## Mathematica 11.3 Integration Test Results

# Test results for the 156 problems in "7.1.2 (d x)^m (a+b arcsinh(c x))^n.m"

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int\!\frac{\text{ArcSinh}\,[\,a\,x\,]^{\,4}}{x^3}\,\text{d}x$$

Optimal (type 4, 108 leaves, 8 steps):

$$\begin{split} &-2 \, a^2 \, \text{ArcSinh} \, [\, a \, x \, ]^{\, 3} \, - \, \frac{2 \, a \, \sqrt{1 + a^2 \, x^2} \, \, \text{ArcSinh} \, [\, a \, x \, ]^{\, 3}}{x} \, - \\ &- \, \frac{\text{ArcSinh} \, [\, a \, x \, ]^{\, 4}}{2 \, x^2} \, + \, 6 \, a^2 \, \, \text{ArcSinh} \, [\, a \, x \, ]^{\, 2} \, \text{Log} \left[ 1 \, - \, e^{2 \, \text{ArcSinh} \, [\, a \, x \, ]} \, \right] \, + \\ &- \, 6 \, a^2 \, \, \text{ArcSinh} \, [\, a \, x \, ] \, \, \text{PolyLog} \left[ 2 \, , \, e^{2 \, \text{ArcSinh} \, [\, a \, x \, ]} \, \right] \, - \, 3 \, a^2 \, \, \text{PolyLog} \left[ 3 \, , \, e^{2 \, \text{ArcSinh} \, [\, a \, x \, ]} \, \right] \end{split}$$

#### Result (type 4, 113 leaves):

$$-\frac{\text{ArcSinh}\left[a\:x\right]^{4}}{2\:x^{2}} + \\ \frac{1}{4}\:a^{2}\left(i\:\pi^{3} - 8\:\text{ArcSinh}\left[a\:x\right]^{3} - \frac{8\:\sqrt{1 + a^{2}\:x^{2}}\:\text{ArcSinh}\left[a\:x\right]^{3}}{a\:x} + 24\:\text{ArcSinh}\left[a\:x\right]^{2}\:\text{Log}\left[1 - e^{2\:\text{ArcSinh}\left[a\:x\right]}\right] + \\ 24\:\text{ArcSinh}\left[a\:x\right]\:\text{PolyLog}\left[2,\ e^{2\:\text{ArcSinh}\left[a\:x\right]}\right] - 12\:\text{PolyLog}\left[3,\ e^{2\:\text{ArcSinh}\left[a\:x\right]}\right]\right)$$

### Problem 119: Unable to integrate problem.

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\int x^m \operatorname{ArcSinh}[ax]^2 dx
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Optimal (type 5, 137 leaves, 2 steps):

$$\frac{x^{1+m} \, \text{ArcSinh} \, [\, a \, x \, ]^{\, 2}}{1 + m} \, - \, \frac{2 \, a \, x^{2+m} \, \text{ArcSinh} \, [\, a \, x \, ] \, \, \text{Hypergeometric2F1} \left[ \, \frac{1}{2} \, , \, \, \frac{2+m}{2} \, , \, \, \frac{4+m}{2} \, , \, - \, a^2 \, x^2 \, \right]}{2 + 3 \, m + m^2} \, + \\ \left( 2 \, a^2 \, x^{3+m} \, \, \text{HypergeometricPFQ} \left[ \, \left\{ \, 1 \, , \, \, \frac{3}{2} \, + \, \frac{m}{2} \, , \, \, \frac{3}{2} \, + \, \frac{m}{2} \, \right\} \, , \, \left\{ \, 2 \, + \, \frac{m}{2} \, , \, \, \frac{5}{2} \, + \, \frac{m}{2} \, \right\} \, , \, - \, a^2 \, x^2 \, \right] \right) \, / \, \left( 6 + \, 11 \, m + \, 6 \, m^2 \, + \, m^3 \right) \, d^2 \, d$$

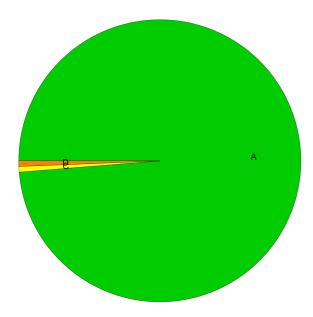
Result (type 9, 133 leaves):

$$\left(4 \operatorname{ArcSinh}\left[a \, x\right] \left(\operatorname{ArcSinh}\left[a \, x\right] - \frac{2 \, a \, x \, \sqrt{1 + a^2 \, x^2} \, \operatorname{Hypergeometric2F1}\left[1, \, \frac{3 + m}{2}, \, \frac{4 + m}{2}, \, -a^2 \, x^2\right]}{2 + m}\right) + 2^{-m} \, a^2 \, a^2 \, a^2 + a^2 \, a^2 \,$$

$$\sqrt{\pi} \ x^2 \ \mathsf{Gamma} \ [2+m] \ \mathsf{HypergeometricPFQRegularized} \left[ \left\{ 1, \ \frac{3+m}{2}, \ \frac{3+m}{2} \right\}, \ \left\{ \frac{4+m}{2}, \ \frac{5+m}{2} \right\}, \ -\mathsf{a}^2 \ x^2 \right] \right]$$

## **Summary of Integration Test Results**

#### 156 integration problems



- A 154 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 1 unnecessarily complex antiderivatives
- D 1 unable to integrate problems
- E 0 integration timeouts