# Rubi 4.16.0.4 Integration Test Results

# on the problems in the test-suite directory "6 Hyperbolic functions"

Test results for the 502 problems in "6.1.1 (c+d x)^m (a+b sinh)^n.m"

Test results for the 102 problems in "6.1.3 (e x) $^m$  (a+b sinh(c+d x $^n$ )) $^p$ .m"

Test results for the 33 problems in "6.1.4 (d+e x) $^n$ m sinh(a+b x+c x $^2$ ) $^n$ n.m"

Test results for the 369 problems in "6.1.5 Hyperbolic sine functions.m"

Test results for the 525 problems in "6.1.7 hyper^m (a+b sinh^n)^p.m"

Test results for the 183 problems in "6.2.1 (c+d x)^m (a+b cosh)^n.m"

Test results for the 111 problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.m"

Test results for the 68 problems in "6.2.3 (e x) $^m$  (a+b cosh(c+d x $^n$ )) $^p$ .m"

Test results for the 33 problems in "6.2.4 (d+e x) $^m$  cosh(a+b x+c x $^2$ ) $^n$ .m"

## Test results for the 336 problems in "6.2.5 Hyperbolic cosine functions.m"

# Test results for the 85 problems in "6.2.7 hyper^m (a+b cosh^n)^p.m"

## Test results for the 77 problems in "6.3.1 (c+d x)^m (a+b tanh)^n.m"

Problem 16: Unable to integrate problem.

$$\int \left(c + dx\right) \left(b \, Tanh \left[\,e + f\,x\,\right]\,\right)^{5/2} \, dx$$

Optimal (type 4, 1392 leaves, 44 steps):

$$\frac{2 \, b^{5/2} \, d \, \text{ArcTan} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big] }{3 \, f^2} - \frac{\left(-b\right)^{5/2} \left(c + d \, x\right) \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{-b}} \Big] }{5} - \frac{\left(-b\right)^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{-b}} \Big] }{2 \, f^2} + \frac{2 \, b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big] }{3 \, f^2} + \frac{b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big]^2}{5 \, f^2} + \frac{b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big]^2}{2 \, f^2} - \frac{b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big] \, \text{Log} \Big[ \frac{2 \, \sqrt{b}}{\sqrt{b} + \sqrt{b \, \text{Tanh} [e+f \, x]}} \Big]}{f^2} + \frac{b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big] \, \text{Log} \Big[ \frac{2 \, \sqrt{b}}{\sqrt{b} + \sqrt{b \, \text{Tanh} [e+f \, x]}} \Big]}{f^2} - \frac{b^{5/2} \, d \, \text{ArcTanh} \Big[ \frac{\sqrt{b \, \text{Tanh} [e+f \, x]}}{\sqrt{b}} \Big] \, \text{Log} \Big[ \frac{2 \, \sqrt{b} \, \left(\sqrt{-b} + \sqrt{b \, \text{Tanh} [e+f \, x]}\right)}{\left(\sqrt{-b} + \sqrt{b} \, \right) \left(\sqrt{b} + \sqrt{b \, \text{Tanh} [e+f \, x]}\right)}} - \frac{1}{2 \, f^2} + \frac{2 \, \sqrt{b} \, \left(\sqrt{-b} + \sqrt{b \, \text{Tanh} [e+f \, x]}\right)}{\sqrt{-b}} \Big] \, \text{Log} \Big[ \frac{2 \, \sqrt{b} \, \left(\sqrt{-b} + \sqrt{b \, \text{Tanh} [e+f \, x]}\right)}}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} [e+f \, x]}} \Big]} - \frac{1}{2 \, f^2} + \frac$$

$$\frac{\left(-b\right)^{5/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \, (e + f \, x)}}{\sqrt{-b}} \right] \, Log \left[ \frac{2 \left[ \sqrt{b} - \sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]}{\sqrt{-b} + \sqrt{b} \, \left[ 1 - \frac{\sqrt{b \, \text{Tanh} \, (e + f \, x)}}{\sqrt{-b}} \right]} - \frac{2 \, f^2}{\left( -b\right)^{5/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \, (e + f \, x)}}{\sqrt{-b}} \right] \, Log \left[ - \frac{2 \left[ \sqrt{b} + \sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]}{\left( \sqrt{-b} - \sqrt{b} \right) \left( 1 - \frac{\sqrt{b \, \text{Tanh} \, (e + f \, x)}}{\sqrt{b}} \right)} - \frac{2 \, f^2}{\sqrt{b} - \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]}{2 \, f^2} - \frac{2 \, f^2}{\sqrt{b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \sqrt{b}}{\sqrt{b} - \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]}{2 \, f^2} - \frac{2 \, f^2}{\sqrt{b} \, \sqrt{-b} - \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} \right]}{2 \, f^2} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \sqrt{b}}{\sqrt{-b} - \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]}{4 \, f^2} - \frac{2 \, \sqrt{b} \, \left( \sqrt{-b} - \sqrt{b} \, \text{Tanh} \, (e + f \, x)} \right)}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \sqrt{b} \, \sqrt{-b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]}{4 \, f^2} - \frac{2 \, \sqrt{b} \, \left( \sqrt{-b} - \sqrt{b} \, \text{Tanh} \, (e + f \, x)} \right)}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \sqrt{b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]} \right]}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}}} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]} {\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]} {\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]} {\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b} \, -\sqrt{b \, \text{Tanh} \, (e + f \, x)} \right]} {\sqrt{-b} \, \sqrt{-b} \, \sqrt{b \, \text{Tanh} \, (e + f \, x)}} \right]} - \frac{b^{5/2} \, d \, \text{PolyLog} \left[ 2 , \, 1 - \frac{2 \, \left[ \sqrt{b}$$

## Problem 17: Unable to integrate problem.

$$\int (c + dx) (b Tanh [e + fx])^{3/2} dx$$

#### Optimal (type 4, 1363 leaves, 43 steps):

$$\frac{2 \, b^{3/2} \, d \, \text{ArcTan} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} = \frac{\left( -b \right)^{3/2} \, \left( c + d \, \mathbf{x} \right) \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{2 \, b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]^{2}}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]}{\sqrt{b}} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} - \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f} \, \mathbf{x} \right)}}{\sqrt{b}} \right]} + \frac{b^{3/2} \, d \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left( \mathbf{e} \cdot \mathbf{f}$$

$$\frac{b^{3/2} \, d \, \text{PolyLog} \Big[ 2, \, 1 - \frac{2 \, \sqrt{b}}{\sqrt{b} \, + \sqrt{b \, \text{Tanh} [e + f \, x]}} \Big)}{2 \, f^2} + \frac{b^{3/2} \, d \, \text{PolyLog} \Big[ 2, \, 1 - \frac{2 \, \sqrt{b} \, \left( \sqrt{-b} \, - \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)}{\sqrt{-b} \, \sqrt{b} \, + \sqrt{b \, \text{Tanh} [e + f \, x]}} \Big)}{4 \, f^2} \\ \frac{b^{3/2} \, d \, \text{PolyLog} \Big[ 2, \, 1 - \frac{2 \, \sqrt{b} \, \left( \sqrt{-b} \, + \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)}{\left( \sqrt{-b} \, + \sqrt{b} \right) \, \left( \sqrt{b} \, + \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)} \\ + \frac{4 \, f^2}{4 \, f^2} + \frac{2 \, \left( -b \right)^{3/2} \, d \, \text{PolyLog} \Big[ 2, \, 1 - \frac{2 \, \left( \sqrt{b} \, - \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)}{\left( \sqrt{-b} \, + \sqrt{b} \right) \, \left( 1 - \frac{\sqrt{b} \, \text{Tanh} [e + f \, x]}}{\sqrt{-b}} \right)} \\ - \frac{2 \, f^2}{4 \, f^2} + \frac{2 \, \left( \sqrt{b} \, + \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{b} \, \left( 1 - \frac{\sqrt{b} \, \text{Tanh} [e + f \, x]}}{\sqrt{-b}} \right)} \\ + \frac{4 \, f^2}{4 \, f^2} + \frac{2 \, \left( \sqrt{b} \, + \sqrt{b} \, \text{Tanh} [e + f \, x]} \right)}{\sqrt{-b} \, \sqrt{-b} \, \sqrt{-b} \, \sqrt{-b} \, \sqrt{b} \, \text{Tanh} [e + f \, x]}} \\ - \frac{2 \, b \, \left( c + d \, x \right) \, \sqrt{b} \, \text{Tanh} [e + f \, x]}}{2 \, b \, \left( c + d \, x \right) \, \sqrt{b} \, \text{Tanh} [e + f \, x]}} \right)}$$

#### Result (type 8, 108 leaves, 6 steps):

2 f<sup>2</sup>

$$-\frac{2\,b^{3/2}\,d\,\text{ArcTan}\Big[\,\frac{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{\sqrt{b}}\,\Big]}{f^2}\,+\,\frac{2\,b^{3/2}\,d\,\text{ArcTanh}\Big[\,\frac{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{\sqrt{b}}\,\Big]}{f^2}\,-\,\\ \frac{2\,b\,\left(c+d\,x\right)\,\sqrt{b\,\text{Tanh}\,[e+f\,x]}}{f}\,+\,b^2\,\text{Unintegrable}\Big[\,\frac{c+d\,x}{\sqrt{b\,\text{Tanh}\,[e+f\,x]}}\,\text{, }x\,\Big]$$

### Problem 18: Unable to integrate problem.

$$\int (c + dx) \sqrt{b \, Tanh [e + fx]} \, dx$$

Optimal (type 4, 1280 leaves, 37 steps):

$$-\frac{\sqrt{-b} \left(c+d\,x\right)\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}\right]}{f} - \frac{\sqrt{-b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}\right]^2}{2\,\,f^2} + \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]^2}{f} + \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]^2}{2\,\,f^2} - \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]^2}{f} - \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\right]^2}{f^2} + \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{b}}{\sqrt{b}}\right]^2}{f^2} + \frac{\sqrt{b} \,\,d\, \text{ArcTanh}\left[\frac{\sqrt{b}$$

$$\frac{\sqrt{b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \times)}} \right] }{ \sqrt{b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b} \left[ \sqrt{-b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}{\left( \sqrt{-b} - \sqrt{b} \right) \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \times)}} \right]} \right] }{ 2 \ f^2}$$

$$\sqrt{b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b} \left[ \sqrt{-b} + \sqrt{b} \operatorname{Tanh} (e + f \times)} \right]}{\left( \sqrt{-b} + \sqrt{b} \right) \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \times)}} \right]} \right]$$

$$2 \ f^2$$

$$\sqrt{-b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{-b}} \right] \operatorname{Log} \left[ \frac{2 \left[ \sqrt{b} - \sqrt{b \operatorname{Tanh} (e + f \times)} \right]}{\sqrt{-b} - \sqrt{b}} \right] } \right]$$

$$2 \ f^2$$

$$\sqrt{-b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{-b}} \right] \operatorname{Log} \left[ \frac{2 \left[ \sqrt{b} - \sqrt{b \operatorname{Tanh} (e + f \times)} \right]}{\sqrt{-b} - \sqrt{b}} \right] } \right]$$

$$2 \ f^2$$

$$\sqrt{-b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{-b}} \right] \operatorname{Log} \left[ \frac{2}{\sqrt{\sqrt{b \operatorname{Tanh} (e + f \times)}}} \right]$$

$$2 \ f^2$$

$$\sqrt{-b} \ d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \times)}}{\sqrt{-b}} \right] \operatorname{Log} \left[ \frac{2}{\sqrt{\sqrt{b \operatorname{Tanh} (e + f \times)}}} \right]$$

$$\sqrt{b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh} (e + f \times)}}} \right]$$

$$2 \ f^2$$

$$\sqrt{b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}} \right]$$

$$4 \ f^2$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}}{\sqrt{-b} - \sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}} \right]$$

$$4 \ f^2$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}{\sqrt{-b} - \sqrt{b} - \sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}} \right]$$

$$2 \ f^2$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}}{\sqrt{-b} - \sqrt{b} - \sqrt{b} - \sqrt{b} - \sqrt{b} - \sqrt{b} - \sqrt{b}}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} - \sqrt{b} \operatorname{Tanh} (e + f \times)}}{\sqrt{-b} - \sqrt{b} - \sqrt{b} - \sqrt{b}}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} - \sqrt{b} - \sqrt{b}}}{\sqrt{-b} - \sqrt{b}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b} - \sqrt{b}}{\sqrt{b}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b}}{\sqrt{b}} \right]$$

$$\sqrt{-b} \ d \operatorname{PolyLog} \left[ 2, 1 - \frac{2\sqrt{b} - \sqrt{b}}{\sqrt{b}} \right]$$

$$\sqrt{$$

$$\frac{\sqrt{-b} \text{ d PolyLog} \Big[ 2\text{, } 1 + \frac{2\left(\sqrt{b} + \sqrt{b \, \text{Tanh} \left[e + f \, x\right]}\right)}{\left(\sqrt{-b} - \sqrt{b}\right) \left(1 - \frac{\sqrt{b \, \text{Tanh} \left[e + f \, x\right]}}{\sqrt{-b}}\right)}}{4 \, f^2} + \frac{\sqrt{-b} \text{ d PolyLog} \Big[ 2\text{, } 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[e + f \, x\right]}}{\sqrt{-b}}} \Big]}{2 \, f^2}$$

Result (type 8, 20 leaves, 0 steps):

Unintegrable 
$$\left[ (c + dx) \sqrt{b Tanh [e + fx]} , x \right]$$

### Problem 19: Unable to integrate problem.

 $2\sqrt{-h} f^{2}$ 

$$\int \frac{c + dx}{\sqrt{b \, Tanh \, [\, e + f\, x\, ]}} \, dx$$

Optimal (type 4, 1280 leaves, 37 steps):

$$\frac{\left(c+d\,x\right)\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{-\mathsf{b}}\,\,f}\right]}{\sqrt{-\mathsf{b}}\,\,f} = \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{-\mathsf{b}}\,\,f^2}\right]}{2\,\sqrt{\mathsf{b}}\,\,f^2} + \frac{\left(c+d\,x\right)\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,\,}\right]}{\sqrt{\mathsf{b}}\,\,f} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]^2}{2\,\sqrt{\mathsf{b}}\,\,f^2} - \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}\right]}{\sqrt{\mathsf{b}}\,\,f^2} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}}{\sqrt{\mathsf{b}}\,+\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}\right]}{\sqrt{\mathsf{b}}\,\,f^2} - \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}\,\left[\sqrt{-\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\left(\sqrt{-\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\right)\left(\sqrt{\mathsf{b}}\,+\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}\right)}\right]}{2\,\sqrt{\mathsf{b}}\,\,f^2} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}\,\left[\sqrt{-\mathsf{b}}\,+\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\left(\sqrt{-\mathsf{b}}\,+\sqrt{\mathsf{b}}\,\right)\left(\sqrt{\mathsf{b}}\,+\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}\right)}\right]}{2\,\sqrt{\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\left(\sqrt{-\mathsf{b}}\,+\sqrt{\mathsf{b}}\,\right)\left(\sqrt{\mathsf{b}}\,+\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}\right)}\right]}{\sqrt{-\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\left(\sqrt{-\mathsf{b}}\,+\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right)}\right]} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}}\right]\,\mathsf{Log}\left[\frac{2\,\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{-\mathsf{b}}\,-\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}\right]}}{\sqrt{-\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\sqrt{\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right]}}{\sqrt{-\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\sqrt{\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\sqrt{\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,\mathsf{Tanh}\left[\mathsf{e}+f\,x\right]}\right]}{\sqrt{\mathsf{b}}\,\,f^2}} + \frac{d\,\mathsf{ArcTanh}\left[\frac{\sqrt{\mathsf{b\,Tanh}\left[\mathsf{e}+f\,x\right]}}{\sqrt{\mathsf{b}}\,-\sqrt{\mathsf{b}}\,+\sqrt$$

$$\frac{\text{d} \, \text{ArcTanh} \left[ \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}} \right] \, \text{Log} \left[ - \frac{2 \left[ \sqrt{b} \, + \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b} - \sqrt{b}} \right]}{\sqrt{-b} - \sqrt{b}} \right] - \frac{2 \sqrt{-b} \, \text{f}^2}{\sqrt{-b} - \sqrt{b} \, \text{formin} \left[ e + f \, x \right]}}{\sqrt{-b} - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}}{\sqrt{-b} - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}}{\sqrt{-b} - \frac{2 \sqrt{b} \, \text{f}^2}{\sqrt{-b} - \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}}{2 \sqrt{b} \, \text{f}^2}$$

$$\frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2 \sqrt{b}}{\sqrt{b} + \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}} \right]}{2 \sqrt{b} \, f^2} + \frac{2 \sqrt{b} \, \left[ \sqrt{-b} - \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}}$$

$$\frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2 \sqrt{b}}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2 \left[ \sqrt{b} - \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]} \right]}{\sqrt{-b}} \right]}{\sqrt{-b}} - \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2 \left[ \sqrt{b} - \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]} \right]}{\sqrt{-b}} \right]}{\sqrt{-b}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}}{\sqrt{-b} + \sqrt{b} \, \text{Tanh} \left[ e + f \, x \right]}} + \frac{\text{d} \, \text{PolyLog} \left[ 2, \, 1 - \frac{2}{$$

Result (type 8, 20 leaves, 0 steps):

Unintegrable 
$$\left[\frac{c+dx}{\sqrt{b \, Tanh \, [e+fx]}}, x\right]$$

### Problem 20: Unable to integrate problem.

$$\int\!\frac{c+d\,x}{\left(b\,\mathsf{Tanh}\,[\,e+f\,x\,]\,\right)^{\,3/2}}\,\mathrm{d}\,x$$

Optimal (type 4, 1365 leaves, 43 steps):

$$\frac{2\,d\,\text{ArcTan}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{b}}\Big]}{b^{3/2}\,f^2} - \frac{\left(c+d\,x\right)\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{-b}}\Big]}{\left(-b\right)^{3/2}\,f} - \frac{d\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{-b}}\Big]^2}{2\,\left(-b\right)^{3/2}\,f^2} + \frac{2\,d\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{b}}\Big]}{b^{3/2}\,f^2} + \frac{\left(c+d\,x\right)\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{b}}\Big]}{b^{3/2}\,f} + \frac{d\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text{Tanh}[e+f\,x]}}{\sqrt{b}}\Big]^2}{2\,b^{3/2}\,f^2} + \frac{d\,\text{ArcTanh}\Big[\frac{\sqrt{b\,\text$$

$$\frac{d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right] + \frac{d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]}{\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]} d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b}} \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]}{\sqrt{b} + \sqrt{b}} \left[ \sqrt{b} + \sqrt{b} \right] \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]} d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b}} \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] \operatorname{Log} \left[ \frac{2\sqrt{b}}{\sqrt{b} + \sqrt{b}} \left[ \sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}} \right]}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] \operatorname{Log} \left[ -\frac{2\left[\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}\right]}}{\sqrt{-b} + \sqrt{b}} \right] \left[ \sqrt{-b} + \sqrt{b} \right] \left[ \sqrt{-b} + \sqrt{b} \right]} d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] \operatorname{Log} \left[ -\frac{2\left[\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}\right]}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] \operatorname{Log} \left[ -\frac{2\left[\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}\right]}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] \operatorname{Log} \left[ -\frac{2\left[\sqrt{b} + \sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}\right]}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}{\sqrt{-b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})}}}{\sqrt{-b} + \sqrt{b}} \right] d \operatorname{ArcTanh} \left[ \frac{\sqrt{b \operatorname{Tanh} (e + f \mathbf{x})$$

$$\frac{\text{d PolyLog} \left[ 2, \ 1 - \frac{2}{1 + \frac{\sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}{\sqrt{-b}}} \right]}{2 \, \left( -b \right)^{3/2} \, f^2} - \frac{2 \, \left( c + d \, x \right)}{b \, f \, \sqrt{b \, \text{Tanh} \left[ e + f \, x \right]}}$$

Result (type 8, 110 leaves, 6 steps):

$$\begin{split} &\frac{2\,\text{d}\,\text{ArcTan}\Big[\,\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\,\Big]}{b^{3/2}\,\,f^2}\,+\,\frac{2\,\,\text{d}\,\text{ArcTanh}\Big[\,\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{b}}\,\Big]}{b^{3/2}\,\,f^2}\,-\,\\ &\frac{2\,\left(\,c+d\,x\right)}{b\,f\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}\,+\,\frac{\text{Unintegrable}\Big[\,\left(\,c+d\,x\right)\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\,\,\text{, }x\,\Big]}{b^2} \end{split}$$

### Problem 21: Result valid but suboptimal antiderivative.

$$\int \left(c+d\,x\right)^{\,2}\,\left(b\,Tanh\left[\,e+f\,x\,\right]\,\right)^{\,3/2}\,\mathrm{d}x$$

Optimal (type 8, 1340 leaves, 38 steps):

$$\frac{4 \left(-b\right)^{3/2} d \left(c+dx\right) ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{-b}}\right]}{f^{2}} + \frac{2 \left(-b\right)^{3/2} d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{-b}}\right]^{2}}{f^{3}} + \frac{4 \, b^{3/2} d \left(c+dx\right) \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right]}{\sqrt{b}} + \frac{2 \, b^{3/2} \, d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right]^{2}}{f^{3}} - \frac{4 \, b^{3/2} \, d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right] \, Log \left[\frac{2 \, \sqrt{b}}{\sqrt{b} \, -\sqrt{b \, Tanh \left(e+fx\right)}}\right]}{f^{3}} + \frac{4 \, b^{3/2} \, d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right] \, Log \left[\frac{2 \, \sqrt{b}}{\sqrt{b} \, +\sqrt{b \, Tanh \left(e+fx\right)}}\right]}{f^{3}} - \frac{2 \, b^{3/2} \, d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right] \, Log \left[\frac{2 \, \sqrt{b} \, \left(\sqrt{-b} \, -\sqrt{b} \, \int b \, Tanh \left(e+fx\right)}\right)}{\left(\sqrt{-b} \, -\sqrt{b}\right) \, \left(\sqrt{b} \, +\sqrt{b \, Tanh \left(e+fx\right)}\right)} - \frac{f^{3}}{f^{3}} - \frac{2 \, b^{3/2} \, d^{2} \, ArcTanh \left[\frac{\sqrt{b \, Tanh \left(e+fx\right)}}{\sqrt{b}}\right] \, Log \left[\frac{2 \, \sqrt{b} \, \left(\sqrt{-b} \, +\sqrt{b} \, Tanh \left(e+fx\right)}\right)}{\left(\sqrt{-b} \, +\sqrt{b} \, Tanh \left(e+fx\right)}\right)} - \frac{f^{3}}{f^{3}} - \frac{f^{$$

$$\frac{2 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}}{\left(\sqrt{-b} + \sqrt{b}\right) \left(1 - \frac{\sqrt{b \operatorname{Tanh}(e+fx)}}{\sqrt{-b}}\right)}\right]}{\sqrt{-b}} + \frac{6^3}{2} \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{-b}}\right] \operatorname{Log}\left[-\frac{2 \left(\sqrt{b} + \sqrt{b \operatorname{Tanh}(e+fx)}}{\left(\sqrt{-b} - \sqrt{b}\right) \left(1 - \frac{\sqrt{b \operatorname{Tanh}(e+fx)}}{\sqrt{-b}}\right)}\right]}{\sqrt{-b}} + \frac{6^3}{4} \frac{4 \left(-b\right)^{3/2} d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{\operatorname{bTanh}(e+fx)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2}{1 + \frac{\sqrt{b \operatorname{Tanh}(e+fx)}}{\sqrt{-b}}}\right]}{\sqrt{-b} - \sqrt{b \operatorname{Tanh}(e+fx)}}} + \frac{6^3}{4} \frac{2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}}\right]}{\sqrt{-b} - \sqrt{b \operatorname{Tanh}(e+fx)}}} + \frac{6^3}{4} \frac{2 b^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{b}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}} + \frac{6^3}{4} \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}} + \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}} + \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}} + \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}} + \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{-b} \operatorname{Tanh}\left[2 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}} + \frac{2 \left(-b\right)^{3/2} d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \left(\sqrt{b} - \sqrt{b \operatorname{Tanh}(e+fx)}\right)}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}(e+fx)}\right]}}{\sqrt{-b} - \sqrt{-b} \operatorname{Tanh}\left[2 - \frac{2}{b} - \frac{2}{b} \operatorname{Tanh}\left[2 - \frac{2}{b} - \frac{2}{b} - \frac{2}{b} \operatorname{Tanh}\left[2 - \frac{2}{b} - \frac{2}{b} - \frac{2}{b} - \frac{2}{b} - \frac{2}{b} - \frac{2}{b} - \frac{2}{b}}\right]}{\sqrt{-b} - \sqrt{-b} \operatorname{Tanh}\left[2 - \frac{2}{b} - \frac{2}$$

## Problem 24: Result valid but suboptimal antiderivative.

$$\int\!\frac{\left(c+d\,x\right)^2}{\left(b\,\text{Tanh}\,[\,e+f\,x\,]\,\right)^{3/2}}\,\text{d}x$$

### Optimal (type 8, 1342 leaves, 38 steps):

 $(-b)^{3/2} f^3$ 

$$\frac{4 \text{ d} \left(c + \text{ d} x\right) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right]}{\left(-b\right)^{3/2} f^{2}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right]^{2}}{\left(-b\right)^{3/2} f^{3}} + \frac{4 \text{ d} \left(c + \text{ d} x\right) \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right]}{\sqrt{b}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right]^{2}}{b^{3/2} f^{3}} - \frac{4 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b}}{\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}{b^{3/2} f^{3}} + \frac{4 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b}}{\sqrt{-b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}{\left(\sqrt{-b} - \sqrt{b}\right) \left(\sqrt{-b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}\right)}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right]^{2}}{b^{3/2} f^{3}}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} \left(\sqrt{-b} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}\right)}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}{b^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b} - \sqrt{b} \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2} \operatorname{ArcTanh}\left[\frac{\sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}}\right] \operatorname{Log}\left[\frac{2\sqrt{b} - \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}{\sqrt{-b}} + \sqrt{b \operatorname{Tanh}\left(e + f x\right)}}\right]}}{\left(-b\right)^{3/2} f^{3}} + \frac{2 \frac{d^{2}$$

h<sup>3/2</sup> f<sup>3</sup>

$$\frac{2\,d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2\,\sqrt{b}}{\sqrt{b}\,+\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}\right]}{b^{3/2}\,f^{3}}+\frac{d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2\,\sqrt{b}\left(\sqrt{-b}\,-\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}{\left(\sqrt{-b}\,-\sqrt{b}\right)\left(\sqrt{b}\,+\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}\right]}{b^{3/2}\,f^{3}}+\frac{d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2\,\sqrt{b}\left(\sqrt{-b}\,+\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}{\left(\sqrt{-b}\,+\sqrt{b}\right)\left(\sqrt{b}\,+\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}\right]}{b^{3/2}\,f^{3}}-\frac{2\,d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2\,\left(\sqrt{b}\,-\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}{\left(\sqrt{-b}\,+\sqrt{b}\right)\left(1-\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}\right)}}{\left(-b\right)^{3/2}\,f^{3}}+\frac{d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2\,\left(\sqrt{b}\,-\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\right)}{\left(\sqrt{-b}\,+\sqrt{b}\right)\left(1-\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}\right)}}{\left(-b\right)^{3/2}\,f^{3}}+\frac{2\,d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2}{1+\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}}\right]}{\left(-b\right)^{3/2}\,f^{3}}-\frac{2\,d^{2}\,\text{PolyLog}\!\left[2,\,1-\frac{2}{1+\frac{\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}{\sqrt{-b}}}\right]}{\left(-b\right)^{3/2}\,f^{3}}-\frac{2\,\left(c+d\,x\right)^{2}}{b\,f\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}}+\frac{\text{Unintegrable}\!\left[\left(c+d\,x\right)^{2}\,\sqrt{b\,\text{Tanh}\left[e+f\,x\right]}\,,\,x\right]}{b^{2}}$$

$$Result (type 8, 83 leaves, 1 step):$$

$$-\frac{2\left(c+d\,x\right)^{2}}{b\,f\,\sqrt{b\,Tanh\left[e+f\,x\right]}}+\frac{4\,d\,Unintegrable\left[\frac{c+d\,x}{\sqrt{b\,Tanh\left[e+f\,x\right]}},\,x\right]}{b\,f}+\frac{Unintegrable\left[\left(c+d\,x\right)^{2}\sqrt{b\,Tanh\left[e+f\,x\right]},\,x\right]}{b^{2}}$$

## Test results for the 247 problems in "6.3.2 Hyperbolic tangent functions.m"

Problem 146: Unable to integrate problem.

$$\int x^3 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 29 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} e^{-2a} Log \left[1 + e^{2a} x^4\right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [a + 2 Log [x]], x]$ 

### Problem 147: Unable to integrate problem.

$$\int x^2 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 151 leaves, 11 steps):

$$\begin{split} \frac{\text{x}^3}{\text{3}} + \frac{\text{e}^{-3\text{ a}/2} \, \text{ArcTan} \left[ 1 - \sqrt{2} \, \, \, \text{e}^{\text{a}/2} \, \text{x} \right]}{\sqrt{2}} - \frac{\text{e}^{-3\text{ a}/2} \, \text{ArcTan} \left[ 1 + \sqrt{2} \, \, \, \text{e}^{\text{a}/2} \, \text{x} \right]}{\sqrt{2}} - \\ \frac{\text{e}^{-3\text{ a}/2} \, \text{Log} \left[ 1 - \sqrt{2} \, \, \, \text{e}^{\text{a}/2} \, \text{x} + \text{e}^{\text{a}} \, \text{x}^2 \right]}{2 \, \sqrt{2}} + \frac{\text{e}^{-3\text{ a}/2} \, \text{Log} \left[ 1 + \sqrt{2} \, \, \, \text{e}^{\text{a}/2} \, \text{x} + \text{e}^{\text{a}} \, \text{x}^2 \right]}{2 \, \sqrt{2}} \end{split}$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [a + 2 Log [x]], x]$ 

### Problem 148: Unable to integrate problem.

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTan} \left[ e^a x^2 \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[x Tanh[a + 2 Log[x]], x]

### Problem 149: Unable to integrate problem.

$$\int Tanh[a + 2 Log[x]] dx$$

Optimal (type 3, 145 leaves, 11 steps):

$$\begin{split} & \text{X} + \frac{\text{e}^{-\text{a}/2}\,\text{ArcTan} \Big[ \, 1 - \sqrt{2} \,\,\, \text{e}^{\text{a}/2}\,\, \text{x} \, \Big]}{\sqrt{2}} - \frac{\text{e}^{-\text{a}/2}\,\,\text{ArcTan} \Big[ \, 1 + \sqrt{2} \,\,\, \text{e}^{\text{a}/2}\,\, \text{x} \, \Big]}{\sqrt{2}} + \\ & \frac{\text{e}^{-\text{a}/2}\,\,\text{Log} \Big[ \, 1 - \sqrt{2} \,\,\, \text{e}^{\text{a}/2}\,\, \text{x} + \text{e}^{\text{a}}\,\, \text{x}^2 \, \Big]}{2\,\sqrt{2}} - \frac{\text{e}^{-\text{a}/2}\,\,\text{Log} \Big[ \, 1 + \sqrt{2} \,\,\, \text{e}^{\text{a}/2}\,\, \text{x} + \text{e}^{\text{a}}\,\, \text{x}^2 \, \Big]}{2\,\sqrt{2}} \end{split}$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate[Tanh[a + 2 Log[x]], x]

## Problem 151: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{a} + \mathsf{2}\,\mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{x}^2}\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 147 leaves, 11 steps):

$$\begin{split} \frac{1}{x} - \frac{e^{a/2} \, \text{ArcTan} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} + \frac{e^{a/2} \, \text{ArcTan} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x \Big]}{\sqrt{2}} + \\ \frac{e^{a/2} \, \text{Log} \Big[ 1 - \sqrt{2} \, e^{a/2} \, x + e^a \, x^2 \Big]}{2 \, \sqrt{2}} - \frac{e^{a/2} \, \text{Log} \Big[ 1 + \sqrt{2} \, e^{a/2} \, x + e^a \, x^2 \Big]}{2 \, \sqrt{2}} \end{split}$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2 Log[x]]}{x^2}, x\right]$$

## Problem 152: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{a} + \mathsf{2}\,\mathsf{Log}\left[\mathsf{x}\right]\right]}{\mathsf{x}^{\mathsf{3}}}\,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 20 leaves, 4 steps):

$$\frac{1}{2 x^2} + e^a \operatorname{ArcTan} \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2Log[x]]}{x^3}, x\right]$$

## Problem 153: Unable to integrate problem.

$$\int x^3 \operatorname{Tanh}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{X^4}{4} - \frac{e^{-2a}}{1 + e^{2a} X^4} - e^{-2a} Log [1 + e^{2a} X^4]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [a + 2 Log [x]]^2, x]$ 

## Problem 154: Unable to integrate problem.

$$\int x^2 \operatorname{Tanh} [a + 2 \operatorname{Log} [x]]^2 dx$$

Optimal (type 3, 173 leaves, 12 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 + e^{2\,a}\,x^4} + \frac{3\,e^{-3\,a/2}\,\text{ArcTan}\big[1 - \sqrt{2}\,e^{a/2}\,x\big]}{2\,\sqrt{2}} - \frac{3\,e^{-3\,a/2}\,\text{ArcTan}\big[1 + \sqrt{2}\,e^{a/2}\,x\big]}{2\,\sqrt{2}} - \frac{3\,e^{-3\,a/2}\,\text{Log}\big[1 - \sqrt{2}\,e^{a/2}\,x + e^{a}\,x^2\big]}{4\,\sqrt{2}} + \frac{3\,e^{-3\,a/2}\,\text{Log}\big[1 + \sqrt{2}\,e^{a/2}\,x + e^{a}\,x^2\big]}{4\,\sqrt{2}}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [a + 2 Log [x]]^2, x]$ 

### Problem 155: Unable to integrate problem.

$$\int x Tanh[a + 2 Log[x]]^2 dx$$

Optimal (type 3, 40 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 + e^{2a} x^4} - e^{-a} ArcTan \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x Tanh [a + 2 Log [x]]^2, x]$ 

### Problem 156: Unable to integrate problem.

Tanh [a + 2 Log [x]] 
$$^2$$
 dx

Optimal (type 3, 165 leaves, 13 steps):

$$\begin{array}{l} x + \dfrac{x}{1 + e^{2\,a}\,x^4} + \dfrac{e^{-a/2}\,\text{ArcTan}\Big[1 - \sqrt{2}\,\,\,e^{a/2}\,x\Big]}{2\,\sqrt{2}} - \dfrac{e^{-a/2}\,\text{ArcTan}\Big[1 + \sqrt{2}\,\,\,e^{a/2}\,x\Big]}{2\,\sqrt{2}} + \\ \\ \dfrac{e^{-a/2}\,\text{Log}\Big[1 - \sqrt{2}\,\,\,e^{a/2}\,x + e^{a}\,x^2\Big]}{4\,\sqrt{2}} - \dfrac{e^{-a/2}\,\text{Log}\Big[1 + \sqrt{2}\,\,\,e^{a/2}\,x + e^{a}\,x^2\Big]}{4\,\sqrt{2}} \end{array} \right. + \\ \end{array}$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + 2 Log[x]]^2, x]$ 

### Problem 158: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh} \left[\mathsf{a} + 2 \,\mathsf{Log} \left[\mathsf{x}\right]\right]^2}{\mathsf{x}^2} \,\mathrm{d}\mathsf{x}$$

Optimal (type 3, 190 leaves, 12 steps):

$$\begin{split} & -\frac{1}{x\left(1+e^{2\,a}\,x^4\right)} - \frac{2\,e^{2\,a}\,x^3}{1+e^{2\,a}\,x^4} + \frac{e^{a/2}\,\text{ArcTan}\!\left[1-\sqrt{2}\,e^{a/2}\,x\right]}{2\,\sqrt{2}} - \\ & \frac{e^{a/2}\,\text{ArcTan}\!\left[1+\sqrt{2}\,e^{a/2}\,x\right]}{2\,\sqrt{2}} - \frac{e^{a/2}\,\text{Log}\!\left[1-\sqrt{2}\,e^{a/2}\,x+e^{a}\,x^2\right]}{4\,\sqrt{2}} + \frac{e^{a/2}\,\text{Log}\!\left[1+\sqrt{2}\,e^{a/2}\,x+e^{a}\,x^2\right]}{4\,\sqrt{2}} \end{split}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2log[x]]^2}{x^2}, x\right]$$

### Problem 159: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh} \left[\mathsf{a} + \mathsf{2} \,\mathsf{Log} \left[\mathsf{x}\right]\right]^2}{\mathsf{x}^3} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 59 leaves, 5 steps):

$$-\,\frac{1}{2\,{{x}^{2}}\,\left( 1+{{e}^{2\,a}}\,{{x}^{4}} \right) }\,-\,\frac{3\,{{e}^{2\,a}}\,{{x}^{2}}}{2\,\left( 1+{{e}^{2\,a}}\,{{x}^{4}} \right) }\,-\,{{e}^{a}}\,\text{ArcTan}\left[ \,{{e}^{a}}\,{{x}^{2}} \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh[a+2log[x]]^2}{x^3}, x\right]$$

### Problem 160: Unable to integrate problem.

$$\int (e x)^m Tanh[a + 2 Log[x]] dx$$

Optimal (type 5, 60 leaves, 3 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} \,-\, \frac{2\,\left(\text{e x}\right)^{\text{1+m}}\,\text{Hypergeometric2F1}\!\left[\text{1, }\frac{1+\text{m}}{4}\text{, }\frac{5+\text{m}}{4}\text{, }-\text{e}^{\text{2 a }}\,\text{x}^{\text{4}}\right]}{\text{e }\left(\text{1+m}\right)}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate[(ex) m Tanh[a + 2 Log[x]], x]

### Problem 161: Unable to integrate problem.

$$\int (e x)^m Tanh [a + 2 Log [x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1+m}\right)}\,+\,\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1+}\,\text{e}^{\text{2}\,\text{a}}\,\text{x}^{\text{4}}\right)}\,-\,\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}\,\text{Hypergeometric2F1}\!\left[\,\text{1,}\,\,\frac{\text{1+m}}{4}\,\text{,}\,\,\frac{\text{5+m}}{4}\,\text{,}\,\,-\,\text{e}^{\text{2}\,\text{a}}\,\text{x}^{\text{4}}\,\right]}}{\text{e}}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + 2 Log [x]]^2, x]$ 

## Problem 162: Unable to integrate problem.

$$\int (e x)^m Tanh [a + 2 Log [x]]^3 dx$$

Optimal (type 5, 176 leaves, 5 steps):

$$\frac{\left(3+m\right) \; \left(5+m\right) \; \left(e\,x\right)^{\,1+m}}{8\; e\; \left(1+m\right)} - \frac{\left(e\,x\right)^{\,1+m} \; \left(1-e^{2\,a}\,x^4\right)^{\,2}}{4\; e\; \left(1+e^{2\,a}\,x^4\right)^{\,2}} - \frac{e^{-2\,a} \; \left(e\,x\right)^{\,1+m} \; \left(e^{2\,a} \; \left(3-m\right) + e^{4\,a} \; \left(5+m\right) \; x^4\right)}{8\; e\; \left(1+e^{2\,a}\,x^4\right)} - \frac{\left(9+2\,m+m^2\right) \; \left(e\,x\right)^{\,1+m} \; Hypergeometric 2F1 \left[1,\, \frac{1+m}{4},\, \frac{5+m}{4},\, -e^{2\,a}\,x^4\right]}{4\; e\; \left(1+m\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + 2 Log[x]]^3, x]$ 

### Problem 163: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left(1-e^{2a}x^{2b}\right)^{-p} \left(-1+e^{2a}x^{2b}\right)^{p} AppellF1\left[\frac{1}{2b}, -p, p, \frac{1}{2}\left(2+\frac{1}{b}\right), e^{2a}x^{2b}, -e^{2a}x^{2b}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh [a + b Log [x]]^p, x]$ 

### Problem 164: Unable to integrate problem.

$$\int (e x)^m Tanh[a + b Log[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{1}{e\,\left(1+m\right)}\,\left(\,e\,x\,\right)^{\,1+m}\,\left(\,1\,-\,\,\mathrm{e}^{\,2\,a}\,\,x^{\,2\,\,b}\,\right)^{\,-p}\,\left(\,-\,1\,+\,\,\mathrm{e}^{\,2\,a}\,\,x^{\,2\,\,b}\,\right)^{\,p}\,\\ \mathsf{AppellF1}\left[\,\frac{\,1\,+\,m\,}{\,2\,\,b}\,,\,\,-\,p\,,\,\,p\,,\,\,1\,+\,\,\frac{\,1\,+\,m\,}{\,2\,\,b}\,,\,\,\,\mathrm{e}^{\,2\,a}\,\,x^{\,2\,\,b}\,,\,\,-\,\mathrm{e}^{\,2\,a}\,\,x^{\,2\,\,b}\,\right]$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Tanh [a + b Log [x]]^p, x]$ 

### Problem 165: Unable to integrate problem.

$$\int Tanh \left[a + \frac{Log[x]}{2}\right]^p dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{1}{1+p} 2^{-p} \; \mathrm{e}^{-2\,a} \; \left(-1 + \mathrm{e}^{2\,a} \; x\right)^{1+p} \; \text{Hypergeometric2F1} \left[\,p , \; 1+p , \; 2+p , \; \frac{1}{2} \; \left(1-\mathrm{e}^{2\,a} \; x\right) \; \right]$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ Tanh \left[ \frac{1}{2} \left( 2 a + Log[x] \right) \right]^p, x \right]$ 

### Problem 166: Unable to integrate problem.

$$\int Tanh \left[a + \frac{Log[x]}{4}\right]^p dx$$

Optimal (type 5, 106 leaves, 4 steps):

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ Tanh \left[ \frac{1}{4} \left( 4 a + Log \left[ x \right] \right) \right]^p$$
,  $x \right]$ 

### Problem 167: Unable to integrate problem.

$$\int Tanh \left[a + \frac{Log[x]}{6}\right]^p dx$$

Optimal (type 5, 158 leaves, 5 steps):

$$-\,\mathrm{e}^{-6\,\mathsf{a}}\,\,\mathsf{p}\,\,\left(-\,1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)^{\,1\,+\,\mathsf{p}}\,\,\left(1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)^{\,1\,-\,\mathsf{p}}\,+\,\mathrm{e}^{-4\,\mathsf{a}}\,\,\left(-\,1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)^{\,1\,+\,\mathsf{p}}\,\,\left(1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)^{\,1\,-\,\mathsf{p}}\,\mathsf{x}^{1/3}\,+\,\frac{1}{1\,+\,\mathsf{p}}$$
 
$$2^{-\mathsf{p}}\,\,\mathrm{e}^{-6\,\mathsf{a}}\,\,\left(1\,+\,2\,\mathsf{p}^2\right)\,\,\left(-\,1\,+\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)^{\,1\,+\,\mathsf{p}}\,\,\mathsf{Hypergeometric}2\mathsf{F1}\!\left[\,\mathsf{p}\,,\,1\,+\,\mathsf{p}\,,\,2\,+\,\mathsf{p}\,,\,\,\frac{1}{2}\,\,\left(1\,-\,\mathrm{e}^{2\,\mathsf{a}}\,\,\mathsf{x}^{1/3}\right)\,\right]$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ Tanh \left[ \frac{1}{6} \left( 6 a + Log[x] \right) \right]^p$$
,  $x \right]$ 

### Problem 168: Unable to integrate problem.

$$\int Tanh \left[a + \frac{Log[x]}{g}\right]^p dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\begin{split} &\frac{1}{3}\;\mathrm{e}^{-12\,\mathsf{a}}\;\left(-\,\mathbf{1}\,+\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)^{\,\mathbf{1}\,+\,\mathsf{p}}\;\left(\mathbf{1}\,+\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)^{\,\mathbf{1}\,-\,\mathsf{p}}\;\left(\mathrm{e}^{4\,\mathsf{a}}\;\left(3\,+\,2\,\,\mathsf{p}^{2}\right)\,-\,2\,\,\mathrm{e}^{6\,\mathsf{a}}\;\mathsf{p}\;x^{1/4}\right)\,+\\ &\,\mathrm{e}^{-4\,\mathsf{a}}\;\left(-\,\mathbf{1}\,+\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)^{\,\mathbf{1}\,+\,\mathsf{p}}\;\left(\mathbf{1}\,+\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)^{\,\mathbf{1}\,-\,\mathsf{p}}\;\sqrt{x}\;-\,\frac{1}{3\;\left(\mathbf{1}\,+\,\mathsf{p}\right)}\\ &\,2^{2-\mathsf{p}}\;\mathrm{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathbf{1}\,+\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)^{\,\mathsf{1}\,+\,\mathsf{p}}\;\mathsf{Hypergeometric}2\mathsf{F1}\!\left[\,\mathsf{p}\,,\,\mathbf{1}\,+\,\mathsf{p}\,,\,2\,+\,\mathsf{p}\,,\,\,\frac{1}{2}\;\left(\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;x^{1/4}\right)\,\right] \end{split}$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ Tanh \left[ \frac{1}{8} \left( 8 a + Log[x] \right) \right]^p, x \right]$$

### Problem 169: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(1-e^{2a}x^{2}\right)^{-p} \left(-1+e^{2a}x^{2}\right)^{p} AppellF1\left[\frac{1}{2}, -p, p, \frac{3}{2}, e^{2a}x^{2}, -e^{2a}x^{2}\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + Log[x]]^p, x]$ 

### Problem 170: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + \mathsf{2} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \, \left( 1 - \mathrm{e}^{2\,a} \, x^4 \right)^{-p} \, \left( -1 + \mathrm{e}^{2\,a} \, x^4 \right)^p \\ \mathsf{AppellF1} \Big[ \, \frac{1}{4} \text{, } -p \text{, } p \text{, } \, \frac{5}{4} \text{, } \, \mathrm{e}^{2\,a} \, x^4 \text{, } -\mathrm{e}^{2\,a} \, x^4 \Big]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + 2 Log[x]]^p, x]$ 

## Problem 171: Unable to integrate problem.

$$\int \mathsf{Tanh} \left[ \mathsf{a} + \mathsf{3} \, \mathsf{Log} \left[ \mathsf{x} \right] \right]^{\mathsf{p}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(1-e^{2a} x^{6}\right)^{-p} \left(-1+e^{2a} x^{6}\right)^{p} AppellF1\left[\frac{1}{6}, -p, p, \frac{7}{6}, e^{2a} x^{6}, -e^{2a} x^{6}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $[Tanh[a + 3 Log[x]]^p, x]$ 

## Problem 172: Unable to integrate problem.

$$\int x^3 Tanh \left[d \left(a + b Log \left[c x^n\right]\right)\right] dx$$

Optimal (type 5, 59 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4$$
 Hypergeometric2F1  $\left[1, \frac{2}{b d n}, 1 + \frac{2}{b d n}, -e^{2 a d} (c x^n)^{2 b d}\right]$ 

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 Tanh [d (a + b Log [c x^n])], x]$ 

### Problem 173: Unable to integrate problem.

$$\left\lceil x^2 \, \mathsf{Tanh} \left[ \mathsf{d} \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[ \, \mathsf{c} \, \, \mathsf{x}^\mathsf{n} \, \right] \, \right) \, \right] \, \mathrm{d} x \right.$$

Optimal (type 5, 63 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3} x^3$$
 Hypergeometric2F1[1,  $\frac{3}{2 \, b \, d \, n}$ ,  $1 + \frac{3}{2 \, b \, d \, n}$ ,  $-e^{2 \, a \, d} \, (c \, x^n)^{2 \, b \, d}$ ]

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^2 Tanh [d (a + b Log [c x^n])], x]$ 

### Problem 174: Unable to integrate problem.

$$\int x Tanh [d (a + b Log [c x^n])] dx$$

Optimal (type 5, 55 leaves, 4 steps):

$$\frac{x^2}{2} - x^2$$
 Hypergeometric2F1[1,  $\frac{1}{b d n}$ , 1 +  $\frac{1}{b d n}$ ,  $-e^{2ad} (c x^n)^{2bd}$ ]

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x Tanh [d (a + b Log [c x^n])], x]$ 

### Problem 175: Unable to integrate problem.

$$\int Tanh \left[ d \left( a + b Log \left[ c x^n \right] \right) \right] dx$$

Optimal (type 5, 53 leaves, 4 steps):

$$x - 2x$$
 Hypergeometric2F1[1,  $\frac{1}{2 h d n}$ ,  $1 + \frac{1}{2 h d n}$ ,  $-e^{2ad} (c x^n)^{2bd}$ ]

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[Tanh[d(a+bLog[cx^n])], x]$ 

## Problem 177: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{d}\,\left(\mathsf{a} + \mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\right]\right)\,\right]}{\mathsf{x}^{\mathsf{2}}}\,\mathsf{d}\mathsf{x}$$

Optimal (type 5, 59 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \, \text{Hypergeometric2F1} \Big[ \, \textbf{1,} \, -\frac{1}{2 \, \text{bdn}} \, , \, \, \textbf{1} - \frac{1}{2 \, \text{bdn}} \, , \, \, - e^{2 \, \text{ad}} \, \left( c \, \, x^n \right)^{2 \, \text{bd}} \Big]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+b\ Log\left[c\ x^{n}\right]\right)\right]}{x^{2}}$$
,  $x\right]$ 

### Problem 178: Unable to integrate problem.

$$\int\! \frac{Tanh\left[d\,\left(a+b\,Log\left[c\,x^{n}\right]\right)\right]}{x^{3}}\,\mathrm{d}x$$

Optimal (type 5, 56 leaves, 4 steps):

$$-\frac{1}{2\,{x}^{2}}+\frac{\text{Hypergeometric2F1}\!\left[1,\,-\frac{1}{b\,d\,n},\,1-\frac{1}{b\,d\,n},\,-{{e}^{2\,a\,d}}\,\left(c\,{\,x^{n}}\right)^{\,2\,b\,d}\right]}{x^{2}}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+b\,Log\left[c\,x^{n}\right]\right)\right]}{x^{3}}$$
,  $x\right]$ 

### Problem 179: Unable to integrate problem.

$$\int x^3 \operatorname{Tanh} \left[ d \left( a + b \operatorname{Log} \left[ c x^n \right] \right) \right]^2 dx$$

Optimal (type 5, 133 leaves, 5 steps):

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[x^{3} \operatorname{Tanh}\left[d\left(a+b \operatorname{Log}\left[c \ x^{n}\right]\right)\right]^{2}$$
,  $x\right]$ 

### Problem 180: Unable to integrate problem.

Optimal (type 5, 137 leaves, 5 steps):

$$\begin{split} &\frac{1}{3} \left(1 + \frac{3}{b \, d \, n}\right) \, x^3 + \frac{x^3 \, \left(1 - \mathrm{e}^{2 \, a \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right)}{b \, d \, n \, \left(1 + \mathrm{e}^{2 \, a \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right)} \, - \\ &\frac{2 \, x^3 \, \text{Hypergeometric} 2 \text{F1} \left[1, \, \frac{3}{2 \, b \, d \, n}, \, 1 + \frac{3}{2 \, b \, d \, n}, \, - \mathrm{e}^{2 \, a \, d} \, \left(c \, x^n\right)^{\, 2 \, b \, d}\right]}{b \, d \, n} \end{split}$$

Result (type 8, 21 leaves, 0 steps):

$$CannotIntegrate \left[\,x^2\,Tanh\left[\,d\,\left(\,a\,+\,b\,Log\left[\,c\,\,x^n\,\right]\,\right)\,\right]^{\,2}\text{, }x\,\right]$$

### Problem 181: Unable to integrate problem.

$$\int x \, Tanh \left[d \left(a + b \, Log \left[c \, x^n\right]\right)\right]^2 dx$$

Optimal (type 5, 131 leaves, 5 steps):

$$\begin{split} &\frac{1}{2} \left( 1 + \frac{2}{b \, d \, n} \right) \, x^2 + \frac{x^2 \, \left( 1 - \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{\, 2 \, b \, d} \right)}{b \, d \, n \, \left( 1 + \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{\, 2 \, b \, d} \right)} \, - \\ &\frac{2 \, x^2 \, \text{Hypergeometric} 2 \text{F1} \left[ \, 1, \, \frac{1}{b \, d \, n}, \, 1 + \frac{1}{b \, d \, n}, \, - \mathrm{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{\, 2 \, b \, d} \right]}{b \, d \, n} \end{split}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x Tanh [d (a + b Log [c x^n])]^2, x]$ 

### Problem 182: Unable to integrate problem.

Optimal (type 5, 127 leaves, 5 steps):

$$\begin{split} \left(1+\frac{1}{b\,d\,n}\right)\,x+\frac{x\,\left(1-{\,\mathrm{e}^{2\,a\,d}}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}{b\,d\,n\,\left(1+{\,\mathrm{e}^{2\,a\,d}}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}\,-\\ \frac{2\,x\,\text{Hypergeometric2F1}\!\left[1\text{,}\,\,\frac{1}{\,2\,b\,d\,n}\text{,}\,\,1+\frac{1}{\,2\,b\,d\,n}\text{,}\,\,-{\,\mathrm{e}^{2\,a\,d}}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n} \end{split}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[Tanh [d (a + b Log [c x^n])]^2, x]$ 

### Problem 184: Unable to integrate problem.

$$\int \frac{\mathsf{Tanh}\left[\mathsf{d}\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\mathsf{x}^{\mathsf{n}}\right]\right)\right]^{2}}{\mathsf{x}^{2}}\,\mathrm{d}\mathsf{x}$$

Optimal (type 5, 135 leaves, 5 steps

$$-\frac{1-\frac{1}{b\,d\,n}}{x}+\frac{1-\,e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}}{b\,d\,n\,x\,\left(1+\,e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,Hypergeometric2F1\!\left[1,\,-\frac{1}{2\,b\,d\,n},\,1-\frac{1}{2\,b\,d\,n},\,1-\frac{1}{2\,b\,d\,n},\,-e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n\,x}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{2}},x\right]$$

### Problem 185: Unable to integrate problem.

$$\int \frac{Tanh \left[d \left(a + b Log \left[c x^{n}\right]\right)\right]^{2}}{x^{3}} dx$$

Optimal (type 5, 136 leaves, 5 steps)

$$\frac{2 - b \, d \, n}{2 \, b \, d \, n \, x^{2}} + \frac{1 - e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}}{b \, d \, n \, x^{2} \, \left(1 + e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right)} - \frac{2 \, \text{Hypergeometric} 2 \text{F1} \left[1, \, -\frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, -e^{2 \, a \, d} \, \left(c \, x^{n}\right)^{2 \, b \, d}\right]}{b \, d \, n \, x^{2}}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{Tanh\left[d\left(a+bLog\left[cx^{n}\right]\right)\right]^{2}}{x^{3}}, x\right]$$

### Problem 189: Unable to integrate problem.

$$\int (e x)^m Tanh [d (a + b Log[c x^n])] dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1+m}\right)} \,-\, \frac{2\,\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}\,\text{Hypergeometric}2\text{F1}\!\left[\text{1,}\,\frac{\text{1+m}}{\text{2}\,\text{b}\,\text{d}\,\text{n}}\text{,}\,\text{1} + \frac{\text{1+m}}{\text{2}\,\text{b}\,\text{d}\,\text{n}}\text{,}\,\text{-}\,\text{e}^{\text{2}\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,\text{2}\,\text{b}\,\text{d}}\right]}{\text{e}\,\left(\text{1+m}\right)}$$

Result (type 8, 21 leaves, 0 steps):

$$CannotIntegrate \left[ \; (e \; x)^{\; m} \; Tanh \left[ d \; \left( a \; + \; b \; Log \left[ c \; x^n \; \right] \right) \; \right] \text{, } \; x \right]$$

### Problem 190: Unable to integrate problem.

$$\left\lceil \left(e\,x\right)^{\,m}\, Tanh \left[\,d\,\left(a+b\, Log \left[\,c\,\,x^{n}\,\right]\,\right)\,\right]^{\,2}\, \mathrm{d}x \right.$$

Optimal (type 5, 169 leaves, 5 steps):

$$\begin{split} \frac{\left(1+\text{m}+\text{bdn}\right) \; \left(\text{ex}\right)^{1+\text{m}}}{\text{bde}\left(1+\text{m}\right) \; n} \; + \; \frac{\left(\text{ex}\right)^{1+\text{m}} \left(1-\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right)}{\text{bden}\left(1+\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right)} \; - \\ & \frac{2 \; \left(\text{ex}\right)^{1+\text{m}} \, \text{Hypergeometric} 2\text{F1}\!\left[1, \, \frac{1+\text{m}}{2\,\text{bdn}}, \, 1+\frac{1+\text{m}}{2\,\text{bdn}}, \, -\text{e}^{2\,\text{ad}} \left(\text{cx}^{\text{n}}\right)^{2\,\text{bd}}\right]}{\text{bden}} \end{split}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate 
$$[(ex)^m Tanh[d(a+bLog[cx^n])]^2$$
,  $x]$ 

## Problem 191: Unable to integrate problem.

$$\left[\,\left(\,e\,x\,\right)^{\,m}\,\mathsf{Tanh}\left[\,\mathsf{d}\,\left(\,\mathsf{a}\,+\,\mathsf{b}\,\mathsf{Log}\left[\,\mathsf{c}\,\,x^{n}\,\right]\,\right)\,\right]^{\,3}\,\mathrm{d}x\right.$$

#### Optimal (type 5, 307 leaves, 6 steps):

$$\begin{split} &\frac{\left(1+\text{m}+\text{b}\,\text{d}\,\text{n}\right)\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}}{2\,\,\text{b}^{2}\,\,\text{d}^{2}\,\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} - \frac{\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}}{2\,\,\text{b}\,\text{d}\,\text{e}\,\text{n}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}} + \\ &\frac{\text{e}^{-2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(1+\text{m}-2\,\text{b}\,\text{d}\,\text{n}\right)}{\text{n}} - \frac{\text{e}^{4\,\text{a}\,\text{d}}\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{n}}\right)}{\text{n}} - \frac{1}{\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} \\ &\frac{2\,\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\text{n}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)}{\text{n}} - \frac{1}{\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} \\ &\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\text{Hypergeometric2F1}\!\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,-\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right] \end{split}$$

### Result (type 8, 23 leaves, 0 steps):

 $CannotIntegrate \left[ \; \left( \; e \; x \right) \, ^m \; Tanh \left[ \; d \; \left( \; a \; + \; b \; Log \left[ \; c \; \; x^n \; \right] \; \right) \; \right]^{\; 3} \text{, } \; x \; \right]$ 

### Problem 192: Unable to integrate problem.

$$\int Tanh \left[ d \left( a + b Log \left[ c x^n \right] \right) \right]^p dx$$

### Optimal (type 6, 115 leaves, 4 steps):

$$\begin{array}{l} x \, \left(1-\, \mathrm{e}^{2\, a\, d} \, \left(c\, x^{n}\right)^{\, 2\, b\, d}\right)^{\, -p} \, \left(-\, 1\, +\, \mathrm{e}^{2\, a\, d} \, \left(c\, x^{n}\right)^{\, 2\, b\, d}\right)^{\, p} \\ \\ \text{AppellF1} \Big[\, \frac{1}{2\, b\, d\, n}\, ,\, -p\, ,\, p\, ,\, 1\, +\, \frac{1}{2\, b\, d\, n}\, ,\, \, \mathrm{e}^{2\, a\, d} \, \left(c\, x^{n}\right)^{\, 2\, b\, d}\, ,\, -\mathrm{e}^{2\, a\, d} \, \left(c\, x^{n}\right)^{\, 2\, b\, d}\, \Big] \end{array}$$

### Result (type 8, 17 leaves, 0 steps):

 $CannotIntegrate \big[ Tanh \big[ d \, \left( a + b \, Log \big[ c \, x^n \big] \, \right) \, \big]^p \text{, } x \big]$ 

### Problem 193: Unable to integrate problem.

$$\label{eq:continuous} \left[ \left. \left( e \, x \right)^{\, \text{m}} \, \mathsf{Tanh} \left[ d \, \left( a + b \, \mathsf{Log} \left[ c \, x^{n} \, \right] \right) \, \right]^{p} \, \mathrm{d}x \right. \right.$$

### Optimal (type 6, 135 leaves, 4 steps):

$$\begin{split} &\frac{1}{e\,\left(1+m\right)}\left(e\,x\right)^{\,1+m}\,\left(1-e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right)^{\,-p}\,\left(-1+e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right)^{\,p}\\ &\text{AppellF1}\Big[\,\frac{1+m}{2\,b\,d\,n}\,\text{, }-p\,\text{, }p\,\text{, }1+\frac{1+m}{2\,b\,d\,n}\,\text{, }e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\,\text{, }-e^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\Big] \end{split}$$

### Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $\left[ \; (e\; x)^{\; m} \; Tanh \left[ \; d \; \left( \; a \; + \; b \; Log \left[ \; c \; \; x^n \; \right] \; \right) \; \right]^{\; p}$ ,  $x \; \right]$ 

# Test results for the 263 problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.m"

## Test results for the 61 problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

## Test results for the 224 problems in "6.4.2 Hyperbolic cotangent functions.m"

### Problem 151: Unable to integrate problem.

```
x^3 \operatorname{Coth}[a + 2 \operatorname{Log}[x]] dx
```

Optimal (type 3, 30 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{1}{2} e^{-2a} Log [1 - e^{2a} x^4]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [a + 2 \text{ Log } [x]], x]$ 

### Problem 152: Unable to integrate problem.

$$\int x^2 \operatorname{Coth}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 45 leaves, 5 steps):

$$\frac{x^3}{3} + e^{-3 a/2} \operatorname{ArcTan} \left[ e^{a/2} x \right] - e^{-3 a/2} \operatorname{ArcTanh} \left[ e^{a/2} x \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Coth } [a + 2 \text{ Log } [x]], x]$ 

### Problem 153: Unable to integrate problem.

$$\int x \operatorname{Coth}[a + 2 \operatorname{Log}[x]] dx$$

Optimal (type 3, 23 leaves, 4 steps):

$$\frac{x^2}{2} - e^{-a} \operatorname{ArcTanh} \left[ e^a x^2 \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate[x Coth[a + 2 Log[x]], x]

## Problem 154: Unable to integrate problem.

Optimal (type 3, 40 leaves, 5 steps):

$$x-\operatorname{e}^{-a/2}\operatorname{ArcTan}\left[\operatorname{e}^{a/2}x\right]-\operatorname{e}^{-a/2}\operatorname{ArcTanh}\left[\operatorname{e}^{a/2}x\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate [Coth[a + 2 Log[x]], x]

### Problem 156: Unable to integrate problem.

$$\int \frac{\mathsf{Coth}[\mathsf{a} + \mathsf{2} \, \mathsf{Log}[\mathsf{x}]]}{\mathsf{x}^2} \, \mathrm{d}\mathsf{x}$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{1}{x} + \operatorname{e}^{a/2} \operatorname{ArcTan} \left[ \operatorname{e}^{a/2} x \right] - \operatorname{e}^{a/2} \operatorname{ArcTanh} \left[ \operatorname{e}^{a/2} x \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}[a+2 Log[x]]}{x^2}, x\right]$$

### Problem 157: Unable to integrate problem.

$$\int \frac{\mathsf{Coth}\,[\,\mathsf{a}\,+\,\mathsf{2}\,\mathsf{Log}\,[\,\mathsf{x}\,]\,\,]}{\mathsf{x}^3}\,\,\mathrm{d}\,\mathsf{x}$$

Optimal (type 3, 21 leaves, 4 steps):

$$\frac{1}{2 x^2}$$
 -  $e^a$  ArcTanh  $\left[e^a x^2\right]$ 

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}[a+2 Log[x]]}{x^3}, x\right]$$

## Problem 158: Unable to integrate problem.

$$\int x^3 \operatorname{Coth}[a + 2 \operatorname{Log}[x]]^2 dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{x^4}{4} + \frac{e^{-2a}}{1 - e^{2a}x^4} + e^{-2a} Log [1 - e^{2a}x^4]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [a + 2 \text{ Log } [x]]^2, x]$ 

### Problem 159: Unable to integrate problem.

$$\int x^2 \operatorname{Coth} [a + 2 \operatorname{Log} [x]]^2 dx$$

Optimal (type 3, 68 leaves, 6 steps):

$$\frac{x^3}{3} + \frac{x^3}{1 - e^{2a}x^4} + \frac{3}{2} e^{-3a/2} \operatorname{ArcTan} \left[ e^{a/2} x \right] - \frac{3}{2} e^{-3a/2} \operatorname{ArcTanh} \left[ e^{a/2} x \right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Coth } [a + 2 \text{ Log } [x]]^2, x]$ 

### Problem 160: Unable to integrate problem.

$$\int x \operatorname{Coth} [a + 2 \operatorname{Log} [x]]^2 dx$$

Optimal (type 3, 41 leaves, 5 steps):

$$\frac{x^2}{2} + \frac{x^2}{1 - e^{2a} x^4} - e^{-a} \operatorname{ArcTanh} \left[ e^a x^2 \right]$$

Result (type 8, 13 leaves, 0 steps):

CannotIntegrate [x Coth[a + 2 Log[x]]<sup>2</sup>, x]

### Problem 161: Unable to integrate problem.

$$\int Coth [a + 2 Log [x]]^2 dx$$

Optimal (type 3, 60 leaves, 7 steps):

$$x + \frac{x}{1 - \operatorname{e}^{2\,a} \, x^4} - \frac{1}{2} \operatorname{e}^{-a/2} \operatorname{ArcTan} \left[ \operatorname{e}^{a/2} x \right] - \frac{1}{2} \operatorname{e}^{-a/2} \operatorname{ArcTanh} \left[ \operatorname{e}^{a/2} x \right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + 2 \text{Log} \left[ x \right] \right]^2, x \right]$ 

## Problem 163: Unable to integrate problem.

$$\int\!\frac{\text{Coth}\left[a+2\,\text{Log}\left[x\right]\,\right]^{\,2}}{x^{2}}\,\text{d}x$$

Optimal (type 3, 86 leaves, 6 steps):

$$-\frac{1}{x\,\left(1-\,{{e}^{2\,a}}\,{{x}^{4}}\right)}\,+\,\frac{2\,{{e}^{2\,a}}\,{{x}^{3}}}{1-\,{{e}^{2\,a}}\,{{x}^{4}}}\,-\,\frac{1}{2}\,\,{{e}^{a/2}}\,\text{ArcTan}\!\left[\,{{e}^{a/2}}\,x\,\right]\,+\,\frac{1}{2}\,\,{{e}^{a/2}}\,\text{ArcTanh}\!\left[\,{{e}^{a/2}}\,x\,\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\mathsf{Coth}\left[\mathsf{a}+2\,\mathsf{Log}\left[\mathsf{x}\right]\right]^2}{\mathsf{x}^2}\right]$$
,  $\mathsf{x}$ 

### Problem 164: Unable to integrate problem.

$$\int \frac{\mathsf{Coth}\, [\,\mathsf{a} + 2\,\mathsf{Log}\,[\,\mathsf{x}\,]\,\,]^{\,2}}{\mathsf{x}^{3}}\,\mathrm{d} \mathsf{x}$$

Optimal (type 3, 60 leaves, 5 steps):

$$-\,\frac{1}{2\,x^{2}\,\left(1-\,\mathrm{e}^{2\,a}\,x^{4}\right)}\,+\,\frac{3\,\,\mathrm{e}^{2\,a}\,x^{2}}{2\,\left(1-\,\mathrm{e}^{2\,a}\,x^{4}\right)}\,+\,\mathrm{e}^{a}\,\,\text{ArcTanh}\left[\,\mathrm{e}^{a}\,x^{2}\,\right]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}[a+2 \log[x]]^2}{x^3}, x\right]$$

### Problem 165: Unable to integrate problem.

$$\int (e x)^m Coth[a + 2 Log[x]] dx$$

Optimal (type 5, 59 leaves, 3 steps):

$$\frac{\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}}{\text{e}\,\left(\text{1+m}\right)}\,-\,\frac{2\,\left(\text{e}\,\text{x}\right)^{\,\text{1+m}}\,\text{Hypergeometric2F1}\!\left[\text{1,}\,\frac{\frac{1+m}{4}}{\text{,}}\,\frac{\frac{5+m}{4}}{\text{,}}\,\,\text{e}^{2\,\text{a}}\,\,\text{x}^{4}\right]}{\text{e}\,\left(\text{1+m}\right)}$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth[a + 2 Log[x]], x]$ 

### Problem 166: Unable to integrate problem.

$$\int (e x)^m Coth[a + 2 Log[x]]^2 dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{(e\,x)^{\,1+m}}{e\,\left(1+m\right)} + \frac{(e\,x)^{\,1+m}}{e\,\left(1-e^{2\,a}\,x^4\right)} - \frac{(e\,x)^{\,1+m}\,\mathsf{Hypergeometric2F1}\!\left[1,\,\frac{1+m}{4},\,\frac{5+m}{4},\,e^{2\,a}\,x^4\right]}{e}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth [a + 2 Log [x]]^2, x]$ 

## Problem 167: Unable to integrate problem.

$$\int (e x)^m Coth[a + 2 Log[x]]^3 dx$$

Optimal (type 5, 177 leaves, 5 steps):

$$\frac{\left(3+m\right) \; \left(5+m\right) \; \left(e\,x\right)^{\,1+m}}{8\; e\; \left(1+m\right)} - \frac{\left(e\,x\right)^{\,1+m} \; \left(1+e^{2\,a}\,x^4\right)^{\,2}}{4\; e\; \left(1-e^{2\,a}\,x^4\right)^{\,2}} - \frac{e^{-2\,a} \; \left(e\,x\right)^{\,1+m} \; \left(e^{2\,a} \; \left(3-m\right) - e^{4\,a} \; \left(5+m\right) \; x^4\right)}{8\; e\; \left(1-e^{2\,a}\,x^4\right)} - \frac{\left(9+2\,m+m^2\right) \; \left(e\,x\right)^{\,1+m} \; Hypergeometric 2F1 \left[1,\, \frac{1+m}{4},\, \frac{5+m}{4},\, e^{2\,a}\,x^4\right]}{4\; e\; \left(1+m\right)}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth [a + 2 Log[x]]^3, x]$ 

### Problem 168: Unable to integrate problem.

$$\int Coth[a + b Log[x]]^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \, \left( -1 - \mathrm{e}^{2\,\mathsf{a}} \, x^{2\,\mathsf{b}} \right)^{\,\mathsf{p}} \, \left( 1 + \mathrm{e}^{2\,\mathsf{a}} \, x^{2\,\mathsf{b}} \right)^{\,-\mathsf{p}} \, \mathsf{AppellF1} \big[ \, \frac{1}{2\,\mathsf{b}} , \, \, \mathsf{p} \, , \, \, -\mathsf{p} \, , \, \, \frac{1}{2} \, \left( 2 + \frac{1}{\mathsf{b}} \right) \, , \, \, \mathrm{e}^{2\,\mathsf{a}} \, x^{2\,\mathsf{b}} \, , \, \, -\mathrm{e}^{2\,\mathsf{a}} \, x^{2\,\mathsf{b}} \, \big]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + b \text{ Log} \left[ x \right] \right]^p, x \right]$ 

### Problem 169: Unable to integrate problem.

$$\int (e x)^m Coth[a + b Log[x]]^p dx$$

Optimal (type 6, 99 leaves, 3 steps):

$$\frac{1}{e\,\left(1+m\right)}\,\left(\,e\,x\,\right)^{\,1+m}\,\left(\,-\,1\,-\,e^{2\,a}\,x^{2\,b}\,\right)^{\,p}\,\left(\,1\,+\,e^{2\,a}\,x^{2\,b}\,\right)^{\,-p}\, \\ \text{AppellF1}\left[\,\frac{1+m}{2\,b}\,\text{, p, -p, 1}\,+\,\frac{1+m}{2\,b}\,\text{, }\,e^{2\,a}\,x^{2\,b}\,\right]^{\,p}\,\left(\,1\,+\,e^{2\,a}\,x^{2\,b}\,\right)^{\,p}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth[a + b Log[x]]^p, x]$ 

### Problem 170: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{2}\right]^p dx$$

Optimal (type 5, 52 leaves, 2 steps):

$$-\frac{1}{1+p}2^{-p}\,\,\mathrm{e}^{-2\,a}\,\left(-1-\mathrm{e}^{2\,a}\,x\right)^{1+p}\, \\ \text{Hypergeometric} \\ 2\text{F1}\!\left[\,p\,,\,\,1+p\,,\,\,2+p\,,\,\,\frac{1}{2}\,\,\left(1+\mathrm{e}^{2\,a}\,x\right)\,\,\right]$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ \text{Coth} \left[ \frac{1}{2} \left( 2 \text{ a} + \text{Log} \left[ x \right] \right) \right]^p$$
,  $x \right]$ 

### Problem 171: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{4}\right]^p dx$$

Optimal (type 5, 108 leaves, 4 steps):

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\Big[ Coth \Big[ \frac{1}{a} (4 a + Log[x]) \Big]^p$ ,  $x \Big]$ 

### Problem 172: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{6}\right]^p dx$$

Optimal (type 5, 162 leaves, 5 steps):

$$\begin{split} & e^{-6\,\mathsf{a}}\;\mathsf{p}\;\left(-\,\mathsf{1}\,-\,e^{2\,\mathsf{a}}\;\mathsf{x}^{1/3}\right)^{\,\mathsf{1}\,+\,\mathsf{p}}\;\left(1\,-\,e^{2\,\mathsf{a}}\;\mathsf{x}^{1/3}\right)^{\,\mathsf{1}\,-\,\mathsf{p}}\,+\,e^{-4\,\mathsf{a}}\;\left(-\,\mathsf{1}\,-\,e^{2\,\mathsf{a}}\;\mathsf{x}^{1/3}\right)^{\,\mathsf{1}\,+\,\mathsf{p}}\;\left(1\,-\,e^{2\,\mathsf{a}}\;\mathsf{x}^{1/3}\right)^{\,\mathsf{1}\,-\,\mathsf{p}}\;\mathsf{x}^{1/3}\,-\,\frac{1}{1\,+\,\mathsf{p}}\;\mathsf{p}^{\,\mathsf{p}}\,\mathsf{p}^{\,\mathsf{p$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate  $\left[ \text{Coth} \left[ \frac{1}{6} \left( 6 \text{ a} + \text{Log} \left[ x \right] \right) \right]^p, x \right]$ 

### Problem 173: Unable to integrate problem.

$$\int Coth \left[a + \frac{Log[x]}{g}\right]^p dx$$

Optimal (type 5, 194 leaves, 5 steps):

$$\begin{split} &\frac{1}{3}\;\mathrm{e}^{-12\,\mathsf{a}}\;\left(-\,\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\left(\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1-\mathsf{p}}\;\left(\mathrm{e}^{4\,\mathsf{a}}\;\left(3\,+\,2\,\mathsf{p}^{2}\right)\,+\,2\,\,\mathrm{e}^{6\,\mathsf{a}}\;\mathsf{p}\;\mathbf{x}^{1/4}\right)\,+\\ &\mathrm{e}^{-4\,\mathsf{a}}\;\left(-\,\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\left(\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1-\mathsf{p}}\;\sqrt{\mathbf{x}}\;-\,\frac{1}{3\;\left(\mathbf{1}\,+\,\mathsf{p}\right)}\\ &2^{2-\mathsf{p}}\;\mathrm{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathbf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{Hypergeometric}\\ &2^{1-\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathsf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{Hypergeometric}\\ &2^{1-\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathsf{1}\,-\,\mathrm{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{Hypergeometric}\\ &2^{1-\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathsf{1}\,-\,\mathsf{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{Hypergeometric}\\ &2^{1-\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathsf{1}\,-\,\mathsf{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{Hypergeometric}\\ &2^{1-\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\left(2\,+\,\mathsf{p}^{2}\right)\;\left(-\,\mathsf{1}\,-\,\mathsf{e}^{2\,\mathsf{a}}\;\mathbf{x}^{1/4}\right)^{\,1+\mathsf{p}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\mathsf{p}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\mathsf{p}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\mathsf{e}^{-8\,\mathsf{a}}\;\mathsf{p}\;\mathsf$$

Result (type 8, 15 leaves, 1 step):

CannotIntegrate 
$$\left[ \text{Coth} \left[ \frac{1}{8} \left( 8 \text{ a} + \text{Log} [x] \right) \right]^p, x \right]$$

### Problem 174: Unable to integrate problem.

$$\int Coth [a + Log[x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(-1 - e^{2a} x^2\right)^p \left(1 + e^{2a} x^2\right)^{-p} AppellF1\left[\frac{1}{2}, p, -p, \frac{3}{2}, e^{2a} x^2, -e^{2a} x^2\right]$$

Result (type 8, 9 leaves, 0 steps):

CannotIntegrate Coth [a + Log[x]]<sup>p</sup>, x

### Problem 175: Unable to integrate problem.

$$\int Coth [a + 2 Log [x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \, \left( -1 - \mathrm{e}^{2\,a} \, x^4 \right)^p \, \left( 1 + \mathrm{e}^{2\,a} \, x^4 \right)^{-p} \, \mathsf{AppellF1} \Big[ \, \frac{1}{4} \text{, p, } -p \text{, } \, \frac{5}{4} \text{, } \, \mathrm{e}^{2\,a} \, x^4 \text{, } -\mathrm{e}^{2\,a} \, x^4 \Big]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate Coth [a + 2 Log[x]]<sup>p</sup>, x

## Problem 176: Unable to integrate problem.

$$\int Coth [a + 3 Log [x]]^p dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$x \left(-1 - e^{2a} x^{6}\right)^{p} \left(1 + e^{2a} x^{6}\right)^{-p} AppellF1\left[\frac{1}{6}, p, -p, \frac{7}{6}, e^{2a} x^{6}, -e^{2a} x^{6}\right]$$

Result (type 8, 11 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ a + 3 \text{ Log} \left[ x \right] \right]^p, x \right]$ 

## Problem 177: Unable to integrate problem.

$$\int x^3 \, \mathsf{Coth} \big[ d \, \left( \mathsf{a} + \mathsf{b} \, \mathsf{Log} \big[ \mathsf{c} \, x^\mathsf{n} \big] \right) \big] \, \, \mathrm{d} x$$

Optimal (type 5, 58 leaves, 4 steps):

$$\frac{x^4}{4} - \frac{1}{2} x^4$$
 Hypergeometric2F1[1,  $\frac{2}{b d n}$ ,  $1 + \frac{2}{b d n}$ ,  $e^{2ad} (c x^n)^{2bd}$ ]

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^3 \text{ Coth } [d (a + b \text{ Log } [c x^n])], x]$ 

### Problem 178: Unable to integrate problem.

Optimal (type 5, 62 leaves, 4 steps):

$$\frac{x^3}{3} - \frac{2}{3}x^3$$
 Hypergeometric2F1[1,  $\frac{3}{2 \text{ b d n}}$ , 1 +  $\frac{3}{2 \text{ b d n}}$ ,  $e^{2 \text{ a d }}(c x^n)^{2 \text{ b d}}$ ]

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x^2 \text{ Coth } [d (a + b \text{ Log } [c x^n])], x]$ 

### Problem 179: Unable to integrate problem.

$$\int x \operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c x^n \right] \right) \right] dx$$

Optimal (type 5, 54 leaves, 4 steps):

$$\frac{x^2}{2}$$
 -  $x^2$  Hypergeometric2F1  $\left[1, \frac{1}{b d n}, 1 + \frac{1}{b d n}, e^{2ad} (c x^n)^{2bd}\right]$ 

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $[x Coth [d (a + b Log [c x^n])], x]$ 

### Problem 180: Unable to integrate problem.

$$\int Coth [d (a + b Log [c x^n])] dx$$

Optimal (type 5, 52 leaves, 4 steps):

$$x - 2 x \text{ Hypergeometric 2F1} \Big[ 1, \frac{1}{2 \text{ h d n}}, 1 + \frac{1}{2 \text{ h d n}}, e^{2 \text{ a d }} (c x^n)^{2 \text{ b d}} \Big]$$

Result (type 8, 15 leaves, 0 steps):

CannotIntegrate  $\lceil Coth \lceil d (a + b Log \lceil c x^n \rceil) \rceil$ ,  $x \rceil$ 

## Problem 182: Unable to integrate problem.

$$\int\!\frac{\mathsf{Coth}\!\left[\mathsf{d}\,\left(\mathsf{a}+\mathsf{b}\,\mathsf{Log}\left[\mathsf{c}\,\,\mathsf{x}^{\mathsf{n}}\right]\right)\,\right]}{\mathsf{x}^{\mathsf{2}}}\,\mathsf{d}\mathsf{x}$$

Optimal (type 5, 58 leaves, 4 steps):

$$-\frac{1}{x} + \frac{2 \text{ Hypergeometric2F1} \left[ 1, -\frac{1}{2 \text{ bdn}}, 1 - \frac{1}{2 \text{ bdn}}, e^{2 \text{ ad}} \left( c \, x^n \right)^{2 \text{ bd}} \right]}{x}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)\right]}{x^{2}}$$
,  $x\right]$ 

### Problem 183: Unable to integrate problem.

$$\int \frac{\mathsf{Coth} \left[\mathsf{d} \left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[\mathsf{c} \, \mathsf{x}^{\mathsf{n}}\right]\right)\right]}{\mathsf{x}^{\mathsf{3}}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 55 leaves, 4 steps):

$$-\frac{1}{2\,{x}^{2}}\,+\,\frac{\text{Hypergeometric2F1}\!\left[\,\mathbf{1}\text{, }-\frac{1}{\,\text{bd}\,\text{n}}\,\text{, }\,\mathbf{1}\,-\,\frac{1}{\,\text{bd}\,\text{n}}\,\text{, }\,\mathbb{e}^{2\,\text{ad}}\,\left(\,\text{c}\,\,x^{n}\right)^{\,2\,\text{bd}}\,\right]}{\,x^{2}}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)\right]}{x^{3}}$$
,  $x\right]$ 

### Problem 184: Unable to integrate problem.

$$\int x^3 \operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c x^n \right] \right) \right]^2 dx$$

Optimal (type 5, 132 leaves, 5 steps):

$$\begin{split} &\frac{1}{4} \left( \mathbf{1} + \frac{4}{b \, d \, n} \right) \, x^4 + \frac{x^4 \, \left( \mathbf{1} + \mathbb{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)}{b \, d \, n \, \left( \mathbf{1} - \mathbb{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)} - \\ &\frac{2 \, x^4 \, \text{Hypergeometric} 2 \text{F1} \left[ \mathbf{1}, \, \frac{2}{b \, d \, n}, \, \mathbf{1} + \frac{2}{b \, d \, n}, \, \mathbb{e}^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right]}{b \, d \, n} \end{split}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[x^{3} \operatorname{Coth}\left[d\left(a+b \operatorname{Log}\left[c \; x^{n}\right]\right)\right]^{2}$$
,  $x\right]$ 

### Problem 185: Unable to integrate problem.

$$\int x^2 \operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c x^n \right] \right) \right]^2 dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{1}{3} \left( 1 + \frac{3}{b \, d \, n} \right) \, x^3 + \frac{x^3 \, \left( 1 + e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)}{b \, d \, n \, \left( 1 - e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right)} - \\ 2 \, x^3 \, \text{Hypergeometric2F1} \left[ 1, \, \frac{3}{2 \, b \, d \, n}, \, 1 + \frac{3}{2 \, b \, d \, n}, \, e^{2 \, a \, d} \, \left( c \, x^n \right)^{2 \, b \, d} \right]$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\begin{bmatrix} x^2 & \text{Coth} & \begin{bmatrix} d & b & \text{Log} & \begin{bmatrix} c & x^n \end{bmatrix} \end{bmatrix} \end{bmatrix}^2$$
,  $x \end{bmatrix}$ 

### Problem 186: Unable to integrate problem.

$$\int x \operatorname{Coth} \left[ d \left( a + b \operatorname{Log} \left[ c x^n \right] \right) \right]^2 dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\begin{split} &\frac{1}{2}\left(1+\frac{2}{b\,d\,n}\right)\,x^2+\frac{x^2\,\left(1+\,e^{2\,a\,d}\,\left(c\,x^n\right)^{\,2\,b\,d}\right)}{b\,d\,n\,\left(1-\,e^{2\,a\,d}\,\left(c\,x^n\right)^{\,2\,b\,d}\right)} -\\ &\frac{2\,x^2\,\text{Hypergeometric} 2\text{F1}\!\left[1,\,\frac{1}{b\,d\,n},\,1+\frac{1}{b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^n\right)^{\,2\,b\,d}\right]}{b\,d\,n} \end{split}$$

Result (type 8, 19 leaves, 0 steps):

CannotIntegrate  $[x Coth [d (a + b Log [c x^n])]^2, x]$ 

### Problem 187: Unable to integrate problem.

Optimal (type 5, 126 leaves, 5 steps

$$\left(1+\frac{1}{b\,d\,n}\right)\,x+\frac{x\,\left(1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}{b\,d\,n\,\left(1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,x\,\text{Hypergeometric}2\text{F1}\!\left[1,\,\frac{1}{2\,b\,d\,n},\,1+\frac{1}{2\,b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n}$$

Result (type 8, 17 leaves, 0 steps):

CannotIntegrate  $\left[ \text{Coth} \left[ d \left( a + b \text{Log} \left[ c x^n \right] \right) \right]^2, x \right]$ 

### Problem 189: Unable to integrate problem.

$$\int \frac{\mathsf{Coth} \left[\mathsf{d} \left(\mathsf{a} + \mathsf{b} \, \mathsf{Log} \left[\mathsf{c} \, \mathsf{x}^{\mathsf{n}}\right]\right)\right]^{2}}{\mathsf{x}^{2}} \, \mathrm{d} \mathsf{x}$$

Optimal (type 5, 134 leaves, 5 step

$$-\frac{1-\frac{1}{b\,d\,n}}{x}+\frac{1+e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}}{b\,d\,n\,x\,\left(1-e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right)}-\frac{2\,\text{Hypergeometric2F1}\!\left[1,-\frac{1}{2\,b\,d\,n},\,1-\frac{1}{2\,b\,d\,n},\,e^{2\,a\,d}\,\left(c\,x^{n}\right)^{\,2\,b\,d}\right]}{b\,d\,n\,x}$$

Result (type 8, 21 leaves, 0 steps):

$$CannotIntegrate \Big[ \frac{Coth \Big[ d \left( a + b Log [ c x^n ] \right) \Big]^2}{x^2}, x \Big]$$

## Problem 190: Unable to integrate problem.

$$\int \frac{\mathsf{Coth} \left[ d \left( a + b \mathsf{Log} \left[ c \, x^n \right] \right) \right]^2}{x^3} \, \mathrm{d} x$$

Optimal (type 5, 135 leaves, 5 steps)

$$\frac{2 - b \, d \, n}{2 \, b \, d \, n \, x^2} + \frac{1 + e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}}{b \, d \, n \, x^2 \, \left(1 - e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right)} - \frac{2 \, \text{Hypergeometric2F1} \left[1, \, -\frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, 1 - \frac{1}{b \, d \, n}, \, e^{2 \, a \, d} \, \left(c \, x^n\right)^{2 \, b \, d}\right]}{b \, d \, n \, x^2}$$

Result (type 8, 21 leaves, 0 steps):

CannotIntegrate 
$$\left[\frac{\text{Coth}\left[d\left(a+b\,\text{Log}\left[c\,x^{n}\right]\right)\right]^{2}}{x^{3}}$$
,  $x\right]$ 

### Problem 194: Unable to integrate problem.

$$\int (e x)^m Coth [d (a + b Log [c x^n])] dx$$

Optimal (type 5, 87 leaves, 4 steps):

$$\frac{\left(\text{e x}\right)^{\text{1+m}}}{\text{e }\left(\text{1+m}\right)} = \frac{2 \; \left(\text{e x}\right)^{\text{1+m}} \, \text{Hypergeometric} 2\text{F1}\left[\text{1, } \frac{1+\text{m}}{2 \, \text{b} \, \text{d} \, \text{n}}, \; 1 + \frac{1+\text{m}}{2 \, \text{b} \, \text{d} \, \text{n}}, \; \text{e}^{2 \, \text{a} \, \text{d}} \; \left(\text{c} \; \text{x}^{\text{n}}\right)^{2 \, \text{b} \, \text{d}}\right]}{\text{e }\left(\text{1+m}\right)}$$

Result (type 8, 21 leaves, 0 steps):

$$CannotIntegrate \left[ \; (e \; x)^{\; m} \; Coth \left[ d \; \left( a \; + \; b \; Log \left[ c \; x^n \; \right] \right) \; \right] \text{, } \; x \right]$$

### Problem 195: Unable to integrate problem.

$$\left\lceil \left( e\,x \right)^{\,m}\, \text{Coth} \left[ \,d\, \left( a + b\, \text{Log} \left[ \,c\,\,x^{n} \,\right] \,\right) \,\right]^{\,2} \, \text{d}x \right.$$

Optimal (type 5, 168 leaves, 5 steps):

$$\frac{\left(\textbf{1} + \textbf{m} + \textbf{b} \, d \, n\right) \ (\textbf{e} \, \textbf{x})^{\, \textbf{1} + \textbf{m}}}{\textbf{b} \, d \, e \, \left(\textbf{1} + \textbf{m}\right) \, n} + \frac{\left(\textbf{e} \, \textbf{x}\right)^{\, \textbf{1} + \textbf{m}} \, \left(\textbf{1} + \textbf{e}^{2 \, \mathsf{a} \, d} \, \left(\textbf{c} \, \textbf{x}^{\mathsf{n}}\right)^{\, 2 \, \mathsf{b} \, d}\right)}{\textbf{b} \, d \, e \, n \, \left(\textbf{1} - \textbf{e}^{2 \, \mathsf{a} \, d} \, \left(\textbf{c} \, \textbf{x}^{\mathsf{n}}\right)^{\, 2 \, \mathsf{b} \, d}\right)} - \\ \\ \frac{2 \, \left(\textbf{e} \, \textbf{x}\right)^{\, \textbf{1} + \textbf{m}} \, \text{Hypergeometric 2F1} \left[\textbf{1}, \, \frac{\textbf{1} + \textbf{m}}{2 \, \mathsf{b} \, d \, \mathsf{n}}, \, \textbf{1} + \frac{\textbf{1} + \textbf{m}}{2 \, \mathsf{b} \, d \, \mathsf{n}}, \, \textbf{e}^{2 \, \mathsf{a} \, d} \, \left(\textbf{c} \, \textbf{x}^{\mathsf{n}}\right)^{\, 2 \, \mathsf{b} \, d}\right]}{\textbf{b} \, d \, e \, n}}$$

Result (type 8, 23 leaves, 0 steps):

CannotIntegrate 
$$[(ex)^m Coth[d(a+bLog[cx^n])]^2$$
,  $x]$ 

## Problem 196: Unable to integrate problem.

$$\left[ \, \left( \, e \, x \, \right) \, ^{m} \, \mathsf{Coth} \left[ \, d \, \left( \, a \, + \, b \, \mathsf{Log} \left[ \, c \, \, x^{n} \, \right] \, \right) \, \right]^{\, 3} \, \mathrm{d} x \, \right.$$

#### Optimal (type 5, 306 leaves, 6 steps):

$$\begin{split} &\frac{\left(1+\text{m}+\text{b}\,\text{d}\,\text{n}\right)\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}}{2\,\,\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} - \frac{\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\left(1+\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}}{2\,\,\text{b}\,\text{d}\,\text{e}\,\text{n}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)^{\,2}} + \\ &\frac{\text{e}^{-2\,\text{a}\,\text{d}}\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\left(\frac{\text{e}^{2\,\text{a}\,\text{d}}\,\left(1+\text{m}-2\,\text{b}\,\text{d}\,\text{n}\right)}{\text{n}} + \frac{\text{e}^{4\,\text{a}\,\text{d}}\,\left(1+\text{m}+2\,\text{b}\,\text{d}\,\text{n}\right)\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}}{\text{n}}\right)}{\text{n}} - \frac{1}{\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} \\ &\frac{2\,\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\text{n}\,\left(1-\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right)}{\text{n}} - \frac{1}{\text{b}^{2}\,\text{d}^{2}\,\text{e}\,\left(1+\text{m}\right)\,\text{n}^{2}} \\ &\left(1+2\,\text{m}+\text{m}^{2}+2\,\text{b}^{2}\,\text{d}^{2}\,\text{n}^{2}\right)\,\left(\text{e}\,\text{x}\right)^{\,1+\text{m}}\,\text{Hypergeometric} \\ \text{2F1}\left[1,\,\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,1+\frac{1+\text{m}}{2\,\text{b}\,\text{d}\,\text{n}},\,\text{e}^{2\,\text{a}\,\text{d}}\,\left(\text{c}\,\text{x}^{\text{n}}\right)^{\,2\,\text{b}\,\text{d}}\right] \end{aligned}$$

### Result (type 8, 23 leaves, 0 steps):

 $CannotIntegrate \left[ \; (e\; x)^{\; m}\; Coth \left[ \; d \; \left( \; a \; + \; b \; Log \left\lceil \; c \; \; x^n \; \right| \; \right) \; \right]^{\; 3} \text{, } \; x \; \right]$ 

### Problem 197: Unable to integrate problem.

$$\int Coth \left[ d \left( a + b Log \left[ c x^n \right] \right) \right]^p dx$$

### Optimal (type 6, 115 leaves, 4 steps):

### Result (type 8, 17 leaves, 0 steps):

 $CannotIntegrate \big[ Coth \big[ d \, \left( a + b \, Log \big[ c \, x^n \big] \, \right) \, \big]^p \text{, } x \big]$ 

### Problem 198: Unable to integrate problem.

$$\label{eq:continuous} \left[ \, \left( \, e \, \, x \, \right) \, ^m \, \text{Coth} \left[ \, d \, \left( \, a \, + \, b \, \, \text{Log} \left[ \, c \, \, x^n \, \right] \, \right) \, \, \right]^p \, \text{d} \, x \, \right.$$

### Optimal (type 6, 135 leaves, 4 steps):

$$\begin{split} &\frac{1}{e\,\left(1+m\right)}\,\left(e\,x\right)^{\,1+m}\,\left(-\,1\,-\,\,\mathrm{e}^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right)^{\,p}\,\left(1\,+\,\,\mathrm{e}^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right)^{\,-p}\\ &\text{AppellF1}\!\left[\,\frac{1+m}{2\,b\,d\,n}\,\text{, p, -p, 1}\,+\,\frac{1+m}{2\,b\,d\,n}\,\text{, }\,\,\mathrm{e}^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\,\text{, }\,-\,\mathrm{e}^{2\,a\,d}\,\left(c\,\,x^{n}\right)^{\,2\,b\,d}\right] \end{split}$$

### Result (type 8, 23 leaves, 0 steps):

CannotIntegrate  $[(ex)^m Coth[d(a+b Log[cx^n])]^p$ , x]

# Test results for the 53 problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Test results for the 16 problems in "6.5.1 (c+d x)^m (a+b sech)^n.m"

Test results for the 84 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

Test results for the 201 problems in "6.5.3 Hyperbolic secant functions.m"

Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\left\lceil \left( \left( 1 - b^2 \, n^2 \right) \, \mathsf{Sech} \left[ \, a + b \, \mathsf{Log} \left[ \, c \, \, x^n \, \right] \, \right] \, + 2 \, b^2 \, n^2 \, \mathsf{Sech} \left[ \, a + b \, \mathsf{Log} \left[ \, c \, \, x^n \, \right] \, \right]^3 \right) \, \mathrm{d} x \right\rangle$$

Optimal (type 3, 40 leaves, ? steps):

$$x\, \mathsf{Sech} \, \big[\, a + b\, \mathsf{Log} \, \big[\, c\, \, x^n \,\big] \,\, \big] \, + b\, n\, x\, \mathsf{Sech} \, \big[\, a + b\, \mathsf{Log} \, \big[\, c\, \, x^n \,\big] \,\, \big] \, \, \mathsf{Tanh} \, \big[\, a + b\, \mathsf{Log} \, \big[\, c\, \, x^n \,\big] \,\, \big]$$

Result (type 5, 139 leaves, 9 steps):

$$2\,\,\mathrm{e}^{a}\,\left(1-b\,n\right)\,x\,\left(c\,x^{n}\right)^{b}\,\text{Hypergeometric2F1}\!\left[1,\,\frac{b+\frac{1}{n}}{2\,b},\,\frac{1}{2}\left(3+\frac{1}{b\,n}\right),\,-\,\mathrm{e}^{2\,a}\,\left(c\,x^{n}\right)^{2\,b}\right]\,+\,\frac{1}{1+3\,b\,n}$$
 
$$16\,b^{2}\,\,\mathrm{e}^{3\,a}\,n^{2}\,x\,\left(c\,x^{n}\right)^{3\,b}\,\text{Hypergeometric2F1}\!\left[3,\,\frac{3\,b+\frac{1}{n}}{2\,b},\,\frac{1}{2}\left(5+\frac{1}{b\,n}\right),\,-\,\mathrm{e}^{2\,a}\,\left(c\,x^{n}\right)^{2\,b}\right]$$

Test results for the 220 problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

Test results for the 29 problems in "6.6.1 (c+d x)^m (a+b csch)^n.m"

Test results for the 83 problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

Test results for the 175 problems in "6.6.3 Hyperbolic cosecant functions.m"

Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

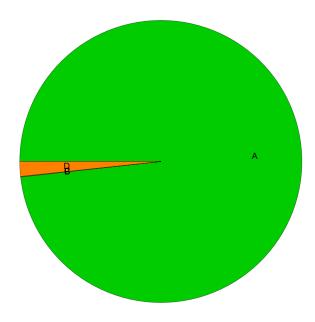
$$\begin{split} &\int \left(-\left(1-b^2\,n^2\right)\,\text{Csch}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right] + 2\,b^2\,n^2\,\text{Csch}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]^3\right)\,\text{d}x \\ &\quad \text{Optimal (type 3, 42 leaves, ? steps):} \\ &\quad -x\,\text{Csch}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right] - b\,n\,x\,\text{Coth}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right]\,\text{Csch}\left[\,a+b\,\text{Log}\left[\,c\,\,x^n\,\right]\,\right] \\ &\quad \text{Result (type 5, 137 leaves, 9 steps):} \\ &\quad 2\,e^a\,\left(\,1-b\,n\,\right)\,x\,\left(\,c\,\,x^n\,\right)^b\,\text{Hypergeometric2F1}\left[\,1,\,\,\frac{b+\frac{1}{n}}{2\,b}\,,\,\,\frac{1}{2}\,\left(\,3+\frac{1}{b\,n}\,\right)\,,\,\,e^{2\,a}\,\left(\,c\,\,x^n\,\right)^{2\,b}\,\right] - \\ &\quad \frac{1}{1+3\,b\,n}16\,b^2\,e^{3\,a}\,n^2\,x\,\left(\,c\,\,x^n\,\right)^{3\,b}\,\text{Hypergeometric2F1}\left[\,3,\,\,\frac{3\,b+\frac{1}{n}}{2\,b}\,,\,\,\frac{1}{2}\,\left(\,5+\frac{1}{b\,n}\,\right)\,,\,\,e^{2\,a}\,\left(\,c\,\,x^n\,\right)^{2\,b}\,\right] \end{split}$$

Test results for the 27 problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

Test results for the 1059 problems in "6.7.1 Hyperbolic functions.m"

# **Summary of Integration Test Results**

### 5166 integration problems



- A 5075 optimal antiderivatives
- B 2 valid but suboptimal antiderivatives
- C 2 unnecessarily complex antiderivatives
- D 87 unable to integrate problems
- E 0 integration timeouts
- F 0 invalid antiderivatives