

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "1 Algebraic functions\1.3 Miscellaneous"

Test results for the 494 problems in "1.3.1 Rational functions.m"

Problem 12: Result is not expressed in closed-form.

$$\int \frac{1}{3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3} dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{b^{1/3} + \frac{2(b+c x)}{(b^2-3 a c)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{\sqrt{3} b^{2/3} (b^2-3 a c)^{2/3}} + \frac{\text{Log}\left[b - b^{1/3} (b^2-3 a c)^{1/3} + c x\right]}{3 b^{2/3} (b^2-3 a c)^{2/3}} - \frac{\text{Log}\left[b^{2/3} (b^2-3 a c)^{2/3} + b^{1/3} c (b^2-3 a c)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{6 b^{2/3} (b^2-3 a c)^{2/3}}$$

Result (type 7, 63 leaves):

$$\frac{1}{3} \text{RootSum}\left[3 a b + 3 b^2 \#1 + 3 b c \#1^2 + c^2 \#1^3 \&, \frac{\text{Log}[x - \#1]}{b^2 + 2 b c \#1 + c^2 \#1^2} \&\right]$$

Problem 13: Result is not expressed in closed-form.

$$\int \frac{1}{(3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^2} dx$$

Optimal (type 3, 245 leaves, 8 steps):

$$\begin{aligned}
& -\frac{c \left(\frac{b}{c} + x\right)}{3 b (b^2 - 3 a c) (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)} + \frac{2 c \operatorname{ArcTan}\left[\frac{b^{1/3} + \frac{2 (b+c x)}{(b^2 - 3 a c)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{3 \sqrt{3} b^{5/3} (b^2 - 3 a c)^{5/3}} - \\
& \frac{2 c \operatorname{Log}\left[b - b^{1/3} (b^2 - 3 a c)^{1/3} + c x\right]}{9 b^{5/3} (b^2 - 3 a c)^{5/3}} + \frac{c \operatorname{Log}\left[b^{2/3} (b^2 - 3 a c)^{2/3} + b^{1/3} c (b^2 - 3 a c)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{9 b^{5/3} (b^2 - 3 a c)^{5/3}}
\end{aligned}$$

Result (type 7, 112 leaves):

$$\begin{aligned}
& -\frac{\frac{3 (b+c x)}{3 a b + x (3 b^2 + 3 b c x + c^2 x^2)} + 2 c \operatorname{RootSum}\left[3 a b + 3 b^2 \#1 + 3 b c \#1^2 + c^2 \#1^3 \&, \frac{\operatorname{Log}[x - \#1]}{b^2 + 2 b c \#1 + c^2 \#1^2} \&\right]}{9 (b^3 - 3 a b c)}
\end{aligned}$$

Problem 14: Result is not expressed in closed-form.

$$\int \frac{1}{(3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^3} dx$$

Optimal (type 3, 305 leaves, 9 steps):

$$\begin{aligned}
& -\frac{c \left(\frac{b}{c} + x\right)}{6 b (b^2 - 3 a c) (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)^2} + \frac{5 c^2 \left(\frac{b}{c} + x\right)}{18 b^2 (b^2 - 3 a c)^2 (3 a b + 3 b^2 x + 3 b c x^2 + c^2 x^3)} - \frac{5 c^2 \operatorname{ArcTan}\left[\frac{b^{1/3} + \frac{2 (b+c x)}{(b^2 - 3 a c)^{1/3}}}{\sqrt{3} b^{1/3}}\right]}{9 \sqrt{3} b^{8/3} (b^2 - 3 a c)^{8/3}} + \\
& \frac{5 c^2 \operatorname{Log}\left[b - b^{1/3} (b^2 - 3 a c)^{1/3} + c x\right]}{27 b^{8/3} (b^2 - 3 a c)^{8/3}} - \frac{5 c^2 \operatorname{Log}\left[b^{2/3} (b^2 - 3 a c)^{2/3} + b^{1/3} c (b^2 - 3 a c)^{1/3} \left(\frac{b}{c} + x\right) + c^2 \left(\frac{b}{c} + x\right)^2\right]}{54 b^{8/3} (b^2 - 3 a c)^{8/3}}
\end{aligned}$$

Result (type 7, 149 leaves):

$$\begin{aligned}
& \frac{1}{54 (b^3 - 3 a b c)^2} \\
& \left(-\frac{3 (b+c x) (3 b^3 - 15 b^2 c x - 5 c^3 x^3 - 3 b c (8 a + 5 c x^2))}{(3 a b + x (3 b^2 + 3 b c x + c^2 x^2))^2} + 10 c^2 \operatorname{RootSum}\left[3 a b + 3 b^2 \#1 + 3 b c \#1^2 + c^2 \#1^3 \&, \frac{\operatorname{Log}[x - \#1]}{b^2 + 2 b c \#1 + c^2 \#1^2} \&\right] \right)
\end{aligned}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int (b x + c x^2 + d x^3)^n dx$$

Optimal (type 6, 132 leaves, 3 steps):

$$\frac{1}{1+n} x \left(1 + \frac{2 dx}{c - \sqrt{c^2 - 4 bd}}\right)^{-n} \left(1 + \frac{2 dx}{c + \sqrt{c^2 - 4 bd}}\right)^{-n} (bx + cx^2 + dx^3)^n \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2 dx}{c - \sqrt{c^2 - 4 bd}}, -\frac{2 dx}{c + \sqrt{c^2 - 4 bd}}\right]$$

Result (type 6, 438 leaves):

$$\begin{aligned} & \left(2^{-1-n} d \left(c + \sqrt{c^2 - 4 bd}\right) (2+n) x^2 \left(\frac{c - \sqrt{c^2 - 4 bd}}{2d} + x\right)^{-n} \left(\frac{c - \sqrt{c^2 - 4 bd} + 2dx}{d}\right)^{1+n} \right. \\ & \left. \left(2b + \left(c - \sqrt{c^2 - 4 bd}\right) x\right)^2 (x (b + x (c + dx)))^{-1+n} \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2 dx}{c + \sqrt{c^2 - 4 bd}}, \frac{2 dx}{-c + \sqrt{c^2 - 4 bd}}\right]\right) / \\ & \left(\left(-c + \sqrt{c^2 - 4 bd}\right) (1+n) \left(c + \sqrt{c^2 - 4 bd} + 2dx\right) \left(-2b (2+n) \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{2 dx}{c + \sqrt{c^2 - 4 bd}}, \frac{2 dx}{-c + \sqrt{c^2 - 4 bd}}\right] + \right. \right. \right. \\ & n x \left(\left(-c + \sqrt{c^2 - 4 bd}\right) \text{AppellF1}\left[2+n, 1-n, -n, 3+n, -\frac{2 dx}{c + \sqrt{c^2 - 4 bd}}, \frac{2 dx}{-c + \sqrt{c^2 - 4 bd}}\right] - \right. \\ & \left. \left. \left. \left(c + \sqrt{c^2 - 4 bd}\right) \text{AppellF1}\left[2+n, -n, 1-n, 3+n, -\frac{2 dx}{c + \sqrt{c^2 - 4 bd}}, \frac{2 dx}{-c + \sqrt{c^2 - 4 bd}}\right]\right)\right) \end{aligned}$$

Problem 32: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + dx^3)^n dx$$

Optimal (type 5, 35 leaves, 2 steps):

$$\frac{x (a + dx^3)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{4}{3} + n, \frac{4}{3}, -\frac{dx^3}{a}\right]}{a}$$

Result (type 6, 196 leaves):

$$\begin{aligned} & \frac{1}{d^{1/3} (1+n)} 2^{-n} \left((-1)^{2/3} a^{1/3} + d^{1/3} x\right) \left(\frac{a^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}\right)^{-n} \left(\frac{\frac{1}{3} \left(1 + \frac{d^{1/3} x}{a^{1/3}}\right)}{3 \frac{1}{3} + \sqrt{3}}\right)^{-n} \\ & (a + dx^3)^n \text{AppellF1}\left[1+n, -n, -n, 2+n, -\frac{\frac{1}{3} \left((-1)^{2/3} a^{1/3} + d^{1/3} x\right)}{\sqrt{3} a^{1/3}}, \frac{\frac{1}{3} + \sqrt{3} - \frac{2 \frac{1}{3} d^{1/3} x}{a^{1/3}}}{3 \frac{1}{3} + \sqrt{3}}\right] \end{aligned}$$

Problem 37: Result is not expressed in closed-form.

$$\int \frac{1}{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} dx$$

Optimal (type 3, 529 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{d \operatorname{ArcTanh} \left[\frac{\sqrt{2} c + c^{1/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} + \sqrt{2} d x}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} \right]}{2 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} + \frac{d \operatorname{ArcTanh} \left[\frac{c^{1/4} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} - \sqrt{2} (c + d x)}{c^{1/4} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} \right]}{2 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}} - \\
 & \frac{d \operatorname{Log} \left[\sqrt{c} \sqrt{c^3 + 4 a d^2} - \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right]}{4 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}} + \\
 & \frac{d \operatorname{Log} \left[\sqrt{c} \sqrt{c^3 + 4 a d^2} + \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right]}{4 \sqrt{2} c^{3/4} \sqrt{c^3 + 4 a d^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}}
 \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{1}{4} \operatorname{RootSum} \left[4 a c + 4 c^2 \#1^2 + 4 c d \#1^3 + d^2 \#1^4 \&, \frac{\operatorname{Log} [x - \#1]}{2 c^2 \#1 + 3 c d \#1^2 + d^2 \#1^3} \& \right]$$

Problem 38: Result is not expressed in closed-form.

$$\int \frac{1}{(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^2} dx$$

Optimal (type 3, 746 leaves, 11 steps):

$$\begin{aligned}
& - \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4 a d^2 - c d^2 \left(\frac{c}{d} + x\right)^2\right)}{16 a c \left(c^3 + 4 a d^2\right) \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4\right)} - \frac{d \left(c^3 + 12 a d^2 + c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \frac{c+c^{1/4} \sqrt{c^{3/2}+\sqrt{c^3+4 a d^2}}}{c^{1/4} \sqrt{c^{3/2}-\sqrt{c^3+4 a d^2}}} + \sqrt{2} d x}{32 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}}\right]}{+} \\
& \frac{d \left(c^3 + 12 a d^2 + c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{ArcTanh}\left[\frac{\frac{c^{1/4} \sqrt{c^{3/2}+\sqrt{c^3+4 a d^2}}}{c^{1/4} \sqrt{c^{3/2}-\sqrt{c^3+4 a d^2}}} - \sqrt{2} (c+d x)}{32 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} - \sqrt{c^3 + 4 a d^2}}}\right]}{-} \\
& \left(d \left(c^3 + 12 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{Log}\left[\sqrt{c} \sqrt{c^3 + 4 a d^2} - \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x\right) + d^2 \left(\frac{c}{d} + x\right)^2\right]\right) / \\
& \left(64 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}\right) + \\
& \left(d \left(c^3 + 12 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2}\right) \operatorname{Log}\left[\sqrt{c} \sqrt{c^3 + 4 a d^2} + \sqrt{2} c^{1/4} d \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}} \left(\frac{c}{d} + x\right) + d^2 \left(\frac{c}{d} + x\right)^2\right]\right) / \\
& \left(64 \sqrt{2} a c^{7/4} \left(c^3 + 4 a d^2\right)^{3/2} \sqrt{c^{3/2} + \sqrt{c^3 + 4 a d^2}}\right)
\end{aligned}$$

Result (type 7, 182 leaves):

$$\begin{aligned}
& \frac{1}{64 a c \left(c^3 + 4 a d^2\right)} \left(\frac{4 (c+d x) (4 a d + c x (2 c + d x))}{4 a c + x^2 (2 c + d x)^2} + \right. \\
& \left. \operatorname{RootSum}\left[4 a c + 4 c^2 \#1^2 + 4 c d \#1^3 + d^2 \#1^4 \&, \frac{2 c^3 \operatorname{Log}[x - \#1] + 12 a d^2 \operatorname{Log}[x - \#1] + 2 c^2 d \operatorname{Log}[x - \#1] \#1 + c d^2 \operatorname{Log}[x - \#1] \#1^2}{2 c^2 \#1 + 3 c d \#1^2 + d^2 \#1^3} \&\right] \right)
\end{aligned}$$

Problem 43: Result is not expressed in closed-form.

$$\int \frac{1}{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} dx$$

Optimal (type 3, 153 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{d+4 e x}{\sqrt{3 d^2-2 \sqrt{d^4-64 a e^3}}}\right]}{\sqrt{d^4-64 a e^3} \sqrt{3 d^2-2 \sqrt{d^4-64 a e^3}}}-\frac{2 \operatorname{ArcTanh}\left[\frac{d+4 e x}{\sqrt{3 d^2+2 \sqrt{d^4-64 a e^3}}}\right]}{\sqrt{d^4-64 a e^3} \sqrt{3 d^2+2 \sqrt{d^4-64 a e^3}}}$$

Result (type 7, 71 leaves):

$$-\operatorname{RootSum}\left[8 a e^2-d^3 \#1+8 d e^2 \#1^3+8 e^3 \#1^4 \&,\frac{\operatorname{Log}[x-\#1]}{d^3-24 d e^2 \#1^2-32 e^3 \#1^3}\&\right]$$

Problem 44: Result is not expressed in closed-form.

$$\int \frac{1}{(8 a e^2-d^3 x+8 d e^2 x^3+8 e^3 x^4)^2} dx$$

Optimal (type 3, 342 leaves, 5 steps):

$$\frac{2 e \left(\frac{d}{4 e}+x\right) \left(13 d^4-256 a e^3-48 d^2 e^2 \left(\frac{d}{4 e}+x\right)^2\right)}{\left(5 d^8-64 a d^4 e^3-16384 a^2 e^6\right) \left(8 a e^2-d^3 x+8 d e^2 x^3+8 e^3 x^4\right)} - \frac{24 e \left(d^4+128 a e^3-d^2 \sqrt{d^4-64 a e^3}\right) \operatorname{ArcTanh}\left[\frac{d+4 e x}{\sqrt{3 d^2-2 \sqrt{d^4-64 a e^3}}}\right]}{\left(d^4-64 a e^3\right)^{3/2} \left(5 d^4+256 a e^3\right) \sqrt{3 d^2-2 \sqrt{d^4-64 a e^3}}} + \frac{24 e \left(d^4+128 a e^3+d^2 \sqrt{d^4-64 a e^3}\right) \operatorname{ArcTanh}\left[\frac{d+4 e x}{\sqrt{3 d^2+2 \sqrt{d^4-64 a e^3}}}\right]}{\left(d^4-64 a e^3\right)^{3/2} \left(5 d^4+256 a e^3\right) \sqrt{3 d^2+2 \sqrt{d^4-64 a e^3}}}$$

Result (type 7, 234 leaves):

$$\frac{(d+4 e x) \left(5 d^4-128 a e^3-12 d^3 e x-24 d^2 e^2 x^2\right)}{\left(d^4-64 a e^3\right) \left(5 d^4+256 a e^3\right) \left(8 a e^2-d^3 x+8 d e^2 x^3+8 e^3 x^4\right)} + \frac{48 e^2 \operatorname{RootSum}\left[8 a e^2-d^3 \#1+8 d e^2 \#1^3+8 e^3 \#1^4 \&,\frac{32 a e^2 \operatorname{Log}[x-\#1]+d^3 \operatorname{Log}[x-\#1] \#1+2 d^2 e \operatorname{Log}[x-\#1] \#1^2}{-d^3+24 d e^2 \#1^2+32 e^3 \#1^3}\&\right]}{-5 d^8+64 a d^4 e^3+16384 a^2 e^6}$$

Problem 49: Result is not expressed in closed-form.

$$\int \frac{1}{8+8 x-x^3+8 x^4} dx$$

Optimal (type 3, 268 leaves, 16 steps):

$$\begin{aligned}
& -\frac{\text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6 \sqrt{7}}\right]}{12 \sqrt{7}}-\frac{1}{12} \sqrt{\frac{109+67 \sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]-\frac{1}{12} \sqrt{\frac{109+67 \sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]- \\
& \frac{1}{24} \sqrt{\frac{-109+67 \sqrt{29}}{1218}} \text{Log}\left[3 \sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]+\frac{1}{24} \sqrt{\frac{-109+67 \sqrt{29}}{1218}} \text{Log}\left[3 \sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 45 leaves) :

$$\text{RootSum}\left[8+8 \#1-\#1^3+8 \#1^4 \&, \frac{\text{Log}[x-\#1]}{8-3 \#1^2+32 \#1^3} \&\right]$$

Problem 50: Result is not expressed in closed-form.

$$\int \frac{1}{(8+8 x-x^3+8 x^4)^2} dx$$

Optimal (type 3, 357 leaves, 18 steps) :

$$\begin{aligned}
& -\frac{\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}-\frac{17 \text{ArcTan}\left[\frac{3-\left(1+\frac{4}{x}\right)^2}{6 \sqrt{7}}\right]}{1008 \sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327 \sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]}{87696}- \\
& \frac{\sqrt{\frac{180983329+45923327 \sqrt{29}}{1218}} \text{ArcTan}\left[\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right]-\sqrt{\frac{-180983329+45923327 \sqrt{29}}{1218}} \text{Log}\left[3 \sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]}{175392}+ \\
& \frac{\sqrt{\frac{-180983329+45923327 \sqrt{29}}{1218}} \text{Log}\left[3 \sqrt{29}+\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right]}{175392}
\end{aligned}$$

Result (type 7, 113 leaves) :

$$\begin{aligned}
& \frac{544+1539 x-1146 x^2+784 x^3}{43848\left(8+8 x-x^3+8 x^4\right)}+\frac{\text{RootSum}\left[8+8 \#1-\#1^3+8 \#1^4 \&, \frac{2243 \text{Log}[x-\#1]-1097 \text{Log}[x-\#1] \#1+392 \text{Log}[x-\#1] \#1^2}{8-3 \#1^2+32 \#1^3} \&\right]}{21924}
\end{aligned}$$

Problem 55: Result is not expressed in closed-form.

$$\int \frac{1}{1 + 4x + 4x^2 + 4x^4} dx$$

Optimal (type 3, 234 leaves, 15 steps):

$$\begin{aligned} & \frac{1}{2} \operatorname{ArcTan}\left[\frac{1}{2} \left(-1 + \left(1 + \frac{1}{x}\right)^2\right)\right] - \frac{1}{2} \sqrt{\frac{1}{5} \left(2 + \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 - \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] - \frac{1}{2} \sqrt{\frac{1}{5} \left(2 + \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 + \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] - \\ & \frac{1}{4} \sqrt{\frac{1}{5} \left(-2 + \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} - \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right] + \frac{1}{4} \sqrt{\frac{1}{5} \left(-2 + \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} + \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right] \end{aligned}$$

Result (type 7, 47 leaves):

$$\frac{1}{4} \operatorname{RootSum}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, \frac{\operatorname{Log}[x - \#1]}{1 + 2 \#1 + 4 \#1^3} \&\right]$$

Problem 56: Result is not expressed in closed-form.

$$\int \frac{1}{(1 + 4x + 4x^2 + 4x^4)^2} dx$$

Optimal (type 3, 317 leaves, 17 steps):

$$\begin{aligned}
& -\frac{17 - \left(1 + \frac{1}{x}\right)^2}{2 \left(5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right)} + \frac{\left(59 - 17 \left(1 + \frac{1}{x}\right)^2\right) \left(1 + \frac{1}{x}\right)}{10 \left(5 - 2 \left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right)} + \frac{7}{4} \operatorname{ArcTan}\left[\frac{1}{2} \left(-1 + \left(1 + \frac{1}{x}\right)^2\right)\right] - \\
& \frac{1}{20} \sqrt{\frac{1}{10} \left(5959 + 2665 \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 - \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] - \frac{1}{20} \sqrt{\frac{1}{10} \left(5959 + 2665 \sqrt{5}\right)} \operatorname{ArcTan}\left[\frac{2 + \sqrt{2 \left(1 + \sqrt{5}\right)} + \frac{2}{x}}{\sqrt{2 \left(-1 + \sqrt{5}\right)}}\right] + \\
& \frac{1}{40} \sqrt{\frac{1}{10} \left(-5959 + 2665 \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} - \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right] - \\
& \frac{1}{40} \sqrt{\frac{1}{10} \left(-5959 + 2665 \sqrt{5}\right)} \operatorname{Log}\left[\sqrt{5} + \sqrt{2 \left(1 + \sqrt{5}\right)} \left(1 + \frac{1}{x}\right) + \left(1 + \frac{1}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 108 leaves):

$$\frac{1}{40} \left(\frac{38 + 84 x - 32 x^2 + 72 x^3}{1 + 4 x + 4 x^2 + 4 x^4} + \operatorname{RootSum}\left[1 + 4 \#1 + 4 \#1^2 + 4 \#1^4 \&, \frac{27 \operatorname{Log}[x - \#1] - 16 \operatorname{Log}[x - \#1] \#1 + 18 \operatorname{Log}[x - \#1] \#1^2}{1 + 2 \#1 + 4 \#1^3} \&\right] \right)$$

Problem 61: Result is not expressed in closed-form.

$$\int \frac{1}{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4} dx$$

Optimal (type 3, 263 leaves, 16 steps):

$$\begin{aligned}
& -\frac{1}{4} \sqrt{\frac{5167 + 235 \sqrt{517}}{40326}} \operatorname{ArcTan}\left[\frac{6 - \sqrt{2 \left(19 + \sqrt{517}\right)} + \frac{8}{x}}{\sqrt{2 \left(-19 + \sqrt{517}\right)}}\right] - \frac{1}{4} \sqrt{\frac{5167 + 235 \sqrt{517}}{40326}} \operatorname{ArcTan}\left[\frac{6 + \sqrt{2 \left(19 + \sqrt{517}\right)} + \frac{8}{x}}{\sqrt{2 \left(-19 + \sqrt{517}\right)}}\right] + \\
& \frac{1}{4} \sqrt{\frac{3}{13}} \operatorname{ArcTan}\left[\frac{8 + 12 x - 5 x^2}{\sqrt{39} x^2}\right] - \frac{1}{8} \sqrt{\frac{-5167 + 235 \sqrt{517}}{40326}} \operatorname{Log}\left[\sqrt{517} - \sqrt{2 \left(19 + \sqrt{517}\right)} \left(3 + \frac{4}{x}\right) + \left(3 + \frac{4}{x}\right)^2\right] + \\
& \frac{1}{8} \sqrt{\frac{-5167 + 235 \sqrt{517}}{40326}} \operatorname{Log}\left[\sqrt{517} + \sqrt{2 \left(19 + \sqrt{517}\right)} \left(3 + \frac{4}{x}\right) + \left(3 + \frac{4}{x}\right)^2\right]
\end{aligned}$$

Result (type 7, 55 leaves):

$$\text{RootSum}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \frac{\text{Log}[x - \#1]}{24 + 16 \#1 - 45 \#1^2 + 32 \#1^3} \&\right]$$

Problem 62: Result is not expressed in closed-form.

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx$$

Optimal (type 3, 366 leaves, 18 steps):

$$\begin{aligned} & -\frac{3 \left(3359 - 107 \left(3 + \frac{4}{x}\right)^2\right)}{208 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right)} - \\ & \frac{\sqrt{\frac{19+\sqrt{517}}{40326}} \left(1678181 + 74897 \sqrt{517}\right) \text{ArcTan}\left[\frac{6-\sqrt{2 \left(19+\sqrt{517}\right)}}{\sqrt{2 \left(-19+\sqrt{517}\right)}}\right] + \sqrt{\frac{19+\sqrt{517}}{40326}} \left(1678181 + 74897 \sqrt{517}\right) \text{ArcTan}\left[\frac{6+\sqrt{2 \left(19+\sqrt{517}\right)}}{\sqrt{2 \left(-19+\sqrt{517}\right)}}\right]}{645216} + \\ & \frac{\frac{73}{208} \sqrt{\frac{3}{13}} \text{ArcTan}\left[\frac{8+12x-5x^2}{\sqrt{39} x^2}\right] - \sqrt{\frac{-59644114671451+2623170438295 \sqrt{517}}{40326}} \text{Log}\left[\sqrt{517}\right] - \sqrt{2 \left(19+\sqrt{517}\right)} \left(3+\frac{4}{x}\right) + \left(3+\frac{4}{x}\right)^2}{645216} + \\ & \frac{\sqrt{\frac{-59644114671451+2623170438295 \sqrt{517}}{40326}} \text{Log}\left[\sqrt{517}\right] + \sqrt{2 \left(19+\sqrt{517}\right)} \left(3+\frac{4}{x}\right) + \left(3+\frac{4}{x}\right)^2}{645216} \end{aligned}$$

Result (type 7, 128 leaves):

$$\frac{72888 + 89033x - 94314x^2 + 39280x^3}{161304 (8 + 24x + 8x^2 - 15x^3 + 8x^4)} + \frac{\text{RootSum}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, \frac{74897 \text{Log}[x - \#1] - 57489 \text{Log}[x - \#1] \#1 + 19640 \text{Log}[x - \#1] \#1^2}{24 + 16 \#1 - 45 \#1^2 + 32 \#1^3} \&\right]}{80652}$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{(a + bx)^6}{6b}$$

Result (type 1, 61 leaves) :

$$a^5 x + \frac{5}{2} a^4 b x^2 + \frac{10}{3} a^3 b^2 x^3 + \frac{5}{2} a^2 b^3 x^4 + a b^4 x^5 + \frac{b^5 x^6}{6}$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - (c + d x)^2} dx$$

Optimal (type 3, 10 leaves, 2 steps) :

$$\frac{\text{ArcTanh}[c + d x]}{d}$$

Result (type 3, 32 leaves) :

$$-\frac{\text{Log}[1 - c - d x]}{2 d} + \frac{\text{Log}[1 + c + d x]}{2 d}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - (1 + x)^2} dx$$

Optimal (type 3, 4 leaves, 2 steps) :

$$\text{ArcTanh}[1 + x]$$

Result (type 3, 15 leaves) :

$$-\frac{\text{Log}[x]}{2} + \frac{1}{2} \text{Log}[2 + x]$$

Problem 103: Result is not expressed in closed-form.

$$\int \frac{x^3}{a + b (c + d x)^3} dx$$

Optimal (type 3, 234 leaves, 11 steps) :

$$\frac{x}{b d^3} + \frac{\left(a - 3 a^{1/3} b^{2/3} c^2 + b c^3\right) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{4/3} d^4} - \frac{\left(a + 3 a^{1/3} b^{2/3} c^2 + b c^3\right) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{4/3} d^4} +$$

$$\frac{\left(a + 3 a^{1/3} b^{2/3} c^2 + b c^3\right) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{4/3} d^4} - \frac{c \operatorname{Log}\left[a + b (c + d x)^3\right]}{b d^4}$$

Result (type 7, 132 leaves):

$$-\frac{1}{3 b^2 d^4} \left(-3 b d x + \operatorname{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{a \operatorname{Log}[x - \#1] + b c^3 \operatorname{Log}[x - \#1] + 3 b c^2 d \operatorname{Log}[x - \#1] \#1 + 3 b c d^2 \operatorname{Log}[x - \#1] \#1^2 \&]\right] \right)$$

Problem 104: Result is not expressed in closed-form.

$$\int \frac{x^2}{a + b (c + d x)^3} dx$$

Optimal (type 3, 210 leaves, 9 steps):

$$\frac{c (2 a^{1/3} - b^{1/3} c) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{2/3} d^3} + \frac{c (2 a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{2/3} d^3} -$$

$$\frac{c (2 a^{1/3} + b^{1/3} c) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{2/3} d^3} + \frac{\operatorname{Log}\left[a + b (c + d x)^3\right]}{3 b d^3}$$

Result (type 7, 81 leaves):

$$\operatorname{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{\operatorname{Log}[x - \#1] \#1^2}{c^2 + 2 c d \#1 + d^2 \#1^2} \&\right]$$

$$\frac{3 b d}{}$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{x}{a + b (c + d x)^3} dx$$

Optimal (type 3, 180 leaves, 7 steps):

$$-\frac{\left(a^{1/3} - b^{1/3} c\right) \operatorname{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} b^{2/3} d^2} - \frac{\left(a^{1/3} + b^{1/3} c\right) \operatorname{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} b^{2/3} d^2} + \frac{\left(a^{1/3} + b^{1/3} c\right) \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} b^{2/3} d^2}$$

Result (type 7, 79 leaves):

$$\frac{\text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{\text{Log}[x - \#1] \#1}{c^2 + 2 c d \#1 + d^2 \#1^2} \&\right]}{3 b d}$$

Problem 107: Result is not expressed in closed-form.

$$\int \frac{1}{x \left(a + b (c + d x)^3\right)} dx$$

Optimal (type 3, 224 leaves, 11 steps):

$$\frac{b^{1/3} c \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a^{2/3} - a^{1/3} b^{1/3} c + b^{2/3} c^2)} + \frac{\text{Log}[x]}{a + b c^3} - \frac{\text{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} (a^{1/3} + b^{1/3} c)} - \frac{(2 a^{1/3} - b^{1/3} c) \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} (a^{2/3} - a^{1/3} b^{1/3} c + b^{2/3} c^2)}$$

Result (type 7, 119 leaves):

$$-\frac{1}{3 (a + b c^3)} \left(-3 \text{Log}[x] + \text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{3 c^2 \text{Log}[x - \#1] + 3 c d \text{Log}[x - \#1] \#1 + d^2 \text{Log}[x - \#1] \#1^2}{c^2 + 2 c d \#1 + d^2 \#1^2} \&\right] \right)$$

Problem 108: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 \left(a + b (c + d x)^3\right)} dx$$

Optimal (type 3, 314 leaves, 11 steps):

$$\begin{aligned} & -\frac{1}{(a + b c^3) x} + \frac{b^{1/3} (a^{1/3} - b^{1/3} c) (a^{1/3} + b^{1/3} c)^3 d \text{ArcTan}\left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3} (a + b c^3)^2} - \\ & \frac{3 b c^2 d \text{Log}[x]}{(a + b c^3)^2} + \frac{b^{1/3} (a^{1/3} (a - 2 b c^3) - b^{1/3} c (2 a - b c^3)) d \text{Log}\left[a^{1/3} + b^{1/3} (c + d x)\right]}{3 a^{2/3} (a + b c^3)^2} - \\ & \frac{b^{1/3} (a^{1/3} (a - 2 b c^3) - b^{1/3} c (2 a - b c^3)) d \text{Log}\left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2\right]}{6 a^{2/3} (a + b c^3)^2} + \frac{b c^2 d \text{Log}\left[a + b (c + d x)^3\right]}{(a + b c^3)^2} \end{aligned}$$

Result (type 7, 173 leaves):

$$\begin{aligned} & \frac{1}{3 (a + b c^3)^2 x} \left(-3 (a + b c^3 + 3 b c^2 d x \text{Log}[x]) + d x \text{RootSum}\left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \right. \right. \\ & \left. \left. \frac{1}{c^2 + 2 c d \#1 + d^2 \#1^2} (-3 a c \text{Log}[x - \#1] + 6 b c^4 \text{Log}[x - \#1] - a d \text{Log}[x - \#1] \#1 + 8 b c^3 d \text{Log}[x - \#1] \#1 + 3 b c^2 d^2 \text{Log}[x - \#1] \#1^2) \&\right] \right) \end{aligned}$$

Problem 109: Result is not expressed in closed-form.

$$\int \frac{1}{x^3 (a + b (c + d x)^3)} dx$$

Optimal (type 3, 393 leaves, 11 steps):

$$\begin{aligned} & -\frac{1}{2 (a + b c^3) x^2} + \frac{3 b c^2 d}{(a + b c^3)^2 x} + \frac{b^{2/3} (a^{1/3} + b^{1/3} c)^3 (a - 3 a^{2/3} b^{1/3} c + b c^3) d^2 \operatorname{ArcTan} \left[\frac{a^{1/3} - 2 b^{1/3} (c + d x)}{\sqrt{3} a^{1/3}} \right]}{\sqrt{3} a^{2/3} (a + b c^3)^3} - \\ & \frac{3 b c (a - 2 b c^3) d^2 \operatorname{Log}[x]}{(a + b c^3)^3} - \frac{b^{2/3} (a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6) d^2 \operatorname{Log} \left[a^{1/3} + b^{1/3} (c + d x) \right]}{3 a^{2/3} (a + b c^3)^3} + \\ & \frac{b^{2/3} (a^2 + 6 a^{4/3} b^{2/3} c^2 - 7 a b c^3 - 3 a^{1/3} b^{5/3} c^5 + b^2 c^6) d^2 \operatorname{Log} \left[a^{2/3} - a^{1/3} b^{1/3} (c + d x) + b^{2/3} (c + d x)^2 \right]}{6 a^{2/3} (a + b c^3)^3} + \frac{b c (a - 2 b c^3) d^2 \operatorname{Log} \left[a + b (c + d x)^3 \right]}{(a + b c^3)^3} \end{aligned}$$

Result (type 7, 244 leaves):

$$\begin{aligned} & -\frac{1}{6 (a + b c^3)^3 x^2} \left(3 (a + b c^3) (a + b c^2 (c - 6 d x)) + 18 b c (a - 2 b c^3) d^2 x^2 \operatorname{Log}[x] + \right. \\ & 2 d^2 x^2 \operatorname{RootSum} \left[a + b c^3 + 3 b c^2 d \#1 + 3 b c d^2 \#1^2 + b d^3 \#1^3 \&, \frac{1}{c^2 + 2 c d \#1 + d^2 \#1^2} (a^2 \operatorname{Log}[x - \#1] - 16 a b c^3 \operatorname{Log}[x - \#1] + \right. \\ & \left. \left. 10 b^2 c^6 \operatorname{Log}[x - \#1] - 12 a b c^2 d \operatorname{Log}[x - \#1] \#1 + 15 b^2 c^5 d \operatorname{Log}[x - \#1] \#1 - 3 a b c d^2 \operatorname{Log}[x - \#1] \#1^2 + 6 b^2 c^4 d^2 \operatorname{Log}[x - \#1] \#1^2) \& \right] \right) \end{aligned}$$

Problem 110: Result is not expressed in closed-form.

$$\int \frac{x^3}{a + b (c + d x)^4} dx$$

Optimal (type 3, 356 leaves, 16 steps):

$$\begin{aligned} & \frac{3 c^2 \operatorname{ArcTan} \left[\frac{\sqrt{b} (c + d x)^2}{\sqrt{a}} \right]}{2 \sqrt{a} \sqrt{b} d^4} + \frac{c (3 \sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} b^{1/4} (c + d x)}{a^{1/4}} \right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \\ & \frac{c (3 \sqrt{a} + \sqrt{b} c^2) \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} b^{1/4} (c + d x)}{a^{1/4}} \right]}{2 \sqrt{2} a^{3/4} b^{3/4} d^4} - \frac{c (3 \sqrt{a} - \sqrt{b} c^2) \operatorname{Log} \left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2 \right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \\ & \frac{c (3 \sqrt{a} - \sqrt{b} c^2) \operatorname{Log} \left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c + d x) + \sqrt{b} (c + d x)^2 \right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{\operatorname{Log} \left[a + b (c + d x)^4 \right]}{4 b d^4} \end{aligned}$$

Result (type 7, 106 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1^3}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \&\right]}{4 b d}$$

Problem 111: Result is not expressed in closed-form.

$$\int \frac{x^2}{a + b (c + d x)^4} dx$$

Optimal (type 3, 318 leaves, 14 steps):

$$\begin{aligned} & \frac{c \text{ArcTan}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a}}\right] - \left(\sqrt{a} + \sqrt{b} c^2\right) \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right] + \left(\sqrt{a} + \sqrt{b} c^2\right) \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{\sqrt{a} \sqrt{b} d^3} + \frac{2 \sqrt{2} a^{3/4} b^{3/4} d^3}{2 \sqrt{2} a^{3/4} b^{3/4} d^3} + \\ & \frac{\left(\sqrt{a} - \sqrt{b} c^2\right) \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right] - \left(\sqrt{a} - \sqrt{b} c^2\right) \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{3/4} d^3} \end{aligned}$$

Result (type 7, 106 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1^2}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \&\right]}{4 b d}$$

Problem 112: Result is not expressed in closed-form.

$$\int \frac{x}{a + b (c + d x)^4} dx$$

Optimal (type 3, 261 leaves, 14 steps):

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a}}\right] + c \text{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right] - c \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{a} \sqrt{b} d^2} + \frac{2 \sqrt{2} a^{3/4} b^{1/4} d^2}{2 \sqrt{2} a^{3/4} b^{1/4} d^2} + \\ & \frac{c \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right] - c \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} b^{1/4} d^2} \end{aligned}$$

Result (type 7, 104 leaves):

$$\frac{\text{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \frac{\text{Log}[x-\#1] \#1}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \&\right]}{4 b d}$$

Problem 114: Result is not expressed in closed-form.

$$\int \frac{1}{x (a + b (c + d x)^4)} dx$$

Optimal (type 3, 393 leaves, 18 steps):

$$\begin{aligned} & -\frac{\sqrt{b} c^2 \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a}}\right]}{2 \sqrt{a} (a+b c^4)} + \frac{b^{1/4} c \left(\sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a+b c^4)} - \\ & \frac{b^{1/4} c \left(\sqrt{a} + \sqrt{b} c^2\right) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a+b c^4)} + \frac{\operatorname{Log}[x]}{a+b c^4} - \frac{b^{1/4} c \left(\sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} (a+b c^4)} + \\ & \frac{b^{1/4} c \left(\sqrt{a} - \sqrt{b} c^2\right) \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} (a+b c^4)} - \frac{\operatorname{Log}[a+b (c+d x)^4]}{4 (a+b c^4)} \end{aligned}$$

Result (type 7, 163 leaves):

$$\begin{aligned} & -\frac{1}{4 (a+b c^4)} \left(-4 \operatorname{Log}[x] + \operatorname{RootSum}\left[a + b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \right. \right. \\ & \left. \left. \frac{4 c^3 \operatorname{Log}[x - \#1] + 6 c^2 d \operatorname{Log}[x - \#1] \#1 + 4 c d^2 \operatorname{Log}[x - \#1] \#1^2 + d^3 \operatorname{Log}[x - \#1] \#1^3}{c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3} \& \right] \right) \end{aligned}$$

Problem 115: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (a + b (c + d x)^4)} dx$$

Optimal (type 3, 496 leaves, 18 steps):

$$\begin{aligned}
& -\frac{1}{(a+b c^4) x} - \frac{\sqrt{b} c (a-b c^4) d \operatorname{ArcTan}\left[\frac{\sqrt{b} (c+d x)^2}{\sqrt{a}}\right]}{\sqrt{a} (a+b c^4)^2} + \frac{b^{1/4} \left(\sqrt{a} (a-3 b c^4) + \sqrt{b} c^2 (3 a-b c^4)\right) d \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a+b c^4)^2} - \\
& \frac{b^{1/4} \left(\sqrt{a} (a-3 b c^4) + \sqrt{b} c^2 (3 a-b c^4)\right) d \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} (c+d x)}{a^{1/4}}\right]}{2 \sqrt{2} a^{3/4} (a+b c^4)^2} - \frac{4 b c^3 d \operatorname{Log}[x]}{(a+b c^4)^2} - \\
& \frac{b^{1/4} \left(\sqrt{a} (a-3 b c^4) - \sqrt{b} c^2 (3 a-b c^4)\right) d \operatorname{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} (a+b c^4)^2} + \\
& \frac{b^{1/4} \left(\sqrt{a} (a-3 b c^4) - \sqrt{b} c^2 (3 a-b c^4)\right) d \operatorname{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} (c+d x) + \sqrt{b} (c+d x)^2\right]}{4 \sqrt{2} a^{3/4} (a+b c^4)^2} + \frac{b c^3 d \operatorname{Log}[a+b (c+d x)^4]}{(a+b c^4)^2}
\end{aligned}$$

Result (type 7, 238 leaves):

$$\begin{aligned}
& \frac{1}{4 (a+b c^4)^2 x} \left(-4 (a+b c^4) + 4 b c^3 d x \operatorname{Log}[x] \right) + \\
& d x \operatorname{RootSum}\left[a+b c^4 + 4 b c^3 d \#1 + 6 b c^2 d^2 \#1^2 + 4 b c d^3 \#1^3 + b d^4 \#1^4 \&, \left(-6 a c^2 \operatorname{Log}[x-\#1] + 10 b c^6 \operatorname{Log}[x-\#1] - 4 a c d \operatorname{Log}[x-\#1] \#1 + 20 b c^5 d \operatorname{Log}[x-\#1] \#1 - a d^2 \operatorname{Log}[x-\#1] \#1^2 + 15 b c^4 d^2 \operatorname{Log}[x-\#1] \#1^2 + 4 b c^3 d^3 \operatorname{Log}[x-\#1] \#1^3 \right) / (c^3 + 3 c^2 d \#1 + 3 c d^2 \#1^2 + d^3 \#1^3) \& \right]
\end{aligned}$$

Problem 120: Result is not expressed in closed-form.

$$\int \frac{1}{a + 8 x - 8 x^2 + 4 x^3 - x^4} dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1-\sqrt{4+a}}} + \frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1+\sqrt{4+a}}}$$

Result (type 7, 57 leaves):

$$-\frac{1}{4} \operatorname{RootSum}\left[a + 8 \#1 - 8 \#1^2 + 4 \#1^3 - \#1^4 \&, \frac{\operatorname{Log}[x-\#1]}{-2 + 4 \#1 - 3 \#1^2 + \#1^3} \& \right]$$

Problem 121: Result is not expressed in closed-form.

$$\int \frac{1}{(a + 8 x - 8 x^2 + 4 x^3 - x^4)^2} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{4 (12 + 7 a + a^2) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)} - \frac{\left(10 + 3 a + \sqrt{4 + a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8 (3 + a) (4 + a)^{3/2} \sqrt{1 - \sqrt{4 + a}}} + \frac{\left(10 + 3 a - \sqrt{4 + a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8 (3 + a) (4 + a)^{3/2} \sqrt{1 + \sqrt{4 + a}}}$$

Result (type 7, 150 leaves):

$$\frac{(-1 + x) (6 + a - 2 x + x^2)}{4 (3 + a) (4 + a) (a - x (-8 + 8 x - 4 x^2 + x^3))} - \frac{\text{RootSum}\left[a + 8 \#1 - 8 \#1^2 + 4 \#1^3 - \#1^4 \&, \frac{12 \text{Log}[x - \#1] + 3 a \text{Log}[x - \#1] - 2 \text{Log}[x - \#1] \#1 + \text{Log}[x - \#1] \#1^2}{-2 + 4 \#1 - 3 \#1^2 + \#1^3} \&\right]}{16 (12 + 7 a + a^2)}$$

Problem 122: Result is not expressed in closed-form.

$$\int \frac{1}{(a + 8 x - 8 x^2 + 4 x^3 - x^4)^3} dx$$

Optimal (type 3, 252 leaves, 6 steps):

$$\frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{8 (12 + 7 a + a^2) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^2} + \frac{\left((6 + a) (25 + 7 a) + 6 (7 + 2 a) (-1 + x)^2\right) (-1 + x)}{32 (3 + a)^2 (4 + a)^2 \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)} - \frac{3 \left(80 + 7 a^2 + 14 \sqrt{4 + a} + a (47 + 4 \sqrt{4 + a})\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64 (3 + a)^2 (4 + a)^{5/2} \sqrt{1 - \sqrt{4 + a}}} - \frac{3 \left(14 + 4 a - \frac{80 + 47 a + 7 a^2}{\sqrt{4+a}}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64 (3 + a)^2 (4 + a)^2 \sqrt{1 + \sqrt{4 + a}}}$$

Result (type 7, 254 leaves):

$$\frac{1}{128} \left(\frac{16 (-1 + x) (6 + a - 2 x + x^2)}{(3 + a) (4 + a) (a - x (-8 + 8 x - 4 x^2 + x^3))^2} + \frac{4 (-1 + x) (7 a^2 + 6 (32 - 14 x + 7 x^2) + a (79 - 24 x + 12 x^2))}{(3 + a)^2 (4 + a)^2 (a - x (-8 + 8 x - 4 x^2 + x^3))} - \frac{1}{(12 + 7 a + a^2)^2} 3 \text{RootSum}\left[a + 8 \#1 - 8 \#1^2 + 4 \#1^3 - \#1^4 \&, \frac{1}{-2 + 4 \#1 - 3 \#1^2 + \#1^3} \right. \right. \\ \left. \left. \left(108 \text{Log}[x - \#1] + 55 a \text{Log}[x - \#1] + 7 a^2 \text{Log}[x - \#1] - 28 \text{Log}[x - \#1] \#1 - 8 a \text{Log}[x - \#1] \#1 + 14 \text{Log}[x - \#1] \#1^2 + 4 a \text{Log}[x - \#1] \#1^2\right) \&\right]$$

Problem 127: Result is not expressed in closed-form.

$$\int \frac{x}{a + 8 x - 8 x^2 + 4 x^3 - x^4} dx$$

Optimal (type 3, 116 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1-\sqrt{4+a}}} + \frac{\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2 \sqrt{4+a} \sqrt{1+\sqrt{4+a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2 \sqrt{4+a}}$$

Result (type 7, 59 leaves):

$$-\frac{1}{4} \text{RootSum}\left[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{\text{Log}[x-\#1] \#1}{-2+4 \#1-3 \#1^2+\#1^3} \&\right]$$

Problem 128: Result is not expressed in closed-form.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal (type 3, 231 leaves, 10 steps):

$$\begin{aligned} & \frac{1+(-1+x)^2}{4 (4+a) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right) (-1+x)}{4 \left(12+7 a+a^2\right) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)} - \\ & \frac{\left(10+3 a+\sqrt{4+a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8 (3+a) (4+a)^{3/2} \sqrt{1-\sqrt{4+a}}} + \frac{\left(10+3 a-\sqrt{4+a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8 (3+a) (4+a)^{3/2} \sqrt{1+\sqrt{4+a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{4 (4+a)^{3/2}} \end{aligned}$$

Result (type 7, 166 leaves):

$$\begin{aligned} & \frac{a+2 x-a x+a x^2+x^3}{4 (3+a) (4+a) \left(a-x \left(-8+8 x-4 x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{6 \text{Log}[x-\#1]+a \text{Log}[x-\#1]+4 \text{Log}[x-\#1] \#1+2 a \text{Log}[x-\#1] \#1+\text{Log}[x-\#1] \#1^2}{-2+4 \#1-3 \#1^2+\#1^3} \&\right]}{16 \left(12+7 a+a^2\right)} \end{aligned}$$

Problem 129: Result is not expressed in closed-form.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal (type 3, 349 leaves, 12 steps):

$$\begin{aligned}
& \frac{1 + (-1+x)^2}{8 (4+a) (3+a-2 (-1+x)^2 - (-1+x)^4)^2} + \frac{3 (1 + (-1+x)^2)}{16 (4+a)^2 (3+a-2 (-1+x)^2 - (-1+x)^4)} + \\
& \frac{(5+a+(-1+x)^2) (-1+x)}{8 (12+7 a+a^2) (3+a-2 (-1+x)^2 - (-1+x)^4)^2} + \frac{((6+a) (25+7 a) + 6 (7+2 a) (-1+x)^2) (-1+x)}{32 (3+a)^2 (4+a)^2 (3+a-2 (-1+x)^2 - (-1+x)^4)} - \\
& \frac{3 (80+7 a^2+14 \sqrt{4+a} + a (47+4 \sqrt{4+a})) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{64 (3+a)^2 (4+a)^{5/2} \sqrt{1-\sqrt{4+a}}} - \frac{3 (14+4 a - \frac{80+47 a+7 a^2}{\sqrt{4+a}}) \operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{64 (3+a)^2 (4+a)^2 \sqrt{1+\sqrt{4+a}}} + \frac{3 \operatorname{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{16 (4+a)^{5/2}}
\end{aligned}$$

Result (type 7, 284 leaves):

$$\begin{aligned}
& \frac{1}{128} \left(\frac{16 (a+2 x - a x + a x^2 + x^3)}{(3+a) (4+a) (a-x (-8+8 x - 4 x^2 + x^3))^2} + \frac{4 (a^2 (5-5 x + 6 x^2) + 6 (-14+28 x - 12 x^2 + 7 x^3) + a (-7+31 x + 12 x^3))}{(3+a)^2 (4+a)^2 (a-x (-8+8 x - 4 x^2 + x^3))} - \right. \\
& \left. \frac{1}{(12+7 a+a^2)^2} 3 \operatorname{RootSum}[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{1}{-2+4 \#1-3 \#1^2+\#1^3} (72 \operatorname{Log}[x-\#1] + 31 a \operatorname{Log}[x-\#1] + \right. \\
& \left. \left. 3 a^2 \operatorname{Log}[x-\#1] + 8 \operatorname{Log}[x-\#1] \#1 + 16 a \operatorname{Log}[x-\#1] \#1 + 4 a^2 \operatorname{Log}[x-\#1] \#1 + 14 \operatorname{Log}[x-\#1] \#1^2 + 4 a \operatorname{Log}[x-\#1] \#1^2) \&] \right)
\end{aligned}$$

Problem 134: Result is not expressed in closed-form.

$$\int \frac{x^2}{a+8 x-8 x^2+4 x^3-x^4} dx$$

Optimal (type 3, 99 leaves, 9 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{2 \sqrt{1-\sqrt{4+a}}} - \frac{\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{2 \sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{\sqrt{4+a}}
\end{aligned}$$

Result (type 7, 61 leaves):

$$-\frac{1}{4} \operatorname{RootSum}[a+8 \#1-8 \#1^2+4 \#1^3-\#1^4 \&, \frac{\operatorname{Log}[x-\#1] \#1^2}{-2+4 \#1-3 \#1^2+\#1^3} \&]$$

Problem 135: Result is not expressed in closed-form.

$$\int \frac{x^2}{(a+8 x-8 x^2+4 x^3-x^4)^2} dx$$

Optimal (type 3, 225 leaves, 11 steps):

$$\frac{1 + (-1 + x)^2}{2 (4 + a) (3 + a - 2 (-1 + x)^2 - (-1 + x)^4)} + \frac{(4 + a) \left(2 + (-1 + x)^2\right) (-1 + x)}{4 (12 + 7 a + a^2) (3 + a - 2 (-1 + x)^2 - (-1 + x)^4)} -$$

$$\frac{\left(4 + a + \sqrt{4 + a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right]}{8 (3 + a) (4 + a) \sqrt{1 - \sqrt{4 + a}}} - \frac{\left(4 + a - \sqrt{4 + a}\right) \text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right]}{8 (3 + a) (4 + a) \sqrt{1 + \sqrt{4 + a}}} + \frac{\text{ArcTanh}\left[\frac{1+(-1+x)^2}{\sqrt{4+a}}\right]}{2 (4 + a)^{3/2}}$$

Result (type 7, 182 leaves):

$$\frac{2 x (4 - 3 x + 2 x^2) + a (1 + x - x^2 + x^3)}{4 (3 + a) (4 + a) (a - x (-8 + 8 x - 4 x^2 + x^3))} - \frac{\text{RootSum}\left[a + 8 \#1 - 8 \#1^2 + 4 \#1^3 - \#1^4 \&, \frac{-a \text{Log}[x - \#1] + 4 \text{Log}[x - \#1] \#1 + 2 a \text{Log}[x - \#1] \#1 + 4 \text{Log}[x - \#1] \#1^2 + a \text{Log}[x - \#1] \#1^2}{-2 + 4 \#1 - 3 \#1^2 + \#1^3} \&\right]}{16 (12 + 7 a + a^2)}$$

Problem 136: Result is not expressed in closed-form.

$$\int \frac{x^4}{27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6} dx$$

Optimal (type 3, 545 leaves, 14 steps):

$$\frac{(-1)^{1/3} \left(2 (-1)^{1/3} b + 3 a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3} - 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{3 \sqrt{3} \left(1 + (-1)^{1/3}\right)^2 a^{5/6} b^2 \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}} c^{2/3}} -$$

$$\frac{\left(2 b - 3 a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} a^{5/6} b^2 \sqrt{4 b - 3 a^{1/3} c^{2/3}} c^{2/3}} - \frac{\left(-1\right)^{2/3} \left(2 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}\right) \text{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{3 \sqrt{3} \left(1 - (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{5/6} b^2 \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}} c^{2/3}} -$$

$$\frac{\text{Log}\left[3 a + 3 a^{2/3} c^{1/3} x + b x^2\right]}{18 a^{2/3} b^2 c^{1/3}} + \frac{\text{Log}\left[3 a - 3 (-1)^{1/3} a^{2/3} c^{1/3} x + b x^2\right]}{6 \left(1 + (-1)^{1/3}\right)^2 a^{2/3} b^2 c^{1/3}} + \frac{\left(-1\right)^{1/3} \text{Log}\left[3 a + 3 (-1)^{2/3} a^{2/3} c^{1/3} x + b x^2\right]}{18 a^{2/3} b^2 c^{1/3}}$$

Result (type 7, 99 leaves):

$$\frac{1}{3} \text{RootSum}\left[27 a^3 + 27 a^2 b \#1^2 + 27 a^2 c \#1^3 + 9 a b^2 \#1^4 + b^3 \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^3}{18 a^2 b + 27 a^2 c \#1 + 12 a b^2 \#1^2 + 2 b^3 \#1^4} \&\right]$$

Problem 137: Result is not expressed in closed-form.

$$\int \frac{x^3}{27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6} dx$$

Optimal (type 3, 487 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{3\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{7/6}b\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{1/3}} - \frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}a^{7/6}b\sqrt{4b-3a^{1/3}c^{2/3}}c^{1/3}} + \\
 & \frac{(-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{3\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{7/6}b\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{1/3}} + \frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}} - \\
 & \frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{18\left(1+(-1)^{1/3}\right)^2a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{54a^{4/3}bc^{2/3}}
 \end{aligned}$$

Result (type 7, 99 leaves):

$$\frac{1}{3} \text{RootSum}\left[27a^3 + 27a^2b\#\!1^2 + 27a^2c\#\!1^3 + 9ab^2\#\!1^4 + b^3\#\!1^6 \&, \frac{\text{Log}[x-\#\!1]\#\!1^2}{18a^2b + 27a^2c\#\!1 + 12ab^2\#\!1^2 + 2b^3\#\!1^4} \&\right]$$

Problem 138: Result is not expressed in closed-form.

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal (type 3, 334 leaves, 8 steps):

$$\begin{aligned}
 & \frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{11/6}\sqrt{4b-3(-1)^{2/3}a^{1/3}c^{2/3}}c^{2/3}} + \\
 & \frac{2\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3a^{1/3}c^{2/3}}}\right]}{27\sqrt{3}a^{11/6}\sqrt{4b-3a^{1/3}c^{2/3}}c^{2/3}} + \frac{2(-1)^{2/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{11/6}\sqrt{4b+3(-1)^{1/3}a^{1/3}c^{2/3}}c^{2/3}}
 \end{aligned}$$

Result (type 7, 97 leaves):

$$\frac{1}{3} \text{RootSum}\left[27a^3 + 27a^2b\#\!1^2 + 27a^2c\#\!1^3 + 9ab^2\#\!1^4 + b^3\#\!1^6 \&, \frac{\text{Log}[x-\#\!1]\#\!1}{18a^2b + 27a^2c\#\!1 + 12ab^2\#\!1^2 + 2b^3\#\!1^4} \&\right]$$

Problem 139: Result is not expressed in closed-form.

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal (type 3, 469 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3}(-1)^{2/3}a^{1/3}c^{2/3}}\right]}{9\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{13/6}\sqrt{4b-3}(-1)^{2/3}a^{1/3}c^{2/3}c^{1/3}} - \frac{\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3}a^{1/3}c^{2/3}}\right]}{27\sqrt{3}a^{13/6}\sqrt{4b-3}a^{1/3}c^{2/3}c^{1/3}} + \\
 & \frac{(-1)^{1/3}\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3}(-1)^{1/3}a^{1/3}c^{2/3}}\right]}{9\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{13/6}\sqrt{4b+3}(-1)^{1/3}a^{1/3}c^{2/3}c^{1/3}} - \frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{162a^{7/3}c^{2/3}} + \\
 & \frac{(-1)^{2/3}\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{54\left(1+(-1)^{1/3}\right)^2a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{162a^{7/3}c^{2/3}}
 \end{aligned}$$

Result (type 7, 95 leaves):

$$\frac{1}{3} \text{RootSum}\left[27a^3 + 27a^2b \#1^2 + 27a^2c \#1^3 + 9ab^2 \#1^4 + b^3 \#1^6 \&, \frac{\text{Log}[x-\#1]}{18a^2b + 27a^2c \#1 + 12ab^2 \#1^2 + 2b^3 \#1^4} \&\right]$$

Problem 140: Result is not expressed in closed-form.

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal (type 3, 522 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{(-1)^{1/3}\left(2(-1)^{1/3}b+3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3(-1)^{1/3}a^{2/3}c^{1/3}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3}(-1)^{2/3}a^{1/3}c^{2/3}}\right]}{27\sqrt{3}\left(1+(-1)^{1/3}\right)^2a^{17/6}\sqrt{4b-3}(-1)^{2/3}a^{1/3}c^{2/3}c^{2/3}} - \\
 & \frac{\left(2b-3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3}a^{1/3}c^{2/3}}\right]}{81\sqrt{3}a^{17/6}\sqrt{4b-3}a^{1/3}c^{2/3}c^{2/3}} - \frac{\left(2(-1)^{2/3}b-3a^{1/3}c^{2/3}\right)\text{ArcTan}\left[\frac{3(-1)^{2/3}a^{2/3}c^{1/3}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3}(-1)^{1/3}a^{1/3}c^{2/3}}\right]}{27\sqrt{3}\left(1-(-1)^{1/3}\right)\left(1+(-1)^{1/3}\right)^2a^{17/6}\sqrt{4b+3}(-1)^{1/3}a^{1/3}c^{2/3}c^{2/3}} + \\
 & \frac{\text{Log}\left[3a+3a^{2/3}c^{1/3}x+bx^2\right]}{162a^{8/3}c^{1/3}} - \frac{\text{Log}\left[3a-3(-1)^{1/3}a^{2/3}c^{1/3}x+bx^2\right]}{54\left(1+(-1)^{1/3}\right)^2a^{8/3}c^{1/3}} - \frac{(-1)^{1/3}\text{Log}\left[3a+3(-1)^{2/3}a^{2/3}c^{1/3}x+bx^2\right]}{162a^{8/3}c^{1/3}}
 \end{aligned}$$

Result (type 7, 99 leaves):

$$\frac{1}{3} \text{RootSum}\left[27a^3 + 27a^2b \#1^2 + 27a^2c \#1^3 + 9ab^2 \#1^4 + b^3 \#1^6 \&, \frac{\text{Log}[x-\#1]}{18a^2b \#1 + 27a^2c \#1^2 + 12ab^2 \#1^3 + 2b^3 \#1^5} \&\right]$$

Problem 141: Result is not expressed in closed-form.

$$\int \frac{1}{x (27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6)} dx$$

Optimal (type 3, 563 leaves, 14 steps):

$$\begin{aligned} & \frac{\left(b - (-1)^{2/3} a^{1/3} c^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3} - 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1 + (-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}} c^{1/3}} + \frac{\left(b - a^{1/3} c^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 a^{1/3} c^{2/3}}}\right]}{27 \sqrt{3} a^{19/6} \sqrt{4 b - 3 a^{1/3} c^{2/3}} c^{1/3}} + \\ & \frac{(-1)^{2/3} \left((-1)^{2/3} b - a^{1/3} c^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{9 \sqrt{3} \left(1 - (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{19/6} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}} c^{1/3}} + \frac{\operatorname{Log}[x]}{27 a^3} - \frac{\left(3 a^{1/3} - \frac{b}{c^{2/3}}\right) \operatorname{Log}[3 a + 3 a^{2/3} c^{1/3} x + b x^2]}{486 a^{10/3}} - \\ & \frac{\left(b + \frac{1}{2} \sqrt{3} b + 6 a^{1/3} c^{2/3}\right) \operatorname{Log}[3 a - 3 (-1)^{1/3} a^{2/3} c^{1/3} x + b x^2]}{972 a^{10/3} c^{2/3}} - \frac{\left(3 a^{1/3} - \frac{(-1)^{2/3} b}{c^{2/3}}\right) \operatorname{Log}[3 a + 3 (-1)^{2/3} a^{2/3} c^{1/3} x + b x^2]}{486 a^{10/3}} \end{aligned}$$

Result (type 7, 157 leaves):

$$\begin{aligned} & -\frac{1}{81 a^3} \left(-3 \operatorname{Log}[x] + \operatorname{RootSum}\left[27 a^3 + 27 a^2 b \#1^2 + 27 a^2 c \#1^3 + 9 a b^2 \#1^4 + b^3 \#1^6 \&, \right. \right. \\ & \left. \left. \frac{27 a^2 b \operatorname{Log}[x - \#1] + 27 a^2 c \operatorname{Log}[x - \#1] \#1 + 9 a b^2 \operatorname{Log}[x - \#1] \#1^2 + b^3 \operatorname{Log}[x - \#1] \#1^4}{18 a^2 b + 27 a^2 c \#1 + 12 a b^2 \#1^2 + 2 b^3 \#1^4} \& \right] \right) \end{aligned}$$

Problem 142: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (27 a^3 + 27 a^2 b x^2 + 27 a^2 c x^3 + 9 a b^2 x^4 + b^3 x^6)} dx$$

Optimal (type 3, 645 leaves, 14 steps):

$$\begin{aligned}
& -\frac{1}{27 a^3 x} + \frac{\left(2 (-1)^{2/3} b^2 + 12 (-1)^{1/3} a^{1/3} b c^{2/3} + 9 a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{1/3} a^{2/3} c^{1/3} - 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}}}\right]}{81 \sqrt{3} \left(1 + (-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}} c^{2/3}} + \\
& \frac{\left(2 b^2 - 12 a^{1/3} b c^{2/3} + 9 a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b - 3 a^{1/3} c^{2/3}}}\right]}{243 \sqrt{3} a^{23/6} \sqrt{4 b - 3 a^{1/3} c^{2/3}} c^{2/3}} + \\
& \frac{\left(-1\right)^{2/3} \left(2 b^2 + 12 (-1)^{1/3} a^{1/3} b c^{2/3} + 9 (-1)^{2/3} a^{2/3} c^{4/3}\right) \operatorname{ArcTan}\left[\frac{3 (-1)^{2/3} a^{2/3} c^{1/3} + 2 b x}{\sqrt{3} \sqrt{a} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}}}\right]}{81 \sqrt{3} \left(1 - (-1)^{1/3}\right) \left(1 + (-1)^{1/3}\right)^2 a^{23/6} \sqrt{4 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}} c^{2/3}} - \frac{\left(2 b - 3 a^{1/3} c^{2/3}\right) \operatorname{Log}\left[3 a + 3 a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{11/3} c^{1/3}} + \\
& \frac{\left(2 b - 3 (-1)^{2/3} a^{1/3} c^{2/3}\right) \operatorname{Log}\left[3 a - 3 (-1)^{1/3} a^{2/3} c^{1/3} x + b x^2\right]}{162 \left(1 + (-1)^{1/3}\right)^2 a^{11/3} c^{1/3}} + \frac{\left(-1\right)^{1/3} \left(2 b + 3 (-1)^{1/3} a^{1/3} c^{2/3}\right) \operatorname{Log}\left[3 a + 3 (-1)^{2/3} a^{2/3} c^{1/3} x + b x^2\right]}{486 a^{11/3} c^{1/3}}
\end{aligned}$$

Result (type 7, 163 leaves):

$$\begin{aligned}
& -\frac{1}{81 a^3 x} \left(3 + x \operatorname{RootSum}\left[27 a^3 + 27 a^2 b \#1^2 + 27 a^2 c \#1^3 + 9 a b^2 \#1^4 + b^3 \#1^6 \&, \right. \right. \\
& \left. \left. \frac{27 a^2 b \operatorname{Log}[x - \#1] + 27 a^2 c \operatorname{Log}[x - \#1] \#1 + 9 a b^2 \operatorname{Log}[x - \#1] \#1^2 + b^3 \operatorname{Log}[x - \#1] \#1^4}{18 a^2 b \#1 + 27 a^2 c \#1^2 + 12 a b^2 \#1^3 + 2 b^3 \#1^5} \& \right] \right)
\end{aligned}$$

Problem 143: Result is not expressed in closed-form.

$$\int \frac{x^5}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 395 leaves, 14 steps):

$$\begin{aligned}
& -\frac{\left(-2\right)^{1/3} \left(1 + (-2)^{1/3} 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)}}\right]}{3^{5/6} \sqrt{8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\left(\frac{3}{2}\right)^{1/6} \left(1 - (-3)^{2/3} 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right)}}\right]}{\left(1 + (-1)^{1/3}\right)^2 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(1 - 2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \times 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)}}\right]}{2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{1}{216} \left(36 + 2^{2/3} \times 3^{1/3} \left(1 + \pm \sqrt{3}\right)\right) \operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right] + \\
& \frac{1}{108} \left(18 - (-2)^{2/3} 3^{1/3}\right) \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right] + \frac{1}{108} \left(18 - 2^{2/3} \times 3^{1/3}\right) \operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]
\end{aligned}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^4}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 144: Result is not expressed in closed-form.

$$\int \frac{x^4}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\begin{aligned} & \frac{(-1)^{2/3} \left(3 (-3)^{2/3} - 2^{2/3}\right) \text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}}\right] + \left(9 - (-2)^{2/3} 3^{1/3}\right) \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{9 \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{2 (4 - 3 (-3)^{2/3} 2^{1/3})}} - \\ & \frac{27 \sqrt{3 (8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3})}}{27 \sqrt{6 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}} \\ & \frac{\left(9 - 2^{2/3} \times 3^{1/3}\right) \text{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \times 3^{2/3})}}\right] + \text{Log}[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2]}{6 \times 2^{2/3} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^2} + \frac{\left(-\frac{1}{3}\right)^{1/3} \text{Log}[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2]}{18 \times 2^{2/3}} - \frac{\text{Log}[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{18 \times 2^{2/3} \times 3^{1/3}} \end{aligned}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^3}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 145: Result is not expressed in closed-form.

$$\int \frac{x^3}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$\begin{aligned} & \frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}}\right] - \left(-1\right)^{1/3} \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right] + \text{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \cdot 2^{1/3} \times 3^{2/3})}}\right]}{6 \times 2^{1/6} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} + \\ & \frac{9 \times 2^{2/3} \times 3^{5/6} \sqrt{8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}}{18 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\left(-1\right)^{2/3} \text{Log}[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2]}{36 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} + \frac{\left(-1\right)^{2/3} \text{Log}[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2]}{108 \times 2^{1/3} \times 3^{2/3}} + \frac{\text{Log}[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{108 \times 2^{1/3} \times 3^{2/3}} \end{aligned}$$

Result (type 7, 61 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1^2}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 146: Result is not expressed in closed-form.

$$\int \frac{x^2}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 248 leaves, 8 steps):

$$\frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{27 \times 2^{5/6} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \frac{(-1)^{2/3} \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{81 \times 2^{1/3} \times 3^{1/6} \sqrt{8+9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \times 3^{2/3})}}\right]}{81 \times 2^{5/6} \times 3^{1/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}}$$

Result (type 7, 59 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1] \#1}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 147: Result is not expressed in closed-form.

$$\int \frac{x}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 361 leaves, 14 steps):

$$\begin{aligned} & - \frac{\text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}}\right]}{36 \times 2^{1/6} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4-3(-3)^{2/3} 2^{1/3}}} + \frac{(-1)^{1/3} \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}}\right]}{54 \times 2^{2/3} \times 3^{5/6} \sqrt{8+9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \cdot 2^{1/3} \times 3^{2/3})}}\right]}{108 \times 2^{1/6} \times 3^{5/6} \sqrt{-4+3 \times 2^{1/3} \times 3^{2/3}}} + \\ & \frac{(-1)^{2/3} \text{Log}[6 - 3(-3)^{1/3} 2^{2/3} x + x^2]}{216 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} - \frac{(-1)^{2/3} \text{Log}[6 + 3(-2)^{2/3} 3^{1/3} x + x^2]}{648 \times 2^{1/3} \times 3^{2/3}} - \frac{\text{Log}[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{648 \times 2^{1/3} \times 3^{2/3}} \end{aligned}$$

Result (type 7, 57 leaves):

$$\frac{1}{6} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\text{Log}[x - \#1]}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&\right]$$

Problem 148: Result is not expressed in closed-form.

$$\int \frac{1}{216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6} dx$$

Optimal (type 3, 377 leaves, 14 steps):

$$\begin{aligned} & \frac{(-1)^{2/3} \left(3 (-3)^{2/3} - 2^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{324 \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{2 (4 - 3 (-3)^{2/3} 2^{1/3})}} + \frac{\left(9 - (-2)^{2/3} 3^{1/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{972 \sqrt{3 (8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3})}} - \\ & \frac{\left(9 - 2^{2/3} \times 3^{1/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{972 \sqrt{6 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}} - \frac{\operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{216 \times 2^{2/3} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^2} - \frac{\left(-\frac{1}{3}\right)^{1/3} \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{648 \times 2^{2/3}} + \frac{\operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{648 \times 2^{2/3} \times 3^{1/3}} \end{aligned}$$

Result (type 7, 62 leaves):

$$\frac{1}{6} \operatorname{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{\operatorname{Log}[x - \#1]}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&\right]$$

Problem 149: Result is not expressed in closed-form.

$$\int \frac{1}{x (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} dx$$

Optimal (type 3, 415 leaves, 14 steps):

$$\begin{aligned} & \frac{(-1)^{2/3} \left((-2)^{2/3} - 2 \times 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{216 \times 2^{1/3} \times 3^{5/6} \sqrt{8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{(-1)^{2/3} \left((-3)^{1/3} + 3 \times 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{216 \times 6^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\ & \frac{\left(1 - 2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{216 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\operatorname{Log}[x]}{216} - \frac{\left(36 + 2^{2/3} \times 3^{1/3} \left(1 + \pm \sqrt{3}\right)\right) \operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{46656} - \\ & \frac{\left(18 - (-2)^{2/3} 3^{1/3}\right) \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{23328} - \frac{\left(18 - 2^{2/3} \times 3^{1/3}\right) \operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{23328} \end{aligned}$$

Result (type 7, 103 leaves):

$$\frac{\text{Log}[x] - \frac{\text{RootSum}[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \text{Log}[x - \#1] + 324 \text{Log}[x - \#1] \#1 + 18 \text{Log}[x - \#1] \#1^2 + \text{Log}[x - \#1] \#1^4}{36 + 162 \#1 + 12 \#1^2 + \#1^4} \&]}{216}}{1296}$$

Problem 150: Result is not expressed in closed-form.

$$\int \frac{1}{x^2 (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} dx$$

Optimal (type 3, 448 leaves, 14 steps):

$$\begin{aligned} \frac{1}{216 x} - \frac{\left(27 (-6)^{1/3} - (-2)^{2/3} + 12 \times 3^{2/3}\right) \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{5832 \times 3^{1/6} \sqrt{8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{(-1)^{2/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3} - 2^{1/3}\right) \text{ArcTan}\left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{1944 \times 6^{1/6} \left(1 + (-1)^{1/3}\right)^2 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\ \frac{\left(2^{1/3} + 27 \times 3^{1/3} - 6 \times 6^{2/3}\right) \text{ArcTanh}\left[\frac{2^{1/6} (3 - 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{5832 \times 6^{1/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} - \frac{(-1)^{2/3} \left(9 + (-3)^{1/3} 2^{2/3}\right) \text{Log}[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2]}{1296 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^2} + \\ \frac{\left(3 (-6)^{2/3} + 2 (-2)^{1/3}\right) \text{Log}[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2]}{7776 \times 3^{1/3}} - \frac{\left(2^{2/3} - 3 \times 3^{2/3}\right) \text{Log}[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{3888 \times 6^{1/3}} \end{aligned}$$

Result (type 7, 109 leaves):

$$\frac{1}{216 x} - \frac{\text{RootSum}[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \text{Log}[x - \#1] + 324 \text{Log}[x - \#1] \#1 + 18 \text{Log}[x - \#1] \#1^2 + \text{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&]}{1296}$$

Problem 151: Result is not expressed in closed-form.

$$\int \frac{x^8}{(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)^2} dx$$

Optimal (type 3, 1064 leaves, 23 steps):

$$\begin{aligned}
& -\frac{\left(-\frac{1}{3}\right)^{1/3} \left(9 \left(6 + (-3)^{1/3} 2^{2/3}\right) + \left(2 - 2^{2/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3}\right)\right) x\right)}{162 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right)} - \\
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(9 \left(6 - (-2)^{2/3} 3^{1/3}\right) + \left(2 + 27 (-2)^{2/3} 3^{1/3} + 12 (-2)^{1/3} 3^{2/3}\right) x\right)}{729 \times 2^{2/3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right)} + \frac{9 \left(6 - 2^{2/3} \times 3^{1/3}\right) + \left(2 + 2^{2/3} (27 \times 3^{1/3} - 6 \times 6^{2/3})\right) x}{1458 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} - \\
& \frac{\pm \left((-2)^{2/3} + 6 \times 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{162 \times 2^{5/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}}} - \frac{(-1)^{1/3} \left(2 + 27 (-2)^{2/3} 3^{1/3} + 12 (-2)^{1/3} 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{162 \times 2^{1/6} \times 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)^{3/2}} - \\
& \frac{(-1)^{1/3} \left(6 (-6)^{2/3} + 27 (-3)^{1/3} - 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{81 \sqrt{2} 3^{5/6} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right)^{3/2}} + \frac{\left(\pm 2^{2/3} - 9 \times 3^{1/6} - 3 \pm 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 (-3)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{162 \times 2^{5/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(1 + 3 \times 2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \times 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{1458 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \frac{\left(2^{1/3} + 27 \times 3^{1/3} - 6 \times 6^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \times 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{81 \sqrt{2} 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \\
& \frac{\operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{972 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^4} + \frac{\pm \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{972 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^5} - \frac{\operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{8748 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-7884 + 324 x - 3990 x^2 - 11610 x^3 - 203 x^4 - 9 x^5}{34182 (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} - \frac{1}{205092} \\
& \operatorname{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{324 \operatorname{Log}[x - \#1] - 96 \operatorname{Log}[x - \#1] \#1 + 324 \operatorname{Log}[x - \#1] \#1^2 + 406 \operatorname{Log}[x - \#1] \#1^3 + 9 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&\right]
\end{aligned}$$

Problem 152: Result is not expressed in closed-form.

$$\int \frac{x^7}{(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)^2} dx$$

Optimal (type 3, 1005 leaves, 23 steps):

$$\begin{aligned}
& -\frac{2 \left(2 \left(-1\right)^{1/3} 3^{2/3}+9 \times 6^{1/3}\right)-9 \left(\left(-2\right)^{2/3}+2 \left(-1\right)^{1/3} 3^{2/3}\right) x}{972 \times 2^{2/3} \left(1+\left(-1\right)^{1/3}\right)^4 \left(4-3 \left(-3\right)^{2/3} 2^{1/3}\right) \left(6-3 \left(-3\right)^{1/3} 2^{2/3} x+x^2\right)} - \\
& \frac{\left(-6\right)^{1/3} \left(9 \left(-2\right)^{1/3}+2 \times 3^{1/3}\right)-9 \left(1+\left(-2\right)^{1/3} 3^{2/3}\right) x}{4374 \left(8+9 \pm 2^{1/3} \times 3^{1/6}+3 \times 2^{1/3} \times 3^{2/3}\right) \left(6+3 \left(-2\right)^{2/3} 3^{1/3} x+x^2\right)} + \frac{2 \left(2-3 \times 2^{1/3} \times 3^{2/3}\right)-3 \left(6-2^{2/3} \times 3^{1/3}\right) x}{2916 \times 2^{2/3} \times 3^{1/3} \left(4-3 \times 2^{1/3} \times 3^{2/3}\right) \left(6+3 \times 2^{2/3} \times 3^{1/3} x+x^2\right)} + \\
& \frac{\left(9 \pm +3^{1/3} \left(2 \pm 2^{2/3}-9 \times 3^{1/6}+2 \times 2^{2/3} \sqrt{3}\right)\right) \operatorname{ArcTan}\left[\frac{3 \left(-3\right)^{1/3} 2^{2/3}-2 x}{\sqrt{6 \left(4-3 \left(-3\right)^{2/3} 2^{1/3}\right)}}\right]}{5832 \left(1+\left(-1\right)^{1/3}\right)^5 \sqrt{2 \left(4-3 \left(-3\right)^{2/3} 2^{1/3}\right)}} + \frac{\left(1+\left(-2\right)^{1/3} 3^{2/3}\right) \operatorname{ArcTan}\left[\frac{3 \left(-2\right)^{2/3} 3^{1/3}+2 x}{\sqrt{6 \left(4+3 \left(-2\right)^{1/3} 3^{2/3}\right)}}\right]}{54 \sqrt{6} \left(1-\left(-1\right)^{1/3}\right)^2 \left(1+\left(-1\right)^{1/3}\right)^4 \left(4+3 \left(-2\right)^{1/3} 3^{2/3}\right)^{3/2}} + \\
& \frac{\left(9 \times 3^{1/6}+\pm \left(4 \times 2^{2/3}-3 \times 3^{2/3}\right)\right) \operatorname{ArcTan}\left[\frac{3 \left(-2\right)^{2/3} 3^{1/3}+2 x}{\sqrt{6 \left(4+3 \left(-2\right)^{1/3} 3^{2/3}\right)}}\right]}{1944 \times 3^{2/3} \left(1+\left(-1\right)^{1/3}\right)^5 \sqrt{2 \left(4+3 \left(-2\right)^{1/3} 3^{2/3}\right)}} - \frac{\left(-1\right)^{1/3} \left(\left(-3\right)^{1/3}+3 \times 2^{1/3}\right) \operatorname{ArcTan}\left[\frac{2^{1/6} \left(3 \left(-3\right)^{1/3}-2^{1/3} x\right)}{\sqrt{3 \left(4-3 \left(-3\right)^{2/3} 2^{1/3}\right)}}\right]}{54 \sqrt{2} 3^{5/6} \left(1+\left(-1\right)^{1/3}\right)^4 \left(4-3 \left(-3\right)^{2/3} 2^{1/3}\right)^{3/2}} + \\
& \frac{\left(1-2^{1/3} \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \times 3^{1/3}+2^{1/3} x\right)}{\sqrt{3 \left(-4+3 \cdot 2^{1/3} \times 3^{2/3}\right)}}\right]}{54 \sqrt{6} \left(1-\left(-1\right)^{1/3}\right)^2 \left(1+\left(-1\right)^{1/3}\right)^4 \left(-4+3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} + \frac{\left(2 \times 2^{2/3}+3 \times 3^{2/3}\right) \operatorname{ArcTanh}\left[\frac{2^{1/6} \left(3 \times 3^{1/3}+2^{1/3} x\right)}{\sqrt{3 \left(-4+3 \cdot 2^{1/3} \times 3^{2/3}\right)}}\right]}{26244 \times 3^{1/6} \sqrt{2 \left(-4+3 \times 2^{1/3} \times 3^{2/3}\right)}} + \\
& \frac{\pm \operatorname{Log}\left[6-3 \left(-3\right)^{1/3} 2^{2/3} x+x^2\right]}{648 \times 2^{2/3} \times 3^{5/6} \left(1+\left(-1\right)^{1/3}\right)^5} - \frac{\left(\pm +\sqrt{3}\right) \operatorname{Log}\left[6+3 \left(-2\right)^{2/3} 3^{1/3} x+x^2\right]}{1296 \times 2^{2/3} \times 3^{5/6} \left(1+\left(-1\right)^{1/3}\right)^5} - \frac{\operatorname{Log}\left[6+3 \times 2^{2/3} \times 3^{1/3} x+x^2\right]}{17496 \times 2^{2/3} \times 3^{1/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{648-96 x+432 x^2+908 x^3-18 x^4+73 x^5}{68364 \left(216+108 x^2+324 x^3+18 x^4+x^6\right)} + \frac{1}{410184} \\
& \text{RootSum}\left[216+108 \#1^2+324 \#1^3+18 \#1^4+\#1^6 \&, \frac{96 \operatorname{Log}\left[x-\#1\right]-216 \operatorname{Log}\left[x-\#1\right] \#1+96 \operatorname{Log}\left[x-\#1\right] \#1^2-36 \operatorname{Log}\left[x-\#1\right] \#1^3+73 \operatorname{Log}\left[x-\#1\right] \#1^4}{36 \#1+162 \#1^2+12 \#1^3+\#1^5} \&\right]
\end{aligned}$$

Problem 153: Result is not expressed in closed-form.

$$\int \frac{x^6}{(216+108 x^2+324 x^3+18 x^4+x^6)^2} dx$$

Optimal (type 3, 677 leaves, 14 steps):

$$\begin{aligned}
& \frac{9(-2)^{2/3} + 6^{1/3} (9 + (-3)^{1/3} 2^{2/3}) x}{2916 \times 2^{2/3} (1 + (-1)^{1/3})^4 (4 - 3(-3)^{2/3} 2^{1/3}) (6 - 3(-3)^{1/3} 2^{2/3} x + x^2)} + \frac{9 \times 2^{2/3} + (-1)^{1/3} 3^{2/3} (2 + 3(-2)^{1/3} 3^{2/3}) x}{13122 \times 2^{2/3} (8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}) (6 + 3(-2)^{2/3} 3^{1/3} x + x^2)} + \\
& \frac{3 \times 2^{2/3} \times 3^{1/3} - (2 - 3 \times 2^{1/3} \times 3^{2/3}) x}{8748 \times 2^{2/3} \times 3^{1/3} (4 - 3 \times 2^{1/3} \times 3^{2/3}) (6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2)} + \frac{(-1)^{1/3} (3(-3)^{2/3} - 2^{2/3}) \operatorname{ArcTan} \left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} 2^{1/3})}} \right]}{486 \times 6^{5/6} (1 + (-1)^{1/3})^4 (4 - 3(-3)^{2/3} 2^{1/3})^{3/2}} + \\
& \frac{(3(-3)^{2/3} + (-1)^{1/3} 2^{2/3}) \operatorname{ArcTan} \left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4+3(-2)^{1/3} 3^{2/3})}} \right]}{486 \times 6^{5/6} (1 - (-1)^{1/3})^2 (1 + (-1)^{1/3})^4 (4 + 3(-2)^{1/3} 3^{2/3})^{3/2}} - \frac{(2^{2/3} - 3 \times 3^{2/3}) \operatorname{ArcTanh} \left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4+3 \times 2^{1/3} \times 3^{2/3})}} \right]}{486 \times 6^{5/6} (1 - (-1)^{1/3})^2 (1 + (-1)^{1/3})^4 (-4 + 3 \times 2^{1/3} \times 3^{2/3})^{3/2}} + \\
& \frac{\left(-\frac{1}{3}\right)^{1/6} \operatorname{Log} [6 - 3(-3)^{1/3} 2^{2/3} x + x^2]}{5832 \times 2^{1/3} (1 + (-1)^{1/3})^5} - \frac{\pm \operatorname{Log} [6 + 3(-2)^{2/3} 3^{1/3} x + x^2]}{5832 \times 2^{1/3} \times 3^{1/6} (1 + (-1)^{1/3})^5} + \frac{\operatorname{Log} [6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2]}{52488 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-96 + 108x - 64x^2 - 72x^3 + 73x^4 - 3x^5}{68364 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} - \frac{1}{410184} \\
& \operatorname{RootSum} [216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{108 \operatorname{Log}[x - \#1] - 32 \operatorname{Log}[x - \#1] \#1 + 108 \operatorname{Log}[x - \#1] \#1^2 - 146 \operatorname{Log}[x - \#1] \#1^3 + 3 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&]
\end{aligned}$$

Problem 154: Result is not expressed in closed-form.

$$\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 682 leaves, 17 steps):

$$\begin{aligned}
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(4 - (-3)^{1/3} 2^{2/3} x\right)}{1944 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right)} + \frac{\left(-\frac{1}{3}\right)^{1/3} \left(4 + (-2)^{2/3} 3^{1/3} x\right)}{8748 \times 2^{2/3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right)} - \\
& \frac{4 + 2^{2/3} \times 3^{1/3} x}{17496 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} - \frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2 x}{\sqrt{6 (4 - 3 (-3)^{2/3} 2^{1/3})}}\right]}{4374 \times 2^{5/6} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^4 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} + \\
& \frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2 x}{\sqrt{6 (4 - 3 (-3)^{2/3} 2^{1/3})}}\right]}{4374 \sqrt{3} \left(8 - 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{\pm \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}}\right]}{1458 \times 2^{5/6} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}}} - \\
& \frac{\text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}}\right]}{4374 \sqrt{3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}}\right]}{8748 \sqrt{6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}}\right]}{39366 \times 2^{5/6} \times 3^{1/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{972 - 144 x + 648 x^2 + 729 x^3 - 27 x^4 + 4 x^5}{615276 (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} + \frac{1}{3691656} \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \right. \\
& \left. \frac{144 \text{Log}[x - \#1] - 324 \text{Log}[x - \#1] \#1 + 2043 \text{Log}[x - \#1] \#1^2 - 54 \text{Log}[x - \#1] \#1^3 + 4 \text{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 155: Result is not expressed in closed-form.

$$\int \frac{x^4}{(216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)^2} dx$$

Optimal (type 3, 850 leaves, 23 steps):

$$\begin{aligned}
& \frac{\left(-\frac{1}{3}\right)^{1/3} \left(3 (-3)^{1/3} 2^{2/3} - 2x\right)}{5832 \times 2^{2/3} \left(1 + (-1)^{1/3}\right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3}\right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right)} - \frac{\left(-\frac{1}{3}\right)^{1/3} \left(3 (-2)^{2/3} 3^{1/3} + 2x\right)}{26244 \times 2^{2/3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right)} - \\
& \frac{3 \times 3^{1/3} + 2^{1/3} x}{52488 \left(9 \times 2^{1/3} - 4 \times 3^{1/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} + \frac{(-1)^{1/3} \operatorname{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4 - 3 (-3)^{2/3} 2^{1/3})}}\right]}{729 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^4 \left(8 - 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \\
& \frac{(-1)^{1/3} \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}}\right]}{2916 \times 2^{1/6} \times 3^{5/6} \left(1 - (-1)^{1/3}\right)^2 \left(1 + (-1)^{1/3}\right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3}\right)^{3/2}} - \frac{\left(\pm + \sqrt{3}\right) \operatorname{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}}\right]}{11664 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}}} - \\
& \frac{\pm \operatorname{ArcTan}\left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4 - 3 (-3)^{2/3} 2^{1/3})}}\right]}{5832 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3}\right)^5 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} + \frac{\operatorname{ArcTanh}\left[\frac{2^{1/6} (3 \cdot 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \cdot 2^{1/3} \times 3^{2/3})}}\right]}{26244 \times 2^{1/6} \times 3^{5/6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} + \frac{\operatorname{ArcTanh}\left[\frac{2^{1/6} (3 \times 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \cdot 2^{1/3} \times 3^{2/3})}}\right]}{52488 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} - \\
& \frac{\operatorname{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{34992 \times 2^{1/3} \times 3^{2/3} \left(1 + (-1)^{1/3}\right)^4} + \frac{\pm \operatorname{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{34992 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3}\right)^5} - \frac{\operatorname{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{314928 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-288 + 324x - 1458x^2 - 216x^3 + 8x^4 - 9x^5}{1230552 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} - \frac{1}{7383312} \operatorname{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \right. \\
& \left. \frac{324 \operatorname{Log}[x - \#1] - 2628 \operatorname{Log}[x - \#1] \#1 + 324 \operatorname{Log}[x - \#1] \#1^2 - 16 \operatorname{Log}[x - \#1] \#1^3 + 9 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 156: Result is not expressed in closed-form.

$$\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 873 leaves, 23 steps):

$$\begin{aligned}
& \frac{(-6)^{1/3} \left(2 (-3)^{1/3} + 9 \times 2^{1/3}\right) - 3x}{157464 \left(8 - 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right)} - \frac{(-6)^{1/3} \left(9 (-2)^{1/3} + 2 \times 3^{1/3}\right) + 3x}{157464 \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right)} - \\
& \frac{2 \times 2^{1/3} - 3 \times 6^{2/3} - 3^{1/3} x}{104976 \left(9 \times 2^{1/3} - 4 \times 3^{1/3}\right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right)} + \frac{\text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{26244 \sqrt{3} \left(8 - 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} - \\
& \frac{\left(9 \pm -3^{1/3} \left(2 \pm 2^{2/3} + 9 \times 3^{1/6} + 2 \times 2^{2/3} \sqrt{3}\right)\right) \text{ArcTan}\left[\frac{3 (-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6 (4-3 (-3)^{2/3} 2^{1/3})}}\right]}{209952 \left(1 + (-1)^{1/3}\right)^5 \sqrt{2 (4-3 (-3)^{2/3} 2^{1/3})}} - \frac{\text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{26244 \sqrt{3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} + \\
& \frac{\left(9 \pm +3^{1/3} \left(4 \pm 2^{2/3} - 9 \times 3^{1/6}\right)\right) \text{ArcTan}\left[\frac{3 (-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6 (4+3 (-2)^{1/3} 3^{2/3})}}\right]}{209952 \left(1 + (-1)^{1/3}\right)^5 \sqrt{2 (4+3 (-2)^{1/3} 3^{2/3})}} - \frac{\text{ArcTanh}\left[\frac{2^{1/6} (3 - 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{52488 \sqrt{6} \left(-4 + 3 \times 2^{1/3} \times 3^{2/3}\right)^{3/2}} + \frac{\left(2 \times 2^{2/3} - 3 \times 3^{2/3}\right) \text{ArcTanh}\left[\frac{2^{1/6} (3 - 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4+3 \times 2^{1/3} \times 3^{2/3})}}\right]}{944784 \times 3^{1/6} \sqrt{2 (-4 + 3 \times 2^{1/3} \times 3^{2/3})}} - \\
& \frac{i \text{Log}\left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2\right]}{23328 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^5} + \frac{\left(\pm \sqrt{3}\right) \text{Log}\left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2\right]}{46656 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3}\right)^5} + \frac{\text{Log}\left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2\right]}{629856 \times 2^{2/3} \times 3^{1/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{972 - 3942x + 648x^2 + 96x^3 - 27x^4 + 4x^5}{3691656 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} + \frac{1}{11074968} \\
& \text{RootSum}\left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \frac{1971 \text{Log}[x - \#1] - 162 \text{Log}[x - \#1] \#1 + 72 \text{Log}[x - \#1] \#1^2 - 27 \text{Log}[x - \#1] \#1^3 + 2 \text{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \&\right]
\end{aligned}$$

Problem 157: Result is not expressed in closed-form.

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Optimal (type 3, 986 leaves, 23 steps):

$$\begin{aligned}
& -\frac{27 \left((-2)^{2/3} + 2 (-1)^{1/3} 3^{2/3} \right) - 6^{1/3} \left(9 + (-3)^{1/3} 2^{2/3} \right) x}{104976 \times 2^{2/3} \left(1 + (-1)^{1/3} \right)^4 \left(4 - 3 (-3)^{2/3} 2^{1/3} \right) \left(6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right)} - \\
& \frac{27 \times 2^{2/3} \left(1 + (-2)^{1/3} 3^{2/3} \right) - (-1)^{1/3} 3^{2/3} \left(2 + 3 (-2)^{1/3} 3^{2/3} \right) x}{472392 \times 2^{2/3} \left(8 + 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right)} + \frac{9 \left(6 - 2^{2/3} \times 3^{1/3} \right) - \left(2 - 3 \times 2^{1/3} \times 3^{2/3} \right) x}{314928 \times 2^{2/3} \times 3^{1/3} \left(4 - 3 \times 2^{1/3} \times 3^{2/3} \right) \left(6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right)} - \\
& \frac{\left(1 + i \sqrt{3} + 3 \times 2^{1/3} \times 3^{2/3} \right) \operatorname{ArcTan} \left[\frac{3 (-3)^{1/3} 2^{2/3} - 2 x}{\sqrt{6 (4 - 3 (-3)^{2/3} 2^{1/3})}} \right]}{8748 \times 2^{2/3} \times 3^{5/6} \left(1 + (-1)^{1/3} \right)^4 \left(8 - 9 \pm 2^{1/3} \times 3^{1/6} + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} + \frac{\left(3 (-3)^{2/3} + (-1)^{1/3} 2^{2/3} \right) \operatorname{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}} \right]}{17496 \times 6^{5/6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(4 + 3 (-2)^{1/3} 3^{2/3} \right)^{3/2}} + \\
& \frac{\left(\pm + \sqrt{3} \right) \operatorname{ArcTan} \left[\frac{3 (-2)^{2/3} 3^{1/3} + 2 x}{\sqrt{6 (4 + 3 (-2)^{1/3} 3^{2/3})}} \right]}{34992 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3} \right)^5 \sqrt{4 + 3 (-2)^{1/3} 3^{2/3}}} + \frac{i \operatorname{ArcTan} \left[\frac{2^{1/6} (3 (-3)^{1/3} - 2^{1/3} x)}{\sqrt{3 (4 - 3 (-3)^{2/3} 2^{1/3})}} \right]}{17496 \times 2^{1/6} \times 3^{1/3} \left(1 + (-1)^{1/3} \right)^5 \sqrt{4 - 3 (-3)^{2/3} 2^{1/3}}} - \\
& \frac{\left(2^{2/3} - 3 \times 3^{2/3} \right) \operatorname{ArcTanh} \left[\frac{2^{1/6} (3 - 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{17496 \times 6^{5/6} \left(1 - (-1)^{1/3} \right)^2 \left(1 + (-1)^{1/3} \right)^4 \left(-4 + 3 \times 2^{1/3} \times 3^{2/3} \right)^{3/2}} - \frac{\operatorname{ArcTanh} \left[\frac{2^{1/6} (3 - 3^{1/3} + 2^{1/3} x)}{\sqrt{3 (-4 + 3 \cdot 2^{1/3} \cdot 3^{2/3})}} \right]}{157464 \times 2^{1/6} \times 3^{5/6} \sqrt{-4 + 3 \times 2^{1/3} \times 3^{2/3}}} + \\
& \frac{\left(\pm + \sqrt{3} \right) \operatorname{Log} \left[6 - 3 (-3)^{1/3} 2^{2/3} x + x^2 \right]}{419904 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3} \right)^5} - \frac{\pm \operatorname{Log} \left[6 + 3 (-2)^{2/3} 3^{1/3} x + x^2 \right]}{209952 \times 2^{1/3} \times 3^{1/6} \left(1 + (-1)^{1/3} \right)^5} + \frac{\operatorname{Log} \left[6 + 3 \times 2^{2/3} \times 3^{1/3} x + x^2 \right]}{1889568 \times 2^{1/3} \times 3^{2/3}}
\end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
& \frac{-7884 + 324 x - 2724 x^2 - 216 x^3 + 8 x^4 - 9 x^5}{7383312 (216 + 108 x^2 + 324 x^3 + 18 x^4 + x^6)} - \frac{1}{44299872} \operatorname{RootSum} \left[216 + 108 \#1^2 + 324 \#1^3 + 18 \#1^4 + \#1^6 \&, \right. \\
& \left. \frac{324 \operatorname{Log}[x - \#1] + 2436 \operatorname{Log}[x - \#1] \#1 + 324 \operatorname{Log}[x - \#1] \#1^2 - 16 \operatorname{Log}[x - \#1] \#1^3 + 9 \operatorname{Log}[x - \#1] \#1^4}{36 \#1 + 162 \#1^2 + 12 \#1^3 + \#1^5} \& \right]
\end{aligned}$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int (b + 2 c x) (b x + c x^2)^{13} dx$$

Optimal (type 1, 15 leaves, 1 step):

$$\frac{1}{14} (b x + c x^2)^{14}$$

Result (type 1, 172 leaves):

$$\begin{aligned} & \frac{b^{14} x^{14}}{14} + b^{13} c x^{15} + \frac{13}{2} b^{12} c^2 x^{16} + 26 b^{11} c^3 x^{17} + \frac{143}{2} b^{10} c^4 x^{18} + 143 b^9 c^5 x^{19} + \frac{429}{2} b^8 c^6 x^{20} + \\ & \frac{1716}{7} b^7 c^7 x^{21} + \frac{429}{2} b^6 c^8 x^{22} + 143 b^5 c^9 x^{23} + \frac{143}{2} b^4 c^{10} x^{24} + 26 b^3 c^{11} x^{25} + \frac{13}{2} b^2 c^{12} x^{26} + b c^{13} x^{27} + \frac{c^{14} x^{28}}{14} \end{aligned}$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int x^{14} (b + 2c x^2) (b x + c x^3)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{28} x^{28} (b + c x^2)^{14}$$

Result (type 1, 182 leaves):

$$\begin{aligned} & \frac{b^{14} x^{28}}{28} + \frac{1}{2} b^{13} c x^{30} + \frac{13}{4} b^{12} c^2 x^{32} + 13 b^{11} c^3 x^{34} + \frac{143}{4} b^{10} c^4 x^{36} + \frac{143}{2} b^9 c^5 x^{38} + \frac{429}{4} b^8 c^6 x^{40} + \\ & \frac{858}{7} b^7 c^7 x^{42} + \frac{429}{4} b^6 c^8 x^{44} + \frac{143}{2} b^5 c^9 x^{46} + \frac{143}{4} b^4 c^{10} x^{48} + 13 b^3 c^{11} x^{50} + \frac{13}{4} b^2 c^{12} x^{52} + \frac{1}{2} b c^{13} x^{54} + \frac{c^{14} x^{56}}{28} \end{aligned}$$

Problem 162: Result more than twice size of optimal antiderivative.

$$\int x^{28} (b + 2c x^3) (b x + c x^4)^{13} dx$$

Optimal (type 1, 16 leaves, 3 steps):

$$\frac{1}{42} x^{42} (b + c x^3)^{14}$$

Result (type 1, 186 leaves):

$$\begin{aligned} & \frac{b^{14} x^{42}}{42} + \frac{1}{3} b^{13} c x^{45} + \frac{13}{6} b^{12} c^2 x^{48} + \frac{26}{3} b^{11} c^3 x^{51} + \frac{143}{6} b^{10} c^4 x^{54} + \frac{143}{3} b^9 c^5 x^{57} + \frac{143}{2} b^8 c^6 x^{60} + \\ & \frac{572}{7} b^7 c^7 x^{63} + \frac{143}{2} b^6 c^8 x^{66} + \frac{143}{3} b^5 c^9 x^{69} + \frac{143}{6} b^4 c^{10} x^{72} + \frac{26}{3} b^3 c^{11} x^{75} + \frac{13}{6} b^2 c^{12} x^{78} + \frac{1}{3} b c^{13} x^{81} + \frac{c^{14} x^{84}}{42} \end{aligned}$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{-7}(-1+n)(b+2cx^n)}{(bx+cx^{1+n})^8} dx$$

Optimal (type 3, 21 leaves, 3 steps):

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7b^{14}n(b+cx^n)^7} x^{-7n} (b^{14} + 1716b^7c^7x^{7n} + 12012b^6c^8x^{8n} + 36036b^5c^9x^{9n} + 60060b^4c^{10}x^{10n} + 60060b^3c^{11}x^{11n} + 36036b^2c^{12}x^{12n} + 12012bc^{13}x^{13n} + 1716c^{14}x^{14n})$$

Problem 192: Result more than twice size of optimal antiderivative.

$$\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^7 dx$$

Optimal (type 1, 21 leaves, 1 step):

$$\frac{1}{8}(a+bx+cx^2+dx^3)^8$$

Result (type 1, 143 leaves):

$$\frac{1}{8}x(b+x(c+dx)) \left(8a^7 + 28a^6x(b+x(c+dx)) + 56a^5x^2(b+x(c+dx))^2 + 70a^4x^3(b+x(c+dx))^3 + 56a^3x^4(b+x(c+dx))^4 + 28a^2x^5(b+x(c+dx))^5 + 8ax^6(b+x(c+dx))^6 + x^7(b+x(c+dx))^7 \right)$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int (b+3dx^2)(a+bx+dx^3)^7 dx$$

Optimal (type 1, 16 leaves, 1 step):

$$\frac{1}{8}(a+bx+dx^3)^8$$

Result (type 1, 127 leaves):

$$\frac{1}{8} x (b + d x^2) \\ (8 a^7 + 28 a^6 x (b + d x^2) + 56 a^5 x^2 (b + d x^2)^2 + 70 a^4 x^3 (b + d x^2)^3 + 56 a^3 x^4 (b + d x^2)^4 + 28 a^2 x^5 (b + d x^2)^5 + 8 a x^6 (b + d x^2)^6 + x^7 (b + d x^2)^7)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int (b + 3 d x^2) (b x + d x^3)^7 dx$$

Optimal (type 1, 15 leaves, 1 step) :

$$\frac{1}{8} (b x + d x^3)^8$$

Result (type 1, 98 leaves) :

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7 b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Problem 197: Result more than twice size of optimal antiderivative.

$$\int x^7 (b + d x^2)^7 (b + 3 d x^2) dx$$

Optimal (type 1, 16 leaves, 2 steps) :

$$\frac{1}{8} x^8 (b + d x^2)^8$$

Result (type 1, 98 leaves) :

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7 b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7 b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Problem 198: Result more than twice size of optimal antiderivative.

$$\int (2 c x + 3 d x^2) (a + c x^2 + d x^3)^7 dx$$

Optimal (type 1, 18 leaves, 1 step) :

$$\frac{1}{8} (a + c x^2 + d x^3)^8$$

Result (type 1, 115 leaves) :

$$\frac{1}{8} x^2 (c + d x) \left(8 a^7 + 28 a^6 x^2 (c + d x) + 56 a^5 x^4 (c + d x)^2 + 70 a^4 x^6 (c + d x)^3 + 56 a^3 x^8 (c + d x)^4 + 28 a^2 x^{10} (c + d x)^5 + 8 a x^{12} (c + d x)^6 + x^{14} (c + d x)^7 \right)$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int (2 c x + 3 d x^2) (c x^2 + d x^3)^7 \, dx$$

Optimal (type 1, 17 leaves, 1 step) :

$$\frac{1}{8} (c x^2 + d x^3)^8$$

Result (type 1, 98 leaves) :

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 200: Result more than twice size of optimal antiderivative.

$$\int x^7 (c x + d x^2)^7 (2 c x + 3 d x^2) \, dx$$

Optimal (type 1, 14 leaves, 2 steps) :

$$\frac{1}{8} x^{16} (c + d x)^8$$

Result (type 1, 98 leaves) :

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 201: Result more than twice size of optimal antiderivative.

$$\int x^{14} (c + d x)^7 (2 c x + 3 d x^2) \, dx$$

Optimal (type 1, 14 leaves, 1 step) :

$$\frac{1}{8} x^{16} (c + d x)^8$$

Result (type 1, 98 leaves) :

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 202: Result more than twice size of optimal antiderivative.

$$\int x (2c + 3dx) (ax^2 + dx^3)^7 dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8} (ax^2 + dx^3)^8$$

Result (type 1, 115 leaves):

$$\frac{1}{8} x^2 (c + dx) (8a^7 + 28a^6 x^2 (c + dx) + 56a^5 x^4 (c + dx)^2 + 70a^4 x^6 (c + dx)^3 + 56a^3 x^8 (c + dx)^4 + 28a^2 x^{10} (c + dx)^5 + 8ax^{12} (c + dx)^6 + x^{14} (c + dx)^7)$$

Problem 203: Result more than twice size of optimal antiderivative.

$$\int x (2c + 3dx) (cx^2 + dx^3)^7 dx$$

Optimal (type 1, 14 leaves, 2 steps):

$$\frac{1}{8} x^{16} (c + dx)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 204: Result more than twice size of optimal antiderivative.

$$\int x^8 (2c + 3dx) (cx + dx^2)^7 dx$$

Optimal (type 1, 18 leaves, 1 step):

$$\frac{1}{8} x^8 (cx + dx^2)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int x^{15} (c + dx)^7 (2c + 3dx) dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{8} x^{16} (c + dx)^8$$

Result (type 1, 98 leaves):

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Problem 206: Result more than twice size of optimal antiderivative.

$$\int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal (type 1, 28 leaves, 2 steps):

$$ax + \frac{bx^2}{2} + \frac{1}{160} x^5 (2a + bx)^5$$

Result (type 1, 80 leaves):

$$ax + \frac{bx^2}{2} + \frac{a^5 x^5}{5} + \frac{1}{2} a^4 b x^6 + \frac{1}{2} a^3 b^2 x^7 + \frac{1}{4} a^2 b^3 x^8 + \frac{1}{16} a b^4 x^9 + \frac{b^5 x^{10}}{160}$$

Problem 207: Result more than twice size of optimal antiderivative.

$$\int (a + bx) \left(1 + \left(cx + ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal (type 1, 31 leaves, 2 steps):

$$ax + \frac{bx^2}{2} + \frac{1}{5} \left(cx + ax + \frac{bx^2}{2} \right)^5$$

Result (type 1, 108 leaves):

$$\frac{1}{160} x (2a + bx) \left(80 + 80c^4 + 16a^4 x^4 + 32a^3 b x^5 + 24a^2 b^2 x^6 + 8ab^3 x^7 + b^4 x^8 + 80c^3 x (2a + bx) + 40c^2 x^2 (2a + bx)^2 + 10c x^3 (2a + bx)^3 \right)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int (a + b x) \left(1 + \left(c + a x + \frac{b x^2}{2} \right)^n \right) dx$$

Optimal (type 3, 35 leaves, 2 steps) :

$$a x + \frac{b x^2}{2} + \frac{\left(c + a x + \frac{b x^2}{2} \right)^{1+n}}{1+n}$$

Result (type 3, 73 leaves) :

$$\frac{2 c \left(c + a x + \frac{b x^2}{2} \right)^n + 2 a x \left(1 + n + \left(c + a x + \frac{b x^2}{2} \right)^n \right) + b x^2 \left(1 + n + \left(c + a x + \frac{b x^2}{2} \right)^n \right)}{2 (1+n)}$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int (a + c x^2) \left(1 + \left(a x + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 30 leaves, 2 steps) :

$$a x + \frac{c x^3}{3} + \frac{1}{6} \left(a x + \frac{c x^3}{3} \right)^6$$

Result (type 1, 93 leaves) :

$$a x + \frac{c x^3}{3} + \frac{a^6 x^6}{6} + \frac{1}{3} a^5 c x^8 + \frac{5}{18} a^4 c^2 x^{10} + \frac{10}{81} a^3 c^3 x^{12} + \frac{5}{162} a^2 c^4 x^{14} + \frac{1}{243} a c^5 x^{16} + \frac{c^6 x^{18}}{4374}$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int (a + c x^2) \left(1 + \left(d + a x + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 31 leaves, 2 steps) :

$$a x + \frac{c x^3}{3} + \frac{1}{6} \left(d + a x + \frac{c x^3}{3} \right)^6$$

Result (type 1, 140 leaves) :

$$\frac{1}{4374} x (3a + cx^2) \left(1458 + 1458d^5 + 243a^5x^5 + 405a^4cx^7 + 270a^3c^2x^9 + 90a^2c^3x^{11} + 15ac^4x^{13} + c^5x^{15} + 1215d^4(3ax + cx^3) + 540d^3(3ax + cx^3)^2 + 135d^2(3ax + cx^3)^3 + 18d(3ax + cx^3)^4 \right)$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 34 leaves, 2 steps):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{x^{12}(3b + 2cx)^6}{279936}$$

Result (type 1, 98 leaves):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{b^6x^{12}}{384} + \frac{1}{96}b^5cx^{13} + \frac{5}{288}b^4c^2x^{14} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{486}bc^5x^{17} + \frac{c^6x^{18}}{4374}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$\frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6$$

Result (type 1, 146 leaves):

$$\frac{1}{279936}x^2(3b + 2cx) \left(46656 + 46656d^5 + 243b^5x^{10} + 810b^4cx^{11} + 1080b^3c^2x^{12} + 720b^2c^3x^{13} + 240bc^4x^{14} + 32c^5x^{15} + 19440d^4x^2(3b + 2cx) + 4320d^3x^4(3b + 2cx)^2 + 540d^2x^6(3b + 2cx)^3 + 36dx^8(3b + 2cx)^4 \right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int (ax + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 46 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^6$$

Result (type 1, 244 leaves):

$$\begin{aligned} & \frac{a^6 x^6}{6} + \frac{1}{6} a^5 x^7 (3b + 2cx) + \frac{5}{72} a^4 x^8 (3b + 2cx)^2 + \frac{5}{324} a^3 x^9 (3b + 2cx)^3 + \\ & \frac{5 a^2 x^{10} (3b + 2cx)^4}{2592} + a \left(x + \frac{b^5 x^{11}}{32} + \frac{5}{48} b^4 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^3 x^{14} + \frac{5}{162} b c^4 x^{15} + \frac{c^5 x^{16}}{243} \right) + \frac{1}{279936} \\ & x^2 (729 b^6 x^{10} + 2916 b^5 c x^{11} + 4860 b^4 c^2 x^{12} + 4320 b^3 c^3 x^{13} + 2160 b^2 c^4 x^{14} + 576 b (243 + c^5 x^{15}) + 64 c x (1458 + c^5 x^{15})) \end{aligned}$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int (a + b x + c x^2) \left(1 + \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^5 \right) dx$$

Optimal (type 1, 47 leaves, 2 steps):

$$a x + \frac{b x^2}{2} + \frac{c x^3}{3} + \frac{1}{6} \left(d + a x + \frac{b x^2}{2} + \frac{c x^3}{3} \right)^6$$

Result (type 1, 248 leaves):

$$\begin{aligned} & \frac{1}{279936} x (6a + x (3b + 2cx)) (46656 + 46656 d^5 + 7776 a^5 x^5 + 243 b^5 x^{10} + 810 b^4 c x^{11} + 1080 b^3 c^2 x^{12} + 720 b^2 c^3 x^{13} + \\ & 240 b c^4 x^{14} + 32 c^5 x^{15} + 6480 a^4 x^6 (3b + 2cx) + 2160 a^3 x^7 (3b + 2cx)^2 + 360 a^2 x^8 (3b + 2cx)^3 + 30 a x^9 (3b + 2cx)^4 + \\ & 19440 d^4 x (6a + x (3b + 2cx)) + 4320 d^3 x^2 (6a + x (3b + 2cx))^2 + 540 d^2 x^3 (6a + x (3b + 2cx))^3 + 36 d x^4 (6a + x (3b + 2cx))^4) \end{aligned}$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int (1 + 2x) (x + x^2)^3 (-18 + 7 (x + x^2)^3)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81 x^4 (1 + x)^4 - 36 x^7 (1 + x)^7 + \frac{49}{10} x^{10} (1 + x)^{10}$$

Result (type 1, 96 leaves):

$$\begin{aligned} & 81 x^4 + 324 x^5 + 486 x^6 + 288 x^7 - 171 x^8 - 756 x^9 - \frac{12551 x^{10}}{10} - 1211 x^{11} - \\ & \frac{1071 x^{12}}{2} + 336 x^{13} + 993 x^{14} + \frac{6174 x^{15}}{5} + 1029 x^{16} + 588 x^{17} + \frac{441 x^{18}}{2} + 49 x^{19} + \frac{49 x^{20}}{10} \end{aligned}$$

Problem 222: Result more than twice size of optimal antiderivative.

$$\int x^3 (1+x)^3 (1+2x) \left(-18 + 7x^3 (1+x)^3\right)^2 dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4 (1+x)^4 - 36x^7 (1+x)^7 + \frac{49}{10}x^{10} (1+x)^{10}$$

Result (type 1, 96 leaves):

$$81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}$$

Problem 227: Result is not expressed in closed-form.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{ax + bx^2 + cx^3 + dx^4} dx$$

Optimal (type 3, 605 leaves, 9 steps):

Result (type 7, 98 leaves):

$$\text{RootSum}\left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, \frac{A \text{Log}[x - \#1] + B \text{Log}[x - \#1] \#1 + C \text{Log}[x - \#1] \#1^2 + D \text{Log}[x - \#1] \#1^3}{b + 2 c \#1 + 3 b \#1^2 + 4 a \#1^3} \&\right]$$

Problem 250: Result is not expressed in closed-form.

$$\int \frac{x^3 (5 + x + 3x^2 + 2x^3)}{2 + x + 5x^2 + x^3 + 2x^4} dx$$

Optimal (type 3, 307 leaves, 13 steps):

$$\begin{aligned}
& -\frac{1}{28} \left(35 - 9 \pm \sqrt{7} \right) x - \frac{1}{28} \left(35 + 9 \pm \sqrt{7} \right) x + \frac{1}{28} \left(7 - 5 \pm \sqrt{7} \right) x^2 + \frac{1}{28} \left(7 + 5 \pm \sqrt{7} \right) x^2 + \frac{1}{42} \left(7 - 5 \pm \sqrt{7} \right) x^3 + \\
& \frac{1}{42} \left(9 \pm 5 \sqrt{7} \right) \operatorname{ArcTan} \left[\frac{1 \pm \sqrt{7} + 8x}{\sqrt{2(35 \pm \sqrt{7})}} \right] - \frac{1}{42} \left(9 \pm 5 \sqrt{7} \right) \operatorname{ArcTan} \left[\frac{1 \pm \sqrt{7} + 8x}{\sqrt{2(35 \mp \sqrt{7})}} \right] + \\
& \frac{1}{42} \left(7 + 5 \pm \sqrt{7} \right) x^3 + \frac{\frac{3}{112} \left(7 - 11 \pm \sqrt{7} \right) \operatorname{Log} \left[4 + \left(1 - \pm \sqrt{7} \right) x + 4x^2 \right] + \frac{3}{112} \left(7 + 11 \pm \sqrt{7} \right) \operatorname{Log} \left[4 + \left(1 + \pm \sqrt{7} \right) x + 4x^2 \right]}{4 \sqrt{14(35 \pm \sqrt{7})}}
\end{aligned}$$

Result (type 7, 109 leaves):

$$\frac{1}{6} \left(x \left(-15 + 3x + 2x^2 \right) + 3 \operatorname{RootSum} \left[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{10 \operatorname{Log}[x - \#1] + \operatorname{Log}[x - \#1] \#1 + 19 \operatorname{Log}[x - \#1] \#1^2 + 3 \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \& \right] \right)$$

Problem 251: Result is not expressed in closed-form.

$$\int \frac{x^2 (5 + x + 3x^2 + 2x^3)}{2 + x + 5x^2 + x^3 + 2x^4} dx$$

Optimal (type 3, 269 leaves, 13 steps):

$$\begin{aligned}
& \frac{1}{14} \left(7 - 5 \pm \sqrt{7} \right) x + \frac{1}{14} \left(7 + 5 \pm \sqrt{7} \right) x + \frac{1}{28} \left(7 - 5 \pm \sqrt{7} \right) x^2 + \frac{1}{28} \left(7 + 5 \pm \sqrt{7} \right) x^2 - \frac{\left(53 \pm \sqrt{7} \right) \operatorname{ArcTan} \left[\frac{1 \pm \sqrt{7} + 8x}{\sqrt{2(35 \pm \sqrt{7})}} \right]}{2 \sqrt{14(35 \pm \sqrt{7})}} + \\
& \frac{\left(53 \pm \sqrt{7} \right) \operatorname{ArcTan} \left[\frac{1 \pm \sqrt{7} + 8x}{\sqrt{2(35 \mp \sqrt{7})}} \right]}{2 \sqrt{14(35 \mp \sqrt{7})}} - \frac{1}{56} \left(35 + 9 \pm \sqrt{7} \right) \operatorname{Log} \left[4 + \left(1 - \pm \sqrt{7} \right) x + 4x^2 \right] - \frac{1}{56} \left(35 - 9 \pm \sqrt{7} \right) \operatorname{Log} \left[4 + \left(1 + \pm \sqrt{7} \right) x + 4x^2 \right]
\end{aligned}$$

Result (type 7, 101 leaves):

$$x + \frac{x^2}{2} - \operatorname{RootSum} \left[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{2 \operatorname{Log}[x - \#1] + 3 \operatorname{Log}[x - \#1] \#1 + \operatorname{Log}[x - \#1] \#1^2 + 5 \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \& \right]$$

Problem 252: Result is not expressed in closed-form.

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal (type 3, 230 leaves, 11 steps):

$$\begin{aligned} & \frac{\frac{1}{14} (7 - 5 \frac{i}{\sqrt{7}}) x + \frac{1}{14} (7 + 5 \frac{i}{\sqrt{7}}) x - \frac{(19 \frac{i}{\sqrt{7}} + 7 \sqrt{7}) \operatorname{ArcTan}[\frac{1 - \frac{i}{\sqrt{7}} + 8x}{\sqrt{2(35 + \frac{i}{\sqrt{7}})}}]}{\sqrt{14(35 + \frac{i}{\sqrt{7}})}} + \\ & \frac{(19 \frac{i}{\sqrt{7}} - 7 \sqrt{7}) \operatorname{ArcTan}[\frac{1 + \frac{i}{\sqrt{7}} + 8x}{\sqrt{2(35 - \frac{i}{\sqrt{7}})}}]}{\sqrt{14(35 - \frac{i}{\sqrt{7}})}} + \frac{1}{28} (7 + 5 \frac{i}{\sqrt{7}}) \operatorname{Log}[4 + (1 - \frac{i}{\sqrt{7}}) x + 4x^2] + \frac{1}{28} (7 - 5 \frac{i}{\sqrt{7}}) \operatorname{Log}[4 + (1 + \frac{i}{\sqrt{7}}) x + 4x^2] \end{aligned}$$

Result (type 7, 94 leaves):

$$x + 2 \operatorname{RootSum}[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{-\operatorname{Log}[x - \#1] + 2 \operatorname{Log}[x - \#1] \#1 - 2 \operatorname{Log}[x - \#1] \#1^2 + \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \&]$$

Problem 253: Result is not expressed in closed-form.

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$$

Optimal (type 3, 198 leaves, 9 steps):

$$\begin{aligned} & \frac{(19 \frac{i}{\sqrt{7}} + 7 \sqrt{7}) \operatorname{ArcTan}[\frac{1 - \frac{i}{\sqrt{7}} + 8x}{\sqrt{2(35 + \frac{i}{\sqrt{7}})}}] - (19 \frac{i}{\sqrt{7}} - 7 \sqrt{7}) \operatorname{ArcTan}[\frac{1 + \frac{i}{\sqrt{7}} + 8x}{\sqrt{2(35 - \frac{i}{\sqrt{7}})}}]}{\sqrt{14(35 + \frac{i}{\sqrt{7}})}} + \\ & \frac{1}{28} (7 + 5 \frac{i}{\sqrt{7}}) \operatorname{Log}[4 + (1 - \frac{i}{\sqrt{7}}) x + 4x^2] + \frac{1}{28} (7 - 5 \frac{i}{\sqrt{7}}) \operatorname{Log}[4 + (1 + \frac{i}{\sqrt{7}}) x + 4x^2] \end{aligned}$$

Result (type 7, 90 leaves):

$$\operatorname{RootSum}[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{5 \operatorname{Log}[x - \#1] + \operatorname{Log}[x - \#1] \#1 + 3 \operatorname{Log}[x - \#1] \#1^2 + 2 \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \&]$$

Problem 254: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 245 leaves, 13 steps):

$$\begin{aligned} & -\frac{\left(53 + \frac{1}{2}\sqrt{7}\right) \operatorname{ArcTanh}\left[\frac{\frac{1}{2}-\sqrt{7}+8\frac{1}{2}x}{\sqrt{2(35-\frac{1}{2}\sqrt{7})}}\right]}{2\sqrt{14(35-\frac{1}{2}\sqrt{7})}} + \frac{\left(53 - \frac{1}{2}\sqrt{7}\right) \operatorname{ArcTanh}\left[\frac{\frac{1}{2}+\sqrt{7}+8\frac{1}{2}x}{\sqrt{2(35+\frac{1}{2}\sqrt{7})}}\right]}{2\sqrt{14(35+\frac{1}{2}\sqrt{7})}} + \frac{1}{28}(35 - 9\frac{1}{2}\sqrt{7}) \operatorname{Log}[x] + \\ & \frac{1}{28}(35 + 9\frac{1}{2}\sqrt{7}) \operatorname{Log}[x] - \frac{1}{56}(35 - 9\frac{1}{2}\sqrt{7}) \operatorname{Log}[4\frac{1}{2} + (\frac{1}{2} - \sqrt{7})x + 4\frac{1}{2}x^2] - \frac{1}{56}(35 + 9\frac{1}{2}\sqrt{7}) \operatorname{Log}[4\frac{1}{2} + (\frac{1}{2} + \sqrt{7})x + 4\frac{1}{2}x^2] \end{aligned}$$

Result (type 7, 101 leaves):

$$\frac{5 \operatorname{Log}[x]}{2} - \frac{1}{2} \operatorname{RootSum}\left[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{3 \operatorname{Log}[x - \#1] + 19 \operatorname{Log}[x - \#1] \#1 + \operatorname{Log}[x - \#1] \#1^2 + 10 \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \&\right]$$

Problem 255: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 281 leaves, 13 steps):

$$\begin{aligned} & -\frac{35 - 9\frac{1}{2}\sqrt{7}}{28x} - \frac{35 + 9\frac{1}{2}\sqrt{7}}{28x} + \frac{11\left(9 + 5\frac{1}{2}\sqrt{7}\right) \operatorname{ArcTanh}\left[\frac{\frac{1}{2}-\sqrt{7}+8\frac{1}{2}x}{\sqrt{2(35-\frac{1}{2}\sqrt{7})}}\right]}{4\sqrt{14(35-\frac{1}{2}\sqrt{7})}} - \frac{11\left(9 - 5\frac{1}{2}\sqrt{7}\right) \operatorname{ArcTanh}\left[\frac{\frac{1}{2}+\sqrt{7}+8\frac{1}{2}x}{\sqrt{2(35+\frac{1}{2}\sqrt{7})}}\right]}{4\sqrt{14(35+\frac{1}{2}\sqrt{7})}} - \frac{3}{56}(7 - 11\frac{1}{2}\sqrt{7}) \operatorname{Log}[x] - \\ & \frac{3}{56}(7 + 11\frac{1}{2}\sqrt{7}) \operatorname{Log}[x] + \frac{3}{112}(7 + 11\frac{1}{2}\sqrt{7}) \operatorname{Log}[4\frac{1}{2} + (\frac{1}{2} - \sqrt{7})x + 4\frac{1}{2}x^2] + \frac{3}{112}(7 - 11\frac{1}{2}\sqrt{7}) \operatorname{Log}[4\frac{1}{2} + (\frac{1}{2} + \sqrt{7})x + 4\frac{1}{2}x^2] \end{aligned}$$

Result (type 7, 109 leaves):

$$\begin{aligned} & -\frac{5}{2x} - \frac{3 \operatorname{Log}[x]}{4} + \frac{1}{4} \operatorname{RootSum}\left[2 + \#1 + 5 \#1^2 + \#1^3 + 2 \#1^4 \&, \frac{-35 \operatorname{Log}[x - \#1] + 13 \operatorname{Log}[x - \#1] \#1 - 17 \operatorname{Log}[x - \#1] \#1^2 + 6 \operatorname{Log}[x - \#1] \#1^3}{1 + 10 \#1 + 3 \#1^2 + 8 \#1^3} \&\right] \end{aligned}$$

Problem 256: Result is not expressed in closed-form.

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^3 (2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Optimal (type 3, 317 leaves, 13 steps):

$$-\frac{35 - 9i\sqrt{7}}{56x^2} - \frac{35 + 9i\sqrt{7}}{56x^2} + \frac{3(7 - 11i\sqrt{7})}{56x} + \frac{3(7 + 11i\sqrt{7})}{56x} + \frac{(355 - 73i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i - \sqrt{7} + 8ix}{\sqrt{2(35 - i\sqrt{7})}}\right]}{8\sqrt{14(35 - i\sqrt{7})}} -$$

$$\frac{(355 + 73i\sqrt{7}) \operatorname{ArcTanh}\left[\frac{i + \sqrt{7} + 8ix}{\sqrt{2(35 + i\sqrt{7})}}\right]}{8\sqrt{14(35 + i\sqrt{7})}} - \frac{1}{16}(35 - 9i\sqrt{7}) \operatorname{Log}[x] - \frac{1}{16}(35 + 9i\sqrt{7}) \operatorname{Log}[x] +$$

$$\frac{1}{32}(35 - 9i\sqrt{7}) \operatorname{Log}[4i + (i - \sqrt{7})x + 4ix^2] + \frac{1}{32}(35 + 9i\sqrt{7}) \operatorname{Log}[4i + (i + \sqrt{7})x + 4ix^2]$$

Result (type 7, 116 leaves):

$$-\frac{5}{4x^2} + \frac{3}{4x} - \frac{35 \operatorname{Log}[x]}{8} + \frac{1}{8} \operatorname{RootSum}\left[2 + \#1 + 5\#1^2 + \#1^3 + 2\#1^4 \&, \frac{61 \operatorname{Log}[x - \#1] + 141 \operatorname{Log}[x - \#1]\#1 + 47 \operatorname{Log}[x - \#1]\#1^2 + 70 \operatorname{Log}[x - \#1]\#1^3}{1 + 10\#1 + 3\#1^2 + 8\#1^3} \&\right]$$

Problem 257: Result is not expressed in closed-form.

$$\int \frac{x^2(3a + bx^2)}{a^2 + 2abx^2 + b^2x^4 + c^2x^6} dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{cx^3}{a+bx^2}\right]}{c}$$

Result (type 7, 87 leaves):

$$\frac{1}{2} \operatorname{RootSum}\left[a^2 + 2ab\#1^2 + b^2\#1^4 + c^2\#1^6 \&, \frac{3a \operatorname{Log}[x - \#1]\#1 + b \operatorname{Log}[x - \#1]\#1^3}{2ab + 2b^2\#1^2 + 3c^2\#1^4} \&\right]$$

Problem 387: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 - (1 + x^2)^4} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{\frac{i \sqrt{1 - i 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 - i 2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{i \sqrt{1 + i 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + i 2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{\sqrt{1 + 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1+2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{\sqrt{-1 + 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{-1+2^{1/4}}}\right]}{4 \times 2^{3/4}}}{}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8} \operatorname{RootSum}\left[-1 + 4 \#1^2 + 6 \#1^4 + 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}\left[x - \#1\right] \#1}{1 + 3 \#1^2 + 3 \#1^4 + \#1^6} \&\right]$$

Problem 388: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 - (1 - x^2)^4} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{\frac{\sqrt{-1 + 2^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{-1+2^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{i \sqrt{1 - i 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 - i 2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{i \sqrt{1 + i 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1+i2^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{\sqrt{1 + 2^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1+2^{1/4}}}\right]}{4 \times 2^{3/4}}}{}$$

Result (type 7, 61 leaves):

$$-\frac{1}{8} \operatorname{RootSum}\left[-1 - 4 \#1^2 + 6 \#1^4 - 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}\left[x - \#1\right] \#1}{-1 + 3 \#1^2 - 3 \#1^4 + \#1^6} \&\right]$$

Problem 389: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 + (1 + x^2)^4} dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\frac{\frac{(-1)^{1/4} \sqrt{1 - (-2)^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 - (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + \frac{i}{2}} (-2)^{1/4} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + \frac{i}{2}} (-2)^{1/4}}\right]}{4 \times 2^{3/4}} - \frac{(-1)^{1/4} \sqrt{1 + (-2)^{1/4}} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{1}{8} \frac{i}{2} \left((-2)^{1/4} + \sqrt{2}\right) \sqrt{\frac{1 + \frac{i}{2}}{(1 + \frac{i}{2}) + 2^{3/4}}} \operatorname{ArcTan}\left[\sqrt{\frac{1 + \frac{i}{2}}{(1 + \frac{i}{2}) + 2^{3/4}}} x\right]}{}$$

Result (type 7, 61 leaves):

$$\frac{1}{8} \operatorname{RootSum}\left[3 + 4 \#1^2 + 6 \#1^4 + 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{1 + 3 \#1^2 + 3 \#1^4 + \#1^6} \&\right]$$

Problem 390: Result is not expressed in closed-form.

$$\int \frac{x^2}{2 + (1 - x^2)^4} dx$$

Optimal (type 3, 188 leaves, 8 steps):

$$\begin{aligned} & - \frac{\frac{(-1)^{1/4} \sqrt{1 - (-2)^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 - (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + \frac{i}{2}} (-2)^{1/4} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + \frac{i}{2}} (-2)^{1/4}}\right]}{4 \times 2^{3/4}} + \\ & \frac{(-1)^{1/4} \sqrt{1 + (-2)^{1/4}} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{1 + (-2)^{1/4}}}\right]}{4 \times 2^{3/4}} - \frac{1}{8} \frac{i}{2} \left((-2)^{1/4} + \sqrt{2}\right) \sqrt{\frac{1 + \frac{i}{2}}{(1 + \frac{i}{2}) + 2^{3/4}}} \operatorname{ArcTanh}\left[\sqrt{\frac{1 + \frac{i}{2}}{(1 + \frac{i}{2}) + 2^{3/4}}} x\right]}{} \end{aligned}$$

Result (type 7, 61 leaves):

$$\frac{1}{8} \operatorname{RootSum}\left[3 - 4 \#1^2 + 6 \#1^4 - 4 \#1^6 + \#1^8 \&, \frac{\operatorname{Log}[x - \#1] \#1}{-1 + 3 \#1^2 - 3 \#1^4 + \#1^6} \&\right]$$

Problem 391: Result is not expressed in closed-form.

$$\int \frac{1 - x^2}{a + b (1 - x^2)^4} dx$$

Optimal (type 3, 663 leaves, 16 steps):

$$\begin{aligned}
& - \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} - b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} - b^{1/4}} b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} - \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} + \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \frac{\text{ArcTanh}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} + b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} + b^{1/4}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}}} - \sqrt{2} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} - \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}}} + \sqrt{2} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}}
\end{aligned}$$

Result (type 7, 63 leaves):

$$\frac{\text{RootSum}\left[a + b - 4 b \#1^2 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{\text{Log}[x - \#1]}{\#1 - 2 \#1^2 + \#1^5} \&\right]}{8 b}$$

Problem 392: Result is not expressed in closed-form.

$$\int \frac{1 - x^2}{a + b (-1 + x^2)^4} dx$$

Optimal (type 3, 663 leaves, 17 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} - b^{1/4}} b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} - \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}} \text{ArcTan}\left[\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} + \sqrt{2} b^{1/8} x}{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} - b^{1/4}}}\right]}{4 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} + \frac{\text{ArcTanh}\left[\frac{b^{1/8} x}{\sqrt{(-a)^{1/4} + b^{1/4}}}\right]}{4 \sqrt{-a} \sqrt{(-a)^{1/4} + b^{1/4}} b^{3/8}} + \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}}} - \sqrt{2} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}} - \\
& \frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} \text{Log}\left[\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}}} + \sqrt{2} \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + b^{1/4}} b^{1/8} x + b^{1/4} x^2\right]}{8 \sqrt{2} \sqrt{-a} \sqrt{\sqrt{-a} + \sqrt{b}} b^{3/8}}
\end{aligned}$$

Result (type 7, 63 leaves):

$$\frac{\text{RootSum}\left[a + b - 4 b \#1^2 + 6 b \#1^4 - 4 b \#1^6 + b \#1^8 \&, \frac{\text{Log}[x - \#1]}{\#1 - 2 \#1^3 + \#1^5} \&\right]}{8 b}$$

Problem 393: Result is not expressed in closed-form.

$$\int \frac{(1+x^2)^2}{a x^6 + b (1+x^2)^3} dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{-(-1)^{1/3} a^{1/3} + b^{1/3}} b^{5/6}} + \frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} x}{b^{1/6}}\right]}{3 \sqrt{(-1)^{2/3} a^{1/3} + b^{1/3}} b^{5/6}}$$

Result (type 7, 95 leaves):

$$\frac{1}{6} \text{RootSum}\left[b + 3 b \#1^2 + 3 b \#1^4 + a \#1^6 + b \#1^6 \&, \frac{\text{Log}[x - \#1] + 2 \text{Log}[x - \#1] \#1^2 + \text{Log}[x - \#1] \#1^4}{\#1 - 2 b \#1^3 + a \#1^5 + b \#1^5} \&\right]$$

Problem 421: Result more than twice size of optimal antiderivative.

$$\int \frac{2}{-1 + 4x^2} dx$$

Optimal (type 3, 6 leaves, 2 steps):

$$-\operatorname{ArcTanh}[2x]$$

Result (type 3, 23 leaves):

$$2 \left(\frac{1}{4} \operatorname{Log}[1 - 2x] - \frac{1}{4} \operatorname{Log}[1 + 2x] \right)$$

Problem 491: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{1 + (-1 + x^2)^2} dx$$

Optimal (type 3, 188 leaves, 10 steps):

$$\begin{aligned} & -\frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \operatorname{ArcTan} \left[\frac{\sqrt{2 (1 + \sqrt{2})} - 2x}{\sqrt{2 (-1 + \sqrt{2})}} \right] + \\ & \frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \operatorname{ArcTan} \left[\frac{\sqrt{2 (1 + \sqrt{2})} + 2x}{\sqrt{2 (-1 + \sqrt{2})}} \right] + \frac{\operatorname{Log}[\sqrt{2} - \sqrt{2 (1 + \sqrt{2})} x + x^2]}{4 \sqrt{2 (1 + \sqrt{2})}} - \frac{\operatorname{Log}[\sqrt{2} + \sqrt{2 (1 + \sqrt{2})} x + x^2]}{4 \sqrt{2 (1 + \sqrt{2})}} \end{aligned}$$

Result (type 3, 39 leaves):

$$-\frac{\operatorname{ArcTan} \left[\frac{x}{\sqrt{-1 - \frac{1}{x}}} \right]}{\left(-1 - \frac{1}{x} \right)^{3/2}} - \frac{\operatorname{ArcTan} \left[\frac{x}{\sqrt{-1 + \frac{1}{x}}} \right]}{\left(-1 + \frac{1}{x} \right)^{3/2}}$$

Test results for the 1025 problems in "1.3.2 Algebraic functions.m"

Problem 1: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{1+x^3}}\right]}{3 \sqrt{3}} + \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 148 leaves):

$$\frac{4 \pm \sqrt{2} \sqrt{\frac{\frac{i (1+x)}{3 \pm \sqrt{3}} \sqrt{1-x+x^2}}{\operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \pm \sqrt{3}}\right]}}}{\left(1+2 \times 2^{2/3}-\pm \sqrt{3}\right) \sqrt{1+x^3}}$$

Problem 2: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} - x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{1-x^3}}\right]}{3 \sqrt{3}} - \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 148 leaves):

$$-\frac{4 \pm \sqrt{2} \sqrt{-\frac{\frac{i (-1+x)}{3 \pm \sqrt{3}} \sqrt{1+x+x^2}}{\operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}+2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \pm \sqrt{3}}\right]}}}{\left(1+2 \times 2^{2/3}-\pm \sqrt{3}\right) \sqrt{1-x^3}}$$

Problem 3: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps):

$$\frac{2 \operatorname{Arctanh} \left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{-1+x^3}} \right] - 2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right]}{3 \sqrt{3} \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 146 leaves):

$$\frac{4 \pm \sqrt{2} \sqrt{-\frac{i (-1+x)}{3 i + \sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}}+2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right]}{(1+2 \times 2^{2/3} - \pm \sqrt{3}) \sqrt{-1+x^3}}$$

Problem 4: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} + x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\frac{2 \operatorname{Arctanh} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}} \right] + 2 \times 2^{1/3} \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right]}{3 \sqrt{3} \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 150 leaves):

$$\frac{4 \pm \sqrt{2} \sqrt{-\frac{i (1+x)}{3 i + \sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{i+2 \pm 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{i+\sqrt{3}}-2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right]}{(1+2 \times 2^{2/3} - \pm \sqrt{3}) \sqrt{-1-x^3}}$$

Problem 5: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 280 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a + b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \frac{2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{a + b x^3}}$$

Result (type 4, 164 leaves):

$$\frac{2 \frac{i}{\sqrt{3}} \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\frac{i}{\sqrt{3}}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\left((-1)^{1/3} + 2^{2/3}\right) b^{1/3} \sqrt{a + b x^3}}$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} - \frac{2 \times 2^{1/3} \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3}}$$

Result (type 4, 166 leaves):

$$\frac{2 \frac{i}{\sqrt{3}} \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\frac{i}{\sqrt{3}}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]}{\left((-1)^{1/3} + 2^{2/3}\right) b^{1/3} \sqrt{a - b x^3}}$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\begin{aligned} & \frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} - \frac{2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3}} \end{aligned}$$

Result (type 4, 167 leaves):

$$\begin{aligned} & \frac{2 \frac{i}{2} \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{-a + b x^3}} \end{aligned}$$

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 293 leaves, 4 steps):

$$\begin{aligned} & \frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{1/3}} + \frac{2 \times 2^{1/3} \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3 \times 3^{1/4} a^{1/3} b^{1/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x)^2}} \sqrt{-a - b x^3}} \end{aligned}$$

Result (type 4, 167 leaves):

$$\begin{aligned} & \frac{2 \frac{i}{2} \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]}{((-1)^{1/3} + 2^{2/3}) b^{1/3} \sqrt{-a - b x^3}} \end{aligned}$$

Problem 9: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 249 leaves, 4 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c+2 d x)}{\sqrt{c^3+4 d^3 x^3}}\right]}{3 \sqrt{3} c^{3/2} d} + \frac{2 \times 2^{1/3} \sqrt{2+\sqrt{3}} (c+2^{2/3} d x) \sqrt{\frac{c^2-2^{2/3} c d x+2^{-2/3} d^2 x^2}{\left(\left(1+\sqrt{3}\right) c+2^{2/3} d x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c+2^{2/3} d x}{\left(1+\sqrt{3}\right) c+2^{2/3} d x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} c d \sqrt{\frac{c (c+2^{2/3} d x)}{\left(\left(1+\sqrt{3}\right) c+2^{2/3} d x\right)^2}} \sqrt{c^3+4 d^3 x^3}}$$

Result (type 4, 169 leaves):

$$-\left(\left(i 2^{5/6} \sqrt{\frac{2^{1/3} c+2 d x}{\left(1+\left(-1\right)^{1/3}\right) c}} \sqrt{2^{2/3}-\frac{2 \times 2^{1/3} d x}{c}+\frac{4 d^2 x^2}{c^2}} \operatorname{EllipticPi}\left[\frac{\frac{i}{2} 2^{1/3} \sqrt{3}}{2+\left(-2\right)^{1/3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{2^{1/3} c+2 \left(-1\right)^{2/3} d x}{\left(1+\left(-1\right)^{1/3}\right) c}}}{2^{1/6}}\right], \left(-1\right)^{1/3}\right]\right) \right. \\ \left. \left(\left(2+\left(-2\right)^{1/3}\right) d \sqrt{c^3+4 d^3 x^3}\right)\right)$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 146 leaves, 4 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{3} \left(3+2 \sqrt{3}\right)} + \frac{\sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}}$$

Result (type 4, 136 leaves):

$$-\frac{4 \sqrt{2} \sqrt{\frac{\frac{1}{2} (1+x)}{3 \frac{1}{2}+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{1}{2}+(1+2 \frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}-2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}}\right]}{\left(3 \frac{1}{2}+(1+2 \frac{1}{2}) \sqrt{3}\right) \sqrt{1+x^3}}$$

Problem 11: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]-\sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{\sqrt{3} \sqrt{3+2 \sqrt{3}} 3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 136 leaves):

$$-\frac{4 \sqrt{2} \sqrt{-\frac{\frac{1}{2} (-1+x)}{3 \frac{1}{2}+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{1}{2}+(1+2 \frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}+2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}}\right]}{\left(3 \frac{1}{2}+(1+2 \frac{1}{2}) \sqrt{3}\right) \sqrt{1-x^3}}$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}-x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 167 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}}\right]-\sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{\sqrt{3} \sqrt{3+2 \sqrt{3}} 3^{3/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 134 leaves):

$$\frac{4 \sqrt{2} \sqrt{-\frac{\frac{i}{3} (-1+x)}{3^{\frac{1}{4}}+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3^{\frac{1}{4}}+(1+2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{3}+\sqrt{3}}+2 \frac{i}{3} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3^{\frac{1}{4}}+\sqrt{3}}\right]}{\left(3^{\frac{1}{4}}+(1+2 i) \sqrt{3}\right) \sqrt{-1+x^3}}$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1+\sqrt{3}+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 157 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{3 (3+2 \sqrt{3})}}+\frac{\sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3^{3/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 138 leaves):

$$\frac{4 \sqrt{2} \sqrt{\frac{\frac{i}{3} (1+x)}{3^{\frac{1}{4}}+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3^{\frac{1}{4}}+(1+2 i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{3}+\sqrt{3}}-2 \frac{i}{3} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3^{\frac{1}{4}}+\sqrt{3}}\right]}{\left(3^{\frac{1}{4}}+(1+2 i) \sqrt{3}\right) \sqrt{-1-x^3}}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right] + 2 \sqrt{26+15 \sqrt{3}} \left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}} + \\
& \frac{4 \times 3^{1/4} \left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 128 leaves):

$$\begin{aligned}
& \frac{4 \sqrt{2} \sqrt{\frac{i (1+x)}{3 i+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{7 i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}-2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i+\sqrt{3}}\right]}{\left(7 i+\sqrt{3}\right) \sqrt{1+x^3}}
\end{aligned}$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 382 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2 \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right] - 2 \sqrt{2+\sqrt{3}} \left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}} + \\
& \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} \left(553+304 \sqrt{3}\right), -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{13 \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}
\end{aligned}$$

Result (type 4, 128 leaves):

$$-\frac{4 \sqrt{2} \sqrt{\frac{\frac{1}{2} (-1+x)}{-3 i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i+\sqrt{3}}\right]}{\left(5 i+\sqrt{3}\right) \sqrt{1-x^3}}$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 376 leaves, 8 steps):

$$\begin{aligned} & \frac{\left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2 \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right]-2 \sqrt{62-35 \sqrt{3}} \left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4 \sqrt{3}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}}+ \\ & \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(1-x\right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} \left(553+304 \sqrt{3}\right),-\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4 \sqrt{3}\right]}{13 \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{-1+x^3}} \end{aligned}$$

Result (type 4, 126 leaves):

$$-\frac{4 \sqrt{2} \sqrt{\frac{\frac{1}{2} (-1+x)}{-3 i+\sqrt{3}}} \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 i+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i+\sqrt{3}-2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i+\sqrt{3}}\right]}{\left(5 i+\sqrt{3}\right) \sqrt{-1+x^3}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(3+x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 342 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right] + 2 \left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}} + \\
& \frac{4 \times 3^{1/4} \left(1+x\right) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}}
\end{aligned}$$

Result (type 4, 130 leaves):

$$-\frac{4 \sqrt{2} \sqrt{\frac{\frac{i}{3} (1+x)}{1+\sqrt{3}}} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{7 \frac{i}{3}+\sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}}-2 \frac{i}{3} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{3}+\sqrt{3}}\right]}{\left(7 \frac{i}{3}+\sqrt{3}\right) \sqrt{-1-x^3}}$$

Problem 18: Unable to integrate problem.

$$\int \frac{1}{(c+d x) \left(-c^3+d^3 x^3\right)^{1/3}} dx$$

Optimal (type 3, 139 leaves, 1 step):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{\frac{1-2^{1/3} (c-d x)}{\left(-c^3+d^3 x^3\right)^{1/3}}}{\sqrt{3}}\right] + \frac{\operatorname{Log}\left[\left(c-d x\right) \left(c+d x\right)^2\right]}{4 \times 2^{1/3} c d} - \frac{3 \operatorname{Log}\left[d \left(c-d x\right)+2^{2/3} d \left(-c^3+d^3 x^3\right)^{1/3}\right]}{4 \times 2^{1/3} c d}}{2 \times 2^{1/3} c d}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(c+d x) \left(-c^3+d^3 x^3\right)^{1/3}} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{1}{(c+d x) \left(2 c^3+d^3 x^3\right)^{1/3}} dx$$

Optimal (type 3, 186 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{1+\frac{2 \text{d} x}{(2 \text{c}^3+\text{d}^3 \text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3} \text{c} \text{d}}-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (2 \text{c}+\text{d} \text{x})}{(2 \text{c}^3+\text{d}^3 \text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2 \text{c} \text{d}}-\frac{\text{Log}[\text{c}+\text{d} \text{x}]}{2 \text{c} \text{d}}-\frac{\text{Log}\left[-\text{d} \text{x}+\left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{1/3}\right]}{4 \text{c} \text{d}}+\frac{3 \text{Log}\left[\text{d} \left(2 \text{c}+\text{d} \text{x}\right)-\text{d} \left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{1/3}\right]}{4 \text{c} \text{d}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(\text{c}+\text{d} \text{x}) \left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{1/3}} \text{d} \text{x}$$

Problem 20: Unable to integrate problem.

$$\int \frac{1}{(\text{c}+\text{d} \text{x}) \left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{2/3}} \text{d} \text{x}$$

Optimal (type 3, 187 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 \text{d} x}{(2 \text{c}^3+\text{d}^3 \text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3} \text{c}^2 \text{d}}+\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (2 \text{c}+\text{d} \text{x})}{(2 \text{c}^3+\text{d}^3 \text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2 \text{c}^2 \text{d}}-\frac{\text{Log}[\text{c}+\text{d} \text{x}]}{2 \text{c}^2 \text{d}}-\frac{\text{Log}\left[\text{d} \text{x}-\left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{1/3}\right]}{4 \text{c}^2 \text{d}}+\frac{3 \text{Log}\left[\text{d} \left(2 \text{c}+\text{d} \text{x}\right)-\text{d} \left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{1/3}\right]}{4 \text{c}^2 \text{d}}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{(\text{c}+\text{d} \text{x}) \left(2 \text{c}^3+\text{d}^3 \text{x}^3\right)^{2/3}} \text{d} \text{x}$$

Problem 21: Unable to integrate problem.

$$\int \frac{1}{(1+2^{1/3} \text{x}) \left(1+\text{x}^3\right)^{2/3}} \text{d} \text{x}$$

Optimal (type 3, 147 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1+\frac{2 \text{x}}{(1+\text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}}+\frac{\sqrt{3} \text{ArcTan}\left[\frac{1+\frac{2 (2^{2/3} \text{x})}{(1+\text{x}^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-\frac{\text{Log}[1+2^{1/3} \text{x}]}{2^{2/3}}-\frac{\text{Log}\left[\text{x}-\left(1+\text{x}^3\right)^{1/3}\right]}{2 \times 2^{2/3}}+\frac{3 \text{Log}\left[2+2^{1/3} \text{x}-2^{1/3} \left(1+\text{x}^3\right)^{1/3}\right]}{2 \times 2^{2/3}}$$

Result (type 8, 23 leaves):

$$\int \frac{1}{(1+2^{1/3} \text{x}) \left(1+\text{x}^3\right)^{2/3}} \text{d} \text{x}$$

Problem 22: Unable to integrate problem.

$$\int \frac{1}{(1 - 2^{1/3} x) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 159 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2 \cdot 2^{2/3}-2 x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}} + \frac{\operatorname{ArcTan}\left[\frac{1-\frac{2 x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3} \sqrt{3}} + \frac{\operatorname{Log}[1-2^{1/3} x]}{2^{2/3}} + \frac{\operatorname{Log}[-x-(1-x^3)^{1/3}]}{2 \times 2^{2/3}} - \frac{3 \operatorname{Log}[-2+2^{1/3} x+2^{1/3} (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 8, 26 leaves):

$$\int \frac{1}{(1 - 2^{1/3} x) (1 - x^3)^{2/3}} dx$$

Problem 27: Unable to integrate problem.

$$\int \frac{(a+b x^3)^{1/3}}{c+d x} dx$$

Optimal (type 6, 435 leaves, 13 steps):

$$\begin{aligned} & \frac{(a+b x^3)^{1/3}}{d} + \frac{x (a+b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c \left(1+\frac{b x^3}{a}\right)^{1/3}} + \frac{b^{1/3} c \operatorname{ArcTan}\left[\frac{1+\frac{2 b^{1/3} x}{(a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} - \\ & \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (b c^3 - a d^3)^{1/3} x}{c (a+b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{ArcTan}\left[\frac{1-\frac{2 d (a+b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}[c^3 + d^3 x^3]}{3 d^2} + \\ & \frac{b^{1/3} c \operatorname{Log}[b^{1/3} x - (a+b x^3)^{1/3}]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a+b x^3)^{1/3}\right]}{2 d^2} - \frac{(b c^3 - a d^3)^{1/3} \operatorname{Log}[(b c^3 - a d^3)^{1/3} + d (a+b x^3)^{1/3}]}{2 d^2} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a+b x^3)^{1/3}}{c+d x} dx$$

Problem 28: Unable to integrate problem.

$$\int \frac{(a + b x^3)^{1/3}}{(c + d x)^2} dx$$

Optimal (type 6, 818 leaves, 20 steps):

$$\begin{aligned} & -\frac{c^2 (a + b x^3)^{1/3}}{d (c^3 + d^3 x^3)} - \frac{d x^2 (a + b x^3)^{1/3}}{c^3 + d^3 x^3} + \frac{x (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 2, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^2 \left(1 + \frac{b x^3}{a}\right)^{1/3}} - \frac{d^3 x^4 (a + b x^3)^{1/3} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^5 \left(1 + \frac{b x^3}{a}\right)^{1/3}} - \\ & \frac{b^{1/3} \operatorname{ArcTan}\left[\frac{1 + 2 b^{1/3} x}{\sqrt{3}}\right]}{\sqrt{3} d^2} + \frac{2 a d \operatorname{ArcTan}\left[\frac{1 + 2 (b c^3 - a d^3)^{1/3} x}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{ArcTan}\left[\frac{1 + 2 (b c^3 - a d^3)^{1/3} x}{\sqrt{3}}\right]}{3 \sqrt{3} c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b c^2 \operatorname{ArcTan}\left[\frac{1 + 2 d (a + b x^3)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3} d^2 (b c^3 - a d^3)^{2/3}} - \\ & \frac{b c^2 \operatorname{Log}[c^3 + d^3 x^3]}{6 d^2 (b c^3 - a d^3)^{2/3}} - \frac{a d \operatorname{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{2/3}} - \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}[c^3 + d^3 x^3]}{18 c d^2 (b c^3 - a d^3)^{2/3}} - \frac{b^{1/3} \operatorname{Log}[b^{1/3} x - (a + b x^3)^{1/3}]}{2 d^2} + \\ & \frac{a d \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{2/3}} + \frac{(3 b c^3 - 2 a d^3) \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c d^2 (b c^3 - a d^3)^{2/3}} + \frac{b c^2 \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 d^2 (b c^3 - a d^3)^{2/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{(a + b x^3)^{1/3}}{(c + d x)^2} dx$$

Problem 33: Unable to integrate problem.

$$\int \frac{1}{(c + d x) (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 333 leaves, 10 steps):

$$\begin{aligned} & -\frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^2 (a + b x^3)^{1/3}} + \frac{\operatorname{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} - \\ & \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{1/3}} + \frac{\operatorname{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{1/3}} - \frac{\operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} - \frac{\operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{1/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x) (a + b x^3)^{1/3}} dx$$

Problem 34: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 761 leaves, 17 steps):

$$\begin{aligned} & \frac{c^2 d^2 (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{c d^3 x (a + b x^3)^{2/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} - \frac{d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 2, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a + b x^3)^{1/3}} + \\ & \frac{d^4 x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^6 (a + b x^3)^{1/3}} + \frac{2 a d^3 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{4/3}} - \\ & \frac{b c^2 \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{4/3}} + \frac{b c^2 \text{Log}[c^3 + d^3 x^3]}{6 (b c^3 - a d^3)^{4/3}} + \frac{a d^3 \text{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{4/3}} + \frac{(3 b c^3 - 2 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 c (b c^3 - a d^3)^{4/3}} - \\ & \frac{a d^3 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{4/3}} - \frac{(3 b c^3 - 2 a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c (b c^3 - a d^3)^{4/3}} - \frac{b c^2 \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{4/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{1/3}} dx$$

Problem 35: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{1/3}} dx$$

Optimal (type 6, 1513 leaves, 32 steps):

$$\begin{aligned}
& \frac{3 c^4 d^2 (a + b x^3)^{2/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} - \frac{3 c^3 d^3 x (a + b x^3)^{2/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{4 b c^4 d^2 (a + b x^3)^{2/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{c d^2 (b c^3 - 3 a d^3) (a + b x^3)^{2/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \\
& \frac{d^3 (3 b c^3 - 7 a d^3) x (a + b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{d^3 (9 b c^3 - 5 a d^3) x (a + b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{7 d^3 (3 b c^3 + a d^3) x (a + b x^3)^{2/3}}{18 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \\
& \frac{3 d x^2 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 3, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^4 (a + b x^3)^{1/3}} + \frac{6 d^4 x^5 \left(1 + \frac{b x^3}{a}\right)^{1/3} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 3, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{5 c^7 (a + b x^3)^{1/3}} + \frac{2 a^2 d^6 \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} + \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{9 \sqrt{3} c^2 (b c^3 - a d^3)^{7/3}} - \frac{4 b^2 c^4 \text{ArcTan}\left[\frac{1 + \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{7/3}} + \\
& \frac{b c (b c^3 - 3 a d^3) \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{7/3}} + \frac{2 b^2 c^4 \text{Log}[c^3 + d^3 x^3]}{9 (b c^3 - a d^3)^{7/3}} + \frac{a^2 d^6 \text{Log}[c^3 + d^3 x^3]}{27 c^2 (b c^3 - a d^3)^{7/3}} - \frac{b c (b c^3 - 3 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 (b c^3 - a d^3)^{7/3}} + \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{54 c^2 (b c^3 - a d^3)^{7/3}} + \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{Log}[c^3 + d^3 x^3]}{54 c^2 (b c^3 - a d^3)^{7/3}} - \frac{a^2 d^6 \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{9 c^2 (b c^3 - a d^3)^{7/3}} - \\
& \frac{7 a d^3 (3 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{18 c^2 (b c^3 - a d^3)^{7/3}} - \frac{(9 b^2 c^6 - 12 a b c^3 d^3 + 5 a^2 d^6) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{18 c^2 (b c^3 - a d^3)^{7/3}} - \\
& \frac{2 b^2 c^4 \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{3 (b c^3 - a d^3)^{7/3}} + \frac{b c (b c^3 - 3 a d^3) \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{6 (b c^3 - a d^3)^{7/3}}
\end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{1/3}} dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(c + d x) (a + b x^3)^{2/3}} dx$$

Optimal (type 6, 332 leaves, 10 steps):

$$\begin{aligned}
& \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c (a + bx^3)^{2/3}} + \frac{d \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} - \\
& \frac{d \operatorname{ArcTan}\left[\frac{1 - \frac{2d(a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{2/3}} - \frac{d \operatorname{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{2/3}} + \frac{d \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}} + \frac{d \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{2 (b c^3 - a d^3)^{2/3}}
\end{aligned}$$

Result (type 8, 21 leaves) :

$$\int \frac{1}{(c + d x) (a + b x^3)^{2/3}} dx$$

Problem 41: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{2/3}} dx$$

Optimal (type 6, 760 leaves, 18 steps) :

$$\begin{aligned}
& \frac{c^2 d^2 (a + b x^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{d^4 x^2 (a + b x^3)^{1/3}}{(b c^3 - a d^3) (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 2, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^2 (a + b x^3)^{2/3}} - \\
& \frac{d^3 x^4 \left(1 + \frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}, -\frac{bx^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{2 c^5 (a + b x^3)^{2/3}} + \frac{2 a d^4 \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} + \frac{2 d (3 b c^3 - a d^3) \operatorname{ArcTan}\left[\frac{1 + \frac{2(b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c (b c^3 - a d^3)^{5/3}} - \\
& \frac{2 b c^2 d \operatorname{ArcTan}\left[\frac{1 - \frac{2d(a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (b c^3 - a d^3)^{5/3}} - \frac{b c^2 d \operatorname{Log}[c^3 + d^3 x^3]}{3 (b c^3 - a d^3)^{5/3}} - \frac{a d^4 \operatorname{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{5/3}} - \frac{d (3 b c^3 - a d^3) \operatorname{Log}[c^3 + d^3 x^3]}{9 c (b c^3 - a d^3)^{5/3}} + \\
& \frac{a d^4 \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{d (3 b c^3 - a d^3) \operatorname{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c (b c^3 - a d^3)^{5/3}} + \frac{b c^2 d \operatorname{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{(b c^3 - a d^3)^{5/3}}
\end{aligned}$$

Result (type 8, 21 leaves) :

$$\int \frac{1}{(c + d x)^2 (a + b x^3)^{2/3}} dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{2/3}} dx$$

