Rules for integrands of the form $(a + b \, \text{ArcCosh}[c \, x])^n$

1: $\int (a + b \, \text{ArcCosh}[c\,x])^n \, dx$ when $n > 0$

Derivation: Integration by parts

Basis: $\partial_x \, (a + b \, \text{ArcCosh}[c\,x])^n == \dfrac{b\,c\,n\,(a + b\,\text{ArcCosh}[c\,x])^{n-1}}{\sqrt{1+c\,x}\,\sqrt{-1+c\,x}}$

Rule: If $n > 0$, then

$$\int (a + b \, \text{ArcCosh}[c\,x])^n \, dx \;\longrightarrow\; x\,(a + b\,\text{ArcCosh}[c\,x])^n - b\,c\,n \int \frac{x\,(a + b\,\text{ArcCosh}[c\,x])^{n-1}}{\sqrt{1+c\,x}\,\sqrt{-1+c\,x}} \, dx$$

Program code:

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_.,x_Symbol] :=
  x*(a+b*ArcCosh[c*x])^n -
  b*c*n*Int[x*(a+b*ArcCosh[c*x])^(n-1)/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c},x] && GtQ[n,0]
```

2: $\int (a + b\, \text{ArcCosh}[c\, x])^n \, dx$ when $n < -1$

Derivation: Integration by parts

Basis: $\dfrac{(a+b\, \text{ArcCosh}[c\, x])^n}{\sqrt{1+c\, x}\ \sqrt{-1+c\, x}} == \partial_x \dfrac{(a+b\, \text{ArcCosh}[c\, x])^{n+1}}{b\, c\ (n+1)}$

Rule: If $n < -1$, then

$$\int (a + b\, \text{ArcCosh}[c\, x])^n \, dx \longrightarrow \frac{\sqrt{1+c\, x}\ \sqrt{-1+c\, x}\ (a + b\, \text{ArcCosh}[c\, x])^{n+1}}{b\, c\ (n+1)} - \frac{c}{b\ (n+1)} \int \frac{x\ (a + b\, \text{ArcCosh}[c\, x])^{n+1}}{\sqrt{1+c\, x}\ \sqrt{-1+c\, x}} \, dx$$

Program code:

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  Sqrt[1+c*x]*Sqrt[-1+c*x]*(a+b*ArcCosh[c*x])^(n+1)/(b*c*(n+1)) -
  c/(b*(n+1))*Int[x*(a+b*ArcCosh[c*x])^(n+1)/(Sqrt[1+c*x]*Sqrt[-1+c*x]),x] /;
FreeQ[{a,b,c},x] && LtQ[n,-1]
```

3: $\int (a + b\, \text{ArcCosh}[c\, x])^n\, dx$

Derivation: Integration by substitution

Basis:

$$(a + b\, \text{ArcCosh}[c\, x])^n == \frac{1}{b\, c}\, \text{Subst}\left[x^n\, \text{Sinh}\left[-\frac{a}{b} + \frac{x}{b}\right],\ x,\ a + b\, \text{ArcCosh}[c\, x]\right] \partial_x\, (a + b\, \text{ArcCosh}[c\, x])$$

Rule:

$$\int (a + b\, \text{ArcCosh}[c\, x])^n\, dx\ \longrightarrow\ \frac{1}{b\, c}\, \text{Subst}\left[\int x^n\, \text{Sinh}\left[-\frac{a}{b} + \frac{x}{b}\right] dx,\ x,\ a + b\, \text{ArcCosh}[c\, x]\right]$$

Program code:

```
Int[(a_.+b_.*ArcCosh[c_.*x_])^n_,x_Symbol] :=
  1/(b*c)*Subst[Int[x^n*Sinh[-a/b+x/b],x],x,a+b*ArcCosh[c*x]] /;
FreeQ[{a,b,c,n},x]
```