

Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "4 Trig functions"

Test results for the 538 problems in "4.1.0 (a sin)^m (b trig)^n.m"

Test results for the 348 problems in "4.1.10 (c+d x)^m (a+b sin)^n.m"

Test results for the 72 problems in "4.1.1.1 (a+b sin)^n.m"

Test results for the 113 problems in "4.1.11 (e x)^m (a+b x^n)^p sin.m"

Test results for the 357 problems in "4.1.12 (e x)^m (a+b sin(c+d x^n))^p.m"

Test results for the 653 problems in "4.1.1.2 (g cos)^p (a+b sin)^m.m"

Problem 648: Result valid but suboptimal antiderivative.

$$\int \left(e \cos [c + d x] \right)^{-3-m} \left(a + b \sin [c + d x] \right)^m dx$$

Optimal (type 5, 311 leaves, ? steps):

$$\begin{aligned}
& \left((e \cos[c+dx])^{-m} \sec[c+dx]^4 (-1+\sin[c+dx]) (1+\sin[c+dx]) (a+b \sin[c+dx])^{1+m} \right) / \\
& \left((a-b) d e^3 (2+m) + (-2b+a(2+m)) (e \cos[c+dx])^{-m} \sec[c+dx]^4 \right. \\
& \quad \left. (-1+\sin[c+dx]) (1+\sin[c+dx])^2 (a+b \sin[c+dx])^{1+m} \right) / \left((a-b)^2 d e^3 m (2+m) \right) - \\
& \left((-b^2+a^2(1+m)) (e \cos[c+dx])^{-m} \text{Hypergeometric2F1}\left[\frac{m}{2}, 1+m, 2+m, \right. \right. \\
& \quad \left. \left. -\frac{2(a+b \sin[c+dx])}{(a-b)(-1+\sin[c+dx])} \right] \sec[c+dx]^4 (1+\sin[c+dx])^3 \right. \\
& \quad \left. \left(\frac{(a+b)(1+\sin[c+dx])}{(a-b)(-1+\sin[c+dx])} \right)^{\frac{1}{2}(-2+m)} (a+b \sin[c+dx])^{1+m} \right) / \left((a-b)^3 d e^3 m (1+m) \right)
\end{aligned}$$

Result (type 5, 420 leaves, 5 steps):

$$\begin{aligned}
& -\frac{(e \cos[c+dx])^{-2-m} (a+b \sin[c+dx])^{1+m}}{(a-b) d e (2+m)} - \\
& \left(b (e \cos[c+dx])^{-2-m} \text{Hypergeometric2F1}\left[1+m, \frac{2+m}{2}, 2+m, \frac{2(a+b \sin[c+dx])}{(a+b)(1+\sin[c+dx])}\right] \right. \\
& \quad \left. (1-\sin[c+dx]) \left(-\frac{(a-b)(1-\sin[c+dx])}{(a+b)(1+\sin[c+dx])} \right)^{m/2} (a+b \sin[c+dx])^{1+m} \right) / \\
& \left((a^2-b^2) d e (1+m) (2+m) + \frac{a (e \cos[c+dx])^{-2-m} (1+\sin[c+dx]) (a+b \sin[c+dx])^{1+m}}{(a^2-b^2) d e (2+m)} + \right. \\
& \quad \left. 2^{-m/2} a (a+b+am) (e \cos[c+dx])^{-2-m} \right. \\
& \quad \left. \text{Hypergeometric2F1}\left[-\frac{m}{2}, \frac{2+m}{2}, \frac{2-m}{2}, \frac{(a-b)(1-\sin[c+dx])}{2(a+b \sin[c+dx])}\right] (1-\sin[c+dx]) \right. \\
& \quad \left. \left(\frac{(a+b)(1+\sin[c+dx])}{a+b \sin[c+dx]} \right)^{\frac{2+m}{2}} (a+b \sin[c+dx])^{1+m} \right) / \left((a-b) (a+b)^2 d e m (2+m) \right)
\end{aligned}$$

Test results for the 36 problems in "4.1.13 (d+e x)^m sin(a+b x+c x^2)^n.m"

Test results for the 208 problems in "4.1.1.3 (g tan)^p (a+b sin)^m.m"

Test results for the 837 problems in "4.1.2.1 (a+b sin)^m (c+d sin)^n.m"

Test results for the 1563 problems in "4.1.2.2 (g cos)^p (a+b sin)^m

$(c+d \sin)^{n.m}$

Problem 1479: Unable to integrate problem.

$$\int \frac{\sec[e+fx]^2 (a+b \sin[e+fx])^{3/2}}{\sqrt{d \sin[e+fx]}} dx$$

Optimal (type 4, 312 leaves, ? steps):

$$\begin{aligned} & \frac{\sec[e+fx] (b+a \sin[e+fx]) \sqrt{a+b \sin[e+fx]}}{f \sqrt{d \sin[e+fx]}} - \frac{1}{\sqrt{d} f} (a+b)^{3/2} \sqrt{-\frac{a(-1+\csc[e+fx])}{a+b}} \\ & \sqrt{\frac{a(1+\csc[e+fx])}{a-b}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \sin[e+fx]}}{\sqrt{a+b} \sqrt{d \sin[e+fx]}}\right], -\frac{a+b}{a-b}\right] \tan[e+fx] - \\ & \left(b(a+b) \sqrt{-\frac{a(-1+\csc[e+fx])}{a+b}} \sqrt{\frac{b+a \csc[e+fx]}{-a+b}} \right. \\ & \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \csc[e+fx]}{a-b}}\right], \frac{-a+b}{a+b}\right] (1+\sin[e+fx]) \tan[e+fx] \right) / \\ & \left(f \sqrt{\frac{a(1+\csc[e+fx])}{a-b}} \sqrt{d \sin[e+fx]} \sqrt{a+b \sin[e+fx]} \right) \end{aligned}$$

Result (type 8, 37 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sec[e+fx]^2 (a+b \sin[e+fx])^{3/2}}{\sqrt{d \sin[e+fx]}}, x\right]$$

Problem 1480: Unable to integrate problem.

$$\int \frac{\sec[e+fx]^4 (a+b \sin[e+fx])^{5/2}}{\sqrt{d \sin[e+fx]}} dx$$

Optimal (type 4, 366 leaves, ? steps):

$$\begin{aligned}
& \frac{5 a \operatorname{Sec}[e+f x] (b+a \sin [e+f x]) \sqrt{a+b \sin [e+f x]}}{6 f \sqrt{d \sin [e+f x]}} + \\
& \frac{\operatorname{Sec}[e+f x]^3 \sqrt{d \sin [e+f x]} (a+b \sin [e+f x])^{5/2}}{3 d f} - \frac{1}{6 \sqrt{d} f} \\
& 5 a (a+b)^{3/2} \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a+b \sin [e+f x]}}{\sqrt{a+b} \sqrt{d \sin [e+f x]}}\right], -\frac{a+b}{a-b}\right] \tan [e+f x] - \\
& \left(5 a b (a+b) \sqrt{-\frac{a(-1+\operatorname{Csc}[e+f x])}{a+b}} \sqrt{\frac{b+a \operatorname{Csc}[e+f x]}{-a+b}}\right. \\
& \left.\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b+a \operatorname{Csc}[e+f x]}{a-b}}\right], \frac{-a+b}{a+b}\right] (1+\sin [e+f x]) \tan [e+f x]\right) / \\
& \left(6 f \sqrt{\frac{a(1+\operatorname{Csc}[e+f x])}{a-b}} \sqrt{d \sin [e+f x]} \sqrt{a+b \sin [e+f x]}\right)
\end{aligned}$$

