

# Rubi 4.16.0.4 Integration Test Results

## on the problems in the test-suite directory "3 Logarithms"

Test results for the 193 problems in "3.1.2 (d x)^m (a+b log(c x^n))^p.m"

Test results for the 456 problems in "3.1.4 (f x)^m (d+e x^r)^q (a+b log(c x^n))^p.m"

**Problem 4: Result valid but suboptimal antiderivative.**

$$\int (d + e x) (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-b d n x - \frac{1}{4} b e n x^2 + d x (a + b \operatorname{Log}[c x^n]) + \frac{1}{2} e x^2 (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 41 leaves, 2 steps):

$$-b d n x - \frac{1}{4} b e n x^2 + \frac{1}{2} (2 d x + e x^2) (a + b \operatorname{Log}[c x^n])$$

**Problem 6: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x) (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 48 leaves, 4 steps):

$$-\frac{b d n}{x} - \frac{d (a + b \operatorname{Log}[c x^n])}{x} + \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 b n}$$

Result (type 3, 43 leaves, 4 steps):

$$-\frac{b d n}{x} - \frac{1}{2} b e n \operatorname{Log}[x]^2 - \left( \frac{d}{x} - e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x) (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{b d n}{9 x^3} - \frac{b e n}{4 x^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{e (a + b \operatorname{Log}[c x^n])}{2 x^2}$$

Result (type 3, 48 leaves, 4 steps):

$$-\frac{b d n}{9 x^3} - \frac{b e n}{4 x^2} - \frac{1}{6} \left( \frac{2 d}{x^3} + \frac{3 e}{x^2} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 13: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x)^2 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$-\frac{1}{4} b n (4 d + e x)^2 - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + 2 d e x (a + b \operatorname{Log}[c x^n]) + \frac{1}{2} e^2 x^2 (a + b \operatorname{Log}[c x^n]) + d^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 63 leaves, 3 steps):

$$-\frac{1}{4} b n (4 d + e x)^2 - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \frac{1}{2} (4 d e x + e^2 x^2 + 2 d^2 \operatorname{Log}[x]) (a + b \operatorname{Log}[c x^n])$$

### Problem 14: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x)^2 (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{b d^2 n}{x} - b e^2 n x - b d e n \operatorname{Log}[x]^2 - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{x} + e^2 x (a + b \operatorname{Log}[c x^n]) + 2 d e \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 61 leaves, 3 steps):

$$-\frac{b d^2 n}{x} - b e^2 n x - b d e n \operatorname{Log}[x]^2 - \left( \frac{d^2}{x} - e^2 x - 2 d e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x)^2 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$-\frac{b n (d + 4 e x)^2}{4 x^2} - \frac{1}{2} b e^2 n \operatorname{Log}[x]^2 - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{2 x^2} - \frac{2 d e (a + b \operatorname{Log}[c x^n])}{x} + e^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 67 leaves, 4 steps):

$$-\frac{b n (d + 4 e x)^2}{4 x^2} - \frac{1}{2} b e^2 n \operatorname{Log}[x]^2 - \frac{1}{2} \left( \frac{d^2}{x^2} + \frac{4 d e}{x} - 2 e^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 17: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x)^2 (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{2 b d e n}{9 x^3} - \frac{b e^2 n}{4 x^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{4 x^4} - \frac{2 d e (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{e^2 (a + b \operatorname{Log}[c x^n])}{2 x^2}$$

Result (type 3, 74 leaves, 4 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{2 b d e n}{9 x^3} - \frac{b e^2 n}{4 x^2} - \frac{1}{12} \left( \frac{3 d^2}{x^4} + \frac{8 d e}{x^3} + \frac{6 e^2}{x^2} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 18: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x)^2 (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{b d e n}{8 x^4} - \frac{b e^2 n}{9 x^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{d e (a + b \operatorname{Log}[c x^n])}{2 x^4} - \frac{e^2 (a + b \operatorname{Log}[c x^n])}{3 x^3}$$

Result (type 3, 74 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{b d e n}{8 x^4} - \frac{b e^2 n}{9 x^3} - \frac{1}{30} \left( \frac{6 d^2}{x^5} + \frac{15 d e}{x^4} + \frac{10 e^2}{x^3} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 23: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x)^3 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 122 leaves, 4 steps):

$$-3 b d^2 e n x - \frac{3}{4} b d e^2 n x^2 - \frac{1}{9} b e^3 n x^3 - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 + 3 d^2 e x (a + b \operatorname{Log}[c x^n]) +$$

$$\frac{3}{2} d e^2 x^2 (a + b \operatorname{Log}[c x^n]) + \frac{1}{3} e^3 x^3 (a + b \operatorname{Log}[c x^n]) + d^3 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 94 leaves, 4 steps):

$$-3 b d^2 e n x - \frac{3}{4} b d e^2 n x^2 - \frac{1}{9} b e^3 n x^3 - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 +$$

$$\frac{1}{6} (18 d^2 e x + 9 d e^2 x^2 + 2 e^3 x^3 + 6 d^3 \operatorname{Log}[x]) (a + b \operatorname{Log}[c x^n])$$

**Problem 24: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x)^3 (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{b d^3 n}{x} - 3 b d e^2 n x - \frac{1}{4} b e^3 n x^2 - \frac{3}{2} b d^2 e n \operatorname{Log}[x]^2 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{x} +$$

$$3 d e^2 x (a + b \operatorname{Log}[c x^n]) + \frac{1}{2} e^3 x^2 (a + b \operatorname{Log}[c x^n]) + 3 d^2 e \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 92 leaves, 3 steps):

$$-\frac{b d^3 n}{x} - 3 b d e^2 n x - \frac{1}{4} b e^3 n x^2 - \frac{3}{2} b d^2 e n \operatorname{Log}[x]^2 -$$

$$\frac{1}{2} \left( \frac{2 d^3}{x} - 6 d e^2 x - e^3 x^2 - 6 d^2 e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 25: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x)^3 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 118 leaves, 3 steps):

$$-\frac{b d^3 n}{4 x^2} - \frac{3 b d^2 e n}{x} - b e^3 n x - \frac{3}{2} b d e^2 n \operatorname{Log}[x]^2 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{2 x^2} -$$

$$\frac{3 d^2 e (a + b \operatorname{Log}[c x^n])}{x} + e^3 x (a + b \operatorname{Log}[c x^n]) + 3 d e^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 91 leaves, 3 steps):

$$-\frac{b d^3 n}{4 x^2} - \frac{3 b d^2 e n}{x} - b e^3 n x - \frac{3}{2} b d e^2 n \operatorname{Log}[x]^2 -$$

$$\frac{1}{2} \left( \frac{d^3}{x^2} + \frac{6 d^2 e}{x} - 2 e^3 x - 6 d e^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{(d + ex)^3 (a + b \operatorname{Log}[cx^n])}{x^4} dx$$

Optimal (type 3, 126 leaves, 7 steps) :

$$\begin{aligned} & -\frac{bd^3n}{9x^3} - \frac{3bd^2en}{4x^2} - \frac{3bde^2n}{x} - \frac{1}{2}be^3n\operatorname{Log}[x]^2 - \frac{d^3(a + b\operatorname{Log}[cx^n])}{3x^3} - \\ & \frac{3d^2e(a + b\operatorname{Log}[cx^n])}{2x^2} - \frac{3de^2(a + b\operatorname{Log}[cx^n])}{x} + e^3\operatorname{Log}[x](a + b\operatorname{Log}[cx^n]) \end{aligned}$$

Result (type 3, 98 leaves, 5 steps) :

$$\begin{aligned} & -\frac{bd^3n}{9x^3} - \frac{3bd^2en}{4x^2} - \frac{3bde^2n}{x} - \frac{1}{2}be^3n\operatorname{Log}[x]^2 - \\ & \frac{1}{6} \left( \frac{2d^3}{x^3} + \frac{9d^2e}{x^2} + \frac{18de^2}{x} - 6e^3\operatorname{Log}[x] \right) (a + b\operatorname{Log}[cx^n]) \end{aligned}$$

### Problem 29: Result valid but suboptimal antiderivative.

$$\int \frac{(d + ex)^3 (a + b \operatorname{Log}[cx^n])}{x^7} dx$$

Optimal (type 3, 133 leaves, 4 steps) :

$$\begin{aligned} & -\frac{bd^3n}{36x^6} - \frac{3bd^2en}{25x^5} - \frac{3bde^2n}{16x^4} - \frac{be^3n}{9x^3} - \frac{d^3(a + b\operatorname{Log}[cx^n])}{6x^6} - \\ & \frac{3d^2e(a + b\operatorname{Log}[cx^n])}{5x^5} - \frac{3de^2(a + b\operatorname{Log}[cx^n])}{4x^4} - \frac{e^3(a + b\operatorname{Log}[cx^n])}{3x^3} \end{aligned}$$

Result (type 3, 100 leaves, 4 steps) :

$$-\frac{bd^3n}{36x^6} - \frac{3bd^2en}{25x^5} - \frac{3bde^2n}{16x^4} - \frac{be^3n}{9x^3} - \frac{1}{60} \left( \frac{10d^3}{x^6} + \frac{36d^2e}{x^5} + \frac{45de^2}{x^4} + \frac{20e^3}{x^3} \right) (a + b\operatorname{Log}[cx^n])$$

### Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{(d + ex)^3 (a + b \operatorname{Log}[cx^n])}{x^8} dx$$

Optimal (type 3, 133 leaves, 4 steps) :

$$\begin{aligned} & -\frac{bd^3n}{49x^7} - \frac{bd^2en}{12x^6} - \frac{3bde^2n}{25x^5} - \frac{be^3n}{16x^4} - \frac{d^3(a + b\operatorname{Log}[cx^n])}{7x^7} - \\ & \frac{d^2e(a + b\operatorname{Log}[cx^n])}{2x^6} - \frac{3de^2(a + b\operatorname{Log}[cx^n])}{5x^5} - \frac{e^3(a + b\operatorname{Log}[cx^n])}{4x^4} \end{aligned}$$

Result (type 3, 100 leaves, 4 steps) :

$$-\frac{b d^3 n}{49 x^7} - \frac{b d^2 e n}{12 x^6} - \frac{3 b d e^2 n}{25 x^5} - \frac{b e^3 n}{16 x^4} - \frac{1}{140} \left( \frac{20 d^3}{x^7} + \frac{70 d^2 e}{x^6} + \frac{84 d e^2}{x^5} + \frac{35 e^3}{x^4} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 35: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$-\frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d}$$

Result (type 4, 66 leaves, 4 steps):

$$\frac{(a + b \operatorname{Log}[c x^n])^2}{2 b d n} - \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d}$$

**Problem 36: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x)} dx$$

Optimal (type 4, 74 leaves, 4 steps):

$$-\frac{b n}{d x} - \frac{a + b \operatorname{Log}[c x^n]}{d x} + \frac{e \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^2} - \frac{b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^2}$$

Result (type 4, 95 leaves, 6 steps):

$$-\frac{b n}{d x} - \frac{a + b \operatorname{Log}[c x^n]}{d x} - \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 b d^2 n} + \frac{e (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} + \frac{b e n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2}$$

**Problem 37: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x)} dx$$

Optimal (type 4, 110 leaves, 6 steps):

$$-\frac{b n}{4 d x^2} + \frac{b e n}{d^2 x} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2} + \frac{e (a + b \operatorname{Log}[c x^n])}{d^2 x} - \frac{e^2 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^3} + \frac{b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^3}$$

Result (type 4, 135 leaves, 7 steps):

$$-\frac{b n}{4 d x^2} + \frac{b e n}{d^2 x} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2} + \frac{e (a + b \operatorname{Log}[c x^n])}{d^2 x} +$$

$$\frac{e^2 (a + b \operatorname{Log}[c x^n])^2}{2 b d^3 n} - \frac{e^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \frac{b e^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3}$$

**Problem 38: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^4 (d + e x)} dx$$

Optimal (type 4, 150 leaves, 8 steps):

$$-\frac{b n}{9 d x^3} + \frac{b e n}{4 d^2 x^2} - \frac{b e^2 n}{d^3 x} - \frac{a + b \operatorname{Log}[c x^n]}{3 d x^3} + \frac{e (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} -$$

$$\frac{e^2 (a + b \operatorname{Log}[c x^n])}{d^3 x} + \frac{e^3 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^4} - \frac{b e^3 n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^4}$$

Result (type 4, 173 leaves, 8 steps):

$$-\frac{b n}{9 d x^3} + \frac{b e n}{4 d^2 x^2} - \frac{b e^2 n}{d^3 x} - \frac{a + b \operatorname{Log}[c x^n]}{3 d x^3} + \frac{e (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} - \frac{e^2 (a + b \operatorname{Log}[c x^n])}{d^3 x} -$$

$$\frac{e^3 (a + b \operatorname{Log}[c x^n])^2}{2 b d^4 n} + \frac{e^3 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^4} + \frac{b e^3 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4}$$

**Problem 39: Result valid but suboptimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])}{(d + e x)^2} dx$$

Optimal (type 4, 152 leaves, 8 steps):

$$\frac{3 b d n x}{e^3} - \frac{d (3 a + b n) x}{e^3} - \frac{3 b n x^2}{4 e^2} - \frac{3 b d x \operatorname{Log}[c x^n]}{e^3} - \frac{x^3 (a + b \operatorname{Log}[c x^n])}{e (d + e x)} +$$

$$\frac{x^2 (3 a + b n + 3 b \operatorname{Log}[c x^n])}{2 e^2} + \frac{d^2 (3 a + b n + 3 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} + \frac{3 b d^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4}$$

Result (type 4, 151 leaves, 9 steps):

$$-\frac{2 a d x}{e^3} + \frac{2 b d n x}{e^3} - \frac{b n x^2}{4 e^2} - \frac{2 b d x \operatorname{Log}[c x^n]}{e^3} + \frac{x^2 (a + b \operatorname{Log}[c x^n])}{2 e^2} - \frac{d^2 x (a + b \operatorname{Log}[c x^n])}{e^3 (d + e x)} +$$

$$\frac{b d^2 n \operatorname{Log}[d + e x]}{e^4} + \frac{3 d^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} + \frac{3 b d^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4}$$

### Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x)^2} dx$$

Optimal (type 4, 98 leaves, 7 steps):

$$-\frac{b n x}{e^2} + \frac{2 x (a + b \operatorname{Log}[c x^n])}{e^2} - \frac{x^2 (a + b \operatorname{Log}[c x^n])}{e (d + e x)} - \frac{d (2 a + b n + 2 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^3} - \frac{2 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3}$$

Result (type 4, 106 leaves, 8 steps):

$$\frac{a x}{e^2} - \frac{b n x}{e^2} + \frac{b x \operatorname{Log}[c x^n]}{e^2} + \frac{d x (a + b \operatorname{Log}[c x^n])}{e^2 (d + e x)} - \frac{b d n \operatorname{Log}[d + e x]}{e^3} - \frac{2 d (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^3} - \frac{2 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3}$$

### Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{Log}[c x^n])}{(d + e x)^2} dx$$

Optimal (type 4, 65 leaves, 3 steps):

$$-\frac{x (a + b \operatorname{Log}[c x^n])}{e (d + e x)} + \frac{(a + b n + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^2} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^2}$$

Result (type 4, 74 leaves, 6 steps):

$$-\frac{x (a + b \operatorname{Log}[c x^n])}{e (d + e x)} + \frac{b n \operatorname{Log}[d + e x]}{e^2} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^2} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^2}$$

### Problem 43: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)^2} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{e x (a + b \operatorname{Log}[c x^n])}{d^2 (d + e x)} - \frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^2} + \frac{b n \operatorname{Log}[d + e x]}{d^2} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^2}$$

Result (type 4, 102 leaves, 7 steps):



$$\begin{aligned}
& - \frac{e x (a + b \operatorname{Log}[c x^n])}{d^2 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 b d^2 n} + \\
& \frac{b n \operatorname{Log}[d + e x]}{d^2} - \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} - \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2}
\end{aligned}$$

**Problem 44: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x)^2} dx$$

Optimal (type 4, 114 leaves, 7 steps):

$$\begin{aligned}
& - \frac{b n}{d^2 x} - \frac{a + b \operatorname{Log}[c x^n]}{d^2 x} + \frac{e^2 x (a + b \operatorname{Log}[c x^n])}{d^3 (d + e x)} + \\
& \frac{2 e \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^3} - \frac{b e n \operatorname{Log}[d + e x]}{d^3} - \frac{2 b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^3}
\end{aligned}$$

Result (type 4, 134 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b n}{d^2 x} - \frac{a + b \operatorname{Log}[c x^n]}{d^2 x} + \frac{e^2 x (a + b \operatorname{Log}[c x^n])}{d^3 (d + e x)} - \frac{e (a + b \operatorname{Log}[c x^n])^2}{b d^3 n} - \\
& \frac{b e n \operatorname{Log}[d + e x]}{d^3} + \frac{2 e (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} + \frac{2 b e n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3}
\end{aligned}$$

**Problem 45: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x)^2} dx$$

Optimal (type 4, 154 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b n}{4 d^2 x^2} + \frac{2 b e n}{d^3 x} - \frac{a + b \operatorname{Log}[c x^n]}{2 d^2 x^2} + \frac{2 e (a + b \operatorname{Log}[c x^n])}{d^3 x} - \frac{e^3 x (a + b \operatorname{Log}[c x^n])}{d^4 (d + e x)} - \\
& \frac{3 e^2 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^4} + \frac{b e^2 n \operatorname{Log}[d + e x]}{d^4} + \frac{3 b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^4}
\end{aligned}$$

Result (type 4, 178 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b n}{4 d^2 x^2} + \frac{2 b e n}{d^3 x} - \frac{a + b \operatorname{Log}[c x^n]}{2 d^2 x^2} + \frac{2 e (a + b \operatorname{Log}[c x^n])}{d^3 x} - \\
& \frac{e^3 x (a + b \operatorname{Log}[c x^n])}{d^4 (d + e x)} + \frac{3 e^2 (a + b \operatorname{Log}[c x^n])^2}{2 b d^4 n} + \frac{b e^2 n \operatorname{Log}[d + e x]}{d^4} - \\
& \frac{3 e^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^4} - \frac{3 b e^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4}
\end{aligned}$$

### Problem 46: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])}{(d + e x)^3} dx$$

Optimal (type 4, 149 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 b n x}{e^3} + \frac{(6 a + 5 b n) x}{2 e^3} + \frac{3 b x \operatorname{Log}[c x^n]}{e^3} - \frac{x^3 (a + b \operatorname{Log}[c x^n])}{2 e (d + e x)^2} - \frac{x^2 (3 a + b n + 3 b \operatorname{Log}[c x^n])}{2 e^2 (d + e x)} \\ & - \frac{d (6 a + 5 b n + 6 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 e^4} - \frac{3 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

Result (type 4, 167 leaves, 11 steps):

$$\begin{aligned} & \frac{a x}{e^3} - \frac{b n x}{e^3} - \frac{b d^2 n}{2 e^4 (d + e x)} - \frac{b d n \operatorname{Log}[x]}{2 e^4} + \frac{b x \operatorname{Log}[c x^n]}{e^3} + \\ & \frac{d^3 (a + b \operatorname{Log}[c x^n])}{2 e^4 (d + e x)^2} + \frac{3 d x (a + b \operatorname{Log}[c x^n])}{e^3 (d + e x)} - \frac{5 b d n \operatorname{Log}[d + e x]}{2 e^4} - \\ & \frac{3 d (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} - \frac{3 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

### Problem 47: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])}{(d + e x)^3} dx$$

Optimal (type 4, 107 leaves, 4 steps):

$$\begin{aligned} & -\frac{x^2 (a + b \operatorname{Log}[c x^n])}{2 e (d + e x)^2} - \frac{x (2 a + b n + 2 b \operatorname{Log}[c x^n])}{2 e^2 (d + e x)} + \\ & \frac{(2 a + 3 b n + 2 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 e^3} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3} \end{aligned}$$

Result (type 4, 132 leaves, 9 steps):

$$\begin{aligned} & \frac{b d n}{2 e^3 (d + e x)} + \frac{b n \operatorname{Log}[x]}{2 e^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{2 e^3 (d + e x)^2} - \frac{2 x (a + b \operatorname{Log}[c x^n])}{e^2 (d + e x)} + \\ & \frac{3 b n \operatorname{Log}[d + e x]}{2 e^3} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^3} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3} \end{aligned}$$

### Problem 50: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)^3} dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$-\frac{bn}{2d^2(d+ex)} - \frac{bn \operatorname{Log}[x]}{2d^3} + \frac{a+b \operatorname{Log}[cx^n]}{2d(d+ex)^2} - \frac{ex(a+b \operatorname{Log}[cx^n])}{d^3(d+ex)} -$$

$$\frac{\operatorname{Log}\left[1+\frac{d}{ex}\right](a+b \operatorname{Log}[cx^n])}{d^3} + \frac{3bn \operatorname{Log}[d+ex]}{2d^3} + \frac{bn \operatorname{PolyLog}\left[2, -\frac{d}{ex}\right]}{d^3}$$

Result (type 4, 156 leaves, 11 steps):

$$-\frac{bn}{2d^2(d+ex)} - \frac{bn \operatorname{Log}[x]}{2d^3} + \frac{a+b \operatorname{Log}[cx^n]}{2d(d+ex)^2} - \frac{ex(a+b \operatorname{Log}[cx^n])}{d^3(d+ex)} + \frac{(a+b \operatorname{Log}[cx^n])^2}{2bd^3n} +$$

$$\frac{3bn \operatorname{Log}[d+ex]}{2d^3} - \frac{(a+b \operatorname{Log}[cx^n]) \operatorname{Log}\left[1+\frac{ex}{d}\right]}{d^3} - \frac{bn \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^3}$$

**Problem 51: Result valid but suboptimal antiderivative.**

$$\int \frac{a+b \operatorname{Log}[cx^n]}{x^2(d+ex)^3} dx$$

Optimal (type 4, 171 leaves, 10 steps):

$$-\frac{bn}{d^3x} + \frac{ben}{2d^3(d+ex)} + \frac{ben \operatorname{Log}[x]}{2d^4} - \frac{a+b \operatorname{Log}[cx^n]}{d^3x} - \frac{e(a+b \operatorname{Log}[cx^n])}{2d^2(d+ex)^2} + \frac{2e^2x(a+b \operatorname{Log}[cx^n])}{d^4(d+ex)} +$$

$$\frac{3e \operatorname{Log}\left[1+\frac{d}{ex}\right](a+b \operatorname{Log}[cx^n])}{d^4} - \frac{5ben \operatorname{Log}[d+ex]}{2d^4} - \frac{3ben \operatorname{PolyLog}\left[2, -\frac{d}{ex}\right]}{d^4}$$

Result (type 4, 193 leaves, 11 steps):

$$-\frac{bn}{d^3x} + \frac{ben}{2d^3(d+ex)} + \frac{ben \operatorname{Log}[x]}{2d^4} - \frac{a+b \operatorname{Log}[cx^n]}{d^3x} -$$

$$\frac{e(a+b \operatorname{Log}[cx^n])}{2d^2(d+ex)^2} + \frac{2e^2x(a+b \operatorname{Log}[cx^n])}{d^4(d+ex)} - \frac{3e(a+b \operatorname{Log}[cx^n])^2}{2bd^4n} -$$

$$\frac{5ben \operatorname{Log}[d+ex]}{2d^4} + \frac{3e(a+b \operatorname{Log}[cx^n]) \operatorname{Log}\left[1+\frac{ex}{d}\right]}{d^4} + \frac{3ben \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^4}$$

**Problem 52: Result valid but suboptimal antiderivative.**

$$\int \frac{a+b \operatorname{Log}[cx^n]}{x^3(d+ex)^3} dx$$

Optimal (type 4, 217 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b n}{4 d^3 x^2} + \frac{3 b e n}{d^4 x} - \frac{b e^2 n}{2 d^4 (d+e x)} - \frac{b e^2 n \operatorname{Log}[x]}{2 d^5} - \frac{a+b \operatorname{Log}[c x^n]}{2 d^3 x^2} + \\
& \frac{3 e (a+b \operatorname{Log}[c x^n])}{d^4 x} + \frac{e^2 (a+b \operatorname{Log}[c x^n])}{2 d^3 (d+e x)^2} - \frac{3 e^3 x (a+b \operatorname{Log}[c x^n])}{d^5 (d+e x)} - \\
& \frac{6 e^2 \operatorname{Log}\left[1+\frac{d}{e x}\right] (a+b \operatorname{Log}[c x^n])}{d^5} + \frac{7 b e^2 n \operatorname{Log}[d+e x]}{2 d^5} + \frac{6 b e^2 n \operatorname{PolyLog}\left[2,-\frac{d}{e x}\right]}{d^5}
\end{aligned}$$

Result (type 4, 239 leaves, 12 steps):

$$\begin{aligned}
& -\frac{b n}{4 d^3 x^2} + \frac{3 b e n}{d^4 x} - \frac{b e^2 n}{2 d^4 (d+e x)} - \frac{b e^2 n \operatorname{Log}[x]}{2 d^5} - \frac{a+b \operatorname{Log}[c x^n]}{2 d^3 x^2} + \frac{3 e (a+b \operatorname{Log}[c x^n])}{d^4 x} + \\
& \frac{e^2 (a+b \operatorname{Log}[c x^n])}{2 d^3 (d+e x)^2} - \frac{3 e^3 x (a+b \operatorname{Log}[c x^n])}{d^5 (d+e x)} + \frac{3 e^2 (a+b \operatorname{Log}[c x^n])^2}{b d^5 n} + \\
& \frac{7 b e^2 n \operatorname{Log}[d+e x]}{2 d^5} - \frac{6 e^2 (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^5} - \frac{6 b e^2 n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{d^5}
\end{aligned}$$

Problem 53: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a+b \operatorname{Log}[c x^n])}{(d+e x)^4} dx$$

Optimal (type 4, 229 leaves, 10 steps):

$$\begin{aligned}
& \frac{10 b d n x}{e^5} - \frac{d (60 a+47 b n) x}{6 e^5} - \frac{5 b n x^2}{2 e^4} - \frac{10 b d x \operatorname{Log}[c x^n]}{e^5} - \frac{x^5 (a+b \operatorname{Log}[c x^n])}{3 e (d+e x)^3} - \\
& \frac{x^4 (5 a+b n+5 b \operatorname{Log}[c x^n])}{6 e^2 (d+e x)^2} - \frac{x^3 (20 a+9 b n+20 b \operatorname{Log}[c x^n])}{6 e^3 (d+e x)} + \frac{x^2 (60 a+47 b n+60 b \operatorname{Log}[c x^n])}{12 e^4} + \\
& \frac{d^2 (60 a+47 b n+60 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{6 e^6} + \frac{10 b d^2 n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{e^6}
\end{aligned}$$

Result (type 4, 260 leaves, 15 steps):

$$\begin{aligned}
& -\frac{4 a d x}{e^5} + \frac{4 b d n x}{e^5} - \frac{b n x^2}{4 e^4} - \frac{b d^4 n}{6 e^6 (d+e x)^2} + \frac{13 b d^3 n}{6 e^6 (d+e x)} + \frac{13 b d^2 n \operatorname{Log}[x]}{6 e^6} - \frac{4 b d x \operatorname{Log}[c x^n]}{e^5} + \\
& \frac{x^2 (a+b \operatorname{Log}[c x^n])}{2 e^4} + \frac{d^5 (a+b \operatorname{Log}[c x^n])}{3 e^6 (d+e x)^3} - \frac{5 d^4 (a+b \operatorname{Log}[c x^n])}{2 e^6 (d+e x)^2} - \frac{10 d^2 x (a+b \operatorname{Log}[c x^n])}{e^5 (d+e x)} + \\
& \frac{47 b d^2 n \operatorname{Log}[d+e x]}{6 e^6} + \frac{10 d^2 (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{e^6} + \frac{10 b d^2 n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{e^6}
\end{aligned}$$

Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 (a+b \operatorname{Log}[c x^n])}{(d+e x)^4} dx$$

Optimal (type 4, 183 leaves, 9 steps):

$$\begin{aligned} & -\frac{4 b n x}{e^4} + \frac{(12 a + 13 b n) x}{3 e^4} + \frac{4 b x \operatorname{Log}[c x^n]}{e^4} - \frac{x^4 (a + b \operatorname{Log}[c x^n])}{3 e (d + e x)^3} - \\ & \frac{x^3 (4 a + b n + 4 b \operatorname{Log}[c x^n])}{6 e^2 (d + e x)^2} - \frac{x^2 (12 a + 7 b n + 12 b \operatorname{Log}[c x^n])}{6 e^3 (d + e x)} - \\ & \frac{d (12 a + 13 b n + 12 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 e^5} - \frac{4 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^5} \end{aligned}$$

Result (type 4, 211 leaves, 14 steps):

$$\begin{aligned} & \frac{a x}{e^4} - \frac{b n x}{e^4} + \frac{b d^3 n}{6 e^5 (d + e x)^2} - \frac{5 b d^2 n}{3 e^5 (d + e x)} - \frac{5 b d n \operatorname{Log}[x]}{3 e^5} + \frac{b x \operatorname{Log}[c x^n]}{e^4} - \\ & \frac{d^4 (a + b \operatorname{Log}[c x^n])}{3 e^5 (d + e x)^3} + \frac{2 d^3 (a + b \operatorname{Log}[c x^n])}{e^5 (d + e x)^2} + \frac{6 d x (a + b \operatorname{Log}[c x^n])}{e^4 (d + e x)} - \\ & \frac{13 b d n \operatorname{Log}[d + e x]}{3 e^5} - \frac{4 d (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^5} - \frac{4 b d n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^5} \end{aligned}$$

**Problem 55: Result valid but suboptimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])}{(d + e x)^4} dx$$

Optimal (type 4, 141 leaves, 5 steps):

$$\begin{aligned} & -\frac{x^3 (a + b \operatorname{Log}[c x^n])}{3 e (d + e x)^3} - \frac{x^2 (3 a + b n + 3 b \operatorname{Log}[c x^n])}{6 e^2 (d + e x)^2} - \frac{x (6 a + 5 b n + 6 b \operatorname{Log}[c x^n])}{6 e^3 (d + e x)} + \\ & \frac{(6 a + 11 b n + 6 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{6 e^4} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

Result (type 4, 178 leaves, 12 steps):

$$\begin{aligned} & -\frac{b d^2 n}{6 e^4 (d + e x)^2} + \frac{7 b d n}{6 e^4 (d + e x)} + \frac{7 b n \operatorname{Log}[x]}{6 e^4} + \frac{d^3 (a + b \operatorname{Log}[c x^n])}{3 e^4 (d + e x)^3} - \frac{3 d^2 (a + b \operatorname{Log}[c x^n])}{2 e^4 (d + e x)^2} - \\ & \frac{3 x (a + b \operatorname{Log}[c x^n])}{e^3 (d + e x)} + \frac{11 b n \operatorname{Log}[d + e x]}{6 e^4} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

**Problem 59: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)^4} dx$$

Optimal (type 4, 174 leaves, 13 steps):

$$\begin{aligned}
& -\frac{b n}{6 d^2 (d+e x)^2} - \frac{5 b n}{6 d^3 (d+e x)} - \frac{5 b n \operatorname{Log}[x]}{6 d^4} + \frac{a+b \operatorname{Log}[c x^n]}{3 d (d+e x)^3} + \frac{a+b \operatorname{Log}[c x^n]}{2 d^2 (d+e x)^2} - \\
& \frac{e x (a+b \operatorname{Log}[c x^n])}{d^4 (d+e x)} - \frac{\operatorname{Log}\left[1+\frac{d}{e x}\right] (a+b \operatorname{Log}[c x^n])}{d^4} + \frac{11 b n \operatorname{Log}[d+e x]}{6 d^4} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^4}
\end{aligned}$$

Result (type 4, 196 leaves, 15 steps):

$$\begin{aligned}
& -\frac{b n}{6 d^2 (d+e x)^2} - \frac{5 b n}{6 d^3 (d+e x)} - \frac{5 b n \operatorname{Log}[x]}{6 d^4} + \frac{a+b \operatorname{Log}[c x^n]}{3 d (d+e x)^3} + \\
& \frac{a+b \operatorname{Log}[c x^n]}{2 d^2 (d+e x)^2} - \frac{e x (a+b \operatorname{Log}[c x^n])}{d^4 (d+e x)} + \frac{(a+b \operatorname{Log}[c x^n])^2}{2 b d^4 n} + \\
& \frac{11 b n \operatorname{Log}[d+e x]}{6 d^4} - \frac{(a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^4} - \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4}
\end{aligned}$$

Problem 60: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x^2 (d+e x)^4} dx$$

Optimal (type 4, 211 leaves, 13 steps):

$$\begin{aligned}
& -\frac{b n}{d^4 x} + \frac{b e n}{6 d^3 (d+e x)^2} + \frac{4 b e n}{3 d^4 (d+e x)} + \frac{4 b e n \operatorname{Log}[x]}{3 d^5} - \frac{a+b \operatorname{Log}[c x^n]}{d^4 x} - \\
& \frac{e (a+b \operatorname{Log}[c x^n])}{3 d^2 (d+e x)^3} - \frac{e (a+b \operatorname{Log}[c x^n])}{d^3 (d+e x)^2} + \frac{3 e^2 x (a+b \operatorname{Log}[c x^n])}{d^5 (d+e x)} + \\
& \frac{4 e \operatorname{Log}\left[1+\frac{d}{e x}\right] (a+b \operatorname{Log}[c x^n])}{d^5} - \frac{13 b e n \operatorname{Log}[d+e x]}{3 d^5} - \frac{4 b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^5}
\end{aligned}$$

Result (type 4, 231 leaves, 14 steps):

$$\begin{aligned}
& -\frac{b n}{d^4 x} + \frac{b e n}{6 d^3 (d+e x)^2} + \frac{4 b e n}{3 d^4 (d+e x)} + \frac{4 b e n \operatorname{Log}[x]}{3 d^5} - \frac{a+b \operatorname{Log}[c x^n]}{d^4 x} - \\
& \frac{e (a+b \operatorname{Log}[c x^n])}{3 d^2 (d+e x)^3} - \frac{e (a+b \operatorname{Log}[c x^n])}{d^3 (d+e x)^2} + \frac{3 e^2 x (a+b \operatorname{Log}[c x^n])}{d^5 (d+e x)} - \frac{2 e (a+b \operatorname{Log}[c x^n])^2}{b d^5 n} - \\
& \frac{13 b e n \operatorname{Log}[d+e x]}{3 d^5} + \frac{4 e (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^5} + \frac{4 b e n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^5}
\end{aligned}$$

Problem 61: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x^3 (d+e x)^4} dx$$

Optimal (type 4, 263 leaves, 14 steps):

$$\begin{aligned}
& -\frac{b n}{4 d^4 x^2} + \frac{4 b e n}{d^5 x} - \frac{b e^2 n}{6 d^4 (d+e x)^2} - \frac{11 b e^2 n}{6 d^5 (d+e x)} - \frac{11 b e^2 n \operatorname{Log}[x]}{6 d^6} - \frac{a+b \operatorname{Log}[c x^n]}{2 d^4 x^2} + \\
& \frac{4 e (a+b \operatorname{Log}[c x^n])}{d^5 x} + \frac{e^2 (a+b \operatorname{Log}[c x^n])}{3 d^3 (d+e x)^3} + \frac{3 e^2 (a+b \operatorname{Log}[c x^n])}{2 d^4 (d+e x)^2} - \frac{6 e^3 x (a+b \operatorname{Log}[c x^n])}{d^6 (d+e x)} - \\
& \frac{10 e^2 \operatorname{Log}\left[1+\frac{d}{e x}\right] (a+b \operatorname{Log}[c x^n])}{d^6} + \frac{47 b e^2 n \operatorname{Log}[d+e x]}{6 d^6} + \frac{10 b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^6}
\end{aligned}$$

Result (type 4, 285 leaves, 15 steps):

$$\begin{aligned}
& -\frac{b n}{4 d^4 x^2} + \frac{4 b e n}{d^5 x} - \frac{b e^2 n}{6 d^4 (d+e x)^2} - \frac{11 b e^2 n}{6 d^5 (d+e x)} - \frac{11 b e^2 n \operatorname{Log}[x]}{6 d^6} - \\
& \frac{a+b \operatorname{Log}[c x^n]}{2 d^4 x^2} + \frac{4 e (a+b \operatorname{Log}[c x^n])}{d^5 x} + \frac{e^2 (a+b \operatorname{Log}[c x^n])}{3 d^3 (d+e x)^3} + \frac{3 e^2 (a+b \operatorname{Log}[c x^n])}{2 d^4 (d+e x)^2} - \\
& \frac{6 e^3 x (a+b \operatorname{Log}[c x^n])}{d^6 (d+e x)} + \frac{5 e^2 (a+b \operatorname{Log}[c x^n])^2}{b d^6 n} + \frac{47 b e^2 n \operatorname{Log}[d+e x]}{6 d^6} - \\
& \frac{10 e^2 (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^6} - \frac{10 b e^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^6}
\end{aligned}$$

Problem 62: Result valid but suboptimal antiderivative.

$$\int \frac{x^8 (a+b \operatorname{Log}[c x^n])}{(d+e x)^7} dx$$

Optimal (type 4, 329 leaves, 13 steps):

$$\begin{aligned}
& \frac{28 b d n x}{e^8} - \frac{d (280 a + 341 b n) x}{10 e^8} - \frac{7 b n x^2}{e^7} - \frac{28 b d x \operatorname{Log}[c x^n]}{e^8} - \\
& \frac{x^8 (a+b \operatorname{Log}[c x^n])}{6 e (d+e x)^6} - \frac{x^7 (8 a + b n + 8 b \operatorname{Log}[c x^n])}{30 e^2 (d+e x)^5} - \frac{x^6 (56 a + 15 b n + 56 b \operatorname{Log}[c x^n])}{120 e^3 (d+e x)^4} - \\
& \frac{x^5 (168 a + 73 b n + 168 b \operatorname{Log}[c x^n])}{180 e^4 (d+e x)^3} + \frac{x^2 (280 a + 341 b n + 280 b \operatorname{Log}[c x^n])}{20 e^7} - \\
& \frac{x^4 (840 a + 533 b n + 840 b \operatorname{Log}[c x^n])}{360 e^5 (d+e x)^2} - \frac{x^3 (840 a + 743 b n + 840 b \operatorname{Log}[c x^n])}{90 e^6 (d+e x)} + \\
& \frac{d^2 (280 a + 341 b n + 280 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{10 e^9} + \frac{28 b d^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^9}
\end{aligned}$$

Result (type 4, 394 leaves, 24 steps):

$$\begin{aligned}
& -\frac{7 a d x}{e^8} + \frac{7 b d n x}{e^8} - \frac{b n x^2}{4 e^7} + \frac{b d^7 n}{30 e^9 (d+e x)^5} - \frac{43 b d^6 n}{120 e^9 (d+e x)^4} + \frac{167 b d^5 n}{90 e^9 (d+e x)^3} - \\
& \frac{131 b d^4 n}{20 e^9 (d+e x)^2} + \frac{219 b d^3 n}{10 e^9 (d+e x)} + \frac{219 b d^2 n \operatorname{Log}[x]}{10 e^9} - \frac{7 b d x \operatorname{Log}[c x^n]}{e^8} + \\
& \frac{x^2 (a+b \operatorname{Log}[c x^n])}{2 e^7} - \frac{d^8 (a+b \operatorname{Log}[c x^n])}{6 e^9 (d+e x)^6} + \frac{8 d^7 (a+b \operatorname{Log}[c x^n])}{5 e^9 (d+e x)^5} - \frac{7 d^6 (a+b \operatorname{Log}[c x^n])}{e^9 (d+e x)^4} + \\
& \frac{56 d^5 (a+b \operatorname{Log}[c x^n])}{3 e^9 (d+e x)^3} - \frac{35 d^4 (a+b \operatorname{Log}[c x^n])}{e^9 (d+e x)^2} - \frac{56 d^2 x (a+b \operatorname{Log}[c x^n])}{e^8 (d+e x)} + \\
& \frac{341 b d^2 n \operatorname{Log}[d+e x]}{10 e^9} + \frac{28 d^2 (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{e^9} + \frac{28 b d^2 n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{e^9}
\end{aligned}$$

**Problem 63: Result valid but suboptimal antiderivative.**

$$\int \frac{x^7 (a+b \operatorname{Log}[c x^n])}{(d+e x)^7} dx$$

Optimal (type 4, 285 leaves, 12 steps):

$$\begin{aligned}
& -\frac{7 b n x}{e^7} + \frac{(140 a+223 b n) x}{20 e^7} + \frac{7 b x \operatorname{Log}[c x^n]}{e^7} - \frac{x^7 (a+b \operatorname{Log}[c x^n])}{6 e (d+e x)^6} - \frac{x^6 (7 a+b n+7 b \operatorname{Log}[c x^n])}{30 e^2 (d+e x)^5} - \\
& \frac{x^5 (42 a+13 b n+42 b \operatorname{Log}[c x^n])}{120 e^3 (d+e x)^4} - \frac{x^2 (140 a+153 b n+140 b \operatorname{Log}[c x^n])}{40 e^6 (d+e x)} - \\
& \frac{x^4 (210 a+107 b n+210 b \operatorname{Log}[c x^n])}{360 e^4 (d+e x)^3} - \frac{x^3 (420 a+319 b n+420 b \operatorname{Log}[c x^n])}{360 e^5 (d+e x)^2} - \\
& \frac{d (140 a+223 b n+140 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{20 e^8} - \frac{7 b d n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{e^8}
\end{aligned}$$

Result (type 4, 351 leaves, 23 steps):

$$\begin{aligned}
& \frac{a x}{e^7} - \frac{b n x}{e^7} - \frac{b d^6 n}{30 e^8 (d+e x)^5} + \frac{37 b d^5 n}{120 e^8 (d+e x)^4} - \frac{241 b d^4 n}{180 e^8 (d+e x)^3} + \frac{153 b d^3 n}{40 e^8 (d+e x)^2} - \\
& \frac{197 b d^2 n}{20 e^8 (d+e x)} - \frac{197 b d n \operatorname{Log}[x]}{20 e^8} + \frac{b x \operatorname{Log}[c x^n]}{e^7} + \frac{d^7 (a+b \operatorname{Log}[c x^n])}{6 e^8 (d+e x)^6} - \frac{7 d^6 (a+b \operatorname{Log}[c x^n])}{5 e^8 (d+e x)^5} + \\
& \frac{21 d^5 (a+b \operatorname{Log}[c x^n])}{4 e^8 (d+e x)^4} - \frac{35 d^4 (a+b \operatorname{Log}[c x^n])}{3 e^8 (d+e x)^3} + \frac{35 d^3 (a+b \operatorname{Log}[c x^n])}{2 e^8 (d+e x)^2} + \frac{21 d x (a+b \operatorname{Log}[c x^n])}{e^7 (d+e x)} - \\
& \frac{223 b d n \operatorname{Log}[d+e x]}{20 e^8} - \frac{7 d (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{e^8} - \frac{7 b d n \operatorname{PolyLog}\left[2,-\frac{e x}{d}\right]}{e^8}
\end{aligned}$$



### Problem 64: Result valid but suboptimal antiderivative.

$$\int \frac{x^6 (a + b \operatorname{Log}[c x^n])}{(d + e x)^7} dx$$

Optimal (type 4, 243 leaves, 8 steps):

$$\begin{aligned} & -\frac{x^6 (a + b \operatorname{Log}[c x^n])}{6 e (d + e x)^6} - \frac{x^5 (6 a + b n + 6 b \operatorname{Log}[c x^n])}{30 e^2 (d + e x)^5} - \\ & \frac{x^2 (20 a + 19 b n + 20 b \operatorname{Log}[c x^n])}{40 e^5 (d + e x)^2} - \frac{x (20 a + 29 b n + 20 b \operatorname{Log}[c x^n])}{20 e^6 (d + e x)} - \\ & \frac{x^4 (30 a + 11 b n + 30 b \operatorname{Log}[c x^n])}{120 e^3 (d + e x)^4} - \frac{x^3 (60 a + 37 b n + 60 b \operatorname{Log}[c x^n])}{180 e^4 (d + e x)^3} + \\ & \frac{(20 a + 49 b n + 20 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{20 e^7} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^7} \end{aligned}$$

Result (type 4, 316 leaves, 21 steps):

$$\begin{aligned} & \frac{b d^5 n}{30 e^7 (d + e x)^5} - \frac{31 b d^4 n}{120 e^7 (d + e x)^4} + \frac{163 b d^3 n}{180 e^7 (d + e x)^3} - \frac{79 b d^2 n}{40 e^7 (d + e x)^2} + \frac{71 b d n}{20 e^7 (d + e x)} + \\ & \frac{71 b n \operatorname{Log}[x]}{20 e^7} - \frac{d^6 (a + b \operatorname{Log}[c x^n])}{6 e^7 (d + e x)^6} + \frac{6 d^5 (a + b \operatorname{Log}[c x^n])}{5 e^7 (d + e x)^5} - \frac{15 d^4 (a + b \operatorname{Log}[c x^n])}{4 e^7 (d + e x)^4} + \\ & \frac{20 d^3 (a + b \operatorname{Log}[c x^n])}{3 e^7 (d + e x)^3} - \frac{15 d^2 (a + b \operatorname{Log}[c x^n])}{2 e^7 (d + e x)^2} - \frac{6 x (a + b \operatorname{Log}[c x^n])}{e^6 (d + e x)} + \\ & \frac{49 b n \operatorname{Log}[d + e x]}{20 e^7} + \frac{(a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^7} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^7} \end{aligned}$$

### Problem 71: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x (d + e x)^7} dx$$

Optimal (type 4, 294 leaves, 25 steps):

$$\begin{aligned} & -\frac{b n}{30 d^2 (d + e x)^5} - \frac{11 b n}{120 d^3 (d + e x)^4} - \frac{37 b n}{180 d^4 (d + e x)^3} - \frac{19 b n}{40 d^5 (d + e x)^2} - \\ & \frac{29 b n}{20 d^6 (d + e x)} - \frac{29 b n \operatorname{Log}[x]}{20 d^7} + \frac{a + b \operatorname{Log}[c x^n]}{6 d (d + e x)^6} + \frac{a + b \operatorname{Log}[c x^n]}{5 d^2 (d + e x)^5} + \\ & \frac{a + b \operatorname{Log}[c x^n]}{4 d^3 (d + e x)^4} + \frac{a + b \operatorname{Log}[c x^n]}{3 d^4 (d + e x)^3} + \frac{a + b \operatorname{Log}[c x^n]}{2 d^5 (d + e x)^2} - \frac{e x (a + b \operatorname{Log}[c x^n])}{d^7 (d + e x)} - \\ & \frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^7} + \frac{49 b n \operatorname{Log}[d + e x]}{20 d^7} + \frac{b n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^7} \end{aligned}$$

Result (type 4, 316 leaves, 27 steps):

$$\begin{aligned}
 & -\frac{b n}{30 d^2 (d+e x)^5} - \frac{11 b n}{120 d^3 (d+e x)^4} - \frac{37 b n}{180 d^4 (d+e x)^3} - \frac{19 b n}{40 d^5 (d+e x)^2} - \\
 & \frac{29 b n}{20 d^6 (d+e x)} - \frac{29 b n \operatorname{Log}[x]}{20 d^7} + \frac{a+b \operatorname{Log}[c x^n]}{6 d (d+e x)^6} + \frac{a+b \operatorname{Log}[c x^n]}{5 d^2 (d+e x)^5} + \frac{a+b \operatorname{Log}[c x^n]}{4 d^3 (d+e x)^4} + \\
 & \frac{a+b \operatorname{Log}[c x^n]}{3 d^4 (d+e x)^3} + \frac{a+b \operatorname{Log}[c x^n]}{2 d^5 (d+e x)^2} - \frac{e x (a+b \operatorname{Log}[c x^n])}{d^7 (d+e x)} + \frac{(a+b \operatorname{Log}[c x^n])^2}{2 b d^7 n} + \\
 & \frac{49 b n \operatorname{Log}[d+e x]}{20 d^7} - \frac{(a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^7} - \frac{b n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^7}
 \end{aligned}$$

Problem 72: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{Log}[c x^n]}{x^2 (d+e x)^7} dx$$

Optimal (type 4, 339 leaves, 22 steps):

$$\begin{aligned}
 & -\frac{b n}{d^7 x} + \frac{b e n}{30 d^3 (d+e x)^5} + \frac{17 b e n}{120 d^4 (d+e x)^4} + \frac{79 b e n}{180 d^5 (d+e x)^3} + \frac{53 b e n}{40 d^6 (d+e x)^2} + \frac{103 b e n}{20 d^7 (d+e x)} + \\
 & \frac{103 b e n \operatorname{Log}[x]}{20 d^8} - \frac{a+b \operatorname{Log}[c x^n]}{d^7 x} - \frac{e (a+b \operatorname{Log}[c x^n])}{6 d^2 (d+e x)^6} - \frac{2 e (a+b \operatorname{Log}[c x^n])}{5 d^3 (d+e x)^5} - \\
 & \frac{3 e (a+b \operatorname{Log}[c x^n])}{4 d^4 (d+e x)^4} - \frac{4 e (a+b \operatorname{Log}[c x^n])}{3 d^5 (d+e x)^3} - \frac{5 e (a+b \operatorname{Log}[c x^n])}{2 d^6 (d+e x)^2} + \frac{6 e^2 x (a+b \operatorname{Log}[c x^n])}{d^8 (d+e x)} + \\
 & \frac{7 e \operatorname{Log}\left[1+\frac{d}{e x}\right] (a+b \operatorname{Log}[c x^n])}{d^8} - \frac{223 b e n \operatorname{Log}[d+e x]}{20 d^8} - \frac{7 b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^8}
 \end{aligned}$$

Result (type 4, 361 leaves, 23 steps):

$$\begin{aligned}
 & -\frac{b n}{d^7 x} + \frac{b e n}{30 d^3 (d+e x)^5} + \frac{17 b e n}{120 d^4 (d+e x)^4} + \frac{79 b e n}{180 d^5 (d+e x)^3} + \frac{53 b e n}{40 d^6 (d+e x)^2} + \\
 & \frac{103 b e n}{20 d^7 (d+e x)} + \frac{103 b e n \operatorname{Log}[x]}{20 d^8} - \frac{a+b \operatorname{Log}[c x^n]}{d^7 x} - \frac{e (a+b \operatorname{Log}[c x^n])}{6 d^2 (d+e x)^6} - \\
 & \frac{2 e (a+b \operatorname{Log}[c x^n])}{5 d^3 (d+e x)^5} - \frac{3 e (a+b \operatorname{Log}[c x^n])}{4 d^4 (d+e x)^4} - \frac{4 e (a+b \operatorname{Log}[c x^n])}{3 d^5 (d+e x)^3} - \\
 & \frac{5 e (a+b \operatorname{Log}[c x^n])}{2 d^6 (d+e x)^2} + \frac{6 e^2 x (a+b \operatorname{Log}[c x^n])}{d^8 (d+e x)} - \frac{7 e (a+b \operatorname{Log}[c x^n])^2}{2 b d^8 n} - \\
 & \frac{223 b e n \operatorname{Log}[d+e x]}{20 d^8} + \frac{7 e (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^8} + \frac{7 b e n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^8}
 \end{aligned}$$

### Problem 73: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x)^7} dx$$

Optimal (type 4, 401 leaves, 23 steps):

$$\begin{aligned} & -\frac{b n}{4 d^7 x^2} + \frac{7 b e n}{d^8 x} - \frac{b e^2 n}{30 d^4 (d + e x)^5} - \frac{23 b e^2 n}{120 d^5 (d + e x)^4} - \frac{34 b e^2 n}{45 d^6 (d + e x)^3} - \frac{14 b e^2 n}{5 d^7 (d + e x)^2} - \\ & \frac{131 b e^2 n}{10 d^8 (d + e x)} - \frac{131 b e^2 n \operatorname{Log}[x]}{10 d^9} - \frac{a + b \operatorname{Log}[c x^n]}{2 d^7 x^2} + \frac{7 e (a + b \operatorname{Log}[c x^n])}{d^8 x} + \\ & \frac{e^2 (a + b \operatorname{Log}[c x^n])}{6 d^3 (d + e x)^6} + \frac{3 e^2 (a + b \operatorname{Log}[c x^n])}{5 d^4 (d + e x)^5} + \frac{3 e^2 (a + b \operatorname{Log}[c x^n])}{2 d^5 (d + e x)^4} + \\ & \frac{10 e^2 (a + b \operatorname{Log}[c x^n])}{3 d^6 (d + e x)^3} + \frac{15 e^2 (a + b \operatorname{Log}[c x^n])}{2 d^7 (d + e x)^2} - \frac{21 e^3 x (a + b \operatorname{Log}[c x^n])}{d^9 (d + e x)} - \\ & \frac{28 e^2 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^9} + \frac{341 b e^2 n \operatorname{Log}[d + e x]}{10 d^9} + \frac{28 b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^9} \end{aligned}$$

Result (type 4, 423 leaves, 24 steps):

$$\begin{aligned} & -\frac{b n}{4 d^7 x^2} + \frac{7 b e n}{d^8 x} - \frac{b e^2 n}{30 d^4 (d + e x)^5} - \frac{23 b e^2 n}{120 d^5 (d + e x)^4} - \frac{34 b e^2 n}{45 d^6 (d + e x)^3} - \frac{14 b e^2 n}{5 d^7 (d + e x)^2} - \\ & \frac{131 b e^2 n}{10 d^8 (d + e x)} - \frac{131 b e^2 n \operatorname{Log}[x]}{10 d^9} - \frac{a + b \operatorname{Log}[c x^n]}{2 d^7 x^2} + \frac{7 e (a + b \operatorname{Log}[c x^n])}{d^8 x} + \\ & \frac{e^2 (a + b \operatorname{Log}[c x^n])}{6 d^3 (d + e x)^6} + \frac{3 e^2 (a + b \operatorname{Log}[c x^n])}{5 d^4 (d + e x)^5} + \frac{3 e^2 (a + b \operatorname{Log}[c x^n])}{2 d^5 (d + e x)^4} + \frac{10 e^2 (a + b \operatorname{Log}[c x^n])}{3 d^6 (d + e x)^3} + \\ & \frac{15 e^2 (a + b \operatorname{Log}[c x^n])}{2 d^7 (d + e x)^2} - \frac{21 e^3 x (a + b \operatorname{Log}[c x^n])}{d^9 (d + e x)} + \frac{14 e^2 (a + b \operatorname{Log}[c x^n])^2}{b d^9 n} + \\ & \frac{341 b e^2 n \operatorname{Log}[d + e x]}{10 d^9} - \frac{28 e^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^9} - \frac{28 b e^2 n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^9} \end{aligned}$$

### Problem 86: Result valid but suboptimal antiderivative.

$$\int (d + e x)^2 (a + b \operatorname{Log}[c x^n])^2 dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$\begin{aligned} & 2 b^2 d^2 n^2 x + \frac{1}{2} b^2 d e n^2 x^2 + \frac{2}{27} b^2 e^2 n^2 x^3 + \frac{b^2 d^3 n^2 \operatorname{Log}[x]^2}{3 e} - \\ & 2 b d^2 n x (a + b \operatorname{Log}[c x^n]) - b d e n x^2 (a + b \operatorname{Log}[c x^n]) - \frac{2}{9} b e^2 n x^3 (a + b \operatorname{Log}[c x^n]) - \\ & \frac{2 b d^3 n \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{3 e} + \frac{(d + e x)^3 (a + b \operatorname{Log}[c x^n])^2}{3 e} \end{aligned}$$

Result (type 3, 141 leaves, 5 steps):

$$2 b^2 d^2 n^2 x + \frac{1}{2} b^2 d e n^2 x^2 + \frac{2}{27} b^2 e^2 n^2 x^3 + \frac{b^2 d^3 n^2 \operatorname{Log}[x]^2}{3 e} -$$

$$\frac{b n (18 d^2 e x + 9 d e^2 x^2 + 2 e^3 x^3 + 6 d^3 \operatorname{Log}[x]) (a + b \operatorname{Log}[c x^n])}{9 e} + \frac{(d + e x)^3 (a + b \operatorname{Log}[c x^n])^2}{3 e}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x (d + e x)} dx$$

Optimal (type 4, 79 leaves, 3 steps):

$$- \frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^2}{d} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d}$$

Result (type 4, 98 leaves, 6 steps):

$$\frac{(a + b \operatorname{Log}[c x^n])^3}{3 b d n} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} -$$

$$\frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d}$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x^2 (d + e x)} dx$$

Optimal (type 4, 135 leaves, 6 steps):

$$- \frac{2 b^2 n^2}{d x} - \frac{2 b n (a + b \operatorname{Log}[c x^n])}{d x} - \frac{(a + b \operatorname{Log}[c x^n])^2}{d x} + \frac{e \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^2}{d^2} -$$

$$\frac{2 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^2} - \frac{2 b^2 e n^2 \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d^2}$$

Result (type 4, 155 leaves, 9 steps):

$$- \frac{2 b^2 n^2}{d x} - \frac{2 b n (a + b \operatorname{Log}[c x^n])}{d x} - \frac{(a + b \operatorname{Log}[c x^n])^2}{d x} -$$

$$\frac{e (a + b \operatorname{Log}[c x^n])^3}{3 b d^2 n} + \frac{e (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} +$$

$$\frac{2 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} - \frac{2 b^2 e n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2}$$

### Problem 98: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x^3 (d + e x)} dx$$

Optimal (type 4, 204 leaves, 9 steps):

$$\begin{aligned} & -\frac{b^2 n^2}{4 d x^2} + \frac{2 b^2 e n^2}{d^2 x} - \frac{b n (a + b \operatorname{Log}[c x^n])}{2 d x^2} + \frac{2 b e n (a + b \operatorname{Log}[c x^n])}{d^2 x} - \\ & \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d x^2} + \frac{e (a + b \operatorname{Log}[c x^n])^2}{d^2 x} - \frac{e^2 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^2}{d^3} + \\ & \frac{2 b e^2 n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^3} + \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d^3} \end{aligned}$$

Result (type 4, 226 leaves, 11 steps):

$$\begin{aligned} & -\frac{b^2 n^2}{4 d x^2} + \frac{2 b^2 e n^2}{d^2 x} - \frac{b n (a + b \operatorname{Log}[c x^n])}{2 d x^2} + \frac{2 b e n (a + b \operatorname{Log}[c x^n])}{d^2 x} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d x^2} + \\ & \frac{e (a + b \operatorname{Log}[c x^n])^2}{d^2 x} + \frac{e^2 (a + b \operatorname{Log}[c x^n])^3}{3 b d^3 n} - \frac{e^2 (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \\ & \frac{2 b e^2 n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} + \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} \end{aligned}$$

### Problem 99: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x^4 (d + e x)} dx$$

Optimal (type 4, 273 leaves, 12 steps):

$$\begin{aligned} & -\frac{2 b^2 n^2}{27 d x^3} + \frac{b^2 e n^2}{4 d^2 x^2} - \frac{2 b^2 e^2 n^2}{d^3 x} - \frac{2 b n (a + b \operatorname{Log}[c x^n])}{9 d x^3} + \\ & \frac{b e n (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} - \frac{2 b e^2 n (a + b \operatorname{Log}[c x^n])}{d^3 x} - \frac{(a + b \operatorname{Log}[c x^n])^2}{3 d x^3} + \\ & \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 d^2 x^2} - \frac{e^2 (a + b \operatorname{Log}[c x^n])^2}{d^3 x} + \frac{e^3 \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^2}{d^4} - \\ & \frac{2 b e^3 n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^4} - \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d^4} \end{aligned}$$

Result (type 4, 295 leaves, 13 steps):

$$\begin{aligned}
& -\frac{2b^2n^2}{27dx^3} + \frac{b^2en^2}{4d^2x^2} - \frac{2b^2e^2n^2}{d^3x} - \frac{2bn(a+b\log[cx^n])}{9dx^3} + \frac{ben(a+b\log[cx^n])}{2d^2x^2} - \\
& \frac{2be^2n(a+b\log[cx^n])}{d^3x} - \frac{(a+b\log[cx^n])^2}{3dx^3} + \frac{e(a+b\log[cx^n])^2}{2d^2x^2} - \\
& \frac{e^2(a+b\log[cx^n])^2}{d^3x} - \frac{e^3(a+b\log[cx^n])^3}{3bd^4n} + \frac{e^3(a+b\log[cx^n])^2\log\left[1+\frac{ex}{d}\right]}{d^4} + \\
& \frac{2be^3n(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^4} - \frac{2b^2e^3n^2\text{PolyLog}\left[3, -\frac{ex}{d}\right]}{d^4}
\end{aligned}$$

**Problem 104: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b\log[cx^n])^2}{x(d+ex)^2} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$\begin{aligned}
& -\frac{ex(a+b\log[cx^n])^2}{d^2(d+ex)} - \frac{\log\left[1+\frac{d}{ex}\right](a+b\log[cx^n])^2}{d^2} + \frac{2bn(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^2} + \\
& \frac{2bn(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{d}{ex}\right]}{d^2} + \frac{2b^2n^2\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^2} + \frac{2b^2n^2\text{PolyLog}\left[3, -\frac{d}{ex}\right]}{d^2}
\end{aligned}$$

Result (type 4, 170 leaves, 10 steps):

$$\begin{aligned}
& -\frac{ex(a+b\log[cx^n])^2}{d^2(d+ex)} + \frac{(a+b\log[cx^n])^3}{3bd^2n} + \frac{2bn(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^2} - \\
& \frac{(a+b\log[cx^n])^2\log\left[1+\frac{ex}{d}\right]}{d^2} + \frac{2b^2n^2\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^2} - \\
& \frac{2bn(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^2} + \frac{2b^2n^2\text{PolyLog}\left[3, -\frac{ex}{d}\right]}{d^2}
\end{aligned}$$

**Problem 105: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b\log[cx^n])^2}{x^2(d+ex)^2} dx$$

Optimal (type 4, 211 leaves, 10 steps):

$$\begin{aligned}
& -\frac{2b^2n^2}{d^2x} - \frac{2bn(a+b\log[cx^n])}{d^2x} - \frac{(a+b\log[cx^n])^2}{d^2x} + \frac{e^2x(a+b\log[cx^n])^2}{d^3(d+ex)} + \\
& \frac{2e\log\left[1+\frac{d}{ex}\right](a+b\log[cx^n])^2}{d^3} - \frac{2ben(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^3} - \\
& \frac{4ben(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{d}{ex}\right]}{d^3} - \frac{2b^2en^2\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^3} - \frac{4b^2en^2\text{PolyLog}\left[3, -\frac{d}{ex}\right]}{d^3}
\end{aligned}$$

Result (type 4, 231 leaves, 12 steps):

$$\begin{aligned}
& -\frac{2b^2n^2}{d^2x} - \frac{2bn(a+b\log[cx^n])}{d^2x} - \frac{(a+b\log[cx^n])^2}{d^2x} + \frac{e^2x(a+b\log[cx^n])^2}{d^3(d+ex)} - \\
& \frac{2e(a+b\log[cx^n])^3}{3bd^3n} - \frac{2ben(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^3} + \frac{2e(a+b\log[cx^n])^2\log\left[1+\frac{ex}{d}\right]}{d^3} - \\
& \frac{2b^2e n^2 \text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^3} + \frac{4ben(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^3} - \frac{4b^2e n^2 \text{PolyLog}\left[3, -\frac{ex}{d}\right]}{d^3}
\end{aligned}$$

Problem 106: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b\log[cx^n])^2}{x^3(d+ex)^2} dx$$

Optimal (type 4, 285 leaves, 12 steps):

$$\begin{aligned}
& -\frac{b^2n^2}{4d^2x^2} + \frac{4b^2en^2}{d^3x} - \frac{bn(a+b\log[cx^n])}{2d^2x^2} + \frac{4ben(a+b\log[cx^n])}{d^3x} - \frac{(a+b\log[cx^n])^2}{2d^2x^2} + \\
& \frac{2e(a+b\log[cx^n])^2}{d^3x} - \frac{e^3x(a+b\log[cx^n])^2}{d^4(d+ex)} - \frac{3e^2\log\left[1+\frac{d}{ex}\right](a+b\log[cx^n])^2}{d^4} + \\
& \frac{2be^2n(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^4} + \frac{6be^2n(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{d}{ex}\right]}{d^4} + \\
& \frac{2b^2e^2n^2\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^4} + \frac{6b^2e^2n^2\text{PolyLog}\left[3, -\frac{d}{ex}\right]}{d^4}
\end{aligned}$$

Result (type 4, 304 leaves, 14 steps):

$$\begin{aligned}
& -\frac{b^2n^2}{4d^2x^2} + \frac{4b^2en^2}{d^3x} - \frac{bn(a+b\log[cx^n])}{2d^2x^2} + \frac{4ben(a+b\log[cx^n])}{d^3x} - \\
& \frac{(a+b\log[cx^n])^2}{2d^2x^2} + \frac{2e(a+b\log[cx^n])^2}{d^3x} - \frac{e^3x(a+b\log[cx^n])^2}{d^4(d+ex)} + \\
& \frac{e^2(a+b\log[cx^n])^3}{bd^4n} + \frac{2be^2n(a+b\log[cx^n])\log\left[1+\frac{ex}{d}\right]}{d^4} - \\
& \frac{3e^2(a+b\log[cx^n])^2\log\left[1+\frac{ex}{d}\right]}{d^4} + \frac{2b^2e^2n^2\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^4} - \\
& \frac{6be^2n(a+b\log[cx^n])\text{PolyLog}\left[2, -\frac{ex}{d}\right]}{d^4} + \frac{6b^2e^2n^2\text{PolyLog}\left[3, -\frac{ex}{d}\right]}{d^4}
\end{aligned}$$

Problem 107: Result optimal but 2 more steps used.

$$\int \frac{x^3(a+b\log[cx^n])^2}{(d+ex)^3} dx$$

Optimal (type 4, 296 leaves, 17 steps):

$$\begin{aligned}
& -\frac{2abnx}{e^3} + \frac{2b^2n^2x}{e^3} - \frac{2b^2nx \operatorname{Log}[cx^n]}{e^3} + \frac{bdnx(a+b \operatorname{Log}[cx^n])}{e^3(d+ex)} - \frac{d(a+b \operatorname{Log}[cx^n])^2}{2e^4} + \\
& \frac{x(a+b \operatorname{Log}[cx^n])^2}{e^3} + \frac{d^3(a+b \operatorname{Log}[cx^n])^2}{2e^4(d+ex)^2} + \frac{3dx(a+b \operatorname{Log}[cx^n])^2}{e^3(d+ex)} - \frac{b^2dn^2 \operatorname{Log}[d+ex]}{e^4} - \\
& \frac{5bdn(a+b \operatorname{Log}[cx^n]) \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^4} - \frac{3d(a+b \operatorname{Log}[cx^n])^2 \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^4} - \\
& \frac{5b^2dn^2 \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^4} - \frac{6bdn(a+b \operatorname{Log}[cx^n]) \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^4} + \frac{6b^2dn^2 \operatorname{PolyLog}\left[3, -\frac{ex}{d}\right]}{e^4}
\end{aligned}$$

Result (type 4, 296 leaves, 19 steps):

$$\begin{aligned}
& -\frac{2abnx}{e^3} + \frac{2b^2n^2x}{e^3} - \frac{2b^2nx \operatorname{Log}[cx^n]}{e^3} + \frac{bdnx(a+b \operatorname{Log}[cx^n])}{e^3(d+ex)} - \frac{d(a+b \operatorname{Log}[cx^n])^2}{2e^4} + \\
& \frac{x(a+b \operatorname{Log}[cx^n])^2}{e^3} + \frac{d^3(a+b \operatorname{Log}[cx^n])^2}{2e^4(d+ex)^2} + \frac{3dx(a+b \operatorname{Log}[cx^n])^2}{e^3(d+ex)} - \frac{b^2dn^2 \operatorname{Log}[d+ex]}{e^4} - \\
& \frac{5bdn(a+b \operatorname{Log}[cx^n]) \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^4} - \frac{3d(a+b \operatorname{Log}[cx^n])^2 \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^4} - \\
& \frac{5b^2dn^2 \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^4} - \frac{6bdn(a+b \operatorname{Log}[cx^n]) \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^4} + \frac{6b^2dn^2 \operatorname{PolyLog}\left[3, -\frac{ex}{d}\right]}{e^4}
\end{aligned}$$

**Problem 108: Result optimal but 2 more steps used.**

$$\int \frac{x^2(a+b \operatorname{Log}[cx^n])^2}{(d+ex)^3} dx$$

Optimal (type 4, 232 leaves, 14 steps):

$$\begin{aligned}
& -\frac{bdnx(a+b \operatorname{Log}[cx^n])}{e^2(d+ex)} + \frac{(a+b \operatorname{Log}[cx^n])^2}{2e^3} - \frac{d^2(a+b \operatorname{Log}[cx^n])^2}{2e^3(d+ex)^2} - \frac{2x(a+b \operatorname{Log}[cx^n])^2}{e^2(d+ex)} + \\
& \frac{b^2n^2 \operatorname{Log}[d+ex]}{e^3} + \frac{3bn(a+b \operatorname{Log}[cx^n]) \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^3} + \frac{(a+b \operatorname{Log}[cx^n])^2 \operatorname{Log}\left[1+\frac{ex}{d}\right]}{e^3} + \\
& \frac{3b^2n^2 \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^3} + \frac{2bn(a+b \operatorname{Log}[cx^n]) \operatorname{PolyLog}\left[2, -\frac{ex}{d}\right]}{e^3} - \frac{2b^2n^2 \operatorname{PolyLog}\left[3, -\frac{ex}{d}\right]}{e^3}
\end{aligned}$$

Result (type 4, 232 leaves, 16 steps):



$$\begin{aligned}
& - \frac{b n x (a + b \operatorname{Log}[c x^n])}{e^2 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])^2}{2 e^3 (d + e x)^2} - \frac{2 x (a + b \operatorname{Log}[c x^n])^2}{e^2 (d + e x)} + \\
& \frac{b^2 n^2 \operatorname{Log}[d + e x]}{e^3} + \frac{3 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^3} + \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^3} + \\
& \frac{3 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^3} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{e^3}
\end{aligned}$$

**Problem 109: Result valid but suboptimal antiderivative.**

$$\int \frac{x (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^3} dx$$

Optimal (type 4, 112 leaves, 4 steps):

$$\begin{aligned}
& \frac{b n x (a + b \operatorname{Log}[c x^n])}{d e (d + e x)} + \frac{x^2 (a + b \operatorname{Log}[c x^n])^2}{2 d (d + e x)^2} - \\
& \frac{b n (a + b n + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d e^2} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d e^2}
\end{aligned}$$

Result (type 4, 176 leaves, 13 steps):

$$\begin{aligned}
& \frac{b n x (a + b \operatorname{Log}[c x^n])}{d e (d + e x)} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d e^2} + \frac{d (a + b \operatorname{Log}[c x^n])^2}{2 e^2 (d + e x)^2} + \frac{x (a + b \operatorname{Log}[c x^n])^2}{d e (d + e x)} - \\
& \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d e^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d e^2} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d e^2}
\end{aligned}$$

**Problem 110: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{(d + e x)^3} dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b n x (a + b \operatorname{Log}[c x^n])}{d^2 (d + e x)} - \frac{b n \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{d^2 e} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e (d + e x)^2} + \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^2 e} + \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^2 e}
\end{aligned}$$

Result (type 4, 145 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b n x (a + b \operatorname{Log}[c x^n])}{d^2 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d^2 e} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e (d + e x)^2} + \\
& \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^2 e} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2 e} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e}
\end{aligned}$$

### Problem 111: Result optimal but 5 more steps used.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x (d + e x)^3} dx$$

Optimal (type 4, 257 leaves, 14 steps):

$$\begin{aligned} & \frac{b e n x (a + b \operatorname{Log}[c x^n])}{d^3 (d + e x)} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d^3} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d (d + e x)^2} - \\ & \frac{e x (a + b \operatorname{Log}[c x^n])^2}{d^3 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^3}{3 b d^3 n} - \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^3} + \\ & \frac{3 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} + \\ & \frac{3 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} \end{aligned}$$

Result (type 4, 257 leaves, 19 steps):

$$\begin{aligned} & \frac{b e n x (a + b \operatorname{Log}[c x^n])}{d^3 (d + e x)} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d^3} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d (d + e x)^2} - \\ & \frac{e x (a + b \operatorname{Log}[c x^n])^2}{d^3 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^3}{3 b d^3 n} - \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^3} + \\ & \frac{3 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} + \\ & \frac{3 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} \end{aligned}$$

### Problem 112: Result optimal but 4 more steps used.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x^2 (d + e x)^3} dx$$

Optimal (type 4, 322 leaves, 16 steps):

$$\begin{aligned} & -\frac{2 b^2 n^2}{d^3 x} - \frac{2 b n (a + b \operatorname{Log}[c x^n])}{d^3 x} - \frac{b e^2 n x (a + b \operatorname{Log}[c x^n])}{d^4 (d + e x)} + \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 d^4} - \\ & \frac{(a + b \operatorname{Log}[c x^n])^2}{d^3 x} - \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 d^2 (d + e x)^2} + \frac{2 e^2 x (a + b \operatorname{Log}[c x^n])^2}{d^4 (d + e x)} - \frac{e (a + b \operatorname{Log}[c x^n])^3}{b d^4 n} + \\ & \frac{b^2 e n^2 \operatorname{Log}[d + e x]}{d^4} - \frac{5 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^4} + \frac{3 e (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^4} - \\ & \frac{5 b^2 e n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4} + \frac{6 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4} - \frac{6 b^2 e n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^4} \end{aligned}$$

Result (type 4, 322 leaves, 20 steps):

$$\begin{aligned}
& -\frac{2b^2n^2}{d^3x} - \frac{2bn(a+b\log[cx^n])}{d^3x} - \frac{be^2nx(a+b\log[cx^n])}{d^4(d+ex)} + \frac{e(a+b\log[cx^n])^2}{2d^4} - \\
& \frac{(a+b\log[cx^n])^2}{d^3x} - \frac{e(a+b\log[cx^n])^2}{2d^2(d+ex)^2} + \frac{2e^2x(a+b\log[cx^n])^2}{d^4(d+ex)} - \frac{e(a+b\log[cx^n])^3}{bd^4n} + \\
& \frac{b^2en^2\log[d+ex]}{d^4} - \frac{5ben(a+b\log[cx^n])\log[1+\frac{ex}{d}]}{d^4} + \frac{3e(a+b\log[cx^n])^2\log[1+\frac{ex}{d}]}{d^4} - \\
& \frac{5b^2en^2\text{PolyLog}[2, -\frac{ex}{d}]}{d^4} + \frac{6ben(a+b\log[cx^n])\text{PolyLog}[2, -\frac{ex}{d}]}{d^4} - \frac{6b^2en^2\text{PolyLog}[3, -\frac{ex}{d}]}{d^4}
\end{aligned}$$

Problem 113: Result optimal but 4 more steps used.

$$\int \frac{x^4 (a+b\log[cx^n])^2}{(d+ex)^4} dx$$

Optimal (type 4, 398 leaves, 27 steps):

$$\begin{aligned}
& -\frac{2abnx}{e^4} + \frac{2b^2n^2x}{e^4} - \frac{b^2d^2n^2}{3e^5(d+ex)} - \frac{b^2dn^2\log[x]}{3e^5} - \frac{2b^2nx\log[cx^n]}{e^4} + \\
& \frac{bd^3n(a+b\log[cx^n])}{3e^5(d+ex)^2} + \frac{10bdnx(a+b\log[cx^n])}{3e^4(d+ex)} - \frac{5d(a+b\log[cx^n])^2}{3e^5} + \\
& \frac{x(a+b\log[cx^n])^2}{e^4} - \frac{d^4(a+b\log[cx^n])^2}{3e^5(d+ex)^3} + \frac{2d^3(a+b\log[cx^n])^2}{e^5(d+ex)^2} + \frac{6dx(a+b\log[cx^n])^2}{e^4(d+ex)} - \\
& \frac{3b^2dn^2\log[d+ex]}{e^5} - \frac{26bdn(a+b\log[cx^n])\log[1+\frac{ex}{d}]}{3e^5} - \frac{4d(a+b\log[cx^n])^2\log[1+\frac{ex}{d}]}{e^5} - \\
& \frac{26b^2dn^2\text{PolyLog}[2, -\frac{ex}{d}]}{3e^5} - \frac{8bdn(a+b\log[cx^n])\text{PolyLog}[2, -\frac{ex}{d}]}{e^5} + \frac{8b^2dn^2\text{PolyLog}[3, -\frac{ex}{d}]}{e^5}
\end{aligned}$$

Result (type 4, 398 leaves, 31 steps):

$$\begin{aligned}
& -\frac{2abnx}{e^4} + \frac{2b^2n^2x}{e^4} - \frac{b^2d^2n^2}{3e^5(d+ex)} - \frac{b^2dn^2\log[x]}{3e^5} - \frac{2b^2nx\log[cx^n]}{e^4} + \\
& \frac{bd^3n(a+b\log[cx^n])}{3e^5(d+ex)^2} + \frac{10bdnx(a+b\log[cx^n])}{3e^4(d+ex)} - \frac{5d(a+b\log[cx^n])^2}{3e^5} + \\
& \frac{x(a+b\log[cx^n])^2}{e^4} - \frac{d^4(a+b\log[cx^n])^2}{3e^5(d+ex)^3} + \frac{2d^3(a+b\log[cx^n])^2}{e^5(d+ex)^2} + \frac{6dx(a+b\log[cx^n])^2}{e^4(d+ex)} - \\
& \frac{3b^2dn^2\log[d+ex]}{e^5} - \frac{26bdn(a+b\log[cx^n])\log[1+\frac{ex}{d}]}{3e^5} - \frac{4d(a+b\log[cx^n])^2\log[1+\frac{ex}{d}]}{e^5} - \\
& \frac{26b^2dn^2\text{PolyLog}[2, -\frac{ex}{d}]}{3e^5} - \frac{8bdn(a+b\log[cx^n])\text{PolyLog}[2, -\frac{ex}{d}]}{e^5} + \frac{8b^2dn^2\text{PolyLog}[3, -\frac{ex}{d}]}{e^5}
\end{aligned}$$

### Problem 114: Result optimal but 4 more steps used.

$$\int \frac{x^3 (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 333 leaves, 24 steps):

$$\begin{aligned} & \frac{b^2 d n^2}{3 e^4 (d + e x)} + \frac{b^2 n^2 \operatorname{Log}[x]}{3 e^4} - \frac{b d^2 n (a + b \operatorname{Log}[c x^n])}{3 e^4 (d + e x)^2} - \frac{7 b n x (a + b \operatorname{Log}[c x^n])}{3 e^3 (d + e x)} + \\ & \frac{7 (a + b \operatorname{Log}[c x^n])^2}{6 e^4} + \frac{d^3 (a + b \operatorname{Log}[c x^n])^2}{3 e^4 (d + e x)^3} - \frac{3 d^2 (a + b \operatorname{Log}[c x^n])^2}{2 e^4 (d + e x)^2} - \frac{3 x (a + b \operatorname{Log}[c x^n])^2}{e^3 (d + e x)} + \\ & \frac{2 b^2 n^2 \operatorname{Log}[d + e x]}{e^4} + \frac{11 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 e^4} + \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} + \\ & \frac{11 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 e^4} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

Result (type 4, 333 leaves, 28 steps):

$$\begin{aligned} & \frac{b^2 d n^2}{3 e^4 (d + e x)} + \frac{b^2 n^2 \operatorname{Log}[x]}{3 e^4} - \frac{b d^2 n (a + b \operatorname{Log}[c x^n])}{3 e^4 (d + e x)^2} - \frac{7 b n x (a + b \operatorname{Log}[c x^n])}{3 e^3 (d + e x)} + \\ & \frac{7 (a + b \operatorname{Log}[c x^n])^2}{6 e^4} + \frac{d^3 (a + b \operatorname{Log}[c x^n])^2}{3 e^4 (d + e x)^3} - \frac{3 d^2 (a + b \operatorname{Log}[c x^n])^2}{2 e^4 (d + e x)^2} - \frac{3 x (a + b \operatorname{Log}[c x^n])^2}{e^3 (d + e x)} + \\ & \frac{2 b^2 n^2 \operatorname{Log}[d + e x]}{e^4} + \frac{11 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 e^4} + \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{e^4} + \\ & \frac{11 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 e^4} + \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{e^4} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{e^4} \end{aligned}$$

### Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 161 leaves, 5 steps):

$$\begin{aligned} & \frac{b n x^2 (a + b \operatorname{Log}[c x^n])}{3 d e (d + e x)^2} + \frac{x^3 (a + b \operatorname{Log}[c x^n])^2}{3 d (d + e x)^3} + \frac{b n x (2 a + b n + 2 b \operatorname{Log}[c x^n])}{3 d e^2 (d + e x)} - \\ & \frac{b n (2 a + 3 b n + 2 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d e^3} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d e^3} \end{aligned}$$

Result (type 4, 274 leaves, 25 steps):

$$\begin{aligned}
& - \frac{b^2 n^2}{3 e^3 (d + e x)} - \frac{b^2 n^2 \operatorname{Log}[x]}{3 d e^3} + \frac{b d n (a + b \operatorname{Log}[c x^n])}{3 e^3 (d + e x)^2} + \frac{4 b n x (a + b \operatorname{Log}[c x^n])}{3 d e^2 (d + e x)} - \\
& \frac{2 (a + b \operatorname{Log}[c x^n])^2}{3 d e^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])^2}{3 e^3 (d + e x)^3} + \frac{d (a + b \operatorname{Log}[c x^n])^2}{e^3 (d + e x)^2} + \frac{x (a + b \operatorname{Log}[c x^n])^2}{d e^2 (d + e x)} - \\
& \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d e^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d e^3} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d e^3}
\end{aligned}$$

**Problem 116: Result valid but suboptimal antiderivative.**

$$\int \frac{x (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 210 leaves, 8 steps):

$$\begin{aligned}
& \frac{b^2 n^2}{3 d e^2 (d + e x)} - \frac{b n (a + b \operatorname{Log}[c x^n])}{3 e^2 (d + e x)^2} + \frac{b n (a + b \operatorname{Log}[c x^n])}{3 d e^2 (d + e x)} + \\
& \frac{(a + b \operatorname{Log}[c x^n])^2}{6 d^2 e^2} + \frac{d (a + b \operatorname{Log}[c x^n])^2}{3 e^2 (d + e x)^3} - \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e^2 (d + e x)^2} - \\
& \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^2 e^2} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^2 e^2}
\end{aligned}$$

Result (type 4, 229 leaves, 22 steps):

$$\begin{aligned}
& \frac{b^2 n^2}{3 d e^2 (d + e x)} + \frac{b^2 n^2 \operatorname{Log}[x]}{3 d^2 e^2} - \frac{b n (a + b \operatorname{Log}[c x^n])}{3 e^2 (d + e x)^2} - \\
& \frac{b n x (a + b \operatorname{Log}[c x^n])}{3 d^2 e (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^2}{6 d^2 e^2} + \frac{d (a + b \operatorname{Log}[c x^n])^2}{3 e^2 (d + e x)^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2}{2 e^2 (d + e x)^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^2 e^2} - \frac{b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^2 e^2}
\end{aligned}$$

**Problem 117: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{(d + e x)^4} dx$$

Optimal (type 4, 203 leaves, 10 steps):

$$\begin{aligned}
& - \frac{b^2 n^2}{3 d^2 e (d + e x)} - \frac{b^2 n^2 \operatorname{Log}[x]}{3 d^3 e} + \frac{b n (a + b \operatorname{Log}[c x^n])}{3 d e (d + e x)^2} - \\
& \frac{2 b n x (a + b \operatorname{Log}[c x^n])}{3 d^3 (d + e x)} - \frac{2 b n \operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])}{3 d^3 e} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2}{3 e (d + e x)^3} + \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^3 e} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{3 d^3 e}
\end{aligned}$$

Result (type 4, 221 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b^2 n^2}{3 d^2 e (d + e x)} - \frac{b^2 n^2 \operatorname{Log}[x]}{3 d^3 e} + \frac{b n (a + b \operatorname{Log}[c x^n])}{3 d e (d + e x)^2} - \\
& \frac{2 b n x (a + b \operatorname{Log}[c x^n])}{3 d^3 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^2}{3 d^3 e} - \frac{(a + b \operatorname{Log}[c x^n])^2}{3 e (d + e x)^3} + \\
& \frac{b^2 n^2 \operatorname{Log}[d + e x]}{d^3 e} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^3 e} - \frac{2 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^3 e}
\end{aligned}$$

Problem 118: Result optimal but 7 more steps used.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2}{x (d + e x)^4} dx$$

Optimal (type 4, 351 leaves, 25 steps):

$$\begin{aligned}
& \frac{b^2 n^2}{3 d^3 (d + e x)} + \frac{b^2 n^2 \operatorname{Log}[x]}{3 d^4} - \frac{b n (a + b \operatorname{Log}[c x^n])}{3 d^2 (d + e x)^2} + \frac{5 b e n x (a + b \operatorname{Log}[c x^n])}{3 d^4 (d + e x)} - \\
& \frac{5 (a + b \operatorname{Log}[c x^n])^2}{6 d^4} + \frac{(a + b \operatorname{Log}[c x^n])^2}{3 d (d + e x)^3} + \frac{(a + b \operatorname{Log}[c x^n])^2}{2 d^2 (d + e x)^2} - \frac{e x (a + b \operatorname{Log}[c x^n])^2}{d^4 (d + e x)} + \\
& \frac{(a + b \operatorname{Log}[c x^n])^3}{3 b d^4 n} - \frac{2 b^2 n^2 \operatorname{Log}[d + e x]}{d^4} + \frac{11 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^4} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^4} + \frac{11 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^4} - \\
& \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^4}
\end{aligned}$$

Result (type 4, 351 leaves, 32 steps):

$$\begin{aligned}
& \frac{b^2 n^2}{3 d^3 (d+e x)} + \frac{b^2 n^2 \operatorname{Log}[x]}{3 d^4} - \frac{b n (a+b \operatorname{Log}[c x^n])}{3 d^2 (d+e x)^2} + \frac{5 b e n x (a+b \operatorname{Log}[c x^n])}{3 d^4 (d+e x)} - \\
& \frac{5 (a+b \operatorname{Log}[c x^n])^2}{6 d^4} + \frac{(a+b \operatorname{Log}[c x^n])^2}{3 d (d+e x)^3} + \frac{(a+b \operatorname{Log}[c x^n])^2}{2 d^2 (d+e x)^2} - \frac{e x (a+b \operatorname{Log}[c x^n])^2}{d^4 (d+e x)} + \\
& \frac{(a+b \operatorname{Log}[c x^n])^3}{3 b d^4 n} - \frac{2 b^2 n^2 \operatorname{Log}[d+e x]}{d^4} + \frac{11 b n (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{3 d^4} - \\
& \frac{(a+b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^4} + \frac{11 b^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^4} - \\
& \frac{2 b n (a+b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^4} + \frac{2 b^2 n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^4}
\end{aligned}$$

Problem 119: Result optimal but 6 more steps used.

$$\int \frac{(a+b \operatorname{Log}[c x^n])^2}{x^2 (d+e x)^4} dx$$

Optimal (type 4, 420 leaves, 26 steps):

$$\begin{aligned}
& -\frac{2 b^2 n^2}{d^4 x} - \frac{b^2 e n^2}{3 d^4 (d+e x)} - \frac{b^2 e n^2 \operatorname{Log}[x]}{3 d^5} - \frac{2 b n (a+b \operatorname{Log}[c x^n])}{d^4 x} + \frac{b e n (a+b \operatorname{Log}[c x^n])}{3 d^3 (d+e x)^2} - \\
& \frac{8 b e^2 n x (a+b \operatorname{Log}[c x^n])}{3 d^5 (d+e x)} + \frac{4 e (a+b \operatorname{Log}[c x^n])^2}{3 d^5} - \frac{(a+b \operatorname{Log}[c x^n])^2}{d^4 x} - \frac{e (a+b \operatorname{Log}[c x^n])^2}{3 d^2 (d+e x)^3} - \\
& \frac{e (a+b \operatorname{Log}[c x^n])^2}{d^3 (d+e x)^2} + \frac{3 e^2 x (a+b \operatorname{Log}[c x^n])^2}{d^5 (d+e x)} - \frac{4 e (a+b \operatorname{Log}[c x^n])^3}{3 b d^5 n} + \frac{3 b^2 e n^2 \operatorname{Log}[d+e x]}{d^5} - \\
& \frac{26 b e n (a+b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1+\frac{e x}{d}\right]}{3 d^5} + \frac{4 e (a+b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1+\frac{e x}{d}\right]}{d^5} - \\
& \frac{26 b^2 e n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^5} + \frac{8 b e n (a+b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^5} - \frac{8 b^2 e n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^5}
\end{aligned}$$

Result (type 4, 420 leaves, 32 steps):

$$\begin{aligned}
& -\frac{2b^2n^2}{d^4x} - \frac{b^2en^2}{3d^4(d+ex)} - \frac{b^2en^2\log[x]}{3d^5} - \frac{2bn(a+b\log[cx^n])}{d^4x} + \frac{ben(a+b\log[cx^n])}{3d^3(d+ex)^2} - \\
& \frac{8be^2nx(a+b\log[cx^n])}{3d^5(d+ex)} + \frac{4e(a+b\log[cx^n])^2}{3d^5} - \frac{(a+b\log[cx^n])^2}{d^4x} - \frac{e(a+b\log[cx^n])^2}{3d^2(d+ex)^3} - \\
& \frac{e(a+b\log[cx^n])^2}{d^3(d+ex)^2} + \frac{3e^2x(a+b\log[cx^n])^2}{d^5(d+ex)} - \frac{4e(a+b\log[cx^n])^3}{3bd^5n} + \frac{3b^2en^2\log[d+ex]}{d^5} - \\
& \frac{26ben(a+b\log[cx^n])\log[1+\frac{ex}{d}]}{3d^5} + \frac{4e(a+b\log[cx^n])^2\log[1+\frac{ex}{d}]}{d^5} - \\
& \frac{26b^2en^2\text{PolyLog}[2, -\frac{ex}{d}]}{3d^5} + \frac{8ben(a+b\log[cx^n])\text{PolyLog}[2, -\frac{ex}{d}]}{d^5} - \frac{8b^2en^2\text{PolyLog}[3, -\frac{ex}{d}]}{d^5}
\end{aligned}$$

**Problem 120: Result valid but suboptimal antiderivative.**

$$\int \frac{x \log[x]^2}{(d+ex)^4} dx$$

Optimal (type 4, 107 leaves, 8 steps):

$$-\frac{x}{3d^2e(d+ex)} + \frac{x\log[x]}{3de(d+ex)^2} + \frac{x^2(3d+ex)\log[x]^2}{6d^2(d+ex)^3} - \frac{\log[x]\log[1+\frac{ex}{d}]}{3d^2e^2} - \frac{\text{PolyLog}[2, -\frac{ex}{d}]}{3d^2e^2}$$

Result (type 4, 157 leaves, 22 steps):

$$\begin{aligned}
& \frac{1}{3d^2e(d+ex)} + \frac{\log[x]}{3d^2e^2} - \frac{\log[x]}{3e^2(d+ex)^2} - \frac{x\log[x]}{3d^2e(d+ex)} + \frac{\log[x]^2}{6d^2e^2} + \\
& \frac{d\log[x]^2}{3e^2(d+ex)^3} - \frac{\log[x]^2}{2e^2(d+ex)^2} - \frac{\log[x]\log[1+\frac{ex}{d}]}{3d^2e^2} - \frac{\text{PolyLog}[2, -\frac{ex}{d}]}{3d^2e^2}
\end{aligned}$$

**Problem 121: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b\log[cx^n])^3}{x(d+ex)} dx$$

Optimal (type 4, 113 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\log[1+\frac{d}{ex}](a+b\log[cx^n])^3}{d} + \frac{3bn(a+b\log[cx^n])^2\text{PolyLog}[2, -\frac{d}{ex}]}{d} + \\
& \frac{6b^2n^2(a+b\log[cx^n])\text{PolyLog}[3, -\frac{d}{ex}]}{d} + \frac{6b^3n^3\text{PolyLog}[4, -\frac{d}{ex}]}{d}
\end{aligned}$$

Result (type 4, 130 leaves, 7 steps):



$$\frac{(a + b \operatorname{Log}[c x^n])^4}{4 b d n} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d} +$$

$$\frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right]}{d}$$

Problem 122: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3}{x (d + e x)^2} dx$$

Optimal (type 4, 217 leaves, 9 steps):

$$-\frac{e x (a + b \operatorname{Log}[c x^n])^3}{d^2 (d + e x)} - \frac{\operatorname{Log}\left[1 + \frac{d}{e x}\right] (a + b \operatorname{Log}[c x^n])^3}{d^2} + \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} +$$

$$\frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d^2} + \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} +$$

$$\frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{d}{e x}\right]}{d^2} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2} + \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{d}{e x}\right]}{d^2}$$

Result (type 4, 234 leaves, 12 steps):

$$-\frac{e x (a + b \operatorname{Log}[c x^n])^3}{d^2 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^4}{4 b d^2 n} + \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} -$$

$$\frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} + \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} -$$

$$\frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2} +$$

$$\frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right]}{d^2}$$

Problem 123: Result optimal but 6 more steps used.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3}{x (d + e x)^3} dx$$

Optimal (type 4, 361 leaves, 18 steps):

$$\begin{aligned}
& \frac{3 b e n x (a + b \operatorname{Log}[c x^n])^2}{2 d^3 (d + e x)} - \frac{(a + b \operatorname{Log}[c x^n])^3}{2 d^3} + \frac{(a + b \operatorname{Log}[c x^n])^3}{2 d (d + e x)^2} - \\
& \frac{e x (a + b \operatorname{Log}[c x^n])^3}{d^3 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^4}{4 b d^3 n} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} + \\
& \frac{9 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^3} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \frac{3 b^3 n^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} + \\
& \frac{9 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \\
& \frac{9 b^3 n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} + \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right]}{d^3}
\end{aligned}$$

Result (type 4, 361 leaves, 24 steps):

$$\begin{aligned}
& \frac{3 b e n x (a + b \operatorname{Log}[c x^n])^2}{2 d^3 (d + e x)} - \frac{(a + b \operatorname{Log}[c x^n])^3}{2 d^3} + \frac{(a + b \operatorname{Log}[c x^n])^3}{2 d (d + e x)^2} - \\
& \frac{e x (a + b \operatorname{Log}[c x^n])^3}{d^3 (d + e x)} + \frac{(a + b \operatorname{Log}[c x^n])^4}{4 b d^3 n} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} + \\
& \frac{9 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^3} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^3} - \frac{3 b^3 n^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} + \\
& \frac{9 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^3} - \\
& \frac{9 b^3 n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} + \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^3} - \frac{6 b^3 n^3 \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right]}{d^3}
\end{aligned}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2) (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 52 leaves, 3 steps):

$$-\frac{b d n}{4 x^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{2 x^2} + \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 b n}$$

Result (type 3, 47 leaves, 3 steps):

$$-\frac{b d n}{4 x^2} - \frac{1}{2} b e n \operatorname{Log}[x]^2 - \frac{1}{2} \left( \frac{d}{x^2} - 2 e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

Problem 176: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2) (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 57 leaves, 4 steps) :

$$-\frac{b d n}{16 x^4} - \frac{b e n}{4 x^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{4 x^4} - \frac{e (a + b \operatorname{Log}[c x^n])}{2 x^2}$$

Result (type 3, 47 leaves, 4 steps) :

$$-\frac{b d n}{16 x^4} - \frac{b e n}{4 x^2} - \frac{1}{4} \left( \frac{d}{x^4} + \frac{2 e}{x^2} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 179: Result valid but suboptimal antiderivative.**

$$\int (d + e x^2) (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 48 leaves, 2 steps) :

$$-b d n x - \frac{1}{9} b e n x^3 + d x (a + b \operatorname{Log}[c x^n]) + \frac{1}{3} e x^3 (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 41 leaves, 2 steps) :

$$-b d n x - \frac{1}{9} b e n x^3 + \frac{1}{3} (3 d x + e x^3) (a + b \operatorname{Log}[c x^n])$$

**Problem 180: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2) (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 44 leaves, 2 steps) :

$$-\frac{b d n}{x} - b e n x - \frac{d (a + b \operatorname{Log}[c x^n])}{x} + e x (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 37 leaves, 2 steps) :

$$-\frac{b d n}{x} - b e n x - \left( \frac{d}{x} - e x \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 181: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2) (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 53 leaves, 4 steps) :

$$-\frac{b d n}{9 x^3} - \frac{b e n}{x} - \frac{d (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{e (a + b \operatorname{Log}[c x^n])}{x}$$

Result (type 3, 45 leaves, 4 steps) :

$$-\frac{b d n}{9 x^3} - \frac{b e n}{x} - \frac{1}{3} \left( \frac{d}{x^3} + \frac{3 e}{x} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 182: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2) (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 57 leaves, 4 steps):

$$-\frac{b d n}{25 x^5} - \frac{b e n}{9 x^3} - \frac{d (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{e (a + b \operatorname{Log}[c x^n])}{3 x^3}$$

Result (type 3, 48 leaves, 4 steps):

$$-\frac{b d n}{25 x^5} - \frac{b e n}{9 x^3} - \frac{1}{15} \left( \frac{3 d}{x^5} + \frac{5 e}{x^3} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 186: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 89 leaves, 3 steps):

$$-\frac{1}{2} b d e n x^2 - \frac{1}{16} b e^2 n x^4 - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \\ d e x^2 (a + b \operatorname{Log}[c x^n]) + \frac{1}{4} e^2 x^4 (a + b \operatorname{Log}[c x^n]) + d^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 73 leaves, 3 steps):

$$-\frac{1}{2} b d e n x^2 - \frac{1}{16} b e^2 n x^4 - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \frac{1}{4} (4 d e x^2 + e^2 x^4 + 4 d^2 \operatorname{Log}[x]) (a + b \operatorname{Log}[c x^n])$$

### Problem 187: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$-\frac{b d^2 n}{4 x^2} - \frac{1}{4} b e^2 n x^2 - b d e n \operatorname{Log}[x]^2 - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{2 x^2} + \\ \frac{1}{2} e^2 x^2 (a + b \operatorname{Log}[c x^n]) + 2 d e \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 71 leaves, 7 steps):

$$-\frac{b d^2 n}{4 x^2} - \frac{1}{4} b e^2 n x^2 - b d e n \operatorname{Log}[x]^2 - \frac{1}{2} \left( \frac{d^2}{x^2} - e^2 x^2 - 4 d e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 188: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 90 leaves, 7 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{b d e n}{2 x^2} - \frac{1}{2} b e^2 n \operatorname{Log}[x]^2 - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{4 x^4} - \frac{d e (a + b \operatorname{Log}[c x^n])}{x^2} + e^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 73 leaves, 5 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{b d e n}{2 x^2} - \frac{1}{2} b e^2 n \operatorname{Log}[x]^2 - \frac{1}{4} \left( \frac{d^2}{x^4} + \frac{4 d e}{x^2} - 4 e^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 191: Result valid but suboptimal antiderivative.

$$\int (d + e x^2)^2 (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 86 leaves, 2 steps):

$$-b d^2 n x - \frac{2}{9} b d e n x^3 - \frac{1}{25} b e^2 n x^5 + d^2 x (a + b \operatorname{Log}[c x^n]) + \frac{2}{3} d e x^3 (a + b \operatorname{Log}[c x^n]) + \frac{1}{5} e^2 x^5 (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 68 leaves, 2 steps):

$$-b d^2 n x - \frac{2}{9} b d e n x^3 - \frac{1}{25} b e^2 n x^5 + \frac{1}{15} (15 d^2 x + 10 d e x^3 + 3 e^2 x^5) (a + b \operatorname{Log}[c x^n])$$

### Problem 192: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 83 leaves, 2 steps):

$$-\frac{b d^2 n}{x} - 2 b d e n x - \frac{1}{9} b e^2 n x^3 - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{x} + 2 d e x (a + b \operatorname{Log}[c x^n]) + \frac{1}{3} e^2 x^3 (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 66 leaves, 2 steps):

$$-\frac{b d^2 n}{x} - 2 b d e n x - \frac{1}{9} b e^2 n x^3 - \frac{1}{3} \left( \frac{3 d^2}{x} - 6 d e x - e^2 x^3 \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{b d^2 n}{9 x^3} - \frac{2 b d e n}{x} - b e^2 n x - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{2 d e (a + b \operatorname{Log}[c x^n])}{x} + e^2 x (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 65 leaves, 2 steps):

$$-\frac{b d^2 n}{9 x^3} - \frac{2 b d e n}{x} - b e^2 n x - \frac{1}{3} \left( \frac{d^2}{x^3} + \frac{6 d e}{x} - 3 e^2 x \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{2 b d e n}{9 x^3} - \frac{b e^2 n}{x} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{2 d e (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{e^2 (a + b \operatorname{Log}[c x^n])}{x}$$

Result (type 3, 72 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{2 b d e n}{9 x^3} - \frac{b e^2 n}{x} - \frac{1}{15} \left( \frac{3 d^2}{x^5} + \frac{10 d e}{x^3} + \frac{15 e^2}{x} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 195: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{Log}[c x^n])}{x^8} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$-\frac{b d^2 n}{49 x^7} - \frac{2 b d e n}{25 x^5} - \frac{b e^2 n}{9 x^3} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{7 x^7} - \frac{2 d e (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{e^2 (a + b \operatorname{Log}[c x^n])}{3 x^3}$$

Result (type 3, 74 leaves, 4 steps):

$$-\frac{b d^2 n}{49 x^7} - \frac{2 b d e n}{25 x^5} - \frac{b e^2 n}{9 x^3} - \frac{1}{105} \left( \frac{15 d^2}{x^7} + \frac{42 d e}{x^5} + \frac{35 e^2}{x^3} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 199: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 130 leaves, 5 steps):

$$-\frac{3}{4} b d^2 e n x^2 - \frac{3}{16} b d e^2 n x^4 - \frac{1}{36} b e^3 n x^6 - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 + \frac{3}{2} d^2 e x^2 (a + b \operatorname{Log}[c x^n]) +$$

$$\frac{3}{4} d e^2 x^4 (a + b \operatorname{Log}[c x^n]) + \frac{1}{6} e^3 x^6 (a + b \operatorname{Log}[c x^n]) + d^3 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 100 leaves, 5 steps):

$$-\frac{3}{4} b d^2 e n x^2 - \frac{3}{16} b d e^2 n x^4 - \frac{1}{36} b e^3 n x^6 - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 +$$

$$\frac{1}{12} (18 d^2 e x^2 + 9 d e^2 x^4 + 2 e^3 x^6 + 12 d^3 \operatorname{Log}[x]) (a + b \operatorname{Log}[c x^n])$$

**Problem 200: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b d^3 n}{4 x^2} - \frac{3}{4} b d e^2 n x^2 - \frac{1}{16} b e^3 n x^4 - \frac{3}{2} b d^2 e n \operatorname{Log}[x]^2 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{2 x^2} +$$

$$\frac{3}{2} d e^2 x^2 (a + b \operatorname{Log}[c x^n]) + \frac{1}{4} e^3 x^4 (a + b \operatorname{Log}[c x^n]) + 3 d^2 e \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 100 leaves, 7 steps):

$$-\frac{b d^3 n}{4 x^2} - \frac{3}{4} b d e^2 n x^2 - \frac{1}{16} b e^3 n x^4 - \frac{3}{2} b d^2 e n \operatorname{Log}[x]^2 -$$

$$\frac{1}{4} \left( \frac{2 d^3}{x^2} - 6 d e^2 x^2 - e^3 x^4 - 12 d^2 e \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 201: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b d^3 n}{16 x^4} - \frac{3 b d^2 e n}{4 x^2} - \frac{1}{4} b e^3 n x^2 - \frac{3}{2} b d e^2 n \operatorname{Log}[x]^2 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{4 x^4} -$$

$$\frac{3 d^2 e (a + b \operatorname{Log}[c x^n])}{2 x^2} + \frac{1}{2} e^3 x^2 (a + b \operatorname{Log}[c x^n]) + 3 d e^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 99 leaves, 7 steps):

$$-\frac{b d^3 n}{16 x^4} - \frac{3 b d^2 e n}{4 x^2} - \frac{1}{4} b e^3 n x^2 - \frac{3}{2} b d e^2 n \operatorname{Log}[x]^2 -$$

$$\frac{1}{4} \left( \frac{d^3}{x^4} + \frac{6 d^2 e}{x^2} - 2 e^3 x^2 - 12 d e^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 204: Result valid but suboptimal antiderivative.

$$\int (d + e x^2)^3 (a + b \operatorname{Log}[c x^n]) \, dx$$

Optimal (type 3, 121 leaves, 2 steps):

$$\begin{aligned} & -b d^3 n x - \frac{1}{3} b d^2 e n x^3 - \frac{3}{25} b d e^2 n x^5 - \frac{1}{49} b e^3 n x^7 + d^3 x (a + b \operatorname{Log}[c x^n]) + \\ & d^2 e x^3 (a + b \operatorname{Log}[c x^n]) + \frac{3}{5} d e^2 x^5 (a + b \operatorname{Log}[c x^n]) + \frac{1}{7} e^3 x^7 (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

Result (type 3, 94 leaves, 2 steps):

$$\begin{aligned} & -b d^3 n x - \frac{1}{3} b d^2 e n x^3 - \frac{3}{25} b d e^2 n x^5 - \frac{1}{49} b e^3 n x^7 + \\ & \frac{1}{35} (35 d^3 x + 35 d^2 e x^3 + 21 d e^2 x^5 + 5 e^3 x^7) (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

### Problem 205: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^2} \, dx$$

Optimal (type 3, 118 leaves, 2 steps):

$$\begin{aligned} & -\frac{b d^3 n}{x} - 3 b d^2 e n x - \frac{1}{3} b d e^2 n x^3 - \frac{1}{25} b e^3 n x^5 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{x} + \\ & 3 d^2 e x (a + b \operatorname{Log}[c x^n]) + d e^2 x^3 (a + b \operatorname{Log}[c x^n]) + \frac{1}{5} e^3 x^5 (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

Result (type 3, 92 leaves, 2 steps):

$$\begin{aligned} & -\frac{b d^3 n}{x} - 3 b d^2 e n x - \frac{1}{3} b d e^2 n x^3 - \frac{1}{25} b e^3 n x^5 - \\ & \frac{1}{5} \left( \frac{5 d^3}{x} - 15 d^2 e x - 5 d e^2 x^3 - e^3 x^5 \right) (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

### Problem 206: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^4} \, dx$$

Optimal (type 3, 121 leaves, 3 steps):

$$\begin{aligned} & -\frac{b d^3 n}{9 x^3} - \frac{3 b d^2 e n}{x} - 3 b d e^2 n x - \frac{1}{9} b e^3 n x^3 - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{3 x^3} - \\ & \frac{3 d^2 e (a + b \operatorname{Log}[c x^n])}{x} + 3 d e^2 x (a + b \operatorname{Log}[c x^n]) + \frac{1}{3} e^3 x^3 (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

Result (type 3, 91 leaves, 3 steps):



$$-\frac{b d^3 n}{9 x^3} - \frac{3 b d^2 e n}{x} - 3 b d e^2 n x - \frac{1}{9} b e^3 n x^3 - \frac{1}{3} \left( \frac{d^3}{x^3} + \frac{9 d^2 e}{x} - 9 d e^2 x - e^3 x^3 \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 207: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 118 leaves, 2 steps):

$$-\frac{b d^3 n}{25 x^5} - \frac{b d^2 e n}{3 x^3} - \frac{3 b d e^2 n}{x} - b e^3 n x - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{d^2 e (a + b \operatorname{Log}[c x^n])}{x^3} - \frac{3 d e^2 (a + b \operatorname{Log}[c x^n])}{x} + e^3 x (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 91 leaves, 2 steps):

$$-\frac{b d^3 n}{25 x^5} - \frac{b d^2 e n}{3 x^3} - \frac{3 b d e^2 n}{x} - b e^3 n x - \frac{1}{5} \left( \frac{d^3}{x^5} + \frac{5 d^2 e}{x^3} + \frac{15 d e^2}{x} - 5 e^3 x \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 208: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^8} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b d^3 n}{49 x^7} - \frac{3 b d^2 e n}{25 x^5} - \frac{b d e^2 n}{3 x^3} - \frac{b e^3 n}{x} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{7 x^7} - \frac{3 d^2 e (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{d e^2 (a + b \operatorname{Log}[c x^n])}{x^3} - \frac{e^3 (a + b \operatorname{Log}[c x^n])}{x}$$

Result (type 3, 98 leaves, 4 steps):

$$-\frac{b d^3 n}{49 x^7} - \frac{3 b d^2 e n}{25 x^5} - \frac{b d e^2 n}{3 x^3} - \frac{b e^3 n}{x} - \frac{1}{35} \left( \frac{5 d^3}{x^7} + \frac{21 d^2 e}{x^5} + \frac{35 d e^2}{x^3} + \frac{35 e^3}{x} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 209: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^2)^3 (a + b \operatorname{Log}[c x^n])}{x^{10}} dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$-\frac{b d^3 n}{81 x^9} - \frac{3 b d^2 e n}{49 x^7} - \frac{3 b d e^2 n}{25 x^5} - \frac{b e^3 n}{9 x^3} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{9 x^9} - \frac{3 d^2 e (a + b \operatorname{Log}[c x^n])}{7 x^7} - \frac{3 d e^2 (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{e^3 (a + b \operatorname{Log}[c x^n])}{3 x^3}$$

Result (type 3, 100 leaves, 4 steps):

$$-\frac{b d^3 n}{81 x^9} - \frac{3 b d^2 e n}{49 x^7} - \frac{3 b d e^2 n}{25 x^5} - \frac{b e^3 n}{9 x^3} - \frac{1}{315} \left( \frac{35 d^3}{x^9} + \frac{135 d^2 e}{x^7} + \frac{189 d e^2}{x^5} + \frac{105 e^3}{x^3} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 214: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$-\frac{b n}{4 d x^2} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2} + \frac{e \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 d^2} - \frac{b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^2}$$

Result (type 4, 109 leaves, 6 steps):

$$-\frac{b n}{4 d x^2} - \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2} - \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 b d^2 n} + \frac{e (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 d^2} + \frac{b e n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 d^2}$$

**Problem 215: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^5 (d + e x^2)} dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$-\frac{b n}{16 d x^4} + \frac{b e n}{4 d^2 x^2} - \frac{a + b \operatorname{Log}[c x^n]}{4 d x^4} + \frac{e (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} - \frac{e^2 \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (a + b \operatorname{Log}[c x^n])}{2 d^3} + \frac{b e^2 n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{4 d^3}$$

Result (type 4, 149 leaves, 7 steps):

$$-\frac{b n}{16 d x^4} + \frac{b e n}{4 d^2 x^2} - \frac{a + b \operatorname{Log}[c x^n]}{4 d x^4} + \frac{e (a + b \operatorname{Log}[c x^n])}{2 d^2 x^2} + \frac{e^2 (a + b \operatorname{Log}[c x^n])^2}{2 b d^3 n} - \frac{e^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{2 d^3} - \frac{b e^2 n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{4 d^3}$$

**Problem 219: Result optimal but 1 more steps used.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x^2)} dx$$

Optimal (type 4, 134 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b n}{d x} - \frac{a + b \operatorname{Log}[c x^n]}{d x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{d^{3/2}} + \\
& \frac{i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{2 d^{3/2}} - \frac{i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{2 d^{3/2}}
\end{aligned}$$

Result (type 4, 134 leaves, 8 steps):

$$\begin{aligned}
& -\frac{b n}{d x} - \frac{a + b \operatorname{Log}[c x^n]}{d x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (a + b \operatorname{Log}[c x^n])}{d^{3/2}} + \\
& \frac{i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{2 d^{3/2}} - \frac{i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{2 d^{3/2}}
\end{aligned}$$

**Problem 225: Result valid but suboptimal antiderivative.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^3 (d + e x^2)^2} dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\begin{aligned}
& -\frac{b n}{2 d^2 x^2} + \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2 (d + e x^2)} - \frac{4 a - b n + 4 b \operatorname{Log}[c x^n]}{4 d^2 x^2} + \\
& \frac{e \operatorname{Log}\left[1 + \frac{d}{e x^2}\right] (4 a - b n + 4 b \operatorname{Log}[c x^n])}{4 d^3} - \frac{b e n \operatorname{PolyLog}\left[2, -\frac{d}{e x^2}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 159 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b n}{2 d^2 x^2} + \frac{a + b \operatorname{Log}[c x^n]}{2 d x^2 (d + e x^2)} - \frac{4 a - b n + 4 b \operatorname{Log}[c x^n]}{4 d^2 x^2} - \frac{e (4 a - b n + 4 b \operatorname{Log}[c x^n])^2}{16 b d^3 n} + \\
& \frac{e (4 a - b n + 4 b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x^2}{d}\right]}{4 d^3} + \frac{b e n \operatorname{PolyLog}\left[2, -\frac{e x^2}{d}\right]}{2 d^3}
\end{aligned}$$

**Problem 229: Result optimal but 1 more steps used.**

$$\int \frac{a + b \operatorname{Log}[c x^n]}{x^2 (d + e x^2)^2} dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3 b n}{2 d^2 x} + \frac{a + b \operatorname{Log}[c x^n]}{2 d x (d + e x^2)} - \frac{3 a - b n + 3 b \operatorname{Log}[c x^n]}{2 d^2 x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (3 a - b n + 3 b \operatorname{Log}[c x^n])}{2 d^{5/2}} + \\
& \frac{3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{5/2}} - \frac{3 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{4 d^{5/2}}
\end{aligned}$$

Result (type 4, 183 leaves, 9 steps):

$$\begin{aligned}
& -\frac{3bn}{2d^2x} + \frac{a+b\operatorname{Log}[cx^n]}{2dx(d+ex^2)} - \frac{3a-bn+3b\operatorname{Log}[cx^n]}{2d^2x} - \frac{\sqrt{e}\operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right](3a-bn+3b\operatorname{Log}[cx^n])}{2d^{5/2}} + \\
& \frac{3ib\sqrt{e}n\operatorname{PolyLog}\left[2, -\frac{i\sqrt{e}x}{\sqrt{d}}\right]}{4d^{5/2}} - \frac{3ib\sqrt{e}n\operatorname{PolyLog}\left[2, \frac{i\sqrt{e}x}{\sqrt{d}}\right]}{4d^{5/2}}
\end{aligned}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int \frac{a+b\operatorname{Log}[cx^n]}{x^3(d+ex^2)^3} dx$$

Optimal (type 4, 162 leaves, 6 steps):

$$\begin{aligned}
& -\frac{3bn}{4d^3x^2} + \frac{a+b\operatorname{Log}[cx^n]}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\operatorname{Log}[cx^n]}{8d^2x^2(d+ex^2)} - \frac{12a-5bn+12b\operatorname{Log}[cx^n]}{8d^3x^2} + \\
& \frac{e\operatorname{Log}\left[1+\frac{d}{ex^2}\right](12a-5bn+12b\operatorname{Log}[cx^n])}{8d^4} - \frac{3ben\operatorname{PolyLog}\left[2, -\frac{d}{ex^2}\right]}{4d^4}
\end{aligned}$$

Result (type 4, 195 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3bn}{4d^3x^2} + \frac{a+b\operatorname{Log}[cx^n]}{4dx^2(d+ex^2)^2} + \frac{6a-bn+6b\operatorname{Log}[cx^n]}{8d^2x^2(d+ex^2)} - \\
& \frac{12a-5bn+12b\operatorname{Log}[cx^n]}{8d^3x^2} - \frac{e(12a-5bn+12b\operatorname{Log}[cx^n])^2}{96bd^4n} + \\
& \frac{e(12a-5bn+12b\operatorname{Log}[cx^n])\operatorname{Log}\left[1+\frac{ex^2}{d}\right]}{8d^4} + \frac{3ben\operatorname{PolyLog}\left[2, -\frac{ex^2}{d}\right]}{4d^4}
\end{aligned}$$

Problem 239: Result optimal but 1 more steps used.

$$\int \frac{a+b\operatorname{Log}[cx^n]}{x^2(d+ex^2)^3} dx$$

Optimal (type 4, 219 leaves, 9 steps):

$$\begin{aligned}
& -\frac{15bn}{8d^3x} + \frac{a+b\operatorname{Log}[cx^n]}{4dx(d+ex^2)^2} + \frac{5a-bn+5b\operatorname{Log}[cx^n]}{8d^2x(d+ex^2)} - \\
& \frac{15a-8bn+15b\operatorname{Log}[cx^n]}{8d^3x} - \frac{\sqrt{e}\operatorname{ArcTan}\left[\frac{\sqrt{e}x}{\sqrt{d}}\right](15a-8bn+15b\operatorname{Log}[cx^n])}{8d^{7/2}} + \\
& \frac{15ib\sqrt{e}n\operatorname{PolyLog}\left[2, -\frac{i\sqrt{e}x}{\sqrt{d}}\right]}{16d^{7/2}} - \frac{15ib\sqrt{e}n\operatorname{PolyLog}\left[2, \frac{i\sqrt{e}x}{\sqrt{d}}\right]}{16d^{7/2}}
\end{aligned}$$

Result (type 4, 219 leaves, 10 steps):

$$\begin{aligned}
& -\frac{15 b n}{8 d^3 x} + \frac{a + b \operatorname{Log}[c x^n]}{4 d x (d + e x^2)^2} + \frac{5 a - b n + 5 b \operatorname{Log}[c x^n]}{8 d^2 x (d + e x^2)} - \\
& \frac{15 a - 8 b n + 15 b \operatorname{Log}[c x^n]}{8 d^3 x} - \frac{\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] (15 a - 8 b n + 15 b \operatorname{Log}[c x^n])}{8 d^{7/2}} + \\
& \frac{15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, -\frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{7/2}} - \frac{15 i b \sqrt{e} n \operatorname{PolyLog}\left[2, \frac{i \sqrt{e} x}{\sqrt{d}}\right]}{16 d^{7/2}}
\end{aligned}$$

**Problem 359: Result valid but suboptimal antiderivative.**

$$\int (f x)^{-1+m} (d + e x^m)^3 (a + b \operatorname{Log}[c x^n])^2 dx$$

Optimal (type 3, 372 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d^3 n^2 x (f x)^{-1+m}}{m^3} + \frac{3 b^2 d^2 e n^2 x^{1+m} (f x)^{-1+m}}{4 m^3} + \\
& \frac{2 b^2 d e^2 n^2 x^{1+2 m} (f x)^{-1+m}}{9 m^3} + \frac{b^2 e^3 n^2 x^{1+3 m} (f x)^{-1+m}}{32 m^3} + \frac{b^2 d^4 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{4 e m} - \\
& \frac{2 b d^3 n x (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{m^2} - \frac{3 b d^2 e n x^{1+m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{2 m^2} - \\
& \frac{2 b d e^2 n x^{1+2 m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{3 m^2} - \frac{b e^3 n x^{1+3 m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{8 m^2} - \\
& \frac{b d^4 n x^{1-m} (f x)^{-1+m} \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{2 e m} + \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^4 (a + b \operatorname{Log}[c x^n])^2}{4 e m}
\end{aligned}$$

Result (type 3, 294 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d^3 n^2 x (f x)^{-1+m}}{m^3} + \frac{3 b^2 d^2 e n^2 x^{1+m} (f x)^{-1+m}}{4 m^3} + \frac{2 b^2 d e^2 n^2 x^{1+2 m} (f x)^{-1+m}}{9 m^3} + \\
& \frac{b^2 e^3 n^2 x^{1+3 m} (f x)^{-1+m}}{32 m^3} + \frac{b^2 d^4 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{4 e m} - \frac{1}{24 e m} \\
& b n x^{1-m} (f x)^{-1+m} \left( \frac{48 d^3 e x^m}{m} + \frac{36 d^2 e^2 x^{2 m}}{m} + \frac{16 d e^3 x^{3 m}}{m} + \frac{3 e^4 x^{4 m}}{m} + 12 d^4 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n]) + \\
& \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^4 (a + b \operatorname{Log}[c x^n])^2}{4 e m}
\end{aligned}$$

**Problem 360: Result valid but suboptimal antiderivative.**

$$\int (f x)^{-1+m} (d + e x^m)^2 (a + b \operatorname{Log}[c x^n])^2 dx$$

Optimal (type 3, 298 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d^2 n^2 x (f x)^{-1+m}}{m^3} + \frac{b^2 d e n^2 x^{1+m} (f x)^{-1+m}}{2 m^3} + \frac{2 b^2 e^2 n^2 x^{1+2 m} (f x)^{-1+m}}{27 m^3} + \\
& \frac{b^2 d^3 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{3 e m} - \frac{2 b d^2 n x (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{m^2} - \\
& \frac{b d e n x^{1+m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{m^2} - \frac{2 b e^2 n x^{1+2 m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{9 m^2} - \\
& \frac{2 b d^3 n x^{1-m} (f x)^{-1+m} \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{3 e m} + \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^3 (a + b \operatorname{Log}[c x^n])^2}{3 e m}
\end{aligned}$$

Result (type 3, 245 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d^2 n^2 x (f x)^{-1+m}}{m^3} + \frac{b^2 d e n^2 x^{1+m} (f x)^{-1+m}}{2 m^3} + \\
& \frac{2 b^2 e^2 n^2 x^{1+2 m} (f x)^{-1+m}}{27 m^3} + \frac{b^2 d^3 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{3 e m} - \frac{1}{9 e m} \\
& b n x^{1-m} (f x)^{-1+m} \left( \frac{18 d^2 e x^m}{m} + \frac{9 d e^2 x^{2 m}}{m} + \frac{2 e^3 x^{3 m}}{m} + 6 d^3 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n]) + \\
& \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^3 (a + b \operatorname{Log}[c x^n])^2}{3 e m}
\end{aligned}$$

**Problem 361: Result valid but suboptimal antiderivative.**

$$\int (f x)^{-1+m} (d + e x^m) (a + b \operatorname{Log}[c x^n])^2 dx$$

Optimal (type 3, 226 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d n^2 x (f x)^{-1+m}}{m^3} + \frac{b^2 e n^2 x^{1+m} (f x)^{-1+m}}{4 m^3} + \frac{b^2 d^2 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{2 e m} - \\
& \frac{2 b d n x (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{m^2} - \frac{b e n x^{1+m} (f x)^{-1+m} (a + b \operatorname{Log}[c x^n])}{2 m^2} - \\
& \frac{b d^2 n x^{1-m} (f x)^{-1+m} \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])}{e m} + \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^2 (a + b \operatorname{Log}[c x^n])^2}{2 e m}
\end{aligned}$$

Result (type 3, 195 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 d n^2 x (f x)^{-1+m}}{m^3} + \frac{b^2 e n^2 x^{1+m} (f x)^{-1+m}}{4 m^3} + \frac{b^2 d^2 n^2 x^{1-m} (f x)^{-1+m} \operatorname{Log}[x]^2}{2 e m} - \\
& \frac{b n x^{1-m} (f x)^{-1+m} \left( \frac{4 d e x^m}{m} + \frac{e^2 x^{2 m}}{m} + 2 d^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])}{2 e m} + \\
& \frac{x^{1-m} (f x)^{-1+m} (d + e x^m)^2 (a + b \operatorname{Log}[c x^n])^2}{2 e m}
\end{aligned}$$

### Problem 371: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r) (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{b d n}{4 x^2} - \frac{b e n x^{-2+r}}{(2-r)^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{2 x^2} - \frac{e x^{-2+r} (a + b \operatorname{Log}[c x^n])}{2-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{b d n}{4 x^2} - \frac{b e n x^{-2+r}}{(2-r)^2} - \frac{1}{2} \left( \frac{d}{x^2} + \frac{2 e x^{-2+r}}{2-r} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 372: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r) (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{b d n}{16 x^4} - \frac{b e n x^{-4+r}}{(4-r)^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{4 x^4} - \frac{e x^{-4+r} (a + b \operatorname{Log}[c x^n])}{4-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{b d n}{16 x^4} - \frac{b e n x^{-4+r}}{(4-r)^2} - \frac{1}{4} \left( \frac{d}{x^4} + \frac{4 e x^{-4+r}}{4-r} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 375: Result valid but suboptimal antiderivative.

$$\int (d + e x^r) (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 57 leaves, 3 steps):

$$-b d n x - \frac{b e n x^{1+r}}{(1+r)^2} + d x (a + b \operatorname{Log}[c x^n]) + \frac{e x^{1+r} (a + b \operatorname{Log}[c x^n])}{1+r}$$

Result (type 3, 49 leaves, 3 steps):

$$-b d n x - \frac{b e n x^{1+r}}{(1+r)^2} + \left( d x + \frac{e x^{1+r}}{1+r} \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 376: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r) (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 67 leaves, 4 steps):

$$-\frac{b d n}{x} - \frac{b e n x^{-1+r}}{(1-r)^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{x} - \frac{e x^{-1+r} (a + b \operatorname{Log}[c x^n])}{1-r}$$

Result (type 3, 58 leaves, 4 steps):

$$-\frac{b d n}{x} - \frac{b e n x^{-1+r}}{(1-r)^2} - \left( \frac{d}{x} + \frac{e x^{-1+r}}{1-r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 377: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r) (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{b d n}{9 x^3} - \frac{b e n x^{-3+r}}{(3-r)^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{3 x^3} - \frac{e x^{-3+r} (a + b \operatorname{Log}[c x^n])}{3-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{b d n}{9 x^3} - \frac{b e n x^{-3+r}}{(3-r)^2} - \frac{1}{3} \left( \frac{d}{x^3} + \frac{3 e x^{-3+r}}{3-r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 378: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r) (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{b d n}{25 x^5} - \frac{b e n x^{-5+r}}{(5-r)^2} - \frac{d (a + b \operatorname{Log}[c x^n])}{5 x^5} - \frac{e x^{-5+r} (a + b \operatorname{Log}[c x^n])}{5-r}$$

Result (type 3, 63 leaves, 2 steps):

$$-\frac{b d n}{25 x^5} - \frac{b e n x^{-5+r}}{(5-r)^2} - \frac{1}{5} \left( \frac{d}{x^5} + \frac{5 e x^{-5+r}}{5-r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 382: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{2 b d e n x^r}{r^2} - \frac{b e^2 n x^{2r}}{4 r^2} - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \frac{2 d e x^r (a + b \operatorname{Log}[c x^n])}{r} + \frac{e^2 x^{2r} (a + b \operatorname{Log}[c x^n])}{2 r} + d^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 87 leaves, 5 steps):



$$-\frac{2 b d e n x^r}{r^2} - \frac{b e^2 n x^{2r}}{4 r^2} - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \frac{1}{2} \left( \frac{4 d e x^r}{r} + \frac{e^2 x^{2r}}{r} + 2 d^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 383: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$-\frac{b d^2 n}{4 x^2} - \frac{b e^2 n x^{-2(1-r)}}{4(1-r)^2} - \frac{2 b d e n x^{-2+r}}{(2-r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{2 x^2} -$$

$$\frac{e^2 x^{-2(1-r)} (a + b \operatorname{Log}[c x^n])}{2(1-r)} - \frac{2 d e x^{-2+r} (a + b \operatorname{Log}[c x^n])}{2-r}$$

Result (type 3, 114 leaves, 4 steps):

$$-\frac{b d^2 n}{4 x^2} - \frac{b e^2 n x^{-2(1-r)}}{4(1-r)^2} - \frac{2 b d e n x^{-2+r}}{(2-r)^2} - \frac{1}{2} \left( \frac{d^2}{x^2} + \frac{e^2 x^{-2(1-r)}}{1-r} + \frac{4 d e x^{-2+r}}{2-r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 384: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{b e^2 n x^{-2(2-r)}}{4(2-r)^2} - \frac{2 b d e n x^{-4+r}}{(4-r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{4 x^4} -$$

$$\frac{e^2 x^{-2(2-r)} (a + b \operatorname{Log}[c x^n])}{2(2-r)} - \frac{2 d e x^{-4+r} (a + b \operatorname{Log}[c x^n])}{4-r}$$

Result (type 3, 115 leaves, 4 steps):

$$-\frac{b d^2 n}{16 x^4} - \frac{b e^2 n x^{-2(2-r)}}{4(2-r)^2} - \frac{2 b d e n x^{-4+r}}{(4-r)^2} - \frac{1}{4} \left( \frac{d^2}{x^4} + \frac{2 e^2 x^{-2(2-r)}}{2-r} + \frac{8 d e x^{-4+r}}{4-r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 387: Result valid but suboptimal antiderivative.**

$$\int (d + e x^r)^2 (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 113 leaves, 2 steps):

$$-b d^2 n x - \frac{2 b d e n x^{1+r}}{(1+r)^2} - \frac{b e^2 n x^{1+2r}}{(1+2r)^2} + d^2 x (a + b \operatorname{Log}[c x^n]) +$$

$$\frac{2 d e x^{1+r} (a + b \operatorname{Log}[c x^n])}{1+r} + \frac{e^2 x^{1+2r} (a + b \operatorname{Log}[c x^n])}{1+2r}$$

Result (type 3, 95 leaves, 2 steps):

$$-b d^2 n x - \frac{2 b d e n x^{1+r}}{(1+r)^2} - \frac{b e^2 n x^{1+2r}}{(1+2r)^2} + \left( d^2 x + \frac{2 d e x^{1+r}}{1+r} + \frac{e^2 x^{1+2r}}{1+2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 388: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$\begin{aligned} & -\frac{b d^2 n}{x} - \frac{2 b d e n x^{-1+r}}{(1-r)^2} - \frac{b e^2 n x^{-1+2r}}{(1-2r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{x} - \\ & \frac{2 d e x^{-1+r} (a + b \operatorname{Log}[c x^n])}{1-r} - \frac{e^2 x^{-1+2r} (a + b \operatorname{Log}[c x^n])}{1-2r} \end{aligned}$$

Result (type 3, 104 leaves, 3 steps):

$$-\frac{b d^2 n}{x} - \frac{2 b d e n x^{-1+r}}{(1-r)^2} - \frac{b e^2 n x^{-1+2r}}{(1-2r)^2} - \left( \frac{d^2}{x} + \frac{2 d e x^{-1+r}}{1-r} + \frac{e^2 x^{-1+2r}}{1-2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 389: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\begin{aligned} & -\frac{b d^2 n}{9 x^3} - \frac{2 b d e n x^{-3+r}}{(3-r)^2} - \frac{b e^2 n x^{-3+2r}}{(3-2r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{3 x^3} - \\ & \frac{2 d e x^{-3+r} (a + b \operatorname{Log}[c x^n])}{3-r} - \frac{e^2 x^{-3+2r} (a + b \operatorname{Log}[c x^n])}{3-2r} \end{aligned}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{b d^2 n}{9 x^3} - \frac{2 b d e n x^{-3+r}}{(3-r)^2} - \frac{b e^2 n x^{-3+2r}}{(3-2r)^2} - \frac{1}{3} \left( \frac{d^2}{x^3} + \frac{6 d e x^{-3+r}}{3-r} + \frac{3 e^2 x^{-3+2r}}{3-2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 390: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{2 b d e n x^{-5+r}}{(5-r)^2} - \frac{b e^2 n x^{-5+2r}}{(5-2r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{5 x^5} -$$

$$\frac{2 d e x^{-5+r} (a + b \operatorname{Log}[c x^n])}{5-r} - \frac{e^2 x^{-5+2r} (a + b \operatorname{Log}[c x^n])}{5-2r}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{b d^2 n}{25 x^5} - \frac{2 b d e n x^{-5+r}}{(5-r)^2} - \frac{b e^2 n x^{-5+2r}}{(5-2r)^2} - \frac{1}{5} \left( \frac{d^2}{x^5} + \frac{10 d e x^{-5+r}}{5-r} + \frac{5 e^2 x^{-5+2r}}{5-2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 391: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x^8} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$-\frac{b d^2 n}{49 x^7} - \frac{2 b d e n x^{-7+r}}{(7-r)^2} - \frac{b e^2 n x^{-7+2r}}{(7-2r)^2} - \frac{d^2 (a + b \operatorname{Log}[c x^n])}{7 x^7} -$$

$$\frac{2 d e x^{-7+r} (a + b \operatorname{Log}[c x^n])}{7-r} - \frac{e^2 x^{-7+2r} (a + b \operatorname{Log}[c x^n])}{7-2r}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{b d^2 n}{49 x^7} - \frac{2 b d e n x^{-7+r}}{(7-r)^2} - \frac{b e^2 n x^{-7+2r}}{(7-2r)^2} - \frac{1}{7} \left( \frac{d^2}{x^7} + \frac{14 d e x^{-7+r}}{7-r} + \frac{7 e^2 x^{-7+2r}}{7-2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 395: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 152 leaves, 5 steps):

$$-\frac{3 b d^2 e n x^r}{r^2} - \frac{3 b d e^2 n x^{2r}}{4 r^2} - \frac{b e^3 n x^{3r}}{9 r^2} - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 + \frac{3 d^2 e x^r (a + b \operatorname{Log}[c x^n])}{r} +$$

$$\frac{3 d e^2 x^{2r} (a + b \operatorname{Log}[c x^n])}{2 r} + \frac{e^3 x^{3r} (a + b \operatorname{Log}[c x^n])}{3 r} + d^3 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 124 leaves, 5 steps):

$$-\frac{3 b d^2 e n x^r}{r^2} - \frac{3 b d e^2 n x^{2r}}{4 r^2} - \frac{b e^3 n x^{3r}}{9 r^2} - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 +$$

$$\frac{1}{6} \left( \frac{18 d^2 e x^r}{r} + \frac{9 d e^2 x^{2r}}{r} + \frac{2 e^3 x^{3r}}{r} + 6 d^3 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 396: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^3} dx$$

Optimal (type 3, 191 leaves, 4 steps):

$$\begin{aligned} & -\frac{b d^3 n}{4 x^2} - \frac{3 b d e^2 n x^{-2(1-r)}}{4(1-r)^2} - \frac{3 b d^2 e n x^{-2+r}}{(2-r)^2} - \frac{b e^3 n x^{-2+3r}}{(2-3r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{2 x^2} \\ & - \frac{3 d e^2 x^{-2(1-r)} (a + b \operatorname{Log}[c x^n])}{2(1-r)} - \frac{3 d^2 e x^{-2+r} (a + b \operatorname{Log}[c x^n])}{2-r} - \frac{e^3 x^{-2+3r} (a + b \operatorname{Log}[c x^n])}{2-3r} \end{aligned}$$

Result (type 3, 161 leaves, 4 steps):

$$\begin{aligned} & -\frac{b d^3 n}{4 x^2} - \frac{3 b d e^2 n x^{-2(1-r)}}{4(1-r)^2} - \frac{3 b d^2 e n x^{-2+r}}{(2-r)^2} - \frac{b e^3 n x^{-2+3r}}{(2-3r)^2} \\ & - \frac{1}{2} \left( \frac{d^3}{x^2} + \frac{3 d e^2 x^{-2(1-r)}}{1-r} + \frac{6 d^2 e x^{-2+r}}{2-r} + \frac{2 e^3 x^{-2+3r}}{2-3r} \right) (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

### Problem 397: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^5} dx$$

Optimal (type 3, 191 leaves, 4 steps):

$$\begin{aligned} & -\frac{b d^3 n}{16 x^4} - \frac{3 b d e^2 n x^{-2(2-r)}}{4(2-r)^2} - \frac{3 b d^2 e n x^{-4+r}}{(4-r)^2} - \frac{b e^3 n x^{-4+3r}}{(4-3r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{4 x^4} \\ & - \frac{3 d e^2 x^{-2(2-r)} (a + b \operatorname{Log}[c x^n])}{2(2-r)} - \frac{3 d^2 e x^{-4+r} (a + b \operatorname{Log}[c x^n])}{4-r} - \frac{e^3 x^{-4+3r} (a + b \operatorname{Log}[c x^n])}{4-3r} \end{aligned}$$

Result (type 3, 161 leaves, 4 steps):

$$\begin{aligned} & -\frac{b d^3 n}{16 x^4} - \frac{3 b d e^2 n x^{-2(2-r)}}{4(2-r)^2} - \frac{3 b d^2 e n x^{-4+r}}{(4-r)^2} - \frac{b e^3 n x^{-4+3r}}{(4-3r)^2} \\ & - \frac{1}{4} \left( \frac{d^3}{x^4} + \frac{6 d e^2 x^{-2(2-r)}}{2-r} + \frac{12 d^2 e x^{-4+r}}{4-r} + \frac{4 e^3 x^{-4+3r}}{4-3r} \right) (a + b \operatorname{Log}[c x^n]) \end{aligned}$$

### Problem 400: Result valid but suboptimal antiderivative.

$$\int (d + e x^r)^3 (a + b \operatorname{Log}[c x^n]) dx$$

Optimal (type 3, 169 leaves, 2 steps):

$$-b d^3 n x - \frac{3 b d^2 e n x^{1+r}}{(1+r)^2} - \frac{3 b d e^2 n x^{1+2r}}{(1+2r)^2} - \frac{b e^3 n x^{1+3r}}{(1+3r)^2} + d^3 x (a + b \operatorname{Log}[c x^n]) +$$

$$\frac{3 d^2 e x^{1+r} (a + b \operatorname{Log}[c x^n])}{1+r} + \frac{3 d e^2 x^{1+2r} (a + b \operatorname{Log}[c x^n])}{1+2r} + \frac{e^3 x^{1+3r} (a + b \operatorname{Log}[c x^n])}{1+3r}$$

Result (type 3, 141 leaves, 2 steps):

$$-b d^3 n x - \frac{3 b d^2 e n x^{1+r}}{(1+r)^2} - \frac{3 b d e^2 n x^{1+2r}}{(1+2r)^2} - \frac{b e^3 n x^{1+3r}}{(1+3r)^2} +$$

$$\left( d^3 x + \frac{3 d^2 e x^{1+r}}{1+r} + \frac{3 d e^2 x^{1+2r}}{1+2r} + \frac{e^3 x^{1+3r}}{1+3r} \right) (a + b \operatorname{Log}[c x^n])$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^2} dx$$

Optimal (type 3, 179 leaves, 3 steps):

$$-\frac{b d^3 n}{x} - \frac{3 b d^2 e n x^{-1+r}}{(1-r)^2} - \frac{3 b d e^2 n x^{-1+2r}}{(1-2r)^2} - \frac{b e^3 n x^{-1+3r}}{(1-3r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{x} -$$

$$\frac{3 d^2 e x^{-1+r} (a + b \operatorname{Log}[c x^n])}{1-r} - \frac{3 d e^2 x^{-1+2r} (a + b \operatorname{Log}[c x^n])}{1-2r} - \frac{e^3 x^{-1+3r} (a + b \operatorname{Log}[c x^n])}{1-3r}$$

Result (type 3, 150 leaves, 3 steps):

$$-\frac{b d^3 n}{x} - \frac{3 b d^2 e n x^{-1+r}}{(1-r)^2} - \frac{3 b d e^2 n x^{-1+2r}}{(1-2r)^2} - \frac{b e^3 n x^{-1+3r}}{(1-3r)^2} -$$

$$\left( \frac{d^3}{x} + \frac{3 d^2 e x^{-1+r}}{1-r} + \frac{3 d e^2 x^{-1+2r}}{1-2r} + \frac{e^3 x^{-1+3r}}{1-3r} \right) (a + b \operatorname{Log}[c x^n])$$

Problem 402: Result valid but suboptimal antiderivative.