Optimal (type 6, 1357 leaves, 30 steps):

$$\begin{aligned} & \frac{3 c^4 d^2 (a + b x^3)^{1/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{3 c^2 d^4 x^2 (a + b x^3)^{1/3}}{2 (b c^3 - a d^3) (c^3 + d^3 x^3)^2} + \frac{5 b c^4 d^2 (a + b x^3)^{1/3}}{3 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} - \frac{c d^2 (b c^3 - 6 a d^3) (a + b x^3)^{1/3}}{6 (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \\ & \frac{d^4 (9 b c^3 - 4 a d^3) x^2 (a + b x^3)^{1/3}}{6 c (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{d^4 (3 b c^3 + 2 a d^3) x^2 (a + b x^3)^{1/3}}{3 c (b c^3 - a d^3)^2 (c^3 + d^3 x^3)} + \frac{x \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 3, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{c^3 (a + b x^3)^{2/3}} - \\ & \frac{7 d^3 x^4 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 3, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{4 c^6 (a + b x^3)^{2/3}} + \frac{d^6 x^7 \left(1 + \frac{b x^3}{a}\right)^{2/3} \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 3, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{d^3 x^3}{c^3}\right]}{7 c^9 (a + b x^3)^{2/3}} + \\ & \frac{2 a d^4 (6 b c^3 - a d^3) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{ArcTan}\left[\frac{1 + \frac{2 (b c^3 - a d^3)^{1/3} x}{c (a + b x^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} c^2 (b c^3 - a d^3)^{8/3}} - \\ & \frac{10 b^2 c^4 d \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} + \frac{b c d (b c^3 - 6 a d^3) \text{ArcTan}\left[\frac{1 - \frac{2 d (a + b x^3)^{1/3}}{(b c^3 - a d^3)^{1/3}}}{\sqrt{3}}\right]}{3 \sqrt{3} (b c^3 - a d^3)^{8/3}} - \frac{5 b^2 c^4 d \text{Log}[c^3 + d^3 x^3]}{9 (b c^3 - a d^3)^{8/3}} + \\ & \frac{b c d (b c^3 - 6 a d^3) \text{Log}[c^3 + d^3 x^3]}{18 (b c^3 - a d^3)^{8/3}} - \frac{a d^4 (6 b c^3 - a d^3) \text{Log}[c^3 + d^3 x^3]}{9 c^2 (b c^3 - a d^3)^{8/3}} - \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}[c^3 + d^3 x^3]}{18 c^2 (b c^3 - a d^3)^{8/3}} + \\ & \frac{a d^4 (6 b c^3 - a d^3) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{3 c^2 (b c^3 - a d^3)^{8/3}} + \frac{d (9 b^2 c^6 - 6 a b c^3 d^3 + 2 a^2 d^6) \text{Log}\left[\frac{(b c^3 - a d^3)^{1/3} x}{c} - (a + b x^3)^{1/3}\right]}{6 c^2 (b c^3 - a d^3)^{8/3}} + \\ & \frac{5 b^2 c^4 d \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{3 (b c^3 - a d^3)^{8/3}} - \frac{b c d (b c^3 - 6 a d^3) \text{Log}\left[(b c^3 - a d^3)^{1/3} + d (a + b x^3)^{1/3}\right]}{6 (b c^3 - a d^3)^{8/3}} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{1}{(c + d x)^3 (a + b x^3)^{2/3}} dx$$

Problem 43: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{1+x^3}} \right]}{\sqrt{3}}$$

Result (type 4, 326 leaves):

$$\begin{aligned} & - \left(\left(4 \times 2^{1/6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \right. \right. \\ & \left. \left. \left(\sqrt{-i + \sqrt{3}} + 2 i x \right) \left(6 i + 3 i 2^{1/3} - 2 \sqrt{3} + 2^{1/3} \sqrt{3} + (-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3}) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] - \right. \\ & \left. \left. 6 i \sqrt{3} \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) \right) / \\ & \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - i \sqrt{3} \right) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1+x^3} \right) \end{aligned}$$

Problem 44: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x) \sqrt{1-x^3}} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan} \left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{1-x^3}} \right]}{\sqrt{3}}$$

Result (type 4, 327 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \right. \\
& \left. \left. \left(\sqrt{-i+\sqrt{3}} - 2ix \right) \left(-6i - 3i2^{1/3} + 2\sqrt{3} - 2^{1/3}\sqrt{3} + (-3i2^{1/3} + 4\sqrt{3} + 2^{1/3}\sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}} + 2ix}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}] + \right. \right. \\
& \left. \left. 6i\sqrt{3}\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}} + 2ix}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \\
& \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - i\sqrt{3} \right) \sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3} \right)
\end{aligned}$$

Problem 45: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x)\sqrt{-1+x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \text{ArcTanh}\left[\frac{\sqrt{3} (1-2^{1/3}x)}{\sqrt{-1+x^3}}\right]}{\sqrt{3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& - \left(\left(4 \times 2^{1/6} \sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}} \right. \right. \\
& \left. \left. \left(\sqrt{-i+\sqrt{3}} - 2ix \right) \left(-6i - 3i2^{1/3} + 2\sqrt{3} - 2^{1/3}\sqrt{3} + (-3i2^{1/3} + 4\sqrt{3} + 2^{1/3}\sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}} + 2ix}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}] + \right. \right. \\
& \left. \left. 6i\sqrt{3}\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i+\sqrt{3}} + 2ix}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3i+\sqrt{3}}\right] \right) \right) / \\
& \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - i\sqrt{3} \right) \sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3} \right)
\end{aligned}$$

Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} - 2x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 39 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTanh} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}} \right]}{\sqrt{3}}$$

Result (type 4, 328 leaves):

$$\begin{aligned} & - \left(\left(4 \times 2^{1/6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \right. \right. \\ & \left. \left. \left(\sqrt{-i + \sqrt{3}} + 2 i x \right) \left(6 i + 3 i 2^{1/3} - 2 \sqrt{3} + 2^{1/3} \sqrt{3} + (-3 i 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3}) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] - \right. \\ & \left. \left. 6 i \sqrt{3} \sqrt{i + \sqrt{3} - 2 i x} \sqrt{1 - x + x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{i + 2 i 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}} \right] \right) \right) / \\ & \left(\sqrt{3} \left(1 + 2 \times 2^{2/3} - i \sqrt{3} \right) \sqrt{i + \sqrt{3} - 2 i x} \sqrt{-1 - x^3} \right) \end{aligned}$$

Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{ArcTan} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a+b x^3}} \right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{3} \ b^{1/3} \ \sqrt{a + b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) \ a^{1/3}}} \left(\frac{2 \times 3^{1/4} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) \ a^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) \ a^{1/3}}}} - \frac{1}{(-1)^{1/3} + 2^{2/3}} \right. \\
& \left. 3 (-1)^{1/3} 2^{2/3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \text{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) \ a^{1/3}}} \right], (-1)^{1/3} \right] \right)
\end{aligned}$$

Problem 48: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \ \sqrt{a - b x^3}} dx$$

Optimal (type 3, 65 leaves, 2 steps) :

$$\frac{2 \times 2^{2/3} \text{ArcTan} \left[\frac{\sqrt{3} \ a^{1/6} \left(a^{1/3} - 2^{1/3} b^{1/3} x \right)}{\sqrt{a - b x^3}} \right]}{\sqrt{3} \ a^{1/6} \ b^{1/3}}$$

Result (type 4, 336 leaves) :

$$\begin{aligned}
& \frac{1}{b^{1/3} \sqrt{a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{2 \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right. \\
& \left. (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{3 + \frac{3 b^{1/3} x}{a^{1/3}} + \frac{3 b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}}\right] \right)
\end{aligned}$$

Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} + 2 b^{1/3} x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}}\right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 390 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. \left(2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] - \right. \\
& \left. (-1)^{1/3} 2^{2/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3} \right] \right) \Bigg) \Bigg/ \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 50: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2^{2/3} a^{1/3} - 2 b^{1/3} x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 66 leaves, 2 steps):

$$\frac{2 \times 2^{2/3} \operatorname{Arctanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3} a^{1/6} b^{1/3}}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. - \frac{1}{3^{1/4}} 2 \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right. \\
& \left. (-1)^{1/3} 2^{2/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \Bigg/ \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 51: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 dx}{(c + dx) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{3} \sqrt{c} (c+2 dx)}{\sqrt{c^3+4 d^3 x^3}} \right]}{\sqrt{3} \sqrt{c} d}$$

Result (type 4, 373 leaves):

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. 2 \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 ((-1)^{1/3} + 2^{2/3}) d x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] - \right. \\
& \left. (-1)^{1/3} 2^{2/3} \sqrt{3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}} \right. \\
& \left. \text{EllipticPi} \left[\frac{\frac{\pm 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3}} \right] \right) / \left((2 + (-2)^{1/3}) d \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 52: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{2 + 3 x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 158 leaves, 4 steps):

$$\frac{2 (2 - 3 \times 2^{2/3}) \text{ArcTan} \left[\frac{\sqrt{3} (1 + 2^{1/3} x)}{\sqrt{1 + x^3}} \right] + 2 (3 + 2 \times 2^{1/3}) \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4 \sqrt{3} \right]}{3 \sqrt{3}}$$

Result (type 4, 336 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{\frac{1}{2} (1+x)}{3 \frac{1}{2} + \sqrt{3}}} \right. \\
& \left. \left(3 \sqrt{-\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \left(-6 - 3 \times 2^{1/3} - 2 \frac{1}{2} \sqrt{3} + \frac{1}{2} 2^{1/3} \sqrt{3} + \left(3 \times 2^{1/3} + 4 \frac{1}{2} \sqrt{3} + \frac{1}{2} 2^{1/3} \sqrt{3} \right) x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}] - \right. \\
& \left. 4 \sqrt{3} (-3 + 2^{1/3}) \sqrt{\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x} \sqrt{1 - x + x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}\right] \right) \Bigg) \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x} \sqrt{1 - x^3} \right)
\end{aligned}$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2 + 3x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 (2 + 3 \times 2^{2/3}) \text{ArcTan}\left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{1 - x^3}}\right]}{3 \sqrt{3}} + \frac{2 (3 - 2 \times 2^{1/3}) \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x}\right], -7 - 4 \sqrt{3}]}{3 \times 3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3}}
\end{aligned}$$

Result (type 4, 335 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{1}{2} (-1 + x)}{3 \frac{1}{2} + \sqrt{3}}} \right. \\
& \left. \left(-3 \frac{1}{2} \sqrt{-\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x} \left(-6 \frac{1}{2} - 3 \frac{1}{2} 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 \frac{1}{2} 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}] + \right. \\
& \left. 4 \sqrt{3} (3 + 2^{1/3}) \sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \sqrt{1 + x + x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}\right] \right) \Bigg) \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \sqrt{1 - x^3} \right)
\end{aligned}$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 176 leaves, 4 steps):

$$-\frac{2 (2+3 \times 2^{2/3}) \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1+x^3}}\right]}{3 \sqrt{3}} + \frac{2 (3-2 \times 2^{1/3}) \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 333 leaves):

$$\begin{aligned} & \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{1}{2} (-1+x)}{3 \frac{1}{2} + \sqrt{3}}} \right. \\ & \left(-3 \frac{1}{2} \sqrt{-\frac{1}{2} + \sqrt{3}} - 2 \frac{1}{2} x \right) \left(-6 \frac{1}{2} - 3 \frac{1}{2} 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 \frac{1}{2} 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3}} + 2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}\right] + \\ & \left. 4 \sqrt{3} (3+2^{1/3}) \sqrt{\frac{2 \sqrt{3}}{\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3}} + 2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}\right] \right) / \\ & \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \sqrt{-1+x^3} \right) \end{aligned}$$

Problem 55: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 169 leaves, 4 steps):

$$-\frac{2 (2-3 \times 2^{2/3}) \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}}\right]}{3 \sqrt{3}} + \frac{2 (3+2 \times 2^{1/3}) \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 338 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{\frac{1}{2}(1+x)}{3\frac{1}{2} + \sqrt{3}}} \right. \\
& \left. \left(3 \sqrt{-\frac{1}{2} + \sqrt{3} + 2\frac{1}{2}x} \left(-6 - 3 \times 2^{1/3} - 2\frac{1}{2}\sqrt{3} + \frac{1}{2}2^{1/3}\sqrt{3} + \left(3 \times 2^{1/3} + 4\frac{1}{2}\sqrt{3} + \frac{1}{2}2^{1/3}\sqrt{3} \right)x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}] - \right. \\
& \left. 4\sqrt{3}(-3 + 2^{1/3})\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{\frac{1}{2} + 2\frac{1}{2}2^{2/3} + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}\right] \right) \Bigg) \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2\frac{1}{2}2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e + f x}{(2^{2/3} + x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 159 leaves, 4 steps):

$$\frac{2(e - 2^{2/3}f) \text{ArcTan}\left[\frac{\sqrt{3}(1+2^{1/3}x)}{\sqrt{1+x^3}}\right]}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(2^{1/3}e+f)(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 340 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{\frac{1}{2}(1+x)}{3\frac{1}{2} + \sqrt{3}}} \right. \\
& \left. \left(f \sqrt{-\frac{1}{2} + \sqrt{3} + 2\frac{1}{2}x} \left(-6 - 3 \times 2^{1/3} - 2\frac{1}{2}\sqrt{3} + \frac{1}{2}2^{1/3}\sqrt{3} + \left(3 \times 2^{1/3} + 4\frac{1}{2}\sqrt{3} + \frac{1}{2}2^{1/3}\sqrt{3} \right)x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}] - \right. \\
& \left. 2\sqrt{3}(2^{1/3}e - 2f)\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}\sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{\frac{1}{2} + 2\frac{1}{2}2^{2/3} + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}\right] \right) \Bigg) \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2\frac{1}{2}2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} - 2\frac{1}{2}x} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 175 leaves, 4 steps):

$$\frac{2 (e + 2^{2/3} f) \operatorname{ArcTan} \left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{1 - x^3}} \right] - 2 \sqrt{2 + \sqrt{3}} (2^{1/3} e - f) (1 - x) \sqrt{\frac{1 + x + x^2}{(1 + \sqrt{3} - x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 - \sqrt{3} - x}{1 + \sqrt{3} - x} \right], -7 - 4\sqrt{3} \right]}{3 \sqrt{3} - 3 \times 3^{1/4} \sqrt{\frac{1 - x}{(1 + \sqrt{3} - x)^2}} \sqrt{1 - x^3}}$$

Result (type 4, 340 leaves):

$$\begin{aligned} & \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{1}{2} (-1 + x)}{3 \frac{1}{2} + \sqrt{3}}} \right. \\ & \left(-\frac{1}{2} f \sqrt{-\frac{1}{2} + \sqrt{3}} - 2 \frac{1}{2} x \right) \left(-6 \frac{1}{2} - 3 \frac{1}{2} 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 \frac{1}{2} 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{1}{2} + \sqrt{3}} + 2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}} \right] + \\ & 2 \sqrt{3} (2^{1/3} e + 2 f) \sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \sqrt{1 + x + x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{1}{2} + \sqrt{3}} + 2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}} \right] \Big) / \\ & \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \sqrt{1 - x^3} \right) \end{aligned}$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 178 leaves, 4 steps):

$$\frac{2 (e + 2^{2/3} f) \operatorname{ArcTanh} \left[\frac{\sqrt{3} (1 - 2^{1/3} x)}{\sqrt{-1 + x^3}} \right] - 2 \sqrt{2 - \sqrt{3}} (2^{1/3} e - f) (1 - x) \sqrt{\frac{1 + x + x^2}{(1 - \sqrt{3} - x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1 + \sqrt{3} - x}{1 - \sqrt{3} - x} \right], -7 + 4\sqrt{3} \right]}{3 \sqrt{3} - 3 \times 3^{1/4} \sqrt{\frac{1 - x}{(1 - \sqrt{3} - x)^2}} \sqrt{-1 + x^3}}$$

Result (type 4, 338 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{-\frac{\frac{1}{2}(-1+x)}{3\frac{1}{2}+\sqrt{3}}} \right. \\
& \left(-\frac{1}{2} f \sqrt{-\frac{1}{2}+\sqrt{3}} - 2 \frac{1}{2} x \right) \left(-6 \frac{1}{2} - 3 \frac{1}{2} 2^{1/3} + 2 \sqrt{3} - 2^{1/3} \sqrt{3} + \left(-3 \frac{1}{2} 2^{1/3} + 4 \sqrt{3} + 2^{1/3} \sqrt{3} \right) x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}+2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}} \right] + \\
& 2 \sqrt{3} (2^{1/3} e + 2 f) \sqrt{\frac{1}{2}+\sqrt{3}} + 2 \frac{1}{2} x \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{\frac{1}{2}+2 \frac{1}{2} 2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}+2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}} \right] \Big) / \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2}+\sqrt{3}} + 2 \frac{1}{2} x \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 59: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e + f x}{(2^{2/3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\frac{2 (e - 2^{2/3} f) \text{ArcTanh} \left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}} \right] + 2 \sqrt{2-\sqrt{3}} (2^{1/3} e + f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4 \sqrt{3} \right]}{3 \sqrt{3}}$$

Result (type 4, 342 leaves):

$$\begin{aligned}
& \left(2 \times 2^{1/6} \sqrt{\frac{\frac{1}{2}(1+x)}{3 \frac{1}{2}+\sqrt{3}}} \right. \\
& \left(f \sqrt{-\frac{1}{2}+\sqrt{3}} + 2 \frac{1}{2} x \right) \left(-6 - 3 \times 2^{1/3} - 2 \frac{1}{2} \sqrt{3} + \frac{1}{2} 2^{1/3} \sqrt{3} + \left(3 \times 2^{1/3} + 4 \frac{1}{2} \sqrt{3} + \frac{1}{2} 2^{1/3} \sqrt{3} \right) x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}-2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}} \right] - \\
& 2 \sqrt{3} (2^{1/3} e - 2 f) \sqrt{\frac{1}{2}+\sqrt{3}} - 2 \frac{1}{2} x \sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{\frac{1}{2}+2 \frac{1}{2} 2^{2/3}+\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}}-2 \frac{1}{2} x}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{1}{2}+\sqrt{3}} \right] \Big) / \\
& \left(\sqrt{3} \left(\frac{1}{2} + 2 \frac{1}{2} 2^{2/3} + \sqrt{3} \right) \sqrt{\frac{1}{2}+\sqrt{3}} - 2 \frac{1}{2} x \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 316 leaves, 4 steps):

$$\begin{aligned} & \frac{2 (b^{1/3} e - 2^{2/3} a^{1/3} f) \operatorname{ArcTan} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{a + b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} + \\ & \left(2 \sqrt{2 + \sqrt{3}} (2^{1/3} b^{1/3} e + a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\ & \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3} \right) \end{aligned}$$

Result (type 4, 336 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{3} b^{2/3} \sqrt{a + b x^3}} \\ & 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{3^{1/4} f \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right. \\ & \left. (-1)^{1/3} (1 + (-1)^{1/3}) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{\frac{i}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \end{aligned}$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 324 leaves, 4 steps):

$$\begin{aligned}
 & \frac{2 \left(b^{1/3} e + 2^{2/3} a^{1/3} f \right) \operatorname{ArcTan} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} - \\
 & \left(2 \sqrt{2 + \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{a - b x^3} \right)
 \end{aligned}$$

Result (type 4, 399 leaves):

$$\begin{aligned}
 & \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
 & \left. - \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right. \\
 & \left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
 & \left. \operatorname{EllipticPi} \left[\frac{\frac{1}{2} \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
 \end{aligned}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 333 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2 \left(b^{1/3} e + 2^{2/3} a^{1/3} f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}}\right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e - a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4\sqrt{3}\right]\right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x\right)^2}} \sqrt{-a + b x^3}\right)
\end{aligned}$$

Result (type 4, 400 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \\
& \left(-\left((-1)^{1/3} + 2^{2/3}\right) f \left((-1)^{1/3} a^{1/3} + b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left.\frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right]\right) / \left(\left((-1)^{1/3} + 2^{2/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3}\right)
\end{aligned}$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 329 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \left(b^{1/3} e - 2^{2/3} a^{1/3} f \right) \operatorname{ArcTanh} \left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{3 \sqrt{3} \sqrt{a} b^{2/3}} + \\
& \left(2 \sqrt{2 - \sqrt{3}} (2^{1/3} b^{1/3} e + a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 + 4\sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 387 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{3^{1/4}} \left((-1)^{1/3} + 2^{2/3} \right) f \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] + \right. \right. \\
& \left. \left. \frac{1}{\sqrt{3}} (-1)^{1/3} (1 + (-1)^{1/3}) (-b^{1/3} e + 2^{2/3} a^{1/3} f) \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) / \left(\left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 265 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (d e - c f) \operatorname{ArcTan} \left[\frac{\sqrt{3} \sqrt{c} (c + 2 d x)}{\sqrt{c^3 + 4 d^3 x^3}} \right]}{3 \sqrt{3} c^{3/2} d^2} + \\
& \left(2^{1/3} \sqrt{2 + \sqrt{3}} (2 d e + c f) (c + 2^{2/3} d x) \sqrt{\frac{c^2 - 2^{2/3} c d x + 2 \times 2^{1/3} d^2 x^2}{\left((1 + \sqrt{3}) c + 2^{2/3} d x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) c + 2^{2/3} d x}{(1 + \sqrt{3}) c + 2^{2/3} d x} \right], -7 - 4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c + 2^{2/3} d x)}{\left((1 + \sqrt{3}) c + 2^{2/3} d x \right)^2}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Result (type 4, 380 leaves):

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. - f \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 ((-1)^{1/3} + 2^{2/3}) d x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] + \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) (-d e + c f) \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}} \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{\frac{i}{2} 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \operatorname{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] \right) / \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{1+x^3}}\right]}{3 \sqrt{3}} + \frac{2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}-x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 207 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{i 2^{2/3} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3}+2^{2/3}} \right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3}-x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 160 leaves, 4 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{1-x^3}}\right]}{3 \sqrt{3}} + \frac{2 \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 209 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
& \left. \frac{\pm 2^{2/3} \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{\pm \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{(-1)^{1/3} + 2^{2/3}} \right)
\end{aligned}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} - x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 163 leaves, 4 steps):

$$\begin{aligned}
& - \frac{2 \times 2^{2/3} \operatorname{ArcTanh} \left[\frac{\sqrt{3} (1-2^{1/3} x)}{\sqrt{-1+x^3}} \right]}{3 \sqrt{3}} + \frac{2 \sqrt{2-\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
\end{aligned}$$

Result (type 4, 207 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
& \left. \frac{\pm 2^{2/3} \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{\pm \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{(-1)^{1/3} + 2^{2/3}} \right)
\end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} + x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} (1+2^{1/3} x)}{\sqrt{-1-x^3}}\right]}{3 \sqrt{3}} + \frac{2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 209 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(-\frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{\pm 2^{2/3} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3}+2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{1/3}+2^{2/3}} \right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$-\frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3}+2^{1/3} b^{1/3} x)}{\sqrt{a+b x^3}}\right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \frac{2 \sqrt{2+\sqrt{3}} (a^{1/3}+b^{1/3} x) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3}+b^{1/3} x)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}}$$

Result (type 4, 324 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{3} \sqrt{b^{2/3} \sqrt{a + b x^3}}} \\
& \frac{2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{3^{1/4} \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{(-1)^{1/3} + 2^{2/3}} \right.}{\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} \\
& \left. (-1)^{1/3} 2^{2/3} \left(1 + (-1)^{1/3}\right) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] \right)
\end{aligned}$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \operatorname{ArcTan}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{a - b x^3}}\right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \frac{2 \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} - b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} - b^{1/3} x\right)^2}} \sqrt{a - b x^3}}
\end{aligned}$$

Result (type 4, 388 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] - \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3} \right] \right) \Bigg) \Bigg/ \left((-1)^{1/3} + 2^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 292 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} - 2^{1/3} b^{1/3} x)}{\sqrt{-a + b x^3}}\right] - 2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4 \sqrt{3}]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \frac{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{-a + b x^3}}
\end{aligned}$$

Result (type 4, 389 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] - \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{\pm \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3} \right] \right) \Bigg) \Bigg/ \left((-1)^{1/3} + 2^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2^{2/3} a^{1/3} + b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 288 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \times 2^{2/3} \operatorname{ArcTanh}\left[\frac{\sqrt{3} a^{1/6} (a^{1/3} + 2^{1/3} b^{1/3} x)}{\sqrt{-a - b x^3}}\right] - 2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 + 4\sqrt{3}\right]}{3 \sqrt{3} a^{1/6} b^{2/3}} + \\
& \frac{3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{-a - b x^3}}
\end{aligned}$$

Result (type 4, 375 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-\frac{1}{3^{1/4}} \left((-1)^{1/3} + 2^{2/3} \right) \left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} \right. \\
& \left. \left(1 + (-1)^{1/3}\right) a^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{(-1)^{1/3} + 2^{2/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}] \right) \right. \\
& \left. \left((-1)^{1/3} + 2^{2/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + d x) \sqrt{c^3 + 4 d^3 x^3}} dx$$

Optimal (type 4, 246 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{c} (c+2 d x)}{\sqrt{c^3+4 d^3 x^3}}\right]}{3 \sqrt{3} \sqrt{c} d^2} + \frac{2^{1/3} \sqrt{2+\sqrt{3}} (c+2^{2/3} d x) \sqrt{\frac{c^2-2^{2/3} c d x+2 \cdot 2^{1/3} d^2 x^2}{\left(\left(1+\sqrt{3}\right) c+2^{2/3} d x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c+2^{2/3} d x}{\left(1+\sqrt{3}\right) c+2^{2/3} d x}\right], -7-4 \sqrt{3}\right]}{3 \times 3^{1/4} d^2 \sqrt{\frac{c (c+2^{2/3} d x)}{\left(\left(1+\sqrt{3}\right) c+2^{2/3} d x\right)^2}} \sqrt{c^3+4 d^3 x^3}}
\end{aligned}$$

Result (type 4, 372 leaves):

$$\begin{aligned}
& \left(2^{1/6} \sqrt{\frac{2^{1/3} c + 2 d x}{(1 + (-1)^{1/3}) c}} \right. \\
& \left. - \sqrt{\frac{(-2)^{1/3} c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \left((-1)^{1/3} (2 + (-2)^{1/3}) c - 2 \left((-1)^{1/3} + 2^{2/3} \right) d x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] + \right. \\
& \left. \frac{1}{\sqrt{3}} (-1)^{1/3} 2^{2/3} (1 + (-1)^{1/3}) c \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{2^{2/3} - \frac{2 \times 2^{1/3} d x}{c} + \frac{4 d^2 x^2}{c^2}} \right. \\
& \left. \text{EllipticPi} \left[\frac{\frac{i}{2} 2^{1/3} \sqrt{3}}{2 + (-2)^{1/3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}}{2^{1/6}} \right], (-1)^{1/3} \right] \right) \Bigg) \Bigg/ \left((2 + (-2)^{1/3}) d^2 \sqrt{\frac{2^{1/3} c + 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 + 4 d^3 x^3} \right)
\end{aligned}$$

Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$$

Optimal (type 3, 23 leaves, 2 steps):

$$\frac{2}{3} \text{ArcTanh} \left[\frac{(1+x)^2}{3\sqrt{1+x^3}} \right]$$

Result (type 4, 265 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{-\frac{\frac{i}{2} (1+x)}{-3i + \sqrt{3}}} \right. \\
& \left(-\frac{i}{2} \sqrt{i + \sqrt{3} - 2ix} \left(-\frac{i}{2} - \sqrt{3} + (-\frac{i}{2} + \sqrt{3})x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right] + 2\sqrt{3} \sqrt{-\frac{i}{2} + \sqrt{3} + 2ix} \right. \\
& \left. \left. \sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right] \right) \right) \Big/ \left(\left(3i + \sqrt{3} \right) \sqrt{-\frac{i}{2} + \sqrt{3} + 2ix} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 75: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x) \sqrt{1-x^3}} dx$$

Optimal (type 3, 27 leaves, 2 steps):

$$-\frac{2}{3} \text{ArcTanh} \left[\frac{(1-x)^2}{3 \sqrt{1-x^3}} \right]$$

Result (type 4, 262 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2} (-1+x)}{-3i + \sqrt{3}}} \left(\sqrt{i + \sqrt{3} + 2ix} \left(-1 + \frac{i}{2} \sqrt{3} + x + \frac{i}{2} \sqrt{3}x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right] + 2\sqrt{3} \sqrt{-\frac{i}{2} + \sqrt{3} - 2ix} \right. \right. \right. \\
& \left. \left. \left. \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right] \right) \right) \Big/ \left(\left(3i + \sqrt{3} \right) \sqrt{-\frac{i}{2} + \sqrt{3} - 2ix} \sqrt{1+x^3} \right)
\end{aligned}$$

Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-x}{(2+x) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$-\frac{2}{3} \text{ArcTan} \left[\frac{(1-x)^2}{3 \sqrt{-1+x^3}} \right]$$

Result (type 4, 260 leaves):

$$-\left(\left(2 \sqrt{6} \sqrt{\frac{\frac{1}{i} (-1+x)}{-3 i + \sqrt{3}}} \left(\sqrt{i + \sqrt{3} + 2 i x} (-1 + i \sqrt{3} + x + i \sqrt{3} x) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}] + 2 \sqrt{3} \sqrt{-i + \sqrt{3} - 2 i x} \right. \right. \right. \\ \left. \left. \left. \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}\right]\right) \right) \right) \Big/ \left((3 i + \sqrt{3}) \sqrt{-i + \sqrt{3} - 2 i x} \sqrt{-1+x^3} \right)$$

Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{(2-x) \sqrt{-1-x^3}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\frac{2}{3} \text{ArcTan}\left[\frac{(1+x)^2}{3 \sqrt{-1-x^3}}\right]$$

Result (type 4, 267 leaves):

$$\left(2 \sqrt{6} \sqrt{-\frac{\frac{1}{i} (1+x)}{-3 i + \sqrt{3}}} \right. \\ \left. -i \sqrt{i + \sqrt{3} - 2 i x} (-i - \sqrt{3} + (-i + \sqrt{3}) x) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} + 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}] + 2 \sqrt{3} \sqrt{-i + \sqrt{3} + 2 i x} \right. \\ \left. \sqrt{1-x+x^2} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} + 2 i x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 i + \sqrt{3}}\right]\right) \Big/ \left((3 i + \sqrt{3}) \sqrt{-i + \sqrt{3} + 2 i x} \sqrt{-1-x^3} \right)$$

Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3} x}{(2 a^{1/3} - b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{2 \text{ArcTanh}\left[\frac{\left(a^{1/3}+b^{1/3} x\right)^2}{3 a^{1/6} \sqrt{a+b x^3}}\right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 407 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-\frac{1}{2\sqrt{2}} 3^{1/4} \left((\frac{1}{2} + \sqrt{3}) a^{1/3} - (-\frac{1}{2} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2\frac{1}{2}b^{1/3}x}{a^{1/3}}}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{2}a^{1/3} + (\frac{1}{2} + \sqrt{3})b^{1/3}x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2}\sqrt{3}\right)] + 3\frac{1}{2} \right. \\
& a^{1/3} \sqrt{\frac{-2\frac{1}{2}a^{1/3} + (\frac{1}{2} + \sqrt{3})b^{1/3}x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \sqrt{1 - \frac{b^{1/3}x}{a^{1/3}} + \frac{b^{2/3}x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{2}a^{1/3} + (\frac{1}{2} + \sqrt{3})b^{1/3}x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2}\sqrt{3}\right)\right] \left. \right) \\
& \left(\left(-2 + (-1)^{1/3}\right) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3}b^{1/3}x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 79: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 52 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\left(a^{1/3} - b^{1/3} x\right)^2}{3 a^{1/6} \sqrt{a - b x^3}}\right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 370 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. \left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] + (-1)^{1/3} \sqrt{3} \right. \\
& \left. \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}]\right) \right) \\
& \left. \left(-2 + (-1)^{1/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} - b^{1/3} x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}}\right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\
& \left. \left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}] + (-1)^{1/3} \sqrt{3} \right. \\
& \left. \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}]\right) \right) \\
& \left(-2 + (-1)^{1/3} \right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3}
\end{aligned}$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} + b^{1/3} x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}}\right]}{3 a^{1/6} b^{1/3}}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-\frac{1}{2\sqrt{2}} 3^{1/4} \left((\frac{1}{2} + \sqrt{3}) a^{1/3} - (-\frac{1}{2} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\frac{2}{3} b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{3} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{1}{3} + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3})] + 3 \frac{1}{3} \right. \\
& a^{1/3} \sqrt{\frac{-2\frac{1}{3} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{1}{3} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{3} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{3} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3\frac{1}{3} + \sqrt{3}) a^{1/3}}}\right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3})\right] \left. \right) \\
& \left((-2 + (-1)^{1/3}) b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{c - 2 d x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} d x$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{(c-2 d x)^2}{3 \sqrt{c} \sqrt{c^3-8 d^3 x^3}}\right]}{3 \sqrt{c} d}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& - \left(2 \sqrt{\frac{c - 2 d x}{(1 + (-1)^{1/3}) c}} \left((-2 + (-1)^{1/3}) \left((-1)^{1/3} c + 2 d x \right) \sqrt{\frac{(-1)^{1/3} (c + 2 (-1)^{1/3} d x)}{(1 + (-1)^{1/3}) c}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}] + \right. \right. \\
& (-1)^{1/3} \sqrt{3} \left(1 + (-1)^{1/3} \right) c \sqrt{\frac{c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{\frac{c^2 + 2 c d x + 4 d^2 x^2}{c^2}} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{3} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}\right] \left. \right) \left. \right) \\
& \left((-2 + (-1)^{1/3}) d \sqrt{\frac{c - 2 (-1)^{2/3} d x}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8 d^3 x^3} \right)
\end{aligned}$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 - x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 139 leaves, 4 steps):

$$\frac{2}{9} (e + 2 f) \operatorname{ArcTanh}\left[\frac{(1+x)^2}{3 \sqrt{1+x^3}}\right] + \frac{2 \sqrt{2+\sqrt{3}} (e-f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 273 leaves):

$$\begin{aligned} & \left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{\frac{i}{2} (1+x)}{-3 \frac{i}{2} + \sqrt{3}}} \right. \\ & \left(-3 \frac{i}{2} f \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \left(-\frac{i}{2} - \sqrt{3} + \left(-\frac{i}{2} + \sqrt{3} \right) x \right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right] + 2 \sqrt{3} (e + 2 f) \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \right. \\ & \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}}\right] \right) \right) \Bigg/ \left(\left(3 \frac{i}{2} + \sqrt{3} \right) \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \sqrt{1+x^3} \right) \end{aligned}$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 + x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$-\frac{2}{9} (e - 2 f) \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3 \sqrt{1-x^3}}\right] - \frac{2 \sqrt{2+\sqrt{3}} (e+f) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 271 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{\mathbf{i}}{2} (-1+x)}{-3 \mathbf{i} + \sqrt{3}}} \right. \\
& \left. \left(3 f \sqrt{\frac{\mathbf{i}}{2} + \sqrt{3} + 2 \mathbf{i} x} (-1 + \frac{\mathbf{i}}{2} \sqrt{3} + x + \frac{\mathbf{i}}{2} \sqrt{3} x) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \mathbf{i} + \sqrt{3}}] - 2 \sqrt{3} (e - 2 f) \sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x} \right. \right. \\
& \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \mathbf{i} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \mathbf{i} + \sqrt{3}}\right]\right) \right) \Big/ \left(\left(3 \mathbf{i} + \sqrt{3} \right) \sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x} \sqrt{1-x^3} \right)
\end{aligned}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 + x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$\begin{aligned}
& -\frac{2}{9} (e - 2 f) \operatorname{ArcTan}\left[\frac{(1-x)^2}{3 \sqrt{-1+x^3}}\right] - \frac{2 \sqrt{2-\sqrt{3}} (e + f) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}]}{3 \times 3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
\end{aligned}$$

Result (type 4, 269 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{\mathbf{i}}{2} (-1+x)}{-3 \mathbf{i} + \sqrt{3}}} \right. \\
& \left. \left(3 f \sqrt{\frac{\mathbf{i}}{2} + \sqrt{3} + 2 \mathbf{i} x} (-1 + \frac{\mathbf{i}}{2} \sqrt{3} + x + \frac{\mathbf{i}}{2} \sqrt{3} x) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \mathbf{i} + \sqrt{3}}] - 2 \sqrt{3} (e - 2 f) \sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x} \right. \right. \\
& \left. \left. \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \mathbf{i} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-3 \mathbf{i} + \sqrt{3}}\right]\right) \right) \Big/ \left(\left(3 \mathbf{i} + \sqrt{3} \right) \sqrt{-\frac{\mathbf{i}}{2} + \sqrt{3} - 2 \mathbf{i} x} \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 - x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 150 leaves, 4 steps):

$$\frac{2}{9} (e + 2f) \operatorname{ArcTan}\left[\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right] + \frac{2\sqrt{2-\sqrt{3}} (e-f)(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4\sqrt{3}]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 275 leaves):

$$\begin{aligned} & \left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{i(1+x)}{-3i + \sqrt{3}}} \right. \\ & \left(-3i f \sqrt{i + \sqrt{3} - 2ix} \left(-i - \sqrt{3} + (-i + \sqrt{3})x \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-3i + \sqrt{3}}] + 2\sqrt{3} (e + 2f) \sqrt{-i + \sqrt{3} + 2ix} \right. \\ & \left. \left. \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3i + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} + 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-3i + \sqrt{3}}\right] \right) \right) \Bigg/ \left((3i + \sqrt{3}) \sqrt{-i + \sqrt{3} + 2ix} \sqrt{-1-x^3} \right) \end{aligned}$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(2a^{1/3}-b^{1/3}x)\sqrt{a+bx^3}} dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\begin{aligned} & \frac{2(b^{1/3}e + 2a^{1/3}f) \operatorname{ArcTanh}\left[\frac{(a^{1/3}+b^{1/3}x)^2}{3a^{1/6}\sqrt{a+bx^3}}\right]}{9\sqrt{a}b^{2/3}} + \\ & \left(2\sqrt{2+\sqrt{3}} (b^{1/3}e - a^{1/3}f) (a^{1/3} + b^{1/3}x) \sqrt{\frac{a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1-\sqrt{3})a^{1/3} + b^{1/3}x}{(1+\sqrt{3})a^{1/3} + b^{1/3}x}\right], -7-4\sqrt{3}] \right) \Bigg/ \\ & \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3}x)}{((1+\sqrt{3})a^{1/3} + b^{1/3}x)^2}} \sqrt{a+bx^3} \right) \end{aligned}$$

Result (type 4, 419 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-\frac{1}{2 \sqrt{2}} 3^{1/4} f \left(\left(\frac{1}{2} + \sqrt{3} \right) a^{1/3} - \left(-\frac{1}{2} + \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)] + \right. \\
& \left. \frac{1}{2} \left(b^{1/3} e + 2 a^{1/3} f \right) \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)\right] \right) \Bigg/ \left(\left(-2 + (-1)^{1/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3} \right) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 304 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e - 2 a^{1/3} f) \operatorname{ArcTanh}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}}\right]}{9 \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 + \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 - 4 \sqrt{3}] \right) \Bigg/ \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 447 leaves):

$$\begin{aligned}
& \frac{1}{\left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \left(-\frac{1}{2} \operatorname{I} f \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3}\right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \left(\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3} - \left(3 \frac{1}{2} + \sqrt{3}\right) b^{1/3} x \right) \right. \\
& \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right) \right] - \frac{1}{2} \left(b^{1/3} e - 2 a^{1/3} f\right) \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right) \right] \right)
\end{aligned}$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 313 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e - 2 a^{1/3} f) \operatorname{ArcTan} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}} \right]}{9 \sqrt{a} b^{2/3}} - \\
& \left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e + a^{1/3} f) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3}\right) a^{1/3} - b^{1/3} x\right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\left(1 + \sqrt{3}\right) a^{1/3} - b^{1/3} x}{\left(1 - \sqrt{3}\right) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left(\left(1 - \sqrt{3}\right) a^{1/3} - b^{1/3} x\right)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 448 leaves):

$$\begin{aligned}
& \frac{1}{\left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{-a + b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \left(-\frac{1}{2} \text{i} f \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3}\right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \left(\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3} - \left(3 \frac{1}{2} + \sqrt{3}\right) b^{1/3} x \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right) \right] - \frac{1}{2} \left(b^{1/3} e - 2 a^{1/3} f\right) \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right) \right] \right)
\end{aligned}$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 310 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (b^{1/3} e + 2 a^{1/3} f) \text{ArcTan} \left[\frac{\left(a^{1/3} + b^{1/3} x\right)^2}{3 a^{1/6} \sqrt{-a - b x^3}} \right]}{9 \sqrt{a} b^{2/3}} + \\
& \left(2 \sqrt{2 - \sqrt{3}} (b^{1/3} e - a^{1/3} f) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3 \times 3^{1/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 422 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-\frac{1}{2\sqrt{2}} 3^{1/4} f \left(\left(\frac{1}{2} + \sqrt{3}\right) a^{1/3} - \left(-\frac{1}{2} + \sqrt{3}\right) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2\frac{1}{2} b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)] + \right. \\
& \left. \frac{1}{2} \left(b^{1/3} e + 2 a^{1/3} f\right) \sqrt{\frac{-2\frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \right. \\
& \left. \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2\frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3\frac{1}{2} + \sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)\right] \right) \Bigg/ \left(\left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 4, 221 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 (d e - c f) \operatorname{ArcTanh}\left[\frac{(c-2 d x)^2}{3 \sqrt{c} \sqrt{c^3 - 8 d^3 x^3}}\right] - \frac{\sqrt{2 + \sqrt{3}} (2 d e + c f) (c - 2 d x) \sqrt{\frac{c^2 + 2 c d x + 4 d^2 x^2}{((1 + \sqrt{3}) c - 2 d x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c - 2 d x}{(1 + \sqrt{3}) c - 2 d x}\right], -7 - 4 \sqrt{3}]}{9 c^{3/2} d^2} \\
& - \frac{3 \times 3^{1/4} c d^2 \sqrt{\frac{c (c - 2 d x)}{\left((1 + \sqrt{3}) c - 2 d x\right)^2}} \sqrt{c^3 - 8 d^3 x^3}}
\end{aligned}$$

Result (type 4, 384 leaves):

$$\begin{aligned}
& - \left(\left(\frac{\frac{1}{2} \sqrt{\frac{c - 2dx}{(1 + (-1)^{1/3})c}}}{\sqrt{\frac{(-\frac{1}{2} + \sqrt{3})c + 2(\frac{1}{2} + \sqrt{3})dx}{(-3\frac{1}{2} + \sqrt{3})c}} \left((-3\frac{1}{2} + \sqrt{3})c - 2(3\frac{1}{2} + \sqrt{3})dx \right) \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{2} \sqrt{\frac{\frac{1}{2}c + \frac{1}{2}dx + \sqrt{3}dx}{3\frac{1}{2}c - \sqrt{3}c}}], \frac{1}{2}(1 + \frac{1}{2}\sqrt{3})] + \right. \right. \\
& \left. \left. 4\sqrt{2}(de - cf) \sqrt{\frac{\frac{1}{2}c + \frac{1}{2}dx + \sqrt{3}dx}{3\frac{1}{2}c - \sqrt{3}c}} \sqrt{\frac{c^2 + 2cdx + 4d^2x^2}{c^2}} \operatorname{EllipticPi}[\frac{2\sqrt{3}}{3\frac{1}{2} + \sqrt{3}}, \right. \right. \\
& \left. \left. \operatorname{ArcSin}[\sqrt{2} \sqrt{\frac{\frac{1}{2}c + \frac{1}{2}dx + \sqrt{3}dx}{3\frac{1}{2}c - \sqrt{3}c}}], \frac{1}{2}(1 + \frac{1}{2}\sqrt{3})] \right) \right) \Big/ \left(2(-2 + (-1)^{1/3})d^2 \sqrt{\frac{c - 2(-1)^{2/3}dx}{(1 + (-1)^{1/3})c}} \sqrt{c^3 - 8d^3x^3} \right)
\end{aligned}$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTanh}\left[\frac{(1+x)^2}{3 \sqrt{1+x^3}}\right] - \frac{2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] + \frac{2 \pm \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2\sqrt{3}}{3i+\sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} - 2 + (-1)^{1/3} \right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTanh}\left[\frac{(1-x)^2}{3\sqrt{1-x^3}}\right] - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4\sqrt{3}\right]}{3\sqrt{3^{1/4}}\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}\sqrt{1-x^3}}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{1-x^3}} 2\sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ \left(\frac{\left((-1)^{1/3}+x\right)\sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{2\pm\sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2\sqrt{3}}{3^{\pm}\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 148 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTan}\left[\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right] - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]}{3\sqrt{3^{1/4}}\sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}}\sqrt{-1+x^3}}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ \left(\frac{\left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2-x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 140 leaves, 4 steps):

$$\frac{4}{9} \operatorname{ArcTan}\left[\frac{(1+x)^2}{3 \sqrt{-1-x^3}}\right] - \frac{2 \sqrt{2-\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3 \times 3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \\ \left(\frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2 \pm \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-2+(-1)^{1/3}} \right)$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3}-b^{1/3} x) \sqrt{a+b x^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTanh} \left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{a + b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \frac{2 \sqrt{2 + \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{a + b x^3}}$$

Result (type 4, 407 leaves):

$$\left(\sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right. \\ \left. - \sqrt{2} 3^{1/4} \left((\frac{1}{2} + \sqrt{3}) a^{1/3} - (-\frac{1}{2} + \sqrt{3}) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] + 8 \frac{1}{2} \right. \\ \left. a^{1/3} \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + (\frac{1}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] \right) \\ \left(2 (-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)$$

Problem 97: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} + b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 268 leaves, 4 steps):

$$\frac{4 \operatorname{ArcTanh} \left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{a - b x^3}} \right]}{9 a^{1/6} b^{2/3}} - \frac{2 \sqrt{2 + \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4 \sqrt{3} \right]}{3 \times 3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{a - b x^3}}$$

Result (type 4, 371 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} + (-1)^{1/3} b^{1/3} x \right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}] + \\
& \frac{1}{\sqrt{3}} 2 (-1)^{1/3} \left(1 + (-1)^{1/3}\right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \\
& \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}]\right) \Bigg/ \left((-2 + (-1)^{1/3}) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} + b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 277 leaves, 4 steps):

$$\begin{aligned}
& \frac{4 \text{ArcTan}\left[\frac{(a^{1/3} - b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a + b x^3}}\right] - \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x\right)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}\right], -7 + 4 \sqrt{3}]}{9 a^{1/6} b^{2/3}} \\
& 3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x\right)^2}} \sqrt{-a + b x^3}
\end{aligned}$$

Result (type 4, 372 leaves):

$$\begin{aligned}
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \\
& \left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} + (-1)^{1/3} b^{1/3} x \right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} 2 (-1)^{1/3} \\
& \left(1 + (-1)^{1/3}\right) a^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \pm \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right] \Bigg) \\
& \left(-2 + (-1)^{1/3} \right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{-a + b x^3}
\end{aligned}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(2 a^{1/3} - b^{1/3} x) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 273 leaves, 4 steps):

$$\begin{aligned}
& \frac{4 \text{ArcTan}\left[\frac{(a^{1/3} + b^{1/3} x)^2}{3 a^{1/6} \sqrt{-a - b x^3}}\right] - \frac{2 \sqrt{2 - \sqrt{3}} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7 + 4 \sqrt{3}]}{9 a^{1/6} b^{2/3}} \\
& 3 \times 3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 - \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{-a - b x^3}
\end{aligned}$$

Result (type 4, 410 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \right. \\
& \left(-\sqrt{2} \, 3^{1/4} \left(\left(\frac{1}{2} + \sqrt{3}\right) a^{1/3} - \left(-\frac{1}{2} + \sqrt{3}\right) b^{1/3} x \right) \sqrt{\frac{\frac{1}{2} + \sqrt{3} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}}}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)] + 8 \frac{1}{2} \right. \\
& \left. a^{1/3} \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{1}{2} + \sqrt{3}}, \operatorname{ArcSin}\left[\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3}\right) a^{1/3}}\right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3}\right)\right] \right) \\
& \left(2 \left(-2 + (-1)^{1/3}\right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(c + d x) \sqrt{c^3 - 8 d^3 x^3}} dx$$

Optimal (type 4, 202 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{(c-2 d x)^2}{3 \sqrt{c} \sqrt{c^3-8 d^3 x^3}}\right] - \frac{\sqrt{2+\sqrt{3}} (c-2 d x)}{\sqrt{\frac{c^2+2 c d x+4 d^2 x^2}{\left(\left(1+\sqrt{3}\right) c-2 d x\right)^2}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c-2 d x}{\left(1+\sqrt{3}\right) c-2 d x}\right], -7-4 \sqrt{3}\right]}{9 \sqrt{c} d^2} \\
& \frac{3 \times 3^{1/4} d^2 \sqrt{\frac{c (c-2 d x)}{\left(\left(1+\sqrt{3}\right) c-2 d x\right)^2}} \sqrt{c^3-8 d^3 x^3}}
\end{aligned}$$

Result (type 4, 295 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{c - 2 dx}{(1 + (-1)^{1/3}) c}} \left(\left(-2 + (-1)^{1/3} \right) \left((-1)^{1/3} c + 2 dx \right) \sqrt{\frac{(-1)^{1/3} (c + 2 (-1)^{1/3} dx)}{(1 + (-1)^{1/3}) c}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}] + \frac{1}{\sqrt{3}} \right. \right. \\
& \left. \left. 2 (-1)^{1/3} (1 + (-1)^{1/3}) c \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{\frac{c^2 + 2 c dx + 4 d^2 x^2}{c^2}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 i + \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}}\right], (-1)^{1/3}\right] \right) \right) / \\
& \left(\left(-2 + (-1)^{1/3} \right) d^2 \sqrt{\frac{c - 2 (-1)^{2/3} dx}{(1 + (-1)^{1/3}) c}} \sqrt{c^3 - 8 d^3 x^3} \right)
\end{aligned}$$

Problem 101: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{6} \sqrt{\frac{i (1+x)}{3 i + \sqrt{3}}} \right. \right. \\
& \left. \left. \left(\sqrt{-i + \sqrt{3}} + 2 i x \right) \left((1+2 i) - i \sqrt{3} + ((-2-i) + \sqrt{3}) x \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}] + 4 i \sqrt{i + \sqrt{3}} - 2 i x \right. \right. \\
& \left. \left. \sqrt{1 - x + x^2} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{-3 + (2+i) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{i + \sqrt{3}} - 2 i x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 i + \sqrt{3}}\right] \right) \right) / \left((-3 + (2+i) \sqrt{3}) \sqrt{i + \sqrt{3}} - 2 i x \sqrt{1 + x^3} \right)
\end{aligned}$$

Problem 102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \operatorname{Arctanh} \left[\frac{\sqrt{-3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}} \right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2} (-1+x)}{-3 \frac{i}{2} + \sqrt{3}}} \right. \\ & \left(\sqrt{\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \left((2 + \frac{i}{2}) - \sqrt{3} + \left((1 + 2 \frac{i}{2}) - \frac{i}{2} \sqrt{3} \right) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}} \right] + 4 \sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{1+x+x^2} \right. \\ & \left. \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-3 \frac{i}{2} + \sqrt{3}} \right] \right) \Bigg/ \left((-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}) \sqrt{-\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{1-x^3} \right) \end{aligned}$$

Problem 103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{Arctan} \left[\frac{\sqrt{-3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}} \right]}{\sqrt{-3+2 \sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2}(-1+x)}{-3\frac{i}{2}+\sqrt{3}}} \left(\sqrt{\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \left((2+\frac{i}{2})-\sqrt{3} + \left((1+2\frac{i}{2})-\frac{i}{2}\sqrt{3} \right)x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] + \right. \right. \\
& \left. \left. 4\sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \sqrt{1+x+x^2} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3\frac{i}{2}+(1+2\frac{i}{2})\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-3\frac{i}{2}+\sqrt{3}} \right] \right) \right) / \\
& \left((-3\frac{i}{2}+(1+2\frac{i}{2})\sqrt{3}) \sqrt{-\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \sqrt{-1+x^3} \right)
\end{aligned}$$

Problem 104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{(1-\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps) :

$$\frac{2 \text{ArcTan} \left[\frac{\sqrt{-3+2\sqrt{3}} (1+x)}{\sqrt{-1-x^3}} \right]}{\sqrt{-3+2\sqrt{3}}}$$

Result (type 4, 269 leaves) :

$$\begin{aligned}
& - \left(\left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2}(1+x)}{3\frac{i}{2}+\sqrt{3}}} \right. \right. \\
& \left. \left. \left(\sqrt{-\frac{i}{2}+\sqrt{3}+2\frac{i}{2}x} \left((1+2\frac{i}{2})-\frac{i}{2}\sqrt{3} + \left((-2-\frac{i}{2})+\sqrt{3} \right)x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] + 4\frac{i}{2} \sqrt{\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \right. \right. \\
& \left. \left. \sqrt{1-x+x^2} \text{EllipticPi} \left[\frac{2\frac{i}{2}\sqrt{3}}{-3+(2+\frac{i}{2})\sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{3\frac{i}{2}+\sqrt{3}} \right] \right) \right) / \left((-3+(2+\frac{i}{2})\sqrt{3}) \sqrt{\frac{i}{2}+\sqrt{3}-2\frac{i}{2}x} \sqrt{-1-x^3} \right)
\end{aligned}$$

Problem 105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} \ a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} \ a^{1/6} \ b^{1/3}}$$

Result (type 4, 322 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{a+b x^3}} \\ & 2 \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \left(- \frac{\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\left((-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]}}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}} + \frac{4 (-1)^{5/6} \left(1+(-1)^{1/3}\right) a^{1/3}}{\sqrt{1-\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]} \right) \Big/ \left(\left(-3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}\right) b^{1/3} \right) \end{aligned}$$

Problem 106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 - \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3} x)}{\sqrt{a-b x^3}} \right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 446 leaves):

$$\frac{1}{\left(-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}\right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \sqrt{a-b x^3}}$$

$$2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1+(-1)^{1/3}) a^{1/3}}} \left(\sqrt{\frac{(-\frac{i}{2} + \sqrt{3}) a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \left((-3 + (2+\frac{i}{2}) \sqrt{3}) a^{1/3} + (-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}) b^{1/3} x \right) \right)$$

$$\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} (2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x)}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right] + 4 \sqrt{3} a^{1/3} \sqrt{-\frac{2 \frac{i}{2} a^{1/3} + (\frac{i}{2} + \sqrt{3}) b^{1/3} x}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}}$$

$$\sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{i}{2} + (1+2 \frac{i}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} (2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x)}{(-3 \frac{i}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{i}{2} \sqrt{3}) \right]$$

Problem 107: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1+\sqrt{3}) a^{1/3} - b^{1/3} x}{\left((1-\sqrt{3}) a^{1/3} - b^{1/3} x\right) \sqrt{-a+b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3} x)}{\sqrt{-a+b x^3}} \right]}{\sqrt{-3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 447 leaves):

$$\begin{aligned}
& \frac{1}{\left(-3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}\right) b^{1/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}} \sqrt{-a + b x^3}}} \\
& 2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\sqrt{\frac{\left(-\frac{i}{2} + \sqrt{3}\right) a^{1/3} + \left(\frac{i}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \left(\left(-3 + (2 + \frac{i}{2}) \sqrt{3}\right) a^{1/3} + \left(-3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}\right) b^{1/3} x \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{i}{2} \sqrt{3}\right) \right] + 4 \sqrt{3} a^{1/3} \sqrt{-\frac{2 \frac{i}{2} a^{1/3} + \left(\frac{i}{2} + \sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3} + (1 - \frac{i}{2} \sqrt{3}) b^{1/3} x\right)}{\left(-3 \frac{i}{2} + \sqrt{3}\right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{i}{2} \sqrt{3}\right) \right] \right)
\end{aligned}$$

Problem 108: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \text{ArcTan} \left[\frac{\sqrt{-3 + 2 \sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a - b x^3}} \right]}{\sqrt{-3 + 2 \sqrt{3}} a^{1/6} b^{1/3}}
\end{aligned}$$

Result (type 4, 325 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{4 (-1)^{5/6} (1 + (-1)^{1/3}) a^{1/3}}{\sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \Big/ \left((-3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}) b^{1/3} \right) \right)
\end{aligned}$$

Problem 109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(1+\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}
\end{aligned}$$

Result (type 6, 862 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3\sqrt{3} \right) \left(2 \left(-5 + 3\sqrt{3} \right) a - b x^3 \right) \sqrt{a + b x^3}} \left(26 - 15\sqrt{3} \right) a x \\
& \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6\sqrt{3} a} \right] \right) \middle/ \left(8 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) + \right. \\
& \left. x \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6\sqrt{3} a} \right] \right) \middle/ \right. \right. \\
& \left. \left. \left(10 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) + \right. \\
& \left. x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6\sqrt{3} a} \right] \right) \middle/ \left(4 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) - \right. \\
& \left. \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6\sqrt{3} a} \right] \right) \middle/ \left(14 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 110: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x}{\left(1 - \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x \right) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \text{ArcTanh} \left[\frac{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a} \right)^{1/3} x \right)}{\sqrt{a-b x^3}} \right]}{\sqrt{-3+2\sqrt{3}} \sqrt{a} \left(\frac{b}{a} \right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3\sqrt{3} \right) \sqrt{a - b x^3} \left(2 \left(-5 + 3\sqrt{3} \right) a + b x^3 \right)} \left(26 - 15\sqrt{3} \right) a x \\
& \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \middle/ \left(8 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \middle/ \left(10 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(16\sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \middle/ \left(4 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) + \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \middle/ \left(14 \left(-5 + 3\sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right. \right. \\
& \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] + \left(5 - 3\sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6\sqrt{3} a} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 111: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x}{\left(1 - \sqrt{3} - \left(\frac{b}{a} \right)^{1/3} x \right) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \text{ArcTan} \left[\frac{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a} \right)^{1/3} x \right)}{\sqrt{-a + b x^3}} \right]}{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(\frac{b}{a} \right)^{1/3}}$$

Result (type 6, 836 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3 \sqrt{3} \right) \sqrt{-a + b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a + b x^3 \right)} \left(26 - 15 \sqrt{3} \right) a x \\
& \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \middle/ \left(8 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \middle/ \left(10 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \middle/ \left(4 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right. \right. \right. \\
& \left. \left. \left. - b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \middle/ \left(14 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right. \right. \\
& \left. \left. - 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 112: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + \sqrt{3} + \left(\frac{b}{a} \right)^{1/3} x}{\left(1 - \sqrt{3} + \left(\frac{b}{a} \right)^{1/3} x \right) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \text{ArcTan} \left[\frac{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a} \right)^{1/3} x \right)}{\sqrt{-a-b x^3}} \right]}{\sqrt{-3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a} \right)^{1/3}}
\end{aligned}$$

Result (type 6, 865 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(-5 + 3 \sqrt{3} \right) \sqrt{-a - b x^3} \left(2 \left(-5 + 3 \sqrt{3} \right) a - b x^3 \right)} \left(26 - 15 \sqrt{3} \right) a x \\
& \left(- \left(\left(96 \left(-1 + \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(8 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(\left(60 \left(-3 + \sqrt{3} \right) a \left(\frac{b}{a} \right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) \middle/ \right. \\
& \left. \left(10 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a} \right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(4 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) - \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, \frac{b x^3}{-10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(14 \left(-5 + 3 \sqrt{3} \right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] + \left(5 - 3 \sqrt{3} \right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a - 6 \sqrt{3} a} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{2 \text{ArcTan} \left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}} \right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 269 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{3} (1+x)}{3i + \sqrt{3}}} \right. \\
& \left. \left(\sqrt{-i + \sqrt{3} + 2ix} \left((-2 - i) - \sqrt{3} + ((1 + 2i) + i\sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}] + 4 \sqrt{i + \sqrt{3} - 2ix} \sqrt{1 - x + x^2} \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}\right] \right) \right) \Big/ \left((3i + (1 + 2i)\sqrt{3}) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1 - x^3} \right)
\end{aligned}$$

Problem 114: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{3+2\sqrt{3}}}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{3} (-1+x)}{-3i + \sqrt{3}}} \right. \\
& \left. \left(\sqrt{i + \sqrt{3} + 2ix} \left((1 + 2i) + i\sqrt{3} + ((2 + i) + \sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-3i + \sqrt{3}}] - 4i \sqrt{-i + \sqrt{3} - 2ix} \right. \right. \\
& \left. \left. \sqrt{1 + x + x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{3 + (2 + i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{-i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{-3i + \sqrt{3}}\right] \right) \right) \Big/ \left((3 + (2 + i)\sqrt{3}) \sqrt{-i + \sqrt{3} - 2ix} \sqrt{1 - x^3} \right)
\end{aligned}$$

Problem 115: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{Arctanh} \left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}} \right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 265 leaves):

$$\begin{aligned} & \left(2 \sqrt{6} \sqrt{\frac{\frac{1}{2} (-1+x)}{-3 \frac{1}{2} + \sqrt{3}}} \right. \\ & \left(\sqrt{\frac{1}{2} + \sqrt{3} + 2 \frac{1}{2} x} \left((1+2 \frac{1}{2}) + \frac{1}{2} \sqrt{3} + \left((2+\frac{1}{2}) + \sqrt{3} \right) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-3 \frac{1}{2} + \sqrt{3}} \right] - 4 \frac{1}{2} \sqrt{-\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x} \right. \\ & \left. \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{2 \frac{1}{2} \sqrt{3}}{3 + (2+\frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\frac{\sqrt{-\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-3 \frac{1}{2} + \sqrt{3}} \right] \right) \Bigg/ \left(\left(3 + (2+\frac{1}{2}) \sqrt{3} \right) \sqrt{-\frac{1}{2} + \sqrt{3} - 2 \frac{1}{2} x} \sqrt{-1+x^3} \right) \end{aligned}$$

Problem 116: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 3, 44 leaves, 2 steps):

$$\frac{2 \operatorname{Arctanh} \left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}} \right]}{\sqrt{3+2 \sqrt{3}}}$$

Result (type 4, 271 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2} + \sqrt{3}}} \right. \\
& \left(\sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \left((-2 - \frac{i}{2}) - \sqrt{3} + \left((1 + 2 \frac{i}{2}) + \frac{i}{2} \sqrt{3} \right) x \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] + 4 \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{1-x+x^2} \right. \\
& \left. \text{EllipticPi} \left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}}, \text{ArcSin} \left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}} \right] \right) \Bigg/ \left(\left(3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3} \right) \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{-1-x^3} \right)
\end{aligned}$$

Problem 117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} + b^{1/3} x) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 69 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \text{ArcTan} \left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{a+b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}} \\
& - \frac{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}
\end{aligned}$$

Result (type 4, 320 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a+b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(- \frac{\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right. \\
& \left. + \frac{1}{(3 + (2 + \frac{i}{2}) \sqrt{3}) b^{1/3}} \right. \\
& 4 (-1)^{1/3} \left(1 + (-1)^{1/3} \right) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \frac{i}{2} \sqrt{3}}{3 + (2 + \frac{i}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right]
\end{aligned}$$

Problem 118: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 71 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTan} \left[\frac{\sqrt{3+2 \sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} a^{1/6} b^{1/3}}$$

Result (type 4, 329 leaves):

$$\frac{1}{b^{1/3} \sqrt{a - b x^3}} + \frac{2 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\begin{array}{l} \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} (a^{1/3} + (-1)^{1/3} b^{1/3} x)}{(1 + (-1)^{1/3}) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \\ - \frac{1}{3 + (2 + i) \sqrt{3}} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \end{array} \right) - 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \pm \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} \ a^{1/6} \left(a^{1/3}-b^{1/3} x\right)}{\sqrt{-a+b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} \ a^{1/6} b^{1/3}}$$

Result (type 4, 330 leaves):

$$\frac{1}{b^{1/3} \sqrt{-a+b x^3}} + \frac{2 \sqrt{\frac{a^{1/3}-b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \left((-1)^{1/3} a^{1/3} + b^{1/3} x \right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3}+(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}} - \frac{1}{3+ \left(2+\frac{1}{2}\right) \sqrt{3}} + 4 (-1)^{1/3} \left(1+(-1)^{1/3}\right) a^{1/3} \sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi} \left[\frac{2 \frac{1}{2} \sqrt{3}}{3+ \left(2+\frac{1}{2}\right) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \right], (-1)^{1/3} \right]$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right) \sqrt{-a-b x^3}} dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} \ a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{-a-b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} \ a^{1/6} b^{1/3}}$$

Result (type 4, 323 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{-a - b x^3}} \\
& 2 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{\left((-1)^{1/3} a^{1/3} - b^{1/3} x \right) \sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}]}{3^{1/4} b^{1/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}} + \frac{1}{(3 + (2 + i) \sqrt{3}) b^{1/3}} \right. \\
& \left. 4 (-1)^{1/3} (1 + (-1)^{1/3}) a^{1/3} \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 i \sqrt{3}}{3 + (2 + i) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 121: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 3, 73 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}
\end{aligned}$$

Result (type 6, 866 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{a + b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a + b x^3\right)} \left(26 + 15 \sqrt{3}\right) a x \\
& \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \middle/ \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right. \right. \right. \\
& \left. \left. \left. + 3 b x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \right) + x \right. \\
& \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \middle/ \left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right. \right. \\
& \left. \left. + 3 b x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \right) + \right. \\
& \left. x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \middle/ \left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right. \right. \right. \right. \\
& \left. \left. \left. \left. + b x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \right) \right) + \right. \\
& \left. \left(21 b x \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \middle/ \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right. \right. \\
& \left. \left. + 3 b x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a}\right] \right) \right) \right) \right)
\end{aligned}$$

Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{a - b x^3}} dx$$

Optimal (type 3, 75 leaves, 2 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1-\left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{a-b x^3}}\right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 835 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{a - b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a - b x^3\right)} \left(26 + 15 \sqrt{3}\right) a x \\
& \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) / \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) / \left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) / \left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) - \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) / \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right)
\end{aligned}$$

Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} - \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{-a + b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\frac{2 \text{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1 - \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a+b x^3}} \right]}{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}$$

Result (type 6, 836 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \left(2 \left(5 + 3 \sqrt{3}\right) a - b x^3\right) \sqrt{-a + b x^3}} \left(26 + 15 \sqrt{3}\right) a x \\
& \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[1, \frac{1}{2}, 1, 2, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \right. \\
& \left. \left. \left. b x^3 \left(\text{AppellF1} \left[2, \frac{1}{2}, 2, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[2, \frac{3}{2}, 1, 3, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) - \\
& \left(21 b x \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, \frac{b x^3}{a}, \frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right)
\end{aligned}$$

Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 - \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x}{\left(1 + \sqrt{3} + \left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{-a - b x^3}} dx$$

Optimal (type 3, 76 leaves, 2 steps):

$$\begin{aligned}
& 2 \text{ArcTanh} \left[\frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(1 + \left(\frac{b}{a}\right)^{1/3} x\right)}{\sqrt{-a - b x^3}} \right] \\
& - \frac{\sqrt{3+2 \sqrt{3}} \sqrt{a} \left(\frac{b}{a}\right)^{1/3}}{\sqrt{-a - b x^3}}
\end{aligned}$$

Result (type 6, 869 leaves):

$$\begin{aligned}
& \frac{1}{3 \left(5 + 3 \sqrt{3}\right) \sqrt{-a - b x^3} \left(2 \left(5 + 3 \sqrt{3}\right) a + b x^3\right)} \left(26 + 15 \sqrt{3}\right) a x \\
& \left(- \left(\left(96 \left(1 + \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(8 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] - \right. \right. \right. \\
& \left. \left. \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) + x \right. \\
& \left(\left(60 \left(3 + \sqrt{3}\right) a \left(\frac{b}{a}\right)^{1/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(10 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) + \\
& x \left(- \left(\left(16 \sqrt{3} a \left(\frac{b}{a}\right)^{2/3} \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(4 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] - \right. \right. \right. \\
& \left. \left. \left. b x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) + \\
& \left(21 b x \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \middle/ \left(14 \left(5 + 3 \sqrt{3}\right) a \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] + \left(5 + 3 \sqrt{3}\right) \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b x^3}{10 a + 6 \sqrt{3} a} \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\begin{aligned}
& \frac{\text{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right] - \sqrt{2+ \sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{3+2 \sqrt{3}}} + \\
& \frac{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}{\sqrt{3+2 \sqrt{3}}}
\end{aligned}$$

Result (type 4, 269 leaves):

$$\begin{aligned}
& \left(2 \sqrt{6} \sqrt{\frac{\frac{i}{3} (1+x)}{3i + \sqrt{3}}} \right. \\
& \left. \left(\sqrt{-i + \sqrt{3} + 2ix} \left((-2 - i) - \sqrt{3} + ((1 + 2i) + i\sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}] + 2\sqrt{i + \sqrt{3} - 2ix} \sqrt{1 - x + x^2} \right. \right. \\
& \left. \left. \text{EllipticPi}\left[\frac{2\sqrt{3}}{3i + (1 + 2i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}\right] \right) \right) \Big/ \left((3i + (1 + 2i)\sqrt{3}) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1 + x^3} \right)
\end{aligned}$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x}{(1-\sqrt{3}+x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\begin{aligned}
& \frac{\text{ArcTanh}\left[\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right] + \sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}]}{\sqrt{-3+2\sqrt{3}}} + \frac{3^{1/4}\sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}\sqrt{1+x^3}}{\sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 267 leaves):

$$\begin{aligned}
& - \left(\left(2 \sqrt{6} \sqrt{\frac{\frac{i}{3} (1+x)}{3i + \sqrt{3}}} \right. \right. \\
& \left. \left. \left(\sqrt{-i + \sqrt{3} + 2ix} \left((1 + 2i) - i\sqrt{3} + ((-2 - i) + \sqrt{3})x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}] + 2i\sqrt{i + \sqrt{3} - 2ix} \right. \right. \\
& \left. \left. \sqrt{1 - x + x^2} \text{EllipticPi}\left[\frac{2i\sqrt{3}}{-3 + (2 + i)\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{i + \sqrt{3} - 2ix}}{\sqrt{2} 3^{1/4}}\right], \frac{2\sqrt{3}}{3i + \sqrt{3}}\right] \right) \right) \Big/ \left((-3 + (2 + i)\sqrt{3}) \sqrt{i + \sqrt{3} - 2ix} \sqrt{1 + x^3} \right)
\end{aligned}$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + fx}{(1 + \sqrt{3} + x)\sqrt{1+x^3}} dx$$

Optimal (type 4, 173 leaves, 4 steps):

$$\frac{\left(e - f - \sqrt{3} f\right) \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}}\right] - \sqrt{2+\sqrt{3}} \left(e - (1-\sqrt{3}) f\right) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{\sqrt{3 (3+2 \sqrt{3})}} + \frac{3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}{\sqrt{1+x^3}}$$

Result (type 4, 291 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2}+\sqrt{3}}} \left(3 f \sqrt{-\frac{i}{2}+\sqrt{3}+2 \frac{i}{2} x} \left((-2-\frac{i}{2})-\sqrt{3}+\left((1+2 \frac{i}{2})+\frac{i}{2} \sqrt{3}\right) x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}}-2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2}+\sqrt{3}}\right]+2 \left(-\sqrt{3} e+\left(3+\sqrt{3}\right) f\right) \sqrt{\frac{i}{2}+\sqrt{3}-2 \frac{i}{2} x} \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}}-2 \frac{i}{2} x}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2}+\sqrt{3}}\right]\right)\right)/\left(\left(3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}\right) \sqrt{\frac{i}{2}+\sqrt{3}-2 \frac{i}{2} x} \sqrt{1+x^3}\right)$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(1 + \sqrt{3} - x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 187 leaves, 4 steps):

$$-\frac{\left(e+f+\sqrt{3} f\right) \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right] - \sqrt{2+\sqrt{3}} \left(e+(1-\sqrt{3}) f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{\sqrt{3 (3+2 \sqrt{3})}} - \frac{3^{3/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}{\sqrt{1-x^3}}$$

Result (type 4, 291 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{\frac{1}{2}(-1+x)}{3\frac{1}{2}+\sqrt{3}}} \left(-3\frac{1}{2}f\sqrt{-\frac{1}{2}+\sqrt{3}-2\frac{1}{2}x} \left(-\frac{1}{2} \left((2+\frac{1}{2})+\sqrt{3} \right) + \left((2-\frac{1}{2})+\sqrt{3} \right)x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2}+\sqrt{3}}] + \right. \right. \\
& \left. \left. 2\left(\sqrt{3}e+(3+\sqrt{3})f\right)\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{2}+(1+2\frac{1}{2})\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2}+\sqrt{3}}\right] \right) \right) / \\
& \left(\left(3\frac{1}{2}+(1+2\frac{1}{2})\sqrt{3} \right) \sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}\sqrt{1-x^3} \right)
\end{aligned}$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}} dx$$

Optimal (type 4, 190 leaves, 4 steps):

$$\begin{aligned}
& \frac{\left(e+f+\sqrt{3}f\right) \text{ArcTanh}\left[\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right] - \sqrt{2-\sqrt{3}} \left(e+(1-\sqrt{3})f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}]}{\sqrt{3(3+2\sqrt{3})}
\end{aligned}$$

Result (type 4, 289 leaves):

$$\begin{aligned}
& \left(2 \sqrt{\frac{2}{3}} \sqrt{-\frac{\frac{1}{2}(-1+x)}{3\frac{1}{2}+\sqrt{3}}} \left(-3\frac{1}{2}f\sqrt{-\frac{1}{2}+\sqrt{3}-2\frac{1}{2}x} \left(-\frac{1}{2} \left((2+\frac{1}{2})+\sqrt{3} \right) + \left((2-\frac{1}{2})+\sqrt{3} \right)x \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2}+\sqrt{3}}] + \right. \right. \\
& \left. \left. 2\left(\sqrt{3}e+(3+\sqrt{3})f\right)\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}\sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2\sqrt{3}}{3\frac{1}{2}+(1+2\frac{1}{2})\sqrt{3}}, \text{ArcSin}\left[\frac{\sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}}{\sqrt{2}3^{1/4}}\right], \frac{2\sqrt{3}}{3\frac{1}{2}+\sqrt{3}}\right] \right) \right) / \\
& \left(\left(3\frac{1}{2}+(1+2\frac{1}{2})\sqrt{3} \right) \sqrt{\frac{1}{2}+\sqrt{3}+2\frac{1}{2}x}\sqrt{1-x^3} \right)
\end{aligned}$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 183 leaves, 4 steps):

$$\frac{\left(e - (1 + \sqrt{3}) f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right] + \sqrt{2-\sqrt{3}} \left(e - (1 - \sqrt{3}) f\right) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{\sqrt{3 (3+2 \sqrt{3})} 3^{3/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 293 leaves):

$$\left(2 \sqrt{\frac{2}{3}} \sqrt{\frac{\frac{i}{2} (1+x)}{3 \frac{i}{2} + \sqrt{3}}} \left(3 f \sqrt{-\frac{i}{2} + \sqrt{3} + 2 \frac{i}{2} x} \left((-2 - \frac{i}{2}) - \sqrt{3} + \left((1 + 2 \frac{i}{2}) + \frac{i}{2} \sqrt{3}\right) x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}\right] + 2 \left(-\sqrt{3} e + (3 + \sqrt{3}) f\right) \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{1 - x + x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}}, \operatorname{ArcSin}\left[\frac{\sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{3 \frac{i}{2} + \sqrt{3}}\right]\right) / \left(\left(3 \frac{i}{2} + (1 + 2 \frac{i}{2}) \sqrt{3}\right) \sqrt{\frac{i}{2} + \sqrt{3} - 2 \frac{i}{2} x} \sqrt{-1 - x^3}\right)$$

Problem 131: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 332 leaves, 4 steps):

$$\begin{aligned} & \frac{\left(b^{1/3} e - (1 - \sqrt{3}) a^{1/3} f\right) \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}+b^{1/3} x)}{\sqrt{a+b x^3}}\right]}{\sqrt{3 (-3+2 \sqrt{3})} \sqrt{a} b^{2/3}} - \\ & \left(\sqrt{2+\sqrt{3}} \left(b^{1/3} e - (1 + \sqrt{3}) a^{1/3} f\right) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) a^{1/3} + b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} + b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) / \\ & \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left((1 + \sqrt{3}) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) \end{aligned}$$

Result (type 4, 438 leaves):

$$\begin{aligned}
& - \left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} \text{EllipticF} \left(\left((-2 - \frac{1}{2}) + \sqrt{3} \right) a^{1/3} + \left((1 + 2 \frac{1}{2}) - \frac{1}{2} \sqrt{3} \right) b^{1/3} x, \sqrt{\frac{1}{2} + \sqrt{3}} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}} \right) \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] + \frac{1}{2} \left(b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f \right) \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right) \right. \\
& \quad \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3 \frac{1}{2} + (1 + 2 \frac{1}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}}, \frac{1}{2} (1 + \frac{1}{2} \sqrt{3}) \right] \right] \right) \\
& \quad \left(\left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a + b x^3} \right)
\end{aligned}$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left((1 - \sqrt{3}) a^{1/3} - b^{1/3} x \right) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 336 leaves, 4 steps):

$$\begin{aligned}
& \frac{\left(b^{1/3} e + (1 - \sqrt{3}) a^{1/3} f \right) \text{ArcTanh} \left[\frac{\sqrt{-3 + 2\sqrt{3}} a^{1/6} (a^{1/3} - b^{1/3} x)}{\sqrt{a - b x^3}} \right]}{\sqrt{3 (-3 + 2\sqrt{3})} \sqrt{a} b^{2/3}} + \\
& \left(\sqrt{2 + \sqrt{3}} \left(b^{1/3} e + (1 + \sqrt{3}) a^{1/3} f \right) (a^{1/3} - b^{1/3} x) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) a^{1/3} - b^{1/3} x}{(1 + \sqrt{3}) a^{1/3} - b^{1/3} x} \right], -7 - 4\sqrt{3} \right] \right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{\frac{a^{1/3} (a^{1/3} - b^{1/3} x)}{((1 + \sqrt{3}) a^{1/3} - b^{1/3} x)^2}} \sqrt{a - b x^3} \right)
\end{aligned}$$

Result (type 4, 466 leaves):

$$\begin{aligned}
& - \frac{1}{\left(3 - (2 - \frac{1}{2}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{a - b x^3}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} f \left(\frac{1}{2} \left(-3 + (2 + \frac{1}{2}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] - \frac{1}{2} \left(b^{1/3} e - \left(-1 + \sqrt{3} \right) a^{1/3} f \right) \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{1}{2} + \left(1 + 2 \frac{1}{2} \right) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] \right)
\end{aligned}$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right) \sqrt{-a + b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps):

$$\begin{aligned}
& \frac{\left(b^{1/3} e + \left(1 - \sqrt{3} \right) a^{1/3} f \right) \text{ArcTan} \left[\frac{\sqrt{-3 + 2 \sqrt{3}} a^{1/6} \left(a^{1/3} - b^{1/3} x \right)}{\sqrt{-a + b x^3}} \right]}{\sqrt{3 \left(-3 + 2 \sqrt{3} \right)} \sqrt{a} b^{2/3}} + \\
& \left(\sqrt{2 - \sqrt{3}} \left(b^{1/3} e + \left(1 + \sqrt{3} \right) a^{1/3} f \right) \left(a^{1/3} - b^{1/3} x \right) \sqrt{\frac{a^{2/3} + a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3} \right) a^{1/3} - b^{1/3} x}{\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3} - b^{1/3} x \right)}{\left(\left(1 - \sqrt{3} \right) a^{1/3} - b^{1/3} x \right)^2}} \sqrt{-a + b x^3} \right)
\end{aligned}$$

Result (type 4, 467 leaves):

$$\begin{aligned}
& -\frac{1}{\left(3 - (2 - \frac{1}{2}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} f \left(\frac{1}{2} \left(-3 + (2 + \frac{1}{2}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] - \frac{1}{2} \left(b^{1/3} e - \left(-1 + \sqrt{3} \right) a^{1/3} f \right) \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{1}{2} + \left(1 + 2 \frac{1}{2} \right) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] \right)
\end{aligned}$$

Problem 134: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 345 leaves, 4 steps):

$$\begin{aligned}
& \frac{\left(b^{1/3} e - \left(1 - \sqrt{3} \right) a^{1/3} f \right) \text{ArcTan} \left[\frac{\sqrt{-3 + 2 \sqrt{3}} a^{1/6} \left(a^{1/3} + b^{1/3} x \right)}{\sqrt{-a - b x^3}} \right]}{\sqrt{3 \left(-3 + 2 \sqrt{3} \right)} \sqrt{a} b^{2/3}} - \\
& \left(\sqrt{2 - \sqrt{3}} \left(b^{1/3} e - \left(1 + \sqrt{3} \right) a^{1/3} f \right) \left(a^{1/3} + b^{1/3} x \right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3} \right) a^{1/3} + b^{1/3} x}{\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right] \right) / \\
& \left(3^{3/4} a^{1/3} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x \right)}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Result (type 4, 441 leaves):

$$\begin{aligned}
& - \left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} \text{EllipticF} \left(\left((-2 - \frac{1}{2}) + \sqrt{3} \right) a^{1/3} + \left((1 + 2 \frac{1}{2}) - \frac{1}{2} \sqrt{3} \right) b^{1/3} x, \sqrt{\frac{1}{2} + \sqrt{3}} - \frac{2 \frac{1}{2} b^{1/3} x}{a^{1/3}} \right) \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] + \frac{1}{2} \left(b^{1/3} e + (-1 + \sqrt{3}) a^{1/3} f \right) \sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right) \right. \\
& \quad \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3 \frac{1}{2} + (1 + 2 \frac{1}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2 \frac{1}{2} a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{(-3 \frac{1}{2} + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] \right) \right) \\
& \quad \left(\left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 135: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 136 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{3+2\sqrt{3}} (1+x)}{\sqrt{1+x^3}} \right]}{3^{3/4}} + \frac{\sqrt{2} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7 - 4\sqrt{3} \right]}{3^{3/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 209 leaves):

$$\frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left\{ - \frac{\left((-1)^{1/3} - x\right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{2 \frac{1}{2} \left(1+\sqrt{3}\right) \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2 \frac{1}{2} \sqrt{3}}{3+(2+\frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{3+\left(2+\frac{1}{2}\right) \sqrt{3}} \right\}$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(1 + \sqrt{3} - x\right) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 152 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{1-x^3}}\right]}{3^{3/4}} + \frac{\sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3^{3/4} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}$$

Result (type 4, 232 leaves):

$$\frac{1}{\left(3 + \left(2 + \frac{1}{2} \right) \sqrt{3} \right) \sqrt{1 - x^3}}$$

$$2 \frac{\text{i}}{\sqrt{1-x}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} \text{i} \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3}x}{1+(-1)^{1/3}}} (3 \frac{\text{i}}{\sqrt{3}} + (1+2 \frac{\text{i}}{\sqrt{3}}) \sqrt{3} + (3+(2+\frac{\text{i}}{\sqrt{3}}) \sqrt{3}) x) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] + \right. \\ \left. 2 (1+\sqrt{3}) \sqrt{1+x+x^2} \text{EllipticPi}\left[\frac{2 \frac{\text{i}}{\sqrt{3}} \sqrt{3}}{3+(2+\frac{\text{i}}{\sqrt{3}}) \sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} - x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 164 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1-x)}{\sqrt{-1+x^3}}\right]}{3^{3/4}} + \frac{2 \sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 230 leaves):

$$\left(2 \frac{1}{i} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \left(3 \frac{1}{i}+(1+2 \frac{1}{i}) \sqrt{3}+\left(3+(2+\frac{1}{i}) \sqrt{3}\right) x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]+2 \left(1+\sqrt{3}\right) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \frac{1}{i} \sqrt{3}}{3+(2+\frac{1}{i}) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]\right) \right) \Bigg/ \left(\left(3+(2+\frac{1}{i}) \sqrt{3}\right) \sqrt{-1+x^3}\right)$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1 + \sqrt{3} + x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 156 leaves, 4 steps):

$$-\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{3+2 \sqrt{3}} (1+x)}{\sqrt{-1-x^3}}\right]}{3^{3/4}} + \frac{2 \sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 211 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\ \left. \frac{2 \frac{1}{2} \left(1 + \sqrt{3} \right) \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2 \frac{1}{2} \sqrt{3}}{3+(2+\frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{3+(2+\frac{1}{2}) \sqrt{3}} \right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(1-\sqrt{3}+x) \sqrt{1+x^3}} dx$$

Optimal (type 4, 147 leaves, 4 steps):

$$- \frac{\sqrt{2} \operatorname{ArcTanh} \left[\frac{\sqrt{-3+2 \sqrt{3}} (1+x)}{\sqrt{1+x^3}} \right]}{3^{3/4}} + \frac{2 \sqrt{\frac{7}{6} + \frac{2}{\sqrt{3}}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 225 leaves):

$$\left(2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(\frac{1}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \left(3 - (2+\frac{1}{2}) \sqrt{3} + (-3\frac{1}{2} + (1+2\frac{1}{2}) \sqrt{3}) x \right) \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] - \right. \right. \\ \left. \left. 2 \left(-1 + \sqrt{3} \right) \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{2 \sqrt{3}}{-3\frac{1}{2} + (1+2\frac{1}{2}) \sqrt{3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right) \right) \Bigg/ \left(\left(-3\frac{1}{2} + (1+2\frac{1}{2}) \sqrt{3} \right) \sqrt{1+x^3} \right)$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x\right) \sqrt{a + b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned} & \frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{a+b x^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{2 \sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]}{3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}} \end{aligned}$$

Result (type 4, 427 leaves):

$$\begin{aligned} & -\left(4 \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}} \left(-\frac{1}{2 \sqrt{2}} \pm 3^{1/4} \left(\left(-2-\frac{i}{2}\right)+\sqrt{3}\right) a^{1/3}+\left(\left(1+2 \frac{i}{2}\right)-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right) \sqrt{\frac{\frac{i}{2}+\sqrt{3}-\frac{2 \pm b^{1/3} x}{a^{1/3}}}{\left(\frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{-2 \pm a^{1/3}+\left(\frac{i}{2}+\sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1+\frac{i}{2} \sqrt{3}\right)\right]+\frac{i}{2} \left(-1+\sqrt{3}\right) a^{1/3} \sqrt{\frac{-2 \pm a^{1/3}+\left(\frac{i}{2}+\sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right. \\ & \left. \sqrt{1-\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{-2 \pm a^{1/3}+\left(\frac{i}{2}+\sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1+\frac{i}{2} \sqrt{3}\right)\right]\right)\right) \\ & \left(3-\left(2-\frac{i}{2}\right) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right) \end{aligned}$$

Problem 141: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1 - \sqrt{3}\right) a^{1/3} - b^{1/3} x\right) \sqrt{a - b x^3}} dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} \operatorname{ArcTanh}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3} x)}{\sqrt{a-b x^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{2 \sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}} (a^{1/3}-b^{1/3} x) \sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}\right], -7-4 \sqrt{3}\right]}{3^{1/4} b^{2/3}} \\
& \sqrt{\frac{a^{1/3} (a^{1/3}-b^{1/3} x)}{\left(\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \sqrt{a-b x^3}
\end{aligned}$$

Result (type 4, 454 leaves):

$$\begin{aligned}
& -\frac{1}{\left(3-\left(2-\frac{i}{2}\right) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}} \sqrt{a-b x^3} \\
& 4 \sqrt{\frac{a^{1/3}-b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}} \left(\frac{1}{2} \left(\frac{i}{2} \left(-3+\left(2+\frac{i}{2}\right) \sqrt{3} \right) a^{1/3} + \left(3-\left(2-\frac{i}{2}\right) \sqrt{3}\right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{i}{2}+\sqrt{3}\right) a^{1/3}+\left(\frac{i}{2}+\sqrt{3}\right) b^{1/3} x}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}} \right. \\
& \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1+\frac{i}{2} \sqrt{3}\right)\right]+\frac{i}{2} \left(-1+\sqrt{3}\right) a^{1/3} \sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right) \\
& \sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-3 \frac{i}{2}+\left(1+2 \frac{i}{2}\right) \sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{-\frac{\frac{i}{2} \left(2 a^{1/3}+\left(1-\frac{i}{2} \sqrt{3}\right) b^{1/3} x\right)}{\left(-3 \frac{i}{2}+\sqrt{3}\right) a^{1/3}}}\right], \frac{1}{2} \left(1+\frac{i}{2} \sqrt{3}\right)\right]
\end{aligned}$$

Problem 142: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x\right) \sqrt{-a+b x^3}} dx$$

Optimal (type 4, 282 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3}-b^{1/3} x)}{\sqrt{-a+b x^3}}\right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{\sqrt{2} (a^{1/3}-b^{1/3} x) \sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}\right], -7+4 \sqrt{3}\right]}{3^{3/4} b^{2/3}} \\
& \sqrt{-\frac{a^{1/3} (a^{1/3}-b^{1/3} x)}{\left(\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x\right)^2}} \sqrt{-a+b x^3}
\end{aligned}$$

Result (type 4, 455 leaves):

$$\begin{aligned}
& - \frac{1}{\left(3 - (2 - \frac{1}{2}) \sqrt{3}\right) b^{2/3} \sqrt{\frac{a^{1/3} - (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a + b x^3}} \\
& 4 \sqrt{\frac{a^{1/3} - b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(\frac{1}{2} \left(\frac{1}{2} \left(-3 + (2 + \frac{1}{2}) \sqrt{3} \right) a^{1/3} + \left(3 - (2 - \frac{1}{2}) \sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{\left(-\frac{1}{2} + \sqrt{3} \right) a^{1/3} + \left(\frac{1}{2} + \sqrt{3} \right) b^{1/3} x}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] + \frac{1}{2} \left(-1 + \sqrt{3} \right) a^{1/3} \sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right. \\
& \left. \sqrt{1 + \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-3 \frac{1}{2} + (1 + 2 \frac{1}{2}) \sqrt{3}}, \text{ArcSin} \left[\sqrt{-\frac{\frac{1}{2} \left(2 a^{1/3} + \left(1 - \frac{1}{2} \sqrt{3} \right) b^{1/3} x \right)}{\left(-3 \frac{1}{2} + \sqrt{3} \right) a^{1/3}}} \right], \frac{1}{2} \left(1 + \frac{1}{2} \sqrt{3} \right) \right] \right)
\end{aligned}$$

Problem 143: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right) \sqrt{-a - b x^3}} dx$$

Optimal (type 4, 278 leaves, 4 steps):

$$\begin{aligned}
& - \frac{\sqrt{2} \text{ArcTan} \left[\frac{\sqrt{-3+2 \sqrt{3}} a^{1/6} (a^{1/3} + b^{1/3} x)}{\sqrt{-a-b x^3}} \right]}{3^{3/4} a^{1/6} b^{2/3}} + \frac{\sqrt{2} (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(1 + \sqrt{3} \right) a^{1/3} + b^{1/3} x}{\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x} \right], -7 + 4 \sqrt{3} \right]}{3^{3/4} b^{2/3} \sqrt{-\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left(\left(1 - \sqrt{3} \right) a^{1/3} + b^{1/3} x \right)^2}} \sqrt{-a - b x^3}}
\end{aligned}$$

Result (type 4, 430 leaves):

$$\begin{aligned}
& - \left(4 \sqrt{\frac{a^{1/3} + b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \left(-\frac{1}{2\sqrt{2}} \sqrt{3}^{1/4} \left(\left((-2 - i) + \sqrt{3} \right) a^{1/3} + \left((1 + 2i) - i\sqrt{3} \right) b^{1/3} x \right) \sqrt{\frac{i + \sqrt{3} - \frac{2i b^{1/3} x}{a^{1/3}}}{i + \sqrt{3}}} \right. \right. \\
& \quad \left. \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] + i (-1 + \sqrt{3}) a^{1/3} \sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right. \right. \\
& \quad \left. \left. \sqrt{1 - \frac{b^{1/3} x}{a^{1/3}} + \frac{b^{2/3} x^2}{a^{2/3}}} \text{EllipticPi} \left[\frac{2\sqrt{3}}{-3i + (1 + 2i)\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{-2i a^{1/3} + (i + \sqrt{3}) b^{1/3} x}{(-3i + \sqrt{3}) a^{1/3}}} \right], \frac{1}{2} (1 + i\sqrt{3}) \right] \right] \right) \\
& \quad \left((3 - (2 - i)\sqrt{3}) b^{2/3} \sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{(1 + (-1)^{1/3}) a^{1/3}}} \sqrt{-a - b x^3} \right)
\end{aligned}$$

Problem 144: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{(c + d x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 319 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\left(c - (1 + \sqrt{3}) d \right) (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{ArcTan} \left[\frac{\sqrt{c^2 + c d + d^2} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}}{\sqrt{c - d} \sqrt{d} \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}} \right]}{\sqrt{c - d} \sqrt{d} \sqrt{c^2 + c d + d^2} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} - \\
& \frac{4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \text{EllipticPi} \left[\frac{(c - (1 + \sqrt{3}) d)^2}{(c - (1 - \sqrt{3}) d)^2}, -\text{ArcSin} \left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x} \right], -7 - 4\sqrt{3} \right]}{\left(c - (1 - \sqrt{3}) d \right) \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}
\end{aligned}$$

Result (type 4, 214 leaves):

$$\begin{aligned}
 & \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
 & \left. \frac{i \left(c - \left(1 + \sqrt{3} \right) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{c+(-1)^{1/3} d} \right)
 \end{aligned}$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{(c+d x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 331 leaves, 6 steps):

$$\begin{aligned}
 & \left(c + d + \sqrt{3} d \right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x \right)^2}} \right] \\
 & - \frac{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2}}{\sqrt{\left(1+\sqrt{3}-x \right)^2}} \sqrt{1-x^3} + \\
 & \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi} \left[\frac{(c+d+\sqrt{3} d)^2}{(c+d-\sqrt{3} d)^2}, -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{\left(c + d - \sqrt{3} d \right) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}
 \end{aligned}$$

Result (type 4, 235 leaves):

$$\frac{1}{3d\sqrt{1-x^3}} 2\sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3}d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{\pm \sqrt{3} d}{-c + (-1)^{1/3}d}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 146: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{(c + d x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 327 leaves, 6 steps):

$$\frac{\left(c + d + \sqrt{3} d \right) \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x \right)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{c^2 - c d + d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x \right)^2}} \right]}{-\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x \right)^2}} \sqrt{-1+x^3}} + \\ \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x \right)^2}} \operatorname{EllipticPi} \left[\frac{\left(c+d+\sqrt{3} d \right)^2}{\left(c+d-\sqrt{3} d \right)^2}, -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{\left(c + d - \sqrt{3} d \right) \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x \right)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 233 leaves):

$$\begin{aligned}
& \frac{1}{3d\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3}d} \right. \\
& \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{\frac{1}{2}\sqrt{3}d}{-c + (-1)^{1/3}d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+x}{(c+d x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 323 leaves, 6 steps):

$$\begin{aligned}
& \frac{\left(c - (1 + \sqrt{3}) d\right) (1 + x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}} - \\
& \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{\left(c - (1 + \sqrt{3}) d\right)^2}{\left(c - (1 - \sqrt{3}) d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{\left(c - (1 - \sqrt{3}) d\right) \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}}
\end{aligned}$$

Result (type 4, 216 leaves):

$$\frac{1}{d \sqrt{-1 - x^3}} 2 \sqrt{\frac{1+x}{1 + (-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}] + \frac{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}}{c + (-1)^{1/3} d} \operatorname{EllipticPi}\left[\frac{\frac{i \sqrt{3} d}{c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}}{c - (1 + \sqrt{3}) d}\right] \right)$$

Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{(c + d x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 360 leaves, 6 steps):

$$\frac{\left(c - (1 - \sqrt{3}) d\right) (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{ArcTanh}\left[\frac{2 \sqrt{2 + \sqrt{3}} \sqrt{c^2 + c d + d^2} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}}}{\sqrt{c - d} \sqrt{d} \sqrt{7 + 4 \sqrt{3} + \frac{(1 + \sqrt{3} + x)^2}{(1 - \sqrt{3} + x)^2}}} + \sqrt{c - d} \sqrt{d} \sqrt{c^2 + c d + d^2} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{1 + x^3}} + \frac{4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticPi}\left[\frac{(c - (1 - \sqrt{3}) d)^2}{(c - (1 + \sqrt{3}) d)^2}, -\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4 \sqrt{3}\right]}{(c - d - \sqrt{3} d) \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{1 + x^3}}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
 & \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
 & \left. \frac{i \left(c + \left(-1 + \sqrt{3} \right) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{c+(-1)^{1/3} d} \right)
 \end{aligned}$$

Problem 149: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{(c + d x) \sqrt{1 - x^3}} dx$$

Optimal (type 4, 348 leaves, 6 steps):

$$\begin{aligned}
 & \left(c + d - \sqrt{3} d \right) \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1 - \sqrt{3} - x \right)^2}} \operatorname{ArcTan} \left[\frac{\sqrt{c^2 - c d + d^2} \sqrt{-\frac{1-x}{\left(1 - \sqrt{3} - x \right)^2}}}{\sqrt{d} \sqrt{c+d} \sqrt{\frac{1+x+x^2}{\left(1 - \sqrt{3} - x \right)^2}}} \right] \\
 & - \frac{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{-\frac{1-x}{\left(1 - \sqrt{3} - x \right)^2}} \sqrt{1-x^3}}{\left(c + d + \sqrt{3} d \right) \sqrt{-\frac{1-x}{\left(1 - \sqrt{3} - x \right)^2}} \sqrt{1-x^3}} - \\
 & \frac{4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1 - \sqrt{3} - x \right)^2}} \operatorname{EllipticPi} \left[\frac{\left(c+d-\sqrt{3} d \right)^2}{\left(c+d+\sqrt{3} d \right)^2}, -\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7 + 4 \sqrt{3} \right]}{\left(c + d + \sqrt{3} d \right) \sqrt{-\frac{1-x}{\left(1 - \sqrt{3} - x \right)^2}} \sqrt{1-x^3}}
 \end{aligned}$$

Result (type 4, 235 leaves):

$$\frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3} d} \right. \\ \left. (-1)^{1/3} \left(1 + (-1)^{1/3} \right) \left(\sqrt{3} c + (-3 + \sqrt{3}) d \right) \sqrt{1+x+x^2} \operatorname{EllipticPi} \left[\frac{\frac{1}{2} \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right] \right)$$

Problem 150: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{(c + d x) \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 344 leaves, 6 steps):

$$\frac{\left(c + d - \sqrt{3} d \right) \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x \right)^2}} \operatorname{ArcTan} \left[\frac{\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x \right)^2}}}{\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x \right)^2}}} \right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2 - c d + d^2} \sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x \right)^2}} \sqrt{-1+x^3}} - \\ \frac{4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} \left(1 - x \right) \sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x \right)^2}} \operatorname{EllipticPi} \left[\frac{\left(c+d-\sqrt{3} d \right)^2}{\left(c+d+\sqrt{3} d \right)^2}, -\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7 + 4 \sqrt{3} \right]}{\left(c + d + \sqrt{3} d \right) \sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x \right)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 233 leaves):

$$\begin{aligned}
& \frac{1}{3d\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(-\frac{3 \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3}x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{1-(-1)^{2/3}x}{1+(-1)^{1/3}}}} + \frac{1}{c - (-1)^{1/3}d} \right. \\
& \left. (-1)^{1/3} (1 + (-1)^{1/3}) (\sqrt{3} c + (-3 + \sqrt{3}) d) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{\frac{1}{2} \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1 - (-1)^{2/3}x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{(c + d x) \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\begin{aligned}
& - \frac{\left(c - (1 - \sqrt{3}) d\right) (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{ArcTanh}\left[\frac{2 \sqrt{2 + \sqrt{3}} \sqrt{c^2 + c d + d^2}}{\sqrt{c - d} \sqrt{d}} \sqrt{\frac{1 + x}{\left(1 - \sqrt{3} + x\right)^2}}\right]}{\sqrt{c - d} \sqrt{d} \sqrt{c^2 + c d + d^2}} + \\
& \frac{4 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticPi}\left[\frac{(c - (1 - \sqrt{3}) d)^2}{(c - (1 + \sqrt{3}) d)^2}, -\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4 \sqrt{3}\right]}{(c - d - \sqrt{3} d) \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}
\end{aligned}$$

Result (type 4, 215 leaves):

$$\begin{aligned}
 & \frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
 & \left. \frac{i \left(c + \left(-1 + \sqrt{3} \right) d \right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3} d} \right)
 \end{aligned}$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+x}{x \sqrt{1+x^3}} dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$-\frac{2}{3} (1+\sqrt{3}) \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] + \frac{2 \sqrt{2+\sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}\right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 149 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right] - \frac{2 \operatorname{ArcTanh}\left[\sqrt{1+x^3}\right]}{\sqrt{3}} - \frac{2 \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{\frac{(-1)^{2/3} \left((-1)^{2/3}+x \right)}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1+x^3}}$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-x}{x \sqrt{1-x^3}} dx$$

Optimal (type 4, 139 leaves, 5 steps):

$$-\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1 - x^3}] + \frac{2 \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}], -7 - 4 \sqrt{3}]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 157 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}[\sqrt{1 - x^3}] - \frac{2 \operatorname{ArcTanh}[\sqrt{1 - x^3}]}{\sqrt{3}} - \frac{2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}], (-1)^{1/3}]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1-x^3}}$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 142 leaves, 5 steps):

$$\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 + x^3}] + \frac{2 \sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}], -7 + 4 \sqrt{3}]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 150 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 + x^3}] + \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 + x^3}] - \frac{3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}], (-1)^{1/3}]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}} \right)$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 + \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 136 leaves, 5 steps):

$$\frac{2}{3} (1 + \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 - x^3}] + \frac{2 \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4 \sqrt{3}]}{3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}$$

Result (type 4, 155 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 - x^3}] + \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 - x^3}] - \frac{3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 - x^3}} \right)$$

Problem 156: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{1 + x^3}} dx$$

Optimal (type 4, 127 leaves, 5 steps):

$$-\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1 + x^3}] + \frac{2 \sqrt{2 + \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}]}{3^{1/4} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}}$$

Result (type 4, 149 leaves):

$$-\frac{2}{3} \operatorname{ArcTanh}[\sqrt{1 + x^3}] + \frac{2 \operatorname{ArcTanh}[\sqrt{1 + x^3}]}{\sqrt{3}} - \frac{2 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{1 + x^3}}$$

Problem 157: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{1 - x^3}} dx$$

Optimal (type 4, 141 leaves, 5 steps):

$$-\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTanh}[\sqrt{1 - x^3}] + \frac{2 \sqrt{2 + \sqrt{3}} (1 - x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7 - 4 \sqrt{3}]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 158 leaves):

$$\frac{2}{3} \left(-\operatorname{ArcTanh}[\sqrt{1 - x^3}] + \sqrt{3} \operatorname{ArcTanh}[\sqrt{1 - x^3}] - \frac{3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1-x^3}} \right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} - x}{x \sqrt{-1 + x^3}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 + x^3}] + \frac{2 \sqrt{2 - \sqrt{3}} (1 - x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7 + 4 \sqrt{3}]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 151 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 + x^3}] - \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 + x^3}] - \frac{3 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}] }{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}} \right)$$

Problem 159: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1 - \sqrt{3} + x}{x \sqrt{-1 - x^3}} dx$$

Optimal (type 4, 138 leaves, 5 steps):

$$\frac{2}{3} (1 - \sqrt{3}) \operatorname{ArcTan}[\sqrt{-1 - x^3}] + \frac{2 \sqrt{2 - \sqrt{3}} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 - \sqrt{3} + x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1 + \sqrt{3} + x}{1 - \sqrt{3} + x}\right], -7 + 4 \sqrt{3}]}{3^{1/4} \sqrt{-\frac{1 + x}{(1 - \sqrt{3} + x)^2}} \sqrt{-1 - x^3}}$$

Result (type 4, 156 leaves):

$$\frac{2}{3} \left(\operatorname{ArcTan}[\sqrt{-1 - x^3}] - \sqrt{3} \operatorname{ArcTan}[\sqrt{-1 - x^3}] - \frac{3 \left((-1)^{1/3} - x \right) \sqrt{\frac{1 + x}{1 + (-1)^{1/3}}} \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1 + (-1)^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}}\right], (-1)^{1/3}]}{\sqrt{\frac{1 + (-1)^{2/3} x}{1 + (-1)^{1/3}}} \sqrt{-1 - x^3}} \right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3 + x) \sqrt{1 + x^3}} dx$$

Optimal (type 4, 334 leaves, 8 steps):

$$\begin{aligned} & \frac{3 (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}}}{\sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}}}\right] - 2 \sqrt{2 (97 + 56 \sqrt{3})} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}]}{\sqrt{26} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} - \\ & \frac{12 \times 3^{1/4} (1 + x) \sqrt{\frac{1 - x + x^2}{(1 + \sqrt{3} + x)^2}} \operatorname{EllipticPi}[97 - 56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1 - \sqrt{3} + x}{1 + \sqrt{3} + x}\right], -7 - 4 \sqrt{3}]}{\sqrt{2 - \sqrt{3}} \sqrt{\frac{1 + x}{(1 + \sqrt{3} + x)^2}} \sqrt{1 + x^3}} \end{aligned}$$

Result (type 4, 194 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3} - x \right) \sqrt{\frac{(-1)^{1/3} - (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
 & \left. \frac{3 \pm \sqrt{1-x+x^2} \operatorname{EllipticPi} \left[\frac{\pm \sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{3+(-1)^{1/3}} \right)
 \end{aligned}$$

Problem 161: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 379 leaves, 8 steps):

$$\begin{aligned}
 & \frac{3 (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh} \left[\frac{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}} \right] - 2 \sqrt{2 (37+20\sqrt{3})} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{2 \sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}} - \\
 & \frac{12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi} \left[\frac{1}{169} (553+304\sqrt{3}), -\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}
 \end{aligned}$$

Result (type 4, 195 leaves):

$$\frac{1}{\sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ \left(\frac{\left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-3+(-1)^{1/3}} \right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 375 leaves, 8 steps):

$$\frac{3 (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}}}{2 \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}}}\right] - 2 \sqrt{2} (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{2 \sqrt{7} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}} - \frac{3^{1/4} (4+\sqrt{3}) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}{12 \times 3^{1/4} \sqrt{2+\sqrt{3}} (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{1}{169} (553+304 \sqrt{3}), -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]} \\ 13 \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}$$

Result (type 4, 193 leaves):

$$\frac{1}{\sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \\ \left(\frac{\left((-1)^{1/3}+x\right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 \pm \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{5 i+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-3+(-1)^{1/3}} \right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}} dx$$

Optimal (type 4, 343 leaves, 8 steps):

$$\frac{3 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{13}{2}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}}}{\sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}}}\right] - 2 \sqrt{14+8 \sqrt{3}} (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{\sqrt{26} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}} - \frac{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}{12 \times 3^{1/4} (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[97-56 \sqrt{3}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]} \sqrt{2-\sqrt{3}} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{-1-x^3}$$

Result (type 4, 196 leaves):

$$\frac{1}{\sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{3 \pm \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3}}{3+(-1)^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{3+(-1)^{1/3}} \right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}} dx$$

Optimal (type 4, 452 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(d e - c f\right) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}} + \\
& \frac{2 \sqrt{2+\sqrt{3}} (e-f-\sqrt{3} f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]}{3^{1/4} (c-d-\sqrt{3} d) \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{\left(c-\left(1+\sqrt{3}\right) d\right)^2}{\left(c-\left(1-\sqrt{3}\right) d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\left(c^2-2 c d-2 d^2\right) \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}\right)
\end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \left(- \frac{f\left((-1)^{1/3}-x\right) \sqrt{\frac{(-1)^{1/3}-(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \right. \\
& \left. \frac{i (-d e + c f) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+(-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{c+(-1)^{1/3} d} \right)
\end{aligned}$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(c+d x) \sqrt{1-x^3}} dx$$

Optimal (type 4, 476 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3} \\
& + \frac{2 \sqrt{2+\sqrt{3}} \left(e+f+\sqrt{3} f\right) (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{3^{1/4} (c+d+\sqrt{3} d) \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}} \\
& + \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(c+d+\sqrt{3} d\right)^2}{\left(c+d-\sqrt{3} d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{\left(c^2+2 c d-2 d^2\right) \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}} \sqrt{1-x^3}}
\end{aligned}$$

Result (type 4, 233 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+\left(-1\right)^{1/3}}} \left(\frac{3 f \left(\left(-1\right)^{1/3} + x \right) \sqrt{\frac{\left(-1\right)^{1/3} + \left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}\right], \left(-1\right)^{1/3}\right]}{\sqrt{\frac{1-\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}} \right. \\
& \left. + \frac{1}{-c + \left(-1\right)^{1/3} d} \right. \\
& \left. \left(-1 \right)^{1/3} \sqrt{3} \left(1 + \left(-1\right)^{1/3} \right) \left(-d e + c f \right) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{\frac{1}{3} \sqrt{3} d}{-c + \left(-1\right)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}\right], \left(-1\right)^{1/3}\right] \right)
\end{aligned}$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{(c + d x) \sqrt{-1+x^3}} dx$$

Optimal (type 4, 477 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2-c d+d^2}}{\sqrt{d} \sqrt{c+d}} \sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\right]}{\sqrt{d} \sqrt{c+d} \sqrt{c^2-c d+d^2}} - \\
& \frac{2 \sqrt{2-\sqrt{3}} \left(e+f+\sqrt{3} f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} (c+d+\sqrt{3} d) \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}} + \\
& \frac{4 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(d e - c f\right) (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticPi}\left[\frac{(c+d+\sqrt{3} d)^2}{(c+d-\sqrt{3} d)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right], -7-4 \sqrt{3}\right]}{(c^2+2 c d-2 d^2) \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{-1+x^3}}
\end{aligned}$$

Result (type 4, 231 leaves):

$$\begin{aligned}
& \frac{1}{3 d \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left(\frac{3 f \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}} + \frac{1}{-c + (-1)^{1/3} d} \right. \\
& \left. (-1)^{1/3} \sqrt{3} \left(1+(-1)^{1/3}\right) (-d e + c f) \sqrt{1+x+x^2} \operatorname{EllipticPi}\left[\frac{\frac{i}{\pi} \sqrt{3} d}{-c + (-1)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 167: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{(c+d x) \sqrt{-1-x^3}} dx$$

Optimal (type 4, 465 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(d e - c f\right) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{ArcTan}\left[\frac{\sqrt{c^2+c d+d^2}}{\sqrt{c-d} \sqrt{d}} \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\right]}{\sqrt{c-d} \sqrt{d} \sqrt{c^2+c d+d^2}} + \\
& \frac{2 \sqrt{2-\sqrt{3}} (e-f-\sqrt{3} f) (1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right], -7+4 \sqrt{3}\right]}{3^{1/4} (c-d-\sqrt{3} d) \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}} + \\
& \left(4 \times 3^{1/4} \sqrt{2+\sqrt{3}} (d e - c f) (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticPi}\left[\frac{\left(c-\left(1+\sqrt{3}\right) d\right)^2}{\left(c-\left(1-\sqrt{3}\right) d\right)^2}, -\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4 \sqrt{3}\right]\right) / \\
& \left(\left(c^2-2 c d-2 d^2\right) \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{-1-x^3}\right)
\end{aligned}$$

Result (type 4, 213 leaves):

$$\begin{aligned}
& \frac{1}{d \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+\left(-1\right)^{1/3}}} \left(- \frac{f\left(\left(-1\right)^{1/3}-x\right) \sqrt{\frac{\left(-1\right)^{1/3}-\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}\right], \left(-1\right)^{1/3}\right]}{\sqrt{\frac{1+\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}} + \right. \\
& \left. \frac{i \left(-d e + c f\right) \sqrt{1-x+x^2} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} d}{c+\left(-1\right)^{1/3} d}, \operatorname{ArcSin}\left[\sqrt{\frac{1+\left(-1\right)^{2/3} x}{1+\left(-1\right)^{1/3}}}\right], \left(-1\right)^{1/3}\right]}{c+\left(-1\right)^{1/3} d} \right)
\end{aligned}$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e+f x}{x \sqrt{1+x^3}} dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$-\frac{2}{3} e \operatorname{ArcTanh} \left[\sqrt{1+x^3} \right] + \frac{2 \sqrt{2+\sqrt{3}} f (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x} \right], -7-4\sqrt{3} \right]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 134 leaves):

$$-\frac{2}{3} e \operatorname{ArcTanh} \left[\sqrt{1+x^3} \right] - \frac{2 f \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} ((-1)^{2/3}+x)}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1+x^3}}$$

Problem 169: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{1-x^3}} dx$$

Optimal (type 4, 134 leaves, 6 steps):

$$-\frac{2}{3} e \operatorname{ArcTanh} \left[\sqrt{1-x^3} \right] - \frac{2 \sqrt{2+\sqrt{3}} f (1-x) \sqrt{\frac{1+x+x^2}{(1+\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x} \right], -7-4\sqrt{3} \right]}{3^{1/4} \sqrt{\frac{1-x}{(1+\sqrt{3}-x)^2}} \sqrt{1-x^3}}$$

Result (type 4, 140 leaves):

$$-\frac{2}{3} e \operatorname{ArcTanh} \left[\sqrt{1-x^3} \right] + \frac{2 f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3}+(-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{1-x^3}}$$

Problem 170: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{-1+x^3}} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$\frac{2}{3} e \operatorname{ArcTan} \left[\sqrt{-1+x^3} \right] - \frac{2 \sqrt{2-\sqrt{3}} f(1-x) \sqrt{\frac{1+x+x^2}{(1-\sqrt{3}-x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x} \right], -7+4\sqrt{3} \right]}{3^{1/4} \sqrt{-\frac{1-x}{(1-\sqrt{3}-x)^2}} \sqrt{-1+x^3}}$$

Result (type 4, 136 leaves):

$$\frac{2}{3} e \operatorname{ArcTan} \left[\sqrt{-1+x^3} \right] + \frac{2 f \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \left((-1)^{1/3} + x \right) \sqrt{\frac{(-1)^{1/3} + (-1)^{2/3} x}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1+x^3}}$$

Problem 171: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e + f x}{x \sqrt{-1-x^3}} dx$$

Optimal (type 4, 131 leaves, 6 steps):

$$\frac{2}{3} e \operatorname{ArcTan} \left[\sqrt{-1-x^3} \right] + \frac{2 \sqrt{2-\sqrt{3}} f(1+x) \sqrt{\frac{1-x+x^2}{(1-\sqrt{3}+x)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x} \right], -7+4\sqrt{3} \right]}{3^{1/4} \sqrt{-\frac{1+x}{(1-\sqrt{3}+x)^2}} \sqrt{-1-x^3}}$$

Result (type 4, 138 leaves):

$$\frac{2}{3} e \operatorname{ArcTan} \left[\sqrt{-1-x^3} \right] - \frac{2 f \left((-1)^{1/3} - x \right) \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{-\frac{(-1)^{2/3} \left((-1)^{2/3} + x \right)}{1+(-1)^{1/3}}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \right], (-1)^{1/3} \right]}{\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}} \sqrt{-1-x^3}}$$

Problem 172: Unable to integrate problem.

$$\int \frac{c - d x}{(c + d x) (2 c^3 + d^3 x^3)^{1/3}} dx$$

Optimal (type 3, 95 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1+\frac{2 (2 c+d x)}{(2 c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{d}-\frac{\operatorname{Log}[c+d x]}{d}+\frac{3 \operatorname{Log}\left[d \left(2 c+d x\right)-d \left(2 c^3+d^3 x^3\right)^{1/3}\right]}{2 d}$$

Result (type 8, 33 leaves) :

$$\int \frac{c-d x}{(c+d x) \left(2 c^3+d^3 x^3\right)^{1/3}} dx$$

Problem 173: Unable to integrate problem.

$$\int \frac{e+f x}{(c+d x) \left(-c^3+d^3 x^3\right)^{1/3}} dx$$

Optimal (type 3, 234 leaves, 3 steps) :

$$\begin{aligned} & \frac{f \operatorname{ArcTan}\left[\frac{1+\frac{2 d x}{(-c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} d^2}+\frac{\sqrt{3} (d e-c f) \operatorname{ArcTan}\left[\frac{1-\frac{2^{1/3} (c-d x)}{(-c^3+d^3 x^3)^{1/3}}}{\sqrt{3}}\right]}{2 \times 2^{1/3} c d^2}+\frac{(d e-c f) \operatorname{Log}\left[(c-d x) \left(c+d x\right)^2\right]}{4 \times 2^{1/3} c d^2}- \\ & \frac{f \operatorname{Log}\left[-d x+\left(-c^3+d^3 x^3\right)^{1/3}\right]}{2 d^2}-\frac{3 (d e-c f) \operatorname{Log}\left[d \left(c-d x\right)+2^{2/3} d \left(-c^3+d^3 x^3\right)^{1/3}\right]}{4 \times 2^{1/3} c d^2} \end{aligned}$$

Result (type 8, 32 leaves) :

$$\int \frac{e+f x}{(c+d x) \left(-c^3+d^3 x^3\right)^{1/3}} dx$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x)^n (c+d x^3)^2}{x} dx$$

Optimal (type 5, 209 leaves, 3 steps) :

$$\begin{aligned} & \frac{a^2 d \left(2 b^3 c-a^3 d\right) \left(a+b x\right)^{1+n}}{b^6 \left(1+n\right)}-\frac{a d \left(4 b^3 c-5 a^3 d\right) \left(a+b x\right)^{2+n}}{b^6 \left(2+n\right)}+\frac{2 d \left(b^3 c-5 a^3 d\right) \left(a+b x\right)^{3+n}}{b^6 \left(3+n\right)}+ \\ & \frac{10 a^2 d^2 \left(a+b x\right)^{4+n}}{b^6 \left(4+n\right)}-\frac{5 a d^2 \left(a+b x\right)^{5+n}}{b^6 \left(5+n\right)}+\frac{d^2 \left(a+b x\right)^{6+n}}{b^6 \left(6+n\right)}-\frac{c^2 \left(a+b x\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1,1+n,2+n,1+\frac{b x}{a}\right]}{a \left(1+n\right)} \end{aligned}$$

Result (type 5, 420 leaves) :

$$\begin{aligned}
& (a + b x)^n \left(\frac{1}{b^3 (1+n) (2+n) (3+n)} \right. \\
& 2 c d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + b^3 (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + \\
& \frac{1}{b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)} d^2 \left(1 + \frac{b x}{a} \right)^{-n} \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& 20 a^3 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - 5 a^2 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + a b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \\
& b^6 (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \left. \right) + \frac{c^2 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{a}{b x}]}{n} \left. \right)
\end{aligned}$$

Problem 182: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 459 leaves, 2 steps):

$$\begin{aligned}
& \frac{a^2 (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{12} (1+n)} - \frac{a (2 b^3 c - 11 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{12} (2+n)} + \frac{(b^3 c - a^3 d) (b^6 c^2 - 29 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{3+n}}{b^{12} (3+n)} + \\
& \frac{3 a^2 d (10 b^6 c^2 - 56 a^3 b^3 c d + 55 a^6 d^2) (a + b x)^{4+n}}{b^{12} (4+n)} - \frac{15 a d (b^6 c^2 - 14 a^3 b^3 c d + 22 a^6 d^2) (a + b x)^{5+n}}{b^{12} (5+n)} + \\
& \frac{3 d (b^6 c^2 - 56 a^3 b^3 c d + 154 a^6 d^2) (a + b x)^{6+n}}{b^{12} (6+n)} + \frac{42 a^2 d^2 (2 b^3 c - 11 a^3 d) (a + b x)^{7+n}}{b^{12} (7+n)} - \frac{6 a d^2 (4 b^3 c - 55 a^3 d) (a + b x)^{8+n}}{b^{12} (8+n)} + \\
& \frac{3 d^2 (b^3 c - 55 a^3 d) (a + b x)^{9+n}}{b^{12} (9+n)} + \frac{55 a^2 d^3 (a + b x)^{10+n}}{b^{12} (10+n)} - \frac{11 a d^3 (a + b x)^{11+n}}{b^{12} (11+n)} + \frac{d^3 (a + b x)^{12+n}}{b^{12} (12+n)}
\end{aligned}$$

Result (type 3, 1134 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} \left(-39916800 a^{11} d^3 + 39916800 a^{10} b d^3 (1+n) x - \right. \right. \\
& \quad 19958400 a^9 b^2 d^3 (2+3n+n^2) x^2 + 120960 a^8 b^3 d^2 (c (1320+362n+33n^2+n^3) + 55d (6+11n+6n^2+n^3) x^3) - \\
& \quad 30240 a^7 b^4 d^2 (1+n) x (4c (1320+362n+33n^2+n^3) + 55d (24+26n+9n^2+n^3) x^3) + \\
& \quad 30240 a^6 b^5 d^2 (2+3n+n^2) x^2 (2c (1320+362n+33n^2+n^3) + 11d (60+47n+12n^2+n^3) x^3) - \\
& \quad 360 a^5 b^6 d (c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \\
& \quad 56 c d (7920+16692n+12100n^2+3861n^3+571n^4+39n^5+n^6) x^3 + 154 d^2 (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^6) + \\
& \quad 360 a^4 b^7 d (1+n) x (c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \\
& \quad 14 c d (31680+43008n+22084n^2+5460n^3+685n^4+42n^5+n^6) x^3 + 22 d^2 (5040+8028n+5104n^2+1665n^3+295n^4+27n^5+n^6) x^6) - \\
& \quad 18 a^3 b^8 d (2+3n+n^2) x^2 (10c^2 (665280+434568n+117454n^2+16815n^3+1345n^4+57n^5+n^6) + \\
& \quad 56 c d (79200+83760n+34834n^2+7275n^3+805n^4+45n^5+n^6) x^3 + 55 d^2 (20160+24552n+12154n^2+3135n^3+445n^4+33n^5+n^6) x^6) + \\
& \quad b^{11} (246400+593520n+541508n^2+251352n^3+66489n^4+10440n^5+962n^6+48n^7+n^8) x^2 (c^3 (648+234n+27n^2+n^3) + \\
& \quad 3c^2 d (324+171n+24n^2+n^3) x^3 + 3c d^2 (216+126n+21n^2+n^3) x^6 + d^3 (162+99n+18n^2+n^3) x^9) - a b^{10} (280+418n+159n^2+22n^3+n^4) x \\
& \quad (2c^3 (285120+221544n+70254n^2+11645n^3+1065n^4+51n^5+n^6) + 15c^2 d (57024+70920n+32574n^2+7115n^3+801n^4+45n^5+n^6) x^3 + \\
& \quad 24 c d^2 (23760+32652n+17160n^2+4421n^3+591n^4+39n^5+n^6) x^6 + 11d^3 (12960+18612n+10404n^2+2915n^3+435n^4+33n^5+n^6) x^9) + \\
& \quad 2 a^2 b^9 (c^3 (79833600+101378880n+56231712n^2+17893196n^3+3602088n^4+476049n^5+41328n^6+2274n^7+72n^8+n^9) + \\
& \quad 30 c^2 d (3991680+9925488n+9476652n^2+4665572n^3+1332327n^4+233481n^5+25518n^6+1698n^7+63n^8+n^9) x^3 + \\
& \quad 84 c d^2 (950400+2589120n+2806008n^2+1617020n^3+552426n^4+116949n^5+15432n^6+1230n^7+54n^8+n^9) x^6 + \\
& \quad 55 d^3 (362880+1026576n+1172700n^2+723680n^3+269325n^4+63273n^5+9450n^6+870n^7+45n^8+n^9) x^9))) / \\
& (b^{12} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) (10+n) \\
& (11+n) \\
& (12+n))
\end{aligned}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 396 leaves, 2 steps):

$$\begin{aligned}
& - \frac{a (b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{11} (1+n)} + \frac{(b^3 c - 10 a^3 d) (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{11} (2+n)} + \frac{9 a^2 d (2 b^3 c - 5 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{11} (3+n)} - \\
& \frac{3 a d (4 b^6 c^2 - 35 a^3 b^3 c d + 40 a^6 d^2) (a + b x)^{4+n}}{b^{11} (4+n)} + \frac{3 d (b^6 c^2 - 35 a^3 b^3 c d + 70 a^6 d^2) (a + b x)^{5+n}}{b^{11} (5+n)} + \frac{63 a^2 d^2 (b^3 c - 4 a^3 d) (a + b x)^{6+n}}{b^{11} (6+n)} - \\
& \frac{21 a d^2 (b^3 c - 10 a^3 d) (a + b x)^{7+n}}{b^{11} (7+n)} + \frac{3 d^2 (b^3 c - 40 a^3 d) (a + b x)^{8+n}}{b^{11} (8+n)} + \frac{45 a^2 d^3 (a + b x)^{9+n}}{b^{11} (9+n)} - \frac{10 a d^3 (a + b x)^{10+n}}{b^{11} (10+n)} + \frac{d^3 (a + b x)^{11+n}}{b^{11} (11+n)}
\end{aligned}$$

Result (type 3, 903 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} (3628800 a^{10} d^3 - 3628800 a^9 b d^3 (1+n) x + \right. \\
& 1814400 a^8 b^2 d^3 (2 + 3 n + n^2) x^2 - 15120 a^7 b^3 d^2 (c (990 + 299 n + 30 n^2 + n^3) + 40 d (6 + 11 n + 6 n^2 + n^3) x^3) + \\
& 15120 a^6 b^4 d^2 (1+n) x (c (990 + 299 n + 30 n^2 + n^3) + 10 d (24 + 26 n + 9 n^2 + n^3) x^3) - 7560 a^5 b^5 d^2 (2 + 3 n + n^2) x^2 \\
& (c (990 + 299 n + 30 n^2 + n^3) + 4 d (60 + 47 n + 12 n^2 + n^3) x^3) + 72 a^4 b^6 d (c^2 (332640 + 245004 n + 74524 n^2 + 11985 n^3 + 1075 n^4 + 51 n^5 + n^6) + \\
& 35 c d (5940 + 12684 n + 9409 n^2 + 3120 n^3 + 490 n^4 + 36 n^5 + n^6) x^3 + 70 d^2 (720 + 1764 n + 1624 n^2 + 735 n^3 + 175 n^4 + 21 n^5 + n^6) x^6) - \\
& 18 a^3 b^7 d (1+n) x (4 c^2 (332640 + 245004 n + 74524 n^2 + 11985 n^3 + 1075 n^4 + 51 n^5 + n^6) + \\
& 35 c d (23760 + 32916 n + 17404 n^2 + 4485 n^3 + 595 n^4 + 39 n^5 + n^6) x^3 + 40 d^2 (5040 + 8028 n + 5104 n^2 + 1665 n^3 + 295 n^4 + 27 n^5 + n^6) x^6) + \\
& 18 a^2 b^8 d (2 + 3 n + n^2) x^2 (2 c^2 (332640 + 245004 n + 74524 n^2 + 11985 n^3 + 1075 n^4 + 51 n^5 + n^6) + \\
& 7 c d (59400 + 64470 n + 27733 n^2 + 6048 n^3 + 706 n^4 + 42 n^5 + n^6) x^3 + 5 d^2 (20160 + 24552 n + 12154 n^2 + 3135 n^3 + 445 n^4 + 33 n^5 + n^6) x^6) + \\
& b^{10} (45360 + 95436 n + 72180 n^2 + 27109 n^3 + 5620 n^4 + 654 n^5 + 40 n^6 + n^7) x (c^3 (440 + 183 n + 24 n^2 + n^3) + \\
& 3 c^2 d (176 + 126 n + 21 n^2 + n^3) x^3 + 3 c d^2 (110 + 87 n + 18 n^2 + n^3) x^6 + d^3 (80 + 66 n + 15 n^2 + n^3) x^9) - a b^9 (162 + 99 n + 18 n^2 + n^3) \\
& (c^3 (123200 + 111960 n + 41214 n^2 + 7875 n^3 + 825 n^4 + 45 n^5 + n^6) + 12 c^2 d (12320 + 24132 n + 15600 n^2 + 4341 n^3 + 591 n^4 + 39 n^5 + n^6) x^3 + \\
& 21 c d^2 (4400 + 9420 n + 7068 n^2 + 2427 n^3 + 411 n^4 + 33 n^5 + n^6) x^6 + 10 d^3 (2240 + 4968 n + 3954 n^2 + 1485 n^3 + 285 n^4 + 27 n^5 + n^6) x^9))) / \\
& (b^{11} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) (10+n) (11+n))
\end{aligned}$$

Problem 184: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^n (c + d x^3)^3 dx$$

Optimal (type 3, 337 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b^3 c - a^3 d)^3 (a + b x)^{1+n}}{b^{10} (1+n)} + \frac{9 a^2 d (b^3 c - a^3 d)^2 (a + b x)^{2+n}}{b^{10} (2+n)} - \frac{9 a d (b^3 c - 4 a^3 d) (b^3 c - a^3 d) (a + b x)^{3+n}}{b^{10} (3+n)} + \\
& \frac{3 d (b^6 c^2 - 20 a^3 b^3 c d + 28 a^6 d^2) (a + b x)^{4+n}}{b^{10} (4+n)} + \frac{9 a^2 d^2 (5 b^3 c - 14 a^3 d) (a + b x)^{5+n}}{b^{10} (5+n)} - \frac{18 a d^2 (b^3 c - 7 a^3 d) (a + b x)^{6+n}}{b^{10} (6+n)} + \\
& \frac{3 d^2 (b^3 c - 28 a^3 d) (a + b x)^{7+n}}{b^{10} (7+n)} + \frac{36 a^2 d^3 (a + b x)^{8+n}}{b^{10} (8+n)} - \frac{9 a d^3 (a + b x)^{9+n}}{b^{10} (9+n)} + \frac{d^3 (a + b x)^{10+n}}{b^{10} (10+n)}
\end{aligned}$$

Result (type 3, 706 leaves):

$$\begin{aligned}
& \left((a + b x)^{1+n} \right. \\
& \left(-362880 a^9 d^3 + 362880 a^8 b d^3 (1+n) x - 181440 a^7 b^2 d^3 (2+3n+n^2) x^2 + 2160 a^6 b^3 d^2 (c (720+242n+27n^2+n^3) + 28d (6+11n+6n^2+n^3) x^3) - \right. \\
& \left. 2160 a^5 b^4 d^2 (1+n) x (c (720+242n+27n^2+n^3) + 7d (24+26n+9n^2+n^3) x^3) + \right. \\
& \left. 216 a^4 b^5 d^2 (2+3n+n^2) x^2 (5c (720+242n+27n^2+n^3) + 14d (60+47n+12n^2+n^3) x^3) - 9ab^8 d (80+146n+81n^2+16n^3+n^4) \right. \\
& \left. x^2 (c^2 (3780+1968n+379n^2+32n^3+n^4) + 2cd (1080+858n+235n^2+26n^3+n^4) x^3 + d^2 (504+450n+145n^2+20n^3+n^4) x^6) - \right. \\
& \left. 18 a^3 b^6 d (c^2 (151200+127860n+44524n^2+8175n^3+835n^4+45n^5+n^6) + 20cd (4320+9372n+7144n^2+2475n^3+415n^4+33n^5+n^6) x^3 + \right. \\
& \left. 28d^2 (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^6) + 18a^2 b^7 d (1+n) x \right. \\
& \left. (c^2 (151200+127860n+44524n^2+8175n^3+835n^4+45n^5+n^6) + 5cd (17280+24528n+13420n^2+3624n^3+511n^4+36n^5+n^6) x^3 + \right. \\
& \left. 4d^2 (5040+8028n+5104n^2+1665n^3+295n^4+27n^5+n^6) x^6) + b^9 (12960+18612n+10404n^2+2915n^3+435n^4+33n^5+n^6) \right. \\
& \left. (c^3 (280+138n+21n^2+n^3) + 3c^2 d (70+87n+18n^2+n^3) x^3 + 3cd^2 (40+54n+15n^2+n^3) x^6 + d^3 (28+39n+12n^2+n^3) x^9)) \right) / \\
& (b^{10} (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n) (10+n))
\end{aligned}$$

Problem 185: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^n (c + d x^3)^3}{x} dx$$

Optimal (type 5, 358 leaves, 3 steps):

$$\begin{aligned}
& \frac{a^2 d (3 b^6 c^2 - 3 a^3 b^3 c d + a^6 d^2) (a + b x)^{1+n}}{b^9 (1+n)} - \frac{a d (6 b^6 c^2 - 15 a^3 b^3 c d + 8 a^6 d^2) (a + b x)^{2+n}}{b^9 (2+n)} + \frac{d (3 b^6 c^2 - 30 a^3 b^3 c d + 28 a^6 d^2) (a + b x)^{3+n}}{b^9 (3+n)} + \\
& \frac{2 a^2 d^2 (15 b^3 c - 28 a^3 d) (a + b x)^{4+n}}{b^9 (4+n)} - \frac{5 a d^2 (3 b^3 c - 14 a^3 d) (a + b x)^{5+n}}{b^9 (5+n)} + \frac{d^2 (3 b^3 c - 56 a^3 d) (a + b x)^{6+n}}{b^9 (6+n)} + \\
& \frac{28 a^2 d^3 (a + b x)^{7+n}}{b^9 (7+n)} - \frac{8 a d^3 (a + b x)^{8+n}}{b^9 (8+n)} + \frac{d^3 (a + b x)^{9+n}}{b^9 (9+n)} - \frac{c^3 (a + b x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, 1 + \frac{b x}{a}]}{a (1+n)}
\end{aligned}$$

Result (type 5, 856 leaves):

$$\begin{aligned}
& \left(a + b x \right)^n \left(\frac{1}{b^3 (1+n) (2+n) (3+n)} \right. \\
& \frac{3 c^2 d \left(1 + \frac{b x}{a} \right)^{-n} \left(-2 a^2 b n x \left(1 + \frac{b x}{a} \right)^n + a b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + b^3 (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + 2 a^3 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \right) + }{b^6 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n)} 3 c d^2 \left(1 + \frac{b x}{a} \right)^{-n} \\
& \left(120 a^5 b n x \left(1 + \frac{b x}{a} \right)^n - 60 a^4 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n + 20 a^3 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n - 5 a^2 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& a b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + b^6 (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 120 a^6 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \left. \right) + \\
& \frac{1}{b^9 (1+n) (2+n) (3+n) (4+n) (5+n) (6+n) (7+n) (8+n) (9+n)} d^3 \left(1 + \frac{b x}{a} \right)^{-n} \\
& \left(-40320 a^8 b n x \left(1 + \frac{b x}{a} \right)^n + 20160 a^7 b^2 n (1+n) x^2 \left(1 + \frac{b x}{a} \right)^n - 6720 a^6 b^3 n (2+3n+n^2) x^3 \left(1 + \frac{b x}{a} \right)^n + \right. \\
& 1680 a^5 b^4 n (6+11n+6n^2+n^3) x^4 \left(1 + \frac{b x}{a} \right)^n - 336 a^4 b^5 n (24+50n+35n^2+10n^3+n^4) x^5 \left(1 + \frac{b x}{a} \right)^n + \\
& 56 a^3 b^6 n (120+274n+225n^2+85n^3+15n^4+n^5) x^6 \left(1 + \frac{b x}{a} \right)^n - 8 a^2 b^7 n (720+1764n+1624n^2+735n^3+175n^4+21n^5+n^6) x^7 \left(1 + \frac{b x}{a} \right)^n + \\
& a b^8 n (5040+13068n+13132n^2+6769n^3+1960n^4+322n^5+28n^6+n^7) x^8 \left(1 + \frac{b x}{a} \right)^n + \\
& b^9 (40320+109584n+118124n^2+67284n^3+22449n^4+4536n^5+546n^6+36n^7+n^8) x^9 \left(1 + \frac{b x}{a} \right)^n + 40320 a^9 \left(-1 + \left(1 + \frac{b x}{a} \right)^n \right) \left. \right) + \\
& \left. \frac{c^3 \left(1 + \frac{a}{b x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{a}{b x}]}{n} \right)
\end{aligned}$$

Problem 186: Result is not expressed in closed-form.

$$\int \frac{x^5 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 324 leaves, 7 steps):

$$\begin{aligned}
& \frac{e^2 (e + f x)^{1+n}}{b f^3 (1+n)} - \frac{2 e (e + f x)^{2+n}}{b f^3 (2+n)} + \frac{(e + f x)^{3+n}}{b f^3 (3+n)} + \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} + \frac{a (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}]}{3 b^{5/3} (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)}
\end{aligned}$$

Result (type 7, 423 leaves):

$$\begin{aligned}
 & \frac{1}{3 b f^3} (e + f x)^n \left(\frac{3 \left(-2 e^2 f n x + e f^2 n (1+n) x^2 + f^3 (2+3 n+n^2) x^3 + e^3 \left(2 - 2 \left(1 + \frac{f x}{e} \right)^{-n} \right) \right)}{6 + 11 n + 6 n^2 + n^3} - \right. \\
 & \frac{1}{b n} a f^3 \left(e^2 \text{RootSum} [b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&] - \right. \\
 & 2 e \text{RootSum} [b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] + \\
 & \left. \text{RootSum} [b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \&] \right)
 \end{aligned}$$

Problem 187: Result is not expressed in closed-form.

$$\int \frac{x^4 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 332 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{e (e + f x)^{1+n}}{b f^2 (1+n)} + \frac{(e + f x)^{2+n}}{b f^2 (2+n)} - \frac{a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 b^{4/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \\
 & \frac{(-1)^{1/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}]}{3 b^{4/3} ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n)} + \\
 & \frac{(-1)^{2/3} a^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}]}{3 b^{4/3} ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n)}
 \end{aligned}$$

Result (type 7, 298 leaves):

$$\begin{aligned}
 & \frac{1}{3 b f^2 (e + f x)^n} \left(-\frac{3 \left(-e f n x - f^2 (1+n) x^2 + e^2 \left(1 - \left(1 + \frac{f x}{e} \right)^{-n} \right) \right)}{2 + 3 n + n^2} + \right. \\
 & \frac{a e f^3 \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \# 1 + 3 b e \# 1^2 - b \# 1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\# 1}{e+f x-\# 1} \right] \left(\frac{e+f x}{e+f x-\# 1} \right)^{-n}}{e^2-2 e \# 1+\# 1^2} \& \right]}{b n} - \\
 & \left. \frac{a f^3 \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \# 1 + 3 b e \# 1^2 - b \# 1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\# 1}{e+f x-\# 1} \right] \left(\frac{e+f x}{e+f x-\# 1} \right)^{-n} \# 1}{e^2-2 e \# 1+\# 1^2} \& \right]}{b n} \right)
 \end{aligned}$$

Problem 188: Result is not expressed in closed-form.

$$\int \frac{x^3 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 293 leaves, 7 steps):

$$\begin{aligned}
 & \frac{(e + f x)^{1+n}}{b f (1+n)} + \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f} \right]}{3 b (b^{1/3} e - a^{1/3} f) (1+n)} + \\
 & \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f} \right]}{3 b \left((-1)^{2/3} b^{1/3} e - a^{1/3} f \right) (1+n)} - \frac{a^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f} \right]}{3 b \left((-1)^{1/3} b^{1/3} e + a^{1/3} f \right) (1+n)}
 \end{aligned}$$

Result (type 7, 142 leaves):

$$\frac{(e + f x)^n \left(\frac{3 b (e+f x)}{1+n} - \frac{a f^3 \text{RootSum} \left[b e^3 - a f^3 - 3 b e^2 \# 1 + 3 b e \# 1^2 - b \# 1^3 \&, \frac{\text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{\# 1}{e+f x-\# 1} \right] \left(\frac{e+f x}{e+f x-\# 1} \right)^{-n}}{e^2-2 e \# 1+\# 1^2} \& \right]}{n} \right)}{3 b^2 f}$$

Problem 189: Result is not expressed in closed-form.

$$\int \frac{x^2 (e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps):

$$\begin{aligned}
& - \frac{(e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e - a^{1/3} f}]}{3 b^{2/3} (b^{1/3} e - a^{1/3} f) (1+n)} - \\
& \frac{(e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}]}{3 b^{2/3} (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} - \frac{(e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}]}{3 b^{2/3} (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)}
\end{aligned}$$

Result (type 7, 337 leaves):

$$\begin{aligned}
& \frac{1}{3 b n} (e + fx)^n \left(e^2 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&] - \right. \\
& 2 e \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] + \\
& \left. \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 190: Result is not expressed in closed-form.

$$\int \frac{x (e + fx)^n}{a + b x^3} dx$$

Optimal (type 5, 288 leaves, 5 steps):

$$\begin{aligned}
& \frac{(e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+fx)}{b^{1/3} e - a^{1/3} f}]}{3 a^{1/3} b^{1/3} (b^{1/3} e - a^{1/3} f) (1+n)} - \\
& \frac{(-1)^{1/3} (e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+fx)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}]}{3 a^{1/3} b^{1/3} ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n)} - \frac{(-1)^{2/3} (e + fx)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+fx)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}]}{3 a^{1/3} b^{1/3} ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n)}
\end{aligned}$$

Result (type 7, 229 leaves):

$$\begin{aligned}
& - \frac{1}{3 b n} f (e + fx)^n \left(e \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&] - \right. \\
& \left. \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+fx-\#1}] \left(\frac{e+fx}{e+fx-\#1}\right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 191: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{a + b x^3} dx$$

Optimal (type 5, 263 leaves, 5 steps):

$$\begin{aligned} & \frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} (b^{1/3} e - a^{1/3} f) (1+n)} - \\ & \frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}\right]}{3 a^{2/3} ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n)} + \frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}\right]}{3 a^{2/3} ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n)} \end{aligned}$$

Result (type 7, 122 leaves):

$$\frac{1}{3 b n} f^2 (e + f x)^n \text{RootSum}\left[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}\left[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}\right] \left(\frac{e+f x}{e+f x-\#1}\right)^{-n}}{e^2 - 2 e \#1 + \#1^2} \&\right]$$

Problem 192: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x (a + b x^3)} dx$$

Optimal (type 5, 300 leaves, 8 steps):

$$\begin{aligned} & \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}\right]}{3 a (b^{1/3} e - a^{1/3} f) (1+n)} + \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e + (-1)^{1/3} a^{1/3} f}\right]}{3 a (b^{1/3} e + (-1)^{1/3} a^{1/3} f) (1+n)} + \\ & \frac{b^{1/3} (e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - (-1)^{2/3} a^{1/3} f}\right]}{3 a (b^{1/3} e - (-1)^{2/3} a^{1/3} f) (1+n)} - \frac{(e + f x)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{f x}{e}\right]}{a e (1+n)} \end{aligned}$$

Result (type 7, 377 leaves):

$$\begin{aligned}
& \frac{1}{3 a n} (e + f x)^n \left(3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{e}{f x}] - \right. \\
& e^2 \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] + \\
& 2 e \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] - \\
& \left. \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1^2}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 193: Result is not expressed in closed-form.

$$\int \frac{(e + f x)^n}{x^2 (a + b x^3)} dx$$

Optimal (type 5, 326 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{b^{1/3} (e+f x)}{b^{1/3} e - a^{1/3} f}]}{3 a^{4/3} (b^{1/3} e - a^{1/3} f) (1+n)} + \frac{(-1)^{1/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{2/3} b^{1/3} (e+f x)}{(-1)^{2/3} b^{1/3} e - a^{1/3} f}]}{3 a^{4/3} ((-1)^{2/3} b^{1/3} e - a^{1/3} f) (1+n)} + \\
& \frac{(-1)^{2/3} b^{2/3} (e + f x)^{1+n} \text{Hypergeometric2F1}[1, 1+n, 2+n, \frac{(-1)^{1/3} b^{1/3} (e+f x)}{(-1)^{1/3} b^{1/3} e + a^{1/3} f}]}{3 a^{4/3} ((-1)^{1/3} b^{1/3} e + a^{1/3} f) (1+n)} + \frac{f (e + f x)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, 1 + \frac{f x}{e}]}{a e^2 (1+n)}
\end{aligned}$$

Result (type 7, 280 leaves):

$$\begin{aligned}
& \frac{1}{3 a} (e + f x)^n \left(\frac{3 \left(1 + \frac{e}{f x} \right)^{-n} \text{Hypergeometric2F1}[1-n, -n, 2-n, -\frac{e}{f x}]}{(-1+n) x} + \right. \\
& e f \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \&}{e^2 - 2 e \#1 + \#1^2} \&] - \\
& \left. f \text{RootSum}[b e^3 - a f^3 - 3 b e^2 \#1 + 3 b e \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{e+f x-\#1}] \left(\frac{e+f x}{e+f x-\#1} \right)^{-n} \#1}{e^2 - 2 e \#1 + \#1^2} \&] \right)
\end{aligned}$$

Problem 194: Result is not expressed in closed-form.

$$\int \frac{x^2 (c + d x)^{1+n}}{a + b x^3} dx$$

Optimal (type 5, 253 leaves, 5 steps):

$$\begin{aligned} & \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c - a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c - a^{1/3} d) (2+n)} - \\ & \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c + (-1)^{1/3} a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c + (-1)^{1/3} a^{1/3} d) (2+n)} - \frac{(c + d x)^{2+n} \text{Hypergeometric2F1}[1, 2+n, 3+n, \frac{b^{1/3} (c+d x)}{b^{1/3} c - (-1)^{2/3} a^{1/3} d}]}{3 b^{2/3} (b^{1/3} c - (-1)^{2/3} a^{1/3} d) (2+n)} \end{aligned}$$

Result (type 7, 375 leaves):

$$\begin{aligned} & \frac{1}{3 b^2 n (1+n)} \\ & (c + d x)^n \left((b c^3 - a d^3) (1+n) \text{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^2 - 2 c \#1 + \#1^2} \&] + \right. \\ & b \left(3 n (c + d x) - 2 c^2 (1+n) \text{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^2 - 2 c \#1 + \#1^2} \&] + \right. \\ & \left. c (1+n) \text{RootSum}[b c^3 - a d^3 - 3 b c^2 \#1 + 3 b c \#1^2 - b \#1^3 \&, \frac{\text{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^2 - 2 c \#1 + \#1^2} \&] \right) \end{aligned}$$

Problem 195: Unable to integrate problem.

$$\int \frac{x^m (e + f x)^n}{a + b x^3} dx$$

Optimal (type 6, 211 leaves, 8 steps):

$$\begin{aligned} & \frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, -\frac{b^{1/3} x}{a^{1/3}}]}{3 a (1+m)} + \frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, \frac{(-1)^{1/3} b^{1/3} x}{a^{1/3}}]}{3 a (1+m)} + \\ & \frac{x^{1+m} (e + f x)^n \left(1 + \frac{f x}{e}\right)^{-n} \text{AppellF1}[1+m, -n, 1, 2+m, -\frac{f x}{e}, -\frac{(-1)^{2/3} b^{1/3} x}{a^{1/3}}]}{3 a (1+m)} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{x^m (e + f x)^n}{a + b x^3} dx$$

Problem 196: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c + d x^3}}{a + b x} dx$$

Optimal (type 4, 1482 leaves, 13 steps):

$$\begin{aligned} & \frac{2 \sqrt{c + d x^3}}{3 b} - \frac{2 a d^{1/3} \sqrt{c + d x^3}}{b^2 \left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)} - \left(c^{1/6} \sqrt{b c^{1/3} - a d^{1/3}} \sqrt{b^2 c^{2/3} + a b c^{1/3} d^{1/3} + a^2 d^{2/3}} (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} \left(1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}} \right)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \right. \\ & \left. \text{ArcTanh} \left[\frac{\sqrt{2 - \sqrt{3}} \sqrt{b^2 c^{2/3} + a b c^{1/3} d^{1/3} + a^2 d^{2/3}} \sqrt{1 - \frac{\left((1 - \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}}}{3^{1/4} \sqrt{b} c^{1/6} \sqrt{b c^{1/3} - a d^{1/3}} \sqrt{7 - 4 \sqrt{3} + \frac{\left((1 - \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}}} \right] \right) \Big/ \left(b^{5/2} \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right) + \\ & \left(3^{1/4} \sqrt{2 - \sqrt{3}} a c^{1/3} d^{1/3} (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) \Big/ \\ & \left(b^2 \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right) + \\ & \left(2 \sqrt{2 + \sqrt{3}} a \left((1 - \sqrt{3}) b c^{1/3} + a d^{1/3} \right) d^{1/3} (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x} \right], -7 - 4 \sqrt{3} \right] \right) \Big/ \\ & \left(3^{1/4} b^3 \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right) \end{aligned}$$

$$\begin{aligned}
& \left(2 \sqrt{2 + \sqrt{3}} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x}\right], -7 - 4\sqrt{3}] \right) / \\
& \left(3^{1/4} b^3 \left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3} \right) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right) - \\
& \left(4 \times 3^{1/4} \sqrt{2 + \sqrt{3}} c^{1/3} (b^3 c - a^3 d) (c^{1/3} + d^{1/3} x) \sqrt{\frac{c^{2/3} \left(1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}} \right)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \operatorname{EllipticPi}\left[\frac{\left((1 + \sqrt{3}) b c^{1/3} - a d^{1/3} \right)^2}{\left((1 - \sqrt{3}) b c^{1/3} - a d^{1/3} \right)^2}, \right. \\
& \left. -\operatorname{ArcSin}\left[\frac{(1 - \sqrt{3}) c^{1/3} + d^{1/3} x}{(1 + \sqrt{3}) c^{1/3} + d^{1/3} x}\right], -7 - 4\sqrt{3} \right] \right) / \left(b^2 (2 b^2 c^{2/3} + 2 a b c^{1/3} d^{1/3} - a^2 d^{2/3}) \sqrt{\frac{c^{1/3} (c^{1/3} + d^{1/3} x)}{\left((1 + \sqrt{3}) c^{1/3} + d^{1/3} x \right)^2}} \sqrt{c + d x^3} \right)
\end{aligned}$$

Result (type 4, 820 leaves):

$$\begin{aligned}
& \frac{1}{3 b \sqrt{c + d x^3}} 2 \left(c + d x^3 - \frac{1}{b^2 \sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}} \right. \\
& \quad \left. 3^{3/4} a^2 d^{2/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{\frac{c^{1/3} + d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}} \sqrt{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}] + \right. \\
& \quad \left. \frac{1}{b \sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}} 3^{3/4} a c^{1/3} d^{1/3} \left((-1)^{1/3} c^{1/3} - d^{1/3} x \right) \sqrt{\frac{i + \sqrt{3} - 2 i d^{1/3} x}{c^{1/3}}} \sqrt{\frac{i \left(1 + \frac{d^{1/3} x}{c^{1/3}}\right)}{3 i + \sqrt{3}}} \right. \\
& \quad \left. \left(-1 + (-1)^{2/3} \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}] + \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{(-1)^{1/6} - \frac{i d^{1/3} x}{c^{1/3}}}{3^{1/4}}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}] \right) - \\
& \quad \frac{1}{(-1)^{1/3} b c^{1/3} + a d^{1/3}} 3^{\frac{i}{2}} b c^{4/3} \sqrt{\frac{c^{1/3} + d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}} \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \\
& \quad \operatorname{EllipticPi}\left[\frac{i \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}\right] + \left((-1)^{1/3} \sqrt{3} (1 + (-1)^{1/3}) a^3 c^{1/3} d \sqrt{\frac{c^{1/3} + d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}} \right. \\
& \quad \left. \sqrt{1 - \frac{d^{1/3} x}{c^{1/3}} + \frac{d^{2/3} x^2}{c^{2/3}}} \operatorname{EllipticPi}\left[\frac{i \sqrt{3} b c^{1/3}}{(-1)^{1/3} b c^{1/3} + a d^{1/3}}, \operatorname{ArcSin}\left[\sqrt{\frac{c^{1/3} + (-1)^{2/3} d^{1/3} x}{(1 + (-1)^{1/3}) c^{1/3}}}\right], (-1)^{1/3}\right] \right) / \left(b^2 \left((-1)^{1/3} b c^{1/3} + a d^{1/3} \right) \right)
\end{aligned}$$

Problem 197: Unable to integrate problem.

$$\int \frac{(d^3 + e^3 x^3)^p}{d + e x} dx$$

Optimal (type 6, 135 leaves, ? steps):

$$\frac{(\mathbf{d}^3 + \mathbf{e}^3 \mathbf{x}^3)^{\mathbf{p}} \left(1 + \frac{2(\mathbf{d} + \mathbf{e} \mathbf{x})}{(-3+i\sqrt{3})\mathbf{d}}\right)^{-\mathbf{p}} \left(1 - \frac{2(\mathbf{d} + \mathbf{e} \mathbf{x})}{(3+i\sqrt{3})\mathbf{d}}\right)^{-\mathbf{p}} \text{AppellF1}[\mathbf{p}, -\mathbf{p}, -\mathbf{p}, 1+\mathbf{p}, -\frac{2(\mathbf{d} + \mathbf{e} \mathbf{x})}{(-3+i\sqrt{3})\mathbf{d}}, \frac{2(\mathbf{d} + \mathbf{e} \mathbf{x})}{(3+i\sqrt{3})\mathbf{d}}]}{\mathbf{e}^{\mathbf{p}}}$$

Result (type 8, 23 leaves):

$$\int \frac{(\mathbf{d}^3 + \mathbf{e}^3 \mathbf{x}^3)^{\mathbf{p}}}{\mathbf{d} + \mathbf{e} \mathbf{x}} d\mathbf{x}$$

Problem 198: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 - 2x - x^2}{(2 + x^2) \sqrt{1 + x^3}} dx$$

Optimal (type 3, 16 leaves, 2 steps):

$$2 \text{ArcTan}\left[\frac{1+x}{\sqrt{1+x^3}}\right]$$

Result (type 4, 296 leaves):

$$\frac{\frac{1}{3\sqrt{1+x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \left(\frac{\sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}]}{1+(-1)^{2/3}x} - \frac{3 \frac{i}{2} \left(-\frac{i}{2}+\sqrt{2}\right) \text{EllipticPi}\left[\frac{2\sqrt{3}}{-\frac{i}{2}-2\sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}+\sqrt{2}} + \frac{3 \left(5+\frac{i}{2}\sqrt{2}+\frac{i}{2}\sqrt{3}+\sqrt{6}\right) \text{EllipticPi}\left[\frac{2\sqrt{3}}{-\frac{i}{2}+2\sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1+(-1)^{2/3}x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{5\frac{i}{2}+2\sqrt{2}+\sqrt{3}+2\frac{i}{2}\sqrt{6}} \right)}{1}$$

Problem 199: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 + 2x - x^2}{(2 + x^2) \sqrt{1 - x^3}} dx$$

Optimal (type 3, 20 leaves, 2 steps):

$$-2 \operatorname{ArcTan}\left[\frac{1-x}{\sqrt{1-x^3}}\right]$$

Result (type 4, 280 leaves):

$$\begin{aligned} & \frac{1}{3 \sqrt{1-x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \left(\frac{\sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3} x} + \right. \\ & \frac{6 \left(1+\frac{1}{2} \sqrt{2}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{1}{2}+2 \sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\frac{1}{2}+2 \sqrt{2}-\sqrt{3}} + \\ & \left. \frac{3 \left(1-\frac{1}{2} \sqrt{2}\right) \operatorname{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{1}{2}+2 \sqrt{2}+\sqrt{3}}, \operatorname{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}-\sqrt{2}} \right) \end{aligned}$$

Problem 200: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2 + 2x - x^2}{(2 + x^2) \sqrt{-1 + x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$-2 \operatorname{ArcTanh}\left[\frac{1-x}{\sqrt{-1+x^3}}\right]$$

Result (type 4, 278 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1+x^3}} 2 \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \left(\frac{\sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}]}{-1+(-1)^{2/3} x} + \right. \\
& \frac{6 \left(1+\frac{i}{2} \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}+2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{\frac{i}{2}+2 \sqrt{2}-\sqrt{3}} + \\
& \left. \frac{3 \left(1-\frac{i}{2} \sqrt{2}\right) \text{EllipticPi}\left[\frac{2 \sqrt{3}}{-\frac{i}{2}+2 \sqrt{2}+\sqrt{3}}, \text{ArcSin}\left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}-\sqrt{2}} \right)
\end{aligned}$$

Problem 201: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2x-x^2}{(2+x^2) \sqrt{-1-x^3}} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$2 \text{ArcTanh}\left[\frac{1+x}{\sqrt{-1-x^3}}\right]$$

Result (type 4, 298 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1-x^3}} 2 \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \left\{ \frac{\sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3} x} - \right. \\
& \frac{3 \frac{1}{2} \left(-\frac{1}{2}+\sqrt{2}\right) \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-\frac{1}{2}-2 \sqrt{2}+\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{(-1)^{5/6}+\sqrt{2}} + \\
& \left. \frac{3 \left(5+\frac{1}{2} \sqrt{2}+\frac{1}{2} \sqrt{3}+\sqrt{6}\right) \text{EllipticPi} \left[\frac{2 \sqrt{3}}{-\frac{1}{2}+2 \sqrt{2}+\sqrt{3}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{5 \frac{1}{2}+2 \sqrt{2}+\sqrt{3}+2 \frac{1}{2} \sqrt{6}} \right\}
\end{aligned}$$

Problem 202: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}} dx$$

Optimal (type 3, 30 leaves, 2 steps):

$$\frac{2 \text{ArcTan} \left[\frac{\sqrt{1+d} (1+x)}{\sqrt{1+x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 424 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{1+x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \\
& \left(\frac{2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3} x} - \frac{1}{\left(2+(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8-4 d+d^2}} \right. \\
& \left. 3 i \left(\left(8+8 (-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2+4 \sqrt{-8-4 d+d^2}-2 (-1)^{1/3} \sqrt{-8-4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8-4 d+d^2}\right)\right) \right. \right. \\
& \left. \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}+d-\sqrt{-8-4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \left. \left(\left(1+(-1)^{1/3}\right) d^2+\left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8-4 d+d^2}\right)-2 \left(4+4 (-1)^{1/3}-2 \sqrt{-8-4 d+d^2}+(-1)^{1/3} \sqrt{-8-4 d+d^2}\right)\right) \right. \right. \\
& \left. \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}+d+\sqrt{-8-4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 203: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2 x-x^2}{(2-d+d x+x^2) \sqrt{1-x^3}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\begin{aligned}
& \frac{2 \text{ArcTan} \left[\frac{\sqrt{1-d} (1-x)}{\sqrt{1-x^3}}\right]}{\sqrt{1-d}}
\end{aligned}$$

Result (type 4, 427 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{1-x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3} x} + \frac{1}{\left(-2-(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8+4 d+d^2}} \right. \\
& 3 i \left(\left(8+8 (-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8+4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}-d+\sqrt{-8+4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \left(-8-8 (-1)^{1/3}+\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8+4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[-\frac{2 i \sqrt{3}}{-2 (-1)^{1/3}+d+\sqrt{-8+4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 204: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2+2 x-x^2}{(2-d+d x+x^2) \sqrt{-1+x^3}} dx$$

Optimal (type 3, 36 leaves, 2 steps):

$$-\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{1-d} (1-x)}{\sqrt{-1+x^3}}\right]}{\sqrt{1-d}}$$

Result (type 4, 425 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1+x^3}} \sqrt{\frac{1-x}{1+(-1)^{1/3}}} \sqrt{1+x+x^2} \\
& \left(\frac{2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}+x\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{-1+(-1)^{2/3} x} + \frac{1}{\left(-2-(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8+4 d+d^2}} \right. \\
& 3 i \left(\left(8+8 (-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8+4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}-d+\sqrt{-8+4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \left(-8-8 (-1)^{1/3}+\left(1+(-1)^{1/3}\right) d^2-4 \sqrt{-8+4 d+d^2}+2 (-1)^{1/3} \sqrt{-8+4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8+4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[-\frac{2 i \sqrt{3}}{-2 (-1)^{1/3}+d+\sqrt{-8+4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1-(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 205: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{2-2 x-x^2}{(2+d+d x+x^2) \sqrt{-1-x^3}} d x$$

Optimal (type 3, 32 leaves, 2 steps):

$$\frac{2 \text{Arctanh} \left[\frac{\sqrt{1+d} (1+x)}{\sqrt{-1-x^3}}\right]}{\sqrt{1+d}}$$

Result (type 4, 426 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{-1 - x^3}} \sqrt{\frac{1+x}{1+(-1)^{1/3}}} \sqrt{1-x+x^2} \\
& \left(\frac{2 \sqrt{3} \left(1+(-1)^{1/3}\right) \left((-1)^{1/3}-x\right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right]}{1+(-1)^{2/3} x} - \frac{1}{\left(2+(-1)^{2/3}+d+(-1)^{1/3} d\right) \sqrt{-8-4 d+d^2}} \right. \\
& 3 i \left(\left(8+8 (-1)^{1/3}-\left(1+(-1)^{1/3}\right) d^2+4 \sqrt{-8-4 d+d^2}-2 (-1)^{1/3} \sqrt{-8-4 d+d^2}+\left(1+(-1)^{1/3}\right) d \left(4+\sqrt{-8-4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}+d-\sqrt{-8-4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] + \right. \\
& \left. \left(\left(1+(-1)^{1/3}\right) d^2+\left(1+(-1)^{1/3}\right) d \left(-4+\sqrt{-8-4 d+d^2}\right)-2 \left(4+4 (-1)^{1/3}-2 \sqrt{-8-4 d+d^2}+(-1)^{1/3} \sqrt{-8-4 d+d^2}\right)\right) \right. \\
& \left. \text{EllipticPi} \left[\frac{2 i \sqrt{3}}{2 (-1)^{1/3}+d+\sqrt{-8-4 d+d^2}}, \text{ArcSin} \left[\sqrt{\frac{1+(-1)^{2/3} x}{1+(-1)^{1/3}}}\right], (-1)^{1/3}\right] \right)
\end{aligned}$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^3 \sqrt{a+c x^4} \, dx$$

Optimal (type 4, 355 leaves, 11 steps):

$$\begin{aligned}
& \frac{3}{4} d^2 e x^2 \sqrt{a + c x^4} + \frac{6 a d e^2 x \sqrt{a + c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{15} d x (5 d^2 + 9 e^2 x^2) \sqrt{a + c x^4} + \frac{e^3 (a + c x^4)^{3/2}}{6 c} + \\
& \frac{3 a d^2 e \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right]}{4 \sqrt{c}} - \frac{6 a^{5/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{5 c^{3/4} \sqrt{a + c x^4}} + \\
& \frac{a^{3/4} d (5 \sqrt{c} d^2 + 9 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{15 c^{3/4} \sqrt{a + c x^4}}
\end{aligned}$$

Result (type 4, 310 leaves):

$$\begin{aligned}
& \frac{1}{60 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a + c x^4}} \\
& \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(10 a^2 e^3 + c^2 x^5 (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3) + a c x (20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 20 e^3 x^3) + 45 a \sqrt{c} d^2 e \sqrt{a + c x^4} \right. \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right] \right) + 72 a^{3/2} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] - \\
& 8 a \sqrt{c} d (5 i \sqrt{c} d^2 + 9 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]
\end{aligned}$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int (d + e x)^2 \sqrt{a + c x^4} dx$$

Optimal (type 4, 326 leaves, 10 steps):

$$\begin{aligned}
& \frac{1}{2} d e x^2 \sqrt{a + c x^4} + \frac{2 a e^2 x \sqrt{a + c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{15} x (5 d^2 + 3 e^2 x^2) \sqrt{a + c x^4} + \\
& \frac{a d e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right] - 2 a^{5/4} e^2 \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 \sqrt{c}} + \\
& \frac{a^{3/4} (5 \sqrt{c} d^2 + 3 \sqrt{a} e^2) \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{15 c^{3/4} \sqrt{a + c x^4}}
\end{aligned}$$

Result (type 4, 247 leaves):

$$\begin{aligned}
& \frac{1}{30 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} \sqrt{a + c x^4}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \left(\sqrt{c} x (10 d^2 + 15 d e x + 6 e^2 x^2) (a + c x^4) + 15 a d e \sqrt{a + c x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}}\right] \right) + \right. \\
& \left. 12 a^{3/2} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - 4 a \left(5 i \sqrt{c} d^2 + 3 \sqrt{a} e^2\right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)
\end{aligned}$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int (d + e x) \sqrt{a + c x^4} dx$$

Optimal (type 4, 158 leaves, 8 steps):

$$\frac{1}{3} d x \sqrt{a + c x^4} + \frac{1}{4} e x^2 \sqrt{a + c x^4} + \frac{a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{4 \sqrt{c}} + \frac{a^{3/4} d \left(\sqrt{a} + \sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 132 leaves):

$$\frac{1}{12} \left(x (4 d + 3 e x) \sqrt{a + c x^4} + \frac{3 a e \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{\sqrt{c}} - \frac{8 i a d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}} \right)$$

Problem 209: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + c x^4} \, dx$$

Optimal (type 4, 105 leaves, 2 steps):

$$\frac{1}{3} x \sqrt{a + c x^4} + \frac{a^{3/4} \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2 \right)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{3 c^{1/4} \sqrt{a + c x^4}}$$

Result (type 4, 89 leaves):

$$\frac{x (a + c x^4) - \frac{2 i a \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 \sqrt{a + c x^4}}$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{d + e x} \, dx$$

Optimal (type 4, 730 leaves, 15 steps):

$$\begin{aligned}
& \frac{\sqrt{a+c x^4}}{2 e} - \frac{\sqrt{c} d x \sqrt{a+c x^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{\sqrt{-c d^4 - a e^4} \operatorname{ArcTan} \left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}} \right]}{2 e^3} + \frac{\sqrt{c} d^2 \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right]}{2 e^3} - \\
& \frac{\sqrt{c d^4 + a e^4} \operatorname{ArcTanh} \left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}} \right]}{2 e^3} + \frac{a^{1/4} c^{1/4} d \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{e^2 \sqrt{a+c x^4}} - \\
& \frac{a^{1/4} c^{1/4} d \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2 \right) \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 e^4 \sqrt{a+c x^4}} + \\
& \frac{c^{1/4} d (c d^4 + a e^4) \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} e^4 \left(\sqrt{c} d^2 + \sqrt{a} e^2 \right) \sqrt{a+c x^4}} - \\
& \left(\left(\sqrt{c} d^2 - \sqrt{a} e^2 \right) (c d^4 + a e^4) \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi} \left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \left(4 a^{1/4} c^{1/4} d e^4 \left(\sqrt{c} d^2 + \sqrt{a} e^2 \right) \sqrt{a+c x^4} \right)
\end{aligned}$$

Result (type 4, 451 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c^{1/4} d e^4 \sqrt{a+c x^4}} \left(-2 \sqrt{a} c^{3/4} d^2 e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE} \left[\operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] + \right. \\
& 2 c^{3/4} d^2 \left(\frac{i \sqrt{c}}{\sqrt{a}} d^2 + \sqrt{a} e^2 \right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[\operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] + \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \\
& \left. \left(-2 (-1)^{1/4} a^{1/4} (c d^4 + a e^4) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi} \left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin} \left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}} \right], -1 \right] + c^{1/4} d e \left(a e^2 + c e^2 x^4 + \sqrt{c d^4 + a e^4} \sqrt{a+c x^4} \right. \right. \right. \\
& \left. \left. \left. \operatorname{Log} \left[-d^2 + e^2 x^2 \right] + \sqrt{c} d^2 \sqrt{a+c x^4} \operatorname{Log} \left[c x^2 + \sqrt{c} \sqrt{a+c x^4} \right] - \sqrt{c d^4 + a e^4} \sqrt{a+c x^4} \operatorname{Log} \left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a+c x^4} \right] \right) \right)
\end{aligned}$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + c x^4}}{(d + e x)^2} dx$$

Optimal (type 4, 1221 leaves, 32 steps):

$$\begin{aligned}
& \frac{2 \sqrt{c} x \sqrt{a+c x^4}}{e^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{d \sqrt{a+c x^4}}{e (d^2 - e^2 x^2)} + \frac{x \sqrt{a+c x^4}}{d^2 - e^2 x^2} + \frac{\sqrt{-c d^4 - a e^4} \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3} - \frac{(c d^4 - a e^4) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 d e^3 \sqrt{-c d^4 - a e^4}} - \\
& \frac{\sqrt{c} d \operatorname{ArcTanh}\left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}}\right]}{e^3} + \frac{c d^3 \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{e^3 \sqrt{c d^4 + a e^4}} - \frac{2 a^{1/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{e^2 \sqrt{a+c x^4}} + \\
& \frac{3 a^{1/4} c^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2\right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 e^4 \sqrt{a+c x^4}} - \\
& \frac{c^{1/4} (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} e^4 \sqrt{a+c x^4}} + \\
& \frac{c^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} e^4 \sqrt{a+c x^4}} - \\
& \frac{c^{1/4} (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+c x^4}} + \\
& \frac{(\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d^2 e^4 \sqrt{a+c x^4}} + \\
& \left((\sqrt{c} d^2 - \sqrt{a} e^2) (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] \right) / \\
& \left(4 a^{1/4} c^{1/4} d^2 e^4 (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+c x^4} \right)
\end{aligned}$$

Result (type 4, 531 leaves):

$$\begin{aligned}
& \frac{1}{e^4 \sqrt{a + c x^4}} \left(-2 \frac{i \sqrt{c}}{\sqrt{a}} e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] - \right. \\
& \frac{2 \sqrt{c} \left(i \sqrt{c} d^2 + \sqrt{a} e^2 \right) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} + \\
& 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi} \left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin} \left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}} \right], -1 \right] - \frac{1}{\sqrt{c d^4 + a e^4} (d + e x)} e \left(a e^2 \sqrt{c d^4 + a e^4} + \right. \\
& c e^2 \sqrt{c d^4 + a e^4} x^4 + c d^3 (d + e x) \sqrt{a + c x^4} \operatorname{Log} \left[-d^2 + e^2 x^2 \right] + \sqrt{c} d \sqrt{c d^4 + a e^4} (d + e x) \sqrt{a + c x^4} \operatorname{Log} \left[c x^2 + \sqrt{c} \sqrt{a + c x^4} \right] - \\
& \left. \left. c d^4 \sqrt{a + c x^4} \operatorname{Log} \left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4} \right] - c d^3 e x \sqrt{a + c x^4} \operatorname{Log} \left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4} \right] \right) \right)
\end{aligned}$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{\sqrt{a + c x^4}} dx$$

Optimal (type 4, 295 leaves, 9 steps):

$$\begin{aligned}
& \frac{e^3 \sqrt{a + c x^4}}{2 c} + \frac{3 d e^2 x \sqrt{a + c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{3 d^2 e \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a + c x^4}} \right]}{2 \sqrt{c}} - \frac{3 a^{1/4} d e^2 (\sqrt{a} + \sqrt{c} x^2)}{\sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] + \\
& \frac{d (\sqrt{c} d^2 + 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} c^{3/4} \sqrt{a + c x^4}}
\end{aligned}$$

Result (type 4, 240 leaves):

$$\begin{aligned}
& \frac{1}{2 \sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}} c \sqrt{a+c x^4}}} \\
& \left(\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} e \left(e^2 (a+c x^4) + 3 \sqrt{c} d^2 \sqrt{a+c x^4} \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right] \right) + 6 \sqrt{a} \sqrt{c} d e^2 \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} x \right], -1 \right] - \right. \\
& \left. 2 \sqrt{c} d \left(\frac{i \sqrt{c}}{\sqrt{a}} d^2 + 3 \sqrt{a} e^2 \right) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} x \right], -1 \right] \right)
\end{aligned}$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d+e x)^2}{\sqrt{a+c x^4}} dx$$

Optimal (type 4, 263 leaves, 8 steps):

$$\begin{aligned}
& \frac{e^2 x \sqrt{a+c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{d e \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right]}{\sqrt{c}} - \frac{a^{1/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{c^{3/4} \sqrt{a+c x^4}} + \\
& \frac{a^{1/4} \left(\frac{\sqrt{c} d^2}{\sqrt{a}} + e^2 \right) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 c^{3/4} \sqrt{a+c x^4}}
\end{aligned}$$

Result (type 4, 204 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}} c \sqrt{a+c x^4}}} \left(\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} d e \sqrt{a+c x^4} \operatorname{ArcTanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right] + \right. \\
& \left. \sqrt{a} e^2 \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} x \right], -1 \right] - \left(\frac{i \sqrt{c}}{\sqrt{a}} d^2 + \sqrt{a} e^2 \right) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{\sqrt{a}}} x \right], -1 \right] \right)
\end{aligned}$$

Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{dx}{\sqrt{a + cx^4}}$$

Optimal (type 4, 121 leaves, 6 steps):

$$\frac{e \operatorname{Arctanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right]}{2 \sqrt{c}} + \frac{d \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2 \right)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 107 leaves):

$$\frac{e \operatorname{Arctanh} \left[\frac{\sqrt{c} x^2}{\sqrt{a+c x^4}} \right]}{2 \sqrt{c}} - \frac{\frac{i}{2} d \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}}$$

Problem 215: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + cx^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a} + \sqrt{c} x^2 \right)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 74 leaves):

$$-\frac{\frac{i}{2} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}}$$

Problem 216: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x) \sqrt{a + cx^4}} dx$$

Optimal (type 4, 405 leaves, 7 steps):

$$\frac{e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4-a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 \sqrt{-c d^4-a e^4}}-\frac{e \operatorname{ArcTanh}\left[\frac{a e^2+c d^2 x^2}{\sqrt{c d^4+a e^4} \sqrt{a+c x^4}}\right]}{2 \sqrt{c d^4+a e^4}}+\frac{c^{1/4} d \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} \left(\sqrt{c} d^2+\sqrt{a} e^2\right) \sqrt{a+c x^4}}-$$

$$\frac{\left(\sqrt{c} d^2-\sqrt{a} e^2\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{c} d^2+\sqrt{a} e^2\right)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d \left(\sqrt{c} d^2+\sqrt{a} e^2\right) \sqrt{a+c x^4}}$$

Result (type 4, 200 leaves):

$$\left(\sqrt{1+\frac{c x^4}{a}}\right. \\ \left.-2 \left(-1\right)^{1/4} a^{1/4} \sqrt{1+\frac{c d^4}{a e^4}} e \operatorname{EllipticPi}\left[\frac{\frac{1}{2} \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{\left(-1\right)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right]+c^{1/4} d \operatorname{Log}\left[\frac{-d^2+e^2 x^2}{c d^2 x^2+a e^2 \left(1+\sqrt{1+\frac{c d^4}{a e^4}} \sqrt{1+\frac{c x^4}{a}}\right)}\right]\right)\Bigg/2 \\ c^{1/4} d \sqrt{1+\frac{c d^4}{a e^4}} e \sqrt{a+c x^4}$$

Problem 217: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d+e x)^2 \sqrt{a+c x^4}} dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$\begin{aligned}
& -\frac{e^3 \sqrt{a+c x^4}}{(c d^4 + a e^4) (d + e x)} + \frac{\sqrt{c} e^2 x \sqrt{a+c x^4}}{(c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2)} - \frac{c d^3 e \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{(-c d^4 - a e^4)^{3/2}} - \frac{c d^3 e \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{(c d^4 + a e^4)^{3/2}} - \\
& \frac{a^{1/4} c^{1/4} e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{(c d^4 + a e^4) \sqrt{a+c x^4}} + \frac{c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) \sqrt{a+c x^4}} - \\
& \frac{c^{3/4} d^2 (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a+c x^4}}
\end{aligned}$$

Result (type 4, 462 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{(c d^4 + a e^4)^{3/2}} (d + e x) \sqrt{a+c x^4}}} \left(\sqrt{a} \sqrt{c} e^2 \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[\frac{i \sqrt{c}}{\sqrt{a}} \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& \left. \frac{i \sqrt{c} (\sqrt{c} d^2 + i \sqrt{a} e^2) \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[\frac{i}{\sqrt{a}} \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \right. \\
& \left. \sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{(c d^4 + a e^4)}} \left(e^3 \sqrt{c d^4 + a e^4} (a + c x^4) + 2 (-1)^{1/4} a^{1/4} c^{3/4} d^2 \sqrt{c d^4 + a e^4} (d + e x) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] - \right. \right. \\
& \left. \left. c d^3 e (d + e x) \sqrt{a+c x^4} \operatorname{Log}\left[-d^2 + e^2 x^2\right] + c d^3 e (d + e x) \sqrt{a+c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a+c x^4}\right] \right) \right)
\end{aligned}$$

Problem 218: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x)^3 \sqrt{a+c x^4}} dx$$

Optimal (type 4, 659 leaves, 12 steps):

$$\begin{aligned}
& -\frac{e^3 \sqrt{a+c x^4}}{2 (c d^4 + a e^4) (d + e x)^2} - \frac{3 c d^3 e^3 \sqrt{a+c x^4}}{(c d^4 + a e^4)^2 (d + e x)} + \frac{3 c^{3/2} d^3 e^2 x \sqrt{a+c x^4}}{(c d^4 + a e^4)^2 (\sqrt{a} + \sqrt{c} x^2)} + \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a+c x^4}}\right]}{2 (-c d^4 - a e^4)^{5/2}} - \\
& \frac{3 c d^2 e (c d^4 - a e^4) \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a+c x^4}}\right]}{2 (c d^4 + a e^4)^{5/2}} - \frac{3 a^{1/4} c^{5/4} d^3 e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{(c d^4 + a e^4)^2 \sqrt{a+c x^4}} + \\
& \frac{c^{3/4} d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (c d^4 + a e^4) \sqrt{a+c x^4}} - \\
& \left(3 c^{3/4} d (\sqrt{c} d^2 - \sqrt{a} e^2)^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) /
\end{aligned}$$

$$\left(4 a^{1/4} (c d^4 + a e^4)^2 \sqrt{a+c x^4}\right)$$

Result (type 4, 884 leaves):

$$\begin{aligned}
& \frac{1}{2 (c d^4 + a e^4)^{5/2} (d + e x)^2 \sqrt{a + c x^4}} \\
& \left\{ -e^3 (c d^4 + a e^4)^{3/2} (a + c x^4) - 6 c d^3 e^3 \sqrt{c d^4 + a e^4} (d + e x) (a + c x^4) - 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \right. \\
& \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] + \frac{4 i c^2 d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} + \\
& 6 i a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c d^3 e^2 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right] - \\
& \frac{2 i a c d e^4 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} - \\
& 6 (-1)^{1/4} a^{1/4} c^{7/4} d^5 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi} \left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin} \left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}} \right], -1 \right] + \\
& 6 (-1)^{1/4} a^{5/4} c^{3/4} d e^4 \sqrt{c d^4 + a e^4} (d + e x)^2 \sqrt{1 + \frac{c x^4}{a}} \text{EllipticPi} \left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \text{ArcSin} \left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}} \right], -1 \right] + \\
& 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \text{Log}[-d^2 + e^2 x^2] - 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \text{Log}[-d^2 + e^2 x^2] - 3 c^2 d^6 e (d + e x)^2 \sqrt{a + c x^4} \\
& \left. \text{Log} \left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4} \right] + 3 a c d^2 e^5 (d + e x)^2 \sqrt{a + c x^4} \text{Log} \left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4} \right] \right\}
\end{aligned}$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^3}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 298 leaves, 4 steps):

$$\begin{aligned} & \frac{3 d e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} - \frac{a e^3 - c x (d^3 + 3 d^2 e x + 3 d e^2 x^2)}{2 a c \sqrt{a + c x^4}} + \frac{3 d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} + \\ & \frac{d (\sqrt{c} d^2 - 3 \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a + c x^4}} \end{aligned}$$

Result (type 4, 215 leaves):

$$\begin{aligned} & \frac{1}{2 a \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} c \sqrt{a + c x^4}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} (-a e^3 + c d x (d^2 + 3 d e x + 3 e^2 x^2)) - 3 \sqrt{a} \sqrt{c} d e^2 \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\ & \left. \sqrt{c} d (-i \sqrt{c} d^2 + 3 \sqrt{a} e^2) \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \end{aligned}$$

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + e x)^2}{(a + c x^4)^{3/2}} dx$$

Optimal (type 4, 270 leaves, 4 steps):

$$\begin{aligned} & \frac{x (d + e x)^2}{2 a \sqrt{a + c x^4}} - \frac{e^2 x \sqrt{a + c x^4}}{2 a \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} c^{3/4} \sqrt{a + c x^4}} + \\ & \frac{(\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{3/4} \sqrt{a + c x^4}} \end{aligned}$$

Result (type 4, 188 leaves):

$$\frac{1}{2 a^{3/2} \left(\frac{i \sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{a+c x^4}} \frac{i}{\sqrt{a}} \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{c} x (d+e x)^2 - \sqrt{a} e^2 \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \left(-i \sqrt{c} d^2 + \sqrt{a} e^2\right) \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right)$$

Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x}{(a+c x^4)^{3/2}} dx$$

Optimal (type 4, 114 leaves, 3 steps):

$$\frac{x (d+e x)}{2 a \sqrt{a+c x^4}} + \frac{d (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 90 leaves):

$$\frac{x (d+e x) - \frac{i d \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{2 a \sqrt{a+c x^4}}$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+c x^4)^{3/2}} dx$$

Optimal (type 4, 108 leaves, 2 steps):

$$\frac{x}{2 a \sqrt{a+c x^4}} + \frac{(\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 102 leaves):

$$\frac{\sqrt{\frac{\frac{1}{2}\sqrt{c}}{\sqrt{a}}} x - \frac{1}{2}\sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcSinh}\left[\sqrt{\frac{\frac{1}{2}\sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{2 a \sqrt{\frac{\frac{1}{2}\sqrt{c}}{\sqrt{a}}} \sqrt{a + c x^4}}$$

Problem 223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(d + e x) (a + c x^4)^{3/2}} dx$$

Optimal (type 4, 818 leaves, 14 steps):

$$\begin{aligned} & \frac{e (a e^2 - c d^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} + \frac{c d x (d^2 + e^2 x^2)}{2 a (c d^4 + a e^4) \sqrt{a + c x^4}} - \frac{\sqrt{c} d e^2 x \sqrt{a + c x^4}}{2 a (c d^4 + a e^4) (\sqrt{a} + \sqrt{c} x^2)} - \frac{e^5 \operatorname{ArcTan}\left[\frac{\sqrt{-c d^4 - a e^4} x}{d e \sqrt{a + c x^4}}\right]}{2 (-c d^4 - a e^4)^{3/2}} - \\ & \frac{e^5 \operatorname{ArcTanh}\left[\frac{a e^2 + c d^2 x^2}{\sqrt{c d^4 + a e^4} \sqrt{a + c x^4}}\right]}{2 (c d^4 + a e^4)^{3/2}} + \frac{c^{1/4} d e^2 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} (c d^4 + a e^4) \sqrt{a + c x^4}} + \\ & \frac{c^{1/4} d (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} (c d^4 + a e^4) \sqrt{a + c x^4}} + \\ & \frac{c^{1/4} d e^4 (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4}} - \\ & \frac{e^4 (\sqrt{c} d^2 - \sqrt{a} e^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticPi}\left[\frac{(\sqrt{c} d^2 + \sqrt{a} e^2)^2}{4 \sqrt{a} \sqrt{c} d^2 e^2}, 2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{1/4} c^{1/4} d (\sqrt{c} d^2 + \sqrt{a} e^2) (c d^4 + a e^4) \sqrt{a + c x^4}} \end{aligned}$$

Result (type 4, 464 leaves):

$$\begin{aligned}
& \frac{1}{2 a \sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{c^{1/4} d (c d^4 + a e^4)^{3/2} \sqrt{a + c x^4}}} \left(-\sqrt{a} c^{3/4} d^2 e^2 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticE}\left[\frac{i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \right. \\
& c^{3/4} d^2 \left(-\frac{i \sqrt{c}}{\sqrt{a}} d^2 + \sqrt{a} e^2 \right) \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticF}\left[\frac{i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] + \\
& \left. \sqrt{\frac{\frac{i \sqrt{c}}{\sqrt{a}}}{c^{1/4} d^2}} \left(-2 (-1)^{1/4} a^{5/4} e^4 \sqrt{c d^4 + a e^4} \sqrt{1 + \frac{c x^4}{a}} \operatorname{EllipticPi}\left[\frac{i \sqrt{a} e^2}{\sqrt{c} d^2}, \operatorname{ArcSin}\left[\frac{(-1)^{3/4} c^{1/4} x}{a^{1/4}}\right], -1\right] + c^{1/4} d \right. \right. \\
& \left. \left. \left(\sqrt{c d^4 + a e^4} (a e^3 + c d x (d^2 - d e x + e^2 x^2)) + a e^5 \sqrt{a + c x^4} \operatorname{Log}\left[-d^2 + e^2 x^2\right] - a e^5 \sqrt{a + c x^4} \operatorname{Log}\left[a e^2 + c d^2 x^2 + \sqrt{c d^4 + a e^4} \sqrt{a + c x^4}\right] \right) \right) \right)
\end{aligned}$$

Problem 224: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^n}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\begin{aligned}
& \frac{\left(c + d x\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}\right] - \left(c + d x\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}\right]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (1+n)} - \\
& \frac{\left(c + d x\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c - (-a)^{1/4} d}\right] - \left(c + d x\right)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{b^{1/4} (c+d x)}{b^{1/4} c + (-a)^{1/4} d}\right]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (1+n)}
\end{aligned}$$

Result (type 7, 526 leaves):

$$\begin{aligned}
& \frac{1}{4 b n} (c + d x)^n \left(c^3 \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& 3 c^2 \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \\
& 3 c \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \\
& \left. \operatorname{RootSum} [b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1 - n, -\frac{\#1}{c + d x - \#1}] \left(\frac{c + d x}{c + d x - \#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] \right)
\end{aligned}$$

Problem 225: Result is not expressed in closed-form.

$$\int \frac{x^3 (c + d x)^{1+n}}{a + b x^4} dx$$

Optimal (type 5, 349 leaves, 10 steps):

$$\begin{aligned}
& \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c - \sqrt{-\sqrt{-a}} d\right) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + \sqrt{-\sqrt{-a}} d}]}{4 b^{3/4} \left(b^{1/4} c + \sqrt{-\sqrt{-a}} d\right) (2 + n)} - \\
& \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c - (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c - (-a)^{1/4} d\right) (2 + n)} - \frac{(c + d x)^{2+n} \operatorname{Hypergeometric2F1}[1, 2 + n, 3 + n, \frac{b^{1/4} (c + d x)}{b^{1/4} c + (-a)^{1/4} d}]}{4 b^{3/4} \left(b^{1/4} c + (-a)^{1/4} d\right) (2 + n)}
\end{aligned}$$

Result (type 7, 691 leaves):

$$\begin{aligned}
& \frac{1}{4 b^2 n (1+n)} (c + d x)^n \\
& \left((b c^4 + a d^4) (1+n) \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n}}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \right. \\
& b \left(-4 c n - 4 d n x + 3 c^3 (1+n) \right. \\
& \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] - \\
& 3 c^2 (1+n) \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^2}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \\
& c \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] + \\
& \left. c n \operatorname{RootSum}[b c^4 + a d^4 - 4 b c^3 \#1 + 6 b c^2 \#1^2 - 4 b c \#1^3 + b \#1^4 \&, \frac{\operatorname{Hypergeometric2F1}[-n, -n, 1-n, -\frac{\#1}{c+d x-\#1}] \left(\frac{c+d x}{c+d x-\#1}\right)^{-n} \#1^3}{c^3 - 3 c^2 \#1 + 3 c \#1^2 - \#1^3} \&] \right) \right)
\end{aligned}$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(c + d x + e x^2) \sqrt{a + b x^4}} dx$$

Optimal (type 4, 1605 leaves, 16 steps):

$$\begin{aligned}
& - \frac{e^2 \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{-b d^4 + 4 b c d^2 e - 2 b c^2 e^2 - 2 a e^4 - b d \sqrt{d^2 - 4 c e}} (d^2 - 2 c e) x}{e (d + \sqrt{d^2 - 4 c e}) \sqrt{a + b x^4}} \right]}{\sqrt{2} \sqrt{d^2 - 4 c e}} \sqrt{-2 a e^4 - b (d^4 - 4 c d^2 e + 2 c^2 e^2 + d^3 \sqrt{d^2 - 4 c e} - 2 c d e \sqrt{d^2 - 4 c e})} + \\
& \frac{e^2 \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{-b d^4 + 4 b c d^2 e - 2 b c^2 e^2 - 2 a e^4 + b d \sqrt{d^2 - 4 c e}} (d^2 - 2 c e) x}{e (d - \sqrt{d^2 - 4 c e}) \sqrt{a + b x^4}} \right]}{\sqrt{2} \sqrt{d^2 - 4 c e}} \sqrt{-2 a e^4 - b (d^4 - 4 c d^2 e + 2 c^2 e^2 - d^3 \sqrt{d^2 - 4 c e} + 2 c d e \sqrt{d^2 - 4 c e})}
\end{aligned}$$

$$\begin{aligned}
& \frac{e^2 \operatorname{ArcTanh} \left[\frac{4 a e^2 + b \left(d - \sqrt{d^2 - 4 c e} \right)^2 x^2}{2 \sqrt{2} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 - b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e) \sqrt{a+b x^4}}} \right]}{\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 - b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e)}} + \\
& \frac{e^2 \operatorname{ArcTanh} \left[\frac{4 a e^2 + b \left(d + \sqrt{d^2 - 4 c e} \right)^2 x^2}{2 \sqrt{2} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 + b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e) \sqrt{a+b x^4}}} \right]}{\sqrt{2} \sqrt{d^2 - 4 c e} \sqrt{b d^4 - 4 b c d^2 e + 2 b c^2 e^2 + 2 a e^4 + b d \sqrt{d^2 - 4 c e} (d^2 - 2 c e)}} + \\
& \frac{b^{1/4} e \left(d - \sqrt{d^2 - 4 c e} \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} \sqrt{d^2 - 4 c e} \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a+b x^4}} - \\
& \frac{b^{1/4} e \left(d + \sqrt{d^2 - 4 c e} \right) \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} \sqrt{d^2 - 4 c e} \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a+b x^4}} + \left(e \left(2 \sqrt{a} e^2 - \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \right. \\
& \left. \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticPi} \left[\frac{\left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right)^2}{4 \sqrt{a} \sqrt{b} e^2 \left(d - \sqrt{d^2 - 4 c e} \right)^2}, 2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \left(2 a^{1/4} b^{1/4} \sqrt{d^2 - 4 c e} \left(d - \sqrt{d^2 - 4 c e} \right) \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e - d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a+b x^4} \right) - \left(e \left(2 \sqrt{a} e^2 - \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \right. \\
& \left. \left(\sqrt{a} + \sqrt{b} x^2 \right) \sqrt{\frac{a+b x^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} \operatorname{EllipticPi} \left[\frac{\left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right)^2}{4 \sqrt{a} \sqrt{b} e^2 \left(d + \sqrt{d^2 - 4 c e} \right)^2}, 2 \operatorname{ArcTan} \left[\frac{b^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right] \right) / \\
& \left(2 a^{1/4} b^{1/4} \sqrt{d^2 - 4 c e} \left(d + \sqrt{d^2 - 4 c e} \right) \left(2 \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \sqrt{a+b x^4} \right)
\end{aligned}$$

Result (type 4, 653 leaves):

$$\begin{aligned}
& - \left(\left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \left(\frac{i}{2} \sqrt{a} + \sqrt{b} x^2 \right) \right) \right. \\
& \left. \left(b^{1/4} \left(-\sqrt{b} c + (-1)^{1/4} a^{1/4} b^{1/4} d - \frac{i}{2} \sqrt{a} e \right) \sqrt{d^2 - 4 c e} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] + \right. \right. \\
& \left. \left. (-1)^{1/4} a^{1/4} \left(- \left(-2 \frac{i}{2} \sqrt{a} e^2 + \sqrt{b} \left(d^2 - 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \operatorname{EllipticPi} \left[\frac{2 (-1)^{3/4} a^{1/4} e - \frac{i}{2} b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)}{2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(-d + \sqrt{d^2 - 4 c e} \right)}, \right. \right. \right. \\
& \left. \left. \left. \operatorname{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] - \left(2 \frac{i}{2} \sqrt{a} e^2 + \sqrt{b} \left(-d^2 + 2 c e + d \sqrt{d^2 - 4 c e} \right) \right) \right. \right. \\
& \left. \left. \operatorname{EllipticPi} \left[-\frac{\frac{i}{2} \left(2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right) \right)}{-2 (-1)^{1/4} a^{1/4} e + b^{1/4} \left(d + \sqrt{d^2 - 4 c e} \right)}, \operatorname{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} x \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} x}} \right], -1 \right] \right] \right) \right) \Bigg) \Bigg) \\
& \left(a^{1/4} \sqrt{d^2 - 4 c e} \left(b c^2 - a e^2 - \frac{i}{2} \sqrt{a} \sqrt{b} \left(d^2 - 2 c e \right) \right) \sqrt{\frac{\frac{i}{2} \sqrt{a} + \sqrt{b} x^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} x \right)^2} \sqrt{a + b x^4}} \right)
\end{aligned}$$

Problem 253: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x} dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$\frac{2}{3} \left(c \sqrt{a + b x^2} \right)^{3/2} + \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTan} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}} - \frac{\left(c \sqrt{a + b x^2} \right)^{3/2} \operatorname{ArcTanh} \left[\left(1 + \frac{b x^2}{a} \right)^{1/4} \right]}{\left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 5, 67 leaves):

$$\frac{2 c^2 \left(a + b x^2 - 3 a \left(1 + \frac{a}{b x^2} \right)^{1/4} \operatorname{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^2} \right] \right)}{3 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 254: Result unnecessarily involves higher level functions.

$$\int \frac{(c \sqrt{a + b x^2})^{3/2}}{x^3} dx$$

Optimal (type 3, 133 leaves, 7 steps):

$$-\frac{(c \sqrt{a + b x^2})^{3/2}}{2 x^2} + \frac{3 b (c \sqrt{a + b x^2})^{3/2} \operatorname{ArcTan}\left[\left(1 + \frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1 + \frac{b x^2}{a}\right)^{3/4}} - \frac{3 b (c \sqrt{a + b x^2})^{3/2} \operatorname{ArcTanh}\left[\left(1 + \frac{b x^2}{a}\right)^{1/4}\right]}{4 a \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 73 leaves):

$$-\frac{c^2 \left(a + b x^2 + 3 b \left(1 + \frac{a}{b x^2}\right)^{1/4} x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, -\frac{a}{b x^2}\right]\right)}{2 x^2 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 255: Result unnecessarily involves higher level functions.

$$\int x^2 (c \sqrt{a + b x^2})^{3/2} dx$$

Optimal (type 4, 152 leaves, 5 steps):

$$\frac{2 a x (c \sqrt{a + b x^2})^{3/2}}{15 b} + \frac{2}{9} x^3 (c \sqrt{a + b x^2})^{3/2} - \frac{4 a^2 x (c \sqrt{a + b x^2})^{3/2}}{15 b (a + b x^2)} + \frac{4 a^{3/2} (c \sqrt{a + b x^2})^{3/2} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a}}\right], 2\right]}{15 b^{3/2} \left(1 + \frac{b x^2}{a}\right)^{3/4}}$$

Result (type 5, 88 leaves):

$$\frac{2 c^2 \left(3 a^2 x + 8 a b x^3 + 5 b^2 x^5 - 3 a^2 x \left(1 + \frac{b x^2}{a}\right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{45 b \sqrt{c \sqrt{a + b x^2}}}$$

Problem 256: Result unnecessarily involves higher level functions.

$$\int (c \sqrt{a + b x^2})^{3/2} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$\frac{2}{5} x \left(c \sqrt{a + b x^2} \right)^{3/2} + \frac{6 a x \left(c \sqrt{a + b x^2} \right)^{3/2}}{5 (a + b x^2)} - \frac{6 \sqrt{a} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x}{\sqrt{a}} \right], 2 \right]}{5 \sqrt{b} \left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 5, 71 leaves) :

$$\frac{c^2 x \left(2 (a + b x^2) + 3 a \left(1 + \frac{b x^2}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a} \right] \right)}{5 \sqrt{c \sqrt{a + b x^2}}}$$

Problem 257: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x^2} dx$$

Optimal (type 4, 115 leaves, 4 steps) :

$$-\frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x} + \frac{3 b x \left(c \sqrt{a + b x^2} \right)^{3/2}}{a + b x^2} - \frac{3 \sqrt{b} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x}{\sqrt{a}} \right], 2 \right]}{\sqrt{a} \left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 5, 76 leaves) :

$$-\frac{c^2 \left(2 (a + b x^2) - 3 b x^2 \left(1 + \frac{b x^2}{a} \right)^{1/4} \text{Hypergeometric2F1} \left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a} \right] \right)}{2 x \sqrt{c \sqrt{a + b x^2}}}$$

Problem 258: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{x^4} dx$$

Optimal (type 4, 154 leaves, 5 steps) :

$$-\frac{\left(c \sqrt{a + b x^2} \right)^{3/2}}{3 x^3} - \frac{b \left(c \sqrt{a + b x^2} \right)^{3/2}}{2 a x} + \frac{b^2 x \left(c \sqrt{a + b x^2} \right)^{3/2}}{2 a (a + b x^2)} - \frac{b^{3/2} \left(c \sqrt{a + b x^2} \right)^{3/2} \text{EllipticE} \left[\frac{1}{2} \text{ArcTan} \left[\frac{\sqrt{b} x}{\sqrt{a}} \right], 2 \right]}{2 a^{3/2} \left(1 + \frac{b x^2}{a} \right)^{3/4}}$$

Result (type 5, 92 leaves) :

$$-\frac{c^2 \left(4 a^2+10 a b x^2+6 b^2 x^4-3 b^2 x^4 \left(1+\frac{b x^2}{a}\right)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{12 a x^3 \sqrt{c \sqrt{a+b x^2}}}$$

Problem 260: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{(1-x^2)(3+x^2)} \, dx$$

Optimal (type 4, 48 leaves, 6 steps):

$$\frac{1}{3} x \sqrt{3-2 x^2-x^4}-\frac{2 \text{EllipticE}[\text{ArcSin}[x],-\frac{1}{3}]}{\sqrt{3}}+\frac{4 \text{EllipticF}[\text{ArcSin}[x],-\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 59 leaves):

$$\frac{1}{3} \left(x \sqrt{3-2 x^2-x^4}-2 \text{i} \text{EllipticE}\left[\text{i} \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]-4 \text{i} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]\right)$$

Problem 262: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}} \, dx$$

Optimal (type 4, 12 leaves, 3 steps):

$$\frac{\text{EllipticF}[\text{ArcSin}[x],-\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 18 leaves):

$$-\text{i} \text{EllipticF}\left[\text{i} \text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]$$

Problem 266: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{x} \, dx$$

Optimal (type 3, 112 leaves, 5 steps):

$$-\frac{\sqrt{a} \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{\sqrt{c}} + \frac{\sqrt{b} \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{b} \sqrt{e}}\right]}{\sqrt{d}}$$

Result (type 6, 245 leaves):

$$\left(5 a (b c - a d) (a + b x^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] \right) / \\ \left(3 b x^2 \left(5 a (b c - a d) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] - \right. \right. \\ \left. \left. (a + b x^2) \left((-2 b c + 2 a d) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] + a d \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a}\right] \right) \right) \right)$$

Problem 267: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{x^3} dx$$

Optimal (type 3, 127 leaves, 4 steps):

$$\frac{(b c - a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{2 c \left(a - \frac{c (a+b x^2)}{c+d x^2}\right)} - \frac{(b c - a d) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{2 \sqrt{a} c^{3/2}}$$

Result (type 6, 190 leaves):

$$\frac{1}{2 c x^2} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(-c - d x^2 + \left(2 b d (b c - a d) x^4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left((a + b x^2) \right. \right. \\ \left. \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 268: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{x^5} dx$$

Optimal (type 3, 208 leaves, 5 steps):

$$-\frac{(b c - a d)^2 \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{4 c^2 \left(a - \frac{c(a+b x^2)}{c+d x^2}\right)^2} + \frac{(b c - 5 a d) (b c - a d) \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{8 a c^2 \left(a - \frac{c(a+b x^2)}{c+d x^2}\right)} + \frac{(b c - a d) (b c + 3 a d) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{8 a^{3/2} c^{5/2}}$$

Result (type 6, 224 leaves):

$$-\frac{1}{8 a c^2 x^4} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \left((c+d x^2) (2 a c + b c x^2 - 3 a d x^2) + \left(2 b d (b^2 c^2 + 2 a b c d - 3 a^2 d^2) x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) \Big/ \left((a+b x^2) \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 269: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{x^7} dx$$

Optimal (type 3, 318 leaves, 6 steps):

$$\begin{aligned} & \frac{(b c - a d)^2 (b c + 3 a d) \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{8 a c^3 \left(a - \frac{c(a+b x^2)}{c+d x^2}\right)^2} - \frac{(b c - a d) (b^2 c^2 + 2 a b c d - 11 a^2 d^2) \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{16 a^2 c^3 \left(a - \frac{c(a+b x^2)}{c+d x^2}\right)} + \\ & \frac{(b c - a d)^3 e^2 \left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}}{6 a c^2 \left(a e - \frac{c e(a+b x^2)}{c+d x^2}\right)^3} - \frac{(b c - a d) (b^2 c^2 + 2 a b c d + 5 a^2 d^2) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{16 a^{5/2} c^{7/2}} \end{aligned}$$

Result (type 6, 272 leaves):

$$\frac{1}{48 a^2 c^3 x^6} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((c + d x^2) (3 b^2 c^2 x^4 - 2 a b c x^2 (c - 2 d x^2) + a^2 (-8 c^2 + 10 c d x^2 - 15 d^2 x^4)) + \right. \\ \left(6 b d (b^3 c^3 + a b^2 c^2 d + 3 a^2 b c d^2 - 5 a^3 d^3) x^8 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \left((a + b x^2) \right. \\ \left. \left(-4 b d x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 270: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} dx$$

Optimal (type 4, 357 leaves, 7 steps):

$$\frac{(8 b^2 c^2 - 3 a b c d - 2 a^2 d^2) x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{15 b^2 d^2} - \frac{(4 b c - a d) x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{15 b d^2} + \frac{x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d} - \\ \frac{\sqrt{c} (8 b^2 c^2 - 3 a b c d - 2 a^2 d^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 b^2 d^{5/2}} + \frac{c^{3/2} (4 b c - a d) \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{15 b d^{5/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}$$

Result (type 4, 255 leaves):

$$\frac{1}{15 b \sqrt{\frac{b}{a}} d^3 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \\ \left(\sqrt{\frac{b}{a}} d x (a + b x^2) (c + d x^2) (-4 b c + a d + 3 b d x^2) + \pm c (-8 b^2 c^2 + 3 a b c d + 2 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] - \right. \\ \left. \pm c (-8 b^2 c^2 + 7 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right)$$

Problem 271: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$$

Optimal (type 4, 266 leaves, 6 steps):

$$\begin{aligned} & \frac{(2bc - ad)x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd} + \frac{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{3d} + \\ & \frac{\sqrt{c} (2bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}]}{3b d^{3/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{c^{3/2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], 1 - \frac{bc}{ad}]}{3d^{3/2} \sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \end{aligned}$$

Result (type 4, 208 leaves):

$$\begin{aligned} & \frac{1}{3 \sqrt{\frac{b}{a}} d^2 (a+bx^2)} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{\frac{b}{a}} dx (a+bx^2) (c+dx^2) - i c (-2bc + ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticE}[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}] + \right. \\ & \left. 2 i c (-bc + ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \operatorname{EllipticF}[i \operatorname{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{ad}{bc}] \right) \end{aligned}$$

Problem 274: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\begin{aligned}
& \frac{d (b c - 2 a d) x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{3 a c^2} - \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{3 c x^3} - \frac{(b c - 2 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{3 a c^2 x} - \\
& \frac{\sqrt{d} (b c - 2 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a c^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \frac{b \sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a \sqrt{c} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 238 leaves):

$$\begin{aligned}
& -\frac{1}{3 b c^2 x^3 (a + b x^2)} \sqrt{\frac{b}{a}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \\
& \left(\sqrt{\frac{b}{a}} (a + b x^2) (c + d x^2) (b c x^2 + a (c - 2 d x^2)) - i b c (-b c + 2 a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] + \right. \\
& \left. i b c (-b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[i \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] \right)
\end{aligned}$$

Problem 275: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{x^6} dx$$

Optimal (type 4, 424 leaves, 8 steps):

$$\begin{aligned}
& -\frac{d (2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{15 a^2 c^3} - \frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{5 c x^5} \\
& + \frac{(b c - 4 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{15 a c^2 x^3} + \frac{(2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{15 a^2 c^3 x} \\
& + \frac{\sqrt{d} (2 b^2 c^2 + 3 a b c d - 8 a^2 d^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{15 a^2 c^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \frac{b \sqrt{d} (b c - 4 a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{15 a^2 c^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 302 leaves):

$$\begin{aligned}
& -\frac{1}{15 a^2 \sqrt{\frac{b}{a} c^3 x^5 (a + b x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} (a + b x^2) (c + d x^2) (-2 b^2 c^2 x^4 + a b c x^2 (c - 3 d x^2) + a^2 (3 c^2 - 4 c d x^2 + 8 d^2 x^4)) + \right. \\
& \left. \pm b c (-2 b^2 c^2 - 3 a b c d + 8 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] - \right. \\
& \left. 2 \pm b c (-b^2 c^2 - a b c d + 2 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] \right)
\end{aligned}$$

Problem 279: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x} dx$$

Optimal (type 3, 151 leaves, 6 steps):

$$\begin{aligned}
& -\frac{(b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d} - \frac{a^{3/2} e^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{c^{3/2}} + \frac{b^{3/2} e^{3/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{b} \sqrt{e}}\right]}{d^{3/2}}
\end{aligned}$$

Result (type 6, 330 leaves):

$$\begin{aligned}
& \frac{1}{c d (a + b x^2)} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((-b c + a d) (a + b x^2) + \left(2 a^2 b d^2 x^2 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right) \right) \Big/ \left(-4 b d x^2 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + \right. \\
& \left. b c \text{AppellF1}[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + a d \text{AppellF1}[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right) - \left(2 a b^2 c^2 x^2 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}] \right) \Big/ \\
& \left(-4 a c \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}] + x^2 \left(a d \text{AppellF1}[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}] + b c \text{AppellF1}[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}] \right) \right)
\end{aligned}$$

Problem 280: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x^3} dx$$

Optimal (type 3, 165 leaves, 5 steps):

$$\frac{3 (b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{2 c^2} + \frac{(b c - a d) \left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{2 c \left(a - \frac{c (a+b x^2)}{c+d x^2}\right)} - \frac{3 \sqrt{a} (b c - a d) e^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{2 c^{5/2}}$$

Result (type 6, 210 leaves):

$$\begin{aligned}
& \frac{1}{2 c^2 x^2 (a + b x^2)} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(- (a + b x^2) (-2 b c x^2 + a (c + 3 d x^2)) + \left(6 a b d (b c - a d) x^4 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right) \right) \Big/ \\
& \left(-4 b d x^2 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + b c \text{AppellF1}[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + a d \text{AppellF1}[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right)
\end{aligned}$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x^5} dx$$

Optimal (type 3, 256 leaves, 6 steps):

$$\begin{aligned}
& - \frac{d (b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c^3} - \frac{a (b c - a d)^2 e^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{4 c^3 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)^2} + \\
& \frac{(5 b c - 9 a d) (b c - a d) e^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{8 c^3 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)} - \frac{3 (b c - 5 a d) (b c - a d) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{8 \sqrt{a} c^{7/2}}
\end{aligned}$$

Result (type 6, 245 leaves):

$$\begin{aligned}
& \frac{1}{8 c^3 x^4 (a + b x^2)} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \\
& \left((a + b x^2) (-b c x^2 (5 c + 13 d x^2) + a (-2 c^2 + 5 c d x^2 + 15 d^2 x^4)) + \left(6 b d (b^2 c^2 - 6 a b c d + 5 a^2 d^2) x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right. \\
& \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)
\end{aligned}$$

Problem 282: Result unnecessarily involves higher level functions.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x^7} dx$$

Optimal (type 3, 366 leaves, 7 steps):

$$\begin{aligned}
& \frac{d^2 (b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c^4} + \frac{(b c - a d)^3 e^2 \left(\frac{e (a+b x^2)}{c+d x^2}\right)^{5/2}}{6 a c^2 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)^3} + \frac{(b c - a d)^2 (b c + 11 a d) e^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{24 c^4 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)^2} - \\
& \frac{(b c - a d) (5 b^2 c^2 + 50 a b c d - 79 a^2 d^2) e^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{48 a c^4 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)} + \frac{(b c - a d) (b^2 c^2 + 10 a b c d - 35 a^2 d^2) e^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{16 a^{3/2} c^{9/2}}
\end{aligned}$$

Result (type 6, 305 leaves):

$$\begin{aligned}
& -\frac{1}{48 a c^4 x^6 (a + b x^2)} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((a + b x^2) (3 b^2 c^2 x^4 (c + d x^2) + 2 a b c x^2 (7 c^2 - 19 c d x^2 - 50 d^2 x^4) + a^2 (8 c^3 - 14 c^2 d x^2 + 35 c d^2 x^4 + 105 d^3 x^6)) + \right. \\
& \left. \left(6 b d (b^3 c^3 + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3) x^8 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right) \right/ \\
& \left(-4 b d x^2 \text{AppellF1}[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + b c \text{AppellF1}[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] + a d \text{AppellF1}[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}] \right)
\end{aligned}$$

Problem 283: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 391 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(16 a c - \frac{16 b c^2}{d} - \frac{a^2 d}{b} \right) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{5 d^2} - \frac{e x^3 (a + b x^2) \sqrt{\frac{e (a + b x^2)}{c + d x^2}}}{d} - \frac{(8 b c - 7 a d) e x \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d^3} + \\
& \frac{6 b e x^3 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}{5 d^2} - \frac{\sqrt{c} (16 b^2 c^2 - 16 a b c d + a^2 d^2) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticE}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}], 1 - \frac{b c}{a d}]}{5 b d^{7/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}} + \\
& \frac{c^{3/2} (8 b c - 7 a d) e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \text{EllipticF}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}], 1 - \frac{b c}{a d}]}{5 d^{7/2} \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 290 leaves):

$$\begin{aligned}
& \frac{1}{5 \sqrt{\frac{b}{a}} d^4 (a + b x^2)} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \\
& \left(\sqrt{\frac{b}{a}} d x (a^2 d (7 c + 2 d x^2) + b^2 x^2 (-8 c^2 - 2 c d x^2 + d^2 x^4) + a b (-8 c^2 + 5 c d x^2 + 3 d^2 x^4)) - \frac{1}{2} c (16 b^2 c^2 - 16 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right. \\
& \left. \text{EllipticE} \left[\frac{1}{2} \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] + 8 \frac{1}{2} c (2 b^2 c^2 - 3 a b c d + a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF} \left[\frac{1}{2} \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] \right)
\end{aligned}$$

Problem 284: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 310 leaves, 7 steps):

$$\begin{aligned}
& - \frac{(8 b c - 7 a d) e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{3 d^2} - \frac{e x (a + b x^2) \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{d} + \frac{4 b e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{3 d^2} + \\
& \frac{\sqrt{c} (8 b c - 7 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE} \left[\text{ArcTan} \left[\frac{\sqrt{d} x}{\sqrt{c}} \right], 1 - \frac{b c}{a d} \right]}{3 d^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \frac{\sqrt{c} (4 b c - 3 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF} \left[\text{ArcTan} \left[\frac{\sqrt{d} x}{\sqrt{c}} \right], 1 - \frac{b c}{a d} \right]}{3 d^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 235 leaves):

$$\begin{aligned}
& -\frac{1}{3 \sqrt[3]{\frac{b}{a} d^3 (a + b x^2)}} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} d x (a + b x^2) (3 a d - b (4 c + d x^2)) + i b c (-8 b c + 7 a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[i \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{a d}{b c}] + \right. \\
& \left. i (8 b^2 c^2 - 11 a b c d + 3 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[i \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{a d}{b c}] \right)
\end{aligned}$$

Problem 285: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 262 leaves, 6 steps):

$$\begin{aligned}
& -\frac{(b c - a d) e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d} + \frac{(2 b c - a d) e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d} - \\
& \frac{(2 b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, 1 - \frac{b c}{a d}]]}{\sqrt{c} d^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} + \frac{b \sqrt{c} e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, 1 - \frac{b c}{a d}]]}{d^{3/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 206 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{\frac{b}{a} c d^2 (a + b x^2)}} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(i b c (-2 b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[i \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{a d}{b c}] + \right. \\
& \left. (-b c + a d) \left(\sqrt{\frac{b}{a}} d x (a + b x^2) - 2 i b c \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[i \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{a d}{b c}] \right) \right)
\end{aligned}$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 307 leaves, 7 steps):

$$\begin{aligned} & \frac{(b c - a d) e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{c d x} - \frac{(b c - 2 a d) e x \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{c^2} + \frac{(b c - 2 a d) e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c + d x^2)}{c^2 d x} + \\ & \frac{(b c - 2 a d) e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{c^{3/2} \sqrt{d} \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}}} + \frac{b e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{\sqrt{c} \sqrt{d} \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}}} \end{aligned}$$

Result (type 4, 228 leaves):

$$\begin{aligned} & -\frac{1}{\sqrt{\frac{b}{a} c^2 d x (a + b x^2)}} \\ & e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \left(\sqrt{\frac{b}{a}} d (a + b x^2) (a c - b c x^2 + 2 a d x^2) + \pm b c (-b c + 2 a d) x \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] - \right. \\ & \left. \pm b c (-b c + a d) x \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] \right) \end{aligned}$$

Problem 287: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 383 leaves, 8 steps):

$$\begin{aligned}
& -\frac{(b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d x^3} + \frac{d (7 b c - 8 a d) e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{3 c^3} + \frac{(3 b c - 4 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{3 c^2 d x^3} - \frac{(7 b c - 8 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{3 c^3 x} - \\
& \frac{\sqrt{d} (7 b c - 8 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 c^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} + \frac{b (3 b c - 4 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a c^{3/2} \sqrt{d} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 275 leaves):

$$\begin{aligned}
& \frac{1}{3 \sqrt{\frac{b}{a}} c^3 x^3 (a + b x^2)} \\
& e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(-\sqrt{\frac{b}{a}} (b^2 c x^4 (4 c + 7 d x^2) + a^2 (c^2 - 4 c d x^2 - 8 d^2 x^4) + a b x^2 (5 c^2 + 3 c d x^2 - 8 d^2 x^4)) + \frac{b c}{a} (-7 b c + 8 a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \right. \\
& \left. \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[\frac{b}{a} x, \frac{a d}{b c}\right] - 4 \frac{b c}{a} (-b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[\frac{b}{a} x, \frac{a d}{b c}\right] \right)
\end{aligned}$$

Problem 288: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}}{x^6} dx$$

Optimal (type 4, 480 leaves, 9 steps):

$$\begin{aligned}
& -\frac{(b c - a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{c d x^5} + \frac{d (b^2 c^2 - 16 a b c d + 16 a^2 d^2) e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{5 a c^4} + \frac{(5 b c - 6 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{5 c^2 d x^5} - \\
& \frac{(7 b c - 8 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{5 c^3 x^3} - \frac{(b^2 c^2 - 16 a b c d + 16 a^2 d^2) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}{5 a c^4 x} - \\
& \frac{\sqrt{d} (b^2 c^2 - 16 a b c d + 16 a^2 d^2) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{5 a c^{7/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}} - \\
& \frac{b \sqrt{d} (7 b c - 8 a d) e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{5 a c^{5/2} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 357 leaves):

$$\begin{aligned}
& -\frac{1}{5 b c^4 x^5 (a + b x^2)} \sqrt{\frac{b}{a}} e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} \right. \\
& \left. (b^3 c^2 x^6 (c + d x^2) + a b^2 c x^4 (3 c^2 - 8 c d x^2 - 16 d^2 x^4) + a^2 b x^2 (3 c^3 - 11 c^2 d x^2 - 8 c d^2 x^4 + 16 d^3 x^6) + a^3 (c^3 - 2 c^2 d x^2 + 8 c d^2 x^4 + 16 d^3 x^6) \right) + \\
& \pm b c (b^2 c^2 - 16 a b c d + 16 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] - \\
& \pm b c (b^2 c^2 - 9 a b c d + 8 a^2 d^2) x^5 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}]
\end{aligned}$$

Problem 299: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} dx$$

Optimal (type 3, 112 leaves, 5 steps):

$$-\frac{\sqrt{c} \operatorname{ArcTanh} \left[\frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{\sqrt{a} \sqrt{e}} + \frac{\sqrt{d} \operatorname{ArcTanh} \left[\frac{\sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{b} \sqrt{e}} \right]}{\sqrt{b} \sqrt{e}}$$

Result (type 6, 244 leaves):

$$\begin{aligned} & \left(3 a (b c - a d) (a + b x^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a} \right] \right) / \\ & \left(b x^2 \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(3 a (b c - a d) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a} \right] - \right. \right. \\ & \left. \left. (a + b x^2) \left((-2 b c + 2 a d) \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a} \right] - a d \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{d (a + b x^2)}{-b c + a d}, 1 + \frac{b x^2}{a} \right] \right) \right) \right) \end{aligned}$$

Problem 300: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} dx$$

Optimal (type 3, 130 leaves, 4 steps):

$$\begin{aligned} & \frac{(b c - a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{2 a \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)} + \frac{(b c - a d) \operatorname{ArcTanh} \left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{2 a^{3/2} \sqrt{c} \sqrt{e}} \end{aligned}$$

Result (type 6, 190 leaves):

$$\begin{aligned} & -a - b x^2 + \frac{2 b d (-b c + a d) x^4 \operatorname{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right]}{(c+d x^2) \left(-4 b d x^2 \operatorname{AppellF1} \left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] + b c \operatorname{AppellF1} \left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] + a d \operatorname{AppellF1} \left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2} \right] \right)} \\ & 2 a x^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \end{aligned}$$

Problem 301: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} dx$$

Optimal (type 3, 218 leaves, 5 steps):

$$-\frac{\left(b c-a d\right)^2 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{4 a c \left(a e-\frac{c e(a+b x^2)}{c+d x^2}\right)^2}-\frac{\left(b c-a d\right) \left(3 b c+a d\right) \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{8 a^2 c \left(a e-\frac{c e(a+b x^2)}{c+d x^2}\right)}-\frac{\left(b c-a d\right) \left(3 b c+a d\right) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{8 a^{5/2} c^{3/2} \sqrt{e}}$$

Result (type 6, 224 leaves):

$$-\frac{1}{8 a^2 c x^4 \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} \left(\left(a+b x^2\right) \left(2 a c-3 b c x^2+a d x^2\right)+\left(2 b d \left(-3 b^2 c^2+2 a b c d+a^2 d^2\right) x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right]\right) \right) \Big/ \left(\left(c+d x^2\right) \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right]+b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right]+a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right]\right)\right)$$

Problem 302: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{\frac{e(a+b x^2)}{c+d x^2}}} dx$$

Optimal (type 4, 403 leaves, 7 steps):

$$\begin{aligned} & \frac{\left(b c-4 a d\right) x \left(a+b x^2\right)}{15 b^2 d \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}+\frac{x^3 \left(a+b x^2\right)}{5 b \sqrt{\frac{e(a+b x^2)}{c+d x^2}}}-\frac{\left(2 b^2 c^2+3 a b c d-8 a^2 d^2\right) x \left(a+b x^2\right)}{15 b^3 d \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \left(c+d x^2\right)}+ \\ & \frac{\sqrt{c} \left(2 b^2 c^2+3 a b c d-8 a^2 d^2\right) \left(a+b x^2\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{15 b^3 d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{\frac{e \left(a+b x^2\right)}{c+d x^2}} \left(c+d x^2\right)}-\frac{c^{3/2} \left(b c-4 a d\right) \left(a+b x^2\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1-\frac{b c}{a d}\right]}{15 b^2 d^{3/2} \sqrt{\frac{c \left(a+b x^2\right)}{a \left(c+d x^2\right)}} \sqrt{\frac{e \left(a+b x^2\right)}{c+d x^2}} \left(c+d x^2\right)} \end{aligned}$$

Result (type 4, 258 leaves):

$$\begin{aligned}
& -\sqrt{\frac{b}{a}} dx (a + bx^2) (c + dx^2) (4ad - b(c + 3dx^2)) - \text{i} c (-2b^2 c^2 - 3abc d + 8a^2 d^2) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}[\text{i} \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{ad}{bc}] + \\
& 2 \text{i} c (-b^2 c^2 - abc d + 2a^2 d^2) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}[\text{i} \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{ad}{bc}] \Bigg/ \left(15a^2 \left(\frac{b}{a}\right)^{5/2} d^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2) \right)
\end{aligned}$$

Problem 303: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$\begin{aligned}
& \frac{x(a+bx^2)}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \\
& \frac{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{\sqrt{c}(bc-2ad)(a+bx^2) \text{EllipticE}[\text{ArcTan}[\frac{\sqrt{d}x}{\sqrt{c}}], 1 - \frac{bc}{ad}]} - \frac{c^{3/2}(a+bx^2)\text{EllipticF}[\text{ArcTan}[\frac{\sqrt{d}x}{\sqrt{c}}], 1 - \frac{bc}{ad}]}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}
\end{aligned}$$

Result (type 4, 212 leaves):

$$\begin{aligned}
& \sqrt{\frac{b}{a}} dx (a + bx^2) (c + dx^2) + \text{i} c (-bc + 2ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticE}[\text{i} \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{ad}{bc}] - \\
& \text{i} c (-bc + ad) \sqrt{1 + \frac{bx^2}{a}} \sqrt{1 + \frac{dx^2}{c}} \text{EllipticF}[\text{i} \text{ArcSinh}[\sqrt{\frac{b}{a}} x], \frac{ad}{bc}] \Bigg/ \left(3b\sqrt{\frac{b}{a}} d \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2) \right)
\end{aligned}$$

Problem 306: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal (type 4, 372 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{a + b x^2}{3 a x^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{(2 b c - a d) (a + b x^2)}{3 a^2 c x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{d (2 b c - a d) x (a + b x^2)}{3 a^2 c \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} + \\
 & \frac{\sqrt{d} (2 b c - a d) (a + b x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a^2 \sqrt{c} \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} - \frac{b \sqrt{c} \sqrt{d} (a + b x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a^2 \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}
 \end{aligned}$$

Result (type 4, 238 leaves):

$$\begin{aligned}
 & \left(-\sqrt{\frac{b}{a}} (a + b x^2) (c + d x^2) (-2 b c x^2 + a (c + d x^2)) - \frac{b c}{a} (-2 b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[\frac{b}{a} x, \frac{a d}{b c}\right] + \right. \\
 & \left. 2 \frac{b c}{a} (-b c + a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[\frac{b}{a} x, \frac{a d}{b c}\right] \right) / \left(3 a^2 \sqrt{\frac{b}{a}} c x^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2) \right)
 \end{aligned}$$

Problem 310: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(\frac{e (a+b x^2)}{c+d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 152 leaves, 6 steps):

$$\begin{aligned}
 & \frac{b c - a d}{a b e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{c^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{a^{3/2} e^{3/2}} + \frac{d^{3/2} \text{ArcTanh}\left[\frac{\sqrt{d} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{b} \sqrt{e}}\right]}{b^{3/2} e^{3/2}}
 \end{aligned}$$

Result (type 6, 332 leaves):

$$\begin{aligned}
& \frac{1}{a b e^2 (a + b x^2)} \\
& \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((b c - a d) (c + d x^2) + \left(2 b^2 c^2 d x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) \Big/ \left(-4 b d x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + \right. \\
& \left. b c \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) - \left(2 a^2 c d^2 x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) \Big/ \\
& \left(-4 a c \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + x^2 \left(a d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] + b c \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{b x^2}{a}, -\frac{d x^2}{c}\right] \right) \right)
\end{aligned}$$

Problem 311: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(\frac{e (a+b x^2)}{c+d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 170 leaves, 5 steps):

$$\begin{aligned}
& -\frac{3 (b c - a d)}{2 a^2 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{b c - a d}{2 a \sqrt{\frac{e (a+b x^2)}{c+d x^2}} \left(a e - \frac{c e (a+b x^2)}{c+d x^2} \right)} + \frac{3 \sqrt{c} (b c - a d) \text{ArcTanh} \left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}} \right]}{2 a^{5/2} e^{3/2}}
\end{aligned}$$

Result (type 6, 212 leaves):

$$\begin{aligned}
& \frac{1}{2 a^2 e^2 x^2 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(- (c + d x^2) (3 b c x^2 + a (c - 2 d x^2)) + \left(6 b c d (-b c + a d) x^4 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right) \Big/ \\
& \left(-4 b d x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right)
\end{aligned}$$

Problem 312: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(\frac{e (a+b x^2)}{c+d x^2} \right)^{3/2}} dx$$

Optimal (type 3, 255 leaves, 6 steps):

$$\frac{b (b c - a d)}{a^3 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{(b c - a d)^2 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{4 a^2 \left(a e - \frac{c e (a+b x^2)}{c+d x^2}\right)^2} - \frac{(7 b c - 3 a d) (b c - a d) \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{8 a^3 \left(a e^2 - \frac{c e^2 (a+b x^2)}{c+d x^2}\right)} - \frac{3 (b c - a d) (5 b c - a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{e (a+b x^2)}{c+d x^2}}}{\sqrt{a} \sqrt{e}}\right]}{8 a^{7/2} \sqrt{c} e^{3/2}}$$

Result (type 6, 247 leaves):

$$\frac{1}{8 a^3 e^2 x^4 (a + b x^2)} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left((c + d x^2) (15 b^2 c x^4 + a b x^2 (5 c - 13 d x^2) - a^2 (2 c + 5 d x^2)) + \left(6 b d (5 b^2 c^2 - 6 a b c d + a^2 d^2) x^6 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) / \right. \\ \left. \left(-4 b d x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + b c \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] + a d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{a}{b x^2}, -\frac{c}{d x^2}\right] \right) \right)$$

Problem 313: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 453 leaves, 8 steps):

$$\frac{(7 b c - 8 a d) x (a + b x^2)}{5 b^3 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{6 d x^3 (a + b x^2)}{5 b^2 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{(b^2 c^2 - 16 a b c d + 16 a^2 d^2) x (a + b x^2)}{5 b^4 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} - \frac{x^3 (c + d x^2)}{b e \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \\ \frac{\sqrt{c} (b^2 c^2 - 16 a b c d + 16 a^2 d^2) (a + b x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 b^4 \sqrt{d} e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} - \frac{c^{3/2} (7 b c - 8 a d) (a + b x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}\right]}{5 b^3 \sqrt{d} e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}$$

Result (type 4, 271 leaves):

$$\begin{aligned}
& \frac{1}{5 b^3 \sqrt{\frac{b}{a}} d e^2 (a + b x^2)} \\
& \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} d x (c + d x^2) (-8 a^2 d + a b (7 c - 2 d x^2) + b^2 x^2 (2 c + d x^2)) - i c (b^2 c^2 - 16 a b c d + 16 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right. \\
& \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] + i c (b^2 c^2 - 9 a b c d + 8 a^2 d^2) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] \right)
\end{aligned}$$

Problem 314: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(\frac{e (a + b x^2)}{c + d x^2} \right)^{3/2}} dx$$

Optimal (type 4, 378 leaves, 7 steps):

$$\begin{aligned}
& \frac{4 d x (a + b x^2)}{3 b^2 e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} + \frac{d (7 b c - 8 a d) x (a + b x^2)}{3 b^3 e \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} - \frac{x (c + d x^2)}{b e \sqrt{\frac{e (a + b x^2)}{c + d x^2}}} - \\
& \frac{\sqrt{c} \sqrt{d} (7 b c - 8 a d) (a + b x^2) \text{EllipticE} \left[\text{ArcTan} \left[\frac{\sqrt{d} x}{\sqrt{c}} \right], 1 - \frac{b c}{a d} \right]}{3 b^3 e \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)} + \frac{c^{3/2} (3 b c - 4 a d) (a + b x^2) \text{EllipticF} \left[\text{ArcTan} \left[\frac{\sqrt{d} x}{\sqrt{c}} \right], 1 - \frac{b c}{a d} \right]}{3 a b^2 \sqrt{d} e \sqrt{\frac{c (a + b x^2)}{a (c + d x^2)}} \sqrt{\frac{e (a + b x^2)}{c + d x^2}} (c + d x^2)}
\end{aligned}$$

Result (type 4, 219 leaves):

$$\begin{aligned}
& \frac{1}{3 a^2 \left(\frac{b}{a} \right)^{5/2} e^2 (a + b x^2)} \\
& \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(\sqrt{\frac{b}{a}} x (c + d x^2) (-3 b c + 4 a d + b d x^2) + i c (-7 b c + 8 a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] - \right. \\
& \left. 4 i c (-b c + a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] \right)
\end{aligned}$$

Problem 315: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 327 leaves, 6 steps):

$$\begin{aligned} & \frac{(b c - a d) x}{a b e \sqrt{\frac{e(a+b x^2)}{c+d x^2}}} - \frac{d(b c - 2 a d) x (a + b x^2)}{a b^2 e \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c + d x^2)} + \\ & \frac{\sqrt{c} \sqrt{d} (b c - 2 a d) (a + b x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{a b^2 e \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c + d x^2)} + \\ & \frac{c^{3/2} \sqrt{d} (a + b x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{a b e \sqrt{\frac{c(a+b x^2)}{a(c+d x^2)}} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} (c + d x^2)} \end{aligned}$$

Result (type 4, 203 leaves):

$$\begin{aligned} & \frac{1}{a^2 \left(\frac{b}{a}\right)^{3/2} e^2 (a + b x^2)} \sqrt{\frac{e(a+b x^2)}{c+d x^2}} \left(-\frac{1}{i} c (-b c + 2 a d) \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] + \right. \\ & \left. (b c - a d) \left(\sqrt{\frac{b}{a}} x (c + d x^2) - \frac{1}{i} c \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}\right] \right) \right) \end{aligned}$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \left(\frac{e(a+b x^2)}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 380 leaves, 7 steps):

$$\begin{aligned}
& \frac{b c - a d}{a b e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{(2 b c - a d) (a + b x^2)}{a^2 b e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{d (2 b c - a d) x (a + b x^2)}{a^2 b e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} - \\
& \frac{\sqrt{c} \sqrt{d} (2 b c - a d) (a + b x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{a^2 b e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} + \frac{c^{3/2} \sqrt{d} (a + b x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{a^2 e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}
\end{aligned}$$

Result (type 4, 223 leaves):

$$\begin{aligned}
& \frac{1}{a^2 \sqrt{\frac{b}{a}} e^2 x (a + b x^2)} \\
& \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(- \sqrt{\frac{b}{a}} (c + d x^2) (a c + 2 b c x^2 - a d x^2) + \text{i} c (-2 b c + a d) x \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\text{i} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] - \right. \\
& \left. 2 \text{i} c (-b c + a d) x \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\text{i} \text{ArcSinh}\left[\sqrt{\frac{b}{a}} x\right], \frac{a d}{b c}] \right)
\end{aligned}$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(\frac{e (a+b x^2)}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 444 leaves, 8 steps):

$$\begin{aligned}
& \frac{b c - a d}{a b e x^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{(4 b c - 3 a d) (a + b x^2)}{3 a^2 b e x^3 \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} + \frac{(8 b c - 7 a d) (a + b x^2)}{3 a^3 e x \sqrt{\frac{e (a+b x^2)}{c+d x^2}}} - \frac{d (8 b c - 7 a d) x (a + b x^2)}{3 a^3 e \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} + \\
& \frac{\sqrt{c} \sqrt{d} (8 b c - 7 a d) (a + b x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a^3 e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)} - \frac{\sqrt{c} \sqrt{d} (4 b c - 3 a d) (a + b x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], 1 - \frac{b c}{a d}]}{3 a^3 e \sqrt{\frac{c (a+b x^2)}{a (c+d x^2)}} \sqrt{\frac{e (a+b x^2)}{c+d x^2}} (c + d x^2)}
\end{aligned}$$

Result (type 4, 266 leaves):

$$\begin{aligned}
& \frac{1}{3 a^3 \sqrt{\frac{b}{a}} e^2 x^3 (a + b x^2)} \\
& \sqrt{\frac{e (a + b x^2)}{c + d x^2}} \left(-\sqrt{\frac{b}{a}} (c + d x^2) (-8 b^2 c x^4 + a^2 (c + 4 d x^2) + a b (-4 c x^2 + 7 d x^4)) - \frac{1}{2} b c (-8 b c + 7 a d) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \right. \\
& \left. \text{EllipticE} \left[\frac{1}{2} \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] - \frac{1}{2} (8 b^2 c^2 - 11 a b c d + 3 a^2 d^2) x^3 \sqrt{1 + \frac{b x^2}{a}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF} \left[\frac{1}{2} \text{ArcSinh} \left[\sqrt{\frac{b}{a}} x \right], \frac{a d}{b c} \right] \right)
\end{aligned}$$

Problem 321: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+d x^2}}}{x} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\sqrt{a} \text{ArcTanh} \left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}} \right] - \frac{\sqrt{b+a c} \text{ArcTanh} \left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}} \right]}{\sqrt{c}}$$

Result (type 3, 210 leaves):

$$\begin{aligned}
& \left(\sqrt{c + d x^2} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left((b + a c) \text{Log}[x^2] + 2 \sqrt{a} \sqrt{c} \sqrt{b + a c} \text{Log}[a \sqrt{c + d x^2} + \sqrt{a} \sqrt{b + a c + a d x^2}] \right. \right. - \\
& \left. \left. (b + a c) \text{Log}[2 a c (c + d x^2) + b (2 c + d x^2) + 2 \sqrt{c} \sqrt{b + a c} \sqrt{c + d x^2} \sqrt{b + a c + a d x^2}] \right) \right) \Big/ \left(2 \sqrt{c} \sqrt{b + a c} \sqrt{b + a (c + d x^2)} \right)
\end{aligned}$$

Problem 322: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + \frac{b}{c+d x^2}}}{x^3} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$-\frac{(c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{2 c x^2} + \frac{b d \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{2 c^{3/2} \sqrt{b+a c}}$$

Result (type 3, 212 leaves):

$$\left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(-2 \sqrt{c(b+a c)} (c+d x^2) (b+a c+a d x^2) - 2 b d x^2 \sqrt{(c+d x^2) (b+a (c+d x^2))} \operatorname{Log}[x] + b d x^2 \sqrt{(c+d x^2) (b+a c+a d x^2)} \right. \right. \\ \left. \left. \operatorname{Log}[2 a c (c+d x^2) + b (2 c+d x^2) + 2 \sqrt{c(b+a c)} \sqrt{(c+d x^2) (b+a c+a d x^2)}] \right) \right) / \left(4 c \sqrt{c(b+a c)} x^2 (b+a (c+d x^2)) \right)$$

Problem 325: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{a + \frac{b}{c+d x^2}} dx$$

Optimal (type 4, 368 leaves, 8 steps):

$$-\frac{(2 b^2 + 7 a b c - 3 a^2 c^2) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a^2 d^2} + \frac{(b - 3 a c) x (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 a d^2} + \frac{x^3 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d} + \\ \frac{\sqrt{c} (2 b^2 + 7 a b c - 3 a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{15 a^2 d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} - \frac{c^{3/2} (b - 3 a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{15 a d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}$$

Result (type 4, 293 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} \times (c + d x^2) (b^2 - 2 a b (c - 2 d x^2) - 3 a^2 (c^2 - d^2 x^4)) + \right. \right. \\
& \left. \left. \pm c (2 b^2 + 7 a b c - 3 a^2 c^2) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] - \right. \\
& \left. \left. \pm b c (b + 9 a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \Big/ \left(15 a d^2 \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 326: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a + \frac{b}{c + d x^2}} dx$$

Optimal (type 4, 282 leaves, 7 steps):

$$\begin{aligned}
& \frac{(b - a c) x \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{3 a d} + \frac{x (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}{3 d} - \\
& \frac{\sqrt{c} (b - a c) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}]}{3 a d^{3/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} - \\
& \frac{c^{3/2} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}]}{3 d^{3/2} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}
\end{aligned}$$

Result (type 4, 250 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} \times (c + d x^2) (b + a c + a d x^2) + \pm c (-b + a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] + \right. \right. \\
& \left. \left. 2 \pm b c \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \Big/ \left(3 d \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^2} dx$$

Optimal (type 4, 265 leaves, 8 steps):

$$\begin{aligned} & \frac{d x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c} - \frac{(c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x} - \\ & \frac{\sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{\sqrt{c}} + \frac{a \sqrt{c} \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{(b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} \end{aligned}$$

Result (type 4, 141 leaves):

$$\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(-\frac{1}{x} - \frac{d x}{c} - \frac{\frac{\pm a d}{\sqrt{b+a c}} \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}\right]}{\sqrt{\frac{a d}{b+a c}} (b+a (c+d x^2))} \right)$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} dx$$

Optimal (type 4, 362 leaves, 8 steps):

$$\begin{aligned}
& - \frac{(2b+ac)d^2 x \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} + \frac{(2b+ac)d(c+dx^2) \sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} + \\
& \frac{(2b+ac)d^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}]}{3c^{3/2}(b+ac)} - \frac{ad^{3/2} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right], \frac{b}{b+ac}]}{3\sqrt{c}(b+ac) \sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}
\end{aligned}$$

Result (type 4, 314 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{ad}{b+ac}} \sqrt{\frac{b+ac+adx^2}{c+dx^2}} \left(\sqrt{\frac{ad}{b+ac}} (c+dx^2) (b^2(c-2dx^2) + a^2c(c^2-d^2x^4) + 2ab(c^2-cdx^2-d^2x^4)) - \right. \right. \right. \\
& \left. \left. \left. \pm a c (2b+ac)d^2 x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}]\right. \right. + \\
& \left. \left. \left. \pm a b c d^2 x^3 \sqrt{\frac{b+ac+adx^2}{b+ac}} \sqrt{1+\frac{dx^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{ad}{b+ac}} x\right], 1+\frac{b}{ac}]\right) \right) \right) \Big/ (3ac^2dx^3(b+a(c+dx^2)))
\end{aligned}$$

Problem 330: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^6} dx$$

Optimal (type 4, 466 leaves, 9 steps):

$$\begin{aligned}
& \frac{\left(8 b^2 + 13 a b c + 3 a^2 c^2\right) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2} - \frac{(c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c x^5} + \\
& \frac{(4 b + 3 a c) d (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^2 (b+a c) x^3} - \frac{(8 b^2 + 13 a b c + 3 a^2 c^2) d^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{15 c^3 (b+a c)^2 x} - \\
& \frac{\left(8 b^2 + 13 a b c + 3 a^2 c^2\right) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{5/2} (b+a c)^2 \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{a (4 b + 3 a c) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 c^{3/2} (b+a c)^2 \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 402 leaves):

$$\begin{aligned}
& - \frac{1}{15 c^3 (b+a c)^2 \sqrt{\frac{a d}{b+a c}} x^5 (b+a (c+d x^2))} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(\sqrt{\frac{a d}{b+a c}} (c+d x^2) \right. \\
& \left(b^3 (3 c^2 - 4 c d x^2 + 8 d^2 x^4) + 3 a^3 c^2 (c^3 + d^3 x^6) + a b^2 (9 c^3 - 8 c^2 d x^2 + 17 c d^2 x^4 + 8 d^3 x^6) + a^2 b c (9 c^3 - 4 c^2 d x^2 + 9 c d^2 x^4 + 13 d^3 x^6) \right) + \\
& \pm a c (8 b^2 + 13 a b c + 3 a^2 c^2) d^3 x^5 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}\right] - \\
& \left. 2 \pm a b c (2 b + 3 a c) d^3 x^5 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}\right] \right)
\end{aligned}$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(a + \frac{b}{c+d x^2}\right)^{3/2} dx$$

Optimal (type 4, 405 leaves, 9 steps):

$$\begin{aligned}
& \frac{(b^2 - 14 a b c + a^2 c^2) \times \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 a d^2} + \frac{(7 b - a c) \times (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d^2} + \frac{6 a x^3 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 d} - \frac{x^3 (b + a c + a d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d} - \\
& \frac{\sqrt{c} (b^2 - 14 a b c + a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticE}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, \frac{b}{b+a c}], \frac{b}{b+a c}]}{5 a d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} - \frac{c^{3/2} (7 b - a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticF}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, \frac{b}{b+a c}], \frac{b}{b+a c}]}{5 d^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 308 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(\sqrt{\frac{a d}{b+a c}} \times \left(-a^2 (c - d x^2) (c + d x^2)^2 + b^2 (7 c + 2 d x^2) + 3 a b (2 c^2 + 3 c d x^2 + d^2 x^4) \right) - \right. \right. \\
& \left. \left. \pm c (b^2 - 14 a b c + a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}[\sqrt{\frac{a d}{b+a c}} x], 1 + \frac{b}{a c}] + \right. \right. \\
& \left. \left. 8 \pm b c (b - a c) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}[\sqrt{\frac{a d}{b+a c}} x], 1 + \frac{b}{a c}] \right) \right) / \left(5 d^2 \sqrt{\frac{a d}{b+a c}} (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(a + \frac{b}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 331 leaves, 8 steps):

$$\begin{aligned}
& \frac{(7 b - a c) \times \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 d} + \frac{4 a x (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 d} - \frac{x (b + a c + a d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{d} - \\
& \frac{\sqrt{c} (7 b - a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticE}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, \frac{b}{b+a c}], \frac{b}{b+a c}]}{3 d^{3/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{\sqrt{c} (3 b - a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticF}[\text{ArcTan}[\frac{\sqrt{d} x}{\sqrt{c}}, \frac{b}{b+a c}], \frac{b}{b+a c}]}{3 d^{3/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 270 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} x \left(-3 b^2 - 2 a b (c + d x^2) + a^2 (c + d x^2)^2 \right) + \right. \right. \\
& \left. \left. \pm a c (-7 b + a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] + \right. \\
& \left. \left. \pm b (-3 b + 5 a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \Big/ \left(3 d \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 340: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + \frac{b}{c + d x^2} \right)^{3/2} dx$$

Optimal (type 4, 260 leaves, 7 steps):

$$\begin{aligned}
& \frac{b x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} - (b - a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c} + \\
& \frac{(b - a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}] + a \sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{\sqrt{c} \sqrt{d} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 243 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(b \sqrt{\frac{a d}{b + a c}} x (b + a (c + d x^2)) - \pm a c (-b + a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] - \right. \right. \\
& \left. \left. 2 \pm a b c \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \Big/ \left(c \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^2} dx$$

Optimal (type 4, 312 leaves, 8 steps):

$$\begin{aligned} & \frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x} + \frac{(2 b+a c) d x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^2} - \frac{(2 b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c^2 x} - \\ & \frac{(2 b+a c) \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{c^{3/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{a \sqrt{d} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{\sqrt{c} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} \end{aligned}$$

Result (type 4, 278 leaves):

$$\begin{aligned} & - \left(\left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(\sqrt{\frac{a d}{b+a c}} (2 a b (c+d x^2)^2 + a^2 c (c+d x^2)^2 + b^2 (c+2 d x^2)) + \right. \right. \right. \\ & \left. \left. \left. \pm a c (2 b+a c) d x \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}] - \right. \right. \\ & \left. \left. \left. \pm a b c d x \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}] \right) \right) \Big/ \left(c^2 \sqrt{\frac{a d}{b+a c}} x (b+a (c+d x^2)) \right) \right) \end{aligned}$$

Problem 342: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^4} dx$$

Optimal (type 4, 388 leaves, 9 steps):

$$\begin{aligned}
& \frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^3} - \frac{(8 b+a c) d^2 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^3} - \frac{(4 b+a c) (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^2 x^3} + \frac{(8 b+a c) d (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{3 c^3 x} + \\
& \frac{(8 b+a c) d^{3/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 c^{5/2} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} - \frac{a (4 b+a c) d^{3/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 c^{3/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 329 leaves):

$$\begin{aligned}
& -\frac{1}{3 c^3 \sqrt{\frac{a d}{b+a c}} x^3 (b+a (c+d x^2))} \\
& -\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \left(\sqrt{\frac{a d}{b+a c}} (a^2 c (c-d x^2) (c+d x^2)^2 + b^2 (c^2 - 4 c d x^2 - 8 d^2 x^4) + a b (2 c^3 - 3 c^2 d x^2 - 13 c d^2 x^4 - 8 d^3 x^6))} - \right. \\
& \left. \pm a c (8 b+a c) d^2 x^3 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}\right] + \right. \\
& \left. 4 \pm a b c d^2 x^3 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}\right] \right)
\end{aligned}$$

Problem 343: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + \frac{b}{c+d x^2}\right)^{3/2}}{x^6} dx$$

Optimal (type 4, 494 leaves, 10 steps):

$$\begin{aligned}
& \frac{b \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{c x^5} + \frac{(16 b^2 + 16 a b c + a^2 c^2) d^3 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c)} - \frac{(6 b + a c) (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^2 x^5} + \\
& \frac{(8 b + a c) d (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^3 x^3} - \frac{(16 b^2 + 16 a b c + a^2 c^2) d^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{5 c^4 (b+a c) x} - \\
& \frac{(16 b^2 + 16 a b c + a^2 c^2) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{5 c^{7/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} + \frac{a (8 b + a c) d^{5/2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{5 c^{5/2} (b+a c) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 430 leaves):

$$\begin{aligned}
& -\frac{1}{5 a c^4 d x^5 (b+a (c+d x^2))} \sqrt{\frac{a d}{b+a c}} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \\
& \left(\sqrt{\frac{a d}{b+a c}} (b^3 (c^3 - 2 c^2 d x^2 + 8 c d^2 x^4 + 16 d^3 x^6) + a^3 c^2 (c^4 + c^3 d x^2 + c d^3 x^6 + d^4 x^8) + a^2 b c (3 c^4 + 5 c^2 d^2 x^4 + 24 c d^3 x^6 + 16 d^4 x^8) + \right. \\
& \left. a b^2 (3 c^4 - 3 c^3 d x^2 + 13 c^2 d^2 x^4 + 40 c d^3 x^6 + 16 d^4 x^8) \right) + \\
& \pm a c (16 b^2 + 16 a b c + a^2 c^2) d^3 x^5 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}] - \\
& \pm a b c (8 b + 7 a c) d^3 x^5 \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1 + \frac{b}{a c}]
\end{aligned}$$

Problem 347: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{a + \frac{b}{c+d x^2}}} dx$$

Optimal (type 3, 96 leaves, 6 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{\sqrt{a}}-\frac{\sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{\sqrt{b+a c}}$$

Result (type 3, 210 leaves):

$$\left(\sqrt{b+a(c+d x^2)}\left(\sqrt{a} \sqrt{c} \operatorname{Log}[x^2]+2 \sqrt{b+a c} \operatorname{Log}[a \sqrt{c+d x^2}]+\sqrt{a} \sqrt{b+a c+a d x^2}\right)-\sqrt{a} \sqrt{c} \operatorname{Log}[2 a c(c+d x^2)+b(2 c+d x^2)+2 \sqrt{c} \sqrt{b+a c} \sqrt{c+d x^2} \sqrt{b+a c+a d x^2}]\right)\right) / \left(2 \sqrt{a} \sqrt{b+a c} \sqrt{c+d x^2} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+d x^2}}} dx$$

Optimal (type 4, 443 leaves, 8 steps):

$$\begin{aligned} & -\frac{(4 b+3 a c) x (b+a c+a d x^2)}{15 a^2 d^2 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{x^3 (b+a c+a d x^2)}{5 a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}+\frac{(8 b^2+13 a b c+3 a^2 c^2) x (b+a c+a d x^2)}{15 a^3 d^2 (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}- \\ & \frac{\sqrt{c} (8 b^2+13 a b c+3 a^2 c^2) (b+a c+a d x^2) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a^3 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}+\frac{c^{3/2} (4 b+3 a c) (b+a c+a d x^2) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}\right]}{15 a^2 d^{5/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} \end{aligned}$$

Result (type 4, 297 leaves):

$$\begin{aligned} & -\left(\left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}\left(\sqrt{\frac{a d}{b+a c}} x(c+d x^2)(4 b^2+a b(7 c+d x^2)+3 a^2(c^2-d^2 x^4))+\right.\right.\right. \\ & \left.\left.\left.\pm c(8 b^2+13 a b c+3 a^2 c^2) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right]-\right.\right.\right. \\ & \left.\left.\left.2 \pm b c(2 b+3 a c) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right]\right)\right) / \left(15 a^2 d^2 \sqrt{\frac{a d}{b+a c}}(b+a(c+d x^2))\right) \right) \end{aligned}$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 354 leaves, 7 steps):

$$\begin{aligned} & \frac{x (b + a c + a d x^2)}{3 a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(2 b + a c) x (b + a c + a d x^2)}{3 a^2 d (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \\ & \frac{\sqrt{c} (2 b + a c) (b + a c + a d x^2) \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 a^2 d^{3/2} (c + d x^2) \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} - \frac{c^{3/2} (b + a c + a d x^2) \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 a d^{3/2} (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} \end{aligned}$$

Result (type 4, 253 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} x (c + d x^2) (b + a c + a d x^2) + \frac{1}{2} c (2 b + a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}\right] - \right. \right. \\ & \left. \left. \frac{1}{2} b c \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}\right] \right) \right) \Big/ \left(3 a d \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right) \end{aligned}$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal (type 4, 431 leaves, 8 steps):

$$\begin{aligned}
& -\frac{b + a c + a d x^2}{3 (b + a c) x^3 \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(b - a c) d (b + a c + a d x^2)}{3 c (b + a c)^2 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(b - a c) d^2 x (b + a c + a d x^2)}{3 c (b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \\
& \frac{(b - a c) d^{3/2} (b + a c + a d x^2) \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 \sqrt{c} (b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{a \sqrt{c} d^{3/2} (b + a c + a d x^2) \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 (b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 314 leaves):

$$\begin{aligned}
& \left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(-\sqrt{\frac{a d}{b + a c}} (c + d x^2) (b^2 (c + d x^2) + a^2 c (c^2 - d^2 x^4) + a b (2 c^2 + c d x^2 + d^2 x^4)) + \right. \right. \\
& \pm a c (-b + a c) d^2 x^3 \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] + \\
& \left. \left. 2 \pm a b c d^2 x^3 \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) / \left(3 c (b + a c)^2 \sqrt{\frac{a d}{b + a c}} x^3 (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 358: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{b}{a (b + a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{a}}\right]}{a^{3/2}} - \frac{c^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}}{\sqrt{b+a c}}\right]}{(b + a c)^{3/2}}$$

Result (type 3, 306 leaves):

$$\frac{1}{2 a^{3/2} (b + a c)^{3/2} (b + a (c + d x^2))} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(a^{3/2} c^{3/2} \sqrt{c + d x^2} \sqrt{b + a (c + d x^2)} \operatorname{Log}[x^2] + \right. \\ \left(b + a c \right)^{3/2} \sqrt{c + d x^2} \sqrt{b + a (c + d x^2)} \operatorname{Log}[b + 2 a (c + d x^2) + 2 \sqrt{a} \sqrt{c + d x^2} \sqrt{b + a (c + d x^2)}] - \sqrt{a} \\ \left. \left(2 b \sqrt{b + a c} (c + d x^2) + a c^{3/2} \sqrt{c + d x^2} \sqrt{b + a (c + d x^2)} \operatorname{Log}[2 a c (c + d x^2) + b (2 c + d x^2) + 2 \sqrt{c} \sqrt{b + a c} \sqrt{c + d x^2} \sqrt{b + a (c + d x^2)}] \right) \right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 482 leaves, 9 steps):

$$- \frac{x^3 (c + d x^2)}{a d \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \frac{(8 b + a c) x (b + a c + a d x^2)}{5 a^3 d^2 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{6 x^3 (b + a c + a d x^2)}{5 a^2 d \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{(16 b^2 + 16 a b c + a^2 c^2) x (b + a c + a d x^2)}{5 a^4 d^2 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \\ \frac{\sqrt{c} (16 b^2 + 16 a b c + a^2 c^2) (b + a c + a d x^2) \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{5 a^4 d^{5/2} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}} + \frac{c^{3/2} (8 b + a c) (b + a c + a d x^2) \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{5 a^3 d^{5/2} (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \sqrt{\frac{c (b + a c + a d x^2)}{(b + a c) (c + d x^2)}}}$$

Result (type 4, 296 leaves):

$$- \left(\left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} x (c + d x^2) (8 b^2 + a b (9 c + 2 d x^2) + a^2 (c^2 - d^2 x^4)) + \right. \right. \right. \\ \left. \left. \left. \pm c (16 b^2 + 16 a b c + a^2 c^2) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] - \right. \right. \\ \left. \left. \left. \pm b c (8 b + 7 a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) / \left(5 a^3 d^2 \sqrt{\frac{a d}{b + a c}} (b + a (c + d x^2)) \right) \right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 409 leaves, 8 steps):

$$\begin{aligned} & -\frac{x(c+dx^2)}{a d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{4 x (b+a c+a d x^2)}{3 a^2 d \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(8 b+a c) x (b+a c+a d x^2)}{3 a^3 d (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \\ & \frac{\sqrt{c} (8 b+a c) (b+a c+a d x^2) \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 a^3 d^{3/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{c^{3/2} (4 b+a c) (b+a c+a d x^2) \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{3 a^2 (b+a c) d^{3/2} (c+d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}} \end{aligned}$$

Result (type 4, 255 leaves):

$$\begin{aligned} & \left(\sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \right. \\ & \left(\sqrt{\frac{a d}{b+a c}} x (c+d x^2) (4 b+a c+a d x^2) + \frac{1}{2} c (8 b+a c) \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticE}\left[\frac{1}{2} \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] - \right. \\ & \left. 4 \frac{1}{2} b c \sqrt{\frac{b+a c+a d x^2}{b+a c}} \sqrt{1+\frac{d x^2}{c}} \operatorname{EllipticF}\left[\frac{1}{2} \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b+a c}} x\right], 1+\frac{b}{a c}\right] \right) \Big/ \left(3 a^2 d \sqrt{\frac{a d}{b+a c}} (b+a (c+d x^2)) \right) \end{aligned}$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal (type 4, 356 leaves, 7 steps):

$$\begin{aligned}
& -\frac{b x}{a (b + a c) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(2 b + a c) x (b + a c + a d x^2)}{a^2 (b + a c) (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \\
& \frac{\sqrt{c} (2 b + a c) (b + a c + a d x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{a^2 (b + a c) \sqrt{d} (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{c^{3/2} (b + a c + a d x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{a (b + a c) \sqrt{d} (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 241 leaves):

$$\begin{aligned}
& -\frac{1}{a^2 d (b + a (c + d x^2))} \\
& \sqrt{\frac{a d}{b + a c}} \sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(b \sqrt{\frac{a d}{b + a c}} x (c + d x^2) + \pm c (2 b + a c) \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] - \right. \\
& \left. \pm b c \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right)
\end{aligned}$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 410 leaves, 8 steps):

$$\begin{aligned}
& -\frac{b}{a (b + a c) x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{(b - a c) (b + a c + a d x^2)}{a (b + a c)^2 x \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} - \frac{(b - a c) d x (b + a c + a d x^2)}{a (b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \\
& \frac{\sqrt{c} (b - a c) \sqrt{d} (b + a c + a d x^2) \text{EllipticE}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{a (b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}}} + \frac{c^{3/2} \sqrt{d} (b + a c + a d x^2) \text{EllipticF}[\text{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b+a c}]}{(b + a c)^2 (c + d x^2) \sqrt{\frac{b+a c+a d x^2}{c+d x^2}} \sqrt{\frac{c (b+a c+a d x^2)}{(b+a c) (c+d x^2)}}}
\end{aligned}$$

Result (type 4, 268 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} (c + d x^2) (b (c - d x^2) + a c (c + d x^2)) + \right. \right. \right. \\
& \left. \left. \left. \pm c (-b + a c) d x \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] + \right. \right. \\
& \left. \left. \left. 2 \pm b c d x \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \right) \Big/ \left((b + a c)^2 \sqrt{\frac{a d}{b + a c}} x (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+d x^2}\right)^{3/2}} dx$$

Optimal (type 4, 490 leaves, 9 steps):

$$\begin{aligned}
& - \frac{b}{a (b + a c) x^3 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{(3 b - a c) (b + a c + a d x^2)}{3 a (b + a c)^2 x^3 \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \frac{(7 b - a c) d (b + a c + a d x^2)}{3 (b + a c)^3 x \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{(7 b - a c) d^2 x (b + a c + a d x^2)}{3 (b + a c)^3 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} - \\
& \frac{\sqrt{c} (7 b - a c) d^{3/2} (b + a c + a d x^2) \operatorname{EllipticE}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}]}{3 (b + a c)^3 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}} + \frac{\sqrt{c} (3 b - a c) d^{3/2} (b + a c + a d x^2) \operatorname{EllipticF}[\operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right], \frac{b}{b + a c}]}{3 (b + a c)^3 (c + d x^2) \sqrt{\frac{b + a c + a d x^2}{c + d x^2}}}
\end{aligned}$$

Result (type 4, 319 leaves):

$$\begin{aligned}
& - \left(\left(\sqrt{\frac{b + a c + a d x^2}{c + d x^2}} \left(\sqrt{\frac{a d}{b + a c}} (c + d x^2) (b^2 (c + 4 d x^2) + a^2 c (c^2 - d^2 x^4) + a b (2 c^2 + 4 c d x^2 + 7 d^2 x^4)) - \right. \right. \right. \\
& \left. \left. \left. \pm a c (-7 b + a c) d^2 x^3 \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticE}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] + \pm b (3 b - 5 a c) d^2 x^3 \right. \right. \\
& \left. \left. \left. \sqrt{\frac{b + a c + a d x^2}{b + a c}} \sqrt{1 + \frac{d x^2}{c}} \operatorname{EllipticF}[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{a d}{b + a c}} x\right], 1 + \frac{b}{a c}] \right) \right) \right) \Big/ \left(3 (b + a c)^3 \sqrt{\frac{a d}{b + a c}} x^3 (b + a (c + d x^2)) \right)
\end{aligned}$$

Problem 366: Unable to integrate problem.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 75 leaves, 6 steps) :

$$-\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\text{ArcSinh}[x^{5/2}]}{20x^{23/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$$

Problem 367: Unable to integrate problem.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 50 leaves, 5 steps) :

$$\frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\text{ArcSinh}[x^{5/2}]}{5x^{13/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$$

Problem 368: Unable to integrate problem.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$$

Optimal (type 3, 24 leaves, 4 steps) :

$$\frac{2\sqrt{ax^3}\text{ArcSinh}[x^{5/2}]}{5x^{3/2}}$$

Result (type 8, 21 leaves) :

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$$

Problem 374: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Optimal (type 3, 49 leaves, 8 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{ax^6} \text{ArcTan}[x]}{2x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{ax^6} \text{ArcTanh}[x]}{2x^3}$$

Result (type 8, 35 leaves):

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Problem 375: Unable to integrate problem.

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$$

Optimal (type 3, 49 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{ax^6} \text{ArcTan}[x]}{2x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{ax^6} \text{ArcTanh}[x]}{2x^3}$$

Result (type 8, 32 leaves):

$$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 83 leaves, 4 steps):

$$\frac{2 \sqrt{a x^3} \sqrt{1+x^2}}{3 x} - \frac{\sqrt{a x^3} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{3 x^{3/2} \sqrt{1+x^2}}$$

Result (type 4, 77 leaves):

$$\frac{2 \sqrt{a x^3} \sqrt{1+x^2} \left(\sqrt{1+\frac{1}{x^2}} x^{3/2} - (-1)^{1/4} \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right] \right)}{3 \sqrt{1+\frac{1}{x^2}} x^{5/2}}$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 131 leaves, 4 steps):

$$\frac{2 \sqrt{a x} \sqrt{1+x^2}}{1+x} - \frac{2 \sqrt{a} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{\sqrt{a x}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} + \frac{\sqrt{a} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{\sqrt{a x}}{\sqrt{a}}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 58 leaves):

$$\frac{2 (-1)^{3/4} \sqrt{a x}}{\sqrt{x}} \left(-\text{EllipticE}\left[\frac{i}{2} \text{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \text{EllipticF}\left[\frac{i}{2} \text{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right)$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{\sqrt{\frac{a}{x}} \sqrt{x} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}}$$

Result (type 4, 57 leaves):

$$\frac{2 (-1)^{1/4} \sqrt{\frac{a}{x}} \sqrt{1+x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{(-1)^{1/4}}{\sqrt{x}}\right], -1\right]}{\sqrt{1+\frac{1}{x^2}} \sqrt{x}}$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}} dx$$

Optimal (type 4, 159 leaves, 6 steps):

$$\begin{aligned} & -2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^2} + \frac{2 \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^2}}{1+x} - \\ & \frac{2 \sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right] + \sqrt{\frac{a}{x^3}} x^{3/2} (1+x) \sqrt{\frac{1+x^2}{(1+x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\sqrt{x}\right], \frac{1}{2}\right]}{\sqrt{1+x^2}} \end{aligned}$$

Result (type 4, 74 leaves):

$$2 \sqrt{\frac{a}{x^3}} x \left(-\sqrt{1+x^2} + (-1)^{3/4} \sqrt{x} \left(-\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] + \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[(-1)^{1/4} \sqrt{x}\right], -1\right] \right) \right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x^3}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 292 leaves, 5 steps):

$$\begin{aligned}
& \frac{\left(1 + \sqrt{3}\right) \sqrt{a x^3} \sqrt{1+x^3}}{x \left(1 + \left(1 + \sqrt{3}\right) x\right)} - \frac{3^{1/4} \sqrt{a x^3} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{1+\left(1-\sqrt{3}\right) x}{1+\left(1+\sqrt{3}\right) x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]}{x \sqrt{\frac{x (1+x)}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \sqrt{1+x^3}} - \\
& \frac{\left(1 - \sqrt{3}\right) \sqrt{a x^3} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{1+\left(1-\sqrt{3}\right) x}{1+\left(1+\sqrt{3}\right) x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]}{2 \times 3^{1/4} x \sqrt{\frac{x (1+x)}{\left(1+\left(1+\sqrt{3}\right) x\right)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 174 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a x^3} \sqrt{1+x^3}} a x \left(1+x^3 + \frac{1}{\sqrt{6}} \left(1 - (-1)^{2/3}\right) x^2 \sqrt{\frac{-(-1)^{1/3} + x}{\left(1 + (-1)^{1/3}\right) x}} \sqrt{\frac{(1+x) (-1 + \sqrt{3} + 2x)}{x^2}}\right. \\
& \left. \left(1 + (-1)^{1/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{(-1 + (-1)^{2/3}) x}}\right], 1 + (-1)^{2/3}\right] - \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{(-1 + (-1)^{2/3}) x}}\right], 1 + (-1)^{2/3}\right]\right)
\end{aligned}$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a x^2}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 260 leaves, 4 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a x^2} \sqrt{1+x^3}}{x \left(1 + \sqrt{3} + x\right)} - \frac{3^{1/4} \sqrt{2 - \sqrt{3}} \sqrt{a x^2} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7 - 4\sqrt{3}\right]}{x \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}} + \\
& \frac{2 \sqrt{2} \sqrt{a x^2} (1+x) \sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7 - 4\sqrt{3}\right]}{3^{1/4} x \sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 134 leaves):

$$\begin{aligned}
& -\frac{1}{3^{1/4} \sqrt{a x^2} \sqrt{1+x^3}} 2 a x \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + x\right)} \sqrt{1+(-1)^{1/3} x+(-1)^{2/3} x^2} \\
& \left(\sqrt{3} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] + (-1)^{5/6} \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] \right)
\end{aligned}$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 116 leaves, 3 steps):

$$\frac{\sqrt{\frac{a}{x}} x (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3}) x)^2}} \operatorname{EllipticF}[\operatorname{ArcCos}\left[\frac{1+(1-\sqrt{3}) x}{1+(1+\sqrt{3}) x}\right], \frac{1}{4} (2+\sqrt{3})]}{3^{1/4} \sqrt{\frac{x (1+x)}{(1+(1+\sqrt{3}) x)^2}} \sqrt{1+x^3}}$$

Result (type 4, 106 leaves):

$$-\frac{1}{3^{1/4} \sqrt{1+x^3}} 2 (-1)^{1/6} \sqrt{-(-1)^{1/6} \left((-1)^{2/3} + \frac{1}{x}\right)} \sqrt{1+\frac{(-1)^{2/3}}{x^2} + \frac{(-1)^{1/3}}{x}} \sqrt{\frac{a}{x}} x^2 \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} \left(1+\frac{1}{x}\right)}}{3^{1/4}}\right], (-1)^{1/3}]$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 312 leaves, 6 steps):

$$\begin{aligned}
& -2 \sqrt{\frac{a}{x^3}} x \sqrt{1+x^3} + \frac{2 (1+\sqrt{3}) \sqrt{\frac{a}{x^3}} x^2 \sqrt{1+x^3}}{1+(1+\sqrt{3}) x} - \frac{2 \times 3^{1/4} \sqrt{\frac{a}{x^3}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3}) x)^2}} \text{EllipticE}[\text{ArcCos}\left[\frac{1+(1-\sqrt{3}) x}{1+(1+\sqrt{3}) x}\right], \frac{1}{4} (2+\sqrt{3})] - \\
& \frac{(1-\sqrt{3}) \sqrt{\frac{a}{x^3}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+(1+\sqrt{3}) x)^2}} \text{EllipticF}[\text{ArcCos}\left[\frac{1+(1-\sqrt{3}) x}{1+(1+\sqrt{3}) x}\right], \frac{1}{4} (2+\sqrt{3})]}{3^{1/4} \sqrt{\frac{x (1+x)}{(1+(1+\sqrt{3}) x)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 165 leaves):

$$\begin{aligned}
& -\frac{1}{\sqrt{\frac{a}{x^3}} \sqrt{1+x^3}} \sqrt{\frac{2}{3} \left(-1+(-1)^{2/3}\right) a} \sqrt{\frac{-(-1)^{1/3}+x}{(1+(-1)^{1/3}) x}} \sqrt{\frac{(1+x) \left(-1+\frac{1}{2} \sqrt{3}+2 x\right)}{x^2}} \\
& \left((1+(-1)^{1/3}) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{(-1+(-1)^{2/3}) x}}\right], 1+(-1)^{2/3}] - \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} (1+x)}{(-1+(-1)^{2/3}) x}}\right], 1+(-1)^{2/3}] \right)
\end{aligned}$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}} dx$$

Optimal (type 4, 281 leaves, 5 steps):

$$\begin{aligned}
& -\sqrt{\frac{a}{x^4}} x \sqrt{1+x^3} + \frac{\sqrt{\frac{a}{x^4}} x^2 \sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticE}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}]}{2 \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}} + \\
& \frac{\sqrt{2} \sqrt{\frac{a}{x^4}} x^2 (1+x) \sqrt{\frac{1-x+x^2}{(1+\sqrt{3}+x)^2}} \text{EllipticF}[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right], -7-4\sqrt{3}]}{3^{1/4} \sqrt{\frac{1+x}{(1+\sqrt{3}+x)^2}} \sqrt{1+x^3}}
\end{aligned}$$

Result (type 4, 146 leaves):

$$\frac{1}{3 \sqrt{1+x^3}} \sqrt{\frac{a}{x^4}} x \left(-3 (1+x^3) - 3^{3/4} x \sqrt{-(-1)^{1/6} ((-1)^{2/3} + x)} \sqrt{1 + (-1)^{1/3} x + (-1)^{2/3} x^2} \right. \\ \left. \left(\sqrt{3} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] + (-1)^{5/6} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} (1+x)}}{3^{1/4}}\right], (-1)^{1/3}] \right) \right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex} \sqrt{e+fx}} dx$$

Optimal (type 4, 114 leaves, 2 steps):

$$\frac{2 \sqrt{-e^2 + df} \sqrt{ax} \sqrt{\frac{e(e+fx)}{e^2 - df}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{f} \sqrt{d+ex}}{\sqrt{-e^2 + df}}\right], 1 - \frac{e^2}{df}]}{e \sqrt{f} \sqrt{-\frac{ex}{d}} \sqrt{e+fx}}$$

Result (type 4, 106 leaves):

$$-\frac{2 \pm e \sqrt{ax} \sqrt{1 + \frac{fx}{e}} \left(\text{EllipticE}[\pm \text{ArcSinh}\left[\sqrt{\frac{ex}{d}}\right], \frac{df}{e^2}] - \text{EllipticF}[\pm \text{ArcSinh}\left[\sqrt{\frac{ex}{d}}\right], \frac{df}{e^2}] \right)}{f \sqrt{\frac{ex}{d+ex}} \sqrt{d+ex} \sqrt{e+fx}}$$

Problem 423: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx$$

Optimal (type 3, 32 leaves, 6 steps):

$$2 \sqrt{1-x^2} - 2 \text{ArcTanh}[\sqrt{1-x^2}] + 2 \text{Log}[x]$$

Result (type 3, 84 leaves):

$$2 \left(\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}] \right)$$

Problem 425: Result more than twice size of optimal antiderivative.

$$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx$$

Optimal (type 3, 34 leaves, 6 steps):

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} + \text{ArcTanh}[\sqrt{1-x^2}]$$

Result (type 3, 88 leaves):

$$-\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} - \text{Log}[1 - \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] + \text{Log}[1 + \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 448: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x} dx$$

Optimal (type 3, 32 leaves, 7 steps):

$$-2\sqrt{1-x^2} + 2\text{ArcTanh}[\sqrt{1-x^2}] - 2\text{Log}[x]$$

Result (type 3, 84 leaves):

$$-2\left(\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]\right)$$

Problem 450: Result more than twice size of optimal antiderivative.

$$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x^3} dx$$

Optimal (type 3, 33 leaves, 7 steps):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} - \text{ArcTanh}[\sqrt{1-x^2}]$$

Result (type 3, 85 leaves):

$$\frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 451: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx$$

Optimal (type 3, 28 leaves, 15 steps):

$$\sqrt{1-x^2} - \text{ArcTanh}[\sqrt{1-x^2}] + \text{Log}[x]$$

Result (type 3, 82 leaves):

$$\sqrt{1-x^2} + \text{Log}[-x] + \text{Log}[1 - \sqrt{1+x}] - \text{Log}[2 + \sqrt{1-x} - \sqrt{1+x}] - \text{Log}[1 + \sqrt{1+x}] + \text{Log}[2 + \sqrt{1-x} + \sqrt{1+x}]$$

Problem 453: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 5, 121 leaves, 4 steps):

$$\frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1+n)} + \frac{a f^2 \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}[2, 1+n, 2+n, \frac{d+e x+f \sqrt{a+\frac{e^2 x^2}{f^2}}}{d}]}{2 d^2 e (1+n)}$$

Result (type 8, 27 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 460: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 225 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a d f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{a d^2 f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \\
& \frac{a f^2 \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}}{3 e} + \frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{7/2}}{7 e} - \frac{5 a d^{3/2} f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d + e x + f} \sqrt{a + \frac{e^2 x^2}{f^2}}}{\sqrt{d}} \right]}{2 e}
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Problem 461: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal (type 3, 183 leaves, 6 steps):

$$\begin{aligned}
& \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{a d f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2}}{5 e} - \frac{3 a \sqrt{d} f^2 \operatorname{ArcTanh} \left[\frac{\sqrt{d + e x + f} \sqrt{a + \frac{e^2 x^2}{f^2}}}{\sqrt{d}} \right]}{2 e}
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 464: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Optimal (type 3, 158 leaves, 5 steps):

$$-\frac{1 + \frac{a f^2}{d^2}}{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^2 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)} + \frac{3 a f^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 d^{5/2} e}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} dx$$

Problem 465: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Optimal (type 3, 199 leaves, 6 steps):

$$-\frac{1 + \frac{a f^2}{d^2}}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{3/2}} - \frac{2 a f^2}{d^3 e \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} - \frac{a f^2 \sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2 d^3 e \left(e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)} + \frac{5 a f^2 \operatorname{ArcTanh}\left[\frac{\sqrt{d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{\sqrt{d}}\right]}{2 d^{7/2} e}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Problem 472: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 5, 166 leaves, 4 steps):

$$\frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1+n)} + \frac{1}{2 e (2 d e - b f^2)^2 (1+n)}$$

$$f^2 (4 a e^2 - b^2 f^2) \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{2 e \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)}{2 d e - b f^2}\right]$$

Result (type 8, 30 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 478: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^3} dx$$

Optimal (type 3, 330 leaves, 3 steps):

$$\begin{aligned}
& - \frac{d^2 e - b d f^2 + a e f^2}{(2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^2} - \frac{2 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)} - \frac{2 e f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right)} + \\
& \frac{6 e f^2 (4 a e^2 - b^2 f^2) \log \left[d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right]}{(2 d e - b f^2)^4} - \frac{6 e f^2 (4 a e^2 - b^2 f^2) \log \left[b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right]}{(2 d e - b f^2)^4}
\end{aligned}$$

Result (type 3, 665 leaves):

$$\begin{aligned}
& \frac{4 e^3 x}{(2 d e - b f^2)^3} - \frac{2 (d^2 e - b d f^2 + a e f^2)^3}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))^2} - \frac{3 (4 a^2 e^3 f^4 + b^2 d f^4 (-d e + b f^2) + a e f^2 (4 d^2 e^2 - 4 b d e f^2 - b^2 f^4))}{(-2 d e + b f^2)^4 (d^2 + 2 d e x - f^2 (a + b x))} - \\
& \left(2 f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} (b^3 f^6 x + b e f^2 (-3 d^3 - a d f^2 + d^2 e x - 9 a e f^2 x + 8 d e^2 x^2) + b^2 (a f^6 - e f^4 x (d + 2 e x))) - \right. \\
& \left. 2 e^2 (3 a^2 f^4 + d^2 e x (3 d + 4 e x) - a d f^2 (5 d + 9 e x)) \right) \Big/ \left((-2 d e + b f^2)^3 (d^2 + 2 d e x - f^2 (a + b x))^2 \right) - \\
& \frac{3 e f^2 (4 a e^2 - b^2 f^2) \log \left[d^2 + 2 d e x - f^2 (a + b x) \right]}{(-2 d e + b f^2)^4} + \frac{3 (4 a e^3 f^2 - b^2 e f^4) \log \left[d^2 + 2 d e x - f^2 (a + b x) \right]}{(-2 d e + b f^2)^4} - \\
& \frac{3 e f^2 (4 a e^2 - b^2 f^2) \log \left[b f^2 + 2 e \left(e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) \right]}{(-2 d e + b f^2)^4} + \frac{1}{(-2 d e + b f^2)^4} 3 e f^2 (4 a e^2 - b^2 f^2) \\
& \log \left[b^2 f^4 x + 2 d^2 e \left(e x - f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) - 2 a e f^2 \left(2 d + e x + f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) + b f^2 \left(d^2 + a f^2 - 2 d e x + 2 d f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} \right) \right]
\end{aligned}$$

Problem 479: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal (type 3, 370 leaves, 6 steps):

$$\begin{aligned}
& \frac{f^2 (2 d e - b f^2) (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{4 e^4} + \frac{f^2 (4 a e^2 - b^2 f^2) \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{3/2}}{12 e^3} + \frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{7/2}}{7 e} - \\
& \frac{f^2 (2 d e - b f^2)^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{16 e^4} - \frac{5 f^2 (2 d e - b f^2)^{3/2} (4 a e^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}}\right]}{16 \sqrt{2} e^{9/2}}
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Problem 480: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal (type 3, 302 leaves, 6 steps):

$$\begin{aligned}
& \frac{f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{4 e^3} + \frac{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}\right)^{5/2}}{5 e} - \\
& \frac{f^2 (2 d e - b f^2) (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{8 e^3} - \frac{3 f^2 \sqrt{2 d e - b f^2} (4 a e^2 - b^2 f^2) \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}}\right]}{8 \sqrt{2} e^{7/2}}
\end{aligned}$$

Result (type 8, 32 leaves):

$$\int \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Problem 483: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Optimal (type 3, 269 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{4 (d^2 e - b d f^2 + a e f^2)}{(2 d e - b f^2)^2 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} - \\
 & \frac{f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{(2 d e - b f^2)^2 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \\
 & \frac{3 f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}} \right]}{\sqrt{2} \sqrt{e} (2 d e - b f^2)^{5/2}}
 \end{aligned}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Problem 484: Unable to integrate problem.

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx$$

Optimal (type 3, 335 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{4 (d^2 e - b d f^2 + a e f^2)}{3 (2 d e - b f^2)^2 \left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{3/2}} - \frac{4 f^2 (4 a e^2 - b^2 f^2)}{(2 d e - b f^2)^3 \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}} - \\
 & \frac{2 e f^2 (4 a e^2 - b^2 f^2) \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{(2 d e - b f^2)^3 \left(b f^2 + 2 e \left(e x + f \sqrt{a + \frac{x (b f^2 + e^2 x)}{f^2}} \right) \right)} + \frac{5 \sqrt{2} \sqrt{e} f^2 (4 a e^2 - b^2 f^2) \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{e} \sqrt{d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}}}}{\sqrt{2 d e - b f^2}} \right]}{(2 d e - b f^2)^{7/2}}
 \end{aligned}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{\left(d + e x + f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx$$

Problem 485: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 \left(x + \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 164 leaves, 3 steps):

$$\begin{aligned}
 & - \frac{a^5 \left(x + \sqrt{a + x^2} \right)^{-5+n}}{32 (5-n)} - \frac{5 a^4 \left(x + \sqrt{a + x^2} \right)^{-3+n}}{32 (3-n)} - \frac{5 a^3 \left(x + \sqrt{a + x^2} \right)^{-1+n}}{16 (1-n)} + \frac{5 a^2 \left(x + \sqrt{a + x^2} \right)^{1+n}}{16 (1+n)} + \frac{5 a \left(x + \sqrt{a + x^2} \right)^{3+n}}{32 (3+n)} + \frac{\left(x + \sqrt{a + x^2} \right)^{5+n}}{32 (5+n)}
 \end{aligned}$$

Result (type 3, 338 leaves):

$$\begin{aligned}
& \frac{1}{2} \left(x + \sqrt{a + x^2} \right)^n \left(-\frac{2 a^2 \left(x - n \sqrt{a + x^2} \right)}{-1 + n^2} + \right. \\
& \frac{1}{16} \left(\frac{a^5}{(-5 + n) \left(x + \sqrt{a + x^2} \right)^5} - \frac{3 a^4}{(-3 + n) \left(x + \sqrt{a + x^2} \right)^3} + \frac{2 a^3}{(-1 + n) \left(x + \sqrt{a + x^2} \right)} + \frac{2 a^2 \left(x + \sqrt{a + x^2} \right)}{1 + n} - \frac{3 a \left(x + \sqrt{a + x^2} \right)^3}{3 + n} + \frac{\left(x + \sqrt{a + x^2} \right)^5}{5 + n} \right) + \\
& \left. \left(4 a \sqrt{a + x^2} \left(2 a^3 n + a^2 (-3 + n) n x \left((-3 + n) x - 2 \sqrt{a + x^2} \right) + 4 (3 - n - 3 n^2 + n^3) x^5 \left(x + \sqrt{a + x^2} \right) + a (3 - 4 n + n^2) x^3 \left((3 + 5 n) x + (1 + 3 n) \sqrt{a + x^2} \right) \right) \right) \right) \Big/ \left((-3 + n) (-1 + n) (1 + n) (3 + n) \left(x + \sqrt{a + x^2} \right)^2 \left(a + x \left(x + \sqrt{a + x^2} \right) \right) \right)
\end{aligned}$$

Problem 488: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a + x^2} \right)^n}{a + x^2} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{2 \left(x + \sqrt{a + x^2} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right]}{a (1+n)}$$

Result (type 8, 23 leaves):

$$\int \frac{\left(x + \sqrt{a + x^2} \right)^n}{(a + x^2)^2} dx$$

Problem 489: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a + x^2} \right)^n}{(a + x^2)^2} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{8 \left(x + \sqrt{a + x^2} \right)^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right]}{a^3 (3+n)}$$

Result (type 8, 23 leaves):

$$\int \frac{(x + \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Problem 490: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^2 \left(x - \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 176 leaves, 3 steps):

$$-\frac{a^5 \left(x - \sqrt{a + x^2} \right)^{-5+n}}{32 (5-n)} - \frac{5 a^4 \left(x - \sqrt{a + x^2} \right)^{-3+n}}{32 (3-n)} - \frac{5 a^3 \left(x - \sqrt{a + x^2} \right)^{-1+n}}{16 (1-n)} + \frac{5 a^2 \left(x - \sqrt{a + x^2} \right)^{1+n}}{16 (1+n)} + \frac{5 a \left(x - \sqrt{a + x^2} \right)^{3+n}}{32 (3+n)} + \frac{\left(x - \sqrt{a + x^2} \right)^{5+n}}{32 (5+n)}$$

Result (type 3, 361 leaves):

$$\begin{aligned} & \frac{1}{2} \left(x - \sqrt{a + x^2} \right)^n \left(-\frac{2 a^2 \left(x + n \sqrt{a + x^2} \right)}{-1 + n^2} + \right. \\ & \frac{1}{16} \left(\frac{a^5}{(-5+n) \left(x - \sqrt{a + x^2} \right)^5} + \frac{2 a^3}{(-1+n) \left(x - \sqrt{a + x^2} \right)} + \frac{2 a^2 \left(x - \sqrt{a + x^2} \right)}{1+n} + \frac{\left(x - \sqrt{a + x^2} \right)^5}{5+n} + \frac{3 a^4}{(-3+n) \left(x - \sqrt{a + x^2} \right)^3} + \frac{3 a \left(-x + \sqrt{a + x^2} \right)^3}{3+n} \right) + \\ & \left(4 a \sqrt{a + x^2} \left(2 a^3 n - 4 (3 - n - 3 n^2 + n^3) x^5 \left(-x + \sqrt{a + x^2} \right) + a^2 (-3 + n) n x \left((-3 + n) x + 2 \sqrt{a + x^2} \right) - \right. \right. \\ & \left. \left. a (3 - 4 n + n^2) x^3 \left(- (3 + 5 n) x + (1 + 3 n) \sqrt{a + x^2} \right) \right) \right) \Big/ \left((-3+n) (-1+n) (1+n) (3+n) \left(x - \sqrt{a + x^2} \right)^2 \left(-a + x \left(-x + \sqrt{a + x^2} \right) \right) \right) \end{aligned}$$

Problem 493: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2} \right)^n}{a + x^2} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{2 \left(x - \sqrt{a + x^2} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, -\frac{\left(x - \sqrt{a + x^2} \right)^2}{a} \right]}{a (1+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{a + x^2} dx$$

Problem 494: Unable to integrate problem.

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{8 (x - \sqrt{a + x^2})^{3+n} \text{Hypergeometric2F1}[3, \frac{3+n}{2}, \frac{5+n}{2}, -\frac{(x - \sqrt{a + x^2})^2}{a}]}{a^3 (3+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^2} dx$$

Problem 495: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx$$

Optimal (type 3, 187 leaves, 3 steps):

$$\begin{aligned} & -\frac{a^6 (x + \sqrt{a + x^2})^{-6+n}}{64 (6 - n)} - \frac{3 a^5 (x + \sqrt{a + x^2})^{-4+n}}{32 (4 - n)} - \frac{15 a^4 (x + \sqrt{a + x^2})^{-2+n}}{64 (2 - n)} + \\ & \frac{5 a^3 (x + \sqrt{a + x^2})^n}{16 n} + \frac{15 a^2 (x + \sqrt{a + x^2})^{2+n}}{64 (2 + n)} + \frac{3 a (x + \sqrt{a + x^2})^{4+n}}{32 (4 + n)} + \frac{(x + \sqrt{a + x^2})^{6+n}}{64 (6 + n)} \end{aligned}$$

Result (type 3, 659 leaves):

$$\begin{aligned}
& \left(\left(x + \sqrt{a + x^2} \right)^{9+n} \left(a + x \left(x + \sqrt{a + x^2} \right) \right) \left(\frac{4 a^3}{n} + \frac{a^6}{(-6+n) (x + \sqrt{a + x^2})^6} - \right. \right. \\
& \left. \left. \frac{2 a^5}{(-4+n) (x + \sqrt{a + x^2})^4} - \frac{a^4}{(-2+n) (x + \sqrt{a + x^2})^2} - \frac{a^2 (x + \sqrt{a + x^2})^2}{2+n} - \frac{2 a (x + \sqrt{a + x^2})^4}{4+n} + \frac{(x + \sqrt{a + x^2})^6}{6+n} \right) \right) / \\
& \left(64 \left(512 x^{10} (x + \sqrt{a + x^2}) + a^5 (10 x + \sqrt{a + x^2}) + 256 a x^8 (6 x + 5 \sqrt{a + x^2}) + 10 a^4 x^2 (17 x + 5 \sqrt{a + x^2}) + \right. \right. \\
& \left. \left. 16 a^3 x^4 (52 x + 25 \sqrt{a + x^2}) + 32 a^2 x^6 (53 x + 35 \sqrt{a + x^2}) \right) \right) + \\
& \left(2 a \sqrt{a + x^2} (x + \sqrt{a + x^2})^{4+n} \left(2 a^4 + a^3 (-4+n) x \left((-4+n) x - 2 \sqrt{a + x^2} \right) + 8 (-4+n) n x^7 (x + \sqrt{a + x^2}) + \right. \right. \\
& \left. \left. 4 a (-4+n) n x^5 (4 x + 3 \sqrt{a + x^2}) + a^2 (-4+n) x^3 \left((-4+9 n) x + 4 (-1+n) \sqrt{a + x^2} \right) \right) \right) / \left((-4+n) n (4+n) \right. \\
& \left. \left(128 x^8 (x + \sqrt{a + x^2}) + a^4 (8 x + \sqrt{a + x^2}) + 64 a x^6 (5 x + 4 \sqrt{a + x^2}) + 8 a^3 x^2 (11 x + 4 \sqrt{a + x^2}) + 16 a^2 x^4 (17 x + 10 \sqrt{a + x^2}) \right) \right) + \\
& \left(a^2 (a + x^2) (x + \sqrt{a + x^2})^n \left(a^2 (-2+n^2) + 2 (-2+n) n x^3 (x + \sqrt{a + x^2}) + a (-2+n) x \left((2+3 n) x + 2 (1+n) \sqrt{a + x^2} \right) \right) \right) / \\
& \left(n (-4+n^2) \left(a + x (x + \sqrt{a + x^2}) \right)^2 \right)
\end{aligned}$$

Problem 496: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{3/2} \left(x + \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 131 leaves, 3 steps):

$$-\frac{a^4 (x + \sqrt{a + x^2})^{-4+n}}{16 (4-n)} - \frac{a^3 (x + \sqrt{a + x^2})^{-2+n}}{4 (2-n)} + \frac{3 a^2 (x + \sqrt{a + x^2})^n}{8 n} + \frac{a (x + \sqrt{a + x^2})^{2+n}}{4 (2+n)} + \frac{(x + \sqrt{a + x^2})^{4+n}}{16 (4+n)}$$

Result (type 3, 355 leaves):

$$\begin{aligned}
& \frac{1}{n} \sqrt{a+x^2} \left(x + \sqrt{a+x^2} \right)^n \\
& \left(\left(\left(x + \sqrt{a+x^2} \right)^4 \left(2 a^4 + a^3 (-4+n) x \left((-4+n) x - 2 \sqrt{a+x^2} \right) + 8 (-4+n) n x^7 \left(x + \sqrt{a+x^2} \right) + 4 a (-4+n) n x^5 \left(4 x + 3 \sqrt{a+x^2} \right) + \right. \right. \right. \\
& \left. \left. \left. a^2 (-4+n) x^3 \left((-4+9n) x + 4 (-1+n) \sqrt{a+x^2} \right) \right) \right) \Big/ \left((-4+n) (4+n) \right. \\
& \left. \left. \left(128 x^8 \left(x + \sqrt{a+x^2} \right) + a^4 \left(8 x + \sqrt{a+x^2} \right) + 64 a x^6 \left(5 x + 4 \sqrt{a+x^2} \right) + 8 a^3 x^2 \left(11 x + 4 \sqrt{a+x^2} \right) + 16 a^2 x^4 \left(17 x + 10 \sqrt{a+x^2} \right) \right) \right) + \\
& \left. \left(a \sqrt{a+x^2} \left(a^2 (-2+n^2) + 2 (-2+n) n x^3 \left(x + \sqrt{a+x^2} \right) + a (-2+n) x \left((2+3n) x + 2 (1+n) \sqrt{a+x^2} \right) \right) \right) \Big/ \left((-4+n^2) \left(a + x \left(x + \sqrt{a+x^2} \right) \right)^2 \right) \right)
\end{aligned}$$

Problem 499: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{3/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{4 \left(x + \sqrt{a+x^2} \right)^{2+n} \text{Hypergeometric2F1}[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a}]}{a^2 (2+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{3/2}} dx$$

Problem 500: Unable to integrate problem.

$$\int \frac{\left(x + \sqrt{a+x^2} \right)^n}{(a+x^2)^{5/2}} dx$$

Optimal (type 5, 59 leaves, 2 steps):

$$\frac{16 \left(x + \sqrt{a+x^2} \right)^{4+n} \text{Hypergeometric2F1}[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{\left(x + \sqrt{a+x^2} \right)^2}{a}]}{a^4 (4+n)}$$

Result (type 8, 25 leaves):

$$\int \frac{(x + \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Problem 501: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{5/2} \left(x - \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 201 leaves, 3 steps):

$$\begin{aligned} & \frac{a^6 \left(x - \sqrt{a + x^2} \right)^{-6+n}}{64 (6-n)} + \frac{3 a^5 \left(x - \sqrt{a + x^2} \right)^{-4+n}}{32 (4-n)} + \frac{15 a^4 \left(x - \sqrt{a + x^2} \right)^{-2+n}}{64 (2-n)} - \\ & \frac{5 a^3 \left(x - \sqrt{a + x^2} \right)^n}{16 n} - \frac{15 a^2 \left(x - \sqrt{a + x^2} \right)^{2+n}}{64 (2+n)} - \frac{3 a \left(x - \sqrt{a + x^2} \right)^{4+n}}{32 (4+n)} - \frac{\left(x - \sqrt{a + x^2} \right)^{6+n}}{64 (6+n)} \end{aligned}$$

Result (type 3, 692 leaves):

$$\begin{aligned} & \left(\left(x - \sqrt{a + x^2} \right)^{9+n} \left(a + x \left(x - \sqrt{a + x^2} \right) \right) \left(\frac{4 a^3}{n} + \frac{a^6}{(-6+n) \left(x - \sqrt{a + x^2} \right)^6} - \right. \right. \\ & \left. \left. \frac{2 a^5}{(-4+n) \left(x - \sqrt{a + x^2} \right)^4} - \frac{a^4}{(-2+n) \left(x - \sqrt{a + x^2} \right)^2} - \frac{a^2 \left(x - \sqrt{a + x^2} \right)^2}{2+n} - \frac{2 a \left(x - \sqrt{a + x^2} \right)^4}{4+n} + \frac{\left(x - \sqrt{a + x^2} \right)^6}{6+n} \right) \right) / \\ & \left(64 \left(a^5 \left(-10 x + \sqrt{a + x^2} \right) + 512 x^{10} \left(-x + \sqrt{a + x^2} \right) + 10 a^4 x^2 \left(-17 x + 5 \sqrt{a + x^2} \right) + 256 a x^8 \left(-6 x + 5 \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. 16 a^3 x^4 \left(-52 x + 25 \sqrt{a + x^2} \right) + 32 a^2 x^6 \left(-53 x + 35 \sqrt{a + x^2} \right) \right) \right) + \\ & \left(2 a \sqrt{a + x^2} \left(x - \sqrt{a + x^2} \right)^{4+n} \left(-2 a^4 + 8 (-4+n) n x^7 \left(-x + \sqrt{a + x^2} \right) - a^3 (-4+n) x \left((-4+n) x + 2 \sqrt{a + x^2} \right) + \right. \right. \\ & \left. \left. 4 a (-4+n) n x^5 \left(-4 x + 3 \sqrt{a + x^2} \right) + a^2 (-4+n) x^3 \left((4-9n) x + 4 (-1+n) \sqrt{a + x^2} \right) \right) \right) / \left((-4+n) n (4+n) \right. \\ & \left. \left(a^4 \left(-8 x + \sqrt{a + x^2} \right) + 128 x^8 \left(-x + \sqrt{a + x^2} \right) + 8 a^3 x^2 \left(-11 x + 4 \sqrt{a + x^2} \right) + 64 a x^6 \left(-5 x + 4 \sqrt{a + x^2} \right) + 16 a^2 x^4 \left(-17 x + 10 \sqrt{a + x^2} \right) \right) \right) + \\ & \left(a^2 (a + x^2) \left(x - \sqrt{a + x^2} \right)^n \left(-a^2 (-2+n^2) + 2 (-2+n) n x^3 \left(-x + \sqrt{a + x^2} \right) + a (-2+n) x \left(-(2+3n) x + 2 (1+n) \sqrt{a + x^2} \right) \right) \right) / \\ & \left(n (-4+n^2) \left(a + x \left(x - \sqrt{a + x^2} \right) \right)^2 \right) \end{aligned}$$

Problem 502: Result more than twice size of optimal antiderivative.

$$\int (a + x^2)^{3/2} \left(x - \sqrt{a + x^2} \right)^n dx$$

Optimal (type 3, 141 leaves, 3 steps):

$$\frac{a^4 \left(x - \sqrt{a + x^2} \right)^{-4+n}}{16 (4-n)} + \frac{a^3 \left(x - \sqrt{a + x^2} \right)^{-2+n}}{4 (2-n)} - \frac{3 a^2 \left(x - \sqrt{a + x^2} \right)^n}{8 n} - \frac{a \left(x - \sqrt{a + x^2} \right)^{2+n}}{4 (2+n)} - \frac{\left(x - \sqrt{a + x^2} \right)^{4+n}}{16 (4+n)}$$

Result (type 3, 366 leaves):

$$\frac{1}{n} \left(x - \sqrt{a + x^2} \right)^n$$

$$\left(\left(\sqrt{a + x^2} \left(x - \sqrt{a + x^2} \right)^4 \left(-2 a^4 + 8 (-4+n) n x^7 \left(-x + \sqrt{a + x^2} \right) - a^3 (-4+n) x \left((-4+n) x + 2 \sqrt{a + x^2} \right) + 4 a (-4+n) n x^5 \left(-4 x + 3 \sqrt{a + x^2} \right) + a^2 (-4+n) x^3 \left((4-9n) x + 4 (-1+n) \sqrt{a + x^2} \right) \right) \right) \right) \bigg/ \left((-4+n) (4+n) \right)$$

$$\left(a^4 \left(-8 x + \sqrt{a + x^2} \right) + 128 x^8 \left(-x + \sqrt{a + x^2} \right) + 8 a^3 x^2 \left(-11 x + 4 \sqrt{a + x^2} \right) + 64 a x^6 \left(-5 x + 4 \sqrt{a + x^2} \right) + 16 a^2 x^4 \left(-17 x + 10 \sqrt{a + x^2} \right) \right) +$$

$$\left(a (a + x^2) \left(-a^2 (-2+n^2) + 2 (-2+n) n x^3 \left(-x + \sqrt{a + x^2} \right) + a (-2+n) x \left(-(2+3n) x + 2 (1+n) \sqrt{a + x^2} \right) \right) \right) \bigg/ \left((-4+n^2) \left(a + x \left(x - \sqrt{a + x^2} \right)^2 \right) \right)$$

Problem 505: Unable to integrate problem.

$$\int \frac{\left(x - \sqrt{a + x^2} \right)^n}{(a + x^2)^{3/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$-\frac{4 \left(x - \sqrt{a + x^2} \right)^{2+n} \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, -\frac{\left(x - \sqrt{a + x^2}\right)^2}{a}\right]}{a^2 (2+n)}$$

Result (type 8, 27 leaves):

$$\int \frac{\left(x - \sqrt{a + x^2} \right)^n}{(a + x^2)^{3/2}} dx$$

Problem 506: Unable to integrate problem.

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Optimal (type 5, 63 leaves, 2 steps):

$$\frac{16 (x - \sqrt{a + x^2})^{4+n} \text{Hypergeometric2F1}\left[4, \frac{4+n}{2}, \frac{6+n}{2}, -\frac{(x - \sqrt{a + x^2})^2}{a}\right]}{a^4 (4+n)}$$

Result (type 8, 27 leaves):

$$\int \frac{(x - \sqrt{a + x^2})^n}{(a + x^2)^{5/2}} dx$$

Problem 507: Unable to integrate problem.

$$\int \left(a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Optimal (type 3, 365 leaves, 4 steps):

$$\frac{(d^2 - a f^2)^5 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-5+n}}{32 e f^4 (5-n)} - \frac{5 (d^2 - a f^2)^4 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-3+n}}{32 e f^4 (3-n)} + \frac{5 (d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{-1+n}}{16 e f^4 (1-n)} +$$

$$\frac{5 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{1+n}}{16 e f^4 (1+n)} - \frac{5 (d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{3+n}}{32 e f^4 (3+n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^{5+n}}{32 e f^4 (5+n)}$$

Result (type 8, 58 leaves):

$$\int \left(a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}\right)^2 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}}\right)^n dx$$

Problem 508: Unable to integrate problem.

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 239 leaves, 4 steps):

$$\frac{(d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-3+n}}{8 e f^2 (3 - n)} - \frac{3 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{8 e f^2 (1 - n)} -$$

$$\frac{3 (d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{8 e f^2 (1 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{3+n}}{8 e f^2 (3 + n)}$$

Result (type 8, 56 leaves):

$$\int \left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 509: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 107 leaves, 4 steps):

$$\frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{2 e (1 - n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1 + n)}$$

Result (type 8, 35 leaves):

$$\int \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 510: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{} dx$$

Optimal (type 5, 122 leaves, 2 steps):

$$- \frac{2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{d+e x+f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}}{d^2-a f^2}\right]}{e (d^2 - a f^2) (1+n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{} dx$$

Problem 511: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^2 dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$- \frac{8 f^4 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{3+n} \text{Hypergeometric2F1}\left[3, \frac{3+n}{2}, \frac{5+n}{2}, \frac{d+e x+f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}}{d^2-a f^2}\right]}{e (d^2 - a f^2)^3 (3+n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^2 dx$$

Problem 512: Unable to integrate problem.

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$\frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-1+n}}{2 e (1 - n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n}}{2 e (1 + n)}$$

Result (type 8, 35 leaves):

$$\int \left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n dx$$

Problem 513: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{2 f^2 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{1+n} \text{Hypergeometric2F1}\left[1, \frac{1+n}{2}, \frac{3+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right]}{e (d^2 - a f^2) (1 + n)}$$

Result (type 8, 58 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} dx$$

Problem 514: Unable to integrate problem.

$$\int \left(a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 297 leaves, 4 steps):

$$\begin{aligned} & - \frac{(d^2 - a f^2)^4 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-4+n}}{16 e f^3 (4 - n)} + \frac{(d^2 - a f^2)^3 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f^3 (2 - n)} + \\ & \frac{3 (d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{8 e f^3 n} - \frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f^3 (2 + n)} + \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{4+n}}{16 e f^3 (4 + n)} \end{aligned}$$

Result (type 8, 60 leaves):

$$\int \left(a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2} \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 515: Unable to integrate problem.

$$\int \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 171 leaves, 4 steps):

$$\begin{aligned} & - \frac{(d^2 - a f^2)^2 \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f (2 - n)} - \frac{(d^2 - a f^2) \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{2 e f n} + \frac{\left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{4 e f (2 + n)} \end{aligned}$$

Result (type 8, 60 leaves):

$$\int \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2d ex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 516: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e^n}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} dx$$

Problem 517: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2}} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{4 f^3 \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n} \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d+e x+f \sqrt{a+\frac{2 d e x}{f^2}+\frac{e^2 x^2}{f^2}}\right)^2}{d^2-a f^2}\right]}{e (d^2 - a f^2)^2 (2+n)}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2} \right)^{3/2}} dx$$

Problem 518: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$\frac{f \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e^n}$$

Result (type 8, 60 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}}} dx$$

Problem 519: Unable to integrate problem.

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Optimal (type 3, 327 leaves, 5 steps):

$$\begin{aligned} & - \frac{\left(d^2 - a f^2 \right)^2 \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{-2+n}}{4 e f (2 - n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} \\ & + \frac{\left(d^2 - a f^2 \right) \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{2 e f n \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} + \frac{4 e f (2 + n) \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n}}{2 e f n \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}} \end{aligned}$$

Result (type 8, 64 leaves) :

$$\int \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Problem 520: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Optimal (type 3, 93 leaves, 4 steps) :

$$\frac{f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Result (type 8, 64 leaves) :

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx}{\sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Problem 521: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx}{\left(a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2} \right)^{3/2}}$$

Optimal (type 5, 177 leaves, 4 steps) :

$$\left(4 f^3 \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^{2+n} \text{Hypergeometric2F1}\left[2, \frac{2+n}{2}, \frac{4+n}{2}, \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}}\right)^2}{d^2 - a f^2}\right] \right) / \\
 \left(e (d^2 - a f^2)^2 g (2+n) \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}} \right)$$

Result (type 8, 64 leaves):

$$\int \frac{\left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{\left(a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2} \right)^{3/2}} dx$$

Problem 522: Unable to integrate problem.

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 g + e g x (2 d + e x)}{f^2}}} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e n \sqrt{a g + \frac{2 d e g x}{f^2} + \frac{e^2 g x^2}{f^2}}}$$

Result (type 8, 62 leaves):

$$\int \frac{\left(d + e x + f \sqrt{\frac{a f^2 + e x (2 d + e x)}{f^2}} \right)^n}{\sqrt{\frac{a f^2 g + e g x (2 d + e x)}{f^2}}} dx$$

Problem 523: Unable to integrate problem.

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Optimal (type 4, 191 leaves, 7 steps):

$$-\frac{b \operatorname{ArcTanh} \left[\frac{\sqrt{b^2 e+a^2 f} \sqrt{c+d x^2}}{\sqrt{b^2 c+a^2 d} \sqrt{e+f x^2}} \right]}{\sqrt{b^2 c+a^2 d} \sqrt{b^2 e+a^2 f}} + \frac{\sqrt{-c} \sqrt{1+\frac{d x^2}{c}} \sqrt{1+\frac{f x^2}{e}} \operatorname{EllipticPi} \left[-\frac{b^2 c}{a^2 d}, \operatorname{ArcSin} \left[\frac{\sqrt{d} x}{\sqrt{-c}} \right], \frac{c f}{d e} \right]}{a \sqrt{d} \sqrt{c+d x^2} \sqrt{e+f x^2}}$$

Result (type 8, 32 leaves):

$$\int \frac{1}{(a + b x) \sqrt{c + d x^2} \sqrt{e + f x^2}} dx$$

Problem 524: Result more than twice size of optimal antiderivative.

$$\int \frac{e - 2 f x^2}{e^2 + 4 d f x^2 + 4 e f x^2 + 4 f^2 x^4} dx$$

Optimal (type 3, 81 leaves, 4 steps):

$$-\frac{\operatorname{Log} \left[e - 2 \sqrt{-d} \sqrt{f} x + 2 f x^2 \right]}{4 \sqrt{-d} \sqrt{f}} + \frac{\operatorname{Log} \left[e + 2 \sqrt{-d} \sqrt{f} x + 2 f x^2 \right]}{4 \sqrt{-d} \sqrt{f}}$$

Result (type 3, 191 leaves):

$$-\frac{\left(-d - 2 e + \sqrt{d} \sqrt{d+2 e} \right) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d+e-\sqrt{d} \sqrt{d+2 e}}} \right]}{\sqrt{d+e-\sqrt{d} \sqrt{d+2 e}}} - \frac{\left(d + 2 e + \sqrt{d} \sqrt{d+2 e} \right) \operatorname{ArcTan} \left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{d+e+\sqrt{d} \sqrt{d+2 e}}} \right]}{\sqrt{d+e+\sqrt{d} \sqrt{d+2 e}}} \\ \frac{2 \sqrt{2} \sqrt{d} \sqrt{d+2 e} \sqrt{f}}{2 \sqrt{2} \sqrt{d} \sqrt{d+2 e} \sqrt{f}}$$

Problem 525: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{e - 2 f x^2}{e^2 - 4 d f x^2 + 4 e f x^2 + 4 f^2 x^4} dx$$

Optimal (type 3, 73 leaves, 4 steps):

$$-\frac{\operatorname{Log} \left[e - 2 \sqrt{d} \sqrt{f} x + 2 f x^2 \right]}{4 \sqrt{d} \sqrt{f}} + \frac{\operatorname{Log} \left[e + 2 \sqrt{d} \sqrt{f} x + 2 f x^2 \right]}{4 \sqrt{d} \sqrt{f}}$$

Result (type 3, 233 leaves):

$$\frac{\left(-\frac{1}{2} d+2 i e+\sqrt{d} \sqrt{-d+2 e}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d+e-i \sqrt{d} \sqrt{-d+2 e}}}\right]-\left(\frac{1}{2} d-2 i e+\sqrt{d} \sqrt{-d+2 e}\right) \operatorname{ArcTan}\left[\frac{\sqrt{2} \sqrt{f} x}{\sqrt{-d+e+i \sqrt{d} \sqrt{-d+2 e}}}\right]}{2 \sqrt{2} \sqrt{d} \sqrt{-d+2 e} \sqrt{f}}$$

Problem 526: Result is not expressed in closed-form.

$$\int \frac{e-4 f x^3}{e^2+4 d f x^2+4 e f x^3+4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e+2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves):

$$\frac{\operatorname{RootSum}\left[e^2+4 d f \#1^2+4 e f \#1^3+4 f^2 \#1^6 \&, \frac{-e \operatorname{Log}[x-\#1]+4 f \operatorname{Log}[x-\#1] \#1^3}{2 d \#1+3 e \#1^2+6 f \#1^5} \&\right]}{4 f}$$

Problem 527: Result is not expressed in closed-form.

$$\int \frac{e-4 f x^3}{e^2-4 d f x^2+4 e f x^3+4 f^2 x^6} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x}{e+2 f x^3}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 7, 87 leaves):

$$\frac{\operatorname{RootSum}\left[e^2-4 d f \#1^2+4 e f \#1^3+4 f^2 \#1^6 \&, \frac{-e \operatorname{Log}[x-\#1]+4 f \operatorname{Log}[x-\#1] \#1^3}{-2 d \#1+3 e \#1^2+6 f \#1^5} \&\right]}{4 f}$$

Problem 528: Unable to integrate problem.

$$\int \frac{e-2 f (-1+n) x^n}{e^2+4 d f x^2+4 e f x^n+4 f^2 x^{2 n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}} dx$$

Problem 529: Unable to integrate problem.

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}} dx$$

Optimal (type 3, 38 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 8, 44 leaves):

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}} dx$$

Problem 532: Result is not expressed in closed-form.

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4+4df\#1^6 \&, \frac{3e\text{Log}[x-\#1]\#1+2f\text{Log}[x-\#1]\#1^3}{e+2f\#1^2+3d\#1^4} \&\right]}{8f}$$

Problem 533: Result is not expressed in closed-form.

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 85 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4ef\#1^2 + 4f^2\#1^4 - 4df\#1^6 \&, \frac{3e\text{Log}[x-\#1]\#1 + 2f\text{Log}[x-\#1]\#1^3}{e+2f\#1^2 - 3d\#1^4} \&\right]}{8f}$$

Problem 536: Result is not expressed in closed-form.

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 86 leaves):

$$\frac{\text{RootSum}\left[e^2 + 4ef\#1^3 + 4df\#1^4 + 4f^2\#1^6 \&, \frac{-e\text{Log}[x-\#1] + f\text{Log}[x-\#1]\#1^3}{3e\#1 + 4d\#1^2 + 6f\#1^4} \&\right]}{2f}$$

Problem 537: Result is not expressed in closed-form.

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 - 4dfx^4 + 4f^2x^6} dx$$

Optimal (type 3, 40 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right]}{2\sqrt{d}\sqrt{f}}$$

Result (type 7, 86 leaves):

$$-\frac{\text{RootSum}\left[e^2+4 e f \#1^3-4 d f \#1^4+4 f^2 \#1^6 \&, \frac{-e \text{Log}[x-\#1]+f \text{Log}[x-\#1] \#1^3}{3 e \#1-4 d \#1^2+6 f \#1^4} \&\right]}{2 f}$$

Problem 542: Unable to integrate problem.

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 + 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{2 \sqrt{d} \sqrt{f} x^{1+m}}{e+2 f x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 + 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 543: Unable to integrate problem.

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Optimal (type 3, 42 leaves, 2 steps):

$$\frac{\text{ArcTanh}\left[\frac{2 \sqrt{d} \sqrt{f} x^{1+m}}{e+2 f x^n}\right]}{2 \sqrt{d} \sqrt{f}}$$

Result (type 8, 58 leaves):

$$\int \frac{x^m (e (1+m) + 2 f (1+m-n) x^n)}{e^2 - 4 d f x^{2+2m} + 4 e f x^n + 4 f^2 x^{2n}} dx$$

Problem 547: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a c+b c x^2+d \sqrt{a+b x^2}\right)} dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{d \operatorname{ArcTanh} \left[\frac{\sqrt{a+b x^2}}{\sqrt{a}} \right]}{\sqrt{a} (a c^2 - d^2)} + \frac{c \operatorname{Log}[x]}{a c^2 - d^2} - \frac{c \operatorname{Log}[d + c \sqrt{a+b x^2}]}{a c^2 - d^2}$$

Result (type 3, 282 leaves):

$$-\frac{1}{2 a c^2 - 2 d^2} \left(c \operatorname{Log}[4] + \left(-2 c + \frac{2 d}{\sqrt{a}} \right) \operatorname{Log}[x] + c \operatorname{Log}[a c^2 - d^2 + b c^2 x^2] - \frac{2 d \operatorname{Log}[a + \sqrt{a} \sqrt{a+b x^2}]}{\sqrt{a}} + c \operatorname{Log} \left[\frac{(a c^2 - d^2) (a c - \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a+b x^2})}{\sqrt{b} c d^2 (\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)} \right] + c \operatorname{Log} \left[\frac{(a c^2 - d^2) (a c + \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a+b x^2})}{\sqrt{b} c d^2 (-\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)} \right] \right)$$

Problem 548: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a c + b c x^2 + d \sqrt{a+b x^2})} dx$$

Optimal (type 3, 151 leaves, 8 steps):

$$-\frac{a c - d \sqrt{a+b x^2}}{2 a (a c^2 - d^2) x^2} - \frac{b d (3 a c^2 - d^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a+b x^2}}{\sqrt{a}} \right]}{2 a^{3/2} (a c^2 - d^2)^2} - \frac{b c^3 \operatorname{Log}[x]}{(a c^2 - d^2)^2} + \frac{b c^3 \operatorname{Log}[d + c \sqrt{a+b x^2}]}{(a c^2 - d^2)^2}$$

Result (type 3, 430 leaves):

$$\frac{1}{2 a^{3/2} (-a c^2 + d^2)^2 x^2} \left(-a^{5/2} c^3 + a^{3/2} c d^2 + a^{3/2} c^2 d \sqrt{a+b x^2} - \sqrt{a} d^3 \sqrt{a+b x^2} - b (2 a^{3/2} c^3 - 3 a c^2 d + d^3) x^2 \operatorname{Log}[x] + a^{3/2} b c^3 x^2 \operatorname{Log}[a c^2 - d^2 + b c^2 x^2] - 3 a b c^2 d x^2 \operatorname{Log}[a + \sqrt{a} \sqrt{a+b x^2}] + b d^3 x^2 \operatorname{Log}[a + \sqrt{a} \sqrt{a+b x^2}] + a^{3/2} b c^3 x^2 \operatorname{Log} \left[-\frac{2 (-a c^2 + d^2)^2 (a c - \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a+b x^2})}{b^{3/2} c^3 d^2 (\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)} \right] + a^{3/2} b c^3 x^2 \operatorname{Log} \left[-\frac{2 (-a c^2 + d^2)^2 (a c + \pm \sqrt{b} \sqrt{a c^2 - d^2} x + d \sqrt{a+b x^2})}{b^{3/2} c^3 d^2 (-\pm \sqrt{a c^2 - d^2} + \sqrt{b} c x)} \right] \right)$$

Problem 555: Unable to integrate problem.

$$\int \frac{1}{x \left(a c + b c x^3 + d \sqrt{a + b x^3} \right)} dx$$

Optimal (type 3, 93 leaves, 7 steps):

$$\frac{2 d \operatorname{ArcTanh} \left[\frac{\sqrt{a+b x^3}}{\sqrt{a}} \right]}{3 \sqrt{a} (a c^2 - d^2)} + \frac{c \operatorname{Log} [x]}{a c^2 - d^2} - \frac{2 c \operatorname{Log} [d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)}$$

Result (type 8, 31 leaves):

$$\int \frac{1}{x \left(a c + b c x^3 + d \sqrt{a + b x^3} \right)} dx$$

Problem 556: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(a c + b c x^3 + d \sqrt{a + b x^3} \right)} dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$-\frac{a c - d \sqrt{a + b x^3}}{3 a (a c^2 - d^2) x^3} - \frac{b d (3 a c^2 - d^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a+b x^3}}{\sqrt{a}} \right]}{3 a^{3/2} (a c^2 - d^2)^2} - \frac{b c^3 \operatorname{Log} [x]}{(a c^2 - d^2)^2} + \frac{2 b c^3 \operatorname{Log} [d + c \sqrt{a + b x^3}]}{3 (a c^2 - d^2)^2}$$

Result (type 6, 596 leaves):

$$\begin{aligned}
& \left(\left(6 b^2 c^2 d x^3 \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \middle/ \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(4 a (a c^2 - d^2) \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right. \right. \right. \\
& \left. \left. \left. + b x^3 \left(-2 a c^2 \text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] + (-a c^2 + d^2) \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right] \right) \right) \right) + \\
& \frac{1}{a x^3} \left(- \left(\left(5 b^2 c^2 d (3 a c^2 - d^2) x^6 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] \right) \middle/ \right. \right. \\
& \left. \left. \left((a c^2 - d^2) \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(5 b c^2 x^3 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] + \right. \right. \right. \\
& \left. \left. \left. (-2 a c^2 + 2 d^2) \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] - a c^2 \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{a}{b x^3}, \frac{-a c^2 + d^2}{b c^2 x^3}\right] \right) \right) \right) + \\
& \left. -3 (a c^2 - d^2) \left(a c - d \sqrt{a + b x^3} \right) - 9 a b c^3 x^3 \text{Log}[x] + 3 a b c^3 x^3 \text{Log}[a c^2 - d^2 + b c^2 x^3] \right) \middle/ \left(-a c^2 + d^2 \right)^2 \right)
\end{aligned}$$

Problem 558: Unable to integrate problem.

$$\int \frac{x}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 304 leaves, 9 steps):

$$\frac{d x^2 \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{2 (a c^2 - d^2) \sqrt{a + b x^3}} - \frac{\operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} - \\
 \frac{\operatorname{Log}\left[\left(a c^2 - d^2\right)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}} + \frac{\operatorname{Log}\left[\left(a c^2 - d^2\right)^{2/3} - b^{1/3} c^{2/3} \left(a c^2 - d^2\right)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 b^{2/3} c^{1/3} (a c^2 - d^2)^{1/3}}$$

Result (type 8, 29 leaves):

$$\int \frac{x}{a + b x^3} dx$$

Problem 559: Unable to integrate problem.

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Optimal (type 6, 300 leaves, 9 steps):

$$\begin{aligned} & \frac{d x \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) \sqrt{a + b x^3}} - \frac{c^{1/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} b^{1/3} (a c^2 - d^2)^{2/3}} + \\ & \frac{c^{1/3} \log\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 b^{1/3} (a c^2 - d^2)^{2/3}} - \frac{c^{1/3} \log\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 b^{1/3} (a c^2 - d^2)^{2/3}} \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{1}{a c + b c x^3 + d \sqrt{a + b x^3}} dx$$

Problem 560: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 319 leaves, 10 steps):

$$\begin{aligned} & -\frac{c}{(a c^2 - d^2) x} + \frac{d \sqrt{1 + \frac{b x^3}{a}} \operatorname{AppellF1}\left[-\frac{1}{3}, \frac{1}{2}, 1, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2}\right]}{(a c^2 - d^2) x \sqrt{a + b x^3}} + \frac{b^{1/3} c^{5/3} \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \\ & \frac{b^{1/3} c^{5/3} \log\left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x\right]}{3 (a c^2 - d^2)^{4/3}} - \frac{b^{1/3} c^{5/3} \log\left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2\right]}{6 (a c^2 - d^2)^{4/3}} \end{aligned}$$

Result (type 6, 1029 leaves):

$$\begin{aligned}
& -\frac{c}{a c^2 x - d^2 x} + \frac{d \sqrt{a + b x^3}}{a^2 c^2 x - a d^2 x} + \left(5 a b c^2 d x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \\
& \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(10 a (a c^2 - d^2) \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) + \\
& \left(5 b d^3 x^2 \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(2 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(10 a (a c^2 - d^2) \text{AppellF1} \left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) + \\
& \left(8 b^2 c^2 d x^5 \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(5 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(-16 a (a c^2 - d^2) \text{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) - \\
& \frac{b^{1/3} c^{5/3} \text{ArcTan} \left[\frac{-1 + \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3} (a c^2 - d^2)^{4/3}} + \frac{b^{1/3} c^{5/3} \text{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 (a c^2 - d^2)^{4/3}} - \frac{b^{1/3} c^{5/3} \text{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 (a c^2 - d^2)^{4/3}}
\end{aligned}$$

Problem 561: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 (a c + b c x^3 + d \sqrt{a + b x^3})} dx$$

Optimal (type 6, 324 leaves, 10 steps):

$$\begin{aligned}
& -\frac{c}{2 (a c^2 - d^2) x^2} + \frac{d \sqrt{1 + \frac{b x^3}{a}} \text{AppellF1} \left[-\frac{2}{3}, \frac{1}{2}, 1, \frac{1}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right]}{2 (a c^2 - d^2) x^2 \sqrt{a + b x^3}} + \frac{b^{2/3} c^{7/3} \text{ArcTan} \left[\frac{1 - \frac{2 b^{1/3} c^{2/3} x}{(a c^2 - d^2)^{1/3}}}{\sqrt{3}} \right]}{\sqrt{3} (a c^2 - d^2)^{5/3}} - \\
& \frac{b^{2/3} c^{7/3} \text{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 (a c^2 - d^2)^{5/3}} + \frac{b^{2/3} c^{7/3} \text{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Result (type 6, 1044 leaves):

$$\begin{aligned}
& -\frac{c}{2(a c^2 - d^2) x^2} + \frac{d \sqrt{a + b x^3}}{2 a (a c^2 - d^2) x^2} + \left(10 a b c^2 d x \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \\
& \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(8 a (a c^2 - d^2) \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) - \\
& \left(2 b d^3 x \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(\sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \left(8 a (a c^2 - d^2) \text{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) + \\
& \left(7 b^2 c^2 d x^4 \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) / \left(8 \sqrt{a + b x^3} (a c^2 - d^2 + b c^2 x^3) \right. \\
& \left. \left(14 a (a c^2 - d^2) \text{AppellF1} \left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] - \right. \right. \\
& \left. \left. 3 b x^3 \left(2 a c^2 \text{AppellF1} \left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] + (a c^2 - d^2) \text{AppellF1} \left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{b x^3}{a}, -\frac{b c^2 x^3}{a c^2 - d^2} \right] \right) \right) - \\
& \frac{b^{2/3} c^{7/3} \text{ArcTan} \left[\frac{-(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x}{\sqrt{3} (a c^2 - d^2)^{1/3}} \right]}{\sqrt{3} (a c^2 - d^2)^{5/3}} - \frac{b^{2/3} c^{7/3} \text{Log} \left[(a c^2 - d^2)^{1/3} + b^{1/3} c^{2/3} x \right]}{3 (a c^2 - d^2)^{5/3}} + \\
& \frac{b^{2/3} c^{7/3} \text{Log} \left[(a c^2 - d^2)^{2/3} - b^{1/3} c^{2/3} (a c^2 - d^2)^{1/3} x + b^{2/3} c^{4/3} x^2 \right]}{6 (a c^2 - d^2)^{5/3}}
\end{aligned}$$

Problem 562: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 135 leaves, 4 steps):

$$\begin{aligned}
& -\frac{d x \sqrt{1 + \frac{b x^n}{a}} \text{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) \sqrt{a + b x^n}} + \frac{c x \text{Hypergeometric2F1} \left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{a c^2 - d^2}
\end{aligned}$$

Result (type 6, 320 leaves):

$$\begin{aligned}
& - \left(\left(2 a d (a c^2 - d^2) (1 + n) \times \text{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right. \\
& \left. \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \left(-2 a b c^2 n x^n \text{AppellF1} \left[1 + \frac{1}{n}, \frac{1}{2}, 2, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a c^2 - d^2) \left(-b n x^n \text{AppellF1} \left[1 + \frac{1}{n}, \frac{3}{2}, 1, 2 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + 2 a (1 + n) \text{AppellF1} \left[\frac{1}{n}, \frac{1}{2}, 1, 1 + \frac{1}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \right) + \\
& \frac{c \times \text{Hypergeometric2F1} \left[1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{a c^2 - d^2}
\end{aligned}$$

Problem 563: Result more than twice size of optimal antiderivative.

$$\int \frac{x^m}{a c + b c x^n + d \sqrt{a + b x^n}} dx$$

Optimal (type 6, 167 leaves, 4 steps):

$$\begin{aligned}
& \frac{d x^{1+m} \sqrt{1 + \frac{b x^n}{a}} \text{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) (1 + m) \sqrt{a + b x^n}} + \frac{c x^{1+m} \text{Hypergeometric2F1} \left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]}{(a c^2 - d^2) (1 + m)}
\end{aligned}$$

Result (type 6, 373 leaves):

$$\begin{aligned}
& \frac{1}{(a c^2 - d^2) (1 + m)} \\
& x^{1+m} \left(- \left(\left(2 a d (-a c^2 + d^2)^2 (1 + m + n) \text{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right. \right. \\
& \left. \left. \left/ \left(\sqrt{a + b x^n} (a c^2 - d^2 + b c^2 x^n) \left(2 a (a c^2 - d^2) \right. \right. \right. \right. \\
& \left. \left. \left. (1 + m + n) \text{AppellF1} \left[\frac{1+m}{n}, \frac{1}{2}, 1, \frac{1+m+n}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] - b n x^n \left(2 a c^2 \text{AppellF1} \left[\frac{1+m+n}{n}, \frac{1}{2}, 2, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] + \right. \right. \right. \\
& \left. \left. \left. (a c^2 - d^2) \text{AppellF1} \left[\frac{1+m+n}{n}, \frac{3}{2}, 1, 2 + \frac{1+m}{n}, -\frac{b x^n}{a}, -\frac{b c^2 x^n}{a c^2 - d^2} \right] \right) \right) \right) + c \text{Hypergeometric2F1} \left[1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{b c^2 x^n}{a c^2 - d^2} \right]
\end{aligned}$$

Problem 566: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} - x^{5/2}} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves) :

$$\text{ArcTan}\left[\sqrt{x}\right] - \frac{1}{2} \text{Log}\left[1 - \sqrt{x}\right] + \frac{1}{2} \text{Log}\left[1 + \sqrt{x}\right]$$

Problem 587: Unable to integrate problem.

$$\int \left(a + \frac{b}{x}\right)^m (c + d x)^n dx$$

Optimal (type 6, 80 leaves, 4 steps) :

$$\frac{\left(a + \frac{b}{x}\right)^m x \left(1 + \frac{ax}{b}\right)^{-m} (c + d x)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[1 - m, -m, -n, 2 - m, -\frac{ax}{b}, -\frac{dx}{c}\right]}{1 - m}$$

Result (type 8, 19 leaves) :

$$\int \left(a + \frac{b}{x}\right)^m (c + d x)^n dx$$

Problem 591: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{c + d x} dx$$

Optimal (type 5, 101 leaves, 5 steps) :

$$-\frac{c \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{d \left(a c - b d\right) \left(1 + m\right)} + \frac{\left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, 1 + \frac{b}{a x}\right]}{a d \left(1 + m\right)}$$

Result (type 8, 19 leaves) :

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{c + d x} dx$$

Problem 592: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^2} dx$$

Optimal (type 5, 56 leaves, 3 steps) :

$$\frac{b \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{(a c - b d)^2 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^2} dx$$

Problem 593: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Optimal (type 5, 112 leaves, 4 steps):

$$\frac{d \left(a + \frac{b}{x}\right)^{1+m}}{2 c \left(a c - b d\right) \left(d + \frac{c}{x}\right)^2} - \frac{b \left(2 a c - b d (1+m)\right) \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{2 c \left(a c - b d\right)^3 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^3} dx$$

Problem 594: Unable to integrate problem.

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^4} dx$$

Optimal (type 5, 185 leaves, 5 steps):

$$\frac{d^2 \left(a + \frac{b}{x}\right)^{1+m}}{3 c^2 \left(a c - b d\right) \left(d + \frac{c}{x}\right)^3} - \frac{d \left(6 a c - b d (4+m)\right) \left(a + \frac{b}{x}\right)^{1+m}}{6 c^2 \left(a c - b d\right)^2 \left(d + \frac{c}{x}\right)^2} - \frac{b \left(6 a^2 c^2 - 6 a b c d (1+m) + b^2 d^2 (2+3 m+m^2)\right) \left(a + \frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{c \left(a + \frac{b}{x}\right)}{a c - b d}\right]}{6 c^2 \left(a c - b d\right)^4 (1+m)}$$

Result (type 8, 19 leaves):

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{(c + d x)^4} dx$$

Problem 598: Unable to integrate problem.

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b x^2}} dx$$

Optimal (type 3, 28 leaves, 3 steps):

$$\frac{\sqrt{b - \frac{a}{x^2}} \times \text{Log}[x]}{\sqrt{a - b x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - b x^2}} dx$$

Problem 601: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c + d x)^{3/2}}{\sqrt{a + \frac{b}{x^2}}} dx$$

Optimal (type 4, 406 leaves, 8 steps):

$$\begin{aligned}
& \frac{2 c \sqrt{c+d x} (b+a x^2)}{5 a \sqrt{a+\frac{b}{x^2}} x} + \frac{2 (c+d x)^{3/2} (b+a x^2)}{5 a \sqrt{a+\frac{b}{x^2}}} + \frac{2 \sqrt{b} (a c^2 - 3 b d^2) \sqrt{c+d x} \sqrt{1+\frac{a x^2}{b}} \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{-a} x}{\sqrt{b}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{-a} \sqrt{b} d}{a c - \sqrt{-a} \sqrt{b} d}]}{5 (-a)^{3/2} d \sqrt{a+\frac{b}{x^2}} x \sqrt{\frac{a (c+d x)}{a c - \sqrt{-a} \sqrt{b} d}}} \\
& \frac{2 \sqrt{b} c (a c^2 + b d^2) \sqrt{\frac{a (c+d x)}{a c - \sqrt{-a} \sqrt{b} d}} \sqrt{1+\frac{a x^2}{b}} \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{-a} x}{\sqrt{b}}}}{\sqrt{2}}\right], -\frac{2 \sqrt{-a} \sqrt{b} d}{a c - \sqrt{-a} \sqrt{b} d}]}{5 (-a)^{3/2} d \sqrt{a+\frac{b}{x^2}} x \sqrt{c+d x}}
\end{aligned}$$

Result (type 4, 540 leaves):

$$\begin{aligned}
& \frac{1}{5 \sqrt{a+\frac{b}{x^2}} x} \sqrt{c+d x} \\
& \left(\frac{2 (2 c + d x) (b + a x^2)}{a} + \left(2 \left(d^2 \sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}} (-3 b^2 d^2 + a^2 c^2 x^2 + a b (c^2 - 3 d^2 x^2)) + \sqrt{a} (-i a^{3/2} c^3 + a \sqrt{b} c^2 d + 3 i \sqrt{a} b c d^2 - 3 b^{3/2} d^3) \right) \right. \right. \\
& \left. \left. \sqrt{\frac{d \left(\frac{i \sqrt{b}}{\sqrt{a}} + x \right)}{c+d x}} \sqrt{-\frac{i \sqrt{b} d - d x}{c+d x}} (c+d x)^{3/2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}}}{\sqrt{c+d x}}\right], \frac{\sqrt{a} c - i \sqrt{b} d}{\sqrt{a} c + i \sqrt{b} d}\right] - \right. \right. \\
& \left. \left. \sqrt{a} \sqrt{b} d (a c^2 + 4 i \sqrt{a} \sqrt{b} c d - 3 b d^2) \sqrt{\frac{d \left(\frac{i \sqrt{b}}{\sqrt{a}} + x \right)}{c+d x}} \sqrt{-\frac{i \sqrt{b} d - d x}{c+d x}} (c+d x)^{3/2} \right. \right. \\
& \left. \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}}}{\sqrt{c+d x}}\right], \frac{\sqrt{a} c - i \sqrt{b} d}{\sqrt{a} c + i \sqrt{b} d}\right] \right) \right) \Big/ \left(a^2 d^2 \sqrt{-c - \frac{i \sqrt{b} d}{\sqrt{a}}} (c+d x) \right)
\end{aligned}$$

Problem 681: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1 + \frac{1}{x^2}}}{(1 + x^2)^2} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} x^2}{1 + x^2}$$

Problem 682: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + \frac{1}{x^2}} x (1 + x^2)} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Result (type 2, 20 leaves):

$$\frac{\sqrt{1 + \frac{1}{x^2}} x^2}{1 + x^2}$$

Problem 737: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$$

Optimal (type 3, 15 leaves, 2 steps):

$$2 \operatorname{ArcTan} \left[\sqrt{-\frac{x}{1+x}} \right]$$

Result (type 3, 32 leaves):

$$\frac{2 \sqrt{-\frac{x}{1+x}} \sqrt{1+x} \operatorname{ArcSinh}[\sqrt{x}]}{\sqrt{x}}$$

Problem 738: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx$$

Optimal (type 3, 18 leaves, 2 steps):

$$2 \operatorname{ArcTan} \left[\sqrt{\frac{1-x}{1+x}} \right]$$

Result (type 3, 47 leaves):

$$\frac{2 \sqrt{\frac{1-x}{1+x}} \sqrt{1-x^2} \operatorname{ArcSin} \left[\frac{\sqrt{1+x}}{\sqrt{2}} \right]}{-1+x}$$

Problem 739: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx$$

Optimal (type 3, 24 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTan} \left[\sqrt{\frac{a+b x}{c-b x}} \right]}{b}$$

Result (type 3, 80 leaves):

$$\frac{\pm \sqrt{c-b x} \sqrt{\frac{a+b x}{c-b x}} \operatorname{Log} \left[2 \sqrt{c-b x} \sqrt{a+b x} - \pm (a-c+2 b x) \right]}{b \sqrt{a+b x}}$$

Problem 740: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{\frac{a+b x}{c+d x}}}{a+b x} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{d} \sqrt{\frac{a+b x}{c+d x}}}{\sqrt{b}} \right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 89 leaves):

$$\frac{\sqrt{\frac{a+b x}{c+d x}} \sqrt{c+d x} \operatorname{Log} \left[b c + a d + 2 b d x + 2 \sqrt{b} \sqrt{d} \sqrt{a+b x} \sqrt{c+d x} \right]}{\sqrt{b} \sqrt{d} \sqrt{a+b x}}$$

Problem 759: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{x + \sqrt{-3 - 4 x - x^2}} dx$$

Optimal (type 3, 108 leaves, 10 steps):

$$-\operatorname{ArcTan} \left[\frac{\sqrt{-1-x}}{\sqrt{3+x}} \right] - \sqrt{2} \operatorname{ArcTan} \left[\frac{1 - \frac{3 \sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}} \right] + \frac{1}{2} \operatorname{Log} [3+x] + \frac{1}{2} \operatorname{Log} \left[\frac{3 \sqrt{-1-x} + \sqrt{-1-x} x + x \sqrt{3+x}}{(3+x)^{3/2}} \right]$$

Result (type 3, 1012 leaves):

$$\begin{aligned}
& \frac{1}{8} \left(4 \operatorname{ArcSin}[2+x] - 4\sqrt{2} \operatorname{ArcTan}[\sqrt{2} (1+x)] + 2\sqrt{1-2\frac{i}{2}\sqrt{2}} \right. \\
& \operatorname{ArcTan} \left[\left(60 + 51\frac{i}{2}\sqrt{2} + (-16 + 6\frac{i}{2}\sqrt{2})x^4 + 54\frac{i}{2}\sqrt{1-2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} + x \left(68 + 176\frac{i}{2}\sqrt{2} + 99\frac{i}{2}\sqrt{1-2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) + \right. \right. \\
& 2\frac{i}{2}x^3 \left(34 \left(\frac{i}{2} + \sqrt{2} \right) + 9\sqrt{1-2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) + \frac{i}{2}x^2 \left(44\frac{i}{2} + 185\sqrt{2} + 72\sqrt{1-2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) \left. \right) \Big/ \\
& \left(93\frac{i}{2} + 150\sqrt{2} + 20 \left(17\frac{i}{2} + 22\sqrt{2} \right)x + \left(493\frac{i}{2} + 466\sqrt{2} \right)x^2 + 16 \left(19\frac{i}{2} + 13\sqrt{2} \right)x^3 + \left(66\frac{i}{2} + 32\sqrt{2} \right)x^4 \right)] - \frac{1}{\sqrt{1+2\frac{i}{2}\sqrt{2}}} 2\frac{i}{2} \left(-\frac{i}{2} + 2\sqrt{2} \right) \\
& \operatorname{ArcTan} \left[\left(-60 + 51\frac{i}{2}\sqrt{2} + 2 \left(8 + 3\frac{i}{2}\sqrt{2} \right)x^4 + 54\frac{i}{2}\sqrt{1+2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} + 2x^3 \left(34 + 34\frac{i}{2}\sqrt{2} + 9\frac{i}{2}\sqrt{1+2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) + \right. \right. \\
& x^2 \left(44 + 185\frac{i}{2}\sqrt{2} + 72\frac{i}{2}\sqrt{1+2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) + \frac{i}{2}x \left(68\frac{i}{2} + 176\sqrt{2} + 99\sqrt{1+2\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) \Big) \Big/ \\
& \left(-93\frac{i}{2} + 150\sqrt{2} + 20 \left(-17\frac{i}{2} + 22\sqrt{2} \right)x + \left(-493\frac{i}{2} + 466\sqrt{2} \right)x^2 + 16 \left(-19\frac{i}{2} + 13\sqrt{2} \right)x^3 + \left(-66\frac{i}{2} + 32\sqrt{2} \right)x^4 \right)] + \\
& 2 \operatorname{Log}[3+4x+2x^2] + \frac{\left(-\frac{i}{2} + 2\sqrt{2} \right) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2\frac{i}{2}\sqrt{2}}} + \frac{\left(\frac{i}{2} + 2\sqrt{2} \right) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2\frac{i}{2}\sqrt{2}}} - \frac{1}{\sqrt{1-2\frac{i}{2}\sqrt{2}}} \\
& \left(\frac{1}{\sqrt{1+2\frac{i}{2}\sqrt{2}}} \left(-\frac{i}{2} + 2\sqrt{2} \right) \operatorname{Log}[(3+4x+2x^2) \left(3+6\frac{i}{2}\sqrt{2} + (2+2\frac{i}{2}\sqrt{2})x^2 - 2\sqrt{2-4\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} + x \left(4+8\frac{i}{2}\sqrt{2} - 2\sqrt{2-4\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) \right)] - \right. \\
& \left. \left. \operatorname{Log}[(3+4x+2x^2) \left(3-6\frac{i}{2}\sqrt{2} + (2-2\frac{i}{2}\sqrt{2})x^2 - 2\sqrt{2+4\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} - 2x \left(-2+4\frac{i}{2}\sqrt{2} + \sqrt{2+4\frac{i}{2}\sqrt{2}}\sqrt{-3-4x-x^2} \right) \right)] \right)
\end{aligned}$$

Problem 760: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x + \sqrt{-3-4x-x^2})^2} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$\begin{aligned}
& \frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\operatorname{ArcTan} \left[\frac{1 - 3\sqrt{-1-x}}{\sqrt{2}} \right]}{\sqrt{2}}
\end{aligned}$$

Result (type 3, 881 leaves):

$$\begin{aligned}
& \frac{1}{16} \left(\frac{8(3+x)}{3+4x+2x^2} + \frac{8(3+2x)\sqrt{-3-4x-x^2}}{3+4x+2x^2} + 4\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] - \frac{1}{\sqrt{1+2\sqrt{2}}} \right. \\
& 2\sqrt{2} \operatorname{ArcTan}[(2+x)(3(5+4\sqrt{2}) + 16(2+\sqrt{2})x + 2(9+2\sqrt{2})x^2)] \Big/ (12\sqrt{2} - 6\sqrt{2} + (8\sqrt{2} + 6\sqrt{2})x^3 - \\
& 9\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2} + x(40\sqrt{2} - 5\sqrt{2} - 12\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2}) + x^2(36\sqrt{2} + 8\sqrt{2} - 6\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2})) + \\
& \frac{1}{\sqrt{1-2\sqrt{2}}} 2(2\sqrt{2}) \operatorname{ArcTanh}[(2+x)(3(5\sqrt{2} + 4\sqrt{2}) + 16(2\sqrt{2})x + 2(9\sqrt{2} + 2\sqrt{2})x^2)] \Big/ (-5(8\sqrt{2})x + (-8\sqrt{2} + 6\sqrt{2})x^3 - \\
& 12\sqrt{1-2\sqrt{2}}x\sqrt{-3-4x-x^2} + x^2(-36\sqrt{2} + 8\sqrt{2} - 6\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2}) - 3(4\sqrt{2} + 2\sqrt{2} + 3\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2})) - \\
& (-2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2] - \frac{(2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2\sqrt{2}}} + \frac{1}{\sqrt{1-2\sqrt{2}}} \\
& (2\sqrt{2}) \operatorname{Log}[(3+4x+2x^2)(3+6\sqrt{2} + (2+2\sqrt{2})x^2 - 2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2}) + x(4+8\sqrt{2} - 2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2})] + \\
& \frac{1}{\sqrt{1+2\sqrt{2}}} (-2\sqrt{2}) \\
& \left. \operatorname{Log}[(3+4x+2x^2)(3-6\sqrt{2} + (2-2\sqrt{2})x^2 - 2\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2} - 2x(-2+4\sqrt{2} + \sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2}))] \right)
\end{aligned}$$

Problem 761: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(x + \sqrt{-3 - 4x - x^2})^3} dx$$

Optimal (type 3, 149 leaves, 6 steps):

$$\begin{aligned}
& -\frac{13 - \frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{\frac{1}{2}\operatorname{ArcTan}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{2\sqrt{2}}
\end{aligned}$$

Result (type 3, 914 leaves):

$$\begin{aligned}
& \frac{1}{32} \left(\frac{8(-3+2x)}{(3+4x+2x^2)^2} - \frac{8(2+3x)}{3+4x+2x^2} - \frac{8\sqrt{-3-4x-x^2}(15+26x+22x^2+8x^3)}{(3+4x+2x^2)^2} - 12\sqrt{2} \operatorname{ArcTan}[\sqrt{2}(1+x)] + \frac{1}{\sqrt{1+2\sqrt{2}}} \right. \\
& 6(2+\sqrt{2}) \operatorname{ArcTan}[(2+x)(3(5+4\sqrt{2})+16(2+\sqrt{2})x+2(9+2\sqrt{2})x^2)] / \left(12\sqrt{2} + (8\sqrt{2})x^3 - \right. \\
& 9\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2} + x \left(40\sqrt{2} - 12\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2} \right) + x^2 \left(36\sqrt{2} - 6\sqrt{1+2\sqrt{2}}\sqrt{-3-4x-x^2} \right) \left. \right) - \\
& \frac{1}{\sqrt{1-2\sqrt{2}}} 6(2\sqrt{2}) \operatorname{ArcTanh}[(2+x)(3(5\sqrt{2})+16(2\sqrt{2})x+2(9\sqrt{2})x^2)] / \left(-5(8\sqrt{2})x^3 - \right. \\
& 12\sqrt{1-2\sqrt{2}}x\sqrt{-3-4x-x^2} + x^2 \left(-36\sqrt{2} - 6\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2} \right) - 3(4\sqrt{2} + 3\sqrt{1-2\sqrt{2}}\sqrt{-3-4x-x^2}) \left. \right) + \\
& \frac{3(-2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1+2\sqrt{2}}} + \frac{3(2\sqrt{2}) \operatorname{Log}[4(3+4x+2x^2)^2]}{\sqrt{1-2\sqrt{2}}} - \frac{1}{\sqrt{1-2\sqrt{2}}} 3(2\sqrt{2}) \\
& \operatorname{Log}[(3+4x+2x^2) \left(3+6\sqrt{2} + (2+2\sqrt{2})x^2 - 2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2} + x(4+8\sqrt{2} - 2\sqrt{2-4\sqrt{2}}\sqrt{-3-4x-x^2}) \right)] - \\
& \frac{1}{\sqrt{1+2\sqrt{2}}} 3(-2\sqrt{2}) \\
& \left. \operatorname{Log}[(3+4x+2x^2) \left(3-6\sqrt{2} + (2-2\sqrt{2})x^2 - 2\sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2} - 2x(-2+4\sqrt{2} + \sqrt{2+4\sqrt{2}}\sqrt{-3-4x-x^2}) \right)] \right)
\end{aligned}$$

Problem 764: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps):

$$\begin{aligned}
& \frac{2}{35} (13 - 3(-1+x)^2) \sqrt{3 - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{7} (3 - 2(-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\
& \frac{16}{5} \sqrt{3} \operatorname{EllipticE}[\operatorname{ArcSin}[1-x], -\frac{1}{3}] - \frac{176}{35} \sqrt{3} \operatorname{EllipticF}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]
\end{aligned}$$

Result (type 4, 278 leaves):

$$\left(\begin{array}{l}
 896 - 1056 x + 352 x^2 + 848 x^3 - 1420 x^4 + 1152 x^5 - 602 x^6 + \\
 112 i \sqrt{2} (-2 + x) x \sqrt{\frac{4 - 2x + x^2}{x^2}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i + \sqrt{3} - 4i}{\sqrt{2} 3^{1/4}}}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-i + \sqrt{3}} \right] \\
 206 x^7 - 45 x^8 + 5 x^9 + \sqrt{-\frac{\frac{i(-2+x)}{(-i + \sqrt{3})x}}{}} \\
 304 i \sqrt{2} \sqrt{-\frac{\frac{i(-2+x)}{(-i + \sqrt{3})x}}{}} x^2 \sqrt{\frac{4 - 2x + x^2}{x^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i + \sqrt{3} - 4i}{\sqrt{2} 3^{1/4}}}}{\sqrt{2} 3^{1/4}} \right], \frac{2\sqrt{3}}{-i + \sqrt{3}} \right] \Bigg) \Bigg/ \left(35 \sqrt{-x(-8 + 8x - 4x^2 + x^3)} \right)
 \end{array} \right)$$

Problem 765: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\frac{1}{3} \sqrt{3 - 2(-1 + x)^2 - (-1 + x)^4} (-1 + x) + \frac{2 \text{EllipticE}[\text{ArcSin}[1 - x], -\frac{1}{3}]}{\sqrt{3}} - \frac{4 \text{EllipticF}[\text{ArcSin}[1 - x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 256 leaves):

$$\begin{aligned}
 & - \left(\left(-16 + 24x - 24x^2 + 14x^3 - 5x^4 + x^5 - \frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}[\operatorname{ArcSin}[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{x}}}{\sqrt{2}3^{1/4}}], \frac{2\sqrt{3}}{-i+\sqrt{3}}]}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}} \right. \right. \\
 & \left. \left. + \frac{8i\sqrt{2}\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}x^2\sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticF}[\operatorname{ArcSin}[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{x}}}{\sqrt{2}3^{1/4}}], \frac{2\sqrt{3}}{-i+\sqrt{3}}]}{3\sqrt{-x(-8+8x-4x^2+x^3)}} \right) \right)
 \end{aligned}$$

Problem 766: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$\frac{\operatorname{EllipticF}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 156 leaves):

$$\frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}}\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}x(-4+x-i\sqrt{3}x)\operatorname{EllipticF}[\operatorname{ArcSin}[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{x}}}{\sqrt{2}3^{1/4}}], \frac{2\sqrt{3}}{-i+\sqrt{3}}]}{\sqrt{2}\sqrt{\frac{i+\sqrt{3}-4i}{x}}\sqrt{-x(-8+8x-4x^2+x^3)}}$$

Problem 767: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$\frac{\left(5 + (-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{\operatorname{EllipticE}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]}{8\sqrt{3}} - \frac{\operatorname{EllipticF}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]}{4\sqrt{3}}$$

Result (type 4, 261 leaves):

$$\frac{1}{24 (-2+x) x} \sqrt{-x (-8+8x-4x^2+x^3)} \left(\begin{array}{l} \frac{\sqrt{2} \left(-\frac{i}{2}+\sqrt{3}\right) \sqrt{-\frac{i (-2+x)}{\left(-\frac{i}{2}+\sqrt{3}\right) x}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}}\right]}{\sqrt{\frac{4-2 x+x^2}{x^2}}} - \\ \frac{2+x^2-4 \frac{i}{2} \sqrt{2} \sqrt{-\frac{i (-2+x)}{\left(-\frac{i}{2}+\sqrt{3}\right) x}} x^2 \sqrt{\frac{4-2 x+x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}}\right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}}\right]}{4-2 x+x^2} \end{array} \right)$$

Problem 768: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(8x^2 - 8x^3 + 4x^4 - x^5)^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\frac{\left(5+(-1+x)^2\right) (-1+x)}{72 \left(3-2 (-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\left(26+7 (-1+x)^2\right) (-1+x)}{432 \sqrt{3-2 (-1+x)^2-(-1+x)^4}} + \frac{7 \text{EllipticE}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{144 \sqrt{3}} - \frac{11 \text{EllipticF}\left[\text{ArcSin}[1-x], -\frac{1}{3}\right]}{144 \sqrt{3}}$$

Result (type 4, 298 leaves):

$$\begin{aligned}
& \frac{7 \pm \sqrt{2} (-2+x) x^2 \sqrt{\frac{4-2x+x^2}{x^2}} \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{\sqrt{2} 3^{1/4}}}{x}}{x}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}]}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}} + \\
& \frac{1}{-8+8x-4x^2+x^3} \left\{ 36 - 232x + 274x^2 - 226x^3 + 115x^4 - 37x^5 + 7x^6 - 19 \pm \sqrt{2} \sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}} x^3 \sqrt{\frac{4-2x+x^2}{x^2}} \right. \\
& \left. (-8+8x-4x^2+x^3) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{i+\sqrt{3}-4i}{\sqrt{2} 3^{1/4}}}{x}}{x}\right], \frac{2\sqrt{3}}{-i+\sqrt{3}}] \right\} \Big/ \left(432x \sqrt{-x(-8+8x-4x^2+x^3)} \right)
\end{aligned}$$

Problem 769: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int ((2-x) \times (4-2x+x^2))^{3/2} dx$$

Optimal (type 4, 102 leaves, 7 steps):

$$\begin{aligned}
& \frac{2}{35} (13 - 3 (-1+x)^2) \sqrt{3 - 2 (-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{7} (3 - 2 (-1+x)^2 - (-1+x)^4)^{3/2} (-1+x) + \\
& \frac{16}{5} \sqrt{3} \operatorname{EllipticE}[\operatorname{ArcSin}[1-x], -\frac{1}{3}] - \frac{176}{35} \sqrt{3} \operatorname{EllipticF}[\operatorname{ArcSin}[1-x], -\frac{1}{3}]
\end{aligned}$$

Result (type 4, 278 leaves):

$$\begin{aligned}
& \frac{1}{35 (-2+x) x \sqrt{\frac{4-2 x+x^2}{x^2}}} \\
& \sqrt{-x (-8+8 x-4 x^2+x^3)} \left(\sqrt{\frac{4-2 x+x^2}{x^2}} (-224+152 x+44 x^2-228 x^3+230 x^4-116 x^5+35 x^6-5 x^7) + 112 \sqrt{2} (-\frac{i}{2}+\sqrt{3}) \sqrt{-\frac{\frac{i}{2} (-2+x)}{(-\frac{i}{2}+\sqrt{3}) x}} \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}} \right] + 304 \frac{i}{2} \sqrt{2} \sqrt{-\frac{\frac{i}{2} (-2+x)}{(-\frac{i}{2}+\sqrt{3}) x}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}} \right] \right)
\end{aligned}$$

Problem 770: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{(2-x) x (4-2 x+x^2)} \, dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\frac{1}{3} \sqrt{3-2 (-1+x)^2-(-1+x)^4} (-1+x) + \frac{2 \text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}} - \frac{4 \text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 256 leaves):

$$\begin{aligned}
& \frac{1}{3 (-2+x) x \sqrt{\frac{4-2 x+x^2}{x^2}}} \sqrt{-x (-8+8 x-4 x^2+x^3)} \\
& \left(\sqrt{\frac{4-2 x+x^2}{x^2}} (-4+4 x-3 x^2+x^3) + 2 \sqrt{2} (-\frac{i}{2}+\sqrt{3}) \sqrt{-\frac{\frac{i}{2} (-2+x)}{(-\frac{i}{2}+\sqrt{3}) x}} \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}} \right] + \right. \\
& \left. 8 \frac{i}{2} \sqrt{2} \sqrt{-\frac{\frac{i}{2} (-2+x)}{(-\frac{i}{2}+\sqrt{3}) x}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{i}{2}+\sqrt{3}-\frac{4 i}{x}}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{-\frac{i}{2}+\sqrt{3}} \right] \right)
\end{aligned}$$

Problem 771: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(2-x) \times (4-2x+x^2)}} dx$$

Optimal (type 4, 17 leaves, 3 steps):

$$-\frac{\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 100 leaves):

$$-\frac{(-1)^{1/3} (-2+x)^2 \sqrt{\frac{x (-1+i \sqrt{3})+x}{(-2+x)^2}} \sqrt{\frac{-2+x-(-1)^{1/3} x}{-2+x}} \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{(-1)^{2/3} x}{-2+x}}\right], (-1)^{2/3}]}{\sqrt{-x (-8+8 x-4 x^2+x^3)}}$$

Problem 772: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{((2-x) \times (4-2x+x^2))^{3/2}} dx$$

Optimal (type 4, 73 leaves, 6 steps):

$$\frac{\left(5+(-1+x)^2\right) (-1+x)}{24 \sqrt{3-2 (-1+x)^2-(-1+x)^4}} + \frac{\text{EllipticE}[\text{ArcSin}[1-x], -\frac{1}{3}]}{8 \sqrt{3}} - \frac{\text{EllipticF}[\text{ArcSin}[1-x], -\frac{1}{3}]}{4 \sqrt{3}}$$

Result (type 4, 298 leaves):

Problem 773: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left((2-x) \times (4-2x+x^2) \right)^{5/2}} dx$$

Optimal (type 4, 109 leaves, 7 steps):

$$\frac{\left(5 + (-1 + x)^2\right) (-1 + x)}{72 \left(3 - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2}} + \frac{\left(26 + 7 (-1 + x)^2\right) (-1 + x)}{432 \sqrt{3 - 2 (-1 + x)^2 - (-1 + x)^4}} + \frac{7 \text{EllipticE}\left[\text{ArcSin}[1 - x], -\frac{1}{3}\right]}{144 \sqrt{3}} - \frac{11 \text{EllipticF}\left[\text{ArcSin}[1 - x], -\frac{1}{3}\right]}{144 \sqrt{3}}$$

Result (type 4, 327 leaves):

$$\begin{aligned}
& \left(-2 + x \right)^3 x^2 \left(4 - 2 x + x^2 \right)^2 \\
& - \frac{7 x \left(4 - 2 x + x^2 \right)}{-2 + x} + \frac{36 + 216 x - 622 x^2 + 670 x^3 - 445 x^4 + 187 x^5 - 49 x^6 + 7 x^7}{\left(-2 + x \right)^2 x \left(4 - 2 x + x^2 \right)} + \frac{7 i \sqrt{2} x \sqrt{\frac{4 - 2 x + x^2}{\left(-2 + x \right)^2}} \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-i + \sqrt{3}} - \frac{4 i}{-2 + x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{i + \sqrt{3}} \right]}{\sqrt{\frac{i x}{\left(i + \sqrt{3} \right) \left(-2 + x \right)}}} - 19 \\
& \left. \frac{i \sqrt{2} \left(-2 + x \right) \sqrt{\frac{i x}{\left(i + \sqrt{3} \right) \left(-2 + x \right)}} \sqrt{\frac{4 - 2 x + x^2}{\left(-2 + x \right)^2}} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{-i + \sqrt{3}} - \frac{4 i}{-2 + x}}{\sqrt{2} 3^{1/4}} \right], \frac{2 \sqrt{3}}{i + \sqrt{3}} \right]}{\left(432 \left(-x \left(-8 + 8 x - 4 x^2 + x^3 \right) \right)^{5/2} \right)} \right)
\end{aligned}$$

Problem 774: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^{3/2} dx$$

Optimal (type 4, 730 leaves, 6 steps):

$$\frac{1}{7} \left(\frac{c}{d} + x \right) \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4 \right)^{3/2} + \frac{2 c \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \left(7 c^3 + 20 a d^2 - 3 c d^2 \left(\frac{c}{d} + x \right)^2 \right)}{35 d^2} - \frac{16 c^3 \left(c^3 + 8 a d^2 \right) \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{35 d^2 \sqrt{c^3 + 4 a d^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)} +$$

$$\left(16 c^{13/4} \left(c^3 + 4 a d^2 \right)^{3/4} \left(c^3 + 8 a d^2 \right) \sqrt{\frac{d^2 \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4 \right)}{\left(c^3 + 4 a d^2 \right) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \right)$$

$$\text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} \left(c^3 + 4 a d^2 \right)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] / \left(35 d^5 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) +$$

$$\left(8 c^{7/4} \left(c^3 + 4 a d^2 \right)^{3/4} \left(\sqrt{c^3 + 4 a d^2} \left(c^3 + 5 a d^2 \right) - c^{3/2} \left(c^3 + 8 a d^2 \right) \right) \sqrt{\frac{d^2 \left(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4 \right)}{\left(c^3 + 4 a d^2 \right) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \right)$$

$$\text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} \left(c^3 + 4 a d^2 \right)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] / \left(35 d^5 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)$$

Result (type 4, 10468 leaves):

$$\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \left(\frac{4 c^2 \left(2 c^3 + 15 a d^2 \right)}{35 d^3} - \frac{4 c \left(c^3 - 15 a d^2 \right) x}{35 d^2} + \frac{2 c^3 x^2}{35 d} + \frac{34 c^2 x^3}{35} + \frac{5}{7} c d x^4 + \frac{d^2 x^5}{7} \right) +$$

$$\frac{1}{35 d^3} 16 c^2 \left(2 a c^3 d \left(\frac{-c - \sqrt{c^2 - 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{\left(\frac{-c + \sqrt{c^2 - 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)} \right. \\ \left. \left(\frac{-c - \sqrt{c^2 - 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{1}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right) \right)$$

$$\begin{aligned}
& \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)} \right], \\
& \left. \frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right)} \right] / \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \\
& \left(40 a^2 d^3 \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{\frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)} \right], \\
& \left. \frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right)} \right] / \\
& \left(\left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - 8 c^5 \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d} \left(\frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(\frac{-c - \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{i}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\sqrt{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \\
& - \frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \\
& \left(\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}} \right)^2 + \frac{1}{d} 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \\
& \left. \text{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \left(\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}} \right)^2 \right] \Bigg/ \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(64 a c^2 d^2 \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \right. \\
& \left. \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right. \\
& \left. \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \right. \\
& \left. \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \right], \right]
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}}, \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \left. d \left(\frac{\left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} \right. \right. \\
& \left. \left. \frac{\left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} \right) \text{EllipticF}\left[\right. \right. \\
& \left. \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right\} \left/ \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right. \right. \\
& \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right) - \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right. \right. \\
& \left. \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \right], \\
& \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right\} \Bigg/ \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \Bigg] - \\
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} \frac{8 a c d^3}{8 a c d^3} \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \\
& \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) + 2 \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}} d \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{ \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + d \left(\frac{\left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} - \right. \right. \\
& \left. \left. \frac{\left(-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} \right) \text{EllipticF} \left[\right. \right. \\
& \left. \left. \text{ArcSin} \left[\sqrt{ \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right/ \left(2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right. \\
& \left. \left. \left. \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) - \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \right. \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right\} \text{EllipticPi} \left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \\
& \left. \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}}, \right. \right. \\
& \left. \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] \right\} \left/ \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right) \right]
\end{aligned}$$

Problem 775: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \, dx$$

Optimal (type 4, 622 leaves, 5 steps):

$$\frac{1}{3} \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} - \frac{2 c^2 \left(\frac{c}{d} + x \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{3 \sqrt{c^3 + 4 a d^2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)} +$$

$$\left(2 c^{9/4} \left(c^3 + 4 a d^2 \right)^{3/4} \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \left(3 d^3 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) +$$

$$\left(c^{3/4} (c^3 + 4 a d^2)^{1/4} \left(c^3 + 4 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2} \right) \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \left(3 d^3 \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)$$

Result (type 4, 5218 leaves):

$$\left(\frac{c}{3 d} + \frac{x}{3} \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} +$$

$$\frac{1}{3 d} 2 c \left(8 a d \left(\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right)$$

$$\begin{aligned}
& \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)} } \right], \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right] / \\
& \left. \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right] \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - \left(8 c^2 \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\sqrt{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \operatorname{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \left(\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right) \right] / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) -
\end{aligned}$$

Problem 776: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} dx$$

Optimal (type 4, 227 leaves, 2 steps):

$$\left(\frac{(c^3 + 4 a d^2)^{1/4}}{(c^3 + 4 a d^2)} \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 (\frac{c}{d} + x)^2}{\sqrt{c^3 + 4 a d^2}} \right)^2}} \left(\sqrt{c} + \frac{d^2 (\frac{c}{d} + x)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}} \right) \right] \right) / \left(2 c^{1/4} d \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right)$$

Result (type 4, 822 leaves):

$$\begin{aligned} & \left(2 \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right) \sqrt{- \frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \right. \\ & \left. - \frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \text{EllipticF} \right. \\ & \left. \text{ArcSin} \left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] / \\ & \left(d \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \sqrt{4 a c + x^2 (2 c + d x)^2} \right) \end{aligned}$$

Problem 777: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{(4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)^{3/2}} dx$$

Optimal (type 4, 674 leaves, 5 steps):

$$\begin{aligned} & - \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4 a d^2 - c d^2 \left(\frac{c}{d} + x\right)^2\right)}{8 a c (c^3 + 4 a d^2) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} - \frac{d^2 \left(\frac{c}{d} + x\right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}}{8 a (c^3 + 4 a d^2)^{3/2} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)} + \\ & \left(c^{1/4} \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}}\right)\right] \right) / \\ & \left(8 a d (c^3 + 4 a d^2)^{1/4} \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) + \left(c^3 + 4 a d^2 - c^{3/2} \sqrt{c^3 + 4 a d^2} \right) \sqrt{\frac{d^2 (4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4)}{(c^3 + 4 a d^2) \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}}\right)^2}} \left(\sqrt{c} + \frac{d^2 \left(\frac{c}{d} + x\right)^2}{\sqrt{c^3 + 4 a d^2}} \right) \\ & \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c + d x}{c^{1/4} (c^3 + 4 a d^2)^{1/4}}\right], \frac{1}{2} \left(1 + \frac{c^{3/2}}{\sqrt{c^3 + 4 a d^2}}\right)\right] / \left(16 a c^{5/4} d (c^3 + 4 a d^2)^{3/4} \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) \end{aligned}$$

Result (type 4, 5276 leaves):

$$\begin{aligned} & \frac{4 a c d + 2 c^3 x + 4 a d^2 x + 3 c^2 d x^2 + c d^2 x^3}{8 a c (c^3 + 4 a d^2) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} + \frac{1}{8 a c (c^3 + 4 a d^2)} \\ & d \left(\left(8 a d \left(\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \right) \sqrt{\frac{\left(\frac{-c + \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(\frac{-c - \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \frac{c}{d} \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \right) \end{aligned}$$

$$\begin{aligned}
& \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)} } \right], \\
& \left. \left(\begin{array}{c} \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \\ \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \end{array} \right) \right] / \\
& \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) - 8 c^2 \left(\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \\
& \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \sqrt{\frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}{\sqrt{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \\
& \left(-\frac{1}{d} \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \frac{1}{d} 2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \operatorname{EllipticPi}\left[\frac{-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}{-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d}}, \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \left(\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right) \right] / \left(\left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \right. \\
& \left. \left(\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{4 a c + 4 c^2 x^2 + 4 c d x^3 + d^2 x^4}} c d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \left(-\frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right) + \\
& 2 \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)^2 \\
& \sqrt{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)} \\
& \sqrt{d \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)} \\
& \sqrt{d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + x \right)} \\
& \sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)}} \left(\frac{1}{2 \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}} d \left(-\frac{-c - \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} + \right. \right. \\
& \left. \left. \frac{-c - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + d x \right)}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - d x \right)} \right], \right. \\
& \left. \left. \frac{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2}{\left(\sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} - \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d} \right)^2} \right] + \left(d \left(\frac{\left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} - \right. \right. \\
& \left. \left. \frac{\left(-c + \sqrt{c^2 - 2 \pm \sqrt{a} \sqrt{c} d} \right) \left(-\frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} - \frac{-c + \sqrt{c^2 + 2 \pm \sqrt{a} \sqrt{c} d}}{d} \right)}{d} \right) \right)
\end{aligned}$$

Problem 778: Result more than twice size of optimal antiderivative.

$$\int \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \, dx$$

Optimal (type 4, 663 leaves, 5 steps):

$$\frac{1}{3} \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} - \frac{2 d^2 \left(\frac{d}{4e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{\sqrt{5 d^4 + 256 a e^3} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} +$$

$$\left. \left(d^2 (5 d^4 + 256 a e^3)^{3/4} \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right. \right. \\ \left. \left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) \right/ \left(8 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) +$$

$$\left. \left((5 d^4 + 256 a e^3)^{1/4} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3} \right) \sqrt{\frac{e (8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)}{(5 d^4 + 256 a e^3) \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right. \right. \\ \left. \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{(5 d^4 + 256 a e^3)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) \right/ \left(48 \sqrt{2} e^2 \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 7543 leaves):

$$\left(\frac{d}{12e} + \frac{x}{3} \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} + \frac{1}{24e}$$

$$\left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \sqrt{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)$$

$$\sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \right],$$

$$\begin{aligned}
& \left(\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right] / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
& \left(\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) + 256ae^3 \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
& \sqrt{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} / \\
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} \\
& \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} / \\
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x}{4e} \right)} \\
& \sqrt{\left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x}{4e} \right)} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x}{4e} \right)}{\left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x}{4e} \right)} \right], \\
& \left. \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \right] / \\
& \left. \left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \right] / \\
& \left(\left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \right. \\
& \left. \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) - \\
& \left. 12d^3e \left(\frac{-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} \right) \left(\frac{-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e}}{4e} + x \right)^2 \right)
\end{aligned}$$

$$\frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}$$

$$\frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right)}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right)}$$

$$\left(-\frac{1}{4e} \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)} \right. \right. \\ \left. \left. \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right] \right) \Big/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \\ \left. \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} + \frac{1}{2e}$$

$$\begin{aligned}
& \frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{\sqrt{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}} \\
& \frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{\sqrt{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}} \\
& \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right)}{\sqrt{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right)}} \\
& \left(2 e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \text{EllipticE}[\text{ArcSin}\left[\sqrt{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) \right] / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \\
& \left. \left. \left. \left. \left. \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right) \right) \right], \frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2} \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) + \left(2 e \left(\frac{\left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right)}{4 e} - \right. \right. \\
& \left. \left. \frac{\left(-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right)}{4 e} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) \right) \right] \left/ \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right) \right] \right], \left. \left. \left. \left. \left. \left. \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \right) \right/ \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right)^2 \right] \right) \right/ \\
& \left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) - \\
& \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi}\left[\frac{-\frac{-d-\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}{4e} + \frac{-d+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}{4e}}{4e}, \text{ArcSin}\left[\frac{-\frac{-d-\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}{4e} + \frac{-d+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}{4e}}{4e}\right]\right] \\
& \sqrt{\left(\left(\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}} - \sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(d + \sqrt{3d^2-2\sqrt{d^4-64ae^3}} + 4ex\right)\right)\right.} \\
& \left.\left(\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}} + \sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d + \sqrt{3d^2-2\sqrt{d^4-64ae^3}} - 4ex\right)\right)\right], \\
& \left.\left.\left.\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}} + \sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)^2\right]\right/ \left(-\frac{-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}{4e} + \frac{-d+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}{4e}\right)\right\}
\end{aligned}$$

Problem 779: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} dx$$

Optimal (type 4, 235 leaves, 2 steps):

$$\left(5 d^4 + 256 a e^3 \right)^{1/4} \sqrt{\frac{e \left(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4 \right)}{\left(5 d^4 + 256 a e^3 \right) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \\ \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{\left(5 d^4 + 256 a e^3 \right)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) \Big/ \left(\sqrt{2} e \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right)$$

Result (type 4, 1065 leaves):

$$\begin{aligned}
& - \left(\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \left(d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right. \\
& \left. - \frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)} \right. \\
& \left. \sqrt{\left(3d^2 - 2\sqrt{d^4 - 64ae^3} - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} + } \right. \\
& \left. d \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) + 4e \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) x \right) / \\
& \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \Bigg) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)} \right], \right. \\
& \left. \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2 \right] / \left(2e \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \\
& \left. \left. \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) / \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right)
\end{aligned}$$

Problem 780: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4)^{3/2}} dx$$

Optimal (type 4, 748 leaves, 5 steps):

$$\begin{aligned} & \frac{4 e \left(\frac{d}{4 e} + x \right) \left(13 d^4 - 256 a e^3 - 48 d^2 e^2 \left(\frac{d}{4 e} + x \right)^2 \right)}{\left(5 d^8 - 64 a d^4 e^3 - 16384 a^2 e^6 \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} + \frac{384 d^2 e^2 \left(\frac{d}{4 e} + x \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}{\left(d^4 - 64 a e^3 \right) \left(5 d^4 + 256 a e^3 \right)^{3/2} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)} - \\ & \left(12 \sqrt{2} d^2 \sqrt{\frac{e \left(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4 \right)}{\left(5 d^4 + 256 a e^3 \right) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{\left(5 d^4 + 256 a e^3 \right)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) / \\ & \left(\left(d^4 - 64 a e^3 \right) \left(5 d^4 + 256 a e^3 \right)^{1/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) - \\ & \left(2 \sqrt{2} \left(5 d^4 + 256 a e^3 - 3 d^2 \sqrt{5 d^4 + 256 a e^3} \right) \sqrt{\frac{e \left(8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4 \right)}{\left(5 d^4 + 256 a e^3 \right) \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right)^2}} \left(1 + \frac{16 e^2 \left(\frac{d}{4 e} + x \right)^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right. \\ & \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{d + 4 e x}{\left(5 d^4 + 256 a e^3 \right)^{1/4}} \right], \frac{1}{2} \left(1 + \frac{3 d^2}{\sqrt{5 d^4 + 256 a e^3}} \right) \right] \right) / \left(\left(d^4 - 64 a e^3 \right) \left(5 d^4 + 256 a e^3 \right)^{3/4} \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4} \right) \end{aligned}$$

Result (type 4, 7629 leaves):

$$\frac{2 \left(-5 d^5 + 128 a d e^3 - 8 d^4 e x + 512 a e^4 x + 72 d^3 e^2 x^2 + 96 d^2 e^3 x^3 \right)}{\left(-d^4 + 64 a e^3 \right) \left(5 d^4 + 256 a e^3 \right) \sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} - \frac{1}{\left(d^4 - 64 a e^3 \right) \left(5 d^4 + 256 a e^3 \right)} 8 e$$

$$\begin{aligned}
& \left(2 d^4 \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \right) \sqrt{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \\
& \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \\
& \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \right], \\
& \sqrt{\left(\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
& \left. \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right] / \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \right) + 256ae^3 \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right. \\
& \left. \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right. \\
& \left. \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \right) \\
& \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right) \\
& \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} \\
& \sqrt{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)} \right], \\
& \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
& \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \\
& \left(\left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \\
& \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} - \left(12d^3e \left(\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)^2 \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}} \\
& \sqrt{\frac{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(-\frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}{e \left(-\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + x \right)}} \\
& \sqrt{\frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right)}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right)}} \\
& \left(-\frac{1}{4e} \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) \right) \right) \left/ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right) \right) \right) \left. \right], \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} + \frac{1}{2e}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \operatorname{EllipticPi}\left[\frac{-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}{\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}}, \operatorname{ArcSin}\left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x\right)\right) / \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x\right)\right) \left(\frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}\right)^2}\right) / \left(\left(-\frac{d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e}\right) \left(\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} - \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e}\right)\right) \left(\frac{1}{\sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}}\right) - \frac{1}{\sqrt{8 a e^2 - d^3 x + 8 d e^2 x^3 + 8 e^3 x^4}} \\
& 24 d^2 e^2 \left(\left(-\frac{d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \left(-\frac{d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) \left(-\frac{d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right) + \right. \\
& \left. \frac{1}{2} \left(-\frac{d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \\
& \frac{\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} \left(-\frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)}{e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \left(-\frac{-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + x \right)} \\
& \sqrt{\left(\frac{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right)}{\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right)} \right)} \\
& \left(2 e \left(-\frac{-d - \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}}}{4 e} + \frac{-d - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}}}{4 e} \right) \text{EllipticE} \right. \\
& \left. \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + 4 e x \right) \right) / \right. \right. \\
& \left. \left. \left(\left(\sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} + \sqrt{3 d^2 + 2 \sqrt{d^4 - 64 a e^3}} \right) \left(-d + \sqrt{3 d^2 - 2 \sqrt{d^4 - 64 a e^3}} - 4 e x \right) \right) \right], \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) + \\
& 2e \left(\frac{\left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} - \right. \\
& \left. \frac{\left(-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \right) \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right)}{4e} \right) \text{EllipticF} \Big[\\
& \text{ArcSin} \left[\sqrt{\left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \right. \\
& \left. \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right], \\
& \left. \frac{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2}{\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2} \right) \Bigg/ \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} \left(\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \right. \right. \\
& \left. \left. \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{-d - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} - \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \text{EllipticPi} \left[\frac{-d - \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e}, \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right. \right. \\
& \quad \left. \left. - \frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right], \right. \\
& \text{ArcSin} \left[\sqrt{ \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + 4ex \right) \right) / \right. \\
& \quad \left. \left(\left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right) \left(-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} - 4ex \right) \right) \right], \\
& \left. \left. \left. \left(\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}} + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}} \right)^2 \right) / \left(-\frac{-d + \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}{4e} + \frac{-d + \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}}{4e} \right) \right) \right]
\end{aligned}$$

Problem 781: Result more than twice size of optimal antiderivative.

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 452 leaves, 8 steps):

$$\begin{aligned}
& -\frac{16 (7+2a) (1-\sqrt{4+a}) (1+\frac{(-1+x)^2}{1-\sqrt{4+a}}) (-1+x)}{35 \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35} (13+5a-3(-1+x)^2) \sqrt{3+a-2(-1+x)^2-(-1+x)^4} (-1+x) + \\
& \frac{1}{7} (3+a-2(-1+x)^2-(-1+x)^4)^{3/2} (-1+x) + \frac{16 (7+2a) (1-\sqrt{4+a}) \sqrt{1+\sqrt{4+a}} (1+\frac{(-1+x)^2}{1-\sqrt{4+a}}) \text{EllipticE}[\text{ArcTan}[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}]}{35 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \\
& \frac{4 (3+a) (16+5a) \sqrt{1+\sqrt{4+a}} (1+\frac{(-1+x)^2}{1-\sqrt{4+a}}) \text{EllipticF}[\text{ArcTan}[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}]}{35 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

Result (type 4, 6287 leaves):

$$\begin{aligned}
& \sqrt{a+8x-8x^2+4x^3-x^4} \left(\frac{1}{7} (-4-3a) + \frac{1}{35} (-32+15a) x + \frac{14x^2}{5} - \frac{66x^3}{35} + \frac{5x^4}{7} - \frac{x^5}{7} \right) + \\
& \frac{4}{35} \left(\left(40 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x \right) \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x \right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1+\sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1-\sqrt{-1-\sqrt{4+a}} + x \right)}}], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right) \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(46a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \left(\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right) \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(10a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}, \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg] \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(32a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right],\right.\right. \\
& \quad \left.\left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]+2\sqrt{-1-\sqrt{4+a}}\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}},\right. \\
& \quad \left.\left.\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\right)\Bigg] \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
28 & \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \frac{1}{2\sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& 4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right] \right)
\end{aligned}$$

Problem 782: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 397 leaves, 7 steps):

$$-\frac{2 \left(1-\sqrt{4+a}\right) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \frac{1}{3} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4} (-1+x) +$$

$$\frac{2 \left(1-\sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt[3]{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} +$$

$$\frac{2 (3+a) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 \sqrt[3]{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}$$

Result (type 4, 3470 leaves):

$$\begin{aligned} & \left(-\frac{1}{3} + \frac{x}{3}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} + \\ & \frac{2}{3} \left(\left(4 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)}}\right. \right. \\ & \left. \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)}}\right) \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)}}\right) \\ & \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x\right)}}\right], \right. \end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right) \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. 4 \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 783: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 144 leaves, 3 steps):

$$\frac{\sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}$$

Result (type 4, 540 leaves):

$$\begin{aligned} & \left(2 \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right) \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(1 + \sqrt{-1 + \sqrt{4 + a}} - x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}\right. \\ & \left. \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right) \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}\right) \\ & \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}\right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}\right] \Big/ \\ & \sqrt{-1 - \sqrt{4 + a}} \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \sqrt{a - x (-8 + 8 x - 4 x^2 + x^3)} \end{aligned}$$

Problem 784: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + 8 x - 8 x^2 + 4 x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 437 leaves, 7 steps):

$$\begin{aligned}
& \frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{2 (12 + 7 a + a^2) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \frac{\left(1 - \sqrt{4 + a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1 + x)}{2 (3 + a) (4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{\left(1 - \sqrt{4 + a}\right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2 (3 + a) (4 + a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{\sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2 (4 + a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}
\end{aligned}$$

Result (type 4, 3526 leaves):

$$\begin{aligned}
& \frac{(6 + a - 8 x - a x + 3 x^2 - x^3) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}}{2 (3 + a) (4 + a) (-a - 8 x + 8 x^2 - 4 x^3 + x^4)} + \\
& \frac{1}{2 (3 + a) (4 + a)} \left(\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \right. \\
& \left. \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \right) \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}\right], \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}\right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right) \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. 4 \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 785: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 517 leaves, 8 steps):

$$\begin{aligned}
& \frac{\left(5 + a + (-1 + x)^2\right) (-1 + x)}{6 (12 + 7 a + a^2) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2}} + \frac{\left(104 + 47 a + 5 a^2 + 4 (7 + 2 a) (-1 + x)^2\right) (-1 + x)}{12 (3 + a)^2 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \\
& \frac{\left(7 + 2 a\right) \left(1 - \sqrt{4 + a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3 (3 + a)^2 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \frac{\left(7 + 2 a\right) \left(1 - \sqrt{4 + a}\right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 (3 + a)^2 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{\left(16 + 5 a\right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12 (3 + a) (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}
\end{aligned}$$

Result (type 4, 6386 leaves):

$$\begin{aligned}
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \left(\frac{-6 - a + 8x + ax - 3x^2 + x^3}{6(3+a)(4+a)(-a - 8x + 8x^2 - 4x^3 + x^4)^2} + \frac{132 + 55a + 5a^2 - 188x - 71ax - 5a^2x + 84x^2 + 24ax^2 - 28x^3 - 8ax^3}{12(3+a)^2(4+a)^2(-a - 8x + 8x^2 - 4x^3 + x^4)} \right) + \\
& \frac{1}{12(3+a)^2(4+a)^2} \\
& \left(\frac{40 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2}{\sqrt{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right], \\
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right) \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(46a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(10a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}, \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg] \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(32a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \quad \left.\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \quad \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \quad \left.\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right],\right.\right. \\
& \quad \left.\left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]+2\sqrt{-1-\sqrt{4+a}}\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}},\right. \\
& \quad \left.\left.\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\right)\Bigg] \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
28 & \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) \right) \left/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \frac{1}{2\sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& 4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right] \right)
\end{aligned}$$

Problem 786: Result more than twice size of optimal antiderivative.

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 558 leaves, 14 steps):

$$\begin{aligned}
& \frac{3}{16} (4+a) \left(1 + (-1+x)^2\right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} + \frac{1}{8} \left(1 + (-1+x)^2\right) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2} - \\
& \frac{16(7+2a)}{35\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x) + \frac{2}{35} \left(13+5a-3(-1+x)^2\right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{1}{7} \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2} (-1+x) + \frac{3}{16} (4+a)^2 \operatorname{ArcTan}\left[\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}\right] + \\
& \frac{16(7+2a)}{\sqrt{1+\sqrt{4+a}}} \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right] + \\
& \frac{35}{\sqrt{\frac{1+(-1+x)^2}{1-\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} \\
& \frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}}{35\sqrt{\frac{1+(-1+x)^2}{1-\sqrt{4+a}}}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]
\end{aligned}$$

Result (type 4, 7235 leaves):

$$\begin{aligned}
& \frac{1}{a+8x-8x^2+4x^3-x^4} \left(\frac{1}{56} (52+11a) - \frac{1}{280} (116+55a)x + \frac{1}{80} (-36+25a)x^2 + \frac{74x^3}{35} - \frac{43x^4}{28} + \frac{17x^5}{28} - \frac{x^6}{8} \right) (a-x(-8+8x-4x^2+x^3))^{3/2} + \\
& \frac{1}{280(a+8x-8x^2+4x^3-x^4)^{3/2}} (a-x(-8+8x-4x^2+x^3))^{3/2} \\
& - \left(\left(2080 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right) \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 + \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right], \\
& \frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\Bigg] \Bigg] \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \\
& \left(208a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right], \\
& \frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\Bigg] \Bigg]
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(\frac{110a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \Bigg] \Bigg) / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(\frac{6944 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg)/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)+ \\
& \left(210a^2\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right.\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}} \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \left(-1-\sqrt{-1-\sqrt{4+a}}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \\
& \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]+2\sqrt{-1-\sqrt{4+a}}\text{EllipticPi}\left[\frac{\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}{-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}}, \right. \\
& \left.\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg)/ \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right)-\frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}
\end{aligned}$$

$$\begin{aligned}
896 & \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} } \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \right. \\
& \left. \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} } \right], \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& \left. \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) - \\
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 256a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \left(\frac{1}{2\sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticE}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}}\right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right]\right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Big] \Big]
\end{aligned}$$

Problem 787: Result more than twice size of optimal antiderivative.

$$\int x \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Optimal (type 4, 466 leaves, 12 steps):

$$\begin{aligned}
& \frac{1}{4} \left(1 + (-1+x)^2 \right) \sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2 \left(1 - \sqrt{4+a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{3 \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{1}{3} \sqrt{3+a-2(-1+x)^2-(-1+x)^4} (-1+x) + \\
& \frac{1}{4} \left(4+a \right) \text{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \right] + \frac{2 \left(1 - \sqrt{4+a} \right) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \text{EllipticE} \left[\text{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{3 \sqrt[3]{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{\frac{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \\
& \frac{2(3+a) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \text{EllipticF} \left[\text{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{3 \sqrt[3]{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{\frac{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

Result (type 4, 4389 leaves):

$$\begin{aligned}
& \left(\frac{1}{6} - \frac{x}{6} + \frac{x^2}{4} \right) \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} + \frac{1}{6 \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} \\
& - \left(\left(8 \left(-\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \right. \\
& \left. \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \right. \\
& \left. \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \right. \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right], \right]
\end{aligned}$$

$$\left(40 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right)$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}$$

$$\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right],$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}},$$

$$\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right]$$

$$\left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \right) +$$

$$\left(6 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right)$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& 4 \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \right. \\
& \left. \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \\
& \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right\} \left/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right\} \right/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right\}
\end{aligned}$$

Problem 788: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 179 leaves, 7 steps):

$$\frac{1}{2} \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \right] + \frac{\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + (-1+x)^2}{1 + \frac{(-1+x)^2}{1 + \sqrt{4+a}}}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}$$

Result (type 4, 865 leaves):

$$\begin{aligned}
& \left(2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a - x (-8 + 8x - 4x^2 + x^3)} \right)
\end{aligned}$$

Problem 789: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 474 leaves, 10 steps):

$$\begin{aligned}
& \frac{1 + (-1+x)^2}{2 (4+a) \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}} + \frac{\left(5+a+(-1+x)^2\right) (-1+x)}{2 (12+7 a+a^2) \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}} - \\
& \frac{\left(1-\sqrt{4+a}\right) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{2 (3+a) (4+a) \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2 (3+a) (4+a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}} + \\
& \frac{\sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{2 (4+a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2 - (-1+x)^4}}
\end{aligned}$$

Result (type 4, 3593 leaves):

$$\begin{aligned}
& \frac{(-a-2 x+a x-a x^2-x^3) (a+8 x-8 x^2+4 x^3-x^4)^2}{2 (3+a) (4+a) (-a-8 x+8 x^2-4 x^3+x^4) (a-x (-8+8 x-4 x^2+x^3))^{3/2}} + \frac{1}{2 (3+a) (4+a) (a-x (-8+8 x-4 x^2+x^3))^{3/2}} (a+8 x-8 x^2+4 x^3-x^4)^{3/2} \\
& \left(4 \left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \\
& \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \right\} / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(2a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \right\} / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) - \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
& \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \operatorname{EllipticE}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \right. \right. \\
& \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. \left(4 \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right) / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right)
\end{aligned}$$

Problem 790: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 591 leaves, 12 steps):

$$\begin{aligned}
& \frac{1 + (-1+x)^2}{6 (4+a) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{1 + (-1+x)^2}{3 (4+a)^2 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \\
& \frac{\left(5+a+(-1+x)^2\right) (-1+x)}{6 (12+7 a+a^2) \left(3+a-2 (-1+x)^2-(-1+x)^4\right)^{3/2}} + \frac{\left(104+47 a+5 a^2+4 (7+2 a) (-1+x)^2\right) (-1+x)}{12 (3+a)^2 (4+a)^2 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} - \\
& \frac{(7+2 a) \left(1-\sqrt{4+a}\right) \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{3 (3+a)^2 (4+a)^2 \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \frac{(7+2 a) \left(1-\sqrt{4+a}\right) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{3 (3+a)^2 (4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}} + \\
& \frac{(16+5 a) \sqrt{1+\sqrt{4+a}} \left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12 (3+a) (4+a)^2 \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2 (-1+x)^2-(-1+x)^4}}
\end{aligned}$$

Result (type 4, 6452 leaves):

$$\begin{aligned}
& \frac{\left(a+8 x-8 x^2+4 x^3-x^4\right)^3 \left(\frac{a+2 x-a x+a x^2+x^3}{6 (3+a) (4+a) (-a-8 x+8 x^2-4 x^3+x^4)^2}+\frac{60+7 a-3 a^2-116 x-23 a x+3 a^2 x+48 x^2-4 a^2 x^2-28 x^3-8 a x^3}{12 (3+a)^2 (4+a)^2 (-a-8 x+8 x^2-4 x^3+x^4)}\right)}{\left(a-x\left(-8+8 x-4 x^2+x^3\right)\right)^{5/2}} + \\
& \frac{1}{12 (3+a)^2 (4+a)^2 \left(a-x\left(-8+8 x-4 x^2+x^3\right)\right)^{5/2}} \left(a+8 x-8 x^2+4 x^3-x^4\right)^{5/2} \\
& \left(40 \left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right) \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right], \\
& \frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\Bigg] \Bigg] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \\
& \left(46a\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\right. \\
& \left.\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right],\right. \\
& \left.\frac{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)}\Bigg] \Bigg] \Bigg] / \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(10 a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \right. \\
& \quad \left. \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \right] \Bigg) \\
& \quad \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \right) + \\
& \quad \left(112 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \\
& \quad \left. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \quad \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(32a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg) \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) - \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& 28 \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right) + \right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2 \sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}} \right. \\
& \left. \frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\sqrt{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}} \right) \frac{1}{2\sqrt{-1-\sqrt{4+a}}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) \\
& \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] +
\end{aligned}$$

$$\left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right)$$

$$\text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}},$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Bigg] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) +$$

$$4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}\right],$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Bigg] \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg\] -$$

$$\frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 8a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right.$$

$$2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& 4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right],
\end{aligned}$$

Problem 791: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} dx$$

Optimal (type 4, 585 leaves, 15 steps):

$$\begin{aligned}
& \frac{3}{8} (4+a) \left(1 + (-1+x)^2\right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} + \frac{1}{4} \left(1 + (-1+x)^2\right) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2} + \\
& \frac{4 (140 + 111 a + 21 a^2) \left(1 - \sqrt{4+a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1+x)}{315 \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \frac{2}{315} \left(2 (80 + 27 a) + 3 (20 + 7 a) (-1+x)^2\right) \sqrt{3+a-2(-1+x)^2 - (-1+x)^4} (-1+x) + \\
& \frac{1}{63} \left(15 + 7 (-1+x)^2\right) \left(3+a-2(-1+x)^2 - (-1+x)^4\right)^{3/2} (-1+x) + \frac{3}{8} (4+a)^2 \operatorname{ArcTan}\left[\frac{1 + (-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}\right] - \\
& \frac{4 (140 + 111 a + 21 a^2) \left(1 - \sqrt{4+a}\right) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticE}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{315 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{4 (3+a) (100 + 33 a) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \operatorname{EllipticF}\left[\operatorname{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right]}{315 \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}}
\end{aligned}$$

Result (type 4, 8500 leaves):

$$\frac{1}{a + 8x - 8x^2 + 4x^3 - x^4}$$

$$\left(\frac{1}{252} (404 + 107a) + \frac{(460 + 81a)x}{1260} - \frac{1}{360} (100 + 39a)x^2 + \frac{1}{315} (-80 + 77a)x^3 + \frac{71x^4}{42} - \frac{163x^5}{126} + \frac{19x^6}{36} - \frac{x^7}{9} \right) (a - x (-8 + 8x - 4x^2 + x^3))^{3/2} +$$

$$\begin{aligned}
& \frac{1}{1260 (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} (a - x (-8 + 8x - 4x^2 + x^3))^{3/2} \\
& - \left(\left(\frac{16160 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2}{\sqrt{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \right. \\
& \left. \left. \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \right. \\
& \left. \left. \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \right. \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right], \\
& \left. \left. \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \right] \right. \\
& \left. \left. \left. \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) \right. \right. \right. \\
& \left. \left. \left. \left. \left(5200 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right) \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}\right], \\
& \left.\left.\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}\right]\right\} \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}\right) - \\
& \left(162a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}\right. \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}\right], \\
& \left.\left.\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}\right]\right\}
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(21280 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2\sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(8016a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \right] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(546 a^2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \right. \\
& \left. \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \right], \right.
\end{aligned}$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \frac{1}{2}\right],$$

$$\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}, \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}\right]\right]/$$

$$\left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}\right) + \frac{1}{\sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}}$$

$$2240 \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right) + \right.$$

$$2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}}$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)\right)$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}, \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}\right]\right] +$$

$$\left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right)$$

$$\text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}},$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Bigg] \Bigg/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) +$$

$$4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}\right],$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} \Bigg] \Bigg/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \Bigg\} +$$

$$\frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 1776 a \left(\left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right.$$

$$2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \\
& \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& 4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right],
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right) + \\
& \left(4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right], \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2}\right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)\right)\Big)
\end{aligned}$$

Problem 792: Result more than twice size of optimal antiderivative.

$$\int x^2 \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \, dx$$

Optimal (type 4, 485 leaves, 13 steps):

$$\begin{aligned}
& \frac{1}{2} \left(1 + (-1+x)^2 \right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} + \frac{2 \left(8+3a \right) \left(1 - \sqrt{4+a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{15 \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} + \\
& \frac{1}{15} \left(7+3(-1+x)^2 \right) \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4} (-1+x) + \frac{1}{2} (4+a) \operatorname{ArcTan} \left[\frac{1+(-1+x)^2}{\sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}} \right] - \\
& \frac{2 \left(8+3a \right) \left(1 - \sqrt{4+a} \right) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{15 \sqrt{\frac{1+(-1+x)^2}{1-\sqrt{4+a}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}} + \\
& \frac{8 \left(3+a \right) \sqrt{1+\sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{15 \sqrt{\frac{1+(-1+x)^2}{1-\sqrt{4+a}} \sqrt{3+a - 2(-1+x)^2 - (-1+x)^4}}}
\end{aligned}$$

Result (type 4, 5647 leaves):

$$\begin{aligned}
& \left(\frac{1}{3} + \frac{x}{15} - \frac{x^2}{10} + \frac{x^3}{5} \right) \sqrt{a-x(-8+8x-4x^2+x^3)} + \frac{1}{15 \sqrt{a+8x-8x^2+4x^3-x^4}} \sqrt{a-x(-8+8x-4x^2+x^3)} \\
& - \left(\left(40 \left(-\sqrt{-1-\sqrt{4+a}} - \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)^2 \right) \sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1-\sqrt{4+a}} \left(-1 - \sqrt{-1+\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right. \\
& \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 + \sqrt{-1-\sqrt{4+a}} + x \right)}{\left(\sqrt{-1-\sqrt{4+a}} + \sqrt{-1+\sqrt{4+a}} \right) \left(-1 - \sqrt{-1-\sqrt{4+a}} + x \right)}} \right], \right]
\end{aligned}$$

$$\left(56 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right) \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}$$

$$\left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right],$$

$$\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}},$$

$$\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right]$$

$$\left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \right) +$$

$$\left(6 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right) \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}$$

$$\sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
16 & \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \right. \\
& \left. \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \\
& \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right\} \left/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. \left(4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right) \right\} \right/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right\} +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 6a \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \frac{1}{2\sqrt{-1 - \sqrt{4+a}}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2}] + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right)^2 \right] \Big/ \left(2 \sqrt{-1 - \sqrt{4+a}} \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) + \\
& \left. \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}{-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \right) \right) \right)
\end{aligned}$$

Problem 793: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} dx$$

Optimal (type 4, 388 leaves, 11 steps):

$$\begin{aligned}
& \frac{\left(1 - \sqrt{4+a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} + \operatorname{ArcTan} \left[\frac{1 + (-1+x)^2}{\sqrt{3+a-2(-1+x)^2 - (-1+x)^4}} \right] - \\
& \frac{\left(1 - \sqrt{4+a} \right) \sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}}} + \\
& \frac{\sqrt{1 + \sqrt{4+a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticF} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2\sqrt{4+a}}{1-\sqrt{4+a}} \right]}{\sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}}} \sqrt{3+a-2(-1+x)^2 - (-1+x)^4}
\end{aligned}$$

Result (type 4, 1247 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{a - x \left(-8 + 8x - 4x^2 + x^3 \right)}} \\
 & \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \\
 & \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
 & \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left\{ \frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
 & \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \left. \right] + \\
 & \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \\
 & \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg/ \\
 & \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) +
 \end{aligned}$$

$$\left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}} \right], \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \\ \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \left/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right)$$

Problem 794: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2}} dx$$

Optimal (type 4, 311 leaves, 10 steps):

$$\frac{1 + (-1 + x)^2}{(4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \frac{(4 + a) (2 + (-1 + x)^2) (-1 + x)}{2 (12 + 7 a + a^2) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \\ \frac{\left(1 - \sqrt{4 + a} \right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) (-1+x)}{2 (3 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \frac{\left(1 - \sqrt{4 + a} \right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}} \right) \operatorname{EllipticE} \left[\operatorname{ArcTan} \left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}} \right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}} \right]}{2 (3 + a) \sqrt{\frac{1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}}{1 + \frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}$$

Result (type 4, 2941 leaves):

$$\frac{(-a - 8x - ax + 6x^2 + ax^2 - 4x^3 - ax^3) (a + 8x - 8x^2 + 4x^3 - x^4)^2}{2 (3 + a) (4 + a) (-a - 8x + 8x^2 - 4x^3 + x^4) (a - x (-8 + 8x - 4x^2 + x^3))^{3/2}} - \frac{1}{2 (3 + a) (a - x (-8 + 8x - 4x^2 + x^3))^{3/2}} (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} \\ \left(2 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)$$

$$\begin{aligned}
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \text{EllipticF}[\text{ArcSin}\left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}\right], \\
& \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)}] \Bigg] \Bigg] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4}\right) - \\
& 4 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}} \\
& \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x\right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}}\right) \text{EllipticF}[\text{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x\right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x\right)}\right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}\right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}}\right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right.
\end{aligned}$$

$$\begin{aligned}
& \text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right]\Bigg) \\
& \left(\sqrt{-1-\sqrt{4+a}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\sqrt{a+8x-8x^2+4x^3-x^4}\right) + \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} \\
& \left(\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right) + \right. \\
& \left. 2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)^2\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right. \\
& \left.\frac{\sqrt{-1-\sqrt{4+a}}\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)}{\sqrt{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1-\sqrt{-1-\sqrt{4+a}}+x\right)}}\right. \\
& \left.\frac{1}{2\sqrt{-1-\sqrt{4+a}}}\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}\right], \right. \right. \\
& \left. \left.\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right] + \left(-\left(-1-\sqrt{-1-\sqrt{4+a}}\right)\left(-2-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right) + \left(-1+\sqrt{-1-\sqrt{4+a}}\right)\right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& 4 \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}}, \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] / \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right] \right)
\end{aligned}$$

Problem 795: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}} dx$$

Optimal (type 4, 582 leaves, 13 steps):

$$\begin{aligned}
& \frac{1 + (-1 + x)^2}{3 (4 + a) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2}} + \frac{2 \left(1 + (-1 + x)^2\right)}{3 (4 + a)^2 \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \frac{(4 + a) \left(2 + (-1 + x)^2\right) (-1 + x)}{6 (12 + 7 a + a^2) \left(3 + a - 2 (-1 + x)^2 - (-1 + x)^4\right)^{3/2}} + \\
& \frac{\left(29 + 7 a + (13 + 3 a) (-1 + x)^2\right) (-1 + x)}{12 (3 + a)^2 (4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} - \frac{(13 + 3 a) \left(1 - \sqrt{4 + a}\right) \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) (-1 + x)}{12 (3 + a)^2 (4 + a) \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{(13 + 3 a) \left(1 - \sqrt{4 + a}\right) \sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticE}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12 (3 + a)^2 (4 + a) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}} + \\
& \frac{\sqrt{1 + \sqrt{4 + a}} \left(1 + \frac{(-1+x)^2}{1-\sqrt{4+a}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right], -\frac{2 \sqrt{4+a}}{1-\sqrt{4+a}}\right]}{12 (12 + 7 a + a^2) \sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}} \sqrt{3 + a - 2 (-1 + x)^2 - (-1 + x)^4}}
\end{aligned}$$

Result (type 4, 5812 leaves):

$$\begin{aligned}
& \left((a + 8 x - 8 x^2 + 4 x^3 - x^4)^3 \right. \\
& \left. \left(\frac{a + 8 x + a x - 6 x^2 - a x^2 + 4 x^3 + a x^3}{6 (3 + a) (4 + a) (-a - 8 x + 8 x^2 - 4 x^3 + x^4)^2} + \frac{24 - 14 a - 6 a^2 - 128 x - 36 a x + 84 x^2 + 27 a x^2 + a^2 x^2 - 52 x^3 - 25 a x^3 - 3 a^2 x^3}{12 (3 + a)^2 (4 + a)^2 (-a - 8 x + 8 x^2 - 4 x^3 + x^4)} \right) \right) / \\
& (a - x (-8 + 8 x - 4 x^2 + x^3))^{5/2} - \frac{1}{12 (3 + a)^2 (4 + a) (a - x (-8 + 8 x - 4 x^2 + x^3))^{5/2}} (a + 8 x - 8 x^2 + 4 x^3 - x^4)^{5/2} \\
& \left(2 \theta \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \\
& \left(4a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)} \right], \right. \\
& \left. \frac{\left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)} \right] \Bigg] \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(52 \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \\
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} + 2 \sqrt{-1 - \sqrt{4 + a}} \text{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8 x - 8 x^2 + 4 x^3 - x^4} \right) - \\
& \left(12 a \left(-\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \right. \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}}
\end{aligned}$$

$$\begin{aligned}
& \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \\
& \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2}] + 2 \sqrt{-1 - \sqrt{4 + a}} \operatorname{EllipticPi}\left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \right. \\
& \left. \operatorname{ArcSin}\left[\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Bigg] / \\
& \left(\sqrt{-1 - \sqrt{4 + a}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \sqrt{a + 8x - 8x^2 + 4x^3 - x^4} \right) + \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} \\
13 & \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 - \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{-1 - \sqrt{4 + a}} \left(-1 + \sqrt{-1 + \sqrt{4 + a}} + x \right)}{\sqrt{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4 + a}} + x \right)}} \left(\frac{1}{2 \sqrt{-1 - \sqrt{4 + a}}} \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] + \right. \\
& \left. \left(- \left(-1 - \sqrt{-1 - \sqrt{4 + a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4 + a}} \right) \left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \right) \right. \\
& \left. \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \right. \\
& \left. 4 \text{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \text{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \right/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{a + 8x - 8x^2 + 4x^3 - x^4}} 3a \left(\left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right) \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right) \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right) + \right. \\
& \left. 2 \left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)^2 \sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 - \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \sqrt{\frac{\sqrt{-1 - \sqrt{4+a}} \left(-1 + \sqrt{-1 + \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 - \sqrt{-1 - \sqrt{4+a}} + x \right)}} \right. \\
& \left. \frac{1}{2\sqrt{-1 - \sqrt{4+a}}} \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticE}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right], \frac{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right)^2}] + \\
& \left(- \left(-1 - \sqrt{-1 - \sqrt{4+a}} \right) \left(-2 - \sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) + \left(-1 + \sqrt{-1 - \sqrt{4+a}} \right) \left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \right) \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4+a}} - \sqrt{-1 + \sqrt{4+a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4+a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4+a}} + \sqrt{-1 + \sqrt{4+a}} \right) \left(1 + \sqrt{-1 - \sqrt{4+a}} - x \right)}} \right],
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right)^2 \right] \Big/ \left(2 \sqrt{-1 - \sqrt{4 + a}} \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) + \\
& \left. \left(4 \operatorname{EllipticPi} \left[\frac{\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}{-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}}}, \operatorname{ArcSin} \left[\sqrt{\frac{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right) \left(-1 + \sqrt{-1 - \sqrt{4 + a}} + x \right)}{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \left(1 + \sqrt{-1 - \sqrt{4 + a}} - x \right)}} \right], \right. \right. \\
& \left. \left. \frac{\left(\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right)^2}{\left(\sqrt{-1 - \sqrt{4 + a}} - \sqrt{-1 + \sqrt{4 + a}} \right)^2} \right] \Big/ \left(-\sqrt{-1 - \sqrt{4 + a}} + \sqrt{-1 + \sqrt{4 + a}} \right) \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 796: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8 + 8x - x^3 + 8x^4}} dx$$

Optimal (type 4, 129 leaves, 4 steps):

$$\begin{aligned}
& \frac{x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right) \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{4+x}{\sqrt{3} 29^{1/4} x} \right], \frac{1}{58} \left(29 + \sqrt{29}\right) \right]}{8 \sqrt{3} 29^{1/4} \sqrt{8 + 8x - x^3 + 8x^4}}
\end{aligned}$$

Result (type 4, 927 leaves):

$$\begin{aligned}
& - \left(\left(2 \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4])}) / \right. \right. \\
& \quad \left. \left((\#1 - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) \right) \right) \right. \\
& \quad \left((\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 3]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) \right) / \\
& \quad \left((\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 3]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) \right) \\
& \quad (x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2])^2 \\
& \quad \sqrt{((\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2]) (x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 3])) /} \\
& \quad ((x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 3])) \\
& \quad (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) \\
& \quad \sqrt{((x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2])} \\
& \quad (x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4])) / \\
& \quad \left((x - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2])^2 (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4])^2 \right) \right) / \\
& \quad \left(\sqrt{8 + 8 x - x^3 + 8 x^4} (-\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 1] + \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2]) \right. \\
& \quad \left. (\operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 2] - \operatorname{Root}[8 + 8 \#1 - \#1^3 + 8 \#1^4, 4]) \right) \Big)
\end{aligned}$$

Problem 797: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 8x - x^3 + 8x^4)^{3/2}} dx$$

Optimal (type 4, 431 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(66 - \left(1 + \frac{4}{x}\right)^2\right) x^2}{1008 \sqrt{8 + 8x - x^3 + 8x^4}} + \frac{\left(216 - 7 \left(1 + \frac{4}{x}\right)^2\right) \left(1 + \frac{4}{x}\right) x^2}{12528 \sqrt{8 + 8x - x^3 + 8x^4}} + \frac{7 \left(261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4\right) \left(1 + \frac{4}{x}\right) x^2}{432 \sqrt{29} \sqrt{8 + 8x - x^3 + 8x^4} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)} - \\
& \quad \frac{7 x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{4+x}{\sqrt{3} 29^{1/4} x}\right], \frac{1}{58} \left(29 + \sqrt{29}\right)\right]}{144 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}} +
\end{aligned}$$

$$\begin{aligned}
& \left(14 - 5 \sqrt{29}\right) x^2 \sqrt{\frac{261 - 6 \left(1 + \frac{4}{x}\right)^2 + \left(1 + \frac{4}{x}\right)^4}{\left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right)^2}} \left(87 + \frac{\sqrt{29} (4+x)^2}{x^2}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{4+x}{\sqrt{3} 29^{1/4} x}\right], \frac{1}{58} \left(29 + \sqrt{29}\right)\right] \\
& \quad 576 \sqrt{3} 29^{3/4} \sqrt{8 + 8x - x^3 + 8x^4}
\end{aligned}$$

Result (type 4, 4865 leaves):

$$\begin{aligned}
& \frac{544 + 1539x - 1146x^2 + 784x^3}{21924\sqrt{8+8x-x^3+8x^4}} + \\
& \frac{1}{6264} \left(\left(28(x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2])^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]))], - ((\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) \right) \right. \\
& \left. \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] + \text{EllipticPi}[\frac{-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]}{-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]}, \right. \\
& \left. \text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])}], - ((\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) \right) \\
& \left. \sqrt{(((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3])}, (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]) \right. \\
& \left. \sqrt{(((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])}, \sqrt{(((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]))} \right) / \\
& \left(\sqrt{8+8x-x^3+8x^4} (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) \right. \\
& \left. (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]) \right) + \\
& (842 \text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])}], \\
& ((\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3]) \\
& (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2])^2 \\
& \sqrt{(((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 3])}, \\
& (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4]) \\
& \sqrt{(((-\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] + \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])) / ((x - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 2]) (\text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 1] - \text{Root}[8+8\#1-\#1^3+8\#1^4 \&, 4])}) /
\end{aligned}$$

$$\begin{aligned}
& \left(\left(x - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 2 \right] \right) \left(\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 1 \right] - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 4 \right] \right) \right) \Big], \\
& - \left(\left(\left(\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 3 \right] \right) \left(\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 1 \right] - \right. \right. \\
& \left. \left. \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 4 \right] \right) \right) / \left(\left(-\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 1 \right] + \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 3 \right] \right) \right. \\
& \left. \left(\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 4 \right] \right) \right) \Big] \left(-\text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 1 \right] - \right. \\
& \left. \left. \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 2 \right] - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 3 \right] - \text{Root} \left[8 + 8 \# 1 - \# 1^3 + 8 \# 1^4 \&, 4 \right] \right) \Big) \Big) \Big)
\end{aligned}$$

Problem 798: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1 + 4x + 4x^2 + 4x^4}} dx$$

Optimal (type 4, 108 leaves, 3 steps):

$$\frac{\left(\sqrt{5} + \left(1 + \frac{1}{x} \right)^2 \right) \sqrt{\frac{5 - 2 \left(1 + \frac{1}{x} \right)^2 + \left(1 + \frac{1}{x} \right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x} \right)^2 \right)^2}} x^2 \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{1 + \frac{1}{x}}{5^{1/4}} \right], \frac{1}{10} \left(5 + \sqrt{5} \right) \right]}{2 \times 5^{1/4} \sqrt{1 + 4x + 4x^2 + 4x^4}}$$

Result (type 4, 249 leaves):

$$\begin{aligned}
& \left(2 - \frac{1}{x} \right) \sqrt{-\frac{1}{10} + \frac{1}{5}} \sqrt{\frac{\left(2 \frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} - \sqrt{-1 + 2 \frac{1}{x}} \right) \left(-\frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} - 2x \right)}{\left(-2 \frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + \sqrt{-1 + 2 \frac{1}{x}} \right) \left(\frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + 2x \right)}} \\
& \left(1 + 2x + 2\frac{1}{x}x^2 \right) \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{\left(2 \frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + \sqrt{-1 + 2 \frac{1}{x}} \right) \left(-\frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + 2x \right)}{\sqrt{-1 + 2 \frac{1}{x}} \left(\frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + 2x \right)}}}{\sqrt{2}} \right], \frac{1}{2} \left(5 - \sqrt{5} \right) \right] \Bigg) \Bigg)
\end{aligned}$$

$$\left(\sqrt{\frac{\left(1 + 2 \frac{1}{x} \right) \left(\left(-1 + \frac{1}{x} \right) + \sqrt{-1 - 2 \frac{1}{x}} \right) \left(1 + 2x + 2\frac{1}{x}x^2 \right)}{\left(\frac{1}{x} + \sqrt{-1 - 2 \frac{1}{x}} + 2x \right)^2}} \sqrt{1 + 4x + 4x^2 + 4x^4} \right)$$

Problem 799: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1 + 4x + 4x^2 + 4x^4)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 9 steps):

$$\begin{aligned}
& -\frac{\left(3 - \left(1 + \frac{1}{x}\right)^2\right) x^2}{\sqrt{1 + 4x + 4x^2 + 4x^4}} + \frac{\left(13 - 9\left(1 + \frac{1}{x}\right)^2\right) \left(1 + \frac{1}{x}\right) x^2}{10 \sqrt{1 + 4x + 4x^2 + 4x^4}} + \frac{9 \left(5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4\right) \left(1 + \frac{1}{x}\right) x^2}{10 \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{1 + 4x + 4x^2 + 4x^4}} \\
& + \frac{9 \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10} \left(5 + \sqrt{5}\right)\right]}{2 \times 5^{3/4} \sqrt{1 + 4x + 4x^2 + 4x^4}} \\
& + \frac{3 \left(3 - \sqrt{5}\right) \left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right) \sqrt{\frac{5 - 2\left(1 + \frac{1}{x}\right)^2 + \left(1 + \frac{1}{x}\right)^4}{\left(\sqrt{5} + \left(1 + \frac{1}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{1 + \frac{1}{x}}{5^{1/4}}\right], \frac{1}{10} \left(5 + \sqrt{5}\right)\right]}{4 \times 5^{3/4} \sqrt{1 + 4x + 4x^2 + 4x^4}}
\end{aligned}$$

Result (type 4, 3334 leaves):

$$\left(-\text{Root}\left[1 + 4 \# 1 + 4 \# 1^2 + 4 \# 1^4 \&, 2 \right] + \text{Root}\left[1 + 4 \# 1 + 4 \# 1^2 + 4 \# 1^4 \&, 4 \right] \right) \right)$$

Problem 800: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} dx$$

Optimal (type 4, 126 leaves, 4 steps):

$$-\frac{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517-38 \left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3+\frac{4}{x}\right)^2\right)^2}} x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4+3 x}{517^{1/4} x}\right], \frac{517+19 \sqrt{517}}{1034}\right]}{8 \times 517^{1/4} \sqrt{8+24 x+8 x^2-15 x^3+8 x^4}}$$

Result (type 4, 1148 leaves):

Problem 801: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^{3/2}} dx$$

Optimal (type 4, 434 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(172 - 7 \left(3 + \frac{4}{x}\right)^2\right) x^2}{208 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \frac{\left(50896 - 2455 \left(3 + \frac{4}{x}\right)^2\right) \left(3 + \frac{4}{x}\right) x^2}{322608 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \frac{2455 \left(517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4\right) \left(3 + \frac{4}{x}\right) x^2}{322608 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} \\
& + \frac{2455 \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{4+3 x}{517^{1/4} x}\right], \frac{517+19 \sqrt{517}}{1034}\right]}{624 \times 517^{3/4} \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \\
& \left(\left(4910 - 203 \sqrt{517}\right) \left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x}\right)^2 + \left(3 + \frac{4}{x}\right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x}\right)^2\right)^2}} x^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{4+3 x}{517^{1/4} x}\right], \frac{517+19 \sqrt{517}}{1034}\right] \right) / \\
& \left(2496 \times 517^{3/4} \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}\right)
\end{aligned}$$

Result (type 4, 6019 leaves):

$$\begin{aligned}
& \frac{72\,888 + 89\,033 x - 94\,314 x^2 + 39\,280 x^3}{80\,652 \sqrt{8 + 24 x + 8 x^2 - 15 x^3 + 8 x^4}} + \\
& \frac{1}{161\,304} \left(\left(147\,300 (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2])^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2])^2)} - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])]) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right. \\
& \left. - ((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right. \\
& \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \\
& \left. ((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \\
& \text{EllipticPi} \left[\frac{-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]}{-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]}, \right. \\
& \left. \text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \\
& \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]))}], \right. \\
& \left. - ((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right)^2 / \left(\left(x - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] \right) \right. \\
& \left. \left(\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \left(\left(\left(\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3 \right] \right) \right. \right. \\
& \left. \left. \left(\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) / \right. \\
& \left. \left(\left(-\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] + \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3 \right] \right) \right. \right. \\
& \left. \left. \left(\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right) \left(\left(-\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] \right. \right. \\
& \left. \left. - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3 \right] - \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) / \right. \\
& \left. \left(-\text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2 \right] + \text{Root}\left[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4 \right] \right) \right) \right)
\end{aligned}$$

Problem 802: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^{5/2}} dx$$

Optimal (type 4, 577 leaves, 12 steps):

$$\begin{aligned}
& -\frac{\left(124\,415 - 6308 \left(3 + \frac{4}{x} \right)^2 \right) x^2}{97\,344 \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} - \frac{\left(64\,489 - 1399 \left(3 + \frac{4}{x} \right)^2 \right) x^2}{624 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& \frac{\left(18\,932\,921\,731 - 1\,086\,525\,994 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right) x^2}{78\,056\,941\,248 \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \frac{\left(11\,921\,698 - 359\,497 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right) x^2}{483\,912 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& -\frac{543\,262\,997 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right) \left(3 + \frac{4}{x} \right) x^2}{39\,028\,470\,624 \left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right) \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} - \\
& \frac{543\,262\,997 \left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right)^2}} x^2 \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{4+3x}{517^{1/4}x} \right], \frac{517+19\sqrt{517}}{1034} \right]}{75\,490\,272 \times 517^{3/4} \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4}} + \\
& \left(4\,346\,103\,976 - 175\,318\,963 \sqrt{517} \right) \left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right) \sqrt{\frac{517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4}{\left(\sqrt{517} + \left(3 + \frac{4}{x} \right)^2 \right)^2}} x^2 \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{4+3x}{517^{1/4}x} \right], \frac{517+19\sqrt{517}}{1034} \right] \Bigg) / \\
& \left(1\,207\,844\,352 \times 517^{3/4} \sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} \right)
\end{aligned}$$

Result (type 4, 6084 leaves):

$$\frac{1}{78056941248} \left(\frac{\sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} \left(\frac{72888 + 89033x - 94314x^2 + 39280x^3}{241956 (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} + \frac{65072399400 + 77274145879x - 83050578336x^2 + 34768831808x^3}{39028470624 (8 + 24x + 8x^2 - 15x^3 + 8x^4)} \right) + \right. \\
 \left(\left(130383119280 (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2])^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) (Root[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]))}], \right. \right. \\
 \left. \left. - ((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right. \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \right. \\
 \left. \left. ((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \right. \\
 \left. \left. 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]))] \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \right. \right. \\
 \left. \left. \text{EllipticPi} \left[\frac{-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]}{-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]}, \right. \right. \\
 \left. \left. \text{ArcSin}[\sqrt{((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \right. \\
 \left. \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]))}], \right. \right. \\
 \left. \left. - ((\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right. \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / \right. \right. \\
 \left. \left. ((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3]) \right. \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]))] \right. \right. \\
 \left. \left. (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right) \right. \\
 \left. \sqrt{(((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 3])) \right) \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4]) \right) \right. \\
 \left. \sqrt{((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1]) (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2] - \right. \right. \\
 \left. \left. \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right. \\
 \left. \sqrt{(((-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) / ((x - \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right. \right. \\
 \left. \left. (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 4])) \right) \right. \\
 \left. \left(\sqrt{8 + 24x + 8x^2 - 15x^3 + 8x^4} (-\text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 1] + \text{Root}[8 + 24 \#1 + 8 \#1^2 - 15 \#1^3 + 8 \#1^4 \&, 2]) \right) \right)$$

Problem 803: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} dx$$

Optimal (type 4, 130 leaves, 4 steps):

$$-\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{\left(6-x\right)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{\left(6-x\right)^2}{x^2}\right)x^2\text{EllipticF}\left[2\text{ArcTan}\left[\frac{6-x}{613^{1/4}x}\right],\frac{613+91\sqrt{613}}{1226}\right]$$

Result (type 4, 826 leaves):

$$\begin{aligned}
& - \left(\left(2 \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\sqrt{ \left(\left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) } \right) / \left(\left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right) \right], \\
& \left(\left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \\
& \sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]} \left(x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] \right)^2} \\
& \sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]}} \\
& \sqrt{\frac{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right]}{x - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right]}} \Bigg) / \\
& \left(\sqrt{\left(\left(9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4 \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \left(\operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \operatorname{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) } \right)
\end{aligned}$$

Problem 804: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(9 - 6x - 44x^2 + 15x^3 + 3x^4)^{3/2}} dx$$

Optimal (type 4, 444 leaves, 10 steps):

$$\begin{aligned}
& - \frac{\left(176 - 23 \left(1 - \frac{6}{x}\right)^2\right) x^2}{51759 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \frac{\left(45401 - 3722 \left(1 - \frac{6}{x}\right)^2\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \frac{3722 \left(613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4\right) \left(1 - \frac{6}{x}\right) x^2}{31728267 \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{3722 \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]}{51759 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} - \\
& \frac{\left(7444 - 145 \sqrt{613}\right) \sqrt{\frac{613 - 182 \left(1 - \frac{6}{x}\right)^2 + \left(-1 + \frac{6}{x}\right)^4}{\left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right)^2}} \left(\sqrt{613} + \frac{(6-x)^2}{x^2}\right) x^2 \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{6-x}{613^{1/4} x}\right], \frac{613 + 91 \sqrt{613}}{1226}\right]}{207036 \times 613^{3/4} \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}}
\end{aligned}$$

Result (type 4, 5428 leaves):

$$\begin{aligned}
& - \frac{2 \left(-106926 - 592639 x + 232005 x^2 + 44664 x^3 \right)}{10576089 \sqrt{9 - 6x - 44x^2 + 15x^3 + 3x^4}} + \\
& \frac{1}{3525363} \left(\left(148880 \left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right)^2 \left(-\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]) / ((x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]))}], \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) / ((x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]) - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]) \right) \right) \right) \right. \\
& \left. \left. \left. \left. - ((\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]) \right. \right. \right. \\
& \left. \left. \left. (\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]) / ((-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]) (\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4])) \right) \right) \right) \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] + \\
& \text{EllipticPi} \left[\frac{-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}, \right. \\
& \left. \left. \left. \left. \left. \text{ArcSin}[\sqrt{((x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]) (\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]) / ((x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]) - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2])} \right) \right) \right) \right) \right. \\
& \left. \left. \left. \left. \left. - ((\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]) \right. \right. \right. \right. \\
& \left. \left. \left. \left. (\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]) / ((-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]) \right. \right. \right. \\
& \left. \left. \left. (\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4])) \right) \right) \right) \right) \right) \\
& \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \\
& \sqrt{\left(\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \right) } \\
& \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right)))) / \\
& \left(\sqrt{9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4} \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right. \\
& \left. \sqrt{\left(\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \right.} \\
& \left. \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right)) - \\
& \left(54294 \text{EllipticF}[\text{ArcSin}[\sqrt{\left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] \right) \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] + \right.} \right. \right. \\
& \left. \left. \left. \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right)) / \left(\left(x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] \right) \right. \right. \\
& \left. \left. \left. \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \right) \right], \\
& \left(\left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \right. \\
& \left. \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) / \\
& \left(\left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) \right. \\
& \left. \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right) \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} (x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2])^2 \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \sqrt{\frac{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4]}{x - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2]}} \\
& \left(\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] - \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right))) / \\
& \left(\sqrt{9 - 6 x - 44 x^2 + 15 x^3 + 3 x^4} \left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4] \right) \right. \\
& \left. \left. \sqrt{\left(-\text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1] + \text{Root}[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3] \right) } \right)
\end{aligned}$$

$$\begin{aligned}
& 9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \]))) \] , - \left(\left(\left(\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + \right. \right. \right. \right. \\
& \left. \left. \left. \left. 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \left(\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(-\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] + \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] \right) \right. \\
& \left. \left(\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right) \\
& \left(-\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 1 \right] - \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] - \right. \\
& \left. \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 3 \right] - \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) / \\
& \left(-\text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 2 \right] + \text{Root} \left[9 - 6 \#1 - 44 \#1^2 + 15 \#1^3 + 3 \#1^4 \&, 4 \right] \right) \right) \right) \right)
\end{aligned}$$

Problem 817: Unable to integrate problem.

$$\int \frac{x - \sqrt{x^6}}{x (1 - x^4)} dx$$

Optimal (type 3, 45 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 26 leaves):

$$\int \frac{x - \sqrt{x^6}}{x (1 - x^4)} dx$$

Problem 818: Unable to integrate problem.

$$\int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx$$

Optimal (type 3, 45 leaves, 9 steps):

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 26 leaves):

$$\int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx$$

Problem 819: Unable to integrate problem.

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Optimal (type 3, 45 leaves, 10 steps) :

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 23 leaves) :

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Problem 820: Unable to integrate problem.

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Optimal (type 3, 45 leaves, 11 steps) :

$$\frac{\text{ArcTan}[x]}{2} + \frac{\sqrt{x^6} \text{ArcTan}[x]}{2 x^3} + \frac{\text{ArcTanh}[x]}{2} - \frac{\sqrt{x^6} \text{ArcTanh}[x]}{2 x^3}$$

Result (type 8, 15 leaves) :

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Problem 821: Unable to integrate problem.

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Optimal (type 3, 52 leaves, 12 steps) :

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 27 leaves) :

$$\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Problem 822: Unable to integrate problem.

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Optimal (type 3, 52 leaves, 13 steps):

$$\text{ArcTan}[\sqrt{x}] + \frac{\sqrt{x^3} \text{ArcTan}[\sqrt{x}]}{x^{3/2}} + \text{ArcTanh}[\sqrt{x}] - \frac{\sqrt{x^3} \text{ArcTanh}[\sqrt{x}]}{x^{3/2}}$$

Result (type 8, 17 leaves):

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Problem 843: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{1}{4}(-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2\sqrt{-3+x}\sqrt{5+x}\text{ArcSinh}\left[\frac{\sqrt{-3+x}}{2\sqrt{2}}\right]}{\sqrt{-(-3+x)(5+x)}}$$

Problem 844: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(3-x)(5+x)}} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$-\text{ArcSin}\left[\frac{1}{4}(-1-x)\right]$$

Result (type 3, 45 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{-3+x}}{2 \sqrt{2}}\right]}{\sqrt{-(-3+x) (5+x)}}$$

Problem 846: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3-x} \sqrt{5+x}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

$$\operatorname{ArcSin}[4+x]$$

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{- (3+x) (5+x)}}$$

Problem 847: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(-3-x) (5+x)}} dx$$

Optimal (type 3, 4 leaves, 3 steps):

$$\operatorname{ArcSin}[4+x]$$

Result (type 3, 42 leaves):

$$\frac{2 \sqrt{3+x} \sqrt{5+x} \operatorname{ArcSinh}\left[\frac{\sqrt{3+x}}{\sqrt{2}}\right]}{\sqrt{- (3+x) (5+x)}}$$

Problem 852: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\frac{1}{-1+x^2}} dx$$

Optimal (type 3, 25 leaves, 2 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \text{ArcSin}[x]$$

Result (type 3, 56 leaves):

$$\frac{1}{2} \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \left(-\text{Log} \left[1 - \frac{x}{\sqrt{-1+x^2}} \right] + \text{Log} \left[1 + \frac{x}{\sqrt{-1+x^2}} \right] \right)$$

Problem 853: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\frac{1+x^2}{-1+x^4}} dx$$

Optimal (type 3, 25 leaves, 3 steps):

$$\sqrt{1-x^2} \sqrt{\frac{1}{-1+x^2}} \text{ArcSin}[x]$$

Result (type 3, 56 leaves):

$$\frac{1}{2} \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \left(-\text{Log} \left[1 - \frac{x}{\sqrt{-1+x^2}} \right] + \text{Log} \left[1 + \frac{x}{\sqrt{-1+x^2}} \right] \right)$$

Problem 855: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 11 leaves, 2 steps):

$$-2 \sqrt{1-x}$$

Result (type 2, 23 leaves):

$$\frac{2 (-1+x) \sqrt{1+x}}{\sqrt{1-x^2}}$$

Problem 857: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx$$

Optimal (type 2, 9 leaves, 2 steps):

$$2 \sqrt{1+x}$$

Result (type 2, 25 leaves):

$$\frac{2 \sqrt{1-x} (1+x)}{\sqrt{1-x^2}}$$

Problem 861: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx$$

Optimal (type 2, 11 leaves, 2 steps):

$$\frac{2}{3} (1+x)^{3/2}$$

Result (type 2, 27 leaves):

$$\frac{2 (1+x) \sqrt{1-x^2}}{3 \sqrt{1-x}}$$

Problem 863: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x} \sqrt{2+3x}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$\sqrt{1+x} \sqrt{2+3x} - \frac{\text{ArcSinh}[\sqrt{2+3x}]}{\sqrt{3}}$$

Result (type 3, 79 leaves):

$$\frac{\sqrt{1-x} \left(3 (1+x) \sqrt{2+3 x}+\sqrt{3} \sqrt{-1-x} \operatorname{ArcTan}\left[\frac{\sqrt{3} \sqrt{-1-x}}{\sqrt{2+3 x}}\right]\right)}{3 \sqrt{1-x^2}}$$

Problem 864: Result more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2} x} dx$$

Optimal (type 3, 43 leaves, 7 steps):

$$\frac{4 \sqrt{1+x}}{\sqrt{1-x}} - \operatorname{ArcSin}[x] - \operatorname{ArcTanh}[\sqrt{1-x} \sqrt{1+x}]$$

Result (type 3, 101 leaves):

$$-\frac{4 \sqrt{1-x^2}}{-1+x} - 2 \operatorname{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{2}}\right] + \operatorname{Log}\left[1-\sqrt{1+x}\right] - \operatorname{Log}\left[2+\sqrt{1-x}-\sqrt{1+x}\right] - \operatorname{Log}\left[1+\sqrt{1+x}\right] + \operatorname{Log}\left[2+\sqrt{1-x}+\sqrt{1+x}\right]$$

Problem 866: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+a x)^{3/2}}{x (1-a x)^{3/2}} dx$$

Optimal (type 3, 51 leaves, 7 steps):

$$\frac{4 \sqrt{1+a x}}{\sqrt{1-a x}} - \operatorname{ArcSin}[a x] - \operatorname{ArcTanh}[\sqrt{1-a x} \sqrt{1+a x}]$$

Result (type 3, 74 leaves):

$$\frac{4 \sqrt{1-a^2 x^2}}{1-a x} + \operatorname{Log}[x] - \operatorname{Log}\left[1+\sqrt{1-a^2 x^2}\right] - i \operatorname{Log}\left[2 \left(-i a x+\sqrt{1-a^2 x^2}\right)\right]$$

Problem 869: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps):

$$\operatorname{ArcSin}[x]$$

Result (type 3, 32 leaves) :

$$-\text{ArcTan}\left[\frac{x \sqrt{1+x^2} \sqrt{1-x^4}}{-1+x^4}\right]$$

Problem 871: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$$

Optimal (type 3, 2 leaves, 2 steps) :

$$\text{ArcSinh}[x]$$

Result (type 3, 42 leaves) :

$$\text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2} \sqrt{1-x^4}]$$

Problem 873: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$$

Optimal (type 3, 23 leaves, 3 steps) :

$$\frac{1}{2} x \sqrt{1-x^2} + \frac{\text{ArcSin}[x]}{2}$$

Result (type 3, 50 leaves) :

$$\frac{1}{2} \left(\frac{x \sqrt{1-x^4}}{\sqrt{1+x^2}} + \text{ArcTan}\left[\frac{x \sqrt{1+x^2}}{\sqrt{1-x^4}}\right] \right)$$

Problem 875: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Optimal (type 3, 21 leaves, 3 steps) :

$$\frac{1}{2} x \sqrt{1+x^2} + \frac{\text{ArcSinh}[x]}{2}$$

Result (type 3, 70 leaves) :

$$\frac{1}{2} \left(\frac{x \sqrt{1-x^4}}{\sqrt{1-x^2}} + \text{Log}[1-x^2] - \text{Log}[-x+x^3+\sqrt{1-x^2} \sqrt{1-x^4}] \right)$$

Problem 911: Unable to integrate problem.

$$\int \frac{\sqrt{b x^2 + \sqrt{a + b^2 x^4}}}{\sqrt{a + b^2 x^4}} dx$$

Optimal (type 3, 47 leaves, 2 steps):

$$\frac{\text{ArcTanh} \left[\frac{\sqrt{2} \sqrt{b} x}{\sqrt{b x^2 + \sqrt{a + b^2 x^4}}} \right]}{\sqrt{2} \sqrt{b}}$$

Result (type 8, 39 leaves):

$$\int \frac{\sqrt{b x^2 + \sqrt{a + b^2 x^4}}}{\sqrt{a + b^2 x^4}} dx$$

Problem 912: Unable to integrate problem.

$$\int \frac{\sqrt{-b x^2 + \sqrt{a + b^2 x^4}}}{\sqrt{a + b^2 x^4}} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{\text{ArcTan} \left[\frac{\sqrt{2} \sqrt{b} x}{\sqrt{-b x^2 + \sqrt{a + b^2 x^4}}} \right]}{\sqrt{2} \sqrt{b}}$$

Result (type 8, 40 leaves):

$$\int \frac{\sqrt{-b x^2 + \sqrt{a + b^2 x^4}}}{\sqrt{a + b^2 x^4}} dx$$

Problem 913: Unable to integrate problem.

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx) \sqrt{3 + 4x^4}} dx$$

Optimal (type 3, 169 leaves, 5 steps):

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \operatorname{ArcTan}\left[\frac{\sqrt{3} d + 2 i c x}{\sqrt{2 i c^2 - \sqrt{3} d^2} \sqrt{\sqrt{3} - 2 i x^2}}\right]}{\sqrt{2 i c^2 - \sqrt{3} d^2}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \operatorname{ArcTanh}\left[\frac{\sqrt{3} d - 2 i c x}{\sqrt{2 i c^2 + \sqrt{3} d^2} \sqrt{\sqrt{3} + 2 i x^2}}\right]}{\sqrt{2 i c^2 + \sqrt{3} d^2}}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx) \sqrt{3 + 4x^4}} dx$$

Problem 914: Unable to integrate problem.

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2 \sqrt{3 + 4x^4}} dx$$

Optimal (type 3, 268 leaves, 7 steps):

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2 i x^2}}{\left(2 i c^2 - \sqrt{3} d^2\right) (c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2 i x^2}}{\left(2 i c^2 + \sqrt{3} d^2\right) (c + dx)} + \frac{(1 + i) c \operatorname{ArcTan}\left[\frac{\sqrt{3} d + 2 i c x}{\sqrt{2 i c^2 - \sqrt{3} d^2} \sqrt{\sqrt{3} - 2 i x^2}}\right]}{\left(2 i c^2 - \sqrt{3} d^2\right)^{3/2}} + \frac{(1 - i) c \operatorname{ArcTanh}\left[\frac{\sqrt{3} d - 2 i c x}{\sqrt{2 i c^2 + \sqrt{3} d^2} \sqrt{\sqrt{3} + 2 i x^2}}\right]}{\left(2 i c^2 + \sqrt{3} d^2\right)^{3/2}}$$

Result (type 8, 42 leaves):

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2 \sqrt{3 + 4x^4}} dx$$

Problem 918: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 + \frac{b}{x^2}}}{b + 2x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps):

$$-\frac{\text{ArcCsch}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 54 leaves) :

$$\frac{\sqrt{2 + \frac{b}{x^2}} \times \left(\text{Log}[x] - \text{Log}[b + \sqrt{b} \sqrt{b + 2 x^2}] \right)}{\sqrt{b} \sqrt{b + 2 x^2}}$$

Problem 919: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 - \frac{b}{x^2}}}{-b + 2 x^2} dx$$

Optimal (type 3, 20 leaves, 3 steps) :

$$-\frac{\text{ArcCsc}\left[\frac{\sqrt{2} x}{\sqrt{b}}\right]}{\sqrt{b}}$$

Result (type 3, 64 leaves) :

$$-\frac{\frac{i}{2} \sqrt{2 - \frac{b}{x^2}} \times \text{Log}\left[\frac{2 \left(-\frac{i}{2} \sqrt{b} + \sqrt{-b+2 x^2}\right)}{x}\right]}{\sqrt{b} \sqrt{-b+2 x^2}}$$

Problem 926: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x) \sqrt{2 x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps) :

$$\text{ArcTan}\left[\sqrt{2 x+x^2}\right]$$

Result (type 3, 37 leaves) :

$$\frac{2 \sqrt{x} \sqrt{2+x} \text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x (2+x)}}$$

Problem 927: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}[2\sqrt{x+x^2}]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{1+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{1+x}}\right]}{\sqrt{x(1+x)}}$$

Problem 929: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x-x^2}}{1+x} dx$$

Optimal (type 3, 54 leaves, 6 steps):

$$\sqrt{x-x^2} - \frac{3}{2}\text{ArcSin}[1-2x] + \sqrt{2}\text{ArcTan}\left[\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right]$$

Result (type 3, 120 leaves):

$$\frac{1}{2\sqrt{-1+x}\sqrt{x}}\sqrt{-(-1+x)x}\left(2\sqrt{-1+x}\sqrt{x} - 6\text{Log}\left[\sqrt{-1+x} + \sqrt{x}\right] + \sqrt{2}\text{Log}\left[1 - 2\sqrt{2}\sqrt{-1+x}\sqrt{x} - 3x\right] - \sqrt{2}\text{Log}\left[1 + 2\sqrt{2}\sqrt{-1+x}\sqrt{x} - 3x\right]\right)$$

Problem 951: Result unnecessarily involves higher level functions.

$$\int \frac{(1+\sqrt{x})^{1/3}}{x} dx$$

Optimal (type 3, 67 leaves, 6 steps):

$$6(1+\sqrt{x})^{1/3} - 2\sqrt{3}\text{ArcTan}\left[\frac{1+2(1+\sqrt{x})^{1/3}}{\sqrt{3}}\right] + 3\text{Log}\left[1 - (1+\sqrt{x})^{1/3}\right] - \frac{\text{Log}[x]}{2}$$

Result (type 5, 51 leaves):

$$\frac{6 + 6\sqrt{x} - 3\left(1 + \frac{1}{\sqrt{x}}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{1}{\sqrt{x}}\right]}{\left(1 + \sqrt{x}\right)^{2/3}}$$

Problem 956: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{(a + b x)(c - d x)}} dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$\frac{\text{ArcTan}\left[\frac{b c - a d - 2 b d x}{2 \sqrt{b} \sqrt{d} \sqrt{a c + (b c - a d) x - b d x^2}}\right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 99 leaves):

$$\frac{\pm \sqrt{a + b x} \sqrt{c - d x} \log\left[2 \sqrt{a + b x} \sqrt{c - d x} - \frac{\pm (-b c + a d + 2 b d x)}{\sqrt{b} \sqrt{d}}\right]}{\sqrt{b} \sqrt{d} \sqrt{(a + b x)(c - d x)}}$$

Problem 957: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{x} (1 - x^2)} dx$$

Optimal (type 3, 13 leaves, 4 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \log[1 - \sqrt{x}] + \frac{1}{2} \log[1 + \sqrt{x}]$$

Problem 958: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x}}{x - x^3} dx$$

Optimal (type 3, 13 leaves, 5 steps):

$$\text{ArcTan}[\sqrt{x}] + \text{ArcTanh}[\sqrt{x}]$$

Result (type 3, 33 leaves):

$$\text{ArcTan}[\sqrt{x}] - \frac{1}{2} \text{Log}[1 - \sqrt{x}] + \frac{1}{2} \text{Log}[1 + \sqrt{x}]$$

Problem 961: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\text{ArcTan}[\sqrt{2x+x^2}]$$

Result (type 3, 37 leaves):

$$\frac{2\sqrt{x}\sqrt{2+x}\text{ArcTan}\left[\frac{\sqrt{x}}{\sqrt{2+x}}\right]}{\sqrt{x}(2+x)}$$

Problem 969: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x}{x - \sqrt{1+2x^2}} dx$$

Optimal (type 3, 31 leaves, 7 steps):

$$-x - \sqrt{1+2x^2} + \text{ArcTan}[x] + \text{ArcTan}[\sqrt{1+2x^2}]$$

Result (type 3, 101 leaves):

$$\frac{1}{4} \left(-4x - 4\sqrt{1+2x^2} + 4\text{ArcTan}[x] - 4\text{ArcTan}\left[\frac{1}{\sqrt{1+2x^2}}\right] + 2i\text{Log}[1+x^2] - i\text{Log}[1+3x^2 - 2x\sqrt{1+2x^2}] - i\text{Log}[1+3x^2 + 2x\sqrt{1+2x^2}] \right)$$

Problem 981: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(1-x)x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\text{ArcSin}[1-2x]$$

Result (type 3, 38 leaves):

$$\frac{2 \sqrt{-1+x} \sqrt{x} \operatorname{Log}[\sqrt{-1+x}+\sqrt{x}]}{\sqrt{-(-1+x) x}}$$

Problem 984: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5} x^2} dx$$

Optimal (type 3, 24 leaves, 2 steps):

$$\frac{1}{2} \operatorname{ArcTan}\left[\sqrt{\frac{1}{2} \left(3-\sqrt{5}\right)} x\right]$$

Result (type 3, 39 leaves):

$$\frac{1}{4} i \operatorname{Log}\left[1+\sqrt{5}-2 i x\right]-\frac{1}{4} i \operatorname{Log}\left[1+\sqrt{5}+2 i x\right]$$

Problem 995: Unable to integrate problem.

$$\int \sqrt{1-x^2+x \sqrt{-1+x^2}} dx$$

Optimal (type 3, 63 leaves, ? steps):

$$\frac{1}{4} \left(3 x+\sqrt{-1+x^2}\right) \sqrt{1-x^2+x \sqrt{-1+x^2}}+\frac{3 \operatorname{ArcSin}\left[x-\sqrt{-1+x^2}\right]}{4 \sqrt{2}}$$

Result (type 8, 24 leaves):

$$\int \sqrt{1-x^2+x \sqrt{-1+x^2}} dx$$

Problem 996: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{-x+\sqrt{x}} \sqrt{1+x}}{\sqrt{1+x}} dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2} \left(\sqrt{x} + 3 \sqrt{1+x} \right) \sqrt{-x + \sqrt{x} \sqrt{1+x}} - \frac{3 \operatorname{ArcSin}[\sqrt{x} - \sqrt{1+x}]}{2 \sqrt{2}}$$

Result (type 3, 180 leaves):

$$- \left(\left((1+x) (1+2x-2\sqrt{x}\sqrt{1+x})^2 \left(2\sqrt{-x+\sqrt{x}\sqrt{1+x}} (-3-2x+2\sqrt{x}\sqrt{1+x}) + 3\sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}} \operatorname{Log}[2\sqrt{-x+\sqrt{x}\sqrt{1+x}} + \sqrt{-2-4x+4\sqrt{x}\sqrt{1+x}}] \right) \right) \right) \Big/ \left(4 (-\sqrt{x} + \sqrt{1+x})^3 (1+x - \sqrt{x}\sqrt{1+x})^2 \right)$$

Problem 997: Unable to integrate problem.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2(1+\sqrt{5})} \operatorname{ArcTan}[\sqrt{-2+\sqrt{5}} (x+\sqrt{1+x^2})] + \sqrt{2(-1+\sqrt{5})} \operatorname{ArcTanh}[\sqrt{2+\sqrt{5}} (x+\sqrt{1+x^2})]$$

Result (type 8, 34 leaves):

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Problem 998: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}} dx$$

Optimal (type 3, 126 leaves, 5 steps):

$$-\sqrt{\frac{1}{2}(1+\sqrt{5})} \operatorname{ArcTan}\left[\frac{2\sqrt{5} - (5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})\sqrt{2+2x+x^2}}}\right] - \sqrt{\frac{1}{2}(-1+\sqrt{5})} \operatorname{ArcTanh}\left[\frac{2\sqrt{5} + (5-\sqrt{5})x}{\sqrt{10(-1+\sqrt{5})\sqrt{2+2x+x^2}}}\right]$$

Result (type 3, 433 leaves):

$$\begin{aligned}
& \frac{1}{4} \left(2 \sqrt{1+2 \text{i}} \operatorname{ArcTan} \left[\left((8+8 \text{i}) - (1-4 \text{i}) x^3 + 5 \text{i} \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} \right) + x^2 \left((-2+13 \text{i}) + 5 \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} \right) + \right. \right. \\
& \left. \left. (1+\text{i}) x \left((9+5 \text{i}) + 5 \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} \right) \right] \Big/ \left((4+14 \text{i}) + (2+2 \text{i}) x + (4-11 \text{i}) x^2 - (3+8 \text{i}) x^3 \right) \right] + \\
& 2 \text{i} \sqrt{1-2 \text{i}} \operatorname{ArcTanh} \left[\left((-8+8 \text{i}) + (1+4 \text{i}) x^3 + 5 \text{i} \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} \right) + x^2 \left((2+13 \text{i}) - 5 \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} \right) + \right. \\
& \left. \left. (1+\text{i}) x \left((5+9 \text{i}) + 5 \text{i} \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} \right) \right] \Big/ \left((-14-4 \text{i}) - (2+2 \text{i}) x + (11-4 \text{i}) x^2 + (8+3 \text{i}) x^3 \right) \right] + \\
& \text{i} \left(\left(\sqrt{1-2 \text{i}} - \sqrt{1+2 \text{i}} \right) \operatorname{Log} [1+x^2] - \sqrt{1-2 \text{i}} \operatorname{Log} \left[(7-4 \text{i}) + (8-4 \text{i}) x + (3-2 \text{i}) x^2 + 4 \sqrt{1-2 \text{i}} \sqrt{2+2x+x^2} + 2 \sqrt{1-2 \text{i}} x \sqrt{2+2x+x^2} \right] + \right. \\
& \left. \left. \sqrt{1+2 \text{i}} \operatorname{Log} \left[(7+4 \text{i}) + (8+4 \text{i}) x + (3+2 \text{i}) x^2 + 4 \sqrt{1+2 \text{i}} \sqrt{2+2x+x^2} + 2 \sqrt{1+2 \text{i}} x \sqrt{2+2x+x^2} \right] \right) \right]
\end{aligned}$$

Problem 1002: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a+b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 184 leaves, 7 steps):

$$\begin{aligned}
& \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2}{\sqrt{a+b d^4 \left(\frac{c}{d} + x \right)^4}} \right] - c \left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a+b d^4 \left(\frac{c}{d} + x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{b^{1/4} (c+d x)}{a^{1/4}} \right], \frac{1}{2} \right]}{2 \sqrt{b} d^2} \\
& - \frac{2 a^{1/4} b^{1/4} d^2 \sqrt{a+b d^4 \left(\frac{c}{d} + x \right)^4}}{2 a^{1/4} b^{1/4} d^2 \sqrt{a+b d^4 \left(\frac{c}{d} + x \right)^4}}
\end{aligned}$$

Result (type 4, 330 leaves):

$$\begin{aligned}
& \left((-1)^{1/4} \sqrt{2} \sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \left(\frac{i}{2} \sqrt{a} + \sqrt{b} (c + d x)^2 \right) \right. \\
& \left(\left((-1)^{1/4} a^{1/4} - b^{1/4} c \right) \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1 \right] - \right. \\
& \left. 2 (-1)^{1/4} a^{1/4} \text{EllipticPi} \left[-\frac{i}{2}, \text{ArcSin} \left[\sqrt{-\frac{\frac{i}{2} \left((-1)^{1/4} a^{1/4} + b^{1/4} (c + d x) \right)}{(-1)^{1/4} a^{1/4} - b^{1/4} (c + d x)}} \right], -1 \right] \right) \Bigg) / \\
& \left(a^{1/4} \sqrt{b} d^2 \sqrt{\frac{\frac{i}{2} \sqrt{a} + \sqrt{b} (c + d x)^2}{\left((-1)^{1/4} a^{1/4} - b^{1/4} (c + d x) \right)^2}} \sqrt{a + b (c + d x)^4} \right)
\end{aligned}$$

Problem 1003: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b c^4 + 4 b c^3 d x + 6 b c^2 d^2 x^2 + 4 b c d^3 x^3 + b d^4 x^4}} dx$$

Optimal (type 4, 131 leaves, 2 steps):

$$\begin{aligned}
& \frac{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right) \sqrt{\frac{a+b d^4 \left(\frac{c}{d}+x \right)^4}{\left(\sqrt{a} + \sqrt{b} d^2 \left(\frac{c}{d} + x \right)^2 \right)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{b^{1/4} (c+d x)}{a^{1/4}} \right], \frac{1}{2} \right]}{2 a^{1/4} b^{1/4} d \sqrt{a + b d^4 \left(\frac{c}{d} + x \right)^4}}
\end{aligned}$$

Result (type 4, 90 leaves):

$$\begin{aligned}
& -\frac{\frac{i}{2} \sqrt{\frac{a+b (c+d x)^4}{a}} \text{EllipticF} \left[\frac{i}{2} \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} (c + d x) \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} d \sqrt{a + b (c + d x)^4}}
\end{aligned}$$

Problem 1004: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a + b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{b d-a e} x}{\sqrt{d} \sqrt{a+b x^2+c x^4}}\right]}{\sqrt{d} \sqrt{b d-a e}}$$

Result (type 4, 419 leaves):

$$\left(\frac{i \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}}{\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}} \left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] - \operatorname{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4 a c}\right) d}{a e-\sqrt{a} \sqrt{-4 c d^2+a e^2}}, i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] - \operatorname{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4 a c}\right) d}{a e+\sqrt{a} \sqrt{-4 c d^2+a e^2}}, i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right) \right) \Big/ \left(\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} d \sqrt{a+b x^2+c x^4} \right)$$

Problem 1005: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{\sqrt{a - b x^2 + c x^4} (a d + a e x^2 + c d x^4)} dx$$

Optimal (type 3, 53 leaves, 2 steps):

$$\frac{\operatorname{ArcTan}\left[\frac{\sqrt{b d+a e} x}{\sqrt{d} \sqrt{a-b x^2+c x^4}}\right]}{\sqrt{d} \sqrt{b d+a e}}$$

Result (type 4, 416 leaves):

$$\begin{aligned} & \frac{\text{i}}{\sqrt{2 + \frac{4 c x^2}{-b + \sqrt{b^2 - 4 a c}}}} \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \left(\text{EllipticF} \left[\text{i} \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] - \right. \\ & \left. \text{EllipticPi} \left[\frac{(b - \sqrt{b^2 - 4 a c}) d}{-a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, \text{i} \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] - \right. \\ & \left. \text{EllipticPi} \left[\frac{(-b + \sqrt{b^2 - 4 a c}) d}{a e + \sqrt{a} \sqrt{-4 c d^2 + a e^2}}, \text{i} \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b - \sqrt{b^2 - 4 a c}}{b + \sqrt{b^2 - 4 a c}} \right] \right) \Big/ \left(2 \sqrt{\frac{c}{-b + \sqrt{b^2 - 4 a c}}} d \sqrt{a - b x^2 + c x^4} \right) \end{aligned}$$

Problem 1009: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e f - e f x^2}{(a d + b d x + a d x^2) \sqrt{a + b x + c x^2 + b x^3 + a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \text{ArcTan} \left[\frac{a b + (4 a^2 + b^2 - 2 a c) x + a b x^2}{2 a \sqrt{2 a - c} \sqrt{a + b x + c x^2 + b x^3 + a x^4}} \right]}{a \sqrt{2 a - c} d}$$

Result (type 4, 13884 leaves):

$$\begin{aligned} & -\frac{1}{d} \\ & e f \left(- \left(8 (x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2])^2 \left(\text{EllipticF} [\text{ArcSin} [\sqrt{((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])} / ((x - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]))], \right. \right. \right. \\ & \left. \left. \left. - ((\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / ((-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])), \right) \right) \right) \left(-b + \sqrt{-4 a^2 + b^2} - 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \right. \\ & \left. 2 a \text{EllipticPi} \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) (-\text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \right. \right. \right. \\ & \left. \left. \left. \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \Big/ \left(\left(b - \sqrt{-4 a^2 + b^2} + 2 a \text{Root} [a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left(8 \left(x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right)^2 \left(\text{EllipticF}[\text{ArcSin}[\sqrt{((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1])} \right. \right. \right. \\
& \quad \left. \left. \left. (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \right) \right) \right. \\
& \quad \left. \left. \left. - ((\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) \right. \right. \\
& \quad \left. \left. \left. (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / \right. \right. \\
& \quad \left. \left. \left. ((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \right. \right. \\
& \quad \left. \left. \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) \right) \right) \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) - \\
& 2 a \text{EllipticPi} \left[\left(\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \right. \right. \\
& \quad \left. \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right) / \left(\left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \right. \\
& \quad \left. \left. (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) \right), \right. \\
& \quad \left. \text{ArcSin}[\sqrt{((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \right. \right. \\
& \quad \left. \left. \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \right. \right. \\
& \quad \left. \left. (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right), \right. \\
& \quad \left. \left((\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, \right. \right. \\
& \quad \left. \left. 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4]) / ((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[\right. \right. \\
& \quad \left. \left. a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right) \right. \\
& \quad \left. \left. (-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \right) \right) \\
& \sqrt{(((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3])) / \\
& ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3] \right) \right) \\
& \sqrt{((((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / \\
& ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) \right) \\
& \sqrt{((((-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) (x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4])) / \\
& ((x - \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2]) \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \right) \right. \\
& \quad \left. \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4] \right) \right) / \\
& \left(\left(\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \right. \\
& \quad \left. \left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] \right) \right. \\
& \quad \left. \left(-\text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1] + \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \right. \\
& \quad \left. \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root}[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) \Big/ \\
& \left(a^2 \left(\frac{-b - \sqrt{-4 a^2 + b^2}}{2 a} - \frac{-b + \sqrt{-4 a^2 + b^2}}{2 a} \right) \sqrt{x (b + c x + b x^2) + a (1 + x^4)} \right. \\
& \left(b + \sqrt{-4 a^2 + b^2} + 2 a \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] \right) \\
& \left(-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] \right) \\
& \left(-b - \sqrt{-4 a^2 + b^2} - 2 a \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] \right) \\
& \left(\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) + \\
& \left(2 \text{EllipticF} \left[\text{ArcSin} \left[\sqrt{((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right]) (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] + \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) / ((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right]) \right) \right) \right], \right. \\
& \left((\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3 \right]) \right. \\
& \left. \left(\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) / \right. \\
& \left. \left((\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3 \right]) \right. \\
& \left. \left(\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) \right] (x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right])^2 \right. \\
& \left. \sqrt{(((-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) (x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3 \right])) / \right. \right. \\
& \left. \left. \left((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) \right. \right. \right. \\
& \left. \left. \left. (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 3 \right]) \right) \right) \right. \\
& \left(\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \left. \sqrt{(((-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) (x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right])) / \right. \right. \\
& \left. \left. \left((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) \right. \right. \right. \\
& \left. \left. \left. (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right]) \right) \right) \right. \\
& \left. \sqrt{(((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right]) (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right])) / \right. \right. \\
& \left. \left. \left((x - \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) \right. \right. \right. \\
& \left. \left. \left. (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right]) \right) \right) \right) \Big/ \\
& \left(a \sqrt{x (b + c x + b x^2) + a (1 + x^4)} (-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right]) \right. \\
& \left. \left(-\text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root} \left[a + b \#1 + c \#1^2 + b \#1^3 + a \#1^4 \&, 4 \right] \right) \right)
\end{aligned}$$

Problem 1010: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e f - e f x^2}{(-a d + b d x - a d x^2) \sqrt{-a + b x + c x^2 + b x^3 - a x^4}} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{e f \operatorname{ArcTanh}\left[\frac{a b - (4 a^2 + b^2 + 2 a c) x + a b x^2}{2 a \sqrt{2 a + c} \sqrt{-a + b x + c x^2 + b x^3 - a x^4}}\right]}{a \sqrt{2 a + c} d}$$

Result (type 4, 15147 leaves):

$$\begin{aligned} & \frac{1}{d} e f \left(- \left(8 \left(x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right)^2 \right. \right. \\ & \left. \left(\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \\ & \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\ & (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])))] \right), \\ & - \left(((\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) \right. \\ & (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\ & \left. ((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) \right) \left(b + \sqrt{-4 a^2 + b^2} - 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) - \\ & 2 a \operatorname{EllipticPi}\left[\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \right. \right. \\ & \left. \left. \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right] / \left(\left(-b - \sqrt{-4 a^2 + b^2} + 2 a \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] \right) \right. \\ & \left. \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right), \\ & \operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \\ & \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\ & (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])))] \right), \\ & - \left(((\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) \right. \\ & (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\ & \left. ((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3]) \right. \\ & \left. \left(\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4] \right) \right) \\ & \left(-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] \right) \right) \\ & \sqrt{(((-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\ & (x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3])) / ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2]) \\ & (-\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1] + \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3])))} \\ & \sqrt{((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1]) (\operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2] - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4])) / \\ & ((x - \operatorname{Root}[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2])}$$

$$\begin{aligned}
& \left(\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right)^2 \\
& \sqrt{\left(\left(\left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \right) / \\
& \left(\left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 3 \right] \right) \right) / \\
& \left(\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \\
& \sqrt{\left(\left(\left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \sqrt{\left(\left(\left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] \right) \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right) / \\
& \left(\left(x - \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \\
& \left. \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right)) / \\
& \left(a \sqrt{x (b + c x + b x^2) - a (1 + x^4)} \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 1 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] \right) \right. \\
& \left. \left(-\text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 2 \right] + \text{Root} \left[a - b \#1 - c \#1^2 - b \#1^3 + a \#1^4 \&, 4 \right] \right) \right)
\end{aligned}$$

Problem 1011: Result more than twice size of optimal antiderivative

$$\frac{\sqrt{ax^2 + bx} \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 2 steps)

$$\frac{\sqrt{2} \ b \ \text{ArcSinh}\left[\frac{a x+b}{\sqrt{-\frac{a}{b^2}+\frac{a^2 x^2}{b^2}}}\right]}{\sqrt{a}}$$

Result (type 3, 199 leaves):

$$\begin{aligned}
& - \left(x \sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right) \right. \\
& \left. \left(\log \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \log \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) \Big/ \left(\sqrt{2} \sqrt{\frac{a (-1 + a x^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)^{3/2} \right) \right)
\end{aligned}$$

Problem 1012: Result more than twice size of optimal antiderivative.

$$\int \frac{-a x^2 + b x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\begin{aligned}
 & \left(b^2 \sqrt{\frac{a(1+ax^2)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \sqrt{x \left(-ax + b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right. \\
 & \left. \left. \left(\frac{\operatorname{Log} \left[1 - \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] - \operatorname{Log} \left[1 + \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right]} \right) \right) \right) \right\} \left/ \left(\sqrt{2} a^2 x \left(-1 - ax^2 + bx \sqrt{\frac{a(1+ax^2)}{b^2}} \right) \right) \right.
 \end{aligned}$$

Problem 1013: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSinh} \left[\frac{ax+b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 199 leaves):

$$\begin{aligned}
& - \left(x \sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)} \left(-1 + a x^2 + b x \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right) \right. \\
& \left. \left(\log \left[1 - \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] - \log \left[1 + \frac{\sqrt{a x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)}}{\sqrt{2} a x} \right] \right) \right) / \left(\sqrt{2} \sqrt{\frac{a (-1 + a x^2)}{b^2}} \left(x \left(a x + b \sqrt{\frac{a (-1 + a x^2)}{b^2}} \right)^{3/2} \right) \right)
\end{aligned}$$

Problem 1014: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(-a x + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal (type 3, 46 leaves, 3 steps):

$$\frac{\sqrt{2} b \operatorname{ArcSin} \left[\frac{a x - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right]}{\sqrt{a}}$$

Result (type 3, 213 leaves):

$$\begin{aligned}
& \left(b^2 \sqrt{\frac{a(1+ax^2)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \sqrt{x \left(-ax + b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right. \\
& \left. \left. \left(\frac{\text{Log} \left[1 - \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right] - \text{Log} \left[1 + \frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(1+ax^2)}{b^2}} \right)}}{\sqrt{2} ax} \right]}{\sqrt{2} a^2 x \left(-1 - ax^2 + bx \sqrt{\frac{a(1+ax^2)}{b^2}} \right)} \right) \right)
\end{aligned}$$

Problem 1015: Result more than twice size of optimal antiderivative.

$$\int \frac{-\sqrt{-4+x} - 4\sqrt{-1+x} + \sqrt{-4+x}x + \sqrt{-1+x}x}{(1+\sqrt{-4+x}+\sqrt{-1+x})(4-5x+x^2)} dx$$

Optimal (type 3, 19 leaves, 3 steps):

$$2 \text{Log} [1 + \sqrt{-4+x} + \sqrt{-1+x}]$$

Result (type 3, 75 leaves):

$$-\text{ArcTanh} [\sqrt{-4+x}] + \text{ArcTanh} \left[\frac{\sqrt{-1+x}}{2} \right] + \frac{1}{2} \text{Log} [17 - 4\sqrt{-4+x}\sqrt{-1+x} - 5x] + \frac{1}{2} \text{Log} [5 - 2\sqrt{-4+x}\sqrt{-1+x} - 2x]$$

Problem 1016: Unable to integrate problem.

$$\int \frac{1}{x(3+3x+x^2)(3+3x+3x^2+x^3)^{1/3}} dx$$

Optimal (type 3, 90 leaves, 3 steps):

$$-\frac{\text{ArcTan} \left[\frac{1 + \frac{2 \cdot 3^{1/3} (1+x)}{(2+(1+x)^3)^{1/3}}}{\sqrt{3}} \right]}{3^{5/6}} - \frac{\text{Log} [1 - (1+x)^3]}{6 \cdot 3^{1/3}} + \frac{\text{Log} [3^{1/3} (1+x) - (2 + (1+x)^3)^{1/3}]}{2 \cdot 3^{1/3}}$$

Result (type 8, 33 leaves):

$$\int \frac{1}{x (3 + 3x + x^2) (3 + 3x + 3x^2 + x^3)^{1/3}} dx$$

Problem 1017: Unable to integrate problem.

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3} \operatorname{ArcTan} \left[\frac{1 - 2^{2/3} (1-x)}{\sqrt{3} (1-x^3)^{1/3}} \right]}{2^{2/3}} - \frac{\operatorname{Log} [1 + 2 (1-x)^3 - x^3]}{2 \times 2^{2/3}} + \frac{3 \operatorname{Log} [2^{1/3} (1-x) + (1-x^3)^{1/3}]}{2 \times 2^{2/3}}$$

Result (type 8, 31 leaves):

$$\int \frac{1 - x^2}{(1 - x + x^2) (1 - x^3)^{2/3}} dx$$

Problem 1018: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\sqrt{-1 + x^4} (1 + x^4)} dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4} \operatorname{ArcTan} \left[\frac{1 + x^2}{x \sqrt{-1 + x^4}} \right] - \frac{1}{4} \operatorname{ArcTanh} \left[\frac{1 - x^2}{x \sqrt{-1 + x^4}} \right]$$

Result (type 6, 114 leaves):

$$-\left(\left(7x^3 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4 \right] \right) / \left(3 \sqrt{-1 + x^4} (1 + x^4) \left(-7 \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, x^4, -x^4 \right] + 2x^4 \left(2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, x^4, -x^4 \right] - \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, x^4, -x^4 \right] \right) \right) \right)$$

Problem 1019: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{a - c x^4}{(a e + c d x^2) (d + e x^2) \sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 3, 80 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c d^2 - b d e + a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a + b x^2 + c x^4}}\right]}{\sqrt{d} \sqrt{e} \sqrt{c d^2 - b d e + a e^2}}$$

Result (type 4, 383 leaves):

$$\left(\frac{i \sqrt{b + \sqrt{b^2 - 4 a c} + 2 c x^2}}{b + \sqrt{b^2 - 4 a c}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) d}{2 a e}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - \text{EllipticPi}\left[\frac{(b + \sqrt{b^2 - 4 a c}) e}{2 c d}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right) \right) / \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} d e \sqrt{a + b x^2 + c x^4} \right)$$

Problem 1021: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\frac{1}{x} + \sqrt{1 - x^2}} dx$$

Optimal (type 3, 122 leaves, 12 steps):

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2 x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}} \sqrt{1-x^2}}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}} x}}{\sqrt{1-x^2}}\right]}{\sqrt{3}}$$

Result (type 3, 2681 leaves):

$$\begin{aligned}
& \frac{\left(1 + x \sqrt{1 - x^2}\right) \operatorname{ArcSin}[x]}{x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)} + \left(\left(-\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \right. \\
& \left. \operatorname{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2\right)\right)\right] \middle/ \left(-6 \frac{i}{2} + 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - \right. \right. \\
& \left. \left. 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2}\right] \right) \middle/ \left(2 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)\right) - \\
& \left(\left(-\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \operatorname{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2\right)\right)\right] \middle/ \left(6 \frac{i}{2} - 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 + 2 \sqrt{3} x^2 - \right. \right. \\
& \left. \left. 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2}\right] \right) \middle/ \\
& \left(2 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)\right) - \left(\left(\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \operatorname{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2\right)\right)\right] \middle/ \right. \\
& \left. \left(-6 \frac{i}{2} - 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 + 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - \right. \right. \\
& \left. \left. 2 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2}\right] \right) \middle/ \left(2 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)\right) + \\
& \left(\left(\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \operatorname{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2\right)\right)\right] \middle/ \left(6 \frac{i}{2} + 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 + \right. \right. \\
& \left. \left. 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2}\right] \right) \middle/ \\
& \left(2 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)\right) + \frac{\frac{i}{2} \left(-\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \operatorname{Log}\left[\left(-\frac{i}{2} + \sqrt{3} - 2 x\right)^2 \left(\frac{i}{2} + \sqrt{3} - 2 x\right)^2\right]}{4 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)} - \\
& \frac{\frac{i}{2} \left(\frac{i}{2} + \sqrt{3}\right) \left(1 + x \sqrt{1 - x^2}\right) \operatorname{Log}\left[\left(-\frac{i}{2} + \sqrt{3} - 2 x\right)^2 \left(\frac{i}{2} + \sqrt{3} - 2 x\right)^2\right]}{4 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \left(\frac{1}{x} + \sqrt{1 - x^2}\right)}
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{1}{2} \left(-\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[\left(-\frac{1}{2} + \sqrt{3} + 2x \right)^2 \left(\frac{1}{2} + \sqrt{3} + 2x \right)^2 \right]}{4 \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} + \\
& \frac{\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[\left(-\frac{1}{2} + \sqrt{3} + 2x \right)^2 \left(\frac{1}{2} + \sqrt{3} + 2x \right)^2 \right]}{4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \frac{\frac{1}{2} \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-\frac{1}{2} - \frac{\frac{1}{2} \sqrt{3}}{2} + x^2 \right]}{2 \sqrt{3} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} + \\
& \frac{\frac{1}{2} \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-\frac{1}{2} + \frac{\frac{1}{2} \sqrt{3}}{2} + x^2 \right]}{2 \sqrt{3} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right)} - \\
& \left(\frac{\frac{1}{2} \left(-\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[3 \frac{1}{2} + \sqrt{3} - 3x - 5 \frac{1}{2} \sqrt{3} x + 10 \frac{1}{2} x^2 + 3x^3 - 3 \frac{1}{2} \sqrt{3} x^3 + \frac{1}{2} x^4 - \sqrt{3} x^4 + 2 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} - \right. \right. \right. \\
& \left. \left. \left. 3 \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} + 5 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} - \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) \Big/ \left(4 \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \right. \\
& \left(\frac{\frac{1}{2} \left(-\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[3 \frac{1}{2} + \sqrt{3} + 3x + 5 \frac{1}{2} \sqrt{3} x + 10 \frac{1}{2} x^2 - 3x^3 + 3 \frac{1}{2} \sqrt{3} x^3 + \frac{1}{2} x^4 - \sqrt{3} x^4 + 2 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} + \right. \right. \right. \\
& \left. \left. \left. 3 \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} + 5 \frac{1}{2} \sqrt{2 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} + \frac{1}{2} \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) \Big/ \left(4 \sqrt{6 \left(1 - \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) - \right. \\
& \left(\frac{\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-3 \frac{1}{2} + \sqrt{3} + 3x - 5 \frac{1}{2} \sqrt{3} x - 10 \frac{1}{2} x^2 - 3x^3 - 3 \frac{1}{2} \sqrt{3} x^3 - \frac{1}{2} x^4 - \sqrt{3} x^4 - 2 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} - \right. \right. \right. \\
& \left. \left. \left. 3 \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} - 5 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} - \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) \Big/ \left(4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right) + \right. \\
& \left(\frac{\frac{1}{2} \left(\frac{1}{2} + \sqrt{3} \right) \left(1 + x \sqrt{1 - x^2} \right) \operatorname{Log} \left[-3 \frac{1}{2} + \sqrt{3} - 3x + 5 \frac{1}{2} \sqrt{3} x - 10 \frac{1}{2} x^2 + 3x^3 + 3 \frac{1}{2} \sqrt{3} x^3 - \frac{1}{2} x^4 - \sqrt{3} x^4 - 2 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} \sqrt{1 - x^2} + \right. \right. \right. \\
& \left. \left. \left. 3 \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \sqrt{1 - x^2} - 5 \frac{1}{2} \sqrt{2 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^2 \sqrt{1 - x^2} + \frac{1}{2} \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x^3 \sqrt{1 - x^2} \right] \right) \Big/ \left(4 \sqrt{6 \left(1 + \frac{1}{2} \sqrt{3} \right)} x \left(\frac{1}{x} + \sqrt{1 - x^2} \right) \right)
\end{aligned}$$

Problem 1022: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx$$

Optimal (type 3, 122 leaves, 13 steps):

$$\text{ArcSin}[x] - \frac{\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}} - \frac{\text{ArcTan}\left[\frac{\sqrt{\frac{i-\sqrt{3}}{i+\sqrt{3}}}x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}$$

Result (type 3, 2155 leaves):

$$\begin{aligned} & \text{ArcSin}[x] + \frac{1}{2 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})}} \\ & \left(-\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] / \left(-6 \frac{i}{2} + 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 - 2 \sqrt{3} x^2 - 3 x^3 - \right. \\ & \left. 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] - \\ & \frac{1}{2 \sqrt{6 (1 - \frac{i}{2} \sqrt{3})}} \left(-\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x + 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] / \left(6 \frac{i}{2} - 2 \sqrt{3} + 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 + \right. \\ & \left. 2 \sqrt{3} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 (1 - \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} + 2 \frac{i}{2} \sqrt{2 (1 - \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] - \\ & \frac{1}{2 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})}} \left(\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} - 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] / \left(-6 \frac{i}{2} - 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x - 18 \frac{i}{2} x^2 + \right. \\ & \left. 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - 2 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} \right] + \\ & \frac{1}{2 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})}} \left(\frac{i}{2} + \sqrt{3} \right) \text{ArcTan}\left[\left(x \left(-7 \frac{i}{2} - \sqrt{3} + 8 \frac{i}{2} \sqrt{3} x - 7 \frac{i}{2} x^2 + \sqrt{3} x^2 \right) \right) \right] / \left(6 \frac{i}{2} + 2 \sqrt{3} - 3 x - 11 \frac{i}{2} \sqrt{3} x + 18 \frac{i}{2} x^2 - \right. \end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{3} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - 2 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2} \Big] + \\
& \frac{\frac{i}{2} \left(-\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[\left(-\frac{i}{2} + \sqrt{3} - 2x\right)^2 \left(\frac{i}{2} + \sqrt{3} - 2x\right)^2\right]}{4 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)}} - \frac{\frac{i}{2} \left(\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[\left(-\frac{i}{2} + \sqrt{3} - 2x\right)^2 \left(\frac{i}{2} + \sqrt{3} - 2x\right)^2\right]}{4 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)}} \\
& + \frac{\frac{i}{2} \left(-\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[\left(-\frac{i}{2} + \sqrt{3} + 2x\right)^2 \left(\frac{i}{2} + \sqrt{3} + 2x\right)^2\right]}{4 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)}} - \\
& \frac{\frac{i}{2} \left(\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[\left(-\frac{i}{2} + \sqrt{3} + 2x\right)^2 \left(\frac{i}{2} + \sqrt{3} + 2x\right)^2\right]}{4 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)}} \\
& + \frac{\frac{i}{2} \text{Log} \left[-\frac{1}{2} - \frac{\frac{i}{2} \sqrt{3}}{2} + x^2\right]}{2 \sqrt{3}} + \\
& \frac{\frac{i}{2} \text{Log} \left[-\frac{1}{2} + \frac{\frac{i}{2} \sqrt{3}}{2} + x^2\right]}{2 \sqrt{3}} - \frac{1}{4 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)}} \\
& \frac{\frac{i}{2} \left(-\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[3 \frac{i}{2} + \sqrt{3} - 3x - 5 \frac{i}{2} \sqrt{3} x + 10 \frac{i}{2} x^2 + 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 + \frac{i}{2} x^4 - \sqrt{3} x^4 + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - \right.} \\
& \left. 3 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 5 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2} - \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^3 \sqrt{1 - x^2}\right] + \frac{1}{4 \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)}} \\
& \frac{\frac{i}{2} \left(-\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[3 \frac{i}{2} + \sqrt{3} + 3x + 5 \frac{i}{2} \sqrt{3} x + 10 \frac{i}{2} x^2 - 3 x^3 + 3 \frac{i}{2} \sqrt{3} x^3 + \frac{i}{2} x^4 - \sqrt{3} x^4 + 2 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} + \right.} \\
& \left. 3 \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 5 \frac{i}{2} \sqrt{2 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2} + \frac{i}{2} \sqrt{6 \left(1 - \frac{i}{2} \sqrt{3}\right)} x^3 \sqrt{1 - x^2}\right] - \frac{1}{4 \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)}} \\
& \frac{\frac{i}{2} \left(\frac{i}{2} + \sqrt{3}\right) \text{Log} \left[-3 \frac{i}{2} + \sqrt{3} + 3x - 5 \frac{i}{2} \sqrt{3} x - 10 \frac{i}{2} x^2 - 3 x^3 - 3 \frac{i}{2} \sqrt{3} x^3 - \frac{i}{2} x^4 - \sqrt{3} x^4 - 2 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} \sqrt{1 - x^2} - \right.} \\
& \left. 3 \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x \sqrt{1 - x^2} + 5 \frac{i}{2} \sqrt{2 \left(1 + \frac{i}{2} \sqrt{3}\right)} x^2 \sqrt{1 - x^2} + \frac{i}{2} \sqrt{6 \left(1 + \frac{i}{2} \sqrt{3}\right)} x^3 \sqrt{1 - x^2}\right]
\end{aligned}$$

$$\begin{aligned}
& 3 \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} - 5 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} - \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x^3 \sqrt{1 - x^2}] + \frac{1}{4 \sqrt{6 (1 + \frac{i}{2} \sqrt{3})}} \\
& \frac{i}{2} (i + \sqrt{3}) \operatorname{Log}[-3 \frac{i}{2} + \sqrt{3} - 3x + 5 \frac{i}{2} \sqrt{3} x - 10 \frac{i}{2} x^2 + 3x^3 + 3 \frac{i}{2} \sqrt{3} x^3 - \frac{i}{2} x^4 - \sqrt{3} x^4 - 2 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} \sqrt{1 - x^2} + \\
& 3 \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x \sqrt{1 - x^2} - 5 \frac{i}{2} \sqrt{2 (1 + \frac{i}{2} \sqrt{3})} x^2 \sqrt{1 - x^2} + \frac{i}{2} \sqrt{6 (1 + \frac{i}{2} \sqrt{3})} x^3 \sqrt{1 - x^2}]
\end{aligned}$$

Problem 1024: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4}} dx$$

Optimal (type 3, 177 leaves, 1 step):

$$\begin{aligned}
& \frac{1}{18432 c^2} \operatorname{Log}[20738073600000000 b^8 c^4 + 597005697024000000 b^6 c^6 x^2 + 2583100705996800000 b^5 c^7 x^3 + \\
& 951050714480640000 b^4 c^8 x^4 + 21641687369515008000 b^3 c^9 x^5 + 32462531054272512000 b^2 c^{10} x^6 + \\
& 149587343098087735296 c^{12} x^8 + 5308416 \sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4} \\
& (12203125 b^6 c^4 + 79200000 b^5 c^5 x + 38880000 b^4 c^6 x^2 + 1105920000 b^3 c^7 x^3 + 1990656000 b^2 c^8 x^4 + 12230590464 c^{10} x^6)]
\end{aligned}$$

Result (type 4, 1671 leaves):

$$\begin{aligned}
& \left(2 \left(x - \frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]}{c} \right)^2 \right. \\
& \left. - \frac{1}{c} b \operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{((c x - b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1])} / \right. \\
& \left. (\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]) (\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] - \right. \\
& \left. \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4])]) \right), \\
& - ((\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] - \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]) / \\
& (\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] - \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4])) / \\
& ((-\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1] + \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 3]) \\
& (\operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] - \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4])) \\
& \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2] + \frac{1}{c} \operatorname{EllipticPi}[]
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]}{c} + \frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]}{c} \right) \Big) \Big) \Big) \\
& \left(\sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4} \right. \\
& \left. \left(-\frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 1]}{c} + \right. \right. \\
& \left. \left. \frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]}{c} \right) \right. \\
& \left. \left(\frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 2]}{c} - \right. \right. \\
& \left. \left. \frac{b \operatorname{Root}[-44375 + 576000 \#1 + 576000 \#1^2 + 5308416 \#1^4 \&, 4]}{c} \right) \right)
\end{aligned}$$

Problem 1025: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1 + 4 x}{\sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4}} dx$$

Optimal (type 3, 100 leaves, 2 steps):

$$\begin{aligned}
& \frac{1}{16} \operatorname{Log}[921 + 2864 x + 9280 x^2 + 13440 x^3 + 17024 x^4 + 19456 x^5 + 12288 x^6 + \\
& 8192 x^7 + 4096 x^8 + \sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4} (179 + 444 x + 744 x^2 + 1280 x^3 + 960 x^4 + 768 x^5 + 512 x^6)]
\end{aligned}$$

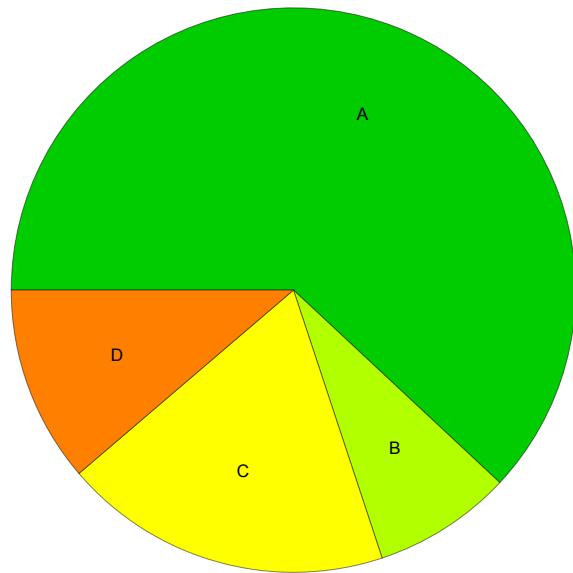
Result (type 4, 2787 leaves):

$$\begin{aligned}
& \left(8 (x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2])^2 \right. \\
& \left(-\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{((x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1]) (\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \right. \\
& \left. \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4])}) / ((x - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2]) \right. \\
& \left. (\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4])) \right), \\
& \left. \left(((\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3]) \right. \right. \\
& \left. \left(\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4]) \right) / \right. \\
& \left. \left((-\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 3]) (\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + \right. \right. \\
& \left. \left. 64 \#1^3 + 64 \#1^4 \&, 2] - \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4])) \right) \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] + \right. \\
& \left. \operatorname{EllipticPi}[(-\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 1] + \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4]) / \right. \\
& \left. \left. (-\operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2] + \operatorname{Root}[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4]), \right. \right)
\end{aligned}$$

$$\left(\sqrt{9 + 120 x + 64 x^2 + 64 x^3 + 64 x^4} \left(-\text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 2\right] + \text{Root}\left[9 + 120 \#1 + 64 \#1^2 + 64 \#1^3 + 64 \#1^4 \&, 4\right] \right) \right)$$

Summary of Integration Test Results

1519 integration problems



A - 941 optimal antiderivatives

B - 121 more than twice size of optimal antiderivatives

C - 286 unnecessarily complex antiderivatives

D - 171 unable to integrate problems

E - 0 integration timeouts