Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "0 Independent test suites"

Test results for the 175 problems in "Apostol Problems.m"

Test results for the 35 problems in "Bondarenko Problems.m"

Problem 7: Unable to integrate problem.

$$\int \frac{\text{Log}[1+x]}{x\sqrt{1+\sqrt{1+x}}}\,dx$$

Optimal (type 4, 291 leaves, ? steps):

$$-8\,\text{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]-\frac{2\,\text{Log}[1+x]}{\sqrt{1+\sqrt{1+x}}}-\sqrt{2}\,\text{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\text{Log}[1+x]+$$
$$2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-2\sqrt{2}\,\text{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\text{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+$$
$$\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-$$
$$\sqrt{2}\,\text{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+\sqrt{2}\,\text{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\text{Log}[1+x]}{x\sqrt{1+\sqrt{1+x}}},x\right]$$

Problem 8: Unable to integrate problem.

$$\int \frac{\sqrt{1+\sqrt{1+x}}\,\text{Log}[1+x]}{x}\,dx$$

Optimal (type 4, 308 leaves, ? steps):

$$-16\sqrt{1+\sqrt{1+x}}+16\,\mathrm{ArcTanh}\left[\sqrt{1+\sqrt{1+x}}\right]+4\sqrt{1+\sqrt{1+x}}\,\mathrm{Log}[1+x]-2\sqrt{2}\,\mathrm{ArcTanh}\left[\frac{\sqrt{1+\sqrt{1+x}}}{\sqrt{2}}\right]\mathrm{Log}[1+x]+4\sqrt{2}\,\mathrm{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\mathrm{Log}\left[1-\sqrt{1+\sqrt{1+x}}\right]-4\sqrt{2}\,\mathrm{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]\mathrm{Log}\left[1+\sqrt{1+\sqrt{1+x}}\right]+2\sqrt{2}\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]-2\sqrt{2}\,\mathrm{PolyLog}\left[2,\frac{\sqrt{2}\left(1-\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]-2\sqrt{2}\,\mathrm{PolyLog}\left[2,-\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2-\sqrt{2}}\right]+2\sqrt{2}\,\mathrm{PolyLog}\left[2,\frac{\sqrt{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)}{2+\sqrt{2}}\right]$$

Result (type 8, 23 leaves, 0 steps):

$$\mathrm{Unintegrable}\left[\frac{\sqrt{1+\sqrt{1+x}}\,\mathrm{Log}[1+x]}{x},x\right]$$

Problem 21: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(\mathrm{Cos}[x]+\mathrm{Cos}[3x])^5}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\frac{523}{256}\mathrm{ArcTanh}[\mathrm{Sin}[x]]+\frac{1483\,\mathrm{ArcTanh}\left[\sqrt{2}\,\mathrm{Sin}[x]\right]}{512\sqrt{2}}+\frac{\mathrm{Sin}[x]}{32\left(1-2\,\mathrm{Sin}[x]^2\right)^4}-\frac{17\,\mathrm{Sin}[x]}{192\left(1-2\,\mathrm{Sin}[x]^2\right)^3}+\frac{203\,\mathrm{Sin}[x]}{768\left(1-2\,\mathrm{Sin}[x]^2\right)^2}-\frac{437\,\mathrm{Sin}[x]}{512\left(1-2\,\mathrm{Sin}[x]^2\right)}-\frac{43}{256}\mathrm{Sec}[x]\,\mathrm{Tan}[x]-\frac{1}{128}\mathrm{Sec}[x]^3\,\mathrm{Tan}[x]$$

Result (type 3, 786 leaves, 45 steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]-\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-$$

$$\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$

$$\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$

$$\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-\frac{1}{128\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^4}+$$

$$\frac{1}{64\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^3}-\frac{47}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^2}+\frac{45}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}+\frac{1}{128\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^4}-$$

$$\frac{1}{64\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^3}+\frac{47}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^2}-\frac{45}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}-\frac{7-17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+$$

$$\frac{119\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{11\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{1-43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}-$$

$$\frac{65\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{451\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-$$

$$\frac{89+15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}+\frac{7+17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{11\left(1-3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-$$

$$\frac{119\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}+\frac{65\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+$$

$$\frac{1+43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{89-15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-\frac{451\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Tanh}[x]}{\sqrt{e^x+e^{2x}}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2\,e^{-x}\sqrt{e^x+e^{2x}}-\frac{\text{ArcTan}\left[\frac{i-(1-2i)\,e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1+i}}+\frac{\text{ArcTan}\left[\frac{i+(1+2i)\,e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1-i}}$$

Result (type 3, 147 leaves, 11 steps):

$$\frac{2\left(1+e^{x}\right)}{\sqrt{e^{x}+e^{2x}}}-\frac{(1-i)^{3/2}\sqrt{e^{x}}\sqrt{1+e^{x}}\,\mathrm{ArcTanh}\left[\frac{\sqrt{1-i}\sqrt{e^{x}}}{\sqrt{1+e^{x}}}\right]}{\sqrt{e^{x}+e^{2x}}}-\frac{(1+i)^{3/2}\sqrt{e^{x}}\sqrt{1+e^{x}}\,\mathrm{ArcTanh}\left[\frac{\sqrt{1+i}\sqrt{e^{x}}}{\sqrt{1+e^{x}}}\right]}{\sqrt{e^{x}+e^{2x}}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \mathrm{Log}\left[x^2+\sqrt{1-x^2}\right]dx$$

Optimal (type 3, 185 leaves, ? steps):

$$-2x-\mathrm{ArcSin}[x]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+x\,\mathrm{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Result (type 3, 349 leaves, 31 steps):

$$-2\,x-\text{ArcSin}[x]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\;\text{ArcTan}\Big[\sqrt{\frac{2}{1+\sqrt{5}}}\;x\Big]+$$

$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\;\text{ArcTan}\Big[\sqrt{\frac{2}{1+\sqrt{5}}}\;x\Big]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\;\text{ArcTan}\Big[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+$$

$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\;\text{ArcTan}\Big[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\;\text{ArcTanh}\Big[\sqrt{\frac{2}{-1+\sqrt{5}}}\;x\Big]+$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\;\text{ArcTanh}\Big[\sqrt{\frac{2}{-1+\sqrt{5}}}\;x\Big]-2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\;\text{ArcTanh}\Big[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]-$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\;\text{ArcTanh}\Big[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\;x}{\sqrt{1-x^2}}\Big]+x\,\text{Log}\Big[x^2+\sqrt{1-x^2}\Big]$$

Test results for the 14 problems in "Bronstein Problems.m"

Problem 12: Unable to integrate problem.

$$\int\frac{x^2+2\,x\,\text{Log}[x]+\text{Log}[x]^2+(1+x)\,\sqrt{x+\text{Log}[x]}}{x^3+2\,x^2\,\text{Log}[x]+x\,\text{Log}[x]^2}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\text{Log}[x]-\frac{2}{\sqrt{x+\text{Log}[x]}}$$

