

# Rubi 4.16.0 Inverse Hyperbolic Integration Test Results

Test results for the 156 problems in "7.1.2 (d x)^m (a+b arcsinh(c x))^n.m"

Test results for the 663 problems in "7.1.4 (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n.m"

Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2) (a + b \operatorname{ArcSinh}[c x])}{x} dx$$

Optimal (type 4, 111 leaves, 8 steps):

$$-\frac{1}{4} b c d x \sqrt{1 + c^2 x^2} - \frac{1}{4} b d \operatorname{ArcSinh}[c x] + \frac{1}{2} d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) + \frac{d (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + d (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b d \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 111 leaves, 8 steps):

$$-\frac{1}{4} b c d x \sqrt{1 + c^2 x^2} - \frac{1}{4} b d \operatorname{ArcSinh}[c x] + \frac{1}{2} d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) - \frac{d (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + d (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + \frac{1}{2} b d \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]$$

Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2) (a + b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 128 leaves, 8 steps):

$$-\frac{b c d \sqrt{1 + c^2 x^2}}{2 x} + \frac{1}{2} b c^2 d \operatorname{ArcSinh}[c x] - \frac{d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])}{2 x^2} + \frac{c^2 d (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + c^2 d (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b c^2 d \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 128 leaves, 8 steps):

$$-\frac{b c d \sqrt{1+c^2 x^2}}{2 x} + \frac{1}{2} b c^2 d \operatorname{ArcSinh}[c x] - \frac{d (1+c^2 x^2) (a+b \operatorname{ArcSinh}[c x])}{2 x^2} - \frac{c^2 d (a+b \operatorname{ArcSinh}[c x])^2}{2 b} + c^2 d (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}[c x]}\right] + \frac{1}{2} b c^2 d \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c x]}\right]$$

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^2 (a+b \operatorname{ArcSinh}[c x])}{x} dx$$

Optimal (type 4, 172 leaves, 12 steps):

$$-\frac{11}{32} b c d^2 x \sqrt{1+c^2 x^2} - \frac{1}{16} b c d^2 x (1+c^2 x^2)^{3/2} - \frac{11}{32} b d^2 \operatorname{ArcSinh}[c x] + \frac{1}{2} d^2 (1+c^2 x^2) (a+b \operatorname{ArcSinh}[c x]) + \frac{1}{4} d^2 (1+c^2 x^2)^2 (a+b \operatorname{ArcSinh}[c x]) + \frac{d^2 (a+b \operatorname{ArcSinh}[c x])^2}{2 b} + d^2 (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{-2 \operatorname{ArcSinh}[c x]}\right] - \frac{1}{2} b d^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}[c x]}\right]$$

Result (type 4, 172 leaves, 12 steps):

$$-\frac{11}{32} b c d^2 x \sqrt{1+c^2 x^2} - \frac{1}{16} b c d^2 x (1+c^2 x^2)^{3/2} - \frac{11}{32} b d^2 \operatorname{ArcSinh}[c x] + \frac{1}{2} d^2 (1+c^2 x^2) (a+b \operatorname{ArcSinh}[c x]) + \frac{1}{4} d^2 (1+c^2 x^2)^2 (a+b \operatorname{ArcSinh}[c x]) - \frac{d^2 (a+b \operatorname{ArcSinh}[c x])^2}{2 b} + d^2 (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}[c x]}\right] + \frac{1}{2} b d^2 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c x]}\right]$$

Problem 17: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^2 (a+b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 187 leaves, 12 steps):

$$\frac{1}{4} b c^3 d^2 x \sqrt{1+c^2 x^2} - \frac{b c d^2 (1+c^2 x^2)^{3/2}}{2 x} + \frac{1}{4} b c^2 d^2 \operatorname{ArcSinh}[c x] + c^2 d^2 (1+c^2 x^2) (a+b \operatorname{ArcSinh}[c x]) - \frac{d^2 (1+c^2 x^2)^2 (a+b \operatorname{ArcSinh}[c x])}{2 x^2} + \frac{c^2 d^2 (a+b \operatorname{ArcSinh}[c x])^2}{b} + 2 c^2 d^2 (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{-2 \operatorname{ArcSinh}[c x]}\right] - b c^2 d^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}[c x]}\right]$$

Result (type 4, 187 leaves, 12 steps):

$$\frac{1}{4} b c^3 d^2 x \sqrt{1 + c^2 x^2} - \frac{b c d^2 (1 + c^2 x^2)^{3/2}}{2 x} + \frac{1}{4} b c^2 d^2 \operatorname{ArcSinh}[c x] + c^2 d^2 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) - \frac{d^2 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])}{2 x^2} - \frac{c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^2}{b} + 2 c^2 d^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + b c^2 d^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^3 (a + b \operatorname{ArcSinh}[c x])}{x} dx$$

Optimal (type 4, 221 leaves, 17 steps):

$$-\frac{19}{48} b c d^3 x \sqrt{1 + c^2 x^2} - \frac{7}{72} b c d^3 x (1 + c^2 x^2)^{3/2} - \frac{1}{36} b c d^3 x (1 + c^2 x^2)^{5/2} - \frac{19}{48} b d^3 \operatorname{ArcSinh}[c x] + \frac{1}{2} d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{4} d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x]) + \frac{d^3 (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b d^3 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 221 leaves, 17 steps):

$$-\frac{19}{48} b c d^3 x \sqrt{1 + c^2 x^2} - \frac{7}{72} b c d^3 x (1 + c^2 x^2)^{3/2} - \frac{1}{36} b c d^3 x (1 + c^2 x^2)^{5/2} - \frac{19}{48} b d^3 \operatorname{ArcSinh}[c x] + \frac{1}{2} d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{4} d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x]) - \frac{d^3 (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + \frac{1}{2} b d^3 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^3 (a + b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 249 leaves, 17 steps):

$$-\frac{3}{32} b c^3 d^3 x \sqrt{1 + c^2 x^2} + \frac{7}{16} b c^3 d^3 x (1 + c^2 x^2)^{3/2} - \frac{b c d^3 (1 + c^2 x^2)^{5/2}}{2 x} - \frac{3}{32} b c^2 d^3 \operatorname{ArcSinh}[c x] + \frac{3}{2} c^2 d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x]) + \frac{3}{4} c^2 d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x]) - \frac{d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])}{2 x^2} + \frac{3 c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + 3 c^2 d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{3}{2} b c^2 d^3 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 249 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{3}{32} b c^3 d^3 x \sqrt{1+c^2 x^2} + \frac{7}{16} b c^3 d^3 x (1+c^2 x^2)^{3/2} - \frac{b c d^3 (1+c^2 x^2)^{5/2}}{2 x} - \frac{3}{32} b c^2 d^3 \operatorname{ArcSinh}[c x] + \\
 & \frac{3}{2} c^2 d^3 (1+c^2 x^2) (a+b \operatorname{ArcSinh}[c x]) + \frac{3}{4} c^2 d^3 (1+c^2 x^2)^2 (a+b \operatorname{ArcSinh}[c x]) - \frac{d^3 (1+c^2 x^2)^3 (a+b \operatorname{ArcSinh}[c x])}{2 x^2} - \\
 & \frac{3 c^2 d^3 (a+b \operatorname{ArcSinh}[c x])^2}{2 b} + 3 c^2 d^3 (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}[c x]}\right] + \frac{3}{2} b c^2 d^3 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c x]}\right]
 \end{aligned}$$

Problem 45: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x^4 (d+c^2 d x^2)^2} dx$$

Optimal (type 4, 239 leaves, 19 steps):

$$\begin{aligned}
 & \frac{b c^3}{3 d^2 \sqrt{1+c^2 x^2}} - \frac{b c}{6 d^2 x^2 \sqrt{1+c^2 x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 d^2 x^3 (1+c^2 x^2)} + \\
 & \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{3 d^2 x (1+c^2 x^2)} + \frac{5 c^4 x (a+b \operatorname{ArcSinh}[c x])}{2 d^2 (1+c^2 x^2)} + \frac{5 c^3 (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2} + \\
 & \frac{13 b c^3 \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{6 d^2} - \frac{5 i b c^3 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{2 d^2} + \frac{5 i b c^3 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{2 d^2}
 \end{aligned}$$

Result (type 4, 264 leaves, 19 steps):

$$\begin{aligned}
 & \frac{5 b c^3}{6 d^2 \sqrt{1+c^2 x^2}} + \frac{b c}{3 d^2 x^2 \sqrt{1+c^2 x^2}} - \frac{b c \sqrt{1+c^2 x^2}}{2 d^2 x^2} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 d^2 x^3 (1+c^2 x^2)} + \\
 & \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{3 d^2 x (1+c^2 x^2)} + \frac{5 c^4 x (a+b \operatorname{ArcSinh}[c x])}{2 d^2 (1+c^2 x^2)} + \frac{5 c^3 (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2} + \\
 & \frac{13 b c^3 \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{6 d^2} - \frac{5 i b c^3 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{2 d^2} + \frac{5 i b c^3 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{2 d^2}
 \end{aligned}$$

Problem 54: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x^4 (d+c^2 d x^2)^3} dx$$

Optimal (type 4, 295 leaves, 23 steps):

$$\begin{aligned}
& - \frac{b c^3}{12 d^3 (1 + c^2 x^2)^{3/2}} - \frac{b c}{6 d^3 x^2 (1 + c^2 x^2)^{3/2}} + \frac{29 b c^3}{24 d^3 \sqrt{1 + c^2 x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{3 d^3 x^3 (1 + c^2 x^2)^2} + \frac{7 c^2 (a + b \operatorname{ArcSinh}[c x])}{3 d^3 x (1 + c^2 x^2)^2} + \\
& \frac{35 c^4 x (a + b \operatorname{ArcSinh}[c x])}{12 d^3 (1 + c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSinh}[c x])}{8 d^3 (1 + c^2 x^2)} + \frac{35 c^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}[e^{\operatorname{ArcSinh}[c x]}]}{4 d^3} + \\
& \frac{19 b c^3 \operatorname{ArcTanh}[\sqrt{1 + c^2 x^2}]}{6 d^3} - \frac{35 i b c^3 \operatorname{PolyLog}[2, -i e^{\operatorname{ArcSinh}[c x]}]}{8 d^3} + \frac{35 i b c^3 \operatorname{PolyLog}[2, i e^{\operatorname{ArcSinh}[c x]}]}{8 d^3}
\end{aligned}$$

Result (type 4, 345 leaves, 23 steps):

$$\begin{aligned}
& \frac{7 b c^3}{36 d^3 (1 + c^2 x^2)^{3/2}} + \frac{b c}{9 d^3 x^2 (1 + c^2 x^2)^{3/2}} + \frac{49 b c^3}{24 d^3 \sqrt{1 + c^2 x^2}} + \frac{5 b c}{9 d^3 x^2 \sqrt{1 + c^2 x^2}} - \frac{5 b c \sqrt{1 + c^2 x^2}}{6 d^3 x^2} - \frac{a + b \operatorname{ArcSinh}[c x]}{3 d^3 x^3 (1 + c^2 x^2)^2} + \\
& \frac{7 c^2 (a + b \operatorname{ArcSinh}[c x])}{3 d^3 x (1 + c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSinh}[c x])}{12 d^3 (1 + c^2 x^2)^2} + \frac{35 c^4 x (a + b \operatorname{ArcSinh}[c x])}{8 d^3 (1 + c^2 x^2)} + \frac{35 c^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}[e^{\operatorname{ArcSinh}[c x]}]}{4 d^3} + \\
& \frac{19 b c^3 \operatorname{ArcTanh}[\sqrt{1 + c^2 x^2}]}{6 d^3} - \frac{35 i b c^3 \operatorname{PolyLog}[2, -i e^{\operatorname{ArcSinh}[c x]}]}{8 d^3} + \frac{35 i b c^3 \operatorname{PolyLog}[2, i e^{\operatorname{ArcSinh}[c x]}]}{8 d^3}
\end{aligned}$$

Problem 56: Result valid but suboptimal antiderivative.

$$\int x^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 119 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b \sqrt{\pi} x^2}{16 c} - \frac{1}{16} b c \sqrt{\pi} x^4 + \frac{\sqrt{\pi} x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c^2} + \frac{1}{4} x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{\sqrt{\pi} (a + b \operatorname{ArcSinh}[c x])^2}{16 b c^3}
\end{aligned}$$

Result (type 3, 181 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b x^2 \sqrt{\pi + c^2 \pi x^2}}{16 c \sqrt{1 + c^2 x^2}} - \frac{b c x^4 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} + \frac{x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c^2} + \\
& \frac{1}{4} x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b c^3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 57: Result valid but suboptimal antiderivative.

$$\int x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 61 leaves, 2 steps):

$$-\frac{b\sqrt{\pi}x}{3c} - \frac{1}{9}bc\sqrt{\pi}x^3 + \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\operatorname{ArcSinh}[cx])}{3c^2\pi}$$

Result (type 3, 105 leaves, 2 steps):

$$-\frac{bx\sqrt{\pi + c^2\pi x^2}}{3c\sqrt{1 + c^2x^2}} - \frac{bcx^3\sqrt{\pi + c^2\pi x^2}}{9\sqrt{1 + c^2x^2}} + \frac{(\pi + c^2\pi x^2)^{3/2}(a + b\operatorname{ArcSinh}[cx])}{3c^2\pi}$$

Problem 58: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi + c^2\pi x^2} (a + b\operatorname{ArcSinh}[cx]) dx$$

Optimal (type 3, 67 leaves, 3 steps):

$$-\frac{1}{4}bc\sqrt{\pi}x^2 + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx]) + \frac{\sqrt{\pi}(a + b\operatorname{ArcSinh}[cx])^2}{4bc}$$

Result (type 3, 111 leaves, 3 steps):

$$-\frac{bcx^2\sqrt{\pi + c^2\pi x^2}}{4\sqrt{1 + c^2x^2}} + \frac{1}{2}x\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx]) + \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx])^2}{4bc\sqrt{1 + c^2x^2}}$$

Problem 59: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx])}{x} dx$$

Optimal (type 4, 89 leaves, 8 steps):

$$-bc\sqrt{\pi}x + \sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx]) - 2\sqrt{\pi}(a + b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[cx]}] - b\sqrt{\pi}\operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[cx]}] + b\sqrt{\pi}\operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[cx]}]$$

Result (type 4, 177 leaves, 8 steps):

$$-\frac{bcx\sqrt{\pi + c^2\pi x^2}}{\sqrt{1 + c^2x^2}} + \sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx]) - \frac{2\sqrt{\pi + c^2\pi x^2}(a + b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[cx]}]}{\sqrt{1 + c^2x^2}} - \frac{b\sqrt{\pi + c^2\pi x^2}\operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[cx]}]}{\sqrt{1 + c^2x^2}} + \frac{b\sqrt{\pi + c^2\pi x^2}\operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[cx]}]}{\sqrt{1 + c^2x^2}}$$

### Problem 60: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x^2} dx$$

Optimal (type 3, 61 leaves, 3 steps):

$$-\frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{c \sqrt{\pi} (a + b \operatorname{ArcSinh}[c x])^2}{2b} + b c \sqrt{\pi} \operatorname{Log}[x]$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{c \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2b \sqrt{1 + c^2 x^2}} + \frac{b c \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{\sqrt{1 + c^2 x^2}}$$

### Problem 61: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 113 leaves, 8 steps):

$$-\frac{b c \sqrt{\pi}}{2x} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2x^2} - c^2 \sqrt{\pi} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}] -$$

$$\frac{1}{2} b c^2 \sqrt{\pi} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}] + \frac{1}{2} b c^2 \sqrt{\pi} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]$$

Result (type 4, 201 leaves, 8 steps):

$$-\frac{b c \sqrt{\pi + c^2 \pi x^2}}{2x \sqrt{1 + c^2 x^2}} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2x^2} - \frac{c^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} -$$

$$\frac{b c^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{1 + c^2 x^2}} + \frac{b c^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{1 + c^2 x^2}}$$

### Problem 62: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x^4} dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{b c \sqrt{\pi}}{6 x^2} - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x^3} + \frac{1}{3} b c^3 \sqrt{\pi} \operatorname{Log}[x]$$

Result (type 3, 106 leaves, 3 steps):

$$-\frac{b c \sqrt{\pi + c^2 \pi x^2}}{6 x^2 \sqrt{1 + c^2 x^2}} - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x^3} + \frac{b c^3 \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{3 \sqrt{1 + c^2 x^2}}$$

**Problem 64: Result valid but suboptimal antiderivative.**

$$\int x^2 (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{b \pi^{3/2} x^2}{32 c} - \frac{7}{96} b c \pi^{3/2} x^4 - \frac{1}{36} b c^3 \pi^{3/2} x^6 + \frac{\pi^{3/2} x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{16 c^2} +$$

$$\frac{1}{8} \pi x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{\pi^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c^3}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{b \pi x^2 \sqrt{\pi + c^2 \pi x^2}}{32 c \sqrt{1 + c^2 x^2}} - \frac{7 b c \pi x^4 \sqrt{\pi + c^2 \pi x^2}}{96 \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x^6 \sqrt{\pi + c^2 \pi x^2}}{36 \sqrt{1 + c^2 x^2}} + \frac{\pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{16 c^2} +$$

$$\frac{1}{8} \pi x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{\pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c^3 \sqrt{1 + c^2 x^2}}$$

**Problem 65: Result valid but suboptimal antiderivative.**

$$\int x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{b \pi^{3/2} x}{5 c} - \frac{2}{15} b c \pi^{3/2} x^3 - \frac{1}{25} b c^3 \pi^{3/2} x^5 + \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{5 c^2 \pi}$$

Result (type 3, 146 leaves, 3 steps):

$$-\frac{b \pi x \sqrt{\pi + c^2 \pi x^2}}{5 c \sqrt{1 + c^2 x^2}} - \frac{2 b c \pi x^3 \sqrt{\pi + c^2 \pi x^2}}{15 \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x^5 \sqrt{\pi + c^2 \pi x^2}}{25 \sqrt{1 + c^2 x^2}} + \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{5 c^2 \pi}$$



### Problem 66: Result valid but suboptimal antiderivative.

$$\int (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 111 leaves, 6 steps):

$$-\frac{5}{16} b c \pi^{3/2} x^2 - \frac{1}{16} b c^3 \pi^{3/2} x^4 + \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{4} x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{3 \pi^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b c}$$

Result (type 3, 180 leaves, 6 steps):

$$-\frac{5 b c \pi x^2 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x^4 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} + \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{4} x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{3 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b c \sqrt{1 + c^2 x^2}}$$

### Problem 67: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x} \, dx$$

Optimal (type 4, 134 leaves, 10 steps):

$$-\frac{4}{3} b c \pi^{3/2} x - \frac{1}{9} b c^3 \pi^{3/2} x^3 + \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{3} (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - 2 \pi^{3/2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}] - b \pi^{3/2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}] + b \pi^{3/2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]$$

Result (type 4, 249 leaves, 10 steps):

$$-\frac{4 b c \pi x \sqrt{\pi + c^2 \pi x^2}}{3 \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x^3 \sqrt{\pi + c^2 \pi x^2}}{9 \sqrt{1 + c^2 x^2}} + \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{3} (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \frac{b \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} + \frac{b \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}$$

### Problem 68: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x^2} \, dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{1}{4} b c^3 \pi^{3/2} x^2 + \frac{3}{2} c^2 \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{3 c \pi^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b} + b c \pi^{3/2} \operatorname{Log}[x]$$

Result (type 3, 177 leaves, 6 steps):

$$-\frac{b c^3 \pi x^2 \sqrt{\pi + c^2 \pi x^2}}{4 \sqrt{1 + c^2 x^2}} + \frac{3}{2} c^2 \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{3 c \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b \sqrt{1 + c^2 x^2}} + \frac{b c \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{\sqrt{1 + c^2 x^2}}$$

Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 155 leaves, 11 steps):

$$-\frac{b c \pi^{3/2}}{2 x} - b c^3 \pi^{3/2} x + \frac{3}{2} c^2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{2 x^2} - 3 c^2 \pi^{3/2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}] - \frac{3}{2} b c^2 \pi^{3/2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}] + \frac{3}{2} b c^2 \pi^{3/2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]$$

Result (type 4, 270 leaves, 11 steps):

$$-\frac{b c \pi \sqrt{\pi + c^2 \pi x^2}}{2 x \sqrt{1 + c^2 x^2}} - \frac{b c^3 \pi x \sqrt{\pi + c^2 \pi x^2}}{\sqrt{1 + c^2 x^2}} + \frac{3}{2} c^2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{2 x^2} - \frac{3 c^2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \frac{3 b c^2 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{1 + c^2 x^2}} + \frac{3 b c^2 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{1 + c^2 x^2}}$$

Problem 70: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x^4} dx$$

Optimal (type 3, 115 leaves, 6 steps):

$$-\frac{b c \pi^{3/2}}{6 x^2} - \frac{c^2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x} - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 x^3} + \frac{c^3 \pi^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b} + \frac{4}{3} b c^3 \pi^{3/2} \operatorname{Log}[x]$$

Result (type 3, 184 leaves, 6 steps):

$$-\frac{b c \pi \sqrt{\pi + c^2 \pi x^2}}{6 x^2 \sqrt{1 + c^2 x^2}} - \frac{c^2 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{x} - \frac{(\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 x^3} + \frac{c^3 \pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b \sqrt{1 + c^2 x^2}} + \frac{4 b c^3 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{3 \sqrt{1 + c^2 x^2}}$$

Problem 72: Result valid but suboptimal antiderivative.

$$\int x^2 (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 3, 213 leaves, 12 steps):

$$-\frac{5 b \pi^{5/2} x^2}{256 c} - \frac{59}{768} b c \pi^{5/2} x^4 - \frac{17}{288} b c^3 \pi^{5/2} x^6 - \frac{1}{64} b c^5 \pi^{5/2} x^8 + \frac{5 \pi^{5/2} x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{128 c^2} + \frac{5}{64} \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5}{48} \pi x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{8} x^3 (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{5 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{256 b c^3}$$

Result (type 3, 337 leaves, 12 steps):

$$-\frac{5 b \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{256 c \sqrt{1 + c^2 x^2}} - \frac{59 b c \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{768 \sqrt{1 + c^2 x^2}} - \frac{17 b c^3 \pi^2 x^6 \sqrt{\pi + c^2 \pi x^2}}{288 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^8 \sqrt{\pi + c^2 \pi x^2}}{64 \sqrt{1 + c^2 x^2}} + \frac{5 \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{128 c^2} + \frac{5}{64} \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5}{48} \pi x^3 (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{8} x^3 (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{256 b c^3 \sqrt{1 + c^2 x^2}}$$

Problem 73: Result valid but suboptimal antiderivative.

$$\int x (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$-\frac{b \pi^{5/2} x}{7 c} - \frac{1}{7} b c \pi^{5/2} x^3 - \frac{3}{35} b c^3 \pi^{5/2} x^5 - \frac{1}{49} b c^5 \pi^{5/2} x^7 + \frac{(\pi + c^2 \pi x^2)^{7/2} (a + b \operatorname{ArcSinh}[c x])}{7 c^2 \pi}$$

Result (type 3, 193 leaves, 3 steps):

$$-\frac{b \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{7 c \sqrt{1 + c^2 x^2}} - \frac{b c \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2}}{7 \sqrt{1 + c^2 x^2}} - \frac{3 b c^3 \pi^2 x^5 \sqrt{\pi + c^2 \pi x^2}}{35 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^7 \sqrt{\pi + c^2 \pi x^2}}{49 \sqrt{1 + c^2 x^2}} + \frac{(\pi + c^2 \pi x^2)^{7/2} (a + b \operatorname{ArcSinh}[c x])}{7 c^2 \pi}$$

**Problem 74: Result valid but suboptimal antiderivative.**

$$\int (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) \, dx$$

Optimal (type 3, 165 leaves, 8 steps):

$$-\frac{25}{96} b c \pi^{5/2} x^2 - \frac{5}{96} b c^3 \pi^{5/2} x^4 - \frac{b \pi^{5/2} (1 + c^2 x^2)^3}{36 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) +$$

$$\frac{5}{24} \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c}$$

Result (type 3, 254 leaves, 8 steps):

$$-\frac{25 b c \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{96 \sqrt{1 + c^2 x^2}} - \frac{5 b c^3 \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{96 \sqrt{1 + c^2 x^2}} - \frac{b \pi^2 (1 + c^2 x^2)^{5/2} \sqrt{\pi + c^2 \pi x^2}}{36 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) +$$

$$\frac{5}{24} \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{6} x (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{32 b c \sqrt{1 + c^2 x^2}}$$

**Problem 75: Result valid but suboptimal antiderivative.**

$$\int \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} \, dx$$

Optimal (type 4, 179 leaves, 13 steps):

$$-\frac{23}{15} b c \pi^{5/2} x - \frac{11}{45} b c^3 \pi^{5/2} x^3 - \frac{1}{25} b c^5 \pi^{5/2} x^5 + \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) +$$

$$\frac{1}{3} \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{5} (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) -$$

$$2 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}] - b \pi^{5/2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}] + b \pi^{5/2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]$$

Result (type 4, 329 leaves, 13 steps):

$$\begin{aligned}
& - \frac{23 b c \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{15 \sqrt{1 + c^2 x^2}} - \frac{11 b c^3 \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2}}{45 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^5 \sqrt{\pi + c^2 \pi x^2}}{25 \sqrt{1 + c^2 x^2}} + \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \\
& \frac{1}{3} \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) + \frac{1}{5} (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{2 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \\
& \frac{b \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} + \frac{b \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

**Problem 76: Result valid but suboptimal antiderivative.**

$$\int \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x^2} dx$$

Optimal (type 3, 157 leaves, 10 steps):

$$\begin{aligned}
& - \frac{9}{16} b c^3 \pi^{5/2} x^2 - \frac{1}{16} b c^5 \pi^{5/2} x^4 + \frac{15}{8} c^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \\
& \frac{5}{4} c^2 \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{15 c \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b} + b c \pi^{5/2} \operatorname{Log}[x]
\end{aligned}$$

Result (type 3, 257 leaves, 10 steps):

$$\begin{aligned}
& - \frac{9 b c^3 \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^4 \sqrt{\pi + c^2 \pi x^2}}{16 \sqrt{1 + c^2 x^2}} + \frac{15}{8} c^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{5}{4} c^2 \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \\
& \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{15 c \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{16 b \sqrt{1 + c^2 x^2}} + \frac{b c \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

**Problem 77: Result valid but suboptimal antiderivative.**

$$\int \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x^3} dx$$

Optimal (type 4, 205 leaves, 13 steps):

$$\begin{aligned}
& - \frac{b c \pi^{5/2}}{2 x} - \frac{7}{3} b c^3 \pi^{5/2} x - \frac{1}{9} b c^5 \pi^{5/2} x^3 + \frac{5}{2} c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \\
& \frac{5}{6} c^2 \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{2 x^2} - \\
& 5 c^2 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}] - \frac{5}{2} b c^2 \pi^{5/2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}] + \frac{5}{2} b c^2 \pi^{5/2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]
\end{aligned}$$

Result (type 4, 355 leaves, 13 steps):

$$\begin{aligned}
 & -\frac{b c \pi^2 \sqrt{\pi + c^2 \pi x^2}}{2 x \sqrt{1 + c^2 x^2}} - \frac{7 b c^3 \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{3 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^3 \sqrt{\pi + c^2 \pi x^2}}{9 \sqrt{1 + c^2 x^2}} + \frac{5}{2} c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) + \\
 & \frac{5}{6} c^2 \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{2 x^2} - \frac{5 c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[\frac{e^{\operatorname{ArcSinh}[c x]}}{\sqrt{1 + c^2 x^2}}\right]}{\sqrt{1 + c^2 x^2}} - \\
 & \frac{5 b c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}} + \frac{5 b c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{2 \sqrt{1 + c^2 x^2}}
 \end{aligned}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x^4} dx$$

Optimal (type 3, 166 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{b c \pi^{5/2}}{6 x^2} - \frac{1}{4} b c^5 \pi^{5/2} x^2 + \frac{5}{2} c^4 \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{5 c^2 \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 x} - \\
 & \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{3 x^3} + \frac{5 c^3 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b} + \frac{7}{3} b c^3 \pi^{5/2} \operatorname{Log}[x]
 \end{aligned}$$

Result (type 3, 266 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{b c \pi^2 \sqrt{\pi + c^2 \pi x^2}}{6 x^2 \sqrt{1 + c^2 x^2}} - \frac{b c^5 \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2}}{4 \sqrt{1 + c^2 x^2}} + \frac{5}{2} c^4 \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{5 c^2 \pi (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{3 x} - \\
 & \frac{(\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{3 x^3} + \frac{5 c^3 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 b \sqrt{1 + c^2 x^2}} + \frac{7 b c^3 \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{Log}[x]}{3 \sqrt{1 + c^2 x^2}}
 \end{aligned}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSinh}[c x])}{\sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$-\frac{8bx}{15c^5\sqrt{\pi}} + \frac{4bx^3}{45c^3\sqrt{\pi}} - \frac{bx^5}{25c\sqrt{\pi}} + \frac{8\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{15c^6\pi} -$$

$$\frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{5c^2\pi}$$

Result (type 3, 215 leaves, 6 steps):

$$-\frac{8bx\sqrt{1+c^2x^2}}{15c^5\sqrt{\pi+c^2\pi x^2}} + \frac{4bx^3\sqrt{1+c^2x^2}}{45c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^5\sqrt{1+c^2x^2}}{25c\sqrt{\pi+c^2\pi x^2}} +$$

$$\frac{8\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{15c^6\pi} - \frac{4x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{15c^4\pi} + \frac{x^4\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{5c^2\pi}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int \frac{x^4(a+b\text{ArcSinh}[cx])}{\sqrt{\pi+c^2\pi x^2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$\frac{3bx^2}{16c^3\sqrt{\pi}} - \frac{bx^4}{16c\sqrt{\pi}} - \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{4c^2\pi} + \frac{3(a+b\text{ArcSinh}[cx])^2}{16bc^5\sqrt{\pi}}$$

Result (type 3, 170 leaves, 5 steps):

$$\frac{3bx^2\sqrt{1+c^2x^2}}{16c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^4\sqrt{1+c^2x^2}}{16c\sqrt{\pi+c^2\pi x^2}} - \frac{3x\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{8c^4\pi} + \frac{x^3\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{4c^2\pi} + \frac{3(a+b\text{ArcSinh}[cx])^2}{16bc^5\sqrt{\pi}}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int \frac{x^3(a+b\text{ArcSinh}[cx])}{\sqrt{\pi+c^2\pi x^2}} dx$$

Optimal (type 3, 98 leaves, 4 steps):

$$\frac{2bx}{3c^3\sqrt{\pi}} - \frac{bx^3}{9c\sqrt{\pi}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{3c^2\pi}$$

Result (type 3, 142 leaves, 4 steps):

$$\frac{2bx\sqrt{1+c^2x^2}}{3c^3\sqrt{\pi+c^2\pi x^2}} - \frac{bx^3\sqrt{1+c^2x^2}}{9c\sqrt{\pi+c^2\pi x^2}} - \frac{2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{3c^4\pi} + \frac{x^2\sqrt{\pi+c^2\pi x^2}(a+b\text{ArcSinh}[cx])}{3c^2\pi}$$

## Problem 83: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])}{\sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$-\frac{b x^2}{4 c \sqrt{\pi}} + \frac{x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^2 \pi} - \frac{(a + b \operatorname{ArcSinh}[c x])^2}{4 b c^3 \sqrt{\pi}}$$

Result (type 3, 97 leaves, 3 steps):

$$-\frac{b x^2 \sqrt{1 + c^2 x^2}}{4 c \sqrt{\pi + c^2 \pi x^2}} + \frac{x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^2 \pi} - \frac{(a + b \operatorname{ArcSinh}[c x])^2}{4 b c^3 \sqrt{\pi}}$$

## Problem 84: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcSinh}[c x])}{\sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$-\frac{b x}{c \sqrt{\pi}} + \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi}$$

Result (type 3, 64 leaves, 2 steps):

$$-\frac{b x \sqrt{1 + c^2 x^2}}{c \sqrt{\pi + c^2 \pi x^2}} + \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi}$$

## Problem 87: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^2 \sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 3, 41 leaves, 2 steps):

$$-\frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{\pi x} + \frac{b c \operatorname{Log}[x]}{\sqrt{\pi}}$$

Result (type 3, 63 leaves, 2 steps):



$$-\frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{\pi x} + \frac{b c \sqrt{1 + c^2 x^2} \operatorname{Log}[x]}{\sqrt{\pi + c^2 \pi x^2}}$$

Problem 88: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^3 \sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 4, 115 leaves, 8 steps):

$$-\frac{b c}{2 \sqrt{\pi} x} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \pi x^2} + \frac{c^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{\pi}} + \frac{b c^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{\pi}} - \frac{b c^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{\pi}}$$

Result (type 4, 137 leaves, 8 steps):

$$-\frac{b c \sqrt{1 + c^2 x^2}}{2 x \sqrt{\pi + c^2 \pi x^2}} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \pi x^2} + \frac{c^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{\pi}} + \frac{b c^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{\pi}} - \frac{b c^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \sqrt{\pi}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^4 \sqrt{\pi + c^2 \pi x^2}} dx$$

Optimal (type 3, 97 leaves, 4 steps):

$$-\frac{b c}{6 \sqrt{\pi} x^2} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x^3} + \frac{2 c^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x} - \frac{2 b c^3 \operatorname{Log}[x]}{3 \sqrt{\pi}}$$

Result (type 3, 141 leaves, 4 steps):

$$-\frac{b c \sqrt{1 + c^2 x^2}}{6 x^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{\sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x^3} + \frac{2 c^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{3 \pi x} - \frac{2 b c^3 \sqrt{1 + c^2 x^2} \operatorname{Log}[x]}{3 \sqrt{\pi + c^2 \pi x^2}}$$

## Problem 91: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{b x^2}{4 c^3 \pi^{3/2}} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{3 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^4 \pi^2} - \frac{3 (a + b \operatorname{ArcSinh}[c x])^2}{4 b c^5 \pi^{3/2}} - \frac{b \operatorname{Log}[1 + c^2 x^2]}{2 c^5 \pi^{3/2}}$$

Result (type 3, 181 leaves, 7 steps):

$$-\frac{b x^2 \sqrt{1 + c^2 x^2}}{4 c^3 \pi \sqrt{\pi + c^2 \pi x^2}} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{3 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^4 \pi^2} - \frac{3 (a + b \operatorname{ArcSinh}[c x])^2}{4 b c^5 \pi^{3/2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 c^5 \pi \sqrt{\pi + c^2 \pi x^2}}$$

## Problem 93: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 3, 80 leaves, 3 steps):

$$-\frac{x (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{(a + b \operatorname{ArcSinh}[c x])^2}{2 b c^3 \pi^{3/2}} + \frac{b \operatorname{Log}[1 + c^2 x^2]}{2 c^3 \pi^{3/2}}$$

Result (type 3, 105 leaves, 3 steps):

$$-\frac{x (a + b \operatorname{ArcSinh}[c x])}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{(a + b \operatorname{ArcSinh}[c x])^2}{2 b c^3 \pi^{3/2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 c^3 \pi \sqrt{\pi + c^2 \pi x^2}}$$

## Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 3, 45 leaves, 2 steps):

$$-\frac{a + b \operatorname{ArcSinh}[c x]}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{b \operatorname{ArcTan}[c x]}{c^2 \pi^{3/2}}$$

Result (type 3, 70 leaves, 2 steps):

$$-\frac{a + b \operatorname{ArcSinh}[c x]}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{c^2 \pi \sqrt{\pi + c^2 \pi x^2}}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{(\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{x (a + b \operatorname{ArcSinh}[c x])}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{b \operatorname{Log}[1 + c^2 x^2]}{2 c \pi^{3/2}}$$

Result (type 3, 76 leaves, 2 steps):

$$\frac{x (a + b \operatorname{ArcSinh}[c x])}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 c \pi \sqrt{\pi + c^2 \pi x^2}}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x (\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 4, 94 leaves, 8 steps):

$$\frac{a + b \operatorname{ArcSinh}[c x]}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{b \operatorname{ArcTan}[c x]}{\pi^{3/2}} - \frac{2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} - \frac{b \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} + \frac{b \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}}$$

Result (type 4, 119 leaves, 8 steps):

$$\frac{a + b \operatorname{ArcSinh}[c x]}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{\pi \sqrt{\pi + c^2 \pi x^2}} - \frac{2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} - \frac{b \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} + \frac{b \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^3 (\pi + c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 4, 162 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b c}{2 \pi^{3/2} x} - \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x])}{2 \pi \sqrt{\pi + c^2 \pi x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{2 \pi x^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{b c^2 \operatorname{ArcTan}[c x]}{\pi^{3/2}} + \\
& \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} + \frac{3 b c^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \pi^{3/2}} - \frac{3 b c^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \pi^{3/2}}
\end{aligned}$$

Result (type 4, 212 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b c \sqrt{1 + c^2 x^2}}{2 \pi x \sqrt{\pi + c^2 \pi x^2}} - \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x])}{2 \pi \sqrt{\pi + c^2 \pi x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{2 \pi x^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{b c^2 \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{\pi \sqrt{\pi + c^2 \pi x^2}} + \\
& \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{3/2}} + \frac{3 b c^2 \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 \pi^{3/2}} - \frac{3 b c^2 \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 \pi^{3/2}}
\end{aligned}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int \frac{x^6 (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 3, 192 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b x^2}{4 c^5 \pi^{5/2}} - \frac{b}{6 c^7 \pi^{5/2} (1 + c^2 x^2)} - \frac{x^5 (a + b \operatorname{ArcSinh}[c x])}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} - \frac{5 x^3 (a + b \operatorname{ArcSinh}[c x])}{3 c^4 \pi^2 \sqrt{\pi + c^2 \pi x^2}} + \\
& \frac{5 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^6 \pi^3} - \frac{5 (a + b \operatorname{ArcSinh}[c x])^2}{4 b c^7 \pi^{5/2}} - \frac{7 b \operatorname{Log}[1 + c^2 x^2]}{6 c^7 \pi^{5/2}}
\end{aligned}$$

Result (type 3, 256 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b}{6 c^7 \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} - \frac{b x^2 \sqrt{1 + c^2 x^2}}{4 c^5 \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{x^5 (a + b \operatorname{ArcSinh}[c x])}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} - \\
& \frac{5 x^3 (a + b \operatorname{ArcSinh}[c x])}{3 c^4 \pi^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{5 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^6 \pi^3} - \frac{5 (a + b \operatorname{ArcSinh}[c x])^2}{4 b c^7 \pi^{5/2}} - \frac{7 b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{6 c^7 \pi^2 \sqrt{\pi + c^2 \pi x^2}}
\end{aligned}$$

Problem 102: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^{5/2} (1 + c^2 x^2)} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} - \frac{x (a + b \operatorname{ArcSinh}[c x])}{c^4 \pi^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{(a + b \operatorname{ArcSinh}[c x])^2}{2 b c^5 \pi^{5/2}} + \frac{2 b \operatorname{Log}[1 + c^2 x^2]}{3 c^5 \pi^{5/2}}$$

Result (type 3, 178 leaves, 7 steps):

$$\frac{b}{6 c^5 \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} - \frac{x (a + b \operatorname{ArcSinh}[c x])}{c^4 \pi^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{(a + b \operatorname{ArcSinh}[c x])^2}{2 b c^5 \pi^{5/2}} + \frac{2 b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{3 c^5 \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^{5/2} (1 + c^2 x^2)} + \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} - \frac{b \operatorname{Log}[1 + c^2 x^2]}{6 c^3 \pi^{5/2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b}{6 c^3 \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{x^3 (a + b \operatorname{ArcSinh}[c x])}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{6 c^3 \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

Problem 105: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcSinh}[c x])}{(\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 3, 75 leaves, 3 steps):

$$\frac{b x}{6 c \pi^{5/2} (1 + c^2 x^2)} - \frac{a + b \operatorname{ArcSinh}[c x]}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{b \operatorname{ArcTan}[c x]}{6 c^2 \pi^{5/2}}$$

Result (type 3, 114 leaves, 3 steps):

$$\frac{b x}{6 c \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{3 c^2 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{6 c^2 \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

## Problem 106: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{(\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 3, 108 leaves, 4 steps):

$$\frac{b}{6 c \pi^{5/2} (1 + c^2 x^2)} + \frac{x (a + b \operatorname{ArcSinh}[c x])}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{2 x (a + b \operatorname{ArcSinh}[c x])}{3 \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{b \operatorname{Log}[1 + c^2 x^2]}{3 c \pi^{5/2}}$$

Result (type 3, 147 leaves, 4 steps):

$$\frac{b}{6 c \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{x (a + b \operatorname{ArcSinh}[c x])}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{2 x (a + b \operatorname{ArcSinh}[c x])}{3 \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{3 c \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

## Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x (\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 4, 148 leaves, 11 steps):

$$-\frac{b c x}{6 \pi^{5/2} (1 + c^2 x^2)} + \frac{a + b \operatorname{ArcSinh}[c x]}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{a + b \operatorname{ArcSinh}[c x]}{\pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{7 b \operatorname{ArcTan}[c x]}{6 \pi^{5/2}} -$$

$$\frac{2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}} - \frac{b \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}} + \frac{b \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}}$$

Result (type 4, 187 leaves, 11 steps):

$$-\frac{b c x}{6 \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{a + b \operatorname{ArcSinh}[c x]}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{a + b \operatorname{ArcSinh}[c x]}{\pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{7 b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{6 \pi^2 \sqrt{\pi + c^2 \pi x^2}} -$$

$$\frac{2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}} - \frac{b \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}} + \frac{b \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\pi^{5/2}}$$

## Problem 109: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^3 (\pi + c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 4, 247 leaves, 15 steps):

$$\begin{aligned}
& -\frac{3bc}{4\pi^{5/2}x} + \frac{bc}{4\pi^{5/2}x(1+c^2x^2)} + \frac{5bc^3x}{12\pi^{5/2}(1+c^2x^2)} - \frac{5c^2(a+b\operatorname{ArcSinh}[cx])}{6\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\operatorname{ArcSinh}[cx]}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}} - \frac{5c^2(a+b\operatorname{ArcSinh}[cx])}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \\
& \frac{13bc^2\operatorname{ArcTan}[cx]}{6\pi^{5/2}} + \frac{5c^2(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[cx]}]}{\pi^{5/2}} + \frac{5bc^2\operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[cx]}]}{2\pi^{5/2}} - \frac{5bc^2\operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[cx]}]}{2\pi^{5/2}}
\end{aligned}$$

Result (type 4, 325 leaves, 15 steps):

$$\begin{aligned}
& \frac{bc}{4\pi^2x\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} + \frac{5bc^3x}{12\pi^2\sqrt{1+c^2x^2}\sqrt{\pi+c^2\pi x^2}} - \frac{3bc\sqrt{1+c^2x^2}}{4\pi^2x\sqrt{\pi+c^2\pi x^2}} - \\
& \frac{5c^2(a+b\operatorname{ArcSinh}[cx])}{6\pi(\pi+c^2\pi x^2)^{3/2}} - \frac{a+b\operatorname{ArcSinh}[cx]}{2\pi x^2(\pi+c^2\pi x^2)^{3/2}} - \frac{5c^2(a+b\operatorname{ArcSinh}[cx])}{2\pi^2\sqrt{\pi+c^2\pi x^2}} + \frac{13bc^2\sqrt{1+c^2x^2}\operatorname{ArcTan}[cx]}{6\pi^2\sqrt{\pi+c^2\pi x^2}} + \\
& \frac{5c^2(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[cx]}]}{\pi^{5/2}} + \frac{5bc^2\operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[cx]}]}{2\pi^{5/2}} - \frac{5bc^2\operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[cx]}]}{2\pi^{5/2}}
\end{aligned}$$

Problem 120: Result optimal but 3 more steps used.

$$\int x^3 \sqrt{d+c^2dx^2} (a+b\operatorname{ArcSinh}[cx]) dx$$

Optimal (type 3, 175 leaves, 3 steps):

$$\frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{ArcSinh}[cx])}{3c^4d} + \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{ArcSinh}[cx])}{5c^4d^2}$$

Result (type 3, 175 leaves, 6 steps):

$$\frac{2bx\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}} - \frac{bx^3\sqrt{d+c^2dx^2}}{45c\sqrt{1+c^2x^2}} - \frac{bcx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{ArcSinh}[cx])}{3c^4d} + \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{ArcSinh}[cx])}{5c^4d^2}$$

Problem 128: Result optimal but 3 more steps used.

$$\int x^3 (d+c^2dx^2)^{3/2} (a+b\operatorname{ArcSinh}[cx]) dx$$

Optimal (type 3, 217 leaves, 4 steps):

$$\begin{aligned}
& \frac{2bdx\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}} - \frac{bdx^3\sqrt{d+c^2dx^2}}{105c\sqrt{1+c^2x^2}} - \frac{8bcdx^5\sqrt{d+c^2dx^2}}{175\sqrt{1+c^2x^2}} - \\
& \frac{bc^3dx^7\sqrt{d+c^2dx^2}}{49\sqrt{1+c^2x^2}} - \frac{(d+c^2dx^2)^{5/2}(a+b\operatorname{ArcSinh}[cx])}{5c^4d} + \frac{(d+c^2dx^2)^{7/2}(a+b\operatorname{ArcSinh}[cx])}{7c^4d^2}
\end{aligned}$$

Result (type 3, 217 leaves, 7 steps):

$$\frac{2 b d x \sqrt{d+c^2 d x^2}}{35 c^3 \sqrt{1+c^2 x^2}} - \frac{b d x^3 \sqrt{d+c^2 d x^2}}{105 c \sqrt{1+c^2 x^2}} - \frac{8 b c d x^5 \sqrt{d+c^2 d x^2}}{175 \sqrt{1+c^2 x^2}} - \frac{b c^3 d x^7 \sqrt{d+c^2 d x^2}}{49 \sqrt{1+c^2 x^2}} - \frac{(d+c^2 d x^2)^{5/2} (a+b \operatorname{ArcSinh}[c x])}{5 c^4 d} + \frac{(d+c^2 d x^2)^{7/2} (a+b \operatorname{ArcSinh}[c x])}{7 c^4 d^2}$$

Problem 136: Result optimal but 3 more steps used.

$$\int x^3 (d+c^2 d x^2)^{5/2} (a+b \operatorname{ArcSinh}[c x]) dx$$

Optimal (type 3, 266 leaves, 4 steps):

$$\frac{2 b d^2 x \sqrt{d+c^2 d x^2}}{63 c^3 \sqrt{1+c^2 x^2}} - \frac{b d^2 x^3 \sqrt{d+c^2 d x^2}}{189 c \sqrt{1+c^2 x^2}} - \frac{b c d^2 x^5 \sqrt{d+c^2 d x^2}}{21 \sqrt{1+c^2 x^2}} - \frac{19 b c^3 d^2 x^7 \sqrt{d+c^2 d x^2}}{441 \sqrt{1+c^2 x^2}} - \frac{b c^5 d^2 x^9 \sqrt{d+c^2 d x^2}}{81 \sqrt{1+c^2 x^2}} - \frac{(d+c^2 d x^2)^{7/2} (a+b \operatorname{ArcSinh}[c x])}{7 c^4 d} + \frac{(d+c^2 d x^2)^{9/2} (a+b \operatorname{ArcSinh}[c x])}{9 c^4 d^2}$$

Result (type 3, 266 leaves, 7 steps):

$$\frac{2 b d^2 x \sqrt{d+c^2 d x^2}}{63 c^3 \sqrt{1+c^2 x^2}} - \frac{b d^2 x^3 \sqrt{d+c^2 d x^2}}{189 c \sqrt{1+c^2 x^2}} - \frac{b c d^2 x^5 \sqrt{d+c^2 d x^2}}{21 \sqrt{1+c^2 x^2}} - \frac{19 b c^3 d^2 x^7 \sqrt{d+c^2 d x^2}}{441 \sqrt{1+c^2 x^2}} - \frac{b c^5 d^2 x^9 \sqrt{d+c^2 d x^2}}{81 \sqrt{1+c^2 x^2}} - \frac{(d+c^2 d x^2)^{7/2} (a+b \operatorname{ArcSinh}[c x])}{7 c^4 d} + \frac{(d+c^2 d x^2)^{9/2} (a+b \operatorname{ArcSinh}[c x])}{9 c^4 d^2}$$

Problem 146: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a+b \operatorname{ArcSinh}[c x])}{\sqrt{d+c^2 d x^2}} dx$$

Optimal (type 3, 192 leaves, 5 steps):

$$\frac{3 b x^2 \sqrt{1+c^2 x^2}}{16 c^3 \sqrt{d+c^2 d x^2}} - \frac{b x^4 \sqrt{1+c^2 x^2}}{16 c \sqrt{d+c^2 d x^2}} - \frac{3 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{8 c^4 d} + \frac{x^3 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{4 c^2 d} + \frac{3 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{16 b c^5 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 192 leaves, 6 steps):



$$\frac{3 b x^2 \sqrt{1+c^2 x^2}}{16 c^3 \sqrt{d+c^2 d x^2}} - \frac{b x^4 \sqrt{1+c^2 x^2}}{16 c \sqrt{d+c^2 d x^2}} - \frac{3 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{8 c^4 d} +$$

$$\frac{x^3 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{4 c^2 d} + \frac{3 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{16 b c^5 \sqrt{d+c^2 d x^2}}$$

Problem 148: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a+b \operatorname{ArcSinh}[c x])}{\sqrt{d+c^2 d x^2}} dx$$

Optimal (type 3, 119 leaves, 3 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c \sqrt{d+c^2 d x^2}} + \frac{x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^2 d} - \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^3 \sqrt{d+c^2 d x^2}}$$

Result (type 3, 119 leaves, 4 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c \sqrt{d+c^2 d x^2}} + \frac{x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^2 d} - \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^3 \sqrt{d+c^2 d x^2}}$$

Problem 150: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{\sqrt{d+c^2 d x^2}} dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c \sqrt{d+c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c \sqrt{d+c^2 d x^2}}$$

Problem 151: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x \sqrt{d+c^2 d x^2}} dx$$

Optimal (type 4, 122 leaves, 6 steps):

$$-\frac{2\sqrt{1+c^2x^2}(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,-e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}$$

Result (type 4, 122 leaves, 7 steps):

$$-\frac{2\sqrt{1+c^2x^2}(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}-\frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,-e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}+\frac{b\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}$$

Problem 153: Result optimal but 1 more steps used.

$$\int \frac{a+b\operatorname{ArcSinh}[cx]}{x^3\sqrt{d+c^2dx^2}} dx$$

Optimal (type 4, 203 leaves, 8 steps):

$$-\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{ArcSinh}[cx])}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}+\frac{bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,-e^{\operatorname{ArcSinh}[cx]}\right]}{2\sqrt{d+c^2dx^2}}-\frac{bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,e^{\operatorname{ArcSinh}[cx]}\right]}{2\sqrt{d+c^2dx^2}}$$

Result (type 4, 203 leaves, 9 steps):

$$-\frac{bc\sqrt{1+c^2x^2}}{2x\sqrt{d+c^2dx^2}}-\frac{\sqrt{d+c^2dx^2}(a+b\operatorname{ArcSinh}[cx])}{2dx^2}+\frac{c^2\sqrt{1+c^2x^2}(a+b\operatorname{ArcSinh}[cx])\operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[cx]}\right]}{\sqrt{d+c^2dx^2}}+\frac{bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,-e^{\operatorname{ArcSinh}[cx]}\right]}{2\sqrt{d+c^2dx^2}}-\frac{bc^2\sqrt{1+c^2x^2}\operatorname{PolyLog}\left[2,e^{\operatorname{ArcSinh}[cx]}\right]}{2\sqrt{d+c^2dx^2}}$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{x^5(a+b\operatorname{ArcSinh}[cx])}{(d+c^2dx^2)^{3/2}} dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$\frac{5bx\sqrt{d+c^2dx^2}}{3c^5d^2\sqrt{1+c^2x^2}}-\frac{bx^3\sqrt{d+c^2dx^2}}{9c^3d^2\sqrt{1+c^2x^2}}-\frac{a+b\operatorname{ArcSinh}[cx]}{c^6d\sqrt{d+c^2dx^2}}-\frac{2\sqrt{d+c^2dx^2}(a+b\operatorname{ArcSinh}[cx])}{c^6d^2}+\frac{(d+c^2dx^2)^{3/2}(a+b\operatorname{ArcSinh}[cx])}{3c^6d^3}+\frac{b\sqrt{d+c^2dx^2}\operatorname{ArcTan}[cx]}{c^6d^2\sqrt{1+c^2x^2}}$$

Result (type 3, 220 leaves, 8 steps):

$$\frac{5 b x \sqrt{1+c^2 x^2}}{3 c^5 d \sqrt{d+c^2 d x^2}} - \frac{b x^3 \sqrt{1+c^2 x^2}}{9 c^3 d \sqrt{d+c^2 d x^2}} - \frac{x^4 (a+b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d+c^2 d x^2}} -$$

$$\frac{8 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{3 c^6 d^2} + \frac{4 x^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{3 c^4 d^2} + \frac{b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{c^6 d \sqrt{d+c^2 d x^2}}$$

Problem 156: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^3 d \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d+c^2 d x^2}} +$$

$$\frac{3 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^4 d^2} - \frac{3 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^5 d \sqrt{d+c^2 d x^2}} - \frac{b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{2 c^5 d \sqrt{d+c^2 d x^2}}$$

Result (type 3, 206 leaves, 8 steps):

$$-\frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^3 d \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d+c^2 d x^2}} +$$

$$\frac{3 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^4 d^2} - \frac{3 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^5 d \sqrt{d+c^2 d x^2}} - \frac{b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{2 c^5 d \sqrt{d+c^2 d x^2}}$$

Problem 157: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 136 leaves, 4 steps):

$$-\frac{b x \sqrt{d+c^2 d x^2}}{c^3 d^2 \sqrt{1+c^2 x^2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{c^4 d \sqrt{d+c^2 d x^2}} + \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{c^4 d^2} - \frac{b \sqrt{d+c^2 d x^2} \operatorname{ArcTan}[c x]}{c^4 d^2 \sqrt{1+c^2 x^2}}$$

Result (type 3, 141 leaves, 5 steps):

$$-\frac{b x \sqrt{1+c^2 x^2}}{c^3 d \sqrt{d+c^2 d x^2}} - \frac{x^2 (a+b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d+c^2 d x^2}} + \frac{2 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{c^4 d^2} - \frac{b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{c^4 d \sqrt{d+c^2 d x^2}}$$

## Problem 158: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])}{(d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$-\frac{x (a + b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d + c^2 d x^2}} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c^3 d \sqrt{d + c^2 d x^2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 c^3 d \sqrt{d + c^2 d x^2}}$$

Result (type 3, 130 leaves, 4 steps):

$$-\frac{x (a + b \operatorname{ArcSinh}[c x])}{c^2 d \sqrt{d + c^2 d x^2}} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c^3 d \sqrt{d + c^2 d x^2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 c^3 d \sqrt{d + c^2 d x^2}}$$

## Problem 161: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 194 leaves, 8 steps):

$$\frac{a + b \operatorname{ArcSinh}[c x]}{d \sqrt{d + c^2 d x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{d \sqrt{d + c^2 d x^2}} - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}}$$

Result (type 4, 194 leaves, 9 steps):

$$\frac{a + b \operatorname{ArcSinh}[c x]}{d \sqrt{d + c^2 d x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{d \sqrt{d + c^2 d x^2}} - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{b \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}}$$

## Problem 162: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^2 (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 143 leaves, 5 steps):

$$-\frac{a + b \operatorname{ArcSinh}[c x]}{d x \sqrt{d + c^2 d x^2}} - \frac{2 c^2 x (a + b \operatorname{ArcSinh}[c x])}{d \sqrt{d + c^2 d x^2}} + \frac{b c \sqrt{d + c^2 d x^2} \operatorname{Log}[x]}{d^2 \sqrt{1 + c^2 x^2}} + \frac{b c \sqrt{d + c^2 d x^2} \operatorname{Log}[1 + c^2 x^2]}{2 d^2 \sqrt{1 + c^2 x^2}}$$

Result (type 3, 143 leaves, 7 steps):

$$-\frac{a + b \operatorname{ArcSinh}[c x]}{d x \sqrt{d + c^2 d x^2}} - \frac{2 c^2 x (a + b \operatorname{ArcSinh}[c x])}{d \sqrt{d + c^2 d x^2}} + \frac{b c \sqrt{1 + c^2 x^2} \operatorname{Log}[x]}{d \sqrt{d + c^2 d x^2}} + \frac{b c \sqrt{1 + c^2 x^2} \operatorname{Log}[1 + c^2 x^2]}{2 d \sqrt{d + c^2 d x^2}}$$

Problem 163: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^3 (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 287 leaves, 11 steps):

$$-\frac{b c \sqrt{1 + c^2 x^2}}{2 d x \sqrt{d + c^2 d x^2}} - \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x])}{2 d \sqrt{d + c^2 d x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{2 d x^2 \sqrt{d + c^2 d x^2}} + \frac{b c^2 \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{d \sqrt{d + c^2 d x^2}} +$$

$$\frac{3 c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{3 b c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 d \sqrt{d + c^2 d x^2}} - \frac{3 b c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 d \sqrt{d + c^2 d x^2}}$$

Result (type 4, 287 leaves, 12 steps):

$$-\frac{b c \sqrt{1 + c^2 x^2}}{2 d x \sqrt{d + c^2 d x^2}} - \frac{3 c^2 (a + b \operatorname{ArcSinh}[c x])}{2 d \sqrt{d + c^2 d x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{2 d x^2 \sqrt{d + c^2 d x^2}} + \frac{b c^2 \sqrt{1 + c^2 x^2} \operatorname{ArcTan}[c x]}{d \sqrt{d + c^2 d x^2}} +$$

$$\frac{3 c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{3 b c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 d \sqrt{d + c^2 d x^2}} - \frac{3 b c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 d \sqrt{d + c^2 d x^2}}$$

Problem 164: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{x^4 (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 228 leaves, 5 steps):

$$-\frac{b c \sqrt{d + c^2 d x^2}}{6 d^2 x^2 \sqrt{1 + c^2 x^2}} - \frac{a + b \operatorname{ArcSinh}[c x]}{3 d x^3 \sqrt{d + c^2 d x^2}} + \frac{4 c^2 (a + b \operatorname{ArcSinh}[c x])}{3 d x \sqrt{d + c^2 d x^2}} +$$

$$\frac{8 c^4 x (a + b \operatorname{ArcSinh}[c x])}{3 d \sqrt{d + c^2 d x^2}} - \frac{5 b c^3 \sqrt{d + c^2 d x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{1 + c^2 x^2}} - \frac{b c^3 \sqrt{d + c^2 d x^2} \operatorname{Log}[1 + c^2 x^2]}{2 d^2 \sqrt{1 + c^2 x^2}}$$

Result (type 3, 228 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1+c^2 x^2}}{6 d x^2 \sqrt{d+c^2 d x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 d x^3 \sqrt{d+c^2 d x^2}} + \frac{4 c^2 (a+b \operatorname{ArcSinh}[c x])}{3 d x \sqrt{d+c^2 d x^2}} + \\
& \frac{8 c^4 x (a+b \operatorname{ArcSinh}[c x])}{3 d \sqrt{d+c^2 d x^2}} - \frac{5 b c^3 \sqrt{1+c^2 x^2} \operatorname{Log}[x]}{3 d \sqrt{d+c^2 d x^2}} - \frac{b c^3 \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{2 d \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 165: Result optimal but 1 more steps used.

$$\int \frac{x^6 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 281 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b}{6 c^7 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^5 d^2 \sqrt{d+c^2 d x^2}} - \frac{x^5 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \frac{5 x^3 (a+b \operatorname{ArcSinh}[c x])}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \\
& \frac{5 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^6 d^3} - \frac{5 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^7 d^2 \sqrt{d+c^2 d x^2}} - \frac{7 b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{6 c^7 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Result (type 3, 281 leaves, 12 steps):

$$\begin{aligned}
& - \frac{b}{6 c^7 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{b x^2 \sqrt{1+c^2 x^2}}{4 c^5 d^2 \sqrt{d+c^2 d x^2}} - \frac{x^5 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \frac{5 x^3 (a+b \operatorname{ArcSinh}[c x])}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \\
& \frac{5 x \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{2 c^6 d^3} - \frac{5 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{4 b c^7 d^2 \sqrt{d+c^2 d x^2}} - \frac{7 b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{6 c^7 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 166: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\begin{aligned}
& \frac{b x \sqrt{d+c^2 d x^2}}{6 c^5 d^3 (1+c^2 x^2)^{3/2}} - \frac{b x \sqrt{d+c^2 d x^2}}{c^5 d^3 \sqrt{1+c^2 x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 c^6 d (d+c^2 d x^2)^{3/2}} + \\
& \frac{2 (a+b \operatorname{ArcSinh}[c x])}{c^6 d^2 \sqrt{d+c^2 d x^2}} + \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{c^6 d^3} - \frac{11 b \sqrt{d+c^2 d x^2} \operatorname{ArcTan}[c x]}{6 c^6 d^3 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 3, 225 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b x^3}{6 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{5 b x \sqrt{1+c^2 x^2}}{6 c^5 d^2 \sqrt{d+c^2 d x^2}} - \frac{x^4 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \\
& \frac{4 x^2 (a+b \operatorname{ArcSinh}[c x])}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{8 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{3 c^6 d^3} - \frac{11 b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 c^6 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 167: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 203 leaves, 7 steps):

$$\begin{aligned}
& \frac{b}{6 c^5 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \frac{x (a+b \operatorname{ArcSinh}[c x])}{c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{2 b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Result (type 3, 203 leaves, 8 steps):

$$\begin{aligned}
& \frac{b}{6 c^5 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \frac{x (a+b \operatorname{ArcSinh}[c x])}{c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{2 b \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 168: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcSinh}[c x])}{(d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 144 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{d+c^2 d x^2}}{6 c^3 d^3 (1+c^2 x^2)^{3/2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{3 c^4 d (d+c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{5 b \sqrt{d+c^2 d x^2} \operatorname{ArcTan}[c x]}{6 c^4 d^3 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 3, 149 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b x}{6 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{x^2 (a+b \operatorname{ArcSinh}[c x])}{3 c^2 d (d+c^2 d x^2)^{3/2}} - \frac{2 (a+b \operatorname{ArcSinh}[c x])}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{5 b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 c^4 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 172: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 262 leaves, 11 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{3 d (d+c^2 d x^2)^{3/2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{7 b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}} -$$

$$\frac{2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{b \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{b \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Result (type 4, 262 leaves, 12 steps):

$$-\frac{b c x}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{3 d (d+c^2 d x^2)^{3/2}} + \frac{a+b \operatorname{ArcSinh}[c x]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{7 b \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}} -$$

$$\frac{2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{b \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{b \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}$$

Problem 173: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x^2 (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 214 leaves, 5 steps):

$$-\frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 (1+c^2 x^2)^{3/2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{d x (d+c^2 d x^2)^{3/2}} - \frac{4 c^2 x (a+b \operatorname{ArcSinh}[c x])}{3 d (d+c^2 d x^2)^{3/2}} -$$

$$\frac{8 c^2 x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}} + \frac{b c \sqrt{d+c^2 d x^2} \operatorname{Log}[x]}{d^3 \sqrt{1+c^2 x^2}} + \frac{5 b c \sqrt{d+c^2 d x^2} \operatorname{Log}[1+c^2 x^2]}{6 d^3 \sqrt{1+c^2 x^2}}$$

Result (type 3, 214 leaves, 8 steps):

$$-\frac{b c}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{d x (d+c^2 d x^2)^{3/2}} - \frac{4 c^2 x (a+b \operatorname{ArcSinh}[c x])}{3 d (d+c^2 d x^2)^{3/2}} -$$

$$\frac{8 c^2 x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}} + \frac{b c \sqrt{1+c^2 x^2} \operatorname{Log}[x]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{5 b c \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{6 d^2 \sqrt{d+c^2 d x^2}}$$

Problem 174: Result optimal but 1 more steps used.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x^3 (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 400 leaves, 15 steps):



$$\begin{aligned}
& \frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}} - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{6 d (d+c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{2 d x^2 (d+c^2 d x^2)^{3/2}} \\
& - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{2 d^2 \sqrt{d+c^2 d x^2}} + \frac{13 b c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}} + \frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d+c^2 d x^2}} + \\
& - \frac{5 b c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 d^2 \sqrt{d+c^2 d x^2}} - \frac{5 b c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Result (type 4, 400 leaves, 16 steps):

$$\begin{aligned}
& \frac{b c}{4 d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{5 b c^3 x}{12 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{3 b c \sqrt{1+c^2 x^2}}{4 d^2 x \sqrt{d+c^2 d x^2}} - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{6 d (d+c^2 d x^2)^{3/2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{2 d x^2 (d+c^2 d x^2)^{3/2}} \\
& - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])}{2 d^2 \sqrt{d+c^2 d x^2}} + \frac{13 b c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTan}[c x]}{6 d^2 \sqrt{d+c^2 d x^2}} + \frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d+c^2 d x^2}} + \\
& - \frac{5 b c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{2 d^2 \sqrt{d+c^2 d x^2}} - \frac{5 b c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{2 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 175: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcSinh}[c x]}{x^4 (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 297 leaves, 5 steps):

$$\begin{aligned}
& \frac{b c^3 \sqrt{d+c^2 d x^2}}{6 d^3 (1+c^2 x^2)^{3/2}} - \frac{b c \sqrt{d+c^2 d x^2}}{6 d^3 x^2 \sqrt{1+c^2 x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 d x^3 (d+c^2 d x^2)^{3/2}} + \frac{2 c^2 (a+b \operatorname{ArcSinh}[c x])}{d x (d+c^2 d x^2)^{3/2}} + \\
& - \frac{8 c^4 x (a+b \operatorname{ArcSinh}[c x])}{3 d (d+c^2 d x^2)^{3/2}} + \frac{16 c^4 x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{8 b c^3 \sqrt{d+c^2 d x^2} \operatorname{Log}[x]}{3 d^3 \sqrt{1+c^2 x^2}} - \frac{4 b c^3 \sqrt{d+c^2 d x^2} \operatorname{Log}[1+c^2 x^2]}{3 d^3 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 3, 297 leaves, 12 steps):

$$\begin{aligned}
& \frac{b c^3}{6 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{b c \sqrt{1+c^2 x^2}}{6 d^2 x^2 \sqrt{d+c^2 d x^2}} - \frac{a+b \operatorname{ArcSinh}[c x]}{3 d x^3 (d+c^2 d x^2)^{3/2}} + \frac{2 c^2 (a+b \operatorname{ArcSinh}[c x])}{d x (d+c^2 d x^2)^{3/2}} + \\
& - \frac{8 c^4 x (a+b \operatorname{ArcSinh}[c x])}{3 d (d+c^2 d x^2)^{3/2}} + \frac{16 c^4 x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{8 b c^3 \sqrt{1+c^2 x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{4 b c^3 \sqrt{1+c^2 x^2} \operatorname{Log}[1+c^2 x^2]}{3 d^2 \sqrt{d+c^2 d x^2}}
\end{aligned}$$

## Problem 194: Result optimal but 1 more steps used.

$$\int \frac{x^m (a + b \operatorname{ArcSinh}[c x])}{\sqrt{d + c^2 d x^2}} dx$$

Optimal (type 5, 161 leaves, 1 step):

$$\frac{x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{(1+m) \sqrt{d + c^2 d x^2}} - \frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{(2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

Result (type 5, 161 leaves, 2 steps):

$$\frac{x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{(1+m) \sqrt{d + c^2 d x^2}} - \frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{(2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

## Problem 195: Result optimal but 1 more steps used.

$$\int \frac{x^m (a + b \operatorname{ArcSinh}[c x])}{(d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 5, 268 leaves, 3 steps):

$$\frac{x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{d \sqrt{d + c^2 d x^2}} - \frac{m x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{d (1+m) \sqrt{d + c^2 d x^2}} - \frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{d (2+m) \sqrt{d + c^2 d x^2}} + \frac{b c m x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{d (2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

Result (type 5, 268 leaves, 4 steps):

$$\frac{x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{d \sqrt{d + c^2 d x^2}} - \frac{m x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{d (1+m) \sqrt{d + c^2 d x^2}} -$$

$$\frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{d (2+m) \sqrt{d + c^2 d x^2}} + \frac{b c m x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{d (2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

Problem 196: Result optimal but 1 more steps used.

$$\int \frac{x^m (a + b \operatorname{ArcSinh}[c x])}{(d + c^2 d x^2)^{5/2}} dx$$

Optimal (type 5, 402 leaves, 5 steps):

$$\frac{x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{3 d (d + c^2 d x^2)^{3/2}} + \frac{(2 - m) x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d + c^2 d x^2}} -$$

$$\frac{(2 - m) m x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{3 d^2 (1 + m) \sqrt{d + c^2 d x^2}} -$$

$$\frac{b c (2 - m) x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d + c^2 d x^2}} - \frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d + c^2 d x^2}} +$$

$$\frac{b c (2 - m) m x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{3 d^2 (2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

Result (type 5, 402 leaves, 6 steps):

$$\frac{x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{3 d (d + c^2 d x^2)^{3/2}} + \frac{(2 - m) x^{1+m} (a + b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{d + c^2 d x^2}} -$$

$$\frac{(2 - m) m x^{1+m} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -c^2 x^2\right]}{3 d^2 (1 + m) \sqrt{d + c^2 d x^2}} -$$

$$\frac{b c (2 - m) x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d + c^2 d x^2}} - \frac{b c x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, -c^2 x^2\right]}{3 d^2 (2 + m) \sqrt{d + c^2 d x^2}} +$$

$$\frac{b c (2 - m) m x^{2+m} \sqrt{1 + c^2 x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, -c^2 x^2\right]}{3 d^2 (2 + 3 m + m^2) \sqrt{d + c^2 d x^2}}$$

## Problem 203: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2) (a + b \operatorname{ArcSinh}[c x])^2}{x} dx$$

Optimal (type 4, 166 leaves, 10 steps):

$$\frac{1}{4} b^2 c^2 d x^2 - \frac{1}{2} b c d x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{1}{4} d (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \frac{d (a + b \operatorname{ArcSinh}[c x])^3}{3 b} +$$

$$d (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - b d (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 165 leaves, 10 steps):

$$\frac{1}{4} b^2 c^2 d x^2 - \frac{1}{2} b c d x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{1}{4} d (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 - \frac{d (a + b \operatorname{ArcSinh}[c x])^3}{3 b} +$$

$$d (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + b d (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}]$$

## Problem 205: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2) (a + b \operatorname{ArcSinh}[c x])^2}{x^3} dx$$

Optimal (type 4, 180 leaves, 10 steps):

$$- \frac{b c d \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{1}{2} c^2 d (a + b \operatorname{ArcSinh}[c x])^2 -$$

$$\frac{d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} + \frac{c^2 d (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + c^2 d (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] +$$

$$b^2 c^2 d \operatorname{Log}[x] - b c^2 d (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 c^2 d \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}]$$

Result (type 4, 179 leaves, 10 steps):

$$- \frac{b c d \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{x} + \frac{1}{2} c^2 d (a + b \operatorname{ArcSinh}[c x])^2 -$$

$$\frac{d (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} - \frac{c^2 d (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + c^2 d (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] +$$

$$b^2 c^2 d \operatorname{Log}[x] + b c^2 d (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 c^2 d \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}]$$

### Problem 212: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2}{x} dx$$

Optimal (type 4, 257 leaves, 17 steps):

$$\begin{aligned} & \frac{13}{32} b^2 c^2 d^2 x^2 + \frac{1}{32} b^2 c^4 d^2 x^4 - \frac{11}{16} b c d^2 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{1}{8} b c d^2 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \\ & \frac{11}{32} d^2 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d^2 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{4} d^2 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{d^2 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + \\ & d^2 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - b d^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d^2 \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Result (type 4, 256 leaves, 17 steps):

$$\begin{aligned} & \frac{13}{32} b^2 c^2 d^2 x^2 + \frac{1}{32} b^2 c^4 d^2 x^4 - \frac{11}{16} b c d^2 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{1}{8} b c d^2 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \\ & \frac{11}{32} d^2 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d^2 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{4} d^2 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 - \frac{d^2 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + \\ & d^2 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + b d^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d^2 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

### Problem 214: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2}{x^3} dx$$

Optimal (type 4, 272 leaves, 17 steps):

$$\begin{aligned} & \frac{1}{4} b^2 c^4 d^2 x^2 + \frac{1}{2} b c^3 d^2 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{b c d^2 (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \\ & \frac{1}{4} c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^2 + c^2 d^2 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 - \frac{d^2 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} + \\ & \frac{2 c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + 2 c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] + b^2 c^2 d^2 \operatorname{Log}[x] - \\ & 2 b c^2 d^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - b^2 c^2 d^2 \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Result (type 4, 272 leaves, 17 steps):

$$\begin{aligned} & \frac{1}{4} b^2 c^4 d^2 x^2 + \frac{1}{2} b c^3 d^2 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{b c d^2 (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])}{x} + \\ & \frac{1}{4} c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^2 + c^2 d^2 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 - \frac{d^2 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} - \\ & \frac{2 c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + 2 c^2 d^2 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + b^2 c^2 d^2 \operatorname{Log}[x] + \\ & 2 b c^2 d^2 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - b^2 c^2 d^2 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Problem 221: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2}{x} dx$$

Optimal (type 4, 337 leaves, 26 steps):

$$\begin{aligned} & \frac{71}{144} b^2 c^2 d^3 x^2 + \frac{7}{144} b^2 c^4 d^3 x^4 + \frac{1}{108} b^2 d^3 (1 + c^2 x^2)^3 - \frac{19}{24} b c d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{7}{36} b c d^3 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \\ & \frac{1}{18} b c d^3 x (1 + c^2 x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{19}{48} d^3 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \\ & \frac{1}{4} d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{6} d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{d^3 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + \\ & d^3 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] - b d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d^3 \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Result (type 4, 336 leaves, 26 steps):

$$\begin{aligned} & \frac{71}{144} b^2 c^2 d^3 x^2 + \frac{7}{144} b^2 c^4 d^3 x^4 + \frac{1}{108} b^2 d^3 (1 + c^2 x^2)^3 - \frac{19}{24} b c d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) - \frac{7}{36} b c d^3 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \\ & \frac{1}{18} b c d^3 x (1 + c^2 x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{19}{48} d^3 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{2} d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \\ & \frac{1}{4} d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{6} d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2 - \frac{d^3 (a + b \operatorname{ArcSinh}[c x])^3}{3 b} + \\ & d^3 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + b d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - \frac{1}{2} b^2 d^3 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Problem 223: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2}{x^3} dx$$

Optimal (type 4, 354 leaves, 28 steps):

$$\begin{aligned} & \frac{21}{32} b^2 c^4 d^3 x^2 + \frac{1}{32} b^2 c^6 d^3 x^4 - \frac{3}{16} b c^3 d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) + \\ & \frac{7}{8} b c^3 d^3 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{b c d^3 (1 + c^2 x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} - \\ & \frac{3}{32} c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{3}{2} c^2 d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \frac{3}{4} c^2 d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 - \\ & \frac{d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} + \frac{c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^3}{b} + 3 c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}] + \\ & b^2 c^2 d^3 \operatorname{Log}[x] - 3 b c^2 d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}] - \frac{3}{2} b^2 c^2 d^3 \operatorname{PolyLog}[3, e^{-2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Result (type 4, 355 leaves, 28 steps):

$$\begin{aligned} & \frac{21}{32} b^2 c^4 d^3 x^2 + \frac{1}{32} b^2 c^6 d^3 x^4 - \frac{3}{16} b c^3 d^3 x \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) + \\ & \frac{7}{8} b c^3 d^3 x (1 + c^2 x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x]) - \frac{b c d^3 (1 + c^2 x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])}{x} - \\ & \frac{3}{32} c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^2 + \frac{3}{2} c^2 d^3 (1 + c^2 x^2) (a + b \operatorname{ArcSinh}[c x])^2 + \frac{3}{4} c^2 d^3 (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])^2 - \\ & \frac{d^3 (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])^2}{2 x^2} - \frac{c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^3}{b} + 3 c^2 d^3 (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}] + \\ & b^2 c^2 d^3 \operatorname{Log}[x] + 3 b c^2 d^3 (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}] - \frac{3}{2} b^2 c^2 d^3 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcSinh}[c x]}] \end{aligned}$$

Problem 252: Result valid but suboptimal antiderivative.

$$\int (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 3, 300 leaves, 16 steps):

$$\begin{aligned} & \frac{245 b^2 \pi^{5/2} x \sqrt{1 + c^2 x^2}}{1152} + \frac{65 b^2 \pi^{5/2} x (1 + c^2 x^2)^{3/2}}{1728} + \frac{1}{108} b^2 \pi^{5/2} x (1 + c^2 x^2)^{5/2} - \frac{115 b^2 \pi^{5/2} \operatorname{ArcSinh}[c x]}{1152 c} - \frac{5}{16} b c \pi^{5/2} x^2 (a + b \operatorname{ArcSinh}[c x]) - \\ & \frac{5 b \pi^{5/2} (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])}{48 c} - \frac{b \pi^{5/2} (1 + c^2 x^2)^3 (a + b \operatorname{ArcSinh}[c x])}{18 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \\ & \frac{5}{24} \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{6} x (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{5 \pi^{5/2} (a + b \operatorname{ArcSinh}[c x])^3}{48 b c} \end{aligned}$$

Result (type 3, 420 leaves, 16 steps):

$$\begin{aligned}
& \frac{245 b^2 \pi^2 x \sqrt{\pi + c^2 \pi x^2}}{1152} + \frac{65 b^2 \pi^2 x (1 + c^2 x^2) \sqrt{\pi + c^2 \pi x^2}}{1728} + \frac{1}{108} b^2 \pi^2 x (1 + c^2 x^2)^2 \sqrt{\pi + c^2 \pi x^2} - \\
& \frac{115 b^2 \pi^2 \sqrt{\pi + c^2 \pi x^2} \operatorname{ArcSinh}[c x]}{1152 c \sqrt{1 + c^2 x^2}} - \frac{5 b c \pi^2 x^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{16 \sqrt{1 + c^2 x^2}} - \frac{5 b \pi^2 (1 + c^2 x^2)^{3/2} \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{48 c} - \\
& \frac{b \pi^2 (1 + c^2 x^2)^{5/2} \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{18 c} + \frac{5}{16} \pi^2 x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \\
& \frac{5}{24} \pi x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{6} x (\pi + c^2 \pi x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{5 \pi^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^3}{48 b c \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 3, 210 leaves, 10 steps):

$$\begin{aligned}
& \frac{15}{64} b^2 \pi^{3/2} x \sqrt{1 + c^2 x^2} + \frac{1}{32} b^2 \pi^{3/2} x (1 + c^2 x^2)^{3/2} - \frac{9 b^2 \pi^{3/2} \operatorname{ArcSinh}[c x]}{64 c} - \\
& \frac{3}{8} b c \pi^{3/2} x^2 (a + b \operatorname{ArcSinh}[c x]) - \frac{b \pi^{3/2} (1 + c^2 x^2)^2 (a + b \operatorname{ArcSinh}[c x])}{8 c} + \\
& \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{4} x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{\pi^{3/2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b c}
\end{aligned}$$

Result (type 3, 294 leaves, 10 steps):

$$\begin{aligned}
& \frac{15}{64} b^2 \pi x \sqrt{\pi + c^2 \pi x^2} + \frac{1}{32} b^2 \pi x (1 + c^2 x^2) \sqrt{\pi + c^2 \pi x^2} - \frac{9 b^2 \pi \sqrt{\pi + c^2 \pi x^2} \operatorname{ArcSinh}[c x]}{64 c \sqrt{1 + c^2 x^2}} - \\
& \frac{3 b c \pi x^2 \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{8 \sqrt{1 + c^2 x^2}} - \frac{b \pi (1 + c^2 x^2)^{3/2} \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c} + \\
& \frac{3}{8} \pi x \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{1}{4} x (\pi + c^2 \pi x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{\pi \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b c \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \sqrt{\pi + c^2 \pi x^2} (a + b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 3, 122 leaves, 5 steps):



$$\frac{1}{4} b^2 \sqrt{\pi} x \sqrt{1+c^2 x^2} - \frac{b^2 \sqrt{\pi} \operatorname{ArcSinh}[c x]}{4 c} - \frac{1}{2} b c \sqrt{\pi} x^2 (a+b \operatorname{ArcSinh}[c x]) + \frac{1}{2} x \sqrt{\pi+c^2 \pi x^2} (a+b \operatorname{ArcSinh}[c x])^2 + \frac{\sqrt{\pi} (a+b \operatorname{ArcSinh}[c x])^3}{6 b c}$$

Result (type 3, 184 leaves, 5 steps):

$$\frac{1}{4} b^2 x \sqrt{\pi+c^2 \pi x^2} - \frac{b^2 \sqrt{\pi+c^2 \pi x^2} \operatorname{ArcSinh}[c x]}{4 c \sqrt{1+c^2 x^2}} - \frac{b c x^2 \sqrt{\pi+c^2 \pi x^2} (a+b \operatorname{ArcSinh}[c x])}{2 \sqrt{1+c^2 x^2}} +$$

$$\frac{1}{2} x \sqrt{\pi+c^2 \pi x^2} (a+b \operatorname{ArcSinh}[c x])^2 + \frac{\sqrt{\pi+c^2 \pi x^2} (a+b \operatorname{ArcSinh}[c x])^3}{6 b c \sqrt{1+c^2 x^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcSinh}[c x])^2}{(\pi+c^2 \pi x^2)^{3/2}} dx$$

Optimal (type 4, 104 leaves, 6 steps):

$$\frac{(a+b \operatorname{ArcSinh}[c x])^2}{c \pi^{3/2}} + \frac{x (a+b \operatorname{ArcSinh}[c x])^2}{\pi \sqrt{\pi+c^2 \pi x^2}} - \frac{2 b (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1+e^{2 \operatorname{ArcSinh}[c x]}]}{c \pi^{3/2}} - \frac{b^2 \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c \pi^{3/2}}$$

Result (type 4, 179 leaves, 6 steps):

$$\frac{x (a+b \operatorname{ArcSinh}[c x])^2}{\pi \sqrt{\pi+c^2 \pi x^2}} + \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{c \pi \sqrt{\pi+c^2 \pi x^2}} -$$

$$\frac{2 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1+e^{2 \operatorname{ArcSinh}[c x]}]}{c \pi \sqrt{\pi+c^2 \pi x^2}} - \frac{b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c \pi \sqrt{\pi+c^2 \pi x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcSinh}[c x])^2}{(\pi+c^2 \pi x^2)^{5/2}} dx$$

Optimal (type 4, 204 leaves, 9 steps):

$$-\frac{b^2 x}{3 \pi^{5/2} \sqrt{1+c^2 x^2}} + \frac{b (a+b \operatorname{ArcSinh}[c x])}{3 c \pi^{5/2} (1+c^2 x^2)} + \frac{2 (a+b \operatorname{ArcSinh}[c x])^2}{3 c \pi^{5/2}} + \frac{x (a+b \operatorname{ArcSinh}[c x])^2}{3 \pi (\pi+c^2 \pi x^2)^{3/2}} +$$

$$\frac{2 x (a+b \operatorname{ArcSinh}[c x])^2}{3 \pi^2 \sqrt{\pi+c^2 \pi x^2}} - \frac{4 b (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1+e^{2 \operatorname{ArcSinh}[c x]}]}{3 c \pi^{5/2}} - \frac{2 b^2 \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{3 c \pi^{5/2}}$$

Result (type 4, 292 leaves, 9 steps):

$$-\frac{b^2 x}{3 \pi^2 \sqrt{\pi + c^2 \pi x^2}} + \frac{b (a + b \operatorname{ArcSinh}[c x])}{3 c \pi^2 \sqrt{1 + c^2 x^2} \sqrt{\pi + c^2 \pi x^2}} + \frac{x (a + b \operatorname{ArcSinh}[c x])^2}{3 \pi (\pi + c^2 \pi x^2)^{3/2}} + \frac{2 x (a + b \operatorname{ArcSinh}[c x])^2}{3 \pi^2 \sqrt{\pi + c^2 \pi x^2}} +$$

$$\frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 c \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{4 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcSinh}[c x]}]}{3 c \pi^2 \sqrt{\pi + c^2 \pi x^2}} - \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{3 c \pi^2 \sqrt{\pi + c^2 \pi x^2}}$$

Problem 263: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{x^2} dx$$

Optimal (type 4, 209 leaves, 7 steps):

$$-\frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{x} + \frac{c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} + \frac{c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b \sqrt{1 + c^2 x^2}} +$$

$$\frac{2 b c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \frac{b^2 c \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}$$

Result (type 4, 209 leaves, 7 steps):

$$-\frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{x} - \frac{c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} + \frac{c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b \sqrt{1 + c^2 x^2}} +$$

$$\frac{2 b c \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} + \frac{b^2 c \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}$$

Problem 265: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{x^4} dx$$

Optimal (type 4, 294 leaves, 9 steps):

$$-\frac{b^2 c^2 \sqrt{d + c^2 d x^2}}{3 x} + \frac{b^2 c^3 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{3 \sqrt{1 + c^2 x^2}} - \frac{b c \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2} + \frac{c^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1 + c^2 x^2}} -$$

$$\frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x^3} + \frac{2 b c^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}} - \frac{b^2 c^3 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}}$$

Result (type 4, 294 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b^2 c^2 \sqrt{d + c^2 d x^2}}{3 x} + \frac{b^2 c^3 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{3 \sqrt{1 + c^2 x^2}} - \frac{b c \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2} - \frac{c^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1 + c^2 x^2}} \\
& + \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x^3} + \frac{2 b c^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}} + \frac{b^2 c^3 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 271: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{x^2} dx$$

Optimal (type 4, 398 leaves, 14 steps):

$$\begin{aligned}
& \frac{1}{4} b^2 c^2 d x \sqrt{d + c^2 d x^2} - \frac{5 b^2 c d \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{4 \sqrt{1 + c^2 x^2}} - \frac{3 b c^3 d x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \sqrt{1 + c^2 x^2}} + \\
& b c d \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{3}{2} c^2 d x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \\
& \frac{c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} - \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{x} + \frac{c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{2 b \sqrt{1 + c^2 x^2}} + \\
& \frac{2 b c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \frac{b^2 c d \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 398 leaves, 14 steps):

$$\begin{aligned}
& \frac{1}{4} b^2 c^2 d x \sqrt{d + c^2 d x^2} - \frac{5 b^2 c d \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{4 \sqrt{1 + c^2 x^2}} - \frac{3 b c^3 d x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \sqrt{1 + c^2 x^2}} + \\
& b c d \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{3}{2} c^2 d x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 - \\
& \frac{c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} - \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{x} + \frac{c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{2 b \sqrt{1 + c^2 x^2}} + \\
& \frac{2 b c d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} + \frac{b^2 c d \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 273: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{x^4} dx$$

Optimal (type 4, 378 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{b^2 c^2 d \sqrt{d+c^2 d x^2}}{3 x} + \frac{b^2 c^3 d \sqrt{d+c^2 d x^2} \operatorname{ArcSinh}[c x]}{3 \sqrt{1+c^2 x^2}} - \\
 & \frac{b c d \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{3 x^2} - \frac{c^2 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{x} + \\
 & \frac{4 c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1+c^2 x^2}} - \frac{(d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2}{3 x^3} + \frac{c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^3}{3 b \sqrt{1+c^2 x^2}} + \\
 & \frac{8 b c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{-2 \operatorname{ArcSinh}[c x]}\right]}{3 \sqrt{1+c^2 x^2}} - \frac{4 b^2 c^3 d \sqrt{d+c^2 d x^2} \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}[c x]}\right]}{3 \sqrt{1+c^2 x^2}}
 \end{aligned}$$

Result (type 4, 378 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{b^2 c^2 d \sqrt{d+c^2 d x^2}}{3 x} + \frac{b^2 c^3 d \sqrt{d+c^2 d x^2} \operatorname{ArcSinh}[c x]}{3 \sqrt{1+c^2 x^2}} - \\
 & \frac{b c d \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{3 x^2} - \frac{c^2 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{x} - \\
 & \frac{4 c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1+c^2 x^2}} - \frac{(d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2}{3 x^3} + \frac{c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^3}{3 b \sqrt{1+c^2 x^2}} + \\
 & \frac{8 b c^3 d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 \sqrt{1+c^2 x^2}} + \frac{4 b^2 c^3 d \sqrt{d+c^2 d x^2} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 \sqrt{1+c^2 x^2}}
 \end{aligned}$$

Problem 279: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^{5/2} (a+b \operatorname{ArcSinh}[c x])^2}{x^2} dx$$

Optimal (type 4, 530 leaves, 23 steps):

$$\begin{aligned}
& \frac{31}{64} b^2 c^2 d^2 x \sqrt{d + c^2 d x^2} + \frac{1}{32} b^2 c^2 d^2 x (1 + c^2 x^2) \sqrt{d + c^2 d x^2} - \frac{89 b^2 c d^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{64 \sqrt{1 + c^2 x^2}} - \\
& \frac{15 b c^3 d^2 x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{8 \sqrt{1 + c^2 x^2}} + b c d^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) - \\
& \frac{1}{8} b c d^2 (1 + c^2 x^2)^{3/2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{15}{8} c^2 d^2 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} + \\
& \frac{5}{4} c^2 d x (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 - \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{x} + \frac{5 c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b \sqrt{1 + c^2 x^2}} + \\
& \frac{2 b c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} - \frac{b^2 c d^2 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 530 leaves, 23 steps):

$$\begin{aligned}
& \frac{31}{64} b^2 c^2 d^2 x \sqrt{d + c^2 d x^2} + \frac{1}{32} b^2 c^2 d^2 x (1 + c^2 x^2) \sqrt{d + c^2 d x^2} - \frac{89 b^2 c d^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{64 \sqrt{1 + c^2 x^2}} - \\
& \frac{15 b c^3 d^2 x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{8 \sqrt{1 + c^2 x^2}} + b c d^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) - \\
& \frac{1}{8} b c d^2 (1 + c^2 x^2)^{3/2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) + \frac{15}{8} c^2 d^2 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 - \frac{c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{1 + c^2 x^2}} + \\
& \frac{5}{4} c^2 d x (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 - \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{x} + \frac{5 c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b \sqrt{1 + c^2 x^2}} + \\
& \frac{2 b c d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}} + \frac{b^2 c d^2 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{\sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 281: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{x^4} dx$$

Optimal (type 4, 561 leaves, 27 steps):

$$\begin{aligned}
& \frac{7}{12} b^2 c^4 d^2 x \sqrt{d + c^2 d x^2} - \frac{b^2 c^2 d^2 (1 + c^2 x^2) \sqrt{d + c^2 d x^2}}{3 x} - \frac{23 b^2 c^3 d^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{12 \sqrt{1 + c^2 x^2}} - \\
& \frac{5 b c^5 d^2 x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \sqrt{1 + c^2 x^2}} + \frac{7}{3} b c^3 d^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) - \\
& \frac{b c d^2 (1 + c^2 x^2)^{3/2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2} + \frac{5}{2} c^4 d^2 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 + \frac{7 c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1 + c^2 x^2}} - \\
& \frac{5 c^2 d (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 x} - \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 x^3} + \frac{5 c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{6 b \sqrt{1 + c^2 x^2}} + \\
& \frac{14 b c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}} - \frac{7 b^2 c^3 d^2 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 561 leaves, 27 steps):

$$\begin{aligned}
& \frac{7}{12} b^2 c^4 d^2 x \sqrt{d + c^2 d x^2} - \frac{b^2 c^2 d^2 (1 + c^2 x^2) \sqrt{d + c^2 d x^2}}{3 x} - \frac{23 b^2 c^3 d^2 \sqrt{d + c^2 d x^2} \operatorname{ArcSinh}[c x]}{12 \sqrt{1 + c^2 x^2}} - \\
& \frac{5 b c^5 d^2 x^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{2 \sqrt{1 + c^2 x^2}} + \frac{7}{3} b c^3 d^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) - \\
& \frac{b c d^2 (1 + c^2 x^2)^{3/2} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2} + \frac{5}{2} c^4 d^2 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2 - \frac{7 c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{1 + c^2 x^2}} - \\
& \frac{5 c^2 d (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 x} - \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{3 x^3} + \frac{5 c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^3}{6 b \sqrt{1 + c^2 x^2}} + \\
& \frac{14 b c^3 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}} + \frac{7 b^2 c^3 d^2 \sqrt{d + c^2 d x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 291: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{d + c^2 d x^2}} dx$$

Optimal (type 3, 323 leaves, 10 steps):

$$\begin{aligned}
& - \frac{15 b^2 x (1 + c^2 x^2)}{64 c^4 \sqrt{d + c^2 d x^2}} + \frac{b^2 x^3 (1 + c^2 x^2)}{32 c^2 \sqrt{d + c^2 d x^2}} + \frac{15 b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{64 c^5 \sqrt{d + c^2 d x^2}} + \\
& \frac{3 b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c^3 \sqrt{d + c^2 d x^2}} - \frac{b x^4 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c \sqrt{d + c^2 d x^2}} - \\
& \frac{3 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{8 c^4 d} + \frac{x^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 c^2 d} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b c^5 \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Result (type 3, 323 leaves, 11 steps):

$$\begin{aligned}
& - \frac{15 b^2 x (1 + c^2 x^2)}{64 c^4 \sqrt{d + c^2 d x^2}} + \frac{b^2 x^3 (1 + c^2 x^2)}{32 c^2 \sqrt{d + c^2 d x^2}} + \frac{15 b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{64 c^5 \sqrt{d + c^2 d x^2}} + \\
& \frac{3 b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c^3 \sqrt{d + c^2 d x^2}} - \frac{b x^4 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{8 c \sqrt{d + c^2 d x^2}} - \\
& \frac{3 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{8 c^4 d} + \frac{x^3 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{4 c^2 d} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{8 b c^5 \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Problem 293: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{d + c^2 d x^2}} dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\begin{aligned}
& \frac{b^2 x (1 + c^2 x^2)}{4 c^2 \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{4 c^3 \sqrt{d + c^2 d x^2}} - \frac{b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c \sqrt{d + c^2 d x^2}} + \\
& \frac{x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 c^2 d} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{6 b c^3 \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Result (type 3, 204 leaves, 6 steps):

$$\begin{aligned}
& \frac{b^2 x (1 + c^2 x^2)}{4 c^2 \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{4 c^3 \sqrt{d + c^2 d x^2}} - \frac{b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c \sqrt{d + c^2 d x^2}} + \\
& \frac{x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 c^2 d} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{6 b c^3 \sqrt{d + c^2 d x^2}}
\end{aligned}$$

## Problem 295: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{d + c^2 d x^2}} dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b c \sqrt{d + c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b c \sqrt{d + c^2 d x^2}}$$

## Problem 296: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{x \sqrt{d + c^2 d x^2}} dx$$

Optimal (type 4, 223 leaves, 8 steps):

$$\begin{aligned} & - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} - \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} + \\ & \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} + \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} - \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} \end{aligned}$$

Result (type 4, 223 leaves, 9 steps):

$$\begin{aligned} & - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} - \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} + \\ & \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} + \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} - \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c x]}]}{\sqrt{d + c^2 d x^2}} \end{aligned}$$

## Problem 297: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{x^2 \sqrt{d + c^2 d x^2}} dx$$



Optimal (type 4, 167 leaves, 6 steps):

$$\frac{c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}} - \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{d x} +$$

$$\frac{2 b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{-2 \operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} - \frac{b^2 c \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}}$$

Result (type 4, 167 leaves, 6 steps):

$$-\frac{c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}} - \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{d x} +$$

$$\frac{2 b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1-e^{2 \operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} + \frac{b^2 c \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}}$$

Problem 298: Result optimal but 1 more steps used.

$$\int \frac{(a+b \operatorname{ArcSinh}[c x])^2}{x^3 \sqrt{d+c^2 d x^2}} dx$$

Optimal (type 4, 360 leaves, 13 steps):

$$-\frac{b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])}{x \sqrt{d+c^2 d x^2}} - \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 d x^2} + \frac{c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} -$$

$$\frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d x^2}} + \frac{b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} -$$

$$\frac{b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} + \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}}$$

Result (type 4, 360 leaves, 14 steps):

$$-\frac{b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])}{x \sqrt{d+c^2 d x^2}} - \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 d x^2} + \frac{c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} -$$

$$\frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{\sqrt{d+c^2 d x^2}} + \frac{b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} -$$

$$\frac{b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}} + \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{\sqrt{d+c^2 d x^2}}$$

## Problem 299: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{x^4 \sqrt{d + c^2 d x^2}} dx$$

Optimal (type 4, 299 leaves, 9 steps):

$$\begin{aligned} & -\frac{b^2 c^2 (1 + c^2 x^2)}{3 x \sqrt{d + c^2 d x^2}} - \frac{b c \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2 \sqrt{d + c^2 d x^2}} - \frac{2 c^3 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{d + c^2 d x^2}} - \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x^3} + \\ & \frac{2 c^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x} - \frac{4 b c^3 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{d + c^2 d x^2}} + \frac{2 b^2 c^3 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{d + c^2 d x^2}} \end{aligned}$$

Result (type 4, 299 leaves, 9 steps):

$$\begin{aligned} & -\frac{b^2 c^2 (1 + c^2 x^2)}{3 x \sqrt{d + c^2 d x^2}} - \frac{b c \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{3 x^2 \sqrt{d + c^2 d x^2}} + \frac{2 c^3 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 \sqrt{d + c^2 d x^2}} - \frac{\sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x^3} + \\ & \frac{2 c^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{3 d x} - \frac{4 b c^3 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 - e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{d + c^2 d x^2}} - \frac{2 b^2 c^3 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, e^{2 \operatorname{ArcSinh}[c x]}]}{3 \sqrt{d + c^2 d x^2}} \end{aligned}$$

## Problem 301: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a + b \operatorname{ArcSinh}[c x])^2}{(d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 400 leaves, 14 steps):

$$\begin{aligned} & \frac{b^2 x (1 + c^2 x^2)}{4 c^4 d \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{4 c^5 d \sqrt{d + c^2 d x^2}} - \frac{b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^3 d \sqrt{d + c^2 d x^2}} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])^2}{c^2 d \sqrt{d + c^2 d x^2}} + \\ & \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{c^5 d \sqrt{d + c^2 d x^2}} + \frac{3 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 c^4 d^2} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{2 b c^5 d \sqrt{d + c^2 d x^2}} - \\ & \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcSinh}[c x]}]}{c^5 d \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c^5 d \sqrt{d + c^2 d x^2}} \end{aligned}$$

Result (type 4, 400 leaves, 15 steps):

$$\begin{aligned}
& \frac{b^2 x (1 + c^2 x^2)}{4 c^4 d \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{ArcSinh}[c x]}{4 c^5 d \sqrt{d + c^2 d x^2}} - \frac{b x^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])}{2 c^3 d \sqrt{d + c^2 d x^2}} - \frac{x^3 (a + b \operatorname{ArcSinh}[c x])^2}{c^2 d \sqrt{d + c^2 d x^2}} + \\
& \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{c^5 d \sqrt{d + c^2 d x^2}} + \frac{3 x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 c^4 d^2} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{2 b c^5 d \sqrt{d + c^2 d x^2}} - \\
& \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcSinh}[c x]}]}{c^5 d \sqrt{d + c^2 d x^2}} - \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c^5 d \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Problem 303: Result optimal but 1 more steps used.

$$\int \frac{x^2 (a + b \operatorname{ArcSinh}[c x])^2}{(d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 233 leaves, 7 steps):

$$\begin{aligned}
& - \frac{x (a + b \operatorname{ArcSinh}[c x])^2}{c^2 d \sqrt{d + c^2 d x^2}} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{c^3 d \sqrt{d + c^2 d x^2}} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b c^3 d \sqrt{d + c^2 d x^2}} + \\
& \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcSinh}[c x]}]}{c^3 d \sqrt{d + c^2 d x^2}} + \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c^3 d \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Result (type 4, 233 leaves, 8 steps):

$$\begin{aligned}
& - \frac{x (a + b \operatorname{ArcSinh}[c x])^2}{c^2 d \sqrt{d + c^2 d x^2}} - \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{c^3 d \sqrt{d + c^2 d x^2}} + \frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^3}{3 b c^3 d \sqrt{d + c^2 d x^2}} + \\
& \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcSinh}[c x]}]}{c^3 d \sqrt{d + c^2 d x^2}} + \frac{b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcSinh}[c x]}]}{c^3 d \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Problem 306: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{x (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 412 leaves, 15 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcSinh}[c x])^2}{d \sqrt{d + c^2 d x^2}} - \frac{4 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& - \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{2 i b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -i e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& - \frac{2 i b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, i e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& + \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Result (type 4, 412 leaves, 16 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcSinh}[c x])^2}{d \sqrt{d + c^2 d x^2}} - \frac{4 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& - \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{2 i b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -i e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& - \frac{2 i b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, i e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} + \frac{2 b \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} \\
& + \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}} - \frac{2 b^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c x]}]}{d \sqrt{d + c^2 d x^2}}
\end{aligned}$$

Problem 308: Result optimal but 1 more steps used.

$$\int \frac{(a + b \operatorname{ArcSinh}[c x])^2}{x^3 (d + c^2 d x^2)^{3/2}} dx$$

Optimal (type 4, 573 leaves, 26 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])}{d x \sqrt{d+c^2 d x^2}} - \frac{3 c^2 (a+b \operatorname{ArcSinh}[c x])^2}{2 d \sqrt{d+c^2 d x^2}} - \\
& \frac{(a+b \operatorname{ArcSinh}[c x])^2}{2 d x^2 \sqrt{d+c^2 d x^2}} + \frac{4 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{3 c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{3 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{2 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{2 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{3 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \\
& \frac{3 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \frac{3 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Result (type 4, 573 leaves, 27 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])}{d x \sqrt{d+c^2 d x^2}} - \frac{3 c^2 (a+b \operatorname{ArcSinh}[c x])^2}{2 d \sqrt{d+c^2 d x^2}} - \\
& \frac{(a+b \operatorname{ArcSinh}[c x])^2}{2 d x^2 \sqrt{d+c^2 d x^2}} + \frac{4 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{3 c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{3 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{2 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \\
& \frac{2 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \frac{3 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} - \\
& \frac{3 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}} + \frac{3 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{d \sqrt{d+c^2 d x^2}}
\end{aligned}$$

Problem 311: Result optimal but 1 more steps used.

$$\int \frac{x^4 (a+b \operatorname{ArcSinh}[c x])^2}{(d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 398 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{b^2 \sqrt{1+c^2 x^2} \operatorname{ArcSinh}[c x]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} - \frac{b x^2 (a+b \operatorname{ArcSinh}[c x])}{3 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])^2}{3 c^2 d (d+c^2 d x^2)^{3/2}} \\
 & - \frac{x (a+b \operatorname{ArcSinh}[c x])^2}{c^4 d^2 \sqrt{d+c^2 d x^2}} - \frac{4 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^3}{3 b c^5 d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{8 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1+e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{4 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2,-e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}
 \end{aligned}$$

Result (type 4, 398 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{b^2 x}{3 c^4 d^2 \sqrt{d+c^2 d x^2}} + \frac{b^2 \sqrt{1+c^2 x^2} \operatorname{ArcSinh}[c x]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} - \frac{b x^2 (a+b \operatorname{ArcSinh}[c x])}{3 c^3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{x^3 (a+b \operatorname{ArcSinh}[c x])^2}{3 c^2 d (d+c^2 d x^2)^{3/2}} \\
 & - \frac{x (a+b \operatorname{ArcSinh}[c x])^2}{c^4 d^2 \sqrt{d+c^2 d x^2}} - \frac{4 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^3}{3 b c^5 d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{8 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{Log}\left[1+e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}} + \frac{4 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2,-e^{2 \operatorname{ArcSinh}[c x]}\right]}{3 c^5 d^2 \sqrt{d+c^2 d x^2}}
 \end{aligned}$$

Problem 316: Result optimal but 1 more steps used.

$$\int \frac{(a+b \operatorname{ArcSinh}[c x])^2}{x (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 518 leaves, 24 steps):

$$\begin{aligned}
 & -\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{b c x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{(a+b \operatorname{ArcSinh}[c x])^2}{3 d (d+c^2 d x^2)^{3/2}} + \frac{(a+b \operatorname{ArcSinh}[c x])^2}{d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{14 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{2 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2,-e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{7 i b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2,-i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{7 i b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2,i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} + \frac{2 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2,e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{2 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3,-e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{2 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3,e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}
 \end{aligned}$$

Result (type 4, 518 leaves, 25 steps):

$$\begin{aligned}
 & -\frac{b^2}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{b c x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} + \frac{(a+b \operatorname{ArcSinh}[c x])^2}{3 d (d+c^2 d x^2)^{3/2}} + \frac{(a+b \operatorname{ArcSinh}[c x])^2}{d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{14 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{2 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{7 i b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{7 i b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} + \frac{2 b \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{2 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{2 b^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}
 \end{aligned}$$

Problem 318: Result optimal but 1 more steps used.

$$\int \frac{(a+b \operatorname{ArcSinh}[c x])^2}{x^3 (d+c^2 d x^2)^{5/2}} dx$$

Optimal (type 4, 687 leaves, 38 steps):

$$\begin{aligned}
 & \frac{b^2 c^2}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{b c (a+b \operatorname{ArcSinh}[c x])}{d^2 x \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{2 b c^3 x (a+b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{1+c^2 x^2} \sqrt{d+c^2 d x^2}} - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])^2}{6 d (d+c^2 d x^2)^{3/2}} - \\
 & \frac{(a+b \operatorname{ArcSinh}[c x])^2}{2 d x^2 (d+c^2 d x^2)^{3/2}} - \frac{5 c^2 (a+b \operatorname{ArcSinh}[c x])^2}{2 d^2 \sqrt{d+c^2 d x^2}} + \frac{26 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{5 c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}\left[e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{ArcTanh}\left[\sqrt{1+c^2 x^2}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{5 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} + \\
 & \frac{13 i b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSinh}[c x]}\right]}{3 d^2 \sqrt{d+c^2 d x^2}} - \frac{5 b c^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} - \\
 & \frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, -e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}} + \frac{5 b^2 c^2 \sqrt{1+c^2 x^2} \operatorname{PolyLog}\left[3, e^{\operatorname{ArcSinh}[c x]}\right]}{d^2 \sqrt{d+c^2 d x^2}}
 \end{aligned}$$

Result (type 4, 687 leaves, 39 steps):

$$\begin{aligned}
& \frac{b^2 c^2}{3 d^2 \sqrt{d + c^2 d x^2}} - \frac{b c (a + b \operatorname{ArcSinh}[c x])}{d^2 x \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2}} - \frac{2 b c^3 x (a + b \operatorname{ArcSinh}[c x])}{3 d^2 \sqrt{1 + c^2 x^2} \sqrt{d + c^2 d x^2}} - \frac{5 c^2 (a + b \operatorname{ArcSinh}[c x])^2}{6 d (d + c^2 d x^2)^{3/2}} - \\
& \frac{(a + b \operatorname{ArcSinh}[c x])^2}{2 d x^2 (d + c^2 d x^2)^{3/2}} - \frac{5 c^2 (a + b \operatorname{ArcSinh}[c x])^2}{2 d^2 \sqrt{d + c^2 d x^2}} + \frac{26 b c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{ArcTan}[e^{\operatorname{ArcSinh}[c x]}]}{3 d^2 \sqrt{d + c^2 d x^2}} + \\
& \frac{5 c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2 \operatorname{ArcTanh}[e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d + c^2 d x^2}} - \frac{b^2 c^2 \sqrt{1 + c^2 x^2} \operatorname{ArcTanh}[\sqrt{1 + c^2 x^2}]}{d^2 \sqrt{d + c^2 d x^2}} + \\
& \frac{5 b c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, -e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d + c^2 d x^2}} - \frac{13 i b^2 c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, -i e^{\operatorname{ArcSinh}[c x]}]}{3 d^2 \sqrt{d + c^2 d x^2}} + \\
& \frac{13 i b^2 c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[2, i e^{\operatorname{ArcSinh}[c x]}]}{3 d^2 \sqrt{d + c^2 d x^2}} - \frac{5 b c^2 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x]) \operatorname{PolyLog}[2, e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d + c^2 d x^2}} - \\
& \frac{5 b^2 c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, -e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d + c^2 d x^2}} + \frac{5 b^2 c^2 \sqrt{1 + c^2 x^2} \operatorname{PolyLog}[3, e^{\operatorname{ArcSinh}[c x]}]}{d^2 \sqrt{d + c^2 d x^2}}
\end{aligned}$$

**Problem 321: Result valid but suboptimal antiderivative.**

$$\int x^m (d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 8, 935 leaves, 12 steps):



$$\begin{aligned}
& \frac{10 b^2 c^2 d^2 x^{3+m} \sqrt{d + c^2 d x^2}}{(4+m)^3 (6+m)} + \frac{2 b^2 c^2 d^2 (52 + 15 m + m^2) x^{3+m} \sqrt{d + c^2 d x^2}}{(4+m)^2 (6+m)^3} + \frac{2 b^2 c^4 d^2 x^{5+m} \sqrt{d + c^2 d x^2}}{(6+m)^3} - \\
& \frac{30 b c d^2 x^{2+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(2+m)^2 (4+m) (6+m) \sqrt{1 + c^2 x^2}} - \frac{10 b c d^2 x^{2+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(6+m) (8 + 6 m + m^2) \sqrt{1 + c^2 x^2}} - \frac{2 b c d^2 x^{2+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(12 + 8 m + m^2) \sqrt{1 + c^2 x^2}} - \\
& \frac{10 b c^3 d^2 x^{4+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(4+m)^2 (6+m) \sqrt{1 + c^2 x^2}} - \frac{4 b c^3 d^2 x^{4+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(4+m) (6+m) \sqrt{1 + c^2 x^2}} - \\
& \frac{2 b c^5 d^2 x^{6+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])}{(6+m)^2 \sqrt{1 + c^2 x^2}} + \frac{15 d^2 x^{1+m} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^2}{(6+m) (8 + 6 m + m^2)} + \frac{5 d x^{1+m} (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2}{(4+m) (6+m)} + \\
& \frac{x^{1+m} (d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2}{6+m} + \frac{30 b^2 c^2 d^2 x^{3+m} \sqrt{d + c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) (6+m) \sqrt{1 + c^2 x^2}} + \\
& \frac{10 b^2 c^2 d^2 (10 + 3 m) x^{3+m} \sqrt{d + c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^3 (6+m) \sqrt{1 + c^2 x^2}} + \\
& \frac{2 b^2 c^2 d^2 (264 + 130 m + 15 m^2) x^{3+m} \sqrt{d + c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^2 (6+m)^3 \sqrt{1 + c^2 x^2}} + \frac{15 d^3 \operatorname{Unintegrable}\left[\frac{x^m (a + b \operatorname{ArcSinh}[c x])^2}{\sqrt{d + c^2 d x^2}}, x\right]}{(6+m) (8 + 6 m + m^2)}
\end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$\operatorname{Unintegrable}\left[x^m (d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^2, x\right]$

**Problem 322: Result valid but suboptimal antiderivative.**

$$\int x^m (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 8, 487 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 b^2 c^2 d x^{3+m} \sqrt{d+c^2 d x^2}}{(4+m)^3} - \frac{6 b c d x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(2+m)^2 (4+m) \sqrt{1+c^2 x^2}} - \frac{2 b c d x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(8+6 m+m^2) \sqrt{1+c^2 x^2}} - \\
& \frac{2 b c^3 d x^{4+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(4+m)^2 \sqrt{1+c^2 x^2}} + \frac{3 d x^{1+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{8+6 m+m^2} + \\
& \frac{x^{1+m} (d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2}{4+m} + \frac{6 b^2 c^2 d x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) (4+m) \sqrt{1+c^2 x^2}} + \\
& \frac{2 b^2 c^2 d (10+3 m) x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m) (3+m) (4+m)^3 \sqrt{1+c^2 x^2}} + \frac{3 d^2 \operatorname{Unintegrable}\left[\frac{x^m (a+b \operatorname{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}, x\right]}{8+6 m+m^2}
\end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^m (d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^2, x\right]$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int x^m \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2 dx$$

Optimal (type 8, 198 leaves, 3 steps):

$$\begin{aligned}
& - \frac{2 b c x^{2+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])}{(2+m)^2 \sqrt{1+c^2 x^2}} + \frac{x^{1+m} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2+m} + \\
& \frac{2 b^2 c^2 x^{3+m} \sqrt{d+c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, -c^2 x^2\right]}{(2+m)^2 (3+m) \sqrt{1+c^2 x^2}} + \frac{d \operatorname{Unintegrable}\left[\frac{x^m (a+b \operatorname{ArcSinh}[c x])^2}{\sqrt{d+c^2 d x^2}}, x\right]}{2+m}
\end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[x^m \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^2, x\right]$$

Problem 337: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSinh}[a x]^3}{\sqrt{c+a^2 c x^2}} dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{\sqrt{1+a^2 x^2} \operatorname{ArcSinh}[a x]^4}{4 a \sqrt{c+a^2 c x^2}}$$

Result (type 3, 40 leaves, 2 steps):

$$\frac{\sqrt{1+a^2 x^2} \operatorname{ArcSinh}[a x]^4}{4 a \sqrt{c+a^2 c x^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1+c^2 x^2}}{(a+b \operatorname{ArcSinh}[c x])^2} dx$$

Optimal (type 4, 149 leaves, 14 steps):

$$\begin{aligned} & -\frac{x(1+c^2 x^2)}{b c (a+b \operatorname{ArcSinh}[c x])} + \frac{\operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{4 b^2 c^2} + \frac{3 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{4 b^2 c^2} \\ & - \frac{\operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{4 b^2 c^2} - \frac{3 \operatorname{Sinh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{4 b^2 c^2} \end{aligned}$$

Result (type 4, 198 leaves, 14 steps):

$$\begin{aligned} & -\frac{x(1+c^2 x^2)}{b c (a+b \operatorname{ArcSinh}[c x])} - \frac{3 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcSinh}[c x]\right]}{4 b^2 c^2} + \\ & \frac{3 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSinh}[c x]\right]}{4 b^2 c^2} + \frac{\operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{b^2 c^2} + \\ & \frac{3 \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcSinh}[c x]\right]}{4 b^2 c^2} - \frac{3 \operatorname{Sinh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSinh}[c x]\right]}{4 b^2 c^2} - \frac{\operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{b^2 c^2} \end{aligned}$$

Problem 474: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\operatorname{ArcSinh}[a x]}}{\sqrt{c+a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1+a^2 x^2} \operatorname{ArcSinh}[a x]^{3/2}}{3 a \sqrt{c+a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1 + a^2 x^2} \operatorname{ArcSinh}[a x]^{3/2}}{3 a \sqrt{c + a^2 c x^2}}$$

Problem 479: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSinh}[a x]^{3/2}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1 + a^2 x^2} \operatorname{ArcSinh}[a x]^{5/2}}{5 a \sqrt{c + a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1 + a^2 x^2} \operatorname{ArcSinh}[a x]^{5/2}}{5 a \sqrt{c + a^2 c x^2}}$$

Problem 483: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSinh}[a x]^{5/2}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$\frac{2 \sqrt{1 + a^2 x^2} \operatorname{ArcSinh}[a x]^{7/2}}{7 a \sqrt{c + a^2 c x^2}}$$

Result (type 3, 42 leaves, 2 steps):

$$\frac{2 \sqrt{1 + a^2 x^2} \operatorname{ArcSinh}[a x]^{7/2}}{7 a \sqrt{c + a^2 c x^2}}$$

Problem 487: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{\operatorname{ArcSinh}\left[\frac{x}{a}\right]}}{\sqrt{a^2 + x^2}} dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2 a \sqrt{1 + \frac{x^2}{a^2}} \operatorname{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2 + x^2}}$$

Result (type 3, 39 leaves, 2 steps):

$$\frac{2 a \sqrt{1 + \frac{x^2}{a^2}} \operatorname{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{3 \sqrt{a^2 + x^2}}$$

Problem 492: Result optimal but 1 more steps used.

$$\int \frac{\operatorname{ArcSinh}\left[\frac{x}{a}\right]^{3/2}}{\sqrt{a^2 + x^2}} dx$$

Optimal (type 3, 39 leaves, 1 step):

$$\frac{2 a \sqrt{1 + \frac{x^2}{a^2}} \operatorname{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2 + x^2}}$$

Result (type 3, 39 leaves, 2 steps):

$$\frac{2 a \sqrt{1 + \frac{x^2}{a^2}} \operatorname{ArcSinh}\left[\frac{x}{a}\right]^{5/2}}{5 \sqrt{a^2 + x^2}}$$

Problem 495: Result optimal but 1 more steps used.

$$\int \frac{(c + a^2 c x^2)^{5/2}}{\sqrt{\operatorname{ArcSinh}[a x]}} dx$$

Optimal (type 4, 396 leaves, 18 steps):

$$\begin{aligned}
& \frac{5 c^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{8 a \sqrt{1+a^2 x^2}} + \frac{3 c^2 \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \\
& \frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{6} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{3 c^2 \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \\
& \frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{6} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}}
\end{aligned}$$

Result (type 4, 396 leaves, 19 steps):

$$\begin{aligned}
& \frac{5 c^2 \sqrt{c+a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{8 a \sqrt{1+a^2 x^2}} + \frac{3 c^2 \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \\
& \frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{6} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{3 c^2 \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \\
& \frac{15 c^2 \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}} + \frac{c^2 \sqrt{\frac{\pi}{6}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{6} \sqrt{\text{ArcSinh}[a x]}\right]}{64 a \sqrt{1+a^2 x^2}}
\end{aligned}$$

Problem 496: Result optimal but 1 more steps used.

$$\int \frac{(c+a^2 c x^2)^{3/2}}{\sqrt{\text{ArcSinh}[a x]}} dx$$

Optimal (type 4, 264 leaves, 13 steps):

$$\begin{aligned}
& \frac{3 c \sqrt{c+a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{4 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{32 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1+a^2 x^2}} + \\
& \frac{c \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{32 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1+a^2 x^2}}
\end{aligned}$$

Result (type 4, 264 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 c \sqrt{c+a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{4 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erf}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{32 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1+a^2 x^2}} + \\
& \frac{c \sqrt{\pi} \sqrt{c+a^2 c x^2} \text{Erfi}\left[2 \sqrt{\text{ArcSinh}[a x]}\right]}{32 a \sqrt{1+a^2 x^2}} + \frac{c \sqrt{\frac{\pi}{2}} \sqrt{c+a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1+a^2 x^2}}
\end{aligned}$$

Problem 497: Result optimal but 1 more steps used.

$$\int \frac{\sqrt{c + a^2 c x^2}}{\sqrt{\text{ArcSinh}[a x]}} dx$$

Optimal (type 4, 156 leaves, 8 steps):

$$\frac{\sqrt{c + a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{1 + a^2 x^2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c + a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1 + a^2 x^2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c + a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1 + a^2 x^2}}$$

Result (type 4, 156 leaves, 9 steps):

$$\frac{\sqrt{c + a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{1 + a^2 x^2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c + a^2 c x^2} \text{Erf}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1 + a^2 x^2}} + \frac{\sqrt{\frac{\pi}{2}} \sqrt{c + a^2 c x^2} \text{Erfi}\left[\sqrt{2} \sqrt{\text{ArcSinh}[a x]}\right]}{4 a \sqrt{1 + a^2 x^2}}$$

Problem 498: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c + a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}} dx$$

Optimal (type 3, 40 leaves, 1 step):

$$\frac{2 \sqrt{1 + a^2 x^2} \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{c + a^2 c x^2}}$$

Result (type 3, 40 leaves, 2 steps):

$$\frac{2 \sqrt{1 + a^2 x^2} \sqrt{\text{ArcSinh}[a x]}}{a \sqrt{c + a^2 c x^2}}$$

Problem 504: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c + a^2 c x^2} \text{ArcSinh}[a x]^{3/2}} dx$$

Optimal (type 3, 40 leaves, 1 step):

$$-\frac{2 \sqrt{1 + a^2 x^2}}{a \sqrt{c + a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}$$

Result (type 3, 40 leaves, 2 steps):

$$- \frac{2 \sqrt{1 + a^2 x^2}}{a \sqrt{c + a^2 c x^2} \sqrt{\text{ArcSinh}[a x]}}$$

Problem 509: Result optimal but 1 more steps used.

$$\int \frac{1}{\sqrt{c + a^2 c x^2} \text{ArcSinh}[a x]^{5/2}} dx$$

Optimal (type 3, 42 leaves, 1 step):

$$- \frac{2 \sqrt{1 + a^2 x^2}}{3 a \sqrt{c + a^2 c x^2} \text{ArcSinh}[a x]^{3/2}}$$

Result (type 3, 42 leaves, 2 steps):

$$- \frac{2 \sqrt{1 + a^2 x^2}}{3 a \sqrt{c + a^2 c x^2} \text{ArcSinh}[a x]^{3/2}}$$

Problem 512: Result optimal but 1 more steps used.

$$\int x^2 \sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^n dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$- \frac{\sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^{1+n}}{8 b c^3 (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-2 (3+n)} e^{-\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^n \left( -\frac{a+b \text{ArcSinh}[c x]}{b} \right)^{-n} \text{Gamma}\left[1+n, -\frac{4(a+b \text{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \frac{2^{-2 (3+n)} e^{\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^n \left( \frac{a+b \text{ArcSinh}[c x]}{b} \right)^{-n} \text{Gamma}\left[1+n, \frac{4(a+b \text{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}}$$

Result (type 4, 235 leaves, 7 steps):

$$- \frac{\sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^{1+n}}{8 b c^3 (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-2 (3+n)} e^{-\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^n \left( -\frac{a+b \text{ArcSinh}[c x]}{b} \right)^{-n} \text{Gamma}\left[1+n, -\frac{4(a+b \text{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \frac{2^{-2 (3+n)} e^{\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \text{ArcSinh}[c x])^n \left( \frac{a+b \text{ArcSinh}[c x]}{b} \right)^{-n} \text{Gamma}\left[1+n, \frac{4(a+b \text{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}}$$



### Problem 513: Result optimal but 1 more steps used.

$$\int x \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 355 leaves, 9 steps):

$$\begin{aligned} & \frac{3^{-1-n} e^{-\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{3(a + b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{e^{-\frac{a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{a + b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{e^{a/b} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{a + b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{3^{-1-n} e^{\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{3(a + b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} \end{aligned}$$

Result (type 4, 355 leaves, 10 steps):

$$\begin{aligned} & \frac{3^{-1-n} e^{-\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{3(a + b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{e^{-\frac{a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, -\frac{a + b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{e^{a/b} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{a + b \operatorname{ArcSinh}[c x]}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} + \\ & \frac{3^{-1-n} e^{\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a + b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1 + n, \frac{3(a + b \operatorname{ArcSinh}[c x])}{b}\right]}{8 c^2 \sqrt{1 + c^2 x^2}} \end{aligned}$$

### Problem 514: Result optimal but 1 more steps used.

$$\int \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 235 leaves, 6 steps):

$$\frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{2 b c (1+n) \sqrt{1+c^2 x^2}} + \frac{2^{-3-n} e^{-\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} -$$

$$\frac{2^{-3-n} e^{\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}$$

Result (type 4, 235 leaves, 7 steps):

$$\frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{2 b c (1+n) \sqrt{1+c^2 x^2}} + \frac{2^{-3-n} e^{-\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} -$$

$$\frac{2^{-3-n} e^{\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}$$

Problem 515: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n}{x} dx$$

Optimal (type 8, 198 leaves, 6 steps):

$$\frac{d e^{-\frac{a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{2 \sqrt{d+c^2 d x^2}} +$$

$$\frac{d e^{a/b} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{2 \sqrt{d+c^2 d x^2}} + d \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x \sqrt{d+c^2 d x^2}}, x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n}{x}, x\right]$$

Problem 516: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2} dx$$

Optimal (type 8, 83 leaves, 3 steps):

$$\frac{c d \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{b (1+n) \sqrt{d+c^2 d x^2}} + d \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x^2 \sqrt{d+c^2 d x^2}}, x\right]$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2}, x\right]$$

Problem 517: Result optimal but 1 more steps used.

$$\int x^2 (d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 616 leaves, 12 steps):

$$\begin{aligned} & -\frac{d \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{16 b c^3 (1+n) \sqrt{1+c^2 x^2}} + \frac{2^{-7-n} \times 3^{-1-n} d e^{-\frac{6a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} + \\ & \frac{2^{-7-2n} d e^{-\frac{4a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} - \\ & \frac{2^{-7-n} d e^{-\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} + \\ & \frac{2^{-7-n} d e^{\frac{2a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} - \\ & \frac{2^{-7-2n} d e^{\frac{4a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} - \\ & \frac{2^{-7-n} \times 3^{-1-n} d e^{\frac{6a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1+c^2 x^2}} \end{aligned}$$

Result (type 4, 616 leaves, 13 steps):

$$\begin{aligned}
& - \frac{d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{16 b c^3 (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-7-n} \times 3^{-1-n} d e^{-\frac{6a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-2n} d e^{-\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} d e^{-\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-n} d e^{\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-2n} d e^{\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} \times 3^{-1-n} d e^{\frac{6a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 518: Result optimal but 1 more steps used.

$$\int x (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 542 leaves, 12 steps):

$$\begin{aligned}
& \frac{5^{-1-n} d e^{-\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{-n} d e^{-\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{d e^{-\frac{a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{d e^{a/b \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{-n} d e^{\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-1-n} d e^{\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 4, 542 leaves, 13 steps):

$$\begin{aligned}
& \frac{5^{-1-n} d e^{-\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{-n} d e^{-\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{d e^{-\frac{a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{d e^{a/b \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{-n} d e^{\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-1-n} d e^{\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 c^2 \sqrt{1+c^2 x^2}}
\end{aligned}$$

## Problem 519: Result optimal but 1 more steps used.

$$\int (d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 420 leaves, 9 steps):

$$\begin{aligned} & \frac{3 d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{8 b c (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-2(3+n)} d e^{-\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} + \\ & \frac{2^{-3-n} d e^{-\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} - \\ & \frac{2^{-3-n} d e^{\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} - \\ & \frac{2^{-2(3+n)} d e^{\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} \end{aligned}$$

Result (type 4, 420 leaves, 10 steps):

$$\begin{aligned} & \frac{3 d \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{8 b c (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-2(3+n)} d e^{-\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} + \\ & \frac{2^{-3-n} d e^{-\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( -\frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} - \\ & \frac{2^{-3-n} d e^{\frac{2a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} - \\ & \frac{2^{-2(3+n)} d e^{\frac{4a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left( \frac{a+b \operatorname{ArcSinh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1 + c^2 x^2}} \end{aligned}$$

## Problem 520: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{3/2} (a + b \operatorname{ArcSinh}[c x])^n}{x} dx$$

Optimal (type 8, 389 leaves, 15 steps):

$$\begin{aligned}
& \frac{3^{-1-n} d^2 e^{-\frac{3a}{b} \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + \\
& \frac{5 d^2 e^{-\frac{a}{b} \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + \\
& \frac{5 d^2 e^{a/b \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + \\
& \frac{3^{-1-n} d^2 e^{\frac{3a}{b} \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + d^2 \operatorname{Unintegrateable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x \sqrt{d+c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{(d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^n}{x}, x\right]$$

Problem 521: Result valid but suboptimal antiderivative.

$$\int \frac{(d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2} dx$$

Optimal (type 8, 272 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 c d^2 \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{2 b (1+n) \sqrt{d+c^2 d x^2}} + \frac{2^{-3-n} c d^2 e^{-\frac{2a}{b} \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}} - \\
& \frac{2^{-3-n} c d^2 e^{\frac{2a}{b} \sqrt{1+c^2 x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d+c^2 d x^2}} + d^2 \operatorname{Unintegrateable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x^2 \sqrt{d+c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{(d+c^2 d x^2)^{3/2} (a+b \operatorname{ArcSinh}[c x])^n}{x^2}, x\right]$$

Problem 522: Result optimal but 1 more steps used.

$$\int x^2 (d+c^2 d x^2)^{5/2} (a+b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 816 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{128 b c^3 (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-11-3 n} d^2 e^{-\frac{8 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{8(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-2(4+n)} d^2 e^{-\frac{4 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-2(4+n)} d^2 e^{\frac{4 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-11-3 n} d^2 e^{\frac{8 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{8(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Result (type 4, 816 leaves, 16 steps):



$$\begin{aligned}
& - \frac{5 d^2 \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{128 b c^3 (1+n) \sqrt{1 + c^2 x^2}} + \frac{2^{-11-3 n} d^2 e^{-\frac{8 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{8(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-2(4+n)} d^2 e^{-\frac{4 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} + \\
& \frac{2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-2(4+n)} d^2 e^{\frac{4 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}} - \\
& \frac{2^{-11-3 n} d^2 e^{\frac{8 a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{8(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c^3 \sqrt{1 + c^2 x^2}}
\end{aligned}$$

Problem 523: Result optimal but 1 more steps used.

$$\int x (d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 745 leaves, 15 steps):

$$\begin{aligned}
& \frac{7^{-1-n} d^2 e^{-\frac{7a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{7(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-n} d^2 e^{-\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{1-n} d^2 e^{-\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5 d^2 e^{-\frac{a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5 d^2 e^{a/b} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{1-n} d^2 e^{\frac{3a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-n} d^2 e^{\frac{5a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{7^{-1-n} d^2 e^{\frac{7a}{b} \sqrt{d+c^2 d x^2}} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{7(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 4, 745 leaves, 16 steps):

$$\begin{aligned}
& \frac{7^{-1-n} d^2 e^{-\frac{7a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{7(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-n} d^2 e^{-\frac{5a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5 d^2 e^{-\frac{a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5 d^2 e^{a/b} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{3^{1-n} d^2 e^{\frac{3a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{5^{-n} d^2 e^{\frac{5a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}} + \\
& \frac{7^{-1-n} d^2 e^{\frac{7a}{b}} \sqrt{d + c^2 d x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{7(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{128 c^2 \sqrt{1+c^2 x^2}}
\end{aligned}$$

Problem 524: Result optimal but 1 more steps used.

$$\int (d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^n dx$$

Optimal (type 4, 632 leaves, 12 steps):

$$\begin{aligned}
& \frac{5 d^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{16 b c (1+n) \sqrt{1+c^2 x^2}} + \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} + \\
& \frac{3 \times 2^{-7-2 n} d^2 e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} + \\
& \frac{15 \times 2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{15 \times 2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{3 \times 2^{-7-2 n} d^2 e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}
\end{aligned}$$

Result (type 4, 632 leaves, 13 steps):

$$\begin{aligned}
& \frac{5 d^2 \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^{1+n}}{16 b c (1+n) \sqrt{1+c^2 x^2}} + \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{-\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} + \\
& \frac{3 \times 2^{-7-2 n} d^2 e^{-\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} + \\
& \frac{15 \times 2^{-7-n} d^2 e^{-\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{15 \times 2^{-7-n} d^2 e^{\frac{2 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{3 \times 2^{-7-2 n} d^2 e^{\frac{4 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}} - \\
& \frac{2^{-7-n} \times 3^{-1-n} d^2 e^{\frac{6 a}{b}} \sqrt{d+c^2 d x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{6(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{c \sqrt{1+c^2 x^2}}
\end{aligned}$$

## Problem 525: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^n}{x} dx$$

Optimal (type 8, 755 leaves, 27 steps):

$$\begin{aligned} & \frac{5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} - \\ & \frac{5 \times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} + \\ & \frac{3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + \\ & \frac{11 d^3 e^{-\frac{a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 \sqrt{d+c^2 d x^2}} + \\ & \frac{11 d^3 e^{a/b} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcSinh}[c x]}{b}\right]}{16 \sqrt{d+c^2 d x^2}} - \\ & \frac{5 \times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} + \\ & \frac{3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{8 \sqrt{d+c^2 d x^2}} + \\ & \frac{5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{32 \sqrt{d+c^2 d x^2}} + d^3 \operatorname{Unintegrateable}\left[\frac{(a+b \operatorname{ArcSinh}[c x])^n}{x \sqrt{d+c^2 d x^2}}, x\right] \end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{(d+c^2 d x^2)^{5/2} (a+b \operatorname{ArcSinh}[c x])^n}{x}, x\right]$$

### Problem 526: Result valid but suboptimal antiderivative.

$$\int \frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^n}{x^2} dx$$

Optimal (type 8, 454 leaves, 18 steps):

$$\begin{aligned} & \frac{15 c d^3 \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^{1+n}}{8 b (1+n) \sqrt{d + c^2 d x^2}} + \frac{2^{-2(3+n)} c d^3 e^{-\frac{4a}{b}} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d + c^2 d x^2}} + \\ & \frac{2^{-2-n} c d^3 e^{-\frac{2a}{b}} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(-\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d + c^2 d x^2}} - \\ & \frac{2^{-2-n} c d^3 e^{\frac{2a}{b}} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d + c^2 d x^2}} - \\ & \frac{2^{-2(3+n)} c d^3 e^{\frac{4a}{b}} \sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^n \left(\frac{a+b \operatorname{ArcSinh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcSinh}[c x])}{b}\right]}{\sqrt{d + c^2 d x^2}} + d^3 \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcSinh}[c x])^n}{x^2 \sqrt{d + c^2 d x^2}}, x\right] \end{aligned}$$

Result (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(d + c^2 d x^2)^{5/2} (a + b \operatorname{ArcSinh}[c x])^n}{x^2}, x\right]$$

## Test results for the 371 problems in "7.1.5 Inverse hyperbolic sine functions.m"

### Problem 50: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcSinh}[c x]}{\sqrt{d + c^2 d x^2}} dx$$

Optimal (type 3, 47 leaves, 1 step):

$$\frac{\sqrt{1 + c^2 x^2} (a + b \operatorname{ArcSinh}[c x])^2}{2 b c \sqrt{d + c^2 d x^2}}$$

Result (type 3, 47 leaves, 2 steps):

$$\frac{\sqrt{1+c^2 x^2} (a+b \operatorname{ArcSinh}[c x])^2}{2 b c \sqrt{d+c^2 d x^2}}$$

Problem 170: Result valid but suboptimal antiderivative.

$$\int \frac{(c e + d e x)^2}{(a + b \operatorname{ArcSinh}[c + d x])^3} dx$$

Optimal (type 4, 246 leaves, 18 steps):

$$\begin{aligned} & -\frac{e^2 (c + d x)^2 \sqrt{1 + (c + d x)^2}}{2 b d (a + b \operatorname{ArcSinh}[c + d x])^2} - \frac{e^2 (c + d x)}{b^2 d (a + b \operatorname{ArcSinh}[c + d x])} - \frac{3 e^2 (c + d x)^3}{2 b^2 d (a + b \operatorname{ArcSinh}[c + d x])} - \frac{e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}\right]}{8 b^3 d} + \\ & \frac{9 e^2 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3 (a + b \operatorname{ArcSinh}[c + d x])}{b}\right]}{8 b^3 d} + \frac{e^2 \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}\right]}{8 b^3 d} - \frac{9 e^2 \operatorname{Sinh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3 (a + b \operatorname{ArcSinh}[c + d x])}{b}\right]}{8 b^3 d} \end{aligned}$$

Result (type 4, 305 leaves, 18 steps):

$$\begin{aligned} & -\frac{e^2 (c + d x)^2 \sqrt{1 + (c + d x)^2}}{2 b d (a + b \operatorname{ArcSinh}[c + d x])^2} - \frac{e^2 (c + d x)}{b^2 d (a + b \operatorname{ArcSinh}[c + d x])} - \frac{3 e^2 (c + d x)^3}{2 b^2 d (a + b \operatorname{ArcSinh}[c + d x])} - \\ & \frac{9 e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcSinh}[c + d x]\right]}{8 b^3 d} + \frac{9 e^2 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSinh}[c + d x]\right]}{8 b^3 d} + \\ & \frac{e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}\right]}{b^3 d} + \frac{9 e^2 \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcSinh}[c + d x]\right]}{8 b^3 d} - \\ & \frac{9 e^2 \operatorname{Sinh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcSinh}[c + d x]\right]}{8 b^3 d} - \frac{e^2 \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a + b \operatorname{ArcSinh}[c + d x]}{b}\right]}{b^3 d} \end{aligned}$$

Problem 163: Result valid but suboptimal antiderivative.

$$\int \sqrt{f x} (a + b \operatorname{ArcCosh}[c x])^2 dx$$

Optimal (type 5, 128 leaves, 2 steps):

$$\begin{aligned} & \frac{2 (f x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 f} - \frac{8 b c (f x)^{5/2} \sqrt{1 - c x} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{15 f^2 \sqrt{-1 + c x}} - \\ & \frac{16 b^2 c^2 (f x)^{7/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{105 f^3} \end{aligned}$$

Result (type 5, 141 leaves, 2 steps):

$$\frac{2 (f x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 f} - \frac{8 b c (f x)^{5/2} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{15 f^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{16 b^2 c^2 (f x)^{7/2} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{105 f^3}$$

Test results for the 569 problems in "7.2.4 (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Problem 63: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{x^6} dx$$

Optimal (type 3, 199 leaves, 4 steps):

$$- \frac{b c \sqrt{d - c^2 d x^2}}{20 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{30 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{5 d x^5} - \frac{2 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{15 d x^3} - \frac{2 b c^5 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 226 leaves, 5 steps):

$$- \frac{b c \sqrt{d - c^2 d x^2}}{20 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{30 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{5 x^5} + \frac{c^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 x^3} + \frac{2 c^4 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 x} - \frac{2 b c^5 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 64: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{x^8} dx$$

Optimal (type 3, 279 leaves, 4 steps):



$$\begin{aligned}
& - \frac{b c \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{140 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^5 \sqrt{d - c^2 d x^2}}{105 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{7 d x^7} - \\
& \frac{4 c^2 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{35 d x^5} - \frac{8 c^4 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{105 d x^3} - \frac{8 b c^7 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{105 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 3, 303 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2}}{140 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^5 \sqrt{d - c^2 d x^2}}{105 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{7 x^7} + \\
& \frac{c^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 x^5} + \frac{4 c^4 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{105 x^3} + \frac{8 c^6 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{105 x} - \frac{8 b c^7 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{105 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 272 leaves, 3 steps):

$$\begin{aligned}
& \frac{8 b x \sqrt{d - c^2 d x^2}}{105 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b x^3 \sqrt{d - c^2 d x^2}}{315 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^5 \sqrt{d - c^2 d x^2}}{175 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{3 c^6 d} + \frac{2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 c^6 d^2} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^6 d^3}
\end{aligned}$$

Result (type 3, 302 leaves, 4 steps):

$$\begin{aligned}
& \frac{8 b x \sqrt{d - c^2 d x^2}}{105 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b x^3 \sqrt{d - c^2 d x^2}}{315 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^5 \sqrt{d - c^2 d x^2}}{175 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{8 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{105 c^6} - \\
& \frac{4 x^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 c^4} - \frac{x^4 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{7 c^2}
\end{aligned}$$

Problem 66: Result valid but suboptimal antiderivative.

$$\int x^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$\frac{2 b x \sqrt{d - c^2 d x^2}}{15 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^3 \sqrt{d - c^2 d x^2}}{45 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^5 \sqrt{d - c^2 d x^2}}{25 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{3 c^4 d} + \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 c^4 d^2}$$

Result (type 3, 214 leaves, 4 steps):

$$\frac{2 b x \sqrt{d - c^2 d x^2}}{15 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b x^3 \sqrt{d - c^2 d x^2}}{45 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c x^5 \sqrt{d - c^2 d x^2}}{25 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 c^4} - \frac{x^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{5 c^2}$$

Problem 77: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{x^8} dx$$

Optimal (type 3, 247 leaves, 5 steps):

$$-\frac{b c d \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^3 d \sqrt{d - c^2 d x^2}}{35 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{70 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{7 d x^7} - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{35 d x^5} + \frac{2 b c^7 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 322 leaves, 6 steps):

$$-\frac{b c d \sqrt{d - c^2 d x^2}}{42 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^3 d \sqrt{d - c^2 d x^2}}{35 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{70 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{3 c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 x^5} - \frac{c^4 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 x^3} - \frac{2 c^6 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 x} - \frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{7 x^7} + \frac{2 b c^7 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{35 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 78: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{x^{10}} dx$$

Optimal (type 3, 328 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{72 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 b c^3 d \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c^5 d \sqrt{d - c^2 d x^2}}{420 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 b c^7 d \sqrt{d - c^2 d x^2}}{315 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{9 d x^9} - \\
& \frac{4 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{63 d x^7} - \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{315 d x^5} + \frac{8 b c^9 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{315 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 3, 401 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{72 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 b c^3 d \sqrt{d - c^2 d x^2}}{189 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{420 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 b c^7 d \sqrt{d - c^2 d x^2}}{315 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{21 x^7} - \frac{c^4 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{105 x^5} - \frac{4 c^6 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{315 x^3} - \\
& \frac{8 c^8 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{315 x} - \frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{9 x^9} + \frac{8 b c^9 d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{315 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 79: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{x^{12}} dx$$

Optimal (type 3, 409 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d \sqrt{d - c^2 d x^2}}{66 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{1386 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^7 d \sqrt{d - c^2 d x^2}}{770 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{4 b c^9 d \sqrt{d - c^2 d x^2}}{1155 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{11 d x^{11}} - \frac{2 c^2 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{33 d x^9} - \\
& \frac{8 c^4 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{231 d x^7} - \frac{16 c^6 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{1155 d x^5} + \frac{16 b c^{11} d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{1155 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 3, 480 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b c d \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d \sqrt{d - c^2 d x^2}}{66 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d \sqrt{d - c^2 d x^2}}{1386 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c^7 d \sqrt{d - c^2 d x^2}}{770 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{4 b c^9 d \sqrt{d - c^2 d x^2}}{1155 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{33 x^9} - \\
& \frac{c^4 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{231 x^7} - \frac{2 c^6 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{385 x^5} - \frac{8 c^8 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{1155 x^3} - \\
& \frac{16 c^{10} d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{1155 x} - \frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{11 x^{11}} + \frac{16 b c^{11} d \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{1155 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 80: Result valid but suboptimal antiderivative.

$$\int x^7 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 399 leaves, 4 steps):

$$\begin{aligned}
& \frac{16 b d x \sqrt{d - c^2 d x^2}}{1155 c^7 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{8 b d x^3 \sqrt{d - c^2 d x^2}}{3465 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b d x^5 \sqrt{d - c^2 d x^2}}{1925 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d x^7 \sqrt{d - c^2 d x^2}}{1617 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{4 b c d x^9 \sqrt{d - c^2 d x^2}}{297 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 c^8 d} + \\
& \frac{3 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^8 d^2} - \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcCosh}[c x])}{3 c^8 d^3} + \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcCosh}[c x])}{11 c^8 d^4}
\end{aligned}$$

Result (type 3, 460 leaves, 5 steps):

$$\begin{aligned}
& \frac{16 b d x \sqrt{d - c^2 d x^2}}{1155 c^7 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{8 b d x^3 \sqrt{d - c^2 d x^2}}{3465 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b d x^5 \sqrt{d - c^2 d x^2}}{1925 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d x^7 \sqrt{d - c^2 d x^2}}{1617 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{4 b c d x^9 \sqrt{d - c^2 d x^2}}{297 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c^3 d x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{16 d (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{1155 c^8} - \frac{8 d x^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{231 c^6} - \\
& \frac{2 d x^4 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{33 c^4} - \frac{d x^6 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{11 c^2}
\end{aligned}$$

Problem 81: Result valid but suboptimal antiderivative.

$$\int x^5 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 321 leaves, 4 steps):

$$\frac{8 b d x \sqrt{d - c^2 d x^2}}{315 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b d x^3 \sqrt{d - c^2 d x^2}}{945 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d x^5 \sqrt{d - c^2 d x^2}}{525 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{10 b c d x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 c^6 d} + \frac{2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^6 d^2} - \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcCosh}[c x])}{9 c^6 d^3}$$

Result (type 3, 366 leaves, 5 steps):

$$\frac{8 b d x \sqrt{d - c^2 d x^2}}{315 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b d x^3 \sqrt{d - c^2 d x^2}}{945 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d x^5 \sqrt{d - c^2 d x^2}}{525 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{10 b c d x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{8 d (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{315 c^6} - \frac{4 d x^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{63 c^4} - \frac{d x^4 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{9 c^2}$$

Problem 82: Result valid but suboptimal antiderivative.

$$\int x^3 (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 243 leaves, 4 steps):

$$\frac{2 b d x \sqrt{d - c^2 d x^2}}{35 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d x^3 \sqrt{d - c^2 d x^2}}{105 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{8 b c d x^5 \sqrt{d - c^2 d x^2}}{175 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 c^4 d} + \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^4 d^2}$$

Result (type 3, 272 leaves, 5 steps):

$$\frac{2 b d x \sqrt{d - c^2 d x^2}}{35 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d x^3 \sqrt{d - c^2 d x^2}}{105 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{8 b c d x^5 \sqrt{d - c^2 d x^2}}{175 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 d x^7 \sqrt{d - c^2 d x^2}}{49 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 d (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{35 c^4} - \frac{d x^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{7 c^2}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 293 leaves, 10 steps):

$$\begin{aligned}
& -\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b d^2 (1-c^2 x^2)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) + \\
& \frac{5}{24} d x (d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x]) + \frac{1}{6} x (d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x]) - \frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Result (type 3, 324 leaves, 9 steps):

$$\begin{aligned}
& -\frac{25 b c d^2 x^2 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{5 b c^3 d^2 x^4 \sqrt{d-c^2 d x^2}}{96 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b d^2 (1-c^2 x^2)^3 \sqrt{d-c^2 d x^2}}{36 c \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) + \frac{5}{24} d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) + \\
& \frac{1}{6} d^2 x (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) - \frac{5 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{32 b c \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])}{x^2} dx$$

Optimal (type 3, 284 leaves, 12 steps):

$$\begin{aligned}
& \frac{9 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c^5 d^2 x^4 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) - \frac{5}{4} c^2 d x (d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x]) - \\
& \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])}{x} + \frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{16 b \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c d^2 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{\sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Result (type 3, 315 leaves, 11 steps):

$$\begin{aligned}
& \frac{9 b c^3 d^2 x^2 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c^5 d^2 x^4 \sqrt{d-c^2 d x^2}}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) - \\
& \frac{5}{4} c^2 d^2 x (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x]) - \frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{x} + \\
& \frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{16 b \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c d^2 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{\sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

### Problem 91: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{x^4} dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$\begin{aligned} & -\frac{b c d^2 \sqrt{d - c^2 d x^2}}{6 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) + \frac{5 c^2 d (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{3 x} \\ & - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 b c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 3, 324 leaves, 11 steps):

$$\begin{aligned} & -\frac{b c d^2 \sqrt{d - c^2 d x^2}}{6 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^2 \sqrt{d - c^2 d x^2}}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) + \\ & \frac{5 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x} - \frac{d^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^3} \\ & - \frac{5 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 b c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

### Problem 92: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{x^6} dx$$

Optimal (type 3, 293 leaves, 12 steps):

$$\begin{aligned} & -\frac{b c d^2 \sqrt{d - c^2 d x^2}}{20 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{11 b c^3 d^2 \sqrt{d - c^2 d x^2}}{30 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{c^4 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{x} + \frac{c^2 d (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{3 x^3} \\ & - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])}{5 x^5} + \frac{c^5 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{23 b c^5 d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 3, 324 leaves, 11 steps):

$$\begin{aligned}
& -\frac{b c d^2 \sqrt{d-c^2 d x^2}}{20 x^4 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{11 b c^3 d^2 \sqrt{d-c^2 d x^2}}{30 x^2 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{c^4 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{x} + \\
& \frac{c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{3 x^3} - \frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{5 x^5} + \\
& \frac{c^5 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{2 b \sqrt{-1+c x} \sqrt{1+c x}} + \frac{23 b c^5 d^2 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{15 \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 94: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])}{x^{10}} dx$$

Optimal (type 3, 314 leaves, 6 steps):

$$\begin{aligned}
& -\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c d^2 (1-c^2 x^2)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{(d-c^2 d x^2)^{7/2} (a+b \operatorname{ArcCosh}[c x])}{9 d x^9} - \frac{2 c^2 (d-c^2 d x^2)^{7/2} (a+b \operatorname{ArcCosh}[c x])}{63 d x^7} - \frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Result (type 3, 448 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b c^3 d^2 \sqrt{d-c^2 d x^2}}{189 x^6 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{b c^5 d^2 \sqrt{d-c^2 d x^2}}{42 x^4 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c^7 d^2 \sqrt{d-c^2 d x^2}}{21 x^2 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{b c d^2 (1-c^2 x^2)^4 \sqrt{d-c^2 d x^2}}{72 x^8 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{c^4 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{21 x^5} + \frac{c^6 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{63 x^3} + \frac{2 c^8 d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{63 x} + \\
& \frac{5 c^2 d^2 (1-c x) (1+c x) \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{63 x^7} - \frac{d^2 (1-c x)^2 (1+c x)^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{9 x^9} - \frac{2 b c^9 d^2 \sqrt{d-c^2 d x^2} \operatorname{Log}[x]}{63 \sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 95: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])}{x^{12}} dx$$

Optimal (type 3, 385 leaves, 5 steps):



$$\begin{aligned}
& - \frac{b c d^2 \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{23 b c^3 d^2 \sqrt{d - c^2 d x^2}}{792 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{113 b c^5 d^2 \sqrt{d - c^2 d x^2}}{4158 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{924 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2}}{693 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{11 d x^{11}} - \\
& \frac{4 c^2 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{99 d x^9} - \frac{8 c^4 (d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{693 d x^7} - \frac{8 b c^{11} d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{693 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 3, 519 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b c d^2 \sqrt{d - c^2 d x^2}}{110 x^{10} \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{23 b c^3 d^2 \sqrt{d - c^2 d x^2}}{792 x^8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{113 b c^5 d^2 \sqrt{d - c^2 d x^2}}{4158 x^6 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c^7 d^2 \sqrt{d - c^2 d x^2}}{924 x^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b c^9 d^2 \sqrt{d - c^2 d x^2}}{693 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5 c^4 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{231 x^7} + \\
& \frac{c^6 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{231 x^5} + \frac{4 c^8 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{693 x^3} + \frac{8 c^{10} d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{693 x} + \\
& \frac{5 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{99 x^9} - \frac{d^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{11 x^{11}} - \frac{8 b c^{11} d^2 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{693 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 96: Result valid but suboptimal antiderivative.

$$\int x^7 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 458 leaves, 4 steps):

$$\begin{aligned}
& \frac{16 b d^2 x \sqrt{d - c^2 d x^2}}{3003 c^7 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{8 b d^2 x^3 \sqrt{d - c^2 d x^2}}{9009 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b d^2 x^5 \sqrt{d - c^2 d x^2}}{5005 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 b d^2 x^7 \sqrt{d - c^2 d x^2}}{21021 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{53 b c d^2 x^9 \sqrt{d - c^2 d x^2}}{3861 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{27 b c^3 d^2 x^{11} \sqrt{d - c^2 d x^2}}{1573 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^{13} \sqrt{d - c^2 d x^2}}{169 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^8 d} + \\
& \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcCosh}[c x])}{3 c^8 d^2} - \frac{3 (d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcCosh}[c x])}{11 c^8 d^3} + \frac{(d - c^2 d x^2)^{13/2} (a + b \operatorname{ArcCosh}[c x])}{13 c^8 d^4}
\end{aligned}$$

Result (type 3, 527 leaves, 5 steps):

$$\begin{aligned}
& \frac{16 b d^2 x \sqrt{d - c^2 d x^2}}{3003 c^7 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{8 b d^2 x^3 \sqrt{d - c^2 d x^2}}{9009 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{2 b d^2 x^5 \sqrt{d - c^2 d x^2}}{5005 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{5 b d^2 x^7 \sqrt{d - c^2 d x^2}}{21021 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{53 b c d^2 x^9 \sqrt{d - c^2 d x^2}}{3861 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{27 b c^3 d^2 x^{11} \sqrt{d - c^2 d x^2}}{1573 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^{13} \sqrt{d - c^2 d x^2}}{169 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{16 d^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3003 c^8} - \frac{8 d^2 x^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{429 c^6} - \\
& \frac{6 d^2 x^4 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{143 c^4} - \frac{d^2 x^6 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{13 c^2}
\end{aligned}$$

Problem 97: Result valid but suboptimal antiderivative.

$$\int x^5 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 378 leaves, 4 steps):

$$\begin{aligned}
& \frac{8 b d^2 x \sqrt{d - c^2 d x^2}}{693 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b d^2 x^3 \sqrt{d - c^2 d x^2}}{2079 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d^2 x^5 \sqrt{d - c^2 d x^2}}{1155 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{113 b c d^2 x^7 \sqrt{d - c^2 d x^2}}{4851 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{23 b c^3 d^2 x^9 \sqrt{d - c^2 d x^2}}{891 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c^5 d^2 x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^6 d} + \frac{2 (d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcCosh}[c x])}{9 c^6 d^2} - \frac{(d - c^2 d x^2)^{11/2} (a + b \operatorname{ArcCosh}[c x])}{11 c^6 d^3}
\end{aligned}$$

Result (type 3, 429 leaves, 5 steps):

$$\begin{aligned}
& \frac{8 b d^2 x \sqrt{d - c^2 d x^2}}{693 c^5 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b d^2 x^3 \sqrt{d - c^2 d x^2}}{2079 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d^2 x^5 \sqrt{d - c^2 d x^2}}{1155 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{113 b c d^2 x^7 \sqrt{d - c^2 d x^2}}{4851 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{23 b c^3 d^2 x^9 \sqrt{d - c^2 d x^2}}{891 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^{11} \sqrt{d - c^2 d x^2}}{121 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{8 d^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{693 c^6} - \\
& \frac{4 d^2 x^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{99 c^4} - \frac{d^2 x^4 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{11 c^2}
\end{aligned}$$

Problem 98: Result valid but suboptimal antiderivative.

$$\int x^3 (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 3, 298 leaves, 4 steps):

$$\frac{2 b d^2 x \sqrt{d - c^2 d x^2}}{63 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d^2 x^3 \sqrt{d - c^2 d x^2}}{189 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c d^2 x^5 \sqrt{d - c^2 d x^2}}{21 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{19 b c^3 d^2 x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{7/2} (a + b \operatorname{ArcCosh}[c x])}{7 c^4 d} + \frac{(d - c^2 d x^2)^{9/2} (a + b \operatorname{ArcCosh}[c x])}{9 c^4 d^2}$$

Result (type 3, 331 leaves, 5 steps):

$$\frac{2 b d^2 x \sqrt{d - c^2 d x^2}}{63 c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d^2 x^3 \sqrt{d - c^2 d x^2}}{189 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c d^2 x^5 \sqrt{d - c^2 d x^2}}{21 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{19 b c^3 d^2 x^7 \sqrt{d - c^2 d x^2}}{441 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^5 d^2 x^9 \sqrt{d - c^2 d x^2}}{81 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 d^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{63 c^4} - \frac{d^2 x^2 (1 - c x)^3 (1 + c x)^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{9 c^2}$$

Problem 103: Result valid but suboptimal antiderivative.

$$\int \sqrt{1 - x^2} \operatorname{ArcCosh}[x] \, dx$$

Optimal (type 3, 66 leaves, 3 steps):

$$-\frac{\sqrt{1 - x^2} x^2}{4 \sqrt{-1 + x}} + \frac{1}{2} x \sqrt{1 - x^2} \operatorname{ArcCosh}[x] - \frac{\sqrt{1 - x^2} \operatorname{ArcCosh}[x]^2}{4 \sqrt{-1 + x}}$$

Result (type 3, 84 leaves, 4 steps):

$$-\frac{x^2 \sqrt{1 - x^2}}{4 \sqrt{-1 + x} \sqrt{1 + x}} + \frac{1}{2} x \sqrt{1 - x^2} \operatorname{ArcCosh}[x] - \frac{\sqrt{1 - x^2} \operatorname{ArcCosh}[x]^2}{4 \sqrt{-1 + x} \sqrt{1 + x}}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCosh}[c x])}{\sqrt{d - c^2 d x^2}} \, dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{8 b x \sqrt{-1 + c x} \sqrt{1 + c x}}{15 c^5 \sqrt{d - c^2 d x^2}} - \frac{4 b x^3 \sqrt{-1 + c x} \sqrt{1 + c x}}{45 c^3 \sqrt{d - c^2 d x^2}} - \frac{b x^5 \sqrt{-1 + c x} \sqrt{1 + c x}}{25 c \sqrt{d - c^2 d x^2}} - \frac{8 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 c^6 d} - \frac{4 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{15 c^4 d} - \frac{x^4 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{5 c^2 d}$$

Result (type 3, 260 leaves, 7 steps):

$$\begin{aligned}
& - \frac{8 b x \sqrt{-1+c x} \sqrt{1+c x}}{15 c^5 \sqrt{d-c^2 d x^2}} - \frac{4 b x^3 \sqrt{-1+c x} \sqrt{1+c x}}{45 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^5 \sqrt{-1+c x} \sqrt{1+c x}}{25 c \sqrt{d-c^2 d x^2}} - \\
& \frac{8 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{15 c^6 \sqrt{d-c^2 d x^2}} - \frac{4 x^2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{15 c^4 \sqrt{d-c^2 d x^2}} - \frac{x^4 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{5 c^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 105: Result valid but suboptimal antiderivative.**

$$\int \frac{x^4 (a+b \operatorname{ArcCosh}[c x])}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 212 leaves, 5 steps):

$$\begin{aligned}
& - \frac{3 b x^2 \sqrt{-1+c x} \sqrt{1+c x}}{16 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^4 \sqrt{-1+c x} \sqrt{1+c x}}{16 c \sqrt{d-c^2 d x^2}} - \frac{3 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{8 c^4 d} - \\
& \frac{x^3 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{4 c^2 d} + \frac{3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2}{16 b c^5 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 228 leaves, 6 steps):

$$\begin{aligned}
& - \frac{3 b x^2 \sqrt{-1+c x} \sqrt{1+c x}}{16 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^4 \sqrt{-1+c x} \sqrt{1+c x}}{16 c \sqrt{d-c^2 d x^2}} - \frac{3 x (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{8 c^4 \sqrt{d-c^2 d x^2}} - \\
& \frac{x^3 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{4 c^2 \sqrt{d-c^2 d x^2}} + \frac{3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2}{16 b c^5 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

**Problem 106: Result valid but suboptimal antiderivative.**

$$\int \frac{x^3 (a+b \operatorname{ArcCosh}[c x])}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 156 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^3 \sqrt{-1+c x} \sqrt{1+c x}}{9 c \sqrt{d-c^2 d x^2}} - \frac{2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{3 c^4 d} - \frac{x^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])}{3 c^2 d}
\end{aligned}$$

Result (type 3, 172 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^3 \sqrt{-1+c x} \sqrt{1+c x}}{9 c \sqrt{d-c^2 d x^2}} - \frac{2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{3 c^4 \sqrt{d-c^2 d x^2}} - \frac{x^2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])}{3 c^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

### Problem 107: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 132 leaves, 3 steps):

$$-\frac{b x^2 \sqrt{-1 + c x} \sqrt{1 + c x}}{4 c \sqrt{d - c^2 d x^2}} - \frac{x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 c^2 d} + \frac{\sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])^2}{4 b c^3 \sqrt{d - c^2 d x^2}}$$

Result (type 3, 140 leaves, 4 steps):

$$-\frac{b x^2 \sqrt{-1 + c x} \sqrt{1 + c x}}{4 c \sqrt{d - c^2 d x^2}} - \frac{x (1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{2 c^2 \sqrt{d - c^2 d x^2}} + \frac{\sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])^2}{4 b c^3 \sqrt{d - c^2 d x^2}}$$

### Problem 108: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcCosh}[c x])}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$-\frac{b x \sqrt{-1 + c x} \sqrt{1 + c x}}{c \sqrt{d - c^2 d x^2}} - \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{c^2 d}$$

Result (type 3, 80 leaves, 3 steps):

$$-\frac{b x \sqrt{-1 + c x} \sqrt{1 + c x}}{c \sqrt{d - c^2 d x^2}} - \frac{(1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{c^2 \sqrt{d - c^2 d x^2}}$$

### Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^2 \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$-\frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{d x} - \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[x]}{\sqrt{d - c^2 d x^2}}$$

Result (type 3, 79 leaves, 3 steps):

$$- \frac{(1 - cx)(1 + cx)(a + b \operatorname{ArcCosh}[cx])}{x \sqrt{d - c^2 dx^2}} - \frac{bc \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{Log}[x]}{\sqrt{d - c^2 dx^2}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[cx]}{x^3 \sqrt{d - c^2 dx^2}} dx$$

Optimal (type 4, 238 leaves, 8 steps):

$$\frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{2x \sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \operatorname{ArcCosh}[cx])}{2dx^2} + \frac{c^2 \sqrt{-1 + cx} \sqrt{1 + cx} (a + b \operatorname{ArcCosh}[cx]) \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{d - c^2 dx^2}} - \frac{i b c^2 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[cx]}\right]}{2 \sqrt{d - c^2 dx^2}} + \frac{i b c^2 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[cx]}\right]}{2 \sqrt{d - c^2 dx^2}}$$

Result (type 4, 246 leaves, 9 steps):

$$\frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{2x \sqrt{d - c^2 dx^2}} - \frac{(1 - cx)(1 + cx)(a + b \operatorname{ArcCosh}[cx])}{2x^2 \sqrt{d - c^2 dx^2}} + \frac{c^2 \sqrt{-1 + cx} \sqrt{1 + cx} (a + b \operatorname{ArcCosh}[cx]) \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[cx]}\right]}{\sqrt{d - c^2 dx^2}} - \frac{i b c^2 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[cx]}\right]}{2 \sqrt{d - c^2 dx^2}} + \frac{i b c^2 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[cx]}\right]}{2 \sqrt{d - c^2 dx^2}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[cx]}{x^4 \sqrt{d - c^2 dx^2}} dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$\frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2 \sqrt{d - c^2 dx^2}} - \frac{\sqrt{d - c^2 dx^2} (a + b \operatorname{ArcCosh}[cx])}{3dx^3} - \frac{2c^2 \sqrt{d - c^2 dx^2} (a + b \operatorname{ArcCosh}[cx])}{3dx} - \frac{2bc^3 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{Log}[x]}{3 \sqrt{d - c^2 dx^2}}$$

Result (type 3, 171 leaves, 5 steps):

$$\frac{bc \sqrt{-1 + cx} \sqrt{1 + cx}}{6x^2 \sqrt{d - c^2 dx^2}} - \frac{(1 - cx)(1 + cx)(a + b \operatorname{ArcCosh}[cx])}{3x^3 \sqrt{d - c^2 dx^2}} - \frac{2c^2 (1 - cx)(1 + cx)(a + b \operatorname{ArcCosh}[cx])}{3x \sqrt{d - c^2 dx^2}} - \frac{2bc^3 \sqrt{-1 + cx} \sqrt{1 + cx} \operatorname{Log}[x]}{3 \sqrt{d - c^2 dx^2}}$$

### Problem 114: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$\begin{aligned} & -\frac{5 b x \sqrt{d - c^2 d x^2}}{3 c^5 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b x^3 \sqrt{d - c^2 d x^2}}{9 c^3 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{a + b \operatorname{ArcCosh}[c x]}{c^6 d \sqrt{d - c^2 d x^2}} + \\ & \frac{2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{c^6 d^2} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{3 c^6 d^3} - \frac{b \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}[c x]}{c^6 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 3, 262 leaves, 5 steps):

$$\begin{aligned} & \frac{5 b x \sqrt{-1 + c x} \sqrt{1 + c x}}{3 c^5 d \sqrt{d - c^2 d x^2}} + \frac{b x^3 \sqrt{-1 + c x} \sqrt{1 + c x}}{9 c^3 d \sqrt{d - c^2 d x^2}} + \frac{x^4 (a + b \operatorname{ArcCosh}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} + \\ & \frac{8 (1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{3 c^6 d \sqrt{d - c^2 d x^2}} + \frac{4 x^2 (1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{3 c^4 d \sqrt{d - c^2 d x^2}} + \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{ArcTanh}[c x]}{c^6 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

### Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 226 leaves, 7 steps):

$$\begin{aligned} & \frac{b x^2 \sqrt{-1 + c x} \sqrt{1 + c x}}{4 c^3 d \sqrt{d - c^2 d x^2}} + \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} + \frac{3 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 c^4 d^2} - \\ & \frac{3 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])^2}{4 b c^5 d \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{2 c^5 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

Result (type 3, 237 leaves, 8 steps):

$$\begin{aligned} & \frac{b x^2 \sqrt{-1 + c x} \sqrt{1 + c x}}{4 c^3 d \sqrt{d - c^2 d x^2}} + \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{c^2 d \sqrt{d - c^2 d x^2}} + \frac{3 x (1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{2 c^4 d \sqrt{d - c^2 d x^2}} - \\ & \frac{3 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])^2}{4 b c^5 d \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{2 c^5 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

## Problem 119: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 84 leaves, 2 steps):

$$\frac{x (a + b \operatorname{ArcCosh}[c x])}{d \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{2 c d \sqrt{d - c^2 d x^2}}$$

Result (type 3, 84 leaves, 3 steps):

$$\frac{x (a + b \operatorname{ArcCosh}[c x])}{d \sqrt{d - c^2 d x^2}} - \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{2 c d \sqrt{d - c^2 d x^2}}$$

## Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$\begin{aligned} & - \frac{b c \sqrt{d - c^2 d x^2}}{6 d^2 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 d x^3 \sqrt{d - c^2 d x^2}} - \frac{4 c^2 (a + b \operatorname{ArcCosh}[c x])}{3 d x \sqrt{d - c^2 d x^2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d \sqrt{d - c^2 d x^2}} + \frac{5 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{2 d^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 3, 250 leaves, 6 steps):

$$\begin{aligned} & \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x}}{6 d x^2 \sqrt{d - c^2 d x^2}} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 d x^3 \sqrt{d - c^2 d x^2}} - \frac{4 c^2 (a + b \operatorname{ArcCosh}[c x])}{3 d x \sqrt{d - c^2 d x^2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d \sqrt{d - c^2 d x^2}} - \frac{5 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[x]}{3 d \sqrt{d - c^2 d x^2}} - \frac{b c^3 \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{2 d \sqrt{d - c^2 d x^2}} \end{aligned}$$

## Problem 124: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 243 leaves, 5 steps):



$$\frac{b x \sqrt{d - c^2 d x^2}}{c^5 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b x \sqrt{d - c^2 d x^2}}{6 c^5 d^3 \sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)} + \frac{a + b \operatorname{ArcCosh}[c x]}{3 c^6 d (d - c^2 d x^2)^{3/2}} - \frac{2 (a + b \operatorname{ArcCosh}[c x])}{c^6 d^2 \sqrt{d - c^2 d x^2}} - \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{c^6 d^3} + \frac{11 b \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}[c x]}{6 c^6 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 280 leaves, 6 steps):

$$-\frac{b x \sqrt{-1 + c x} \sqrt{1 + c x}}{c^5 d^2 \sqrt{d - c^2 d x^2}} + \frac{b x \sqrt{-1 + c x} \sqrt{1 + c x}}{6 c^5 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2}} - \frac{4 x^2 (a + b \operatorname{ArcCosh}[c x])}{3 c^4 d^2 \sqrt{d - c^2 d x^2}} + \frac{x^4 (a + b \operatorname{ArcCosh}[c x])}{3 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}} - \frac{8 (1 - c x) (1 + c x) (a + b \operatorname{ArcCosh}[c x])}{3 c^6 d^2 \sqrt{d - c^2 d x^2}} - \frac{11 b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{ArcTanh}[c x]}{6 c^6 d^2 \sqrt{d - c^2 d x^2}}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 158 leaves, 4 steps):

$$\frac{b x \sqrt{d - c^2 d x^2}}{6 c^3 d^3 (-1 + c x)^{3/2} (1 + c x)^{3/2}} + \frac{a + b \operatorname{ArcCosh}[c x]}{3 c^4 d (d - c^2 d x^2)^{3/2}} - \frac{a + b \operatorname{ArcCosh}[c x]}{c^4 d^2 \sqrt{d - c^2 d x^2}} + \frac{5 b \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}[c x]}{6 c^4 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

Result (type 3, 243 leaves, 5 steps):

$$\frac{b x \sqrt{-1 + c x} \sqrt{1 + c x}}{6 c^3 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2}} - \frac{a + b \operatorname{ArcCosh}[c x]}{c^4 d^2 (1 + c x) \sqrt{d - c^2 d x^2}} + \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{3 c d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}} + \frac{(1 - c x)^2 (a + b \operatorname{ArcCosh}[c x])}{3 c^4 d^2 (1 + c x) \sqrt{d - c^2 d x^2}} - \frac{5 b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{ArcTanh}[c x]}{6 c^4 d^2 \sqrt{d - c^2 d x^2}}$$

Problem 127: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$\frac{b \sqrt{-1 + c x} \sqrt{1 + c x}}{6 c^3 d (d - c^2 d x^2)^{3/2}} + \frac{x^3 (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{b \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{6 c^3 d^2 \sqrt{d - c^2 d x^2}}$$

Result (type 3, 160 leaves, 5 steps):

$$\frac{b \sqrt{-1+cx} \sqrt{1+cx}}{6c^3 d^2 (1-c^2 x^2) \sqrt{d-c^2 dx^2}} + \frac{x^3 (a+b \operatorname{ArcCosh}[cx])}{3d^2 (1-cx)(1+cx) \sqrt{d-c^2 dx^2}} + \frac{b \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Log}[1-c^2 x^2]}{6c^3 d^2 \sqrt{d-c^2 dx^2}}$$

Problem 128: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \operatorname{ArcCosh}[cx])}{(d-c^2 dx^2)^{5/2}} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\frac{bx \sqrt{-1+cx} \sqrt{1+cx}}{6cd (d-c^2 dx^2)^{3/2}} + \frac{a+b \operatorname{ArcCosh}[cx]}{3c^2 d (d-c^2 dx^2)^{3/2}} + \frac{b \sqrt{-1+cx} \sqrt{1+cx} \operatorname{ArcTanh}[cx]}{6c^2 d^2 \sqrt{d-c^2 dx^2}}$$

Result (type 3, 154 leaves, 4 steps):

$$\frac{bx \sqrt{-1+cx} \sqrt{1+cx}}{6cd^2 (1-c^2 x^2) \sqrt{d-c^2 dx^2}} + \frac{a+b \operatorname{ArcCosh}[cx]}{3c^2 d^2 (1-cx)(1+cx) \sqrt{d-c^2 dx^2}} + \frac{b \sqrt{-1+cx} \sqrt{1+cx} \operatorname{ArcTanh}[cx]}{6c^2 d^2 \sqrt{d-c^2 dx^2}}$$

Problem 131: Result valid but suboptimal antiderivative.

$$\int \frac{a+b \operatorname{ArcCosh}[cx]}{x^2 (d-c^2 dx^2)^{5/2}} dx$$

Optimal (type 3, 248 leaves, 5 steps):

$$-\frac{bc \sqrt{d-c^2 dx^2}}{6d^3 \sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)} - \frac{a+b \operatorname{ArcCosh}[cx]}{dx (d-c^2 dx^2)^{3/2}} + \frac{4c^2 x (a+b \operatorname{ArcCosh}[cx])}{3d (d-c^2 dx^2)^{3/2}} + \frac{8c^2 x (a+b \operatorname{ArcCosh}[cx])}{3d^2 \sqrt{d-c^2 dx^2}} + \frac{bc \sqrt{d-c^2 dx^2} \operatorname{Log}[x]}{d^3 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{5bc \sqrt{d-c^2 dx^2} \operatorname{Log}[1-c^2 x^2]}{6d^3 \sqrt{-1+cx} \sqrt{1+cx}}$$

Result (type 3, 279 leaves, 6 steps):

$$\frac{bc \sqrt{-1+cx} \sqrt{1+cx}}{6d^2 (1-c^2 x^2) \sqrt{d-c^2 dx^2}} + \frac{8c^2 x (a+b \operatorname{ArcCosh}[cx])}{3d^2 \sqrt{d-c^2 dx^2}} - \frac{a+b \operatorname{ArcCosh}[cx]}{d^2 x (1-cx)(1+cx) \sqrt{d-c^2 dx^2}} + \frac{4c^2 x (a+b \operatorname{ArcCosh}[cx])}{3d^2 (1-cx)(1+cx) \sqrt{d-c^2 dx^2}} - \frac{bc \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Log}[x]}{d^2 \sqrt{d-c^2 dx^2}} - \frac{5bc \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Log}[1-c^2 x^2]}{6d^2 \sqrt{d-c^2 dx^2}}$$

### Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^4 (d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 3, 338 leaves, 5 steps):

$$\begin{aligned} & -\frac{b c \sqrt{d - c^2 d x^2}}{6 d^3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^3 \sqrt{d - c^2 d x^2}}{6 d^3 \sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)} - \frac{a + b \operatorname{ArcCosh}[c x]}{3 d x^3 (d - c^2 d x^2)^{3/2}} - \frac{2 c^2 (a + b \operatorname{ArcCosh}[c x])}{d x (d - c^2 d x^2)^{3/2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d (d - c^2 d x^2)^{3/2}} + \frac{16 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} + \frac{8 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[x]}{3 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b c^3 \sqrt{d - c^2 d x^2} \operatorname{Log}[1 - c^2 x^2]}{3 d^3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 3, 383 leaves, 6 steps):

$$\begin{aligned} & \frac{b c \sqrt{-1 + c x} \sqrt{1 + c x}}{6 d^2 x^2 \sqrt{d - c^2 d x^2}} + \frac{b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}{6 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2}} + \frac{16 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 \sqrt{d - c^2 d x^2}} - \\ & \frac{a + b \operatorname{ArcCosh}[c x]}{3 d^2 x^3 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}} - \frac{2 c^2 (a + b \operatorname{ArcCosh}[c x])}{d^2 x (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}} + \\ & \frac{8 c^4 x (a + b \operatorname{ArcCosh}[c x])}{3 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}} - \frac{8 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[x]}{3 d^2 \sqrt{d - c^2 d x^2}} - \frac{4 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Log}[1 - c^2 x^2]}{3 d^2 \sqrt{d - c^2 d x^2}} \end{aligned}$$

### Problem 134: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 246 leaves, 8 steps):

$$\begin{aligned} & \frac{\sqrt{-1 + a x} \sqrt{1 + a x}}{20 a c^3 (1 - a^2 x^2)^2 \sqrt{c - a^2 c x^2}} + \frac{2 \sqrt{-1 + a x} \sqrt{1 + a x}}{15 a c^3 (1 - a^2 x^2) \sqrt{c - a^2 c x^2}} + \\ & \frac{x \operatorname{ArcCosh}[a x]}{5 c (c - a^2 c x^2)^{5/2}} + \frac{4 x \operatorname{ArcCosh}[a x]}{15 c^2 (c - a^2 c x^2)^{3/2}} + \frac{8 x \operatorname{ArcCosh}[a x]}{15 c^3 \sqrt{c - a^2 c x^2}} - \frac{4 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{Log}[1 - a^2 x^2]}{15 a c^3 \sqrt{c - a^2 c x^2}} \end{aligned}$$

Result (type 3, 276 leaves, 7 steps):

$$\frac{\sqrt{-1+ax}\sqrt{1+ax}}{20a^3(1-a^2x^2)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\sqrt{1+ax}}{15a^3(1-a^2x^2)\sqrt{c-a^2cx^2}} + \frac{8x\text{ArcCosh}[ax]}{15c^3\sqrt{c-a^2cx^2}} +$$

$$\frac{x\text{ArcCosh}[ax]}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\text{ArcCosh}[ax]}{15c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{4\sqrt{-1+ax}\sqrt{1+ax}\text{Log}[1-a^2x^2]}{15a^3\sqrt{c-a^2cx^2}}$$

Problem 135: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \text{ArcCosh}[ax]}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 145 leaves, 5 steps):

$$-\frac{3x^2\sqrt{-1+ax}}{16a^3\sqrt{1-ax}} - \frac{x^4\sqrt{-1+ax}}{16a\sqrt{1-ax}} - \frac{3x\sqrt{1-a^2x^2}\text{ArcCosh}[ax]}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\text{ArcCosh}[ax]}{4a^2} + \frac{3\sqrt{-1+ax}\text{ArcCosh}[ax]^2}{16a^5\sqrt{1-ax}}$$

Result (type 3, 206 leaves, 6 steps):

$$-\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{16a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\text{ArcCosh}[ax]}{8a^4\sqrt{1-a^2x^2}} -$$

$$\frac{x^3(1-ax)(1+ax)\text{ArcCosh}[ax]}{4a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\sqrt{1+ax}\text{ArcCosh}[ax]^2}{16a^5\sqrt{1-a^2x^2}}$$

Problem 136: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \text{ArcCosh}[ax]}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{2x\sqrt{-1+ax}}{3a^3\sqrt{1-ax}} - \frac{x^3\sqrt{-1+ax}}{9a\sqrt{1-ax}} - \frac{2\sqrt{1-a^2x^2}\text{ArcCosh}[ax]}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\text{ArcCosh}[ax]}{3a^2}$$

Result (type 3, 158 leaves, 5 steps):

$$-\frac{2x\sqrt{-1+ax}\sqrt{1+ax}}{3a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\text{ArcCosh}[ax]}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\text{ArcCosh}[ax]}{3a^2\sqrt{1-a^2x^2}}$$

### Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 88 leaves, 3 steps):

$$-\frac{x^2 \sqrt{-1+ax}}{4a\sqrt{1-ax}} - \frac{x\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}{2a^2} + \frac{\sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2}{4a^3\sqrt{1-ax}}$$

Result (type 3, 125 leaves, 4 steps):

$$-\frac{x^2 \sqrt{-1+ax} \sqrt{1+ax}}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax) \operatorname{ArcCosh}[ax]}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2}{4a^3\sqrt{1-a^2x^2}}$$

### Problem 138: Result valid but suboptimal antiderivative.

$$\int \frac{x \operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$-\frac{x\sqrt{-1+ax}}{a\sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}{a^2}$$

Result (type 3, 73 leaves, 3 steps):

$$-\frac{x\sqrt{-1+ax} \sqrt{1+ax}}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]}{a^2\sqrt{1-a^2x^2}}$$

### Problem 139: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2}{2a\sqrt{1-ax}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2}{2a\sqrt{1-a^2x^2}}$$

Problem 140: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]}{x\sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 103 leaves, 6 steps):

$$\frac{2\sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{i\sqrt{-1+ax} \operatorname{PolyLog}\left[2, -ie^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \frac{i\sqrt{-1+ax} \operatorname{PolyLog}\left[2, ie^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}}$$

Result (type 4, 142 leaves, 7 steps):

$$\frac{2\sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{i\sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[2, -ie^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{i\sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[2, ie^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}}$$

Problem 141: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]}{x^2\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$-\frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}{x} - \frac{a\sqrt{-1+ax} \operatorname{Log}[x]}{\sqrt{1-ax}}$$

Result (type 3, 72 leaves, 3 steps):

$$-\frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]}{x\sqrt{1-a^2x^2}} - \frac{a\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Log}[x]}{\sqrt{1-a^2x^2}}$$

Problem 142: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]}{x^3\sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 167 leaves, 8 steps):

$$\frac{a \sqrt{-1+ax}}{2x \sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}{2x^2} + \frac{a^2 \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{i a^2 \sqrt{-1+ax} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-ax}} + \frac{i a^2 \sqrt{-1+ax} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-ax}}$$

Result (type 4, 230 leaves, 9 steps):

$$\frac{a \sqrt{-1+ax} \sqrt{1+ax}}{2x \sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]}{2x^2 \sqrt{1-a^2x^2}} + \frac{a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-a^2x^2}} + \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-a^2x^2}}$$

Problem 143: Result valid but suboptimal antiderivative.

$$\int \frac{(fx)^{3/2} (a + b \operatorname{ArcCosh}[cx])}{\sqrt{1-c^2x^2}} dx$$

Optimal (type 5, 98 leaves, 1 step):

$$\frac{2 (fx)^{5/2} (a + b \operatorname{ArcCosh}[cx]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{5 f} + \frac{4 b c (fx)^{7/2} \sqrt{-1+cx} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{35 f^2 \sqrt{1-cx}}$$

Result (type 5, 111 leaves, 2 steps):

$$\frac{2 (fx)^{5/2} (a + b \operatorname{ArcCosh}[cx]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{4}, \frac{9}{4}, c^2 x^2\right]}{5 f} + \frac{4 b c (fx)^{7/2} \sqrt{-1+cx} \sqrt{1+cx} \operatorname{HypergeometricPFQ}\left[\left\{1, \frac{7}{4}, \frac{7}{4}\right\}, \left\{\frac{9}{4}, \frac{11}{4}\right\}, c^2 x^2\right]}{35 f^2 \sqrt{1-c^2x^2}}$$

Problem 153: Result valid but suboptimal antiderivative.

$$\int (fx)^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[cx]) dx$$

Optimal (type 5, 278 leaves, 3 steps):

$$\begin{aligned}
& - \frac{b c (f x)^{2+m} \sqrt{d - c^2 d x^2}}{f^2 (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f (2+m)} + \\
& \frac{(f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (2+3m+m^2) \sqrt{1 - c x} \sqrt{1 + c x}} - \\
& \frac{b c (f x)^{2+m} \sqrt{d - c^2 d x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 5, 288 leaves, 4 steps):

$$\begin{aligned}
& - \frac{b c (f x)^{2+m} \sqrt{d - c^2 d x^2}}{f^2 (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{(f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f (2+m)} + \\
& \frac{(f x)^{1+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{f (2+3m+m^2) (1 - c x) (1 + c x)} - \\
& \frac{b c (f x)^{2+m} \sqrt{d - c^2 d x^2} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, c^2 x^2\right]}{f^2 (1+m) (2+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 163: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m \operatorname{ArcCosh}[a x]}{\sqrt{1 - a^2 x^2}} dx$$

Optimal (type 5, 128 leaves, 1 step):

$$\frac{(f x)^{1+m} \operatorname{ArcCosh}[a x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{f (1+m)} + \frac{a (f x)^{2+m} \sqrt{-1 + a x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, a^2 x^2\right]}{f^2 (1+m) (2+m) \sqrt{1 - a x}}$$

Result (type 5, 141 leaves, 2 steps):

$$\begin{aligned}
& \frac{(f x)^{1+m} \operatorname{ArcCosh}[a x] \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{f (1+m)} + \\
& \frac{a (f x)^{2+m} \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{HypergeometricPFQ}\left[\left\{1, 1 + \frac{m}{2}, 1 + \frac{m}{2}\right\}, \left\{\frac{3}{2} + \frac{m}{2}, 2 + \frac{m}{2}\right\}, a^2 x^2\right]}{f^2 (1+m) (2+m) \sqrt{1 - a^2 x^2}}
\end{aligned}$$



### Problem 177: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{x^4} dx$$

Optimal (type 4, 336 leaves, 11 steps):

$$\begin{aligned} & \frac{b^2 c^2 \sqrt{d - c^2 d x^2}}{3 x} - \frac{b^2 c^3 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{c^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 d x^3} - \\ & \frac{2 b c^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{-2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b^2 c^3 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

Result (type 4, 344 leaves, 11 steps):

$$\begin{aligned} & \frac{b^2 c^2 \sqrt{d - c^2 d x^2}}{3 x} - \frac{b^2 c^3 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\ & \frac{c^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x^3} - \\ & \frac{2 b c^3 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b^2 c^3 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$

### Problem 183: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^2} dx$$

Optimal (type 4, 453 leaves, 15 steps):

$$\begin{aligned}
& -\frac{1}{4} b^2 c^2 d x \sqrt{d - c^2 d x^2} - \frac{5 b^2 c d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{3 b c^3 d x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c d (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{3}{2} c^2 d x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 + \\
& \frac{c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{x} + \frac{c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{2 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{2 b c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b^2 c d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 465 leaves, 15 steps):

$$\begin{aligned}
& -\frac{1}{4} b^2 c^2 d x \sqrt{d - c^2 d x^2} - \frac{5 b^2 c d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{3 b c^3 d x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b c d (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{3}{2} c^2 d x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{x} + \frac{c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{2 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{2 b c d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b^2 c d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 185: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^4} dx$$

Optimal (type 4, 426 leaves, 18 steps):

$$\begin{aligned}
& \frac{b^2 c^2 d \sqrt{d - c^2 d x^2}}{3 x} - \frac{b^2 c^3 d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c d (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{x} - \\
& \frac{4 c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x^3} - \frac{c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{3 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{8 b c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{4 b^2 c^3 d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcCosh}[c x]}\right]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 438 leaves, 18 steps):

$$\begin{aligned}
& \frac{b^2 c^2 d \sqrt{d - c^2 d x^2}}{3 x} - \frac{b^2 c^3 d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c d (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c^2 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{x} \\
& \frac{4 c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{d (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x^3} - \frac{c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{3 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{8 b c^3 d \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{4 b^2 c^3 d \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

**Problem 191: Result valid but suboptimal antiderivative.**

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^2} dx$$

Optimal (type 4, 607 leaves, 25 steps):

$$\begin{aligned}
& -\frac{31}{64} b^2 c^2 d^2 x \sqrt{d - c^2 d x^2} - \frac{1}{32} b^2 c^2 d^2 x (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} - \frac{89 b^2 c d^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{64 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{15 b c^3 d^2 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 + \frac{c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{5}{4} c^2 d x (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{x} + \frac{5 c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{8 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{2 b c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{-2 \operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b^2 c d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 638 leaves, 24 steps):

$$\begin{aligned}
& -\frac{31}{64} b^2 c^2 d^2 x \sqrt{d - c^2 d x^2} - \frac{1}{32} b^2 c^2 d^2 x (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} - \frac{89 b^2 c d^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{64 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{15 b c^3 d^2 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b c d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{8 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{15}{8} c^2 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \\
& \frac{c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5}{4} c^2 d^2 x (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \\
& \frac{d^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{x} + \frac{5 c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{8 b \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{2 b c d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b^2 c d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcCosh}[c x]}]}{\sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 193: Result valid but suboptimal antiderivative.

$$\int \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{x^4} dx$$

Optimal (type 4, 638 leaves, 30 steps):

$$\begin{aligned}
& \frac{7}{12} b^2 c^4 d^2 x \sqrt{d - c^2 d x^2} + \frac{b^2 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}}{3 x} + \frac{23 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{12 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{5 b c^5 d^2 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 b c^3 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 - \frac{7 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{5 c^2 d (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x} - \frac{(d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{6 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{14 b c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{-2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{7 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 4, 669 leaves, 29 steps):

$$\begin{aligned}
& \frac{7}{12} b^2 c^4 d^2 x \sqrt{d - c^2 d x^2} + \frac{b^2 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2}}{3 x} + \frac{23 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{12 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{5 b c^5 d^2 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 b c^3 d^2 (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b c d^2 (1 - c^2 x^2)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{3 x^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5}{2} c^4 d^2 x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 + \\
& \frac{7 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{5 c^2 d^2 (1 - c x) (1 + c x) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x} - \\
& \frac{d^2 (1 - c x)^2 (1 + c x)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{3 x^3} - \frac{5 c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^3}{6 b \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{14 b c^3 d^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 b^2 c^3 d^2 \sqrt{d - c^2 d x^2} \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcCosh}[c x]}]}{3 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 194: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 3, 421 leaves, 16 steps):

$$\begin{aligned}
& - \frac{16 a b x \sqrt{-1 + c x} \sqrt{1 + c x}}{15 c^5 \sqrt{d - c^2 d x^2}} - \frac{4144 b^2 (1 - c x) (1 + c x)}{3375 c^6 \sqrt{d - c^2 d x^2}} - \frac{272 b^2 x^2 (1 - c x) (1 + c x)}{3375 c^4 \sqrt{d - c^2 d x^2}} - \frac{2 b^2 x^4 (1 - c x) (1 + c x)}{125 c^2 \sqrt{d - c^2 d x^2}} - \\
& \frac{16 b^2 x \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{ArcCosh}[c x]}{15 c^5 \sqrt{d - c^2 d x^2}} - \frac{8 b x^3 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])}{45 c^3 \sqrt{d - c^2 d x^2}} - \frac{2 b x^5 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])}{25 c \sqrt{d - c^2 d x^2}} - \\
& \frac{8 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{15 c^6 d} - \frac{4 x^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{15 c^4 d} - \frac{x^4 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{5 c^2 d}
\end{aligned}$$

Result (type 3, 445 leaves, 17 steps):

$$\begin{aligned}
& - \frac{16 a b x \sqrt{-1+c x} \sqrt{1+c x}}{15 c^5 \sqrt{d-c^2 d x^2}} - \frac{4144 b^2 (1-c x) (1+c x)}{3375 c^6 \sqrt{d-c^2 d x^2}} - \frac{272 b^2 x^2 (1-c x) (1+c x)}{3375 c^4 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x^4 (1-c x) (1+c x)}{125 c^2 \sqrt{d-c^2 d x^2}} - \\
& \frac{16 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{15 c^5 \sqrt{d-c^2 d x^2}} - \frac{8 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{45 c^3 \sqrt{d-c^2 d x^2}} - \frac{2 b x^5 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{25 c \sqrt{d-c^2 d x^2}} - \\
& \frac{8 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{15 c^6 \sqrt{d-c^2 d x^2}} - \frac{4 x^2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{15 c^4 \sqrt{d-c^2 d x^2}} - \frac{x^4 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{5 c^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 195: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 355 leaves, 11 steps):

$$\begin{aligned}
& - \frac{15 b^2 x (1-c x) (1+c x)}{64 c^4 \sqrt{d-c^2 d x^2}} - \frac{b^2 x^3 (1-c x) (1+c x)}{32 c^2 \sqrt{d-c^2 d x^2}} + \frac{15 b^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{64 c^5 \sqrt{d-c^2 d x^2}} - \\
& \frac{3 b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{8 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^4 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{8 c \sqrt{d-c^2 d x^2}} - \\
& \frac{3 x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{8 c^4 d} - \frac{x^3 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{4 c^2 d} + \frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^3}{8 b c^5 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 371 leaves, 12 steps):

$$\begin{aligned}
& - \frac{15 b^2 x (1-c x) (1+c x)}{64 c^4 \sqrt{d-c^2 d x^2}} - \frac{b^2 x^3 (1-c x) (1+c x)}{32 c^2 \sqrt{d-c^2 d x^2}} + \frac{15 b^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{64 c^5 \sqrt{d-c^2 d x^2}} - \\
& \frac{3 b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{8 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^4 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{8 c \sqrt{d-c^2 d x^2}} - \\
& \frac{3 x (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{8 c^4 \sqrt{d-c^2 d x^2}} - \frac{x^3 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{4 c^2 \sqrt{d-c^2 d x^2}} + \frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^3}{8 b c^5 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 196: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 292 leaves, 9 steps):

$$\begin{aligned}
& - \frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}} - \frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}} - \frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}} - \\
& \frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{9 c \sqrt{d-c^2 d x^2}} - \frac{2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{3 c^4 d} - \frac{x^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{3 c^2 d}
\end{aligned}$$

Result (type 3, 308 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 a b x \sqrt{-1+c x} \sqrt{1+c x}}{3 c^3 \sqrt{d-c^2 d x^2}} - \frac{40 b^2 (1-c x) (1+c x)}{27 c^4 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x^2 (1-c x) (1+c x)}{27 c^2 \sqrt{d-c^2 d x^2}} - \frac{4 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{3 c^3 \sqrt{d-c^2 d x^2}} - \\
& \frac{2 b x^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{9 c \sqrt{d-c^2 d x^2}} - \frac{2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{3 c^4 \sqrt{d-c^2 d x^2}} - \frac{x^2 (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{3 c^2 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 197: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 226 leaves, 5 steps):

$$\begin{aligned}
& - \frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}} + \frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{2 c \sqrt{d-c^2 d x^2}} - \\
& \frac{x \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{2 c^2 d} + \frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^3}{6 b c^3 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 3, 234 leaves, 6 steps):

$$\begin{aligned}
& - \frac{b^2 x (1-c x) (1+c x)}{4 c^2 \sqrt{d-c^2 d x^2}} + \frac{b^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{4 c^3 \sqrt{d-c^2 d x^2}} - \frac{b x^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{2 c \sqrt{d-c^2 d x^2}} - \\
& \frac{x (1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{2 c^2 \sqrt{d-c^2 d x^2}} + \frac{\sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^3}{6 b c^3 \sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 198: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} dx$$

Optimal (type 3, 155 leaves, 4 steps):

$$-\frac{2 a b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}} - \frac{2 b^2 (1-c x) (1+c x)}{c^2 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{c \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{c^2 d}$$

Result (type 3, 163 leaves, 5 steps):

$$-\frac{2 a b x \sqrt{-1+c x} \sqrt{1+c x}}{c \sqrt{d-c^2 d x^2}} - \frac{2 b^2 (1-c x) (1+c x)}{c^2 \sqrt{d-c^2 d x^2}} - \frac{2 b^2 x \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcCosh}[c x]}{c \sqrt{d-c^2 d x^2}} - \frac{(1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{c^2 \sqrt{d-c^2 d x^2}}$$

Problem 201: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[c x])^2}{x^2 \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 186 leaves, 6 steps):

$$-\frac{c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{d x} - \frac{2 b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+e^{-2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \frac{b^2 c \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}}$$

Result (type 4, 194 leaves, 7 steps):

$$\frac{c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d-c^2 d x^2}} - \frac{(1-c x) (1+c x) (a+b \operatorname{ArcCosh}[c x])^2}{x \sqrt{d-c^2 d x^2}} - \frac{2 b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{Log}\left[1+e^{2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{b^2 c \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[2,-e^{2 \operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}}$$

Problem 202: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[c x])^2}{x^3 \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 430 leaves, 12 steps):



$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{x \sqrt{d-c^2 d x^2}} - \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^2}{2 d x^2} + \frac{c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Result (type 4, 438 leaves, 13 steps):

$$\begin{aligned}
& \frac{b c \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}{x \sqrt{d-c^2 d x^2}} - \frac{(1-c x)(1+c x)(a+b \operatorname{ArcCosh}[c x])^2}{2 x^2 \sqrt{d-c^2 d x^2}} + \\
& \frac{c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{ArcTan}\left[\sqrt{-1+c x} \sqrt{1+c x}\right]}{\sqrt{d-c^2 d x^2}} - \\
& \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b c^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} + \\
& \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}} - \frac{i b^2 c^2 \sqrt{-1+c x} \sqrt{1+c x} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[c x]}\right]}{\sqrt{d-c^2 d x^2}}
\end{aligned}$$

Problem 203: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[c x])^2}{x^4 \sqrt{d-c^2 d x^2}} dx$$

Optimal (type 4, 328 leaves, 9 steps):

$$\frac{b^2 c^2 (1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])}{3x^2\sqrt{d-c^2dx^2}} - \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])^2}{3\sqrt{d-c^2dx^2}} - \frac{\sqrt{d-c^2dx^2}(a+b\text{ArcCosh}[cx])^2}{3dx^3} - \frac{2c^2\sqrt{d-c^2dx^2}(a+b\text{ArcCosh}[cx])^2}{3dx} - \frac{4bc^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])\text{Log}[1+e^{-2\text{ArcCosh}[cx]}]}{3\sqrt{d-c^2dx^2}} + \frac{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}\text{PolyLog}[2, -e^{-2\text{ArcCosh}[cx]}]}{3\sqrt{d-c^2dx^2}}$$

Result (type 4, 344 leaves, 10 steps):

$$\frac{b^2 c^2 (1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])^2}{3\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)(a+b\text{ArcCosh}[cx])^2}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+cx)(a+b\text{ArcCosh}[cx])^2}{3x\sqrt{d-c^2dx^2}} - \frac{4bc^3\sqrt{-1+cx}\sqrt{1+cx}(a+b\text{ArcCosh}[cx])\text{Log}[1+e^{2\text{ArcCosh}[cx]}]}{3\sqrt{d-c^2dx^2}} - \frac{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}\text{PolyLog}[2, -e^{2\text{ArcCosh}[cx]}]}{3\sqrt{d-c^2dx^2}}$$

Problem 225: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \text{ArcCosh}[ax]^2}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 243 leaves, 11 steps):

$$-\frac{15x\sqrt{1-ax}\sqrt{1+ax}}{64a^4} - \frac{x^3\sqrt{1-ax}\sqrt{1+ax}}{32a^2} + \frac{15\sqrt{-1+ax}\text{ArcCosh}[ax]}{64a^5\sqrt{1-ax}} - \frac{3x^2\sqrt{-1+ax}\text{ArcCosh}[ax]}{8a^3\sqrt{1-ax}} - \frac{x^4\sqrt{-1+ax}\text{ArcCosh}[ax]}{8a\sqrt{1-ax}} - \frac{3x\sqrt{1-a^2x^2}\text{ArcCosh}[ax]^2}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\text{ArcCosh}[ax]^2}{4a^2} + \frac{\sqrt{-1+ax}\text{ArcCosh}[ax]^3}{8a^5\sqrt{1-ax}}$$

Result (type 3, 329 leaves, 12 steps):

$$-\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{15\sqrt{-1+ax}\sqrt{1+ax}\text{ArcCosh}[ax]}{64a^5\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}\text{ArcCosh}[ax]}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}\text{ArcCosh}[ax]}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\text{ArcCosh}[ax]^2}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\text{ArcCosh}[ax]^2}{4a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\text{ArcCosh}[ax]^3}{8a^5\sqrt{1-a^2x^2}}$$

### Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 177 leaves, 8 steps):

$$\begin{aligned} & -\frac{40 \sqrt{1-a x} \sqrt{1+a x}}{27 a^4} - \frac{2 x^2 \sqrt{1-a x} \sqrt{1+a x}}{27 a^2} - \frac{4 x \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{3 a^3 \sqrt{1-a x}} - \\ & \frac{2 x^3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{9 a \sqrt{1-a x}} - \frac{2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{3 a^4} - \frac{x^2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{3 a^2} \end{aligned}$$

Result (type 3, 237 leaves, 9 steps):

$$\begin{aligned} & -\frac{40 (1-a x) (1+a x)}{27 a^4 \sqrt{1-a^2 x^2}} - \frac{2 x^2 (1-a x) (1+a x)}{27 a^2 \sqrt{1-a^2 x^2}} - \frac{4 x \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{3 a^3 \sqrt{1-a^2 x^2}} - \\ & \frac{2 x^3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a \sqrt{1-a^2 x^2}} - \frac{2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{3 a^4 \sqrt{1-a^2 x^2}} - \frac{x^2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{3 a^2 \sqrt{1-a^2 x^2}} \end{aligned}$$

### Problem 227: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 151 leaves, 5 steps):

$$\begin{aligned} & -\frac{x \sqrt{1-a x} \sqrt{1+a x}}{4 a^2} + \frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{4 a^3 \sqrt{1-a x}} - \frac{x^2 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]}{2 a \sqrt{1-a x}} - \frac{x \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^2}{2 a^2} + \frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^3}{6 a^3 \sqrt{1-a x}} \end{aligned}$$

Result (type 3, 207 leaves, 6 steps):

$$\begin{aligned} & -\frac{x (1-a x) (1+a x)}{4 a^2 \sqrt{1-a^2 x^2}} + \frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{4 a^3 \sqrt{1-a^2 x^2}} - \\ & \frac{x^2 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{2 a \sqrt{1-a^2 x^2}} - \frac{x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^2}{2 a^2 \sqrt{1-a^2 x^2}} + \frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^3}{6 a^3 \sqrt{1-a^2 x^2}} \end{aligned}$$

## Problem 228: Result valid but suboptimal antiderivative.

$$\int \frac{x \operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 79 leaves, 3 steps):

$$-\frac{2\sqrt{1-ax}\sqrt{1+ax}}{a^2} - \frac{2x\sqrt{-1+ax}\operatorname{ArcCosh}[ax]}{a\sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2}\operatorname{ArcCosh}[ax]^2}{a^2}$$

Result (type 3, 109 leaves, 4 steps):

$$-\frac{2(1-ax)(1+ax)}{a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}\operatorname{ArcCosh}[ax]}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\operatorname{ArcCosh}[ax]^2}{a^2\sqrt{1-a^2x^2}}$$

## Problem 229: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]^2}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+ax}\operatorname{ArcCosh}[ax]^3}{3a\sqrt{1-ax}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+ax}\sqrt{1+ax}\operatorname{ArcCosh}[ax]^3}{3a\sqrt{1-a^2x^2}}$$

## Problem 230: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]^2}{x\sqrt{1-a^2 x^2}} dx$$

Optimal (type 4, 183 leaves, 8 steps):

$$\frac{2\sqrt{-1+ax}\operatorname{ArcCosh}[ax]^2\operatorname{ArcTan}[e^{\operatorname{ArcCosh}[ax]}]}{\sqrt{1-ax}} - \frac{2i\sqrt{-1+ax}\operatorname{ArcCosh}[ax]\operatorname{PolyLog}[2, -ie^{\operatorname{ArcCosh}[ax]}]}{\sqrt{1-ax}} +$$

$$\frac{2i\sqrt{-1+ax}\operatorname{ArcCosh}[ax]\operatorname{PolyLog}[2, ie^{\operatorname{ArcCosh}[ax]}]}{\sqrt{1-ax}} + \frac{2i\sqrt{-1+ax}\operatorname{PolyLog}[3, -ie^{\operatorname{ArcCosh}[ax]}]}{\sqrt{1-ax}} - \frac{2i\sqrt{-1+ax}\operatorname{PolyLog}[3, ie^{\operatorname{ArcCosh}[ax]}]}{\sqrt{1-ax}}$$

Result (type 4, 248 leaves, 9 steps):

$$\frac{2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{2 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{2 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{2 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{2 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]^2}{x^2 \sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{a \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2}{\sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]^2}{x} - \frac{2 a \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{Log}\left[1+e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{a \sqrt{-1+ax} \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}}$$

Result (type 4, 174 leaves, 7 steps):

$$\frac{a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]^2}{x \sqrt{1-a^2x^2}} - \frac{2 a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{Log}\left[1+e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]^2}{x^3 \sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 296 leaves, 12 steps):

$$\begin{aligned}
& \frac{a \sqrt{-1+ax} \operatorname{ArcCosh}[ax]}{x \sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]^2}{2x^2} + \frac{a^2 \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \\
& \frac{a^2 \sqrt{-1+ax} \operatorname{ArcTan}\left[\sqrt{-1+ax} \sqrt{1+ax}\right]}{\sqrt{1-ax}} - \frac{i a^2 \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \\
& \frac{i a^2 \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \frac{i a^2 \sqrt{-1+ax} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{i a^2 \sqrt{-1+ax} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}}
\end{aligned}$$

Result (type 4, 398 leaves, 13 steps):

$$\begin{aligned}
& \frac{a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]}{x \sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]^2}{2x^2 \sqrt{1-a^2x^2}} + \\
& \frac{a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcTan}\left[\sqrt{-1+ax} \sqrt{1+ax}\right]}{\sqrt{1-a^2x^2}} - \\
& \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \\
& \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{i a^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int (fx)^m (d - c^2 dx^2)^{5/2} (a + b \operatorname{ArcCosh}[cx])^2 dx$$

Optimal (type 8, 1153 leaves, 22 steps):

$$\begin{aligned}
& - \frac{10 b^2 c^2 d^2 (f x)^{3+m} \sqrt{d - c^2 d x^2}}{f^3 (4+m)^3 (6+m)} - \frac{2 b^2 c^2 d^2 (52 + 15 m + m^2) (f x)^{3+m} (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{f^3 (4+m)^2 (6+m)^3 (1 - c x) (1 + c x)} + \frac{2 b^2 c^4 d^2 (f x)^{5+m} (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{f^5 (6+m)^3 (1 - c x) (1 + c x)} - \\
& \frac{2 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m) (6+m) \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{30 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) (6+m) \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{10 b c d^2 (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m) (4+m) (6+m) \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{10 b c^3 d^2 (f x)^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^4 (4+m)^2 (6+m) \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{4 b c^3 d^2 (f x)^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^4 (4+m) (6+m) \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{2 b c^5 d^2 (f x)^{6+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^6 (6+m)^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{15 d^2 (f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{f (6+m) (8 + 6 m + m^2)} + \frac{5 d (f x)^{1+m} (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{f (4+m) (6+m)} + \\
& \frac{(f x)^{1+m} (d - c^2 d x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}{f (6+m)} - \frac{30 b^2 c^2 d^2 (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (4+m) (6+m) (1 - c x) (1 + c x)} - \\
& \frac{10 b^2 c^2 d^2 (10 + 3 m) (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^3 (6+m) (1 - c x) (1 + c x)} - \\
& \frac{2 b^2 c^2 d^2 (264 + 130 m + 15 m^2) (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^2 (6+m)^3 (1 - c x) (1 + c x)} + \\
& \frac{15 d^3 \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{(6+m) (8 + 6 m + m^2)}
\end{aligned}$$

Result (type 8, 73 leaves, 1 step):

$$\frac{d^2 \sqrt{d - c^2 d x^2} \operatorname{Unintegrable}\left[(f x)^m (-1 + c x)^{5/2} (1 + c x)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int (f x)^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2 dx$$

Optimal (type 8, 583 leaves, 13 steps):

$$\begin{aligned}
& - \frac{2 b^2 c^2 d (f x)^{3+m} \sqrt{d - c^2 d x^2}}{f^3 (4+m)^3} - \frac{6 b c d (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 (4+m) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{2 b c d (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m) (4+m) \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{2 b c^3 d (f x)^{4+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^4 (4+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{3 d (f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{f (8+6 m+m^2)} + \\
& \frac{(f x)^{1+m} (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}{f (4+m)} - \frac{6 b^2 c^2 d (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (4+m) (1-c x) (1+c x)} - \\
& \frac{2 b^2 c^2 d (10+3 m) (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m) (3+m) (4+m)^3 (1-c x) (1+c x)} + \frac{3 d^2 \operatorname{Unintegrateable}\left[\frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{8+6 m+m^2}
\end{aligned}$$

Result (type 8, 72 leaves, 1 step):

$$- \frac{d \sqrt{d - c^2 d x^2} \operatorname{Unintegrateable}\left[(f x)^m (-1+c x)^{3/2} (1+c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int (f x)^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2 dx$$

Optimal (type 8, 239 leaves, 5 steps):

$$\begin{aligned}
& - \frac{2 b c (f x)^{2+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{f^2 (2+m)^2 \sqrt{-1+c x} \sqrt{1+c x}} + \frac{(f x)^{1+m} \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{f (2+m)} - \\
& \frac{2 b^2 c^2 (f x)^{3+m} \sqrt{1 - c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, c^2 x^2\right]}{f^3 (2+m)^2 (3+m) (1-c x) (1+c x)} + \frac{d \operatorname{Unintegrateable}\left[\frac{(f x)^m (a+b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]}{2+m}
\end{aligned}$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{d - c^2 d x^2} \operatorname{Unintegrateable}\left[(f x)^m \sqrt{-1+c x} \sqrt{1+c x} (a + b \operatorname{ArcCosh}[c x])^2, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}} dx$$



Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{\sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{d - c^2 d x^2}}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(-1 + c x)^{3/2} (1 + c x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d x^2}}$$

Problem 238: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(d - c^2 d x^2)^{5/2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(d - c^2 d x^2)^{5/2}}, x\right]$$

Result (type 8, 73 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^2}{(-1 + c x)^{5/2} (1 + c x)^{5/2}}, x\right]}{d^2 \sqrt{d - c^2 d x^2}}$$

## Problem 239: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{1 - c^2 x^2}}, x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(f x)^m \operatorname{ArcCosh}[c x]^2}{\sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{1 - c^2 x^2}}$$

## Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \operatorname{ArcCosh}[a x]^3}{\sqrt{1 - a^2 x^2}} dx$$

Optimal (type 3, 315 leaves, 13 steps):

$$\begin{aligned} & -\frac{45 x^2 \sqrt{-1 + a x}}{128 a^3 \sqrt{1 - a x}} - \frac{3 x^4 \sqrt{-1 + a x}}{128 a \sqrt{1 - a x}} - \frac{45 x \sqrt{1 - a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]}{64 a^4} - \\ & \frac{3 x^3 \sqrt{1 - a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]}{32 a^2} + \frac{45 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2}{128 a^5 \sqrt{1 - a x}} - \frac{9 x^2 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2}{16 a^3 \sqrt{1 - a x}} - \\ & \frac{3 x^4 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^2}{16 a \sqrt{1 - a x}} - \frac{3 x \sqrt{1 - a^2 x^2} \operatorname{ArcCosh}[a x]^3}{8 a^4} - \frac{x^3 \sqrt{1 - a^2 x^2} \operatorname{ArcCosh}[a x]^3}{4 a^2} + \frac{3 \sqrt{-1 + a x} \operatorname{ArcCosh}[a x]^4}{32 a^5 \sqrt{1 - a x}} \end{aligned}$$

Result (type 3, 427 leaves, 14 steps):

$$\begin{aligned} & -\frac{45 x^2 \sqrt{-1 + a x} \sqrt{1 + a x}}{128 a^3 \sqrt{1 - a^2 x^2}} - \frac{3 x^4 \sqrt{-1 + a x} \sqrt{1 + a x}}{128 a \sqrt{1 - a^2 x^2}} - \frac{45 x (1 - a x) (1 + a x) \operatorname{ArcCosh}[a x]}{64 a^4 \sqrt{1 - a^2 x^2}} - \frac{3 x^3 (1 - a x) (1 + a x) \operatorname{ArcCosh}[a x]}{32 a^2 \sqrt{1 - a^2 x^2}} + \\ & \frac{45 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2}{128 a^5 \sqrt{1 - a^2 x^2}} - \frac{9 x^2 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2}{16 a^3 \sqrt{1 - a^2 x^2}} - \frac{3 x^4 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^2}{16 a \sqrt{1 - a^2 x^2}} - \\ & \frac{3 x (1 - a x) (1 + a x) \operatorname{ArcCosh}[a x]^3}{8 a^4 \sqrt{1 - a^2 x^2}} - \frac{x^3 (1 - a x) (1 + a x) \operatorname{ArcCosh}[a x]^3}{4 a^2 \sqrt{1 - a^2 x^2}} + \frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \operatorname{ArcCosh}[a x]^4}{32 a^5 \sqrt{1 - a^2 x^2}} \end{aligned}$$

### Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \operatorname{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 243 leaves, 10 steps):

$$\begin{aligned} & -\frac{40 x \sqrt{-1+a x}}{9 a^3 \sqrt{1-a x}} - \frac{2 x^3 \sqrt{-1+a x}}{27 a \sqrt{1-a x}} - \frac{40 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a^4} - \frac{2 x^2 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{9 a^2} \\ & - \frac{2 x \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{a^3 \sqrt{1-a x}} - \frac{x^3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{3 a \sqrt{1-a x}} - \frac{2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{3 a^4} - \frac{x^2 \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{3 a^2} \end{aligned}$$

Result (type 3, 329 leaves, 11 steps):

$$\begin{aligned} & -\frac{40 x \sqrt{-1+a x} \sqrt{1+a x}}{9 a^3 \sqrt{1-a^2 x^2}} - \frac{2 x^3 \sqrt{-1+a x} \sqrt{1+a x}}{27 a \sqrt{1-a^2 x^2}} - \frac{40 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{9 a^4 \sqrt{1-a^2 x^2}} \\ & - \frac{2 x^2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{9 a^2 \sqrt{1-a^2 x^2}} - \frac{2 x \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{a^3 \sqrt{1-a^2 x^2}} \\ & - \frac{x^3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{3 a \sqrt{1-a^2 x^2}} - \frac{2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{3 a^4 \sqrt{1-a^2 x^2}} - \frac{x^2 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{3 a^2 \sqrt{1-a^2 x^2}} \end{aligned}$$

### Problem 255: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 188 leaves, 6 steps):

$$\begin{aligned} & -\frac{3 x^2 \sqrt{-1+a x}}{8 a \sqrt{1-a x}} - \frac{3 x \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{4 a^2} + \frac{3 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{8 a^3 \sqrt{1-a x}} \\ & - \frac{3 x^2 \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{4 a \sqrt{1-a x}} - \frac{x \sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{2 a^2} + \frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^4}{8 a^3 \sqrt{1-a x}} \end{aligned}$$

Result (type 3, 257 leaves, 7 steps):

$$\begin{aligned}
& - \frac{3 x^2 \sqrt{-1+a x} \sqrt{1+a x}}{8 a \sqrt{1-a^2 x^2}} - \frac{3 x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{4 a^2 \sqrt{1-a^2 x^2}} + \frac{3 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{8 a^3 \sqrt{1-a^2 x^2}} - \\
& \frac{3 x^2 \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{4 a \sqrt{1-a^2 x^2}} - \frac{x (1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{2 a^2 \sqrt{1-a^2 x^2}} + \frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^4}{8 a^3 \sqrt{1-a^2 x^2}}
\end{aligned}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \frac{x \operatorname{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$\begin{aligned}
& - \frac{6 x \sqrt{-1+a x}}{a \sqrt{1-a x}} - \frac{6 \sqrt{1-a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]}{a^2} - \frac{3 x \sqrt{-1+a x} \operatorname{ArcCosh}[a x]^2}{a \sqrt{1-a x}} - \frac{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]^3}{a^2}
\end{aligned}$$

Result (type 3, 153 leaves, 5 steps):

$$\begin{aligned}
& - \frac{6 x \sqrt{-1+a x} \sqrt{1+a x}}{a \sqrt{1-a^2 x^2}} - \frac{6 (1-a x) (1+a x) \operatorname{ArcCosh}[a x]}{a^2 \sqrt{1-a^2 x^2}} - \frac{3 x \sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^2}{a \sqrt{1-a^2 x^2}} - \frac{(1-a x) (1+a x) \operatorname{ArcCosh}[a x]^3}{a^2 \sqrt{1-a^2 x^2}}
\end{aligned}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{\sqrt{1-a^2 x^2}} dx$$

Optimal (type 3, 32 leaves, 1 step):

$$\frac{\sqrt{-1+a x} \operatorname{ArcCosh}[a x]^4}{4 a \sqrt{1-a x}}$$

Result (type 3, 45 leaves, 2 steps):

$$\frac{\sqrt{-1+a x} \sqrt{1+a x} \operatorname{ArcCosh}[a x]^4}{4 a \sqrt{1-a^2 x^2}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[a x]^3}{x \sqrt{1-a^2 x^2}} dx$$

Optimal (type 4, 265 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^3 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{3 i \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \\
& \frac{3 i \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \frac{6 i \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \\
& \frac{6 i \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \frac{6 i \sqrt{-1+ax} \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \frac{6 i \sqrt{-1+ax} \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}}
\end{aligned}$$

Result (type 4, 356 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^3 \operatorname{ArcTan}\left[e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \\
& \frac{3 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{3 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \\
& \frac{6 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \frac{6 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \\
& \frac{6 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{6 i \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Problem 259: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCosh}[ax]^3}{x^2 \sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 166 leaves, 7 steps):

$$\begin{aligned}
& \frac{a \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^3}{\sqrt{1-ax}} - \frac{\sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]^3}{x} - \frac{3 a \sqrt{-1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{Log}\left[1+e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} - \\
& \frac{3 a \sqrt{-1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-ax}} + \frac{3 a \sqrt{-1+ax} \operatorname{PolyLog}\left[3, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-ax}}
\end{aligned}$$

Result (type 4, 229 leaves, 8 steps):

$$\begin{aligned}
& \frac{a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax) \operatorname{ArcCosh}[ax]^3}{x \sqrt{1-a^2x^2}} - \frac{3 a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]^2 \operatorname{Log}\left[1+e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} - \\
& \frac{3 a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax] \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{\sqrt{1-a^2x^2}} + \frac{3 a \sqrt{-1+ax} \sqrt{1+ax} \operatorname{PolyLog}\left[3, -e^{2 \operatorname{ArcCosh}[ax]}\right]}{2 \sqrt{1-a^2x^2}}
\end{aligned}$$

## Problem 260: Result valid but suboptimal antiderivative.

$$\int \frac{\text{ArcCosh}[a x]^3}{x^3 \sqrt{1-a^2 x^2}} dx$$

Optimal (type 4, 460 leaves, 18 steps):

$$\begin{aligned} & \frac{3 a \sqrt{-1+a x} \text{ArcCosh}[a x]^2}{2 x \sqrt{1-a x}} - \frac{\sqrt{1-a^2 x^2} \text{ArcCosh}[a x]^3}{2 x^2} - \frac{6 a^2 \sqrt{-1+a x} \text{ArcCosh}[a x] \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} + \\ & \frac{a^2 \sqrt{-1+a x} \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} + \frac{3 i a^2 \sqrt{-1+a x} \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} - \\ & \frac{3 i a^2 \sqrt{-1+a x} \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right]}{2 \sqrt{1-a x}} - \frac{3 i a^2 \sqrt{-1+a x} \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} + \\ & \frac{3 i a^2 \sqrt{-1+a x} \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{2 \sqrt{1-a x}} + \frac{3 i a^2 \sqrt{-1+a x} \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} - \\ & \frac{3 i a^2 \sqrt{-1+a x} \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} - \frac{3 i a^2 \sqrt{-1+a x} \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} + \frac{3 i a^2 \sqrt{-1+a x} \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a x}} \end{aligned}$$

Result (type 4, 614 leaves, 19 steps):

$$\begin{aligned} & \frac{3 a \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^2}{2 x \sqrt{1-a^2 x^2}} - \frac{(1-a x)(1+a x) \text{ArcCosh}[a x]^3}{2 x^2 \sqrt{1-a^2 x^2}} - \\ & \frac{6 a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x] \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} + \frac{a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^3 \text{ArcTan}\left[e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} + \\ & \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} - \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, -i e^{\text{ArcCosh}[a x]}\right]}{2 \sqrt{1-a^2 x^2}} - \\ & \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} + \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x]^2 \text{PolyLog}\left[2, i e^{\text{ArcCosh}[a x]}\right]}{2 \sqrt{1-a^2 x^2}} + \\ & \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x] \text{PolyLog}\left[3, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} - \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{ArcCosh}[a x] \text{PolyLog}\left[3, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} - \\ & \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{PolyLog}\left[4, -i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} + \frac{3 i a^2 \sqrt{-1+a x} \sqrt{1+a x} \text{PolyLog}\left[4, i e^{\text{ArcCosh}[a x]}\right]}{\sqrt{1-a^2 x^2}} \end{aligned}$$

### Problem 261: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^3}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^3}{\sqrt{1 - c^2 x^2}}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^3}{\sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{1 - c^2 x^2}}$$

### Problem 267: Result valid but suboptimal antiderivative.

$$\int \frac{x^4 \sqrt{1 - c^2 x^2}}{a + b \operatorname{ArcCosh}[c x]} dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\begin{aligned} & - \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{32 b c^5 \sqrt{-1 + c x}} + \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{16 b c^5 \sqrt{-1 + c x}} + \\ & \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{32 b c^5 \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{16 b c^5 \sqrt{-1 + c x}} + \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{32 b c^5 \sqrt{-1 + c x}} - \\ & \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{16 b c^5 \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{32 b c^5 \sqrt{-1 + c x}} \end{aligned}$$

Result (type 4, 430 leaves, 13 steps):

$$\begin{aligned}
& - \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{16 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{16 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{16 b c^5 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32 b c^5 \sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 268: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \sqrt{1-c^2x^2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$\begin{aligned}
& - \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b c^4 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b c^4 \sqrt{-1+cx}} + \\
& \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b c^4 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b c^4 \sqrt{-1+cx}} - \\
& \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b c^4 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b c^4 \sqrt{-1+cx}}
\end{aligned}$$

Result (type 4, 371 leaves, 13 steps):

$$\begin{aligned}
& - \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16 b c^4 \sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$



### Problem 269: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \sqrt{1 - c^2 x^2}}{a + b \operatorname{ArcCosh}[c x]} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 b c^3 \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{8 b c^3 \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 b c^3 \sqrt{-1 + c x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[c x]\right]}{8 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{8 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[c x]\right]}{8 b c^3 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 270: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1 - c^2 x^2}}{a + b \operatorname{ArcCosh}[c x]} dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$-\frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{4 b c^2 \sqrt{-1 + c x}} + \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{4 b c^2 \sqrt{-1 + c x}} +$$

$$\frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{4 b c^2 \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{4 b c^2 \sqrt{-1 + c x}}$$

Result (type 4, 245 leaves, 10 steps):

$$-\frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{4 b c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right]}{4 b c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} +$$

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{4 b c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right]}{4 b c^2 \sqrt{-1 + c x} \sqrt{1 + c x}}$$

## Problem 271: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1 - c^2 x^2}}{a + b \operatorname{ArcCosh}[c x]} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{2 b c \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{2 b c \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{2 b c \sqrt{-1 + c x}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[c x]\right]}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[c x]\right]}{2 b c \sqrt{-1 + c x} \sqrt{1 + c x}}$$

## Problem 272: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 116 leaves, 6 steps):

$$- \frac{\sqrt{-1 + c x} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{b \sqrt{1 - c x}} + \frac{\sqrt{-1 + c x} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{b \sqrt{1 - c x}} + \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 176 leaves, 7 steps):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{b \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{b \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 273: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^2 (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 65 leaves, 3 steps):

$$-\frac{c \sqrt{-1 + c x} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{b \sqrt{1 - c x}} + \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 115 leaves, 4 steps):

$$\frac{c \sqrt{1 - c^2 x^2} \operatorname{Log}[a + b \operatorname{ArcCosh}[c x]]}{b \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 274: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{1 - c^2 x^2}}{x^3 (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{\sqrt{-1 + c x} \sqrt{1 + c x}}{x^3 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1 - c^2 x^2}}{x^4 (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{1 - c^2 x^2}}{x^4 (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{\sqrt{-1+cx} \sqrt{1+cx}}{x^4 (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (1-c^2 x^2)^{3/2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$\begin{aligned} & -\frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{64 b c^4 \sqrt{-1+cx}} + \frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} + \\ & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} + \\ & \frac{3\sqrt{1-cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{64 b c^4 \sqrt{-1+cx}} - \frac{3\sqrt{1-cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} - \\ & \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^4 \sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 497 leaves, 16 steps):

$$\begin{aligned} & -\frac{3\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{3\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{3\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{3\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{64 b c^4 \sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (1-c^2 x^2)^{3/2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\begin{aligned} & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{32bc^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^3\sqrt{-1+cx}} - \\ & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{32bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Log}[a+b\operatorname{ArcCosh}[cx]]}{16bc^3\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{32bc^3\sqrt{-1+cx}} - \\ & \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{32bc^3\sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 430 leaves, 13 steps):

$$\begin{aligned} & \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2\operatorname{ArcCosh}[cx]\right]}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \\ & \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4\operatorname{ArcCosh}[cx]\right]}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6\operatorname{ArcCosh}[cx]\right]}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \\ & \frac{\sqrt{1-c^2x^2} \operatorname{Log}[a+b\operatorname{ArcCosh}[cx]]}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2\operatorname{ArcCosh}[cx]\right]}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} - \\ & \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4\operatorname{ArcCosh}[cx]\right]}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6\operatorname{ArcCosh}[cx]\right]}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} \end{aligned}$$

Problem 278: Result valid but suboptimal antiderivative.

$$\int \frac{x(1-c^2x^2)^{3/2}}{a+b\operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 297 leaves, 12 steps):

$$\begin{aligned} & -\frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b\operatorname{ArcCosh}[cx]}{b}\right]}{8bc^2\sqrt{-1+cx}} + \frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^2\sqrt{-1+cx}} - \\ & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b\operatorname{ArcCosh}[cx]}{b}\right]}{8bc^2\sqrt{-1+cx}} - \\ & \frac{3\sqrt{1-cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^2\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{16bc^2\sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 371 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{\sqrt{1-c^2x^2} \cosh\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2} \cosh\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3\operatorname{ArcCosh}[cx]\right]}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}} - \\
 & \frac{\sqrt{1-c^2x^2} \cosh\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5\operatorname{ArcCosh}[cx]\right]}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \sinh\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8bc^2\sqrt{-1+cx}\sqrt{1+cx}} - \\
 & \frac{3\sqrt{1-c^2x^2} \sinh\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3\operatorname{ArcCosh}[cx]\right]}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \sinh\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5\operatorname{ArcCosh}[cx]\right]}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}}
 \end{aligned}$$

Problem 279: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2x^2)^{3/2}}{a+b\operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 239 leaves, 9 steps):

$$\begin{aligned}
 & \frac{\sqrt{1-cx} \cosh\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{2bc\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \cosh\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{8bc\sqrt{-1+cx}} - \\
 & \frac{3\sqrt{1-cx} \log[a+b\operatorname{ArcCosh}[cx]]}{8bc\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \sinh\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{2bc\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \sinh\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{8bc\sqrt{-1+cx}}
 \end{aligned}$$

Result (type 4, 304 leaves, 10 steps):

$$\begin{aligned}
 & \frac{\sqrt{1-c^2x^2} \cosh\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2\operatorname{ArcCosh}[cx]\right]}{2bc\sqrt{-1+cx}\sqrt{1+cx}} - \\
 & \frac{\sqrt{1-c^2x^2} \cosh\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4\operatorname{ArcCosh}[cx]\right]}{8bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2} \log[a+b\operatorname{ArcCosh}[cx]]}{8bc\sqrt{-1+cx}\sqrt{1+cx}} - \\
 & \frac{\sqrt{1-c^2x^2} \sinh\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2\operatorname{ArcCosh}[cx]\right]}{2bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \sinh\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4\operatorname{ArcCosh}[cx]\right]}{8bc\sqrt{-1+cx}\sqrt{1+cx}}
 \end{aligned}$$

Problem 280: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 215 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4 b \sqrt{1-cx}} + \\
& \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b \sqrt{1-cx}} + \frac{5 \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4 b \sqrt{1-cx}} - \\
& \frac{\sqrt{-1+cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b \sqrt{1-cx}} + \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])}, x\right]
\end{aligned}$$

Result (type 8, 301 leaves, 16 steps):

$$\begin{aligned}
& \frac{5 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4 b \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4 b \sqrt{-1+cx} \sqrt{1+cx}} - \frac{5 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4 b \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4 b \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 281: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2 x^2)^{3/2}}{x^2 (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 163 leaves, 9 steps):

$$\begin{aligned}
& \frac{c \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2 b \sqrt{1-cx}} - \frac{3 c \sqrt{-1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{2 b \sqrt{1-cx}} - \\
& \frac{c \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2 b \sqrt{1-cx}} + \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])}, x\right]
\end{aligned}$$

Result (type 8, 240 leaves, 10 steps):

$$\begin{aligned}
& - \frac{c \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{2 b \sqrt{-1+cx} \sqrt{1+cx}} + \frac{3 c \sqrt{1-c^2 x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{2 b \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{c \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{2 b \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

## Problem 282: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + c x)^{3/2} (1 + c x)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

## Problem 283: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1 - c^2 x^2)^{3/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + c x)^{3/2} (1 + c x)^{3/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

## Problem 284: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (1 - c^2 x^2)^{5/2}}{a + b \operatorname{ArcCosh}[c x]} dx$$

Optimal (type 4, 397 leaves, 15 steps):



$$\begin{aligned}
& - \frac{3 \sqrt{1-cx} \cosh\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{128 b^4 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \cosh\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b^4 \sqrt{-1+cx}} - \\
& \frac{3 \sqrt{1-cx} \cosh\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{256 b^4 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \cosh\left[\frac{9a}{b}\right] \operatorname{CoshIntegral}\left[\frac{9(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{256 b^4 \sqrt{-1+cx}} + \\
& \frac{3 \sqrt{1-cx} \sinh\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{128 b^4 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \sinh\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b^4 \sqrt{-1+cx}} + \\
& \frac{3 \sqrt{1-cx} \sinh\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{256 b^4 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \sinh\left[\frac{9a}{b}\right] \operatorname{SinhIntegral}\left[\frac{9(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{256 b^4 \sqrt{-1+cx}}
\end{aligned}$$

Result (type 4, 497 leaves, 16 steps):

$$\begin{aligned}
& - \frac{3 \sqrt{1-c^2 x^2} \cosh\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{128 b^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \cosh\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{32 b^4 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{3 \sqrt{1-c^2 x^2} \cosh\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{256 b^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \cosh\left[\frac{9a}{b}\right] \operatorname{CoshIntegral}\left[\frac{9a}{b} + 9 \operatorname{ArcCosh}[cx]\right]}{256 b^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{3 \sqrt{1-c^2 x^2} \sinh\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{128 b^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \sinh\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{32 b^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{3 \sqrt{1-c^2 x^2} \sinh\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{256 b^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \sinh\left[\frac{9a}{b}\right] \operatorname{SinhIntegral}\left[\frac{9a}{b} + 9 \operatorname{ArcCosh}[cx]\right]}{256 b^4 \sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 285: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (1-c^2 x^2)^{5/2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 439 leaves, 15 steps):

$$\begin{aligned}
& \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} - \\
& \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{8a}{b}\right] \operatorname{CoshIntegral}\left[\frac{8(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{128 b c^3 \sqrt{-1+cx}} - \frac{5 \sqrt{1-cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{128 b c^3 \sqrt{-1+cx}} - \\
& \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} + \\
& \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32 b c^3 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{8a}{b}\right] \operatorname{SinhIntegral}\left[\frac{8(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{128 b c^3 \sqrt{-1+cx}}
\end{aligned}$$

Result (type 4, 556 leaves, 16 steps):

$$\begin{aligned}
& \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{8a}{b}\right] \operatorname{CoshIntegral}\left[\frac{8a}{b} + 8 \operatorname{ArcCosh}[cx]\right]}{128 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{5 \sqrt{1-c^2x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{128 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32 b c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{8a}{b}\right] \operatorname{SinhIntegral}\left[\frac{8a}{b} + 8 \operatorname{ArcCosh}[cx]\right]}{128 b c^3 \sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 286: Result valid but suboptimal antiderivative.

$$\int \frac{x (1-c^2x^2)^{5/2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 397 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{64 b c^2 \sqrt{-1+cx}} + \frac{9 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}} - \\
& \frac{5 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}} + \\
& \frac{5 \sqrt{1-cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{64 b c^2 \sqrt{-1+cx}} - \frac{9 \sqrt{1-cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}} + \\
& \frac{5 \sqrt{1-cx} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{64 b c^2 \sqrt{-1+cx}}
\end{aligned}$$

Result (type 4, 497 leaves, 16 steps):

$$\begin{aligned}
& - \frac{5 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{9 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{5 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{CoshIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{5 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{9 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{5 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[cx]\right]}{64 b c^2 \sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 287: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2 x^2)^{5/2}}{a+b \operatorname{ArcCosh}[cx]} dx$$

Optimal (type 4, 339 leaves, 12 steps):

$$\begin{aligned}
& \frac{15 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32bc\sqrt{-1+cx}} - \frac{3 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16bc\sqrt{-1+cx}} + \\
& \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32bc\sqrt{-1+cx}} - \frac{5 \sqrt{1-cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{16bc\sqrt{-1+cx}} - \frac{15 \sqrt{1-cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32bc\sqrt{-1+cx}} + \\
& \frac{3 \sqrt{1-cx} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16bc\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{32bc\sqrt{-1+cx}}
\end{aligned}$$

Result (type 4, 430 leaves, 13 steps):

$$\begin{aligned}
& \frac{15 \sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32bc\sqrt{-1+cx}\sqrt{1+cx}} - \\
& \frac{3 \sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{16bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32bc\sqrt{-1+cx}\sqrt{1+cx}} - \\
& \frac{5 \sqrt{1-c^2x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{16bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{15 \sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{32bc\sqrt{-1+cx}\sqrt{1+cx}} + \\
& \frac{3 \sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{16bc\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Sinh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{32bc\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2x^2)^{5/2}}{x(a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 309 leaves, 27 steps):

$$\begin{aligned}
& - \frac{11 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b \sqrt{1-cx}} + \\
& \frac{7 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b \sqrt{1-cx}} - \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b \sqrt{1-cx}} + \\
& \frac{11 \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b \sqrt{1-cx}} - \frac{7 \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b \sqrt{1-cx}} + \\
& \frac{\sqrt{-1+cx} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b \sqrt{1-cx}} + \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])}, x\right]
\end{aligned}$$

Result (type 8, 421 leaves, 28 steps):

$$\begin{aligned}
& \frac{11 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8 b \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{7 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16 b \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16 b \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{11 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8 b \sqrt{-1+cx} \sqrt{1+cx}} + \frac{7 \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16 b \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{\sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16 b \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 289: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2 x^2)^{5/2}}{x^2 (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 254 leaves, 18 steps):

$$\begin{aligned}
& \frac{c \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{b \sqrt{1-cx}} - \frac{c \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8b \sqrt{1-cx}} - \\
& \frac{15c \sqrt{-1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{8b \sqrt{1-cx}} - \frac{c \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{b \sqrt{1-cx}} + \\
& \frac{c \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8b \sqrt{1-cx}} + \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])}, x\right]
\end{aligned}$$

Result (type 8, 357 leaves, 19 steps):

$$\begin{aligned}
& - \frac{c \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{b \sqrt{-1+cx} \sqrt{1+cx}} + \frac{c \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{8b \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{15c \sqrt{1-c^2 x^2} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{8b \sqrt{-1+cx} \sqrt{1+cx}} + \frac{c \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{b \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{c \sqrt{1-c^2 x^2} \operatorname{Sinh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{8b \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 290: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2 x^2)^{5/2}}{x^3 (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1-c^2 x^2)^{5/2}}{x^3 (a+b \operatorname{ArcCosh}[cx])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1+cx)^{5/2} (1+cx)^{5/2}}{x^3 (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 291: Result valid but suboptimal antiderivative.

$$\int \frac{(1-c^2 x^2)^{5/2}}{x^4 (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(1 - c^2 x^2)^{5/2}}{x^4 (a + b \text{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} \text{Unintegrable}\left[\frac{(-1 + c x)^{5/2} (1 + c x)^{5/2}}{x^4 (a + b \text{ArcCosh}[c x])}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 292: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1 - a^2 x^2} \text{ArcCosh}[a x]} dx$$

Optimal (type 4, 98 leaves, 5 steps):

$$\frac{\sqrt{-1 + a x} \text{CoshIntegral}[2 \text{ArcCosh}[a x]]}{2 a^5 \sqrt{1 - a x}} + \frac{\sqrt{-1 + a x} \text{CoshIntegral}[4 \text{ArcCosh}[a x]]}{8 a^5 \sqrt{1 - a x}} + \frac{3 \sqrt{-1 + a x} \text{Log}[\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1 - a x}}$$

Result (type 4, 137 leaves, 6 steps):

$$\frac{\sqrt{-1 + a x} \sqrt{1 + a x} \text{CoshIntegral}[2 \text{ArcCosh}[a x]]}{2 a^5 \sqrt{1 - a^2 x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \text{CoshIntegral}[4 \text{ArcCosh}[a x]]}{8 a^5 \sqrt{1 - a^2 x^2}} + \frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \text{Log}[\text{ArcCosh}[a x]]}{8 a^5 \sqrt{1 - a^2 x^2}}$$

Problem 293: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1 - a^2 x^2} \text{ArcCosh}[a x]} dx$$

Optimal (type 4, 65 leaves, 5 steps):

$$\frac{3 \sqrt{-1 + a x} \text{CoshIntegral}[\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1 - a x}} + \frac{\sqrt{-1 + a x} \text{CoshIntegral}[3 \text{ArcCosh}[a x]]}{4 a^4 \sqrt{1 - a x}}$$

Result (type 4, 91 leaves, 6 steps):

$$\frac{3 \sqrt{-1 + a x} \sqrt{1 + a x} \text{CoshIntegral}[\text{ArcCosh}[a x]]}{4 a^4 \sqrt{1 - a^2 x^2}} + \frac{\sqrt{-1 + a x} \sqrt{1 + a x} \text{CoshIntegral}[3 \text{ArcCosh}[a x]]}{4 a^4 \sqrt{1 - a^2 x^2}}$$

## Problem 294: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]} dx$$

Optimal (type 4, 65 leaves, 4 steps):

$$\frac{\sqrt{-1+ax} \operatorname{CoshIntegral}[2 \operatorname{ArcCosh}[ax]]}{2 a^3 \sqrt{1-ax}} + \frac{\sqrt{-1+ax} \operatorname{Log}[\operatorname{ArcCosh}[ax]]}{2 a^3 \sqrt{1-ax}}$$

Result (type 4, 91 leaves, 5 steps):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{CoshIntegral}[2 \operatorname{ArcCosh}[ax]]}{2 a^3 \sqrt{1-a^2 x^2}} + \frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Log}[\operatorname{ArcCosh}[ax]]}{2 a^3 \sqrt{1-a^2 x^2}}$$

## Problem 295: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]} dx$$

Optimal (type 4, 28 leaves, 2 steps):

$$\frac{\sqrt{-1+ax} \operatorname{CoshIntegral}[\operatorname{ArcCosh}[ax]]}{a^2 \sqrt{1-ax}}$$

Result (type 4, 41 leaves, 3 steps):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{CoshIntegral}[\operatorname{ArcCosh}[ax]]}{a^2 \sqrt{1-a^2 x^2}}$$

## Problem 296: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-a^2 x^2} \operatorname{ArcCosh}[a x]} dx$$

Optimal (type 3, 28 leaves, 1 step):

$$\frac{\sqrt{-1+ax} \operatorname{Log}[\operatorname{ArcCosh}[ax]]}{a \sqrt{1-ax}}$$

Result (type 3, 41 leaves, 2 steps):



$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Log}[\operatorname{ArcCosh}[ax]]}{a \sqrt{1-a^2x^2}}$$

Problem 297: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{x \sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}, x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]}, x\right]}{\sqrt{1-a^2x^2}}$$

Problem 298: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{1-a^2x^2} \operatorname{ArcCosh}[ax]}, x\right]$$

Result (type 8, 63 leaves, 1 step):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1+ax} \sqrt{1+ax} \operatorname{ArcCosh}[ax]}, x\right]}{\sqrt{1-a^2x^2}}$$

Problem 299: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2x^2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 4, 197 leaves, 9 steps):

$$\frac{3 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4 b c^4 \sqrt{1-cx}} + \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b c^4 \sqrt{1-cx}} -$$

$$\frac{3 \sqrt{-1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4 b c^4 \sqrt{1-cx}} - \frac{\sqrt{-1+cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b c^4 \sqrt{1-cx}}$$

Result (type 4, 245 leaves, 10 steps):

$$\frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} + \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} -$$

$$\frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4 b c^4 \sqrt{1-c^2 x^2}} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Sinh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4 b c^4 \sqrt{1-c^2 x^2}}$$

Problem 300: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2 b c^3 \sqrt{1-cx}} + \frac{\sqrt{-1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{2 b c^3 \sqrt{1-cx}} - \frac{\sqrt{-1+cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2 b c^3 \sqrt{1-cx}}$$

Result (type 4, 178 leaves, 7 steps):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{2 b c^3 \sqrt{1-c^2 x^2}} +$$

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{2 b c^3 \sqrt{1-c^2 x^2}} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{2 b c^3 \sqrt{1-c^2 x^2}}$$

Problem 301: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{b c^2 \sqrt{1-cx}} - \frac{\sqrt{-1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{b c^2 \sqrt{1-cx}}$$

Result (type 4, 114 leaves, 5 steps):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{b c^2 \sqrt{1-c^2 x^2}} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Sinh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{b c^2 \sqrt{1-c^2 x^2}}$$

Problem 302: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 3, 35 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{b c \sqrt{1-cx}}$$

Result (type 3, 48 leaves, 2 steps):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Log}[a+b \operatorname{ArcCosh}[cx]]}{b c \sqrt{1-c^2 x^2}}$$

Problem 303: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{x \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{1}{x \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{1-c^2 x^2}}$$

## Problem 304: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{1}{x^2 \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1 - c^2 x^2}}$$

## Problem 305: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{x^2}{(-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1 - c^2 x^2}}$$

## Problem 306: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{(1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 66 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrateable}\left[\frac{x}{(-1+cx)^{3/2} (1+cx)^{3/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 307: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 27 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{1}{(1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]$$

Result (type 8, 65 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrateable}\left[\frac{1}{(-1+cx)^{3/2} (1+cx)^{3/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 308: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x (1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrateable}\left[\frac{1}{x (1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrateable}\left[\frac{1}{x (-1+cx)^{3/2} (1+cx)^{3/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 309: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 (1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \text{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{1}{x^2 (-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \text{ArcCosh}[c x])}, x\right]}{\sqrt{1 - c^2 x^2}}$$

Problem 310: Result valid but suboptimal antiderivative.

$$\int \frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \text{ArcCosh}[c x]} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m (1 - c^2 x^2)^{3/2}}{a + b \text{ArcCosh}[c x]}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1 - c^2 x^2} \text{Unintegrable}\left[\frac{x^m (-1 + c x)^{3/2} (1 + c x)^{3/2}}{a + b \text{ArcCosh}[c x]}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

Problem 311: Result valid but suboptimal antiderivative.

$$\int \frac{x^m \sqrt{1 - c^2 x^2}}{a + b \text{ArcCosh}[c x]} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m \sqrt{1 - c^2 x^2}}{a + b \text{ArcCosh}[c x]}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} \text{Unintegrable}\left[\frac{x^m \sqrt{-1 + c x} \sqrt{1 + c x}}{a + b \text{ArcCosh}[c x]}, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 312: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{Unintegrable}\left[\frac{x^m}{\sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

### Problem 313: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{(1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{(1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1+c x} \sqrt{1+c x} \text{Unintegrable}\left[\frac{x^m}{(-1+c x)^{3/2} (1+c x)^{3/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]}{\sqrt{1-c^2 x^2}}$$

### Problem 314: Result valid but suboptimal antiderivative.

$$\int \frac{x^m}{(1-c^2 x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^m}{(1-c^2 x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{x^m}{(-1+cx)^{5/2} (1+cx)^{5/2} (a+b \operatorname{ArcCosh}[cx])}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 320: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 350 leaves, 22 steps):

$$\begin{aligned} & -\frac{x^3 \sqrt{-1+cx} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{bc(a+b \operatorname{ArcCosh}[cx])} + \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8b^2 c^4 \sqrt{-1+cx}} - \frac{3\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16b^2 c^4 \sqrt{-1+cx}} \\ & - \frac{5\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16b^2 c^4 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8b^2 c^4 \sqrt{-1+cx}} + \\ & + \frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16b^2 c^4 \sqrt{-1+cx}} + \frac{5\sqrt{1-cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16b^2 c^4 \sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 429 leaves, 23 steps):

$$\begin{aligned} & \frac{x^3 (1-cx) \sqrt{1+cx} \sqrt{1-c^2 x^2}}{bc \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])} + \frac{\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \\ & - \frac{3\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \\ & - \frac{5\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \\ & + \frac{3\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{5\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16b^2 c^4 \sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 154 leaves, 16 steps):



$$-\frac{x^2 \sqrt{-1+cx} \sqrt{1+cx} \sqrt{1-c^2x^2}}{bc(a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2b^2c^3\sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2b^2c^3\sqrt{-1+cx}}$$

Result (type 4, 185 leaves, 17 steps):

$$\frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}(a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-c^2x^2} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int \frac{x \sqrt{1-c^2x^2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 248 leaves, 14 steps):

$$-\frac{x\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b \operatorname{ArcCosh}[cx])} + \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{4b^2c^2\sqrt{-1+cx}} - \frac{3\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{4b^2c^2\sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4b^2c^2\sqrt{-1+cx}} + \frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4b^2c^2\sqrt{-1+cx}}$$

Result (type 4, 418 leaves, 15 steps):

$$\frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}(a+b \operatorname{ArcCosh}[cx])} - \frac{3\sqrt{1-c^2x^2} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3\sqrt{1-c^2x^2} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2x^2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 146 leaves, 7 steps):

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc(a+b\operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-cx}\operatorname{CoshIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]\operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2c\sqrt{-1+cx}} + \frac{\sqrt{1-cx}\operatorname{Cosh}\left[\frac{2a}{b}\right]\operatorname{SinhIntegral}\left[\frac{2(a+b\operatorname{ArcCosh}[cx])}{b}\right]}{b^2c\sqrt{-1+cx}}$$

Result (type 4, 177 leaves, 8 steps):

$$\frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}(a+b\operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-c^2x^2}\operatorname{CoshIntegral}\left[\frac{2a}{b}+2\operatorname{ArcCosh}[cx]\right]\operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\operatorname{Cosh}\left[\frac{2a}{b}\right]\operatorname{SinhIntegral}\left[\frac{2a}{b}+2\operatorname{ArcCosh}[cx]\right]}{b^2c\sqrt{-1+cx}\sqrt{1+cx}}$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2x^2}}{x^2(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 97 leaves, 1 step):

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx^2(a+b\operatorname{ArcCosh}[cx])} + \frac{2\sqrt{1-cx}\operatorname{Unintegrable}\left[\frac{1}{x^3(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{-1+cx}}$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}(a+b\operatorname{ArcCosh}[cx])} + \frac{2\sqrt{1-c^2x^2}\operatorname{Unintegrable}\left[\frac{1}{x^3(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{1-c^2x^2}}{x^3(a+b\operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{\sqrt{-1+cx} \sqrt{1+cx}}{x^3 (a+b \operatorname{ArcCosh}[cx])^2}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-c^2 x^2}}{x^4 (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\sqrt{1-c^2 x^2}}{x^4 (a+b \operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{\sqrt{-1+cx} \sqrt{1+cx}}{x^4 (a+b \operatorname{ArcCosh}[cx])^2}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (1-c^2 x^2)^{3/2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 354 leaves, 21 steps):

$$\begin{aligned} & -\frac{x^2 \sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{3/2}}{b c (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c^3 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c^3 \sqrt{-1+cx}} + \\ & \frac{3 \sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c^3 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^3 \sqrt{-1+cx}} + \\ & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b^2 c^3 \sqrt{-1+cx}} - \frac{3 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^3 \sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 439 leaves, 20 steps):

$$\begin{aligned}
& \frac{x^2 (1 - cx)^2 (1 + cx)^{3/2} \sqrt{1 - c^2 x^2}}{bc \sqrt{-1 + cx} (a + b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}} - \\
& \frac{\sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}} + \\
& \frac{3 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}} + \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}} + \\
& \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{4 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}} - \frac{3 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c^3 \sqrt{-1 + cx} \sqrt{1 + cx}}
\end{aligned}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{x (1 - c^2 x^2)^{3/2}}{(a + b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 348 leaves, 24 steps):

$$\begin{aligned}
& - \frac{x \sqrt{-1 + cx} \sqrt{1 + cx} (1 - c^2 x^2)^{3/2}}{bc (a + b \operatorname{ArcCosh}[cx])} + \frac{\sqrt{1 - cx} \operatorname{CoshIntegral}\left[\frac{a + b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^2 \sqrt{-1 + cx}} - \frac{9 \sqrt{1 - cx} \operatorname{CoshIntegral}\left[\frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16 b^2 c^2 \sqrt{-1 + cx}} + \\
& \frac{5 \sqrt{1 - cx} \operatorname{CoshIntegral}\left[\frac{5(a + b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16 b^2 c^2 \sqrt{-1 + cx}} - \frac{\sqrt{1 - cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a + b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b^2 c^2 \sqrt{-1 + cx}} + \\
& \frac{9 \sqrt{1 - cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^2 \sqrt{-1 + cx}} - \frac{5 \sqrt{1 - cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^2 \sqrt{-1 + cx}}
\end{aligned}$$

Result (type 4, 429 leaves, 23 steps):

$$\begin{aligned}
& \frac{x (1 - c x)^2 (1 + c x)^{3/2} \sqrt{1 - c^2 x^2}}{b c \sqrt{-1 + c x} (a + b \operatorname{ArcCosh}[c x])} + \frac{\sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{9 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{5 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{8 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{9 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right]}{16 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{5 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[c x]\right]}{16 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{(a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 4, 246 leaves, 11 steps):

$$\begin{aligned}
& - \frac{\sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)^{3/2}}{b c (a + b \operatorname{ArcCosh}[c x])} - \frac{\sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{2(a + b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c \sqrt{-1 + c x}} + \frac{\sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{4(a + b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2 b^2 c \sqrt{-1 + c x}} + \\
& \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{b^2 c \sqrt{-1 + c x}} - \frac{\sqrt{1 - c x} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{2 b^2 c \sqrt{-1 + c x}}
\end{aligned}$$

Result (type 4, 305 leaves, 11 steps):

$$\begin{aligned}
& \frac{(1 - c x)^2 (1 + c x)^{3/2} \sqrt{1 - c^2 x^2}}{b c \sqrt{-1 + c x} (a + b \operatorname{ArcCosh}[c x])} - \frac{\sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{\sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2 b^2 c \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[c x]\right]}{b^2 c \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{\sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[c x]\right]}{2 b^2 c \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

## Problem 332: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^2 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 156 leaves, 3 steps):

$$-\frac{\sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{3/2}}{b c x^2 (a+b \operatorname{ArcCosh}[c x])} - \frac{2 \sqrt{1-cx} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+cx}} - \frac{2 c \sqrt{1-cx} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+cx}}$$

Result (type 8, 189 leaves, 2 steps):

$$\frac{(1-cx)^2 (1+cx)^{3/2} \sqrt{1-c^2 x^2}}{b c x^2 \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[c x])} - \frac{2 \sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x^3 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+cx} \sqrt{1+cx}} - \frac{2 c \sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b \sqrt{-1+cx} \sqrt{1+cx}}$$

## Problem 333: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1 - c^2 x^2)^{3/2}}{x^3 (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1+cx)^{3/2} (1+cx)^{3/2}}{x^3 (a+b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

## Problem 334: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{3/2}}{x^4 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{3/2}}{b c x^4 (a+b \operatorname{ArcCosh}[c x])} - \frac{4 \sqrt{1-cx} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5 (a+b \operatorname{ArcCosh}[c x])}, x\right]}{b c \sqrt{-1+cx}}$$

Result (type 8, 126 leaves, 2 steps):

$$\frac{(1-cx)^2 (1+cx)^{3/2} \sqrt{1-c^2 x^2}}{bcx^4 \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{4 \sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{-1+c^2 x^2}{x^5 (a+b \operatorname{ArcCosh}[cx])}, x\right]}{bc \sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (1-c^2 x^2)^{5/2}}{(a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 454 leaves, 30 steps):

$$\begin{aligned} & -\frac{x^2 \sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{5/2}}{bc (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{8b^2 c^3 \sqrt{-1+cx}} + \\ & \frac{3\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} - \frac{\sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{8(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{8a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} + \\ & \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8b^2 c^3 \sqrt{-1+cx}} - \\ & \frac{3\sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} + \frac{\sqrt{1-cx} \operatorname{Cosh}\left[\frac{8a}{b}\right] \operatorname{SinhIntegral}\left[\frac{8(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16b^2 c^3 \sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 565 leaves, 29 steps):

$$\begin{aligned} & \frac{x^2 (1-cx)^3 (1+cx)^{5/2} \sqrt{1-c^2 x^2}}{bc \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{8b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{3\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{8a}{b} + 8 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{8a}{b}\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{8b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{3\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} + \frac{\sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{8a}{b}\right] \operatorname{SinhIntegral}\left[\frac{8a}{b} + 8 \operatorname{ArcCosh}[cx]\right]}{16b^2 c^3 \sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

## Problem 336: Result valid but suboptimal antiderivative.

$$\int \frac{x (1 - c^2 x^2)^{5/2}}{(a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 4, 448 leaves, 30 steps):

$$\begin{aligned} & - \frac{x \sqrt{-1 + c x} \sqrt{1 + c x} (1 - c^2 x^2)^{5/2}}{b c (a + b \operatorname{ArcCosh}[c x])} + \frac{5 \sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} - \frac{27 \sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{3(a + b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} + \\ & \frac{25 \sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{5(a + b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} - \frac{7 \sqrt{1 - c x} \operatorname{CoshIntegral}\left[\frac{7(a + b \operatorname{ArcCosh}[c x])}{b}\right] \operatorname{Sinh}\left[\frac{7a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} - \\ & \frac{5 \sqrt{1 - c x} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a + b \operatorname{ArcCosh}[c x]}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} + \frac{27 \sqrt{1 - c x} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} - \\ & \frac{25 \sqrt{1 - c x} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} + \frac{7 \sqrt{1 - c x} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7(a + b \operatorname{ArcCosh}[c x])}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x}} \end{aligned}$$

Result (type 4, 555 leaves, 29 steps):

$$\begin{aligned} & \frac{x (1 - c x)^3 (1 + c x)^{5/2} \sqrt{1 - c^2 x^2}}{b c \sqrt{-1 + c x} (a + b \operatorname{ArcCosh}[c x])} + \frac{5 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{27 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\ & \frac{25 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{7 \sqrt{1 - c^2 x^2} \operatorname{CoshIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[c x]\right] \operatorname{Sinh}\left[\frac{7a}{b}\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{5 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c x]\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{27 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[c x]\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\ & \frac{25 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[c x]\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{7 \sqrt{1 - c^2 x^2} \operatorname{Cosh}\left[\frac{7a}{b}\right] \operatorname{SinhIntegral}\left[\frac{7a}{b} + 7 \operatorname{ArcCosh}[c x]\right]}{64 b^2 c^2 \sqrt{-1 + c x} \sqrt{1 + c x}} \end{aligned}$$



### Problem 337: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{(a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 4, 351 leaves, 14 steps):

$$\begin{aligned} & - \frac{\sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{5/2}}{bc (a+b \operatorname{ArcCosh}[cx])} - \frac{15 \sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c \sqrt{-1+cx}} + \frac{3 \sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c \sqrt{-1+cx}} \\ & - \frac{3 \sqrt{1-cx} \operatorname{CoshIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c \sqrt{-1+cx}} + \frac{15 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c \sqrt{-1+cx}} \\ & - \frac{3 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b^2 c \sqrt{-1+cx}} + \frac{3 \sqrt{1-cx} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c \sqrt{-1+cx}} \end{aligned}$$

Result (type 4, 436 leaves, 14 steps):

$$\begin{aligned} & \frac{(1-cx)^3 (1+cx)^{5/2} \sqrt{1-c^2 x^2}}{bc \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{15 \sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{16 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{3 \sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{4 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} - \frac{3 \sqrt{1-c^2 x^2} \operatorname{CoshIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{6a}{b}\right]}{16 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} + \\ & \frac{15 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} - \\ & \frac{3 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{4 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} + \frac{3 \sqrt{1-c^2 x^2} \operatorname{Cosh}\left[\frac{6a}{b}\right] \operatorname{SinhIntegral}\left[\frac{6a}{b} + 6 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c \sqrt{-1+cx} \sqrt{1+cx}} \end{aligned}$$

### Problem 339: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^2 (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 160 leaves, 3 steps):

$$\begin{aligned} & - \frac{\sqrt{-1+cx} \sqrt{1+cx} (1-c^2 x^2)^{5/2}}{b c x^2 (a+b \operatorname{ArcCosh}[cx])} + \frac{2 \sqrt{1-cx} \operatorname{Unintegrable}\left[\frac{(-1+c^2 x^2)^2}{x^3 (a+b \operatorname{ArcCosh}[cx])}, x\right]}{b c \sqrt{-1+cx}} + \frac{4 c \sqrt{1-cx} \operatorname{Unintegrable}\left[\frac{(-1+c^2 x^2)^2}{x (a+b \operatorname{ArcCosh}[cx])}, x\right]}{b \sqrt{-1+cx}} \end{aligned}$$

Result (type 8, 193 leaves, 2 steps):

$$\frac{(1 - cx)^3 (1 + cx)^{5/2} \sqrt{1 - c^2 x^2}}{b c x^2 \sqrt{-1 + cx} (a + b \operatorname{ArcCosh}[cx])} + \frac{2 \sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + c^2 x^2)^2}{x^3 (a + b \operatorname{ArcCosh}[cx])}, x\right]}{b c \sqrt{-1 + cx} \sqrt{1 + cx}} + \frac{4 c \sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + c^2 x^2)^2}{x (a + b \operatorname{ArcCosh}[cx])}, x\right]}{b \sqrt{-1 + cx} \sqrt{1 + cx}}$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1 - c^2 x^2)^{5/2}}{x^3 (a + b \operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + cx)^{5/2} (1 + cx)^{5/2}}{x^3 (a + b \operatorname{ArcCosh}[cx])^2}, x\right]}{\sqrt{-1 + cx} \sqrt{1 + cx}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int \frac{(1 - c^2 x^2)^{5/2}}{x^4 (a + b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(1 - c^2 x^2)^{5/2}}{x^4 (a + b \operatorname{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{1 - c^2 x^2} \operatorname{Unintegrable}\left[\frac{(-1 + cx)^{5/2} (1 + cx)^{5/2}}{x^4 (a + b \operatorname{ArcCosh}[cx])^2}, x\right]}{\sqrt{-1 + cx} \sqrt{1 + cx}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 337 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{x^5 \sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{5 \sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^6 \sqrt{1-cx}} - \frac{15 \sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16 b^2 c^6 \sqrt{1-cx}} \\
 & + \frac{5 \sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16 b^2 c^6 \sqrt{1-cx}} + \frac{5 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 b^2 c^6 \sqrt{1-cx}} + \\
 & + \frac{15 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^6 \sqrt{1-cx}} + \frac{5 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{16 b^2 c^6 \sqrt{1-cx}}
 \end{aligned}$$

Result (type 4, 424 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{x^5 \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} - \frac{5 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^2 c^6 \sqrt{1-c^2 x^2}} \\
 & - \frac{15 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{16 b^2 c^6 \sqrt{1-c^2 x^2}} \\
 & + \frac{5 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{5a}{b}\right]}{16 b^2 c^6 \sqrt{1-c^2 x^2}} + \frac{5 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{8 b^2 c^6 \sqrt{1-c^2 x^2}} \\
 & + \frac{15 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c^6 \sqrt{1-c^2 x^2}} + \frac{5 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{5a}{b}\right] \operatorname{SinhIntegral}\left[\frac{5a}{b} + 5 \operatorname{ArcCosh}[cx]\right]}{16 b^2 c^6 \sqrt{1-c^2 x^2}}
 \end{aligned}$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 236 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{x^4 \sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c^5 \sqrt{1-cx}} - \frac{\sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2 b^2 c^5 \sqrt{1-cx}} \\
 & + \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{b^2 c^5 \sqrt{1-cx}} + \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{2 b^2 c^5 \sqrt{1-cx}}
 \end{aligned}$$

Result (type 4, 301 leaves, 11 steps):

$$\begin{aligned}
& - \frac{x^4 \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{1-c^2x^2} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c^5 \sqrt{1-c^2x^2}} - \\
& \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{4a}{b}\right]}{2 b^2 c^5 \sqrt{1-c^2x^2}} + \\
& \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{b^2 c^5 \sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{4a}{b}\right] \operatorname{SinhIntegral}\left[\frac{4a}{b} + 4 \operatorname{ArcCosh}[cx]\right]}{2 b^2 c^5 \sqrt{1-c^2x^2}}
\end{aligned}$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{1-c^2x^2} (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 237 leaves, 10 steps):

$$\begin{aligned}
& - \frac{x^3 \sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{3 \sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^4 \sqrt{1-cx}} - \frac{3 \sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{4 b^2 c^4 \sqrt{1-cx}} + \\
& \frac{3 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{4 b^2 c^4 \sqrt{1-cx}} + \frac{3 \sqrt{-1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{4 b^2 c^4 \sqrt{1-cx}}
\end{aligned}$$

Result (type 4, 298 leaves, 11 steps):

$$\begin{aligned}
& - \frac{x^3 \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{1-c^2x^2} (a+b \operatorname{ArcCosh}[cx])} - \frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{4 b^2 c^4 \sqrt{1-c^2x^2}} - \\
& \frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{3a}{b}\right]}{4 b^2 c^4 \sqrt{1-c^2x^2}} + \\
& \frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[cx]\right]}{4 b^2 c^4 \sqrt{1-c^2x^2}} + \frac{3 \sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{3a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3a}{b} + 3 \operatorname{ArcCosh}[cx]\right]}{4 b^2 c^4 \sqrt{1-c^2x^2}}
\end{aligned}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{1-c^2x^2} (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 136 leaves, 7 steps):

$$-\frac{x^2 \sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c^3 \sqrt{1-cx}} + \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{b^2 c^3 \sqrt{1-cx}}$$

Result (type 4, 175 leaves, 8 steps):

$$-\frac{x^2 \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right] \operatorname{Sinh}\left[\frac{2a}{b}\right]}{b^2 c^3 \sqrt{1-c^2 x^2}} +$$

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcCosh}[cx]\right]}{b^2 c^3 \sqrt{1-c^2 x^2}}$$

Problem 346: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$-\frac{x \sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{b^2 c^2 \sqrt{1-cx}} + \frac{\sqrt{-1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{b^2 c^2 \sqrt{1-cx}}$$

Result (type 4, 169 leaves, 6 steps):

$$-\frac{x \sqrt{-1+cx} \sqrt{1+cx}}{bc \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{b^2 c^2 \sqrt{1-c^2 x^2}} +$$

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{b^2 c^2 \sqrt{1-c^2 x^2}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 3, 37 leaves, 1 step):

$$-\frac{\sqrt{-1+cx}}{bc \sqrt{1-cx} (a+b \operatorname{ArcCosh}[cx])}$$

Result (type 3, 50 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+cx}}{bcx\sqrt{1-cx}(a+b\operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx} \operatorname{Unintegrable}\left[\frac{1}{x^2(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{1-cx}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bcx\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])} - \frac{\sqrt{-1+cx}\sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{1}{x^2(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{1-c^2x^2}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 84 leaves, 1 step):

$$-\frac{\sqrt{-1+cx}}{bcx^2\sqrt{1-cx}(a+b\operatorname{ArcCosh}[cx])} - \frac{2\sqrt{-1+cx} \operatorname{Unintegrable}\left[\frac{1}{x^3(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{1-cx}}$$

Result (type 8, 110 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bcx^2\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])} - \frac{2\sqrt{-1+cx}\sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{1}{x^3(a+b\operatorname{ArcCosh}[cx])}, x\right]}{bc\sqrt{1-c^2x^2}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^3}{(1-c^2 x^2)^{3/2} (a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x^3}{(-1+cx)^{3/2} (1+cx)^{3/2} (a+b \text{ArcCosh}[cx])^2}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{(1-c^2 x^2)^{3/2} (a+b \text{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 106 leaves, 2 steps):

$$-\frac{x^2 \sqrt{-1+cx} \sqrt{1+cx}}{bc (1-c^2 x^2)^{3/2} (a+b \text{ArcCosh}[cx])} + \frac{2 \sqrt{-1+cx} \text{Unintegrable}\left[\frac{x}{(-1+c^2 x^2)^2 (a+b \text{ArcCosh}[cx])}, x\right]}{bc \sqrt{1-cx}}$$

Result (type 8, 127 leaves, 2 steps):

$$-\frac{x^2 \sqrt{-1+cx}}{bc (1-cx) \sqrt{1+cx} \sqrt{1-c^2 x^2} (a+b \text{ArcCosh}[cx])} + \frac{2 \sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x}{(-1+c^2 x^2)^2 (a+b \text{ArcCosh}[cx])}, x\right]}{bc \sqrt{1-c^2 x^2}}$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1-c^2 x^2)^{3/2} (a+b \text{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{(1-c^2 x^2)^{3/2} (a+b \text{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 66 leaves, 1 step):

$$-\frac{\sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x}{(-1+cx)^{3/2} (1+cx)^{3/2} (a+b \text{ArcCosh}[cx])^2}, x\right]}{\sqrt{1-c^2 x^2}}$$

## Problem 353: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x}}{b c (1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])} + \frac{2 c \sqrt{-1 + c x} \operatorname{Unintegrable}\left[\frac{x}{(-1 + c^2 x^2)^2 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{b \sqrt{1 - c x}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1 + c x}}{b c (1 - c x) \sqrt{1 + c x} \sqrt{1 - c^2 x^2} (a + b \operatorname{ArcCosh}[c x])} + \frac{2 c \sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{x}{(-1 + c^2 x^2)^2 (a + b \operatorname{ArcCosh}[c x])}, x\right]}{b \sqrt{1 - c^2 x^2}}$$

## Problem 354: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x (1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{x (1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{1}{x (-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{1 - c^2 x^2}}$$

## Problem 355: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{1}{x^2 (1 - c^2 x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 68 leaves, 1 step):



$$-\frac{\sqrt{-1+cx}\sqrt{1+cx}\operatorname{Unintegrable}\left[\frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\operatorname{ArcCosh}[cx])^2},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{(1-c^2x^2)^{5/2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 108 leaves, 2 steps):

$$-\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc(1-c^2x^2)^{5/2}(a+b\operatorname{ArcCosh}[cx])} - \frac{4\sqrt{-1+cx}\operatorname{Unintegrable}\left[\frac{x^3}{(-1+c^2x^2)^3(a+b\operatorname{ArcCosh}[cx])},x\right]}{bc\sqrt{1-cx}}$$

Result (type 8, 129 leaves, 2 steps):

$$-\frac{x^4\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}(a+b\operatorname{ArcCosh}[cx])} - \frac{4\sqrt{-1+cx}\sqrt{1+cx}\operatorname{Unintegrable}\left[\frac{x^3}{(-1+c^2x^2)^3(a+b\operatorname{ArcCosh}[cx])},x\right]}{bc\sqrt{1-c^2x^2}}$$

Problem 357: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(1-c^2x^2)^{5/2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{x^3}{(1-c^2x^2)^{5/2}(a+b\operatorname{ArcCosh}[cx])^2},x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx}\sqrt{1+cx}\operatorname{Unintegrable}\left[\frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\operatorname{ArcCosh}[cx])^2},x\right]}{\sqrt{1-c^2x^2}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{(1-c^2x^2)^{5/2}(a+b\operatorname{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(1-c^2 x^2)^{5/2} (a+b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x^2}{(-1+cx)^{5/2} (1+cx)^{5/2} (a+b \text{ArcCosh}[cx])^2}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1-c^2 x^2)^{5/2} (a+b \text{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x}{(1-c^2 x^2)^{5/2} (a+b \text{ArcCosh}[cx])^2}, x\right]$$

Result (type 8, 65 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x}{(-1+cx)^{5/2} (1+cx)^{5/2} (a+b \text{ArcCosh}[cx])^2}, x\right]}{\sqrt{1-c^2 x^2}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-c^2 x^2)^{5/2} (a+b \text{ArcCosh}[cx])^2} dx$$

Optimal (type 8, 101 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx} \sqrt{1+cx}}{bc (1-c^2 x^2)^{5/2} (a+b \text{ArcCosh}[cx])} - \frac{4c \sqrt{-1+cx} \text{Unintegrable}\left[\frac{x}{(-1+c^2 x^2)^3 (a+b \text{ArcCosh}[cx])}, x\right]}{b \sqrt{1-cx}}$$

Result (type 8, 122 leaves, 2 steps):

$$-\frac{\sqrt{-1+cx}}{bc (1-cx)^2 (1+cx)^{3/2} \sqrt{1-c^2 x^2} (a+b \text{ArcCosh}[cx])} - \frac{4c \sqrt{-1+cx} \sqrt{1+cx} \text{Unintegrable}\left[\frac{x}{(-1+c^2 x^2)^3 (a+b \text{ArcCosh}[cx])}, x\right]}{b \sqrt{1-c^2 x^2}}$$

### Problem 361: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x (1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x (1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{1}{x (-1 + c x)^{5/2} (1 + c x)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{1 - c^2 x^2}}$$

### Problem 362: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{x^2 (1 - c^2 x^2)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{1}{x^2 (-1 + c x)^{5/2} (1 + c x)^{5/2} (a + b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{1 - c^2 x^2}}$$

### Problem 363: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (1 - c^2 x^2)^{3/2}}{(a + b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (1 - c^2 x^2)^{3/2}}{(a + b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{(f x)^m (-1+c x)^{3/2} (1+c x)^{3/2}}{(a+b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(f x)^m \sqrt{1-c^2 x^2}}{(a+b \operatorname{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{1-c^2 x^2} \operatorname{Unintegrable}\left[\frac{(f x)^m \sqrt{-1+c x} \sqrt{1+c x}}{(a+b \operatorname{ArcCosh}[c x])^2}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{\sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 91 leaves, 1 step):

$$-\frac{(f x)^m \sqrt{-1+c x}}{b c \sqrt{1-c x} (a+b \operatorname{ArcCosh}[c x])} + \frac{f m \sqrt{-1+c x} \operatorname{Unintegrable}\left[\frac{(f x)^{-1+m}}{a+b \operatorname{ArcCosh}[c x]}, x\right]}{b c \sqrt{1-c x}}$$

Result (type 8, 117 leaves, 2 steps):

$$-\frac{(f x)^m \sqrt{-1+c x} \sqrt{1+c x}}{b c \sqrt{1-c^2 x^2} (a+b \operatorname{ArcCosh}[c x])} + \frac{f m \sqrt{-1+c x} \sqrt{1+c x} \operatorname{Unintegrable}\left[\frac{(f x)^{-1+m}}{a+b \operatorname{ArcCosh}[c x]}, x\right]}{b c \sqrt{1-c^2 x^2}}$$

Problem 366: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{(1-c^2 x^2)^{3/2} (a+b \operatorname{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m}{(1 - c^2 x^2)^{3/2} (a + b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{(f x)^m}{(-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \text{ArcCosh}[c x])^2}, x\right]}{\sqrt{1 - c^2 x^2}}$$

Problem 367: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m}{(1 - c^2 x^2)^{5/2} (a + b \text{ArcCosh}[c x])^2} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m}{(1 - c^2 x^2)^{5/2} (a + b \text{ArcCosh}[c x])^2}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \text{Unintegrable}\left[\frac{(f x)^m}{(-1 + c x)^{5/2} (1 + c x)^{5/2} (a + b \text{ArcCosh}[c x])^2}, x\right]}{\sqrt{1 - c^2 x^2}}$$

Problem 368: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{1 - a^2 x^2} \text{ArcCosh}[a x]^3} dx$$

Optimal (type 3, 32 leaves, 1 step):

$$-\frac{\sqrt{-1 + a x}}{2 a \sqrt{1 - a x} \text{ArcCosh}[a x]^2}$$

Result (type 3, 45 leaves, 2 steps):

$$-\frac{\sqrt{-1 + a x} \sqrt{1 + a x}}{2 a \sqrt{1 - a^2 x^2} \text{ArcCosh}[a x]^2}$$

## Problem 401: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{1-x^2} \sqrt{\text{ArcCosh}[x]}} dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$\frac{\sqrt{\pi} \sqrt{-1+x} \text{Erf}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2 \sqrt{1-x}} + \frac{\sqrt{\pi} \sqrt{-1+x} \text{Erfi}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2 \sqrt{1-x}}$$

Result (type 4, 83 leaves, 7 steps):

$$\frac{\sqrt{\pi} \sqrt{-1+x} \sqrt{1+x} \text{Erf}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2 \sqrt{1-x^2}} + \frac{\sqrt{\pi} \sqrt{-1+x} \sqrt{1+x} \text{Erfi}\left[\sqrt{\text{ArcCosh}[x]}\right]}{2 \sqrt{1-x^2}}$$

## Problem 406: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(c - a^2 c x^2)^{3/2} \sqrt{\text{ArcCosh}[a x]}} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c - a^2 c x^2)^{3/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$-\frac{\sqrt{-1+ax} \sqrt{1+ax} \text{Unintegrable}\left[\frac{1}{(-1+ax)^{3/2} (1+ax)^{3/2} \sqrt{\text{ArcCosh}[ax]}}, x\right]}{c \sqrt{c - a^2 c x^2}}$$

## Problem 407: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(c - a^2 c x^2)^{5/2} \sqrt{\text{ArcCosh}[a x]}} dx$$

Optimal (type 8, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c - a^2 c x^2)^{5/2} \sqrt{\text{ArcCosh}[a x]}}, x\right]$$

Result (type 8, 66 leaves, 1 step):

$$\frac{\sqrt{-1+ax} \sqrt{1+ax} \operatorname{Unintegrable}\left[\frac{1}{(-1+ax)^{5/2} (1+ax)^{5/2} \sqrt{\operatorname{ArcCosh}[ax]}}, x\right]}{c^2 \sqrt{c-a^2cx^2}}$$

Problem 410: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{c-a^2cx^2}}{\operatorname{ArcCosh}[ax]^{3/2}} dx$$

Optimal (type 4, 170 leaves, 9 steps):

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{\operatorname{ArcCosh}[ax]}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erf}\left[\sqrt{2}\sqrt{\operatorname{ArcCosh}[ax]}\right]}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erfi}\left[\sqrt{2}\sqrt{\operatorname{ArcCosh}[ax]}\right]}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

Result (type 4, 176 leaves, 10 steps):

$$\frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\operatorname{ArcCosh}[ax]}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erf}\left[\sqrt{2}\sqrt{\operatorname{ArcCosh}[ax]}\right]}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erfi}\left[\sqrt{2}\sqrt{\operatorname{ArcCosh}[ax]}\right]}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

Problem 412: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(c-a^2cx^2)^{3/2} \operatorname{ArcCosh}[ax]^{3/2}} dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a(c-a^2cx^2)^{3/2}\sqrt{\operatorname{ArcCosh}[ax]}} + \frac{4a\sqrt{-1+ax}\sqrt{1+ax}\operatorname{Unintegrable}\left[\frac{x}{(-1+a^2x^2)^2\sqrt{\operatorname{ArcCosh}[ax]}}, x\right]}{c\sqrt{c-a^2cx^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2\sqrt{-1+ax}}{ac(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}\sqrt{\operatorname{ArcCosh}[ax]}} + \frac{4a\sqrt{-1+ax}\sqrt{1+ax}\operatorname{Unintegrable}\left[\frac{x}{(-1+a^2x^2)^2\sqrt{\operatorname{ArcCosh}[ax]}}, x\right]}{c\sqrt{c-a^2cx^2}}$$

Problem 413: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(c-a^2cx^2)^{5/2} \operatorname{ArcCosh}[ax]^{3/2}} dx$$

Optimal (type 8, 109 leaves, 2 steps):

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a(c-a^2cx^2)^{5/2}\sqrt{\text{ArcCosh}[ax]}} - \frac{8a\sqrt{-1+ax}\sqrt{1+ax}\text{Unintegrable}\left[\frac{x}{(-1+a^2x^2)^3\sqrt{\text{ArcCosh}[ax]}}, x\right]}{c^2\sqrt{c-a^2cx^2}}$$

Result (type 8, 120 leaves, 2 steps):

$$-\frac{2\sqrt{-1+ax}}{a c^2 (1-ax)^2 (1+ax)^{3/2} \sqrt{c-a^2cx^2} \sqrt{\text{ArcCosh}[ax]}} - \frac{8a\sqrt{-1+ax}\sqrt{1+ax}\text{Unintegrable}\left[\frac{x}{(-1+a^2x^2)^3\sqrt{\text{ArcCosh}[ax]}}, x\right]}{c^2\sqrt{c-a^2cx^2}}$$

Problem 415: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{c-a^2cx^2}}{\text{ArcCosh}[ax]^{5/2}} dx$$

Optimal (type 4, 201 leaves, 7 steps):

$$-\frac{2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\text{ArcCosh}[ax]^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\text{ArcCosh}[ax]}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcCosh}[ax]}\right]}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcCosh}[ax]}\right]}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

Result (type 4, 207 leaves, 8 steps):

$$\frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\text{ArcCosh}[ax]^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\text{ArcCosh}[ax]}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\text{Erf}\left[\sqrt{2}\sqrt{\text{ArcCosh}[ax]}\right]}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\text{Erfi}\left[\sqrt{2}\sqrt{\text{ArcCosh}[ax]}\right]}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

Problem 422: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\text{ArcCosh}[cx])^n}{x} dx$$

Optimal (type 8, 211 leaves, 6 steps):



$$\begin{aligned}
& - \frac{d e^{-\frac{a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 \sqrt{d-c^2 d x^2}} + \\
& \frac{d e^{a/b} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 \sqrt{d-c^2 d x^2}} + d \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x \sqrt{d-c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 245 leaves, 7 steps):

$$\begin{aligned}
& \frac{e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{e^{a/b} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x \sqrt{-1+cx} \sqrt{1+cx}}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 423: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n}{x^2} dx$$

Optimal (type 8, 91 leaves, 3 steps):

$$- \frac{c d \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{b (1+n) \sqrt{d-c^2 d x^2}} + d \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x^2 \sqrt{d-c^2 d x^2}}, x\right]$$

Result (type 8, 125 leaves, 4 steps):

$$\frac{c \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{b (1+n) \sqrt{-1+cx} \sqrt{1+cx}} - \frac{\sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x^2 \sqrt{-1+cx} \sqrt{1+cx}}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}$$

Problem 427: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])^n}{x} dx$$

Optimal (type 8, 414 leaves, 15 steps):

$$\begin{aligned}
& \frac{3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{d-c^2 d x^2}} - \\
& \frac{5 d^2 e^{-\frac{a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 \sqrt{d-c^2 d x^2}} + \\
& \frac{5 d^2 e^{a/b} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 \sqrt{d-c^2 d x^2}} - \\
& \frac{3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{d-c^2 d x^2}} + d^2 \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x \sqrt{d-c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 441 leaves, 16 steps):

$$\begin{aligned}
& - \frac{3^{-1-n} d e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{5 d e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 \sqrt{-1+cx} \sqrt{1+cx}} - \\
& \frac{5 d e^{a/b} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 \sqrt{-1+cx} \sqrt{1+cx}} + \\
& \frac{3^{-1-n} d e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{-1+cx} \sqrt{1+cx}} - \frac{d \sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x \sqrt{-1+cx} \sqrt{1+cx}}, x\right]}{\sqrt{-1+cx} \sqrt{1+cx}}
\end{aligned}$$

Problem 428: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{3/2} (a+b \operatorname{ArcCosh}[cx])^n}{x^2} dx$$

Optimal (type 8, 291 leaves, 9 steps):

$$\begin{aligned}
& - \frac{3 c d^2 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^{1+n}}{2 b (1+n) \sqrt{d-c^2 d x^2}} + \frac{1}{\sqrt{d-c^2 d x^2}} \\
& 2^{-3-n} c d^2 e^{-\frac{2 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right] - \\
& \frac{2^{-3-n} c d^2 e^{\frac{2 a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{d-c^2 d x^2}} + \\
& d^2 \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{d-c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 320 leaves, 10 steps):

$$\begin{aligned}
& \frac{3 c d \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^{1+n}}{2 b (1+n) \sqrt{-1+c x} \sqrt{1+c x}} - \frac{2^{-3-n} c d e^{-\frac{2 a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{2^{-3-n} c d e^{\frac{2 a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} - \frac{d \sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 432: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])^n}{x} dx$$

Optimal (type 8, 804 leaves, 27 steps):

$$\begin{aligned}
& - \frac{1}{32 \sqrt{d - c^2 d x^2}} 5^{-1-n} d^3 e^{-\frac{5a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( -\frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a + b \operatorname{ArcCosh}[cx])}{b}\right] - \\
& \frac{1}{32 \sqrt{d - c^2 d x^2}} 5 \times 3^{-1-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( -\frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right] + \\
& \frac{3^{-n} d^3 e^{-\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( -\frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{d - c^2 d x^2}} - \\
& \frac{11 d^3 e^{-\frac{a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( -\frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a + b \operatorname{ArcCosh}[cx]}{b}\right]}{16 \sqrt{d - c^2 d x^2}} + \\
& \frac{11 d^3 e^{a/b} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( \frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a + b \operatorname{ArcCosh}[cx]}{b}\right]}{16 \sqrt{d - c^2 d x^2}} + \\
& \frac{5 \times 3^{-1-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( \frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{32 \sqrt{d - c^2 d x^2}} - \\
& \frac{3^{-n} d^3 e^{\frac{3a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( \frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{8 \sqrt{d - c^2 d x^2}} + \\
& \frac{5^{-1-n} d^3 e^{\frac{5a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a + b \operatorname{ArcCosh}[cx])^n \left( \frac{a + b \operatorname{ArcCosh}[cx]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a + b \operatorname{ArcCosh}[cx])}{b}\right]}{32 \sqrt{d - c^2 d x^2}} + d^3 \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[cx])^n}{x \sqrt{d - c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 841 leaves, 28 steps):

$$\begin{aligned}
& \frac{5^{-1-n} d^2 e^{-\frac{5a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{5(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{5 \times 3^{-1-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{3^{-n} d^2 e^{-\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{11 d^2 e^{-\frac{a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(-\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{11 d^2 e^{a/b} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[c x]}{b}\right]}{16 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{5 \times 3^{-1-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{3^{-n} d^2 e^{\frac{3a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{8 \sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{5^{-1-n} d^2 e^{\frac{5a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left(\frac{a+b \operatorname{ArcCosh}[c x]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{5(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{32 \sqrt{-1+c x} \sqrt{1+c x}} - \frac{d^2 \sqrt{d-c^2 d x^2} \operatorname{Unintegrateable}\left[\frac{(a+b \operatorname{ArcCosh}[c x])^n}{x \sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 433: Result valid but suboptimal antiderivative.

$$\int \frac{(d-c^2 d x^2)^{5/2} (a+b \operatorname{ArcCosh}[c x])^n}{x^2} dx$$

Optimal (type 8, 485 leaves, 18 steps):

$$\begin{aligned}
& - \frac{15 c d^3 \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^{1+n}}{8 b (1+n) \sqrt{d-c^2 d x^2}} - \frac{1}{\sqrt{d-c^2 d x^2}} \\
& 2^{-2(3+n)} c d^3 e^{-\frac{4a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right] + \\
& \frac{1}{\sqrt{d-c^2 d x^2}} 2^{-2-n} c d^3 e^{-\frac{2a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right] - \\
& \frac{2^{-2-n} c d^3 e^{\frac{2a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{d-c^2 d x^2}} + \frac{1}{\sqrt{d-c^2 d x^2}} \\
& 2^{-2(3+n)} c d^3 e^{\frac{4a}{b}} \sqrt{-1+c x} \sqrt{1+c x} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right] + \\
& d^3 \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{d-c^2 d x^2}}, x\right]
\end{aligned}$$

Result (type 8, 522 leaves, 19 steps):

$$\begin{aligned}
& \frac{15 c d^2 \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^{1+n}}{8 b (1+n) \sqrt{-1+c x} \sqrt{1+c x}} + \frac{2^{-2(3+n)} c d^2 e^{-\frac{4a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{2^{-2-n} c d^2 e^{-\frac{2a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( -\frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} + \\
& \frac{2^{-2-n} c d^2 e^{\frac{2a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{2^{-2(3+n)} c d^2 e^{\frac{4a}{b}} \sqrt{d-c^2 d x^2} (a+b \operatorname{ArcCosh}[c x])^n \left( \frac{a+b \operatorname{ArcCosh}[c x]}{b} \right)^{-n} \operatorname{Gamma}\left[1+n, \frac{4(a+b \operatorname{ArcCosh}[c x])}{b}\right]}{\sqrt{-1+c x} \sqrt{1+c x}} - \\
& \frac{d^2 \sqrt{d-c^2 d x^2} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{-1+c x} \sqrt{1+c x}}, x\right]}{\sqrt{-1+c x} \sqrt{1+c x}}
\end{aligned}$$

Problem 434: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcCosh}[c x])^n}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 4, 323 leaves, 9 steps):

$$\begin{aligned}
& \frac{3^{-1-n} e^{-\frac{3a}{b} \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 c^4 \sqrt{1-cx}} + \\
& \frac{3 e^{-\frac{a}{b} \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 c^4 \sqrt{1-cx}} - \\
& \frac{3 e^{a/b \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 c^4 \sqrt{1-cx}} - \\
& \frac{3^{-1-n} e^{\frac{3a}{b} \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 c^4 \sqrt{1-cx}}
\end{aligned}$$

Result (type 4, 375 leaves, 10 steps):

$$\begin{aligned}
& \frac{3^{-1-n} e^{-\frac{3a}{b} \sqrt{-1+cx}} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} + \\
& \frac{3 e^{-\frac{a}{b} \sqrt{-1+cx}} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} - \\
& \frac{3 e^{a/b \sqrt{-1+cx}} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}} - \\
& \frac{3^{-1-n} e^{\frac{3a}{b} \sqrt{-1+cx}} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{3(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{8 c^4 \sqrt{1-c^2 x^2}}
\end{aligned}$$

Problem 435: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a+b \operatorname{ArcCosh}[cx])^n}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$\begin{aligned}
& \frac{\sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{2 b c^3 (1+n) \sqrt{1-cx}} + \frac{2^{-3-n} e^{-\frac{2a}{b} \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{c^3 \sqrt{1-cx}} - \\
& \frac{2^{-3-n} e^{\frac{2a}{b} \sqrt{-1+cx}} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{c^3 \sqrt{1-cx}}
\end{aligned}$$

Result (type 4, 250 leaves, 7 steps):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{2 b c^3 (1+n) \sqrt{1-c^2 x^2}} +$$

$$\frac{2^{-3-n} e^{-\frac{2a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{c^3 \sqrt{1-c^2 x^2}} -$$

$$\frac{2^{-3-n} e^{\frac{2a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{2(a+b \operatorname{ArcCosh}[cx])}{b}\right]}{c^3 \sqrt{1-c^2 x^2}}$$

Problem 436: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \operatorname{ArcCosh}[cx])^n}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 4, 154 leaves, 4 steps):

$$\frac{e^{-\frac{a}{b}} \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 c^2 \sqrt{1-cx}} -$$

$$\frac{e^{a/b} \sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 c^2 \sqrt{1-cx}}$$

Result (type 4, 180 leaves, 5 steps):

$$\frac{e^{-\frac{a}{b}} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(-\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, -\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 c^2 \sqrt{1-c^2 x^2}} -$$

$$\frac{e^{a/b} \sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^n \left(\frac{a+b \operatorname{ArcCosh}[cx]}{b}\right)^{-n} \operatorname{Gamma}\left[1+n, \frac{a+b \operatorname{ArcCosh}[cx]}{b}\right]}{2 c^2 \sqrt{1-c^2 x^2}}$$

Problem 437: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^n}{\sqrt{1-c^2 x^2}} dx$$

Optimal (type 3, 43 leaves, 1 step):



$$\frac{\sqrt{-1+cx} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{bc(1+n)\sqrt{1-cx}}$$

Result (type 3, 56 leaves, 2 steps):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} (a+b \operatorname{ArcCosh}[cx])^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}$$

Problem 438: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^n}{x\sqrt{1-c^2x^2}} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x\sqrt{1-c^2x^2}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x\sqrt{-1+cx} \sqrt{1+cx}}, x\right]}{\sqrt{1-c^2x^2}}$$

Problem 439: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^n}{x^2\sqrt{1-c^2x^2}} dx$$

Optimal (type 8, 30 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x^2\sqrt{1-c^2x^2}}, x\right]$$

Result (type 8, 67 leaves, 1 step):

$$\frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrable}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{x^2\sqrt{-1+cx} \sqrt{1+cx}}, x\right]}{\sqrt{1-c^2x^2}}$$

## Problem 444: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^n}{x \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x \sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x \sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{d - c^2 d x^2}}$$

## Problem 445: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{d - c^2 d x^2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 68 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 \sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{d - c^2 d x^2}}$$

## Problem 446: Result valid but suboptimal antiderivative.

$$\int \frac{x^2 (a + b \operatorname{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{x^2 (a + b \operatorname{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 72 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrate}\left[\frac{x^2 (a+b \operatorname{ArcCosh}[cx])^n}{(-1+cx)^{3/2} (1+cx)^{3/2}}, x\right]}{d \sqrt{d-c^2 dx^2}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int \frac{x (a+b \operatorname{ArcCosh}[cx])^n}{(d-c^2 dx^2)^{3/2}} dx$$

Optimal (type 8, 29 leaves, 0 steps):

$$\operatorname{Unintegrate}\left[\frac{x (a+b \operatorname{ArcCosh}[cx])^n}{(d-c^2 dx^2)^{3/2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrate}\left[\frac{x (a+b \operatorname{ArcCosh}[cx])^n}{(-1+cx)^{3/2} (1+cx)^{3/2}}, x\right]}{d \sqrt{d-c^2 dx^2}}$$

Problem 448: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^n}{(d-c^2 dx^2)^{3/2}} dx$$

Optimal (type 8, 28 leaves, 0 steps):

$$\operatorname{Unintegrate}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{(d-c^2 dx^2)^{3/2}}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$- \frac{\sqrt{-1+cx} \sqrt{1+cx} \operatorname{Unintegrate}\left[\frac{(a+b \operatorname{ArcCosh}[cx])^n}{(-1+cx)^{3/2} (1+cx)^{3/2}}, x\right]}{d \sqrt{d-c^2 dx^2}}$$

Problem 449: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCosh}[cx])^n}{x (d-c^2 dx^2)^{3/2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x (d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x (-1 + c x)^{3/2} (1 + c x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d x^2}}$$

Problem 450: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 (d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 8, 31 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 (d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcCosh}[c x])^n}{x^2 (-1 + c x)^{3/2} (1 + c x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d x^2}}$$

Problem 451: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{1 - c^2 x^2}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{1 - c^2 x^2}}, x\right]$$

Result (type 8, 69 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{1 - c^2 x^2}}$$

### Problem 457: Result valid but suboptimal antiderivative.

$$\int (f x)^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^n dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[(f x)^m (d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])^n, x\right]$$

Result (type 8, 72 leaves, 1 step):

$$-\frac{d \sqrt{d - c^2 d x^2} \operatorname{Unintegrable}\left[(f x)^m (-1 + c x)^{3/2} (1 + c x)^{3/2} (a + b \operatorname{ArcCosh}[c x])^n, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 458: Result valid but suboptimal antiderivative.

$$\int (f x)^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^n dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[(f x)^m \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^n, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{d - c^2 d x^2} \operatorname{Unintegrable}\left[(f x)^m \sqrt{-1 + c x} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])^n, x\right]}{\sqrt{-1 + c x} \sqrt{1 + c x}}$$

### Problem 459: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{d - c^2 d x^2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{d - c^2 d x^2}}, x\right]$$

Result (type 8, 70 leaves, 1 step):

$$\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{\sqrt{-1 + c x} \sqrt{1 + c x}}, x\right]}{\sqrt{d - c^2 d x^2}}$$

### Problem 460: Result valid but suboptimal antiderivative.

$$\int \frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}} dx$$

Optimal (type 8, 33 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{(d - c^2 d x^2)^{3/2}}, x\right]$$

Result (type 8, 74 leaves, 1 step):

$$-\frac{\sqrt{-1 + c x} \sqrt{1 + c x} \operatorname{Unintegrable}\left[\frac{(f x)^m (a + b \operatorname{ArcCosh}[c x])^n}{(-1 + c x)^{3/2} (1 + c x)^{3/2}}, x\right]}{d \sqrt{d - c^2 d x^2}}$$

## Test results for the 296 problems in "7.2.5 Inverse hyperbolic cosine functions.m"

### Problem 61: Unable to integrate problem.

$$\int \frac{(d - c^2 d x^2)^{3/2} (a + b \operatorname{ArcCosh}[c x])}{f + g x} dx$$

Optimal (type 4, 1270 leaves, ? steps):

$$\begin{aligned}
& - \frac{a d (c f - g) (c f + g) \sqrt{d - c^2 d x^2}}{g^3} + \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 d (c f - g) x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{a d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2}}{6 g} + \frac{b c d x (-12 - 9 c x + 4 c^2 x^2) \sqrt{d - c^2 d x^2}}{36 g \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^3} - \\
& \frac{a d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{2 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (2 + 3 c x - 2 c^2 x^2) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{6 g} - \frac{b d \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]^2}{4 g \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{c d (c f - g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^2} - \frac{d (c f - g) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{d (c f - g)^2 (c f + g)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \\
& \frac{d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^2 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} - \frac{2 a d (c f - g)^{3/2} (c f + g)^{3/2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{c f + g} \sqrt{1 + c x}}{\sqrt{c f - g} \sqrt{-1 + c x}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Result (type 8, 1150 leaves, 28 steps):

$$\begin{aligned}
& \frac{b c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2}}{g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{b c^2 d (c f - g) x^2 \sqrt{d - c^2 d x^2}}{4 g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} - \frac{a d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2}}{g^3 (1 - c x) (1 + c x)} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x]}{g^3} + \frac{c d (c f - g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])}{2 g^2} - \\
& \frac{d (c f - g) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{4 b g^2 \sqrt{-1 + c x} \sqrt{1 + c x}} + \frac{c d (c f - g) (c f + g) x \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b g^3 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{d (c f - g)^2 (c f + g)^2 \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^4 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \frac{d (c f - g) (c f + g) (1 - c^2 x^2) \sqrt{d - c^2 d x^2} (a + b \operatorname{ArcCosh}[c x])^2}{2 b c g^2 \sqrt{-1 + c x} \sqrt{1 + c x} (f + g x)} + \\
& \frac{a d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{-1 + c^2 x^2} \sqrt{d - c^2 d x^2} \operatorname{ArcTanh}\left[\frac{g + c^2 f x}{\sqrt{c^2 f^2 - g^2} \sqrt{-1 + c^2 x^2}}\right]}{g^4 (1 - c x) (1 + c x)} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{ArcCosh}[c x] \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f - \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} + \\
& \frac{b d (c f - g) (c f + g) \sqrt{c^2 f^2 - g^2} \sqrt{d - c^2 d x^2} \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCosh}[c x]} g}{c f + \sqrt{c^2 f^2 - g^2}}\right]}{g^4 \sqrt{-1 + c x} \sqrt{1 + c x}} - \\
& \frac{c d \sqrt{d - c^2 d x^2} \operatorname{Unintegrateable}\left[\left(-1 + c x\right)^{3/2} \sqrt{1 + c x} (a + b \operatorname{ArcCosh}[c x])\right], x]}{g \sqrt{-1 + c x} \sqrt{1 + c x}}
\end{aligned}$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int \frac{(c e + d e x)^2}{(a + b \operatorname{ArcCosh}[c + d x])^3} dx$$

Optimal (type 4, 252 leaves, 18 steps):



$$\begin{aligned}
& - \frac{e^2 \sqrt{-1+c+d x} (c+d x)^2 \sqrt{1+c+d x}}{2 b d (a+b \operatorname{ArcCosh}[c+d x])^2} + \frac{e^2 (c+d x)}{b^2 d (a+b \operatorname{ArcCosh}[c+d x])} - \frac{3 e^2 (c+d x)^3}{2 b^2 d (a+b \operatorname{ArcCosh}[c+d x])} - \\
& \frac{e^2 \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c+d x]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^3 d} - \frac{9 e^2 \operatorname{CoshIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[c+d x])}{b}\right] \operatorname{Sinh}\left[\frac{3 a}{b}\right]}{8 b^3 d} + \\
& \frac{e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c+d x]}{b}\right]}{8 b^3 d} + \frac{9 e^2 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3(a+b \operatorname{ArcCosh}[c+d x])}{b}\right]}{8 b^3 d}
\end{aligned}$$

Result (type 4, 311 leaves, 18 steps):

$$\begin{aligned}
& - \frac{e^2 \sqrt{-1+c+d x} (c+d x)^2 \sqrt{1+c+d x}}{2 b d (a+b \operatorname{ArcCosh}[c+d x])^2} + \frac{e^2 (c+d x)}{b^2 d (a+b \operatorname{ArcCosh}[c+d x])} - \frac{3 e^2 (c+d x)^3}{2 b^2 d (a+b \operatorname{ArcCosh}[c+d x])} - \\
& \frac{9 e^2 \operatorname{CoshIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c+d x]\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{8 b^3 d} + \frac{e^2 \operatorname{CoshIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c+d x]}{b}\right] \operatorname{Sinh}\left[\frac{a}{b}\right]}{b^3 d} - \\
& \frac{9 e^2 \operatorname{CoshIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcCosh}[c+d x]\right] \operatorname{Sinh}\left[\frac{3 a}{b}\right]}{8 b^3 d} + \frac{9 e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a}{b} + \operatorname{ArcCosh}[c+d x]\right]}{8 b^3 d} + \\
& \frac{9 e^2 \operatorname{Cosh}\left[\frac{3 a}{b}\right] \operatorname{SinhIntegral}\left[\frac{3 a}{b} + 3 \operatorname{ArcCosh}[c+d x]\right]}{8 b^3 d} - \frac{e^2 \operatorname{Cosh}\left[\frac{a}{b}\right] \operatorname{SinhIntegral}\left[\frac{a+b \operatorname{ArcCosh}[c+d x]}{b}\right]}{b^3 d}
\end{aligned}$$

## Test results for the 243 problems in "7.3.2 (d x)^m (a+b arctanh(c x^n))^p.m"

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^7 (a+b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 3, 125 leaves, 12 steps):

$$\frac{a b x^2}{4 c^3} + \frac{b^2 x^4}{24 c^2} + \frac{b^2 x^2 \operatorname{ArcTanh}[c x^2]}{4 c^3} + \frac{b x^6 (a+b \operatorname{ArcTanh}[c x^2])}{12 c} - \frac{(a+b \operatorname{ArcTanh}[c x^2])^2}{8 c^4} + \frac{1}{8} x^8 (a+b \operatorname{ArcTanh}[c x^2])^2 + \frac{b^2 \operatorname{Log}[1-c^2 x^4]}{6 c^4}$$

Result (type 4, 636 leaves, 62 steps):

$$\begin{aligned}
& \frac{a b x^2}{8 c^3} + \frac{23 b^2 x^2}{192 c^3} + \frac{b^2 x^4}{128 c^2} - \frac{7 b^2 x^6}{576 c} - \frac{b^2 x^8}{256} + \frac{3 b^2 (1 - c x^2)^2}{32 c^4} - \frac{b^2 (1 - c x^2)^3}{36 c^4} + \frac{b^2 (1 - c x^2)^4}{256 c^4} - \frac{5 b^2 \operatorname{Log}[1 - c x^2]}{192 c^4} + \\
& \frac{b^2 (1 - c x^2) \operatorname{Log}[1 - c x^2]}{16 c^4} + \frac{b^2 \operatorname{Log}[1 - c x^2]^2}{32 c^4} - \frac{b x^4 (2 a - b \operatorname{Log}[1 - c x^2])}{32 c^2} + \frac{b x^6 (2 a - b \operatorname{Log}[1 - c x^2])}{48 c} - \frac{1}{64} b x^8 (2 a - b \operatorname{Log}[1 - c x^2]) + \\
& \frac{1}{32} x^8 (2 a - b \operatorname{Log}[1 - c x^2])^2 - \frac{1}{192} b (2 a - b \operatorname{Log}[1 - c x^2]) \left( \frac{48 (1 - c x^2)}{c^4} - \frac{36 (1 - c x^2)^2}{c^4} + \frac{16 (1 - c x^2)^3}{c^4} - \frac{3 (1 - c x^2)^4}{c^4} - \frac{12 \operatorname{Log}[1 - c x^2]}{c^4} \right) - \\
& \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right]}{16 c^4} + \frac{b^2 \operatorname{Log}[1 + c x^2]}{24 c^4} + \frac{b^2 x^6 \operatorname{Log}[1 + c x^2]}{24 c} + \frac{b^2 (1 + c x^2) \operatorname{Log}[1 + c x^2]}{8 c^4} + \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2]}{16 c^4} + \\
& \frac{1}{16} b x^8 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2] - \frac{b^2 \operatorname{Log}[1 + c x^2]^2}{32 c^4} + \frac{1}{32} b^2 x^8 \operatorname{Log}[1 + c x^2]^2 + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right]}{16 c^4} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right]}{16 c^4}
\end{aligned}$$

Problem 65: Result valid but suboptimal antiderivative.

$$\int x^5 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\begin{aligned}
& \frac{b^2 x^2}{6 c^2} - \frac{b^2 \operatorname{ArcTanh}[c x^2]}{6 c^3} + \frac{b x^4 (a + b \operatorname{ArcTanh}[c x^2])}{6 c} + \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{6 c^3} + \\
& \frac{1}{6} x^6 (a + b \operatorname{ArcTanh}[c x^2])^2 - \frac{b (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}\left[\frac{2}{1 - c x^2}\right]}{3 c^3} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^2}\right]}{6 c^3}
\end{aligned}$$

Result (type 4, 536 leaves, 53 steps):

$$\begin{aligned}
& -\frac{a b x^2}{6 c^2} + \frac{19 b^2 x^2}{72 c^2} - \frac{5 b^2 x^4}{144 c} - \frac{b^2 x^6}{108} + \frac{b^2 (1 - c x^2)^2}{16 c^3} - \frac{b^2 (1 - c x^2)^3}{108 c^3} + \frac{b^2 \operatorname{Log}[1 - c x^2]}{72 c^3} - \frac{b^2 (1 - c x^2) \operatorname{Log}[1 - c x^2]}{12 c^3} + \\
& \frac{b^2 \operatorname{Log}[1 - c x^2]^2}{24 c^3} + \frac{b x^4 (2 a - b \operatorname{Log}[1 - c x^2])}{24 c} - \frac{1}{36} b x^6 (2 a - b \operatorname{Log}[1 - c x^2]) + \frac{1}{24} x^6 (2 a - b \operatorname{Log}[1 - c x^2])^2 - \\
& \frac{1}{72} b (2 a - b \operatorname{Log}[1 - c x^2]) \left( \frac{18 (1 - c x^2)}{c^3} - \frac{9 (1 - c x^2)^2}{c^3} + \frac{2 (1 - c x^2)^3}{c^3} - \frac{6 \operatorname{Log}[1 - c x^2]}{c^3} \right) + \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right]}{12 c^3} - \\
& \frac{b^2 \operatorname{Log}[1 + c x^2]}{12 c^3} + \frac{b^2 x^4 \operatorname{Log}[1 + c x^2]}{12 c} + \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2]}{12 c^3} + \frac{1}{12} b x^6 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2] + \\
& \frac{b^2 \operatorname{Log}[1 + c x^2]^2}{24 c^3} + \frac{1}{24} b^2 x^6 \operatorname{Log}[1 + c x^2]^2 - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right]}{12 c^3} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right]}{12 c^3}
\end{aligned}$$

**Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^3 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a b x^2}{2 c} + \frac{b^2 x^2 \operatorname{ArcTanh}[c x^2]}{2 c} - \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{4 c^2} + \frac{1}{4} x^4 (a + b \operatorname{ArcTanh}[c x^2])^2 + \frac{b^2 \operatorname{Log}[1 - c^2 x^4]}{4 c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\begin{aligned} & \frac{3 a b x^2}{4 c} - \frac{b^2 x^4}{16} + \frac{b^2 (1 - c x^2)^2}{32 c^2} + \frac{b^2 (1 + c x^2)^2}{32 c^2} - \frac{b^2 \operatorname{Log}[1 - c x^2]}{16 c^2} + \frac{3 b^2 (1 - c x^2) \operatorname{Log}[1 - c x^2]}{8 c^2} - \frac{1}{16} b x^4 (2 a - b \operatorname{Log}[1 - c x^2]) + \\ & \frac{b (1 - c x^2)^2 (2 a - b \operatorname{Log}[1 - c x^2])}{16 c^2} - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^2}{8 c^2} + \frac{(1 - c x^2)^2 (2 a - b \operatorname{Log}[1 - c x^2])^2}{16 c^2} - \\ & \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[\frac{1}{2} (1 + c x^2)]}{8 c^2} - \frac{b^2 \operatorname{Log}[1 + c x^2]}{16 c^2} + \frac{1}{16} b^2 x^4 \operatorname{Log}[1 + c x^2] + \frac{3 b^2 (1 + c x^2) \operatorname{Log}[1 + c x^2]}{8 c^2} - \\ & \frac{b^2 (1 + c x^2)^2 \operatorname{Log}[1 + c x^2]}{16 c^2} + \frac{b^2 \operatorname{Log}[\frac{1}{2} (1 - c x^2)] \operatorname{Log}[1 + c x^2]}{8 c^2} + \frac{1}{8} b x^4 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2] - \\ & \frac{b^2 (1 + c x^2) \operatorname{Log}[1 + c x^2]^2}{8 c^2} + \frac{b^2 (1 + c x^2)^2 \operatorname{Log}[1 + c x^2]^2}{16 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - c x^2)]}{8 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + c x^2)]}{8 c^2} \end{aligned}$$

**Problem 67: Result valid but suboptimal antiderivative.**

$$\int x (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 94 leaves, 6 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{2 c} + \frac{1}{2} x^2 (a + b \operatorname{ArcTanh}[c x^2])^2 - \frac{b (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}[\frac{2}{1 - c x^2}]}{c} - \frac{b^2 \operatorname{PolyLog}[2, 1 - \frac{2}{1 - c x^2}]}{2 c}$$

Result (type 4, 207 leaves, 28 steps):

$$\begin{aligned} & - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^2}{8 c} + \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[\frac{1}{2} (1 + c x^2)]}{4 c} + \frac{b^2 \operatorname{Log}[\frac{1}{2} (1 - c x^2)] \operatorname{Log}[1 + c x^2]}{4 c} + \\ & \frac{1}{4} b x^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2] + \frac{b^2 (1 + c x^2) \operatorname{Log}[1 + c x^2]^2}{8 c} - \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - c x^2)]}{4 c} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + c x^2)]}{4 c} \end{aligned}$$

## Problem 69: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^3} dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{1}{2} c (a + b \operatorname{ArcTanh}[c x^2])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{2 x^2} + b c (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}\left[2 - \frac{2}{1 + c x^2}\right] - \frac{1}{2} b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^2}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$\begin{aligned} & 2 a b c \operatorname{Log}[x] - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^2}{8 x^2} - \frac{1}{4} b c (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right] - \\ & \frac{1}{4} b^2 c \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2] - \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]}{4 x^2} - \frac{b^2 (1 + c x^2) \operatorname{Log}[1 + c x^2]^2}{8 x^2} - \\ & \frac{1}{2} b^2 c \operatorname{PolyLog}[2, -c x^2] + \frac{1}{2} b^2 c \operatorname{PolyLog}[2, c x^2] + \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right] - \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right] \end{aligned}$$

## Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^5} dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$-\frac{b c (a + b \operatorname{ArcTanh}[c x^2])}{2 x^2} + \frac{1}{4} c^2 (a + b \operatorname{ArcTanh}[c x^2])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{4 x^4} + b^2 c^2 \operatorname{Log}[x] - \frac{1}{4} b^2 c^2 \operatorname{Log}[1 - c^2 x^4]$$

Result (type 4, 360 leaves, 46 steps):

$$\begin{aligned} & b^2 c^2 \operatorname{Log}[x] - \frac{1}{8} b^2 c^2 \operatorname{Log}[1 - c x^2] - \frac{b c (2 a - b \operatorname{Log}[1 - c x^2])}{8 x^2} - \frac{b c (1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])}{8 x^2} + \\ & \frac{1}{16} c^2 (2 a - b \operatorname{Log}[1 - c x^2])^2 - \frac{(2 a - b \operatorname{Log}[1 - c x^2])^2}{16 x^4} + \frac{1}{8} b c^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right] - \\ & \frac{1}{4} b^2 c^2 \operatorname{Log}[1 + c x^2] - \frac{b^2 c \operatorname{Log}[1 + c x^2]}{4 x^2} - \frac{1}{8} b^2 c^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2] - \frac{b (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]}{8 x^4} + \\ & \frac{1}{16} b^2 c^2 \operatorname{Log}[1 + c x^2]^2 - \frac{b^2 \operatorname{Log}[1 + c x^2]^2}{16 x^4} - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right] \end{aligned}$$

### Problem 77: Result valid but suboptimal antiderivative.

$$\int x^3 (a + b \operatorname{ArcTanh}[c x^2])^3 dx$$

Optimal (type 4, 141 leaves, 9 steps):

$$\frac{3 b (a + b \operatorname{ArcTanh}[c x^2])^2}{4 c^2} + \frac{3 b x^2 (a + b \operatorname{ArcTanh}[c x^2])^2}{4 c} - \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{4 c^2} +$$

$$\frac{1}{4} x^4 (a + b \operatorname{ArcTanh}[c x^2])^3 - \frac{3 b^2 (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}\left[\frac{2}{1-c x^2}\right]}{2 c^2} - \frac{3 b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x^2}\right]}{4 c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$- \frac{3 b (1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^2}{16 c^2} - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^3}{16 c^2} + \frac{(1 - c x^2)^2 (2 a - b \operatorname{Log}[1 - c x^2])^3}{32 c^2} +$$

$$\frac{3 b^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right]}{8 c^2} + \frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2]}{8 c^2} + \frac{3 b^2 x^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]}{8 c} -$$

$$\frac{3 b (2 a - b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2]}{32 c^2} + \frac{3}{32} b x^4 (2 a - b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2] + \frac{3 b^3 (1 + c x^2) \operatorname{Log}[1 + c x^2]^2}{16 c^2} -$$

$$\frac{3 b^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2}{32 c^2} + \frac{3}{32} b^2 x^4 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2 - \frac{b^3 (1 + c x^2) \operatorname{Log}[1 + c x^2]^3}{16 c^2} +$$

$$\frac{b^3 (1 + c x^2)^2 \operatorname{Log}[1 + c x^2]^3}{32 c^2} - \frac{3 b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right]}{8 c^2} + \frac{3 b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right]}{8 c^2}$$

### Problem 78: Result valid but suboptimal antiderivative.

$$\int x (a + b \operatorname{ArcTanh}[c x^2])^3 dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{2 c} + \frac{1}{2} x^2 (a + b \operatorname{ArcTanh}[c x^2])^3 - \frac{3 b (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{Log}\left[\frac{2}{1-c x^2}\right]}{2 c} -$$

$$\frac{3 b^2 (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x^2}\right]}{2 c} + \frac{3 b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x^2}\right]}{4 c}$$

Result (type 4, 390 leaves, 82 steps):

$$\begin{aligned}
& - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^3}{16 c} + \frac{3 b (2 a - b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}\left[\frac{1}{2} (1 + c x^2)\right]}{8 c} - \frac{3 b (2 a - b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2]}{16 c} + \\
& \frac{3}{16} b x^2 (2 a - b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2] + \frac{3 b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^2)\right] \operatorname{Log}[1 + c x^2]^2}{8 c} + \frac{3 b^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2}{16 c} + \\
& \frac{3}{16} b^2 x^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2 + \frac{b^3 (1 + c x^2) \operatorname{Log}[1 + c x^2]^3}{16 c} - \frac{3 b^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^2)\right]}{4 c} + \\
& \frac{3 b^3 \operatorname{Log}[1 + c x^2] \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^2)\right]}{4 c} - \frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 - c x^2)\right]}{4 c} - \frac{3 b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 + c x^2)\right]}{4 c}
\end{aligned}$$

Problem 80: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{x^3} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{2} c (a + b \operatorname{ArcTanh}[c x^2])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{2 x^2} + \frac{3}{2} b c (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x^2}\right] - \\
& \frac{3}{2} b^2 c (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^2}\right] - \frac{3}{4} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x^2}\right]
\end{aligned}$$

Result (type 8, 284 leaves, 16 steps):

$$\begin{aligned}
& \frac{3}{16} b c \operatorname{Log}[c x^2] (2 a - b \operatorname{Log}[1 - c x^2])^2 - \frac{(1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^3}{16 x^2} + \\
& \frac{3}{16} b^3 c \operatorname{Log}[-c x^2] \operatorname{Log}[1 + c x^2]^2 - \frac{b^3 (1 + c x^2) \operatorname{Log}[1 + c x^2]^3}{16 x^2} - \frac{3}{8} b^2 c (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{PolyLog}[2, 1 - c x^2] + \\
& \frac{3}{8} b^3 c \operatorname{Log}[1 + c x^2] \operatorname{PolyLog}[2, 1 + c x^2] - \frac{3}{8} b^3 c \operatorname{PolyLog}[3, 1 - c x^2] - \frac{3}{8} b^3 c \operatorname{PolyLog}[3, 1 + c x^2] + \\
& \frac{3}{8} b \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2]}{x^3}, x\right] - \frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2}{x^3}, x\right]
\end{aligned}$$

Problem 81: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{x^5} dx$$

Optimal (type 4, 139 leaves, 8 steps):

$$\frac{3}{4} b c^2 (a + b \operatorname{ArcTanh}[c x^2])^2 - \frac{3 b c (a + b \operatorname{ArcTanh}[c x^2])^2}{4 x^2} + \frac{1}{4} c^2 (a + b \operatorname{ArcTanh}[c x^2])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{4 x^4} + \frac{3}{2} b^2 c^2 (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}\left[2 - \frac{2}{1 + c x^2}\right] - \frac{3}{4} b^3 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^2}\right]$$

Result (type 8, 437 leaves, 29 steps):

$$\begin{aligned} & \frac{3}{4} a b^2 c^2 \operatorname{Log}[x] - \frac{3 b c (1 - c x^2) (2 a - b \operatorname{Log}[1 - c x^2])^2}{32 x^2} + \frac{3}{32} b c^2 \operatorname{Log}[c x^2] (2 a - b \operatorname{Log}[1 - c x^2])^2 + \frac{1}{32} c^2 (2 a - b \operatorname{Log}[1 - c x^2])^3 - \\ & \frac{(2 a - b \operatorname{Log}[1 - c x^2])^3}{32 x^4} - \frac{3 b^3 c (1 + c x^2) \operatorname{Log}[1 + c x^2]^2}{32 x^2} - \frac{3}{32} b^3 c^2 \operatorname{Log}[-c x^2] \operatorname{Log}[1 + c x^2]^2 + \frac{1}{32} b^3 c^2 \operatorname{Log}[1 + c x^2]^3 - \\ & \frac{b^3 \operatorname{Log}[1 + c x^2]^3}{32 x^4} - \frac{3}{16} b^3 c^2 \operatorname{PolyLog}[2, -c x^2] + \frac{3}{16} b^3 c^2 \operatorname{PolyLog}[2, c x^2] - \frac{3}{16} b^2 c^2 (2 a - b \operatorname{Log}[1 - c x^2]) \operatorname{PolyLog}[2, 1 - c x^2] - \\ & \frac{3}{16} b^3 c^2 \operatorname{Log}[1 + c x^2] \operatorname{PolyLog}[2, 1 + c x^2] - \frac{3}{16} b^3 c^2 \operatorname{PolyLog}[3, 1 - c x^2] + \frac{3}{16} b^3 c^2 \operatorname{PolyLog}[3, 1 + c x^2] + \\ & \frac{3}{8} b \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^2])^2 \operatorname{Log}[1 + c x^2]}{x^5}, x\right] - \frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^2]) \operatorname{Log}[1 + c x^2]^2}{x^5}, x\right] \end{aligned}$$

Problem 82: Result optimal but 1 more steps used.

$$\int (d x)^{5/2} (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned} & \frac{8 b d (d x)^{3/2}}{21 c} + \frac{2 b d^{5/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} - \frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{2 (d x)^{7/2} (a + b \operatorname{ArcTanh}[c x^2])}{7 d} - \\ & \frac{2 b d^{5/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} - \frac{b d^{5/2} \operatorname{Log}[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}]}{7 \sqrt{2} c^{7/4}} + \frac{b d^{5/2} \operatorname{Log}[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}]}{7 \sqrt{2} c^{7/4}} \end{aligned}$$

Result (type 3, 317 leaves, 17 steps):

$$\begin{aligned} & \frac{8 b d (d x)^{3/2}}{21 c} + \frac{2 b d^{5/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} - \frac{\sqrt{2} b d^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} + \frac{2 (d x)^{7/2} (a + b \operatorname{ArcTanh}[c x^2])}{7 d} - \\ & \frac{2 b d^{5/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{7 c^{7/4}} - \frac{b d^{5/2} \operatorname{Log}[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}]}{7 \sqrt{2} c^{7/4}} + \frac{b d^{5/2} \operatorname{Log}[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}]}{7 \sqrt{2} c^{7/4}} \end{aligned}$$

## Problem 83: Result optimal but 1 more steps used.

$$\int (d x)^{3/2} (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{8 b d \sqrt{d x}}{5 c} - \frac{2 b d^{3/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{\sqrt{2} b d^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} - \frac{\sqrt{2} b d^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{2 (d x)^{5/2} (a + b \operatorname{ArcTanh}[c x^2])}{5 d} -$$

$$\frac{2 b d^{3/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{b d^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{5 \sqrt{2} c^{5/4}} - \frac{b d^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{5 \sqrt{2} c^{5/4}}$$

Result (type 3, 317 leaves, 17 steps):

$$\frac{8 b d \sqrt{d x}}{5 c} - \frac{2 b d^{3/2} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{\sqrt{2} b d^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} - \frac{\sqrt{2} b d^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{2 (d x)^{5/2} (a + b \operatorname{ArcTanh}[c x^2])}{5 d} -$$

$$\frac{2 b d^{3/2} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{5 c^{5/4}} + \frac{b d^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{5 \sqrt{2} c^{5/4}} - \frac{b d^{3/2} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{5 \sqrt{2} c^{5/4}}$$

## Problem 84: Result optimal but 1 more steps used.

$$\int \sqrt{d x} (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\frac{2 b \sqrt{d} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} - \frac{\sqrt{2} b \sqrt{d} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{\sqrt{2} b \sqrt{d} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{2 (d x)^{3/2} (a + b \operatorname{ArcTanh}[c x^2])}{3 d} -$$

$$\frac{2 b \sqrt{d} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{b \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} c^{3/4}} - \frac{b \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} c^{3/4}}$$

Result (type 3, 301 leaves, 16 steps):

$$\frac{2 b \sqrt{d} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} - \frac{\sqrt{2} b \sqrt{d} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{\sqrt{2} b \sqrt{d} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{2 (d x)^{3/2} (a + b \operatorname{ArcTanh}[c x^2])}{3 d} -$$

$$\frac{2 b \sqrt{d} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 c^{3/4}} + \frac{b \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} c^{3/4}} - \frac{b \sqrt{d} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} c^{3/4}}$$



### Problem 85: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{\sqrt{d x}} dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 b \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} - \frac{\sqrt{2} b \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} + \frac{\sqrt{2} b \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} + \frac{2 \sqrt{d x} (a + b \operatorname{ArcTanh}[c x^2])}{d} \\ & - \frac{2 b \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} - \frac{b \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} c^{1/4} \sqrt{d}} + \frac{b \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} c^{1/4} \sqrt{d}} \end{aligned}$$

Result (type 3, 285 leaves, 16 steps):

$$\begin{aligned} & -\frac{2 b \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} - \frac{\sqrt{2} b \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} + \frac{\sqrt{2} b \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} + \frac{2 \sqrt{d x} (a + b \operatorname{ArcTanh}[c x^2])}{d} \\ & - \frac{2 b \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{c^{1/4} \sqrt{d}} - \frac{b \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} c^{1/4} \sqrt{d}} + \frac{b \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} c^{1/4} \sqrt{d}} \end{aligned}$$

### Problem 86: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{(d x)^{3/2}} dx$$

Optimal (type 3, 285 leaves, 15 steps):

$$\begin{aligned} & -\frac{2 b c^{1/4} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} - \frac{\sqrt{2} b c^{1/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{\sqrt{2} b c^{1/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} - \frac{2 (a + b \operatorname{ArcTanh}[c x^2])}{d \sqrt{d x}} \\ & + \frac{2 b c^{1/4} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{b c^{1/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} d^{3/2}} - \frac{b c^{1/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} d^{3/2}} \end{aligned}$$

Result (type 3, 285 leaves, 16 steps):

$$\begin{aligned}
& - \frac{2 b c^{1/4} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} - \frac{\sqrt{2} b c^{1/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{\sqrt{2} b c^{1/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} - \frac{2 (a + b \operatorname{ArcTanh}[c x^2])}{d \sqrt{d x}} + \\
& \frac{2 b c^{1/4} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{b c^{1/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} d^{3/2}} - \frac{b c^{1/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{\sqrt{2} d^{3/2}}
\end{aligned}$$

Problem 87: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{(d x)^{5/2}} dx$$

Optimal (type 3, 301 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 b c^{3/4} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{\sqrt{2} b c^{3/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} + \frac{\sqrt{2} b c^{3/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{2 (a + b \operatorname{ArcTanh}[c x^2])}{3 d (d x)^{3/2}} + \\
& \frac{2 b c^{3/4} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{b c^{3/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} d^{5/2}} + \frac{b c^{3/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} d^{5/2}}
\end{aligned}$$

Result (type 3, 301 leaves, 16 steps):

$$\begin{aligned}
& \frac{2 b c^{3/4} \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{\sqrt{2} b c^{3/4} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} + \frac{\sqrt{2} b c^{3/4} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{2 (a + b \operatorname{ArcTanh}[c x^2])}{3 d (d x)^{3/2}} + \\
& \frac{2 b c^{3/4} \operatorname{ArcTanh}\left[\frac{c^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{3 d^{5/2}} - \frac{b c^{3/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x - \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} d^{5/2}} + \frac{b c^{3/4} \operatorname{Log}\left[\sqrt{d} + \sqrt{c} \sqrt{d} x + \sqrt{2} c^{1/4} \sqrt{d x}\right]}{3 \sqrt{2} d^{5/2}}
\end{aligned}$$

Problem 88: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{(d x)^{7/2}} dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned}
& - \frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} + \frac{\sqrt{2}bc^{5/4}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{\sqrt{2}bc^{5/4}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{2(a+b\operatorname{ArcTanh}[cx^2])}{5d(dx)^{5/2}} + \\
& \frac{2bc^{5/4}\operatorname{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{bc^{5/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x - \sqrt{2}c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}d^{7/2}} + \frac{bc^{5/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x + \sqrt{2}c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}d^{7/2}}
\end{aligned}$$

Result (type 3, 317 leaves, 17 steps):

$$\begin{aligned}
& - \frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} + \frac{\sqrt{2}bc^{5/4}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{\sqrt{2}bc^{5/4}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{2(a+b\operatorname{ArcTanh}[cx^2])}{5d(dx)^{5/2}} + \\
& \frac{2bc^{5/4}\operatorname{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{5d^{7/2}} - \frac{bc^{5/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x - \sqrt{2}c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}d^{7/2}} + \frac{bc^{5/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x + \sqrt{2}c^{1/4}\sqrt{dx}\right]}{5\sqrt{2}d^{7/2}}
\end{aligned}$$

Problem 89: Result optimal but 1 more steps used.

$$\int \frac{a+b\operatorname{ArcTanh}[cx^2]}{(dx)^{9/2}} dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned}
& - \frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4}\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} + \frac{\sqrt{2}bc^{7/4}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} - \frac{\sqrt{2}bc^{7/4}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} - \frac{2(a+b\operatorname{ArcTanh}[cx^2])}{7d(dx)^{7/2}} + \\
& \frac{2bc^{7/4}\operatorname{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} + \frac{bc^{7/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x - \sqrt{2}c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}d^{9/2}} - \frac{bc^{7/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x + \sqrt{2}c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}d^{9/2}}
\end{aligned}$$

Result (type 3, 317 leaves, 17 steps):

$$\begin{aligned}
& - \frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4}\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} + \frac{\sqrt{2}bc^{7/4}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} - \frac{\sqrt{2}bc^{7/4}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} - \frac{2(a+b\operatorname{ArcTanh}[cx^2])}{7d(dx)^{7/2}} + \\
& \frac{2bc^{7/4}\operatorname{ArcTanh}\left[\frac{c^{1/4}\sqrt{dx}}{\sqrt{d}}\right]}{7d^{9/2}} + \frac{bc^{7/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x - \sqrt{2}c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}d^{9/2}} - \frac{bc^{7/4}\operatorname{Log}\left[\sqrt{d} + \sqrt{c}\sqrt{d}x + \sqrt{2}c^{1/4}\sqrt{dx}\right]}{7\sqrt{2}d^{9/2}}
\end{aligned}$$

## Problem 90: Unable to integrate problem.

$$\int \sqrt{d x} \left( a + b \operatorname{ArcTanh} \left[ c x^2 \right] \right)^2 dx$$

Optimal (type 4, 6327 leaves, 238 steps):

$$\begin{aligned} & -\frac{8}{9} a b x \sqrt{d x} - \frac{2 \sqrt{2} a b \sqrt{d x} \operatorname{ArcTan} \left[ 1 - \sqrt{2} c^{1/4} \sqrt{x} \right]}{3 c^{3/4} \sqrt{x}} + \frac{2 \sqrt{2} a b \sqrt{d x} \operatorname{ArcTan} \left[ 1 + \sqrt{2} c^{1/4} \sqrt{x} \right]}{3 c^{3/4} \sqrt{x}} - \\ & \frac{2 i b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right]^2}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{ArcTan} \left[ c^{1/4} \sqrt{x} \right]^2}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right]^2}{3 (-c)^{3/4} \sqrt{x}} - \\ & \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ c^{1/4} \sqrt{x} \right]^2}{3 c^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2}{1 - (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2}{1 - i (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} - \\ & \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ -\frac{2 (-c)^{1/4} \left( 1 - \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( i \sqrt{-\sqrt{-c}} - (-c)^{1/4} \right) \left( 1 - i (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2 (-c)^{1/4} \left( 1 + \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( i \sqrt{-\sqrt{-c}} + (-c)^{1/4} \right) \left( 1 - i (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} + \\ & \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{(1+i) \left( 1 - (-c)^{1/4} \sqrt{x} \right)}{1 - i (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2}{1 + i (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} - \\ & \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2}{1 + (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ -\frac{2 (-c)^{1/4} \left( 1 - \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( \sqrt{-\sqrt{-c}} - (-c)^{1/4} \right) \left( 1 + (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} - \\ & \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2 (-c)^{1/4} \left( 1 + \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( \sqrt{-\sqrt{-c}} + (-c)^{1/4} \right) \left( 1 + (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ -\frac{2 (-c)^{1/4} \left( 1 - \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( \sqrt{-\sqrt{-c}} - (-c)^{1/4} \right) \left( 1 + (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} + \\ & \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{2 (-c)^{1/4} \left( 1 + \sqrt{-\sqrt{-c}} \sqrt{x} \right)}{\left( \sqrt{-\sqrt{-c}} + (-c)^{1/4} \right) \left( 1 + (-c)^{1/4} \sqrt{x} \right)} \right]}{3 (-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan} \left[ (-c)^{1/4} \sqrt{x} \right] \operatorname{Log} \left[ \frac{(1-i) \left( 1 + (-c)^{1/4} \sqrt{x} \right)}{1 - i (-c)^{1/4} \sqrt{x}} \right]}{3 (-c)^{3/4} \sqrt{x}} + \end{aligned}$$

$$\begin{aligned}
& \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}}\left(1+(-c)^{1/4} \sqrt{x}\right)\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{\sqrt{2} a b \sqrt{d x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{3 c^{3/4} \sqrt{x}} - \frac{\sqrt{2} a b \sqrt{d x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{3 c^{3/4} \sqrt{x}} + \frac{4}{9} b^2 x \sqrt{d x} \operatorname{Log}\left[1-c x^2\right] + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{4}{9} b x \sqrt{d x}\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) + \\
& \frac{2 b \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}} - \frac{2 b \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}} + \frac{1}{6} x \sqrt{d x}\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 + \\
& \frac{2}{3} a b x \sqrt{d x} \operatorname{Log}\left[1+c x^2\right] - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}} - \frac{1}{3} b^2 x \sqrt{d x} \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right] + \\
& \frac{1}{6} b^2 x \sqrt{d x} \operatorname{Log}\left[1+c x^2\right]^2 + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{((-c)^{1/4}-i c^{1/4}) \left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{((-c)^{1/4}-c^{1/4}) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{((-c)^{1/4}-c^{1/4}) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{((-c)^{1/4}+c^{1/4}) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{((-c)^{1/4}+i c^{1/4}) \left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{((-c)^{1/4}+c^{1/4}) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}
\end{aligned}$$

Result (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\sqrt{d x} \left(a + b \operatorname{ArcTanh}\left[c x^2\right]\right)^2, x\right]$$

## Problem 91: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{\sqrt{d x}} dx$$

Optimal (type 4, 6177 leaves, 241 steps):

$$\begin{aligned} & \frac{2 a^2 x}{\sqrt{d x}} - \frac{2 \sqrt{2} a b \sqrt{x} \operatorname{ArcTan}\left[1 - \sqrt{2} c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 \sqrt{2} a b \sqrt{x} \operatorname{ArcTan}\left[1 + \sqrt{2} c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{(-c)^{1/4} \sqrt{d x}} - \\ & \frac{4 a b \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{c^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{(-c)^{1/4} \sqrt{d x}} - \frac{4 a b \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} - \\ & \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{c^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \\ & \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \\ & \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i) \left(1 - (-c)^{1/4} \sqrt{x}\right)}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \\ & \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \\ & \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \\ & \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i) \left(1 + (-c)^{1/4} \sqrt{x}\right)}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \end{aligned}$$



$$\begin{aligned}
& \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} - \frac{\sqrt{2} a b \sqrt{x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{c^{1/4} \sqrt{d x}} + \frac{\sqrt{2} a b \sqrt{x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 a b x \operatorname{Log}\left[1-c x^2\right]}{\sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}} + \frac{b^2 x \operatorname{Log}\left[1-c x^2\right]^2}{2 \sqrt{d x}} + \frac{2 a b x \operatorname{Log}\left[1+c x^2\right]}{\sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}} - \frac{b^2 x \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right]}{\sqrt{d x}} + \frac{b^2 x \operatorname{Log}\left[1+c x^2\right]^2}{2 \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 - c^{1/4} \sqrt{x})}{((-c)^{1/4} - i c^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 - c^{1/4} \sqrt{x})}{((-c)^{1/4} - c^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} - c^{1/4}) (1 - i c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} + c^{1/4}) (1 - i c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} (1 - (-c)^{1/4} \sqrt{x})}{(i (-c)^{1/4} - c^{1/4}) (1 - i c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + (-c)^{1/4} \sqrt{x})}{(i (-c)^{1/4} + c^{1/4}) (1 - i c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) (1 - c^{1/4} \sqrt{x})}{1 - i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} - c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} + c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} + \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} - c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} + c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} (1 - (-c)^{1/4} \sqrt{x})}{((-c)^{1/4} - c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + (-c)^{1/4} \sqrt{x})}{((-c)^{1/4} + c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + c^{1/4} \sqrt{x})}{((-c)^{1/4} + i c^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + c^{1/4} \sqrt{x})}{((-c)^{1/4} + c^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) (1 + c^{1/4} \sqrt{x})}{1 - i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}}
\end{aligned}$$

Result (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{\sqrt{d x}}, x\right]$$

Problem 92: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d x)^{3/2}} dx$$

Optimal (type 4, 6334 leaves, 197 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[1 - \sqrt{2} c^{1/4} \sqrt{x}\right]}{d \sqrt{d x}} + \frac{2 \sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[1 + \sqrt{2} c^{1/4} \sqrt{x}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{d \sqrt{d x}} + \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{d \sqrt{d x}} + \\
& \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{d \sqrt{d x}} - \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \\
& \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i) \left(1 - (-c)^{1/4} \sqrt{x}\right)}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} \\
& - \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} \\
& - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& - \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}}\left(1+c^{1/4} \sqrt{x}\right)\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+i c^{1/4}}\left(1-i(-c)^{1/4} \sqrt{x}\right)\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}}\left(1+(-c)^{1/4} \sqrt{x}\right)\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{\sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{d \sqrt{d x}} - \frac{\sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{d \sqrt{d x}} - \\
& \frac{2 b c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] (2 a-b \operatorname{Log}\left[1-c x^2\right])}{d \sqrt{d x}} + \frac{2 b c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] (2 a-b \operatorname{Log}\left[1-c x^2\right])}{d \sqrt{d x}} - \frac{(2 a-b \operatorname{Log}\left[1-c x^2\right])^2}{2 d \sqrt{d x}} - \\
& \frac{2 a b \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \\
& \frac{b^2 \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \frac{b^2 \operatorname{Log}\left[1+c x^2\right]^2}{2 d \sqrt{d x}} - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2 (-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2 (-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} \left(1 - \sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1 + \sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1 + c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + i c^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1 + c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + c^{1/4} \sqrt{x}\right)}{1 - i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}
\end{aligned}$$

Result (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(a + b \operatorname{ArcTanh}\left[c x^2\right]\right)^2}{(d x)^{3/2}}, x\right]$$

Problem 93: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c x^2\right]\right)^2}{(d x)^{5/2}} dx$$

Optimal (type 4, 6520 leaves, 197 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[1 - \sqrt{2} c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} + \frac{2 \sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[1 + \sqrt{2} c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} +
\end{aligned}$$



$$\begin{aligned}
& \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} \\
& + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1-c^{1/4} \sqrt{x})}{(-c)^{1/4}-c^{1/4}} \frac{1}{(1+(-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} (1-\sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}}-c^{1/4}) (1-i c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} (1+\sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}}+c^{1/4}) (1-i c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} (1-(-c)^{1/4} \sqrt{x})}{(i (-c)^{1/4}-c^{1/4}) (1-i c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} (1+(-c)^{1/4} \sqrt{x})}{(i (-c)^{1/4}+c^{1/4}) (1-i c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i) (1-c^{1/4} \sqrt{x})}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} (1-\sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}}-c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} (1+\sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}}+c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} (1-\sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}}-c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} (1+\sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}}+c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} (1-(-c)^{1/4} \sqrt{x})}{((-c)^{1/4}-c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} (1+(-c)^{1/4} \sqrt{x})}{((-c)^{1/4}+c^{1/4}) (1+c^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1+c^{1/4} \sqrt{x})}{((-c)^{1/4}+i c^{1/4}) (1-i (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1+c^{1/4} \sqrt{x})}{((-c)^{1/4}+c^{1/4}) (1+(-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i) (1+c^{1/4} \sqrt{x})}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{Log}\left[1 - \sqrt{2} c^{1/4} \sqrt{x} + \sqrt{c} x\right]}{3 d^2 \sqrt{d x}} + \frac{\sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{Log}\left[1 + \sqrt{2} c^{1/4} \sqrt{x} + \sqrt{c} x\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 - c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 - c x^2\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] (2 a - b \operatorname{Log}\left[1 - c x^2\right])}{3 d^2 \sqrt{d x}} + \frac{2 b c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] (2 a - b \operatorname{Log}\left[1 - c x^2\right])}{3 d^2 \sqrt{d x}} - \frac{(2 a - b \operatorname{Log}\left[1 - c x^2\right])^2}{6 d^2 x \sqrt{d x}} - \\
& \frac{2 a b \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 x \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 \operatorname{Log}\left[1 - c x^2\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 x \sqrt{d x}} - \frac{b^2 \operatorname{Log}\left[1 + c x^2\right]^2}{6 d^2 x \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} - (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} + (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) (1 - (-c)^{1/4} \sqrt{x})}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(\sqrt{-c} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(\sqrt{-c} + (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(\sqrt{-c} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(\sqrt{-c} + (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}-i c^{1/4}} \left(1-i (-c)^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}-c^{1/4}} \left(1+(-c)^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{(-c)^{1/4}-c^{1/4}} \left(1+c^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}} \left(1+c^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+i c^{1/4}} \left(1-i (-c)^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} +
\end{aligned}$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1+c^{1/4} \sqrt{x})}{(-c)^{1/4}+c^{1/4}}\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) (1+c^{1/4} \sqrt{x})}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}$$

Result (type 8, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d x)^{5/2}}, x\right]$$

Problem 96: Result optimal but 1 more steps used.

$$\int (d x)^m (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTanh}[c x^2])}{d (1+m)} - \frac{2 b c (d x)^{3+m} \operatorname{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Result (type 5, 74 leaves, 3 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTanh}[c x^2])}{d (1+m)} - \frac{2 b c (d x)^{3+m} \operatorname{Hypergeometric2F1}\left[1, \frac{3+m}{4}, \frac{7+m}{4}, c^2 x^4\right]}{d^3 (1+m) (3+m)}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^{11} (a + b \operatorname{ArcTanh}[c x^3])^2 dx$$

Optimal (type 3, 125 leaves, 12 steps):

$$\frac{a b x^3}{6 c^3} + \frac{b^2 x^6}{36 c^2} + \frac{b^2 x^3 \operatorname{ArcTanh}[c x^3]}{6 c^3} + \frac{b x^9 (a + b \operatorname{ArcTanh}[c x^3])}{18 c} - \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{12 c^4} + \frac{1}{12} x^{12} (a + b \operatorname{ArcTanh}[c x^3])^2 + \frac{b^2 \operatorname{Log}[1 - c^2 x^6]}{9 c^4}$$

Result (type 4, 636 leaves, 62 steps):

$$\begin{aligned}
& \frac{a b x^3}{12 c^3} + \frac{23 b^2 x^3}{288 c^3} + \frac{b^2 x^6}{192 c^2} - \frac{7 b^2 x^9}{864 c} - \frac{b^2 x^{12}}{384} + \frac{b^2 (1 - c x^3)^2}{16 c^4} - \frac{b^2 (1 - c x^3)^3}{54 c^4} + \frac{b^2 (1 - c x^3)^4}{384 c^4} - \frac{5 b^2 \operatorname{Log}[1 - c x^3]}{288 c^4} + \\
& \frac{b^2 (1 - c x^3) \operatorname{Log}[1 - c x^3]}{24 c^4} + \frac{b^2 \operatorname{Log}[1 - c x^3]^2}{48 c^4} - \frac{b x^6 (2 a - b \operatorname{Log}[1 - c x^3])}{48 c^2} + \frac{b x^9 (2 a - b \operatorname{Log}[1 - c x^3])}{72 c} - \frac{1}{96} b x^{12} (2 a - b \operatorname{Log}[1 - c x^3]) + \\
& \frac{1}{48} x^{12} (2 a - b \operatorname{Log}[1 - c x^3])^2 - \frac{1}{288} b (2 a - b \operatorname{Log}[1 - c x^3]) \left( \frac{48 (1 - c x^3)}{c^4} - \frac{36 (1 - c x^3)^2}{c^4} + \frac{16 (1 - c x^3)^3}{c^4} - \frac{3 (1 - c x^3)^4}{c^4} - \frac{12 \operatorname{Log}[1 - c x^3]}{c^4} \right) - \\
& \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{24 c^4} + \frac{b^2 \operatorname{Log}[1 + c x^3]}{36 c^4} + \frac{b^2 x^9 \operatorname{Log}[1 + c x^3]}{36 c} + \frac{b^2 (1 + c x^3) \operatorname{Log}[1 + c x^3]}{12 c^4} + \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]}{24 c^4} + \\
& \frac{1}{24} b x^{12} (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3] - \frac{b^2 \operatorname{Log}[1 + c x^3]^2}{48 c^4} + \frac{1}{48} b^2 x^{12} \operatorname{Log}[1 + c x^3]^2 + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{24 c^4} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{24 c^4}
\end{aligned}$$

**Problem 117: Result valid but suboptimal antiderivative.**

$$\int x^8 (a + b \operatorname{ArcTanh}[c x^3])^2 dx$$

Optimal (type 4, 146 leaves, 10 steps):

$$\begin{aligned}
& \frac{b^2 x^3}{9 c^2} - \frac{b^2 \operatorname{ArcTanh}[c x^3]}{9 c^3} + \frac{b x^6 (a + b \operatorname{ArcTanh}[c x^3])}{9 c} + \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{9 c^3} + \\
& \frac{1}{9} x^9 (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{2 b (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}\left[\frac{2}{1 - c x^3}\right]}{9 c^3} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^3}\right]}{9 c^3}
\end{aligned}$$

Result (type 4, 536 leaves, 53 steps):

$$\begin{aligned}
& -\frac{a b x^3}{9 c^2} + \frac{19 b^2 x^3}{108 c^2} - \frac{5 b^2 x^6}{216 c} - \frac{b^2 x^9}{162} + \frac{b^2 (1 - c x^3)^2}{24 c^3} - \frac{b^2 (1 - c x^3)^3}{162 c^3} + \frac{b^2 \operatorname{Log}[1 - c x^3]}{108 c^3} - \frac{b^2 (1 - c x^3) \operatorname{Log}[1 - c x^3]}{18 c^3} + \\
& \frac{b^2 \operatorname{Log}[1 - c x^3]^2}{36 c^3} + \frac{b x^6 (2 a - b \operatorname{Log}[1 - c x^3])}{36 c} - \frac{1}{54} b x^9 (2 a - b \operatorname{Log}[1 - c x^3]) + \frac{1}{36} x^9 (2 a - b \operatorname{Log}[1 - c x^3])^2 - \\
& \frac{1}{108} b (2 a - b \operatorname{Log}[1 - c x^3]) \left( \frac{18 (1 - c x^3)}{c^3} - \frac{9 (1 - c x^3)^2}{c^3} + \frac{2 (1 - c x^3)^3}{c^3} - \frac{6 \operatorname{Log}[1 - c x^3]}{c^3} \right) + \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{18 c^3} - \\
& \frac{b^2 \operatorname{Log}[1 + c x^3]}{18 c^3} + \frac{b^2 x^6 \operatorname{Log}[1 + c x^3]}{18 c} + \frac{b^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]}{18 c^3} + \frac{1}{18} b x^9 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3] + \\
& \frac{b^2 \operatorname{Log}[1 + c x^3]^2}{36 c^3} + \frac{1}{36} b^2 x^9 \operatorname{Log}[1 + c x^3]^2 - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{18 c^3} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{18 c^3}
\end{aligned}$$

**Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^5 (a + b \operatorname{ArcTanh}[c x^3])^2 dx$$

Optimal (type 3, 91 leaves, 7 steps):

$$\frac{a b x^3}{3 c} + \frac{b^2 x^3 \operatorname{ArcTanh}[c x^3]}{3 c} - \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{6 c^2} + \frac{1}{6} x^6 (a + b \operatorname{ArcTanh}[c x^3])^2 + \frac{b^2 \operatorname{Log}[1 - c^2 x^6]}{6 c^2}$$

Result (type 4, 524 leaves, 44 steps):

$$\begin{aligned} & \frac{a b x^3}{2 c} - \frac{b^2 x^6}{24} + \frac{b^2 (1 - c x^3)^2}{48 c^2} + \frac{b^2 (1 + c x^3)^2}{48 c^2} - \frac{b^2 \operatorname{Log}[1 - c x^3]}{24 c^2} + \frac{b^2 (1 - c x^3) \operatorname{Log}[1 - c x^3]}{4 c^2} - \frac{1}{24} b x^6 (2 a - b \operatorname{Log}[1 - c x^3]) + \\ & \frac{b (1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])}{24 c^2} - \frac{(1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{12 c^2} + \frac{(1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])^2}{24 c^2} - \\ & \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[\frac{1}{2} (1 + c x^3)]}{12 c^2} - \frac{b^2 \operatorname{Log}[1 + c x^3]}{24 c^2} + \frac{1}{24} b^2 x^6 \operatorname{Log}[1 + c x^3] + \frac{b^2 (1 + c x^3) \operatorname{Log}[1 + c x^3]}{4 c^2} - \\ & \frac{b^2 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]}{24 c^2} + \frac{b^2 \operatorname{Log}[\frac{1}{2} (1 - c x^3)] \operatorname{Log}[1 + c x^3]}{12 c^2} + \frac{1}{12} b x^6 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3] - \\ & \frac{b^2 (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{12 c^2} + \frac{b^2 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]^2}{24 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - c x^3)]}{12 c^2} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + c x^3)]}{12 c^2} \end{aligned}$$

**Problem 119: Result valid but suboptimal antiderivative.**

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^3])^2 dx$$

Optimal (type 4, 96 leaves, 6 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{3 c} + \frac{1}{3} x^3 (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{2 b (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}[\frac{2}{1 - c x^3}]}{3 c} - \frac{b^2 \operatorname{PolyLog}[2, 1 - \frac{2}{1 - c x^3}]}{3 c}$$

Result (type 4, 207 leaves, 28 steps):

$$\begin{aligned} & - \frac{(1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{12 c} + \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[\frac{1}{2} (1 + c x^3)]}{6 c} + \frac{b^2 \operatorname{Log}[\frac{1}{2} (1 - c x^3)] \operatorname{Log}[1 + c x^3]}{6 c} + \\ & \frac{1}{6} b x^3 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3] + \frac{b^2 (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{12 c} - \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 - c x^3)]}{6 c} + \frac{b^2 \operatorname{PolyLog}[2, \frac{1}{2} (1 + c x^3)]}{6 c} \end{aligned}$$

### Problem 121: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{x^4} dx$$

Optimal (type 4, 90 leaves, 5 steps):

$$\frac{1}{3} c (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{3 x^3} + \frac{2}{3} b c (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 + c x^3}\right] - \frac{1}{3} b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^3}\right]$$

Result (type 4, 237 leaves, 24 steps):

$$\begin{aligned} & 2 a b c \operatorname{Log}[x] - \frac{(1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{12 x^3} - \frac{1}{6} b c (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right] - \\ & \frac{1}{6} b^2 c \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3] - \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]}{6 x^3} - \frac{b^2 (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{12 x^3} - \\ & \frac{1}{3} b^2 c \operatorname{PolyLog}[2, -c x^3] + \frac{1}{3} b^2 c \operatorname{PolyLog}[2, c x^3] + \frac{1}{6} b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right] - \frac{1}{6} b^2 c \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right] \end{aligned}$$

### Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{x^7} dx$$

Optimal (type 3, 88 leaves, 9 steps):

$$- \frac{b c (a + b \operatorname{ArcTanh}[c x^3])}{3 x^3} + \frac{1}{6} c^2 (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{6 x^6} + b^2 c^2 \operatorname{Log}[x] - \frac{1}{6} b^2 c^2 \operatorname{Log}[1 - c^2 x^6]$$

Result (type 4, 360 leaves, 46 steps):

$$\begin{aligned} & b^2 c^2 \operatorname{Log}[x] - \frac{1}{12} b^2 c^2 \operatorname{Log}[1 - c x^3] - \frac{b c (2 a - b \operatorname{Log}[1 - c x^3])}{12 x^3} - \frac{b c (1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])}{12 x^3} + \\ & \frac{1}{24} c^2 (2 a - b \operatorname{Log}[1 - c x^3])^2 - \frac{(2 a - b \operatorname{Log}[1 - c x^3])^2}{24 x^6} + \frac{1}{12} b c^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right] - \\ & \frac{1}{6} b^2 c^2 \operatorname{Log}[1 + c x^3] - \frac{b^2 c \operatorname{Log}[1 + c x^3]}{6 x^3} - \frac{1}{12} b^2 c^2 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3] - \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]}{12 x^6} + \\ & \frac{1}{24} b^2 c^2 \operatorname{Log}[1 + c x^3]^2 - \frac{b^2 \operatorname{Log}[1 + c x^3]^2}{24 x^6} - \frac{1}{12} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right] - \frac{1}{12} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right] \end{aligned}$$



### Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{x^{10}} dx$$

Optimal (type 4, 144 leaves, 9 steps):

$$-\frac{b^2 c^2}{9 x^3} + \frac{1}{9} b^2 c^3 \operatorname{ArcTanh}[c x^3] - \frac{b c (a + b \operatorname{ArcTanh}[c x^3])}{9 x^6} + \frac{1}{9} c^3 (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{9 x^9} + \frac{2}{9} b c^3 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 + c x^3}\right] - \frac{1}{9} b^2 c^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^3}\right]$$

Result (type 4, 420 leaves, 59 steps):

$$-\frac{b^2 c^2}{9 x^3} + \frac{2}{3} a b c^3 \operatorname{Log}[x] - \frac{b c (2 a - b \operatorname{Log}[1 - c x^3])}{18 x^6} + \frac{b c^2 (2 a - b \operatorname{Log}[1 - c x^3])}{18 x^3} - \frac{b c^2 (1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])}{18 x^3} + \frac{1}{36} c^3 (2 a - b \operatorname{Log}[1 - c x^3])^2 - \frac{(2 a - b \operatorname{Log}[1 - c x^3])^2}{36 x^9} - \frac{1}{18} b c^3 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right] + \frac{1}{18} b^2 c^3 \operatorname{Log}[1 + c x^3] - \frac{b^2 c \operatorname{Log}[1 + c x^3]}{18 x^6} - \frac{1}{18} b^2 c^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3] - \frac{b (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]}{18 x^9} - \frac{1}{36} b^2 c^3 \operatorname{Log}[1 + c x^3]^2 - \frac{b^2 \operatorname{Log}[1 + c x^3]^2}{36 x^9} - \frac{1}{9} b^2 c^3 \operatorname{PolyLog}[2, -c x^3] + \frac{1}{9} b^2 c^3 \operatorname{PolyLog}[2, c x^3] + \frac{1}{18} b^2 c^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right] - \frac{1}{18} b^2 c^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]$$

### Problem 124: Result valid but suboptimal antiderivative.

$$\int x^8 (a + b \operatorname{ArcTanh}[c x^3])^3 dx$$

Optimal (type 4, 231 leaves, 13 steps):

$$\frac{a b^2 x^3}{3 c^2} + \frac{b^3 x^3 \operatorname{ArcTanh}[c x^3]}{3 c^2} - \frac{b (a + b \operatorname{ArcTanh}[c x^3])^2}{6 c^3} + \frac{b x^6 (a + b \operatorname{ArcTanh}[c x^3])^2}{6 c} + \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{9 c^3} + \frac{1}{9} x^9 (a + b \operatorname{ArcTanh}[c x^3])^3 - \frac{b (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{Log}\left[\frac{2}{1 - c x^3}\right]}{3 c^3} + \frac{b^3 \operatorname{Log}\left[1 - c^2 x^6\right]}{6 c^3} - \frac{b^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^3}\right]}{3 c^3} + \frac{b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^3}\right]}{6 c^3}$$

Result (type 4, 1421 leaves, 239 steps):

$$\begin{aligned}
& \frac{2 a b^2 x^3}{3 c^2} - \frac{7 b^3 x^3}{216 c^2} - \frac{23 b^3 x^6}{432 c} + \frac{b^3 x^9}{324} + \frac{b^3 (1 - c x^3)^2}{48 c^3} + \frac{b^3 (1 + c x^3)^2}{24 c^3} - \frac{b^3 (1 + c x^3)^3}{324 c^3} - \frac{b^3 \operatorname{Log}[1 - c x^3]}{24 c^3} + \frac{b^3 (1 - c x^3) \operatorname{Log}[1 - c x^3]}{3 c^3} - \\
& \frac{b^3 \operatorname{Log}[1 - c x^3]^2}{72 c^3} - \frac{b^2 x^6 (2 a - b \operatorname{Log}[1 - c x^3])}{24 c} + \frac{b^2 (1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])}{12 c^3} - \frac{b^2 (1 - c x^3)^3 (2 a - b \operatorname{Log}[1 - c x^3])}{108 c^3} - \\
& \frac{1}{72} b x^9 (2 a - b \operatorname{Log}[1 - c x^3])^2 - \frac{b (1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{8 c^3} + \frac{b (1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])^2}{12 c^3} - \frac{b (1 - c x^3)^3 (2 a - b \operatorname{Log}[1 - c x^3])^2}{72 c^3} - \\
& \frac{(1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^3}{24 c^3} + \frac{(1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])^3}{24 c^3} - \frac{(1 - c x^3)^3 (2 a - b \operatorname{Log}[1 - c x^3])^3}{72 c^3} + \\
& \frac{1}{216} b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \left( \frac{18 (1 - c x^3)}{c^3} - \frac{9 (1 - c x^3)^2}{c^3} + \frac{2 (1 - c x^3)^3}{c^3} - \frac{6 \operatorname{Log}[1 - c x^3]}{c^3} \right) - \frac{b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{12 c^3} + \\
& \frac{b (2 a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{12 c^3} - \frac{7 b^3 \operatorname{Log}[1 + c x^3]}{108 c^3} + \frac{b^3 x^6 \operatorname{Log}[1 + c x^3]}{18 c} - \frac{1}{108} b^3 x^9 \operatorname{Log}[1 + c x^3] + \frac{11 b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]}{36 c^3} - \\
& \frac{b^3 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]}{12 c^3} + \frac{b^3 (1 + c x^3)^3 \operatorname{Log}[1 + c x^3]}{108 c^3} + \frac{b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]}{12 c^3} + \frac{b^2 x^6 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]}{12 c} - \\
& \frac{b (2 a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3]}{24 c^3} + \frac{1}{24} b x^9 (2 a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3] + \frac{b^3 \operatorname{Log}[1 + c x^3]^2}{72 c^3} + \frac{1}{72} b^3 x^9 \operatorname{Log}[1 + c x^3]^2 - \\
& \frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{8 c^3} + \frac{b^3 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]^2}{12 c^3} - \frac{b^3 (1 + c x^3)^3 \operatorname{Log}[1 + c x^3]^2}{72 c^3} + \frac{b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]^2}{12 c^3} + \\
& \frac{b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2}{24 c^3} + \frac{1}{24} b^2 x^9 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2 + \frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^3}{24 c^3} - \\
& \frac{b^3 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]^3}{24 c^3} + \frac{b^3 (1 + c x^3)^3 \operatorname{Log}[1 + c x^3]^3}{72 c^3} + \frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{12 c^3} - \frac{b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{6 c^3} + \\
& \frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{12 c^3} + \frac{b^3 \operatorname{Log}[1 + c x^3] \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{6 c^3} - \frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 - c x^3)\right]}{6 c^3} - \frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 + c x^3)\right]}{6 c^3}
\end{aligned}$$

Problem 125: Result valid but suboptimal antiderivative.

$$\int x^5 (a + b \operatorname{ArcTanh}[c x^3])^3 dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$\frac{b (a + b \operatorname{ArcTanh}[c x^3])^2}{2 c^2} + \frac{b x^3 (a + b \operatorname{ArcTanh}[c x^3])^2}{2 c} - \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{6 c^2} +$$

$$\frac{1}{6} x^6 (a + b \operatorname{ArcTanh}[c x^3])^3 - \frac{b^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}\left[\frac{2}{1-c x^3}\right]}{c^2} - \frac{b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x^3}\right]}{2 c^2}$$

Result (type 4, 479 leaves, 155 steps):

$$-\frac{b (1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{8 c^2} - \frac{(1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^3}{24 c^2} +$$

$$\frac{(1 - c x^3)^2 (2 a - b \operatorname{Log}[1 - c x^3])^3}{48 c^2} + \frac{b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{4 c^2} + \frac{b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]}{4 c^2} +$$

$$\frac{b^2 x^3 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]}{4 c} - \frac{b (2 a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3]}{16 c^2} + \frac{1}{16} b x^6 (2 a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3] +$$

$$\frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{8 c^2} - \frac{b^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2}{16 c^2} + \frac{1}{16} b^2 x^6 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2 -$$

$$\frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^3}{24 c^2} + \frac{b^3 (1 + c x^3)^2 \operatorname{Log}[1 + c x^3]^3}{48 c^2} - \frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{4 c^2} + \frac{b^3 \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{4 c^2}$$

Problem 126: Result valid but suboptimal antiderivative.

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^3])^3 dx$$

Optimal (type 4, 130 leaves, 6 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{3 c} + \frac{1}{3} x^3 (a + b \operatorname{ArcTanh}[c x^3])^3 -$$

$$\frac{b (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{Log}\left[\frac{2}{1-c x^3}\right]}{c} - \frac{b^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x^3}\right]}{c} + \frac{b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x^3}\right]}{2 c}$$

Result (type 4, 390 leaves, 82 steps):

$$\begin{aligned}
& - \frac{(1 - c x^3) (2a - b \operatorname{Log}[1 - c x^3])^3}{24 c} + \frac{b (2a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}\left[\frac{1}{2} (1 + c x^3)\right]}{4 c} - \frac{b (2a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3]}{8 c} + \\
& \frac{1}{8} b x^3 (2a - b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3] + \frac{b^3 \operatorname{Log}\left[\frac{1}{2} (1 - c x^3)\right] \operatorname{Log}[1 + c x^3]^2}{4 c} + \frac{b^2 (2a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2}{8 c} + \\
& \frac{1}{8} b^2 x^3 (2a - b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2 + \frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^3}{24 c} - \frac{b^2 (2a - b \operatorname{Log}[1 - c x^3]) \operatorname{PolyLog}\left[2, \frac{1}{2} (1 - c x^3)\right]}{2 c} + \\
& \frac{b^3 \operatorname{Log}[1 + c x^3] \operatorname{PolyLog}\left[2, \frac{1}{2} (1 + c x^3)\right]}{2 c} - \frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 - c x^3)\right]}{2 c} - \frac{b^3 \operatorname{PolyLog}\left[3, \frac{1}{2} (1 + c x^3)\right]}{2 c}
\end{aligned}$$

Problem 128: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{x^4} dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{3} c (a + b \operatorname{ArcTanh}[c x^3])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{3 x^3} + b c (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x^3}\right] - \\
& b^2 c (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^3}\right] - \frac{1}{2} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x^3}\right]
\end{aligned}$$

Result (type 8, 284 leaves, 16 steps):

$$\begin{aligned}
& \frac{1}{8} b c \operatorname{Log}[c x^3] (2a - b \operatorname{Log}[1 - c x^3])^2 - \frac{(1 - c x^3) (2a - b \operatorname{Log}[1 - c x^3])^3}{24 x^3} + \\
& \frac{1}{8} b^3 c \operatorname{Log}[-c x^3] \operatorname{Log}[1 + c x^3]^2 - \frac{b^3 (1 + c x^3) \operatorname{Log}[1 + c x^3]^3}{24 x^3} - \frac{1}{4} b^2 c (2a - b \operatorname{Log}[1 - c x^3]) \operatorname{PolyLog}[2, 1 - c x^3] + \\
& \frac{1}{4} b^3 c \operatorname{Log}[1 + c x^3] \operatorname{PolyLog}[2, 1 + c x^3] - \frac{1}{4} b^3 c \operatorname{PolyLog}[3, 1 - c x^3] - \frac{1}{4} b^3 c \operatorname{PolyLog}[3, 1 + c x^3] + \\
& \frac{3}{8} b \operatorname{Unintegrable}\left[\frac{(-2a + b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3]}{x^4}, x\right] - \frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{(-2a + b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2}{x^4}, x\right]
\end{aligned}$$

Problem 129: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{x^7} dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$\frac{1}{2} b c^2 (a + b \operatorname{ArcTanh}[c x^3])^2 - \frac{b c (a + b \operatorname{ArcTanh}[c x^3])^2}{2 x^3} + \frac{1}{6} c^2 (a + b \operatorname{ArcTanh}[c x^3])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{6 x^6} + b^2 c^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}\left[2 - \frac{2}{1 + c x^3}\right] - \frac{1}{2} b^3 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^3}\right]$$

Result (type 8, 437 leaves, 29 steps):

$$\begin{aligned} & \frac{3}{4} a b^2 c^2 \operatorname{Log}[x] - \frac{b c (1 - c x^3) (2 a - b \operatorname{Log}[1 - c x^3])^2}{16 x^3} + \frac{1}{16} b c^2 \operatorname{Log}[c x^3] (2 a - b \operatorname{Log}[1 - c x^3])^2 + \frac{1}{48} c^2 (2 a - b \operatorname{Log}[1 - c x^3])^3 - \\ & \frac{(2 a - b \operatorname{Log}[1 - c x^3])^3}{48 x^6} - \frac{b^3 c (1 + c x^3) \operatorname{Log}[1 + c x^3]^2}{16 x^3} - \frac{1}{16} b^3 c^2 \operatorname{Log}[-c x^3] \operatorname{Log}[1 + c x^3]^2 + \frac{1}{48} b^3 c^2 \operatorname{Log}[1 + c x^3]^3 - \\ & \frac{b^3 \operatorname{Log}[1 + c x^3]^3}{48 x^6} - \frac{1}{8} b^3 c^2 \operatorname{PolyLog}[2, -c x^3] + \frac{1}{8} b^3 c^2 \operatorname{PolyLog}[2, c x^3] - \frac{1}{8} b^2 c^2 (2 a - b \operatorname{Log}[1 - c x^3]) \operatorname{PolyLog}[2, 1 - c x^3] - \\ & \frac{1}{8} b^3 c^2 \operatorname{Log}[1 + c x^3] \operatorname{PolyLog}[2, 1 + c x^3] - \frac{1}{8} b^3 c^2 \operatorname{PolyLog}[3, 1 - c x^3] + \frac{1}{8} b^3 c^2 \operatorname{PolyLog}[3, 1 + c x^3] + \\ & \frac{3}{8} b \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^3])^2 \operatorname{Log}[1 + c x^3]}{x^7}, x\right] - \frac{3}{8} b^2 \operatorname{Unintegrable}\left[\frac{(-2 a + b \operatorname{Log}[1 - c x^3]) \operatorname{Log}[1 + c x^3]^2}{x^7}, x\right] \end{aligned}$$

Problem 132: Result optimal but 1 more steps used.

$$\int (d x)^m (a + b \operatorname{ArcTanh}[c x^3]) dx$$

Optimal (type 5, 74 leaves, 2 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTanh}[c x^3])}{d (1+m)} - \frac{3 b c (d x)^{4+m} \operatorname{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, c^2 x^6\right]}{d^4 (1+m) (4+m)}$$

Result (type 5, 74 leaves, 3 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcTanh}[c x^3])}{d (1+m)} - \frac{3 b c (d x)^{4+m} \operatorname{Hypergeometric2F1}\left[1, \frac{4+m}{6}, \frac{10+m}{6}, c^2 x^6\right]}{d^4 (1+m) (4+m)}$$

Problem 143: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^2 dx$$

Optimal (type 3, 123 leaves, 14 steps):

$$\frac{1}{12} b^2 c^2 x^2 + \frac{1}{2} b c^3 x \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) + \frac{1}{6} b c x^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) -$$

$$\frac{1}{4} c^4 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + \frac{1}{4} x^4 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + \frac{1}{3} b^2 c^4 \operatorname{Log} \left[ 1 - \frac{c^2}{x^2} \right] + \frac{2}{3} b^2 c^4 \operatorname{Log} [x]$$

Result (type 4, 812 leaves, 88 steps):

$$\begin{aligned} & \frac{1}{4} a b c^3 x - \frac{1}{8} a b c^2 x^2 + \frac{1}{12} b^2 c^2 x^2 + \frac{1}{12} a b c x^3 + \frac{5}{48} b^2 c^4 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] - \frac{1}{8} b^2 c^3 x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] + \frac{1}{16} b^2 c^2 x^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] - \\ & \frac{1}{24} b^2 c x^3 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] + \frac{1}{8} b c^3 \left( 1 - \frac{c}{x} \right) x \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) + \frac{1}{16} b c^2 x^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) + \frac{1}{24} b c x^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) - \\ & \frac{1}{16} c^4 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 + \frac{1}{16} x^4 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 + \frac{1}{8} b^2 c^3 x \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{1}{16} b^2 c^2 x^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{1}{24} b^2 c x^3 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \\ & \frac{1}{4} a b x^4 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] - \frac{1}{8} b^2 x^4 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{5}{48} b^2 c^4 \operatorname{Log} [c - x] + \frac{1}{8} b^2 c^4 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] \operatorname{Log} [c - x] + \frac{1}{4} a b c^4 \operatorname{Log} [x] + \\ & \frac{11}{24} b^2 c^4 \operatorname{Log} [x] + \frac{1}{8} b^2 c^4 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{x}{c} \right] - \frac{1}{4} a b c^4 \operatorname{Log} [c + x] + \frac{5}{48} b^2 c^4 \operatorname{Log} [c + x] + \frac{1}{8} b^2 c^4 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} [c + x] - \\ & \frac{1}{8} b^2 c^4 \operatorname{Log} \left[ \frac{c - x}{2c} \right] \operatorname{Log} [c + x] + \frac{1}{8} b^2 c^4 \operatorname{Log} \left[ -\frac{x}{c} \right] \operatorname{Log} [c + x] - \frac{1}{8} b^2 c^4 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{c + x}{2c} \right] + \frac{11}{48} b^2 c^4 \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{1}{8} b^2 c^3 x \operatorname{Log} \left[ \frac{c + x}{x} \right] - \\ & \frac{1}{16} b^2 c^2 x^2 \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{1}{24} b^2 c x^3 \operatorname{Log} \left[ \frac{c + x}{x} \right] - \frac{1}{16} b^2 c^4 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{1}{16} b^2 x^4 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 - \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, \frac{c - x}{2c} \right] - \\ & \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, \frac{c}{x} \right] - \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, \frac{c + x}{2c} \right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, 1 - \frac{x}{c} \right] + \frac{1}{8} b^2 c^4 \operatorname{PolyLog} \left[ 2, 1 + \frac{x}{c} \right] \end{aligned}$$

Problem 144: Result valid but suboptimal antiderivative.

$$\int x^2 \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x} \right] \right)^2 dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\begin{aligned} & \frac{1}{3} b^2 c^2 x - \frac{1}{3} b^2 c^3 \operatorname{ArcCoth} \left[ \frac{x}{c} \right] + \frac{1}{3} b c x^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) - \frac{1}{3} c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + \\ & \frac{1}{3} x^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 - \frac{2}{3} b c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ 2 - \frac{2}{1 + \frac{c}{x}} \right] + \frac{1}{3} b^2 c^3 \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 + \frac{c}{x}} \right] \end{aligned}$$

Result (type 4, 695 leaves, 73 steps):

$$\begin{aligned}
& -\frac{1}{3} a b c^2 x + \frac{1}{3} b^2 c^2 x + \frac{1}{6} a b c x^2 + \frac{1}{12} b^2 c^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{6} b^2 c^2 x \operatorname{Log}\left[1 - \frac{c}{x}\right] - \frac{1}{12} b^2 c x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] + \\
& \frac{1}{6} b c^2 \left(1 - \frac{c}{x}\right) x \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{12} b c x^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{1}{12} c^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{12} x^3 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \\
& \frac{1}{6} b^2 c^2 x \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{12} b^2 c x^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{3} a b x^3 \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{6} b^2 x^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{12} b^2 c^3 \operatorname{Log}[c - x] + \\
& \frac{1}{6} b^2 c^3 \operatorname{Log}\left[1 + \frac{c}{x}\right] \operatorname{Log}[c - x] + \frac{1}{3} a b c^3 \operatorname{Log}[x] + \frac{1}{6} b^2 c^3 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{x}{c}\right] + \frac{1}{3} a b c^3 \operatorname{Log}[c + x] + \frac{1}{12} b^2 c^3 \operatorname{Log}[c + x] - \\
& \frac{1}{6} b^2 c^3 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}[c + x] + \frac{1}{6} b^2 c^3 \operatorname{Log}\left[\frac{c - x}{2c}\right] \operatorname{Log}[c + x] - \frac{1}{6} b^2 c^3 \operatorname{Log}\left[-\frac{x}{c}\right] \operatorname{Log}[c + x] - \frac{1}{6} b^2 c^3 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{c + x}{2c}\right] - \\
& \frac{1}{4} b^2 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right] - \frac{1}{6} b^2 c^2 x \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{12} b^2 c x^2 \operatorname{Log}\left[\frac{c + x}{x}\right] + \frac{1}{12} b^2 c^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{12} b^2 x^3 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c - x}{2c}\right] + \\
& \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c}{x}\right] + \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, \frac{c + x}{2c}\right] + \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right] - \frac{1}{6} b^2 c^3 \operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right]
\end{aligned}$$

**Problem 145: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x \left( a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right] \right)^2 dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$b c x \left( a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right) - \frac{1}{2} c^2 \left( a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right)^2 + \frac{1}{2} x^2 \left( a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right)^2 + \frac{1}{2} b^2 c^2 \operatorname{Log}\left[1 - \frac{c^2}{x^2}\right] + b^2 c^2 \operatorname{Log}[x]$$

Result (type 4, 574 leaves, 58 steps):

$$\begin{aligned}
& \frac{1}{2} a b c x - \frac{1}{4} b^2 c x \operatorname{Log}\left[1 - \frac{c}{x}\right] + \frac{1}{4} b c \left(1 - \frac{c}{x}\right) x \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{1}{8} c^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{8} x^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \\
& \frac{1}{4} b^2 c x \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{2} a b x^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] - \frac{1}{4} b^2 x^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{4} b^2 c^2 \operatorname{Log}[c - x] + \frac{1}{4} b^2 c^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] \operatorname{Log}[c - x] + \\
& \frac{1}{2} a b c^2 \operatorname{Log}[x] + \frac{1}{2} b^2 c^2 \operatorname{Log}[x] + \frac{1}{4} b^2 c^2 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{x}{c}\right] - \frac{1}{2} a b c^2 \operatorname{Log}[c + x] + \frac{1}{4} b^2 c^2 \operatorname{Log}[c + x] + \frac{1}{4} b^2 c^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}[c + x] - \\
& \frac{1}{4} b^2 c^2 \operatorname{Log}\left[\frac{c - x}{2c}\right] \operatorname{Log}[c + x] + \frac{1}{4} b^2 c^2 \operatorname{Log}\left[-\frac{x}{c}\right] \operatorname{Log}[c + x] - \frac{1}{4} b^2 c^2 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{c + x}{2c}\right] + \frac{1}{4} b^2 c^2 \operatorname{Log}\left[\frac{c + x}{x}\right] + \\
& \frac{1}{4} b^2 c x \operatorname{Log}\left[\frac{c + x}{x}\right] - \frac{1}{8} b^2 c^2 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 + \frac{1}{8} b^2 x^2 \operatorname{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c - x}{2c}\right] - \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \\
& \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c}{x}\right] - \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, \frac{c + x}{2c}\right] + \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right] + \frac{1}{4} b^2 c^2 \operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right]
\end{aligned}$$

## Problem 146: Result valid but suboptimal antiderivative.

$$\int \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x} \right] \right)^2 dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$c \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + x \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 - 2 b c \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ \frac{2 c}{c - x} \right] - b^2 c \operatorname{PolyLog} \left[ 2, -\frac{c+x}{c-x} \right]$$

Result (type 4, 370 leaves, 31 steps):

$$\begin{aligned} & a^2 x - a b x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] - \frac{1}{4} b^2 (c - x) \operatorname{Log} \left[ 1 - \frac{c}{x} \right]^2 + a b x \operatorname{Log} \left[ 1 + \frac{c}{x} \right] - \frac{1}{2} b^2 x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{1}{4} b^2 (c + x) \operatorname{Log} \left[ 1 + \frac{c}{x} \right]^2 - \\ & \frac{1}{2} b^2 c \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} [-c - x] + a b c \operatorname{Log} [c - x] + \frac{1}{2} b^2 c \operatorname{Log} [-c - x] \operatorname{Log} \left[ \frac{c - x}{2 c} \right] - \frac{1}{2} b^2 c \operatorname{Log} [-c - x] \operatorname{Log} \left[ -\frac{x}{c} \right] + \\ & \frac{1}{2} b^2 c \operatorname{Log} \left[ 1 + \frac{c}{x} \right] \operatorname{Log} [-c + x] + \frac{1}{2} b^2 c \operatorname{Log} \left[ \frac{x}{c} \right] \operatorname{Log} [-c + x] + a b c \operatorname{Log} [c + x] - \frac{1}{2} b^2 c \operatorname{Log} [-c + x] \operatorname{Log} \left[ \frac{c+x}{2 c} \right] - \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, \frac{c-x}{2 c} \right] + \\ & \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] - \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, \frac{c}{x} \right] + \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, \frac{c+x}{2 c} \right] + \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, 1 - \frac{x}{c} \right] - \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, 1 + \frac{x}{c} \right] \end{aligned}$$

## Problem 148: Result valid but suboptimal antiderivative.

$$\int \frac{\left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x} \right] \right)^2}{x^2} dx$$

Optimal (type 4, 87 leaves, 6 steps):

$$-\frac{\left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2}{c} - \frac{\left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2}{x} + \frac{2 b \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ \frac{2}{1 - \frac{c}{x}} \right]}{c} + \frac{b^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{2}{1 - \frac{c}{x}} \right]}{c}$$

Result (type 4, 205 leaves, 28 steps):

$$\begin{aligned} & \frac{\left( 1 - \frac{c}{x} \right) \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2}{4 c} - \frac{b \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) \operatorname{Log} \left[ \frac{c+x}{2 x} \right]}{2 c} - \frac{b \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) \operatorname{Log} \left[ \frac{c+x}{x} \right]}{2 x} - \\ & \frac{b^2 \operatorname{Log} \left[ -\frac{c-x}{2 x} \right] \operatorname{Log} \left[ \frac{c+x}{x} \right]}{2 c} - \frac{b^2 \left( 1 + \frac{c}{x} \right) \operatorname{Log} \left[ \frac{c+x}{x} \right]^2}{4 c} + \frac{b^2 \operatorname{PolyLog} \left[ 2, -\frac{c-x}{2 x} \right]}{2 c} - \frac{b^2 \operatorname{PolyLog} \left[ 2, \frac{c+x}{2 x} \right]}{2 c} \end{aligned}$$



**Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x^3} dx$$

Optimal (type 3, 87 leaves, 7 steps):

$$-\frac{a b}{c x} - \frac{b^2 \operatorname{ArcCoth}\left[\frac{x}{c}\right]}{c x} + \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2 c^2} - \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2 x^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c^2}{x^2}\right]}{2 c^2}$$

Result (type 4, 707 leaves, 66 steps):

$$\begin{aligned} & -\frac{b^2 \left(1 - \frac{c}{x}\right)^2}{16 c^2} - \frac{b^2 \left(1 + \frac{c}{x}\right)^2}{16 c^2} + \frac{a b}{4 x^2} + \frac{b^2}{8 x^2} - \frac{3 a b}{2 c x} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x}\right]}{8 c^2} - \frac{3 b^2 \left(1 - \frac{c}{x}\right) \operatorname{Log}\left[1 - \frac{c}{x}\right]}{4 c^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x}\right]}{8 x^2} - \frac{b \left(1 - \frac{c}{x}\right)^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)}{8 c^2} + \\ & \frac{\left(1 - \frac{c}{x}\right) \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2}{4 c^2} - \frac{\left(1 - \frac{c}{x}\right)^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2}{8 c^2} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}\left[1 + \frac{c}{x}\right]}{4 x^2} - \frac{b^2 \operatorname{Log}\left[1 + \frac{c}{x}\right] \operatorname{Log}[c - x]}{4 c^2} - \frac{b^2 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{x}{c}\right]}{4 c^2} - \\ & \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x}\right] \operatorname{Log}[c + x]}{4 c^2} + \frac{b^2 \operatorname{Log}\left[\frac{c-x}{2c}\right] \operatorname{Log}[c + x]}{4 c^2} - \frac{b^2 \operatorname{Log}\left[-\frac{x}{c}\right] \operatorname{Log}[c + x]}{4 c^2} + \frac{b^2 \operatorname{Log}[c - x] \operatorname{Log}\left[\frac{c+x}{2c}\right]}{4 c^2} + \frac{a b \operatorname{Log}\left[\frac{c+x}{x}\right]}{2 c^2} + \frac{b^2 \operatorname{Log}\left[\frac{c+x}{x}\right]}{8 c^2} - \\ & \frac{3 b^2 \left(1 + \frac{c}{x}\right) \operatorname{Log}\left[\frac{c+x}{x}\right]}{4 c^2} + \frac{b^2 \left(1 + \frac{c}{x}\right)^2 \operatorname{Log}\left[\frac{c+x}{x}\right]}{8 c^2} - \frac{a b \operatorname{Log}\left[\frac{c+x}{x}\right]}{2 x^2} - \frac{b^2 \operatorname{Log}\left[\frac{c+x}{x}\right]}{8 x^2} + \frac{b^2 \left(1 + \frac{c}{x}\right) \operatorname{Log}\left[\frac{c+x}{x}\right]}{4 c^2} - \frac{b^2 \left(1 + \frac{c}{x}\right)^2 \operatorname{Log}\left[\frac{c+x}{x}\right]}{8 c^2} + \\ & \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c-x}{2c}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{c}{x}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c}{x}\right]}{4 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c+x}{2c}\right]}{4 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right]}{4 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right]}{4 c^2} \end{aligned}$$

**Problem 150: Unable to integrate problem.**

$$\int x^3 \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 203 leaves, 17 steps):

$$\begin{aligned} & \frac{1}{4} b^3 c^3 x - \frac{1}{4} b^3 c^4 \operatorname{ArcCoth}\left[\frac{x}{c}\right] + \frac{1}{4} b^2 c^2 x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) - b c^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \\ & \frac{3}{4} b c^3 x \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{4} b c x^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 - \frac{1}{4} c^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 + \\ & \frac{1}{4} x^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 - 2 b^2 c^4 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 + \frac{c}{x}}\right] + b^3 c^4 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x}}\right] \end{aligned}$$

Result (type 8, 1398 leaves, 139 steps):

$$\begin{aligned}
& \frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{c}{x}\right]^2 \text{Log}\left[1 + \frac{c}{x}\right], x\right] - \\
& \frac{3}{8} b^3 \text{CannotIntegrate}\left[x^3 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right]^2, x\right] + \frac{1}{32} b^3 c^4 \text{Log}\left[1 - \frac{c}{x}\right] - \frac{3}{8} a b^2 c^3 x \text{Log}\left[1 - \frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1 - \frac{c}{x}\right] - \\
& \frac{1}{8} a b^2 c x^3 \text{Log}\left[1 - \frac{c}{x}\right] + \frac{5}{32} b^2 c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) + \frac{1}{32} b^2 c^2 x^2 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) - \frac{5}{64} b c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \\
& \frac{3}{32} b c^3 \left(1 - \frac{c}{x}\right) x \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{3}{64} b c^2 x^2 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 + \frac{1}{32} b c x^3 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 - \frac{1}{32} c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^3 + \\
& \frac{1}{32} x^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^3 + \frac{3}{8} a b^2 c^3 x \text{Log}\left[1 + \frac{c}{x}\right] + \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[1 + \frac{c}{x}\right] + \frac{1}{8} a b^2 c x^3 \text{Log}\left[1 + \frac{c}{x}\right] + \frac{3}{8} a^2 b x^4 \text{Log}\left[1 + \frac{c}{x}\right] - \\
& \frac{3}{8} a b^2 x^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right] + \frac{5}{16} a b^2 c^4 \text{Log}[c - x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[1 + \frac{c}{x}\right] \text{Log}[c - x] - \frac{3}{32} b c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c}{x}\right] + \\
& \frac{11}{8} a b^2 c^4 \text{Log}[x] + \frac{3}{8} a b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{x}{c}\right] - \frac{3}{8} a^2 b c^4 \text{Log}[c + x] + \frac{5}{16} a b^2 c^4 \text{Log}[c + x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}[c + x] - \\
& \frac{3}{8} a b^2 c^4 \text{Log}\left[\frac{c - x}{2 c}\right] \text{Log}[c + x] + \frac{3}{8} a b^2 c^4 \text{Log}\left[-\frac{x}{c}\right] \text{Log}[c + x] - \frac{3}{8} a b^2 c^4 \text{Log}[c - x] \text{Log}\left[\frac{c + x}{2 c}\right] + \frac{11}{16} a b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right] - \\
& \frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right] + \frac{3}{8} a b^2 c^3 x \text{Log}\left[\frac{c + x}{x}\right] - \frac{5}{32} b^3 c^3 \left(1 + \frac{c}{x}\right) x \text{Log}\left[\frac{c + x}{x}\right] - \frac{3}{16} a b^2 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right] + \frac{1}{32} b^3 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right] + \\
& \frac{1}{8} a b^2 c x^3 \text{Log}\left[\frac{c + x}{x}\right] - \frac{3}{16} a b^2 c^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{5}{64} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{3}{32} b^3 c^3 \left(1 + \frac{c}{x}\right) x \text{Log}\left[\frac{c + x}{x}\right]^2 - \frac{3}{64} b^3 c^2 x^2 \text{Log}\left[\frac{c + x}{x}\right]^2 + \\
& \frac{1}{32} b^3 c x^3 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{3}{16} a b^2 x^4 \text{Log}\left[\frac{c + x}{x}\right]^2 + \frac{3}{32} b^3 c^4 \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c + x}{x}\right]^2 - \frac{1}{32} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right]^3 + \frac{1}{32} b^3 x^4 \text{Log}\left[\frac{c + x}{x}\right]^3 + \\
& \frac{3}{16} b^2 c^4 \left(2 a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) \text{PolyLog}\left[2, 1 - \frac{c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c - x}{2 c}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] + \\
& \frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{11}{32} b^3 c^4 \text{PolyLog}\left[2, \frac{c}{x}\right] - \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, \frac{c + x}{2 c}\right] + \frac{3}{16} b^3 c^4 \text{Log}\left[\frac{c + x}{x}\right] \text{PolyLog}\left[2, \frac{c + x}{x}\right] + \\
& \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 - \frac{x}{c}\right] + \frac{3}{8} a b^2 c^4 \text{PolyLog}\left[2, 1 + \frac{x}{c}\right] + \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, 1 - \frac{c}{x}\right] - \frac{3}{16} b^3 c^4 \text{PolyLog}\left[3, \frac{c + x}{x}\right]
\end{aligned}$$

Problem 151: Unable to integrate problem.

$$\int x^2 \left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 217 leaves, 15 steps):

$$\begin{aligned}
& b^2 c^2 x \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) - \frac{1}{2} b c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + \frac{1}{2} b c x^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 - \\
& \frac{1}{3} c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 + \frac{1}{3} x^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 - b c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 \operatorname{Log} \left[ 2 - \frac{2}{1 + \frac{c}{x}} \right] + \\
& \frac{1}{2} b^3 c^3 \operatorname{Log} \left[ 1 - \frac{c^2}{x^2} \right] + b^3 c^3 \operatorname{Log} [x] + b^2 c^3 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 + \frac{c}{x}} \right] + \frac{1}{2} b^3 c^3 \operatorname{PolyLog} \left[ 3, -1 + \frac{2}{1 + \frac{c}{x}} \right]
\end{aligned}$$

Result (type 8, 1152 leaves, 103 steps):

$$\begin{aligned}
& -\frac{1}{2} a^2 b c^2 x + \frac{3}{4} a b^2 c^2 x + \frac{1}{4} a^2 b c x^2 + \frac{3}{8} b^3 \operatorname{CannotIntegrate} \left[ x^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right]^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right], x \right] - \\
& \frac{3}{8} b^3 \operatorname{CannotIntegrate} \left[ x^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right]^2, x \right] + \frac{1}{2} a b^2 c^2 x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] - \frac{1}{4} a b^2 c x^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] + \\
& \frac{1}{8} b^2 c^2 \left( 1 - \frac{c}{x} \right) x \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) - \frac{1}{16} b c^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 + \frac{1}{8} b c^2 \left( 1 - \frac{c}{x} \right) x \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 + \frac{1}{16} b c x^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 - \\
& \frac{1}{24} c^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^3 + \frac{1}{24} x^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^3 + \frac{1}{2} a b^2 c^2 x \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{1}{4} a b^2 c x^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{1}{2} a^2 b x^3 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] - \\
& \frac{1}{2} a b^2 x^3 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right] - \frac{1}{4} a b^2 c^3 \operatorname{Log} [c - x] + \frac{1}{2} a b^2 c^3 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] \operatorname{Log} [c - x] - \frac{1}{8} b c^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 \operatorname{Log} \left[ \frac{c}{x} \right] + \\
& \frac{1}{4} b^3 c^3 \operatorname{Log} [x] + \frac{1}{2} a b^2 c^3 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{x}{c} \right] + \frac{1}{2} a^2 b c^3 \operatorname{Log} [c + x] + \frac{1}{4} a b^2 c^3 \operatorname{Log} [c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} [c + x] + \\
& \frac{1}{2} a b^2 c^3 \operatorname{Log} \left[ \frac{c - x}{2 c} \right] \operatorname{Log} [c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log} \left[ -\frac{x}{c} \right] \operatorname{Log} [c + x] - \frac{1}{2} a b^2 c^3 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{c + x}{2 c} \right] - \frac{3}{4} a b^2 c^3 \operatorname{Log} \left[ \frac{c + x}{x} \right] - \\
& \frac{1}{2} a b^2 c^2 x \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{1}{8} b^3 c^2 \left( 1 + \frac{c}{x} \right) x \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{1}{4} a b^2 c x^2 \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{1}{4} a b^2 c^3 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 - \frac{1}{16} b^3 c^3 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 - \\
& \frac{1}{8} b^3 c^2 \left( 1 + \frac{c}{x} \right) x \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{1}{16} b^3 c x^2 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{1}{4} a b^2 x^3 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 - \frac{1}{8} b^3 c^3 \operatorname{Log} \left[ -\frac{c}{x} \right] \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{1}{24} b^3 c^3 \operatorname{Log} \left[ \frac{c + x}{x} \right]^3 + \\
& \frac{1}{24} b^3 x^3 \operatorname{Log} \left[ \frac{c + x}{x} \right]^3 + \frac{1}{4} b^2 c^3 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{c}{x} \right] - \frac{1}{2} a b^2 c^3 \operatorname{PolyLog} \left[ 2, \frac{c - x}{2 c} \right] + \frac{1}{2} a b^2 c^3 \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] - \\
& \frac{3}{8} b^3 c^3 \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] - \frac{3}{8} b^3 c^3 \operatorname{PolyLog} \left[ 2, \frac{c}{x} \right] + \frac{1}{2} a b^2 c^3 \operatorname{PolyLog} \left[ 2, \frac{c + x}{2 c} \right] - \frac{1}{4} b^3 c^3 \operatorname{Log} \left[ \frac{c + x}{x} \right] \operatorname{PolyLog} \left[ 2, \frac{c + x}{x} \right] + \\
& \frac{1}{2} a b^2 c^3 \operatorname{PolyLog} \left[ 2, 1 - \frac{x}{c} \right] - \frac{1}{2} a b^2 c^3 \operatorname{PolyLog} \left[ 2, 1 + \frac{x}{c} \right] + \frac{1}{4} b^3 c^3 \operatorname{PolyLog} \left[ 3, 1 - \frac{c}{x} \right] + \frac{1}{4} b^3 c^3 \operatorname{PolyLog} \left[ 3, \frac{c + x}{x} \right]
\end{aligned}$$

Problem 152: Unable to integrate problem.

$$\int x \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x} \right] \right)^3 dx$$

Optimal (type 4, 135 leaves, 8 steps):

$$-\frac{3}{2} b c^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 + \frac{3}{2} b c x \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 - \frac{1}{2} c^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 +$$

$$\frac{1}{2} x^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 - 3 b^2 c^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{Log} \left[ 2 - \frac{2}{1 + \frac{c}{x}} \right] + \frac{3}{2} b^3 c^2 \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 + \frac{c}{x}} \right]$$

Result (type 8, 897 leaves, 75 steps):

$$\begin{aligned} & \frac{3}{4} a^2 b c x + \frac{3}{8} b^3 \operatorname{CannotIntegrate} \left[ x \operatorname{Log} \left[ 1 - \frac{c}{x} \right]^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right], x \right] - \frac{3}{8} b^3 \operatorname{CannotIntegrate} \left[ x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right]^2, x \right] - \frac{3}{4} a b^2 c x \operatorname{Log} \left[ 1 - \frac{c}{x} \right] + \\ & \frac{3}{16} b c \left( 1 - \frac{c}{x} \right) x \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 - \frac{1}{16} c^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^3 + \frac{1}{16} x^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^3 + \frac{3}{4} a b^2 c x \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{3}{4} a^2 b x^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] - \\ & \frac{3}{4} a b^2 x^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} \left[ 1 + \frac{c}{x} \right] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ 1 + \frac{c}{x} \right] \operatorname{Log} [c - x] - \frac{3}{16} b c^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right)^2 \operatorname{Log} \left[ \frac{c}{x} \right] + \\ & \frac{3}{2} a b^2 c^2 \operatorname{Log} [x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{x}{c} \right] - \frac{3}{4} a^2 b c^2 \operatorname{Log} [c + x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} [c + x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \operatorname{Log} [c + x] - \\ & \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ \frac{c - x}{2 c} \right] \operatorname{Log} [c + x] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ -\frac{x}{c} \right] \operatorname{Log} [c + x] - \frac{3}{4} a b^2 c^2 \operatorname{Log} [c - x] \operatorname{Log} \left[ \frac{c + x}{2 c} \right] + \frac{3}{4} a b^2 c^2 \operatorname{Log} \left[ \frac{c + x}{x} \right] + \frac{3}{4} a b^2 c x \operatorname{Log} \left[ \frac{c + x}{x} \right] - \\ & \frac{3}{8} a b^2 c^2 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{3}{16} b^3 c \left( 1 + \frac{c}{x} \right) x \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{3}{8} a b^2 x^2 \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 + \frac{3}{16} b^3 c^2 \operatorname{Log} \left[ -\frac{c}{x} \right] \operatorname{Log} \left[ \frac{c + x}{x} \right]^2 - \frac{1}{16} b^3 c^2 \operatorname{Log} \left[ \frac{c + x}{x} \right]^3 + \\ & \frac{1}{16} b^3 x^2 \operatorname{Log} \left[ \frac{c + x}{x} \right]^3 + \frac{3}{8} b^2 c^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{c}{x} \right] - \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c - x}{2 c} \right] - \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] + \\ & \frac{3}{8} b^3 c^2 \operatorname{PolyLog} \left[ 2, -\frac{c}{x} \right] - \frac{3}{8} b^3 c^2 \operatorname{PolyLog} \left[ 2, \frac{c}{x} \right] - \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c + x}{2 c} \right] + \frac{3}{8} b^3 c^2 \operatorname{Log} \left[ \frac{c + x}{x} \right] \operatorname{PolyLog} \left[ 2, \frac{c + x}{x} \right] + \\ & \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{x}{c} \right] + \frac{3}{4} a b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 + \frac{x}{c} \right] + \frac{3}{8} b^3 c^2 \operatorname{PolyLog} \left[ 3, 1 - \frac{c}{x} \right] - \frac{3}{8} b^3 c^2 \operatorname{PolyLog} \left[ 3, \frac{c + x}{x} \right] \end{aligned}$$

Problem 153: Unable to integrate problem.

$$\int \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x} \right] \right)^3 dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$c \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 + x \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^3 - 3 b c \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right)^2 \operatorname{Log} \left[ \frac{2 c}{c - x} \right] -$$

$$3 b^2 c \left( a + b \operatorname{ArcCoth} \left[ \frac{x}{c} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{2 c}{c - x} \right] + \frac{3}{2} b^3 c \operatorname{PolyLog} \left[ 3, 1 - \frac{2 c}{c - x} \right]$$

Result (type 8, 642 leaves, 43 steps):

$$\begin{aligned}
& a^3 x + \frac{3}{8} b^3 \text{CannotIntegrate}\left[\text{Log}\left[1 - \frac{c}{x}\right]^2 \text{Log}\left[1 + \frac{c}{x}\right], x\right] - \frac{3}{8} b^3 \text{CannotIntegrate}\left[\text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right]^2, x\right] - \frac{3}{2} a^2 b x \text{Log}\left[1 - \frac{c}{x}\right] - \\
& \frac{3}{4} a b^2 (c - x) \text{Log}\left[1 - \frac{c}{x}\right]^2 + \frac{1}{8} b^3 (c - x) \text{Log}\left[1 - \frac{c}{x}\right]^3 + \frac{3}{2} a^2 b x \text{Log}\left[1 + \frac{c}{x}\right] - \frac{3}{2} a b^2 x \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}\left[1 + \frac{c}{x}\right] + \frac{3}{4} a b^2 (c + x) \text{Log}\left[1 + \frac{c}{x}\right]^2 + \\
& \frac{1}{8} b^3 (c + x) \text{Log}\left[1 + \frac{c}{x}\right]^3 - \frac{3}{2} a b^2 c \text{Log}\left[1 - \frac{c}{x}\right] \text{Log}[-c - x] + \frac{3}{2} a^2 b c \text{Log}[c - x] + \frac{3}{2} a b^2 c \text{Log}[-c - x] \text{Log}\left[\frac{c - x}{2c}\right] - \frac{3}{8} b^3 c \text{Log}\left[1 - \frac{c}{x}\right]^2 \text{Log}\left[\frac{c}{x}\right] - \\
& \frac{3}{2} a b^2 c \text{Log}[-c - x] \text{Log}\left[-\frac{x}{c}\right] + \frac{3}{2} a b^2 c \text{Log}\left[1 + \frac{c}{x}\right] \text{Log}[-c + x] + \frac{3}{2} a b^2 c \text{Log}\left[\frac{x}{c}\right] \text{Log}[-c + x] + \frac{3}{2} a^2 b c \text{Log}[c + x] - \\
& \frac{3}{2} a b^2 c \text{Log}[-c + x] \text{Log}\left[\frac{c + x}{2c}\right] - \frac{3}{8} b^3 c \text{Log}\left[-\frac{c}{x}\right] \text{Log}\left[\frac{c + x}{x}\right]^2 - \frac{3}{4} b^3 c \text{Log}\left[1 - \frac{c}{x}\right] \text{PolyLog}\left[2, 1 - \frac{c}{x}\right] - \frac{3}{2} a b^2 c \text{PolyLog}\left[2, \frac{c - x}{2c}\right] + \\
& \frac{3}{2} a b^2 c \text{PolyLog}\left[2, -\frac{c}{x}\right] - \frac{3}{2} a b^2 c \text{PolyLog}\left[2, \frac{c}{x}\right] + \frac{3}{2} a b^2 c \text{PolyLog}\left[2, \frac{c + x}{2c}\right] - \frac{3}{4} b^3 c \text{Log}\left[\frac{c + x}{x}\right] \text{PolyLog}\left[2, \frac{c + x}{x}\right] + \\
& \frac{3}{2} a b^2 c \text{PolyLog}\left[2, 1 - \frac{x}{c}\right] - \frac{3}{2} a b^2 c \text{PolyLog}\left[2, 1 + \frac{x}{c}\right] + \frac{3}{4} b^3 c \text{PolyLog}\left[3, 1 - \frac{c}{x}\right] + \frac{3}{4} b^3 c \text{PolyLog}\left[3, \frac{c + x}{x}\right]
\end{aligned}$$

Problem 155: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \text{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^2} dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$\begin{aligned}
& -\frac{\left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{c} - \frac{\left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{x} + \frac{3b \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \text{Log}\left[\frac{2}{1 - \frac{c}{x}}\right]}{c} + \\
& \frac{3b^2 \left(a + b \text{ArcCoth}\left[\frac{x}{c}\right]\right) \text{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{c}{x}}\right]}{c} - \frac{3b^3 \text{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{c}{x}}\right]}{2c}
\end{aligned}$$

Result (type 4, 387 leaves, 82 steps):

$$\begin{aligned}
& \frac{\left(1 - \frac{c}{x}\right) \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^3}{8c} - \frac{3b \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c + x}{2x}\right]}{4c} + \frac{3b \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c + x}{x}\right]}{8c} - \frac{3b \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right)^2 \text{Log}\left[\frac{c + x}{x}\right]}{8x} - \\
& \frac{3b^2 \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) \text{Log}\left[\frac{c + x}{x}\right]^2}{8c} - \frac{3b^2 \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) \text{Log}\left[\frac{c + x}{x}\right]^2}{8x} - \frac{3b^3 \text{Log}\left[-\frac{c - x}{2x}\right] \text{Log}\left[\frac{c + x}{x}\right]^2}{4c} - \frac{b^3 \left(1 + \frac{c}{x}\right) \text{Log}\left[\frac{c + x}{x}\right]^3}{8c} + \\
& \frac{3b^2 \left(2a - b \text{Log}\left[1 - \frac{c}{x}\right]\right) \text{PolyLog}\left[2, -\frac{c - x}{2x}\right]}{2c} - \frac{3b^3 \text{Log}\left[\frac{c + x}{x}\right] \text{PolyLog}\left[2, \frac{c + x}{2x}\right]}{2c} + \frac{3b^3 \text{PolyLog}\left[3, -\frac{c - x}{2x}\right]}{2c} + \frac{3b^3 \text{PolyLog}\left[3, \frac{c + x}{2x}\right]}{2c}
\end{aligned}$$

## Problem 156: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x^3} dx$$

Optimal (type 4, 139 leaves, 9 steps):

$$-\frac{3b\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2c^2} - \frac{3b\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2}{2cx} + \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2c^2} - \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3}{2x^2} + \frac{3b^2\left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)\operatorname{Log}\left[\frac{2}{1-\frac{c}{x}}\right]}{c^2} + \frac{3b^3\operatorname{PolyLog}\left[2, 1 - \frac{2}{1-\frac{c}{x}}\right]}{2c^2}$$

Result (type 8, 1098 leaves, 81 steps):

$$\begin{aligned} & -\frac{3b^3\left(1 - \frac{c}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1 + \frac{c}{x}\right)^2}{16c^2} + \frac{3b^3\left(1 + \frac{c}{x}\right)^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3}{8}b^3\operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{c}{x}\right]^2\operatorname{Log}\left[1 + \frac{c}{x}\right]}{x^3}, x\right] - \\ & \frac{3}{8}b^3\operatorname{CannotIntegrate}\left[\frac{\operatorname{Log}\left[1 - \frac{c}{x}\right]\operatorname{Log}\left[1 + \frac{c}{x}\right]^2}{x^3}, x\right] + \frac{3ab^2\operatorname{Log}\left[1 - \frac{c}{x}\right]}{8c^2} - \frac{3ab^2\left(1 - \frac{c}{x}\right)\operatorname{Log}\left[1 - \frac{c}{x}\right]}{4c^2} - \frac{3b^3\left(1 - \frac{c}{x}\right)\operatorname{Log}\left[1 - \frac{c}{x}\right]}{4c^2} - \\ & \frac{3ab^2\operatorname{Log}\left[1 - \frac{c}{x}\right]}{8x^2} - \frac{3b^2\left(1 - \frac{c}{x}\right)^2\left(2a - b\operatorname{Log}\left[1 - \frac{c}{x}\right]\right)}{32c^2} + \frac{3b\left(1 - \frac{c}{x}\right)\left(2a - b\operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2}{8c^2} - \frac{3b\left(1 - \frac{c}{x}\right)^2\left(2a - b\operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^2}{32c^2} + \\ & \frac{\left(1 - \frac{c}{x}\right)\left(2a - b\operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^3}{8c^2} - \frac{\left(1 - \frac{c}{x}\right)^2\left(2a - b\operatorname{Log}\left[1 - \frac{c}{x}\right]\right)^3}{16c^2} + \frac{3ab^2\operatorname{Log}\left[1 - \frac{c}{x}\right]\operatorname{Log}\left[1 + \frac{c}{x}\right]}{4x^2} - \frac{3ab^2\operatorname{Log}\left[1 + \frac{c}{x}\right]\operatorname{Log}[c-x]}{4c^2} - \\ & \frac{3ab^2\operatorname{Log}[c-x]\operatorname{Log}\left[\frac{x}{c}\right]}{4c^2} - \frac{3ab^2\operatorname{Log}\left[1 - \frac{c}{x}\right]\operatorname{Log}[c+x]}{4c^2} + \frac{3ab^2\operatorname{Log}\left[\frac{c-x}{2c}\right]\operatorname{Log}[c+x]}{4c^2} - \frac{3ab^2\operatorname{Log}\left[-\frac{x}{c}\right]\operatorname{Log}[c+x]}{4c^2} + \\ & \frac{3ab^2\operatorname{Log}[c-x]\operatorname{Log}\left[\frac{c+x}{2c}\right]}{4c^2} + \frac{3a^2b\operatorname{Log}\left[\frac{c+x}{x}\right]}{4c^2} + \frac{3ab^2\operatorname{Log}\left[\frac{c+x}{x}\right]}{8c^2} - \frac{9ab^2\left(1 + \frac{c}{x}\right)\operatorname{Log}\left[\frac{c+x}{x}\right]}{4c^2} + \frac{3b^3\left(1 + \frac{c}{x}\right)\operatorname{Log}\left[\frac{c+x}{x}\right]}{4c^2} + \\ & \frac{3ab^2\left(1 + \frac{c}{x}\right)^2\operatorname{Log}\left[\frac{c+x}{x}\right]}{8c^2} - \frac{3b^3\left(1 + \frac{c}{x}\right)^2\operatorname{Log}\left[\frac{c+x}{x}\right]}{32c^2} - \frac{3a^2b\operatorname{Log}\left[\frac{c+x}{x}\right]}{4x^2} - \frac{3ab^2\operatorname{Log}\left[\frac{c+x}{x}\right]}{8x^2} + \frac{3ab^2\left(1 + \frac{c}{x}\right)\operatorname{Log}\left[\frac{c+x}{x}\right]^2}{4c^2} - \frac{3b^3\left(1 + \frac{c}{x}\right)\operatorname{Log}\left[\frac{c+x}{x}\right]^2}{8c^2} - \\ & \frac{3ab^2\left(1 + \frac{c}{x}\right)^2\operatorname{Log}\left[\frac{c+x}{x}\right]^2}{8c^2} + \frac{3b^3\left(1 + \frac{c}{x}\right)^2\operatorname{Log}\left[\frac{c+x}{x}\right]^2}{32c^2} + \frac{b^3\left(1 + \frac{c}{x}\right)\operatorname{Log}\left[\frac{c+x}{x}\right]^3}{8c^2} - \frac{b^3\left(1 + \frac{c}{x}\right)^2\operatorname{Log}\left[\frac{c+x}{x}\right]^3}{16c^2} + \frac{3ab^2\operatorname{PolyLog}\left[2, \frac{c-x}{2c}\right]}{4c^2} + \\ & \frac{3ab^2\operatorname{PolyLog}\left[2, -\frac{c}{x}\right]}{4c^2} + \frac{3ab^2\operatorname{PolyLog}\left[2, \frac{c}{x}\right]}{4c^2} + \frac{3ab^2\operatorname{PolyLog}\left[2, \frac{c+x}{2c}\right]}{4c^2} - \frac{3ab^2\operatorname{PolyLog}\left[2, 1 - \frac{x}{c}\right]}{4c^2} - \frac{3ab^2\operatorname{PolyLog}\left[2, 1 + \frac{x}{c}\right]}{4c^2} \end{aligned}$$

**Problem 171: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x^3 \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x^2} \right] \right)^2 dx$$

Optimal (type 3, 94 leaves, 9 steps):

$$\frac{1}{2} b c x^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right) - \frac{1}{4} c^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right)^2 + \frac{1}{4} x^4 \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right)^2 + \frac{1}{4} b^2 c^2 \operatorname{Log} \left[ 1 - \frac{c^2}{x^4} \right] + b^2 c^2 \operatorname{Log} [x]$$

Result (type 4, 599 leaves, 59 steps):

$$\begin{aligned} & \frac{1}{4} a b c x^2 - \frac{1}{8} b^2 c x^2 \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] + \frac{1}{8} b c \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] \right) - \frac{1}{16} c^2 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] \right)^2 + \frac{1}{16} x^4 \left( 2 a - b \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] \right)^2 + \\ & \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right] + \frac{1}{4} b^2 c x^2 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right] + \frac{1}{4} a b x^4 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right] - \frac{1}{8} b^2 x^4 \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right] - \frac{1}{16} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right]^2 + \\ & \frac{1}{16} b^2 x^4 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right]^2 + \frac{1}{2} a b c^2 \operatorname{Log} [x] + \frac{1}{2} b^2 c^2 \operatorname{Log} [x] + \frac{1}{8} b^2 c^2 \operatorname{Log} [c - x^2] + \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ 1 + \frac{c}{x^2} \right] \operatorname{Log} [c - x^2] + \\ & \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ \frac{x^2}{c} \right] \operatorname{Log} [c - x^2] - \frac{1}{4} a b c^2 \operatorname{Log} [c + x^2] + \frac{1}{8} b^2 c^2 \operatorname{Log} [c + x^2] + \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ 1 - \frac{c}{x^2} \right] \operatorname{Log} [c + x^2] + \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ -\frac{x^2}{c} \right] \operatorname{Log} [c + x^2] - \\ & \frac{1}{8} b^2 c^2 \operatorname{Log} \left[ \frac{c - x^2}{2 c} \right] \operatorname{Log} [c + x^2] - \frac{1}{8} b^2 c^2 \operatorname{Log} [c - x^2] \operatorname{Log} \left[ \frac{c + x^2}{2 c} \right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, -\frac{c}{x^2} \right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c}{x^2} \right] - \\ & \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c - x^2}{2 c} \right] - \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c + x^2}{2 c} \right] + \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, \frac{c + x^2}{c} \right] + \frac{1}{8} b^2 c^2 \operatorname{PolyLog} \left[ 2, 1 - \frac{x^2}{c} \right] \end{aligned}$$

**Problem 172: Result valid but suboptimal antiderivative.**

$$\int x \left( a + b \operatorname{ArcTanh} \left[ \frac{c}{x^2} \right] \right)^2 dx$$

Optimal (type 4, 94 leaves, 5 steps):

$$-\frac{1}{2} c \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right)^2 + \frac{1}{2} x^2 \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right)^2 - b c \left( a + b \operatorname{ArcCoth} \left[ \frac{x^2}{c} \right] \right) \operatorname{Log} \left[ 2 - \frac{2}{1 + \frac{c}{x^2}} \right] + \frac{1}{2} b^2 c \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 + \frac{c}{x^2}} \right]$$

Result (type 4, 404 leaves, 34 steps):

$$\begin{aligned} & \frac{1}{8} \left(1 - \frac{c}{x^2}\right) x^2 \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 + \frac{1}{2} a b x^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{4} b^2 x^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{8} b^2 \left(1 + \frac{c}{x^2}\right) x^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2 + a b c \operatorname{Log}[x] - \\ & \frac{1}{4} b^2 c \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}[-c - x^2] - \frac{1}{4} b^2 c \operatorname{Log}\left[-\frac{x^2}{c}\right] \operatorname{Log}[-c - x^2] + \frac{1}{4} b^2 c \operatorname{Log}[-c - x^2] \operatorname{Log}\left[\frac{c - x^2}{2c}\right] + \frac{1}{4} b^2 c \operatorname{Log}\left[1 + \frac{c}{x^2}\right] \operatorname{Log}[-c + x^2] + \\ & \frac{1}{4} b^2 c \operatorname{Log}\left[\frac{x^2}{c}\right] \operatorname{Log}[-c + x^2] + \frac{1}{2} a b c \operatorname{Log}[c + x^2] - \frac{1}{4} b^2 c \operatorname{Log}[-c + x^2] \operatorname{Log}\left[\frac{c + x^2}{2c}\right] + \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, -\frac{c}{x^2}\right] - \\ & \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{c}{x^2}\right] - \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{c - x^2}{2c}\right] + \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{c + x^2}{2c}\right] - \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, \frac{c + x^2}{c}\right] + \frac{1}{4} b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{x^2}{c}\right] \end{aligned}$$

Problem 174: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^3} dx$$

Optimal (type 4, 99 leaves, 6 steps):

$$-\frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2c} - \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{2x^2} + \frac{b\left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right) \operatorname{Log}\left[\frac{2}{1 - \frac{c}{x^2}}\right]}{c} + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{c}{x^2}}\right]}{2c}$$

Result (type 4, 207 leaves, 28 steps):

$$\begin{aligned} & \frac{\left(1 - \frac{c}{x^2}\right) \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2}{8c} - \frac{b\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right) \operatorname{Log}\left[\frac{1}{2} \left(1 + \frac{c}{x^2}\right)\right]}{4c} - \frac{b^2 \operatorname{Log}\left[\frac{1}{2} \left(1 - \frac{c}{x^2}\right)\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{4c} - \\ & \frac{b\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right) \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{4x^2} - \frac{b^2 \left(1 + \frac{c}{x^2}\right) \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2}{8c} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 - \frac{c}{x^2}\right)\right]}{4c} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{1}{2} \left(1 + \frac{c}{x^2}\right)\right]}{4c} \end{aligned}$$

Problem 175: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^5} dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$-\frac{ab}{2cx^2} - \frac{b^2 \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]}{2cx^2} + \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4c^2} - \frac{\left(a + b \operatorname{ArcCoth}\left[\frac{x^2}{c}\right]\right)^2}{4x^4} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^4}\right]}{4c^2}$$

Result (type 4, 770 leaves, 67 steps):



$$\begin{aligned}
& -\frac{b^2 \left(1 - \frac{c}{x^2}\right)^2}{32 c^2} - \frac{b^2 \left(1 + \frac{c}{x^2}\right)^2}{32 c^2} + \frac{a b}{8 x^4} + \frac{b^2}{16 x^4} - \frac{3 a b}{4 c x^2} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{16 c^2} - \frac{3 b^2 \left(1 - \frac{c}{x^2}\right) \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{8 c^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{16 x^4} - \\
& \frac{b \left(1 - \frac{c}{x^2}\right)^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{16 c^2} + \frac{\left(1 - \frac{c}{x^2}\right) \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2}{8 c^2} - \frac{\left(1 - \frac{c}{x^2}\right)^2 \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2}{16 c^2} - \frac{b^2 \left(1 + \frac{c}{x^2}\right) \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{4 c^2} + \\
& \frac{b^2 \left(1 + \frac{c}{x^2}\right)^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{16 c^2} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{8 x^4} + \frac{b^2 \left(1 + \frac{c}{x^2}\right) \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2}{8 c^2} - \frac{b^2 \left(1 + \frac{c}{x^2}\right)^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2}{16 c^2} - \\
& \frac{b^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] \operatorname{Log}\left[c - x^2\right]}{8 c^2} - \frac{b^2 \operatorname{Log}\left[\frac{x^2}{c}\right] \operatorname{Log}\left[c - x^2\right]}{8 c^2} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[c + x^2\right]}{8 c^2} - \frac{b^2 \operatorname{Log}\left[-\frac{x^2}{c}\right] \operatorname{Log}\left[c + x^2\right]}{8 c^2} + \frac{b^2 \operatorname{Log}\left[\frac{c-x^2}{2c}\right] \operatorname{Log}\left[c + x^2\right]}{8 c^2} + \\
& \frac{b^2 \operatorname{Log}\left[c - x^2\right] \operatorname{Log}\left[\frac{c+x^2}{2c}\right]}{8 c^2} + \frac{a b \operatorname{Log}\left[\frac{c+x^2}{x^2}\right]}{4 c^2} + \frac{b^2 \operatorname{Log}\left[\frac{c+x^2}{x^2}\right]}{16 c^2} - \frac{b^2 \left(1 + \frac{c}{x^2}\right) \operatorname{Log}\left[\frac{c+x^2}{x^2}\right]}{8 c^2} - \frac{a b \operatorname{Log}\left[\frac{c+x^2}{x^2}\right]}{4 x^4} - \frac{b^2 \operatorname{Log}\left[\frac{c+x^2}{x^2}\right]}{16 x^4} + \\
& \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{c}{x^2}\right]}{8 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c}{x^2}\right]}{8 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c-x^2}{2c}\right]}{8 c^2} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c+x^2}{2c}\right]}{8 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, \frac{c+x^2}{c}\right]}{8 c^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{x^2}{c}\right]}{8 c^2}
\end{aligned}$$

Problem 184: Result optimal but 1 more steps used.

$$\int (d x)^m \left( a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right] \right) dx$$

Optimal (type 5, 75 leaves, 3 steps):

$$\frac{(d x)^{1+m} \left( a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right] \right)}{d (1+m)} - \frac{2 b c d (d x)^{-1+m} \operatorname{Hypergeometric2F1}\left[1, \frac{1-m}{4}, \frac{5-m}{4}, \frac{c^2}{x^4}\right]}{1-m^2}$$

Result (type 5, 75 leaves, 4 steps):

$$\frac{(d x)^{1+m} \left( a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right] \right)}{d (1+m)} - \frac{2 b c d (d x)^{-1+m} \operatorname{Hypergeometric2F1}\left[1, \frac{1-m}{4}, \frac{5-m}{4}, \frac{c^2}{x^4}\right]}{1-m^2}$$

Problem 195: Unable to integrate problem.

$$\int x^3 \left( a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right] \right)^2 dx$$

Optimal (type 3, 211 leaves, 22 steps):

$$\frac{a b \sqrt{x}}{2 c^7} + \frac{71 b^2 x}{420 c^6} + \frac{3 b^2 x^2}{70 c^4} + \frac{b^2 x^3}{84 c^2} + \frac{b^2 \sqrt{x} \operatorname{ArcTanh}[c \sqrt{x}]}{2 c^7} + \frac{b x^{3/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{6 c^5} + \frac{b x^{5/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{10 c^3} + \frac{b x^{7/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{14 c} - \frac{(a + b \operatorname{ArcTanh}[c \sqrt{x}])^2}{4 c^8} + \frac{1}{4} x^4 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 + \frac{44 b^2 \operatorname{Log}[1 - c^2 x]}{105 c^8}$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}[x^3 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2, x]$$

### Problem 196: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 dx$$

Optimal (type 3, 173 leaves, 17 steps):

$$\frac{2 a b \sqrt{x}}{3 c^5} + \frac{8 b^2 x}{45 c^4} + \frac{b^2 x^2}{30 c^2} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}[c \sqrt{x}]}{3 c^5} + \frac{2 b x^{3/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{9 c^3} + \frac{2 b x^{5/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{15 c} - \frac{(a + b \operatorname{ArcTanh}[c \sqrt{x}])^2}{3 c^6} + \frac{1}{3} x^3 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 + \frac{23 b^2 \operatorname{Log}[1 - c^2 x]}{45 c^6}$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}[x^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2, x]$$

### Problem 197: Unable to integrate problem.

$$\int x (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 dx$$

Optimal (type 3, 129 leaves, 12 steps):

$$\frac{a b \sqrt{x}}{c^3} + \frac{b^2 x}{6 c^2} + \frac{b^2 \sqrt{x} \operatorname{ArcTanh}[c \sqrt{x}]}{c^3} + \frac{b x^{3/2} (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{3 c} - \frac{(a + b \operatorname{ArcTanh}[c \sqrt{x}])^2}{2 c^4} + \frac{1}{2} x^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 + \frac{2 b^2 \operatorname{Log}[1 - c^2 x]}{3 c^4}$$

Result (type 8, 18 leaves, 0 steps):

$$\operatorname{Unintegrable}[x (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2, x]$$

### Problem 198: Unable to integrate problem.

$$\int \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2 dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{2 a b \sqrt{x}}{c} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh} \left[ c \sqrt{x} \right]}{c} - \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{c^2} + x \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2 + \frac{b^2 \operatorname{Log} \left[ 1 - c^2 x \right]}{c^2}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable} \left[ \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2, x \right]$$

### Problem 200: Unable to integrate problem.

$$\int \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{x^2} dx$$

Optimal (type 3, 85 leaves, 9 steps):

$$- \frac{2 b c \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)}{\sqrt{x}} + c^2 \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2 - \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{x} + b^2 c^2 \operatorname{Log} [x] - b^2 c^2 \operatorname{Log} [1 - c^2 x]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable} \left[ \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{x^2}, x \right]$$

### Problem 201: Unable to integrate problem.

$$\int \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{x^3} dx$$

Optimal (type 3, 133 leaves, 14 steps):

$$- \frac{b^2 c^2}{6 x} - \frac{b c \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)}{3 x^{3/2}} - \frac{b c^3 \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)}{\sqrt{x}} + \frac{1}{2} c^4 \left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2 - \frac{\left( a + b \operatorname{ArcTanh} \left[ c \sqrt{x} \right] \right)^2}{2 x^2} + \frac{2}{3} b^2 c^4 \operatorname{Log} [x] - \frac{2}{3} b^2 c^4 \operatorname{Log} [1 - c^2 x]$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{x^3}, x\right]$$

Problem 202: Unable to integrate problem.

$$\int x^3 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 374 leaves, 54 steps):

$$\begin{aligned} & \frac{47 b^3 \sqrt{x}}{70 c^7} + \frac{23 b^3 x^{3/2}}{420 c^5} + \frac{b^3 x^{5/2}}{140 c^3} - \frac{47 b^3 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}{70 c^8} + \frac{71 b^2 x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{140 c^6} + \\ & \frac{9 b^2 x^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{70 c^4} + \frac{b^2 x^3 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{28 c^2} + \frac{44 b \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{35 c^8} + \frac{3 b \sqrt{x} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{4 c^7} + \\ & \frac{b x^{3/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{4 c^5} + \frac{3 b x^{5/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{20 c^3} + \frac{3 b x^{7/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{28 c} - \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{4 c^8} + \\ & \frac{1}{4} x^4 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{88 b^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[\frac{2}{1-c \sqrt{x}}\right]}{35 c^8} - \frac{44 b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c \sqrt{x}}\right]}{35 c^8} \end{aligned}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^3 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3, x\right]$$

Problem 203: Unable to integrate problem.

$$\int x^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 304 leaves, 34 steps):

$$\begin{aligned} & \frac{19 b^3 \sqrt{x}}{30 c^5} + \frac{b^3 x^{3/2}}{30 c^3} - \frac{19 b^3 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}{30 c^6} + \frac{8 b^2 x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{15 c^4} + \frac{b^2 x^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{10 c^2} + \\ & \frac{23 b \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{15 c^6} + \frac{b \sqrt{x} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{c^5} + \frac{b x^{3/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{3 c^3} + \frac{b x^{5/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{5 c} - \\ & \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{3 c^6} + \frac{1}{3} x^3 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{46 b^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[\frac{2}{1-c \sqrt{x}}\right]}{15 c^6} - \frac{23 b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c \sqrt{x}}\right]}{15 c^6} \end{aligned}$$

Result (type 8, 20 leaves, 0 steps):

$$\text{Unintegrable}\left[x^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3, x\right]$$

Problem 204: Unable to integrate problem.

$$\int x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 234 leaves, 19 steps):

$$\begin{aligned} & \frac{b^3 \sqrt{x}}{2 c^3} - \frac{b^3 \operatorname{ArcTanh}\left[c \sqrt{x}\right]}{2 c^4} + \frac{b^2 x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{2 c^2} + \frac{2 b \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{c^4} + \\ & \frac{3 b \sqrt{x} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{2 c^3} + \frac{b x^{3/2} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{2 c} - \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{2 c^4} + \\ & \frac{1}{2} x^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{4 b^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[\frac{2}{1-c \sqrt{x}}\right]}{c^4} - \frac{2 b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c \sqrt{x}}\right]}{c^4} \end{aligned}$$

Result (type 8, 18 leaves, 0 steps):

$$\text{Unintegrable}\left[x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3, x\right]$$

Problem 205: Unable to integrate problem.

$$\int \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\begin{aligned} & \frac{3 b \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{c^2} + \frac{3 b \sqrt{x} \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{c} - \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{c^2} + \\ & x \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{6 b^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[\frac{2}{1-c \sqrt{x}}\right]}{c^2} - \frac{3 b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c \sqrt{x}}\right]}{c^2} \end{aligned}$$

Result (type 8, 16 leaves, 0 steps):

$$\text{Unintegrable}\left[\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3, x\right]$$

## Problem 207: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{x^2} dx$$

Optimal (type 4, 142 leaves, 8 steps):

$$3 b c^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2 - \frac{3 b c \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{\sqrt{x}} + c^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{x} + 6 b^2 c^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 + c \sqrt{x}}\right] - 3 b^3 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c \sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{x^2}, x\right]$$

## Problem 208: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{x^3} dx$$

Optimal (type 4, 234 leaves, 17 steps):

$$-\frac{b^3 c^3}{2 \sqrt{x}} + \frac{1}{2} b^3 c^4 \operatorname{ArcTanh}\left[c \sqrt{x}\right] - \frac{b^2 c^2 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)}{2 x} + 2 b c^4 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2 - \frac{b c \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{2 x^{3/2}} - \frac{3 b c^3 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^2}{2 \sqrt{x}} + \frac{1}{2} c^4 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3 - \frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{2 x^2} + 4 b^2 c^4 \left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right) \operatorname{Log}\left[2 - \frac{2}{1 + c \sqrt{x}}\right] - 2 b^3 c^4 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c \sqrt{x}}\right]$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{\left(a + b \operatorname{ArcTanh}\left[c \sqrt{x}\right]\right)^3}{x^3}, x\right]$$

### Problem 221: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^{3/2}])^2 dx$$

Optimal (type 3, 101 leaves, 7 steps):

$$\frac{2 a b x^{3/2}}{3 c} + \frac{2 b^2 x^{3/2} \operatorname{ArcTanh}[c x^{3/2}]}{3 c} - \frac{(a + b \operatorname{ArcTanh}[c x^{3/2}])^2}{3 c^2} + \frac{1}{3} x^3 (a + b \operatorname{ArcTanh}[c x^{3/2}])^2 + \frac{b^2 \operatorname{Log}[1 - c^2 x^3]}{3 c^2}$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}[x^2 (a + b \operatorname{ArcTanh}[c x^{3/2}])^2, x]$$

### Problem 223: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^{3/2}])^2}{x^4} dx$$

Optimal (type 3, 96 leaves, 9 steps):

$$-\frac{2 b c (a + b \operatorname{ArcTanh}[c x^{3/2}])}{3 x^{3/2}} + \frac{1}{3} c^2 (a + b \operatorname{ArcTanh}[c x^{3/2}])^2 - \frac{(a + b \operatorname{ArcTanh}[c x^{3/2}])^2}{3 x^3} + b^2 c^2 \operatorname{Log}[x] - \frac{1}{3} b^2 c^2 \operatorname{Log}[1 - c^2 x^3]$$

Result (type 8, 20 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^{3/2}])^2}{x^4}, x\right]$$

## Test results for the 49 problems in "7.3.3 (d+e x)^m (a+b arctanh(c x^n))^p.m"

### Problem 23: Result valid but suboptimal antiderivative.

$$\int (d + e x)^3 (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 182 leaves, 13 steps):

$$\frac{2 b d e^2 x}{c} + \frac{b e^3 x^2}{4 c} + \frac{b d (c d^2 - e^2) \operatorname{ArcTan}[\sqrt{c} x]}{c^{3/2}} - \frac{b d (c d^2 + e^2) \operatorname{ArcTanh}[\sqrt{c} x]}{c^{3/2}} +$$

$$\frac{(d + e x)^4 (a + b \operatorname{ArcTanh}[c x^2])}{4 e} + \frac{b (c^2 d^4 + 6 c d^2 e^2 + e^4) \operatorname{Log}[1 - c x^2]}{8 c^2 e} - \frac{b (c^2 d^4 - 6 c d^2 e^2 + e^4) \operatorname{Log}[1 + c x^2]}{8 c^2 e}$$

Result (type 3, 220 leaves, 19 steps):

$$\frac{2 b d e^2 x}{c} + \frac{b e^3 x^2}{4 c} + \frac{a (d + e x)^4}{4 e} + \frac{b d^3 \operatorname{ArcTan}[\sqrt{c} x]}{\sqrt{c}} - \frac{b d e^2 \operatorname{ArcTan}[\sqrt{c} x]}{c^{3/2}} - \frac{b d^3 \operatorname{ArcTanh}[\sqrt{c} x]}{\sqrt{c}} - \frac{b d e^2 \operatorname{ArcTanh}[\sqrt{c} x]}{c^{3/2}} - \frac{b e^3 \operatorname{ArcTanh}[c x^2]}{4 c^2} +$$

$$b d^3 x \operatorname{ArcTanh}[c x^2] + \frac{3}{2} b d^2 e x^2 \operatorname{ArcTanh}[c x^2] + b d e^2 x^3 \operatorname{ArcTanh}[c x^2] + \frac{1}{4} b e^3 x^4 \operatorname{ArcTanh}[c x^2] + \frac{3 b d^2 e \operatorname{Log}[1 - c^2 x^4]}{4 c}$$

Problem 24: Result optimal but 1 more steps used.

$$\int (d + e x)^2 (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 158 leaves, 11 steps):

$$\frac{2 b e^2 x}{3 c} + \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[\sqrt{c} x]}{3 c^{3/2}} - \frac{b (3 c d^2 + e^2) \operatorname{ArcTanh}[\sqrt{c} x]}{3 c^{3/2}} +$$

$$\frac{(d + e x)^3 (a + b \operatorname{ArcTanh}[c x^2])}{3 e} + \frac{b d (c d^2 + 3 e^2) \operatorname{Log}[1 - c x^2]}{6 c e} - \frac{b d (c d^2 - 3 e^2) \operatorname{Log}[1 + c x^2]}{6 c e}$$

Result (type 3, 158 leaves, 12 steps):

$$\frac{2 b e^2 x}{3 c} + \frac{b (3 c d^2 - e^2) \operatorname{ArcTan}[\sqrt{c} x]}{3 c^{3/2}} - \frac{b (3 c d^2 + e^2) \operatorname{ArcTanh}[\sqrt{c} x]}{3 c^{3/2}} +$$

$$\frac{(d + e x)^3 (a + b \operatorname{ArcTanh}[c x^2])}{3 e} + \frac{b d (c d^2 + 3 e^2) \operatorname{Log}[1 - c x^2]}{6 c e} - \frac{b d (c d^2 - 3 e^2) \operatorname{Log}[1 + c x^2]}{6 c e}$$

Problem 25: Result valid but suboptimal antiderivative.

$$\int (d + e x) (a + b \operatorname{ArcTanh}[c x^2]) dx$$

Optimal (type 3, 117 leaves, 10 steps):

$$\frac{b d \operatorname{ArcTan}[\sqrt{c} x]}{\sqrt{c}} - \frac{b d \operatorname{ArcTanh}[\sqrt{c} x]}{\sqrt{c}} + \frac{(d + e x)^2 (a + b \operatorname{ArcTanh}[c x^2])}{2 e} + \frac{b (c d^2 + e^2) \operatorname{Log}[1 - c x^2]}{4 c e} - \frac{b (c d^2 - e^2) \operatorname{Log}[1 + c x^2]}{4 c e}$$

Result (type 3, 94 leaves, 10 steps):

$$\frac{a (d + e x)^2}{2 e} + \frac{b d \operatorname{ArcTan}[\sqrt{c} x]}{\sqrt{c}} - \frac{b d \operatorname{ArcTanh}[\sqrt{c} x]}{\sqrt{c}} + b d x \operatorname{ArcTanh}[c x^2] + \frac{1}{2} b e x^2 \operatorname{ArcTanh}[c x^2] + \frac{b e \operatorname{Log}[1 - c^2 x^4]}{4 c}$$



### Problem 26: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{d + e x} dx$$

Optimal (type 4, 325 leaves, 19 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}[d + e x]}{e} - \frac{b \operatorname{Log}\left[\frac{e(1 - \sqrt{-c} x)}{\sqrt{-c} d + e}\right] \operatorname{Log}[d + e x]}{2 e} - \frac{b \operatorname{Log}\left[-\frac{e(1 + \sqrt{-c} x)}{\sqrt{-c} d - e}\right] \operatorname{Log}[d + e x]}{2 e} + \frac{b \operatorname{Log}\left[\frac{e(1 - \sqrt{c} x)}{\sqrt{c} d + e}\right] \operatorname{Log}[d + e x]}{2 e} + \\ & \frac{b \operatorname{Log}\left[-\frac{e(1 + \sqrt{c} x)}{\sqrt{c} d - e}\right] \operatorname{Log}[d + e x]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(d + e x)}{\sqrt{-c} d - e}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(d + e x)}{\sqrt{-c} d + e}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d + e}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d - e}\right]}{2 e} \end{aligned}$$

Result (type 8, 30 leaves, 2 steps):

$$b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^2]}{d + e x}, x\right] + \frac{a \operatorname{Log}[d + e x]}{e}$$

### Problem 27: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{(d + e x)^2} dx$$

Optimal (type 3, 166 leaves, 9 steps):

$$\frac{b \sqrt{c} \operatorname{ArcTan}[\sqrt{c} x]}{c d^2 + e^2} - \frac{b \sqrt{c} \operatorname{ArcTanh}[\sqrt{c} x]}{c d^2 - e^2} - \frac{a + b \operatorname{ArcTanh}[c x^2]}{e(d + e x)} + \frac{2 b c d e \operatorname{Log}[d + e x]}{c^2 d^4 - e^4} - \frac{b c d \operatorname{Log}[1 - c x^2]}{2 e(c d^2 - e^2)} + \frac{b c d \operatorname{Log}[1 + c x^2]}{2 e(c d^2 + e^2)}$$

Result (type 3, 166 leaves, 10 steps):

$$\frac{b \sqrt{c} \operatorname{ArcTan}[\sqrt{c} x]}{c d^2 + e^2} - \frac{b \sqrt{c} \operatorname{ArcTanh}[\sqrt{c} x]}{c d^2 - e^2} - \frac{a + b \operatorname{ArcTanh}[c x^2]}{e(d + e x)} + \frac{2 b c d e \operatorname{Log}[d + e x]}{c^2 d^4 - e^4} - \frac{b c d \operatorname{Log}[1 - c x^2]}{2 e(c d^2 - e^2)} + \frac{b c d \operatorname{Log}[1 + c x^2]}{2 e(c d^2 + e^2)}$$

### Problem 28: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{(d + e x)^3} dx$$

Optimal (type 3, 226 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b c d e}{(c^2 d^4 - e^4) (d + e x)} + \frac{b c^{3/2} d \operatorname{ArcTan}[\sqrt{c} x]}{(c d^2 + e^2)^2} - \frac{b c^{3/2} d \operatorname{ArcTanh}[\sqrt{c} x]}{(c d^2 - e^2)^2} - \\
& \frac{a + b \operatorname{ArcTanh}[c x^2]}{2 e (d + e x)^2} + \frac{b c e (3 c^2 d^4 + e^4) \operatorname{Log}[d + e x]}{(c^2 d^4 - e^4)^2} - \frac{b c (c d^2 + e^2) \operatorname{Log}[1 - c x^2]}{4 e (c d^2 - e^2)^2} + \frac{b c (c d^2 - e^2) \operatorname{Log}[1 + c x^2]}{4 e (c d^2 + e^2)^2}
\end{aligned}$$

Result (type 8, 34 leaves, 2 steps):

$$- \frac{a}{2 e (d + e x)^2} + b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^2]}{(d + e x)^3}, x\right]$$

**Problem 29: Result valid but suboptimal antiderivative.**

$$\int (d + e x) (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 1085 leaves, 77 steps):

$$\begin{aligned}
& a^2 d x + \frac{2 a b d \operatorname{ArcTan}[\sqrt{c} x]}{\sqrt{c}} + \frac{i b^2 d \operatorname{ArcTan}[\sqrt{c} x]^2}{\sqrt{c}} - \frac{2 a b d \operatorname{ArcTanh}[\sqrt{c} x]}{\sqrt{c}} - \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x]^2}{\sqrt{c}} + \\
& \frac{e (a + b \operatorname{ArcTanh}[c x^2])^2}{2 c} + \frac{1}{2} e x^2 (a + b \operatorname{ArcTanh}[c x^2])^2 + \frac{2 b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-\sqrt{c} x}\right]}{\sqrt{c}} - \frac{2 b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-i \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{\sqrt{c}} + \frac{2 b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+i \sqrt{c} x}\right]}{\sqrt{c}} - \frac{2 b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+\sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[-\frac{2 \sqrt{c}(1-\sqrt{-c} x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2 \sqrt{c}(1+\sqrt{-c} x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{b e (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}\left[\frac{2}{1-c x^2}\right]}{c} - a b d x \operatorname{Log}[1-c x^2] - \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{\sqrt{c}} + \\
& \frac{1}{4} b^2 d x \operatorname{Log}[1-c x^2]^2 + a b d x \operatorname{Log}[1+c x^2] + \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{\sqrt{c}} - \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{\sqrt{c}} - \\
& \frac{1}{2} b^2 d x \operatorname{Log}[1-c x^2] \operatorname{Log}[1+c x^2] + \frac{1}{4} b^2 d x \operatorname{Log}[1+c x^2]^2 + \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\sqrt{c} x}\right]}{\sqrt{c}} + \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i \sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{2 \sqrt{c}} + \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i \sqrt{c} x}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\sqrt{c} x}\right]}{\sqrt{c}} - \frac{b^2 d \operatorname{PolyLog}\left[2, 1+\frac{2 \sqrt{c}(1-\sqrt{-c} x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{2 \sqrt{c}} - \\
& \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{c}(1+\sqrt{-c} x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{2 \sqrt{c}} - \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{2 \sqrt{c}} - \frac{b^2 e \operatorname{PolyLog}\left[2, 1-\frac{2}{1-c x^2}\right]}{2 c}
\end{aligned}$$

Result (type 4, 1216 leaves, 104 steps):

$$\begin{aligned}
& \frac{a^2 (d + e x)^2}{2 e} + \frac{2 a b d \operatorname{ArcTan}[\sqrt{c} x]}{\sqrt{c}} + \frac{i b^2 d \operatorname{ArcTan}[\sqrt{c} x]^2}{\sqrt{c}} - \frac{2 a b d \operatorname{ArcTanh}[\sqrt{c} x]}{\sqrt{c}} - \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x]^2}{\sqrt{c}} + 2 a b d x \operatorname{ArcTanh}[c x^2] + \\
& a b e x^2 \operatorname{ArcTanh}[c x^2] + \frac{2 b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-\sqrt{c} x}\right]}{\sqrt{c}} - \frac{2 b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-i \sqrt{c} x}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{2 b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+i \sqrt{c} x}\right]}{\sqrt{c}} - \frac{2 b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+\sqrt{c} x}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[-\frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{\sqrt{c}} + \\
& \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{\sqrt{c}} - \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{\sqrt{c}} + \\
& \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{\sqrt{c}} + \frac{1}{4} b^2 d x \operatorname{Log}[1-c x^2]^2 - \frac{b^2 e (1-c x^2) \operatorname{Log}[1-c x^2]^2}{8 c} - \frac{b^2 e \operatorname{Log}[1-c x^2] \operatorname{Log}\left[\frac{1}{2}(1+c x^2)\right]}{4 c} + \\
& \frac{b^2 d \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{\sqrt{c}} - \frac{b^2 d \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{\sqrt{c}} + \frac{b^2 e \operatorname{Log}\left[\frac{1}{2}(1-c x^2)\right] \operatorname{Log}[1+c x^2]}{4 c} - \\
& \frac{1}{2} b^2 d x \operatorname{Log}[1-c x^2] \operatorname{Log}[1+c x^2] - \frac{1}{4} b^2 e x^2 \operatorname{Log}[1-c x^2] \operatorname{Log}[1+c x^2] + \frac{1}{4} b^2 d x \operatorname{Log}[1+c x^2]^2 + \frac{b^2 e (1+c x^2) \operatorname{Log}[1+c x^2]^2}{8 c} + \\
& \frac{a b e \operatorname{Log}[1-c^2 x^4]}{2 c} - \frac{b^2 e \operatorname{PolyLog}\left[2, \frac{1}{2}(1-c x^2)\right]}{4 c} + \frac{b^2 e \operatorname{PolyLog}\left[2, \frac{1}{2}(1+c x^2)\right]}{4 c} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-\sqrt{c} x}\right]}{\sqrt{c}} + \\
& \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i \sqrt{c} x}\right]}{\sqrt{c}} - \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{2 \sqrt{c}} + \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i \sqrt{c} x}\right]}{\sqrt{c}} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2}{1+\sqrt{c} x}\right]}{\sqrt{c}} - \\
& \frac{b^2 d \operatorname{PolyLog}\left[2, 1+\frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{2 \sqrt{c}} - \frac{b^2 d \operatorname{PolyLog}\left[2, 1-\frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{2 \sqrt{c}} - \frac{i b^2 d \operatorname{PolyLog}\left[2, 1-\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{2 \sqrt{c}}
\end{aligned}$$

Problem 30: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{d + e x} dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{d + e x}, x\right]$$

Result (type 8, 56 leaves, 2 steps):

$$2 a b \text{ CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^2]}{d + e x}, x\right] + b^2 \text{ CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^2]^2}{d + e x}, x\right] + \frac{a^2 \operatorname{Log}[d + e x]}{e}$$

Problem 31: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d + e x)^2} dx$$

Optimal (type 8, 22 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d + e x)^2}, x\right]$$

Result (type 8, 202 leaves, 12 steps):

$$-\frac{a^2}{e(d + e x)} + \frac{2 a b \sqrt{c} \operatorname{ArcTan}[\sqrt{c} x]}{c d^2 + e^2} - \frac{2 a b \sqrt{c} \operatorname{ArcTanh}[\sqrt{c} x]}{c d^2 - e^2} - \frac{2 a b \operatorname{ArcTanh}[c x^2]}{e(d + e x)} +$$

$$b^2 \text{ CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^2]^2}{(d + e x)^2}, x\right] + \frac{4 a b c d e \operatorname{Log}[d + e x]}{c^2 d^4 - e^4} - \frac{a b c d \operatorname{Log}[1 - c x^2]}{e(c d^2 - e^2)} + \frac{a b c d \operatorname{Log}[1 + c x^2]}{e(c d^2 + e^2)}$$

Problem 32: Result valid but suboptimal antiderivative.

$$\int (d + e x)^2 (a + b \operatorname{ArcTanh}[c x^3]) dx$$

Optimal (type 3, 336 leaves, 24 steps):

$$-\frac{\sqrt{3} b d e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{2 c^{2/3}} + \frac{\sqrt{3} b d e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{2 c^{2/3}} + \frac{\sqrt{3} b d^2 \operatorname{ArcTan}\left[\frac{1 + 2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} -$$

$$\frac{b d e \operatorname{ArcTanh}[c^{1/3} x]}{c^{2/3}} + \frac{(d + e x)^3 (a + b \operatorname{ArcTanh}[c x^3])}{3 e} + \frac{b d^2 \operatorname{Log}[1 - c^{2/3} x^2]}{2 c^{1/3}} + \frac{b d e \operatorname{Log}[1 - c^{1/3} x + c^{2/3} x^2]}{4 c^{2/3}} -$$

$$\frac{b d e \operatorname{Log}[1 + c^{1/3} x + c^{2/3} x^2]}{4 c^{2/3}} + \frac{b (c d^3 + e^3) \operatorname{Log}[1 - c x^3]}{6 c e} - \frac{b (c d^3 - e^3) \operatorname{Log}[1 + c x^3]}{6 c e} - \frac{b d^2 \operatorname{Log}[1 + c^{2/3} x^2 + c^{4/3} x^4]}{4 c^{1/3}}$$

Result (type 3, 332 leaves, 25 steps):

$$\frac{a (d + e x)^3}{3 e} - \frac{\sqrt{3} b d e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{2 c^{2/3}} + \frac{\sqrt{3} b d e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{2 c^{2/3}} + \frac{\sqrt{3} b d^2 \operatorname{ArcTan}\left[\frac{1+2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} -$$

$$\frac{b d e \operatorname{ArcTanh}\left[c^{1/3} x\right]}{c^{2/3}} + b d^2 x \operatorname{ArcTanh}\left[c x^3\right] + b d e x^2 \operatorname{ArcTanh}\left[c x^3\right] + \frac{1}{3} b e^2 x^3 \operatorname{ArcTanh}\left[c x^3\right] + \frac{b d^2 \operatorname{Log}\left[1 - c^{2/3} x^2\right]}{2 c^{1/3}} +$$

$$\frac{b d e \operatorname{Log}\left[1 - c^{1/3} x + c^{2/3} x^2\right]}{4 c^{2/3}} - \frac{b d e \operatorname{Log}\left[1 + c^{1/3} x + c^{2/3} x^2\right]}{4 c^{2/3}} - \frac{b d^2 \operatorname{Log}\left[1 + c^{2/3} x^2 + c^{4/3} x^4\right]}{4 c^{1/3}} + \frac{b e^2 \operatorname{Log}\left[1 - c^2 x^6\right]}{6 c}$$

Problem 33: Result optimal but 1 more steps used.

$$\int (d + e x) (a + b \operatorname{ArcTanh}[c x^3]) dx$$

Optimal (type 3, 285 leaves, 22 steps):

$$- \frac{\sqrt{3} b e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{4 c^{2/3}} + \frac{\sqrt{3} b e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{4 c^{2/3}} + \frac{\sqrt{3} b d \operatorname{ArcTan}\left[\frac{1+2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} - \frac{b e \operatorname{ArcTanh}\left[c^{1/3} x\right]}{2 c^{2/3}} - \frac{b d^2 \operatorname{ArcTanh}\left[c x^3\right]}{2 e} +$$

$$\frac{(d + e x)^2 (a + b \operatorname{ArcTanh}[c x^3])}{2 e} + \frac{b d \operatorname{Log}\left[1 - c^{2/3} x^2\right]}{2 c^{1/3}} + \frac{b e \operatorname{Log}\left[1 - c^{1/3} x + c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b e \operatorname{Log}\left[1 + c^{1/3} x + c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b d \operatorname{Log}\left[1 + c^{2/3} x^2 + c^{4/3} x^4\right]}{4 c^{1/3}}$$

Result (type 3, 285 leaves, 23 steps):

$$- \frac{\sqrt{3} b e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{4 c^{2/3}} + \frac{\sqrt{3} b e \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 c^{1/3} x}{\sqrt{3}}\right]}{4 c^{2/3}} + \frac{\sqrt{3} b d \operatorname{ArcTan}\left[\frac{1+2 c^{2/3} x^2}{\sqrt{3}}\right]}{2 c^{1/3}} - \frac{b e \operatorname{ArcTanh}\left[c^{1/3} x\right]}{2 c^{2/3}} - \frac{b d^2 \operatorname{ArcTanh}\left[c x^3\right]}{2 e} +$$

$$\frac{(d + e x)^2 (a + b \operatorname{ArcTanh}[c x^3])}{2 e} + \frac{b d \operatorname{Log}\left[1 - c^{2/3} x^2\right]}{2 c^{1/3}} + \frac{b e \operatorname{Log}\left[1 - c^{1/3} x + c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b e \operatorname{Log}\left[1 + c^{1/3} x + c^{2/3} x^2\right]}{8 c^{2/3}} - \frac{b d \operatorname{Log}\left[1 + c^{2/3} x^2 + c^{4/3} x^4\right]}{4 c^{1/3}}$$

Problem 34: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^3]}{d + e x} dx$$

Optimal (type 4, 523 leaves, 25 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}[d + e x]}{e} + \frac{b \operatorname{Log}\left[\frac{e(1 - c^{1/3} x)}{c^{1/3} d + e}\right] \operatorname{Log}[d + e x]}{2e} - \frac{b \operatorname{Log}\left[-\frac{e(1 + c^{1/3} x)}{c^{1/3} d - e}\right] \operatorname{Log}[d + e x]}{2e} + \\
& \frac{b \operatorname{Log}\left[-\frac{e(-1)^{1/3} + c^{1/3} x}{c^{1/3} d - (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2e} - \frac{b \operatorname{Log}\left[-\frac{e(-1)^{2/3} + c^{1/3} x}{c^{1/3} d - (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2e} + \frac{b \operatorname{Log}\left[\frac{(-1)^{2/3} e(1 + (-1)^{1/3} c^{1/3} x)}{c^{1/3} d + (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2e} - \\
& \frac{b \operatorname{Log}\left[\frac{(-1)^{1/3} e(1 + (-1)^{2/3} c^{1/3} x)}{c^{1/3} d + (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - e}\right]}{2e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + e}\right]}{2e} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - (-1)^{1/3} e}\right]}{2e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + (-1)^{1/3} e}\right]}{2e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - (-1)^{2/3} e}\right]}{2e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + (-1)^{2/3} e}\right]}{2e}
\end{aligned}$$

Result (type 8, 30 leaves, 2 steps):

$$b \operatorname{CannotIntegrate}\left[\frac{\operatorname{ArcTanh}[c x^3]}{d + e x}, x\right] + \frac{a \operatorname{Log}[d + e x]}{e}$$

Problem 35: Result optimal but 1 more steps used.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^3]}{(d + e x)^2} dx$$

Optimal (type 3, 414 leaves, 19 steps):

$$\begin{aligned}
& -\frac{\sqrt{3} b c^{1/3} \operatorname{ArcTan}\left[\frac{1 - 2 c^{1/3} x}{\sqrt{3}}\right]}{2(c^{2/3} d^2 + c^{1/3} d e + e^2)} - \frac{\sqrt{3} b c^{1/3} (c^{1/3} d + e) \operatorname{ArcTan}\left[\frac{1 + 2 c^{1/3} x}{\sqrt{3}}\right]}{2(c d^3 + e^3)} - \frac{a + b \operatorname{ArcTanh}[c x^3]}{e(d + e x)} + \\
& \frac{b c^{1/3} (c^{1/3} d - e) \operatorname{Log}[1 - c^{1/3} x]}{2(c d^3 + e^3)} + \frac{b c^{1/3} (c^{1/3} d + e) \operatorname{Log}[1 + c^{1/3} x]}{2(c d^3 - e^3)} - \frac{3 b c d^2 e^2 \operatorname{Log}[d + e x]}{c^2 d^6 - e^6} - \\
& \frac{b c^{1/3} (c^{1/3} d + e) \operatorname{Log}[1 - c^{1/3} x + c^{2/3} x^2]}{4(c d^3 - e^3)} - \frac{b c^{1/3} (c^{1/3} d - e) \operatorname{Log}[1 + c^{1/3} x + c^{2/3} x^2]}{4(c d^3 + e^3)} - \frac{b c d^2 \operatorname{Log}[1 - c x^3]}{2e(c d^3 + e^3)} + \frac{b c d^2 \operatorname{Log}[1 + c x^3]}{2e(c d^3 - e^3)}
\end{aligned}$$

Result (type 3, 414 leaves, 20 steps):

$$\begin{aligned}
& - \frac{\sqrt{3} b c^{1/3} \operatorname{ArcTan}\left[\frac{1-2 c^{1/3} x}{\sqrt{3}}\right]}{2\left(c^{2/3} d^2 + c^{1/3} d e + e^2\right)} - \frac{\sqrt{3} b c^{1/3}\left(c^{1/3} d + e\right) \operatorname{ArcTan}\left[\frac{1+2 c^{1/3} x}{\sqrt{3}}\right]}{2\left(c d^3 + e^3\right)} - \frac{a + b \operatorname{ArcTanh}\left[c x^3\right]}{e(d + e x)} + \\
& \frac{b c^{1/3}\left(c^{1/3} d - e\right) \operatorname{Log}\left[1 - c^{1/3} x\right]}{2\left(c d^3 + e^3\right)} + \frac{b c^{1/3}\left(c^{1/3} d + e\right) \operatorname{Log}\left[1 + c^{1/3} x\right]}{2\left(c d^3 - e^3\right)} - \frac{3 b c d^2 e^2 \operatorname{Log}[d + e x]}{c^2 d^6 - e^6} - \\
& \frac{b c^{1/3}\left(c^{1/3} d + e\right) \operatorname{Log}\left[1 - c^{1/3} x + c^{2/3} x^2\right]}{4\left(c d^3 - e^3\right)} - \frac{b c^{1/3}\left(c^{1/3} d - e\right) \operatorname{Log}\left[1 + c^{1/3} x + c^{2/3} x^2\right]}{4\left(c d^3 + e^3\right)} - \frac{b c d^2 \operatorname{Log}\left[1 - c x^3\right]}{2 e\left(c d^3 + e^3\right)} + \frac{b c d^2 \operatorname{Log}\left[1 + c x^3\right]}{2 e\left(c d^3 - e^3\right)}
\end{aligned}$$

## Test results for the 538 problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Problem 528: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^2} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c e (a + b \operatorname{ArcTanh}[c x])^2}{b} - \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} + \frac{1}{2} b c (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{2} b c e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{c e (a + b \operatorname{ArcTanh}[c x])^2}{b} + b c d \operatorname{Log}[x] - \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])^2}{4 e} - \frac{1}{2} b c e \operatorname{PolyLog}\left[2, c^2 x^2\right]$$

Problem 530: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^4} dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 c^2 e (a + b \operatorname{ArcTanh}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcTanh}[c x])^2}{3 b} - b c^3 e \operatorname{Log}[x] + \frac{1}{3} b c^3 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{6 x^2} - \\
& \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{3 x^3} + \frac{1}{6} b c^3 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{6} b c^3 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]
\end{aligned}$$

Result (type 4, 191 leaves, 17 steps):



$$\frac{2 c^2 e (a + b \operatorname{ArcTanh}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcTanh}[c x])^2}{3 b} + \frac{1}{3} b c^3 d \operatorname{Log}[x] - b c^3 e \operatorname{Log}[x] + \frac{1}{3} b c^3 e \operatorname{Log}[1 - c^2 x^2] -$$

$$\frac{b c (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{6 x^2} - \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{3 x^3} - \frac{b c^3 (d + e \operatorname{Log}[1 - c^2 x^2])^2}{12 e} - \frac{1}{6} b c^3 e \operatorname{PolyLog}[2, c^2 x^2]$$

Problem 532: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^6} dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2} + \frac{2 c^2 e (a + b \operatorname{ArcTanh}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcTanh}[c x])}{5 x} - \frac{c^5 e (a + b \operatorname{ArcTanh}[c x])^2}{5 b} -$$

$$\frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])}{20 x^4} - \frac{b c^3 (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{10 x^2} -$$

$$\frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{5 x^5} + \frac{1}{10} b c^5 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{10} b c^5 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\frac{7 b c^3 e}{60 x^2} + \frac{2 c^2 e (a + b \operatorname{ArcTanh}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcTanh}[c x])}{5 x} - \frac{c^5 e (a + b \operatorname{ArcTanh}[c x])^2}{5 b} + \frac{1}{5} b c^5 d \operatorname{Log}[x] -$$

$$\frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])}{20 x^4} - \frac{b c^3 (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{10 x^2} -$$

$$\frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{5 x^5} - \frac{b c^5 (d + e \operatorname{Log}[1 - c^2 x^2])^2}{20 e} - \frac{1}{10} b c^5 e \operatorname{PolyLog}[2, c^2 x^2]$$

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Test results for the 62 problems in "7.3.5 u (a+b arctanh(c+d x))^p.m"

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Test results for the 1378 problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.m"

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Problem 620: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x}\right)^2 dx$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{4 c^2 (1 - a x)^{-n/2} (1 + a x)^{n/2} \operatorname{Hypergeometric2F1}\left[2, \frac{n}{2}, \frac{2+n}{2}, \frac{1+ax}{1-ax}\right]}{a n} + \frac{2^{n/2} c^2 (1 - a x)^{2-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[1 - \frac{n}{2}, 2 - \frac{n}{2}, 3 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (4 - n)}$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{2^{3-\frac{n}{2}} c^2 (1 + a x)^{\frac{2+n}{2}} \operatorname{AppellF1}\left[\frac{2+n}{2}, \frac{1}{2} (-4 + n), 2, \frac{4+n}{2}, \frac{1}{2} (1 + a x), 1 + a x\right]}{a (2 + n)}$$

**Problem 621: Result valid but suboptimal antiderivative.**

$$\int e^{n \operatorname{ArcTanh}[a x]} \left( c - \frac{c}{a x} \right) dx$$

Optimal (type 5, 187 leaves, 6 steps):

$$\frac{c (1 - a x)^{2-\frac{n}{2}} (1 + a x)^{\frac{1}{2} (-2+n)} }{a (2 - n)} - \frac{2 c (1 - a x)^{1-\frac{n}{2}} (1 + a x)^{\frac{1}{2} (-2+n)} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} (-2 + n), \frac{n}{2}, \frac{1+ax}{1-ax}\right]}{a (2 - n)} + \frac{2^{n/2} c (1 - n) (1 - a x)^{2-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[\frac{2-n}{2}, 2 - \frac{n}{2}, 3 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (2 - n) (4 - n)}$$

Result (type 5, 184 leaves, 7 steps):

$$- \frac{2 c (1 - a x)^{-n/2} (1 + a x)^{n/2} \operatorname{Hypergeometric2F1}\left[1, -\frac{n}{2}, 1 - \frac{n}{2}, \frac{1-ax}{1+ax}\right]}{a n} - \frac{2^{1+\frac{n}{2}} c (1 - a x)^{1-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[1 - \frac{n}{2}, -\frac{n}{2}, 2 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (2 - n)} + \frac{2^{1+\frac{n}{2}} c (1 - a x)^{-n/2} \operatorname{Hypergeometric2F1}\left[-\frac{n}{2}, -\frac{n}{2}, 1 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a n}$$

**Problem 791: Result unnecessarily involves higher level functions.**

$$\int e^{n \operatorname{ArcTanh}[a x]} \left( c - \frac{c}{a^2 x^2} \right)^2 dx$$

Optimal (type 5, 331 leaves, 10 steps):

$$\begin{aligned}
& - \frac{4 c^2 (1 - a x)^{3 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-4 + n)}}{a (4 - n)} - \frac{c^2 (1 - a x)^{3 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-4 + n)}}{3 a^4 x^3} - \frac{c^2 (10 + n) (1 - a x)^{3 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-4 + n)}}{6 a^3 x^2} - \\
& \frac{c^2 (14 + 5 n + n^2) (1 - a x)^{3 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-4 + n)}}{6 a^2 x} - \frac{c^2 n (10 - n^2) (1 - a x)^{2 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-4 + n)}}{3 a (4 - n)} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} (-4 + n), \frac{1}{2} (-2 + n), \frac{1 + a x}{1 - a x}\right] + \\
& \frac{2^{-1 + \frac{n}{2}} c^2 n (1 - a x)^{3 - \frac{n}{2}} \operatorname{Hypergeometric2F1}\left[\frac{4 + n}{2}, 3 - \frac{n}{2}, 4 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (24 - 10 n + n^2)}
\end{aligned}$$

Result (type 6, 71 leaves, 3 steps):

$$\frac{2^{3 - \frac{n}{2}} c^2 (1 + a x)^{\frac{6 + n}{2}} \operatorname{AppellF1}\left[\frac{6 + n}{2}, \frac{1}{2} (-4 + n), 4, \frac{8 + n}{2}, \frac{1}{2} (1 + a x), 1 + a x\right]}{a (6 + n)}$$

Problem 792: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcTanh}[a x]} \left( c - \frac{c}{a^2 x^2} \right) dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{4 c (1 - a x)^{1 - \frac{n}{2}} (1 + a x)^{\frac{1}{2} (-2 + n)} \operatorname{Hypergeometric2F1}\left[2, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{1 - a x}{1 + a x}\right]}{a (2 - n)} - \frac{2^{1 + \frac{n}{2}} c (1 - a x)^{1 - \frac{n}{2}} \operatorname{Hypergeometric2F1}\left[1 - \frac{n}{2}, -\frac{n}{2}, 2 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (2 - n)}$$

Result (type 6, 70 leaves, 3 steps):

$$- \frac{2^{2 - \frac{n}{2}} c (1 + a x)^{\frac{4 + n}{2}} \operatorname{AppellF1}\left[\frac{4 + n}{2}, \frac{1}{2} (-2 + n), 2, \frac{6 + n}{2}, \frac{1}{2} (1 + a x), 1 + a x\right]}{a (4 + n)}$$

Problem 795: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcTanh}[a x]} \left( c - \frac{c}{a^2 x^2} \right)^{3/2} dx$$

Optimal (type 5, 430 leaves, 9 steps):

$$\begin{aligned}
& - \frac{\left(c - \frac{c}{a^2 x^2}\right)^{3/2} x (1 - a x)^{\frac{5-n}{2}} (1 + a x)^{\frac{1}{2}(-3+n)}}{2 (1 - a^2 x^2)^{3/2}} - \frac{a (4 + n) \left(c - \frac{c}{a^2 x^2}\right)^{3/2} x^2 (1 - a x)^{\frac{5-n}{2}} (1 + a x)^{\frac{1}{2}(-3+n)}}{2 (1 - a^2 x^2)^{3/2}} - \frac{3 a^2 \left(c - \frac{c}{a^2 x^2}\right)^{3/2} x^3 (1 - a x)^{\frac{5-n}{2}} (1 + a x)^{\frac{1}{2}(-3+n)}}{(3 - n) (1 - a^2 x^2)^{3/2}} \\
& - \frac{1}{(3 - n) (1 - a^2 x^2)^{3/2}} a^2 (3 - n^2) \left(c - \frac{c}{a^2 x^2}\right)^{3/2} x^3 (1 - a x)^{\frac{3-n}{2}} (1 + a x)^{\frac{1}{2}(-3+n)} \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-3 + n), \frac{1}{2}(-1 + n), \frac{1 + a x}{1 - a x}\right] + \\
& \frac{2^{\frac{1}{2}(-1+n)} a^2 n \left(c - \frac{c}{a^2 x^2}\right)^{3/2} x^3 (1 - a x)^{\frac{5-n}{2}} \text{Hypergeometric2F1}\left[\frac{3-n}{2}, \frac{5-n}{2}, \frac{7-n}{2}, \frac{1}{2}(1 - a x)\right]}{(3 - n) (5 - n) (1 - a^2 x^2)^{3/2}}
\end{aligned}$$

Result (type 6, 103 leaves, 3 steps):

$$- \frac{2^{\frac{5}{2} - \frac{n}{2}} a^2 \left(c - \frac{c}{a^2 x^2}\right)^{3/2} x^3 (1 + a x)^{\frac{5+n}{2}} \text{AppellF1}\left[\frac{5+n}{2}, \frac{1}{2}(-3 + n), 3, \frac{7+n}{2}, \frac{1}{2}(1 + a x), 1 + a x\right]}{(5 + n) (1 - a^2 x^2)^{3/2}}$$

Problem 796: Result valid but suboptimal antiderivative.

$$\int e^{n \text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a^2 x^2}} \, dx$$

Optimal (type 5, 272 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{3-n}{2}} (1 + a x)^{\frac{1}{2}(-1+n)}}{(1 - n) \sqrt{1 - a^2 x^2}} + \frac{2 \sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{1-n}{2}} (1 + a x)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \frac{1+a x}{1-a x}\right]}{(1 - n) \sqrt{1 - a^2 x^2}} + \\
& \frac{2^{\frac{1+n}{2}} n \sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{3-n}{2}} \text{Hypergeometric2F1}\left[\frac{1-n}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1}{2}(1 - a x)\right]}{(3 - 4 n + n^2) \sqrt{1 - a^2 x^2}}
\end{aligned}$$

Result (type 5, 302 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 \sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{1}{2}(-1-n)} (1 + a x)^{\frac{1+n}{2}} \text{Hypergeometric2F1}\left[1, \frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{1-ax}{1+ax}\right]}{(1+n) \sqrt{1-a^2 x^2}} - \\
& \frac{2^{\frac{3+n}{2}} \sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{1}{2}(-1-n)} \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{1}{2}(1-ax)\right]}{(1+n) \sqrt{1-a^2 x^2}} + \\
& \frac{2^{\frac{3+n}{2}} \sqrt{c - \frac{c}{a^2 x^2}} x (1 - a x)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{3-n}{2}, \frac{1}{2}(1-ax)\right]}{(1-n) \sqrt{1-a^2 x^2}}
\end{aligned}$$

Problem 1316: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{x (c - a^2 c x^2)} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{(1-ax)^{-n/2} (1+ax)^{n/2}}{c n} - \frac{2 (1-ax)^{-n/2} (1+ax)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{1+ax}{1-ax}\right]}{c n}$$

Result (type 5, 100 leaves, 3 steps):

$$\frac{(1-ax)^{-n/2} (1+ax)^{n/2}}{c n} - \frac{2 (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{1-ax}{1+ax}\right]}{c (2-n)}$$

Problem 1317: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{x^2 (c - a^2 c x^2)} dx$$

Optimal (type 5, 123 leaves, 5 steps):

$$\frac{a (1+n) (1-ax)^{-n/2} (1+ax)^{n/2}}{c n} - \frac{(1-ax)^{-n/2} (1+ax)^{n/2}}{c x} - \frac{2 a (1-ax)^{-n/2} (1+ax)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{1+ax}{1-ax}\right]}{c}$$

Result (type 5, 137 leaves, 5 steps):

$$\frac{a (1+n) (1-ax)^{-n/2} (1+ax)^{n/2}}{cn} - \frac{(1-ax)^{-n/2} (1+ax)^{n/2}}{cx} - \frac{2an (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{1-ax}{1+ax}\right]}{c(2-n)}$$

Problem 1323: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{x (c - a^2 c x^2)^2} dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$\frac{(1-ax)^{-1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 (2+n)} - \frac{(4-n-n^2) (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (4-n^2)} +$$

$$\frac{(4+n) (1-ax)^{-n/2} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (2+n)} - \frac{2 (1-ax)^{-n/2} (1+ax)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{1+ax}{1-ax}\right]}{c^2 n}$$

Result (type 5, 200 leaves, 6 steps):

$$\frac{(1-ax)^{-1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 (2+n)} - \frac{(4-n-n^2) (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (4-n^2)} +$$

$$\frac{(4+n) (1-ax)^{-n/2} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (2+n)} - \frac{2 (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{1-ax}{1+ax}\right]}{c^2 (2-n)}$$

Problem 1324: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{x^2 (c - a^2 c x^2)^2} dx$$

Optimal (type 5, 239 leaves, 7 steps):

$$\frac{a (3+n) (1-ax)^{-1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 (2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 x} - \frac{a (6+4n-n^2-n^3) (1-ax)^{1-\frac{n}{2}} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (4-n^2)} +$$

$$\frac{a (6+4n+n^2) (1-ax)^{-n/2} (1+ax)^{\frac{1}{2}(-2+n)}}{c^2 n (2+n)} - \frac{2a (1-ax)^{-n/2} (1+ax)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{1+ax}{1-ax}\right]}{c^2}$$

Result (type 5, 253 leaves, 7 steps):

$$\frac{a(3+n)(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2(2+n)} - \frac{(1-ax)^{-1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2x} - \frac{a(6+4n-n^2-n^3)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n(4-n^2)} +$$

$$\frac{a(6+4n+n^2)(1-ax)^{-n/2}(1+ax)^{\frac{1}{2}(-2+n)}}{c^2n(2+n)} - \frac{2an(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)} \operatorname{Hypergeometric2F1}\left[1, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{1-ax}{1+ax}\right]}{c^2(2-n)}$$

Problem 1331: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]} \sqrt{c-a^2cx^2}}{x} dx$$

Optimal (type 5, 269 leaves, 6 steps):

$$-\frac{(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{c-a^2cx^2}}{(1-n)\sqrt{1-a^2x^2}} + \frac{2(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{(1-n)\sqrt{1-a^2x^2}} +$$

$$\frac{2^{\frac{1+n}{2}}n(1-ax)^{\frac{3-n}{2}}\sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[\frac{1-n}{2}, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1}{2}(1-ax)\right]}{(3-4n+n^2)\sqrt{1-a^2x^2}}$$

Result (type 5, 299 leaves, 7 steps):

$$\frac{2(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1+n}{2}}\sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{1-ax}{1+ax}\right]}{(1+n)\sqrt{1-a^2x^2}} -$$

$$\frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{1}{2}(-1-n)}\sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{1}{2}(1-ax)\right]}{(1+n)\sqrt{1-a^2x^2}} +$$

$$\frac{2^{\frac{3+n}{2}}(1-ax)^{\frac{1-n}{2}}\sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), \frac{1-n}{2}, \frac{3-n}{2}, \frac{1}{2}(1-ax)\right]}{(1-n)\sqrt{1-a^2x^2}}$$

Problem 1332: Result unnecessarily involves higher level functions.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]} \sqrt{c-a^2cx^2}}{x^2} dx$$

Optimal (type 5, 268 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1+n}{2}} \sqrt{c-a^2cx^2}}{x \sqrt{1-a^2x^2}} - \frac{2an(1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1-n}{2}, \frac{3-n}{2}, \frac{1-ax}{1+ax}\right]}{(1-n) \sqrt{1-a^2x^2}} + \\
& \frac{2^{\frac{1+n}{2}} a (1-ax)^{\frac{1-n}{2}} \sqrt{c-a^2cx^2} \operatorname{Hypergeometric2F1}\left[\frac{1-n}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \frac{1}{2}(1-ax)\right]}{(1-n) \sqrt{1-a^2x^2}}
\end{aligned}$$

Result (type 6, 97 leaves, 3 steps):

$$\frac{2^{\frac{3}{2}} \frac{n}{2} a (1+ax)^{\frac{3+n}{2}} \sqrt{c-a^2cx^2} \operatorname{AppellF1}\left[\frac{3+n}{2}, \frac{1}{2}(-1+n), 2, \frac{5+n}{2}, \frac{1}{2}(1+ax), 1+ax\right]}{(3+n) \sqrt{1-a^2x^2}}$$

Problem 1345: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]}}{x (c-a^2cx^2)^{3/2}} dx$$

Optimal (type 5, 243 leaves, 6 steps):

$$\begin{aligned}
& \frac{(1-ax)^{\frac{1}{2}(-1-n)} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2x^2}}{c(1+n) \sqrt{c-a^2cx^2}} - \frac{(2+n) (1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2x^2}}{c(1-n^2) \sqrt{c-a^2cx^2}} + \\
& \frac{2(1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c(1-n) \sqrt{c-a^2cx^2}}
\end{aligned}$$

Result (type 5, 247 leaves, 6 steps):

$$\begin{aligned}
& \frac{(1-ax)^{\frac{1}{2}(-1-n)} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2x^2}}{c(1+n) \sqrt{c-a^2cx^2}} - \frac{(2+n) (1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2x^2}}{c(1-n^2) \sqrt{c-a^2cx^2}} - \\
& \frac{2(1-ax)^{\frac{3-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c(3-n) \sqrt{c-a^2cx^2}}
\end{aligned}$$

Problem 1346: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]}}{x^2 (c-a^2cx^2)^{3/2}} dx$$

Optimal (type 5, 321 leaves, 7 steps):



$$\frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} -$$

$$\frac{a(2+2n+n^2)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1-n^2)\sqrt{c-a^2cx^2}} + \frac{2an(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c(1-n)\sqrt{c-a^2cx^2}}$$

Result (type 5, 325 leaves, 7 steps):

$$\frac{a(2+n)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{cx\sqrt{c-a^2cx^2}} -$$

$$\frac{a(2+2n+n^2)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{c(1-n^2)\sqrt{c-a^2cx^2}} - \frac{2an(1-ax)^{\frac{3-n}{2}}(1+ax)^{\frac{1}{2}(-3+n)}\sqrt{1-a^2x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c(3-n)\sqrt{c-a^2cx^2}}$$

Problem 1347: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]}}{x^3(c-a^2cx^2)^{3/2}} dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\frac{a^2(3+2n+n^2)(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1+n)\sqrt{c-a^2cx^2}} - \frac{(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx^2\sqrt{c-a^2cx^2}} -$$

$$\frac{an(1-ax)^{\frac{1}{2}(-1-n)}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2cx\sqrt{c-a^2cx^2}} - \frac{a^2(6+5n+2n^2+n^3)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2}}{2c(1-n^2)\sqrt{c-a^2cx^2}} +$$

$$\frac{a^2(3+n^2)(1-ax)^{\frac{1-n}{2}}(1+ax)^{\frac{1}{2}(-1+n)}\sqrt{1-a^2x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c(1-n)\sqrt{c-a^2cx^2}}$$

Result (type 5, 422 leaves, 8 steps):

$$\begin{aligned}
& \frac{a^2 (3 + 2n + n^2) (1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2}}{2c(1+n) \sqrt{c - a^2 cx^2}} - \frac{(1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2}}{2cx^2 \sqrt{c - a^2 cx^2}} - \\
& \frac{an(1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2}}{2cx \sqrt{c - a^2 cx^2}} - \frac{a^2(6 + 5n + 2n^2 + n^3) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2}}{2c(1 - n^2) \sqrt{c - a^2 cx^2}} - \\
& \frac{a^2(3 + n^2) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c(3 - n) \sqrt{c - a^2 cx^2}}
\end{aligned}$$

Problem 1352: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]}}{x(c - a^2 cx^2)^{5/2}} dx$$

Optimal (type 5, 417 leaves, 8 steps):

$$\begin{aligned}
& \frac{(1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(3 + n) \sqrt{c - a^2 cx^2}} + \frac{(6 + n) (1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(1 + n) (3 + n) \sqrt{c - a^2 cx^2}} - \\
& \frac{(15 + 6n + n^2) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(3 + n) (1 - n^2) \sqrt{c - a^2 cx^2}} + \frac{(18 + 7n - 2n^2 - n^3) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(9 - 10n^2 + n^4) \sqrt{c - a^2 cx^2}} + \\
& \frac{2(1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c^2(1 - n) \sqrt{c - a^2 cx^2}}
\end{aligned}$$

Result (type 5, 421 leaves, 8 steps):

$$\begin{aligned}
& \frac{(1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(3 + n) \sqrt{c - a^2 cx^2}} + \frac{(6 + n) (1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(1 + n) (3 + n) \sqrt{c - a^2 cx^2}} - \\
& \frac{(15 + 6n + n^2) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(3 + n) (1 - n^2) \sqrt{c - a^2 cx^2}} + \frac{(18 + 7n - 2n^2 - n^3) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{c^2(9 - 10n^2 + n^4) \sqrt{c - a^2 cx^2}} - \\
& \frac{2(1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c^2(3 - n) \sqrt{c - a^2 cx^2}}
\end{aligned}$$

### Problem 1353: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{x^2 (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 5, 507 leaves, 9 steps):

$$\begin{aligned} & \frac{a (4+n) (1-ax)^{\frac{1}{2}(-3-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \sqrt{c-a^2 c x^2}} - \\ & \frac{(1-ax)^{\frac{1}{2}(-3-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 x \sqrt{c-a^2 c x^2}} + \frac{a (12+6n+n^2) (1-ax)^{\frac{1}{2}(-1-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}} - \\ & \frac{a (24+15n+6n^2+n^3) (1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) (1-n^2) \sqrt{c-a^2 c x^2}} + \frac{a (24+18n+7n^2-2n^3-n^4) (1-ax)^{\frac{3-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (9-10n^2+n^4) \sqrt{c-a^2 c x^2}} + \\ & \frac{2 a n (1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-1+n)} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c^2 (1-n) \sqrt{c-a^2 c x^2}} \end{aligned}$$

Result (type 5, 511 leaves, 9 steps):

$$\begin{aligned} & \frac{a (4+n) (1-ax)^{\frac{1}{2}(-3-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) \sqrt{c-a^2 c x^2}} - \\ & \frac{(1-ax)^{\frac{1}{2}(-3-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 x \sqrt{c-a^2 c x^2}} + \frac{a (12+6n+n^2) (1-ax)^{\frac{1}{2}(-1-n)} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (1+n) (3+n) \sqrt{c-a^2 c x^2}} - \\ & \frac{a (24+15n+6n^2+n^3) (1-ax)^{\frac{1-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (3+n) (1-n^2) \sqrt{c-a^2 c x^2}} + \frac{a (24+18n+7n^2-2n^3-n^4) (1-ax)^{\frac{3-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2}}{c^2 (9-10n^2+n^4) \sqrt{c-a^2 c x^2}} - \\ & \frac{2 a n (1-ax)^{\frac{3-n}{2}} (1+ax)^{\frac{1}{2}(-3+n)} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c^2 (3-n) \sqrt{c-a^2 c x^2}} \end{aligned}$$

### Problem 1354: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{x^3 (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 5, 623 leaves, 10 steps):

$$\frac{a^2 (5 + 4n + n^2) (1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (3 + n) \sqrt{c - a^2 c x^2}} - \frac{(1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 x^2 \sqrt{c - a^2 c x^2}} -$$

$$\frac{a n (1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 x \sqrt{c - a^2 c x^2}} + \frac{a^2 (30 + 17n + 6n^2 + n^3) (1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (1 + n) (3 + n) \sqrt{c - a^2 c x^2}} -$$

$$\frac{a^2 (75 + 54n + 20n^2 + 6n^3 + n^4) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (3 + n) (1 - n^2) \sqrt{c - a^2 c x^2}} + \frac{a^2 (90 + 59n + 8n^2 + 2n^3 - 2n^4 - n^5) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (9 - 10n^2 + n^4) \sqrt{c - a^2 c x^2}} +$$

$$\frac{a^2 (5 + n^2) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-1+n)} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \frac{1+ax}{1-ax}\right]}{c^2 (1 - n) \sqrt{c - a^2 c x^2}}$$

Result (type 5, 628 leaves, 10 steps):

$$\frac{a^2 (5 + 4n + n^2) (1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (3 + n) \sqrt{c - a^2 c x^2}} - \frac{(1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 x^2 \sqrt{c - a^2 c x^2}} -$$

$$\frac{a n (1 - ax)^{\frac{1}{2}(-3-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 x \sqrt{c - a^2 c x^2}} + \frac{a^2 (30 + 17n + 6n^2 + n^3) (1 - ax)^{\frac{1}{2}(-1-n)} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (1 + n) (3 + n) \sqrt{c - a^2 c x^2}} -$$

$$\frac{a^2 (75 + 54n + 20n^2 + 6n^3 + n^4) (1 - ax)^{\frac{1-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (3 + n) (1 - n^2) \sqrt{c - a^2 c x^2}} + \frac{a^2 (90 + 59n + 8n^2 + 2n^3 - 2n^4 - n^5) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2}}{2c^2 (9 - 10n^2 + n^4) \sqrt{c - a^2 c x^2}} -$$

$$\frac{a^2 (5 + n^2) (1 - ax)^{\frac{3-n}{2}} (1 + ax)^{\frac{1}{2}(-3+n)} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{1-ax}{1+ax}\right]}{c^2 (3 - n) \sqrt{c - a^2 c x^2}}$$

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Test results for the 361 problems in "7.3.7 Inverse hyperbolic tangent functions.m"

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Test results for the 300 problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

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Problem 76: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{ArcCoth}[a + bx]}{c + dx^2} dx$$

Optimal (type 4, 673 leaves, 15 steps):

$$\frac{\frac{\text{Log}\left[-\frac{1-a-bx}{a+bx}\right] \text{Log}\left[1 + \frac{(b^2c+a^2d)(1-a-bx)}{(b^2c-b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}\left[-\frac{1-a-bx}{a+bx}\right] \text{Log}\left[1 + \frac{(b^2c+a^2d)(1-a-bx)}{(b^2c+b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{Log}\left[\frac{1+a+bx}{a+bx}\right] \text{Log}\left[1 - \frac{(b^2c+a^2d)(1+a+bx)}{(b^2c-b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}\left[\frac{1+a+bx}{a+bx}\right] \text{Log}\left[1 - \frac{(b^2c+a^2d)(1+a+bx)}{(b^2c+b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, -\frac{(b^2c+a^2d)(1-a-bx)}{(b^2c-b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{(b^2c+a^2d)(1-a-bx)}{(b^2c+b\sqrt{-c}\sqrt{d}-(1-a)ad)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{(b^2c+a^2d)(1+a+bx)}{(b^2c-b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, \frac{(b^2c+a^2d)(1+a+bx)}{(b^2c+b\sqrt{-c}\sqrt{d}+a(1+a)d)(a+bx)}\right]}{4\sqrt{-c}\sqrt{d}}}$$

Result (type 4, 597 leaves, 37 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \left(\text{Log}[-1+a+bx] - \text{Log}\left[-\frac{1-a-bx}{a+bx}\right] - \text{Log}[a+bx]\right)}{2\sqrt{c}\sqrt{d}} + \frac{\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \left(\text{Log}[a+bx] - \text{Log}[1+a+bx] + \text{Log}\left[\frac{1+a+bx}{a+bx}\right]\right)}{2\sqrt{c}\sqrt{d}} - \frac{\text{Log}[-1+a+bx] \text{Log}\left[\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{Log}[1+a+bx] \text{Log}\left[\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{Log}[-1+a+bx] \text{Log}\left[\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}[1+a+bx] \text{Log}\left[\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}$$

Problem 275: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b \text{ArcCoth}[cx]) (d+e \text{Log}[1-c^2x^2])}{x^2} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{ce(a+b \text{ArcCoth}[cx])^2}{b} - \frac{(a+b \text{ArcCoth}[cx]) (d+e \text{Log}[1-c^2x^2])}{x} + \frac{1}{2}bc(d+e \text{Log}[1-c^2x^2]) \text{Log}\left[1-\frac{1}{1-c^2x^2}\right] - \frac{1}{2}bce \text{PolyLog}\left[2, \frac{1}{1-c^2x^2}\right]$$

Result (type 4, 94 leaves, 8 steps):

$$-\frac{ce(a+b \text{ArcCoth}[cx])^2}{b} + bcd \text{Log}[x] - \frac{(a+b \text{ArcCoth}[cx]) (d+e \text{Log}[1-c^2x^2])}{x} - \frac{bc(d+e \text{Log}[1-c^2x^2])^2}{4e} - \frac{1}{2}bce \text{PolyLog}\left[2, c^2x^2\right]$$

## Problem 276: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^4} dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\frac{2 c^2 e (a + b \operatorname{ArcCoth}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcCoth}[c x])^2}{3 b} - b c^3 e \operatorname{Log}[x] + \frac{1}{3} b c^3 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{6 x^2} -$$

$$\frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{3 x^3} + \frac{1}{6} b c^3 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{6} b c^3 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 4, 191 leaves, 17 steps):

$$\frac{2 c^2 e (a + b \operatorname{ArcCoth}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcCoth}[c x])^2}{3 b} + \frac{1}{3} b c^3 d \operatorname{Log}[x] - b c^3 e \operatorname{Log}[x] + \frac{1}{3} b c^3 e \operatorname{Log}[1 - c^2 x^2] -$$

$$\frac{b c (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{6 x^2} - \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{3 x^3} - \frac{b c^3 (d + e \operatorname{Log}[1 - c^2 x^2])^2}{12 e} - \frac{1}{6} b c^3 e \operatorname{PolyLog}\left[2, c^2 x^2\right]$$

## Problem 277: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^6} dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7 b c^3 e}{60 x^2} + \frac{2 c^2 e (a + b \operatorname{ArcCoth}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcCoth}[c x])}{5 x} - \frac{c^5 e (a + b \operatorname{ArcCoth}[c x])^2}{5 b} -$$

$$\frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])}{20 x^4} - \frac{b c^3 (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{10 x^2} -$$

$$\frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{5 x^5} + \frac{1}{10} b c^5 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{10} b c^5 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 4, 250 leaves, 26 steps):

$$\begin{aligned} & \frac{7 b c^3 e}{60 x^2} + \frac{2 c^2 e (a + b \operatorname{ArcCoth}[c x])}{15 x^3} + \frac{2 c^4 e (a + b \operatorname{ArcCoth}[c x])}{5 x} - \frac{c^5 e (a + b \operatorname{ArcCoth}[c x])^2}{5 b} + \frac{1}{5} b c^5 d \operatorname{Log}[x] - \\ & \frac{5}{6} b c^5 e \operatorname{Log}[x] + \frac{19}{60} b c^5 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (d + e \operatorname{Log}[1 - c^2 x^2])}{20 x^4} - \frac{b c^3 (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{10 x^2} - \\ & \frac{(a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{5 x^5} - \frac{b c^5 (d + e \operatorname{Log}[1 - c^2 x^2])^2}{20 e} - \frac{1}{10} b c^5 e \operatorname{PolyLog}[2, c^2 x^2] \end{aligned}$$

## Test results for the 935 problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.m"

Problem 542: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcCoth}[a x]} \left( c - \frac{c}{a x} \right) dx$$

Optimal (type 5, 185 leaves, 5 steps):

$$\begin{aligned} & c \left( 1 - \frac{1}{a x} \right)^{1 - \frac{n}{2}} \left( 1 + \frac{1}{a x} \right)^{n/2} x - \frac{2 c (1 - n) \left( 1 - \frac{1}{a x} \right)^{-n/2} \left( 1 + \frac{1}{a x} \right)^{n/2} \operatorname{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a n} - \\ & \frac{2^{n/2} c \left( 1 - \frac{1}{a x} \right)^{1 - \frac{n}{2}} \operatorname{Hypergeometric2F1}\left[1 - \frac{n}{2}, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right]}{a (2 - n)} \end{aligned}$$

Result (type 6, 81 leaves, 2 steps):

$$- \frac{2^{2 - \frac{n}{2}} c \left( 1 + \frac{1}{a x} \right)^{\frac{2+n}{2}} \operatorname{AppellF1}\left[\frac{2+n}{2}, \frac{1}{2} (-2+n), 2, \frac{4+n}{2}, \frac{a+\frac{1}{x}}{2a}, 1 + \frac{1}{a x}\right]}{a (2 + n)}$$

Problem 751: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcCoth}[a x]} x^3}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 5, 359 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(2+n) \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1+n)(c - a^2 c x^2)^{3/2}} + \frac{(2+2n+n^2) \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1-n)(1+n)(c - a^2 c x^2)^{3/2}} + \\
& \frac{\left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{(c - a^2 c x^2)^{3/2}} - \frac{2n \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a(1-n)(c - a^2 c x^2)^{3/2}}
\end{aligned}$$

Result (type 5, 363 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(2+n) \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1+n)(c - a^2 c x^2)^{3/2}} + \frac{(2+2n+n^2) \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^3}{a(1-n)(1+n)(c - a^2 c x^2)^{3/2}} + \\
& \frac{\left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^4}{(c - a^2 c x^2)^{3/2}} + \frac{2n \left(1 - \frac{1}{a^2 x^2}\right)^{3/2} \left(1 - \frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^3 \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a(3-n)(c - a^2 c x^2)^{3/2}}
\end{aligned}$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcCoth}[a x]} x^4}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 5, 463 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-3-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n)(c - a^2 c x^2)^{5/2}} - \frac{(6+n) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n)(3+n)(c - a^2 c x^2)^{5/2}} + \\
& \frac{(15+6n+n^2) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n)(1+n)(3+n)(c - a^2 c x^2)^{5/2}} - \frac{(18+7n-2n^2-n^3) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(9-10n^2+n^4)(c - a^2 c x^2)^{5/2}} - \\
& \frac{2 \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} x^5 \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}(-1+n), \frac{1+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{(1-n)(c - a^2 c x^2)^{5/2}}
\end{aligned}$$

Result (type 5, 467 leaves, 8 steps):



$$\begin{aligned}
& - \frac{\left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-3-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(3+n) (c - a^2 c x^2)^{5/2}} - \frac{(6+n) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1}{2}(-1-n)} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1+n) (3+n) (c - a^2 c x^2)^{5/2}} + \\
& \frac{(15+6n+n^2) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(1-n) (1+n) (3+n) (c - a^2 c x^2)^{5/2}} - \frac{(18+7n-2n^2-n^3) \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5}{(9-10n^2+n^4) (c - a^2 c x^2)^{5/2}} + \\
& \frac{2 \left(1 - \frac{1}{a^2 x^2}\right)^{5/2} \left(1 - \frac{1}{a x}\right)^{\frac{3-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-3+n)} x^5 \operatorname{Hypergeometric2F1}\left[1, \frac{3-n}{2}, \frac{5-n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{(3-n) (c - a^2 c x^2)^{5/2}}
\end{aligned}$$

Problem 928: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcCoth}[a x]} \left(c - \frac{c}{a^2 x^2}\right) dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$\frac{4 c \left(1 - \frac{1}{a x}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-2+n)} \operatorname{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{a (2-n)} - \frac{2^{1+\frac{n}{2}} c \left(1 - \frac{1}{a x}\right)^{1-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[1-\frac{n}{2}, -\frac{n}{2}, 2-\frac{n}{2}, \frac{a-\frac{1}{x}}{2a}\right]}{a (2-n)}$$

Result (type 6, 81 leaves, 2 steps):

$$- \frac{2^{2-\frac{n}{2}} c \left(1 + \frac{1}{a x}\right)^{\frac{4+n}{2}} \operatorname{AppellF1}\left[\frac{4+n}{2}, \frac{1}{2}(-2+n), 2, \frac{6+n}{2}, \frac{a+\frac{1}{x}}{2a}, 1+\frac{1}{a x}\right]}{a (4+n)}$$

Problem 929: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcCoth}[a x]}}{c - \frac{c}{a^2 x^2}} dx$$

Optimal (type 5, 150 leaves, 5 steps):

$$- \frac{(1+n) \left(1 - \frac{1}{a x}\right)^{-n/2} \left(1 + \frac{1}{a x}\right)^{n/2}}{a c n} + \frac{\left(1 - \frac{1}{a x}\right)^{-n/2} \left(1 + \frac{1}{a x}\right)^{n/2} x}{c} + \frac{2 \left(1 - \frac{1}{a x}\right)^{-n/2} \left(1 + \frac{1}{a x}\right)^{n/2} \operatorname{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right]}{a c}$$

Result (type 5, 164 leaves, 5 steps):

$$-\frac{(1+n) \left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{n/2}}{a c n} + \frac{\left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{n/2} x}{c} + \frac{2 n \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right]}{a c (2 - n)}$$

Problem 930: Result valid but suboptimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcCoth}[ax]}}{\left(c - \frac{c}{a^2 x^2}\right)^2} dx$$

Optimal (type 5, 289 leaves, 7 steps):

$$-\frac{(3+n) \left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)} + \frac{(6+4n-n^2-n^3) \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)} - \frac{(6+4n+n^2) \left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)} +$$

$$\frac{\left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} x}{c^2} + \frac{2 \left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{n/2} \text{Hypergeometric2F1}\left[1, \frac{n}{2}, \frac{2+n}{2}, \frac{a + \frac{1}{x}}{a - \frac{1}{x}}\right]}{a c^2}$$

Result (type 5, 303 leaves, 7 steps):

$$-\frac{(3+n) \left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2+n)} + \frac{(6+4n-n^2-n^3) \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 (2-n) n (2+n)} - \frac{(6+4n+n^2) \left(1 - \frac{1}{ax}\right)^{-n/2} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{a c^2 n (2+n)} +$$

$$\frac{\left(1 - \frac{1}{ax}\right)^{-1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} x}{c^2} + \frac{2 n \left(1 - \frac{1}{ax}\right)^{1-\frac{n}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[1, 1 - \frac{n}{2}, 2 - \frac{n}{2}, \frac{a - \frac{1}{x}}{a + \frac{1}{x}}\right]}{a c^2 (2 - n)}$$

Problem 931: Result unnecessarily involves higher level functions.

$$\int e^{n \operatorname{ArcCoth}[ax]} \sqrt{c - \frac{c}{a^2 x^2}} dx$$

Optimal (type 5, 295 leaves, 6 steps):

$$\frac{\sqrt{c - \frac{c}{a^2 x^2}} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1+n}{2}} x^{2n} \sqrt{c - \frac{c}{a^2 x^2}} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \left(1 + \frac{1}{a x}\right)^{\frac{1}{2}(-1+n)} \text{Hypergeometric2F1}\left[1, \frac{1-n}{2}, \frac{3-n}{2}, \frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right]}{\sqrt{1 - \frac{1}{a^2 x^2}} + a(1-n) \sqrt{1 - \frac{1}{a^2 x^2}}} -$$

$$\frac{2^{\frac{1+n}{2}} \sqrt{c - \frac{c}{a^2 x^2}} \left(1 - \frac{1}{a x}\right)^{\frac{1-n}{2}} \text{Hypergeometric2F1}\left[\frac{1-n}{2}, \frac{1-n}{2}, \frac{3-n}{2}, \frac{a-\frac{1}{x}}{2a}\right]}{a(1-n) \sqrt{1 - \frac{1}{a^2 x^2}}}$$

Result (type 6, 111 leaves, 3 steps):

$$\frac{2^{\frac{3-n}{2}} \sqrt{c - \frac{c}{a^2 x^2}} \left(1 + \frac{1}{a x}\right)^{\frac{3+n}{2}} \text{AppellF1}\left[\frac{3+n}{2}, \frac{1}{2}(-1+n), 2, \frac{5+n}{2}, \frac{a+\frac{1}{x}}{2a}, 1 + \frac{1}{a x}\right]}{a(3+n) \sqrt{1 - \frac{1}{a^2 x^2}}}$$

## Test results for the 190 problems in "7.5.1 u (a+b arcsech(c x))^n.m"

Problem 26: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \text{ArcSech}[c x]}{x} dx$$

Optimal (type 4, 56 leaves, 6 steps):

$$-\frac{(a + b \text{ArcSech}[c x])^2}{2b} - (a + b \text{ArcSech}[c x]) \text{Log}\left[1 + e^{-2 \text{ArcSech}[c x]}\right] + \frac{1}{2} b \text{PolyLog}\left[2, -e^{-2 \text{ArcSech}[c x]}\right]$$

Result (type 4, 56 leaves, 6 steps):

$$\frac{(a + b \text{ArcSech}[c x])^2}{2b} - (a + b \text{ArcSech}[c x]) \text{Log}\left[1 + e^{2 \text{ArcSech}[c x]}\right] - \frac{1}{2} b \text{PolyLog}\left[2, -e^{2 \text{ArcSech}[c x]}\right]$$

Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \text{ArcSech}[c x])}{d + e x^2} dx$$

Optimal (type 4, 459 leaves, 26 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcSech}[c x])^2}{b e} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right]}{e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e}
\end{aligned}$$

Result (type 4, 441 leaves, 26 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{2 e}
\end{aligned}$$

Problem 115: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 631 leaves, 32 steps):

$$\begin{aligned}
& - \frac{b \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x}{2 c e^2} + \frac{d (a + b \operatorname{ArcSech}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \operatorname{ArcSech}[c x])}{2 e^2} + \frac{2 d (a + b \operatorname{ArcSech}[c x])^2}{b e^3} - \frac{b d \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \\
& \frac{2 d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \\
& \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right]}{e^3} - \\
& \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3}
\end{aligned}$$

Result (type 4, 611 leaves, 32 steps):

$$\begin{aligned}
& - \frac{b \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x}{2 c e^2} + \frac{d (a + b \operatorname{ArcSech}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \operatorname{ArcSech}[c x])}{2 e^2} - \frac{b d \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \\
& \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \\
& \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} + \frac{2 d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \\
& \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} + \frac{b d \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{e^3}
\end{aligned}$$

Problem 116: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 580 leaves, 30 steps):

$$\begin{aligned}
& -\frac{a + b \operatorname{ArcSech}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} - \frac{(a + b \operatorname{ArcSech}[c x])^2}{b e^2} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{2 e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right]}{e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right]}{2 e^2} + \\
& \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2}
\end{aligned}$$

Result (type 4, 562 leaves, 30 steps):

$$\begin{aligned}
& -\frac{a + b \operatorname{ArcSech}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{2 e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^2} - \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{2 e^2}
\end{aligned}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 778 leaves, 35 steps):

$$\begin{aligned}
& \frac{b d \left( c^2 - \frac{1}{x^2} \right)}{8 c e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x} - \frac{a + b \operatorname{ArcSech}[c x]}{4 e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{ArcSech}[c x]}{2 e^2 \left( e + \frac{d}{x^2} \right)} - \\
& \frac{(a + b \operatorname{ArcSech}[c x])^2}{b e^3} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \frac{b (c^2 d + 2 e) \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{8 e^{5/2} (c^2 d + e)^{3/2} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right]}{e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right]}{2 e^3} + \\
& \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3}
\end{aligned}$$

Result (type 4, 760 leaves, 35 steps):

$$\begin{aligned}
& \frac{b d \left( c^2 - \frac{1}{x^2} \right)}{8 c e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x} - \frac{a + b \operatorname{ArcSech}[c x]}{4 e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{ArcSech}[c x]}{2 e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh} \left[ \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \\
& \frac{b (c^2 d + 2 e) \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh} \left[ \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right]}{8 e^{5/2} (c^2 d + e)^{3/2} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log} \left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^3} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log} \left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log} \left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^3} + \\
& \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log} \left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^3} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log} \left[ 1 + e^{2 \operatorname{ArcSech}[c x]} \right]}{e^3} + \frac{b \operatorname{PolyLog} \left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^3} + \\
& \frac{b \operatorname{PolyLog} \left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^3} + \frac{b \operatorname{PolyLog} \left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^3} + \frac{b \operatorname{PolyLog} \left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^3} - \frac{b \operatorname{PolyLog} \left[ 2, -e^{2 \operatorname{ArcSech}[c x]} \right]}{2 e^3}
\end{aligned}$$

Test results for the 100 problems in "7.5.2 Inverse hyperbolic secant functions.m"

Test results for the 178 problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Problem 8: Result valid but suboptimal antiderivative.

$$\int \frac{a + b \operatorname{ArcCsch}[c x]}{x} dx$$

Optimal (type 4, 56 leaves, 6 steps):

$$-\frac{(a + b \operatorname{ArcCsch}[c x])^2}{2 b} - (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log} \left[ 1 - e^{-2 \operatorname{ArcCsch}[c x]} \right] + \frac{1}{2} b \operatorname{PolyLog} \left[ 2, e^{-2 \operatorname{ArcCsch}[c x]} \right]$$

Result (type 4, 56 leaves, 6 steps):



$$\frac{(a + b \operatorname{ArcCsch}[c x])^2}{2 b} - (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsch}[c x]}\right] - \frac{1}{2} b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsch}[c x]}\right]$$

Problem 99: Result valid but suboptimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcCsch}[c x])}{d + e x^2} dx$$

Optimal (type 4, 467 leaves, 26 steps):

$$\begin{aligned} & -\frac{(a + b \operatorname{ArcCsch}[c x])^2}{2 b e} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCsch}[c x]}\right]}{e} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \\ & \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} + \\ & \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCsch}[c x]}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \\ & \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} \end{aligned}$$

Result (type 4, 449 leaves, 26 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} + \\ & \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsch}[c x]}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \\ & \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsch}[c x]}\right]}{2 e} \end{aligned}$$

Problem 103: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCsch}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 591 leaves, 31 steps):

$$\begin{aligned}
 & \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2 c e^2} + \frac{d (a + b \operatorname{ArcSch}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \operatorname{ArcSch}[c x])}{2 e^2} + \frac{2 d (a + b \operatorname{ArcSch}[c x])^2}{b e^3} - \frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{2 \sqrt{c^2 d - e} e^{5/2}} + \\
 & \frac{2 d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcSch}[c x]}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \\
 & \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcSch}[c x]}\right]}{e^3} - \\
 & \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3}
 \end{aligned}$$

Result (type 4, 571 leaves, 31 steps):

$$\begin{aligned}
 & \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2 c e^2} + \frac{d (a + b \operatorname{ArcSch}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \operatorname{ArcSch}[c x])}{2 e^2} - \frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{2 \sqrt{c^2 d - e} e^{5/2}} - \\
 & \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} - \\
 & \frac{d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} + \frac{2 d (a + b \operatorname{ArcSch}[c x]) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcSch}[c x]}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \\
 & \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{e^3} + \frac{b d \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcSch}[c x]}\right]}{e^3}
 \end{aligned}$$

Problem 104: Result valid but suboptimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{ArcSch}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 553 leaves, 29 steps):

$$\begin{aligned}
& -\frac{a + b \operatorname{ArcCsch}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} - \frac{(a + b \operatorname{ArcCsch}[c x])^2}{b e^2} + \frac{b \operatorname{ArcTan}\left[ \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x} \right]}{2 \sqrt{c^2 d - e} e^{3/2}} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - e^{-2 \operatorname{ArcCsch}[c x]} \right]}{e^2} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, e^{-2 \operatorname{ArcCsch}[c x]} \right]}{2 e^2} + \\
& \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2}
\end{aligned}$$

Result (type 4, 535 leaves, 29 steps):

$$\begin{aligned}
& -\frac{a + b \operatorname{ArcCsch}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} + \frac{b \operatorname{ArcTan}\left[ \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x} \right]}{2 \sqrt{c^2 d - e} e^{3/2}} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[ 1 - e^{2 \operatorname{ArcCsch}[c x]} \right]}{e^2} + \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \\
& \frac{b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}} \right]}{2 e^2} - \frac{b \operatorname{PolyLog}\left[ 2, e^{2 \operatorname{ArcCsch}[c x]} \right]}{2 e^2}
\end{aligned}$$

Problem 111: Result valid but suboptimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcCsch}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 694 leaves, 33 steps):

$$\begin{aligned}
& \frac{b c d \sqrt{1 + \frac{1}{c^2 x^2}}}{8 (c^2 d - e) e^2 \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \operatorname{ArcCsch}[c x]}{4 e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \operatorname{ArcCsch}[c x]}{2 e^2 \left(e + \frac{d}{x^2}\right)} - \frac{(a + b \operatorname{ArcCsch}[c x])^2}{b e^3} + \\
& \frac{b (c^2 d - 2 e) \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{8 (c^2 d - e)^{3/2} e^{5/2}} + \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{2 \sqrt{c^2 d - e} e^{5/2}} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCsch}[c x]}\right]}{e^3} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCsch}[c x]}\right]}{2 e^3} + \\
& \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3}
\end{aligned}$$

Result (type 4, 676 leaves, 33 steps):

$$\begin{aligned}
& \frac{b c d \sqrt{1 + \frac{1}{c^2 x^2}}}{8 (c^2 d - e) e^2 \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \operatorname{ArcCsch}[c x]}{4 e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \operatorname{ArcCsch}[c x]}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{b (c^2 d - 2 e) \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{8 (c^2 d - e)^{3/2} e^{5/2}} + \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}} x}\right]}{2 \sqrt{c^2 d - e} e^{5/2}} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} + \\
& \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} - \frac{(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsch}[c x]}\right]}{e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \\
& \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right]}{2 e^3} - \frac{b \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsch}[c x]}\right]}{2 e^3}
\end{aligned}$$

Test results for the 71 problems in "7.6.2 Inverse hyperbolic cosecant functions.m"