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^4} dx$$

Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b d^3 n}{9 x^3} - \frac{b e^3 n x^{-3(1-r)}}{9(1-r)^2} - \frac{3 b d^2 e n x^{-3+r}}{(3-r)^2} - \frac{3 b d e^2 n x^{-3+2r}}{(3-2r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{3 x^3} -$$

$$\frac{e^3 x^{-3(1-r)} (a + b \operatorname{Log}[c x^n])}{3(1-r)} - \frac{3 d^2 e x^{-3+r} (a + b \operatorname{Log}[c x^n])}{3-r} - \frac{3 d e^2 x^{-3+2r} (a + b \operatorname{Log}[c x^n])}{3-2r}$$

Result (type 3, 160 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b d^3 n}{9 x^3} - \frac{b e^3 n x^{-3(1-r)}}{9(1-r)^2} - \frac{3 b d^2 e n x^{-3+r}}{(3-r)^2} - \frac{3 b d e^2 n x^{-3+2r}}{(3-2r)^2} - \\
& \frac{1}{3} \left( \frac{d^3}{x^3} + \frac{e^3 x^{-3(1-r)}}{1-r} + \frac{9 d^2 e x^{-3+r}}{3-r} + \frac{9 d e^2 x^{-3+2r}}{3-2r} \right) (a + b \operatorname{Log}[c x^n])
\end{aligned}$$

**Problem 403: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^6} dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b d^3 n}{25 x^5} - \frac{3 b d^2 e n x^{-5+r}}{(5-r)^2} - \frac{3 b d e^2 n x^{-5+2r}}{(5-2r)^2} - \frac{b e^3 n x^{-5+3r}}{(5-3r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{5 x^5} - \\
& \frac{3 d^2 e x^{-5+r} (a + b \operatorname{Log}[c x^n])}{5-r} - \frac{3 d e^2 x^{-5+2r} (a + b \operatorname{Log}[c x^n])}{5-2r} - \frac{e^3 x^{-5+3r} (a + b \operatorname{Log}[c x^n])}{5-3r}
\end{aligned}$$

Result (type 3, 155 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b d^3 n}{25 x^5} - \frac{3 b d^2 e n x^{-5+r}}{(5-r)^2} - \frac{3 b d e^2 n x^{-5+2r}}{(5-2r)^2} - \frac{b e^3 n x^{-5+3r}}{(5-3r)^2} - \\
& \frac{1}{5} \left( \frac{d^3}{x^5} + \frac{15 d^2 e x^{-5+r}}{5-r} + \frac{15 d e^2 x^{-5+2r}}{5-2r} + \frac{5 e^3 x^{-5+3r}}{5-3r} \right) (a + b \operatorname{Log}[c x^n])
\end{aligned}$$

**Problem 404: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^8} dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b d^3 n}{49 x^7} - \frac{3 b d^2 e n x^{-7+r}}{(7-r)^2} - \frac{3 b d e^2 n x^{-7+2r}}{(7-2r)^2} - \frac{b e^3 n x^{-7+3r}}{(7-3r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{7 x^7} - \\
& \frac{3 d^2 e x^{-7+r} (a + b \operatorname{Log}[c x^n])}{7-r} - \frac{3 d e^2 x^{-7+2r} (a + b \operatorname{Log}[c x^n])}{7-2r} - \frac{e^3 x^{-7+3r} (a + b \operatorname{Log}[c x^n])}{7-3r}
\end{aligned}$$

Result (type 3, 155 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b d^3 n}{49 x^7} - \frac{3 b d^2 e n x^{-7+r}}{(7-r)^2} - \frac{3 b d e^2 n x^{-7+2r}}{(7-2r)^2} - \frac{b e^3 n x^{-7+3r}}{(7-3r)^2} - \\
& \frac{1}{7} \left( \frac{d^3}{x^7} + \frac{21 d^2 e x^{-7+r}}{7-r} + \frac{21 d e^2 x^{-7+2r}}{7-2r} + \frac{7 e^3 x^{-7+3r}}{7-3r} \right) (a + b \operatorname{Log}[c x^n])
\end{aligned}$$

**Problem 405: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x^{10}} dx$$

Optimal (type 3, 191 leaves, 4 steps):

$$-\frac{b d^3 n}{81 x^9} - \frac{b e^3 n x^{-3(3-r)}}{9(3-r)^2} - \frac{3 b d^2 e n x^{-9+r}}{(9-r)^2} - \frac{3 b d e^2 n x^{-9+2r}}{(9-2r)^2} - \frac{d^3 (a + b \operatorname{Log}[c x^n])}{9 x^9} -$$

$$\frac{e^3 x^{-3(3-r)} (a + b \operatorname{Log}[c x^n])}{3(3-r)} - \frac{3 d^2 e x^{-9+r} (a + b \operatorname{Log}[c x^n])}{9-r} - \frac{3 d e^2 x^{-9+2r} (a + b \operatorname{Log}[c x^n])}{9-2r}$$

Result (type 3, 161 leaves, 4 steps):

$$-\frac{b d^3 n}{81 x^9} - \frac{b e^3 n x^{-3(3-r)}}{9(3-r)^2} - \frac{3 b d^2 e n x^{-9+r}}{(9-r)^2} - \frac{3 b d e^2 n x^{-9+2r}}{(9-2r)^2} -$$

$$\frac{1}{9} \left( \frac{d^3}{x^9} + \frac{3 e^3 x^{-3(3-r)}}{3-r} + \frac{27 d^2 e x^{-9+r}}{9-r} + \frac{27 d e^2 x^{-9+2r}}{9-2r} \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 421: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^3 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 152 leaves, 5 steps):

$$-\frac{3 b d^2 e n x^r}{r^2} - \frac{3 b d e^2 n x^{2r}}{4 r^2} - \frac{b e^3 n x^{3r}}{9 r^2} - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 + \frac{3 d^2 e x^r (a + b \operatorname{Log}[c x^n])}{r} +$$

$$\frac{3 d e^2 x^{2r} (a + b \operatorname{Log}[c x^n])}{2 r} + \frac{e^3 x^{3r} (a + b \operatorname{Log}[c x^n])}{3 r} + d^3 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 124 leaves, 5 steps):

$$-\frac{3 b d^2 e n x^r}{r^2} - \frac{3 b d e^2 n x^{2r}}{4 r^2} - \frac{b e^3 n x^{3r}}{9 r^2} - \frac{1}{2} b d^3 n \operatorname{Log}[x]^2 +$$

$$\frac{1}{6} \left( \frac{18 d^2 e x^r}{r} + \frac{9 d e^2 x^{2r}}{r} + \frac{2 e^3 x^{3r}}{r} + 6 d^3 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

**Problem 422: Result valid but suboptimal antiderivative.**

$$\int \frac{(d + e x^r)^2 (a + b \operatorname{Log}[c x^n])}{x} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{2 b d e n x^r}{r^2} - \frac{b e^2 n x^{2r}}{4 r^2} - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 +$$

$$\frac{2 d e x^r (a + b \operatorname{Log}[c x^n])}{r} + \frac{e^2 x^{2r} (a + b \operatorname{Log}[c x^n])}{2 r} + d^2 \operatorname{Log}[x] (a + b \operatorname{Log}[c x^n])$$

Result (type 3, 87 leaves, 5 steps):

$$-\frac{2 b d e n x^r}{r^2} - \frac{b e^2 n x^{2r}}{4 r^2} - \frac{1}{2} b d^2 n \operatorname{Log}[x]^2 + \frac{1}{2} \left( \frac{4 d e x^r}{r} + \frac{e^2 x^{2r}}{r} + 2 d^2 \operatorname{Log}[x] \right) (a + b \operatorname{Log}[c x^n])$$

### Problem 454: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x) (a + b \operatorname{Log}[c x^n])}{(d + e x)^3} dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$\frac{b (e f - d g) n}{2 d e^2 (d + e x)} + \frac{b f^2 n \operatorname{Log}[x]}{2 d^2 (e f - d g)} - \frac{(f + g x)^2 (a + b \operatorname{Log}[c x^n])}{2 (e f - d g) (d + e x)^2} - \frac{b (e f + d g) n \operatorname{Log}[d + e x]}{2 d^2 e^2}$$

Result (type 3, 151 leaves, 7 steps):

$$\frac{b (e f - d g) n}{2 d e^2 (d + e x)} + \frac{b (e f - d g) n \operatorname{Log}[x]}{2 d^2 e^2} - \frac{(e f - d g) (a + b \operatorname{Log}[c x^n])}{2 e^2 (d + e x)^2} + \frac{g x (a + b \operatorname{Log}[c x^n])}{d e (d + e x)} - \frac{b g n \operatorname{Log}[d + e x]}{d e^2} - \frac{b (e f - d g) n \operatorname{Log}[d + e x]}{2 d^2 e^2}$$

### Problem 455: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x) (a + b \operatorname{Log}[c x^n])^2}{(d + e x)^3} dx$$

Optimal (type 4, 202 leaves, 8 steps):

$$\begin{aligned} & - \frac{b (e f - d g) n x (a + b \operatorname{Log}[c x^n])}{d^2 e (d + e x)} + \frac{f^2 (a + b \operatorname{Log}[c x^n])^2}{2 d^2 (e f - d g)} - \\ & \frac{(f + g x)^2 (a + b \operatorname{Log}[c x^n])^2}{2 (e f - d g) (d + e x)^2} + \frac{b^2 (e f - d g) n^2 \operatorname{Log}[d + e x]}{d^2 e^2} - \\ & \frac{b (e f + d g) n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2 e^2} - \frac{b^2 (e f + d g) n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} \end{aligned}$$

Result (type 4, 278 leaves, 13 steps):

$$\begin{aligned} & - \frac{b (e f - d g) n x (a + b \operatorname{Log}[c x^n])}{d^2 e (d + e x)} + \frac{(e f - d g) (a + b \operatorname{Log}[c x^n])^2}{2 d^2 e^2} - \\ & \frac{(e f - d g) (a + b \operatorname{Log}[c x^n])^2}{2 e^2 (d + e x)^2} + \frac{g x (a + b \operatorname{Log}[c x^n])^2}{d e (d + e x)} + \frac{b^2 (e f - d g) n^2 \operatorname{Log}[d + e x]}{d^2 e^2} - \\ & \frac{2 b g n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d e^2} - \frac{b (e f - d g) n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2 e^2} - \\ & \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d e^2} - \frac{b^2 (e f - d g) n^2 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} \end{aligned}$$



### Problem 456: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x) (a + b \operatorname{Log}[c x^n])^3}{(d + e x)^3} dx$$

Optimal (type 4, 295 leaves, 11 steps):

$$\begin{aligned} & -\frac{3 b (e f - d g) n x (a + b \operatorname{Log}[c x^n])^2}{2 d^2 e (d + e x)} + \frac{f^2 (a + b \operatorname{Log}[c x^n])^3}{2 d^2 (e f - d g)} - \\ & \frac{(f + g x)^2 (a + b \operatorname{Log}[c x^n])^3}{2 (e f - d g) (d + e x)^2} + \frac{3 b^2 (e f - d g) n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2 e^2} - \\ & \frac{3 b (e f + d g) n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^2 e^2} + \frac{3 b^3 (e f - d g) n^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} - \\ & \frac{3 b^2 (e f + d g) n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} + \frac{3 b^3 (e f + d g) n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2 e^2} \end{aligned}$$

Result (type 4, 408 leaves, 17 steps):

$$\begin{aligned} & -\frac{3 b (e f - d g) n x (a + b \operatorname{Log}[c x^n])^2}{2 d^2 e (d + e x)} + \frac{(e f - d g) (a + b \operatorname{Log}[c x^n])^3}{2 d^2 e^2} - \frac{(e f - d g) (a + b \operatorname{Log}[c x^n])^3}{2 e^2 (d + e x)^2} + \\ & \frac{g x (a + b \operatorname{Log}[c x^n])^3}{d e (d + e x)} + \frac{3 b^2 (e f - d g) n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2 e^2} - \\ & \frac{3 b g n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d e^2} - \frac{3 b (e f - d g) n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^2 e^2} + \\ & \frac{3 b^3 (e f - d g) n^3 \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} - \frac{6 b^2 g n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d e^2} - \\ & \frac{3 b^2 (e f - d g) n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2 e^2} + \\ & \frac{6 b^3 g n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d e^2} + \frac{3 b^3 (e f - d g) n^3 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2 e^2} \end{aligned}$$

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## Test results for the 249 problems in "3.1.5 u (a+b log(c x^n))^p.m"

### Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x^2} dx$$

Optimal (type 4, 203 leaves, 10 steps):

$$\begin{aligned}
& 2 b^2 e n^2 \operatorname{Log}[x] - 2 b e n \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n]) - e \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^2 - \\
& \frac{2 b^2 e n^2 \operatorname{Log}[1 + e x]}{x} - \frac{2 b^2 n^2 \operatorname{Log}[1 + e x]}{x} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{x} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x} + 2 b^2 e n^2 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] + \\
& 2 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] + 2 b^2 e n^2 \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right]
\end{aligned}$$

Result (type 4, 220 leaves, 15 steps):

$$\begin{aligned}
& 2 b^2 e n^2 \operatorname{Log}[x] + e (a + b \operatorname{Log}[c x^n])^2 + \frac{e (a + b \operatorname{Log}[c x^n])^3}{3 b n} - 2 b^2 e n^2 \operatorname{Log}[1 + e x] - \\
& \frac{2 b^2 n^2 \operatorname{Log}[1 + e x]}{x} - 2 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x] - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{x} - \\
& e (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x] - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x} - 2 b^2 e n^2 \operatorname{PolyLog}[2, -e x] - \\
& 2 b e n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -e x] + 2 b^2 e n^2 \operatorname{PolyLog}[3, -e x]
\end{aligned}$$

**Problem 16: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x^3} dx$$

Optimal (type 4, 287 leaves, 14 steps):

$$\begin{aligned}
& -\frac{7 b^2 e n^2}{4 x} - \frac{1}{4} b^2 e^2 n^2 \operatorname{Log}[x] - \frac{3 b e n (a + b \operatorname{Log}[c x^n])}{2 x} + \frac{1}{2} b e^2 n \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n]) - \\
& \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 x} + \frac{1}{2} e^2 \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^2 + \frac{1}{4} b^2 e^2 n^2 \operatorname{Log}[1 + e x] - \\
& \frac{b^2 n^2 \operatorname{Log}[1 + e x]}{4 x^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{2 x^2} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{2 x^2} - \\
& \frac{1}{2} b^2 e^2 n^2 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] - b e^2 n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] - b^2 e^2 n^2 \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right]
\end{aligned}$$

Result (type 4, 310 leaves, 19 steps):

$$\begin{aligned}
& -\frac{7 b^2 e n^2}{4 x} - \frac{1}{4} b^2 e^2 n^2 \operatorname{Log}[x] - \frac{3 b e n (a + b \operatorname{Log}[c x^n])}{2 x} - \frac{1}{4} e^2 (a + b \operatorname{Log}[c x^n])^2 - \\
& \frac{e (a + b \operatorname{Log}[c x^n])^2}{2 x} - \frac{e^2 (a + b \operatorname{Log}[c x^n])^3}{6 b n} + \frac{1}{4} b^2 e^2 n^2 \operatorname{Log}[1 + e x] - \frac{b^2 n^2 \operatorname{Log}[1 + e x]}{4 x^2} + \\
& \frac{1}{2} b e^2 n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x] - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{2 x^2} + \\
& \frac{1}{2} e^2 (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x] - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{2 x^2} + \\
& \frac{1}{2} b^2 e^2 n^2 \operatorname{PolyLog}[2, -e x] + b e^2 n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -e x] - b^2 e^2 n^2 \operatorname{PolyLog}[3, -e x]
\end{aligned}$$

### Problem 22: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{x^2} dx$$

Optimal (type 4, 342 leaves, 14 steps):

$$\begin{aligned} & 6 b^3 e n^3 \operatorname{Log}[x] - 6 b^2 e n^2 \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n]) - \\ & 3 b e n \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^2 - e \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^3 - \\ & 6 b^3 e n^3 \operatorname{Log}[1 + e x] - \frac{6 b^3 n^3 \operatorname{Log}[1 + e x]}{x} - \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{x} - \\ & \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{x} + \\ & 6 b^3 e n^3 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] + 6 b^2 e n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] + \\ & 3 b e n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] + 6 b^3 e n^3 \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right] + \\ & 6 b^2 e n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right] + 6 b^3 e n^3 \operatorname{PolyLog}\left[4, -\frac{1}{e x}\right] \end{aligned}$$

Result (type 4, 360 leaves, 22 steps):

$$\begin{aligned} & 6 b^3 e n^3 \operatorname{Log}[x] + 3 b e n (a + b \operatorname{Log}[c x^n])^2 + e (a + b \operatorname{Log}[c x^n])^3 + \\ & \frac{e (a + b \operatorname{Log}[c x^n])^4}{4 b n} - 6 b^3 e n^3 \operatorname{Log}[1 + e x] - \frac{6 b^3 n^3 \operatorname{Log}[1 + e x]}{x} - \\ & 6 b^2 e n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x] - \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{x} - \\ & 3 b e n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x] - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{x} - \\ & e (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x] - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{x} - 6 b^3 e n^3 \operatorname{PolyLog}[2, -e x] - \\ & 6 b^2 e n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -e x] - 3 b e n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -e x] + \\ & 6 b^3 e n^3 \operatorname{PolyLog}[3, -e x] + 6 b^2 e n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -e x] - 6 b^3 e n^3 \operatorname{PolyLog}[4, -e x] \end{aligned}$$

### Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{x^3} dx$$

Optimal (type 4, 470 leaves, 22 steps):

$$\begin{aligned}
& -\frac{45 b^3 e n^3}{8 x} - \frac{3}{8} b^3 e^2 n^3 \operatorname{Log}[x] - \frac{21 b^2 e n^2 (a + b \operatorname{Log}[c x^n])}{4 x} + \\
& \frac{3}{4} b^2 e^2 n^2 \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n]) - \frac{9 b e n (a + b \operatorname{Log}[c x^n])^2}{4 x} + \\
& \frac{3}{4} b e^2 n \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^2 - \frac{e (a + b \operatorname{Log}[c x^n])^3}{2 x} + \frac{1}{2} e^2 \operatorname{Log}\left[1 + \frac{1}{e x}\right] (a + b \operatorname{Log}[c x^n])^3 + \\
& \frac{3}{8} b^3 e^2 n^3 \operatorname{Log}[1 + e x] - \frac{3 b^3 n^3 \operatorname{Log}[1 + e x]}{8 x^2} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{4 x^2} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{4 x^2} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{2 x^2} - \\
& \frac{3}{4} b^3 e^2 n^3 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] - \frac{3}{2} b^2 e^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] - \\
& \frac{3}{2} b e^2 n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{1}{e x}\right] - \frac{3}{2} b^3 e^2 n^3 \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right] - \\
& 3 b^2 e^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{1}{e x}\right] - 3 b^3 e^2 n^3 \operatorname{PolyLog}\left[4, -\frac{1}{e x}\right]
\end{aligned}$$

Result (type 4, 499 leaves, 30 steps):

$$\begin{aligned}
& -\frac{45 b^3 e n^3}{8 x} - \frac{3}{8} b^3 e^2 n^3 \operatorname{Log}[x] - \frac{21 b^2 e n^2 (a + b \operatorname{Log}[c x^n])}{4 x} - \\
& \frac{3}{8} b e^2 n (a + b \operatorname{Log}[c x^n])^2 - \frac{9 b e n (a + b \operatorname{Log}[c x^n])^2}{4 x} - \frac{1}{4} e^2 (a + b \operatorname{Log}[c x^n])^3 - \\
& \frac{e (a + b \operatorname{Log}[c x^n])^3}{2 x} - \frac{e^2 (a + b \operatorname{Log}[c x^n])^4}{8 b n} + \frac{3}{8} b^3 e^2 n^3 \operatorname{Log}[1 + e x] - \frac{3 b^3 n^3 \operatorname{Log}[1 + e x]}{8 x^2} + \\
& \frac{3}{4} b^2 e^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x] - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + e x]}{4 x^2} + \\
& \frac{3}{4} b e^2 n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x] - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + e x]}{4 x^2} + \\
& \frac{1}{2} e^2 (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x] - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[1 + e x]}{2 x^2} + \\
& \frac{3}{4} b^3 e^2 n^3 \operatorname{PolyLog}[2, -e x] + \frac{3}{2} b^2 e^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -e x] + \\
& \frac{3}{2} b e^2 n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}[2, -e x] - \frac{3}{2} b^3 e^2 n^3 \operatorname{PolyLog}[3, -e x] - \\
& 3 b^2 e^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[3, -e x] + 3 b^3 e^2 n^3 \operatorname{PolyLog}[4, -e x]
\end{aligned}$$

**Problem 39: Result optimal but 2 more steps used.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[d \left(\frac{1}{d} + f x^2\right)\right]}{x^4} dx$$

Optimal (type 4, 543 leaves, 22 steps):

$$\begin{aligned}
& -\frac{52 b^2 d f n^2}{27 x} - \frac{4}{27} b^2 d^{3/2} f^{3/2} n^2 \operatorname{ArcTan}[\sqrt{d} \sqrt{f} x] - \\
& \frac{16 b d f n (a + b \operatorname{Log}[c x^n])}{9 x} - \frac{4}{9} b d^{3/2} f^{3/2} n \operatorname{ArcTan}[\sqrt{d} \sqrt{f} x] (a + b \operatorname{Log}[c x^n]) - \\
& \frac{2 d f (a + b \operatorname{Log}[c x^n])^2}{3 x} + \frac{1}{3} (-d)^{3/2} f^{3/2} (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 - \sqrt{-d} \sqrt{f} x] - \\
& \frac{1}{3} (-d)^{3/2} f^{3/2} (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + \sqrt{-d} \sqrt{f} x] - \frac{2 b^2 n^2 \operatorname{Log}[1 + d f x^2]}{27 x^3} - \\
& \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + d f x^2]}{9 x^3} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + d f x^2]}{3 x^3} - \\
& \frac{2}{3} b (-d)^{3/2} f^{3/2} n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\sqrt{-d} \sqrt{f} x] + \\
& \frac{2}{3} b (-d)^{3/2} f^{3/2} n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \sqrt{-d} \sqrt{f} x] + \\
& \frac{2}{9} i b^2 d^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - \frac{2}{9} i b^2 d^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] + \\
& \frac{2}{3} b^2 (-d)^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[3, -\sqrt{-d} \sqrt{f} x] - \frac{2}{3} b^2 (-d)^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[3, \sqrt{-d} \sqrt{f} x]
\end{aligned}$$

Result (type 4, 543 leaves, 24 steps):

$$\begin{aligned}
& -\frac{52 b^2 d f n^2}{27 x} - \frac{4}{27} b^2 d^{3/2} f^{3/2} n^2 \operatorname{ArcTan}[\sqrt{d} \sqrt{f} x] - \\
& \frac{16 b d f n (a + b \operatorname{Log}[c x^n])}{9 x} - \frac{4}{9} b d^{3/2} f^{3/2} n \operatorname{ArcTan}[\sqrt{d} \sqrt{f} x] (a + b \operatorname{Log}[c x^n]) - \\
& \frac{2 d f (a + b \operatorname{Log}[c x^n])^2}{3 x} + \frac{1}{3} (-d)^{3/2} f^{3/2} (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 - \sqrt{-d} \sqrt{f} x] - \\
& \frac{1}{3} (-d)^{3/2} f^{3/2} (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + \sqrt{-d} \sqrt{f} x] - \frac{2 b^2 n^2 \operatorname{Log}[1 + d f x^2]}{27 x^3} - \\
& \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[1 + d f x^2]}{9 x^3} - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[1 + d f x^2]}{3 x^3} - \\
& \frac{2}{3} b (-d)^{3/2} f^{3/2} n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, -\sqrt{-d} \sqrt{f} x] + \\
& \frac{2}{3} b (-d)^{3/2} f^{3/2} n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}[2, \sqrt{-d} \sqrt{f} x] + \\
& \frac{2}{9} i b^2 d^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[2, -i \sqrt{d} \sqrt{f} x] - \frac{2}{9} i b^2 d^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[2, i \sqrt{d} \sqrt{f} x] + \\
& \frac{2}{3} b^2 (-d)^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[3, -\sqrt{-d} \sqrt{f} x] - \frac{2}{3} b^2 (-d)^{3/2} f^{3/2} n^2 \operatorname{PolyLog}[3, \sqrt{-d} \sqrt{f} x]
\end{aligned}$$

**Problem 82: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x^2} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 b^2 f m n^2 \operatorname{Log}[x]}{e} - \frac{2 b f m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{e} - \frac{f m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{e} \\
& - \frac{2 b^2 f m n^2 \operatorname{Log}[e + f x]}{e} - \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} \\
& - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} + \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \\
& + \frac{2 b f m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e}
\end{aligned}$$

Result (type 4, 283 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 b^2 f m n^2 \operatorname{Log}[x]}{e} + \frac{f m (a + b \operatorname{Log}[c x^n])^2}{e} + \frac{f m (a + b \operatorname{Log}[c x^n])^3}{3 b e n} - \\
& - \frac{2 b^2 f m n^2 \operatorname{Log}[e + f x]}{e} - \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} \\
& - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{2 b f m n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{e} - \\
& - \frac{f m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{e} - \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e} - \\
& - \frac{2 b f m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e} + \frac{2 b^2 f m n^2 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{e}
\end{aligned}$$

**Problem 83: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x^3} dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$\begin{aligned}
& - \frac{7 b^2 f m n^2}{4 e x} - \frac{b^2 f^2 m n^2 \operatorname{Log}[x]}{4 e^2} - \frac{3 b f m n (a + b \operatorname{Log}[c x^n])}{2 e x} + \frac{b f^2 m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{2 e^2} - \\
& - \frac{f m (a + b \operatorname{Log}[c x^n])^2}{2 e x} + \frac{f^2 m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{2 e^2} + \frac{b^2 f^2 m n^2 \operatorname{Log}[e + f x]}{4 e^2} - \\
& - \frac{b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{2 x^2} - \\
& - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{2 x^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{2 e^2} - \\
& - \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e^2}
\end{aligned}$$

Result (type 4, 385 leaves, 19 steps):

$$\begin{aligned}
& - \frac{7 b^2 f m n^2}{4 e x} - \frac{b^2 f^2 m n^2 \operatorname{Log}[x]}{4 e^2} - \frac{3 b f m n (a + b \operatorname{Log}[c x^n])}{2 e x} - \\
& \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2}{4 e^2} - \frac{f m (a + b \operatorname{Log}[c x^n])^2}{2 e x} - \frac{f^2 m (a + b \operatorname{Log}[c x^n])^3}{6 b e^2 n} + \\
& \frac{b^2 f^2 m n^2 \operatorname{Log}[e + f x]}{4 e^2} - \frac{b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{2 x^2} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{2 x^2} + \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{2 e^2} + \\
& \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{2 e^2} + \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{2 e^2} + \\
& \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{e^2}
\end{aligned}$$

**Problem 84: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x^4} dx$$

Optimal (type 4, 420 leaves, 19 steps):

$$\begin{aligned}
& - \frac{19 b^2 f m n^2}{108 e x^2} + \frac{26 b^2 f^2 m n^2}{27 e^2 x} + \frac{2 b^2 f^3 m n^2 \operatorname{Log}[x]}{27 e^3} - \frac{5 b f m n (a + b \operatorname{Log}[c x^n])}{18 e x^2} + \\
& \frac{8 b f^2 m n (a + b \operatorname{Log}[c x^n])}{9 e^2 x} - \frac{2 b f^3 m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{9 e^3} - \\
& \frac{f m (a + b \operatorname{Log}[c x^n])^2}{6 e x^2} + \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2}{3 e^2 x} - \frac{f^3 m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{3 e^3} - \\
& \frac{2 b^2 f^3 m n^2 \operatorname{Log}[e + f x]}{27 e^3} - \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{27 x^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{9 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{3 x^3} + \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{9 e^3} + \\
& \frac{2 b f^3 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{3 e^3} + \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{3 e^3}
\end{aligned}$$

Result (type 4, 462 leaves, 22 steps):

$$\begin{aligned}
& -\frac{19 b^2 f m n^2}{108 e x^2} + \frac{26 b^2 f^2 m n^2}{27 e^2 x} + \frac{2 b^2 f^3 m n^2 \operatorname{Log}[x]}{27 e^3} - \frac{5 b f m n (a + b \operatorname{Log}[c x^n])}{18 e x^2} + \\
& \frac{8 b f^2 m n (a + b \operatorname{Log}[c x^n])}{9 e^2 x} + \frac{f^3 m (a + b \operatorname{Log}[c x^n])^2}{9 e^3} - \frac{f m (a + b \operatorname{Log}[c x^n])^2}{6 e x^2} + \\
& \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2}{3 e^2 x} + \frac{f^3 m (a + b \operatorname{Log}[c x^n])^3}{9 b e^3 n} - \frac{2 b^2 f^3 m n^2 \operatorname{Log}[e + f x]}{27 e^3} - \\
& \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x)^m]}{27 x^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{9 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{3 x^3} - \frac{2 b f^3 m n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{9 e^3} - \\
& \frac{f^3 m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{3 e^3} - \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{9 e^3} - \\
& \frac{2 b f^3 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{3 e^3} + \frac{2 b^2 f^3 m n^2 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{3 e^3}
\end{aligned}$$

**Problem 88: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x^2} dx$$

Optimal (type 4, 411 leaves, 14 steps):

$$\begin{aligned}
& \frac{6 b^3 f m n^3 \operatorname{Log}[x]}{e} - \frac{6 b^2 f m n^2 \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{e} - \\
& \frac{3 b f m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{e} - \frac{f m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^3}{e} - \\
& \frac{6 b^3 f m n^3 \operatorname{Log}[e + f x]}{e} - \frac{6 b^3 n^3 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x} + \\
& \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \\
& \frac{3 b f m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{e} + \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e} + \\
& \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e} + \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[4, -\frac{e}{f x}\right]}{e}
\end{aligned}$$

Result (type 4, 459 leaves, 22 steps):



$$\begin{aligned}
& \frac{6 b^3 f m n^3 \operatorname{Log}[x]}{e} + \frac{3 b f m n (a + b \operatorname{Log}[c x^n])^2}{e} + \frac{f m (a + b \operatorname{Log}[c x^n])^3}{e} + \\
& \frac{f m (a + b \operatorname{Log}[c x^n])^4}{4 b e n} - \frac{6 b^3 f m n^3 \operatorname{Log}[e + f x]}{e} - \frac{6 b^3 n^3 \operatorname{Log}[d (e + f x)^m]}{x} - \\
& \frac{6 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{x} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{x} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x} - \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{e} - \\
& \frac{3 b f m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{e} - \frac{f m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{e} - \\
& \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e} - \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e} - \\
& \frac{3 b f m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{e} + \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{e} + \\
& \frac{6 b^2 f m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{e} - \frac{6 b^3 f m n^3 \operatorname{PolyLog}\left[4, -\frac{f x}{e}\right]}{e}
\end{aligned}$$

**Problem 89: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{x^3} dx$$

Optimal (type 4, 555 leaves, 22 steps):

$$\begin{aligned}
& - \frac{45 b^3 f m n^3}{8 e x} - \frac{3 b^3 f^2 m n^3 \operatorname{Log}[x]}{8 e^2} - \frac{21 b^2 f m n^2 (a + b \operatorname{Log}[c x^n])}{4 e x} + \\
& \frac{3 b^2 f^2 m n^2 \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])}{4 e^2} - \frac{9 b f m n (a + b \operatorname{Log}[c x^n])^2}{4 e x} + \\
& \frac{3 b f^2 m n \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^2}{4 e^2} - \frac{f m (a + b \operatorname{Log}[c x^n])^3}{2 e x} + \\
& \frac{f^2 m \operatorname{Log}\left[1 + \frac{e}{f x}\right] (a + b \operatorname{Log}[c x^n])^3}{2 e^2} + \frac{3 b^3 f^2 m n^3 \operatorname{Log}[e + f x]}{8 e^2} - \\
& \frac{3 b^3 n^3 \operatorname{Log}[d (e + f x)^m]}{8 x^2} - \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \\
& \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{2 x^2} - \\
& \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{4 e^2} - \frac{3 b^2 f^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{2 e^2} - \\
& \frac{3 b f^2 m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x}\right]}{2 e^2} - \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{2 e^2} - \\
& \frac{3 b^2 f^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{e}{f x}\right]}{e^2} - \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[4, -\frac{e}{f x}\right]}{e^2}
\end{aligned}$$

Result (type 4, 614 leaves, 30 steps):

$$\begin{aligned}
& - \frac{45 b^3 f m n^3}{8 e x} - \frac{3 b^3 f^2 m n^3 \operatorname{Log}[x]}{8 e^2} - \frac{21 b^2 f m n^2 (a + b \operatorname{Log}[c x^n])}{4 e x} - \frac{3 b f^2 m n (a + b \operatorname{Log}[c x^n])^2}{8 e^2} - \\
& \frac{9 b f m n (a + b \operatorname{Log}[c x^n])^2}{4 e x} - \frac{f^2 m (a + b \operatorname{Log}[c x^n])^3}{4 e^2} - \frac{f m (a + b \operatorname{Log}[c x^n])^3}{2 e x} - \\
& \frac{f^2 m (a + b \operatorname{Log}[c x^n])^4}{8 b e^2 n} + \frac{3 b^3 f^2 m n^3 \operatorname{Log}[e + f x]}{8 e^2} - \frac{3 b^3 n^3 \operatorname{Log}[d (e + f x)^m]}{8 x^2} - \\
& \frac{3 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \frac{3 b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x)^m]}{4 x^2} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x)^m]}{2 x^2} + \frac{3 b^2 f^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{4 e^2} + \\
& \frac{3 b f^2 m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{4 e^2} + \frac{f^2 m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{f x}{e}\right]}{2 e^2} + \\
& \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{4 e^2} + \frac{3 b^2 f^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{2 e^2} + \\
& \frac{3 b f^2 m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{f x}{e}\right]}{2 e^2} - \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{2 e^2} - \\
& \frac{3 b^2 f^2 m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{f x}{e}\right]}{e^2} + \frac{3 b^3 f^2 m n^3 \operatorname{PolyLog}\left[4, -\frac{f x}{e}\right]}{e^2}
\end{aligned}$$

### Problem 103: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{x^5} dx$$

Optimal (type 4, 356 leaves, 15 steps):

$$\begin{aligned} & -\frac{7 b^2 f m n^2}{32 e x^2} - \frac{b^2 f^2 m n^2 \operatorname{Log}[x]}{16 e^2} - \frac{3 b f m n (a + b \operatorname{Log}[c x^n])}{8 e x^2} + \frac{b f^2 m n \operatorname{Log}\left[1 + \frac{e}{f x^2}\right] (a + b \operatorname{Log}[c x^n])}{8 e^2} \\ & - \frac{f m (a + b \operatorname{Log}[c x^n])^2}{4 e x^2} + \frac{f^2 m \operatorname{Log}\left[1 + \frac{e}{f x^2}\right] (a + b \operatorname{Log}[c x^n])^2}{4 e^2} + \frac{b^2 f^2 m n^2 \operatorname{Log}[e + f x^2]}{32 e^2} \\ & - \frac{b^2 n^2 \operatorname{Log}[d (e + f x^2)^m]}{32 x^4} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{8 x^4} \\ & - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{4 x^4} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[2, -\frac{e}{f x^2}\right]}{16 e^2} \\ & - \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{e}{f x^2}\right]}{4 e^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{e}{f x^2}\right]}{8 e^2} \end{aligned}$$

Result (type 4, 408 leaves, 20 steps):

$$\begin{aligned} & -\frac{7 b^2 f m n^2}{32 e x^2} - \frac{b^2 f^2 m n^2 \operatorname{Log}[x]}{16 e^2} - \frac{3 b f m n (a + b \operatorname{Log}[c x^n])}{8 e x^2} - \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2}{8 e^2} \\ & - \frac{f m (a + b \operatorname{Log}[c x^n])^2}{4 e x^2} - \frac{f^2 m (a + b \operatorname{Log}[c x^n])^3}{6 b e^2 n} + \frac{b^2 f^2 m n^2 \operatorname{Log}[e + f x^2]}{32 e^2} \\ & - \frac{b^2 n^2 \operatorname{Log}[d (e + f x^2)^m]}{32 x^4} - \frac{b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{8 x^4} \\ & - \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{4 x^4} + \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}\left[1 + \frac{f x^2}{e}\right]}{8 e^2} \\ & + \frac{f^2 m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{f x^2}{e}\right]}{4 e^2} + \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[2, -\frac{f x^2}{e}\right]}{16 e^2} \\ & - \frac{b f^2 m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{f x^2}{e}\right]}{4 e^2} - \frac{b^2 f^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{f x^2}{e}\right]}{8 e^2} \end{aligned}$$

### Problem 107: Result optimal but 2 more steps used.

$$\int \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{x^4} dx$$

Optimal (type 4, 571 leaves, 22 steps):

$$\begin{aligned}
& - \frac{52 b^2 f m n^2}{27 e x} - \frac{4 b^2 f^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 e^{3/2}} - \frac{16 b f m n (a + b \operatorname{Log}[c x^n])}{9 e x} - \\
& \frac{4 b f^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 e^{3/2}} - \frac{2 f m (a + b \operatorname{Log}[c x^n])^2}{3 e x} + \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x^2)^m]}{27 x^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{9 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \frac{2 b f^{3/2} m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 b f^{3/2} m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 i b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} - \frac{2 i b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} + \\
& \frac{2 b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{-e}}\right]}{3 (-e)^{3/2}}
\end{aligned}$$

Result (type 4, 571 leaves, 24 steps):

$$\begin{aligned}
& - \frac{52 b^2 f m n^2}{27 e x} - \frac{4 b^2 f^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 e^{3/2}} - \frac{16 b f m n (a + b \operatorname{Log}[c x^n])}{9 e x} - \\
& \frac{4 b f^{3/2} m n \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 e^{3/2}} - \frac{2 f m (a + b \operatorname{Log}[c x^n])^2}{3 e x} + \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{2 b^2 n^2 \operatorname{Log}[d (e + f x^2)^m]}{27 x^3} - \frac{2 b n (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{9 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \frac{2 b f^{3/2} m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 b f^{3/2} m n (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 i b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} - \frac{2 i b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} + \\
& \frac{2 b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^2 f^{3/2} m n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}}
\end{aligned}$$

**Problem 114: Result optimal but 3 more steps used.**

$$\int \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{x^4} dx$$

Optimal (type 4, 1007 leaves, 36 steps):

$$\begin{aligned}
& - \frac{160 b^3 f m n^3}{27 e x} - \frac{4 b^3 f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 e^{3/2}} - \frac{52 b^2 f m n^2 (a + b \operatorname{Log}[c x^n])}{9 e x} - \\
& \frac{4 b^2 f^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 e^{3/2}} - \frac{8 b f m n (a + b \operatorname{Log}[c x^n])^2}{3 e x} - \\
& \frac{2 f m (a + b \operatorname{Log}[c x^n])^3}{3 e x} + \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^3 n^3 \operatorname{Log}[d (e + f x^2)^m]}{27 x^3} - \\
& \frac{2 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{9 x^3} - \frac{b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} - \\
& \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[4, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[4, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}}
\end{aligned}$$

Result (type 4, 1007 leaves, 39 steps):

$$\begin{aligned}
& - \frac{160 b^3 f m n^3}{27 e x} - \frac{4 b^3 f^{3/2} m n^3 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right]}{27 e^{3/2}} - \frac{52 b^2 f m n^2 (a + b \operatorname{Log}[c x^n])}{9 e x} - \\
& \frac{4 b^2 f^{3/2} m n^2 \operatorname{ArcTan}\left[\frac{\sqrt{f} x}{\sqrt{e}}\right] (a + b \operatorname{Log}[c x^n])}{9 e^{3/2}} - \frac{8 b f m n (a + b \operatorname{Log}[c x^n])^2}{3 e x} - \\
& \frac{2 f m (a + b \operatorname{Log}[c x^n])^3}{3 e x} + \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 - \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{f^{3/2} m (a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}\left[1 + \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^3 n^3 \operatorname{Log}[d (e + f x^2)^m]}{27 x^3} - \\
& \frac{2 b^2 n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{Log}[d (e + f x^2)^m]}{9 x^3} - \frac{b n (a + b \operatorname{Log}[c x^n])^2 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \\
& \frac{(a + b \operatorname{Log}[c x^n])^3 \operatorname{Log}[d (e + f x^2)^m]}{3 x^3} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{b f^{3/2} m n (a + b \operatorname{Log}[c x^n])^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[2, -\frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} - \\
& \frac{2 i b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[2, \frac{i \sqrt{f} x}{\sqrt{e}}\right]}{9 e^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} + \\
& \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{3 (-e)^{3/2}} - \frac{2 b^2 f^{3/2} m n^2 (a + b \operatorname{Log}[c x^n]) \operatorname{PolyLog}\left[3, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} - \\
& \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[4, -\frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}} + \frac{2 b^3 f^{3/2} m n^3 \operatorname{PolyLog}\left[4, \frac{\sqrt{f} x}{\sqrt{e}}\right]}{(-e)^{3/2}}
\end{aligned}$$

## Test results for the 314 problems in "3.2.1 (f+g x)^m (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{a g + b g x} dx$$

Optimal (type 4, 84 leaves, 5 steps):

$$-\frac{\operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{bg} + \frac{B n \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{bg}$$

Result (type 4, 126 leaves, 9 steps):

$$-\frac{B n \operatorname{Log}\left[g(a+bx)\right]^2}{2bg} + \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[a g + b g x]}{bg} + \frac{B n \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[a g + b g x]}{bg} + \frac{B n \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{bg}$$

Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(a g + b g x)^2} dx$$

Optimal (type 3, 67 leaves, 2 steps):

$$-\frac{B n}{b g^2 (a+bx)} - \frac{(c+dx) \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad) g^2 (a+bx)}$$

Result (type 3, 108 leaves, 4 steps):

$$-\frac{B n}{b g^2 (a+bx)} - \frac{B d n \operatorname{Log}[a+bx]}{b(bc-ad) g^2} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{b g^2 (a+bx)} + \frac{B d n \operatorname{Log}[c+dx]}{b(bc-ad) g^2}$$

Problem 10: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 396 leaves, 8 steps):



$$\begin{aligned}
& - \frac{B (b c - a d) g^4 n (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b d} + \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b} + \\
& \frac{B (b c - a d)^2 g^4 n (a + b x)^3 \left( 4 A + B n + 4 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b d^2} - \\
& \frac{B (b c - a d)^3 g^4 n (a + b x)^2 \left( 12 A + 7 B n + 12 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 b d^3} + \\
& \frac{B (b c - a d)^4 g^4 n (a + b x) \left( 12 A + 13 B n + 12 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b d^4} + \\
& \frac{B (b c - a d)^5 g^4 n \left( 12 A + 25 B n + 12 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{30 b d^5} + \\
& \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{5 b d^5}
\end{aligned}$$

Result (type 4, 602 leaves, 27 steps):

$$\begin{aligned}
& \frac{2 A B (b c - a d)^4 g^4 n x}{5 d^4} + \frac{13 B^2 (b c - a d)^4 g^4 n^2 x}{30 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 n^2 (a + b x)^2}{60 b d^3} + \\
& \frac{B^2 (b c - a d)^2 g^4 n^2 (a + b x)^3}{30 b d^2} + \frac{2 B^2 (b c - a d)^4 g^4 n (a + b x) \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{5 b d^4} - \\
& \frac{B (b c - a d)^3 g^4 n (a + b x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 b d^3} + \\
& \frac{2 B (b c - a d)^2 g^4 n (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 b d^2} - \\
& \frac{B (b c - a d) g^4 n (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b d} + \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b} - \\
& \frac{5 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log} [c + d x]}{6 b d^5} + \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{5 b d^5} - \\
& \frac{2 B (b c - a d)^5 g^4 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} [c + d x]}{5 b d^5} - \\
& \frac{B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log} [c + d x]^2}{5 b d^5} + \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{5 b d^5}
\end{aligned}$$

**Problem 11: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 335 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g^3 n (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b d} + \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b} + \\
& \frac{B (b c - a d)^2 g^3 n (a + b x)^2 \left( 3 A + B n + 3 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{12 b d^2} - \\
& \frac{B (b c - a d)^3 g^3 n (a + b x) \left( 6 A + 5 B n + 6 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{12 b d^3} - \\
& \frac{B (b c - a d)^4 g^3 n \left( 6 A + 11 B n + 6 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{12 b d^4} - \\
& \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{2 b d^4}
\end{aligned}$$

Result (type 4, 512 leaves, 23 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^3 g^3 n x}{2 d^3} - \frac{5 B^2 (b c - a d)^3 g^3 n^2 x}{12 d^3} + \frac{B^2 (b c - a d)^2 g^3 n^2 (a + b x)^2}{12 b d^2} - \\
& \frac{B^2 (b c - a d)^3 g^3 n (a + b x) \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{2 b d^3} + \frac{B (b c - a d)^2 g^3 n (a + b x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b d^2} - \\
& \frac{B (b c - a d) g^3 n (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b d} + \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log} [c + d x]}{12 b d^4} - \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{2 b d^4} + \\
& \frac{B (b c - a d)^4 g^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} [c + d x]}{2 b d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log} [c + d x]^2}{4 b d^4} - \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{2 b d^4}
\end{aligned}$$

**Problem 12: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 274 leaves, 6 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g^2 n (a + b x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b d} + \frac{g^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b} + \\
& \frac{B (b c - a d)^2 g^2 n (a + b x) \left( 2 A + B n + 2 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b d^2} + \\
& \frac{B (b c - a d)^3 g^2 n \left( 2 A + 3 B n + 2 B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{3 b d^3} + \\
& \frac{2 B^2 (b c - a d)^3 g^2 n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 420 leaves, 19 steps):

$$\begin{aligned}
& \frac{2 A B (b c - a d)^2 g^2 n x}{3 d^2} + \frac{B^2 (b c - a d)^2 g^2 n^2 x}{3 d^2} + \frac{2 B^2 (b c - a d)^2 g^2 n (a + b x) \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{3 b d^2} - \\
& \frac{B (b c - a d) g^2 n (a + b x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b d} + \frac{g^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b} - \\
& \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log} [c + d x]}{b d^3} + \frac{2 B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{3 b d^3} - \\
& \frac{2 B (b c - a d)^3 g^2 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} [c + d x]}{3 b d^3} - \\
& \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log} [c + d x]^2}{3 b d^3} + \frac{2 B^2 (b c - a d)^3 g^2 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{3 b d^3}
\end{aligned}$$

### Problem 13: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 196 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g n (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b d} + \frac{g (a + b x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b} - \\
& \frac{B (b c - a d)^2 g n \left( A + B n + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{b d^2} - \frac{B^2 (b c - a d)^2 g n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b d^2}
\end{aligned}$$

Result (type 4, 309 leaves, 15 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d) g n x}{d} - \frac{B^2 (b c - a d) g n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b d} + \\
& \frac{g (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b} + \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^2} + \\
& \frac{B (b c - a d)^2 g n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b d^2} + \\
& \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log}[c + d x]^2}{2 b d^2} - \frac{B^2 (b c - a d)^2 g n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b d^2}
\end{aligned}$$

**Problem 14: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 138 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b g} + \\
& \frac{2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b g} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b g}
\end{aligned}$$

Result (type 4, 789 leaves, 45 steps):

$$\begin{aligned}
& - \frac{A B n \operatorname{Log}\left[g(a+b x)\right]^2}{b g} + \frac{B^2 n^2 \operatorname{Log}\left[g(a+b x)\right]^3}{3 b g} - \frac{B^2 n^2 \operatorname{Log}\left[g(a+b x)\right]^2 \operatorname{Log}[-c-d x]}{b g} + \\
& \frac{2 B^2 n \operatorname{Log}\left[g(a+b x)\right] \operatorname{Log}\left[(a+b x)^n\right] \operatorname{Log}[-c-d x]}{b g} - \frac{B^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}[-c-d x]}{b g} + \\
& \frac{B^2 n^2 \operatorname{Log}\left[g(a+b x)\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b g} + \frac{B^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b g} + \\
& \frac{B^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}\left[(c+d x)^{-n}\right]^2}{b g} - \frac{B^2 \operatorname{Log}\left[g(a+b x)\right] \operatorname{Log}\left[(c+d x)^{-n}\right]^2}{b g} + \\
& \frac{\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 \operatorname{Log}[a g+b g x]}{b g} + \frac{2 A B n \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[a g+b g x]}{b g} - \frac{1}{b g} \\
& 2 B^2 n \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \left(\operatorname{Log}\left[(a+b x)^n\right] - \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] + \operatorname{Log}\left[(c+d x)^{-n}\right]\right) \operatorname{Log}[a g+b g x] - \\
& \frac{B^2 n \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] \operatorname{Log}[a g+b g x]^2}{b g} - \frac{B^2 n^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[a g+b g x]^2}{b g} + \\
& \frac{2 A B n \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{b g} + \frac{2 B^2 n \operatorname{Log}\left[(a+b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{b g} - \frac{1}{b g} \\
& 2 B^2 n \left(\operatorname{Log}\left[(a+b x)^n\right] - \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] + \operatorname{Log}\left[(c+d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right] - \\
& \frac{2 B^2 n \operatorname{Log}\left[(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{b g} - \\
& \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{b g} - \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{b g}
\end{aligned}$$

**Problem 15: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(a g+b g x)^2} d x$$

Optimal (type 3, 136 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2 (c+d x)}{(b c-a d) g^2 (a+b x)} - \frac{2 B n (c+d x) \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c-a d) g^2 (a+b x)} - \frac{(c+d x) \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(b c-a d) g^2 (a+b x)}
\end{aligned}$$

Result (type 4, 512 leaves, 24 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2}{b g^2 (a + b x)} - \frac{2 B^2 d n^2 \operatorname{Log}[a + b x]}{b (b c - a d) g^2} + \frac{B^2 d n^2 \operatorname{Log}[a + b x]^2}{b (b c - a d) g^2} - \\
& \frac{2 B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b g^2 (a + b x)} - \frac{2 B d n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b (b c - a d) g^2} - \\
& \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b g^2 (a + b x)} + \frac{2 B^2 d n^2 \operatorname{Log}[c + d x]}{b (b c - a d) g^2} - \frac{2 B^2 d n^2 \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \\
& \frac{2 B d n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{B^2 d n^2 \operatorname{Log}[c + d x]^2}{b (b c - a d) g^2} - \\
& \frac{2 B^2 d n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{b (b c - a d) g^2} - \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b (b c - a d) g^2} - \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{b (b c - a d) g^2}
\end{aligned}$$

**Problem 16: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 288 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 B^2 d n^2 (c + d x)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B^2 n^2 (c + d x)^2}{4 (b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{2 B d n (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B n (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{d (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d)^2 g^3 (a + b x)} - \frac{b (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 (b c - a d)^2 g^3 (a + b x)^2}
\end{aligned}$$

Result (type 4, 626 leaves, 28 steps):

$$\begin{aligned}
& - \frac{B^2 n^2}{4 b g^3 (a + b x)^2} + \frac{3 B^2 d n^2}{2 b (b c - a d) g^3 (a + b x)} + \frac{3 B^2 d^2 n^2 \operatorname{Log}[a + b x]}{2 b (b c - a d)^2 g^3} - \\
& \frac{B^2 d^2 n^2 \operatorname{Log}[a + b x]^2}{2 b (b c - a d)^2 g^3} - \frac{B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b g^3 (a + b x)^2} + \frac{B d n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b (b c - a d) g^3 (a + b x)} + \\
& \frac{B d^2 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b (b c - a d)^2 g^3} - \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b g^3 (a + b x)^2} - \\
& \frac{3 B^2 d^2 n^2 \operatorname{Log}[c + d x]}{2 b (b c - a d)^2 g^3} + \frac{B^2 d^2 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \\
& \frac{B d^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{B^2 d^2 n^2 \operatorname{Log}[c + d x]^2}{2 b (b c - a d)^2 g^3} + \\
& \frac{B^2 d^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B^2 d^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B^2 d^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3}
\end{aligned}$$

**Problem 17: Result unnecessarily involves higher level functions.**

$$\int \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 448 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 n^2 (c + d x)}{(b c - a d)^3 g^4 (a + b x)} + \frac{b B^2 d n^2 (c + d x)^2}{2 (b c - a d)^3 g^4 (a + b x)^2} - \frac{2 b^2 B^2 n^2 (c + d x)^3}{27 (b c - a d)^3 g^4 (a + b x)^3} - \\
& \frac{2 B d^2 n (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^4 (a + b x)} + \frac{b B d n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^4 (a + b x)^2} - \\
& \frac{2 b^2 B n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 (b c - a d)^3 g^4 (a + b x)^3} - \frac{d^2 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d)^3 g^4 (a + b x)} + \\
& \frac{b d (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d)^3 g^4 (a + b x)^2} - \frac{b^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 (b c - a d)^3 g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 736 leaves, 32 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2}{27 b g^4 (a + b x)^3} + \frac{5 B^2 d n^2}{18 b (b c - a d) g^4 (a + b x)^2} - \frac{11 B^2 d^2 n^2}{9 b (b c - a d)^2 g^4 (a + b x)} - \\
& \frac{11 B^2 d^3 n^2 \operatorname{Log}[a + b x]}{9 b (b c - a d)^3 g^4} + \frac{B^2 d^3 n^2 \operatorname{Log}[a + b x]^2}{3 b (b c - a d)^3 g^4} - \frac{2 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 b g^4 (a + b x)^3} + \\
& \frac{B d n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b (b c - a d) g^4 (a + b x)^2} - \frac{2 B d^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b (b c - a d)^2 g^4 (a + b x)} - \\
& \frac{2 B d^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b (b c - a d)^3 g^4} - \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b g^4 (a + b x)^3} + \frac{11 B^2 d^3 n^2 \operatorname{Log}[c + d x]}{9 b (b c - a d)^3 g^4} - \\
& \frac{2 B^2 d^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b (b c - a d)^3 g^4} + \frac{2 B d^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{3 b (b c - a d)^3 g^4} + \\
& \frac{B^2 d^3 n^2 \operatorname{Log}[c + d x]^2}{3 b (b c - a d)^3 g^4} - \frac{2 B^2 d^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4} - \\
& \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4} - \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4}
\end{aligned}$$

**Problem 18: Result unnecessarily involves higher level functions.**

$$\int \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 615 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 B^2 d^3 n^2 (c + d x)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B^2 d^2 n^2 (c + d x)^2}{4 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{2 b^2 B^2 d n^2 (c + d x)^3}{9 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B^2 n^2 (c + d x)^4}{32 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{2 B d^3 n (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B d^2 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{2 b^2 B d n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{d^3 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b d^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{b^2 d (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 (b c - a d)^4 g^5 (a + b x)^4}
\end{aligned}$$



Result (type 4, 826 leaves, 36 steps):

$$\begin{aligned}
& -\frac{B^2 n^2}{32 b g^5 (a + b x)^4} + \frac{7 B^2 d n^2}{72 b (b c - a d) g^5 (a + b x)^3} - \frac{13 B^2 d^2 n^2}{48 b (b c - a d)^2 g^5 (a + b x)^2} + \\
& \frac{25 B^2 d^3 n^2}{24 b (b c - a d)^3 g^5 (a + b x)} + \frac{25 B^2 d^4 n^2 \operatorname{Log}[a + b x]}{24 b (b c - a d)^4 g^5} - \frac{B^2 d^4 n^2 \operatorname{Log}[a + b x]^2}{4 b (b c - a d)^4 g^5} - \\
& \frac{B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 b g^5 (a + b x)^4} + \frac{B d n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b (b c - a d) g^5 (a + b x)^3} - \frac{B d^2 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b (b c - a d)^2 g^5 (a + b x)^2} + \\
& \frac{B d^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b (b c - a d)^3 g^5 (a + b x)} + \frac{B d^4 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b (b c - a d)^4 g^5} - \\
& \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b g^5 (a + b x)^4} - \frac{25 B^2 d^4 n^2 \operatorname{Log}[c + d x]}{24 b (b c - a d)^4 g^5} + \frac{B^2 d^4 n^2 \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{2 b (b c - a d)^4 g^5} - \\
& \frac{B d^4 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{2 b (b c - a d)^4 g^5} - \frac{B^2 d^4 n^2 \operatorname{Log}[c + d x]^2}{4 b (b c - a d)^4 g^5} + \\
& \frac{B^2 d^4 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{2 b (b c - a d)^4 g^5} + \frac{B^2 d^4 n^2 \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{2 b (b c - a d)^4 g^5} + \frac{B^2 d^4 n^2 \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{2 b (b c - a d)^4 g^5}
\end{aligned}$$

Problem 19: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[ \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x \right]$$

Result (type 8, 106 leaves, 2 steps):

$$\begin{aligned}
& a^2 g^2 \text{CannotIntegrate}\left[ \frac{1}{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x \right] + \\
& 2 a b g^2 \text{CannotIntegrate}\left[ \frac{x}{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x \right] + \\
& b^2 g^2 \text{CannotIntegrate}\left[ \frac{x^2}{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x \right]
\end{aligned}$$

### Problem 20: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]} dx$$

Optimal (type 8, 35 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

$$a g \operatorname{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}, x\right] + b g \operatorname{CannotIntegrate}\left[\frac{x}{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}, x\right]$$

### Problem 21: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}, x\right]$$

Result (type 8, 37 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}, x\right]$$

### Problem 22: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)} dx$$

Optimal (type 4, 94 leaves, 3 steps):

$$\frac{e^{\frac{A}{B n}} \left(e \left(\frac{a+b x}{c+d x}\right)^n\right)^{\frac{1}{n}} (c+d x) \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{B n}\right]}{B (b c - a d) g^2 n (a+b x)}$$

Result (type 8, 37 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}, x\right]$$

### Problem 23: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)} dx$$

Optimal (type 4, 197 leaves, 7 steps):

$$\frac{b e^{\frac{2A}{Bn}} \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{2/n} (c+d x)^2 \operatorname{ExpIntegralEi}\left[ -\frac{2 \left( A+B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{Bn} \right]}{B (b c - a d)^2 g^3 n (a+b x)^2} -$$

$$\frac{d e^{\frac{A}{Bn}} \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{\frac{1}{n}} (c+d x) \operatorname{ExpIntegralEi}\left[ -\frac{A+B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{Bn} \right]}{B (b c - a d)^2 g^3 n (a+b x)}$$

Result (type 8, 37 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[ \frac{1}{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}, x \right]$$

### Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[ \frac{(a g + b g x)^2}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

Result (type 8, 106 leaves, 2 steps):

$$a^2 g^2 \operatorname{CannotIntegrate}\left[ \frac{1}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right] +$$

$$2 a b g^2 \operatorname{CannotIntegrate}\left[ \frac{x}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right] +$$

$$b^2 g^2 \operatorname{CannotIntegrate}\left[ \frac{x^2}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

### Problem 25: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal (type 8, 35 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{\left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

$$a g \text{ CannotIntegrate}\left[\frac{1}{\left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right] + b g \text{ CannotIntegrate}\left[\frac{x}{\left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right]$$

**Problem 26: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(a g + b g x) \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right]$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right]$$

**Problem 27: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$-\frac{\frac{A}{B n} \left(e^{\left(\frac{a+b x}{c+d x}\right)^n}\right)^{\frac{1}{n}} (c+d x) \text{ExpIntegralEi}\left[-\frac{A+B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]}{B n}\right]}{B^2 (b c - a d) g^2 n^2 (a+b x)} - \frac{B (b c - a d) g^2 n (a+b x) \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{c+d x}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}, x\right]$$

**Problem 28: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \text{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2} dx$$

Optimal (type 4, 314 leaves, 9 steps):

$$\begin{aligned}
 & - \left( \left( 2 b e^{\frac{2A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{2/n} (c+dx)^2 \text{ExpIntegralEi} \left[ -\frac{2 \left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{Bn} \right] \right) \right) / \\
 & \quad \left( B^2 (bc-ad)^2 g^3 n^2 (a+bx)^2 \right) + \\
 & \quad \frac{d e^{\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} (c+dx) \text{ExpIntegralEi} \left[ -\frac{A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right]}{Bn} \right]}{B^2 (bc-ad)^2 g^3 n^2 (a+bx)} + \\
 & \quad \frac{d (c+dx)}{B (bc-ad)^2 g^3 n (a+bx) \left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)} - \\
 & \quad \frac{b (c+dx)^2}{B (bc-ad)^2 g^3 n (a+bx)^2 \left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}
 \end{aligned}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(ag+bgx)^3 \left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}, x \right]$$

**Problem 33: Result valid but suboptimal antiderivative.**

$$\int \frac{A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right]}{cg+dgx} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$- \frac{\left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \text{Log} \left[ \frac{bc-ad}{b(c+dx)} \right]}{dg} - \frac{Bn \text{PolyLog} \left[ 2, \frac{d(a+bx)}{b(c+dx)} \right]}{dg}$$

Result (type 4, 128 leaves, 9 steps):

$$\begin{aligned}
 & \frac{Bn \text{Log} [g(c+dx)]^2}{2dg} - \frac{Bn \text{Log} \left[ -\frac{d(a+bx)}{b(c+dx)} \right] \text{Log} [cg+dgx]}{dg} + \\
 & \frac{\left( A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \text{Log} [cg+dgx]}{dg} - \frac{Bn \text{PolyLog} \left[ 2, \frac{b(c+dx)}{b(c+dx)} \right]}{dg}
 \end{aligned}$$

**Problem 34: Result valid but suboptimal antiderivative.**

$$\int \frac{A+B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right]}{(cg+dgx)^2} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{A (a + b x)}{(b c - a d) g^2 (c + d x)} - \frac{B n (a + b x)}{(b c - a d) g^2 (c + d x)} + \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d) g^2 (c + d x)}$$

Result (type 3, 107 leaves, 4 steps):

$$\frac{B n}{d g^2 (c + d x)} + \frac{b B n \operatorname{Log}[a + b x]}{d (b c - a d) g^2} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d g^2 (c + d x)} - \frac{b B n \operatorname{Log}[c + d x]}{d (b c - a d) g^2}$$

### Problem 38: Result valid but suboptimal antiderivative.

$$\int (c g + d g x)^4 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2 dx$$

Optimal (type 4, 544 leaves, 19 steps):

$$\begin{aligned} & \frac{13 B^2 (b c - a d)^4 g^4 n^2 x}{30 b^4} + \frac{7 B^2 (b c - a d)^3 g^4 n^2 (c + d x)^2}{60 b^3 d} + \\ & \frac{B^2 (b c - a d)^2 g^4 n^2 (c + d x)^3}{30 b^2 d} - \frac{2 B (b c - a d)^4 g^4 n (a + b x) \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{5 b^5} - \\ & \frac{B (b c - a d)^3 g^4 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{5 b^3 d} - \\ & \frac{2 B (b c - a d)^2 g^4 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{15 b^2 d} - \\ & \frac{B (b c - a d) g^4 n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{10 b d} + \frac{g^4 (c + d x)^5 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2}{5 d} + \\ & \frac{13 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{30 b^5 d} + \frac{5 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}[c + d x]}{6 b^5 d} + \\ & \frac{2 B (b c - a d)^5 g^4 n \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{5 b^5 d} - \\ & \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{5 b^5 d} \end{aligned}$$

Result (type 4, 634 leaves, 27 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d)^4 g^4 n x}{5 b^4} + \frac{13 B^2 (b c - a d)^4 g^4 n^2 x}{30 b^4} + \frac{7 B^2 (b c - a d)^3 g^4 n^2 (c + d x)^2}{60 b^3 d} + \\
& \frac{B^2 (b c - a d)^2 g^4 n^2 (c + d x)^3}{30 b^2 d} + \frac{13 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}[a + b x]}{30 b^5 d} + \\
& \frac{B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}[a + b x]^2}{5 b^5 d} - \frac{2 B^2 (b c - a d)^4 g^4 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{5 b^5} - \\
& \frac{B (b c - a d)^3 g^4 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{5 b^3 d} - \\
& \frac{2 B (b c - a d)^2 g^4 n (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{15 b^2 d} - \\
& \frac{B (b c - a d) g^4 n (c + d x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{10 b d} - \\
& \frac{2 B (b c - a d)^5 g^4 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{5 b^5 d} + \\
& \frac{g^4 (c + d x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{5 d} + \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}[c + d x]}{5 b^5 d} - \\
& \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{5 b^5 d} - \frac{2 B^2 (b c - a d)^5 g^4 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{5 b^5 d}
\end{aligned}$$

**Problem 39: Result valid but suboptimal antiderivative.**

$$\int (c g + d g x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 454 leaves, 15 steps):

$$\begin{aligned}
& \frac{5 B^2 (b c - a d)^3 g^3 n^2 x}{12 b^3} + \frac{B^2 (b c - a d)^2 g^3 n^2 (c + d x)^2}{12 b^2 d} - \\
& \frac{B (b c - a d)^3 g^3 n (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 b^4} - \\
& \frac{B (b c - a d)^2 g^3 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{4 b^2 d} - \\
& \frac{B (b c - a d) g^3 n (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{6 b d} + \frac{g^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{4 d} + \\
& \frac{5 B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{12 b^4 d} + \frac{11 B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}[c + d x]}{12 b^4 d} + \\
& \frac{B (b c - a d)^4 g^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{2 b^4 d} - \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{2 b^4 d}
\end{aligned}$$

Result (type 4, 544 leaves, 23 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^3 g^3 n x}{2 b^3} + \frac{5 B^2 (b c - a d)^3 g^3 n^2 x}{12 b^3} + \frac{B^2 (b c - a d)^2 g^3 n^2 (c + d x)^2}{12 b^2 d} + \\
& \frac{5 B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}[a + b x]}{12 b^4 d} + \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}[a + b x]^2}{4 b^4 d} - \\
& \frac{B^2 (b c - a d)^3 g^3 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{2 b^4} - \frac{B (b c - a d)^2 g^3 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{4 b^2 d} - \\
& \frac{B (b c - a d) g^3 n (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{6 b d} - \\
& \frac{B (b c - a d)^4 g^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 b^4 d} + \\
& \frac{g^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{4 d} + \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}[c + d x]}{2 b^4 d} - \\
& \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 b^4 d} - \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 b^4 d}
\end{aligned}$$

**Problem 40: Result valid but suboptimal antiderivative.**

$$\int (c g + d g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 361 leaves, 11 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 n^2 x}{3 b^2} - \frac{2 B (b c - a d)^2 g^2 n (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b^3} - \\
& \frac{B (b c - a d) g^2 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b d} + \frac{g^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 d} + \\
& \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{3 b^3 d} + \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log}[c + d x]}{b^3 d} + \\
& \frac{2 B (b c - a d)^3 g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{3 b^3 d} - \\
& \frac{2 B^2 (b c - a d)^3 g^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{3 b^3 d}
\end{aligned}$$

Result (type 4, 454 leaves, 19 steps):



$$\begin{aligned}
& - \frac{2AB(b c - a d)^2 g^2 n x}{3 b^2} + \frac{B^2(b c - a d)^2 g^2 n^2 x}{3 b^2} + \\
& \frac{B^2(b c - a d)^3 g^2 n^2 \operatorname{Log}[a + b x]}{3 b^3 d} + \frac{B^2(b c - a d)^3 g^2 n^2 \operatorname{Log}[a + b x]^2}{3 b^3 d} - \\
& \frac{2 B^2(b c - a d)^2 g^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{3 b^3} - \frac{B(b c - a d) g^2 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b d} - \\
& \frac{2 B(b c - a d)^3 g^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b^3 d} + \\
& \frac{g^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 d} + \frac{2 B^2(b c - a d)^3 g^2 n^2 \operatorname{Log}[c + d x]}{3 b^3 d} - \\
& \frac{2 B^2(b c - a d)^3 g^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 b^3 d} - \frac{2 B^2(b c - a d)^3 g^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 b^3 d}
\end{aligned}$$

**Problem 41: Result valid but suboptimal antiderivative.**

$$\int (c g + d g x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g n (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2} + \\
& \frac{g(c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 d} + \frac{B^2(b c - a d)^2 g n^2 \operatorname{Log}[c + d x]}{b^2 d} + \\
& \frac{B(b c - a d)^2 g n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{b^2 d} - \frac{B^2(b c - a d)^2 g n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{b^2 d}
\end{aligned}$$

Result (type 4, 307 leaves, 15 steps):

$$\begin{aligned}
& - \frac{AB(b c - a d) g n x}{b} + \frac{B^2(b c - a d)^2 g n^2 \operatorname{Log}[a + b x]^2}{2 b^2 d} - \\
& \frac{B^2(b c - a d) g n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b^2} - \frac{B(b c - a d)^2 g n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 d} + \\
& \frac{g(c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 d} + \frac{B^2(b c - a d)^2 g n^2 \operatorname{Log}[c + d x]}{b^2 d} - \\
& \frac{B^2(b c - a d)^2 g n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b^2 d} - \frac{B^2(b c - a d)^2 g n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^2 d}
\end{aligned}$$

### Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{c g + d g x} dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$\begin{aligned} & - \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c+dx)}\right]}{d g} \\ & - \frac{2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a+bx)}{b (c+dx)}\right]}{d g} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{d (a+bx)}{b (c+dx)}\right]}{d g} \end{aligned}$$

Result (type 4, 782 leaves, 45 steps):

$$\begin{aligned} & \frac{B^2 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{b c - a d}\right]}{d g} - \frac{B^2 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}\left[g(c+dx)\right]}{d g} + \frac{A B n \operatorname{Log}\left[g(c+dx)\right]^2}{d g} - \\ & \frac{B^2 n^2 \operatorname{Log}\left[a+bx\right] \operatorname{Log}\left[g(c+dx)\right]^2}{d g} + \frac{B^2 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \operatorname{Log}\left[g(c+dx)\right]^2}{d g} + \\ & \frac{B^2 n^2 \operatorname{Log}\left[g(c+dx)\right]^3}{3 d g} - \frac{2 B^2 n \operatorname{Log}\left[a+bx\right] \operatorname{Log}\left[g(c+dx)\right] \operatorname{Log}\left[(c+dx)^{-n}\right]}{d g} - \\ & \frac{B^2 \operatorname{Log}\left[a+bx\right] \operatorname{Log}\left[(c+dx)^{-n}\right]^2}{d g} + \frac{B^2 \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \operatorname{Log}\left[(c+dx)^{-n}\right]^2}{d g} - \\ & \frac{2 A B n \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \operatorname{Log}\left[c g + d g x\right]}{d g} + \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}\left[c g + d g x\right]}{d g} + \frac{1}{d g} \\ & 2 B^2 n \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \left(\operatorname{Log}\left[(a+bx)^n\right] - \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c+dx)^{-n}\right]\right) \operatorname{Log}\left[c g + d g x\right] - \\ & \frac{B^2 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \operatorname{Log}\left[c g + d g x\right]^2}{d g} + \frac{B^2 n \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}\left[c g + d g x\right]^2}{d g} + \\ & \frac{2 B^2 n \operatorname{Log}\left[(a+bx)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b c - a d}\right]}{d g} - \frac{2 A B n \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d g} - \\ & \frac{2 B^2 n \operatorname{Log}\left[(c+dx)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d g} + \frac{1}{d g} \\ & 2 B^2 n \left(\operatorname{Log}\left[(a+bx)^n\right] - \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c+dx)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right] - \\ & \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{b c - a d}\right]}{d g} - \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{b c - a d}\right]}{d g} \end{aligned}$$

**Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(cg + dgx)^2} dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$\begin{aligned} & -\frac{2ABn(a+bx)}{(bc-ad)g^2(c+dx)} + \frac{2B^2n^2(a+bx)}{(bc-ad)g^2(c+dx)} - \\ & \frac{2B^2n(a+bx)\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)g^2(c+dx)} + \frac{(a+bx)\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)g^2(c+dx)} \end{aligned}$$

Result (type 4, 514 leaves, 24 steps):

$$\begin{aligned} & -\frac{2B^2n^2}{dg^2(c+dx)} - \frac{2bB^2n^2\operatorname{Log}[a+bx]}{d(bc-ad)g^2} - \frac{bB^2n^2\operatorname{Log}[a+bx]^2}{d(bc-ad)g^2} + \\ & \frac{2Bn\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{dg^2(c+dx)} + \frac{2bBn\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{d(bc-ad)g^2} - \\ & \frac{\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{dg^2(c+dx)} + \frac{2bB^2n^2\operatorname{Log}[c+dx]}{d(bc-ad)g^2} + \frac{2bB^2n^2\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{d(bc-ad)g^2} - \\ & \frac{2bBn\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)\operatorname{Log}[c+dx]}{d(bc-ad)g^2} - \frac{bB^2n^2\operatorname{Log}[c+dx]^2}{d(bc-ad)g^2} + \\ & \frac{2bB^2n^2\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)g^2} + \frac{2bB^2n^2\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{d(bc-ad)g^2} + \frac{2bB^2n^2\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)g^2} \end{aligned}$$

**Problem 44: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(cg + dgx)^3} dx$$

Optimal (type 3, 317 leaves, 8 steps):

$$\begin{aligned} & -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2g^3(c+dx)^2} - \frac{2ABn(a+bx)}{(bc-ad)^2g^3(c+dx)} + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2g^3(c+dx)} - \\ & \frac{2bB^2n(a+bx)\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^2g^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2(bc-ad)^2g^3(c+dx)^2} - \\ & \frac{d(a+bx)^2\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2(bc-ad)^2g^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^2g^3(c+dx)} \end{aligned}$$

Result (type 4, 626 leaves, 28 steps):

$$\begin{aligned}
& -\frac{B^2 n^2}{4 d g^3 (c+d x)^2} - \frac{3 b B^2 n^2}{2 d (b c - a d) g^3 (c+d x)} - \frac{3 b^2 B^2 n^2 \operatorname{Log}[a+b x]}{2 d (b c - a d)^2 g^3} - \\
& \frac{b^2 B^2 n^2 \operatorname{Log}[a+b x]^2}{2 d (b c - a d)^2 g^3} + \frac{B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d g^3 (c+d x)^2} + \frac{b B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d (b c - a d) g^3 (c+d x)} + \\
& \frac{b^2 B n \operatorname{Log}[a+b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d (b c - a d)^2 g^3} - \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{2 d g^3 (c+d x)^2} + \\
& \frac{3 b^2 B^2 n^2 \operatorname{Log}[c+d x]}{2 d (b c - a d)^2 g^3} + \frac{b^2 B^2 n^2 \operatorname{Log}\left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log}[c+d x]}{d (b c - a d)^2 g^3} - \\
& \frac{b^2 B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}[c+d x]}{d (b c - a d)^2 g^3} - \frac{b^2 B^2 n^2 \operatorname{Log}[c+d x]^2}{2 d (b c - a d)^2 g^3} + \\
& \frac{b^2 B^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[ \frac{b (c+d x)}{b c - a d} \right]}{d (b c - a d)^2 g^3} + \frac{b^2 B^2 n^2 \operatorname{PolyLog}\left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d (b c - a d)^2 g^3} + \frac{b^2 B^2 n^2 \operatorname{PolyLog}\left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d (b c - a d)^2 g^3}
\end{aligned}$$

**Problem 45: Result unnecessarily involves higher level functions.**

$$\int \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(c g + d g x)^4} dx$$

Optimal (type 3, 429 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 B^2 d^2 n^2 (a+b x)^3}{27 (b c - a d)^3 g^4 (c+d x)^3} - \frac{b B^2 d n^2 (a+b x)^2}{2 (b c - a d)^3 g^4 (c+d x)^2} + \\
& \frac{2 b^2 B^2 n^2 (a+b x)}{(b c - a d)^3 g^4 (c+d x)} - \frac{2 B d^2 n (a+b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{9 (b c - a d)^3 g^4 (c+d x)^3} + \\
& \frac{b B d n (a+b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(b c - a d)^3 g^4 (c+d x)^2} - \frac{2 b^2 B n (a+b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(b c - a d)^3 g^4 (c+d x)} - \\
& \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{3 d g^4 (c+d x)^3} + \frac{2 b^3 B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{a+b x}{c+d x} \right]}{3 d (b c - a d)^3 g^4} - \frac{b^3 B^2 n^2 \operatorname{Log}\left[ \frac{a+b x}{c+d x} \right]^2}{3 d (b c - a d)^3 g^4}
\end{aligned}$$

Result (type 4, 736 leaves, 32 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2}{27 d g^4 (c + d x)^3} - \frac{5 b B^2 n^2}{18 d (b c - a d) g^4 (c + d x)^2} - \frac{11 b^2 B^2 n^2}{9 d (b c - a d)^2 g^4 (c + d x)} - \\
& \frac{11 b^3 B^2 n^2 \operatorname{Log}[a + b x]}{9 d (b c - a d)^3 g^4} - \frac{b^3 B^2 n^2 \operatorname{Log}[a + b x]^2}{3 d (b c - a d)^3 g^4} + \frac{2 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 d g^4 (c + d x)^3} + \\
& \frac{b B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d (b c - a d) g^4 (c + d x)^2} + \frac{2 b^2 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d (b c - a d)^2 g^4 (c + d x)} + \\
& \frac{2 b^3 B n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d (b c - a d)^3 g^4} - \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 d g^4 (c + d x)^3} + \frac{11 b^3 B^2 n^2 \operatorname{Log}[c + d x]}{9 d (b c - a d)^3 g^4} + \\
& \frac{2 b^3 B^2 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 d (b c - a d)^3 g^4} - \frac{2 b^3 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{3 d (b c - a d)^3 g^4} - \\
& \frac{b^3 B^2 n^2 \operatorname{Log}[c + d x]^2}{3 d (b c - a d)^3 g^4} + \frac{2 b^3 B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 d (b c - a d)^3 g^4} + \\
& \frac{2 b^3 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 d (b c - a d)^3 g^4} + \frac{2 b^3 B^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{3 d (b c - a d)^3 g^4}
\end{aligned}$$

**Problem 46: Result unnecessarily involves higher level functions.**

$$\int \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(c g + d g x)^5} dx$$

Optimal (type 3, 536 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B^2 d^3 n^2 (a + b x)^4}{32 (b c - a d)^4 g^5 (c + d x)^4} + \frac{2 b B^2 d^2 n^2 (a + b x)^3}{9 (b c - a d)^4 g^5 (c + d x)^3} - \frac{3 b^2 B^2 d n^2 (a + b x)^2}{4 (b c - a d)^4 g^5 (c + d x)^2} + \\
& \frac{2 b^3 B^2 n^2 (a + b x)}{(b c - a d)^4 g^5 (c + d x)} + \frac{B d^3 n (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 (b c - a d)^4 g^5 (c + d x)^4} - \\
& \frac{2 b B d^2 n (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^4 g^5 (c + d x)^3} + \frac{3 b^2 B d n (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^4 g^5 (c + d x)^2} - \\
& \frac{2 b^3 B n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^5 (c + d x)} - \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 d g^5 (c + d x)^4} + \\
& \frac{b^4 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{2 d (b c - a d)^4 g^5} - \frac{b^4 B^2 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{4 d (b c - a d)^4 g^5}
\end{aligned}$$

Result (type 4, 826 leaves, 36 steps):

$$\begin{aligned}
& - \frac{B^2 n^2}{32 d g^5 (c + d x)^4} - \frac{7 b B^2 n^2}{72 d (b c - a d) g^5 (c + d x)^3} - \frac{13 b^2 B^2 n^2}{48 d (b c - a d)^2 g^5 (c + d x)^2} - \\
& \frac{25 b^3 B^2 n^2}{24 d (b c - a d)^3 g^5 (c + d x)} - \frac{25 b^4 B^2 n^2 \operatorname{Log}[a + b x]}{24 d (b c - a d)^4 g^5} - \frac{b^4 B^2 n^2 \operatorname{Log}[a + b x]^2}{4 d (b c - a d)^4 g^5} + \\
& \frac{B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 d g^5 (c + d x)^4} + \frac{b B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 d (b c - a d) g^5 (c + d x)^3} + \frac{b^2 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d (b c - a d)^2 g^5 (c + d x)^2} + \\
& \frac{b^3 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 d (b c - a d)^3 g^5 (c + d x)} + \frac{b^4 B n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 d (b c - a d)^4 g^5} - \\
& \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 d g^5 (c + d x)^4} + \frac{25 b^4 B^2 n^2 \operatorname{Log}[c + d x]}{24 d (b c - a d)^4 g^5} + \frac{b^4 B^2 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 d (b c - a d)^4 g^5} - \\
& \frac{b^4 B n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{2 d (b c - a d)^4 g^5} - \frac{b^4 B^2 n^2 \operatorname{Log}[c + d x]^2}{4 d (b c - a d)^4 g^5} + \\
& \frac{b^4 B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{2 d (b c - a d)^4 g^5} + \frac{b^4 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{2 d (b c - a d)^4 g^5} + \frac{b^4 B^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{2 d (b c - a d)^4 g^5}
\end{aligned}$$

**Problem 47: Result valid but suboptimal antiderivative.**

$$\int \frac{(c g + d g x)^2}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(c g + d g x)^2}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x\right]$$

Result (type 8, 106 leaves, 2 steps):

$$\begin{aligned}
& c^2 g^2 \operatorname{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x\right] + \\
& 2 c d g^2 \operatorname{CannotIntegrate}\left[\frac{x}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x\right] + \\
& d^2 g^2 \operatorname{CannotIntegrate}\left[\frac{x^2}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}, x\right]
\end{aligned}$$

**Problem 48: Result valid but suboptimal antiderivative.**

$$\int \frac{c g + d g x}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]} dx$$

Optimal (type 8, 35 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{c g + d g x}{A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

Result (type 8, 61 leaves, 2 steps):

$$c g \text{ CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right] + d g \text{ CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

**Problem 49: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(c g + d g x) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c g + d g x) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c g + d g x) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

**Problem 50: Unable to integrate problem.**

$$\int \frac{1}{(c g + d g x)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 4, 96 leaves, 3 steps):

$$\frac{e^{-\frac{A}{Bn}} (a + b x) \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{Bn}\right]}{B (b c - a d) g^2 n (c + d x)}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c g + d g x)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

**Problem 51: Unable to integrate problem.**

$$\int \frac{1}{(c g + d g x)^3 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 4, 199 leaves, 7 steps):

$$\frac{b e^{-\frac{A}{B n}} (a + b x) \left( e \left( \frac{a + b x}{c + d x} \right)^n \right)^{-1/n} \text{ExpIntegralEi} \left[ \frac{A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{B n} \right]}{B (b c - a d)^2 g^3 n (c + d x)} -$$

$$\frac{d e^{-\frac{2 A}{B n}} (a + b x)^2 \left( e \left( \frac{a + b x}{c + d x} \right)^n \right)^{-2/n} \text{ExpIntegralEi} \left[ \frac{2 (A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right])}{B n} \right]}{B (b c - a d)^2 g^3 n (c + d x)^2}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(c g + d g x)^3 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}, x \right]$$

**Problem 52: Result valid but suboptimal antiderivative.**

$$\int \frac{(c g + d g x)^2}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{(c g + d g x)^2}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right]$$

Result (type 8, 106 leaves, 2 steps):

$$c^2 g^2 \text{CannotIntegrate} \left[ \frac{1}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right] +$$

$$2 c d g^2 \text{CannotIntegrate} \left[ \frac{x}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right] +$$

$$d^2 g^2 \text{CannotIntegrate} \left[ \frac{x^2}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right]$$

**Problem 53: Result valid but suboptimal antiderivative.**

$$\int \frac{c g + d g x}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2} dx$$

Optimal (type 8, 35 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{c g + d g x}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right]$$

Result (type 8, 61 leaves, 2 steps):

$$c g \text{CannotIntegrate} \left[ \frac{1}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right] + d g \text{CannotIntegrate} \left[ \frac{x}{\left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right]$$



### Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(c g + d g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal (type 8, 37 leaves, 0 steps):

$$\text{Unintegrable}\left[ \frac{1}{(c g + d g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[ \frac{1}{(c g + d g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

### Problem 55: Unable to integrate problem.

$$\int \frac{1}{(c g + d g x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal (type 4, 154 leaves, 4 steps):

$$\frac{e^{-\frac{A}{B n}} (a + b x) \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{-1/n} \operatorname{ExpIntegralEi}\left[ \frac{A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{B n} \right]}{B^2 (b c - a d) g^2 n^2 (c + d x)} - \frac{a + b x}{B (b c - a d) g^2 n (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate}\left[ \frac{1}{(c g + d g x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

### Problem 56: Unable to integrate problem.

$$\int \frac{1}{(c g + d g x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal (type 4, 256 leaves, 10 steps):

$$\frac{b e^{-\frac{A}{B n}} (a + b x) \left( e \left( \frac{a + b x}{c + d x} \right)^n \right)^{-1/n} \text{ExpIntegralEi} \left[ \frac{A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{B n} \right]}{B^2 (b c - a d)^2 g^3 n^2 (c + d x)} -$$

$$\frac{2 d e^{-\frac{2 A}{B n}} (a + b x)^2 \left( e \left( \frac{a + b x}{c + d x} \right)^n \right)^{-2/n} \text{ExpIntegralEi} \left[ \frac{2 (A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right])}{B n} \right]}{B^2 (b c - a d)^2 g^3 n^2 (c + d x)^2}$$

$$\frac{a + b x}{B (b c - a d) g^3 n (c + d x)^2 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}$$

Result (type 8, 37 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(c g + d g x)^3 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}, x \right]$$

**Problem 57: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^4 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 364 leaves, 3 steps):

$$\frac{1}{5 b^4 d^4} B (b c - a d) g (a^3 d^3 g^3 - a^2 b d^2 g^2 (5 d f - c g) +$$

$$a b^2 d g (10 d^2 f^2 - 5 c d f g + c^2 g^2) - b^3 (10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3)) n x -$$

$$\frac{1}{10 b^3 d^3} B (b c - a d) g^2 (a^2 d^2 g^2 - a b d g (5 d f - c g) + b^2 (10 d^2 f^2 - 5 c d f g + c^2 g^2)) n x^2 -$$

$$\frac{B (b c - a d) g^3 (5 b d f - b c g - a d g) n x^3}{15 b^2 d^2} - \frac{B (b c - a d) g^4 n x^4}{20 b d} -$$

$$\frac{B (b f - a g)^5 n \text{Log}[a + b x]}{5 b^5 g} + \frac{(f + g x)^5 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 g} + \frac{B (d f - c g)^5 n \text{Log}[c + d x]}{5 d^5 g}$$

Result (type 3, 348 leaves, 4 steps):

$$\frac{1}{5 b^4 d^4} B g (10 a b^3 d^4 f^3 - 10 a^2 b^2 d^4 f^2 g + 5 a^3 b d^4 f g^2 -$$

$$a^4 d^4 g^3 - b^4 c (10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3)) n x - \frac{1}{10 b^3 d^3}$$

$$B (b c - a d) g^2 (a^2 d^2 g^2 - a b d g (5 d f - c g) + b^2 (10 d^2 f^2 - 5 c d f g + c^2 g^2)) n x^2 -$$

$$\frac{B (b c - a d) g^3 (5 b d f - b c g - a d g) n x^3}{15 b^2 d^2} - \frac{B (b c - a d) g^4 n x^4}{20 b d} -$$

$$\frac{B (b f - a g)^5 n \text{Log}[a + b x]}{5 b^5 g} + \frac{(f + g x)^5 \left( A + B \text{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 g} + \frac{B (d f - c g)^5 n \text{Log}[c + d x]}{5 d^5 g}$$

### Problem 58: Result optimal but 1 more steps used.

$$\int (f + g x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 235 leaves, 3 steps):

$$\begin{aligned} & - \frac{1}{4 b^3 d^3} B (b c - a d) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) n x - \\ & \frac{B (b c - a d) g^2 (4 b d f - b c g - a d g) n x^2}{8 b^2 d^2} - \frac{B (b c - a d) g^3 n x^3}{12 b d} - \\ & \frac{B (b f - a g)^4 n \operatorname{Log}[a + b x]}{4 b^4 g} + \frac{(f + g x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 g} + \frac{B (d f - c g)^4 n \operatorname{Log}[c + d x]}{4 d^4 g} \end{aligned}$$

Result (type 3, 235 leaves, 4 steps):

$$\begin{aligned} & - \frac{1}{4 b^3 d^3} B (b c - a d) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) n x - \\ & \frac{B (b c - a d) g^2 (4 b d f - b c g - a d g) n x^2}{8 b^2 d^2} - \frac{B (b c - a d) g^3 n x^3}{12 b d} - \\ & \frac{B (b f - a g)^4 n \operatorname{Log}[a + b x]}{4 b^4 g} + \frac{(f + g x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 g} + \frac{B (d f - c g)^4 n \operatorname{Log}[c + d x]}{4 d^4 g} \end{aligned}$$

### Problem 59: Result optimal but 1 more steps used.

$$\int (f + g x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 157 leaves, 3 steps):

$$\begin{aligned} & - \frac{B (b c - a d) g (3 b d f - b c g - a d g) n x}{3 b^2 d^2} - \frac{B (b c - a d) g^2 n x^2}{6 b d} - \\ & \frac{B (b f - a g)^3 n \operatorname{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 g} + \frac{B (d f - c g)^3 n \operatorname{Log}[c + d x]}{3 d^3 g} \end{aligned}$$

Result (type 3, 157 leaves, 4 steps):

$$\begin{aligned} & - \frac{B (b c - a d) g (3 b d f - b c g - a d g) n x}{3 b^2 d^2} - \frac{B (b c - a d) g^2 n x^2}{6 b d} - \\ & \frac{B (b f - a g)^3 n \operatorname{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 g} + \frac{B (d f - c g)^3 n \operatorname{Log}[c + d x]}{3 d^3 g} \end{aligned}$$

### Problem 60: Result optimal but 1 more steps used.

$$\int (f + g x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g n x}{2 b d} - \frac{B (b f - a g)^2 n \operatorname{Log}[a + b x]}{2 b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 g} + \frac{B (d f - c g)^2 n \operatorname{Log}[c + d x]}{2 d^2 g}
\end{aligned}$$

Result (type 3, 115 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g n x}{2 b d} - \frac{B (b f - a g)^2 n \operatorname{Log}[a + b x]}{2 b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 g} + \frac{B (d f - c g)^2 n \operatorname{Log}[c + d x]}{2 d^2 g}
\end{aligned}$$

Problem 62: Result optimal but 2 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{f + g x} dx$$

Optimal (type 4, 147 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B n \operatorname{Log}\left[ -\frac{g(a + b x)}{b f - a g} \right] \operatorname{Log}[f + g x]}{g} + \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{B n \operatorname{Log}\left[ -\frac{g(c + d x)}{d f - c g} \right] \operatorname{Log}[f + g x]}{g} - \frac{B n \operatorname{PolyLog}\left[ 2, \frac{b(f + g x)}{b f - a g} \right]}{g} + \frac{B n \operatorname{PolyLog}\left[ 2, \frac{d(f + g x)}{d f - c g} \right]}{g}
\end{aligned}$$

Result (type 4, 147 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B n \operatorname{Log}\left[ -\frac{g(a + b x)}{b f - a g} \right] \operatorname{Log}[f + g x]}{g} + \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{B n \operatorname{Log}\left[ -\frac{g(c + d x)}{d f - c g} \right] \operatorname{Log}[f + g x]}{g} - \frac{B n \operatorname{PolyLog}\left[ 2, \frac{b(f + g x)}{b f - a g} \right]}{g} + \frac{B n \operatorname{PolyLog}\left[ 2, \frac{d(f + g x)}{d f - c g} \right]}{g}
\end{aligned}$$

Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(f + g x)^2} dx$$

Optimal (type 3, 91 leaves, 3 steps):

$$\frac{(a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b f - a g) (f + g x)} + \frac{B (b c - a d) n \operatorname{Log}\left[ \frac{f + g x}{c + d x} \right]}{(b f - a g) (d f - c g)}$$

Result (type 3, 119 leaves, 4 steps):

$$\frac{b B n \operatorname{Log}[a + b x]}{g (b f - a g)} - \frac{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{g (f + g x)} - \frac{B d n \operatorname{Log}[c + d x]}{g (d f - c g)} + \frac{B (b c - a d) n \operatorname{Log}[f + g x]}{(b f - a g) (d f - c g)}$$

### Problem 64: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(f+gx)^3} dx$$

Optimal (type 3, 190 leaves, 3 steps):

$$\begin{aligned} & -\frac{B(b c - a d) n}{2(b f - a g)(d f - c g)(f + g x)} + \frac{b^2 B n \operatorname{Log}[a + b x]}{2 g(b f - a g)^2} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{2 g(f + g x)^2} - \\ & \frac{B d^2 n \operatorname{Log}[c + d x]}{2 g(d f - c g)^2} + \frac{B(b c - a d)(2 b d f - b c g - a d g) n \operatorname{Log}[f + g x]}{2(b f - a g)^2(d f - c g)^2} \end{aligned}$$

Result (type 3, 190 leaves, 4 steps):

$$\begin{aligned} & -\frac{B(b c - a d) n}{2(b f - a g)(d f - c g)(f + g x)} + \frac{b^2 B n \operatorname{Log}[a + b x]}{2 g(b f - a g)^2} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{2 g(f + g x)^2} - \\ & \frac{B d^2 n \operatorname{Log}[c + d x]}{2 g(d f - c g)^2} + \frac{B(b c - a d)(2 b d f - b c g - a d g) n \operatorname{Log}[f + g x]}{2(b f - a g)^2(d f - c g)^2} \end{aligned}$$

### Problem 65: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(f+gx)^4} dx$$

Optimal (type 3, 283 leaves, 3 steps):

$$\begin{aligned} & -\frac{B(b c - a d) n}{6(b f - a g)(d f - c g)(f + g x)^2} - \frac{B(b c - a d)(2 b d f - b c g - a d g) n}{3(b f - a g)^2(d f - c g)^2(f + g x)} + \\ & \frac{b^3 B n \operatorname{Log}[a + b x]}{3 g(b f - a g)^3} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{3 g(f + g x)^3} - \frac{B d^3 n \operatorname{Log}[c + d x]}{3 g(d f - c g)^3} + \\ & \frac{(B(b c - a d)(a^2 d^2 g^2 - a b d g(3 d f - c g) + b^2(3 d^2 f^2 - 3 c d f g + c^2 g^2)) n \operatorname{Log}[f + g x])}{(3(b f - a g)^3(d f - c g)^3)} \end{aligned}$$

Result (type 3, 283 leaves, 4 steps):

$$\begin{aligned} & -\frac{B(b c - a d) n}{6(b f - a g)(d f - c g)(f + g x)^2} - \frac{B(b c - a d)(2 b d f - b c g - a d g) n}{3(b f - a g)^2(d f - c g)^2(f + g x)} + \\ & \frac{b^3 B n \operatorname{Log}[a + b x]}{3 g(b f - a g)^3} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{3 g(f + g x)^3} - \frac{B d^3 n \operatorname{Log}[c + d x]}{3 g(d f - c g)^3} + \\ & \frac{(B(b c - a d)(a^2 d^2 g^2 - a b d g(3 d f - c g) + b^2(3 d^2 f^2 - 3 c d f g + c^2 g^2)) n \operatorname{Log}[f + g x])}{(3(b f - a g)^3(d f - c g)^3)} \end{aligned}$$

### Problem 66: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]}{(f+gx)^5} dx$$

Optimal (type 3, 388 leaves, 3 steps):

$$\begin{aligned} & -\frac{B(b c-a d) n}{12(b f-a g)(d f-c g)(f+g x)^3}-\frac{B(b c-a d)(2 b d f-b c g-a d g) n}{8(b f-a g)^2(d f-c g)^2(f+g x)^2}- \\ & (B(b c-a d)\left(a^2 d^2 g^2-a b d g(3 d f-c g)+b^2\left(3 d^2 f^2-3 c d f g+c^2 g^2\right)\right) n) / \\ & \left(4(b f-a g)^3(d f-c g)^3(f+g x)\right)+\frac{b^4 B n \operatorname{Log}[a+b x]}{4 g(b f-a g)^4}- \\ & \frac{A+B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]}{4 g(f+g x)^4}-\frac{B d^4 n \operatorname{Log}[c+d x]}{4 g(d f-c g)^4}-\left(B(b c-a d)(2 b d f-b c g-a d g)\right. \\ & \left.(2 a b d^2 f g-a^2 d^2 g^2-b^2\left(2 d^2 f^2-2 c d f g+c^2 g^2\right)\right) n \operatorname{Log}[f+g x]) /\left(4(b f-a g)^4(d f-c g)^4\right) \end{aligned}$$

Result (type 3, 388 leaves, 4 steps):

$$\begin{aligned} & -\frac{B(b c-a d) n}{12(b f-a g)(d f-c g)(f+g x)^3}-\frac{B(b c-a d)(2 b d f-b c g-a d g) n}{8(b f-a g)^2(d f-c g)^2(f+g x)^2}- \\ & (B(b c-a d)\left(a^2 d^2 g^2-a b d g(3 d f-c g)+b^2\left(3 d^2 f^2-3 c d f g+c^2 g^2\right)\right) n) / \\ & \left(4(b f-a g)^3(d f-c g)^3(f+g x)\right)+\frac{b^4 B n \operatorname{Log}[a+b x]}{4 g(b f-a g)^4}- \\ & \frac{A+B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]}{4 g(f+g x)^4}-\frac{B d^4 n \operatorname{Log}[c+d x]}{4 g(d f-c g)^4}-\left(B(b c-a d)(2 b d f-b c g-a d g)\right. \\ & \left.(2 a b d^2 f g-a^2 d^2 g^2-b^2\left(2 d^2 f^2-2 c d f g+c^2 g^2\right)\right) n \operatorname{Log}[f+g x]) /\left(4(b f-a g)^4(d f-c g)^4\right) \end{aligned}$$

### Problem 67: Result valid but suboptimal antiderivative.

$$\int (f+g x)^3\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2 dx$$

Optimal (type 4, 923 leaves, 15 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^3 g^3 n^2 x}{6 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^2 (4 b d f - 3 b c g - a d g) n^2 x}{4 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^3 n^2 (c + d x)^2}{12 b^2 d^4} - \\
& \frac{1}{2 b^4 d^3} B (b c - a d) g (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) \\
& n (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) - \frac{1}{4 b^2 d^4} \\
& B (b c - a d) g^2 (4 b d f - 3 b c g - a d g) n (c + d x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) - \\
& \frac{B (b c - a d) g^3 n (c + d x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b d^4} - \frac{(b f - a g)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b^4 g} + \\
& \frac{(f + g x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 g} - \frac{1}{2 b^4 d^4} B (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] + \\
& \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{6 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) n^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{4 b^4 d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 n^2 \operatorname{Log} [c + d x]}{6 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) n^2 \operatorname{Log} [c + d x]}{4 b^4 d^4} + \frac{1}{2 b^4 d^4} \\
& B^2 (b c - a d)^2 g (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) n^2 \operatorname{Log} [c + d x] - \\
& \frac{1}{2 b^4 d^4} B^2 (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]
\end{aligned}$$

Result (type 4, 1060 leaves, 31 steps):

$$\begin{aligned}
& -\frac{1}{2b^3d^3}AB(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))nx- \\
& \frac{B^2(bc-ad)^2(bc+ad)g^3n^2x}{6b^3d^3} + \frac{B^2(bc-ad)^2g^2(4bdf-bcg-adg)n^2x}{4b^3d^3} + \\
& \frac{B^2(bc-ad)^2g^3n^2x^2}{12b^2d^2} - \frac{a^3B^2(bc-ad)g^3n^2\text{Log}[a+bx]}{6b^4d} + \\
& \frac{a^2B^2(bc-ad)g^2(4bdf-bcg-adg)n^2\text{Log}[a+bx]}{4b^4d^2} + \frac{B^2(bf-ag)^4n^2\text{Log}[a+bx]^2}{4b^4g} - \\
& \frac{1}{2b^4d^3}B^2(bc-ad)g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))n(a+bx) \\
& \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - \frac{B(bc-ad)g^2(4bdf-bcg-adg)nx^2\left(A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{4b^2d^2} - \\
& \frac{B(bc-ad)g^3nx^3\left(A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{6bd} - \frac{B(bf-ag)^4n\text{Log}[a+bx]\left(A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2b^4g} + \\
& \frac{(f+gx)^4\left(A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{4g} + \frac{B^2c^3(bc-ad)g^3n^2\text{Log}[c+dx]}{6bd^4} - \\
& \frac{B^2c^2(bc-ad)g^2(4bdf-bcg-adg)n^2\text{Log}[c+dx]}{4b^2d^4} + \frac{1}{2b^4d^4} \\
& B^2(bc-ad)^2g(a^2d^2g^2-abdg(4df-cg)+b^2(6d^2f^2-4cdfg+c^2g^2))n^2\text{Log}[c+dx] - \\
& \frac{B^2(df-cg)^4n^2\text{Log}\left[-\frac{d(a+bx)}{b(c-ad)}\right]\text{Log}[c+dx]}{2d^4g} + \frac{B(df-cg)^4n\left(A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)\text{Log}[c+dx]}{2d^4g} + \\
& \frac{B^2(df-cg)^4n^2\text{Log}[c+dx]^2}{4d^4g} - \frac{B^2(bf-ag)^4n^2\text{Log}[a+bx]\text{Log}\left[\frac{b(c+dx)}{b(c-ad)}\right]}{2b^4g} - \\
& \frac{B^2(bf-ag)^4n^2\text{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-ad)}\right]}{2b^4g} - \frac{B^2(df-cg)^4n^2\text{PolyLog}\left[2, \frac{b(c+dx)}{b(c-ad)}\right]}{2d^4g}
\end{aligned}$$

**Problem 68: Result valid but suboptimal antiderivative.**

$$\int (f+gx)^2 \left( A+B\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \right)^2 dx$$

Optimal (type 4, 565 leaves, 12 steps):



$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 n^2 x}{3 b^2 d^2} - \frac{1}{3 b^3 d^2} \\
& 2 B (b c - a d) g (3 b d f - 2 b c g - a d g) n (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) - \\
& \frac{B (b c - a d) g^2 n (c + d x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b d^3} - \\
& \frac{(b f - a g)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 g} + \frac{1}{3 b^3 d^3} \\
& 2 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \\
& n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] + \\
& \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{3 b^3 d^3} + \frac{B^2 (b c - a d)^3 g^2 n^2 \operatorname{Log} [c + d x]}{3 b^3 d^3} + \\
& \frac{2 B^2 (b c - a d)^2 g (3 b d f - 2 b c g - a d g) n^2 \operatorname{Log} [c + d x]}{3 b^3 d^3} + \frac{1}{3 b^3 d^3} 2 B^2 (b c - a d) \\
& (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) n^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]
\end{aligned}$$

Result (type 4, 699 leaves, 27 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d) g (3 b d f - b c g - a d g) n x}{3 b^2 d^2} + \frac{B^2 (b c - a d)^2 g^2 n^2 x}{3 b^2 d^2} + \\
& \frac{a^2 B^2 (b c - a d) g^2 n^2 \text{Log}[a + b x]}{3 b^3 d} + \frac{B^2 (b f - a g)^3 n^2 \text{Log}[a + b x]^2}{3 b^3 g} - \\
& \frac{2 B^2 (b c - a d) g (3 b d f - b c g - a d g) n (a + b x) \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{3 b^3 d^2} - \\
& \frac{B (b c - a d) g^2 n x^2 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b d} - \frac{2 B (b f - a g)^3 n \text{Log}[a + b x] \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b^3 g} + \\
& \frac{(f + g x)^3 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 g} - \frac{B^2 c^2 (b c - a d) g^2 n^2 \text{Log}[c + d x]}{3 b d^3} + \\
& \frac{2 B^2 (b c - a d)^2 g (3 b d f - b c g - a d g) n^2 \text{Log}[c + d x]}{3 b^3 d^3} - \\
& \frac{2 B^2 (d f - c g)^3 n^2 \text{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \text{Log}[c + d x]}{3 d^3 g} + \\
& \frac{2 B (d f - c g)^3 n \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \text{Log}[c + d x]}{3 d^3 g} + \\
& \frac{B^2 (d f - c g)^3 n^2 \text{Log}[c + d x]^2}{3 d^3 g} - \frac{2 B^2 (b f - a g)^3 n^2 \text{Log}[a + b x] \text{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 b^3 g} - \\
& \frac{2 B^2 (b f - a g)^3 n^2 \text{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 b^3 g} - \frac{2 B^2 (d f - c g)^3 n^2 \text{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{3 d^3 g}
\end{aligned}$$

**Problem 69: Result valid but suboptimal antiderivative.**

$$\int (f + g x) \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 290 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g n (a + b x) \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 d} - \\
& \frac{(b f - a g)^2 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b^2 g} + \frac{(f + g x)^2 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 g} + \frac{1}{b^2 d^2} \\
& B (b c - a d) (2 b d f - b c g - a d g) n \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \text{Log}\left[\frac{b c - a d}{b (c + d x)}\right] + \\
& \frac{B^2 (b c - a d)^2 g n^2 \text{Log}[c + d x]}{b^2 d^2} + \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \text{PolyLog}\left[2, \frac{d(a + b x)}{b(c + d x)}\right]}{b^2 d^2}
\end{aligned}$$

Result (type 4, 481 leaves, 23 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d) g n x}{b d} + \frac{B^2 (b f - a g)^2 n^2 \operatorname{Log}[a + b x]^2}{2 b^2 g} - \\
& \frac{B^2 (b c - a d) g n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b^2 d} - \frac{B (b f - a g)^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 g} + \\
& \frac{(f + g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 g} + \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log}[c + d x]}{b^2 d^2} - \\
& \frac{B^2 (d f - c g)^2 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 g} + \frac{B (d f - c g)^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^2 g} + \\
& \frac{B^2 (d f - c g)^2 n^2 \operatorname{Log}[c + d x]^2}{2 d^2 g} - \frac{B^2 (b f - a g)^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b^2 g} - \\
& \frac{B^2 (b f - a g)^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^2 g} - \frac{B^2 (d f - c g)^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 g}
\end{aligned}$$

Problem 70: Result valid but suboptimal antiderivative.

$$\int \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2 dx$$

Optimal (type 4, 135 leaves, 6 steps):

$$\begin{aligned}
& \frac{(a + b x) \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2}{b} + \\
& \frac{2 B (b c - a d) n \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) \operatorname{Log}\left[\frac{b c - a d}{b(c + d x)}\right]}{b d} + \frac{2 B^2 (b c - a d) n^2 \operatorname{PolyLog}\left[2, \frac{d(a + b x)}{b(c + d x)}\right]}{b d}
\end{aligned}$$

Result (type 4, 275 leaves, 20 steps):

$$\begin{aligned}
& - \frac{a B^2 n^2 \operatorname{Log}[a + b x]^2}{b} + \frac{2 a B n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{b} + \\
& x \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2 + \frac{2 B^2 c n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d} - \\
& \frac{2 B c n \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) \operatorname{Log}[c + d x]}{d} - \frac{B^2 c n^2 \operatorname{Log}[c + d x]^2}{d} + \\
& \frac{2 a B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b} + \frac{2 a B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b} + \frac{2 B^2 c n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d}
\end{aligned}$$

Problem 71: Result valid but suboptimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^2}{f + g x} dx$$

Optimal (type 4, 297 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}\left[ \frac{bc-ad}{b(c+dx)} \right]}{g} + \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}\left[ 1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)} \right]}{g} - \\
 & \frac{2 B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{d(a+bx)}{b(c+dx)} \right]}{g} + \\
 & \frac{2 B n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)} \right]}{g} + \\
 & \frac{2 B^2 n^2 \operatorname{PolyLog}\left[ 3, \frac{d(a+bx)}{b(c+dx)} \right]}{g} - \frac{2 B^2 n^2 \operatorname{PolyLog}\left[ 3, \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)} \right]}{g}
 \end{aligned}$$

Result (type 4, 2233 leaves, 43 steps):

$$\begin{aligned}
 & - \frac{2 A B n \operatorname{Log}\left[ -\frac{g(a+bx)}{bf-ag} \right] \operatorname{Log}[f+gx]}{g} - \frac{B^2 \operatorname{Log}\left[ (a+bx)^n \right]^2 \operatorname{Log}[f+gx]}{g} + \\
 & \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}[f+gx]}{g} + \frac{2 B^2 n^2 \operatorname{Log}\left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx] \operatorname{Log}[f+gx]}{g} + \\
 & \frac{2 B^2 n^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] \operatorname{Log}[f+gx]}{g} + \frac{2 A B n \operatorname{Log}\left[ -\frac{g(c+dx)}{df-cg} \right] \operatorname{Log}[f+gx]}{g} - \\
 & \frac{2 B^2 n \left( n \operatorname{Log}[a+bx] - \operatorname{Log}\left[ (a+bx)^n \right] \right) \operatorname{Log}\left[ -\frac{g(c+dx)}{df-cg} \right] \operatorname{Log}[f+gx]}{g} - \\
 & \frac{B^2 \operatorname{Log}\left[ (c+dx)^{-n} \right]^2 \operatorname{Log}[f+gx]}{g} + \frac{1}{g} \\
 & 2 B^2 n \operatorname{Log}\left[ -\frac{g(a+bx)}{bf-ag} \right] \left( \operatorname{Log}\left[ (a+bx)^n \right] - \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}\left[ (c+dx)^{-n} \right] \right) \operatorname{Log}[f+gx] - \\
 & \frac{1}{g} 2 B^2 n \operatorname{Log}\left[ -\frac{g(c+dx)}{df-cg} \right] \left( \operatorname{Log}\left[ (a+bx)^n \right] - \operatorname{Log}\left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}\left[ (c+dx)^{-n} \right] \right) \operatorname{Log}[f+gx] - \\
 & \frac{2 B^2 n \operatorname{Log}\left[ -\frac{g(a+bx)}{bf-ag} \right] \left( n \operatorname{Log}[c+dx] + \operatorname{Log}\left[ (c+dx)^{-n} \right] \right) \operatorname{Log}[f+gx]}{g} + \\
 & \frac{B^2 \operatorname{Log}\left[ (a+bx)^n \right]^2 \operatorname{Log}\left[ \frac{b(f+gx)}{bf-ag} \right]}{g} + \frac{B^2 \operatorname{Log}\left[ (c+dx)^{-n} \right]^2 \operatorname{Log}\left[ \frac{d(f+gx)}{df-cg} \right]}{g} + \frac{1}{g} B^2 n^2 \\
 & \left( \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] + \operatorname{Log}\left[ \frac{bf-ag}{b(f+gx)} \right] - \operatorname{Log}\left[ \frac{(bf-ag)(c+dx)}{(bc-ad)(f+gx)} \right] \right) \operatorname{Log}\left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right]^2 - \frac{1}{g} \\
 & B^2 n^2 \left( \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] - \operatorname{Log}\left[ -\frac{g(c+dx)}{df-cg} \right] \right) \left( \operatorname{Log}[a+bx] + \operatorname{Log}\left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right)^2 + \frac{1}{g} B^2 n^2 \\
 & \left( \operatorname{Log}\left[ -\frac{d(a+bx)}{bc-ad} \right] + \operatorname{Log}\left[ \frac{df-cg}{d(f+gx)} \right] - \operatorname{Log}\left[ -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right] \right) \operatorname{Log}\left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right]^2 -
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{g} B^2 n^2 \left( \log \left[ -\frac{d(a+bx)}{bc-ad} \right] - \log \left[ -\frac{g(a+bx)}{bf-ag} \right] \right) \left( \log[c+dx] + \log \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right)^2 + \\
& \frac{2 B^2 n^2 \left( \log[f+gx] - \log \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right) \text{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{g} + \\
& \frac{2 B^2 n \log[(a+bx)^n] \text{PolyLog} \left[ 2, -\frac{g(a+bx)}{bf-ag} \right]}{g} + \\
& \frac{2 B^2 n^2 \left( \log[f+gx] - \log \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right) \text{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{g} - \\
& \frac{2 B^2 n \log[(c+dx)^{-n}] \text{PolyLog} \left[ 2, -\frac{g(c+dx)}{df-cg} \right]}{g} - \frac{2 B^2 n^2 \log \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \text{PolyLog} \left[ 2, \frac{g(a+bx)}{b(f+gx)} \right]}{g} + \\
& \frac{2 B^2 n^2 \log \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \text{PolyLog} \left[ 2, -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right]}{g} - \\
& \frac{2 B^2 n^2 \log \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \text{PolyLog} \left[ 2, \frac{g(c+dx)}{d(f+gx)} \right]}{g} + \\
& \frac{2 B^2 n^2 \log \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \text{PolyLog} \left[ 2, \frac{(bf-ag)(c+dx)}{(bc-ad)(f+gx)} \right]}{g} - \frac{2 A B n \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \frac{1}{g} \\
& 2 B^2 n \left( \log[(a+bx)^n] - \log \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \log[(c+dx)^{-n}] \right) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right] - \\
& \frac{2 B^2 n (n \log[c+dx] + \log[(c+dx)^{-n}]) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \\
& \frac{2 B^2 n^2 \left( \log[c+dx] + \log \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \\
& \frac{2 A B n \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} - \frac{2 B^2 n (n \log[a+bx] - \log[(a+bx)^n]) \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} - \\
& \frac{1}{g} 2 B^2 n \left( \log[(a+bx)^n] - \log \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \log[(c+dx)^{-n}] \right) \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right] + \\
& \frac{2 B^2 n^2 \left( \log[a+bx] + \log \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right) \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, -\frac{d(a+bx)}{bc-ad} \right]}{g} - \\
& \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, -\frac{g(a+bx)}{bf-ag} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{b(c+dx)}{bc-ad} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, -\frac{g(c+dx)}{df-cg} \right]}{g} - \\
& \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{g(a+bx)}{b(f+gx)} \right]}{g} + \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{g(c+dx)}{d(f+gx)} \right]}{g} + \\
& \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{(bf-ag)(c+dx)}{(bc-ad)(f+gx)} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{b(f+gx)}{bf-ag} \right]}{g} - \frac{2 B^2 n^2 \text{PolyLog} \left[ 3, \frac{d(f+gx)}{df-cg} \right]}{g}
\end{aligned}$$

### Problem 72: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(f+gx)^2} dx$$

Optimal (type 4, 206 leaves, 4 steps):

$$\frac{(a+bx) \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad)n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right]}{(bf-ag)(df-cg)} +$$

$$\frac{2B^2(bc-ad)n^2 \operatorname{PolyLog}\left[2, \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right]}{(bf-ag)(df-cg)}$$

Result (type 4, 657 leaves, 29 steps):

$$-\frac{bB^2n^2 \operatorname{Log}[a+bx]^2}{g(bf-ag)} + \frac{2bBn \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{g(bf-ag)} -$$

$$\frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{g(f+gx)} + \frac{2B^2d n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{g(df-cg)} -$$

$$\frac{2Bdn \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[c+dx]}{g(df-cg)} - \frac{B^2d n^2 \operatorname{Log}[c+dx]^2}{g(df-cg)} +$$

$$\frac{2bB^2n^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{g(bf-ag)} - \frac{2B^2(bc-ad)n^2 \operatorname{Log}\left[-\frac{g(a+bx)}{bf-ag}\right] \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} +$$

$$\frac{2B(bc-ad)n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)n^2 \operatorname{Log}\left[-\frac{g(c+dx)}{df-cg}\right] \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} +$$

$$\frac{2bB^2n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{g(bf-ag)} + \frac{2B^2d n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{g(df-cg)} -$$

$$\frac{2B^2(bc-ad)n^2 \operatorname{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right]}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)n^2 \operatorname{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right]}{(bf-ag)(df-cg)}$$

### Problem 73: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(f+gx)^3} dx$$

Optimal (type 4, 389 leaves, 9 steps):

$$\begin{aligned}
& \frac{B (b c - a d) g n (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b f - a g)^2 (d f - c g) (f + g x)} + \\
& \frac{b^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 g (b f - a g)^2} - \frac{\left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 g (f + g x)^2} + \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log} \left[ \frac{f + g x}{c + d x} \right]}{(b f - a g)^2 (d f - c g)^2} + \\
& \left( B (b c - a d) (2 b d f - b c g - a d g) n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)} \right] \right) / \\
& \left( (b f - a g)^2 (d f - c g)^2 \right) + \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{PolyLog} \left[ 2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)} \right]}{(b f - a g)^2 (d f - c g)^2}
\end{aligned}$$

Result (type 4, 941 leaves, 33 steps):

$$\begin{aligned}
& \frac{b B^2 (b c - a d) n^2 \operatorname{Log} [a + b x]}{(b f - a g)^2 (d f - c g)} - \frac{b^2 B^2 n^2 \operatorname{Log} [a + b x]^2}{2 g (b f - a g)^2} - \frac{B (b c - a d) n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b f - a g) (d f - c g) (f + g x)} + \\
& \frac{b^2 B n \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{g (b f - a g)^2} - \frac{\left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 g (f + g x)^2} - \\
& \frac{B^2 d (b c - a d) n^2 \operatorname{Log} [c + d x]}{(b f - a g) (d f - c g)^2} + \frac{B^2 d^2 n^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{g (d f - c g)^2} - \\
& \frac{B d^2 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} [c + d x]}{g (d f - c g)^2} - \frac{B^2 d^2 n^2 \operatorname{Log} [c + d x]^2}{2 g (d f - c g)^2} + \\
& \frac{b^2 B^2 n^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{g (b f - a g)^2} + \frac{B^2 (b c - a d)^2 g n^2 \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} - \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{Log} \left[ -\frac{g (a + b x)}{b f - a g} \right] \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \left( B (b c - a d) (2 b d f - b c g - a d g) n \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} [f + g x] \right) / \\
& \left( (b f - a g)^2 (d f - c g)^2 \right) + \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{Log} \left[ -\frac{g (c + d x)}{d f - c g} \right] \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{b^2 B^2 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{g (b f - a g)^2} + \frac{B^2 d^2 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{g (d f - c g)^2} - \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{PolyLog} \left[ 2, \frac{b (f + g x)}{b f - a g} \right]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{PolyLog} \left[ 2, \frac{d (f + g x)}{d f - c g} \right]}{(b f - a g)^2 (d f - c g)^2}
\end{aligned}$$

### Problem 74: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(f+gx)^4} dx$$

Optimal (type 4, 747 leaves, 12 steps):

$$\begin{aligned} & \frac{B^2 (bc - ad)^2 g^2 n^2 (c + dx)}{3 (bf - ag)^2 (df - cg)^3 (f + gx)} - \frac{B (bc - ad) g^2 n (c + dx)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3 (bf - ag) (df - cg)^3 (f + gx)^2} + \\ & \left(2 B (bc - ad) g (3 bdf - bcg - 2 adg) n (a + bx) \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)\right) / \\ & \left(3 (bf - ag)^3 (df - cg)^2 (f + gx)\right) + \frac{b^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3 g (bf - ag)^3} - \\ & \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3 g (f + gx)^3} + \frac{B^2 (bc - ad)^3 g^2 n^2 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{3 (bf - ag)^3 (df - cg)^3} - \frac{B^2 (bc - ad)^3 g^2 n^2 \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{3 (bf - ag)^3 (df - cg)^3} + \\ & \frac{2 B^2 (bc - ad)^2 g (3 bdf - bcg - 2 adg) n^2 \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{3 (bf - ag)^3 (df - cg)^3} + \\ & \left(2 B (bc - ad) (a^2 d^2 g^2 - abdg (3 df - cg) + b^2 (3 d^2 f^2 - 3 cdfg + c^2 g^2)) n \right. \\ & \left. \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)}\right]\right) / \left(3 (bf - ag)^3 (df - cg)^3\right) + \\ & \left(2 B^2 (bc - ad) (a^2 d^2 g^2 - abdg (3 df - cg) + b^2 (3 d^2 f^2 - 3 cdfg + c^2 g^2)) \right. \\ & \left. n^2 \operatorname{PolyLog}\left[2, \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)}\right]\right) / \left(3 (bf - ag)^3 (df - cg)^3\right) \end{aligned}$$

Result (type 4, 1427 leaves, 37 steps):

$$\begin{aligned} & - \frac{B^2 (bc - ad)^2 g n^2}{3 (bf - ag)^2 (df - cg)^2 (f + gx)} + \frac{b^2 B^2 (bc - ad) n^2 \operatorname{Log}[a + bx]}{3 (bf - ag)^3 (df - cg)} + \\ & \frac{2 b B^2 (bc - ad) (2 bdf - bcg - adg) n^2 \operatorname{Log}[a + bx]}{3 (bf - ag)^3 (df - cg)^2} - \\ & \frac{b^3 B^2 n^2 \operatorname{Log}[a + bx]^2}{3 g (bf - ag)^3} - \frac{B (bc - ad) n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3 (bf - ag) (df - cg) (f + gx)^2} - \\ & \frac{2 B (bc - ad) (2 bdf - bcg - adg) n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3 (bf - ag)^2 (df - cg)^2 (f + gx)} + \\ & \frac{2 b^3 B n \operatorname{Log}[a + bx] \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3 g (bf - ag)^3} - \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3 g (f + gx)^3} - \end{aligned}$$



$$\begin{aligned}
& \frac{B^2 d^2 (b c - a d) n^2 \operatorname{Log}[c + d x]}{3 (b f - a g) (d f - c g)^3} - \frac{2 B^2 d (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{Log}[c + d x]}{3 (b f - a g)^2 (d f - c g)^3} + \\
& \frac{2 B^2 d^3 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \frac{2 B d^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \\
& \frac{B^2 d^3 n^2 \operatorname{Log}[c + d x]^2}{3 g (d f - c g)^3} + \frac{2 b^3 B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{3 g (b f - a g)^3} + \\
& \frac{B^2 (b c - a d)^2 g (2 b d f - b c g - a d g) n^2 \operatorname{Log}[f + g x]}{(b f - a g)^3 (d f - c g)^3} - \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{Log}\left[-\frac{g(a+b x)}{b f - a g}\right] \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) n \right. \\
& \quad \left. \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{Log}\left[-\frac{g(c+d x)}{d f - c g}\right] \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \frac{2 b^3 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{3 g (b f - a g)^3} + \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{3 g (d f - c g)^3} - \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{PolyLog}\left[2, \frac{b(f+g x)}{b f - a g}\right] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{PolyLog}\left[2, \frac{d(f+g x)}{d f - c g}\right] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right)
\end{aligned}$$

**Problem 75: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(f + g x)^5} dx$$

Optimal (type 4, 1208 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g^3 n^2 (c + dx)^2}{12 (bf - ag)^2 (df - cg)^4 (f + gx)^2} - \frac{B^2 (bc - ad)^3 g^3 n^2 (c + dx)}{6 (bf - ag)^3 (df - cg)^4 (f + gx)} + \\
& \frac{B^2 (bc - ad)^2 g^2 (4bdf - bcg - 3adg) n^2 (c + dx)}{4 (bf - ag)^3 (df - cg)^4 (f + gx)} + \\
& \frac{B (bc - ad) g^3 n (c + dx)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{6 (bf - ag) (df - cg)^4 (f + gx)^3} - \\
& \left( B (bc - ad) g^2 (4bdf - bcg - 3adg) n (c + dx)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \right) / \\
& \left( 4 (bf - ag)^2 (df - cg)^4 (f + gx)^2 \right) + \\
& \left( B (bc - ad) g (3a^2 d^2 g^2 - 2abd g (4df - cg) + b^2 (6d^2 f^2 - 4cdfg + c^2 g^2)) \right. \\
& \left. n (a + bx) \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \right) / \left( 2 (bf - ag)^4 (df - cg)^3 (f + gx) \right) + \\
& \frac{b^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{4 g (bf - ag)^4} - \frac{\left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{4 g (f + gx)^4} - \frac{B^2 (bc - ad)^4 g^3 n^2 \operatorname{Log} \left[ \frac{a+bx}{c+dx} \right]}{6 (bf - ag)^4 (df - cg)^4} + \\
& \frac{B^2 (bc - ad)^3 g^2 (4bdf - bcg - 3adg) n^2 \operatorname{Log} \left[ \frac{a+bx}{c+dx} \right]}{4 (bf - ag)^4 (df - cg)^4} + \\
& \frac{B^2 (bc - ad)^4 g^3 n^2 \operatorname{Log} \left[ \frac{f+gx}{c+dx} \right]}{6 (bf - ag)^4 (df - cg)^4} - \frac{B^2 (bc - ad)^3 g^2 (4bdf - bcg - 3adg) n^2 \operatorname{Log} \left[ \frac{f+gx}{c+dx} \right]}{4 (bf - ag)^4 (df - cg)^4} + \\
& \left( B^2 (bc - ad)^2 g (3a^2 d^2 g^2 - 2abd g (4df - cg) + b^2 (6d^2 f^2 - 4cdfg + c^2 g^2)) n^2 \operatorname{Log} \left[ \frac{f+gx}{c+dx} \right] \right) / \\
& \left( 2 (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B (bc - ad) (2bdf - bcg - adg) (2abd^2 fg - a^2 d^2 g^2 - b^2 (2d^2 f^2 - 2cdfg + c^2 g^2)) n \right. \\
& \left. \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)} \right] \right) / \left( 2 (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B^2 (bc - ad) (2bdf - bcg - adg) (2abd^2 fg - a^2 d^2 g^2 - b^2 (2d^2 f^2 - 2cdfg + c^2 g^2)) \right. \\
& \left. n^2 \operatorname{PolyLog} \left[ 2, \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)} \right] \right) / \left( 2 (bf - ag)^4 (df - cg)^4 \right)
\end{aligned}$$

Result (type 4, 1968 leaves, 41 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g n^2}{12 (bf - ag)^2 (df - cg)^2 (f + gx)^2} - \frac{5 B^2 (bc - ad)^2 g (2bdf - bcg - adg) n^2}{12 (bf - ag)^3 (df - cg)^3 (f + gx)} + \\
& \frac{b^3 B^2 (bc - ad) n^2 \operatorname{Log} [a + bx]}{6 (bf - ag)^4 (df - cg)} + \frac{b^2 B^2 (bc - ad) (2bdf - bcg - adg) n^2 \operatorname{Log} [a + bx]}{4 (bf - ag)^4 (df - cg)^2} + \\
& \left( b B^2 (bc - ad) (a^2 d^2 g^2 - abd g (3df - cg) + b^2 (3d^2 f^2 - 3cdfg + c^2 g^2)) n^2 \operatorname{Log} [a + bx] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( 2 (b f - a g)^4 (d f - c g)^3 \right) - \frac{b^4 B^2 n^2 \operatorname{Log}[a + b x]^2}{4 g (b f - a g)^4} - \frac{B (b c - a d) n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 (b f - a g) (d f - c g) (f + g x)^3} - \\
& \frac{B (b c - a d) (2 b d f - b c g - a d g) n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 (b f - a g)^2 (d f - c g)^2 (f + g x)^2} - \\
& \left( B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \right) / \left( 2 (b f - a g)^3 (d f - c g)^3 (f + g x) \right) + \\
& \frac{b^4 B n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 g (b f - a g)^4} - \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 g (f + g x)^4} - \\
& \frac{B^2 d^3 (b c - a d) n^2 \operatorname{Log}[c + d x]}{6 (b f - a g) (d f - c g)^4} - \frac{B^2 d^2 (b c - a d) (2 b d f - b c g - a d g) n^2 \operatorname{Log}[c + d x]}{4 (b f - a g)^2 (d f - c g)^4} - \\
& \left( B^2 d (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) n^2 \operatorname{Log}[c + d x] \right) / \\
& \left( 2 (b f - a g)^3 (d f - c g)^4 \right) + \frac{B^2 d^4 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 g (d f - c g)^4} - \\
& \frac{B d^4 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{2 g (d f - c g)^4} - \frac{B^2 d^4 n^2 \operatorname{Log}[c + d x]^2}{4 g (d f - c g)^4} + \\
& \frac{b^4 B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 g (b f - a g)^4} + \frac{B^2 (b c - a d)^2 g (2 b d f - b c g - a d g)^2 n^2 \operatorname{Log}[f + g x]}{4 (b f - a g)^4 (d f - c g)^4} + \\
& \left( 2 B^2 (b c - a d)^2 g (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) n^2 \operatorname{Log}[f + g x] \right) / \\
& \left( 3 (b f - a g)^4 (d f - c g)^4 \right) + \\
& \left( B^2 (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x] \right) / \left( 2 (b f - a g)^4 (d f - c g)^4 \right) - \\
& \left( B (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[f + g x] \right) / \left( 2 (b f - a g)^4 (d f - c g)^4 \right) - \\
& \left( B^2 (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. n^2 \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x] \right) / \left( 2 (b f - a g)^4 (d f - c g)^4 \right) + \\
& \frac{b^4 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 g (b f - a g)^4} + \frac{B^2 d^4 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{2 g (d f - c g)^4} + \\
& \left( B^2 (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right)
\end{aligned}$$

$$\begin{aligned} & n^2 \text{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right] \Big/ \left(2(bf-ag)^4(df-cg)^4\right) - \\ & \left(B^2(bc-ad)(2bdf-bcg-adg)(2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2))\right) \\ & n^2 \text{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right] \Big/ \left(2(bf-ag)^4(df-cg)^4\right) \end{aligned}$$

**Problem 76: Result valid but suboptimal antiderivative.**

$$\int \frac{(f+gx)^2}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f+gx)^2}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$\begin{aligned} & f^2 \text{CannotIntegrate}\left[\frac{1}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right] + \\ & 2fg \text{CannotIntegrate}\left[\frac{x}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right] + g^2 \text{CannotIntegrate}\left[\frac{x^2}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right] \end{aligned}$$

**Problem 77: Result valid but suboptimal antiderivative.**

$$\int \frac{f+gx}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{f+gx}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$f \text{CannotIntegrate}\left[\frac{1}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right] + g \text{CannotIntegrate}\left[\frac{x}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

**Problem 78: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{A+B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

Result (type 8, 26 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}, x\right]$$

**Problem 79: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f+gx) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+gx) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+gx) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

**Problem 80: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f+gx)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+gx)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+gx)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

**Problem 81: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f+gx)^3 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+gx)^3 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f + g x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}, x\right]$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x)^2}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f + g x)^2}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$\begin{aligned} & f^2 \text{ CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right] + \\ & 2 f g \text{ CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right] + \\ & g^2 \text{ CannotIntegrate}\left[\frac{x^2}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right] \end{aligned}$$

Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{f + g x}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$f \text{ CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right] + g \text{ CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}, x\right]$$

Problem 84: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

Result (type 8, 26 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

**Problem 85: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

**Problem 86: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f + g x)^2 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x)^2 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f + g x)^2 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2}, x\right]$$

**Problem 87: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(f + g x)^3 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+g x)^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+g x)^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}, x\right]$$

**Problem 92: Result valid but suboptimal antiderivative.**

$$\int \frac{A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{a g+b g x} d x$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{\operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{b g}+\frac{B \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{b g}$$

Result (type 4, 120 leaves, 10 steps):

$$-\frac{B \operatorname{Log}\left[g(a+b x)\right]^2}{2 b g}+\frac{\left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[a g+b g x]}{b g}+\frac{B \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[a g+b g x]}{b g}+\frac{B \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{b g}$$

**Problem 93: Result valid but suboptimal antiderivative.**

$$\int \frac{A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{(a g+b g x)^2} d x$$

Optimal (type 3, 63 leaves, 2 steps):

$$-\frac{B}{b g^2(a+b x)}-\frac{(c+d x)\left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{(b c-a d) g^2(a+b x)}$$

Result (type 3, 102 leaves, 4 steps):

$$-\frac{B}{b g^2(a+b x)}-\frac{B d \operatorname{Log}[a+b x]}{b(b c-a d) g^2}-\frac{A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{b g^2(a+b x)}+\frac{B d \operatorname{Log}[c+d x]}{b(b c-a d) g^2}$$

**Problem 97: Result valid but suboptimal antiderivative.**

$$\int (a g+b g x)^4 \left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2 d x$$

Optimal (type 4, 365 leaves, 8 steps):



$$\begin{aligned}
& - \frac{B (b c - a d) g^4 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b d} + \\
& \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 b} + \frac{B (b c - a d)^2 g^4 (a + b x)^3 \left( 4 A + B + 4 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{30 b d^2} - \\
& \frac{B (b c - a d)^3 g^4 (a + b x)^2 \left( 12 A + 7 B + 12 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{60 b d^3} + \\
& \frac{B (b c - a d)^4 g^4 (a + b x) \left( 12 A + 13 B + 12 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{30 b d^4} + \\
& \frac{B (b c - a d)^5 g^4 \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( 12 A + 25 B + 12 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{30 b d^5} + \\
& \frac{2 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{5 b d^5}
\end{aligned}$$

Result (type 4, 557 leaves, 28 steps):

$$\begin{aligned}
& \frac{2 A B (b c - a d)^4 g^4 x}{5 d^4} + \frac{13 B^2 (b c - a d)^4 g^4 x}{30 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 (a + b x)^2}{60 b d^3} + \\
& \frac{B^2 (b c - a d)^2 g^4 (a + b x)^3}{30 b d^2} + \frac{2 B^2 (b c - a d)^4 g^4 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{5 b d^4} - \\
& \frac{B (b c - a d)^3 g^4 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b d^3} + \frac{2 B (b c - a d)^2 g^4 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{15 b d^2} - \\
& \frac{B (b c - a d) g^4 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b d} + \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 b} - \\
& \frac{5 B^2 (b c - a d)^5 g^4 \operatorname{Log} [c + d x]}{6 b d^5} + \frac{2 B^2 (b c - a d)^5 g^4 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{5 b d^5} - \\
& \frac{2 B (b c - a d)^5 g^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{5 b d^5} - \\
& \frac{B^2 (b c - a d)^5 g^4 \operatorname{Log} [c + d x]^2}{5 b d^5} + \frac{2 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{5 b d^5}
\end{aligned}$$

**Problem 98: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 309 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g^3 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b d} + \\
& \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b} + \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left( 3 A + B + 3 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b d^2} - \\
& \frac{B (b c - a d)^3 g^3 (a + b x) \left( 6 A + 5 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b d^3} - \\
& \frac{B (b c - a d)^4 g^3 \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( 6 A + 11 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b d^4} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{2 b d^4}
\end{aligned}$$

Result (type 4, 474 leaves, 24 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^3 g^3 x}{2 d^3} - \frac{5 B^2 (b c - a d)^3 g^3 x}{12 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{12 b d^2} - \\
& \frac{B^2 (b c - a d)^3 g^3 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{2 b d^3} + \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b d^2} - \\
& \frac{B (b c - a d) g^3 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b d} + \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log} [c + d x]}{12 b d^4} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{2 b d^4} + \\
& \frac{B (b c - a d)^4 g^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{2 b d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log} [c + d x]^2}{4 b d^4} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{2 b d^4}
\end{aligned}$$

Problem 99: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 253 leaves, 6 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g^2 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b d} + \\
& \frac{g^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 b} + \frac{B (b c - a d)^2 g^2 (a + b x) \left( 2 A + B + 2 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b d^2} + \\
& \frac{B (b c - a d)^3 g^2 \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( 2 A + 3 B + 2 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b d^3} + \frac{2 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 389 leaves, 20 steps):

$$\begin{aligned}
& \frac{2AB(b c - a d)^2 g^2 x}{3 d^2} + \frac{B^2(b c - a d)^2 g^2 x}{3 d^2} + \frac{2B^2(b c - a d)^2 g^2 (a + b x) \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{3 b d^2} - \\
& \frac{B(b c - a d) g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{3 b d} + \frac{g^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{3 b} - \\
& \frac{B^2(b c - a d)^3 g^2 \operatorname{Log}[c + d x]}{b d^3} + \frac{2B^2(b c - a d)^3 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b d^3} - \\
& \frac{2B(b c - a d)^3 g^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[c + d x]}{3 b d^3} - \\
& \frac{B^2(b c - a d)^3 g^2 \operatorname{Log}[c + d x]^2}{3 b d^3} + \frac{2B^2(b c - a d)^3 g^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{3 b d^3}
\end{aligned}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2 dx$$

Optimal (type 4, 180 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g(a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{b d} + \frac{g(a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{2 b} - \\
& \frac{B(b c - a d)^2 g \operatorname{Log}\left[\frac{b c - a d}{b(c+d x)}\right] \left(A + B + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{b d^2} - \frac{B^2(b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{b d^2}
\end{aligned}$$

Result (type 4, 285 leaves, 16 steps):

$$\begin{aligned}
& - \frac{AB(b c - a d) g x}{d} - \frac{B^2(b c - a d) g(a + b x) \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{b d} + \\
& \frac{g(a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{2 b} + \frac{B^2(b c - a d)^2 g \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{B^2(b c - a d)^2 g \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^2} + \frac{B(b c - a d)^2 g \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[c + d x]}{b d^2} + \\
& \frac{B^2(b c - a d)^2 g \operatorname{Log}[c + d x]^2}{2 b d^2} - \frac{B^2(b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b d^2}
\end{aligned}$$

Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 128 leaves, 4 steps):

$$- \frac{\left( A + B \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right)^2 \operatorname{Log}\left[ 1 - \frac{b(c+dx)}{d(a+bx)} \right]}{bg} + \frac{2B \left( A + B \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right) \operatorname{PolyLog}\left[ 2, \frac{b(c+dx)}{d(a+bx)} \right]}{bg} + \frac{2B^2 \operatorname{PolyLog}\left[ 3, \frac{b(c+dx)}{d(a+bx)} \right]}{bg}$$

Result (type 4, 728 leaves, 46 steps):

$$\begin{aligned} & - \frac{AB \operatorname{Log}\left[ g(a+bx) \right]^2}{bg} + \frac{B^2 \operatorname{Log}\left[ g(a+bx) \right]^3}{3bg} - \frac{B^2 \operatorname{Log}\left[ a+bx \right]^2 \operatorname{Log}\left[ -c-dx \right]}{bg} + \\ & \frac{2B^2 \operatorname{Log}\left[ a+bx \right] \operatorname{Log}\left[ g(a+bx) \right] \operatorname{Log}\left[ -c-dx \right]}{bg} - \frac{B^2 \operatorname{Log}\left[ g(a+bx) \right]^2 \operatorname{Log}\left[ -c-dx \right]}{bg} + \\ & \frac{B^2 \operatorname{Log}\left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}\left[ \frac{1}{c+dx} \right]^2}{bg} - \frac{B^2 \operatorname{Log}\left[ g(a+bx) \right] \operatorname{Log}\left[ \frac{1}{c+dx} \right]^2}{bg} + \\ & \frac{B^2 \operatorname{Log}\left[ a+bx \right]^2 \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right]}{bg} + \frac{B^2 \operatorname{Log}\left[ g(a+bx) \right]^2 \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right]}{bg} + \\ & \frac{\left( A + B \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right)^2 \operatorname{Log}\left[ ag+bgx \right]}{bg} + \frac{2AB \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] \operatorname{Log}\left[ ag+bgx \right]}{bg} - \frac{1}{bg} \\ & 2B^2 \left( \operatorname{Log}\left[ a+bx \right] + \operatorname{Log}\left[ \frac{1}{c+dx} \right] - \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right) \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] \operatorname{Log}\left[ ag+bgx \right] - \\ & \frac{B^2 \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \operatorname{Log}\left[ ag+bgx \right]^2}{bg} - \frac{B^2 \operatorname{Log}\left[ \frac{b(c+dx)}{bc-ad} \right] \operatorname{Log}\left[ ag+bgx \right]^2}{bg} + \\ & \frac{2AB \operatorname{PolyLog}\left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{bg} + \frac{2B^2 \operatorname{Log}\left[ a+bx \right] \operatorname{PolyLog}\left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{bg} - \\ & \frac{2B^2 \left( \operatorname{Log}\left[ a+bx \right] + \operatorname{Log}\left[ \frac{1}{c+dx} \right] - \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right) \operatorname{PolyLog}\left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{bg} - \\ & \frac{2B^2 \operatorname{Log}\left[ \frac{1}{c+dx} \right] \operatorname{PolyLog}\left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{bg} - \frac{2B^2 \operatorname{PolyLog}\left[ 3, -\frac{d(a+bx)}{bc-ad} \right]}{bg} - \frac{2B^2 \operatorname{PolyLog}\left[ 3, \frac{b(c+dx)}{bc-ad} \right]}{bg} \end{aligned}$$

**Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[ \frac{e(a+bx)}{c+dx} \right] \right)^2}{(ag+bgx)^2} dx$$

Optimal (type 3, 126 leaves, 3 steps):

$$- \frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)g^2(a+bx)}$$

Result (type 4, 470 leaves, 26 steps):

$$\begin{aligned}
& - \frac{2 B^2}{b g^2 (a + b x)} - \frac{2 B^2 d \operatorname{Log}[a + b x]}{b (b c - a d) g^2} + \frac{B^2 d \operatorname{Log}[a + b x]^2}{b (b c - a d) g^2} - \frac{2 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b g^2 (a + b x)} - \\
& \frac{2 B d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b (b c - a d) g^2} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{b g^2 (a + b x)} + \frac{2 B^2 d \operatorname{Log}[c + d x]}{b (b c - a d) g^2} - \\
& \frac{2 B^2 d \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{2 B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{B^2 d \operatorname{Log}[c + d x]^2}{b (b c - a d) g^2} - \\
& \frac{2 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d) g^2} - \frac{2 B^2 d \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b (b c - a d) g^2} - \frac{2 B^2 d \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d) g^2}
\end{aligned}$$

**Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 268 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 B^2 d (c + d x)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B^2 (c + d x)^2}{4 (b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{2 B d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^2 g^3 (a + b x)} - \frac{b (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^2 g^3 (a + b x)^2}
\end{aligned}$$

Result (type 4, 577 leaves, 30 steps):

$$\begin{aligned}
& - \frac{B^2}{4 b g^3 (a + b x)^2} + \frac{3 B^2 d}{2 b (b c - a d) g^3 (a + b x)} + \frac{3 B^2 d^2 \operatorname{Log}[a + b x]}{2 b (b c - a d)^2 g^3} - \\
& \frac{B^2 d^2 \operatorname{Log}[a + b x]^2}{2 b (b c - a d)^2 g^3} - \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 b g^3 (a + b x)^2} + \frac{B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b (b c - a d) g^3 (a + b x)} + \\
& \frac{B d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b (b c - a d)^2 g^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 b g^3 (a + b x)^2} - \frac{3 B^2 d^2 \operatorname{Log}[c + d x]}{2 b (b c - a d)^2 g^3} + \\
& \frac{B^2 d^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{B d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{B^2 d^2 \operatorname{Log}[c + d x]^2}{2 b (b c - a d)^2 g^3} + \\
& \frac{B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3}
\end{aligned}$$

### Problem 104: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag + bgx)^4} dx$$

Optimal (type 3, 418 leaves, 9 steps):

$$\begin{aligned} & -\frac{2B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{bB^2d(c+dx)^2}{2(bc-ad)^3g^4(a+bx)^2} - \frac{2b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \\ & \frac{2Bd^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3g^4(a+bx)} + \frac{bBd(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3g^4(a+bx)^2} - \\ & \frac{2b^2B(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3g^4(a+bx)} + \\ & \frac{bd(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3(bc-ad)^3g^4(a+bx)^3} \end{aligned}$$

Result (type 4, 680 leaves, 34 steps):

$$\begin{aligned} & -\frac{2B^2}{27bg^4(a+bx)^3} + \frac{5B^2d}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \\ & \frac{11B^2d^3\operatorname{Log}[a+bx]}{9b(bc-ad)^3g^4} + \frac{B^2d^3\operatorname{Log}[a+bx]^2}{3b(bc-ad)^3g^4} - \frac{2B\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9bg^4(a+bx)^3} + \frac{Bd\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3b(bc-ad)g^4(a+bx)^2} - \\ & \frac{2Bd^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3b(bc-ad)^2g^4(a+bx)} - \frac{2Bd^3\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3b(bc-ad)^3g^4} - \\ & \frac{\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3bg^4(a+bx)^3} + \frac{11B^2d^3\operatorname{Log}[c+dx]}{9b(bc-ad)^3g^4} - \frac{2B^2d^3\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{3b(bc-ad)^3g^4} + \\ & \frac{2Bd^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)\operatorname{Log}[c+dx]}{3b(bc-ad)^3g^4} + \frac{B^2d^3\operatorname{Log}[c+dx]^2}{3b(bc-ad)^3g^4} - \\ & \frac{2B^2d^3\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{3b(bc-ad)^3g^4} - \frac{2B^2d^3\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{3b(bc-ad)^3g^4} - \frac{2B^2d^3\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{3b(bc-ad)^3g^4} \end{aligned}$$

### Problem 105: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag + bgx)^5} dx$$

Optimal (type 3, 575 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 B^2 d^3 (c + d x)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B^2 d^2 (c + d x)^2}{4 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{2 b^2 B^2 d (c + d x)^3}{9 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B^2 (c + d x)^4}{32 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{2 B d^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B d^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{2 b^2 B d (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{8 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{d^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b d^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{b^2 d (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 (b c - a d)^4 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 763 leaves, 38 steps):

$$\begin{aligned}
& - \frac{B^2}{32 b g^5 (a + b x)^4} + \frac{7 B^2 d}{72 b (b c - a d) g^5 (a + b x)^3} - \frac{13 B^2 d^2}{48 b (b c - a d)^2 g^5 (a + b x)^2} + \\
& \frac{25 B^2 d^3}{24 b (b c - a d)^3 g^5 (a + b x)} + \frac{25 B^2 d^4 \operatorname{Log}[a + b x]}{24 b (b c - a d)^4 g^5} - \frac{B^2 d^4 \operatorname{Log}[a + b x]^2}{4 b (b c - a d)^4 g^5} - \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{8 b g^5 (a + b x)^4} + \\
& \frac{B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{6 b (b c - a d) g^5 (a + b x)^3} - \frac{B d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 b (b c - a d)^2 g^5 (a + b x)^2} + \frac{B d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 b (b c - a d)^3 g^5 (a + b x)} + \\
& \frac{B d^4 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 b (b c - a d)^4 g^5} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 b g^5 (a + b x)^4} - \frac{25 B^2 d^4 \operatorname{Log}[c + d x]}{24 b (b c - a d)^4 g^5} + \\
& \frac{B^2 d^4 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 b (b c - a d)^4 g^5} - \frac{B d^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{2 b (b c - a d)^4 g^5} - \frac{B^2 d^4 \operatorname{Log}[c + d x]^2}{4 b (b c - a d)^4 g^5} + \\
& \frac{B^2 d^4 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5} + \frac{B^2 d^4 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5} + \frac{B^2 d^4 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5}
\end{aligned}$$

Problem 109: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$a^2 g^2 \text{ CannotIntegrate} \left[ \frac{1}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right] +$$

$$2 a b g^2 \text{ CannotIntegrate} \left[ \frac{x}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right] + b^2 g^2 \text{ CannotIntegrate} \left[ \frac{x^2}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right]$$

Problem 110: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{a g + b g x}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right]$$

Result (type 8, 55 leaves, 2 steps):

$$a g \text{ CannotIntegrate} \left[ \frac{1}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right] + b g \text{ CannotIntegrate} \left[ \frac{x}{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}, x \right]$$

Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left( A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(a g + b g x) \left( A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}, x \right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(a g + b g x) \left( A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}, x \right]$$

Problem 112: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left( A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)} dx$$

Optimal (type 4, 50 leaves, 3 steps):

$$\frac{e^{A/B} \text{ExpIntegralEi} \left[ -\frac{A + B \text{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}{B} \right]}{B (b c - a d) g^2}$$



Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}, x\right]$$

Problem 113: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)} dx$$

Optimal (type 4, 107 leaves, 7 steps):

$$\frac{b e^2 e^{\frac{2A}{B}} \operatorname{ExpIntegralEi}\left[-\frac{2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{B}\right]}{B (b c - a d)^2 g^3} - \frac{d e e^{A/B} \operatorname{ExpIntegralEi}\left[-\frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{B}\right]}{B (b c - a d)^2 g^3}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}, x\right]$$

Problem 114: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a g + b g x)^2}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$\begin{aligned} & a^2 g^2 \text{CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}, x\right] + \\ & 2 a b g^2 \text{CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}, x\right] + \\ & b^2 g^2 \text{CannotIntegrate}\left[\frac{x^2}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}, x\right] \end{aligned}$$

Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$a g \text{ CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right] + b g \text{ CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

**Problem 116: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

**Problem 117: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2} dx$$

Optimal (type 4, 103 leaves, 4 steps):

$$-\frac{e^{A/B} \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{B}\right]}{B^2 (b c - a d) g^2} - \frac{c + d x}{B (b c - a d) g^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}, x\right]$$

**Problem 118: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2} dx$$

Optimal (type 4, 212 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 b e^2 e^{\frac{2A}{B}} \text{ExpIntegralEi} \left[ -\frac{2 \left( A+B \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)}{B} \right]}{B^2 (bc-ad)^2 g^3} + \frac{d e e^{A/B} \text{ExpIntegralEi} \left[ -\frac{A+B \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right]}{B} \right]}{B^2 (bc-ad)^2 g^3} + \\
& \frac{d (c+dx)}{B (bc-ad)^2 g^3 (a+bx) \left( A+B \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)} - \frac{b (c+dx)^2}{B (bc-ad)^2 g^3 (a+bx)^2 \left( A+B \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)}
\end{aligned}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(ag+bgx)^3 \left( A+B \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)^2}, x \right]$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right]}{ag+bgx} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$\begin{aligned}
& - \frac{\text{Log} \left[ -\frac{bc-ad}{d(a+bx)} \right] \left( A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right] \right)}{bg} + \frac{2 B \text{PolyLog} \left[ 2, 1 + \frac{bc-ad}{d(a+bx)} \right]}{bg}
\end{aligned}$$

Result (type 4, 122 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B \text{Log} [g(a+bx)]^2}{bg} + \frac{\left( A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right] \right) \text{Log} [ag+bgx]}{bg} + \\
& \frac{2 B \text{Log} \left[ \frac{b(c+dx)}{bc-ad} \right] \text{Log} [ag+bgx]}{bg} + \frac{2 B \text{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{bg}
\end{aligned}$$

Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right]}{(ag+bgx)^2} dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\begin{aligned}
& - \frac{2 B}{bg^2 (a+bx)} - \frac{(c+dx) \left( A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right] \right)}{(bc-ad) g^2 (a+bx)}
\end{aligned}$$

Result (type 3, 105 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 B}{bg^2 (a+bx)} - \frac{2 B d \text{Log} [a+bx]}{b (bc-ad) g^2} - \frac{A+B \text{Log} \left[ \frac{e(a+bx)^2}{(c+dx)^2} \right]}{bg^2 (a+bx)} + \frac{2 B d \text{Log} [c+dx]}{b (bc-ad) g^2}
\end{aligned}$$

### Problem 128: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 377 leaves, 8 steps):

$$\begin{aligned} & - \frac{B (b c - a d) g^4 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{5 b d} + \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{5 b} + \\ & - \frac{2 B (b c - a d)^2 g^4 (a + b x)^3 \left( 2 A + B + 2 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{15 b d^2} - \\ & + \frac{B (b c - a d)^3 g^4 (a + b x)^2 \left( 6 A + 7 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{15 b d^3} + \\ & + \frac{2 B (b c - a d)^4 g^4 (a + b x) \left( 6 A + 13 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{15 b d^4} + \\ & + \frac{2 B (b c - a d)^5 g^4 \left( 6 A + 25 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{15 b d^5} + \\ & + \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{5 b d^5} \end{aligned}$$

Result (type 4, 569 leaves, 28 steps):

$$\begin{aligned} & \frac{4 A B (b c - a d)^4 g^4 x}{5 d^4} + \frac{26 B^2 (b c - a d)^4 g^4 x}{15 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 (a + b x)^2}{15 b d^3} + \\ & + \frac{2 B^2 (b c - a d)^2 g^4 (a + b x)^3}{15 b d^2} + \frac{4 B^2 (b c - a d)^4 g^4 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right]}{5 b d^4} - \\ & + \frac{2 B (b c - a d)^3 g^4 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{5 b d^3} + \\ & - \frac{4 B (b c - a d)^2 g^4 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{15 b d^2} - \\ & + \frac{B (b c - a d) g^4 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{5 b d} + \frac{g^4 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{5 b} - \\ & - \frac{10 B^2 (b c - a d)^5 g^4 \operatorname{Log} [c + d x]}{3 b d^5} + \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{5 b d^5} - \\ & - \frac{4 B (b c - a d)^5 g^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} [c + d x]}{5 b d^5} - \\ & + \frac{4 B^2 (b c - a d)^5 g^4 \operatorname{Log} [c + d x]^2}{5 b d^5} + \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{5 b d^5} \end{aligned}$$

### Problem 129: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 319 leaves, 7 steps):

$$\begin{aligned} & - \frac{B (b c - a d) g^3 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d} + \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{4 b} + \\ & \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left( 3 A + 2 B + 3 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{6 b d^2} - \\ & \frac{B (b c - a d)^3 g^3 (a + b x) \left( 3 A + 5 B + 3 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d^3} - \\ & \frac{B (b c - a d)^4 g^3 \left( 3 A + 11 B + 3 B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{3 b d^4} - \\ & \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b d^4} \end{aligned}$$

Result (type 4, 470 leaves, 24 steps):

$$\begin{aligned} & - \frac{A B (b c - a d)^3 g^3 x}{d^3} - \frac{5 B^2 (b c - a d)^3 g^3 x}{3 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{3 b d^2} - \\ & \frac{B^2 (b c - a d)^3 g^3 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right]}{b d^3} + \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{2 b d^2} - \\ & \frac{B (b c - a d) g^3 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d} + \frac{g^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{4 b} + \\ & \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log} [c + d x]}{3 b d^4} - \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{b d^4} + \\ & \frac{B (b c - a d)^4 g^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} [c + d x]}{b d^4} + \\ & \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log} [c + d x]^2}{b d^4} - \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{b d^4} \end{aligned}$$

### Problem 130: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 255 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g^2 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d} + \\
& \frac{g^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{3 b} + \frac{4 B (b c - a d)^2 g^2 (a + b x) \left( A + B + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d^2} + \\
& \frac{4 B (b c - a d)^3 g^2 \left( A + 3 B + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{3 b d^3} + \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 397 leaves, 20 steps):

$$\begin{aligned}
& \frac{4 A B (b c - a d)^2 g^2 x}{3 d^2} + \frac{4 B^2 (b c - a d)^2 g^2 x}{3 d^2} + \frac{4 B^2 (b c - a d)^2 g^2 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right]}{3 b d^2} - \\
& \frac{2 B (b c - a d) g^2 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d} + \frac{g^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{3 b} - \\
& \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log} [c + d x]}{b d^3} + \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{3 b d^3} - \\
& \frac{4 B (b c - a d)^3 g^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} [c + d x]}{3 b d^3} - \\
& \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log} [c + d x]^2}{3 b d^3} + \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{3 b d^3}
\end{aligned}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 188 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{b d} + \frac{g (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{2 b} - \\
& \frac{2 B (b c - a d)^2 g \left( A + 2 B + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{b d^2} - \frac{4 B^2 (b c - a d)^2 g \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b d^2}
\end{aligned}$$

Result (type 4, 291 leaves, 16 steps):

$$\begin{aligned}
& - \frac{2 A B (b c - a d) g x}{d} - \frac{2 B^2 (b c - a d) g (a + b x) \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]}{b d} + \\
& \frac{g (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{2 b} + \frac{4 B^2 (b c - a d)^2 g \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{4 B^2 (b c - a d)^2 g \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^2} + \frac{2 B (b c - a d)^2 g \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \operatorname{Log}[c + d x]}{b d^2} + \\
& \frac{2 B^2 (b c - a d)^2 g \operatorname{Log}[c + d x]^2}{b d^2} - \frac{4 B^2 (b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b d^2}
\end{aligned}$$

**Problem 132: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 132 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b g} + \\
& \frac{4 B \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b g} + \frac{8 B^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b g}
\end{aligned}$$

Result (type 4, 749 leaves, 46 steps):

$$\begin{aligned}
& - \frac{2AB \operatorname{Log}\left[g(a+bx)\right]^2}{bg} + \frac{4B^2 \operatorname{Log}\left[g(a+bx)\right]^3}{3bg} - \frac{4B^2 \operatorname{Log}\left[g(a+bx)\right]^2 \operatorname{Log}[-c-dx]}{bg} + \\
& \frac{4B^2 \operatorname{Log}\left[g(a+bx)\right] \operatorname{Log}\left[(a+bx)^2\right] \operatorname{Log}[-c-dx]}{bg} - \frac{B^2 \operatorname{Log}\left[(a+bx)^2\right]^2 \operatorname{Log}[-c-dx]}{bg} + \\
& \frac{B^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[\frac{1}{(c+dx)^2}\right]^2}{bg} - \frac{B^2 \operatorname{Log}\left[g(a+bx)\right] \operatorname{Log}\left[\frac{1}{(c+dx)^2}\right]^2}{bg} + \\
& \frac{4B^2 \operatorname{Log}\left[g(a+bx)\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{bg} + \frac{B^2 \operatorname{Log}\left[(a+bx)^2\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{bg} + \\
& \frac{\left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)^2 \operatorname{Log}[ag+bgx]}{bg} + \frac{4AB \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag+bgx]}{bg} - \frac{1}{bg} \\
& 4B^2 \left( \operatorname{Log}\left[(a+bx)^2\right] + \operatorname{Log}\left[\frac{1}{(c+dx)^2}\right] - \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \right) \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag+bgx] - \\
& \frac{2B^2 \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \operatorname{Log}[ag+bgx]^2}{bg} - \frac{4B^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag+bgx]^2}{bg} + \\
& \frac{4AB \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{bg} + \frac{4B^2 \operatorname{Log}\left[(a+bx)^2\right] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{bg} - \frac{1}{bg} \\
& 4B^2 \left( \operatorname{Log}\left[(a+bx)^2\right] + \operatorname{Log}\left[\frac{1}{(c+dx)^2}\right] - \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \right) \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right] - \\
& \frac{4B^2 \operatorname{Log}\left[\frac{1}{(c+dx)^2}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{bg} - \frac{8B^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{bg} - \frac{8B^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{bg}
\end{aligned}$$

**Problem 133:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)^2}{(ag+bgx)^2} dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$- \frac{8B^2(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{4B(c+dx)\left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)^2}{(bc-ad)g^2(a+bx)}$$

Result (type 4, 480 leaves, 26 steps):



$$\begin{aligned}
& - \frac{8 B^2}{b g^2 (a + b x)} - \frac{8 B^2 d \operatorname{Log}[a + b x]}{b (b c - a d) g^2} + \frac{4 B^2 d \operatorname{Log}[a + b x]^2}{b (b c - a d) g^2} - \frac{4 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b g^2 (a + b x)} - \\
& \frac{4 B d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b (b c - a d) g^2} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{b g^2 (a + b x)} + \frac{8 B^2 d \operatorname{Log}[c + d x]}{b (b c - a d) g^2} - \\
& \frac{8 B^2 d \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{4 B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{4 B^2 d \operatorname{Log}[c + d x]^2}{b (b c - a d) g^2} - \\
& \frac{8 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d) g^2} - \frac{8 B^2 d \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b (b c - a d) g^2} - \frac{8 B^2 d \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d) g^2}
\end{aligned}$$

**Problem 134: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 272 leaves, 7 steps):

$$\begin{aligned}
& \frac{8 B^2 d (c + d x)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B^2 (c + d x)^2}{(b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{4 B d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{(b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{(b c - a d)^2 g^3 (a + b x)} - \frac{b (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{2 (b c - a d)^2 g^3 (a + b x)^2}
\end{aligned}$$

Result (type 4, 579 leaves, 30 steps):

$$\begin{aligned}
& - \frac{B^2}{b g^3 (a + b x)^2} + \frac{6 B^2 d}{b (b c - a d) g^3 (a + b x)} + \frac{6 B^2 d^2 \operatorname{Log}[a + b x]}{b (b c - a d)^2 g^3} - \\
& \frac{2 B^2 d^2 \operatorname{Log}[a + b x]^2}{b (b c - a d)^2 g^3} - \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b g^3 (a + b x)^2} + \frac{2 B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b (b c - a d) g^3 (a + b x)} + \\
& \frac{2 B d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b (b c - a d)^2 g^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{2 b g^3 (a + b x)^2} - \frac{6 B^2 d^2 \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} + \\
& \frac{4 B^2 d^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{2 B d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{2 B^2 d^2 \operatorname{Log}[c + d x]^2}{b (b c - a d)^2 g^3} + \\
& \frac{4 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{4 B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{4 B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3}
\end{aligned}$$

### Problem 135: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{(ag + bgx)^4} dx$$

Optimal (type 3, 429 leaves, 9 steps):

$$\begin{aligned} & -\frac{8B^2d^2(c+dx)}{(bc-ad)^3g^4(a+bx)} + \frac{2bB^2d(c+dx)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{8b^2B^2(c+dx)^3}{27(bc-ad)^3g^4(a+bx)^3} - \\ & \frac{4Bd^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{(bc-ad)^3g^4(a+bx)} + \frac{2bBd(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{(bc-ad)^3g^4(a+bx)^2} - \\ & \frac{4b^2B(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{9(bc-ad)^3g^4(a+bx)^3} - \frac{d^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{(bc-ad)^3g^4(a+bx)} + \\ & \frac{bd(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{(bc-ad)^3g^4(a+bx)^2} - \frac{b^2(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{3(bc-ad)^3g^4(a+bx)^3} \end{aligned}$$

Result (type 4, 692 leaves, 34 steps):

$$\begin{aligned} & -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3\operatorname{Log}[a+bx]}{9b(bc-ad)^3g^4} + \\ & \frac{4B^2d^3\operatorname{Log}[a+bx]^2}{3b(bc-ad)^3g^4} - \frac{4B\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{3b(bc-ad)g^4(a+bx)^2} - \\ & \frac{4Bd^2\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{3b(bc-ad)^2g^4(a+bx)} - \frac{4Bd^3\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{3b(bc-ad)^3g^4} - \\ & \frac{\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{3bg^4(a+bx)^3} + \frac{44B^2d^3\operatorname{Log}[c+dx]}{9b(bc-ad)^3g^4} - \frac{8B^2d^3\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-ad)}\right]\operatorname{Log}[c+dx]}{3b(bc-ad)^3g^4} + \\ & \frac{4Bd^3\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)\operatorname{Log}[c+dx]}{3b(bc-ad)^3g^4} + \frac{4B^2d^3\operatorname{Log}[c+dx]^2}{3b(bc-ad)^3g^4} - \\ & \frac{8B^2d^3\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{b(c-ad)}\right]}{3b(bc-ad)^3g^4} - \frac{8B^2d^3\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-ad)}\right]}{3b(bc-ad)^3g^4} - \frac{8B^2d^3\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b(c-ad)}\right]}{3b(bc-ad)^3g^4} \end{aligned}$$

### Problem 136: Result unnecessarily involves higher level functions.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{(ag + bgx)^5} dx$$

Optimal (type 3, 587 leaves, 11 steps):

$$\begin{aligned}
& \frac{8 B^2 d^3 (c + d x)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B^2 d^2 (c + d x)^2}{(b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{8 b^2 B^2 d (c + d x)^3}{9 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B^2 (c + d x)^4}{8 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{4 B d^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B d^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{(b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{4 b^2 B d (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 (b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 B (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{4 (b c - a d)^4 g^5 (a + b x)^4} + \\
& \frac{d^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b d^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{2 (b c - a d)^4 g^5 (a + b x)^2} + \\
& \frac{b^2 d (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{(b c - a d)^4 g^5 (a + b x)^3} - \frac{b^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{4 (b c - a d)^4 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 757 leaves, 38 steps):

$$\begin{aligned}
& -\frac{B^2}{8 b g^5 (a + b x)^4} + \frac{7 B^2 d}{18 b (b c - a d) g^5 (a + b x)^3} - \frac{13 B^2 d^2}{12 b (b c - a d)^2 g^5 (a + b x)^2} + \\
& \frac{25 B^2 d^3}{6 b (b c - a d)^3 g^5 (a + b x)} + \frac{25 B^2 d^4 \operatorname{Log}[a + b x]}{6 b (b c - a d)^4 g^5} - \frac{B^2 d^4 \operatorname{Log}[a + b x]^2}{b (b c - a d)^4 g^5} - \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{4 b g^5 (a + b x)^4} + \\
& \frac{B d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 b (b c - a d) g^5 (a + b x)^3} - \frac{B d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{2 b (b c - a d)^2 g^5 (a + b x)^2} + \frac{B d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b (b c - a d)^3 g^5 (a + b x)} + \\
& \frac{B d^4 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b (b c - a d)^4 g^5} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{4 b g^5 (a + b x)^4} - \frac{25 B^2 d^4 \operatorname{Log}[c + d x]}{6 b (b c - a d)^4 g^5} + \\
& \frac{2 B^2 d^4 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^4 g^5} - \frac{B d^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{b (b c - a d)^4 g^5} - \frac{B^2 d^4 \operatorname{Log}[c + d x]^2}{b (b c - a d)^4 g^5} + \\
& \frac{2 B^2 d^4 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^4 g^5} + \frac{2 B^2 d^4 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b (b c - a d)^4 g^5} + \frac{2 B^2 d^4 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b (b c - a d)^4 g^5}
\end{aligned}$$

**Problem 137: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a g + b g x)^2}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$a^2 g^2 \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right] +$$

$$2 a b g^2 \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right] + b^2 g^2 \text{CannotIntegrate}\left[\frac{x^2}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right]$$

Problem 138: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$a g \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right] + b g \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}, x\right]$$

Problem 139: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)}, x\right]$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)}, x\right]$$

Problem 140: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)} dx$$

Optimal (type 4, 91 leaves, 3 steps):

$$\frac{e^{\frac{A}{2B}} \sqrt{\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}} (c+dx) \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]}{2B}\right]}{2B (bc-ad) g^2 (a+bx)}$$

Result (type 8, 36 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(ag+bgx)^2 \left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

**Problem 141: Unable to integrate problem.**

$$\int \frac{1}{(ag+bgx)^3 \left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)} dx$$

Optimal (type 4, 149 leaves, 7 steps):

$$\frac{b e^{A/B} \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]}{B}\right]}{2B (bc-ad)^2 g^3} - \frac{d e^{\frac{A}{2B}} \sqrt{\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}} (c+dx) \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]}{2B}\right]}{2B (bc-ad)^2 g^3 (a+bx)}$$

Result (type 8, 36 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(ag+bgx)^3 \left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

**Problem 142: Result valid but suboptimal antiderivative.**

$$\int \frac{(ag+bgx)^2}{\left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)^2} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(ag+bgx)^2}{\left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)^2}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$\begin{aligned} & a^2 g^2 \operatorname{CannotIntegrate}\left[\frac{1}{\left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)^2}, x\right] + \\ & 2 a b g^2 \operatorname{CannotIntegrate}\left[\frac{x}{\left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)^2}, x\right] + \\ & b^2 g^2 \operatorname{CannotIntegrate}\left[\frac{x^2}{\left(A+B \operatorname{Log}\left[\frac{e \frac{(a+bx)^2}{(c+dx)^2}}{(c+dx)^2}\right]\right)^2}, x\right] \end{aligned}$$

### Problem 143: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$a g \operatorname{CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right] + b g \operatorname{CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

### Problem 144: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 36 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

### Problem 145: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$-\frac{e^{\frac{A}{2B}} \sqrt{\frac{e (a+b x)^2}{(c+d x)^2}} (c+d x) \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}{2B}\right]}{4 B^2 (b c - a d) g^2 (a+b x)} - \frac{c+d x}{2 B (b c - a d) g^2 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Problem 146: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 4, 263 leaves, 9 steps):

$$\begin{aligned} & -\frac{b e^{A/B} \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}{B}\right]}{2 B^2 (b c - a d)^2 g^3} + \\ & \frac{d e^{\frac{A}{2B}} \sqrt{\frac{e (a+b x)^2}{(c+d x)^2}} (c+d x) \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]}{2 B}\right]}{4 B^2 (b c - a d)^2 g^3 (a+b x)} + \\ & \frac{d (c+d x)}{2 B (b c - a d)^2 g^3 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)} - \frac{b (c+d x)^2}{2 B (b c - a d)^2 g^3 (a+b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)} \end{aligned}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Problem 147: Result valid but suboptimal antiderivative.

$$\int (a+b x)^4 \left(A + B \operatorname{Log}\left[e (a+b x)^n (c+d x)^{-n}\right]\right) dx$$

Optimal (type 3, 171 leaves, 3 steps):

$$\begin{aligned} & \frac{B (b c - a d)^4 n x}{5 d^4} - \frac{B (b c - a d)^3 n (a+b x)^2}{10 b d^3} + \frac{B (b c - a d)^2 n (a+b x)^3}{15 b d^2} - \frac{B (b c - a d) n (a+b x)^4}{20 b d} \\ & \frac{B (b c - a d)^5 n \operatorname{Log}[c+d x]}{5 b d^5} + \frac{(a+b x)^5 \left(A + B \operatorname{Log}\left[e (a+b x)^n (c+d x)^{-n}\right]\right)}{5 b} \end{aligned}$$

Result (type 3, 183 leaves, 5 steps):

$$\begin{aligned} & \frac{B (b c - a d)^4 n x}{5 d^4} - \frac{B (b c - a d)^3 n (a+b x)^2}{10 b d^3} + \frac{B (b c - a d)^2 n (a+b x)^3}{15 b d^2} - \frac{B (b c - a d) n (a+b x)^4}{20 b d} \\ & \frac{A (a+b x)^5}{5 b} - \frac{B (b c - a d)^5 n \operatorname{Log}[c+d x]}{5 b d^5} + \frac{B (a+b x)^5 \operatorname{Log}\left[e (a+b x)^n (c+d x)^{-n}\right]}{5 b} \end{aligned}$$

### Problem 148: Result valid but suboptimal antiderivative.

$$\int (a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 142 leaves, 3 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^3 n x}{4 d^3} + \frac{B (b c - a d)^2 n (a + b x)^2}{8 b d^2} - \frac{B (b c - a d) n (a + b x)^3}{12 b d} \\ & + \frac{B (b c - a d)^4 n \operatorname{Log}[c + d x]}{4 b d^4} + \frac{(a + b x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{4 b} \end{aligned}$$

Result (type 3, 154 leaves, 5 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^3 n x}{4 d^3} + \frac{B (b c - a d)^2 n (a + b x)^2}{8 b d^2} - \frac{B (b c - a d) n (a + b x)^3}{12 b d} \\ & + \frac{A (a + b x)^4}{4 b} + \frac{B (b c - a d)^4 n \operatorname{Log}[c + d x]}{4 b d^4} + \frac{B (a + b x)^4 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 b} \end{aligned}$$

### Problem 149: Result valid but suboptimal antiderivative.

$$\int (a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 113 leaves, 3 steps):

$$\begin{aligned} & \frac{B (b c - a d)^2 n x}{3 d^2} - \frac{B (b c - a d) n (a + b x)^2}{6 b d} \\ & + \frac{B (b c - a d)^3 n \operatorname{Log}[c + d x]}{3 b d^3} + \frac{(a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 b} \end{aligned}$$

Result (type 3, 125 leaves, 5 steps):

$$\begin{aligned} & \frac{B (b c - a d)^2 n x}{3 d^2} - \frac{B (b c - a d) n (a + b x)^2}{6 b d} + \frac{A (a + b x)^3}{3 b} \\ & + \frac{B (b c - a d)^3 n \operatorname{Log}[c + d x]}{3 b d^3} + \frac{B (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b} \end{aligned}$$

### Problem 150: Result valid but suboptimal antiderivative.

$$\int (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 84 leaves, 3 steps):

$$\begin{aligned} & -\frac{B (b c - a d) n x}{2 d} + \frac{B (b c - a d)^2 n \operatorname{Log}[c + d x]}{2 b d^2} + \frac{(a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{2 b} \end{aligned}$$

Result (type 3, 96 leaves, 5 steps):



$$-\frac{B(b c - a d) n x}{2 d} + \frac{A(a + b x)^2}{2 b} + \frac{B(b c - a d)^2 n \operatorname{Log}[c + d x]}{2 b d^2} + \frac{B(a + b x)^2 \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{2 b}$$

**Problem 151: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{a + b x} dx$$

Optimal (type 4, 79 leaves, 5 steps):

$$-\frac{\operatorname{Log}\left[-\frac{b c - a d}{d(a + b x)}\right] (A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}])}{b} + \frac{B n \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{b}$$

Result (type 4, 87 leaves, 7 steps):

$$\frac{A \operatorname{Log}[a + b x]}{b} - \frac{B \operatorname{Log}\left[-\frac{b c - a d}{d(a + b x)}\right] \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{b} + \frac{B n \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{b}$$

**Problem 152: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{(a + b x)^2} dx$$

Optimal (type 3, 97 leaves, 3 steps):

$$-\frac{B n}{b(a + b x)} - \frac{B d n \operatorname{Log}[a + b x]}{b(b c - a d)} + \frac{B d n \operatorname{Log}[c + d x]}{b(b c - a d)} - \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{b(a + b x)}$$

Result (type 3, 72 leaves, 4 steps):

$$-\frac{A}{b(a + b x)} - \frac{B n}{b(a + b x)} - \frac{B(c + d x) \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{(b c - a d)(a + b x)}$$

**Problem 153: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{(a + b x)^3} dx$$

Optimal (type 3, 137 leaves, 3 steps):

$$-\frac{B n}{4 b(a + b x)^2} + \frac{B d n}{2 b(b c - a d)(a + b x)} + \frac{B d^2 n \operatorname{Log}[a + b x]}{2 b(b c - a d)^2} - \frac{B d^2 n \operatorname{Log}[c + d x]}{2 b(b c - a d)^2} - \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{2 b(a + b x)^2}$$

Result (type 3, 149 leaves, 5 steps):

$$\begin{aligned}
& - \frac{A}{2b(a+bx)^2} - \frac{Bn}{4b(a+bx)^2} + \frac{Bdn}{2b(bc-ad)(a+bx)} + \\
& \frac{Bd^2n \operatorname{Log}[a+bx]}{2b(bc-ad)^2} - \frac{Bd^2n \operatorname{Log}[c+dx]}{2b(bc-ad)^2} - \frac{B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2b(a+bx)^2}
\end{aligned}$$

**Problem 154: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{(a+bx)^4} dx$$

Optimal (type 3, 166 leaves, 3 steps):

$$\begin{aligned}
& - \frac{Bn}{9b(a+bx)^3} + \frac{Bdn}{6b(bc-ad)(a+bx)^2} - \frac{Bd^2n}{3b(bc-ad)^2(a+bx)} - \\
& \frac{Bd^3n \operatorname{Log}[a+bx]}{3b(bc-ad)^3} + \frac{Bd^3n \operatorname{Log}[c+dx]}{3b(bc-ad)^3} - \frac{A + B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{3b(a+bx)^3}
\end{aligned}$$

Result (type 3, 178 leaves, 5 steps):

$$\begin{aligned}
& - \frac{A}{3b(a+bx)^3} - \frac{Bn}{9b(a+bx)^3} + \frac{Bdn}{6b(bc-ad)(a+bx)^2} - \frac{Bd^2n}{3b(bc-ad)^2(a+bx)} - \\
& \frac{Bd^3n \operatorname{Log}[a+bx]}{3b(bc-ad)^3} + \frac{Bd^3n \operatorname{Log}[c+dx]}{3b(bc-ad)^3} - \frac{B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{3b(a+bx)^3}
\end{aligned}$$

**Problem 155: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{(a+bx)^5} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$\begin{aligned}
& - \frac{Bn}{16b(a+bx)^4} + \frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2} + \frac{Bd^3n}{4b(bc-ad)^3(a+bx)} + \\
& \frac{Bd^4n \operatorname{Log}[a+bx]}{4b(bc-ad)^4} - \frac{Bd^4n \operatorname{Log}[c+dx]}{4b(bc-ad)^4} - \frac{A + B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{4b(a+bx)^4}
\end{aligned}$$

Result (type 3, 207 leaves, 5 steps):

$$\begin{aligned}
& - \frac{A}{4b(a+bx)^4} - \frac{Bn}{16b(a+bx)^4} + \frac{Bdn}{12b(bc-ad)(a+bx)^3} - \frac{Bd^2n}{8b(bc-ad)^2(a+bx)^2} + \\
& \frac{Bd^3n}{4b(bc-ad)^3(a+bx)} + \frac{Bd^4n \operatorname{Log}[a+bx]}{4b(bc-ad)^4} - \frac{Bd^4n \operatorname{Log}[c+dx]}{4b(bc-ad)^4} - \frac{B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{4b(a+bx)^4}
\end{aligned}$$

### Problem 156: Result valid but suboptimal antiderivative.

$$\int (a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 dx$$

Optimal (type 4, 322 leaves, 8 steps):

$$\begin{aligned} & - \frac{B (b c - a d) n (a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{6 b d} + \\ & \frac{(a + b x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{4 b} + \\ & \frac{B (b c - a d)^2 n (a + b x)^2 (3 A + B n + 3 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{12 b d^2} - \\ & \frac{B (b c - a d)^3 n (a + b x) (6 A + 5 B n + 6 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{12 b d^3} - \frac{1}{12 b d^4} \\ & \frac{B (b c - a d)^4 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (6 A + 11 B n + 6 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{2 b d^4} - \\ & \frac{B^2 (b c - a d)^4 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{2 b d^4} \end{aligned}$$

Result (type 4, 542 leaves, 21 steps):

$$\begin{aligned} & - \frac{A B (b c - a d)^3 n x}{2 d^3} - \frac{5 B^2 (b c - a d)^3 n^2 x}{12 d^3} + \frac{A B (b c - a d)^2 n (a + b x)^2}{4 b d^2} + \frac{B^2 (b c - a d)^2 n^2 (a + b x)^2}{12 b d^2} - \\ & \frac{A B (b c - a d) n (a + b x)^3}{6 b d} + \frac{A^2 (a + b x)^4}{4 b} + \frac{A B (b c - a d)^4 n \operatorname{Log}[c + d x]}{2 b d^4} + \\ & \frac{11 B^2 (b c - a d)^4 n^2 \operatorname{Log}[c + d x]}{12 b d^4} - \frac{B^2 (b c - a d)^3 n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b d^3} + \\ & \frac{B^2 (b c - a d)^2 n (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 b d^2} - \\ & \frac{B^2 (b c - a d) n (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{6 b d} + \frac{A B (a + b x)^4 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b} - \\ & \frac{B^2 (b c - a d)^4 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b d^4} + \\ & \frac{B^2 (a + b x)^4 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{4 b} - \frac{B^2 (b c - a d)^4 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{2 b d^4} \end{aligned}$$

### Problem 157: Result valid but suboptimal antiderivative.

$$\int (a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 dx$$

Optimal (type 4, 263 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) n (a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 b d} + \\
& \frac{(a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{3 b} + \\
& \frac{B (b c - a d)^2 n (a + b x) (2 A + B n + 2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 b d^2} + \frac{1}{3 b d^3} \\
& B (b c - a d)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (2 A + 3 B n + 2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) + \\
& \frac{2 B^2 (b c - a d)^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 427 leaves, 18 steps):

$$\begin{aligned}
& \frac{2 A B (b c - a d)^2 n x}{3 d^2} + \frac{B^2 (b c - a d)^2 n^2 x}{3 d^2} - \frac{A B (b c - a d) n (a + b x)^2}{3 b d} + \\
& \frac{A^2 (a + b x)^3}{3 b} - \frac{2 A B (b c - a d)^3 n \operatorname{Log}[c + d x]}{3 b d^3} - \frac{B^2 (b c - a d)^3 n^2 \operatorname{Log}[c + d x]}{b d^3} + \\
& \frac{2 B^2 (b c - a d)^2 n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b d^2} - \\
& \frac{B^2 (b c - a d) n (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b d} + \frac{2 A B (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b} + \\
& \frac{2 B^2 (b c - a d)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b d^3} + \\
& \frac{B^2 (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{3 b} + \frac{2 B^2 (b c - a d)^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 b d^3}
\end{aligned}$$

**Problem 158: Result valid but suboptimal antiderivative.**

$$\int (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 dx$$

Optimal (type 4, 195 leaves, 6 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b d} + \\
& \frac{(a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 b} - \\
& \frac{B (b c - a d)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B n + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b d^2} - \\
& \frac{B^2 (b c - a d)^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2}
\end{aligned}$$

Result (type 4, 308 leaves, 15 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d) n x}{d} + \frac{A^2 (a + b x)^2}{2 b} + \frac{A B (b c - a d)^2 n \operatorname{Log}[c + d x]}{b d^2} + \frac{B^2 (b c - a d)^2 n^2 \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{B^2 (b c - a d) n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d} + \frac{A B (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b} - \\
& \frac{B^2 (b c - a d)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^2} + \\
& \frac{B^2 (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b} - \frac{B^2 (b c - a d)^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2}
\end{aligned}$$

**Problem 159: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{a + b x} dx$$

Optimal (type 4, 131 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b} + \\
& \frac{2 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b}
\end{aligned}$$

Result (type 4, 227 leaves, 10 steps):

$$\begin{aligned}
& \frac{A^2 \operatorname{Log}[a + b x]}{b} - \frac{2 A B \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b} - \\
& \frac{B^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b} + \frac{2 A B n \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \\
& \frac{2 B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b}
\end{aligned}$$

**Problem 160: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(a + b x)^2} dx$$

Optimal (type 3, 129 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2 (c + d x)}{(b c - a d) (a + b x)} - \frac{2 B n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d) (a + b x)} - \\
& \frac{(c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d) (a + b x)}
\end{aligned}$$

Result (type 3, 189 leaves, 7 steps):

$$\begin{aligned}
& - \frac{A^2}{b(a+bx)} - \frac{2ABn}{b(a+bx)} - \frac{2B^2n^2}{b(a+bx)} - \frac{2AB(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{(bc-ad)(a+bx)} - \\
& \frac{2B^2n(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{(bc-ad)(a+bx)} - \frac{B^2(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]^2}{(bc-ad)(a+bx)}
\end{aligned}$$

**Problem 161: Result valid but suboptimal antiderivative.**

$$\int \frac{(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])^2}{(a+bx)^3} dx$$

Optimal (type 3, 274 leaves, 8 steps):

$$\begin{aligned}
& \frac{2B^2dn^2(c+dx)}{(bc-ad)^2(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} + \frac{2Bdn(c+dx)(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])}{(bc-ad)^2(a+bx)} - \\
& \frac{bBn(c+dx)^2(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])}{2(bc-ad)^2(a+bx)^2} + \\
& \frac{d(c+dx)(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])^2}{(bc-ad)^2(a+bx)} - \frac{b(c+dx)^2(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])^2}{2(bc-ad)^2(a+bx)^2}
\end{aligned}$$

Result (type 3, 411 leaves, 12 steps):

$$\begin{aligned}
& - \frac{A^2}{2b(a+bx)^2} - \frac{ABn}{2b(a+bx)^2} + \frac{ABdn}{b(bc-ad)(a+bx)} + \\
& \frac{2B^2dn^2}{b(bc-ad)(a+bx)} - \frac{bB^2n^2(c+dx)^2}{4(bc-ad)^2(a+bx)^2} + \frac{ABd^2n \operatorname{Log}[a+bx]}{b(bc-ad)^2} - \\
& \frac{ABd^2n \operatorname{Log}[c+dx]}{b(bc-ad)^2} - \frac{AB \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{b(a+bx)^2} + \\
& \frac{2B^2dn(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{(bc-ad)^2(a+bx)} - \frac{bB^2n(c+dx)^2 \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2(bc-ad)^2(a+bx)^2} + \\
& \frac{B^2d(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]^2}{(bc-ad)^2(a+bx)} - \frac{bB^2(c+dx)^2 \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]^2}{2(bc-ad)^2(a+bx)^2}
\end{aligned}$$

**Problem 162: Result unnecessarily involves higher level functions.**

$$\int \frac{(A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])^2}{(a+bx)^4} dx$$

Optimal (type 3, 427 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 n^2 (c + d x)}{(b c - a d)^3 (a + b x)} + \frac{b B^2 d n^2 (c + d x)^2}{2 (b c - a d)^3 (a + b x)^2} - \\
& \frac{2 b^2 B^2 n^2 (c + d x)^3}{27 (b c - a d)^3 (a + b x)^3} - \frac{2 B d^2 n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^3 (a + b x)} + \\
& \frac{b B d n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^3 (a + b x)^2} - \\
& \frac{2 b^2 B n (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{9 (b c - a d)^3 (a + b x)^3} - \\
& \frac{d^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^3 (a + b x)} + \\
& \frac{b d (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^3 (a + b x)^2} - \\
& \frac{b^2 (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{3 (b c - a d)^3 (a + b x)^3}
\end{aligned}$$