Result (type 8, 65 leaves, 3 steps):

$$\text{CannotIntegrate}\Big[\frac{1}{(x+\text{Log}[x])^{3/2}},x\Big]-\text{CannotIntegrate}\Big[\frac{1}{\text{Log}[x]\,(x+\text{Log}[x])^{3/2}},x\Big]-$$

$$\text{CannotIntegrate}\Big[\frac{1}{\text{Log}[x]^2\,\sqrt{x+\text{Log}[x]}},x\Big]+\text{CannotIntegrate}\Big[\frac{\sqrt{x+\text{Log}[x]}}{x\,\text{Log}[x]^2},x\Big]+\text{Log}[x]$$

Test results for the 50 problems in "Charlwood Problems.m"

Problem 3: Unable to integrate problem.

$$\int -\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right] \, dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{4\sqrt{2}}-\left(\frac{3}{8}+x\right)\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]$$

Result (type 8, 60 leaves, 3 steps) :

$$-x\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]+\frac{\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}},x\right]}{2\sqrt{2}}$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[1+x\sqrt{1+x^2}\right] \, dx$$

Optimal (type 3, 97 leaves, ? steps):

$$-2x+\sqrt{2\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right]-\sqrt{2\left(-1+\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Result (type 3, 332 leaves, 32 steps) :

$$-2x-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+$$

$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+$$

$$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-$$

$$\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3}+\frac{1}{3}\text{ArcTan}\left[\frac{\text{Cos}[x]\left(1+\text{Cos}[x]^2\right)\text{Sin}[x]}{1+\text{Cos}[x]^2\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\right]$$

Result (type 4, 289 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{\text{Tan}[x]}{\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}\right]\text{Cos}[x]^2\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}{2\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}-$$

$$\left(\left(1+\sqrt{3}\right)\text{Cos}[x]^2\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\left(\sqrt{3}+\text{Tan}[x]^2\right)\right.$$

$$\left.\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/\left(4\times3^{1/4}\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)+$$

$$\left(\left(2+\sqrt{3}\right)\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{1}{6}\left(3-2\sqrt{3}\right),2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\right.$$

$$\left.\left(\sqrt{3}+\text{Tan}[x]^2\right)\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/\left(4\times3^{1/4}\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int\text{ArcTan}\left[x+\sqrt{1-x^2}\right]dx$$

Optimal (type 3, 141 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 269 leaves, 40 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,\sqrt{1-x^2}}\right]}{\sqrt{3}}+$$
$$\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}-$$
$$\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}}\,dx$$

Optimal (type 3, 152 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-$$
$$\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 286 leaves, 32 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,\sqrt{1-x^2}}\right]}{2\sqrt{3}}-$$
$$\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{2\sqrt{3}}+$$
$$\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]}{\sqrt{-1+\text{Sec}[x]^4}}\,dx$$

Optimal (type 3, 28 leaves, ? steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]\,\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 59 leaves, 5 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Sin}[x]}{\sqrt{2\,\text{Sin}[x]^2-\text{Sin}[x]^4}}\right]\sqrt{1-\text{Cos}[x]^4}\,\text{Sec}[x]^2}{\sqrt{2}\,\sqrt{-1+\text{Sec}[x]^4}}$$

Problem 45: Unable to integrate problem.

$$\int \sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\,dx$$

Optimal (type 3, 337 leaves, ? steps):

$$\sqrt{2}\left(\sqrt{-1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]-\right.$$
$$\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left[\frac{\sqrt{2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}\right)}{2\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}\right]-$$
$$\sqrt{1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{-2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]+$$
$$\left.\sqrt{-1+\sqrt{2}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}}{\sqrt{2}-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}}\right]\right)$$
$$\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]}\,\sqrt{1+\text{Sec}[x]}$$

Result (type 8, 25 leaves, 0 steps):

$$\text{CannotIntegrate}\left[\sqrt{-\sqrt{-1+\text{Sec}[x]}+\sqrt{1+\text{Sec}[x]}},\,x\right]$$

Test results for the 284 problems in "Hearn Problems.m"

Problem 169: Unable to integrate problem.

$$\int \frac{e^{1-e^{x^2} x+2 x^2} \left(x+2 x^3\right)}{\left(1-e^{x^2} x\right)^2} \, dx$$

Optimal (type 3, 25 leaves, ? steps):

$$-\frac{e^{1-e^{x^2} x}}{-1+e^{x^2} x}$$

Result (type 8, 69 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x}{\left(-1+e^{x^2} x\right)^2}, x\right]+2 \, \text{CannotIntegrate}\left[\frac{e^{1-e^{x^2} x+2 x^2} x^3}{\left(-1+e^{x^2} x\right)^2}, x\right]$$

Problem 278: Unable to integrate problem.

$$\int \frac{-8-8 x-x^2-3 x^3+7 x^4+4 x^5+2 x^6}{\left(-1+2 x^2\right)^2 \sqrt{1+2 x^2+4 x^3+x^4}} \, dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1+2 x) \sqrt{1+2 x^2+4 x^3+x^4}}{2 \left(-1+2 x^2\right)}-\text{ArcTanh}\left[\frac{x (2+x) \left(7-x+27 x^2+33 x^3\right)}{\left(2+37 x^2+31 x^3\right) \sqrt{1+2 x^2+4 x^3+x^4}}\right]$$

Result (type 8, 354 leaves, 10 steps):

$$\frac{9}{4}\,\text{CannotIntegrate}\left[\frac{1}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{13}{4}\,\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}-2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]+$$
$$\text{CannotIntegrate}\left[\frac{x}{\sqrt{1+2x^2+4x^3+x^4}},x\right]+\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{13}{4}\,\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}+2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{13}{8}\,\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{1}{8}\left(15+\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{13}{8}\,\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{1}{8}\left(15-\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-$$
$$\frac{17}{2}\,\text{CannotIntegrate}\left[\frac{x}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]$$

Problem 279: Unable to integrate problem.

$$\int\frac{(1+2y)\sqrt{1-5y-5y^2}}{y\,(1+y)\,(2+y)\,\sqrt{1-y-y^2}}\,dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4}\,\text{ArcTanh}\left[\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right]-$$
$$\frac{1}{2}\,\text{ArcTanh}\left[\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right]+\frac{9}{4}\,\text{ArcTanh}\left[\frac{(11+7y)\sqrt{1-5y-5y^2}}{3\,(7+5y)\sqrt{1-y-y^2}}\right]$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{1}{2}\,\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}},y\right]+$$
$$\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}},y\right]-\frac{3}{2}\,\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}},y\right]$$

Problem 281: Unable to integrate problem.

