Mathematica 11.3 Integration Test Results

Test results for the 12 problems in "5.4.2 Exponentials of inverse cotangent.m"

Problem 8: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a \, x]}}{\left(c + a^2 \, c \, x^2\right)^{1/3}} \, \mathrm{d} x$$

Optimal (type 5, 147 leaves, 3 steps):

$$\frac{1}{\left(\,c\,+\,a^{2}\,c\,\,x^{2}\,\right)^{\,1/3}}3\,\left(1\,+\,\frac{1}{\,a^{2}\,\,x^{2}}\,\right)^{1/3}\,\left(\frac{\,a\,-\,\frac{\,\mathrm{i}}{\,x}\,}{\,a\,+\,\frac{\,\mathrm{i}}{\,x}\,}\right)^{\,\frac{1}{\,6}\,\,(2\,-\,3\,\,\mathrm{i}\,\,n\,)}\,\left(1\,-\,\frac{\,\mathrm{i}}{\,a\,\,x}\,\right)^{\,\frac{1}{\,6}\,\,(-\,2\,+\,3\,\,\mathrm{i}\,\,n\,)}$$

$$\left(1+\frac{\mathrm{i}}{\mathsf{a}\,\mathsf{x}}\right)^{\frac{1}{6}\,(4-3\,\mathrm{i}\,\mathsf{n})}\,\mathsf{x}\,\mathsf{Hypergeometric}2\mathsf{F1}\!\left[\,-\,\frac{1}{3}\,,\,\frac{1}{6}\,\left(2\,-\,3\,\,\mathrm{i}\,\,\mathsf{n}\right)\,,\,\,\frac{2}{3}\,,\,\,\frac{2\,\,\mathrm{i}}{\left(\mathsf{a}\,+\,\frac{\mathrm{i}}{\mathsf{x}}\right)\,\mathsf{x}}\,\right]$$

Result (type 8, 25 leaves):

$$\int \frac{ \, e^{n \, \text{ArcCot} \, [\, a \, x \,]}}{ \, \left(\, c \, + \, a^2 \, c \, \, x^2 \,\right)^{\, 1/3}} \, \, \mathrm{d} x$$

Problem 9: Unable to integrate problem.

$$\int \frac{\, {\textstyle \mathop{\mathbb{e}}}^{n \, \mathsf{ArcCot} \, [\, a \, x \,]}}{\, \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{\, 2/3}} \, \, \mathrm{d} x$$

Optimal (type 5, 147 leaves, 3 steps):

$$-\frac{1}{\left(c+a^2\;c\;x^2\right)^{2/3}} 3\; \left(1+\frac{1}{a^2\;x^2}\right)^{2/3} \; \left(\frac{a-\frac{\dot{1}}{x}}{a+\frac{\dot{1}}{x}}\right)^{\frac{1}{6}\;(4-3\;\dot{1}\;n)} \; \left(1-\frac{\dot{1}}{a\;x}\right)^{\frac{1}{6}\;(-4+3\;\dot{1}\;n)}$$

$$\left(1+\frac{\dot{\mathbb{I}}}{\mathsf{a}\,\mathsf{x}}\right)^{\frac{1}{6}\,(2-3\,\dot{\mathbb{I}}\,\mathsf{n})}\,\mathsf{x}\,\mathsf{Hypergeometric2F1}\!\left[\,\frac{1}{3}\,,\,\,\frac{1}{6}\,\left(4-3\,\dot{\mathbb{I}}\,\mathsf{n}\right)\,,\,\,\frac{4}{3}\,,\,\,\frac{2\,\dot{\mathbb{I}}}{\left(\mathsf{a}+\frac{\dot{\mathbb{I}}}{\mathsf{x}}\right)\,\mathsf{x}}\,\right]$$

Result (type 8, 25 leaves):

$$\int \frac{ \, {\textstyle \mathop{\mathbb{C}}}^{ n \, \mathsf{ArcCot} \, [\, a \, x \,] } }{ \, \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{\, 2/3} } \, \, \mathrm{d} \, x$$

Problem 10: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcCot}[a \, x]}}{\left(c + a^2 \, c \, x^2\right)^{4/3}} \, \mathrm{d} x$$

Optimal (type 5, 207 leaves, 4 steps):

$$-\frac{3 e^{n \operatorname{ArcCot}[a \, x]} \left(3 \, n - 2 \, a \, x\right)}{a \, c \, \left(4 + 9 \, n^2\right) \, \left(c + a^2 \, c \, x^2\right)^{1/3}} - \\ \left[6 \left(1 + \frac{1}{a^2 \, x^2}\right)^{1/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}}\right)^{\frac{1}{6} \, (2 - 3 \, i \, n)} \, \left(1 - \frac{i}{a \, x}\right)^{\frac{1}{6} \, (-2 + 3 \, i \, n)} \, \left(1 + \frac{i}{a \, x}\right)^{\frac{1}{6} \, (4 - 3 \, i \, n)} \, x\right]$$

$$\text{Hypergeometric2F1} \Big[-\frac{1}{3} \text{, } \frac{1}{6} \left(2 - 3 \text{ in} \right) \text{, } \frac{2}{3} \text{, } \frac{2 \text{ in}}{\left(a + \frac{\text{in}}{x} \right) x} \Big] \Bigg] \Bigg/ \left(c \left(4 + 9 \text{ n}^2 \right) \left(c + a^2 \text{ c } x^2 \right)^{1/3} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{ \, e^{ n \, \text{ArcCot} \left[\, a \, x \, \right] }}{ \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{\, 4/3}} \, \, \mathrm{d} \, x$$

Problem 11: Unable to integrate problem.

$$\int \frac{ \, {\textstyle \mathop{\mathbb{C}}}^{n \, \mathsf{ArcCot} \, [\, a \, x \,]}}{ \, \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{\, 5/3}} \, \, \mathrm{d} \, x$$

Optimal (type 5, 207 leaves, 4 steps):

$$-\frac{3 e^{n \operatorname{ArcCot}[a \, x]} \left(3 \, n - 4 \, a \, x\right)}{a \, c \, \left(16 + 9 \, n^2\right) \, \left(c + a^2 \, c \, x^2\right)^{2/3}} - \\ \left[12 \left(1 + \frac{1}{a^2 \, x^2}\right)^{2/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}}\right)^{\frac{1}{6} \, (4 - 3 \, i \, n)} \, \left(1 - \frac{i}{a \, x}\right)^{\frac{1}{6} \, (-4 + 3 \, i \, n)} \, \left(1 + \frac{i}{a \, x}\right)^{\frac{1}{6} \, (2 - 3 \, i \, n)} \, x\right] \right]$$

Hypergeometric2F1
$$\left[\frac{1}{3}, \frac{1}{6} \left(4-3 \pm n\right), \frac{4}{3}, \frac{2 \pm \left(a+\frac{1}{2} \times x\right)}{\left(a+\frac{1}{2} \times x\right)}\right] / \left(c \left(16+9 n^2\right) \left(c+a^2 c x^2\right)^{2/3}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{ \, e^{n \, \text{ArcCot} \, [\, a \, x \,]}}{ \, \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{5/3}} \, \, \mathrm{d} x$$

Problem 12: Unable to integrate problem.

$$\int \frac{\, {\textstyle \mathop{\mathbb{C}}}^{n \, \mathsf{ArcCot} \, [\, a \, x \,]}}{\, \left(\, c \, + \, a^2 \, c \, \, x^2 \, \right)^{\, 7/3}} \, \, \mathrm{d} x$$

Optimal (type 5, 272 leaves, 5 steps):

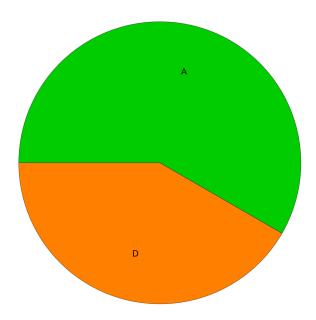
$$-\frac{3 e^{n \operatorname{ArcCot}[a \, x]} \, \left(3 \, n - 8 \, a \, x\right)}{a \, c \, \left(64 + 9 \, n^2\right) \, \left(c + a^2 \, c \, x^2\right)^{4/3}} - \frac{120 \, e^{n \operatorname{ArcCot}[a \, x]} \, \left(3 \, n - 2 \, a \, x\right)}{a \, c^2 \, \left(4 + 9 \, n^2\right) \, \left(64 + 9 \, n^2\right) \, \left(c + a^2 \, c \, x^2\right)^{1/3}} - \\ \left[240 \left(1 + \frac{1}{a^2 \, x^2}\right)^{1/3} \left(\frac{a - \frac{i}{x}}{a + \frac{i}{x}}\right)^{\frac{1}{6} \, (2 - 3 \, i \, n)} \, \left(1 - \frac{i}{a \, x}\right)^{\frac{1}{6} \, (-2 + 3 \, i \, n)} \, \left(1 + \frac{i}{a \, x}\right)^{\frac{1}{6} \, (4 - 3 \, i \, n)} \, x \, \text{Hypergeometric2F1} \right[- \frac{1}{3} \, \frac{1}{6} \, \left(2 - 3 \, i \, n\right) \, \frac{2}{3} \, \frac{2 \, i}{\left(a + \frac{i}{x}\right) \, x}\right] \left/ \left(c^2 \, \left(4 + 9 \, n^2\right) \, \left(64 + 9 \, n^2\right) \, \left(c + a^2 \, c \, x^2\right)^{1/3}\right)$$

Result (type 8, 25 leaves):

$$\int\!\frac{\,{\textstyle\mathop{\mathrm{e}}}^{n\,\mathsf{ArcCot}\,[\,a\,x\,]}}{\,\left(\,c\,+\,a^2\;c\;x^2\right)^{\,7/3}}\;{\rm d}x$$

Summary of Integration Test Results

12 integration problems



- A 7 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 0 unnecessarily complex antiderivatives
- D 5 unable to integrate problems
- E 0 integration timeouts