Result (type 8, 87 leaves, 1 step):

$$\begin{aligned}
& \frac{\operatorname{Sec}[e+f x]^3 \sqrt{d \sin [e+f x]} (a+b \sin [e+f x])^{5/2}}{3 d f} + \\
& \frac{5}{6} a \operatorname{Unintegrable}\left[\frac{\operatorname{Sec}[e+f x]^2 (a+b \sin [e+f x])^{3/2}}{\sqrt{d \sin [e+f x]}}, x\right]
\end{aligned}$$

Problem 1515: Unable to integrate problem.

$$\int \frac{\operatorname{Sec}[e+f x]^6 (a+b \sin [e+f x])^{9/2}}{\sqrt{d \sin [e+f x]}} dx$$

Optimal (type 4, 502 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 a b (-2 a^2 + b^2) \cos[e + f x] \sqrt{a + b \sin[e + f x]}}{5 f \sqrt{d \sin[e + f x]}} + \\
& \frac{\sec[e + f x]^5 \sqrt{d \sin[e + f x]} (a + b \sin[e + f x])^{9/2}}{5 d f} - \frac{1}{20 d f} \\
& 3 a \sec[e + f x]^3 \sqrt{d \sin[e + f x]} \sqrt{a + b \sin[e + f x]} (-a (7 a^2 + b^2) + \\
& 2 b (-7 a^2 + b^2) \sin[e + f x] + 5 a (a^2 - b^2) \sin[e + f x]^2 + (8 a^2 b - 4 b^3) \sin[e + f x]^3) - \\
& \frac{1}{20 \sqrt{d} f} 3 a (a + b)^{3/2} (5 a^2 + 3 a b - 4 b^2) \sqrt{-\frac{a (-1 + \csc[e + f x])}{a + b}} \sqrt{\frac{a (1 + \csc[e + f x])}{a - b}} \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d} \sqrt{a + b \sin[e + f x]}}{\sqrt{a + b} \sqrt{d \sin[e + f x]}}\right], -\frac{a + b}{a - b}\right] \tan[e + f x] - \\
& \left(3 b (2 a^4 - 3 a^2 b^2 + b^4) \sqrt{-\frac{a (-1 + \csc[e + f x])}{a + b}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{b + a \csc[e + f x]}{a - b}}\right], \right. \right. \\
& \left. \left. 1 - \frac{2 a}{a + b}\right] \sqrt{d \sin[e + f x]} \sqrt{-\frac{a \csc[e + f x]^2 (1 + \sin[e + f x]) (a + b \sin[e + f x])}{(a - b)^2}} \right. \\
& \left. \left. \tan[e + f x] \right) \right] / (5 d f \sqrt{a + b \sin[e + f x]})
\end{aligned}$$

Result (type 8, 87 leaves, 1 step):

$$\begin{aligned}
& \frac{\sec[e + f x]^5 \sqrt{d \sin[e + f x]} (a + b \sin[e + f x])^{9/2}}{5 d f} + \\
& \frac{9}{10} a \operatorname{Unintegrable}\left[\frac{\sec[e + f x]^4 (a + b \sin[e + f x])^{7/2}}{\sqrt{d \sin[e + f x]}}, x\right]
\end{aligned}$$

Test results for the 51 problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

Test results for the 358 problems in "4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin).m"

Test results for the 19 problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

Test results for the 34 problems in "4.1.4.2 (a+b sin)^m (c+d sin)^n

$(A+B \sin+C \sin^2).m''$

Test results for the 594 problems in "4.1.7 (d trig)^m (a+b (c sin)^n)^p.m"

Problem 391: Unable to integrate problem.

$$\int \frac{\sec [c+d x]^2}{a+b \sin [c+d x]^3} d x$$

Optimal (type 3, 299 leaves, ? steps):

$$\frac{2 (-1)^{2/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \left(a^{2/3} - (-1)^{2/3} b^{2/3}\right)^{3/2} d} - \frac{2 b^{2/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \left(a^{2/3} - b^{2/3}\right)^{3/2} d} +$$

$$\frac{2 (-1)^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \left(a^{2/3} + (-1)^{1/3} b^{2/3}\right)^{3/2} d} + \frac{\sec [c+d x] (b - a \sin [c+d x])}{(-a^2 + b^2) d}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sec [c+d x]^2}{a+b \sin [c+d x]^3}, x\right]$$

Problem 392: Unable to integrate problem.

$$\int \frac{\sec [c+d x]^4}{a+b \sin [c+d x]^3} d x$$

Optimal (type 3, 1093 leaves, ? steps):

$$\begin{aligned}
& - \frac{2 (-1)^{2/3} a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{(-1)^{1/3} b^{1/3} - a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - (-1)^{2/3} b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{\sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} + \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} - b^{2/3}}}\right]}{3 \sqrt{a^{2/3} - b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 (-1)^{1/3} a^{2/3} b^{8/3} \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} + \\
& \frac{2 b^2 (2 a^2 + b^2) \operatorname{ArcTan}\left[\frac{(-1)^{2/3} b^{1/3} + a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}}}\right]}{3 a^{2/3} \sqrt{a^{2/3} + (-1)^{1/3} b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b^{1/3} - (-1)^{1/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{-(-1)^{2/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} - \\
& \frac{2 b^{4/3} (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b^{1/3} + (-1)^{2/3} a^{1/3} \operatorname{Tan}\left[\frac{1}{2}(c+dx)\right]}{\sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}}}\right]}{3 \sqrt{(-1)^{1/3} a^{2/3} + b^{2/3}} (a^2 - b^2)^2 d} + \frac{\cos[c+dx]}{12 (a+b) d (1 - \sin[c+dx])^2} + \\
& \frac{\cos[c+dx]}{12 (a+b) d (1 - \sin[c+dx])} + \frac{(a+4b) \cos[c+dx]}{4 (a+b)^2 d (1 - \sin[c+dx])} - \\
& \frac{\cos[c+dx]}{12 (a-b) d (1 + \sin[c+dx])^2} - \frac{(a-4b) \cos[c+dx]}{4 (a-b)^2 d (1 + \sin[c+dx])} - \frac{\cos[c+dx]}{12 (a-b) d (1 + \sin[c+dx])}
\end{aligned}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\sec[c+dx]^4}{a+b \sin[c+dx]^3}, x\right]$$

Test results for the 9 problems in "4.1.8 (a+b sin)^m (c+d trig)^n.m"

Test results for the 19 problems in "4.1.9 trig^m (a+b sin^n+c sin^(2n))^p.m"

Test results for the 294 problems in "4.2.0 $(a \cos)^m (b \operatorname{trg})^n.m$ "

Test results for the 189 problems in "4.2.10 $(c+d x)^m (a+b \cos)^n.m$ "

Test results for the 62 problems in "4.2.1.1 $(a+b \cos)^n.m$ "

Test results for the 99 problems in "4.2.12 $(e x)^m (a+b \cos(c+d x^n))^p.m$ "

Test results for the 88 problems in "4.2.1.2 $(g \sin)^p (a+b \cos)^m.m$ "

Test results for the 34 problems in "4.2.13 $(d+e x)^m \cos(a+b x+c x^2)^n.m$ "

Test results for the 22 problems in "4.2.1.3 $(g \tan)^p (a+b \cos)^m.m$ "

Test results for the 932 problems in "4.2.2.1 $(a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 4 problems in "4.2.2.2 $(g \sin)^p (a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 1 problems in "4.2.2.3 $(g \cos)^p (a+b \cos)^m (c+d \cos)^n.m$ "

Test results for the 644 problems in "4.2.3.1 $(a+b \cos)^m (c+d \cos)^n (A+B \cos).m$ "

Test results for the 393 problems in "4.2.4.1 $(a+b \cos)^m (A+B \cos+C \cos^2).m$ "

Test results for the 1541 problems in "4.2.4.2 $(a+b \cos)^m (c+d \cos)^n (A+B \cos+C \cos^2).m$ "

Test results for the 98 problems in "4.2.7 (d trig)^m (a+b (c cos)^n)^p.m"

Test results for the 21 problems in "4.2.8 (a+b cos)^m (c+d trig)^n.m"

Test results for the 20 problems in "4.2.9 trig^m (a+b cos^n+c cos^(2 n))^p.m"

Test results for the 387 problems in "4.3.0 (a trig)^m (b tan)^n.m"

Test results for the 63 problems in "4.3.10 (c+d x)^m (a+b tan)^n.m"

Problem 17: Unable to integrate problem.

$$\int \left(\frac{x^2}{\sqrt{\tan[a + b x^2]}} + \frac{\sqrt{\tan[a + b x^2]}}{b} + x^2 \tan[a + b x^2]^{3/2} \right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$\frac{x \sqrt{\tan[a + b x^2]}}{b}$$

Result (type 8, 55 leaves, 1 step):

$$\text{Unintegrable}\left[\frac{x^2}{\sqrt{\tan[a + b x^2]}}, x\right] + \frac{\text{Unintegrable}\left[\sqrt{\tan[a + b x^2]}, x\right]}{b} + \text{Unintegrable}\left[x^2 \tan[a + b x^2]^{3/2}, x\right]$$

Test results for the 66 problems in "4.3.11 (e x)^m (a+b tan(c+d x^n))^p.m"

Test results for the 700 problems in "4.3.1.2 (d sec)^m (a+b tan)^n.m"

Test results for the 91 problems in "4.3.1.3 (d sin)^m (a+b tan)^n.m"

Test results for the 1328 problems in "4.3.2.1 $(a+b \tan)^m (c+d \tan)^n.m$ "

Test results for the 855 problems in "4.3.3.1 $(a+b \tan)^m (c+d \tan)^n (A+B \tan).m$ "

Test results for the 171 problems in "4.3.4.2 $(a+b \tan)^m (c+d \tan)^n (A+B \tan+C \tan^2).m$ "

Test results for the 499 problems in "4.3.7 $(d \operatorname{trig})^m (a+b (c \tan)^n)^p.m$ "

Test results for the 51 problems in "4.3.9 $\operatorname{trig}^m (a+b \tan^n+c \tan^{(2n)})^p.m$ "

Test results for the 52 problems in "4.4.0 $(a \operatorname{trg})^m (b \cot)^n.m$ "

Test results for the 61 problems in "4.4.10 $(c+d x)^m (a+b \cot)^n.m$ "

Test results for the 23 problems in "4.4.1.2 $(d \operatorname{csc})^m (a+b \cot)^n.m$ "

Test results for the 19 problems in "4.4.1.3 $(d \operatorname{cos})^m (a+b \cot)^n.m$ "

Test results for the 106 problems in "4.4.2.1 $(a+b \cot)^m (c+d \cot)^n.m$ "

Test results for the 64 problems in "4.4.7 $(d \operatorname{trig})^m (a+b (c \cot)^n)^p.m$ "

Test results for the 32 problems in "4.4.9 $\operatorname{trig}^m (a+b \cot^n+c \cot^{(2n)})^p.m$ "

Test results for the 299 problems in "4.5.0 $(a \operatorname{sec})^m (b \operatorname{trg})^n.m$ "

Test results for the 46 problems in "4.5.10 (c+d x)^m (a+b sec)^n.m"

Test results for the 83 problems in "4.5.11 (e x)^m (a+b sec(c+d x^n))^p.m"

Test results for the 879 problems in "4.5.1.2 (d sec)^n (a+b sec)^m.m"

Problem 286: Result unnecessarily involves higher level functions.

$$\int \sec[c + d x]^{5/3} (a + a \sec[c + d x])^{2/3} dx$$

Optimal (type 5, 327 leaves, ? steps):

$$\begin{aligned} & -\frac{3 a \sec[c + d x]^{5/3} \sin[c + d x]}{2 d (a (1 + \sec[c + d x]))^{1/3}} + \\ & \frac{9 \sec[c + d x]^{2/3} (a (1 + \sec[c + d x]))^{2/3} \sin[c + d x]}{4 d} - \frac{9 (a (1 + \sec[c + d x]))^{2/3} \tan[c + d x]}{4 d \left(\frac{1}{1 + \cos[c + d x]}\right)^{1/3} (1 + \sec[c + d x])^{7/3}} + \\ & \left(\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{3}, \frac{5}{4}, \tan\left[\frac{1}{2}(c + d x)\right]^4\right] \left(\cos[c + d x] \sec\left[\frac{1}{2}(c + d x)\right]^4\right)^{1/3} \right. \\ & \quad \left. (a (1 + \sec[c + d x]))^{2/3} \tan[c + d x] \right) / \left(8 d \left(\frac{1}{1 + \cos[c + d x]}\right)^{1/3} (1 + \sec[c + d x])^{4/3} \right) - \\ & \left(5 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{3}{4}, \frac{7}{4}, \tan\left[\frac{1}{2}(c + d x)\right]^4\right] \left(\cos[c + d x] \sec\left[\frac{1}{2}(c + d x)\right]^4\right)^{1/3} \right. \\ & \quad \left. (a (1 + \sec[c + d x]))^{2/3} \tan[c + d x]^3 \right) / \left(8 d \left(\frac{1}{1 + \cos[c + d x]}\right)^{1/3} (1 + \sec[c + d x])^{10/3} \right) \end{aligned}$$

Result (type 6, 79 leaves, 3 steps):

$$\begin{aligned} & \left(2 \times 2^{1/6} \text{AppellF1}\left[\frac{1}{2}, -\frac{2}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \sec[c + d x], \frac{1}{2}(1 - \sec[c + d x])\right] \right) \\ & (a + a \sec[c + d x])^{2/3} \tan[c + d x] / \left(d (1 + \sec[c + d x])^{7/6} \right) \end{aligned}$$

Test results for the 306 problems in "4.5.1.3 (d sin)^n (a+b sec)^m.m"

Problem 276: Unable to integrate problem.

$$\int \csc[c + d x]^4 (a + b \sec[c + d x])^n dx$$

Optimal (type 6, 424 leaves, ? steps):

$$\begin{aligned}
 & -\frac{1}{2\sqrt{2}d} {}_3\text{AppellF1}\left[-\frac{1}{2}, \frac{5}{2}, -n, \frac{1}{2}, \frac{1}{2} (1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] \\
 & \quad \text{Cot}[c + dx] \sqrt{1 + \text{Sec}[c + dx]} (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a+b}\right)^{-n} - \\
 & \frac{1}{6\sqrt{2}d} {}_3\text{AppellF1}\left[-\frac{3}{2}, \frac{5}{2}, -n, -\frac{1}{2}, \frac{1}{2} (1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] \\
 & \quad \text{Cot}[c + dx]^3 (1 + \text{Sec}[c + dx])^{3/2} (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a+b}\right)^{-n} + \\
 & \left({}_3\text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] \right. \\
 & \quad \left. (a + b \text{Sec}[c + dx])^n \left(\frac{a + b \text{Sec}[c + dx]}{a+b}\right)^{-n} \text{Tan}[c + dx] \right) / \left(\sqrt{2}d \sqrt{1 + \text{Sec}[c + dx]} \right) + \\
 & \left({}_3\text{AppellF1}\left[\frac{1}{2}, \frac{5}{2}, -n, \frac{3}{2}, \frac{1}{2} (1 - \text{Sec}[c + dx]), \frac{b(1 - \text{Sec}[c + dx])}{a+b}\right] (a + b \text{Sec}[c + dx])^n \right. \\
 & \quad \left. \left(\frac{a + b \text{Sec}[c + dx]}{a+b}\right)^{-n} \text{Tan}[c + dx] \right) / \left(2\sqrt{2}d \sqrt{1 + \text{Sec}[c + dx]} \right)
 \end{aligned}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{Unintegrate}\left[\text{Csc}[c + dx]^4 (a + b \text{Sec}[c + dx])^n, x\right]$$

Test results for the 365 problems in "4.5.1.4 (d tan)^n (a+b sec)^m.m"

Problem 207: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Tan}[e + fx]^2}{(a + a \text{Sec}[e + fx])^{9/2}} dx$$

Optimal (type 3, 177 leaves, ? steps):

$$\begin{aligned}
 & -\frac{2 \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + fx]}{\sqrt{a + a \text{Sec}[e + fx]}}\right]}{a^{9/2} f} + \frac{91 \text{ArcTan}\left[\frac{\sqrt{a} \text{Tan}[e + fx]}{\sqrt{2} \sqrt{a + a \text{Sec}[e + fx]}}\right]}{32 \sqrt{2} a^{9/2} f} + \\
 & \frac{\text{Tan}[e + fx]}{3 a f (a + a \text{Sec}[e + fx])^{7/2}} + \frac{11 \text{Tan}[e + fx]}{24 a^2 f (a + a \text{Sec}[e + fx])^{5/2}} + \frac{27 \text{Tan}[e + fx]}{32 a^3 f (a + a \text{Sec}[e + fx])^{3/2}}
 \end{aligned}$$

Result (type 3, 227 leaves, 7 steps):

$$\begin{aligned}
& - \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{a^{9/2} f} + \frac{91 \operatorname{ArcTan}\left[\frac{\sqrt{a} \operatorname{Tan}[e+fx]}{\sqrt{2} \sqrt{a+a \operatorname{Sec}[e+fx]}}\right]}{32 \sqrt{2} a^{9/2} f} + \frac{27 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^2 \operatorname{Sin}[e+fx]}{64 a^4 f \sqrt{a+a \operatorname{Sec}[e+fx]}} + \\
& \frac{11 \operatorname{Cos}[e+fx] \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^4 \operatorname{Sin}[e+fx]}{96 a^4 f \sqrt{a+a \operatorname{Sec}[e+fx]}} + \frac{\operatorname{Cos}[e+fx]^2 \operatorname{Sec}\left[\frac{1}{2}(e+fx)\right]^6 \operatorname{Sin}[e+fx]}{24 a^4 f \sqrt{a+a \operatorname{Sec}[e+fx]}}
\end{aligned}$$

Problem 347: Unable to integrate problem.

$$\int \frac{(d \operatorname{Tan}[e+fx])^n}{a+b \operatorname{Sec}[e+fx]} dx$$

Optimal (type 6, 266 leaves, ? steps):

$$\begin{aligned}
& \frac{1}{a f (1-n)} d \operatorname{AppellF1}\left[1-n, \frac{1-n}{2}, \frac{1-n}{2}, 2-n, \frac{a+b}{a+b \operatorname{Sec}[e+fx]}, \frac{a-b}{a+b \operatorname{Sec}[e+fx]}\right] \\
& \left(-\frac{b(1-\operatorname{Sec}[e+fx])}{a+b \operatorname{Sec}[e+fx]}\right)^{\frac{1-n}{2}} \left(\frac{b(1+\operatorname{Sec}[e+fx])}{a+b \operatorname{Sec}[e+fx]}\right)^{\frac{1-n}{2}} \\
& (d \operatorname{Tan}[e+fx])^{-1+n} (-\operatorname{Tan}[e+fx]^2)^{\frac{1-n}{2}+\frac{1}{2}(-1+n)} - \frac{1}{a f (1+n)} \\
& d \operatorname{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\operatorname{Tan}[e+fx]^2\right] (d \operatorname{Tan}[e+fx])^{-1+n} (-\operatorname{Tan}[e+fx]^2)^{\frac{1-n}{2}+\frac{1+n}{2}}
\end{aligned}$$

Result (type 8, 25 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(d \operatorname{Tan}[e+fx])^n}{a+b \operatorname{Sec}[e+fx]}, x\right]$$

Test results for the 241 problems in "4.5.2.1 (a+b sec)^m (c+d sec)^n.m"

Problem 217: Unable to integrate problem.

$$\int \frac{(c+d \operatorname{Sec}[e+fx])^{3/2}}{\sqrt{a+b \operatorname{Sec}[e+fx]}} dx$$

Optimal (type 4, 652 leaves, ? steps):

$$\begin{aligned}
& - \left(\left(2 c (c+d) \cot [e+f x] \operatorname{EllipticPi} \left[\frac{a (c+d)}{(a+b) c}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec [e+f x])}{(c+d) (a+b \sec [e+f x])}} \right], \right. \right. \right. \\
& \quad \left. \left. \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \sqrt{\frac{(b c-a d) (1+\sec [e+f x])}{(c-d) (a+b \sec [e+f x])}} (a+b \sec [e+f x])^{3/2} \right. \\
& \quad \left. \left. \sqrt{\frac{(a+b) (b c-a d) (-1+\sec [e+f x]) (c+d \sec [e+f x])}{(c+d)^2 (a+b \sec [e+f x])^2}} \right) / \right. \\
& \quad \left. \left(a (a+b) f \sqrt{c+d \sec [e+f x]} \right) \right) + \left(2 d (c+d) \cot [e+f x] \right. \\
& \quad \left. \operatorname{EllipticPi} \left[\frac{b (c+d)}{(a+b) d}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec [e+f x])}{(c+d) (a+b \sec [e+f x])}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \right. \\
& \quad \left. \sqrt{\frac{(b c-a d) (1+\sec [e+f x])}{(c-d) (a+b \sec [e+f x])}} (a+b \sec [e+f x])^{3/2} \right. \\
& \quad \left. \sqrt{-\frac{(a+b) (-b c+a d) (-1+\sec [e+f x]) (c+d \sec [e+f x])}{(c+d)^2 (a+b \sec [e+f x])^2}} \right) / \\
& \quad \left(b (a+b) f \sqrt{c+d \sec [e+f x]} \right) + \\
& \quad \left(2 (b c-a d) \cot [e+f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec [e+f x])}{(c+d) (a+b \sec [e+f x])}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \right. \\
& \quad \left. \sqrt{\frac{(b c-a d) (-1+\sec [e+f x])}{(c+d) (a+b \sec [e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec [e+f x])}{(c-d) (a+b \sec [e+f x])}} \right. \\
& \quad \left. \left. \sqrt{a+b \sec [e+f x]} \sqrt{c+d \sec [e+f x]} \right) / \left(a b f \sqrt{\frac{(a+b) (c+d \sec [e+f x])}{(c+d) (a+b \sec [e+f x])}} \right)
\end{aligned}$$

Result (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable} \left[\frac{(c+d \sec [e+f x])^{3/2}}{\sqrt{a+b \sec [e+f x]}}, x \right]$$

Test results for the 286 problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n.m"

Test results for the 634 problems in "4.5.3.1 (a+b sec)^m (d sec)^n

$(A+B \sec).m^n$

Test results for the 70 problems in "4.5.4.1 $(a+b \sec)^m (A+B \sec+C \sec^2).m^n$ "

Test results for the 1373 problems in "4.5.4.2 $(a+b \sec)^m (d \sec)^n (A+B \sec+C \sec^2).m^n$ "

Test results for the 470 problems in "4.5.7 $(d \operatorname{trig})^m (a+b (c \sec)^n)^p.m^n$ "

Problem 132: Unable to integrate problem.

$$\int (a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m dx$$

Optimal (type 6, 123 leaves, ? steps):

$$\frac{1}{f(1+m)} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}+p, -p, \frac{3+m}{2}, \operatorname{Sin}[e + f x]^2, \frac{a \operatorname{Sin}[e + f x]^2}{a+b}\right] (\operatorname{Cos}[e + f x]^2)^{\frac{1}{2}+p} \\ (a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m \left(\frac{a+b - a \operatorname{Sin}[e + f x]^2}{a+b}\right)^{-p} \operatorname{Tan}[e + f x]$$

Result (type 8, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[(a + b \operatorname{Sec}[e + f x]^2)^p (d \operatorname{Sin}[e + f x])^m, x\right]$$

Problem 298: Unable to integrate problem.

$$\int (d \operatorname{Sec}[e + f x])^m (a + b \operatorname{Sec}[e + f x]^2)^p dx$$

Optimal (type 6, 111 leaves, ? steps):

$$\frac{1}{f m} \operatorname{AppellF1}\left[\frac{m}{2}, \frac{1}{2}, -p, \frac{2+m}{2}, \operatorname{Sec}[e + f x]^2, -\frac{b \operatorname{Sec}[e + f x]^2}{a}\right] \operatorname{Cot}[e + f x] \\ (d \operatorname{Sec}[e + f x])^m (a + b \operatorname{Sec}[e + f x]^2)^p \left(1 + \frac{b \operatorname{Sec}[e + f x]^2}{a}\right)^{-p} \sqrt{-\operatorname{Tan}[e + f x]^2}$$

Result (type 8, 27 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[(d \operatorname{Sec}[e + f x])^m (a + b \operatorname{Sec}[e + f x]^2)^p, x\right]$$

Test results for the 70 problems in "4.6.0 $(a \operatorname{csc})^m (b \operatorname{trg})^n.m^n$ "

Test results for the 84 problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.m"

Test results for the 59 problems in "4.6.1.2 (d csc)^n (a+b csc)^m.m"

Test results for the 16 problems in "4.6.1.3 (d cos)^n (a+b csc)^m.m"

Test results for the 23 problems in "4.6.1.4 (d cot)^n (a+b csc)^m.m"

Test results for the 24 problems in "4.6.3.1 (a+b csc)^m (d csc)^n (A+B csc).m"

Test results for the 1 problems in "4.6.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2).m"

Test results for the 27 problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.m"

Test results for the 254 problems in "4.7.1 (c trig)^m (d trig)^n.m"

Test results for the 294 problems in "4.7.2 trig^m (a trig+b trig)^n.m"

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^3}{(a \cos[x] + b \sin[x])^2} dx$$

Optimal (type 3, 107 leaves, ? steps):

$$\frac{6 a^2 b \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{(a^2+b^2)^{5/2}} + \frac{3 a (a^2-b^2) + a (a^2+b^2) \cos[2 x] - b (a^2+b^2) \sin[2 x]}{2 (a^2+b^2)^2 (a \cos[x] + b \sin[x])}$$

Result (type 3, 283 leaves, 19 steps):

$$\begin{aligned}
& - \frac{3 a^2 \operatorname{ArcTanh}\left[\frac{b \cos [x]-a \sin [x]}{\sqrt{a^2+b^2}}\right]}{b\left(a^2+b^2\right)^{3 / 2}} - \frac{2 a^2 b \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5 / 2}} + \\
& \frac{2 a^2\left(3 a^2+b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b\left(a^2+b^2\right)^{5 / 2}} - \frac{\cos [x]}{b^2} + \frac{3 a^2 \cos [x]}{b^2\left(a^2+b^2\right)} - \frac{2 a \sin [x]}{b^3} + \frac{3 a^3 \sin [x]}{b^3\left(a^2+b^2\right)} - \\
& \frac{2 a^3 \cos \left[\frac{x}{2}\right]^2\left(2 a b+\left(a^2-b^2\right) \tan \left[\frac{x}{2}\right]\right)}{b^3\left(a^2+b^2\right)^2} + \frac{2 a^2\left(a+b \tan \left[\frac{x}{2}\right]\right)}{\left(a^2+b^2\right)^2\left(a+2 b \tan \left[\frac{x}{2}\right]-a \tan \left[\frac{x}{2}\right]^2\right)}
\end{aligned}$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\sin [x]^2}{\left(a \cos [x]+b \sin [x]\right)^3} d x$$

Optimal (type 3, 92 leaves, ? steps):

$$\begin{aligned}
& - \frac{\left(a^2-2 b^2\right) \operatorname{ArcTanh}\left[\frac{-b+a \tan \left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5 / 2}} + \frac{a\left(3 a b \cos [x]+\left(a^2+4 b^2\right) \sin [x]\right)}{2\left(a^2+b^2\right)^2\left(a \cos [x]+b \sin [x]\right)^2}
\end{aligned}$$

Result (type 3, 300 leaves, 13 steps):

$$\begin{aligned}
& \frac{2 a^2 \operatorname{ArcTanh}\left[\frac{b \cos [x]-a \sin [x]}{\sqrt{a^2+b^2}}\right]}{b^2\left(a^2+b^2\right)^{3 / 2}} - \frac{\operatorname{ArcTanh}\left[\frac{b \cos [x]-a \sin [x]}{\sqrt{a^2+b^2}}\right]}{b^2 \sqrt{a^2+b^2}} - \\
& \frac{a^2\left(2 a^2-b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b^2\left(a^2+b^2\right)^{5 / 2}} + \frac{2 a}{b\left(a^2+b^2\right)\left(a \cos [x]+b \sin [x]\right)} + \\
& \frac{2\left(a b+\left(a^2+2 b^2\right) \tan \left[\frac{x}{2}\right]\right)}{a\left(a^2+b^2\right)\left(a+2 b \tan \left[\frac{x}{2}\right]-a \tan \left[\frac{x}{2}\right]^2\right)^2} - \frac{4 a^4+3 a^2 b^2+2 b^4+a b\left(5 a^2+2 b^2\right) \tan \left[\frac{x}{2}\right]}{a b\left(a^2+b^2\right)^2\left(a+2 b \tan \left[\frac{x}{2}\right]-a \tan \left[\frac{x}{2}\right]^2\right)}
\end{aligned}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c+d x]^3}{\left(a \cos [c+d x]+b \sin [c+d x]\right)^2} d x$$

Optimal (type 3, 138 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 a b^2 \operatorname{ArcTanh}\left[\frac{b \cos [c+d x]-a \sin [c+d x]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5 / 2} d} + \frac{2 a b \cos [c+d x]}{\left(a^2+b^2\right)^2 d} + \\
& \frac{\left(a^2-b^2\right) \sin [c+d x]}{\left(a^2+b^2\right)^2 d} - \frac{b^3}{\left(a^2+b^2\right)^2 d\left(a \cos [c+d x]+b \sin [c+d x]\right)}
\end{aligned}$$

Result (type 3, 231 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 b^4 \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a\left(a^2+b^2\right)^{5 / 2} d} - \frac{2 b^2\left(3 a^2+b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a\left(a^2+b^2\right)^{5 / 2} d} + \\
& \frac{2\left(2 a b+\left(a^2-b^2\right) \tan \left[\frac{1}{2}(c+d x)\right]\right)}{\left(a^2+b^2\right)^2 d\left(1+\tan \left[\frac{1}{2}(c+d x)\right]^2\right)} - \frac{2 b^3\left(a+b \tan \left[\frac{1}{2}(c+d x)\right]\right)}{a\left(a^2+b^2\right)^2 d\left(a+2 b \tan \left[\frac{1}{2}(c+d x)\right]-a \tan \left[\frac{1}{2}(c+d x)\right]^2\right)}
\end{aligned}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c+d x]^4}{\left(a \cos [c+d x]+b \sin [c+d x]\right)^3} d x$$

Optimal (type 3, 216 leaves, ? steps):

$$\begin{aligned}
& - \frac{3 b^2\left(4 a^2-b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \tan \left[\frac{1}{2}(c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{7 / 2} d} + \frac{b\left(3 a^2-b^2\right) \cos [c+d x]}{\left(a^2+b^2\right)^3 d} + \frac{a\left(a^2-3 b^2\right) \sin [c+d x]}{\left(a^2+b^2\right)^3 d} + \\
& \frac{b^4 \sin [c+d x]}{2 a\left(a^2+b^2\right)^2 d\left(a \cos [c+d x]+b \sin [c+d x]\right)^2} - \frac{b^3\left(8 a^2+b^2\right)}{2 a\left(a^2+b^2\right)^3 d\left(a \cos [c+d x]+b \sin [c+d x]\right)}
\end{aligned}$$

Result (type 3, 492 leaves, 15 steps):

$$\begin{aligned}
& - \frac{3 b^4 (a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \frac{4 b^4 (3 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} - \\
& \frac{2 b^2 (6 a^4 + 3 a^2 b^2 + b^4) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{a^2 (a^2 + b^2)^{7/2} d} + \frac{2 \left(b (3 a^2 - b^2) + a (a^2 - 3 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{(a^2 + b^2)^3 d \left(1 + \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)} + \\
& \frac{2 b^4 \left(a b + (a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{a^3 (a^2 + b^2)^2 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^2} - \\
& \frac{3 b^4 (a^2 + 2 b^2) \left(b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{a^3 (a^2 + b^2)^3 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)} - \\
& \frac{4 b^3 \left(2 a^4 - b^4 + a b (3 a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{a^3 (a^2 + b^2)^3 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)}
\end{aligned}$$

Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c + d x]^2}{(a \cos [c + d x] + b \sin [c + d x])^3} dx$$

Optimal (type 3, 119 leaves, ? steps):

$$\frac{(2 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{-b + a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{5/2} d} - \frac{b \left((4 a^2 + b^2) \cos [c + d x] + 3 a b \sin [c + d x]\right)}{2 (a^2 + b^2)^2 d (a \cos [c + d x] + b \sin [c + d x])^2}$$

Result (type 3, 225 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(2 a^2 - b^2) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{5/2} d} + \\
& \frac{2 b^2 \left(a b + (a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{a^3 (a^2 + b^2) d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^2} - \\
& \frac{b \left(4 a^4 + 3 a^2 b^2 + 2 b^4 + a b (5 a^2 + 2 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{a^3 (a^2 + b^2)^2 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)}
\end{aligned}$$

Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\cos [c + d x]^3}{(a \cos [c + d x] + b \sin [c + d x])^4} dx$$

Optimal (type 3, 157 leaves, ? steps):

$$\frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh}\left[\frac{-b + a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2} d} + \frac{\left(-3 (3 a^4 b - a^2 b^3 + b^5) \cos[2 (c + d x)] + \frac{1}{2} b (-9 a^2 + b^2) (2 (a^2 + b^2) + 3 a b \sin[2 (c + d x)])\right)}{(6 (a^2 + b^2)^3 d (a \cos[c + d x] + b \sin[c + d x]))^3}$$

Result (type 3, 362 leaves, 7 steps):

$$-\frac{a (2 a^2 - 3 b^2) \operatorname{ArcTanh}\left[\frac{b - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]}{\sqrt{a^2 + b^2}}\right]}{(a^2 + b^2)^{7/2} d} - \frac{8 b^3 \left(a (a^2 + 2 b^2) + b (3 a^2 + 4 b^2) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{3 a^5 (a^2 + b^2) d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^3} + \frac{\left(2 b^2 \left(b (15 a^4 + 18 a^2 b^2 + 8 b^4) + a (9 a^4 + 30 a^2 b^2 + 16 b^4) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)\right)}{\left(3 a^5 (a^2 + b^2)^2 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)^2\right)} - \frac{b \left(6 a^6 + 9 a^4 b^2 + 12 a^2 b^4 + 4 b^6 + a b (9 a^4 + 6 a^2 b^2 + 2 b^4) \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]\right)}{\left(a^4 (a^2 + b^2)^3 d \left(a + 2 b \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right] - a \operatorname{Tan}\left[\frac{1}{2} (c + d x)\right]^2\right)\right)}$$

Test results for the 397 problems in "4.7.3 (c+d x)^m trig^n trig^p.m"

Test results for the 9 problems in "4.7.4 x^m (a+b trig^n)^p.m"

Test results for the 330 problems in "4.7.5 x^m trig(a+b log(c x^n))^p.m"

Problem 259: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(- (1 + b^2 n^2) \operatorname{Sec}[a + b \operatorname{Log}[c x^n]] + 2 b^2 n^2 \operatorname{Sec}[a + b \operatorname{Log}[c x^n]]^3 \right) dx$$

Optimal (type 3, 41 leaves, ? steps):

$$-x \operatorname{Sec}[a + b \operatorname{Log}[c x^n]] + b n x \operatorname{Sec}[a + b \operatorname{Log}[c x^n]] \operatorname{Tan}[a + b \operatorname{Log}[c x^n]]$$

Result (type 5, 175 leaves, 7 steps):

$$\begin{aligned}
& -2 e^{i a} (1 - i b n) x (c x^n)^{i b} \\
& \text{Hypergeometric2F1}\left[1, \frac{1}{2} \left(1 - \frac{i}{b n}\right), \frac{1}{2} \left(3 - \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 i b}\right] + \frac{1}{1 + 3 i b n} \\
& 16 b^2 e^{3 i a} n^2 x (c x^n)^{3 i b} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i}{b n}\right), \frac{1}{2} \left(5 - \frac{i}{b n}\right), -e^{2 i a} (c x^n)^{2 i b}\right]
\end{aligned}$$

Problem 260: Result unnecessarily involves higher level functions.

$$\int x^m \sec\left[a + 2 \log\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]^3 dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m} \sec\left[a + 2 \log\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]}{2 (1+m)} + \frac{x^{1+m} \sec\left[a + 2 \log\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right] \tan\left[a + 2 \log\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]}{2 \sqrt{-(1+m)^2}}$$

Result (type 5, 146 leaves, 3 steps):

$$\begin{aligned}
& \left(8 e^{3 i a} x^{1+m} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}} \right)^{6 i} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), \right. \right. \\
& \left. \left. \frac{1}{2} \left(5 - \frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), -e^{2 i a} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}} \right)^{4 i}\right] \right) / \left(1 - i \left(i m - 3 \sqrt{-(1+m)^2} \right) \right)
\end{aligned}$$

Problem 301: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(- (1 + b^2 n^2) \csc\left[a + b \log\left[c x^n\right]\right] + 2 b^2 n^2 \csc\left[a + b \log\left[c x^n\right]\right]^3 \right) dx$$

Optimal (type 3, 42 leaves, ? steps):

$$-x \csc\left[a + b \log\left[c x^n\right]\right] - b n x \cot\left[a + b \log\left[c x^n\right]\right] \csc\left[a + b \log\left[c x^n\right]\right]$$

Result (type 5, 172 leaves, 7 steps):

$$\begin{aligned}
& 2 e^{i a} (i + b n) x (c x^n)^{i b} \text{Hypergeometric2F1}\left[1, \frac{1}{2} \left(1 - \frac{i}{b n}\right), \frac{1}{2} \left(3 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right] - \frac{1}{i - 3 b n} \\
& 16 b^2 e^{3 i a} n^2 x (c x^n)^{3 i b} \text{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i}{b n}\right), \frac{1}{2} \left(5 - \frac{i}{b n}\right), e^{2 i a} (c x^n)^{2 i b}\right]
\end{aligned}$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int x^m \csc\left[a + 2 \log\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]^3 dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m} \operatorname{Csc}\left[a + 2 \operatorname{Log}\left[c x^{\frac{1}{2}} \sqrt{-(1+m)^2}\right]\right]}{2(1+m)} - \frac{x^{1+m} \operatorname{Cot}\left[a + 2 \operatorname{Log}\left[c x^{\frac{1}{2}} \sqrt{-(1+m)^2}\right]\right] \operatorname{Csc}\left[a + 2 \operatorname{Log}\left[c x^{\frac{1}{2}} \sqrt{-(1+m)^2}\right]\right]}{2 \sqrt{-(1+m)^2}}$$

Result (type 5, 142 leaves, 3 steps):

$$-\left(\left(8 e^{3 i a} x^{1+m} \left(c x^{\frac{1}{2}} \sqrt{-(1+m)^2}\right)^{6 i} \operatorname{Hypergeometric2F1}\left[3, \frac{1}{2} \left(3 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right), \frac{1}{2} \left(5 - \frac{i(1+m)}{\sqrt{-(1+m)^2}}\right), e^{2 i a} \left(c x^{\frac{1}{2}} \sqrt{-(1+m)^2}\right)^{4 i}\right]\right) / \left(i + i m - 3 \sqrt{-(1+m)^2}\right)\right)$$

Test results for the 142 problems in "4.7.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n.m"

Problem 28: Unable to integrate problem.

$$\int F^{c(a+b x)} (f x)^m \sin[d+e x] dx$$

Optimal (type 4, 139 leaves, ? steps):

$$-\left(\left(e^{-i d} F^{a c} (f x)^m \operatorname{Gamma}\left[1+m, x(i e - b c \operatorname{Log}[F])\right]\right) (x(i e - b c \operatorname{Log}[F]))^{-m} / \left(2(e + i b c \operatorname{Log}[F])\right) - \left(e^{i d} F^{a c} (f x)^m \operatorname{Gamma}\left[1+m, -x(i e + b c \operatorname{Log}[F])\right]\right) (-x(i e + b c \operatorname{Log}[F]))^{-m} / \left(2(e - i b c \operatorname{Log}[F])\right)\right)$$

Result (type 8, 24 leaves, 1 step):

$$\operatorname{CannotIntegrate}\left[F^{a c+b c x} (f x)^m \sin[d+e x], x\right]$$

Problem 32: Unable to integrate problem.

$$\int f F^{c(a+b x)} (f x)^m (e x \cos[d+e x] + (1+m+b c x \operatorname{Log}[F]) \sin[d+e x]) dx$$

Optimal (type 3, 23 leaves, ? steps):

$$f F^{c(a+b x)} x (f x)^m \sin[d+e x]$$

Result (type 8, 89 leaves, 6 steps):

$$e \operatorname{CannotIntegrate}\left[F^{a c+b c x} (f x)^{1+m} \cos[d+e x], x\right] + f(1+m) \operatorname{CannotIntegrate}\left[F^{a c+b c x} (f x)^m \sin[d+e x], x\right] + b c \operatorname{CannotIntegrate}\left[F^{a c+b c x} (f x)^{1+m} \sin[d+e x], x\right] \operatorname{Log}[F]$$

Test results for the 950 problems in "4.7.7 Trig functions.m"

Problem 759: Result valid but suboptimal antiderivative.

$$\int (\cos[x]^{12} \sin[x]^{10} - \cos[x]^{10} \sin[x]^{12}) dx$$

Optimal (type 3, 12 leaves, ? steps):

$$\frac{1}{11} \cos[x]^{11} \sin[x]^{11}$$

Result (type 3, 129 leaves, 25 steps):

$$\begin{aligned} & \frac{3 \cos[x]^{11} \sin[x]}{5632} - \frac{3 \cos[x]^{13} \sin[x]}{5632} + \frac{1}{512} \cos[x]^{11} \sin[x]^3 - \\ & \frac{7 \cos[x]^{13} \sin[x]^3}{2816} + \frac{7 \cos[x]^{11} \sin[x]^5}{1280} - \frac{7}{880} \cos[x]^{13} \sin[x]^5 + \frac{1}{80} \cos[x]^{11} \sin[x]^7 - \\ & \frac{9}{440} \cos[x]^{13} \sin[x]^7 + \frac{1}{40} \cos[x]^{11} \sin[x]^9 - \frac{1}{22} \cos[x]^{13} \sin[x]^9 + \frac{1}{22} \cos[x]^{11} \sin[x]^{11} \end{aligned}$$

Problem 796: Unable to integrate problem.

$$\int e^{\sin[x]} \sec[x]^2 (x \cos[x]^3 - \sin[x]) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\sin[x]} (-1 + x \cos[x]) \sec[x]$$

Result (type 8, 24 leaves, 2 steps):

$$\text{CannotIntegrate}[e^{\sin[x]} x \cos[x], x] - \text{CannotIntegrate}[e^{\sin[x]} \sec[x] \tan[x], x]$$

Problem 858: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\cos[x]^{3/2} \sqrt{3 \cos[x] + \sin[x]}} dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{2 \sqrt{3 \cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

Result (type 3, 88 leaves, 5 steps):

$$\frac{2 \cos\left[\frac{x}{2}\right]^2 \left(3 + 2 \tan\left[\frac{x}{2}\right] - 3 \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{\cos\left[\frac{x}{2}\right]^2 \left(3 + 2 \tan\left[\frac{x}{2}\right] - 3 \tan\left[\frac{x}{2}\right]^2\right)} \sqrt{\cos\left[\frac{x}{2}\right]^2 \left(1 - \tan\left[\frac{x}{2}\right]^2\right)}}$$

Problem 859: Unable to integrate problem.

$$\int \frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}} dx$$

Optimal (type 3, 44 leaves, ? steps):

$$-\log[\sin[x]] + 2 \log\left[-\sqrt{\cos[x]} + \sqrt{\cos[x] + \sin[x]}\right] + \frac{2 \sqrt{\cos[x] + \sin[x]}}{\sqrt{\cos[x]}}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{\csc[x] \sqrt{\cos[x] + \sin[x]}}{\cos[x]^{3/2}}, x\right]$$

Problem 860: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{1 + \sin[2x]}} dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{x \sqrt{1 + \sin[2x]}}{\cos[x] + \sin[x]}$$

Result (type 3, 72 leaves, 17 steps):

$$\frac{2 \operatorname{ArcTan}\left[\tan\left[\frac{x}{2}\right]\right] \cos\left[\frac{x}{2}\right]^2 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)}{\sqrt{\cos\left[\frac{x}{2}\right]^4 \left(1 + 2 \tan\left[\frac{x}{2}\right] - \tan\left[\frac{x}{2}\right]^2\right)^2}}$$

Problem 912: Result valid but suboptimal antiderivative.

$$\int \frac{\cos[x] + \sin[x]}{\sqrt{\cos[x]} \sqrt{\sin[x]}} dx$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]$$

Result (type 3, 243 leaves, 22 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{\sqrt{2}} - \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} + \\
& \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}}\right]}{\sqrt{2}} - \frac{\text{Log}\left[1 + \text{Cot}[x] - \frac{\sqrt{2}\sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \frac{\text{Log}\left[1 + \text{Cot}[x] + \frac{\sqrt{2}\sqrt{\cos[x]}}{\sqrt{\sin[x]}}\right]}{2\sqrt{2}} + \\
& \frac{\text{Log}\left[1 - \frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \text{Tan}[x]\right]}{2\sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{2}\sqrt{\sin[x]}}{\sqrt{\cos[x]}} + \text{Tan}[x]\right]}{2\sqrt{2}}
\end{aligned}$$

Problem 914: Unable to integrate problem.

$$\int \left(10 x^9 \cos\left[x^5 \log[x]\right] - x^{10} \left(x^4 + 5 x^4 \log[x]\right) \sin\left[x^5 \log[x]\right]\right) dx$$

Optimal (type 3, 11 leaves, ? steps):

$$x^{10} \cos\left[x^5 \log[x]\right]$$

Result (type 8, 48 leaves, 4 steps):

$$10 \text{ CannotIntegrate}\left[x^9 \cos\left[x^5 \log[x]\right], x\right] - \text{CannotIntegrate}\left[x^{14} \sin\left[x^5 \log[x]\right], x\right] - 5 \text{ CannotIntegrate}\left[x^{14} \log[x] \sin\left[x^5 \log[x]\right], x\right]$$

Problem 915: Unable to integrate problem.

$$\int \cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right] dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x}{2} - \frac{\cos[x]}{2} - \text{Log}\left[\cos\left[\frac{\pi}{4} + \frac{x}{2}\right]\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\cos\left[\frac{x}{2}\right]^2 \tan\left[\frac{\pi}{4} + \frac{x}{2}\right], x\right]$$

Problem 931: Unable to integrate problem.

$$\int \left(\frac{x^4}{b \sqrt{x^3 + 3 \sin[a + b x]}} + \frac{x^2 \cos[a + b x]}{\sqrt{x^3 + 3 \sin[a + b x]}} + \frac{4 x \sqrt{x^3 + 3 \sin[a + b x]}}{3 b} \right) dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^2 \sqrt{x^3 + 3 \sin[a + b x]}}{3 b}$$

Result (type 8, 82 leaves, 1 step):

$$\frac{\text{CannotIntegrate}\left[\frac{x^4}{\sqrt{x^3+3\sin[a+bx]}}, x\right]}{b} +$$

$$\text{CannotIntegrate}\left[\frac{x^2 \cos[a+bx]}{\sqrt{x^3+3\sin[a+bx]}}, x\right] + \frac{4 \text{ CannotIntegrate}\left[x \sqrt{x^3+3\sin[a+bx]}, x\right]}{3b}$$

Problem 933: Unable to integrate problem.

$$\int \frac{\cos[x] + \sin[x]}{e^{-x} + \sin[x]} dx$$

Optimal (type 3, 9 leaves, ? steps):

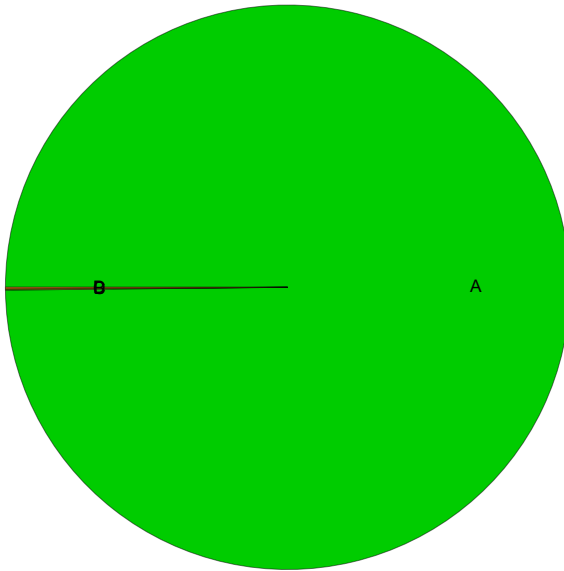
$$\text{Log}[1 + e^x \sin[x]]$$

Result (type 8, 36 leaves, 5 steps):

$$x - \text{CannotIntegrate}\left[\frac{1}{1 + e^x \sin[x]}, x\right] - \text{CannotIntegrate}\left[\frac{\cot[x]}{1 + e^x \sin[x]}, x\right] + \text{Log}[\sin[x]]$$

Summary of Integration Test Results

22551 integration problems



- A - 22515 optimal antiderivatives
- B - 12 valid but suboptimal antiderivatives
- C - 5 unnecessarily complex antiderivatives
- D - 19 unable to integrate problems
- E - 0 integration timeouts
- F - 0 invalid antiderivatives