$$\int \Big(\sqrt{9-4\sqrt{2}}\ x-\sqrt{2}\ \sqrt{1+4x+2x^2+x^4}\Big)\, dx$$

Optimal (type 4, 4030 leaves, ? steps):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\ x^2-\sqrt{2}\ \Bigg(-\frac{1}{3}\sqrt{1+4x+2x^2+x^4}\ +$$

$$\frac{1}{3}(1+x)\sqrt{1+4x+2x^2+x^4}+\Big(4\,i\,\big(-13+3\sqrt{33}\big)^{1/3}\sqrt{1+4x+2x^2+x^4}\Big)\Big/$$

$$\Big(4\times 2^{2/3}\big(-i+\sqrt{3}\big)-2\,i\,\big(-13+3\sqrt{33}\big)^{1/3}+2^{1/3}\big(i+\sqrt{3}\big)\big(-13+3\sqrt{33}\big)^{2/3}+$$

$$6\,i\,\big(-13+3\sqrt{33}\big)^{1/3}x\Big)-\Bigg(8\times 2^{2/3}\sqrt{\frac{3}{-13+3\sqrt{33}+4\big(-26+6\sqrt{33}\big)^{1/3}}}$$

$$\sqrt{}\Big(\Big(i\,\Big(-19899+3445\sqrt{33}+\big(-26+6\sqrt{33}\big)^{2/3}\big(-2574+466\sqrt{33}\big)+$$

$$\big(-26+6\sqrt{33}\big)^{1/3}\big(-19899+3445\sqrt{33}\big)+\big(59697-10335\sqrt{33}\big)x\Big)\Big)\Big/$$

$$\Big(\Big(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\,\big(3\,i+\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{1/3}\Big)$$

$$\Big(26-6\sqrt{33}+\big(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\big)\big(-26+6\sqrt{33}\big)^{1/3}+$$

$$\big(-4-4\,i\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{2/3}+6\big(-13+3\sqrt{33}\big)x\Big)\Big)\Big)\ \sqrt{1+4x+2x^2+x^4}$$

$$\text{EllipticE}\Big[\text{ArcSin}\Big[\Big(\sqrt{}\Big(26-6\sqrt{33}+\big(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\big)$$

$$\big(-26+6\sqrt{33}\big)^{1/3}+4\,i\,\big(i+\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{2/3}+6\big(-13+3\sqrt{33}\big)x\Big)\Big)\Big/$$

$$\Big(\sqrt{}\Big(\Big(39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\big(3-i\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{1/3}\Big)\Big/$$

$$\Big(39-13\,i\sqrt{3}+9\,i\sqrt{11}-9\sqrt{33}+4\big(3+i\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{1/3}\Big)\Big)$$

$$\sqrt{}\Big(26-6\sqrt{33}+\big(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\big)\big(-26+6\sqrt{33}\big)^{1/3}+$$

$$\big(-4-4\,i\sqrt{3}\big)\big(-26+6\sqrt{33}\big)^{2/3}+6\big(-13+3\sqrt{33}\big)x\Big)\Big)\Big],$$

$$\Big(4\big(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\big)+\big(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\big)\big(-26+6\sqrt{33}\big)^{1/3}\Big)\Big/$$

$$\Big(4\big(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\big)+$$

$$\big(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\big)\big(-26+6\sqrt{33}\big)^{1/3}\Big)\Big]\Bigg)\Big/$$

$$\Big(\Big(4\times 2^{2/3}-\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}+3\left(-13+3\sqrt{33}\right)^{1/3}x\Big)$$

$$\sqrt{\Big(\big(\mathbb{i}\,(1+x)\big)\Big/\Big(\Big(104-24\sqrt{33}+\left(-13-13\,\mathbb{i}\sqrt{3}+9\,\mathbb{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$4\,\mathbb{i}\left(\mathbb{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big(26-6\sqrt{33}+\left(-13+13\,\mathbb{i}\sqrt{3}-9\,\mathbb{i}\sqrt{11}+3\sqrt{33}\right)$$

$$\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,\mathbb{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)\Big)\Big)$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13+13\,\mathbb{i}\sqrt{3}-9\,\mathbb{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$\left(-4-4\,\mathbb{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13-13\,\mathbb{i}\sqrt{3}+9\,\mathbb{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$4\,\mathbb{i}\left(\mathbb{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)+$$

$$\Big(\Big(2^{1/3}\left(13-13\,\mathbb{i}\sqrt{3}+9\,\mathbb{i}\sqrt{11}-3\sqrt{33}\right)+4\times 2^{2/3}\left(1+\mathbb{i}\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{1/3}+$$

$$20\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\Big(4\times 2^{2/3}\left(\mathbb{i}+\sqrt{3}\right)+8\,\mathbb{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-\mathbb{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\sqrt{\frac{52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}$$

$$\sqrt{\Big(\frac{1}{1+x}\Big(-8\,\mathbb{i}\left(-13+3\sqrt{33}\right)+\left(-43\,\mathbb{i}-13\sqrt{3}+9\sqrt{11}+5\,\mathbb{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+}$$

$$\left(2\,\mathbb{i}+4\sqrt{3}-2\,\mathbb{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+$$

$$\Big(8\,\mathbb{i}\left(-13+3\sqrt{33}\right)+\left(13\,\mathbb{i}-13\sqrt{3}+9\sqrt{11}-3\,\mathbb{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+$$

$$4\left(\mathbb{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)x\Big)\Big)\sqrt{1+4x+2x^2+x^4}$$

$$\text{EllipticF}\Big[\text{ArcSin}\Big[\Big(\sqrt{\Big(52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}\Big)}$$

$$\sqrt{\Big(26-6\sqrt{33}+\left(-13-13\,\mathbb{i}\sqrt{3}+9\,\mathbb{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,\mathbb{i}\left(\mathbb{i}+\sqrt{3}\right)}$$

$$\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\Big)\Big)\Big/\Big(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}$$

$$\sqrt{\Big(39+13\,\mathbb{i}\sqrt{3}-9\,\mathbb{i}\sqrt{11}-9\sqrt{33}+4\left(3-\mathbb{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)}\sqrt{1+x}\Big)\Big],$$

$$\Big(4\left(21\,\mathbb{i}-7\sqrt{3}+3\sqrt{11}-3\,\mathbb{i}\sqrt{33}\right)+\left(3\,\mathbb{i}+\sqrt{3}+3\sqrt{11}+3\,\mathbb{i}\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/$$

$$\Big(-56\sqrt{3}+24\sqrt{11}+2\left(\sqrt{3}+3\sqrt{11}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big]\Big)\Big/\Big(3\times 2^{2/3}\times 3^{3/4}$$

$$\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{\left(39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)}$$
$$\sqrt{1+x}\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+\right.$$
$$\left.2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$
$$\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.}$$
$$\left.4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)$$
$$\sqrt{\left(\left(8\left(-13+3\sqrt{33}\right)-\left(5-3\,i\sqrt{3}+3\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+\left(-26+6\sqrt{33}\right)^{1/3}\right.\right.}$$
$$\left(-41+15\,i\sqrt{3}-3\,i\sqrt{11}+7\sqrt{33}\right)+\left(104-24\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+\right.\right.$$
$$\left.\left.3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)x\Big)\Big/$$
$$\left.\left(\left(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)+$$
$$\left(\left(4\times 2^{2/3}+2\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}\right)\right.$$
$$\left(4\times 2^{2/3}\left(i+\sqrt{3}\right)-4\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)$$
$$\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)+4\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)$$
$$\sqrt{\left(\left(-39+13\,i\sqrt{3}-9\,i\sqrt{11}+9\sqrt{33}-4\,i\left(-3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big/\right.}$$
$$\left(104-24\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.$$
$$\left.\left.\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)\right)\sqrt{1+x}$$
$$\sqrt{\left(\left(104-24\sqrt{33}+2\left(1+14\,i\sqrt{3}-6\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.\right.}$$
$$\left(-7-i\sqrt{3}-3\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+$$
$$2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\Big)\Big/$$
$$\left.\left(\left(-39+13\,i\sqrt{3}-9\,i\sqrt{11}+9\sqrt{33}-4\,i\left(-3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)$$
$$\sqrt{\left(\left(104-24\sqrt{33}+2\left(1-14\,i\sqrt{3}+6\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.\right.}$$
$$\left(-7+i\sqrt{3}+3\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+$$
$$2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\Big)\Big/\left(\left(-39-13\,i\sqrt{3}+\right.\right.$$
$$\left.\left.\left.9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)\sqrt{1+4x+2x^2+x^4}$$
$$\text{EllipticPi}\left[\left(2^{1/3}\left(4\times 2^{1/3}\left(-3\,i+\sqrt{3}\right)+\left(3\,i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)\right)\Big/\right.$$
$$\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)-8\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right),$$

$$\text{ArcSin}\Big[\Big(\sqrt{\Big(13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x\Big)}\Big)\Big/$$
$$\Big(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{\Big(\Big(\Big(-39+13\,i\sqrt{3}-9\,i\sqrt{11}+9\sqrt{33}-4\,i\left(-3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/\Big(104-24\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\Big)\Big)\sqrt{1+x}\Big)}\Big)\Big],$$
$$\Big(4\left(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big/$$
$$\Big(4\left(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}\Big)\Big]\Big)\Big/$$
$$\Big(2^{1/6}\sqrt{3}\left(4\times2^{2/3}\left(i+\sqrt{3}\right)+2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}-6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)\left(4\times2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$
$$\sqrt{\Big(13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x\Big)}\Big)$$

Result (type 8, 47 leaves, 1 step):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\;x^2-\sqrt{2}\;\text{CannotIntegrate}\left[\sqrt{1+4x+2x^2+x^4},\,x\right]$$

Problem 284: Unable to integrate problem.

$$\int\frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Optimal (type 3, 71 leaves, ? steps):

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\text{Log}\left[1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right]-\left(-1+\sqrt{2}\right)\text{Log}\left[-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right]\right)$$

Result (type 8, 248 leaves, 5 steps):

$$2\,\text{CannotIntegrate}\left[\frac{1}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{x}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{x^2}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$12\,\text{CannotIntegrate}\left[\frac{x^7}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$10\,\text{CannotIntegrate}\left[\frac{x^8}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},x\right]+$$
$$\frac{1}{2}\,\text{Log}\left[1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right]$$

Test results for the 7 problems in "Hebisch Problems.m"

Problem 2: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}}\left(2-x^2\right)}{2x+x^3}\,dx$$

Optimal (type 4, 10 leaves, ? steps):

$$\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 76 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}-x},x\right]+\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x},x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}+x},x\right]$$

Problem 3: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}}\left(2+2x+3x^2-x^3+2x^4\right)}{2x+x^3}\,dx$$

Optimal (type 4, 28 leaves, ? steps):

$$e^{\frac{x}{2+x^2}}\left(2+x^2\right)+\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 131 leaves, 5 steps):

$$-\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}}, x\right] + \left(1 + i\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2} - x}, x\right] + \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x}, x\right] + 2\,\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}}\,x, x\right] - \left(1 - i\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2} + x}, x\right]$$

Problem 5: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{-1+x^2}}\left(1 - 3x - x^2 + x^3\right)}{1 - x - x^2 + x^3}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\frac{1}{-1+x^2}}\,(1 + x)$$

Result (type 8, 75 leaves, 6 steps):

$$\text{CannotIntegrate}\left[e^{\frac{1}{-1+x^2}}, x\right] + \frac{1}{2}\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1 - x}, x\right] - \text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{(-1 + x)^2}, x\right] + \frac{1}{2}\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1 + x}, x\right]$$

Problem 7: Unable to integrate problem.

$$\int \frac{e^{x+\frac{1}{\text{Log}[x]}}\left(-1 + (1 + x)\,\text{Log}[x]^2\right)}{\text{Log}[x]^2}\,dx$$

Optimal (type 3, 10 leaves, ? steps):

$$e^{x+\frac{1}{\text{Log}[x]}}\,x$$

Result (type 8, 40 leaves, 2 steps):

$$\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}, x\right] + \text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}\,x, x\right] - \text{CannotIntegrate}\left[\frac{e^{x+\frac{1}{\text{Log}[x]}}}{\text{Log}[x]^2}, x\right]$$

Test results for the 9 problems in "Jeffrey Problems.m"

Problem 2: Result valid but suboptimal antiderivative.

$$\int \frac{1 + \text{Cos}[x] + 2\,\text{Sin}[x]}{3 + \text{Cos}[x]^2 + 2\,\text{Sin}[x] - 2\,\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 38 leaves, 43 steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]+\text{Cot}\left[\frac{x}{2}\right]-\frac{\text{Sin}[x]}{1-\text{Cos}[x]}$$

Problem 3: Result valid but suboptimal antiderivative.

$$\int\frac{2+\text{Cos}[x]+5\,\text{Sin}[x]}{4\,\text{Cos}[x]-2\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[x]-2\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1-3\,\text{Cos}[x]+\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 42 leaves, 25 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[1+\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int\frac{3+7\,\text{Cos}[x]+2\,\text{Sin}[x]}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-5\,\text{Sin}[x]-\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1+\text{Cos}[x]-2\,\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 31 leaves, 32 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 5: Unable to integrate problem.

$$\int\frac{-1+4\,\text{Cos}[x]+5\,\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 43 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{3+\text{Cos}[x]}\right]-2\,\text{ArcTan}\left[\frac{3\,\text{Sin}[x]+7\,\text{Cos}[x]\,\text{Sin}[x]}{1+2\,\text{Cos}[x]+5\,\text{Cos}[x]^2}\right]$$

Result (type 8, 79 leaves, 2 steps):

$$\text{CannotIntegrate}\left[\frac{1}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-4\,\text{Cos}[x]^3},x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+$$
$$5\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]$$

Problem 6: Unable to integrate problem.

$$\int \frac{-5+2\,\text{Cos}[x]+7\,\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{2\,\text{Cos}[x]\,\text{Sin}[x]}{1-\text{Cos}[x]+2\,\text{Cos}[x]^2}\right]$$

Result (type 8, 81 leaves, 2 steps):

$$-5\,\text{CannotIntegrate}\left[\frac{1}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+2\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]+7\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},x\right]$$

Test results for the 113 problems in "Moses Problems.m"

Test results for the 376 problems in "Stewart Problems.m"

Test results for the 705 problems in "Timofeev Problems.m"

