48 c^2}+\frac{b^2\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[\frac{1}{2}\left(1+c x^3\right)\right]}{4 c^2}+\frac{b^3 \operatorname{Log}\left[\frac{1}{2}\left(1-c x^3\right)\right] \operatorname{Log}\left[1+c x^3\right]}{4 c^2}+\frac{b^2 x^3\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]}{4 c}-\frac{b\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[1+c x^3\right]}{16 c^2}+\frac{1}{16} b x^6\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[1+c x^3\right]+\frac{b^3\left(1+c x^3\right) \operatorname{Log}\left[1+c x^3\right]^2}{8 c^2}-\frac{b^2\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]^2}{16 c^2}+\frac{1}{16} b^2 x^6\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]^2-\frac{b^3\left(1+c x^3\right) \operatorname{Log}\left[1+c x^3\right]^3}{24 c^2}+\frac{b^3\left(1+c x^3\right)^2 \operatorname{Log}\left[1+c x^3\right]^3}{48 c^2}-\frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2}\left(1-c x^3\right)\right]}{4 c^2}+\frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2}\left(1+c x^3\right)\right]}{4 c^2}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int x^2\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^3 dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$\frac{\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^3}{3 c}+\frac{1}{3} x^3\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^3-\frac{b\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right)^2 \operatorname{Log}\left[\frac{2}{1-c x^3}\right]}{c}-\frac{b^2\left(a+b \operatorname{ArcTanh}\left[c x^3\right]\right) \operatorname{PolyLog}\left[2,1-\frac{2}{1-c x^3}\right]}{c}+\frac{b^3 \operatorname{PolyLog}\left[3,1-\frac{2}{1-c x^3}\right]}{2 c}$$

Result (type 4, 390 leaves, 82 steps):

$$-\frac{\left(1-c x^3\right)\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^3}{24 c}+\frac{b\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[\frac{1}{2}\left(1+c x^3\right)\right]}{4 c}-\frac{b\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[1+c x^3\right]}{8 c}+\frac{1}{8} b x^3\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right)^2 \operatorname{Log}\left[1+c x^3\right]+\frac{b^3 \operatorname{Log}\left[\frac{1}{2}\left(1-c x^3\right)\right] \operatorname{Log}\left[1+c x^3\right]^2}{4 c}+\frac{b^2\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]^2}{8 c}+\frac{1}{8} b^2 x^3\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{Log}\left[1+c x^3\right]^2+\frac{b^3\left(1+c x^3\right) \operatorname{Log}\left[1+c x^3\right]^3}{24 c}-\frac{b^2\left(2 a-b \operatorname{Log}\left[1-c x^3\right]\right) \operatorname{PolyLog}\left[2, \frac{1}{2}\left(1-c x^3\right)\right]}{2 c}+\frac{b^3 \operatorname{Log}\left[1+c x^3\right] \operatorname{PolyLog}\left[2, \frac{1}{2}\left(1+c x^3\right)\right]}{2 c}-\frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2}\left(1-c x^3\right)\right]}{2 c}-\frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2}\left(1+c x^3\right)\right]}{2 c}$$

Problem 128: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x^3])^3}{x^4}\,dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$\frac{1}{3}c\,(a+b\,\text{ArcTanh}[c\,x^3])^3-\frac{(a+b\,\text{ArcTanh}[c\,x^3])^3}{3\,x^3}+b\,c\,(a+b\,\text{ArcTanh}[c\,x^3])^2\,\text{Log}\left[2-\frac{2}{1+c\,x^3}\right]-b^2\,c\,(a+b\,\text{ArcTanh}[c\,x^3])\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^3}\right]-\frac{1}{2}b^3\,c\,\text{PolyLog}\left[3,-1+\frac{2}{1+c\,x^3}\right]$$

Result (type 8, 284 leaves, 16 steps):

$$\frac{1}{8}b\,c\,\text{Log}[c\,x^3]\,(2\,a-b\,\text{Log}[1-c\,x^3])^2-\frac{(1-c\,x^3)\,(2\,a-b\,\text{Log}[1-c\,x^3])^3}{24\,x^3}+\frac{1}{8}b^3\,c\,\text{Log}[-c\,x^3]\,\text{Log}[1+c\,x^3]^2-\frac{b^3\,(1+c\,x^3)\,\text{Log}[1+c\,x^3]^3}{24\,x^3}-\frac{1}{4}b^2\,c\,(2\,a-b\,\text{Log}[1-c\,x^3])\,\text{PolyLog}[2,1-c\,x^3]+\frac{1}{4}b^3\,c\,\text{Log}[1+c\,x^3]\,\text{PolyLog}[2,1+c\,x^3]-\frac{1}{4}b^3\,c\,\text{PolyLog}[3,1-c\,x^3]-\frac{1}{4}b^3\,c\,\text{PolyLog}[3,1+c\,x^3]+\frac{3}{8}b\,\text{Unintegrable}\left[\frac{(-2\,a+b\,\text{Log}[1-c\,x^3])^2\,\text{Log}[1+c\,x^3]}{x^4},x\right]-\frac{3}{8}b^2\,\text{Unintegrable}\left[\frac{(-2\,a+b\,\text{Log}[1-c\,x^3])\,\text{Log}[1+c\,x^3]^2}{x^4},x\right]$$

Problem 129: Unable to integrate problem.

$$\int \frac{(a+b\,\text{ArcTanh}[c\,x^3])^3}{x^7}\,dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$\frac{1}{2}b\,c^2\,(a+b\,\text{ArcTanh}[c\,x^3])^2-\frac{b\,c\,(a+b\,\text{ArcTanh}[c\,x^3])^2}{2\,x^3}+\frac{1}{6}c^2\,(a+b\,\text{ArcTanh}[c\,x^3])^3-\frac{(a+b\,\text{ArcTanh}[c\,x^3])^3}{6\,x^6}+b^2\,c^2\,(a+b\,\text{ArcTanh}[c\,x^3])\,\text{Log}\left[2-\frac{2}{1+c\,x^3}\right]-\frac{1}{2}b^3\,c^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+c\,x^3}\right]$$

Result (type 8, 437 leaves, 29 steps):

$$\frac{3}{4} a b^2 c^2 \text{Log}[x] - \frac{b c \left(1 - c x^3\right) \left(2 a - b \text{Log}\left[1 - c x^3\right]\right)^2}{16 x^3} + \frac{1}{16} b c^2 \text{Log}\left[c x^3\right] \left(2 a - b \text{Log}\left[1 - c x^3\right]\right)^2 + \frac{1}{48} c^2 \left(2 a - b \text{Log}\left[1 - c x^3\right]\right)^3 - \frac{\left(2 a - b \text{Log}\left[1 - c x^3\right]\right)^3}{48 x^6} - \frac{b^3 c \left(1 + c x^3\right) \text{Log}\left[1 + c x^3\right]^2}{16 x^3} - \frac{1}{16} b^3 c^2 \text{Log}\left[-c x^3\right] \text{Log}\left[1 + c x^3\right]^2 + \frac{1}{48} b^3 c^2 \text{Log}\left[1 + c x^3\right]^3 - \frac{b^3 \text{Log}\left[1 + c x^3\right]^3}{48 x^6} - \frac{1}{8} b^3 c^2 \text{PolyLog}\left[2, -c x^3\right] + \frac{1}{8} b^3 c^2 \text{PolyLog}\left[2, c x^3\right] - \frac{1}{8} b^2 c^2 \left(2 a - b \text{Log}\left[1 - c x^3\right]\right) \text{PolyLog}\left[2, 1 - c x^3\right] - \frac{1}{8} b^3 c^2 \text{Log}\left[1 + c x^3\right] \text{PolyLog}\left[2, 1 + c x^3\right] - \frac{1}{8} b^3 c^2 \text{PolyLog}\left[3, 1 - c x^3\right] + \frac{1}{8} b^3 c^2 \text{PolyLog}\left[3, 1 + c x^3\right] + \frac{3}{8} b\, \text{Unintegrable}\left[\frac{\left(-2 a + b \text{Log}\left[1 - c x^3\right]\right)^2 \text{Log}\left[1 + c x^3\right]}{x^7}, x\right] - \frac{3}{8} b^2\, \text{Unintegrable}\left[\frac{\left(-2 a + b \text{Log}\left[1 - c x^3\right]\right) \text{Log}\left[1 + c x^3\right]^2}{x^7}, x\right]$$

Problem 132: Result optimal but 1 more steps used.

$$\int (d x)^m \left(a + b \text{ArcTanh}\left[c x^3\right]\right) dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{(d x)^{1+m} \left(a + b \text{ArcTanh}\left[c x^3\right]\right)}{d (1 + m)} - \frac{3 b c (d x)^{4+m} \text{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, c^2 x^6\right]}{d^4 (1 + m) (4 + m)}$$

Result (type 5, 74 leaves, 3 steps):

$$\frac{(d x)^{1+m} \left(a + b \text{ArcTanh}\left[c x^3\right]\right)}{d (1 + m)} - \frac{3 b c (d x)^{4+m} \text{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, c^2 x^6\right]}{d^4 (1 + m) (4 + m)}$$

Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 3, 123 leaves, 14 steps):

$$\frac{1}{12} b^2 c^2 x^2 + \frac{1}{2} b c^3 x \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) + \frac{1}{6} b c x^3 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{4} c^4 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{4} x^4 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{3} b^2 c^4 \text{Log}\left[1 - \frac{c^2}{x^2}\right] + \frac{2}{3} b^2 c^4 \text{Log}[x]$$

Result (type 4, 812 leaves, 88 steps):

$$\frac{1}{4} a b c^3 x - \frac{1}{8} a b c^2 x^2 + \frac{1}{12} b^2 c^2 x^2 + \frac{1}{12} a b c x^3 + \frac{5}{48} b^2 c^4 \text{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{8} b^2 c^3 x \text{Log}\left[1 - \frac{c}{x}\right] +$$
$$\frac{1}{16} b^2 c^2 x^2 \text{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{24} b^2 c x^3 \text{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{8} b c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) +$$
$$\frac{1}{16} b c^2 x^2 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{24} b c x^3 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{1}{16} c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 +$$
$$\frac{1}{16} x^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{8} b^2 c^3 x \text{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{16} b^2 c^2 x^2 \text{Log}\left[1 + \frac{c}{x}\right] +$$
$$\frac{1}{24} b^2 c x^3 \text{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{4} a b x^4 \text{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{8} b^2 x^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right] +$$
$$\frac{5}{48} b^2 c^4 \text{Log}[c - x] + \frac{1}{8} b^2 c^4 \text{Log}\left[1 + \frac{c}{x}\right] \text{Log}[c - x] + \frac{1}{4} a b c^4 \text{Log}[x] +$$
$$\frac{11}{24} b^2 c^4 \text{Log}[x] + \frac{1}{8} b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{x}{c}\right] - \frac{1}{4} a b c^4 \text{Log}[c + x] + \frac{5}{48} b^2 c^4 \text{Log}[c + x] +$$
$$\frac{1}{8} b^2 c^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}[c + x] - \frac{1}{8} b^2 c^4 \text{Log}\left[\frac{c - x}{2 c}\right] \text{Log}[c + x] + \frac{1}{8} b^2 c^4 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c + x] -$$
$$\frac{1}{8} b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{c + x}{2 c}\right] + \frac{11}{48} b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right] + \frac{1}{8} b^2 c^3 x \text{Log}\left[\frac{c + x}{x}\right] -$$
$$\frac{1}{16} b^2 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right] + \frac{1}{24} b^2 c x^3 \text{Log}\left[\frac{c + x}{x}\right] - \frac{1}{16} b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{16} b^2 x^4 \text{Log}\left[\frac{c + x}{x}\right]^2 -$$
$$\frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, \frac{c - x}{2 c}\right] - \frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, \frac{c}{x}\right] -$$
$$\frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, \frac{c + x}{2 c}\right] + \frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, 1 - \frac{x}{c}\right] + \frac{1}{8} b^2 c^4 \text{PolyLog}\left[2, 1 + \frac{x}{c}\right]$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{1}{3} b^2 c^2 x - \frac{1}{3} b^2 c^3 \text{ArcCoth}\left[\frac{x}{c}\right] + \frac{1}{3} b c x^2 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) -$$
$$\frac{1}{3} c^3 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{3} x^3 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 -$$
$$\frac{2}{3} b c^3 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{Log}\left[2 - \frac{2}{1 + \frac{c}{x}}\right] + \frac{1}{3} b^2 c^3 \text{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x}}\right]$$

Result (type 4, 695 leaves, 73 steps):

$$-\frac{1}{3} a b c^2 x+\frac{1}{3} b^2 c^2 x+\frac{1}{6} a b c x^2+\frac{1}{12} b^2 c^3 \text{Log}\left[1-\frac{c}{x}\right]+\frac{1}{6} b^2 c^2 x \text{Log}\left[1-\frac{c}{x}\right]-\frac{1}{12} b^2 c x^2 \text{Log}\left[1-\frac{c}{x}\right]+\frac{1}{6} b c^2 \left(1-\frac{c}{x}\right) x \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)+\frac{1}{12} b c x^2 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)-\frac{1}{12} c^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2+\frac{1}{12} x^3 \left(2 a-b \text{Log}\left[1-\frac{c}{x}\right]\right)^2+\frac{1}{6} b^2 c^2 x \text{Log}\left[1+\frac{c}{x}\right]+\frac{1}{12} b^2 c x^2 \text{Log}\left[1+\frac{c}{x}\right]+\frac{1}{3} a b x^3 \text{Log}\left[1+\frac{c}{x}\right]-\frac{1}{6} b^2 x^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]-\frac{1}{12} b^2 c^3 \text{Log}[c-x]+\frac{1}{6} b^2 c^3 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x]+\frac{1}{3} a b c^3 \text{Log}[x]+\frac{1}{6} b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right]+\frac{1}{3} a b c^3 \text{Log}[c+x]+\frac{1}{12} b^2 c^3 \text{Log}[c+x]-\frac{1}{6} b^2 c^3 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x]+\frac{1}{6} b^2 c^3 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x]-\frac{1}{6} b^2 c^3 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x]-\frac{1}{6} b^2 c^3 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right]-\frac{1}{4} b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right]-\frac{1}{6} b^2 c^2 x \text{Log}\left[\frac{c+x}{x}\right]+\frac{1}{12} b^2 c x^2 \text{Log}\left[\frac{c+x}{x}\right]+\frac{1}{12} b^2 c^3 \text{Log}\left[\frac{c+x}{x}\right]^2+\frac{1}{12} b^2 x^3 \text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,\frac{c-x}{2 c}\right]+\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,\frac{c}{x}\right]+\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,\frac{c+x}{2 c}\right]+\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,1-\frac{x}{c}\right]-\frac{1}{6} b^2 c^3 \text{PolyLog}\left[2,1+\frac{x}{c}\right]$$

Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \left(a+b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$b c x \left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)-\frac{1}{2} c^2 \left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{1}{2} x^2 \left(a+b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{1}{2} b^2 c^2 \text{Log}\left[1-\frac{c^2}{x^2}\right]+b^2 c^2 \text{Log}[x]$$

Result (type 4, 574 leaves, 58 steps):

$$\frac{1}{2} a b c x - \frac{1}{4} b^2 c x \text{Log}\left[1-\frac{c}{x}\right] + \frac{1}{4} b c \left(1-\frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right) - \frac{1}{8} c^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{1}{8} x^2 \left(2 a - b \text{Log}\left[1-\frac{c}{x}\right]\right)^2 + \frac{1}{4} b^2 c x \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{2} a b x^2 \text{Log}\left[1+\frac{c}{x}\right] - \frac{1}{4} b^2 x^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{4} b^2 c^2 \text{Log}[c-x] + \frac{1}{4} b^2 c^2 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x] + \frac{1}{2} a b c^2 \text{Log}[x] + \frac{1}{2} b^2 c^2 \text{Log}[x] + \frac{1}{4} b^2 c^2 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right] - \frac{1}{2} a b c^2 \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x] - \frac{1}{4} b^2 c^2 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x] + \frac{1}{4} b^2 c^2 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x] - \frac{1}{4} b^2 c^2 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right] + \frac{1}{4} b^2 c^2 \text{Log}\left[\frac{c+x}{x}\right] + \frac{1}{4} b^2 c x \text{Log}\left[\frac{c+x}{x}\right] - \frac{1}{8} b^2 c^2 \text{Log}\left[\frac{c+x}{x}\right]^2 + \frac{1}{8} b^2 x^2 \text{Log}\left[\frac{c+x}{x}\right]^2 - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c-x}{2 c}\right] - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c}{x}\right] - \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, \frac{c+x}{2 c}\right] + \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, 1-\frac{x}{c}\right] + \frac{1}{4} b^2 c^2 \text{PolyLog}\left[2, 1+\frac{x}{c}\right]$$

Problem 146: Result valid but suboptimal antiderivative.

$$\int \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$c \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + x \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 - 2 b c \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{Log}\left[\frac{2 c}{c-x}\right] - b^2 c \text{PolyLog}\left[2, -\frac{c+x}{c-x}\right]$$

Result (type 4, 370 leaves, 31 steps):

$$a^2 x - a b x \text{Log}\left[1-\frac{c}{x}\right] - \frac{1}{4} b^2 (c-x) \text{Log}\left[1-\frac{c}{x}\right]^2 + a b x \text{Log}\left[1+\frac{c}{x}\right] - \frac{1}{2} b^2 x \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right] + \frac{1}{4} b^2 (c+x) \text{Log}\left[1+\frac{c}{x}\right]^2 - \frac{1}{2} b^2 c \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[-c-x] + a b c \text{Log}[c-x] + \frac{1}{2} b^2 c \text{Log}[-c-x] \text{Log}\left[\frac{c-x}{2 c}\right] - \frac{1}{2} b^2 c \text{Log}[-c-x] \text{Log}\left[-\frac{x}{c}\right] + \frac{1}{2} b^2 c \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[-c+x] + \frac{1}{2} b^2 c \text{Log}\left[\frac{x}{c}\right] \text{Log}[-c+x] + a b c \text{Log}[c+x] - \frac{1}{2} b^2 c \text{Log}[-c+x] \text{Log}\left[\frac{c+x}{2 c}\right] - \frac{1}{2} b^2 c \text{PolyLog}\left[2, \frac{c-x}{2 c}\right] + \frac{1}{2} b^2 c \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{2} b^2 c \text{PolyLog}\left[2, \frac{c}{x}\right] + \frac{1}{2} b^2 c \text{PolyLog}\left[2, \frac{c+x}{2 c}\right] + \frac{1}{2} b^2 c \text{PolyLog}\left[2, 1-\frac{x}{c}\right] - \frac{1}{2} b^2 c \text{PolyLog}\left[2, 1+\frac{x}{c}\right]$$

Problem 148: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x^2} dx$$

Optimal (type 4, 87 leaves, 6 steps):

$$-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{c}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{x}+\frac{2\,b\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\,\text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c}+\frac{b^2\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\frac{c}{x}}\right]}{c}$$

Result (type 4, 205 leaves, 28 steps):

$$\frac{\left(1-\frac{c}{x}\right)\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{4\,c}-\frac{b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{2\,x}\right]}{2\,c}-\frac{b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,x}-\frac{b^2\,\text{Log}\left[-\frac{c-x}{2\,x}\right]\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,c}-\frac{b^2\,\left(1+\frac{c}{x}\right)\,\text{Log}\left[\frac{c+x}{x}\right]^2}{4\,c}+\frac{b^2\,\text{PolyLog}\left[2,\,-\frac{c-x}{2\,x}\right]}{2\,c}-\frac{b^2\,\text{PolyLog}\left[2,\,\frac{c+x}{2\,x}\right]}{2\,c}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x^3}\,dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$-\frac{a\,b}{c\,x}-\frac{b^2\,\text{ArcCoth}\left[\frac{x}{c}\right]}{c\,x}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,c^2}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,x^2}-\frac{b^2\,\text{Log}\left[1-\frac{c^2}{x^2}\right]}{2\,c^2}$$

Result (type 4, 707 leaves, 66 steps):

$$-\frac{b^2\left(1-\frac{c}{x}\right)^2}{16\,c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2}{16\,c^2}+\frac{a\,b}{4\,x^2}+\frac{b^2}{8\,x^2}-\frac{3\,a\,b}{2\,c\,x}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]}{8\,c^2}-\frac{3\,b^2\left(1-\frac{c}{x}\right)\text{Log}\left[1-\frac{c}{x}\right]}{4\,c^2}-$$
$$\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]}{8\,x^2}-\frac{b\left(1-\frac{c}{x}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)}{8\,c^2}+\frac{\left(1-\frac{c}{x}\right)\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{4\,c^2}-$$
$$\frac{\left(1-\frac{c}{x}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{8\,c^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]}{4\,x^2}-\frac{b^2\,\text{Log}\left[1+\frac{c}{x}\right]\text{Log}[c-x]}{4\,c^2}-$$
$$\frac{b^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{x}{c}\right]}{4\,c^2}-\frac{b^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}[c+x]}{4\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c-x}{2\,c}\right]\text{Log}[c+x]}{4\,c^2}-$$
$$\frac{b^2\,\text{Log}\left[-\frac{x}{c}\right]\text{Log}[c+x]}{4\,c^2}+\frac{b^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{c+x}{2\,c}\right]}{4\,c^2}+\frac{a\,b\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,c^2}-$$
$$\frac{3\,b^2\left(1+\frac{c}{x}\right)\text{Log}\left[\frac{c+x}{x}\right]}{4\,c^2}+\frac{b^2\left(1+\frac{c}{x}\right)^2\text{Log}\left[\frac{c+x}{x}\right]}{8\,c^2}-\frac{a\,b\,\text{Log}\left[\frac{c+x}{x}\right]}{2\,x^2}-\frac{b^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,x^2}+$$
$$\frac{b^2\left(1+\frac{c}{x}\right)\text{Log}\left[\frac{c+x}{x}\right]^2}{4\,c^2}-\frac{b^2\left(1+\frac{c}{x}\right)^2\text{Log}\left[\frac{c+x}{x}\right]^2}{8\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c-x}{2\,c}\right]}{4\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,-\frac{c}{x}\right]}{4\,c^2}+$$
$$\frac{b^2\,\text{PolyLog}\left[2,\frac{c}{x}\right]}{4\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x}{2\,c}\right]}{4\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{x}{c}\right]}{4\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,1+\frac{x}{c}\right]}{4\,c^2}$$

Problem 150: Unable to integrate problem.

$$\int x^3\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3\,dx$$

Optimal (type 4, 203 leaves, 17 steps):

$$\frac{1}{4}\,b^3\,c^3\,x-\frac{1}{4}\,b^3\,c^4\,\text{ArcCoth}\left[\frac{x}{c}\right]+\frac{1}{4}\,b^2\,c^2\,x^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)-b\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+$$
$$\frac{3}{4}\,b\,c^3\,x\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{1}{4}\,b\,c\,x^3\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-\frac{1}{4}\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+$$
$$\frac{1}{4}\,x^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-2\,b^2\,c^4\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\text{Log}\left[2-\frac{2}{1+\frac{c}{x}}\right]+b^3\,c^4\,\text{PolyLog}\left[2,-1+\frac{2}{1+\frac{c}{x}}\right]$$

Result (type 8, 1398 leaves, 139 steps):

$$\frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 +$$
$$\frac{1}{8} a^2 b c x^3 + \frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{c}{x}\right]^2 \text{Log}\left[1 + \frac{c}{x}\right], x\right] -$$
$$\frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right]^2, x\right] + \frac{1}{32} b^3 c^4 \text{Log}\left[1 - \frac{c}{x}\right] -$$
$$\frac{3}{8} a b^2 c^3 x \text{Log}\left[1 - \frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{8} a b^2 c x^3 \text{Log}\left[1 - \frac{c}{x}\right] +$$
$$\frac{5}{32} b^2 c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{32} b^2 c^2 x^2 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) -$$
$$\frac{5}{64} b c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{3}{32} b c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 +$$
$$\frac{3}{64} b c^2 x^2 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{32} b c x^3 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 - \frac{1}{32} c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^3 +$$
$$\frac{1}{32} x^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^3 + \frac{3}{8} a b^2 c^3 x \text{Log}\left[1 + \frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1 + \frac{c}{x}\right] +$$
$$\frac{1}{8} a b^2 c x^3 \text{Log}\left[1 + \frac{c}{x}\right] + \frac{3}{8} a^2 b x^4 \text{Log}\left[1 + \frac{c}{x}\right] - \frac{3}{8} a b^2 x^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right] +$$
$$\frac{5}{16} a b^2 c^4 \text{Log}[c - x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[1 + \frac{c}{x}\right] \text{Log}[c - x] - \frac{3}{32} b c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c}{x}\right] +$$
$$\frac{11}{8} a b^2 c^4 \text{Log}[x] + \frac{3}{8} a b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{x}{c}\right] - \frac{3}{8} a^2 b c^4 \text{Log}[c + x] + \frac{5}{16} a b^2 c^4 \text{Log}[c + x] +$$
$$\frac{3}{8} a b^2 c^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}[c + x] - \frac{3}{8} a b^2 c^4 \text{Log}\left[\frac{c - x}{2 c}\right] \text{Log}[c + x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c + x] -$$
$$\frac{3}{8} a b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{c + x}{2 c}\right] + \frac{11}{16} a b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right] - \frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right] +$$
$$\frac{3}{8} a b^2 c^3 x \text{Log}\left[\frac{c + x}{x}\right] - \frac{5}{32} b^3 c^3 \left(1 + \frac{c}{x}\right) x \text{Log}\left[\frac{c + x}{x}\right] - \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right] +$$
$$\frac{1}{32} b^3 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right] + \frac{1}{8} a b^2 c x^3 \text{Log}\left[\frac{c + x}{x}\right] - \frac{3}{16} a b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{5}{64} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right]^2 +$$
$$\frac{3}{32} b^3 c^3 \left(1 + \frac{c}{x}\right) x \text{Log}\left[\frac{c + x}{x}\right]^2 - \frac{3}{64} b^3 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{32} b^3 c x^3 \text{Log}\left[\frac{c + x}{x}\right]^2 +$$
$$\frac{3}{16} a b^2 x^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{3}{32} b^3 c^4 \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right]^3 + \frac{1}{32} b^3 x^4 \text{Log}\left[\frac{c + x}{x}\right]^3 +$$
$$\frac{3}{16} b^2 c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) \text{PolyLog}\left[2, 1 - \frac{c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c - x}{2 c}\right] -$$
$$\frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] + \frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, \frac{c}{x}\right] -$$
$$\frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c + x}{2 c}\right] + \frac{3}{16} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right] \text{PolyLog}\left[2, \frac{c + x}{x}\right] + \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 - \frac{x}{c}\right] +$$
$$\frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 + \frac{x}{c}\right] + \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, 1 - \frac{c}{x}\right] - \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, \frac{c + x}{x}\right]$$

Problem 151: Unable to integrate problem.

$$\int x^2 \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 \, dx$$

Optimal (type 4, 217 leaves, 15 steps):

$$b^2 c^2 x \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{2} b c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{2} b c x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 - \frac{1}{3} c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 + \frac{1}{3} x^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 - b c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \operatorname{Log}\left[2 - \frac{2}{1 + \frac{c}{x}}\right] + \frac{1}{2} b^3 c^3 \operatorname{Log}\left[1 - \frac{c^2}{x^2}\right] + b^3 c^3 \operatorname{Log}[x] + b^2 c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x}}\right] + \frac{1}{2} b^3 c^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + \frac{c}{x}}\right]$$

Result (type 8, 1152 leaves, 103 steps):

$$-\frac{1}{2} a^2 b c^2 x + \frac{3}{4} a b^2 c^2 x + \frac{1}{4} a^2 b c x^2 + \frac{3}{8} b^3 \operatorname{CannotIntegrate}\left[x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right]^2 \operatorname{Log}\left[1 + \frac{c}{x}\right], x\right] - \frac{3}{8} b^3 \operatorname{CannotIntegrate}\left[x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right]^2, x\right] + \frac{1}{2} a b^2 c^2 x \operatorname{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{4} a b^2 c x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{8} b^2 c^2 \left(1 - \frac{c}{x}\right) x \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{1}{16} b c^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{8} b c^2 \left(1 - \frac{c}{x}\right) x \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{16} b c x^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 - \frac{1}{24} c^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^3 + \frac{1}{24} x^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^3 + \frac{1}{2} a b^2 c^2 x \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{4} a b^2 c x^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{2} a^2 b x^3 \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{2} a b^2 x^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{4} a b^2 c^3 \operatorname{Log}[c - x] + \frac{1}{2} a b^2 c^3 \operatorname{Log}\left[1 + \frac{c}{x}\right] \operatorname{Log}[c - x] - \frac{1}{8} b c^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 \operatorname{Log}\left[\frac{c}{x}\right] + \frac{1}{4} b^3 c^3 \operatorname{Log}[x] + \frac{1}{2} a b^2 c^3 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{x}{c}\right] + \frac{1}{2} a^2 b c^3 \operatorname{Log}[c + x] + \frac{1}{4} a b^2 c^3 \operatorname{Log}[c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}[c + x] + \frac{1}{2} a b^2 c^3 \operatorname{Log}\left[\frac{c - x}{2 c}\right] \operatorname{Log}[c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log}\left[-\frac{x}{c}\right] \operatorname{Log}[c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{c + x}{2 c}\right] - \frac{3}{4} a b^2 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right] - \frac{1}{2} a b^2 c^2 x \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{8} b^3 c^2 \left(1 + \frac{c}{x}\right) x \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{4} a b^2 c x^2 \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{4} a b^2 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{16} b^3 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{8} b^3 c^2 \left(1 + \frac{c}{x}\right) x \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{16} b^3 c x^2 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{4} a b^2 x^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{8} b^3 c^3 \operatorname{Log}\left[-\frac{c}{x}\right] \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{24} b^3 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^3 + \frac{1}{24} b^3 x^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^3 + \frac{1}{4} b^2 c^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{c}{x}\right] - \frac{1}{2} a b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c - x}{2 c}\right] + \frac{1}{2} a b^2 c^3 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{3}{8} b^3 c^3 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{3}{8} b^3 c^3 \operatorname{PolyLog}\left[2, \frac{c}{x}\right] + \frac{1}{2} a b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c + x}{2 c}\right] - \frac{1}{4} b^3 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right] \operatorname{PolyLog}\left[2, \frac{c + x}{x}\right] + \frac{1}{2} a b^2 c^3 \operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right] - \frac{1}{2} a b^2 c^3 \operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right] + \frac{1}{4} b^3 c^3 \operatorname{PolyLog}\left[3, 1 - \frac{c}{x}\right] + \frac{1}{4} b^3 c^3 \operatorname{PolyLog}\left[3, \frac{c + x}{x}\right]$$

Problem 152: Unable to integrate problem.

$$\int x\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$-\frac{3}{2}b\,c^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2+\frac{3}{2}b\,c\,x\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2-\frac{1}{2}c^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+\frac{1}{2}x^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-3\,b^2c^2\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\text{Log}\left[2-\frac{2}{1+\frac{c}{x}}\right]+\frac{3}{2}b^3c^2\,\text{PolyLog}\left[2,-1+\frac{2}{1+\frac{c}{x}}\right]$$

Result (type 8, 897 leaves, 75 steps):

$$\frac{3}{4}a^2b\,c\,x+\frac{3}{8}b^3\,\text{CannotIntegrate}\left[x\,\text{Log}\left[1-\frac{c}{x}\right]^2\text{Log}\left[1+\frac{c}{x}\right],x\right]-\frac{3}{8}b^3\,\text{CannotIntegrate}\left[x\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]^2,x\right]-\frac{3}{4}a\,b^2c\,x\,\text{Log}\left[1-\frac{c}{x}\right]+\frac{3}{16}b\,c\left(1-\frac{c}{x}\right)x\left(2a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2-\frac{1}{16}c^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{1}{16}x^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^3+\frac{3}{4}a\,b^2c\,x\,\text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4}a^2b\,x^2\,\text{Log}\left[1+\frac{c}{x}\right]-\frac{3}{4}a\,b^2x^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4}a\,b^2c^2\,\text{Log}[c-x]+\frac{3}{4}a\,b^2c^2\,\text{Log}\left[1+\frac{c}{x}\right]\text{Log}[c-x]-\frac{3}{16}b\,c^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2\text{Log}\left[\frac{c}{x}\right]+\frac{3}{2}a\,b^2c^2\,\text{Log}[x]+\frac{3}{4}a\,b^2c^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{x}{c}\right]-\frac{3}{4}a^2b\,c^2\,\text{Log}[c+x]+\frac{3}{4}a\,b^2c^2\,\text{Log}[c+x]+\frac{3}{4}a\,b^2c^2\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}[c+x]-\frac{3}{4}a\,b^2c^2\,\text{Log}\left[\frac{c-x}{2c}\right]\text{Log}[c+x]+\frac{3}{4}a\,b^2c^2\,\text{Log}\left[-\frac{x}{c}\right]\text{Log}[c+x]-\frac{3}{4}a\,b^2c^2\,\text{Log}[c-x]\,\text{Log}\left[\frac{c+x}{2c}\right]+\frac{3}{4}a\,b^2c^2\,\text{Log}\left[\frac{c+x}{x}\right]+\frac{3}{4}a\,b^2c\,x\,\text{Log}\left[\frac{c+x}{x}\right]-\frac{3}{8}a\,b^2c^2\,\text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{16}b^3c\left(1+\frac{c}{x}\right)x\,\text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{8}a\,b^2x^2\,\text{Log}\left[\frac{c+x}{x}\right]^2+\frac{3}{16}b^3c^2\,\text{Log}\left[-\frac{c}{x}\right]\text{Log}\left[\frac{c+x}{x}\right]^2-\frac{1}{16}b^3c^2\,\text{Log}\left[\frac{c+x}{x}\right]^3+\frac{1}{16}b^3x^2\,\text{Log}\left[\frac{c+x}{x}\right]^3+\frac{3}{8}b^2c^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\text{PolyLog}\left[2,1-\frac{c}{x}\right]-\frac{3}{4}a\,b^2c^2\,\text{PolyLog}\left[2,\frac{c-x}{2c}\right]-\frac{3}{4}a\,b^2c^2\,\text{PolyLog}\left[2,-\frac{c}{x}\right]+\frac{3}{8}b^3c^2\,\text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{3}{8}b^3c^2\,\text{PolyLog}\left[2,\frac{c}{x}\right]-\frac{3}{4}a\,b^2c^2\,\text{PolyLog}\left[2,\frac{c+x}{2c}\right]+\frac{3}{8}b^3c^2\,\text{Log}\left[\frac{c+x}{x}\right]\text{PolyLog}\left[2,\frac{c+x}{x}\right]+\frac{3}{4}a\,b^2c^2\,\text{PolyLog}\left[2,1-\frac{x}{c}\right]+\frac{3}{4}a\,b^2c^2\,\text{PolyLog}\left[2,1+\frac{x}{c}\right]+\frac{3}{8}b^3c^2\,\text{PolyLog}\left[3,1-\frac{c}{x}\right]-\frac{3}{8}b^3c^2\,\text{PolyLog}\left[3,\frac{c+x}{x}\right]$$

Problem 153: Unable to integrate problem.

$$\int \left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$c\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3+x\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3-3\,b\,c\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2\text{Log}\left[\frac{2\,c}{c-x}\right]-3\,b^2\,c\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\text{PolyLog}\left[2,1-\frac{2\,c}{c-x}\right]+\frac{3}{2}\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{2\,c}{c-x}\right]$$

Result (type 8, 642 leaves, 43 steps):

$$a^3\,x+\frac{3}{8}\,b^3\,\text{CannotIntegrate}\left[\text{Log}\left[1-\frac{c}{x}\right]^2\text{Log}\left[1+\frac{c}{x}\right],x\right]-\frac{3}{8}\,b^3\,\text{CannotIntegrate}\left[\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]^2,x\right]-\frac{3}{2}\,a^2\,b\,x\,\text{Log}\left[1-\frac{c}{x}\right]-\frac{3}{4}\,a\,b^2\,(c-x)\,\text{Log}\left[1-\frac{c}{x}\right]^2+\frac{1}{8}\,b^3\,(c-x)\,\text{Log}\left[1-\frac{c}{x}\right]^3+\frac{3}{2}\,a^2\,b\,x\,\text{Log}\left[1+\frac{c}{x}\right]-\frac{3}{2}\,a\,b^2\,x\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}\left[1+\frac{c}{x}\right]+\frac{3}{4}\,a\,b^2\,(c+x)\,\text{Log}\left[1+\frac{c}{x}\right]^2+\frac{1}{8}\,b^3\,(c+x)\,\text{Log}\left[1+\frac{c}{x}\right]^3-\frac{3}{2}\,a\,b^2\,c\,\text{Log}\left[1-\frac{c}{x}\right]\text{Log}[-c-x]+\frac{3}{2}\,a^2\,b\,c\,\text{Log}[c-x]+\frac{3}{2}\,a\,b^2\,c\,\text{Log}[-c-x]\,\text{Log}\left[\frac{c-x}{2\,c}\right]-\frac{3}{8}\,b^3\,c\,\text{Log}\left[1-\frac{c}{x}\right]^2\text{Log}\left[\frac{c}{x}\right]-\frac{3}{2}\,a\,b^2\,c\,\text{Log}[-c-x]\,\text{Log}\left[-\frac{x}{c}\right]+\frac{3}{2}\,a\,b^2\,c\,\text{Log}\left[1+\frac{c}{x}\right]\text{Log}[-c+x]+\frac{3}{2}\,a\,b^2\,c\,\text{Log}\left[\frac{x}{c}\right]\text{Log}[-c+x]+\frac{3}{2}\,a^2\,b\,c\,\text{Log}[c+x]-\frac{3}{2}\,a\,b^2\,c\,\text{Log}[-c+x]\,\text{Log}\left[\frac{c+x}{2\,c}\right]-\frac{3}{8}\,b^3\,c\,\text{Log}\left[-\frac{c}{x}\right]\text{Log}\left[\frac{c+x}{x}\right]^2-\frac{3}{4}\,b^3\,c\,\text{Log}\left[1-\frac{c}{x}\right]\text{PolyLog}\left[2,1-\frac{c}{x}\right]-\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,\frac{c-x}{2\,c}\right]+\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,-\frac{c}{x}\right]-\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,\frac{c}{x}\right]+\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,\frac{c+x}{2\,c}\right]-\frac{3}{4}\,b^3\,c\,\text{Log}\left[\frac{c+x}{x}\right]\text{PolyLog}\left[2,\frac{c+x}{x}\right]+\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,1-\frac{x}{c}\right]-\frac{3}{2}\,a\,b^2\,c\,\text{PolyLog}\left[2,1+\frac{x}{c}\right]+\frac{3}{4}\,b^3\,c\,\text{PolyLog}\left[3,1-\frac{c}{x}\right]+\frac{3}{4}\,b^3\,c\,\text{PolyLog}\left[3,\frac{c+x}{x}\right]$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^2}\,dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{c}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{x}+\frac{3\,b\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2\,\text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c}+$$
$$\frac{3\,b^2\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\frac{c}{x}}\right]}{c}-\frac{3\,b^3\,\text{PolyLog}\left[3,\,1-\frac{2}{1-\frac{c}{x}}\right]}{2\,c}$$

Result (type 4, 387 leaves, 82 steps):

$$\frac{\left(1-\frac{c}{x}\right)\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^3}{8\,c}-\frac{3\,b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2\,\text{Log}\left[\frac{c+x}{2\,x}\right]}{4\,c}+$$
$$\frac{3\,b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,c}-\frac{3\,b\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)^2\,\text{Log}\left[\frac{c+x}{x}\right]}{8\,x}-$$
$$\frac{3\,b^2\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{x}\right]^2}{8\,c}-\frac{3\,b^2\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{Log}\left[\frac{c+x}{x}\right]^2}{8\,x}-$$
$$\frac{3\,b^3\,\text{Log}\left[-\frac{c-x}{2\,x}\right]\,\text{Log}\left[\frac{c+x}{x}\right]^2}{4\,c}-\frac{b^3\,\left(1+\frac{c}{x}\right)\,\text{Log}\left[\frac{c+x}{x}\right]^3}{8\,c}+\frac{3\,b^2\,\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x}\right]\right)\,\text{PolyLog}\left[2,\,-\frac{c-x}{2\,x}\right]}{2\,c}-$$
$$\frac{3\,b^3\,\text{Log}\left[\frac{c+x}{x}\right]\,\text{PolyLog}\left[2,\,\frac{c+x}{2\,x}\right]}{2\,c}+\frac{3\,b^3\,\text{PolyLog}\left[3,\,-\frac{c-x}{2\,x}\right]}{2\,c}+\frac{3\,b^3\,\text{PolyLog}\left[3,\,\frac{c+x}{2\,x}\right]}{2\,c}$$

Problem 156: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^3}\,dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$-\frac{3\,b\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,c^2}-\frac{3\,b\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2\,c\,x}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2\,c^2}-$$
$$\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2\,x^2}+\frac{3\,b^2\,\left(a+b\,\text{ArcCoth}\left[\frac{x}{c}\right]\right)\,\text{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c^2}+\frac{3\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-\frac{c}{x}}\right]}{2\,c^2}$$

Result (type 8, 1098 leaves, 81 steps):

$$-\frac{3 b^3 \left(1-\frac{c}{x}\right)^2}{64 c^2}-\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2}{16 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right)^2}{64 c^2}+\frac{3 a^2 b}{8 x^2}+\frac{3 a b^2}{8 x^2}-$$

$$\frac{3 a^2 b}{4 c x}-\frac{3 b^3}{2 c x}+\frac{3}{8} b^3 \,\text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c}{x}\right]^2 \text{Log}\left[1+\frac{c}{x}\right]}{x^3},x\right]-$$

$$\frac{3}{8} b^3 \,\text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]^2}{x^3},x\right]+\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right]}{8 c^2}-\frac{3 a b^2 \left(1-\frac{c}{x}\right) \text{Log}\left[1-\frac{c}{x}\right]}{4 c^2}-$$

$$\frac{3 b^3 \left(1-\frac{c}{x}\right) \text{Log}\left[1-\frac{c}{x}\right]}{4 c^2}-\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right]}{8 x^2}-\frac{3 b^2 \left(1-\frac{c}{x}\right)^2 \left(2 a-b \,\text{Log}\left[1-\frac{c}{x}\right]\right)}{32 c^2}+$$

$$\frac{3 b \left(1-\frac{c}{x}\right) \left(2 a-b \,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{8 c^2}-\frac{3 b \left(1-\frac{c}{x}\right)^2 \left(2 a-b \,\text{Log}\left[1-\frac{c}{x}\right]\right)^2}{32 c^2}+\frac{\left(1-\frac{c}{x}\right) \left(2 a-b \,\text{Log}\left[1-\frac{c}{x}\right]\right)^3}{8 c^2}-$$

$$\frac{\left(1-\frac{c}{x}\right)^2 \left(2 a-b \,\text{Log}\left[1-\frac{c}{x}\right]\right)^3}{16 c^2}+\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}\left[1+\frac{c}{x}\right]}{4 x^2}-\frac{3 a b^2 \text{Log}\left[1+\frac{c}{x}\right] \text{Log}[c-x]}{4 c^2}-$$

$$\frac{3 a b^2 \text{Log}[c-x] \text{Log}\left[\frac{x}{c}\right]}{4 c^2}-\frac{3 a b^2 \text{Log}\left[1-\frac{c}{x}\right] \text{Log}[c+x]}{4 c^2}+\frac{3 a b^2 \text{Log}\left[\frac{c-x}{2 c}\right] \text{Log}[c+x]}{4 c^2}-$$

$$\frac{3 a b^2 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c+x]}{4 c^2}+\frac{3 a b^2 \text{Log}[c-x] \text{Log}\left[\frac{c+x}{2 c}\right]}{4 c^2}+\frac{3 a^2 b \,\text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+\frac{3 a b^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 c^2}-$$

$$\frac{9 a b^2 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]}{4 c^2}+\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 c^2}-$$

$$\frac{3 b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]}{32 c^2}-\frac{3 a^2 b \,\text{Log}\left[\frac{c+x}{x}\right]}{4 x^2}-\frac{3 a b^2 \text{Log}\left[\frac{c+x}{x}\right]}{8 x^2}+\frac{3 a b^2 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{4 c^2}-$$

$$\frac{3 b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^2}{8 c^2}-\frac{3 a b^2 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^2}{8 c^2}+\frac{3 b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^2}{32 c^2}+$$

$$\frac{b^3 \left(1+\frac{c}{x}\right) \text{Log}\left[\frac{c+x}{x}\right]^3}{8 c^2}-\frac{b^3 \left(1+\frac{c}{x}\right)^2 \text{Log}\left[\frac{c+x}{x}\right]^3}{16 c^2}+\frac{3 a b^2 \text{PolyLog}\left[2,\frac{c-x}{2 c}\right]}{4 c^2}+\frac{3 a b^2 \text{PolyLog}\left[2,-\frac{c}{x}\right]}{4 c^2}+$$

$$\frac{3 a b^2 \text{PolyLog}\left[2,\frac{c}{x}\right]}{4 c^2}+\frac{3 a b^2 \text{PolyLog}\left[2,\frac{c+x}{2 c}\right]}{4 c^2}-\frac{3 a b^2 \text{PolyLog}\left[2,1-\frac{x}{c}\right]}{4 c^2}-\frac{3 a b^2 \text{PolyLog}\left[2,1+\frac{x}{c}\right]}{4 c^2}$$

Problem 171: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b \,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2 dx$$

Optimal (type 3, 94 leaves, 9 steps):

$$\frac{1}{2} b c x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right) - \frac{1}{4} c^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{4} x^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{4} b^2 c^2 \operatorname{Log}\left[1 - \frac{c^2}{x^4}\right] + b^2 c^2 \operatorname{Log}[x]$$

Result (type 4, 599 leaves, 59 steps):

$$\frac{1}{4} a b c x^2 - \frac{1}{8} b^2 c x^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] + \frac{1}{8} b c \left(1 - \frac{c}{x^2}\right) x^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right) - \frac{1}{16} c^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 + \frac{1}{16} x^4 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{4} b^2 c x^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{4} a b x^4 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{8} b^2 x^4 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{16} b^2 c^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2 + \frac{1}{16} b^2 x^4 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2 + \frac{1}{2} a b c^2 \operatorname{Log}[x] + \frac{1}{2} b^2 c^2 \operatorname{Log}[x] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[c - x^2\right] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] \operatorname{Log}\left[c - x^2\right] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[\frac{x^2}{c}\right] \operatorname{Log}\left[c - x^2\right] - \frac{1}{4} a b c^2 \operatorname{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[c + x^2\right] + \frac{1}{8} b^2 c^2 \operatorname{Log}\left[-\frac{x^2}{c}\right] \operatorname{Log}\left[c + x^2\right] - \frac{1}{8} b^2 c^2 \operatorname{Log}\left[\frac{c - x^2}{2 c}\right] \operatorname{Log}\left[c + x^2\right] - \frac{1}{8} b^2 c^2 \operatorname{Log}\left[c - x^2\right] \operatorname{Log}\left[\frac{c + x^2}{2 c}\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, -\frac{c}{x^2}\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c}{x^2}\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c - x^2}{2 c}\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c + x^2}{2 c}\right] + \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c + x^2}{c}\right] + \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{x^2}{c}\right]$$

Problem 172: Result valid but suboptimal antiderivative.

$$\int x \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2 dx$$

Optimal (type 4, 94 leaves, 5 steps):

$$-\frac{1}{2} c \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 + \frac{1}{2} x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2 - b c \left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 + \frac{c}{x^2}}\right] + \frac{1}{2} b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x^2}}\right]$$

Result (type 4, 404 leaves, 34 steps):

$$\frac{1}{8}\left(1-\frac{c}{x^2}\right)x^2\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2+\frac{1}{2}a\,b\,x^2\,\text{Log}\left[1+\frac{c}{x^2}\right]-\frac{1}{4}b^2\,x^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[1+\frac{c}{x^2}\right]+$$
$$\frac{1}{8}b^2\left(1+\frac{c}{x^2}\right)x^2\,\text{Log}\left[1+\frac{c}{x^2}\right]^2+a\,b\,c\,\text{Log}[x]-\frac{1}{4}b^2\,c\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[-c-x^2\right]-$$
$$\frac{1}{4}b^2\,c\,\text{Log}\left[-\frac{x^2}{c}\right]\text{Log}\left[-c-x^2\right]+\frac{1}{4}b^2\,c\,\text{Log}\left[-c-x^2\right]\text{Log}\left[\frac{c-x^2}{2c}\right]+\frac{1}{4}b^2\,c\,\text{Log}\left[1+\frac{c}{x^2}\right]\text{Log}\left[-c+x^2\right]+$$
$$\frac{1}{4}b^2\,c\,\text{Log}\left[\frac{x^2}{c}\right]\text{Log}\left[-c+x^2\right]+\frac{1}{2}a\,b\,c\,\text{Log}\left[c+x^2\right]-\frac{1}{4}b^2\,c\,\text{Log}\left[-c+x^2\right]\text{Log}\left[\frac{c+x^2}{2c}\right]+$$
$$\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,-\frac{c}{x^2}\right]-\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{c}{x^2}\right]-\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{c-x^2}{2c}\right]+$$
$$\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{c+x^2}{2c}\right]-\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,\frac{c+x^2}{c}\right]+\frac{1}{4}b^2\,c\,\text{PolyLog}\left[2,1-\frac{x^2}{c}\right]$$

Problem 174: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^3}\,dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2c}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2x^2}+$$
$$\frac{b\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)\text{Log}\left[\frac{2}{1-\frac{c}{x^2}}\right]}{c}+\frac{b^2\,\text{PolyLog}\left[2,1-\frac{2}{1-\frac{c}{x^2}}\right]}{2c}$$

Result (type 4, 207 leaves, 28 steps):

$$\frac{\left(1-\frac{c}{x^2}\right)\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{8c}-\frac{b\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)\text{Log}\left[\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right]}{4c}-$$
$$\frac{b^2\,\text{Log}\left[\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right]\text{Log}\left[1+\frac{c}{x^2}\right]}{4c}-\frac{b\left(2a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)\text{Log}\left[1+\frac{c}{x^2}\right]}{4x^2}-$$
$$\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]^2}{8c}+\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1-\frac{c}{x^2}\right)\right]}{4c}-\frac{b^2\,\text{PolyLog}\left[2,\frac{1}{2}\left(1+\frac{c}{x^2}\right)\right]}{4c}$$

Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^5}\,dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{a\,b}{2\,c\,x^2}-\frac{b^2\,\text{ArcCoth}\left[\frac{x^2}{c}\right]}{2\,c\,x^2}+\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4\,c^2}-\frac{\left(a+b\,\text{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4\,x^4}-\frac{b^2\,\text{Log}\left[1-\frac{c^2}{x^4}\right]}{4\,c^2}$$

Result (type 4, 770 leaves, 67 steps):

$$-\frac{b^2\left(1-\frac{c}{x^2}\right)^2}{32\,c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)^2}{32\,c^2}+\frac{a\,b}{8\,x^4}+\frac{b^2}{16\,x^4}-\frac{3\,a\,b}{4\,c\,x^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]}{16\,c^2}-$$
$$\frac{3\,b^2\left(1-\frac{c}{x^2}\right)\text{Log}\left[1-\frac{c}{x^2}\right]}{8\,c^2}-\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]}{16\,x^4}-\frac{b\left(1-\frac{c}{x^2}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)}{16\,c^2}+$$
$$\frac{\left(1-\frac{c}{x^2}\right)\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{8\,c^2}-\frac{\left(1-\frac{c}{x^2}\right)^2\left(2\,a-b\,\text{Log}\left[1-\frac{c}{x^2}\right]\right)^2}{16\,c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]}{4\,c^2}+$$
$$\frac{b^2\left(1+\frac{c}{x^2}\right)^2\text{Log}\left[1+\frac{c}{x^2}\right]}{16\,c^2}+\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[1+\frac{c}{x^2}\right]}{8\,x^4}+\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[1+\frac{c}{x^2}\right]^2}{8\,c^2}-$$
$$\frac{b^2\left(1+\frac{c}{x^2}\right)^2\text{Log}\left[1+\frac{c}{x^2}\right]^2}{16\,c^2}-\frac{b^2\,\text{Log}\left[1+\frac{c}{x^2}\right]\text{Log}\left[c-x^2\right]}{8\,c^2}-\frac{b^2\,\text{Log}\left[\frac{x^2}{c}\right]\text{Log}\left[c-x^2\right]}{8\,c^2}-$$
$$\frac{b^2\,\text{Log}\left[1-\frac{c}{x^2}\right]\text{Log}\left[c+x^2\right]}{8\,c^2}-\frac{b^2\,\text{Log}\left[-\frac{x^2}{c}\right]\text{Log}\left[c+x^2\right]}{8\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c-x^2}{2\,c}\right]\text{Log}\left[c+x^2\right]}{8\,c^2}+$$
$$\frac{b^2\,\text{Log}\left[c-x^2\right]\text{Log}\left[\frac{c+x^2}{2\,c}\right]}{8\,c^2}+\frac{a\,b\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{4\,c^2}+\frac{b^2\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{16\,c^2}-\frac{b^2\left(1+\frac{c}{x^2}\right)\text{Log}\left[\frac{c+x^2}{x^2}\right]}{8\,c^2}-$$
$$\frac{a\,b\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{4\,x^4}-\frac{b^2\,\text{Log}\left[\frac{c+x^2}{x^2}\right]}{16\,x^4}+\frac{b^2\,\text{PolyLog}\left[2,-\frac{c}{x^2}\right]}{8\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c}{x^2}\right]}{8\,c^2}+$$
$$\frac{b^2\,\text{PolyLog}\left[2,\frac{c-x^2}{2\,c}\right]}{8\,c^2}+\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x^2}{2\,c}\right]}{8\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,\frac{c+x^2}{c}\right]}{8\,c^2}-\frac{b^2\,\text{PolyLog}\left[2,1-\frac{x^2}{c}\right]}{8\,c^2}$$

Problem 184: Result optimal but 1 more steps used.

$$\int(d\,x)^m\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)dx$$

Optimal (type 5, 75 leaves, 3 steps):

$$\frac{(d\,x)^{1+m}\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)}{d\,(1+m)}-\frac{2\,b\,c\,d\,(d\,x)^{-1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right]}{1-m^2}$$

Result (type 5, 75 leaves, 4 steps):

$$\frac{(d\,x)^{1+m}\left(a+b\,\text{ArcTanh}\left[\frac{c}{x^2}\right]\right)}{d\,(1+m)}-\frac{2\,b\,c\,d\,(d\,x)^{-1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right]}{1-m^2}$$

Problem 195: Unable to integrate problem.

$$\int x^3 \left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2 dx$$

Optimal (type 3, 211 leaves, 22 steps):

$$\frac{a\,b\sqrt{x}}{2\,c^7}+\frac{71\,b^2\,x}{420\,c^6}+\frac{3\,b^2\,x^2}{70\,c^4}+\frac{b^2\,x^3}{84\,c^2}+\frac{b^2\sqrt{x}\,\text{ArcTanh}\left[c\sqrt{x}\right]}{2\,c^7}+\frac{b\,x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{6\,c^5}+\frac{b\,x^{5/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{10\,c^3}+\frac{b\,x^{7/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{14\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{4\,c^8}+\frac{1}{4}x^4\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2+\frac{44\,b^2\,\text{Log}\left[1-c^2\,x\right]}{105\,c^8}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2,\ x\right]$$

Problem 196: Unable to integrate problem.

$$\int x^2 \left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2 dx$$

Optimal (type 3, 173 leaves, 17 steps):

$$\frac{2\,a\,b\sqrt{x}}{3\,c^5}+\frac{8\,b^2\,x}{45\,c^4}+\frac{b^2\,x^2}{30\,c^2}+\frac{2\,b^2\sqrt{x}\,\text{ArcTanh}\left[c\sqrt{x}\right]}{3\,c^5}+\frac{2\,b\,x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{9\,c^3}+\frac{2\,b\,x^{5/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{15\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{3\,c^6}+\frac{1}{3}x^3\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2+\frac{23\,b^2\,\text{Log}\left[1-c^2\,x\right]}{45\,c^6}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2,\ x\right]$$

Problem 197: Unable to integrate problem.

$$\int x \left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2 dx$$

Optimal (type 3, 129 leaves, 12 steps):

$$\frac{a\,b\sqrt{x}}{c^3}+\frac{b^2\,x}{6\,c^2}+\frac{b^2\sqrt{x}\,\text{ArcTanh}\left[c\sqrt{x}\right]}{c^3}+\frac{b\,x^{3/2}\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)}{3\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2}{2\,c^4}+\frac{1}{2}x^2\left(a+b\,\text{ArcTanh}\left[c\sqrt{x}\right]\right)^2+\frac{2\,b^2\,\text{Log}\left[1-c^2\,x\right]}{3\,c^4}$$

Result (type 8, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[x \left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2, x\right]$$

Problem 198: Unable to integrate problem.

$$\int \left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2 dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{2 a b \sqrt{x}}{c}+\frac{2 b^2 \sqrt{x} \,\text{ArcTanh}\left[c \sqrt{x}\right]}{c}-\frac{\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{c^2}+x \left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2+\frac{b^2 \,\text{Log}\left[1-c^2 x\right]}{c^2}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable}\left[\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2, x\right]$$

Problem 200: Unable to integrate problem.

$$\int \frac{\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x^2} dx$$

Optimal (type 3, 85 leaves, 9 steps):

$$-\frac{2 b c \left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)}{\sqrt{x}}+c^2 \left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2-\frac{\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x}+b^2 c^2 \,\text{Log}[x]-b^2 c^2 \,\text{Log}\left[1-c^2 x\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x^2}, x\right]$$

Problem 201: Unable to integrate problem.

$$\int \frac{\left(a+b \,\text{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x^3} dx$$

Optimal (type 3, 133 leaves, 14 steps):

$$-\frac{b^2 c^2}{6 x}-\frac{b c \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{3 x^{3/2}}-\frac{b c^3 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{\sqrt{x}}+\frac{1}{2} c^4 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2-\frac{\left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{2 x^2}+\frac{2}{3} b^2 c^4 \operatorname{Log}[x]-\frac{2}{3} b^2 c^4 \operatorname{Log}\left[1-c^2 x\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x^3}, x\right]$$

Problem 202: Unable to integrate problem.

$$\int x^3 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 374 leaves, 54 steps):

$$\frac{47 b^3 \sqrt{x}}{70 c^7}+\frac{23 b^3 x^{3/2}}{420 c^5}+\frac{b^3 x^{5/2}}{140 c^3}-\frac{47 b^3 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}{70 c^8}+\frac{71 b^2 x \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{140 c^6}+\frac{9 b^2 x^2 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{70 c^4}+\frac{b^2 x^3 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{28 c^2}+\frac{44 b \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{35 c^8}+\frac{3 b \sqrt{x} \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{4 c^7}+\frac{b x^{3/2} \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{4 c^5}+\frac{3 b x^{5/2} \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{20 c^3}+\frac{3 b x^{7/2} \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{28 c}-\frac{\left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{4 c^8}+\frac{1}{4} x^4 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3-\frac{88 b^2 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[\frac{2}{1-c \sqrt{x}}\right]}{35 c^8}-\frac{44 b^3 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-c \sqrt{x}}\right]}{35 c^8}$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^3 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3, x\right]$$

Problem 203: Unable to integrate problem.

$$\int x^2 \left(a+b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 304 leaves, 34 steps):

$$\frac{19\,b^3\sqrt{x}}{30\,c^5}+\frac{b^3\,x^{3/2}}{30\,c^3}-\frac{19\,b^3\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{30\,c^6}+\frac{8\,b^2\,x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{15\,c^4}+$$
$$\frac{b^2\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{10\,c^2}+\frac{23\,b\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{15\,c^6}+$$
$$\frac{b\,\sqrt{x}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{c^5}+\frac{b\,x^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{3\,c^3}+$$
$$\frac{b\,x^{5/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{5\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{3\,c^6}+\frac{1}{3}\,x^3\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-$$
$$\frac{46\,b^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2}{1-c\,\sqrt{x}}\right]}{15\,c^6}-\frac{23\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,\sqrt{x}}\right]}{15\,c^6}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3,\,x\right]$$

Problem 204: Unable to integrate problem.

$$\int x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3\,dx$$

Optimal (type 4, 234 leaves, 19 steps):

$$\frac{b^3\sqrt{x}}{2\,c^3}-\frac{b^3\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]}{2\,c^4}+\frac{b^2\,x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{2\,c^2}+$$
$$\frac{2\,b\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{c^4}+\frac{3\,b\,\sqrt{x}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,c^3}+$$
$$\frac{b\,x^{3/2}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{2\,c^4}+\frac{1}{2}\,x^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-$$
$$\frac{4\,b^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2}{1-c\,\sqrt{x}}\right]}{c^4}-\frac{2\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,\sqrt{x}}\right]}{c^4}$$

Result (type 8, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3,\,x\right]$$

Problem 205: Unable to integrate problem.

$$\int \left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3\,dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{3\,b\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{c^2}+\frac{3\,b\,\sqrt{x}\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{c^2}+$$
$$x\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-\frac{6\,b^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[\frac{2}{1-c\,\sqrt{x}}\right]}{c^2}-\frac{3\,b^3\,\text{PolyLog}\left[2,\,1-\frac{2}{1-c\,\sqrt{x}}\right]}{c^2}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable}\left[\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3,\,x\right]$$

Problem 207: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^2}\,dx$$

Optimal (type 4, 142 leaves, 8 steps):

$$3\,b\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2-\frac{3\,b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{\sqrt{x}}+$$
$$c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x}+$$
$$6\,b^2\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[2-\frac{2}{1+c\,\sqrt{x}}\right]-3\,b^3\,c^2\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,\sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^2},\,x\right]$$

Problem 208: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{x^3}\,dx$$

Optimal (type 4, 234 leaves, 17 steps):

$$-\frac{b^3\,c^3}{2\,\sqrt{x}}+\frac{1}{2}\,b^3\,c^4\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]-\frac{b^2\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)}{2\,x}+$$
$$2\,b\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2-\frac{b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,x^{3/2}}-$$
$$\frac{3\,b\,c^3\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^2}{2\,\sqrt{x}}+\frac{1}{2}\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3-\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)^3}{2\,x^2}+$$
$$4\,b^2\,c^4\,\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\,\right]\right)\,\text{Log}\left[2-\frac{2}{1+c\,\sqrt{x}}\right]-2\,b^3\,c^4\,\text{PolyLog}\left[2,\,-1+\frac{2}{1+c\,\sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,\sqrt{x}\right]\right)^3}{x^3},\,x\right]$$

Problem 221: Unable to integrate problem.

$$\int x^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2\,dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$\frac{2\,a\,b\,x^{3/2}}{3\,c}+\frac{2\,b^2\,x^{3/2}\,\text{ArcTanh}\left[c\,x^{3/2}\right]}{3\,c}-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2}{3\,c^2}+\frac{1}{3}\,x^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2+\frac{b^2\,\text{Log}\left[1-c^2\,x^3\right]}{3\,c^2}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2,\,x\right]$$

Problem 223: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2}{x^4}\,dx$$

Optimal (type 3, 96 leaves, 9 steps):

$$-\frac{2\,b\,c\,\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)}{3\,x^{3/2}}+\frac{1}{3}\,c^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2-\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2}{3\,x^3}+b^2\,c^2\,\text{Log}[x]-\frac{1}{3}\,b^2\,c^2\,\text{Log}\left[1-c^2\,x^3\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^{3/2}\right]\right)^2}{x^4},\,x\right]$$

Test results for the 49 problems in "7.3.3 (d+e x)^m (a+b arctanh(c x^n))^p.m"

Problem 23: Result valid but suboptimal antiderivative.

$$\int(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 182 leaves, 13 steps):

$$\frac{2\,b\,d\,e^2\,x}{c}+\frac{b\,e^3\,x^2}{4\,c}+\frac{b\,d\,\left(c\,d^2-e^2\right)\,\text{ArcTan}\left[\sqrt{c}\,\,x\right]}{c^{3/2}}-\frac{b\,d\,\left(c\,d^2+e^2\right)\,\text{ArcTanh}\left[\sqrt{c}\,\,x\right]}{c^{3/2}}+\frac{(d+e\,x)^4\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{4\,e}+\frac{b\,\left(c^2\,d^4+6\,c\,d^2\,e^2+e^4\right)\,\text{Log}\left[1-c\,x^2\right]}{8\,c^2\,e}-\frac{b\,\left(c^2\,d^4-6\,c\,d^2\,e^2+e^4\right)\,\text{Log}\left[1+c\,x^2\right]}{8\,c^2\,e}$$

Result (type 3, 220 leaves, 19 steps):

$$\frac{2\,b\,d\,e^2\,x}{c}+\frac{b\,e^3\,x^2}{4\,c}+\frac{a\,(d+e\,x)^4}{4\,e}+\frac{b\,d^3\,\text{ArcTan}\left[\sqrt{c}\,\,x\right]}{\sqrt{c}}-\frac{b\,d\,e^2\,\text{ArcTan}\left[\sqrt{c}\,\,x\right]}{c^{3/2}}-\frac{b\,d^3\,\text{ArcTanh}\left[\sqrt{c}\,\,x\right]}{\sqrt{c}}-\frac{b\,d\,e^2\,\text{ArcTanh}\left[\sqrt{c}\,\,x\right]}{c^{3/2}}-\frac{b\,e^3\,\text{ArcTanh}\left[c\,x^2\right]}{4\,c^2}+b\,d^3\,x\,\text{ArcTanh}\left[c\,x^2\right]+\frac{3}{2}\,b\,d^2\,e\,x^2\,\text{ArcTanh}\left[c\,x^2\right]+b\,d\,e^2\,x^3\,\text{ArcTanh}\left[c\,x^2\right]+\frac{1}{4}\,b\,e^3\,x^4\,\text{ArcTanh}\left[c\,x^2\right]+\frac{3\,b\,d^2\,e\,\text{Log}\left[1-c^2\,x^4\right]}{4\,c}$$

Problem 24: Result optimal but 1 more steps used.

$$\int (d+e\,x)^2\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,dx$$

Optimal (type 3, 158 leaves, 11 steps):

$$\frac{2\,b\,e^2\,x}{3\,c}+\frac{b\,\left(3\,c\,d^2-e^2\right)\,\text{ArcTan}\left[\sqrt{c}\,\,x\right]}{3\,c^{3/2}}-\frac{b\,\left(3\,c\,d^2+e^2\right)\,\text{ArcTanh}\left[\sqrt{c}\,\,x\right]}{3\,c^{3/2}}+\frac{(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,e}+\frac{b\,d\,\left(c\,d^2+3\,e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{6\,c\,e}-\frac{b\,d\,\left(c\,d^2-3\,e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{6\,c\,e}$$

Result (type 3, 158 leaves, 12 steps):

$$\frac{2\,b\,e^2\,x}{3\,c}+\frac{b\,\left(3\,c\,d^2-e^2\right)\,\text{ArcTan}\left[\sqrt{c}\,\,x\right]}{3\,c^{3/2}}-\frac{b\,\left(3\,c\,d^2+e^2\right)\,\text{ArcTanh}\left[\sqrt{c}\,\,x\right]}{3\,c^{3/2}}+\frac{(d+e\,x)^3\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)}{3\,e}+\frac{b\,d\,\left(c\,d^2+3\,e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{6\,c\,e}-\frac{b\,d\,\left(c\,d^2-3\,e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{6\,c\,e}$$

Problem 26: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{d+e\,x}\,dx$$

Optimal (type 4, 325 leaves, 19 steps):

$$\frac{\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)\,\text{Log}[d+e\,x]}{e}-\frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{-c}\,x\right)}{\sqrt{-c}\,d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}-$$
$$\frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{-c}\,x\right)}{\sqrt{-c}\,d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}+\frac{b\,\text{Log}\left[\frac{e\left(1-\sqrt{c}\,x\right)}{\sqrt{c}\,d+e}\right]\,\text{Log}[d+e\,x]}{2\,e}+$$
$$\frac{b\,\text{Log}\left[-\frac{e\left(1+\sqrt{c}\,x\right)}{\sqrt{c}\,d-e}\right]\,\text{Log}[d+e\,x]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\,\frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d-e}\right]}{2\,e}+$$
$$\frac{b\,\text{PolyLog}\left[2,\,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d-e}\right]}{2\,e}-\frac{b\,\text{PolyLog}\left[2,\,\frac{\sqrt{-c}\,(d+e\,x)}{\sqrt{-c}\,d+e}\right]}{2\,e}+\frac{b\,\text{PolyLog}\left[2,\,\frac{\sqrt{c}\,(d+e\,x)}{\sqrt{c}\,d+e}\right]}{2\,e}$$

Result (type 8, 30 leaves, 2 steps):

$$b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]}{d+e\,x},\,x\right]+\frac{a\,\text{Log}[d+e\,x]}{e}$$

Problem 27: Result optimal but 1 more steps used.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^2}\,dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{b\,\sqrt{c}\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c\,d^2+e^2}-\frac{b\,\sqrt{c}\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c\,d^2-e^2}-\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{e\,(d+e\,x)}+$$
$$\frac{2\,b\,c\,d\,e\,\text{Log}[d+e\,x]}{c^2\,d^4-e^4}-\frac{b\,c\,d\,\text{Log}\left[1-c\,x^2\right]}{2\,e\,\left(c\,d^2-e^2\right)}+\frac{b\,c\,d\,\text{Log}\left[1+c\,x^2\right]}{2\,e\,\left(c\,d^2+e^2\right)}$$

Result (type 3, 166 leaves, 10 steps):

$$\frac{b\,\sqrt{c}\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{c\,d^2+e^2}-\frac{b\,\sqrt{c}\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{c\,d^2-e^2}-\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{e\,(d+e\,x)}+$$
$$\frac{2\,b\,c\,d\,e\,\text{Log}[d+e\,x]}{c^2\,d^4-e^4}-\frac{b\,c\,d\,\text{Log}\left[1-c\,x^2\right]}{2\,e\,\left(c\,d^2-e^2\right)}+\frac{b\,c\,d\,\text{Log}\left[1+c\,x^2\right]}{2\,e\,\left(c\,d^2+e^2\right)}$$

Problem 28: Unable to integrate problem.

$$\int\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^3}\,dx$$

Optimal (type 3, 226 leaves, 9 steps):

$$-\frac{b\,c\,d\,e}{\left(c^2 d^4-e^4\right)\,(d+e\,x)}+\frac{b\,c^{3/2}\,d\,\text{ArcTan}\left[\sqrt{c}\,x\right]}{\left(c\,d^2+e^2\right)^2}-\frac{b\,c^{3/2}\,d\,\text{ArcTanh}\left[\sqrt{c}\,x\right]}{\left(c\,d^2-e^2\right)^2}-\frac{a+b\,\text{ArcTanh}\left[c\,x^2\right]}{2\,e\,(d+e\,x)^2}+\frac{b\,c\,e\,\left(3\,c^2 d^4+e^4\right)\,\text{Log}[d+e\,x]}{\left(c^2 d^4-e^4\right)^2}-\frac{b\,c\,\left(c\,d^2+e^2\right)\,\text{Log}\left[1-c\,x^2\right]}{4\,e\,\left(c\,d^2-e^2\right)^2}+\frac{b\,c\,\left(c\,d^2-e^2\right)\,\text{Log}\left[1+c\,x^2\right]}{4\,e\,\left(c\,d^2+e^2\right)^2}$$

Result (type 8, 34 leaves, 2 steps):

$$-\frac{a}{2\,e\,(d+e\,x)^2}+b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\,x^2\right]}{(d+e\,x)^3},\,x\right]$$

Problem 29: Result valid but suboptimal antiderivative.

$$\int (d+e\,x)\,\left(a+b\,\text{ArcTanh}\left[c\,x^2\right]\right)^2\,dx$$

Optimal (type 4, 1085 leaves, 77 steps):

$$a^2 d x+\frac{2 a b d \operatorname{ArcTan}\left[\sqrt{c}\, x\right]}{\sqrt{c}}+\frac{i b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right]^2}{\sqrt{c}}-\frac{2 a b d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right]}{\sqrt{c}}-$$
$$\frac{b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right]^2}{\sqrt{c}}+\frac{e\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2}{2 c}+\frac{1}{2} e x^2\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right)^2+$$
$$\frac{2 b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{2}{1-\sqrt{c}\, x}\right]}{\sqrt{c}}-\frac{2 b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{2}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}+$$
$$\frac{b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{(1+i)\left(1-\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{2 b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{2}{1+i \sqrt{c}\, x}\right]}{\sqrt{c}}-$$
$$\frac{2 b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{2}{1+\sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[-\frac{2 \sqrt{c}\left(1-\sqrt{-c}\, x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(1+\sqrt{c}\, x\right)}\right]}{\sqrt{c}}+$$
$$\frac{b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{2 \sqrt{c}\left(1+\sqrt{-c}\, x\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(1+\sqrt{c}\, x\right)}\right]}{\sqrt{c}}+\frac{b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[\frac{(1-i)\left(1+\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}-$$
$$\frac{b e\left(a+b \operatorname{ArcTanh}\left[c x^2\right]\right) \operatorname{Log}\left[\frac{2}{1-c x^2}\right]}{c}-a b d x \operatorname{Log}\left[1-c x^2\right]-\frac{b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[1-c x^2\right]}{\sqrt{c}}+$$
$$\frac{b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[1-c x^2\right]}{\sqrt{c}}+\frac{1}{4} b^2 d x \operatorname{Log}\left[1-c x^2\right]^2+a b d x \operatorname{Log}\left[1+c x^2\right]+$$
$$\frac{b^2 d \operatorname{ArcTan}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[1+c x^2\right]}{\sqrt{c}}-\frac{b^2 d \operatorname{ArcTanh}\left[\sqrt{c}\, x\right] \operatorname{Log}\left[1+c x^2\right]}{\sqrt{c}}-$$
$$\frac{1}{2} b^2 d x \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right]+\frac{1}{4} b^2 d x \operatorname{Log}\left[1+c x^2\right]^2+\frac{b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1-\sqrt{c}\, x}\right]}{\sqrt{c}}+$$
$$\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}-\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{(1+i)\left(1-\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{2 \sqrt{c}}+$$
$$\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1+i \sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{b^2 d \operatorname{PolyLog}\left[2,1-\frac{2}{1+\sqrt{c}\, x}\right]}{\sqrt{c}}-$$
$$\frac{b^2 d \operatorname{PolyLog}\left[2,1+\frac{2 \sqrt{c}\left(1-\sqrt{-c}\, x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(1+\sqrt{c}\, x\right)}\right]}{2 \sqrt{c}}-\frac{b^2 d \operatorname{PolyLog}\left[2,1-\frac{2 \sqrt{c}\left(1+\sqrt{-c}\, x\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(1+\sqrt{c}\, x\right)}\right]}{2 \sqrt{c}}-$$
$$\frac{i b^2 d \operatorname{PolyLog}\left[2,1-\frac{(1-i)\left(1+\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{2 \sqrt{c}}-\frac{b^2 e \operatorname{PolyLog}\left[2,1-\frac{2}{1-c x^2}\right]}{2 c}$$

Result (type 4, 1216 leaves, 104 steps):

$$\frac{a^2 (d+e x)^2}{2 e}+\frac{2 a b d \,\text{ArcTan}\left[\sqrt{c}\, x\right]}{\sqrt{c}}+\frac{i\, b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right]^2}{\sqrt{c}}-\frac{2 a b d \,\text{ArcTanh}\left[\sqrt{c}\, x\right]}{\sqrt{c}}-$$

$$\frac{b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right]^2}{\sqrt{c}}+2 a b d x \,\text{ArcTanh}\left[c x^2\right]+a b e x^2 \,\text{ArcTanh}\left[c x^2\right]+$$

$$\frac{2 b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{2}{1-\sqrt{c}\, x}\right]}{\sqrt{c}}-\frac{2 b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{2}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}+$$

$$\frac{b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{(1+i) \left(1-\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{2 b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{2}{1+i \sqrt{c}\, x}\right]}{\sqrt{c}}-$$

$$\frac{2 b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{2}{1+\sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[-\frac{2 \sqrt{c} \left(1-\sqrt{-c}\, x\right)}{\left(\sqrt{-c}-\sqrt{c}\right) \left(1+\sqrt{c}\, x\right)}\right]}{\sqrt{c}}+$$

$$\frac{b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{2 \sqrt{c} \left(1+\sqrt{-c}\, x\right)}{\left(\sqrt{-c}+\sqrt{c}\right) \left(1+\sqrt{c}\, x\right)}\right]}{\sqrt{c}}+\frac{b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[\frac{(1-i) \left(1+\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}-$$

$$\frac{b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[1-c x^2\right]}{\sqrt{c}}+\frac{b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[1-c x^2\right]}{\sqrt{c}}+$$

$$\frac{1}{4} b^2 d x \,\text{Log}\left[1-c x^2\right]^2-\frac{b^2 e \left(1-c x^2\right) \text{Log}\left[1-c x^2\right]^2}{8 c}-\frac{b^2 e \,\text{Log}\left[1-c x^2\right] \text{Log}\left[\frac{1}{2} \left(1+c x^2\right)\right]}{4 c}+$$

$$\frac{b^2 d \,\text{ArcTan}\left[\sqrt{c}\, x\right] \text{Log}\left[1+c x^2\right]}{\sqrt{c}}-\frac{b^2 d \,\text{ArcTanh}\left[\sqrt{c}\, x\right] \text{Log}\left[1+c x^2\right]}{\sqrt{c}}+$$

$$\frac{b^2 e \,\text{Log}\left[\frac{1}{2} \left(1-c x^2\right)\right] \text{Log}\left[1+c x^2\right]}{4 c}-\frac{1}{2} b^2 d x \,\text{Log}\left[1-c x^2\right] \text{Log}\left[1+c x^2\right]-$$

$$\frac{1}{4} b^2 e x^2 \,\text{Log}\left[1-c x^2\right] \text{Log}\left[1+c x^2\right]+\frac{1}{4} b^2 d x \,\text{Log}\left[1+c x^2\right]^2+\frac{b^2 e \left(1+c x^2\right) \text{Log}\left[1+c x^2\right]^2}{8 c}+$$

$$\frac{a b e \,\text{Log}\left[1-c^2 x^4\right]}{2 c}-\frac{b^2 e \,\text{PolyLog}\left[2,\frac{1}{2} \left(1-c x^2\right)\right]}{4 c}+\frac{b^2 e \,\text{PolyLog}\left[2,\frac{1}{2} \left(1+c x^2\right)\right]}{4 c}+$$

$$\frac{b^2 d \,\text{PolyLog}\left[2,1-\frac{2}{1-\sqrt{c}\, x}\right]}{\sqrt{c}}+\frac{i\, b^2 d \,\text{PolyLog}\left[2,1-\frac{2}{1-i \sqrt{c}\, x}\right]}{\sqrt{c}}-$$

$$\frac{i\, b^2 d \,\text{PolyLog}\left[2,1-\frac{(1+i) \left(1-\sqrt{c}\, x\right)}{1-i \sqrt{c}\, x}\right]}{2 \sqrt{c}}+\frac{i\, b^2 d \,\text{PolyLog}\left[2,1-\frac{2}{1+i \sqrt{c}\, x}\right]}{\sqrt{c}}+$$

$$\frac{b^2 d \,\text{PolyLog}\left[2,1-\frac{2}{1+\sqrt{c}\, x}\right]}{\sqrt{c}}-\frac{b^2 d \,\text{PolyLog}\left[2,1+\frac{2 \sqrt{c} \left(1-\sqrt{-c}\, x\right)}{\left(\sqrt{-c}-\sqrt{c}\right) \left(1+\sqrt{c}\, x\right)}\right]}{2 \sqrt{c}}-$$

$$\frac{b^2\, d\, \text{PolyLog}\left[2,\, 1 - \frac{2\sqrt{c}\,\left(1+\sqrt{-c}\, x\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(1+\sqrt{c}\, x\right)}\right]}{2\sqrt{c}} - \frac{i\, b^2\, d\, \text{PolyLog}\left[2,\, 1 - \frac{(1-i)\left(1+\sqrt{c}\, x\right)}{1-i\sqrt{c}\, x}\right]}{2\sqrt{c}}$$

Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{d + e\, x}\, dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{d + e\, x},\, x\right]$$

Result (type 8, 56 leaves, 2 steps):

$$2\, a\, b\, \text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\, x^2\right]}{d + e\, x},\, x\right] + b^2\, \text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\, x^2\right]^2}{d + e\, x},\, x\right] + \frac{a^2\, \text{Log}[d + e\, x]}{e}$$

Problem 31: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{(d + e\, x)^2}\, dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b\, \text{ArcTanh}\left[c\, x^2\right]\right)^2}{(d + e\, x)^2},\, x\right]$$

Result (type 8, 202 leaves, 12 steps):

$$-\frac{a^2}{e\,(d + e\, x)} + \frac{2\, a\, b\, \sqrt{c}\, \text{ArcTan}\left[\sqrt{c}\, x\right]}{c\, d^2 + e^2} - \frac{2\, a\, b\, \sqrt{c}\, \text{ArcTanh}\left[\sqrt{c}\, x\right]}{c\, d^2 - e^2} - \frac{2\, a\, b\, \text{ArcTanh}\left[c\, x^2\right]}{e\,(d + e\, x)} + b^2\, \text{CannotIntegrate}\left[\frac{\text{ArcTanh}\left[c\, x^2\right]^2}{(d + e\, x)^2},\, x\right] + \frac{4\, a\, b\, c\, d\, e\, \text{Log}[d + e\, x]}{c^2\, d^4 - e^4} - \frac{a\, b\, c\, d\, \text{Log}\left[1 - c\, x^2\right]}{e\left(c\, d^2 - e^2\right)} + \frac{a\, b\, c\, d\, \text{Log}\left[1 + c\, x^2\right]}{e\left(c\, d^2 + e^2\right)}$$

Problem 32: Result valid but suboptimal antiderivative.

$$\int (d + e\, x)^2\left(a + b\, \text{ArcTanh}\left[c\, x^3\right]\right) dx$$

Optimal (type 3, 336 leaves, 24 steps):

$$-\frac{\sqrt{3}\ b\ d\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ c^{2/3}}+\frac{\sqrt{3}\ b\ d\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ c^{2/3}}+\frac{\sqrt{3}\ b\ d^2\ \text{ArcTan}\left[\frac{1+2\ c^{2/3}\ x^2}{\sqrt{3}}\right]}{2\ c^{1/3}}-\frac{b\ d\ e\ \text{ArcTanh}\left[c^{1/3}\ x\right]}{c^{2/3}}+\frac{(d+e\ x)^3\ \left(a+b\ \text{ArcTanh}\left[c\ x^3\right]\right)}{3\ e}+\frac{b\ d^2\ \text{Log}\left[1-c^{2/3}\ x^2\right]}{2\ c^{1/3}}+\frac{b\ d\ e\ \text{Log}\left[1-c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ c^{2/3}}-\frac{b\ d\ e\ \text{Log}\left[1+c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ c^{2/3}}+\frac{b\ \left(c\ d^3+e^3\right)\ \text{Log}\left[1-c\ x^3\right]}{6\ c\ e}-\frac{b\ \left(c\ d^3-e^3\right)\ \text{Log}\left[1+c\ x^3\right]}{6\ c\ e}-\frac{b\ d^2\ \text{Log}\left[1+c^{2/3}\ x^2+c^{4/3}\ x^4\right]}{4\ c^{1/3}}$$

Result (type 3, 332 leaves, 25 steps):

$$\frac{a\ (d+e\ x)^3}{3\ e}-\frac{\sqrt{3}\ b\ d\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ c^{2/3}}+\frac{\sqrt{3}\ b\ d\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ c^{2/3}}+\frac{\sqrt{3}\ b\ d^2\ \text{ArcTan}\left[\frac{1+2\ c^{2/3}\ x^2}{\sqrt{3}}\right]}{2\ c^{1/3}}-\frac{b\ d\ e\ \text{ArcTanh}\left[c^{1/3}\ x\right]}{c^{2/3}}+b\ d^2\ x\ \text{ArcTanh}\left[c\ x^3\right]+b\ d\ e\ x^2\ \text{ArcTanh}\left[c\ x^3\right]+\frac{1}{3}\ b\ e^2\ x^3\ \text{ArcTanh}\left[c\ x^3\right]+\frac{b\ d^2\ \text{Log}\left[1-c^{2/3}\ x^2\right]}{2\ c^{1/3}}+\frac{b\ d\ e\ \text{Log}\left[1-c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ c^{2/3}}-\frac{b\ d\ e\ \text{Log}\left[1+c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ c^{2/3}}-\frac{b\ d^2\ \text{Log}\left[1+c^{2/3}\ x^2+c^{4/3}\ x^4\right]}{4\ c^{1/3}}+\frac{b\ e^2\ \text{Log}\left[1-c^2\ x^6\right]}{6\ c}$$

Problem 33: Result optimal but 1 more steps used.

$$\int (d+e\ x)\ \left(a+b\ \text{ArcTanh}\left[c\ x^3\right]\right)\, dx$$

Optimal (type 3, 285 leaves, 22 steps):

$$-\frac{\sqrt{3}\ b\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{4\ c^{2/3}}+\frac{\sqrt{3}\ b\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{4\ c^{2/3}}+\frac{\sqrt{3}\ b\ d\ \text{ArcTan}\left[\frac{1+2\ c^{2/3}\ x^2}{\sqrt{3}}\right]}{2\ c^{1/3}}-\frac{b\ e\ \text{ArcTanh}\left[c^{1/3}\ x\right]}{2\ c^{2/3}}-\frac{b\ d^2\ \text{ArcTanh}\left[c\ x^3\right]}{2\ e}+\frac{(d+e\ x)^2\ \left(a+b\ \text{ArcTanh}\left[c\ x^3\right]\right)}{2\ e}+\frac{b\ d\ \text{Log}\left[1-c^{2/3}\ x^2\right]}{2\ c^{1/3}}+\frac{b\ e\ \text{Log}\left[1-c^{1/3}\ x+c^{2/3}\ x^2\right]}{8\ c^{2/3}}-\frac{b\ e\ \text{Log}\left[1+c^{1/3}\ x+c^{2/3}\ x^2\right]}{8\ c^{2/3}}-\frac{b\ d\ \text{Log}\left[1+c^{2/3}\ x^2+c^{4/3}\ x^4\right]}{4\ c^{1/3}}$$

Result (type 3, 285 leaves, 23 steps):

$$-\frac{\sqrt{3}\ b\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{4\ c^{2/3}}+\frac{\sqrt{3}\ b\ e\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ c^{1/3}\ x}{\sqrt{3}}\right]}{4\ c^{2/3}}+\frac{\sqrt{3}\ b\ d\ \text{ArcTan}\left[\frac{1+2\ c^{2/3}\ x^2}{\sqrt{3}}\right]}{2\ c^{1/3}}-\frac{b\ e\ \text{ArcTanh}\left[c^{1/3}\ x\right]}{2\ c^{2/3}}-\frac{b\ d^2\ \text{ArcTanh}\left[c\ x^3\right]}{2\ e}+\frac{(d+e\ x)^2\ \left(a+b\ \text{ArcTanh}\left[c\ x^3\right]\right)}{2\ e}+\frac{b\ d\ \text{Log}\left[1-c^{2/3}\ x^2\right]}{2\ c^{1/3}}+\frac{b\ e\ \text{Log}\left[1-c^{1/3}\ x+c^{2/3}\ x^2\right]}{8\ c^{2/3}}-\frac{b\ e\ \text{Log}\left[1+c^{1/3}\ x+c^{2/3}\ x^2\right]}{8\ c^{2/3}}-\frac{b\ d\ \text{Log}\left[1+c^{2/3}\ x^2+c^{4/3}\ x^4\right]}{4\ c^{1/3}}$$

Problem 34: Unable to integrate problem.

$$\int \frac{a + b\,\text{ArcTanh}[c\,x^3]}{d + e\,x}\,dx$$

Optimal (type 4, 523 leaves, 25 steps):

$$\frac{(a + b\,\text{ArcTanh}[c\,x^3])\,\text{Log}[d + e\,x]}{e} + \frac{b\,\text{Log}\left[\frac{e\,(1-c^{1/3}\,x)}{c^{1/3}\,d+e}\right]\,\text{Log}[d + e\,x]}{2\,e} - \frac{b\,\text{Log}\left[-\frac{e\,(1+c^{1/3}\,x)}{c^{1/3}\,d-e}\right]\,\text{Log}[d + e\,x]}{2\,e} + \frac{b\,\text{Log}\left[-\frac{e\,((-1)^{1/3}+c^{1/3}\,x)}{c^{1/3}\,d-(-1)^{1/3}\,e}\right]\,\text{Log}[d + e\,x]}{2\,e} - \frac{b\,\text{Log}\left[-\frac{e\,((-1)^{2/3}+c^{1/3}\,x)}{c^{1/3}\,d-(-1)^{2/3}\,e}\right]\,\text{Log}[d + e\,x]}{2\,e} + \frac{b\,\text{Log}\left[\frac{(-1)^{2/3}\,e\,(1+(-1)^{1/3}\,c^{1/3}\,x)}{c^{1/3}\,d+(-1)^{2/3}\,e}\right]\,\text{Log}[d + e\,x]}{2\,e} - \frac{b\,\text{Log}\left[\frac{(-1)^{1/3}\,e\,(1+(-1)^{2/3}\,c^{1/3}\,x)}{c^{1/3}\,d+(-1)^{1/3}\,e}\right]\,\text{Log}[d + e\,x]}{2\,e} - \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d-e}\right]}{2\,e} + \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d+e}\right]}{2\,e} + \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d-(-1)^{1/3}\,e}\right]}{2\,e} - \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d+(-1)^{1/3}\,e}\right]}{2\,e} - \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d-(-1)^{2/3}\,e}\right]}{2\,e} + \frac{b\,\text{PolyLog}\left[2, \frac{c^{1/3}\,(d+e\,x)}{c^{1/3}\,d+(-1)^{2/3}\,e}\right]}{2\,e}$$

Result (type 8, 30 leaves, 2 steps):

$$b\,\text{CannotIntegrate}\left[\frac{\text{ArcTanh}[c\,x^3]}{d + e\,x}, x\right] + \frac{a\,\text{Log}[d + e\,x]}{e}$$

Problem 35: Result optimal but 1 more steps used.

$$\int \frac{a + b\,\text{ArcTanh}[c\,x^3]}{(d + e\,x)^2}\,dx$$

Optimal (type 3, 414 leaves, 19 steps):

$$-\frac{\sqrt{3}\,b\,c^{1/3}\,\text{ArcTan}\left[\frac{1-2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,(c^{2/3}\,d^2 + c^{1/3}\,d\,e + e^2)} - \frac{\sqrt{3}\,b\,c^{1/3}\,(c^{1/3}\,d + e)\,\text{ArcTan}\left[\frac{1+2\,c^{1/3}\,x}{\sqrt{3}}\right]}{2\,(c\,d^3 + e^3)} - \frac{a + b\,\text{ArcTanh}[c\,x^3]}{e\,(d + e\,x)} + \frac{b\,c^{1/3}\,(c^{1/3}\,d - e)\,\text{Log}[1 - c^{1/3}\,x]}{2\,(c\,d^3 + e^3)} + \frac{b\,c^{1/3}\,(c^{1/3}\,d + e)\,\text{Log}[1 + c^{1/3}\,x]}{2\,(c\,d^3 - e^3)} - \frac{3\,b\,c\,d^2\,e^2\,\text{Log}[d + e\,x]}{c^2\,d^6 - e^6} - \frac{b\,c^{1/3}\,(c^{1/3}\,d + e)\,\text{Log}[1 - c^{1/3}\,x + c^{2/3}\,x^2]}{4\,(c\,d^3 - e^3)} - \frac{b\,c^{1/3}\,(c^{1/3}\,d - e)\,\text{Log}[1 + c^{1/3}\,x + c^{2/3}\,x^2]}{4\,(c\,d^3 + e^3)} - \frac{b\,c\,d^2\,\text{Log}[1 - c\,x^3]}{2\,e\,(c\,d^3 + e^3)} + \frac{b\,c\,d^2\,\text{Log}[1 + c\,x^3]}{2\,e\,(c\,d^3 - e^3)}$$

Result (type 3, 414 leaves, 20 steps):

$$-\frac{\sqrt{3}\ b\ c^{1/3}\ \text{ArcTan}\left[\frac{1-2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ \left(c^{2/3}\ d^2+c^{1/3}\ d\ e+e^2\right)}-\frac{\sqrt{3}\ b\ c^{1/3}\ \left(c^{1/3}\ d+e\right)\ \text{ArcTan}\left[\frac{1+2\ c^{1/3}\ x}{\sqrt{3}}\right]}{2\ \left(c\ d^3+e^3\right)}-$$
$$\frac{a+b\ \text{ArcTanh}\left[c\ x^3\right]}{e\ (d+e\ x)}+\frac{b\ c^{1/3}\ \left(c^{1/3}\ d-e\right)\ \text{Log}\left[1-c^{1/3}\ x\right]}{2\ \left(c\ d^3+e^3\right)}+\frac{b\ c^{1/3}\ \left(c^{1/3}\ d+e\right)\ \text{Log}\left[1+c^{1/3}\ x\right]}{2\ \left(c\ d^3-e^3\right)}-$$
$$\frac{3\ b\ c\ d^2\ e^2\ \text{Log}[d+e\ x]}{c^2\ d^6-e^6}-\frac{b\ c^{1/3}\ \left(c^{1/3}\ d+e\right)\ \text{Log}\left[1-c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ \left(c\ d^3-e^3\right)}-$$
$$\frac{b\ c^{1/3}\ \left(c^{1/3}\ d-e\right)\ \text{Log}\left[1+c^{1/3}\ x+c^{2/3}\ x^2\right]}{4\ \left(c\ d^3+e^3\right)}-\frac{b\ c\ d^2\ \text{Log}\left[1-c\ x^3\right]}{2\ e\ \left(c\ d^3+e^3\right)}+\frac{b\ c\ d^2\ \text{Log}\left[1+c\ x^3\right]}{2\ e\ \left(c\ d^3-e^3\right)}$$

Test results for the 538 problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Problem 528: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{x^2}\, dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{b}-\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{x}+$$
$$\frac{1}{2}\ b\ c\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)\ \text{Log}\left[1-\frac{1}{1-c^2\ x^2}\right]-\frac{1}{2}\ b\ c\ e\ \text{PolyLog}\left[2,\ \frac{1}{1-c^2\ x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{c\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{b}+b\ c\ d\ \text{Log}[x]-\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{x}-$$
$$\frac{b\ c\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)^2}{4\ e}-\frac{1}{2}\ b\ c\ e\ \text{PolyLog}\left[2,\ c^2\ x^2\right]$$

Problem 530: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{x^4}\, dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2\ c^2\ e\ (a+b\ \text{ArcTanh}[c\ x])}{3\ x}-\frac{c^3\ e\ (a+b\ \text{ArcTanh}[c\ x])^2}{3\ b}-b\ c^3\ e\ \text{Log}[x]+\frac{1}{3}\ b\ c^3\ e\ \text{Log}\left[1-c^2\ x^2\right]-$$
$$\frac{b\ c\ \left(1-c^2\ x^2\right)\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{6\ x^2}-\frac{(a+b\ \text{ArcTanh}[c\ x])\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)}{3\ x^3}+$$
$$\frac{1}{6}\ b\ c^3\ \left(d+e\ \text{Log}\left[1-c^2\ x^2\right]\right)\ \text{Log}\left[1-\frac{1}{1-c^2\ x^2}\right]-\frac{1}{6}\ b\ c^3\ e\ \text{PolyLog}\left[2,\ \frac{1}{1-c^2\ x^2}\right]$$

Result (type 4, 191 leaves, 17 steps):

$$\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{3 x}-\frac{c^3 e (a+b \text{ArcTanh}[c x])^2}{3 b}+\frac{1}{3} b c^3 d \text{Log}[x]-b c^3 e \text{Log}[x]+\frac{1}{3} b c^3 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{6 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{3 x^3}-\frac{b c^3 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{12 e}-\frac{1}{6} b c^3 e \text{PolyLog}\left[2, c^2 x^2\right]$$

Problem 532: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{x^6} \, dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcTanh}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcTanh}[c x])^2}{5 b}-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}+\frac{1}{10} b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e \text{PolyLog}\left[2, \frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e (a+b \text{ArcTanh}[c x])}{15 x^3}+\frac{2 c^4 e (a+b \text{ArcTanh}[c x])}{5 x}-\frac{c^5 e (a+b \text{ArcTanh}[c x])^2}{5 b}+\frac{1}{5} b c^5 d \text{Log}[x]-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{(a+b \text{ArcTanh}[c x]) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}-\frac{b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{20 e}-\frac{1}{10} b c^5 e \text{PolyLog}\left[2, c^2 x^2\right]$$

Test results for the 62 problems in "7.3.5 u (a+b arctanh(c+d x))^p.m"

Test results for the 1378 problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.m"

Problem 620: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcTanh}[a x]} \left(c-\frac{c}{a x}\right)^2 dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{4\,c^2\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[2,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{a\,n}+\frac{1}{a\,(4-n)}2^{n/2}\,c^2\,(1-a\,x)^{2-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[1-\frac{n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-a\,x)\right]$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{1}{a\,(2+n)}2^{3-\frac{n}{2}}\,c^2\,(1+a\,x)^{\frac{2+n}{2}}\,\text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2}(-4+n),2,\frac{4+n}{2},\frac{1}{2}(1+a\,x),1+a\,x\right]$$

Problem 621: Result valid but suboptimal antiderivative.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a\,x}\right)dx$$

Optimal (type 5, 187 leaves, 6 steps):

$$\frac{c\,(1-a\,x)^{2-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{a\,(2-n)}-\frac{1}{a\,(2-n)}2\,c\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-2+n),\frac{n}{2},\frac{1+a\,x}{1-a\,x}\right]+\frac{1}{a\,(2-n)\,(4-n)}2^{n/2}\,c\,(1-n)\,(1-a\,x)^{2-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[\frac{2-n}{2},2-\frac{n}{2},3-\frac{n}{2},\frac{1}{2}(1-a\,x)\right]$$

Result (type 5, 184 leaves, 7 steps):

$$-\frac{2\,c\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,-\frac{n}{2},1-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]}{a\,n}-\frac{2^{1+\frac{n}{2}}\,c\,(1-a\,x)^{1-\frac{n}{2}}\,\text{Hypergeometric2F1}\left[1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-a\,x)\right]}{a\,(2-n)}+\frac{2^{1+\frac{n}{2}}\,c\,(1-a\,x)^{-n/2}\,\text{Hypergeometric2F1}\left[-\frac{n}{2},-\frac{n}{2},1-\frac{n}{2},\frac{1}{2}(1-a\,x)\right]}{a\,n}$$

Problem 791: Result unnecessarily involves higher level functions.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a^2\,x^2}\right)^2dx$$

Optimal (type 5, 331 leaves, 10 steps):

$$-\frac{4c^2(1-ax)^{3-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-4+n)}}{a(4-n)}-\frac{c^2(1-ax)^{3-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-4+n)}}{3a^4x^3}-\frac{c^2(10+n)(1-ax)^{3-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-4+n)}}{6a^3x^2}-\frac{c^2(14+5n+n^2)(1-ax)^{3-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-4+n)}}{6a^2x}-\frac{1}{3a(4-n)}c^2n(10-n^2)(1-ax)^{2-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-4+n)}\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-4+n),\frac{1}{2}(-2+n),\frac{1+ax}{1-ax}\right]+\frac{1}{a(24-10n+n^2)}2^{-1+\frac{n}{2}}c^2n(1-ax)^{3-\frac{n}{2}}\text{Hypergeometric2F1}\left[\frac{4-n}{2},3-\frac{n}{2},4-\frac{n}{2},\frac{1}{2}(1-ax)\right]$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{1}{a(6+n)}2^{3-\frac{n}{2}}c^2(1+ax)^{\frac{6+n}{2}}\text{AppellF1}\left[\frac{6+n}{2},\frac{1}{2}(-4+n),4,\frac{8+n}{2},\frac{1}{2}(1+ax),1+ax\right]$$

Problem 792: Result unnecessarily involves higher level functions.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a^2x^2}\right)dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{1}{a(2-n)}4c(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left[2,1-\frac{n}{2},2-\frac{n}{2},\frac{1-ax}{1+ax}\right]-\frac{2^{1+\frac{n}{2}}c(1-ax)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left[1-\frac{n}{2},-\frac{n}{2},2-\frac{n}{2},\frac{1}{2}(1-ax)\right]}{a(2-n)}$$

Result (type 6, 70 leaves, 3 steps):

$$-\frac{1}{a(4+n)}2^{2-\frac{n}{2}}c(1+ax)^{\frac{4+n}{2}}\text{AppellF1}\left[\frac{4+n}{2},\frac{1}{2}(-2+n),2,\frac{6+n}{2},\frac{1}{2}(1+ax),1+ax\right]$$

Problem 795: Result unnecessarily involves higher level functions.

$$\int e^{n\,\text{ArcTanh}[a\,x]}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

Optimal (type 5, 430 leaves, 9 steps):

$$-\frac{\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{2\,\left(1-a^2 x^2\right)^{3/2}}-\frac{a\,(4+n)\,\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^2\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{2\,\left(1-a^2 x^2\right)^{3/2}}-$$

$$\frac{3\,a^2\,\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{5-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}}{(3-n)\,\left(1-a^2 x^2\right)^{3/2}}-$$

$$\left(a^2\,(3-n^2)\,\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\right.$$
$$\left.\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\frac{1+a x}{1-a x}\right]\right)\Big/\left((3-n)\,\left(1-a^2 x^2\right)^{3/2}\right)+$$

$$\left(2^{\frac{1}{2}(-1+n)}\,a^2\,n\,\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1-a x)^{\frac{5-n}{2}}\,\text{Hypergeometric2F1}\left[\frac{3-n}{2},\frac{5-n}{2},\frac{7-n}{2},\frac{1}{2}(1-a x)\right]\right)\Big/$$
$$\left((3-n)\,(5-n)\,\left(1-a^2 x^2\right)^{3/2}\right)$$

Result (type 6, 103 leaves, 3 steps):

$$-\left(\left(2^{\frac{5}{2}-\frac{n}{2}}\,a^2\,\left(c-\frac{c}{a^2 x^2}\right)^{3/2} x^3\,(1+a x)^{\frac{5+n}{2}}\,\text{AppellF1}\left[\frac{5+n}{2},\frac{1}{2}(-3+n),3,\frac{7+n}{2},\frac{1}{2}(1+a x),1+a x\right]\right)\Big/\right.$$
$$\left.\left((5+n)\,\left(1-a^2 x^2\right)^{3/2}\right)\right)$$

Problem 796: Result valid but suboptimal antiderivative.

$$\int e^{n\,\text{ArcTanh}[a x]}\,\sqrt{c-\frac{c}{a^2 x^2}}\,dx$$

Optimal (type 5, 272 leaves, 6 steps):

$$-\frac{\sqrt{c-\frac{c}{a^2 x^2}}\,x\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-1+n)}}{(1-n)\,\sqrt{1-a^2 x^2}}+$$

$$\left(2\,\sqrt{c-\frac{c}{a^2 x^2}}\,x\,(1-a x)^{\frac{1-n}{2}}\,(1+a x)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a x}{1-a x}\right]\right)\Big/$$
$$\left((1-n)\,\sqrt{1-a^2 x^2}\right)+$$

$$\left(2^{\frac{1+n}{2}}\,n\,\sqrt{c-\frac{c}{a^2 x^2}}\,x\,(1-a x)^{\frac{3-n}{2}}\,\text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-a x)\right]\right)\Big/$$
$$\left(\left(3-4\,n+n^2\right)\,\sqrt{1-a^2 x^2}\right)$$

Result (type 5, 302 leaves, 7 steps):

$$\left(2\sqrt{c-\frac{c}{a^2 x^2}}\ x\ (1-a x)^{\frac{1}{2}(-1-n)}\ (1+a x)^{\frac{1+n}{2}}\ \text{Hypergeometric2F1}\left[1,\ \frac{1}{2}(-1-n),\ \frac{1-n}{2},\ \frac{1-a x}{1+a x}\right]\right)\Big/$$
$$\left((1+n)\sqrt{1-a^2 x^2}\right)-\left(2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\ x\ (1-a x)^{\frac{1}{2}(-1-n)}\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\ \frac{1}{2}(-1-n),\ \frac{1-n}{2},\ \frac{1}{2}(1-a x)\right]\right)\Big/\left((1+n)\sqrt{1-a^2 x^2}\right)+$$
$$\left(2^{\frac{3+n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\ x\ (1-a x)^{\frac{1-n}{2}}\ \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n),\ \frac{1-n}{2},\ \frac{3-n}{2},\ \frac{1}{2}(1-a x)\right]\right)\Big/$$
$$\left((1-n)\sqrt{1-a^2 x^2}\right)$$

Problem 1316: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x\,(c-a^2 c x^2)}\,dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{(1-a x)^{-n/2}(1+a x)^{n/2}}{c n}-\frac{2(1-a x)^{-n/2}(1+a x)^{n/2}\ \text{Hypergeometric2F1}\left[1,\ \frac{n}{2},\ \frac{2+n}{2},\ \frac{1+a x}{1-a x}\right]}{c n}$$

Result (type 5, 100 leaves, 3 steps):

$$\frac{(1-a x)^{-n/2}(1+a x)^{n/2}}{c n}-\frac{1}{c(2-n)}$$
$$2(1-a x)^{1-\frac{n}{2}}(1+a x)^{\frac{1}{2}(-2+n)}\ \text{Hypergeometric2F1}\left[1,\ 1-\frac{n}{2},\ 2-\frac{n}{2},\ \frac{1-a x}{1+a x}\right]$$

Problem 1317: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x^2\,(c-a^2 c x^2)}\,dx$$

Optimal (type 5, 123 leaves, 5 steps):

$$\frac{a(1+n)(1-a x)^{-n/2}(1+a x)^{n/2}}{c n}-\frac{(1-a x)^{-n/2}(1+a x)^{n/2}}{c x}-$$
$$\frac{2 a(1-a x)^{-n/2}(1+a x)^{n/2}\ \text{Hypergeometric2F1}\left[1,\ \frac{n}{2},\ \frac{2+n}{2},\ \frac{1+a x}{1-a x}\right]}{c}$$

Result (type 5, 137 leaves, 5 steps):

$$\frac{a(1+n)(1-a x)^{-n/2}(1+a x)^{n/2}}{c n}-\frac{(1-a x)^{-n/2}(1+a x)^{n/2}}{c x}-\frac{1}{c(2-n)}$$
$$2 a n(1-a x)^{1-\frac{n}{2}}(1+a x)^{\frac{1}{2}(-2+n)}\ \text{Hypergeometric2F1}\left[1,\ 1-\frac{n}{2},\ 2-\frac{n}{2},\ \frac{1-a x}{1+a x}\right]$$

Problem 1323: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}}{x\,\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{\left(4-n-n^2\right)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,\left(4-n^2\right)}+\frac{(4+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c^2\,n}$$

Result (type 5, 200 leaves, 6 steps):

$$\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{\left(4-n-n^2\right)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,\left(4-n^2\right)}+\frac{(4+n)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{1}{c^2\,(2-n)}2\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{1-a\,x}{1+a\,x}\right]$$

Problem 1324: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^2\,\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Optimal (type 5, 239 leaves, 7 steps):

$$\frac{a\,(3+n)\,(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,x}-\frac{a\,\left(6+4\,n-n^2-n^3\right)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,\left(4-n^2\right)}+\frac{a\,\left(6+4\,n+n^2\right)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-\frac{2\,a\,(1-a\,x)^{-n/2}\,(1+a\,x)^{n/2}\,\text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{1+a\,x}{1-a\,x}\right]}{c^2}$$

Result (type 5, 253 leaves, 7 steps):

$$\frac{a\,(3+n)\,(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,(2+n)}-\frac{(1-a\,x)^{-1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,x}-$$
$$\frac{a\,(6+4\,n-n^2-n^3)\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(4-n^2)}+\frac{a\,(6+4\,n+n^2)\,(1-a\,x)^{-n/2}\,(1+a\,x)^{\frac{1}{2}(-2+n)}}{c^2\,n\,(2+n)}-$$
$$\frac{1}{c^2\,(2-n)}2\,a\,n\,(1-a\,x)^{1-\frac{n}{2}}\,(1+a\,x)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left[1,\,1-\frac{n}{2},\,2-\frac{n}{2},\,\frac{1-a\,x}{1+a\,x}\right]$$

Problem 1331: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}\,\sqrt{c-a^2\,c\,x^2}}{x}\,dx$$

Optimal (type 5, 269 leaves, 6 steps):

$$-\frac{(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{c-a^2\,c\,x^2}}{(1-n)\,\sqrt{1-a^2\,x^2}}+$$
$$\left(2\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[1,\,\frac{1}{2}\,(-1+n),\,\frac{1+n}{2},\,\frac{1+a\,x}{1-a\,x}\right]\right)\Big/$$
$$\left((1-n)\,\sqrt{1-a^2\,x^2}\right)+$$
$$\left(2^{\frac{1+n}{2}}\,n\,(1-a\,x)^{\frac{3-n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1-n}{2},\,\frac{3-n}{2},\,\frac{5-n}{2},\,\frac{1}{2}\,(1-a\,x)\right]\right)\Big/$$
$$\left((3-4\,n+n^2)\,\sqrt{1-a^2\,x^2}\right)$$

Result (type 5, 299 leaves, 7 steps):

$$\left(2\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1+n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[1,\,\frac{1}{2}\,(-1-n),\,\frac{1-n}{2},\,\frac{1-a\,x}{1+a\,x}\right]\right)\Big/$$
$$\left((1+n)\,\sqrt{1-a^2\,x^2}\right)-\left(2^{\frac{3+n}{2}}\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,\sqrt{c-a^2\,c\,x^2}\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1}{2}\,(-1-n),\,\frac{1}{2}\,(-1-n),\,\frac{1-n}{2},\,\frac{1}{2}\,(1-a\,x)\right]\right)\Big/\left((1+n)\,\sqrt{1-a^2\,x^2}\right)+$$
$$\left(2^{\frac{3+n}{2}}\,(1-a\,x)^{\frac{1-n}{2}}\,\sqrt{c-a^2\,c\,x^2}\,\text{Hypergeometric2F1}\left[\frac{1}{2}\,(-1-n),\,\frac{1-n}{2},\,\frac{3-n}{2},\,\frac{1}{2}\,(1-a\,x)\right]\right)\Big/$$
$$\left((1-n)\,\sqrt{1-a^2\,x^2}\right)$$

Problem 1332: Result unnecessarily involves higher level functions.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}\,\sqrt{c-a^2\,c\,x^2}}{x^2}\,dx$$

Optimal (type 5, 268 leaves, 6 steps):

$$-\frac{(1-a x)^{\frac{1-n}{2}}(1+a x)^{\frac{1+n}{2}}\sqrt{c-a^2 c x^2}}{x\sqrt{1-a^2 x^2}}-$$
$$\left(2 a n (1-a x)^{\frac{1-n}{2}}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{c-a^2 c x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1-n}{2},\frac{3-n}{2},\frac{1-a x}{1+a x}\right]\right)\Big/$$
$$\left((1-n)\sqrt{1-a^2 x^2}\right)+$$
$$\left(2^{\frac{1+n}{2}} a (1-a x)^{\frac{1-n}{2}}\sqrt{c-a^2 c x^2}\,\text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-a x)\right]\right)\Big/$$
$$\left((1-n)\sqrt{1-a^2 x^2}\right)$$

Result (type 6, 97 leaves, 3 steps):

$$\left(2^{\frac{3}{2}-\frac{n}{2}} a (1+a x)^{\frac{3+n}{2}}\sqrt{c-a^2 c x^2}\,\text{AppellF1}\left[\frac{3+n}{2},\frac{1}{2}(-1+n),2,\frac{5+n}{2},\frac{1}{2}(1+a x),1+a x\right]\right)\Big/$$
$$\left((3+n)\sqrt{1-a^2 x^2}\right)$$

Problem 1345: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x\left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 243 leaves, 6 steps):

$$\frac{(1-a x)^{\frac{1}{2}(-1-n)}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2 x^2}}{c(1+n)\sqrt{c-a^2 c x^2}}-\frac{(2+n)(1-a x)^{\frac{1-n}{2}}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2 x^2}}{c\left(1-n^2\right)\sqrt{c-a^2 c x^2}}+$$
$$\left(2(1-a x)^{\frac{1-n}{2}}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2 x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a x}{1-a x}\right]\right)\Big/$$
$$\left(c(1-n)\sqrt{c-a^2 c x^2}\right)$$

Result (type 5, 247 leaves, 6 steps):

$$\frac{(1-a x)^{\frac{1}{2}(-1-n)}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2 x^2}}{c(1+n)\sqrt{c-a^2 c x^2}}-\frac{(2+n)(1-a x)^{\frac{1-n}{2}}(1+a x)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2 x^2}}{c\left(1-n^2\right)\sqrt{c-a^2 c x^2}}-$$
$$\left(2(1-a x)^{\frac{3-n}{2}}(1+a x)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2 x^2}\,\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a x}{1+a x}\right]\right)\Big/$$
$$\left(c(3-n)\sqrt{c-a^2 c x^2}\right)$$

Problem 1346: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x^2\left(c-a^2 c x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 321 leaves, 7 steps):

$$\frac{a\,(2+n)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,x\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,(2+2\,n+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+\left(2\,a\,n\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]\right)\Big/\left(c\,(1-n)\,\sqrt{c-a^2\,c\,x^2}\right)$$

Result (type 5, 325 leaves, 7 steps):

$$\frac{a\,(2+n)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,x\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,(2+2\,n+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}-\left(2\,a\,n\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a\,x}{1+a\,x}\right]\right)\Big/\left(c\,(3-n)\,\sqrt{c-a^2\,c\,x^2}\right)$$

Problem 1347: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^3\,(c-a^2\,c\,x^2)^{3/2}}\,dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\frac{a^2\,(3+2\,n+n^2)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,(1+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,x^2\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,n\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,x\,\sqrt{c-a^2\,c\,x^2}}-\frac{a^2\,(6+5\,n+2\,n^2+n^3)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}}{2\,c\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+\left(a^2\,(3+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]\right)\Big/\left(c\,(1-n)\,\sqrt{c-a^2\,c\,x^2}\right)$$

Result (type 5, 422 leaves, 8 steps):

$$\frac{a^2\left(3+2n+n^2\right)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}}-$$

$$\frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}}-\frac{an(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx\sqrt{c-a^2cx^2}}-$$

$$\frac{a^2\left(6+5n+2n^2+n^3\right)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-$$

$$\left(a^2\left(3+n^2\right)(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}\ \text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-ax}{1+ax}\right]\right)\Big/$$
$$\left(c(3-n)\sqrt{c-a^2cx^2}\right)$$

Problem 1352: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\frac{(1-ax)^{\frac{1}{2}(-3-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\sqrt{c-a^2cx^2}}+\frac{(6+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(1+n)(3+n)\sqrt{c-a^2cx^2}}-$$

$$\frac{\left(15+6n+n^2\right)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2(3+n)\left(1-n^2\right)\sqrt{c-a^2cx^2}}+$$

$$\frac{\left(18+7n-2n^2-n^3\right)(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2}}{c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}+$$

$$\left(2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}\ \text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+ax}{1-ax}\right]\right)\Big/$$
$$\left(c^2(1-n)\sqrt{c-a^2cx^2}\right)$$

Result (type 5, 421 leaves, 8 steps):

$$\frac{(1-a x)^{\frac{1}{2}(-3-n)}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(3+n)\,\sqrt{c-a^2 c x^2}}+\frac{(6+n)\,(1-a x)^{\frac{1}{2}(-1-n)}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2 c x^2}}-$$
$$\frac{\left(15+6 n+n^2\right)\,(1-a x)^{\frac{1-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(3+n)\,\left(1-n^2\right)\,\sqrt{c-a^2 c x^2}}+$$
$$\frac{\left(18+7 n-2 n^2-n^3\right)\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,\left(9-10 n^2+n^4\right)\,\sqrt{c-a^2 c x^2}}-$$
$$\left(2\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}\;\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a x}{1+a x}\right]\right)\Big/$$
$$\left(c^2\,(3-n)\,\sqrt{c-a^2 c x^2}\right)$$

Problem 1353: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcTanh}[a x]}}{x^2\,\left(c-a^2 c x^2\right)^{5/2}}\,dx$$

Optimal (type 5, 507 leaves, 9 steps):

$$\frac{a\,(4+n)\,(1-a x)^{\frac{1}{2}(-3-n)}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(3+n)\,\sqrt{c-a^2 c x^2}}-\frac{(1-a x)^{\frac{1}{2}(-3-n)}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,x\,\sqrt{c-a^2 c x^2}}+$$
$$\frac{a\,\left(12+6 n+n^2\right)\,(1-a x)^{\frac{1}{2}(-1-n)}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2 c x^2}}-$$
$$\frac{a\,\left(24+15 n+6 n^2+n^3\right)\,(1-a x)^{\frac{1-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,(3+n)\,\left(1-n^2\right)\,\sqrt{c-a^2 c x^2}}+$$
$$\frac{a\,\left(24+18 n+7 n^2-2 n^3-n^4\right)\,(1-a x)^{\frac{3-n}{2}}\,(1+a x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2 x^2}}{c^2\,\left(9-10 n^2+n^4\right)\,\sqrt{c-a^2 c x^2}}+$$
$$\left(2\,a\,n\,(1-a x)^{\frac{1-n}{2}}\,(1+a x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2 x^2}\;\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a x}{1-a x}\right]\right)\Big/$$
$$\left(c^2\,(1-n)\,\sqrt{c-a^2 c x^2}\right)$$

Result (type 5, 511 leaves, 9 steps):

$$\frac{a\,(4+n)\,(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-\frac{(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,x\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{a\,(12+6\,n+n^2)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{a\,(24+15\,n+6\,n^2+n^3)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(3+n)\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{a\,(24+18\,n+7\,n^2-2\,n^3-n^4)\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{c^2\,(9-10\,n^2+n^4)\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\left(2\,a\,n\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}\;\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a\,x}{1+a\,x}\right]\right)\Big/$$
$$\left(c^2\,(3-n)\,\sqrt{c-a^2\,c\,x^2}\right)$$

Problem 1354: Result valid but suboptimal antiderivative.

$$\int\frac{e^{n\,\text{ArcTanh}[a\,x]}}{x^3\,(c-a^2\,c\,x^2)^{5/2}}\,dx$$

Optimal (type 5, 623 leaves, 10 steps):

$$\frac{a^2\,(5+4\,n+n^2)\,(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{2\,c^2\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{2\,c^2\,x^2\,\sqrt{c-a^2\,c\,x^2}}-\frac{a\,n\,(1-a\,x)^{\frac{1}{2}(-3-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{2\,c^2\,x\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\frac{a^2\,(30+17\,n+6\,n^2+n^3)\,(1-a\,x)^{\frac{1}{2}(-1-n)}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{2\,c^2\,(1+n)\,(3+n)\,\sqrt{c-a^2\,c\,x^2}}-$$
$$\frac{a^2\,(75+54\,n+20\,n^2+6\,n^3+n^4)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}}{2\,c^2\,(3+n)\,(1-n^2)\,\sqrt{c-a^2\,c\,x^2}}+$$
$$\left(a^2\,(90+59\,n+8\,n^2+2\,n^3-2\,n^4-n^5)\,(1-a\,x)^{\frac{3-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-3+n)}\,\sqrt{1-a^2\,x^2}\right)\Big/$$
$$\left(2\,c^2\,(9-10\,n^2+n^4)\,\sqrt{c-a^2\,c\,x^2}\right)+\left(a^2\,(5+n^2)\,(1-a\,x)^{\frac{1-n}{2}}\,(1+a\,x)^{\frac{1}{2}(-1+n)}\,\sqrt{1-a^2\,x^2}\right.$$
$$\left.\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{1+a\,x}{1-a\,x}\right]\right)\Big/\left(c^2\,(1-n)\,\sqrt{c-a^2\,c\,x^2}\right)$$

Result (type 5, 628 leaves, 10 steps):

$$\frac{a^2 \left(5+4 n+n^2\right) (1-a x)^{\frac{1}{2} (-3-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) \sqrt{c-a^2 c x^2}}-$$

$$\frac{(1-a x)^{\frac{1}{2} (-3-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x^2 \sqrt{c-a^2 c x^2}}-\frac{a n (1-a x)^{\frac{1}{2} (-3-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 x \sqrt{c-a^2 c x^2}}+$$

$$\frac{a^2 \left(30+17 n+6 n^2+n^3\right) (1-a x)^{\frac{1}{2} (-1-n)} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}}-$$

$$\frac{a^2 \left(75+54 n+20 n^2+6 n^3+n^4\right) (1-a x)^{\frac{1-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}}{2 c^2 (3+n) \left(1-n^2\right) \sqrt{c-a^2 c x^2}}+$$

$$\left(a^2 \left(90+59 n+8 n^2+2 n^3-2 n^4-n^5\right) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}\right) \Big/ \left(2 c^2 \left(9-10 n^2+n^4\right) \sqrt{c-a^2 c x^2}\right)-$$

$$\left(a^2 \left(5+n^2\right) (1-a x)^{\frac{3-n}{2}} (1+a x)^{\frac{1}{2} (-3+n)} \sqrt{1-a^2 x^2}\ \text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{1-a x}{1+a x}\right]\right) \Big/ \left(c^2 (3-n) \sqrt{c-a^2 c x^2}\right)$$

Test results for the 361 problems in "7.3.7 Inverse hyperbolic tangent functions.m"

Test results for the 300 problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

Problem 76: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCoth}[a+b x]}{c+d x^2} \, dx$$

Optimal (type 4, 673 leaves, 15 steps):

$$\frac{\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]\text{Log}\left[1+\frac{\left(b^2 c+a^2 d\right)(1-a-b x)}{\left(b^2 c-b\sqrt{-c}\sqrt{d}-(1-a) a d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}-$$

$$\frac{\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]\text{Log}\left[1+\frac{\left(b^2 c+a^2 d\right)(1-a-b x)}{\left(b^2 c+b\sqrt{-c}\sqrt{d}-(1-a) a d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}+$$

$$\frac{\text{Log}\left[\frac{1+a+b x}{a+b x}\right]\text{Log}\left[1-\frac{\left(b^2 c+a^2 d\right)(1+a+b x)}{\left(b^2 c-b\sqrt{-c}\sqrt{d}+a(1+a) d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}-$$

$$\frac{\text{Log}\left[\frac{1+a+b x}{a+b x}\right]\text{Log}\left[1-\frac{\left(b^2 c+a^2 d\right)(1+a+b x)}{\left(b^2 c+b\sqrt{-c}\sqrt{d}+a(1+a) d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}+$$

$$\frac{\text{PolyLog}\left[2,-\frac{\left(b^2 c+a^2 d\right)(1-a-b x)}{\left(b^2 c-b\sqrt{-c}\sqrt{d}-(1-a) a d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left[2,-\frac{\left(b^2 c+a^2 d\right)(1-a-b x)}{\left(b^2 c+b\sqrt{-c}\sqrt{d}-(1-a) a d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}+$$

$$\frac{\text{PolyLog}\left[2,\frac{\left(b^2 c+a^2 d\right)(1+a+b x)}{\left(b^2 c-b\sqrt{-c}\sqrt{d}+a(1+a) d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left[2,\frac{\left(b^2 c+a^2 d\right)(1+a+b x)}{\left(b^2 c+b\sqrt{-c}\sqrt{d}+a(1+a) d\right)(a+b x)}\right]}{4\sqrt{-c}\sqrt{d}}$$

Result (type 4, 597 leaves, 37 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\left(\text{Log}[-1+a+b x]-\text{Log}\left[-\frac{1-a-b x}{a+b x}\right]-\text{Log}[a+b x]\right)}{2\sqrt{c}\sqrt{d}}+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{d}\,x}{\sqrt{c}}\right]\left(\text{Log}[a+b x]-\text{Log}[1+a+b x]+\text{Log}\left[\frac{1+a+b x}{a+b x}\right]\right)}{2\sqrt{c}\sqrt{d}}-$$

$$\frac{\text{Log}[-1+a+b x]\,\text{Log}\left[\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}+\frac{\text{Log}[1+a+b x]\,\text{Log}\left[\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}+$$

$$\frac{\text{Log}[-1+a+b x]\,\text{Log}\left[\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}-\frac{\text{Log}[1+a+b x]\,\text{Log}\left[\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}-$$

$$\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\,(1-a-b x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\,(1-a-b x)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}-$$

$$\frac{\text{PolyLog}\left[2,-\frac{\sqrt{d}\,(1+a+b x)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left[2,\frac{\sqrt{d}\,(1+a+b x)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}$$

Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{x^2}\,dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c\,e\,(a+b\,\text{ArcCoth}[c\,x])^2}{b}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{x}+\frac{1}{2}\,b\,c\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)\,\text{Log}\left[1-\frac{1}{1-c^2\,x^2}\right]-\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,\frac{1}{1-c^2\,x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{c\,e\,(a+b\,\text{ArcCoth}[c\,x])^2}{b}+b\,c\,d\,\text{Log}[x]-\frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{x}-\frac{b\,c\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)^2}{4\,e}-\frac{1}{2}\,b\,c\,e\,\text{PolyLog}\left[2,c^2\,x^2\right]$$

Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{x^4}\,dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2\,c^2\,e\,(a+b\,\text{ArcCoth}[c\,x])}{3\,x}-\frac{c^3\,e\,(a+b\,\text{ArcCoth}[c\,x])^2}{3\,b}-b\,c^3\,e\,\text{Log}[x]+\frac{1}{3}\,b\,c^3\,e\,\text{Log}\left[1-c^2\,x^2\right]-\frac{b\,c\,\left(1-c^2\,x^2\right)\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{6\,x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{3\,x^3}+\frac{1}{6}\,b\,c^3\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)\,\text{Log}\left[1-\frac{1}{1-c^2\,x^2}\right]-\frac{1}{6}\,b\,c^3\,e\,\text{PolyLog}\left[2,\frac{1}{1-c^2\,x^2}\right]$$

Result (type 4, 191 leaves, 17 steps):

$$\frac{2\,c^2\,e\,(a+b\,\text{ArcCoth}[c\,x])}{3\,x}-\frac{c^3\,e\,(a+b\,\text{ArcCoth}[c\,x])^2}{3\,b}+\frac{1}{3}\,b\,c^3\,d\,\text{Log}[x]-b\,c^3\,e\,\text{Log}[x]+\frac{1}{3}\,b\,c^3\,e\,\text{Log}\left[1-c^2\,x^2\right]-\frac{b\,c\,\left(1-c^2\,x^2\right)\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{6\,x^2}-\frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{3\,x^3}-\frac{b\,c^3\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)^2}{12\,e}-\frac{1}{6}\,b\,c^3\,e\,\text{PolyLog}\left[2,c^2\,x^2\right]$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCoth}[c\,x])\,\left(d+e\,\text{Log}\left[1-c^2\,x^2\right]\right)}{x^6}\,dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e \left(a+b \text{ArcCoth}[c x]\right)}{15 x^3}+\frac{2 c^4 e \left(a+b \text{ArcCoth}[c x]\right)}{5 x}-\frac{c^5 e \left(a+b \text{ArcCoth}[c x]\right)^2}{5 b}-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{\left(a+b \text{ArcCoth}[c x]\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}+\frac{1}{10} b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right) \text{Log}\left[1-\frac{1}{1-c^2 x^2}\right]-\frac{1}{10} b c^5 e \text{PolyLog}\left[2,\frac{1}{1-c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\frac{7 b c^3 e}{60 x^2}+\frac{2 c^2 e \left(a+b \text{ArcCoth}[c x]\right)}{15 x^3}+\frac{2 c^4 e \left(a+b \text{ArcCoth}[c x]\right)}{5 x}-\frac{c^5 e \left(a+b \text{ArcCoth}[c x]\right)^2}{5 b}+\frac{1}{5} b c^5 d \text{Log}[x]-\frac{5}{6} b c^5 e \text{Log}[x]+\frac{19}{60} b c^5 e \text{Log}\left[1-c^2 x^2\right]-\frac{b c \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{20 x^4}-\frac{b c^3 \left(1-c^2 x^2\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{10 x^2}-\frac{\left(a+b \text{ArcCoth}[c x]\right) \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)}{5 x^5}-\frac{b c^5 \left(d+e \text{Log}\left[1-c^2 x^2\right]\right)^2}{20 e}-\frac{1}{10} b c^5 e \text{PolyLog}\left[2,c^2 x^2\right]$$

Test results for the 935 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.m"

Problem 542: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcCoth}[a x]} \left(c-\frac{c}{a x}\right) dx$$

Optimal (type 5, 185 leaves, 5 steps):

$$c \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1+\frac{1}{a x}\right)^{n/2} x-\frac{1}{a n} 2 c (1-n) \left(1-\frac{1}{a x}\right)^{-n/2} \left(1+\frac{1}{a x}\right)^{n/2} \text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]-\frac{2^{n/2} c \left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}} \text{Hypergeometric2F1}\left[1-\frac{n}{2},1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{2 a}\right]}{a (2-n)}$$

Result (type 6, 81 leaves, 2 steps):

$$-\frac{2^{2-\frac{n}{2}} c \left(1+\frac{1}{a x}\right)^{\frac{2+n}{2}} \text{AppellF1}\left[\frac{2+n}{2},\frac{1}{2} (-2+n),2,\frac{4+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]}{a (2+n)}$$

Problem 751: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}\,x^3}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Optimal (type 5, 359 leaves, 7 steps):

$$-\frac{(2+n)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a\,(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(2+2 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a\,(1-n)\,(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{\left(c-a^2 c x^2\right)^{3/2}}-\left(2\,n\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3\,\text{Hypergeometric2F1}\left[1,\frac{1}{2}(-1+n),\frac{1+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]\right)\Big/\left(a\,(1-n)\left(c-a^2 c x^2\right)^{3/2}\right)$$

Result (type 5, 363 leaves, 7 steps):

$$-\frac{(2+n)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a\,(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(2+2 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a\,(1-n)\,(1+n)\left(c-a^2 c x^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{\left(c-a^2 c x^2\right)^{3/2}}+\left(2\,n\left(1-\frac{1}{a^2 x^2}\right)^{3/2}\left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^3\,\text{Hypergeometric2F1}\left[1,\frac{3-n}{2},\frac{5-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]\right)\Big/\left(a\,(3-n)\left(c-a^2 c x^2\right)^{3/2}\right)$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}\,x^4}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Optimal (type 5, 463 leaves, 8 steps):

$$-\frac{\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\frac{(6+n)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\frac{\left(15+6 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n)(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\frac{\left(18+7 n-2 n^2-n^3\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{\left(9-10 n^2+n^4\right)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\left(2\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^5 \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]\right) \Big/ \left((1-n)\left(c-a^2 c x^2\right)^{5/2}\right)$$

Result (type 5, 467 leaves, 8 steps):

$$-\frac{\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-3-n)} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\frac{(6+n)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\frac{\left(15+6 n+n^2\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n)(1+n)(3+n)\left(c-a^2 c x^2\right)^{5/2}}-$$

$$\frac{\left(18+7 n-2 n^2-n^3\right)\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{\left(9-10 n^2+n^4\right)\left(c-a^2 c x^2\right)^{5/2}}+$$

$$\left(2\left(1-\frac{1}{a^2 x^2}\right)^{5/2} \left(1-\frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5 \text{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]\right) \Big/ \left((3-n)\left(c-a^2 c x^2\right)^{5/2}\right)$$

Problem 928: Result unnecessarily involves higher level functions.

$$\int e^{n \,\text{ArcCoth}[a x]} \left(c-\frac{c}{a^2 x^2}\right) dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$\frac{1}{a\,(2-n)}4\,c\left(1-\frac{1}{a\,x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left[2,\,1-\frac{n}{2},\,2-\frac{n}{2},\,\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]-$$

$$\frac{2^{1+\frac{n}{2}}\,c\left(1-\frac{1}{a\,x}\right)^{1-\frac{n}{2}}\text{Hypergeometric2F1}\left[1-\frac{n}{2},\,-\frac{n}{2},\,2-\frac{n}{2},\,\frac{a-\frac{1}{x}}{2\,a}\right]}{a\,(2-n)}$$

Result (type 6, 81 leaves, 2 steps):

$$-\frac{2^{2-\frac{n}{2}}\,c\left(1+\frac{1}{a\,x}\right)^{\frac{4+n}{2}}\text{AppellF1}\left[\frac{4+n}{2},\,\frac{1}{2}\,(-2+n),\,2,\,\frac{6+n}{2},\,\frac{a+\frac{1}{x}}{2\,a},\,1+\frac{1}{a\,x}\right]}{a\,(4+n)}$$

Problem 929: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{c-\frac{c}{a^2\,x^2}}\,dx$$

Optimal (type 5, 150 leaves, 5 steps):

$$-\frac{(1+n)\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{n/2}}{a\,c\,n}+\frac{\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{n/2}x}{c}+$$

$$\frac{2\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{n/2}\text{Hypergeometric2F1}\left[1,\,\frac{n}{2},\,\frac{2+n}{2},\,\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a\,c}$$

Result (type 5, 164 leaves, 5 steps):

$$-\frac{(1+n)\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{n/2}}{a\,c\,n}+\frac{\left(1-\frac{1}{a\,x}\right)^{-n/2}\left(1+\frac{1}{a\,x}\right)^{n/2}x}{c}+\frac{1}{a\,c\,(2-n)}$$

$$2\,n\left(1-\frac{1}{a\,x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a\,x}\right)^{\frac{1}{2}(-2+n)}\text{Hypergeometric2F1}\left[1,\,1-\frac{n}{2},\,2-\frac{n}{2},\,\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]$$

Problem 930: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n\,\text{ArcCoth}[a\,x]}}{\left(c-\frac{c}{a^2\,x^2}\right)^2}\,dx$$

Optimal (type 5, 289 leaves, 7 steps):

$$-\frac{(3+n)\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)}+\frac{\left(6+4 n-n^2-n^3\right)\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)}-$$
$$\frac{\left(6+4 n+n^2\right)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)}+\frac{\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} x}{c^2}+$$
$$\frac{2\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{n/2} \text{Hypergeometric2F1}\left[1,\frac{n}{2},\frac{2+n}{2},\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a c^2}$$

Result (type 5, 303 leaves, 7 steps):

$$-\frac{(3+n)\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)}+\frac{\left(6+4 n-n^2-n^3\right)\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)}-$$
$$\frac{\left(6+4 n+n^2\right)\left(1-\frac{1}{a x}\right)^{-n/2}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)}+\frac{\left(1-\frac{1}{a x}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} x}{c^2}+\frac{1}{a c^2 (2-n)}$$
$$2 n\left(1-\frac{1}{a x}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1,1-\frac{n}{2},2-\frac{n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]$$

Problem 931: Result unnecessarily involves higher level functions.

$$\int e^{n \text{ArcCoth}[a x]} \sqrt{c-\frac{c}{a^2 x^2}}\, dx$$

Optimal (type 5, 295 leaves, 6 steps):

$$\frac{\sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1+n}{2}} x}{\sqrt{1-\frac{1}{a^2 x^2}}}+$$
$$\left(2 n \sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left[1,\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]\right)\Big/$$
$$\left(a(1-n)\sqrt{1-\frac{1}{a^2 x^2}}\right)-\frac{2^{\frac{1+n}{2}} \sqrt{c-\frac{c}{a^2 x^2}}\left(1-\frac{1}{a x}\right)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left[\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{a-\frac{1}{x}}{2 a}\right]}{a(1-n)\sqrt{1-\frac{1}{a^2 x^2}}}$$

Result (type 6, 111 leaves, 3 steps):

$$-\left(\left(2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2 x^2}}\left(1+\frac{1}{a x}\right)^{\frac{3+n}{2}}\text{AppellF1}\left[\frac{3+n}{2},\frac{1}{2}(-1+n),2,\frac{5+n}{2},\frac{a+\frac{1}{x}}{2 a},1+\frac{1}{a x}\right]\right)\Big/\left(a(3+n)\sqrt{1-\frac{1}{a^2 x^2}}\right)\right)$$

Test results for the 190 problems in "7.5.1 u (a+b arcsech(c x))^n.m"

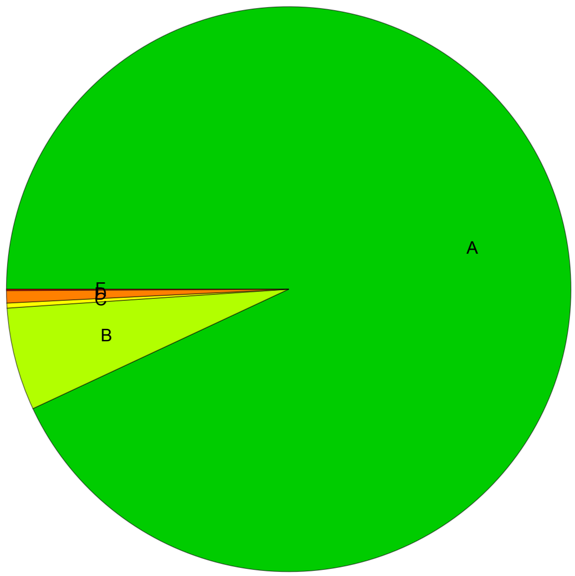
Test results for the 100 problems in "7.5.2 Inverse hyperbolic secant functions.m"

Test results for the 178 problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Test results for the 71 problems in "7.6.2 Inverse hyperbolic cosecant functions.m"

Summary of Integration Test Results

6650 integration problems



A - 6193 optimal antiderivatives

B - 391 valid but suboptimal antiderivatives

C - 19 unnecessarily complex antiderivatives

D - 47 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives