

Rubi 4.16.0 Algebraic Function Integration Test Suite Results

Test results for the 1917 problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b x^m}{2 (a + b x)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a + b x}} \right) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a + b x}}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^m \left(-\frac{b x}{a} \right)^{-m} \text{Hypergeometric2F1} \left[-\frac{1}{2}, -m, \frac{1}{2}, 1 + \frac{b x}{a} \right]}{\sqrt{a + b x}} - \frac{2 m x^m \left(-\frac{b x}{a} \right)^{-m} \sqrt{a + b x} \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - m, \frac{3}{2}, 1 + \frac{b x}{a} \right]}{a}$$

Test results for the 3201 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 957: Result valid but suboptimal antiderivative.

$$\int (e x)^m (a - b x)^{2+n} (a + b x)^n dx$$

Optimal (type 5, 211 leaves, ? steps):

$$-\frac{(e x)^{1+m} (a - b x)^{1+n} (a + b x)^{1+n}}{e (3 + m + 2 n)} + \frac{2 a^2 (2 + m + n) (e x)^{1+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2} \right)^{-n} \text{Hypergeometric2F1} \left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2} \right]}{e (1 + m) (3 + m + 2 n)} - \frac{2 a b (e x)^{2+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2} \right)^{-n} \text{Hypergeometric2F1} \left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2} \right]}{e^2 (2 + m)}$$

Result (type 5, 238 leaves, 11 steps):

$$\frac{a^2 (e x)^{1+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, -n, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e (1+m)} -$$

$$\frac{2 a b (e x)^{2+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, -n, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)} +$$

$$\frac{b^2 (e x)^{3+m} (a - b x)^n (a + b x)^n \left(1 - \frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{3+m}{2}, -n, \frac{5+m}{2}, \frac{b^2 x^2}{a^2}\right]}{e^3 (3+m)}$$

Problem 1001: Result unnecessarily involves higher level functions.

$$\int \frac{(1 - a x)^{1-n} (1 + a x)^{1+n}}{x^2} dx$$

Optimal (type 5, 106 leaves, 3 steps):

$$- \frac{2 a (1 - a x)^{1-n} (1 + a x)^{-1+n} \text{Hypergeometric2F1}\left[2, 1-n, 2-n, \frac{1-ax}{1+ax}\right]}{1-n} + \frac{2^n a (1 - a x)^{1-n} \text{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{1}{2} (1 - a x)\right]}{1-n}$$

Result (type 6, 48 leaves, 1 step):

$$\frac{2^{1-n} a (1 + a x)^{2+n} \text{AppellF1}\left[2+n, -1+n, 2, 3+n, \frac{1}{2} (1 + a x), 1 + a x\right]}{2+n}$$

Problem 1006: Result valid but suboptimal antiderivative.

$$\int \frac{(a - b x)^{-n} (a + b x)^{1+n}}{x} dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{(a - b x)^{1-n} (a + b x)^n}{2n} - \frac{a (a - b x)^{-n} (a + b x)^n \text{Hypergeometric2F1}\left[1, n, 1+n, \frac{a+bx}{a-bx}\right]}{n} +$$

$$\frac{2^{-1-n} (1+2n) (a - b x)^{-n} \left(\frac{a-bx}{a}\right)^n (a + b x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{a+bx}{2a}\right]}{n (1+n)}$$

Result (type 5, 173 leaves, 7 steps):

$$\frac{a (a - b x)^{-n} (a + b x)^n \operatorname{Hypergeometric2F1}\left[1, -n, 1 - n, \frac{a - b x}{a + b x}\right]}{n} - \frac{2^n a (a - b x)^{-n} (a + b x)^n \left(\frac{a + b x}{a}\right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, \frac{a - b x}{2a}\right]}{n} +$$

$$\frac{2^{-n} (a - b x)^{-n} \left(\frac{a - b x}{a}\right)^n (a + b x)^{1+n} \operatorname{Hypergeometric2F1}\left[n, 1 + n, 2 + n, \frac{a + b x}{2a}\right]}{1 + n}$$

Problem 1007: Result unnecessarily involves higher level functions.

$$\int \frac{(a - b x)^{-n} (a + b x)^{1+n}}{x^2} dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$-\frac{(a - b x)^{-n} (a + b x)^{1+n}}{x} + \frac{b (1 + 2n) (a - b x)^{-n} (a + b x)^n \operatorname{Hypergeometric2F1}\left[1, -n, 1 - n, \frac{a - b x}{a + b x}\right]}{n}$$

$$-\frac{2^n b (a - b x)^{-n} (a + b x)^n \left(\frac{a + b x}{a}\right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, \frac{a - b x}{2a}\right]}{n}$$

Result (type 6, 76 leaves, 2 steps):

$$\frac{2^{-n} b (a - b x)^{-n} \left(\frac{a - b x}{a}\right)^n (a + b x)^{2+n} \operatorname{AppellF1}\left[2 + n, n, 2, 3 + n, \frac{a + b x}{2a}, \frac{a + b x}{a}\right]}{a^2 (2 + n)}$$

Problem 2121: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1 - 2x)^{3/2} (3 + 5x)^2} dx$$

Optimal (type 3, 63 leaves, 4 steps):

$$\frac{6}{121 \sqrt{1 - 2x}} - \frac{1}{11 \sqrt{1 - 2x} (3 + 5x)} - \frac{6}{121} \sqrt{\frac{5}{11}} \operatorname{ArcTanh}\left[\sqrt{\frac{5}{11}} \sqrt{1 - 2x}\right]$$

Result (type 3, 70 leaves, 4 steps):

$$\frac{2}{11 \sqrt{1 - 2x} (3 + 5x)} - \frac{15 \sqrt{1 - 2x}}{121 (3 + 5x)} - \frac{6}{121} \sqrt{\frac{5}{11}} \operatorname{ArcTanh}\left[\sqrt{\frac{5}{11}} \sqrt{1 - 2x}\right]$$

Problem 2144: Result valid but suboptimal antiderivative.

$$\int \frac{3 + 5x}{(1 - 2x)^{5/2} (2 + 3x)^4} dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$\frac{160}{3087 (1 - 2x)^{3/2}} + \frac{160}{2401 \sqrt{1 - 2x}} + \frac{1}{63 (1 - 2x)^{3/2} (2 + 3x)^3} - \frac{16}{147 (1 - 2x)^{3/2} (2 + 3x)^2} - \frac{16}{147 (1 - 2x)^{3/2} (2 + 3x)} - \frac{160 \sqrt{\frac{3}{7}} \operatorname{ArcTanh}\left[\sqrt{\frac{3}{7}} \sqrt{1 - 2x}\right]}{2401}$$

Result (type 3, 130 leaves, 7 steps):

$$\frac{1}{63 (1 - 2x)^{3/2} (2 + 3x)^3} + \frac{64}{441 (1 - 2x)^{3/2} (2 + 3x)^2} + \frac{64}{147 \sqrt{1 - 2x} (2 + 3x)^2} - \frac{80 \sqrt{1 - 2x}}{343 (2 + 3x)^2} - \frac{240 \sqrt{1 - 2x}}{2401 (2 + 3x)} - \frac{160 \sqrt{\frac{3}{7}} \operatorname{ArcTanh}\left[\sqrt{\frac{3}{7}} \sqrt{1 - 2x}\right]}{2401}$$

Problem 2145: Result valid but suboptimal antiderivative.

$$\int \frac{3 + 5x}{(1 - 2x)^{5/2} (2 + 3x)^5} dx$$

Optimal (type 3, 136 leaves, 8 steps):

$$\frac{215}{9604 (1 - 2x)^{3/2}} + \frac{1935}{67228 \sqrt{1 - 2x}} + \frac{1}{84 (1 - 2x)^{3/2} (2 + 3x)^4} - \frac{43}{588 (1 - 2x)^{3/2} (2 + 3x)^3} - \frac{129}{2744 (1 - 2x)^{3/2} (2 + 3x)^2} - \frac{129}{2744 (1 - 2x)^{3/2} (2 + 3x)} - \frac{1935 \sqrt{\frac{3}{7}} \operatorname{ArcTanh}\left[\sqrt{\frac{3}{7}} \sqrt{1 - 2x}\right]}{67228}$$

Result (type 3, 150 leaves, 8 steps):

$$\frac{1}{84 (1-2x)^{3/2} (2+3x)^4} + \frac{43}{294 (1-2x)^{3/2} (2+3x)^3} + \frac{387}{686 \sqrt{1-2x} (2+3x)^3} -$$

$$\frac{387 \sqrt{1-2x}}{1372 (2+3x)^3} - \frac{1935 \sqrt{1-2x}}{19208 (2+3x)^2} - \frac{5805 \sqrt{1-2x}}{134456 (2+3x)} - \frac{1935 \sqrt{\frac{3}{7}} \operatorname{ArcTanh}\left[\sqrt{\frac{3}{7}} \sqrt{1-2x}\right]}{67228}$$

Problem 2196: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-2x)^{5/2} (3+5x)^3} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{35}{3993 (1-2x)^{3/2}} + \frac{175}{14641 \sqrt{1-2x}} - \frac{1}{22 (1-2x)^{3/2} (3+5x)^2} - \frac{7}{242 (1-2x)^{3/2} (3+5x)} - \frac{175 \sqrt{\frac{5}{11}} \operatorname{ArcTanh}\left[\sqrt{\frac{5}{11}} \sqrt{1-2x}\right]}{14641}$$

Result (type 3, 110 leaves, 6 steps):

$$\frac{2}{33 (1-2x)^{3/2} (3+5x)^2} + \frac{70}{363 \sqrt{1-2x} (3+5x)^2} - \frac{875 \sqrt{1-2x}}{7986 (3+5x)^2} - \frac{875 \sqrt{1-2x}}{29282 (3+5x)} - \frac{175 \sqrt{\frac{5}{11}} \operatorname{ArcTanh}\left[\sqrt{\frac{5}{11}} \sqrt{1-2x}\right]}{14641}$$

Problem 3075: Result valid but suboptimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{-1-m}}{e+fx} dx$$

Optimal (type 5, 72 leaves, 1 step):

$$-\frac{(a+bx)^m (c+dx)^{-m} \operatorname{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{(de-cf)m}$$

Result (type 5, 75 leaves, 1 step):

$$\frac{(a+bx)^{1+m} (c+dx)^{-1-m} \operatorname{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(1+m)}$$

Problem 3077: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-1-m}}{(e + f x)^3} dx$$

Optimal (type 5, 283 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a + b x)^{1+m} (c + d x)^{-m}}{2 (b e - a f) (d e - c f) (e + f x)^2} - \frac{f (b (3 d e - c f (1 - m)) - a d f (2 + m)) (a + b x)^{1+m} (c + d x)^{-m}}{2 (b e - a f)^2 (d e - c f)^2 (e + f x)} + \\ & \left((2 a b d f (1 + m) (2 d e + c f m) - b^2 (2 d^2 e^2 + 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (2 + 3 m + m^2)) \right. \\ & \left. (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] \right) / \left(2 (b e - a f)^2 (d e - c f)^3 m \right) \end{aligned}$$

Result (type 5, 300 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a d f (2 + m) - b (2 d e + c f m)) (a + b x)^{1+m} (c + d x)^{1-m}}{2 (b c - a d) (b e - a f) (d e - c f)^2 m (e + f x)^2} + \frac{d (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) (d e - c f) m (e + f x)^2} + \\ & \left((2 a b d f (1 + m) (2 d e + c f m) - b^2 (2 d^2 e^2 + 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (2 + 3 m + m^2)) (a + b x)^{1+m} \right. \\ & \left. (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[2, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left(2 (b e - a f)^3 (d e - c f)^2 m (1 + m) \right) \end{aligned}$$

Problem 3078: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-1-m}}{(e + f x)^4} dx$$

Optimal (type 5, 498 leaves, 5 steps):

$$\begin{aligned}
& - \frac{f(a+bx)^{1+m}(c+dx)^{-m}}{3(b e - a f)(d e - c f)(e + f x)^3} - \frac{f(b(5 d e - c f(2 - m)) - a d f(3 + m))(a + b x)^{1+m}(c + d x)^{-m}}{6(b e - a f)^2(d e - c f)^2(e + f x)^2} - \\
& \left(f(a^2 d^2 f^2(6 + 5 m + m^2) - a b d f(d e(15 + 8 m) - c f(3 - 2 m - 2 m^2)) + b^2(11 d^2 e^2 - c d e f(7 - 8 m) + c^2 f^2(2 - 3 m + m^2))) (a + b x)^{1+m}(c + d x)^{-m} \right) / \\
& \left(6(b e - a f)^3(d e - c f)^3(e + f x) \right) + \\
& \left((3 a b^2 d f(1 + m)(6 d^2 e^2 + 6 c d e f m - c^2 f^2(1 - m) m) - 3 a^2 b d^2 f^2(3 d e + c f m)(2 + 3 m + m^2) + a^3 d^3 f^3(6 + 11 m + 6 m^2 + m^3) - \right. \\
& \quad \left. b^3(6 d^3 e^3 + 18 c d^2 e^2 f m - 9 c^2 d e f^2(1 - m) m + c^3 f^3 m(2 - 3 m + m^2))) (a + b x)^m \right. \\
& \quad \left. (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f)(c + d x)}{(d e - c f)(a + b x)}\right] \right) / (6(b e - a f)^3(d e - c f)^4 m)
\end{aligned}$$

Result (type 5, 520 leaves, 5 steps):

$$\begin{aligned}
& - \frac{f(a d f(3 + m) - b(3 d e + c f m))(a + b x)^{1+m}(c + d x)^{1-m}}{3(b c - a d)(b e - a f)(d e - c f)^2 m(e + f x)^3} + \frac{d(a + b x)^{1+m}(c + d x)^{-m}}{(b c - a d)(d e - c f) m(e + f x)^3} + \\
& \left(f(b^2(6 d^2 e^2 + 7 c d e f m - c^2 f^2(2 - m) m) + a^2 d^2 f^2(6 + 5 m + m^2) - a b d f(c f m(3 + 2 m) + d e(12 + 7 m))) (a + b x)^{1+m}(c + d x)^{1-m} \right) / \\
& \left(6(b c - a d)(b e - a f)^2(d e - c f)^3 m(e + f x)^2 \right) + \\
& \left((3 a b^2 d f(1 + m)(6 d^2 e^2 + 6 c d e f m - c^2 f^2(1 - m) m) - 3 a^2 b d^2 f^2(3 d e + c f m)(2 + 3 m + m^2) + a^3 d^3 f^3(6 + 11 m + 6 m^2 + m^3) - \right. \\
& \quad \left. b^3(6 d^3 e^3 + 18 c d^2 e^2 f m - 9 c^2 d e f^2(1 - m) m + c^3 f^3 m(2 - 3 m + m^2))) (a + b x)^{1+m} \right. \\
& \quad \left. (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[2, 1 + m, 2 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right] \right) / (6(b e - a f)^4(d e - c f)^3 m(1 + m))
\end{aligned}$$

Problem 3084: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{e + f x} dx$$

Optimal (type 5, 120 leaves, 2 steps):

$$\frac{d(a + b x)^{1+m}(c + d x)^{-1-m}}{(b c - a d)(d e - c f)(1 + m)} + \frac{f(a + b x)^m(c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f)(c + d x)}{(d e - c f)(a + b x)}\right]}{(d e - c f)^2 m}$$

Result (type 5, 135 leaves, 2 steps):

$$\frac{d(a + b x)^{1+m}(c + d x)^{-1-m}}{(b c - a d)(d e - c f)(1 + m)} - \frac{f(a + b x)^{1+m}(c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}\right]}{(b e - a f)(d e - c f)(1 + m)}$$

Problem 3085: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{(e + f x)^2} dx$$

Optimal (type 5, 233 leaves, 4 steps):

$$\begin{aligned} & - \frac{d (a d f (2+m) - b (d e + c f (1+m))) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (1+m)} - \frac{f (a + b x)^{1+m} (c + d x)^{-1-m}}{(b e - a f) (d e - c f) (e + f x)} - \\ & \frac{f (a d f (2+m) - b (2 d e + c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{(b e - a f) (d e - c f)^3 m} \end{aligned}$$

Result (type 5, 243 leaves, 4 steps):

$$\begin{aligned} & \frac{d (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d) (d e - c f) (1+m) (e + f x)} + \frac{f (b d e + b c f (1+m) - a d f (2+m)) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (1+m) (e + f x)} + \\ & \left(\frac{f (a d f (2+m) - b (2 d e + c f m)) (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f)^2 (d e - c f)^2 (1+m)} \right) / \end{aligned}$$

Problem 3086: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{(e + f x)^3} dx$$

Optimal (type 5, 432 leaves, 5 steps):

$$\begin{aligned} & \left(d (a^2 d^2 f^2 (6 + 5 m + m^2) + b^2 (2 d^2 e^2 + 5 c d e f (1+m) - c^2 f^2 (1-m^2))) - a b d f (d e (9 + 5 m) + c f (3 + 5 m + 2 m^2)) \right) (a + b x)^{1+m} (c + d x)^{-1-m} / \\ & \left(2 (b c - a d) (b e - a f)^2 (d e - c f)^3 (1+m) \right) - \frac{f (a + b x)^{1+m} (c + d x)^{-1-m}}{2 (b e - a f) (d e - c f) (e + f x)^2} - \frac{f (b (4 d e - c f (1-m)) - a d f (3+m)) (a + b x)^{1+m} (c + d x)^{-1-m}}{2 (b e - a f)^2 (d e - c f)^2 (e + f x)} - \\ & \left(f (2 a b d f (2+m) (3 d e + c f m) - b^2 (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1-m) m) - a^2 d^2 f^2 (6 + 5 m + m^2)) (a + b x)^m \right. \\ & \left. (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] \right) / \left(2 (b e - a f)^2 (d e - c f)^4 m \right) \end{aligned}$$

Result (type 5, 452 leaves, 5 steps):

$$\frac{d (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d) (d e - c f) (1+m) (e + f x)^2} + \frac{f (2 b d e + b c f (1+m) - a d f (3+m)) (a + b x)^{1+m} (c + d x)^{-m}}{2 (b c - a d) (b e - a f) (d e - c f)^2 (1+m) (e + f x)^2} +$$

$$\left(f (a^2 d^2 f^2 (6 + 5 m + m^2) + b^2 (2 d^2 e^2 + 5 c d e f (1+m) - c^2 f^2 (1 - m^2))) - a b d f (d e (9 + 5 m) + c f (3 + 5 m + 2 m^2)) \right) (a + b x)^{1+m} (c + d x)^{-m} \Big/$$

$$\left(2 (b c - a d) (b e - a f)^2 (d e - c f)^3 (1+m) (e + f x) \right) +$$

$$\left(f (2 a b d f (2+m) (3 d e + c f m) - b^2 (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (6 + 5 m + m^2)) \right) (a + b x)^{1+m}$$

$$(c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \Big/ \left(2 (b e - a f)^3 (d e - c f)^3 (1+m) \right)$$

Problem 3093: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-3-m}}{e + f x} dx$$

Optimal (type 5, 196 leaves, 4 steps):

$$\frac{d (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d) (d e - c f) (2+m)} + \frac{d (a d f (2+m) + b (d e - c f (3+m))) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^2 (d e - c f)^2 (1+m) (2+m)} -$$

$$\frac{f^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{(d e - c f)^3 m}$$

Result (type 5, 208 leaves, 4 steps):

$$\frac{d (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d) (d e - c f) (2+m)} + \frac{d (b d e + a d f (2+m) - b c f (3+m)) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^2 (d e - c f)^2 (1+m) (2+m)} +$$

$$\frac{f^2 (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (d e - c f)^2 (1+m)}$$

Problem 3094: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-3-m}}{(e + f x)^2} dx$$

Optimal (type 5, 384 leaves, 5 steps):

$$\begin{aligned}
& - \frac{d (a d f (3+m) - b (d e + c f (2+m))) (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (2+m)} - \\
& \left(d (a^2 d^2 f^2 (6+5 m+m^2) - b^2 (d^2 e^2 - c d e f (5+2 m) - c^2 f^2 (2+3 m+m^2))) - a b d f (d e (3+2 m) + c f (9+8 m+2 m^2)) \right) (a+b x)^{1+m} (c+d x)^{-1-m} \Big/ \\
& \left((b c - a d)^2 (b e - a f) (d e - c f)^3 (1+m) (2+m) \right) - \frac{f (a+b x)^{1+m} (c+d x)^{-2-m}}{(b e - a f) (d e - c f) (e+f x)} + \\
& \frac{f^2 (a d f (3+m) - b (3 d e + c f m)) (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}\right]}{(b e - a f) (d e - c f)^4 m}
\end{aligned}$$

Result (type 5, 398 leaves, 5 steps):

$$\begin{aligned}
& - \left(\left(d (a^2 d^2 f^2 (6+5 m+m^2) - b^2 (d^2 e^2 - c d e f (5+2 m) - c^2 f^2 (2+3 m+m^2))) - a b d f (d e (3+2 m) + c f (9+8 m+2 m^2)) \right) (a+b x)^{1+m} (c+d x)^{-1-m} \right) \Big/ \\
& \left((b c - a d)^2 (b e - a f) (d e - c f)^3 (1+m) (2+m) \right) + \\
& \frac{d (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c - a d) (d e - c f) (2+m) (e+f x)} + \frac{f (b d e + b c f (2+m) - a d f (3+m)) (a+b x)^{1+m} (c+d x)^{-1-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (2+m) (e+f x)} - \\
& \left(f^2 (a d f (3+m) - b (3 d e + c f m)) (a+b x)^{1+m} (c+d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{(d e - c f) (a+b x)}{(b e - a f) (c+d x)}\right] \right) \Big/ \\
& \left((b e - a f)^2 (d e - c f)^3 (1+m) \right)
\end{aligned}$$

Problem 3101: Result valid but suboptimal antiderivative.

$$\int \frac{(a+b x)^m (c+d x)^{-4-m}}{e+f x} dx$$

Optimal (type 5, 330 leaves, 5 steps):

$$\begin{aligned}
& \frac{d (a+b x)^{1+m} (c+d x)^{-3-m}}{(b c - a d) (d e - c f) (3+m)} + \frac{d (a d f (3+m) + b (2 d e - c f (5+m))) (a+b x)^{1+m} (c+d x)^{-2-m}}{(b c - a d)^2 (d e - c f)^2 (2+m) (3+m)} + \\
& \left(d (a^2 d^2 f^2 (6+5 m+m^2) + a b d f (3+m) (d e - c f (5+2 m)) + b^2 (2 d^2 e^2 - c d e f (7+m) + c^2 f^2 (11+6 m+m^2))) \right) (a+b x)^{1+m} (c+d x)^{-1-m} \Big/ \\
& \left((b c - a d)^3 (d e - c f)^3 (1+m) (2+m) (3+m) \right) + \frac{f^3 (a+b x)^m (c+d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}\right]}{(d e - c f)^4 m}
\end{aligned}$$

Result (type 5, 344 leaves, 5 steps):

$$\frac{d (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d) (d e - c f) (3 + m)} + \frac{d (2 b d e + a d f (3 + m) - b c f (5 + m)) (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d)^2 (d e - c f)^2 (2 + m) (3 + m)} +$$

$$\left(d (a^2 d^2 f^2 (6 + 5 m + m^2) + a b d f (3 + m) (d e - c f (5 + 2 m)) + b^2 (2 d^2 e^2 - c d e f (7 + m) + c^2 f^2 (11 + 6 m + m^2)) \right) (a + b x)^{1+m} (c + d x)^{-1-m} \Big/$$

$$\left((b c - a d)^3 (d e - c f)^3 (1 + m) (2 + m) (3 + m) \right) - \frac{f^3 (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (d e - c f)^3 (1 + m)}$$

Problem 3102: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-4-m}}{(e + f x)^2} dx$$

Optimal (type 5, 634 leaves, 6 steps):

$$- \frac{d (a d f (4 + m) - b (d e + c f (3 + m))) (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (3 + m)} -$$

$$\left(d (a^2 d^2 f^2 (12 + 7 m + m^2) - b^2 (2 d^2 e^2 - 2 c d e f (4 + m) - c^2 f^2 (6 + 5 m + m^2)) - 2 a b d f (d e (2 + m) + c f (10 + 6 m + m^2))) (a + b x)^{1+m} (c + d x)^{-2-m} \right) \Big/$$

$$\left((b c - a d)^2 (b e - a f) (d e - c f)^3 (2 + m) (3 + m) \right) -$$

$$\left(d (a^3 d^3 f^3 (24 + 26 m + 9 m^2 + m^3) - a^2 b d^2 f^2 (3 + m) (d e (4 + 3 m) + c f (20 + 15 m + 3 m^2)) - b^3 (2 d^3 e^3 - 2 c d^2 e^2 f (5 + m) + c^2 d e f^2 (26 + 17 m + 3 m^2) + c^3 f^3 (6 + 11 m + 6 m^2 + m^3)) - a b^2 d f (2 d^2 e^2 (2 + m) - 2 c d e f (16 + 15 m + 3 m^2) - c^2 f^2 (44 + 50 m + 21 m^2 + 3 m^3))) (a + b x)^{1+m} (c + d x)^{-1-m} \right) \Big/$$

$$\left((b c - a d)^3 (b e - a f) (d e - c f)^4 (1 + m) (2 + m) (3 + m) \right) - \frac{f (a + b x)^{1+m} (c + d x)^{-3-m}}{(b e - a f) (d e - c f) (e + f x)} -$$

$$\frac{f^3 (a d f (4 + m) - b (4 d e + c f m)) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{(b e - a f) (d e - c f)^5 m}$$

Result (type 5, 646 leaves, 6 steps):

$$\begin{aligned}
& - \left(\left(d \left(a^2 d^2 f^2 (12 + 7m + m^2) - b^2 (2 d^2 e^2 - 2 c d e f (4 + m) - c^2 f^2 (6 + 5m + m^2)) - 2 a b d f (d e (2 + m) + c f (10 + 6m + m^2)) \right) (a + b x)^{1+m} (c + d x)^{-2-m} \right) / \right. \\
& \quad \left. \left((b c - a d)^2 (b e - a f) (d e - c f)^3 (2 + m) (3 + m) \right) \right) - \\
& \quad \left(d \left(a^3 d^3 f^3 (24 + 26m + 9m^2 + m^3) - a^2 b d^2 f^2 (3 + m) (d e (4 + 3m) + c f (20 + 15m + 3m^2)) - b^3 (2 d^3 e^3 - 2 c d^2 e^2 f (5 + m) + c^2 d e f^2 (26 + 17m + 3m^2) + \right. \right. \\
& \quad \left. \left. c^3 f^3 (6 + 11m + 6m^2 + m^3)) - a b^2 d f (2 d^2 e^2 (2 + m) - 2 c d e f (16 + 15m + 3m^2) - c^2 f^2 (44 + 50m + 21m^2 + 3m^3)) \right) (a + b x)^{1+m} (c + d x)^{-1-m} \right) / \\
& \quad \left((b c - a d)^3 (b e - a f) (d e - c f)^4 (1 + m) (2 + m) (3 + m) \right) + \frac{d (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d) (d e - c f) (3 + m) (e + f x)} + \\
& \quad \frac{f (b d e + b c f (3 + m) - a d f (4 + m)) (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (3 + m) (e + f x)} + \\
& \quad \left(f^3 (a d f (4 + m) - b (4 d e + c f m)) (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
& \quad \left((b e - a f)^2 (d e - c f)^4 (1 + m) \right)
\end{aligned}$$

Problem 3110: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{-5-m}}{e + f x} dx$$

Optimal (type 5, 557 leaves, 6 steps):

$$\begin{aligned}
& \frac{d (a + b x)^{1+m} (c + d x)^{-4-m}}{(b c - a d) (d e - c f) (4 + m)} + \frac{d (a d f (4 + m) + b (3 d e - c f (7 + m))) (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d)^2 (d e - c f)^2 (3 + m) (4 + m)} + \\
& \left(d \left(a^2 d^2 f^2 (12 + 7m + m^2) + 2 a b d f (4 + m) (d e - c f (4 + m)) + b^2 (6 d^2 e^2 - 2 c d e f (10 + m) + c^2 f^2 (26 + 9m + m^2)) \right) (a + b x)^{1+m} (c + d x)^{-2-m} \right) / \\
& \quad \left((b c - a d)^3 (d e - c f)^3 (2 + m) (3 + m) (4 + m) \right) + \\
& \quad \left(d \left(a^3 d^3 f^3 (24 + 26m + 9m^2 + m^3) + a^2 b d^2 f^2 (12 + 7m + m^2) (d e - c f (7 + 3m)) + a b^2 d f (4 + m) (2 d^2 e^2 - 2 c d e f (5 + m) + c^2 f^2 (26 + 17m + 3m^2)) + \right. \right. \\
& \quad \left. \left. b^3 (6 d^3 e^3 - 2 c d^2 e^2 f (13 + m) + c^2 d e f^2 (46 + 11m + m^2) - c^3 f^3 (50 + 35m + 10m^2 + m^3)) \right) (a + b x)^{1+m} (c + d x)^{-1-m} \right) / \\
& \quad \left((b c - a d)^4 (d e - c f)^4 (1 + m) (2 + m) (3 + m) (4 + m) \right) - \frac{f^4 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{(d e - c f)^5 m}
\end{aligned}$$

Result (type 5, 569 leaves, 6 steps):

$$\begin{aligned}
& \frac{d (a + b x)^{1+m} (c + d x)^{-4-m}}{(b c - a d) (d e - c f) (4 + m)} + \frac{d (3 b d e + a d f (4 + m) - b c f (7 + m)) (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d)^2 (d e - c f)^2 (3 + m) (4 + m)} + \\
& \left(d (a^2 d^2 f^2 (12 + 7 m + m^2) + 2 a b d f (4 + m) (d e - c f (4 + m)) + b^2 (6 d^2 e^2 - 2 c d e f (10 + m) + c^2 f^2 (26 + 9 m + m^2))) (a + b x)^{1+m} (c + d x)^{-2-m} \right) / \\
& \left((b c - a d)^3 (d e - c f)^3 (2 + m) (3 + m) (4 + m) \right) + \\
& \left(d (a^3 d^3 f^3 (24 + 26 m + 9 m^2 + m^3) + a^2 b d^2 f^2 (12 + 7 m + m^2) (d e - c f (7 + 3 m)) + a b^2 d f (4 + m) (2 d^2 e^2 - 2 c d e f (5 + m) + c^2 f^2 (26 + 17 m + 3 m^2)) + \right. \\
& \quad \left. b^3 (6 d^3 e^3 - 2 c d^2 e^2 f (13 + m) + c^2 d e f^2 (46 + 11 m + m^2) - c^3 f^3 (50 + 35 m + 10 m^2 + m^3))) (a + b x)^{1+m} (c + d x)^{-1-m} \right) / \\
& \left((b c - a d)^4 (d e - c f)^4 (1 + m) (2 + m) (3 + m) (4 + m) \right) + \frac{f^4 (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) (d e - c f)^4 (1 + m)}
\end{aligned}$$

Problem 3116: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{e + f x} dx$$

Optimal (type 5, 230 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{(d e - c f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{f^2 m} + \frac{1}{b (b c - a d) f^2 m (1 + m)} \\
& d (b (d e - c f (1 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, - \frac{d (a + b x)}{b c - a d}\right]
\end{aligned}$$

Result (type 5, 220 leaves, 7 steps):

$$\begin{aligned}
& \frac{(d e - c f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, m, 1 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{f^2 m} - \\
& \frac{(d e - c f) (a + b x)^m (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[m, m, 1 + m, - \frac{d (a + b x)}{b c - a d}\right]}{f^2 m} + \\
& \frac{d (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, - \frac{d (a + b x)}{b c - a d}\right]}{b f (1 + m)}
\end{aligned}$$

Problem 3117: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^2} dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(a+bx)^m (c+dx)^{1-m}}{f(e+fx)} + \frac{(adf(1-m) - b(de-cfm)) (a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1}\left[1, m, 1+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{f^2 (be-af) m} \\
& + \frac{d(a+bx)^m (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[m, m, 1+m, -\frac{d(a+bx)}{bc-ad}\right]}{f^2 m}
\end{aligned}$$

Result (type 6, 108 leaves, 2 steps):

$$\frac{(bc-ad) (a+bx)^{1+m} (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{AppellF1}\left[1+m, -1+m, 2, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{(be-af)^2 (1+m)}$$

Problem 3127: Result valid but suboptimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{e+fx} dx$$

Optimal (type 5, 370 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d(2abcd f^2 m - a^2 d^2 f^2 m - b^2(2d^2 e^2 - 4cdef + c^2 f^2(2+m))) (a+bx)^{1+m} (c+dx)^{-m}}{2b^2 (bc-ad) f^3 m} + \\
& \frac{d^2 (a+bx)^{2+m} (c+dx)^{-m}}{2b^2 f} + \frac{(de-cf)^2 (a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]}{f^3 m} + \\
& \left(d(2abd f(de-cf(2-m))m + a^2 d^2 f^2(1-m)m - b^2(2d^2 e^2 - 2cdef(2-m) + c^2 f^2(2-3m+m^2))) \right. \\
& \left. (a+bx)^{1+m} (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right] \right) / (2b^2 (bc-ad) f^3 m (1+m))
\end{aligned}$$

Result (type 5, 319 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(de-cf)^2 (a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1}\left[1, m, 1+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{f^3 m} + \\
& \frac{d(bc-ad) (a+bx)^{1+m} (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[-1+m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right]}{b^2 f (1+m)} + \\
& \frac{(de-cf)^2 (a+bx)^m (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[m, m, 1+m, -\frac{d(a+bx)}{bc-ad}\right]}{f^3 m} - \\
& \frac{d(de-cf) (a+bx)^{1+m} (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right]}{b f^2 (1+m)}
\end{aligned}$$

Problem 3128: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(e+fx)^2} dx$$

Optimal (type 5, 316 leaves, 7 steps):

$$\begin{aligned} & -\frac{2d^2(de-cf)(a+bx)^{1+m}(c+dx)^{-m}}{(bc-ad)f^3m} + \frac{(de-cf)^2(a+bx)^{1+m}(c+dx)^{-m}}{f^2(be-af)(e+fx)} + \frac{1}{f^3(be-af)m} \\ & (de-cf)(adf(2-m) - b(2de-cfm))(a+bx)^m(c+dx)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right] + \frac{1}{b(bc-ad)f^3m(1+m)} \\ & d^2(b(2de-cf(2-m)) - adfm)(a+bx)^{1+m}(c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right] \end{aligned}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(bc-ad)^2(a+bx)^{1+m}(c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{AppellF1}\left[1+m, -2+m, 2, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{b(be-af)^2(1+m)}$$

Problem 3129: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(e+fx)^3} dx$$

Optimal (type 5, 362 leaves, 7 steps):

$$\begin{aligned} & \frac{(be-af)(a+bx)^{-1+m}(c+dx)^{2-m}}{2f^2(e+fx)^2} + \frac{(adf(2-m) - b(3de-cf(1+m)))(a+bx)^{-1+m}(c+dx)^{2-m}}{2f^2(de-cf)(e+fx)} - \\ & \left((2abd f(2-m)(de-cfm) - b^2(2d^2e^2 - 2cdefm - c^2f^2(1-m)m) - a^2d^2f^2(2-3m+m^2))(a+bx)^{-1+m} \right. \\ & \left. (c+dx)^{1-m} \text{Hypergeometric2F1}\left[1, -1+m, m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / (2f^3(be-af)(de-cf)(1-m)) - \\ & \frac{d(bc-ad)(a+bx)^{-1+m}(c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \text{Hypergeometric2F1}\left[-1+m, -1+m, m, -\frac{d(a+bx)}{bc-ad}\right]}{f^3(1-m)} \end{aligned}$$

Result (type 6, 110 leaves, 2 steps):

$$\frac{(b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{AppellF1}\left[1 + m, -2 + m, 3, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}\right]}{(b e - a f)^3 (1 + m)}$$

Problem 3134: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{e + f x} dx$$

Optimal (type 5, 488 leaves, 7 steps):

$$\begin{aligned} & \frac{b (b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{4-m}}{(b c - a d) f^4 (3 - m)} - \frac{b (b (3 d e - c f (1 - m)) - a d f (2 + m)) (a + b x)^{-2+m} (c + d x)^{4-m}}{6 d^2 f^2} + \\ & \frac{b (a + b x)^{-1+m} (c + d x)^{4-m}}{3 d f} - \frac{(b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{3-m} \text{Hypergeometric2F1}\left[1, -3 + m, -2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{f^4 (3 - m)} - \\ & \frac{1}{6 b^3 d^2 f^4 (2 - m) (3 - m)} (b c - a d)^2 (3 a^2 b d^2 f^2 (d e - c f (3 - m)) (1 - m) m + a^3 d^3 f^3 m (2 - 3 m + m^2) + \\ & 3 a b^2 d f m (2 d^2 e^2 - 2 c d e f (3 - m) + c^2 f^2 (6 - 5 m + m^2)) - b^3 (6 d^3 e^3 - 6 c d^2 e^2 f (3 - m) + 3 c^2 d e f^2 (6 - 5 m + m^2) - c^3 f^3 (6 - 11 m + 6 m^2 - m^3))) \\ & (a + b x)^{-2+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[-3 + m, -2 + m, -1 + m, -\frac{d (a + b x)}{b c - a d}\right] \end{aligned}$$

Result (type 5, 417 leaves, 13 steps):

$$\begin{aligned} & \frac{(d e - c f)^3 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, m, 1 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{f^4 m} + \\ & \frac{d (b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[-2 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{b^3 f (1 + m)} - \frac{1}{b^2 f^2 (1 + m)} \\ & d (b c - a d) (d e - c f) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[-1 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] - \\ & \frac{(d e - c f)^3 (a + b x)^m (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[m, m, 1 + m, -\frac{d (a + b x)}{b c - a d}\right]}{f^4 m} + \\ & \frac{d (d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{b f^3 (1 + m)} \end{aligned}$$

Problem 3136: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{(e + f x)^3} dx$$

Optimal (type 5, 453 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 d^3 (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^4 m} - \frac{(d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-m}}{2 f^3 (b e - a f) (e + f x)^2} + \\ & \frac{(d e - c f)^2 (b (5 d e + c f (1 - m)) - a d f (6 - m)) (a + b x)^{1+m} (c + d x)^{-m}}{2 f^3 (b e - a f)^2 (e + f x)} + \frac{1}{2 f^4 (b e - a f)^2 m} \\ & (d e - c f) (2 a b d f (3 - m) (2 d e - c f m) - b^2 (6 d^2 e^2 - 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (6 - 5 m + m^2)) \\ & (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] + \frac{1}{b (b c - a d) f^4 m (1 + m)} \\ & d^3 (b (3 d e - c f (3 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \end{aligned}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(b c - a d)^3 (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{AppellF1}\left[1 + m, -3 + m, 3, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}\right]}{b (b e - a f)^3 (1 + m)}$$

Problem 3137: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{b c + a d + 2 b d x} dx$$

Optimal (type 5, 245 leaves, 6 steps):

$$\begin{aligned} & \frac{(b c - a d) (3 - 2 n) (a + b x)^{2-n} (c + d x)^{-1+n}}{8 b^3 (1 - n)} + \frac{d (a + b x)^{3-n} (c + d x)^{-1+n}}{4 b^3} + \\ & \frac{(b c - a d)^2 (a + b x)^{1-n} (c + d x)^{-1+n} \text{Hypergeometric2F1}\left[1, -1 + n, n, -\frac{b (c + d x)}{d (a + b x)}\right]}{8 b^3 d (1 - n)} - \\ & \frac{(b c - a d)^2 (1 - 2 n^2) (a + b x)^{-n} \left(-\frac{d (a + b x)}{b c - a d}\right)^n (c + d x)^n \text{Hypergeometric2F1}\left[-1 + n, n, 1 + n, \frac{b (c + d x)}{b c - a d}\right]}{8 b^2 d^2 (1 - n) n} \end{aligned}$$

Result (type 5, 319 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(b c - a d)^2 (a + b x)^{-n} (c + d x)^n \operatorname{Hypergeometric2F1}\left[1, -n, 1 - n, -\frac{d(a + b x)}{b(c + d x)}\right]}{8 b^2 d^2 n} + \\
& \frac{(b c - a d)^2 (a + b x)^{-n} (c + d x)^n \left(\frac{b(c + d x)}{b c - a d}\right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1 - n, -\frac{d(a + b x)}{b c - a d}\right]}{8 b^2 d^2 n} - \\
& \frac{(b c - a d) (a + b x)^{-n} \left(-\frac{d(a + b x)}{b c - a d}\right)^n (c + d x)^{1+n} \operatorname{Hypergeometric2F1}\left[n, 1 + n, 2 + n, \frac{b(c + d x)}{b c - a d}\right]}{4 b d^2 (1 + n)} + \\
& \frac{(a + b x)^{-n} \left(-\frac{d(a + b x)}{b c - a d}\right)^n (c + d x)^{2+n} \operatorname{Hypergeometric2F1}\left[n, 2 + n, 3 + n, \frac{b(c + d x)}{b c - a d}\right]}{2 d^2 (2 + n)}
\end{aligned}$$

Problem 3138: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{(b c + a d + 2 b d x)^2} dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(b c - a d) (a + b x)^{1-n} (c + d x)^{-1+n} \operatorname{Hypergeometric2F1}\left[2, 1 - n, 2 - n, -\frac{d(a + b x)}{b(c + d x)}\right]}{4 b^3 d (1 - n)} + \\
& \frac{(a + b x)^{-n} \left(-\frac{d(a + b x)}{b c - a d}\right)^n (c + d x)^{1+n} \operatorname{Hypergeometric2F1}\left[n, 1 + n, 2 + n, \frac{b(c + d x)}{b c - a d}\right]}{4 b d^2 (1 + n)}
\end{aligned}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(a + b x)^{2-n} (c + d x)^n \left(\frac{b(c + d x)}{b c - a d}\right)^{-n} \operatorname{AppellF1}\left[2 - n, -1 - n, 2, 3 - n, -\frac{d(a + b x)}{b c - a d}, -\frac{2 d(a + b x)}{b c - a d}\right]}{b^2 (b c - a d) (2 - n)}$$

Problem 3139: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{(b c + a d + 2 b d x)^3} dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(b c - a d) (a + b x)^{1-n} (c + d x)^n}{8 b^2 d (b c + a d + 2 b d x)^2} - \frac{(1 + 2 n) (a + b x)^{1-n} (c + d x)^n}{8 b^2 d (b c + a d + 2 b d x)} - \frac{(1 - 2 n^2) (a + b x)^{-n} (c + d x)^n \text{Hypergeometric2F1}\left[1, n, 1 + n, -\frac{b (c + d x)}{d (a + b x)}\right]}{8 b^2 d^2 n} + \\
& \frac{(a + b x)^{-n} \left(-\frac{d (a + b x)}{b c - a d}\right)^n (c + d x)^n \text{Hypergeometric2F1}\left[n, n, 1 + n, \frac{b (c + d x)}{b c - a d}\right]}{8 b^2 d^2 n}
\end{aligned}$$

Result (type 6, 113 leaves, 2 steps):

$$\frac{(a + b x)^{2-n} (c + d x)^n \left(\frac{b (c + d x)}{b c - a d}\right)^{-n} \text{AppellF1}\left[2 - n, -1 - n, 3, 3 - n, -\frac{d (a + b x)}{b c - a d}, -\frac{2 d (a + b x)}{b c - a d}\right]}{b^2 (b c - a d)^2 (2 - n)}$$

Problem 3141: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{b c + a d + 2 b d x} dx$$

Optimal (type 5, 231 leaves, 6 steps):

$$\begin{aligned}
& \frac{(b c - a d) (1 + 2 m) (a + b x)^{1+m} (c + d x)^{-m}}{8 b^3 m} + \frac{d (a + b x)^{2+m} (c + d x)^{-m}}{4 b^3} + \frac{(b c - a d)^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, -\frac{b (c + d x)}{d (a + b x)}\right]}{8 b^3 d m} - \\
& \frac{(b c - a d) (1 - 4 m + 2 m^2) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{8 b^3 m (1 + m)}
\end{aligned}$$

Result (type 5, 314 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(b c - a d)^2 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, m, 1 + m, -\frac{d (a + b x)}{b (c + d x)}\right]}{8 b^3 d m} + \\
& \frac{(b c - a d) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[-1 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{2 b^3 (1 + m)} + \\
& \frac{(b c - a d)^2 (a + b x)^m (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, m, 1 + m, -\frac{d (a + b x)}{b c - a d}\right]}{8 b^3 d m} + \\
& \frac{(b c - a d) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{4 b^3 (1 + m)}
\end{aligned}$$

Problem 3142: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(b c + a d + 2 b d x)^2} dx$$

Optimal (type 5, 144 leaves, 4 steps):

$$-\frac{(b c - a d) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[2, m, 1 + m, -\frac{d (a + b x)}{b (c + d x)}\right]}{4 b^3 d m} + \frac{(b c - a d) (a + b x)^m (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[-1 + m, m, 1 + m, -\frac{d (a + b x)}{b c - a d}\right]}{4 b^3 d m}$$

Result (type 6, 93 leaves, 2 steps):

$$\frac{(a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{AppellF1}\left[1 + m, -2 + m, 2, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{2 d (a + b x)}{b c - a d}\right]}{b^3 (1 + m)}$$

Problem 3143: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{(b c + a d + 2 b d x)^3} dx$$

Optimal (type 5, 261 leaves, 7 steps):

$$\frac{(b c - a d) (a + b x)^{-1+m} (c + d x)^{2-m}}{8 b d^2 (b c + a d + 2 b d x)^2} + \frac{(1 - 2 m) (a + b x)^{-1+m} (c + d x)^{2-m}}{8 b d^2 (b c + a d + 2 b d x)} - \frac{(1 - 4 m + 2 m^2) (a + b x)^{-1+m} (c + d x)^{1-m} \text{Hypergeometric2F1}\left[1, -1 + m, m, -\frac{d (a + b x)}{b (c + d x)}\right]}{8 b^2 d^2 (1 - m)} - \frac{(b c - a d) (a + b x)^{-1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[-1 + m, -1 + m, m, -\frac{d (a + b x)}{b c - a d}\right]}{8 b^3 d^2 (1 - m)}$$

Result (type 6, 103 leaves, 2 steps):

$$\frac{(a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{AppellF1}\left[1 + m, -2 + m, 3, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{2 d (a + b x)}{b c - a d}\right]}{b^3 (b c - a d) (1 + m)}$$

Test results for the 159 problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Problem 137: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x)^m (A + B x) (c + d x)^{-m}}{e + f x} dx$$

Optimal (type 5, 233 leaves, 5 steps):

$$\begin{aligned} & -\frac{d (B e - A f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{(B e - A f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right]}{f^2 m} - \frac{1}{b (b c - a d) f^2 m (1 + m)} \\ & (a B d f m - b (B d e - A d f + B c f m)) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \end{aligned}$$

Result (type 5, 220 leaves, 7 steps):

$$\begin{aligned} & \frac{(B e - A f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, m, 1 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{f^2 m} - \\ & \frac{(B e - A f) (a + b x)^m (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, m, 1 + m, -\frac{d (a + b x)}{b c - a d}\right]}{f^2 m} + \\ & \frac{B (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{b f (1 + m)} \end{aligned}$$

Test results for the 34 problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Test results for the 78 problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n (e+f x)^p.m"

Test results for the 35 problems in "1.1.1.7 P(x) (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Test results for the 1071 problems in "1.1.2.2 (c x)^m (a+b x^2)^p.m"

Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{a (2 + m) x^{1+m}}{\sqrt{a + b x^2}} + \frac{b (3 + m) x^{3+m}}{\sqrt{a + b x^2}} \right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m} \sqrt{a + b x^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{a x^{2+m} \sqrt{1 + \frac{b x^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{b x^2}{a}\right]}{\sqrt{a + b x^2}} + \frac{b (3+m) x^{4+m} \sqrt{1 + \frac{b x^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, -\frac{b x^2}{a}\right]}{(4+m) \sqrt{a + b x^2}}$$

Problem 664: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b x^{1+m}}{(a + b x^2)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a + b x^2}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a + b x^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{b x^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, -\frac{b x^2}{a}\right]}{\sqrt{a + b x^2}} - \frac{b x^{2+m} \sqrt{1 + \frac{b x^2}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{b x^2}{a}\right]}{a (2+m) \sqrt{a + b x^2}}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int (c x)^{13/3} (a + b x^2)^{1/3} dx$$

Optimal (type 3, 195 leaves, 6 steps):

$$\begin{aligned} & -\frac{5 a^2 c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{108 b^2} + \frac{a c (c x)^{10/3} (a + b x^2)^{1/3}}{36 b} + \\ & \frac{(c x)^{16/3} (a + b x^2)^{1/3}}{6 c} - \frac{5 a^3 c^{13/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{54 \sqrt{3} b^{8/3}} - \frac{5 a^3 c^{13/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{108 b^{8/3}} \end{aligned}$$

Result (type 3, 275 leaves, 12 steps):

$$\begin{aligned}
& - \frac{5 a^2 c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{108 b^2} + \frac{a c (c x)^{10/3} (a + b x^2)^{1/3}}{36 b} + \frac{(c x)^{16/3} (a + b x^2)^{1/3}}{6 c} - \\
& \frac{5 a^3 c^{13/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + 2 b^{1/3} (c x)^{2/3}}{\sqrt{3} c^{2/3}}\right]}{54 \sqrt{3} b^{8/3}} - \frac{5 a^3 c^{13/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{162 b^{8/3}} + \frac{5 a^3 c^{13/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{324 b^{8/3}}
\end{aligned}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int (c x)^{7/3} (a + b x^2)^{1/3} dx$$

Optimal (type 3, 164 leaves, 5 steps):

$$\begin{aligned}
& \frac{a c (c x)^{4/3} (a + b x^2)^{1/3}}{12 b} + \frac{(c x)^{10/3} (a + b x^2)^{1/3}}{4 c} + \frac{a^2 c^{7/3} \operatorname{ArcTan}\left[\frac{1 + 2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}\right]}{6 \sqrt{3} b^{5/3}} + \frac{a^2 c^{7/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{12 b^{5/3}}
\end{aligned}$$

Result (type 3, 244 leaves, 11 steps):

$$\begin{aligned}
& \frac{a c (c x)^{4/3} (a + b x^2)^{1/3}}{12 b} + \frac{(c x)^{10/3} (a + b x^2)^{1/3}}{4 c} + \frac{a^2 c^{7/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + 2 b^{1/3} (c x)^{2/3}}{\sqrt{3} c^{2/3}}\right]}{6 \sqrt{3} b^{5/3}} + \\
& \frac{a^2 c^{7/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{18 b^{5/3}} - \frac{a^2 c^{7/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{36 b^{5/3}}
\end{aligned}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int (c x)^{1/3} (a + b x^2)^{1/3} dx$$

Optimal (type 3, 133 leaves, 4 steps):

$$\begin{aligned}
& \frac{(c x)^{4/3} (a + b x^2)^{1/3}}{2 c} - \frac{a c^{1/3} \operatorname{ArcTan}\left[\frac{1 + 2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}\right]}{2 \sqrt{3} b^{2/3}} - \frac{a c^{1/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{4 b^{2/3}}
\end{aligned}$$

Result (type 3, 211 leaves, 10 steps):

$$\begin{aligned}
& \frac{(c x)^{4/3} (a + b x^2)^{1/3}}{2 c} - \frac{a c^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + 2 b^{1/3} (c x)^{2/3}}{\sqrt{3} c^{2/3}}\right]}{2 \sqrt{3} b^{2/3}} - \frac{a c^{1/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{6 b^{2/3}} + \frac{a c^{1/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{12 b^{2/3}}
\end{aligned}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^2)^{1/3}}{(c x)^{5/3}} dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$-\frac{3 (a + b x^2)^{1/3}}{2 c (c x)^{2/3}} - \frac{\sqrt{3} b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{2 c^{5/3}} - \frac{3 b^{1/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{4 c^{5/3}}$$

Result (type 3, 208 leaves, 10 steps):

$$-\frac{3 (a + b x^2)^{1/3}}{2 c (c x)^{2/3}} - \frac{\sqrt{3} b^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{2 c^{5/3}} - \frac{b^{1/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{2 c^{5/3}} + \frac{b^{1/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{4 c^{5/3}}$$

Problem 754: Result valid but suboptimal antiderivative.

$$\int (c x)^{13/3} (a + b x^2)^{4/3} dx$$

Optimal (type 3, 223 leaves, 7 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{324 b^2} + \frac{a^2 c (c x)^{10/3} (a + b x^2)^{1/3}}{108 b} + \frac{a (c x)^{16/3} (a + b x^2)^{1/3}}{18 c} +$$

$$\frac{(c x)^{16/3} (a + b x^2)^{4/3}}{8 c} - \frac{5 a^4 c^{13/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{162 \sqrt{3} b^{8/3}} - \frac{5 a^4 c^{13/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{324 b^{8/3}}$$

Result (type 3, 303 leaves, 13 steps):

$$-\frac{5 a^3 c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{324 b^2} + \frac{a^2 c (c x)^{10/3} (a + b x^2)^{1/3}}{108 b} + \frac{a (c x)^{16/3} (a + b x^2)^{1/3}}{18 c} + \frac{(c x)^{16/3} (a + b x^2)^{4/3}}{8 c} -$$

$$\frac{5 a^4 c^{13/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{162 \sqrt{3} b^{8/3}} - \frac{5 a^4 c^{13/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{486 b^{8/3}} + \frac{5 a^4 c^{13/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{972 b^{8/3}}$$

Problem 755: Result valid but suboptimal antiderivative.

$$\int (c x)^{7/3} (a + b x^2)^{4/3} dx$$

Optimal (type 3, 192 leaves, 6 steps):

$$\frac{a^2 c (c x)^{4/3} (a + b x^2)^{1/3}}{27 b} + \frac{a (c x)^{10/3} (a + b x^2)^{1/3}}{9 c} + \frac{(c x)^{10/3} (a + b x^2)^{4/3}}{6 c} +$$

$$\frac{2 a^3 c^{7/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{27 \sqrt{3} b^{5/3}} + \frac{a^3 c^{7/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{27 b^{5/3}}$$

Result (type 3, 272 leaves, 12 steps):

$$\frac{a^2 c (c x)^{4/3} (a + b x^2)^{1/3}}{27 b} + \frac{a (c x)^{10/3} (a + b x^2)^{1/3}}{9 c} + \frac{(c x)^{10/3} (a + b x^2)^{4/3}}{6 c} +$$

$$\frac{2 a^3 c^{7/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{27 \sqrt{3} b^{5/3}} + \frac{2 a^3 c^{7/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{81 b^{5/3}} - \frac{a^3 c^{7/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{81 b^{5/3}}$$

Problem 756: Result valid but suboptimal antiderivative.

$$\int (c x)^{1/3} (a + b x^2)^{4/3} dx$$

Optimal (type 3, 163 leaves, 5 steps):

$$\frac{a (c x)^{4/3} (a + b x^2)^{1/3}}{3 c} + \frac{(c x)^{4/3} (a + b x^2)^{4/3}}{4 c} - \frac{a^2 c^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} - \frac{a^2 c^{1/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 3, 243 leaves, 11 steps):

$$\frac{a (c x)^{4/3} (a + b x^2)^{1/3}}{3 c} + \frac{(c x)^{4/3} (a + b x^2)^{4/3}}{4 c} - \frac{a^2 c^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{3 \sqrt{3} b^{2/3}} -$$

$$\frac{a^2 c^{1/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{9 b^{2/3}} + \frac{a^2 c^{1/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^2)^{4/3}}{(c x)^{5/3}} dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$\frac{2 b (c x)^{4/3} (a + b x^2)^{1/3}}{c^3} - \frac{3 (a + b x^2)^{4/3}}{2 c (c x)^{2/3}} - \frac{2 a b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3}} - \frac{a b^{1/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{c^{5/3}}$$

Result (type 3, 233 leaves, 11 steps):

$$\frac{2 b (c x)^{4/3} (a + b x^2)^{1/3}}{c^3} - \frac{3 (a + b x^2)^{4/3}}{2 c (c x)^{2/3}} - \frac{2 a b^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{\sqrt{3} c^{5/3}} - \frac{2 a b^{1/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{3 c^{5/3}} + \frac{a b^{1/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{3 c^{5/3}}$$

Problem 758: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^2)^{4/3}}{(c x)^{11/3}} dx$$

Optimal (type 3, 157 leaves, 5 steps):

$$-\frac{3 b (a + b x^2)^{1/3}}{2 c^3 (c x)^{2/3}} - \frac{3 (a + b x^2)^{4/3}}{8 c (c x)^{8/3}} - \frac{\sqrt{3} b^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{2 c^{11/3}} - \frac{3 b^{4/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{4 c^{11/3}}$$

Result (type 3, 234 leaves, 11 steps):

$$-\frac{3 b (a + b x^2)^{1/3}}{2 c^3 (c x)^{2/3}} - \frac{3 (a + b x^2)^{4/3}}{8 c (c x)^{8/3}} - \frac{\sqrt{3} b^{4/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{2 c^{11/3}} - \frac{b^{4/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{2 c^{11/3}} + \frac{b^{4/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{4 c^{11/3}}$$

Problem 771: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{19/3}}{(a + b x^2)^{2/3}} dx$$

Optimal (type 3, 198 leaves, 6 steps):

$$\frac{10 a^2 c^5 (c x)^{4/3} (a + b x^2)^{1/3}}{27 b^3} - \frac{2 a c^3 (c x)^{10/3} (a + b x^2)^{1/3}}{9 b^2} + \frac{c (c x)^{16/3} (a + b x^2)^{1/3}}{6 b} +$$

$$\frac{20 a^3 c^{19/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{27 \sqrt{3} b^{11/3}} + \frac{10 a^3 c^{19/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{27 b^{11/3}}$$

Result (type 3, 278 leaves, 12 steps):

$$\frac{10 a^2 c^5 (c x)^{4/3} (a + b x^2)^{1/3}}{27 b^3} - \frac{2 a c^3 (c x)^{10/3} (a + b x^2)^{1/3}}{9 b^2} + \frac{c (c x)^{16/3} (a + b x^2)^{1/3}}{6 b} +$$

$$\frac{20 a^3 c^{19/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{27 \sqrt{3} b^{11/3}} + \frac{20 a^3 c^{19/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{81 b^{11/3}} - \frac{10 a^3 c^{19/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{81 b^{11/3}}$$

Problem 772: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{13/3}}{(a + b x^2)^{2/3}} dx$$

Optimal (type 3, 167 leaves, 5 steps):

$$- \frac{5 a c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{12 b^2} + \frac{c (c x)^{10/3} (a + b x^2)^{1/3}}{4 b} - \frac{5 a^2 c^{13/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{6 \sqrt{3} b^{8/3}} - \frac{5 a^2 c^{13/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{12 b^{8/3}}$$

Result (type 3, 247 leaves, 11 steps):

$$- \frac{5 a c^3 (c x)^{4/3} (a + b x^2)^{1/3}}{12 b^2} + \frac{c (c x)^{10/3} (a + b x^2)^{1/3}}{4 b} - \frac{5 a^2 c^{13/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{6 \sqrt{3} b^{8/3}} -$$

$$\frac{5 a^2 c^{13/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{18 b^{8/3}} + \frac{5 a^2 c^{13/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{36 b^{8/3}}$$

Problem 773: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{7/3}}{(a + b x^2)^{2/3}} dx$$

Optimal (type 3, 131 leaves, 4 steps):

$$\frac{c (c x)^{4/3} (a + b x^2)^{1/3}}{2 b} + \frac{a c^{7/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3}} + \frac{a c^{7/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{2 b^{5/3}}$$

Result (type 3, 209 leaves, 10 steps):

$$\frac{c (c x)^{4/3} (a + b x^2)^{1/3}}{2 b} + \frac{a c^{7/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{\sqrt{3} b^{5/3}} + \frac{a c^{7/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{3 b^{5/3}} - \frac{a c^{7/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{6 b^{5/3}}$$

Problem 774: Result valid but suboptimal antiderivative.

$$\int \frac{(c x)^{1/3}}{(a + b x^2)^{2/3}} dx$$

Optimal (type 3, 106 leaves, 3 steps):

$$- \frac{\sqrt{3} c^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} (c x)^{2/3}}{c^{2/3} (a + b x^2)^{1/3}}}{\sqrt{3}}\right]}{2 b^{2/3}} - \frac{3 c^{1/3} \operatorname{Log}\left[b^{1/3} (c x)^{2/3} - c^{2/3} (a + b x^2)^{1/3}\right]}{4 b^{2/3}}$$

Result (type 3, 183 leaves, 9 steps):

$$- \frac{\sqrt{3} c^{1/3} \operatorname{ArcTan}\left[\frac{c^{2/3} + \frac{2 b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}}{\sqrt{3} c^{2/3}}\right]}{2 b^{2/3}} - \frac{c^{1/3} \operatorname{Log}\left[c^{2/3} - \frac{b^{1/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{2 b^{2/3}} + \frac{c^{1/3} \operatorname{Log}\left[c^{4/3} + \frac{b^{2/3} (c x)^{4/3}}{(a + b x^2)^{2/3}} + \frac{b^{1/3} c^{2/3} (c x)^{2/3}}{(a + b x^2)^{1/3}}\right]}{4 b^{2/3}}$$

Test results for the 349 problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{(1 - 2 x^2)^m}{\sqrt{1 - x^2}} dx$$

Optimal (type 5, 62 leaves, ? steps):

$$- \frac{2^{-2-m} \sqrt{x^2} (2 - 4 x^2)^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, (1 - 2 x^2)^2\right]}{(1 + m) x}$$

Result (type 6, 23 leaves, 1 step):

$$x \operatorname{AppellF1}\left[\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right]$$

Test results for the 1156 problems in "1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q.m"

Test results for the 115 problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 51 problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 174 problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3078 problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 516: Result valid but suboptimal antiderivative.

$$\int x^4 (a + b x^3)^{1/3} dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\frac{a x^2 (a + b x^3)^{1/3}}{18 b} + \frac{1}{6} x^5 (a + b x^3)^{1/3} + \frac{a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{5/3}} + \frac{a^2 \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{18 b^{5/3}}$$

Result (type 3, 173 leaves, 9 steps):

$$\frac{a x^2 (a + b x^3)^{1/3}}{18 b} + \frac{1}{6} x^5 (a + b x^3)^{1/3} + \frac{a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{5/3}} + \frac{a^2 \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{27 b^{5/3}} - \frac{a^2 \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{54 b^{5/3}}$$

Problem 517: Result valid but suboptimal antiderivative.

$$\int x (a + b x^3)^{1/3} dx$$

Optimal (type 3, 94 leaves, 2 steps):

$$\frac{1}{3} x^2 (a + b x^3)^{1/3} - \frac{a \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} - \frac{a \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 3, 145 leaves, 8 steps):

$$\frac{1}{3} x^2 (a + b x^3)^{1/3} - \frac{a \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} - \frac{a \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{2/3}} + \frac{a \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 518: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^3)^{1/3}}{x^2} dx$$

Optimal (type 3, 88 leaves, 2 steps):

$$-\frac{(a + b x^3)^{1/3}}{x} - \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{2} b^{1/3} \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]$$

Result (type 3, 138 leaves, 8 steps):

$$-\frac{(a + b x^3)^{1/3}}{x} - \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{3} b^{1/3} \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right] + \frac{1}{6} b^{1/3} \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{(a + b x^3)^{2/3}} dx$$

Optimal (type 3, 123 leaves, 3 steps):

$$-\frac{5 a x^2 (a + b x^3)^{1/3}}{18 b^2} + \frac{x^5 (a + b x^3)^{1/3}}{6 b} - \frac{5 a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}} - \frac{5 a^2 \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{18 b^{8/3}}$$

Result (type 3, 176 leaves, 9 steps):

$$-\frac{5 a x^2 (a+b x^3)^{1/3}}{18 b^2} + \frac{x^5 (a+b x^3)^{1/3}}{6 b} - \frac{5 a^2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}} - \frac{5 a^2 \operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{27 b^{8/3}} + \frac{5 a^2 \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{54 b^{8/3}}$$

Problem 568: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{(a+b x^3)^{2/3}} dx$$

Optimal (type 3, 97 leaves, 2 steps):

$$\frac{x^2 (a+b x^3)^{1/3}}{3 b} + \frac{2 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}} + \frac{a \operatorname{Log}\left[b^{1/3} x - (a+b x^3)^{1/3}\right]}{3 b^{5/3}}$$

Result (type 3, 148 leaves, 8 steps):

$$\frac{x^2 (a+b x^3)^{1/3}}{3 b} + \frac{2 a \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}} + \frac{2 a \operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{9 b^{5/3}} - \frac{a \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{9 b^{5/3}}$$

Problem 569: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(a+b x^3)^{2/3}} dx$$

Optimal (type 3, 72 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{\operatorname{Log}\left[b^{1/3} x - (a+b x^3)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{\operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 b^{2/3}} + \frac{\operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 b^{2/3}}$$

Problem 581: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1-x^3)^{2/3}} dx$$

Optimal (type 3, 53 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{2} \text{Log}\left[-x - (1-x^3)^{1/3}\right]$$

Result (type 3, 87 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{1}{6} \text{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] - \frac{1}{3} \text{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right]$$

Problem 2271: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(a+b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$-\frac{5 a x (a+b x^{3/2})^{1/3}}{9 b^2} + \frac{x^{5/2} (a+b x^{3/2})^{1/3}}{3 b} - \frac{10 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}} - \frac{5 a^2 \text{Log}\left[b^{1/3} \sqrt{x} - (a+b x^{3/2})^{1/3}\right]}{9 b^{8/3}}$$

Result (type 3, 198 leaves, 10 steps):

$$-\frac{5 a x (a+b x^{3/2})^{1/3}}{9 b^2} + \frac{x^{5/2} (a+b x^{3/2})^{1/3}}{3 b} - \frac{10 a^2 \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{8/3}} - \frac{10 a^2 \text{Log}\left[1 - \frac{b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}\right]}{27 b^{8/3}} + \frac{5 a^2 \text{Log}\left[1 + \frac{b^{2/3} x}{(a+b x^{3/2})^{2/3}} + \frac{b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}\right]}{27 b^{8/3}}$$

Problem 2272: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a+b x^{3/2})^{2/3}} dx$$

Optimal (type 3, 82 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{\operatorname{Log}\left[b^{1/3} \sqrt{x} - (a+b x^{3/2})^{1/3}\right]}{b^{2/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} - \frac{2 \operatorname{Log}\left[1 - \frac{b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}\right]}{3 b^{2/3}} + \frac{\operatorname{Log}\left[1 + \frac{b^{2/3} x}{(a+b x^{3/2})^{2/3}} + \frac{b^{1/3} \sqrt{x}}{(a+b x^{3/2})^{1/3}}\right]}{3 b^{2/3}}$$

Problem 2686: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b n x^{-1+m+n}}{2 (a+b x^n)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a+b x^n}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{b x^n}{a}\right]}{\sqrt{a+b x^n}} - \frac{b n x^{m+n} \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2 + \frac{m}{n}, -\frac{b x^n}{a}\right]}{2 a (m+n) \sqrt{a+b x^n}}$$

Problem 2697: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{6 a x^2}{b (4+m) \sqrt{a+b x^{-2+m}}} + \frac{x^m}{\sqrt{a+b x^{-2+m}}} \right) dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^3 \sqrt{a+b x^{-2+m}}}{b (4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2 a x^3 \sqrt{1 + \frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{3}{2-m}, -\frac{1+m}{2-m}, -\frac{b x^{-2+m}}{a}\right]}{b (4+m) \sqrt{a + b x^{-2+m}}} + \frac{x^{1+m} \sqrt{1 + \frac{b x^{-2+m}}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1+m}{2-m}, \frac{1-2m}{2-m}, -\frac{b x^{-2+m}}{a}\right]}{(1+m) \sqrt{a + b x^{-2+m}}}$$

Problem 2699: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b n x^{-1+m+n}}{2 (a + b x^n)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a + b x^n}} \right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a + b x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{m}{n}, \frac{m+n}{n}, -\frac{b x^n}{a}\right]}{\sqrt{a + b x^n}} - \frac{b n x^{m+n} \sqrt{1 + \frac{b x^n}{a}} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{m+n}{n}, 2 + \frac{m}{n}, -\frac{b x^n}{a}\right]}{2 a (m+n) \sqrt{a + b x^n}}$$

Test results for the 385 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Problem 34: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{7/3}}{a - b x^3} dx$$

Optimal (type 5, 483 leaves, 22 steps):

$$\begin{aligned}
& -\frac{7}{5} a x (a + b x^3)^{1/3} - \frac{1}{5} x (a + b x^3)^{4/3} - \frac{4 \times 2^{1/3} a^{5/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3}} - \frac{2 \times 2^{1/3} a^{5/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3}} - \\
& \frac{7 a^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{5 (a + b x^3)^{2/3}} - \frac{2 \times 2^{1/3} a^{5/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} + \\
& \frac{2 \times 2^{1/3} a^{5/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} - \frac{4 \times 2^{1/3} a^{5/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} + \frac{2^{1/3} a^{5/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}}
\end{aligned}$$

Result (type 6, 56 leaves, 2 steps):

$$\frac{a x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, 1, -\frac{7}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{\left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 35: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{4/3}}{a - b x^3} dx$$

Optimal (type 5, 464 leaves, 21 steps):

$$\begin{aligned}
& -\frac{1}{2} x (a + b x^3)^{1/3} - \frac{2 \times 2^{1/3} a^{2/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3}} - \frac{2^{1/3} a^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3}} - \\
& \frac{a x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}} - \frac{2^{1/3} a^{2/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} + \\
& \frac{2^{1/3} a^{2/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} - \frac{2 \times 2^{1/3} a^{2/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3}} + \frac{a^{2/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} b^{1/3}}
\end{aligned}$$

Result (type 6, 55 leaves, 2 steps):

$$\frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, 1, -\frac{4}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{\left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 36: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{a - b x^3} dx$$

Optimal (type 3, 398 leaves, 14 steps):

$$\begin{aligned} & - \frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^{1/3} b^{1/3}} - \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3} a^{1/3} b^{1/3}} - \frac{\operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3}} + \\ & \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3}} - \frac{2^{1/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 a^{1/3} b^{1/3}} + \frac{\operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{1/3} b^{1/3}} \end{aligned}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, 1, -\frac{1}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{a \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 37: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - b x^3) (a + b x^3)^{2/3}} dx$$

Optimal (type 5, 452 leaves, 17 steps):

$$\begin{aligned} & - \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3} a^{4/3} b^{1/3}} - \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{2/3} \sqrt{3} a^{4/3} b^{1/3}} + \frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 a (a + b x^3)^{2/3}} - \\ & \frac{\operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{4/3} b^{1/3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{4/3} b^{1/3}} - \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{4/3} b^{1/3}} + \frac{\operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{12 \times 2^{2/3} a^{4/3} b^{1/3}} \end{aligned}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, 1, \frac{2}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{a (a + b x^3)^{2/3}}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - b x^3) (a + b x^3)^{5/3}} dx$$

Optimal (type 5, 473 leaves, 21 steps):

$$\frac{x}{4 a^2 (a + b x^3)^{2/3}} - \frac{\text{ArcTan}\left[\frac{1 - \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{2/3} \sqrt{3} a^{7/3} b^{1/3}} - \frac{\text{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{4 \times 2^{2/3} \sqrt{3} a^{7/3} b^{1/3}} + \frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 a^2 (a + b x^3)^{2/3}} -$$

$$\frac{\text{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{12 \times 2^{2/3} a^{7/3} b^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{12 \times 2^{2/3} a^{7/3} b^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{7/3} b^{1/3}} + \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{24 \times 2^{2/3} a^{7/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, 1, \frac{5}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{a^2 (a + b x^3)^{2/3}}$$

Problem 39: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - b x^3) (a + b x^3)^{8/3}} dx$$

Optimal (type 5, 492 leaves, 22 steps):

$$\frac{x}{10 a^2 (a + b x^3)^{5/3}} + \frac{13 x}{40 a^3 (a + b x^3)^{2/3}} - \frac{\text{ArcTan}\left[\frac{1 - \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{4 \times 2^{2/3} \sqrt{3} a^{10/3} b^{1/3}} - \frac{\text{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{8 \times 2^{2/3} \sqrt{3} a^{10/3} b^{1/3}} + \frac{9 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{20 a^3 (a + b x^3)^{2/3}} -$$

$$\frac{\text{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{24 \times 2^{2/3} a^{10/3} b^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{24 \times 2^{2/3} a^{10/3} b^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{12 \times 2^{2/3} a^{10/3} b^{1/3}} + \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{48 \times 2^{2/3} a^{10/3} b^{1/3}}$$

Result (type 6, 58 leaves, 2 steps):

$$\frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, 1, \frac{8}{3}, \frac{4}{3}, \frac{b x^3}{a}, -\frac{b x^3}{a}\right]}{a^3 (a + b x^3)^{2/3}}$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{8/3}}{c + d x^3} dx$$

Optimal (type 3, 331 leaves, 5 steps):

$$\begin{aligned} & -\frac{b(6bc - 11ad)x(a + bx^3)^{2/3}}{18d^2} + \frac{bx(a + bx^3)^{5/3}}{6d} + \frac{b^{2/3}(9b^2c^2 - 24abcd + 20a^2d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}d^3} - \frac{(bc - ad)^{8/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{2/3}d^3} \\ & - \frac{(bc - ad)^{8/3} \operatorname{Log}[c + dx^3]}{6c^{2/3}d^3} + \frac{(bc - ad)^{8/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{2c^{2/3}d^3} - \frac{b^{2/3}(9b^2c^2 - 24abcd + 20a^2d^2) \operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{18d^3} \end{aligned}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{8}{3}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{c \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{5/3}}{c + d x^3} dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\begin{aligned} & \frac{bx(a + bx^3)^{2/3}}{3d} - \frac{b^{2/3}(3bc - 5ad) \operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}d^2} + \frac{(bc - ad)^{5/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{2/3}d^2} + \\ & \frac{(bc - ad)^{5/3} \operatorname{Log}[c + dx^3]}{6c^{2/3}d^2} - \frac{(bc - ad)^{5/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{2c^{2/3}d^2} + \frac{b^{2/3}(3bc - 5ad) \operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{6d^2} \end{aligned}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{5}{3}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{c \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 88: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{c + d x^3} dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d} - \frac{(b c - a d)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} d} - \frac{(b c - a d)^{2/3} \operatorname{Log}[c + d x^3]}{6 c^{2/3} d} + \frac{(b c - a d)^{2/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} d} - \frac{b^{2/3} \operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{2 d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 89: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{1/3}} + \frac{\operatorname{Log}[c + d x^3]}{6 c^{2/3} (b c - a d)^{1/3}} - \frac{\operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} (b c - a d)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{1/3}} - \frac{\operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{2/3} (b c - a d)^{1/3}} + \frac{\operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{2/3} (b c - a d)^{1/3}}$$

Problem 90: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{b x}{a (b c - a d) (a + b x^3)^{1/3}} - \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{4/3}} - \frac{d \operatorname{Log}[c + d x^3]}{6 c^{2/3} (b c - a d)^{4/3}} + \frac{d \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} (b c - a d)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps):

$$\frac{b x}{a (b c - a d) (a + b x^3)^{1/3}} - \frac{d \operatorname{ArcTan}\left[\frac{c^{1/3} + 2 (b c - a d)^{1/3} x}{\sqrt{3} c^{1/3} (a + b x^3)^{1/3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{4/3}} + \frac{d \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{2/3} (b c - a d)^{4/3}} - \frac{d \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{2/3} (b c - a d)^{4/3}}$$

Problem 91: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{7/3} (c + d x^3)} dx$$

Optimal (type 3, 226 leaves, 4 steps):

$$\frac{b x}{4 a (b c - a d) (a + b x^3)^{4/3}} + \frac{b (3 b c - 7 a d) x}{4 a^2 (b c - a d)^2 (a + b x^3)^{1/3}} + \frac{d^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{7/3}} + \frac{d^2 \operatorname{Log}[c + d x^3]}{6 c^{2/3} (b c - a d)^{7/3}} - \frac{d^2 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} (b c - a d)^{7/3}}$$

Result (type 5, 621 leaves, 2 steps):

$$\begin{aligned} & - \frac{1}{40 c^4 (b c - a d)^2 x^5 (a + b x^3)^{10/3}} \left(70 c^4 (b c - a d) x^3 (a + b x^3)^2 + 105 c^3 d (b c - a d) x^6 (a + b x^3)^2 + \right. \\ & 45 c^2 d^2 (b c - a d) x^9 (a + b x^3)^2 + 280 c^5 (a + b x^3)^3 + 420 c^4 d x^3 (a + b x^3)^3 + 180 c^3 d^2 x^6 (a + b x^3)^3 - \\ & 280 c^5 (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 420 c^4 d x^3 (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\ & 180 c^3 d^2 x^6 (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 33 c^2 (b c - a d)^3 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{10}{3}, \frac{13}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\ & 60 c d (b c - a d)^3 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{10}{3}, \frac{13}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 27 d^2 (b c - a d)^3 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{10}{3}, \frac{13}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\ & 9 c^2 (b c - a d)^3 x^9 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 18 c d (b c - a d)^3 x^{12} \\ & \left. \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 9 d^2 (b c - a d)^3 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right) \end{aligned}$$

Problem 92: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{10/3} (c + d x^3)} dx$$

Optimal (type 3, 280 leaves, 5 steps):

$$\frac{b x}{7 a (b c - a d) (a + b x^3)^{7/3}} + \frac{b (6 b c - 13 a d) x}{28 a^2 (b c - a d)^2 (a + b x^3)^{4/3}} + \frac{b (18 b^2 c^2 - 57 a b c d + 67 a^2 d^2) x}{28 a^3 (b c - a d)^3 (a + b x^3)^{1/3}} -$$

$$\frac{d^3 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{10/3}} - \frac{d^3 \operatorname{Log}[c + d x^3]}{6 c^{2/3} (b c - a d)^{10/3}} + \frac{d^3 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} (b c - a d)^{10/3}}$$

Result (type 5, 1172 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{5096 c^5 (b c - a d)^3 x^8 (a + b x^3)^{13/3}} \left(7280 c^5 (b c - a d)^2 x^6 (a + b x^3)^2 + 16380 c^4 d (b c - a d)^2 x^9 (a + b x^3)^2 + 14040 c^3 d^2 (b c - a d)^2 x^{12} (a + b x^3)^2 + \right. \\
& 4212 c^2 d^3 (b c - a d)^2 x^{15} (a + b x^3)^2 + 12740 c^6 (b c - a d) x^3 (a + b x^3)^3 + 28665 c^5 d (b c - a d) x^6 (a + b x^3)^3 + \\
& 24570 c^4 d^2 (b c - a d) x^9 (a + b x^3)^3 + 7371 c^3 d^3 (b c - a d) x^{12} (a + b x^3)^3 + 50960 c^7 (a + b x^3)^4 + 114660 c^6 d x^3 (a + b x^3)^4 + \\
& 98280 c^5 d^2 x^6 (a + b x^3)^4 + 29484 c^4 d^3 x^9 (a + b x^3)^4 - 50960 c^7 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 114660 c^6 d x^3 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 98280 c^5 d^2 x^6 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 29484 c^4 d^3 x^9 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 5796 c^3 (b c - a d)^4 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{13}{3}, \frac{16}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 15246 c^2 d (b c - a d)^4 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{13}{3}, \frac{16}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 13608 c d^2 (b c - a d)^4 x^{18} \operatorname{Hypergeometric2F1}\left[2, \frac{13}{3}, \frac{16}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 4158 d^3 (b c - a d)^4 x^{21} \\
& \operatorname{Hypergeometric2F1}\left[2, \frac{13}{3}, \frac{16}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 2646 c^3 (b c - a d)^4 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{13}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 7560 c^2 d (b c - a d)^4 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{13}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 7182 c d^2 (b c - a d)^4 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{13}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 2268 d^3 (b c - a d)^4 x^{21} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{13}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 378 c^3 (b c - a d)^4 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{13}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 1134 c^2 d (b c - a d)^4 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{13}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 1134 c d^2 (b c - a d)^4 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{13}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& \left. 378 d^3 (b c - a d)^4 x^{21} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{13}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]\right)
\end{aligned}$$

Problem 98: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{8/3}}{(c + d x^3)^2} dx$$

Optimal (type 3, 351 leaves, 5 steps):

$$\begin{aligned} & \frac{b(2bc - ad)x(a + bx^3)^{2/3}}{3cd^2} - \frac{(bc - ad)x(a + bx^3)^{5/3}}{3cd(c + dx^3)} - \frac{2b^{5/3}(3bc - 4ad)\operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}d^3} + \frac{2(bc - ad)^{5/3}(3bc + ad)\operatorname{ArcTan}\left[\frac{1 + \frac{2(bc - ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}c^{5/3}d^3} + \\ & \frac{(bc - ad)^{5/3}(3bc + ad)\operatorname{Log}[c + dx^3]}{9c^{5/3}d^3} - \frac{(bc - ad)^{5/3}(3bc + ad)\operatorname{Log}\left[\frac{(bc - ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{3c^{5/3}d^3} + \frac{b^{5/3}(3bc - 4ad)\operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{3d^3} \end{aligned}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{8}{3}, 2, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{c^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 99: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{5/3}}{(c + d x^3)^2} dx$$

Optimal (type 3, 301 leaves, 4 steps):

$$\begin{aligned} & -\frac{(bc - ad)x(a + bx^3)^{2/3}}{3cd(c + dx^3)} + \frac{b^{5/3}\operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}d^2} - \frac{(bc - ad)^{2/3}(3bc + 2ad)\operatorname{ArcTan}\left[\frac{1 + \frac{2(bc - ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}c^{5/3}d^2} - \\ & \frac{(bc - ad)^{2/3}(3bc + 2ad)\operatorname{Log}[c + dx^3]}{18c^{5/3}d^2} + \frac{(bc - ad)^{2/3}(3bc + 2ad)\operatorname{Log}\left[\frac{(bc - ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{6c^{5/3}d^2} - \frac{b^{5/3}\operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{2d^2} \end{aligned}$$

Result (type 6, 60 leaves, 2 steps):

$$\frac{a x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{5}{3}, 2, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{c^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^3)^{2/3}}{(c + d x^3)^2} dx$$

Optimal (type 3, 182 leaves, 2 steps):

$$\frac{x (a + b x^3)^{2/3}}{3 c (c + d x^3)} + \frac{2 a \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{1/3}} + \frac{a \operatorname{Log}[c + d x^3]}{9 c^{5/3} (b c - a d)^{1/3}} - \frac{a \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{3 c^{5/3} (b c - a d)^{1/3}}$$

Result (type 3, 241 leaves, 8 steps):

$$\frac{x (a + b x^3)^{2/3}}{3 c (c + d x^3)} + \frac{2 a \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{1/3}} - \frac{2 a \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 c^{5/3} (b c - a d)^{1/3}} + \frac{a \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 c^{5/3} (b c - a d)^{1/3}}$$

Problem 101: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{1/3} (c + d x^3)^2} dx$$

Optimal (type 3, 217 leaves, 2 steps):

$$-\frac{d x (a + b x^3)^{2/3}}{3 c (b c - a d) (c + d x^3)} + \frac{(3 b c - 2 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{4/3}} + \frac{(3 b c - 2 a d) \operatorname{Log}[c + d x^3]}{18 c^{5/3} (b c - a d)^{4/3}} - \frac{(3 b c - 2 a d) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{6 c^{5/3} (b c - a d)^{4/3}}$$

Result (type 3, 276 leaves, 8 steps):

$$-\frac{d x (a + b x^3)^{2/3}}{3 c (b c - a d) (c + d x^3)} + \frac{(3 b c - 2 a d) \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{4/3}} - \frac{(3 b c - 2 a d) \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 c^{5/3} (b c - a d)^{4/3}} + \frac{(3 b c - 2 a d) \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 c^{5/3} (b c - a d)^{4/3}}$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{4/3} (c + d x^3)^2} dx$$

Optimal (type 3, 261 leaves, 4 steps):

$$\frac{b (3 b c + a d) x}{3 a c (b c - a d)^2 (a + b x^3)^{1/3}} - \frac{d x}{3 c (b c - a d) (a + b x^3)^{1/3} (c + d x^3)} - \frac{2 d (3 b c - a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{7/3}} - \frac{d (3 b c - a d) \operatorname{Log}[c + d x^3]}{9 c^{5/3} (b c - a d)^{7/3}} + \frac{d (3 b c - a d) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{3 c^{5/3} (b c - a d)^{7/3}}$$

Result (type 5, 625 leaves, 2 steps):

$$\begin{aligned} & - \frac{1}{420 (b c - a d)^2 x^5 (c + d x^3)} \\ & c (a + b x^3)^{2/3} \left(6860 + \frac{13720 d x^3}{c} + \frac{6300 d^2 x^6}{c^2} - \frac{525 (b c - a d) x^3}{c (a + b x^3)} - \frac{1890 d (b c - a d) x^6}{c^2 (a + b x^3)} - \frac{945 d^2 (b c - a d) x^9}{c^3 (a + b x^3)} - 6860 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \right. \right. \\ & \quad \left. \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \frac{13720 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c} - \frac{6300 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^2} + \\ & \quad \frac{2240 (b c - a d) x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c (a + b x^3)} + \frac{5320 d (b c - a d) x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^2 (a + b x^3)} + \\ & \quad \frac{2520 d^2 (b c - a d) x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^3 (a + b x^3)} - \frac{54 (b c - a d)^3 x^9 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{7}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^3 (a + b x^3)^3} - \\ & \quad \frac{108 d (b c - a d)^3 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{7}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^4 (a + b x^3)^3} - \\ & \quad \left. \frac{54 d^2 (b c - a d)^3 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{7}{3}\right\}, \left\{1, \frac{13}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]}{c^5 (a + b x^3)^3} \right) \end{aligned}$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{7/3} (c + d x^3)^2} dx$$

Optimal (type 3, 324 leaves, 5 steps):

$$\frac{b (3 b c + 4 a d) x}{12 a c (b c - a d)^2 (a + b x^3)^{4/3}} + \frac{b (9 b^2 c^2 - 33 a b c d - 4 a^2 d^2) x}{12 a^2 c (b c - a d)^3 (a + b x^3)^{1/3}} - \frac{d x}{3 c (b c - a d) (a + b x^3)^{4/3} (c + d x^3)} +$$

$$\frac{d^2 (9 b c - 2 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^{5/3} (b c - a d)^{10/3}} + \frac{d^2 (9 b c - 2 a d) \operatorname{Log}[c + d x^3]}{18 c^{5/3} (b c - a d)^{10/3}} - \frac{d^2 (9 b c - 2 a d) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{6 c^{5/3} (b c - a d)^{10/3}}$$

Result (type 5, 1214 leaves, 2 steps):

1

$$\begin{aligned}
& 21840 c^5 (b c - a d)^3 x^8 (a + b x^3)^{10/3} (c + d x^3) \\
& \left(26130 c^5 (b c - a d)^2 x^6 (a + b x^3)^2 + 89505 c^4 d (b c - a d)^2 x^9 (a + b x^3)^2 + 84240 c^3 d^2 (b c - a d)^2 x^{12} (a + b x^3)^2 + \right. \\
& 26325 c^2 d^3 (b c - a d)^2 x^{15} (a + b x^3)^2 + 748020 c^6 (b c - a d) x^3 (a + b x^3)^3 + 2113020 c^5 d (b c - a d) x^6 (a + b x^3)^3 + \\
& 1916460 c^4 d^2 (b c - a d) x^9 (a + b x^3)^3 + 589680 c^3 d^3 (b c - a d) x^{12} (a + b x^3)^3 - 2002000 c^7 (a + b x^3)^4 - 5460000 c^6 d x^3 (a + b x^3)^4 - \\
& 4914000 c^5 d^2 x^6 (a + b x^3)^4 - 1506960 c^4 d^3 x^9 (a + b x^3)^4 - 1248520 c^6 (b c - a d) x^3 (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 3478020 c^5 d (b c - a d) x^6 (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 3144960 c^4 d^2 (b c - a d) x^9 (a + b x^3)^3 \\
& \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 966420 c^3 d^3 (b c - a d) x^{12} (a + b x^3)^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 2002000 c^7 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 5460000 c^6 d x^3 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 4914000 c^5 d^2 x^6 (a + b x^3)^4 \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 1506960 c^4 d^3 x^9 (a + b x^3)^4 \\
& \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 7938 c^3 (b c - a d)^4 x^{12} \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 22680 c^2 d (b c - a d)^4 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 21546 c d^2 (b c - a d)^4 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 6804 d^3 (b c - a d)^4 x^{21} \text{HypergeometricPFQ}\left[\left\{2, 2, \frac{10}{3}\right\}, \left\{1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 1134 c^3 (b c - a d)^4 x^{12} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 3402 c^2 d (b c - a d)^4 x^{15} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 3402 c d^2 (b c - a d)^4 x^{18} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& \left. 1134 d^3 (b c - a d)^4 x^{21} \text{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]\right)
\end{aligned}$$

Problem 109: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{14/3}}{(c + d x^3)^3} dx$$

Optimal (type 3, 541 leaves, 7 steps):

$$\begin{aligned} & - \frac{b (2 b c - a d) (18 b^2 c^2 - 18 a b c d - 5 a^2 d^2) x (a + b x^3)^{2/3}}{18 c^2 d^4} + \frac{b (18 b^2 c^2 - 10 a b c d - 5 a^2 d^2) x (a + b x^3)^{5/3}}{18 c^2 d^3} - \\ & \frac{(b c - a d) x (a + b x^3)^{11/3}}{6 c d (c + d x^3)^2} - \frac{(b c - a d) (12 b c + 5 a d) x (a + b x^3)^{8/3}}{18 c^2 d^2 (c + d x^3)} + \frac{b^{8/3} (54 b^2 c^2 - 126 a b c d + 77 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} d^5} - \\ & \frac{(b c - a d)^{8/3} (54 b^2 c^2 + 18 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^5} - \frac{(b c - a d)^{8/3} (54 b^2 c^2 + 18 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} d^5} + \\ & \frac{(b c - a d)^{8/3} (54 b^2 c^2 + 18 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} d^5} - \frac{b^{8/3} (54 b^2 c^2 - 126 a b c d + 77 a^2 d^2) \operatorname{Log}\left[-b^{1/3} x + (a + b x^3)^{1/3}\right]}{18 d^5} \end{aligned}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^4 x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{14}{3}, 3, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c^3 \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 110: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{11/3}}{(c + d x^3)^3} dx$$

Optimal (type 3, 458 leaves, 6 steps):

$$\begin{aligned}
& \frac{b (18 b^2 c^2 - 7 a b c d - 5 a^2 d^2) x (a + b x^3)^{2/3}}{18 c^2 d^3} - \frac{(b c - a d) x (a + b x^3)^{8/3}}{6 c d (c + d x^3)^2} - \\
& \frac{(b c - a d) (9 b c + 5 a d) x (a + b x^3)^{5/3}}{18 c^2 d^2 (c + d x^3)} - \frac{b^{8/3} (9 b c - 11 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^4} + \\
& \frac{(b c - a d)^{5/3} (27 b^2 c^2 + 12 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^4} + \frac{(b c - a d)^{5/3} (27 b^2 c^2 + 12 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} d^4} - \\
& \frac{(b c - a d)^{5/3} (27 b^2 c^2 + 12 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} d^4} + \frac{b^{8/3} (9 b c - 11 a d) \operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{6 d^4}
\end{aligned}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^3 x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{11}{3}, 3, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c^3 \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 111: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{8/3}}{(c + d x^3)^3} dx$$

Optimal (type 3, 391 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(b c - a d) x (a + b x^3)^{5/3}}{6 c d (c + d x^3)^2} - \frac{(b c - a d) (6 b c + 5 a d) x (a + b x^3)^{2/3}}{18 c^2 d^2 (c + d x^3)} + \frac{b^{8/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3} - \\
& \frac{(b c - a d)^{2/3} (9 b^2 c^2 + 6 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} d^3} - \frac{(b c - a d)^{2/3} (9 b^2 c^2 + 6 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} d^3} + \\
& \frac{(b c - a d)^{2/3} (9 b^2 c^2 + 6 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} d^3} - \frac{b^{8/3} \operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{2 d^3}
\end{aligned}$$

Result (type 6, 62 leaves, 2 steps):

$$\frac{a^2 x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{8}{3}, 3, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c^3 \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 112: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^3)^{5/3}}{(c + d x^3)^3} dx$$

Optimal (type 3, 217 leaves, 3 steps):

$$\frac{x (a + b x^3)^{5/3}}{6 c (c + d x^3)^2} + \frac{5 a x (a + b x^3)^{2/3}}{18 c^2 (c + d x^3)} + \frac{5 a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{1/3}} + \frac{5 a^2 \operatorname{Log}[c + d x^3]}{54 c^{8/3} (b c - a d)^{1/3}} - \frac{5 a^2 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} (b c - a d)^{1/3}}$$

Result (type 3, 276 leaves, 9 steps):

$$\frac{x (a + b x^3)^{5/3}}{6 c (c + d x^3)^2} + \frac{5 a x (a + b x^3)^{2/3}}{18 c^2 (c + d x^3)} + \frac{5 a^2 \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{1/3}} - \frac{5 a^2 \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{27 c^{8/3} (b c - a d)^{1/3}} + \frac{5 a^2 \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{54 c^{8/3} (b c - a d)^{1/3}}$$

Problem 113: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x^3)^{2/3}}{(c + d x^3)^3} dx$$

Optimal (type 3, 267 leaves, 3 steps):

$$-\frac{d x (a + b x^3)^{5/3}}{6 c (b c - a d) (c + d x^3)^2} + \frac{(6 b c - 5 a d) x (a + b x^3)^{2/3}}{18 c^2 (b c - a d) (c + d x^3)} + \frac{a (6 b c - 5 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{4/3}} + \frac{a (6 b c - 5 a d) \operatorname{Log}[c + d x^3]}{54 c^{8/3} (b c - a d)^{4/3}} - \frac{a (6 b c - 5 a d) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} (b c - a d)^{4/3}}$$

Result (type 3, 326 leaves, 9 steps):

$$\begin{aligned}
& - \frac{d x (a + b x^3)^{5/3}}{6 c (b c - a d) (c + d x^3)^2} + \frac{(6 b c - 5 a d) x (a + b x^3)^{2/3}}{18 c^2 (b c - a d) (c + d x^3)} + \frac{a (6 b c - 5 a d) \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{4/3}} - \\
& \frac{a (6 b c - 5 a d) \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{27 c^{8/3} (b c - a d)^{4/3}} + \frac{a (6 b c - 5 a d) \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{54 c^{8/3} (b c - a d)^{4/3}}
\end{aligned}$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^3)^{1/3} (c + d x^3)^3} dx$$

Optimal (type 3, 307 leaves, 4 steps):

$$\begin{aligned}
& - \frac{d x (a + b x^3)^{2/3}}{6 c (b c - a d) (c + d x^3)^2} - \frac{d (9 b c - 5 a d) x (a + b x^3)^{2/3}}{18 c^2 (b c - a d)^2 (c + d x^3)} + \frac{(9 b^2 c^2 - 12 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{7/3}} + \\
& \frac{(9 b^2 c^2 - 12 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} (b c - a d)^{7/3}} - \frac{(9 b^2 c^2 - 12 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} (b c - a d)^{7/3}}
\end{aligned}$$

Result (type 5, 167 leaves, 2 steps):

$$\begin{aligned}
& - \left(\left(x \left(c d (3 b^2 c x^3 (4 c + 3 d x^3) - a^2 d (8 c + 5 d x^3) + a b (12 c^2 + c d x^3 - 5 d^2 x^6) \right) - \right. \right. \\
& \left. \left. 2 (9 b^2 c^2 - 12 a b c d + 5 a^2 d^2) (c + d x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right) \right) / \left(18 c^3 (b c - a d)^2 (a + b x^3)^{1/3} (c + d x^3)^2 \right)
\end{aligned}$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x^3)^{4/3} (c + d x^3)^3} dx$$

Optimal (type 3, 377 leaves, 5 steps):

$$\begin{aligned}
& - \frac{d x}{6 c (b c - a d) (a + b x^3)^{1/3} (c + d x^3)^2} + \frac{b (6 b c + a d) x}{6 a c (b c - a d)^2 (a + b x^3)^{1/3} (c + d x^3)} + \\
& \frac{d (18 b^2 c^2 + 15 a b c d - 5 a^2 d^2) x (a + b x^3)^{2/3}}{18 a c^2 (b c - a d)^3 (c + d x^3)} - \frac{d (27 b^2 c^2 - 18 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{10/3}} - \\
& \frac{d (27 b^2 c^2 - 18 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} (b c - a d)^{10/3}} + \frac{d (27 b^2 c^2 - 18 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} (b c - a d)^{10/3}}
\end{aligned}$$

Result (type 5, 428 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{16380 c^5 (b c - a d)^3 x^8 (a + b x^3)^{7/3} (c + d x^3)^2} \\
& \left(65 c^2 (a + b x^3)^2 \left(14000 a^2 c^5 + 21896 a b c^5 x^3 + 48104 a^2 c^4 d x^3 + 8391 b^2 c^5 x^6 + 70802 a b c^4 d x^6 + 60807 a^2 c^3 d^2 x^6 + 24417 b^2 c^4 d x^9 + \right. \right. \\
& 81534 a b c^3 d^2 x^9 + 33657 a^2 c^2 d^3 x^9 + 23409 b^2 c^3 d^2 x^{12} + 38652 a b c^2 d^3 x^{12} + 7155 a^2 c d^4 x^{12} + 7425 b^2 c^2 d^3 x^{15} + 5940 a b c d^4 x^{15} + \\
& 243 a^2 d^5 x^{15} - 28 (c + d x^3)^2 (27 b^2 c^2 x^6 (7 c + 6 d x^3) + 9 a b c x^3 (73 c^2 + 104 c d x^3 + 33 d^2 x^6) + a^2 (500 c^3 + 843 c^2 d x^3 + 375 c d^2 x^6 + 27 d^3 x^9)) \\
& \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right) - \\
& 486 (b c - a d)^4 x^{12} (c + d x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{16}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]
\end{aligned}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{7/3} (c + d x^3)^3} dx$$

Optimal (type 3, 463 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d x}{6 c (b c - a d) (a + b x^3)^{4/3} (c + d x^3)^2} + \frac{b (3 b c + 2 a d) x}{12 a c (b c - a d)^2 (a + b x^3)^{4/3} (c + d x^3)} + \frac{b (9 b^2 c^2 - 42 a b c d - 2 a^2 d^2) x}{12 a^2 c (b c - a d)^3 (a + b x^3)^{1/3} (c + d x^3)} + \\
& \frac{d (27 b^3 c^3 - 135 a b^2 c^2 d - 42 a^2 b c d^2 + 10 a^3 d^3) x (a + b x^3)^{2/3}}{36 a^2 c^2 (b c - a d)^4 (c + d x^3)} + \frac{d^2 (54 b^2 c^2 - 24 a b c d + 5 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^{8/3} (b c - a d)^{13/3}} + \\
& \frac{d^2 (54 b^2 c^2 - 24 a b c d + 5 a^2 d^2) \operatorname{Log}[c + d x^3]}{54 c^{8/3} (b c - a d)^{13/3}} - \frac{d^2 (54 b^2 c^2 - 24 a b c d + 5 a^2 d^2) \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{18 c^{8/3} (b c - a d)^{13/3}}
\end{aligned}$$

Result (type 5, 1990 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{524160 c^6 (b c - a d)^4 x^{11} (a + b x^3)^{10/3} (c + d x^3)^2} \\
& \left(522756 c^6 (b c - a d)^3 x^9 (a + b x^3)^2 + 1516320 c^5 d (b c - a d)^3 x^{12} (a + b x^3)^2 + 2198664 c^4 d^2 (b c - a d)^3 x^{15} (a + b x^3)^2 + \right. \\
& 1415232 c^3 d^3 (b c - a d)^3 x^{18} (a + b x^3)^2 + 341172 c^2 d^4 (b c - a d)^3 x^{21} (a + b x^3)^2 + 28042560 c^7 (b c - a d)^2 x^6 (a + b x^3)^3 + \\
& 107602560 c^6 d (b c - a d)^2 x^9 (a + b x^3)^3 + 157697280 c^5 d^2 (b c - a d)^2 x^{12} (a + b x^3)^3 + 101088000 c^4 d^3 (b c - a d)^2 x^{15} (a + b x^3)^3 + \\
& 24261120 c^3 d^4 (b c - a d)^2 x^{18} (a + b x^3)^3 - 265470660 c^8 (b c - a d) x^3 (a + b x^3)^4 - 1019636800 c^7 d (b c - a d) x^6 (a + b x^3)^4 - \\
& 1466086440 c^6 d^2 (b c - a d) x^9 (a + b x^3)^4 - 930252960 c^5 d^3 (b c - a d) x^{12} (a + b x^3)^4 - 221899860 c^4 d^4 (b c - a d) x^{15} (a + b x^3)^4 + \\
& 335877360 c^9 (a + b x^3)^5 + 1279532800 c^8 d x^3 (a + b x^3)^5 + 1823334240 c^7 d^2 x^6 (a + b x^3)^5 + 1151579520 c^6 d^3 x^9 (a + b x^3)^5 + \\
& 273939120 c^5 d^4 x^{12} (a + b x^3)^5 - 67420080 c^7 (b c - a d)^2 x^6 (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 259692160 c^6 d (b c - a d)^2 x^9 (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 377700960 c^5 d^2 (b c - a d)^2 x^{12} (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 241113600 c^4 d^3 (b c - a d)^2 x^{15} (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 57723120 c^3 d^4 (b c - a d)^2 x^{18} (a + b x^3)^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 349440000 c^8 (b c - a d) x^3 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 1339520000 c^7 d (b c - a d) x^6 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +
\end{aligned}$$

$$\begin{aligned}
& 1921920000 c^6 d^2 (b c - a d) x^9 (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 1218147840 c^5 d^3 (b c - a d) x^{12} (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 290384640 c^4 d^4 (b c - a d) x^{15} (a + b x^3)^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 335877360 c^9 (a + b x^3)^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 1279532800 c^8 d x^3 (a + b x^3)^5 \\
& \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 1823334240 c^7 d^2 x^6 (a + b x^3)^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 1151579520 c^6 d^3 x^9 (a + b x^3)^5 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 273939120 c^5 d^4 x^{12} (a + b x^3)^5 \\
& \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 57834 c^4 (b c - a d)^5 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 224532 c^3 d (b c - a d)^5 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 326592 c^2 d^2 (b c - a d)^5 x^{21} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 210924 c d^3 (b c - a d)^5 x^{24} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 51030 d^4 (b c - a d)^5 x^{27} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 5103 c^4 (b c - a d)^5 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 20412 c^3 d (b c - a d)^5 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 30618 c^2 d^2 (b c - a d)^5 x^{21} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 20412 c d^3 (b c - a d)^5 x^{24} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 5103 d^4 (b c - a d)^5 x^{27} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{10}{3}\right\}, \left\{1, 1, 1, \frac{19}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]
\end{aligned}$$

Test results for the 1081 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(8c - dx^3)^2 (c + dx^3)^{3/2}} dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2x(4c + dx^3)}{81c^2(8c - dx^3)\sqrt{c + dx^3}} - \frac{2\sqrt{2 + \sqrt{3}}(c^{1/3} + d^{1/3}x) \sqrt{\frac{c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2}{((1 + \sqrt{3})c^{1/3} + d^{1/3}x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3})c^{1/3} + d^{1/3}x}{(1 + \sqrt{3})c^{1/3} + d^{1/3}x}\right], -7 - 4\sqrt{3}\right]}{81 \times 3^{1/4} c d^{7/3} \sqrt{\frac{c^{1/3}(c^{1/3} + d^{1/3}x)}{((1 + \sqrt{3})c^{1/3} + d^{1/3}x)^2}} \sqrt{c + dx^3}}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 \sqrt{1 + \frac{dx^3}{c}} \operatorname{AppellF1}\left[\frac{7}{3}, 2, \frac{3}{2}, \frac{10}{3}, \frac{dx^3}{8c}, -\frac{dx^3}{c}\right]}{448c^3 \sqrt{c + dx^3}}$$

Problem 574: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 (a + bx^3)^{1/3}}{ad - bdx^3} dx$$

Optimal (type 3, 268 leaves, 6 steps):

$$-\frac{7ax^2(a + bx^3)^{1/3}}{18b^2d} - \frac{x^5(a + bx^3)^{1/3}}{6bd} + \frac{11a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a + bx^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}b^{8/3}d} - \frac{2^{1/3}a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 \cdot 2^{1/3}b^{1/3}x}{(a + bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}b^{8/3}d} +$$

$$\frac{a^2 \operatorname{Log}[ad - bdx^3]}{3 \times 2^{2/3}b^{8/3}d} + \frac{11a^2 \operatorname{Log}[b^{1/3}x - (a + bx^3)^{1/3}]}{18b^{8/3}d} - \frac{a^2 \operatorname{Log}[2^{1/3}b^{1/3}x - (a + bx^3)^{1/3}]}{2^{2/3}b^{8/3}d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 (a + bx^3)^{1/3} \operatorname{AppellF1}\left[\frac{8}{3}, -\frac{1}{3}, 1, \frac{11}{3}, -\frac{bx^3}{a}, \frac{bx^3}{a}\right]}{8ad \left(1 + \frac{bx^3}{a}\right)^{1/3}}$$

Problem 575: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (a + b x^3)^{1/3}}{a d - b d x^3} dx$$

Optimal (type 3, 233 leaves, 5 steps):

$$\begin{aligned} & -\frac{x^2 (a + b x^3)^{1/3}}{3 b d} + \frac{4 a \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d} - \frac{2^{1/3} a \operatorname{ArcTan}\left[\frac{1 + \frac{2 \cdot 2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3} d} + \\ & \frac{a \operatorname{Log}[a d - b d x^3]}{3 \times 2^{2/3} b^{5/3} d} + \frac{2 a \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{3 b^{5/3} d} - \frac{a \operatorname{Log}[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}]}{2^{2/3} b^{5/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3}, -\frac{1}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{5 a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 576: Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x^3)^{1/3}}{a d - b d x^3} dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\begin{aligned} & \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d} - \frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \cdot 2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d} + \frac{\operatorname{Log}[a d - b d x^3]}{3 \times 2^{2/3} b^{2/3} d} + \frac{\operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 b^{2/3} d} - \frac{\operatorname{Log}[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}]}{2^{2/3} b^{2/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{2 a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 577: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^2 (a d - b d x^3)} dx$$

Optimal (type 3, 156 leaves, 3 steps):

$$-\frac{(a+bx^3)^{1/3}}{adx} - \frac{2^{1/3} b^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2}{3} 2^{1/3} b^{1/3} x}{\frac{(a+bx^3)^{1/3}}{\sqrt{3}}}\right]}{\sqrt{3} ad} + \frac{b^{1/3} \operatorname{Log}[ad-bdx^3]}{3 \times 2^{2/3} ad} - \frac{b^{1/3} \operatorname{Log}[2^{1/3} b^{1/3} x - (a+bx^3)^{1/3}]}{2^{2/3} ad}$$

Result (type 5, 77 leaves, 2 steps):

$$-\frac{(a+bx^3)^{1/3} \left(1 - \frac{bx^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{2bx^3}{a-bx^3}\right]}{adx \left(1 + \frac{bx^3}{a}\right)^{1/3}}$$

Problem 578: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{1/3}}{x^5 (ad-bdx^3)} dx$$

Optimal (type 3, 183 leaves, 4 steps):

$$-\frac{(a+bx^3)^{1/3}}{4adx^4} - \frac{5b(a+bx^3)^{1/3}}{4a^2dx} - \frac{2^{1/3} b^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2}{3} 2^{1/3} b^{1/3} x}{\frac{(a+bx^3)^{1/3}}{\sqrt{3}}}\right]}{\sqrt{3} a^2d} + \frac{b^{4/3} \operatorname{Log}[ad-bdx^3]}{3 \times 2^{2/3} a^2d} - \frac{b^{4/3} \operatorname{Log}[2^{1/3} b^{1/3} x - (a+bx^3)^{1/3}]}{2^{2/3} a^2d}$$

Result (type 5, 117 leaves, 2 steps):

$$-\frac{1}{4a^2dx^4 (a+bx^3)^{2/3}} \left(a^2 + 4abx^3 + 3b^2x^6 - bx^3(a+3bx^3) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2bx^3}{a+bx^3}\right] + 3bx^3(a-bx^3) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2bx^3}{a+bx^3}\right] \right)$$

Problem 579: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{1/3}}{x^8 (ad-bdx^3)} dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$-\frac{(a+bx^3)^{1/3}}{7adx^7} - \frac{2b(a+bx^3)^{1/3}}{7a^2dx^4} - \frac{8b^2(a+bx^3)^{1/3}}{7a^3dx} - \frac{2^{1/3} b^{7/3} \operatorname{ArcTan}\left[\frac{1+\frac{2}{3} 2^{1/3} b^{1/3} x}{\frac{(a+bx^3)^{1/3}}{\sqrt{3}}}\right]}{\sqrt{3} a^3d} + \frac{b^{7/3} \operatorname{Log}[ad-bdx^3]}{3 \times 2^{2/3} a^3d} - \frac{b^{7/3} \operatorname{Log}[2^{1/3} b^{1/3} x - (a+bx^3)^{1/3}]}{2^{2/3} a^3d}$$

Result (type 5, 244 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{28 a^3 d x^7 (a + b x^3)^{2/3}} \left(4 a^3 + 10 a^2 b x^3 + 24 a b^2 x^6 + 18 b^3 x^9 - 2 b x^3 (2 a^2 + 3 a b x^3 + 9 b^2 x^6) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] + \right. \\
& 15 a^2 b x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] + 12 a b^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - \\
& \left. 27 b^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - 9 b x^3 (a - b x^3)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2\right\}, \left\{1, \frac{5}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] \right)
\end{aligned}$$

Problem 580: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^{11} (a d - b d x^3)} dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(a + b x^3)^{1/3}}{10 a d x^{10}} - \frac{11 b (a + b x^3)^{1/3}}{70 a^2 d x^7} - \frac{37 b^2 (a + b x^3)^{1/3}}{140 a^3 d x^4} - \frac{169 b^3 (a + b x^3)^{1/3}}{140 a^4 d x} - \\
& \frac{2^{1/3} b^{10/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 \cdot 2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^4 d} + \frac{b^{10/3} \operatorname{Log}[a d - b d x^3]}{3 \times 2^{2/3} a^4 d} - \frac{b^{10/3} \operatorname{Log}[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}]}{2^{2/3} a^4 d}
\end{aligned}$$

Result (type 5, 423 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{280 a^4 d x^{10} (a + b x^3)^{2/3}} \left(28 a^4 + 64 a^3 b x^3 + 90 a^2 b^2 x^6 + 216 a b^3 x^9 + 162 b^4 x^{12} - 28 a^3 b x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - \right. \\
& 36 a^2 b^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - 54 a b^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - \\
& 162 b^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] + 117 a^3 b x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] + \\
& 99 a^2 b^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] + 81 a b^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - \\
& 297 b^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{2 b x^3}{a + b x^3}\right] - 54 b x^3 (a - b x^3)^2 (2 a + 3 b x^3) \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2\right\}, \left\{1, \frac{5}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] + \\
& \left. 27 b x^3 (a - b x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2, 2\right\}, \left\{1, 1, \frac{5}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] \right)
\end{aligned}$$

Problem 581: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 (a + b x^3)^{1/3}}{a d - b d x^3} dx$$

Optimal (type 5, 521 leaves, 22 steps):

$$\begin{aligned} & -\frac{3 a x (a + b x^3)^{1/3}}{5 b^2 d} - \frac{x^4 (a + b x^3)^{1/3}}{5 b d} - \frac{2^{1/3} a^{5/3} \operatorname{ArcTan}\left[\frac{1 - 2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{\sqrt{3} b^{7/3} d} - \\ & \frac{a^{5/3} \operatorname{ArcTan}\left[\frac{1 + 2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3} b^{7/3} d} - \frac{2 a^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{5 b^2 d (a + b x^3)^{2/3}} - \frac{a^{5/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} b^{7/3} d} + \\ & \frac{a^{5/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} b^{7/3} d} - \frac{2^{1/3} a^{5/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{7/3} d} + \frac{a^{5/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} b^{7/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{7}{3}, -\frac{1}{3}, 1, \frac{10}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{7 a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 582: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (a + b x^3)^{1/3}}{a d - b d x^3} dx$$

Optimal (type 5, 494 leaves, 21 steps):

$$\begin{aligned}
& - \frac{x (a + b x^3)^{1/3}}{2 b d} - \frac{2^{1/3} a^{2/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d} - \frac{a^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3} b^{4/3} d} - \\
& \frac{a x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 b d (a + b x^3)^{2/3}} - \frac{a^{2/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} b^{4/3} d} + \\
& \frac{a^{2/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} b^{4/3} d} - \frac{2^{1/3} a^{2/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{4/3} d} + \frac{a^{2/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} b^{4/3} d}
\end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{4 a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 583: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{a d - b d x^3} dx$$

Optimal (type 3, 416 leaves, 14 steps):

$$\begin{aligned}
& - \frac{2^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^{1/3} b^{1/3} d} - \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3} a^{1/3} b^{1/3} d} - \frac{\operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3} d} + \\
& \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{1/3} b^{1/3} d} - \frac{2^{1/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 a^{1/3} b^{1/3} d} + \frac{\operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{1/3} b^{1/3} d}
\end{aligned}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{a d \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 584: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^3 (a d - b d x^3)} dx$$

Optimal (type 5, 496 leaves, 21 steps):

$$\begin{aligned} & -\frac{(a + b x^3)^{1/3}}{2 a d x^2} - \frac{2^{1/3} b^{2/3} \operatorname{ArcTan}\left[\frac{1 - 2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{\sqrt{3} a^{4/3} d} - \frac{b^{2/3} \operatorname{ArcTan}\left[\frac{1 + 2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3} a^{4/3} d} + \\ & \frac{b x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 a d (a + b x^3)^{2/3}} - \frac{b^{2/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{4/3} d} + \\ & \frac{b^{2/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{4/3} d} - \frac{2^{1/3} b^{2/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 a^{4/3} d} + \frac{b^{2/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{4/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{(a + b x^3)^{1/3} \operatorname{AppellF1}\left[-\frac{2}{3}, -\frac{1}{3}, 1, \frac{1}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{2 a d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 585: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^6 (a d - b d x^3)} dx$$

Optimal (type 5, 523 leaves, 22 steps):

$$\begin{aligned}
& - \frac{(a + b x^3)^{1/3}}{5 a d x^5} - \frac{3 b (a + b x^3)^{1/3}}{5 a^2 d x^2} - \frac{2^{1/3} b^{5/3} \operatorname{ArcTan}\left[\frac{1 - 2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{\sqrt{3} a^{7/3} d} \\
& - \frac{b^{5/3} \operatorname{ArcTan}\left[\frac{1 + 2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{2^{2/3} \sqrt{3} a^{7/3} d} + \frac{2 b^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{5 a^2 d (a + b x^3)^{2/3}} - \frac{b^{5/3} \operatorname{Log}\left[2^{2/3} - \frac{a^{1/3} + b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{7/3} d} + \\
& \frac{b^{5/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{2/3} a^{7/3} d} - \frac{2^{1/3} b^{5/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 a^{7/3} d} + \frac{b^{5/3} \operatorname{Log}\left[2 \times 2^{1/3} + \frac{(a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{6 \times 2^{2/3} a^{7/3} d}
\end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$- \frac{(a + b x^3)^{1/3} \operatorname{AppellF1}\left[-\frac{5}{3}, -\frac{1}{3}, 1, -\frac{2}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{5 a d x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 593: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 (a + b x^3)^{2/3}}{a d - b d x^3} dx$$

Optimal (type 3, 264 leaves, 5 steps):

$$\begin{aligned}
& - \frac{4 a x (a + b x^3)^{2/3}}{9 b^2 d} - \frac{x^4 (a + b x^3)^{2/3}}{6 b d} - \frac{14 a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{7/3} d} + \frac{2^{2/3} a^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2^{2/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{7/3} d} + \\
& \frac{a^2 \operatorname{Log}[a d - b d x^3]}{3 \times 2^{1/3} b^{7/3} d} - \frac{a^2 \operatorname{Log}\left[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}\right]}{2^{1/3} b^{7/3} d} + \frac{7 a^2 \operatorname{Log}\left[-b^{1/3} x + (a + b x^3)^{1/3}\right]}{9 b^{7/3} d}
\end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7 (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{7}{3}, -\frac{2}{3}, 1, \frac{10}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{7 a d \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 594: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (a + b x^3)^{2/3}}{a d - b d x^3} dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$-\frac{x(a+bx^3)^{2/3}}{3bd} - \frac{5a \operatorname{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a \operatorname{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3}b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}b^{4/3}d} +$$

$$\frac{a \operatorname{Log}[ad-bdx^3]}{3 \times 2^{1/3}b^{4/3}d} - \frac{a \operatorname{Log}\left[2^{1/3}b^{1/3}x - (a+bx^3)^{1/3}\right]}{2^{1/3}b^{4/3}d} + \frac{5a \operatorname{Log}\left[-b^{1/3}x + (a+bx^3)^{1/3}\right]}{6b^{4/3}d}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^4(a+bx^3)^{2/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{2}{3}, 1, \frac{7}{3}, -\frac{bx^3}{a}, \frac{bx^3}{a}\right]}{4ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Problem 595: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3} dx$$

Optimal (type 3, 200 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}b^{1/3}d} + \frac{2^{2/3}\operatorname{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3}b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}b^{1/3}d} + \frac{\operatorname{Log}[ad-bdx^3]}{3 \times 2^{1/3}b^{1/3}d} - \frac{\operatorname{Log}\left[2^{1/3}b^{1/3}x - (a+bx^3)^{1/3}\right]}{2^{1/3}b^{1/3}d} + \frac{\operatorname{Log}\left[-b^{1/3}x + (a+bx^3)^{1/3}\right]}{2b^{1/3}d}$$

Result (type 6, 61 leaves, 2 steps):

$$\frac{x(a+bx^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{bx^3}{a}, \frac{bx^3}{a}\right]}{ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Problem 596: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)} dx$$

Optimal (type 3, 157 leaves, 3 steps):

$$-\frac{(a+bx^3)^{2/3}}{2adx^2} + \frac{2^{2/3}b^{2/3}\operatorname{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3}b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}ad} + \frac{b^{2/3}\operatorname{Log}[ad-bdx^3]}{3 \times 2^{1/3}ad} - \frac{b^{2/3}\operatorname{Log}\left[2^{1/3}b^{1/3}x - (a+bx^3)^{1/3}\right]}{2^{1/3}ad}$$

Result (type 5, 79 leaves, 2 steps):

$$- \frac{(a + b x^3)^{2/3} \left(1 - \frac{b x^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{2 b x^3}{a - b x^3}\right]}{2 a d x^2 \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 597: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^6 (a d - b d x^3)} dx$$

Optimal (type 3, 182 leaves, 4 steps):

$$- \frac{(a + b x^3)^{2/3}}{5 a d x^5} - \frac{7 b (a + b x^3)^{2/3}}{10 a^2 d x^2} + \frac{2^{2/3} b^{5/3} \text{ArcTan}\left[\frac{1 + \frac{2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^2 d} + \frac{b^{5/3} \text{Log}[a d - b d x^3]}{3 \times 2^{1/3} a^2 d} - \frac{b^{5/3} \text{Log}\left[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}\right]}{2^{1/3} a^2 d}$$

Result (type 5, 121 leaves, 2 steps):

$$- \frac{1}{10 a^2 d x^5 (a + b x^3)^{1/3}} \left(2 a^2 + 5 a b x^3 + 3 b^2 x^6 - 4 b x^3 (2 a + 3 b x^3) \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] + 12 b x^3 (a - b x^3) \text{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] \right)$$

Problem 598: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^9 (a d - b d x^3)} dx$$

Optimal (type 3, 209 leaves, 5 steps):

$$- \frac{(a + b x^3)^{2/3}}{8 a d x^8} - \frac{b (a + b x^3)^{2/3}}{4 a^2 d x^5} - \frac{5 b^2 (a + b x^3)^{2/3}}{8 a^3 d x^2} + \frac{2^{2/3} b^{8/3} \text{ArcTan}\left[\frac{1 + \frac{2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^3 d} + \frac{b^{8/3} \text{Log}[a d - b d x^3]}{3 \times 2^{1/3} a^3 d} - \frac{b^{8/3} \text{Log}\left[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}\right]}{2^{1/3} a^3 d}$$

Result (type 5, 244 leaves, 2 steps):

$$- \frac{1}{40 a^3 d x^8 (a + b x^3)^{1/3}} \left(5 a^3 + 11 a^2 b x^3 + 15 a b^2 x^6 + 9 b^3 x^9 - 4 b x^3 (5 a^2 + 6 a b x^3 + 9 b^2 x^6) \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] + \right. \\ \left. 42 a^2 b x^3 \text{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] + 12 a b^2 x^6 \text{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] - \right. \\ \left. 54 b^3 x^9 \text{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] - 18 b x^3 (a - b x^3)^2 \text{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2\right\}, \left\{1, \frac{4}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] \right)$$

Problem 599: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^{12} (a d - b d x^3)} dx$$

Optimal (type 3, 236 leaves, 6 steps):

$$-\frac{(a + b x^3)^{2/3}}{11 a d x^{11}} - \frac{13 b (a + b x^3)^{2/3}}{88 a^2 d x^8} - \frac{49 b^2 (a + b x^3)^{2/3}}{220 a^3 d x^5} - \frac{293 b^3 (a + b x^3)^{2/3}}{440 a^4 d x^2} +$$

$$\frac{2^{2/3} b^{11/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^4 d} + \frac{b^{11/3} \operatorname{Log}[a d - b d x^3]}{3 \times 2^{1/3} a^4 d} - \frac{b^{11/3} \operatorname{Log}[2^{1/3} b^{1/3} x - (a + b x^3)^{1/3}]}{2^{1/3} a^4 d}$$

Result (type 5, 391 leaves, 2 steps):

$$-\frac{1}{440 a^4 d x^{11} (a + b x^3)^{1/3}} \left(40 a^4 + 85 a^3 b x^3 + 99 a^2 b^2 x^6 + 135 a b^3 x^9 + 81 b^4 x^{12} - \right.$$

$$160 a^3 b x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] - 180 a^2 b^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] -$$

$$216 a b^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] - 324 b^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] +$$

$$396 a^3 b x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] + 198 a^2 b^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] -$$

$$594 b^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{2 b x^3}{a + b x^3}\right] - 54 b x^3 (a - b x^3)^2 (5 a + 6 b x^3) \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2\right\}, \left\{1, \frac{4}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] +$$

$$54 b x^3 (a - b x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2, 2\right\}, \left\{1, 1, \frac{4}{3}\right\}, \frac{2 b x^3}{a + b x^3}\right] \Big)$$

Problem 600: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 (a + b x^3)^{2/3}}{a d - b d x^3} dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$\begin{aligned}
& -\frac{9 a x^2 (a+b x^3)^{2/3}}{28 b^2 d} - \frac{x^5 (a+b x^3)^{2/3}}{7 b d} + \frac{2^{2/3} a^{7/3} \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{8/3} d} + \frac{a^{7/3} \operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} b^{8/3} d} - \\
& \frac{19 a^2 x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{28 b^2 d (a+b x^3)^{1/3}} + \frac{a^{7/3} \operatorname{Log}\left[\frac{(a^{1/3}-b^{1/3} x)^2 (a^{1/3}+b^{1/3} x)}{a}\right]}{6 \times 2^{1/3} b^{8/3} d} + \\
& \frac{a^{7/3} \operatorname{Log}\left[1+\frac{2^{2/3}(a^{1/3}+b^{1/3} x)^2}{(a+b x^3)^{2/3}}-\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}\right]}{3 \times 2^{1/3} b^{8/3} d} - \frac{2^{2/3} a^{7/3} \operatorname{Log}\left[1+\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}\right]}{3 b^{8/3} d} - \frac{a^{7/3} \operatorname{Log}\left[\frac{b^{1/3}(a^{1/3}+b^{1/3} x)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}(a+b x^3)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} b^{8/3} d}
\end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^8 (a+b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{8}{3}, -\frac{2}{3}, 1, \frac{11}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{8 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 601: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (a+b x^3)^{2/3}}{a d - b d x^3} dx$$

Optimal (type 5, 485 leaves, 13 steps):

$$\begin{aligned}
& -\frac{x^2 (a+b x^3)^{2/3}}{4 b d} + \frac{2^{2/3} a^{4/3} \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{5/3} d} + \frac{a^{4/3} \operatorname{ArcTan}\left[\frac{1+\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} b^{5/3} d} - \\
& \frac{3 a x^2 \left(1+\frac{b x^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{4 b d (a+b x^3)^{1/3}} + \frac{a^{4/3} \operatorname{Log}\left[\frac{(a^{1/3}-b^{1/3} x)^2 (a^{1/3}+b^{1/3} x)}{a}\right]}{6 \times 2^{1/3} b^{5/3} d} + \\
& \frac{a^{4/3} \operatorname{Log}\left[1+\frac{2^{2/3}(a^{1/3}+b^{1/3} x)^2}{(a+b x^3)^{2/3}}-\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}\right]}{3 \times 2^{1/3} b^{5/3} d} - \frac{2^{2/3} a^{4/3} \operatorname{Log}\left[1+\frac{2^{1/3}(a^{1/3}+b^{1/3} x)}{(a+b x^3)^{1/3}}\right]}{3 b^{5/3} d} - \frac{a^{4/3} \operatorname{Log}\left[\frac{b^{1/3}(a^{1/3}+b^{1/3} x)}{a^{1/3}}-\frac{2^{2/3} b^{1/3}(a+b x^3)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} b^{5/3} d}
\end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^5 (a+b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{5}{3}, -\frac{2}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{5 a d \left(1+\frac{b x^3}{a}\right)^{2/3}}$$

Problem 602: Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x^3)^{2/3}}{a d - b d x^3} dx$$

Optimal (type 5, 457 leaves, 11 steps):

$$\begin{aligned} & \frac{2^{2/3} a^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2}{3} \frac{(a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d} + \frac{a^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2}{3} \frac{(a^{1/3} - b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} b^{2/3} d} - \frac{x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{2 d (a + b x^3)^{1/3}} + \frac{a^{1/3} \operatorname{Log}\left[\frac{(a^{1/3} - b^{1/3} x)^2 (a^{1/3} + b^{1/3} x)}{a}\right]}{6 \times 2^{1/3} b^{2/3} d} + \\ & \frac{a^{1/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{1/3} b^{2/3} d} - \frac{2^{2/3} a^{1/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3} d} - \frac{a^{1/3} \operatorname{Log}\left[\frac{b^{1/3} (a^{1/3} + b^{1/3} x)}{a^{1/3}} - \frac{2^{2/3} b^{1/3} (a + b x^3)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} b^{2/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^2 (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{2}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{a}\right]}{2 a d \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 603: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^2 (a d - b d x^3)} dx$$

Optimal (type 5, 483 leaves, 13 steps):

$$\begin{aligned} & -\frac{(a + b x^3)^{2/3}}{a d x} + \frac{2^{2/3} b^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2}{3} \frac{(a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^{2/3} d} + \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2}{3} \frac{(a^{1/3} - b^{1/3} x)}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} a^{2/3} d} + \\ & \frac{b x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{b x^3}{a}\right]}{2 a d (a + b x^3)^{1/3}} + \frac{b^{1/3} \operatorname{Log}\left[\frac{(a^{1/3} - b^{1/3} x)^2 (a^{1/3} + b^{1/3} x)}{a}\right]}{6 \times 2^{1/3} a^{2/3} d} + \\ & \frac{b^{1/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a + b x^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 \times 2^{1/3} a^{2/3} d} - \frac{2^{2/3} b^{1/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a + b x^3)^{1/3}}\right]}{3 a^{2/3} d} - \frac{b^{1/3} \operatorname{Log}\left[\frac{b^{1/3} (a^{1/3} + b^{1/3} x)}{a^{1/3}} - \frac{2^{2/3} b^{1/3} (a + b x^3)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} a^{2/3} d} \end{aligned}$$

Result (type 6, 64 leaves, 2 steps):

$$-\frac{(a+bx^3)^{2/3} \operatorname{AppellF1}\left[-\frac{1}{3}, -\frac{2}{3}, 1, \frac{2}{3}, -\frac{bx^3}{a}, \frac{bx^3}{a}\right]}{a d x \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 604: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{2/3}}{x^5 (a d - b d x^3)} dx$$

Optimal (type 5, 512 leaves, 14 steps):

$$\begin{aligned} & -\frac{(a+bx^3)^{2/3}}{4 a d x^4} - \frac{3 b (a+bx^3)^{2/3}}{2 a^2 d x} + \frac{2^{2/3} b^{4/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} a^{5/3} d} + \frac{b^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3} a^{5/3} d} + \\ & \frac{3 b^2 x^2 \left(1 + \frac{bx^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{bx^3}{a}\right]}{4 a^2 d (a+bx^3)^{1/3}} + \frac{b^{4/3} \operatorname{Log}\left[\frac{(a^{1/3} - b^{1/3} x)^2 (a^{1/3} + b^{1/3} x)}{a}\right]}{6 \times 2^{1/3} a^{5/3} d} + \\ & \frac{b^{4/3} \operatorname{Log}\left[1 + \frac{2^{2/3} (a^{1/3} + b^{1/3} x)^2}{(a+bx^3)^{2/3}} - \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a+bx^3)^{1/3}}\right]}{3 \times 2^{1/3} a^{5/3} d} - \frac{2^{2/3} b^{4/3} \operatorname{Log}\left[1 + \frac{2^{1/3} (a^{1/3} + b^{1/3} x)}{(a+bx^3)^{1/3}}\right]}{3 a^{5/3} d} - \frac{b^{4/3} \operatorname{Log}\left[\frac{b^{1/3} (a^{1/3} + b^{1/3} x)}{a^{1/3}} - \frac{2^{2/3} b^{1/3} (a+bx^3)^{1/3}}{a^{1/3}}\right]}{2 \times 2^{1/3} a^{5/3} d} \end{aligned}$$

Result (type 6, 66 leaves, 2 steps):

$$-\frac{(a+bx^3)^{2/3} \operatorname{AppellF1}\left[-\frac{4}{3}, -\frac{2}{3}, 1, -\frac{1}{3}, -\frac{bx^3}{a}, \frac{bx^3}{a}\right]}{4 a d x^4 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 612: Result valid but suboptimal antiderivative.

$$\int \frac{x^6}{(1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 154 leaves, 4 steps):

$$-\frac{1}{3} x (1-x^3)^{2/3} + \frac{2 \operatorname{ArcTan}\left[\frac{1 - \frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}} - \frac{\operatorname{Log}[1+x^3]}{6 \times 2^{1/3}} + \frac{\operatorname{Log}\left[-2^{1/3} x - (1-x^3)^{1/3}\right]}{2 \times 2^{1/3}} - \frac{1}{3} \operatorname{Log}\left[x + (1-x^3)^{1/3}\right]$$

Result (type 3, 226 leaves, 15 steps):

$$\begin{aligned}
& -\frac{1}{3} x (1-x^3)^{2/3} + \frac{2 \operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} + \\
& \frac{1}{9} \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] - \frac{2}{9} \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] - \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}
\end{aligned}$$

Problem 613: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 135 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\operatorname{Log}[1+x^3]}{6 \times 2^{1/3}} - \frac{\operatorname{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{2 \times 2^{1/3}} + \frac{1}{2} \operatorname{Log}[x + (1-x^3)^{1/3}]$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} - \frac{1}{6} \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{3} \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] + \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}$$

Problem 614: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\operatorname{Log}[1+x^3]}{6 \times 2^{1/3}} + \frac{\operatorname{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{2 \times 2^{1/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}$$

Problem 615: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 (1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 105 leaves, 3 steps):

$$-\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{Log}[1+x^3]}{6 \times 2^{1/3}} - \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{1/3}}$$

Result (type 3, 139 leaves, 8 steps):

$$-\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}$$

Problem 616: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 (1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\text{Log}[1+x^3]}{6 \times 2^{1/3}} + \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{1/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{(1-x^3)^{5/3}}{5x^5} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}$$

Problem 617: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 (1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 3, 141 leaves, 5 steps):

$$-\frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{Log}[1+x^3]}{6 \times 2^{1/3}} - \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{1/3}}$$

Result (type 3, 175 leaves, 9 steps):

$$-\frac{(1-x^3)^{2/3}}{2x^2} - \frac{(1-x^3)^{5/3}}{5x^5} - \frac{(1-x^3)^{8/3}}{8x^8} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}}$$

Problem 618: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{(1-x^3)^{1/3}(1+x^3)} dx$$

Optimal (type 5, 271 leaves, 12 steps):

$$-\frac{1}{4}x^2(1-x^3)^{2/3} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3}\sqrt{3}} - \frac{1}{4}x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] +$$

$$\frac{\text{Log}\left[\frac{(1-x)(1+x)^2}{12 \times 2^{1/3}}\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4 \times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{8}x^8 \text{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 1, \frac{11}{3}, x^3, -x^3\right]$$

Problem 619: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(1-x^3)^{1/3}(1+x^3)} dx$$

Optimal (type 5, 254 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{1 - 2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1 + 2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}\sqrt{3}} + \frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \\
& \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}} + \frac{\text{Log}\left[-1 + x + 2^{2/3}(1-x^3)^{1/3}\right]}{4 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5} x^5 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, x^3, -x^3\right]$$

Problem 620: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(1-x^3)^{1/3}(1+x^3)} dx$$

Optimal (type 3, 233 leaves, 8 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{1 - 2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1 + 2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}\sqrt{3}} + \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} \\
& - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}} - \frac{\text{Log}\left[-1 + x + 2^{2/3}(1-x^3)^{1/3}\right]}{4 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2} x^2 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, x^3, -x^3\right]$$

Problem 621: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2(1-x^3)^{1/3}(1+x^3)} dx$$

Optimal (type 5, 270 leaves, 12 steps):

$$\begin{aligned}
& - \frac{(1-x^3)^{2/3}}{x} - \frac{\text{ArcTan}\left[\frac{1 - 2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1 + 2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}\sqrt{3}} - \frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \\
& \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}} + \frac{\text{Log}\left[-1 + x + 2^{2/3}(1-x^3)^{1/3}\right]}{4 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3}, \frac{1}{3}, 1, \frac{2}{3}, x^3, -x^3\right]}{x}$$

Problem 622: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (1-x^3)^{1/3} (1+x^3)} dx$$

Optimal (type 5, 289 leaves, 14 steps):

$$-\frac{(1-x^3)^{2/3}}{4x^4} + \frac{(1-x^3)^{2/3}}{2x} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3}\sqrt{3}} + \frac{1}{4}x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] +$$

$$\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{4 \times 2^{1/3}}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3}, \frac{1}{3}, 1, -\frac{1}{3}, x^3, -x^3\right]}{4x^4}$$

Problem 629: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 160 leaves, 5 steps):

$$-\frac{1}{3}x^2(1-x^3)^{1/3} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{(1-x^3)^{1/3}}\right]}{3\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{1/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{Log}[1+x^3]}{6 \times 2^{2/3}} + \frac{1}{6}\text{Log}[-x - (1-x^3)^{1/3}] - \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 3, 228 leaves, 14 steps):

$$-\frac{1}{3}x^2(1-x^3)^{1/3} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{(1-x^3)^{1/3}}\right]}{3\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{1/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} -$$

$$\frac{1}{18}\text{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] + \frac{1}{9}\text{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] + \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}}$$

Problem 630: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 139 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{2^{2/3} \sqrt{3}}\right]}{2^{2/3} \sqrt{3}} - \frac{\operatorname{Log}[1+x^3]}{6 \times 2^{2/3}} - \frac{1}{2} \operatorname{Log}[-x - (1-x^3)^{1/3}] + \frac{\operatorname{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 3, 207 leaves, 14 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{2^{2/3} \sqrt{3}}\right]}{2^{2/3} \sqrt{3}} + \frac{1}{6} \operatorname{Log}\left[1 + \frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{(1-x^3)^{1/3}}\right] - \frac{1}{3} \operatorname{Log}\left[1 + \frac{x}{(1-x^3)^{1/3}}\right] - \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}}$$

Problem 631: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{2^{2/3} \sqrt{3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{Log}[1+x^3]}{6 \times 2^{2/3}} - \frac{\operatorname{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 3, 122 leaves, 7 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{2^{2/3} \sqrt{3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\operatorname{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}}$$

Problem 632: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 103 leaves, 2 steps):

$$-\frac{(1-x^3)^{1/3}}{x} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\text{Log}[1+x^3]}{6 \times 2^{2/3}} + \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 3, 137 leaves, 8 steps):

$$-\frac{(1-x^3)^{1/3}}{x} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}}$$

Problem 633: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$-\frac{(1-x^3)^{1/3}}{4x^4} + \frac{(1-x^3)^{1/3}}{4x} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{Log}[1+x^3]}{6 \times 2^{2/3}} - \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 3, 140 leaves, 9 steps):

$$-\frac{(1-x^3)^{4/3}}{4x^4} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}}$$

Problem 634: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 291 leaves, 15 steps):

$$-\frac{1}{2}x(1-x^3)^{1/3} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+\frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{2/3}\sqrt{3}} + \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} -$$

$$\frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} - \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{2/3}}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7} x^7 \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 1, \frac{10}{3}, x^3, -x^3\right]$$

Problem 635: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 5, 294 leaves, 18 steps):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{1 - \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{2/3}\sqrt{3}} + \frac{1}{2} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right] - \\ & \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} + \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{2/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{4} x^4 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]$$

Problem 636: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 5, 293 leaves, 16 steps):

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{1 - \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{2/3}\sqrt{3}} + \frac{1}{2} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right] + \\ & \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} - \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{2/3}} \end{aligned}$$

Result (type 6, 21 leaves, 1 step):

$$x \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, x^3, -x^3\right]$$

Problem 637: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx$$

Optimal (type 3, 294 leaves, 16 steps):

$$\begin{aligned} & -\frac{(1-x^3)^{1/3}}{2x^2} - \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{2/3}\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{2/3}\sqrt{3}} - \frac{\text{Log}\left[2^{2/3} - \frac{1-x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} + \\ & \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{2/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3 \times 2^{2/3}} + \frac{\text{Log}\left[2 \times 2^{1/3} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{2/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{2}{3}, \frac{2}{3}, 1, \frac{1}{3}, x^3, -x^3\right]}{2x^2}$$

Problem 645: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$\begin{aligned} & \frac{x^4}{2(1-x^3)^{1/3}} + \frac{5}{6}x(1-x^3)^{2/3} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{(1-x^3)^{1/3}}\right]}{3\sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3}\sqrt{3}} + \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} - \frac{\text{Log}[-2^{1/3}x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}} - \frac{1}{6}\text{Log}[x + (1-x^3)^{1/3}] \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{10}x^{10}\text{AppellF1}\left[\frac{10}{3}, \frac{4}{3}, 1, \frac{13}{3}, x^3, -x^3\right]$$

Problem 646: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{x}{2(1-x^3)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} + \frac{\text{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}} - \frac{1}{2} \text{Log}[x + (1-x^3)^{1/3}]$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{7} x^7 \text{AppellF1}\left[\frac{7}{3}, \frac{4}{3}, 1, \frac{10}{3}, x^3, -x^3\right]$$

Problem 647: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2(1-x^3)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} - \frac{\text{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

Result (type 5, 38 leaves, 1 step):

$$\frac{x^4 \text{Hypergeometric2F1}\left[\frac{4}{3}, \frac{4}{3}, \frac{7}{3}, \frac{2x^3}{1+x^3}\right]}{4(1+x^3)^{4/3}}$$

Problem 648: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 106 leaves, 2 steps):

$$\frac{x}{2(1-x^3)^{1/3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} + \frac{\text{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

Result (type 3, 140 leaves, 8 steps):

$$\frac{x}{2(1-x^3)^{1/3}} - \frac{\text{ArcTan}\left[\frac{1-\frac{2 \cdot 2^{1/3} x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3} x^2}{(1-x^3)^{2/3}} - \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3} x}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}}$$

Problem 649: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 124 leaves, 4 steps):

$$\frac{1}{2 x^2 (1-x^3)^{1/3}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} - \frac{\text{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

Result (type 5, 204 leaves, 1 step):

$$-\frac{1}{14 x^5 (1-x^3)^{7/3}} \left(14 + 56 x^3 - 91 x^6 - 42 x^9 + 63 x^{12} - 7 (1-x^3)^2 (2 + 12 x^3 + 9 x^6) \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - \right. \\ \left. 30 x^6 \text{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - 84 x^9 \text{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - \right. \\ \left. 54 x^{12} \text{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - 18 x^6 (1+x^3)^2 \text{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, -\frac{2 x^3}{1-x^3}\right] \right)$$

Problem 650: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{1}{2 x^5 (1-x^3)^{1/3}} - \frac{7 (1-x^3)^{2/3}}{10 x^5} - \frac{4 (1-x^3)^{2/3}}{5 x^2} - \frac{\text{ArcTan}\left[\frac{1-\frac{2}{3}x}{\sqrt{3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{Log}[1+x^3]}{12 \times 2^{1/3}} + \frac{\text{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

Result (type 5, 397 leaves, 1 step):

$$\begin{aligned}
& - \frac{1}{70 x^8 (1-x^3)^{7/3}} \\
& \left(28 - 182 x^3 - 476 x^6 + 819 x^9 + 378 x^{12} - 567 x^{15} - 28 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + 182 x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + \right. \\
& 476 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - 819 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - 378 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + \\
& 567 x^{15} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - 36 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + \\
& 342 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + 972 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + \\
& 594 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + 54 x^6 (1+x^3)^2 (1+6 x^3) \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{7}{3}\right\}, \left\{1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] + \\
& \left. 54 x^6 (1+x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] \right)
\end{aligned}$$

Problem 651: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 3, 162 leaves, 6 steps):

$$\frac{1}{2 x^8 (1-x^3)^{1/3}} - \frac{5 (1-x^3)^{2/3}}{8 x^8} - \frac{13 (1-x^3)^{2/3}}{20 x^5} - \frac{49 (1-x^3)^{2/3}}{40 x^2} + \frac{\operatorname{ArcTan}\left[\frac{1-2^{2/3} x}{\sqrt{3} (1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\operatorname{Log}[1+x^3]}{12 \times 2^{1/3}} - \frac{\operatorname{Log}[-2^{1/3} x - (1-x^3)^{1/3}]}{4 \times 2^{1/3}}$$

Result (type 5, 643 leaves, 1 step):

$$\begin{aligned}
& - \frac{1}{280 x^{11} (1-x^3)^{7/3}} \left(70 - 308 x^3 + 1162 x^6 + 2856 x^9 - 4914 x^{12} - 2268 x^{15} + 3402 x^{18} - 70 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + \right. \\
& 308 x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - 1162 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - \\
& 2856 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + 4914 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] + \\
& 2268 x^{15} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - 3402 x^{18} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, -\frac{2 x^3}{1-x^3}\right] - \\
& 66 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + 312 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - \\
& 2268 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - 6696 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] - \\
& 4050 x^{18} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, -\frac{2 x^3}{1-x^3}\right] + 27 x^6 (1+x^3)^2 (7-18 x^3-105 x^6) \operatorname{HypergeometricPFQ}\left[\left\{2, 2, \frac{7}{3}\right\}, \left\{1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] + \\
& 54 x^6 (1-15 x^3) (1+x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] - \\
& 81 x^6 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] - 324 x^9 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] - \\
& 486 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] - \\
& \left. 324 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] - 81 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, -\frac{2 x^3}{1-x^3}\right] \right)
\end{aligned}$$

Problem 652: Result unnecessarily involves higher level functions.

$$\int \frac{x^{10}}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\begin{aligned}
& \frac{x^5}{2 (1-x^3)^{1/3}} + \frac{3}{4} x^2 (1-x^3)^{2/3} - \frac{\operatorname{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\operatorname{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{1}{2} x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \\
& \frac{\operatorname{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} - \frac{\operatorname{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} + \frac{\operatorname{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\operatorname{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}}
\end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{11} x^{11} \text{AppellF1}\left[\frac{11}{3}, \frac{4}{3}, 1, \frac{14}{3}, x^3, -x^3\right]$$

Problem 653: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\begin{aligned} & \frac{x^2}{2(1-x^3)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{3}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] + \\ & \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{8} x^8 \text{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, x^3, -x^3\right]$$

Problem 654: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 274 leaves, 12 steps):

$$\begin{aligned} & \frac{x^2}{2(1-x^3)^{1/3}} - \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{1}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \\ & \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{5} x^5 \text{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, x^3, -x^3\right]$$

Problem 655: Result unnecessarily involves higher level functions.

$$\int \frac{x}{(1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 274 leaves, 11 steps):

$$\begin{aligned} & \frac{x^2}{2(1-x^3)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{1}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] + \\ & \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$\frac{1}{2} x^2 \text{AppellF1}\left[\frac{2}{3}, \frac{4}{3}, 1, \frac{5}{3}, x^3, -x^3\right]$$

Problem 656: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 292 leaves, 13 steps):

$$\begin{aligned} & \frac{1}{2x(1-x^3)^{1/3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} - \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{3}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] - \\ & \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} + \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}} \end{aligned}$$

Result (type 6, 24 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{1}{3}, \frac{4}{3}, 1, \frac{2}{3}, x^3, -x^3\right]}{x}$$

Problem 657: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} dx$$

Optimal (type 5, 308 leaves, 14 steps):

$$\begin{aligned} & \frac{1}{2 x^4 (1-x^3)^{1/3}} - \frac{3 (1-x^3)^{2/3}}{4 x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\text{ArcTan}\left[\frac{1-2^{2/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \frac{\text{ArcTan}\left[\frac{1+2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{4 \times 2^{1/3} \sqrt{3}} - \frac{1}{2} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, x^3\right] + \\ & \frac{\text{Log}\left[(1-x)(1+x)^2\right]}{24 \times 2^{1/3}} + \frac{\text{Log}\left[1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{12 \times 2^{1/3}} - \frac{\text{Log}\left[1 + \frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{6 \times 2^{1/3}} - \frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{8 \times 2^{1/3}} \end{aligned}$$

Result (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{4}{3}, \frac{4}{3}, 1, -\frac{1}{3}, x^3, -x^3\right]}{4 x^4}$$

Problem 665: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 (a+b x^3)^{1/3}}{c+d x^3} dx$$

Optimal (type 3, 336 leaves, 6 steps):

$$\begin{aligned} & -\frac{(6 b c-a d) x^2 (a+b x^3)^{1/3}}{18 b d^2} + \frac{x^5 (a+b x^3)^{1/3}}{6 d} - \frac{(9 b^2 c^2-3 a b c d-a^2 d^2) \text{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{5/3} d^3} + \frac{c^{5/3} (b c-a d)^{1/3} \text{ArcTan}\left[\frac{1+\frac{2 (b c-a d)^{1/3} x}{c^{1/3} (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3} - \\ & \frac{c^{5/3} (b c-a d)^{1/3} \text{Log}[c+d x^3]}{6 d^3} - \frac{(9 b^2 c^2-3 a b c d-a^2 d^2) \text{Log}[b^{1/3} x-(a+b x^3)^{1/3}]}{18 b^{5/3} d^3} + \frac{c^{5/3} (b c-a d)^{1/3} \text{Log}\left[\frac{(b c-a d)^{1/3} x}{c^{1/3}}-(a+b x^3)^{1/3}\right]}{2 d^3} \end{aligned}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^8 (a+b x^3)^{1/3} \text{AppellF1}\left[\frac{8}{3}, -\frac{1}{3}, 1, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{8 c \left(1+\frac{b x^3}{a}\right)^{1/3}}$$

Problem 666: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (a + b x^3)^{1/3}}{c + d x^3} dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{x^2 (a + b x^3)^{1/3}}{3 d} + \frac{(3 b c - a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3} d^2} - \frac{c^{2/3} (b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} +$$

$$\frac{c^{2/3} (b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 d^2} + \frac{(3 b c - a d) \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{6 b^{2/3} d^2} - \frac{c^{2/3} (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d^2}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^5 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3}, -\frac{1}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{5 c \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 667: Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x^3)^{1/3}}{c + d x^3} dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$- \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d} + \frac{(b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d} -$$

$$\frac{(b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 c^{1/3} d} - \frac{b^{1/3} \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d} + \frac{(b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{1/3} d}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^2 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 c \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 668: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^2 (c + d x^3)} dx$$

Optimal (type 3, 168 leaves, 3 steps):

$$-\frac{(a + b x^3)^{1/3}}{c x} - \frac{(b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3}(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3}} + \frac{(b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 c^{4/3}} - \frac{(b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{4/3}}$$

Result (type 5, 87 leaves, 2 steps):

$$-\frac{(a + b x^3)^{1/3} \left(1 + \frac{d x^3}{c}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{c \left(\frac{b x^3}{a} - \frac{d x^3}{c}\right)}{c + d x^3}\right]}{c x \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 669: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^5 (c + d x^3)} dx$$

Optimal (type 3, 204 leaves, 4 steps):

$$-\frac{(a + b x^3)^{1/3}}{4 c x^4} - \frac{(b c - 4 a d) (a + b x^3)^{1/3}}{4 a c^2 x} + \frac{d (b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3}(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3}} - \frac{d (b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 c^{7/3}} + \frac{d (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 145 leaves, 2 steps):

$$-\frac{1}{8 c^3 x^4 (a + b x^3)^{2/3}} \left(2 c (a + b x^3) (c - 3 d x^3) - (b c - a d) x^3 (c - 3 d x^3) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 3 (b c - a d) x^3 (c + d x^3) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right)$$

Problem 670: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{1/3}}{x^8 (c + d x^3)} dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$-\frac{(a + b x^3)^{1/3}}{7 c x^7} - \frac{(b c - 7 a d) (a + b x^3)^{1/3}}{28 a c^2 x^4} + \frac{(3 b^2 c^2 + 7 a b c d - 28 a^2 d^2) (a + b x^3)^{1/3}}{28 a^2 c^3 x} -$$

$$\frac{d^2 (b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}} + \frac{d^2 (b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 c^{10/3}} - \frac{d^2 (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$-\frac{1}{56 c^4 x^7 (a + b x^3)^{2/3}} \left(8 a c^3 + 8 b c^3 x^3 - 12 a c^2 d x^3 - 12 b c^2 d x^6 + 36 a c d^2 x^6 + 36 b c d^2 x^9 - \right.$$

$$2 (b c - a d) x^3 (2 c^2 - 3 c d x^3 + 9 d^2 x^6) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 15 b c^3 x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$15 a c^2 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 12 b c^2 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$12 a c d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 27 b c d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$27 a d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 9 (b c - a d) x^3 (c + d x^3)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2\right\}, \left\{1, \frac{5}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \Bigg)$$

Problem 671: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^{1/3}}{x^{11} (c + d x^3)} dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(a + b x^3)^{1/3}}{10 c x^{10}} - \frac{(b c - 10 a d) (a + b x^3)^{1/3}}{70 a c^2 x^7} + \frac{(3 b^2 c^2 + 5 a b c d - 35 a^2 d^2) (a + b x^3)^{1/3}}{140 a^2 c^3 x^4} - \frac{(9 b^3 c^3 + 15 a b^2 c^2 d + 35 a^2 b c d^2 - 140 a^3 d^3) (a + b x^3)^{1/3}}{140 a^3 c^4 x} + \\
& \frac{d^3 (b c - a d)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{13/3}} - \frac{d^3 (b c - a d)^{1/3} \operatorname{Log}[c + d x^3]}{6 c^{13/3}} + \frac{d^3 (b c - a d)^{1/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{13/3}}
\end{aligned}$$

Result (type 5, 905 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{560 c^5 x^{10} (a + b x^3)^{2/3}} \left(56 a c^4 + 56 b c^4 x^3 - 72 a c^3 d x^3 - 72 b c^3 d x^6 + 108 a c^2 d^2 x^6 + 108 b c^2 d^2 x^9 - 324 a c d^3 x^9 - 324 b c d^3 x^{12} - \right. \\
& 28 b c^4 x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 28 a c^3 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 36 b c^3 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 36 a c^2 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 54 b c^2 d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 54 a c d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 162 b c d^3 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 162 a d^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 117 b c^4 x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 117 a c^3 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 99 b c^3 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 99 a c^2 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 81 b c^2 d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 81 a c d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 297 b c d^3 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 297 a d^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 2, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 54 (b c - a d) x^3 (2 c - 3 d x^3) (c + d x^3)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2\right\}, \left\{1, \frac{5}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& \left. 27 (b c - a d) x^3 (c + d x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{\frac{2}{3}, 2, 2, 2\right\}, \left\{1, 1, \frac{5}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right)
\end{aligned}$$

Problem 684: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 (a + b x^3)^{2/3}}{c + d x^3} dx$$

Optimal (type 3, 334 leaves, 5 steps):

$$\begin{aligned} & - \frac{(3bc - ad)x(a + bx^3)^{2/3}}{9bd^2} + \frac{x^4(a + bx^3)^{2/3}}{6d} + \frac{(9b^2c^2 - 6abcd - a^2d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}b^{4/3}d^3} - \frac{c^{4/3}(bc - ad)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}d^3} \\ & - \frac{c^{4/3}(bc - ad)^{2/3} \operatorname{Log}[c + dx^3]}{6d^3} + \frac{c^{4/3}(bc - ad)^{2/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{2d^3} - \frac{(9b^2c^2 - 6abcd - a^2d^2) \operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{18b^{4/3}d^3} \end{aligned}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^7(a + bx^3)^{2/3} \operatorname{AppellF1}\left[\frac{7}{3}, -\frac{2}{3}, 1, \frac{10}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{7c\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 685: Result unnecessarily involves higher level functions.

$$\int \frac{x^3 (a + b x^3)^{2/3}}{c + d x^3} dx$$

Optimal (type 3, 272 leaves, 4 steps):

$$\begin{aligned} & \frac{x(a + bx^3)^{2/3}}{3d} - \frac{(3bc - 2ad) \operatorname{ArcTan}\left[\frac{1 + \frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}b^{1/3}d^2} + \frac{c^{1/3}(bc - ad)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}d^2} \\ & - \frac{c^{1/3}(bc - ad)^{2/3} \operatorname{Log}[c + dx^3]}{6d^2} - \frac{c^{1/3}(bc - ad)^{2/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a + bx^3)^{1/3}\right]}{2d^2} + \frac{(3bc - 2ad) \operatorname{Log}[-b^{1/3}x + (a + bx^3)^{1/3}]}{6b^{1/3}d^2} \end{aligned}$$

Result (type 6, 64 leaves, 2 steps):

$$\frac{x^4(a + bx^3)^{2/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{2}{3}, 1, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right]}{4c\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

Problem 686: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{c + d x^3} dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{b^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d} - \frac{(b c - a d)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} d} - \frac{(b c - a d)^{2/3} \operatorname{Log}[c + d x^3]}{6 c^{2/3} d} + \frac{(b c - a d)^{2/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} d} - \frac{b^{2/3} \operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{2 d}$$

Result (type 6, 59 leaves, 2 steps):

$$\frac{x (a + b x^3)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{2}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 687: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^3 (c + d x^3)} dx$$

Optimal (type 3, 169 leaves, 3 steps):

$$-\frac{(a + b x^3)^{2/3}}{2 c x^2} + \frac{(b c - a d)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3}} + \frac{(b c - a d)^{2/3} \operatorname{Log}[c + d x^3]}{6 c^{5/3}} - \frac{(b c - a d)^{2/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{5/3}}$$

Result (type 5, 89 leaves, 2 steps):

$$-\frac{(a + b x^3)^{2/3} \left(1 + \frac{d x^3}{c}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{c \left(\frac{b x^3}{a} - \frac{d x^3}{c}\right)}{c + d x^3}\right]}{2 c x^2 \left(1 + \frac{b x^3}{a}\right)^{2/3}}$$

Problem 688: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{2/3}}{x^6 (c + d x^3)} dx$$

Optimal (type 3, 206 leaves, 4 steps):

$$-\frac{(a+bx^3)^{2/3}}{5cx^5} - \frac{(2bc-5ad)(a+bx^3)^{2/3}}{10a^2c^2x^2} - \frac{d(bc-ad)^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{8/3}} - \frac{d(bc-ad)^{2/3} \operatorname{Log}[c+dx^3]}{6c^{8/3}} + \frac{d(bc-ad)^{2/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a+bx^3)^{1/3}\right]}{2c^{8/3}}$$

Result (type 5, 148 leaves, 2 steps):

$$-\frac{1}{10c^3x^5(a+bx^3)^{1/3}} \left(c(a+bx^3)(2c-3dx^3) - 2(bc-ad)x^3(2c-3dx^3) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(bc-ad)x^3}{c(a+bx^3)}\right] + 6(bc-ad)x^3(c+dx^3) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(bc-ad)x^3}{c(a+bx^3)}\right] \right)$$

Problem 689: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)} dx$$

Optimal (type 3, 257 leaves, 5 steps):

$$-\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5} + \frac{(3b^2c^2+8ab cd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3} \operatorname{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{11/3}} + \frac{d^2(bc-ad)^{2/3} \operatorname{Log}[c+dx^3]}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a+bx^3)^{1/3}\right]}{2c^{11/3}}$$

Result (type 5, 451 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{40 c^4 x^8 (a + b x^3)^{1/3}} \left(5 a c^3 + 5 b c^3 x^3 - 6 a c^2 d x^3 - 6 b c^2 d x^6 + 9 a c d^2 x^6 + 9 b c d^2 x^9 - \right. \\
& 2 (b c - a d) x^3 (5 c^2 - 6 c d x^3 + 9 d^2 x^6) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 21 b c^3 x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 21 a c^2 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 6 b c^2 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 6 a c d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 27 b c d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& \left. 27 a d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 9 (b c - a d) x^3 (c + d x^3)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2\right\}, \left\{1, \frac{4}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right)
\end{aligned}$$

Problem 690: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^{2/3}}{x^{12} (c + d x^3)} dx$$

Optimal (type 3, 320 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(a + b x^3)^{2/3}}{11 c x^{11}} - \frac{(2 b c - 11 a d) (a + b x^3)^{2/3}}{88 a c^2 x^8} + \frac{(6 b^2 c^2 + 11 a b c d - 44 a^2 d^2) (a + b x^3)^{2/3}}{220 a^2 c^3 x^5} - \\
& \frac{(18 b^3 c^3 + 33 a b^2 c^2 d + 88 a^2 b c d^2 - 220 a^3 d^3) (a + b x^3)^{2/3}}{440 a^3 c^4 x^2} - \frac{d^3 (b c - a d)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{14/3}} - \\
& \frac{d^3 (b c - a d)^{2/3} \operatorname{Log}[c + d x^3]}{6 c^{14/3}} + \frac{d^3 (b c - a d)^{2/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{14/3}}
\end{aligned}$$

Result (type 5, 819 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{440 c^5 x^{11} (a + b x^3)^{1/3}} \left(40 a c^4 + 40 b c^4 x^3 - 45 a c^3 d x^3 - 45 b c^3 d x^6 + 54 a c^2 d^2 x^6 + 54 b c^2 d^2 x^9 - 81 a c d^3 x^9 - 81 b c d^3 x^{12} - \right. \\
& 80 b c^4 x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 80 a c^3 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 90 b c^3 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 90 a c^2 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 108 b c^2 d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 108 a c d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 162 b c d^3 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 162 a d^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 198 b c^4 x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 198 a c^3 d x^3 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 99 b c^3 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 99 a c^2 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 297 b c d^3 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 297 a d^4 x^{12} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 2, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 27 (b c - a d) x^3 (5 c - 6 d x^3) (c + d x^3)^2 \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2\right\}, \left\{1, \frac{4}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 27 (b c - a d) x^3 (c + d x^3)^3 \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{3}, 2, 2, 2\right\}, \left\{1, 1, \frac{4}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \Big)
\end{aligned}$$

Problem 702: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 (a + b x^3)^{4/3}}{c + d x^3} dx$$

Optimal (type 3, 334 leaves, 6 steps):

$$\begin{aligned}
& - \frac{(6 b c - 7 a d) x^2 (a + b x^3)^{1/3}}{18 d^2} + \frac{b x^5 (a + b x^3)^{1/3}}{6 d} - \frac{(9 b^2 c^2 - 12 a b c d + 2 a^2 d^2) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{2/3} d^3} + \frac{c^{2/3} (b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^3} - \\
& \frac{c^{2/3} (b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 d^3} - \frac{(9 b^2 c^2 - 12 a b c d + 2 a^2 d^2) \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{18 b^{2/3} d^3} + \frac{c^{2/3} (b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d^3}
\end{aligned}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^5 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{5}{3}, -\frac{4}{3}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{5 c \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 703: Result unnecessarily involves higher level functions.

$$\int \frac{x (a + b x^3)^{4/3}}{c + d x^3} dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\begin{aligned} & \frac{b x^2 (a + b x^3)^{1/3}}{3 d} + \frac{b^{1/3} (3 b c - 4 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} d^2} - \frac{(b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} d^2} + \\ & \frac{(b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 c^{1/3} d^2} + \frac{b^{1/3} (3 b c - 4 a d) \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{6 d^2} - \frac{(b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{1/3} d^2} \end{aligned}$$

Result (type 6, 65 leaves, 2 steps):

$$\frac{a x^2 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, -\frac{4}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 c \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 704: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{4/3}}{x^2 (c + d x^3)} dx$$

Optimal (type 3, 254 leaves, 5 steps):

$$\begin{aligned} & -\frac{a (a + b x^3)^{1/3}}{c x} - \frac{b^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d} + \frac{(b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3} d} - \\ & \frac{(b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 c^{4/3} d} - \frac{b^{4/3} \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d} + \frac{(b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{4/3} d} \end{aligned}$$

Result (type 6, 63 leaves, 2 steps):

$$-\frac{a \left(a + b x^3\right)^{1/3} \operatorname{AppellF1}\left[-\frac{1}{3}, -\frac{4}{3}, 1, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{c x \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 705: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{4/3}}{x^5 (c + d x^3)} dx$$

Optimal (type 3, 201 leaves, 4 steps):

$$-\frac{a \left(a + b x^3\right)^{1/3}}{4 c x^4} - \frac{(5 b c - 4 a d) \left(a + b x^3\right)^{1/3}}{4 c^2 x} - \frac{(b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3}} +$$

$$\frac{(b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 c^{7/3}} - \frac{(b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{7/3}}$$

Result (type 5, 90 leaves, 2 steps):

$$-\frac{a \left(a + b x^3\right)^{1/3} \left(1 + \frac{d x^3}{c}\right)^{4/3} \operatorname{Hypergeometric2F1}\left[-\frac{4}{3}, -\frac{4}{3}, -\frac{1}{3}, -\frac{c \left(\frac{b x^3}{a} - \frac{d x^3}{c}\right)}{c + d x^3}\right]}{4 c x^4 \left(1 + \frac{b x^3}{a}\right)^{1/3}}$$

Problem 706: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{4/3}}{x^8 (c + d x^3)} dx$$

Optimal (type 3, 250 leaves, 5 steps):

$$-\frac{a \left(a + b x^3\right)^{1/3}}{7 c x^7} - \frac{(8 b c - 7 a d) \left(a + b x^3\right)^{1/3}}{28 c^2 x^4} - \frac{(4 b^2 c^2 - 35 a b c d + 28 a^2 d^2) \left(a + b x^3\right)^{1/3}}{28 a c^3 x} +$$

$$\frac{d (b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{10/3}} - \frac{d (b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 c^{10/3}} + \frac{d (b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{10/3}}$$

Result (type 5, 169 leaves, 2 steps):

$$\frac{1}{28 c^4 x^7 (a + b x^3)^{2/3}} \left(12 c (b c - a d) x^3 (a + b x^3) (c + d x^3) \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)} \right] - \right. \\ \left. (4 c - 3 d x^3) \left(c (a + b x^3) (5 b c x^3 + a (c - 4 d x^3)) - 2 (b c - a d)^2 x^6 \operatorname{Hypergeometric2F1} \left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)} \right] \right) \right)$$

Problem 707: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x^3)^{4/3}}{x^{11} (c + d x^3)} dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$-\frac{a (a + b x^3)^{1/3}}{10 c x^{10}} - \frac{(11 b c - 10 a d) (a + b x^3)^{1/3}}{70 c^2 x^7} - \frac{(2 b^2 c^2 - 40 a b c d + 35 a^2 d^2) (a + b x^3)^{1/3}}{140 a c^3 x^4} + \\ \frac{(6 b^3 c^3 + 20 a b^2 c^2 d - 175 a^2 b c d^2 + 140 a^3 d^3) (a + b x^3)^{1/3}}{140 a^2 c^4 x} - \frac{d^2 (b c - a d)^{4/3} \operatorname{ArcTan} \left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3} c^{13/3}} + \\ \frac{d^2 (b c - a d)^{4/3} \operatorname{Log} [c + d x^3]}{6 c^{13/3}} - \frac{d^2 (b c - a d)^{4/3} \operatorname{Log} \left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3} \right]}{2 c^{13/3}}$$

Result (type 5, 260 leaves, 2 steps):

$$\frac{1}{140 c^5 x^{10} (a + b x^3)^{2/3}} \left(6 c (b c - a d) x^3 (a + b x^3) (11 c^2 + 2 c d x^3 - 9 d^2 x^6) \operatorname{Hypergeometric2F1} \left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)} \right] - \right. \\ \left. (14 c^2 - 12 c d x^3 + 9 d^2 x^6) \left(c (a + b x^3) (5 b c x^3 + a (c - 4 d x^3)) - 2 (b c - a d)^2 x^6 \operatorname{Hypergeometric2F1} \left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)} \right] \right) - \right. \\ \left. 18 c (b c - a d) x^3 (a + b x^3) (c + d x^3)^2 \operatorname{HypergeometricPFQ} \left[\left\{ -\frac{1}{3}, 2, 2 \right\}, \left\{ \frac{2}{3}, 1 \right\}, \frac{(b c - a d) x^3}{c (a + b x^3)} \right] \right)$$

Problem 708: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^3)^{4/3}}{x^{14} (c + d x^3)} dx$$

Optimal (type 3, 392 leaves, 7 steps):

$$\begin{aligned}
& - \frac{a (a + b x^3)^{1/3}}{13 c x^{13}} - \frac{(14 b c - 13 a d) (a + b x^3)^{1/3}}{130 c^2 x^{10}} - \frac{(4 b^2 c^2 - 143 a b c d + 130 a^2 d^2) (a + b x^3)^{1/3}}{910 a c^3 x^7} + \\
& \frac{(12 b^3 c^3 + 26 a b^2 c^2 d - 520 a^2 b c d^2 + 455 a^3 d^3) (a + b x^3)^{1/3}}{1820 a^2 c^4 x^4} - \frac{(36 b^4 c^4 + 78 a b^3 c^3 d + 260 a^2 b^2 c^2 d^2 - 2275 a^3 b c d^3 + 1820 a^4 d^4) (a + b x^3)^{1/3}}{1820 a^3 c^5 x} + \\
& \frac{d^3 (b c - a d)^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{16/3}} - \frac{d^3 (b c - a d)^{4/3} \operatorname{Log}[c + d x^3]}{6 c^{16/3}} + \frac{d^3 (b c - a d)^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{16/3}}
\end{aligned}$$

Result (type 5, 1446 leaves, 2 steps):

$$\begin{aligned}
& - \frac{1}{1820 c^6 x^{13} (a + b x^3)^{2/3}} \\
& \left(140 a^2 c^5 + 840 a b c^5 x^3 - 686 a^2 c^4 d x^3 + 700 b^2 c^5 x^6 - 1316 a b c^4 d x^6 + 612 a^2 c^3 d^2 x^6 - 630 b^2 c^4 d x^9 + 1152 a b c^3 d^2 x^9 - 513 a^2 c^2 d^3 x^9 + \right. \\
& 540 b^2 c^3 d^2 x^{12} - 918 a b c^2 d^3 x^{12} + 324 a^2 c d^4 x^{12} - 405 b^2 c^2 d^3 x^{15} + 324 a b c d^4 x^{15} - 828 a b c^5 x^3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 828 a^2 c^4 d x^3 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 828 b^2 c^5 x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 918 a b c^4 d x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 90 a^2 c^3 d^2 x^6 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 90 b^2 c^4 d x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 234 a b c^3 d^2 x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 324 a^2 c^2 d^3 x^9 \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 324 b^2 c^3 d^2 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 918 a b c^2 d^3 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 594 a^2 c d^4 x^{12} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 594 b^2 c^2 d^3 x^{15} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 594 a b c d^4 x^{15} \operatorname{Hypergeometric2F1}\left[-\frac{1}{3}, 2, \frac{2}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 280 b^2 c^5 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 560 a b c^4 d x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 280 a^2 c^3 d^2 x^6 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 252 b^2 c^4 d x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 504 a b c^3 d^2 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 252 a^2 c^2 d^3 x^9 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -
\end{aligned}$$

$$\begin{aligned}
& 216 b^2 c^3 d^2 x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 432 a b c^2 d^3 x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 216 a^2 c d^4 x^{12} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 162 b^2 c^2 d^3 x^{15} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 324 a b c d^4 x^{15} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 162 a^2 d^5 x^{15} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 54 c (b c - a d) x^3 (a + b x^3) (7 c - 6 d x^3) (c + d x^3)^2 \text{HypergeometricPFQ}\left[\left\{-\frac{1}{3}, 2, 2\right\}, \left\{\frac{2}{3}, 1\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 54 c (b c - a d) x^3 (a + b x^3) (c + d x^3)^3 \text{HypergeometricPFQ}\left[\left\{-\frac{1}{3}, 2, 2, 2\right\}, \left\{\frac{2}{3}, 1, 1\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]
\end{aligned}$$

Problem 721: Result valid but suboptimal antiderivative.

$$\int \frac{x^6}{(a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 273 leaves, 4 steps):

$$\begin{aligned}
& \frac{x (a + b x^3)^{2/3}}{3 b d} - \frac{(3 b c + a d) \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{4/3} d^2} + \frac{c^{4/3} \text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c - a d)^{1/3}} + \\
& \frac{c^{4/3} \text{Log}[c + d x^3]}{6 d^2 (b c - a d)^{1/3}} - \frac{c^{4/3} \text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d^2 (b c - a d)^{1/3}} + \frac{(3 b c + a d) \text{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{6 b^{4/3} d^2}
\end{aligned}$$

Result (type 3, 394 leaves, 15 steps):

$$\begin{aligned}
& \frac{x (a + b x^3)^{2/3}}{3 b d} - \frac{(3 b c + a d) \text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{4/3} d^2} + \frac{c^{4/3} \text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} d^2 (b c - a d)^{1/3}} + \frac{(3 b c + a d) \text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{4/3} d^2} - \\
& \frac{(3 b c + a d) \text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 b^{4/3} d^2} - \frac{c^{4/3} \text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 d^2 (b c - a d)^{1/3}} + \frac{c^{4/3} \text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 d^2 (b c - a d)^{1/3}}
\end{aligned}$$

Problem 722: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{(a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 233 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} d} - \frac{c^{1/3} \text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d (b c - a d)^{1/3}} - \frac{c^{1/3} \text{Log}[c + d x^3]}{6 d (b c - a d)^{1/3}} + \frac{c^{1/3} \text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d (b c - a d)^{1/3}} - \frac{\text{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{2 b^{1/3} d}$$

Result (type 3, 346 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} d} - \frac{c^{1/3} \text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} d (b c - a d)^{1/3}} - \frac{\text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{1/3} d} +$$

$$\frac{\text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 b^{1/3} d} + \frac{c^{1/3} \text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 d (b c - a d)^{1/3}} - \frac{c^{1/3} \text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 d (b c - a d)^{1/3}}$$

Problem 723: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 148 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{1/3}} + \frac{\text{Log}[c + d x^3]}{6 c^{2/3} (b c - a d)^{1/3}} - \frac{\text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{2/3} (b c - a d)^{1/3}}$$

Result (type 3, 207 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} (b c - a d)^{1/3}} - \frac{\text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{2/3} (b c - a d)^{1/3}} + \frac{\text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{2/3} (b c - a d)^{1/3}}$$

Problem 724: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 (a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{(a + b x^3)^{2/3}}{2 a c x^2} - \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{5/3} (b c - a d)^{1/3}} - \frac{d \operatorname{Log}[c + d x^3]}{6 c^{5/3} (b c - a d)^{1/3}} + \frac{d \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{5/3} (b c - a d)^{1/3}}$$

Result (type 3, 235 leaves, 8 steps):

$$-\frac{(a + b x^3)^{2/3}}{2 a c x^2} - \frac{d \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{5/3} (b c - a d)^{1/3}} + \frac{d \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{5/3} (b c - a d)^{1/3}} - \frac{d \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{5/3} (b c - a d)^{1/3}}$$

Problem 725: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^6 (a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 214 leaves, 4 steps):

$$-\frac{(a + b x^3)^{2/3}}{5 a c x^5} + \frac{(3 b c + 5 a d) (a + b x^3)^{2/3}}{10 a^2 c^2 x^2} + \frac{d^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{8/3} (b c - a d)^{1/3}} + \frac{d^2 \operatorname{Log}[c + d x^3]}{6 c^{8/3} (b c - a d)^{1/3}} - \frac{d^2 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{8/3} (b c - a d)^{1/3}}$$

Result (type 3, 271 leaves, 9 steps):

$$\frac{(b c + a d) (a + b x^3)^{2/3}}{2 a^2 c^2 x^2} - \frac{(a + b x^3)^{5/3}}{5 a^2 c x^5} + \frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{8/3} (b c - a d)^{1/3}} - \frac{d^2 \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{8/3} (b c - a d)^{1/3}} + \frac{d^2 \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{8/3} (b c - a d)^{1/3}}$$

Problem 726: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^9 (a + b x^3)^{1/3} (c + d x^3)} dx$$

Optimal (type 3, 262 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(a + b x^3)^{2/3}}{8 a c x^8} + \frac{(3 b c + 4 a d) (a + b x^3)^{2/3}}{20 a^2 c^2 x^5} - \frac{(9 b^2 c^2 + 12 a b c d + 20 a^2 d^2) (a + b x^3)^{2/3}}{40 a^3 c^3 x^2} - \\
& \frac{d^3 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{11/3} (b c - a d)^{1/3}} - \frac{d^3 \operatorname{Log}[c + d x^3]}{6 c^{11/3} (b c - a d)^{1/3}} + \frac{d^3 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{11/3} (b c - a d)^{1/3}}
\end{aligned}$$

Result (type 3, 317 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(b^2 c^2 + a b c d + a^2 d^2) (a + b x^3)^{2/3}}{2 a^3 c^3 x^2} + \frac{(2 b c + a d) (a + b x^3)^{5/3}}{5 a^3 c^2 x^5} - \frac{(a + b x^3)^{8/3}}{8 a^3 c x^8} - \\
& \frac{d^3 \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{11/3} (b c - a d)^{1/3}} + \frac{d^3 \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{11/3} (b c - a d)^{1/3}} - \frac{d^3 \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{11/3} (b c - a d)^{1/3}}
\end{aligned}$$

Problem 738: Result valid but suboptimal antiderivative.

$$\int \frac{x^7}{(a + b x^3)^{2/3} (c + d x^3)} dx$$

Optimal (type 3, 279 leaves, 5 steps):

$$\begin{aligned}
& \frac{x^2 (a + b x^3)^{1/3}}{3 b d} + \frac{(3 b c + 2 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d^2} - \frac{c^{5/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c - a d)^{2/3}} + \\
& \frac{c^{5/3} \operatorname{Log}[c + d x^3]}{6 d^2 (b c - a d)^{2/3}} + \frac{(3 b c + 2 a d) \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{6 b^{5/3} d^2} - \frac{c^{5/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d^2 (b c - a d)^{2/3}}
\end{aligned}$$

Result (type 3, 400 leaves, 16 steps):

$$\begin{aligned}
& \frac{x^2 (a + b x^3)^{1/3}}{3 b d} + \frac{(3 b c + 2 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3} d^2} - \frac{c^{5/3} \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} d^2 (b c - a d)^{2/3}} + \frac{(3 b c + 2 a d) \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{5/3} d^2} - \\
& \frac{(3 b c + 2 a d) \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 b^{5/3} d^2} - \frac{c^{5/3} \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 d^2 (b c - a d)^{2/3}} + \frac{c^{5/3} \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 d^2 (b c - a d)^{2/3}}
\end{aligned}$$

Problem 739: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{(a + b x^3)^{2/3} (c + d x^3)} dx$$

Optimal (type 3, 234 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d} + \frac{c^{2/3} \text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d (b c - a d)^{2/3}} - \frac{c^{2/3} \text{Log}[c + d x^3]}{6 d (b c - a d)^{2/3}} - \frac{\text{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 b^{2/3} d} + \frac{c^{2/3} \text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d (b c - a d)^{2/3}}$$

Result (type 3, 346 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} d} + \frac{c^{2/3} \text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} d (b c - a d)^{2/3}} - \frac{\text{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3} d} +$$

$$\frac{\text{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 b^{2/3} d} + \frac{c^{2/3} \text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 d (b c - a d)^{2/3}} - \frac{c^{2/3} \text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 d (b c - a d)^{2/3}}$$

Problem 740: Result valid but suboptimal antiderivative.

$$\int \frac{x}{(a + b x^3)^{2/3} (c + d x^3)} dx$$

Optimal (type 3, 149 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{1/3} (b c - a d)^{2/3}} + \frac{\text{Log}[c + d x^3]}{6 c^{1/3} (b c - a d)^{2/3}} - \frac{\text{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{1/3} (b c - a d)^{2/3}}$$

Result (type 3, 208 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{c^{1/3} + \frac{2 (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{1/3} (b c - a d)^{2/3}} - \frac{\text{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{1/3} (b c - a d)^{2/3}} + \frac{\text{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{1/3} (b c - a d)^{2/3}}$$

Problem 741: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 (a + b x^3)^{2/3} (c + d x^3)} dx$$

Optimal (type 3, 173 leaves, 3 steps):

$$-\frac{(a + b x^3)^{1/3}}{a c x} + \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{4/3} (b c - a d)^{2/3}} - \frac{d \operatorname{Log}[c + d x^3]}{6 c^{4/3} (b c - a d)^{2/3}} + \frac{d \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{4/3} (b c - a d)^{2/3}}$$

Result (type 3, 232 leaves, 8 steps):

$$-\frac{(a + b x^3)^{1/3}}{a c x} + \frac{d \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{4/3} (b c - a d)^{2/3}} + \frac{d \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{4/3} (b c - a d)^{2/3}} - \frac{d \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{4/3} (b c - a d)^{2/3}}$$

Problem 742: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 (a + b x^3)^{2/3} (c + d x^3)} dx$$

Optimal (type 3, 215 leaves, 4 steps):

$$-\frac{(a + b x^3)^{1/3}}{4 a c x^4} + \frac{(3 b c + 4 a d) (a + b x^3)^{1/3}}{4 a^2 c^2 x} - \frac{d^2 \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{7/3} (b c - a d)^{2/3}} + \frac{d^2 \operatorname{Log}[c + d x^3]}{6 c^{7/3} (b c - a d)^{2/3}} - \frac{d^2 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{7/3} (b c - a d)^{2/3}}$$

Result (type 3, 269 leaves, 9 steps):

$$\frac{(b c + a d) (a + b x^3)^{1/3}}{a^2 c^2 x} - \frac{(a + b x^3)^{4/3}}{4 a^2 c x^4} - \frac{d^2 \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{7/3} (b c - a d)^{2/3}} - \frac{d^2 \operatorname{Log}\left[c^{1/3} - \frac{(b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 c^{7/3} (b c - a d)^{2/3}} + \frac{d^2 \operatorname{Log}\left[c^{2/3} + \frac{(b c - a d)^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{c^{1/3} (b c - a d)^{1/3} x}{(a + b x^3)^{1/3}}\right]}{6 c^{7/3} (b c - a d)^{2/3}}$$

Problem 754: Result unnecessarily involves higher level functions.

$$\int \frac{x^9}{(a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 322 leaves, 5 steps):

$$\frac{a x^4}{b (b c - a d) (a + b x^3)^{1/3}} + \frac{(b c - 4 a d) x (a + b x^3)^{2/3}}{3 b^2 d (b c - a d)} - \frac{(3 b c + 4 a d) \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{7/3} d^2} +$$

$$\frac{c^{7/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c - a d)^{4/3}} + \frac{c^{7/3} \operatorname{Log}[c + d x^3]}{6 d^2 (b c - a d)^{4/3}} - \frac{c^{7/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d^2 (b c - a d)^{4/3}} + \frac{(3 b c + 4 a d) \operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{6 b^{7/3} d^2}$$

Result (type 6, 67 leaves, 2 steps):

$$\frac{x^{10} \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{10}{3}, \frac{4}{3}, 1, \frac{13}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{10 a c (a + b x^3)^{1/3}}$$

Problem 755: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{(a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 260 leaves, 4 steps):

$$\frac{a x}{b (b c - a d) (a + b x^3)^{1/3}} + \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{4/3} d} - \frac{c^{4/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d (b c - a d)^{4/3}} -$$

$$\frac{c^{4/3} \operatorname{Log}[c + d x^3]}{6 d (b c - a d)^{4/3}} + \frac{c^{4/3} \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 d (b c - a d)^{4/3}} - \frac{\operatorname{Log}[-b^{1/3} x + (a + b x^3)^{1/3}]}{2 b^{4/3} d}$$

Result (type 6, 67 leaves, 2 steps):

$$\frac{x^7 \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{7}{3}, \frac{4}{3}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{7 a c (a + b x^3)^{1/3}}$$

Problem 756: Result unnecessarily involves higher level functions.

$$\int \frac{x^3}{(a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 172 leaves, 3 steps):

$$-\frac{x}{(bc-ad)(a+bx^3)^{1/3}} + \frac{c^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}(bc-ad)^{4/3}} + \frac{c^{1/3} \operatorname{Log}[c+dx^3]}{6(bc-ad)^{4/3}} - \frac{c^{1/3} \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a+bx^3)^{1/3}\right]}{2(bc-ad)^{4/3}}$$

Result (type 5, 92 leaves, 2 steps):

$$\frac{x^4 \left(1 + \frac{bx^3}{a}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{4}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{c\left(\frac{bx^3}{a} - \frac{dx^3}{c}\right)}{c+dx^3}\right]}{4ac(a+bx^3)^{1/3}\left(1 + \frac{dx^3}{c}\right)^{4/3}}$$

Problem 757: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal (type 3, 179 leaves, 2 steps):

$$\frac{bx}{a(bc-ad)(a+bx^3)^{1/3}} - \frac{d \operatorname{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d \operatorname{Log}[c+dx^3]}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a+bx^3)^{1/3}\right]}{2c^{2/3}(bc-ad)^{4/3}}$$

Result (type 3, 238 leaves, 8 steps):

$$\frac{bx}{a(bc-ad)(a+bx^3)^{1/3}} - \frac{d \operatorname{ArcTan}\left[\frac{c^{1/3} + \frac{2(bc-ad)^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}c^{1/3}}\right]}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{d \operatorname{Log}\left[c^{1/3} - \frac{(bc-ad)^{1/3}x}{(a+bx^3)^{1/3}}\right]}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \operatorname{Log}\left[c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{c^{1/3}(bc-ad)^{1/3}x}{(a+bx^3)^{1/3}}\right]}{6c^{2/3}(bc-ad)^{4/3}}$$

Problem 758: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)} dx$$

Optimal (type 3, 229 leaves, 4 steps):

$$\frac{b}{a(bc-ad)x^2(a+bx^3)^{1/3}} - \frac{(3bc-ad)(a+bx^3)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2 \operatorname{ArcTan}\left[\frac{1+\frac{2(bc-ad)^{1/3}x}{c^{1/3}(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{d^2 \operatorname{Log}[c+dx^3]}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2 \operatorname{Log}\left[\frac{(bc-ad)^{1/3}x}{c^{1/3}} - (a+bx^3)^{1/3}\right]}{2c^{5/3}(bc-ad)^{4/3}}$$

Result (type 5, 542 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{14 c^4 (b c - a d) x^5 (a + b x^3)^{7/3}} \\
& \left(28 c^4 (a + b x^3)^2 + 168 c^3 d x^3 (a + b x^3)^2 + 126 c^2 d^2 x^6 (a + b x^3)^2 - 28 c^4 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \right. \\
& 168 c^3 d x^3 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 126 c^2 d^2 x^6 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 15 c^2 (b c - a d)^2 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 42 c d (b c - a d)^2 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 27 d^2 (b c - a d)^2 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 9 c^2 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 18 c d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& \left. 9 d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right)
\end{aligned}$$

Problem 759: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 (a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 287 leaves, 5 steps):

$$\begin{aligned}
& \frac{b}{a (b c - a d) x^5 (a + b x^3)^{1/3}} - \frac{(6 b c - a d) (a + b x^3)^{2/3}}{5 a^2 c (b c - a d) x^5} + \frac{(18 b^2 c^2 - 3 a b c d - 5 a^2 d^2) (a + b x^3)^{2/3}}{10 a^3 c^2 (b c - a d) x^2} - \\
& \frac{d^3 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{8/3} (b c - a d)^{4/3}} - \frac{d^3 \operatorname{Log}[c + d x^3]}{6 c^{8/3} (b c - a d)^{4/3}} + \frac{d^3 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{8/3} (b c - a d)^{4/3}}
\end{aligned}$$

Result (type 5, 950 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{70 c^5 (b c - a d) x^8 (a + b x^3)^{7/3}} \left(56 c^5 (a + b x^3)^2 - 252 c^4 d x^3 (a + b x^3)^2 - 1512 c^3 d^2 x^6 (a + b x^3)^2 - 1134 c^2 d^3 x^9 (a + b x^3)^2 - \right. \\
& 56 c^5 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 252 c^4 d x^3 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 1512 c^3 d^2 x^6 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 1134 c^2 d^3 x^9 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 18 c^3 (b c - a d)^2 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 171 c^2 d (b c - a d)^2 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 486 c d^2 (b c - a d)^2 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 297 d^3 (b c - a d)^2 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 27 c^3 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 216 c^2 d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 351 c d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 162 d^3 (b c - a d)^2 x^{15} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 27 c^3 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 81 c^2 d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& 81 c d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + \\
& \left. 27 d^3 (b c - a d)^2 x^{15} \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] \right)
\end{aligned}$$

Problem 760: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^9 (a + b x^3)^{4/3} (c + d x^3)} dx$$

Optimal (type 3, 351 leaves, 6 steps):

$$\frac{b}{a (b c - a d) x^8 (a + b x^3)^{1/3}} - \frac{(9 b c - a d) (a + b x^3)^{2/3}}{8 a^2 c (b c - a d) x^8} + \frac{(9 b c - 4 a d) (3 b c + a d) (a + b x^3)^{2/3}}{20 a^3 c^2 (b c - a d) x^5} -$$

$$\frac{(81 b^3 c^3 - 9 a b^2 c^2 d - 12 a^2 b c d^2 - 20 a^3 d^3) (a + b x^3)^{2/3}}{40 a^4 c^3 (b c - a d) x^2} + \frac{d^4 \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c - a d)^{1/3} x}{c^{1/3} (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} c^{11/3} (b c - a d)^{4/3}} + \frac{d^4 \operatorname{Log}[c + d x^3]}{6 c^{11/3} (b c - a d)^{4/3}} - \frac{d^4 \operatorname{Log}\left[\frac{(b c - a d)^{1/3} x}{c^{1/3}} - (a + b x^3)^{1/3}\right]}{2 c^{11/3} (b c - a d)^{4/3}}$$

Result (type 5, 1486 leaves, 2 steps):

$$\frac{1}{560 c^6 (b c - a d) x^{11} (a + b x^3)^{7/3}}$$

$$\left(280 c^6 (a + b x^3)^2 - 672 c^5 d x^3 (a + b x^3)^2 + 3024 c^4 d^2 x^6 (a + b x^3)^2 + 18144 c^3 d^3 x^9 (a + b x^3)^2 + 13608 c^2 d^4 x^{12} (a + b x^3)^2 - \right.$$

$$280 c^6 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] + 672 c^5 d x^3 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$3024 c^4 d^2 x^6 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 18144 c^3 d^3 x^9 (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$13608 c^2 d^4 x^{12} (a + b x^3)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 66 c^4 (b c - a d)^2 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$312 c^3 d (b c - a d)^2 x^9 \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 2268 c^2 d^2 (b c - a d)^2 x^{12} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$6696 c d^3 (b c - a d)^2 x^{15} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - 4050 d^4 (b c - a d)^2 x^{18} \operatorname{Hypergeometric2F1}\left[2, \frac{7}{3}, \frac{10}{3}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$189 c^4 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$108 c^3 d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$3618 c^2 d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$6156 c d^3 (b c - a d)^2 x^{15} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] -$$

$$2835 d^4 (b c - a d)^2 x^{18} \operatorname{HypergeometricPFQ}\left[\{2, 2, \frac{7}{3}\}, \{1, \frac{10}{3}\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] +$$

$$\begin{aligned}
& 54 c^4 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 648 c^3 d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 2268 c^2 d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 2376 c d^3 (b c - a d)^2 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 810 d^4 (b c - a d)^2 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 81 c^4 (b c - a d)^2 x^6 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 324 c^3 d (b c - a d)^2 x^9 \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 486 c^2 d^2 (b c - a d)^2 x^{12} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 324 c d^3 (b c - a d)^2 x^{15} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right] - \\
& 81 d^4 (b c - a d)^2 x^{18} \operatorname{HypergeometricPFQ}\left[\left\{2, 2, 2, 2, \frac{7}{3}\right\}, \left\{1, 1, 1, \frac{10}{3}\right\}, \frac{(b c - a d) x^3}{c (a + b x^3)}\right]
\end{aligned}$$

Test results for the 46 problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Test results for the 298 problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Test results for the 143 problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Problem 1412: Result valid but suboptimal antiderivative.

$$\int \frac{(a + b x + c x^2)^{4/3}}{(b d + 2 c d x)^{11/3}} dx$$

Optimal (type 3, 320 leaves, 8 steps):

$$\begin{aligned} & -\frac{3 (d (b + 2 c x))^{4/3} (a + b x + c x^2)^{1/3}}{16 c^2 (b^2 - 4 a c) d^5} + \frac{9 (d (b + 2 c x))^{4/3} (a + b x + c x^2)^{4/3}}{16 c (b^2 - 4 a c)^2 d^5} + \frac{3 (a + b x + c x^2)^{7/3}}{4 (b^2 - 4 a c) d (b d + 2 c d x)^{8/3}} \\ & - \frac{9 (a + b x + c x^2)^{7/3}}{4 (b^2 - 4 a c)^2 d^3 (b d + 2 c d x)^{2/3}} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2^{1/3} (d (b + 2 c x))^{2/3}}{c^{1/3} d^{2/3} (a + b x + c x^2)^{1/3}}}{\sqrt{3}}\right]}{16 \times 2^{2/3} c^{7/3} d^{11/3}} - \frac{3 \operatorname{Log}\left[(d (b + 2 c x))^{2/3} - 2^{2/3} c^{1/3} d^{2/3} (a + b x + c x^2)^{1/3}\right]}{32 \times 2^{2/3} c^{7/3} d^{11/3}} \end{aligned}$$

Result (type 3, 468 leaves, 14 steps):

$$\begin{aligned} & -\frac{3 (d (b + 2 c x))^{4/3} (a + b x + c x^2)^{1/3}}{16 c^2 (b^2 - 4 a c) d^5} + \frac{9 (d (b + 2 c x))^{4/3} (a + b x + c x^2)^{4/3}}{16 c (b^2 - 4 a c)^2 d^5} + \\ & \frac{3 (a + b x + c x^2)^{7/3}}{4 (b^2 - 4 a c) d (b d + 2 c d x)^{8/3}} - \frac{9 (a + b x + c x^2)^{7/3}}{4 (b^2 - 4 a c)^2 d^3 (b d + 2 c d x)^{2/3}} - \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{c^{1/3} d^{2/3} + \frac{2^{1/3} (d (b + 2 c x))^{2/3}}{(a + b x + c x^2)^{1/3}}}{\sqrt{3} c^{1/3} d^{2/3}}\right]}{16 \times 2^{2/3} c^{7/3} d^{11/3}} \\ & - \frac{\operatorname{Log}\left[\frac{2^{1/3} (d (b + 2 c x))^{2/3} - 2 c^{1/3} d^{2/3} (a + b x + c x^2)^{1/3}}{(a + b x + c x^2)^{1/3}}\right]}{16 \times 2^{2/3} c^{7/3} d^{11/3}} + \frac{\operatorname{Log}\left[\frac{(d (b + 2 c x))^{4/3} + 2^{2/3} c^{1/3} d^{2/3} (d (b + 2 c x))^{2/3} (a + b x + c x^2)^{1/3} + 2 \cdot 2^{1/3} c^{2/3} d^{4/3} (a + b x + c x^2)^{2/3}}{(a + b x + c x^2)^{2/3}}\right]}{32 \times 2^{2/3} c^{7/3} d^{11/3}} \end{aligned}$$

Test results for the 2646 problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

Test results for the 958 problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

Problem 833: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{-1+x} \sqrt{1+x}}{1+x-x^2} dx$$

Optimal (type 3, 91 leaves, ? steps):

$$-\text{ArcCosh}[x] + \sqrt{\frac{2}{5}(-1+\sqrt{5})} \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\sqrt{-1+x}}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\sqrt{-1+x}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10}(-1+\sqrt{5})}\sqrt{-1+x}\sqrt{1+x}\text{ArcTan}\left[\frac{2-(1-\sqrt{5})x}{\sqrt{2(-1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}} - \frac{\sqrt{-1+x}\sqrt{1+x}\text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})}\sqrt{-1+x}\sqrt{1+x}\text{ArcTanh}\left[\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

Test results for the 123 problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 143 problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 400 problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 374: Result valid but suboptimal antiderivative.

$$\int \frac{(d+e x^2)^{3/2}}{x^2 (a+b x^2+c x^4)} dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\sqrt{d+ex^2}}{ax} - \frac{(2cd - (b - \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac}(b - \sqrt{b^2 - 4ac})^{3/2}} + \frac{(2cd - (b + \sqrt{b^2 - 4ac})e)^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{\sqrt{b^2 - 4ac}(b + \sqrt{b^2 - 4ac})^{3/2}}$$

Result (type 3, 432 leaves, 16 steps):

$$-\frac{d\sqrt{d+ex^2}}{ax} - \frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}x}{\sqrt{b - \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{2a\sqrt{b - \sqrt{b^2 - 4ac}}} -$$

$$\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}x}{\sqrt{b + \sqrt{b^2 - 4ac}}\sqrt{d+ex^2}}\right]}{2a\sqrt{b + \sqrt{b^2 - 4ac}}} + \frac{d\sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{a} -$$

$$\frac{\sqrt{e} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{2a} - \frac{\sqrt{e} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{2a}$$

Test results for the 111 problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Test results for the 145 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 42 problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Test results for the 4 problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Test results for the 664 problems in "1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 (a^2 + 2 a b x^3 + b^2 x^6)^{3/2} dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^2 (a + b x^3)^3 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{12 b^3} - \frac{2 a (a + b x^3)^4 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{15 b^3} + \frac{(a + b x^3)^5 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{18 b^3}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 x^9 \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{9 (a + b x^3)} + \frac{a^2 b x^{12} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{4 (a + b x^3)} + \frac{a b^2 x^{15} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{5 (a + b x^3)} + \frac{b^3 x^{18} \sqrt{a^2 + 2 a b x^3 + b^2 x^6}}{18 (a + b x^3)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \left(\frac{(a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{x^2} - \frac{2 b^3 (1 - 2 p) (1 - p) p (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{3 a^3 x} \right) dx$$

Optimal (type 3, 146 leaves, ? steps):

$$- \frac{(a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a x} + \frac{b (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^2 x^{2/3}} - \frac{b^2 (1 - 2 p) (1 - p) (a + b x^{1/3}) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p}{a^3 x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{\frac{1}{a^3 (1+2p)} 2 b^3 (1-2p) (1-p) p \left(1 + \frac{b x^{1/3}}{a}\right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1}\left[1, 1+2p, 2(1+p), 1 + \frac{b x^{1/3}}{a}\right] + 3 b^3 \left(1 + \frac{b x^{1/3}}{a}\right) (a^2 + 2 a b x^{1/3} + b^2 x^{2/3})^p \text{Hypergeometric2F1}\left[4, 1+2p, 2(1+p), 1 + \frac{b x^{1/3}}{a}\right]}{a^3 (1+2p)}$$

Test results for the 96 problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 156 problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 17 problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.m"

Test results for the 140 problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (b x^{1+p} (b x + c x^3)^p + 2 c x^{3+p} (b x + c x^3)^p) dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p} (b x + c x^3)^{1+p}}{2 (1+p)}$$

Result (type 5, 116 leaves, 7 steps):

$$\frac{b x^{2+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \text{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{c x^2}{b}\right]}{2 (1+p)} + \frac{c x^{4+p} \left(1 + \frac{c x^2}{b}\right)^{-p} (b x + c x^3)^p \text{Hypergeometric2F1}\left[-p, 2+p, 3+p, -\frac{c x^2}{b}\right]}{2+p}$$

Problem 221: Result valid but suboptimal antiderivative.

$$\int (1+2x) (x+x^2)^3 (-18+7(x+x^2)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int x^3 (1+x)^3 (1+2x) \left(-18 + 7 x^3 (1+x)^3 \right)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 x^4 (1+x)^4 - 36 x^7 (1+x)^7 + \frac{49}{10} x^{10} (1+x)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} dx$$

Optimal (type 3, 31 leaves, ? steps):

$$\text{Log}[1-x] - \frac{1}{2} \text{Log}[3-x] + \frac{3}{2} \text{Log}[1+x] - 2 \text{Log}[3+x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2} \text{ArcTanh}\left[\frac{x}{3}\right] + \frac{\text{ArcTanh}[x]}{2} + \frac{5}{4} \text{Log}[1-x^2] - \frac{5}{4} \text{Log}[9-x^2]$$

Problem 393: Unable to integrate problem.

$$\int \frac{(1+x^2)^2}{a x^6 + b (1+x^2)^3} dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3}+b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3}+b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3}+b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3}+b^{1/3}} b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{1}{a x^6 + b (1+x^2)^3}, x\right] + 2 \text{CannotIntegrate}\left[\frac{x^2}{a x^6 + b (1+x^2)^3}, x\right] + \text{CannotIntegrate}\left[\frac{x^4}{a x^6 + b (1+x^2)^3}, x\right]$$

Problem 493: Unable to integrate problem.

$$\int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3+x+x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3+x+x^4)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$-\frac{19}{4(3+x+x^4)^3} + \frac{1}{(3+x+x^4)^2} - \frac{621}{4} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] +$$

$$684 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] + 360 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] + 44 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] -$$

$$320 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right] - 75 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^3}, x\right] + 30 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^2}, x\right]$$

Problem 494: Unable to integrate problem.

$$\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2 - 3x + 5x^2 + x^4 - 5x^6}{(3 + x + x^4)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\begin{aligned} & \frac{7}{2(3+x+x^4)^3} - \frac{63x}{22(3+x+x^4)^3} - \frac{12x^2}{(3+x+x^4)^3} - \frac{5x^3}{(3+x+x^4)^3} + \frac{3x^4}{2(3+x+x^4)^3} - \frac{10x^6}{(3+x+x^4)^3} - \\ & \frac{1}{2(3+x+x^4)^2} + \frac{5x^2}{(3+x+x^4)^2} + \frac{144}{11} \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^4}, x\right] + \frac{828}{11} \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^4}, x\right] + \\ & 18 \text{CannotIntegrate}\left[\frac{x^2}{(3+x+x^4)^4}, x\right] - 4 \text{CannotIntegrate}\left[\frac{1}{(3+x+x^4)^3}, x\right] - 20 \text{CannotIntegrate}\left[\frac{x}{(3+x+x^4)^3}, x\right] \end{aligned}$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 20: Unable to integrate problem.

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Optimal (type 3, 187 leaves, 1 step):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{1+\frac{2dx}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\text{ArcTan}\left[\frac{1+\frac{2(2c+dx)}{(2c^3+d^3x^3)^{1/3}}}{\sqrt{3}}\right]}{2c^2d} - \frac{\text{Log}[c+dx]}{2c^2d} - \frac{\text{Log}[dx-(2c^3+d^3x^3)^{1/3}]}{4c^2d} + \frac{3\text{Log}[d(2c+dx)-d(2c^3+d^3x^3)^{1/3}]}{4c^2d} \end{aligned}$$

Result (type 8, 27 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}}, x\right]$$

Problem 21: Unable to integrate problem.

$$\int \frac{1}{(1+2^{1/3}x)(1+x^3)^{2/3}} dx$$

Optimal (type 3, 147 leaves, 1 step):

$$-\frac{\operatorname{ArcTan}\left[\frac{1+\frac{2x}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\operatorname{ArcTan}\left[\frac{1+\frac{2(2^{2/3}+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[1+2^{1/3}x\right]}{2^{2/3}} - \frac{\operatorname{Log}\left[x-(1+x^3)^{1/3}\right]}{2\times 2^{2/3}} + \frac{3\operatorname{Log}\left[2+2^{1/3}x-2^{1/3}(1+x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 23 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(1+2^{1/3}x)(1+x^3)^{2/3}}, x\right]$$

Problem 22: Unable to integrate problem.

$$\int \frac{1}{(1-2^{1/3}x)(1-x^3)^{2/3}} dx$$

Optimal (type 3, 159 leaves, 1 step):

$$-\frac{\sqrt{3}\operatorname{ArcTan}\left[\frac{1+\frac{2\cdot 2^{2/3}-2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}} + \frac{\operatorname{Log}\left[1-2^{1/3}x\right]}{2^{2/3}} + \frac{\operatorname{Log}\left[-x-(1-x^3)^{1/3}\right]}{2\times 2^{2/3}} - \frac{3\operatorname{Log}\left[-2+2^{1/3}x+2^{1/3}(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 26 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(1-2^{1/3}x)(1-x^3)^{2/3}}, x\right]$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int (c+dx)^4 (a+bx^3)^{1/3} dx$$

Optimal (type 5, 387 leaves, 11 steps):

$$\begin{aligned} & \frac{3ac^2d^2(a+bx^3)^{1/3}}{2b} + \frac{ad^4x^2(a+bx^3)^{1/3}}{18b} + \frac{1}{30}(a+bx^3)^{1/3}(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5) - \\ & \frac{4ac^3d\operatorname{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}b^{2/3}} + \frac{a^2d^4\operatorname{ArcTan}\left[\frac{1+\frac{2b^{1/3}x}{(a+bx^3)^{1/3}}}{\sqrt{3}}\right]}{9\sqrt{3}b^{5/3}} + \frac{ac^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{bx^3}{a}\right]}{2(a+bx^3)^{2/3}} + \\ & \frac{ac^3d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{bx^3}{a}\right]}{5(a+bx^3)^{2/3}} - \frac{2ac^3d\operatorname{Log}\left[b^{1/3}x-(a+bx^3)^{1/3}\right]}{3b^{2/3}} + \frac{a^2d^4\operatorname{Log}\left[b^{1/3}x-(a+bx^3)^{1/3}\right]}{18b^{5/3}} \end{aligned}$$

Result (type 5, 498 leaves, 23 steps):

$$\begin{aligned}
& \frac{3 a c^2 d^2 (a+b x^3)^{1/3}}{2 b} + \frac{a d^4 x^2 (a+b x^3)^{1/3}}{18 b} + \frac{1}{30} (a+b x^3)^{1/3} (15 c^4 x + 40 c^3 d x^2 + 45 c^2 d^2 x^3 + 24 c d^3 x^4 + 5 d^4 x^5) - \frac{4 a c^3 d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} + \\
& \frac{a^2 d^4 \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} b^{5/3}} + \frac{a c^4 x \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a+b x^3)^{2/3}} + \frac{a c d^3 x^4 \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{b x^3}{a}\right]}{5 (a+b x^3)^{2/3}} - \\
& \frac{4 a c^3 d \operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{9 b^{2/3}} + \frac{a^2 d^4 \operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{27 b^{5/3}} + \frac{2 a c^3 d \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{9 b^{2/3}} - \frac{a^2 d^4 \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{54 b^{5/3}}
\end{aligned}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int (c+d x)^3 (a+b x^3)^{1/3} dx$$

Optimal (type 5, 242 leaves, 9 steps):

$$\begin{aligned}
& \frac{3 a c d^2 (a+b x^3)^{1/3}}{4 b} + \frac{a d^3 x (a+b x^3)^{1/3}}{10 b} + \frac{1}{20} (a+b x^3)^{1/3} (10 c^3 x + 20 c^2 d x^2 + 15 c d^2 x^3 + 4 d^3 x^4) - \\
& \frac{a c^2 d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{a (5 b c^3 - a d^3) x \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{10 b (a+b x^3)^{2/3}} - \frac{a c^2 d \operatorname{Log}\left[b^{1/3} x - (a+b x^3)^{1/3}\right]}{2 b^{2/3}}
\end{aligned}$$

Result (type 5, 297 leaves, 15 steps):

$$\begin{aligned}
& \frac{3 a c d^2 (a+b x^3)^{1/3}}{4 b} + \frac{a d^3 x (a+b x^3)^{1/3}}{10 b} + \frac{1}{20} (a+b x^3)^{1/3} (10 c^3 x + 20 c^2 d x^2 + 15 c d^2 x^3 + 4 d^3 x^4) - \frac{a c^2 d \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \\
& \frac{a (5 b c^3 - a d^3) x \left(1+\frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{10 b (a+b x^3)^{2/3}} - \frac{a c^2 d \operatorname{Log}\left[1-\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 b^{2/3}} + \frac{a c^2 d \operatorname{Log}\left[1+\frac{b^{2/3} x^2}{(a+b x^3)^{2/3}}+\frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 b^{2/3}}
\end{aligned}$$

Problem 25: Result valid but suboptimal antiderivative.

$$\int (c+d x)^2 (a+b x^3)^{1/3} dx$$

Optimal (type 5, 192 leaves, 8 steps):

$$\frac{a d^2 (a + b x^3)^{1/3}}{4 b} + \frac{1}{12} (a + b x^3)^{1/3} (6 c^2 x + 8 c d x^2 + 3 d^2 x^3) - \frac{2 a c d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} +$$

$$\frac{a c^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}} - \frac{a c d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{3 b^{2/3}}$$

Result (type 5, 245 leaves, 14 steps):

$$\frac{a d^2 (a + b x^3)^{1/3}}{4 b} + \frac{1}{12} (a + b x^3)^{1/3} (6 c^2 x + 8 c d x^2 + 3 d^2 x^3) - \frac{2 a c d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} +$$

$$\frac{a c^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}} - \frac{2 a c d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{2/3}} + \frac{a c d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{2/3}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int (c + d x) (a + b x^3)^{1/3} dx$$

Optimal (type 5, 155 leaves, 6 steps):

$$\frac{1}{6} (3 c x + 2 d x^2) (a + b x^3)^{1/3} - \frac{a d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} + \frac{a c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}} - \frac{a d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{6 b^{2/3}}$$

Result (type 5, 207 leaves, 12 steps):

$$\frac{1}{6} (3 c x + 2 d x^2) (a + b x^3)^{1/3} - \frac{a d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{2/3}} +$$

$$\frac{a c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 (a + b x^3)^{2/3}} - \frac{a d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{2/3}} + \frac{a d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{18 b^{2/3}}$$

Problem 27: Unable to integrate problem.

$$\int \frac{(a + b x^3)^{1/3}}{c + d x} dx$$

Optimal (type 6, 435 leaves, 13 steps):

$$\begin{aligned} & \frac{(a + b x^3)^{1/3}}{d} + \frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c \left(1 + \frac{b x^3}{a}\right)^{1/3}} + \frac{b^{1/3} c \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} - \\ & \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}[c^3 + d^3 x^3]}{3 d^2} + \\ & \frac{b^{1/3} c \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}]}{2 d^2} \end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{(a + b x^3)^{1/3}}{c + d x}, x\right]$$

Problem 28: Unable to integrate problem.

$$\int \frac{(a + b x^3)^{1/3}}{(c + d x)^2} dx$$

Optimal (type 6, 818 leaves, 20 steps):

$$\begin{aligned} & -\frac{c^2 (a + b x^3)^{1/3}}{d (c^3 + d^3 x^3)} - \frac{d x^2 (a + b x^3)^{1/3}}{c^3 + d^3 x^3} + \frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 2, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^2 \left(1 + \frac{b x^3}{a}\right)^{1/3}} - \frac{d^3 x^4 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^5 \left(1 + \frac{b x^3}{a}\right)^{1/3}} - \\ & \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{2 a d \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b c^2 \operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c^3 - a d^3)^{2/3}} - \\ & \frac{b c^2 \operatorname{Log}[c^3 + d^3 x^3]}{6 d^2 (b c^3 - a d^3)^{2/3}} - \frac{a d \operatorname{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{2/3}} - \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}[c^3 + d^3 x^3]}{18 c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b^{1/3} \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d^2} + \\ & \frac{a d \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c d^2 (b c^3 - a d^3)^{2/3}} + \frac{b c^2 \operatorname{Log}[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}]}{2 d^2 (b c^3 - a d^3)^{2/3}} \end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{(a + b x^3)^{1/3}}{(c + d x)^2}, x\right]$$

Problem 33: Unable to integrate problem.

$$\int \frac{1}{(c + d x) (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 333 leaves, 10 steps):

$$\begin{aligned} & - \frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^2 (a + b x^3)^{1/3}} + \frac{\text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} - \\ & \frac{\text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} + \frac{\text{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{1/3}} - \frac{\text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} - \frac{\text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} \end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{1}{(c + d x) (a + b x^3)^{1/3}}, x\right]$$

Problem 34: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 761 leaves, 17 steps):

$$\begin{aligned}
& \frac{c^2 d^2 (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{c d^3 x (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 2, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a + b x^3)^{1/3}} + \\
& \frac{d^4 x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^6 (a + b x^3)^{1/3}} + \frac{2 a d^3 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} - \\
& \frac{b c^2 \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{4/3}} + \frac{b c^2 \text{Log}[c^3 + d^3 x^3]}{6 (b c^3 - a d^3)^{4/3}} + \frac{a d^3 \text{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 c (b c^3 - a d^3)^{4/3}} - \\
& \frac{a d^3 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{4/3}} - \frac{(3 b c^3 - 2 a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c (b c^3 - a d^3)^{4/3}} - \frac{b c^2 \text{Log}[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}]}{2 (b c^3 - a d^3)^{4/3}}
\end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c + d x)^2 (a + b x^3)^{1/3}}, x\right]$$

Problem 35: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 1513 leaves, 32 steps):

$$\begin{aligned}
& \frac{3 c^4 d^2 (a+b x^3)^{2/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} - \frac{3 c^3 d^3 x (a+b x^3)^{2/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{4 b c^4 d^2 (a+b x^3)^{2/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{c d^2 (b c^3 - 3 a d^3) (a+b x^3)^{2/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \\
& \frac{d^3 (3 b c^3 - 7 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{d^3 (9 b c^3 - 5 a d^3) x (a+b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{7 d^3 (3 b c^3 + a d^3) x (a+b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \\
& \frac{3 d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 3, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^4 (a+b x^3)^{1/3}} + \frac{6 d^4 x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 3, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^7 (a+b x^3)^{1/3}} + \frac{2 a^2 d^6 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} + \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} - \frac{4 b^2 c^4 \text{ArcTan}\left[\frac{1 - \frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{7/3}} + \\
& \frac{b c (b c^3 - 3 a d^3) \text{ArcTan}\left[\frac{1 - \frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{7/3}} + \frac{2 b^2 c^4 \text{Log}[c^3 + d^3 x^3]}{9 (b c^3 - a d^3)^{7/3}} + \frac{a^2 d^6 \text{Log}[c^3 + d^3 x^3]}{27 c^2 (b c^3 - a d^3)^{7/3}} - \frac{b c (b c^3 - 3 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 (b c^3 - a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{54 c^2 (b c^3 - a d^3)^{7/3}} + \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{Log}[c^3 + d^3 x^3]}{54 c^2 (b c^3 - a d^3)^{7/3}} - \frac{a^2 d^6 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{9 c^2 (b c^3 - a d^3)^{7/3}} - \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{18 c^2 (b c^3 - a d^3)^{7/3}} - \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{18 c^2 (b c^3 - a d^3)^{7/3}} - \\
& \frac{2 b^2 c^4 \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}\right]}{3 (b c^3 - a d^3)^{7/3}} + \frac{b c (b c^3 - 3 a d^3) \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}\right]}{6 (b c^3 - a d^3)^{7/3}}
\end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^3 (a+b x^3)^{1/3}}, x\right]$$

Problem 36: Result valid but suboptimal antiderivative.

$$\int \frac{(c+d x)^4}{(a+b x^3)^{2/3}} dx$$

Optimal (type 5, 306 leaves, 10 steps):

$$\frac{6 c^2 d^2 (a + b x^3)^{1/3}}{b} + \frac{d^4 x^2 (a + b x^3)^{1/3}}{3 b} - \frac{4 c^3 d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{2 a d^4 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}} + \frac{c^4 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} +$$

$$\frac{c d^3 x^4 \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} - \frac{2 c^3 d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{b^{2/3}} + \frac{a d^4 \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{3 b^{5/3}}$$

Result (type 5, 416 leaves, 22 steps):

$$\frac{6 c^2 d^2 (a + b x^3)^{1/3}}{b} + \frac{d^4 x^2 (a + b x^3)^{1/3}}{3 b} - \frac{4 c^3 d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{2 a d^4 \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} b^{5/3}} +$$

$$\frac{c^4 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} + \frac{c d^3 x^4 \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{4}{3}, \frac{7}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} -$$

$$\frac{4 c^3 d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3}} + \frac{2 a d^4 \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{5/3}} + \frac{2 c^3 d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3}} - \frac{a d^4 \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{9 b^{5/3}}$$

Problem 37: Result valid but suboptimal antiderivative.

$$\int \frac{(c + d x)^3}{(a + b x^3)^{2/3}} dx$$

Optimal (type 5, 187 leaves, 8 steps):

$$\frac{3 c d^2 (a + b x^3)^{1/3}}{b} + \frac{d^3 x (a + b x^3)^{1/3}}{2 b} - \frac{\sqrt{3} c^2 d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}} +$$

$$\frac{(2 b c^3 - a d^3) x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 b (a + b x^3)^{2/3}} - \frac{3 c^2 d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 239 leaves, 14 steps):

$$\frac{3 c d^2 (a + b x^3)^{1/3}}{b} + \frac{d^3 x (a + b x^3)^{1/3}}{2 b} - \frac{\sqrt{3} c^2 d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}} +$$

$$\frac{(2 b c^3 - a d^3) x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{2 b (a + b x^3)^{2/3}} - \frac{c^2 d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{b^{2/3}} + \frac{c^2 d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{2 b^{2/3}}$$

Problem 38: Result valid but suboptimal antiderivative.

$$\int \frac{(c + d x)^2}{(a + b x^3)^{2/3}} dx$$

Optimal (type 5, 141 leaves, 7 steps):

$$\frac{d^2 (a + b x^3)^{1/3}}{b} - \frac{2 c d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{c^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} - \frac{c d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{b^{2/3}}$$

Result (type 5, 195 leaves, 13 steps):

$$\frac{d^2 (a + b x^3)^{1/3}}{b} - \frac{2 c d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} +$$

$$\frac{c^2 x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} - \frac{2 c d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3}} + \frac{c d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a + b x^3)^{2/3}} + \frac{b^{1/3} x}{(a + b x^3)^{1/3}}\right]}{3 b^{2/3}}$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{c + d x}{(a + b x^3)^{2/3}} dx$$

Optimal (type 5, 121 leaves, 5 steps):

$$-\frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a + b x^3)^{2/3}} - \frac{d \operatorname{Log}\left[b^{1/3} x - (a + b x^3)^{1/3}\right]}{2 b^{2/3}}$$

Result (type 5, 172 leaves, 11 steps):

$$- \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3}} + \frac{c x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{b x^3}{a}\right]}{(a+b x^3)^{2/3}} - \frac{d \operatorname{Log}\left[1 - \frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{3 b^{2/3}} + \frac{d \operatorname{Log}\left[1 + \frac{b^{2/3} x^2}{(a+b x^3)^{2/3}} + \frac{b^{1/3} x}{(a+b x^3)^{1/3}}\right]}{6 b^{2/3}}$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(c+d x) (a+b x^3)^{2/3}} dx$$

Optimal (type 6, 332 leaves, 10 steps):

$$\frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \operatorname{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c (a+b x^3)^{2/3}} + \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} - \frac{d \operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} - \frac{d \operatorname{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{2/3}} + \frac{d \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}} + \frac{d \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}}$$

Result (type 8, 21 leaves, 0 steps):

$$\operatorname{CannotIntegrate}\left[\frac{1}{(c+d x) (a+b x^3)^{2/3}}, x\right]$$

Problem 41: Unable to integrate problem.

$$\int \frac{1}{(c+d x)^2 (a+b x^3)^{2/3}} dx$$

Optimal (type 6, 760 leaves, 18 steps):

$$\begin{aligned}
& \frac{c^2 d^2 (a + b x^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{d^4 x^2 (a + b x^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 2, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^2 (a + b x^3)^{2/3}} - \\
& \frac{d^3 x^4 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^5 (a + b x^3)^{2/3}} + \frac{2 a d^4 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} + \frac{2 d (3 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} - \\
& \frac{2 b c^2 d \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{5/3}} - \frac{b c^2 d \text{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{5/3}} - \frac{a d^4 \text{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{5/3}} - \frac{d (3 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{5/3}} + \\
& \frac{a d^4 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{d (3 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{b c^2 d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{(b c^3 - a d^3)^{5/3}}
\end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c + d x)^2 (a + b x^3)^{2/3}}, x\right]$$

Problem 42: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{2/3}} dx$$

Optimal (type 6, 1357 leaves, 30 steps):

$$\begin{aligned}
& \frac{3 c^4 d^2 (a+b x^3)^{1/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{3 c^2 d^4 x^2 (a+b x^3)^{1/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{5 b c^4 d^2 (a+b x^3)^{1/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{c d^2 (b c^3 - 6 a d^3) (a+b x^3)^{1/3}}{6 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \\
& \frac{d^4 (9 b c^3 - 4 a d^3) x^2 (a+b x^3)^{1/3}}{6 c (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{d^4 (3 b c^3 + 2 a d^3) x^2 (a+b x^3)^{1/3}}{3 c (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 3, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a+b x^3)^{2/3}} - \\
& \frac{7 d^3 x^4 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 3, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{4 c^6 (a+b x^3)^{2/3}} + \frac{d^6 x^7 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 3, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{7 c^9 (a+b x^3)^{2/3}} + \\
& \frac{2 a d^4 (6 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} - \\
& \frac{10 b^2 c^4 d \text{ArcTan}\left[\frac{1 - \frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} + \frac{b c d (b c^3 - 6 a d^3) \text{ArcTan}\left[\frac{1 - \frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} - \frac{5 b^2 c^4 d \text{Log}[c^3 + d^3 x^3]}{9 (b c^3 - a d^3)^{8/3}} + \\
& \frac{b c d (b c^3 - 6 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 (b c^3 - a d^3)^{8/3}} - \frac{a d^4 (6 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{9 c^2 (b c^3 - a d^3)^{8/3}} - \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}[c^3 + d^3 x^3]}{18 c^2 (b c^3 - a d^3)^{8/3}} + \\
& \frac{a d^4 (6 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{3 c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{6 c^2 (b c^3 - a d^3)^{8/3}} + \\
& \frac{5 b^2 c^4 d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}\right]}{3 (b c^3 - a d^3)^{8/3}} - \frac{b c d (b c^3 - 6 a d^3) \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}\right]}{6 (b c^3 - a d^3)^{8/3}}
\end{aligned}$$

Result (type 8, 21 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(c+d x)^3 (a+b x^3)^{2/3}}, x\right]$$

Problem 227: Result valid but suboptimal antiderivative.

$$\int x^m \left(c (a+b x^2)^2\right)^{3/2} dx$$

Optimal (type 3, 161 leaves, 3 steps):

$$\frac{a^3 c x^{1+m} \sqrt{c (a+b x^2)^2}}{(1+m) (a+b x^2)} + \frac{3 a^2 b c x^{3+m} \sqrt{c (a+b x^2)^2}}{(3+m) (a+b x^2)} + \frac{3 a b^2 c x^{5+m} \sqrt{c (a+b x^2)^2}}{(5+m) (a+b x^2)} + \frac{b^3 c x^{7+m} \sqrt{c (a+b x^2)^2}}{(7+m) (a+b x^2)}$$

Result (type 3, 205 leaves, 4 steps):

$$\frac{a^3 c x^{1+m} \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{(1+m) (a + b x^2)} + \frac{3 a^2 b c x^{3+m} \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{(3+m) (a + b x^2)} +$$

$$\frac{3 a b^2 c x^{5+m} \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{(5+m) (a + b x^2)} + \frac{b^3 c x^{7+m} \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{(7+m) (a + b x^2)}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int x^5 \left(c (a + b x^2)^2 \right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 4 steps):

$$\frac{a^3 c x^6 \sqrt{c (a + b x^2)^2}}{6 (a + b x^2)} + \frac{3 a^2 b c x^8 \sqrt{c (a + b x^2)^2}}{8 (a + b x^2)} + \frac{3 a b^2 c x^{10} \sqrt{c (a + b x^2)^2}}{10 (a + b x^2)} + \frac{b^3 c x^{12} \sqrt{c (a + b x^2)^2}}{12 (a + b x^2)}$$

Result (type 2, 134 leaves, 4 steps):

$$\frac{a^2 c (a + b x^2)^3 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{8 b^3} - \frac{a c (a + b x^2)^4 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{5 b^3} + \frac{c (a + b x^2)^5 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{12 b^3}$$

Problem 229: Result valid but suboptimal antiderivative.

$$\int x^4 \left(c (a + b x^2)^2 \right)^{3/2} dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^5 \sqrt{c (a + b x^2)^2}}{5 (a + b x^2)} + \frac{3 a^2 b c x^7 \sqrt{c (a + b x^2)^2}}{7 (a + b x^2)} + \frac{a b^2 c x^9 \sqrt{c (a + b x^2)^2}}{3 (a + b x^2)} + \frac{b^3 c x^{11} \sqrt{c (a + b x^2)^2}}{11 (a + b x^2)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^5 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{5 (a + b x^2)} + \frac{3 a^2 b c x^7 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{7 (a + b x^2)} + \frac{a b^2 c x^9 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{3 (a + b x^2)} + \frac{b^3 c x^{11} \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{11 (a + b x^2)}$$

Problem 230: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c (a + b x^2)^2 \right)^{3/2} dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{a c (a+b x^2)^3 \sqrt{c (a+b x^2)^2}}{8 b^2} + \frac{c (a+b x^2)^4 \sqrt{c (a+b x^2)^2}}{10 b^2}$$

Result (type 2, 78 leaves, 4 steps):

$$-\frac{a (a+b x^2) (a^2 c + 2 a b c x^2 + b^2 c x^4)^{3/2}}{8 b^2} + \frac{(a^2 c + 2 a b c x^2 + b^2 c x^4)^{5/2}}{10 b^2 c}$$

Problem 231: Result valid but suboptimal antiderivative.

$$\int x^2 (c (a+b x^2)^2)^{3/2} dx$$

Optimal (type 2, 143 leaves, 3 steps):

$$\frac{a^3 c x^3 \sqrt{c (a+b x^2)^2}}{3 (a+b x^2)} + \frac{3 a^2 b c x^5 \sqrt{c (a+b x^2)^2}}{5 (a+b x^2)} + \frac{3 a b^2 c x^7 \sqrt{c (a+b x^2)^2}}{7 (a+b x^2)} + \frac{b^3 c x^9 \sqrt{c (a+b x^2)^2}}{9 (a+b x^2)}$$

Result (type 2, 187 leaves, 4 steps):

$$\frac{a^3 c x^3 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{3 (a+b x^2)} + \frac{3 a^2 b c x^5 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{5 (a+b x^2)} + \frac{3 a b^2 c x^7 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{7 (a+b x^2)} + \frac{b^3 c x^9 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{9 (a+b x^2)}$$

Problem 233: Result valid but suboptimal antiderivative.

$$\int (c (a+b x^2)^2)^{3/2} dx$$

Optimal (type 2, 135 leaves, 3 steps):

$$\frac{a^3 c x \sqrt{c (a+b x^2)^2}}{a+b x^2} + \frac{a^2 b c x^3 \sqrt{c (a+b x^2)^2}}{a+b x^2} + \frac{3 a b^2 c x^5 \sqrt{c (a+b x^2)^2}}{5 (a+b x^2)} + \frac{b^3 c x^7 \sqrt{c (a+b x^2)^2}}{7 (a+b x^2)}$$

Result (type 2, 175 leaves, 4 steps):

$$\frac{a^3 x (a^2 c + 2 a b c x^2 + b^2 c x^4)^{3/2}}{(a+b x^2)^3} + \frac{a^2 b x^3 (a^2 c + 2 a b c x^2 + b^2 c x^4)^{3/2}}{(a+b x^2)^3} + \frac{3 a b^2 x^5 (a^2 c + 2 a b c x^2 + b^2 c x^4)^{3/2}}{5 (a+b x^2)^3} + \frac{b^3 x^7 (a^2 c + 2 a b c x^2 + b^2 c x^4)^{3/2}}{7 (a+b x^2)^3}$$

Problem 234: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a+b x^2)^2)^{3/2}}{x} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{3 a^2 b c x^2 \sqrt{c (a + b x^2)^2}}{2 (a + b x^2)} + \frac{3 a b^2 c x^4 \sqrt{c (a + b x^2)^2}}{4 (a + b x^2)} + \frac{b^3 c x^6 \sqrt{c (a + b x^2)^2}}{6 (a + b x^2)} + \frac{a^3 c \sqrt{c (a + b x^2)^2} \operatorname{Log}[x]}{a + b x^2}$$

Result (type 3, 183 leaves, 5 steps):

$$\frac{3 a^2 b c x^2 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{2 (a + b x^2)} + \frac{3 a b^2 c x^4 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{4 (a + b x^2)} + \frac{b^3 c x^6 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{6 (a + b x^2)} + \frac{a^3 c \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4} \operatorname{Log}[x]}{a + b x^2}$$

Problem 235: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a + b x^2)^2)^{3/2}}{x^2} dx$$

Optimal (type 2, 134 leaves, 3 steps):

$$-\frac{a^3 c \sqrt{c (a + b x^2)^2}}{x (a + b x^2)} + \frac{3 a^2 b c x \sqrt{c (a + b x^2)^2}}{a + b x^2} + \frac{a b^2 c x^3 \sqrt{c (a + b x^2)^2}}{a + b x^2} + \frac{b^3 c x^5 \sqrt{c (a + b x^2)^2}}{5 (a + b x^2)}$$

Result (type 2, 178 leaves, 4 steps):

$$-\frac{a^3 c \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{x (a + b x^2)} + \frac{3 a^2 b c x \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{a + b x^2} + \frac{a b^2 c x^3 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{a + b x^2} + \frac{b^3 c x^5 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{5 (a + b x^2)}$$

Problem 236: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a + b x^2)^2)^{3/2}}{x^3} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{a^3 c \sqrt{c (a + b x^2)^2}}{2 x^2 (a + b x^2)} + \frac{3 a b^2 c x^2 \sqrt{c (a + b x^2)^2}}{2 (a + b x^2)} + \frac{b^3 c x^4 \sqrt{c (a + b x^2)^2}}{4 (a + b x^2)} + \frac{3 a^2 b c \sqrt{c (a + b x^2)^2} \operatorname{Log}[x]}{a + b x^2}$$

Result (type 3, 184 leaves, 5 steps):

$$-\frac{a^3 c \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{2 x^2 (a + b x^2)} + \frac{3 a b^2 c x^2 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{2 (a + b x^2)} + \frac{b^3 c x^4 \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4}}{4 (a + b x^2)} + \frac{3 a^2 b c \sqrt{a^2 c + 2 a b c x^2 + b^2 c x^4} \operatorname{Log}[x]}{a + b x^2}$$

Problem 237: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c (a + b x^2)^3 \right)^{3/2} dx$$

Optimal (type 3, 253 leaves, 8 steps):

$$\begin{aligned} & \frac{7}{128} a^3 c x^3 \sqrt{c (a + b x^2)^3} + \frac{21 a^5 c x \sqrt{c (a + b x^2)^3}}{1024 b (a + b x^2)} + \frac{21 a^4 c x^3 \sqrt{c (a + b x^2)^3}}{512 (a + b x^2)} + \frac{21}{320} a^2 c x^3 (a + b x^2) \sqrt{c (a + b x^2)^3} + \\ & \frac{3}{40} a c x^3 (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{12} c x^3 (a + b x^2)^3 \sqrt{c (a + b x^2)^3} - \frac{21 a^{9/2} c \sqrt{c (a + b x^2)^3} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{1024 b^{3/2} \left(1 + \frac{b x^2}{a}\right)^{3/2}} \end{aligned}$$

Result (type 3, 254 leaves, 9 steps):

$$\begin{aligned} & \frac{7}{128} a^3 c x^3 \sqrt{c (a + b x^2)^3} + \frac{21 a^5 c x \sqrt{c (a + b x^2)^3}}{1024 b (a + b x^2)} + \frac{21 a^4 c x^3 \sqrt{c (a + b x^2)^3}}{512 (a + b x^2)} + \frac{21}{320} a^2 c x^3 (a + b x^2) \sqrt{c (a + b x^2)^3} + \\ & \frac{3}{40} a c x^3 (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{12} c x^3 (a + b x^2)^3 \sqrt{c (a + b x^2)^3} - \frac{21 a^6 c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a + b x^2}}\right]}{1024 b^{3/2} (a + b x^2)^{3/2}} \end{aligned}$$

Problem 239: Result valid but suboptimal antiderivative.

$$\int \left(c (a + b x^2)^3 \right)^{3/2} dx$$

Optimal (type 3, 207 leaves, 7 steps):

$$\begin{aligned} & \frac{21}{128} a^3 c x \sqrt{c (a + b x^2)^3} + \frac{63 a^4 c x \sqrt{c (a + b x^2)^3}}{256 (a + b x^2)} + \frac{21}{160} a^2 c x (a + b x^2) \sqrt{c (a + b x^2)^3} + \\ & \frac{9}{80} a c x (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{10} c x (a + b x^2)^3 \sqrt{c (a + b x^2)^3} + \frac{63 a^{7/2} c \sqrt{c (a + b x^2)^3} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{256 \sqrt{b} \left(1 + \frac{b x^2}{a}\right)^{3/2}} \end{aligned}$$

Result (type 3, 208 leaves, 8 steps):

$$\frac{21}{128} a^3 c x \sqrt{c (a + b x^2)^3} + \frac{63 a^4 c x \sqrt{c (a + b x^2)^3}}{256 (a + b x^2)} + \frac{21}{160} a^2 c x (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{9}{80} a c x (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{10} c x (a + b x^2)^3 \sqrt{c (a + b x^2)^3} + \frac{63 a^5 c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a + b x^2}}\right]}{256 \sqrt{b} (a + b x^2)^{3/2}}$$

Problem 240: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a + b x^2)^3)^{3/2}}{x} dx$$

Optimal (type 3, 192 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c (a + b x^2)^3} + \frac{a^4 c \sqrt{c (a + b x^2)^3}}{a + b x^2} + \frac{1}{5} a^2 c (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{1}{7} a c (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{9} c (a + b x^2)^3 \sqrt{c (a + b x^2)^3} - \frac{a^3 c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\sqrt{1 + \frac{b x^2}{a}}\right]}{\left(1 + \frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 194 leaves, 9 steps):

$$\frac{1}{3} a^3 c \sqrt{c (a + b x^2)^3} + \frac{a^4 c \sqrt{c (a + b x^2)^3}}{a + b x^2} + \frac{1}{5} a^2 c (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{1}{7} a c (a + b x^2)^2 \sqrt{c (a + b x^2)^3} + \frac{1}{9} c (a + b x^2)^3 \sqrt{c (a + b x^2)^3} - \frac{a^{9/2} c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b x^2}}{\sqrt{a}}\right]}{(a + b x^2)^{3/2}}$$

Problem 241: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a + b x^2)^3)^{3/2}}{x^2} dx$$

Optimal (type 3, 208 leaves, 7 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c (a + b x^2)^3} + \frac{315 a^3 b c x \sqrt{c (a + b x^2)^3}}{128 (a + b x^2)} + \frac{21}{16} a b c x (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{9}{8} b c x (a + b x^2)^2 \sqrt{c (a + b x^2)^3} - \frac{c (a + b x^2)^3 \sqrt{c (a + b x^2)^3}}{x} + \frac{315 a^{5/2} \sqrt{b} c \sqrt{c (a + b x^2)^3} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{128 \left(1 + \frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 209 leaves, 8 steps):

$$\frac{105}{64} a^2 b c x \sqrt{c (a + b x^2)^3} + \frac{315 a^3 b c x \sqrt{c (a + b x^2)^3}}{128 (a + b x^2)} + \frac{21}{16} a b c x (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{9}{8} b c x (a + b x^2)^2 \sqrt{c (a + b x^2)^3} - \frac{c (a + b x^2)^3 \sqrt{c (a + b x^2)^3}}{x} + \frac{315 a^4 \sqrt{b} c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a + b x^2}}\right]}{128 (a + b x^2)^{3/2}}$$

Problem 242: Result valid but suboptimal antiderivative.

$$\int \frac{(c (a + b x^2)^3)^{3/2}}{x^3} dx$$

Optimal (type 3, 202 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c (a + b x^2)^3} + \frac{9 a^3 b c \sqrt{c (a + b x^2)^3}}{2 (a + b x^2)} + \frac{9}{10} a b c (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{9}{14} b c (a + b x^2)^2 \sqrt{c (a + b x^2)^3} - \frac{c (a + b x^2)^3 \sqrt{c (a + b x^2)^3}}{2 x^2} - \frac{9 a^2 b c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\sqrt{1 + \frac{b x^2}{a}}\right]}{2 \left(1 + \frac{b x^2}{a}\right)^{3/2}}$$

Result (type 3, 204 leaves, 9 steps):

$$\frac{3}{2} a^2 b c \sqrt{c (a + b x^2)^3} + \frac{9 a^3 b c \sqrt{c (a + b x^2)^3}}{2 (a + b x^2)} + \frac{9}{10} a b c (a + b x^2) \sqrt{c (a + b x^2)^3} +$$

$$\frac{9}{14} b c (a + b x^2)^2 \sqrt{c (a + b x^2)^3} - \frac{c (a + b x^2)^3 \sqrt{c (a + b x^2)^3}}{2 x^2} - \frac{9 a^{7/2} b c \sqrt{c (a + b x^2)^3} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b x^2}}{\sqrt{a}}\right]}{2 (a + b x^2)^{3/2}}$$

Problem 243: Result valid but suboptimal antiderivative.

$$\int x^2 \left(\frac{c}{a + b x^2} \right)^{3/2} dx$$

Optimal (type 3, 77 leaves, 3 steps):

$$-\frac{c x \sqrt{\frac{c}{a + b x^2}}}{b} + \frac{\sqrt{a} c \sqrt{\frac{c}{a + b x^2}} \sqrt{1 + \frac{b x^2}{a}} \operatorname{ArcSinh}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right]}{b^{3/2}}$$

Result (type 3, 75 leaves, 4 steps):

$$-\frac{c x \sqrt{\frac{c}{a + b x^2}}}{b} + \frac{c \sqrt{\frac{c}{a + b x^2}} \sqrt{a + b x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x}{\sqrt{a + b x^2}}\right]}{b^{3/2}}$$

Problem 246: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a + b x^2} \right)^{3/2}}{x} dx$$

Optimal (type 3, 71 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a + b x^2}}}{a} - \frac{c \sqrt{\frac{c}{a + b x^2}} \sqrt{1 + \frac{b x^2}{a}} \operatorname{ArcTanh}\left[\sqrt{1 + \frac{b x^2}{a}}\right]}{a}$$

Result (type 3, 73 leaves, 5 steps):

$$\frac{c \sqrt{\frac{c}{a + b x^2}}}{a} - \frac{c \sqrt{\frac{c}{a + b x^2}} \sqrt{a + b x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b x^2}}{\sqrt{a}}\right]}{a^{3/2}}$$

Problem 248: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\frac{c}{a + b x^2} \right)^{3/2}}{x^3} dx$$

Optimal (type 3, 104 leaves, 6 steps):

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\operatorname{ArcTanh}\left[\sqrt{1+\frac{bx^2}{a}}\right]}{2a^2}$$

Result (type 3, 112 leaves, 6 steps):

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}(a+bx^2)}{2a^2x^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\operatorname{ArcTanh}\left[\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right]}{2a^{5/2}}$$

Problem 249: Result valid but suboptimal antiderivative.

$$\int x^7 \left(c \sqrt{a+bx^2} \right)^{3/2} dx$$

Optimal (type 2, 138 leaves, 4 steps):

$$-\frac{2a^3 \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)}{7b^4} + \frac{6a^2 \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)^2}{11b^4} - \frac{2a \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)^3}{5b^4} + \frac{2 \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)^4}{19b^4}$$

Result (type 2, 152 leaves, 4 steps):

$$-\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{3/2}}{7b^4} + \frac{6a^2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{5/2}}{11b^4} - \frac{2ac\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{7/2}}{5b^4} + \frac{2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{9/2}}{19b^4}$$

Problem 250: Result valid but suboptimal antiderivative.

$$\int x^5 \left(c \sqrt{a+bx^2} \right)^{3/2} dx$$

Optimal (type 2, 102 leaves, 4 steps):

$$\frac{2a^2 \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)}{7b^3} - \frac{4a \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)^2}{11b^3} + \frac{2 \left(c \sqrt{a+bx^2} \right)^{3/2} (a+bx^2)^3}{15b^3}$$

Result (type 2, 113 leaves, 4 steps):

$$\frac{2a^2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{3/2}}{7b^3} - \frac{4ac\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{5/2}}{11b^3} + \frac{2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{7/2}}{15b^3}$$

Problem 251: Result valid but suboptimal antiderivative.

$$\int x^3 \left(c \sqrt{a + b x^2} \right)^{3/2} dx$$

Optimal (type 2, 66 leaves, 4 steps):

$$-\frac{2 a \left(c \sqrt{a + b x^2} \right)^{3/2} (a + b x^2)}{7 b^2} + \frac{2 \left(c \sqrt{a + b x^2} \right)^{3/2} (a + b x^2)^2}{11 b^2}$$

Result (type 2, 74 leaves, 4 steps):

$$-\frac{2 a c \sqrt{c \sqrt{a + b x^2}} (a + b x^2)^{3/2}}{7 b^2} + \frac{2 c \sqrt{c \sqrt{a + b x^2}} (a + b x^2)^{5/2}}{11 b^2}$$

Problem 253: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{2}{3} \left(c \sqrt{a + b x^2} \right)^{3/2} + \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTan} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}} - \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTanh} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 3, 141 leaves, 7 steps):

$$\frac{2}{3} c \sqrt{c \sqrt{a + b x^2}} \sqrt{a + b x^2} + \frac{a^{3/4} c \sqrt{c \sqrt{a + b x^2}} \operatorname{ArcTan} \left[\frac{(a + b x^2)^{1/4}}{a^{1/4}} \right]}{(a + b x^2)^{1/4}} - \frac{a^{3/4} c \sqrt{c \sqrt{a + b x^2}} \operatorname{ArcTanh} \left[\frac{(a + b x^2)^{1/4}}{a^{1/4}} \right]}{(a + b x^2)^{1/4}}$$

Problem 254: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x^3} dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\text{ArcTan}\left[\left(1+\frac{bx^2}{a}\right)^{1/4}\right]}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\text{ArcTanh}\left[\left(1+\frac{bx^2}{a}\right)^{1/4}\right]}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Result (type 3, 151 leaves, 7 steps):

$$-\frac{c\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{2x^2} + \frac{3bc\sqrt{c\sqrt{a+bx^2}}\text{ArcTan}\left[\frac{(a+bx^2)^{1/4}}{a^{1/4}}\right]}{4a^{1/4}(a+bx^2)^{1/4}} - \frac{3bc\sqrt{c\sqrt{a+bx^2}}\text{ArcTanh}\left[\frac{(a+bx^2)^{1/4}}{a^{1/4}}\right]}{4a^{1/4}(a+bx^2)^{1/4}}$$

Problem 255: Result valid but suboptimal antiderivative.

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b} + \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} - \frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b(a+bx^2)} + \frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}x}{\sqrt{a}}\right], 2\right]}{15b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Result (type 4, 191 leaves, 6 steps):

$$-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}} + \frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b} + \frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2} + \frac{4a^{5/2}c\sqrt{c\sqrt{a+bx^2}}\left(1+\frac{bx^2}{a}\right)^{1/4}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}x}{\sqrt{a}}\right], 2\right]}{15b^{3/2}\sqrt{a+bx^2}}$$

Problem 256: Result valid but suboptimal antiderivative.

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2} + \frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5(a+bx^2)} - \frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}\text{EllipticE}\left[\frac{1}{2}\text{ArcTan}\left[\frac{\sqrt{b}x}{\sqrt{a}}\right], 2\right]}{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Result (type 4, 146 leaves, 5 steps):

$$\frac{6 a c x \sqrt{c \sqrt{a+b x^2}}}{5 \sqrt{a+b x^2}} + \frac{2}{5} c x \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2} - \frac{6 a^{3/2} c \sqrt{c \sqrt{a+b x^2}} \left(1 + \frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{5 \sqrt{b} \sqrt{a+b x^2}}$$

Problem 257: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 115 leaves, 4 steps):

$$-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x} + \frac{3 b x \left(c \sqrt{a+b x^2}\right)^{3/2}}{a+b x^2} - \frac{3 \sqrt{b} \left(c \sqrt{a+b x^2}\right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{\sqrt{a} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 142 leaves, 5 steps):

$$\frac{3 b c x \sqrt{c \sqrt{a+b x^2}}}{\sqrt{a+b x^2}} - \frac{c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}}{x} - \frac{3 \sqrt{a} \sqrt{b} c \sqrt{c \sqrt{a+b x^2}} \left(1 + \frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{\sqrt{a+b x^2}}$$

Problem 258: Result valid but suboptimal antiderivative.

$$\int \frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 154 leaves, 5 steps):

$$-\frac{\left(c \sqrt{a+b x^2}\right)^{3/2}}{3 x^3} - \frac{b \left(c \sqrt{a+b x^2}\right)^{3/2}}{2 a x} + \frac{b^2 x \left(c \sqrt{a+b x^2}\right)^{3/2}}{2 a (a+b x^2)} - \frac{b^{3/2} \left(c \sqrt{a+b x^2}\right)^{3/2} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{2 a^{3/2} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 4, 193 leaves, 6 steps):

$$\frac{b^2 c x \sqrt{c \sqrt{a+b x^2}}}{2 a \sqrt{a+b x^2}} - \frac{c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}}{3 x^3} - \frac{b c \sqrt{c \sqrt{a+b x^2}} \sqrt{a+b x^2}}{2 a x} - \frac{b^{3/2} c \sqrt{c \sqrt{a+b x^2}} \left(1 + \frac{b x^2}{a}\right)^{1/4} \text{EllipticE}\left[\frac{1}{2} \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{2 \sqrt{a} \sqrt{a+b x^2}}$$

Problem 318: Result valid but suboptimal antiderivative.

$$\int x^5 \sqrt{a + \frac{b}{c + d x^2}} dx$$

Optimal (type 3, 216 leaves, 7 steps):

$$\begin{aligned} & - \frac{(b^2 + 4 a b c - 8 a^2 c^2) (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{16 a^2 d^3} - \frac{(b + 4 a c) (c + d x^2)^2 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{8 a d^3} + \\ & \frac{(c + d x^2)^3 \left(\frac{b + a c + a d x^2}{c + d x^2}\right)^{3/2}}{6 a d^3} + \frac{b (b^2 + 4 a b c + 8 a^2 c^2) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{a}}\right]}{16 a^{5/2} d^3} \end{aligned}$$

Result (type 3, 259 leaves, 9 steps):

$$\begin{aligned} & \frac{(b^2 + 4 a b c + 8 a^2 c^2) (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{16 a^2 d^3} - \frac{(3 b + 8 a c) (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}} (b + a (c + d x^2))}{24 a^2 d^3} + \\ & \frac{x^2 (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}} (b + a (c + d x^2))}{6 a d^2} + \frac{b (b^2 + 4 a b c + 8 a^2 c^2) \sqrt{c + d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c + d x^2}}{\sqrt{b + a (c + d x^2)}}\right]}{16 a^{5/2} d^3 \sqrt{b + a (c + d x^2)}} \end{aligned}$$

Problem 319: Result valid but suboptimal antiderivative.

$$\int x^3 \sqrt{a + \frac{b}{c + d x^2}} dx$$

Optimal (type 3, 141 leaves, 6 steps):

$$\begin{aligned} & \frac{(b - 4 a c) (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{8 a d^2} + \frac{(c + d x^2)^2 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{4 d^2} - \frac{b (b + 4 a c) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{a}}\right]}{8 a^{3/2} d^2} \end{aligned}$$

Result (type 3, 181 leaves, 8 steps):

$$- \frac{(b + 4ac)(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}}{8ad^2} + \frac{(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}(b + a(c + dx^2))}{4ad^2} - \frac{b(b + 4ac)\sqrt{c + dx^2}\sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{8a^{3/2}d^2\sqrt{b+a(c+dx^2)}}$$

Problem 321: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right] - \frac{\sqrt{b+ac} \operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{\sqrt{c}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{\sqrt{b+a(c+dx^2)}} - \frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{\sqrt{c}\sqrt{b+a(c+dx^2)}}$$

Problem 322: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$- \frac{(c + dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} + \frac{bd \operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2c^{3/2}\sqrt{b+ac}}$$

Result (type 3, 140 leaves, 6 steps):

$$- \frac{(c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{2 c x^2} + \frac{b d \sqrt{c + d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b + a c} \sqrt{c + d x^2}}{\sqrt{c} \sqrt{b + a (c + d x^2)}}\right]}{2 c^{3/2} \sqrt{b + a c} \sqrt{b + a (c + d x^2)}}$$

Problem 323: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c + d x^2}}}{x^5} dx$$

Optimal (type 3, 174 leaves, 6 steps):

$$\frac{(5 b + 4 a c) d (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{8 c^2 (b + a c) x^2} - \frac{(c + d x^2)^2 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{4 c^2 x^4} - \frac{b (3 b + 4 a c) d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{b + a c}}\right]}{8 c^{5/2} (b + a c)^{3/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(3 b + 4 a c) d (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{8 c^2 (b + a c) x^2} - \frac{(c + d x^2) \sqrt{a + \frac{b}{c + d x^2}} (b + a (c + d x^2))}{4 c (b + a c) x^4} - \frac{b (3 b + 4 a c) d^2 \sqrt{c + d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b + a c} \sqrt{c + d x^2}}{\sqrt{c} \sqrt{b + a (c + d x^2)}}\right]}{8 c^{5/2} (b + a c)^{3/2} \sqrt{b + a (c + d x^2)}}$$

Problem 324: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c + d x^2}}}{x^7} dx$$

Optimal (type 3, 265 leaves, 7 steps):

$$- \frac{(11 b^2 + 20 a b c + 8 a^2 c^2) d^2 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{16 c^3 (b + a c)^2 x^2} + \frac{(3 b + 4 a c) d (c + d x^2)^2 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{8 c^3 (b + a c) x^4} - \frac{(c + d x^2)^3 \left(\frac{b + a c + a d x^2}{c + d x^2}\right)^{3/2}}{6 c^2 (b + a c) x^6} + \frac{b (5 b^2 + 12 a b c + 8 a^2 c^2) d^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{\sqrt{b + a c}}\right]}{16 c^{7/2} (b + a c)^{5/2}}$$

Result (type 3, 271 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{(c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{6 c x^6} + \frac{(5 b + 4 a c) d (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{24 c^2 (b + a c) x^4} - \\
 & \frac{(5 b + 2 a c) (3 b + 4 a c) d^2 (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}}{48 c^3 (b + a c)^2 x^2} + \frac{b (5 b^2 + 12 a b c + 8 a^2 c^2) d^3 \sqrt{c + d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b + a c} \sqrt{c + d x^2}}{\sqrt{c} \sqrt{b + a (c + d x^2)}}\right]}{16 c^{7/2} (b + a c)^{5/2} \sqrt{b + a (c + d x^2)}}
 \end{aligned}$$

Problem 325: Result valid but suboptimal antiderivative.

$$\int x^4 \sqrt{a + \frac{b}{c + d x^2}} dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(2 b^2 + 7 a b c - 3 a^2 c^2) x \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{15 a^2 d^2} + \frac{(b - 3 a c) x (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{15 a d^2} + \frac{x^3 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{5 d} + \\
 & \frac{\sqrt{c} (2 b^2 + 7 a b c - 3 a^2 c^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{15 a^2 d^{5/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} - \frac{c^{3/2} (b - 3 a c) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{15 a d^{5/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}
 \end{aligned}$$

Result (type 4, 478 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(2b^2 + 7abc - 3a^2c^2)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a(c+dx^2)}} + \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a(c+dx^2)}} + \\
& \frac{x^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a(c+dx^2)}} + \frac{\sqrt{c}(2b^2 + 7abc - 3a^2c^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+dx^2)}} - \\
& \frac{c^{3/2}(b-3ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+dx^2)}}
\end{aligned}$$

Problem 326: Result valid but suboptimal antiderivative.

$$\int x^2 \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\begin{aligned}
& \frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad} + \frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \\
& \frac{\sqrt{c}(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3ad^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 370 leaves, 7 steps):

$$\frac{(b - a c) x \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}}}{3 a d \sqrt{b + a (c + d x^2)}} + \frac{x (c + d x^2) \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}}}{3 d \sqrt{b + a (c + d x^2)}} -$$

$$\frac{\sqrt{c} (b - a c) \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 a d^{3/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{b + a (c + d x^2)}} - \frac{c^{3/2} \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 d^{3/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{b + a (c + d x^2)}}$$

Problem 327: Result valid but suboptimal antiderivative.

$$\int \sqrt{a + \frac{b}{c + d x^2}} dx$$

Optimal (type 4, 213 leaves, 6 steps):

$$x \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} - \frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{\sqrt{d} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} + \frac{\sqrt{c} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{\sqrt{d} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}$$

Result (type 4, 279 leaves, 6 steps):

$$\frac{x \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}}}{\sqrt{b + a (c + d x^2)}} - \frac{\sqrt{c} \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{\sqrt{d} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{b + a (c + d x^2)}} +$$

$$\frac{\sqrt{c} \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{\sqrt{d} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{b + a (c + d x^2)}}$$

Problem 328: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^2} dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\frac{dx \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} -$$

$$\frac{\sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] + a\sqrt{c} \sqrt{d} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{c} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} + (b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 353 leaves, 8 steps):

$$\frac{dx \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{c \sqrt{b+ac+dx^2}} - \frac{(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{cx \sqrt{b+ac+dx^2}} -$$

$$\frac{\sqrt{d} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right] + a\sqrt{c} \sqrt{d} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{c} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac+dx^2} + (b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac+dx^2}}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} dx$$

Optimal (type 4, 362 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} + \\
& \frac{(2b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 472 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a(c+dx^2)}} + \frac{(2b+ac)d(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)x\sqrt{b+a(c+dx^2)}} + \\
& \frac{(2b+ac)d^{3/2}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+dx^2)}} - \frac{ad^{3/2}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+dx^2)}}
\end{aligned}$$

Problem 330: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\begin{aligned}
& \frac{(8b^2 + 13abc + 3a^2c^2)d^3x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2} - \frac{(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} + \\
& \frac{(4b+3ac)d(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3} - \frac{(8b^2 + 13abc + 3a^2c^2)d^2(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x} - \\
& \frac{(8b^2 + 13abc + 3a^2c^2)d^{5/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{5/2}(b+ac)^2 \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a(4b+3ac)d^{5/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{3/2}(b+ac)^2 \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 598 leaves, 9 steps):

$$\begin{aligned}
& \frac{(8b^2 + 13abc + 3a^2c^2)d^3x \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{15c^3(b+ac)^2 \sqrt{b+ac(c+dx^2)}} - \frac{(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{5cx^5 \sqrt{b+ac(c+dx^2)}} + \\
& \frac{(4b+3ac)d(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{15c^2(b+ac)x^3 \sqrt{b+ac(c+dx^2)}} - \frac{(8b^2 + 13abc + 3a^2c^2)d^2(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{15c^3(b+ac)^2x \sqrt{b+ac(c+dx^2)}} - \\
& \frac{(8b^2 + 13abc + 3a^2c^2)d^{5/2} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{5/2}(b+ac)^2 \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac(c+dx^2)}} + \\
& \frac{a(4b+3ac)d^{5/2} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15c^{3/2}(b+ac)^2 \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac(c+dx^2)}}
\end{aligned}$$

Problem 331: Result valid but suboptimal antiderivative.

$$\int x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$$

Optimal (type 3, 249 leaves, 8 steps):

$$-\frac{b c^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d^3} - \frac{(5 b^2 + 60 a b c - 24 a^2 c^2) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{48 a d^3} -$$

$$\frac{(b+12 a c) (c+d x^2)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 d^3} + \frac{(c+d x^2)^3 \left(\frac{b+a c+a d x^2}{c+d x^2}\right)^{5/2}}{6 a d^3} - \frac{b (b^2 + 12 a b c - 24 a^2 c^2) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{16 a^{3/2} d^3}$$

Result (type 3, 311 leaves, 10 steps):

$$-\frac{(b^2 + 12 a b c - 24 a^2 c^2) (c+d x^2) \sqrt{a + \frac{b}{c+d x^2}}}{16 a d^3} - \frac{(b^2 + 12 a b c - 24 a^2 c^2) (c+d x^2) \sqrt{a + \frac{b}{c+d x^2}} (b+a (c+d x^2))}{24 a b d^3} - \frac{c^2 \sqrt{a + \frac{b}{c+d x^2}} (b+a (c+d x^2))^2}{b d^3} +$$

$$\frac{(c+d x^2) \sqrt{a + \frac{b}{c+d x^2}} (b+a (c+d x^2))^2}{6 a d^3} - \frac{b (b^2 + 12 a b c - 24 a^2 c^2) \sqrt{c+d x^2} \sqrt{a + \frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a (c+d x^2)}}\right]}{16 a^{3/2} d^3 \sqrt{b+a (c+d x^2)}}$$

Problem 332: Result valid but suboptimal antiderivative.

$$\int x^3 \left(a + \frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\frac{b c \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d^2} + \frac{(5 b - 4 a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 d^2} + \frac{a (c+d x^2)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 d^2} + \frac{3 b (b - 4 a c) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{8 \sqrt{a} d^2}$$

Result (type 3, 222 leaves, 9 steps):

$$\frac{3 (b - 4 a c) (c+d x^2) \sqrt{a + \frac{b}{c+d x^2}}}{8 d^2} + \frac{(b - 4 a c) (c+d x^2) \sqrt{a + \frac{b}{c+d x^2}} (b+a (c+d x^2))}{4 b d^2} +$$

$$\frac{c \sqrt{a + \frac{b}{c+d x^2}} (b+a (c+d x^2))^2}{b d^2} + \frac{3 b (b - 4 a c) \sqrt{c+d x^2} \sqrt{a + \frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+d x^2}}{\sqrt{b+a (c+d x^2)}}\right]}{8 \sqrt{a} d^2 \sqrt{b+a (c+d x^2)}}$$

Problem 334: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x} dx$$

Optimal (type 3, 126 leaves, 7 steps):

$$\frac{b \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right] - \frac{(b+ac)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{c^{3/2}}$$

Result (type 3, 206 leaves, 10 steps):

$$\frac{b \sqrt{a + \frac{b}{c+dx^2}}}{c} + \frac{a^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{b+ac} \sqrt{c+dx^2}}\right]}{\sqrt{b+ac} (c+dx^2)} - \frac{(b+ac)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+ac} \sqrt{c+dx^2}}\right]}{c^{3/2} \sqrt{b+ac} (c+dx^2)}$$

Problem 335: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$$

Optimal (type 3, 138 leaves, 6 steps):

$$-\frac{3bd \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2} - \frac{(c+dx^2) \left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2} + \frac{3b \sqrt{b+ac} d \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2c^{5/2}}$$

Result (type 3, 170 leaves, 7 steps):

$$-\frac{3bd \sqrt{a + \frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+ac+dx^2)}{2cx^2} + \frac{3b \sqrt{b+ac} d \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+ac} \sqrt{c+dx^2}}\right]}{2c^{5/2} \sqrt{b+ac} (c+dx^2)}$$

Problem 336: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^5} dx$$

Optimal (type 3, 205 leaves, 7 steps):

$$\frac{b d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^3} + \frac{(9 b+4 a c) d (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 c^3 x^2} - \frac{(b+a c) (c+d x^2)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 c^3 x^4} - \frac{3 b (5 b+4 a c) d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{8 c^{7/2} \sqrt{b+a c}}$$

Result (type 3, 260 leaves, 8 steps):

$$\frac{3 b (5 b+4 a c) d^2 \sqrt{a+\frac{b}{c+d x^2}}}{8 c^3 (b+a c)} + \frac{(5 b+4 a c) d \sqrt{a+\frac{b}{c+d x^2}} (b+a (c+d x^2))}{8 c^2 (b+a c) x^2} - \frac{\sqrt{a+\frac{b}{c+d x^2}} (b+a (c+d x^2))^2}{4 c (b+a c) x^4} - \frac{3 b (5 b+4 a c) d^2 \sqrt{c+d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{-\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a (c+d x^2)}}\right]}{8 c^{7/2} \sqrt{b+a c} \sqrt{b+a (c+d x^2)}}$$

Problem 337: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^7} dx$$

Optimal (type 3, 292 leaves, 8 steps):

$$-\frac{b d^3 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^4} - \frac{(79 b^2+108 a b c+24 a^2 c^2) d^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{48 c^4 (b+a c) x^2} + \frac{(11 b+12 a c) d (c+d x^2)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{24 c^4 x^4} - \frac{(c+d x^2)^3 \left(\frac{b+a c+a d x^2}{c+d x^2}\right)^{5/2}}{6 c^2 (b+a c) x^6} + \frac{b (35 b^2+60 a b c+24 a^2 c^2) d^3 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{16 c^{9/2} (b+a c)^{3/2}}$$

Result (type 3, 287 leaves, 10 steps):

$$\begin{aligned}
& - \frac{(105 b^2 + 110 a b c + 8 a^2 c^2) d^3 \sqrt{a + \frac{b}{c+d x^2}}}{48 c^4 (b+a c)} - \frac{(b+a c) \sqrt{a + \frac{b}{c+d x^2}}}{6 c x^6} + \frac{7 b d \sqrt{a + \frac{b}{c+d x^2}}}{24 c^2 x^4} - \\
& \frac{b (35 b + 32 a c) d^2 \sqrt{a + \frac{b}{c+d x^2}}}{48 c^3 (b+a c) x^2} + \frac{b (35 b^2 + 60 a b c + 24 a^2 c^2) d^3 \sqrt{c+d x^2} \sqrt{a + \frac{b}{c+d x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a c} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a (c+d x^2)}}\right]}{16 c^{9/2} (b+a c)^{3/2} \sqrt{b+a (c+d x^2)}}
\end{aligned}$$

Problem 338: Result valid but suboptimal antiderivative.

$$\int x^4 \left(a + \frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$\begin{aligned}
& \frac{(b^2 - 14 a b c + a^2 c^2) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 a d^2} + \frac{(7 b - a c) x (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d^2} + \frac{6 a x^3 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d} - \frac{x^3 (b+a c+a d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d} - \\
& \frac{\sqrt{c} (b^2 - 14 a b c + a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} - \frac{c^{3/2} (7 b - a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 526 leaves, 9 steps):

$$\begin{aligned}
& \frac{(b^2 - 14abc + a^2c^2) x \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{5a^2 \sqrt{b+a(c+dx^2)}} + \frac{(7b-ac) x (c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{5d^2 \sqrt{b+a(c+dx^2)}} + \\
& \frac{6ax^3 (c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{5d \sqrt{b+a(c+dx^2)}} - \frac{x^3 (b+ac+adx^2)^{3/2} \sqrt{a + \frac{b}{c+dx^2}}}{d \sqrt{b+a(c+dx^2)}} - \\
& \frac{\sqrt{c} (b^2 - 14abc + a^2c^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5ad^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+a(c+dx^2)}} - \\
& \frac{c^{3/2} (7b-ac) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{5d^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+a(c+dx^2)}}
\end{aligned}$$

Problem 339: Result valid but suboptimal antiderivative.

$$\int x^2 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\begin{aligned}
& \frac{(7b-ac) x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} + \frac{4ax (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d} - \frac{x (b+ac+adx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} - \\
& \frac{\sqrt{c} (7b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c} (3b-ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 430 leaves, 8 steps):

$$\begin{aligned}
& \frac{(7b - ac) x \sqrt{b + ac + adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{3d \sqrt{b + a(c + dx^2)}} + \frac{4ax(c + dx^2) \sqrt{b + ac + adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{3d \sqrt{b + a(c + dx^2)}} - \\
& \frac{x(b + ac + adx^2)^{3/2} \sqrt{a + \frac{b}{c+dx^2}}}{d \sqrt{b + a(c + dx^2)}} - \frac{\sqrt{c}(7b - ac) \sqrt{b + ac + adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b + a(c + dx^2)}} + \\
& \frac{\sqrt{c}(3b - ac) \sqrt{b + ac + adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3d^{3/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b + a(c + dx^2)}}
\end{aligned}$$

Problem 340: Result valid but suboptimal antiderivative.

$$\int \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\begin{aligned}
& \frac{bx \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(b - ac) x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + \\
& \frac{(b - ac) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{c} \sqrt{d} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a \sqrt{c} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{\sqrt{d} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 348 leaves, 7 steps):

$$\frac{b x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c \sqrt{b+a(c+d x^2)}} - \frac{(b-a c) x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c \sqrt{b+a(c+d x^2)}} +$$

$$\frac{(b-a c) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{c} \sqrt{d} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}} \sqrt{b+a(c+d x^2)}} + \frac{a \sqrt{c} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{d} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}} \sqrt{b+a(c+d x^2)}}$$

Problem 341: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a+\frac{b}{c+d x^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 312 leaves, 8 steps):

$$\frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x} + \frac{(2 b+a c) d x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^2} - \frac{(2 b+a c)(c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^2 x} -$$

$$\frac{(2 b+a c) \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{c^{3/2} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}}} + \frac{a \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{c} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}}}$$

Result (type 4, 422 leaves, 8 steps):

$$\frac{b \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c x \sqrt{b+a(c+d x^2)}} + \frac{(2 b+a c) d x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c^2 \sqrt{b+a(c+d x^2)}} - \frac{(2 b+a c)(c+d x^2) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c^2 x \sqrt{b+a(c+d x^2)}} -$$

$$\frac{(2 b+a c) \sqrt{d} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{c^{3/2} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}} \sqrt{b+a(c+d x^2)}} + \frac{a \sqrt{d} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{c} \sqrt{\frac{c(b+a c+a d x^2)}{(b+a c)(c+d x^2)}} \sqrt{b+a(c+d x^2)}}$$

Problem 342: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 388 leaves, 9 steps):

$$\frac{b \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3} - \frac{(8b+ac)d^2x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3} - \frac{(4b+ac)(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3} + \frac{(8b+ac)d(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x} +$$

$$\frac{(8b+ac)d^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{a(4b+ac)d^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{3/2}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 520 leaves, 9 steps):

$$\frac{b \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{cx^3 \sqrt{b+ac(c+dx^2)}} - \frac{(8b+ac)d^2x \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{3c^3 \sqrt{b+ac(c+dx^2)}} - \frac{(4b+ac)(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{3c^2x^3 \sqrt{b+ac(c+dx^2)}} +$$

$$\frac{(8b+ac)d(c+dx^2) \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}}}{3c^3x \sqrt{b+ac(c+dx^2)}} + \frac{(8b+ac)d^{3/2} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{5/2} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac(c+dx^2)}} -$$

$$\frac{a(4b+ac)d^{3/2} \sqrt{b+ac+adx^2} \sqrt{a + \frac{b}{c+dx^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3c^{3/2}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{b+ac(c+dx^2)}}$$

Problem 343: Result valid but suboptimal antiderivative.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^6} dx$$

Optimal (type 4, 494 leaves, 10 steps):

$$\begin{aligned}
& \frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^5} + \frac{(16 b^2+16 a b c+a^2 c^2) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c)} - \frac{(6 b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^2 x^5} + \\
& \frac{(8 b+a c) d (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^3 x^3} - \frac{(16 b^2+16 a b c+a^2 c^2) d^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c) x} - \\
& \frac{(16 b^2+16 a b c+a^2 c^2) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 c^{7/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{a (8 b+a c) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 c^{5/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 648 leaves, 10 steps):

$$\begin{aligned}
& \frac{b \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{c x^5 \sqrt{b+a (c+d x^2)}} + \frac{(16 b^2+16 a b c+a^2 c^2) d^3 x \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c^4 (b+a c) \sqrt{b+a (c+d x^2)}} - \frac{(6 b+a c) (c+d x^2) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c^2 x^5 \sqrt{b+a (c+d x^2)}} + \\
& \frac{(8 b+a c) d (c+d x^2) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c^3 x^3 \sqrt{b+a (c+d x^2)}} - \frac{(16 b^2+16 a b c+a^2 c^2) d^2 (c+d x^2) \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}}}{5 c^4 (b+a c) x \sqrt{b+a (c+d x^2)}} - \\
& \frac{(16 b^2+16 a b c+a^2 c^2) d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 c^{7/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}} \sqrt{b+a (c+d x^2)}} + \\
& \frac{a (8 b+a c) d^{5/2} \sqrt{b+a c+a d x^2} \sqrt{a+\frac{b}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 c^{5/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}} \sqrt{b+a (c+d x^2)}}
\end{aligned}$$

Problem 344: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 225 leaves, 7 steps):

$$\frac{(5b^2 + 12abc + 8a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3} - \frac{(5b+8ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} +$$

$$\frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{b(5b^2 + 12abc + 8a^2c^2)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{16a^{7/2}d^3}$$

Result (type 3, 267 leaves, 9 steps):

$$\frac{(5b^2 + 12abc + 8a^2c^2)(b+a(c+dx^2))}{16a^3d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2d^3\sqrt{a + \frac{b}{c+dx^2}}} +$$

$$\frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2\sqrt{a + \frac{b}{c+dx^2}}} - \frac{b(5b^2 + 12abc + 8a^2c^2)\sqrt{b+a(c+dx^2)}\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{16a^{7/2}d^3\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}}}$$

Problem 345: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 148 leaves, 6 steps):

$$-\frac{(3b+4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2} + \frac{b(3b+4ac)\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{8a^{5/2}d^2}$$

Result (type 3, 189 leaves, 8 steps):

$$- \frac{(3b + 4ac)(b + a(c + dx^2))}{8a^2d^2\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c + dx^2)(b + a(c + dx^2))}{4ad^2\sqrt{a + \frac{b}{c+dx^2}}} + \frac{b(3b + 4ac)\sqrt{b + a(c + dx^2)} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{8a^{5/2}d^2\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}}}$$

Problem 347: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{\sqrt{a}} - \frac{\sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{\sqrt{b+ac}}$$

Result (type 3, 184 leaves, 9 steps):

$$\frac{\sqrt{b+ac+dx^2} \operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+ac+dx^2} \operatorname{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}}}$$

Problem 348: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 108 leaves, 5 steps):

$$- \frac{(c + dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2} - \frac{bd \operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2\sqrt{c}(b+ac)^{3/2}}$$

Result (type 3, 148 leaves, 6 steps):

$$-\frac{b+a(c+dx^2)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{bd\sqrt{b+a(c+dx^2)}\operatorname{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{2\sqrt{c}(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 349: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 3, 177 leaves, 6 steps):

$$\frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4}+\frac{b(b+4ac)d^2\operatorname{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{8c^{3/2}(b+ac)^{5/2}}$$

Result (type 3, 218 leaves, 7 steps):

$$\frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}}-\frac{(c+dx^2)(b+a(c+dx^2))}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}}+\frac{b(b+4ac)d^2\sqrt{b+a(c+dx^2)}\operatorname{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right]}{8c^{3/2}(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 350: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 443 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(4b + 3ac)x(b + ac + adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{x^3(b + ac + adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(8b^2 + 13abc + 3a^2c^2)x(b + ac + adx^2)}{15a^3d^2(c + dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \\
& \frac{\sqrt{c}(8b^2 + 13abc + 3a^2c^2)(b + ac + adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^3d^{5/2}(c + dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(4b + 3ac)(b + ac + adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^2d^{5/2}(c + dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 498 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(4b + 3ac)x\sqrt{b + ac + adx^2}\sqrt{b + a(c + dx^2)}}{15a^2d^2\sqrt{a + \frac{b}{c+dx^2}}} + \\
& \frac{x^3\sqrt{b + ac + adx^2}\sqrt{b + a(c + dx^2)}}{5ad\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(8b^2 + 13abc + 3a^2c^2)x\sqrt{b + ac + adx^2}\sqrt{b + a(c + dx^2)}}{15a^3d^2(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}} - \\
& \frac{\sqrt{c}(8b^2 + 13abc + 3a^2c^2)\sqrt{b + ac + adx^2}\sqrt{b + a(c + dx^2)}\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^3d^{5/2}(c + dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a + \frac{b}{c+dx^2}}} + \\
& \frac{c^{3/2}(4b + 3ac)\sqrt{b + ac + adx^2}\sqrt{b + a(c + dx^2)}\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{15a^2d^{5/2}(c + dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 351: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 354 leaves, 7 steps):

$$\frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(2b+ac)x(b+ac+adx^2)}{3a^2d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} +$$

$$\frac{\sqrt{c}(2b+ac)(b+ac+adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3a^2d^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{c^{3/2}(b+ac+adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3ad^{3/2}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 398 leaves, 7 steps):

$$\frac{x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{3a^2d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$\frac{\sqrt{c}(2b+ac)\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3a^2d^{3/2}(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}} -$$

$$\frac{c^{3/2}\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3ad^{3/2}(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 352: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 286 leaves, 6 steps):

$$\frac{x(b+ac+adx^2)}{a(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(b+ac+adx^2)\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{a\sqrt{d}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} +$$

$$\frac{c^{3/2}(b+ac+adx^2)\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{(b+ac)\sqrt{d}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

Result (type 4, 319 leaves, 6 steps):

$$\frac{x \sqrt{b+a c+a d x^2} \sqrt{b+a (c+d x^2)}}{a (c+d x^2) \sqrt{a+\frac{b}{c+d x^2}}} - \frac{\sqrt{c} \sqrt{b+a c+a d x^2} \sqrt{b+a (c+d x^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{a \sqrt{d} (c+d x^2) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}} \sqrt{a+\frac{b}{c+d x^2}}} +$$

$$\frac{c^{3/2} \sqrt{b+a c+a d x^2} \sqrt{b+a (c+d x^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \sqrt{d} (c+d x^2) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}} \sqrt{a+\frac{b}{c+d x^2}}}$$

Problem 353: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \sqrt{a+\frac{b}{c+d x^2}}} dx$$

Optimal (type 4, 343 leaves, 8 steps):

$$-\frac{b+a c+a d x^2}{(b+a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{d x (b+a c+a d x^2)}{(b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} -$$

$$\frac{\sqrt{c} \sqrt{d} (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{\sqrt{c} \sqrt{d} (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}$$

Result (type 4, 387 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{(b+ac) x \sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{(b+ac)(c+dx^2) \sqrt{a+\frac{b}{c+dx^2}}} - \\
& \frac{\sqrt{c} \sqrt{d} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{(b+ac)(c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{\sqrt{c} \sqrt{d} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{(b+ac)(c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 354: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^4 \sqrt{a+\frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 431 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b+ac+adx^2}{3(b+ac)x^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)d^2 x(b+ac+adx^2)}{3c(b+ac)^2 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \\
& \frac{(b-ac)d^{3/2}(b+ac+adx^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3\sqrt{c}(b+ac)^2 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} - \frac{a\sqrt{c}d^{3/2}(b+ac+adx^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^2 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 486 leaves, 8 steps):

$$\begin{aligned}
& - \frac{\sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3(b+ac)x^3 \sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3c(b+ac)^2 x \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{(b-ac)d^2 x \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3c(b+ac)^2 (c+dx^2) \sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d^{3/2} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3\sqrt{c}(b+ac)^2 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}} - \\
& \frac{a\sqrt{c}d^{3/2} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^2 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 355: Result valid but suboptimal antiderivative.

$$\int \frac{x^5}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 3, 310 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(b+ac)^2 (c+dx^2)^3}{ab^2 d^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(35b^2 + 60abc + 24a^2 c^2) (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4 d^3} - \frac{(35b^2 + 60abc + 24a^2 c^2) (c+dx^2)^2 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3 b d^3} + \\
& \frac{(7b^2 + 12abc + 6a^2 c^2) (c+dx^2)^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2 b^2 d^3} - \frac{b(35b^2 + 60abc + 24a^2 c^2) \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{16a^{9/2} d^3}
\end{aligned}$$

Result (type 3, 323 leaves, 10 steps):

$$\frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(b+a(c+dx^2))}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)(b+a(c+dx^2))}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$\frac{(c+dx^2)^2(b+a(c+dx^2))}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{b(35b^2+60abc+24a^2c^2)\sqrt{b+a(c+dx^2)}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{16a^{9/2}d^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 356: Result valid but suboptimal antiderivative.

$$\int \frac{x^3}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 3, 187 leaves, 7 steps):

$$-\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b+4ac)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} + \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{3b(5b+4ac)\operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{8a^{7/2}d^2}$$

Result (type 3, 242 leaves, 9 steps):

$$-\frac{(b+ac)(c+dx^2)^2}{ab^2d^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)(b+a(c+dx^2))}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$\frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3b(5b+4ac)\sqrt{b+a(c+dx^2)}\operatorname{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right]}{8a^{7/2}d^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 358: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{c^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{(b+ac)^{3/2}}$$

Result (type 3, 214 leaves, 10 steps):

$$-\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+ac(c+dx^2)} \text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+ac(c+dx^2)}}\right]}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+ac(c+dx^2)} \text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+ac(c+dx^2)}}\right]}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 359: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 3, 146 leaves, 6 steps):

$$\frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{3b\sqrt{c}d \text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right]}{2(b+ac)^{5/2}}$$

Result (type 3, 174 leaves, 7 steps):

$$\frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}d\sqrt{b+ac(c+dx^2)} \text{ArcTanh}\left[\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+ac(c+dx^2)}}\right]}{2(b+ac)^{5/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Problem 360: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 3, 212 leaves, 7 steps):

$$-\frac{a b d^2}{(b+a c)^3 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(3 b-4 a c) d (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{8 (b+a c)^3 x^2} - \frac{(c+d x^2)^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{4 (b+a c)^2 x^4} - \frac{3 b (b-4 a c) d^2 \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{8 \sqrt{c} (b+a c)^{7/2}}$$

Result (type 3, 246 leaves, 8 steps):

$$\frac{3 b (b-4 a c) d^2}{8 c (b+a c)^3 \sqrt{a + \frac{b}{c+d x^2}}} - \frac{(b-4 a c) d (c+d x^2)}{8 c (b+a c)^2 x^2 \sqrt{a + \frac{b}{c+d x^2}}} - \frac{(c+d x^2)^2}{4 c (b+a c) x^4 \sqrt{a + \frac{b}{c+d x^2}}} - \frac{3 b (b-4 a c) d^2 \sqrt{b+a} \sqrt{c+d x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{b+a} \sqrt{c+d x^2}}{\sqrt{c} \sqrt{b+a} (c+d x^2)}\right]}{8 \sqrt{c} (b+a c)^{7/2} \sqrt{c+d x^2} \sqrt{a + \frac{b}{c+d x^2}}}$$

Problem 361: Result valid but suboptimal antiderivative.

$$\int \frac{x^4}{\left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 482 leaves, 9 steps):

$$-\frac{x^3 (c+d x^2)}{a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(8 b+a c) x (b+a c+a d x^2)}{5 a^3 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{6 x^3 (b+a c+a d x^2)}{5 a^2 d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(16 b^2+16 a b c+a^2 c^2) x (b+a c+a d x^2)}{5 a^4 d^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{\sqrt{c} (16 b^2+16 a b c+a^2 c^2) (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a^4 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{c^{3/2} (8 b+a c) (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{5 a^3 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}$$

Result (type 4, 559 leaves, 9 steps):

$$\begin{aligned}
& - \frac{x^3 (c + d x^2) \sqrt{b + a (c + d x^2)}}{a d \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}}} - \frac{(8 b + a c) x \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)}}{5 a^3 d^2 \sqrt{a + \frac{b}{c + d x^2}}} + \\
& \frac{6 x^3 \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)}}{5 a^2 d \sqrt{a + \frac{b}{c + d x^2}}} + \frac{(16 b^2 + 16 a b c + a^2 c^2) x \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)}}{5 a^4 d^2 (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}} - \\
& \frac{\sqrt{c} (16 b^2 + 16 a b c + a^2 c^2) \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{5 a^4 d^{5/2} (c + d x^2) \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{a + \frac{b}{c + d x^2}}} + \\
& \frac{c^{3/2} (8 b + a c) \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{5 a^3 d^{5/2} (c + d x^2) \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{a + \frac{b}{c + d x^2}}}
\end{aligned}$$

Problem 362: Result valid but suboptimal antiderivative.

$$\int \frac{x^2}{\left(a + \frac{b}{c + d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 409 leaves, 8 steps):

$$\begin{aligned}
& - \frac{x (c + d x^2)}{a d \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{4 x (b + a c + a d x^2)}{3 a^2 d \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \frac{(8 b + a c) x (b + a c + a d x^2)}{3 a^3 d (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \\
& \frac{\sqrt{c} (8 b + a c) (b + a c + a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 a^3 d^{3/2} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} - \frac{c^{3/2} (4 b + a c) (b + a c + a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 a^2 (b + a c) d^{3/2} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 475 leaves, 8 steps):

$$\begin{aligned}
& - \frac{x (c + d x^2) \sqrt{b + a (c + d x^2)}}{a d \sqrt{b + a c + a d x^2} \sqrt{a + \frac{b}{c + d x^2}}} + \frac{4 x \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)}}{3 a^2 d \sqrt{a + \frac{b}{c + d x^2}}} - \\
& \frac{(8 b + a c) x \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)}}{3 a^3 d (c + d x^2) \sqrt{a + \frac{b}{c + d x^2}}} + \frac{\sqrt{c} (8 b + a c) \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 a^3 d^{3/2} (c + d x^2) \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{a + \frac{b}{c + d x^2}}} - \\
& \frac{c^{3/2} (4 b + a c) \sqrt{b + a c + a d x^2} \sqrt{b + a (c + d x^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{3 a^2 (b + a c) d^{3/2} (c + d x^2) \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}} \sqrt{a + \frac{b}{c + d x^2}}}
\end{aligned}$$

Problem 363: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(a + \frac{b}{c + d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$\begin{aligned}
& - \frac{b x}{a (b + a c) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{(2 b + a c) x (b + a c + a d x^2)}{a^2 (b + a c) (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \\
& \frac{\sqrt{c} (2 b + a c) (b + a c + a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{a^2 (b + a c) \sqrt{d} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} + \frac{c^{3/2} (b + a c + a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}\right]}{a (b + a c) \sqrt{d} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 411 leaves, 7 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{b+a(c+dx^2)}}{a(b+ac) \sqrt{b+ac+adx^2} \sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{a^2(b+ac)(c+dx^2) \sqrt{a+\frac{b}{c+dx^2}}} - \\
& \frac{\sqrt{c}(2b+ac) \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{a^2(b+ac) \sqrt{d}(c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{c^{3/2} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{a(b+ac) \sqrt{d}(c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 364: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^2 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 410 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b}{a(b+ac)x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(b-ac)(b+ac+adx^2)}{a(b+ac)^2 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \\
& \frac{\sqrt{c}(b-ac) \sqrt{d}(b+ac+adx^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{a(b+ac)^2(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2} \sqrt{d}(b+ac+adx^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{(b+ac)^2(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 476 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b \sqrt{b+a(c+dx^2)}}{a(b+ac)x \sqrt{b+ac+adx^2} \sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac) \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{a(b+ac)^2 x \sqrt{a+\frac{b}{c+dx^2}}} - \\
& \frac{(b-ac) dx \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{a(b+ac)^2 (c+dx^2) \sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{c} (b-ac) \sqrt{d} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{a(b+ac)^2 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{c^{3/2} \sqrt{d} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{(b+ac)^2 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 365: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 490 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b}{a(b+ac)x^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(3b-ac)(b+ac+adx^2)}{3a(b+ac)^2 x^3 \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(7b-ac)d^2 x(b+ac+adx^2)}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \\
& \frac{\sqrt{c} (7b-ac) d^{3/2} (b+ac+adx^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c} (3b-ac) d^{3/2} (b+ac+adx^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 567 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b \sqrt{b+a(c+dx^2)}}{a(b+ac)x^3 \sqrt{b+ac+adx^2} \sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac) \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3a(b+ac)^2 x^3 \sqrt{a+\frac{b}{c+dx^2}}} - \frac{(7b-ac)d \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3(b+ac)^3 x \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{(7b-ac)d^2 x \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)}}{3(b+ac)^3 (c+dx^2) \sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c} (7b-ac) d^{3/2} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}} + \\
& \frac{\sqrt{c} (3b-ac) d^{3/2} \sqrt{b+ac+adx^2} \sqrt{b+a(c+dx^2)} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}\right]}{3(b+ac)^3 (c+dx^2) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \sqrt{a+\frac{b}{c+dx^2}}}
\end{aligned}$$

Problem 396: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n} \sqrt{ax^{2n}}}{(2+n) \sqrt{1+x^n}} \right) dx$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2x^{1-n} \sqrt{ax^{2n}} \sqrt{1+x^n}}{2+n}$$

Result (type 5, 80 leaves, 5 steps):

$$\frac{x \sqrt{ax^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, 1 + \frac{1}{n}, 2 + \frac{1}{n}, -x^n\right]}{1+n} + \frac{2x^{1-n} \sqrt{ax^{2n}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{n}, 1 + \frac{1}{n}, -x^n\right]}{2+n}$$

Problem 616: Unable to integrate problem.

$$\int \frac{1}{x^2} (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad + (b dm + a en) x + (cd + be + af + 2cdm + bem + ben + 2afn) x^2 + (2ce + 2bf + 2ag + 2cem + bfm + cen + 2bf n + 3agn) x^3 + (3cf + 3bg + 2cfm + bgm + 2cf n + 3bgn) x^4 + cg(4+2m+3n) x^5) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a+bx+cx^2)^{1+m} (d+ex+fx^2+gx^3)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$\begin{aligned} & (c(d+2dm) + be(1+m+n) + af(1+2n)) \text{ CannotIntegrate}[(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] - \\ & ad \text{ CannotIntegrate}\left[\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n}{x^2}, x\right] + (bdm + aen) \text{ CannotIntegrate}\left[\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n}{x}, x\right] + \\ & (ce(2+2m+n) + bf(2+m+2n) + ag(2+3n)) \text{ CannotIntegrate}[x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] + \\ & (cf(3+2m+2n) + bg(3+m+3n)) \text{ CannotIntegrate}[x^2(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] + \\ & cg(4+2m+3n) \text{ CannotIntegrate}[x^3(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] \end{aligned}$$

Problem 617: Unable to integrate problem.

$$\begin{aligned} & \int \frac{1}{x^3} (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad + (-bd - ae + bdm + aen)x + (2cdm + bem + ben + 2afn)x^2 + \\ & (ce + bf + ag + 2cem + bfm + c en + 2bf n + 3agn)x^3 + (2cf + 2bg + 2cfm + bgm + 2cf n + 3bg n)x^4 + cg(3+2m+3n)x^5) dx \end{aligned}$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a+bx+cx^2)^{1+m} (d+ex+fx^2+gx^3)^{1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$\begin{aligned} & (ce(1+2m+n) + bf(1+m+2n) + ag(1+3n)) \text{ CannotIntegrate}[(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] - \\ & 2ad \text{ CannotIntegrate}\left[\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n}{x^3}, x\right] - \\ & (bd(1-m) + ae(1-n)) \text{ CannotIntegrate}\left[\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n}{x^2}, x\right] + \\ & (2cdm + 2afn + be(m+n)) \text{ CannotIntegrate}\left[\frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n}{x}, x\right] + \\ & (2cf(1+m+n) + bg(2+m+3n)) \text{ CannotIntegrate}[x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] + \\ & cg(3+2m+3n) \text{ CannotIntegrate}[x^2(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n, x] \end{aligned}$$

Problem 852: Result valid but suboptimal antiderivative.

$$\int \sqrt{\frac{1}{-1+x^2}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \operatorname{ArcSin}[x]$$

Result (type 3, 33 leaves, 3 steps):

$$\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 853: Result valid but suboptimal antiderivative.

$$\int \sqrt{\frac{1+x^2}{-1+x^4}} dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \operatorname{ArcSin}[x]$$

Result (type 3, 33 leaves, 4 steps):

$$\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]$$

Problem 941: Result unnecessarily involves higher level functions.

$$\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{(1-x^6)^{2/3}}{5x^5} + \frac{1}{5}x(1-x^6)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, -\frac{2}{3}, \frac{1}{6}, x^6\right]}{5x^5} + x \operatorname{Hypergeometric2F1}\left[-\frac{2}{3}, \frac{1}{6}, \frac{7}{6}, x^6\right]$$

Problem 996: Unable to integrate problem.

$$\int \frac{\sqrt{-x + \sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}} dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left(\sqrt{x} + 3 \sqrt{1+x} \right) \sqrt{-x + \sqrt{x}} \sqrt{1+x} - \frac{3 \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2 \sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\sqrt{-x + \sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}}, x\right]$$

Problem 997: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{-2+\sqrt{5}}(x+\sqrt{1+x^2})\right] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{2+\sqrt{5}}(x+\sqrt{1+x^2})\right]$$

Result (type 3, 319 leaves, 25 steps):

$$\begin{aligned} & -2 \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}} x\right] - \sqrt{\frac{1}{10}(1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}} x\right] - \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right] - \\ & \sqrt{\frac{2}{5}(-1+\sqrt{5})} \operatorname{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} \sqrt{1+x^2}\right] - 2 \sqrt{\frac{2}{5(-1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} x\right] + \\ & \sqrt{\frac{1}{10}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}} x\right] - \sqrt{\frac{2}{5(1+\sqrt{5})}} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2}\right] + \sqrt{\frac{2}{5}(1+\sqrt{5})} \operatorname{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{1+x^2}\right] \end{aligned}$$

Problem 1016: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x (3 + 3x + x^2) (3 + 3x + 3x^2 + x^3)^{1/3}} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{1 + \frac{2 \cdot 3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}}}{\sqrt{3}}\right]}{3^{5/6}} - \frac{\text{Log}\left[1 - (1+x)^3\right]}{6 \times 3^{1/3}} + \frac{\text{Log}\left[3^{1/3} (1+x) - (2 + (1+x)^3)^{1/3}\right]}{2 \times 3^{1/3}}$$

Result (type 3, 123 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1+x)}{3^{1/6} (2+(1+x)^3)^{1/3}}\right]}{3^{5/6}} + \frac{\text{Log}\left[1 - \frac{3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}}\right]}{3 \times 3^{1/3}} - \frac{\text{Log}\left[1 + \frac{3^{2/3} (1+x)^2}{(2+(1+x)^3)^{2/3}} + \frac{3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}}\right]}{6 \times 3^{1/3}}$$

Problem 1017: Unable to integrate problem.

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1 - \frac{2 \cdot 2^{1/3} (1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} - \frac{\text{Log}\left[1 + 2(1-x)^3 - x^3\right]}{2 \times 2^{2/3}} + \frac{3 \text{Log}\left[2^{1/3} (1-x) + (1-x^3)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 8, 103 leaves, 5 steps):

$$-\left(1 + i\sqrt{3}\right) \text{CannotIntegrate}\left[\frac{1}{(-1 - i\sqrt{3} + 2x) (1 - x^3)^{2/3}}, x\right] - \\ (1 - i\sqrt{3}) \text{CannotIntegrate}\left[\frac{1}{(-1 + i\sqrt{3} + 2x) (1 - x^3)^{2/3}}, x\right] - x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, x^3\right]$$

Problem 1018: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{-1 + x^4} (1 + x^4)} dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTan}\left[\frac{1+x^2}{x\sqrt{-1+x^4}}\right] - \frac{1}{4} \operatorname{ArcTanh}\left[\frac{1-x^2}{x\sqrt{-1+x^4}}\right]$$

Result (type 3, 47 leaves, 9 steps):

$$\left(-\frac{1}{8} - \frac{i}{8}\right) \operatorname{ArcTan}\left[\frac{(1+i)x}{\sqrt{-1+x^4}}\right] + \left(\frac{1}{8} + \frac{i}{8}\right) \operatorname{ArcTanh}\left[\frac{(1+i)x}{\sqrt{-1+x^4}}\right]$$