Result (type 4, 730 leaves, 26 steps):

$$\begin{aligned}
& - \frac{A^2}{3 b (a + b x)^3} - \frac{2 A B n}{9 b (a + b x)^3} - \frac{2 B^2 n^2}{27 b (a + b x)^3} + \frac{A B d n}{3 b (b c - a d) (a + b x)^2} + \\
& \frac{5 B^2 d n^2}{18 b (b c - a d) (a + b x)^2} - \frac{2 A B d^2 n}{3 b (b c - a d)^2 (a + b x)} - \frac{11 B^2 d^2 n^2}{9 b (b c - a d)^2 (a + b x)} - \\
& \frac{2 A B d^3 n \operatorname{Log}[a + b x]}{3 b (b c - a d)^3} - \frac{5 B^2 d^3 n^2 \operatorname{Log}[a + b x]}{9 b (b c - a d)^3} + \frac{2 A B d^3 n \operatorname{Log}[c + d x]}{3 b (b c - a d)^3} + \frac{5 B^2 d^3 n^2 \operatorname{Log}[c + d x]}{9 b (b c - a d)^3} - \\
& \frac{2 A B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b (a + b x)^3} - \frac{2 B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{9 b (a + b x)^3} + \\
& \frac{B^2 d n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b (b c - a d) (a + b x)^2} - \frac{2 B^2 d^2 n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 (b c - a d)^3 (a + b x)} + \\
& \frac{2 B^2 d^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b (b c - a d)^3} - \\
& \frac{2 B^2 d^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b (b c - a d)^3} - \frac{B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{3 b (a + b x)^3} - \\
& \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 b (b c - a d)^3} - \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{3 b (b c - a d)^3}
\end{aligned}$$

### Problem 163: Result unnecessarily involves higher level functions.

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(a + b x)^5} dx$$

Optimal (type 3, 587 leaves, 12 steps):

$$\begin{aligned} & \frac{2 B^2 d^3 n^2 (c + d x)}{(b c - a d)^4 (a + b x)} - \frac{3 b B^2 d^2 n^2 (c + d x)^2}{4 (b c - a d)^4 (a + b x)^2} + \frac{2 b^2 B^2 d n^2 (c + d x)^3}{9 (b c - a d)^4 (a + b x)^3} - \\ & \frac{b^3 B^2 n^2 (c + d x)^4}{32 (b c - a d)^4 (a + b x)^4} + \frac{2 B d^3 n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^4 (a + b x)} - \\ & \frac{3 b B d^2 n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{2 (b c - a d)^4 (a + b x)^2} + \\ & \frac{2 b^2 B d n (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 (b c - a d)^4 (a + b x)^3} - \\ & \frac{b^3 B n (c + d x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{8 (b c - a d)^4 (a + b x)^4} + \\ & \frac{d^3 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^4 (a + b x)} - \\ & \frac{3 b d^2 (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 (b c - a d)^4 (a + b x)^2} + \\ & \frac{b^2 d (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^4 (a + b x)^3} - \\ & \frac{b^3 (c + d x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{4 (b c - a d)^4 (a + b x)^4} \end{aligned}$$

Result (type 4, 843 leaves, 29 steps):



$$\begin{aligned}
& - \frac{A^2}{4b(a+bx)^4} - \frac{ABn}{8b(a+bx)^4} - \frac{B^2n^2}{32b(a+bx)^4} + \frac{ABdn}{6b(bc-ad)(a+bx)^3} + \frac{7B^2dn^2}{72b(bc-ad)(a+bx)^3} - \\
& \frac{ABd^2n}{4b(bc-ad)^2(a+bx)^2} - \frac{13B^2d^2n^2}{48b(bc-ad)^2(a+bx)^2} + \frac{ABd^3n}{2b(bc-ad)^3(a+bx)} + \\
& \frac{25B^2d^3n^2}{24b(bc-ad)^3(a+bx)} + \frac{ABd^4n \operatorname{Log}[a+bx]}{2b(bc-ad)^4} + \frac{13B^2d^4n^2 \operatorname{Log}[a+bx]}{24b(bc-ad)^4} - \frac{ABd^4n \operatorname{Log}[c+dx]}{2b(bc-ad)^4} - \\
& \frac{13B^2d^4n^2 \operatorname{Log}[c+dx]}{24b(bc-ad)^4} - \frac{AB \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2b(a+bx)^4} - \frac{B^2n \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{8b(a+bx)^4} + \\
& \frac{B^2dn \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{6b(bc-ad)(a+bx)^3} - \frac{B^2d^2n \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{4b(bc-ad)^2(a+bx)^2} + \\
& \frac{B^2d^3n(c+dx) \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2(bc-ad)^4(a+bx)} - \frac{B^2d^4n \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2b(bc-ad)^4} + \\
& \frac{B^2d^4n \operatorname{Log}\left[\frac{bc-ad}{b(c+dx)}\right] \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]}{2b(bc-ad)^4} - \frac{B^2 \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}]^2}{4b(a+bx)^4} + \\
& \frac{B^2d^4n^2 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{2b(bc-ad)^4} + \frac{B^2d^4n^2 \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{2b(bc-ad)^4}
\end{aligned}$$

**Problem 164: Result valid but suboptimal antiderivative.**

$$\int (a+bx)^3 (A+B \operatorname{Log}[e(a+bx)^n(c+dx)^{-n}])^3 dx$$

Optimal (type 4, 809 leaves, 27 steps):

$$\begin{aligned}
& - \frac{B^3 (bc - ad)^3 n^3 x}{4 d^3} - \frac{B^3 (bc - ad)^4 n^3 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{4 b d^4} + \frac{3 B^3 (bc - ad)^4 n^3 \operatorname{Log}[c + dx]}{2 b d^4} - \\
& \frac{7 B^2 (bc - ad)^3 n^2 (a + bx) (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])}{4 b d^3} + \\
& \frac{b B^2 (bc - ad)^2 n^2 (c + dx)^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])}{4 d^4} - \\
& \frac{9 B^2 (bc - ad)^4 n^2 \operatorname{Log}\left[\frac{bc-ad}{b(c+dx)}\right] (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])}{2 b d^4} - \\
& \frac{9 B (bc - ad)^3 n (a + bx) (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{4 b d^3} + \\
& \frac{9 b B (bc - ad)^2 n (c + dx)^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{8 d^4} - \\
& \frac{b^2 B (bc - ad) n (c + dx)^3 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{4 d^4} - \\
& \frac{3 B (bc - ad)^4 n \operatorname{Log}\left[\frac{bc-ad}{b(c+dx)}\right] (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{4 b d^4} + \\
& \frac{(a + bx)^4 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^3}{4 b} + \frac{1}{4 b d^4} \\
& 7 B^2 (bc - ad)^4 n^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]) \operatorname{Log}\left[1 - \frac{b(c + dx)}{d(a + bx)}\right] - \\
& \frac{9 B^3 (bc - ad)^4 n^3 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{2 b d^4} - \frac{1}{2 b d^4} \\
& 3 B^2 (bc - ad)^4 n^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right] - \\
& \frac{7 B^3 (bc - ad)^4 n^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{4 b d^4} + \frac{3 B^3 (bc - ad)^4 n^3 \operatorname{PolyLog}\left[3, \frac{d(a+bx)}{b(c+dx)}\right]}{2 b d^4}
\end{aligned}$$

Result (type 4, 1203 leaves, 56 steps):

$$\begin{aligned}
& - \frac{3 A^2 B (bc - ad)^3 n x}{4 d^3} - \frac{5 A B^2 (bc - ad)^3 n^2 x}{4 d^3} - \frac{B^3 (bc - ad)^3 n^3 x}{4 d^3} + \\
& \frac{3 A^2 B (bc - ad)^2 n (a + bx)^2}{8 b d^2} + \frac{A B^2 (bc - ad)^2 n^2 (a + bx)^2}{4 b d^2} - \frac{A^2 B (bc - ad) n (a + bx)^3}{4 b d} + \\
& \frac{A^3 (a + bx)^4}{4 b} + \frac{3 A^2 B (bc - ad)^4 n \operatorname{Log}[c + dx]}{4 b d^4} + \frac{11 A B^2 (bc - ad)^4 n^2 \operatorname{Log}[c + dx]}{4 b d^4} + \\
& \frac{3 B^3 (bc - ad)^4 n^3 \operatorname{Log}[c + dx]}{2 b d^4} - \frac{3 A B^2 (bc - ad)^3 n (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{2 b d^3} - \\
& \frac{5 B^3 (bc - ad)^3 n^2 (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{4 b d^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 A B^2 (b c - a d)^2 n (a + b x)^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{4 b d^2} + \\
& \frac{B^3 (b c - a d)^2 n^2 (a + b x)^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{4 b d^2} - \\
& \frac{A B^2 (b c - a d) n (a + b x)^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{2 b d} + \\
& \frac{3 A^2 B (a + b x)^4 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{4 b} - \\
& \frac{3 A B^2 (b c - a d)^4 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{2 b d^4} - \\
& \frac{11 B^3 (b c - a d)^4 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{4 b d^4} - \\
& \frac{3 B^3 (b c - a d)^3 n (a + b x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{4 b d^3} + \\
& \frac{3 B^3 (b c - a d)^2 n (a + b x)^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{8 b d^2} - \\
& \frac{B^3 (b c - a d) n (a + b x)^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{4 b d} + \\
& \frac{3 A B^2 (a + b x)^4 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{4 b} - \\
& \frac{3 B^3 (b c - a d)^4 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{4 b d^4} + \\
& \frac{B^3 (a + b x)^4 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3}{4 b} - \frac{3 A B^2 (b c - a d)^4 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{2 b d^4} - \\
& \frac{11 B^3 (b c - a d)^4 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{4 b d^4} - \frac{1}{2 b d^4} \\
& \frac{3 B^3 (b c - a d)^4 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right]}{2 b d^4} + \\
& \frac{3 B^3 (b c - a d)^4 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{2 b d^4}
\end{aligned}$$

**Problem 165: Result valid but suboptimal antiderivative.**

$$\int (a + b x)^2 (A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right])^3 dx$$

Optimal (type 4, 614 leaves, 17 steps):

$$\begin{aligned}
& - \frac{B^3 (b c - a d)^3 n^3 \operatorname{Log}[c + d x]}{b d^3} + \frac{B^2 (b c - a d)^2 n^2 (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b d^2} + \\
& \frac{4 B^2 (b c - a d)^3 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b d^3} + \\
& \frac{2 B (b c - a d)^2 n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{b d^2} - \\
& \frac{b B (b c - a d) n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 d^3} + \\
& \frac{B (b c - a d)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{b d^3} + \\
& \frac{(a + b x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{3 b} - \\
& \frac{B^2 (b c - a d)^3 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b d^3} + \\
& \frac{4 B^3 (b c - a d)^3 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3} + \frac{1}{b d^3} \\
& \frac{2 B^2 (b c - a d)^3 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3} + \\
& \frac{B^3 (b c - a d)^3 n^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b d^3} - \frac{2 B^3 (b c - a d)^3 n^3 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3}
\end{aligned}$$

Result (type 4, 915 leaves, 40 steps):

$$\begin{aligned}
& \frac{A^2 B (b c - a d)^2 n x}{d^2} + \frac{A B^2 (b c - a d)^2 n^2 x}{d^2} - \frac{A^2 B (b c - a d) n (a + b x)^2}{2 b d} + \\
& \frac{A^3 (a + b x)^3}{3 b} - \frac{A^2 B (b c - a d)^3 n \operatorname{Log}[c + d x]}{b d^3} - \frac{3 A B^2 (b c - a d)^3 n^2 \operatorname{Log}[c + d x]}{b d^3} - \\
& \frac{B^3 (b c - a d)^3 n^3 \operatorname{Log}[c + d x]}{b d^3} + \frac{2 A B^2 (b c - a d)^2 n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^2} + \\
& \frac{B^3 (b c - a d)^2 n^2 (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^2} - \\
& \frac{A B^2 (b c - a d) n (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d} + \\
& \frac{A^2 B (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b} + \\
& \frac{2 A B^2 (b c - a d)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^3} + \\
& \frac{3 B^3 (b c - a d)^3 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^3} + \\
& \frac{B^3 (b c - a d)^2 n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b d^2} - \\
& \frac{B^3 (b c - a d) n (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b d} + \\
& \frac{A B^2 (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b} + \\
& \frac{B^3 (b c - a d)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b d^3} + \\
& \frac{B^3 (a + b x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{3 b} + \frac{2 A B^2 (b c - a d)^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3} + \\
& \frac{3 B^3 (b c - a d)^3 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3} + \frac{1}{b d^3} \\
& 2 B^3 (b c - a d)^3 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right] - \\
& \frac{2 B^3 (b c - a d)^3 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b d^3}
\end{aligned}$$

**Problem 166: Result valid but suboptimal antiderivative.**

$$\int (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 dx$$

Optimal (type 4, 376 leaves, 11 steps):

$$\begin{aligned}
& - \frac{3 B^2 (b c - a d)^2 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b d^2} - \\
& \frac{3 B (b c - a d) n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 b d} - \\
& \frac{3 B (b c - a d)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 b d^2} + \\
& \frac{(a + b x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 b} - \frac{3 B^3 (b c - a d)^2 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2} - \\
& \frac{1}{b d^2} 3 B^2 (b c - a d)^2 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right] + \\
& \frac{3 B^3 (b c - a d)^2 n^3 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2}
\end{aligned}$$

Result (type 4, 700 leaves, 27 steps):

$$\begin{aligned}
& - \frac{3 A^2 B (b c - a d) n x}{2 d} + \frac{A^3 (a + b x)^2}{2 b} + \frac{3 A^2 B (b c - a d)^2 n \operatorname{Log}[c + d x]}{2 b d^2} + \\
& \frac{3 A B^2 (b c - a d)^2 n^2 \operatorname{Log}[c + d x]}{b d^2} - \frac{3 A B^2 (b c - a d) n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d} + \\
& \frac{3 A^2 B (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b} - \\
& \frac{3 A B^2 (b c - a d)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^2} - \\
& \frac{3 B^3 (b c - a d)^2 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d^2} - \\
& \frac{3 B^3 (b c - a d) n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b d} + \\
& \frac{3 A B^2 (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b} - \\
& \frac{3 B^3 (b c - a d)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b d^2} + \\
& \frac{B^3 (a + b x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{2 b} - \frac{3 A B^2 (b c - a d)^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2} - \\
& \frac{3 B^3 (b c - a d)^2 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^2} - \frac{1}{b d^2} \\
& 3 B^3 (b c - a d)^2 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right] + \\
& \frac{3 B^3 (b c - a d)^2 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b d^2}
\end{aligned}$$

**Problem 167: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{a + b x} dx$$

Optimal (type 4, 186 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b} + \\
& \frac{3 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b} + \\
& \frac{6 B^2 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b} + \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, \frac{b (c + d x)}{d (a + b x)}\right]}{b}
\end{aligned}$$

Result (type 4, 424 leaves, 14 steps):

$$\begin{aligned}
& \frac{A^3 \operatorname{Log}[a + b x]}{b} - \frac{3 A^2 B \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b} - \\
& \frac{3 A B^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{b} - \frac{B^3 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3}{b} + \\
& \frac{3 A^2 B n \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \frac{6 A B^2 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \\
& \frac{3 B^3 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \frac{6 A B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \\
& \frac{6 B^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b} + \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b}
\end{aligned}$$

**Problem 168: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(a + b x)^2} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\begin{aligned}
& -\frac{6 B^3 n^3 (c + d x)}{(b c - a d) (a + b x)} - \frac{6 B^2 n^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d) (a + b x)} - \\
& \frac{3 B n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d) (a + b x)} - \frac{(c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d) (a + b x)}
\end{aligned}$$

Result (type 3, 360 leaves, 11 steps):

$$\begin{aligned}
& -\frac{A^3}{b (a + b x)} - \frac{3 A^2 B n}{b (a + b x)} - \frac{6 A B^2 n^2}{b (a + b x)} - \frac{6 B^3 n^3}{b (a + b x)} - \\
& \frac{3 A^2 B (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b c - a d) (a + b x)} - \frac{6 A B^2 n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b c - a d) (a + b x)} - \\
& \frac{6 B^3 n^2 (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b c - a d) (a + b x)} - \frac{3 A B^2 (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{(b c - a d) (a + b x)} - \\
& \frac{3 B^3 n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{(b c - a d) (a + b x)} - \frac{B^3 (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{(b c - a d) (a + b x)}
\end{aligned}$$

**Problem 169: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(a + b x)^3} dx$$

Optimal (type 3, 390 leaves, 10 steps):



$$\begin{aligned}
& \frac{6 B^3 d n^3 (c + d x)}{(b c - a d)^2 (a + b x)} - \frac{3 b B^3 n^3 (c + d x)^2}{8 (b c - a d)^2 (a + b x)^2} + \\
& \frac{6 B^2 d n^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^2 (a + b x)} - \\
& \frac{3 b B^2 n^2 (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{4 (b c - a d)^2 (a + b x)^2} + \\
& \frac{3 B d n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^2 (a + b x)} - \\
& \frac{3 b B n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{4 (b c - a d)^2 (a + b x)^2} + \\
& \frac{d (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d)^2 (a + b x)} - \frac{b (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 (b c - a d)^2 (a + b x)^2}
\end{aligned}$$

Result (type 3, 811 leaves, 21 steps):

$$\begin{aligned}
& -\frac{A^3}{2 b (a + b x)^2} - \frac{3 A^2 B n}{4 b (a + b x)^2} + \frac{3 A^2 B d n}{2 b (b c - a d) (a + b x)} + \frac{6 A B^2 d n^2}{b (b c - a d) (a + b x)} + \\
& \frac{6 B^3 d n^3}{b (b c - a d) (a + b x)} - \frac{3 A b B^2 n^2 (c + d x)^2}{4 (b c - a d)^2 (a + b x)^2} - \frac{3 b B^3 n^3 (c + d x)^2}{8 (b c - a d)^2 (a + b x)^2} + \\
& \frac{3 A^2 B d^2 n \operatorname{Log}[a + b x]}{2 b (b c - a d)^2} - \frac{3 A^2 B d^2 n \operatorname{Log}[c + d x]}{2 b (b c - a d)^2} - \frac{3 A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b (a + b x)^2} + \\
& \frac{6 A B^2 d n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b c - a d)^2 (a + b x)} + \frac{6 B^3 d n^2 (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b c - a d)^2 (a + b x)} - \\
& \frac{3 A b B^2 n (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 (b c - a d)^2 (a + b x)^2} - \frac{3 b B^3 n^2 (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 (b c - a d)^2 (a + b x)^2} + \\
& \frac{3 A B^2 d (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{(b c - a d)^2 (a + b x)} + \frac{3 B^3 d n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{(b c - a d)^2 (a + b x)} - \\
& \frac{3 A b B^2 (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 (b c - a d)^2 (a + b x)^2} - \frac{3 b B^3 n (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{4 (b c - a d)^2 (a + b x)^2} + \\
& \frac{B^3 d (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{(b c - a d)^2 (a + b x)} - \frac{b B^3 (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{2 (b c - a d)^2 (a + b x)^2}
\end{aligned}$$

**Problem 170:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(a + b x)^4} dx$$

Optimal (type 3, 611 leaves, 13 steps):

$$\begin{aligned}
& - \frac{6 B^3 d^2 n^3 (c + d x)}{(b c - a d)^3 (a + b x)} + \frac{3 b B^3 d n^3 (c + d x)^2}{4 (b c - a d)^3 (a + b x)^2} - \\
& \frac{2 b^2 B^3 n^3 (c + d x)^3}{27 (b c - a d)^3 (a + b x)^3} - \frac{6 B^2 d^2 n^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^3 (a + b x)} + \\
& \frac{3 b B^2 d n^2 (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{2 (b c - a d)^3 (a + b x)^2} - \\
& \frac{2 b^2 B^2 n^2 (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{9 (b c - a d)^3 (a + b x)^3} - \\
& \frac{3 B d^2 n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^3 (a + b x)} + \\
& \frac{3 b B d n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 (b c - a d)^3 (a + b x)^2} - \\
& \frac{b^2 B n (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{3 (b c - a d)^3 (a + b x)^3} - \\
& \frac{d^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d)^3 (a + b x)} + \\
& \frac{b d (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d)^3 (a + b x)^2} - \\
& \frac{b^2 (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{3 (b c - a d)^3 (a + b x)^3}
\end{aligned}$$

Result (type 4, 1876 leaves, 66 steps):

$$\begin{aligned}
& - \frac{A^3}{3 b (a + b x)^3} - \frac{A^2 B n}{3 b (a + b x)^3} - \frac{2 A B^2 n^2}{9 b (a + b x)^3} - \frac{2 B^3 n^3}{27 b (a + b x)^3} + \frac{A^2 B d n}{2 b (b c - a d) (a + b x)^2} + \\
& \frac{5 A B^2 d n^2}{6 b (b c - a d) (a + b x)^2} + \frac{5 B^3 d n^3}{18 b (b c - a d) (a + b x)^2} - \frac{A^2 B d^2 n}{b (b c - a d)^2 (a + b x)} - \\
& \frac{11 A B^2 d^2 n^2}{3 b (b c - a d)^2 (a + b x)} - \frac{47 B^3 d^2 n^3}{9 b (b c - a d)^2 (a + b x)} + \frac{b B^3 d n^3 (c + d x)^2}{4 (b c - a d)^3 (a + b x)^2} - \frac{A^2 B d^3 n \operatorname{Log}[a + b x]}{b (b c - a d)^3} - \\
& \frac{5 A B^2 d^3 n^2 \operatorname{Log}[a + b x]}{3 b (b c - a d)^3} - \frac{5 B^3 d^3 n^3 \operatorname{Log}[a + b x]}{9 b (b c - a d)^3} + \frac{A^2 B d^3 n \operatorname{Log}[c + d x]}{b (b c - a d)^3} + \frac{5 A B^2 d^3 n^2 \operatorname{Log}[c + d x]}{3 b (b c - a d)^3} + \\
& \frac{5 B^3 d^3 n^3 \operatorname{Log}[c + d x]}{9 b (b c - a d)^3} - \frac{A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b (a + b x)^3} - \frac{2 A B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b (a + b x)^3} - \\
& \frac{2 B^3 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{9 b (a + b x)^3} + \frac{A B^2 d n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b (b c - a d) (a + b x)^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{B^3 d n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{3 b (b c - a d) (a + b x)^2} - \frac{2 A B^2 d^2 n (c + d x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{(b c - a d)^3 (a + b x)} - \\
& \frac{14 B^3 d^2 n^2 (c + d x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{3 (b c - a d)^3 (a + b x)} + \frac{b B^3 d n^2 (c + d x)^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{2 (b c - a d)^3 (a + b x)^2} + \\
& \frac{2 A B^2 d^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b (b c - a d)^3} + \\
& \frac{2 B^3 d^3 n^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{3 b (b c - a d)^3} - \\
& \frac{2 A B^2 d^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b (b c - a d)^3} - \\
& \frac{2 B^3 d^3 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{3 b (b c - a d)^3} - \\
& \frac{A B^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{b (a + b x)^3} - \frac{B^3 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{3 b (a + b x)^3} - \\
& \frac{2 B^3 d^2 n (c + d x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{(b c - a d)^3 (a + b x)} + \frac{b B^3 d n (c + d x)^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{2 (b c - a d)^3 (a + b x)^2} + \\
& \frac{B^3 d^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{b (b c - a d)^3} - \\
& \frac{B^3 d^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{b (b c - a d)^3} - \frac{B^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3}{3 b (a + b x)^3} - \\
& \frac{2 A B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b (b c - a d)^3} - \frac{2 B^3 d^3 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 b (b c - a d)^3} - \\
& \frac{2 A B^2 d^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b (b c - a d)^3} - \frac{2 B^3 d^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{3 b (b c - a d)^3} - \\
& \frac{2 B^3 d^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b (b c - a d)^3} - \\
& \frac{2 B^3 d^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b (b c - a d)^3} - \\
& \frac{2 B^3 d^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b (b c - a d)^3} + \frac{2 B^3 d^3 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b (b c - a d)^3}
\end{aligned}$$

**Problem 171:** Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(a + b x)^5} dx$$

Optimal (type 3, 830 leaves, 16 steps):

$$\begin{aligned} & \frac{6 B^3 d^3 n^3 (c + d x)}{(b c - a d)^4 (a + b x)} - \frac{9 b B^3 d^2 n^3 (c + d x)^2}{8 (b c - a d)^4 (a + b x)^2} + \frac{2 b^2 B^3 d n^3 (c + d x)^3}{9 (b c - a d)^4 (a + b x)^3} - \\ & \frac{3 b^3 B^3 n^3 (c + d x)^4}{128 (b c - a d)^4 (a + b x)^4} + \frac{6 B^2 d^3 n^2 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{(b c - a d)^4 (a + b x)} - \\ & \frac{9 b B^2 d^2 n^2 (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{4 (b c - a d)^4 (a + b x)^2} + \\ & \frac{2 b^2 B^2 d n^2 (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 (b c - a d)^4 (a + b x)^3} - \\ & \frac{3 b^3 B^2 n^2 (c + d x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{32 (b c - a d)^4 (a + b x)^4} + \\ & \frac{3 B d^3 n (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^4 (a + b x)} - \\ & \frac{9 b B d^2 n (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{4 (b c - a d)^4 (a + b x)^2} + \\ & \frac{b^2 B d n (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(b c - a d)^4 (a + b x)^3} - \\ & \frac{3 b^3 B n (c + d x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{16 (b c - a d)^4 (a + b x)^4} + \\ & \frac{d^3 (c + d x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d)^4 (a + b x)} - \\ & \frac{3 b d^2 (c + d x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 (b c - a d)^4 (a + b x)^2} + \\ & \frac{b^2 d (c + d x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(b c - a d)^4 (a + b x)^3} - \\ & \frac{b^3 (c + d x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{4 (b c - a d)^4 (a + b x)^4} \end{aligned}$$

Result (type 4, 2173 leaves, 93 steps):

$$-\frac{A^3}{4 b (a + b x)^4} - \frac{3 A^2 B n}{16 b (a + b x)^4} - \frac{3 A B^2 n^2}{32 b (a + b x)^4} - \frac{3 B^3 n^3}{128 b (a + b x)^4} + \frac{A^2 B d n}{4 b (b c - a d) (a + b x)^3} +$$

$$\begin{aligned}
& \frac{7 A B^2 d n^2}{24 b (b c - a d) (a + b x)^3} + \frac{37 B^3 d n^3}{288 b (b c - a d) (a + b x)^3} - \frac{3 A^2 B d^2 n}{8 b (b c - a d)^2 (a + b x)^2} - \\
& \frac{13 A B^2 d^2 n^2}{16 b (b c - a d)^2 (a + b x)^2} - \frac{79 B^3 d^2 n^3}{192 b (b c - a d)^2 (a + b x)^2} + \frac{3 A^2 B d^3 n}{4 b (b c - a d)^3 (a + b x)} + \\
& \frac{25 A B^2 d^3 n^2}{8 b (b c - a d)^3 (a + b x)} + \frac{451 B^3 d^3 n^3}{96 b (b c - a d)^3 (a + b x)} - \frac{3 b B^3 d^2 n^3 (c + d x)^2}{16 (b c - a d)^4 (a + b x)^2} + \\
& \frac{3 A^2 B d^4 n \operatorname{Log}[a + b x]}{4 b (b c - a d)^4} + \frac{13 A B^2 d^4 n^2 \operatorname{Log}[a + b x]}{8 b (b c - a d)^4} + \frac{79 B^3 d^4 n^3 \operatorname{Log}[a + b x]}{96 b (b c - a d)^4} - \\
& \frac{3 A^2 B d^4 n \operatorname{Log}[c + d x]}{4 b (b c - a d)^4} - \frac{13 A B^2 d^4 n^2 \operatorname{Log}[c + d x]}{8 b (b c - a d)^4} - \frac{79 B^3 d^4 n^3 \operatorname{Log}[c + d x]}{96 b (b c - a d)^4} - \\
& \frac{3 A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 b (a + b x)^4} - \frac{3 A B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{8 b (a + b x)^4} - \\
& \frac{3 B^3 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{32 b (a + b x)^4} + \frac{A B^2 d n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b (b c - a d) (a + b x)^3} + \\
& \frac{7 B^3 d n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{24 b (b c - a d) (a + b x)^3} - \frac{3 A B^2 d^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 b (b c - a d)^2 (a + b x)^2} - \\
& \frac{7 B^3 d^2 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{16 b (b c - a d)^2 (a + b x)^2} + \frac{3 A B^2 d^3 n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 (b c - a d)^4 (a + b x)} + \\
& \frac{31 B^3 d^3 n^2 (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{8 (b c - a d)^4 (a + b x)} - \frac{3 b B^3 d^2 n^2 (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{8 (b c - a d)^4 (a + b x)^2} - \\
& \frac{3 A B^2 d^4 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b (b c - a d)^4} - \\
& \frac{7 B^3 d^4 n^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{8 b (b c - a d)^4} + \\
& \frac{3 A B^2 d^4 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 b (b c - a d)^4} + \\
& \frac{7 B^3 d^4 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{8 b (b c - a d)^4} - \frac{3 A B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{4 b (a + b x)^4} - \\
& \frac{3 B^3 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{16 b (a + b x)^4} + \frac{B^3 d n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{4 b (b c - a d) (a + b x)^3} + \\
& \frac{3 B^3 d^3 n (c + d x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 (b c - a d)^4 (a + b x)} - \frac{3 b B^3 d^2 n (c + d x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{8 (b c - a d)^4 (a + b x)^2} - \\
& \frac{3 B^3 d^4 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{4 b (b c - a d)^4} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 B^3 d^4 n \operatorname{Log}\left[\frac{b c-a d}{b(c+d x)}\right] \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2}{4 b(b c-a d)^4} - \frac{B^3 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^3}{4 b(a+b x)^4} + \\
& \frac{3 A B^2 d^4 n^2 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{2 b(b c-a d)^4} + \frac{7 B^3 d^4 n^3 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{8 b(b c-a d)^4} + \\
& \frac{3 A B^2 d^4 n^2 \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{2 b(b c-a d)^4} + \frac{7 B^3 d^4 n^3 \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{8 b(b c-a d)^4} + \\
& \frac{3 B^3 d^4 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{2 b(b c-a d)^4} + \\
& \frac{3 B^3 d^4 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1-\frac{b c-a d}{b(c+d x)}\right]}{2 b(b c-a d)^4} + \\
& \frac{3 B^3 d^4 n^3 \operatorname{PolyLog}\left[3, 1+\frac{b c-a d}{d(a+b x)}\right]}{2 b(b c-a d)^4} - \frac{3 B^3 d^4 n^3 \operatorname{PolyLog}\left[3, 1-\frac{b c-a d}{b(c+d x)}\right]}{2 b(b c-a d)^4}
\end{aligned}$$

Problem 172: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 (A + B \operatorname{Log}[e(a+b x)^n(c+d x)^{-n}])} dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$\left( e^{\frac{A}{B n}} (c+d x) (e(a+b x)^n(c+d x)^{-n})^{\frac{1}{n}} \operatorname{ExpIntegralEi}\left[-\frac{A+B \operatorname{Log}[e(a+b x)^n(c+d x)^{-n}]}{B n}\right] \right) / (B(b c-a d) g^2 n(a+b x))$$

Result (type 8, 38 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 (A + B \operatorname{Log}[e(a+b x)^n(c+d x)^{-n}])}, x\right]$$

Problem 177: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]}{a g + b g x} dx$$

Optimal (type 4, 81 leaves, 5 steps):

$$-\frac{\operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] (A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right])}{b g} - \frac{B \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{b g}$$

Result (type 4, 122 leaves, 10 steps):

$$\frac{B \operatorname{Log}[g(a+bx)]^2}{2bg} - \frac{B \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag+bgx]}{bg} +$$

$$\frac{\left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right) \operatorname{Log}[ag+bgx]}{bg} - \frac{B \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b(c+dx)}\right]}{bg}$$

**Problem 178: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]}{(ag+bgx)^2} dx$$

Optimal (type 3, 64 leaves, 3 steps):

$$-\frac{A-B}{bg^2(a+bx)} - \frac{B(c+dx) \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]}{(bc-ad)g^2(a+bx)}$$

Result (type 3, 101 leaves, 4 steps):

$$\frac{B}{bg^2(a+bx)} + \frac{Bd \operatorname{Log}[a+bx]}{b(bc-ad)g^2} - \frac{Bd \operatorname{Log}[c+dx]}{b(bc-ad)g^2} - \frac{A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]}{bg^2(a+bx)}$$

**Problem 182: Result valid but suboptimal antiderivative.**

$$\int (ag+bgx)^4 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2 dx$$

Optimal (type 4, 503 leaves, 19 steps):

$$\frac{13B^2(bc-ad)^4g^4x}{30d^4} - \frac{7B^2(bc-ad)^3g^4(a+bx)^2}{60bd^3} +$$

$$\frac{B^2(bc-ad)^2g^4(a+bx)^3}{30bd^2} - \frac{5B^2(bc-ad)^5g^4 \operatorname{Log}[a+bx]}{6bd^5} -$$

$$\frac{13B^2(bc-ad)^5g^4 \operatorname{Log}\left[\frac{c+dx}{a+bx}\right]}{30bd^5} + \frac{B(bc-ad)^3g^4(a+bx)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{5bd^3} -$$

$$\frac{2B(bc-ad)^2g^4(a+bx)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{15bd^2} + \frac{B(bc-ad)g^4(a+bx)^4 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{10bd} -$$

$$\frac{2B(bc-ad)^4g^4(c+dx) \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{5d^5} + \frac{g^4(a+bx)^5 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{5b} -$$

$$\frac{2B(bc-ad)^5g^4 \left(A + B \operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right) \operatorname{Log}\left[1 - \frac{d(a+bx)}{b(c+dx)}\right]}{5bd^5} + \frac{2B^2(bc-ad)^5g^4 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{5bd^5}$$

Result (type 4, 557 leaves, 28 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d)^4 g^4 x}{5 d^4} + \frac{13 B^2 (b c - a d)^4 g^4 x}{30 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 (a + b x)^2}{60 b d^3} + \\
& \frac{B^2 (b c - a d)^2 g^4 (a + b x)^3}{30 b d^2} - \frac{5 B^2 (b c - a d)^5 g^4 \operatorname{Log}[c + d x]}{6 b d^5} + \\
& \frac{2 B^2 (b c - a d)^5 g^4 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{5 b d^5} - \frac{B^2 (b c - a d)^5 g^4 \operatorname{Log}[c + d x]^2}{5 b d^5} - \\
& \frac{2 B^2 (b c - a d)^4 g^4 (a + b x) \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]}{5 b d^4} + \frac{B (b c - a d)^3 g^4 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{5 b d^3} - \\
& \frac{2 B (b c - a d)^2 g^4 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{15 b d^2} + \frac{B (b c - a d) g^4 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{10 b d} + \\
& \frac{2 B (b c - a d)^5 g^4 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{5 b d^5} + \\
& \frac{g^4 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{5 b} + \frac{2 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{5 b d^5}
\end{aligned}$$

**Problem 183: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right] \right)^2 dx$$

Optimal (type 4, 420 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 B^2 (b c - a d)^3 g^3 x}{12 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{12 b d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log}[a + b x]}{12 b d^4} + \frac{5 B^2 (b c - a d)^4 g^3 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right]}{12 b d^4} - \\
& \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{4 b d^2} + \frac{B (b c - a d) g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{6 b d} + \\
& \frac{B (b c - a d)^3 g^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{2 d^4} + \frac{g^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{4 b} + \\
& \frac{B (b c - a d)^4 g^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right) \operatorname{Log}\left[1 - \frac{d(a+b x)}{b(c+d x)}\right]}{2 b d^4} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{2 b d^4}
\end{aligned}$$

Result (type 4, 474 leaves, 24 steps):



$$\begin{aligned}
& \frac{A B (b c - a d)^3 g^3 x}{2 d^3} - \frac{5 B^2 (b c - a d)^3 g^3 x}{12 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{12 b d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log}[c + d x]}{12 b d^4} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 b d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log}[c + d x]^2}{4 b d^4} + \frac{B^2 (b c - a d)^3 g^3 (a + b x) \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]}{2 b d^3} - \\
& \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{4 b d^2} + \frac{B (b c - a d) g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{6 b d} - \\
& \frac{B (b c - a d)^4 g^3 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{2 b d^4} + \\
& \frac{g^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{4 b} - \frac{B^2 (b c - a d)^4 g^3 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{2 b d^4}
\end{aligned}$$

**Problem 184: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2 dx$$

Optimal (type 4, 335 leaves, 11 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 x}{3 d^2} - \frac{B^2 (b c - a d)^3 g^2 \operatorname{Log}[a + b x]}{b d^3} - \\
& \frac{B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right]}{3 b d^3} + \frac{B (b c - a d) g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b d} - \\
& \frac{2 B (b c - a d)^2 g^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 d^3} + \frac{g^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{3 b} - \\
& \frac{2 B (b c - a d)^3 g^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right) \operatorname{Log}\left[1 - \frac{d(a+b x)}{b(c+d x)}\right]}{3 b d^3} + \frac{2 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 389 leaves, 20 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d)^2 g^2 x}{3 d^2} + \frac{B^2(b c - a d)^2 g^2 x}{3 d^2} - \frac{B^2(b c - a d)^3 g^2 \operatorname{Log}[c + d x]}{b d^3} + \\
& \frac{2B^2(b c - a d)^3 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b d^3} - \frac{B^2(b c - a d)^3 g^2 \operatorname{Log}[c + d x]^2}{3 b d^3} - \\
& \frac{2B^2(b c - a d)^2 g^2 (a + b x) \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]}{3 b d^2} + \frac{B(b c - a d) g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b d} + \\
& \frac{2B(b c - a d)^3 g^2 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b d^3} + \\
& \frac{g^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{3 b} + \frac{2B^2(b c - a d)^3 g^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{3 b d^3}
\end{aligned}$$

**Problem 185: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) \left( A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right] \right)^2 dx$$

Optimal (type 4, 202 leaves, 7 steps):

$$\begin{aligned}
& \frac{B^2(b c - a d)^2 g \operatorname{Log}[a + b x]}{b d^2} + \\
& \frac{B(b c - a d) g(c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{d^2} + \frac{g(a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{2 b} + \\
& \frac{B(b c - a d)^2 g \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right) \operatorname{Log}\left[1 - \frac{d(a+b x)}{b(c+d x)}\right]}{b d^2} - \frac{B^2(b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{b d^2}
\end{aligned}$$

Result (type 4, 284 leaves, 16 steps):

$$\begin{aligned}
& \frac{AB(b c - a d) g x}{d} + \frac{B^2(b c - a d)^2 g \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{B^2(b c - a d)^2 g \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^2} + \frac{B^2(b c - a d)^2 g \operatorname{Log}[c + d x]^2}{2 b d^2} + \\
& \frac{B^2(b c - a d) g(a + b x) \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]}{b d} - \frac{B(b c - a d)^2 g \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{b d^2} + \\
& \frac{g(a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{2 b} - \frac{B^2(b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b d^2}
\end{aligned}$$

**Problem 186: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 128 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \left(A + B \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right)^2}{b g} - \\
& \frac{2 B \left(A + B \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right) \text{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b g} + \frac{2 B^2 \text{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b g}
\end{aligned}$$

Result (type 4, 719 leaves, 47 steps):

$$\begin{aligned}
& \frac{A B \text{Log}\left[g (a + b x)\right]^2}{b g} + \frac{B^2 \text{Log}\left[g (a + b x)\right]^3}{3 b g} - \frac{B^2 \text{Log}\left[\frac{1}{a + b x}\right]^2 \text{Log}[c + d x]}{b g} - \\
& \frac{2 B^2 \text{Log}\left[\frac{1}{a + b x}\right] \text{Log}\left[g (a + b x)\right] \text{Log}[c + d x]}{b g} - \frac{B^2 \text{Log}\left[g (a + b x)\right]^2 \text{Log}[c + d x]}{b g} + \\
& \frac{B^2 \text{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \text{Log}[c + d x]^2}{b g} - \frac{B^2 \text{Log}\left[g (a + b x)\right] \text{Log}[c + d x]^2}{b g} + \frac{B^2 \text{Log}\left[\frac{1}{a + b x}\right]^2 \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b g} + \\
& \frac{B^2 \text{Log}\left[g (a + b x)\right]^2 \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b g} - \frac{2 A B \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \text{Log}[a g + b g x]}{b g} + \frac{1}{b g} \\
& 2 B^2 \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \left(\text{Log}\left[\frac{1}{a + b x}\right] + \text{Log}[c + d x] - \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right) \text{Log}[a g + b g x] + \\
& \frac{\left(A + B \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right)^2 \text{Log}[a g + b g x]}{b g} - \\
& \frac{B^2 \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \text{Log}[a g + b g x]^2}{b g} + \frac{B^2 \text{Log}\left[\frac{e (c + d x)}{a + b x}\right] \text{Log}[a g + b g x]^2}{b g} - \\
& \frac{2 A B \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b g} - \frac{2 B^2 \text{Log}\left[\frac{1}{a + b x}\right] \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b g} + \\
& \frac{2 B^2 \left(\text{Log}\left[\frac{1}{a + b x}\right] + \text{Log}[c + d x] - \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right) \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b g} + \\
& \frac{2 B^2 \text{Log}[c + d x] \text{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b g} - \frac{2 B^2 \text{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{b g} - \frac{2 B^2 \text{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{b g}
\end{aligned}$$

**Problem 187:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \text{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right)^2}{(a g + b g x)^2} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{2AB(c+dx)}{(bc-ad)g^2(a+bx)} - \frac{2B^2(c+dx)}{(bc-ad)g^2(a+bx)} +$$

$$\frac{2B^2(c+dx)\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]}{(bc-ad)g^2(a+bx)} - \frac{(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{(bc-ad)g^2(a+bx)}$$

Result (type 4, 470 leaves, 26 steps):

$$-\frac{2B^2}{bg^2(a+bx)} - \frac{2B^2d\operatorname{Log}[a+bx]}{b(bc-ad)g^2} + \frac{B^2d\operatorname{Log}[a+bx]^2}{b(bc-ad)g^2} +$$

$$\frac{2B^2d\operatorname{Log}[c+dx]}{b(bc-ad)g^2} - \frac{2B^2d\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{b(bc-ad)g^2} + \frac{B^2d\operatorname{Log}[c+dx]^2}{b(bc-ad)g^2} -$$

$$\frac{2B^2d\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b(bc-ad)g^2} + \frac{2B\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{bg^2(a+bx)} +$$

$$\frac{2Bd\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{b(bc-ad)g^2} - \frac{2Bd\operatorname{Log}[c+dx]\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{b(bc-ad)g^2} -$$

$$\frac{\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{bg^2(a+bx)} - \frac{2B^2d\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b(bc-ad)g^2} - \frac{2B^2d\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b(bc-ad)g^2}$$

**Problem 188: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{(ag+bgx)^3} dx$$

Optimal (type 3, 296 leaves, 8 steps):

$$-\frac{2ABd(c+dx)}{(bc-ad)^2g^3(a+bx)} + \frac{2B^2d(c+dx)}{(bc-ad)^2g^3(a+bx)} - \frac{bB^2(c+dx)^2}{4(bc-ad)^2g^3(a+bx)^2} -$$

$$\frac{2B^2d(c+dx)\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]}{(bc-ad)^2g^3(a+bx)} + \frac{bB(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)}{2(bc-ad)^2g^3(a+bx)^2} +$$

$$\frac{d(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{(bc-ad)^2g^3(a+bx)} - \frac{b(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(c+dx)}{a+bx}\right]\right)^2}{2(bc-ad)^2g^3(a+bx)^2}$$

Result (type 4, 578 leaves, 30 steps):

$$\begin{aligned}
& - \frac{B^2}{4 b g^3 (a + b x)^2} + \frac{3 B^2 d}{2 b (b c - a d) g^3 (a + b x)} + \frac{3 B^2 d^2 \operatorname{Log}[a + b x]}{2 b (b c - a d)^2 g^3} - \frac{B^2 d^2 \operatorname{Log}[a + b x]^2}{2 b (b c - a d)^2 g^3} - \\
& \frac{3 B^2 d^2 \operatorname{Log}[c + d x]}{2 b (b c - a d)^2 g^3} + \frac{B^2 d^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{B^2 d^2 \operatorname{Log}[c + d x]^2}{2 b (b c - a d)^2 g^3} + \\
& \frac{B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{2 b g^3 (a + b x)^2} - \frac{B d \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{b (b c - a d) g^3 (a + b x)} - \\
& \frac{B d^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{b (b c - a d)^2 g^3} + \frac{B d^2 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{b (b c - a d)^2 g^3} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)^2}{2 b g^3 (a + b x)^2} + \frac{B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3}
\end{aligned}$$

**Problem 189: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 399 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 (c + d x)}{(b c - a d)^3 g^4 (a + b x)} + \frac{b B^2 d (c + d x)^2}{2 (b c - a d)^3 g^4 (a + b x)^2} - \\
& \frac{2 b^2 B^2 (c + d x)^3}{27 (b c - a d)^3 g^4 (a + b x)^3} + \frac{B^2 d^3 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right]^2}{3 b (b c - a d)^3 g^4} + \frac{2 B d^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{(b c - a d)^3 g^4 (a + b x)} - \\
& \frac{b B d (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{(b c - a d)^3 g^4 (a + b x)^2} + \frac{2 b^2 B (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{9 (b c - a d)^3 g^4 (a + b x)^3} - \\
& \frac{2 B d^3 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{3 b (b c - a d)^3 g^4} - \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)^2}{3 b g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 680 leaves, 34 steps):

$$\begin{aligned}
& - \frac{2 B^2}{27 b g^4 (a+b x)^3} + \frac{5 B^2 d}{18 b (b c-a d) g^4 (a+b x)^2} - \frac{11 B^2 d^2}{9 b (b c-a d)^2 g^4 (a+b x)} - \\
& \frac{11 B^2 d^3 \operatorname{Log}[a+b x]}{9 b (b c-a d)^3 g^4} + \frac{B^2 d^3 \operatorname{Log}[a+b x]^2}{3 b (b c-a d)^3 g^4} + \frac{11 B^2 d^3 \operatorname{Log}[c+d x]}{9 b (b c-a d)^3 g^4} - \\
& \frac{2 B^2 d^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{3 b (b c-a d)^3 g^4} + \frac{B^2 d^3 \operatorname{Log}[c+d x]^2}{3 b (b c-a d)^3 g^4} - \frac{2 B^2 d^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{3 b (b c-a d)^3 g^4} + \\
& \frac{2 B \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{9 b g^4 (a+b x)^3} - \frac{B d \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b (b c-a d) g^4 (a+b x)^2} + \frac{2 B d^2 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b (b c-a d)^2 g^4 (a+b x)} + \\
& \frac{2 B d^3 \operatorname{Log}[a+b x] \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b (b c-a d)^3 g^4} - \frac{2 B d^3 \operatorname{Log}[c+d x] \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 b (b c-a d)^3 g^4} - \\
& \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{3 b g^4 (a+b x)^3} - \frac{2 B^2 d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{3 b (b c-a d)^3 g^4} - \frac{2 B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{3 b (b c-a d)^3 g^4}
\end{aligned}$$

**Problem 190: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{(a g+b g x)^5} dx$$

Optimal (type 3, 498 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 B^2 d^3 (c+d x)}{(b c-a d)^4 g^5 (a+b x)} - \frac{3 b B^2 d^2 (c+d x)^2}{4 (b c-a d)^4 g^5 (a+b x)^2} + \frac{2 b^2 B^2 d (c+d x)^3}{9 (b c-a d)^4 g^5 (a+b x)^3} - \\
& \frac{b^3 B^2 (c+d x)^4}{32 (b c-a d)^4 g^5 (a+b x)^4} - \frac{B^2 d^4 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right]^2}{4 b (b c-a d)^4 g^5} - \frac{2 B d^3 (c+d x) \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{(b c-a d)^4 g^5 (a+b x)} + \\
& \frac{3 b B d^2 (c+d x)^2 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{2 (b c-a d)^4 g^5 (a+b x)^2} - \frac{2 b^2 B d (c+d x)^3 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{3 (b c-a d)^4 g^5 (a+b x)^3} + \\
& \frac{b^3 B (c+d x)^4 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{8 (b c-a d)^4 g^5 (a+b x)^4} + \frac{B d^4 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right] \left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)}{2 b (b c-a d)^4 g^5} - \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)}{a+b x}\right]\right)^2}{4 b g^5 (a+b x)^4}
\end{aligned}$$

Result (type 4, 763 leaves, 38 steps):

$$\begin{aligned}
& - \frac{B^2}{32 b g^5 (a + b x)^4} + \frac{7 B^2 d}{72 b (b c - a d) g^5 (a + b x)^3} - \\
& \frac{13 B^2 d^2}{48 b (b c - a d)^2 g^5 (a + b x)^2} + \frac{25 B^2 d^3}{24 b (b c - a d)^3 g^5 (a + b x)} + \frac{25 B^2 d^4 \operatorname{Log}[a + b x]}{24 b (b c - a d)^4 g^5} - \\
& \frac{B^2 d^4 \operatorname{Log}[a + b x]^2}{4 b (b c - a d)^4 g^5} - \frac{25 B^2 d^4 \operatorname{Log}[c + d x]}{24 b (b c - a d)^4 g^5} + \frac{B^2 d^4 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 b (b c - a d)^4 g^5} - \\
& \frac{B^2 d^4 \operatorname{Log}[c + d x]^2}{4 b (b c - a d)^4 g^5} + \frac{B^2 d^4 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5} + \frac{B \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{8 b g^5 (a + b x)^4} - \\
& \frac{B d \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{6 b (b c - a d) g^5 (a + b x)^3} + \frac{B d^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{4 b (b c - a d)^2 g^5 (a + b x)^2} - \frac{B d^3 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{2 b (b c - a d)^3 g^5 (a + b x)} - \\
& \frac{B d^4 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{2 b (b c - a d)^4 g^5} + \frac{B d^4 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)}{2 b (b c - a d)^4 g^5} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]\right)^2}{4 b g^5 (a + b x)^4} + \frac{B^2 d^4 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5} + \frac{B^2 d^4 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{2 b (b c - a d)^4 g^5}
\end{aligned}$$

Problem 191: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$\begin{aligned}
& a^2 g^2 \operatorname{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]}, x\right] + \\
& 2 a b g^2 \operatorname{CannotIntegrate}\left[\frac{x}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]}, x\right] + b^2 g^2 \operatorname{CannotIntegrate}\left[\frac{x^2}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]}, x\right]
\end{aligned}$$

Problem 192: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{A + B \operatorname{Log}\left[\frac{e(c + d x)}{a + b x}\right]} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrateable}\left[\frac{a g + b g x}{A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$a g \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]}, x\right] + b g \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]}, x\right]$$

Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrateable}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)}, x\right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)}, x\right]$$

Problem 194: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)} dx$$

Optimal (type 4, 53 leaves, 3 steps):

$$-\frac{e^{-\frac{A}{B}} \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]}{B}\right]}{B (b c - a d) e g^2}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)}, x\right]$$

Problem 195: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \text{Log}\left[\frac{e^{(c+dx)}}{a+bx}\right]\right)} dx$$

Optimal (type 4, 109 leaves, 7 steps):



$$\frac{d e^{-\frac{A}{B}} \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]}{B}\right]}{B (b c - a d)^2 e g^3} - \frac{b e^{-\frac{2A}{B}} \text{ExpIntegralEi}\left[\frac{2 (A+B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right])}{B}\right]}{B (b c - a d)^2 e^2 g^3}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)}, x\right]$$

Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a g + b g x)^2}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right]$$

Result (type 8, 97 leaves, 2 steps):

$$\begin{aligned} & a^2 g^2 \text{CannotIntegrate}\left[\frac{1}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right] + \\ & 2 a b g^2 \text{CannotIntegrate}\left[\frac{x}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right] + \\ & b^2 g^2 \text{CannotIntegrate}\left[\frac{x^2}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right] \end{aligned}$$

Problem 197: Result valid but suboptimal antiderivative.

$$\int \frac{a g + b g x}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right]$$

Result (type 8, 55 leaves, 2 steps):

$$a g \text{CannotIntegrate}\left[\frac{1}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right] + b g \text{CannotIntegrate}\left[\frac{x}{\left(A + B \text{Log}\left[\frac{e^{(c+d x)}}{a+b x}\right]\right)^2}, x\right]$$

### Problem 198: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2}, x \right]$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2}, x \right]$$

### Problem 199: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$-\frac{e^{-\frac{A}{B}} \operatorname{ExpIntegralEi} \left[ \frac{A+B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right]}{B} \right]}{B^2 (b c - a d) e g^2} + \frac{c + d x}{B (b c - a d) g^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2}, x \right]$$

### Problem 200: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)^2} dx$$

Optimal (type 4, 159 leaves, 10 steps):

$$\frac{d e^{-\frac{A}{B}} \operatorname{ExpIntegralEi} \left[ \frac{A+B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right]}{B} \right]}{B^2 (b c - a d)^2 e g^3} - \frac{2 b e^{-\frac{2A}{B}} \operatorname{ExpIntegralEi} \left[ \frac{2 \left( A+B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)}{B} \right]}{B^2 (b c - a d)^2 e^2 g^3} + \frac{c + d x}{B (b c - a d) g^3 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)}}{a+bx} \right] \right)}$$

Result (type 8, 34 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)}{a + b x}\right]\right)^2}, x\right]$$

**Problem 205: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{a g + b g x} dx$$

Optimal (type 4, 83 leaves, 5 steps):

$$-\frac{\operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{b g} - \frac{2 B \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b g}$$

Result (type 4, 121 leaves, 10 steps):

$$\frac{B \operatorname{Log}\left[g (a + b x)\right]^2}{b g} - \frac{2 B \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]}{b g} +$$

$$\frac{\left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right) \operatorname{Log}[a g + b g x]}{b g} - \frac{2 B \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b g}$$

**Problem 206: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{(a g + b g x)^2} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$-\frac{A (c + d x)}{(b c - a d) g^2 (a + b x)} + \frac{2 B (c + d x)}{(b c - a d) g^2 (a + b x)} - \frac{B (c + d x) \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{(b c - a d) g^2 (a + b x)}$$

Result (type 3, 105 leaves, 4 steps):

$$\frac{2 B}{b g^2 (a + b x)} + \frac{2 B d \operatorname{Log}[a + b x]}{b (b c - a d) g^2} - \frac{2 B d \operatorname{Log}[c + d x]}{b (b c - a d) g^2} - \frac{A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{b g^2 (a + b x)}$$

**Problem 210: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^4 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2 dx$$

Optimal (type 4, 515 leaves, 19 steps):

$$\begin{aligned}
& \frac{26 B^2 (b c - a d)^4 g^4 x}{15 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 (a + b x)^2}{15 b d^3} + \\
& \frac{2 B^2 (b c - a d)^2 g^4 (a + b x)^3}{15 b d^2} - \frac{10 B^2 (b c - a d)^5 g^4 \operatorname{Log}[a + b x]}{3 b d^5} - \\
& \frac{26 B^2 (b c - a d)^5 g^4 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right]}{15 b d^5} + \frac{2 B (b c - a d)^3 g^4 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 b d^3} - \\
& \frac{4 B (b c - a d)^2 g^4 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{15 b d^2} + \frac{B (b c - a d) g^4 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 b d} - \\
& \frac{4 B (b c - a d)^4 g^4 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 d^5} + \frac{g^4 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}{5 b} - \\
& \frac{4 B (b c - a d)^5 g^4 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right) \operatorname{Log}\left[1 - \frac{d (a+b x)}{b (c+d x)}\right]}{5 b d^5} + \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{5 b d^5}
\end{aligned}$$

Result(type 4, 569 leaves, 28 steps):

$$\begin{aligned}
& -\frac{4 A B (b c - a d)^4 g^4 x}{5 d^4} + \frac{26 B^2 (b c - a d)^4 g^4 x}{15 d^4} - \frac{7 B^2 (b c - a d)^3 g^4 (a + b x)^2}{15 b d^3} + \\
& \frac{2 B^2 (b c - a d)^2 g^4 (a + b x)^3}{15 b d^2} - \frac{10 B^2 (b c - a d)^5 g^4 \operatorname{Log}[c + d x]}{3 b d^5} + \\
& \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{5 b d^5} - \frac{4 B^2 (b c - a d)^5 g^4 \operatorname{Log}[c + d x]^2}{5 b d^5} - \\
& \frac{4 B^2 (b c - a d)^4 g^4 (a + b x) \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]}{5 b d^4} + \frac{2 B (b c - a d)^3 g^4 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 b d^3} - \\
& \frac{4 B (b c - a d)^2 g^4 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{15 b d^2} + \frac{B (b c - a d) g^4 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 b d} + \\
& \frac{4 B (b c - a d)^5 g^4 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{5 b d^5} + \\
& \frac{g^4 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}{5 b} + \frac{8 B^2 (b c - a d)^5 g^4 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{5 b d^5}
\end{aligned}$$

Problem 211: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2 dx$$

Optimal (type 4, 422 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 B^2 (b c - a d)^3 g^3 x}{3 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{3 b d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log}[a + b x]}{3 b d^4} + \frac{5 B^2 (b c - a d)^4 g^3 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right]}{3 b d^4} - \\
& \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{2 b d^2} + \frac{B (b c - a d) g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{3 b d} + \\
& \frac{B (b c - a d)^3 g^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{d^4} + \frac{g^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2}{4 b} + \\
& \frac{B (b c - a d)^4 g^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right) \operatorname{Log}\left[1 - \frac{d (a + b x)}{b (c + d x)}\right]}{b d^4} - \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^4}
\end{aligned}$$

Result (type 4, 469 leaves, 24 steps):

$$\begin{aligned}
& \frac{A B (b c - a d)^3 g^3 x}{d^3} - \frac{5 B^2 (b c - a d)^3 g^3 x}{3 d^3} + \frac{B^2 (b c - a d)^2 g^3 (a + b x)^2}{3 b d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 \operatorname{Log}[c + d x]}{3 b d^4} - \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log}[c + d x]^2}{b d^4} + \frac{B^2 (b c - a d)^3 g^3 (a + b x) \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{b d^3} - \\
& \frac{B (b c - a d)^2 g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{2 b d^2} + \frac{B (b c - a d) g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{3 b d} - \\
& \frac{B (b c - a d)^4 g^3 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}{b d^4} + \\
& \frac{g^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2}{4 b} - \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b d^4}
\end{aligned}$$

**Problem 212: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2 dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\begin{aligned}
& \frac{4 B^2 (b c - a d)^2 g^2 x}{3 d^2} - \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}[a + b x]}{b d^3} - \\
& \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[\frac{c+d x}{a+b x}\right]}{3 b d^3} + \frac{2 B (b c - a d) g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{3 b d} - \\
& \frac{4 B (b c - a d)^2 g^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{3 d^3} + \frac{g^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}{3 b} - \\
& \frac{4 B (b c - a d)^3 g^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right) \operatorname{Log}\left[1 - \frac{d (a+b x)}{b (c+d x)}\right]}{3 b d^3} + \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{3 b d^3}
\end{aligned}$$

Result (type 4, 397 leaves, 20 steps):

$$\begin{aligned}
& - \frac{4 A B (b c - a d)^2 g^2 x}{3 d^2} + \frac{4 B^2 (b c - a d)^2 g^2 x}{3 d^2} - \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}[c + d x]}{b d^3} + \\
& \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b d^3} - \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}[c + d x]^2}{3 b d^3} - \\
& \frac{4 B^2 (b c - a d)^2 g^2 (a + b x) \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]}{3 b d^2} + \frac{2 B (b c - a d) g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{3 b d} + \\
& \frac{4 B (b c - a d)^3 g^2 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{3 b d^3} + \\
& \frac{g^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}{3 b} + \frac{8 B^2 (b c - a d)^3 g^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{3 b d^3}
\end{aligned}$$

**Problem 213: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) \left( A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right] \right)^2 dx$$

Optimal (type 4, 211 leaves, 7 steps):

$$\begin{aligned}
& \frac{4 B^2 (b c - a d)^2 g \operatorname{Log}[a + b x]}{b d^2} + \\
& \frac{2 B (b c - a d) g (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)}{d^2} + \frac{g (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right)^2}{2 b} + \\
& \frac{2 B (b c - a d)^2 g \left(A + B \operatorname{Log}\left[\frac{e (c+d x)^2}{(a+b x)^2}\right]\right) \operatorname{Log}\left[1 - \frac{d (a+b x)}{b (c+d x)}\right]}{b d^2} - \frac{4 B^2 (b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{b d^2}
\end{aligned}$$

Result (type 4, 291 leaves, 16 steps):

$$\begin{aligned}
& \frac{2AB(b c - a d) g x}{d} + \frac{4B^2(b c - a d)^2 g \operatorname{Log}[c + d x]}{b d^2} - \\
& \frac{4B^2(b c - a d)^2 g \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b d^2} + \frac{2B^2(b c - a d)^2 g \operatorname{Log}[c + d x]^2}{b d^2} + \\
& \frac{2B^2(b c - a d) g (a + b x) \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]}{b d} - \frac{2B(b c - a d)^2 g \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]\right)}{b d^2} + \\
& \frac{g(a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]\right)^2}{2 b} - \frac{4B^2(b c - a d)^2 g \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b d^2}
\end{aligned}$$

Problem 214: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 132 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\operatorname{Log}\left[-\frac{b c - a d}{d(a+b x)}\right] \left(A + B \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]\right)^2}{b g} - \\
& \frac{4B \left(A + B \operatorname{Log}\left[\frac{e^{(c+d x)^2}}{(a+b x)^2}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{d(a+b x)}\right]}{b g} + \frac{8B^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{d(a+b x)}\right]}{b g}
\end{aligned}$$

Result (type 4, 740 leaves, 46 steps):

$$\begin{aligned}
& \frac{2 A B \operatorname{Log}\left[g(a+b x)\right]^2}{b g} + \frac{4 B^2 \operatorname{Log}\left[g(a+b x)\right]^3}{3 b g} - \\
& \frac{B^2 \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right]^2 \operatorname{Log}[c+d x]}{b g} - \frac{4 B^2 \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right] \operatorname{Log}[g(a+b x)] \operatorname{Log}[c+d x]}{b g} - \\
& \frac{4 B^2 \operatorname{Log}\left[g(a+b x)\right]^2 \operatorname{Log}[c+d x]}{b g} + \frac{B^2 \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b g} + \\
& \frac{4 B^2 \operatorname{Log}\left[g(a+b x)\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b g} + \frac{B^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}\left[(c+d x)^2\right]^2}{b g} - \\
& \frac{B^2 \operatorname{Log}\left[g(a+b x)\right] \operatorname{Log}\left[(c+d x)^2\right]^2}{b g} - \frac{4 A B \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[a g+b g x]}{b g} + \frac{1}{b g} \\
& 4 B^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \left( \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right] + \operatorname{Log}\left[(c+d x)^2\right] - \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right] \right) \operatorname{Log}[a g+b g x] + \\
& \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)^2 \operatorname{Log}[a g+b g x]}{b g} - \frac{4 B^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[a g+b g x]^2}{b g} + \\
& \frac{2 B^2 \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right] \operatorname{Log}[a g+b g x]^2}{b g} - \frac{4 A B \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{b g} - \\
& \frac{4 B^2 \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{b g} + \frac{1}{b g} \\
& 4 B^2 \left( \operatorname{Log}\left[\frac{1}{(a+b x)^2}\right] + \operatorname{Log}\left[(c+d x)^2\right] - \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right] \right) \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right] + \\
& \frac{4 B^2 \operatorname{Log}\left[(c+d x)^2\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{b g} - \frac{8 B^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{b g} - \frac{8 B^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{b g}
\end{aligned}$$

**Problem 215:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)^2}{(a g+b g x)^2} d x$$

Optimal (type 3, 157 leaves, 4 steps):

$$\begin{aligned}
& \frac{4 A B (c+d x)}{(b c-a d) g^2 (a+b x)} - \frac{8 B^2 (c+d x)}{(b c-a d) g^2 (a+b x)} + \\
& \frac{4 B^2 (c+d x) \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]}{(b c-a d) g^2 (a+b x)} - \frac{(c+d x) \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)^2}{(b c-a d) g^2 (a+b x)}
\end{aligned}$$

Result (type 4, 480 leaves, 26 steps):



$$\begin{aligned}
& - \frac{8 B^2}{b g^2 (a + b x)} - \frac{8 B^2 d \operatorname{Log}[a + b x]}{b (b c - a d) g^2} + \frac{4 B^2 d \operatorname{Log}[a + b x]^2}{b (b c - a d) g^2} + \\
& \frac{8 B^2 d \operatorname{Log}[c + d x]}{b (b c - a d) g^2} - \frac{8 B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d) g^2} + \frac{4 B^2 d \operatorname{Log}[c + d x]^2}{b (b c - a d) g^2} - \\
& \frac{8 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d) g^2} + \frac{4 B \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b g^2 (a + b x)} + \\
& \frac{4 B d \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d) g^2} - \frac{4 B d \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d) g^2} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{b g^2 (a + b x)} - \frac{8 B^2 d \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b (b c - a d) g^2} - \frac{8 B^2 d \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d) g^2}
\end{aligned}$$

**Problem 216: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 299 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 A B d (c + d x)}{(b c - a d)^2 g^3 (a + b x)} + \frac{8 B^2 d (c + d x)}{(b c - a d)^2 g^3 (a + b x)} - \frac{b B^2 (c + d x)^2}{(b c - a d)^2 g^3 (a + b x)^2} - \\
& \frac{4 B^2 d (c + d x) \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]}{(b c - a d)^2 g^3 (a + b x)} + \frac{b B (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{(b c - a d)^2 g^3 (a + b x)^2} + \\
& \frac{d (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{(b c - a d)^2 g^3 (a + b x)} - \frac{b (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{2 (b c - a d)^2 g^3 (a + b x)^2}
\end{aligned}$$

Result (type 4, 578 leaves, 30 steps):

$$\begin{aligned}
& - \frac{B^2}{b g^3 (a + b x)^2} + \frac{6 B^2 d}{b (b c - a d) g^3 (a + b x)} + \frac{6 B^2 d^2 \operatorname{Log}[a + b x]}{b (b c - a d)^2 g^3} - \frac{2 B^2 d^2 \operatorname{Log}[a + b x]^2}{b (b c - a d)^2 g^3} - \\
& \frac{6 B^2 d^2 \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} + \frac{4 B^2 d^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b (b c - a d)^2 g^3} - \frac{2 B^2 d^2 \operatorname{Log}[c + d x]^2}{b (b c - a d)^2 g^3} + \\
& \frac{4 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{B \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b g^3 (a + b x)^2} - \frac{2 B d \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d) g^3 (a + b x)} - \\
& \frac{2 B d^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d)^2 g^3} + \frac{2 B d^2 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d)^2 g^3} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{2 b g^3 (a + b x)^2} + \frac{4 B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b (b c - a d)^2 g^3} + \frac{4 B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b (b c - a d)^2 g^3}
\end{aligned}$$

**Problem 217: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 407 leaves, 6 steps):

$$\begin{aligned}
& - \frac{8 B^2 d^2 (c + d x)}{(b c - a d)^3 g^4 (a + b x)} + \frac{2 b B^2 d (c + d x)^2}{(b c - a d)^3 g^4 (a + b x)^2} - \\
& \frac{8 b^2 B^2 (c + d x)^3}{27 (b c - a d)^3 g^4 (a + b x)^3} + \frac{4 B^2 d^3 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right]^2}{3 b (b c - a d)^3 g^4} + \frac{4 B d^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{(b c - a d)^3 g^4 (a + b x)} - \\
& \frac{2 b B d (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{(b c - a d)^3 g^4 (a + b x)^2} + \frac{4 b^2 B (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{9 (b c - a d)^3 g^4 (a + b x)^3} - \\
& \frac{4 B d^3 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 b (b c - a d)^3 g^4} - \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{3 b g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 692 leaves, 34 steps):

$$\begin{aligned}
& - \frac{8 B^2}{27 b g^4 (a + b x)^3} + \frac{10 B^2 d}{9 b (b c - a d) g^4 (a + b x)^2} - \frac{44 B^2 d^2}{9 b (b c - a d)^2 g^4 (a + b x)} - \\
& \frac{44 B^2 d^3 \operatorname{Log}[a + b x]}{9 b (b c - a d)^3 g^4} + \frac{4 B^2 d^3 \operatorname{Log}[a + b x]^2}{3 b (b c - a d)^3 g^4} + \frac{44 B^2 d^3 \operatorname{Log}[c + d x]}{9 b (b c - a d)^3 g^4} - \\
& \frac{8 B^2 d^3 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b (b c - a d)^3 g^4} + \frac{4 B^2 d^3 \operatorname{Log}[c + d x]^2}{3 b (b c - a d)^3 g^4} - \frac{8 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4} + \\
& \frac{4 B \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{9 b g^4 (a + b x)^3} - \frac{2 B d \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 b (b c - a d) g^4 (a + b x)^2} + \frac{4 B d^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 b (b c - a d)^2 g^4 (a + b x)} + \\
& \frac{4 B d^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 b (b c - a d)^3 g^4} - \frac{4 B d^3 \operatorname{Log}[c + d x] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 b (b c - a d)^3 g^4} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{3 b g^4 (a + b x)^3} - \frac{8 B^2 d^3 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4} - \frac{8 B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{3 b (b c - a d)^3 g^4}
\end{aligned}$$

**Problem 218: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 501 leaves, 5 steps):

$$\begin{aligned}
& \frac{8 B^2 d^3 (c + d x)}{(b c - a d)^4 g^5 (a + b x)} - \frac{3 b B^2 d^2 (c + d x)^2}{(b c - a d)^4 g^5 (a + b x)^2} + \frac{8 b^2 B^2 d (c + d x)^3}{9 (b c - a d)^4 g^5 (a + b x)^3} - \\
& \frac{b^3 B^2 (c + d x)^4}{8 (b c - a d)^4 g^5 (a + b x)^4} - \frac{B^2 d^4 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right]^2}{b (b c - a d)^4 g^5} - \frac{4 B d^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{(b c - a d)^4 g^5 (a + b x)} + \\
& \frac{3 b B d^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{(b c - a d)^4 g^5 (a + b x)^2} - \frac{4 b^2 B d (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{3 (b c - a d)^4 g^5 (a + b x)^3} + \\
& \frac{b^3 B (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{4 (b c - a d)^4 g^5 (a + b x)^4} + \frac{B d^4 \operatorname{Log}\left[\frac{c + d x}{a + b x}\right] \left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)}{b (b c - a d)^4 g^5} - \frac{\left(A + B \operatorname{Log}\left[\frac{e(c + d x)^2}{(a + b x)^2}\right]\right)^2}{4 b g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 758 leaves, 38 steps):

$$\begin{aligned}
& -\frac{B^2}{8 b g^5 (a+b x)^4} + \frac{7 B^2 d}{18 b (b c-a d) g^5 (a+b x)^3} - \\
& \frac{13 B^2 d^2}{12 b (b c-a d)^2 g^5 (a+b x)^2} + \frac{25 B^2 d^3}{6 b (b c-a d)^3 g^5 (a+b x)} + \frac{25 B^2 d^4 \operatorname{Log}[a+b x]}{6 b (b c-a d)^4 g^5} - \\
& \frac{B^2 d^4 \operatorname{Log}[a+b x]^2}{b (b c-a d)^4 g^5} - \frac{25 B^2 d^4 \operatorname{Log}[c+d x]}{6 b (b c-a d)^4 g^5} + \frac{2 B^2 d^4 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{b (b c-a d)^4 g^5} - \\
& \frac{B^2 d^4 \operatorname{Log}[c+d x]^2}{b (b c-a d)^4 g^5} + \frac{2 B^2 d^4 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b (b c-a d)^4 g^5} + \frac{B \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{4 b g^5 (a+b x)^4} - \\
& \frac{B d \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{3 b (b c-a d) g^5 (a+b x)^3} + \frac{B d^2 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{2 b (b c-a d)^2 g^5 (a+b x)^2} - \frac{B d^3 \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{b (b c-a d)^3 g^5 (a+b x)} - \\
& \frac{B d^4 \operatorname{Log}[a+b x] \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{b (b c-a d)^4 g^5} + \frac{B d^4 \operatorname{Log}[c+d x] \left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)}{b (b c-a d)^4 g^5} - \\
& \frac{\left(A+B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]\right)^2}{4 b g^5 (a+b x)^4} + \frac{2 B^2 d^4 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{b (b c-a d)^4 g^5} + \frac{2 B^2 d^4 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{b (b c-a d)^4 g^5}
\end{aligned}$$

**Problem 219: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a g + b g x)^2}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$\begin{aligned}
& a^2 g^2 \text{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]}, x\right] + \\
& 2 a b g^2 \text{CannotIntegrate}\left[\frac{x}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]}, x\right] + b^2 g^2 \text{CannotIntegrate}\left[\frac{x^2}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]}, x\right]
\end{aligned}$$

**Problem 220: Result valid but suboptimal antiderivative.**

$$\int \frac{a g + b g x}{A + B \operatorname{Log}\left[\frac{e(c+d x)^2}{(a+b x)^2}\right]} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$a g \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]}, x\right] + b g \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]}, x\right]$$

**Problem 221: Result valid but suboptimal antiderivative.**

$$\int \frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)}, x\right]$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x) \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)}, x\right]$$

**Problem 222: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)} dx$$

Optimal (type 4, 91 leaves, 3 steps):

$$-\frac{e^{-\frac{A}{2B}} (c+dx) \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]}{2B}\right]}{2B (bc-ad) g^2 (a+bx) \sqrt{\frac{e^{(c+dx)^2}}{(a+bx)^2}}}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)}, x\right]$$

**Problem 223: Unable to integrate problem.**

$$\int \frac{1}{(a g + b g x)^3 \left(A + B \text{Log}\left[\frac{e^{(c+dx)^2}}{(a+bx)^2}\right]\right)} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$\frac{d e^{-\frac{A}{2B}} (c + d x) \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]}{2B}\right]}{2B (bc - ad)^2 g^3 (a + b x) \sqrt{\frac{e (c+dx)^2}{(a+bx)^2}}} - \frac{b e^{-\frac{A}{B}} \text{ExpIntegralEi}\left[\frac{A+B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]}{B}\right]}{2B (bc - ad)^2 e g^3}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)}, x\right]$$

**Problem 224: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a g + b g x)^2}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right]$$

Result (type 8, 103 leaves, 2 steps):

$$\begin{aligned} & a^2 g^2 \text{CannotIntegrate}\left[\frac{1}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right] + \\ & 2 a b g^2 \text{CannotIntegrate}\left[\frac{x}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right] + \\ & b^2 g^2 \text{CannotIntegrate}\left[\frac{x^2}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right] \end{aligned}$$

**Problem 225: Result valid but suboptimal antiderivative.**

$$\int \frac{a g + b g x}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2} dx$$

Optimal (type 8, 34 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{a g + b g x}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right]$$

Result (type 8, 59 leaves, 2 steps):

$$a g \text{CannotIntegrate}\left[\frac{1}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right] + b g \text{CannotIntegrate}\left[\frac{x}{\left(A + B \text{Log}\left[\frac{e (c+dx)^2}{(a+bx)^2}\right]\right)^2}, x\right]$$

### Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2} dx$$

Optimal (type 8, 36 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2}, x \right]$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2}, x \right]$$

### Problem 227: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$-\frac{e^{-\frac{A}{2B}} (c+dx) \operatorname{ExpIntegralEi} \left[ \frac{A+B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right]}{2B} \right]}{4 B^2 (bc-ad) g^2 (a+bx) \sqrt{\frac{e^{(c+dx)^2}}{(a+bx)^2}}} + \frac{c+dx}{2 B (bc-ad) g^2 (a+bx) \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)}$$

Result (type 8, 36 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2}, x \right]$$

### Problem 228: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(c+dx)^2}}{(a+bx)^2} \right] \right)^2} dx$$

Optimal (type 4, 206 leaves, 10 steps):

$$\frac{d e^{-\frac{A}{2B}} (c + d x) \operatorname{ExpIntegralEi}\left[\frac{A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{2B}\right]}{4 B^2 (b c - a d)^2 g^3 (a + b x) \sqrt{\frac{e (c + d x)^2}{(a + b x)^2}}} -$$

$$\frac{b e^{-\frac{A}{B}} \operatorname{ExpIntegralEi}\left[\frac{A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]}{B}\right]}{2 B^2 (b c - a d)^2 e g^3} + \frac{c + d x}{2 B (b c - a d) g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)}$$

Result(type 8, 36 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (c + d x)^2}{(a + b x)^2}\right]\right)^2}, x\right]$$

Problem 229: Unable to integrate problem.

$$\int \frac{1}{(a g + b g x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])} dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$\left( e^{\frac{A}{Bn}} (c + d x) (e (a + b x)^n (c + d x)^{-n})^{\frac{1}{n}} \operatorname{ExpIntegralEi}\left[-\frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{Bn}\right] \right) /$$

$$(B (b c - a d) g^2 n (a + b x))$$

Result(type 8, 38 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(a g + b g x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}, x\right]$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int (f + g x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) dx$$

Optimal (type 3, 355 leaves, 3 steps):

$$\frac{1}{5 b^4 d^4} B (b c - a d) g (a^3 d^3 g^3 - a^2 b d^2 g^2 (5 d f - c g) +$$

$$a b^2 d g (10 d^2 f^2 - 5 c d f g + c^2 g^2) - b^3 (10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3)) x -$$

$$\frac{1}{10 b^3 d^3} B (b c - a d) g^2 (a^2 d^2 g^2 - a b d g (5 d f - c g) + b^2 (10 d^2 f^2 - 5 c d f g + c^2 g^2)) x^2 -$$

$$\frac{B (b c - a d) g^3 (5 b d f - b c g - a d g) x^3}{15 b^2 d^2} - \frac{B (b c - a d) g^4 x^4}{20 b d}$$

$$\frac{B (b f - a g)^5 \operatorname{Log}[a + b x]}{5 b^5 g} + \frac{(f + g x)^5 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{5 g} + \frac{B (d f - c g)^5 \operatorname{Log}[c + d x]}{5 d^5 g}$$

Result(type 3, 339 leaves, 4 steps):



$$\begin{aligned}
& \frac{1}{5 b^4 d^4} B g \left( 10 a b^3 d^4 f^3 - 10 a^2 b^2 d^4 f^2 g + 5 a^3 b d^4 f g^2 - \right. \\
& \quad \left. a^4 d^4 g^3 - b^4 c \left( 10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3 \right) \right) x - \frac{1}{10 b^3 d^3} \\
& B (b c - a d) g^2 \left( a^2 d^2 g^2 - a b d g (5 d f - c g) + b^2 (10 d^2 f^2 - 5 c d f g + c^2 g^2) \right) x^2 - \\
& \frac{B (b c - a d) g^3 (5 b d f - b c g - a d g) x^3}{15 b^2 d^2} - \frac{B (b c - a d) g^4 x^4}{20 b d} - \\
& \frac{B (b f - a g)^5 \text{Log}[a + b x]}{5 b^5 g} + \frac{(f + g x)^5 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{5 g} + \frac{B (d f - c g)^5 \text{Log}[c + d x]}{5 d^5 g}
\end{aligned}$$

Problem 231: Result optimal but 1 more steps used.

$$\int (f + g x)^3 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) dx$$

Optimal (type 3, 227 leaves, 3 steps):

$$\begin{aligned}
& - \frac{1}{4 b^3 d^3} B (b c - a d) g \left( a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2) \right) x - \\
& \frac{B (b c - a d) g^2 (4 b d f - b c g - a d g) x^2}{8 b^2 d^2} - \frac{B (b c - a d) g^3 x^3}{12 b d} - \\
& \frac{B (b f - a g)^4 \text{Log}[a + b x]}{4 b^4 g} + \frac{(f + g x)^4 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 g} + \frac{B (d f - c g)^4 \text{Log}[c + d x]}{4 d^4 g}
\end{aligned}$$

Result (type 3, 227 leaves, 4 steps):

$$\begin{aligned}
& - \frac{1}{4 b^3 d^3} B (b c - a d) g \left( a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2) \right) x - \\
& \frac{B (b c - a d) g^2 (4 b d f - b c g - a d g) x^2}{8 b^2 d^2} - \frac{B (b c - a d) g^3 x^3}{12 b d} - \\
& \frac{B (b f - a g)^4 \text{Log}[a + b x]}{4 b^4 g} + \frac{(f + g x)^4 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 g} + \frac{B (d f - c g)^4 \text{Log}[c + d x]}{4 d^4 g}
\end{aligned}$$

Problem 232: Result optimal but 1 more steps used.

$$\int (f + g x)^2 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) dx$$

Optimal (type 3, 150 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} - \frac{B (b c - a d) g^2 x^2}{6 b d} - \\
& \frac{B (b f - a g)^3 \text{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 g} + \frac{B (d f - c g)^3 \text{Log}[c + d x]}{3 d^3 g}
\end{aligned}$$

Result (type 3, 150 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} - \frac{B(b c - a d) g^2 x^2}{6 b d} \\
& - \frac{B(b f - a g)^3 \operatorname{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right)}{3 g} + \frac{B(d f - c g)^3 \operatorname{Log}[c + d x]}{3 d^3 g}
\end{aligned}$$

**Problem 233: Result optimal but 1 more steps used.**

$$\int (f + g x) \left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right) dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g x}{2 b d} - \frac{B(b f - a g)^2 \operatorname{Log}[a + b x]}{2 b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right)}{2 g} + \frac{B(d f - c g)^2 \operatorname{Log}[c + d x]}{2 d^2 g}
\end{aligned}$$

Result (type 3, 109 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g x}{2 b d} - \frac{B(b f - a g)^2 \operatorname{Log}[a + b x]}{2 b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right)}{2 g} + \frac{B(d f - c g)^2 \operatorname{Log}[c + d x]}{2 d^2 g}
\end{aligned}$$

**Problem 235: Result optimal but 3 more steps used.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]}{f + g x} dx$$

Optimal (type 4, 140 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B \operatorname{Log}\left[-\frac{g(a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{g} + \frac{\left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{B \operatorname{Log}\left[-\frac{g(c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \frac{B \operatorname{PolyLog}\left[2, \frac{b(f + g x)}{b f - a g}\right]}{g} + \frac{B \operatorname{PolyLog}\left[2, \frac{d(f + g x)}{d f - c g}\right]}{g}
\end{aligned}$$

Result (type 4, 140 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B \operatorname{Log}\left[-\frac{g(a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{g} + \frac{\left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{B \operatorname{Log}\left[-\frac{g(c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \frac{B \operatorname{PolyLog}\left[2, \frac{b(f + g x)}{b f - a g}\right]}{g} + \frac{B \operatorname{PolyLog}\left[2, \frac{d(f + g x)}{d f - c g}\right]}{g}
\end{aligned}$$

### Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(f+gx)^2} dx$$

Optimal (type 3, 87 leaves, 3 steps):

$$\frac{(a+bx) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{(bf-ag)(f+gx)} + \frac{B(bc-ad) \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{(bf-ag)(df-cg)}$$

Result (type 3, 113 leaves, 4 steps):

$$\frac{b B \operatorname{Log}[a+bx]}{g(bf-ag)} - \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{g(f+gx)} - \frac{B d \operatorname{Log}[c+dx]}{g(df-cg)} + \frac{B(bc-ad) \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)}$$

### Problem 237: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(f+gx)^3} dx$$

Optimal (type 3, 183 leaves, 3 steps):

$$-\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B \operatorname{Log}[a+bx]}{2g(bf-ag)^2} - \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{2g(f+gx)^2} - \frac{B d^2 \operatorname{Log}[c+dx]}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg) \operatorname{Log}[f+gx]}{2(bf-ag)^2(df-cg)^2}$$

Result (type 3, 183 leaves, 4 steps):

$$-\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B \operatorname{Log}[a+bx]}{2g(bf-ag)^2} - \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{2g(f+gx)^2} - \frac{B d^2 \operatorname{Log}[c+dx]}{2g(df-cg)^2} + \frac{B(bc-ad)(2bdf-bcg-adg) \operatorname{Log}[f+gx]}{2(bf-ag)^2(df-cg)^2}$$

### Problem 238: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(f+gx)^4} dx$$

Optimal (type 3, 275 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)}{6 (b f - a g) (d f - c g) (f + g x)^2} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \\
& \frac{b^3 B \operatorname{Log}[a + b x]}{3 g (b f - a g)^3} - \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{3 g (f + g x)^3} - \frac{B d^3 \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} + \\
& \left( \frac{B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[f + g x]}{(3 (b f - a g)^3 (d f - c g)^3)} \right) /
\end{aligned}$$

Result (type 3, 275 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)}{6 (b f - a g) (d f - c g) (f + g x)^2} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \\
& \frac{b^3 B \operatorname{Log}[a + b x]}{3 g (b f - a g)^3} - \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{3 g (f + g x)^3} - \frac{B d^3 \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} + \\
& \left( \frac{B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[f + g x]}{(3 (b f - a g)^3 (d f - c g)^3)} \right) /
\end{aligned}$$

Problem 239: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(f + g x)^5} dx$$

Optimal (type 3, 379 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)}{12 (b f - a g) (d f - c g) (f + g x)^3} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{8 (b f - a g)^2 (d f - c g)^2 (f + g x)^2} - \\
& \left( \frac{B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2))}{(4 (b f - a g)^3 (d f - c g)^3 (f + g x))} + \frac{b^4 B \operatorname{Log}[a + b x]}{4 g (b f - a g)^4} - \right. \\
& \left. \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{4 g (f + g x)^4} - \frac{B d^4 \operatorname{Log}[c + d x]}{4 g (d f - c g)^4} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{(2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{Log}[f + g x]} \right) / (4 (b f - a g)^4 (d f - c g)^4)
\end{aligned}$$

Result (type 3, 379 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)}{12 (b f - a g) (d f - c g) (f + g x)^3} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{8 (b f - a g)^2 (d f - c g)^2 (f + g x)^2} - \\
& (B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2))) / \\
& (4 (b f - a g)^3 (d f - c g)^3 (f + g x)) + \frac{b^4 B \operatorname{Log}[a + b x]}{4 g (b f - a g)^4} - \\
& \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{4 g (f + g x)^4} - \frac{B d^4 \operatorname{Log}[c + d x]}{4 g (d f - c g)^4} - (B (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{Log}[f + g x]) / (4 (b f - a g)^4 (d f - c g)^4)
\end{aligned}$$

**Problem 240: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 874 leaves, 15 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^3 g^3 x}{6 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^2 (4 b d f - 3 b c g - a d g) x}{4 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^3 (c + d x)^2}{12 b^2 d^4} + \\
& \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{6 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{4 b^4 d^4} - \frac{1}{2 b^4 d^3} \\
& B (b c - a d) g (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) (a + b x) \\
& \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) - \frac{B (b c - a d) g^2 (4 b d f - 3 b c g - a d g) (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 b^2 d^4} - \\
& \frac{B (b c - a d) g^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{6 b d^4} - \frac{1}{2 b^4 d^4} B (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) - \\
& \frac{(b f - a g)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 b^4 g} + \frac{(f + g x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 g} + \\
& \frac{B^2 (b c - a d)^4 g^3 \operatorname{Log}[c + d x]}{6 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) \operatorname{Log}[c + d x]}{4 b^4 d^4} + \frac{1}{2 b^4 d^4} \\
& B^2 (b c - a d)^2 g (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) \operatorname{Log}[c + d x] - \\
& \frac{1}{2 b^4 d^4} B^2 (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]
\end{aligned}$$

Result (type 4, 994 leaves, 33 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 (bc + ad) g^3 x}{6 b^3 d^3} + \frac{B^2 (bc - ad)^2 g^2 (4 b d f - b c g - a d g) x}{4 b^3 d^3} - \frac{1}{2 b^3 d^3} \\
& A B (bc - ad) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) x + \\
& \frac{B^2 (bc - ad)^2 g^3 x^2}{12 b^2 d^2} - \frac{a^3 B^2 (bc - ad) g^3 \operatorname{Log}[a + b x]}{6 b^4 d} + \\
& \frac{a^2 B^2 (bc - ad) g^2 (4 b d f - b c g - a d g) \operatorname{Log}[a + b x]}{4 b^4 d^2} + \frac{B^2 (b f - a g)^4 \operatorname{Log}[a + b x]^2}{4 b^4 g} - \\
& \frac{1}{2 b^4 d^3} B^2 (bc - ad) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) (a + b x) \\
& \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] - \frac{B (bc - ad) g^2 (4 b d f - b c g - a d g) x^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{4 b^2 d^2} - \\
& \frac{B (bc - ad) g^3 x^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{6 b d} - \frac{B (b f - a g)^4 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^4 g} + \\
& \frac{(f + g x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{4 g} + \frac{B^2 c^3 (bc - ad) g^3 \operatorname{Log}[c + d x]}{6 b d^4} - \\
& \frac{B^2 c^2 (bc - ad) g^2 (4 b d f - b c g - a d g) \operatorname{Log}[c + d x]}{4 b^2 d^4} + \frac{1}{2 b^4 d^4} \\
& B^2 (bc - ad)^2 g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) \operatorname{Log}[c + d x] - \\
& \frac{B^2 (d f - c g)^4 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{2 d^4 g} + \frac{B (d f - c g)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{2 d^4 g} + \\
& \frac{B^2 (d f - c g)^4 \operatorname{Log}[c + d x]^2}{4 d^4 g} - \frac{B^2 (b f - a g)^4 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 b^4 g} - \\
& \frac{B^2 (b f - a g)^4 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 b^4 g} - \frac{B^2 (d f - c g)^4 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{2 d^4 g}
\end{aligned}$$

**Problem 241: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 532 leaves, 12 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 x}{3 b^2 d^2} + \frac{B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{3 b^3 d^3} - \\
& \frac{2 B (b c - a d) g (3 b d f - 2 b c g - a d g) (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b^3 d^2} - \\
& \frac{B (b c - a d) g^2 (c+d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b d^3} + \frac{1}{3 b^3 d^3} \\
& 2 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \\
& \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2 - \frac{(b f - a g)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 b^3 g} + \frac{(f+g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 g} + \\
& \frac{B^2 (b c - a d)^3 g^2 \operatorname{Log}[c+d x]}{3 b^3 d^3} + \frac{2 B^2 (b c - a d)^2 g (3 b d f - 2 b c g - a d g) \operatorname{Log}[c+d x]}{3 b^3 d^3} + \frac{1}{3 b^3 d^3} \\
& 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]
\end{aligned}$$

Result (type 4, 649 leaves, 29 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 x}{3 b^2 d^2} - \frac{2 A B (b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} + \frac{a^2 B^2 (b c - a d) g^2 \operatorname{Log}[a+b x]}{3 b^3 d} + \\
& \frac{B^2 (b f - a g)^3 \operatorname{Log}[a+b x]^2}{3 b^3 g} - \frac{2 B^2 (b c - a d) g (3 b d f - b c g - a d g) (a+b x) \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{3 b^3 d^2} - \\
& \frac{B (b c - a d) g^2 x^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b d} - \frac{2 B (b f - a g)^3 \operatorname{Log}[a+b x] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b^3 g} + \\
& \frac{(f+g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 g} - \frac{B^2 c^2 (b c - a d) g^2 \operatorname{Log}[c+d x]}{3 b d^3} + \\
& \frac{2 B^2 (b c - a d)^2 g (3 b d f - b c g - a d g) \operatorname{Log}[c+d x]}{3 b^3 d^3} - \\
& \frac{2 B^2 (d f - c g)^3 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c+d x]}{3 d^3 g} + \frac{2 B (d f - c g)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x]}{3 d^3 g} + \\
& \frac{B^2 (d f - c g)^3 \operatorname{Log}[c+d x]^2}{3 d^3 g} - \frac{2 B^2 (b f - a g)^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c - a d}\right]}{3 b^3 g} - \\
& \frac{2 B^2 (b f - a g)^3 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c - a d}\right]}{3 b^3 g} - \frac{2 B^2 (d f - c g)^3 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{3 d^3 g}
\end{aligned}$$

**Problem 242: Result valid but suboptimal antiderivative.**

$$\int (f+g x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2 dx$$

Optimal (type 4, 270 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 d} + \\
& \frac{B (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 d^2} - \\
& \frac{(b f - a g)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 b^2 g} + \frac{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 g} + \\
& \frac{B^2 (b c - a d)^2 g \operatorname{Log} [c + d x]}{b^2 d^2} + \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b^2 d^2}
\end{aligned}$$

Result (type 4, 444 leaves, 25 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d) g x}{b d} + \frac{B^2 (b f - a g)^2 \operatorname{Log} [a + b x]^2}{2 b^2 g} - \\
& \frac{B^2 (b c - a d) g (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{b^2 d} - \frac{B (b f - a g)^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 g} + \frac{B^2 (b c - a d)^2 g \operatorname{Log} [c + d x]}{b^2 d^2} - \\
& \frac{B^2 (d f - c g)^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{d^2 g} + \frac{B (d f - c g)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{d^2 g} + \\
& \frac{B^2 (d f - c g)^2 \operatorname{Log} [c + d x]^2}{2 d^2 g} - \frac{B^2 (b f - a g)^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{b^2 g} - \\
& \frac{B^2 (b f - a g)^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^2 g} - \frac{B^2 (d f - c g)^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{d^2 g}
\end{aligned}$$

**Problem 243: Result valid but suboptimal antiderivative.**

$$\int \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 B (b c - a d) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b d} + \\
& \frac{(a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{b} + \frac{2 B^2 (b c - a d) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b d}
\end{aligned}$$

Result (type 4, 246 leaves, 22 steps):



$$\begin{aligned}
& - \frac{a B^2 \operatorname{Log}[a + b x]^2}{b} + \frac{2 a B \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b} + x \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 + \\
& \frac{2 B^2 c \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d} - \frac{2 B c \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d} - \frac{B^2 c \operatorname{Log}[c + d x]^2}{d} + \\
& \frac{2 a B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b} + \frac{2 a B^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b} + \frac{2 B^2 c \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d}
\end{aligned}$$

Problem 244: Result valid but suboptimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{f + g x} dx$$

Optimal (type 4, 277 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{g} + \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 \operatorname{Log}\left[1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g} - \\
& \frac{2 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{g} + \frac{2 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g} + \\
& \frac{2 B^2 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{g} - \frac{2 B^2 \operatorname{PolyLog}\left[3, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g}
\end{aligned}$$

Result (type 4, 1998 leaves, 41 steps):

$$\begin{aligned}
& - \frac{B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[f + g x]}{g} - \frac{2 A B \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{g} - \frac{B^2 \operatorname{Log}\left[\frac{1}{c + d x}\right]^2 \operatorname{Log}[f + g x]}{g} + \\
& \frac{1}{g} 2 B^2 \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[f + g x] + \\
& \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 \operatorname{Log}[f + g x]}{g} + \frac{2 B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \operatorname{Log}[f + g x]}{g} - \\
& \frac{2 B^2 \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \left( \operatorname{Log}\left[\frac{1}{c + d x}\right] + \operatorname{Log}[c + d x] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{2 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[f + g x]}{g} + \frac{2 A B \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \frac{1}{g} \\
& 2 B^2 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x] + \\
& \frac{B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (f + g x)}{b f - a g}\right]}{g} + \frac{B^2 \operatorname{Log}\left[\frac{1}{c + d x}\right]^2 \operatorname{Log}\left[\frac{d (f + g x)}{d f - c g}\right]}{g} + \frac{1}{g} \\
& B^2 \left( \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] + \operatorname{Log}\left[\frac{b f - a g}{b (f + g x)}\right] - \operatorname{Log}\left[\frac{(b f - a g) (c + d x)}{(b c - a d) (f + g x)}\right] \right) \operatorname{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{g} B^2 \left( \text{Log} \left[ \frac{b(c+dx)}{bc-ad} \right] - \text{Log} \left[ -\frac{g(c+dx)}{df-cg} \right] \right) \left( \text{Log}[a+bx] + \text{Log} \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right)^2 + \frac{1}{g} \\
& B^2 \left( \text{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] + \text{Log} \left[ \frac{df-cg}{d(f+gx)} \right] - \text{Log} \left[ -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right] \right) \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right]^2 - \\
& \frac{1}{g} B^2 \left( \text{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] - \text{Log} \left[ -\frac{g(a+bx)}{bf-ag} \right] \right) \left( \text{Log}[c+dx] + \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right)^2 + \\
& \frac{2 B^2 \left( \text{Log}[f+gx] - \text{Log} \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right) \text{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{g} + \\
& \frac{2 B^2 \text{Log}[a+bx] \text{PolyLog} \left[ 2, -\frac{g(a+bx)}{bf-ag} \right]}{g} + \\
& \frac{2 B^2 \left( \text{Log}[f+gx] - \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right) \text{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{g} - \\
& \frac{2 B^2 \text{Log} \left[ \frac{1}{c+dx} \right] \text{PolyLog} \left[ 2, -\frac{g(c+dx)}{df-cg} \right]}{g} - \frac{2 B^2 \text{Log} \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \text{PolyLog} \left[ 2, \frac{g(a+bx)}{b(f+gx)} \right]}{g} + \\
& \frac{2 B^2 \text{Log} \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \text{PolyLog} \left[ 2, -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right]}{g} - \\
& \frac{2 B^2 \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \text{PolyLog} \left[ 2, \frac{g(c+dx)}{d(f+gx)} \right]}{g} + \frac{2 B^2 \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \text{PolyLog} \left[ 2, \frac{(bf-ag)(c+dx)}{(bc-ad)(f+gx)} \right]}{g} - \\
& \frac{2 A B \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \frac{2 B^2 \left( \text{Log}[a+bx] + \text{Log} \left[ \frac{1}{c+dx} \right] - \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} - \\
& \frac{2 B^2 \left( \text{Log} \left[ \frac{1}{c+dx} \right] + \text{Log}[c+dx] \right) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \\
& \frac{2 B^2 \left( \text{Log}[c+dx] + \text{Log} \left[ \frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)} \right] \right) \text{PolyLog} \left[ 2, \frac{b(f+gx)}{bf-ag} \right]}{g} + \frac{2 A B \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} - \\
& \frac{2 B^2 \left( \text{Log}[a+bx] + \text{Log} \left[ \frac{1}{c+dx} \right] - \text{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right) \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} + \\
& \frac{2 B^2 \left( \text{Log}[a+bx] + \text{Log} \left[ -\frac{(bc-ad)(f+gx)}{(df-cg)(a+bx)} \right] \right) \text{PolyLog} \left[ 2, \frac{d(f+gx)}{df-cg} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, -\frac{d(a+bx)}{bc-ad} \right]}{g} - \\
& \frac{2 B^2 \text{PolyLog} \left[ 3, -\frac{g(a+bx)}{bf-ag} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{b(c+dx)}{bc-ad} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, -\frac{g(c+dx)}{df-cg} \right]}{g} - \\
& \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{g(a+bx)}{b(f+gx)} \right]}{g} + \frac{2 B^2 \text{PolyLog} \left[ 3, -\frac{(df-cg)(a+bx)}{(bc-ad)(f+gx)} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{g(c+dx)}{d(f+gx)} \right]}{g} + \\
& \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{(bf-ag)(c+dx)}{(bc-ad)(f+gx)} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{b(f+gx)}{bf-ag} \right]}{g} - \frac{2 B^2 \text{PolyLog} \left[ 3, \frac{d(f+gx)}{df-cg} \right]}{g}
\end{aligned}$$

### Problem 245: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(f+gx)^2} dx$$

Optimal (type 4, 196 leaves, 4 steps):

$$\frac{(a+bx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bf-ag)(f+gx)} + \frac{2B(bc-ad) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right) \operatorname{Log}\left[1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right]}{(bf-ag)(df-cg)} +$$

$$\frac{2B^2(bc-ad) \operatorname{PolyLog}\left[2, \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right]}{(bf-ag)(df-cg)}$$

Result (type 4, 612 leaves, 32 steps):

$$-\frac{bB^2 \operatorname{Log}[a+bx]^2}{g(bf-ag)} + \frac{2bB \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{g(bf-ag)} - \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{g(f+gx)} +$$

$$\frac{2B^2 d \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{g(df-cg)} - \frac{2Bd \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right) \operatorname{Log}[c+dx]}{g(df-cg)} - \frac{B^2 d \operatorname{Log}[c+dx]^2}{g(df-cg)} +$$

$$\frac{2bB^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{g(bf-ag)} - \frac{2B^2(bc-ad) \operatorname{Log}\left[-\frac{g(a+bx)}{bf-ag}\right] \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} +$$

$$\frac{2B(bc-ad) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right) \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad) \operatorname{Log}\left[-\frac{g(c+dx)}{df-cg}\right] \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)} +$$

$$\frac{2bB^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{g(bf-ag)} + \frac{2B^2 d \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{g(df-cg)} -$$

$$\frac{2B^2(bc-ad) \operatorname{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right]}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad) \operatorname{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right]}{(bf-ag)(df-cg)}$$

### Problem 246: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(f+gx)^3} dx$$

Optimal (type 4, 369 leaves, 9 steps):

$$\begin{aligned}
& \frac{B (b c - a d) g (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(b f - a g)^2 (d f - c g) (f + g x)} + \\
& \frac{b^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 g (b f - a g)^2} - \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 g (f + g x)^2} + \frac{B^2 (b c - a d)^2 g \operatorname{Log} \left[ \frac{f + g x}{c + d x} \right]}{(b f - a g)^2 (d f - c g)^2} + \\
& \left( B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)} \right] \right) / \\
& \left( (b f - a g)^2 (d f - c g)^2 \right) + \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog} \left[ 2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)} \right]}{(b f - a g)^2 (d f - c g)^2}
\end{aligned}$$

Result (type 4, 883 leaves, 36 steps):

$$\begin{aligned}
& \frac{b B^2 (b c - a d) \operatorname{Log} [a + b x]}{(b f - a g)^2 (d f - c g)} - \frac{b^2 B^2 \operatorname{Log} [a + b x]^2}{2 g (b f - a g)^2} - \frac{B (b c - a d) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(b f - a g) (d f - c g) (f + g x)} + \\
& \frac{b^2 B \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{g (b f - a g)^2} - \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 g (f + g x)^2} - \frac{B^2 d (b c - a d) \operatorname{Log} [c + d x]}{(b f - a g) (d f - c g)^2} + \\
& \frac{B^2 d^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{g (d f - c g)^2} - \frac{B d^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{g (d f - c g)^2} - \\
& \frac{B^2 d^2 \operatorname{Log} [c + d x]^2}{2 g (d f - c g)^2} + \frac{b^2 B^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{g (b f - a g)^2} + \frac{B^2 (b c - a d)^2 g \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} - \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log} \left[ -\frac{g (a + b x)}{b f - a g} \right] \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log} \left[ -\frac{g (c + d x)}{d f - c g} \right] \operatorname{Log} [f + g x]}{(b f - a g)^2 (d f - c g)^2} + \frac{b^2 B^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{g (b f - a g)^2} + \\
& \frac{B^2 d^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{g (d f - c g)^2} - \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog} \left[ 2, \frac{b (f + g x)}{b f - a g} \right]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog} \left[ 2, \frac{d (f + g x)}{d f - c g} \right]}{(b f - a g)^2 (d f - c g)^2}
\end{aligned}$$

Problem 247: Result valid but suboptimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(f + g x)^4} dx$$

Optimal (type 4, 714 leaves, 12 steps):

$$\begin{aligned}
& \frac{B^2 (bc - ad)^2 g^2 (c + dx)}{3 (bf - ag)^2 (df - cg)^3 (f + gx)} + \\
& \frac{B^2 (bc - ad)^3 g^2 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right] - B (bc - ad) g^2 (c + dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3 (bf - ag)^3 (df - cg)^3 - 3 (bf - ag) (df - cg)^3 (f + gx)^2} + \\
& \frac{2 B (bc - ad) g (3 bdf - bcg - 2 adg) (a + bx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3 (bf - ag)^3 (df - cg)^2 (f + gx)} + \\
& \frac{b^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3 g (bf - ag)^3} - \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3 g (f + gx)^3} - \frac{B^2 (bc - ad)^3 g^2 \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{3 (bf - ag)^3 (df - cg)^3} + \\
& \frac{2 B^2 (bc - ad)^2 g (3 bdf - bcg - 2 adg) \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{3 (bf - ag)^3 (df - cg)^3} + \\
& \left( 2 B (bc - ad) (a^2 d^2 g^2 - abdg (3 df - cg) + b^2 (3 d^2 f^2 - 3 cdfg + c^2 g^2)) \right. \\
& \quad \left. \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{Log}\left[1 - \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)}\right] \right) / (3 (bf - ag)^3 (df - cg)^3) + \\
& \left( 2 B^2 (bc - ad) (a^2 d^2 g^2 - abdg (3 df - cg) + b^2 (3 d^2 f^2 - 3 cdfg + c^2 g^2)) \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, \frac{(df - cg)(a + bx)}{(bf - ag)(c + dx)}\right] \right) / (3 (bf - ag)^3 (df - cg)^3)
\end{aligned}$$

Result (type 4, 1356 leaves, 40 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^2 g}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \frac{b^2 B^2 (b c - a d) \operatorname{Log}[a + b x]}{3 (b f - a g)^3 (d f - c g)} + \\
& \frac{2 b B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}[a + b x]}{3 (b f - a g)^3 (d f - c g)^2} - \frac{b^3 B^2 \operatorname{Log}[a + b x]^2}{3 g (b f - a g)^3} - \\
& \frac{B (b c - a d) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b f - a g) (d f - c g) (f + g x)^2} - \frac{2 B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \\
& \frac{2 b^3 B \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 g (b f - a g)^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 g (f + g x)^3} - \\
& \frac{B^2 d^2 (b c - a d) \operatorname{Log}[c + d x]}{3 (b f - a g) (d f - c g)^3} - \frac{2 B^2 d (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}[c + d x]}{3 (b f - a g)^2 (d f - c g)^3} + \\
& \frac{2 B^2 d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \frac{2 B d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \frac{B^2 d^3 \operatorname{Log}[c + d x]^2}{3 g (d f - c g)^3} + \\
& \frac{2 b^3 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 g (b f - a g)^3} + \frac{B^2 (b c - a d)^2 g (2 b d f - b c g - a d g) \operatorname{Log}[f + g x]}{(b f - a g)^3 (d f - c g)^3} - \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \frac{2 b^3 B^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 g (b f - a g)^3} + \frac{2 B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 g (d f - c g)^3} - \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right) + \\
& \left( 2 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right] \right) / \left( 3 (b f - a g)^3 (d f - c g)^3 \right)
\end{aligned}$$

### Problem 248: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(f+gx)^5} dx$$

Optimal (type 4, 1159 leaves, 15 steps):

$$\begin{aligned} & -\frac{B^2 (bc-ad)^2 g^3 (c+dx)^2}{12 (bf-ag)^2 (df-cg)^4 (f+gx)^2} - \frac{B^2 (bc-ad)^3 g^3 (c+dx)}{6 (bf-ag)^3 (df-cg)^4 (f+gx)} + \\ & \frac{B^2 (bc-ad)^2 g^2 (4bdf-bcg-3adg) (c+dx)}{4 (bf-ag)^3 (df-cg)^4 (f+gx)} - \frac{B^2 (bc-ad)^4 g^3 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{6 (bf-ag)^4 (df-cg)^4} + \\ & \frac{B^2 (bc-ad)^3 g^2 (4bdf-bcg-3adg) \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{4 (bf-ag)^4 (df-cg)^4} + \\ & \frac{B (bc-ad) g^3 (c+dx)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{6 (bf-ag) (df-cg)^4 (f+gx)^3} - \\ & \frac{B (bc-ad) g^2 (4bdf-bcg-3adg) (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{4 (bf-ag)^2 (df-cg)^4 (f+gx)^2} + \\ & \left( B (bc-ad) g (3a^2 d^2 g^2 - 2abd g (4df-cg) + b^2 (6d^2 f^2 - 4cdfg + c^2 g^2)) \right. \\ & \quad \left. (a+bx) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \right) / \left( 2 (bf-ag)^4 (df-cg)^3 (f+gx) \right) + \\ & \frac{b^4 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{4 g (bf-ag)^4} - \frac{\left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{4 g (f+gx)^4} + \frac{B^2 (bc-ad)^4 g^3 \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{6 (bf-ag)^4 (df-cg)^4} - \\ & \frac{B^2 (bc-ad)^3 g^2 (4bdf-bcg-3adg) \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{4 (bf-ag)^4 (df-cg)^4} + \\ & \left( B^2 (bc-ad)^2 g (3a^2 d^2 g^2 - 2abd g (4df-cg) + b^2 (6d^2 f^2 - 4cdfg + c^2 g^2)) \operatorname{Log}\left[\frac{f+gx}{c+dx}\right] \right) / \\ & \left( 2 (bf-ag)^4 (df-cg)^4 \right) - \\ & \left( B (bc-ad) (2bdf-bcg-adg) (2abd^2 fg - a^2 d^2 g^2 - b^2 (2d^2 f^2 - 2cdfg + c^2 g^2)) \right. \\ & \quad \left. \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}\left[1 - \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right] \right) / \left( 2 (bf-ag)^4 (df-cg)^4 \right) - \\ & \left( B^2 (bc-ad) (2bdf-bcg-adg) (2abd^2 fg - a^2 d^2 g^2 - b^2 (2d^2 f^2 - 2cdfg + c^2 g^2)) \right. \\ & \quad \left. \operatorname{PolyLog}\left[2, \frac{(df-cg)(a+bx)}{(bf-ag)(c+dx)}\right] \right) / \left( 2 (bf-ag)^4 (df-cg)^4 \right) \end{aligned}$$

Result (type 4, 1881 leaves, 44 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g}{12 (bf - ag)^2 (df - cg)^2 (f + gx)^2} - \frac{5 B^2 (bc - ad)^2 g (2 bdf - b c g - a d g)}{12 (bf - ag)^3 (df - cg)^3 (f + gx)} + \\
& \frac{b^3 B^2 (bc - ad) \operatorname{Log}[a + bx]}{6 (bf - ag)^4 (df - cg)} + \frac{b^2 B^2 (bc - ad) (2 bdf - b c g - a d g) \operatorname{Log}[a + bx]}{4 (bf - ag)^4 (df - cg)^2} + \\
& (b B^2 (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[a + bx]) / \\
& \left( 2 (bf - ag)^4 (df - cg)^3 - \frac{b^4 B^2 \operatorname{Log}[a + bx]^2}{4 g (bf - ag)^4} - \frac{B (bc - ad) (A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}])}{6 (bf - ag) (df - cg) (f + gx)^3} - \right. \\
& \left. \frac{B (bc - ad) (2 bdf - b c g - a d g) (A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}])}{4 (bf - ag)^2 (df - cg)^2 (f + gx)^2} - \right. \\
& \left. \left( B (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \right. \\
& \left. \left. \left( A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}] \right) \right) \right) / \left( 2 (bf - ag)^3 (df - cg)^3 (f + gx) \right) + \\
& \frac{b^4 B \operatorname{Log}[a + bx] (A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}])}{2 g (bf - ag)^4} - \frac{(A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}])^2}{4 g (f + gx)^4} - \frac{B^2 d^3 (bc - ad) \operatorname{Log}[c + dx]}{6 (bf - ag) (df - cg)^4} - \\
& \frac{B^2 d^2 (bc - ad) (2 bdf - b c g - a d g) \operatorname{Log}[c + dx]}{4 (bf - ag)^2 (df - cg)^4} - \\
& (B^2 d (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[c + dx]) / \\
& \left( 2 (bf - ag)^3 (df - cg)^4 + \frac{B^2 d^4 \operatorname{Log}[-\frac{d(a+bx)}{bc-ad}] \operatorname{Log}[c + dx]}{2 g (df - cg)^4} - \right. \\
& \left. \frac{B d^4 (A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}]) \operatorname{Log}[c + dx]}{2 g (df - cg)^4} - \frac{B^2 d^4 \operatorname{Log}[c + dx]^2}{4 g (df - cg)^4} + \frac{b^4 B^2 \operatorname{Log}[a + bx] \operatorname{Log}[\frac{b(c+dx)}{bc-ad}]}{2 g (bf - ag)^4} + \right. \\
& \left. \frac{B^2 (bc - ad)^2 g (2 bdf - b c g - a d g)^2 \operatorname{Log}[f + gx]}{4 (bf - ag)^4 (df - cg)^4} + \right. \\
& \left. (2 B^2 (bc - ad)^2 g (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[f + gx]) / \right. \\
& \left. (3 (bf - ag)^4 (df - cg)^4) + \right. \\
& \left( B^2 (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \left. \operatorname{Log}[-\frac{g(a+bx)}{bf-ag}] \operatorname{Log}[f + gx] \right) / \left( 2 (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \left. \left( A + B \operatorname{Log}[\frac{e(a+bx)}{c+dx}] \right) \operatorname{Log}[f + gx] \right) / \left( 2 (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B^2 (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right.
\end{aligned}$$



$$\begin{aligned}
& \left. \frac{\text{Log}\left[-\frac{g(c+dx)}{df-cg}\right] \text{Log}[f+gx]}{df-cg} \right) / \left( 2(bf-ag)^4 (df-cg)^4 \right) + \\
& \frac{b^4 B^2 \text{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{2g(bf-ag)^4} + \frac{B^2 d^4 \text{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{2g(df-cg)^4} + \\
& \left( B^2 (bc-ad) (2bdf-bcg-adg) (2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)) \right. \\
& \left. \text{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right] \right) / \left( 2(bf-ag)^4 (df-cg)^4 \right) - \\
& \left( B^2 (bc-ad) (2bdf-bcg-adg) (2abd^2fg-a^2d^2g^2-b^2(2d^2f^2-2cdfg+c^2g^2)) \right. \\
& \left. \text{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right] \right) / \left( 2(bf-ag)^4 (df-cg)^4 \right)
\end{aligned}$$

**Problem 249: Result valid but suboptimal antiderivative.**

$$\int \frac{\text{Log}\left[\frac{1+x}{-1+x}\right]}{x^2} dx$$

Optimal (type 3, 35 leaves, 3 steps):

$$2 \text{Log}\left[-\frac{x}{1-x}\right] - \frac{(1+x) \text{Log}\left[-\frac{1+x}{1-x}\right]}{x}$$

Result (type 3, 34 leaves, 4 steps):

$$2 \text{Log}[x] - 2 \text{Log}[1+x] - \frac{(1-x) \text{Log}\left[-\frac{1+x}{1-x}\right]}{x}$$

**Problem 250: Result valid but suboptimal antiderivative.**

$$\int \frac{(f+gx)^2}{A+B \text{Log}\left[\frac{e(a+bx)}{c+dx}\right]} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f+gx)^2}{A+B \text{Log}\left[\frac{e(a+bx)}{c+dx}\right]}, x\right]$$

Result (type 8, 88 leaves, 2 steps):

$$\begin{aligned}
& f^2 \text{CannotIntegrate}\left[\frac{1}{A+B \text{Log}\left[\frac{e(a+bx)}{c+dx}\right]}, x\right] + \\
& 2fg \text{CannotIntegrate}\left[\frac{x}{A+B \text{Log}\left[\frac{e(a+bx)}{c+dx}\right]}, x\right] + g^2 \text{CannotIntegrate}\left[\frac{x^2}{A+B \text{Log}\left[\frac{e(a+bx)}{c+dx}\right]}, x\right]
\end{aligned}$$

### Problem 251: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]} dx$$

Optimal (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{f + g x}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}, x\right]$$

Result (type 8, 53 leaves, 2 steps):

$$f \operatorname{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}, x\right] + g \operatorname{CannotIntegrate}\left[\frac{x}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}, x\right]$$

### Problem 252: Result valid but suboptimal antiderivative.

$$\int \frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]} dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}, x\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}, x\right]$$

### Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}, x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}, x\right]$$

### Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}, x \right]$$

Result (type 8, 31 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}, x \right]$$

### Problem 255: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}, x \right]$$

Result (type 8, 31 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}, x \right]$$

### Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x)^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{(f + g x)^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}, x \right]$$

Result (type 8, 88 leaves, 2 steps):

$$\begin{aligned} & f^2 \text{CannotIntegrate} \left[ \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}, x \right] + \\ & 2 f g \text{CannotIntegrate} \left[ \frac{x}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}, x \right] + g^2 \text{CannotIntegrate} \left[ \frac{x^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}, x \right] \end{aligned}$$

### Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2} dx$$

Optimal (type 8, 29 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{f + g x}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

Result (type 8, 53 leaves, 2 steps):

$$f \operatorname{CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right] + g \operatorname{CannotIntegrate}\left[\frac{x}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

### Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2} dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

### Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(f + g x) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}, x\right]$$

### Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}, x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}, x\right]$$

### Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}, x\right]$$

Result (type 8, 31 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}, x\right]$$

### Problem 262: Result valid but suboptimal antiderivative.

$$\int (f + g x)^4 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \right) dx$$

Optimal (type 3, 357 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{5 b^4 d^4} 2 B (b c - a d) g (a^3 d^3 g^3 - a^2 b d^2 g^2 (5 d f - c g) + \\ & \quad a b^2 d g (10 d^2 f^2 - 5 c d f g + c^2 g^2) - b^3 (10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3)) x - \\ & \frac{1}{5 b^3 d^3} B (b c - a d) g^2 (a^2 d^2 g^2 - a b d g (5 d f - c g) + b^2 (10 d^2 f^2 - 5 c d f g + c^2 g^2)) x^2 - \\ & \frac{2 B (b c - a d) g^3 (5 b d f - b c g - a d g) x^3}{15 b^2 d^2} - \frac{B (b c - a d) g^4 x^4}{10 b d} - \\ & \frac{2 B (b f - a g)^5 \operatorname{Log}[a + b x]}{5 b^5 g} + \frac{(f + g x)^5 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \right)}{5 g} + \frac{2 B (d f - c g)^5 \operatorname{Log}[c + d x]}{5 d^5 g} \end{aligned}$$

Result (type 3, 341 leaves, 4 steps):

$$\begin{aligned} & \frac{1}{5 b^4 d^4} 2 B g \left( 10 a b^3 d^4 f^3 - 10 a^2 b^2 d^4 f^2 g + 5 a^3 b d^4 f g^2 - \right. \\ & \quad \left. a^4 d^4 g^3 - b^4 c \left( 10 d^3 f^3 - 10 c d^2 f^2 g + 5 c^2 d f g^2 - c^3 g^3 \right) \right) x - \frac{1}{5 b^3 d^3} \\ & B \left( b c - a d \right) g^2 \left( a^2 d^2 g^2 - a b d g \left( 5 d f - c g \right) + b^2 \left( 10 d^2 f^2 - 5 c d f g + c^2 g^2 \right) \right) x^2 - \\ & \frac{2 B \left( b c - a d \right) g^3 \left( 5 b d f - b c g - a d g \right) x^3}{15 b^2 d^2} - \frac{B \left( b c - a d \right) g^4 x^4}{10 b d} - \\ & \frac{2 B \left( b f - a g \right)^5 \text{Log}[a + b x]}{5 b^5 g} + \frac{\left( f + g x \right)^5 \left( A + B \text{Log}\left[\frac{e \left( a + b x \right)^2}{\left( c + d x \right)^2}\right] \right)}{5 g} + \frac{2 B \left( d f - c g \right)^5 \text{Log}[c + d x]}{5 d^5 g} \end{aligned}$$

Problem 263: Result optimal but 1 more steps used.

$$\int \left( f + g x \right)^3 \left( A + B \text{Log}\left[\frac{e \left( a + b x \right)^2}{\left( c + d x \right)^2}\right] \right) dx$$

Optimal (type 3, 229 leaves, 3 steps):

$$\begin{aligned} & - \frac{1}{2 b^3 d^3} B \left( b c - a d \right) g \left( a^2 d^2 g^2 - a b d g \left( 4 d f - c g \right) + b^2 \left( 6 d^2 f^2 - 4 c d f g + c^2 g^2 \right) \right) x - \\ & \frac{B \left( b c - a d \right) g^2 \left( 4 b d f - b c g - a d g \right) x^2}{4 b^2 d^2} - \frac{B \left( b c - a d \right) g^3 x^3}{6 b d} - \\ & \frac{B \left( b f - a g \right)^4 \text{Log}[a + b x]}{2 b^4 g} + \frac{\left( f + g x \right)^4 \left( A + B \text{Log}\left[\frac{e \left( a + b x \right)^2}{\left( c + d x \right)^2}\right] \right)}{4 g} + \frac{B \left( d f - c g \right)^4 \text{Log}[c + d x]}{2 d^4 g} \end{aligned}$$

Result (type 3, 229 leaves, 4 steps):

$$\begin{aligned} & - \frac{1}{2 b^3 d^3} B \left( b c - a d \right) g \left( a^2 d^2 g^2 - a b d g \left( 4 d f - c g \right) + b^2 \left( 6 d^2 f^2 - 4 c d f g + c^2 g^2 \right) \right) x - \\ & \frac{B \left( b c - a d \right) g^2 \left( 4 b d f - b c g - a d g \right) x^2}{4 b^2 d^2} - \frac{B \left( b c - a d \right) g^3 x^3}{6 b d} - \\ & \frac{B \left( b f - a g \right)^4 \text{Log}[a + b x]}{2 b^4 g} + \frac{\left( f + g x \right)^4 \left( A + B \text{Log}\left[\frac{e \left( a + b x \right)^2}{\left( c + d x \right)^2}\right] \right)}{4 g} + \frac{B \left( d f - c g \right)^4 \text{Log}[c + d x]}{2 d^4 g} \end{aligned}$$

Problem 264: Result optimal but 1 more steps used.

$$\int \left( f + g x \right)^2 \left( A + B \text{Log}\left[\frac{e \left( a + b x \right)^2}{\left( c + d x \right)^2}\right] \right) dx$$

Optimal (type 3, 152 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} - \frac{B (b c - a d) g^2 x^2}{3 b d} - \\
& \frac{2 B (b f - a g)^3 \operatorname{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 g} + \frac{2 B (d f - c g)^3 \operatorname{Log}[c + d x]}{3 d^3 g}
\end{aligned}$$

Result (type 3, 152 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} - \frac{B (b c - a d) g^2 x^2}{3 b d} - \\
& \frac{2 B (b f - a g)^3 \operatorname{Log}[a + b x]}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 g} + \frac{2 B (d f - c g)^3 \operatorname{Log}[c + d x]}{3 d^3 g}
\end{aligned}$$

Problem 265: Result optimal but 1 more steps used.

$$\int (f + g x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g x}{b d} - \frac{B (b f - a g)^2 \operatorname{Log}[a + b x]}{b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{2 g} + \frac{B (d f - c g)^2 \operatorname{Log}[c + d x]}{d^2 g}
\end{aligned}$$

Result (type 3, 104 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g x}{b d} - \frac{B (b f - a g)^2 \operatorname{Log}[a + b x]}{b^2 g} + \\
& \frac{(f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{2 g} + \frac{B (d f - c g)^2 \operatorname{Log}[c + d x]}{d^2 g}
\end{aligned}$$

Problem 267: Result optimal but 3 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]}{f + g x} dx$$

Optimal (type 4, 144 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 B \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{g} + \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{2 B \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \frac{2 B \operatorname{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right]}{g} + \frac{2 B \operatorname{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right]}{g}
\end{aligned}$$

Result (type 4, 144 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 B \operatorname{Log}\left[-\frac{g(a+bx)}{bf-ag}\right] \operatorname{Log}[f+gx]}{g} + \frac{\left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right) \operatorname{Log}[f+gx]}{g} + \\
& \frac{2 B \operatorname{Log}\left[-\frac{g(c+dx)}{df-cg}\right] \operatorname{Log}[f+gx]}{g} - \frac{2 B \operatorname{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right]}{g} + \frac{2 B \operatorname{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right]}{g}
\end{aligned}$$

**Problem 268: Result valid but suboptimal antiderivative.**

$$\int \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{(f+gx)^2} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$\frac{(a+bx) \left(A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)}{(bf-ag)(f+gx)} + \frac{2 B (bc-ad) \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{(bf-ag)(df-cg)}$$

Result (type 3, 117 leaves, 4 steps):

$$\frac{2 b B \operatorname{Log}[a+bx]}{g(bf-ag)} - \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{g(f+gx)} - \frac{2 B d \operatorname{Log}[c+dx]}{g(df-cg)} + \frac{2 B (bc-ad) \operatorname{Log}[f+gx]}{(bf-ag)(df-cg)}$$

**Problem 269: Result optimal but 1 more steps used.**

$$\int \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{(f+gx)^3} dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B \operatorname{Log}[a+bx]}{g(bf-ag)^2} - \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{2 g(f+gx)^2} - \\
& \frac{B d^2 \operatorname{Log}[c+dx]}{g(df-cg)^2} + \frac{B(bc-ad)(2 b d f - b c g - a d g) \operatorname{Log}[f+gx]}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$

Result (type 3, 175 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B(bc-ad)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B \operatorname{Log}[a+bx]}{g(bf-ag)^2} - \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{2 g(f+gx)^2} - \\
& \frac{B d^2 \operatorname{Log}[c+dx]}{g(df-cg)^2} + \frac{B(bc-ad)(2 b d f - b c g - a d g) \operatorname{Log}[f+gx]}{(bf-ag)^2(df-cg)^2}
\end{aligned}$$



Problem 270: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{(f+gx)^4} dx$$

Optimal (type 3, 277 leaves, 3 steps):

$$\begin{aligned} & -\frac{B(b c-a d)}{3(b f-a g)(d f-c g)(f+g x)^2}-\frac{2 B(b c-a d)(2 b d f-b c g-a d g)}{3(b f-a g)^2(d f-c g)^2(f+g x)}+ \\ & \frac{2 b^3 B \operatorname{Log}[a+b x]}{3 g(b f-a g)^3}-\frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{3 g(f+g x)^3}-\frac{2 B d^3 \operatorname{Log}[c+d x]}{3 g(d f-c g)^3}+ \\ & \frac{(2 B(b c-a d)\left(a^2 d^2 g^2-a b d g(3 d f-c g)+b^2\left(3 d^2 f^2-3 c d f g+c^2 g^2\right)\right) \operatorname{Log}[f+g x])}{\left(3(b f-a g)^3(d f-c g)^3\right)} \end{aligned}$$

Result (type 3, 277 leaves, 4 steps):

$$\begin{aligned} & -\frac{B(b c-a d)}{3(b f-a g)(d f-c g)(f+g x)^2}-\frac{2 B(b c-a d)(2 b d f-b c g-a d g)}{3(b f-a g)^2(d f-c g)^2(f+g x)}+ \\ & \frac{2 b^3 B \operatorname{Log}[a+b x]}{3 g(b f-a g)^3}-\frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{3 g(f+g x)^3}-\frac{2 B d^3 \operatorname{Log}[c+d x]}{3 g(d f-c g)^3}+ \\ & \frac{(2 B(b c-a d)\left(a^2 d^2 g^2-a b d g(3 d f-c g)+b^2\left(3 d^2 f^2-3 c d f g+c^2 g^2\right)\right) \operatorname{Log}[f+g x])}{\left(3(b f-a g)^3(d f-c g)^3\right)} \end{aligned}$$

Problem 271: Result optimal but 1 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{(f+gx)^5} dx$$

Optimal (type 3, 381 leaves, 3 steps):

$$\begin{aligned} & -\frac{B(b c-a d)}{6(b f-a g)(d f-c g)(f+g x)^3}-\frac{B(b c-a d)(2 b d f-b c g-a d g)}{4(b f-a g)^2(d f-c g)^2(f+g x)^2}- \\ & \frac{(B(b c-a d)\left(a^2 d^2 g^2-a b d g(3 d f-c g)+b^2\left(3 d^2 f^2-3 c d f g+c^2 g^2\right)\right))}{\left(2(b f-a g)^3(d f-c g)^3(f+g x)\right)}+\frac{b^4 B \operatorname{Log}[a+b x]}{2 g(b f-a g)^4}- \\ & \frac{A+B \operatorname{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}{4 g(f+g x)^4}-\frac{B d^4 \operatorname{Log}[c+d x]}{2 g(d f-c g)^4}-\frac{(B(b c-a d)(2 b d f-b c g-a d g))}{\left(2 a b d^2 f g-a^2 d^2 g^2-b^2\left(2 d^2 f^2-2 c d f g+c^2 g^2\right)\right) \operatorname{Log}[f+g x]} \end{aligned}$$

Result (type 3, 381 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)}{6 (b f - a g) (d f - c g) (f + g x)^3} - \frac{B (b c - a d) (2 b d f - b c g - a d g)}{4 (b f - a g)^2 (d f - c g)^2 (f + g x)^2} - \\
& (B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2))) / \\
& (2 (b f - a g)^3 (d f - c g)^3 (f + g x)) + \frac{b^4 B \operatorname{Log}[a + b x]}{2 g (b f - a g)^4} - \\
& \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]}{4 g (f + g x)^4} - \frac{B d^4 \operatorname{Log}[c + d x]}{2 g (d f - c g)^4} - (B (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{Log}[f + g x]) / (2 (b f - a g)^4 (d f - c g)^4)
\end{aligned}$$

**Problem 272: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 dx$$

Optimal (type 4, 869 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 B^2 (b c - a d)^3 g^3 x}{3 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^2 (4 b d f - 3 b c g - a d g) x}{b^3 d^3} + \\
& \frac{B^2 (b c - a d)^2 g^3 (c + d x)^2}{3 b^2 d^4} - \frac{1}{b^4 d^3} B (b c - a d) g \\
& (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) - \\
& \frac{1}{2 b^2 d^4} B (b c - a d) g^2 (4 b d f - 3 b c g - a d g) (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) - \\
& \frac{B (b c - a d) g^3 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d^4} - \frac{(b f - a g)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{4 b^4 g} + \\
& \frac{(f + g x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{4 g} - \frac{1}{b^4 d^4} B (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] + \\
& \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{3 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{b^4 d^4} + \\
& \frac{2 B^2 (b c - a d)^4 g^3 \operatorname{Log} [c + d x]}{3 b^4 d^4} + \frac{B^2 (b c - a d)^3 g^2 (4 b d f - 3 b c g - a d g) \operatorname{Log} [c + d x]}{b^4 d^4} + \frac{1}{b^4 d^4} \\
& 2 B^2 (b c - a d)^2 g (a^2 d^2 g^2 - 2 a b d g (2 d f - c g) + b^2 (6 d^2 f^2 - 8 c d f g + 3 c^2 g^2)) \operatorname{Log} [c + d x] - \\
& \frac{1}{b^4 d^4} 2 B^2 (b c - a d) (2 b d f - b c g - a d g) \\
& (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]
\end{aligned}$$

Result (type 4, 973 leaves, 33 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 (b c + a d) g^3 x}{3 b^3 d^3} + \frac{B^2 (b c - a d)^2 g^2 (4 b d f - b c g - a d g) x}{b^3 d^3} - \\
& \frac{1}{b^3 d^3} A B (b c - a d) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) x + \\
& \frac{B^2 (b c - a d)^2 g^3 x^2}{3 b^2 d^2} - \frac{2 a^3 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x]}{3 b^4 d} + \\
& \frac{a^2 B^2 (b c - a d) g^2 (4 b d f - b c g - a d g) \operatorname{Log}[a + b x]}{b^4 d^2} + \frac{B^2 (b f - a g)^4 \operatorname{Log}[a + b x]^2}{b^4 g} - \\
& \frac{1}{b^4 d^3} B^2 (b c - a d) g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) (a + b x) \\
& \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] - \frac{B (b c - a d) g^2 (4 b d f - b c g - a d g) x^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)}{2 b^2 d^2} - \\
& \frac{B (b c - a d) g^3 x^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)}{3 b d} - \frac{B (b f - a g)^4 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)}{b^4 g} + \\
& \frac{(f + g x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{4 g} + \frac{2 B^2 c^3 (b c - a d) g^3 \operatorname{Log}[c + d x]}{3 b d^4} - \\
& \frac{B^2 c^2 (b c - a d) g^2 (4 b d f - b c g - a d g) \operatorname{Log}[c + d x]}{b^2 d^4} + \frac{1}{b^4 d^4} \\
& \frac{2 B^2 (b c - a d)^2 g (a^2 d^2 g^2 - a b d g (4 d f - c g) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) \operatorname{Log}[c + d x]}{d^4 g} - \\
& \frac{2 B^2 (d f - c g)^4 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 g} + \frac{B (d f - c g)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \operatorname{Log}[c + d x]}{d^4 g} + \\
& \frac{B^2 (d f - c g)^4 \operatorname{Log}[c + d x]^2}{d^4 g} - \frac{2 B^2 (b f - a g)^4 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g} - \\
& \frac{2 B^2 (b f - a g)^4 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} - \frac{2 B^2 (d f - c g)^4 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 g}
\end{aligned}$$

**Problem 273: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 dx$$

Optimal (type 4, 542 leaves, 12 steps):

$$\begin{aligned}
& \frac{4 B^2 (b c - a d)^2 g^2 x}{3 b^2 d^2} - \frac{4 B (b c - a d) g (3 b d f - 2 b c g - a d g) (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b^3 d^2} - \\
& \frac{2 B (b c - a d) g^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d^3} - \\
& \frac{(b f - a g)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{3 b^3 g} + \frac{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{3 g} + \frac{1}{3 b^3 d^3} \\
& \frac{4 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2))}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] + \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{3 b^3 d^3} + \\
& \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log} [c + d x]}{3 b^3 d^3} + \frac{8 B^2 (b c - a d)^2 g (3 b d f - 2 b c g - a d g) \operatorname{Log} [c + d x]}{3 b^3 d^3} + \frac{1}{3 b^3 d^3} \\
& 8 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]
\end{aligned}$$

Result (type 4, 659 leaves, 29 steps):

$$\begin{aligned}
& \frac{4 B^2 (b c - a d)^2 g^2 x}{3 b^2 d^2} - \frac{4 A B (b c - a d) g (3 b d f - b c g - a d g) x}{3 b^2 d^2} + \frac{4 a^2 B^2 (b c - a d) g^2 \operatorname{Log} [a + b x]}{3 b^3 d} + \\
& \frac{4 B^2 (b f - a g)^3 \operatorname{Log} [a + b x]^2}{3 b^3 g} - \frac{4 B^2 (b c - a d) g (3 b d f - b c g - a d g) (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right]}{3 b^3 d^2} - \\
& \frac{2 B (b c - a d) g^2 x^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b d} - \frac{4 B (b f - a g)^3 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{3 b^3 g} + \\
& \frac{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{3 g} - \frac{4 B^2 c^2 (b c - a d) g^2 \operatorname{Log} [c + d x]}{3 b d^3} + \\
& \frac{8 B^2 (b c - a d)^2 g (3 b d f - b c g - a d g) \operatorname{Log} [c + d x]}{3 b^3 d^3} - \\
& \frac{8 B^2 (d f - c g)^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{3 d^3 g} + \frac{4 B (d f - c g)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} [c + d x]}{3 d^3 g} + \\
& \frac{4 B^2 (d f - c g)^3 \operatorname{Log} [c + d x]^2}{3 d^3 g} - \frac{8 B^2 (b f - a g)^3 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{3 b^3 g} - \\
& \frac{8 B^2 (b f - a g)^3 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{3 b^3 g} - \frac{8 B^2 (d f - c g)^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{3 d^3 g}
\end{aligned}$$

**Problem 274: Result valid but suboptimal antiderivative.**

$$\int (f + g x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 281 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{2 B (b c - a d) g (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{b^2 d} - \\
 & \frac{(b f - a g)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{2 b^2 g} + \frac{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{2 g} + \frac{1}{b^2 d^2} \\
 & 2 B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] + \\
 & \frac{4 B^2 (b c - a d)^2 g \operatorname{Log} [c + d x]}{b^2 d^2} + \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b^2 d^2}
 \end{aligned}$$

Result (type 4, 450 leaves, 25 steps):

$$\begin{aligned}
 & - \frac{2 A B (b c - a d) g x}{b d} + \frac{2 B^2 (b f - a g)^2 \operatorname{Log} [a + b x]^2}{b^2 g} - \\
 & \frac{2 B^2 (b c - a d) g (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right]}{b^2 d} - \frac{2 B (b f - a g)^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}{b^2 g} + \\
 & \frac{(f + g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{2 g} + \frac{4 B^2 (b c - a d)^2 g \operatorname{Log} [c + d x]}{b^2 d^2} - \\
 & \frac{4 B^2 (d f - c g)^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{d^2 g} + \frac{2 B (d f - c g)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} [c + d x]}{d^2 g} + \\
 & \frac{2 B^2 (d f - c g)^2 \operatorname{Log} [c + d x]^2}{d^2 g} - \frac{4 B^2 (b f - a g)^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{b^2 g} - \\
 & \frac{4 B^2 (b f - a g)^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^2 g} - \frac{4 B^2 (d f - c g)^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{d^2 g}
 \end{aligned}$$

**Problem 275: Result valid but suboptimal antiderivative.**

$$\int \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2 dx$$

Optimal (type 4, 129 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}{b} + \\
 & \frac{4 B (b c - a d) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right) \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right]}{b d} + \frac{8 B^2 (b c - a d) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b d}
 \end{aligned}$$

Result (type 4, 252 leaves, 22 steps):

$$\begin{aligned}
& - \frac{4 a B^2 \operatorname{Log}[a + b x]^2}{b} + \frac{4 a B \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{b} + x \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 + \\
& \frac{8 B^2 c \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d} - \frac{4 B c \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{d} - \frac{4 B^2 c \operatorname{Log}[c + d x]^2}{d} + \\
& \frac{8 a B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b} + \frac{8 a B^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b} + \frac{8 B^2 c \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d}
\end{aligned}$$

**Problem 276: Result valid but suboptimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{f + g x} dx$$

Optimal (type 4, 285 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{g} + \\
& \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 \operatorname{Log}\left[1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g} - \frac{4 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{g} + \\
& \frac{4 B \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{PolyLog}\left[2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g} + \\
& \frac{8 B^2 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{g} - \frac{8 B^2 \operatorname{PolyLog}\left[3, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{g}
\end{aligned}$$

Result (type 4, 2126 leaves, 44 steps):

$$\begin{aligned}
& - \frac{4 A B \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{g} - \frac{B^2 \operatorname{Log}\left[(a + b x)^2\right]^2 \operatorname{Log}[f + g x]}{g} - \frac{B^2 \operatorname{Log}\left[\frac{1}{(c + d x)^2}\right]^2 \operatorname{Log}[f + g x]}{g} + \\
& \frac{1}{g} 4 B^2 \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \left( \operatorname{Log}\left[(a + b x)^2\right] + \operatorname{Log}\left[\frac{1}{(c + d x)^2}\right] - \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[f + g x] + \\
& \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2 \operatorname{Log}[f + g x]}{g} + \frac{8 B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \operatorname{Log}[f + g x]}{g} - \\
& \frac{4 B^2 \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \left( \operatorname{Log}\left[\frac{1}{(c + d x)^2}\right] + 2 \operatorname{Log}[c + d x] \right) \operatorname{Log}[f + g x]}{g} + \\
& \frac{8 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[f + g x]}{g} + \frac{4 A B \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \\
& \frac{4 B^2 \left( 2 \operatorname{Log}[a + b x] - \operatorname{Log}\left[(a + b x)^2\right] \right) \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{g} - \frac{1}{g}
\end{aligned}$$

$$\begin{aligned}
& 4 B^2 \left( \text{Log}[(a + b x)^2] + \text{Log}\left[\frac{1}{(c + d x)^2}\right] - \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \text{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \text{Log}[f + g x] + \\
& \frac{B^2 \text{Log}[(a + b x)^2]^2 \text{Log}\left[\frac{b (f + g x)}{b f - a g}\right] - B^2 \text{Log}\left[\frac{1}{(c + d x)^2}\right]^2 \text{Log}\left[\frac{d (f + g x)}{d f - c g}\right] + \frac{1}{g}}{g} \\
& 4 B^2 \left( \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] + \text{Log}\left[\frac{b f - a g}{b (f + g x)}\right] - \text{Log}\left[\frac{(b f - a g) (c + d x)}{(b c - a d) (f + g x)}\right] \right) \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]^2 - \\
& \frac{1}{g} 4 B^2 \left( \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] - \text{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \right) \left( \text{Log}[a + b x] + \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right] \right)^2 + \frac{1}{g} 4 B^2 \\
& \left( \text{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] + \text{Log}\left[\frac{d f - c g}{d (f + g x)}\right] - \text{Log}\left[-\frac{(d f - c g) (a + b x)}{(b c - a d) (f + g x)}\right] \right) \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right]^2 - \\
& \frac{1}{g} 4 B^2 \left( \text{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] - \text{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \right) \left( \text{Log}[c + d x] + \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right] \right)^2 + \\
& \frac{8 B^2 \left( \text{Log}[f + g x] - \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right] \right) \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{g} + \\
& \frac{4 B^2 \text{Log}[(a + b x)^2] \text{PolyLog}\left[2, -\frac{g (a + b x)}{b f - a g}\right]}{g} + \\
& \frac{8 B^2 \left( \text{Log}[f + g x] - \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right] \right) \text{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{g} - \\
& \frac{4 B^2 \text{Log}\left[\frac{1}{(c + d x)^2}\right] \text{PolyLog}\left[2, -\frac{g (c + d x)}{d f - c g}\right] - 8 B^2 \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right] \text{PolyLog}\left[2, \frac{g (a + b x)}{b (f + g x)}\right]}{g} + \\
& \frac{8 B^2 \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right] \text{PolyLog}\left[2, -\frac{(d f - c g) (a + b x)}{(b c - a d) (f + g x)}\right]}{g} - \\
& \frac{8 B^2 \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right] \text{PolyLog}\left[2, \frac{g (c + d x)}{d (f + g x)}\right]}{g} + \\
& \frac{8 B^2 \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right] \text{PolyLog}\left[2, \frac{(b f - a g) (c + d x)}{(b c - a d) (f + g x)}\right] - 4 A B \text{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right] + \frac{1}{g}}{g} \\
& 4 B^2 \left( \text{Log}[(a + b x)^2] + \text{Log}\left[\frac{1}{(c + d x)^2}\right] - \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \text{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right] - \\
& \frac{4 B^2 \left( \text{Log}\left[\frac{1}{(c + d x)^2}\right] + 2 \text{Log}[c + d x] \right) \text{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right]}{g} + \\
& \frac{8 B^2 \left( \text{Log}[c + d x] + \text{Log}\left[\frac{(b c - a d) (f + g x)}{(b f - a g) (c + d x)}\right] \right) \text{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right]}{g} + \\
& \frac{4 A B \text{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right] - 4 B^2 \left( 2 \text{Log}[a + b x] - \text{Log}[(a + b x)^2] \right) \text{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right]}{g} -
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{g} 4 B^2 \left( \text{Log}[(a + b x)^2] + \text{Log}\left[\frac{1}{(c + d x)^2}\right] - \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \text{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right] + \\
& \frac{8 B^2 \left( \text{Log}[a + b x] + \text{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right] \right) \text{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right] - 8 B^2 \text{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right] - \\
& \frac{8 B^2 \text{PolyLog}\left[3, -\frac{g (a + b x)}{b f - a g}\right] - 8 B^2 \text{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right] - 8 B^2 \text{PolyLog}\left[3, -\frac{g (c + d x)}{d f - c g}\right] - \\
& \frac{8 B^2 \text{PolyLog}\left[3, \frac{g (a + b x)}{b (f + g x)}\right] - 8 B^2 \text{PolyLog}\left[3, -\frac{(d f - c g) (a + b x)}{(b c - a d) (f + g x)}\right] - 8 B^2 \text{PolyLog}\left[3, \frac{g (c + d x)}{d (f + g x)}\right] + \\
& \frac{8 B^2 \text{PolyLog}\left[3, \frac{(b f - a g) (c + d x)}{(b c - a d) (f + g x)}\right] - 8 B^2 \text{PolyLog}\left[3, \frac{b (f + g x)}{b f - a g}\right] - 8 B^2 \text{PolyLog}\left[3, \frac{d (f + g x)}{d f - c g}\right]}{g}
\end{aligned}$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{(f + g x)^2} dx$$

Optimal (type 4, 200 leaves, 4 steps):

$$\begin{aligned}
& \frac{(a + b x) \left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{(b f - a g) (f + g x)} + \frac{4 B (b c - a d) \left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \text{Log}\left[1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{(b f - a g) (d f - c g)} + \\
& \frac{8 B^2 (b c - a d) \text{PolyLog}\left[2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{(b f - a g) (d f - c g)}
\end{aligned}$$

Result (type 4, 620 leaves, 32 steps):

$$\begin{aligned}
& -\frac{4 b B^2 \text{Log}[a + b x]^2}{g (b f - a g)} + \frac{4 b B \text{Log}[a + b x] \left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)}{g (b f - a g)} - \frac{\left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right)^2}{g (f + g x)} + \\
& \frac{8 B^2 d \text{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \text{Log}[c + d x]}{g (d f - c g)} - \frac{4 B d \left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \text{Log}[c + d x]}{g (d f - c g)} - \frac{4 B^2 d \text{Log}[c + d x]^2}{g (d f - c g)} + \\
& \frac{8 b B^2 \text{Log}[a + b x] \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{g (b f - a g)} - \frac{8 B^2 (b c - a d) \text{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \text{Log}[f + g x]}{(b f - a g) (d f - c g)} + \\
& \frac{4 B (b c - a d) \left(A + B \text{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right]\right) \text{Log}[f + g x]}{(b f - a g) (d f - c g)} + \frac{8 B^2 (b c - a d) \text{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \text{Log}[f + g x]}{(b f - a g) (d f - c g)} + \\
& \frac{8 b B^2 \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{g (b f - a g)} + \frac{8 B^2 d \text{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{g (d f - c g)} - \\
& \frac{8 B^2 (b c - a d) \text{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right]}{(b f - a g) (d f - c g)} + \frac{8 B^2 (b c - a d) \text{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right]}{(b f - a g) (d f - c g)}
\end{aligned}$$

### Problem 278: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{(f+gx)^3} dx$$

Optimal (type 4, 381 leaves, 9 steps):

$$\begin{aligned} & \frac{2 B (b c - a d) g (a + b x) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}{(b f - a g)^2 (d f - c g) (f + g x)} + \\ & \frac{b^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{2 g (b f - a g)^2} - \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)^2}{2 g (f + g x)^2} + \frac{4 B^2 (b c - a d)^2 g \operatorname{Log}\left[\frac{f+gx}{c+dx}\right]}{(b f - a g)^2 (d f - c g)^2} + \\ & \left(2 B (b c - a d) (2 b d f - b c g - a d g) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right) \operatorname{Log}\left[1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]\right) / \\ & \left((b f - a g)^2 (d f - c g)^2\right) + \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog}\left[2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right]}{(b f - a g)^2 (d f - c g)^2} \end{aligned}$$

Result (type 4, 899 leaves, 36 steps):

$$\begin{aligned}
& \frac{4 b B^2 (b c - a d) \operatorname{Log}[a + b x]}{(b f - a g)^2 (d f - c g)} - \frac{2 b^2 B^2 \operatorname{Log}[a + b x]^2}{g (b f - a g)^2} - \frac{2 B (b c - a d) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{(b f - a g) (d f - c g) (f + g x)} + \\
& \frac{2 b^2 B \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{g (b f - a g)^2} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{2 g (f + g x)^2} - \frac{4 B^2 d (b c - a d) \operatorname{Log}[c + d x]}{(b f - a g) (d f - c g)^2} + \\
& \frac{4 B^2 d^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{g (d f - c g)^2} - \frac{2 B d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{g (d f - c g)^2} - \\
& \frac{2 B^2 d^2 \operatorname{Log}[c + d x]^2}{g (d f - c g)^2} + \frac{4 b^2 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{g (b f - a g)^2} + \frac{4 B^2 (b c - a d)^2 g \operatorname{Log}[f + g x]}{(b f - a g)^2 (d f - c g)^2} - \\
& \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}\left[-\frac{g (a + b x)}{b f - a g}\right] \operatorname{Log}[f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \left( 2 B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[f + g x] \right) / \\
& \left( (b f - a g)^2 (d f - c g)^2 \right) + \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}\left[-\frac{g (c + d x)}{d f - c g}\right] \operatorname{Log}[f + g x]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{4 b^2 B^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{g (b f - a g)^2} + \frac{4 B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{g (d f - c g)^2} - \\
& \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog}\left[2, \frac{b (f + g x)}{b f - a g}\right]}{(b f - a g)^2 (d f - c g)^2} + \\
& \frac{4 B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{PolyLog}\left[2, \frac{d (f + g x)}{d f - c g}\right]}{(b f - a g)^2 (d f - c g)^2}
\end{aligned}$$

**Problem 279: Result valid but suboptimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{(f + g x)^4} dx$$

Optimal (type 4, 724 leaves, 12 steps):

$$\begin{aligned}
& \frac{4 B^2 (b c - a d)^2 g^2 (c + d x)}{3 (b f - a g)^2 (d f - c g)^3 (f + g x)} - \frac{2 B (b c - a d) g^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 (b f - a g) (d f - c g)^3 (f + g x)^2} + \\
& \frac{4 B (b c - a d) g (3 b d f - b c g - 2 a d g) (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 (b f - a g)^3 (d f - c g)^2 (f + g x)} + \\
& \frac{b^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{3 g (b f - a g)^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{3 g (f + g x)^3} + \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{3 (b f - a g)^3 (d f - c g)^3} - \\
& \frac{4 B^2 (b c - a d)^3 g^2 \operatorname{Log}\left[\frac{f + g x}{c + d x}\right]}{3 (b f - a g)^3 (d f - c g)^3} + \frac{8 B^2 (b c - a d)^2 g (3 b d f - b c g - 2 a d g) \operatorname{Log}\left[\frac{f + g x}{c + d x}\right]}{3 (b f - a g)^3 (d f - c g)^3} + \\
& \left( 4 B (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}\left[1 - \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right] \right) / (3 (b f - a g)^3 (d f - c g)^3) + \\
& \left( 8 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, \frac{(d f - c g) (a + b x)}{(b f - a g) (c + d x)}\right] \right) / (3 (b f - a g)^3 (d f - c g)^3)
\end{aligned}$$

Result (type 4, 1369 leaves, 40 steps):

$$\begin{aligned}
& - \frac{4 B^2 (b c - a d)^2 g}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \frac{4 b^2 B^2 (b c - a d) \operatorname{Log}[a + b x]}{3 (b f - a g)^3 (d f - c g)} + \\
& \frac{8 b B^2 (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}[a + b x]}{3 (b f - a g)^3 (d f - c g)^2} - \frac{4 b^3 B^2 \operatorname{Log}[a + b x]^2}{3 g (b f - a g)^3} - \\
& \frac{2 B (b c - a d) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 (b f - a g) (d f - c g) (f + g x)^2} - \frac{4 B (b c - a d) (2 b d f - b c g - a d g) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 (b f - a g)^2 (d f - c g)^2 (f + g x)} + \\
& \frac{4 b^3 B \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)}{3 g (b f - a g)^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right)^2}{3 g (f + g x)^3} - \\
& \frac{4 B^2 d^2 (b c - a d) \operatorname{Log}[c + d x]}{3 (b f - a g) (d f - c g)^3} - \frac{8 B^2 d (b c - a d) (2 b d f - b c g - a d g) \operatorname{Log}[c + d x]}{3 (b f - a g)^2 (d f - c g)^3} + \\
& \frac{8 B^2 d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \frac{4 B d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)^2}{(c + d x)^2}\right] \right) \operatorname{Log}[c + d x]}{3 g (d f - c g)^3} - \frac{4 B^2 d^3 \operatorname{Log}[c + d x]^2}{3 g (d f - c g)^3} + \\
& \frac{8 b^3 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 g (b f - a g)^3} + \frac{4 B^2 (b c - a d)^2 g (2 b d f - b c g - a d g) \operatorname{Log}[f + g x]}{(b f - a g)^3 (d f - c g)^3} - \\
& \left( 8 B^2 (b c - a d) (a^2 d^2 g^2 - a b d g (3 d f - c g) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{Log}\left[-\frac{g(a+bx)}{bf-ag}\right] \text{Log}[f+gx] \right) \Bigg/ \left(3(bf-ag)^3(df-cg)^3\right) + \\
& \left(4B(bc-ad)(a^2d^2g^2 - abd g(3df-cg) + b^2(3d^2f^2 - 3cdfg + c^2g^2)) \right. \\
& \quad \left. \left(A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right] \text{Log}[f+gx] \right) \right) \Bigg/ \left(3(bf-ag)^3(df-cg)^3\right) + \\
& \left(8B^2(bc-ad)(a^2d^2g^2 - abd g(3df-cg) + b^2(3d^2f^2 - 3cdfg + c^2g^2)) \right. \\
& \quad \left. \text{Log}\left[-\frac{g(c+dx)}{df-cg}\right] \text{Log}[f+gx] \right) \Bigg/ \left(3(bf-ag)^3(df-cg)^3\right) + \\
& \frac{8b^3B^2 \text{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{3g(bf-ag)^3} + \frac{8B^2d^3 \text{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{3g(df-cg)^3} - \\
& \left(8B^2(bc-ad)(a^2d^2g^2 - abd g(3df-cg) + b^2(3d^2f^2 - 3cdfg + c^2g^2)) \right. \\
& \quad \left. \text{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right] \right) \Bigg/ \left(3(bf-ag)^3(df-cg)^3\right) + \\
& \left(8B^2(bc-ad)(a^2d^2g^2 - abd g(3df-cg) + b^2(3d^2f^2 - 3cdfg + c^2g^2)) \right. \\
& \quad \left. \text{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right] \right) \Bigg/ \left(3(bf-ag)^3(df-cg)^3\right)
\end{aligned}$$

**Problem 280: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]\right)^2}{(f+gx)^5} dx$$

Optimal (type 4, 1154 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g^3 (c + dx)^2}{3 (bf - ag)^2 (df - cg)^4 (f + gx)^2} - \frac{2 B^2 (bc - ad)^3 g^3 (c + dx)}{3 (bf - ag)^3 (df - cg)^4 (f + gx)} + \\
& \frac{B^2 (bc - ad)^2 g^2 (4 bdf - b c g - 3 a d g) (c + dx)}{(bf - ag)^3 (df - cg)^4 (f + gx)} + \frac{B (bc - ad) g^3 (c + dx)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right)}{3 (bf - ag) (df - cg)^4 (f + gx)^3} - \\
& \left( B (bc - ad) g^2 (4 bdf - b c g - 3 a d g) (c + dx)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right) \right) / \\
& \left( 2 (bf - ag)^2 (df - cg)^4 (f + gx)^2 \right) + \\
& \left( B (bc - ad) g (3 a^2 d^2 g^2 - 2 a b d g (4 df - cg) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) \right. \\
& \left. (a + bx) \left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right) \right) / \left( (bf - ag)^4 (df - cg)^3 (f + gx) \right) + \\
& \frac{b^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right)^2}{4 g (bf - ag)^4} - \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right)^2}{4 g (f + gx)^4} - \frac{2 B^2 (bc - ad)^4 g^3 \operatorname{Log} \left[ \frac{a + bx}{c + dx} \right]}{3 (bf - ag)^4 (df - cg)^4} + \\
& \frac{B^2 (bc - ad)^3 g^2 (4 bdf - b c g - 3 a d g) \operatorname{Log} \left[ \frac{a + bx}{c + dx} \right]}{(bf - ag)^4 (df - cg)^4} + \\
& \frac{2 B^2 (bc - ad)^4 g^3 \operatorname{Log} \left[ \frac{f + gx}{c + dx} \right]}{3 (bf - ag)^4 (df - cg)^4} - \frac{B^2 (bc - ad)^3 g^2 (4 bdf - b c g - 3 a d g) \operatorname{Log} \left[ \frac{f + gx}{c + dx} \right]}{(bf - ag)^4 (df - cg)^4} + \\
& \left( 2 B^2 (bc - ad)^2 g (3 a^2 d^2 g^2 - 2 a b d g (4 df - cg) + b^2 (6 d^2 f^2 - 4 c d f g + c^2 g^2)) \operatorname{Log} \left[ \frac{f + gx}{c + dx} \right] \right) / \\
& \left( (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \left. \left( A + B \operatorname{Log} \left[ \frac{e (a + bx)^2}{(c + dx)^2} \right] \right) \operatorname{Log} \left[ 1 - \frac{(df - cg) (a + bx)}{(bf - ag) (c + dx)} \right] \right) / \left( (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( 2 B^2 (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \left. \operatorname{PolyLog} \left[ 2, \frac{(df - cg) (a + bx)}{(bf - ag) (c + dx)} \right] \right) / \left( (bf - ag)^4 (df - cg)^4 \right)
\end{aligned}$$

Result (type 4, 1854 leaves, 44 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g}{3 (bf - ag)^2 (df - cg)^2 (f + gx)^2} - \frac{5 B^2 (bc - ad)^2 g (2 bdf - b c g - a d g)}{3 (bf - ag)^3 (df - cg)^3 (f + gx)} + \\
& \frac{2 b^3 B^2 (bc - ad) \operatorname{Log} [a + bx]}{3 (bf - ag)^4 (df - cg)} + \frac{b^2 B^2 (bc - ad) (2 bdf - b c g - a d g) \operatorname{Log} [a + bx]}{(bf - ag)^4 (df - cg)^2} + \\
& \left( 2 b B^2 (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log} [a + bx] \right) /
\end{aligned}$$

$$\begin{aligned}
& \left( (bf - ag)^4 (df - cg)^3 \right) - \frac{b^4 B^2 \operatorname{Log}[a + bx]^2}{g (bf - ag)^4} - \frac{B (bc - ad) \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right)}{3 (bf - ag) (df - cg) (f + gx)^3} - \\
& \frac{B (bc - ad) (2 bdf - b c g - a d g) \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right)}{2 (bf - ag)^2 (df - cg)^2 (f + gx)^2} - \\
& \left( B (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \right. \\
& \quad \left. \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right) \right) / \left( (bf - ag)^3 (df - cg)^3 (f + gx) \right) + \\
& \frac{b^4 B \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right)}{g (bf - ag)^4} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right)^2}{4 g (f + gx)^4} - \frac{2 B^2 d^3 (bc - ad) \operatorname{Log}[c + dx]}{3 (bf - ag) (df - cg)^4} - \\
& \frac{B^2 d^2 (bc - ad) (2 bdf - b c g - a d g) \operatorname{Log}[c + dx]}{(bf - ag)^2 (df - cg)^4} - \\
& (2 B^2 d (bc - ad) (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[c + dx]) / \\
& \left( (bf - ag)^3 (df - cg)^4 \right) + \frac{2 B^2 d^4 \operatorname{Log}\left[-\frac{d (a+bx)}{b c - a d}\right] \operatorname{Log}[c + dx]}{g (df - cg)^4} - \\
& \frac{B d^4 \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right) \operatorname{Log}[c + dx]}{g (df - cg)^4} - \frac{B^2 d^4 \operatorname{Log}[c + dx]^2}{g (df - cg)^4} + \\
& \frac{2 b^4 B^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b (c+dx)}{b c - a d}\right]}{g (bf - ag)^4} + \frac{B^2 (bc - ad)^2 g (2 bdf - b c g - a d g)^2 \operatorname{Log}[f + gx]}{(bf - ag)^4 (df - cg)^4} + \\
& \left( 8 B^2 (bc - ad)^2 g (a^2 d^2 g^2 - a b d g (3 df - cg) + b^2 (3 d^2 f^2 - 3 c d f g + c^2 g^2)) \operatorname{Log}[f + gx] \right) / \\
& \left( 3 (bf - ag)^4 (df - cg)^4 \right) + \\
& \left( 2 B^2 (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{Log}\left[-\frac{g (a + bx)}{b f - a g}\right] \operatorname{Log}[f + gx] \right) / \left( (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( B (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. \left( A + B \operatorname{Log}\left[\frac{e (a+bx)^2}{(c+dx)^2}\right] \right) \operatorname{Log}[f + gx] \right) / \left( (bf - ag)^4 (df - cg)^4 \right) - \\
& \left( 2 B^2 (bc - ad) (2 bdf - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. \operatorname{Log}\left[-\frac{g (c + dx)}{d f - c g}\right] \operatorname{Log}[f + gx] \right) / \left( (bf - ag)^4 (df - cg)^4 \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^4 B^2 \text{PolyLog}\left[2, -\frac{d(a+bx)}{b c - a d}\right]}{g(b f - a g)^4} + \frac{2 B^2 d^4 \text{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{g(d f - c g)^4} + \\
& \left( 2 B^2 (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. \text{PolyLog}\left[2, \frac{b(f+gx)}{b f - a g}\right] \right) / \left( (b f - a g)^4 (d f - c g)^4 \right) - \\
& \left( 2 B^2 (b c - a d) (2 b d f - b c g - a d g) (2 a b d^2 f g - a^2 d^2 g^2 - b^2 (2 d^2 f^2 - 2 c d f g + c^2 g^2)) \right. \\
& \quad \left. \text{PolyLog}\left[2, \frac{d(f+gx)}{d f - c g}\right] \right) / \left( (b f - a g)^4 (d f - c g)^4 \right)
\end{aligned}$$

**Problem 281: Result valid but suboptimal antiderivative.**

$$\int \frac{(f+gx)^2}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f+gx)^2}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right]$$

Result (type 8, 94 leaves, 2 steps):

$$\begin{aligned}
& f^2 \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right] + \\
& 2 f g \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right] + g^2 \text{CannotIntegrate}\left[\frac{x^2}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right]
\end{aligned}$$

**Problem 282: Result valid but suboptimal antiderivative.**

$$\int \frac{f+gx}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{f+gx}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right]$$

Result (type 8, 57 leaves, 2 steps):

$$f \text{CannotIntegrate}\left[\frac{1}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right] + g \text{CannotIntegrate}\left[\frac{x}{A + B \text{Log}\left[\frac{e(a+bx)^2}{(c+dx)^2}\right]}, x\right]$$



### Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]} dx$$

Optimal (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]}, x\right]$$

Result (type 8, 25 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]}, x\right]$$

### Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f+gx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+gx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+gx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

### Problem 285: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f+gx)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+gx)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+gx)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]\right)}, x\right]$$

### Problem 286: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}, x \right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(f + g x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)}, x \right]$$

### Problem 287: Result valid but suboptimal antiderivative.

$$\int \frac{(f + g x)^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{(f + g x)^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}, x \right]$$