Problem 222: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-x}\,x\,(1+x)^{2/3}}{-(1-x)^{5/6}\,(1+x)^{1/3}+(1-x)^{2/3}\,\sqrt{1+x}}\,dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12}\,(1-3\,x)\,(1-x)^{2/3}\,(1+x)^{1/3}+\frac{1}{4}\,\sqrt{1-x}\,x\,\sqrt{1+x}-\frac{1}{4}\,(1-x)\,(3+x)+\frac{1}{12}\,(1-x)^{1/3}\,(1+x)^{2/3}\,(1+3\,x)+\frac{1}{12}\,(1-x)^{1/6}\,(1+x)^{5/6}\,(2+3\,x)-\frac{1}{12}\,(1-x)^{5/6}\,(1+x)^{1/6}\,(10+3\,x)+\frac{1}{6}\,\text{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right]-\frac{4\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-2\,(1+x)^{1/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{3\,\sqrt{3}}-\frac{5}{6}\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-(1+x)^{1/3}}{(1-x)^{1/6}\,(1+x)^{1/6}}\right]+\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,(1-x)^{1/6}\,(1+x)^{1/6}}{(1-x)^{1/3}+(1+x)^{1/3}}\right]}{6\,\sqrt{3}}$$

Result (type 3, 522 leaves, 46 steps):

$$\frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}(1+x)^{1/6}+\frac{1}{6}(1-x)^{2/3}(1+x)^{1/3}-\frac{1}{4}(1-x)^{5/3}(1+x)^{1/3}+\frac{1}{3}(1-x)^{1/3}(1+x)^{2/3}-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3}+\frac{5}{12}(1-x)^{1/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{7/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{5/6}(1+x)^{7/6}+\frac{1}{4}x\sqrt{1-x^2}+\frac{\text{ArcSin}[x]}{4}-\frac{2}{3}\text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right]+\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{3\sqrt{3}}+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{\sqrt{3}(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{9}\text{Log}[1-x]+\frac{1}{9}\text{Log}[1+x]+\frac{1}{3}\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}-\frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}+\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}+\frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}-\frac{1}{3}\text{Log}\left[1+\frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right]$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left((-1+x)^2(1+x)\right)^{1/3}}\,dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(3-3x)^{1/3}}\right]}{3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[-\frac{8}{3}(-1+x)\right]}{2\times 3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[1+\frac{3^{1/3}(1+x)^{1/3}}{(3-3x)^{1/3}}\right]}{2\left(1-x-x^2+x^3\right)^{1/3}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int \frac{\left((-1+x)^2(1+x)\right)^{1/3}}{x^2}\,dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{\left(\left(-1+x\right)^2\left(1+x\right)\right)^{1/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]+\frac{\text{Log}[x]}{6}-\frac{2}{3}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]-\frac{1}{2}\text{Log}\left[1+\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 3, 404 leaves, 6 steps):

$$-\frac{\left(1-x-x^2+x^3\right)^{1/3}}{x}-\frac{3\times 3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\right]}{(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(3-3x)^{1/3}}{3^{5/6}(1+x)^{1/3}}\right]}{(3-3x)^{2/3}(1+x)^{1/3}}+\frac{\left(1-x-x^2+x^3\right)^{1/3}\text{Log}[x]}{2\times 3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\text{Log}\left[\frac{4(1+x)}{3}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3\times 3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\text{Log}\left[1+\frac{(3-3x)^{1/3}}{3^{1/3}(1+x)^{1/3}}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}-\frac{3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}\text{Log}\left[\left(\frac{2}{3}\right)^{2/3}(3-3x)^{1/3}-\frac{2^{2/3}(1+x)^{1/3}}{3^{1/3}}\right]}{2(3-3x)^{2/3}(1+x)^{1/3}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(9+3x-5x^2+x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-3+x)}{\left(9+3x-5x^2+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-3+x}{\left(9+3x-5x^2+x^3\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(9-3x)^{2/3}(1+x)^{1/3}\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(9-3x)^{1/3}}\right]}{3^{1/6}\left(9+3x-5x^2+x^3\right)^{1/3}}-\frac{(9-3x)^{2/3}(1+x)^{1/3}\text{Log}\left[-\frac{32}{3}(-3+x)\right]}{2\times 3^{2/3}\left(9+3x-5x^2+x^3\right)^{1/3}}-\frac{3^{1/3}(9-3x)^{2/3}(1+x)^{1/3}\text{Log}\left[1+\frac{3^{1/3}(1+x)^{1/3}}{(9-3x)^{1/3}}\right]}{2\left(9+3x-5x^2+x^3\right)^{1/3}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int\frac{1}{x+\sqrt{1+x+x^2}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-x+\sqrt{1+x+x^2}-\frac{3}{2}\,\text{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]$$

Result (type 3, 59 leaves, 3 steps):

$$\frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)}+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]-\frac{3}{2}\,\text{Log}\left[1+2\left(x+\sqrt{1+x+x^2}\right)\right]$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Tan}[x]}{\left(-1+\sqrt{\text{Tan}[x]}\right)^2}\,dx$$

Optimal (type 3, 84 leaves, ? steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[\frac{1-\text{Tan}[x]}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{1+\text{Tan}[x]}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{1}{2}\,\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Result (type 3, 133 leaves, 19 steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}+\frac{1}{2}\,\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]-\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Problem 416: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Cos}[2x]-\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2}\,\text{Log}\left[\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sec}[x]\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\right]-\frac{\text{ArcSin}[\text{Cos}[x]-\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{ArcTanh}[\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{Sin}[2x]}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}$$

Result (type 3, 234 leaves, 27 steps):

$$-2\,\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\;-$$
$$\sqrt{2}\,\text{ArcSinh}[\text{Tan}[x]]\,\text{Cot}[x]\,\left(\text{Sec}[x]^2\right)^{3/2}\sqrt{\text{Cos}[x]\,\text{Sin}[x]}\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\;-$$
$$\frac{\sqrt{2}\,\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}+$$
$$\frac{\sqrt{2}\,\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}-$$
$$\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}+$$
$$\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Sec}[x]^2\left(-\text{Cos}[2\,x]+2\,\text{Tan}[x]^2\right)}{\left(\text{Tan}[x]\,\text{Tan}[2\,x]\right)^{3/2}}\,dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2\,\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2\,x]}}\right]-\frac{11\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2\,x]}}\right]}{4\sqrt{2}}+$$
$$\frac{\text{Tan}[x]}{2\left(\text{Tan}[x]\,\text{Tan}[2\,x]\right)^{3/2}}+\frac{2\,\text{Tan}[x]^3}{3\left(\text{Tan}[x]\,\text{Tan}[2\,x]\right)^{3/2}}+\frac{3\,\text{Tan}[x]}{4\sqrt{\text{Tan}[x]\,\text{Tan}[2\,x]}}$$

Result (type 3, 208 leaves, 22 steps):

$$\frac{3\,\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Cot}[x]\left(1-\text{Tan}[x]^2\right)}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Tan}[x]\left(1-\text{Tan}[x]^2\right)}{3\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}-$$
$$\frac{11\,\text{ArcTan}\left[\sqrt{-1+\text{Tan}[x]^2}\right]\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}+\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-1+\text{Tan}[x]^2}}{\sqrt{2}}\right]\text{Tan}[x]}{\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}$$

Problem 455: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Sin}[x]^6\,\text{Tan}[x]}{\text{Cos}[2\,x]^{3/4}}\,dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{1-\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{1+\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+\frac{1}{36}\text{Cos}[2x]^{9/4}$$

Result (type 3, 154 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+\frac{1}{36}\text{Cos}[2x]^{9/4}+\frac{\text{Log}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int e^{x/2}\,x^2\,\text{Cos}[x]^3\,dx$$

Optimal (type 3, 187 leaves, ? steps):

$$-\frac{132}{125}e^{x/2}\text{Cos}[x]+\frac{18}{25}e^{x/2}x\,\text{Cos}[x]+\frac{48}{185}e^{x/2}x^2\text{Cos}[x]+\frac{2}{37}e^{x/2}x^2\text{Cos}[x]^3-\frac{428\,e^{x/2}\text{Cos}[3x]}{50653}+\frac{70\,e^{x/2}x\,\text{Cos}[3x]}{1369}-\frac{24}{125}e^{x/2}\text{Sin}[x]-\frac{24}{25}e^{x/2}x\,\text{Sin}[x]+\frac{96}{185}e^{x/2}x^2\text{Sin}[x]+\frac{12}{37}e^{x/2}x^2\text{Cos}[x]^2\text{Sin}[x]-\frac{792\,e^{x/2}\text{Sin}[3x]}{50653}-\frac{24\,e^{x/2}x\,\text{Sin}[3x]}{1369}$$

Result (type 3, 253 leaves, 31 steps):

$$-\frac{6687696\,e^{x/2}\text{Cos}[x]}{6331625}+\frac{24792\,e^{x/2}x\,\text{Cos}[x]}{34225}+\frac{48}{185}e^{x/2}x^2\text{Cos}[x]+\frac{16\,e^{x/2}\text{Cos}[x]^3}{50653}-\frac{8\,e^{x/2}x\,\text{Cos}[x]^3}{1369}+\frac{2}{37}e^{x/2}x^2\text{Cos}[x]^3-\frac{432\,e^{x/2}\text{Cos}[3x]}{50653}+\frac{72\,e^{x/2}x\,\text{Cos}[3x]}{1369}-\frac{1218672\,e^{x/2}\text{Sin}[x]}{6331625}-\frac{32556\,e^{x/2}x\,\text{Sin}[x]}{34225}+\frac{96}{185}e^{x/2}x^2\text{Sin}[x]+\frac{96\,e^{x/2}\text{Cos}[x]^2\text{Sin}[x]}{50653}-\frac{48\,e^{x/2}x\,\text{Cos}[x]^2\text{Sin}[x]}{1369}+\frac{12}{37}e^{x/2}x^2\text{Cos}[x]^2\text{Sin}[x]-\frac{816\,e^{x/2}\text{Sin}[3x]}{50653}-\frac{12\,e^{x/2}x\,\text{Sin}[3x]}{1369}$$

Problem 695: Result valid but suboptimal antiderivative.

$$\int \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]dx$$

Optimal (type 3, 55 leaves, ? steps):

$$-\frac{\sqrt{2}\,a\sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}}+(a+x)\,\text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

Result (type 3, 125 leaves, 8 steps):

$$-\sqrt{2}\, a \sqrt{\frac{a}{a+x}} \sqrt{-\frac{a-x}{a+x}} \sqrt{\frac{a+x}{a}} \sqrt{1+\frac{x}{a}} +$$

$$x\, \text{ArcSin}\left[\sqrt{-\frac{a-x}{a+x}}\right] + \frac{a^2 \sqrt{\frac{a+x}{a}} \sqrt{1+\frac{x}{a}}\, \text{ArcSin}\left[\sqrt{-\frac{a-x}{a+x}}\right]}{a+x}$$

Test results for the 116 problems in "Welz Problems.m"

Problem 9: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\, dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)} + \frac{1}{25}\sqrt{-110+50\sqrt{5}}\, \text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\, \sqrt{x}\right] -$$

$$\frac{1}{50}\sqrt{-110+50\sqrt{5}}\, \text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\, \sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right] -$$

$$\frac{1}{25}\sqrt{110+50\sqrt{5}}\, \text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\, \sqrt{x}\right] -$$

$$\frac{1}{50}\sqrt{110+50\sqrt{5}}\, \text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\, \sqrt{-1+x^2}}{2-x-\sqrt{5}\, x}\right]$$

Result (type 3, 365 leaves, 18 steps):

$$\frac{2\left(1-2x\right)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{2\left(1-2x\right)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]+$$
$$\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\mathrm{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{2}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\mathrm{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-$$
$$\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 10: Result valid but suboptimal antiderivative.

$$\int\frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\mathrm{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-$$
$$\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\mathrm{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-$$
$$\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\mathrm{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-$$
$$\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\mathrm{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 3, 541 leaves, 25 steps):

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)}-\frac{(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)}-\frac{(3-x)\sqrt{-1+x^2}}{5(1+x-x^2)}+$$
$$\frac{(2+x)\sqrt{-1+x^2}}{5(1+x-x^2)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{x}\right]-$$
$$\frac{1}{5}\sqrt{\frac{1}{10}\left(-11+5\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]-$$
$$\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]+$$
$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]-$$
$$\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{x}\right]-$$
$$\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]-$$
$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]+$$
$$\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(2-3x+x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}(2-x)}{\sqrt{3}\left(2-3x+x^2\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\left(2-3x+x^2\right)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 176 leaves, 2 steps) :

$$-\frac{\sqrt{3}\ (-2+x)^{1/3}\ (-1+x)^{1/3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{1/3}\ (-2+x)^{2/3}}{\sqrt{3}\ (-1+x)^{1/3}}\right]}{2\times 2^{1/3}\ \left(2-3\ x+x^2\right)^{1/3}}+$$

$$\frac{3\ (-2+x)^{1/3}\ (-1+x)^{1/3}\ \text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}}-2^{1/3}\ (-1+x)^{1/3}\right]}{4\times 2^{1/3}\ \left(2-3\ x+x^2\right)^{1/3}}-\frac{(-2+x)^{1/3}\ (-1+x)^{1/3}\ \text{Log}[x]}{2\times 2^{1/3}\ \left(2-3\ x+x^2\right)^{1/3}}$$

Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(-5+7\ x-3\ x^2+x^3\right)^{1/3}}\, dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ (-1+x)}{\sqrt{3}\ \left(-5+7\ x-3\ x^2+x^3\right)^{1/3}}\right]+$$

$$\frac{1}{4}\text{Log}[1-x]-\frac{3}{4}\text{Log}\left[1-x+\left(-5+7\ x-3\ x^2+x^3\right)^{1/3}\right]$$

Result (type 3, 131 leaves, 5 steps) :

$$\frac{\sqrt{3}\ \left(4+(-1+x)^2\right)^{1/3}\ (-1+x)^{1/3}\ \text{ArcTan}\left[\frac{1+\frac{2\ (-1+x)^{2/3}}{\left(4+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\ \left(4\ (-1+x)+(-1+x)^3\right)^{1/3}}-$$

$$\left(3\ \left(4+(-1+x)^2\right)^{1/3}\ (-1+x)^{1/3}\ \text{Log}\left[-\left(4+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]\right)\Big/$$
$$\left(4\ \left(4\ (-1+x)+(-1+x)^3\right)^{1/3}\right)$$

Problem 41: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(x\ \left(-q+x^2\right)\right)^{1/3}}\, dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\ \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\ x}{\sqrt{3}\ \left(x\ \left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\text{Log}\left[-x+\left(x\ \left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 117 leaves, 5 steps) :

$$\frac{\sqrt{3}\ x^{1/3}\ \left(-q+x^2\right)^{1/3}\ \text{ArcTan}\left[\frac{1+\frac{2\ x^{2/3}}{\left(-q+x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\ \left(-q\ x+x^3\right)^{1/3}}-\frac{3\ x^{1/3}\ \left(-q+x^2\right)^{1/3}\ \text{Log}\left[x^{2/3}-\left(-q+x^2\right)^{1/3}\right]}{4\ \left(-q\ x+x^3\right)^{1/3}}$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 145 leaves, 5 steps) :

$$\frac{\sqrt{3}\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{\left(-1+q+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}}-\left(3\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\left(-1+q+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]\right)\Big/\left(4\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}\right)$$

Problem 43: Unable to integrate problem.

$$\int \frac{1}{x\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2q^{1/3}(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\right]}{2q^{1/3}}+\frac{\text{Log}[1-x]}{4q^{1/3}}+\frac{\text{Log}[x]}{2q^{1/3}}-\frac{3\,\text{Log}\left[-q^{1/3}(-1+x)+\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}\right]}{4q^{1/3}}$$

Result (type 8, 677 leaves, 2 steps) :

$$\frac{1}{3\left(-q+3\,q\,x+(-1-2\,q)\,x^2+x^3\right)^{1/3}}\left(-1-2\,q-\frac{1-5\,q+4\,q^2+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{-(-1+q)^3\,q}\right)^{2/3}}{\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{-(-1+q)^3\,q}\right)^{1/3}}+3\,x\right)^{1/3}$$

$$\left(-1+5\,q-4\,q^2+\frac{(1-4\,q)^2\,(1-q)^2}{\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}}+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}+\left(3\left(1-5\,q+4\,q^2+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}\right)\left(\frac{1}{3}(-1-2\,q)+x\right)\right)\Big/\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{1/3}+9\left(\frac{1}{3}(-1-2\,q)+x\right)^2\right)^{1/3}$$

$$\texttt{Unintegrable}\Big[3\Big/\Bigg(x\left(-1-2\,q-\left(1-5\,q+4\,q^2+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{-(-1+q)^3\,q}\right)^{2/3}\right)\Big/\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{-(-1+q)^3\,q}\right)^{1/3}+3\,x\right)^{1/3}$$
$$\left(-1+5\,q-4\,q^2+\frac{(1-4\,q)^2\,(1-q)^2}{\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}}+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}+9\left(\frac{1}{3}(-1-2\,q)+x\right)^2+\left(\left(1-5\,q+4\,q^2+\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{2/3}\right)(-1-2\,q+3\,x)\right)\Big/\left(1+6\,q-15\,q^2+8\,q^3+3\,\sqrt{3}\,\sqrt{(1-q)^3\,q}\right)^{1/3}\right)^{1/3}\Bigg),\;x\Big]$$

Problem 44: Unable to integrate problem.

$$\int\frac{2-(1+k)\,x}{\left((1-x)\,x\,(1-k\,x)\right)^{1/3}\,\left(1-(1+k)\,x\right)}\,dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,k^{1/3}\,x}{\left((1-x)\,x\,(1-k\,x)\right)^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2\,k^{1/3}}+\frac{\text{Log}\left[1-(1+k)\,x\right]}{2\,k^{1/3}}-\frac{3\,\text{Log}\left[-k^{1/3}\,x+\left((1-x)\,x\,(1-k\,x)\right)^{1/3}\right]}{2\,k^{1/3}}$$

Result (type 8, 139 leaves, 3 steps):

$$\frac{3\,(1-x)^{1/3}\,x\,(1-k\,x)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},x,k\,x\right]}{2\,\left((1-x)\,x\,(1-k\,x)\right)^{1/3}}+$$

$$\left((1-x)^{1/3}\,x^{1/3}\,(1-k\,x)^{1/3}\,\text{CannotIntegrate}\left[\frac{1}{(1-x)^{1/3}\,x^{1/3}\,(1+(-1-k)\,x)\,(1-k\,x)^{1/3}},x\right]\right)\Big/$$
$$\left((1-x)\,x\,(1-k\,x)\right)^{1/3}$$

Problem 45: Unable to integrate problem.

$$\int\frac{1-k\,x}{(1+(-2+k)\,x)\,\left((1-x)\,x\,(1-k\,x)\right)^{2/3}}\,dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-k\,x)}{(1-k)^{1/3}\,\left((1-x)\,x\,(1-k\,x)\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}\left[1-(2-k)\,x\right]}{2^{2/3}\,(1-k)^{1/3}}+$$

$$\frac{\text{Log}[1-k\,x]}{2\times 2^{2/3}\,(1-k)^{1/3}}-\frac{3\,\text{Log}\left[-1+k\,x+2^{2/3}\,(1-k)^{1/3}\,\left((1-x)\,x\,(1-k\,x)\right)^{1/3}\right]}{2\times 2^{2/3}\,(1-k)^{1/3}}$$

Result (type 8, 78 leaves, 1 step):

$$\frac{(1-x)^{2/3}\,x^{2/3}\,(1-k\,x)^{2/3}\,\text{CannotIntegrate}\left[\frac{(1-k\,x)^{1/3}}{(1-x)^{2/3}\,x^{2/3}\,(1+(-2+k)\,x)},x\right]}{\left((1-x)\,x\,(1-k\,x)\right)^{2/3}}$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\,\sqrt{(-a+x)\,(1+x^2)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\,\sqrt{a+\sqrt{1+a^2}}\,\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\,(1+x^2)}}\right]$$

Result (type 4, 204 leaves, 9 steps):

$$\frac{2\,\mathrm{i}\,\sqrt{\frac{a-x}{\mathrm{i}+a}}\,\sqrt{1+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\mathrm{i}\,x}}{\sqrt{2}}\right],\,\frac{2}{1-\mathrm{i}\,a}\right]}{\sqrt{-(a-x)\,\left(1+x^2\right)}}+$$

$$\left(4\,\sqrt{1+a^2}\,\sqrt{\frac{a-x}{\mathrm{i}+a}}\,\sqrt{1+x^2}\,\text{EllipticPi}\left[\frac{2}{1-\mathrm{i}\,\left(a-\sqrt{1+a^2}\right)},\,\text{ArcSin}\left[\frac{\sqrt{1-\mathrm{i}\,x}}{\sqrt{2}}\right],\,\frac{2}{1-\mathrm{i}\,a}\right]\right)\Big/$$

$$\left(\left(1-\mathrm{i}\,\left(a-\sqrt{1+a^2}\right)\right)\,\sqrt{-(a-x)\,\left(1+x^2\right)}\right)$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-2+a+x}{(-a+x)\,\sqrt{(2-a)\,a\,x+\left(-1-2\,a+a^2\right)\,x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

0

Result (type 4, 529 leaves, 5 steps) :

$$\left(2\,(1-a)\,\sqrt{x}\,\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}\;\text{ArcTan}\left[\frac{\sqrt{-1+2\,a-a^2}\,\sqrt{x}}{\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}}\right]\right)\Big/$$
$$\left(a\,\sqrt{-1+2\,a-a^2}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)+$$
$$\left(((2-a)\,a)^{3/4}\,\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\;\text{EllipticF}\left[\right.\right.$$
$$\left.\left.2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/\left(a\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)+$$
$$\left((2-a)\,\left(1-\sqrt{(2-a)\,a}\right)\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\right.$$
$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\,\sqrt{(2-a)\,a}},2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/$$
$$\left(((2-a)\,a)^{3/4}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a+(-1+2\,a)\,x}{(-a+x)\,\sqrt{a^2\,x-(-1+2\,a+a^2)\,x^2+(-1+2\,a)\,x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\left[\frac{-a^2+2\,a\,x+x^2-2\left(x+\sqrt{(1-x)\,x\,(a^2+x-2\,a\,x)}\right)}{(a-x)^2}\right]$$

Result (type 4, 180 leaves, 7 steps):

$$-\left(\left(2\,(1-2a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)\,x}{a^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{x}\right],-\frac{1-2a}{a^2}\right]\right)\Big/\left(\sqrt{a^2x+(1-2a-a^2)\,x^2-(1-2a)\,x^3}\right)\right)+$$
$$\left(4\,(1-a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)\,x}{a^2}}\,\text{EllipticPi}\left[\frac{1}{a},\text{ArcSin}\left[\sqrt{x}\right],-\frac{1-2a}{a^2}\right]\right)\Big/\left(\sqrt{a^2x+(1-2a-a^2)\,x^2-(1-2a)\,x^3}\right)$$

Problem 100: Result valid but suboptimal antiderivative.

$$\int\frac{1+x}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 383 leaves, 16 steps):

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+$$
$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times2^{1/3}}-\frac{1}{3}\times2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]+$$
$$\frac{\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2\times2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times2^{1/3}}$$

Problem 101: Result valid but suboptimal antiderivative.

$$\int\frac{(1+x)^2}{(1-x^3)^{1/3}\,(1+x^3)}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2\times2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 383 leaves, 17 steps):

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\,(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}+$$

$$\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}\right]+$$

$$\frac{\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[-2^{1/3}\,x-(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Problem 102: Result valid but suboptimal antiderivative.

$$\int\frac{1-x}{(1+x+x^2)\,(1+x^3)^{1/3}}\,dx$$

Optimal (type 3, 119 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1+x)^2}{(1+x^3)^{2/3}}-\frac{2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 357 leaves, 16 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}\,x}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\,(1+x^3)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\,\sqrt{3}}-$$

$$\frac{\text{Log}\left[(1-x)^2\,(1+x)\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1-x^3\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}\,(1+x)^2}{(1+x^3)^{2/3}}-\frac{2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}\,(1+x)}{(1+x^3)^{1/3}}\right]-$$

$$\frac{\text{Log}\left[2^{1/3}-(1+x^3)^{1/3}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[2^{1/3}\,x-(1+x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1+x-2^{2/3}\,(1+x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(1-x)\,(1-x^3)^{2/3}}{1+x^3}\,dx$$

Optimal (type 5, 383 leaves, ? steps):

$$-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-$$

$$\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[(1-x)(1+x)^2\right]}{6\times 2^{1/3}}-$$

$$\frac{\text{Log}\left[1+x^3\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{1}{3}\times 2^{2/3}\,\text{Log}\left[1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}\right]+$$

$$\frac{\text{Log}\left[-2^{1/3}x-(1-x^3)^{1/3}\right]}{2^{1/3}}-\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]+\frac{\text{Log}\left[-1+x+2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 5, 648 leaves, 17 steps):

$$-\frac{2^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{2\,\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\left(1-(-1)^{1/3}\right)\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}+$$

$$\frac{\left(1+(-1)^{2/3}\right)\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\left(1-(-1)^{1/3}\right)\text{ArcTan}\left[\frac{1-\frac{2^{1/3}\left((-1)^{1/3}+x\right)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-$$

$$\frac{\left(1+(-1)^{2/3}\right)\text{ArcTan}\left[\frac{1+\frac{(-1)^{2/3}2^{1/3}\left(1+(-1)^{1/3}x\right)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{1}{3}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$

$$\frac{1}{6}\left(1-(-1)^{1/3}\right)x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+$$

$$\frac{1}{6}\left(1+(-1)^{2/3}\right)x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[-(1-x)(1+x)^2\right]}{3\times 2^{1/3}}-$$

$$\frac{\left(1+(-1)^{2/3}\right)\text{Log}\left[-(-1)^{2/3}\left((-1)^{2/3}+x\right)^2\left(1+(-1)^{1/3}x\right)\right]}{6\times 2^{1/3}}-$$

$$\frac{\left(1-(-1)^{1/3}\right)\text{Log}\left[(-1)^{2/3}\left((-1)^{1/3}+x\right)\left(1+(-1)^{2/3}x\right)^2\right]}{6\times 2^{1/3}}-\frac{1}{3}\text{Log}\left[x+(1-x^3)^{1/3}\right]-$$

$$\frac{1}{6}\left(1-(-1)^{1/3}\right)\text{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{1}{6}\left(1+(-1)^{2/3}\right)\text{Log}\left[x+(1-x^3)^{1/3}\right]+$$

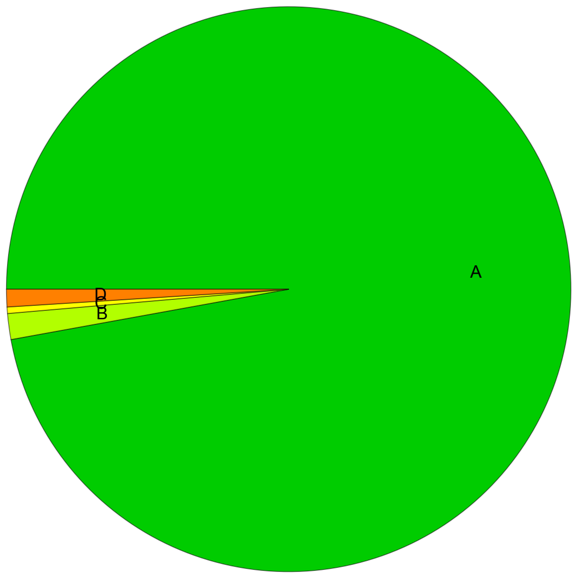
$$\frac{\left(1-(-1)^{1/3}\right)\text{Log}\left[1-(-1)^{2/3}x-(-2)^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}+\frac{\text{Log}\left[1-x-2^{2/3}(1-x^3)^{1/3}\right]}{2^{1/3}}+$$

$$\frac{\left(1+(-1)^{2/3}\right)\text{Log}\left[1+(-1)^{1/3}x+(-1)^{1/3}2^{2/3}(1-x^3)^{1/3}\right]}{2\times 2^{1/3}}$$

Test results for the 8 problems in "Wester Problems.m"

Summary of Integration Test Results

1892 integration problems



A - 1838 optimal antiderivatives

B - 28 valid but suboptimal antiderivatives

C - 7 unnecessarily complex antiderivatives

D - 19 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives