Mathematica 11.3 Integration Test Results

Test results for the 243 problems in "7.3.2 (d x) m (a+b arctanh(c x^n) p .m"

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \ x\right]\right)^2}{x} \, \mathrm{d} x$$

Optimal (type 4, 117 leaves, 6 steps):

$$2 \left(a + b \operatorname{ArcTanh} \left[c \, x \right] \right)^2 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \, x} \right] - \\ b \left(a + b \operatorname{ArcTanh} \left[c \, x \right] \right) \operatorname{PolyLog} \left[2, \, 1 - \frac{2}{1 - c \, x} \right] + b \left(a + b \operatorname{ArcTanh} \left[c \, x \right] \right) \operatorname{PolyLog} \left[2, \, -1 + \frac{2}{1 - c \, x} \right] + \\ \frac{1}{2} b^2 \operatorname{PolyLog} \left[3, \, 1 - \frac{2}{1 - c \, x} \right] - \frac{1}{2} b^2 \operatorname{PolyLog} \left[3, \, -1 + \frac{2}{1 - c \, x} \right]$$

Result (type 4, 151 leaves):

$$\begin{split} & a^2 \, \text{Log}[\text{c} \, x] \, + a \, b \, \left(-\text{PolyLog}[2\text{, -c} \, x] \, + \text{PolyLog}[2\text{, c} \, x] \, \right) \, + \\ & b^2 \, \left(\frac{\text{i} \, \pi^3}{24} - \frac{2}{3} \, \text{ArcTanh}[\text{c} \, x]^3 - \text{ArcTanh}[\text{c} \, x]^2 \, \text{Log} \Big[1 + \text{e}^{-2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, + \\ & \quad \text{ArcTanh}[\text{c} \, x]^2 \, \text{Log} \Big[1 - \text{e}^{2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, + \text{ArcTanh}[\text{c} \, x] \, \text{PolyLog} \Big[2\text{, -e}^{-2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, + \text{ArcTanh}[\text{c} \, x] \\ & \quad \text{PolyLog} \Big[2\text{, } \text{e}^{2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, + \frac{1}{2} \, \text{PolyLog} \Big[3\text{, -e}^{-2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, - \frac{1}{2} \, \text{PolyLog} \Big[3\text{, } \text{e}^{2 \, \text{ArcTanh}[\text{c} \, x]} \, \Big] \, \end{split}$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \times\right]\right)^{3}}{x} \, dx$$

Optimal (type 4, 184 leaves, 8 steps):

$$2 \left(a + b \operatorname{ArcTanh}[c \, x] \right)^3 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \, x} \right] - \frac{3}{2} \, b \, \left(a + b \operatorname{ArcTanh}[c \, x] \right)^2 \operatorname{PolyLog} \left[2, \, 1 - \frac{2}{1 - c \, x} \right] + \frac{3}{2} \, b \, \left(a + b \operatorname{ArcTanh}[c \, x] \right)^2 \operatorname{PolyLog} \left[2, \, -1 + \frac{2}{1 - c \, x} \right] + \frac{3}{2} \, b^2 \, \left(a + b \operatorname{ArcTanh}[c \, x] \right) \operatorname{PolyLog} \left[3, \, 1 - \frac{2}{1 - c \, x} \right] - \frac{3}{2} \, b^2 \, \left(a + b \operatorname{ArcTanh}[c \, x] \right) \operatorname{PolyLog} \left[3, \, -1 + \frac{2}{1 - c \, x} \right] - \frac{3}{4} \, b^3 \operatorname{PolyLog} \left[4, \, 1 - \frac{2}{1 - c \, x} \right] + \frac{3}{4} \, b^3 \operatorname{PolyLog} \left[4, \, -1 + \frac{2}{1 - c \, x} \right]$$

Result (type 4, 315 leaves):

$$a^{3} \log [c\,x] + \frac{3}{2} \, a^{2} \, b \, \left(-\text{PolyLog}[2, -c\,x] + \text{PolyLog}[2, c\,x] \right) + \\ 3 \, a \, b^{2} \, \left(\frac{i \, \pi^{3}}{24} - \frac{2}{3} \, \text{ArcTanh}[c\,x]^{3} - \text{ArcTanh}[c\,x]^{2} \, \log \left[1 + e^{-2 \, \text{ArcTanh}[c\,x]} \right] + \\ \text{ArcTanh}[c\,x]^{2} \, \log \left[1 - e^{2 \, \text{ArcTanh}[c\,x]} \right] + \text{ArcTanh}[c\,x] \, \text{PolyLog}[2, -e^{-2 \, \text{ArcTanh}[c\,x]} \right] + \text{ArcTanh}[c\,x] \\ \text{PolyLog}[2, \, e^{2 \, \text{ArcTanh}[c\,x]} \right] + \frac{1}{2} \, \text{PolyLog}[3, -e^{-2 \, \text{ArcTanh}[c\,x]} \right] - \frac{1}{2} \, \text{PolyLog}[3, \, e^{2 \, \text{ArcTanh}[c\,x]} \right] + \\ \frac{1}{64} \, b^{3} \, \left(\pi^{4} - 32 \, \text{ArcTanh}[c\,x]^{4} - 64 \, \text{ArcTanh}[c\,x]^{3} \, \log \left[1 + e^{-2 \, \text{ArcTanh}[c\,x]} \right] + \\ 64 \, \text{ArcTanh}[c\,x]^{3} \, \text{Log}[1 - e^{2 \, \text{ArcTanh}[c\,x]}] + 96 \, \text{ArcTanh}[c\,x]^{2} \, \text{PolyLog}[2, -e^{-2 \, \text{ArcTanh}[c\,x]}] + \\ 96 \, \text{ArcTanh}[c\,x]^{2} \, \text{PolyLog}[2, e^{2 \, \text{ArcTanh}[c\,x]}] + \\ 96 \, \text{ArcTanh}[c\,x] \, \text{PolyLog}[3, -e^{-2 \, \text{ArcTanh}[c\,x]}] - 96 \, \text{ArcTanh}[c\,x] \, \text{PolyLog}[3, e^{2 \, \text{ArcTanh}[c\,x]}] + \\ 48 \, \text{PolyLog}[4, -e^{-2 \, \text{ArcTanh}[c\,x]}] + 48 \, \text{PolyLog}[4, e^{2 \, \text{ArcTanh}[c\,x]}] \right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \ x\right]\right)^{3}}{x^{2}} \, dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$c \left(a + b \operatorname{ArcTanh}\left[c \; x \right] \right)^{3} - \frac{\left(a + b \operatorname{ArcTanh}\left[c \; x \right] \right)^{3}}{x} + 3 \; b \; c \; \left(a + b \operatorname{ArcTanh}\left[c \; x \right] \right)^{2} \operatorname{Log}\left[2 - \frac{2}{1 + c \; x} \right] - 3 \; b^{2} \; c \; \left(a + b \operatorname{ArcTanh}\left[c \; x \right] \right) \operatorname{PolyLog}\left[2 \right] - 1 + \frac{2}{1 + c \; x} \right] - \frac{3}{2} \; b^{3} \; c \; \operatorname{PolyLog}\left[3 \right] - 1 + \frac{2}{1 + c \; x} \right]$$

Result (type 4, 196 leaves):

$$-\frac{a^3}{x} - \frac{3 \, a^2 \, b \, \text{ArcTanh} \, [c \, x]}{x} + 3 \, a^2 \, b \, c \, \text{Log} \, [x] - \frac{3}{2} \, a^2 \, b \, c \, \text{Log} \, \Big[1 - c^2 \, x^2 \Big] + \\ 3 \, a \, b^2 \, c \, \left(\text{ArcTanh} \, [c \, x] \, \left(\text{ArcTanh} \, [c \, x] - \frac{\text{ArcTanh} \, [c \, x]}{c \, x} + 2 \, \text{Log} \, \Big[1 - e^{-2 \, \text{ArcTanh} \, [c \, x]} \, \Big] \right) - \\ \text{PolyLog} \, \Big[2 \, , \, e^{-2 \, \text{ArcTanh} \, [c \, x]} \, \Big] \Big) + \\ b^3 \, c \, \left(\frac{\dot{\mathbb{I}} \, \pi^3}{8} - \text{ArcTanh} \, [c \, x]^3 - \frac{\text{ArcTanh} \, [c \, x]^3}{c \, x} + 3 \, \text{ArcTanh} \, [c \, x]^2 \, \text{Log} \, \Big[1 - e^{2 \, \text{ArcTanh} \, [c \, x]} \, \Big] + \\ 3 \, \text{ArcTanh} \, [c \, x] \, \text{PolyLog} \, \Big[2 \, , \, e^{2 \, \text{ArcTanh} \, [c \, x]} \, \Big] - \frac{3}{2} \, \text{PolyLog} \, \Big[3 \, , \, e^{2 \, \text{ArcTanh} \, [c \, x]} \, \Big] \Big)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \ x\right]\right)^3}{x^4} \, \mathrm{d} x$$

Optimal (type 4, 200 leaves, 14 steps):

$$-\frac{b^2\,c^2\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)}{x} + \frac{1}{2}\,b\,\,c^3\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)^2 - \\ \frac{b\,c\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)^2}{2\,x^2} + \frac{1}{3}\,c^3\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)^3 - \frac{\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)^3}{3\,x^3} + \\ b^3\,c^3\,\text{Log}\,[\,x\,] - \frac{1}{2}\,b^3\,c^3\,\text{Log}\,\big[\,1-c^2\,x^2\,\big] + b\,c^3\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)^2\,\text{Log}\,\big[\,2-\frac{2}{1+c\,x}\,\big] - \\ b^2\,c^3\,\left(a+b\,\text{ArcTanh}\,[\,c\,\,x\,]\,\right)\,\text{PolyLog}\,\big[\,2\,,\,\,-1+\frac{2}{1+c\,x}\,\big] - \frac{1}{2}\,b^3\,c^3\,\text{PolyLog}\,\big[\,3\,,\,\,-1+\frac{2}{1+c\,x}\,\big]$$

Result (type 4, 323 leaves):

$$-\frac{1}{24\,x^3}\left(8\,a^3+12\,a^2\,b\,c\,x+24\,a^2\,b\,\text{ArcTanh}[c\,x]-24\,a^2\,b\,c^3\,x^3\,\text{Log}[x]+\right.\\ -\frac{1}{24\,x^3}\left(8\,a^3+12\,a^2\,b\,c\,x+24\,a^2\,b\,\text{ArcTanh}[c\,x]-24\,a^2\,b\,c^3\,x^3\,\text{Log}[x]+\right.\\ -\frac{1}{24\,x^3}\left(-c^2\,x^2\right)+24\,a\,b^2\left(c^2\,x^2+\left(1-c^3\,x^3\right)\,\text{ArcTanh}[c\,x]^2-c\,x\,\text{ArcTanh}[c\,x]-\left(-1+c^2\,x^2+2\,c^2\,x^2\,\text{Log}\left[1-e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+c^3\,x^3\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcTanh}[c\,x]}\right]\right)+\\ -\frac{1}{24}\,c^3\,x^3\,x^3+24\,c^2\,x^2\,\text{ArcTanh}[c\,x]+12\,c\,x\,\text{ArcTanh}[c\,x]^2-12\,c^3\,x^3\,\text{ArcTanh}[c\,x]^2+\\ -\frac{1}{24}\,c^3\,x^3\,\text{ArcTanh}[c\,x]^3+8\,c^3\,x^3\,\text{ArcTanh}[c\,x]^3-\\ -24\,c^3\,x^3\,\text{ArcTanh}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcTanh}[c\,x]}\right]-24\,c^3\,x^3\,\text{Log}\left[\frac{c\,x}{\sqrt{1-c^2\,x^2}}\right]-\\ -24\,c^3\,x^3\,\text{ArcTanh}[c\,x]\,\text{PolyLog}\left[2,\,e^{2\,\text{ArcTanh}[c\,x]}\right]+12\,c^3\,x^3\,\text{PolyLog}\left[3,\,e^{2\,\text{ArcTanh}[c\,x]}\right]\right)$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\,\mathsf{a} + \mathsf{b}\,\mathsf{ArcTanh}\left[\,\mathsf{c}\,\,\mathsf{x}^2\,\right]\,\right)^{\,2}}{\mathsf{x}}\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 4, 137 leaves, 7 steps):

$$\left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^2 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \ x^2} \right] - \frac{1}{2} \ b \ \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right) \ \operatorname{PolyLog} \left[2 \ , \ 1 - \frac{2}{1 - c \ x^2} \right] + \frac{1}{2} \ b \ \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right) \ \operatorname{PolyLog} \left[2 \ , \ -1 + \frac{2}{1 - c \ x^2} \right] + \frac{1}{4} \ b^2 \ \operatorname{PolyLog} \left[3 \ , \ 1 - \frac{2}{1 - c \ x^2} \right] - \frac{1}{4} \ b^2 \ \operatorname{PolyLog} \left[3 \ , \ -1 + \frac{2}{1 - c \ x^2} \right]$$

Result (type 4, 181 leaves):

$$\begin{split} & a^2 \, \text{Log} \, [\, x\,] \, + \frac{1}{2} \, a \, b \, \left(- \text{PolyLog} \big[\, 2 \, , \, - \, c \, \, x^2 \,\big] \, + \text{PolyLog} \big[\, 2 \, , \, c \, \, x^2 \,\big] \, \right) \, + \\ & \frac{1}{2} \, b^2 \, \left(\frac{\text{i} \, \pi^3}{24} \, - \frac{2}{3} \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]^3 \, - \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]^2 \, \text{Log} \big[\, 1 \, + \, e^{-2 \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]} \,\big] \, + \\ & \quad \text{ArcTanh} \big[\, c \, \, x^2 \,\big]^2 \, \text{Log} \big[\, 1 \, - \, e^{2 \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]} \,\big] \, + \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big] \, \, \text{PolyLog} \big[\, 2 \, , \, - \, e^{-2 \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]} \,\big] \, + \\ & \quad \text{ArcTanh} \big[\, c \, \, x^2 \,\big] \, \, \text{PolyLog} \big[\, 2 \, , \, - \, e^{-2 \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]} \,\big] \, - \, \frac{1}{2} \, \, \text{PolyLog} \big[\, 3 \, , \, e^{2 \, \text{ArcTanh} \big[\, c \, \, x^2 \,\big]} \,\big] \, \end{split}$$

Problem 71: Unable to integrate problem.

$$\int x^4 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 1173 leaves, 102 steps):

Result (type 8, 18 leaves):

$$\int x^4 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Problem 72: Unable to integrate problem.

$$\int x^2 \, \left(a + b \, \text{ArcTanh} \left[\, c \, \, x^2 \, \right] \, \right)^2 \, \text{d} \, x$$

Optimal (type 4, 1129 leaves, 86 steps):

$$\frac{4 \text{ a b } x}{3 \text{ c}} - \frac{2}{9} \text{ a b } x^3 - \frac{2 \text{ a b ArcTan}[\sqrt{c} \ x]}{3 \text{ c}^{3/2}} + \frac{4 \text{ b}^2 \text{ ArcTan}[\sqrt{c} \ x]}{3 \text{ c}^{3/2}} - \frac{\text{ i b}^2 \text{ ArcTan}[\sqrt{c} \ x]}{3 \text{ c}^{3/2}} - \frac{3 \text{ c}^{3/2}}{3 \text{ c}^{3/2}} - \frac{4 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2}{1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} + \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2}{1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{(1 + 1) \left(1 \cdot \sqrt{c} \ x\right)}{1 \cdot 1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2}{1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2}{1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2}{1 \cdot \sqrt{c} \ x}\right]}{3 \text{ c}^{3/2}} + \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2 \sqrt{c} \left(1 \cdot \sqrt{c} \ x\right)}{\sqrt{\sqrt{c} \cdot c \cdot \sqrt{c}} \left(1 \cdot \sqrt{c} \ x\right)}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2 \sqrt{c} \left(1 \cdot \sqrt{c} \ x\right)}{\sqrt{\sqrt{c} \cdot c \cdot \sqrt{c}} \left(1 \cdot \sqrt{c} \ x\right)}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2 \sqrt{c} \left(1 \cdot \sqrt{c} \ x\right)}{\sqrt{\sqrt{c} \cdot c \cdot \sqrt{c}} \left(1 \cdot \sqrt{c} \ x\right)}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[\frac{2 \sqrt{c} \left(1 \cdot \sqrt{c} \ x\right)}{\sqrt{c} \cdot c \cdot \sqrt{c}} \left(1 \cdot \sqrt{c} \ x\right)}\right]}{3 \text{ c}^{3/2}} - \frac{2 \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right]}{3 \text{ c}} + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right]} + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c} \ x] \log \left[1 - \text{ c} \ x^2\right] + \frac{1}{9} \text{ b}^2 \text{ ArcTanh}[\sqrt{c}$$

Result (type 8, 18 leaves):

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Problem 75: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c x^{2}\right]\right)^{2}}{v^{4}} dx$$

Optimal (type 4, 1102 leaves, 64 steps):

$$\begin{split} & -\frac{2 \operatorname{ab} c}{3 \operatorname{av}} - \frac{2}{3} \operatorname{ab} \operatorname{c}^{3/2} \operatorname{ArcTan} \left[\sqrt{c} \ x \right] + \frac{4}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTan} \left[\sqrt{c} \ x \right] - \frac{1}{3} \operatorname{1b}^2 \operatorname{c}^{3/2} \operatorname{ArcTan} \left[\sqrt{c} \ x \right]^2 + \frac{4}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] + \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right]^2 - \frac{2}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 - i \sqrt{c} \ x} \right] + \frac{2}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 - i \sqrt{c} \ x} \right] + \frac{2}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 - i \sqrt{c} \ x} \right] - \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 + i \sqrt{c} \ x} \right] + \frac{2}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 + i \sqrt{c} \ x} \right] - \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 + i \sqrt{c} \ x} \right] + \frac{2}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2}{1 + i \sqrt{c} \ x} \right] - \frac{2 \sqrt{c} \left(1 + \sqrt{-c} \ x \right)}{\left(\sqrt{-c} - \sqrt{c} \right) \left(1 + \sqrt{c} \ x \right)} \right] - \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{2 \sqrt{c} \left(1 + \sqrt{-c} \ x \right)}{\left(\sqrt{-c} - \sqrt{c} \right) \left(1 + \sqrt{c} \ x \right)} \right] - \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[\frac{1 - i \right) \left(1 + \sqrt{c} \ x \right)}{1 - i \sqrt{c} \ x} \right] + \frac{b^2 \operatorname{c} \operatorname{Log} \left[1 - \operatorname{c} x^2 \right]}{3 \times 3} + \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 - \operatorname{c} x^2 \right] - \frac{b^2 \operatorname{c} \left(2 \operatorname{a} - \operatorname{b} \operatorname{Log} \left[1 - \operatorname{c} x^2 \right] \right)}{1 2 x^3} - \frac{1}{3} \operatorname{b}^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \left(2 \operatorname{a} - \operatorname{b} \operatorname{Log} \left[1 - \operatorname{c} x^2 \right] \right) - \frac{b^2 \operatorname{c} \operatorname{Log} \left[1 + \operatorname{c} x^2 \right]}{1 2 x^3} - \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 + \operatorname{c} x^2 \right] + \frac{b^2 \operatorname{c} \operatorname{Log} \left[1 + \operatorname{c} x^2 \right]}{1 \times 3} - \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 + \operatorname{c} x^2 \right] + \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 + \operatorname{c} x^2 \right] - \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 + \operatorname{c} x^2 \right] + \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 + \operatorname{c} x^2 \right] + \frac{b^2 \operatorname{c}^{3/2} \operatorname{ArcTanh} \left[\sqrt{c} \ x \right] \operatorname{Log} \left[1 +$$

Result (type 8, 18 leaves):

$$\int \frac{\left(a+b\, Arc Tanh \left[\, c\,\, x^2\,\right]\,\right)^{\,2}}{x^4}\, \mathrm{d}x$$

Problem 76: Unable to integrate problem.

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\; x^2\,\right]\,\right)^{\,2}}{x^6}\, \mathrm{d}x$$

Optimal (type 4, 1176 leaves, 77 steps):

$$\begin{aligned} & \frac{2 \, a \, b \, c}{15 \, x^2} + \frac{2 \, a \, b \, c^2}{5 \, x} - \frac{8 \, b^2 \, c^2}{15 \, x} + \frac{2}{5} \, a \, b \, c^{5/2} \, \text{ArcTan} \left[\sqrt{c} \, \, x \right] - \frac{4}{15} \, b^2 \, c^{5/2} \, \text{ArcTan} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x \right] + \frac{1}{5} \, b^2 \, c^{5/2} \, \text{ArcTanh} \left[\sqrt{c} \, \, x$$

Result (type 8, 18 leaves):

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\; x^2\,\right]\,\right)^{\,2}}{x^6}\, \mathrm{d}x$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b \operatorname{ArcTanh}\left[c \ x^{2}\right]\right)^{3}}{x} \, dx$$

Optimal (type 4, 207 leaves, 9 steps):

$$\left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^3 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \ x^2} \right] - \frac{3}{4} b \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^2 \operatorname{PolyLog} \left[2 , \ 1 - \frac{2}{1 - c \ x^2} \right] + \frac{3}{4} b \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^2 \operatorname{PolyLog} \left[2 , \ -1 + \frac{2}{1 - c \ x^2} \right] + \frac{3}{4} b^2 \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right) \operatorname{PolyLog} \left[3 , \ 1 - \frac{2}{1 - c \ x^2} \right] - \frac{3}{4} b^2 \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right) \operatorname{PolyLog} \left[3 , \ -1 + \frac{2}{1 - c \ x^2} \right] - \frac{3}{8} b^3 \operatorname{PolyLog} \left[4 , \ 1 - \frac{2}{1 - c \ x^2} \right] + \frac{3}{8} b^3 \operatorname{PolyLog} \left[4 , \ -1 + \frac{2}{1 - c \ x^2} \right]$$

Result (type 4, 371 leaves):

$$a^{3} \, \text{Log} \, [x] \, + \, \frac{3}{4} \, a^{2} \, \text{b} \, \left(-\text{PolyLog} \big[2 \text{, } -\text{c} \, x^{2} \big] + \text{PolyLog} \big[2 \text{, } \text{c} \, x^{2} \big] \right) \, + \\ \frac{3}{2} \, a \, b^{2} \, \left(\frac{\text{i} \, \pi^{3}}{24} - \frac{2}{3} \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]^{3} - \text{ArcTanh} \big[\text{c} \, x^{2} \big]^{2} \, \text{Log} \big[1 + \text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ \text{ArcTanh} \, \big[\text{c} \, x^{2} \big]^{2} \, \text{Log} \big[1 - \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \text{ArcTanh} \, \big[\text{c} \, x^{2} \big] \, \text{PolyLog} \big[2 \text{, } -\text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ \text{ArcTanh} \, \big[\text{c} \, x^{2} \big] \, \text{PolyLog} \big[2 \text{, } \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ \frac{1}{2} \, \text{PolyLog} \big[3 \text{, } -\text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, - \, \frac{1}{2} \, \text{PolyLog} \big[3 \text{, } \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ \frac{1}{128} \, b^{3} \, \left(\pi^{4} - 32 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]^{4} - 64 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]^{3} \, \text{Log} \big[1 + \text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ 64 \, \text{ArcTanh} \, \big[\text{c} \, x^{2} \big]^{3} \, \text{Log} \big[1 - \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] + 96 \, \text{ArcTanh} \, \big[\text{c} \, x^{2} \big]^{2} \, \text{PolyLog} \big[2 \text{, } -\text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ 96 \, \text{ArcTanh} \, \big[\text{c} \, x^{2} \big] \, \text{PolyLog} \big[3 \text{, } -\text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] - 96 \, \text{ArcTanh} \, \big[\text{c} \, x^{2} \big] \, \text{PolyLog} \big[3 \text{, } \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + \\ 48 \, \text{PolyLog} \big[4 \text{, } -\text{e}^{-2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, + 48 \, \text{PolyLog} \big[4 \text{, } \text{e}^{2 \, \text{ArcTanh} \big[\text{c} \, x^{2} \big]} \big] \, \big)$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\; x^2\,\right]\,\right)^{\,3}}{x^3}\, \mathrm{d}x$$

Optimal (type 4, 125 leaves, 6 steps):

$$\frac{1}{2} c \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^3 - \frac{\left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^3}{2 \ x^2} + \frac{3}{2} b c \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right)^2 \operatorname{Log} \left[2 - \frac{2}{1 + c \ x^2} \right] - \frac{3}{2} b^2 c \left(a + b \operatorname{ArcTanh} \left[c \ x^2 \right] \right) \operatorname{PolyLog} \left[2 \right] - \frac{1}{1 + c \ x^2} - \frac{3}{1 + c \ x^2} \right]$$

Result (type 4, 222 leaves):

$$\begin{split} &\frac{1}{4} \left(-\frac{2\,\mathsf{a}^3}{\mathsf{x}^2} - \frac{6\,\mathsf{a}^2\,\mathsf{b}\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}{\mathsf{x}^2} + 12\,\mathsf{a}^2\,\mathsf{b}\,\mathsf{c}\,\mathsf{Log}\big[\mathsf{x}\big] - 3\,\mathsf{a}^2\,\mathsf{b}\,\mathsf{c}\,\mathsf{Log}\big[\mathsf{1} - \mathsf{c}^2\,\mathsf{x}^4\big] + \\ & 6\,\mathsf{a}\,\mathsf{b}^2\,\mathsf{c}\,\left(\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]\,\left(\left(\mathsf{1} - \frac{\mathsf{1}}{\mathsf{c}\,\mathsf{x}^2}\right)\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big] + 2\,\mathsf{Log}\big[\mathsf{1} - \mathsf{e}^{-2\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}\big]\right) - \\ & \qquad \mathsf{PolyLog}\big[\mathsf{2}\,,\,\,\mathsf{e}^{-2\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}\big]\right) + \\ & 2\,\mathsf{b}^3\,\mathsf{c}\,\left(\frac{\mathrm{i}\,\pi^3}{8} - \mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]^3 - \frac{\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]^3}{\mathsf{c}\,\mathsf{x}^2} + 3\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]^2\,\mathsf{Log}\big[\mathsf{1} - \mathsf{e}^{2\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}\big] + \\ & 3\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]\,\mathsf{PolyLog}\big[\mathsf{2}\,,\,\,\mathsf{e}^{2\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}\big] - \frac{3}{2}\,\mathsf{PolyLog}\big[\mathsf{3}\,,\,\,\mathsf{e}^{2\,\mathsf{ArcTanh}\big[\mathsf{c}\,\mathsf{x}^2\big]}\big] \bigg) \end{split}$$

Problem 90: Attempted integration timed out after 120 seconds.

$$\int \sqrt{d x} \left(a + b \operatorname{ArcTanh} \left[c x^2 \right] \right)^2 dx$$

Optimal (type 4, 6327 leaves, 238 steps

$$\frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTan} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{\, 1/4} \, \left[1 + \sqrt{-c} \, - \sqrt{x} \, \right]}{\left[1 \, \sqrt{-\sqrt{c}} \, + (-c)^{\, 1/4} \, \right] \left[1 - i \, (-c)^{\, 1/4} \, \sqrt{x} \, \right]} \right] }{3 \, (-c)^{\, 3/4} \, \sqrt{x}} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTan} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{(1+i) \, \left[1 - (-c)^{\, 1/4} \, \sqrt{x} \, \right]}{1 - i \, (-c)^{\, 1/4} \, \sqrt{x}} \right]} - \frac{3 \, (-c)^{\, 3/4} \, \sqrt{x}}{3 \, (-c)^{\, 3/4} \, \sqrt{x}} + \frac{4 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2}{1 + i \, (-c)^{\, 1/4} \, \sqrt{x}} \right]} - \frac{2 \, (-c)^{\, 1/4} \, \left[1 - \sqrt{-c} \, - \sqrt{x} \, \right]}{3 \, (-c)^{\, 3/4} \, \sqrt{x}} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{\, 1/4} \, \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{-c}} \, + (-c)^{\, 1/4} \, \left[1 + (-c)^{\, 1/4} \, \sqrt{x} \, \right]} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{\, 1/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} \, + (-c)^{\, 1/4} \, \left[1 + (-c)^{\, 1/4} \, \sqrt{x} \, \right]} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{\, 1/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} \, + (-c)^{\, 1/4} \, \left[1 + (-c)^{\, 1/4} \, \sqrt{x} \, \right]} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{\, 1/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} \, + (-c)^{\, 1/4} \, \left[1 + (-c)^{\, 1/4} \, \sqrt{x} \, \right]} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{\, 1/4} \, \left[1 - (-c)^{\, 1/4} \, \sqrt{x} \, \right]}{1 - i \, (-c)^{\, 1/4} \, \sqrt{x}} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{\, 1/4} \, \left[1 - (-c)^{\, 1/4} \, \sqrt{x} \, \right]}{1 - i \, (-c)^{\, 1/4} \, \sqrt{x}} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{\, 1/4} \, \left[(-c)^{\, 1/4} \, \sqrt{x} \, \right]}{1 - i \, (-c)^{\, 1/4} \, \sqrt{x}} \right]} + \frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{\, 1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{\, 1/4} \, \left[(-c)^{\, 1/4} \, \sqrt{x} \, \right]}{1 - i$$

$$\frac{2 \, b^2 \, \sqrt{d \, x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \log \left[\frac{2 \, (-c)^{1/4} \, (x-c)^{1/4} \, \sqrt{x}}{\left[\, (-c)^{1/4} \, \sqrt{x} \, \right]} \right]}{3 \, (-c)^{3/4} \, \sqrt{x}} + \frac{3 \, (-c)^{3/4} \, \sqrt{x}}{3 \, c^{3/4} \, \sqrt{x}} - \frac{3 \, (-c)^{3/4} \, \sqrt{x}}{3 \, c^{3/4} \, \sqrt{x}} - \frac{3 \, c^{3/4} \, \sqrt{x}}{3 \, c^{3/4} \, \sqrt{x}} - \frac{3 \, c^{3/4} \, \sqrt{x}}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[\frac{1}{2} \cdot \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[1 \, \sqrt{-\sqrt{-c}} \, - c^{2/4} \right] \left[1 \cdot 1 \, c^{1/4} \, \sqrt{x} \right]} - \frac{3 \, c^{3/4} \, \sqrt{x}}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3 \, c^{3/4} \, \sqrt{x}} - \frac{2 \, c^{1/4} \left[1 \cdot (-c)^{1/4} \, \sqrt{x} \right]}{3$$

$$2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[\frac{2c^{1/4} \left[1 \sqrt{-\sqrt{c} - c^{1/4}} \right]}{\left[\sqrt{-\sqrt{c} - c^{1/4}} \right]} \left[1 c^{1/4} \sqrt{x} \right]} \right] \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2c^{1/4} \left[1 + (-c)^{3/4} \sqrt{x} \right]}{\left[(-c)^{3/4} \sqrt{x} \right]} \right]}{3 \, c^{3/4} \sqrt{x}} \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2c^{1/4} \left[1 + (-c)^{3/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right] \left[1 + c^{1/4} \sqrt{x} \right]} \right]} \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2c^{1/4} \left[1 + (-c)^{3/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right] \left[1 + (-c)^{3/4} \sqrt{x} \right]} \right]} \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[(-c)^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2(-c)^{1/4} \left[1 + c^{1/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right] \left[1 + (-c)^{3/4} \sqrt{x} \right]} \right]} \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[(-c)^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2(-c)^{1/4} \left[1 + c^{1/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right] \left[1 + (-c)^{3/4} \sqrt{x} \right]} \right]} \\ + \\ 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2(-c)^{1/4} \left[1 + c^{1/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right]} \right] \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2(-c)^{1/4} \left[1 + c^{1/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right]} \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[-\frac{2(-c)^{1/4} \left[1 + c^{1/4} \sqrt{x} \right]}{\left[(-c)^{3/4} + c^{1/4}} \right]} \right] \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[1 + c^{1/4} \sqrt{x} \right]} \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \log \left[1 + c^{2/2} \right] - 2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[(-c)^{1/4} \sqrt{x} \right] \, \log \left[1 + c^{2/2} \right] \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b \log \left[1 + c^{2/2} \right] \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b \log \left[1 + c^{2/2} \right] \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b \log \left[1 + c^{2/2} \right] \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b \log \left[1 + c^{2/2} \right] \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b \log \left[1 + c^{2/2} \right]} \\ + \frac{2b^2 \sqrt{d \, x} \, \operatorname{ArcTanh} \left[c^{1/4} \sqrt{x} \right] \, \left[2a - b$$

$$\frac{i \ b^2 \sqrt{d \ x} \ PolyLog}{3 \ (-c)^{3/4} \sqrt{x}} + \frac{2 \ (-c)^{3/4} \left[1 \sqrt{-\sqrt{c}} - (-c)^{3/4} \sqrt{x} \right]}{\left[1 \sqrt{-\sqrt{c}} - (-c)^{3/4} \sqrt{x} \right]} + \frac{3 \ (-c)^{3/4} \sqrt{x}}{3 \ (-c)^{3/4} \sqrt{x}} + \frac{2 \ (-c)^{3/4} \left[1 \sqrt{-\sqrt{c}} - (-c)^{3/4} \sqrt{x} \right]}{\left[1 \sqrt{-\sqrt{c}} + (-c)^{3/4} \sqrt{x} \right]} + \frac{1}{3 \ (-c)^{3/4} \sqrt{x}} + \frac{1$$

$$\frac{i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 + \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{\sqrt{-c} \cdot \sqrt{x}}\right]}{\left[1 \cdot \sqrt{-c} \cdot c^{3/4}\right] \left[1 \cdot 1 \, c^{3/4} \sqrt{x}\right]} + \\ \frac{i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{-c} \cdot \sqrt{x}\right]}{\left[1 \cdot \sqrt{-c} \cdot c^{3/4}\right] \left[1 \cdot 1 \, c^{3/4} \sqrt{x}\right]}}{3 \, c^{3/4} \sqrt{x}} + \\ \frac{i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot (-c)^{1/4} \sqrt{x}\right]}{\left[1 \cdot (-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot 1 \, c^{3/4} \sqrt{x}\right]}}{3 \, c^{3/4} \sqrt{x}} + \\ \frac{i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot (-c)^{1/4} \sqrt{x}\right]}{\left[1 \cdot (-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot 1 \, c^{3/4} \sqrt{x}\right]}}{3 \, c^{3/4} \sqrt{x}} - \\ \frac{2 \, i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot (-c)^{3/4} \sqrt{x}\right]}{\left[1 \cdot (-c)^{3/4} \cdot \sqrt{x}\right]}} - \frac{i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{(1 \cdot 1) \left[1 \cdot c^{3/4} \sqrt{x}\right]}{1 \cdot 1 \cdot c^{3/4} \sqrt{x}}} - \\ \frac{2 \, i \ b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{-c} \cdot \sqrt{x}\right]}{\left[\sqrt{-\sqrt{-c}} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{-c} \cdot \sqrt{x}\right]}{\left[\sqrt{-\sqrt{-c}} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \\ \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{-c} \cdot \sqrt{x}\right]}{\left[\sqrt{-\sqrt{-c}} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot \sqrt{-c} \cdot \sqrt{x}\right]}{\left[\sqrt{-\sqrt{-c}} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \\ \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot \sqrt{x}\right]}} + \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot c^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]}{\left[(-c)^{3/4} \cdot c^{3/4}\right] \left[1 \cdot (-c)^{3/4} \sqrt{x}\right]}} - \frac{b^2 \sqrt{d \, x} \ PolyLog}[2, \ 1 - \frac{2 \, c^{3/4} \left[1 \cdot c^{3/4} \sqrt{x}\right]$$

Result (type 1, 1 leaves):

Problem 91: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \ x^{2}\right]\right)^{2}}{\sqrt{d \ x}} \, dx$$

Optimal (type 4, 6177 leaves, 241 steps):

$$\begin{array}{c} \text{Spinital (type 4), } & \text{ArcTan} \Big[1 + \sqrt{2} \ c^{1/4} \sqrt{x} \Big] \\ \text{$\sqrt{d\,x}$} & \text{$c^{1/4} \sqrt{d\,x}$} \\ \text{$c^{1/4} \sqrt{d\,x}$} & \text{$c^{1/4} \sqrt{d\,x}$} \Big] \\ \text{$\frac{2\,i\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]^2 - \frac{4\,a\,b\,\sqrt{x}\,\,\text{ArcTan} \Big[c^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} + \frac{2\,i\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[c^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} - \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]^2}{c^{1/4}\,\sqrt{d\,x}} - \frac{4\,a\,b\,\sqrt{x}\,\,\text{ArcTanh} \Big[c^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} - \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTanh} \Big[(-c)^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} + \frac{4\,b^2\,\sqrt{x}\,\,\text{ArcTanh} \Big[(-c)^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} - \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTanh} \Big[(-c)^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTanh} \Big[(-c)^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]}{c^{1/4}\,\sqrt{d\,x}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{\left|i\sqrt{-\sqrt{c}}\,\,-(-c)^{1/4}\,\sqrt{x} \right|} \Big]} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{\left|i\sqrt{-\sqrt{c}}\,\,+(-c)^{1/4}\,\sqrt{x} \right|} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{d\,x}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{\left|i\sqrt{-\sqrt{c}}\,\,+(-c)^{1/4}\,\sqrt{x} \right|} \Big]} - \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{(1+1)\,\left|i-(-c)^{1/4}\,\sqrt{x} \Big|}{\left|i\sqrt{-\sqrt{c}}\,\,+(-c)^{1/4}\,\sqrt{x} \Big|} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{d\,x}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{2\,(-c)^{1/4}\,\sqrt{x}}{\left|i\sqrt{-c}\,\,-(-c)^{1/4}\,\sqrt{x} \Big|} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{d\,x}} + \frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{d\,x}} - \frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{d\,x}}} + \frac{2\,b^2\,\sqrt{x}\,\,\text{ArcTan} \Big[(-c)^{1/4}\,\sqrt{x} \Big]\,\,\text{Log} \Big[\frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}} - \frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}} - \frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}} - \frac{2\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x}}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}} \Big]}{c^{-2}\,(-c)^{1/4}\,\sqrt{x}\,\sqrt{x$$

 $2 \ b^2 \ \sqrt{x} \ \text{ArcTan} \Big[\ c^{1/4} \ \sqrt{x} \ \Big] \ \text{Log} \Big[\ \frac{2 \ c^{1/4} \ \left(1 + (-c)^{1/4} \ \sqrt{x} \ \right)}{\left(i \ (-c)^{1/4} + c^{1/4}\right) \ \left(1 - i \ c^{1/4} \ \sqrt{x} \ \right)} \ \Big]$

$$\frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[\frac{2 \, (-c)^{1/4} \, \left[1 \, x \sqrt{-\sqrt{c}} - \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} - (-c)^{1/4} \, \sqrt{x} \, \right]} \right] }{ (-c)^{1/4} \, \sqrt{d \, x}} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, x \sqrt{-\sqrt{c}} - \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} - (-c)^{1/4} \, \sqrt{x} \, \right]} \right]}{ (-c)^{1/4} \, \sqrt{d \, x}} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, x \sqrt{-\sqrt{c}} - \sqrt{x} \, \right]}{\left[\sqrt{-\sqrt{c}} + (-c)^{1/4} \, \sqrt{x} \, \right]} \right]}{ (-c)^{1/4} \, \sqrt{d \, x}} - \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, x \sqrt{x} \, \right]}{\left[(-c)^{1/4} \, \sqrt{x} \, \right]} \right]} + \frac{4 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2}{1 \, c^{1/4} \, \sqrt{x}} \right]}{ (-c)^{1/4} \, \sqrt{d \, x}} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(-c)^{1/4} \, \sqrt{x} \, \right]} + \frac{4 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(-c)^{1/4} \, \sqrt{x} \, \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, (-c)^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(-c)^{1/4} \, \sqrt{x} \, \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, (-c)^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, c^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(-c)^{1/4} \, \sqrt{x} \, \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, c^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(1 \, \sqrt{-\sqrt{c}} \, c^{1/4} \, \sqrt{x} \, \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, c^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(1 \, \sqrt{-\sqrt{c}} \, c^{1/4} \, \sqrt{x} \, \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, c^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(1 \, c^{1/4} \, \sqrt{x} \, \right]} \right]} + \\ \frac{2 \, b^2 \, \sqrt{x} \, \operatorname{ArcTanh} \left[\, c^{1/4} \, \sqrt{x} \, \right] \, \operatorname{Log} \left[-\frac{2 \, c^{1/4} \, \left[1 \, c^{1/4} \, \sqrt{x} \, \right]}{ \left[(-c)^{1/4} \, \sqrt{x} \, \right]} \right]} + \\ \frac{$$

$$\frac{2b^2 \sqrt{x} \ \text{ArcTan} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[\frac{(1+1) \left[\frac{1+c^{1/4} \sqrt{x}}{\sqrt{x}} \right]}{1-1 e^{1/4} \sqrt{x}} \Big] } + \frac{4b^2 \sqrt{x} \ \text{ArcTan} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[\frac{2}{1+e^{1/4} \sqrt{x}} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{4b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[\frac{2}{1+e^{1/4} \sqrt{x}} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-\sqrt{\sqrt{-c}} - c^{1/4} \right] \left[1-e^{1/4} \sqrt{x}}{\left| \sqrt{-c} - c^{1/4} \right| \left| 1-e^{1/4} \sqrt{x}} \right|} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-\sqrt{-c} - c^{1/4} \right] \left[1-e^{1/4} \sqrt{x}}{\left| \sqrt{-c} - c^{1/4} \right| \left[1-e^{1/4} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-\sqrt{-c} - c^{1/4} \right] \left[1-e^{1/4} \sqrt{x}}{\left| \sqrt{-c} - c^{1/4} \right| \left[1-e^{1/4} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-c^{1/4} \sqrt{x} \right]}{\left((-c)^{1/4} \sqrt{x} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-e^{1/4} \sqrt{x} \right]}{\left((-c)^{1/4} \sqrt{x} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-e^{1/4} \sqrt{x} \right]}{\left((-c)^{1/4} \sqrt{x} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[e^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-e^{1/4} \sqrt{x} \right]}{\left((-c)^{1/4} \sqrt{x} \sqrt{x}} \right]} \Big]}{e^{1/4} \sqrt{d \, x}} + \frac{2b^2 \sqrt{x} \ \text{ArcTanh} \Big[(-c)^{1/4} \sqrt{x} \Big] \ \log \Big[-\frac{2e^{1/4} \left[1-e^{1/4} \sqrt{x} \right]}{\left((-c)^{1/4} \sqrt{x} \right)} \Big]} - \frac{e^{1/4} \sqrt{d \, x}}{e^{1/4} \sqrt{d \, x}} + \frac{e^{1/4} \sqrt{d \, x}}{e$$

$$\frac{\sqrt{2} \text{ a b } \sqrt{x} \text{ Log} \left[1 + \sqrt{2} \text{ } \frac{c^{1/4} \sqrt{x} + \sqrt{c} \text{ } x \right] }{c^{1/4} \sqrt{dx}} - \frac{2 \text{ a b } x \text{ Log} \left[1 - c \text{ } x^2 \right] }{\sqrt{dx}} - \frac{2 \text{ b}^2 \sqrt{x} \text{ ArcTan} \left[c^{1/4} \sqrt{x} \right] \text{ Log} \left[1 - c \text{ } x^2 \right] }{c^{1/4} \sqrt{dx}} - \frac{2^{1/4} \sqrt{dx}}{c^{1/4} \sqrt{dx}} - \frac{c^{1/4} \sqrt{dx}}{c^{1/4} \sqrt{d$$

$$\frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1+\frac{2\left(-c\right)^{1/4}\left|1+\sqrt{-c}\right|\sqrt{x}}{\sqrt{\sqrt{c}}\left(-c\right)^{3/4}\sqrt{x}}\big]}{(-c)^{3/4}\sqrt{d\,x}} - \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\left|1+\sqrt{-c}\right|\sqrt{x}}{\sqrt{\sqrt{c}}\left(-c\right)^{3/4}\sqrt{x}}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{i\;b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{(1-i)\left|\ln\left(-c\right)^{3/4}\sqrt{x}\right|}{i+1\left(-c\right)^{3/4}\sqrt{x}}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{i\;b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{(1-i)\left|\ln\left(-c\right)^{3/4}\sqrt{x}\right|}{i+1\left(-c\right)^{3/4}\sqrt{x}}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{(1-i)\left|\ln\left(-c\right)^{3/4}\sqrt{x}\right|}{(-c)^{3/4}\sqrt{d\,x}}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\left|\ln\left(-c\right)^{3/4}\sqrt{x}\right|}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\left|-c\right|^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\right)}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\right]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{d\,x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{x}\sqrt{x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{x}\sqrt{x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4}\sqrt{x}\big]}{(-c)^{3/4}\sqrt{x}} + \frac{b^{2}\sqrt{x}\; \mathsf{PolyLog}\big[2,\,1-\frac{2\left(-c\right)^{3/4$$

$$\frac{b^2 \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 + \frac{2 \, c^{1/4} \, \Big(1 - \sqrt{-\sqrt{c}} \, \, \sqrt{x}\,\Big)}{\Big(\sqrt{-\sqrt{c}} \, - c^{1/4}\Big) \, \Big(1 + c^{1/4} \, \sqrt{x}\,\Big)}}{c^{1/4} \, \sqrt{d \, x}} + \frac{b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{2 \, c^{1/4} \, \Big(1 + \sqrt{-\sqrt{c}} \, \, \sqrt{x}\,\Big)}{\Big(\sqrt{-\sqrt{c}} \, + c^{1/4}\Big) \, \Big(1 + c^{1/4} \, \sqrt{x}\,\Big)}}{c^{1/4} \, \sqrt{d \, x}} - \frac{b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{2 \, c^{1/4} \, \Big(1 + (-c)^{1/4} \, \sqrt{x}\,\Big)}{\Big((-c)^{1/4} - c^{1/4}\Big) \, \Big(1 + c^{1/4} \, \sqrt{x}\,\Big)}}{c^{1/4} \, \sqrt{d \, x}} - \frac{b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{2 \, c^{1/4} \, \Big(1 + (-c)^{1/4} \, \sqrt{x}\,\Big)}{\Big((-c)^{1/4} + c^{1/4} \, \sqrt{x}\,\Big)}\Big]}{c^{1/4} \, \sqrt{d \, x}} - \frac{b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{2 \, c^{1/4} \, \Big(1 + (-c)^{1/4} \, \sqrt{x}\,\Big)}{\Big((-c)^{1/4} + c^{1/4} \, \Big) \, \Big(1 - i \, (-c)^{1/4} \, \sqrt{x}\,\Big)}\Big]}{c^{1/4} \, \sqrt{d \, x}} - \frac{b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{2 \, c^{1/4} \, \Big(1 + c^{1/4} \, \sqrt{x}\,\Big)}{\Big((-c)^{1/4} \, \sqrt{d \, x}}\Big)} + \frac{i \, b^2 \, \sqrt{x} \; \mathsf{PolyLog}\big[2, \, 1 - \frac{(1 - i) \, \Big(1 + c^{1/4} \, \sqrt{x}\,\Big)}{1 - i \, c^{1/4} \, \sqrt{d \, x}}\Big]}{c^{1/4} \, \sqrt{d \, x}}$$

Result (type 1, 1 leaves):

???

Problem 92: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(a+b\, Arc Tanh \left[c\, x^2\right]\right)^2}{\left(d\, x\right)^{3/2}}\, \mathrm{d} x$$

Optimal (type 4, 6334 leaves, 197 steps):

$$\frac{2\sqrt{2} \text{ a b } c^{1/4} \sqrt{x} \text{ ArcTan} \Big[1 - \sqrt{2} \text{ } c^{1/4} \sqrt{x} \Big]}{d\sqrt{dx}} + \frac{2\sqrt{2} \text{ a b } c^{1/4} \sqrt{x} \text{ ArcTan} \Big[1 + \sqrt{2} \text{ } c^{1/4} \sqrt{x} \Big]}{d\sqrt{dx}} + \frac{2 \text{ i b}^2 \left(-c \right)^{1/4} \sqrt{x} \text{ ArcTan} \Big[\left(-c \right)^{1/4} \sqrt{x} \Big]^2}{d\sqrt{dx}} + \frac{2 \text{ i b}^2 c^{1/4} \sqrt{x} \text{ ArcTan} \Big[c^{1/4} \sqrt{x} \Big]^2}{d\sqrt{dx}} + \frac{2 \text{ b}^2 c^{1/4} \sqrt{x} \text{ ArcTanh} \Big[c^{1/4} \sqrt{x} \Big]^2}{d\sqrt{dx}} - \frac{2 \text{ b}^2 \left(-c \right)^{1/4} \sqrt{x} \text{ ArcTanh} \Big[\left(-c \right)^{1/4} \sqrt{x} \Big] \log \Big[\frac{2}{1 - \left(-c \right)^{1/4} \sqrt{x}} \Big]}{d\sqrt{dx}} - \frac{4 \text{ b}^2 \left(-c \right)^{1/4} \sqrt{x} \text{ ArcTanh} \Big[\left(-c \right)^{1/4} \sqrt{x} \Big] \log \Big[\frac{2}{1 - \left(-c \right)^{1/4} \sqrt{x}} \Big]} - \frac{4 \text{ b}^2 \left(-c \right)^{1/4} \sqrt{x} \text{ ArcTanh} \Big[\left(-c \right)^{1/4} \sqrt{x} \Big] \log \Big[\frac{2}{1 - \left(-c \right)^{1/4} \sqrt{x}} \Big]} + \frac{1}{d\sqrt{dx}} - \frac{2}{d\sqrt{dx}} + \frac{1}{d\sqrt{dx}} + \frac{1}$$

$$\frac{1}{d\sqrt{d\,x}} 2\,b^2 \, (-c)^{1/4} \, \sqrt{x} \, \operatorname{ArcTan} \big[\, (-c)^{3/4} \, \sqrt{x} \, \big] \, \operatorname{Log} \bigg[\frac{1}{i \, \sqrt{-\sqrt{c}} \, + (-c)^{3/4}} \, \bigg[1 + i \, (-c)^{3/4} \, \sqrt{x} \bigg] \, \\ -\frac{2\,b^2 \, (-c)^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \big[\, (-c)^{3/4} \, \sqrt{x} \, \big] \, \operatorname{Log} \bigg[\frac{(1+i) \, \big[1 + (-c)^{3/4} \, \sqrt{x} \big]}{1 + i \, (-c)^{3/4} \, \sqrt{x}} \bigg] \, + \\ -\frac{2\,b^2 \, (-c)^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \big[\, (-c)^{3/4} \, \sqrt{x} \, \big] \, \operatorname{Log} \bigg[\frac{2}{1 + i \, (-c)^{3/4} \, \sqrt{x}} \bigg] \, + \\ -\frac{4\,b^2 \, (-c)^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \big[\, (-c)^{3/4} \, \sqrt{x} \, \big] \, \operatorname{Log} \bigg[\frac{2}{1 + i \, (-c)^{3/4} \, \sqrt{x}} \bigg] \, + \\ -\frac{1}{d\sqrt{d\,x}} \, \frac{1}{d\sqrt{d\,x}} \, \frac{1}{$$

$$\frac{4\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,\frac{2}{1+c^{1/4}\,\sqrt{x}}\,]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+\sqrt{-\sqrt{-c}}\,\,\sqrt{x}\,\right]}{\left[1\,\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/2}\,\sqrt{x}\,\right]}}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,\frac{2\,c^{1/4}\,\left[1+\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/2}\,\sqrt{x}\,\right]}{\left[1+\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/2}\,\sqrt{x}\,\right]}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+(-c)^{1/4}\,\sqrt{x}\,\right]}{\left[1+c^{1/4}\,\sqrt{x}\,\right]}\,\right]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,\frac{2\,c^{1/4}\,\left[1+(-c)^{1/4}\,\sqrt{x}\,\right]}{\left[1+c^{1/4}\,\sqrt{x}\,\right]}\,\right]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,\frac{2\,c^{1/4}\,\left[1+(-c)^{1/4}\,\sqrt{x}\,\right]}{\left[1+c^{1/4}\,\sqrt{x}\,\right]}\,\right]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTan[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,\frac{2\,c^{1/4}\,\left[1+(-c)^{1/4}\,\sqrt{x}\,\right]}{\left[1+c^{1/4}\,\sqrt{x}\,\right]}\,\right]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}\,\right]}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}}{d\,\sqrt{d\,x}} + \frac{2\,b^{2}\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}} + \frac{2\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}} + \frac{2\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}} + \frac{2\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}} + \frac{2\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{1/4}\,\sqrt{x}\,]\,\,Log[\,-\frac{2\,c^{1/4}\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}{\left[\sqrt{-\sqrt{-c}}\,\,-c^{1/4}\,\right]\,\left[1+c^{1/4}\,\sqrt{x}\,\right]}} + \frac{2\,c^{1/4}\,\sqrt{x}\,\,ArcTanh[\,c^{$$

$$\frac{2\,b^2\,c^{1/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{1/4}\,\sqrt{x}\,\right]\,\log\left[-\frac{2\,c^{3/4}\left[1\,\cdot\,(-c)^{1/4}\,\sqrt{x}\,\right]}{\left((-c)^{1/4}\,\cdot\,c^{1/4}\right)\left[1\,\cdot\,c^{1/4}\,\sqrt{x}\,\right]}\right]}{d\,\sqrt{d\,x}} = \frac{2\,c^{3/4}\left[1\,\cdot\,(-c)^{3/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{1/4}\,\sqrt{x}\,\right]\,\left(1\,\cdot\,c^{1/4}\,\sqrt{x}\,\right)} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,c^{1/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{1/4}\,\sqrt{x}\,\right]\,\log\left[\frac{2\,c^{3/4}\left[1\,\cdot\,(-c)^{3/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{3/4}\,+\,i\,c^{3/4}\,\sqrt{x}\,\right)}\right]}{\left(\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[\left(-c\right)^{1/4}\,\sqrt{x}\,\right]\,\log\left[\frac{2\,(-c)^{1/4}\left[1\,+\,c^{1/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{3/4}\,+\,i\,c^{3/4}\,\sqrt{x}\,\right)}\right]} - \frac{2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[\left(-c\right)^{3/4}\,\sqrt{x}\,\right]\,\log\left[\frac{2\,(-c)^{3/4}\,\left[1\,+\,c^{3/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{3/4}\,\sqrt{x}\,\right)}\right]}$$

$$d\,\sqrt{d\,x}$$

$$2\,b^2\,c^{1/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{1/4}\,\sqrt{x}\,\right]\,\log\left[\frac{(3-1)\left[1\,-\,c^{3/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{3/4}\,\sqrt{x}\,\right)\left[1\,-\,c^{3/4}\,\sqrt{x}\,\right]}\right]} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,c^{1/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{1/4}\,\sqrt{x}\,\right]\,\log\left[\frac{(3-1)\left[1\,-\,c^{3/4}\,\sqrt{x}\,\right]}{\left(\left(-c\right)^{3/4}\,\sqrt{x}\,\right)\,\left(\left(-c\right)^{3/4}\,\sqrt{x}\,\right)}\right]} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[\left(-c\right)^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,-\,c\,x^2\right]\right)} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\left(2\,a\,-\,b\,\log\left[1\,-\,c\,x^2\right]\right)} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\left(2\,a\,-\,b\,\log\left[1\,-\,c\,x^2\right]\right)} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\left(2\,a\,-\,b\,\log\left[1\,-\,c\,x^2\right]\right)} + \frac{1}{d\,\sqrt{d\,x}}$$

$$2\,b\,d\,d\,x$$

$$2\,a\,b\,\log\left[1\,+\,c\,x^2\right] + \frac{2\,b^2\,\left(-\,c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b\,d\,d\,x}{d\,\sqrt{d\,x}}$$

$$2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right] + \frac{2\,b^2\,\left(-\,c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[-\,c\right)^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]\,\log\left[1\,+\,c\,x^2\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{3/4}\,\sqrt{x}\,\right]} + \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\!\left[c^{$$

 $d\sqrt{dx}$

$$\begin{array}{c} 2 \text{ i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2}{1 + c^{1/4} \, \sqrt{x}}] \\ d \, \sqrt{d \, x} \end{array}) \\ \text{i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x}} \right]} \\ d \, \sqrt{d \, x} \\ \text{i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \\ d \, \sqrt{d \, x} \\ \text{i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 + \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{i } b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{2} b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{3} b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{4} b^2 \, c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{5} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{5} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{5} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{6} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{6} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{6} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 + c^{1/4} \, \sqrt{x} \right]} \right]} \\ d \, \sqrt{d \, x} \\ \text{7} c^{1/4} \, \sqrt{x} \, \operatorname{PolyLog}[2, 1 - \frac{2 \, c^{1/4} \, \left[1 + c^{1/4} \, \sqrt{x} \right]}{\left[1 +$$

$$\frac{b^2\,c^{1/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,c^{1/4}\,\left(1+(-c)^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+c^{\,1/4}\,\right)\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{d\,\sqrt{d\,x}}-\frac{i\,\,b^2\,\left(-c\,\right)^{\,1/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,(-c)^{\,1/4}\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+i\,\,c^{\,1/4}\,\right)\,\left(1-i\,\,(-c)^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{d\,\sqrt{d\,x}}+\frac{b^2\,\left(-c\,\right)^{\,1/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,(-c)^{\,1/4}\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+c^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{d\,\sqrt{d\,x}}+\frac{i\,\,b^2\,c^{\,1/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{(1-i)\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{1-i\,\,c^{\,1/4}\,\sqrt{x}}\,\right]}{d\,\sqrt{d\,x}}$$

Result (type 1, 1 leaves):

???

Problem 93: Attempted integration timed out after 120 seconds.

$$\int \frac{\left(\,a + b\, \text{ArcTanh}\left[\,c\,\,x^2\,\right]\,\right)^{\,2}}{\left(\,d\,\,x\,\right)^{\,5/2}}\,\,\mathrm{d}x$$

Optimal (type 4, 6520 leaves, 197 steps):

$$\frac{2\sqrt{2} \text{ a b } c^{3/4} \sqrt{x} \text{ ArcTan} \left[1 - \sqrt{2} \ c^{1/4} \sqrt{x} \ \right]}{3 \ d^2 \sqrt{d \, x}} + \frac{2\sqrt{2} \text{ a b } c^{3/4} \sqrt{x} \text{ ArcTan} \left[1 + \sqrt{2} \ c^{1/4} \sqrt{x} \ \right]}{3 \ d^2 \sqrt{d \, x}} - \frac{2 \ i \ b^2 \ (-c)^{3/4} \sqrt{x} \text{ ArcTan} \left[(-c)^{1/4} \sqrt{x} \ \right]^2}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ c^{3/4} \sqrt{x} \text{ ArcTan} \left[c^{1/4} \sqrt{x} \ \right]^2}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ c^{3/4} \sqrt{x} \text{ ArcTanh} \left[c^{1/4} \sqrt{x} \ \right]^2}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ c^{3/4} \sqrt{x} \text{ ArcTanh} \left[c^{1/4} \sqrt{x} \ \right]^2}{3 \ d^2 \sqrt{d \, x}} + \frac{4 \ b^2 \ (-c)^{3/4} \sqrt{x} \text{ ArcTanh} \left[(-c)^{1/4} \sqrt{x} \ \right] \text{ Log} \left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}} \right]}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ c^{3/4} \sqrt{x} \text{ ArcTanh} \left[c^{1/4} \sqrt{x} \ \right]^2}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ c^{3/4} \sqrt{x} \text{ ArcTanh} \left[(-c)^{1/4} \sqrt{x} \ \right] \text{ Log} \left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}} \right]} + \frac{2 \ b^2 \ (-c)^{3/4} \sqrt{x} \text{ ArcTanh} \left[(-c)^{1/4} \sqrt{x} \ \right] \text{ Log} \left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}} \right]} - \frac{1}{3 \ d^2 \sqrt{d \, x}} + \frac{2 \ b^2 \ (-c)^{3/4} \sqrt{x} \text{ ArcTanh} \left[(-c)^{1/4} \sqrt{x} \ \right] \text{ Log} \left[-\frac{2}{(-c)^{1/4} \left(1 - \sqrt{-\sqrt{c}} \ \sqrt{x} \ \right)} \right]} - \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \left(1 - i \ (-c)^{1/4} \sqrt{x} \ \right)} - \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} - \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt{-\sqrt{c}} \ - (-c)^{1/4} \sqrt{x} \)} + \frac{1}{(i \sqrt$$

$$\frac{1}{3\,d^{2}\sqrt{dx}}2\,b^{2}\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTan}\left[\left(-c\right)^{1/4}\,\sqrt{x}\,\right]\,\operatorname{Log}\left[\frac{1}{\left[\pm\sqrt{-\sqrt{c}}+\left(-c\right)^{3/4}\right]}\left(1-i\,\left(-c\right)^{1/4}\,\sqrt{x}\,\right)\right]}{\left[\pm\sqrt{-\sqrt{c}}+\left(-c\right)^{3/4}\right]}\left(1-i\,\left(-c\right)^{1/4}\,\sqrt{x}\,\right)\right]}+\frac{2\,b^{2}\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTan}\left[\left(-c\right)^{1/4}\,\sqrt{x}\,\right]\,\operatorname{Log}\left[\frac{3+1}{1+\left(-c\right)^{3/4}\sqrt{x}}\right]}{3\,d^{2}\,\sqrt{dx}}+\frac{3\,d^{2}\,\sqrt{dx}}{3\,d^{2}\,\sqrt{dx}}+\frac{3\,d^{2}\,\sqrt{dx}}{3\,d^{2}\,\sqrt{dx}}+\frac{3\,d^{2}\,\sqrt{dx}}{3\,d^{2}\,\sqrt{dx}}+\frac{1}{3\,d^{2}\,\sqrt{dx}}$$

 $3 d^2 \sqrt{d x}$

$$\frac{4 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2}{1 + e^{1/4} \, \sqrt{x}} \right]}{3 \, d^2 \, \sqrt{d \, x}} = 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{1/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[1 \, \sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left[1 + e^{1/4} \, \sqrt{x} \right]}} \right] = 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{1/4} \left[1 + \sqrt{-c} \, \sqrt{x} \, \sqrt{x} \right]}{\left[1 \, \sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left[1 + e^{1/4} \, \sqrt{x} \right]}} \right] = 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{1/4} \left[1 + (-c)^{3/4} \, \sqrt{x} \right]}{\left[1 + (-c)^{3/4} \, \sqrt{x} \right]}} \right] = 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + (-c)^{3/4} \, \sqrt{x} \right]}{\left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + (-c)^{3/4} \, \sqrt{x} \right]}{1 + c^{3/4} \, \sqrt{x}} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTan} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + c^{3/4} \, \sqrt{x} \right]}{1 + c^{3/4} \, \sqrt{x}} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{1/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{3/4} \right] \left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{3/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{3/4} \right] \left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{3/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{3/4} \right] \left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{3/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{3/4} \right] \left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{3/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-\sqrt{-c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{3/4} \right] \left[1 + c^{3/4} \, \sqrt{x} \right]} \right] + 2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{ArcTanh} \left[c^{3/4} \, \sqrt{x} \right] \, \operatorname{Log} \left[\frac{2 \, c^{3/4} \left[1 + \sqrt{-c} \, \sqrt{x} \, \right]}{\left[\sqrt{-c} \, - c^{3/4} \, \right] \left[1$$

$$\frac{2b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,c^{3/4}\,\left[1-\,c\,c^{3/4}\,\sqrt{x}\right]}{\left\{(-c)^{3/4}\,\sqrt{x}\right\}}\right]}{3\,d^2\,\sqrt{d\,x}} } \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,c^{3/4}\,\left[1+\,c\,c^{3/4}\,\sqrt{x}\right]}{\left\{(-c)^{3/4}\,c^{3/4}\right\}}\right]}{3\,d^2\,\sqrt{d\,x}} } \\ \frac{2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,\left(-c\right)^{3/4}\,\left\{1+\,c^{1/4}\,\sqrt{x}\right\}}{\left((-c\right)^{3/4}\,\sqrt{x}\right)}\right]}{\left\{(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,\left(-c\right)^{3/4}\,\left\{1+\,c^{1/4}\,\sqrt{x}\right\}}{\left((-c\right)^{3/4}\,\sqrt{x}\right)}\right]} - \frac{1}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[(-c)^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,\left(-c\right)^{3/4}\,\left\{1+\,c^{3/4}\,\sqrt{x}\right\}}{\left[(-c\right)^{3/4}\,\sqrt{x}\,\sqrt{x}\right]}\right]} \\ \frac{3\,d^2\,\sqrt{d\,x}}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[-\frac{2\,\left(-c\right)^{3/4}\,\left\{1+\,c^{3/4}\,\sqrt{x}\right\}}{\left[1+\,c^{3/4}\,\sqrt{x}\right]}\right]} \\ \frac{3\,d^2\,\sqrt{d\,x}}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\operatorname{Log}\left[1-\,c\,x^2\right]}{3\,d^2\,\sqrt{d\,x}} + \frac{\sqrt{2}\,a\,b\,c^{3/4}\,\sqrt{x}\,\operatorname{Log}\left[1+\sqrt{2}\,c^{3/4}\,\sqrt{x}+\sqrt{c}\,x\right]}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,\left(-c\right)^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[-c\right)^{1/4}\,\sqrt{x}\,\right]\,\operatorname{Log}\left[1-\,c\,x^2\right]}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\right]\,\left(2\,a-b\,\operatorname{Log}\left[1-\,c\,x^2\right]\right)}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\,\right]\,\left(2\,a-b\,\operatorname{Log}\left[1-\,c\,x^2\right]\right)}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\,\right]\,\operatorname{Log}\left[1+\,c\,x^2\right]}{3\,d^2\,\sqrt{d\,x}} \\ \frac{2\,b^2\,c^{3/4}\,\sqrt{x}\,\operatorname{ArcTanh}\left[c^{1/4}\,\sqrt{x}\,\right]\,\operatorname{Log}\left[1+\,$$

$$\begin{split} & \|b^2 (-c)^{3/4} \sqrt{x} \ \text{PolyLog} \Big[2, \, 1 + \frac{2 \, (-c)^{3/4} \left[1 - c \, (-c)^{3/4} \sqrt{x} \right]}{\left[i \, \sqrt{-\sqrt{c}} - (-c)^{3/4} \right] \left[1 - c \, (-c)^{3/4} \sqrt{x} \right]} + \\ & \frac{3 \, d^2 \, \sqrt{d \, x}}{\left[i \, \sqrt{-\sqrt{c}} + (-c)^{3/4} \right] \left[1 - c \, (-c)^{3/4} \sqrt{x} \right]}}{3 \, d^2 \, \sqrt{d \, x}} + \\ & \frac{1}{6} \, b^2 \, (-c)^{3/4} \, \sqrt{x} \ \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \sqrt{x}}{1 + i \, (-c)^{3/4} \sqrt{x}} \Big]}{3 \, d^2 \, \sqrt{d \, x}} - \frac{2 \, b^2 \, (-c)^{3/4} \, \sqrt{x} \ \text{PolyLog} \Big[2, \, 1 - \frac{1 \, (-c)^{3/4} \, \sqrt{x}}{1 + i \, (-c)^{3/4} \, \sqrt{x}} \Big]}{3 \, d^2 \, \sqrt{d \, x}} - \frac{2 \, b^2 \, (-c)^{3/4} \, \sqrt{x} \ \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \sqrt{x}}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \Big]} - \frac{2 \, b^2 \, (-c)^{3/4} \, \sqrt{x} \ \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \Big]} - \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} - \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} - \frac{2 \, (-c)^{3/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \right]} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{2 \, (-c)^{3/4} \, \sqrt{x} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \, \Big]} - \frac{2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \left[1 - \sqrt{-\sqrt{c}} \, \sqrt{x} \right]}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \, \Big]} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{2 \, b^2 \, (-c)^{3/4} \, \sqrt{x} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \sqrt{x}}{\left[\sqrt{-\sqrt{c}} \, (-c)^{3/4} \, \sqrt{x}} \, \Big]} + \frac{2 \, b^2 \, (-c)^{3/4} \, \sqrt{x} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \sqrt{x}}{\left[\sqrt{-c} \, (-c)^{3/4} \, \sqrt{x}} \, \Big]} - \frac{2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \text{PolyLog} \Big[2, \, 1 - \frac{2 \, (-c)^{3/4} \, \sqrt{x}}{\left[(-c)^{3/4} \, \sqrt{x}} \, \Big]} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}}} + \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \,$$

$$\begin{array}{c} 2 \text{ i } b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2}{1 + \left[c^{1/4} \, \sqrt{x} \right]} \right] \\ 3 \, d^2 \, \sqrt{d \, x} \\ \\ \text{i } b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left[1 \cdot \sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[i \, \sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left[\left[1 \cdot \sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]} \\ \\ \text{i } b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - c \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]} \right]} \\ \\ \text{3 } d^2 \, \sqrt{d \, x} \\ \\ \text{i } b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - c \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]} \right]} \\ \\ \text{3 } d^2 \, \sqrt{d \, x} \\ \\ \text{i } b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]} \right]} \\ \\ \text{3 } d^2 \, \sqrt{d \, x} \\ \\ \frac{2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]} \right]} \\ \\ \frac{2 \, b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}} \\ \\ \frac{3 \, d^2 \, \sqrt{d \, x}}{3 \, d^2 \, \sqrt{d \, x}} \\ \\ b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\sqrt{-\sqrt{-c}} \, - c^{1/4} \right] \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right)} \right]} \\ \\ 3 \, d^2 \, \sqrt{d \, x} \\ \\ b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\sqrt{-\sqrt{c}} \, - c^{1/4} \right] \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right)} \right]} \\ \\ 3 \, d^2 \, \sqrt{d \, x} \\ \\ b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\sqrt{-\sqrt{c}} \, - c^{1/4} \right] \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right)} \right]} \\ \\ \\ 3 \, d^2 \, \sqrt{d \, x} \\ \\ b^2 \, c^{3/4} \, \sqrt{x} \, \operatorname{Polytog} \left[2, \, 1 - \frac{2 \, c^{1/4} \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right) \right]}{\left[\sqrt{-\sqrt{c}} \, - c^{1/4} \right] \left[\left(1 - i \, c^{1/4} \, \sqrt{x} \right)} \right]} \\ \\ \\ + \frac{1 \, c^{1/4} \, \left(1 - i \, c^{1/4} \, \left(1 - i \, c^{1/4} \, \left(1 - i \, c^{1/4} \,$$

$$\frac{b^2\,c^{3/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,c^{1/4}\,\left(1+(-c)^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+c^{\,1/4}\,\right)\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{3\,d^2\,\sqrt{d\,x}}+\\ \frac{i\,\,b^2\,\left(-c\,\right)^{\,3/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,\left(-c\right)^{\,1/4}\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+i\,\,c^{\,1/4}\,\right)\,\left(1-i\,\,\left(-c\right)^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{3\,d^2\,\sqrt{d\,x}}+\\ \frac{b^2\,\left(-c\,\right)^{\,3/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{2\,\left(-c\right)^{\,1/4}\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{\left(\,(-c)^{\,1/4}+c^{\,1/4}\,\sqrt{x}\,\right)}\,\right]}{3\,d^2\,\sqrt{d\,x}}-\\ \frac{i\,\,b^2\,c^{\,3/4}\,\sqrt{x}\,\,\text{PolyLog}\!\left[2\,,\,1-\frac{(1-i)\,\left(1+c^{\,1/4}\,\sqrt{x}\,\right)}{1-i\,\,c^{\,1/4}\,\sqrt{x}}\,\right]}{3\,d^2\,\sqrt{d\,x}}$$

Result (type 1, 1 leaves): ???

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\; x^3\,\right]\,\right)^{\,2}}{x}\, \mathrm{d}x$$

Optimal (type 4, 140 leaves, 7 steps):

$$\begin{split} &\frac{2}{3} \, \left(a + b \, \text{ArcTanh} \left[c \, x^3 \right] \right)^2 \, \text{ArcTanh} \left[1 - \frac{2}{1 - c \, x^3} \right] - \frac{1}{3} \, b \, \left(a + b \, \text{ArcTanh} \left[c \, x^3 \right] \right) \, \text{PolyLog} \left[2 \, , \, 1 - \frac{2}{1 - c \, x^3} \right] + \\ &\frac{1}{3} \, b \, \left(a + b \, \text{ArcTanh} \left[c \, x^3 \right] \right) \, \text{PolyLog} \left[2 \, , \, -1 + \frac{2}{1 - c \, x^3} \right] + \\ &\frac{1}{6} \, b^2 \, \text{PolyLog} \left[3 \, , \, 1 - \frac{2}{1 - c \, x^3} \right] - \frac{1}{6} \, b^2 \, \text{PolyLog} \left[3 \, , \, -1 + \frac{2}{1 - c \, x^3} \right] \end{split}$$

Result (type 4, 181 leaves):

$$\begin{split} &a^2 \, \text{Log} \, [\, x\,] \, + \frac{1}{3} \, a \, b \, \left(- \, \text{PolyLog} \, \big[\, 2 \, , \, - \, c \, x^3 \, \big] \, + \, \text{PolyLog} \, \big[\, 2 \, , \, c \, x^3 \, \big] \, \right) \, + \\ &\frac{1}{3} \, b^2 \, \left(\frac{\text{i} \, \pi^3}{24} \, - \frac{2}{3} \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]^3 \, - \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]^2 \, \text{Log} \, \big[\, 1 \, + \, e^{-2 \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]} \, \big] \, + \\ &\text{ArcTanh} \, \big[\, c \, x^3 \, \big]^2 \, \text{Log} \, \big[\, 1 \, - \, e^{2 \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]} \, \big] \, + \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big] \, \, \text{PolyLog} \, \big[\, 2 \, , \, - \, e^{-2 \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]} \, \big] \, + \\ &\text{ArcTanh} \, \big[\, c \, x^3 \, \big] \, \, \text{PolyLog} \, \big[\, 2 \, , \, - \, e^{-2 \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]} \, \big] \, - \, \frac{1}{2} \, \, \text{PolyLog} \, \big[\, 3 \, , \, e^{2 \, \text{ArcTanh} \, \big[\, c \, x^3 \, \big]} \, \big] \, \bigg) \end{split}$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\operatorname{ArcTanh}\left[c\;x^{3}\right]\right)^{3}}{x}\,\mathrm{d}x$$

Optimal (type 4, 210 leaves, 9 steps):

$$\frac{2}{3} \left(a + b \operatorname{ArcTanh} \left[c \, x^3 \right] \right)^3 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \, x^3} \right] - \frac{1}{2} \, b \, \left(a + b \operatorname{ArcTanh} \left[c \, x^3 \right] \right)^2 \operatorname{PolyLog} \left[2 \, , \, 1 - \frac{2}{1 - c \, x^3} \right] + \\ \frac{1}{2} \, b \, \left(a + b \operatorname{ArcTanh} \left[c \, x^3 \right] \right)^2 \operatorname{PolyLog} \left[2 \, , \, -1 + \frac{2}{1 - c \, x^3} \right] + \\ \frac{1}{2} \, b^2 \, \left(a + b \operatorname{ArcTanh} \left[c \, x^3 \right] \right) \operatorname{PolyLog} \left[3 \, , \, 1 - \frac{2}{1 - c \, x^3} \right] - \\ \frac{1}{2} \, b^2 \, \left(a + b \operatorname{ArcTanh} \left[c \, x^3 \right] \right) \operatorname{PolyLog} \left[3 \, , \, -1 + \frac{2}{1 - c \, x^3} \right] - \\ \frac{1}{4} \, b^3 \operatorname{PolyLog} \left[4 \, , \, 1 - \frac{2}{1 - c \, x^3} \right] + \frac{1}{4} \, b^3 \operatorname{PolyLog} \left[4 \, , \, -1 + \frac{2}{1 - c \, x^3} \right]$$

Result (type 4, 368 leaves):

$$a^{3} \log [x] + \frac{1}{2} a^{2} b \left(- \text{PolyLog} \left[2, -c \, x^{3} \right] + \text{PolyLog} \left[2, c \, x^{3} \right] \right) + \\ a b^{2} \left(\frac{i \, \pi^{3}}{24} - \frac{2}{3} \operatorname{ArcTanh} \left[c \, x^{3} \right]^{3} - \operatorname{ArcTanh} \left[c \, x^{3} \right]^{2} \operatorname{Log} \left[1 + e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ \operatorname{ArcTanh} \left[c \, x^{3} \right]^{2} \operatorname{Log} \left[1 - e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \operatorname{ArcTanh} \left[c \, x^{3} \right] \operatorname{PolyLog} \left[2, -e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ \operatorname{ArcTanh} \left[c \, x^{3} \right] \operatorname{PolyLog} \left[2, e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ \frac{1}{2} \operatorname{PolyLog} \left[3, -e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] - \frac{1}{2} \operatorname{PolyLog} \left[3, e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] \right) + \\ \frac{1}{192} b^{3} \left(\pi^{4} - 32\operatorname{ArcTanh} \left[c \, x^{3} \right]^{4} - 64\operatorname{ArcTanh} \left[c \, x^{3} \right]^{3} \operatorname{Log} \left[1 + e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ 64\operatorname{ArcTanh} \left[c \, x^{3} \right]^{3} \operatorname{Log} \left[1 - e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + 96\operatorname{ArcTanh} \left[c \, x^{3} \right]^{2} \operatorname{PolyLog} \left[2, -e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ 96\operatorname{ArcTanh} \left[c \, x^{3} \right]^{2} \operatorname{PolyLog} \left[2, e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] - 96\operatorname{ArcTanh} \left[c \, x^{3} \right] \operatorname{PolyLog} \left[3, e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + \\ 48\operatorname{PolyLog} \left[4, -e^{-2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] + 48\operatorname{PolyLog} \left[4, e^{2\operatorname{ArcTanh} \left[c \, x^{3} \right]} \right] \right)$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\; x^3\,\right]\,\right)^3}{x^4}\; \text{d}\, x$$

Optimal (type 4, 120 leaves, 6 steps):

$$\frac{1}{3} c \left(a + b \operatorname{ArcTanh}\left[c \ x^{3}\right]\right)^{3} - \frac{\left(a + b \operatorname{ArcTanh}\left[c \ x^{3}\right]\right)^{3}}{3 \ x^{3}} + b c \left(a + b \operatorname{ArcTanh}\left[c \ x^{3}\right]\right)^{2} \operatorname{Log}\left[2 - \frac{2}{1 + c \ x^{3}}\right] - b^{2} c \left(a + b \operatorname{ArcTanh}\left[c \ x^{3}\right]\right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c \ x^{3}}\right] - \frac{1}{2} b^{3} c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c \ x^{3}}\right]$$

Result (type 4, 223 leaves):

$$\begin{split} &-\frac{a^3}{3\,x^3}-\frac{a^2\,b\,\text{ArcTanh}\big[c\,x^3\big]}{x^3}+3\,a^2\,b\,c\,\text{Log}\,[\,x\,]\,-\frac{1}{2}\,a^2\,b\,c\,\text{Log}\big[\,1-c^2\,x^6\,\big]\,+\\ &a\,b^2\,c\,\left(\text{ArcTanh}\big[c\,x^3\big]\,\left(\left(1-\frac{1}{c\,x^3}\right)\,\text{ArcTanh}\big[c\,x^3\big]\,+2\,\text{Log}\,\big[\,1-\text{e}^{-2\,\text{ArcTanh}\big[c\,x^3\big]}\,\big]\right)-\\ &\quad \text{PolyLog}\big[\,2\,,\,\,\text{e}^{-2\,\text{ArcTanh}\big[c\,x^3\big]}\,\big]\,\right)\,+\\ &\frac{1}{3}\,b^3\,c\,\left(\frac{i\,\pi^3}{8}-\text{ArcTanh}\big[c\,x^3\big]^3-\frac{\text{ArcTanh}\big[c\,x^3\big]^3}{c\,x^3}+3\,\text{ArcTanh}\big[c\,x^3\big]^2\,\text{Log}\,\big[\,1-\text{e}^{2\,\text{ArcTanh}\big[c\,x^3\big]}\,\big]\,+\\ &3\,\text{ArcTanh}\big[c\,x^3\big]\,\text{PolyLog}\big[\,2\,,\,\,\text{e}^{2\,\text{ArcTanh}\big[c\,x^3\big]}\,\big]\,-\frac{3}{2}\,\text{PolyLog}\big[\,3\,,\,\,\text{e}^{2\,\text{ArcTanh}\big[c\,x^3\big]}\,\big]\,\right) \end{split}$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^{2}}{x} dx$$

Optimal (type 4, 133 leaves, 7 steps):

$$-2\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)^2\mathsf{ArcTanh}\left[1-\frac{2}{1-\frac{\mathsf{c}}{\mathsf{x}}}\right]+\\ \mathsf{b}\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)\mathsf{PolyLog}\left[2,\,1-\frac{2}{1-\frac{\mathsf{c}}{\mathsf{x}}}\right]-\mathsf{b}\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)\mathsf{PolyLog}\left[2,\,-1+\frac{2}{1-\frac{\mathsf{c}}{\mathsf{x}}}\right]-\\ \frac{1}{2}\,\mathsf{b}^2\,\mathsf{PolyLog}\left[3,\,1-\frac{2}{1-\frac{\mathsf{c}}{\mathsf{x}}}\right]+\frac{1}{2}\,\mathsf{b}^2\,\mathsf{PolyLog}\left[3,\,-1+\frac{2}{1-\frac{\mathsf{c}}{\mathsf{x}}}\right]$$

Result (type 4, 177 leaves):

$$\begin{split} & a^2 \, \text{Log} \, [\, x \,] \, + a \, b \, \left(\text{PolyLog} \, \big[\, 2 \, , \, -\frac{c}{x} \, \big] \, - \, \text{PolyLog} \, \big[\, 2 \, , \, \frac{c}{x} \, \big] \, \big) \, + \, b^2 \\ & \left(-\frac{\mathrm{i} \, \pi^3}{24} + \frac{2}{3} \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]^3 \, + \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]^2 \, \text{Log} \, \big[\, 1 \, + \, \mathrm{e}^{-2 \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]} \, \big] \, - \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big] \, 2 \, \text{Log} \, \big[\, 1 \, - \, \mathrm{e}^{2 \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]} \, \big] \, - \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big] \, \big] \, - \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big] \, \big] \, - \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big] \, \big] \, - \, \frac{1}{2} \, \text{PolyLog} \, \big[\, 3 \, , \, - \, \mathrm{e}^{-2 \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]} \, \big] \, + \, \frac{1}{2} \, \, \text{PolyLog} \, \big[\, 3 \, , \, \, \mathrm{e}^{2 \, \text{ArcTanh} \, \big[\, \frac{c}{x} \, \big]} \, \big] \, \big) \, \end{split}$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(a + b \operatorname{ArcTanh} \left[\frac{c}{x} \right] \right)^3 dx$$

Optimal (type 4, 217 leaves, 15 steps):

$$\begin{split} b^2 & \ c^2 \ x \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right) - \frac{1}{2} \ b \ c^3 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right)^2 + \\ & \frac{1}{2} \ b \ c \ x^2 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right)^2 - \frac{1}{3} \ c^3 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right)^3 + \frac{1}{3} \ x^3 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right)^3 - \\ & b \ c^3 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right)^2 \ \text{Log} \left[2 - \frac{2}{1 + \frac{c}{x}} \right] + \frac{1}{2} \ b^3 \ c^3 \ \text{Log} \left[1 - \frac{c^2}{x^2} \right] + b^3 \ c^3 \ \text{Log} \left[x \right] + \\ & b^2 \ c^3 \ \left(a + b \ \text{ArcCoth} \left[\frac{x}{c} \right] \right) \ \text{PolyLog} \left[2 \text{,} \ -1 + \frac{2}{1 + \frac{c}{x}} \right] + \frac{1}{2} \ b^3 \ c^3 \ \text{PolyLog} \left[3 \text{,} \ -1 + \frac{2}{1 + \frac{c}{x}} \right] \end{split}$$

$$\frac{1}{6} \left(3 \, a^2 \, b \, c \, x^2 + 2 \, a^3 \, x^3 + 6 \, a^2 \, b \, x^3 \, \text{ArcTanh} \left[\frac{c}{x} \right] + \right.$$

$$3 \, a^2 \, b \, c \, x^2 + 2 \, a^3 \, x^3 + 6 \, a^2 \, b \, x^3 \, \text{ArcTanh} \left[\frac{c}{x} \right] +$$

$$c \, \text{ArcTanh} \left[\frac{c}{x} \right] \left(-c^2 + x^2 \right] + 6 \, a \, b^2 \left(c^2 \, x + \left(-c^3 + x^3 \right) \, \text{ArcTanh} \left[\frac{c}{x} \right]^2 +$$

$$c \, \text{ArcTanh} \left[\frac{c}{x} \right] \left(-c^2 + x^2 - 2 \, c^2 \, \text{Log} \left[1 - e^{-2 \, \text{ArcTanh} \left[\frac{c}{x} \right]} \right] \right) + c^3 \, \text{PolyLog} \left[2, \, e^{-2 \, \text{ArcTanh} \left[\frac{c}{x} \right]} \right] \right) + \frac{1}{4} \, b^3$$

$$\left(-i \, c^3 \, \pi^3 + 24 \, c^2 \, x \, \text{ArcTanh} \left[\frac{c}{x} \right] - 12 \, c^3 \, \text{ArcTanh} \left[\frac{c}{x} \right]^2 + 12 \, c \, x^2 \, \text{ArcTanh} \left[\frac{c}{x} \right]^2 + 8 \, c^3 \, \text{ArcTanh} \left[\frac{c}{x} \right]^3 +$$

$$8 \, x^3 \, \text{ArcTanh} \left[\frac{c}{x} \right]^3 - 24 \, c^3 \, \text{ArcTanh} \left[\frac{c}{x} \right]^2 \, \text{Log} \left[1 - e^{2 \, \text{ArcTanh} \left[\frac{c}{x} \right]} \right] - 24 \, c^3 \, \text{Log} \left[\frac{c}{1 - \frac{c^2}{x^2}} \, x \right]$$

$$24 \, c^3 \, \text{ArcTanh} \left[\frac{c}{x} \right] \, \text{PolyLog} \left[2, \, e^{2 \, \text{ArcTanh} \left[\frac{c}{x} \right]} \right] + 12 \, c^3 \, \text{PolyLog} \left[3, \, e^{2 \, \text{ArcTanh} \left[\frac{c}{x} \right]} \right]$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + b \, \text{ArcTanh} \left[\, \frac{c}{x} \, \right] \, \right)^3 \, \text{d}x$$

Optimal (type 4, 108 leaves, 6 steps):

$$c\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)^3 + \mathsf{x}\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)^3 - 3 \, \mathsf{b} \, \mathsf{c}\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right)^2 \, \mathsf{Log}\left[\frac{2 \, \mathsf{c}}{\mathsf{c} - \mathsf{x}}\right] - 3 \, \mathsf{b}^2 \, \mathsf{c}\left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcCoth}\left[\frac{\mathsf{x}}{\mathsf{c}}\right]\right) \, \mathsf{PolyLog}\left[2, \, 1 - \frac{2 \, \mathsf{c}}{\mathsf{c} - \mathsf{x}}\right] + \frac{3}{2} \, \mathsf{b}^3 \, \mathsf{c} \, \mathsf{PolyLog}\left[3, \, 1 - \frac{2 \, \mathsf{c}}{\mathsf{c} - \mathsf{x}}\right]$$

Result (type 4, 198 leaves):

$$\begin{aligned} & \mathsf{a}^3 \; \mathsf{x} + \mathsf{3} \; \mathsf{a}^2 \; \mathsf{b} \; \mathsf{x} \, \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big] \, + \frac{3}{2} \; \mathsf{a}^2 \; \mathsf{b} \; \mathsf{c} \; \mathsf{Log}\Big[-\mathsf{c}^2 + \mathsf{x}^2\Big] \, - \\ & \mathsf{3} \; \mathsf{a} \; \mathsf{b}^2 \; \left(\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big] \; \left(\; (\mathsf{c} - \mathsf{x}) \; \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big] \, + 2 \; \mathsf{c} \; \mathsf{Log}\Big[1 - \mathsf{e}^{-2\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]}\Big] \right) - \mathsf{c} \; \mathsf{PolyLog}\Big[2 \text{,} \; \mathsf{e}^{-2\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]}\Big] \right) + \\ & \frac{1}{8} \; \mathsf{b}^3 \; \left(- \, \mathrm{i} \; \mathsf{c} \; \mathsf{x}^3 + 8 \; \mathsf{c} \; \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]^3 + 8 \; \mathsf{x} \; \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]^3 - 24 \; \mathsf{c} \; \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]^2 \; \mathsf{Log}\Big[1 - \mathsf{e}^{2\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]}\Big] - \\ & 24 \; \mathsf{c} \; \mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big] \; \mathsf{PolyLog}\Big[2 \text{,} \; \mathsf{e}^{2\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]} \Big] + 12 \; \mathsf{c} \; \mathsf{PolyLog}\Big[3 \text{,} \; \mathsf{e}^{2\mathsf{ArcTanh}\Big[\frac{\mathsf{c}}{\mathsf{x}}\Big]} \Big] \right) \end{aligned}$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x} \, dx$$

Optimal (type 4, 208 leaves, 9 steps):

$$-2\left(a+b\operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^{3}\operatorname{ArcTanh}\left[1-\frac{2}{1-\frac{c}{x}}\right]+\\ \frac{3}{2}b\left(a+b\operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^{2}\operatorname{PolyLog}\left[2,1-\frac{2}{1-\frac{c}{x}}\right]-\frac{3}{2}b\left(a+b\operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^{2}\operatorname{PolyLog}\left[2,-1+\frac{2}{1-\frac{c}{x}}\right]-\\ \frac{3}{2}b^{2}\left(a+b\operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)\operatorname{PolyLog}\left[3,1-\frac{2}{1-\frac{c}{x}}\right]+\frac{3}{2}b^{2}\left(a+b\operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)\operatorname{PolyLog}\left[3,-1+\frac{2}{1-\frac{c}{x}}\right]+\\ \frac{3}{4}b^{3}\operatorname{PolyLog}\left[4,1-\frac{2}{1-\frac{c}{x}}\right]-\frac{3}{4}b^{3}\operatorname{PolyLog}\left[4,-1+\frac{2}{1-\frac{c}{x}}\right]$$

Result (type 4, 373 leaves):

$$a^{3} \text{ Log}[x] + \frac{3}{2} a^{2} b \left(\text{PolyLog}[2, -\frac{c}{x}] - \text{PolyLog}[2, \frac{c}{x}] \right) + 3 a b^{2}$$

$$\left(-\frac{i \pi^{3}}{24} + \frac{2}{3} \text{ArcTanh} \left[\frac{c}{x} \right]^{3} + \text{ArcTanh} \left[\frac{c}{x} \right]^{2} \text{Log}[1 + e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] - \text{ArcTanh} \left[\frac{c}{x} \right]^{2} \text{Log}[1 - e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] - \text{ArcTanh} \left[\frac{c}{x} \right] \right) - \text{ArcTanh} \left[\frac{c}{x} \right] \text{PolyLog}[2, -e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] - \text{ArcTanh} \left[\frac{c}{x} \right] \text{PolyLog}[3, -e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] + \frac{1}{2} \text{PolyLog}[3, e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] \right) +$$

$$\frac{1}{64} b^{3} \left(-\pi^{4} + 32 \text{ArcTanh} \left[\frac{c}{x} \right]^{4} + 64 \text{ArcTanh} \left[\frac{c}{x} \right]^{3} \text{Log}[1 + e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] -$$

$$64 \text{ArcTanh} \left[\frac{c}{x} \right]^{3} \text{Log}[1 - e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] - 96 \text{ArcTanh} \left[\frac{c}{x} \right]^{2} \text{PolyLog}[2, -e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] -$$

$$96 \text{ArcTanh} \left[\frac{c}{x} \right] \text{PolyLog}[2, e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] -$$

$$96 \text{ArcTanh} \left[\frac{c}{x} \right] \text{PolyLog}[3, -e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] + 96 \text{ArcTanh} \left[\frac{c}{x} \right] \text{PolyLog}[3, e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] -$$

$$48 \text{PolyLog}[4, -e^{-2 \text{ArcTanh} \left[\frac{c}{x} \right]}] - 48 \text{PolyLog}[4, e^{2 \text{ArcTanh} \left[\frac{c}{x} \right]}] \right)$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x} \, dx$$

Optimal (type 4, 144 leaves, 7 steps):

$$-\left(a+b\operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2\operatorname{ArcTanh}\left[1-\frac{2}{1-\frac{c}{x^2}}\right]+\\ \frac{1}{2}\left(b\left(a+b\operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)\operatorname{PolyLog}\left[2,\ 1-\frac{2}{1-\frac{c}{x^2}}\right]-\frac{1}{2}\left(b\left(a+b\operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)\operatorname{PolyLog}\left[2,\ -1+\frac{2}{1-\frac{c}{x^2}}\right]-\frac{1}{2}\left(b\left(a+b\operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)\operatorname{PolyLog}\left[2,\ -1+\frac{2}{1-\frac{c}{x^2}}\right]-\frac$$

Result (type 4, 183 leaves):

$$\begin{split} & a^2 \, \text{Log} \, [\, x \,] \, + \frac{1}{2} \, a \, b \, \left(\text{PolyLog} \, \big[\, 2 \, , \, - \frac{c}{x^2} \, \big] \, - \text{PolyLog} \, \big[\, 2 \, , \, \frac{c}{x^2} \, \big] \right) \, + \\ & \frac{1}{2} \, b^2 \, \left(- \frac{\dot{\mathbf{i}} \, \pi^3}{24} \, + \frac{2}{3} \, \text{ArcTanh} \, \big[\, \frac{c}{x^2} \, \big]^3 \, + \, \text{ArcTanh} \, \big[\, \frac{c}{x^2} \, \big]^2 \, \text{Log} \, \big[\, 1 \, + \, \mathrm{e}^{-2 \, \text{ArcTanh} \, \left[\, \frac{c}{x^2} \, \big]} \, \big] \, - \\ & \text{ArcTanh} \, \big[\, \frac{c}{x^2} \, \big]^2 \, \, \text{Log} \, \big[\, 1 \, - \, \mathrm{e}^{2 \, \text{ArcTanh} \, \left[\, \frac{c}{x^2} \, \big]} \, \big] \, - \, \text{ArcTanh} \, \big[\, \frac{c}{x^2} \, \big] \, \\ & \text{PolyLog} \, \big[\, 2 \, , \, \, \mathrm{e}^{2 \, \text{ArcTanh} \, \left[\, \frac{c}{x^2} \, \big]} \, \big] \, - \, \frac{1}{2} \, \, \text{PolyLog} \, \big[\, 3 \, , \, \, - \, \mathrm{e}^{-2 \, \text{ArcTanh} \, \left[\, \frac{c}{x^2} \, \big]} \, \big] \, + \, \frac{1}{2} \, \, \text{PolyLog} \, \big[\, 3 \, , \, \, \, \, \, \mathrm{e}^{2 \, \, \text{ArcTanh} \, \left[\, \frac{c}{x^2} \, \big]} \, \big] \, \right) \end{split}$$

Problem 176: Unable to integrate problem.

$$\int \! x^4 \, \left(\text{a} + \text{b} \, \text{ArcTanh} \left[\, \frac{\text{c}}{\text{x}^2} \, \right] \, \right)^2 \, \text{d} \, x$$

Optimal (type 4, 1214 leaves, 98 steps):

$$\begin{split} &\frac{8}{15} b^2 c^5 x + \frac{2}{15} a b c x^2 + \frac{2}{5} a b c^{5/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{4}{15} b^2 c^{5/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{4}{15} b^2 c^{5/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{4}{15} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{4}{15} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{4}{15} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{4}{15} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{1}{15} b^2 c x^3 \log \Big[1 - \frac{c}{x^2} \Big] - \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] \log \Big[1 - \frac{c}{x^2} \Big] + \frac{1}{15} b c x^3 \Big[2 a - b \log \Big[1 - \frac{c}{x^2} \Big] \Big] - \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] \Big[2 a - b \log \Big[1 - \frac{c}{x^2} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] - \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{5} b^2 c^{5/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] - \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \log \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1 + \frac{c}{x^2} \Big] + \frac{1}{20} b^2 x^5 \exp \Big[1$$

$$\int \! x^4 \, \left(\text{a} + \text{b} \, \text{ArcTanh} \, \big[\, \frac{\text{c}}{\text{x}^2} \, \big] \, \right)^2 \, \text{d} x$$

Problem 177: Unable to integrate problem.

$$\int \! x^2 \, \left(\text{a + b ArcTanh} \left[\, \frac{c}{x^2} \, \right] \right)^2 \, \text{d} \, x$$

Optimal (type 4, 1172 leaves, 80 steps):

$$\frac{4}{3} \text{ a b c x } - \frac{2}{3} \text{ a b c}^{3/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{4}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ i b}^2 \text{ c}^{3/2} \text{ ArcTan} \Big[\frac{x}{\sqrt{c}} \Big]^2 - \frac{4}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{2}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{2}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] - \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ a b x}^3 \text{ Log} \Big[1 - \frac{c}{x^2} \Big] + \frac{1}{12} x^3 \left(2 \text{ a - b Log} \Big[1 - \frac{c}{x^2} \Big] \right)^2 + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ a b x}^3 \text{ Log} \Big[1 - \frac{c}{x^2} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^{3/2} \text{ ArcTanh} \Big[\frac{x}{\sqrt{c}} \Big] + \frac{1}{3} \text{ b}^2 \text{ c}^3 \text{ arcTanh}$$

$$\int \! x^2 \, \left(\text{a} + \text{b} \, \text{ArcTanh} \left[\, \frac{c}{x^2} \, \right] \, \right)^2 \, \text{d} \, x$$

Problem 180: Unable to integrate problem.

$$\int \frac{\left(a+b \, ArcTanh\left[\frac{c}{x^2}\right]\right)^2}{x^4} \, dx$$

Optimal (type 4, 1263 leaves, 105 steps):

$$\begin{array}{c} \frac{2 \, a \, b}{9 \, x^{2}} - \frac{2 \, a \, b}{3 \, c \, x} - \frac{2 \, a \, b \, A \, c \, c \, T \, n \, \left[\frac{x}{\sqrt{c}} \right]}{3 \, c^{3/2}} + \frac{4 \, b^{2} \, A \, c \, c \, T \, n \, \left[\frac{x}{\sqrt{c}} \right]}{3 \, c^{3/2}} + \frac{4 \, b^{2} \, A \, c \, c \, c \, n \, n \, \left[\frac{x}{\sqrt{c}} \right]}{3 \, c^{3/2}} + \frac{4 \, b^{2} \, A \, c \, c \, c \, n \, n \, \left[\frac{x}{\sqrt{c}} \right]}{3 \, c^{3/2}} + \frac{3 \, c^{3/2}}{3 \, c^{3/2}} - \frac{3 \, c^{3/2}}{3 \, c^{3/2}} + \frac{3 \, c^{3/2}}{3 \, c^{3/2}} + \frac{3 \, c^{3/2}}{3 \, c \, x} - \frac{3 \, c^{3/2}}{3 \, c \, x} + \frac{3 \, c^{3/2}}{3 \, c^{3/2}} + \frac{3 \, c^{3/2}}{3 \, c \, x} + \frac{3 \, c^{3/2}}{3 \, c^{3/2}} + \frac{3 \, c^{3$$

$$\int \frac{\left(a + b \, \text{ArcTanh} \left[\frac{c}{x^2}\right]\right)^2}{x^4} \, \text{d} x$$

Problem 181: Unable to integrate problem.

$$\int \frac{\left(a + b \, \text{ArcTanh} \left[\, \frac{c}{x^2} \, \right] \, \right)^2}{x^6} \, \mathrm{d} x$$

Optimal (type 4, 1337 leaves, 130 steps):

$$\begin{array}{c} 2a\,b \\ 25\,x^5 - 15\,c\,x^3 \\ 5\,c^2\,x \\ 25\,x^5 - 15\,c\,x^3 \\ 25\,x^5 - 15\,c\,x^5 - 15\,c\,x^5 \\ 25\,x^5 - 15\,c\,x^5 - 15\,c\,x^5 - 15\,c\,x^5 \\ 25\,x^5 - 15\,c\,x^5 - 15$$

$$\int \frac{\left(a+b \, Arc Tanh \left[\frac{c}{x^2}\right]\right)^2}{x^6} \, dx$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^{2}}{x} dx$$

Optimal (type 4, 145 leaves, 7 steps):

$$4 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - c \sqrt{x}} \right] \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right)^2 - \\ 2 b \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right) \operatorname{PolyLog} \left[2, 1 - \frac{2}{1 - c \sqrt{x}} \right] + \\ 2 b \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right) \operatorname{PolyLog} \left[2, -1 + \frac{2}{1 - c \sqrt{x}} \right] + \\ b^2 \operatorname{PolyLog} \left[3, 1 - \frac{2}{1 - c \sqrt{x}} \right] - b^2 \operatorname{PolyLog} \left[3, -1 + \frac{2}{1 - c \sqrt{x}} \right]$$

Result (type 4, 203 leaves):

$$\begin{split} & a^2 \, \text{Log} \, [\, x \,] \, + 2 \, a \, b \, \left(- \, \text{PolyLog} \, \big[\, 2 \, , \, - \, c \, \sqrt{x} \, \, \big] \, + \, \text{PolyLog} \, \big[\, 2 \, , \, c \, \sqrt{x} \, \, \big] \, \right) \, + \\ & 2 \, b^2 \, \left(\frac{\mathrm{i}}{24} \, - \, \frac{\pi^3}{2} \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]^3 \, - \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]^2 \, \operatorname{Log} \, \big[\, 1 \, + \, \mathrm{e}^{-2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, + \\ & \quad \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]^2 \, \operatorname{Log} \, \big[\, 1 \, - \, \mathrm{e}^{2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, + \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big] \, \operatorname{PolyLog} \, \big[\, 2 \, , \, - \, \mathrm{e}^{-2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, + \\ & \quad \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big] \, \operatorname{PolyLog} \, \big[\, 2 \, , \, - \, \mathrm{e}^{-2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, + \\ & \quad \frac{1}{2} \, \operatorname{PolyLog} \, \big[\, 3 \, , \, - \, \mathrm{e}^{-2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, - \, \frac{1}{2} \, \operatorname{PolyLog} \, \big[\, 3 \, , \, \, \mathrm{e}^{2 \, \operatorname{ArcTanh} \, \big[\, c \, \sqrt{x} \, \, \big]} \, \big] \, \right) \end{split}$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^{3}}{x} dx$$

Optimal (type 4, 224 leaves, 9 steps):

$$\begin{tabular}{l} 4 & ArcTanh \Big[1 - \frac{2}{1 - c \sqrt{x}} \Big] & (a + b ArcTanh \Big[c \sqrt{x} \Big] \Big)^3 - 3b & (a + b ArcTanh \Big[c \sqrt{x} \Big] \Big)^2 & PolyLog \Big[2, \ 1 - \frac{2}{1 - c \sqrt{x}} \Big] + 3b & (a + b ArcTanh \Big[c \sqrt{x} \Big] \Big)^2 & PolyLog \Big[2, \ -1 + \frac{2}{1 - c \sqrt{x}} \Big] + 3b^2 & (a + b ArcTanh \Big[c \sqrt{x} \Big] \Big) & PolyLog \Big[3, \ 1 - \frac{2}{1 - c \sqrt{x}} \Big] - 3b^2 & (a + b ArcTanh \Big[c \sqrt{x} \Big] \Big) & PolyLog \Big[3, \ -1 + \frac{2}{1 - c \sqrt{x}} \Big] - 3b^3 & PolyLog \Big[4, \ 1 - \frac{2}{1 - c \sqrt{x}} \Big] + \frac{3}{2}b^3 & PolyLog \Big[4, \ -1 + \frac{2}{1 - c \sqrt{x}} \Big] \\ & Result (type 4, \ 423 \, leaves): \\ & a^3 Log \Big[x \Big] + 3a^2 b & \Big(-PolyLog \Big[2, \ -c \sqrt{x} \Big] + PolyLog \Big[2, \ c \sqrt{x} \Big] \Big) + \\ & 6ab^2 & \Big(\frac{i \pi^3}{24} - \frac{2}{3} ArcTanh \Big[c \sqrt{x} \Big]^3 - ArcTanh \Big[c \sqrt{x} \Big]^2 & Log \Big[1 + e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] + ArcTanh \Big[c \sqrt{x} \Big] & PolyLog \Big[2, \ -e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] + ArcTanh \Big[c \sqrt{x} \Big] & PolyLog \Big[2, \ -e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] + \frac{1}{2} & PolyLog \Big[3, \ -e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] - \frac{1}{2} & PolyLog \Big[3, \ -e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] + \frac{1}{2} & PolyLog \Big[3, \ -e^{-2 ArcTanh \Big[c \sqrt{x} \Big]} \Big] + 96 \, ArcTanh \Big[c \sqrt{x} \Big] + 90 \, ArcTanh \Big[c \sqrt{x} \Big] + 96 \, ArcTanh \Big[c \sqrt{x} \Big] +$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, ArcTanh\left[\, c\, \, x^{3/2}\,\right]\,\right)^{\,2}}{x}\, \mathrm{d}x$$

Optimal (type 4, 156 leaves, 7 steps):

$$\begin{split} &\frac{4}{3}\,\left(a+b\,\text{ArcTanh}\left[c\,\,x^{3/2}\right]\right)^2\,\text{ArcTanh}\left[1-\frac{2}{1-c\,\,x^{3/2}}\right] - \\ &\frac{2}{3}\,b\,\left(a+b\,\text{ArcTanh}\left[c\,\,x^{3/2}\right]\right)\,\text{PolyLog}\!\left[2\,\text{, }1-\frac{2}{1-c\,\,x^{3/2}}\right] + \\ &\frac{2}{3}\,b\,\left(a+b\,\text{ArcTanh}\left[c\,\,x^{3/2}\right]\right)\,\text{PolyLog}\!\left[2\,\text{, }-1+\frac{2}{1-c\,\,x^{3/2}}\right] + \\ &\frac{1}{3}\,b^2\,\text{PolyLog}\!\left[3\,\text{, }1-\frac{2}{1-c\,\,x^{3/2}}\right] - \frac{1}{3}\,b^2\,\text{PolyLog}\!\left[3\,\text{, }-1+\frac{2}{1-c\,\,x^{3/2}}\right] \end{split}$$

Result (type 4, 207 leaves):

$$\begin{split} & a^2 \, \text{Log} \, [\, x \,] \, + \frac{2}{3} \, a \, b \, \left(- \text{PolyLog} \left[\, 2 \, , \, - c \, x^{3/2} \, \right] \, + \text{PolyLog} \left[\, 2 \, , \, c \, x^{3/2} \, \right] \, \right) \, + \\ & \frac{2}{3} \, b^2 \, \left(\frac{\mathrm{i} \, \pi^3}{24} \, - \frac{2}{3} \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]^3 \, - \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]^2 \, \text{Log} \left[\, 1 \, + \, e^{-2 \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]} \, \right] \, + \\ & \quad \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]^2 \, \text{Log} \left[\, 1 \, - \, e^{2 \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]} \, \right] \, + \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right] \, \text{PolyLog} \left[\, 2 \, , \, - \, e^{-2 \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]} \, \right] \, + \\ & \quad \frac{1}{2} \, \text{PolyLog} \left[\, 3 \, , \, - \, e^{-2 \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]} \, \right] \, - \, \frac{1}{2} \, \text{PolyLog} \left[\, 3 \, , \, e^{2 \, \text{ArcTanh} \left[\, c \, x^{3/2} \, \right]} \, \right] \, \right) \end{split}$$

Problem 227: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{ArcTanh} \left[c \ x^n \right]}{x} \, \mathrm{d}x$$

Optimal (type 4, 36 leaves, 2 steps):

$$a \, \mathsf{Log} \, [\, x \,] \, - \, \frac{b \, \mathsf{PolyLog} \, [\, 2 \, , \, -c \, \, x^n \,]}{2 \, n} \, + \, \frac{b \, \mathsf{PolyLog} \, [\, 2 \, , \, c \, \, x^n \,]}{2 \, n}$$

Result (type 5, 39 leaves):

$$\frac{b c x^{n} \text{ HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^{2} x^{2 n}\right]}{n} + a \text{ Log}\left[x\right]$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\,a\,+\,b\,\,ArcTanh\,[\,c\,\,x^{n}\,]\,\right)^{\,2}}{x}\,\,\mathrm{d}x$$

Optimal (type 4, 148 leaves, 7 steps):

$$\frac{2\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcTanh}\left[\mathsf{c}\,\mathsf{x}^{\mathsf{n}}\right]\right)^{2}\,\mathsf{ArcTanh}\left[1-\frac{2}{1-\mathsf{c}\,\mathsf{x}^{\mathsf{n}}}\right]}{\mathsf{n}}-\frac{\mathsf{b}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcTanh}\left[\mathsf{c}\,\mathsf{x}^{\mathsf{n}}\right]\right)\,\mathsf{PolyLog}\left[2\,\mathsf{,}\,1-\frac{2}{1-\mathsf{c}\,\mathsf{x}^{\mathsf{n}}}\right]}{\mathsf{n}}+\frac{\mathsf{b}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{ArcTanh}\left[\mathsf{c}\,\mathsf{x}^{\mathsf{n}}\right]\right)\,\mathsf{PolyLog}\left[2\,\mathsf{,}\,-1+\frac{2}{1-\mathsf{c}\,\mathsf{x}^{\mathsf{n}}}\right]}{\mathsf{n}}+\frac{\mathsf{b}^{2}\,\mathsf{PolyLog}\left[3\,\mathsf{,}\,1-\frac{2}{1-\mathsf{c}\,\mathsf{x}^{\mathsf{n}}}\right]}{\mathsf{n}}-\frac{\mathsf{b}^{2}\,\mathsf{PolyLog}\left[3\,\mathsf{,}\,-1+\frac{2}{1-\mathsf{c}\,\mathsf{x}^{\mathsf{n}}}\right]}{2\,\mathsf{n}}$$

Result (type 4, 181 leaves):

$$\begin{split} & a^2 \, \text{Log}\,[\,x\,] \, + \frac{a\,\, b\, \left(-\, \text{PolyLog}\,[\,2\,,\, - c\,\, x^n\,] \, + \text{PolyLog}\,[\,2\,,\, c\,\, x^n\,]\,\right)}{n} \, + \frac{1}{n} \\ & b^2 \, \left(\frac{\mathrm{i}\,\,\pi^3}{24} \, - \, \frac{2}{3}\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]^3 \, - \, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]^2 \, \text{Log}\,\big[\,1 \, + \, \mathrm{e}^{-2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, + \\ & \quad \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]^2 \, \text{Log}\,\big[\,1 \, - \, \mathrm{e}^{2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, + \, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big] \, \, \text{PolyLog}\,\big[\,2\,,\, - \, \mathrm{e}^{-2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, + \\ & \quad \text{ArcTanh}\,\big[\,c\,\, x^n\,\big] \, \, \text{PolyLog}\,\big[\,2\,,\, - \, \mathrm{e}^{-2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, + \\ & \quad \frac{1}{2}\, \text{PolyLog}\,\big[\,3\,,\, - \, \mathrm{e}^{-2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, - \, \frac{1}{2}\, \, \text{PolyLog}\,\big[\,3\,,\, \mathrm{e}^{2\, \text{ArcTanh}\,\big[\,c\,\, x^n\,\big]}\,\big] \, \right) \end{split}$$

Problem 236: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcTanh}\,[\,a\,\,x^n\,]}{x}\,\text{d}\,x$$

Optimal (type 4, 30 leaves, 2 steps):

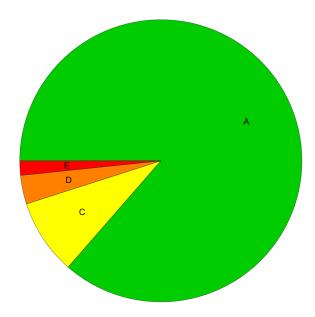
$$-\frac{\text{PolyLog[2, -a } x^{n}]}{2 \, n} + \frac{\text{PolyLog[2, a } x^{n}]}{2 \, n}$$

Result (type 5, 33 leaves):

$$\frac{\text{a } x^{\text{n}} \text{ HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \text{ a}^{2} \text{ x}^{2 \text{ n}}\right]}{\text{n}}$$

Summary of Integration Test Results

243 integration problems



- A 210 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 21 unnecessarily complex antiderivatives
- D 8 unable to integrate problems
- E 4 integration timeouts