Result (type 8, 94 leaves, 2 steps):

$$\begin{aligned} & f^2 \text{ CannotIntegrate} \left[ \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}, x \right] + \\ & 2 f g \text{ CannotIntegrate} \left[ \frac{x}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}, x \right] + g^2 \text{ CannotIntegrate} \left[ \frac{x^2}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}, x \right] \end{aligned}$$

### Problem 288: Result valid but suboptimal antiderivative.

$$\int \frac{f + g x}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{f + g x}{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)^2}{(c + d x)^2} \right] \right)^2}, x \right]$$

Result (type 8, 57 leaves, 2 steps):

$$f \text{ CannotIntegrate} \left[ \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right] + g \text{ CannotIntegrate} \left[ \frac{x}{\left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Problem 289: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2} dx$$

Optimal (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Result (type 8, 25 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{\left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Problem 290: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f+gx) \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(f+gx) \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{1}{(f+gx) \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Problem 291: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f+gx)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{1}{(f+gx)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)^2}}{(c+dx)^2} \right] \right)^2}, x \right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+g x)^2 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Problem 292: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(f+g x)^3 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(f+g x)^3 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Result (type 8, 33 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(f+g x)^3 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)^2}{(c+d x)^2}\right]\right)^2}, x\right]$$

Problem 293: Result valid but suboptimal antiderivative.

$$\int (g+h x)^4 \left(A+B \operatorname{Log}\left[e (a+b x)^n (c+d x)^{-n}\right]\right) dx$$

Optimal (type 3, 365 leaves, 3 steps):

$$\begin{aligned} & \frac{1}{5 b^4 d^4} B (b c - a d) h \left( a^3 d^3 h^3 - a^2 b d^2 h^2 (5 d g - c h) + \right. \\ & \quad \left. a b^2 d h (10 d^2 g^2 - 5 c d g h + c^2 h^2) - b^3 (10 d^3 g^3 - 10 c d^2 g^2 h + 5 c^2 d g h^2 - c^3 h^3) \right) n x - \\ & \frac{1}{10 b^3 d^3} B (b c - a d) h^2 \left( a^2 d^2 h^2 - a b d h (5 d g - c h) + b^2 (10 d^2 g^2 - 5 c d g h + c^2 h^2) \right) n x^2 - \\ & \frac{B (b c - a d) h^3 (5 b d g - b c h - a d h) n x^3}{15 b^2 d^2} - \frac{B (b c - a d) h^4 n x^4}{20 b d} - \\ & \frac{B (b g - a h)^5 n \operatorname{Log}[a + b x]}{5 b^5 h} + \frac{B (d g - c h)^5 n \operatorname{Log}[c + d x]}{5 d^5 h} + \\ & \frac{(g + h x)^5 \left( A + B \operatorname{Log}\left[ e (a + b x)^n (c + d x)^{-n} \right] \right)}{5 h} \end{aligned}$$

Result (type 3, 377 leaves, 5 steps):

$$\begin{aligned}
& \frac{1}{5 b^4 d^4} B (b c - a d) h (a^3 d^3 h^3 - a^2 b d^2 h^2 (5 d g - c h) + \\
& \quad a b^2 d h (10 d^2 g^2 - 5 c d g h + c^2 h^2) - b^3 (10 d^3 g^3 - 10 c d^2 g^2 h + 5 c^2 d g h^2 - c^3 h^3)) n x - \\
& \frac{1}{10 b^3 d^3} B (b c - a d) h^2 (a^2 d^2 h^2 - a b d h (5 d g - c h) + b^2 (10 d^2 g^2 - 5 c d g h + c^2 h^2)) n x^2 - \\
& \frac{B (b c - a d) h^3 (5 b d g - b c h - a d h) n x^3}{15 b^2 d^2} - \\
& \frac{B (b c - a d) h^4 n x^4}{20 b d} + \frac{A (g + h x)^5}{5 h} - \frac{B (b g - a h)^5 n \operatorname{Log}[a + b x]}{5 b^5 h} + \\
& \frac{B (d g - c h)^5 n \operatorname{Log}[c + d x]}{5 d^5 h} + \frac{B (g + h x)^5 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{5 h}
\end{aligned}$$

**Problem 294: Result valid but suboptimal antiderivative.**

$$\int (g + h x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 236 leaves, 3 steps):

$$\begin{aligned}
& - \frac{1}{4 b^3 d^3} B (b c - a d) h (a^2 d^2 h^2 - a b d h (4 d g - c h) + b^2 (6 d^2 g^2 - 4 c d g h + c^2 h^2)) n x - \\
& \frac{B (b c - a d) h^2 (4 b d g - b c h - a d h) n x^2}{8 b^2 d^2} - \frac{B (b c - a d) h^3 n x^3}{12 b d} - \frac{B (b g - a h)^4 n \operatorname{Log}[a + b x]}{4 b^4 h} + \\
& \frac{B (d g - c h)^4 n \operatorname{Log}[c + d x]}{4 d^4 h} + \frac{(g + h x)^4 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{4 h}
\end{aligned}$$

Result (type 3, 248 leaves, 5 steps):

$$\begin{aligned}
& - \frac{1}{4 b^3 d^3} B (b c - a d) h (a^2 d^2 h^2 - a b d h (4 d g - c h) + b^2 (6 d^2 g^2 - 4 c d g h + c^2 h^2)) n x - \\
& \frac{B (b c - a d) h^2 (4 b d g - b c h - a d h) n x^2}{8 b^2 d^2} - \frac{B (b c - a d) h^3 n x^3}{12 b d} + \frac{A (g + h x)^4}{4 h} - \\
& \frac{B (b g - a h)^4 n \operatorname{Log}[a + b x]}{4 b^4 h} + \frac{B (d g - c h)^4 n \operatorname{Log}[c + d x]}{4 d^4 h} + \frac{B (g + h x)^4 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 h}
\end{aligned}$$

**Problem 295: Result valid but suboptimal antiderivative.**

$$\int (g + h x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 158 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) h (3 b d g - b c h - a d h) n x}{3 b^2 d^2} - \frac{B (b c - a d) h^2 n x^2}{6 b d} - \frac{B (b g - a h)^3 n \operatorname{Log}[a + b x]}{3 b^3 h} + \\
& \frac{B (d g - c h)^3 n \operatorname{Log}[c + d x]}{3 d^3 h} + \frac{(g + h x)^3 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{3 h}
\end{aligned}$$

Result (type 3, 170 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) h (3 b d g - b c h - a d h) n x}{3 b^2 d^2} - \frac{B(b c - a d) h^2 n x^2}{6 b d} + \frac{A(g + h x)^3}{3 h} - \\
& \frac{B(b g - a h)^3 n \operatorname{Log}[a + b x]}{3 b^3 h} + \frac{B(d g - c h)^3 n \operatorname{Log}[c + d x]}{3 d^3 h} + \frac{B(g + h x)^3 \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{3 h}
\end{aligned}$$

**Problem 296: Result valid but suboptimal antiderivative.**

$$\int (g + h x) (A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]) dx$$

Optimal (type 3, 116 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) h n x}{2 b d} - \frac{B(b g - a h)^2 n \operatorname{Log}[a + b x]}{2 b^2 h} + \\
& \frac{B(d g - c h)^2 n \operatorname{Log}[c + d x]}{2 d^2 h} + \frac{(g + h x)^2 (A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}])}{2 h}
\end{aligned}$$

Result (type 3, 128 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) h n x}{2 b d} + \frac{A(g + h x)^2}{2 h} - \frac{B(b g - a h)^2 n \operatorname{Log}[a + b x]}{2 b^2 h} + \\
& \frac{B(d g - c h)^2 n \operatorname{Log}[c + d x]}{2 d^2 h} + \frac{B(g + h x)^2 \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{2 h}
\end{aligned}$$

**Problem 298: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]}{g + h x} dx$$

Optimal (type 4, 148 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B n \operatorname{Log}\left[-\frac{h(a + b x)}{b g - a h}\right] \operatorname{Log}[g + h x]}{h} + \frac{B n \operatorname{Log}\left[-\frac{h(c + d x)}{d g - c h}\right] \operatorname{Log}[g + h x]}{h} + \\
& \frac{(A + B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}]) \operatorname{Log}[g + h x]}{h} - \\
& \frac{B n \operatorname{PolyLog}\left[2, \frac{b(g + h x)}{b g - a h}\right]}{h} + \frac{B n \operatorname{PolyLog}\left[2, \frac{d(g + h x)}{d g - c h}\right]}{h}
\end{aligned}$$

Result (type 4, 156 leaves, 9 steps):

$$\begin{aligned}
& \frac{A \operatorname{Log}[g + h x]}{h} - \frac{B n \operatorname{Log}\left[-\frac{h(a + b x)}{b g - a h}\right] \operatorname{Log}[g + h x]}{h} + \frac{B n \operatorname{Log}\left[-\frac{h(c + d x)}{d g - c h}\right] \operatorname{Log}[g + h x]}{h} + \\
& \frac{B \operatorname{Log}[e(a + b x)^n (c + d x)^{-n}] \operatorname{Log}[g + h x]}{h} - \frac{B n \operatorname{PolyLog}\left[2, \frac{b(g + h x)}{b g - a h}\right]}{h} + \frac{B n \operatorname{PolyLog}\left[2, \frac{d(g + h x)}{d g - c h}\right]}{h}
\end{aligned}$$

**Problem 299: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{(g + h x)^2} dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{b B n \operatorname{Log}[a + b x]}{h (b g - a h)} - \frac{B d n \operatorname{Log}[c + d x]}{h (d g - c h)} - \frac{A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{h (g + h x)} + \frac{B (b c - a d) n \operatorname{Log}[g + h x]}{(b g - a h) (d g - c h)}$$

Result (type 3, 132 leaves, 6 steps):

$$-\frac{A}{h (g + h x)} - \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{(b g - a h) (d g - c h)} + \frac{B (a + b x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{(b g - a h) (g + h x)} + \frac{B (b c - a d) n \operatorname{Log}[g + h x]}{(b g - a h) (d g - c h)}$$

**Problem 300: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{(g + h x)^3} dx$$

Optimal (type 3, 191 leaves, 3 steps):

$$-\frac{B (b c - a d) n}{2 (b g - a h) (d g - c h) (g + h x)} + \frac{b^2 B n \operatorname{Log}[a + b x]}{2 h (b g - a h)^2} - \frac{B d^2 n \operatorname{Log}[c + d x]}{2 h (d g - c h)^2} - \frac{A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{2 h (g + h x)^2} + \frac{B (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}[g + h x]}{2 (b g - a h)^2 (d g - c h)^2}$$

Result (type 3, 203 leaves, 5 steps):

$$-\frac{A}{2 h (g + h x)^2} - \frac{B (b c - a d) n}{2 (b g - a h) (d g - c h) (g + h x)} + \frac{b^2 B n \operatorname{Log}[a + b x]}{2 h (b g - a h)^2} - \frac{B d^2 n \operatorname{Log}[c + d x]}{2 h (d g - c h)^2} - \frac{B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{2 h (g + h x)^2} + \frac{B (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}[g + h x]}{2 (b g - a h)^2 (d g - c h)^2}$$

**Problem 301: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{(g + h x)^4} dx$$

Optimal (type 3, 284 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) n}{6 (b g - a h) (d g - c h) (g + h x)^2} - \frac{B (b c - a d) (2 b d g - b c h - a d h) n}{3 (b g - a h)^2 (d g - c h)^2 (g + h x)} + \\
& \frac{b^3 B n \operatorname{Log}[a + b x]}{3 h (b g - a h)^3} - \frac{B d^3 n \operatorname{Log}[c + d x]}{3 h (d g - c h)^3} - \frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 h (g + h x)^3} + \\
& \frac{(B (b c - a d) (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) n \operatorname{Log}[g + h x])}{(3 (b g - a h)^3 (d g - c h)^3)} \Big/
\end{aligned}$$

Result (type 3, 296 leaves, 5 steps):

$$\begin{aligned}
& - \frac{A}{3 h (g + h x)^3} - \frac{B (b c - a d) n}{6 (b g - a h) (d g - c h) (g + h x)^2} - \frac{B (b c - a d) (2 b d g - b c h - a d h) n}{3 (b g - a h)^2 (d g - c h)^2 (g + h x)} + \\
& \frac{b^3 B n \operatorname{Log}[a + b x]}{3 h (b g - a h)^3} - \frac{B d^3 n \operatorname{Log}[c + d x]}{3 h (d g - c h)^3} - \frac{B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 h (g + h x)^3} + \\
& \frac{(B (b c - a d) (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) n \operatorname{Log}[g + h x])}{(3 (b g - a h)^3 (d g - c h)^3)} \Big/
\end{aligned}$$

**Problem 302: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(g + h x)^5} dx$$

Optimal (type 3, 389 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) n}{12 (b g - a h) (d g - c h) (g + h x)^3} - \frac{B (b c - a d) (2 b d g - b c h - a d h) n}{8 (b g - a h)^2 (d g - c h)^2 (g + h x)^2} - \\
& \frac{(B (b c - a d) (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) n)}{(4 (b g - a h)^3 (d g - c h)^3 (g + h x))} + \frac{b^4 B n \operatorname{Log}[a + b x]}{4 h (b g - a h)^4} - \frac{B d^4 n \operatorname{Log}[c + d x]}{4 h (d g - c h)^4} - \\
& \frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{4 h (g + h x)^4} - \frac{(B (b c - a d) (2 b d g - b c h - a d h) (2 a b d^2 g h - a^2 d^2 h^2 - b^2 (2 d^2 g^2 - 2 c d g h + c^2 h^2)) n \operatorname{Log}[g + h x])}{(4 (b g - a h)^4 (d g - c h)^4)}
\end{aligned}$$

Result (type 3, 401 leaves, 5 steps):



$$\begin{aligned}
& - \frac{A}{4h(g+hx)^4} - \frac{B(bc-ad)n}{12(bg-ah)(dg-ch)(g+hx)^3} - \frac{B(bc-ad)(2bdg-bch-adh)n}{8(bg-ah)^2(dg-ch)^2(g+hx)^2} - \\
& \frac{(B(bc-ad)(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2))n)}{(4(bg-ah)^3(dg-ch)^3(g+hx))} + \frac{b^4Bn\log[a+bx]}{4h(bg-ah)^4} - \frac{Bd^4n\log[c+dx]}{4h(dg-ch)^4} - \\
& \frac{B\log[e(a+bx)^n(c+dx)^{-n}]}{4h(g+hx)^4} - (B(bc-ad)(2bdg-bch-adh) \\
& (2abd^2gh-a^2d^2h^2-b^2(2d^2g^2-2cdgh+c^2h^2))n\log[g+hx]) / (4(bg-ah)^4(dg-ch)^4)
\end{aligned}$$

**Problem 303: Result valid but suboptimal antiderivative.**

$$\int (g+hx)^2 (A+B\log[e(a+bx)^n(c+dx)^{-n}])^2 dx$$

Optimal (type 4, 570 leaves, 13 steps):

$$\begin{aligned}
& \frac{B^2(bc-ad)^2h^2n^2x}{3b^2d^2} + \frac{B^2(bc-ad)^3h^2n^2\log\left[\frac{a+bx}{c+dx}\right]}{3b^3d^3} + \frac{B^2(bc-ad)^3h^2n^2\log[c+dx]}{3b^3d^3} + \\
& \frac{2B^2(bc-ad)^2h(3bdg-2bch-adh)n^2\log[c+dx]}{3b^3d^3} - \frac{1}{3b^3d^2} \\
& 2B(bc-ad)h(3bdg-2bch-adh)n(a+bx)(A+B\log[e(a+bx)^n(c+dx)^{-n}]) - \\
& \frac{B(bc-ad)h^2n(c+dx)^2(A+B\log[e(a+bx)^n(c+dx)^{-n}])}{3bd^3} + \frac{1}{3b^3d^3} \\
& 2B(bc-ad)(a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2))n\log\left[\frac{bc-ad}{b(c+dx)}\right] \\
& (A+B\log[e(a+bx)^n(c+dx)^{-n}]) - \frac{(bg-ah)^3(A+B\log[e(a+bx)^n(c+dx)^{-n}])^2}{3b^3h} + \\
& \frac{(g+hx)^3(A+B\log[e(a+bx)^n(c+dx)^{-n}])^2}{3h} + \frac{1}{3b^3d^3} 2B^2(bc-ad) \\
& (a^2d^2h^2-abdh(3dg-ch)+b^2(3d^2g^2-3cdgh+c^2h^2))n^2\text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]
\end{aligned}$$

Result (type 4, 697 leaves, 23 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d) h (3 b d g - b c h - a d h) n x}{3 b^2 d^2} + \frac{B^2 (b c - a d)^2 h^2 n^2 x}{3 b^2 d^2} - \\
& \frac{AB(b c - a d) h^2 n x^2}{3 b d} + \frac{A^2 (g + h x)^3}{3 h} - \frac{2AB(b g - a h)^3 n \operatorname{Log}[a + b x]}{3 b^3 h} + \\
& \frac{a^2 B^2 (b c - a d) h^2 n^2 \operatorname{Log}[a + b x]}{3 b^3 d} + \frac{2AB(d g - c h)^3 n \operatorname{Log}[c + d x]}{3 d^3 h} - \\
& \frac{B^2 c^2 (b c - a d) h^2 n^2 \operatorname{Log}[c + d x]}{3 b d^3} + \frac{2B^2 (b c - a d)^2 h (3 b d g - b c h - a d h) n^2 \operatorname{Log}[c + d x]}{3 b^3 d^3} - \\
& \frac{B^2 (b c - a d) h^2 n x^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b d} - \frac{1}{3 b^3 d^2} \\
& 2B^2 (b c - a d) h (3 b d g - b c h - a d h) n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] + \\
& \frac{2AB(g + h x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 h} + \\
& \frac{2B^2 (b g - a h)^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 b^3 h} - \\
& \frac{2B^2 (d g - c h)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{3 d^3 h} + \\
& \frac{B^2 (g + h x)^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{3 h} - \\
& \frac{2B^2 (d g - c h)^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 d^3 h} - \frac{2B^2 (b g - a h)^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{3 b^3 h}
\end{aligned}$$

**Problem 304: Result valid but suboptimal antiderivative.**

$$\int (g + h x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 dx$$

Optimal (type 4, 294 leaves, 10 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 h n^2 \operatorname{Log}[c + d x]}{b^2 d^2} - \frac{B (b c - a d) h n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])}{b^2 d} + \frac{1}{b^2 d^2} \\
& B (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) - \\
& \frac{(b g - a h)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 b^2 h} + \frac{(g + h x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 h} + \\
& \frac{B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^2 d^2}
\end{aligned}$$

Result (type 4, 449 leaves, 20 steps):

$$\begin{aligned}
& - \frac{AB (bc - ad) h n x}{bd} + \frac{A^2 (g + hx)^2}{2h} - \frac{AB (bg - ah)^2 n \operatorname{Log}[a + bx]}{b^2 h} + \\
& \frac{AB (dg - ch)^2 n \operatorname{Log}[c + dx]}{d^2 h} + \frac{B^2 (bc - ad)^2 h n^2 \operatorname{Log}[c + dx]}{b^2 d^2} - \\
& \frac{B^2 (bc - ad) h n (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{b^2 d} + \frac{AB (g + hx)^2 \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{h} + \\
& \frac{B^2 (bg - ah)^2 n \operatorname{Log}\left[-\frac{bc - ad}{d(a + bx)}\right] \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{b^2 h} - \\
& \frac{B^2 (dg - ch)^2 n \operatorname{Log}\left[\frac{bc - ad}{b(c + dx)}\right] \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{d^2 h} + \\
& \frac{B^2 (g + hx)^2 \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]^2}{2h} - \\
& \frac{B^2 (dg - ch)^2 n^2 \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right]}{d^2 h} - \frac{B^2 (bg - ah)^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{bc - ad}{d(a + bx)}\right]}{b^2 h}
\end{aligned}$$

**Problem 305: Result valid but suboptimal antiderivative.**

$$\int (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2 dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$\begin{aligned}
& \frac{2B (bc - ad) n \operatorname{Log}\left[\frac{bc - ad}{b(c + dx)}\right] (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])}{bd} + \\
& \frac{(a + bx) (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{b} + \frac{2B^2 (bc - ad) n^2 \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right]}{bd}
\end{aligned}$$

Result (type 4, 195 leaves, 10 steps):

$$\begin{aligned}
& A^2 x - \frac{2AB (bc - ad) n \operatorname{Log}[c + dx]}{bd} + \frac{2AB (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{b} + \\
& \frac{2B^2 (bc - ad) n \operatorname{Log}\left[\frac{bc - ad}{b(c + dx)}\right] \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{bd} + \\
& \frac{B^2 (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]^2}{b} + \frac{2B^2 (bc - ad) n^2 \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right]}{bd}
\end{aligned}$$

**Problem 306: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{g + hx} dx$$

Optimal (type 4, 301 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^2}{h} + \\
& \frac{\left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^2 \text{Log}\left[1 - \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h} - \\
& \frac{2 B n \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right) \text{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{h} + \\
& \frac{2 B n \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right) \text{PolyLog}\left[2, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h} + \\
& \frac{2 B^2 n^2 \text{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{h} - \frac{2 B^2 n^2 \text{PolyLog}\left[3, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h}
\end{aligned}$$

Result (type 4, 473 leaves, 16 steps):

$$\begin{aligned}
& - \frac{B^2 \text{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{h} + \\
& \frac{A^2 \text{Log}[g + h x]}{h} - \frac{2 A B n \text{Log}\left[-\frac{h (a + b x)}{b g - a h}\right] \text{Log}[g + h x]}{h} + \\
& \frac{2 A B n \text{Log}\left[-\frac{h (c + d x)}{d g - c h}\right] \text{Log}[g + h x]}{h} + \frac{2 A B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \text{Log}[g + h x]}{h} + \\
& \frac{B^2 \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 \text{Log}\left[\frac{(b c - a d) (g + h x)}{(b g - a h) (c + d x)}\right]}{h} - \frac{2 A B n \text{PolyLog}\left[2, \frac{b (g + h x)}{b g - a h}\right]}{h} + \\
& \frac{2 A B n \text{PolyLog}\left[2, \frac{d (g + h x)}{d g - c h}\right]}{h} - \frac{2 B^2 n \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \text{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right]}{h} + \\
& \frac{2 B^2 n \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \text{PolyLog}\left[2, 1 - \frac{(b c - a d) (g + h x)}{(b g - a h) (c + d x)}\right]}{h} + \\
& \frac{2 B^2 n^2 \text{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{h} - \frac{2 B^2 n^2 \text{PolyLog}\left[3, 1 - \frac{(b c - a d) (g + h x)}{(b g - a h) (c + d x)}\right]}{h}
\end{aligned}$$

Problem 307: Result valid but suboptimal antiderivative.

$$\int \frac{\left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^2}{(g + h x)^2} dx$$

Optimal (type 4, 208 leaves, 5 steps):

$$\frac{(a+bx) (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^2}{(bg-ah) (g+hx)} +$$

$$\left( 2B (bc-ad) n (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]) \operatorname{Log}\left[1 - \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right] \right) /$$

$$((bg-ah) (dg-ch)) + \frac{2B^2 (bc-ad) n^2 \operatorname{PolyLog}\left[2, \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)}$$

Result (type 4, 343 leaves, 10 steps):

$$-\frac{A^2}{h (g+hx)} - \frac{2AB (bc-ad) n \operatorname{Log}[c+dx]}{(bg-ah) (dg-ch)} + \frac{2AB (a+bx) \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]}{(bg-ah) (g+hx)} +$$

$$\frac{B^2 (a+bx) \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]^2}{(bg-ah) (g+hx)} + \frac{2AB (bc-ad) n \operatorname{Log}[g+hx]}{(bg-ah) (dg-ch)} +$$

$$\frac{2B^2 (bc-ad) n \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}] \operatorname{Log}\left[\frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} +$$

$$\frac{2B^2 (bc-ad) n^2 \operatorname{PolyLog}\left[2, 1 - \frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)}$$

**Problem 308: Result valid but suboptimal antiderivative.**

$$\int \frac{(A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^2}{(g+hx)^3} dx$$

Optimal (type 4, 393 leaves, 10 steps):

$$\frac{B (bc-ad) h n (a+bx) (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])}{(bg-ah)^2 (dg-ch) (g+hx)} +$$

$$\frac{b^2 (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^2}{2h (bg-ah)^2} -$$

$$\frac{(A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^2}{2h (g+hx)^2} + \frac{B^2 (bc-ad)^2 h n^2 \operatorname{Log}\left[\frac{g+hx}{c+dx}\right]}{(bg-ah)^2 (dg-ch)^2} +$$

$$\left( B (bc-ad) (2bdg-bch-adh) n (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]) \right.$$

$$\left. \operatorname{Log}\left[1 - \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right] \right) / ((bg-ah)^2 (dg-ch)^2) +$$

$$\frac{B^2 (bc-ad) (2bdg-bch-adh) n^2 \operatorname{PolyLog}\left[2, \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right]}{(bg-ah)^2 (dg-ch)^2}$$

Result (type 4, 968 leaves, 29 steps):

$$\begin{aligned}
& - \frac{A^2}{2 h (g + h x)^2} - \frac{A B (b c - a d) n}{(b g - a h) (d g - c h) (g + h x)} + \\
& \frac{A b^2 B n \operatorname{Log}[a + b x]}{h (b g - a h)^2} - \frac{A B d^2 n \operatorname{Log}[c + d x]}{h (d g - c h)^2} - \frac{B^2 (b c - a d)^2 h n^2 \operatorname{Log}[c + d x]}{(b g - a h)^2 (d g - c h)^2} - \\
& \frac{A B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{h (g + h x)^2} + \frac{B^2 (b c - a d) h n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b g - a h)^2 (d g - c h) (g + h x)} - \\
& \frac{b^2 B^2 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{h (b g - a h)^2} + \\
& \frac{B^2 d^2 n \operatorname{Log}\left[-\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{h (d g - c h)^2} - \frac{B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 h (g + h x)^2} + \\
& \frac{A B (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}[g + h x]}{(b g - a h)^2 (d g - c h)^2} + \frac{B^2 (b c - a d)^2 h n^2 \operatorname{Log}[g + h x]}{(b g - a h)^2 (d g - c h)^2} - \\
& \frac{B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{Log}\left[-\frac{h (a + b x)}{b g - a h}\right] \operatorname{Log}[g + h x]}{(b g - a h)^2 (d g - c h)^2} + \\
& \frac{B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{Log}\left[-\frac{h (c + d x)}{d g - c h}\right] \operatorname{Log}[g + h x]}{(b g - a h)^2 (d g - c h)^2} + \\
& (B^2 (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}[g + h x]) / \\
& \left( (b g - a h)^2 (d g - c h)^2 \right) + \frac{B^2 d^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{h (d g - c h)^2} - \\
& \frac{B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{PolyLog}\left[2, \frac{b (g + h x)}{b g - a h}\right]}{(b g - a h)^2 (d g - c h)^2} + \\
& \frac{B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{PolyLog}\left[2, \frac{d (g + h x)}{d g - c h}\right]}{(b g - a h)^2 (d g - c h)^2} + \frac{b^2 B^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{h (b g - a h)^2}
\end{aligned}$$

**Problem 309: Result valid but suboptimal antiderivative.**

$$\int (g + h x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 dx$$

Optimal (type 4, 875 leaves, 19 steps):

$$\begin{aligned}
& - \frac{B^3 (bc - ad)^3 h^2 n^3 \operatorname{Log}[c + dx]}{b^3 d^3} + \\
& \frac{B^2 (bc - ad)^2 h^2 n^2 (a + bx) (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])}{b^3 d^2} - \frac{1}{b^3 d^3} \\
& 2 B^2 (bc - ad)^2 h (3 b d g - 2 b c h - a d h) n^2 \operatorname{Log}\left[\frac{bc - ad}{b(c + dx)}\right] (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]) - \\
& \frac{1}{b^3 d^2} B (bc - ad) h (3 b d g - 2 b c h - a d h) n (a + bx) (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2 - \\
& \frac{B (bc - ad) h^2 n (c + dx)^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2}{2 b d^3} + \frac{1}{b^3 d^3} \\
& B (bc - ad) (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) \\
& n \operatorname{Log}\left[\frac{bc - ad}{b(c + dx)}\right] (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^2 - \\
& \frac{(bg - ah)^3 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^3}{3 b^3 h} + \frac{(g + hx)^3 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}])^3}{3 h} - \\
& \frac{1}{b^3 d^3} B^2 (bc - ad)^3 h^2 n^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]) \operatorname{Log}\left[1 - \frac{b(c + dx)}{d(a + bx)}\right] - \\
& \frac{2 B^3 (bc - ad)^2 h (3 b d g - 2 b c h - a d h) n^3 \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right]}{b^3 d^3} + \frac{1}{b^3 d^3} \\
& 2 B^2 (bc - ad) (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) \\
& n^2 (A + B \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d(a + bx)}{b(c + dx)}\right] + \\
& \frac{B^3 (bc - ad)^3 h^2 n^3 \operatorname{PolyLog}\left[2, \frac{b(c + dx)}{d(a + bx)}\right]}{b^3 d^3} - \frac{1}{b^3 d^3} 2 B^3 (bc - ad) \\
& (a^2 d^2 h^2 - a b d h (3 d g - c h) + b^2 (3 d^2 g^2 - 3 c d g h + c^2 h^2)) n^3 \operatorname{PolyLog}\left[3, \frac{d(a + bx)}{b(c + dx)}\right]
\end{aligned}$$

Result(type 4, 1640 leaves, 53 steps):

$$\begin{aligned}
& - \frac{A^2 B (bc - ad) h (3 b d g - b c h - a d h) n x}{b^2 d^2} + \frac{A B^2 (bc - ad)^2 h^2 n^2 x}{b^2 d^2} - \\
& \frac{A^2 B (bc - ad) h^2 n x^2}{2 b d} + \frac{A^3 (g + hx)^3}{3 h} - \frac{A^2 B (bg - ah)^3 n \operatorname{Log}[a + bx]}{b^3 h} + \\
& \frac{a^2 A B^2 (bc - ad) h^2 n^2 \operatorname{Log}[a + bx]}{b^3 d} + \frac{A^2 B (dg - ch)^3 n \operatorname{Log}[c + dx]}{d^3 h} - \\
& \frac{A B^2 c^2 (bc - ad) h^2 n^2 \operatorname{Log}[c + dx]}{b d^3} + \frac{2 A B^2 (bc - ad)^2 h (3 b d g - b c h - a d h) n^2 \operatorname{Log}[c + dx]}{b^3 d^3} - \\
& \frac{B^3 (bc - ad)^3 h^2 n^3 \operatorname{Log}[c + dx]}{b^3 d^3} - \frac{A B^2 (bc - ad) h^2 n x^2 \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}]}{b d} - \\
& \frac{1}{b^3 d^2} 2 A B^2 (bc - ad) h (3 b d g - b c h - a d h) n (a + bx) \operatorname{Log}[e (a + bx)^n (c + dx)^{-n}] +
\end{aligned}$$

$$\begin{aligned}
& \frac{B^3 (b c - a d)^2 h^2 n^2 (a + b x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b^3 d^2} + \\
& \frac{A^2 B (g + h x)^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{h} + \\
& \frac{2 A B^2 (b g - a h)^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b^3 h} - \\
& \frac{a^2 B^3 (b c - a d) h^2 n^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b^3 d} - \\
& \frac{2 A B^2 (d g - c h)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{d^3 h} + \\
& \frac{B^3 c^2 (b c - a d) h^2 n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]}{b d^3} - \frac{1}{b^3 d^3} \\
& 2 B^3 (b c - a d)^2 h (3 b d g - b c h - a d h) n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] - \\
& \frac{B^3 (b c - a d) h^2 n x^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{2 b d} - \frac{1}{b^3 d^2} \\
& B^3 (b c - a d) h (3 b d g - b c h - a d h) n (a + b x) \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 + \\
& \frac{A B^2 (g + h x)^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{h} + \\
& \frac{B^3 (b g - a h)^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{b^3 h} - \\
& \frac{B^3 (d g - c h)^3 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2}{d^3 h} + \\
& \frac{B^3 (g + h x)^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3}{3 h} - \frac{2 A B^2 (d g - c h)^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 h} + \\
& \frac{B^3 c^2 (b c - a d) h^2 n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d^3} - \\
& \frac{2 B^3 (b c - a d)^2 h (3 b d g - b c h - a d h) n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^3 d^3} - \\
& \frac{2 A B^2 (b g - a h)^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 h} + \frac{a^2 B^3 (b c - a d) h^2 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 d} - \\
& \frac{1}{b^3 h} 2 B^3 (b g - a h)^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right] - \\
& \frac{1}{d^3 h} 2 B^3 (d g - c h)^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right] - \\
& \frac{2 B^3 (b g - a h)^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 h} + \frac{2 B^3 (d g - c h)^3 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{d^3 h}
\end{aligned}$$



### Problem 310: Result valid but suboptimal antiderivative.

$$\int (g + h x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 dx$$

Optimal (type 4, 466 leaves, 13 steps):

$$\begin{aligned} & -\frac{1}{b^2 d^2} 3 B^2 (b c - a d)^2 h n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) - \\ & \frac{3 B (b c - a d) h n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 b^2 d} + \frac{1}{2 b^2 d^2} \\ & 3 B (b c - a d) (2 b d g - b c h - a d h) n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 - \\ & \frac{(b g - a h)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 b^2 h} + \\ & \frac{(g + h x)^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 h} - \frac{3 B^3 (b c - a d)^2 h n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^2 d^2} + \frac{1}{b^2 d^2} \\ & 3 B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right] - \\ & \frac{3 B^3 (b c - a d) (2 b d g - b c h - a d h) n^3 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{b^2 d^2} \end{aligned}$$

Result (type 4, 1030 leaves, 35 steps):

$$\begin{aligned}
& - \frac{3 A^2 B (b c - a d) h n x}{2 b d} + \frac{A^3 (g + h x)^2}{2 h} - \frac{3 A^2 B (b g - a h)^2 n \operatorname{Log}[a + b x]}{2 b^2 h} + \\
& \frac{3 A^2 B (d g - c h)^2 n \operatorname{Log}[c + d x]}{2 d^2 h} + \frac{3 A B^2 (b c - a d)^2 h n^2 \operatorname{Log}[c + d x]}{b^2 d^2} - \\
& \frac{3 A B^2 (b c - a d) h n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b^2 d} + \\
& \frac{3 A^2 B (g + h x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 h} + \\
& \frac{3 A B^2 (b g - a h)^2 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b^2 h} - \\
& \frac{3 A B^2 (d g - c h)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{d^2 h} - \\
& \frac{3 B^3 (b c - a d)^2 h n^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b^2 d^2} - \\
& \frac{3 B^3 (b c - a d) h n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b^2 d} + \\
& \frac{3 A B^2 (g + h x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 h} + \\
& \frac{3 B^3 (b g - a h)^2 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 b^2 h} - \\
& \frac{3 B^3 (d g - c h)^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 d^2 h} + \\
& \frac{B^3 (g + h x)^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{2 h} - \frac{3 A B^2 (d g - c h)^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^2 h} - \\
& \frac{3 B^3 (b c - a d)^2 h n^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^2 d^2} - \frac{3 A B^2 (b g - a h)^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 h} - \\
& \frac{1}{b^2 h} 3 B^3 (b g - a h)^2 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right] - \\
& \frac{1}{d^2 h} 3 B^3 (d g - c h)^2 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right] - \\
& \frac{3 B^3 (b g - a h)^2 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 h} + \frac{3 B^3 (d g - c h)^2 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{d^2 h}
\end{aligned}$$

**Problem 311: Result valid but suboptimal antiderivative.**

$$\int (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 dx$$

Optimal (type 4, 203 leaves, 6 steps):

$$\begin{aligned}
& \frac{3 B (b c - a d) n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{b d} + \\
& \frac{(a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{b} + \frac{1}{b d} \\
& 6 B^2 (b c - a d) n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right] - \\
& \frac{6 B^3 (b c - a d) n^3 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{b d}
\end{aligned}$$

Result (type 4, 408 leaves, 14 steps):

$$\begin{aligned}
& A^3 x - \frac{3 A^2 B (b c - a d) n \operatorname{Log}[c + d x]}{b d} + \frac{3 A^2 B (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b} + \\
& \frac{6 A B^2 (b c - a d) n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{b d} + \\
& \frac{3 A B^2 (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b} + \\
& \frac{3 B^3 (b c - a d) n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{b d} + \\
& \frac{B^3 (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3}{b} + \frac{6 A B^2 (b c - a d) n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b d} + \\
& \frac{6 B^3 (b c - a d) n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b d} - \\
& \frac{6 B^3 (b c - a d) n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{b d}
\end{aligned}$$

**Problem 312: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{g + h x} dx$$

Optimal (type 4, 425 leaves, 12 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^3}{h} + \\
& \frac{\left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^3 \text{Log}\left[1 - \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h} - \\
& \frac{3 B n \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^2 \text{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{h} + \\
& \frac{3 B n \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right)^2 \text{PolyLog}\left[2, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h} + \\
& \frac{6 B^2 n^2 \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right) \text{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{h} - \\
& \frac{6 B^2 n^2 \left(A + B \text{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]\right) \text{PolyLog}\left[3, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h} - \\
& \frac{6 B^3 n^3 \text{PolyLog}\left[4, \frac{d (a + b x)}{b (c + d x)}\right]}{h} + \frac{6 B^3 n^3 \text{PolyLog}\left[4, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{h}
\end{aligned}$$

Result (type 4, 921 leaves, 25 steps):

$$\begin{aligned}
& - \frac{3 A B^2 \operatorname{Log}\left[\frac{b c-a d}{b(c+d x)}\right] \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2}{h} - \frac{B^3 \operatorname{Log}\left[\frac{b c-a d}{b(c+d x)}\right] \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^3}{h} + \\
& \frac{A^3 \operatorname{Log}[g+h x]}{h} - \frac{3 A^2 B n \operatorname{Log}\left[-\frac{h(a+b x)}{b g-a h}\right] \operatorname{Log}[g+h x]}{h} + \\
& \frac{3 A^2 B n \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right] \operatorname{Log}[g+h x]}{h} + \frac{3 A^2 B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{Log}[g+h x]}{h} + \\
& \frac{3 A B^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2 \operatorname{Log}\left[\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} + \\
& \frac{B^3 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^3 \operatorname{Log}\left[\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} - \frac{3 A^2 B n \operatorname{PolyLog}\left[2, \frac{b(g+h x)}{b g-a h}\right]}{h} + \\
& \frac{3 A^2 B n \operatorname{PolyLog}\left[2, \frac{d(g+h x)}{d g-c h}\right]}{h} - \frac{6 A B^2 n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1-\frac{b c-a d}{b(c+d x)}\right]}{h} - \\
& \frac{3 B^3 n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2 \operatorname{PolyLog}\left[2, 1-\frac{b c-a d}{b(c+d x)}\right]}{h} + \\
& \frac{6 A B^2 n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} + \\
& \frac{3 B^3 n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2 \operatorname{PolyLog}\left[2, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} + \\
& \frac{6 A B^2 n^2 \operatorname{PolyLog}\left[3, 1-\frac{b c-a d}{b(c+d x)}\right]}{h} + \frac{6 B^3 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[3, 1-\frac{b c-a d}{b(c+d x)}\right]}{h} - \\
& \frac{6 A B^2 n^2 \operatorname{PolyLog}\left[3, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} - \\
& \frac{6 B^3 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[3, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h} - \\
& \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, 1-\frac{b c-a d}{b(c+d x)}\right]}{h} + \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{h}
\end{aligned}$$

**Problem 313: Result valid but suboptimal antiderivative.**

$$\int \frac{(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right])^3}{(g+h x)^2} dx$$

Optimal (type 4, 302 leaves, 6 steps):

$$\begin{aligned}
& \frac{(a+bx) (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^3}{(bg-ah) (g+hx)} + \\
& \left( \frac{3B (bc-ad) n (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^2 \operatorname{Log}\left[1 - \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} + \right. \\
& \left. \frac{6B^2 (bc-ad) n^2 (A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]) \operatorname{PolyLog}\left[2, \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} \right) / \\
& ((bg-ah) (dg-ch)) - \frac{6B^3 (bc-ad) n^3 \operatorname{PolyLog}\left[3, \frac{(dg-ch) (a+bx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)}
\end{aligned}$$

Result (type 4, 650 leaves, 14 steps):

$$\begin{aligned}
& -\frac{A^3}{h (g+hx)} - \frac{3A^2 B (bc-ad) n \operatorname{Log}[c+dx]}{(bg-ah) (dg-ch)} + \frac{3A^2 B (a+bx) \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]}{(bg-ah) (g+hx)} + \\
& \frac{3AB^2 (a+bx) \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]^2}{(bg-ah) (g+hx)} + \frac{B^3 (a+bx) \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]^3}{(bg-ah) (g+hx)} + \\
& \frac{3A^2 B (bc-ad) n \operatorname{Log}[g+hx]}{(bg-ah) (dg-ch)} + \frac{6AB^2 (bc-ad) n \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}] \operatorname{Log}\left[\frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} + \\
& \frac{3B^3 (bc-ad) n \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}]^2 \operatorname{Log}\left[\frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} + \\
& \frac{6AB^2 (bc-ad) n^2 \operatorname{PolyLog}\left[2, 1 - \frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} + \\
& \left( \frac{6B^3 (bc-ad) n^2 \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}] \operatorname{PolyLog}\left[2, 1 - \frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)} \right) / \\
& ((bg-ah) (dg-ch)) - \frac{6B^3 (bc-ad) n^3 \operatorname{PolyLog}\left[3, 1 - \frac{(bc-ad) (g+hx)}{(bg-ah) (c+dx)}\right]}{(bg-ah) (dg-ch)}
\end{aligned}$$

**Problem 314: Result valid but suboptimal antiderivative.**

$$\int \frac{(A+B \operatorname{Log}[e (a+bx)^n (c+dx)^{-n}])^3}{(g+hx)^3} dx$$

Optimal (type 4, 629 leaves, 13 steps):

$$\begin{aligned}
& \frac{3 B (b c - a d) h n (a + b x) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{2 (b g - a h)^2 (d g - c h) (g + h x)} + \\
& \frac{b^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 h (b g - a h)^2} - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{2 h (g + h x)^2} + \\
& \left( 3 B^2 (b c - a d)^2 h n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{Log}\left[1 - \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right] \right) / \\
& \left( (b g - a h)^2 (d g - c h)^2 \right) + \left( 3 B (b c - a d) (2 b d g - b c h - a d h) n \right. \\
& \left. (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{Log}\left[1 - \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right] \right) / \\
& \left( 2 (b g - a h)^2 (d g - c h)^2 \right) + \frac{3 B^3 (b c - a d)^2 h n^3 \operatorname{PolyLog}\left[2, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{(b g - a h)^2 (d g - c h)^2} + \\
& \left( 3 B^2 (b c - a d) (2 b d g - b c h - a d h) n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right] \right) / \left( (b g - a h)^2 (d g - c h)^2 \right) - \\
& \frac{3 B^3 (b c - a d) (2 b d g - b c h - a d h) n^3 \operatorname{PolyLog}\left[3, \frac{(d g - c h) (a + b x)}{(b g - a h) (c + d x)}\right]}{(b g - a h)^2 (d g - c h)^2}
\end{aligned}$$

Result (type 4, 2207 leaves, 49 steps):

$$\begin{aligned}
& - \frac{A^3}{2 h (g + h x)^2} - \frac{3 A^2 B (b c - a d) n}{2 (b g - a h) (d g - c h) (g + h x)} + \frac{3 A^2 b^2 B n \operatorname{Log}[a + b x]}{2 h (b g - a h)^2} - \\
& \frac{3 A^2 B d^2 n \operatorname{Log}[c + d x]}{2 h (d g - c h)^2} - \frac{3 A B^2 (b c - a d)^2 h n^2 \operatorname{Log}[c + d x]}{(b g - a h)^2 (d g - c h)^2} - \\
& \frac{3 A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{2 h (g + h x)^2} + \frac{3 A B^2 (b c - a d) h n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(b g - a h)^2 (d g - c h) (g + h x)} - \\
& \frac{3 A b^2 B^2 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{h (b g - a h)^2} + \\
& \frac{3 A B^2 d^2 n \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{h (d g - c h)^2} - \frac{3 A B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 h (g + h x)^2} + \\
& \frac{3 B^3 (b c - a d) h n (a + b x) \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 (b g - a h)^2 (d g - c h) (g + h x)} - \\
& \frac{3 b^2 B^3 n \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2}{2 h (b g - a h)^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 B^3 d^2 n \operatorname{Log}\left[\frac{b c-a d}{b(c+d x)}\right] \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2}{2 h(d g-c h)^2}- \\
& \left(3 B^3(b c-a d)(2 b d g-b c h-a d h) n \operatorname{Log}\left[\frac{b c-a d}{b(c+d x)}\right] \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2\right) / \\
& \left(2(b g-a h)^2(d g-c h)^2\right)-\frac{B^3 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^3}{2 h(g+h x)^2}+ \\
& \frac{3 A^2 B(b c-a d)(2 b d g-b c h-a d h) n \operatorname{Log}[g+h x]}{2(b g-a h)^2(d g-c h)^2}+\frac{3 A B^2(b c-a d)^2 h n^2 \operatorname{Log}[g+h x]}{(b g-a h)^2(d g-c h)^2}- \\
& \frac{3 A B^2(b c-a d)(2 b d g-b c h-a d h) n^2 \operatorname{Log}\left[-\frac{h(a+b x)}{b g-a h}\right] \operatorname{Log}[g+h x]}{(b g-a h)^2(d g-c h)^2}+ \\
& \frac{3 A B^2(b c-a d)(2 b d g-b c h-a d h) n^2 \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right] \operatorname{Log}[g+h x]}{(b g-a h)^2(d g-c h)^2}+ \\
& \left(3 A B^2(b c-a d)(2 b d g-b c h-a d h) n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{Log}[g+h x]\right) / \\
& \left((b g-a h)^2(d g-c h)^2\right)+\frac{3 B^3(b c-a d)^2 h n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{Log}\left[\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{(b g-a h)^2(d g-c h)^2}+ \\
& \left(3 B^3(b c-a d)(2 b d g-b c h-a d h) n \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]^2 \operatorname{Log}\left[\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]\right) / \\
& \left(2(b g-a h)^2(d g-c h)^2\right)+\frac{3 A B^2 d^2 n^2 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{h(d g-c h)^2}- \\
& \frac{3 A B^2(b c-a d)(2 b d g-b c h-a d h) n^2 \operatorname{PolyLog}\left[2, \frac{b(g+h x)}{b g-a h}\right]}{(b g-a h)^2(d g-c h)^2}+ \\
& \frac{3 A B^2(b c-a d)(2 b d g-b c h-a d h) n^2 \operatorname{PolyLog}\left[2, \frac{d(g+h x)}{d g-c h}\right]}{(b g-a h)^2(d g-c h)^2}+ \\
& \frac{3 A b^2 B^2 n^2 \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{h(b g-a h)^2}+ \\
& \frac{3 b^2 B^3 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{h(b g-a h)^2}+ \\
& \frac{3 B^3 d^2 n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1-\frac{b c-a d}{b(c+d x)}\right]}{h(d g-c h)^2}- \\
& \left(3 B^3(b c-a d)(2 b d g-b c h-a d h) n^2 \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1-\frac{b c-a d}{b(c+d x)}\right]\right) / \\
& \left((b g-a h)^2(d g-c h)^2\right)+\frac{3 B^3(b c-a d)^2 h n^3 \operatorname{PolyLog}\left[2, 1-\frac{(b c-a d)(g+h x)}{(b g-a h)(c+d x)}\right]}{(b g-a h)^2(d g-c h)^2}+
\end{aligned}$$



$$\begin{aligned}
& \left( 3 B^3 (b c - a d) (2 b d g - b c h - a d h) n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \right. \\
& \quad \left. \operatorname{PolyLog}\left[2, 1 - \frac{(b c - a d) (g + h x)}{(b g - a h) (c + d x)}\right] \right) / \left( (b g - a h)^2 (d g - c h)^2 \right) + \\
& \quad \frac{3 b^2 B^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{h (b g - a h)^2} - \frac{3 B^3 d^2 n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{h (d g - c h)^2} + \\
& \quad \frac{3 B^3 (b c - a d) (2 b d g - b c h - a d h) n^3 \operatorname{PolyLog}\left[3, 1 - \frac{b c - a d}{b (c + d x)}\right]}{(b g - a h)^2 (d g - c h)^2} - \\
& \quad \frac{3 B^3 (b c - a d) (2 b d g - b c h - a d h) n^3 \operatorname{PolyLog}\left[3, 1 - \frac{(b c - a d) (g + h x)}{(b g - a h) (c + d x)}\right]}{(b g - a h)^2 (d g - c h)^2}
\end{aligned}$$

Test results for the 263 problems in "3.2.2 (f+g x)^m (h+i x)^q (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

Problem 1: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^3 (c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^4 g^3 i x}{20 b d^3} + \frac{B (b c - a d)^3 g^3 i (a + b x)^2}{40 b^2 d^2} - \\
& \frac{B (b c - a d)^2 g^3 i (a + b x)^3}{60 b^2 d} + \frac{g^3 i (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{5 b} + \\
& \frac{(b c - a d) g^3 i (a + b x)^4 \left( A - B + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{20 b^2} + \frac{B (b c - a d)^5 g^3 i \operatorname{Log}[c + d x]}{20 b^2 d^4}
\end{aligned}$$

Result (type 3, 232 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^4 g^3 i x}{20 b d^3} + \frac{B (b c - a d)^3 g^3 i (a + b x)^2}{40 b^2 d^2} - \frac{B (b c - a d)^2 g^3 i (a + b x)^3}{60 b^2 d} - \\
& \frac{B (b c - a d) g^3 i (a + b x)^4}{20 b^2} + \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 b^2} + \\
& \frac{d g^3 i (a + b x)^5 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{5 b^2} + \frac{B (b c - a d)^5 g^3 i \operatorname{Log}[c + d x]}{20 b^2 d^4}
\end{aligned}$$

### Problem 2: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 (c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 180 leaves, 5 steps):

$$\frac{B (b c - a d)^3 g^2 i x}{12 b d^2} - \frac{B (b c - a d)^2 g^2 i (a + b x)^2}{24 b^2 d} + \frac{g^2 i (a + b x)^3 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b} +$$

$$\frac{(b c - a d) g^2 i (a + b x)^3 \left( A - B + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b^2} - \frac{B (b c - a d)^4 g^2 i \operatorname{Log} [c + d x]}{12 b^2 d^3}$$

Result (type 3, 200 leaves, 10 steps):

$$\frac{B (b c - a d)^3 g^2 i x}{12 b d^2} - \frac{B (b c - a d)^2 g^2 i (a + b x)^2}{24 b^2 d} -$$

$$\frac{B (b c - a d) g^2 i (a + b x)^3}{12 b^2} + \frac{(b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^2} +$$

$$\frac{d g^2 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^2} - \frac{B (b c - a d)^4 g^2 i \operatorname{Log} [c + d x]}{12 b^2 d^3}$$

### Problem 3: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) (c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$- \frac{B (b c - a d)^2 g i x}{6 b d} + \frac{g i (a + b x)^2 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b} +$$

$$\frac{(b c - a d) g i (a + b x)^2 \left( A - B + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b^2} + \frac{B (b c - a d)^3 g i \operatorname{Log} [c + d x]}{6 b^2 d^2}$$

Result (type 3, 294 leaves, 13 steps):

$$a A c g i x - \frac{1}{3} b B \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) d g i x - \frac{B (b c - a d) (b c + a d) g i x}{2 b d} - \frac{1}{6} B (b c - a d) g i x^2 +$$

$$\frac{a^3 B d g i \operatorname{Log} [a + b x]}{3 b^2} - \frac{a^2 B (b c + a d) g i \operatorname{Log} [a + b x]}{2 b^2} + \frac{a B c g i (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{b} +$$

$$\frac{1}{2} (b c + a d) g i x^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) + \frac{1}{3} b d g i x^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) -$$

$$\frac{b B c^3 g i \operatorname{Log} [c + d x]}{3 d^2} - \frac{a B c (b c - a d) g i \operatorname{Log} [c + d x]}{b d} + \frac{B c^2 (b c + a d) g i \operatorname{Log} [c + d x]}{2 d^2}$$

### Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{a g + b g x} dx$$

Optimal (type 4, 133 leaves, 6 steps):

$$\frac{i(c+dx) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b g} - \frac{(bc-ad) i \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \left( A - B + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^2 g} + \frac{B(bc-ad) i \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{b^2 g}$$

Result (type 4, 213 leaves, 14 steps):

$$\frac{A d i x}{b g} - \frac{B(bc-ad) i \operatorname{Log}[a+bx]^2}{2 b^2 g} + \frac{B d i (a+bx) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{b^2 g} + \frac{(bc-ad) i \operatorname{Log}[a+bx] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^2 g} - \frac{B(bc-ad) i \operatorname{Log}[c+dx]}{b^2 g} + \frac{B(bc-ad) i \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^2 g} + \frac{B(bc-ad) i \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^2 g}$$

### Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 142 leaves, 5 steps):

$$-\frac{B i(c+dx)}{b g^2 (a+bx)} - \frac{i(c+dx) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b g^2 (a+bx)} - \frac{d i \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^2 g^2} + \frac{B d i \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^2 g^2}$$

Result (type 4, 221 leaves, 15 steps):

$$-\frac{B(bc-ad) i}{b^2 g^2 (a+bx)} - \frac{B d i \operatorname{Log}[a+bx]}{b^2 g^2} - \frac{B d i \operatorname{Log}[a+bx]^2}{2 b^2 g^2} - \frac{(bc-ad) i \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^2 g^2 (a+bx)} + \frac{d i \operatorname{Log}[a+bx] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^2 g^2} + \frac{B d i \operatorname{Log}[c+dx]}{b^2 g^2} + \frac{B d i \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^2 g^2} + \frac{B d i \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^2 g^2}$$

### Problem 7: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(a g + b g x)^3} dx$$

Optimal (type 3, 85 leaves, 2 steps):

$$-\frac{B i (c + d x)^2}{4 (b c - a d) g^3 (a + b x)^2} - \frac{i (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 (b c - a d) g^3 (a + b x)^2}$$

Result (type 3, 191 leaves, 10 steps):

$$-\frac{B (b c - a d) i}{4 b^2 g^3 (a + b x)^2} - \frac{B d i}{2 b^2 g^3 (a + b x)} - \frac{B d^2 i \operatorname{Log} [a + b x]}{2 b^2 (b c - a d) g^3} - \frac{(b c - a d) i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^2 g^3 (a + b x)^2} - \frac{d i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 g^3 (a + b x)} + \frac{B d^2 i \operatorname{Log} [c + d x]}{2 b^2 (b c - a d) g^3}$$

### Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(a g + b g x)^4} dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$\frac{B d i (c + d x)^2}{4 (b c - a d)^2 g^4 (a + b x)^2} - \frac{b B i (c + d x)^3}{9 (b c - a d)^2 g^4 (a + b x)^3} + \frac{d i (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 (b c - a d)^2 g^4 (a + b x)^2} - \frac{b i (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 (b c - a d)^2 g^4 (a + b x)^3}$$

Result (type 3, 225 leaves, 10 steps):

$$-\frac{B (b c - a d) i}{9 b^2 g^4 (a + b x)^3} - \frac{B d i}{12 b^2 g^4 (a + b x)^2} + \frac{B d^2 i}{6 b^2 (b c - a d) g^4 (a + b x)} + \frac{B d^3 i \operatorname{Log} [a + b x]}{6 b^2 (b c - a d)^2 g^4} - \frac{(b c - a d) i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^2 g^4 (a + b x)^3} - \frac{d i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^2 g^4 (a + b x)^2} - \frac{B d^3 i \operatorname{Log} [c + d x]}{6 b^2 (b c - a d)^2 g^4}$$

### Problem 9: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(a g + b g x)^5} dx$$

Optimal (type 3, 269 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^2 i (c + d x)^2}{4 (b c - a d)^3 g^5 (a + b x)^2} + \frac{2 b B d i (c + d x)^3}{9 (b c - a d)^3 g^5 (a + b x)^3} - \\
& \frac{b^2 B i (c + d x)^4}{16 (b c - a d)^3 g^5 (a + b x)^4} - \frac{d^2 i (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 (b c - a d)^3 g^5 (a + b x)^2} + \\
& \frac{2 b d i (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 i (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 (b c - a d)^3 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 3, 257 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) i}{16 b^2 g^5 (a + b x)^4} - \frac{B d i}{36 b^2 g^5 (a + b x)^3} + \\
& \frac{B d^2 i}{24 b^2 (b c - a d) g^5 (a + b x)^2} - \frac{B d^3 i}{12 b^2 (b c - a d)^2 g^5 (a + b x)} - \frac{B d^4 i \operatorname{Log} [a + b x]}{12 b^2 (b c - a d)^3 g^5} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^2 g^5 (a + b x)^4} - \frac{d i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^2 g^5 (a + b x)^3} + \frac{B d^4 i \operatorname{Log} [c + d x]}{12 b^2 (b c - a d)^3 g^5}
\end{aligned}$$

**Problem 10: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 423 leaves, 5 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^5 g^3 i^2 x}{60 b^2 d^3} + \frac{B (b c - a d)^4 g^3 i^2 (c + d x)^2}{120 b d^4} - \frac{19 B (b c - a d)^3 g^3 i^2 (c + d x)^3}{180 d^4} + \\
& \frac{13 b B (b c - a d)^2 g^3 i^2 (c + d x)^4}{120 d^4} - \frac{b^2 B (b c - a d) g^3 i^2 (c + d x)^5}{30 d^4} + \\
& \frac{B (b c - a d)^6 g^3 i^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{60 b^3 d^4} - \frac{(b c - a d)^3 g^3 i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 d^4} + \\
& \frac{3 b (b c - a d)^2 g^3 i^2 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^4} - \\
& \frac{3 b^2 (b c - a d) g^3 i^2 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^4} + \\
& \frac{b^3 g^3 i^2 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 d^4} + \frac{B (b c - a d)^6 g^3 i^2 \operatorname{Log} [c + d x]}{60 b^3 d^4}
\end{aligned}$$

Result (type 3, 330 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^3 i^2 x}{60 b^2 d^3} + \frac{B (b c - a d)^4 g^3 i^2 (a + b x)^2}{120 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^3 i^2 (a + b x)^3}{180 b^3 d} - \frac{7 B (b c - a d)^2 g^3 i^2 (a + b x)^4}{120 b^3} - \\
& \frac{B d (b c - a d) g^3 i^2 (a + b x)^5}{30 b^3} + \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^3} + \\
& \frac{2 d (b c - a d) g^3 i^2 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b^3} + \\
& \frac{d^2 g^3 i^2 (a + b x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b^3} + \frac{B (b c - a d)^6 g^3 i^2 \operatorname{Log} [c + d x]}{60 b^3 d^4}
\end{aligned}$$

**Problem 11: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 337 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^4 g^2 i^2 x}{30 b^2 d^2} - \frac{B (b c - a d)^3 g^2 i^2 (c + d x)^2}{60 b d^3} + \frac{B (b c - a d)^2 g^2 i^2 (c + d x)^3}{10 d^3} - \\
& \frac{b B (b c - a d) g^2 i^2 (c + d x)^4}{20 d^3} - \frac{B (b c - a d)^5 g^2 i^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{30 b^3 d^3} + \\
& \frac{(b c - a d)^2 g^2 i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 d^3} - \frac{b (b c - a d) g^2 i^2 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 d^3} + \\
& \frac{b^2 g^2 i^2 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^3} - \frac{B (b c - a d)^5 g^2 i^2 \operatorname{Log} [c + d x]}{30 b^3 d^3}
\end{aligned}$$

Result (type 3, 296 leaves, 14 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^4 g^2 i^2 x}{30 b^2 d^2} - \frac{B (b c - a d)^3 g^2 i^2 (a + b x)^2}{60 b^3 d} - \frac{B (b c - a d)^2 g^2 i^2 (a + b x)^3}{10 b^3} - \\
& \frac{B d (b c - a d) g^2 i^2 (a + b x)^4}{20 b^3} + \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^3} + \\
& \frac{d (b c - a d) g^2 i^2 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^3} + \\
& \frac{d^2 g^2 i^2 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b^3} - \frac{B (b c - a d)^5 g^2 i^2 \operatorname{Log} [c + d x]}{30 b^3 d^3}
\end{aligned}$$

**Problem 12: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 239 leaves, 5 steps):

$$\begin{aligned} & \frac{B (b c - a d)^3 g i^2 x}{12 b^2 d} + \frac{B (b c - a d)^2 g i^2 (c + d x)^2}{24 b d^2} - \frac{B (b c - a d) g i^2 (c + d x)^3}{12 d^2} + \\ & \frac{B (b c - a d)^4 g i^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{12 b^3 d^2} - \frac{(b c - a d) g i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 d^2} + \\ & \frac{b g i^2 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{4 d^2} + \frac{B (b c - a d)^4 g i^2 \operatorname{Log}[c + d x]}{12 b^3 d^2} \end{aligned}$$

Result (type 3, 200 leaves, 10 steps):

$$\begin{aligned} & \frac{B (b c - a d)^3 g i^2 x}{12 b^2 d} + \frac{B (b c - a d)^2 g i^2 (c + d x)^2}{24 b d^2} - \\ & \frac{B (b c - a d) g i^2 (c + d x)^3}{12 d^2} + \frac{B (b c - a d)^4 g i^2 \operatorname{Log}[a + b x]}{12 b^3 d^2} - \\ & \frac{(b c - a d) g i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 d^2} + \frac{b g i^2 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{4 d^2} \end{aligned}$$

**Problem 14: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{a g + b g x} dx$$

Optimal (type 4, 276 leaves, 10 steps):

$$\begin{aligned} & -\frac{B d (b c - a d) i^2 x}{2 b^2 g} - \frac{B (b c - a d)^2 i^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{2 b^3 g} + \frac{d (b c - a d) i^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{b^3 g} + \\ & \frac{i^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{2 b g} - \frac{3 B (b c - a d)^2 i^2 \operatorname{Log}[c + d x]}{2 b^3 g} - \\ & \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{b^3 g} + \frac{B (b c - a d)^2 i^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{b^3 g} \end{aligned}$$

Result (type 4, 354 leaves, 19 steps):

$$\begin{aligned}
& \frac{A d (b c - a d) i^2 x}{b^2 g} - \frac{B d (b c - a d) i^2 x}{2 b^2 g} - \frac{B (b c - a d)^2 i^2 \text{Log}[a + b x]}{2 b^3 g} - \\
& \frac{B (b c - a d)^2 i^2 \text{Log}\left[g (a + b x)\right]^2}{2 b^3 g} + \frac{B d (b c - a d) i^2 (a + b x) \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{b^3 g} + \\
& \frac{i^2 (c + d x)^2 \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b g} - \frac{B (b c - a d)^2 i^2 \text{Log}[c + d x]}{b^3 g} + \\
& \frac{(b c - a d)^2 i^2 \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \text{Log}[a g + b g x]}{b^3 g} + \\
& \frac{B (b c - a d)^2 i^2 \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \text{Log}[a g + b g x]}{b^3 g} + \frac{B (b c - a d)^2 i^2 \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g}
\end{aligned}$$

**Problem 15: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 247 leaves, 8 steps):

$$\begin{aligned}
& -\frac{B (b c - a d) i^2 (c + d x)}{b^2 g^2 (a + b x)} + \frac{d^2 i^2 (a + b x) \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2} - \\
& \frac{(b c - a d) i^2 (c + d x) \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^2 g^2 (a + b x)} - \frac{B d (b c - a d) i^2 \text{Log}[c + d x]}{b^3 g^2} - \\
& \frac{2 d (b c - a d) i^2 \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \text{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^3 g^2} + \frac{2 B d (b c - a d) i^2 \text{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^3 g^2}
\end{aligned}$$

Result (type 4, 313 leaves, 18 steps):

$$\begin{aligned}
& \frac{A d^2 i^2 x}{b^2 g^2} - \frac{B (b c - a d)^2 i^2}{b^3 g^2 (a + b x)} - \frac{B d (b c - a d) i^2 \text{Log}[a + b x]}{b^3 g^2} - \\
& \frac{B d (b c - a d) i^2 \text{Log}[a + b x]^2}{b^3 g^2} + \frac{B d^2 i^2 (a + b x) \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{b^3 g^2} - \\
& \frac{(b c - a d)^2 i^2 \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2 (a + b x)} + \frac{2 d (b c - a d) i^2 \text{Log}[a + b x] \left(A + B \text{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2} + \\
& \frac{2 B d (b c - a d) i^2 \text{Log}[a + b x] \text{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \frac{2 B d (b c - a d) i^2 \text{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2}
\end{aligned}$$



### Problem 16: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{(a g + b g x)^3} dx$$

Optimal (type 4, 230 leaves, 7 steps):

$$\begin{aligned} & -\frac{B d i^2 (c + d x)}{b^2 g^3 (a + b x)} - \frac{B i^2 (c + d x)^2}{4 b g^3 (a + b x)^2} - \\ & \frac{d i^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{b^2 g^3 (a + b x)} - \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{2 b g^3 (a + b x)^2} - \\ & \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right) \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{b^3 g^3} + \frac{B d^2 i^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{b^3 g^3} \end{aligned}$$

Result (type 4, 338 leaves, 19 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^2 i^2}{4 b^3 g^3 (a + b x)^2} - \frac{3 B d (b c - a d) i^2}{2 b^3 g^3 (a + b x)} - \frac{3 B d^2 i^2 \operatorname{Log}[a + b x]}{2 b^3 g^3} - \\ & \frac{B d^2 i^2 \operatorname{Log}[a + b x]^2}{2 b^3 g^3} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{2 b^3 g^3 (a + b x)^2} - \\ & \frac{2 d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{b^3 g^3 (a + b x)} + \frac{d^2 i^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{b^3 g^3} + \\ & \frac{3 B d^2 i^2 \operatorname{Log}[c + d x]}{2 b^3 g^3} + \frac{B d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c - a d}\right]}{b^3 g^3} + \frac{B d^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c - a d}\right]}{b^3 g^3} \end{aligned}$$

### Problem 17: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{(a g + b g x)^4} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{B i^2 (c + d x)^3}{9 (b c - a d) g^4 (a + b x)^3} - \frac{i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{3 (b c - a d) g^4 (a + b x)^3}$$

Result (type 3, 287 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2}{9 b^3 g^4 (a + b x)^3} - \frac{B d (b c - a d) i^2}{3 b^3 g^4 (a + b x)^2} - \frac{B d^2 i^2}{3 b^3 g^4 (a + b x)} - \\
& \frac{B d^3 i^2 \operatorname{Log}[a + b x]}{3 b^3 (b c - a d) g^4} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^3 g^4 (a + b x)^3} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b^3 g^4 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b^3 g^4 (a + b x)} + \frac{B d^3 i^2 \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4}
\end{aligned}$$

**Problem 18: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(a g + b g x)^5} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned}
& \frac{B d i^2 (c + d x)^3}{9 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B i^2 (c + d x)^4}{16 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{d i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 (b c - a d)^2 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 3, 325 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2}{16 b^3 g^5 (a + b x)^4} - \frac{5 B d (b c - a d) i^2}{36 b^3 g^5 (a + b x)^3} - \frac{B d^2 i^2}{24 b^3 g^5 (a + b x)^2} + \\
& \frac{B d^3 i^2}{12 b^3 (b c - a d) g^5 (a + b x)} + \frac{B d^4 i^2 \operatorname{Log}[a + b x]}{12 b^3 (b c - a d)^2 g^5} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{4 b^3 g^5 (a + b x)^4} - \\
& \frac{2 d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^3 g^5 (a + b x)^3} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 b^3 g^5 (a + b x)^2} - \frac{B d^4 i^2 \operatorname{Log}[c + d x]}{12 b^3 (b c - a d)^2 g^5}
\end{aligned}$$

**Problem 19: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(a g + b g x)^6} dx$$

Optimal (type 3, 281 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^2 i^2 (c + d x)^3}{9 (b c - a d)^3 g^6 (a + b x)^3} + \frac{b B d i^2 (c + d x)^4}{8 (b c - a d)^3 g^6 (a + b x)^4} - \\
& \frac{b^2 B i^2 (c + d x)^5}{25 (b c - a d)^3 g^6 (a + b x)^5} - \frac{d^2 i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 (b c - a d)^3 g^6 (a + b x)^3} + \\
& \frac{b d i^2 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 (b c - a d)^3 g^6 (a + b x)^4} - \frac{b^2 i^2 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 (b c - a d)^3 g^6 (a + b x)^5}
\end{aligned}$$

Result (type 3, 359 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2}{25 b^3 g^6 (a + b x)^5} - \frac{3 B d (b c - a d) i^2}{40 b^3 g^6 (a + b x)^4} - \frac{B d^2 i^2}{90 b^3 g^6 (a + b x)^3} + \frac{B d^3 i^2}{60 b^3 (b c - a d) g^6 (a + b x)^2} - \\
& \frac{B d^4 i^2}{30 b^3 (b c - a d)^2 g^6 (a + b x)} - \frac{B d^5 i^2 \operatorname{Log} [a + b x]}{30 b^3 (b c - a d)^3 g^6} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b^3 g^6 (a + b x)^5} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^3 g^6 (a + b x)^4} - \frac{d^2 i^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^3 g^6 (a + b x)^3} + \frac{B d^5 i^2 \operatorname{Log} [c + d x]}{30 b^3 (b c - a d)^3 g^6}
\end{aligned}$$

**Problem 20: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 457 leaves, 5 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^6 g^3 i^3 x}{140 b^3 d^3} + \frac{B (b c - a d)^5 g^3 i^3 (c + d x)^2}{280 b^2 d^4} + \frac{B (b c - a d)^4 g^3 i^3 (c + d x)^3}{420 b d^4} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 (c + d x)^4}{280 d^4} + \frac{b B (b c - a d)^2 g^3 i^3 (c + d x)^5}{14 d^4} - \frac{b^2 B (b c - a d) g^3 i^3 (c + d x)^6}{42 d^4} + \\
& \frac{B (b c - a d)^7 g^3 i^3 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{140 b^4 d^4} - \frac{(b c - a d)^3 g^3 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^4} + \\
& \frac{3 b (b c - a d)^2 g^3 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^4} - \\
& \frac{b^2 (b c - a d) g^3 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 d^4} + \\
& \frac{b^3 g^3 i^3 (c + d x)^7 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{7 d^4} + \frac{B (b c - a d)^7 g^3 i^3 \operatorname{Log} [c + d x]}{140 b^4 d^4}
\end{aligned}$$

Result (type 3, 416 leaves, 18 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^6 g^3 i^3 x}{140 b^3 d^3} + \frac{B (b c - a d)^5 g^3 i^3 (a + b x)^2}{280 b^4 d^2} - \frac{B (b c - a d)^4 g^3 i^3 (a + b x)^3}{420 b^4 d} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 (a + b x)^4}{280 b^4} - \frac{B d (b c - a d)^2 g^3 i^3 (a + b x)^5}{14 b^4} - \\
& \frac{B d^2 (b c - a d) g^3 i^3 (a + b x)^6}{42 b^4} + \frac{(b c - a d)^3 g^3 i^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^4} + \\
& \frac{3 d (b c - a d)^2 g^3 i^3 (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b^4} + \\
& \frac{d^2 (b c - a d) g^3 i^3 (a + b x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4} + \\
& \frac{d^3 g^3 i^3 (a + b x)^7 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{7 b^4} + \frac{B (b c - a d)^7 g^3 i^3 \operatorname{Log} [c + d x]}{140 b^4 d^4}
\end{aligned}$$

**Problem 21: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 371 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^2 i^3 x}{60 b^3 d^2} - \frac{B (b c - a d)^4 g^2 i^3 (c + d x)^2}{120 b^2 d^3} - \frac{B (b c - a d)^3 g^2 i^3 (c + d x)^3}{180 b d^3} + \\
& \frac{7 B (b c - a d)^2 g^2 i^3 (c + d x)^4}{120 d^3} - \frac{b B (b c - a d) g^2 i^3 (c + d x)^5}{30 d^3} - \\
& \frac{B (b c - a d)^6 g^2 i^3 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{60 b^4 d^3} + \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^3} - \\
& \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^3} + \\
& \frac{b^2 g^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 d^3} - \frac{B (b c - a d)^6 g^2 i^3 \operatorname{Log} [c + d x]}{60 b^4 d^3}
\end{aligned}$$

Result (type 3, 330 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^2 i^3 x}{60 b^3 d^2} - \frac{B (b c - a d)^4 g^2 i^3 (c + d x)^2}{120 b^2 d^3} - \frac{B (b c - a d)^3 g^2 i^3 (c + d x)^3}{180 b d^3} + \\
& \frac{7 B (b c - a d)^2 g^2 i^3 (c + d x)^4}{120 d^3} - \frac{b B (b c - a d) g^2 i^3 (c + d x)^5}{30 d^3} - \\
& \frac{B (b c - a d)^6 g^2 i^3 \operatorname{Log} [a + b x]}{60 b^4 d^3} + \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^3} - \\
& \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^3} + \frac{b^2 g^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 d^3}
\end{aligned}$$

### Problem 22: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) (c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) dx$$

Optimal (type 3, 271 leaves, 5 steps):

$$\begin{aligned} & \frac{B (b c - a d)^4 g i^3 x}{20 b^3 d} + \frac{B (b c - a d)^3 g i^3 (c + d x)^2}{40 b^2 d^2} + \\ & \frac{B (b c - a d)^2 g i^3 (c + d x)^3}{60 b d^2} - \frac{B (b c - a d) g i^3 (c + d x)^4}{20 d^2} + \\ & \frac{B (b c - a d)^5 g i^3 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{20 b^4 d^2} - \frac{(b c - a d) g i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^2} + \\ & \frac{b g i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^2} + \frac{B (b c - a d)^5 g i^3 \operatorname{Log} [c + d x]}{20 b^4 d^2} \end{aligned}$$

Result (type 3, 232 leaves, 10 steps):

$$\begin{aligned} & \frac{B (b c - a d)^4 g i^3 x}{20 b^3 d} + \frac{B (b c - a d)^3 g i^3 (c + d x)^2}{40 b^2 d^2} + \frac{B (b c - a d)^2 g i^3 (c + d x)^3}{60 b d^2} - \\ & \frac{B (b c - a d) g i^3 (c + d x)^4}{20 d^2} + \frac{B (b c - a d)^5 g i^3 \operatorname{Log} [a + b x]}{20 b^4 d^2} - \\ & \frac{(b c - a d) g i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 d^2} + \frac{b g i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 d^2} \end{aligned}$$

### Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{a g + b g x} dx$$

Optimal (type 4, 356 leaves, 14 steps):

$$\begin{aligned} & - \frac{5 B d (b c - a d)^2 i^3 x}{6 b^3 g} - \frac{B (b c - a d) i^3 (c + d x)^2}{6 b^2 g} - \frac{5 B (b c - a d)^3 i^3 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{6 b^4 g} + \\ & \frac{d (b c - a d)^2 i^3 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^4 g} + \frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^2 g} + \\ & \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b g} - \frac{11 B (b c - a d)^3 i^3 \operatorname{Log} [c + d x]}{6 b^4 g} - \\ & \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g} + \frac{B (b c - a d)^3 i^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g} \end{aligned}$$

Result (type 4, 436 leaves, 23 steps):

$$\begin{aligned}
& \frac{A d (b c - a d)^2 i^3 x}{b^3 g} - \frac{5 B d (b c - a d)^2 i^3 x}{6 b^3 g} - \\
& \frac{B (b c - a d) i^3 (c + d x)^2}{6 b^2 g} - \frac{5 B (b c - a d)^3 i^3 \operatorname{Log}[a + b x]}{6 b^4 g} - \\
& \frac{B (b c - a d)^3 i^3 \operatorname{Log}[g(a + b x)]^2}{2 b^4 g} + \frac{B d (b c - a d)^2 i^3 (a + b x) \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]}{b^4 g} + \\
& \frac{(b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{2 b^2 g} + \frac{i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{3 b g} - \\
& \frac{B (b c - a d)^3 i^3 \operatorname{Log}[c + d x]}{b^4 g} + \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[a g + b g x]}{b^4 g} + \\
& \frac{B (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]}{b^4 g} + \frac{B (b c - a d)^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^4 g}
\end{aligned}$$

**Problem 25: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 373 leaves, 11 steps):

$$\begin{aligned}
& -\frac{B d^2 (b c - a d) i^3 x}{2 b^3 g^2} - \frac{B (b c - a d)^2 i^3 (c + d x)}{b^3 g^2 (a + b x)} - \frac{B d (b c - a d)^2 i^3 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{2 b^4 g^2} + \\
& \frac{2 d^2 (b c - a d) i^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{b^4 g^2} - \frac{(b c - a d)^2 i^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{b^3 g^2 (a + b x)} + \\
& \frac{d i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{2 b^2 g^2} - \frac{5 B d (b c - a d)^2 i^3 \operatorname{Log}[c + d x]}{2 b^4 g^2} - \\
& \frac{3 d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2} + \frac{3 B d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2}
\end{aligned}$$

Result (type 4, 521 leaves, 22 steps):

$$\begin{aligned}
& \frac{A d^2 (3 b c - 2 a d) i^3 x}{b^3 g^2} - \frac{B d^2 (b c - a d) i^3 x}{2 b^3 g^2} - \frac{B (b c - a d)^3 i^3}{b^4 g^2 (a + b x)} - \\
& \frac{a^2 B d^3 i^3 \operatorname{Log}[a + b x]}{2 b^4 g^2} - \frac{B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x]}{b^4 g^2} - \frac{3 B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x]^2}{2 b^4 g^2} + \\
& \frac{B d^2 (3 b c - 2 a d) i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{b^4 g^2} + \frac{d^3 i^3 x^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^2 g^2} - \\
& \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2 (a + b x)} + \frac{3 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2} + \\
& \frac{B c^2 d i^3 \operatorname{Log}[c + d x]}{2 b^2 g^2} - \frac{B d (3 b c - 2 a d) (b c - a d) i^3 \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B d (b c - a d)^2 i^3 \operatorname{Log}[c + d x]}{b^4 g^2} + \\
& \frac{3 B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} + \frac{3 B d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^2}
\end{aligned}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(a g + b g x)^3} dx$$

Optimal (type 4, 345 leaves, 9 steps):

$$\begin{aligned}
& -\frac{2 B d (b c - a d) i^3 (c + d x)}{b^3 g^3 (a + b x)} - \frac{B (b c - a d) i^3 (c + d x)^2}{4 b^2 g^3 (a + b x)^2} + \\
& \frac{d^3 i^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^3} - \frac{2 d (b c - a d) i^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^3 (a + b x)} - \\
& \frac{(b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^2 g^3 (a + b x)^2} - \frac{B d^2 (b c - a d) i^3 \operatorname{Log}[c + d x]}{b^4 g^3} - \\
& \frac{3 d^2 (b c - a d) i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^3} + \frac{3 B d^2 (b c - a d) i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^3}
\end{aligned}$$

Result (type 4, 442 leaves, 22 steps):

$$\begin{aligned}
& \frac{A d^3 i^3 x}{b^3 g^3} - \frac{B (b c - a d)^3 i^3}{4 b^4 g^3 (a + b x)^2} - \frac{5 B d (b c - a d)^2 i^3}{2 b^4 g^3 (a + b x)} - \frac{5 B d^2 (b c - a d) i^3 \operatorname{Log}[a + b x]}{2 b^4 g^3} - \\
& \frac{3 B d^2 (b c - a d) i^3 \operatorname{Log}[a + b x]^2}{2 b^4 g^3} + \frac{B d^3 i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{b^4 g^3} - \\
& \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^4 g^3 (a + b x)^2} - \frac{3 d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^3 (a + b x)} + \\
& \frac{3 d^2 (b c - a d) i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^3} + \frac{3 B d^2 (b c - a d) i^3 \operatorname{Log}[c + d x]}{2 b^4 g^3} + \\
& \frac{3 B d^2 (b c - a d) i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^3} + \frac{3 B d^2 (b c - a d) i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^3}
\end{aligned}$$

**Problem 27: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(a g + b g x)^4} dx$$

Optimal (type 4, 310 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B d^2 i^3 (c + d x)}{b^3 g^4 (a + b x)} - \frac{B d i^3 (c + d x)^2}{4 b^2 g^4 (a + b x)^2} - \frac{B i^3 (c + d x)^3}{9 b g^4 (a + b x)^3} - \frac{d^2 i^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^4 (a + b x)} - \\
& \frac{d i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^2 g^4 (a + b x)^2} - \frac{i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 b g^4 (a + b x)^3} - \\
& \frac{d^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^4} + \frac{B d^3 i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^4}
\end{aligned}$$

Result (type 4, 424 leaves, 23 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^3 i^3}{9 b^4 g^4 (a + b x)^3} - \frac{7 B d (b c - a d)^2 i^3}{12 b^4 g^4 (a + b x)^2} - \frac{11 B d^2 (b c - a d) i^3}{6 b^4 g^4 (a + b x)} - \frac{11 B d^3 i^3 \operatorname{Log}[a + b x]}{6 b^4 g^4} - \\
& \frac{B d^3 i^3 \operatorname{Log}[a + b x]^2}{2 b^4 g^4} - \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 b^4 g^4 (a + b x)^3} - \frac{3 d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^4 g^4 (a + b x)^2} - \\
& \frac{3 d^2 (b c - a d) i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^4 (a + b x)} + \frac{d^3 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^4} + \\
& \frac{11 B d^3 i^3 \operatorname{Log}[c + d x]}{6 b^4 g^4} + \frac{B d^3 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^4} + \frac{B d^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^4}
\end{aligned}$$



### Problem 28: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(a g + b g x)^5} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{B i^3 (c + d x)^4}{16 (b c - a d) g^5 (a + b x)^4} - \frac{i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 (b c - a d) g^5 (a + b x)^4}$$

Result (type 3, 373 leaves, 18 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^3 i^3}{16 b^4 g^5 (a + b x)^4} - \frac{B d (b c - a d)^2 i^3}{4 b^4 g^5 (a + b x)^3} - \frac{3 B d^2 (b c - a d) i^3}{8 b^4 g^5 (a + b x)^2} - \frac{B d^3 i^3}{4 b^4 g^5 (a + b x)} - \\ & \frac{B d^4 i^3 \operatorname{Log}[a + b x]}{4 b^4 (b c - a d) g^5} - \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^4 g^5 (a + b x)^4} - \frac{d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^4 g^5 (a + b x)^3} - \\ & \frac{3 d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4 g^5 (a + b x)^2} - \frac{d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^4 g^5 (a + b x)} + \frac{B d^4 i^3 \operatorname{Log}[c + d x]}{4 b^4 (b c - a d) g^5} \end{aligned}$$

### Problem 29: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(a g + b g x)^6} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned} & \frac{B d i^3 (c + d x)^4}{16 (b c - a d)^2 g^6 (a + b x)^4} - \frac{b B i^3 (c + d x)^5}{25 (b c - a d)^2 g^6 (a + b x)^5} + \\ & \frac{d i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 (b c - a d)^2 g^6 (a + b x)^4} - \frac{b i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 (b c - a d)^2 g^6 (a + b x)^5} \end{aligned}$$

Result (type 3, 409 leaves, 18 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^3 i^3}{25 b^4 g^6 (a + b x)^5} - \frac{11 B d (b c - a d)^2 i^3}{80 b^4 g^6 (a + b x)^4} - \frac{3 B d^2 (b c - a d) i^3}{20 b^4 g^6 (a + b x)^3} - \\ & \frac{B d^3 i^3}{40 b^4 g^6 (a + b x)^2} + \frac{B d^4 i^3}{20 b^4 (b c - a d) g^6 (a + b x)} + \frac{B d^5 i^3 \operatorname{Log}[a + b x]}{20 b^4 (b c - a d)^2 g^6} - \\ & \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{5 b^4 g^6 (a + b x)^5} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^4 g^6 (a + b x)^4} - \\ & \frac{d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^4 g^6 (a + b x)^3} - \frac{d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4 g^6 (a + b x)^2} - \frac{B d^5 i^3 \operatorname{Log}[c + d x]}{20 b^4 (b c - a d)^2 g^6} \end{aligned}$$

### Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{(a g + b g x)^7} dx$$

Optimal (type 3, 281 leaves, 5 steps):

$$\begin{aligned} & -\frac{B d^2 i^3 (c + d x)^4}{16 (b c - a d)^3 g^7 (a + b x)^4} + \frac{2 b B d i^3 (c + d x)^5}{25 (b c - a d)^3 g^7 (a + b x)^5} - \\ & \frac{b^2 B i^3 (c + d x)^6}{36 (b c - a d)^3 g^7 (a + b x)^6} - \frac{d^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{4 (b c - a d)^3 g^7 (a + b x)^4} + \\ & \frac{2 b d i^3 (c + d x)^5 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{5 (b c - a d)^3 g^7 (a + b x)^5} - \frac{b^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 (b c - a d)^3 g^7 (a + b x)^6} \end{aligned}$$

Result (type 3, 445 leaves, 18 steps):

$$\begin{aligned} & -\frac{B (b c - a d)^3 i^3}{36 b^4 g^7 (a + b x)^6} - \frac{13 B d (b c - a d)^2 i^3}{150 b^4 g^7 (a + b x)^5} - \frac{19 B d^2 (b c - a d) i^3}{240 b^4 g^7 (a + b x)^4} - \frac{B d^3 i^3}{180 b^4 g^7 (a + b x)^3} + \\ & \frac{B d^4 i^3}{120 b^4 (b c - a d) g^7 (a + b x)^2} - \frac{B d^5 i^3}{60 b^4 (b c - a d)^2 g^7 (a + b x)} - \frac{B d^6 i^3 \operatorname{Log}[a + b x]}{60 b^4 (b c - a d)^3 g^7} - \\ & \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 b^4 g^7 (a + b x)^6} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{5 b^4 g^7 (a + b x)^5} - \\ & \frac{3 d^2 (b c - a d) i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{4 b^4 g^7 (a + b x)^4} - \frac{d^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{3 b^4 g^7 (a + b x)^3} + \frac{B d^6 i^3 \operatorname{Log}[c + d x]}{60 b^4 (b c - a d)^3 g^7} \end{aligned}$$

### Problem 31: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{c i + d i x} dx$$

Optimal (type 4, 252 leaves, 6 steps):

$$\begin{aligned} & \frac{g^3 (a + b x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{3 d i} - \frac{(b c - a d) g^3 (a + b x)^2 \left( 3 A + B + 3 B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 d^2 i} + \\ & \frac{(b c - a d)^2 g^3 (a + b x) \left( 6 A + 5 B + 6 B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 d^3 i} + \\ & \frac{(b c - a d)^3 g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( 6 A + 11 B + 6 B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 d^4 i} + \frac{B (b c - a d)^3 g^3 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{d^4 i} \end{aligned}$$

Result (type 4, 408 leaves, 23 steps):

$$\begin{aligned}
& \frac{A b (b c - a d)^2 g^3 x}{d^3 i} + \frac{5 b B (b c - a d)^2 g^3 x}{6 d^3 i} - \frac{B (b c - a d) g^3 (a + b x)^2}{6 d^2 i} + \\
& \frac{B (b c - a d)^2 g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i} - \frac{(b c - a d) g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i} + \\
& \frac{g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 d i} - \frac{11 B (b c - a d)^3 g^3 \operatorname{Log}[c + d x]}{6 d^4 i} - \\
& \frac{B (b c - a d)^3 g^3 \operatorname{Log}[i (c + d x)]^2}{2 d^4 i} + \frac{B (b c - a d)^3 g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d^4 i} - \\
& \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c i + d i x]}{d^4 i} + \frac{B (b c - a d)^3 g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i}
\end{aligned}$$

**Problem 32: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{c i + d i x} dx$$

Optimal (type 4, 198 leaves, 5 steps):

$$\begin{aligned}
& \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d i} - \frac{(b c - a d) g^2 (a + b x) \left(2 A + B + 2 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i} - \\
& \frac{(b c - a d)^2 g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(2 A + 3 B + 2 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^3 i} - \frac{B (b c - a d)^2 g^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i}
\end{aligned}$$

Result (type 4, 329 leaves, 19 steps):

$$\begin{aligned}
& -\frac{A b (b c - a d) g^2 x}{d^2 i} - \frac{b B (b c - a d) g^2 x}{2 d^2 i} - \frac{B (b c - a d) g^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^2 i} + \\
& \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d i} + \frac{3 B (b c - a d)^2 g^2 \operatorname{Log}[c + d x]}{2 d^3 i} + \\
& \frac{B (b c - a d)^2 g^2 \operatorname{Log}[i (c + d x)]^2}{2 d^3 i} - \frac{B (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d^3 i} + \\
& \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c i + d i x]}{d^3 i} - \frac{B (b c - a d)^2 g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i}
\end{aligned}$$

**Problem 33: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{c i + d i x} dx$$

Optimal (type 4, 125 leaves, 4 steps):

$$\frac{g(a+bx) \left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)}{di} + \frac{(bc-ad) g \operatorname{Log} \left[ \frac{bc-ad}{b(c+dx)} \right] \left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)}{d^2 i} + \frac{B(bc-ad) g \operatorname{PolyLog} \left[ 2, \frac{d(a+bx)}{b(c+dx)} \right]}{d^2 i}$$

Result (type 4, 213 leaves, 14 steps):

$$\frac{A b g x}{di} + \frac{B g(a+bx) \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right]}{di} - \frac{B(bc-ad) g \operatorname{Log}[c+dx]}{d^2 i} + \frac{B(bc-ad) g \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^2 i} - \frac{(bc-ad) g \left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right) \operatorname{Log}[c+dx]}{d^2 i} - \frac{B(bc-ad) g \operatorname{Log}[c+dx]^2}{2 d^2 i} + \frac{B(bc-ad) g \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{d^2 i}$$

Problem 34: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right]}{ci + di x} dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$- \frac{\operatorname{Log} \left[ \frac{bc-ad}{b(c+dx)} \right] \left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)}{di} - \frac{B \operatorname{PolyLog} \left[ 2, \frac{d(a+bx)}{b(c+dx)} \right]}{di}$$

Result (type 4, 122 leaves, 10 steps):

$$\frac{B \operatorname{Log}[i(c+dx)]^2}{2 di} - \frac{B \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[ci + di x]}{di} + \frac{\left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right) \operatorname{Log}[ci + di x]}{di} - \frac{B \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{di}$$

Problem 35: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right]}{(ag + bgx)(ci + di x)} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{\left( A + B \operatorname{Log} \left[ \frac{e(a+bx)}{c+dx} \right] \right)^2}{2 B(bc-ad) gi}$$

Result (type 4, 304 leaves, 20 steps):

$$\begin{aligned}
& - \frac{B \operatorname{Log}[a + b x]^2}{2 (b c - a d) g i} + \frac{\operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d) g i} + \\
& \frac{B \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d) g i} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d) g i} - \frac{B \operatorname{Log}[c + d x]^2}{2 (b c - a d) g i} + \\
& \frac{B \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{B \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{B \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i}
\end{aligned}$$

**Problem 36:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^2 (c i + d i x)} dx$$

Optimal (type 3, 173 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b B (c + d x)}{(b c - a d)^2 g^2 i (a + b x)} + \frac{B d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^2 g^2 i} - \\
& \frac{b (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^2 g^2 i (a + b x)} - \frac{d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^2 g^2 i}
\end{aligned}$$

Result (type 4, 437 leaves, 24 steps):

$$\begin{aligned}
& - \frac{B}{(b c - a d) g^2 i (a + b x)} - \frac{B d \operatorname{Log}[a + b x]}{(b c - a d)^2 g^2 i} + \frac{B d \operatorname{Log}[a + b x]^2}{2 (b c - a d)^2 g^2 i} - \\
& \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(b c - a d) g^2 i (a + b x)} - \frac{d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^2 g^2 i} + \frac{B d \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} - \\
& \frac{B d \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} + \frac{d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} + \frac{B d \operatorname{Log}[c + d x]^2}{2 (b c - a d)^2 g^2 i} - \\
& \frac{B d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{B d \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{B d \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i}
\end{aligned}$$

**Problem 37:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^3 (c i + d i x)} dx$$

Optimal (type 3, 255 leaves, 7 steps):

$$\begin{aligned}
& - \frac{B (c+d x)^2 \left(b - \frac{4 d (a+b x)}{c+d x}\right)^2}{4 (b c-a d)^3 g^3 i (a+b x)^2} - \frac{B d^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]^2}{2 (b c-a d)^3 g^3 i} + \frac{2 b d (c+d x) \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^3 g^3 i (a+b x)} - \\
& \frac{b^2 (c+d x)^2 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{2 (b c-a d)^3 g^3 i (a+b x)^2} + \frac{d^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right] \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^3 g^3 i}
\end{aligned}$$

Result (type 4, 535 leaves, 28 steps):

$$\begin{aligned}
& - \frac{B}{4 (b c-a d) g^3 i (a+b x)^2} + \frac{3 B d}{2 (b c-a d)^2 g^3 i (a+b x)} + \\
& \frac{3 B d^2 \operatorname{Log}[a+b x]}{2 (b c-a d)^3 g^3 i} - \frac{B d^2 \operatorname{Log}[a+b x]^2}{2 (b c-a d)^3 g^3 i} - \frac{A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{2 (b c-a d) g^3 i (a+b x)^2} + \\
& \frac{d \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^2 g^3 i (a+b x)} + \frac{d^2 \operatorname{Log}[a+b x] \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^3 g^3 i} - \frac{3 B d^2 \operatorname{Log}[c+d x]}{2 (b c-a d)^3 g^3 i} + \\
& \frac{B d^2 \operatorname{Log}\left[-\frac{d (a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} - \frac{d^2 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} - \frac{B d^2 \operatorname{Log}[c+d x]^2}{2 (b c-a d)^3 g^3 i} + \\
& \frac{B d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{B d^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{B d^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i}
\end{aligned}$$

**Problem 38: Result unnecessarily involves higher level functions.**

$$\int \frac{A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{(a g+b g x)^4 (c i+d i x)} dx$$

Optimal (type 3, 373 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 b B d^2 (c+d x)}{(b c-a d)^4 g^4 i (a+b x)} + \frac{3 b^2 B d (c+d x)^2}{4 (b c-a d)^4 g^4 i (a+b x)^2} - \frac{b^3 B (c+d x)^3}{9 (b c-a d)^4 g^4 i (a+b x)^3} + \\
& \frac{B d^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]^2}{2 (b c-a d)^4 g^4 i} - \frac{3 b d^2 (c+d x) \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^4 g^4 i (a+b x)} + \frac{3 b^2 d (c+d x)^2 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{2 (b c-a d)^4 g^4 i (a+b x)^2} - \\
& \frac{b^3 (c+d x)^3 \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 (b c-a d)^4 g^4 i (a+b x)^3} - \frac{d^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right] \left(A+B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{(b c-a d)^4 g^4 i}
\end{aligned}$$

Result (type 4, 620 leaves, 32 steps):

$$\begin{aligned}
& - \frac{B}{9 (b c - a d) g^4 i (a + b x)^3} + \frac{5 B d}{12 (b c - a d)^2 g^4 i (a + b x)^2} - \frac{11 B d^2}{6 (b c - a d)^3 g^4 i (a + b x)} - \\
& \frac{11 B d^3 \operatorname{Log}[a + b x]}{6 (b c - a d)^4 g^4 i} + \frac{B d^3 \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^4 i} - \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{3 (b c - a d) g^4 i (a + b x)^3} + \frac{d \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 (b c - a d)^2 g^4 i (a + b x)^2} \\
& \frac{d^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^3 g^4 i (a + b x)} - \frac{d^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^4 g^4 i} + \frac{11 B d^3 \operatorname{Log}[c + d x]}{6 (b c - a d)^4 g^4 i} - \\
& \frac{B d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^4 i} + \frac{d^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^4 i} + \frac{B d^3 \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^4 i} - \\
& \frac{B d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i} - \frac{B d^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i} - \frac{B d^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i}
\end{aligned}$$

**Problem 39: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 341 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 B (b c - a d)^2 g^3 (a + b x)}{d^3 i^2 (c + d x)} - \frac{(6 A + 5 B) (b c - a d)^2 g^3 (a + b x)}{2 d^3 i^2 (c + d x)} - \\
& \frac{3 B (b c - a d)^2 g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i^2 (c + d x)} + \frac{g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d i^2 (c + d x)} - \\
& \frac{(b c - a d) g^3 (a + b x)^2 \left(3 A + B + 3 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i^2 (c + d x)} - \\
& \frac{b (b c - a d)^2 g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(6 A + 5 B + 6 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^4 i^2} - \\
& \frac{3 b B (b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i^2}
\end{aligned}$$

Result (type 4, 519 leaves, 22 steps):

$$\begin{aligned}
& - \frac{A b^2 (2 b c - 3 a d) g^3 x}{d^3 i^2} - \frac{b^2 B (b c - a d) g^3 x}{2 d^3 i^2} - \frac{B (b c - a d)^3 g^3}{d^4 i^2 (c + d x)} - \frac{a^2 b B g^3 \operatorname{Log}[a + b x]}{2 d^2 i^2} - \\
& \frac{b B (b c - a d)^2 g^3 \operatorname{Log}[a + b x]}{d^4 i^2} - \frac{b B (2 b c - 3 a d) g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i^2} + \\
& \frac{b^3 g^3 x^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i^2} + \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^2 (c + d x)} + \\
& \frac{b^3 B c^2 g^3 \operatorname{Log}[c + d x]}{2 d^4 i^2} + \frac{b B (2 b c - 3 a d) (b c - a d) g^3 \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{b B (b c - a d)^2 g^3 \operatorname{Log}[c + d x]}{d^4 i^2} - \frac{3 b B (b c - a d)^2 g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{3 b (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{3 b B (b c - a d)^2 g^3 \operatorname{Log}[c + d x]^2}{2 d^4 i^2} - \frac{3 b B (b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^2}
\end{aligned}$$

**Problem 40: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 260 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g^2 (a + b x)}{d^2 i^2 (c + d x)} + \frac{(2 A + B) (b c - a d) g^2 (a + b x)}{d^2 i^2 (c + d x)} + \\
& \frac{2 B (b c - a d) g^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^2 i^2 (c + d x)} + \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d i^2 (c + d x)} + \\
& \frac{b (b c - a d) g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(2 A + B + 2 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^3 i^2} + \frac{2 b B (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i^2}
\end{aligned}$$

Result (type 4, 336 leaves, 18 steps):

$$\begin{aligned}
& \frac{A b^2 g^2 x}{d^2 i^2} + \frac{B (b c - a d)^2 g^2}{d^3 i^2 (c + d x)} + \frac{b B (b c - a d) g^2 \operatorname{Log}[a + b x]}{d^3 i^2} + \frac{b B g^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^2 i^2} - \\
& \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^3 i^2 (c + d x)} - \frac{2 b B (b c - a d) g^2 \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{2 b B (b c - a d) g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} - \frac{2 b (b c - a d) g^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^3 i^2} - \\
& \frac{b B (b c - a d) g^2 \operatorname{Log}[c + d x]^2}{d^3 i^2} + \frac{2 b B (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2}
\end{aligned}$$



### Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 160 leaves, 7 steps):

$$\begin{aligned} & -\frac{A g (a + b x)}{d i^2 (c + d x)} + \frac{B g (a + b x)}{d i^2 (c + d x)} - \frac{B g (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{d i^2 (c + d x)} - \\ & \frac{b g \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{d^2 i^2} - \frac{b B g \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{d^2 i^2} \end{aligned}$$

Result (type 4, 222 leaves, 15 steps):

$$\begin{aligned} & -\frac{B (b c - a d) g}{d^2 i^2 (c + d x)} - \frac{b B g \operatorname{Log} [a + b x]}{d^2 i^2} + \frac{(b c - a d) g \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{d^2 i^2 (c + d x)} + \\ & \frac{b B g \operatorname{Log} [c + d x]}{d^2 i^2} - \frac{b B g \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{d^2 i^2} + \\ & \frac{b g \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{d^2 i^2} + \frac{b B g \operatorname{Log} [c + d x]^2}{2 d^2 i^2} - \frac{b B g \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{d^2 i^2} \end{aligned}$$

### Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{(c i + d i x)^2} dx$$

Optimal (type 3, 98 leaves, 3 steps):

$$\frac{A (a + b x)}{(b c - a d) i^2 (c + d x)} - \frac{B (a + b x)}{(b c - a d) i^2 (c + d x)} + \frac{B (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{(b c - a d) i^2 (c + d x)}$$

Result (type 3, 101 leaves, 4 steps):

$$\frac{B}{d i^2 (c + d x)} + \frac{b B \operatorname{Log} [a + b x]}{d (b c - a d) i^2} - \frac{A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{d i^2 (c + d x)} - \frac{b B \operatorname{Log} [c + d x]}{d (b c - a d) i^2}$$

### Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{(a g + b g x) (c i + d i x)^2} dx$$

Optimal (type 3, 156 leaves, 5 steps):

$$\begin{aligned}
& - \frac{A d (a + b x)}{(b c - a d)^2 g i^2 (c + d x)} + \frac{B d (a + b x)}{(b c - a d)^2 g i^2 (c + d x)} - \\
& \frac{B d (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(b c - a d)^2 g i^2 (c + d x)} + \frac{b \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 B (b c - a d)^2 g i^2}
\end{aligned}$$

Result (type 4, 432 leaves, 24 steps):

$$\begin{aligned}
& - \frac{B}{(b c - a d) g i^2 (c + d x)} - \frac{b B \operatorname{Log}[a + b x]}{(b c - a d)^2 g i^2} - \frac{b B \operatorname{Log}[a + b x]^2}{2 (b c - a d)^2 g i^2} + \\
& \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(b c - a d) g i^2 (c + d x)} + \frac{b \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g i^2} + \frac{b B \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \\
& \frac{b B \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{b \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{b B \operatorname{Log}[c + d x]^2}{2 (b c - a d)^2 g i^2} + \\
& \frac{b B \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{b B \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{b B \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2}
\end{aligned}$$

**Problem 44: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^2 (c i + d i x)^2} dx$$

Optimal (type 3, 261 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B d^2 (a + b x)}{(b c - a d)^3 g^2 i^2 (c + d x)} - \frac{b^2 B (c + d x)}{(b c - a d)^3 g^2 i^2 (a + b x)} + \\
& \frac{b B d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} + \frac{d^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^3 g^2 i^2 (c + d x)} - \\
& \frac{b^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^3 g^2 i^2 (a + b x)} - \frac{2 b d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^3 g^2 i^2}
\end{aligned}$$

Result (type 4, 462 leaves, 28 steps):

$$\begin{aligned}
& - \frac{b B}{(b c - a d)^2 g^2 i^2 (a + b x)} + \frac{B d}{(b c - a d)^2 g^2 i^2 (c + d x)} + \frac{b B d \operatorname{Log}[a + b x]^2}{(b c - a d)^3 g^2 i^2} - \\
& \frac{b \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^2 g^2 i^2 (a + b x)} - \frac{d \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^2 g^2 i^2 (c + d x)} - \frac{2 b d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B d \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \frac{2 b d \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \frac{b B d \operatorname{Log}[c + d x]^2}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{2 b B d \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{2 b B d \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2}
\end{aligned}$$

**Problem 45: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{(a g + b g x)^3 (c i + d i x)^2} dx$$

Optimal (type 3, 364 leaves, 8 steps):

$$\begin{aligned}
& \frac{B d^3 (a + b x)}{(b c - a d)^4 g^3 i^2 (c + d x)} + \frac{3 b^2 B d (c + d x)}{(b c - a d)^4 g^3 i^2 (a + b x)} - \frac{b^3 B (c + d x)^2}{4 (b c - a d)^4 g^3 i^2 (a + b x)^2} - \\
& \frac{3 b B d^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]^2}{2 (b c - a d)^4 g^3 i^2} - \frac{d^3 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^4 g^3 i^2 (c + d x)} + \frac{3 b^2 d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^4 g^3 i^2 (a + b x)} - \\
& \frac{b^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{2 (b c - a d)^4 g^3 i^2 (a + b x)^2} + \frac{3 b d^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right] \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^4 g^3 i^2}
\end{aligned}$$

Result (type 4, 630 leaves, 32 steps):

$$\begin{aligned}
& - \frac{b B}{4 (b c - a d)^2 g^3 i^2 (a + b x)^2} + \frac{5 b B d}{2 (b c - a d)^3 g^3 i^2 (a + b x)} - \\
& \frac{B d^2}{(b c - a d)^3 g^3 i^2 (c + d x)} + \frac{3 b B d^2 \operatorname{Log}[a + b x]}{2 (b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^3 i^2} - \\
& \frac{b \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{2 (b c - a d)^2 g^3 i^2 (a + b x)^2} + \frac{2 b d \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^3 g^3 i^2 (a + b x)} + \frac{d^2 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^3 g^3 i^2 (c + d x)} + \\
& \frac{3 b d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)}{(b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 \operatorname{Log}[c + d x]}{2 (b c - a d)^4 g^3 i^2} + \frac{3 b B d^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^3 i^2} - \\
& \frac{3 b d^2 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^3 i^2} + \\
& \frac{3 b B d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B d^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B d^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2}
\end{aligned}$$

### Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(ag+bgx)^4 (ci+dix)^2} dx$$

Optimal (type 3, 457 leaves, 4 steps):

$$\begin{aligned} & -\frac{B d^4 (a+bx)}{(bc-ad)^5 g^4 i^2 (c+dx)} - \frac{6 b^2 B d^2 (c+dx)}{(bc-ad)^5 g^4 i^2 (a+bx)} + \frac{b^3 B d (c+dx)^2}{(bc-ad)^5 g^4 i^2 (a+bx)^2} - \\ & \frac{b^4 B (c+dx)^3}{9 (bc-ad)^5 g^4 i^2 (a+bx)^3} + \frac{2 b B d^3 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}{(bc-ad)^5 g^4 i^2} + \frac{d^4 (a+bx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^2 (c+dx)} - \\ & \frac{6 b^2 d^2 (c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^2 (a+bx)} + \frac{2 b^3 d (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^2 (a+bx)^2} - \\ & \frac{b^4 (c+dx)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3 (bc-ad)^5 g^4 i^2 (a+bx)^3} - \frac{4 b d^3 \operatorname{Log}\left[\frac{a+bx}{c+dx}\right] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^2} \end{aligned}$$

Result (type 4, 705 leaves, 36 steps):

$$\begin{aligned} & -\frac{b B}{9 (bc-ad)^2 g^4 i^2 (a+bx)^3} + \frac{2 b B d}{3 (bc-ad)^3 g^4 i^2 (a+bx)^2} - \frac{13 b B d^2}{3 (bc-ad)^4 g^4 i^2 (a+bx)} + \\ & \frac{B d^3}{(bc-ad)^4 g^4 i^2 (c+dx)} - \frac{10 b B d^3 \operatorname{Log}[a+bx]}{3 (bc-ad)^5 g^4 i^2} + \frac{2 b B d^3 \operatorname{Log}[a+bx]^2}{(bc-ad)^5 g^4 i^2} - \\ & \frac{b \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3 (bc-ad)^2 g^4 i^2 (a+bx)^3} + \frac{b d \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3 g^4 i^2 (a+bx)^2} - \frac{3 b d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^4 i^2 (a+bx)} - \\ & \frac{d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^4 i^2 (c+dx)} - \frac{4 b d^3 \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^2} + \frac{10 b B d^3 \operatorname{Log}[c+dx]}{3 (bc-ad)^5 g^4 i^2} - \\ & \frac{4 b B d^3 \operatorname{Log}\left[-\frac{d(a+bx)}{b c-a d}\right] \operatorname{Log}[c+dx]}{(bc-ad)^5 g^4 i^2} + \frac{4 b d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c+dx]}{(bc-ad)^5 g^4 i^2} + \\ & \frac{2 b B d^3 \operatorname{Log}[c+dx]^2}{(bc-ad)^5 g^4 i^2} - \frac{4 b B d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{b c-a d}\right]}{(bc-ad)^5 g^4 i^2} - \\ & \frac{4 b B d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b c-a d}\right]}{(bc-ad)^5 g^4 i^2} - \frac{4 b B d^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c-a d}\right]}{(bc-ad)^5 g^4 i^2} \end{aligned}$$

### Problem 47: Result valid but suboptimal antiderivative.

$$\int \frac{(ag+bgx)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(ci+dix)^3} dx$$

Optimal (type 4, 361 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{3 B (b c - a d) g^3 (a + b x)^2}{4 d^2 i^3 (c + d x)^2} - \frac{3 b B (b c - a d) g^3 (a + b x)}{d^3 i^3 (c + d x)} + \\
 & \frac{b (3 A + B) (b c - a d) g^3 (a + b x)}{d^3 i^3 (c + d x)} + \frac{3 b B (b c - a d) g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i^3 (c + d x)} + \\
 & \frac{g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d i^3 (c + d x)^2} + \frac{(b c - a d) g^3 (a + b x)^2 \left(3 A + B + 3 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i^3 (c + d x)^2} + \\
 & \frac{b^2 (b c - a d) g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(3 A + B + 3 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^3} + \frac{3 b^2 B (b c - a d) g^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i^3}
 \end{aligned}$$

Result (type 4, 442 leaves, 22 steps):

$$\begin{aligned}
 & \frac{A b^3 g^3 x}{d^3 i^3} - \frac{B (b c - a d)^3 g^3}{4 d^4 i^3 (c + d x)^2} + \frac{5 b B (b c - a d)^2 g^3}{2 d^4 i^3 (c + d x)} + \\
 & \frac{5 b^2 B (b c - a d) g^3 \operatorname{Log}[a + b x]}{2 d^4 i^3} + \frac{b^2 B g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i^3} + \\
 & \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^4 i^3 (c + d x)^2} - \frac{3 b (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^3 (c + d x)} - \\
 & \frac{7 b^2 B (b c - a d) g^3 \operatorname{Log}[c + d x]}{2 d^4 i^3} + \frac{3 b^2 B (b c - a d) g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^3} - \\
 & \frac{3 b^2 (b c - a d) g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^4 i^3} - \\
 & \frac{3 b^2 B (b c - a d) g^3 \operatorname{Log}[c + d x]^2}{2 d^4 i^3} + \frac{3 b^2 B (b c - a d) g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3}
 \end{aligned}$$

**Problem 48: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(c i + d i x)^3} dx$$

Optimal (type 4, 251 leaves, 8 steps):

$$\begin{aligned}
 & \frac{B g^2 (a + b x)^2}{4 d i^3 (c + d x)^2} - \frac{A b g^2 (a + b x)}{d^2 i^3 (c + d x)} + \frac{b B g^2 (a + b x)}{d^2 i^3 (c + d x)} - \\
 & \frac{b B g^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^2 i^3 (c + d x)} - \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d i^3 (c + d x)^2} - \\
 & \frac{b^2 g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^3 i^3} - \frac{b^2 B g^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i^3}
 \end{aligned}$$

Result (type 4, 340 leaves, 19 steps):

$$\begin{aligned} & \frac{B (b c - a d)^2 g^2}{4 d^3 i^3 (c + d x)^2} - \frac{3 b B (b c - a d) g^2}{2 d^3 i^3 (c + d x)} - \frac{3 b^2 B g^2 \operatorname{Log}[a + b x]}{2 d^3 i^3} - \frac{(b c - a d)^2 g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 d^3 i^3 (c + d x)^2} + \\ & \frac{2 b (b c - a d) g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{d^3 i^3 (c + d x)} + \frac{3 b^2 B g^2 \operatorname{Log}[c + d x]}{2 d^3 i^3} - \frac{b^2 B g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^3} + \\ & \frac{b^2 g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d^3 i^3} + \frac{b^2 B g^2 \operatorname{Log}[c + d x]^2}{2 d^3 i^3} - \frac{b^2 B g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^3} \end{aligned}$$

Problem 49: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(c i + d i x)^3} dx$$

Optimal (type 3, 85 leaves, 2 steps):

$$-\frac{B g (a + b x)^2}{4 (b c - a d) i^3 (c + d x)^2} + \frac{g (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d) i^3 (c + d x)^2}$$

Result (type 3, 191 leaves, 10 steps):

$$\begin{aligned} & -\frac{B (b c - a d) g}{4 d^2 i^3 (c + d x)^2} + \frac{b B g}{2 d^2 i^3 (c + d x)} + \frac{b^2 B g \operatorname{Log}[a + b x]}{2 d^2 (b c - a d) i^3} + \\ & \frac{(b c - a d) g \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 d^2 i^3 (c + d x)^2} - \frac{b g \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{d^2 i^3 (c + d x)} - \frac{b^2 B g \operatorname{Log}[c + d x]}{2 d^2 (b c - a d) i^3} \end{aligned}$$

Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x) (c i + d i x)^3} dx$$

Optimal (type 3, 243 leaves, 4 steps):

$$\begin{aligned} & -\frac{B \left( 4 b - \frac{d (a + b x)}{c + d x} \right)^2}{4 (b c - a d)^3 g i^3} - \frac{b^2 B \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^3 g i^3} + \frac{d^2 (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^3 g i^3 (c + d x)^2} - \\ & \frac{2 b d (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^3 g i^3 (c + d x)} + \frac{b^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^3 g i^3} \end{aligned}$$

Result (type 4, 535 leaves, 28 steps):

$$\begin{aligned}
& - \frac{B}{4 (b c - a d) g i^3 (c + d x)^2} - \frac{3 b B}{2 (b c - a d)^2 g i^3 (c + d x)} - \\
& \frac{3 b^2 B \operatorname{Log}[a + b x]}{2 (b c - a d)^3 g i^3} - \frac{b^2 B \operatorname{Log}[a + b x]^2}{2 (b c - a d)^3 g i^3} + \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{2 (b c - a d) g i^3 (c + d x)^2} + \\
& \frac{b \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g i^3 (c + d x)} + \frac{b^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^3 g i^3} + \frac{3 b^2 B \operatorname{Log}[c + d x]}{2 (b c - a d)^3 g i^3} + \\
& \frac{b^2 B \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g i^3} - \frac{b^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^3 g i^3} - \frac{b^2 B \operatorname{Log}[c + d x]^2}{2 (b c - a d)^3 g i^3} + \\
& \frac{b^2 B \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{b^2 B \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{b^2 B \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3}
\end{aligned}$$

**Problem 52: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^2 (c i + d i x)^3} dx$$

Optimal (type 3, 365 leaves, 4 steps):

$$\begin{aligned}
& \frac{B d^3 (a + b x)^2}{4 (b c - a d)^4 g^2 i^3 (c + d x)^2} - \frac{3 b B d^2 (a + b x)}{(b c - a d)^4 g^2 i^3 (c + d x)} - \frac{b^3 B (c + d x)}{(b c - a d)^4 g^2 i^3 (a + b x)} + \\
& \frac{3 b^2 B d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^4 g^2 i^3} - \frac{d^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 (b c - a d)^4 g^2 i^3 (c + d x)^2} + \frac{3 b d^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^4 g^2 i^3 (c + d x)} - \\
& \frac{b^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^4 g^2 i^3 (a + b x)} - \frac{3 b^2 d \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{(b c - a d)^4 g^2 i^3}
\end{aligned}$$

Result (type 4, 631 leaves, 32 steps):

$$\begin{aligned}
& - \frac{b^2 B}{(b c - a d)^3 g^2 i^3 (a + b x)} + \frac{B d}{4 (b c - a d)^2 g^2 i^3 (c + d x)^2} + \\
& \frac{5 b B d}{2 (b c - a d)^3 g^2 i^3 (c + d x)} + \frac{3 b^2 B d \operatorname{Log}[a + b x]}{2 (b c - a d)^4 g^2 i^3} + \frac{3 b^2 B d \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^2 i^3} - \\
& \frac{b^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^3 g^2 i^3 (a + b x)} - \frac{d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^2 g^2 i^3 (c + d x)^2} - \frac{2 b d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^3 g^2 i^3 (c + d x)} - \\
& \frac{3 b^2 d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d \operatorname{Log}[c + d x]}{2 (b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B d \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3}
\end{aligned}$$

**Problem 53: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^3 (c i + d i x)^3} dx$$

Optimal (type 3, 463 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^4 (a + b x)^2}{4 (b c - a d)^5 g^3 i^3 (c + d x)^2} + \frac{4 b B d^3 (a + b x)}{(b c - a d)^5 g^3 i^3 (c + d x)} + \frac{4 b^3 B d (c + d x)}{(b c - a d)^5 g^3 i^3 (a + b x)} - \\
& \frac{b^4 B (c + d x)^2}{4 (b c - a d)^5 g^3 i^3 (a + b x)^2} - \frac{3 b^2 B d^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^5 g^3 i^3} + \frac{d^4 (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^5 g^3 i^3 (c + d x)^2} - \\
& \frac{4 b d^3 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^3 i^3 (c + d x)} + \frac{4 b^3 d (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^3 i^3 (a + b x)} - \\
& \frac{b^4 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^5 g^3 i^3 (a + b x)^2} + \frac{6 b^2 d^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

Result (type 4, 673 leaves, 36 steps):



$$\begin{aligned}
& - \frac{b^2 B}{4 (b c - a d)^3 g^3 i^3 (a + b x)^2} + \frac{7 b^2 B d}{2 (b c - a d)^4 g^3 i^3 (a + b x)} - \frac{B d^2}{4 (b c - a d)^3 g^3 i^3 (c + d x)^2} - \\
& \frac{7 b B d^2}{2 (b c - a d)^4 g^3 i^3 (c + d x)} - \frac{3 b^2 B d^2 \operatorname{Log}[a + b x]^2}{(b c - a d)^5 g^3 i^3} - \frac{b^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^3 g^3 i^3 (a + b x)^2} + \\
& \frac{3 b^2 d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^4 g^3 i^3 (a + b x)} + \frac{d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^3 g^3 i^3 (c + d x)^2} + \frac{3 b d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^4 g^3 i^3 (c + d x)} + \\
& \frac{6 b^2 d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B d^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \\
& \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \frac{3 b^2 B d^2 \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} + \\
& \frac{6 b^2 B d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B d^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B d^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

**Problem 54: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{(a g + b g x)^4 (c i + d i x)^3} dx$$

Optimal (type 3, 563 leaves, 8 steps):

$$\begin{aligned}
& \frac{B d^5 (a + b x)^2}{4 (b c - a d)^6 g^4 i^3 (c + d x)^2} - \frac{5 b B d^4 (a + b x)}{(b c - a d)^6 g^4 i^3 (c + d x)} - \frac{10 b^3 B d^2 (c + d x)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \\
& \frac{5 b^4 B d (c + d x)^2}{4 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \frac{b^5 B (c + d x)^3}{9 (b c - a d)^6 g^4 i^3 (a + b x)^3} + \frac{5 b^2 B d^3 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^6 g^4 i^3} - \\
& \frac{d^5 (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^6 g^4 i^3 (c + d x)^2} + \frac{5 b d^4 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^6 g^4 i^3 (c + d x)} - \\
& \frac{10 b^3 d^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \frac{5 b^4 d (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \\
& \frac{b^5 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \frac{10 b^2 d^3 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$

Result (type 4, 825 leaves, 40 steps):

$$\begin{aligned}
& - \frac{b^2 B}{9 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \frac{11 b^2 B d}{12 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{47 b^2 B d^2}{6 (b c - a d)^5 g^4 i^3 (a + b x)} + \\
& \frac{B d^3}{4 (b c - a d)^4 g^4 i^3 (c + d x)^2} + \frac{9 b B d^3}{2 (b c - a d)^5 g^4 i^3 (c + d x)} - \frac{10 b^2 B d^3 \operatorname{Log}[a + b x]}{3 (b c - a d)^6 g^4 i^3} + \\
& \frac{5 b^2 B d^3 \operatorname{Log}[a + b x]^2}{(b c - a d)^6 g^4 i^3} - \frac{b^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \frac{3 b^2 d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \\
& \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^4 i^3 (a + b x)} - \frac{d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^4 g^4 i^3 (c + d x)^2} - \frac{4 b d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^5 g^4 i^3 (c + d x)} - \\
& \frac{10 b^2 d^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B d^3 \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{10 b^2 B d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \\
& \frac{5 b^2 B d^3 \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} - \frac{10 b^2 B d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{10 b^2 B d^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{10 b^2 B d^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$

**Problem 55: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 539 leaves, 11 steps):

$$\begin{aligned}
& \frac{3 B^2 (b c - a d)^4 g^3 i x}{10 b d^3} - \frac{3 B^2 (b c - a d)^3 g^3 i (c + d x)^2}{20 d^4} + \frac{b B^2 (b c - a d)^2 g^3 i (c + d x)^3}{30 d^4} - \\
& \frac{B (b c - a d)^2 g^3 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{30 b^2 d} - \frac{B (b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b^2} + \\
& \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{20 b^2} + \frac{g^3 i (a + b x)^4 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 b} + \\
& \frac{B (b c - a d)^3 g^3 i (a + b x)^2 \left( 3 A + B + 3 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{60 b^2 d^2} - \\
& \frac{B (b c - a d)^4 g^3 i (a + b x) \left( 6 A + 5 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{60 b^2 d^3} - \\
& \frac{B (b c - a d)^5 g^3 i \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( 6 A + 11 B + 6 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{60 b^2 d^4} - \\
& \frac{B^2 (b c - a d)^5 g^3 i \operatorname{Log} [c + d x]}{10 b^2 d^4} - \frac{B^2 (b c - a d)^5 g^3 i \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{10 b^2 d^4}
\end{aligned}$$

Result (type 4, 622 leaves, 54 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^4 g^3 i x}{10 b d^3} + \frac{B^2 (b c - a d)^4 g^3 i x}{60 b d^3} - \frac{B^2 (b c - a d)^3 g^3 i (a + b x)^2}{30 b^2 d^2} + \\
& \frac{B^2 (b c - a d)^2 g^3 i (a + b x)^3}{30 b^2 d} - \frac{B^2 (b c - a d)^4 g^3 i (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{10 b^2 d^3} + \\
& \frac{B (b c - a d)^3 g^3 i (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{20 b^2 d^2} - \frac{B (b c - a d)^2 g^3 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{30 b^2 d} - \\
& \frac{B (b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b^2} + \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b^2} + \\
& \frac{d g^3 i (a + b x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 b^2} + \frac{B^2 (b c - a d)^5 g^3 i \operatorname{Log} [c + d x]}{12 b^2 d^4} - \\
& \frac{B^2 (b c - a d)^5 g^3 i \operatorname{Log} \left[ - \frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{10 b^2 d^4} + \frac{B (b c - a d)^5 g^3 i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{10 b^2 d^4} + \\
& \frac{B^2 (b c - a d)^5 g^3 i \operatorname{Log} [c + d x]^2}{20 b^2 d^4} - \frac{B^2 (b c - a d)^5 g^3 i \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{10 b^2 d^4}
\end{aligned}$$

**Problem 56: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 450 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^3 g^2 i x}{3 b d^2} + \frac{B^2 (b c - a d)^2 g^2 i (c + d x)^2}{12 d^3} - \\
& \frac{B (b c - a d)^2 g^2 i (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b^2 d} - \frac{B (b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b^2} + \\
& \frac{(b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{12 b^2} + \frac{g^2 i (a + b x)^3 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b} + \\
& \frac{B (b c - a d)^3 g^2 i (a + b x) \left( 2 A + B + 2 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b^2 d^2} + \\
& \frac{B (b c - a d)^4 g^2 i \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( 2 A + 3 B + 2 B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b^2 d^3} + \\
& \frac{B^2 (b c - a d)^4 g^2 i \operatorname{Log} [c + d x]}{6 b^2 d^3} + \frac{B^2 (b c - a d)^4 g^2 i \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{6 b^2 d^3}
\end{aligned}$$

Result (type 4, 537 leaves, 46 steps):

$$\begin{aligned}
& \frac{A B (b c - a d)^3 g^2 i x}{6 b d^2} - \frac{B^2 (b c - a d)^3 g^2 i x}{12 b d^2} + \frac{B^2 (b c - a d)^2 g^2 i (a + b x)^2}{12 b^2 d} + \\
& \frac{B^2 (b c - a d)^3 g^2 i (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{6 b^2 d^2} - \frac{B (b c - a d)^2 g^2 i (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{12 b^2 d} - \\
& \frac{B (b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 b^2} + \frac{(b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 b^2} + \\
& \frac{d g^2 i (a + b x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b^2} - \frac{B^2 (b c - a d)^4 g^2 i \operatorname{Log} [c + d x]}{12 b^2 d^3} + \\
& \frac{B^2 (b c - a d)^4 g^2 i \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{6 b^2 d^3} - \frac{B (b c - a d)^4 g^2 i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{6 b^2 d^3} - \\
& \frac{B^2 (b c - a d)^4 g^2 i \operatorname{Log} [c + d x]^2}{12 b^2 d^3} + \frac{B^2 (b c - a d)^4 g^2 i \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{6 b^2 d^3}
\end{aligned}$$

**Problem 57: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 343 leaves, 9 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g i x}{3 b d} - \frac{B (b c - a d)^2 g i (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2 d} - \\
& \frac{B (b c - a d) g i (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2} + \\
& \frac{(b c - a d) g i (a + b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{6 b^2} + \frac{g i (a + b x)^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 b} - \\
& \frac{B (b c - a d)^3 g i \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2 d^2} - \\
& \frac{B^2 (b c - a d)^3 g i \operatorname{Log}[c + d x]}{3 b^2 d^2} - \frac{B^2 (b c - a d)^3 g i \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{3 b^2 d^2}
\end{aligned}$$

Result (type 4, 1214 leaves, 78 steps):

$$\begin{aligned}
& -\frac{2}{3} A b B \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) d g i x + \frac{B^2 (b c - a d)^2 g i x}{3 b d} - \frac{A B (b c - a d) (b c + a d) g i x}{b d} + \\
& \frac{a^2 B^2 (b c - a d) g i \operatorname{Log}[a + b x]}{3 b^2} - \frac{a^2 B^2 c g i \operatorname{Log}[a + b x]^2}{b} - \frac{a^3 B^2 d g i \operatorname{Log}[a + b x]^2}{3 b^2} + \\
& \frac{a^2 B^2 (b c + a d) g i \operatorname{Log}[a + b x]^2}{2 b^2} - \frac{B^2 (b c - a d) (b c + a d) g i (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{3 b^2 d} - \\
& \frac{1}{3} B (b c - a d) g i x^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) + \frac{2 a^2 B c g i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b} + \\
& \frac{2 a^3 B d g i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2} - \frac{a^2 B (b c + a d) g i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b^2} + \\
& a c g i x \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 + \frac{1}{2} (b c + a d) g i x^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 + \\
& \frac{1}{3} b d g i x^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 - \frac{B^2 c^2 (b c - a d) g i \operatorname{Log}[c + d x]}{3 d^2} + \\
& \frac{B^2 (b c - a d)^2 (b c + a d) g i \operatorname{Log}[c + d x]}{3 b^2 d^2} + \frac{2 b B^2 c^3 g i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 d^2} + \\
& \frac{2 a B^2 c^2 g i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d} - \frac{B^2 c^2 (b c + a d) g i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2} - \\
& \frac{2 b B c^3 g i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{3 d^2} - \frac{2 a B c^2 g i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d} + \\
& \frac{B c^2 (b c + a d) g i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d^2} - \frac{b B^2 c^3 g i \operatorname{Log}[c + d x]^2}{3 d^2} - \\
& \frac{a B^2 c^2 g i \operatorname{Log}[c + d x]^2}{d} + \frac{B^2 c^2 (b c + a d) g i \operatorname{Log}[c + d x]^2}{2 d^2} + \frac{2 a^2 B^2 c g i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b} + \\
& \frac{2 a^3 B^2 d g i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^2} - \frac{a^2 B^2 (b c + a d) g i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2} + \\
& \frac{2 a^2 B^2 c g i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b} + \frac{2 a^3 B^2 d g i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^2} - \\
& \frac{a^2 B^2 (b c + a d) g i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2} + \frac{2 b B^2 c^3 g i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 d^2} + \\
& \frac{2 a B^2 c^2 g i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d} - \frac{B^2 c^2 (b c + a d) g i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^2}
\end{aligned}$$

**Problem 58: Result valid but suboptimal antiderivative.**

$$\int (c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{B (b c - a d) i (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2} + \\
 & \frac{i (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 d} + \frac{B^2 (b c - a d)^2 i \operatorname{Log} [c + d x]}{b^2 d} + \\
 & \frac{B (b c - a d)^2 i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^2 d} - \frac{B^2 (b c - a d)^2 i \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^2 d}
 \end{aligned}$$

Result (type 4, 283 leaves, 16 steps):

$$\begin{aligned}
 & - \frac{A B (b c - a d) i x}{b} + \frac{B^2 (b c - a d)^2 i \operatorname{Log} [a + b x]^2}{2 b^2 d} - \\
 & \frac{B^2 (b c - a d) i (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{b^2} - \frac{B (b c - a d)^2 i \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 d} + \\
 & \frac{i (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 d} + \frac{B^2 (b c - a d)^2 i \operatorname{Log} [c + d x]}{b^2 d} - \\
 & \frac{B^2 (b c - a d)^2 i \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{b^2 d} - \frac{B^2 (b c - a d)^2 i \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^2 d}
 \end{aligned}$$

**Problem 59: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{a g + b g x} dx$$

Optimal (type 4, 286 leaves, 8 steps):

$$\begin{aligned}
 & \frac{2 B (b c - a d) i \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{b^2 g} + \frac{d i (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{b^2 g} - \\
 & \frac{(b c - a d) i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^2 g} + \frac{2 B^2 (b c - a d) i \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b^2 g} + \\
 & \frac{2 B (b c - a d) i \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^2 g} + \frac{2 B^2 (b c - a d) i \operatorname{PolyLog} \left[ 3, \frac{b (c + d x)}{d (a + b x)} \right]}{b^2 g}
 \end{aligned}$$

Result (type 4, 644 leaves, 39 steps):

$$\begin{aligned}
& - \frac{a B^2 d i \operatorname{Log}[a + b x]^2}{b^2 g} - \frac{A B (b c - a d) i \operatorname{Log}[a + b x]^2}{b^2 g} - \\
& \frac{B^2 (b c - a d) i \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^2 g} - \frac{B^2 (b c - a d) i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^2 g} + \\
& \frac{2 a B d i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^2 g} + \frac{d i x \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b g} + \\
& \frac{(b c - a d) i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^2 g} + \frac{2 B^2 c i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b g} - \\
& \frac{2 B c i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b g} - \frac{B^2 c i \operatorname{Log}[c + d x]^2}{b g} + \frac{2 a B^2 d i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g} + \\
& \frac{2 A B (b c - a d) i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g} + \frac{2 a B^2 d i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g} + \\
& \frac{2 A B (b c - a d) i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g} + \frac{2 B^2 c i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b g} + \\
& \frac{2 B^2 (b c - a d) i \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g} + \frac{2 B^2 (b c - a d) i \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g}
\end{aligned}$$

**Problem 60: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{(a g + b g x)^2} dx$$

Optimal (type 4, 241 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 B^2 i (c + d x)}{b g^2 (a + b x)} - \frac{2 B i (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b g^2 (a + b x)} - \\
& \frac{i (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b g^2 (a + b x)} - \frac{d i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2} + \\
& \frac{2 B d i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2} + \frac{2 B^2 d i \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2}
\end{aligned}$$

Result (type 4, 705 leaves, 43 steps):



$$\begin{aligned}
& - \frac{2 B^2 (b c - a d) i}{b^2 g^2 (a + b x)} - \frac{2 B^2 d i \operatorname{Log}[a + b x]}{b^2 g^2} - \frac{A B d i \operatorname{Log}[a + b x]^2}{b^2 g^2} + \frac{B^2 d i \operatorname{Log}[a + b x]^2}{b^2 g^2} - \\
& \frac{B^2 d i \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^2 g^2} - \frac{B^2 d i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^2 g^2} - \\
& \frac{2 B (b c - a d) i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^2 g^2 (a + b x)} - \frac{2 B d i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^2 g^2} - \\
& \frac{(b c - a d) i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^2 g^2 (a + b x)} + \frac{d i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^2 g^2} + \frac{2 B^2 d i \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B^2 d i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} + \frac{2 B d i \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} + \\
& \frac{B^2 d i \operatorname{Log}[c + d x]^2}{b^2 g^2} + \frac{2 A B d i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} + \\
& \frac{2 A B d i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} + \\
& \frac{2 B^2 d i \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g^2} + \frac{2 B^2 d i \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g^2}
\end{aligned}$$

**Problem 61:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$- \frac{B^2 i (c + d x)^2}{4 (b c - a d) g^3 (a + b x)^2} - \frac{B i (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 (b c - a d) g^3 (a + b x)^2} - \frac{i (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 (b c - a d) g^3 (a + b x)^2}$$

Result (type 4, 639 leaves, 58 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d) i}{4 b^2 g^3 (a + b x)^2} - \frac{B^2 d i}{2 b^2 g^3 (a + b x)} - \frac{B^2 d^2 i \operatorname{Log}[a + b x]}{2 b^2 (b c - a d) g^3} + \\
& \frac{B^2 d^2 i \operatorname{Log}[a + b x]^2}{2 b^2 (b c - a d) g^3} - \frac{B (b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 b^2 g^3 (a + b x)^2} - \frac{B d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b^2 g^3 (a + b x)} - \\
& \frac{B d^2 i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{b^2 (b c - a d) g^3} - \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 b^2 g^3 (a + b x)^2} - \\
& \frac{d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{b^2 g^3 (a + b x)} + \frac{B^2 d^2 i \operatorname{Log}[c + d x]}{2 b^2 (b c - a d) g^3} - \frac{B^2 d^2 i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 (b c - a d) g^3} + \\
& \frac{B d^2 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{b^2 (b c - a d) g^3} + \frac{B^2 d^2 i \operatorname{Log}[c + d x]^2}{2 b^2 (b c - a d) g^3} - \\
& \frac{B^2 d^2 i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 (b c - a d) g^3} - \frac{B^2 d^2 i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 (b c - a d) g^3} - \frac{B^2 d^2 i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 (b c - a d) g^3}
\end{aligned}$$

**Problem 62: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 287 leaves, 7 steps):

$$\begin{aligned}
& \frac{B^2 d i (c + d x)^2}{4 (b c - a d)^2 g^4 (a + b x)^2} - \frac{2 b B^2 i (c + d x)^3}{27 (b c - a d)^2 g^4 (a + b x)^3} + \\
& \frac{B d i (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^2 g^4 (a + b x)^2} - \frac{2 b B i (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{9 (b c - a d)^2 g^4 (a + b x)^3} + \\
& \frac{d i (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^2 g^4 (a + b x)^2} - \frac{b i (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 (b c - a d)^2 g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 741 leaves, 66 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d) i}{27 b^2 g^4 (a + b x)^3} + \frac{B^2 d i}{36 b^2 g^4 (a + b x)^2} + \frac{5 B^2 d^2 i}{18 b^2 (b c - a d) g^4 (a + b x)} + \frac{5 B^2 d^3 i \operatorname{Log}[a + b x]}{18 b^2 (b c - a d)^2 g^4} - \\
& \frac{B^2 d^3 i \operatorname{Log}[a + b x]^2}{6 b^2 (b c - a d)^2 g^4} - \frac{2 B (b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{9 b^2 g^4 (a + b x)^3} - \frac{B d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{6 b^2 g^4 (a + b x)^2} + \\
& \frac{B d^2 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2 (b c - a d) g^4 (a + b x)} + \frac{B d^3 i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^2 (b c - a d)^2 g^4} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 b^2 g^4 (a + b x)^3} - \frac{d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 b^2 g^4 (a + b x)^2} - \frac{5 B^2 d^3 i \operatorname{Log}[c + d x]}{18 b^2 (b c - a d)^2 g^4} + \\
& \frac{B^2 d^3 i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b^2 (b c - a d)^2 g^4} - \frac{B d^3 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{3 b^2 (b c - a d)^2 g^4} - \frac{B^2 d^3 i \operatorname{Log}[c + d x]^2}{6 b^2 (b c - a d)^2 g^4} + \\
& \frac{B^2 d^3 i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4} + \frac{B^2 d^3 i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4} + \frac{B^2 d^3 i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4}
\end{aligned}$$

**Problem 63: Result unnecessarily involves higher level functions.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 445 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B^2 d^2 i (c + d x)^2}{4 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B^2 d i (c + d x)^3}{27 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 B^2 i (c + d x)^4}{32 (b c - a d)^3 g^5 (a + b x)^4} - \\
& \frac{B d^2 i (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{2 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B d i (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{9 (b c - a d)^3 g^5 (a + b x)^3} - \\
& \frac{b^2 B i (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{8 (b c - a d)^3 g^5 (a + b x)^4} - \frac{d^2 i (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^3 g^5 (a + b x)^2} + \\
& \frac{2 b d i (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 i (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 (b c - a d)^3 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 826 leaves, 74 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d) i}{32 b^2 g^5 (a + b x)^4} + \frac{5 B^2 d i}{216 b^2 g^5 (a + b x)^3} + \frac{B^2 d^2 i}{144 b^2 (b c - a d) g^5 (a + b x)^2} - \\
& \frac{13 B^2 d^3 i}{72 b^2 (b c - a d)^2 g^5 (a + b x)} - \frac{13 B^2 d^4 i \operatorname{Log}[a + b x]}{72 b^2 (b c - a d)^3 g^5} + \frac{B^2 d^4 i \operatorname{Log}[a + b x]^2}{12 b^2 (b c - a d)^3 g^5} - \\
& \frac{B (b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{8 b^2 g^5 (a + b x)^4} - \frac{B d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{18 b^2 g^5 (a + b x)^3} + \frac{B d^2 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{12 b^2 (b c - a d) g^5 (a + b x)^2} - \\
& \frac{B d^3 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{6 b^2 (b c - a d)^2 g^5 (a + b x)} - \frac{B d^4 i \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{6 b^2 (b c - a d)^3 g^5} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 b^2 g^5 (a + b x)^4} - \frac{d i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 b^2 g^5 (a + b x)^3} + \frac{13 B^2 d^4 i \operatorname{Log}[c + d x]}{72 b^2 (b c - a d)^3 g^5} - \\
& \frac{B^2 d^4 i \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{6 b^2 (b c - a d)^3 g^5} + \frac{B d^4 i \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{6 b^2 (b c - a d)^3 g^5} + \frac{B^2 d^4 i \operatorname{Log}[c + d x]^2}{12 b^2 (b c - a d)^3 g^5} - \\
& \frac{B^2 d^4 i \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{6 b^2 (b c - a d)^3 g^5} - \frac{B^2 d^4 i \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{6 b^2 (b c - a d)^3 g^5} - \frac{B^2 d^4 i \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{6 b^2 (b c - a d)^3 g^5}
\end{aligned}$$

**Problem 64: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 711 leaves, 17 steps):

$$\begin{aligned}
& \frac{3 B^2 (b c - a d)^5 g^3 i^2 x}{20 b^2 d^3} + \frac{B^2 (b c - a d)^2 g^3 i^2 (a + b x)^4}{60 b^3} - \frac{3 B^2 (b c - a d)^4 g^3 i^2 (c + d x)^2}{40 b d^4} + \\
& \frac{B^2 (b c - a d)^3 g^3 i^2 (c + d x)^3}{60 d^4} - \frac{B (b c - a d)^3 g^3 i^2 (a + b x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{90 b^3 d} - \\
& \frac{B (b c - a d)^2 g^3 i^2 (a + b x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{20 b^3} - \\
& \frac{B (b c - a d) g^3 i^2 (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{15 b^2} + \\
& \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{60 b^3} + \\
& \frac{(b c - a d) g^3 i^2 (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{15 b^2} + \\
& \frac{g^3 i^2 (a + b x)^4 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{6 b} + \\
& \frac{B (b c - a d)^4 g^3 i^2 (a + b x)^2 \left( 3 A + B + 3 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{180 b^3 d^2} - \\
& \frac{B (b c - a d)^5 g^3 i^2 (a + b x) \left( 6 A + 5 B + 6 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{180 b^3 d^3} - \\
& \frac{B (b c - a d)^6 g^3 i^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( 6 A + 11 B + 6 B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{180 b^3 d^4} - \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{Log}[c + d x]}{20 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{30 b^3 d^4}
\end{aligned}$$

Result (type 4, 790 leaves, 86 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^5 g^3 i^2 x}{30 b^2 d^3} + \frac{B^2 (b c - a d)^5 g^3 i^2 x}{45 b^2 d^3} - \frac{7 B^2 (b c - a d)^4 g^3 i^2 (a + b x)^2}{360 b^3 d^2} + \\
& \frac{B^2 (b c - a d)^3 g^3 i^2 (a + b x)^3}{60 b^3 d} + \frac{B^2 (b c - a d)^2 g^3 i^2 (a + b x)^4}{60 b^3} - \\
& \frac{B^2 (b c - a d)^5 g^3 i^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{30 b^3 d^3} + \frac{B (b c - a d)^4 g^3 i^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{60 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^3 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{90 b^3 d} - \\
& \frac{7 B (b c - a d)^2 g^3 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{60 b^3} - \\
& \frac{B d (b c - a d) g^3 i^2 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{15 b^3} + \\
& \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{4 b^3} + \\
& \frac{2 d (b c - a d) g^3 i^2 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{5 b^3} + \frac{d^2 g^3 i^2 (a + b x)^6 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{6 b^3} + \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{Log}[c + d x]}{90 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{30 b^3 d^4} + \\
& \frac{B (b c - a d)^6 g^3 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{30 b^3 d^4} + \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{Log}[c + d x]^2}{60 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{30 b^3 d^4}
\end{aligned}$$

**Problem 65: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 dx$$

Optimal (type 4, 761 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^4 g^2 i^2 x}{10 b^2 d^2} - \frac{B^2 (b c - a d)^3 g^2 i^2 (c + d x)^2}{20 b d^3} + \frac{B^2 (b c - a d)^2 g^2 i^2 (c + d x)^3}{30 d^3} + \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{30 b^3 d^3} - \frac{B (b c - a d)^3 g^2 i^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{30 b^3 d} - \\
& \frac{B (b c - a d)^2 g^2 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{15 b^3} - \\
& \frac{B (b c - a d)^3 g^2 i^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{5 b d^3} + \\
& \frac{4 B (b c - a d)^2 g^2 i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{15 d^3} - \\
& \frac{b B (b c - a d) g^2 i^2 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 d^3} + \\
& \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{30 b^3} + \\
& \frac{(b c - a d) g^2 i^2 (a + b x)^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{10 b^2} + \\
& \frac{g^2 i^2 (a + b x)^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{5 b} + \\
& \frac{B (b c - a d)^4 g^2 i^2 (a + b x) \left(2 A + B + 2 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{30 b^3 d^2} + \\
& \frac{B (b c - a d)^5 g^2 i^2 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(2 A + 3 B + 2 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{30 b^3 d^3} + \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{Log}[c + d x]}{10 b^3 d^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{15 b^3 d^3}
\end{aligned}$$

Result (type 4, 666 leaves, 74 steps):

$$\begin{aligned}
& \frac{A B (b c - a d)^4 g^2 i^2 x}{15 b^2 d^2} - \frac{B^2 (b c - a d)^4 g^2 i^2 x}{15 b^2 d^2} + \frac{B^2 (b c - a d)^3 g^2 i^2 (a + b x)^2}{20 b^3 d} + \\
& \frac{B^2 (b c - a d)^2 g^2 i^2 (a + b x)^3}{30 b^3} + \frac{B^2 (b c - a d)^4 g^2 i^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{15 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^2 i^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{30 b^3 d} - \\
& \frac{B (b c - a d)^2 g^2 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{5 b^3} - \\
& \frac{B d (b c - a d) g^2 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{10 b^3} + \\
& \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{3 b^3} + \\
& \frac{d (b c - a d) g^2 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 b^3} + \\
& \frac{d^2 g^2 i^2 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{5 b^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{15 b^3 d^3} - \\
& \frac{B (b c - a d)^5 g^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{15 b^3 d^3} - \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{Log}[c + d x]^2}{30 b^3 d^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{15 b^3 d^3}
\end{aligned}$$

**Problem 66: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 dx$$

Optimal (type 4, 589 leaves, 14 steps):



$$\begin{aligned}
& \frac{B^2 (b c - a d)^3 g i^2 x}{12 b^2 d} + \frac{B^2 (b c - a d)^2 g i^2 (c + d x)^2}{12 b d^2} - \\
& \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{12 b^3 d^2} - \frac{B (b c - a d)^3 g i^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 b^3 d} - \\
& \frac{B (b c - a d)^2 g i^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 b^3} + \frac{B (b c - a d)^2 g i^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{4 b d^2} - \\
& \frac{B (b c - a d) g i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 d^2} + \frac{(b c - a d)^2 g i^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{12 b^3} + \\
& \frac{(b c - a d) g i^2 (a + b x)^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{6 b^2} + \\
& \frac{g i^2 (a + b x)^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{4 b} - \\
& \frac{B (b c - a d)^4 g i^2 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(A + B + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 b^3 d^2} - \\
& \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}[c + d x]}{4 b^3 d^2} - \frac{B^2 (b c - a d)^4 g i^2 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{6 b^3 d^2}
\end{aligned}$$

Result (type 4, 570 leaves, 46 steps):

$$\begin{aligned}
& \frac{A B (b c - a d)^3 g i^2 x}{6 b^2 d} + \frac{B^2 (b c - a d)^3 g i^2 x}{12 b^2 d} + \\
& \frac{B^2 (b c - a d)^2 g i^2 (c + d x)^2}{12 b d^2} + \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}[a + b x]}{12 b^3 d^2} - \\
& \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}[a + b x]^2}{12 b^3 d^2} + \frac{B^2 (b c - a d)^3 g i^2 (a + b x) \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{6 b^3 d} + \\
& \frac{B (b c - a d)^2 g i^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{12 b d^2} - \frac{B (b c - a d) g i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 d^2} + \\
& \frac{B (b c - a d)^4 g i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{6 b^3 d^2} - \\
& \frac{(b c - a d) g i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 d^2} + \\
& \frac{b g i^2 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{4 d^2} - \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}[c + d x]}{6 b^3 d^2} + \\
& \frac{B^2 (b c - a d)^4 g i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c - a d}\right]}{6 b^3 d^2} + \frac{B^2 (b c - a d)^4 g i^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c - a d}\right]}{6 b^3 d^2}
\end{aligned}$$

### Problem 67: Result valid but suboptimal antiderivative.

$$\int (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2 dx$$

Optimal (type 4, 334 leaves, 11 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d)^2 i^2 x}{3 b^2} + \frac{B^2 (b c - a d)^3 i^2 \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{3 b^3 d} - \\ & \frac{2 B (b c - a d)^2 i^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^3} - \frac{B (b c - a d) i^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b d} + \\ & \frac{i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 d} + \frac{B^2 (b c - a d)^3 i^2 \operatorname{Log} [c + d x]}{b^3 d} + \\ & \frac{2 B (b c - a d)^3 i^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{3 b^3 d} - \frac{2 B^2 (b c - a d)^3 i^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{3 b^3 d} \end{aligned}$$

Result (type 4, 420 leaves, 20 steps):

$$\begin{aligned} & -\frac{2 A B (b c - a d)^2 i^2 x}{3 b^2} + \frac{B^2 (b c - a d)^2 i^2 x}{3 b^2} + \frac{B^2 (b c - a d)^3 i^2 \operatorname{Log} [a + b x]}{3 b^3 d} + \\ & \frac{B^2 (b c - a d)^3 i^2 \operatorname{Log} [a + b x]^2}{3 b^3 d} - \frac{2 B^2 (b c - a d)^2 i^2 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{3 b^3} - \\ & \frac{B (b c - a d) i^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b d} - \frac{2 B (b c - a d)^3 i^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 b^3 d} + \\ & \frac{i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 d} + \frac{2 B^2 (b c - a d)^3 i^2 \operatorname{Log} [c + d x]}{3 b^3 d} - \\ & \frac{2 B^2 (b c - a d)^3 i^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{3 b^3 d} - \frac{2 B^2 (b c - a d)^3 i^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{3 b^3 d} \end{aligned}$$

### Problem 68: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{a g + b g x} dx$$

Optimal (type 4, 535 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B d (b c - a d) i^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)}{b^3 g} + \\
& \frac{2 B (b c - a d)^2 i^2 \operatorname{Log} \left[ \frac{b c - a d}{b (c+d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)}{b^3 g} + \\
& \frac{d (b c - a d) i^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)^2}{b^3 g} + \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)^2}{2 b g} + \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [c + d x]}{b^3 g} + \frac{B (b c - a d)^2 i^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g} - \\
& \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)^2 \operatorname{Log} \left[ 1 - \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g} + \\
& \frac{2 B^2 (b c - a d)^2 i^2 \operatorname{PolyLog} \left[ 2, \frac{d (a+b x)}{b (c+d x)} \right]}{b^3 g} - \frac{B^2 (b c - a d)^2 i^2 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g} + \\
& \frac{2 B (b c - a d)^2 i^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right) \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g} + \\
& \frac{2 B^2 (b c - a d)^2 i^2 \operatorname{PolyLog} \left[ 3, \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g}
\end{aligned}$$

Result(type 4, 1676 leaves, 86 steps):

$$\begin{aligned}
& - \frac{A B d (b c - a d) i^2 x}{b^2 g} - \frac{a B^2 d (b c - a d) i^2 \operatorname{Log} [a + b x]^2}{b^3 g} + \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [a + b x]^2}{2 b^3 g} - \frac{A B (b c - a d)^2 i^2 \operatorname{Log} [g (a + b x)]^2}{b^3 g} + \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [g (a + b x)]^3}{3 b^3 g} - \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [a + b x]^2 \operatorname{Log} [-c - d x]}{b^3 g} + \\
& \frac{2 B^2 (b c - a d)^2 i^2 \operatorname{Log} [a + b x] \operatorname{Log} [g (a + b x)] \operatorname{Log} [-c - d x]}{b^3 g} - \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [g (a + b x)]^2 \operatorname{Log} [-c - d x]}{b^3 g} + \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log} \left[ \frac{1}{c+d x} \right]^2}{b^3 g} - \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log} [g (a + b x)] \operatorname{Log} \left[ \frac{1}{c+d x} \right]^2}{b^3 g} - \frac{B^2 d (b c - a d) i^2 (a + b x) \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right]}{b^3 g} + \\
& \frac{2 a B d (b c - a d) i^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)}{b^3 g} - \\
& \frac{B (b c - a d)^2 i^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)}{b^3 g} + \\
& \frac{d (b c - a d) i^2 x \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)^2}{b^2 g} + \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+b x)}}{c+d x} \right] \right)^2}{2 b g} +
\end{aligned}$$

$$\begin{aligned}
& \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}[c + dx]}{b^3 g} + \frac{2 B^2 c (bc - ad) i^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{b^2 g} - \\
& \frac{2 B c (bc - ad) i^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c + dx]}{b^2 g} - \frac{B^2 c (bc - ad) i^2 \operatorname{Log}[c + dx]^2}{b^2 g} + \\
& \frac{2 a B^2 d (bc - ad) i^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^3 g} - \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^3 g} + \\
& \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}[a + bx]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^3 g} + \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}[g(a + bx)]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^3 g} + \\
& \frac{(bc - ad)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2 \operatorname{Log}[ag + bgx]}{b^3 g} + \\
& \frac{2 A B (bc - ad)^2 i^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag + bgx]}{b^3 g} - \frac{1}{b^3 g} \\
& 2 B^2 (bc - ad)^2 i^2 \left(\operatorname{Log}[a + bx] + \operatorname{Log}\left[\frac{1}{c + dx}\right] - \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right) \operatorname{Log}\left[\frac{b(c + dx)}{bc - ad}\right] \operatorname{Log}[ag + bgx] - \\
& \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \operatorname{Log}[ag + bgx]^2}{b^3 g} - \frac{B^2 (bc - ad)^2 i^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag + bgx]^2}{b^3 g} + \\
& \frac{2 a B^2 d (bc - ad) i^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} + \frac{2 A B (bc - ad)^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \\
& \frac{B^2 (bc - ad)^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} + \frac{2 B^2 (bc - ad)^2 i^2 \operatorname{Log}[a + bx] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \\
& \frac{1}{b^3 g} 2 B^2 (bc - ad)^2 i^2 \left(\operatorname{Log}[a + bx] + \operatorname{Log}\left[\frac{1}{c + dx}\right] - \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right) \operatorname{PolyLog}\left[2, -\frac{d(a + bx)}{bc - ad}\right] + \\
& \frac{2 B^2 c (bc - ad) i^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b^2 g} - \frac{2 B^2 (bc - ad)^2 i^2 \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b^3 g} - \\
& \frac{2 B^2 (bc - ad)^2 i^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \frac{2 B^2 (bc - ad)^2 i^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{b^3 g}
\end{aligned}$$

**Problem 69: Result valid but suboptimal antiderivative.**

$$\int \frac{(ci + di x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag + bgx)^2} dx$$

Optimal (type 4, 442 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d) i^2 (c + d x)}{b^2 g^2 (a + b x)} - \frac{2 B (b c - a d) i^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right)}{b^2 g^2 (a + b x)} + \\
& \frac{2 B d (b c - a d) i^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right)}{b^3 g^2} + \\
& \frac{d^2 i^2 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right)^2}{b^3 g^2} - \frac{(b c - a d) i^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right)^2}{b^2 g^2 (a + b x)} - \\
& \frac{2 d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^3 g^2} + \frac{2 B^2 d (b c - a d) i^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^3 g^2} + \\
& \frac{4 B d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+b x)}}{c+d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^3 g^2} + \\
& \frac{4 B^2 d (b c - a d) i^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^3 g^2}
\end{aligned}$$

Result (type 4, 1219 leaves, 65 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 i^2}{b^3 g^2 (a + b x)} - \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}[a + b x]}{b^3 g^2} - \frac{a B^2 d^2 i^2 \operatorname{Log}[a + b x]^2}{b^3 g^2} - \\
& \frac{2 A B d (b c - a d) i^2 \operatorname{Log}[a + b x]^2}{b^3 g^2} + \frac{B^2 d (b c - a d) i^2 \operatorname{Log}[a + b x]^2}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^3 g^2} - \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^3 g^2} - \\
& \frac{2 B (b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2 (a + b x)} + \frac{2 a B d^2 i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2} - \\
& \frac{2 B d (b c - a d) i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^2} + \frac{d^2 i^2 x \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^2 g^2} - \\
& \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g^2 (a + b x)} + \frac{2 d (b c - a d) i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g^2} + \\
& \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}[c + d x]}{b^3 g^2} + \frac{2 B^2 c d i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^2} - \frac{2 B c d i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} + \\
& \frac{2 B d (b c - a d) i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^3 g^2} - \frac{B^2 c d i^2 \operatorname{Log}[c + d x]^2}{b^2 g^2} + \\
& \frac{B^2 d (b c - a d) i^2 \operatorname{Log}[c + d x]^2}{b^3 g^2} + \frac{2 a B^2 d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{4 A B d (b c - a d) i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \frac{2 a B^2 d^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{4 A B d (b c - a d) i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} - \frac{2 B^2 d (b c - a d) i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{2 B^2 c d i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d (b c - a d) i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{4 B^2 d (b c - a d) i^2 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^2} + \\
& \frac{4 B^2 d (b c - a d) i^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^2}
\end{aligned}$$

### Problem 70: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2}{(a g + b g x)^3} dx$$

Optimal (type 4, 387 leaves, 10 steps):

$$\begin{aligned} & -\frac{2 B^2 d i^2 (c + d x)}{b^2 g^3 (a + b x)} - \frac{B^2 i^2 (c + d x)^2}{4 b g^3 (a + b x)^2} - \frac{2 B d i^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)}{b^2 g^3 (a + b x)} - \\ & \frac{B i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)}{2 b g^3 (a + b x)^2} - \frac{d i^2 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2}{b^2 g^3 (a + b x)} - \\ & \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2}{2 b g^3 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2 \operatorname{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^3} + \\ & \frac{2 B d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^3} + \frac{2 B^2 d^2 i^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^3} \end{aligned}$$

Result (type 4, 932 leaves, 73 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^2 i^2}{4 b^3 g^3 (a + b x)^2} - \frac{5 B^2 d (b c - a d) i^2}{2 b^3 g^3 (a + b x)} - \frac{5 B^2 d^2 i^2 \operatorname{Log}[a + b x]}{2 b^3 g^3} - \\
& \frac{A B d^2 i^2 \operatorname{Log}[a + b x]^2}{b^3 g^3} + \frac{3 B^2 d^2 i^2 \operatorname{Log}[a + b x]^2}{2 b^3 g^3} - \frac{B^2 d^2 i^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^3 g^3} - \\
& \frac{B^2 d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^3 g^3} - \frac{B (b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^3 g^3 (a + b x)^2} - \\
& \frac{3 B d (b c - a d) i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^3 (a + b x)} - \frac{3 B d^2 i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^3 g^3} - \\
& \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 b^3 g^3 (a + b x)^2} - \frac{2 d (b c - a d) i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g^3 (a + b x)} + \\
& \frac{d^2 i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g^3} + \frac{5 B^2 d^2 i^2 \operatorname{Log}[c + d x]}{2 b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^3} + \frac{3 B d^2 i^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^3 g^3} + \\
& \frac{3 B^2 d^2 i^2 \operatorname{Log}[c + d x]^2}{2 b^3 g^3} + \frac{2 A B d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} + \frac{2 A B d^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^3} - \frac{3 B^2 d^2 i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} + \\
& \frac{2 B^2 d^2 i^2 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^3} + \frac{2 B^2 d^2 i^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^3}
\end{aligned}$$

**Problem 71:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 147 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 B^2 i^2 (c + d x)^3}{27 (b c - a d) g^4 (a + b x)^3} - \\
& \frac{2 B i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{9 (b c - a d) g^4 (a + b x)^3} - \frac{i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{3 (b c - a d) g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 827 leaves, 92 steps):



$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 i^2}{27 b^3 g^4 (a + b x)^3} - \frac{2 B^2 d (b c - a d) i^2}{9 b^3 g^4 (a + b x)^2} - \frac{2 B^2 d^2 i^2}{9 b^3 g^4 (a + b x)} - \\
& \frac{2 B^2 d^3 i^2 \operatorname{Log}[a + b x]}{9 b^3 (b c - a d) g^4} + \frac{B^2 d^3 i^2 \operatorname{Log}[a + b x]^2}{3 b^3 (b c - a d) g^4} - \frac{2 B (b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{9 b^3 g^4 (a + b x)^3} - \\
& \frac{2 B d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^3 g^4 (a + b x)^2} - \frac{2 B d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^3 g^4 (a + b x)} - \\
& \frac{2 B d^3 i^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 b^3 (b c - a d) g^4} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 b^3 g^4 (a + b x)^3} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{b^3 g^4 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{b^3 g^4 (a + b x)} + \frac{2 B^2 d^3 i^2 \operatorname{Log}[c + d x]}{9 b^3 (b c - a d) g^4} - \\
& \frac{2 B^2 d^3 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4} + \frac{2 B d^3 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4} + \\
& \frac{B^2 d^3 i^2 \operatorname{Log}[c + d x]^2}{3 b^3 (b c - a d) g^4} - \frac{2 B^2 d^3 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4} - \\
& \frac{2 B^2 d^3 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4} - \frac{2 B^2 d^3 i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4}
\end{aligned}$$

**Problem 72: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 299 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 B^2 d i^2 (c + d x)^3}{27 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B^2 i^2 (c + d x)^4}{32 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{2 B d i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{9 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{8 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{d i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{4 (b c - a d)^2 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 920 leaves, 104 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 i^2}{32 b^3 g^5 (a + bx)^4} - \frac{11 B^2 d (bc - ad) i^2}{216 b^3 g^5 (a + bx)^3} + \frac{5 B^2 d^2 i^2}{144 b^3 g^5 (a + bx)^2} + \frac{7 B^2 d^3 i^2}{72 b^3 (bc - ad) g^5 (a + bx)} + \\
& \frac{7 B^2 d^4 i^2 \operatorname{Log}[a + bx]}{72 b^3 (bc - ad)^2 g^5} - \frac{B^2 d^4 i^2 \operatorname{Log}[a + bx]^2}{12 b^3 (bc - ad)^2 g^5} - \frac{B (bc - ad)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{8 b^3 g^5 (a + bx)^4} - \\
& \frac{5 B d (bc - ad) i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{18 b^3 g^5 (a + bx)^3} - \frac{B d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{12 b^3 g^5 (a + bx)^2} + \\
& \frac{B d^3 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 b^3 (bc - ad) g^5 (a + bx)} + \frac{B d^4 i^2 \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{6 b^3 (bc - ad)^2 g^5} - \\
& \frac{(bc - ad)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{4 b^3 g^5 (a + bx)^4} - \frac{2 d (bc - ad) i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{3 b^3 g^5 (a + bx)^3} - \\
& \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{2 b^3 g^5 (a + bx)^2} - \frac{7 B^2 d^4 i^2 \operatorname{Log}[c + dx]}{72 b^3 (bc - ad)^2 g^5} + \frac{B^2 d^4 i^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{6 b^3 (bc - ad)^2 g^5} - \\
& \frac{B d^4 i^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{Log}[c + dx]}{6 b^3 (bc - ad)^2 g^5} - \frac{B^2 d^4 i^2 \operatorname{Log}[c + dx]^2}{12 b^3 (bc - ad)^2 g^5} + \\
& \frac{B^2 d^4 i^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{6 b^3 (bc - ad)^2 g^5} + \frac{B^2 d^4 i^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{6 b^3 (bc - ad)^2 g^5} + \frac{B^2 d^4 i^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{6 b^3 (bc - ad)^2 g^5}
\end{aligned}$$

**Problem 73:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(ci + di x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{(ag + bg x)^6} dx$$

Optimal (type 3, 463 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 i^2 (c + dx)^3}{27 (bc - ad)^3 g^6 (a + bx)^3} + \frac{b B^2 d i^2 (c + dx)^4}{16 (bc - ad)^3 g^6 (a + bx)^4} - \frac{2 b^2 B^2 i^2 (c + dx)^5}{125 (bc - ad)^3 g^6 (a + bx)^5} - \\
& \frac{2 B d^2 i^2 (c + dx)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{9 (bc - ad)^3 g^6 (a + bx)^3} + \frac{b B d i^2 (c + dx)^4 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{4 (bc - ad)^3 g^6 (a + bx)^4} - \\
& \frac{2 b^2 B i^2 (c + dx)^5 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{25 (bc - ad)^3 g^6 (a + bx)^5} - \frac{d^2 i^2 (c + dx)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{3 (bc - ad)^3 g^6 (a + bx)^3} + \\
& \frac{b d i^2 (c + dx)^4 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{2 (bc - ad)^3 g^6 (a + bx)^4} - \frac{b^2 i^2 (c + dx)^5 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{5 (bc - ad)^3 g^6 (a + bx)^5}
\end{aligned}$$

Result (type 4, 1009 leaves, 116 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 i^2}{125 b^3 g^6 (a + b x)^5} - \frac{7 B^2 d (b c - a d) i^2}{400 b^3 g^6 (a + b x)^4} + \frac{43 B^2 d^2 i^2}{2700 b^3 g^6 (a + b x)^3} - \frac{13 B^2 d^3 i^2}{1800 b^3 (b c - a d) g^6 (a + b x)^2} - \\
& \frac{47 B^2 d^4 i^2}{900 b^3 (b c - a d)^2 g^6 (a + b x)} - \frac{47 B^2 d^5 i^2 \operatorname{Log}[a + b x]}{900 b^3 (b c - a d)^3 g^6} + \frac{B^2 d^5 i^2 \operatorname{Log}[a + b x]^2}{30 b^3 (b c - a d)^3 g^6} - \\
& \frac{2 B (b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{25 b^3 g^6 (a + b x)^5} - \frac{3 B d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{20 b^3 g^6 (a + b x)^4} - \\
& \frac{B d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{45 b^3 g^6 (a + b x)^3} + \frac{B d^3 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{30 b^3 (b c - a d) g^6 (a + b x)^2} - \\
& \frac{B d^4 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{15 b^3 (b c - a d)^2 g^6 (a + b x)} - \frac{B d^5 i^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{15 b^3 (b c - a d)^3 g^6} - \\
& \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{5 b^3 g^6 (a + b x)^5} - \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 b^3 g^6 (a + b x)^4} - \\
& \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 b^3 g^6 (a + b x)^3} + \frac{47 B^2 d^5 i^2 \operatorname{Log}[c + d x]}{900 b^3 (b c - a d)^3 g^6} - \frac{B^2 d^5 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{15 b^3 (b c - a d)^3 g^6} + \\
& \frac{B d^5 i^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{15 b^3 (b c - a d)^3 g^6} + \frac{B^2 d^5 i^2 \operatorname{Log}[c + d x]^2}{30 b^3 (b c - a d)^3 g^6} - \\
& \frac{B^2 d^5 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6} - \frac{B^2 d^5 i^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6} - \frac{B^2 d^5 i^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6}
\end{aligned}$$

**Problem 74: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 dx$$

Optimal (type 4, 1089 leaves, 22 steps):

$$\begin{aligned}
& \frac{5 B^2 (b c - a d)^6 g^3 i^3 x}{84 b^3 d^3} + \frac{B^2 (b c - a d)^3 g^3 i^3 (a + b x)^4}{140 b^4} - \frac{29 B^2 (b c - a d)^5 g^3 i^3 (c + d x)^2}{840 b^2 d^4} + \\
& \frac{47 B^2 (b c - a d)^4 g^3 i^3 (c + d x)^3}{1260 b d^4} - \frac{13 B^2 (b c - a d)^3 g^3 i^3 (c + d x)^4}{420 d^4} + \frac{b B^2 (b c - a d)^2 g^3 i^3 (c + d x)^5}{105 d^4} - \\
& \frac{B^2 (b c - a d)^7 g^3 i^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{210 b^4 d^4} - \frac{B (b c - a d)^4 g^3 i^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{210 b^4 d} - \\
& \frac{3 B (b c - a d)^3 g^3 i^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{140 b^4} - \\
& \frac{B (b c - a d)^2 g^3 i^3 (a + b x)^4 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{35 b^3} + \\
& \frac{2 B (b c - a d)^4 g^3 i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{21 b d^4} - \\
& \frac{3 B (b c - a d)^3 g^3 i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{14 d^4} + \\
& \frac{6 b B (b c - a d)^2 g^3 i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{35 d^4} - \\
& \frac{b^2 B (b c - a d) g^3 i^3 (c + d x)^6 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{21 d^4} + \\
& \frac{(b c - a d)^3 g^3 i^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{140 b^4} + \\
& \frac{(b c - a d)^2 g^3 i^3 (a + b x)^4 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{35 b^3} + \\
& \frac{(b c - a d) g^3 i^3 (a + b x)^4 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{14 b^2} + \\
& \frac{g^3 i^3 (a + b x)^4 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{7 b} + \\
& \frac{B (b c - a d)^5 g^3 i^3 (a + b x)^2 \left(3 A + B + 3 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{420 b^4 d^2} - \\
& \frac{B (b c - a d)^6 g^3 i^3 (a + b x) \left(6 A + 5 B + 6 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{420 b^4 d^3} - \\
& \frac{B (b c - a d)^7 g^3 i^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(6 A + 11 B + 6 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{420 b^4 d^4} - \\
& \frac{11 B^2 (b c - a d)^7 g^3 i^3 \operatorname{Log}[c + d x]}{420 b^4 d^4} - \frac{B^2 (b c - a d)^7 g^3 i^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c + d x)}\right]}{70 b^4 d^4}
\end{aligned}$$

Result (type 4, 896 leaves, 122 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^6 g^3 i^3 x}{70 b^3 d^3} + \frac{B^2 (b c - a d)^6 g^3 i^3 x}{70 b^3 d^3} - \frac{3 B^2 (b c - a d)^5 g^3 i^3 (a + b x)^2}{280 b^4 d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 i^3 (a + b x)^3}{1260 b^4 d} + \frac{B^2 (b c - a d)^3 g^3 i^3 (a + b x)^4}{42 b^4} + \frac{B^2 d (b c - a d)^2 g^3 i^3 (a + b x)^5}{105 b^4} - \\
& \frac{B^2 (b c - a d)^6 g^3 i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{70 b^4 d^3} + \frac{B (b c - a d)^5 g^3 i^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{140 b^4 d^2} - \\
& \frac{B (b c - a d)^4 g^3 i^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{210 b^4 d} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{140 b^4} - \\
& \frac{B d (b c - a d)^2 g^3 i^3 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{7 b^4} - \\
& \frac{B d^2 (b c - a d) g^3 i^3 (a + b x)^6 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{21 b^4} + \\
& \frac{(b c - a d)^3 g^3 i^3 (a + b x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{4 b^4} + \\
& \frac{3 d (b c - a d)^2 g^3 i^3 (a + b x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{5 b^4} + \\
& \frac{d^2 (b c - a d) g^3 i^3 (a + b x)^6 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 b^4} + \\
& \frac{d^3 g^3 i^3 (a + b x)^7 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{7 b^4} - \frac{B^2 (b c - a d)^7 g^3 i^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{70 b^4 d^4} + \\
& \frac{B (b c - a d)^7 g^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{70 b^4 d^4} + \\
& \frac{B^2 (b c - a d)^7 g^3 i^3 \operatorname{Log}[c + d x]^2}{140 b^4 d^4} - \frac{B^2 (b c - a d)^7 g^3 i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{70 b^4 d^4}
\end{aligned}$$

**Problem 75: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 dx$$

Optimal (type 4, 908 leaves, 20 steps):

$$\begin{aligned}
& - \frac{7 B^2 (b c - a d)^5 g^2 i^3 x}{180 b^3 d^2} - \frac{7 B^2 (b c - a d)^4 g^2 i^3 (c + d x)^2}{360 b^2 d^3} - \\
& \frac{B^2 (b c - a d)^3 g^2 i^3 (c + d x)^3}{60 b d^3} + \frac{B^2 (b c - a d)^2 g^2 i^3 (c + d x)^4}{60 d^3} + \\
& \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{36 b^4 d^3} - \frac{B (b c - a d)^4 g^2 i^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{60 b^4 d} - \\
& \frac{B (b c - a d)^3 g^2 i^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{30 b^4} - \\
& \frac{B (b c - a d)^4 g^2 i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 b^2 d^3} + \\
& \frac{B (b c - a d)^3 g^2 i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{45 b d^3} + \\
& \frac{7 B (b c - a d)^2 g^2 i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{60 d^3} - \\
& \frac{b B (b c - a d) g^2 i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{15 d^3} + \\
& \frac{(b c - a d)^3 g^2 i^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{60 b^4} + \\
& \frac{(b c - a d)^2 g^2 i^3 (a + b x)^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{20 b^3} + \\
& \frac{(b c - a d) g^2 i^3 (a + b x)^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{10 b^2} + \\
& \frac{g^2 i^3 (a + b x)^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{6 b} + \\
& \frac{B (b c - a d)^5 g^2 i^3 (a + b x) \left(2 A + B + 2 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{60 b^4 d^2} + \\
& \frac{B (b c - a d)^6 g^2 i^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(2 A + 3 B + 2 B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{60 b^4 d^3} + \\
& \frac{11 B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}[c + d x]}{180 b^4 d^3} + \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c + d x)}\right]}{30 b^4 d^3}
\end{aligned}$$

Result (type 4, 825 leaves, 86 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^5 g^2 i^3 x}{30 b^3 d^2} - \frac{B^2 (b c - a d)^5 g^2 i^3 x}{45 b^3 d^2} - \frac{7 B^2 (b c - a d)^4 g^2 i^3 (c + d x)^2}{360 b^2 d^3} - \\
& \frac{B^2 (b c - a d)^3 g^2 i^3 (c + d x)^3}{60 b d^3} + \frac{B^2 (b c - a d)^2 g^2 i^3 (c + d x)^4}{60 d^3} - \\
& \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}[a + b x]}{45 b^4 d^3} + \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}[a + b x]^2}{60 b^4 d^3} - \\
& \frac{B^2 (b c - a d)^5 g^2 i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{30 b^4 d^2} - \frac{B (b c - a d)^4 g^2 i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{60 b^2 d^3} - \\
& \frac{B (b c - a d)^3 g^2 i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{90 b d^3} + \\
& \frac{7 B (b c - a d)^2 g^2 i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{60 d^3} - \\
& \frac{b B (b c - a d) g^2 i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{15 d^3} - \\
& \frac{B (b c - a d)^6 g^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{30 b^4 d^3} + \\
& \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{4 d^3} - \\
& \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{5 d^3} + \\
& \frac{b^2 g^2 i^3 (c + d x)^6 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{6 d^3} + \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}[c + d x]}{30 b^4 d^3} - \\
& \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{30 b^4 d^3} - \frac{B^2 (b c - a d)^6 g^2 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{30 b^4 d^3}
\end{aligned}$$

**Problem 76: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 dx$$

Optimal (type 4, 730 leaves, 19 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^4 g i^3 x}{60 b^3 d} + \frac{B^2 (b c - a d)^3 g i^3 (c + d x)^2}{30 b^2 d^2} + \frac{B^2 (b c - a d)^2 g i^3 (c + d x)^3}{30 b d^2} - \\
& \frac{B^2 (b c - a d)^5 g i^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{12 b^4 d^2} - \frac{B (b c - a d)^4 g i^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 b^4 d} - \\
& \frac{B (b c - a d)^3 g i^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 b^4} + \\
& \frac{3 B (b c - a d)^3 g i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{20 b^2 d^2} + \\
& \frac{B (b c - a d)^2 g i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{30 b d^2} - \\
& \frac{B (b c - a d) g i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 d^2} + \frac{(b c - a d)^3 g i^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{20 b^4} + \\
& \frac{(b c - a d)^2 g i^3 (a + b x)^2 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{10 b^3} + \\
& \frac{3 (b c - a d) g i^3 (a + b x)^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{20 b^2} + \\
& \frac{g i^3 (a + b x)^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{5 b} - \\
& \frac{B (b c - a d)^5 g i^3 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(A + B + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{10 b^4 d^2} - \\
& \frac{11 B^2 (b c - a d)^5 g i^3 \operatorname{Log}[c + d x]}{60 b^4 d^2} - \frac{B^2 (b c - a d)^5 g i^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{10 b^4 d^2}
\end{aligned}$$

Result(type 4, 655 leaves, 54 steps):



$$\begin{aligned}
& \frac{AB(b c - a d)^4 g i^3 x}{10 b^3 d} + \frac{B^2(b c - a d)^4 g i^3 x}{60 b^3 d} + \frac{B^2(b c - a d)^3 g i^3 (c + d x)^2}{30 b^2 d^2} + \\
& \frac{B^2(b c - a d)^2 g i^3 (c + d x)^3}{30 b d^2} + \frac{B^2(b c - a d)^5 g i^3 \operatorname{Log}[a + b x]}{60 b^4 d^2} - \frac{B^2(b c - a d)^5 g i^3 \operatorname{Log}[a + b x]^2}{20 b^4 d^2} + \\
& \frac{B^2(b c - a d)^4 g i^3 (a + b x) \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]}{10 b^4 d} + \frac{B(b c - a d)^3 g i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{20 b^2 d^2} + \\
& \frac{B(b c - a d)^2 g i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{30 b d^2} - \frac{B(b c - a d) g i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{10 d^2} + \\
& \frac{B(b c - a d)^5 g i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{10 b^4 d^2} - \\
& \frac{(b c - a d) g i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{4 d^2} + \\
& \frac{b g i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{5 d^2} - \frac{B^2(b c - a d)^5 g i^3 \operatorname{Log}[c + d x]}{10 b^4 d^2} + \\
& \frac{B^2(b c - a d)^5 g i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{10 b^4 d^2} + \frac{B^2(b c - a d)^5 g i^3 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{10 b^4 d^2}
\end{aligned}$$

**Problem 77: Result valid but suboptimal antiderivative.**

$$\int (c i + d i x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2 dx$$

Optimal (type 4, 420 leaves, 15 steps):

$$\begin{aligned}
& \frac{5 B^2(b c - a d)^3 i^3 x}{12 b^3} + \frac{B^2(b c - a d)^2 i^3 (c + d x)^2}{12 b^2 d} + \\
& \frac{5 B^2(b c - a d)^4 i^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{12 b^4 d} - \frac{B(b c - a d)^3 i^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{2 b^4} - \\
& \frac{B(b c - a d)^2 i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{4 b^2 d} - \frac{B(b c - a d) i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)}{6 b d} + \\
& \frac{i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{4 d} + \frac{11 B^2(b c - a d)^4 i^3 \operatorname{Log}[c + d x]}{12 b^4 d} + \\
& \frac{B(b c - a d)^4 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}\left[1 - \frac{b(c+d x)}{d(a+b x)}\right]}{2 b^4 d} - \frac{B^2(b c - a d)^4 i^3 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{d(a+b x)}\right]}{2 b^4 d}
\end{aligned}$$

Result (type 4, 503 leaves, 24 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^3 i^3 x}{2 b^3} + \frac{5 B^2 (b c - a d)^3 i^3 x}{12 b^3} + \frac{B^2 (b c - a d)^2 i^3 (c + d x)^2}{12 b^2 d} + \\
& \frac{5 B^2 (b c - a d)^4 i^3 \operatorname{Log}[a + b x]}{12 b^4 d} + \frac{B^2 (b c - a d)^4 i^3 \operatorname{Log}[a + b x]^2}{4 b^4 d} - \\
& \frac{B^2 (b c - a d)^3 i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{2 b^4} - \frac{B (b c - a d)^2 i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{4 b^2 d} - \\
& \frac{B (b c - a d) i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{6 b d} - \frac{B (b c - a d)^4 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 b^4 d} + \\
& \frac{i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{4 d} + \frac{B^2 (b c - a d)^4 i^3 \operatorname{Log}[c + d x]}{2 b^4 d} - \\
& \frac{B^2 (b c - a d)^4 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 b^4 d} - \frac{B^2 (b c - a d)^4 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 b^4 d}
\end{aligned}$$

**Problem 78: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{a g + b g x} dx$$

Optimal (type 4, 712 leaves, 26 steps):

$$\begin{aligned}
& \frac{B^2 d (b c - a d)^2 i^3 x}{3 b^3 g} + \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{3 b^4 g} - \\
& \frac{5 B d (b c - a d)^2 i^3 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b^4 g} - \frac{B (b c - a d) i^3 (c+d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 b^2 g} + \\
& \frac{2 B (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{b^4 g} + \\
& \frac{d (b c - a d)^2 i^3 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{b^4 g} + \frac{(b c - a d) i^3 (c+d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{2 b^2 g} + \\
& \frac{i^3 (c+d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 b g} + \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{Log}[c+d x]}{b^4 g} + \\
& \frac{5 B (b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{3 b^4 g} - \\
& \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{b^4 g} - \frac{5 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{3 b^4 g} + \\
& \frac{2 B (b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[3, \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g}
\end{aligned}$$

Result (type 4, 1868 leaves, 106 steps):

$$\begin{aligned}
& - \frac{5 A B d (b c - a d)^2 i^3 x}{3 b^3 g} + \frac{B^2 d (b c - a d)^2 i^3 x}{3 b^3 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[a+b x]}{3 b^4 g} - \frac{a B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a+b x]^2}{b^4 g} + \\
& \frac{5 B^2 (b c - a d)^3 i^3 \operatorname{Log}[a+b x]^2}{6 b^4 g} - \frac{A B (b c - a d)^3 i^3 \operatorname{Log}[g (a+b x)]^2}{b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[g (a+b x)]^3}{3 b^4 g} - \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[a+b x]^2 \operatorname{Log}[-c-d x]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{Log}[a+b x] \operatorname{Log}[g (a+b x)] \operatorname{Log}[-c-d x]}{b^4 g} - \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[g (a+b x)]^2 \operatorname{Log}[-c-d x]}{b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{b^4 g} - \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[g (a+b x)] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{b^4 g} -
\end{aligned}$$

$$\begin{aligned}
& \frac{5 B^2 d (b c - a d)^2 i^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{3 b^4 g} - \frac{B (b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 b^2 g} + \\
& \frac{2 a B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g} - \\
& \frac{5 B (b c - a d)^3 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{3 b^4 g} + \frac{d (b c - a d)^2 i^3 x \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g} + \\
& \frac{(b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 b^2 g} + \frac{i^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{3 b g} + \\
& \frac{5 B^2 (b c - a d)^3 i^3 \operatorname{Log}[c + d x]}{3 b^4 g} + \frac{2 B^2 c (b c - a d)^2 i^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g} - \\
& \frac{2 B c (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^3 g} - \frac{B^2 c (b c - a d)^2 i^3 \operatorname{Log}[c + d x]^2}{b^3 g} + \\
& \frac{2 a B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g} - \frac{5 B^2 (b c - a d)^3 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g} + \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[g (a + b x)]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g} + \\
& \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[a g + b g x]}{b^4 g} + \\
& \frac{2 A B (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]}{b^4 g} - \frac{1}{b^4 g} \\
& \frac{2 B^2 (b c - a d)^3 i^3 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]}{b^4 g} - \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{Log}[a g + b g x]^2}{b^4 g} - \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]^2}{b^4 g} + \\
& \frac{2 a B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} + \frac{2 A B (b c - a d)^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} - \\
& \frac{5 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^4 g} + \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} - \\
& \frac{\frac{1}{b^4 g} 2 B^2 (b c - a d)^3 i^3 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} + \\
& \frac{2 B^2 c (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g} - \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^4 g} - \\
& \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g} - \frac{2 B^2 (b c - a d)^3 i^3 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{b^4 g}
\end{aligned}$$

### Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{(a g + b g x)^2} dx$$

Optimal (type 4, 692 leaves, 17 steps):

$$\begin{aligned} & -\frac{2 B^2 (b c - a d)^2 i^3 (c + d x)}{b^3 g^2 (a + b x)} - \frac{B d^2 (b c - a d) i^3 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^4 g^2} - \\ & \frac{2 B (b c - a d)^2 i^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^3 g^2 (a + b x)} + \\ & \frac{4 B d (b c - a d)^2 i^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{b^4 g^2} + \\ & \frac{2 d^2 (b c - a d) i^3 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{b^4 g^2} - \frac{(b c - a d)^2 i^3 (c + d x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{b^3 g^2 (a + b x)} + \\ & \frac{d i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{2 b^2 g^2} + \frac{B^2 d (b c - a d)^2 i^3 \operatorname{Log}[c + d x]}{b^4 g^2} + \\ & \frac{B d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^4 g^2} - \\ & \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2 \operatorname{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^4 g^2} + \\ & \frac{4 B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{b^4 g^2} - \frac{B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^4 g^2} + \\ & \frac{6 B d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^4 g^2} + \\ & \frac{6 B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{b^4 g^2} \end{aligned}$$

Result (type 4, 1751 leaves, 90 steps):

$$\begin{aligned} & -\frac{A B d^2 (b c - a d) i^3 x}{b^3 g^2} - \frac{2 B^2 (b c - a d)^3 i^3}{b^4 g^2 (a + b x)} - \frac{2 B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x]}{b^4 g^2} + \\ & \frac{a^2 B^2 d^3 i^3 \operatorname{Log}[a + b x]^2}{2 b^4 g^2} - \frac{a B^2 d^2 (3 b c - 2 a d) i^3 \operatorname{Log}[a + b x]^2}{b^4 g^2} - \frac{3 A B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x]^2}{b^4 g^2} + \\ & \frac{B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x]^2}{b^4 g^2} - \frac{B^2 d^2 (b c - a d) i^3 (a + b x) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{b^4 g^2} - \end{aligned}$$

$$\begin{aligned}
& \frac{3 B^2 d (b c - a d)^2 i^3 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^4 g^2} - \frac{3 B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]^2}{b^4 g^2} - \\
& \frac{2 B (b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2 (a + b x)} - \frac{a^2 B d^3 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2} + \\
& \frac{2 a B d^2 (3 b c - 2 a d) i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2} - \\
& \frac{2 B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{b^4 g^2} + \frac{d^2 (3 b c - 2 a d) i^3 x \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^3 g^2} + \\
& \frac{d^3 i^3 x^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 b^2 g^2} - \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^4 g^2 (a + b x)} + \\
& \frac{3 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{b^4 g^2} + \frac{3 B^2 d (b c - a d)^2 i^3 \operatorname{Log}[c + d x]}{b^4 g^2} - \\
& \frac{B^2 c^2 d i^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} + \frac{2 B^2 c d (3 b c - 2 a d) i^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B c^2 d i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B c d (3 b c - 2 a d) i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^3 g^2} + \\
& \frac{2 B d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B^2 c^2 d i^3 \operatorname{Log}[c + d x]^2}{2 b^2 g^2} - \\
& \frac{B^2 c d (3 b c - 2 a d) i^3 \operatorname{Log}[c + d x]^2}{b^3 g^2} + \frac{B^2 d (b c - a d)^2 i^3 \operatorname{Log}[c + d x]^2}{b^4 g^2} - \\
& \frac{a^2 B^2 d^3 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} + \frac{2 a B^2 d^2 (3 b c - 2 a d) i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} + \\
& \frac{6 A B d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} - \frac{a^2 B^2 d^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^2} + \\
& \frac{2 a B^2 d^2 (3 b c - 2 a d) i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^2} + \frac{6 A B d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^2} - \frac{B^2 c^2 d i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} + \\
& \frac{2 B^2 c d (3 b c - 2 a d) i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} - \frac{2 B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^4 g^2} +
\end{aligned}$$

$$\frac{6 B^2 d (b c - a d)^2 i^3 \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d (a+b x)}\right]}{b^4 g^2} +$$

$$\frac{6 B^2 d (b c - a d)^2 i^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d (a+b x)}\right]}{b^4 g^2}$$

**Problem 80: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{(a g + b g x)^3} dx$$

Optimal (type 4, 604 leaves, 13 steps):

$$\begin{aligned} & -\frac{4 B^2 d (b c - a d) i^3 (c + d x)}{b^3 g^3 (a + b x)} - \frac{B^2 (b c - a d) i^3 (c + d x)^2}{4 b^2 g^3 (a + b x)^2} - \\ & \frac{4 B d (b c - a d) i^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{b^3 g^3 (a + b x)} - \frac{B (b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{2 b^2 g^3 (a + b x)^2} + \\ & \frac{2 B d^2 (b c - a d) i^3 \operatorname{Log}\left[\frac{b c-a d}{b (c+d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{b^4 g^3} + \frac{d^3 i^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{b^4 g^3} - \\ & \frac{2 d (b c - a d) i^3 (c + d x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{b^3 g^3 (a + b x)} - \frac{(b c - a d) i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{2 b^2 g^3 (a + b x)^2} - \\ & \frac{3 d^2 (b c - a d) i^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g^3} + \\ & \frac{2 B^2 d^2 (b c - a d) i^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{b^4 g^3} + \\ & \frac{6 B d^2 (b c - a d) i^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g^3} + \\ & \frac{6 B^2 d^2 (b c - a d) i^3 \operatorname{PolyLog}\left[3, \frac{b (c+d x)}{d (a+b x)}\right]}{b^4 g^3} \end{aligned}$$

Result (type 4, 1412 leaves, 95 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^3 i^3}{4 b^4 g^3 (a + bx)^2} - \frac{9 B^2 d (bc - ad)^2 i^3}{2 b^4 g^3 (a + bx)} - \frac{9 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[a + bx]}{2 b^4 g^3} - \\
& \frac{a B^2 d^3 i^3 \operatorname{Log}[a + bx]^2}{b^4 g^3} - \frac{3 A B d^2 (bc - ad) i^3 \operatorname{Log}[a + bx]^2}{b^4 g^3} + \\
& \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[a + bx]^2}{2 b^4 g^3} - \frac{3 B^2 d^2 (bc - ad) i^3 \operatorname{Log}\left[-\frac{bc - ad}{d(a + bx)}\right] \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]^2}{b^4 g^3} - \\
& \frac{3 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]^2}{b^4 g^3} - \frac{B (bc - ad)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)}{2 b^4 g^3 (a + bx)^2} - \\
& \frac{5 B d (bc - ad)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)}{b^4 g^3 (a + bx)} + \frac{2 a B d^3 i^3 \operatorname{Log}[a + bx] \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)}{b^4 g^3} - \\
& \frac{5 B d^2 (bc - ad) i^3 \operatorname{Log}[a + bx] \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)}{b^4 g^3} + \frac{d^3 i^3 x \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)^2}{b^3 g^3} - \\
& \frac{(bc - ad)^3 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)^2}{2 b^4 g^3 (a + bx)^2} - \frac{3 d (bc - ad)^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)^2}{b^4 g^3 (a + bx)} + \\
& \frac{3 d^2 (bc - ad) i^3 \operatorname{Log}[a + bx] \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right)^2}{b^4 g^3} + \frac{9 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[c + dx]}{2 b^4 g^3} + \\
& \frac{2 B^2 c d^2 i^3 \operatorname{Log}\left[-\frac{d(a + bx)}{b(c - ad)}\right] \operatorname{Log}[c + dx]}{b^3 g^3} - \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{Log}\left[-\frac{d(a + bx)}{b(c - ad)}\right] \operatorname{Log}[c + dx]}{b^4 g^3} - \\
& \frac{2 B c d^2 i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right) \operatorname{Log}[c + dx]}{b^3 g^3} + \frac{5 B d^2 (bc - ad) i^3 \left(A + B \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right]\right) \operatorname{Log}[c + dx]}{b^4 g^3} - \\
& \frac{B^2 c d^2 i^3 \operatorname{Log}[c + dx]^2}{b^3 g^3} + \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[c + dx]^2}{2 b^4 g^3} + \\
& \frac{2 a B^2 d^3 i^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c + dx)}{b(c - ad)}\right]}{b^4 g^3} + \frac{6 A B d^2 (bc - ad) i^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c + dx)}{b(c - ad)}\right]}{b^4 g^3} - \\
& \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c + dx)}{b(c - ad)}\right]}{b^4 g^3} + \frac{2 a B^2 d^3 i^3 \operatorname{PolyLog}\left[2, -\frac{d(a + bx)}{b(c - ad)}\right]}{b^4 g^3} + \\
& \frac{6 A B d^2 (bc - ad) i^3 \operatorname{PolyLog}\left[2, -\frac{d(a + bx)}{b(c - ad)}\right]}{b^4 g^3} - \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{PolyLog}\left[2, -\frac{d(a + bx)}{b(c - ad)}\right]}{b^4 g^3} + \\
& \frac{2 B^2 c d^2 i^3 \operatorname{PolyLog}\left[2, \frac{b(c + dx)}{b(c - ad)}\right]}{b^3 g^3} - \frac{5 B^2 d^2 (bc - ad) i^3 \operatorname{PolyLog}\left[2, \frac{b(c + dx)}{b(c - ad)}\right]}{b^4 g^3} + \\
& \frac{6 B^2 d^2 (bc - ad) i^3 \operatorname{Log}\left[\frac{e(a + bx)}{c + dx}\right] \operatorname{PolyLog}\left[2, 1 + \frac{bc - ad}{d(a + bx)}\right]}{b^4 g^3} + \\
& \frac{6 B^2 d^2 (bc - ad) i^3 \operatorname{PolyLog}\left[3, 1 + \frac{bc - ad}{d(a + bx)}\right]}{b^4 g^3}
\end{aligned}$$



Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 147 leaves, 3 steps):

$$-\frac{B^2 i^3 (c + d x)^4}{32 (b c - a d) g^5 (a + b x)^4} - \frac{B i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{8 (b c - a d) g^5 (a + b x)^4} - \frac{i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 (b c - a d) g^5 (a + b x)^4}$$

Result (type 4, 970 leaves, 130 steps):

$$\begin{aligned} & -\frac{B^2 (b c - a d)^3 i^3}{32 b^4 g^5 (a + b x)^4} - \frac{B^2 d (b c - a d)^2 i^3}{8 b^4 g^5 (a + b x)^3} - \frac{3 B^2 d^2 (b c - a d) i^3}{16 b^4 g^5 (a + b x)^2} - \\ & -\frac{B^2 d^3 i^3}{8 b^4 g^5 (a + b x)} - \frac{B^2 d^4 i^3 \operatorname{Log}[a + b x]}{8 b^4 (b c - a d) g^5} + \frac{B^2 d^4 i^3 \operatorname{Log}[a + b x]^2}{4 b^4 (b c - a d) g^5} - \\ & -\frac{B (b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{8 b^4 g^5 (a + b x)^4} - \frac{B d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4 g^5 (a + b x)^3} - \\ & -\frac{3 B d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{4 b^4 g^5 (a + b x)^2} - \frac{B d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4 g^5 (a + b x)} - \\ & -\frac{B d^4 i^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 b^4 (b c - a d) g^5} - \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b^4 g^5 (a + b x)^4} - \\ & -\frac{d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{b^4 g^5 (a + b x)^3} - \frac{3 d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 b^4 g^5 (a + b x)^2} - \\ & -\frac{d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{b^4 g^5 (a + b x)} + \frac{B^2 d^4 i^3 \operatorname{Log}[c + d x]}{8 b^4 (b c - a d) g^5} - \frac{B^2 d^4 i^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{2 b^4 (b c - a d) g^5} + \\ & + \frac{B d^4 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log}[c + d x]}{2 b^4 (b c - a d) g^5} + \frac{B^2 d^4 i^3 \operatorname{Log}[c + d x]^2}{4 b^4 (b c - a d) g^5} - \\ & -\frac{B^2 d^4 i^3 \operatorname{Log}[a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{2 b^4 (b c - a d) g^5} - \frac{B^2 d^4 i^3 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{2 b^4 (b c - a d) g^5} - \frac{B^2 d^4 i^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{2 b^4 (b c - a d) g^5} \end{aligned}$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(a g + b g x)^6} dx$$

Optimal (type 3, 299 leaves, 7 steps):

$$\begin{aligned} & \frac{B^2 d i^3 (c + d x)^4}{32 (b c - a d)^2 g^6 (a + b x)^4} - \frac{2 b B^2 i^3 (c + d x)^5}{125 (b c - a d)^2 g^6 (a + b x)^5} + \\ & \frac{B d i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{8 (b c - a d)^2 g^6 (a + b x)^4} - \frac{2 b B i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{25 (b c - a d)^2 g^6 (a + b x)^5} + \\ & \frac{d i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 (b c - a d)^2 g^6 (a + b x)^4} - \frac{b i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 (b c - a d)^2 g^6 (a + b x)^5} \end{aligned}$$

Result (type 4, 1061 leaves, 146 steps):

$$\begin{aligned} & - \frac{2 B^2 (b c - a d)^3 i^3}{125 b^4 g^6 (a + b x)^5} - \frac{39 B^2 d (b c - a d)^2 i^3}{800 b^4 g^6 (a + b x)^4} - \frac{7 B^2 d^2 (b c - a d) i^3}{200 b^4 g^6 (a + b x)^3} + \frac{11 B^2 d^3 i^3}{400 b^4 g^6 (a + b x)^2} + \\ & \frac{9 B^2 d^4 i^3}{200 b^4 (b c - a d) g^6 (a + b x)} + \frac{9 B^2 d^5 i^3 \operatorname{Log}[a + b x]}{200 b^4 (b c - a d)^2 g^6} - \frac{B^2 d^5 i^3 \operatorname{Log}[a + b x]^2}{20 b^4 (b c - a d)^2 g^6} - \\ & \frac{2 B (b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{25 b^4 g^6 (a + b x)^5} - \frac{11 B d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{40 b^4 g^6 (a + b x)^4} - \\ & \frac{3 B d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b^4 g^6 (a + b x)^3} - \frac{B d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{20 b^4 g^6 (a + b x)^2} + \\ & \frac{B d^4 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b^4 (b c - a d) g^6 (a + b x)} + \frac{B d^5 i^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{10 b^4 (b c - a d)^2 g^6} - \\ & \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{5 b^4 g^6 (a + b x)^5} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{4 b^4 g^6 (a + b x)^4} - \\ & \frac{d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{b^4 g^6 (a + b x)^3} - \frac{d^3 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 b^4 g^6 (a + b x)^2} - \frac{9 B^2 d^5 i^3 \operatorname{Log}[c + d x]}{200 b^4 (b c - a d)^2 g^6} + \\ & \frac{B^2 d^5 i^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{10 b^4 (b c - a d)^2 g^6} - \frac{B d^5 i^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log}[c + d x]}{10 b^4 (b c - a d)^2 g^6} - \\ & \frac{B^2 d^5 i^3 \operatorname{Log}[c + d x]^2}{20 b^4 (b c - a d)^2 g^6} + \frac{B^2 d^5 i^3 \operatorname{Log}[a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{10 b^4 (b c - a d)^2 g^6} + \\ & \frac{B^2 d^5 i^3 \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{10 b^4 (b c - a d)^2 g^6} + \frac{B^2 d^5 i^3 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{10 b^4 (b c - a d)^2 g^6} \end{aligned}$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{(a g + b g x)^7} dx$$

Optimal (type 3, 463 leaves, 9 steps):

$$\begin{aligned} & - \frac{B^2 d^2 i^3 (c + d x)^4}{32 (b c - a d)^3 g^7 (a + b x)^4} + \frac{4 b B^2 d i^3 (c + d x)^5}{125 (b c - a d)^3 g^7 (a + b x)^5} - \frac{b^2 B^2 i^3 (c + d x)^6}{108 (b c - a d)^3 g^7 (a + b x)^6} - \\ & \frac{B d^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)}{8 (b c - a d)^3 g^7 (a + b x)^4} + \frac{4 b B d i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)}{25 (b c - a d)^3 g^7 (a + b x)^5} - \\ & \frac{b^2 B i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)}{18 (b c - a d)^3 g^7 (a + b x)^6} - \frac{d^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{4 (b c - a d)^3 g^7 (a + b x)^4} + \\ & \frac{2 b d i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{5 (b c - a d)^3 g^7 (a + b x)^5} - \frac{b^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{6 (b c - a d)^3 g^7 (a + b x)^6} \end{aligned}$$

Result (type 4, 1152 leaves, 162 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^3 i^3}{108 b^4 g^7 (a + bx)^6} - \frac{53 B^2 d (bc - ad)^2 i^3}{2250 b^4 g^7 (a + bx)^5} - \frac{73 B^2 d^2 (bc - ad) i^3}{7200 b^4 g^7 (a + bx)^4} + \\
& \frac{53 B^2 d^3 i^3}{5400 b^4 g^7 (a + bx)^3} - \frac{23 B^2 d^4 i^3}{3600 b^4 (bc - ad) g^7 (a + bx)^2} - \frac{37 B^2 d^5 i^3}{1800 b^4 (bc - ad)^2 g^7 (a + bx)} - \\
& \frac{37 B^2 d^6 i^3 \operatorname{Log}[a + bx]}{1800 b^4 (bc - ad)^3 g^7} + \frac{B^2 d^6 i^3 \operatorname{Log}[a + bx]^2}{60 b^4 (bc - ad)^3 g^7} - \frac{B (bc - ad)^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{18 b^4 g^7 (a + bx)^6} - \\
& \frac{13 B d (bc - ad)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{75 b^4 g^7 (a + bx)^5} - \frac{19 B d^2 (bc - ad) i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{120 b^4 g^7 (a + bx)^4} - \\
& \frac{B d^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{90 b^4 g^7 (a + bx)^3} + \frac{B d^4 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{60 b^4 (bc - ad) g^7 (a + bx)^2} - \frac{B d^5 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{30 b^4 (bc - ad)^2 g^7 (a + bx)} - \\
& \frac{B d^6 i^3 \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{30 b^4 (bc - ad)^3 g^7} - \frac{(bc - ad)^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{6 b^4 g^7 (a + bx)^6} - \\
& \frac{3 d (bc - ad)^2 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{5 b^4 g^7 (a + bx)^5} - \frac{3 d^2 (bc - ad) i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{4 b^4 g^7 (a + bx)^4} - \\
& \frac{d^3 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{3 b^4 g^7 (a + bx)^3} + \frac{37 B^2 d^6 i^3 \operatorname{Log}[c + dx]}{1800 b^4 (bc - ad)^3 g^7} - \frac{B^2 d^6 i^3 \operatorname{Log}\left[-\frac{d(a+bx)}{b(c-a)d}\right] \operatorname{Log}[c + dx]}{30 b^4 (bc - ad)^3 g^7} + \\
& \frac{B d^6 i^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{Log}[c + dx]}{30 b^4 (bc - ad)^3 g^7} + \frac{B^2 d^6 i^3 \operatorname{Log}[c + dx]^2}{60 b^4 (bc - ad)^3 g^7} - \\
& \frac{B^2 d^6 i^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{b(c-a)d}\right]}{30 b^4 (bc - ad)^3 g^7} - \frac{B^2 d^6 i^3 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-a)d}\right]}{30 b^4 (bc - ad)^3 g^7} - \frac{B^2 d^6 i^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b(c-a)d}\right]}{30 b^4 (bc - ad)^3 g^7}
\end{aligned}$$

**Problem 84: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{c i + d i x} dx$$

Optimal (type 4, 718 leaves, 25 steps):

$$\begin{aligned}
& \frac{b B^2 (b c - a d)^2 g^3 x}{3 d^3 i} + \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{3 d^4 i} + \\
& \frac{7 B (b c - a d)^2 g^3 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 d^3 i} - \frac{b^2 B (b c - a d) g^3 (c+d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 d^4 i} + \\
& \frac{6 B (b c - a d)^3 g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{d^4 i} + \\
& \frac{3 (b c - a d)^2 g^3 (a+b x) \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{d^3 i} - \\
& \frac{3 b^2 (b c - a d) g^3 (c+d x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{2 d^4 i} + \frac{b^3 g^3 (c+d x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 d^4 i} + \\
& \frac{(b c - a d)^3 g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{d^4 i} - \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{Log}[c+d x]}{d^4 i} - \\
& \frac{7 B (b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{3 d^4 i} + \frac{6 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{d^4 i} + \\
& \frac{2 B (b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{d^4 i} + \\
& \frac{7 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{3 d^4 i} - \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog}\left[3, \frac{d (a+b x)}{b (c+d x)}\right]}{d^4 i}
\end{aligned}$$

Result (type 4, 1828 leaves, 106 steps):

$$\begin{aligned}
& \frac{5 A b B (b c - a d)^2 g^3 x}{3 d^3 i} + \frac{b B^2 (b c - a d)^2 g^3 x}{3 d^3 i} - \frac{a B^2 (b c - a d)^2 g^3 \operatorname{Log}[a+b x]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{d^4 i} - \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{d^4 i} + \\
& \frac{5 B^2 (b c - a d)^2 g^3 (a+b x) \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{3 d^3 i} - \frac{B (b c - a d) g^3 (a+b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{3 d^2 i} + \\
& \frac{2 a B (b c - a d)^2 g^3 \operatorname{Log}[a+b x] \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)}{d^3 i} + \\
& \frac{b (b c - a d)^2 g^3 x \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{d^3 i} - \frac{(b c - a d) g^3 (a+b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{2 d^2 i} + \\
& \frac{g^3 (a+b x)^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{3 d i} - \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{Log}[c+d x]}{d^4 i} + \\
& \frac{2 b B^2 c (b c - a d)^2 g^3 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c+d x]}{d^4 i} + \frac{5 B^2 (b c - a d)^3 g^3 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c+d x]}{3 d^4 i} - \\
& \frac{2 b B c (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x]}{d^4 i} -
\end{aligned}$$

$$\begin{aligned}
& \frac{5 B (b c - a d)^3 g^3 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right) \operatorname{Log} [c+d x]}{3 d^4 i} - \frac{b B^2 c (b c - a d)^2 g^3 \operatorname{Log} [c+d x]^2}{d^4 i} - \\
& \frac{5 B^2 (b c - a d)^3 g^3 \operatorname{Log} [c+d x]^2}{6 d^4 i} + \frac{2 a B^2 (b c - a d)^2 g^3 \operatorname{Log} [a+b x] \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} [a+b x]^2 \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^4 i} + \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} [a+b x]^2 \operatorname{Log} [i (c+d x)]}{d^4 i} + \\
& \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{Log} [a+b x] \operatorname{Log} \left[ \frac{1}{c+d x} \right] \operatorname{Log} [i (c+d x)]}{d^4 i} - \frac{A B (b c - a d)^3 g^3 \operatorname{Log} [i (c+d x)]^2}{d^4 i} + \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} [a+b x] \operatorname{Log} [i (c+d x)]^2}{d^4 i} - \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log} [i (c+d x)]^2}{d^4 i} - \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} [i (c+d x)]^3}{3 d^4 i} + \frac{2 A B (b c - a d)^3 g^3 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log} [c i + d i x]}{d^4 i} - \\
& \frac{1}{d^4 i} 2 B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \left( \operatorname{Log} [a+b x] + \operatorname{Log} \left[ \frac{1}{c+d x} \right] - \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right) \\
& \operatorname{Log} [c i + d i x] - \frac{(b c - a d)^3 g^3 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2 \operatorname{Log} [c i + d i x]}{d^4 i} + \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log} [c i + d i x]^2}{d^4 i} - \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \operatorname{Log} [c i + d i x]^2}{d^4 i} + \\
& \frac{2 a B^2 (b c - a d)^2 g^3 \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^3 i} - \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{Log} [a+b x] \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^4 i} + \\
& \frac{2 b B^2 c (b c - a d)^2 g^3 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^4 i} + \frac{2 A B (b c - a d)^3 g^3 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^4 i} + \\
& \frac{5 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{3 d^4 i} + \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ \frac{1}{c+d x} \right] \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^4 i} - \\
& \frac{1}{d^4 i} 2 B^2 (b c - a d)^3 g^3 \left( \operatorname{Log} [a+b x] + \operatorname{Log} \left[ \frac{1}{c+d x} \right] - \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right) \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right] + \\
& \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog} \left[ 3, -\frac{d (a+b x)}{b c - a d} \right]}{d^4 i} + \frac{2 B^2 (b c - a d)^3 g^3 \operatorname{PolyLog} \left[ 3, \frac{b (c+d x)}{b c - a d} \right]}{d^4 i}
\end{aligned}$$

**Problem 85: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{c i + d i x} dx$$

Optimal (type 4, 536 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B (b c - a d) g^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{d^2 i} - \\
& \frac{4 B (b c - a d)^2 g^2 \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{d^3 i} - \\
& \frac{2 (b c - a d) g^2 (a + b x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{d^2 i} + \frac{b^2 g^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 d^3 i} - \\
& \frac{(b c - a d)^2 g^2 \operatorname{Log} \left[ \frac{b c - a d}{b (c + d x)} \right] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [c + d x]}{d^3 i} + \\
& \frac{B (b c - a d)^2 g^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{d^3 i} - \frac{4 B^2 (b c - a d)^2 g^2 \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{d^3 i} - \\
& \frac{2 B (b c - a d)^2 g^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{PolyLog} \left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{d^3 i} + \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{PolyLog} \left[ 3, \frac{d (a + b x)}{b (c + d x)} \right]}{d^3 i}
\end{aligned}$$

Result (type 4, 1666 leaves, 86 steps):

$$\begin{aligned}
& - \frac{A B B (b c - a d) g^2 x}{d^2 i} + \frac{a B^2 (b c - a d) g^2 \operatorname{Log} [a + b x]^2}{d^2 i} - \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{1}{c + d x} \right]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} \left[ \frac{1}{c + d x} \right]^2}{d^3 i} - \frac{B^2 (b c - a d) g^2 (a + b x) \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]}{d^2 i} - \\
& \frac{2 a B (b c - a d) g^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{d^2 i} - \frac{b (b c - a d) g^2 x \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{d^2 i} + \\
& \frac{g^2 (a + b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 d i} + \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [c + d x]}{d^3 i} - \\
& \frac{2 b B^2 c (b c - a d) g^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{d^3 i} - \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{d^3 i} + \\
& \frac{2 b B c (b c - a d) g^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{d^3 i} + \\
& \frac{B (b c - a d)^2 g^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x]}{d^3 i} + \frac{b B^2 c (b c - a d) g^2 \operatorname{Log} [c + d x]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [c + d x]^2}{2 d^3 i} - \frac{2 a B^2 (b c - a d) g^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{d^2 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [a + b x]^2 \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{d^3 i} - \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log} [a + b x]^2 \operatorname{Log} [i (c + d x)]}{d^3 i} - \\
& \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{1}{c + d x} \right] \operatorname{Log} [i (c + d x)]}{d^3 i} + \frac{A B (b c - a d)^2 g^2 \operatorname{Log} [i (c + d x)]^2}{d^3 i} -
\end{aligned}$$

$$\begin{aligned}
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}[i (c + d x)]^2}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[i (c + d x)]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}[i (c + d x)]^3}{3 d^3 i} - \frac{2 A B (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d^3 i} + \\
& \frac{1}{d^3 i} 2 B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right] \right) \\
& \operatorname{Log}[c i + d i x] + \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c i + d i x]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]^2}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right] \operatorname{Log}[c i + d i x]^2}{d^3 i} - \\
& \frac{2 a B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^2 i} + \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^3 i} - \\
& \frac{2 b B^2 c (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \frac{2 A B (b c - a d)^2 g^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} + \\
& \frac{1}{d^3 i} 2 B^2 (b c - a d)^2 g^2 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right] - \\
& \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c - a d}\right]}{d^3 i} - \frac{2 B^2 (b c - a d)^2 g^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i}
\end{aligned}$$

**Problem 86: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x) \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right)^2}{c i + d i x} dx$$

Optimal (type 4, 283 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 B (b c - a d) g \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right)}{d^2 i} + \frac{g (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right)^2}{d i} + \\
& \frac{(b c - a d) g \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right)^2}{d^2 i} + \frac{2 B^2 (b c - a d) g \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b (c + d x)}\right]}{d^2 i} + \\
& \frac{2 B (b c - a d) g \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b (c + d x)}\right]}{d^2 i} - \frac{2 B^2 (b c - a d) g \operatorname{PolyLog}\left[3, \frac{d(a+b x)}{b (c + d x)}\right]}{d^2 i}
\end{aligned}$$

Result (type 4, 1072 leaves, 68 steps):



$$\begin{aligned}
& - \frac{a B^2 g \operatorname{Log}[a + b x]^2}{d i} + \frac{B^2 (b c - a d) g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^2 i} - \\
& \frac{B^2 (b c - a d) g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^2 i} + \frac{2 a B g \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d i} + \\
& \frac{b g x \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d i} + \frac{B^2 (b c - a d) g \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{d^2 i} + \\
& \frac{2 b B^2 c g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i} + \frac{2 A B (b c - a d) g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i} + \\
& \frac{2 B^2 (b c - a d) g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{d^2 i} - \frac{1}{d^2 i} \\
& 2 B^2 (b c - a d) g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] - \\
& \frac{2 b B c g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^2 i} - \frac{(b c - a d) g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{d^2 i} - \\
& \frac{b B^2 c g \operatorname{Log}[c + d x]^2}{d^2 i} - \frac{A B (b c - a d) g \operatorname{Log}[c + d x]^2}{d^2 i} + \frac{B^2 (b c - a d) g \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^2 i} - \\
& \frac{B^2 (b c - a d) g \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{d^2 i} - \frac{B^2 (b c - a d) g \operatorname{Log}[c + d x]^3}{3 d^2 i} + \\
& \frac{2 a B^2 g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d i} - \frac{B^2 (b c - a d) g \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 a B^2 g \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d i} - \frac{2 B^2 (b c - a d) g \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 b B^2 c g \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \frac{2 A B (b c - a d) g \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 B^2 (b c - a d) g \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} - \frac{1}{d^2 i} \\
& 2 B^2 (b c - a d) g \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] + \\
& \frac{2 B^2 (b c - a d) g \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i} + \frac{2 B^2 (b c - a d) g \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i}
\end{aligned}$$

**Problem 87: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{c i + d i x} dx$$

Optimal (type 4, 127 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d i} - \\
& \frac{2 B \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d i} + \frac{2 B^2 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{d i}
\end{aligned}$$

Result (type 4, 721 leaves, 46 steps):

$$\begin{aligned}
& - \frac{B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d i} + \frac{B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d i} + \frac{B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d i} - \\
& \frac{B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[i (c + d x)]}{d i} - \frac{2 B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[i (c + d x)]}{d i} + \\
& \frac{A B \operatorname{Log}[i (c + d x)]^2}{d i} - \frac{B^2 \operatorname{Log}[a + b x] \operatorname{Log}[i (c + d x)]^2}{d i} + \frac{B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[i (c + d x)]^2}{d i} + \\
& \frac{B^2 \operatorname{Log}[i (c + d x)]^3}{3 d i} - \frac{2 A B \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d i} + \frac{1}{d i} \\
& 2 B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c i + d i x] + \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c i + d i x]}{d i} - \frac{B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]^2}{d i} + \\
& \frac{B^2 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{Log}[c i + d i x]^2}{d i} + \frac{2 B^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d i} - \\
& \frac{2 A B \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d i} - \frac{2 B^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d i} + \\
& \frac{2 B^2 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d i} - \\
& \frac{2 B^2 \operatorname{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{d i} - \frac{2 B^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{d i}
\end{aligned}$$

**Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{(a g + b g x) (c i + d i x)} dx$$

Optimal (type 3, 44 leaves, 3 steps):

$$\frac{\left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^3}{3 B (b c - a d) g i}$$

Result (type 4, 1163 leaves, 61 steps):

$$\begin{aligned}
& - \frac{A B \operatorname{Log}[a+b x]^2}{(b c-a d) g i} + \frac{B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{(b c-a d) g i} - \frac{B^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{(b c-a d) g i} - \\
& \frac{B^2 \operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]^2}{(b c-a d) g i} - \frac{B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]^2}{(b c-a d) g i} + \\
& \frac{\operatorname{Log}[a+b x] \left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{(b c-a d) g i} + \frac{B^2 \operatorname{Log}[a+b x]^2 \operatorname{Log}[c+d x]}{(b c-a d) g i} + \\
& \frac{2 A B \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d) g i} + \frac{2 B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{Log}[c+d x]}{(b c-a d) g i} - \frac{1}{(b c-a d) g i} \\
& 2 B^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \left(\operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x] - \\
& \frac{\left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2 \operatorname{Log}[c+d x]}{(b c-a d) g i} - \frac{A B \operatorname{Log}[c+d x]^2}{(b c-a d) g i} + \\
& \frac{B^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d) g i} - \frac{B^2 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{Log}[c+d x]^2}{(b c-a d) g i} - \frac{B^2 \operatorname{Log}[c+d x]^3}{3(b c-a d) g i} + \\
& \frac{2 A B \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} - \frac{B^2 \operatorname{Log}[a+b x]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} + \\
& \frac{2 A B \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d) g i} - \frac{2 B^2 \operatorname{Log}[a+b x] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d) g i} + \\
& \frac{2 A B \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} + \frac{2 B^2 \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} - \\
& \frac{2 B^2 \left(\operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} + \\
& \frac{2 B^2 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d) g i} + \frac{2 B^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d) g i} + \\
& \frac{2 B^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d) g i} + \frac{2 B^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d) g i}
\end{aligned}$$

**Problem 89: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{(a g+b g x)^2(c i+d i x)} d x$$

Optimal (type 3, 183 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 b B^2 (c + d x)}{(b c - a d)^2 g^2 i (a + b x)} - \frac{2 b B (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{(b c - a d)^2 g^2 i (a + b x)} - \\
& \frac{b (c + d x) \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^2 g^2 i (a + b x)} - \frac{d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^3}{3 B (b c - a d)^2 g^2 i}
\end{aligned}$$

Result (type 4, 1684 leaves, 87 steps):

$$\begin{aligned}
& - \frac{2 B^2}{(b c - a d) g^2 i (a + b x)} - \frac{2 B^2 d \operatorname{Log}[a + b x]}{(b c - a d)^2 g^2 i} + \frac{A B d \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g^2 i} - \\
& \frac{B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}\left[-\frac{b c - a d}{d(a + b x)}\right] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^2 g^2 i} + \\
& \frac{B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^2 g^2 i} - \frac{2 B \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d) g^2 i (a + b x)} - \frac{2 B d \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g^2 i} - \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d) g^2 i (a + b x)} - \frac{d \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d)^2 g^2 i} + \frac{2 B^2 d \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} - \\
& \frac{B^2 d \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} - \frac{2 A B d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} + \frac{1}{(b c - a d)^2 g^2 i} \\
& 2 B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] + \\
& \frac{2 B d \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} + \frac{d \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g^2 i} + \\
& \frac{A B d \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g^2 i} - \frac{B^2 d \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g^2 i} + \\
& \frac{B^2 d \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}[c + d x]^3}{3 (b c - a d)^2 g^2 i} - \frac{2 A B d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 A B d \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} + \frac{2 B^2 d \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 A B d \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} + \\
& \frac{2 B^2 d \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g^2 i}
\end{aligned}$$

Problem 90: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag+bx)^3 (ci+dx)} dx$$

Optimal (type 3, 343 leaves, 9 steps):

$$\begin{aligned} & \frac{4bB^2d(c+dx)}{(bc-ad)^3 g^3 i (a+bx)} - \frac{b^2 B^2 (c+dx)^2}{4(bc-ad)^3 g^3 i (a+bx)^2} + \frac{4bBd(c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3 g^3 i (a+bx)} - \\ & \frac{b^2 B (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2(bc-ad)^3 g^3 i (a+bx)^2} + \frac{2bd(c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3 g^3 i (a+bx)} - \\ & \frac{b^2 (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2(bc-ad)^3 g^3 i (a+bx)^2} + \frac{d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{3B(bc-ad)^3 g^3 i} \end{aligned}$$

Result (type 4, 1899 leaves, 117 steps):

$$\begin{aligned} & -\frac{B^2}{4(bc-ad)g^3 i (a+bx)^2} + \frac{7B^2 d}{2(bc-ad)^2 g^3 i (a+bx)} + \frac{7B^2 d^2 \operatorname{Log}[a+bx]}{2(bc-ad)^3 g^3 i} - \frac{ABd^2 \operatorname{Log}[a+bx]^2}{(bc-ad)^3 g^3 i} - \\ & \frac{3B^2 d^2 \operatorname{Log}[a+bx]^2}{2(bc-ad)^3 g^3 i} + \frac{B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^3 g^3 i} - \\ & \frac{B^2 d^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^3 g^3 i} - \\ & \frac{B \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2(bc-ad)g^3 i (a+bx)^2} + \frac{3Bd \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^2 g^3 i (a+bx)} + \frac{3Bd^2 \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3 g^3 i} - \\ & \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2(bc-ad)g^3 i (a+bx)^2} + \frac{d \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^2 g^3 i (a+bx)} + \frac{d^2 \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3 g^3 i} - \\ & \frac{7B^2 d^2 \operatorname{Log}[c+dx]}{2(bc-ad)^3 g^3 i} + \frac{B^2 d^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} + \frac{2ABd^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} + \\ & \frac{3B^2 d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} + \frac{2B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} - \frac{1}{(bc-ad)^3 g^3 i} - \\ & 2B^2 d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \left(\operatorname{Log}[a+bx] + \operatorname{Log}\left[\frac{1}{c+dx}\right] - \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c+dx] - \\ & \frac{3Bd^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} - \frac{d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^3 g^3 i} - \end{aligned}$$

$$\begin{aligned}
& \frac{A B d^2 \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \frac{3 B^2 d^2 \operatorname{Log}[c+d x]^2}{2 (b c-a d)^3 g^3 i} + \frac{B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \\
& \frac{B^2 d^2 \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}[c+d x]^3}{3 (b c-a d)^3 g^3 i} + \frac{2 A B d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{3 B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}[a+b x]^2 \operatorname{Log}\left[\frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{2 A B d^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{3 B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \\
& \frac{2 B^2 d^2 \operatorname{Log}[a+b x] \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 A B d^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{3 B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \\
& \frac{2 B^2 d^2 \left(\operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{2 B^2 d^2 \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d (a+b x)}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 \operatorname{PolyLog}\left[3, -\frac{d (a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{2 B^2 d^2 \operatorname{PolyLog}\left[3, \frac{b (c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d (a+b x)}\right]}{(b c-a d)^3 g^3 i}
\end{aligned}$$

**Problem 91:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]\right)^2}{(a g + b g x)^4 (c i + d i x)} dx$$

Optimal (type 3, 507 leaves, 11 steps):

$$\begin{aligned}
& - \frac{6 b B^2 d^2 (c + d x)}{(b c - a d)^4 g^4 i (a + b x)} + \frac{3 b^2 B^2 d (c + d x)^2}{4 (b c - a d)^4 g^4 i (a + b x)^2} - \\
& \frac{2 b^3 B^2 (c + d x)^3}{27 (b c - a d)^4 g^4 i (a + b x)^3} - \frac{6 b B d^2 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{(b c - a d)^4 g^4 i (a + b x)} + \\
& \frac{3 b^2 B d (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{2 (b c - a d)^4 g^4 i (a + b x)^2} - \frac{2 b^3 B (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{9 (b c - a d)^4 g^4 i (a + b x)^3} - \\
& \frac{3 b d^2 (c + d x) \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(b c - a d)^4 g^4 i (a + b x)} + \frac{3 b^2 d (c + d x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 (b c - a d)^4 g^4 i (a + b x)^2} - \\
& \frac{b^3 (c + d x)^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 (b c - a d)^4 g^4 i (a + b x)^3} - \frac{d^3 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^3}{3 B (b c - a d)^4 g^4 i}
\end{aligned}$$

Result (type 4, 2044 leaves, 151 steps):

$$\begin{aligned}
& - \frac{2 B^2}{27 (b c - a d) g^4 i (a + b x)^3} + \frac{19 B^2 d}{36 (b c - a d)^2 g^4 i (a + b x)^2} - \\
& \frac{85 B^2 d^2}{18 (b c - a d)^3 g^4 i (a + b x)} - \frac{85 B^2 d^3 \operatorname{Log} [a + b x]}{18 (b c - a d)^4 g^4 i} + \frac{A B d^3 \operatorname{Log} [a + b x]^2}{(b c - a d)^4 g^4 i} + \\
& \frac{11 B^2 d^3 \operatorname{Log} [a + b x]^2}{6 (b c - a d)^4 g^4 i} - \frac{B^2 d^3 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{1}{c + d x} \right]^2}{(b c - a d)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} \left[ \frac{1}{c + d x} \right]^2}{(b c - a d)^4 g^4 i} + \\
& \frac{B^2 d^3 \operatorname{Log} \left[ -\frac{b c - a d}{d (a + b x)} \right] \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]^2}{(b c - a d)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right]^2}{(b c - a d)^4 g^4 i} - \\
& \frac{2 B \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{9 (b c - a d) g^4 i (a + b x)^3} + \frac{5 B d \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{6 (b c - a d)^2 g^4 i (a + b x)^2} - \frac{11 B d^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 (b c - a d)^3 g^4 i (a + b x)} - \\
& \frac{11 B d^3 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)}{3 (b c - a d)^4 g^4 i} - \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{3 (b c - a d) g^4 i (a + b x)^3} + \\
& \frac{d \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{2 (b c - a d)^2 g^4 i (a + b x)^2} - \frac{d^2 \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(b c - a d)^3 g^4 i (a + b x)} - \frac{d^3 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right)^2}{(b c - a d)^4 g^4 i} + \\
& \frac{85 B^2 d^3 \operatorname{Log} [c + d x]}{18 (b c - a d)^4 g^4 i} - \frac{B^2 d^3 \operatorname{Log} [a + b x]^2 \operatorname{Log} [c + d x]}{(b c - a d)^4 g^4 i} - \frac{2 A B d^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{(b c - a d)^4 g^4 i} - \\
& \frac{11 B^2 d^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log} [c + d x]}{3 (b c - a d)^4 g^4 i} - \frac{2 B^2 d^3 \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{1}{c + d x} \right] \operatorname{Log} [c + d x]}{(b c - a d)^4 g^4 i} + \frac{1}{(b c - a d)^4 g^4 i} \\
& 2 B^2 d^3 \operatorname{Log} \left[ -\frac{d (a + b x)}{b c - a d} \right] \left( \operatorname{Log} [a + b x] + \operatorname{Log} \left[ \frac{1}{c + d x} \right] - \operatorname{Log} \left[ \frac{e (a + b x)}{c + d x} \right] \right) \operatorname{Log} [c + d x] +
\end{aligned}$$



$$\begin{aligned}
& \frac{11 B d^3 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right) \operatorname{Log}[c+dx]}{3 (bc-ad)^4 g^4 i} + \frac{d^3 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^4 g^4 i} + \\
& \frac{A B d^3 \operatorname{Log}[c+dx]^2}{(bc-ad)^4 g^4 i} + \frac{11 B^2 d^3 \operatorname{Log}[c+dx]^2}{6 (bc-ad)^4 g^4 i} - \frac{B^2 d^3 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{(bc-ad)^4 g^4 i} + \\
& \frac{B^2 d^3 \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \operatorname{Log}[c+dx]^2}{(bc-ad)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log}[c+dx]^3}{3 (bc-ad)^4 g^4 i} - \frac{2 A B d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \\
& \frac{11 B^2 d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{3 (bc-ad)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log}[a+bx]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \\
& \frac{2 A B d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \frac{11 B^2 d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{3 (bc-ad)^4 g^4 i} + \\
& \frac{2 B^2 d^3 \operatorname{Log}[a+bx] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \frac{2 A B d^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \\
& \frac{11 B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{3 (bc-ad)^4 g^4 i} - \frac{2 B^2 d^3 \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} + \\
& \frac{2 B^2 d^3 \left( \operatorname{Log}[a+bx] + \operatorname{Log}\left[\frac{1}{c+dx}\right] - \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \\
& \frac{2 B^2 d^3 \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^4 g^4 i} - \frac{2 B^2 d^3 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \\
& \frac{2 B^2 d^3 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^4 g^4 i} - \frac{2 B^2 d^3 \operatorname{PolyLog}\left[3, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^4 g^4 i}
\end{aligned}$$

**Problem 92: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 722 leaves, 18 steps):

$$\begin{aligned}
& \frac{2AB(b c - a d)^2 g^3 (a + b x)}{d^3 i^2 (c + d x)} - \frac{2B^2(b c - a d)^2 g^3 (a + b x)}{d^3 i^2 (c + d x)} + \\
& \frac{2B^2(b c - a d)^2 g^3 (a + b x) \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]}{d^3 i^2 (c + d x)} - \frac{b B(b c - a d) g^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^3 i^2} - \\
& \frac{6b B(b c - a d)^2 g^3 \operatorname{Log}\left[\frac{b c - a d}{b(c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^4 i^2} - \\
& \frac{3b(b c - a d) g^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^3 i^2} - \\
& \frac{(b c - a d)^2 g^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^3 i^2 (c + d x)} + \frac{b^3 g^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{2 d^4 i^2} - \\
& \frac{3b(b c - a d)^2 g^3 \operatorname{Log}\left[\frac{b c - a d}{b(c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^4 i^2} + \frac{b B^2(b c - a d)^2 g^3 \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{b B(b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{d^4 i^2} - \frac{6b B^2(b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{d(a + b x)}{b(c + d x)}\right]}{d^4 i^2} - \\
& \frac{6b B(b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d(a + b x)}{b(c + d x)}\right]}{d^4 i^2} - \\
& \frac{b B^2(b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{d^4 i^2} + \frac{6b B^2(b c - a d)^2 g^3 \operatorname{PolyLog}\left[3, \frac{d(a + b x)}{b(c + d x)}\right]}{d^4 i^2}
\end{aligned}$$

Result(type 4, 2224 leaves, 119 steps):

$$\begin{aligned}
& - \frac{A b^2 B(b c - a d) g^3 x}{d^3 i^2} + \frac{2B^2(b c - a d)^3 g^3}{d^4 i^2 (c + d x)} + \frac{2b B^2(b c - a d)^2 g^3 \operatorname{Log}[a + b x]}{d^4 i^2} + \\
& \frac{a^2 b B^2 g^3 \operatorname{Log}[a + b x]^2}{2 d^2 i^2} + \frac{a b B^2(2b c - 3a d) g^3 \operatorname{Log}[a + b x]^2}{d^3 i^2} + \\
& \frac{b B^2(b c - a d)^2 g^3 \operatorname{Log}[a + b x]^2}{d^4 i^2} - \frac{3b B^2(b c - a d)^2 g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^4 i^2} + \\
& \frac{3b B^2(b c - a d)^2 g^3 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^4 i^2} - \frac{b B^2(b c - a d) g^3 (a + b x) \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]}{d^3 i^2} - \\
& \frac{2B(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^4 i^2 (c + d x)} - \frac{a^2 b B g^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^2 i^2} - \\
& \frac{2a b B(2b c - 3a d) g^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^3 i^2} - \\
& \frac{2b B(b c - a d)^2 g^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^4 i^2} - \frac{b^2(2b c - 3a d) g^3 x \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^3 i^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b^3 g^3 x^2 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{2 d^2 i^2} + \frac{(bc-ad)^3 g^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2}{d^4 i^2 (c+dx)} - \\
& \frac{b B^2 (bc-ad)^2 g^3 \operatorname{Log}[c+dx]}{d^4 i^2} - \frac{3 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx]^2 \operatorname{Log}[c+dx]}{d^4 i^2} - \\
& \frac{b^3 B^2 c^2 g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^2} - \frac{2 b^2 B^2 c (2bc-3ad) g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^2} - \\
& \frac{6 A b B (bc-ad)^2 g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^2} - \frac{2 b B^2 (bc-ad)^2 g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^2} - \\
& \frac{6 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{1}{c+dx} \right] \operatorname{Log}[c+dx]}{d^4 i^2} + \frac{1}{d^4 i^2} \\
& 6 b B^2 (bc-ad)^2 g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \left( \operatorname{Log}[a+bx] + \operatorname{Log} \left[ \frac{1}{c+dx} \right] - \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right) \operatorname{Log}[c+dx] + \\
& \frac{b^3 B c^2 g^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right) \operatorname{Log}[c+dx]}{d^4 i^2} + \\
& \frac{2 b^2 B c (2bc-3ad) g^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right) \operatorname{Log}[c+dx]}{d^4 i^2} + \\
& \frac{2 b B (bc-ad)^2 g^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right) \operatorname{Log}[c+dx]}{d^4 i^2} + \\
& \frac{3 b (bc-ad)^2 g^3 \left( A + B \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \right)^2 \operatorname{Log}[c+dx]}{d^4 i^2} + \frac{b^3 B^2 c^2 g^3 \operatorname{Log}[c+dx]^2}{2 d^4 i^2} + \\
& \frac{b^2 B^2 c (2bc-3ad) g^3 \operatorname{Log}[c+dx]^2}{d^4 i^2} + \frac{3 A b B (bc-ad)^2 g^3 \operatorname{Log}[c+dx]^2}{d^4 i^2} + \\
& \frac{b B^2 (bc-ad)^2 g^3 \operatorname{Log}[c+dx]^2}{d^4 i^2} - \frac{3 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{d^4 i^2} + \\
& \frac{3 b B^2 (bc-ad)^2 g^3 \operatorname{Log} \left[ \frac{e^{(a+bx)}}{c+dx} \right] \operatorname{Log}[c+dx]^2}{d^4 i^2} + \frac{b B^2 (bc-ad)^2 g^3 \operatorname{Log}[c+dx]^3}{d^4 i^2} - \\
& \frac{a^2 b B^2 g^3 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^2 i^2} - \frac{2 a b B^2 (2bc-3ad) g^3 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^3 i^2} - \\
& \frac{2 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^2} + \\
& \frac{3 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx]^2 \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^2} - \frac{a^2 b B^2 g^3 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^2 i^2} - \\
& \frac{2 a b B^2 (2bc-3ad) g^3 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^3 i^2} - \frac{2 b B^2 (bc-ad)^2 g^3 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^4 i^2} + \\
& \frac{6 b B^2 (bc-ad)^2 g^3 \operatorname{Log}[a+bx] \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^4 i^2} - \frac{b^3 B^2 c^2 g^3 \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 B^2 c (2 b c - 3 a d) g^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i^2} - \frac{6 A b B (b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i^2} - \\
& \frac{2 b B^2 (b c - a d)^2 g^3 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i^2} - \frac{6 b B^2 (b c - a d)^2 g^3 \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i^2} + \\
& \frac{1}{d^4 i^2} 6 b B^2 (b c - a d)^2 g^3 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c+dx}\right] - \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b c - a d}\right] - \\
& \frac{6 b B^2 (b c - a d)^2 g^3 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{b c - a d}\right]}{d^4 i^2} - \frac{6 b B^2 (b c - a d)^2 g^3 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i^2}
\end{aligned}$$

**Problem 93: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 469 leaves, 12 steps):

$$\begin{aligned}
& -\frac{2 A B (b c - a d) g^2 (a + b x)}{d^2 i^2 (c + d x)} + \frac{2 B^2 (b c - a d) g^2 (a + b x)}{d^2 i^2 (c + d x)} - \\
& \frac{2 B^2 (b c - a d) g^2 (a + b x) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{d^2 i^2 (c + d x)} + \frac{2 b B (b c - a d) g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{d^3 i^2} + \\
& \frac{b g^2 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{d^2 i^2} + \frac{(b c - a d) g^2 (a + b x) \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{d^2 i^2 (c + d x)} + \\
& \frac{2 b (b c - a d) g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)^2}{d^3 i^2} + \frac{2 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b (c + d x)}\right]}{d^3 i^2} + \\
& \frac{4 b B (b c - a d) g^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right) \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b (c + d x)}\right]}{d^3 i^2} - \\
& \frac{4 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[3, \frac{d(a+bx)}{b (c + d x)}\right]}{d^3 i^2}
\end{aligned}$$

Result (type 4, 1681 leaves, 94 steps):

$$\begin{aligned}
& -\frac{2 B^2 (b c - a d)^2 g^2}{d^3 i^2 (c + d x)} - \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x]}{d^3 i^2} - \\
& \frac{a b B^2 g^2 \operatorname{Log}[a + b x]^2}{d^2 i^2} - \frac{b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x]^2}{d^3 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{d^3 i^2} - \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{d^3 i^2} + \\
& \frac{2 B (b c - a d)^2 g^2 \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{d^3 i^2 (c + d x)} + \frac{2 a b B g^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \right)}{d^2 i^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b B (b c - a d) g^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{d^3 i^2} + \frac{b^2 g^2 x \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{d^2 i^2} - \\
& \frac{(b c - a d)^2 g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{d^3 i^2 (c + d x)} + \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{d^3 i^2} + \frac{2 b^2 B^2 c g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{4 A b B (b c - a d) g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{4 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{d^3 i^2} - \frac{1}{d^3 i^2} \\
& 4 b B^2 (b c - a d) g^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x] - \\
& \frac{2 b^2 B c g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d^3 i^2} - \frac{2 b B (b c - a d) g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{d^3 i^2} - \\
& \frac{2 b (b c - a d) g^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 \operatorname{Log}[c + d x]}{d^3 i^2} - \frac{b^2 B^2 c g^2 \operatorname{Log}[c + d x]^2}{d^3 i^2} - \\
& \frac{2 A b B (b c - a d) g^2 \operatorname{Log}[c + d x]^2}{d^3 i^2} - \frac{b B^2 (b c - a d) g^2 \operatorname{Log}[c + d x]^2}{d^3 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^3 i^2} - \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[c + d x]^3}{3 d^3 i^2} + \frac{2 a b B^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^2 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} - \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} + \\
& \frac{2 a b B^2 g^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^2 i^2} + \frac{2 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^3 i^2} - \\
& \frac{4 b B^2 (b c - a d) g^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^3 i^2} + \frac{2 b^2 B^2 c g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} + \\
& \frac{4 A b B (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} + \frac{2 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} + \\
& \frac{4 b B^2 (b c - a d) g^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2} - \frac{1}{d^3 i^2} \\
& 4 b B^2 (b c - a d) g^2 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right] + \\
& \frac{4 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{d^3 i^2} + \frac{4 b B^2 (b c - a d) g^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2}
\end{aligned}$$

### Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 261 leaves, 9 steps):

$$\begin{aligned} & \frac{2 A B g (a + b x)}{d i^2 (c + d x)} - \frac{2 B^2 g (a + b x)}{d i^2 (c + d x)} + \frac{2 B^2 g (a + b x) \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{d i^2 (c + d x)} - \\ & \frac{g (a + b x) \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{d i^2 (c + d x)} - \frac{b g \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right] \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{d^2 i^2} - \\ & \frac{2 b B g \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{d^2 i^2} + \frac{2 b B^2 g \operatorname{PolyLog}\left[3, \frac{d (a+b x)}{b (c+d x)}\right]}{d^2 i^2} \end{aligned}$$

Result (type 4, 1060 leaves, 72 steps):

$$\begin{aligned}
& \frac{2 B^2 (b c - a d) g}{d^2 i^2 (c + d x)} + \frac{2 b B^2 g \operatorname{Log}[a + b x]}{d^2 i^2} + \frac{b B^2 g \operatorname{Log}[a + b x]^2}{d^2 i^2} - \frac{b B^2 g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^2 i^2} + \\
& \frac{b B^2 g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^2 i^2} - \frac{2 B (b c - a d) g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^2 i^2 (c + d x)} - \\
& \frac{2 b B g \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^2 i^2} + \frac{(b c - a d) g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^2 i^2 (c + d x)} - \\
& \frac{2 b B^2 g \operatorname{Log}[c + d x]}{d^2 i^2} - \frac{b B^2 g \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{d^2 i^2} - \frac{2 A b B g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i^2} - \\
& \frac{2 b B^2 g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i^2} - \frac{2 b B^2 g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{d^2 i^2} + \frac{1}{d^2 i^2} \\
& 2 b B^2 g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] + \\
& \frac{2 b B g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^2 i^2} + \frac{b g \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{d^2 i^2} + \\
& \frac{A b B g \operatorname{Log}[c + d x]^2}{d^2 i^2} + \frac{b B^2 g \operatorname{Log}[c + d x]^2}{d^2 i^2} - \frac{b B^2 g \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^2 i^2} + \\
& \frac{b B^2 g \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{d^2 i^2} + \frac{b B^2 g \operatorname{Log}[c + d x]^3}{3 d^2 i^2} - \frac{2 b B^2 g \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} + \\
& \frac{b B^2 g \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} + \\
& \frac{2 b B^2 g \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 A b B g \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \\
& \frac{2 b B^2 g \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} + \frac{1}{d^2 i^2} \\
& 2 b B^2 g \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] - \\
& \frac{2 b B^2 g \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2}
\end{aligned}$$

**Problem 95:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(c i + d i x)^2} dx$$

Optimal (type 3, 152 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2(a+bx)}{(bc-ad)i^2(c+dx)} - \\
& \frac{2B^2(a+bx)\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)i^2(c+dx)}
\end{aligned}$$

Result (type 4, 472 leaves, 26 steps):

$$\begin{aligned}
& - \frac{2B^2}{di^2(c+dx)} - \frac{2bB^2\operatorname{Log}[a+bx]}{d(bc-ad)i^2} - \frac{bB^2\operatorname{Log}[a+bx]^2}{d(bc-ad)i^2} + \frac{2B\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{di^2(c+dx)} + \\
& \frac{2bB\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{d(bc-ad)i^2} - \frac{\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{di^2(c+dx)} + \frac{2bB^2\operatorname{Log}[c+dx]}{d(bc-ad)i^2} + \\
& \frac{2bB^2\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{d(bc-ad)i^2} - \frac{2bB\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)\operatorname{Log}[c+dx]}{d(bc-ad)i^2} - \frac{bB^2\operatorname{Log}[c+dx]^2}{d(bc-ad)i^2} + \\
& \frac{2bB^2\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)i^2} + \frac{2bB^2\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{d(bc-ad)i^2} + \frac{2bB^2\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)i^2}
\end{aligned}$$

**Problem 96: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag+bgx)(ci+dix)^2} dx$$

Optimal (type 3, 214 leaves, 7 steps):

$$\begin{aligned}
& \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2d(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \\
& \frac{2B^2d(a+bx)\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{3B(bc-ad)^2gi^2}
\end{aligned}$$

Result (type 4, 1687 leaves, 87 steps):



$$\begin{aligned}
& \frac{2 B^2}{(b c - a d) g i^2 (c + d x)} + \frac{2 b B^2 \operatorname{Log}[a + b x]}{(b c - a d)^2 g i^2} - \frac{A b B \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g i^2} + \frac{b B^2 \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g i^2} + \\
& \frac{b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}\left[-\frac{b c - a d}{d(a + b x)}\right] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^2 g i^2} - \\
& \frac{b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^2 g i^2} - \frac{2 B \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d) g i^2 (c + d x)} - \frac{2 b B \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g i^2} + \\
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d) g i^2 (c + d x)} + \frac{b \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d)^2 g i^2} - \frac{2 b B^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \\
& \frac{b B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \frac{2 A b B \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{1}{(b c - a d)^2 g i^2} \\
& 2 b B^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] + \\
& \frac{2 b B \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{b \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \\
& \frac{A b B \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} + \frac{b B^2 \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} + \frac{b B^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} - \\
& \frac{b B^2 \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}[c + d x]^3}{3 (b c - a d)^2 g i^2} + \frac{2 A b B \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 A b B \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{2 b B^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \\
& \frac{2 A b B \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{2 b B^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \\
& \frac{2 b B^2 \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \\
& \frac{2 b B^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g i^2}
\end{aligned}$$

Problem 97: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(ag+bgx)^2 (ci+di x)^2} dx$$

Optimal (type 3, 365 leaves, 10 steps):

$$\begin{aligned} & -\frac{2ABd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} + \frac{2B^2d^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{2b^2B^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} - \\ & \frac{2B^2d^2(a+bx)\operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}{(bc-ad)^3g^2i^2(c+dx)} - \frac{2b^2B(c+dx)\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{(bc-ad)^3g^2i^2(a+bx)} + \\ & \frac{d^2(a+bx)\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\left(A+B\operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^3}{3B(bc-ad)^3g^2i^2} \end{aligned}$$

Result (type 4, 1521 leaves, 113 steps):

$$\begin{aligned}
& - \frac{2 b B^2}{(b c - a d)^2 g^2 i^2 (a + b x)} - \frac{2 B^2 d}{(b c - a d)^2 g^2 i^2 (c + d x)} - \frac{4 b B^2 d \operatorname{Log}[a + b x]}{(b c - a d)^3 g^2 i^2} + \\
& \frac{2 A b B d \operatorname{Log}[a + b x]^2}{(b c - a d)^3 g^2 i^2} - \frac{2 b B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} + \\
& \frac{2 b B^2 d \operatorname{Log}\left[-\frac{b c - a d}{d(a + b x)}\right] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g^2 i^2 (a + b x)} + \frac{2 B d \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{(b c - a d)^2 g^2 i^2 (c + d x)} - \frac{b \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d)^2 g^2 i^2 (a + b x)} - \\
& \frac{d \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d)^2 g^2 i^2 (c + d x)} - \frac{2 b d \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{(b c - a d)^3 g^2 i^2} + \frac{4 b B^2 d \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B^2 d \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} - \frac{4 A b B d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} - \\
& \frac{4 b B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \frac{1}{(b c - a d)^3 g^2 i^2} \\
& 4 b B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] + \\
& \frac{2 b d \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \frac{2 A b B d \operatorname{Log}[c + d x]^2}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B^2 d \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}[c + d x]^3}{3 (b c - a d)^3 g^2 i^2} - \\
& \frac{4 A b B d \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \\
& \frac{4 A b B d \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} + \frac{4 b B^2 d \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \\
& \frac{4 A b B d \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{4 b B^2 d \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} + \frac{1}{(b c - a d)^3 g^2 i^2} \\
& 4 b B^2 d \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] - \\
& \frac{4 b B^2 d \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^3 g^2 i^2} - \frac{4 b B^2 d \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \\
& \frac{4 b B^2 d \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{4 b B^2 d \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^3 g^2 i^2}
\end{aligned}$$

Problem 98: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(ag+bgx)^3 (ci+dix)^2} dx$$

Optimal (type 3, 523 leaves, 12 steps):

$$\begin{aligned} & \frac{2ABd^3(a+bx)}{(bc-ad)^4 g^3 i^2 (c+dx)} - \frac{2B^2 d^3 (a+bx)}{(bc-ad)^4 g^3 i^2 (c+dx)} + \\ & \frac{6b^2 B^2 d (c+dx)}{(bc-ad)^4 g^3 i^2 (a+bx)} - \frac{b^3 B^2 (c+dx)^2}{4(bc-ad)^4 g^3 i^2 (a+bx)^2} + \frac{2B^2 d^3 (a+bx) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)^4 g^3 i^2 (c+dx)} + \\ & \frac{6b^2 B d (c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^3 i^2 (a+bx)} - \frac{b^3 B (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2(bc-ad)^4 g^3 i^2 (a+bx)^2} - \\ & \frac{d^3 (a+bx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4 g^3 i^2 (c+dx)} + \frac{3b^2 d (c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4 g^3 i^2 (a+bx)} - \\ & \frac{b^3 (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2(bc-ad)^4 g^3 i^2 (a+bx)^2} + \frac{b d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{B(bc-ad)^4 g^3 i^2} \end{aligned}$$

Result (type 4, 2071 leaves, 143 steps):

$$\begin{aligned} & -\frac{bB^2}{4(bc-ad)^2 g^3 i^2 (a+bx)^2} + \frac{11bB^2 d}{2(bc-ad)^3 g^3 i^2 (a+bx)} + \frac{2B^2 d^2}{(bc-ad)^3 g^3 i^2 (c+dx)} + \\ & \frac{15bB^2 d^2 \operatorname{Log}[a+bx]}{2(bc-ad)^4 g^3 i^2} - \frac{3ABd^2 \operatorname{Log}[a+bx]^2}{(bc-ad)^4 g^3 i^2} - \frac{3bB^2 d^2 \operatorname{Log}[a+bx]^2}{2(bc-ad)^4 g^3 i^2} + \\ & \frac{3bB^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^4 g^3 i^2} - \frac{3bB^2 d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^4 g^3 i^2} - \\ & \frac{3bB^2 d^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^4 g^3 i^2} - \frac{3bB^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^4 g^3 i^2} - \\ & \frac{bB \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2(bc-ad)^2 g^3 i^2 (a+bx)^2} + \frac{5bBd \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3 g^3 i^2 (a+bx)} - \frac{2Bd^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^3 g^3 i^2 (c+dx)} + \\ & \frac{3bBd^2 \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^3 i^2} - \frac{b \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2(bc-ad)^2 g^3 i^2 (a+bx)^2} + \\ & \frac{2bd \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3 g^3 i^2 (a+bx)} + \frac{d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3 g^3 i^2 (c+dx)} + \frac{3bd^2 \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4 g^3 i^2} - \end{aligned}$$

$$\begin{aligned}
& \frac{15 b B^2 d^2 \operatorname{Log}[c+d x]}{2 (b c-a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 \operatorname{Log}[a+b x]^2 \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{6 A b B d^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{6 b B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} - \frac{1}{(b c-a d)^4 g^3 i^2} \\
& 6 b B^2 d^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \left( \operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c+d x] - \\
& \frac{3 b B d^2 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} - \frac{3 b d^2 \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2 \operatorname{Log}[c+d x]}{(b c-a d)^4 g^3 i^2} - \\
& \frac{3 A b B d^2 \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^3 i^2} - \frac{3 b B^2 d^2 \operatorname{Log}[c+d x]^2}{2 (b c-a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^3 i^2} - \\
& \frac{3 b B^2 d^2 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^3 i^2} - \frac{b B^2 d^2 \operatorname{Log}[c+d x]^3}{(b c-a d)^4 g^3 i^2} + \frac{6 A b B d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{3 b B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} - \frac{3 b B^2 d^2 \operatorname{Log}[a+b x]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{6 A b B d^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} - \\
& \frac{6 b B^2 d^2 \operatorname{Log}[a+b x] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \frac{6 A b B d^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{3 b B^2 d^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} - \frac{1}{(b c-a d)^4 g^3 i^2} \\
& 6 b B^2 d^2 \left( \operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right] + \\
& \frac{6 b B^2 d^2 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \\
& \frac{6 b B^2 d^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^3 i^2}
\end{aligned}$$

**Problem 99:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2}{(a g + b g x)^4 (c i + d i x)^2} dx$$

Optimal (type 3, 682 leaves, 14 steps):

$$\begin{aligned}
& - \frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} + \frac{2B^2d^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \\
& \frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{2B^2d^4(a+bx)\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)^5g^4i^2(c+dx)} - \\
& \frac{12b^2Bd^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bd(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \\
& \frac{2b^4B(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \\
& \frac{6b^2d^2(c+dx)\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \\
& \frac{b^4(c+dx)^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{3B(bc-ad)^5g^4i^2}
\end{aligned}$$

Result (type 4, 2222 leaves, 177 steps):

$$\begin{aligned}
& - \frac{2bB^2}{27(bc-ad)^2g^4i^2(a+bx)^3} + \frac{7bB^2d}{9(bc-ad)^3g^4i^2(a+bx)^2} - \frac{92bB^2d^2}{9(bc-ad)^4g^4i^2(a+bx)} - \\
& \frac{2B^2d^3}{(bc-ad)^4g^4i^2(c+dx)} - \frac{110bB^2d^3\operatorname{Log}[a+bx]}{9(bc-ad)^5g^4i^2} + \frac{4AbBd^3\operatorname{Log}[a+bx]^2}{(bc-ad)^5g^4i^2} + \\
& \frac{10bB^2d^3\operatorname{Log}[a+bx]^2}{3(bc-ad)^5g^4i^2} - \frac{4bB^2d^3\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^5g^4i^2} + \frac{4bB^2d^3\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^5g^4i^2} + \\
& \frac{4bB^2d^3\operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right]\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^5g^4i^2} + \frac{4bB^2d^3\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^5g^4i^2} - \\
& \frac{2bB\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9(bc-ad)^2g^4i^2(a+bx)^3} + \frac{4bBd\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3(bc-ad)^3g^4i^2(a+bx)^2} - \frac{26bBd^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3(bc-ad)^4g^4i^2(a+bx)} + \\
& \frac{2Bd^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4g^4i^2(c+dx)} - \frac{20bBd^3\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3(bc-ad)^5g^4i^2} - \\
& \frac{b\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3(bc-ad)^2g^4i^2(a+bx)^3} + \frac{bd\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3g^4i^2(a+bx)^2} - \frac{3bd^2\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4g^4i^2(a+bx)} - \\
& \frac{d^3\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4g^4i^2(c+dx)} - \frac{4bd^3\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5g^4i^2} + \\
& \frac{110bB^2d^3\operatorname{Log}[c+dx]}{9(bc-ad)^5g^4i^2} - \frac{4bB^2d^3\operatorname{Log}[a+bx]^2\operatorname{Log}[c+dx]}{(bc-ad)^5g^4i^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{8 A b B d^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^5 g^4 i^2} - \frac{20 b B^2 d^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{3(b c-a d)^5 g^4 i^2} - \\
& \frac{8 b B^2 d^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{Log}[c+d x]}{(b c-a d)^5 g^4 i^2} + \frac{1}{(b c-a d)^5 g^4 i^2} \\
& \frac{8 b B^2 d^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \left(\operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x]}{(b c-a d)^5 g^4 i^2} + \\
& \frac{20 b B d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{Log}[c+d x]}{3(b c-a d)^5 g^4 i^2} + \frac{4 b d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2 \operatorname{Log}[c+d x]}{(b c-a d)^5 g^4 i^2} + \\
& \frac{4 A b B d^3 \operatorname{Log}[c+d x]^2}{(b c-a d)^5 g^4 i^2} + \frac{10 b B^2 d^3 \operatorname{Log}[c+d x]^2}{3(b c-a d)^5 g^4 i^2} - \frac{4 b B^2 d^3 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^5 g^4 i^2} + \\
& \frac{4 b B^2 d^3 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{Log}[c+d x]^2}{(b c-a d)^5 g^4 i^2} + \frac{4 b B^2 d^3 \operatorname{Log}[c+d x]^3}{3(b c-a d)^5 g^4 i^2} - \frac{8 A b B d^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \\
& \frac{20 b B^2 d^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{3(b c-a d)^5 g^4 i^2} + \frac{4 b B^2 d^3 \operatorname{Log}[a+b x]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \\
& \frac{8 A b B d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \frac{20 b B^2 d^3 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{3(b c-a d)^5 g^4 i^2} + \\
& \frac{8 b B^2 d^3 \operatorname{Log}[a+b x] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \frac{8 A b B d^3 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \\
& \frac{20 b B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{3(b c-a d)^5 g^4 i^2} - \frac{8 b B^2 d^3 \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} + \frac{1}{(b c-a d)^5 g^4 i^2} \\
& \frac{8 b B^2 d^3 \left(\operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \\
& \frac{8 b B^2 d^3 \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^5 g^4 i^2} - \frac{8 b B^2 d^3 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \\
& \frac{8 b B^2 d^3 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^5 g^4 i^2} - \frac{8 b B^2 d^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^5 g^4 i^2}
\end{aligned}$$

**Problem 100: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right]\right)^2}{(c i + d i x)^3} dx$$

Optimal (type 4, 635 leaves, 14 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d) g^3 (a + b x)^2}{4 d^2 i^3 (c + d x)^2} - \frac{4 A b B (b c - a d) g^3 (a + b x)}{d^3 i^3 (c + d x)} + \frac{4 b B^2 (b c - a d) g^3 (a + b x)}{d^3 i^3 (c + d x)} - \\
& \frac{4 b B^2 (b c - a d) g^3 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^3 i^3 (c + d x)} - \frac{B (b c - a d) g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^2 i^3 (c + d x)^2} + \\
& \frac{2 b^2 B (b c - a d) g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^3} + \frac{b^2 g^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^3 i^3} + \\
& \frac{(b c - a d) g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 d^2 i^3 (c + d x)^2} + \frac{2 b (b c - a d) g^3 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^3 i^3 (c + d x)} + \\
& \frac{3 b^2 (b c - a d) g^3 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^4 i^3} + \frac{2 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i^3} + \\
& \frac{6 b^2 B (b c - a d) g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i^3} - \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i^3}
\end{aligned}$$

Result (type 4, 1890 leaves, 124 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^3 g^3}{4 d^4 i^3 (c + d x)^2} - \frac{9 b B^2 (b c - a d)^2 g^3}{2 d^4 i^3 (c + d x)} - \frac{9 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x]}{2 d^4 i^3} - \\
& \frac{a b^2 B^2 g^3 \operatorname{Log}[a + b x]^2}{d^3 i^3} - \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x]^2}{2 d^4 i^3} + \\
& \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^4 i^3} - \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^4 i^3} - \\
& \frac{B (b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d^4 i^3 (c + d x)^2} + \frac{5 b B (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^3 (c + d x)} + \\
& \frac{2 a b^2 B g^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^3 i^3} + \frac{5 b^2 B (b c - a d) g^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{d^4 i^3} + \\
& \frac{b^3 g^3 x \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^3 i^3} + \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 d^4 i^3 (c + d x)^2} - \\
& \frac{3 b (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^4 i^3 (c + d x)} + \frac{9 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[c + d x]}{2 d^4 i^3} + \\
& \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{d^4 i^3} + \frac{2 b^3 B^2 c g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^3} + \\
& \frac{6 A b^2 B (b c - a d) g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^3} + \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^3} +
\end{aligned}$$



$$\begin{aligned}
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{d^4 i^3} - \frac{1}{d^4 i^3} \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^4 i^3} - \\
& \frac{2 b^3 B c g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^4 i^3} - \frac{5 b^2 B (b c - a d) g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^4 i^3} - \\
& \frac{3 b^2 (b c - a d) g^3 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{d^4 i^3} - \frac{b^3 B^2 c g^3 \operatorname{Log}[c + d x]^2}{d^4 i^3} - \\
& \frac{3 A b^2 B (b c - a d) g^3 \operatorname{Log}[c + d x]^2}{d^4 i^3} - \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[c + d x]^2}{2 d^4 i^3} + \\
& \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^4 i^3} - \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{d^4 i^3} - \\
& \frac{b^2 B^2 (b c - a d) g^3 \operatorname{Log}[c + d x]^3}{d^4 i^3} + \frac{2 a b^2 B^2 g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^3 i^3} + \\
& \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} - \frac{3 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} + \\
& \frac{2 a b^2 B^2 g^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^3 i^3} + \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^4 i^3} - \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d^4 i^3} + \frac{2 b^3 B^2 c g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} + \\
& \frac{6 A b^2 B (b c - a d) g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} + \frac{5 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} + \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} - \frac{1}{d^4 i^3} \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3} + \\
& \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{d^4 i^3} + \frac{6 b^2 B^2 (b c - a d) g^3 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^3}
\end{aligned}$$

**Problem 101: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{(c i + d i x)^3} dx$$

Optimal (type 4, 410 leaves, 11 steps):

$$\begin{aligned}
& - \frac{B^2 g^2 (a + b x)^2}{4 d i^3 (c + d x)^2} + \frac{2 A b B g^2 (a + b x)}{d^2 i^3 (c + d x)} - \frac{2 b B^2 g^2 (a + b x)}{d^2 i^3 (c + d x)} + \frac{2 b B^2 g^2 (a + b x) \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]}{d^2 i^3 (c + d x)} + \\
& \frac{B g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)}{2 d i^3 (c + d x)^2} - \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{2 d i^3 (c + d x)^2} - \\
& \frac{b g^2 (a + b x) \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^2 i^3 (c + d x)} - \frac{b^2 g^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right] \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right)^2}{d^3 i^3} - \\
& \frac{2 b^2 B g^2 \left(A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i^3} + \frac{2 b^2 B^2 g^2 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i^3}
\end{aligned}$$

Result (type 4, 1328 leaves, 102 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^2 g^2}{4 d^3 i^3 (c + d x)^2} + \frac{5 b B^2 (b c - a d) g^2}{2 d^3 i^3 (c + d x)} + \frac{5 b^2 B^2 g^2 \operatorname{Log}[a + b x]}{2 d^3 i^3} + \frac{3 b^2 B^2 g^2 \operatorname{Log}[a + b x]^2}{2 d^3 i^3} - \\
& \frac{b^2 B^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^3 i^3} + \frac{b^2 B^2 g^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[\frac{1}{c + d x}\right]^2}{d^3 i^3} + \\
& \frac{B (b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{2 d^3 i^3 (c + d x)^2} - \frac{3 b B (b c - a d) g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^3 i^3 (c + d x)} - \\
& \frac{3 b^2 B g^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)}{d^3 i^3} - \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{2 d^3 i^3 (c + d x)^2} + \\
& \frac{2 b (b c - a d) g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2}{d^3 i^3 (c + d x)} - \frac{5 b^2 B^2 g^2 \operatorname{Log}[c + d x]}{2 d^3 i^3} - \\
& \frac{b^2 B^2 g^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{d^3 i^3} - \frac{2 A b^2 B g^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^3} - \\
& \frac{3 b^2 B^2 g^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^3} - \frac{2 b^2 B^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{d^3 i^3} + \frac{1}{d^3 i^3} \\
& 2 b^2 B^2 g^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x] + \\
& \frac{3 b^2 B g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{Log}[c + d x]}{d^3 i^3} + \frac{b^2 g^2 \left(A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right)^2 \operatorname{Log}[c + d x]}{d^3 i^3} + \\
& \frac{A b^2 B g^2 \operatorname{Log}[c + d x]^2}{d^3 i^3} + \frac{3 b^2 B^2 g^2 \operatorname{Log}[c + d x]^2}{2 d^3 i^3} - \frac{b^2 B^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^3 i^3} + \\
& \frac{b^2 B^2 g^2 \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{d^3 i^3} + \frac{b^2 B^2 g^2 \operatorname{Log}[c + d x]^3}{3 d^3 i^3} - \frac{3 b^2 B^2 g^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3} + \\
& \frac{b^2 B^2 g^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3} - \frac{3 b^2 B^2 g^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^3 i^3} + \\
& \frac{2 b^2 B^2 g^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^3 i^3} - \frac{2 A b^2 B g^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3} - \\
& \frac{3 b^2 B^2 g^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3} - \frac{2 b^2 B^2 g^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3} + \frac{1}{d^3 i^3} \\
& 2 b^2 B^2 g^2 \left(\operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] - \\
& \frac{2 b^2 B^2 g^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{d^3 i^3} - \frac{2 b^2 B^2 g^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3}
\end{aligned}$$

**Problem 102:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{(c i + d i x)^3} dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{B^2 g (a+b x)^2}{4 (b c - a d) i^3 (c+d x)^2} - \frac{B g (a+b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}{2 (b c - a d) i^3 (c+d x)^2} + \frac{g (a+b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{2 (b c - a d) i^3 (c+d x)^2}$$

Result (type 4, 634 leaves, 58 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d) g}{4 d^2 i^3 (c+d x)^2} - \frac{b B^2 g}{2 d^2 i^3 (c+d x)} - \frac{b^2 B^2 g \operatorname{Log}[a+b x]}{2 d^2 (b c - a d) i^3} - \\ & \frac{b^2 B^2 g \operatorname{Log}[a+b x]^2}{2 d^2 (b c - a d) i^3} - \frac{B (b c - a d) g \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}{2 d^2 i^3 (c+d x)^2} + \frac{b B g \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}{d^2 i^3 (c+d x)} + \\ & \frac{b^2 B g \operatorname{Log}[a+b x] \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}{d^2 (b c - a d) i^3} + \frac{(b c - a d) g \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{2 d^2 i^3 (c+d x)^2} - \\ & \frac{b g \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{d^2 i^3 (c+d x)} + \frac{b^2 B^2 g \operatorname{Log}[c+d x]}{2 d^2 (b c - a d) i^3} + \frac{b^2 B^2 g \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log}[c+d x]}{d^2 (b c - a d) i^3} - \\ & \frac{b^2 B g \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right) \operatorname{Log}[c+d x]}{d^2 (b c - a d) i^3} - \frac{b^2 B^2 g \operatorname{Log}[c+d x]^2}{2 d^2 (b c - a d) i^3} + \\ & \frac{b^2 B^2 g \operatorname{Log}[a+b x] \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} + \frac{b^2 B^2 g \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} + \frac{b^2 B^2 g \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} \end{aligned}$$

**Problem 103:** Result unnecessarily involves higher level functions.

$$\int \frac{\left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{(c i + d i x)^3} dx$$

Optimal (type 3, 296 leaves, 8 steps):

$$\begin{aligned} & -\frac{B^2 d (a+b x)^2}{4 (b c - a d)^2 i^3 (c+d x)^2} - \frac{2 A b B (a+b x)}{(b c - a d)^2 i^3 (c+d x)} + \frac{2 b B^2 (a+b x)}{(b c - a d)^2 i^3 (c+d x)} - \\ & \frac{2 b B^2 (a+b x) \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right]}{(b c - a d)^2 i^3 (c+d x)} + \frac{B d (a+b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)}{2 (b c - a d)^2 i^3 (c+d x)^2} - \\ & \frac{d (a+b x)^2 \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{2 (b c - a d)^2 i^3 (c+d x)^2} + \frac{b (a+b x) \left( A + B \operatorname{Log} \left[ \frac{e (a+b x)}{c+d x} \right] \right)^2}{(b c - a d)^2 i^3 (c+d x)} \end{aligned}$$

Result (type 4, 577 leaves, 30 steps):

$$\begin{aligned}
 & -\frac{B^2}{4 d i^3 (c+d x)^2} - \frac{3 b B^2}{2 d (b c-a d) i^3 (c+d x)} - \frac{3 b^2 B^2 \operatorname{Log}[a+b x]}{2 d (b c-a d)^2 i^3} - \\
 & \frac{b^2 B^2 \operatorname{Log}[a+b x]^2}{2 d (b c-a d)^2 i^3} + \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{2 d i^3 (c+d x)^2} + \frac{b B \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{d (b c-a d) i^3 (c+d x)} + \\
 & \frac{b^2 B \operatorname{Log}[a+b x] \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{d (b c-a d)^2 i^3} - \frac{\left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{2 d i^3 (c+d x)^2} + \frac{3 b^2 B^2 \operatorname{Log}[c+d x]}{2 d (b c-a d)^2 i^3} + \\
 & \frac{b^2 B^2 \operatorname{Log}\left[-\frac{d (a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{d (b c-a d)^2 i^3} - \frac{b^2 B \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right) \operatorname{Log}[c+d x]}{d (b c-a d)^2 i^3} - \frac{b^2 B^2 \operatorname{Log}[c+d x]^2}{2 d (b c-a d)^2 i^3} + \\
 & \frac{b^2 B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c-a d}\right]}{d (b c-a d)^2 i^3} + \frac{b^2 B^2 \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c-a d}\right]}{d (b c-a d)^2 i^3} + \frac{b^2 B^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c-a d}\right]}{d (b c-a d)^2 i^3}
 \end{aligned}$$

**Problem 104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{(a g + b g x) (c i + d i x)^3} dx$$

Optimal (type 3, 375 leaves, 15 steps):

$$\begin{aligned}
 & \frac{B^2 d^2 (a+b x)^2}{4 (b c-a d)^3 g i^3 (c+d x)^2} + \frac{4 A b B d (a+b x)}{(b c-a d)^3 g i^3 (c+d x)} - \frac{4 b B^2 d (a+b x)}{(b c-a d)^3 g i^3 (c+d x)} + \\
 & \frac{4 b B^2 d (a+b x) \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]}{(b c-a d)^3 g i^3 (c+d x)} - \frac{B d^2 (a+b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{2 (b c-a d)^3 g i^3 (c+d x)^2} + \\
 & \frac{d^2 (a+b x)^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{2 (b c-a d)^3 g i^3 (c+d x)^2} - \frac{2 b d (a+b x) \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^2}{(b c-a d)^3 g i^3 (c+d x)} + \frac{b^2 \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)^3}{3 B (b c-a d)^3 g i^3}
 \end{aligned}$$

Result (type 4, 1899 leaves, 117 steps):

$$\begin{aligned}
 & \frac{B^2}{4 (b c-a d) g i^3 (c+d x)^2} + \frac{7 b B^2}{2 (b c-a d)^2 g i^3 (c+d x)} + \frac{7 b^2 B^2 \operatorname{Log}[a+b x]}{2 (b c-a d)^3 g i^3} - \frac{A b^2 B \operatorname{Log}[a+b x]^2}{(b c-a d)^3 g i^3} + \\
 & \frac{3 b^2 B^2 \operatorname{Log}[a+b x]^2}{2 (b c-a d)^3 g i^3} + \frac{b^2 B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{(b c-a d)^3 g i^3} - \frac{b^2 B^2 \operatorname{Log}\left[-\frac{d (a+b x)}{b c-a d}\right] \operatorname{Log}\left[\frac{1}{c+d x}\right]^2}{(b c-a d)^3 g i^3} - \\
 & \frac{b^2 B^2 \operatorname{Log}\left[-\frac{b c-a d}{d (a+b x)}\right] \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]^2}{(b c-a d)^3 g i^3} - \frac{b^2 B^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right]^2}{(b c-a d)^3 g i^3} - \\
 & \frac{B \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{2 (b c-a d) g i^3 (c+d x)^2} - \frac{3 b B \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{(b c-a d)^2 g i^3 (c+d x)} - \frac{3 b^2 B \operatorname{Log}[a+b x] \left( A + B \operatorname{Log}\left[\frac{e (a+b x)}{c+d x}\right] \right)}{(b c-a d)^3 g i^3} +
 \end{aligned}$$

$$\begin{aligned}
& \frac{\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2(bc-ad)gi^3(c+dx)^2} + \frac{b\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^2gi^3(c+dx)} + \frac{b^2 \operatorname{Log}[a+bx]\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^3gi^3} - \\
& \frac{7b^2B^2 \operatorname{Log}[c+dx]}{2(bc-ad)^3gi^3} + \frac{b^2B^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} + \frac{2Ab^2B \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} - \\
& \frac{3b^2B^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} + \frac{2b^2B^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} - \frac{1}{(bc-ad)^3gi^3} \\
& 2b^2B^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \left(\operatorname{Log}[a+bx] + \operatorname{Log}\left[\frac{1}{c+dx}\right] - \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c+dx] + \\
& \frac{3b^2B\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} - \frac{b^2\left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^3gi^3} - \\
& \frac{Ab^2B \operatorname{Log}[c+dx]^2}{(bc-ad)^3gi^3} + \frac{3b^2B^2 \operatorname{Log}[c+dx]^2}{2(bc-ad)^3gi^3} + \frac{b^2B^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{(bc-ad)^3gi^3} - \\
& \frac{b^2B^2 \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \operatorname{Log}[c+dx]^2}{(bc-ad)^3gi^3} - \frac{b^2B^2 \operatorname{Log}[c+dx]^3}{3(bc-ad)^3gi^3} + \frac{2Ab^2B \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \\
& \frac{3b^2B^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \frac{b^2B^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \\
& \frac{2Ab^2B \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \frac{3b^2B^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \\
& \frac{2b^2B^2 \operatorname{Log}[a+bx] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \frac{2Ab^2B \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \\
& \frac{3b^2B^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \frac{2b^2B^2 \operatorname{Log}\left[\frac{1}{c+dx}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} - \\
& \frac{2b^2B^2 \left(\operatorname{Log}[a+bx] + \operatorname{Log}\left[\frac{1}{c+dx}\right] - \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \\
& \frac{2b^2B^2 \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^3gi^3} + \frac{2b^2B^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \\
& \frac{2b^2B^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3gi^3} + \frac{2b^2B^2 \operatorname{PolyLog}\left[3, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^3gi^3}
\end{aligned}$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(ag+bgx)^2 (ci+dix)^3} dx$$

Optimal (type 3, 525 leaves, 12 steps):

$$\begin{aligned} & -\frac{B^2 d^3 (a+bx)^2}{4 (bc-ad)^4 g^2 i^3 (c+dx)^2} - \frac{6AbBd^2 (a+bx)}{(bc-ad)^4 g^2 i^3 (c+dx)} + \\ & \frac{6bB^2 d^2 (a+bx)}{(bc-ad)^4 g^2 i^3 (c+dx)} - \frac{2b^3 B^2 (c+dx)}{(bc-ad)^4 g^2 i^3 (a+bx)} - \frac{6bB^2 d^2 (a+bx) \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]}{(bc-ad)^4 g^2 i^3 (c+dx)} + \\ & \frac{Bd^3 (a+bx)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{2 (bc-ad)^4 g^2 i^3 (c+dx)^2} - \frac{2b^3 B (c+dx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{(bc-ad)^4 g^2 i^3 (a+bx)} - \\ & \frac{d^3 (a+bx)^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{2 (bc-ad)^4 g^2 i^3 (c+dx)^2} + \frac{3bd^2 (a+bx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^4 g^2 i^3 (c+dx)} - \\ & \frac{b^3 (c+dx) \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^4 g^2 i^3 (a+bx)} - \frac{b^2 d \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^3}{B (bc-ad)^4 g^2 i^3} \end{aligned}$$

Result (type 4, 2071 leaves, 143 steps):

$$\begin{aligned} & -\frac{2b^2 B^2}{(bc-ad)^3 g^2 i^3 (a+bx)} - \frac{B^2 d}{4 (bc-ad)^2 g^2 i^3 (c+dx)^2} - \frac{11bB^2 d}{2 (bc-ad)^3 g^2 i^3 (c+dx)} - \\ & \frac{15b^2 B^2 d \operatorname{Log}[a+bx]}{2 (bc-ad)^4 g^2 i^3} + \frac{3Ab^2 B d \operatorname{Log}[a+bx]^2}{(bc-ad)^4 g^2 i^3} - \frac{3b^2 B^2 d \operatorname{Log}[a+bx]^2}{2 (bc-ad)^4 g^2 i^3} - \\ & \frac{3b^2 B^2 d \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^4 g^2 i^3} + \frac{3b^2 B^2 d \operatorname{Log}\left[-\frac{d(a+bx)}{b c-a d}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^4 g^2 i^3} + \\ & \frac{3b^2 B^2 d \operatorname{Log}\left[-\frac{b c-a d}{d(a+bx)}\right] \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]^2}{(bc-ad)^4 g^2 i^3} + \frac{3b^2 B^2 d \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]^2}{(bc-ad)^4 g^2 i^3} - \\ & \frac{2b^2 B \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{(bc-ad)^3 g^2 i^3 (a+bx)} + \frac{B d \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{2 (bc-ad)^2 g^2 i^3 (c+dx)^2} + \\ & \frac{5bB d \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{(bc-ad)^3 g^2 i^3 (c+dx)} + \frac{3b^2 B d \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)}{(bc-ad)^4 g^2 i^3} - \\ & \frac{b^2 \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^3 g^2 i^3 (a+bx)} - \frac{d \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{2 (bc-ad)^2 g^2 i^3 (c+dx)^2} - \\ & \frac{2bd \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^3 g^2 i^3 (c+dx)} - \frac{3b^2 d \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[\frac{e^{(a+bx)}}{c+dx}\right]\right)^2}{(bc-ad)^4 g^2 i^3} + \\ & \frac{15b^2 B^2 d \operatorname{Log}[c+dx]}{2 (bc-ad)^4 g^2 i^3} - \frac{3b^2 B^2 d \operatorname{Log}[a+bx]^2 \operatorname{Log}[c+dx]}{(bc-ad)^4 g^2 i^3} - \end{aligned}$$

$$\begin{aligned}
& \frac{6 A b^2 B d \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^2 i^3} - \\
& \frac{6 b^2 B^2 d \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{Log}[c+d x]}{(b c-a d)^4 g^2 i^3} + \frac{1}{(b c-a d)^4 g^2 i^3} \\
& 6 b^2 B^2 d \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \left( \operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c+d x] - \\
& \frac{3 b^2 B d \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{Log}[c+d x]}{(b c-a d)^4 g^2 i^3} + \frac{3 b^2 d \left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2 \operatorname{Log}[c+d x]}{(b c-a d)^4 g^2 i^3} + \\
& \frac{3 A b^2 B d \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^2 i^3} - \frac{3 b^2 B^2 d \operatorname{Log}[c+d x]^2}{2 (b c-a d)^4 g^2 i^3} - \frac{3 b^2 B^2 d \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{Log}[c+d x]^2}{(b c-a d)^4 g^2 i^3} + \frac{b^2 B^2 d \operatorname{Log}[c+d x]^3}{(b c-a d)^4 g^2 i^3} - \frac{6 A b^2 B d \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d \operatorname{Log}[a+b x]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} - \\
& \frac{6 A b^2 B d \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \\
& \frac{6 b^2 B^2 d \operatorname{Log}[a+b x] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} - \frac{6 A b^2 B d \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} - \frac{6 b^2 B^2 d \operatorname{Log}\left[\frac{1}{c+d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} + \frac{1}{(b c-a d)^4 g^2 i^3} \\
& 6 b^2 B^2 d \left( \operatorname{Log}[a+b x] + \operatorname{Log}\left[\frac{1}{c+d x}\right] - \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c-a d}\right] - \\
& \frac{6 b^2 B^2 d \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^2 i^3} - \frac{6 b^2 B^2 d \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} - \\
& \frac{6 b^2 B^2 d \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3} - \frac{6 b^2 B^2 d \operatorname{PolyLog}\left[3, 1 + \frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^2 i^3}
\end{aligned}$$

**Problem 106:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e(a+b x)}{c+d x}\right] \right)^2}{(a g + b g x)^3 (c i + d i x)^3} dx$$

Optimal (type 3, 685 leaves, 14 steps):



$$\begin{aligned}
& \frac{B^2 d^4 (a+bx)^2}{4 (bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8AbBd^3 (a+bx)}{(bc-ad)^5 g^3 i^3 (c+dx)} - \frac{8bB^2 d^3 (a+bx)}{(bc-ad)^5 g^3 i^3 (c+dx)} + \\
& \frac{8b^3 B^2 d (c+dx)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4 B^2 (c+dx)^2}{4 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{8bB^2 d^3 (a+bx) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)^5 g^3 i^3 (c+dx)} - \\
& \frac{Bd^4 (a+bx)^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8b^3 Bd (c+dx) \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \\
& \frac{b^4 B (c+dx)^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{d^4 (a+bx)^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^5 g^3 i^3 (c+dx)^2} - \\
& \frac{4bd^3 (a+bx) \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5 g^3 i^3 (c+dx)} + \frac{4b^3 d (c+dx) \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5 g^3 i^3 (a+bx)} - \\
& \frac{b^4 (c+dx)^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{2b^2 d^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{B (bc-ad)^5 g^3 i^3}
\end{aligned}$$

Result (type 4, 1921 leaves, 173 steps):

$$\begin{aligned}
& -\frac{b^2 B^2}{4 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \frac{15b^2 B^2 d}{2 (bc-ad)^4 g^3 i^3 (a+bx)} + \frac{B^2 d^2}{4 (bc-ad)^3 g^3 i^3 (c+dx)^2} + \\
& \frac{15bB^2 d^2}{2 (bc-ad)^4 g^3 i^3 (c+dx)} + \frac{15b^2 B^2 d^2 \operatorname{Log}[a+bx]}{(bc-ad)^5 g^3 i^3} - \frac{6Ab^2 B d^2 \operatorname{Log}[a+bx]^2}{(bc-ad)^5 g^3 i^3} + \\
& \frac{6b^2 B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^5 g^3 i^3} - \frac{6b^2 B^2 d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^5 g^3 i^3} - \\
& \frac{6b^2 B^2 d^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^5 g^3 i^3} - \frac{6b^2 B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^5 g^3 i^3} - \\
& \frac{b^2 B \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \frac{7b^2 Bd \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^3 i^3 (a+bx)} - \\
& \frac{Bd^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^3 g^3 i^3 (c+dx)^2} - \frac{7bBd^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^4 g^3 i^3 (c+dx)} - \frac{b^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \\
& \frac{3b^2 d \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4 g^3 i^3 (a+bx)} + \frac{d^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^3 g^3 i^3 (c+dx)^2} + \frac{3bd^2 \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^4 g^3 i^3 (c+dx)} + \\
& \frac{6b^2 d^2 \operatorname{Log}[a+bx] \left(A+B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^5 g^3 i^3} - \frac{15b^2 B^2 d^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} + \\
& \frac{6b^2 B^2 d^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} + \frac{12Ab^2 B d^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{12 b^2 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \frac{1}{(b c - a d)^5 g^3 i^3} \\
& 12 b^2 B^2 d^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x] - \\
& \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \frac{6 A b^2 B d^2 \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} + \\
& \frac{6 b^2 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} - \frac{6 b^2 B^2 d^2 \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} - \frac{2 b^2 B^2 d^2 \operatorname{Log}[c + d x]^3}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 A b^2 B d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \frac{6 b^2 B^2 d^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 A b^2 B d^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \frac{12 b^2 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 A b^2 B d^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 b^2 B^2 d^2 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \frac{1}{(b c - a d)^5 g^3 i^3} \\
& 12 b^2 B^2 d^2 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] + \\
& \frac{12 b^2 B^2 d^2 \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 b^2 B^2 d^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 b^2 B^2 d^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 b^2 B^2 d^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

**Problem 107:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[\frac{e(a + b x)}{c + d x}\right] \right)^2}{(a g + b g x)^4 (c i + d i x)^3} dx$$

Optimal (type 3, 851 leaves, 16 steps):

$$\begin{aligned}
& - \frac{B^2 d^5 (a+bx)^2}{4 (bc-ad)^6 g^4 i^3 (c+dx)^2} - \frac{10 Ab B d^4 (a+bx)}{(bc-ad)^6 g^4 i^3 (c+dx)} + \\
& \frac{10 b B^2 d^4 (a+bx)}{(bc-ad)^6 g^4 i^3 (c+dx)} - \frac{20 b^3 B^2 d^2 (c+dx)}{(bc-ad)^6 g^4 i^3 (a+bx)} + \frac{5 b^4 B^2 d (c+dx)^2}{4 (bc-ad)^6 g^4 i^3 (a+bx)^2} - \\
& \frac{2 b^5 B^2 (c+dx)^3}{27 (bc-ad)^6 g^4 i^3 (a+bx)^3} - \frac{10 b B^2 d^4 (a+bx) \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]}{(bc-ad)^6 g^4 i^3 (c+dx)} + \\
& \frac{B d^5 (a+bx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^6 g^4 i^3 (c+dx)^2} - \frac{20 b^3 B d^2 (c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^6 g^4 i^3 (a+bx)} + \\
& \frac{5 b^4 B d (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^6 g^4 i^3 (a+bx)^2} - \frac{2 b^5 B (c+dx)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9 (bc-ad)^6 g^4 i^3 (a+bx)^3} - \\
& \frac{d^5 (a+bx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^6 g^4 i^3 (c+dx)^2} + \frac{5 b d^4 (a+bx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^6 g^4 i^3 (c+dx)} - \\
& \frac{10 b^3 d^2 (c+dx) \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{(bc-ad)^6 g^4 i^3 (a+bx)} + \frac{5 b^4 d (c+dx)^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{2 (bc-ad)^6 g^4 i^3 (a+bx)^2} - \\
& \frac{b^5 (c+dx)^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^2}{3 (bc-ad)^6 g^4 i^3 (a+bx)^3} - \frac{10 b^2 d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)^3}{3 B (bc-ad)^6 g^4 i^3}
\end{aligned}$$

Result (type 4, 2454 leaves, 207 steps):

$$\begin{aligned}
& - \frac{2 b^2 B^2}{27 (bc-ad)^3 g^4 i^3 (a+bx)^3} + \frac{37 b^2 B^2 d}{36 (bc-ad)^4 g^4 i^3 (a+bx)^2} - \\
& \frac{319 b^2 B^2 d^2}{18 (bc-ad)^5 g^4 i^3 (a+bx)} - \frac{B^2 d^3}{4 (bc-ad)^4 g^4 i^3 (c+dx)^2} - \frac{19 b B^2 d^3}{2 (bc-ad)^5 g^4 i^3 (c+dx)} - \\
& \frac{245 b^2 B^2 d^3 \operatorname{Log}[a+bx]}{9 (bc-ad)^6 g^4 i^3} + \frac{10 A b^2 B d^3 \operatorname{Log}[a+bx]^2}{(bc-ad)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}[a+bx]^2}{3 (bc-ad)^6 g^4 i^3} - \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}\left[-\frac{d(a+bx)}{b c-a d}\right] \operatorname{Log}\left[\frac{1}{c+dx}\right]^2}{(bc-ad)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log}\left[-\frac{b c-a d}{d(a+bx)}\right] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]^2}{(bc-ad)^6 g^4 i^3} - \\
& \frac{2 b^2 B \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{9 (bc-ad)^3 g^4 i^3 (a+bx)^3} + \frac{11 b^2 B d \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{6 (bc-ad)^4 g^4 i^3 (a+bx)^2} - \\
& \frac{47 b^2 B d^2 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{3 (bc-ad)^5 g^4 i^3 (a+bx)} + \frac{B d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{2 (bc-ad)^4 g^4 i^3 (c+dx)^2} + \frac{9 b B d^3 \left(A + B \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right]\right)}{(bc-ad)^5 g^4 i^3 (c+dx)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{20 b^2 B d^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)}{3 (b c - a d)^6 g^4 i^3} - \frac{b^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{3 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \\
& \frac{3 b^2 d \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^5 g^4 i^3 (a + b x)} - \frac{d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{2 (b c - a d)^4 g^4 i^3 (c + d x)^2} - \\
& \frac{4 b d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^5 g^4 i^3 (c + d x)} - \frac{10 b^2 d^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{245 b^2 B^2 d^3 \operatorname{Log}[c + d x]}{9 (b c - a d)^6 g^4 i^3} - \frac{10 b^2 B^2 d^3 \operatorname{Log}[a + b x]^2 \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 A b^2 B d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \frac{1}{(b c - a d)^6 g^4 i^3} + \\
& 20 b^2 B^2 d^3 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x] + \\
& \frac{20 b^2 B d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} + \frac{10 b^2 d^3 \left( A + B \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 A b^2 B d^3 \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}[c + d x]^2}{3 (b c - a d)^6 g^4 i^3} - \frac{10 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}[c + d x]^3}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{20 A b^2 B d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 (b c - a d)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 A b^2 B d^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 (b c - a d)^6 g^4 i^3} + \frac{20 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 A b^2 B d^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 \operatorname{Log}\left[\frac{1}{c + d x}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} + \frac{1}{(b c - a d)^6 g^4 i^3} + \\
& 20 b^2 B^2 d^3 \left( \operatorname{Log}[a + b x] + \operatorname{Log}\left[\frac{1}{c + d x}\right] - \operatorname{Log}\left[\frac{e (a + b x)}{c + d x}\right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right] -
\end{aligned}$$

$$\frac{20 b^2 B^2 d^3 \operatorname{Log}\left[\frac{e(a+bx)}{c+dx}\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a+bx)}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} -$$

$$\frac{20 b^2 B^2 d^3 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a+bx)}\right]}{(b c - a d)^6 g^4 i^3}$$

Problem 108: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^3 (c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) dx$$

Optimal (type 3, 223 leaves, 5 steps):

$$-\frac{B(b c - a d)^4 g^3 i n x}{20 b d^3} + \frac{B(b c - a d)^3 g^3 i n (a + b x)^2}{40 b^2 d^2} -$$

$$\frac{B(b c - a d)^2 g^3 i n (a + b x)^3}{60 b^2 d} + \frac{g^3 i (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{5 b} +$$

$$\frac{(b c - a d) g^3 i (a + b x)^4 \left( A - B n + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{20 b^2} + \frac{B(b c - a d)^5 g^3 i n \operatorname{Log}[c + d x]}{20 b^2 d^4}$$

Result (type 3, 243 leaves, 10 steps):

$$-\frac{B(b c - a d)^4 g^3 i n x}{20 b d^3} + \frac{B(b c - a d)^3 g^3 i n (a + b x)^2}{40 b^2 d^2} - \frac{B(b c - a d)^2 g^3 i n (a + b x)^3}{60 b^2 d} -$$

$$\frac{B(b c - a d) g^3 i n (a + b x)^4}{20 b^2} + \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{4 b^2} +$$

$$\frac{d g^3 i (a + b x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{5 b^2} + \frac{B(b c - a d)^5 g^3 i n \operatorname{Log}[c + d x]}{20 b^2 d^4}$$

Problem 109: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 (c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) dx$$

Optimal (type 3, 190 leaves, 5 steps):

$$\frac{B(b c - a d)^3 g^2 i n x}{12 b d^2} - \frac{B(b c - a d)^2 g^2 i n (a + b x)^2}{24 b^2 d} + \frac{g^2 i (a + b x)^3 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{4 b} +$$

$$\frac{(b c - a d) g^2 i (a + b x)^3 \left( A - B n + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{12 b^2} - \frac{B(b c - a d)^4 g^2 i n \operatorname{Log}[c + d x]}{12 b^2 d^3}$$

Result (type 3, 210 leaves, 10 steps):

$$\begin{aligned} & \frac{B (b c - a d)^3 g^2 i n x}{12 b d^2} - \frac{B (b c - a d)^2 g^2 i n (a + b x)^2}{24 b^2 d} - \\ & \frac{B (b c - a d) g^2 i n (a + b x)^3}{12 b^2} + \frac{(b c - a d) g^2 i (a + b x)^3 (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{3 b^2} + \\ & \frac{d g^2 i (a + b x)^4 (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{4 b^2} - \frac{B (b c - a d)^4 g^2 i n \operatorname{Log}[c + d x]}{12 b^2 d^3} \end{aligned}$$

**Problem 110: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x) \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) dx$$

Optimal (type 3, 149 leaves, 5 steps):

$$\begin{aligned} & - \frac{B (b c - a d)^2 g i n x}{6 b d} + \frac{g i (a + b x)^2 (c + d x) (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{3 b} + \\ & \frac{(b c - a d) g i (a + b x)^2 (A - B n + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{6 b^2} + \frac{B (b c - a d)^3 g i n \operatorname{Log}[c + d x]}{6 b^2 d^2} \end{aligned}$$

Result (type 3, 311 leaves, 13 steps):

$$\begin{aligned} & a A c g i x - \frac{1}{3} b B \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) d g i n x - \frac{B (b c - a d) (b c + a d) g i n x}{2 b d} - \frac{1}{6} B (b c - a d) g i n x^2 + \\ & \frac{a^3 B d g i n \operatorname{Log}[a + b x]}{3 b^2} - \frac{a^2 B (b c + a d) g i n \operatorname{Log}[a + b x]}{2 b^2} + \frac{a B c g i (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} + \\ & \frac{1}{2} (b c + a d) g i x^2 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) + \frac{1}{3} b d g i x^3 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) - \\ & \frac{b B c^3 g i n \operatorname{Log}[c + d x]}{3 d^2} - \frac{a B c (b c - a d) g i n \operatorname{Log}[c + d x]}{b d} + \frac{B c^2 (b c + a d) g i n \operatorname{Log}[c + d x]}{2 d^2} \end{aligned}$$

**Problem 112: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{a g + b g x} dx$$

Optimal (type 4, 141 leaves, 6 steps):

$$\begin{aligned} & \frac{i (c + d x) \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{b g} - \\ & \frac{(b c - a d) i \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \left( A - B n + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{b^2 g} + \frac{B (b c - a d) i n \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g} \end{aligned}$$

Result (type 4, 223 leaves, 13 steps):

$$\begin{aligned} & \frac{A d i x}{b g} - \frac{B (b c - a d) i n \operatorname{Log}[a + b x]^2}{2 b^2 g} + \frac{B d i (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b^2 g} + \\ & \frac{(b c - a d) i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^2 g} - \frac{B (b c - a d) i n \operatorname{Log}[c + d x]}{b^2 g} + \\ & \frac{B (b c - a d) i n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c - a d}\right]}{b^2 g} + \frac{B (b c - a d) i n \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c - a d}\right]}{b^2 g} \end{aligned}$$

**Problem 113: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 150 leaves, 5 steps):

$$\begin{aligned} & -\frac{B i n (c + d x)}{b g^2 (a + b x)} - \frac{i (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b g^2 (a + b x)} - \\ & \frac{d i \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{b (c+d x)}{d (a+b x)}\right]}{b^2 g^2} + \frac{B d i n \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{d (a+b x)}\right]}{b^2 g^2} \end{aligned}$$

Result (type 4, 233 leaves, 14 steps):

$$\begin{aligned} & -\frac{B (b c - a d) i n}{b^2 g^2 (a + b x)} - \frac{B d i n \operatorname{Log}[a + b x]}{b^2 g^2} - \frac{B d i n \operatorname{Log}[a + b x]^2}{2 b^2 g^2} - \\ & \frac{(b c - a d) i \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^2 g^2 (a + b x)} + \frac{d i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^2 g^2} + \\ & \frac{B d i n \operatorname{Log}[c + d x]}{b^2 g^2} + \frac{B d i n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c+d x)}{b c - a d}\right]}{b^2 g^2} + \frac{B d i n \operatorname{PolyLog}\left[2, -\frac{d (a+b x)}{b c - a d}\right]}{b^2 g^2} \end{aligned}$$

**Problem 114: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(a g + b g x)^3} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{B i n (c + d x)^2}{4 (b c - a d) g^3 (a + b x)^2} - \frac{i (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 (b c - a d) g^3 (a + b x)^2}$$

Result (type 3, 201 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) \ln}{4 b^2 g^3 (a + b x)^2} - \frac{B d \ln}{2 b^2 g^3 (a + b x)} - \frac{B d^2 \ln \operatorname{Log}[a + b x]}{2 b^2 (b c - a d) g^3} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^3 (a + b x)^2} - \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^2 g^3 (a + b x)} + \frac{B d^2 \ln \operatorname{Log}[c + d x]}{2 b^2 (b c - a d) g^3}
\end{aligned}$$

**Problem 115: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^4} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned}
& \frac{B d \ln (c + d x)^2}{4 (b c - a d)^2 g^4 (a + b x)^2} - \frac{b B \ln (c + d x)^3}{9 (b c - a d)^2 g^4 (a + b x)^3} + \\
& \frac{d i (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^2 g^4 (a + b x)^2} - \frac{b i (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^2 g^4 (a + b x)^3}
\end{aligned}$$

Result (type 3, 236 leaves, 10 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) \ln}{9 b^2 g^4 (a + b x)^3} - \frac{B d \ln}{12 b^2 g^4 (a + b x)^2} + \frac{B d^2 \ln}{6 b^2 (b c - a d) g^4 (a + b x)} + \frac{B d^3 \ln \operatorname{Log}[a + b x]}{6 b^2 (b c - a d)^2 g^4} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 g^4 (a + b x)^3} - \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^4 (a + b x)^2} - \frac{B d^3 \ln \operatorname{Log}[c + d x]}{6 b^2 (b c - a d)^2 g^4}
\end{aligned}$$

**Problem 116: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^5} dx$$

Optimal (type 3, 281 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^2 \ln (c + d x)^2}{4 (b c - a d)^3 g^5 (a + b x)^2} + \frac{2 b B d \ln (c + d x)^3}{9 (b c - a d)^3 g^5 (a + b x)^3} - \\
& \frac{b^2 B \ln (c + d x)^4}{16 (b c - a d)^3 g^5 (a + b x)^4} - \frac{d^2 i (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^3 g^5 (a + b x)^2} + \\
& \frac{2 b d i (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 i (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 (b c - a d)^3 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 3, 269 leaves, 10 steps):



$$\begin{aligned}
& - \frac{B (b c - a d) i n}{16 b^2 g^5 (a + b x)^4} - \frac{B d i n}{36 b^2 g^5 (a + b x)^3} + \\
& \frac{B d^2 i n}{24 b^2 (b c - a d) g^5 (a + b x)^2} - \frac{B d^3 i n}{12 b^2 (b c - a d)^2 g^5 (a + b x)} - \frac{B d^4 i n \operatorname{Log}[a + b x]}{12 b^2 (b c - a d)^3 g^5} \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b^2 g^5 (a + b x)^4} - \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 g^5 (a + b x)^3} + \frac{B d^4 i n \operatorname{Log}[c + d x]}{12 b^2 (b c - a d)^3 g^5}
\end{aligned}$$

**Problem 117: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 442 leaves, 5 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^5 g^3 i^2 n x}{60 b^2 d^3} + \frac{B (b c - a d)^4 g^3 i^2 n (c + d x)^2}{120 b d^4} - \\
& \frac{19 B (b c - a d)^3 g^3 i^2 n (c + d x)^3}{180 d^4} + \frac{13 b B (b c - a d)^2 g^3 i^2 n (c + d x)^4}{120 d^4} - \\
& \frac{b^2 B (b c - a d) g^3 i^2 n (c + d x)^5}{30 d^4} - \frac{(b c - a d)^3 g^3 i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^4} + \\
& \frac{3 b (b c - a d)^2 g^3 i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^4} - \\
& \frac{3 b^2 (b c - a d) g^3 i^2 (c + d x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^4} + \frac{b^3 g^3 i^2 (c + d x)^6 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 d^4} + \\
& \frac{B (b c - a d)^6 g^3 i^2 n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{60 b^3 d^4} + \frac{B (b c - a d)^6 g^3 i^2 n \operatorname{Log}[c + d x]}{60 b^3 d^4}
\end{aligned}$$

Result (type 3, 345 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^3 i^2 n x}{60 b^2 d^3} + \frac{B (b c - a d)^4 g^3 i^2 n (a + b x)^2}{120 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^3 i^2 n (a + b x)^3}{180 b^3 d} - \frac{7 B (b c - a d)^2 g^3 i^2 n (a + b x)^4}{120 b^3} - \\
& \frac{B d (b c - a d) g^3 i^2 n (a + b x)^5}{30 b^3} + \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b^3} + \\
& \frac{2 d (b c - a d) g^3 i^2 (a + b x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 b^3} + \\
& \frac{d^2 g^3 i^2 (a + b x)^6 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^3} + \frac{B (b c - a d)^6 g^3 i^2 n \operatorname{Log}[c + d x]}{60 b^3 d^4}
\end{aligned}$$

### Problem 118: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 352 leaves, 5 steps):

$$\begin{aligned} & - \frac{B (b c - a d)^4 g^2 i^2 n x}{30 b^2 d^2} - \frac{B (b c - a d)^3 g^2 i^2 n (c + d x)^2}{60 b d^3} + \frac{B (b c - a d)^2 g^2 i^2 n (c + d x)^3}{10 d^3} - \\ & \frac{b B (b c - a d) g^2 i^2 n (c + d x)^4}{20 d^3} + \frac{(b c - a d)^2 g^2 i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^3} - \\ & \frac{b (b c - a d) g^2 i^2 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 d^3} + \frac{b^2 g^2 i^2 (c + d x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^3} - \\ & \frac{B (b c - a d)^5 g^2 i^2 n \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{30 b^3 d^3} - \frac{B (b c - a d)^5 g^2 i^2 n \operatorname{Log} [c + d x]}{30 b^3 d^3} \end{aligned}$$

Result (type 3, 310 leaves, 14 steps):

$$\begin{aligned} & \frac{B (b c - a d)^4 g^2 i^2 n x}{30 b^2 d^2} - \frac{B (b c - a d)^3 g^2 i^2 n (a + b x)^2}{60 b^3 d} - \frac{B (b c - a d)^2 g^2 i^2 n (a + b x)^3}{10 b^3} - \\ & \frac{B d (b c - a d) g^2 i^2 n (a + b x)^4}{20 b^3} + \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^3} + \\ & \frac{d (b c - a d) g^2 i^2 (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^3} + \\ & \frac{d^2 g^2 i^2 (a + b x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 b^3} - \frac{B (b c - a d)^5 g^2 i^2 n \operatorname{Log} [c + d x]}{30 b^3 d^3} \end{aligned}$$

### Problem 119: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) (c i + d i x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$\begin{aligned} & \frac{B (b c - a d)^3 g i^2 n x}{12 b^2 d} + \frac{B (b c - a d)^2 g i^2 n (c + d x)^2}{24 b d^2} - \frac{B (b c - a d) g i^2 n (c + d x)^3}{12 d^2} - \\ & \frac{(b c - a d) g i^2 (c + d x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^2} + \frac{b g i^2 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^2} + \\ & \frac{B (b c - a d)^4 g i^2 n \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{12 b^3 d^2} + \frac{B (b c - a d)^4 g i^2 n \operatorname{Log} [c + d x]}{12 b^3 d^2} \end{aligned}$$

Result (type 3, 210 leaves, 10 steps):

$$\begin{aligned} & \frac{B (b c - a d)^3 g i^2 n x}{12 b^2 d} + \frac{B (b c - a d)^2 g i^2 n (c + d x)^2}{24 b d^2} - \\ & \frac{B (b c - a d) g i^2 n (c + d x)^3}{12 d^2} + \frac{B (b c - a d)^4 g i^2 n \operatorname{Log}[a + b x]}{12 b^3 d^2} - \\ & \frac{(b c - a d) g i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^2} + \frac{b g i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^2} \end{aligned}$$

**Problem 121: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{a g + b g x} dx$$

Optimal (type 4, 289 leaves, 10 steps):

$$\begin{aligned} & - \frac{B d (b c - a d) i^2 n x}{2 b^2 g} + \frac{d (b c - a d) i^2 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g} + \\ & \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b g} - \frac{B (b c - a d)^2 i^2 n \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]}{2 b^3 g} - \\ & \frac{3 B (b c - a d)^2 i^2 n \operatorname{Log}[c + d x]}{2 b^3 g} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g} + \\ & \frac{B (b c - a d)^2 i^2 n \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g} \end{aligned}$$

Result (type 4, 369 leaves, 18 steps):

$$\begin{aligned} & \frac{A d (b c - a d) i^2 x}{b^2 g} - \frac{B d (b c - a d) i^2 n x}{2 b^2 g} - \frac{B (b c - a d)^2 i^2 n \operatorname{Log}[a + b x]}{2 b^3 g} - \\ & \frac{B (b c - a d)^2 i^2 n \operatorname{Log}\left[ g (a + b x) \right]^2}{2 b^3 g} + \frac{B d (b c - a d) i^2 (a + b x) \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{b^3 g} + \\ & \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b g} - \frac{B (b c - a d)^2 i^2 n \operatorname{Log}[c + d x]}{b^3 g} + \\ & \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[a g + b g x]}{b^3 g} + \\ & \frac{B (b c - a d)^2 i^2 n \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right] \operatorname{Log}[a g + b g x]}{b^3 g} + \frac{B (b c - a d)^2 i^2 n \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^3 g} \end{aligned}$$

**Problem 122: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 259 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{B (b c - a d) i^2 n (c + d x)}{b^2 g^2 (a + b x)} + \frac{d^2 i^2 (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^2} - \\
 & \frac{(b c - a d) i^2 (c + d x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^2 g^2 (a + b x)} - \frac{B d (b c - a d) i^2 n \operatorname{Log} [c + d x]}{b^3 g^2} - \\
 & \frac{2 d (b c - a d) i^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g^2} + \frac{2 B d (b c - a d) i^2 n \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g^2}
 \end{aligned}$$

Result (type 4, 327 leaves, 17 steps):

$$\begin{aligned}
 & \frac{A d^2 i^2 x}{b^2 g^2} - \frac{B (b c - a d)^2 i^2 n}{b^3 g^2 (a + b x)} - \frac{B d (b c - a d) i^2 n \operatorname{Log} [a + b x]}{b^3 g^2} - \\
 & \frac{B d (b c - a d) i^2 n \operatorname{Log} [a + b x]^2}{b^3 g^2} + \frac{B d^2 i^2 (a + b x) \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{b^3 g^2} - \\
 & \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^2 (a + b x)} + \frac{2 d (b c - a d) i^2 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^2} + \\
 & \frac{2 B d (b c - a d) i^2 n \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{b^3 g^2} + \frac{2 B d (b c - a d) i^2 n \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^3 g^2}
 \end{aligned}$$

**Problem 123: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^3} dx$$

Optimal (type 4, 242 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{B d i^2 n (c + d x)}{b^2 g^3 (a + b x)} - \frac{B i^2 n (c + d x)^2}{4 b g^3 (a + b x)^2} - \\
 & \frac{d i^2 (c + d x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^2 g^3 (a + b x)} - \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b g^3 (a + b x)^2} - \\
 & \frac{d^2 i^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g^3} + \frac{B d^2 i^2 n \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^3 g^3}
 \end{aligned}$$

Result (type 4, 354 leaves, 18 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2 n}{4 b^3 g^3 (a + b x)^2} - \frac{3 B d (b c - a d) i^2 n}{2 b^3 g^3 (a + b x)} - \frac{3 B d^2 i^2 n \operatorname{Log}[a + b x]}{2 b^3 g^3} - \frac{B d^2 i^2 n \operatorname{Log}[a + b x]^2}{2 b^3 g^3} - \\
& \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^3 g^3 (a + b x)^2} - \frac{2 d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^3 (a + b x)} + \\
& \frac{d^2 i^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^3} + \frac{3 B d^2 i^2 n \operatorname{Log}[c + d x]}{2 b^3 g^3} + \\
& \frac{B d^2 i^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{b^3 g^3} + \frac{B d^2 i^2 n \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^3 g^3}
\end{aligned}$$

**Problem 124: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^4} dx$$

Optimal (type 3, 93 leaves, 2 steps):

$$- \frac{B i^2 n (c + d x)^3}{9 (b c - a d) g^4 (a + b x)^3} - \frac{i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d) g^4 (a + b x)^3}$$

Result (type 3, 301 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2 n}{9 b^3 g^4 (a + b x)^3} - \frac{B d (b c - a d) i^2 n}{3 b^3 g^4 (a + b x)^2} - \frac{B d^2 i^2 n}{3 b^3 g^4 (a + b x)} - \\
& \frac{B d^3 i^2 n \operatorname{Log}[a + b x]}{3 b^3 (b c - a d) g^4} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^3 g^4 (a + b x)^3} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^4 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^4 (a + b x)} + \frac{B d^3 i^2 n \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4}
\end{aligned}$$

**Problem 125: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^5} dx$$

Optimal (type 3, 189 leaves, 5 steps):

$$\begin{aligned}
& \frac{B d i^2 n (c + d x)^3}{9 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B i^2 n (c + d x)^4}{16 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{d i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 (b c - a d)^2 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 3, 340 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2 n}{16 b^3 g^5 (a + b x)^4} - \frac{5 B d (b c - a d) i^2 n}{36 b^3 g^5 (a + b x)^3} - \frac{B d^2 i^2 n}{24 b^3 g^5 (a + b x)^2} + \\
& \frac{B d^3 i^2 n}{12 b^3 (b c - a d) g^5 (a + b x)} + \frac{B d^4 i^2 n \operatorname{Log}[a + b x]}{12 b^3 (b c - a d)^2 g^5} - \frac{(b c - a d)^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{4 b^3 g^5 (a + b x)^4} - \\
& \frac{2 d (b c - a d) i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 b^3 g^5 (a + b x)^3} - \frac{d^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 b^3 g^5 (a + b x)^2} - \frac{B d^4 i^2 n \operatorname{Log}[c + d x]}{12 b^3 (b c - a d)^2 g^5}
\end{aligned}$$

**Problem 126: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(a g + b g x)^6} dx$$

Optimal (type 3, 293 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^2 i^2 n (c + d x)^3}{9 (b c - a d)^3 g^6 (a + b x)^3} + \frac{b B d i^2 n (c + d x)^4}{8 (b c - a d)^3 g^6 (a + b x)^4} - \\
& \frac{b^2 B i^2 n (c + d x)^5}{25 (b c - a d)^3 g^6 (a + b x)^5} - \frac{d^2 i^2 (c + d x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 (b c - a d)^3 g^6 (a + b x)^3} + \\
& \frac{b d i^2 (c + d x)^4 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 (b c - a d)^3 g^6 (a + b x)^4} - \frac{b^2 i^2 (c + d x)^5 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{5 (b c - a d)^3 g^6 (a + b x)^5}
\end{aligned}$$

Result (type 3, 375 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^2 i^2 n}{25 b^3 g^6 (a + b x)^5} - \frac{3 B d (b c - a d) i^2 n}{40 b^3 g^6 (a + b x)^4} - \frac{B d^2 i^2 n}{90 b^3 g^6 (a + b x)^3} + \frac{B d^3 i^2 n}{60 b^3 (b c - a d) g^6 (a + b x)^2} - \\
& \frac{B d^4 i^2 n}{30 b^3 (b c - a d)^2 g^6 (a + b x)} - \frac{B d^5 i^2 n \operatorname{Log}[a + b x]}{30 b^3 (b c - a d)^3 g^6} - \frac{(b c - a d)^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{5 b^3 g^6 (a + b x)^5} - \\
& \frac{d (b c - a d) i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 b^3 g^6 (a + b x)^4} - \frac{d^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 b^3 g^6 (a + b x)^3} + \frac{B d^5 i^2 n \operatorname{Log}[c + d x]}{30 b^3 (b c - a d)^3 g^6}
\end{aligned}$$

**Problem 127: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) dx$$

Optimal (type 3, 477 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^6 g^3 i^3 n x}{140 b^3 d^3} + \frac{B (b c - a d)^5 g^3 i^3 n (c + d x)^2}{280 b^2 d^4} + \frac{B (b c - a d)^4 g^3 i^3 n (c + d x)^3}{420 b d^4} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 n (c + d x)^4}{280 d^4} + \frac{b B (b c - a d)^2 g^3 i^3 n (c + d x)^5}{14 d^4} - \\
& \frac{b^2 B (b c - a d) g^3 i^3 n (c + d x)^6}{42 d^4} - \frac{(b c - a d)^3 g^3 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^4} + \\
& \frac{3 b (b c - a d)^2 g^3 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^4} - \\
& \frac{b^2 (b c - a d) g^3 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 d^4} + \frac{b^3 g^3 i^3 (c + d x)^7 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{7 d^4} + \\
& \frac{B (b c - a d)^7 g^3 i^3 n \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{140 b^4 d^4} + \frac{B (b c - a d)^7 g^3 i^3 n \operatorname{Log} [c + d x]}{140 b^4 d^4}
\end{aligned}$$

Result (type 3, 435 leaves, 18 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^6 g^3 i^3 n x}{140 b^3 d^3} + \frac{B (b c - a d)^5 g^3 i^3 n (a + b x)^2}{280 b^4 d^2} - \frac{B (b c - a d)^4 g^3 i^3 n (a + b x)^3}{420 b^4 d} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 n (a + b x)^4}{280 b^4} - \frac{B d (b c - a d)^2 g^3 i^3 n (a + b x)^5}{14 b^4} - \\
& \frac{B d^2 (b c - a d) g^3 i^3 n (a + b x)^6}{42 b^4} + \frac{(b c - a d)^3 g^3 i^3 (a + b x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b^4} + \\
& \frac{3 d (b c - a d)^2 g^3 i^3 (a + b x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 b^4} + \\
& \frac{d^2 (b c - a d) g^3 i^3 (a + b x)^6 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^4} + \\
& \frac{d^3 g^3 i^3 (a + b x)^7 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{7 b^4} + \frac{B (b c - a d)^7 g^3 i^3 n \operatorname{Log} [c + d x]}{140 b^4 d^4}
\end{aligned}$$

**Problem 128: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 387 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^2 i^3 n x}{60 b^3 d^2} - \frac{B (b c - a d)^4 g^2 i^3 n (c + d x)^2}{120 b^2 d^3} - \\
& \frac{B (b c - a d)^3 g^2 i^3 n (c + d x)^3}{180 b d^3} + \frac{7 B (b c - a d)^2 g^2 i^3 n (c + d x)^4}{120 d^3} - \\
& \frac{b B (b c - a d) g^2 i^3 n (c + d x)^5}{30 d^3} + \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^3} - \\
& \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^3} + \frac{b^2 g^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 d^3} - \\
& \frac{B (b c - a d)^6 g^2 i^3 n \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{60 b^4 d^3} - \frac{B (b c - a d)^6 g^2 i^3 n \operatorname{Log} [c + d x]}{60 b^4 d^3}
\end{aligned}$$

Result (type 3, 345 leaves, 14 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^5 g^2 i^3 n x}{60 b^3 d^2} - \frac{B (b c - a d)^4 g^2 i^3 n (c + d x)^2}{120 b^2 d^3} - \frac{B (b c - a d)^3 g^2 i^3 n (c + d x)^3}{180 b d^3} + \\
& \frac{7 B (b c - a d)^2 g^2 i^3 n (c + d x)^4}{120 d^3} - \frac{b B (b c - a d) g^2 i^3 n (c + d x)^5}{30 d^3} - \\
& \frac{B (b c - a d)^6 g^2 i^3 n \operatorname{Log} [a + b x]}{60 b^4 d^3} + \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^3} - \\
& \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^3} + \frac{b^2 g^2 i^3 (c + d x)^6 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 d^3}
\end{aligned}$$

**Problem 129: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 283 leaves, 5 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^4 g i^3 n x}{20 b^3 d} + \frac{B (b c - a d)^3 g i^3 n (c + d x)^2}{40 b^2 d^2} + \\
& \frac{B (b c - a d)^2 g i^3 n (c + d x)^3}{60 b d^2} - \frac{B (b c - a d) g i^3 n (c + d x)^4}{20 d^2} - \\
& \frac{(b c - a d) g i^3 (c + d x)^4 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^2} + \frac{b g i^3 (c + d x)^5 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^2} + \\
& \frac{B (b c - a d)^5 g i^3 n \operatorname{Log} \left[ \frac{a + b x}{c + d x} \right]}{20 b^4 d^2} + \frac{B (b c - a d)^5 g i^3 n \operatorname{Log} [c + d x]}{20 b^4 d^2}
\end{aligned}$$

Result (type 3, 243 leaves, 10 steps):



$$\frac{B(b c - a d)^4 g i^3 n x}{20 b^3 d} + \frac{B(b c - a d)^3 g i^3 n (c + d x)^2}{40 b^2 d^2} + \frac{B(b c - a d)^2 g i^3 n (c + d x)^3}{60 b d^2} -$$

$$\frac{B(b c - a d) g i^3 n (c + d x)^4}{20 d^2} + \frac{B(b c - a d)^5 g i^3 n \operatorname{Log}[a + b x]}{20 b^4 d^2} -$$

$$\frac{(b c - a d) g i^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 d^2} + \frac{b g i^3 (c + d x)^5 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 d^2}$$

**Problem 131: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{a g + b g x} dx$$

Optimal (type 4, 373 leaves, 14 steps):

$$-\frac{5 B d (b c - a d)^2 i^3 n x}{6 b^3 g} - \frac{B (b c - a d) i^3 n (c + d x)^2}{6 b^2 g} +$$

$$\frac{d (b c - a d)^2 i^3 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g} +$$

$$\frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g} + \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b g} -$$

$$\frac{5 B (b c - a d)^3 i^3 n \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]}{6 b^4 g} - \frac{11 B (b c - a d)^3 i^3 n \operatorname{Log}[c + d x]}{6 b^4 g} -$$

$$\frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ 1 - \frac{b(c + d x)}{d(a + b x)} \right]}{b^4 g} + \frac{B (b c - a d)^3 i^3 n \operatorname{PolyLog}\left[ 2, \frac{b(c + d x)}{d(a + b x)} \right]}{b^4 g}$$

Result (type 4, 455 leaves, 22 steps):

$$\frac{A d (b c - a d)^2 i^3 x}{b^3 g} - \frac{5 B d (b c - a d)^2 i^3 n x}{6 b^3 g} -$$

$$\frac{B (b c - a d) i^3 n (c + d x)^2}{6 b^2 g} - \frac{5 B (b c - a d)^3 i^3 n \operatorname{Log}[a + b x]}{6 b^4 g} -$$

$$\frac{B (b c - a d)^3 i^3 n \operatorname{Log}[g(a + b x)]^2}{2 b^4 g} + \frac{B d (b c - a d)^2 i^3 (a + b x) \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{b^4 g} +$$

$$\frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g} + \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b g} -$$

$$\frac{B (b c - a d)^3 i^3 n \operatorname{Log}[c + d x]}{b^4 g} + \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[a g + b g x]}{b^4 g} +$$

$$\frac{B (b c - a d)^3 i^3 n \operatorname{Log}\left[ \frac{b(c + d x)}{b c - a d} \right] \operatorname{Log}[a g + b g x]}{b^4 g} + \frac{B (b c - a d)^3 i^3 n \operatorname{PolyLog}\left[ 2, -\frac{d(a + b x)}{b c - a d} \right]}{b^4 g}$$

### Problem 132: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(a g + b g x)^2} dx$$

Optimal (type 4, 390 leaves, 11 steps):

$$\begin{aligned} & -\frac{B d^2 (b c - a d) i^3 n x}{2 b^3 g^2} - \frac{B (b c - a d)^2 i^3 n (c + d x)}{b^3 g^2 (a + b x)} + \\ & \frac{2 d^2 (b c - a d) i^3 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{b^4 g^2} - \frac{(b c - a d)^2 i^3 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{b^3 g^2 (a + b x)} + \\ & \frac{d i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 b^2 g^2} - \frac{B d (b c - a d)^2 i^3 n \operatorname{Log}\left[ \frac{a+b x}{c+d x} \right]}{2 b^4 g^2} - \\ & \frac{5 B d (b c - a d)^2 i^3 n \operatorname{Log}[c + d x]}{2 b^4 g^2} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ 1 - \frac{b (c+d x)}{d (a+b x)} \right]}{b^4 g^2} + \\ & \frac{3 B d (b c - a d)^2 i^3 n \operatorname{PolyLog}\left[ 2, \frac{b (c+d x)}{d (a+b x)} \right]}{b^4 g^2} \end{aligned}$$

Result (type 4, 543 leaves, 21 steps):

$$\begin{aligned} & \frac{A d^2 (3 b c - 2 a d) i^3 x}{b^3 g^2} - \frac{B d^2 (b c - a d) i^3 n x}{2 b^3 g^2} - \frac{B (b c - a d)^3 i^3 n}{b^4 g^2 (a + b x)} - \\ & \frac{a^2 B d^3 i^3 n \operatorname{Log}[a + b x]}{2 b^4 g^2} - \frac{B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x]}{b^4 g^2} - \\ & \frac{3 B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x]^2}{2 b^4 g^2} + \frac{B d^2 (3 b c - 2 a d) i^3 (a + b x) \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{b^4 g^2} + \\ & \frac{d^3 i^3 x^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 b^2 g^2} - \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{b^4 g^2 (a + b x)} + \\ & \frac{3 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{b^4 g^2} + \frac{B c^2 d i^3 n \operatorname{Log}[c + d x]}{2 b^2 g^2} - \\ & \frac{B d (3 b c - 2 a d) (b c - a d) i^3 n \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B d (b c - a d)^2 i^3 n \operatorname{Log}[c + d x]}{b^4 g^2} + \\ & \frac{3 B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c+d x)}{b c - a d} \right]}{b^4 g^2} + \frac{3 B d (b c - a d)^2 i^3 n \operatorname{PolyLog}\left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{b^4 g^2} \end{aligned}$$

### Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(a g + b g x)^3} dx$$

Optimal (type 4, 361 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{2 B d (b c - a d) i^3 n (c + d x)}{b^3 g^3 (a + b x)} - \frac{B (b c - a d) i^3 n (c + d x)^2}{4 b^2 g^3 (a + b x)^2} + \frac{d^3 i^3 (a + b x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3} - \\
 & \frac{2 d (b c - a d) i^3 (c + d x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^3 (a + b x)} - \frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^3 (a + b x)^2} - \\
 & \frac{B d^2 (b c - a d) i^3 n \operatorname{Log} [c + d x]}{b^4 g^3} - \frac{3 d^2 (b c - a d) i^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^3} + \\
 & \frac{3 B d^2 (b c - a d) i^3 n \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^3}
 \end{aligned}$$

Result (type 4, 461 leaves, 21 steps):

$$\begin{aligned}
 & \frac{A d^3 i^3 x}{b^3 g^3} - \frac{B (b c - a d)^3 i^3 n}{4 b^4 g^3 (a + b x)^2} - \frac{5 B d (b c - a d)^2 i^3 n}{2 b^4 g^3 (a + b x)} - \frac{5 B d^2 (b c - a d) i^3 n \operatorname{Log} [a + b x]}{2 b^4 g^3} - \\
 & \frac{3 B d^2 (b c - a d) i^3 n \operatorname{Log} [a + b x]^2}{2 b^4 g^3} + \frac{B d^3 i^3 (a + b x) \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{b^4 g^3} - \\
 & \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^4 g^3 (a + b x)^2} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3 (a + b x)} + \\
 & \frac{3 d^2 (b c - a d) i^3 \operatorname{Log} [a + b x] \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3} + \frac{3 B d^2 (b c - a d) i^3 n \operatorname{Log} [c + d x]}{2 b^4 g^3} + \\
 & \frac{3 B d^2 (b c - a d) i^3 n \operatorname{Log} [a + b x] \operatorname{Log} \left[ \frac{b (c + d x)}{b c - a d} \right]}{b^4 g^3} + \frac{3 B d^2 (b c - a d) i^3 n \operatorname{PolyLog} \left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^4 g^3}
 \end{aligned}$$

**Problem 134: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(a g + b g x)^4} dx$$

Optimal (type 4, 326 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{B d^2 i^3 n (c + d x)}{b^3 g^4 (a + b x)} - \frac{B d i^3 n (c + d x)^2}{4 b^2 g^4 (a + b x)^2} - \frac{B i^3 n (c + d x)^3}{9 b g^4 (a + b x)^3} - \frac{d^2 i^3 (c + d x) \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^4 (a + b x)} - \\
 & \frac{d i^3 (c + d x)^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^4 (a + b x)^2} - \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b g^4 (a + b x)^3} - \\
 & \frac{d^3 i^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log} \left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^4} + \frac{B d^3 i^3 n \operatorname{PolyLog} \left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^4}
 \end{aligned}$$

Result (type 4, 444 leaves, 22 steps):

$$\begin{aligned}
& - \frac{B (b c - a d)^3 i^3 n}{9 b^4 g^4 (a + b x)^3} - \frac{7 B d (b c - a d)^2 i^3 n}{12 b^4 g^4 (a + b x)^2} - \frac{11 B d^2 (b c - a d) i^3 n}{6 b^4 g^4 (a + b x)} - \\
& \frac{11 B d^3 i^3 n \operatorname{Log}[a + b x]}{6 b^4 g^4} - \frac{B d^3 i^3 n \operatorname{Log}[a + b x]^2}{2 b^4 g^4} - \frac{(b c - a d)^3 i^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 b^4 g^4 (a + b x)^3} - \\
& \frac{3 d (b c - a d)^2 i^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 b^4 g^4 (a + b x)^2} - \frac{3 d^2 (b c - a d) i^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{b^4 g^4 (a + b x)} + \\
& \frac{d^3 i^3 \operatorname{Log}[a + b x] (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{b^4 g^4} + \frac{11 B d^3 i^3 n \operatorname{Log}[c + d x]}{6 b^4 g^4} + \\
& \frac{B d^3 i^3 n \operatorname{Log}[a + b x] \operatorname{Log}[\frac{b (c+d x)}{b c - a d}]}{b^4 g^4} + \frac{B d^3 i^3 n \operatorname{PolyLog}[2, -\frac{d (a+b x)}{b c - a d}]}{b^4 g^4}
\end{aligned}$$

**Problem 135: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{c i + d i x} dx$$

Optimal (type 4, 269 leaves, 6 steps):

$$\begin{aligned}
& \frac{g^3 (a + b x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 d i} - \frac{(b c - a d) g^3 (a + b x)^2 (3 A + B n + 3 B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{6 d^2 i} + \\
& \frac{(b c - a d)^2 g^3 (a + b x) (6 A + 5 B n + 6 B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{6 d^3 i} + \\
& \frac{(b c - a d)^3 g^3 (6 A + 11 B n + 6 B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) \operatorname{Log}[\frac{b c - a d}{b (c+d x)}]}{6 d^4 i} + \\
& \frac{B (b c - a d)^3 g^3 n \operatorname{PolyLog}[2, \frac{d (a+b x)}{b (c+d x)}]}{d^4 i}
\end{aligned}$$

Result (type 4, 426 leaves, 22 steps):

$$\begin{aligned}
& \frac{A b (b c - a d)^2 g^3 x}{d^3 i} + \frac{5 b B (b c - a d)^2 g^3 n x}{6 d^3 i} - \frac{B (b c - a d) g^3 n (a + b x)^2}{6 d^2 i} + \\
& \frac{B (b c - a d)^2 g^3 (a + b x) \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]}{d^3 i} - \frac{(b c - a d) g^3 (a + b x)^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 d^2 i} + \\
& \frac{g^3 (a + b x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 d i} - \frac{11 B (b c - a d)^3 g^3 n \operatorname{Log}[c + d x]}{6 d^4 i} - \\
& \frac{B (b c - a d)^3 g^3 n \operatorname{Log}[i (c + d x)]^2}{2 d^4 i} + \frac{B (b c - a d)^3 g^3 n \operatorname{Log}[-\frac{d (a+b x)}{b c - a d}] \operatorname{Log}[c i + d i x]}{d^4 i} - \\
& \frac{(b c - a d)^3 g^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) \operatorname{Log}[c i + d i x]}{d^4 i} + \frac{B (b c - a d)^3 g^3 n \operatorname{PolyLog}[2, \frac{b (c+d x)}{b c - a d}]}{d^4 i}
\end{aligned}$$

### Problem 136: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{c i + d i x} dx$$

Optimal (type 4, 211 leaves, 5 steps):

$$\begin{aligned} & \frac{g^2 (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d i} - \frac{(b c - a d) g^2 (a + b x) \left( 2 A + B n + 2 B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d^2 i} - \\ & \frac{(b c - a d)^2 g^2 \left( 2 A + 3 B n + 2 B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c+d x)} \right]}{2 d^3 i} - \\ & \frac{B (b c - a d)^2 g^2 n \operatorname{PolyLog}\left[ 2, \frac{d (a+b x)}{b (c+d x)} \right]}{d^3 i} \end{aligned}$$

Result (type 4, 343 leaves, 18 steps):

$$\begin{aligned} & - \frac{A b (b c - a d) g^2 x}{d^2 i} - \frac{b B (b c - a d) g^2 n x}{2 d^2 i} - \frac{B (b c - a d) g^2 (a + b x) \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{d^2 i} + \\ & \frac{g^2 (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d i} + \frac{3 B (b c - a d)^2 g^2 n \operatorname{Log}[c + d x]}{2 d^3 i} + \\ & \frac{B (b c - a d)^2 g^2 n \operatorname{Log}\left[ i (c + d x) \right]^2}{2 d^3 i} - \frac{B (b c - a d)^2 g^2 n \operatorname{Log}\left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log}[c i + d i x]}{d^3 i} + \\ & \frac{(b c - a d)^2 g^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}[c i + d i x]}{d^3 i} - \frac{B (b c - a d)^2 g^2 n \operatorname{PolyLog}\left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i} \end{aligned}$$

### Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{c i + d i x} dx$$

Optimal (type 4, 134 leaves, 4 steps):

$$\begin{aligned} & \frac{g (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d i} + \\ & \frac{(b c - a d) g \left( A + B n + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c+d x)} \right]}{d^2 i} + \frac{B (b c - a d) g n \operatorname{PolyLog}\left[ 2, \frac{d (a+b x)}{b (c+d x)} \right]}{d^2 i} \end{aligned}$$

Result (type 4, 223 leaves, 13 steps):

$$\begin{aligned}
& \frac{A b g x}{d i} + \frac{B g (a + b x) \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]}{d i} - \frac{B (b c - a d) g n \operatorname{Log}[c + d x]}{d^2 i} + \\
& \frac{B (b c - a d) g n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i} - \frac{(b c - a d) g \left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^2 i} - \\
& \frac{B (b c - a d) g n \operatorname{Log}[c + d x]^2}{2 d^2 i} + \frac{B (b c - a d) g n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^2 i}
\end{aligned}$$

**Problem 138: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]}{c i + d i x} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{\left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b(c+d x)}\right]}{d i} - \frac{B n \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{d i}$$

Result (type 4, 128 leaves, 9 steps):

$$\begin{aligned}
& \frac{B n \operatorname{Log}[i(c+d x)]^2}{2 d i} - \frac{B n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d i} + \\
& \frac{\left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c i + d i x]}{d i} - \frac{B n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d i}
\end{aligned}$$

**Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]}{(a g + b g x)(c i + d i x)} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{\left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 B (b c - a d) g i n}$$

Result (type 4, 316 leaves, 18 steps):

$$\begin{aligned}
& -\frac{B n \operatorname{Log}[a + b x]^2}{2 (b c - a d) g i} + \frac{\operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c - a d) g i} + \\
& \frac{B n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d) g i} - \frac{\left(A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d) g i} - \frac{B n \operatorname{Log}[c + d x]^2}{2 (b c - a d) g i} + \\
& \frac{B n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{B n \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{B n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{(b c - a d) g i}
\end{aligned}$$

**Problem 140:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(ag+bgx)^2 (ci+dix)} dx$$

Optimal (type 3, 181 leaves, 5 steps):

$$\begin{aligned} & -\frac{b B n (c+dx)}{(bc-ad)^2 g^2 i (a+bx)} - \frac{b (c+dx) \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^2 g^2 i (a+bx)} - \\ & \frac{d \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{(bc-ad)^2 g^2 i} + \frac{B d n \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}{2 (bc-ad)^2 g^2 i} \end{aligned}$$

Result (type 4, 455 leaves, 22 steps):

$$\begin{aligned} & -\frac{B n}{(bc-ad) g^2 i (a+bx)} - \frac{B d n \operatorname{Log}[a+bx]}{(bc-ad)^2 g^2 i} + \frac{B d n \operatorname{Log}[a+bx]^2}{2 (bc-ad)^2 g^2 i} - \\ & \frac{A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad) g^2 i (a+bx)} - \frac{d \operatorname{Log}[a+bx] \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^2 g^2 i} + \frac{B d n \operatorname{Log}[c+dx]}{(bc-ad)^2 g^2 i} - \\ & \frac{B d n \operatorname{Log}\left[-\frac{d(a+bx)}{b(c-ad)}\right] \operatorname{Log}[c+dx]}{(bc-ad)^2 g^2 i} + \frac{d \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[c+dx]}{(bc-ad)^2 g^2 i} + \frac{B d n \operatorname{Log}[c+dx]^2}{2 (bc-ad)^2 g^2 i} - \\ & \frac{B d n \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{b(c-ad)}\right]}{(bc-ad)^2 g^2 i} - \frac{B d n \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-ad)}\right]}{(bc-ad)^2 g^2 i} - \frac{B d n \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b(c-ad)}\right]}{(bc-ad)^2 g^2 i} \end{aligned}$$

**Problem 141:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(ag+bgx)^3 (ci+dix)} dx$$

Optimal (type 3, 266 leaves, 7 steps):

$$\begin{aligned} & -\frac{B n (c+dx)^2 \left(b - \frac{4d(a+bx)}{c+dx}\right)^2}{4 (bc-ad)^3 g^3 i (a+bx)^2} + \frac{2 b d (c+dx) \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^3 g^3 i (a+bx)} - \\ & \frac{b^2 (c+dx)^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^3 g^3 i (a+bx)^2} + \frac{d^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]}{(bc-ad)^3 g^3 i} - \frac{B d^2 n \operatorname{Log}\left[\frac{a+bx}{c+dx}\right]^2}{2 (bc-ad)^3 g^3 i} \end{aligned}$$

Result (type 4, 557 leaves, 26 steps):

$$\begin{aligned}
& - \frac{B n}{4 (b c - a d) g^3 i (a + b x)^2} + \frac{3 B d n}{2 (b c - a d)^2 g^3 i (a + b x)} + \\
& \frac{3 B d^2 n \operatorname{Log}[a + b x]}{2 (b c - a d)^3 g^3 i} - \frac{B d^2 n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^3 g^3 i} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{2 (b c - a d) g^3 i (a + b x)^2} + \\
& \frac{d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^2 g^3 i (a + b x)} + \frac{d^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^3 g^3 i} - \frac{3 B d^2 n \operatorname{Log}[c + d x]}{2 (b c - a d)^3 g^3 i} + \\
& \frac{B d^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g^3 i} - \frac{d^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^3 g^3 i} - \frac{B d^2 n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^3 g^3 i} + \\
& \frac{B d^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^3 i} + \frac{B d^2 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g^3 i} + \frac{B d^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^3 i}
\end{aligned}$$

**Problem 142: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(a g + b g x)^4 (c i + d i x)} dx$$

Optimal (type 3, 389 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 b B d^2 n (c + d x)}{(b c - a d)^4 g^4 i (a + b x)} + \frac{3 b^2 B d n (c + d x)^2}{4 (b c - a d)^4 g^4 i (a + b x)^2} - \frac{b^3 B n (c + d x)^3}{9 (b c - a d)^4 g^4 i (a + b x)^3} - \\
& \frac{3 b d^2 (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^4 g^4 i (a + b x)} + \frac{3 b^2 d (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^4 g^4 i (a + b x)^2} - \\
& \frac{b^3 (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 (b c - a d)^4 g^4 i (a + b x)^3} - \frac{d^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^4 g^4 i} + \frac{B d^3 n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^4 g^4 i}
\end{aligned}$$

Result (type 4, 646 leaves, 30 steps):



$$\begin{aligned}
& - \frac{B n}{9 (b c - a d) g^4 i (a + b x)^3} + \frac{5 B d n}{12 (b c - a d)^2 g^4 i (a + b x)^2} - \\
& \frac{11 B d^2 n}{6 (b c - a d)^3 g^4 i (a + b x)} - \frac{11 B d^3 n \operatorname{Log}[a + b x]}{6 (b c - a d)^4 g^4 i} + \frac{B d^3 n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^4 i} - \\
& \frac{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{3 (b c - a d) g^4 i (a + b x)^3} + \frac{d \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 (b c - a d)^2 g^4 i (a + b x)^2} - \frac{d^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c - a d)^3 g^4 i (a + b x)} - \\
& \frac{d^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c - a d)^4 g^4 i} + \frac{11 B d^3 n \operatorname{Log}[c + d x]}{6 (b c - a d)^4 g^4 i} - \\
& \frac{B d^3 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^4 i} + \frac{d^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^4 i} + \frac{B d^3 n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^4 i} - \\
& \frac{B d^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i} - \frac{B d^3 n \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i} - \frac{B d^3 n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{(b c - a d)^4 g^4 i}
\end{aligned}$$

**Problem 143: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 359 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 B (b c - a d)^2 g^3 n (a + b x)}{d^3 i^2 (c + d x)} - \frac{(b c - a d)^2 g^3 (6 A + 5 B n) (a + b x)}{2 d^3 i^2 (c + d x)} - \\
& \frac{3 B (b c - a d)^2 g^3 (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{d^3 i^2 (c + d x)} + \frac{g^3 (a + b x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 d i^2 (c + d x)} - \\
& \frac{(b c - a d) g^3 (a + b x)^2 \left(3 A + B n + 3 B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 d^2 i^2 (c + d x)} - \\
& \frac{b (b c - a d)^2 g^3 \left(6 A + 5 B n + 6 B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{2 d^4 i^2} - \\
& \frac{3 b B (b c - a d)^2 g^3 n \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{d^4 i^2}
\end{aligned}$$

Result (type 4, 541 leaves, 21 steps):

$$\begin{aligned}
& - \frac{A b^2 (2 b c - 3 a d) g^3 x}{d^3 i^2} - \frac{b^2 B (b c - a d) g^3 n x}{2 d^3 i^2} - \frac{B (b c - a d)^3 g^3 n}{d^4 i^2 (c + d x)} - \frac{a^2 b B g^3 n \operatorname{Log}[a + b x]}{2 d^2 i^2} - \\
& \frac{b B (b c - a d)^2 g^3 n \operatorname{Log}[a + b x]}{d^4 i^2} - \frac{b B (2 b c - 3 a d) g^3 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d^3 i^2} + \\
& \frac{b^3 g^3 x^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 d^2 i^2} + \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d^4 i^2 (c + d x)} + \\
& \frac{b^3 B c^2 g^3 n \operatorname{Log}[c + d x]}{2 d^4 i^2} + \frac{b B (2 b c - 3 a d) (b c - a d) g^3 n \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{b B (b c - a d)^2 g^3 n \operatorname{Log}[c + d x]}{d^4 i^2} - \frac{3 b B (b c - a d)^2 g^3 n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{3 b (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{3 b B (b c - a d)^2 g^3 n \operatorname{Log}[c + d x]^2}{2 d^4 i^2} - \frac{3 b B (b c - a d)^2 g^3 n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^4 i^2}
\end{aligned}$$

**Problem 144: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 275 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 B (b c - a d) g^2 n (a + b x)}{d^2 i^2 (c + d x)} + \frac{(b c - a d) g^2 (2 A + B n) (a + b x)}{d^2 i^2 (c + d x)} + \\
& \frac{2 B (b c - a d) g^2 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d^2 i^2 (c + d x)} + \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d i^2 (c + d x)} + \\
& \frac{b (b c - a d) g^2 \left(2 A + B n + 2 B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^3 i^2} + \\
& \frac{2 b B (b c - a d) g^2 n \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^3 i^2}
\end{aligned}$$

Result (type 4, 351 leaves, 17 steps):

$$\begin{aligned}
& \frac{A b^2 g^2 x}{d^2 i^2} + \frac{B (b c - a d)^2 g^2 n}{d^3 i^2 (c + d x)} + \frac{b B (b c - a d) g^2 n \operatorname{Log}[a + b x]}{d^3 i^2} + \\
& \frac{b B g^2 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d^2 i^2} - \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d^3 i^2 (c + d x)} - \\
& \frac{2 b B (b c - a d) g^2 n \operatorname{Log}[c + d x]}{d^3 i^2} + \frac{2 b B (b c - a d) g^2 n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} - \\
& \frac{2 b (b c - a d) g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^3 i^2} - \\
& \frac{b B (b c - a d) g^2 n \operatorname{Log}[c + d x]^2}{d^3 i^2} + \frac{2 b B (b c - a d) g^2 n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^3 i^2}
\end{aligned}$$

**Problem 145: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(c i + d i x)^2} dx$$

Optimal (type 4, 168 leaves, 7 steps):

$$\begin{aligned}
& -\frac{A g (a + b x)}{d i^2 (c + d x)} + \frac{B g n (a + b x)}{d i^2 (c + d x)} - \frac{B g (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d i^2 (c + d x)} - \\
& \frac{b g \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^2 i^2} - \frac{b B g n \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^2 i^2}
\end{aligned}$$

Result (type 4, 234 leaves, 14 steps):

$$\begin{aligned}
& -\frac{B (b c - a d) g n}{d^2 i^2 (c + d x)} - \frac{b B g n \operatorname{Log}[a + b x]}{d^2 i^2} + \frac{(b c - a d) g \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d^2 i^2 (c + d x)} + \\
& \frac{b B g n \operatorname{Log}[c + d x]}{d^2 i^2} - \frac{b B g n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i^2} + \\
& \frac{b g \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^2 i^2} + \frac{b B g n \operatorname{Log}[c + d x]^2}{2 d^2 i^2} - \frac{b B g n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^2 i^2}
\end{aligned}$$

**Problem 146: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(c i + d i x)^2} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{A (a + b x)}{(b c - a d) i^2 (c + d x)} - \frac{B n (a + b x)}{(b c - a d) i^2 (c + d x)} + \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d) i^2 (c + d x)}$$

Result (type 3, 107 leaves, 4 steps):

$$\frac{B n}{d i^2 (c + d x)} + \frac{b B n \operatorname{Log}[a + b x]}{d (b c - a d) i^2} - \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d i^2 (c + d x)} - \frac{b B n \operatorname{Log}[c + d x]}{d (b c - a d) i^2}$$

**Problem 147:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(a g + b g x) (c i + d i x)^2} dx$$

Optimal (type 3, 166 leaves, 5 steps):

$$-\frac{A d (a + b x)}{(b c - a d)^2 g i^2 (c + d x)} + \frac{B d n (a + b x)}{(b c - a d)^2 g i^2 (c + d x)} - \frac{B d (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d)^2 g i^2 (c + d x)} + \frac{b (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])^2}{2 B (b c - a d)^2 g i^2 n}$$

Result (type 4, 450 leaves, 22 steps):

$$\begin{aligned} & -\frac{B n}{(b c - a d) g i^2 (c + d x)} - \frac{b B n \operatorname{Log}[a + b x]}{(b c - a d)^2 g i^2} - \frac{b B n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^2 g i^2} + \\ & \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d) g i^2 (c + d x)} + \frac{b \operatorname{Log}[a + b x] (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{(b c - a d)^2 g i^2} + \frac{b B n \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \\ & \frac{b B n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{b (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]) \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{b B n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^2 g i^2} + \\ & \frac{b B n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{b B n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{b B n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} \end{aligned}$$

**Problem 148:** Result unnecessarily involves higher level functions.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(a g + b g x)^2 (c i + d i x)^2} dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\begin{aligned} & -\frac{B d^2 n (a + b x)}{(b c - a d)^3 g^2 i^2 (c + d x)} - \frac{b^2 B n (c + d x)}{(b c - a d)^3 g^2 i^2 (a + b x)} + \frac{d^2 (a + b x) (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{(b c - a d)^3 g^2 i^2 (c + d x)} - \\ & \frac{b^2 (c + d x) (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right])}{(b c - a d)^3 g^2 i^2 (a + b x)} - \frac{2 b d (A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^3 g^2 i^2} + \frac{b B d n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^3 g^2 i^2} \end{aligned}$$

Result (type 4, 482 leaves, 26 steps):

$$\begin{aligned}
& - \frac{b B n}{(b c - a d)^2 g^2 i^2 (a + b x)} + \frac{B d n}{(b c - a d)^2 g^2 i^2 (c + d x)} + \\
& \frac{b B d n \operatorname{Log}[a + b x]^2}{(b c - a d)^3 g^2 i^2} - \frac{b \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^2 g^2 i^2 (a + b x)} - \frac{d \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^2 g^2 i^2 (c + d x)} - \\
& \frac{2 b d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^2 i^2} - \frac{2 b B d n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \\
& \frac{2 b d \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^3 g^2 i^2} + \frac{b B d n \operatorname{Log}[c + d x]^2}{(b c - a d)^3 g^2 i^2} - \\
& \frac{2 b B d n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{2 b B d n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2} - \frac{2 b B d n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g^2 i^2}
\end{aligned}$$

**Problem 149: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(a g + b g x)^3 (c i + d i x)^2} dx$$

Optimal (type 3, 380 leaves, 8 steps):

$$\begin{aligned}
& \frac{B d^3 n (a + b x)}{(b c - a d)^4 g^3 i^2 (c + d x)} + \frac{3 b^2 B d n (c + d x)}{(b c - a d)^4 g^3 i^2 (a + b x)} - \frac{b^3 B n (c + d x)^2}{4 (b c - a d)^4 g^3 i^2 (a + b x)^2} - \\
& \frac{d^3 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^3 i^2 (c + d x)} + \frac{3 b^2 d (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^3 i^2 (a + b x)} - \\
& \frac{b^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^4 g^3 i^2 (a + b x)^2} + \frac{3 b d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^4 g^3 i^2}
\end{aligned}$$

Result (type 4, 656 leaves, 30 steps):

$$\begin{aligned}
& - \frac{b B n}{4 (b c - a d)^2 g^3 i^2 (a + b x)^2} + \frac{5 b B d n}{2 (b c - a d)^3 g^3 i^2 (a + b x)} - \\
& \frac{B d^2 n}{(b c - a d)^3 g^3 i^2 (c + d x)} + \frac{3 b B d^2 n \operatorname{Log}[a + b x]}{2 (b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^3 i^2} - \\
& \frac{b \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^2 g^3 i^2 (a + b x)^2} + \frac{2 b d \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^3 i^2 (a + b x)} + \frac{d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^3 i^2 (c + d x)} + \\
& \frac{3 b d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^3 i^2} - \frac{3 b B d^2 n \operatorname{Log}[c + d x]}{2 (b c - a d)^4 g^3 i^2} + \\
& \frac{3 b B d^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^3 i^2} - \frac{3 b d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^3 i^2} - \\
& \frac{3 b B d^2 n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^3 i^2} + \frac{3 b B d^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \\
& \frac{3 b B d^2 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B d^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2}
\end{aligned}$$

**Problem 150: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(a g + b g x)^4 (c i + d i x)^2} dx$$

Optimal (type 3, 477 leaves, 4 steps):

$$\begin{aligned}
& - \frac{B d^4 n (a + b x)}{(b c - a d)^5 g^4 i^2 (c + d x)} - \frac{6 b^2 B d^2 n (c + d x)}{(b c - a d)^5 g^4 i^2 (a + b x)} + \frac{b^3 B d n (c + d x)^2}{(b c - a d)^5 g^4 i^2 (a + b x)^2} - \\
& \frac{b^4 B n (c + d x)^3}{9 (b c - a d)^5 g^4 i^2 (a + b x)^3} + \frac{d^4 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^4 i^2 (c + d x)} - \\
& \frac{6 b^2 d^2 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^4 i^2 (a + b x)} + \frac{2 b^3 d (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^4 i^2 (a + b x)^2} - \\
& \frac{b^4 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^5 g^4 i^2 (a + b x)^3} - \frac{4 b d^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^5 g^4 i^2} + \frac{2 b B d^3 n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^5 g^4 i^2}
\end{aligned}$$

Result (type 4, 735 leaves, 34 steps):

$$\begin{aligned}
& - \frac{b B n}{9 (b c - a d)^2 g^4 i^2 (a + b x)^3} + \frac{2 b B d n}{3 (b c - a d)^3 g^4 i^2 (a + b x)^2} - \frac{13 b B d^2 n}{3 (b c - a d)^4 g^4 i^2 (a + b x)} + \\
& \frac{B d^3 n}{(b c - a d)^4 g^4 i^2 (c + d x)} - \frac{10 b B d^3 n \operatorname{Log}[a + b x]}{3 (b c - a d)^5 g^4 i^2} + \frac{2 b B d^3 n \operatorname{Log}[a + b x]^2}{(b c - a d)^5 g^4 i^2} - \\
& \frac{b (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{3 (b c - a d)^2 g^4 i^2 (a + b x)^3} + \frac{b d (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(b c - a d)^3 g^4 i^2 (a + b x)^2} - \frac{3 b d^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(b c - a d)^4 g^4 i^2 (a + b x)} - \\
& \frac{d^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(b c - a d)^4 g^4 i^2 (c + d x)} - \frac{4 b d^3 \operatorname{Log}[a + b x] (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(b c - a d)^5 g^4 i^2} + \frac{10 b B d^3 n \operatorname{Log}[c + d x]}{3 (b c - a d)^5 g^4 i^2} - \\
& \frac{4 b B d^3 n \operatorname{Log}[-\frac{d(a+b x)}{b c - a d}] \operatorname{Log}[c + d x]}{(b c - a d)^5 g^4 i^2} + \frac{4 b d^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) \operatorname{Log}[c + d x]}{(b c - a d)^5 g^4 i^2} + \\
& \frac{2 b B d^3 n \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^4 i^2} - \frac{4 b B d^3 n \operatorname{Log}[a + b x] \operatorname{Log}[\frac{b(c+d x)}{b c - a d}]}{(b c - a d)^5 g^4 i^2} - \\
& \frac{4 b B d^3 n \operatorname{PolyLog}[2, -\frac{d(a+b x)}{b c - a d}]}{(b c - a d)^5 g^4 i^2} - \frac{4 b B d^3 n \operatorname{PolyLog}[2, \frac{b(c+d x)}{b c - a d}]}{(b c - a d)^5 g^4 i^2}
\end{aligned}$$

**Problem 151: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{(c i + d i x)^3} dx$$

Optimal (type 4, 382 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3 B (b c - a d) g^3 n (a + b x)^2}{4 d^2 i^3 (c + d x)^2} - \frac{3 b B (b c - a d) g^3 n (a + b x)}{d^3 i^3 (c + d x)} + \\
& \frac{b (b c - a d) g^3 (3 A + B n) (a + b x)}{d^3 i^3 (c + d x)} + \frac{3 b B (b c - a d) g^3 (a + b x) \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]}{d^3 i^3 (c + d x)} + \\
& \frac{g^3 (a + b x)^3 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{d i^3 (c + d x)^2} + \frac{(b c - a d) g^3 (a + b x)^2 (3 A + B n + 3 B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{2 d^2 i^3 (c + d x)^2} + \\
& \frac{b^2 (b c - a d) g^3 (3 A + B n + 3 B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) \operatorname{Log}[\frac{b c - a d}{b (c + d x)}]}{d^4 i^3} + \\
& \frac{3 b^2 B (b c - a d) g^3 n \operatorname{PolyLog}[2, \frac{d(a+b x)}{b (c + d x)}]}{d^4 i^3}
\end{aligned}$$

Result (type 4, 461 leaves, 21 steps):

$$\begin{aligned}
& \frac{A b^3 g^3 x}{d^3 i^3} - \frac{B (b c - a d)^3 g^3 n}{4 d^4 i^3 (c + d x)^2} + \frac{5 b B (b c - a d)^2 g^3 n}{2 d^4 i^3 (c + d x)} + \\
& \frac{5 b^2 B (b c - a d) g^3 n \operatorname{Log}[a + b x]}{2 d^4 i^3} + \frac{b^2 B g^3 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d^3 i^3} + \\
& \frac{(b c - a d)^3 g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 d^4 i^3 (c + d x)^2} - \frac{3 b (b c - a d)^2 g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d^4 i^3 (c + d x)} - \\
& \frac{7 b^2 B (b c - a d) g^3 n \operatorname{Log}[c + d x]}{2 d^4 i^3} + \frac{3 b^2 B (b c - a d) g^3 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^3} - \\
& \frac{3 b^2 (b c - a d) g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^4 i^3} - \\
& \frac{3 b^2 B (b c - a d) g^3 n \operatorname{Log}[c + d x]^2}{2 d^4 i^3} + \frac{3 b^2 B (b c - a d) g^3 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^4 i^3}
\end{aligned}$$

**Problem 152: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(c i + d i x)^3} dx$$

Optimal (type 4, 263 leaves, 8 steps):

$$\begin{aligned}
& \frac{B g^2 n (a + b x)^2}{4 d i^3 (c + d x)^2} - \frac{A b g^2 (a + b x)}{d^2 i^3 (c + d x)} + \frac{b B g^2 n (a + b x)}{d^2 i^3 (c + d x)} - \\
& \frac{b B g^2 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{d^2 i^3 (c + d x)} - \frac{g^2 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 d i^3 (c + d x)^2} - \\
& \frac{b^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[-\frac{b c - a d}{b (c + d x)}\right]}{d^3 i^3} - \frac{b^2 B g^2 n \operatorname{PolyLog}\left[2, \frac{d(a + b x)}{b (c + d x)}\right]}{d^3 i^3}
\end{aligned}$$

Result (type 4, 356 leaves, 18 steps):

$$\begin{aligned}
& \frac{B (b c - a d)^2 g^2 n}{4 d^3 i^3 (c + d x)^2} - \frac{3 b B (b c - a d) g^2 n}{2 d^3 i^3 (c + d x)} - \frac{3 b^2 B g^2 n \operatorname{Log}[a + b x]}{2 d^3 i^3} - \\
& \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 d^3 i^3 (c + d x)^2} + \frac{2 b (b c - a d) g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d^3 i^3 (c + d x)} + \\
& \frac{3 b^2 B g^2 n \operatorname{Log}[c + d x]}{2 d^3 i^3} - \frac{b^2 B g^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^3} + \\
& \frac{b^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^3 i^3} + \frac{b^2 B g^2 n \operatorname{Log}[c + d x]^2}{2 d^3 i^3} - \frac{b^2 B g^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^3 i^3}
\end{aligned}$$



**Problem 153: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(c i + d i x)^3} dx$$

Optimal (type 3, 89 leaves, 2 steps):

$$-\frac{B g n (a + b x)^2}{4 (b c - a d) i^3 (c + d x)^2} + \frac{g (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 (b c - a d) i^3 (c + d x)^2}$$

Result (type 3, 201 leaves, 10 steps):

$$-\frac{B (b c - a d) g n}{4 d^2 i^3 (c + d x)^2} + \frac{b B g n}{2 d^2 i^3 (c + d x)} + \frac{b^2 B g n \operatorname{Log}[a + b x]}{2 d^2 (b c - a d) i^3} +$$

$$\frac{(b c - a d) g \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d^2 i^3 (c + d x)^2} - \frac{b g \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^2 i^3 (c + d x)} - \frac{b^2 B g n \operatorname{Log}[c + d x]}{2 d^2 (b c - a d) i^3}$$

**Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{(a g + b g x) (c i + d i x)^3} dx$$

Optimal (type 3, 254 leaves, 4 steps):

$$-\frac{B n \left( 4 b - \frac{d (a+b x)}{c+d x} \right)^2}{4 (b c - a d)^3 g i^3} + \frac{d^2 (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 (b c - a d)^3 g i^3 (c + d x)^2} -$$

$$\frac{2 b d (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(b c - a d)^3 g i^3 (c + d x)} + \frac{b^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{a+b x}{c+d x} \right]}{(b c - a d)^3 g i^3} - \frac{b^2 B n \operatorname{Log}\left[ \frac{a+b x}{c+d x} \right]^2}{2 (b c - a d)^3 g i^3}$$

Result (type 4, 557 leaves, 26 steps):

$$\begin{aligned}
& - \frac{B n}{4 (b c - a d) g i^3 (c + d x)^2} - \frac{3 b B n}{2 (b c - a d)^2 g i^3 (c + d x)} - \\
& \frac{3 b^2 B n \operatorname{Log}[a + b x]}{2 (b c - a d)^3 g i^3} - \frac{b^2 B n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^3 g i^3} + \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{2 (b c - a d) g i^3 (c + d x)^2} + \\
& \frac{b \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^2 g i^3 (c + d x)} + \frac{b^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^3 g i^3} + \frac{3 b^2 B n \operatorname{Log}[c + d x]}{2 (b c - a d)^3 g i^3} + \\
& \frac{b^2 B n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^3 g i^3} - \frac{b^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^3 g i^3} - \frac{b^2 B n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^3 g i^3} + \\
& \frac{b^2 B n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{b^2 B n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{b^2 B n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3}
\end{aligned}$$

**Problem 156: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(a g + b g x)^2 (c i + d i x)^3} dx$$

Optimal (type 3, 381 leaves, 4 steps):

$$\begin{aligned}
& \frac{B d^3 n (a + b x)^2}{4 (b c - a d)^4 g^2 i^3 (c + d x)^2} - \frac{3 b B d^2 n (a + b x)}{(b c - a d)^4 g^2 i^3 (c + d x)} - \frac{b^3 B n (c + d x)}{(b c - a d)^4 g^2 i^3 (a + b x)} - \\
& \frac{d^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^4 g^2 i^3 (c + d x)^2} + \frac{3 b d^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^4 g^2 i^3 (c + d x)} - \\
& \frac{b^3 (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^4 g^2 i^3 (a + b x)} - \frac{3 b^2 d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B d n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{2 (b c - a d)^4 g^2 i^3}
\end{aligned}$$

Result (type 4, 657 leaves, 30 steps):

$$\begin{aligned}
& - \frac{b^2 B n}{(b c - a d)^3 g^2 i^3 (a + b x)} + \frac{B d n}{4 (b c - a d)^2 g^2 i^3 (c + d x)^2} + \\
& \frac{5 b B d n}{2 (b c - a d)^3 g^2 i^3 (c + d x)} + \frac{3 b^2 B d n \operatorname{Log}[a + b x]}{2 (b c - a d)^4 g^2 i^3} + \frac{3 b^2 B d n \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^2 i^3} - \\
& \frac{b^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^2 i^3 (a + b x)} - \frac{d \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^2 g^2 i^3 (c + d x)^2} - \frac{2 b d \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^3 g^2 i^3 (c + d x)} - \\
& \frac{3 b^2 d \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d n \operatorname{Log}[c + d x]}{2 (b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B d n \operatorname{Log}\left[ -\frac{d(a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 d \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B d n \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d n \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b(c + d x)}{b c - a d} \right]}{(b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B d n \operatorname{PolyLog}\left[ 2, -\frac{d(a + b x)}{b c - a d} \right]}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d n \operatorname{PolyLog}\left[ 2, \frac{b(c + d x)}{b c - a d} \right]}{(b c - a d)^4 g^2 i^3}
\end{aligned}$$

**Problem 157: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(a g + b g x)^3 (c i + d i x)^3} dx$$

Optimal (type 3, 483 leaves, 5 steps):

$$\begin{aligned}
& - \frac{B d^4 n (a + b x)^2}{4 (b c - a d)^5 g^3 i^3 (c + d x)^2} + \frac{4 b B d^3 n (a + b x)}{(b c - a d)^5 g^3 i^3 (c + d x)} + \\
& \frac{4 b^3 B d n (c + d x)}{(b c - a d)^5 g^3 i^3 (a + b x)} - \frac{b^4 B n (c + d x)^2}{4 (b c - a d)^5 g^3 i^3 (a + b x)^2} + \\
& \frac{d^4 (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^5 g^3 i^3 (c + d x)^2} - \frac{4 b d^3 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^3 i^3 (c + d x)} + \\
& \frac{4 b^3 d (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^3 i^3 (a + b x)} - \frac{b^4 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^5 g^3 i^3 (a + b x)^2} + \\
& \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]}{(b c - a d)^5 g^3 i^3} - \frac{3 b^2 B d^2 n \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]^2}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

Result (type 4, 701 leaves, 34 steps):

$$\begin{aligned}
& - \frac{b^2 B n}{4 (b c - a d)^3 g^3 i^3 (a + b x)^2} + \frac{7 b^2 B d n}{2 (b c - a d)^4 g^3 i^3 (a + b x)} - \\
& \frac{B d^2 n}{4 (b c - a d)^3 g^3 i^3 (c + d x)^2} - \frac{7 b B d^2 n}{2 (b c - a d)^4 g^3 i^3 (c + d x)} - \frac{3 b^2 B d^2 n \operatorname{Log}[a + b x]^2}{(b c - a d)^5 g^3 i^3} - \\
& \frac{b^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^3 g^3 i^3 (a + b x)^2} + \frac{3 b^2 d \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^3 i^3 (a + b x)} + \frac{d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^3 g^3 i^3 (c + d x)^2} + \\
& \frac{3 b d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^4 g^3 i^3 (c + d x)} + \frac{6 b^2 d^2 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^3 i^3} + \\
& \frac{6 b^2 B d^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^5 g^3 i^3} - \\
& \frac{3 b^2 B d^2 n \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B d^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{6 b^2 B d^2 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B d^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

**Problem 158: Result unnecessarily involves higher level functions.**

$$\int \frac{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(a g + b g x)^4 (c i + d i x)^3} dx$$

Optimal (type 3, 587 leaves, 8 steps):

$$\begin{aligned}
& \frac{B d^5 n (a + b x)^2}{4 (b c - a d)^6 g^4 i^3 (c + d x)^2} - \frac{5 b B d^4 n (a + b x)}{(b c - a d)^6 g^4 i^3 (c + d x)} - \frac{10 b^3 B d^2 n (c + d x)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \\
& \frac{5 b^4 B d n (c + d x)^2}{4 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \frac{b^5 B n (c + d x)^3}{9 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \frac{d^5 (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^6 g^4 i^3 (c + d x)^2} + \\
& \frac{5 b d^4 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^6 g^4 i^3 (c + d x)} - \frac{10 b^3 d^2 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \\
& \frac{5 b^4 d (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \frac{b^5 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \\
& \frac{10 b^2 d^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{(b c - a d)^6 g^4 i^3} + \frac{5 b^2 B d^3 n \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]^2}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$

Result (type 4, 859 leaves, 38 steps):

$$\begin{aligned}
& - \frac{b^2 B n}{9 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \frac{11 b^2 B d n}{12 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{47 b^2 B d^2 n}{6 (b c - a d)^5 g^4 i^3 (a + b x)} + \\
& \frac{B d^3 n}{4 (b c - a d)^4 g^4 i^3 (c + d x)^2} + \frac{9 b B d^3 n}{2 (b c - a d)^5 g^4 i^3 (c + d x)} - \frac{10 b^2 B d^3 n \operatorname{Log}[a + b x]}{3 (b c - a d)^6 g^4 i^3} + \\
& \frac{5 b^2 B d^3 n \operatorname{Log}[a + b x]^2}{(b c - a d)^6 g^4 i^3} - \frac{b^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \frac{3 b^2 d \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \\
& \frac{6 b^2 d^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^4 i^3 (a + b x)} - \frac{d^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^4 g^4 i^3 (c + d x)^2} - \frac{4 b d^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^5 g^4 i^3 (c + d x)} - \\
& \frac{10 b^2 d^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B d^3 n \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{10 b^2 B d^3 n \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 d^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \\
& \frac{5 b^2 B d^3 n \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} - \frac{10 b^2 B d^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{10 b^2 B d^3 n \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \frac{10 b^2 B d^3 n \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$

**Problem 159: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^3 (c i + d i x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 584 leaves, 11 steps):

$$\begin{aligned}
& \frac{3 B^2 (b c - a d)^4 g^3 i n^2 x}{10 b d^3} - \frac{3 B^2 (b c - a d)^3 g^3 i n^2 (c + d x)^2}{20 d^4} + \\
& \frac{b B^2 (b c - a d)^2 g^3 i n^2 (c + d x)^3}{30 d^4} - \frac{B (b c - a d)^2 g^3 i n (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^2 d} - \\
& \frac{B (b c - a d) g^3 i n (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b^2} + \\
& \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{20 b^2} + \frac{g^3 i (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b} + \\
& \frac{B (b c - a d)^3 g^3 i n (a + b x)^2 \left( 3 A + B n + 3 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 b^2 d^2} - \\
& \frac{B (b c - a d)^4 g^3 i n (a + b x) \left( 6 A + 5 B n + 6 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 b^2 d^3} - \\
& \frac{B (b c - a d)^5 g^3 i n \left( 6 A + 11 B n + 6 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{60 b^2 d^4} - \\
& \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{Log}[c + d x]}{10 b^2 d^4} - \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{10 b^2 d^4}
\end{aligned}$$

Result (type 4, 670 leaves, 52 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^4 g^3 i n x}{10 b d^3} + \frac{B^2 (b c - a d)^4 g^3 i n^2 x}{60 b d^3} - \frac{B^2 (b c - a d)^3 g^3 i n^2 (a + b x)^2}{30 b^2 d^2} + \\
& \frac{B^2 (b c - a d)^2 g^3 i n^2 (a + b x)^3}{30 b^2 d} - \frac{B^2 (b c - a d)^4 g^3 i n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{10 b^2 d^3} + \\
& \frac{B (b c - a d)^3 g^3 i n (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{20 b^2 d^2} - \\
& \frac{B (b c - a d)^2 g^3 i n (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^2 d} - \\
& \frac{B (b c - a d) g^3 i n (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b^2} + \\
& \frac{(b c - a d) g^3 i (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b^2} + \frac{d g^3 i (a + b x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b^2} + \\
& \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{Log}[c + d x]}{12 b^2 d^4} - \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{Log}\left[ - \frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{10 b^2 d^4} + \\
& \frac{B (b c - a d)^5 g^3 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{10 b^2 d^4} + \\
& \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{Log}[c + d x]^2}{20 b^2 d^4} - \frac{B^2 (b c - a d)^5 g^3 i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{10 b^2 d^4}
\end{aligned}$$

### Problem 160: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 (c i + d i x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 487 leaves, 10 steps):

$$\begin{aligned} & - \frac{B^2 (b c - a d)^3 g^2 i n^2 x}{3 b d^2} + \frac{B^2 (b c - a d)^2 g^2 i n^2 (c + d x)^2}{12 d^3} - \\ & \frac{B (b c - a d)^2 g^2 i n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{12 b^2 d} - \\ & \frac{B (b c - a d) g^2 i n (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^2} + \\ & \frac{(b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{12 b^2} + \frac{g^2 i (a + b x)^3 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b} + \\ & \frac{B (b c - a d)^3 g^2 i n (a + b x) \left( 2 A + B n + 2 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{12 b^2 d^2} + \\ & \frac{B (b c - a d)^4 g^2 i n \left( 2 A + 3 B n + 2 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{12 b^2 d^3} + \\ & \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{Log}[c + d x]}{6 b^2 d^3} + \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{6 b^2 d^3} \end{aligned}$$

Result (type 4, 578 leaves, 44 steps):

$$\begin{aligned} & \frac{A B (b c - a d)^3 g^2 i n x}{6 b d^2} - \frac{B^2 (b c - a d)^3 g^2 i n^2 x}{12 b d^2} + \frac{B^2 (b c - a d)^2 g^2 i n^2 (a + b x)^2}{12 b^2 d} + \\ & \frac{B^2 (b c - a d)^3 g^2 i n (a + b x) \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]}{6 b^2 d^2} - \frac{B (b c - a d)^2 g^2 i n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{12 b^2 d} - \\ & \frac{B (b c - a d) g^2 i n (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^2} + \\ & \frac{(b c - a d) g^2 i (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b^2} + \frac{d g^2 i (a + b x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b^2} - \\ & \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{Log}[c + d x]}{12 b^2 d^3} + \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{6 b^2 d^3} - \\ & \frac{B (b c - a d)^4 g^2 i n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{6 b^2 d^3} - \\ & \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{Log}[c + d x]^2}{12 b^2 d^3} + \frac{B^2 (b c - a d)^4 g^2 i n^2 \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{b c - a d} \right]}{6 b^2 d^3} \end{aligned}$$

### Problem 161: Result valid but suboptimal antiderivative.

$$\int (a g + b g x) (c i + d i x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 372 leaves, 9 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d)^2 g i n^2 x}{3 b d} - \frac{B (b c - a d)^2 g i n (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 d} - \\ & \frac{B (b c - a d) g i n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2} + \\ & \frac{(b c - a d) g i (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{6 b^2} + \frac{g i (a + b x)^2 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b} - \\ & \frac{B (b c - a d)^3 g i n \left( A + B n + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{3 b^2 d^2} - \\ & \frac{B^2 (b c - a d)^3 g i n^2 \operatorname{Log}[c + d x]}{3 b^2 d^2} - \frac{B^2 (b c - a d)^3 g i n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{3 b^2 d^2} \end{aligned}$$

Result (type 4, 1323 leaves, 72 steps):



$$\begin{aligned}
& -\frac{2}{3} A b B \left( \frac{a^2}{b^2} - \frac{c^2}{d^2} \right) d g i n x - \frac{A B (b c - a d) (b c + a d) g i n x}{b d} + \frac{B^2 (b c - a d)^2 g i n^2 x}{3 b d} + \\
& \frac{a^2 B^2 (b c - a d) g i n^2 \operatorname{Log}[a + b x]}{3 b^2} - \frac{a^2 B^2 c g i n^2 \operatorname{Log}[a + b x]^2}{b} - \frac{a^3 B^2 d g i n^2 \operatorname{Log}[a + b x]^2}{3 b^2} + \\
& \frac{a^2 B^2 (b c + a d) g i n^2 \operatorname{Log}[a + b x]^2}{2 b^2} - \frac{B^2 (b c - a d) (b c + a d) g i n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right]}{3 b^2 d} - \\
& \frac{1}{3} B (b c - a d) g i n x^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right) + \frac{2 a^2 B c g i n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)}{b} + \\
& \frac{2 a^3 B d g i n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)}{3 b^2} - \\
& \frac{a^2 B (b c + a d) g i n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)}{b^2} + \\
& a c g i x \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)^2 + \frac{1}{2} (b c + a d) g i x^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)^2 + \\
& \frac{1}{3} b d g i x^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)^2 - \frac{B^2 c^2 (b c - a d) g i n^2 \operatorname{Log}[c + d x]}{3 d^2} + \\
& \frac{B^2 (b c - a d)^2 (b c + a d) g i n^2 \operatorname{Log}[c + d x]}{3 b^2 d^2} + \frac{2 b B^2 c^3 g i n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 d^2} + \\
& \frac{2 a B^2 c^2 g i n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d} - \frac{B^2 c^2 (b c + a d) g i n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2} - \\
& \frac{2 b B c^3 g i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right) \operatorname{Log}[c + d x]}{3 d^2} - \frac{2 a B c^2 g i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right) \operatorname{Log}[c + d x]}{d} + \\
& \frac{B c^2 (b c + a d) g i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right) \operatorname{Log}[c + d x]}{d^2} - \\
& \frac{b B^2 c^3 g i n^2 \operatorname{Log}[c + d x]^2}{3 d^2} - \frac{a B^2 c^2 g i n^2 \operatorname{Log}[c + d x]^2}{d} + \\
& \frac{B^2 c^2 (b c + a d) g i n^2 \operatorname{Log}[c + d x]^2}{2 d^2} + \frac{2 a^2 B^2 c g i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b} + \\
& \frac{2 a^3 B^2 d g i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^2} - \frac{a^2 B^2 (b c + a d) g i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2} + \\
& \frac{2 a^2 B^2 c g i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b} + \frac{2 a^3 B^2 d g i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^2} - \\
& \frac{a^2 B^2 (b c + a d) g i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2} + \frac{2 b B^2 c^3 g i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 d^2} + \\
& \frac{2 a B^2 c^2 g i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d} - \frac{B^2 c^2 (b c + a d) g i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d^2}
\end{aligned}$$

### Problem 162: Result valid but suboptimal antiderivative.

$$\int (c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$\begin{aligned} & - \frac{B (b c - a d) i n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^2} + \\ & \frac{i (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 d} + \frac{B^2 (b c - a d)^2 i n^2 \operatorname{Log}[c + d x]}{b^2 d} + \\ & \frac{B (b c - a d)^2 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 d} - \frac{B^2 (b c - a d)^2 i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 d} \end{aligned}$$

Result (type 4, 307 leaves, 15 steps):

$$\begin{aligned} & - \frac{A B (b c - a d) i n x}{b} + \frac{B^2 (b c - a d)^2 i n^2 \operatorname{Log}[a + b x]^2}{2 b^2 d} - \\ & \frac{B^2 (b c - a d) i n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{b^2} - \frac{B (b c - a d)^2 i n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^2 d} + \\ & \frac{i (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 d} + \frac{B^2 (b c - a d)^2 i n^2 \operatorname{Log}[c + d x]}{b^2 d} - \\ & \frac{B^2 (b c - a d)^2 i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 d} - \frac{B^2 (b c - a d)^2 i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 d} \end{aligned}$$

### Problem 163: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{a g + b g x} dx$$

Optimal (type 4, 306 leaves, 8 steps):

$$\begin{aligned} & \frac{d i (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^2 g} + \frac{2 B (b c - a d) i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{b^2 g} - \\ & \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g} + \frac{2 B^2 (b c - a d) i n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^2 g} + \\ & \frac{2 B (b c - a d) i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g} + \\ & \frac{2 B^2 (b c - a d) i n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g} \end{aligned}$$

Result (type 4, 692 leaves, 36 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d) i n \operatorname{Log}[a + b x]^2}{b^2 g} - \frac{a B^2 d i n^2 \operatorname{Log}[a + b x]^2}{b^2 g} - \\
& \frac{B^2 (b c - a d) i \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^2 g} - \frac{B^2 (b c - a d) i \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^2 g} + \\
& \frac{2 a B d i n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 g} + \frac{d i x \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b g} + \\
& \frac{(b c - a d) i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^2 g} + \frac{2 B^2 c i n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b g} - \\
& \frac{2 B c i n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b g} - \frac{B^2 c i n^2 \operatorname{Log}[c + d x]^2}{b g} + \\
& \frac{2 A B (b c - a d) i n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g} + \frac{2 a B^2 d i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g} + \\
& \frac{2 A B (b c - a d) i n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g} + \frac{2 a B^2 d i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g} + \\
& \frac{2 B^2 c i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b g} + \frac{2 B^2 (b c - a d) i n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g} + \\
& \frac{2 B^2 (b c - a d) i n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g}
\end{aligned}$$

Problem 164: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^2} dx$$

Optimal (type 4, 261 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 B^2 i n^2 (c + d x)}{b g^2 (a + b x)} - \frac{2 B i n (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b g^2 (a + b x)} - \\
& \frac{i (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b g^2 (a + b x)} - \frac{d i \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2} + \\
& \frac{2 B d i n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2} + \frac{2 B^2 d i n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^2 g^2}
\end{aligned}$$

Result (type 4, 766 leaves, 40 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d) i n^2}{b^2 g^2 (a + b x)} - \frac{2 B^2 d i n^2 \operatorname{Log}[a + b x]}{b^2 g^2} - \frac{A B d i n \operatorname{Log}[a + b x]^2}{b^2 g^2} + \\
& \frac{B^2 d i n^2 \operatorname{Log}[a + b x]^2}{b^2 g^2} - \frac{B^2 d i \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^2 g^2} - \\
& \frac{B^2 d i \operatorname{Log}[a + b x] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^2 g^2} - \frac{2 B (b c - a d) i n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 g^2 (a + b x)} - \\
& \frac{2 B d i n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^2 g^2} - \frac{(b c - a d) i \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^2 g^2 (a + b x)} + \\
& \frac{d i \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^2 g^2} + \frac{2 B^2 d i n^2 \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B^2 d i n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} + \frac{2 B d i n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} + \\
& \frac{B^2 d i n^2 \operatorname{Log}[c + d x]^2}{b^2 g^2} + \frac{2 A B d i n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} - \\
& \frac{2 B^2 d i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} + \frac{2 A B d i n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g^2} - \\
& \frac{2 B^2 d i n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d i n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} + \\
& \frac{2 B^2 d i n \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g^2} + \frac{2 B^2 d i n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^2 g^2}
\end{aligned}$$

**Problem 165: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c i + d i x) \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^3} dx$$

Optimal (type 3, 151 leaves, 3 steps):

$$\begin{aligned}
& - \frac{B^2 i n^2 (c + d x)^2}{4 (b c - a d) g^3 (a + b x)^2} - \\
& \frac{B i n (c + d x)^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d) g^3 (a + b x)^2} - \frac{i (c + d x)^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d) g^3 (a + b x)^2}
\end{aligned}$$

Result (type 4, 691 leaves, 54 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad) i n^2}{4 b^2 g^3 (a + bx)^2} - \frac{B^2 d i n^2}{2 b^2 g^3 (a + bx)} - \frac{B^2 d^2 i n^2 \operatorname{Log}[a + bx]}{2 b^2 (bc - ad) g^3} + \\
& \frac{B^2 d^2 i n^2 \operatorname{Log}[a + bx]^2}{2 b^2 (bc - ad) g^3} - \frac{B (bc - ad) i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{2 b^2 g^3 (a + bx)^2} - \\
& \frac{B d i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{b^2 g^3 (a + bx)} - \frac{B d^2 i n \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{b^2 (bc - ad) g^3} - \\
& \frac{(bc - ad) i \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 b^2 g^3 (a + bx)^2} - \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{b^2 g^3 (a + bx)} + \frac{B^2 d^2 i n^2 \operatorname{Log}[c + dx]}{2 b^2 (bc - ad) g^3} - \\
& \frac{B^2 d^2 i n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{b^2 (bc - ad) g^3} + \frac{B d^2 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c + dx]}{b^2 (bc - ad) g^3} + \\
& \frac{B^2 d^2 i n^2 \operatorname{Log}[c + dx]^2}{2 b^2 (bc - ad) g^3} - \frac{B^2 d^2 i n^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{b^2 (bc - ad) g^3} - \\
& \frac{B^2 d^2 i n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^2 (bc - ad) g^3} - \frac{B^2 d^2 i n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b^2 (bc - ad) g^3}
\end{aligned}$$

**Problem 166:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 307 leaves, 7 steps):

$$\begin{aligned}
& \frac{B^2 d i n^2 (c + dx)^2}{4 (bc - ad)^2 g^4 (a + bx)^2} - \frac{2 b B^2 i n^2 (c + dx)^3}{27 (bc - ad)^2 g^4 (a + bx)^3} + \\
& \frac{B d i n (c + dx)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{2 (bc - ad)^2 g^4 (a + bx)^2} - \frac{2 b B i n (c + dx)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{9 (bc - ad)^2 g^4 (a + bx)^3} + \\
& \frac{d i (c + dx)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 (bc - ad)^2 g^4 (a + bx)^2} - \frac{b i (c + dx)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{3 (bc - ad)^2 g^4 (a + bx)^3}
\end{aligned}$$

Result (type 4, 800 leaves, 62 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d) i n^2}{27 b^2 g^4 (a + b x)^3} + \frac{B^2 d i n^2}{36 b^2 g^4 (a + b x)^2} + \frac{5 B^2 d^2 i n^2}{18 b^2 (b c - a d) g^4 (a + b x)} + \frac{5 B^2 d^3 i n^2 \operatorname{Log}[a + b x]}{18 b^2 (b c - a d)^2 g^4} - \\
& \frac{B^2 d^3 i n^2 \operatorname{Log}[a + b x]^2}{6 b^2 (b c - a d)^2 g^4} - \frac{2 B (b c - a d) i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 b^2 g^4 (a + b x)^3} - \frac{B d i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^2 g^4 (a + b x)^2} + \\
& \frac{B d^2 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 (b c - a d) g^4 (a + b x)} + \frac{B d^3 i n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 (b c - a d)^2 g^4} - \\
& \frac{(b c - a d) i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b^2 g^4 (a + b x)^3} - \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^2 g^4 (a + b x)^2} - \frac{5 B^2 d^3 i n^2 \operatorname{Log}[c + d x]}{18 b^2 (b c - a d)^2 g^4} + \\
& \frac{B^2 d^3 i n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b^2 (b c - a d)^2 g^4} - \frac{B d^3 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{3 b^2 (b c - a d)^2 g^4} - \\
& \frac{B^2 d^3 i n^2 \operatorname{Log}[c + d x]^2}{6 b^2 (b c - a d)^2 g^4} + \frac{B^2 d^3 i n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4} + \\
& \frac{B^2 d^3 i n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4} + \frac{B^2 d^3 i n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{3 b^2 (b c - a d)^2 g^4}
\end{aligned}$$

**Problem 167: Result unnecessarily involves higher level functions.**

$$\int \frac{(c i + d i x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 475 leaves, 9 steps):

$$\begin{aligned}
& - \frac{B^2 d^2 i n^2 (c + d x)^2}{4 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B^2 d i n^2 (c + d x)^3}{27 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 B^2 i n^2 (c + d x)^4}{32 (b c - a d)^3 g^5 (a + b x)^4} - \\
& \frac{B d^2 i n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 (b c - a d)^3 g^5 (a + b x)^2} + \frac{4 b B d i n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 (b c - a d)^3 g^5 (a + b x)^3} - \\
& \frac{b^2 B i n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 (b c - a d)^3 g^5 (a + b x)^4} - \frac{d^2 i (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 (b c - a d)^3 g^5 (a + b x)^2} + \\
& \frac{2 b d i (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 (b c - a d)^3 g^5 (a + b x)^3} - \frac{b^2 i (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 (b c - a d)^3 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 892 leaves, 70 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad) i n^2}{32 b^2 g^5 (a + bx)^4} + \frac{5 B^2 d i n^2}{216 b^2 g^5 (a + bx)^3} + \frac{B^2 d^2 i n^2}{144 b^2 (bc - ad) g^5 (a + bx)^2} - \\
& \frac{13 B^2 d^3 i n^2}{72 b^2 (bc - ad)^2 g^5 (a + bx)} - \frac{13 B^2 d^4 i n^2 \operatorname{Log}[a + bx]}{72 b^2 (bc - ad)^3 g^5} + \frac{B^2 d^4 i n^2 \operatorname{Log}[a + bx]^2}{12 b^2 (bc - ad)^3 g^5} - \\
& \frac{B (bc - ad) i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{8 b^2 g^5 (a + bx)^4} - \frac{B d i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{18 b^2 g^5 (a + bx)^3} + \\
& \frac{B d^2 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{12 b^2 (bc - ad) g^5 (a + bx)^2} - \frac{B d^3 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{6 b^2 (bc - ad)^2 g^5 (a + bx)} - \\
& \frac{B d^4 i n \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{6 b^2 (bc - ad)^3 g^5} - \frac{(bc - ad) i \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{4 b^2 g^5 (a + bx)^4} - \\
& \frac{d i \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{3 b^2 g^5 (a + bx)^3} + \frac{13 B^2 d^4 i n^2 \operatorname{Log}[c + dx]}{72 b^2 (bc - ad)^3 g^5} - \\
& \frac{B^2 d^4 i n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{b(c-ad)}\right] \operatorname{Log}[c + dx]}{6 b^2 (bc - ad)^3 g^5} + \frac{B d^4 i n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c + dx]}{6 b^2 (bc - ad)^3 g^5} + \\
& \frac{B^2 d^4 i n^2 \operatorname{Log}[c + dx]^2}{12 b^2 (bc - ad)^3 g^5} - \frac{B^2 d^4 i n^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{b(c-ad)}\right]}{6 b^2 (bc - ad)^3 g^5} - \\
& \frac{B^2 d^4 i n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-ad)}\right]}{6 b^2 (bc - ad)^3 g^5} - \frac{B^2 d^4 i n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{b(c-ad)}\right]}{6 b^2 (bc - ad)^3 g^5}
\end{aligned}$$

**Problem 168: Result valid but suboptimal antiderivative.**

$$\int (ag + bgx)^3 (ci + dix)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 766 leaves, 17 steps):

$$\begin{aligned}
& \frac{3 B^2 (b c - a d)^5 g^3 i^2 n^2 x}{20 b^2 d^3} + \frac{B^2 (b c - a d)^2 g^3 i^2 n^2 (a + b x)^4}{60 b^3} - \frac{3 B^2 (b c - a d)^4 g^3 i^2 n^2 (c + d x)^2}{40 b d^4} + \\
& \frac{B^2 (b c - a d)^3 g^3 i^2 n^2 (c + d x)^3}{60 d^4} - \frac{B (b c - a d)^3 g^3 i^2 n (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{90 b^3 d} - \\
& \frac{B (b c - a d)^2 g^3 i^2 n (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{20 b^3} - \\
& \frac{B (b c - a d) g^3 i^2 n (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 b^2} + \\
& \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{60 b^3} + \\
& \frac{(b c - a d) g^3 i^2 (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{15 b^2} + \\
& \frac{g^3 i^2 (a + b x)^4 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{6 b} + \\
& \frac{B (b c - a d)^4 g^3 i^2 n (a + b x)^2 \left( 3 A + B n + 3 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{180 b^3 d^2} - \\
& \frac{B (b c - a d)^5 g^3 i^2 n (a + b x) \left( 6 A + 5 B n + 6 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{180 b^3 d^3} - \\
& \frac{B (b c - a d)^6 g^3 i^2 n \left( 6 A + 11 B n + 6 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{180 b^3 d^4} - \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{Log}[c + d x]}{20 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{30 b^3 d^4}
\end{aligned}$$

Result(type 4, 848 leaves, 83 steps):



$$\begin{aligned}
& - \frac{A B (b c - a d)^5 g^3 i^2 n x}{30 b^2 d^3} + \frac{B^2 (b c - a d)^5 g^3 i^2 n^2 x}{45 b^2 d^3} - \\
& \frac{7 B^2 (b c - a d)^4 g^3 i^2 n^2 (a + b x)^2}{360 b^3 d^2} + \frac{B^2 (b c - a d)^3 g^3 i^2 n^2 (a + b x)^3}{60 b^3 d} + \\
& \frac{B^2 (b c - a d)^2 g^3 i^2 n^2 (a + b x)^4}{60 b^3} - \frac{B^2 (b c - a d)^5 g^3 i^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{30 b^3 d^3} + \\
& \frac{B (b c - a d)^4 g^3 i^2 n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{60 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^3 i^2 n (a + b x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{90 b^3 d} - \\
& \frac{7 B (b c - a d)^2 g^3 i^2 n (a + b x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{60 b^3} - \\
& \frac{B d (b c - a d) g^3 i^2 n (a + b x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{15 b^3} + \\
& \frac{(b c - a d)^2 g^3 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{4 b^3} + \\
& \frac{2 d (b c - a d) g^3 i^2 (a + b x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{5 b^3} + \frac{d^2 g^3 i^2 (a + b x)^6 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{6 b^3} + \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{Log}[c + d x]}{90 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{30 b^3 d^4} + \\
& \frac{B (b c - a d)^6 g^3 i^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{30 b^3 d^4} + \\
& \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{Log}[c + d x]^2}{60 b^3 d^4} - \frac{B^2 (b c - a d)^6 g^3 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{30 b^3 d^4}
\end{aligned}$$

**Problem 169: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^2 (c i + d i x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 819 leaves, 15 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^4 g^2 i^2 n^2 x}{10 b^2 d^2} - \frac{B^2 (b c - a d)^3 g^2 i^2 n^2 (c + d x)^2}{20 b d^3} + \\
& \frac{B^2 (b c - a d)^2 g^2 i^2 n^2 (c + d x)^3}{30 d^3} - \frac{B (b c - a d)^3 g^2 i^2 n (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^3 d} - \\
& \frac{B (b c - a d)^2 g^2 i^2 n (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 b^3} - \\
& \frac{B (b c - a d)^3 g^2 i^2 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{5 b d^3} + \\
& \frac{4 B (b c - a d)^2 g^2 i^2 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 d^3} - \\
& \frac{b B (b c - a d) g^2 i^2 n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 d^3} + \\
& \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{30 b^3} + \\
& \frac{(b c - a d) g^2 i^2 (a + b x)^3 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{10 b^2} + \\
& \frac{g^2 i^2 (a + b x)^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b} + \\
& \frac{B (b c - a d)^4 g^2 i^2 n (a + b x) \left( 2 A + B n + 2 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^3 d^2} + \\
& \frac{B (b c - a d)^5 g^2 i^2 n \left( 2 A + 3 B n + 2 B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{30 b^3 d^3} + \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{30 b^3 d^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{Log}[c + d x]}{10 b^3 d^3} + \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{15 b^3 d^3}
\end{aligned}$$

Result(type 4, 714 leaves, 71 steps):

$$\begin{aligned}
& \frac{A B (b c - a d)^4 g^2 i^2 n x}{15 b^2 d^2} - \frac{B^2 (b c - a d)^4 g^2 i^2 n^2 x}{15 b^2 d^2} + \frac{B^2 (b c - a d)^3 g^2 i^2 n^2 (a + b x)^2}{20 b^3 d} + \\
& \frac{B^2 (b c - a d)^2 g^2 i^2 n^2 (a + b x)^3}{30 b^3} + \frac{B^2 (b c - a d)^4 g^2 i^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{15 b^3 d^2} - \\
& \frac{B (b c - a d)^3 g^2 i^2 n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{30 b^3 d} - \\
& \frac{B (b c - a d)^2 g^2 i^2 n (a + b x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{5 b^3} - \\
& \frac{B d (b c - a d) g^2 i^2 n (a + b x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{10 b^3} + \\
& \frac{(b c - a d)^2 g^2 i^2 (a + b x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 b^3} + \\
& \frac{d (b c - a d) g^2 i^2 (a + b x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b^3} + \\
& \frac{d^2 g^2 i^2 (a + b x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{5 b^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{15 b^3 d^3} - \\
& \frac{B (b c - a d)^5 g^2 i^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{15 b^3 d^3} - \\
& \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{Log}[c + d x]^2}{30 b^3 d^3} + \frac{B^2 (b c - a d)^5 g^2 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{15 b^3 d^3}
\end{aligned}$$

**Problem 170: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 635 leaves, 14 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^3 g i^2 n^2 x}{12 b^2 d} + \frac{B^2 (b c - a d)^2 g i^2 n^2 (c + d x)^2}{12 b d^2} - \\
& \frac{B (b c - a d)^3 g i^2 n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^3 d} - \\
& \frac{B (b c - a d)^2 g i^2 n (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b^3} + \\
& \frac{B (b c - a d)^2 g i^2 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b d^2} - \\
& \frac{B (b c - a d) g i^2 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 d^2} + \\
& \frac{(b c - a d)^2 g i^2 (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{12 b^3} + \\
& \frac{(b c - a d) g i^2 (a + b x)^2 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{6 b^2} + \\
& \frac{g i^2 (a + b x)^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 b} - \\
& \frac{B (b c - a d)^4 g i^2 n \left( A + B n + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{6 b^3 d^2} - \frac{B^2 (b c - a d)^4 g i^2 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{12 b^3 d^2} - \\
& \frac{B^2 (b c - a d)^4 g i^2 n^2 \operatorname{Log}[c + d x]}{4 b^3 d^2} - \frac{B^2 (b c - a d)^4 g i^2 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{6 b^3 d^2}
\end{aligned}$$

Result (type 4, 614 leaves, 44 steps):

$$\begin{aligned}
& \frac{AB(b c - a d)^3 g i^2 n x}{6 b^2 d} + \frac{B^2(b c - a d)^3 g i^2 n^2 x}{12 b^2 d} + \frac{B^2(b c - a d)^2 g i^2 n^2 (c + d x)^2}{12 b d^2} + \\
& \frac{B^2(b c - a d)^4 g i^2 n^2 \text{Log}[a + b x]}{12 b^3 d^2} - \frac{B^2(b c - a d)^4 g i^2 n^2 \text{Log}[a + b x]^2}{12 b^3 d^2} + \\
& \frac{B^2(b c - a d)^3 g i^2 n (a + b x) \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{6 b^3 d} + \frac{B(b c - a d)^2 g i^2 n (c + d x)^2 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{12 b d^2} - \\
& \frac{B(b c - a d) g i^2 n (c + d x)^3 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{6 d^2} + \\
& \frac{B(b c - a d)^4 g i^2 n \text{Log}[a + b x] \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{6 b^3 d^2} - \\
& \frac{(b c - a d) g i^2 (c + d x)^3 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{3 d^2} + \\
& \frac{b g i^2 (c + d x)^4 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{4 d^2} - \frac{B^2(b c - a d)^4 g i^2 n^2 \text{Log}[c + d x]}{6 b^3 d^2} + \\
& \frac{B^2(b c - a d)^4 g i^2 n^2 \text{Log}[a + b x] \text{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{6 b^3 d^2} + \frac{B^2(b c - a d)^4 g i^2 n^2 \text{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{6 b^3 d^2}
\end{aligned}$$

**Problem 171: Result valid but suboptimal antiderivative.**

$$\int (c i + d i x)^2 \left( A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right] \right)^2 dx$$

Optimal (type 4, 361 leaves, 11 steps):

$$\begin{aligned}
& \frac{B^2(b c - a d)^2 i^2 n^2 x}{3 b^2} - \frac{2 B(b c - a d)^2 i^2 n (a + b x) \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{3 b^3} - \\
& \frac{B(b c - a d) i^2 n (c + d x)^2 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{3 b d} + \frac{i^2 (c + d x)^3 \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{3 d} + \\
& \frac{B^2(b c - a d)^3 i^2 n^2 \text{Log}\left[\frac{a+b x}{c+d x}\right]}{3 b^3 d} + \frac{B^2(b c - a d)^3 i^2 n^2 \text{Log}[c + d x]}{b^3 d} + \\
& \frac{2 B(b c - a d)^3 i^2 n \left(A + B \text{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \text{Log}\left[1 - \frac{b(c+d x)}{d(a+b x)}\right]}{3 b^3 d} - \\
& \frac{2 B^2(b c - a d)^3 i^2 n^2 \text{PolyLog}\left[2, \frac{b(c+d x)}{d(a+b x)}\right]}{3 b^3 d}
\end{aligned}$$

Result (type 4, 454 leaves, 19 steps):

$$\begin{aligned}
& - \frac{2AB(bcd - ad)^2 i^2 n x}{3b^2} + \frac{B^2(bcd - ad)^2 i^2 n^2 x}{3b^2} + \\
& \frac{B^2(bcd - ad)^3 i^2 n^2 \text{Log}[a + bx]}{3b^3 d} + \frac{B^2(bcd - ad)^3 i^2 n^2 \text{Log}[a + bx]^2}{3b^3 d} - \\
& \frac{2B^2(bcd - ad)^2 i^2 n(a + bx) \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{3b^3} - \frac{B(bcd - ad) i^2 n(c + dx)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3bd} - \\
& \frac{2B(bcd - ad)^3 i^2 n \text{Log}[a + bx] \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3b^3 d} + \\
& \frac{i^2(c + dx)^3 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3d} + \frac{2B^2(bcd - ad)^3 i^2 n^2 \text{Log}[c + dx]}{3b^3 d} - \\
& \frac{2B^2(bcd - ad)^3 i^2 n^2 \text{Log}[a + bx] \text{Log}\left[\frac{b(c+dx)}{b(c-d)}\right]}{3b^3 d} - \frac{2B^2(bcd - ad)^3 i^2 n^2 \text{PolyLog}\left[2, -\frac{d(a+bx)}{b(c-d)}\right]}{3b^3 d}
\end{aligned}$$

**Problem 172: Result valid but suboptimal antiderivative.**

$$\int \frac{(ci + di x)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{ag + bgx} dx$$

Optimal (type 4, 572 leaves, 15 steps):

$$\begin{aligned}
& - \frac{Bd(bcd - ad) i^2 n(a + bx) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{b^3 g} + \\
& \frac{d(bcd - ad) i^2(a + bx) \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{b^3 g} + \frac{i^2(c + dx)^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2bg} + \\
& \frac{2B(bcd - ad)^2 i^2 n \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{Log}\left[\frac{b(c-d)}{b(c+dx)}\right]}{b^3 g} + \frac{B^2(bcd - ad)^2 i^2 n^2 \text{Log}[c + dx]}{b^3 g} + \\
& \frac{B(bcd - ad)^2 i^2 n \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g} - \\
& \frac{(bcd - ad)^2 i^2 \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \text{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g} + \\
& \frac{2B^2(bcd - ad)^2 i^2 n^2 \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{b^3 g} - \frac{B^2(bcd - ad)^2 i^2 n^2 \text{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g} + \\
& \frac{2B(bcd - ad)^2 i^2 n \left(A + B \text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g} + \\
& \frac{2B^2(bcd - ad)^2 i^2 n^2 \text{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g}
\end{aligned}$$

Result (type 4, 1790 leaves, 82 steps):

$$\begin{aligned}
& - \frac{A B d (b c - a d) i^2 n x}{b^2 g} - \frac{a B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[a + b x]^2}{b^3 g} + \\
& \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[a + b x]^2}{2 b^3 g} - \frac{A B (b c - a d)^2 i^2 n \operatorname{Log}[g (a + b x)]^2}{b^3 g} + \\
& \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[g (a + b x)]^3}{3 b^3 g} - \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[g (a + b x)]^2 \operatorname{Log}[-c - d x]}{b^3 g} + \\
& \frac{2 B^2 (b c - a d)^2 i^2 n \operatorname{Log}[g (a + b x)] \operatorname{Log}[(a + b x)^n] \operatorname{Log}[-c - d x]}{b^3 g} - \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log}[-c - d x]}{b^3 g} - \frac{B^2 d (b c - a d) i^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b^3 g} + \\
& \frac{2 a B d (b c - a d) i^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g} - \\
& \frac{B (b c - a d)^2 i^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g} + \\
& \frac{d (b c - a d) i^2 x \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^2 g} + \frac{i^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b g} + \\
& \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[c + d x]}{b^3 g} + \frac{2 B^2 c (b c - a d) i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g} - \\
& \frac{2 B c (b c - a d) i^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^2 g} - \\
& \frac{B^2 c (b c - a d) i^2 n^2 \operatorname{Log}[c + d x]^2}{b^2 g} + \frac{2 a B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g} - \\
& \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g} + \frac{B^2 (b c - a d)^2 i^2 n^2 \operatorname{Log}[g (a + b x)]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g} + \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g} + \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[(c + d x)^{-n}]^2}{b^3 g} - \\
& \frac{B^2 (b c - a d)^2 i^2 \operatorname{Log}[g (a + b x)] \operatorname{Log}[(c + d x)^{-n}]^2}{b^3 g} + \\
& \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}[a g + b g x]}{b^3 g} + \\
& \frac{2 A B (b c - a d)^2 i^2 n \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \operatorname{Log}[a g + b g x]}{b^3 g} - \frac{1}{b^3 g} \\
& 2 B^2 (b c - a d)^2 i^2 n \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right] \left(\operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}]\right) \\
& \operatorname{Log}[a g + b g x] - \frac{B^2 (b c - a d)^2 i^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[a g + b g x]^2}{b^3 g} -
\end{aligned}$$

$$\begin{aligned}
& \frac{B^2 (bc - ad)^2 i^2 n^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[ag + bgx]^2}{b^3 g} + \frac{2AB (bc - ad)^2 i^2 n \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} + \\
& \frac{2aB^2 d (bc - ad) i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \frac{B^2 (bc - ad)^2 i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} + \\
& \frac{2B^2 (bc - ad)^2 i^2 n \operatorname{Log}[(a+bx)^n] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \frac{1}{b^3 g} \\
& 2B^2 (bc - ad)^2 i^2 n \left( \operatorname{Log}[(a+bx)^n] - \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}[(c+dx)^{-n}] \right) \\
& \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right] + \frac{2B^2 c (bc - ad) i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b^2 g} - \\
& \frac{2B^2 (bc - ad)^2 i^2 n \operatorname{Log}[(c+dx)^{-n}] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{b^3 g} - \\
& \frac{2B^2 (bc - ad)^2 i^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{b^3 g} - \frac{2B^2 (bc - ad)^2 i^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{b^3 g}
\end{aligned}$$

**Problem 173: Result valid but suboptimal antiderivative.**

$$\int \frac{(ci + di x)^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(ag + bgx)^2} dx$$

Optimal (type 4, 472 leaves, 11 steps):

$$\begin{aligned}
& -\frac{2B^2 (bc - ad) i^2 n^2 (c+dx)}{b^2 g^2 (a+bx)} - \frac{2B (bc - ad) i^2 n (c+dx) \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{b^2 g^2 (a+bx)} + \\
& \frac{d^2 i^2 (a+bx) \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{b^3 g^2} - \frac{(bc - ad) i^2 (c+dx) \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{b^2 g^2 (a+bx)} + \\
& \frac{2Bd (bc - ad) i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}\left[\frac{bc-ad}{b(c+dx)}\right]}{b^3 g^2} - \\
& \frac{2d (bc - ad) i^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}\left[1 - \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^2} + \\
& \frac{2B^2 d (bc - ad) i^2 n^2 \operatorname{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{b^3 g^2} + \\
& \frac{4Bd (bc - ad) i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^2} + \\
& \frac{4B^2 d (bc - ad) i^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{b^3 g^2}
\end{aligned}$$

Result (type 4, 1309 leaves, 60 steps):



$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 i^2 n^2}{b^3 g^2 (a + b x)} - \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[a + b x]}{b^3 g^2} - \\
& \frac{2 A B d (b c - a d) i^2 n \operatorname{Log}[a + b x]^2}{b^3 g^2} - \frac{a B^2 d^2 i^2 n^2 \operatorname{Log}[a + b x]^2}{b^3 g^2} + \\
& \frac{B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[a + b x]^2}{b^3 g^2} - \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^3 g^2} - \\
& \frac{2 B (b c - a d)^2 i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^2 (a + b x)} + \frac{2 a B d^2 i^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^2} - \\
& \frac{2 B d (b c - a d) i^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^2} + \frac{d^2 i^2 x \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^2 g^2} - \\
& \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^3 g^2 (a + b x)} + \frac{2 d (b c - a d) i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^3 g^2} + \\
& \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[c + d x]}{b^3 g^2} + \frac{2 B^2 c d i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^2} - \frac{2 B c d i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} + \\
& \frac{2 B d (b c - a d) i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^3 g^2} - \\
& \frac{B^2 c d i^2 n^2 \operatorname{Log}[c + d x]^2}{b^2 g^2} + \frac{B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[c + d x]^2}{b^3 g^2} + \\
& \frac{4 A B d (b c - a d) i^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \frac{2 a B^2 d^2 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \frac{4 A B d (b c - a d) i^2 n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{2 a B^2 d^2 i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} - \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{2 B^2 c d i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^2 g^2} - \frac{2 B^2 d (b c - a d) i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g^2} + \\
& \frac{4 B^2 d (b c - a d) i^2 n \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^2} + \\
& \frac{4 B^2 d (b c - a d) i^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^2}
\end{aligned}$$

### Problem 174: Result valid but suboptimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(a g + b g x)^3} dx$$

Optimal (type 4, 417 leaves, 10 steps):

$$\begin{aligned} & -\frac{2 B^2 d i^2 n^2 (c + d x)}{b^2 g^3 (a + b x)} - \frac{B^2 i^2 n^2 (c + d x)^2}{4 b g^3 (a + b x)^2} - \frac{2 B d i^2 n (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{b^2 g^3 (a + b x)} - \\ & \frac{B i^2 n (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 b g^3 (a + b x)^2} - \frac{d i^2 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{b^2 g^3 (a + b x)} - \\ & \frac{i^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{2 b g^3 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 \operatorname{Log}\left[ 1 - \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g^3} + \\ & \frac{2 B d^2 i^2 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g^3} + \frac{2 B^2 d^2 i^2 n^2 \operatorname{PolyLog}\left[ 3, \frac{b (c+d x)}{d (a+b x)} \right]}{b^3 g^3} \end{aligned}$$

Result (type 4, 1003 leaves, 68 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^2 i^2 n^2}{4 b^3 g^3 (a + b x)^2} - \frac{5 B^2 d (b c - a d) i^2 n^2}{2 b^3 g^3 (a + b x)} - \frac{5 B^2 d^2 i^2 n^2 \operatorname{Log}[a + b x]}{2 b^3 g^3} - \\
& \frac{A B d^2 i^2 n \operatorname{Log}[a + b x]^2}{b^3 g^3} + \frac{3 B^2 d^2 i^2 n^2 \operatorname{Log}[a + b x]^2}{2 b^3 g^3} - \frac{B^2 d^2 i^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^3 g^3} - \\
& \frac{B^2 d^2 i^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^3 g^3} - \frac{B (b c - a d)^2 i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 b^3 g^3 (a + b x)^2} - \\
& \frac{3 B d (b c - a d) i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^3 (a + b x)} - \frac{3 B d^2 i^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^3} - \\
& \frac{(b c - a d)^2 i^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b^3 g^3 (a + b x)^2} - \frac{2 d (b c - a d) i^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^3 g^3 (a + b x)} + \\
& \frac{d^2 i^2 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^3 g^3} + \frac{5 B^2 d^2 i^2 n^2 \operatorname{Log}[c + d x]}{2 b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^3} + \frac{3 B d^2 i^2 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^3 g^3} + \\
& \frac{3 B^2 d^2 i^2 n^2 \operatorname{Log}[c + d x]^2}{2 b^3 g^3} + \frac{2 A B d^2 i^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} + \frac{2 A B d^2 i^2 n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^3} - \\
& \frac{3 B^2 d^2 i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 g^3} - \frac{3 B^2 d^2 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{b^3 g^3} + \\
& \frac{2 B^2 d^2 i^2 n \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^3} + \frac{2 B^2 d^2 i^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^3 g^3}
\end{aligned}$$

**Problem 175:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^2 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^4} dx$$

Optimal (type 3, 157 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 B^2 i^2 n^2 (c + d x)^3}{27 (b c - a d) g^4 (a + b x)^3} - \\
& \frac{2 B i^2 n (c + d x)^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{9 (b c - a d) g^4 (a + b x)^3} - \frac{i^2 (c + d x)^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 (b c - a d) g^4 (a + b x)^3}
\end{aligned}$$

Result (type 4, 889 leaves, 86 steps):

$$\begin{aligned}
& -\frac{2 B^2 (b c - a d)^2 i^2 n^2}{27 b^3 g^4 (a + b x)^3} - \frac{2 B^2 d (b c - a d) i^2 n^2}{9 b^3 g^4 (a + b x)^2} - \frac{2 B^2 d^2 i^2 n^2}{9 b^3 g^4 (a + b x)} - \frac{2 B^2 d^3 i^2 n^2 \operatorname{Log}[a + b x]}{9 b^3 (b c - a d) g^4} + \\
& \frac{B^2 d^3 i^2 n^2 \operatorname{Log}[a + b x]^2}{3 b^3 (b c - a d) g^4} - \frac{2 B (b c - a d)^2 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 b^3 g^4 (a + b x)^3} - \\
& \frac{2 B d (b c - a d) i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^3 g^4 (a + b x)^2} - \frac{2 B d^2 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^3 g^4 (a + b x)} - \\
& \frac{2 B d^3 i^2 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^3 (b c - a d) g^4} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b^3 g^4 (a + b x)^3} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^3 g^4 (a + b x)^2} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^3 g^4 (a + b x)} + \frac{2 B^2 d^3 i^2 n^2 \operatorname{Log}[c + d x]}{9 b^3 (b c - a d) g^4} - \\
& \frac{2 B^2 d^3 i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4} + \frac{2 B d^3 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{3 b^3 (b c - a d) g^4} + \\
& \frac{B^2 d^3 i^2 n^2 \operatorname{Log}[c + d x]^2}{3 b^3 (b c - a d) g^4} - \frac{2 B^2 d^3 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4} - \\
& \frac{2 B^2 d^3 i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4} - \frac{2 B^2 d^3 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 b^3 (b c - a d) g^4}
\end{aligned}$$

**Problem 176: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(c i + d i x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^5} dx$$

Optimal (type 3, 319 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 B^2 d i^2 n^2 (c + d x)^3}{27 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B^2 i^2 n^2 (c + d x)^4}{32 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{2 B d i^2 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b B i^2 n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{8 (b c - a d)^2 g^5 (a + b x)^4} + \\
& \frac{d i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 (b c - a d)^2 g^5 (a + b x)^3} - \frac{b i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 (b c - a d)^2 g^5 (a + b x)^4}
\end{aligned}$$

Result (type 4, 989 leaves, 98 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^2 i^2 n^2}{32 b^3 g^5 (a + b x)^4} - \frac{11 B^2 d (b c - a d) i^2 n^2}{216 b^3 g^5 (a + b x)^3} + \frac{5 B^2 d^2 i^2 n^2}{144 b^3 g^5 (a + b x)^2} + \\
& \frac{7 B^2 d^3 i^2 n^2}{72 b^3 (b c - a d) g^5 (a + b x)} + \frac{7 B^2 d^4 i^2 n^2 \operatorname{Log}[a + b x]}{72 b^3 (b c - a d)^2 g^5} - \frac{B^2 d^4 i^2 n^2 \operatorname{Log}[a + b x]^2}{12 b^3 (b c - a d)^2 g^5} - \\
& \frac{B (b c - a d)^2 i^2 n (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{8 b^3 g^5 (a + b x)^4} - \frac{5 B d (b c - a d) i^2 n (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{18 b^3 g^5 (a + b x)^3} - \\
& \frac{B d^2 i^2 n (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{12 b^3 g^5 (a + b x)^2} + \frac{B d^3 i^2 n (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{6 b^3 (b c - a d) g^5 (a + b x)} + \\
& \frac{B d^4 i^2 n \operatorname{Log}[a + b x] (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])}{6 b^3 (b c - a d)^2 g^5} - \frac{(b c - a d)^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])^2}{4 b^3 g^5 (a + b x)^4} - \\
& \frac{2 d (b c - a d) i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])^2}{3 b^3 g^5 (a + b x)^3} - \frac{d^2 i^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])^2}{2 b^3 g^5 (a + b x)^2} - \frac{7 B^2 d^4 i^2 n^2 \operatorname{Log}[c + d x]}{72 b^3 (b c - a d)^2 g^5} + \\
& \frac{B^2 d^4 i^2 n^2 \operatorname{Log}[-\frac{d (a+b x)}{b c - a d}] \operatorname{Log}[c + d x]}{6 b^3 (b c - a d)^2 g^5} - \frac{B d^4 i^2 n (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n]) \operatorname{Log}[c + d x]}{6 b^3 (b c - a d)^2 g^5} - \\
& \frac{B^2 d^4 i^2 n^2 \operatorname{Log}[c + d x]^2}{12 b^3 (b c - a d)^2 g^5} + \frac{B^2 d^4 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[\frac{b (c+d x)}{b c - a d}]}{6 b^3 (b c - a d)^2 g^5} + \\
& \frac{B^2 d^4 i^2 n^2 \operatorname{PolyLog}[2, -\frac{d (a+b x)}{b c - a d}]}{6 b^3 (b c - a d)^2 g^5} + \frac{B^2 d^4 i^2 n^2 \operatorname{PolyLog}[2, \frac{b (c+d x)}{b c - a d}]}{6 b^3 (b c - a d)^2 g^5}
\end{aligned}$$

**Problem 177:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(c i + d i x)^2 (A + B \operatorname{Log}[e (\frac{a+b x}{c+d x})^n])^2}{(a g + b g x)^6} dx$$

Optimal (type 3, 493 leaves, 9 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 i^2 n^2 (c + d x)^3}{27 (b c - a d)^3 g^6 (a + b x)^3} + \frac{b B^2 d i^2 n^2 (c + d x)^4}{16 (b c - a d)^3 g^6 (a + b x)^4} - \frac{2 b^2 B^2 i^2 n^2 (c + d x)^5}{125 (b c - a d)^3 g^6 (a + b x)^5} - \\
& \frac{2 B d^2 i^2 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 (b c - a d)^3 g^6 (a + b x)^3} + \frac{b B d i^2 n (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 (b c - a d)^3 g^6 (a + b x)^4} - \\
& \frac{2 b^2 B i^2 n (c + d x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{25 (b c - a d)^3 g^6 (a + b x)^5} - \frac{d^2 i^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 (b c - a d)^3 g^6 (a + b x)^3} + \\
& \frac{b d i^2 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 (b c - a d)^3 g^6 (a + b x)^4} - \frac{b^2 i^2 (c + d x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 (b c - a d)^3 g^6 (a + b x)^5}
\end{aligned}$$

Result (type 4, 1085 leaves, 110 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^2 i^2 n^2}{125 b^3 g^6 (a + b x)^5} - \frac{7 B^2 d (b c - a d) i^2 n^2}{400 b^3 g^6 (a + b x)^4} + \frac{43 B^2 d^2 i^2 n^2}{2700 b^3 g^6 (a + b x)^3} - \\
& \frac{13 B^2 d^3 i^2 n^2}{1800 b^3 (b c - a d) g^6 (a + b x)^2} - \frac{47 B^2 d^4 i^2 n^2}{900 b^3 (b c - a d)^2 g^6 (a + b x)} - \frac{47 B^2 d^5 i^2 n^2 \operatorname{Log}[a + b x]}{900 b^3 (b c - a d)^3 g^6} + \\
& \frac{B^2 d^5 i^2 n^2 \operatorname{Log}[a + b x]^2}{30 b^3 (b c - a d)^3 g^6} - \frac{2 B (b c - a d)^2 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{25 b^3 g^6 (a + b x)^5} - \\
& \frac{3 B d (b c - a d) i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{20 b^3 g^6 (a + b x)^4} - \frac{B d^2 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{45 b^3 g^6 (a + b x)^3} + \\
& \frac{B d^3 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^3 (b c - a d) g^6 (a + b x)^2} - \frac{B d^4 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 b^3 (b c - a d)^2 g^6 (a + b x)} - \\
& \frac{B d^5 i^2 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 b^3 (b c - a d)^3 g^6} - \frac{(b c - a d)^2 i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b^3 g^6 (a + b x)^5} - \\
& \frac{d (b c - a d) i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^3 g^6 (a + b x)^4} - \frac{d^2 i^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b^3 g^6 (a + b x)^3} + \frac{47 B^2 d^5 i^2 n^2 \operatorname{Log}[c + d x]}{900 b^3 (b c - a d)^3 g^6} - \\
& \frac{B^2 d^5 i^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{15 b^3 (b c - a d)^3 g^6} + \frac{B d^5 i^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{15 b^3 (b c - a d)^3 g^6} + \\
& \frac{B^2 d^5 i^2 n^2 \operatorname{Log}[c + d x]^2}{30 b^3 (b c - a d)^3 g^6} - \frac{B^2 d^5 i^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6} - \\
& \frac{B^2 d^5 i^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6} - \frac{B^2 d^5 i^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{15 b^3 (b c - a d)^3 g^6}
\end{aligned}$$

### Problem 178: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^3 (c i + d i x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 1172 leaves, 22 steps):

$$\begin{aligned} & \frac{5 B^2 (b c - a d)^6 g^3 i^3 n^2 x}{84 b^3 d^3} + \frac{B^2 (b c - a d)^3 g^3 i^3 n^2 (a + b x)^4}{140 b^4} - \frac{29 B^2 (b c - a d)^5 g^3 i^3 n^2 (c + d x)^2}{840 b^2 d^4} + \\ & \frac{47 B^2 (b c - a d)^4 g^3 i^3 n^2 (c + d x)^3}{1260 b d^4} - \frac{13 B^2 (b c - a d)^3 g^3 i^3 n^2 (c + d x)^4}{420 d^4} + \\ & \frac{b B^2 (b c - a d)^2 g^3 i^3 n^2 (c + d x)^5}{105 d^4} - \frac{B (b c - a d)^4 g^3 i^3 n (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{210 b^4 d} - \\ & \frac{3 B (b c - a d)^3 g^3 i^3 n (a + b x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{140 b^4} - \\ & \frac{B (b c - a d)^2 g^3 i^3 n (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{35 b^3} + \\ & \frac{2 B (b c - a d)^4 g^3 i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{21 b d^4} - \\ & \frac{3 B (b c - a d)^3 g^3 i^3 n (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{14 d^4} + \\ & \frac{6 b B (b c - a d)^2 g^3 i^3 n (c + d x)^5 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{35 d^4} - \\ & \frac{b^2 B (b c - a d) g^3 i^3 n (c + d x)^6 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{21 d^4} + \\ & \frac{(b c - a d)^3 g^3 i^3 (a + b x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{140 b^4} + \\ & \frac{(b c - a d)^2 g^3 i^3 (a + b x)^4 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{35 b^3} + \\ & \frac{(b c - a d) g^3 i^3 (a + b x)^4 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{14 b^2} + \\ & \frac{g^3 i^3 (a + b x)^4 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{7 b} + \\ & \frac{B (b c - a d)^5 g^3 i^3 n (a + b x)^2 \left( 3 A + B n + 3 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{420 b^4 d^2} - \\ & \frac{B (b c - a d)^6 g^3 i^3 n (a + b x) \left( 6 A + 5 B n + 6 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{420 b^4 d^3} - \end{aligned}$$

$$\begin{aligned}
& \frac{B (b c - a d)^7 g^3 i^3 n \left( 6 A + 11 B n + 6 B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right]}{420 b^4 d^4} - \\
& \frac{B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{Log}\left[\frac{a+b x}{c+d x}\right]}{210 b^4 d^4} - \frac{11 B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{Log}[c+d x]}{420 b^4 d^4} - \\
& \frac{B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{70 b^4 d^4}
\end{aligned}$$

Result(type 4, 961 leaves, 118 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^6 g^3 i^3 n x}{70 b^3 d^3} + \frac{B^2 (b c - a d)^6 g^3 i^3 n^2 x}{70 b^3 d^3} - \frac{3 B^2 (b c - a d)^5 g^3 i^3 n^2 (a+b x)^2}{280 b^4 d^2} + \\
& \frac{11 B^2 (b c - a d)^4 g^3 i^3 n^2 (a+b x)^3}{1260 b^4 d} + \frac{B^2 (b c - a d)^3 g^3 i^3 n^2 (a+b x)^4}{42 b^4} + \\
& \frac{B^2 d (b c - a d)^2 g^3 i^3 n^2 (a+b x)^5}{105 b^4} - \frac{B^2 (b c - a d)^6 g^3 i^3 n (a+b x) \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right]}{70 b^4 d^3} + \\
& \frac{B (b c - a d)^5 g^3 i^3 n (a+b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)}{140 b^4 d^2} - \\
& \frac{B (b c - a d)^4 g^3 i^3 n (a+b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)}{210 b^4 d} - \\
& \frac{17 B (b c - a d)^3 g^3 i^3 n (a+b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)}{140 b^4} - \\
& \frac{B d (b c - a d)^2 g^3 i^3 n (a+b x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)}{7 b^4} - \\
& \frac{B d^2 (b c - a d) g^3 i^3 n (a+b x)^6 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)}{21 b^4} + \\
& \frac{(b c - a d)^3 g^3 i^3 (a+b x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)^2}{4 b^4} + \\
& \frac{3 d (b c - a d)^2 g^3 i^3 (a+b x)^5 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)^2}{5 b^4} + \\
& \frac{d^2 (b c - a d) g^3 i^3 (a+b x)^6 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)^2}{2 b^4} + \\
& \frac{d^3 g^3 i^3 (a+b x)^7 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)^2}{7 b^4} - \frac{B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{Log}\left[-\frac{d (a+b x)}{b c - a d}\right] \operatorname{Log}[c+d x]}{70 b^4 d^4} + \\
& \frac{B (b c - a d)^7 g^3 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right) \operatorname{Log}[c+d x]}{70 b^4 d^4} + \\
& \frac{B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{Log}[c+d x]^2}{140 b^4 d^4} - \frac{B^2 (b c - a d)^7 g^3 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{70 b^4 d^4}
\end{aligned}$$



### Problem 179: Result valid but suboptimal antiderivative.

$$\int (a g + b g x)^2 (c i + d i x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 976 leaves, 20 steps):

$$\begin{aligned} & - \frac{7 B^2 (b c - a d)^5 g^2 i^3 n^2 x}{180 b^3 d^2} - \frac{7 B^2 (b c - a d)^4 g^2 i^3 n^2 (c + d x)^2}{360 b^2 d^3} - \frac{B^2 (b c - a d)^3 g^2 i^3 n^2 (c + d x)^3}{60 b d^3} + \\ & \frac{B^2 (b c - a d)^2 g^2 i^3 n^2 (c + d x)^4}{60 d^3} - \frac{B (b c - a d)^4 g^2 i^3 n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 b^4 d} - \\ & \frac{B (b c - a d)^3 g^2 i^3 n (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b^4} - \\ & \frac{B (b c - a d)^4 g^2 i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b^2 d^3} + \\ & \frac{B (b c - a d)^3 g^2 i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{45 b d^3} + \\ & \frac{7 B (b c - a d)^2 g^2 i^3 n (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 d^3} - \\ & \frac{b B (b c - a d) g^2 i^3 n (c + d x)^5 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{15 d^3} + \\ & \frac{(b c - a d)^3 g^2 i^3 (a + b x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{60 b^4} + \\ & \frac{(b c - a d)^2 g^2 i^3 (a + b x)^3 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{20 b^3} + \\ & \frac{(b c - a d) g^2 i^3 (a + b x)^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{10 b^2} + \\ & \frac{g^2 i^3 (a + b x)^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{6 b} + \\ & \frac{B (b c - a d)^5 g^2 i^3 n (a + b x) \left( 2 A + B n + 2 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{60 b^4 d^2} + \\ & \frac{B (b c - a d)^6 g^2 i^3 n \left( 2 A + 3 B n + 2 B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{60 b^4 d^3} + \\ & \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]}{36 b^4 d^3} + \frac{11 B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}[c + d x]}{180 b^4 d^3} + \\ & \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{30 b^4 d^3} \end{aligned}$$

Result (type 4, 886 leaves, 83 steps):

$$\begin{aligned}
 & - \frac{A B (b c - a d)^5 g^2 i^3 n x}{30 b^3 d^2} - \frac{B^2 (b c - a d)^5 g^2 i^3 n^2 x}{45 b^3 d^2} - \\
 & \frac{7 B^2 (b c - a d)^4 g^2 i^3 n^2 (c + d x)^2}{360 b^2 d^3} - \frac{B^2 (b c - a d)^3 g^2 i^3 n^2 (c + d x)^3}{60 b d^3} + \\
 & \frac{B^2 (b c - a d)^2 g^2 i^3 n^2 (c + d x)^4}{60 d^3} - \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}[a + b x]}{45 b^4 d^3} + \\
 & \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}[a + b x]^2}{60 b^4 d^3} - \frac{B^2 (b c - a d)^5 g^2 i^3 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{30 b^4 d^2} - \\
 & \frac{B (b c - a d)^4 g^2 i^3 n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{60 b^2 d^3} - \\
 & \frac{B (b c - a d)^3 g^2 i^3 n (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{90 b d^3} + \\
 & \frac{7 B (b c - a d)^2 g^2 i^3 n (c + d x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{60 d^3} - \\
 & \frac{b B (b c - a d) g^2 i^3 n (c + d x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{15 d^3} - \\
 & \frac{B (b c - a d)^6 g^2 i^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{30 b^4 d^3} + \\
 & \frac{(b c - a d)^2 g^2 i^3 (c + d x)^4 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{4 d^3} - \\
 & \frac{2 b (b c - a d) g^2 i^3 (c + d x)^5 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{5 d^3} + \\
 & \frac{b^2 g^2 i^3 (c + d x)^6 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{6 d^3} + \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}[c + d x]}{30 b^4 d^3} - \\
 & \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{30 b^4 d^3} - \frac{B^2 (b c - a d)^6 g^2 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{30 b^4 d^3}
 \end{aligned}$$

**Problem 180: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x) (c i + d i x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 786 leaves, 19 steps):

$$\begin{aligned}
& \frac{B^2 (b c - a d)^4 g i^3 n^2 x}{60 b^3 d} + \frac{B^2 (b c - a d)^3 g i^3 n^2 (c + d x)^2}{30 b^2 d^2} + \\
& \frac{B^2 (b c - a d)^2 g i^3 n^2 (c + d x)^3}{30 b d^2} - \frac{B (b c - a d)^4 g i^3 n (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b^4 d} - \\
& \frac{B (b c - a d)^3 g i^3 n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 b^4} + \\
& \frac{3 B (b c - a d)^3 g i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{20 b^2 d^2} + \\
& \frac{B (b c - a d)^2 g i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{30 b d^2} - \\
& \frac{B (b c - a d) g i^3 n (c + d x)^4 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{10 d^2} + \\
& \frac{(b c - a d)^3 g i^3 (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{20 b^4} + \\
& \frac{(b c - a d)^2 g i^3 (a + b x)^2 (c + d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{10 b^3} + \\
& \frac{3 (b c - a d) g i^3 (a + b x)^2 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{20 b^2} + \\
& \frac{g i^3 (a + b x)^2 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{5 b} - \\
& \frac{B (b c - a d)^5 g i^3 n \left( A + B n + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{10 b^4 d^2} - \frac{B^2 (b c - a d)^5 g i^3 n^2 \operatorname{Log}\left[ \frac{a + b x}{c + d x} \right]}{12 b^4 d^2} - \\
& \frac{11 B^2 (b c - a d)^5 g i^3 n^2 \operatorname{Log}[c + d x]}{60 b^4 d^2} - \frac{B^2 (b c - a d)^5 g i^3 n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{10 b^4 d^2}
\end{aligned}$$

Result (type 4, 706 leaves, 52 steps):

$$\begin{aligned}
& \frac{AB(b c - a d)^4 g i^3 n x}{10 b^3 d} + \frac{B^2(b c - a d)^4 g i^3 n^2 x}{60 b^3 d} + \frac{B^2(b c - a d)^3 g i^3 n^2 (c + d x)^2}{30 b^2 d^2} + \\
& \frac{B^2(b c - a d)^2 g i^3 n^2 (c + d x)^3}{30 b d^2} + \frac{B^2(b c - a d)^5 g i^3 n^2 \text{Log}[a + b x]}{60 b^4 d^2} - \\
& \frac{B^2(b c - a d)^5 g i^3 n^2 \text{Log}[a + b x]^2}{20 b^4 d^2} + \frac{B^2(b c - a d)^4 g i^3 n (a + b x) \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{10 b^4 d} + \\
& \frac{B(b c - a d)^3 g i^3 n (c + d x)^2 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{20 b^2 d^2} + \\
& \frac{B(b c - a d)^2 g i^3 n (c + d x)^3 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{30 b d^2} - \\
& \frac{B(b c - a d) g i^3 n (c + d x)^4 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{10 d^2} + \\
& \frac{B(b c - a d)^5 g i^3 n \text{Log}[a + b x] \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{10 b^4 d^2} - \\
& \frac{(b c - a d) g i^3 (c + d x)^4 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{4 d^2} + \\
& \frac{b g i^3 (c + d x)^5 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{5 d^2} - \frac{B^2(b c - a d)^5 g i^3 n^2 \text{Log}[c + d x]}{10 b^4 d^2} + \\
& \frac{B^2(b c - a d)^5 g i^3 n^2 \text{Log}[a + b x] \text{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{10 b^4 d^2} + \frac{B^2(b c - a d)^5 g i^3 n^2 \text{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{10 b^4 d^2}
\end{aligned}$$

**Problem 181: Result valid but suboptimal antiderivative.**

$$\int (c i + d i x)^3 \left(A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 454 leaves, 15 steps):

$$\begin{aligned}
& \frac{5 B^2 (b c - a d)^3 i^3 n^2 x}{12 b^3} + \frac{B^2 (b c - a d)^2 i^3 n^2 (c + d x)^2}{12 b^2 d} - \\
& \frac{B (b c - a d)^3 i^3 n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^4} - \\
& \frac{B (b c - a d)^2 i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b^2 d} - \\
& \frac{B (b c - a d) i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b d} + \frac{i^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 d} + \\
& \frac{5 B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{12 b^4 d} + \frac{11 B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}[c + d x]}{12 b^4 d} + \\
& \frac{B (b c - a d)^4 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{2 b^4 d} - \frac{B^2 (b c - a d)^4 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{2 b^4 d}
\end{aligned}$$

Result (type 4, 544 leaves, 23 steps):

$$\begin{aligned}
& - \frac{A B (b c - a d)^3 i^3 n x}{2 b^3} + \frac{5 B^2 (b c - a d)^3 i^3 n^2 x}{12 b^3} + \frac{B^2 (b c - a d)^2 i^3 n^2 (c + d x)^2}{12 b^2 d} + \\
& \frac{5 B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}[a + b x]}{12 b^4 d} + \frac{B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}[a + b x]^2}{4 b^4 d} - \\
& \frac{B^2 (b c - a d)^3 i^3 n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{2 b^4} - \frac{B (b c - a d)^2 i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{4 b^2 d} - \\
& \frac{B (b c - a d) i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{6 b d} - \\
& \frac{B (b c - a d)^4 i^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^4 d} + \\
& \frac{i^3 (c + d x)^4 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{4 d} + \frac{B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}[c + d x]}{2 b^4 d} - \\
& \frac{B^2 (b c - a d)^4 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{2 b^4 d} - \frac{B^2 (b c - a d)^4 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{2 b^4 d}
\end{aligned}$$

**Problem 182: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{a g + b g x} dx$$

Optimal (type 4, 762 leaves, 26 steps):

$$\begin{aligned}
& \frac{B^2 d (b c - a d)^2 i^3 n^2 x}{3 b^3 g} - \frac{5 B d (b c - a d)^2 i^3 n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^4 g} - \\
& \frac{B (b c - a d) i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 g} + \\
& \frac{d (b c - a d)^2 i^3 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^4 g} + \frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^2 g} + \\
& \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b g} + \frac{2 B (b c - a d)^3 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{3 b^4 g} + \frac{2 B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[c + d x]}{b^4 g} + \\
& \frac{5 B (b c - a d)^3 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{3 b^4 g} - \\
& \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{b^4 g} - \frac{5 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{3 b^4 g} + \\
& \frac{2 B (b c - a d)^3 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g}
\end{aligned}$$

Result (type 4, 1995 leaves, 101 steps):

$$\begin{aligned}
& - \frac{5 A B d (b c - a d)^2 i^3 n x}{3 b^3 g} + \frac{B^2 d (b c - a d)^2 i^3 n^2 x}{3 b^3 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[a + b x]}{3 b^4 g} - \frac{a B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[a + b x]^2}{b^4 g} + \\
& \frac{5 B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[a + b x]^2}{6 b^4 g} - \frac{A B (b c - a d)^3 i^3 n \operatorname{Log}[g (a + b x)]^2}{b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[g (a + b x)]^3}{3 b^4 g} - \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[g (a + b x)]^2 \operatorname{Log}[-c - d x]}{b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 n \operatorname{Log}[g (a + b x)] \operatorname{Log}[(a + b x)^n] \operatorname{Log}[-c - d x]}{b^4 g} - \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log}[-c - d x]}{b^4 g} - \frac{5 B^2 d (b c - a d)^2 i^3 n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{3 b^4 g} - \\
& \frac{B (b c - a d) i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 b^2 g} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)}{b^4 g} - \\
& \frac{5 B (b c - a d)^3 i^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)}{3 b^4 g} + \frac{d (b c - a d)^2 i^3 x \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)^2}{b^3 g} + \\
& \frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)^2}{2 b^2 g} + \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)^2}{3 b g} + \\
& \frac{5 B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[c + d x]}{3 b^4 g} + \frac{2 B^2 c (b c - a d)^2 i^3 n^2 \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{b^3 g} - \\
& \frac{2 B c (b c - a d)^2 i^3 n \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right) \operatorname{Log}[c + d x]}{b^3 g} - \\
& \frac{B^2 c (b c - a d)^2 i^3 n^2 \operatorname{Log}[c + d x]^2}{b^3 g} + \frac{2 a B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{b^4 g} - \\
& \frac{5 B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{3 b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}[g (a + b x)]^2 \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{b^4 g} + \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right]}{b^4 g} + \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}\left[ -\frac{d (a + b x)}{b c - a d} \right] \operatorname{Log}[(c + d x)^{-n}]^2}{b^4 g} - \\
& \frac{B^2 (b c - a d)^3 i^3 \operatorname{Log}[g (a + b x)] \operatorname{Log}[(c + d x)^{-n}]^2}{b^4 g} + \\
& \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \right)^2 \operatorname{Log}[a g + b g x]}{b^4 g} + \\
& \frac{2 A B (b c - a d)^3 i^3 n \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right] \operatorname{Log}[a g + b g x]}{b^4 g} - \frac{1}{b^4 g} \\
& 2 B^2 (b c - a d)^3 i^3 n \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right] \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] + \operatorname{Log}[(c + d x)^{-n}] \right) \\
& \operatorname{Log}[a g + b g x] - \frac{B^2 (b c - a d)^3 i^3 n \operatorname{Log}\left[ e^{\left( \frac{a + b x}{c + d x} \right)^n} \right] \operatorname{Log}[a g + b g x]^2}{b^4 g} - \\
& \frac{B^2 (b c - a d)^3 i^3 n^2 \operatorname{Log}\left[ \frac{b (c + d x)}{b c - a d} \right] \operatorname{Log}[a g + b g x]^2}{b^4 g} + \frac{2 A B (b c - a d)^3 i^3 n \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^4 g} + \\
& \frac{2 a B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^4 g} - \frac{5 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{3 b^4 g} + \\
& \frac{2 B^2 (b c - a d)^3 i^3 n \operatorname{Log}[(a + b x)^n] \operatorname{PolyLog}\left[ 2, -\frac{d (a + b x)}{b c - a d} \right]}{b^4 g} - \frac{1}{b^4 g}
\end{aligned}$$

$$\begin{aligned}
& 2 B^2 (b c - a d)^3 i^3 n \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}] \right) \\
& \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right] + \frac{2 B^2 c (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b^3 g} - \\
& \frac{2 B^2 (b c - a d)^3 i^3 n \operatorname{Log}[(c + d x)^{-n}] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b^4 g} - \\
& \frac{2 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{b^4 g} - \frac{2 B^2 (b c - a d)^3 i^3 n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{b^4 g}
\end{aligned}$$

**Problem 183: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^2} dx$$

Optimal (type 4, 739 leaves, 17 steps):

$$\begin{aligned}
& -\frac{2 B^2 (b c - a d)^2 i^3 n^2 (c + d x)}{b^3 g^2 (a + b x)} - \frac{B d^2 (b c - a d) i^3 n (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^4 g^2} - \\
& \frac{2 B (b c - a d)^2 i^3 n (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{b^3 g^2 (a + b x)} + \\
& \frac{2 d^2 (b c - a d) i^3 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^4 g^2} - \\
& \frac{(b c - a d)^2 i^3 (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^3 g^2 (a + b x)} + \frac{d i^3 (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b^2 g^2} + \\
& \frac{4 B d (b c - a d)^2 i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{b^4 g^2} + \frac{B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[c + d x]}{b^4 g^2} + \\
& \frac{B d (b c - a d)^2 i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2} - \\
& \frac{3 d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}\left[1 - \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2} + \\
& \frac{4 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{d(a + b x)}{b(c + d x)}\right]}{b^4 g^2} - \frac{B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2} + \\
& \frac{6 B d (b c - a d)^2 i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2} + \\
& \frac{6 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{d(a + b x)}\right]}{b^4 g^2}
\end{aligned}$$



Result (type 4, 1875 leaves, 83 steps):

$$\begin{aligned}
& - \frac{A B d^2 (b c - a d) i^3 n x}{b^3 g^2} - \frac{2 B^2 (b c - a d)^3 i^3 n^2}{b^4 g^2 (a + b x)} - \frac{2 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[a + b x]}{b^4 g^2} - \\
& \frac{3 A B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x]^2}{b^4 g^2} + \frac{a^2 B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x]^2}{2 b^4 g^2} - \\
& \frac{a B^2 d^2 (3 b c - 2 a d) i^3 n^2 \operatorname{Log}[a + b x]^2}{b^4 g^2} + \frac{B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[a + b x]^2}{b^4 g^2} - \\
& \frac{B^2 d^2 (b c - a d) i^3 n (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b^4 g^2} - \frac{3 B^2 d (b c - a d)^2 i^3 \operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]^2}{b^4 g^2} - \\
& \frac{3 B^2 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]^2}{b^4 g^2} - \\
& \frac{2 B (b c - a d)^3 i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^4 g^2 (a + b x)} - \frac{a^2 B d^3 i^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^4 g^2} + \\
& \frac{2 a B d^2 (3 b c - 2 a d) i^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^4 g^2} - \\
& \frac{2 B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{b^4 g^2} + \\
& \frac{d^2 (3 b c - 2 a d) i^3 x \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{b^3 g^2} + \frac{d^3 i^3 x^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 b^2 g^2} - \\
& \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{b^4 g^2 (a + b x)} + \frac{3 d (b c - a d)^2 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{b^4 g^2} + \\
& \frac{3 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[c + d x]}{b^4 g^2} - \frac{B^2 c^2 d i^3 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c + d x]}{b^2 g^2} + \\
& \frac{2 B^2 c d (3 b c - 2 a d) i^3 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c + d x]}{b^3 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B c^2 d i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^2 g^2} - \\
& \frac{2 B c d (3 b c - 2 a d) i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^3 g^2} + \\
& \frac{2 B d (b c - a d)^2 i^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{b^4 g^2} + \frac{B^2 c^2 d i^3 n^2 \operatorname{Log}[c + d x]^2}{2 b^2 g^2} - \\
& \frac{B^2 c d (3 b c - 2 a d) i^3 n^2 \operatorname{Log}[c + d x]^2}{b^3 g^2} + \frac{B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[c + d x]^2}{b^4 g^2} + \\
& \frac{6 A B d (b c - a d)^2 i^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b^4 g^2} - \frac{a^2 B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{b^4 g^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 a B^2 d^2 (3 b c - 2 a d) i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{b^4 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{b^4 g^2} + \frac{6 A B d (b c - a d)^2 i^3 n \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{b^4 g^2} - \\
& \frac{a^2 B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{b^4 g^2} + \frac{2 a B^2 d^2 (3 b c - 2 a d) i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{b^4 g^2} - \\
& \frac{2 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{b^4 g^2} - \frac{B^2 c^2 d i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b^2 g^2} + \\
& \frac{2 B^2 c d (3 b c - 2 a d) i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b^3 g^2} - \frac{2 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{b^4 g^2} + \\
& \frac{6 B^2 d (b c - a d)^2 i^3 n \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a+b x)}\right]}{b^4 g^2} + \\
& \frac{6 B^2 d (b c - a d)^2 i^3 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a+b x)}\right]}{b^4 g^2}
\end{aligned}$$

**Problem 184: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(a g + b g x)^3} dx$$

Optimal (type 4, 644 leaves, 13 steps):

$$\begin{aligned}
& - \frac{4 B^2 d (b c - a d) i^3 n^2 (c + d x)}{b^3 g^3 (a + b x)} - \frac{B^2 (b c - a d) i^3 n^2 (c + d x)^2}{4 b^2 g^3 (a + b x)^2} - \\
& \frac{4 B d (b c - a d) i^3 n (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^3 (a + b x)} - \\
& \frac{B (b c - a d) i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^3 (a + b x)^2} + \frac{d^3 i^3 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^4 g^3} - \\
& \frac{2 d (b c - a d) i^3 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^3 g^3 (a + b x)} - \frac{(b c - a d) i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^2 g^3 (a + b x)^2} + \\
& \frac{2 B d^2 (b c - a d) i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{b^4 g^3} - \\
& \frac{3 d^2 (b c - a d) i^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}\left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^3} + \\
& \frac{2 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{b^4 g^3} + \\
& \frac{6 B d^2 (b c - a d) i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^3} + \\
& \frac{6 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{PolyLog}\left[ 3, \frac{b (c + d x)}{d (a + b x)} \right]}{b^4 g^3}
\end{aligned}$$

Result (type 4, 1512 leaves, 88 steps):

$$\begin{aligned}
& - \frac{B^2 (b c - a d)^3 i^3 n^2}{4 b^4 g^3 (a + b x)^2} - \frac{9 B^2 d (b c - a d)^2 i^3 n^2}{2 b^4 g^3 (a + b x)} - \frac{9 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}[a + b x]}{2 b^4 g^3} - \\
& \frac{3 A B d^2 (b c - a d) i^3 n \operatorname{Log}[a + b x]^2}{b^4 g^3} - \frac{a B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x]^2}{b^4 g^3} + \\
& \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}[a + b x]^2}{2 b^4 g^3} - \frac{3 B^2 d^2 (b c - a d) i^3 \operatorname{Log}\left[ -\frac{b c - a d}{d (a + b x)} \right] \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]^2}{b^4 g^3} - \\
& \frac{3 B^2 d^2 (b c - a d) i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right]^2}{b^4 g^3} - \frac{B (b c - a d)^3 i^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^4 g^3 (a + b x)^2} - \\
& \frac{5 B d (b c - a d)^2 i^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3 (a + b x)} + \frac{2 a B d^3 i^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3} - \\
& \frac{5 B d^2 (b c - a d) i^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^4 g^3} + \frac{d^3 i^3 x \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^3 g^3} - \\
& \frac{(b c - a d)^3 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^4 g^3 (a + b x)^2} - \frac{3 d (b c - a d)^2 i^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^4 g^3 (a + b x)} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 d^2 (b c - a d) i^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^4 g^3} + \frac{9 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}[c + d x]}{2 b^4 g^3} + \\
& \frac{2 B^2 c d^2 i^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^3 g^3} - \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{b^4 g^3} - \\
& \frac{2 B c d^2 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{b^3 g^3} + \\
& \frac{5 B d^2 (b c - a d) i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{b^4 g^3} - \\
& \frac{B^2 c d^2 i^3 n^2 \operatorname{Log}[c + d x]^2}{b^3 g^3} + \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}[c + d x]^2}{2 b^4 g^3} + \\
& \frac{6 A B d^2 (b c - a d) i^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b^4 g^3} + \frac{2 a B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b^4 g^3} - \\
& \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{b^4 g^3} + \frac{6 A B d^2 (b c - a d) i^3 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^4 g^3} + \\
& \frac{2 a B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^4 g^3} - \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{b^4 g^3} + \\
& \frac{2 B^2 c d^2 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b^3 g^3} - \frac{5 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{b^4 g^3} + \\
& \frac{6 B^2 d^2 (b c - a d) i^3 n \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{b^4 g^3} + \\
& \frac{6 B^2 d^2 (b c - a d) i^3 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{b^4 g^3}
\end{aligned}$$

**Problem 185: Result valid but suboptimal antiderivative.**

$$\int \frac{(c i + d i x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(a g + b g x)^4} dx$$

Optimal (type 4, 561 leaves, 13 steps):

$$\begin{aligned}
& - \frac{2 B^2 d^2 i^3 n^2 (c + d x)}{b^3 g^4 (a + b x)} - \frac{B^2 d i^3 n^2 (c + d x)^2}{4 b^2 g^4 (a + b x)^2} - \\
& \frac{2 B^2 i^3 n^2 (c + d x)^3}{27 b g^4 (a + b x)^3} - \frac{2 B d^2 i^3 n (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{b^3 g^4 (a + b x)} - \\
& \frac{B d i^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{2 b^2 g^4 (a + b x)^2} - \frac{2 B i^3 n (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{9 b g^4 (a + b x)^3} - \\
& \frac{d^2 i^3 (c + d x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{b^3 g^4 (a + b x)} - \frac{d i^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 b^2 g^4 (a + b x)^2} - \\
& \frac{i^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 b g^4 (a + b x)^3} - \frac{d^3 i^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^4} + \\
& \frac{2 B d^3 i^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^4} + \frac{2 B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{d (a + b x)}\right]}{b^4 g^4}
\end{aligned}$$

Result (type 4, 1170 leaves, 100 steps):

$$\begin{aligned}
& - \frac{2 B^2 (b c - a d)^3 i^3 n^2}{27 b^4 g^4 (a + b x)^3} - \frac{17 B^2 d (b c - a d)^2 i^3 n^2}{36 b^4 g^4 (a + b x)^2} - \\
& \frac{49 B^2 d^2 (b c - a d) i^3 n^2}{18 b^4 g^4 (a + b x)} - \frac{49 B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x]}{18 b^4 g^4} - \frac{A B d^3 i^3 n \operatorname{Log}[a + b x]^2}{b^4 g^4} + \\
& \frac{11 B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x]^2}{6 b^4 g^4} - \frac{B^2 d^3 i^3 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^4 g^4} - \\
& \frac{B^2 d^3 i^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2}{b^4 g^4} - \frac{2 B (b c - a d)^3 i^3 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{9 b^4 g^4 (a + b x)^3} - \\
& \frac{7 B d (b c - a d)^2 i^3 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{6 b^4 g^4 (a + b x)^2} - \frac{11 B d^2 (b c - a d) i^3 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b^4 g^4 (a + b x)} - \\
& \frac{11 B d^3 i^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 b^4 g^4} - \frac{(b c - a d)^3 i^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 b^4 g^4 (a + b x)^3} - \\
& \frac{3 d (b c - a d)^2 i^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 b^4 g^4 (a + b x)^2} - \frac{3 d^2 (b c - a d) i^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^4 g^4 (a + b x)} + \\
& \frac{d^3 i^3 \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{b^4 g^4} + \frac{49 B^2 d^3 i^3 n^2 \operatorname{Log}[c + d x]}{18 b^4 g^4} - \\
& \frac{11 B^2 d^3 i^3 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 b^4 g^4} + \frac{11 B d^3 i^3 n \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{3 b^4 g^4} + \\
& \frac{11 B^2 d^3 i^3 n^2 \operatorname{Log}[c + d x]^2}{6 b^4 g^4} + \frac{2 A B d^3 i^3 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{b^4 g^4} - \\
& \frac{11 B^2 d^3 i^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 b^4 g^4} + \frac{2 A B d^3 i^3 n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{b^4 g^4} - \\
& \frac{11 B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 b^4 g^4} - \frac{11 B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 b^4 g^4} + \\
& \frac{2 B^2 d^3 i^3 n \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^4 g^4} + \frac{2 B^2 d^3 i^3 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{b^4 g^4}
\end{aligned}$$

**Problem 186: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{c i + d i x} dx$$

Optimal (type 4, 768 leaves, 25 steps):

$$\begin{aligned}
& \frac{b B^2 (b c - a d)^2 g^3 n^2 x}{3 d^3 i} + \frac{7 B (b c - a d)^2 g^3 n (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^3 i} - \\
& \frac{b^2 B (b c - a d) g^3 n (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^4 i} + \\
& \frac{3 (b c - a d)^2 g^3 (a + b x) \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{d^3 i} - \\
& \frac{3 b^2 (b c - a d) g^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 d^4 i} + \frac{b^3 g^3 (c + d x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 d^4 i} + \\
& \frac{6 B (b c - a d)^3 g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^4 i} + \\
& \frac{(b c - a d)^3 g^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^4 i} + \frac{B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}\left[\frac{a + b x}{c + d x}\right]}{3 d^4 i} - \\
& \frac{2 B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}[c + d x]}{d^4 i} - \frac{7 B (b c - a d)^3 g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{3 d^4 i} + \\
& \frac{6 B^2 (b c - a d)^3 g^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i} + \\
& \frac{2 B (b c - a d)^3 g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{PolyLog}\left[2, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i} + \\
& \frac{7 B^2 (b c - a d)^3 g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{d (a + b x)}\right]}{3 d^4 i} - \frac{2 B^2 (b c - a d)^3 g^3 n^2 \operatorname{PolyLog}\left[3, \frac{d (a + b x)}{b (c + d x)}\right]}{d^4 i}
\end{aligned}$$

Result (type 4, 1952 leaves, 101 steps):

$$\begin{aligned}
& \frac{5 A b B (b c - a d)^2 g^3 n x}{3 d^3 i} + \frac{b B^2 (b c - a d)^2 g^3 n^2 x}{3 d^3 i} - \frac{a B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a + b x]^2}{d^3 i} + \\
& \frac{5 B^2 (b c - a d)^2 g^3 n (a + b x) \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{3 d^3 i} - \frac{B (b c - a d) g^3 n (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{3 d^2 i} + \\
& \frac{2 a B (b c - a d)^2 g^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{d^3 i} + \\
& \frac{b (b c - a d)^2 g^3 x \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{d^3 i} - \frac{(b c - a d) g^3 (a + b x)^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 d^2 i} + \\
& \frac{g^3 (a + b x)^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{3 d i} - \frac{2 B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}[c + d x]}{d^4 i} + \\
& \frac{2 b B^2 c (b c - a d)^2 g^3 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i} + \\
& \frac{5 B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 d^4 i} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b B c (b c - a d)^2 g^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}[c+d x]}{d^4 i} - \\
& \frac{5 B (b c - a d)^3 g^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}[c+d x]}{3 d^4 i} - \frac{b B^2 c (b c - a d)^2 g^3 n^2 \operatorname{Log}[c+d x]^2}{d^4 i} - \\
& \frac{5 B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}[c+d x]^2}{6 d^4 i} + \frac{2 a B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a+b x] \operatorname{Log} \left[ \frac{b(c+d x)}{b c - a d} \right]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}[(a+b x)^n]^2 \operatorname{Log} \left[ \frac{b(c+d x)}{b c - a d} \right]}{d^4 i} + \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}[(a+b x)^n]^2 \operatorname{Log}[i(c+d x)]}{d^4 i} - \\
& \frac{A B (b c - a d)^3 g^3 n \operatorname{Log}[i(c+d x)]^2}{d^4 i} + \frac{B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}[a+b x] \operatorname{Log}[i(c+d x)]^2}{d^4 i} - \\
& \frac{B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log} \left[ -\frac{d(a+b x)}{b c - a d} \right] \operatorname{Log}[i(c+d x)]^2}{d^4 i} - \frac{B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log}[i(c+d x)]^3}{3 d^4 i} + \\
& \frac{2 B^2 (b c - a d)^3 g^3 n \operatorname{Log}[a+b x] \operatorname{Log}[i(c+d x)] \operatorname{Log}[(c+d x)^{-n}]}{d^4 i} + \\
& \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log}[a+b x] \operatorname{Log}[(c+d x)^{-n}]^2}{d^4 i} - \frac{B^2 (b c - a d)^3 g^3 \operatorname{Log} \left[ -\frac{d(a+b x)}{b c - a d} \right] \operatorname{Log}[(c+d x)^{-n}]^2}{d^4 i} + \\
& \frac{2 A B (b c - a d)^3 g^3 n \operatorname{Log} \left[ -\frac{d(a+b x)}{b c - a d} \right] \operatorname{Log}[c i + d i x]}{d^4 i} - \\
& \frac{(b c - a d)^3 g^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 \operatorname{Log}[c i + d i x]}{d^4 i} - \frac{1}{d^4 i} \\
& 2 B^2 (b c - a d)^3 g^3 n \operatorname{Log} \left[ -\frac{d(a+b x)}{b c - a d} \right] \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] + \operatorname{Log}[(c+d x)^{-n}] \right) \\
& \operatorname{Log}[c i + d i x] + \frac{B^2 (b c - a d)^3 g^3 n^2 \operatorname{Log} \left[ -\frac{d(a+b x)}{b c - a d} \right] \operatorname{Log}[c i + d i x]^2}{d^4 i} - \\
& \frac{B^2 (b c - a d)^3 g^3 n \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \operatorname{Log}[c i + d i x]^2}{d^4 i} + \frac{2 a B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d(a+b x)}{b c - a d} \right]}{d^3 i} - \\
& \frac{2 B^2 (b c - a d)^3 g^3 n \operatorname{Log}[(a+b x)^n] \operatorname{PolyLog} \left[ 2, -\frac{d(a+b x)}{b c - a d} \right]}{d^4 i} + \\
& \frac{2 A B (b c - a d)^3 g^3 n \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c - a d} \right]}{d^4 i} + \frac{2 b B^2 c (b c - a d)^2 g^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c - a d} \right]}{d^4 i} + \\
& \frac{5 B^2 (b c - a d)^3 g^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c - a d} \right]}{3 d^4 i} + \\
& \frac{2 B^2 (b c - a d)^3 g^3 n \operatorname{Log}[(c+d x)^{-n}] \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c - a d} \right]}{d^4 i} - \frac{1}{d^4 i} 2 B^2 (b c - a d)^3 g^3 n \\
& \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] + \operatorname{Log}[(c+d x)^{-n}] \right) \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c - a d} \right] +
\end{aligned}$$



$$\frac{2 B^2 (b c - a d)^3 g^3 n^2 \text{PolyLog}\left[3, -\frac{d(a+bx)}{b c - a d}\right]}{d^4 i} + \frac{2 B^2 (b c - a d)^3 g^3 n^2 \text{PolyLog}\left[3, \frac{b(c+dx)}{b c - a d}\right]}{d^4 i}$$

**Problem 187: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{c i + d i x} dx$$

Optimal (type 4, 573 leaves, 15 steps):

$$\begin{aligned} & - \frac{B (b c - a d) g^2 n (a + b x) \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{d^2 i} - \frac{2 (b c - a d) g^2 (a + b x) \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{d^2 i} + \\ & \frac{b^2 g^2 (c + d x)^2 \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 d^3 i} - \frac{4 B (b c - a d)^2 g^2 n \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^3 i} - \\ & \frac{(b c - a d)^2 g^2 \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \text{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 n^2 \text{Log}[c + d x]}{d^3 i} + \\ & \frac{B (b c - a d)^2 g^2 n \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{Log}\left[1 - \frac{b (c + d x)}{d (a + b x)}\right]}{d^3 i} - \\ & \frac{4 B^2 (b c - a d)^2 g^2 n^2 \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{d^3 i} - \\ & \frac{2 B (b c - a d)^2 g^2 n \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{d^3 i} - \\ & \frac{B^2 (b c - a d)^2 g^2 n^2 \text{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{d^3 i} + \frac{2 B^2 (b c - a d)^2 g^2 n^2 \text{PolyLog}\left[3, \frac{d(a+bx)}{b(c+dx)}\right]}{d^3 i} \end{aligned}$$

Result (type 4, 1780 leaves, 82 steps):

$$\begin{aligned} & - \frac{A b B (b c - a d) g^2 n x}{d^2 i} + \frac{a B^2 (b c - a d) g^2 n^2 \text{Log}[a + b x]^2}{d^2 i} - \\ & \frac{B^2 (b c - a d) g^2 n (a + b x) \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{d^2 i} - \\ & \frac{2 a B (b c - a d) g^2 n \text{Log}[a + b x] \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{d^2 i} - \\ & \frac{b (b c - a d) g^2 x \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{d^2 i} + \frac{g^2 (a + b x)^2 \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 d i} + \\ & \frac{B^2 (b c - a d)^2 g^2 n^2 \text{Log}[c + d x]}{d^3 i} - \frac{2 b B^2 c (b c - a d) g^2 n^2 \text{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \text{Log}[c + d x]}{d^3 i} - \\ & \frac{B^2 (b c - a d)^2 g^2 n^2 \text{Log}\left[-\frac{d(a+bx)}{b c - a d}\right] \text{Log}[c + d x]}{d^3 i} + \end{aligned}$$

$$\begin{aligned}
& \frac{2 b B c (b c - a d) g^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right) \operatorname{Log}[c+d x]}{d^3 i} + \\
& \frac{B (b c - a d)^2 g^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right) \operatorname{Log}[c+d x]}{d^3 i} + \frac{b B^2 c (b c - a d) g^2 n^2 \operatorname{Log}[c+d x]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{Log}[c+d x]^2}{2 d^3 i} - \frac{2 a B^2 (b c - a d) g^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}[i(c+d x)]}{d^3 i} + \\
& \frac{A B (b c - a d)^2 g^2 n \operatorname{Log}[i(c+d x)]^2}{d^3 i} - \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}[i(c+d x)]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[i(c+d x)]^2}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{Log}[i(c+d x)]^3}{3 d^3 i} - \\
& \frac{2 B^2 (b c - a d)^2 g^2 n \operatorname{Log}[a+b x] \operatorname{Log}[i(c+d x)] \operatorname{Log}[(c+d x)^{-n}]}{d^3 i} - \\
& \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}[a+b x] \operatorname{Log}[(c+d x)^{-n}]^2}{d^3 i} + \frac{B^2 (b c - a d)^2 g^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[(c+d x)^{-n}]^2}{d^3 i} - \\
& \frac{2 A B (b c - a d)^2 g^2 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]}{d^3 i} + \\
& \frac{(b c - a d)^2 g^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \right)^2 \operatorname{Log}[c i + d i x]}{d^3 i} + \frac{1}{d^3 i} \\
& 2 B^2 (b c - a d)^2 g^2 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] + \operatorname{Log}[(c+d x)^{-n}] \right) \\
& \operatorname{Log}[c i + d i x] - \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c i + d i x]^2}{d^3 i} + \\
& \frac{B^2 (b c - a d)^2 g^2 n \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] \operatorname{Log}[c i + d i x]^2}{d^3 i} - \frac{2 a B^2 (b c - a d) g^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 B^2 (b c - a d)^2 g^2 n \operatorname{Log}[(a+b x)^n] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^3 i} - \\
& \frac{2 A B (b c - a d)^2 g^2 n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \\
& \frac{2 b B^2 c (b c - a d) g^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \frac{B^2 (b c - a d)^2 g^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} - \\
& \frac{2 B^2 (b c - a d)^2 g^2 n \operatorname{Log}[(c+d x)^{-n}] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^3 i} + \frac{1}{d^3 i} 2 B^2 (b c - a d)^2 g^2 n \\
& \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log}\left[e \left( \frac{a+b x}{c+d x} \right)^n\right] + \operatorname{Log}[(c+d x)^{-n}] \right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right] -
\end{aligned}$$

$$\frac{2 B^2 (b c - a d)^2 g^2 n^2 \text{PolyLog}\left[3, -\frac{d(a+bx)}{b c - a d}\right]}{d^3 i} - \frac{2 B^2 (b c - a d)^2 g^2 n^2 \text{PolyLog}\left[3, \frac{b(c+dx)}{b c - a d}\right]}{d^3 i}$$

Problem 188: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x) \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{c i + d i x} dx$$

Optimal (type 4, 303 leaves, 9 steps):

$$\begin{aligned} & \frac{g(a+bx) \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{d i} + \frac{2 B (b c - a d) g n \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{Log}\left[\frac{b c - a d}{b(c+dx)}\right]}{d^2 i} + \\ & \frac{(b c - a d) g \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \text{Log}\left[\frac{b c - a d}{b(c+dx)}\right]}{d^2 i} + \frac{2 B^2 (b c - a d) g n^2 \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{d^2 i} + \\ & \frac{2 B (b c - a d) g n \left(A + B \text{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{d^2 i} - \\ & \frac{2 B^2 (b c - a d) g n^2 \text{PolyLog}\left[3, \frac{d(a+bx)}{b(c+dx)}\right]}{d^2 i} \end{aligned}$$

Result (type 4, 1156 leaves, 65 steps):

$$\begin{aligned}
& - \frac{a B^2 g n^2 \operatorname{Log}[a + b x]^2}{d i} + \frac{2 a B g n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{d i} + \\
& \frac{b g x \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{d i} + \frac{2 A B (b c - a d) g n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i} + \\
& \frac{2 b B^2 c g n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i} + \frac{B^2 (b c - a d) g \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}[c + d x]}{d^2 i} - \\
& \frac{2 b B c g n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{d^2 i} - \\
& \frac{(b c - a d) g \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}[c + d x]}{d^2 i} - \frac{A B (b c - a d) g n \operatorname{Log}[c + d x]^2}{d^2 i} - \\
& \frac{b B^2 c g n^2 \operatorname{Log}[c + d x]^2}{d^2 i} + \frac{B^2 (b c - a d) g n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^2 i} - \\
& \frac{B^2 (b c - a d) g n \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \operatorname{Log}[c + d x]^2}{d^2 i} - \frac{B^2 (b c - a d) g n^2 \operatorname{Log}[c + d x]^3}{3 d^2 i} + \\
& \frac{2 a B^2 g n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d i} - \frac{B^2 (b c - a d) g \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 B^2 (b c - a d) g n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{d^2 i} + \\
& \frac{B^2 (b c - a d) g \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{d^2 i} - \frac{B^2 (b c - a d) g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{d^2 i} - \\
& \frac{1}{d^2 i} 2 B^2 (b c - a d) g n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \\
& \left( \operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] + \operatorname{Log}\left[(c + d x)^{-n}\right] \right) + \frac{2 a B^2 g n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d i} - \\
& \frac{2 B^2 (b c - a d) g n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i} + \frac{2 A B (b c - a d) g n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \\
& \frac{2 b B^2 c g n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} + \frac{2 B^2 (b c - a d) g n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i} - \frac{1}{d^2 i} \\
& 2 B^2 (b c - a d) g n \left( \operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] + \operatorname{Log}\left[(c + d x)^{-n}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] + \\
& \frac{2 B^2 (b c - a d) g n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i} + \frac{2 B^2 (b c - a d) g n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i}
\end{aligned}$$

Problem 189: Result valid but suboptimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{c i + d i x} dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$-\frac{\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2\operatorname{Log}\left[\frac{b(c-d)}{b(c+dx)}\right]}{di}-\frac{2Bn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)\operatorname{PolyLog}\left[2,\frac{d(a+bx)}{b(c+dx)}\right]}{di}+\frac{2B^2n^2\operatorname{PolyLog}\left[3,\frac{d(a+bx)}{b(c+dx)}\right]}{di}$$

Result (type 4, 782 leaves, 45 steps):

$$\begin{aligned} & \frac{B^2\operatorname{Log}\left[(a+bx)^n\right]^2\operatorname{Log}\left[\frac{b(c+dx)}{b(c-d)}\right]}{di}-\frac{B^2\operatorname{Log}\left[(a+bx)^n\right]^2\operatorname{Log}\left[i(c+dx)\right]}{di}+\frac{ABn\operatorname{Log}\left[i(c+dx)\right]^2}{di}- \\ & \frac{B^2n^2\operatorname{Log}\left[a+bx\right]\operatorname{Log}\left[i(c+dx)\right]^2}{di}+\frac{B^2n^2\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-d)}\right]\operatorname{Log}\left[i(c+dx)\right]^2}{di}+ \\ & \frac{B^2n^2\operatorname{Log}\left[i(c+dx)\right]^3}{3di}-\frac{2B^2n\operatorname{Log}\left[a+bx\right]\operatorname{Log}\left[i(c+dx)\right]\operatorname{Log}\left[(c+dx)^{-n}\right]}{di}- \\ & \frac{B^2\operatorname{Log}\left[a+bx\right]\operatorname{Log}\left[(c+dx)^{-n}\right]^2}{di}+\frac{B^2\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-d)}\right]\operatorname{Log}\left[(c+dx)^{-n}\right]^2}{di}- \\ & \frac{2ABn\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-d)}\right]\operatorname{Log}\left[ci+di\right]}{di}+\frac{\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2\operatorname{Log}\left[ci+di\right]}{di}+\frac{1}{di}- \\ & 2B^2n\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-d)}\right]\left(\operatorname{Log}\left[(a+bx)^n\right]-\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]+\operatorname{Log}\left[(c+dx)^{-n}\right]\right)\operatorname{Log}\left[ci+di\right]- \\ & \frac{B^2n^2\operatorname{Log}\left[-\frac{d(a+bx)}{b(c-d)}\right]\operatorname{Log}\left[ci+di\right]^2}{di}+\frac{B^2n\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\operatorname{Log}\left[ci+di\right]^2}{di}+ \\ & \frac{2B^2n\operatorname{Log}\left[(a+bx)^n\right]\operatorname{PolyLog}\left[2,-\frac{d(a+bx)}{b(c-d)}\right]}{di}-\frac{2ABn\operatorname{PolyLog}\left[2,\frac{b(c+dx)}{b(c-d)}\right]}{di}- \\ & \frac{2B^2n\operatorname{Log}\left[(c+dx)^{-n}\right]\operatorname{PolyLog}\left[2,\frac{b(c+dx)}{b(c-d)}\right]}{di}+\frac{1}{di}- \\ & 2B^2n\left(\operatorname{Log}\left[(a+bx)^n\right]-\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]+\operatorname{Log}\left[(c+dx)^{-n}\right]\right)\operatorname{PolyLog}\left[2,\frac{b(c+dx)}{b(c-d)}\right]- \\ & \frac{2B^2n^2\operatorname{PolyLog}\left[3,-\frac{d(a+bx)}{b(c-d)}\right]}{di}-\frac{2B^2n^2\operatorname{PolyLog}\left[3,\frac{b(c+dx)}{b(c-d)}\right]}{di} \end{aligned}$$

**Problem 190:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(ag+bgx)(ci+di)}dx$$

Optimal (type 3, 50 leaves, 3 steps):

$$\frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{3 B (b c - a d) g i n}$$

Result (type 4, 1237 leaves, 59 steps):

$$\begin{aligned} & -\frac{A B n \operatorname{Log}[a + b x]^2}{(b c - a d) g i} - \frac{B^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d) g i} - \\ & \frac{B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d) g i} + \frac{\operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d) g i} + \\ & \frac{2 A B n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d) g i} + \frac{B^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}[c + d x]}{(b c - a d) g i} - \\ & \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d) g i} - \frac{A B n \operatorname{Log}[c + d x]^2}{(b c - a d) g i} + \frac{B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d) g i} - \\ & \frac{B^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[c + d x]^2}{(b c - a d) g i} - \frac{B^2 n^2 \operatorname{Log}[c + d x]^3}{3 (b c - a d) g i} + \frac{2 A B n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} - \\ & \frac{B^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{2 B^2 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{(b c - a d) g i} + \\ & \frac{B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d) g i} - \frac{B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d) g i} - \frac{1}{(b c - a d) g i} \\ & \frac{2 B^2 n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right)}{(b c - a d) g i} + \\ & \frac{2 A B n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d) g i} - \frac{2 B^2 n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d) g i} + \\ & \frac{2 A B n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{2 B^2 n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} - \frac{1}{(b c - a d) g i} \\ & \frac{2 B^2 n \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} + \\ & \frac{2 B^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d (a + b x)}\right]}{(b c - a d) g i} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d) g i} + \\ & \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d) g i} + \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d (a + b x)}\right]}{(b c - a d) g i} \end{aligned}$$

**Problem 191:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(ag+bgx)^2 (ci+di x)} dx$$

Optimal (type 3, 199 leaves, 7 steps):

$$\begin{aligned} & -\frac{2bB^2n^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bBn(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^2g^2i(a+bx)} - \\ & \frac{b(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{3B(bc-ad)^2g^2in} \end{aligned}$$

Result (type 4, 1800 leaves, 83 steps):

$$\begin{aligned} & -\frac{2B^2n^2}{(bc-ad)g^2i(a+bx)} - \frac{2B^2dn^2\operatorname{Log}[a+bx]}{(bc-ad)^2g^2i} + \frac{ABdn\operatorname{Log}[a+bx]^2}{(bc-ad)^2g^2i} + \frac{B^2dn^2\operatorname{Log}[a+bx]^2}{(bc-ad)^2g^2i} + \\ & \frac{B^2d\operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^2g^2i} + \frac{B^2d\operatorname{Log}[a+bx]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^2g^2i} - \\ & \frac{2Bn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)g^2i(a+bx)} - \frac{2Bdn\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^2g^2i} - \\ & \frac{\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)g^2i(a+bx)} - \frac{d\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^2g^2i} + \frac{2B^2dn^2\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} - \\ & \frac{2ABdn\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} - \frac{2B^2dn^2\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} - \\ & \frac{B^2d\operatorname{Log}\left[(a+bx)^n\right]^2\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} + \frac{2Bdn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} + \\ & \frac{d\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2\operatorname{Log}[c+dx]}{(bc-ad)^2g^2i} + \frac{ABdn\operatorname{Log}[c+dx]^2}{(bc-ad)^2g^2i} + \frac{B^2dn^2\operatorname{Log}[c+dx]^2}{(bc-ad)^2g^2i} - \\ & \frac{B^2dn^2\operatorname{Log}[a+bx]\operatorname{Log}[c+dx]^2}{(bc-ad)^2g^2i} + \frac{B^2dn\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\operatorname{Log}[c+dx]^2}{(bc-ad)^2g^2i} + \frac{B^2dn^2\operatorname{Log}[c+dx]^3}{3(bc-ad)^2g^2i} - \\ & \frac{2ABdn\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^2g^2i} - \frac{2B^2dn^2\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^2g^2i} + \\ & \frac{B^2d\operatorname{Log}\left[(a+bx)^n\right]^2\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^2g^2i} - \frac{2B^2dn\operatorname{Log}[a+bx]\operatorname{Log}[c+dx]\operatorname{Log}\left[(c+dx)^{-n}\right]}{(bc-ad)^2g^2i} - \end{aligned}$$

$$\begin{aligned}
& \frac{B^2 d \operatorname{Log}[a + b x] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^2 g^2 i} + \frac{B^2 d \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^2 g^2 i} + \frac{1}{(b c - a d)^2 g^2 i} \\
& 2 B^2 d n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}] \right) - \\
& \frac{2 A B d n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} + \\
& \frac{2 B^2 d n \operatorname{Log}[(a + b x)^n] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 A B d n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d n \operatorname{Log}[(c + d x)^{-n}] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} + \frac{1}{(b c - a d)^2 g^2 i} \\
& 2 B^2 d n \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}] \right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] - \\
& \frac{2 B^2 d n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \\
& \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^2 g^2 i} - \frac{2 B^2 d n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^2 g^2 i}
\end{aligned}$$

**Problem 192: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^3 (c i + d i x)} dx$$

Optimal (type 3, 369 leaves, 9 steps):

$$\begin{aligned}
& \frac{4 b B^2 d n^2 (c + d x)}{(b c - a d)^3 g^3 i (a + b x)} - \frac{b^2 B^2 n^2 (c + d x)^2}{4 (b c - a d)^3 g^3 i (a + b x)^2} + \frac{4 b B d n (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^3 g^3 i (a + b x)} - \\
& \frac{b^2 B n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^3 g^3 i (a + b x)^2} + \frac{2 b d (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^3 g^3 i (a + b x)} - \\
& \frac{b^2 (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^3 g^3 i (a + b x)^2} + \frac{d^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^3}{3 B (b c - a d)^3 g^3 i n}
\end{aligned}$$

Result (type 4, 2025 leaves, 111 steps):

$$- \frac{B^2 n^2}{4 (b c - a d) g^3 i (a + b x)^2} + \frac{7 B^2 d n^2}{2 (b c - a d)^2 g^3 i (a + b x)} + \frac{7 B^2 d^2 n^2 \operatorname{Log}[a + b x]}{2 (b c - a d)^3 g^3 i} -$$



$$\begin{aligned}
& \frac{A B d^2 n \operatorname{Log}[a+b x]^2}{(b c-a d)^3 g^3 i} - \frac{3 B^2 d^2 n^2 \operatorname{Log}[a+b x]^2}{2 (b c-a d)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]^2}{(b c-a d)^3 g^3 i} - \\
& \frac{B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]^2}{(b c-a d)^3 g^3 i} - \frac{B n\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 (b c-a d) g^3 i(a+b x)^2} + \frac{3 B d n\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c-a d)^2 g^3 i(a+b x)} + \\
& \frac{3 B d^2 n \operatorname{Log}[a+b x]\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c-a d)^3 g^3 i} - \frac{\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 (b c-a d) g^3 i(a+b x)^2} + \\
& \frac{d\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(b c-a d)^2 g^3 i(a+b x)} + \frac{d^2 \operatorname{Log}[a+b x]\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(b c-a d)^3 g^3 i} - \frac{7 B^2 d^2 n^2 \operatorname{Log}[c+d x]}{2 (b c-a d)^3 g^3 i} + \\
& \frac{2 A B d^2 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} + \frac{3 B^2 d^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} + \\
& \frac{B^2 d^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} - \frac{3 B d^2 n\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} - \\
& \frac{d^2\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 \operatorname{Log}[c+d x]}{(b c-a d)^3 g^3 i} - \frac{A B d^2 n \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \frac{3 B^2 d^2 n^2 \operatorname{Log}[c+d x]^2}{2 (b c-a d)^3 g^3 i} + \\
& \frac{B^2 d^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \frac{B^2 d^2 n \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^3 i} - \frac{B^2 d^2 n^2 \operatorname{Log}[c+d x]^3}{3 (b c-a d)^3 g^3 i} + \\
& \frac{2 A B d^2 n \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{3 B^2 d^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \\
& \frac{B^2 d^2 \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 n \operatorname{Log}[a+b x] \operatorname{Log}[c+d x] \operatorname{Log}\left[(c+d x)^{-n}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{B^2 d^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[(c+d x)^{-n}\right]^2}{(b c-a d)^3 g^3 i} - \frac{B^2 d^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}\left[(c+d x)^{-n}\right]^2}{(b c-a d)^3 g^3 i} - \frac{1}{(b c-a d)^3 g^3 i} \\
& \frac{2 B^2 d^2 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]\left(\operatorname{Log}\left[(a+b x)^n\right]-\operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]+\operatorname{Log}\left[(c+d x)^{-n}\right]\right)}{(b c-a d)^3 g^3 i} + \\
& \frac{2 A B d^2 n \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{3 B^2 d^2 n^2 \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \\
& \frac{2 B^2 d^2 n \operatorname{Log}\left[(a+b x)^n\right] \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 A B d^2 n \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \\
& \frac{3 B^2 d^2 n^2 \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 n \operatorname{Log}\left[(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} - \frac{1}{(b c-a d)^3 g^3 i} \\
& \frac{2 B^2 d^2 n\left(\operatorname{Log}\left[(a+b x)^n\right]-\operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]+\operatorname{Log}\left[(c+d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} +
\end{aligned}$$

$$\frac{2 B^2 d^2 n \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} +$$

$$\frac{2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^3 g^3 i} + \frac{2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, 1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^3 g^3 i}$$

**Problem 193: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(a g+b g x)^4 (c i+d i x)} d x$$

Optimal (type 3, 543 leaves, 11 steps):

$$-\frac{6 b B^2 d^2 n^2 (c+d x)}{(b c-a d)^4 g^4 i (a+b x)} + \frac{3 b^2 B^2 d n^2 (c+d x)^2}{4 (b c-a d)^4 g^4 i (a+b x)^2} -$$

$$\frac{2 b^3 B^2 n^2 (c+d x)^3}{27 (b c-a d)^4 g^4 i (a+b x)^3} - \frac{6 b B d^2 n (c+d x) \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{(b c-a d)^4 g^4 i (a+b x)} +$$

$$\frac{3 b^2 B d n (c+d x)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 (b c-a d)^4 g^4 i (a+b x)^2} - \frac{2 b^3 B n (c+d x)^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{9 (b c-a d)^4 g^4 i (a+b x)^3} -$$

$$\frac{3 b d^2 (c+d x) \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(b c-a d)^4 g^4 i (a+b x)} + \frac{3 b^2 d (c+d x)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 (b c-a d)^4 g^4 i (a+b x)^2} -$$

$$\frac{b^3 (c+d x)^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{3 (b c-a d)^4 g^4 i (a+b x)^3} - \frac{d^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^3}{3 B (b c-a d)^4 g^4 i n}$$

Result (type 4, 2180 leaves, 143 steps):

$$-\frac{2 B^2 n^2}{27 (b c-a d) g^4 i (a+b x)^3} + \frac{19 B^2 d n^2}{36 (b c-a d)^2 g^4 i (a+b x)^2} - \frac{85 B^2 d^2 n^2}{18 (b c-a d)^3 g^4 i (a+b x)} -$$

$$\frac{85 B^2 d^3 n^2 \operatorname{Log}[a+b x]}{18 (b c-a d)^4 g^4 i} + \frac{A B d^3 n \operatorname{Log}[a+b x]^2}{(b c-a d)^4 g^4 i} + \frac{11 B^2 d^3 n^2 \operatorname{Log}[a+b x]^2}{6 (b c-a d)^4 g^4 i} +$$

$$\frac{B^2 d^3 \operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]^2}{(b c-a d)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log}[a+b x] \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]^2}{(b c-a d)^4 g^4 i} -$$

$$\frac{2 B n \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{9 (b c-a d) g^4 i (a+b x)^3} + \frac{5 B d n \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{6 (b c-a d)^2 g^4 i (a+b x)^2} -$$

$$\frac{11 B d^2 n \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{3 (b c-a d)^3 g^4 i (a+b x)} - \frac{11 B d^3 n \operatorname{Log}[a+b x] \left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{3 (b c-a d)^4 g^4 i} -$$

$$\begin{aligned}
& \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3 (bc - ad) g^4 i (a + bx)^3} + \frac{d \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc - ad)^2 g^4 i (a + bx)^2} - \frac{d^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc - ad)^3 g^4 i (a + bx)} - \\
& \frac{d^3 \operatorname{Log}[a + bx] \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc - ad)^4 g^4 i} + \frac{85 B^2 d^3 n^2 \operatorname{Log}[c + dx]}{18 (bc - ad)^4 g^4 i} - \\
& \frac{2 A B d^3 n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{(bc - ad)^4 g^4 i} - \frac{11 B^2 d^3 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{3 (bc - ad)^4 g^4 i} - \\
& \frac{B^2 d^3 \operatorname{Log}\left[(a + bx)^n\right]^2 \operatorname{Log}[c + dx]}{(bc - ad)^4 g^4 i} + \frac{11 B d^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[c + dx]}{3 (bc - ad)^4 g^4 i} + \\
& \frac{d^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}[c + dx]}{(bc - ad)^4 g^4 i} + \frac{A B d^3 n \operatorname{Log}[c + dx]^2}{(bc - ad)^4 g^4 i} + \frac{11 B^2 d^3 n^2 \operatorname{Log}[c + dx]^2}{6 (bc - ad)^4 g^4 i} - \\
& \frac{B^2 d^3 n^2 \operatorname{Log}[a + bx] \operatorname{Log}[c + dx]^2}{(bc - ad)^4 g^4 i} + \frac{B^2 d^3 n \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}[c + dx]^2}{(bc - ad)^4 g^4 i} + \frac{B^2 d^3 n^2 \operatorname{Log}[c + dx]^3}{3 (bc - ad)^4 g^4 i} - \\
& \frac{2 A B d^3 n \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \frac{11 B^2 d^3 n^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{3 (bc - ad)^4 g^4 i} + \\
& \frac{B^2 d^3 \operatorname{Log}\left[(a + bx)^n\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \frac{2 B^2 d^3 n \operatorname{Log}[a + bx] \operatorname{Log}[c + dx] \operatorname{Log}\left[(c + dx)^{-n}\right]}{(bc - ad)^4 g^4 i} - \\
& \frac{B^2 d^3 \operatorname{Log}[a + bx] \operatorname{Log}\left[(c + dx)^{-n}\right]^2}{(bc - ad)^4 g^4 i} + \frac{B^2 d^3 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[(c + dx)^{-n}\right]^2}{(bc - ad)^4 g^4 i} + \frac{1}{(bc - ad)^4 g^4 i} \\
& \frac{2 B^2 d^3 n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx] \left(\operatorname{Log}\left[(a + bx)^n\right] - \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c + dx)^{-n}\right]\right)}{(bc - ad)^4 g^4 i} - \\
& \frac{2 A B d^3 n \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \frac{11 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{3 (bc - ad)^4 g^4 i} + \\
& \frac{2 B^2 d^3 n \operatorname{Log}\left[(a + bx)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \frac{2 A B d^3 n \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \\
& \frac{11 B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{3 (bc - ad)^4 g^4 i} - \frac{2 B^2 d^3 n \operatorname{Log}\left[(c + dx)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} + \frac{1}{(bc - ad)^4 g^4 i} \\
& \frac{2 B^2 d^3 n \left(\operatorname{Log}\left[(a + bx)^n\right] - \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c + dx)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \\
& \frac{2 B^2 d^3 n \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc - ad)^4 g^4 i} - \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \\
& \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{(bc - ad)^4 g^4 i} - \frac{2 B^2 d^3 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc - ad)^4 g^4 i}
\end{aligned}$$

### Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 770 leaves, 18 steps):

$$\begin{aligned} & \frac{2 A B (b c - a d)^2 g^3 n (a + b x)}{d^3 i^2 (c + d x)} - \frac{2 B^2 (b c - a d)^2 g^3 n^2 (a + b x)}{d^3 i^2 (c + d x)} + \\ & \frac{2 B^2 (b c - a d)^2 g^3 n (a + b x) \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{d^3 i^2 (c + d x)} - \frac{b B (b c - a d) g^3 n (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^3 i^2} - \\ & \frac{3 b (b c - a d) g^3 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{d^3 i^2} - \frac{(b c - a d)^2 g^3 (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{d^3 i^2 (c + d x)} + \\ & \frac{b^3 g^3 (c + d x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{2 d^4 i^2} - \frac{6 b B (b c - a d)^2 g^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{d^4 i^2} - \\ & \frac{3 b (b c - a d)^2 g^3 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{d^4 i^2} + \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[c + d x]}{d^4 i^2} + \\ & \frac{b B (b c - a d)^2 g^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}\left[ 1 - \frac{b (c + d x)}{d (a + b x)} \right]}{d^4 i^2} - \\ & \frac{6 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{d^4 i^2} - \\ & \frac{6 b B (b c - a d)^2 g^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{d (a + b x)}{b (c + d x)} \right]}{d^4 i^2} - \\ & \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[ 2, \frac{b (c + d x)}{d (a + b x)} \right]}{d^4 i^2} + \frac{6 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[ 3, \frac{d (a + b x)}{b (c + d x)} \right]}{d^4 i^2} \end{aligned}$$

Result (type 4, 2384 leaves, 112 steps):

$$\begin{aligned} & - \frac{A b^2 B (b c - a d) g^3 n x}{d^3 i^2} + \frac{2 B^2 (b c - a d)^3 g^3 n^2}{d^4 i^2 (c + d x)} + \frac{2 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a + b x]}{d^4 i^2} + \\ & \frac{a^2 b B^2 g^3 n^2 \operatorname{Log}[a + b x]^2}{2 d^2 i^2} + \frac{a b B^2 (2 b c - 3 a d) g^3 n^2 \operatorname{Log}[a + b x]^2}{d^3 i^2} + \\ & \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a + b x]^2}{d^4 i^2} - \frac{b B^2 (b c - a d) g^3 n (a + b x) \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{d^3 i^2} - \\ & \frac{2 B (b c - a d)^3 g^3 n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^4 i^2 (c + d x)} - \frac{a^2 b B g^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^2 i^2} - \\ & \frac{2 a b B (2 b c - 3 a d) g^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^3 i^2} - \end{aligned}$$

$$\begin{aligned}
& \frac{2 b B (b c - a d)^2 g^3 n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{d^4 i^2} - \\
& \frac{b^2 (2 b c - 3 a d) g^3 x \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{d^3 i^2} + \frac{b^3 g^3 x^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{2 d^2 i^2} + \\
& \frac{(b c - a d)^3 g^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{d^4 i^2 (c + d x)} - \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[c + d x]}{d^4 i^2} - \\
& \frac{6 A b B (b c - a d)^2 g^3 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} - \frac{b^3 B^2 c^2 g^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} - \\
& \frac{2 b^2 B^2 c (2 b c - 3 a d) g^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} - \\
& \frac{2 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^4 i^2} - \\
& \frac{3 b B^2 (b c - a d)^2 g^3 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}[c + d x]}{d^4 i^2} + \frac{b^3 B c^2 g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{2 b^2 B c (2 b c - 3 a d) g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{2 b B (b c - a d)^2 g^3 n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{d^4 i^2} + \\
& \frac{3 b (b c - a d)^2 g^3 \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2 \operatorname{Log}[c + d x]}{d^4 i^2} + \frac{3 A b B (b c - a d)^2 g^3 n \operatorname{Log}[c + d x]^2}{d^4 i^2} + \\
& \frac{b^3 B^2 c^2 g^3 n^2 \operatorname{Log}[c + d x]^2}{2 d^4 i^2} + \frac{b^2 B^2 c (2 b c - 3 a d) g^3 n^2 \operatorname{Log}[c + d x]^2}{d^4 i^2} + \\
& \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[c + d x]^2}{d^4 i^2} - \frac{3 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^4 i^2} + \\
& \frac{3 b B^2 (b c - a d)^2 g^3 n \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \operatorname{Log}[c + d x]^2}{d^4 i^2} + \frac{b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[c + d x]^3}{d^4 i^2} - \\
& \frac{a^2 b B^2 g^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 a b B^2 (2 b c - 3 a d) g^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^4 i^2} + \\
& \frac{3 b B^2 (b c - a d)^2 g^3 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^4 i^2} - \\
& \frac{6 b B^2 (b c - a d)^2 g^3 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{d^4 i^2} - \\
& \frac{3 b B^2 (b c - a d)^2 g^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{d^4 i^2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 b B^2 (b c - a d)^2 g^3 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}\left[(c+d x)^{-n}\right]^2}{d^4 i^2} + \frac{1}{d^4 i^2} 6 b B^2 (b c - a d)^2 g^3 n \\
& \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c+d x] \left( \operatorname{Log}\left[(a+b x)^n\right] - \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] + \operatorname{Log}\left[(c+d x)^{-n}\right] \right) - \\
& \frac{a^2 b B^2 g^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 a b B^2 (2 b c - 3 a d) g^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^4 i^2} + \\
& \frac{6 b B^2 (b c - a d)^2 g^3 n \operatorname{Log}\left[(a+b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{d^4 i^2} - \\
& \frac{6 A b B (b c - a d)^2 g^3 n \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2} - \frac{b^3 B^2 c^2 g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2} - \\
& \frac{2 b^2 B^2 c (2 b c - 3 a d) g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2} - \frac{2 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2} - \\
& \frac{6 b B^2 (b c - a d)^2 g^3 n \operatorname{Log}\left[(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2} + \frac{1}{d^4 i^2} 6 b B^2 (b c - a d)^2 g^3 \\
& n \left( \operatorname{Log}\left[(a+b x)^n\right] - \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] + \operatorname{Log}\left[(c+d x)^{-n}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right] - \\
& \frac{6 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c - a d}\right]}{d^4 i^2} - \frac{6 b B^2 (b c - a d)^2 g^3 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c - a d}\right]}{d^4 i^2}
\end{aligned}$$

**Problem 195: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left( A + B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 500 leaves, 12 steps):

$$\begin{aligned}
& - \frac{2AB(b c - a d) g^2 n (a + b x)}{d^2 i^2 (c + d x)} + \frac{2B^2(b c - a d) g^2 n^2 (a + b x)}{d^2 i^2 (c + d x)} - \\
& \frac{2B^2(b c - a d) g^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{d^2 i^2 (c + d x)} + \frac{b g^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^2 i^2} + \\
& \frac{(b c - a d) g^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^2 i^2 (c + d x)} + \\
& \frac{2bB(b c - a d) g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^3 i^2} + \\
& \frac{2b(b c - a d) g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^3 i^2} + \frac{2bB^2(b c - a d) g^2 n^2 \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{d^3 i^2} + \\
& \frac{4bB(b c - a d) g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d(a+b x)}{b(c+d x)}\right]}{d^3 i^2} - \\
& \frac{4bB^2(b c - a d) g^2 n^2 \operatorname{PolyLog}\left[3, \frac{d(a+b x)}{b(c+d x)}\right]}{d^3 i^2}
\end{aligned}$$

Result(type 4, 1807 leaves, 89 steps):

$$\begin{aligned}
& - \frac{2B^2(b c - a d)^2 g^2 n^2}{d^3 i^2 (c + d x)} - \frac{2bB^2(b c - a d) g^2 n^2 \operatorname{Log}[a + b x]}{d^3 i^2} - \\
& \frac{a b B^2 g^2 n^2 \operatorname{Log}[a + b x]^2}{d^2 i^2} - \frac{b B^2(b c - a d) g^2 n^2 \operatorname{Log}[a + b x]^2}{d^3 i^2} + \\
& \frac{2B(b c - a d)^2 g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^3 i^2 (c + d x)} + \frac{2a b B g^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^2 i^2} + \\
& \frac{2bB(b c - a d) g^2 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^3 i^2} + \\
& \frac{b^2 g^2 x \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^2 i^2} - \frac{(b c - a d)^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^3 i^2 (c + d x)} + \\
& \frac{2bB^2(b c - a d) g^2 n^2 \operatorname{Log}[c + d x]}{d^3 i^2} + \frac{4A b B(b c - a d) g^2 n \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{2b^2 B^2 c g^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \frac{2bB^2(b c - a d) g^2 n^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^3 i^2} + \\
& \frac{2bB^2(b c - a d) g^2 \operatorname{Log}\left[\left(a + b x\right)^n\right]^2 \operatorname{Log}[c + d x]}{d^3 i^2} - \frac{2b^2 B c g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^3 i^2} - \\
& \frac{2bB(b c - a d) g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d^3 i^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b (b c - a d) g^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 \operatorname{Log}[c+d x]}{d^3 i^2} - \\
& \frac{2 A b B (b c - a d) g^2 n \operatorname{Log}[c+d x]^2}{d^3 i^2} - \frac{b^2 B^2 c g^2 n^2 \operatorname{Log}[c+d x]^2}{d^3 i^2} - \\
& \frac{b B^2 (b c - a d) g^2 n^2 \operatorname{Log}[c+d x]^2}{d^3 i^2} + \frac{2 b B^2 (b c - a d) g^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d) g^2 n \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \operatorname{Log}[c+d x]^2}{d^3 i^2} - \frac{2 b B^2 (b c - a d) g^2 n^2 \operatorname{Log}[c+d x]^3}{3 d^3 i^2} + \\
& \frac{2 a b B^2 g^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^2 i^2} + \frac{2 b B^2 (b c - a d) g^2 n^2 \operatorname{Log}[a+b x] \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log} \left[ (a+b x)^n \right]^2 \operatorname{Log} \left[ \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} + \\
& \frac{4 b B^2 (b c - a d) g^2 n \operatorname{Log}[a+b x] \operatorname{Log}[c+d x] \operatorname{Log}[(c+d x)^{-n}]}{d^3 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log}[a+b x] \operatorname{Log}[(c+d x)^{-n}]^2}{d^3 i^2} - \\
& \frac{2 b B^2 (b c - a d) g^2 \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log}[(c+d x)^{-n}]^2}{d^3 i^2} - \frac{1}{d^3 i^2} 4 b B^2 (b c - a d) g^2 n \\
& \operatorname{Log} \left[ -\frac{d (a+b x)}{b c - a d} \right] \operatorname{Log}[c+d x] \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] + \operatorname{Log}[(c+d x)^{-n}] \right) + \\
& \frac{2 a b B^2 g^2 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^2 i^2} + \frac{2 b B^2 (b c - a d) g^2 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^3 i^2} - \\
& \frac{4 b B^2 (b c - a d) g^2 n \operatorname{Log}[(a+b x)^n] \operatorname{PolyLog} \left[ 2, -\frac{d (a+b x)}{b c - a d} \right]}{d^3 i^2} + \\
& \frac{4 A b B (b c - a d) g^2 n \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} + \frac{2 b^2 B^2 c g^2 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} + \\
& \frac{2 b B^2 (b c - a d) g^2 n^2 \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} + \\
& \frac{4 b B^2 (b c - a d) g^2 n \operatorname{Log}[(c+d x)^{-n}] \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2} - \frac{1}{d^3 i^2} 4 b B^2 (b c - a d) g^2 \\
& n \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] + \operatorname{Log}[(c+d x)^{-n}] \right) \operatorname{PolyLog} \left[ 2, \frac{b (c+d x)}{b c - a d} \right] + \\
& \frac{4 b B^2 (b c - a d) g^2 n^2 \operatorname{PolyLog} \left[ 3, -\frac{d (a+b x)}{b c - a d} \right]}{d^3 i^2} + \frac{4 b B^2 (b c - a d) g^2 n^2 \operatorname{PolyLog} \left[ 3, \frac{b (c+d x)}{b c - a d} \right]}{d^3 i^2}
\end{aligned}$$

Problem 196: Result valid but suboptimal antiderivative.



$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(c i + d i x)^2} dx$$

Optimal (type 4, 282 leaves, 9 steps):

$$\begin{aligned} & \frac{2 A B g n (a + b x)}{d i^2 (c + d x)} - \frac{2 B^2 g n^2 (a + b x)}{d i^2 (c + d x)} + \frac{2 B^2 g n (a + b x) \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{d i^2 (c + d x)} - \\ & \frac{g (a + b x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{d i^2 (c + d x)} - \frac{b g \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 \operatorname{Log}\left[ \frac{b c - a d}{b (c + d x)} \right]}{d^2 i^2} - \\ & \frac{2 b B g n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{PolyLog}\left[ 2, \frac{d (a+b x)}{b (c + d x)} \right]}{d^2 i^2} + \frac{2 b B^2 g n^2 \operatorname{PolyLog}\left[ 3, \frac{d (a+b x)}{b (c + d x)} \right]}{d^2 i^2} \end{aligned}$$

Result (type 4, 1157 leaves, 69 steps):

$$\begin{aligned}
& \frac{2 B^2 (b c - a d) g n^2}{d^2 i^2 (c + d x)} + \frac{2 b B^2 g n^2 \operatorname{Log}[a + b x]}{d^2 i^2} + \frac{b B^2 g n^2 \operatorname{Log}[a + b x]^2}{d^2 i^2} - \\
& \frac{2 B (b c - a d) g n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)}{d^2 i^2 (c + d x)} - \frac{2 b B g n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)}{d^2 i^2} + \\
& \frac{(b c - a d) g \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)^2}{d^2 i^2 (c + d x)} - \frac{2 b B^2 g n^2 \operatorname{Log}[c + d x]}{d^2 i^2} - \\
& \frac{2 A b B g n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i^2} - \frac{2 b B^2 g n^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d^2 i^2} - \\
& \frac{b B^2 g \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}[c + d x]}{d^2 i^2} + \frac{2 b B g n \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right) \operatorname{Log}[c + d x]}{d^2 i^2} + \\
& \frac{b g \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \right)^2 \operatorname{Log}[c + d x]}{d^2 i^2} + \frac{A b B g n \operatorname{Log}[c + d x]^2}{d^2 i^2} + \frac{b B^2 g n^2 \operatorname{Log}[c + d x]^2}{d^2 i^2} - \\
& \frac{b B^2 g n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{d^2 i^2} + \frac{b B^2 g n \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] \operatorname{Log}[c + d x]^2}{d^2 i^2} + \\
& \frac{b B^2 g n^2 \operatorname{Log}[c + d x]^3}{3 d^2 i^2} - \frac{2 b B^2 g n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} + \\
& \frac{b B^2 g \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{d^2 i^2} - \\
& \frac{b B^2 g \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{d^2 i^2} + \frac{b B^2 g \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{d^2 i^2} + \frac{1}{d^2 i^2} \\
& \frac{2 b B^2 g n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left( \operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right] \right)}{d^2 i^2} - \\
& \frac{2 b B^2 g n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} + \frac{2 b B^2 g n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} - \\
& \frac{2 A b B g n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \\
& \frac{2 b B^2 g n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} + \frac{1}{d^2 i^2} \\
& \frac{2 b B^2 g n \left( \operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2} - \\
& \frac{2 b B^2 g n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{d^2 i^2} - \frac{2 b B^2 g n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{d^2 i^2}
\end{aligned}$$

**Problem 197: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(ci+di x)^2} dx$$

Optimal (type 3, 163 leaves, 4 steps):

$$\begin{aligned} & -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2n^2(a+bx)}{(bc-ad)i^2(c+dx)} - \\ & \frac{2B^2n(a+bx)\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)i^2(c+dx)} \end{aligned}$$

Result (type 4, 514 leaves, 24 steps):

$$\begin{aligned} & -\frac{2B^2n^2}{di^2(c+dx)} - \frac{2bB^2n^2\operatorname{Log}[a+bx]}{d(bc-ad)i^2} - \frac{bB^2n^2\operatorname{Log}[a+bx]^2}{d(bc-ad)i^2} + \\ & \frac{2Bn\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{di^2(c+dx)} + \frac{2bBn\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{d(bc-ad)i^2} - \\ & \frac{\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{di^2(c+dx)} + \frac{2bB^2n^2\operatorname{Log}[c+dx]}{d(bc-ad)i^2} + \frac{2bB^2n^2\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{d(bc-ad)i^2} - \\ & \frac{2bBn\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)\operatorname{Log}[c+dx]}{d(bc-ad)i^2} - \frac{bB^2n^2\operatorname{Log}[c+dx]^2}{d(bc-ad)i^2} + \\ & \frac{2bB^2n^2\operatorname{Log}[a+bx]\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)i^2} + \frac{2bB^2n^2\operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{d(bc-ad)i^2} + \frac{2bB^2n^2\operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d(bc-ad)i^2} \end{aligned}$$

**Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(ag+bg x)(ci+di x)^2} dx$$

Optimal (type 3, 231 leaves, 7 steps):

$$\begin{aligned} & \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2dn^2(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2dn(a+bx)\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^2gi^2(c+dx)} - \\ & \frac{d(a+bx)\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{3B(bc-ad)^2gi^2n} \end{aligned}$$

Result (type 4, 1803 leaves, 83 steps):

$$\begin{aligned}
& \frac{2 B^2 n^2}{(b c - a d) g i^2 (c + d x)} + \frac{2 b B^2 n^2 \operatorname{Log}[a + b x]}{(b c - a d)^2 g i^2} - \frac{A b B n \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g i^2} + \frac{b B^2 n^2 \operatorname{Log}[a + b x]^2}{(b c - a d)^2 g i^2} - \\
& \frac{b B^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^2 g i^2} - \\
& \frac{2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d) g i^2 (c + d x)} - \frac{2 b B n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^2 g i^2} + \\
& \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d) g i^2 (c + d x)} + \frac{b \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^2 g i^2} - \frac{2 b B^2 n^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \\
& \frac{2 A b B n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{2 b B^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \\
& \frac{b B^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} + \frac{2 b B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \\
& \frac{b \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^2 g i^2} - \frac{A b B n \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} + \frac{b B^2 n^2 \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} + \\
& \frac{b B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 n^2 \operatorname{Log}[c + d x]^3}{3 (b c - a d)^2 g i^2} + \\
& \frac{2 A b B n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{2 b B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{b B^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{(b c - a d)^2 g i^2} + \\
& \frac{b B^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^2 g i^2} - \frac{b B^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^2 g i^2} - \frac{1}{(b c - a d)^2 g i^2} \\
& \frac{2 b B^2 n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right)}{(b c - a d)^2 g i^2} + \\
& \frac{2 A b B n \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{2 b B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 A b B n \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \\
& \frac{2 b B^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} + \frac{2 b B^2 n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 g i^2} - \frac{1}{(b c - a d)^2 g i^2} \\
& 2 b B^2 n \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right] +
\end{aligned}$$

$$\frac{2 b B^2 n \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right] \operatorname{PolyLog}\left[2, 1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^2 g i^2} + \frac{2 b B^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^2 g i^2} +$$

$$\frac{2 b B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^2 g i^2} + \frac{2 b B^2 n^2 \operatorname{PolyLog}\left[3, 1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^2 g i^2}$$

**Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(a g+b g x)^2(c i+d i x)^2} d x$$

Optimal (type 3, 392 leaves, 10 steps):

$$-\frac{2 A B d^2 n(a+b x)}{(b c-a d)^3 g^2 i^2(c+d x)} + \frac{2 B^2 d^2 n^2(a+b x)}{(b c-a d)^3 g^2 i^2(c+d x)} -$$

$$\frac{2 b^2 B^2 n^2(c+d x)}{(b c-a d)^3 g^2 i^2(a+b x)} - \frac{2 B^2 d^2 n(a+b x) \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]}{(b c-a d)^3 g^2 i^2(c+d x)} -$$

$$\frac{2 b^2 B n(c+d x)\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{(b c-a d)^3 g^2 i^2(a+b x)} + \frac{d^2(a+b x)\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(b c-a d)^3 g^2 i^2(c+d x)} -$$

$$\frac{b^2(c+d x)\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(b c-a d)^3 g^2 i^2(a+b x)} - \frac{2 b d\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^3}{3 B(b c-a d)^3 g^2 i^2 n}$$

Result (type 4, 1621 leaves, 107 steps):

$$-\frac{2 b B^2 n^2}{(b c-a d)^2 g^2 i^2(a+b x)} - \frac{2 B^2 d n^2}{(b c-a d)^2 g^2 i^2(c+d x)} - \frac{4 b B^2 d n^2 \operatorname{Log}[a+b x]}{(b c-a d)^3 g^2 i^2} +$$

$$\frac{2 A b B d n \operatorname{Log}[a+b x]^2}{(b c-a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]^2}{(b c-a d)^3 g^2 i^2} +$$

$$\frac{2 b B^2 d \operatorname{Log}[a+b x] \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]^2}{(b c-a d)^3 g^2 i^2} - \frac{2 b B n\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{(b c-a d)^2 g^2 i^2(a+b x)} +$$

$$\frac{2 B d n\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{(b c-a d)^2 g^2 i^2(c+d x)} - \frac{b\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(b c-a d)^2 g^2 i^2(a+b x)} - \frac{d\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(b c-a d)^2 g^2 i^2(c+d x)} -$$

$$\frac{2 b d \operatorname{Log}[a+b x]\left(A+B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)^2}{(b c-a d)^3 g^2 i^2} + \frac{4 b B^2 d n^2 \operatorname{Log}[c+d x]}{(b c-a d)^3 g^2 i^2} -$$

$$\frac{4 A b B d n \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]}{(b c-a d)^3 g^2 i^2} - \frac{2 b B^2 d \operatorname{Log}\left[(a+b x)^n\right]^2 \operatorname{Log}[c+d x]}{(b c-a d)^3 g^2 i^2} +$$

$$\begin{aligned}
& \frac{2 b d \left( A + B \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] \right)^2 \operatorname{Log}[c+d x]}{(b c-a d)^3 g^2 i^2} + \frac{2 A b B d n \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^2 i^2} - \\
& \frac{2 b B^2 d n^2 \operatorname{Log}[a+b x] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^2 i^2} + \frac{2 b B^2 d n \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] \operatorname{Log}[c+d x]^2}{(b c-a d)^3 g^2 i^2} + \\
& \frac{2 b B^2 d n^2 \operatorname{Log}[c+d x]^3}{3 (b c-a d)^3 g^2 i^2} - \frac{4 A b B d n \operatorname{Log}[a+b x] \operatorname{Log} \left[ \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} + \\
& \frac{2 b B^2 d \operatorname{Log} \left[ \left( a+b x \right)^n \right]^2 \operatorname{Log} \left[ \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \frac{4 b B^2 d n \operatorname{Log}[a+b x] \operatorname{Log}[c+d x] \operatorname{Log}[(c+d x)^{-n}]}{(b c-a d)^3 g^2 i^2} - \\
& \frac{2 b B^2 d \operatorname{Log}[a+b x] \operatorname{Log}[(c+d x)^{-n}]^2}{(b c-a d)^3 g^2 i^2} + \frac{2 b B^2 d \operatorname{Log} \left[ -\frac{d(a+b x)}{b c-a d} \right] \operatorname{Log}[(c+d x)^{-n}]^2}{(b c-a d)^3 g^2 i^2} + \frac{1}{(b c-a d)^3 g^2 i^2} \\
& \frac{4 b B^2 d n \operatorname{Log} \left[ -\frac{d(a+b x)}{b c-a d} \right] \operatorname{Log}[c+d x] \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] + \operatorname{Log}[(c+d x)^{-n}] \right)}{(b c-a d)^3 g^2 i^2} - \\
& \frac{4 A b B d n \operatorname{PolyLog} \left[ 2, -\frac{d(a+b x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} + \frac{4 b B^2 d n \operatorname{Log}[(a+b x)^n] \operatorname{PolyLog} \left[ 2, -\frac{d(a+b x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \\
& \frac{4 A b B d n \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \frac{4 b B^2 d n \operatorname{Log}[(c+d x)^{-n}] \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} + \frac{1}{(b c-a d)^3 g^2 i^2} \\
& \frac{4 b B^2 d n \left( \operatorname{Log}[(a+b x)^n] - \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] + \operatorname{Log}[(c+d x)^{-n}] \right) \operatorname{PolyLog} \left[ 2, \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \\
& \frac{4 b B^2 d n \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] \operatorname{PolyLog} \left[ 2, 1 + \frac{b c-a d}{d(a+b x)} \right]}{(b c-a d)^3 g^2 i^2} - \frac{4 b B^2 d n^2 \operatorname{PolyLog} \left[ 3, -\frac{d(a+b x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \\
& \frac{4 b B^2 d n^2 \operatorname{PolyLog} \left[ 3, \frac{b(c+d x)}{b c-a d} \right]}{(b c-a d)^3 g^2 i^2} - \frac{4 b B^2 d n^2 \operatorname{PolyLog} \left[ 3, 1 + \frac{b c-a d}{d(a+b x)} \right]}{(b c-a d)^3 g^2 i^2}
\end{aligned}$$

**Problem 200:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left( A + B \operatorname{Log} \left[ e^{\left( \frac{a+b x}{c+d x} \right)^n} \right] \right)^2}{(a g + b g x)^3 (c i + d i x)^2} dx$$

Optimal (type 3, 560 leaves, 12 steps):

$$\begin{aligned}
& \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3n^2(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} + \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \\
& \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} + \frac{2B^2d^3n(a+bx)\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^4g^3i^2(c+dx)} + \\
& \frac{6b^2Bdn(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3Bn(c+dx)^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} - \\
& \frac{d^3(a+bx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4g^3i^2(c+dx)} + \frac{3b^2d(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4g^3i^2(a+bx)} - \\
& \frac{b^3(c+dx)^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} + \frac{bd^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{B(bc-ad)^4g^3i^2n}
\end{aligned}$$

Result (type 4, 2207 leaves, 135 steps):

$$\begin{aligned}
& -\frac{bB^2n^2}{4(bc-ad)^2g^3i^2(a+bx)^2} + \frac{11bB^2dn^2}{2(bc-ad)^3g^3i^2(a+bx)} + \frac{2B^2d^2n^2}{(bc-ad)^3g^3i^2(c+dx)} + \\
& \frac{15bB^2d^2n^2\operatorname{Log}[a+bx]}{2(bc-ad)^4g^3i^2} - \frac{3AbB^2dn\operatorname{Log}[a+bx]^2}{(bc-ad)^4g^3i^2} - \frac{3bB^2d^2n^2\operatorname{Log}[a+bx]^2}{2(bc-ad)^4g^3i^2} - \\
& \frac{3bB^2d^2\operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^4g^3i^2} - \frac{3bB^2d^2\operatorname{Log}[a+bx]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^4g^3i^2} - \\
& \frac{bBn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2(bc-ad)^2g^3i^2(a+bx)^2} + \frac{5bBdn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^3g^3i^2(a+bx)} - \\
& \frac{2Bd^2n\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^3g^3i^2(c+dx)} + \frac{3bBd^2n\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^4g^3i^2} - \\
& \frac{b\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2(bc-ad)^2g^3i^2(a+bx)^2} + \frac{2bd\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^3g^3i^2(a+bx)} + \frac{d^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^3g^3i^2(c+dx)} + \\
& \frac{3bd^2\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4g^3i^2} - \frac{15bB^2d^2n^2\operatorname{Log}[c+dx]}{2(bc-ad)^4g^3i^2} + \\
& \frac{6AbBd^2n\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{(bc-ad)^4g^3i^2} + \frac{3bB^2d^2n^2\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{(bc-ad)^4g^3i^2} + \\
& \frac{3bB^2d^2\operatorname{Log}\left[(a+bx)^n\right]^2\operatorname{Log}[c+dx]}{(bc-ad)^4g^3i^2} - \frac{3bBd^2n\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)\operatorname{Log}[c+dx]}{(bc-ad)^4g^3i^2} - \\
& \frac{3bd^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2\operatorname{Log}[c+dx]}{(bc-ad)^4g^3i^2} - \frac{3AbBd^2n\operatorname{Log}[c+dx]^2}{(bc-ad)^4g^3i^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{3 b B^2 d^2 n^2 \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^4 g^3 i^2} - \\
& \frac{3 b B^2 d^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^4 g^3 i^2} - \frac{b B^2 d^2 n^2 \operatorname{Log}[c + d x]^3}{(b c - a d)^4 g^3 i^2} + \\
& \frac{6 A b B d^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} - \\
& \frac{3 b B^2 d^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{(b c - a d)^4 g^3 i^2} + \\
& \frac{3 b B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^4 g^3 i^2} - \frac{3 b B^2 d^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^4 g^3 i^2} - \frac{1}{(b c - a d)^4 g^3 i^2} \\
& 6 b B^2 d^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right) + \\
& \frac{6 A b B d^2 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} - \\
& \frac{6 b B^2 d^2 n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \\
& \frac{6 A b B d^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{3 b B^2 d^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \\
& \frac{6 b B^2 d^2 n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} - \frac{1}{(b c - a d)^4 g^3 i^2} \\
& 6 b B^2 d^2 n \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] + \\
& \frac{6 b B^2 d^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \\
& \frac{6 b B^2 d^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^3 i^2} + \frac{6 b B^2 d^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^4 g^3 i^2}
\end{aligned}$$

**Problem 201:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^4 (c i + d i x)^2} dx$$

Optimal (type 3, 729 leaves, 14 steps):



$$\begin{aligned}
& - \frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} + \frac{2B^2d^4n^2(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} - \frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} + \\
& \frac{b^3B^2d^2n^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3} - \frac{2B^2d^4n(a+bx)\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^5g^4i^2(c+dx)} - \\
& \frac{12b^2B^2d^2n(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3Bdn(c+dx)^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^5g^4i^2(a+bx)^2} - \\
& \frac{2b^4Bn(c+dx)^3\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} + \frac{d^4(a+bx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5g^4i^2(c+dx)} - \\
& \frac{6b^2d^2(c+dx)\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5g^4i^2(a+bx)} + \frac{2b^3d(c+dx)^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} - \\
& \frac{b^4(c+dx)^3\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} - \frac{4bd^3\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{3B(bc-ad)^5g^4i^2n}
\end{aligned}$$

Result (type 4, 2368 leaves, 167 steps):

$$\begin{aligned}
& - \frac{2bB^2n^2}{27(bc-ad)^2g^4i^2(a+bx)^3} + \frac{7bB^2dn^2}{9(bc-ad)^3g^4i^2(a+bx)^2} - \frac{92bB^2d^2n^2}{9(bc-ad)^4g^4i^2(a+bx)} - \\
& \frac{2B^2d^3n^2}{(bc-ad)^4g^4i^2(c+dx)} - \frac{110bB^2d^3n^2\operatorname{Log}[a+bx]}{9(bc-ad)^5g^4i^2} + \frac{4AbBd^3n\operatorname{Log}[a+bx]^2}{(bc-ad)^5g^4i^2} + \\
& \frac{10bB^2d^3n^2\operatorname{Log}[a+bx]^2}{3(bc-ad)^5g^4i^2} + \frac{4bB^2d^3\operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^5g^4i^2} + \\
& \frac{4bB^2d^3\operatorname{Log}[a+bx]\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^5g^4i^2} - \frac{2bBn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{9(bc-ad)^2g^4i^2(a+bx)^3} + \\
& \frac{4bBdn\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3(bc-ad)^3g^4i^2(a+bx)^2} - \frac{26bBd^2n\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3(bc-ad)^4g^4i^2(a+bx)} + \\
& \frac{2Bd^3n\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^4g^4i^2(c+dx)} - \frac{20bBd^3n\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{3(bc-ad)^5g^4i^2} - \\
& \frac{b\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{3(bc-ad)^2g^4i^2(a+bx)^3} + \frac{bd\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^3g^4i^2(a+bx)^2} - \frac{3bd^2\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4g^4i^2(a+bx)} - \\
& \frac{d^3\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4g^4i^2(c+dx)} - \frac{4bd^3\operatorname{Log}[a+bx]\left(A+B\operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5g^4i^2} + \\
& \frac{110bB^2d^3n^2\operatorname{Log}[c+dx]}{9(bc-ad)^5g^4i^2} - \frac{8AbBd^3n\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right]\operatorname{Log}[c+dx]}{(bc-ad)^5g^4i^2} -
\end{aligned}$$

$$\begin{aligned}
& \frac{20 b B^2 d^3 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{3 (bc-ad)^5 g^4 i^2} - \frac{4 b B^2 d^3 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^4 i^2} + \\
& \frac{20 b B d^3 n \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[c+dx]}{3 (bc-ad)^5 g^4 i^2} + \frac{4 b d^3 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^4 i^2} + \\
& \frac{4 A b B d^3 n \operatorname{Log}[c+dx]^2}{(bc-ad)^5 g^4 i^2} + \frac{10 b B^2 d^3 n^2 \operatorname{Log}[c+dx]^2}{3 (bc-ad)^5 g^4 i^2} - \frac{4 b B^2 d^3 n^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{(bc-ad)^5 g^4 i^2} + \\
& \frac{4 b B^2 d^3 n \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}[c+dx]^2}{(bc-ad)^5 g^4 i^2} + \frac{4 b B^2 d^3 n^2 \operatorname{Log}[c+dx]^3}{3 (bc-ad)^5 g^4 i^2} - \\
& \frac{8 A b B d^3 n \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \frac{20 b B^2 d^3 n^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{3 (bc-ad)^5 g^4 i^2} + \\
& \frac{4 b B^2 d^3 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \frac{8 b B^2 d^3 n \operatorname{Log}[a+bx] \operatorname{Log}[c+dx] \operatorname{Log}\left[(c+dx)^{-n}\right]}{(bc-ad)^5 g^4 i^2} - \\
& \frac{4 b B^2 d^3 \operatorname{Log}[a+bx] \operatorname{Log}\left[(c+dx)^{-n}\right]^2}{(bc-ad)^5 g^4 i^2} + \frac{4 b B^2 d^3 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[(c+dx)^{-n}\right]^2}{(bc-ad)^5 g^4 i^2} + \frac{1}{(bc-ad)^5 g^4 i^2} \\
& 8 b B^2 d^3 n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx] \left(\operatorname{Log}\left[(a+bx)^n\right] - \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c+dx)^{-n}\right]\right) - \\
& \frac{8 A b B d^3 n \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \frac{20 b B^2 d^3 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{3 (bc-ad)^5 g^4 i^2} + \\
& \frac{8 b B^2 d^3 n \operatorname{Log}\left[(a+bx)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \\
& \frac{8 A b B d^3 n \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \frac{20 b B^2 d^3 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{3 (bc-ad)^5 g^4 i^2} - \\
& \frac{8 b B^2 d^3 n \operatorname{Log}\left[(c+dx)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} + \frac{1}{(bc-ad)^5 g^4 i^2} \\
& 8 b B^2 d^3 n \left(\operatorname{Log}\left[(a+bx)^n\right] - \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] + \operatorname{Log}\left[(c+dx)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right] - \\
& \frac{8 b B^2 d^3 n \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^5 g^4 i^2} - \frac{8 b B^2 d^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \\
& \frac{8 b B^2 d^3 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^5 g^4 i^2} - \frac{8 b B^2 d^3 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{bc-ad}{d(a+bx)}\right]}{(bc-ad)^5 g^4 i^2}
\end{aligned}$$

Problem 202: Result valid but suboptimal antiderivative.

$$\int \frac{(a g + b g x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(c i + d i x)^3} dx$$

Optimal (type 4, 676 leaves, 14 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d) g^3 n^2 (a + b x)^2}{4 d^2 i^3 (c + d x)^2} - \frac{4 A b B (b c - a d) g^3 n (a + b x)}{d^3 i^3 (c + d x)} + \\ & \frac{4 b B^2 (b c - a d) g^3 n^2 (a + b x)}{d^3 i^3 (c + d x)} - \frac{4 b B^2 (b c - a d) g^3 n (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{d^3 i^3 (c + d x)} - \\ & \frac{B (b c - a d) g^3 n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 d^2 i^3 (c + d x)^2} + \frac{b^2 g^3 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^3 i^3} + \\ & \frac{(b c - a d) g^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 d^2 i^3 (c + d x)^2} + \frac{2 b (b c - a d) g^3 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^3 i^3 (c + d x)} + \\ & \frac{2 b^2 B (b c - a d) g^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^4 i^3} + \\ & \frac{3 b^2 (b c - a d) g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c + d x)}\right]}{d^4 i^3} + \\ & \frac{2 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c + d x)}\right]}{d^4 i^3} + \\ & \frac{6 b^2 B (b c - a d) g^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c + d x)}\right]}{d^4 i^3} - \\ & \frac{6 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{PolyLog}\left[3, \frac{d (a+b x)}{b (c + d x)}\right]}{d^4 i^3} \end{aligned}$$

Result (type 4, 2026 leaves, 117 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d)^3 g^3 n^2}{4 d^4 i^3 (c + d x)^2} - \frac{9 b B^2 (b c - a d)^2 g^3 n^2}{2 d^4 i^3 (c + d x)} - \\ & \frac{9 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{Log}[a + b x]}{2 d^4 i^3} - \frac{a b^2 B^2 g^3 n^2 \operatorname{Log}[a + b x]^2}{d^3 i^3} - \\ & \frac{5 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{Log}[a + b x]^2}{2 d^4 i^3} - \frac{B (b c - a d)^3 g^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 d^4 i^3 (c + d x)^2} + \\ & \frac{5 b B (b c - a d)^2 g^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^4 i^3 (c + d x)} + \frac{2 a b^2 B g^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^3 i^3} + \\ & \frac{5 b^2 B (b c - a d) g^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{d^4 i^3} + \frac{b^3 g^3 x \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^3 i^3} + \end{aligned}$$

$$\begin{aligned}
& \frac{(bc - ad)^3 g^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 d^4 i^3 (c+dx)^2} - \frac{3 b (bc - ad)^2 g^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{d^4 i^3 (c+dx)} + \\
& \frac{9 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log}[c+dx]}{2 d^4 i^3} + \frac{6 A b^2 B (bc - ad) g^3 n \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^3} + \\
& \frac{2 b^3 B^2 c g^3 n^2 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^3} + \frac{5 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx]}{d^4 i^3} + \\
& \frac{3 b^2 B^2 (bc - ad) g^3 \operatorname{Log}[(a+bx)^n]^2 \operatorname{Log}[c+dx]}{d^4 i^3} - \frac{2 b^3 B c g^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c+dx]}{d^4 i^3} - \\
& \frac{5 b^2 B (bc - ad) g^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c+dx]}{d^4 i^3} - \\
& \frac{3 b^2 (bc - ad) g^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}[c+dx]}{d^4 i^3} - \\
& \frac{3 A b^2 B (bc - ad) g^3 n \operatorname{Log}[c+dx]^2}{d^4 i^3} - \frac{b^3 B^2 c g^3 n^2 \operatorname{Log}[c+dx]^2}{d^4 i^3} - \\
& \frac{5 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log}[c+dx]^2}{2 d^4 i^3} + \frac{3 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{d^4 i^3} - \\
& \frac{3 b^2 B^2 (bc - ad) g^3 n \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \operatorname{Log}[c+dx]^2}{d^4 i^3} - \frac{b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log}[c+dx]^3}{d^4 i^3} + \\
& \frac{2 a b^2 B^2 g^3 n^2 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^3 i^3} + \frac{5 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{Log}[a+bx] \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^3} - \\
& \frac{3 b^2 B^2 (bc - ad) g^3 \operatorname{Log}[(a+bx)^n]^2 \operatorname{Log} \left[ \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^3} + \\
& \frac{6 b^2 B^2 (bc - ad) g^3 n \operatorname{Log}[a+bx] \operatorname{Log}[c+dx] \operatorname{Log}[(c+dx)^{-n}]}{d^4 i^3} + \\
& \frac{3 b^2 B^2 (bc - ad) g^3 \operatorname{Log}[a+bx] \operatorname{Log}[(c+dx)^{-n}]^2}{d^4 i^3} - \\
& \frac{3 b^2 B^2 (bc - ad) g^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[(c+dx)^{-n}]^2}{d^4 i^3} - \frac{1}{d^4 i^3} 6 b^2 B^2 (bc - ad) g^3 n \\
& \operatorname{Log} \left[ -\frac{d(a+bx)}{bc-ad} \right] \operatorname{Log}[c+dx] \left( \operatorname{Log}[(a+bx)^n] - \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}[(c+dx)^{-n}] \right) + \\
& \frac{2 a b^2 B^2 g^3 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^3 i^3} + \frac{5 b^2 B^2 (bc - ad) g^3 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^4 i^3} - \\
& \frac{6 b^2 B^2 (bc - ad) g^3 n \operatorname{Log}[(a+bx)^n] \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{bc-ad} \right]}{d^4 i^3} + \\
& \frac{6 A b^2 B (bc - ad) g^3 n \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^3} + \frac{2 b^3 B^2 c g^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{bc-ad} \right]}{d^4 i^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{5 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{d^4 i^3} + \\
& \frac{6 b^2 B^2 (b c - a d) g^3 n \operatorname{Log}\left[(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right]}{d^4 i^3} - \frac{1}{d^4 i^3} 6 b^2 B^2 (b c - a d) g^3 \\
& n \left( \operatorname{Log}\left[(a+b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right] + \operatorname{Log}\left[(c+d x)^{-n}\right] \right) \operatorname{PolyLog}\left[2, \frac{b (c+d x)}{b c - a d}\right] + \\
& \frac{6 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{PolyLog}\left[3, -\frac{d (a+b x)}{b c - a d}\right]}{d^4 i^3} + \frac{6 b^2 B^2 (b c - a d) g^3 n^2 \operatorname{PolyLog}\left[3, \frac{b (c+d x)}{b c - a d}\right]}{d^4 i^3}
\end{aligned}$$

**Problem 203: Result valid but suboptimal antiderivative.**

$$\int \frac{(a g + b g x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(c i + d i x)^3} dx$$

Optimal (type 4, 441 leaves, 11 steps):

$$\begin{aligned}
& -\frac{B^2 g^2 n^2 (a+b x)^2}{4 d i^3 (c+d x)^2} + \frac{2 A b B g^2 n (a+b x)}{d^2 i^3 (c+d x)} - \frac{2 b B^2 g^2 n^2 (a+b x)}{d^2 i^3 (c+d x)} + \frac{2 b B^2 g^2 n (a+b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{d^2 i^3 (c+d x)} + \\
& \frac{B g^2 n (a+b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 d i^3 (c+d x)^2} - \frac{g^2 (a+b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{2 d i^3 (c+d x)^2} - \\
& \frac{b g^2 (a+b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{d^2 i^3 (c+d x)} - \frac{b^2 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2 \operatorname{Log}\left[\frac{b c - a d}{b (c+d x)}\right]}{d^3 i^3} - \\
& \frac{2 b^2 B g^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d (a+b x)}{b (c+d x)}\right]}{d^3 i^3} + \frac{2 b^2 B^2 g^2 n^2 \operatorname{PolyLog}\left[3, \frac{d (a+b x)}{b (c+d x)}\right]}{d^3 i^3}
\end{aligned}$$

Result (type 4, 1435 leaves, 97 steps):

$$\begin{aligned}
& - \frac{B^2 (bc - ad)^2 g^2 n^2}{4 d^3 i^3 (c + dx)^2} + \frac{5 b B^2 (bc - ad) g^2 n^2}{2 d^3 i^3 (c + dx)} + \frac{5 b^2 B^2 g^2 n^2 \operatorname{Log}[a + bx]}{2 d^3 i^3} + \frac{3 b^2 B^2 g^2 n^2 \operatorname{Log}[a + bx]^2}{2 d^3 i^3} + \\
& \frac{B (bc - ad)^2 g^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{2 d^3 i^3 (c + dx)^2} - \frac{3 b B (bc - ad) g^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{d^3 i^3 (c + dx)} - \\
& \frac{3 b^2 B g^2 n \operatorname{Log}[a + bx] \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{d^3 i^3} - \frac{(bc - ad)^2 g^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 d^3 i^3 (c + dx)^2} + \\
& \frac{2 b (bc - ad) g^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{d^3 i^3 (c + dx)} - \frac{5 b^2 B^2 g^2 n^2 \operatorname{Log}[c + dx]}{2 d^3 i^3} - \\
& \frac{2 A b^2 B g^2 n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{d^3 i^3} - \frac{3 b^2 B^2 g^2 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx]}{d^3 i^3} - \\
& \frac{b^2 B^2 g^2 \operatorname{Log}\left[(a + bx)^n\right]^2 \operatorname{Log}[c + dx]}{d^3 i^3} + \frac{3 b^2 B g^2 n \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c + dx]}{d^3 i^3} + \\
& \frac{b^2 g^2 \left( A + B \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}[c + dx]}{d^3 i^3} + \frac{A b^2 B g^2 n \operatorname{Log}[c + dx]^2}{d^3 i^3} + \frac{3 b^2 B^2 g^2 n^2 \operatorname{Log}[c + dx]^2}{2 d^3 i^3} - \\
& \frac{b^2 B^2 g^2 n^2 \operatorname{Log}[a + bx] \operatorname{Log}[c + dx]^2}{d^3 i^3} + \frac{b^2 B^2 g^2 n \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] \operatorname{Log}[c + dx]^2}{d^3 i^3} + \\
& \frac{b^2 B^2 g^2 n^2 \operatorname{Log}[c + dx]^3}{3 d^3 i^3} - \frac{3 b^2 B^2 g^2 n^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} + \\
& \frac{b^2 B^2 g^2 \operatorname{Log}\left[(a + bx)^n\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} - \frac{2 b^2 B^2 g^2 n \operatorname{Log}[a + bx] \operatorname{Log}[c + dx] \operatorname{Log}\left[(c + dx)^{-n}\right]}{d^3 i^3} - \\
& \frac{b^2 B^2 g^2 \operatorname{Log}[a + bx] \operatorname{Log}\left[(c + dx)^{-n}\right]^2}{d^3 i^3} + \frac{b^2 B^2 g^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}\left[(c + dx)^{-n}\right]^2}{d^3 i^3} + \frac{1}{d^3 i^3} \\
& \frac{2 b^2 B^2 g^2 n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c + dx] \left( \operatorname{Log}\left[(a + bx)^n\right] - \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}\left[(c + dx)^{-n}\right] \right)}{d^3 i^3} - \\
& \frac{3 b^2 B^2 g^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{d^3 i^3} + \frac{2 b^2 B^2 g^2 n \operatorname{Log}\left[(a + bx)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{d^3 i^3} - \\
& \frac{2 A b^2 B g^2 n \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} - \frac{3 b^2 B^2 g^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} - \\
& \frac{2 b^2 B^2 g^2 n \operatorname{Log}\left[(c + dx)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} + \frac{1}{d^3 i^3} \\
& \frac{2 b^2 B^2 g^2 n \left( \operatorname{Log}\left[(a + bx)^n\right] - \operatorname{Log}\left[e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}\left[(c + dx)^{-n}\right] \right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3} - \\
& \frac{2 b^2 B^2 g^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{d^3 i^3} - \frac{2 b^2 B^2 g^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{d^3 i^3}
\end{aligned}$$

**Problem 204:** Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{(a g + b g x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(c i + d i x)^3} dx$$

Optimal (type 3, 151 leaves, 3 steps):

$$\frac{B^2 g n^2 (a + b x)^2}{4 (b c - a d) i^3 (c + d x)^2} - \frac{B g n (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 (b c - a d) i^3 (c + d x)^2} + \frac{g (a + b x)^2 \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{2 (b c - a d) i^3 (c + d x)^2}$$

Result (type 4, 686 leaves, 54 steps):

$$\begin{aligned} & \frac{B^2 (b c - a d) g n^2}{4 d^2 i^3 (c + d x)^2} - \frac{b B^2 g n^2}{2 d^2 i^3 (c + d x)} - \frac{b^2 B^2 g n^2 \operatorname{Log}[a + b x]}{2 d^2 (b c - a d) i^3} - \\ & \frac{b^2 B^2 g n^2 \operatorname{Log}[a + b x]^2}{2 d^2 (b c - a d) i^3} - \frac{B (b c - a d) g n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{2 d^2 i^3 (c + d x)^2} + \\ & \frac{b B g n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^2 i^3 (c + d x)} + \frac{b^2 B g n \operatorname{Log}[a + b x] \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{d^2 (b c - a d) i^3} + \\ & \frac{(b c - a d) g \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{2 d^2 i^3 (c + d x)^2} - \frac{b g \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{d^2 i^3 (c + d x)} + \frac{b^2 B^2 g n^2 \operatorname{Log}[c + d x]}{2 d^2 (b c - a d) i^3} + \\ & \frac{b^2 B^2 g n^2 \operatorname{Log}\left[ -\frac{d(a+b x)}{b c - a d} \right] \operatorname{Log}[c + d x]}{d^2 (b c - a d) i^3} - \frac{b^2 B g n \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) \operatorname{Log}[c + d x]}{d^2 (b c - a d) i^3} - \\ & \frac{b^2 B^2 g n^2 \operatorname{Log}[c + d x]^2}{2 d^2 (b c - a d) i^3} + \frac{b^2 B^2 g n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[ \frac{b(c+d x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} + \\ & \frac{b^2 B^2 g n^2 \operatorname{PolyLog}\left[ 2, -\frac{d(a+b x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} + \frac{b^2 B^2 g n^2 \operatorname{PolyLog}\left[ 2, \frac{b(c+d x)}{b c - a d} \right]}{d^2 (b c - a d) i^3} \end{aligned}$$

**Problem 205:** Result unnecessarily involves higher level functions.

$$\int \frac{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(c i + d i x)^3} dx$$

Optimal (type 3, 317 leaves, 8 steps):

$$\begin{aligned}
& - \frac{B^2 d n^2 (a + b x)^2}{4 (b c - a d)^2 i^3 (c + d x)^2} - \frac{2 A b B n (a + b x)}{(b c - a d)^2 i^3 (c + d x)} + \frac{2 b B^2 n^2 (a + b x)}{(b c - a d)^2 i^3 (c + d x)} - \\
& \frac{2 b B^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d)^2 i^3 (c + d x)} + \frac{B d n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^2 i^3 (c + d x)^2} - \\
& \frac{d (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^2 i^3 (c + d x)^2} + \frac{b (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^2 i^3 (c + d x)}
\end{aligned}$$

Result (type 4, 626 leaves, 28 steps):

$$\begin{aligned}
& - \frac{B^2 n^2}{4 d i^3 (c + d x)^2} - \frac{3 b B^2 n^2}{2 d (b c - a d) i^3 (c + d x)} - \frac{3 b^2 B^2 n^2 \operatorname{Log}[a + b x]}{2 d (b c - a d)^2 i^3} - \\
& \frac{b^2 B^2 n^2 \operatorname{Log}[a + b x]^2}{2 d (b c - a d)^2 i^3} + \frac{B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 d i^3 (c + d x)^2} + \frac{b B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d (b c - a d) i^3 (c + d x)} + \\
& \frac{b^2 B n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{d (b c - a d)^2 i^3} - \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 d i^3 (c + d x)^2} + \\
& \frac{3 b^2 B^2 n^2 \operatorname{Log}[c + d x]}{2 d (b c - a d)^2 i^3} + \frac{b^2 B^2 n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{d (b c - a d)^2 i^3} - \\
& \frac{b^2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{d (b c - a d)^2 i^3} - \frac{b^2 B^2 n^2 \operatorname{Log}[c + d x]^2}{2 d (b c - a d)^2 i^3} + \\
& \frac{b^2 B^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{d (b c - a d)^2 i^3} + \frac{b^2 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{d (b c - a d)^2 i^3} + \frac{b^2 B^2 n^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{d (b c - a d)^2 i^3}
\end{aligned}$$

**Problem 206:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x) (c i + d i x)^3} dx$$

Optimal (type 3, 402 leaves, 15 steps):



$$\begin{aligned}
& \frac{B^2 d^2 n^2 (a+bx)^2}{4 (bc-ad)^3 g i^3 (c+dx)^2} + \frac{4 Ab B d n (a+bx)}{(bc-ad)^3 g i^3 (c+dx)} - \\
& \frac{4 b B^2 d n^2 (a+bx)}{(bc-ad)^3 g i^3 (c+dx)} + \frac{4 b B^2 d n (a+bx) \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^3 g i^3 (c+dx)} - \\
& \frac{B d^2 n (a+bx)^2 \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^3 g i^3 (c+dx)^2} + \frac{d^2 (a+bx)^2 \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad)^3 g i^3 (c+dx)^2} - \\
& \frac{2 b d (a+bx) \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^3 g i^3 (c+dx)} + \frac{b^2 \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{3 B (bc-ad)^3 g i^3 n}
\end{aligned}$$

Result(type 4, 2025 leaves, 111 steps):

$$\begin{aligned}
& \frac{B^2 n^2}{4 (bc-ad) g i^3 (c+dx)^2} + \frac{7 b B^2 n^2}{2 (bc-ad)^2 g i^3 (c+dx)} + \frac{7 b^2 B^2 n^2 \operatorname{Log}[a+bx]}{2 (bc-ad)^3 g i^3} - \\
& \frac{A b^2 B n \operatorname{Log}[a+bx]^2}{(bc-ad)^3 g i^3} + \frac{3 b^2 B^2 n^2 \operatorname{Log}[a+bx]^2}{2 (bc-ad)^3 g i^3} - \frac{b^2 B^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^3 g i^3} - \\
& \frac{b^2 B^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^3 g i^3} - \frac{B n \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad) g i^3 (c+dx)^2} - \frac{3 b B n \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^2 g i^3 (c+dx)} - \\
& \frac{3 b^2 B n \operatorname{Log}[a+bx] \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^3 g i^3} + \frac{\left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad) g i^3 (c+dx)^2} + \\
& \frac{b \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^2 g i^3 (c+dx)} + \frac{b^2 \operatorname{Log}[a+bx] \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^3 g i^3} - \frac{7 b^2 B^2 n^2 \operatorname{Log}[c+dx]}{2 (bc-ad)^3 g i^3} + \\
& \frac{2 A b^2 B n \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3 g i^3} - \frac{3 b^2 B^2 n^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx]}{(bc-ad)^3 g i^3} + \\
& \frac{b^2 B^2 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}[c+dx]}{(bc-ad)^3 g i^3} + \frac{3 b^2 B n \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right) \operatorname{Log}[c+dx]}{(bc-ad)^3 g i^3} - \\
& \frac{b^2 \left(A+B \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^3 g i^3} - \frac{A b^2 B n \operatorname{Log}[c+dx]^2}{(bc-ad)^3 g i^3} + \frac{3 b^2 B^2 n^2 \operatorname{Log}[c+dx]^2}{2 (bc-ad)^3 g i^3} + \\
& \frac{b^2 B^2 n^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx]^2}{(bc-ad)^3 g i^3} - \frac{b^2 B^2 n \operatorname{Log}\left[e \left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}[c+dx]^2}{(bc-ad)^3 g i^3} - \frac{b^2 B^2 n^2 \operatorname{Log}[c+dx]^3}{3 (bc-ad)^3 g i^3} + \\
& \frac{2 A b^2 B n \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3 g i^3} - \frac{3 b^2 B^2 n^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3 g i^3} - \\
& \frac{b^2 B^2 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3 g i^3} + \frac{2 b^2 B^2 n \operatorname{Log}[a+bx] \operatorname{Log}[c+dx] \operatorname{Log}\left[(c+dx)^{-n}\right]}{(bc-ad)^3 g i^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 B^2 \operatorname{Log}[a + b x] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^3 g i^3} - \frac{b^2 B^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^3 g i^3} - \frac{1}{(b c - a d)^3 g i^3} \\
& 2 b^2 B^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}] \right) + \\
& \frac{2 A b^2 B n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} - \frac{3 b^2 B^2 n^2 \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} - \\
& \frac{2 b^2 B^2 n \operatorname{Log}[(a + b x)^n] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{2 A b^2 B n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} - \\
& \frac{3 b^2 B^2 n^2 \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{2 b^2 B^2 n \operatorname{Log}[(c + d x)^{-n}] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} - \frac{1}{(b c - a d)^3 g i^3} \\
& 2 b^2 B^2 n \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}[(c + d x)^{-n}] \right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] + \\
& \frac{2 b^2 B^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^3 g i^3} + \frac{2 b^2 B^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \\
& \frac{2 b^2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^3 g i^3} + \frac{2 b^2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^3 g i^3}
\end{aligned}$$

**Problem 207: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^2 (c i + d i x)^3} dx$$

Optimal (type 3, 562 leaves, 12 steps):

$$\begin{aligned}
& -\frac{B^2 d^3 n^2 (a + b x)^2}{4 (b c - a d)^4 g^2 i^3 (c + d x)^2} - \frac{6 A b B d^2 n (a + b x)}{(b c - a d)^4 g^2 i^3 (c + d x)} + \\
& \frac{6 b B^2 d^2 n^2 (a + b x)}{(b c - a d)^4 g^2 i^3 (c + d x)} - \frac{2 b^3 B^2 n^2 (c + d x)}{(b c - a d)^4 g^2 i^3 (a + b x)} - \frac{6 b B^2 d^2 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d)^4 g^2 i^3 (c + d x)} + \\
& \frac{B d^3 n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^4 g^2 i^3 (c + d x)^2} - \frac{2 b^3 B n (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^4 g^2 i^3 (a + b x)} - \\
& \frac{d^3 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^4 g^2 i^3 (c + d x)^2} + \frac{3 b d^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^4 g^2 i^3 (c + d x)} - \\
& \frac{b^3 (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^4 g^2 i^3 (a + b x)} - \frac{b^2 d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^3}{B (b c - a d)^4 g^2 i^3 n}
\end{aligned}$$

Result (type 4, 2207 leaves, 135 steps):

$$\begin{aligned}
& - \frac{2 b^2 B^2 n^2}{(b c - a d)^3 g^2 i^3 (a + b x)} - \frac{B^2 d n^2}{4 (b c - a d)^2 g^2 i^3 (c + d x)^2} - \frac{11 b B^2 d n^2}{2 (b c - a d)^3 g^2 i^3 (c + d x)} - \\
& \frac{15 b^2 B^2 d n^2 \operatorname{Log}[a + b x]}{2 (b c - a d)^4 g^2 i^3} + \frac{3 A b^2 B d n \operatorname{Log}[a + b x]^2}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B^2 d n^2 \operatorname{Log}[a + b x]^2}{2 (b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^4 g^2 i^3} - \\
& \frac{2 b^2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^3 g^2 i^3 (a + b x)} + \frac{B d n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^2 g^2 i^3 (c + d x)^2} + \\
& \frac{5 b B d n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^3 g^2 i^3 (c + d x)} + \frac{3 b^2 B d n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^4 g^2 i^3} - \\
& \frac{b^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^3 g^2 i^3 (a + b x)} - \frac{d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^2 g^2 i^3 (c + d x)^2} - \frac{2 b d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^3 g^2 i^3 (c + d x)} - \\
& \frac{3 b^2 d \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^4 g^2 i^3} + \frac{15 b^2 B^2 d n^2 \operatorname{Log}[c + d x]}{2 (b c - a d)^4 g^2 i^3} - \\
& \frac{6 A b^2 B d n \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d n^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B^2 d \operatorname{Log}\left[\left(a + b x\right)^n\right]^2 \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} - \frac{3 b^2 B d n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 d \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^4 g^2 i^3} + \frac{3 A b^2 B d n \operatorname{Log}[c + d x]^2}{(b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B^2 d n^2 \operatorname{Log}[c + d x]^2}{2 (b c - a d)^4 g^2 i^3} - \frac{3 b^2 B^2 d n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^4 g^2 i^3} + \frac{b^2 B^2 d n^2 \operatorname{Log}[c + d x]^3}{(b c - a d)^4 g^2 i^3} - \\
& \frac{6 A b^2 B d n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d n^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3} + \\
& \frac{3 b^2 B^2 d \operatorname{Log}\left[\left(a + b x\right)^n\right]^2 \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^4 g^2 i^3} - \frac{6 b^2 B^2 d n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{(b c - a d)^4 g^2 i^3} - \\
& \frac{3 b^2 B^2 d \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^4 g^2 i^3} + \frac{3 b^2 B^2 d \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^4 g^2 i^3} + \frac{1}{(b c - a d)^4 g^2 i^3}
\end{aligned}$$

$$\begin{aligned}
& \frac{6 b^2 B^2 d n \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x]\left(\operatorname{Log}\left[(a+b x)^n\right]-\operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]+\operatorname{Log}\left[(c+d x)^{-n}\right]\right)}{(b c-a d)^4 g^2 i^3}- \\
& \frac{6 A b^2 B d n \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}+\frac{3 b^2 B^2 d n^2 \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}+ \\
& \frac{6 b^2 B^2 d n \operatorname{Log}\left[(a+b x)^n\right] \operatorname{PolyLog}\left[2,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}- \\
& \frac{6 A b^2 B d n \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}+\frac{3 b^2 B^2 d n^2 \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}- \\
& \frac{6 b^2 B^2 d n \operatorname{Log}\left[(c+d x)^{-n}\right] \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}+\frac{1}{(b c-a d)^4 g^2 i^3} \\
& \frac{6 b^2 B^2 d n\left(\operatorname{Log}\left[(a+b x)^n\right]-\operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]+\operatorname{Log}\left[(c+d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}- \\
& \frac{6 b^2 B^2 d n \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right] \operatorname{PolyLog}\left[2,1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^2 i^3}-\frac{6 b^2 B^2 d n^2 \operatorname{PolyLog}\left[3,-\frac{d(a+b x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}- \\
& \frac{6 b^2 B^2 d n^2 \operatorname{PolyLog}\left[3,\frac{b(c+d x)}{b c-a d}\right]}{(b c-a d)^4 g^2 i^3}-\frac{6 b^2 B^2 d n^2 \operatorname{PolyLog}\left[3,1+\frac{b c-a d}{d(a+b x)}\right]}{(b c-a d)^4 g^2 i^3}
\end{aligned}$$

**Problem 208:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}{(a g+b g x)^3(c i+d i x)^3} d x$$

Optimal (type 3, 732 leaves, 14 steps):

$$\begin{aligned}
& \frac{B^2 d^4 n^2 (a+bx)^2}{4 (bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8 AbBd^3 n (a+bx)}{(bc-ad)^5 g^3 i^3 (c+dx)} - \frac{8 b B^2 d^3 n^2 (a+bx)}{(bc-ad)^5 g^3 i^3 (c+dx)} + \\
& \frac{8 b^3 B^2 d n^2 (c+dx)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \frac{b^4 B^2 n^2 (c+dx)^2}{4 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{8 b B^2 d^3 n (a+bx) \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{(bc-ad)^5 g^3 i^3 (c+dx)} - \\
& \frac{B d^4 n (a+bx)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^5 g^3 i^3 (c+dx)^2} + \frac{8 b^3 B d n (c+dx) \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^5 g^3 i^3 (a+bx)} - \\
& \frac{b^4 B n (c+dx)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{d^4 (a+bx)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad)^5 g^3 i^3 (c+dx)^2} - \\
& \frac{4 b d^3 (a+bx) \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5 g^3 i^3 (c+dx)} + \frac{4 b^3 d (c+dx) \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5 g^3 i^3 (a+bx)} - \\
& \frac{b^4 (c+dx)^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad)^5 g^3 i^3 (a+bx)^2} + \frac{2 b^2 d^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^3}{B (bc-ad)^5 g^3 i^3 n}
\end{aligned}$$

Result (type 4, 2041 leaves, 163 steps):

$$\begin{aligned}
& -\frac{b^2 B^2 n^2}{4 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \frac{15 b^2 B^2 d n^2}{2 (bc-ad)^4 g^3 i^3 (a+bx)} + \frac{B^2 d^2 n^2}{4 (bc-ad)^3 g^3 i^3 (c+dx)^2} + \\
& \frac{15 b B^2 d^2 n^2}{2 (bc-ad)^4 g^3 i^3 (c+dx)} + \frac{15 b^2 B^2 d^2 n^2 \operatorname{Log}[a+bx]}{(bc-ad)^5 g^3 i^3} - \frac{6 A b^2 B d^2 n \operatorname{Log}[a+bx]^2}{(bc-ad)^5 g^3 i^3} - \\
& \frac{6 b^2 B^2 d^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^5 g^3 i^3} - \frac{6 b^2 B^2 d^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]^2}{(bc-ad)^5 g^3 i^3} - \\
& \frac{b^2 B n \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \frac{7 b^2 B d n \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^4 g^3 i^3 (a+bx)} - \\
& \frac{B d^2 n \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{2 (bc-ad)^3 g^3 i^3 (c+dx)^2} - \frac{7 b B d^2 n \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)}{(bc-ad)^4 g^3 i^3 (c+dx)} - \\
& \frac{b^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad)^3 g^3 i^3 (a+bx)^2} + \frac{3 b^2 d \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4 g^3 i^3 (a+bx)} + \frac{d^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{2 (bc-ad)^3 g^3 i^3 (c+dx)^2} + \\
& \frac{3 b d^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^4 g^3 i^3 (c+dx)} + \frac{6 b^2 d^2 \operatorname{Log}[a+bx] \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2}{(bc-ad)^5 g^3 i^3} - \\
& \frac{15 b^2 B^2 d^2 n^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} + \frac{12 A b^2 B d^2 n \operatorname{Log}\left[-\frac{d(a+bx)}{b c-a d}\right] \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} + \\
& \frac{6 b^2 B^2 d^2 \operatorname{Log}\left[(a+bx)^n\right]^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} - \frac{6 b^2 d^2 \left(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\right)^2 \operatorname{Log}[c+dx]}{(bc-ad)^5 g^3 i^3} -
\end{aligned}$$

$$\begin{aligned}
& \frac{6 A b^2 B d^2 n \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B^2 d^2 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} - \\
& \frac{6 b^2 B^2 d^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{Log}[c + d x]^2}{(b c - a d)^5 g^3 i^3} - \frac{2 b^2 B^2 d^2 n^2 \operatorname{Log}[c + d x]^3}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 A b^2 B d^2 n \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \frac{6 b^2 B^2 d^2 \operatorname{Log}\left[(a + b x)^n\right]^2 \operatorname{Log}\left[\frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 b^2 B^2 d^2 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[(c + d x)^{-n}\right]}{(b c - a d)^5 g^3 i^3} + \frac{6 b^2 B^2 d^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^5 g^3 i^3} - \\
& \frac{6 b^2 B^2 d^2 \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}\left[(c + d x)^{-n}\right]^2}{(b c - a d)^5 g^3 i^3} - \frac{1}{(b c - a d)^5 g^3 i^3} \\
& \frac{12 b^2 B^2 d^2 n \operatorname{Log}\left[-\frac{d(a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x] \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right)}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 A b^2 B d^2 n \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \\
& \frac{12 b^2 B^2 d^2 n \operatorname{Log}\left[(a + b x)^n\right] \operatorname{PolyLog}\left[2, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 A b^2 B d^2 n \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 b^2 B^2 d^2 n \operatorname{Log}\left[(c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} - \frac{1}{(b c - a d)^5 g^3 i^3} \\
& \frac{12 b^2 B^2 d^2 n \left(\operatorname{Log}\left[(a + b x)^n\right] - \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \operatorname{Log}\left[(c + d x)^{-n}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 b^2 B^2 d^2 n \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 b^2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \\
& \frac{12 b^2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^5 g^3 i^3} + \frac{12 b^2 B^2 d^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^5 g^3 i^3}
\end{aligned}$$

**Problem 209: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(a g + b g x)^4 (c i + d i x)^3} dx$$

Optimal (type 3, 908 leaves, 16 steps):

$$\begin{aligned}
& - \frac{B^2 d^5 n^2 (a + b x)^2}{4 (b c - a d)^6 g^4 i^3 (c + d x)^2} - \frac{10 A b B d^4 n (a + b x)}{(b c - a d)^6 g^4 i^3 (c + d x)} + \\
& \frac{10 b B^2 d^4 n^2 (a + b x)}{(b c - a d)^6 g^4 i^3 (c + d x)} - \frac{20 b^3 B^2 d^2 n^2 (c + d x)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \frac{5 b^4 B^2 d n^2 (c + d x)^2}{4 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \\
& \frac{2 b^5 B^2 n^2 (c + d x)^3}{27 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \frac{10 b B^2 d^4 n (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d)^6 g^4 i^3 (c + d x)} + \\
& \frac{B d^5 n (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^6 g^4 i^3 (c + d x)^2} - \frac{20 b^3 B d^2 n (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^6 g^4 i^3 (a + b x)} + \\
& \frac{5 b^4 B d n (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \frac{2 b^5 B n (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{9 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \\
& \frac{d^5 (a + b x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^6 g^4 i^3 (c + d x)^2} + \frac{5 b d^4 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^6 g^4 i^3 (c + d x)} - \\
& \frac{10 b^3 d^2 (c + d x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d)^6 g^4 i^3 (a + b x)} + \frac{5 b^4 d (c + d x)^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{2 (b c - a d)^6 g^4 i^3 (a + b x)^2} - \\
& \frac{b^5 (c + d x)^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 (b c - a d)^6 g^4 i^3 (a + b x)^3} - \frac{10 b^2 d^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^3}{3 B (b c - a d)^6 g^4 i^3 n}
\end{aligned}$$

Result (type 4, 2610 leaves, 195 steps):

$$\begin{aligned}
& - \frac{2 b^2 B^2 n^2}{27 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \frac{37 b^2 B^2 d n^2}{36 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{319 b^2 B^2 d^2 n^2}{18 (b c - a d)^5 g^4 i^3 (a + b x)} - \\
& \frac{B^2 d^3 n^2}{4 (b c - a d)^4 g^4 i^3 (c + d x)^2} - \frac{19 b B^2 d^3 n^2}{2 (b c - a d)^5 g^4 i^3 (c + d x)} - \frac{245 b^2 B^2 d^3 n^2 \operatorname{Log}[a + b x]}{9 (b c - a d)^6 g^4 i^3} + \\
& \frac{10 A b^2 B d^3 n \operatorname{Log}[a + b x]^2}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 n^2 \operatorname{Log}[a + b x]^2}{3 (b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]^2}{(b c - a d)^6 g^4 i^3} - \frac{2 b^2 B n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{9 (b c - a d)^3 g^4 i^3 (a + b x)^3} + \\
& \frac{11 b^2 B d n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{6 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{47 b^2 B d^2 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 (b c - a d)^5 g^4 i^3 (a + b x)} + \\
& \frac{B d^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 (b c - a d)^4 g^4 i^3 (c + d x)^2} + \frac{9 b B d^3 n \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d)^5 g^4 i^3 (c + d x)} - \\
& \frac{20 b^2 B d^3 n \operatorname{Log}[a + b x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{3 (b c - a d)^6 g^4 i^3} - \frac{b^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{3 (b c - a d)^3 g^4 i^3 (a + b x)^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{3 b^2 d \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 (b c - a d)^4 g^4 i^3 (a + b x)^2} - \frac{6 b^2 d^2 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{(b c - a d)^5 g^4 i^3 (a + b x)} - \frac{d^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{2 (b c - a d)^4 g^4 i^3 (c + d x)^2} - \\
& \frac{4 b d^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{(b c - a d)^5 g^4 i^3 (c + d x)} - \frac{10 b^2 d^3 \operatorname{Log}[a + b x] \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{245 b^2 B^2 d^3 n^2 \operatorname{Log}[c + d x]}{9 (b c - a d)^6 g^4 i^3} - \frac{20 A b^2 B d^3 n \operatorname{Log} \left[ -\frac{d(a+bx)}{b c - a d} \right] \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 n^2 \operatorname{Log} \left[ -\frac{d(a+bx)}{b c - a d} \right] \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} - \frac{10 b^2 B^2 d^3 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \\
& \frac{20 b^2 B d^3 n \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) \operatorname{Log}[c + d x]}{3 (b c - a d)^6 g^4 i^3} + \frac{10 b^2 d^3 \left( A + B \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2 \operatorname{Log}[c + d x]}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 A b^2 B d^3 n \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 n^2 \operatorname{Log}[c + d x]^2}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{10 b^2 B^2 d^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 n \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \operatorname{Log}[c + d x]^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 n^2 \operatorname{Log}[c + d x]^3}{3 (b c - a d)^6 g^4 i^3} - \frac{20 A b^2 B d^3 n \operatorname{Log}[a + b x] \operatorname{Log} \left[ \frac{b(c+dx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 n^2 \operatorname{Log}[a + b x] \operatorname{Log} \left[ \frac{b(c+dx)}{b c - a d} \right]}{3 (b c - a d)^6 g^4 i^3} + \frac{10 b^2 B^2 d^3 \operatorname{Log}[(a + b x)^n]^2 \operatorname{Log} \left[ \frac{b(c+dx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 n \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}[(c + d x)^{-n}]}{(b c - a d)^6 g^4 i^3} - \frac{10 b^2 B^2 d^3 \operatorname{Log}[a + b x] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^6 g^4 i^3} + \\
& \frac{10 b^2 B^2 d^3 \operatorname{Log} \left[ -\frac{d(a+bx)}{b c - a d} \right] \operatorname{Log}[(c + d x)^{-n}]^2}{(b c - a d)^6 g^4 i^3} + \frac{1}{(b c - a d)^6 g^4 i^3} \\
& \frac{20 b^2 B^2 d^3 n \operatorname{Log} \left[ -\frac{d(a+bx)}{b c - a d} \right] \operatorname{Log}[c + d x] \left( \operatorname{Log}[(a + b x)^n] - \operatorname{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] + \operatorname{Log}[(c + d x)^{-n}] \right)}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 A b^2 B d^3 n \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 n^2 \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{b c - a d} \right]}{3 (b c - a d)^6 g^4 i^3} + \\
& \frac{20 b^2 B^2 d^3 n \operatorname{Log}[(a + b x)^n] \operatorname{PolyLog} \left[ 2, -\frac{d(a+bx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 A b^2 B d^3 n \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 n^2 \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{b c - a d} \right]}{3 (b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 n \operatorname{Log}[(c + d x)^{-n}] \operatorname{PolyLog} \left[ 2, \frac{b(c+dx)}{b c - a d} \right]}{(b c - a d)^6 g^4 i^3} + \frac{1}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$



$$\begin{aligned}
& 20 b^2 B^2 d^3 n \left( \text{Log}[(a + b x)^n] - \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] + \text{Log}[(c + d x)^{-n}] \right) \text{PolyLog}\left[2, \frac{b(c + d x)}{b c - a d}\right] - \\
& \frac{20 b^2 B^2 d^3 n \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \text{PolyLog}\left[2, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 n^2 \text{PolyLog}\left[3, -\frac{d(a + b x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \\
& \frac{20 b^2 B^2 d^3 n^2 \text{PolyLog}\left[3, \frac{b(c + d x)}{b c - a d}\right]}{(b c - a d)^6 g^4 i^3} - \frac{20 b^2 B^2 d^3 n^2 \text{PolyLog}\left[3, 1 + \frac{b c - a d}{d(a + b x)}\right]}{(b c - a d)^6 g^4 i^3}
\end{aligned}$$

### Problem 210: Unable to integrate problem.

$$\int (a g + b g x)^m (c i + d i x)^{-2-m} \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^p dx$$

Optimal (type 4, 189 leaves, 3 steps):

$$\begin{aligned}
& \left( e^{-\frac{A(1+m)}{Bn}} (a + b x) (g(a + b x))^m \left( e \left(\frac{a + b x}{c + d x}\right)^n \right)^{-\frac{1+m}{n}} (i(c + d x))^{-m} \right. \\
& \quad \text{Gamma}\left[1 + p, -\frac{(1+m) \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{Bn}\right] \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^p \\
& \quad \left. \left( -\frac{(1+m) \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{Bn} \right)^{-p} \right) / ((b c - a d) i^2 (1+m) (c + d x))
\end{aligned}$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(a g + b g x)^m (c i + d i x)^{-2-m} \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^p, x\right]$$

### Problem 211: Unable to integrate problem.

$$\int (a g + b g x)^{-2-m} (c i + d i x)^m \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^p dx$$

Optimal (type 4, 190 leaves, 3 steps):

$$\begin{aligned}
& - \left( \left( e^{\frac{A(1+m)}{Bn}} (a + b x) (g(a + b x))^{-2-m} \left( e \left(\frac{a + b x}{c + d x}\right)^n \right)^{\frac{1+m}{n}} (i(c + d x))^{2+m} \right. \right. \\
& \quad \text{Gamma}\left[1 + p, \frac{(1+m) \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{Bn}\right] \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)^p \\
& \quad \left. \left( \frac{(1+m) \left( A + B \text{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right)}{Bn} \right)^{-p} \right) / ((b c - a d) i^2 (1+m) (c + d x))
\end{aligned}$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\left(a g + b g x\right)^{-2-m}\left(c i + d i x\right)^m\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^p, x\right]$$

### Problem 212: Unable to integrate problem.

$$\int\left(a g + b g x\right)^m\left(c i + d i x\right)^{-2-m}\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^3 d x$$

Optimal (type 3, 292 leaves, 4 steps):

$$\begin{aligned} & -\frac{6 B^3 n^3 (a + b x) (g (a + b x))^m (i (c + d x))^{-m}}{(b c - a d) i^2 (1 + m)^4 (c + d x)} + \\ & \frac{6 B^2 n^2 (a + b x) (g (a + b x))^m (i (c + d x))^{-m} \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{(b c - a d) i^2 (1 + m)^3 (c + d x)} - \\ & \frac{3 B n (a + b x) (g (a + b x))^m (i (c + d x))^{-m} \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2}{(b c - a d) i^2 (1 + m)^2 (c + d x)} + \\ & \frac{(a + b x) (g (a + b x))^m (i (c + d x))^{-m} \left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^3}{(b c - a d) i^2 (1 + m) (c + d x)} \end{aligned}$$

Result (type 8, 281 leaves, 6 steps):

$$\begin{aligned} & \frac{A^3 (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)} - \frac{3 A^2 B n (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)^2} + \\ & 3 A B^2 \text{CannotIntegrate}\left[\left(a g + b g x\right)^m\left(c i + d i x\right)^{-2-m} \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^2, x\right] + \\ & B^3 \text{CannotIntegrate}\left[\left(a g + b g x\right)^m\left(c i + d i x\right)^{-2-m} \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]^3, x\right] + \\ & \frac{3 A^2 B (a g + b g x)^{1+m} (c i + d i x)^{-1-m} \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]}{(b c - a d) g i (1 + m)} \end{aligned}$$

### Problem 213: Unable to integrate problem.

$$\int\left(a g + b g x\right)^m\left(c i + d i x\right)^{-2-m}\left(A + B \operatorname{Log}\left[e\left(\frac{a + b x}{c + d x}\right)^n\right]\right)^2 d x$$

Optimal (type 3, 210 leaves, 3 steps):

$$\begin{aligned}
& \frac{2 B^2 n^2 (a + b x) (g(a + b x))^m (i(c + d x))^{-m}}{(b c - a d) i^2 (1 + m)^3 (c + d x)} - \\
& \frac{2 B n (a + b x) (g(a + b x))^m (i(c + d x))^{-m} \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d) i^2 (1 + m)^2 (c + d x)} + \\
& \frac{(a + b x) (g(a + b x))^m (i(c + d x))^{-m} \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)^2}{(b c - a d) i^2 (1 + m) (c + d x)}
\end{aligned}$$

Result (type 8, 224 leaves, 6 steps):

$$\begin{aligned}
& \frac{A^2 (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)} - \frac{2 A B n (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)^2} + \\
& B^2 \operatorname{CannotIntegrate}\left[ (a g + b g x)^m (c i + d i x)^{-2-m} \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]^2, x \right] + \\
& \frac{2 A B (a g + b g x)^{1+m} (c i + d i x)^{-1-m} \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(b c - a d) g i (1 + m)}
\end{aligned}$$

**Problem 214: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^m (c i + d i x)^{-2-m} \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right) dx$$

Optimal (type 3, 128 leaves, 2 steps):

$$\begin{aligned}
& - \frac{B n (a + b x) (g(a + b x))^m (i(c + d x))^{-m}}{(b c - a d) i^2 (1 + m)^2 (c + d x)} + \\
& \frac{(a + b x) (g(a + b x))^m (i(c + d x))^{-m} \left( A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right] \right)}{(b c - a d) i^2 (1 + m) (c + d x)}
\end{aligned}$$

Result (type 3, 168 leaves, 6 steps):

$$\begin{aligned}
& \frac{A (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)} - \frac{B n (a g + b g x)^{1+m} (c i + d i x)^{-1-m}}{(b c - a d) g i (1 + m)^2} + \\
& \frac{B (a g + b g x)^{1+m} (c i + d i x)^{-1-m} \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]}{(b c - a d) g i (1 + m)}
\end{aligned}$$

**Problem 215: Unable to integrate problem.**

$$\int \frac{(a g + b g x)^m (c i + d i x)^{-2-m}}{A + B \operatorname{Log}\left[e \left( \frac{a + b x}{c + d x} \right)^n \right]} dx$$

Optimal (type 4, 125 leaves, 3 steps):

$$\left( e^{-\frac{A(1+m)}{Bn}} (a+bx) (g(a+bx))^m \left( e^{\left( \frac{a+bx}{c+dx} \right)^n} \right)^{-\frac{1+m}{n}} (i(c+dx))^{-m} \right. \\ \left. \text{ExpIntegralEi} \left[ \frac{(1+m) \left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right]}{Bn} \right] \right) \right] / (B(b c - a d) i^2 n (c+dx))$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{(a g + b g x)^m (c i + d i x)^{-2-m}}{A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right]}, x \right]$$

Problem 216: Unable to integrate problem.

$$\int \frac{(a g + b g x)^m (c i + d i x)^{-2-m}}{\left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right] \right)^2} dx$$

Optimal (type 4, 206 leaves, 4 steps):

$$\left( e^{-\frac{A(1+m)}{Bn}} (1+m) (a+bx) (g(a+bx))^m \left( e^{\left( \frac{a+bx}{c+dx} \right)^n} \right)^{-\frac{1+m}{n}} \right. \\ \left. (i(c+dx))^{-m} \text{ExpIntegralEi} \left[ \frac{(1+m) \left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right]}{Bn} \right] \right) \right] / \\ (B^2 (b c - a d) i^2 n^2 (c+dx)) - \frac{(a+bx) (g(a+bx))^m (i(c+dx))^{-m}}{B(b c - a d) i^2 n (c+dx) \left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right] \right)}$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{(a g + b g x)^m (c i + d i x)^{-2-m}}{\left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right] \right)^2}, x \right]$$

Problem 217: Unable to integrate problem.

$$\int \frac{(a g + b g x)^m (c i + d i x)^{-2-m}}{\left( A + B \text{Log} \left[ e^{\left( \frac{a+bx}{c+dx} \right)^n} \right] \right)^3} dx$$

Optimal (type 4, 295 leaves, 5 steps):

$$\begin{aligned}
& \left( e^{-\frac{A(1+m)}{Bn}} (1+m)^2 (a+bx) (g(a+bx))^m \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-\frac{1+m}{n}} (i(c+dx))^{-m} \right. \\
& \quad \left. \text{ExpIntegralEi} \left[ \frac{(1+m) \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{Bn} \right] \right) / (2B^3 (bc-ad) i^2 n^3 (c+dx)) - \\
& \quad \frac{(a+bx) (g(a+bx))^m (i(c+dx))^{-m}}{2B (bc-ad) i^2 n (c+dx) \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2} - \\
& \quad \frac{(1+m) (a+bx) (g(a+bx))^m (i(c+dx))^{-m}}{2B^2 (bc-ad) i^2 n^2 (c+dx) \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}
\end{aligned}$$

Result (type 8, 51 leaves, 0 steps):

$$\text{CannotIntegrate} \left[ \frac{(ag+bgx)^m (ci+dix)^{-2-m}}{\left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^3}, x \right]$$

### Problem 218: Unable to integrate problem.

$$\int (ag+bgx)^{-2-m} (ci+dix)^m \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^3 dx$$

Optimal (type 3, 309 leaves, 4 steps):

$$\begin{aligned}
& - \frac{6B^3 n^3 (a+bx) (g(a+bx))^{-2-m} (i(c+dx))^{2+m}}{(bc-ad) i^2 (1+m)^4 (c+dx)} - \\
& \left( \frac{6B^2 n^2 (a+bx) (g(a+bx))^{-2-m} (i(c+dx))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}{(bc-ad) i^2 (1+m)^3 (c+dx)} - \right. \\
& \left. \frac{3Bn (a+bx) (g(a+bx))^{-2-m} (i(c+dx))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}{(bc-ad) i^2 (1+m)^2 (c+dx)} - \right. \\
& \left. \frac{(a+bx) (g(a+bx))^{-2-m} (i(c+dx))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^3}{(bc-ad) i^2 (1+m) (c+dx)} \right)
\end{aligned}$$

Result (type 8, 282 leaves, 6 steps):

$$\begin{aligned}
& - \frac{A^3 (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)} - \frac{3 A^2 B n (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)^2} + \\
& 3 A B^2 \text{CannotIntegrate} \left[ (a g + b g x)^{-2-m} (c i + d i x)^m \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]^2, x \right] + \\
& B^3 \text{CannotIntegrate} \left[ (a g + b g x)^{-2-m} (c i + d i x)^m \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]^3, x \right] - \\
& \frac{3 A^2 B (a g + b g x)^{-1-m} (c i + d i x)^{1+m} \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{(b c - a d) g i (1+m)}
\end{aligned}$$

**Problem 219: Unable to integrate problem.**

$$\int (a g + b g x)^{-2-m} (c i + d i x)^m \left( A + B \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2 dx$$

Optimal (type 3, 223 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 B^2 n^2 (a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m}}{(b c - a d) i^2 (1+m)^3 (c+d x)} - \\
& \frac{2 B n (a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(b c - a d) i^2 (1+m)^2 (c+d x)} - \\
& \frac{(a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}{(b c - a d) i^2 (1+m) (c+d x)}
\end{aligned}$$

Result (type 8, 225 leaves, 6 steps):

$$\begin{aligned}
& - \frac{A^2 (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)} - \frac{2 A B n (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)^2} + \\
& B^2 \text{CannotIntegrate} \left[ (a g + b g x)^{-2-m} (c i + d i x)^m \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]^2, x \right] - \\
& \frac{2 A B (a g + b g x)^{-1-m} (c i + d i x)^{1+m} \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}{(b c - a d) g i (1+m)}
\end{aligned}$$

**Problem 220: Result valid but suboptimal antiderivative.**

$$\int (a g + b g x)^{-2-m} (c i + d i x)^m \left( A + B \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right) dx$$

Optimal (type 3, 137 leaves, 2 steps):

$$\begin{aligned}
& - \frac{B n (a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m}}{(b c - a d) i^2 (1+m)^2 (c+d x)} - \\
& \frac{(a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m} \left( A + B \text{Log} \left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{(b c - a d) i^2 (1+m) (c+d x)}
\end{aligned}$$

Result(type 3, 170 leaves, 6 steps):

$$-\frac{A (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)} - \frac{B n (a g + b g x)^{-1-m} (c i + d i x)^{1+m}}{(b c - a d) g i (1+m)^2} - \frac{B (a g + b g x)^{-1-m} (c i + d i x)^{1+m} \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{(b c - a d) g i (1+m)}$$

Problem 221: Unable to integrate problem.

$$\int \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]} dx$$

Optimal(type 4, 128 leaves, 3 steps):

$$\left( e^{\frac{A(1+m)}{B n}} (a+b x) (g(a+b x))^{-2-m} \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{\frac{1+m}{n}} (i(c+d x))^{2+m} \right. \\ \left. \operatorname{ExpIntegralEi}\left[ -\frac{(1+m) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{B n} \right] \right) / (B (b c - a d) i^2 n (c+d x))$$

Result(type 8, 51 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[ \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right]}, x \right]$$

Problem 222: Unable to integrate problem.

$$\int \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} dx$$

Optimal(type 4, 214 leaves, 4 steps):

$$-\left( e^{\frac{A(1+m)}{B n}} (1+m) (a+b x) (g(a+b x))^{-2-m} \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{\frac{1+m}{n}} (i(c+d x))^{2+m} \right. \\ \left. \operatorname{ExpIntegralEi}\left[ -\frac{(1+m) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{B n} \right] \right) / (B^2 (b c - a d) i^2 n^2 (c+d x)) - \frac{(a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m}}{B (b c - a d) i^2 n (c+d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}$$

Result(type 8, 51 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[ \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{\left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2}, x \right]$$

### Problem 223: Unable to integrate problem.

$$\int \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{\left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^3} dx$$

Optimal (type 4, 306 leaves, 5 steps):

$$\begin{aligned} & \left( e^{\frac{A(1+m)}{Bn}} (1+m)^2 (a+b x) (g(a+b x))^{-2-m} \left( e \left( \frac{a+b x}{c+d x} \right)^n \right)^{\frac{1+m}{n}} \right. \\ & \quad \left. (i(c+d x))^{2+m} \operatorname{ExpIntegralEi} \left[ -\frac{(1+m) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)}{Bn} \right] \right) / \\ & \quad \left( 2 B^3 (b c - a d) i^2 n^3 (c+d x) \right) - \frac{(a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m}}{2 B (b c - a d) i^2 n (c+d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)^2} + \\ & \quad \frac{(1+m) (a+b x) (g(a+b x))^{-2-m} (i(c+d x))^{2+m}}{2 B^2 (b c - a d) i^2 n^2 (c+d x) \left( A + B \operatorname{Log}\left[ e \left( \frac{a+b x}{c+d x} \right)^n \right] \right)} \end{aligned}$$

Result (type 8, 51 leaves, 0 steps):

$$\operatorname{CannotIntegrate} \left[ \frac{(a g + b g x)^{-2-m} (c i + d i x)^m}{\left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)^3}, x \right]$$

### Problem 226: Unable to integrate problem.

$$\int (a g + b g x)^m (c i + d i x)^{-2-m} (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])^p dx$$

Optimal (type 4, 193 leaves, 4 steps):

$$\begin{aligned} & \left( e^{-\frac{A(1+m)}{Bn}} (a+b x) (g(a+b x))^m (i(c+d x))^{-m} (e (a+b x)^n (c+d x)^{-n})^{-\frac{1+m}{n}} \right. \\ & \quad \left. \operatorname{Gamma} \left[ 1+p, -\frac{(1+m) (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])}{Bn} \right] (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])^p \right. \\ & \quad \left. \left( -\frac{(1+m) (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])}{Bn} \right)^{-p} \right) / ((b c - a d) i^2 (1+m) (c+d x)) \end{aligned}$$

Result (type 8, 52 leaves, 0 steps):

$$\operatorname{CannotIntegrate} \left[ (a g + b g x)^m (c i + d i x)^{-2-m} (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])^p, x \right]$$

### Problem 227: Unable to integrate problem.

$$\int (a g + b g x)^{-2-m} (c i + d i x)^m (A + B \operatorname{Log}[e (a+b x)^n (c+d x)^{-n}])^p dx$$



Optimal (type 4, 194 leaves, 4 steps):

$$- \left( \left( e^{\frac{A(1+m)}{Bn}} (a+bx) (g(a+bx))^{-2-m} (i(c+dx))^{2+m} (e(a+bx)^n (c+dx)^{-n})^{\frac{1+m}{n}} \right. \right. \\ \left. \left. \text{Gamma}\left[1+p, \frac{(1+m)(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])}{Bn}\right] (A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^p \right. \right. \\ \left. \left. \left( \frac{(1+m)(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])}{Bn} \right)^{-p} \right) / ((bc-ad) i^2 (1+m) (c+dx)) \right)$$

Result (type 8, 52 leaves, 0 steps):

$$\text{CannotIntegrate}\left[(ag+bgx)^{-2-m} (ci+di x)^m (A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^p, x\right]$$

**Problem 249: Result valid but suboptimal antiderivative.**

$$\int \frac{(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^4}{(f+gx)(ah+bhx)} dx$$

Optimal (type 4, 361 leaves, 8 steps):

$$- \frac{(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^4 \text{Log}\left[1 - \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right]}{(bf-ag)h} + \\ \frac{4Bn(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^3 \text{PolyLog}\left[2, \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right]}{(bf-ag)h} + \frac{1}{(bf-ag)h} \\ 12B^2n^2(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}])^2 \text{PolyLog}\left[3, \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right] + \\ \frac{24B^3n^3(A+B\text{Log}[e(a+bx)^n (c+dx)^{-n}]) \text{PolyLog}\left[4, \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right]}{(bf-ag)h} + \\ \frac{24B^4n^4 \text{PolyLog}\left[5, \frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right]}{(bf-ag)h}$$

Result (type 4, 1021 leaves, 20 steps):

$$\begin{aligned}
& \frac{A^4 \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A^4 \operatorname{Log}[f + g x]}{(b f - a g) h} - \frac{4 A^3 B \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} - \\
& \frac{6 A^2 B^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 \operatorname{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} - \\
& \frac{4 A B^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3 \operatorname{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} - \\
& \frac{B^4 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^4 \operatorname{Log}\left[-\frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \frac{4 A^3 B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{12 A^2 B^2 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{12 A B^3 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{4 B^4 n \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^3 \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{12 A^2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{24 A B^3 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{12 B^4 n^2 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right]^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{24 A B^3 n^3 \operatorname{PolyLog}\left[4, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{24 B^4 n^3 \operatorname{Log}\left[e (a + b x)^n (c + d x)^{-n}\right] \operatorname{PolyLog}\left[4, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h} + \\
& \frac{24 B^4 n^4 \operatorname{PolyLog}\left[5, 1 + \frac{(b c - a d) (f + g x)}{(d f - c g) (a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

**Problem 250: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3}{(f + g x) (a h + b h x)} dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{3 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^2 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[3, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Result (type 4, 656 leaves, 15 steps):

$$\begin{aligned}
& \frac{A^3 \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A^3 \operatorname{Log}[f + g x]}{(b f - a g) h} - \frac{3 A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{3 A B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{B^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{3 A^2 B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 A B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{3 B^3 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 A B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

**Problem 251: Result valid but suboptimal antiderivative.**

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{(f + g x)(a h + b h x)} dx$$

Optimal (type 4, 203 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Result (type 4, 371 leaves, 11 steps):

$$\begin{aligned}
& \frac{A^2 \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A^2 \operatorname{Log}[f + g x]}{(b f - a g) h} - \frac{2 A B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{2 A B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Problem 252: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{(f + g x)(a h + b h x)} dx$$

Optimal (type 4, 123 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{B n \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Result (type 4, 163 leaves, 8 steps):

$$\begin{aligned}
& \frac{A \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A \operatorname{Log}[f + g x]}{(b f - a g) h} - \\
& \frac{B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Problem 253: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{1}{(f + g x)(a h + b h x)(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])} dx$$

Optimal (type 9, 81 leaves, 1 step):

$$\text{Subst}\left[\text{Unintegrable}\left[\frac{1}{(f+g x)(a h+b h x)\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)}, x\right],\right. \\ \left.e\left(\frac{a+b x}{c+d x}\right)^n, e(a+b x)^n(c+d x)^{-n}\right]$$

Result (type 8, 102 leaves, 2 steps):

$$\frac{b \operatorname{CannotIntegrate}\left[\frac{1}{(a+b x)\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)}, x\right]}{(b f-a g) h}- \\ \frac{g \operatorname{CannotIntegrate}\left[\frac{1}{(f+g x)\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)}, x\right]}{(b f-a g) h}$$

**Problem 254: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{1}{(f+g x)(a h+b h x)\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)^2} d x$$

Optimal (type 9, 81 leaves, 1 step):

$$\text{Subst}\left[\text{Unintegrable}\left[\frac{1}{(f+g x)(a h+b h x)\left(A+B \operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]\right)^2}, x\right],\right. \\ \left.e\left(\frac{a+b x}{c+d x}\right)^n, e(a+b x)^n(c+d x)^{-n}\right]$$

Result (type 8, 102 leaves, 2 steps):

$$\frac{b \operatorname{CannotIntegrate}\left[\frac{1}{(a+b x)\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)^2}, x\right]}{(b f-a g) h}- \\ \frac{g \operatorname{CannotIntegrate}\left[\frac{1}{(f+g x)\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)^2}, x\right]}{(b f-a g) h}$$

**Problem 259: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(A+B \operatorname{Log}\left[e(a+b x)^n(c+d x)^{-n}\right]\right)^3}{a f h+b g h x^2+h(b f x+a g x)} d x$$

Optimal (type 4, 282 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^3 \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{3 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^2 n^2 (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[3, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Result (type 4, 656 leaves, 17 steps):

$$\begin{aligned}
& \frac{A^3 \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A^3 \operatorname{Log}[f + g x]}{(b f - a g) h} - \frac{3 A^2 B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{3 A B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{B^3 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^3 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{3 A^2 B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 A B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{3 B^3 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 A B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{6 B^3 n^3 \operatorname{PolyLog}\left[4, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2}{a f h + b g h x^2 + h (b f x + a g x)} dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])^2 \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B n (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

Result (type 4, 371 leaves, 13 steps):

$$\begin{aligned}
& \frac{A^2 \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A^2 \operatorname{Log}[f + g x]}{(b f - a g) h} - \frac{2 A B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} - \\
& \frac{B^2 \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]^2 \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{2 A B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \\
& \frac{2 B^2 n^2 \operatorname{PolyLog}\left[3, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

**Problem 261: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]}{a f h + b g h x^2 + h (b f x + a g x)} dx$$

Optimal (type 4, 123 leaves, 6 steps):

$$- \frac{(A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}]) \operatorname{Log}\left[1 - \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{B n \operatorname{PolyLog}\left[2, \frac{(b f - a g)(c + d x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}$$

Result (type 4, 163 leaves, 10 steps):

$$\begin{aligned}
& \frac{A \operatorname{Log}[a + b x]}{(b f - a g) h} - \frac{A \operatorname{Log}[f + g x]}{(b f - a g) h} - \\
& \frac{B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}] \operatorname{Log}\left[-\frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h} + \frac{B n \operatorname{PolyLog}\left[2, 1 + \frac{(b c - a d)(f + g x)}{(d f - c g)(a + b x)}\right]}{(b f - a g) h}
\end{aligned}$$

**Problem 262: Rubi result verified and simpler than optimal antiderivative.**

$$\int \frac{1}{(a f h + b g h x^2 + h (b f x + a g x)) (A + B \operatorname{Log}[e (a + b x)^n (c + d x)^{-n}])} dx$$

Optimal (type 9, 82 leaves, 3 steps):

$$\frac{1}{h} \text{Subst} \left[ \text{Unintegrable} \left[ \frac{1}{(a+bx)(f+gx) \left( A+B \log \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)}, x \right], \right. \\ \left. e \left( \frac{a+bx}{c+dx} \right)^n, e(a+bx)^n(c+dx)^{-n} \right]$$

Result (type 8, 102 leaves, 4 steps):

$$\frac{b \text{ CannotIntegrate} \left[ \frac{1}{(a+bx) (A+B \log [e (a+bx)^n (c+dx)^{-n}])}, x \right]}{(bf-ag)h} - \\ \frac{g \text{ CannotIntegrate} \left[ \frac{1}{(f+gx) (A+B \log [e (a+bx)^n (c+dx)^{-n}])}, x \right]}{(bf-ag)h}$$

Problem 263: Rubi result verified and simpler than optimal antiderivative.

$$\int \left( 1 / \left( (afh + bghx^2 + h(bfx + agx)) (A+B \log [e(a+bx)^n(c+dx)^{-n}])^2 \right) \right) dx$$

Optimal (type 9, 82 leaves, 3 steps):

$$\frac{1}{h} \text{Subst} \left[ \text{Unintegrable} \left[ \frac{1}{(a+bx)(f+gx) \left( A+B \log \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right)^2}, x \right], \right. \\ \left. e \left( \frac{a+bx}{c+dx} \right)^n, e(a+bx)^n(c+dx)^{-n} \right]$$

Result (type 8, 102 leaves, 4 steps):

$$\frac{b \text{ CannotIntegrate} \left[ \frac{1}{(a+bx) (A+B \log [e (a+bx)^n (c+dx)^{-n}])^2}, x \right]}{(bf-ag)h} - \\ \frac{g \text{ CannotIntegrate} \left[ \frac{1}{(f+gx) (A+B \log [e (a+bx)^n (c+dx)^{-n}])^2}, x \right]}{(bf-ag)h}$$

Test results for the 108 problems in "3.2.3 u log(e (f (a+bx)^p (c+dx)^q)^r)^s.m"

Problem 1: Result valid but suboptimal antiderivative.

$$\int \left( f + \frac{g}{x} \right)^3 \left( A+B \log \left[ e \left( \frac{a+bx}{c+dx} \right)^n \right] \right) dx$$

Optimal (type 4, 404 leaves, 16 steps):



$$\begin{aligned}
& - \frac{B(b c - a d) g^3 n}{2 a c x} + A f^3 x - \frac{1}{2} B \left( \frac{b^2}{a^2} - \frac{d^2}{c^2} \right) g^3 n \operatorname{Log}[x] + \frac{b^2 B g^3 n \operatorname{Log}[a + b x]}{2 a^2} - \\
& 3 B f^2 g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f^3 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} - \frac{g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 x^2} + \\
& \frac{3(b c - a d) f g^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{a(c + d x) \left(a - \frac{c(a + b x)}{c + d x}\right)} + 3 f^2 g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \\
& \frac{B(b c - a d) f^3 n \operatorname{Log}[c + d x]}{b d} - \frac{B d^2 g^3 n \operatorname{Log}[c + d x]}{2 c^2} + 3 B f^2 g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] + \\
& \frac{3 B(b c - a d) f g^2 n \operatorname{Log}\left[a - \frac{c(a + b x)}{c + d x}\right]}{a c} - 3 B f^2 g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + 3 B f^2 g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
\end{aligned}$$

Result (type 4, 385 leaves, 20 steps):

$$\begin{aligned}
& - \frac{B(b c - a d) g^3 n}{2 a c x} + A f^3 x + \frac{3 B(b c - a d) f g^2 n \operatorname{Log}[x]}{a c} - \\
& \frac{1}{2} B \left( \frac{b^2}{a^2} - \frac{d^2}{c^2} \right) g^3 n \operatorname{Log}[x] - \frac{3 b B f g^2 n \operatorname{Log}[a + b x]}{a} + \frac{b^2 B g^3 n \operatorname{Log}[a + b x]}{2 a^2} - \\
& 3 B f^2 g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f^3 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} - \frac{g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{2 x^2} - \\
& \frac{3 f g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{x} + 3 f^2 g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \\
& \frac{B(b c - a d) f^3 n \operatorname{Log}[c + d x]}{b d} + \frac{3 B d f g^2 n \operatorname{Log}[c + d x]}{c} - \frac{B d^2 g^3 n \operatorname{Log}[c + d x]}{2 c^2} + \\
& 3 B f^2 g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] - 3 B f^2 g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + 3 B f^2 g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
\end{aligned}$$

Problem 2: Result valid but suboptimal antiderivative.

$$\int \left( f + \frac{g}{x} \right)^2 \left( A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right] \right) dx$$

Optimal (type 4, 263 leaves, 13 steps):

$$\begin{aligned}
& A f^2 x - 2 B f g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f^2 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} + \\
& \frac{(b c - a d) g^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{a(c + d x) \left(a - \frac{c(a + b x)}{c + d x}\right)} + 2 f g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \\
& \frac{B(b c - a d) f^2 n \operatorname{Log}[c + d x]}{b d} + 2 B f g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] + \\
& \frac{B(b c - a d) g^2 n \operatorname{Log}\left[a - \frac{c(a + b x)}{c + d x}\right]}{a c} - 2 B f g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + 2 B f g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
\end{aligned}$$

Result (type 4, 242 leaves, 16 steps):

$$\begin{aligned}
 & A f^2 x + \frac{B (b c - a d) g^2 n \operatorname{Log}[x]}{a c} - \frac{b B g^2 n \operatorname{Log}[a + b x]}{a} - \\
 & 2 B f g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f^2 (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} - \frac{g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right)}{x} + \\
 & 2 f g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \frac{B (b c - a d) f^2 n \operatorname{Log}[c + d x]}{b d} + \frac{B d g^2 n \operatorname{Log}[c + d x]}{c} + \\
 & 2 B f g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] - 2 B f g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + 2 B f g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
 \end{aligned}$$

Problem 3: Result optimal but 2 more steps used.

$$\int \left(f + \frac{g}{x}\right) \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) dx$$

Optimal (type 4, 143 leaves, 10 steps):

$$\begin{aligned}
 & A f x - B g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} + \\
 & g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \frac{B (b c - a d) f n \operatorname{Log}[c + d x]}{b d} + \\
 & B g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] - B g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + B g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
 \end{aligned}$$

Result (type 4, 143 leaves, 12 steps):

$$\begin{aligned}
 & A f x - B g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \frac{B f (a + b x) \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{b} + \\
 & g \operatorname{Log}[x] \left(A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]\right) - \frac{B (b c - a d) f n \operatorname{Log}[c + d x]}{b d} + \\
 & B g n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{d x}{c}\right] - B g n \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right] + B g n \operatorname{PolyLog}\left[2, -\frac{d x}{c}\right]
 \end{aligned}$$

Problem 4: Result optimal but 2 more steps used.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a + b x}{c + d x}\right)^n\right]}{f + \frac{g}{x}} dx$$

Optimal (type 4, 217 leaves, 12 steps):

$$\begin{aligned}
& \frac{A x}{f} + \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b f} - \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f} + \\
& \frac{B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^2} - \frac{g \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[g + f x]}{f^2} - \\
& \frac{B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^2} + \frac{B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^2} - \frac{B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^2}
\end{aligned}$$

Result (type 4, 217 leaves, 14 steps):

$$\begin{aligned}
& \frac{A x}{f} + \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b f} - \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f} + \\
& \frac{B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^2} - \frac{g \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[g + f x]}{f^2} - \\
& \frac{B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^2} + \frac{B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^2} - \frac{B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^2}
\end{aligned}$$

Problem 5: Result valid but suboptimal antiderivative.

$$\int \frac{A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{\left(f + \frac{g}{x}\right)^2} dx$$

Optimal (type 4, 322 leaves, 15 steps):

$$\begin{aligned}
& \frac{A x}{f^2} + \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b f^2} - \frac{g^2 (a + b x) \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{f^2 (a f - b g) (g + f x)} - \\
& \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f^2} + \frac{2 B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^3} - \\
& \frac{2 g \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[g + f x]}{f^3} - \frac{2 B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^3} + \\
& \frac{B (b c - a d) g^2 n \operatorname{Log}\left[\frac{g+f x}{c+d x}\right]}{f^2 (a f - b g) (c f - d g)} + \frac{2 B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^3} - \frac{2 B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^3}
\end{aligned}$$

Result (type 4, 352 leaves, 18 steps):

$$\begin{aligned}
& \frac{A x}{f^2} - \frac{b B g^2 n \operatorname{Log}[a + b x]}{f^3 (a f - b g)} + \frac{B (a + b x) \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]}{b f^2} - \frac{g^2 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{f^3 (g + f x)} - \\
& \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f^2} + \frac{B d g^2 n \operatorname{Log}[c + d x]}{f^3 (c f - d g)} + \frac{B (b c - a d) g^2 n \operatorname{Log}[g + f x]}{f^2 (a f - b g) (c f - d g)} + \\
& \frac{2 B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^3} - \frac{2 g \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right) \operatorname{Log}[g + f x]}{f^3} - \\
& \frac{2 B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^3} + \frac{2 B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^3} - \frac{2 B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^3}
\end{aligned}$$

**Problem 6: Result valid but suboptimal antiderivative.**

$$\int \frac{A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]}{\left(f + \frac{g}{x}\right)^3} dx$$

Optimal (type 4, 531 leaves, 18 steps):

$$\begin{aligned}
& \frac{A x}{f^3} + \frac{B (b c - a d) g^3 n}{2 f^3 (a f - b g) (c f - d g) (g + f x)} - \frac{b^2 B g^3 n \operatorname{Log}[a + b x]}{2 f^4 (a f - b g)^2} + \\
& \frac{B (a + b x) \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]}{b f^3} + \frac{g^3 \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{2 f^4 (g + f x)^2} - \\
& \frac{3 g^2 (a + b x) \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right)}{f^3 (a f - b g) (g + f x)} - \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f^3} + \frac{B d^2 g^3 n \operatorname{Log}[c + d x]}{2 f^4 (c f - d g)^2} + \\
& \frac{B (b c - a d) g^3 (b c f + a d f - 2 b d g) n \operatorname{Log}[g + f x]}{2 f^3 (a f - b g)^2 (c f - d g)^2} + \frac{3 B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^4} - \\
& \frac{3 g \left(A + B \operatorname{Log}\left[e^{\left(\frac{a+b x}{c+d x}\right)^n}\right]\right) \operatorname{Log}[g + f x]}{f^4} - \frac{3 B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^4} + \\
& \frac{3 B (b c - a d) g^2 n \operatorname{Log}\left[\frac{g+f x}{c+d x}\right]}{f^3 (a f - b g) (c f - d g)} + \frac{3 B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^4} - \frac{3 B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^4}
\end{aligned}$$

Result (type 4, 562 leaves, 22 steps):

$$\begin{aligned}
& \frac{A x}{f^3} + \frac{B (b c - a d) g^3 n}{2 f^3 (a f - b g) (c f - d g) (g + f x)} - \frac{b^2 B g^3 n \operatorname{Log}[a + b x]}{2 f^4 (a f - b g)^2} - \frac{3 b B g^2 n \operatorname{Log}[a + b x]}{f^4 (a f - b g)} + \\
& \frac{B (a + b x) \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]}{b f^3} + \frac{g^3 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{2 f^4 (g + f x)^2} - \frac{3 g^2 \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right)}{f^4 (g + f x)} - \\
& \frac{B (b c - a d) n \operatorname{Log}[c + d x]}{b d f^3} + \frac{B d^2 g^3 n \operatorname{Log}[c + d x]}{2 f^4 (c f - d g)^2} + \frac{3 B d g^2 n \operatorname{Log}[c + d x]}{f^4 (c f - d g)} + \\
& \frac{3 B (b c - a d) g^2 n \operatorname{Log}[g + f x]}{f^3 (a f - b g) (c f - d g)} + \frac{B (b c - a d) g^3 (b c f + a d f - 2 b d g) n \operatorname{Log}[g + f x]}{2 f^3 (a f - b g)^2 (c f - d g)^2} + \\
& \frac{3 B g n \operatorname{Log}\left[\frac{f (a+b x)}{a f - b g}\right] \operatorname{Log}[g + f x]}{f^4} - \frac{3 g \left(A + B \operatorname{Log}\left[e \left(\frac{a+b x}{c+d x}\right)^n\right]\right) \operatorname{Log}[g + f x]}{f^4} - \\
& \frac{3 B g n \operatorname{Log}\left[\frac{f (c+d x)}{c f - d g}\right] \operatorname{Log}[g + f x]}{f^4} + \frac{3 B g n \operatorname{PolyLog}\left[2, -\frac{b (g+f x)}{a f - b g}\right]}{f^4} - \frac{3 B g n \operatorname{PolyLog}\left[2, -\frac{d (g+f x)}{c f - d g}\right]}{f^4}
\end{aligned}$$

**Problem 39: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}\left[e \left(f (a + b x)^p (c + d x)^q\right)^r\right]^2}{g + h x} dx$$

Optimal (type 4, 1471 leaves, ? steps):

$$\begin{aligned}
& \frac{p q r^2 \operatorname{Log}\left[-\frac{b c - a d}{d (a + b x)}\right] \operatorname{Log}\left[\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}\right]^2}{h} + \frac{p^2 r^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}[g + h x]}{h} + \\
& \frac{2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}[g + h x]}{h} + \frac{q^2 r^2 \operatorname{Log}[c + d x]^2 \operatorname{Log}[g + h x]}{h} - \\
& \frac{2 p r \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(f (a + b x)^p (c + d x)^q\right)^r\right] \operatorname{Log}[g + h x]}{h} - \\
& \frac{2 q r \operatorname{Log}[c + d x] \operatorname{Log}\left[e \left(f (a + b x)^p (c + d x)^q\right)^r\right] \operatorname{Log}[g + h x]}{h} + \\
& \frac{\operatorname{Log}\left[e \left(f (a + b x)^p (c + d x)^q\right)^r\right]^2 \operatorname{Log}[g + h x]}{h} - \frac{p^2 r^2 \operatorname{Log}[a + b x]^2 \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[-\frac{h (c + d x)}{d g - c h}\right] \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} + \frac{p q r^2 \operatorname{Log}\left[-\frac{h (c + d x)}{d g - c h}\right]^2 \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{Log}\left[-\frac{h (c + d x)}{d g - c h}\right] \operatorname{Log}\left[\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}\right] \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} + \frac{p q r^2 \operatorname{Log}\left[\frac{(b g - a h) (c + d x)}{(d g - c h) (a + b x)}\right]^2 \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} + \\
& \frac{2 p r \operatorname{Log}[a + b x] \operatorname{Log}\left[e \left(f (a + b x)^p (c + d x)^q\right)^r\right] \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d (g + h x)}{d g - c h}\right]}{h} - \frac{q^2 r^2 \operatorname{Log}[c + d x]^2 \operatorname{Log}\left[\frac{d (g + h x)}{d g - c h}\right]}{h} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 p q r^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right] \operatorname{Log}\left[\frac{d(g+h x)}{d g-c h}\right]}{h}-\frac{p q r^2 \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right]^2 \operatorname{Log}\left[\frac{d(g+h x)}{d g-c h}\right]}{h}+ \\
& \frac{2 p q r^2 \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right] \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right] \operatorname{Log}\left[\frac{d(g+h x)}{d g-c h}\right]}{h}+ \\
& \frac{2 q r \operatorname{Log}[c+d x] \operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right] \operatorname{Log}\left[\frac{d(g+h x)}{d g-c h}\right]}{h}- \\
& \frac{p q r^2 \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right]^2 \operatorname{Log}\left[-\frac{(b c-a d)(g+h x)}{(d g-c h)(a+b x)}\right]}{h}-\frac{1}{h} \\
& 2 p r\left(q r \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right]-\operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right]\right) \operatorname{PolyLog}\left[2,-\frac{h(a+b x)}{b g-a h}\right]+ \\
& \frac{1}{h} 2 q r\left(p r \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right]+\operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right]\right) \operatorname{PolyLog}\left[2,-\frac{h(c+d x)}{d g-c h}\right]+ \\
& \frac{2 p q r^2 \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right] \operatorname{PolyLog}\left[2,\frac{b(c+d x)}{d(a+b x)}\right]}{h}- \\
& \frac{2 p q r^2 \operatorname{Log}\left[\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right] \operatorname{PolyLog}\left[2,\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right]}{h}-\frac{2 p^2 r^2 \operatorname{PolyLog}\left[3,-\frac{h(a+b x)}{b g-a h}\right]}{h} \\
& \frac{2 p q r^2 \operatorname{PolyLog}\left[3,-\frac{h(a+b x)}{b g-a h}\right]}{h}-\frac{2 p q r^2 \operatorname{PolyLog}\left[3,-\frac{h(c+d x)}{d g-c h}\right]}{h}-\frac{2 q^2 r^2 \operatorname{PolyLog}\left[3,-\frac{h(c+d x)}{d g-c h}\right]}{h}- \\
& \frac{2 p q r^2 \operatorname{PolyLog}\left[3,\frac{b(c+d x)}{d(a+b x)}\right]}{h}+\frac{2 p q r^2 \operatorname{PolyLog}\left[3,\frac{(b g-a h)(c+d x)}{(d g-c h)(a+b x)}\right]}{h}
\end{aligned}$$

Result (type 4, 2096 leaves, 29 steps):

$$\begin{aligned}
& -\frac{\operatorname{Log}\left[(a+b x)^{p r}\right]^2 \operatorname{Log}[g+h x]}{h}-\frac{2 p q r^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c-a d}\right] \operatorname{Log}[c+d x] \operatorname{Log}[g+h x]}{h}- \\
& \frac{2 p q r^2 \operatorname{Log}[a+b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c-a d}\right] \operatorname{Log}[g+h x]}{h}+ \\
& \frac{2 q r\left(p r \operatorname{Log}[a+b x]-\operatorname{Log}\left[(a+b x)^{p r}\right]\right) \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right] \operatorname{Log}[g+h x]}{h}+ \\
& \frac{2 p r \operatorname{Log}\left[-\frac{h(a+b x)}{b g-a h}\right]\left(q r \operatorname{Log}[c+d x]-\operatorname{Log}\left[(c+d x)^{q r}\right]\right) \operatorname{Log}[g+h x]}{h}- \\
& \frac{\operatorname{Log}\left[(c+d x)^{q r}\right]^2 \operatorname{Log}[g+h x]}{h}+\frac{1}{h} 2 p r \operatorname{Log}\left[-\frac{h(a+b x)}{b g-a h}\right] \\
& \left(\operatorname{Log}\left[(a+b x)^{p r}\right]+\operatorname{Log}\left[(c+d x)^{q r}\right]-\operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right]\right) \operatorname{Log}[g+h x]+\frac{1}{h} \\
& 2 q r \operatorname{Log}\left[-\frac{h(c+d x)}{d g-c h}\right]\left(\operatorname{Log}\left[(a+b x)^{p r}\right]+\operatorname{Log}\left[(c+d x)^{q r}\right]-\operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right]\right) \\
& \operatorname{Log}[g+h x]+\frac{\operatorname{Log}\left[e\left(f(a+b x)^p(c+d x)^q\right)^r\right]^2 \operatorname{Log}[g+h x]}{h}+
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{Log}[(a+bx)^{pr}]^2 \text{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{\text{Log}[(c+dx)^{qr}]^2 \text{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} - \frac{1}{h} p q r^2 \\
& \left( \text{Log}\left[\frac{b(c+dx)}{bc-ad}\right] + \text{Log}\left[\frac{bg-ah}{b(g+hx)}\right] - \text{Log}\left[\frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right] \right) \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]^2 + \frac{1}{h} \\
& p q r^2 \left( \text{Log}\left[\frac{b(c+dx)}{bc-ad}\right] - \text{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \right) \left( \text{Log}[a+bx] + \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \right)^2 - \frac{1}{h} p q r^2 \\
& \left( \text{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] + \text{Log}\left[\frac{dg-ch}{d(g+hx)}\right] - \text{Log}\left[-\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right] \right) \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right]^2 + \\
& \frac{1}{h} p q r^2 \left( \text{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] - \text{Log}\left[-\frac{h(a+bx)}{bg-ah}\right] \right) \left( \text{Log}[c+dx] + \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \right)^2 - \\
& \frac{2 p q r^2 \left( \text{Log}[g+hx] - \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \right) \text{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{h} + \\
& \frac{2 p r \text{Log}[(a+bx)^{pr}] \text{PolyLog}\left[2, -\frac{h(a+bx)}{bg-ah}\right]}{h} - \\
& \frac{2 p q r^2 \left( \text{Log}[g+hx] - \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \right) \text{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{h} + \\
& \frac{2 q r \text{Log}[(c+dx)^{qr}] \text{PolyLog}\left[2, -\frac{h(c+dx)}{dg-ch}\right]}{h} + \frac{2 p q r^2 \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \text{PolyLog}\left[2, \frac{h(a+bx)}{b(g+hx)}\right]}{h} - \\
& \frac{2 p q r^2 \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \text{PolyLog}\left[2, -\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right]}{h} + \\
& \frac{2 p q r^2 \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \text{PolyLog}\left[2, \frac{h(c+dx)}{d(g+hx)}\right]}{h} - \\
& \frac{2 p q r^2 \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \text{PolyLog}\left[2, \frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right]}{h} + \\
& \frac{2 p r (q r \text{Log}[c+dx] - \text{Log}[(c+dx)^{qr}]) \text{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{1}{h} \\
& 2 p r (\text{Log}[(a+bx)^{pr}] + \text{Log}[(c+dx)^{qr}] - \text{Log}[e (f (a+bx)^p (c+dx)^q)^r]) \\
& \text{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right] - \frac{2 p q r^2 \left( \text{Log}[c+dx] + \text{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \right) \text{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} + \\
& \frac{2 q r (p r \text{Log}[a+bx] - \text{Log}[(a+bx)^{pr}]) \text{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} + \frac{1}{h} 2 q r \\
& (\text{Log}[(a+bx)^{pr}] + \text{Log}[(c+dx)^{qr}] - \text{Log}[e (f (a+bx)^p (c+dx)^q)^r]) \text{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right] - \\
& \frac{2 p q r^2 \left( \text{Log}[a+bx] + \text{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \right) \text{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{h} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 p^2 r^2 \text{PolyLog}\left[3, -\frac{h(a+bx)}{bg-ah}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{h} - \frac{2 q^2 r^2 \text{PolyLog}\left[3, -\frac{h(c+dx)}{dg-ch}\right]}{h} + \\
& \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{h(a+bx)}{b(g+hx)}\right]}{h} - \frac{2 p q r^2 \text{PolyLog}\left[3, -\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{h(c+dx)}{d(g+hx)}\right]}{h} - \\
& \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{d(g+hx)}{dg-ch}\right]}{h}
\end{aligned}$$

**Problem 40: Result valid but suboptimal antiderivative.**

$$\int \frac{\text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]^2}{(g+hx)^2} dx$$

Optimal (type 4, 832 leaves, 31 steps):

$$\begin{aligned}
& \frac{2 b p q r^2 \text{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \text{Log}[c+dx]}{h(bg-ah)} + \frac{2 d p q r^2 \text{Log}[a+bx] \text{Log}\left[\frac{b(c+dx)}{bc-ad}\right]}{h(dg-ch)} - \frac{1}{h(bg-ah)} 2 b p r \\
& \text{Log}[a+bx] \left(p r \text{Log}[a+bx] + q r \text{Log}[c+dx] - \text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) - \frac{1}{h(dg-ch)} \\
& 2 d q r \text{Log}[c+dx] \left(p r \text{Log}[a+bx] + q r \text{Log}[c+dx] - \text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) - \\
& \frac{\text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]^2}{h(g+hx)} + \frac{1}{h(bg-ah)} 2 b p r \\
& \left(p r \text{Log}[a+bx] + q r \text{Log}[c+dx] - \text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) \text{Log}[g+hx] + \frac{1}{h(dg-ch)} \\
& 2 d q r \left(p r \text{Log}[a+bx] + q r \text{Log}[c+dx] - \text{Log}\left[e \left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) \text{Log}[g+hx] - \\
& \frac{2 d p q r^2 \text{Log}[a+bx] \text{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h(dg-ch)} - \frac{2 b p q r^2 \text{Log}[c+dx] \text{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h(bg-ah)} - \\
& \frac{2 b p^2 r^2 \text{Log}[a+bx] \text{Log}\left[1 + \frac{bg-ah}{h(a+bx)}\right]}{h(bg-ah)} - \frac{2 d q^2 r^2 \text{Log}[c+dx] \text{Log}\left[1 + \frac{dg-ch}{h(c+dx)}\right]}{h(dg-ch)} + \\
& \frac{2 b p^2 r^2 \text{PolyLog}\left[2, -\frac{bg-ah}{h(a+bx)}\right]}{h(bg-ah)} + \frac{2 d p q r^2 \text{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{h(dg-ch)} - \frac{2 d p q r^2 \text{PolyLog}\left[2, -\frac{h(a+bx)}{bg-ah}\right]}{h(dg-ch)} + \\
& \frac{2 d q^2 r^2 \text{PolyLog}\left[2, -\frac{dg-ch}{h(c+dx)}\right]}{h(dg-ch)} + \frac{2 b p q r^2 \text{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{h(bg-ah)} - \frac{2 b p q r^2 \text{PolyLog}\left[2, -\frac{h(c+dx)}{dg-ch}\right]}{h(bg-ah)}
\end{aligned}$$

Result (type 4, 878 leaves, 35 steps):



$$\begin{aligned}
& \frac{b p^2 r^2 \operatorname{Log}[a + b x]^2}{h (b g - a h)} + \frac{2 b p q r^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{h (b g - a h)} + \frac{d q^2 r^2 \operatorname{Log}[c + d x]^2}{h (d g - c h)} + \\
& \frac{2 d p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{h (d g - c h)} - \frac{1}{h (b g - a h)} 2 b p r \operatorname{Log}[a + b x] \\
& \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) - \frac{1}{h (d g - c h)} \\
& 2 d q r \operatorname{Log}[c + d x] \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) - \\
& \frac{\operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right]^2}{h (g + h x)} + \frac{1}{h (b g - a h)} 2 b p r \\
& \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \operatorname{Log}[g + h x] + \frac{1}{h (d g - c h)} \\
& 2 d q r \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \operatorname{Log}[g + h x] - \\
& \frac{2 b p^2 r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h (b g - a h)} - \frac{2 d p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (g + h x)}{b g - a h}\right]}{h (d g - c h)} - \\
& \frac{2 b p q r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d (g + h x)}{d g - c h}\right]}{h (b g - a h)} - \frac{2 d q^2 r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d (g + h x)}{d g - c h}\right]}{h (d g - c h)} + \\
& \frac{2 d p q r^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{h (d g - c h)} - \frac{2 b p^2 r^2 \operatorname{PolyLog}\left[2, -\frac{h (a + b x)}{b g - a h}\right]}{h (b g - a h)} - \frac{2 d p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (a + b x)}{b g - a h}\right]}{h (d g - c h)} + \\
& \frac{2 b p q r^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{h (b g - a h)} - \frac{2 b p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (c + d x)}{d g - c h}\right]}{h (b g - a h)} - \frac{2 d q^2 r^2 \operatorname{PolyLog}\left[2, -\frac{h (c + d x)}{d g - c h}\right]}{h (d g - c h)}
\end{aligned}$$

**Problem 41: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}\left[e \left( f (a + b x)^p (c + d x)^q \right)^r \right]^2}{(g + h x)^3} dx$$

Optimal (type 4, 1304 leaves, 43 steps):

$$\begin{aligned}
& - \frac{b d p q r^2 \operatorname{Log}[a + b x]}{h (b g - a h) (d g - c h)} + \frac{d p q r^2 \operatorname{Log}[a + b x]}{h (d g - c h) (g + h x)} - \frac{b p^2 r^2 (a + b x) \operatorname{Log}[a + b x]}{(b g - a h)^2 (g + h x)} - \\
& \frac{b d p q r^2 \operatorname{Log}[c + d x]}{h (b g - a h) (d g - c h)} + \frac{b p q r^2 \operatorname{Log}[c + d x]}{h (b g - a h) (g + h x)} - \frac{d q^2 r^2 (c + d x) \operatorname{Log}[c + d x]}{(d g - c h)^2 (g + h x)} + \\
& \frac{b^2 p q r^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{h (b g - a h)^2} + \frac{d^2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{h (d g - c h)^2} - \\
& \left( b p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \right) / \\
& \left( h (b g - a h) (g + h x) \right) - \\
& \left( d q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \right) / \\
& \left( h (d g - c h) (g + h x) \right) - \frac{1}{h (b g - a h)^2} b^2 p r \operatorname{Log}[a + b x] \\
& \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r] \right) - \frac{1}{h (d g - c h)^2} \\
& d^2 q r \operatorname{Log}[c + d x] (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) - \\
& \frac{\operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]^2}{2 h (g + h x)^2} + \frac{b^2 p^2 r^2 \operatorname{Log}[g + h x]}{h (b g - a h)^2} + \\
& \frac{2 b d p q r^2 \operatorname{Log}[g + h x]}{h (b g - a h) (d g - c h)} + \frac{d^2 q^2 r^2 \operatorname{Log}[g + h x]}{h (d g - c h)^2} + \frac{1}{h (b g - a h)^2} \\
& b^2 p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \operatorname{Log}[g + h x] + \\
& \frac{1}{h (d g - c h)^2} d^2 q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \\
& \operatorname{Log}[g + h x] - \frac{d^2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(g+h x)}{b g - a h}\right]}{h (d g - c h)^2} - \frac{b^2 p q r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d(g+h x)}{d g - c h}\right]}{h (b g - a h)^2} - \\
& \frac{b^2 p^2 r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[1 + \frac{b g - a h}{h (a + b x)}\right]}{h (b g - a h)^2} - \frac{d^2 q^2 r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[1 + \frac{d g - c h}{h (c + d x)}\right]}{h (d g - c h)^2} + \\
& \frac{b^2 p^2 r^2 \operatorname{PolyLog}\left[2, -\frac{b g - a h}{h (a + b x)}\right]}{h (b g - a h)^2} + \frac{d^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{h (d g - c h)^2} - \frac{d^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (a + b x)}{b g - a h}\right]}{h (d g - c h)^2} + \\
& \frac{d^2 q^2 r^2 \operatorname{PolyLog}\left[2, -\frac{d g - c h}{h (c + d x)}\right]}{h (d g - c h)^2} + \frac{b^2 p q r^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{h (b g - a h)^2} - \frac{b^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (c + d x)}{d g - c h}\right]}{h (b g - a h)^2}
\end{aligned}$$

Result (type 4, 1362 leaves, 47 steps):

$$\begin{aligned}
& - \frac{b d p q r^2 \operatorname{Log}[a + b x]}{h (b g - a h) (d g - c h)} + \frac{d p q r^2 \operatorname{Log}[a + b x]}{h (d g - c h) (g + h x)} - \frac{b p^2 r^2 (a + b x) \operatorname{Log}[a + b x]}{(b g - a h)^2 (g + h x)} + \\
& \frac{b^2 p^2 r^2 \operatorname{Log}[a + b x]^2}{2 h (b g - a h)^2} - \frac{b d p q r^2 \operatorname{Log}[c + d x]}{h (b g - a h) (d g - c h)} + \frac{b p q r^2 \operatorname{Log}[c + d x]}{h (b g - a h) (g + h x)} - \\
& \frac{d q^2 r^2 (c + d x) \operatorname{Log}[c + d x]}{(d g - c h)^2 (g + h x)} + \frac{b^2 p q r^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{h (b g - a h)^2} + \\
& \frac{d^2 q^2 r^2 \operatorname{Log}[c + d x]^2}{2 h (d g - c h)^2} + \frac{d^2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{h (d g - c h)^2} - \\
& \frac{(b p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r])}{(h (b g - a h) (g + h x))} - \\
& \frac{(d q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r])}{(h (d g - c h) (g + h x))} - \frac{1}{h (b g - a h)^2} b^2 p r \operatorname{Log}[a + b x] \\
& (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) - \frac{1}{h (d g - c h)^2} \\
& d^2 q r \operatorname{Log}[c + d x] (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) - \\
& \frac{\operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]^2}{2 h (g + h x)^2} + \frac{b^2 p^2 r^2 \operatorname{Log}[g + h x]}{h (b g - a h)^2} + \\
& \frac{2 b d p q r^2 \operatorname{Log}[g + h x]}{h (b g - a h) (d g - c h)} + \frac{d^2 q^2 r^2 \operatorname{Log}[g + h x]}{h (d g - c h)^2} + \frac{1}{h (b g - a h)^2} \\
& b^2 p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \operatorname{Log}[g + h x] + \\
& \frac{1}{h (d g - c h)^2} d^2 q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \\
& \operatorname{Log}[g + h x] - \frac{b^2 p^2 r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(g+h x)}{b g - a h}\right]}{h (b g - a h)^2} - \frac{d^2 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(g+h x)}{b g - a h}\right]}{h (d g - c h)^2} - \\
& \frac{b^2 p q r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d(g+h x)}{d g - c h}\right]}{h (b g - a h)^2} - \frac{d^2 q^2 r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d(g+h x)}{d g - c h}\right]}{h (d g - c h)^2} + \\
& \frac{d^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{d(a+b x)}{b c - a d}\right]}{h (d g - c h)^2} - \frac{b^2 p^2 r^2 \operatorname{PolyLog}\left[2, -\frac{h(a+b x)}{b g - a h}\right]}{h (b g - a h)^2} - \frac{d^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h(a+b x)}{b g - a h}\right]}{h (d g - c h)^2} + \\
& \frac{b^2 p q r^2 \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{b c - a d}\right]}{h (b g - a h)^2} - \frac{b^2 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h(c+d x)}{d g - c h}\right]}{h (b g - a h)^2} - \frac{d^2 q^2 r^2 \operatorname{PolyLog}\left[2, -\frac{h(c+d x)}{d g - c h}\right]}{h (d g - c h)^2}
\end{aligned}$$

**Problem 42: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]^2}{(g + h x)^4} dx$$

Optimal (type 4, 1957 leaves, 57 steps):

$$\begin{aligned}
& - \frac{b^2 p^2 r^2}{3 h (b g - a h)^2 (g + h x)} - \frac{2 b d p q r^2}{3 h (b g - a h) (d g - c h) (g + h x)} - \\
& \frac{d^2 q^2 r^2}{3 h (d g - c h)^2 (g + h x)} - \frac{b^3 p^2 r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h)^3} - \frac{2 b d^2 p q r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h) (d g - c h)^2} - \\
& \frac{b^2 d p q r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h)^2 (d g - c h)} + \frac{b p^2 r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h) (g + h x)^2} + \frac{d p q r^2 \operatorname{Log}[a + b x]}{3 h (d g - c h) (g + h x)^2} + \\
& \frac{2 d^2 p q r^2 \operatorname{Log}[a + b x]}{3 h (d g - c h)^2 (g + h x)} - \frac{2 b^2 p^2 r^2 (a + b x) \operatorname{Log}[a + b x]}{3 (b g - a h)^3 (g + h x)} - \frac{b d^2 p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h) (d g - c h)^2} - \\
& \frac{2 b^2 d p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h)^2 (d g - c h)} - \frac{d^3 q^2 r^2 \operatorname{Log}[c + d x]}{3 h (d g - c h)^3} + \frac{b p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h) (g + h x)^2} + \\
& \frac{d q^2 r^2 \operatorname{Log}[c + d x]}{3 h (d g - c h) (g + h x)^2} + \frac{2 b^2 p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h)^2 (g + h x)} - \frac{2 d^2 q^2 r^2 (c + d x) \operatorname{Log}[c + d x]}{3 (d g - c h)^3 (g + h x)} + \\
& \frac{2 b^3 p q r^2 \operatorname{Log}\left[-\frac{d(a+b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 h (b g - a h)^3} + \frac{2 d^3 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(c+d x)}{b c - a d}\right]}{3 h (d g - c h)^3} - \\
& \frac{(b p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]))}{(3 h (b g - a h) (g + h x)^2)} - \\
& \frac{(d q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]))}{(3 h (d g - c h) (g + h x)^2)} - \\
& \frac{(2 b^2 p r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]))}{(3 h (b g - a h)^2 (g + h x))} - \\
& \frac{(2 d^2 q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]))}{(3 h (d g - c h)^2 (g + h x))} - \frac{1}{3 h (b g - a h)^3} 2 b^3 p r \operatorname{Log}[a + b x] \\
& (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) - \frac{1}{3 h (d g - c h)^3} \\
& 2 d^3 q r \operatorname{Log}[c + d x] (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) - \\
& \frac{\operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]^2}{3 h (g + h x)^3} + \frac{b^3 p^2 r^2 \operatorname{Log}[g + h x]}{h (b g - a h)^3} + \frac{b d^2 p q r^2 \operatorname{Log}[g + h x]}{h (b g - a h) (d g - c h)^2} + \\
& \frac{b^2 d p q r^2 \operatorname{Log}[g + h x]}{h (b g - a h)^2 (d g - c h)} + \frac{d^3 q^2 r^2 \operatorname{Log}[g + h x]}{h (d g - c h)^3} + \frac{1}{3 h (b g - a h)^3} 2 b^3 p r \\
& (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \operatorname{Log}[g + h x] + \frac{1}{3 h (d g - c h)^3} \\
& 2 d^3 q r (p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}[e (f(a + b x)^p (c + d x)^q)^r]) \operatorname{Log}[g + h x] - \\
& \frac{2 d^3 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b(g+h x)}{b g - a h}\right]}{3 h (d g - c h)^3} - \frac{2 b^3 p q r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[\frac{d(g+h x)}{d g - c h}\right]}{3 h (b g - a h)^3} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^3 p^2 r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[1 + \frac{b g - a h}{h (a + b x)}\right]}{3 h (b g - a h)^3} - \frac{2 d^3 q^2 r^2 \operatorname{Log}[c + d x] \operatorname{Log}\left[1 + \frac{d g - c h}{h (c + d x)}\right]}{3 h (d g - c h)^3} + \\
& \frac{2 b^3 p^2 r^2 \operatorname{PolyLog}\left[2, -\frac{b g - a h}{h (a + b x)}\right]}{3 h (b g - a h)^3} + \frac{2 d^3 p q r^2 \operatorname{PolyLog}\left[2, -\frac{d (a + b x)}{b c - a d}\right]}{3 h (d g - c h)^3} - \\
& \frac{2 d^3 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (a + b x)}{b g - a h}\right]}{3 h (d g - c h)^3} + \frac{2 d^3 q^2 r^2 \operatorname{PolyLog}\left[2, -\frac{d g - c h}{h (c + d x)}\right]}{3 h (d g - c h)^3} + \\
& \frac{2 b^3 p q r^2 \operatorname{PolyLog}\left[2, \frac{b (c + d x)}{b c - a d}\right]}{3 h (b g - a h)^3} - \frac{2 b^3 p q r^2 \operatorname{PolyLog}\left[2, -\frac{h (c + d x)}{d g - c h}\right]}{3 h (b g - a h)^3}
\end{aligned}$$

Result (type 4, 2013 leaves, 61 steps):

$$\begin{aligned}
& - \frac{b^2 p^2 r^2}{3 h (b g - a h)^2 (g + h x)} - \frac{2 b d p q r^2}{3 h (b g - a h) (d g - c h) (g + h x)} - \frac{d^2 q^2 r^2}{3 h (d g - c h)^2 (g + h x)} - \\
& \frac{b^3 p^2 r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h)^3} - \frac{2 b d^2 p q r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h) (d g - c h)^2} - \frac{b^2 d p q r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h)^2 (d g - c h)} + \\
& \frac{b p^2 r^2 \operatorname{Log}[a + b x]}{3 h (b g - a h) (g + h x)^2} + \frac{d p q r^2 \operatorname{Log}[a + b x]}{3 h (d g - c h) (g + h x)^2} + \frac{2 d^2 p q r^2 \operatorname{Log}[a + b x]}{3 h (d g - c h)^2 (g + h x)} - \\
& \frac{2 b^2 p^2 r^2 (a + b x) \operatorname{Log}[a + b x]}{3 (b g - a h)^3 (g + h x)} + \frac{b^3 p^2 r^2 \operatorname{Log}[a + b x]^2}{3 h (b g - a h)^3} - \frac{b d^2 p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h) (d g - c h)^2} - \\
& \frac{2 b^2 d p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h)^2 (d g - c h)} - \frac{d^3 q^2 r^2 \operatorname{Log}[c + d x]}{3 h (d g - c h)^3} + \frac{b p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h) (g + h x)^2} + \\
& \frac{d q^2 r^2 \operatorname{Log}[c + d x]}{3 h (d g - c h) (g + h x)^2} + \frac{2 b^2 p q r^2 \operatorname{Log}[c + d x]}{3 h (b g - a h)^2 (g + h x)} - \frac{2 d^2 q^2 r^2 (c + d x) \operatorname{Log}[c + d x]}{3 (d g - c h)^3 (g + h x)} + \\
& \frac{2 b^3 p q r^2 \operatorname{Log}\left[-\frac{d (a + b x)}{b c - a d}\right] \operatorname{Log}[c + d x]}{3 h (b g - a h)^3} + \frac{d^3 q^2 r^2 \operatorname{Log}[c + d x]^2}{3 h (d g - c h)^3} + \frac{2 d^3 p q r^2 \operatorname{Log}[a + b x] \operatorname{Log}\left[\frac{b (c + d x)}{b c - a d}\right]}{3 h (d g - c h)^3} - \\
& \left( b p r \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \right) / \\
& \left( 3 h (b g - a h) (g + h x)^2 \right) - \\
& \left( d q r \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \right) / \\
& \left( 3 h (d g - c h) (g + h x)^2 \right) - \\
& \left( 2 b^2 p r \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \right) / \\
& \left( 3 h (b g - a h)^2 (g + h x) \right) - \\
& \left( 2 d^2 q r \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) \right) / \\
& \left( 3 h (d g - c h)^2 (g + h x) \right) - \frac{1}{3 h (b g - a h)^3} 2 b^3 p r \operatorname{Log}[a + b x] \\
& \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) - \frac{1}{3 h (d g - c h)^3} \\
& 2 d^3 q r \operatorname{Log}[c + d x] \left( p r \operatorname{Log}[a + b x] + q r \operatorname{Log}[c + d x] - \operatorname{Log}\left[ e \left( f (a + b x)^p (c + d x)^q \right)^r \right] \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{Log}\left[e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right]^2}{3h\left(g+hx\right)^3} + \frac{b^3p^2r^2\text{Log}\left[g+hx\right]}{h\left(bg-ah\right)^3} + \frac{bd^2pq r^2\text{Log}\left[g+hx\right]}{h\left(bg-ah\right)\left(dg-ch\right)^2} + \\
& \frac{b^2dpqr^2\text{Log}\left[g+hx\right]}{h\left(bg-ah\right)^2\left(dg-ch\right)} + \frac{d^3q^2r^2\text{Log}\left[g+hx\right]}{h\left(dg-ch\right)^3} + \frac{1}{3h\left(bg-ah\right)^3} \\
& 2b^3pr\left(pr\text{Log}\left[a+bx\right]+qr\text{Log}\left[c+dx\right]-\text{Log}\left[e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right]\right)\text{Log}\left[g+hx\right]+ \\
& \frac{1}{3h\left(dg-ch\right)^3}2d^3qr\left(pr\text{Log}\left[a+bx\right]+qr\text{Log}\left[c+dx\right]-\text{Log}\left[e\left(f\left(a+bx\right)^p\left(c+dx\right)^q\right)^r\right]\right) \\
& \text{Log}\left[g+hx\right]-\frac{2b^3p^2r^2\text{Log}\left[a+bx\right]\text{Log}\left[\frac{b\left(g+hx\right)}{bg-ah}\right]}{3h\left(bg-ah\right)^3}-\frac{2d^3pq r^2\text{Log}\left[a+bx\right]\text{Log}\left[\frac{b\left(g+hx\right)}{bg-ah}\right]}{3h\left(dg-ch\right)^3}- \\
& \frac{2b^3pq r^2\text{Log}\left[c+dx\right]\text{Log}\left[\frac{d\left(g+hx\right)}{dg-ch}\right]}{3h\left(bg-ah\right)^3}-\frac{2d^3q^2r^2\text{Log}\left[c+dx\right]\text{Log}\left[\frac{d\left(g+hx\right)}{dg-ch}\right]}{3h\left(dg-ch\right)^3}+ \\
& \frac{2d^3pq r^2\text{PolyLog}\left[2,-\frac{d\left(a+bx\right)}{bc-ad}\right]}{3h\left(dg-ch\right)^3}-\frac{2b^3p^2r^2\text{PolyLog}\left[2,-\frac{h\left(a+bx\right)}{bg-ah}\right]}{3h\left(bg-ah\right)^3}- \\
& \frac{2d^3pq r^2\text{PolyLog}\left[2,-\frac{h\left(a+bx\right)}{bg-ah}\right]}{3h\left(dg-ch\right)^3}+\frac{2b^3pq r^2\text{PolyLog}\left[2,\frac{b\left(c+dx\right)}{bc-ad}\right]}{3h\left(bg-ah\right)^3}- \\
& \frac{2b^3pq r^2\text{PolyLog}\left[2,-\frac{h\left(c+dx\right)}{dg-ch}\right]}{3h\left(bg-ah\right)^3}-\frac{2d^3q^2r^2\text{PolyLog}\left[2,-\frac{h\left(c+dx\right)}{dg-ch}\right]}{3h\left(dg-ch\right)^3}
\end{aligned}$$

**Problem 43: Result optimal but 1 more steps used.**

$$\int \frac{\left(a+b\text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^n}{1-c^2x^2}dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{\left(a+b\text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^{1+n}}{bc(1+n)}$$

Result (type 3, 42 leaves, 3 steps):

$$-\frac{\left(a+b\text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^{1+n}}{bc(1+n)}$$

**Problem 47: Result optimal but 1 more steps used.**

$$\int \frac{1}{\left(1-c^2x^2\right)\left(a+b\text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)}dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{\text{Log}\left[a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right]}{bc}$$

Result (type 3, 34 leaves, 3 steps):

$$\frac{\text{Log}\left[a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right]}{bc}$$

Problem 48: Result optimal but 1 more steps used.

$$\int \frac{1}{(1-c^2x^2) \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2} dx$$

Optimal (type 3, 34 leaves, 2 steps):

$$\frac{1}{bc \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)}$$

Result (type 3, 34 leaves, 3 steps):

$$\frac{1}{bc \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)}$$

Problem 49: Result optimal but 1 more steps used.

$$\int \frac{1}{(1-c^2x^2) \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{1}{2bc \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}$$

Result (type 3, 37 leaves, 3 steps):

$$\frac{1}{2bc \left(a + b \text{Log}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}$$

Problem 74: Unable to integrate problem.

$$\int \left( \frac{1}{(c+dx) (-a+c+(-b+d)x) \text{Log}\left[\frac{a+bx}{c+dx}\right]} + \frac{\text{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(a+bx) (c+dx) \text{Log}\left[\frac{a+bx}{c+dx}\right]^2} \right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(bc-ad)\text{Log}\left[\frac{a+bx}{c+dx}\right]}$$

Result (type 8, 152 leaves, 3 steps):

$$\frac{b \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(a+bx)\text{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} - \frac{d \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1 - \frac{a+bx}{c+dx}\right]}{(c+dx)\text{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} +$$

$$\text{Unintegrable}\left[\frac{1}{(c+dx)(-a+c+(-b+d)x)\text{Log}\left[\frac{a+bx}{c+dx}\right]}, x\right]$$

Problem 75: Unable to integrate problem.

$$\int \left( -\frac{1}{(a+bx)(a-c+(b-d)x)\text{Log}\left[\frac{a+bx}{c+dx}\right]} + \frac{\text{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(a+bx)(c+dx)\text{Log}\left[\frac{a+bx}{c+dx}\right]^2} \right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(bc-ad)\text{Log}\left[\frac{a+bx}{c+dx}\right]}$$

Result (type 8, 154 leaves, 3 steps):

$$\frac{b \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(a+bx)\text{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} - \frac{d \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1 - \frac{c+dx}{a+bx}\right]}{(c+dx)\text{Log}\left[\frac{a+bx}{c+dx}\right]^2}, x\right]}{bc-ad} -$$

$$\text{Unintegrable}\left[\frac{1}{(a+bx)(a-c+(b-d)x)\text{Log}\left[\frac{a+bx}{c+dx}\right]}, x\right]$$

Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{f-gx^2} dx$$

Optimal (type 4, 291 leaves, 7 steps):

$$\frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\text{Log}\left[1 - \frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right]}{2\sqrt{f}\sqrt{g}} - \frac{\text{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]\text{Log}\left[1 - \frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right]}{2\sqrt{f}\sqrt{g}} +$$

$$\frac{n \text{ PolyLog}\left[2, \frac{(d\sqrt{f}-c\sqrt{g})(a+bx)}{(b\sqrt{f}-a\sqrt{g})(c+dx)}\right]}{2\sqrt{f}\sqrt{g}} - \frac{n \text{ PolyLog}\left[2, \frac{(d\sqrt{f}+c\sqrt{g})(a+bx)}{(b\sqrt{f}+a\sqrt{g})(c+dx)}\right]}{2\sqrt{f}\sqrt{g}}$$

Result (type 4, 468 leaves, 18 steps):



$$\begin{aligned}
& - \frac{\text{ArcTanh}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right] \left(n \log[a+bx] - \log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] - n \log[c+dx]\right)}{\sqrt{f}\sqrt{g}} - \\
& \frac{n \log[a+bx] \log\left[\frac{b(\sqrt{f}-\sqrt{g}x)}{b\sqrt{f}+a\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} + \frac{n \log[c+dx] \log\left[\frac{d(\sqrt{f}-\sqrt{g}x)}{d\sqrt{f}+c\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} + \\
& \frac{n \log[a+bx] \log\left[\frac{b(\sqrt{f}+\sqrt{g}x)}{b\sqrt{f}-a\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} - \frac{n \log[c+dx] \log\left[\frac{d(\sqrt{f}+\sqrt{g}x)}{d\sqrt{f}-c\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} + \frac{n \text{PolyLog}\left[2, -\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} - \\
& \frac{n \text{PolyLog}\left[2, \frac{\sqrt{g}(a+bx)}{b\sqrt{f}+a\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} - \frac{n \text{PolyLog}\left[2, -\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}} + \frac{n \text{PolyLog}\left[2, \frac{\sqrt{g}(c+dx)}{d\sqrt{f}+c\sqrt{g}}\right]}{2\sqrt{f}\sqrt{g}}
\end{aligned}$$

Problem 85: Result valid but suboptimal antiderivative.

$$\int \frac{\log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{f+gx+hx^2} dx$$

Optimal (type 4, 401 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \log\left[1 - \frac{2(d^2f-cdg+c^2h)(a+bx)}{(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh})(c+dx)}\right]}{\sqrt{g^2-4fh}} + \\
& \frac{\log\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \log\left[1 - \frac{2(d^2f-cdg+c^2h)(a+bx)}{(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh})(c+dx)}\right]}{\sqrt{g^2-4fh}} - \\
& \frac{n \text{PolyLog}\left[2, \frac{2(d^2f-cdg+c^2h)(a+bx)}{(2bdf-bcg-adg+2ach-(bc-ad)\sqrt{g^2-4fh})(c+dx)}\right]}{\sqrt{g^2-4fh}} + \\
& \frac{n \text{PolyLog}\left[2, \frac{2(d^2f-cdg+c^2h)(a+bx)}{(2bdf-bcg-adg+2ach+(bc-ad)\sqrt{g^2-4fh})(c+dx)}\right]}{\sqrt{g^2-4fh}}
\end{aligned}$$

Result (type 4, 545 leaves, 19 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{g+2 h x}{\sqrt{g^2-4 f h}}\right]\left(n \operatorname{Log}[a+b x]-\operatorname{Log}\left[e\left(\frac{a+b x}{c+d x}\right)^n\right]-n \operatorname{Log}[c+d x]\right)}{\sqrt{g^2-4 f h}}+ \\
& \frac{n \operatorname{Log}[a+b x] \operatorname{Log}\left[-\frac{b\left(g-\sqrt{g^2-4 f h}+2 h x\right)}{2 a h-b\left(g-\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}-\frac{n \operatorname{Log}[c+d x] \operatorname{Log}\left[-\frac{d\left(g-\sqrt{g^2-4 f h}+2 h x\right)}{2 c h-d\left(g-\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}- \\
& \frac{n \operatorname{Log}[a+b x] \operatorname{Log}\left[-\frac{b\left(g+\sqrt{g^2-4 f h}+2 h x\right)}{2 a h-b\left(g+\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}+\frac{n \operatorname{Log}[c+d x] \operatorname{Log}\left[-\frac{d\left(g+\sqrt{g^2-4 f h}+2 h x\right)}{2 c h-d\left(g+\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}+ \\
& \frac{n \operatorname{PolyLog}\left[2, \frac{2 h(a+b x)}{2 a h-b\left(g-\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}-\frac{n \operatorname{PolyLog}\left[2, \frac{2 h(a+b x)}{2 a h-b\left(g+\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}- \\
& \frac{n \operatorname{PolyLog}\left[2, \frac{2 h(c+d x)}{2 c h-d\left(g-\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}+\frac{n \operatorname{PolyLog}\left[2, \frac{2 h(c+d x)}{2 c h-d\left(g+\sqrt{g^2-4 f h}\right)}\right]}{\sqrt{g^2-4 f h}}
\end{aligned}$$

Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Log}\left[\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]^2}{e+f x} d x$$

Optimal (type 4, 322 leaves, 9 steps):

$$\begin{aligned}
& -\frac{\operatorname{Log}\left[-\frac{b c-a d}{d(a+b x)}\right] \operatorname{Log}\left[\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]^2}{f}+\frac{\operatorname{Log}\left[\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]^2 \operatorname{Log}\left[1-\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]}{f}- \\
& \frac{2 \operatorname{Log}\left[\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right] \operatorname{PolyLog}\left[2, \frac{b(c+d x)}{d(a+b x)}\right]}{f}+\frac{2 \operatorname{Log}\left[\frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right] \operatorname{PolyLog}\left[2, \frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]}{f}+ \\
& \frac{2 \operatorname{PolyLog}\left[3, \frac{b(c+d x)}{d(a+b x)}\right]}{f}-\frac{2 \operatorname{PolyLog}\left[3, \frac{(b e-a f)(c+d x)}{(d e-c f)(a+b x)}\right]}{f}
\end{aligned}$$

Result (type 4, 334 leaves, 7 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[\frac{bc-ad}{b(c+dx)}\right] \text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2}{f} + \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2 \text{Log}\left[\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{f} + \\
& \frac{2 \text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, 1 - \frac{bc-ad}{b(c+dx)}\right]}{f} - \frac{2 \text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, 1 - \frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{f} + \\
& \frac{2 \text{PolyLog}\left[3, 1 - \frac{bc-ad}{b(c+dx)}\right]}{f} - \frac{2 \text{PolyLog}\left[3, 1 - \frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{f}
\end{aligned}$$

Problem 108: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{Log}\left[\frac{b(e+fx)}{be-af}\right]}{(a+bx)(c+dx)} dx$$

Optimal (type 4, 433 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2}{2(bc-ad)} - \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2 \text{Log}\left[\frac{b(e+fx)}{be-af}\right]}{2(bc-ad)} + \\
& \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2 \text{Log}\left[1 - \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{2(bc-ad)} - \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{bc-ad} + \\
& \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{bc-ad} + \frac{\text{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{bc-ad} - \frac{\text{PolyLog}\left[3, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{bc-ad}
\end{aligned}$$

Result (type 4, 445 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\text{Log}\left[\frac{bc-ad}{b(c+dx)}\right] \text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2}{2(bc-ad)} - \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2 \text{Log}\left[\frac{b(e+fx)}{be-af}\right]}{2(bc-ad)} + \\
& \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]^2 \text{Log}\left[\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{2(bc-ad)} + \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, 1 - \frac{bc-ad}{b(c+dx)}\right]}{bc-ad} - \\
& \frac{\text{Log}\left[\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] \text{PolyLog}\left[2, 1 - \frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{bc-ad} + \\
& \frac{\text{PolyLog}\left[3, 1 - \frac{bc-ad}{b(c+dx)}\right]}{bc-ad} - \frac{\text{PolyLog}\left[3, 1 - \frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right]}{bc-ad}
\end{aligned}$$

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Test results for the 547 problems in "3.3 u (a+b log(c (d+e x)^n))^p.m"

Problem 44: Result valid but suboptimal antiderivative.

$$\int (f+gx)^3 (a+b \text{Log}[c(d+ex)^n])^2 dx$$

Optimal (type 3, 365 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 b^2 (e f - d g)^3 n^2 x}{e^3} + \frac{3 b^2 g (e f - d g)^2 n^2 (d + e x)^2}{4 e^4} + \\
& \frac{2 b^2 g^2 (e f - d g) n^2 (d + e x)^3}{9 e^4} + \frac{b^2 g^3 n^2 (d + e x)^4}{32 e^4} + \frac{b^2 (e f - d g)^4 n^2 \operatorname{Log}[d + e x]^2}{4 e^4 g} - \\
& \frac{2 b (e f - d g)^3 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{e^4} - \\
& \frac{3 b g (e f - d g)^2 n (d + e x)^2 (a + b \operatorname{Log}[c (d + e x)^n])}{2 e^4} - \\
& \frac{2 b g^2 (e f - d g) n (d + e x)^3 (a + b \operatorname{Log}[c (d + e x)^n])}{3 e^4} - \frac{b g^3 n (d + e x)^4 (a + b \operatorname{Log}[c (d + e x)^n])}{8 e^4} - \\
& \frac{b (e f - d g)^4 n \operatorname{Log}[d + e x] (a + b \operatorname{Log}[c (d + e x)^n])}{2 e^4 g} + \frac{(f + g x)^4 (a + b \operatorname{Log}[c (d + e x)^n])^2}{4 g}
\end{aligned}$$

Result (type 3, 301 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 b^2 (e f - d g)^3 n^2 x}{e^3} + \frac{3 b^2 g (e f - d g)^2 n^2 (d + e x)^2}{4 e^4} + \\
& \frac{2 b^2 g^2 (e f - d g) n^2 (d + e x)^3}{9 e^4} + \frac{b^2 g^3 n^2 (d + e x)^4}{32 e^4} + \frac{b^2 (e f - d g)^4 n^2 \operatorname{Log}[d + e x]^2}{4 e^4 g} - \\
& \frac{1}{24 g} b n \left( \frac{48 g (e f - d g)^3 (d + e x)}{e^4} + \frac{36 g^2 (e f - d g)^2 (d + e x)^2}{e^4} + \right. \\
& \quad \left. \frac{16 g^3 (e f - d g) (d + e x)^3}{e^4} + \frac{3 g^4 (d + e x)^4}{e^4} + \frac{12 (e f - d g)^4 \operatorname{Log}[d + e x]}{e^4} \right) \\
& (a + b \operatorname{Log}[c (d + e x)^n]) + \frac{(f + g x)^4 (a + b \operatorname{Log}[c (d + e x)^n])^2}{4 g}
\end{aligned}$$

**Problem 45: Result valid but suboptimal antiderivative.**

$$\int (f + g x)^2 (a + b \operatorname{Log}[c (d + e x)^n])^2 dx$$

Optimal (type 3, 287 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 b^2 (e f - d g)^2 n^2 x}{e^2} + \frac{b^2 g (e f - d g) n^2 (d + e x)^2}{2 e^3} + \frac{2 b^2 g^2 n^2 (d + e x)^3}{27 e^3} + \\
& \frac{b^2 (e f - d g)^3 n^2 \operatorname{Log}[d + e x]^2}{3 e^3 g} - \frac{2 b (e f - d g)^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{e^3} - \\
& \frac{b g (e f - d g) n (d + e x)^2 (a + b \operatorname{Log}[c (d + e x)^n])}{e^3} - \frac{2 b g^2 n (d + e x)^3 (a + b \operatorname{Log}[c (d + e x)^n])}{9 e^3} - \\
& \frac{2 b (e f - d g)^3 n \operatorname{Log}[d + e x] (a + b \operatorname{Log}[c (d + e x)^n])}{3 e^3 g} + \frac{(f + g x)^3 (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 g}
\end{aligned}$$

Result (type 3, 243 leaves, 8 steps):

$$\begin{aligned} & \frac{2 b^2 (e f - d g)^2 n^2 x}{e^2} + \frac{b^2 g (e f - d g) n^2 (d + e x)^2}{2 e^3} + \\ & \frac{2 b^2 g^2 n^2 (d + e x)^3}{27 e^3} + \frac{b^2 (e f - d g)^3 n^2 \operatorname{Log}[d + e x]^2}{3 e^3 g} - \frac{1}{9 g} b n \\ & \left( \frac{18 g (e f - d g)^2 (d + e x)}{e^3} + \frac{9 g^2 (e f - d g) (d + e x)^2}{e^3} + \frac{2 g^3 (d + e x)^3}{e^3} + \frac{6 (e f - d g)^3 \operatorname{Log}[d + e x]}{e^3} \right) \\ & (a + b \operatorname{Log}[c (d + e x)^n]) + \frac{(f + g x)^3 (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 g} \end{aligned}$$

Problem 50: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{(f + g x)^3} dx$$

Optimal (type 4, 202 leaves, 7 steps):

$$\begin{aligned} & - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{(e f - d g)^2 (f + g x)} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (f + g x)^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[f + g x]}{g (e f - d g)^2} - \\ & \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 + \frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^2} + \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^2} \end{aligned}$$

Result (type 4, 233 leaves, 9 steps):

$$\begin{aligned} & - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{(e f - d g)^2 (f + g x)} + \frac{e^2 (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (e f - d g)^2} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (f + g x)^2} + \\ & \frac{b^2 e^2 n^2 \operatorname{Log}[f + g x]}{g (e f - d g)^2} - \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^2} - \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^2} \end{aligned}$$

Problem 51: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{(f + g x)^4} dx$$

Optimal (type 4, 317 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b^2 e^2 n^2}{3 g (e f - d g)^2 (f + g x)} - \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x]}{3 g (e f - d g)^3} + \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])}{3 g (e f - d g) (f + g x)^2} - \\
& \frac{2 b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{3 (e f - d g)^3 (f + g x)} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{3 g (f + g x)^3} + \frac{b^2 e^3 n^2 \operatorname{Log}[f + g x]}{g (e f - d g)^3} - \\
& \frac{2 b e^3 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 + \frac{e f - d g}{g (d + e x)}\right]}{3 g (e f - d g)^3} + \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, -\frac{e f - d g}{g (d + e x)}\right]}{3 g (e f - d g)^3}
\end{aligned}$$

Result (type 4, 347 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b^2 e^2 n^2}{3 g (e f - d g)^2 (f + g x)} - \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x]}{3 g (e f - d g)^3} + \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])}{3 g (e f - d g) (f + g x)^2} - \frac{2 b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{3 (e f - d g)^3 (f + g x)} + \\
& \frac{e^3 (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 g (e f - d g)^3} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{3 g (f + g x)^3} + \frac{b^2 e^3 n^2 \operatorname{Log}[f + g x]}{g (e f - d g)^3} - \\
& \frac{2 b e^3 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{3 g (e f - d g)^3} - \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{3 g (e f - d g)^3}
\end{aligned}$$

**Problem 58: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{(f + g x)^3} dx$$

Optimal (type 4, 342 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3 b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 (e f - d g)^2 (f + g x)} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{2 g (f + g x)^2} + \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^2} - \\
& \frac{3 b e^2 n (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[1 + \frac{e f - d g}{g (d + e x)}\right]}{2 g (e f - d g)^2} + \\
& \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^2} + \\
& \frac{3 b^3 e^2 n^3 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^2} + \frac{3 b^3 e^2 n^3 \operatorname{PolyLog}\left[3, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^2}
\end{aligned}$$

Result (type 4, 370 leaves, 12 steps):

$$\begin{aligned}
& - \frac{3 b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 (e f - d g)^2 (f + g x)} + \frac{e^2 (a + b \operatorname{Log}[c (d + e x)^n])^3}{2 g (e f - d g)^2} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{2 g (f + g x)^2} + \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^2} - \\
& \frac{3 b e^2 n (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{2 g (e f - d g)^2} + \frac{3 b^3 e^2 n^3 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^2} - \\
& \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^2} + \frac{3 b^3 e^2 n^3 \operatorname{PolyLog}\left[3, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^2}
\end{aligned}$$

**Problem 59: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{(f + g x)^4} dx$$

Optimal (type 4, 564 leaves, 16 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{(e f - d g)^3 (f + g x)} + \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (e f - d g) (f + g x)^2} - \\
& \frac{b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{(e f - d g)^3 (f + g x)} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{3 g (f + g x)^3} - \\
& \frac{b^3 e^3 n^3 \operatorname{Log}[f + g x]}{g (e f - d g)^3} + \frac{2 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^3} + \\
& \frac{b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 + \frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^3} - \\
& \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[1 + \frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^3} - \frac{b^3 e^3 n^3 \operatorname{PolyLog}\left[2, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^3} + \\
& \frac{2 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^3} + \\
& \frac{2 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^3} + \frac{2 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, -\frac{e f - d g}{g (d + e x)}\right]}{g (e f - d g)^3}
\end{aligned}$$

Result (type 4, 525 leaves, 21 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{(e f - d g)^3 (f + g x)} - \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (e f - d g)^3} + \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 g (e f - d g) (f + g x)^2} - \frac{b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{(e f - d g)^3 (f + g x)} + \\
& \frac{e^3 (a + b \operatorname{Log}[c (d + e x)^n])^3}{3 g (e f - d g)^3} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^3}{3 g (f + g x)^3} - \\
& \frac{b^3 e^3 n^3 \operatorname{Log}[f + g x]}{g (e f - d g)^3} + \frac{3 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^3} - \\
& \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (f + g x)}{e f - d g}\right]}{g (e f - d g)^3} + \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^3} - \\
& \frac{2 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^3} + \frac{2 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, -\frac{g (d + e x)}{e f - d g}\right]}{g (e f - d g)^3}
\end{aligned}$$

Problem 85: Result valid but suboptimal antiderivative.

$$\int x^2 \operatorname{Log}[c (a + b x)^n]^2 dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 a^2 n^2 x}{b^2} - \frac{a n^2 (a + b x)^2}{2 b^3} + \frac{2 n^2 (a + b x)^3}{27 b^3} - \frac{a^3 n^2 \operatorname{Log}[a + b x]^2}{3 b^3} - \\
& \frac{2 a^2 n (a + b x) \operatorname{Log}[c (a + b x)^n]}{b^3} + \frac{a n (a + b x)^2 \operatorname{Log}[c (a + b x)^n]}{b^3} - \\
& \frac{2 n (a + b x)^3 \operatorname{Log}[c (a + b x)^n]}{9 b^3} + \frac{2 a^3 n \operatorname{Log}[a + b x] \operatorname{Log}[c (a + b x)^n]}{3 b^3} + \frac{1}{3} x^3 \operatorname{Log}[c (a + b x)^n]^2
\end{aligned}$$

Result (type 3, 156 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 a^2 n^2 x}{b^2} - \frac{a n^2 (a + b x)^2}{2 b^3} + \frac{2 n^2 (a + b x)^3}{27 b^3} - \frac{a^3 n^2 \operatorname{Log}[a + b x]^2}{3 b^3} - \\
& \frac{1}{9} n \left( \frac{18 a^2 (a + b x)}{b^3} - \frac{9 a (a + b x)^2}{b^3} + \frac{2 (a + b x)^3}{b^3} - \frac{6 a^3 \operatorname{Log}[a + b x]}{b^3} \right) \operatorname{Log}[c (a + b x)^n] + \\
& \frac{1}{3} x^3 \operatorname{Log}[c (a + b x)^n]^2
\end{aligned}$$

Problem 86: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Log}[c (a + b x)^n]^2}{x^4} dx$$

Optimal (type 4, 177 leaves, 11 steps):



$$\begin{aligned}
& - \frac{b^2 n^2}{3 a^2 x} - \frac{b^3 n^2 \operatorname{Log}[x]}{a^3} + \frac{b^3 n^2 \operatorname{Log}[a + b x]}{3 a^3} - \frac{b n \operatorname{Log}[c (a + b x)^n]}{3 a x^2} + \frac{2 b^2 n (a + b x) \operatorname{Log}[c (a + b x)^n]}{3 a^3 x} \\
& - \frac{\operatorname{Log}[c (a + b x)^n]^2}{3 x^3} + \frac{2 b^3 n \operatorname{Log}[c (a + b x)^n] \operatorname{Log}\left[1 - \frac{a}{a + b x}\right]}{3 a^3} - \frac{2 b^3 n^2 \operatorname{PolyLog}\left[2, \frac{a}{a + b x}\right]}{3 a^3}
\end{aligned}$$

Result (type 4, 193 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b^2 n^2}{3 a^2 x} - \frac{b^3 n^2 \operatorname{Log}[x]}{a^3} + \frac{b^3 n^2 \operatorname{Log}[a + b x]}{3 a^3} - \frac{b n \operatorname{Log}[c (a + b x)^n]}{3 a x^2} + \\
& \frac{2 b^2 n (a + b x) \operatorname{Log}[c (a + b x)^n]}{3 a^3 x} + \frac{2 b^3 n \operatorname{Log}\left[-\frac{b x}{a}\right] \operatorname{Log}[c (a + b x)^n]}{3 a^3} - \\
& \frac{b^3 \operatorname{Log}[c (a + b x)^n]^2}{3 a^3} - \frac{\operatorname{Log}[c (a + b x)^n]^2}{3 x^3} + \frac{2 b^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{b x}{a}\right]}{3 a^3}
\end{aligned}$$

**Problem 175: Result valid but suboptimal antiderivative.**

$$\int \frac{(h + i x)^4 (a + b \operatorname{Log}[c (e + f x)])}{d e + d f x} dx$$

Optimal (type 3, 315 leaves, 8 steps):

$$\begin{aligned}
& - \frac{4 b i (f h - e i)^3 x}{d f^4} - \frac{3 b i^2 (f h - e i)^2 (e + f x)^2}{2 d f^5} - \frac{4 b i^3 (f h - e i) (e + f x)^3}{9 d f^5} - \frac{b i^4 (e + f x)^4}{16 d f^5} \\
& - \frac{b (f h - e i)^4 \operatorname{Log}[e + f x]^2}{2 d f^5} + \frac{4 i (f h - e i)^3 (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d f^5} + \\
& \frac{3 i^2 (f h - e i)^2 (e + f x)^2 (a + b \operatorname{Log}[c (e + f x)])}{d f^5} + \\
& \frac{4 i^3 (f h - e i) (e + f x)^3 (a + b \operatorname{Log}[c (e + f x)])}{3 d f^5} + \\
& \frac{i^4 (e + f x)^4 (a + b \operatorname{Log}[c (e + f x)])}{4 d f^5} + \frac{(f h - e i)^4 \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (e + f x)])}{d f^5}
\end{aligned}$$

Result (type 3, 260 leaves, 6 steps):

$$\begin{aligned}
& - \frac{4 b i (f h - e i)^3 x}{d f^4} - \frac{3 b i^2 (f h - e i)^2 (e + f x)^2}{2 d f^5} - \\
& \frac{4 b i^3 (f h - e i) (e + f x)^3}{9 d f^5} - \frac{b i^4 (e + f x)^4}{16 d f^5} - \frac{b (f h - e i)^4 \operatorname{Log}[e + f x]^2}{2 d f^5} + \frac{1}{12 d f} \\
& \left( \frac{48 i (f h - e i)^3 (e + f x)}{f^4} + \frac{36 i^2 (f h - e i)^2 (e + f x)^2}{f^4} + \frac{16 i^3 (f h - e i) (e + f x)^3}{f^4} + \right. \\
& \left. \frac{3 i^4 (e + f x)^4}{f^4} + \frac{12 (f h - e i)^4 \operatorname{Log}[e + f x]}{f^4} \right) (a + b \operatorname{Log}[c (e + f x)])
\end{aligned}$$

### Problem 176: Result valid but suboptimal antiderivative.

$$\int \frac{(h + i x)^3 (a + b \operatorname{Log}[c (e + f x)])}{d e + d f x} dx$$

Optimal (type 3, 244 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 b i (f h - e i)^2 x}{d f^3} - \frac{3 b i^2 (f h - e i) (e + f x)^2}{4 d f^4} - \frac{b i^3 (e + f x)^3}{9 d f^4} - \frac{b (f h - e i)^3 \operatorname{Log}[e + f x]^2}{2 d f^4} + \\ & \frac{3 i (f h - e i)^2 (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d f^4} + \frac{3 i^2 (f h - e i) (e + f x)^2 (a + b \operatorname{Log}[c (e + f x)])}{2 d f^4} + \\ & \frac{i^3 (e + f x)^3 (a + b \operatorname{Log}[c (e + f x)])}{3 d f^4} + \frac{(f h - e i)^3 \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (e + f x)])}{d f^4} \end{aligned}$$

Result (type 3, 204 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 b i (f h - e i)^2 x}{d f^3} - \frac{3 b i^2 (f h - e i) (e + f x)^2}{4 d f^4} - \frac{b i^3 (e + f x)^3}{9 d f^4} - \frac{b (f h - e i)^3 \operatorname{Log}[e + f x]^2}{2 d f^4} + \frac{1}{6 d f} \\ & \left( \frac{18 i (f h - e i)^2 (e + f x)}{f^3} + \frac{9 i^2 (f h - e i) (e + f x)^2}{f^3} + \frac{2 i^3 (e + f x)^3}{f^3} + \frac{6 (f h - e i)^3 \operatorname{Log}[e + f x]}{f^3} \right) \\ & (a + b \operatorname{Log}[c (e + f x)]) \end{aligned}$$

### Problem 177: Result valid but suboptimal antiderivative.

$$\int \frac{(h + i x)^2 (a + b \operatorname{Log}[c (e + f x)])}{d e + d f x} dx$$

Optimal (type 3, 157 leaves, 7 steps):

$$\begin{aligned} & -\frac{b (4 f h - 3 e i + f i x)^2}{4 d f^3} - \frac{b (f h - e i)^2 \operatorname{Log}[e + f x]^2}{2 d f^3} + \\ & \frac{2 i (f h - e i) (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d f^3} + \\ & \frac{i^2 (e + f x)^2 (a + b \operatorname{Log}[c (e + f x)])}{2 d f^3} + \frac{(f h - e i)^2 \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (e + f x)])}{d f^3} \end{aligned}$$

Result (type 3, 133 leaves, 7 steps):

$$\begin{aligned} & -\frac{b (4 f h - 3 e i + f i x)^2}{4 d f^3} - \frac{b (f h - e i)^2 \operatorname{Log}[e + f x]^2}{2 d f^3} + \frac{1}{2 d f} \\ & \left( \frac{4 i (f h - e i) (e + f x)}{f^2} + \frac{i^2 (e + f x)^2}{f^2} + \frac{2 (f h - e i)^2 \operatorname{Log}[e + f x]}{f^2} \right) (a + b \operatorname{Log}[c (e + f x)]) \end{aligned}$$

### Problem 180: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c(e + f x)]}{(d e + d f x)(h + i x)} dx$$

Optimal (type 4, 87 leaves, 4 steps):

$$-\frac{(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[1 + \frac{f h - e i}{i(e + f x)}\right]}{d(f h - e i)} + \frac{b \operatorname{PolyLog}\left[2, -\frac{f h - e i}{i(e + f x)}\right]}{d(f h - e i)}$$

Result (type 4, 116 leaves, 6 steps):

$$\frac{(a + b \operatorname{Log}[c(e + f x)])^2}{2 b d(f h - e i)} - \frac{(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[\frac{f(h + i x)}{f h - e i}\right]}{d(f h - e i)} - \frac{b \operatorname{PolyLog}\left[2, -\frac{i(e + f x)}{f h - e i}\right]}{d(f h - e i)}$$

### Problem 181: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c(e + f x)]}{(d e + d f x)(h + i x)^2} dx$$

Optimal (type 4, 151 leaves, 7 steps):

$$-\frac{i(e + f x)(a + b \operatorname{Log}[c(e + f x)])}{d(f h - e i)^2(h + i x)} + \frac{b f \operatorname{Log}[h + i x]}{d(f h - e i)^2} - \frac{f(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[1 + \frac{f h - e i}{i(e + f x)}\right]}{d(f h - e i)^2} + \frac{b f \operatorname{PolyLog}\left[2, -\frac{f h - e i}{i(e + f x)}\right]}{d(f h - e i)^2}$$

Result (type 4, 181 leaves, 9 steps):

$$-\frac{i(e + f x)(a + b \operatorname{Log}[c(e + f x)])}{d(f h - e i)^2(h + i x)} + \frac{f(a + b \operatorname{Log}[c(e + f x)])^2}{2 b d(f h - e i)^2} + \frac{b f \operatorname{Log}[h + i x]}{d(f h - e i)^2} - \frac{f(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[\frac{f(h + i x)}{f h - e i}\right]}{d(f h - e i)^2} - \frac{b f \operatorname{PolyLog}\left[2, -\frac{i(e + f x)}{f h - e i}\right]}{d(f h - e i)^2}$$

### Problem 182: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{Log}[c(e + f x)]}{(d e + d f x)(h + i x)^3} dx$$

Optimal (type 4, 250 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b f}{2 d (f h - e i)^2 (h + i x)} - \frac{b f^2 \operatorname{Log}[e + f x]}{2 d (f h - e i)^3} + \frac{a + b \operatorname{Log}[c (e + f x)]}{2 d (f h - e i) (h + i x)^2} - \\
& \frac{f i (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d (f h - e i)^3 (h + i x)} + \frac{3 b f^2 \operatorname{Log}[h + i x]}{2 d (f h - e i)^3} - \\
& \frac{f^2 (a + b \operatorname{Log}[c (e + f x)]) \operatorname{Log}\left[1 + \frac{f h - e i}{i (e + f x)}\right]}{d (f h - e i)^3} + \frac{b f^2 \operatorname{PolyLog}\left[2, -\frac{f h - e i}{i (e + f x)}\right]}{d (f h - e i)^3}
\end{aligned}$$

Result (type 4, 282 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b f}{2 d (f h - e i)^2 (h + i x)} - \frac{b f^2 \operatorname{Log}[e + f x]}{2 d (f h - e i)^3} + \frac{a + b \operatorname{Log}[c (e + f x)]}{2 d (f h - e i) (h + i x)^2} - \\
& \frac{f i (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d (f h - e i)^3 (h + i x)} + \frac{f^2 (a + b \operatorname{Log}[c (e + f x)])^2}{2 b d (f h - e i)^3} + \frac{3 b f^2 \operatorname{Log}[h + i x]}{2 d (f h - e i)^3} - \\
& \frac{f^2 (a + b \operatorname{Log}[c (e + f x)]) \operatorname{Log}\left[\frac{f (h + i x)}{f h - e i}\right]}{d (f h - e i)^3} - \frac{b f^2 \operatorname{PolyLog}\left[2, -\frac{i (e + f x)}{f h - e i}\right]}{d (f h - e i)^3}
\end{aligned}$$

**Problem 183: Result valid but suboptimal antiderivative.**

$$\int \frac{(h + i x)^4 (a + b \operatorname{Log}[c (e + f x)])^2}{d e + d f x} dx$$

Optimal (type 3, 579 leaves, 32 steps):

$$\begin{aligned}
& - \frac{4 a b i (f h - e i)^3 x}{d f^4} + \frac{8 b^2 i (f h - e i)^3 x}{d f^4} + \frac{3 b^2 i^2 (f h - e i)^2 (e + f x)^2}{2 d f^5} + \\
& \frac{8 b^2 i^3 (f h - e i) (e + f x)^3}{27 d f^5} + \frac{b^2 i^4 (e + f x)^4}{32 d f^5} + \frac{7 b^2 (f h - e i)^4 \operatorname{Log}[e + f x]^2}{12 d f^5} - \\
& \frac{4 b^2 i (f h - e i)^3 (e + f x) \operatorname{Log}[c (e + f x)]}{d f^5} - \frac{4 b i (f h - e i)^3 (e + f x) (a + b \operatorname{Log}[c (e + f x)])}{d f^5} - \\
& \frac{3 b i^2 (f h - e i)^2 (e + f x)^2 (a + b \operatorname{Log}[c (e + f x)])}{d f^5} - \\
& \frac{8 b i^3 (f h - e i) (e + f x)^3 (a + b \operatorname{Log}[c (e + f x)])}{9 d f^5} - \frac{b i^4 (e + f x)^4 (a + b \operatorname{Log}[c (e + f x)])}{8 d f^5} - \\
& \frac{7 b (f h - e i)^4 \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (e + f x)])}{6 d f^5} + \\
& \frac{2 i (f h - e i)^3 (e + f x) (a + b \operatorname{Log}[c (e + f x)])^2}{d f^5} + \\
& \frac{i^2 (f h - e i)^2 (e + f x)^2 (a + b \operatorname{Log}[c (e + f x)])^2}{2 d f^5} + \frac{(f h - e i) (h + i x)^3 (a + b \operatorname{Log}[c (e + f x)])^2}{3 d f^2} + \\
& \frac{(h + i x)^4 (a + b \operatorname{Log}[c (e + f x)])^2}{4 d f} + \frac{(f h - e i)^4 (a + b \operatorname{Log}[c (e + f x)])^3}{3 b d f^5}
\end{aligned}$$

Result (type 3, 672 leaves, 30 steps):

$$\begin{aligned}
& -\frac{4 a b i (f h-e i)^3 x}{d f^4} + \frac{8 b^2 i (f h-e i)^3 x}{d f^4} + \\
& \frac{3 b^2 i^2 (f h-e i)^2 (e+f x)^2}{2 d f^5} + \frac{8 b^2 i^3 (f h-e i) (e+f x)^3}{27 d f^5} + \frac{b^2 i^4 (e+f x)^4}{32 d f^5} + \\
& \frac{7 b^2 (f h-e i)^4 \operatorname{Log}[e+f x]^2}{12 d f^5} - \frac{4 b^2 i (f h-e i)^3 (e+f x) \operatorname{Log}[c (e+f x)]}{d f^5} - \\
& \frac{b i^2 (f h-e i)^2 (e+f x)^2 (a+b \operatorname{Log}[c (e+f x)])}{2 d f^5} - \frac{1}{9 d f^3} b (f h-e i) \\
& \left( \frac{18 i (f h-e i)^2 (e+f x)}{f^2} + \frac{9 i^2 (f h-e i) (e+f x)^2}{f^2} + \frac{2 i^3 (e+f x)^3}{f^2} + \frac{6 (f h-e i)^3 \operatorname{Log}[e+f x]}{f^2} \right) \\
& (a+b \operatorname{Log}[c (e+f x)]) - \frac{1}{24 d f^2} b \left( \frac{48 i (f h-e i)^3 (e+f x)}{f^3} + \frac{36 i^2 (f h-e i)^2 (e+f x)^2}{f^3} + \right. \\
& \left. \frac{16 i^3 (f h-e i) (e+f x)^3}{f^3} + \frac{3 i^4 (e+f x)^4}{f^3} + \frac{12 (f h-e i)^4 \operatorname{Log}[e+f x]}{f^3} \right) \\
& (a+b \operatorname{Log}[c (e+f x)]) + \frac{2 i (f h-e i)^3 (e+f x) (a+b \operatorname{Log}[c (e+f x)])^2}{d f^5} + \\
& \frac{i^2 (f h-e i)^2 (e+f x)^2 (a+b \operatorname{Log}[c (e+f x)])^2}{2 d f^5} + \frac{(f h-e i) (h+i x)^3 (a+b \operatorname{Log}[c (e+f x)])^2}{3 d f^2} + \\
& \frac{(h+i x)^4 (a+b \operatorname{Log}[c (e+f x)])^2}{4 d f} + \frac{(f h-e i)^4 (a+b \operatorname{Log}[c (e+f x)])^3}{3 b d f^5}
\end{aligned}$$

**Problem 188: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b \operatorname{Log}[c (e+f x)])^2}{(d e+d f x) (h+i x)} dx$$

Optimal (type 4, 142 leaves, 5 steps):

$$\begin{aligned}
& -\frac{(a+b \operatorname{Log}[c (e+f x)])^2 \operatorname{Log}\left[1+\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)} + \\
& \frac{2 b (a+b \operatorname{Log}[c (e+f x)]) \operatorname{PolyLog}\left[2,-\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)} + \frac{2 b^2 \operatorname{PolyLog}\left[3,-\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)}
\end{aligned}$$

Result (type 4, 168 leaves, 8 steps):

$$\begin{aligned}
& \frac{(a+b \operatorname{Log}[c (e+f x)])^3}{3 b d (f h-e i)} - \frac{(a+b \operatorname{Log}[c (e+f x)])^2 \operatorname{Log}\left[\frac{f (h+i x)}{f h-e i}\right]}{d (f h-e i)} - \\
& \frac{2 b (a+b \operatorname{Log}[c (e+f x)]) \operatorname{PolyLog}\left[2,-\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)} + \frac{2 b^2 \operatorname{PolyLog}\left[3,-\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)}
\end{aligned}$$

### Problem 189: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c(e + f x)])^2}{(d e + d f x)(h + i x)^2} dx$$

Optimal (type 4, 273 leaves, 9 steps):

$$\begin{aligned} & -\frac{i(e + f x)(a + b \operatorname{Log}[c(e + f x)])^2}{d(fh - ei)^2(h + i x)} + \frac{2bf(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[\frac{f(h + i x)}{fh - ei}\right]}{d(fh - ei)^2} - \\ & \frac{f(a + b \operatorname{Log}[c(e + f x)])^2 \operatorname{Log}\left[1 + \frac{fh - ei}{i(e + f x)}\right]}{d(fh - ei)^2} + \frac{2bf(a + b \operatorname{Log}[c(e + f x)]) \operatorname{PolyLog}\left[2, -\frac{fh - ei}{i(e + f x)}\right]}{d(fh - ei)^2} + \\ & \frac{2b^2f \operatorname{PolyLog}\left[2, -\frac{i(e + f x)}{fh - ei}\right]}{d(fh - ei)^2} + \frac{2b^2f \operatorname{PolyLog}\left[3, -\frac{fh - ei}{i(e + f x)}\right]}{d(fh - ei)^2} \end{aligned}$$

Result (type 4, 300 leaves, 12 steps):

$$\begin{aligned} & -\frac{i(e + f x)(a + b \operatorname{Log}[c(e + f x)])^2}{d(fh - ei)^2(h + i x)} + \\ & \frac{f(a + b \operatorname{Log}[c(e + f x)])^3}{3bd(fh - ei)^2} + \frac{2bf(a + b \operatorname{Log}[c(e + f x)]) \operatorname{Log}\left[\frac{f(h + i x)}{fh - ei}\right]}{d(fh - ei)^2} - \\ & \frac{f(a + b \operatorname{Log}[c(e + f x)])^2 \operatorname{Log}\left[\frac{f(h + i x)}{fh - ei}\right]}{d(fh - ei)^2} + \frac{2b^2f \operatorname{PolyLog}\left[2, -\frac{i(e + f x)}{fh - ei}\right]}{d(fh - ei)^2} - \\ & \frac{2bf(a + b \operatorname{Log}[c(e + f x)]) \operatorname{PolyLog}\left[2, -\frac{i(e + f x)}{fh - ei}\right]}{d(fh - ei)^2} + \frac{2b^2f \operatorname{PolyLog}\left[3, -\frac{i(e + f x)}{fh - ei}\right]}{d(fh - ei)^2} \end{aligned}$$

### Problem 190: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c(e + f x)])^2}{(d e + d f x)(h + i x)^3} dx$$

Optimal (type 4, 485 leaves, 16 steps):

$$\begin{aligned}
& \frac{b f i (e+f x) (a+b \operatorname{Log}[c (e+f x)])}{d (f h-e i)^3 (h+i x)} + \frac{(a+b \operatorname{Log}[c (e+f x)])^2}{2 d (f h-e i) (h+i x)^2} - \\
& \frac{f i (e+f x) (a+b \operatorname{Log}[c (e+f x)])^2}{d (f h-e i)^3 (h+i x)} - \frac{b^2 f^2 \operatorname{Log}[h+i x]}{d (f h-e i)^3} + \\
& \frac{2 b f^2 (a+b \operatorname{Log}[c (e+f x)]) \operatorname{Log}\left[\frac{f(h+i x)}{f h-e i}\right]}{d (f h-e i)^3} + \frac{b f^2 (a+b \operatorname{Log}[c (e+f x)]) \operatorname{Log}\left[1+\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)^3} - \\
& \frac{f^2 (a+b \operatorname{Log}[c (e+f x)])^2 \operatorname{Log}\left[1+\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)^3} - \frac{b^2 f^2 \operatorname{PolyLog}\left[2, -\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)^3} + \\
& \frac{2 b f^2 (a+b \operatorname{Log}[c (e+f x)]) \operatorname{PolyLog}\left[2, -\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)^3} + \\
& \frac{2 b^2 f^2 \operatorname{PolyLog}\left[2, -\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)^3} + \frac{2 b^2 f^2 \operatorname{PolyLog}\left[3, -\frac{f h-e i}{i (e+f x)}\right]}{d (f h-e i)^3}
\end{aligned}$$

Result (type 4, 453 leaves, 21 steps):

$$\begin{aligned}
& \frac{b f i (e+f x) (a+b \operatorname{Log}[c (e+f x)])}{d (f h-e i)^3 (h+i x)} - \frac{f^2 (a+b \operatorname{Log}[c (e+f x)])^2}{2 d (f h-e i)^3} + \\
& \frac{(a+b \operatorname{Log}[c (e+f x)])^2}{2 d (f h-e i) (h+i x)^2} - \frac{f i (e+f x) (a+b \operatorname{Log}[c (e+f x)])^2}{d (f h-e i)^3 (h+i x)} + \\
& \frac{f^2 (a+b \operatorname{Log}[c (e+f x)])^3}{3 b d (f h-e i)^3} - \frac{b^2 f^2 \operatorname{Log}[h+i x]}{d (f h-e i)^3} + \frac{3 b f^2 (a+b \operatorname{Log}[c (e+f x)]) \operatorname{Log}\left[\frac{f(h+i x)}{f h-e i}\right]}{d (f h-e i)^3} - \\
& \frac{f^2 (a+b \operatorname{Log}[c (e+f x)])^2 \operatorname{Log}\left[\frac{f(h+i x)}{f h-e i}\right]}{d (f h-e i)^3} + \frac{3 b^2 f^2 \operatorname{PolyLog}\left[2, -\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)^3} - \\
& \frac{2 b f^2 (a+b \operatorname{Log}[c (e+f x)]) \operatorname{PolyLog}\left[2, -\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)^3} + \frac{2 b^2 f^2 \operatorname{PolyLog}\left[3, -\frac{i (e+f x)}{f h-e i}\right]}{d (f h-e i)^3}
\end{aligned}$$

**Problem 314: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b \operatorname{Log}[c (d+e x)^n])^2}{x^3 (f+g x^2)} dx$$

Optimal (type 4, 551 leaves, 23 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2 f} - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f x} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f x^2} - \\
& \frac{g \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])^2}{f^2} + \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^2} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 f^2} - \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 - \frac{d}{d + e x}\right]}{d^2 f} + \\
& \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x}\right]}{d^2 f} + \frac{b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^2} + \\
& \frac{b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^2} - \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{f^2} - \frac{b^2 g n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^2} - \\
& \frac{b^2 g n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^2} + \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{f^2}
\end{aligned}$$

Result (type 4, 575 leaves, 25 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2 f} - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f x} - \frac{b e^2 n \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f} + \\
& \frac{e^2 (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 d^2 f} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f x^2} - \frac{g \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])^2}{f^2} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^2} + \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 f^2} + \\
& \frac{b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^2} + \\
& \frac{b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^2} - \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d^2 f} - \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{f^2} - \frac{b^2 g n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^2} - \\
& \frac{b^2 g n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^2} + \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{f^2}
\end{aligned}$$



### Problem 319: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{x^4 (f + g x^2)} dx$$

Optimal (type 4, 694 leaves, 26 steps):

$$\begin{aligned} & -\frac{b^2 e^2 n^2}{3 d^2 f x} - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3 f} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x]}{3 d^3 f} - \\ & \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])}{3 d f x^2} + \frac{2 b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{3 d^3 f x} - \\ & \frac{2 b e g n \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])}{d f^2} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{3 f x^3} + \\ & \frac{g (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{d f^2 x} + \frac{g^{3/2} (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 (-f)^{5/2}} - \\ & \frac{g^{3/2} (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 (-f)^{5/2}} + \frac{2 b e^3 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 - \frac{d}{d + e x}\right]}{3 d^3 f} - \\ & \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x}\right]}{3 d^3 f} - \frac{b g^{3/2} n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{(-f)^{5/2}} + \\ & \frac{b g^{3/2} n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{(-f)^{5/2}} - \frac{2 b^2 e g n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d f^2} + \\ & \frac{b^2 g^{3/2} n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{(-f)^{5/2}} - \frac{b^2 g^{3/2} n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{(-f)^{5/2}} \end{aligned}$$

Result (type 4, 717 leaves, 28 steps):

$$\begin{aligned}
& - \frac{b^2 e^2 n^2}{3 d^2 f x} - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3 f} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x]}{3 d^3 f} - \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e x)^n])}{3 d f x^2} + \frac{2 b e^2 n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{3 d^3 f x} + \\
& \frac{2 b e^3 n \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])}{3 d^3 f} - \frac{2 b e g n \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])}{d f^2} - \\
& \frac{e^3 (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 d^3 f} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{3 f x^3} + \\
& \frac{g (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])^2}{d f^2 x} + \frac{g^{3/2} (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 (-f)^{5/2}} - \\
& \frac{g^{3/2} (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 (-f)^{5/2}} - \\
& \frac{b g^{3/2} n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{(-f)^{5/2}} + \\
& \frac{b g^{3/2} n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{(-f)^{5/2}} + \\
& \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{3 d^3 f} - \frac{2 b^2 e g n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d f^2} + \\
& \frac{b^2 g^{3/2} n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{(-f)^{5/2}} - \frac{b^2 g^{3/2} n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{(-f)^{5/2}}
\end{aligned}$$

**Problem 324: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{x^3 (f + g x^2)^2} dx$$

Optimal (type 4, 970 leaves, 36 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2 f^2} - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f^2 x} + \frac{e^2 g (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 (e^2 f + d^2 g)} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 x^2} - \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 (f + g x^2)} - \frac{2 g \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])^2}{f^3} - \\
& \frac{b e (e f + d \sqrt{-f} \sqrt{g}) g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} - \\
& \frac{b e (e f - d \sqrt{-f} \sqrt{g}) g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[1 - \frac{d}{d + e x}\right]}{d^2 f^2} + \\
& \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x}\right]}{d^2 f^2} - \frac{b^2 e (e \sqrt{-f} + d \sqrt{g}) g n^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 (-f)^{5/2} (e^2 f + d^2 g)} + \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \\
& \frac{b^2 e (e f + d \sqrt{-f} \sqrt{g}) g n^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} - \\
& \frac{4 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{f^3} - \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \\
& \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} + \frac{4 b^2 g n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{f^3}
\end{aligned}$$

Result (type 4, 994 leaves, 38 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2 f^2} - \frac{b e n (d + e x) (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f^2 x} - \frac{b e^2 n \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])}{d^2 f^2} + \\
& \frac{e^2 (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 d^2 f^2} + \frac{e^2 g (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 (e^2 f + d^2 g)} - \frac{(a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 x^2} - \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 f^2 (f + g x^2)} - \frac{2 g \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])^2}{f^3} - \\
& \frac{b e (e f + d \sqrt{-f} \sqrt{g}) g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} - \sqrt{g} x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} - \\
& \frac{b e (e f - d \sqrt{-f} \sqrt{g}) g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{g (a + b \operatorname{Log}[c (d + e x)^n])^2 \operatorname{Log}\left[\frac{e (\sqrt{-f} + \sqrt{g} x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \\
& \frac{b^2 e (e \sqrt{-f} + d \sqrt{g}) g n^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{2 (-f)^{5/2} (e^2 f + d^2 g)} + \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \\
& \frac{b^2 e (e f + d \sqrt{-f} \sqrt{g}) g n^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{2 f^3 (e^2 f + d^2 g)} + \\
& \frac{2 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} - \frac{b^2 e^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d^2 f^2} - \\
& \frac{4 b g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{f^3} - \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, -\frac{\sqrt{g} (d + e x)}{e \sqrt{-f} - d \sqrt{g}}\right]}{f^3} - \\
& \frac{2 b^2 g n^2 \operatorname{PolyLog}\left[3, \frac{\sqrt{g} (d + e x)}{e \sqrt{-f} + d \sqrt{g}}\right]}{f^3} + \frac{4 b^2 g n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{f^3}
\end{aligned}$$

**Problem 363: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])}{x^2} dx$$

Optimal (type 4, 102 leaves, 6 steps):

$$\frac{b e m n \operatorname{Log}[x]}{d} - \frac{b e n \operatorname{Log}\left[1 + \frac{d}{e x}\right] \operatorname{Log}[f x^m]}{d} - \frac{b e m n \operatorname{Log}[d + e x]}{d} -$$

$$\left(\frac{m}{x} + \frac{\operatorname{Log}[f x^m]}{x}\right) (a + b \operatorname{Log}[c (d + e x)^n]) + \frac{b e m n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{d}$$

Result (type 4, 120 leaves, 8 steps):

$$\frac{b e m n \operatorname{Log}[x]}{d} + \frac{b e n \operatorname{Log}[f x^m]^2}{2 d m} - \frac{b e m n \operatorname{Log}[d + e x]}{d} -$$

$$\left(\frac{m}{x} + \frac{\operatorname{Log}[f x^m]}{x}\right) (a + b \operatorname{Log}[c (d + e x)^n]) - \frac{b e n \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \frac{b e m n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d}$$

**Problem 364: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])}{x^3} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$-\frac{3 b e m n}{4 d x} - \frac{b e^2 m n \operatorname{Log}[x]}{4 d^2} - \frac{b e n \operatorname{Log}[f x^m]}{2 d x} + \frac{b e^2 n \operatorname{Log}\left[1 + \frac{d}{e x}\right] \operatorname{Log}[f x^m]}{2 d^2} +$$

$$\frac{b e^2 m n \operatorname{Log}[d + e x]}{4 d^2} - \frac{1}{4} \left(\frac{m}{x^2} + \frac{2 \operatorname{Log}[f x^m]}{x^2}\right) (a + b \operatorname{Log}[c (d + e x)^n]) - \frac{b e^2 m n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{2 d^2}$$

Result (type 4, 175 leaves, 9 steps):

$$-\frac{3 b e m n}{4 d x} - \frac{b e^2 m n \operatorname{Log}[x]}{4 d^2} - \frac{b e n \operatorname{Log}[f x^m]}{2 d x} - \frac{b e^2 n \operatorname{Log}[f x^m]^2}{4 d^2 m} +$$

$$\frac{b e^2 m n \operatorname{Log}[d + e x]}{4 d^2} - \frac{1}{4} \left(\frac{m}{x^2} + \frac{2 \operatorname{Log}[f x^m]}{x^2}\right) (a + b \operatorname{Log}[c (d + e x)^n]) +$$

$$\frac{b e^2 n \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^2} + \frac{b e^2 m n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{2 d^2}$$

**Problem 365: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])}{x^4} dx$$

Optimal (type 4, 193 leaves, 9 steps):

$$-\frac{5 b e m n}{36 d x^2} + \frac{4 b e^2 m n}{9 d^2 x} + \frac{b e^3 m n \operatorname{Log}[x]}{9 d^3} - \frac{b e n \operatorname{Log}[f x^m]}{6 d x^2} +$$

$$\frac{b e^2 n \operatorname{Log}[f x^m]}{3 d^2 x} - \frac{b e^3 n \operatorname{Log}\left[1 + \frac{d}{e x}\right] \operatorname{Log}[f x^m]}{3 d^3} - \frac{b e^3 m n \operatorname{Log}[d + e x]}{9 d^3} -$$

$$\frac{1}{9} \left(\frac{m}{x^3} + \frac{3 \operatorname{Log}[f x^m]}{x^3}\right) (a + b \operatorname{Log}[c (d + e x)^n]) + \frac{b e^3 m n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{3 d^3}$$

Result (type 4, 212 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{5 b e m n}{36 d x^2} + \frac{4 b e^2 m n}{9 d^2 x} + \frac{b e^3 m n \operatorname{Log}[x]}{9 d^3} - \frac{b e n \operatorname{Log}[f x^m]}{6 d x^2} + \frac{b e^2 n \operatorname{Log}[f x^m]}{3 d^2 x} + \\
 & \frac{b e^3 n \operatorname{Log}[f x^m]^2}{6 d^3 m} - \frac{b e^3 m n \operatorname{Log}[d + e x]}{9 d^3} - \frac{1}{9} \left( \frac{m}{x^3} + \frac{3 \operatorname{Log}[f x^m]}{x^3} \right) (a + b \operatorname{Log}[c (d + e x)^n]) - \\
 & \frac{b e^3 n \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{3 d^3} - \frac{b e^3 m n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{3 d^3}
 \end{aligned}$$

Problem 366: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])}{x^5} dx$$

Optimal (type 4, 230 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{7 b e m n}{144 d x^3} + \frac{3 b e^2 m n}{32 d^2 x^2} - \frac{5 b e^3 m n}{16 d^3 x} - \frac{b e^4 m n \operatorname{Log}[x]}{16 d^4} - \frac{b e n \operatorname{Log}[f x^m]}{12 d x^3} + \\
 & \frac{b e^2 n \operatorname{Log}[f x^m]}{8 d^2 x^2} - \frac{b e^3 n \operatorname{Log}[f x^m]}{4 d^3 x} + \frac{b e^4 n \operatorname{Log}\left[1 + \frac{d}{e x}\right] \operatorname{Log}[f x^m]}{4 d^4} + \frac{b e^4 m n \operatorname{Log}[d + e x]}{16 d^4} - \\
 & \frac{1}{16} \left( \frac{m}{x^4} + \frac{4 \operatorname{Log}[f x^m]}{x^4} \right) (a + b \operatorname{Log}[c (d + e x)^n]) - \frac{b e^4 m n \operatorname{PolyLog}\left[2, -\frac{d}{e x}\right]}{4 d^4}
 \end{aligned}$$

Result (type 4, 249 leaves, 11 steps):

$$\begin{aligned}
 & -\frac{7 b e m n}{144 d x^3} + \frac{3 b e^2 m n}{32 d^2 x^2} - \frac{5 b e^3 m n}{16 d^3 x} - \frac{b e^4 m n \operatorname{Log}[x]}{16 d^4} - \\
 & \frac{b e n \operatorname{Log}[f x^m]}{12 d x^3} + \frac{b e^2 n \operatorname{Log}[f x^m]}{8 d^2 x^2} - \frac{b e^3 n \operatorname{Log}[f x^m]}{4 d^3 x} - \frac{b e^4 n \operatorname{Log}[f x^m]^2}{8 d^4 m} + \\
 & \frac{b e^4 m n \operatorname{Log}[d + e x]}{16 d^4} - \frac{1}{16} \left( \frac{m}{x^4} + \frac{4 \operatorname{Log}[f x^m]}{x^4} \right) (a + b \operatorname{Log}[c (d + e x)^n]) + \\
 & \frac{b e^4 n \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{4 d^4} + \frac{b e^4 m n \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{4 d^4}
 \end{aligned}$$

Problem 367: Result valid but suboptimal antiderivative.

$$\int x^2 \operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2 dx$$

Optimal (type 4, 705 leaves, 52 steps):

$$\begin{aligned}
& \frac{2 a b d^2 m n x}{9 e^2} - \frac{71 b^2 d^2 m n^2 x}{54 e^2} + \frac{b d^2 m n (6 a - 11 b n) x}{9 e^2} + \frac{19 b^2 d m n^2 x^2}{54 e} - \\
& \frac{2}{27} b^2 m n^2 x^3 - \frac{2 a b d^2 n x \operatorname{Log}[f x^m]}{3 e^2} + \frac{11 b^2 d^2 n^2 x \operatorname{Log}[f x^m]}{9 e^2} - \frac{5 b^2 d n^2 x^2 \operatorname{Log}[f x^m]}{18 e} + \\
& \frac{2}{27} b^2 n^2 x^3 \operatorname{Log}[f x^m] + \frac{23 b^2 d^3 m n^2 \operatorname{Log}[d + e x]}{54 e^3} + \frac{5 b^2 d^3 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{9 e^3} - \\
& \frac{5 b^2 d^3 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{9 e^3} + \frac{8 b^2 d^2 m n (d + e x) \operatorname{Log}[c (d + e x)^n]}{9 e^3} + \\
& \frac{2 b^2 d^3 m n \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[c (d + e x)^n]}{3 e^3} - \frac{2 b^2 d^2 n (d + e x) \operatorname{Log}[f x^m] \operatorname{Log}[c (d + e x)^n]}{3 e^3} - \\
& \frac{5 b d m n x^2 (a + b \operatorname{Log}[c (d + e x)^n])}{18 e} + \frac{4}{27} b m n x^3 (a + b \operatorname{Log}[c (d + e x)^n]) + \\
& \frac{b d n x^2 \operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])}{3 e} - \frac{2}{9} b n x^3 \operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n]) - \\
& \frac{d^3 m (a + b \operatorname{Log}[c (d + e x)^n])^2}{9 e^3} - \frac{1}{9} m x^3 (a + b \operatorname{Log}[c (d + e x)^n])^2 - \\
& \frac{d^3 m \operatorname{Log}\left[-\frac{e x}{d}\right] (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 e^3} + \frac{d^3 \operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2}{3 e^3} + \\
& \frac{1}{3} x^3 \operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2 + \frac{11 b^2 d^3 m n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{9 e^3} - \\
& \frac{2 b d^3 m n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{3 e^3} + \frac{2 b^2 d^3 m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{3 e^3}
\end{aligned}$$

Result (type 4, 902 leaves, 50 steps):

$$\begin{aligned}
& \frac{2 a b d^2 m n x}{3 e^2} - \frac{151 b^2 d^2 m n^2 x}{54 e^2} - \frac{a b d m n x^2}{6 e} + \frac{7 b^2 d m n^2 x^2}{27 e} + \frac{2}{27} a b m n x^3 - \frac{4}{81} b^2 m n^2 x^3 + \\
& \frac{b^2 d m n^2 (d + e x)^2}{6 e^3} - \frac{2 b^2 m n^2 (d + e x)^3}{81 e^3} + \frac{11 a b d^3 m n \operatorname{Log}[x]}{9 e^3} + \frac{23 b^2 d^3 m n^2 \operatorname{Log}[x]}{54 e^3} + \\
& \frac{2 b^2 d^2 n^2 x \operatorname{Log}[f x^m]}{e^2} - \frac{b^2 d n^2 (d + e x)^2 \operatorname{Log}[f x^m]}{2 e^3} + \frac{2 b^2 n^2 (d + e x)^3 \operatorname{Log}[f x^m]}{27 e^3} + \\
& \frac{13 b^2 d^3 m n^2 \operatorname{Log}[d + e x]}{54 e^3} - \frac{2 a b d^3 m n \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{3 e^3} + \frac{b^2 d^3 m n^2 \operatorname{Log}[d + e x]^2}{9 e^3} + \\
& \frac{b^2 d^3 m n^2 \operatorname{Log}[x] \operatorname{Log}[d + e x]^2}{3 e^3} - \frac{b^2 d^3 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{3 e^3} - \frac{b^2 d m n x^2 \operatorname{Log}\left[c (d + e x)^n\right]}{6 e} + \\
& \frac{2}{27} b^2 m n x^3 \operatorname{Log}\left[c (d + e x)^n\right] + \frac{2 b^2 d^2 m n (d + e x) \operatorname{Log}\left[c (d + e x)^n\right]}{3 e^3} + \\
& \frac{11 b^2 d^3 m n \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}\left[c (d + e x)^n\right]}{9 e^3} - \frac{2 b^2 d^3 m n \operatorname{Log}[x] \operatorname{Log}[d + e x] \operatorname{Log}\left[c (d + e x)^n\right]}{3 e^3} + \\
& \frac{b^2 d^3 m \operatorname{Log}[x] \operatorname{Log}\left[c (d + e x)^n\right]^2}{3 e^3} - \frac{b^2 d^3 m \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}\left[c (d + e x)^n\right]^2}{3 e^3} + \\
& \frac{1}{27} b m n \left( \frac{18 d^2 (d + e x)}{e^3} - \frac{9 d (d + e x)^2}{e^3} + \frac{2 (d + e x)^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d + e x]}{e^3} \right) (a + b \operatorname{Log}\left[c (d + e x)^n\right]) - \\
& \frac{1}{9} b n \operatorname{Log}[f x^m] \left( \frac{18 d^2 (d + e x)}{e^3} - \frac{9 d (d + e x)^2}{e^3} + \frac{2 (d + e x)^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d + e x]}{e^3} \right) \\
& (a + b \operatorname{Log}\left[c (d + e x)^n\right]) - \frac{1}{9} m x^3 (a + b \operatorname{Log}\left[c (d + e x)^n\right])^2 + \\
& \frac{1}{3} x^3 \operatorname{Log}[f x^m] (a + b \operatorname{Log}\left[c (d + e x)^n\right])^2 - \frac{2 a b d^3 m n \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{3 e^3} + \\
& \frac{11 b^2 d^3 m n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{9 e^3} - \\
& \frac{2 b^2 d^3 m n \operatorname{Log}\left[c (d + e x)^n\right] \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{3 e^3} + \frac{2 b^2 d^3 m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{3 e^3}
\end{aligned}$$

**Problem 370: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2}{x} dx$$

Optimal (type 4, 823 leaves, ? steps):



$$\begin{aligned}
& \frac{1}{2} m \operatorname{Log}[x]^2 \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[c (d + e x)^n\right] \right)^2 + \\
& \operatorname{Log}[x] \left( -m \operatorname{Log}[x] + \operatorname{Log}[f x^m] \right) \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[c (d + e x)^n\right] \right)^2 + \\
& 2 b n \left( -m \operatorname{Log}[x] + \operatorname{Log}[f x^m] \right) \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[c (d + e x)^n\right] \right) \\
& \left( \operatorname{Log}[x] \left( \operatorname{Log}[d + e x] - \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) - \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \\
& 2 b m n \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[c (d + e x)^n\right] \right) \\
& \left( \frac{1}{2} \operatorname{Log}[x]^2 \left( \operatorname{Log}[d + e x] - \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) - \operatorname{Log}[x] \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] + \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] \right) - \\
& b^2 n^2 \left( m \operatorname{Log}[x] - \operatorname{Log}[f x^m] \right) \\
& \left( \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]^2 + 2 \operatorname{Log}[d + e x] \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right] - 2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right] \right) + \\
& \frac{1}{12} b^2 m n^2 \left( \operatorname{Log}\left[-\frac{e x}{d}\right]^4 + 6 \operatorname{Log}\left[-\frac{e x}{d}\right]^2 \operatorname{Log}\left[-\frac{e x}{d + e x}\right]^2 - \right. \\
& 4 \left( \operatorname{Log}\left[-\frac{e x}{d}\right] + \operatorname{Log}\left[\frac{d}{d + e x}\right] \right) \operatorname{Log}\left[-\frac{e x}{d + e x}\right]^3 + \operatorname{Log}\left[-\frac{e x}{d + e x}\right]^4 + \\
& 6 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x]^2 + 4 \left( 2 \operatorname{Log}\left[-\frac{e x}{d}\right]^3 - 3 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x] \right) \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \\
& 6 \left( \operatorname{Log}[x] - \operatorname{Log}\left[-\frac{e x}{d}\right] \right) \left( \operatorname{Log}[x] + 3 \operatorname{Log}\left[-\frac{e x}{d}\right] \right) \operatorname{Log}\left[1 + \frac{e x}{d}\right]^2 - 4 \operatorname{Log}\left[-\frac{e x}{d}\right]^2 \operatorname{Log}\left[-\frac{e x}{d + e x}\right] \\
& \left( \operatorname{Log}\left[-\frac{e x}{d}\right] + 3 \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) + 12 \left( \operatorname{Log}\left[-\frac{e x}{d}\right]^2 - 2 \operatorname{Log}\left[-\frac{e x}{d}\right] \left( \operatorname{Log}\left[-\frac{e x}{d + e x}\right] + \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) \right) + \\
& 2 \operatorname{Log}[x] \left( -\operatorname{Log}[d + e x] + \operatorname{Log}\left[1 + \frac{e x}{d}\right] \right) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] - \\
& 12 \operatorname{Log}\left[-\frac{e x}{d + e x}\right]^2 \operatorname{PolyLog}\left[2, \frac{e x}{d + e x}\right] + 12 \left( \operatorname{Log}\left[-\frac{e x}{d}\right] - \operatorname{Log}\left[-\frac{e x}{d + e x}\right] \right)^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + \\
& 24 \left( \operatorname{Log}[x] - \operatorname{Log}\left[-\frac{e x}{d}\right] \right) \operatorname{Log}\left[1 + \frac{e x}{d}\right] \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + \\
& 24 \left( \operatorname{Log}\left[-\frac{e x}{d + e x}\right] + \operatorname{Log}[d + e x] \right) \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right] + \\
& 24 \operatorname{Log}\left[-\frac{e x}{d + e x}\right] \operatorname{PolyLog}\left[3, \frac{e x}{d + e x}\right] + 24 \left( -\operatorname{Log}[x] + \operatorname{Log}\left[-\frac{e x}{d + e x}\right] \right) \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right] - \\
& 24 \left( \operatorname{PolyLog}\left[4, -\frac{e x}{d}\right] + \operatorname{PolyLog}\left[4, \frac{e x}{d + e x}\right] - \operatorname{PolyLog}\left[4, 1 + \frac{e x}{d}\right] \right) \Big)
\end{aligned}$$

Result (type 8, 72 leaves, 1 step):

$$\frac{\operatorname{Log}[f x^m]^2 \left( a + b \operatorname{Log}\left[c (d + e x)^n\right] \right)^2}{2 m} - \frac{b e n \operatorname{Unintegrable}\left[\frac{\operatorname{Log}[f x^m]^2 \left( a + b \operatorname{Log}\left[c (d + e x)^n\right] \right)}{d + e x}, x\right]}{m}$$

**Problem 371: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[f x^m] \left( a + b \operatorname{Log}\left[c (d + e x)^n\right] \right)^2}{x^2} dx$$

Optimal (type 4, 607 leaves, ? steps):

$$\begin{aligned}
& - \frac{b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x]}{d} + \frac{2 b^2 e m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{d} + \\
& \frac{2 b^2 e n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d} - \frac{b^2 e m n^2 \operatorname{Log}[d + e x]^2}{d} - \frac{b^2 m n^2 \operatorname{Log}[d + e x]^2}{x} + \\
& \frac{b^2 e m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]^2}{d} - \frac{b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{d} - \frac{b^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{x} - \\
& \frac{1}{d x} 2 b n \left( m \operatorname{Log}[x] - \operatorname{Log}[f x^m] \right) \left( e x \operatorname{Log}\left[-\frac{e x}{d}\right] - (d + e x) \operatorname{Log}[d + e x] \right) \\
& \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right) - \frac{m \operatorname{Log}[x] \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2}{x} - \\
& \frac{1}{x} \left( m - m \operatorname{Log}[x] + \operatorname{Log}[f x^m] \right) \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2 + \\
& \frac{b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \frac{2 b^2 e n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \\
& \frac{2 b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d} + \frac{1}{d x} \\
& b m n \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right) \left( 2 e x \operatorname{Log}\left[-\frac{e x}{d}\right] - 2 (d + e x) \operatorname{Log}[d + e x] - \right. \\
& \quad \left. 2 d \operatorname{Log}[x] \operatorname{Log}[d + e x] + e x \left( \operatorname{Log}[x]^2 - 2 \left( \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) \right) \right) + \\
& \frac{2 b^2 e m n^2 \left( 1 + \operatorname{Log}[d + e x] \right) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d} + \frac{2 b^2 e m n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d} - \\
& \frac{2 b^2 e m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{d}
\end{aligned}$$

Result (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Log}[f x^m] \left( a + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2}{x^2}, x\right]$$

**Problem 372: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[f x^m] \left( a + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2}{x^3} dx$$

Optimal (type 4, 939 leaves, ? steps):

$$\begin{aligned}
& \frac{b^2 e^2 m n^2 \operatorname{Log}[x]}{d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2}{2 d^2} + \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right]}{2 d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m]}{d^2} - \\
& \frac{3 b^2 e^2 m n^2 \operatorname{Log}[d + e x]}{2 d^2} - \frac{3 b^2 e m n^2 \operatorname{Log}[d + e x]}{2 d x} + \frac{b^2 e^2 m n^2 \operatorname{Log}[x] \operatorname{Log}[d + e x]}{d^2} + \\
& \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x]}{2 d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{2 d^2} - \frac{b^2 e^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d^2} - \\
& \frac{b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d x} - \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d^2} + \\
& \frac{b^2 e^2 m n^2 \operatorname{Log}[d + e x]^2}{4 d^2} - \frac{b^2 m n^2 \operatorname{Log}[d + e x]^2}{4 x^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]^2}{2 d^2} + \\
& \frac{b^2 e^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{2 d^2} - \frac{b^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{2 x^2} + \frac{1}{d^2 x^2} \\
& b n \left( m \operatorname{Log}[x] - \operatorname{Log}[f x^m] \right) \left( e^2 x^2 \operatorname{Log}\left[-\frac{e x}{d}\right] + (d + e x) \left( e x + (d - e x) \operatorname{Log}[d + e x] \right) \right) \\
& \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right) - \frac{m \operatorname{Log}[x] \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2}{2 x^2} - \\
& \frac{1}{4 x^2} \left( m - 2 m \operatorname{Log}[x] + 2 \operatorname{Log}[f x^m] \right) \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2 - \\
& \frac{b^2 e^2 m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^2} + \\
& \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} - \frac{b^2 e^2 n^2 \left( m - \operatorname{Log}[f x^m] \right) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} - \\
& \frac{1}{2 d^2 x^2} b m n \left( a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}\left[ c (d + e x)^n \right] \right) \\
& \left( e x (d + e x) + e^2 x^2 \operatorname{Log}\left[-\frac{e x}{d}\right] + (d^2 - e^2 x^2) \operatorname{Log}[d + e x] + 2 d^2 \operatorname{Log}[x] \operatorname{Log}[d + e x] + \right. \\
& \left. e x \left( e x \operatorname{Log}[x]^2 + 2 d (1 + \operatorname{Log}[x]) - 2 e x \left( \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) \right) \right) - \\
& \frac{b^2 e^2 m n^2 \left( 1 + 2 \operatorname{Log}[d + e x] \right) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{2 d^2} - \frac{b^2 e^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2} + \\
& \frac{b^2 e^2 m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{d^2}
\end{aligned}$$

Result (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Log}[f x^m] \left( a + b \operatorname{Log}\left[ c (d + e x)^n \right] \right)^2}{x^3}, x\right]$$

**Problem 374: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[x] \operatorname{Log}[a + b x]^2}{x} dx$$

Optimal (type 4, 519 leaves, ? steps):

$$\begin{aligned}
& \frac{1}{12} \left( \text{Log}\left[-\frac{bx}{a}\right]^4 + 6 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 - 4 \left( \text{Log}\left[-\frac{bx}{a}\right] + \text{Log}\left[\frac{a}{a+bx}\right] \right) \text{Log}\left[-\frac{bx}{a+bx}\right]^3 + \right. \\
& \quad \left. \text{Log}\left[-\frac{bx}{a+bx}\right]^4 + 6 \text{Log}[x]^2 \text{Log}[a+bx]^2 + 4 \left( 2 \text{Log}\left[-\frac{bx}{a}\right]^3 - 3 \text{Log}[x]^2 \text{Log}[a+bx] \right) \text{Log}\left[1 + \frac{bx}{a}\right] + \right. \\
& \quad 6 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \left( \text{Log}[x] + 3 \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right]^2 - \\
& \quad 4 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right] \left( \text{Log}\left[-\frac{bx}{a}\right] + 3 \text{Log}\left[1 + \frac{bx}{a}\right] \right) + \\
& \quad 12 \left( \text{Log}\left[-\frac{bx}{a}\right]^2 - 2 \text{Log}\left[-\frac{bx}{a}\right] \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) + \right. \\
& \quad \left. 2 \text{Log}[x] \left( -\text{Log}[a+bx] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) \right) \text{PolyLog}\left[2, -\frac{bx}{a}\right] - \\
& \quad 12 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 \text{PolyLog}\left[2, \frac{bx}{a+bx}\right] + 12 \left( \text{Log}\left[-\frac{bx}{a}\right] - \text{Log}\left[-\frac{bx}{a+bx}\right] \right)^2 \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + \\
& \quad 24 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right] \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + \\
& \quad 24 \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}[a+bx] \right) \text{PolyLog}\left[3, -\frac{bx}{a}\right] + \\
& \quad 24 \text{Log}\left[-\frac{bx}{a+bx}\right] \text{PolyLog}\left[3, \frac{bx}{a+bx}\right] + 24 \left( -\text{Log}[x] + \text{Log}\left[-\frac{bx}{a+bx}\right] \right) \text{PolyLog}\left[3, 1 + \frac{bx}{a}\right] - \\
& \quad \left. 24 \left( \text{PolyLog}\left[4, -\frac{bx}{a}\right] + \text{PolyLog}\left[4, \frac{bx}{a+bx}\right] - \text{PolyLog}\left[4, 1 + \frac{bx}{a}\right] \right) \right)
\end{aligned}$$

Result (type 8, 40 leaves, 1 step):

$$\frac{1}{2} \text{Log}[x]^2 \text{Log}[a+bx]^2 - b \text{Unintegrable}\left[\frac{\text{Log}[x]^2 \text{Log}[a+bx]}{a+bx}, x\right]$$

**Problem 379: Result valid but suboptimal antiderivative.**

$$\int x^2 (a + b \text{Log}[c (d + ex)^n]) (f + g \text{Log}[c (d + ex)^n]) dx$$

Optimal (type 3, 258 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b d^2 g n^2 x}{e^2} - \frac{b d g n^2 (d + ex)^2}{2 e^3} + \frac{2 b g n^2 (d + ex)^3}{27 e^3} - \\
& \frac{b d^3 g n^2 \text{Log}[d + ex]^2}{3 e^3} + \frac{1}{3} x^3 (a + b \text{Log}[c (d + ex)^n]) (f + g \text{Log}[c (d + ex)^n]) - \\
& \frac{d^2 n (d + ex) (b f + a g + 2 b g \text{Log}[c (d + ex)^n])}{e^3} + \frac{d n (d + ex)^2 (b f + a g + 2 b g \text{Log}[c (d + ex)^n])}{2 e^3} - \\
& \frac{n (d + ex)^3 (b f + a g + 2 b g \text{Log}[c (d + ex)^n])}{9 e^3} + \frac{d^3 n \text{Log}[d + ex] (b f + a g + 2 b g \text{Log}[c (d + ex)^n])}{3 e^3}
\end{aligned}$$

Result (type 3, 258 leaves, 13 steps):

$$\begin{aligned}
& -\frac{2 b d^2 g n^2 x}{e^2} - \frac{b d g n^2 (d+e x)^2}{2 e^3} + \frac{2 b g n^2 (d+e x)^3}{27 e^3} - \frac{b d^3 g n^2 \operatorname{Log}[d+e x]^2}{3 e^3} - \\
& \frac{1}{18} g n \left( \frac{18 d^2 (d+e x)}{e^3} - \frac{9 d (d+e x)^2}{e^3} + \frac{2 (d+e x)^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d+e x]}{e^3} \right) (a+b \operatorname{Log}[c (d+e x)^n]) - \\
& \frac{1}{18} b n \left( \frac{18 d^2 (d+e x)}{e^3} - \frac{9 d (d+e x)^2}{e^3} + \frac{2 (d+e x)^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d+e x]}{e^3} \right) (f+g \operatorname{Log}[c (d+e x)^n]) + \\
& \frac{1}{3} x^3 (a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])
\end{aligned}$$

**Problem 380: Result valid but suboptimal antiderivative.**

$$\int x (a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n]) dx$$

Optimal (type 3, 196 leaves, 7 steps):

$$\begin{aligned}
& -\frac{2 b d g n^2 x}{e} + \frac{b g n^2 (d+e x)^2}{4 e^2} + \frac{b d^2 g n^2 \operatorname{Log}[d+e x]^2}{2 e^2} + \\
& \frac{1}{2} x^2 (a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n]) + \\
& \frac{d n (d+e x) (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{e^2} - \frac{n (d+e x)^2 (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{4 e^2} - \\
& \frac{d^2 n \operatorname{Log}[d+e x] (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{2 e^2}
\end{aligned}$$

Result (type 3, 206 leaves, 13 steps):

$$\begin{aligned}
& -\frac{2 b d g n^2 x}{e} + \frac{b g n^2 (d+e x)^2}{4 e^2} + \frac{b d^2 g n^2 \operatorname{Log}[d+e x]^2}{2 e^2} + \\
& \frac{1}{4} g n \left( \frac{4 d (d+e x)}{e^2} - \frac{(d+e x)^2}{e^2} - \frac{2 d^2 \operatorname{Log}[d+e x]}{e^2} \right) (a+b \operatorname{Log}[c (d+e x)^n]) + \\
& \frac{1}{4} b n \left( \frac{4 d (d+e x)}{e^2} - \frac{(d+e x)^2}{e^2} - \frac{2 d^2 \operatorname{Log}[d+e x]}{e^2} \right) (f+g \operatorname{Log}[c (d+e x)^n]) + \\
& \frac{1}{2} x^2 (a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])
\end{aligned}$$

**Problem 381: Result valid but suboptimal antiderivative.**

$$\int (a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n]) dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$- (b f + a g) n x + 2 b g n^2 x - \frac{2 b g n (d + e x) \operatorname{Log}[c (d + e x)^n]}{e} +$$

$$x (a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n]) + \frac{d (b f + a g + 2 b g \operatorname{Log}[c (d + e x)^n])^2}{4 b e g}$$

Result (type 3, 130 leaves, 11 steps):

$$- b f n x - a g n x + 2 b g n^2 x - \frac{2 b g n (d + e x) \operatorname{Log}[c (d + e x)^n]}{e} + \frac{d g (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 b e} +$$

$$x (a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n]) + \frac{b d (f + g \operatorname{Log}[c (d + e x)^n])^2}{2 e g}$$

**Problem 382: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n])}{x} dx$$

Optimal (type 4, 158 leaves, 6 steps):

$$\operatorname{Log}[x] (a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n]) -$$

$$\frac{\operatorname{Log}[x] (b f + a g + 2 b g \operatorname{Log}[c (d + e x)^n])^2}{4 b g} + \frac{\operatorname{Log}[-\frac{e x}{d}] (b f + a g + 2 b g \operatorname{Log}[c (d + e x)^n])^2}{4 b g} +$$

$$n (b f + a g + 2 b g \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}[2, 1 + \frac{e x}{d}] - 2 b g n^2 \operatorname{PolyLog}[3, 1 + \frac{e x}{d}]$$

Result (type 4, 219 leaves, 11 steps):

$$- \frac{g \operatorname{Log}[x] (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 b} + \frac{g \operatorname{Log}[-\frac{e x}{d}] (a + b \operatorname{Log}[c (d + e x)^n])^2}{2 b} +$$

$$\operatorname{Log}[x] (a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n]) - \frac{b \operatorname{Log}[x] (f + g \operatorname{Log}[c (d + e x)^n])^2}{2 g} +$$

$$\frac{b \operatorname{Log}[-\frac{e x}{d}] (f + g \operatorname{Log}[c (d + e x)^n])^2}{2 g} + g n (a + b \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}[2, 1 + \frac{e x}{d}] +$$

$$b n (f + g \operatorname{Log}[c (d + e x)^n]) \operatorname{PolyLog}[2, 1 + \frac{e x}{d}] - 2 b g n^2 \operatorname{PolyLog}[3, 1 + \frac{e x}{d}]$$

**Problem 383: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n])}{x^2} dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$- \frac{(a + b \operatorname{Log}[c (d + e x)^n]) (f + g \operatorname{Log}[c (d + e x)^n])}{x} +$$

$$\frac{e n (b f + a g + 2 b g \operatorname{Log}[c (d + e x)^n]) \operatorname{Log}[1 - \frac{d}{d + e x}]}{d} - \frac{2 b e g n^2 \operatorname{PolyLog}[2, \frac{d}{d + e x}]}{d}$$

Result (type 4, 169 leaves, 11 steps):

$$\frac{e g n \operatorname{Log}\left[-\frac{e x}{d}\right] (a+b \operatorname{Log}[c (d+e x)^n])}{d} - \frac{e g (a+b \operatorname{Log}[c (d+e x)^n])^2}{2 b d} +$$

$$\frac{b e n \operatorname{Log}\left[-\frac{e x}{d}\right] (f+g \operatorname{Log}[c (d+e x)^n])}{d} - \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{x} -$$

$$\frac{b e (f+g \operatorname{Log}[c (d+e x)^n])^2}{2 d g} + \frac{2 b e g n^2 \operatorname{PolyLog}\left[2, 1+\frac{e x}{d}\right]}{d}$$

**Problem 384: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{x^3} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$\frac{b e^2 g n^2 \operatorname{Log}[x]}{d^2} - \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{2 x^2} -$$

$$\frac{e n (d+e x) (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{2 d^2 x} -$$

$$\frac{e^2 n (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n]) \operatorname{Log}\left[1-\frac{d}{d+e x}\right]}{2 d^2} + \frac{b e^2 g n^2 \operatorname{PolyLog}\left[2, \frac{d}{d+e x}\right]}{d^2}$$

Result (type 4, 265 leaves, 17 steps):

$$\frac{b e^2 g n^2 \operatorname{Log}[x]}{d^2} - \frac{e g n (d+e x) (a+b \operatorname{Log}[c (d+e x)^n])}{2 d^2 x} - \frac{e^2 g n \operatorname{Log}\left[-\frac{e x}{d}\right] (a+b \operatorname{Log}[c (d+e x)^n])}{2 d^2} +$$

$$\frac{e^2 g (a+b \operatorname{Log}[c (d+e x)^n])^2}{4 b d^2} - \frac{b e n (d+e x) (f+g \operatorname{Log}[c (d+e x)^n])}{2 d^2 x} -$$

$$\frac{b e^2 n \operatorname{Log}\left[-\frac{e x}{d}\right] (f+g \operatorname{Log}[c (d+e x)^n])}{2 d^2} - \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{2 x^2} +$$

$$\frac{b e^2 (f+g \operatorname{Log}[c (d+e x)^n])^2}{4 d^2 g} - \frac{b e^2 g n^2 \operatorname{PolyLog}\left[2, 1+\frac{e x}{d}\right]}{d^2}$$

**Problem 385: Result valid but suboptimal antiderivative.**

$$\int \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{x^4} dx$$

Optimal (type 4, 234 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b e^2 g n^2}{3 d^2 x} - \frac{b e^3 g n^2 \operatorname{Log}[x]}{d^3} + \frac{b e^3 g n^2 \operatorname{Log}[d+e x]}{3 d^3} - \\
& \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{3 x^3} - \\
& \frac{e n (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{6 d x^2} + \frac{e^2 n (d+e x) (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n])}{3 d^3 x} + \\
& \frac{e^3 n (b f+a g+2 b g \operatorname{Log}[c (d+e x)^n]) \operatorname{Log}\left[1-\frac{d}{d+e x}\right]}{3 d^3} - \frac{2 b e^3 g n^2 \operatorname{PolyLog}\left[2, \frac{d}{d+e x}\right]}{3 d^3}
\end{aligned}$$

Result (type 4, 365 leaves, 25 steps):

$$\begin{aligned}
& -\frac{b e^2 g n^2}{3 d^2 x} - \frac{b e^3 g n^2 \operatorname{Log}[x]}{d^3} + \frac{b e^3 g n^2 \operatorname{Log}[d+e x]}{3 d^3} - \\
& \frac{e g n (a+b \operatorname{Log}[c (d+e x)^n])}{6 d x^2} + \frac{e^2 g n (d+e x) (a+b \operatorname{Log}[c (d+e x)^n])}{3 d^3 x} + \\
& \frac{e^3 g n \operatorname{Log}\left[-\frac{e x}{d}\right] (a+b \operatorname{Log}[c (d+e x)^n])}{3 d^3} - \frac{e^3 g (a+b \operatorname{Log}[c (d+e x)^n])^2}{6 b d^3} - \\
& \frac{b e n (f+g \operatorname{Log}[c (d+e x)^n])}{6 d x^2} + \frac{b e^2 n (d+e x) (f+g \operatorname{Log}[c (d+e x)^n])}{3 d^3 x} + \\
& \frac{b e^3 n \operatorname{Log}\left[-\frac{e x}{d}\right] (f+g \operatorname{Log}[c (d+e x)^n])}{3 d^3} - \frac{(a+b \operatorname{Log}[c (d+e x)^n]) (f+g \operatorname{Log}[c (d+e x)^n])}{3 x^3} - \\
& \frac{b e^3 (f+g \operatorname{Log}[c (d+e x)^n])^2}{6 d^3 g} + \frac{2 b e^3 g n^2 \operatorname{PolyLog}\left[2, 1+\frac{e x}{d}\right]}{3 d^3}
\end{aligned}$$

**Problem 428: Result valid but suboptimal antiderivative.**

$$\int (g+hx)^3 (a+b \operatorname{Log}[c (d(e+fx)^p)^q])^2 dx$$

Optimal (type 3, 409 leaves, 9 steps):



$$\begin{aligned}
& \frac{2 b^2 (f g - e h)^3 p^2 q^2 x}{f^3} + \frac{3 b^2 h (f g - e h)^2 p^2 q^2 (e + f x)^2}{4 f^4} + \\
& \frac{2 b^2 h^2 (f g - e h) p^2 q^2 (e + f x)^3}{9 f^4} + \frac{b^2 h^3 p^2 q^2 (e + f x)^4}{32 f^4} + \frac{b^2 (f g - e h)^4 p^2 q^2 \operatorname{Log}[e + f x]^2}{4 f^4 h} - \\
& \frac{2 b (f g - e h)^3 p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{f^4} - \\
& \frac{3 b h (f g - e h)^2 p q (e + f x)^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{2 f^4} - \\
& \frac{2 b h^2 (f g - e h) p q (e + f x)^3 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{3 f^4} - \\
& \frac{b h^3 p q (e + f x)^4 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{8 f^4} - \\
& \frac{b (f g - e h)^4 p q \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{2 f^4 h} + \\
& \frac{(g + h x)^4 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{4 h}
\end{aligned}$$

Result (type 3, 325 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 (f g - e h)^3 p^2 q^2 x}{f^3} + \frac{3 b^2 h (f g - e h)^2 p^2 q^2 (e + f x)^2}{4 f^4} + \\
& \frac{2 b^2 h^2 (f g - e h) p^2 q^2 (e + f x)^3}{9 f^4} + \frac{b^2 h^3 p^2 q^2 (e + f x)^4}{32 f^4} + \frac{b^2 (f g - e h)^4 p^2 q^2 \operatorname{Log}[e + f x]^2}{4 f^4 h} - \\
& \frac{1}{24 h} b p q \left( \frac{48 h (f g - e h)^3 (e + f x)}{f^4} + \frac{36 h^2 (f g - e h)^2 (e + f x)^2}{f^4} + \right. \\
& \quad \left. \frac{16 h^3 (f g - e h) (e + f x)^3}{f^4} + \frac{3 h^4 (e + f x)^4}{f^4} + \frac{12 (f g - e h)^4 \operatorname{Log}[e + f x]}{f^4} \right) \\
& (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) + \frac{(g + h x)^4 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{4 h}
\end{aligned}$$

**Problem 429: Result valid but suboptimal antiderivative.**

$$\int (g + h x)^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2 dx$$

Optimal (type 3, 323 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 b^2 (f g - e h)^2 p^2 q^2 x}{f^2} + \frac{b^2 h (f g - e h) p^2 q^2 (e + f x)^2}{2 f^3} + \frac{2 b^2 h^2 p^2 q^2 (e + f x)^3}{27 f^3} + \\
& \frac{b^2 (f g - e h)^3 p^2 q^2 \operatorname{Log}[e + f x]^2}{3 f^3 h} - \frac{2 b (f g - e h)^2 p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{f^3} - \\
& \frac{b h (f g - e h) p q (e + f x)^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{f^3} - \\
& \frac{2 b h^2 p q (e + f x)^3 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{9 f^3} - \\
& \frac{2 b (f g - e h)^3 p q \operatorname{Log}[e + f x] (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{3 f^3 h} + \\
& \frac{(g + h x)^3 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{3 h}
\end{aligned}$$

Result (type 3, 264 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 b^2 (f g - e h)^2 p^2 q^2 x}{f^2} + \frac{b^2 h (f g - e h) p^2 q^2 (e + f x)^2}{2 f^3} + \\
& \frac{2 b^2 h^2 p^2 q^2 (e + f x)^3}{27 f^3} + \frac{b^2 (f g - e h)^3 p^2 q^2 \operatorname{Log}[e + f x]^2}{3 f^3 h} - \frac{1}{9 h} b p q \\
& \left( \frac{18 h (f g - e h)^2 (e + f x)}{f^3} + \frac{9 h^2 (f g - e h) (e + f x)^2}{f^3} + \frac{2 h^3 (e + f x)^3}{f^3} + \frac{6 (f g - e h)^3 \operatorname{Log}[e + f x]}{f^3} \right) \\
& (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) + \frac{(g + h x)^3 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{3 h}
\end{aligned}$$

**Problem 434: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{(g + h x)^3} dx$$

Optimal (type 4, 222 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b f p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{(f g - e h)^2 (g + h x)} - \\
& \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{2 h (g + h x)^2} + \frac{b^2 f^2 p^2 q^2 \operatorname{Log}[g + h x]}{h (f g - e h)^2} - \\
& \frac{b f^2 p q (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{Log}\left[1 + \frac{f g - e h}{h (e + f x)}\right]}{h (f g - e h)^2} + \frac{b^2 f^2 p^2 q^2 \operatorname{PolyLog}\left[2, -\frac{f g - e h}{h (e + f x)}\right]}{h (f g - e h)^2}
\end{aligned}$$

Result (type 4, 257 leaves, 10 steps):

$$\begin{aligned}
& - \frac{b f p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])}{(f g - e h)^2 (g + h x)} + \frac{f^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{2 h (f g - e h)^2} - \\
& \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{2 h (g + h x)^2} + \frac{b^2 f^2 p^2 q^2 \operatorname{Log}[g + h x]}{h (f g - e h)^2} - \\
& \frac{b f^2 p q (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{Log}\left[\frac{f(g+hx)}{fg-eh}\right]}{h (f g - e h)^2} - \frac{b^2 f^2 p^2 q^2 \operatorname{PolyLog}\left[2, -\frac{h(e+fx)}{fg-eh}\right]}{h (f g - e h)^2}
\end{aligned}$$

**Problem 440: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^3}{(g + h x)^3} dx$$

Optimal (type 4, 376 leaves, 10 steps):

$$\begin{aligned}
& - \frac{3 b f p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{2 (f g - e h)^2 (g + h x)} - \\
& \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^3}{2 h (g + h x)^2} + \frac{3 b^2 f^2 p^2 q^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{Log}\left[\frac{f(g+hx)}{fg-eh}\right]}{h (f g - e h)^2} - \\
& \frac{3 b f^2 p q (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2 \operatorname{Log}\left[1 + \frac{fg-eh}{h(e+fx)}\right]}{2 h (f g - e h)^2} + \\
& \frac{3 b^2 f^2 p^2 q^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{PolyLog}\left[2, -\frac{fg-eh}{h(e+fx)}\right]}{h (f g - e h)^2} + \\
& \frac{3 b^3 f^2 p^3 q^3 \operatorname{PolyLog}\left[2, -\frac{h(e+fx)}{fg-eh}\right]}{h (f g - e h)^2} + \frac{3 b^3 f^2 p^3 q^3 \operatorname{PolyLog}\left[3, -\frac{fg-eh}{h(e+fx)}\right]}{h (f g - e h)^2}
\end{aligned}$$

Result (type 4, 408 leaves, 13 steps):

$$\begin{aligned}
& - \frac{3 b f p q (e + f x) (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2}{2 (f g - e h)^2 (g + h x)} + \frac{f^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^3}{2 h (f g - e h)^2} - \\
& \frac{(a + b \operatorname{Log}[c (d (e + f x)^p)^q])^3}{2 h (g + h x)^2} + \frac{3 b^2 f^2 p^2 q^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{Log}\left[\frac{f(g+hx)}{fg-eh}\right]}{h (f g - e h)^2} - \\
& \frac{3 b f^2 p q (a + b \operatorname{Log}[c (d (e + f x)^p)^q])^2 \operatorname{Log}\left[\frac{f(g+hx)}{fg-eh}\right]}{2 h (f g - e h)^2} + \frac{3 b^3 f^2 p^3 q^3 \operatorname{PolyLog}\left[2, -\frac{h(e+fx)}{fg-eh}\right]}{h (f g - e h)^2} - \\
& \frac{3 b^2 f^2 p^2 q^2 (a + b \operatorname{Log}[c (d (e + f x)^p)^q]) \operatorname{PolyLog}\left[2, -\frac{h(e+fx)}{fg-eh}\right]}{h (f g - e h)^2} + \\
& \frac{3 b^3 f^2 p^3 q^3 \operatorname{PolyLog}\left[3, -\frac{h(e+fx)}{fg-eh}\right]}{h (f g - e h)^2}
\end{aligned}$$

## Test results for the 641 problems in "3.4 u (a+b log(c (d+e x^m)^n))^p.m"

Problem 77: Result valid but suboptimal antiderivative.

$$\int x^5 \operatorname{Log}\left[c \left(a + b x^2\right)^p\right]^2 dx$$

Optimal (type 3, 215 leaves, 8 steps):

$$\begin{aligned} & \frac{a^2 p^2 x^2}{b^2} - \frac{a p^2 (a + b x^2)^2}{4 b^3} + \frac{p^2 (a + b x^2)^3}{27 b^3} - \frac{a^3 p^2 \operatorname{Log}[a + b x^2]^2}{6 b^3} - \\ & \frac{a^2 p (a + b x^2) \operatorname{Log}[c (a + b x^2)^p]}{b^3} + \frac{a p (a + b x^2)^2 \operatorname{Log}[c (a + b x^2)^p]}{2 b^3} - \\ & \frac{p (a + b x^2)^3 \operatorname{Log}[c (a + b x^2)^p]}{9 b^3} + \frac{a^3 p \operatorname{Log}[a + b x^2] \operatorname{Log}[c (a + b x^2)^p]}{3 b^3} + \frac{1}{6} x^6 \operatorname{Log}[c (a + b x^2)^p]^2 \end{aligned}$$

Result (type 3, 175 leaves, 8 steps):

$$\begin{aligned} & \frac{a^2 p^2 x^2}{b^2} - \frac{a p^2 (a + b x^2)^2}{4 b^3} + \frac{p^2 (a + b x^2)^3}{27 b^3} - \frac{a^3 p^2 \operatorname{Log}[a + b x^2]^2}{6 b^3} - \\ & \frac{1}{18} p \left( \frac{18 a^2 (a + b x^2)}{b^3} - \frac{9 a (a + b x^2)^2}{b^3} + \frac{2 (a + b x^2)^3}{b^3} - \frac{6 a^3 \operatorname{Log}[a + b x^2]}{b^3} \right) \operatorname{Log}[c (a + b x^2)^p] + \\ & \frac{1}{6} x^6 \operatorname{Log}[c (a + b x^2)^p]^2 \end{aligned}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Log}[c (a + b x^2)^p]^2}{x^5} dx$$

Optimal (type 4, 129 leaves, 8 steps):

$$\begin{aligned} & \frac{b^2 p^2 \operatorname{Log}[x]}{a^2} - \frac{b p (a + b x^2) \operatorname{Log}[c (a + b x^2)^p]}{2 a^2 x^2} - \frac{\operatorname{Log}[c (a + b x^2)^p]^2}{4 x^4} - \\ & \frac{b^2 p \operatorname{Log}[c (a + b x^2)^p] \operatorname{Log}\left[1 - \frac{a}{a + b x^2}\right]}{2 a^2} + \frac{b^2 p^2 \operatorname{PolyLog}\left[2, \frac{a}{a + b x^2}\right]}{2 a^2} \end{aligned}$$

Result (type 4, 147 leaves, 10 steps):

$$\begin{aligned} & \frac{b^2 p^2 \operatorname{Log}[x]}{a^2} - \frac{b p (a + b x^2) \operatorname{Log}[c (a + b x^2)^p]}{2 a^2 x^2} - \frac{b^2 p \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}[c (a + b x^2)^p]}{2 a^2} + \\ & \frac{b^2 \operatorname{Log}[c (a + b x^2)^p]^2}{4 a^2} - \frac{\operatorname{Log}[c (a + b x^2)^p]^2}{4 x^4} - \frac{b^2 p^2 \operatorname{PolyLog}\left[2, 1 + \frac{b x^2}{a}\right]}{2 a^2} \end{aligned}$$

### Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}[c (a + b x^2)^p]^2}{x^7} dx$$

Optimal (type 4, 193 leaves, 12 steps):

$$\begin{aligned} & -\frac{b^2 p^2}{6 a^2 x^2} - \frac{b^3 p^2 \text{Log}[x]}{a^3} + \frac{b^3 p^2 \text{Log}[a + b x^2]}{6 a^3} - \\ & \frac{b p \text{Log}[c (a + b x^2)^p]}{6 a x^4} + \frac{b^2 p (a + b x^2) \text{Log}[c (a + b x^2)^p]}{3 a^3 x^2} - \frac{\text{Log}[c (a + b x^2)^p]^2}{6 x^6} + \\ & \frac{b^3 p \text{Log}[c (a + b x^2)^p] \text{Log}\left[1 - \frac{a}{a + b x^2}\right]}{3 a^3} - \frac{b^3 p^2 \text{PolyLog}\left[2, \frac{a}{a + b x^2}\right]}{3 a^3} \end{aligned}$$

Result (type 4, 211 leaves, 14 steps):

$$\begin{aligned} & -\frac{b^2 p^2}{6 a^2 x^2} - \frac{b^3 p^2 \text{Log}[x]}{a^3} + \frac{b^3 p^2 \text{Log}[a + b x^2]}{6 a^3} - \frac{b p \text{Log}[c (a + b x^2)^p]}{6 a x^4} + \\ & \frac{b^2 p (a + b x^2) \text{Log}[c (a + b x^2)^p]}{3 a^3 x^2} + \frac{b^3 p \text{Log}\left[-\frac{b x^2}{a}\right] \text{Log}[c (a + b x^2)^p]}{3 a^3} - \\ & \frac{b^3 \text{Log}[c (a + b x^2)^p]^2}{6 a^3} - \frac{\text{Log}[c (a + b x^2)^p]^2}{6 x^6} + \frac{b^3 p^2 \text{PolyLog}\left[2, 1 + \frac{b x^2}{a}\right]}{3 a^3} \end{aligned}$$

### Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}[c (a + b x^2)^p]^3}{x^5} dx$$

Optimal (type 4, 219 leaves, 10 steps):

$$\begin{aligned} & \frac{3 b^2 p^2 \text{Log}\left[-\frac{b x^2}{a}\right] \text{Log}[c (a + b x^2)^p]}{2 a^2} - \frac{3 b p (a + b x^2) \text{Log}[c (a + b x^2)^p]^2}{4 a^2 x^2} - \frac{\text{Log}[c (a + b x^2)^p]^3}{4 x^4} - \\ & \frac{3 b^2 p \text{Log}[c (a + b x^2)^p]^2 \text{Log}\left[1 - \frac{a}{a + b x^2}\right]}{4 a^2} + \frac{3 b^2 p^2 \text{Log}[c (a + b x^2)^p] \text{PolyLog}\left[2, \frac{a}{a + b x^2}\right]}{2 a^2} + \\ & \frac{3 b^2 p^3 \text{PolyLog}\left[2, 1 + \frac{b x^2}{a}\right]}{2 a^2} + \frac{3 b^2 p^3 \text{PolyLog}\left[3, \frac{a}{a + b x^2}\right]}{2 a^2} \end{aligned}$$

Result (type 4, 236 leaves, 13 steps):

$$\begin{aligned}
& \frac{3 b^2 p^2 \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]}{2 a^2} - \\
& \frac{3 b p\left(a+b x^2\right) \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{4 a^2 x^2} - \frac{3 b^2 p \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{4 a^2} + \\
& \frac{b^2 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{4 a^2} - \frac{\operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{4 x^4} + \frac{3 b^2 p^3 \operatorname{PolyLog}\left[2, 1+\frac{b x^2}{a}\right]}{2 a^2} - \\
& \frac{3 b^2 p^2 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right] \operatorname{PolyLog}\left[2, 1+\frac{b x^2}{a}\right]}{2 a^2} + \frac{3 b^2 p^3 \operatorname{PolyLog}\left[3, 1+\frac{b x^2}{a}\right]}{2 a^2}
\end{aligned}$$

**Problem 97: Result valid but suboptimal antiderivative.**

$$\int \frac{\operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{x^7} dx$$

Optimal (type 4, 352 leaves, 17 steps):

$$\begin{aligned}
& \frac{b^3 p^3 \operatorname{Log}[x]}{a^3} - \frac{b^2 p^2 (a+b x^2) \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]}{2 a^3 x^2} - \frac{b^3 p^2 \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]}{a^3} - \\
& \frac{b p \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{4 a x^4} + \frac{b^2 p (a+b x^2) \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{2 a^3 x^2} - \frac{\operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{6 x^6} - \\
& \frac{b^3 p^2 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right] \operatorname{Log}\left[1-\frac{a}{a+b x^2}\right]}{2 a^3} + \frac{b^3 p \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2 \operatorname{Log}\left[1-\frac{a}{a+b x^2}\right]}{2 a^3} + \\
& \frac{b^3 p^3 \operatorname{PolyLog}\left[2, \frac{a}{a+b x^2}\right]}{2 a^3} - \frac{b^3 p^2 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right] \operatorname{PolyLog}\left[2, \frac{a}{a+b x^2}\right]}{a^3} - \\
& \frac{b^3 p^3 \operatorname{PolyLog}\left[2, 1+\frac{b x^2}{a}\right]}{a^3} - \frac{b^3 p^3 \operatorname{PolyLog}\left[3, \frac{a}{a+b x^2}\right]}{a^3}
\end{aligned}$$

Result (type 4, 331 leaves, 22 steps):

$$\begin{aligned}
& \frac{b^3 p^3 \operatorname{Log}[x]}{a^3} - \frac{b^2 p^2 (a+b x^2) \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]}{2 a^3 x^2} - \\
& \frac{3 b^3 p^2 \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]}{2 a^3} + \frac{b^3 p \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{4 a^3} - \frac{b p \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{4 a x^4} + \\
& \frac{b^2 p (a+b x^2) \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{2 a^3 x^2} + \frac{b^3 p \operatorname{Log}\left[-\frac{b x^2}{a}\right] \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^2}{2 a^3} - \\
& \frac{b^3 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{6 a^3} - \frac{\operatorname{Log}\left[c\left(a+b x^2\right)^p\right]^3}{6 x^6} - \frac{3 b^3 p^3 \operatorname{PolyLog}\left[2, 1+\frac{b x^2}{a}\right]}{2 a^3} + \\
& \frac{b^3 p^2 \operatorname{Log}\left[c\left(a+b x^2\right)^p\right] \operatorname{PolyLog}\left[2, 1+\frac{b x^2}{a}\right]}{a^3} - \frac{b^3 p^3 \operatorname{PolyLog}\left[3, 1+\frac{b x^2}{a}\right]}{a^3}
\end{aligned}$$

### Problem 163: Result valid but suboptimal antiderivative.

$$\int (f x)^{-1+3n} \operatorname{Log}[c (d + e x^n)^p]^2 dx$$

Optimal (type 3, 372 leaves, 9 steps):

$$\begin{aligned} & \frac{2 d^2 p^2 x^{1-2n} (f x)^{-1+3n}}{e^2 n} - \frac{d p^2 x^{1-3n} (f x)^{-1+3n} (d + e x^n)^2}{2 e^3 n} + \frac{2 p^2 x^{1-3n} (f x)^{-1+3n} (d + e x^n)^3}{27 e^3 n} - \\ & \frac{d^3 p^2 x^{1-3n} (f x)^{-1+3n} \operatorname{Log}[d + e x^n]^2}{3 e^3 n} - \frac{2 d^2 p x^{1-3n} (f x)^{-1+3n} (d + e x^n) \operatorname{Log}[c (d + e x^n)^p]}{e^3 n} + \\ & \frac{d p x^{1-3n} (f x)^{-1+3n} (d + e x^n)^2 \operatorname{Log}[c (d + e x^n)^p]}{e^3 n} - \frac{2 p x^{1-3n} (f x)^{-1+3n} (d + e x^n)^3 \operatorname{Log}[c (d + e x^n)^p]}{9 e^3 n} + \\ & \frac{2 d^3 p x^{1-3n} (f x)^{-1+3n} \operatorname{Log}[d + e x^n] \operatorname{Log}[c (d + e x^n)^p]}{3 e^3 n} + \frac{x (f x)^{-1+3n} \operatorname{Log}[c (d + e x^n)^p]^2}{3 n} \end{aligned}$$

Result (type 3, 278 leaves, 9 steps):

$$\begin{aligned} & \frac{2 d^2 p^2 x^{1-2n} (f x)^{-1+3n}}{e^2 n} - \frac{d p^2 x^{1-3n} (f x)^{-1+3n} (d + e x^n)^2}{2 e^3 n} + \\ & \frac{2 p^2 x^{1-3n} (f x)^{-1+3n} (d + e x^n)^3}{27 e^3 n} - \frac{d^3 p^2 x^{1-3n} (f x)^{-1+3n} \operatorname{Log}[d + e x^n]^2}{3 e^3 n} - \frac{1}{9 n} \\ & p x^{1-3n} (f x)^{-1+3n} \left( \frac{18 d^2 (d + e x^n)}{e^3} - \frac{9 d (d + e x^n)^2}{e^3} + \frac{2 (d + e x^n)^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d + e x^n]}{e^3} \right) \\ & \operatorname{Log}[c (d + e x^n)^p] + \frac{x (f x)^{-1+3n} \operatorname{Log}[c (d + e x^n)^p]^2}{3 n} \end{aligned}$$

### Problem 168: Result valid but suboptimal antiderivative.

$$\int (f x)^{-1-2n} \operatorname{Log}[c (d + e x^n)^p]^2 dx$$

Optimal (type 4, 200 leaves, 9 steps):

$$\begin{aligned} & \frac{e^2 p^2 x^{1+2n} (f x)^{-1-2n} \operatorname{Log}[x]}{d^2} - \\ & \frac{e p x^{1+n} (f x)^{-1-2n} (d + e x^n) \operatorname{Log}[c (d + e x^n)^p]}{d^2 n} - \frac{x (f x)^{-1-2n} \operatorname{Log}[c (d + e x^n)^p]^2}{2 n} - \\ & \frac{e^2 p x^{1+2n} (f x)^{-1-2n} \operatorname{Log}[c (d + e x^n)^p] \operatorname{Log}\left[1 - \frac{d}{d + e x^n}\right]}{d^2 n} + \frac{e^2 p^2 x^{1+2n} (f x)^{-1-2n} \operatorname{PolyLog}\left[2, \frac{d}{d + e x^n}\right]}{d^2 n} \end{aligned}$$

Result (type 4, 238 leaves, 11 steps):

$$\begin{aligned}
& \frac{e^2 p^2 x^{1+2n} (f x)^{-1-2n} \operatorname{Log}[x]}{d^2} - \frac{e p x^{1+n} (f x)^{-1-2n} (d + e x^n) \operatorname{Log}[c (d + e x^n)^p]}{d^2 n} - \\
& \frac{e^2 p x^{1+2n} (f x)^{-1-2n} \operatorname{Log}\left[-\frac{e x^n}{d}\right] \operatorname{Log}[c (d + e x^n)^p]}{d^2 n} - \frac{x (f x)^{-1-2n} \operatorname{Log}[c (d + e x^n)^p]^2}{2 n} + \\
& \frac{e^2 x^{1+2n} (f x)^{-1-2n} \operatorname{Log}[c (d + e x^n)^p]^2}{2 d^2 n} - \frac{e^2 p^2 x^{1+2n} (f x)^{-1-2n} \operatorname{PolyLog}\left[2, 1 + \frac{e x^n}{d}\right]}{d^2 n}
\end{aligned}$$

**Problem 408: Result valid but suboptimal antiderivative.**

$$\int x^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2 dx$$

Optimal (type 3, 480 leaves, 8 steps):

$$\begin{aligned}
& \frac{5 b^2 d^4 n^2 (d + e \sqrt{x})^2}{2 e^6} - \frac{40 b^2 d^3 n^2 (d + e \sqrt{x})^3}{27 e^6} + \frac{5 b^2 d^2 n^2 (d + e \sqrt{x})^4}{8 e^6} - \\
& \frac{4 b^2 d n^2 (d + e \sqrt{x})^5}{25 e^6} + \frac{b^2 n^2 (d + e \sqrt{x})^6}{54 e^6} - \frac{4 b^2 d^5 n^2 \sqrt{x}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e \sqrt{x}]^2}{3 e^6} + \\
& \frac{4 b d^5 n (d + e \sqrt{x}) (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{e^6} - \frac{5 b d^4 n (d + e \sqrt{x})^2 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{e^6} + \\
& \frac{40 b d^3 n (d + e \sqrt{x})^3 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{9 e^6} - \frac{5 b d^2 n (d + e \sqrt{x})^4 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{2 e^6} + \\
& \frac{4 b d n (d + e \sqrt{x})^5 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{5 e^6} - \frac{b n (d + e \sqrt{x})^6 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{9 e^6} - \\
& \frac{2 b d^6 n \operatorname{Log}[d + e \sqrt{x}] (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{3 e^6} + \frac{1}{3} x^3 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& \frac{5 b^2 d^4 n^2 (d + e \sqrt{x})^2}{2 e^6} - \frac{40 b^2 d^3 n^2 (d + e \sqrt{x})^3}{27 e^6} + \frac{5 b^2 d^2 n^2 (d + e \sqrt{x})^4}{8 e^6} - \\
& \frac{4 b^2 d n^2 (d + e \sqrt{x})^5}{25 e^6} + \frac{b^2 n^2 (d + e \sqrt{x})^6}{54 e^6} - \frac{4 b^2 d^5 n^2 \sqrt{x}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e \sqrt{x}]^2}{3 e^6} + \\
& \frac{1}{90} b n \left( \frac{360 d^5 (d + e \sqrt{x})}{e^6} - \frac{450 d^4 (d + e \sqrt{x})^2}{e^6} + \frac{400 d^3 (d + e \sqrt{x})^3}{e^6} - \right. \\
& \left. \frac{225 d^2 (d + e \sqrt{x})^4}{e^6} + \frac{72 d (d + e \sqrt{x})^5}{e^6} - \frac{10 (d + e \sqrt{x})^6}{e^6} - \frac{60 d^6 \operatorname{Log}[d + e \sqrt{x}]}{e^6} \right) \\
& (a + b \operatorname{Log}[c (d + e \sqrt{x})^n]) + \frac{1}{3} x^3 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2
\end{aligned}$$



### Problem 409: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2 dx$$

Optimal (type 3, 342 leaves, 8 steps):

$$\begin{aligned} & \frac{3 b^2 d^2 n^2 \left( d + e \sqrt{x} \right)^2}{2 e^4} - \frac{4 b^2 d n^2 \left( d + e \sqrt{x} \right)^3}{9 e^4} + \frac{b^2 n^2 \left( d + e \sqrt{x} \right)^4}{16 e^4} - \frac{4 b^2 d^3 n^2 \sqrt{x}}{e^3} + \\ & \frac{b^2 d^4 n^2 \operatorname{Log} \left[ d + e \sqrt{x} \right]^2}{2 e^4} + \frac{4 b d^3 n \left( d + e \sqrt{x} \right) \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{e^4} - \\ & \frac{3 b d^2 n \left( d + e \sqrt{x} \right)^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{e^4} + \\ & \frac{4 b d n \left( d + e \sqrt{x} \right)^3 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{3 e^4} - \frac{b n \left( d + e \sqrt{x} \right)^4 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{4 e^4} - \\ & \frac{b d^4 n \operatorname{Log} \left[ d + e \sqrt{x} \right] \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{e^4} + \frac{1}{2} x^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2 \end{aligned}$$

Result (type 3, 263 leaves, 8 steps):

$$\begin{aligned} & \frac{3 b^2 d^2 n^2 \left( d + e \sqrt{x} \right)^2}{2 e^4} - \frac{4 b^2 d n^2 \left( d + e \sqrt{x} \right)^3}{9 e^4} + \\ & \frac{b^2 n^2 \left( d + e \sqrt{x} \right)^4}{16 e^4} - \frac{4 b^2 d^3 n^2 \sqrt{x}}{e^3} + \frac{b^2 d^4 n^2 \operatorname{Log} \left[ d + e \sqrt{x} \right]^2}{2 e^4} + \frac{1}{12} b n \\ & \left( \frac{48 d^3 \left( d + e \sqrt{x} \right)}{e^4} - \frac{36 d^2 \left( d + e \sqrt{x} \right)^2}{e^4} + \frac{16 d \left( d + e \sqrt{x} \right)^3}{e^4} - \frac{3 \left( d + e \sqrt{x} \right)^4}{e^4} - \frac{12 d^4 \operatorname{Log} \left[ d + e \sqrt{x} \right]}{e^4} \right) \\ & \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right) + \frac{1}{2} x^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2 \end{aligned}$$

### Problem 412: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{x^2} dx$$

Optimal (type 4, 155 leaves, 8 steps):

$$\begin{aligned}
& - \frac{2 b e n (d + e \sqrt{x}) (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{d^2 \sqrt{x}} - \\
& \frac{2 b e^2 n \operatorname{Log}\left[1 - \frac{d}{d + e \sqrt{x}}\right] (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{d^2} - \\
& \frac{(a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2}{x} + \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2} + \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e \sqrt{x}}\right]}{d^2}
\end{aligned}$$

Result (type 4, 176 leaves, 10 steps):

$$\begin{aligned}
& - \frac{2 b e n (d + e \sqrt{x}) (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{d^2 \sqrt{x}} + \\
& \frac{e^2 (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2}{d^2} - \frac{(a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2}{x} - \\
& \frac{2 b e^2 n (a + b \operatorname{Log}[c (d + e \sqrt{x})^n]) \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2} - \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{d^2}
\end{aligned}$$

**Problem 413: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2}{x^3} dx$$

Optimal (type 4, 293 leaves, 16 steps):

$$\begin{aligned}
& - \frac{b^2 e^2 n^2}{6 d^2 x} + \frac{5 b^2 e^3 n^2}{6 d^3 \sqrt{x}} - \frac{5 b^2 e^4 n^2 \operatorname{Log}[d + e \sqrt{x}]}{6 d^4} - \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{3 d x^{3/2}} + \frac{b e^2 n (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{2 d^2 x} - \\
& \frac{b e^3 n (d + e \sqrt{x}) (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{d^4 \sqrt{x}} - \frac{b e^4 n \operatorname{Log}\left[1 - \frac{d}{d + e \sqrt{x}}\right] (a + b \operatorname{Log}[c (d + e \sqrt{x})^n])}{d^4} - \\
& \frac{(a + b \operatorname{Log}[c (d + e \sqrt{x})^n])^2}{2 x^2} + \frac{11 b^2 e^4 n^2 \operatorname{Log}[x]}{12 d^4} + \frac{b^2 e^4 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e \sqrt{x}}\right]}{d^4}
\end{aligned}$$

Result (type 4, 318 leaves, 18 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{6 d^2 x} + \frac{5 b^2 e^3 n^2}{6 d^3 \sqrt{x}} - \frac{5 b^2 e^4 n^2 \operatorname{Log}[d + e \sqrt{x}]}{6 d^4} - \frac{b e n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d x^{3/2}} + \\
& \frac{b e^2 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{2 d^2 x} - \frac{b e^3 n (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{d^4 \sqrt{x}} + \\
& \frac{e^4 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{2 d^4} - \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{2 x^2} - \\
& \frac{b e^4 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{d^4} + \frac{11 b^2 e^4 n^2 \operatorname{Log}[x]}{12 d^4} - \frac{b^2 e^4 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{d^4}
\end{aligned}$$

Problem 414: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{x^4} dx$$

Optimal (type 4, 408 leaves, 24 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{30 d^2 x^2} + \frac{b^2 e^3 n^2}{10 d^3 x^{3/2}} - \frac{47 b^2 e^4 n^2}{180 d^4 x} + \frac{77 b^2 e^5 n^2}{90 d^5 \sqrt{x}} - \\
& \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e \sqrt{x}]}{90 d^6} - \frac{2 b e n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{15 d x^{5/2}} + \\
& \frac{b e^2 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{6 d^2 x^2} - \frac{2 b e^3 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{9 d^3 x^{3/2}} + \\
& \frac{b e^4 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d^4 x} - \frac{2 b e^5 n (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d^6 \sqrt{x}} - \\
& \frac{2 b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + e \sqrt{x}}\right] \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d^6} - \\
& \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{3 x^3} + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} + \frac{2 b^2 e^6 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e \sqrt{x}}\right]}{3 d^6}
\end{aligned}$$

Result (type 4, 432 leaves, 26 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{30 d^2 x^2} + \frac{b^2 e^3 n^2}{10 d^3 x^{3/2}} - \frac{47 b^2 e^4 n^2}{180 d^4 x} + \frac{77 b^2 e^5 n^2}{90 d^5 \sqrt{x}} - \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e \sqrt{x}]}{90 d^6} - \\
& \frac{2 b e n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{15 d x^{5/2}} + \frac{b e^2 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{6 d^2 x^2} - \\
& \frac{2 b e^3 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{9 d^3 x^{3/2}} + \frac{b e^4 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d^4 x} - \\
& \frac{2 b e^5 n (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{3 d^6 \sqrt{x}} + \frac{e^6 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{3 d^6} - \\
& \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{3 x^3} - \frac{2 b e^6 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{3 d^6} + \\
& \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} - \frac{2 b^2 e^6 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{3 d^6}
\end{aligned}$$

**Problem 419: Result valid but suboptimal antiderivative.**

$$\int \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^3}{x^2} dx$$

Optimal (type 4, 263 leaves, 10 steps):

$$\begin{aligned}
& -\frac{3 b e n (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{d^2 \sqrt{x}} - \\
& \frac{3 b e^2 n \operatorname{Log}\left[1 - \frac{d}{d + e \sqrt{x}}\right] \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{d^2} - \\
& \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^3}{x} + \frac{6 b^2 e^2 n^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{d^2} + \\
& \frac{6 b^2 e^2 n^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{PolyLog}\left[2, \frac{d}{d + e \sqrt{x}}\right]}{d^2} + \\
& \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{d^2} + \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + e \sqrt{x}}\right]}{d^2}
\end{aligned}$$

Result (type 4, 283 leaves, 13 steps):

$$\begin{aligned}
& - \frac{3 b e n \left( d + e \sqrt{x} \right) \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{d^2 \sqrt{x}} + \frac{e^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^3}{d^2} - \\
& \frac{\left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^3}{x} + \frac{6 b^2 e^2 n^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right) \operatorname{Log} \left[ -\frac{e \sqrt{x}}{d} \right]}{d^2} - \\
& \frac{3 b e^2 n \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2 \operatorname{Log} \left[ -\frac{e \sqrt{x}}{d} \right]}{d^2} + \frac{6 b^3 e^2 n^3 \operatorname{PolyLog} \left[ 2, 1 + \frac{e \sqrt{x}}{d} \right]}{d^2} - \\
& \frac{6 b^2 e^2 n^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right) \operatorname{PolyLog} \left[ 2, 1 + \frac{e \sqrt{x}}{d} \right]}{d^2} + \frac{6 b^3 e^2 n^3 \operatorname{PolyLog} \left[ 3, 1 + \frac{e \sqrt{x}}{d} \right]}{d^2}
\end{aligned}$$

Problem 420: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^3}{x^3} dx$$

Optimal (type 4, 573 leaves, 28 steps):

$$\begin{aligned}
& - \frac{b^3 e^3 n^3}{2 d^3 \sqrt{x}} + \frac{b^3 e^4 n^3 \operatorname{Log} \left[ d + e \sqrt{x} \right]}{2 d^4} - \frac{b^2 e^2 n^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{2 d^2 x} + \\
& \frac{5 b^2 e^3 n^2 \left( d + e \sqrt{x} \right) \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{2 d^4 \sqrt{x}} + \\
& \frac{5 b^2 e^4 n^2 \operatorname{Log} \left[ 1 - \frac{d}{d + e \sqrt{x}} \right] \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)}{2 d^4} - \frac{b e n \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{2 d x^{3/2}} + \\
& \frac{3 b e^2 n \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{4 d^2 x} - \frac{3 b e^3 n \left( d + e \sqrt{x} \right) \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{2 d^4 \sqrt{x}} - \\
& \frac{3 b e^4 n \operatorname{Log} \left[ 1 - \frac{d}{d + e \sqrt{x}} \right] \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^2}{2 d^4} - \frac{\left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right)^3}{2 x^2} + \\
& \frac{3 b^2 e^4 n^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right) \operatorname{Log} \left[ -\frac{e \sqrt{x}}{d} \right]}{d^4} - \frac{3 b^3 e^4 n^3 \operatorname{Log} [x]}{2 d^4} - \\
& \frac{5 b^3 e^4 n^3 \operatorname{PolyLog} \left[ 2, \frac{d}{d + e \sqrt{x}} \right]}{2 d^4} + \frac{3 b^2 e^4 n^2 \left( a + b \operatorname{Log} \left[ c \left( d + e \sqrt{x} \right)^n \right] \right) \operatorname{PolyLog} \left[ 2, \frac{d}{d + e \sqrt{x}} \right]}{d^4} + \\
& \frac{3 b^3 e^4 n^3 \operatorname{PolyLog} \left[ 2, 1 + \frac{e \sqrt{x}}{d} \right]}{d^4} + \frac{3 b^3 e^4 n^3 \operatorname{PolyLog} \left[ 3, \frac{d}{d + e \sqrt{x}} \right]}{d^4}
\end{aligned}$$

Result (type 4, 550 leaves, 35 steps):

$$\begin{aligned}
& -\frac{b^3 e^3 n^3}{2 d^3 \sqrt{x}} + \frac{b^3 e^4 n^3 \operatorname{Log}[d + e \sqrt{x}]}{2 d^4} - \frac{b^2 e^2 n^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{2 d^2 x} + \\
& \frac{5 b^2 e^3 n^2 (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)}{2 d^4 \sqrt{x}} - \frac{5 b e^4 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{4 d^4} - \\
& \frac{b e n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{2 d x^{3/2}} + \frac{3 b e^2 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{4 d^2 x} - \\
& \frac{3 b e^3 n (d + e \sqrt{x}) \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2}{2 d^4 \sqrt{x}} + \frac{e^4 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^3}{2 d^4} - \\
& \frac{\left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^3}{2 x^2} + \frac{11 b^2 e^4 n^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{2 d^4} - \\
& \frac{3 b e^4 n \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right)^2 \operatorname{Log}\left[-\frac{e \sqrt{x}}{d}\right]}{2 d^4} - \\
& \frac{3 b^3 e^4 n^3 \operatorname{Log}[x]}{2 d^4} + \frac{11 b^3 e^4 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{2 d^4} - \\
& \frac{3 b^2 e^4 n^2 \left( a + b \operatorname{Log}[c (d + e \sqrt{x})^n] \right) \operatorname{PolyLog}\left[2, 1 + \frac{e \sqrt{x}}{d}\right]}{d^4} + \frac{3 b^3 e^4 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e \sqrt{x}}{d}\right]}{d^4}
\end{aligned}$$

**Problem 429: Result valid but suboptimal antiderivative.**

$$\int x^2 \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 404 leaves, 24 steps):

$$\begin{aligned}
& -\frac{77 b^2 e^5 n^2 \sqrt{x}}{90 d^5} + \frac{47 b^2 e^4 n^2 x}{180 d^4} - \frac{b^2 e^3 n^2 x^{3/2}}{10 d^3} + \frac{b^2 e^2 n^2 x^2}{30 d^2} + \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{90 d^6} + \\
& \frac{2 b e^5 n \left( d + \frac{e}{\sqrt{x}} \right) \sqrt{x} \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{3 d^6} - \frac{b e^4 n x \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{3 d^4} + \\
& \frac{2 b e^3 n x^{3/2} \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{9 d^3} - \frac{b e^2 n x^2 \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{6 d^2} + \\
& \frac{2 b e n x^{5/2} \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{15 d} + \frac{2 b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)}{3 d^6} + \\
& \frac{1}{3} x^3 \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} - \frac{2 b^2 e^6 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{3 d^6}
\end{aligned}$$

Result (type 4, 428 leaves, 26 steps):

$$\begin{aligned}
& -\frac{77 b^2 e^5 n^2 \sqrt{x}}{90 d^5} + \frac{47 b^2 e^4 n^2 x}{180 d^4} - \frac{b^2 e^3 n^2 x^{3/2}}{10 d^3} + \frac{b^2 e^2 n^2 x^2}{30 d^2} + \\
& \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{90 d^6} + \frac{2 b e^5 n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 d^6} - \\
& \frac{b e^4 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 d^4} + \frac{2 b e^3 n x^{3/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{9 d^3} - \\
& \frac{b e^2 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{6 d^2} + \frac{2 b e n x^{5/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{15 d} - \\
& \frac{e^6 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{3 d^6} + \frac{1}{3} x^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + \\
& \frac{2 b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{3 d^6} + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} + \frac{2 b^2 e^6 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{3 d^6}
\end{aligned}$$

**Problem 430: Result valid but suboptimal antiderivative.**

$$\int x \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{\sqrt{x}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 288 leaves, 16 steps):

$$\begin{aligned}
& -\frac{5 b^2 e^3 n^2 \sqrt{x}}{6 d^3} + \frac{b^2 e^2 n^2 x}{6 d^2} + \frac{5 b^2 e^4 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{6 d^4} + \\
& \frac{b e^3 n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^4} - \frac{b e^2 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^2} + \\
& \frac{b e n x^{3/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 d} + \frac{b e^4 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^4} + \\
& \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + \frac{11 b^2 e^4 n^2 \operatorname{Log}[x]}{12 d^4} - \frac{b^2 e^4 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^4}
\end{aligned}$$

Result (type 4, 311 leaves, 18 steps):

$$\begin{aligned}
& - \frac{5 b^2 e^3 n^2 \sqrt{x}}{6 d^3} + \frac{b^2 e^2 n^2 x}{6 d^2} + \frac{5 b^2 e^4 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{6 d^4} + \frac{b e^3 n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^4} \\
& - \frac{b e^2 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^2} + \frac{b e n x^{3/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 d} - \\
& \frac{e^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d^4} + \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + \\
& \frac{b e^4 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^4} + \frac{11 b^2 e^4 n^2 \operatorname{Log}[x]}{12 d^4} + \frac{b^2 e^4 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^4}
\end{aligned}$$

**Problem 431: Result valid but suboptimal antiderivative.**

$$\int \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 152 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 b e n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^2} + \frac{2 b e^2 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^2} + \\
& x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2} - \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^2}
\end{aligned}$$

Result (type 4, 174 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 b e n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{d^2} - \\
& \frac{e^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} + x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 + \\
& \frac{2 b e^2 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[x]}{d^2} + \frac{2 b^2 e^2 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^2}
\end{aligned}$$

**Problem 434: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{x^3} dx$$

Optimal (type 3, 341 leaves, 8 steps):



$$\begin{aligned}
& - \frac{3 b^2 d^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^2}{2 e^4} + \frac{4 b^2 d n^2 \left(d + \frac{e}{\sqrt{x}}\right)^3}{9 e^4} - \frac{b^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^4}{16 e^4} + \frac{4 b^2 d^3 n^2}{e^3 \sqrt{x}} - \frac{b^2 d^4 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]^2}{2 e^4} \\
& - \frac{4 b d^3 n \left(d + \frac{e}{\sqrt{x}}\right) \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{e^4} + \frac{3 b d^2 n \left(d + \frac{e}{\sqrt{x}}\right)^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{e^4} - \\
& - \frac{4 b d n \left(d + \frac{e}{\sqrt{x}}\right)^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 e^4} + \frac{b n \left(d + \frac{e}{\sqrt{x}}\right)^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{4 e^4} + \\
& + \frac{b d^4 n \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{e^4} - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 x^2}
\end{aligned}$$

Result (type 3, 263 leaves, 8 steps):

$$\begin{aligned}
& - \frac{3 b^2 d^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^2}{2 e^4} + \frac{4 b^2 d n^2 \left(d + \frac{e}{\sqrt{x}}\right)^3}{9 e^4} - \frac{b^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^4}{16 e^4} + \frac{4 b^2 d^3 n^2}{e^3 \sqrt{x}} - \frac{b^2 d^4 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]^2}{2 e^4} \\
& - \frac{1}{12} b n \left( \frac{48 d^3 \left(d + \frac{e}{\sqrt{x}}\right)}{e^4} - \frac{36 d^2 \left(d + \frac{e}{\sqrt{x}}\right)^2}{e^4} + \frac{16 d \left(d + \frac{e}{\sqrt{x}}\right)^3}{e^4} - \frac{3 \left(d + \frac{e}{\sqrt{x}}\right)^4}{e^4} - \frac{12 d^4 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{e^4} \right) \\
& - \left( a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right] \right) - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 x^2}
\end{aligned}$$

**Problem 435: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{x^4} dx$$

Optimal (type 3, 480 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5 b^2 d^4 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^2}{2 e^6} + \frac{40 b^2 d^3 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^3}{27 e^6} - \frac{5 b^2 d^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^4}{8 e^6} + \\
& \frac{4 b^2 d n^2 \left(d + \frac{e}{\sqrt{x}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^6}{54 e^6} + \frac{4 b^2 d^5 n^2}{e^5 \sqrt{x}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]^2}{3 e^6} - \\
& \frac{4 b d^5 n \left(d + \frac{e}{\sqrt{x}}\right) \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{e^6} + \frac{5 b d^4 n \left(d + \frac{e}{\sqrt{x}}\right)^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{e^6} - \\
& \frac{40 b d^3 n \left(d + \frac{e}{\sqrt{x}}\right)^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{9 e^6} + \frac{5 b d^2 n \left(d + \frac{e}{\sqrt{x}}\right)^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 e^6} - \\
& \frac{4 b d n \left(d + \frac{e}{\sqrt{x}}\right)^5 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{5 e^6} + \frac{b n \left(d + \frac{e}{\sqrt{x}}\right)^6 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{9 e^6} + \\
& \frac{2 b d^6 n \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{3 e^6} - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{3 x^3}
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5 b^2 d^4 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^2}{2 e^6} + \frac{40 b^2 d^3 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^3}{27 e^6} - \frac{5 b^2 d^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^4}{8 e^6} + \\
& \frac{4 b^2 d n^2 \left(d + \frac{e}{\sqrt{x}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{\sqrt{x}}\right)^6}{54 e^6} + \frac{4 b^2 d^5 n^2}{e^5 \sqrt{x}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]^2}{3 e^6} - \\
& \frac{1}{90} b n \left( \frac{360 d^5 \left(d + \frac{e}{\sqrt{x}}\right)}{e^6} - \frac{450 d^4 \left(d + \frac{e}{\sqrt{x}}\right)^2}{e^6} + \frac{400 d^3 \left(d + \frac{e}{\sqrt{x}}\right)^3}{e^6} - \right. \\
& \left. \frac{225 d^2 \left(d + \frac{e}{\sqrt{x}}\right)^4}{e^6} + \frac{72 d \left(d + \frac{e}{\sqrt{x}}\right)^5}{e^6} - \frac{10 \left(d + \frac{e}{\sqrt{x}}\right)^6}{e^6} - \frac{60 d^6 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{e^6} \right) \\
& \left( a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right] \right) - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{3 x^3}
\end{aligned}$$

**Problem 436: Result valid but suboptimal antiderivative.**

$$\int x \left( a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right] \right)^3 dx$$

Optimal (type 4, 569 leaves, 28 steps):

$$\begin{aligned}
& \frac{b^3 e^3 n^3 \sqrt{x}}{2 d^3} - \frac{b^3 e^4 n^3 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{2 d^4} - \frac{5 b^2 e^3 n^2 \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^4} + \\
& \frac{b^2 e^2 n^2 x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^2} - \frac{5 b^2 e^4 n^2 \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^4} + \\
& \frac{3 b e^3 n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d^4} - \\
& \frac{3 b e^2 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{4 d^2} + \frac{b e n x^{3/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d} + \\
& \frac{3 b e^4 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d^4} + \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 - \\
& \frac{3 b^2 e^4 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^4} - \frac{3 b^3 e^4 n^3 \operatorname{Log}[x]}{2 d^4} + \\
& \frac{5 b^3 e^4 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{2 d^4} - \frac{3 b^2 e^4 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^4} - \\
& \frac{3 b^3 e^4 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^4} - \frac{3 b^3 e^4 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^4}
\end{aligned}$$

Result (type 4, 546 leaves, 35 steps):

$$\begin{aligned}
& \frac{b^3 e^3 n^3 \sqrt{x}}{2 d^3} - \frac{b^3 e^4 n^3 \operatorname{Log}\left[d + \frac{e}{\sqrt{x}}\right]}{2 d^4} - \frac{5 b^2 e^3 n^2 \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^4} + \\
& \frac{b^2 e^2 n^2 x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)}{2 d^2} + \frac{5 b e^4 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{4 d^4} + \\
& \frac{3 b e^3 n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d^4} - \frac{3 b e^2 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{4 d^2} + \\
& \frac{b e n x^{3/2} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{2 d} - \frac{e^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3}{2 d^4} + \\
& \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 - \frac{11 b^2 e^4 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{2 d^4} + \\
& \frac{3 b e^4 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{2 d^4} - \\
& \frac{3 b^3 e^4 n^3 \operatorname{Log}[x]}{2 d^4} - \frac{11 b^3 e^4 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{2 d^4} + \\
& \frac{3 b^2 e^4 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^4} - \frac{3 b^3 e^4 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d \sqrt{x}}\right]}{d^4}
\end{aligned}$$

**Problem 437: Result valid but suboptimal antiderivative.**

$$\int \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 dx$$

Optimal (type 4, 260 leaves, 11 steps):

$$\begin{aligned}
& \frac{3 b e n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} + \frac{3 b e^2 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{\sqrt{x}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} + \\
& x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 - \frac{6 b^2 e^2 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^2} - \\
& \frac{6 b^2 e^2 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^2} - \\
& \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^2} - \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{\sqrt{x}}}\right]}{d^2}
\end{aligned}$$

Result (type 4, 281 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 b e n \left(d + \frac{e}{\sqrt{x}}\right) \sqrt{x} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2}{d^2} - \frac{e^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3}{d^2} + \\
& x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^3 - \frac{6 b^2 e^2 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^2} + \\
& \frac{3 b e^2 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d \sqrt{x}}\right]}{d^2} - \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^2} + \\
& \frac{6 b^2 e^2 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{\sqrt{x}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d \sqrt{x}}\right]}{d^2} - \frac{6 b^3 e^2 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d \sqrt{x}}\right]}{d^2}
\end{aligned}$$

Problem 450: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)^2 dx$$

Optimal (type 3, 680 leaves, 8 steps):

$$\begin{aligned}
& -\frac{6 b^2 d^7 n^2 \left(d + e x^{1/3}\right)^2}{e^9} + \frac{56 b^2 d^6 n^2 \left(d + e x^{1/3}\right)^3}{9 e^9} - \frac{21 b^2 d^5 n^2 \left(d + e x^{1/3}\right)^4}{4 e^9} + \\
& \frac{84 b^2 d^4 n^2 \left(d + e x^{1/3}\right)^5}{25 e^9} - \frac{14 b^2 d^3 n^2 \left(d + e x^{1/3}\right)^6}{9 e^9} + \frac{24 b^2 d^2 n^2 \left(d + e x^{1/3}\right)^7}{49 e^9} - \\
& \frac{3 b^2 d n^2 \left(d + e x^{1/3}\right)^8}{32 e^9} + \frac{2 b^2 n^2 \left(d + e x^{1/3}\right)^9}{243 e^9} + \frac{6 b^2 d^8 n^2 x^{1/3}}{e^8} - \frac{b^2 d^9 n^2 \operatorname{Log}\left[d + e x^{1/3}\right]^2}{3 e^9} - \\
& \frac{6 b d^8 n \left(d + e x^{1/3}\right) \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{e^9} + \frac{12 b d^7 n \left(d + e x^{1/3}\right)^2 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{e^9} - \\
& \frac{56 b d^6 n \left(d + e x^{1/3}\right)^3 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{3 e^9} + \\
& \frac{21 b d^5 n \left(d + e x^{1/3}\right)^4 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{e^9} - \\
& \frac{84 b d^4 n \left(d + e x^{1/3}\right)^5 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{5 e^9} + \\
& \frac{28 b d^3 n \left(d + e x^{1/3}\right)^6 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{3 e^9} - \\
& \frac{24 b d^2 n \left(d + e x^{1/3}\right)^7 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{7 e^9} + \\
& \frac{3 b d n \left(d + e x^{1/3}\right)^8 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{4 e^9} - \frac{2 b n \left(d + e x^{1/3}\right)^9 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{27 e^9} + \\
& \frac{2 b d^9 n \operatorname{Log}\left[d + e x^{1/3}\right] \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)}{3 e^9} + \frac{1}{3} x^3 \left(a + b \operatorname{Log}\left[c \left(d + e x^{1/3}\right)^n\right]\right)^2
\end{aligned}$$

Result (type 3, 491 leaves, 8 steps):

$$\begin{aligned}
& - \frac{6 b^2 d^7 n^2 (d + e x^{1/3})^2}{e^9} + \frac{56 b^2 d^6 n^2 (d + e x^{1/3})^3}{9 e^9} - \frac{21 b^2 d^5 n^2 (d + e x^{1/3})^4}{4 e^9} + \\
& \frac{84 b^2 d^4 n^2 (d + e x^{1/3})^5}{25 e^9} - \frac{14 b^2 d^3 n^2 (d + e x^{1/3})^6}{9 e^9} + \frac{24 b^2 d^2 n^2 (d + e x^{1/3})^7}{49 e^9} - \\
& \frac{3 b^2 d n^2 (d + e x^{1/3})^8}{32 e^9} + \frac{2 b^2 n^2 (d + e x^{1/3})^9}{243 e^9} + \frac{6 b^2 d^8 n^2 x^{1/3}}{e^8} - \frac{b^2 d^9 n^2 \operatorname{Log}[d + e x^{1/3}]^2}{3 e^9} - \\
& \frac{1}{3780} b n \left( \frac{22 680 d^8 (d + e x^{1/3})}{e^9} - \frac{45 360 d^7 (d + e x^{1/3})^2}{e^9} + \frac{70 560 d^6 (d + e x^{1/3})^3}{e^9} - \right. \\
& \quad \frac{79 380 d^5 (d + e x^{1/3})^4}{e^9} + \frac{63 504 d^4 (d + e x^{1/3})^5}{e^9} - \frac{35 280 d^3 (d + e x^{1/3})^6}{e^9} + \\
& \quad \left. \frac{12 960 d^2 (d + e x^{1/3})^7}{e^9} - \frac{2835 d (d + e x^{1/3})^8}{e^9} + \frac{280 (d + e x^{1/3})^9}{e^9} - \frac{2520 d^9 \operatorname{Log}[d + e x^{1/3}]}{e^9} \right) \\
& (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) + \frac{1}{3} x^3 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2
\end{aligned}$$

**Problem 451: Result valid but suboptimal antiderivative.**

$$\int x (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2 dx$$

Optimal (type 3, 480 leaves, 8 steps):

$$\begin{aligned}
& \frac{15 b^2 d^4 n^2 (d + e x^{1/3})^2}{4 e^6} - \frac{20 b^2 d^3 n^2 (d + e x^{1/3})^3}{9 e^6} + \frac{15 b^2 d^2 n^2 (d + e x^{1/3})^4}{16 e^6} - \\
& \frac{6 b^2 d n^2 (d + e x^{1/3})^5}{25 e^6} + \frac{b^2 n^2 (d + e x^{1/3})^6}{36 e^6} - \frac{6 b^2 d^5 n^2 x^{1/3}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e x^{1/3}]^2}{2 e^6} + \\
& \frac{6 b d^5 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{e^6} - \frac{15 b d^4 n (d + e x^{1/3})^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{2 e^6} + \\
& \frac{20 b d^3 n (d + e x^{1/3})^3 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{3 e^6} - \\
& \frac{15 b d^2 n (d + e x^{1/3})^4 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{4 e^6} + \\
& \frac{6 b d n (d + e x^{1/3})^5 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{5 e^6} - \frac{b n (d + e x^{1/3})^6 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{6 e^6} - \\
& \frac{b d^6 n \operatorname{Log}[d + e x^{1/3}] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{e^6} + \frac{1}{2} x^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& \frac{15 b^2 d^4 n^2 (d + e x^{1/3})^2}{4 e^6} - \frac{20 b^2 d^3 n^2 (d + e x^{1/3})^3}{9 e^6} + \frac{15 b^2 d^2 n^2 (d + e x^{1/3})^4}{16 e^6} - \\
& \frac{6 b^2 d n^2 (d + e x^{1/3})^5}{25 e^6} + \frac{b^2 n^2 (d + e x^{1/3})^6}{36 e^6} - \frac{6 b^2 d^5 n^2 x^{1/3}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e x^{1/3}]^2}{2 e^6} + \\
& \frac{1}{60} b n \left( \frac{360 d^5 (d + e x^{1/3})}{e^6} - \frac{450 d^4 (d + e x^{1/3})^2}{e^6} + \frac{400 d^3 (d + e x^{1/3})^3}{e^6} - \right. \\
& \quad \left. \frac{225 d^2 (d + e x^{1/3})^4}{e^6} + \frac{72 d (d + e x^{1/3})^5}{e^6} - \frac{10 (d + e x^{1/3})^6}{e^6} - \frac{60 d^6 \operatorname{Log}[d + e x^{1/3}]}{e^6} \right) \\
& (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) + \frac{1}{2} x^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2
\end{aligned}$$

**Problem 452: Result valid but suboptimal antiderivative.**

$$\int (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2 dx$$

Optimal (type 3, 267 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3 b^2 d n^2 (d + e x^{1/3})^2}{2 e^3} + \frac{2 b^2 n^2 (d + e x^{1/3})^3}{9 e^3} + \frac{6 b^2 d^2 n^2 x^{1/3}}{e^2} - \\
& \frac{b^2 d^3 n^2 \operatorname{Log}[d + e x^{1/3}]^2}{e^3} - \frac{6 b d^2 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{e^3} + \\
& \frac{3 b d n (d + e x^{1/3})^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{e^3} - \frac{2 b n (d + e x^{1/3})^3 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{3 e^3} + \\
& \frac{2 b d^3 n \operatorname{Log}[d + e x^{1/3}] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{e^3} + x (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2
\end{aligned}$$

Result (type 3, 210 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3 b^2 d n^2 (d + e x^{1/3})^2}{2 e^3} + \frac{2 b^2 n^2 (d + e x^{1/3})^3}{9 e^3} + \frac{6 b^2 d^2 n^2 x^{1/3}}{e^2} - \frac{b^2 d^3 n^2 \operatorname{Log}[d + e x^{1/3}]^2}{e^3} - \\
& \frac{1}{3} b n \left( \frac{18 d^2 (d + e x^{1/3})}{e^3} - \frac{9 d (d + e x^{1/3})^2}{e^3} + \frac{2 (d + e x^{1/3})^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d + e x^{1/3}]}{e^3} \right) \\
& (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) + x (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2
\end{aligned}$$

**Problem 454: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{x^2} dx$$

Optimal (type 4, 231 leaves, 12 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{d^2 x^{1/3}} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x^{1/3}]}{d^3} - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d x^{2/3}} + \\
& \frac{2 b e^2 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3 x^{1/3}} + \\
& \frac{2 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{x} - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} - \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 253 leaves, 14 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{d^2 x^{1/3}} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x^{1/3}]}{d^3} - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d x^{2/3}} + \\
& \frac{2 b e^2 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3 x^{1/3}} - \frac{e^3 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{d^3} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{x} + \frac{2 b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}\left[-\frac{e x^{1/3}}{d}\right]}{d^3} - \\
& \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} + \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^3}
\end{aligned}$$

**Problem 455: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{x^3} dx$$

Optimal (type 4, 405 leaves, 24 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{20 d^2 x^{4/3}} + \frac{3 b^2 e^3 n^2}{20 d^3 x} - \frac{47 b^2 e^4 n^2}{120 d^4 x^{2/3}} + \frac{77 b^2 e^5 n^2}{60 d^5 x^{1/3}} - \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e x^{1/3}]}{60 d^6} - \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{5 d x^{5/3}} + \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{4 d^2 x^{4/3}} - \\
& \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{3 d^3 x} + \frac{b e^4 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{2 d^4 x^{2/3}} - \\
& \frac{b e^5 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^6 x^{1/3}} - \frac{b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^6} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 x^2} + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} + \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{d^6}
\end{aligned}$$

Result (type 4, 430 leaves, 26 steps):



$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{20 d^2 x^{4/3}} + \frac{3 b^2 e^3 n^2}{20 d^3 x} - \frac{47 b^2 e^4 n^2}{120 d^4 x^{2/3}} + \frac{77 b^2 e^5 n^2}{60 d^5 x^{1/3}} - \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e x^{1/3}]}{60 d^6} - \\
& \frac{b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{5 d x^{5/3}} + \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{4 d^2 x^{4/3}} - \\
& \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{3 d^3 x} + \frac{b e^4 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{2 d^4 x^{2/3}} - \\
& \frac{b e^5 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^6 x^{1/3}} + \frac{e^6 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^6} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 x^2} - \frac{b e^6 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}\left[-\frac{e x^{1/3}}{d}\right]}{d^6} + \\
& \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} - \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^6}
\end{aligned}$$

**Problem 461: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{x^2} dx$$

Optimal (type 4, 439 leaves, 17 steps):

$$\begin{aligned}
& -\frac{3 b^2 e^2 n^2 (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3 x^{1/3}} - \\
& \frac{3 b^2 e^3 n^2 \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3} - \\
& \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d x^{2/3}} + \frac{3 b e^2 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{d^3 x^{1/3}} + \\
& \frac{3 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{d^3} - \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{x} - \\
& \frac{6 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}\left[-\frac{e x^{1/3}}{d}\right]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} + \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{d^3} - \frac{6 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{d^3} - \\
& \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^3} - \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + e x^{1/3}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 414 leaves, 22 steps):

$$\begin{aligned}
& - \frac{3 b^2 e^2 n^2 (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{d^3 x^{1/3}} + \\
& \frac{3 b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^3} - \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d x^{2/3}} + \\
& \frac{3 b e^2 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{d^3 x^{1/3}} - \frac{e^3 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{d^3} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{x} - \frac{9 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}[-\frac{e x^{1/3}}{d}]}{d^3} + \\
& \frac{3 b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2 \operatorname{Log}[-\frac{e x^{1/3}}{d}]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} - \frac{9 b^3 e^3 n^3 \operatorname{PolyLog}[2, 1 + \frac{e x^{1/3}}{d}]}{d^3} + \\
& \frac{6 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{PolyLog}[2, 1 + \frac{e x^{1/3}}{d}]}{d^3} - \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}[3, 1 + \frac{e x^{1/3}}{d}]}{d^3}
\end{aligned}$$

**Problem 462: Result valid but suboptimal antiderivative.**

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{x^3} dx$$

Optimal (type 4, 765 leaves, 62 steps):

$$\begin{aligned}
& -\frac{b^3 e^3 n^3}{20 d^3 x} + \frac{3 b^3 e^4 n^3}{10 d^4 x^{2/3}} - \frac{71 b^3 e^5 n^3}{40 d^5 x^{1/3}} + \frac{71 b^3 e^6 n^3 \operatorname{Log}[d + e x^{1/3}]}{40 d^6} - \\
& \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^2 x^{4/3}} + \frac{9 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^3 x} - \\
& \frac{47 b^2 e^4 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{40 d^4 x^{2/3}} + \frac{77 b^2 e^5 n^2 (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^6 x^{1/3}} + \\
& \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^6} - \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{10 d x^{5/3}} + \\
& \frac{3 b e^2 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{8 d^2 x^{4/3}} - \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^3 x} + \\
& \frac{3 b e^4 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{4 d^4 x^{2/3}} - \frac{3 b e^5 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^6 x^{1/3}} - \\
& \frac{3 b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{1/3}}\right] (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^6} - \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{2 x^2} + \\
& \frac{3 b^2 e^6 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}\left[-\frac{e x^{1/3}}{d}\right]}{d^6} - \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} - \\
& \frac{77 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{20 d^6} + \frac{3 b^2 e^6 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{1/3}}\right]}{d^6} + \\
& \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{1/3}}{d}\right]}{d^6} + \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + e x^{1/3}}\right]}{d^6}
\end{aligned}$$

Result (type 4, 742 leaves, 73 steps):

$$\begin{aligned}
& -\frac{b^3 e^3 n^3}{20 d^3 x} + \frac{3 b^3 e^4 n^3}{10 d^4 x^{2/3}} - \frac{71 b^3 e^5 n^3}{40 d^5 x^{1/3}} + \frac{71 b^3 e^6 n^3 \operatorname{Log}[d + e x^{1/3}]}{40 d^6} - \frac{3 b^2 e^2 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^2 x^{4/3}} + \\
& \frac{9 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^3 x} - \frac{47 b^2 e^4 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{40 d^4 x^{2/3}} + \\
& \frac{77 b^2 e^5 n^2 (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])}{20 d^6 x^{1/3}} - \frac{77 b e^6 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{40 d^6} - \\
& \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{10 d x^{5/3}} + \frac{3 b e^2 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{8 d^2 x^{4/3}} - \\
& \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^3 x} + \frac{3 b e^4 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{4 d^4 x^{2/3}} - \\
& \frac{3 b e^5 n (d + e x^{1/3}) (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2}{2 d^6 x^{1/3}} + \frac{e^6 (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{2 d^6} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{1/3})^n])^3}{2 x^2} + \frac{137 b^2 e^6 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{Log}[-\frac{e x^{1/3}}{d}]}{20 d^6} - \\
& \frac{3 b e^6 n (a + b \operatorname{Log}[c (d + e x^{1/3})^n])^2 \operatorname{Log}[-\frac{e x^{1/3}}{d}]}{2 d^6} - \\
& \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} + \frac{137 b^3 e^6 n^3 \operatorname{PolyLog}[2, 1 + \frac{e x^{1/3}}{d}]}{20 d^6} - \\
& \frac{3 b^2 e^6 n^2 (a + b \operatorname{Log}[c (d + e x^{1/3})^n]) \operatorname{PolyLog}[2, 1 + \frac{e x^{1/3}}{d}]}{d^6} + \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}[3, 1 + \frac{e x^{1/3}}{d}]}{d^6}
\end{aligned}$$

**Problem 471: Result valid but suboptimal antiderivative.**

$$\int x^3 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2 dx$$

Optimal (type 3, 482 leaves, 8 steps):

$$\begin{aligned}
& \frac{15 b^2 d^4 n^2 (d + e x^{2/3})^2}{8 e^6} - \frac{10 b^2 d^3 n^2 (d + e x^{2/3})^3}{9 e^6} + \frac{15 b^2 d^2 n^2 (d + e x^{2/3})^4}{32 e^6} - \\
& \frac{3 b^2 d n^2 (d + e x^{2/3})^5}{25 e^6} + \frac{b^2 n^2 (d + e x^{2/3})^6}{72 e^6} - \frac{3 b^2 d^5 n^2 x^{2/3}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e x^{2/3}]^2}{4 e^6} + \\
& \frac{3 b d^5 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{e^6} - \frac{15 b d^4 n (d + e x^{2/3})^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{4 e^6} + \\
& \frac{10 b d^3 n (d + e x^{2/3})^3 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{3 e^6} - \\
& \frac{15 b d^2 n (d + e x^{2/3})^4 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{8 e^6} + \\
& \frac{3 b d n (d + e x^{2/3})^5 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{5 e^6} - \frac{b n (d + e x^{2/3})^6 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{12 e^6} - \\
& \frac{b d^6 n \operatorname{Log}[d + e x^{2/3}] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 e^6} + \frac{1}{4} x^4 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& \frac{15 b^2 d^4 n^2 (d + e x^{2/3})^2}{8 e^6} - \frac{10 b^2 d^3 n^2 (d + e x^{2/3})^3}{9 e^6} + \frac{15 b^2 d^2 n^2 (d + e x^{2/3})^4}{32 e^6} - \\
& \frac{3 b^2 d n^2 (d + e x^{2/3})^5}{25 e^6} + \frac{b^2 n^2 (d + e x^{2/3})^6}{72 e^6} - \frac{3 b^2 d^5 n^2 x^{2/3}}{e^5} + \frac{b^2 d^6 n^2 \operatorname{Log}[d + e x^{2/3}]^2}{4 e^6} + \\
& \frac{1}{120} b n \left( \frac{360 d^5 (d + e x^{2/3})}{e^6} - \frac{450 d^4 (d + e x^{2/3})^2}{e^6} + \frac{400 d^3 (d + e x^{2/3})^3}{e^6} - \right. \\
& \quad \left. \frac{225 d^2 (d + e x^{2/3})^4}{e^6} + \frac{72 d (d + e x^{2/3})^5}{e^6} - \frac{10 (d + e x^{2/3})^6}{e^6} - \frac{60 d^6 \operatorname{Log}[d + e x^{2/3}]}{e^6} \right) \\
& (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) + \frac{1}{4} x^4 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2
\end{aligned}$$

**Problem 472: Result valid but suboptimal antiderivative.**

$$\int x (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2 dx$$

Optimal (type 3, 275 leaves, 8 steps):

$$\begin{aligned}
& -\frac{3 b^2 d n^2 (d + e x^{2/3})^2}{4 e^3} + \frac{b^2 n^2 (d + e x^{2/3})^3}{9 e^3} + \frac{3 b^2 d^2 n^2 x^{2/3}}{e^2} - \\
& \frac{b^2 d^3 n^2 \operatorname{Log}[d + e x^{2/3}]^2}{2 e^3} - \frac{3 b d^2 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{e^3} + \\
& \frac{3 b d n (d + e x^{2/3})^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 e^3} - \frac{b n (d + e x^{2/3})^3 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{3 e^3} + \\
& \frac{b d^3 n \operatorname{Log}[d + e x^{2/3}] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{e^3} + \frac{1}{2} x^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2
\end{aligned}$$

Result (type 3, 217 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{3 b^2 d n^2 (d + e x^{2/3})^2}{4 e^3} + \frac{b^2 n^2 (d + e x^{2/3})^3}{9 e^3} + \frac{3 b^2 d^2 n^2 x^{2/3}}{e^2} - \frac{b^2 d^3 n^2 \operatorname{Log}[d + e x^{2/3}]^2}{2 e^3} - \\
 & \frac{1}{6} b n \left( \frac{18 d^2 (d + e x^{2/3})}{e^3} - \frac{9 d (d + e x^{2/3})^2}{e^3} + \frac{2 (d + e x^{2/3})^3}{e^3} - \frac{6 d^3 \operatorname{Log}[d + e x^{2/3}]}{e^3} \right) \\
 & (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) + \frac{1}{2} x^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2
 \end{aligned}$$

Problem 474: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{x^3} dx$$

Optimal (type 4, 238 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{b^2 e^2 n^2}{2 d^2 x^{2/3}} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x^{2/3}]}{2 d^3} - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d x^{4/3}} + \\
 & \frac{b e^2 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{d^3 x^{2/3}} + \frac{b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{2/3}}\right] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{d^3} - \\
 & \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 x^2} - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} - \frac{b^2 e^3 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{2/3}}\right]}{d^3}
 \end{aligned}$$

Result (type 4, 261 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{b^2 e^2 n^2}{2 d^2 x^{2/3}} + \frac{b^2 e^3 n^2 \operatorname{Log}[d + e x^{2/3}]}{2 d^3} - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d x^{4/3}} + \\
 & \frac{b e^2 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{d^3 x^{2/3}} - \frac{e^3 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 d^3} - \\
 & \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 x^2} + \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{Log}\left[-\frac{e x^{2/3}}{d}\right]}{d^3} - \\
 & \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} + \frac{b^2 e^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{2/3}}{d}\right]}{d^3}
 \end{aligned}$$

Problem 475: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{x^5} dx$$

Optimal (type 4, 412 leaves, 24 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{40 d^2 x^{8/3}} + \frac{3 b^2 e^3 n^2}{40 d^3 x^2} - \frac{47 b^2 e^4 n^2}{240 d^4 x^{4/3}} + \frac{77 b^2 e^5 n^2}{120 d^5 x^{2/3}} - \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e x^{2/3}]}{120 d^6} \\
& - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{10 d x^{10/3}} + \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{8 d^2 x^{8/3}} - \\
& - \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{6 d^3 x^2} + \frac{b e^4 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{4 d^4 x^{4/3}} - \\
& - \frac{b e^5 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^6 x^{2/3}} - \frac{b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{2/3}}\right] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^6} \\
& + \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 x^4} + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} + \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{2/3}}\right]}{2 d^6}
\end{aligned}$$

Result (type 4, 436 leaves, 26 steps):

$$\begin{aligned}
& -\frac{b^2 e^2 n^2}{40 d^2 x^{8/3}} + \frac{3 b^2 e^3 n^2}{40 d^3 x^2} - \frac{47 b^2 e^4 n^2}{240 d^4 x^{4/3}} + \frac{77 b^2 e^5 n^2}{120 d^5 x^{2/3}} - \frac{77 b^2 e^6 n^2 \operatorname{Log}[d + e x^{2/3}]}{120 d^6} \\
& - \frac{b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{10 d x^{10/3}} + \frac{b e^2 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{8 d^2 x^{8/3}} - \\
& - \frac{b e^3 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{6 d^3 x^2} + \frac{b e^4 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{4 d^4 x^{4/3}} - \\
& - \frac{b e^5 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^6 x^{2/3}} + \frac{e^6 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 d^6} \\
& - \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 x^4} - \frac{b e^6 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{Log}\left[-\frac{e x^{2/3}}{d}\right]}{2 d^6} + \\
& + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} - \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{2/3}}{d}\right]}{2 d^6}
\end{aligned}$$

Problem 484: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^3}{x^3} dx$$

Optimal (type 4, 451 leaves, 17 steps):

$$\begin{aligned}
& - \frac{3 b^2 e^2 n^2 (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^3 x^{2/3}} - \\
& \frac{3 b^2 e^3 n^2 \operatorname{Log}\left[1 - \frac{d}{d + e x^{2/3}}\right] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^3} - \\
& \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 d x^{4/3}} + \frac{3 b e^2 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 d^3 x^{2/3}} + \\
& \frac{3 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + e x^{2/3}}\right] (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 d^3} - \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^3}{2 x^2} - \\
& \frac{3 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{Log}\left[-\frac{e x^{2/3}}{d}\right]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} + \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{2/3}}\right]}{2 d^3} - \frac{3 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{PolyLog}\left[2, \frac{d}{d + e x^{2/3}}\right]}{d^3} - \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{2/3}}{d}\right]}{d^3} - \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + e x^{2/3}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 428 leaves, 22 steps):

$$\begin{aligned}
& - \frac{3 b^2 e^2 n^2 (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])}{2 d^3 x^{2/3}} + \\
& \frac{3 b e^3 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 d^3} - \frac{3 b e n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{4 d x^{4/3}} + \\
& \frac{3 b e^2 n (d + e x^{2/3}) (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2}{2 d^3 x^{2/3}} - \frac{e^3 (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^3}{2 d^3} - \\
& \frac{(a + b \operatorname{Log}[c (d + e x^{2/3})^n])^3}{2 x^2} - \frac{9 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{Log}\left[-\frac{e x^{2/3}}{d}\right]}{2 d^3} + \\
& \frac{3 b e^3 n (a + b \operatorname{Log}[c (d + e x^{2/3})^n])^2 \operatorname{Log}\left[-\frac{e x^{2/3}}{d}\right]}{2 d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} - \frac{9 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e x^{2/3}}{d}\right]}{2 d^3} + \\
& \frac{3 b^2 e^3 n^2 (a + b \operatorname{Log}[c (d + e x^{2/3})^n]) \operatorname{PolyLog}\left[2, 1 + \frac{e x^{2/3}}{d}\right]}{d^3} - \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e x^{2/3}}{d}\right]}{d^3}
\end{aligned}$$

**Problem 497: Result valid but suboptimal antiderivative.**

$$\int x^2 \left( a + b \operatorname{Log}\left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 572 leaves, 36 steps):



$$\begin{aligned}
& \frac{481 b^2 e^8 n^2 x^{1/3}}{420 d^8} - \frac{341 b^2 e^7 n^2 x^{2/3}}{840 d^7} + \frac{743 b^2 e^6 n^2 x}{3780 d^6} - \frac{533 b^2 e^5 n^2 x^{4/3}}{5040 d^5} + \frac{73 b^2 e^4 n^2 x^{5/3}}{1260 d^4} - \frac{5 b^2 e^3 n^2 x^2}{168 d^3} + \\
& \frac{b^2 e^2 n^2 x^{7/3}}{84 d^2} - \frac{481 b^2 e^9 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{420 d^9} - \frac{2 b e^8 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 d^9} + \\
& \frac{b e^7 n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 d^7} - \frac{2 b e^6 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{9 d^6} + \\
& \frac{b e^5 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{6 d^5} - \frac{2 b e^4 n x^{5/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{15 d^4} + \\
& \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{9 d^3} - \frac{2 b e^2 n x^{7/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{21 d^2} + \\
& \frac{b e n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 d} - \frac{2 b e^9 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 d^9} + \\
& \frac{\frac{1}{3} x^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{3} - \frac{761 b^2 e^9 n^2 \operatorname{Log}[x]}{1260 d^9} + \frac{2 b^2 e^9 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{3 d^9}
\end{aligned}$$

Result (type 4, 596 leaves, 38 steps):

$$\begin{aligned}
& \frac{481 b^2 e^8 n^2 x^{1/3}}{420 d^8} - \frac{341 b^2 e^7 n^2 x^{2/3}}{840 d^7} + \frac{743 b^2 e^6 n^2 x}{3780 d^6} - \frac{533 b^2 e^5 n^2 x^{4/3}}{5040 d^5} + \\
& \frac{73 b^2 e^4 n^2 x^{5/3}}{1260 d^4} - \frac{5 b^2 e^3 n^2 x^2}{168 d^3} + \frac{b^2 e^2 n^2 x^{7/3}}{84 d^2} - \frac{481 b^2 e^9 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{420 d^9} - \\
& \frac{2 b e^8 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 d^9} + \frac{b e^7 n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 d^7} - \\
& \frac{2 b e^6 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{9 d^6} + \frac{b e^5 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{6 d^5} - \\
& \frac{2 b e^4 n x^{5/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{15 d^4} + \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{9 d^3} - \\
& \frac{2 b e^2 n x^{7/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{21 d^2} + \frac{b e n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{12 d} + \\
& \frac{e^9 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{3 d^9} + \frac{\frac{1}{3} x^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{3} - \\
& \frac{2 b e^9 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{3 d^9} - \frac{761 b^2 e^9 n^2 \operatorname{Log}[x]}{1260 d^9} - \frac{2 b^2 e^9 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{3 d^9}
\end{aligned}$$

### Problem 498: Result valid but suboptimal antiderivative.

$$\int x \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 400 leaves, 24 steps):

$$\begin{aligned} & -\frac{77 b^2 e^5 n^2 x^{1/3}}{60 d^5} + \frac{47 b^2 e^4 n^2 x^{2/3}}{120 d^4} - \frac{3 b^2 e^3 n^2 x}{20 d^3} + \frac{b^2 e^2 n^2 x^{4/3}}{20 d^2} + \frac{77 b^2 e^6 n^2 \operatorname{Log} \left[ d + \frac{e}{x^{1/3}} \right]}{60 d^6} + \\ & \frac{b e^5 n \left( d + \frac{e}{x^{1/3}} \right) x^{1/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{d^6} - \frac{b e^4 n x^{2/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{2 d^4} + \\ & \frac{b e^3 n x \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{3 d^3} - \frac{b e^2 n x^{4/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{4 d^2} + \\ & \frac{b e n x^{5/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{5 d} + \frac{b e^6 n \operatorname{Log} \left[ 1 - \frac{d}{d + \frac{e}{x^{1/3}}} \right] \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{d^6} + \\ & \frac{1}{2} x^2 \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 + \frac{137 b^2 e^6 n^2 \operatorname{Log} [x]}{180 d^6} - \frac{b^2 e^6 n^2 \operatorname{PolyLog} \left[ 2, \frac{d}{d + \frac{e}{x^{1/3}}} \right]}{d^6} \end{aligned}$$

Result (type 4, 423 leaves, 26 steps):

$$\begin{aligned} & -\frac{77 b^2 e^5 n^2 x^{1/3}}{60 d^5} + \frac{47 b^2 e^4 n^2 x^{2/3}}{120 d^4} - \frac{3 b^2 e^3 n^2 x}{20 d^3} + \frac{b^2 e^2 n^2 x^{4/3}}{20 d^2} + \\ & \frac{77 b^2 e^6 n^2 \operatorname{Log} \left[ d + \frac{e}{x^{1/3}} \right]}{60 d^6} + \frac{b e^5 n \left( d + \frac{e}{x^{1/3}} \right) x^{1/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{d^6} - \\ & \frac{b e^4 n x^{2/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{2 d^4} + \frac{b e^3 n x \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{3 d^3} - \\ & \frac{b e^2 n x^{4/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{4 d^2} + \frac{b e n x^{5/3} \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)}{5 d} - \\ & \frac{e^6 \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2}{2 d^6} + \frac{1}{2} x^2 \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 + \\ & \frac{b e^6 n \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right) \operatorname{Log} \left[ -\frac{e}{d x^{1/3}} \right]}{d^6} + \frac{137 b^2 e^6 n^2 \operatorname{Log} [x]}{180 d^6} + \frac{b^2 e^6 n^2 \operatorname{PolyLog} \left[ 2, 1 + \frac{e}{d x^{1/3}} \right]}{d^6} \end{aligned}$$

### Problem 499: Result valid but suboptimal antiderivative.

$$\int \left( a + b \operatorname{Log} \left[ c \left( d + \frac{e}{x^{1/3}} \right)^n \right] \right)^2 dx$$

Optimal (type 4, 227 leaves, 13 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 x^{1/3}}{d^2} - \frac{b^2 e^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{d^3} - \frac{2 b e^2 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} + \\
& \frac{b e n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d} - \frac{2 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} + \\
& x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2 - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} + \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 248 leaves, 15 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 x^{1/3}}{d^2} - \frac{b^2 e^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{d^3} - \frac{2 b e^2 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} + \\
& \frac{b e n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d} + \frac{e^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{d^3} + x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2 - \\
& \frac{2 b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{d^3} - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} - \frac{2 b^2 e^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^3}
\end{aligned}$$

**Problem 501: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{x^2} dx$$

Optimal (type 3, 269 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{1/3}}\right)^2}{2 e^3} - \frac{2 b^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^3}{9 e^3} - \frac{6 b^2 d^2 n^2}{e^2 x^{1/3}} + \\
& \frac{b^2 d^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]^2}{e^3} + \frac{6 b d^2 n \left(d + \frac{e}{x^{1/3}}\right) \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \\
& \frac{3 b d n \left(d + \frac{e}{x^{1/3}}\right)^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} + \frac{2 b n \left(d + \frac{e}{x^{1/3}}\right)^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{3 e^3} - \\
& \frac{2 b d^3 n \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{e^3} - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{x}
\end{aligned}$$

Result (type 3, 212 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{1/3}}\right)^2}{2 e^3} - \frac{2 b^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^3}{9 e^3} - \frac{6 b^2 d^2 n^2}{e^2 x^{1/3}} + \frac{b^2 d^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]^2}{e^3} + \\
& \frac{1}{3} b n \left( \frac{18 d^2 \left(d + \frac{e}{x^{1/3}}\right)}{e^3} - \frac{9 d \left(d + \frac{e}{x^{1/3}}\right)^2}{e^3} + \frac{2 \left(d + \frac{e}{x^{1/3}}\right)^3}{e^3} - \frac{6 d^3 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{e^3} \right) \\
& \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right) - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)^2}{x}
\end{aligned}$$

**Problem 502: Result valid but suboptimal antiderivative.**

$$\int \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)^2}{x^3} dx$$

Optimal (type 3, 479 leaves, 8 steps):

$$\begin{aligned}
& -\frac{15 b^2 d^4 n^2 \left(d + \frac{e}{x^{1/3}}\right)^2}{4 e^6} + \frac{20 b^2 d^3 n^2 \left(d + \frac{e}{x^{1/3}}\right)^3}{9 e^6} - \frac{15 b^2 d^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^4}{16 e^6} + \\
& \frac{6 b^2 d n^2 \left(d + \frac{e}{x^{1/3}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^6}{36 e^6} + \frac{6 b^2 d^5 n^2}{e^5 x^{1/3}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]^2}{2 e^6} - \\
& \frac{6 b d^5 n \left(d + \frac{e}{x^{1/3}}\right) \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{e^6} + \frac{15 b d^4 n \left(d + \frac{e}{x^{1/3}}\right)^2 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{2 e^6} - \\
& \frac{20 b d^3 n \left(d + \frac{e}{x^{1/3}}\right)^3 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{3 e^6} + \frac{15 b d^2 n \left(d + \frac{e}{x^{1/3}}\right)^4 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{4 e^6} - \\
& \frac{6 b d n \left(d + \frac{e}{x^{1/3}}\right)^5 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{5 e^6} + \frac{b n \left(d + \frac{e}{x^{1/3}}\right)^6 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{6 e^6} + \\
& \frac{b d^6 n \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right] \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)}{e^6} - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)^2}{2 x^2}
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& - \frac{15 b^2 d^4 n^2 \left(d + \frac{e}{x^{1/3}}\right)^2}{4 e^6} + \frac{20 b^2 d^3 n^2 \left(d + \frac{e}{x^{1/3}}\right)^3}{9 e^6} - \frac{15 b^2 d^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^4}{16 e^6} + \\
& \frac{6 b^2 d n^2 \left(d + \frac{e}{x^{1/3}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{x^{1/3}}\right)^6}{36 e^6} + \frac{6 b^2 d^5 n^2}{e^5 x^{1/3}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]^2}{2 e^6} - \\
& \frac{1}{60} b n \left( \frac{360 d^5 \left(d + \frac{e}{x^{1/3}}\right)}{e^6} - \frac{450 d^4 \left(d + \frac{e}{x^{1/3}}\right)^2}{e^6} + \frac{400 d^3 \left(d + \frac{e}{x^{1/3}}\right)^3}{e^6} - \frac{225 d^2 \left(d + \frac{e}{x^{1/3}}\right)^4}{e^6} + \frac{72 d \left(d + \frac{e}{x^{1/3}}\right)^5}{e^6} - \right. \\
& \left. \frac{10 \left(d + \frac{e}{x^{1/3}}\right)^6}{e^6} - \frac{60 d^6 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{e^6} \right) \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right) - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)^2}{2 x^2}
\end{aligned}$$

**Problem 503: Result valid but suboptimal antiderivative.**

$$\int x \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{1/3}}\right)^n \right] \right)^3 dx$$

Optimal (type 4, 759 leaves, 62 steps):

$$\begin{aligned}
& \frac{71 b^3 e^5 n^3 x^{1/3}}{40 d^5} - \frac{3 b^3 e^4 n^3 x^{2/3}}{10 d^4} + \frac{b^3 e^3 n^3 x}{20 d^3} - \\
& \frac{71 b^3 e^6 n^3 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{40 d^6} - \frac{77 b^2 e^5 n^2 \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^6} + \\
& \frac{47 b^2 e^4 n^2 x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{40 d^4} - \frac{9 b^2 e^3 n^2 x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^3} + \\
& \frac{3 b^2 e^2 n^2 x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^2} - \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^6} + \\
& \frac{3 b e^5 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^6} - \\
& \frac{3 b e^4 n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{4 d^4} + \frac{b e^3 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^3} - \\
& \frac{3 b e^2 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{8 d^2} + \frac{3 b e n x^{5/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{10 d} + \\
& \frac{3 b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^6} + \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 - \\
& \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{d^6} - \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} + \\
& \frac{77 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{20 d^6} - \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^6} - \\
& \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^6} - \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^6}
\end{aligned}$$

Result (type 4, 736 leaves, 73 steps):

$$\begin{aligned}
& \frac{71 b^3 e^5 n^3 x^{1/3}}{40 d^5} - \frac{3 b^3 e^4 n^3 x^{2/3}}{10 d^4} + \frac{b^3 e^3 n^3 x}{20 d^3} - \frac{71 b^3 e^6 n^3 \operatorname{Log}\left[d + \frac{e}{x^{1/3}}\right]}{40 d^6} - \\
& \frac{77 b^2 e^5 n^2 \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^6} + \frac{47 b^2 e^4 n^2 x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{40 d^4} - \\
& \frac{9 b^2 e^3 n^2 x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^3} + \frac{3 b^2 e^2 n^2 x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{20 d^2} + \\
& \frac{77 b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{40 d^6} + \frac{3 b e^5 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^6} - \\
& \frac{3 b e^4 n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{4 d^4} + \frac{b e^3 n x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^3} - \\
& \frac{3 b e^2 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{8 d^2} + \frac{3 b e n x^{5/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{10 d} - \\
& \frac{e^6 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3}{2 d^6} + \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 - \\
& \frac{137 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{20 d^6} + \frac{3 b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{2 d^6} - \\
& \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} - \frac{137 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{20 d^6} + \\
& \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^6} - \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d x^{1/3}}\right]}{d^6}
\end{aligned}$$

**Problem 504: Result valid but suboptimal antiderivative.**

$$\int \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 dx$$

Optimal (type 4, 436 leaves, 18 steps):

$$\begin{aligned}
& \frac{3 b^2 e^2 n^2 \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} + \frac{3 b^2 e^3 n^2 \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} - \\
& \frac{3 b e^2 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{d^3} + \frac{3 b e n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d} - \\
& \frac{3 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{1/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{d^3} + x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 + \\
& \frac{6 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} - \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^3} + \frac{6 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^3} + \\
& \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^3} + \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{x^{1/3}}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 410 leaves, 23 steps):

$$\begin{aligned}
& \frac{3 b^2 e^2 n^2 \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)}{d^3} - \\
& \frac{3 b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d^3} - \frac{3 b e^2 n \left(d + \frac{e}{x^{1/3}}\right) x^{1/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{d^3} + \\
& \frac{3 b e n x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2}{2 d} + \frac{e^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3}{d^3} + \\
& x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^3 + \frac{9 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{d^3} - \\
& \frac{3 b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d x^{1/3}}\right]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} + \frac{9 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^3} - \\
& \frac{6 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{1/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{1/3}}\right]}{d^3} + \frac{6 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d x^{1/3}}\right]}{d^3}
\end{aligned}$$

**Problem 516: Result valid but suboptimal antiderivative.**

$$\int x^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 412 leaves, 24 steps):



$$\begin{aligned}
& -\frac{77 b^2 e^5 n^2 x^{2/3}}{120 d^5} + \frac{47 b^2 e^4 n^2 x^{4/3}}{240 d^4} - \frac{3 b^2 e^3 n^2 x^2}{40 d^3} + \frac{b^2 e^2 n^2 x^{8/3}}{40 d^2} + \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{120 d^6} + \\
& \frac{b e^5 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^6} - \frac{b e^4 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{4 d^4} + \\
& \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{6 d^3} - \frac{b e^2 n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{8 d^2} + \\
& \frac{b e n x^{10/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{10 d} + \frac{b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^6} + \\
& \frac{1}{4} x^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} - \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{2 d^6}
\end{aligned}$$

Result (type 4, 436 leaves, 26 steps):

$$\begin{aligned}
& -\frac{77 b^2 e^5 n^2 x^{2/3}}{120 d^5} + \frac{47 b^2 e^4 n^2 x^{4/3}}{240 d^4} - \frac{3 b^2 e^3 n^2 x^2}{40 d^3} + \frac{b^2 e^2 n^2 x^{8/3}}{40 d^2} + \\
& \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{120 d^6} + \frac{b e^5 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^6} - \\
& \frac{b e^4 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{4 d^4} + \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{6 d^3} - \\
& \frac{b e^2 n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{8 d^2} + \frac{b e n x^{10/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{10 d} - \\
& \frac{e^6 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^6} + \frac{1}{4} x^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 + \\
& \frac{b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{2 d^6} + \frac{137 b^2 e^6 n^2 \operatorname{Log}[x]}{180 d^6} + \frac{b^2 e^6 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{2 d^6}
\end{aligned}$$

**Problem 517: Result valid but suboptimal antiderivative.**

$$\int x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 dx$$

Optimal (type 4, 239 leaves, 12 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 x^{2/3}}{2 d^2} - \frac{b^2 e^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{2 d^3} - \frac{b e^2 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{d^3} + \\
& \frac{b e n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d} - \frac{b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{d^3} + \\
& \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 - \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} + \frac{b^2 e^3 n^2 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3}
\end{aligned}$$

Result (type 4, 264 leaves, 14 steps):

$$\begin{aligned}
& \frac{b^2 e^2 n^2 x^{2/3}}{2 d^2} - \frac{b^2 e^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{2 d^3} - \frac{b e^2 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{d^3} + \\
& \frac{b e n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d} + \frac{e^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 d^3} + \\
& \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 - \frac{b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{d^3} - \\
& \frac{b^2 e^3 n^2 \operatorname{Log}[x]}{d^3} - \frac{b^2 e^3 n^2 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{d^3}
\end{aligned}$$

**Problem 519: Result valid but suboptimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{x^3} dx$$

Optimal (type 3, 276 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{2/3}}\right)^2}{4 e^3} - \frac{b^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^3}{9 e^3} - \frac{3 b^2 d^2 n^2}{e^2 x^{2/3}} + \\
& \frac{b^2 d^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]^2}{2 e^3} + \frac{3 b d^2 n \left(d + \frac{e}{x^{2/3}}\right) \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{e^3} - \\
& \frac{3 b d n \left(d + \frac{e}{x^{2/3}}\right)^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 e^3} + \frac{b n \left(d + \frac{e}{x^{2/3}}\right)^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{3 e^3} - \\
& \frac{b d^3 n \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{e^3} - \frac{\left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 x^2}
\end{aligned}$$

Result (type 3, 217 leaves, 8 steps):

$$\begin{aligned}
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{2/3}}\right)^2}{4 e^3} - \frac{b^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^3}{9 e^3} - \frac{3 b^2 d^2 n^2}{e^2 x^{2/3}} + \frac{b^2 d^3 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]^2}{2 e^3} + \\
& \frac{1}{6} b n \left( \frac{18 d^2 \left(d + \frac{e}{x^{2/3}}\right)}{e^3} - \frac{9 d \left(d + \frac{e}{x^{2/3}}\right)^2}{e^3} + \frac{2 \left(d + \frac{e}{x^{2/3}}\right)^3}{e^3} - \frac{6 d^3 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{e^3} \right) \\
& \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right) - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)^2}{2 x^2}
\end{aligned}$$

**Problem 520: Result valid but suboptimal antiderivative.**

$$\int \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)^2}{x^5} dx$$

Optimal (type 3, 482 leaves, 8 steps):

$$\begin{aligned}
& -\frac{15 b^2 d^4 n^2 \left(d + \frac{e}{x^{2/3}}\right)^2}{8 e^6} + \frac{10 b^2 d^3 n^2 \left(d + \frac{e}{x^{2/3}}\right)^3}{9 e^6} - \frac{15 b^2 d^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^4}{32 e^6} + \\
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{2/3}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^6}{72 e^6} + \frac{3 b^2 d^5 n^2}{e^5 x^{2/3}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]^2}{4 e^6} - \\
& \frac{3 b d^5 n \left(d + \frac{e}{x^{2/3}}\right) \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{e^6} + \frac{15 b d^4 n \left(d + \frac{e}{x^{2/3}}\right)^2 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{4 e^6} - \\
& \frac{10 b d^3 n \left(d + \frac{e}{x^{2/3}}\right)^3 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{3 e^6} + \frac{15 b d^2 n \left(d + \frac{e}{x^{2/3}}\right)^4 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{8 e^6} - \\
& \frac{3 b d n \left(d + \frac{e}{x^{2/3}}\right)^5 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{5 e^6} + \frac{b n \left(d + \frac{e}{x^{2/3}}\right)^6 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{12 e^6} + \\
& \frac{b d^6 n \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right] \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)}{2 e^6} - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)^2}{4 x^4}
\end{aligned}$$

Result (type 3, 355 leaves, 8 steps):

$$\begin{aligned}
& - \frac{15 b^2 d^4 n^2 \left(d + \frac{e}{x^{2/3}}\right)^2}{8 e^6} + \frac{10 b^2 d^3 n^2 \left(d + \frac{e}{x^{2/3}}\right)^3}{9 e^6} - \frac{15 b^2 d^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^4}{32 e^6} + \\
& \frac{3 b^2 d n^2 \left(d + \frac{e}{x^{2/3}}\right)^5}{25 e^6} - \frac{b^2 n^2 \left(d + \frac{e}{x^{2/3}}\right)^6}{72 e^6} + \frac{3 b^2 d^5 n^2}{e^5 x^{2/3}} - \frac{b^2 d^6 n^2 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]^2}{4 e^6} - \frac{1}{120} b n \\
& \left( \frac{360 d^5 \left(d + \frac{e}{x^{2/3}}\right)}{e^6} - \frac{450 d^4 \left(d + \frac{e}{x^{2/3}}\right)^2}{e^6} + \frac{400 d^3 \left(d + \frac{e}{x^{2/3}}\right)^3}{e^6} - \frac{225 d^2 \left(d + \frac{e}{x^{2/3}}\right)^4}{e^6} + \frac{72 d \left(d + \frac{e}{x^{2/3}}\right)^5}{e^6} - \right. \\
& \left. \frac{10 \left(d + \frac{e}{x^{2/3}}\right)^6}{e^6} - \frac{60 d^6 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{e^6} \right) \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right) - \frac{\left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)^2}{4 x^4}
\end{aligned}$$

**Problem 524: Result valid but suboptimal antiderivative.**

$$\int x^3 \left( a + b \operatorname{Log}\left[ c \left(d + \frac{e}{x^{2/3}}\right)^n \right] \right)^3 dx$$

Optimal (type 4, 773 leaves, 62 steps):

$$\begin{aligned}
& \frac{71 b^3 e^5 n^3 x^{2/3}}{80 d^5} - \frac{3 b^3 e^4 n^3 x^{4/3}}{20 d^4} + \frac{b^3 e^3 n^3 x^2}{40 d^3} - \\
& \frac{71 b^3 e^6 n^3 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{80 d^6} - \frac{77 b^2 e^5 n^2 \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^6} + \\
& \frac{47 b^2 e^4 n^2 x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{80 d^4} - \frac{9 b^2 e^3 n^2 x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^3} + \\
& \frac{3 b^2 e^2 n^2 x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^2} - \frac{77 b^2 e^6 n^2 \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^6} + \\
& \frac{3 b e^5 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^6} - \\
& \frac{3 b e^4 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{8 d^4} + \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^3} - \\
& \frac{3 b e^2 n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{16 d^2} + \frac{3 b e n x^{10/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{20 d} + \\
& \frac{3 b e^6 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^6} + \frac{1}{4} x^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3 - \\
& \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{2 d^6} - \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} + \\
& \frac{77 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{40 d^6} - \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{2 d^6} - \\
& \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{2 d^6} - \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{2 d^6}
\end{aligned}$$

Result (type 4, 746 leaves, 73 steps):

$$\begin{aligned}
& \frac{71 b^3 e^5 n^3 x^{2/3}}{80 d^5} - \frac{3 b^3 e^4 n^3 x^{4/3}}{20 d^4} + \frac{b^3 e^3 n^3 x^2}{40 d^3} - \frac{71 b^3 e^6 n^3 \operatorname{Log}\left[d + \frac{e}{x^{2/3}}\right]}{80 d^6} - \\
& \frac{77 b^2 e^5 n^2 \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^6} + \frac{47 b^2 e^4 n^2 x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{80 d^4} - \\
& \frac{9 b^2 e^3 n^2 x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^3} + \frac{3 b^2 e^2 n^2 x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{40 d^2} + \\
& \frac{77 b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{80 d^6} + \frac{3 b e^5 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^6} - \\
& \frac{3 b e^4 n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{8 d^4} + \frac{b e^3 n x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^3} - \\
& \frac{3 b e^2 n x^{8/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{16 d^2} + \frac{3 b e n x^{10/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{20 d} - \\
& \frac{e^6 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3}{4 d^6} + \frac{1}{4} x^4 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3 - \\
& \frac{137 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{40 d^6} + \frac{3 b e^6 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{4 d^6} - \\
& \frac{15 b^3 e^6 n^3 \operatorname{Log}[x]}{8 d^6} - \frac{137 b^3 e^6 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{40 d^6} + \\
& \frac{3 b^2 e^6 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{2 d^6} - \frac{3 b^3 e^6 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d x^{2/3}}\right]}{2 d^6}
\end{aligned}$$

**Problem 525: Result valid but suboptimal antiderivative.**

$$\int x \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3 dx$$

Optimal (type 4, 451 leaves, 17 steps):

$$\begin{aligned}
& \frac{3 b^2 e^2 n^2 \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^3} + \frac{3 b^2 e^3 n^2 \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^3} - \\
& \frac{3 b e^2 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 d^3} + \frac{3 b e n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d} - \\
& \frac{3 b e^3 n \operatorname{Log}\left[1 - \frac{d}{d + \frac{e}{x^{2/3}}}\right] \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 d^3} + \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3 + \\
& \frac{3 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} - \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{2 d^3} + \frac{3 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3} + \\
& \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{d^3} + \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, \frac{d}{d + \frac{e}{x^{2/3}}}\right]}{d^3}
\end{aligned}$$

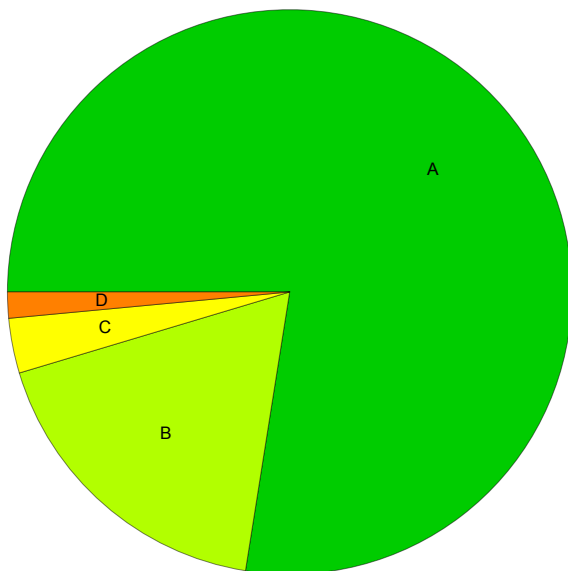
Result (type 4, 428 leaves, 22 steps):

$$\begin{aligned}
& \frac{3 b^2 e^2 n^2 \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)}{2 d^3} - \\
& \frac{3 b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d^3} - \frac{3 b e^2 n \left(d + \frac{e}{x^{2/3}}\right) x^{2/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{2 d^3} + \\
& \frac{3 b e n x^{4/3} \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2}{4 d} + \frac{e^3 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3}{2 d^3} + \\
& \frac{1}{2} x^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^3 + \frac{9 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{2 d^3} - \\
& \frac{3 b e^3 n \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right)^2 \operatorname{Log}\left[-\frac{e}{d x^{2/3}}\right]}{2 d^3} + \frac{b^3 e^3 n^3 \operatorname{Log}[x]}{d^3} + \frac{9 b^3 e^3 n^3 \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{2 d^3} - \\
& \frac{3 b^2 e^3 n^2 \left(a + b \operatorname{Log}\left[c \left(d + \frac{e}{x^{2/3}}\right)^n\right]\right) \operatorname{PolyLog}\left[2, 1 + \frac{e}{d x^{2/3}}\right]}{d^3} + \frac{3 b^3 e^3 n^3 \operatorname{PolyLog}\left[3, 1 + \frac{e}{d x^{2/3}}\right]}{d^3}
\end{aligned}$$

Test results for the 314 problems in "3.5 Logarithm functions.m"

## Summary of Integration Test Results

3085 integration problems



A - 2391 optimal antiderivatives

B - 551 valid but suboptimal antiderivatives

C - 97 unnecessarily complex antiderivatives

D - 46 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives