# Mathematica 11.3 Integration Test Results

Test results for the 109 problems in "7.2.4b (f x) $^m$  (d+e x $^2$ ) $^p$  (a+b arccosh(c x))^n.m"

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(a + b \operatorname{ArcCosh}[c x]\right)}{d + e x^2} dx$$

Optimal (type 4, 627 leaves, 27 steps):

$$\begin{array}{c} -\frac{\text{d d x}}{\text{e}^2} + \frac{\text{b d }\sqrt{-1 + \text{c x}}}{\text{c e}^2} - \frac{2 \text{ b }\sqrt{-1 + \text{c x}}}{\text{9 c}^3 \text{ e}} \\ \frac{\text{b }x^2 \sqrt{-1 + \text{c x}}}{\text{9 c e}} - \frac{\text{b d x AncCosh}[\text{c x}]}{\text{e}^2} + \frac{x^3 \left(\text{a + b AncCosh}[\text{c x}]\right)}{\text{3 e}} \\ \frac{\text{b }x^2 \sqrt{-1 + \text{c x}}}{\text{9 c e}} - \frac{\text{b d x AncCosh}[\text{c x}]}{\text{e}^2} + \frac{x^3 \left(\text{a + b AncCosh}[\text{c x}]\right)}{\text{3 e}} \\ \frac{\left(-\text{d}\right)^{3/2} \left(\text{a + b AncCosh}[\text{c x}]\right) \text{Log} \left[1 - \frac{\sqrt{\text{e e}^{\text{AncCosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d - }\sqrt{-\text{c}^2 \text{d - e}}}}\right]}{\text{2 e}^{5/2}} \\ \frac{\left(-\text{d}\right)^{3/2} \left(\text{a + b AncCosh}[\text{c x}]\right) \text{Log} \left[1 + \frac{\sqrt{\text{e e}^{\text{AncCosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d + }\sqrt{-\text{c}^2 \text{d - e}}}}\right]}{\text{2 e}^{5/2}} \\ \frac{\left(-\text{d}\right)^{3/2} \left(\text{a + b AncCosh}[\text{c x}]\right) \text{Log} \left[1 + \frac{\sqrt{\text{e e}^{\text{AncCosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d + }\sqrt{-\text{c}^2 \text{d - e}}}}\right]}{\text{2 e}^{5/2}} \\ \frac{\text{b }\left(-\text{d}\right)^{3/2} \text{PolyLog} \left[2, -\frac{\sqrt{\text{e e}^{\text{AncCosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d - }\sqrt{-\text{c}^2 \text{d - e}}}}\right]}{\text{2 e}^{5/2}} + \frac{\text{b }\left(-\text{d}\right)^{3/2} \text{PolyLog} \left[2, \frac{\sqrt{\text{e e}^{\text{Anccosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d - }\sqrt{-\text{c}^2 \text{d - e}}}}\right]}{\text{2 e}^{5/2}} \\ \frac{\text{b }\left(-\text{d}\right)^{3/2} \text{PolyLog} \left[2, -\frac{\sqrt{\text{e e}^{\text{Anccosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d + }\sqrt{-\text{c}^2 \text{d - e}}}}\right]} + \frac{\text{b }\left(-\text{d}\right)^{3/2} \text{PolyLog} \left[2, \frac{\sqrt{\text{e e}^{\text{Anccosh}[\text{c x}]}}}{\text{c }\sqrt{-\text{d - }\sqrt{-\text{c}^2 \text{d - e}}}}}\right]} \\ \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}}} \\ \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}}} + \frac{\text{2 c}^{5/2}}{\text{2 c}^{5/2}} + \frac{\text{2 c}$$

Result (type 4, 956 leaves):

$$-\frac{a\,d\,x}{e^2} + \frac{a\,x^3}{3\,e} + \frac{a\,d^{3/2}\,\text{ArcTan}\!\left[\frac{\sqrt{e}\,\,x}{\sqrt{d}}\right]}{e^{5/2}} + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{4\,d\,\sqrt{e}\,\,\left(\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)}{c}\right) + \frac{1}{4\,e^{5/2}}\,b\,\left(\frac{1+c\,x}{1+c\,x}\,\,\left(1+c\,x\right)\,-\,c\,\,x\,\text{ArcCosh}\,[\,c\,\,x\,]\right)$$

$$\frac{4 \, e^{3/2} \, \left( \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x} \, \left( 2 + c^2 \, x^2 \right) \, - \, 3 \, c^3 \, x^3 \, \text{ArcCosh} \left[ \, c \, x \, \right] \, \right)}{9 \, c^3} \, + \, \text{i} \, \, d^{3/2} \, \left( \text{ArcCosh} \left[ \, c \, x \, \right] \, \right) + \, \text{i} \, \, d^{3/2} \, d^{3/2}$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{d} \;$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,\frac{\,\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,$$

$$4\,\, \text{$\stackrel{1}{\text{ArcSin}}$} \Big[\, \frac{\sqrt{1+\frac{\text{$\stackrel{1}{\text{c}}}\, \text{$\sqrt{d}$}}{\sqrt{e}}}}{\sqrt{2}}\, \Big]\, \, \text{Log} \, \Big[\, 1\, -\, \frac{\,\, \text{$\stackrel{1}{\text{c}}}\, \left(-\, \text{$c}\,\, \sqrt{\,\text{$d}\,}\, +\, \sqrt{\,\text{$c^2\,\,\text{$d}}\, +\, \text{$e}\,}\,\right)}{\sqrt{\,\text{$e}\,}}\, \, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,]}}\, \Big]\, + \, \frac{\,\, \text{$c^2\,\,\text{$d}\,\, +\, \text{$e}\,\, }}{\sqrt{\,\text{$e}\,\, }}\, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,\,]}}\, \Big]\, + \, \frac{\,\, \text{$c^2\,\,\text{$d}\,\, +\, \text{$e}\,\, }}{\sqrt{\,\text{$e}\,\, }}\, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,\,]}}\, \Big]\, + \, \frac{\,\, \text{$c^2\,\,\text{$d}\,\, +\, \text{$e}\,\, }}{\sqrt{\,\text{$e}\,\, }}\, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,\,]}\, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,\,]}}\, \Big]\, + \, \frac{\,\, \text{$c^2\,\,\text{$d}\,\, +\, \text{$e}\,\, }}{\sqrt{\,\text{$e}\,\, }}\, e^{-\text{ArcCosh}\, [\,\text{$c}\,\,\text{$x}\,\,]}\, e^{-\text{ArcCosh}\, [\,\text{$c}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\dot{\mathbb{I}}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,$$

$$4\,\,\dot{\mathbb{1}}\,\,\text{ArcSin}\Big[\,\frac{\sqrt{\,1+\frac{\dot{\mathbb{1}}\,c\,\sqrt{d}\,}{\sqrt{e}}\,}}{\sqrt{2}}\,\Big]\,\,\text{Log}\,\Big[\,1+\frac{\dot{\mathbb{1}}\,\,\Big(\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\Big)\,\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-$$

$$2 \, \text{PolyLog} \Big[ 2 \text{,} \quad \frac{\text{i} \left( -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \mathbb{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right] \, - \frac{1}{\sqrt{e}} \, \left[ -c \, \sqrt{d} \, + \sqrt{c^2 \, d$$

$$2\,\text{PolyLog}\!\left[\,2\,\text{, }-\frac{\,\text{i}\,\left(\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\right)\,\,\text{e}^{-\text{ArcCosh}\,\left[\,c\,\,x\,\right]}}{\sqrt{e}}\,\right]\,-\,\,\text{i}\,\,d^{3/2}\,\left(\text{ArcCosh}\,\left[\,c\,\,x\,\right]^{\,2}\,+\,\right)$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; - \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; - \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{c} \; \mathsf{d} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \Big] \; + \; \mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d}} \; \mathsf{e}} \; \mathsf{e}\; \mathsf{e}} \; \mathsf{e} \; \mathsf{e}} \; \mathsf{e}\; \mathsf{e}\; \mathsf{e} \; \mathsf{e}\; \mathsf$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\,\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{2}\,\,d\,+\,e^{\,}}\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e^{\,}}}\,\Big]\,\,-\,\,\frac{\,}{}$$

$$4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i } \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] + \\ 2 \text{ ArcCosh}[c \, x] \text{ Log} \Big[ 1 - \frac{\text{i } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i } c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 - \frac{\text{i } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ 2 \text{ PolyLog} \Big[ 2 \text{, } - \frac{\text{i } \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ 2 \text{ PolyLog} \Big[ 2 \text{, } \frac{\text{i } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ \\ 2 \text{ PolyLog} \Big[ 2 \text{, } \frac{\text{i } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] \right]$$

### Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{d + e \, x^2} \, dx$$

$$\operatorname{Optimal} (type \, 4, \, 521 \, leaves, \, 23 \, steps):$$

$$- \frac{b \, x \, \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}}{4 \, c \, e} - \frac{b \operatorname{ArcCosh}[c \, x]}{4 \, c^2 \, e} + \frac{x^2 \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{2 \, e} + \frac{2 \, e}{2 \, e} + \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, e^2} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right)} - \frac{d \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{d \left(a + b \operatorname{ArcCosh}[c$$

Result (type 4, 893 leaves):

$$\frac{1}{4 c^2 e^2} \left( 2 a c^2 e x^2 - 2 a c^2 d Log [d + e x^2] + \right)$$

$$b \left[ 2 c^2 e x^2 \operatorname{ArcCosh}[c x] - e \left[ c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} \sqrt{1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1 + c x}}{\sqrt{2}}\right] \right] - e \left[ -1 + c x \sqrt{-1 + c x} + 2 \operatorname{ArcSi$$

$$c^2\,d\,\left(\text{ArcCosh}\left[\,c\;x\,\right]^{\,2} + 8\,\,\dot{\mathbb{1}}\,\,\text{ArcSin}\left[\,\,\frac{\sqrt{\,1\,+\,\,\frac{\dot{\mathbb{1}}\,c\,\sqrt{d}\,\,}{\sqrt{e}\,\,}}}{\sqrt{2}}\,\right]$$

$$\label{eq:arcTanh} \operatorname{ArcTanh}\Big[\, \frac{\left(c\,\,\sqrt{d}\,\,+\,i\,\,\sqrt{e}\,\,\right)\,\, \operatorname{Tanh}\Big[\,\frac{1}{2}\,\operatorname{ArcCosh}\left[\,c\,\,x\,\right]\,\,\Big]}{\sqrt{c^2\,\,d\,+\,e}}\,\Big]\,\,+\,$$

$$2\operatorname{ArcCosh}[c \ x] \ \operatorname{Log}\left[1 - \frac{\operatorname{i}\left(-c \ \sqrt{d} + \sqrt{c^2 \ d + e}\right) \ \operatorname{e}^{-\operatorname{ArcCosh}[c \ x]}}{\sqrt{e}}\right] -$$

$$4 \; \text{$\mathbb{1}$ ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{$\mathbb{1}$ } c \; \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{$Log} \Big[ 1 - \frac{\text{$\mathbb{1}$ } \left( -c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}{\sqrt{e}} \Big] \; + \\ \frac{1}{\sqrt{e}} \left[ \frac{1}{\sqrt{e}} \right] \; \frac{1}{\sqrt{e}} \; \frac{1$$

$$4\,\, \dot{\mathbb{1}}\, \text{ArcSin} \Big[ \, \frac{\sqrt{1 + \frac{\dot{\mathbb{1}}\, c\, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \, \Big[ \, 1 + \frac{\dot{\mathbb{1}}\, \left( c\, \sqrt{d} \, + \sqrt{c^2\, d + e} \, \right) \, \, e^{-\text{ArcCosh} \, [c\, x]}}{\sqrt{e}} \, \Big] \, - \, \frac{1}{\sqrt{e}} \, \, \frac{1}{\sqrt{e}} \, \left( -\frac{1}{\sqrt{e}} \, \frac{1}{\sqrt{e}} \, \frac{1$$

$$2 \, \text{PolyLog} \, \Big[ \, 2 \, , \, \, \frac{ \, \mathbb{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]} }{\sqrt{e}} \, \Big] \, - \,$$

$$2 \, \text{PolyLog} \left[ 2, - \frac{\dot{\mathbb{I}} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] \right) - c^2 \, d \, \left[ \text{ArcCosh} \left[ c \, x \right]^2 + \right] = c^2 \, d \, \left[ \frac{1}{2} \left( \frac{1}{2$$

$$8 \ \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \ \mathsf{ArcTanh} \Big[ \ \frac{\left( \mathsf{c} \ \sqrt{\mathsf{d}} \ - \ \verb"i" \sqrt{\mathsf{e}} \ \right) \ \mathsf{Tanh} \left[ \ \frac{1}{2} \ \mathsf{ArcCosh} \left[ \mathsf{c} \ \mathsf{x} \right] \ \right]}{\sqrt{\mathsf{c}^2 \ \mathsf{d} + \mathsf{e}}} \Big] \ + \\ = \frac{1}{2} \ \mathsf{ArcTanh} \Big[ \ \frac{1}{2} \ \mathsf{ArcCosh} \left[ \ \mathsf{c} \ \mathsf{x} \right] \ \mathsf{deg} \Big] \ + \\ = \frac{1}{2} \ \mathsf{ArcSin} \Big[ \ \mathsf{arcTanh} \Big[ \ \mathsf{arcT$$

$$\begin{split} & 2\operatorname{ArcCosh}[c\,x]\,\operatorname{Log}\Big[1 + \frac{\mathrm{i}\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] - \\ & 4\,\mathrm{i}\,\operatorname{ArcSin}\Big[\frac{\sqrt{1 - \frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\operatorname{Log}\Big[1 + \frac{\mathrm{i}\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] + \\ & 2\operatorname{ArcCosh}[c\,x]\,\operatorname{Log}\Big[1 - \frac{\mathrm{i}\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] + \\ & 4\,\mathrm{i}\,\operatorname{ArcSin}\Big[\frac{\sqrt{1 - \frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\operatorname{Log}\Big[1 - \frac{\mathrm{i}\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] - \\ & 2\operatorname{PolyLog}\Big[2 \text{, } - \frac{\mathrm{i}\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] - \\ & 2\operatorname{PolyLog}\Big[2 \text{, } \frac{\mathrm{i}\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\mathrm{e}^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\,\Big] - \end{split}$$

# Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(a + b \operatorname{ArcCosh}[c x]\right)}{d + e x^2} \, dx$$

Optimal (type 4, 544 leaves, 23 steps):

$$\frac{a\,x}{e} - \frac{b\,\sqrt{-1 + c\,x}\,\,\sqrt{1 + c\,x}}{c\,e} + \frac{b\,x\,\text{ArcCosh}[c\,x]}{e} + \frac{\sqrt{-d}\,\,\left(a + b\,\text{ArcCosh}[c\,x]\,\right)\,\text{Log}\left[1 - \frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} - \frac{\sqrt{-d}\,\,\left(a + b\,\text{ArcCosh}[c\,x]\,\right)\,\text{Log}\left[1 + \frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} + \frac{\sqrt{-d}\,\,\left(a + b\,\text{ArcCosh}[c\,x]\,\right)\,\text{Log}\left[1 - \frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} - \frac{2\,e^{3/2}}{2\,e^{3/2}} - \frac{\sqrt{-d}\,\,\left(a + b\,\text{ArcCosh}[c\,x]\,\right)\,\text{Log}\left[1 + \frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} - \frac{2\,e^{3/2}}{2\,e^{3/2}} + \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}}{2\,e^{3/2}} - \frac{b\,\sqrt{-d}\,\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,\,e^{\text{Ar$$

#### Result (type 4, 893 leaves):

$$\begin{split} \frac{a\,x}{e} &- \frac{a\,\sqrt{d}\,\operatorname{ArcTan}\left[\frac{\sqrt{e}\,\,x}{\sqrt{d}}\right]}{e^{3/2}} + \\ b\, \left( -\frac{\sqrt{\frac{-1+c\,x}{1+c\,x}}\,\,\left(1+c\,x\right) + c\,x\,\operatorname{ArcCosh}\left[c\,x\right]}{c\,e} - \frac{1}{4\,e^{3/2}}\,\,i\,\sqrt{d}\,\,\left[\operatorname{ArcCosh}\left[c\,x\right]^2 + 8\,i\right] \\ &- \frac{1}{4\,e^{3/2}}\,\,i\,\sqrt{d}\,\,\left[\operatorname{ArcCosh}\left[c\,x\right]^2 + 8\,i\right] \\ &- \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\operatorname{ArcTanh}\left[\frac{\left(c\,\sqrt{d}\,+i\,\sqrt{e}\,\right)\,\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcCosh}\left[c\,x\right]\right]}{\sqrt{c^2\,d}+e}\right] + \\ &- 2\operatorname{ArcCosh}\left[c\,x\right]\operatorname{Log}\left[1 - \frac{i\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\operatorname{Log}\left[1 - \frac{i\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] + \\ &- 2\operatorname{ArcCosh}\left[c\,x\right]\operatorname{Log}\left[1 + \frac{i\,c\,\sqrt{d}}{\sqrt{e}}\right]\operatorname{Log}\left[1 + \frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\operatorname{Log}\left[1 + \frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{e}}\right]\operatorname{Log}\left[1 + \frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{e}}\right]\operatorname{Log}\left[1 + \frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{e}}\right]\operatorname{Log}\left[1 + \frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d}+e\right)\,e^{-\operatorname{ArcCosh}\left[c\,x\right)}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{e}}\right] - \\ &- 4\,i\,\operatorname{Arc$$

$$2 \, \text{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, - \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, + \\ \frac{1}{4 \, \mathrm{e}^{3/2}} \, \mathrm{i} \, \sqrt{d} \, \left[ \mathrm{ArcCosh} \left[ c \, x \right]^2 + 8 \, \mathrm{i} \, \mathrm{ArcSin} \Big[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \right] \, + \\ \mathrm{ArcTanh} \Big[ \frac{\left( c \, \sqrt{d} - \mathrm{i} \, \sqrt{e} \, \right) \, \mathrm{Tanh} \Big[ \frac{1}{2} \, \mathrm{ArcCosh} \left[ c \, x \right] \, \Big]}{\sqrt{c^2 \, d + e}} \Big] \, + \\ 2 \, \mathrm{ArcCosh} \Big[ c \, x \Big] \, \mathrm{Log} \Big[ 1 + \frac{\mathrm{i} \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 4 \, \mathrm{i} \, \mathrm{ArcSin} \Big[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \mathrm{Log} \Big[ 1 + \frac{\mathrm{i} \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, + \\ 4 \, \mathrm{i} \, \mathrm{ArcSin} \Big[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \mathrm{Log} \Big[ 1 - \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, - \frac{\mathrm{i} \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2 \, \mathrm{PolyLog} \Big[ 2, \, \frac{\mathrm{i} \, \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, \mathrm{e}^{- \mathrm{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] \, - \\ 2$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \left(a + b \operatorname{ArcCosh} \left[c x\right]\right)}{d + e x^{2}} dx$$

Optimal (type 4, 449 leaves, 18 steps):

$$-\frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right)^{2}}{2 \, b \, e} + \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, - \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, - \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{2 \, e}{2 \, e} + \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, - \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} \, + \sqrt{-c^{2} \, d - e}}\right]}}{2 \, e} + \frac{b \operatorname{PolyLog}\left[2, - \frac{$$

### Result (type 4, 808 leaves):

$$\frac{1}{2 \, e} \left[ b \, \operatorname{ArcCosh}\left[\, c \, x \,\right]^{\, 2} + 4 \, \, \dot{\mathbb{1}} \, \, b \, \operatorname{ArcSin}\left[\, \frac{\sqrt{1 - \frac{\dot{\mathbb{1}} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \,\right] \, \operatorname{ArcTanh}\left[\, \frac{\left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \,\right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \operatorname{Tanh}\left[\, \frac{1}{2} \, \operatorname{ArcCosh}\left[\, c \, x \,\right] \,\right]}{\sqrt{c^{\, 2} \, d + e}} \, \right] + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \left(\, c \, \sqrt{d}\,\right) \, \right]} + \frac{1}{2} \, \left[\, c \, \sqrt{d}\,\right]} + \frac{1}{$$

$$4 \pm b \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{\pm c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \operatorname{ArcTanh} \left[ \frac{\left( c \sqrt{d} + \pm \sqrt{e} \right) \operatorname{Tanh} \left[ \frac{\pm}{2} \operatorname{ArcCosh} \left[ c \times \right] \right]}{\sqrt{c^2 d + e}} \right] + \frac{1}{2} \left( - c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcCosh} \left[ c \times \right]}$$

$$b\, \operatorname{ArcCosh}\, [\, c\,\, x\, ] \,\, \operatorname{Log}\, \Big[\, 1\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i}\,\, \left( -\, c\,\, \sqrt{\,d\,} \, + \, \sqrt{\,c^{\,2}\,\, d \, + \, e\,} \,\, \right) \,\, \mathrm{e}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}} \,\, -\,\, \frac{\, \mathrm{i}\,\, c\,\, \sqrt{\,c\,}}{\sqrt{\,c\,}} \,\, \Big] \,\, -\,\, \frac{\, \mathrm{i$$

$$2\,\,\dot{\mathbb{1}}\,\,b\,\,\text{ArcSin}\,\Big[\,\frac{\sqrt{\,1+\frac{\dot{\mathbb{1}}\,c\,\,\sqrt{d}\,}{\sqrt{e}\,}}}{\sqrt{2}}\,\Big]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\dot{\mathbb{1}}\,\,\Big(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\Big)}{\sqrt{e}}\,\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}\,\,\Big]\,\,+\,\,\frac{\dot{\mathbb{1}}\,\,c\,\,\sqrt{d}\,\,}{\sqrt{e}}\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}$$

$$b\, \text{ArcCosh}\, [\, c\,\, x\, ] \,\, \text{Log}\, \Big[\, 1\, +\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, \, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big]\, -\,\, \frac{\, \dot{\mathbb{1}} \,\, \left( -\, c\,\, \sqrt{\,d\,} \,\, +\,\, \sqrt{\,c^{\,2}\,\, d\, +\, e\,}\,\, \right) \,\, e^{-\text{ArcCosh}\, [\, c\,\, x\, ]}}$$

$$2 \; \text{$\dot{\text{1}}$ b ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{$\dot{\text{1}}$ c $\sqrt{d}$}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{Log} \Big[ 1 + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}}{\sqrt{e}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right)}}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right)}}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; x \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \; + \sqrt{c^2 \; d + e} \; \right)}} \Big] \; + \frac{\text{$\dot{\text{1}}$ $\left( - c \; x \; + \sqrt{c^2 \; d +$$

$$b\, \operatorname{ArcCosh}\, [\, c\,\, x\, ] \,\, \operatorname{Log}\, \Big[\, 1 \,-\,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{\mathfrak{C}}^{-\operatorname{ArcCosh}\, [\, c\,\, x\, ]}}{\sqrt{\,e\,}}\, \Big] \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, +\, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right) \,\, + \,\, \frac{\dot{\mathbb{I}}\,\, \left(\, c\,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2}\,\, d \,+\, e\,}\,\,\right$$

$$2 \; \text{$\dot{\text{1}}$ b ArcSin} \Big[ \; \frac{\sqrt{1 - \frac{\text{$\dot{\text{1}}$ c} \; \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \; \text{Log} \Big[ 1 - \frac{\text{$\dot{\text{1}}$ } \left( c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; \text{$e^{-\text{ArcCosh} [c \; x]}$}}{\sqrt{e}} \, \Big] \; + \\$$

$$2 \text{ ib ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{ic} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{ii} \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] + \\ a \text{ Log} \Big[ d + e \, x^2 \Big] - b \text{ PolyLog} \Big[ 2 \text{,} - \frac{\text{ii} \left( -c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}}} \Big] - \\ b \text{ PolyLog} \Big[ 2 \text{,} - \frac{\text{ii} \left( -c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}}} \Big] - \\ b \text{ PolyLog} \Big[ 2 \text{,} - \frac{\text{ii} \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}}} \Big] - \\ b \text{ PolyLog} \Big[ 2 \text{,} - \frac{\text{ii} \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}}} \Big]$$

### Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\mathsf{a} + \mathsf{b} \, \mathsf{ArcCosh} \, [\, \mathsf{c} \, \, \mathsf{x} \, ]}{\mathsf{d} + \mathsf{e} \, \, \mathsf{x}^2} \, \, \mathrm{d} \, \mathsf{x}$$

Optimal (type 4, 501 leaves, 18 steps)

Result (type 4, 821 leaves):

$$\frac{1}{2\sqrt{d}\sqrt{e}}$$

$$\left[ 2 \text{ a ArcTan} \Big[ \frac{\sqrt{e} \ x}{\sqrt{d}} \Big] + 4 \text{ b ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ ArcTanh} \Big[ \frac{\left( \text{c} \sqrt{d} - \text{i} \sqrt{e} \right) \text{ Tanh} \Big[ \frac{1}{2} \text{ ArcCosh} \left[ \text{c} x \right] \Big]}{\sqrt{c^2 \, d + e}} \Big] - \frac{1}{2} \left( -\frac{1}{2} \sqrt{e} \right) \left( -\frac{1}{2} \sqrt{e} \right$$

$$4 \text{ b ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{ c} \sqrt{\text{d}}}{\sqrt{\text{e}}}}}{\sqrt{2}} \Big] \text{ ArcTanh} \Big[ \frac{\left(\text{c} \sqrt{\text{d}} + \text{i} \sqrt{\text{e}}\right) \text{ Tanh} \Big[\frac{1}{2} \text{ ArcCosh} [\text{c} \text{ x}] \Big]}{\sqrt{\text{c}^2 \text{d} + \text{e}}} \Big] + \frac{1}{\sqrt{\text{c}^2 \text{d} + \text{e}}} \frac{1$$

$$\label{eq:log_loss} \dot{\texttt{l}} \,\, b \, \text{ArcCosh} \, [\, c \,\, x \,] \,\, \text{Log} \, \Big[ \, 1 \, - \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} }{\sqrt{e}} \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} } \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} } \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \mathbb{e}^{-\text{ArcCosh} \, [\, c \,\, x \,]} } \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c \,\, \sqrt{d} \,\, + \, \sqrt{c^2 \,\, d + e} \,\, \right) \,\, \Big] \,\, + \,\, \frac{\dot{\texttt{l}} \,\, \left( - \, c$$

$$2\,b\,\text{ArcSin}\Big[\,\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\,\text{Log}\Big[1\,-\,\frac{i\,\left(-\,c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\text{$\mathbb{e}^{-\text{ArcCosh}\,[\,c\,x\,]}$}}{\sqrt{e}}\,\Big]\,-\,\frac{i\,\left(-\,c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\text{$\mathbb{e}^{-\text{ArcCosh}\,[\,c\,x\,]}$}}{\sqrt{e}}\,\Big]$$

$$\label{eq:log_loss} \dot{\mathbb{1}} \ b \ \text{ArcCosh} \ [ \ c \ x \ ] \ \ Log \Big[ 1 + \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \, \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ + \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ + \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}{\sqrt{e}} \Big] \ + \frac{\dot{\mathbb{1}} \ \left( - \ c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ \mathbb{e}^{-\text{ArcCosh} \ [ \ c \ x \ ]}}$$

$$2\,b\,\text{ArcSin}\Big[\,\frac{\sqrt{1-\frac{\text{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\,\text{Log}\Big[\,1\,+\,\,\frac{\text{i}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{2}\,d\,+\,e}\,\,\right)\,\,\text{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,-\,$$

$$\label{eq:log_loss} \dot{\mathbb{I}} \ b \ ArcCosh \ [ \ c \ x \ ] \ \ Log \left[ 1 - \frac{\dot{\mathbb{I}} \ \left( c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh \ [ \ c \ x \ ]}}{\sqrt{e}} \right] \ + \\$$

$$2\,b\,\text{ArcSin}\Big[\,\frac{\sqrt{1-\frac{\text{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\,\text{Log}\,\Big[\,1-\frac{\text{i}\,\,\Big(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\,\Big)\,\,\text{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,+$$

$$\label{eq:log_loss} \dot{\mathbb{I}} \ b \ ArcCosh \ [ c \ x \ ] \ Log \Big[ 1 + \frac{\dot{\mathbb{I}} \ \left( c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh \ [ c \ x \ ]}}{\sqrt{e}} \, \Big] \ - \\$$

$$2 \text{ b ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{$\underline{i}$ c $\sqrt{d}$}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{$\underline{i}$ $\left(c \sqrt{d}$ + $\sqrt{c^2 d} + e$}\right) \ e^{-\text{ArcCosh}[c \, x]}}}{\sqrt{e}} \Big] + \frac{\sqrt{e}}{\sqrt{e}} \Big] + \frac{e$$

$$\label{eq:log_log_log} \dot{\mathbb{1}} \ b \ PolyLog \Big[ 2 \text{, } - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ - \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \Big] \ + \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{e} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}$$

$$\label{eq:log_log_log} \dot{\mathbb{1}} \ b \ PolyLog \Big[ 2 \text{,} \ \ \, \frac{\dot{\mathbb{1}} \ \left( - c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \ \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \, \Big] \ -$$

$$\label{eq:log_log_log} \dot{\mathbb{I}} \text{ b PolyLog} \left[ 2 \text{, } -\frac{\dot{\mathbb{I}} \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \right] \, + \, \left( \frac{1}{2} \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \left( c \, \sqrt{e} \, \right) \right) \, + \, \left( c \, \sqrt{e} \, \right) \, + \, \left( c \, \sqrt{$$

$$i \ b \ PolyLog \left[ 2, \ \frac{i \left( c \ \sqrt{d} \ + \sqrt{c^2 \ d + e} \right) \ e^{-ArcCosh[c \ x]}}{\sqrt{e}} \right]$$

### Problem 34: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c \ x]}{x \ (d + e \ x^2)} \ dx$$

### Optimal (type 4, 472 leaves, 25 steps):

$$\frac{\left(a+b\operatorname{ArcCosh}[c\,x]\right)\operatorname{Log}\left[1-\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{2\,d} = \frac{\left(a+b\operatorname{ArcCosh}[c\,x]\right)\operatorname{Log}\left[1+\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{2\,d} = \frac{2\,d}{2\,d}$$

$$\frac{\left(a+b\operatorname{ArcCosh}[c\,x]\right)\operatorname{Log}\left[1-\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,d} = \frac{\left(a+b\operatorname{ArcCosh}[c\,x]\right)\operatorname{Log}\left[1+\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,d} + \frac{2\,d}{2\,d}$$

$$\frac{\left(a+b\operatorname{ArcCosh}[c\,x]\right)\operatorname{Log}\left[1+\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,d} = \frac{b\operatorname{PolyLog}\left[2,-\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,d}$$

$$\frac{b\operatorname{PolyLog}\left[2,\frac{\sqrt{e}\ e^{\operatorname{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{2\,d} + \frac{b\operatorname{PolyLog}\left[2,-e^{\operatorname{2ArcCosh}[c\,x]}\right]}{2\,d}$$

#### Result (type 4, 837 leaves):

$$-\frac{1}{2\,\mathsf{d}}\left(4\,\,\dot{\mathbb{1}}\,\,\mathsf{b}\,\mathsf{ArcSin}\Big[\,\frac{\sqrt{1-\frac{\dot{\mathbb{1}}\,\mathsf{c}\,\sqrt{\mathsf{d}}}{\sqrt{\mathsf{e}}}}}{\sqrt{2}}\,\Big]\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{Tanh}\,\Big[\,\frac{1}{2}\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,]\,\,\Big]}{\sqrt{\mathsf{c}^2\,\,\mathsf{d}\,+\,\mathsf{e}}}\,\Big]\,\,+\,\,\mathsf{ArcTanh}\,\Big[\,\frac{\left(\mathsf{c}\,\,\sqrt{\mathsf{d}}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{\mathsf{e}}\,\,\right)\,\,\mathsf{d}\,\mathsf{c}}{\sqrt{\mathsf{e}^2}\,\,\mathsf{d}\,+\,\mathsf{e}}\,\,\mathsf{d}\,\mathsf{c}\,\mathsf{d}\,\mathsf{c}}\,\Big]\,\,+\,\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{c}\,\,\mathsf{d}\,\,\mathsf{c}\,\,\mathsf{c}\,\,\mathsf{d}\,\,\mathsf{d}\,\,\mathsf{c}}\,\mathsf{d}\,\,\mathsf{c}\,\mathsf{d}\,\,\mathsf{c}\,\,\mathsf{d}\,\,\mathsf{d}\,\,\mathsf{c}\,\,\mathsf{d}\,\,\mathsf{c}\,\,\mathsf{d}\,\,\mathsf{d}\,\,\mathsf{c}\,\,\mathsf{d$$

$$4 \pm b \operatorname{ArcSin} \Big[ \frac{\sqrt{1 + \frac{\pm c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \operatorname{ArcTanh} \Big[ \frac{\left( c \sqrt{d} + \pm \sqrt{e} \right) \operatorname{Tanh} \Big[ \frac{1}{2} \operatorname{ArcCosh} [c \, x] \Big]}{\sqrt{c^2 \, d + e}} \Big] - 2 \, b \operatorname{ArcCosh} [c \, x] \\ \operatorname{Log} \Big[ 1 + e^{-2 \operatorname{ArcCosh} [c \, x]} \Big] + b \operatorname{ArcCosh} [c \, x] \operatorname{Log} \Big[ 1 - \frac{\pm \left( -c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\operatorname{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \frac{\pm c \sqrt{d}}{\sqrt{e}} \Big]$$

$$2\,\,\dot{\mathbb{1}}\,\,b\,\,\mathsf{ArcSin}\,\Big[\,\frac{\sqrt{1+\frac{\dot{\mathbb{1}}\,c\,\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\,\mathsf{Log}\,\Big[\,1\,-\,\frac{\dot{\mathbb{1}}\,\,\Big(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{c^2\,d+e}\,\,\Big)\,\,\,\mathrm{e}^{-\mathsf{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,+\,$$

$$\begin{split} & \text{b} \, \text{ArcCosh} \, [\text{c} \, x] \, \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( -\text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 - \frac{\text{i} \, c \, \sqrt{d}}{\sqrt{\text{e}}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( -\text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, + \\ & b \, \text{ArcCosh} \, [\text{c} \, x] \, \, \text{Log} \, \Big[ 1 - \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, + \\ & b \, \text{ArcCosh} \, [\text{c} \, x] \, \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, + \\ & b \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, \text{c} \, \sqrt{d}}{\sqrt{\text{e}}}}}{\sqrt{2}} \, \Big] \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, \text{c} \, \sqrt{d}}{\sqrt{\text{e}}}}}{\sqrt{2}} \, \Big] \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, \text{c} \, \sqrt{d}}{\sqrt{\text{e}}}}}{\sqrt{2}} \, \Big] \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, \text{c} \, \sqrt{d}}{\sqrt{\text{e}}}}}{\sqrt{2}} \, \Big] \, \text{Log} \, \Big[ 1 + \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{ArcSin} \, \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, \text{c} \, \sqrt{d}}}{\sqrt{2}} \, \Big] \, \text{b} \, \text{PolyLog} \, \Big[ 2 \, , \, - \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}}{\sqrt{\text{e}}} \, \Big] \, - \\ & 2 \, \text{i} \, \text{b} \, \text{PolyLog} \, \Big[ 2 \, , \, - \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text{c} \, x)}}{\sqrt{\text{e}}} \, \Big] \, - \\ & \text{b} \, \text{PolyLog} \, \Big[ 2 \, , \, - \frac{\text{i} \, \left( \text{c} \, \sqrt{d} \, + \sqrt{\text{c}^2 \, d + \text{e}} \, \right) \, \text{e}^{-\text{ArcCosh} \, (\text$$

## Problem 35: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\, ArcCosh\, [\, c\,\, x\,]}{x^2\, \left(d+e\, x^2\right)}\, \, \mathrm{d}x$$

Optimal (type 4, 543 leaves, 23 steps):

$$-\frac{a + b \operatorname{ArcCosh}[c \ x]}{d \ x} + \frac{b \operatorname{c ArcTan}\left[\sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}\right]}{d} + \frac{\sqrt{e} \left(a + b \operatorname{ArcCosh}[c \ x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} - \sqrt{-c^2 \ d - e}}\right]}{c \sqrt{-d} - \sqrt{-c^2 \ d - e}} - \frac{2 \left(-d\right)^{3/2}}{2 \left(-d\right)^{3/2}} + \frac{\sqrt{e} \ \left(a + b \operatorname{ArcCosh}[c \ x]\right) \operatorname{Log}\left[1 + \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} - \sqrt{-c^2 \ d - e}}\right]}{c \sqrt{-d} + \sqrt{-c^2 \ d - e}} + \frac{\sqrt{e} \ \left(a + b \operatorname{ArcCosh}[c \ x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} + \sqrt{-c^2 \ d - e}}\right]}{2 \left(-d\right)^{3/2}} - \frac{\sqrt{e} \ \left(a + b \operatorname{ArcCosh}[c \ x]\right) \operatorname{Log}\left[1 + \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} + \sqrt{-c^2 \ d - e}}\right]}{2 \left(-d\right)^{3/2}} + \frac{b \sqrt{e} \ \operatorname{PolyLog}\left[2, \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} - \sqrt{-c^2 \ d - e}}\right]}{2 \left(-d\right)^{3/2}} - \frac{b \sqrt{e} \ \operatorname{PolyLog}\left[2, -\frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} + \sqrt{-c^2 \ d - e}}}\right]}{2 \left(-d\right)^{3/2}} + \frac{b \sqrt{e} \ \operatorname{PolyLog}\left[2, \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \sqrt{-d} + \sqrt{-c^2 \ d - e}}}\right]}{2 \left(-d\right)^{3/2}}$$

#### Result (type 4, 887 leaves):

Result (type 4, 887 leaves): 
$$\frac{1}{4 \, d^{3/2} \, x} \left[ -4 \, a \, \sqrt{d} - 4 \, a \, \sqrt{e} \, x \, \text{ArcTan} \Big[ \frac{\sqrt{e} \, x}{\sqrt{d}} \Big] - 4 \, a \, \sqrt{d} - 4 \, a \, \sqrt{e} \, x \, \text{ArcTan} \Big[ \frac{1}{\sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}} \Big] \right) - i \, b \, \sqrt{e} \, x$$

$$\left[ \text{ArcCosh} \left[ c \, x \right]^2 + 8 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 + \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{ArcTanh} \Big[ \frac{\left( c \, \sqrt{d} + i \, \sqrt{e} \right) \, \text{Tanh} \Big[ \frac{1}{2} \, \text{ArcCosh} \left[ c \, x \right] \Big]}{\sqrt{c^2 \, d + e}} \Big] + 2 \, \text{ArcCosh} \left[ c \, x \right] \, \log \Big[ 1 - \frac{i \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{c^2 \, d + e}} \Big] - \frac{i \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{c^2 \, d + e}} \right] - \frac{i \, \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{c^2 \, d + e}}$$

 $4 \pm \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{\pm c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{\pm \left(-c\sqrt{d} + \sqrt{c^2d + e}\right) e^{-\operatorname{ArcCosh}[cx]}}{\sqrt{2}}\right] + \frac{1}{\sqrt{2}}$ 

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x^3 (d + e x^2)} dx$$

### Optimal (type 4, 531 leaves, 27 steps):

$$\frac{b \ c \ \sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}}{2 \ d \ x} - \frac{a + b \ ArcCosh[c \ x]}{2 \ d \ x^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ -\sqrt{-c^2 d - e}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ -\sqrt{-c^2 d - e}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}}} {e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}}} \right]}{2 \ d^2} + \frac{e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}}} {e \ (a + b \ ArcCosh[c \ x]) \ Log \left[1 - \frac{\sqrt{e \ e^{ArcCosh[c \ x]}}}{c \ \sqrt{-d} \ +\sqrt{-c^2 d - e}}}} \right]}$$

### Result (type 4, 913 leaves):

$$\begin{split} \frac{1}{4\,d^2\,x^2} \\ & \left[ -2\,a\,d - 4\,a\,e\,x^2\,\text{Log}\big[x\big] + 2\,a\,e\,x^2\,\text{Log}\big[d + e\,x^2\big] + b\, \left[ 2\,d\,\left(c\,x\,\sqrt{-1 + c\,x}\,\,\sqrt{1 + c\,x}\,\,-\,\text{ArcCosh}[c\,x]\,\right) - 2\,e\,x^2\,\left(\text{ArcCosh}[c\,x]\,\left(\text{ArcCosh}[c\,x] + 2\,\text{Log}\big[1 + e^{-2\,\text{ArcCosh}[c\,x]}\,\right]\right) - \text{PolyLog}\big[2,\,-e^{-2\,\text{ArcCosh}[c\,x]}\,\big]\right) + \\ & e\,x^2\,\left[ \text{ArcCosh}[c\,x]\,^2 + 8\,\frac{i}{2}\,\text{ArcSin}\Big[\frac{\sqrt{1 + \frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]} \right] \\ & - \text{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}\,+i\,\sqrt{e}\,\right)\,\text{Tanh}\Big[\frac{1}{2}\,\text{ArcCosh}[c\,x]\,\Big]}{\sqrt{c^2\,d + e}}\Big] + \\ & - 2\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1 - \frac{i\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d + e}\,\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ & - 4\,i\,\text{ArcSin}\Big[\frac{\sqrt{1 + \frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1 - \frac{i\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d + e}\,\right)\,e^{-\text{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] + \end{split}$$

$$\begin{split} & 2\operatorname{ArcCosh}[c\,x]\, \text{Log}\Big[1 + \frac{\mathrm{i}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \\ & 4\,\mathrm{i}\,\operatorname{ArcSin}\Big[\frac{\sqrt{1 + \frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\,\operatorname{Log}\Big[1 + \frac{\mathrm{i}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] - \\ & 2\operatorname{PolyLog}\Big[2, \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] - \\ & 2\operatorname{PolyLog}\Big[2, -\frac{\mathrm{i}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \mathrm{e}\,x^2\left[\operatorname{ArcCosh}[c\,x]^2 + \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \mathrm{e}\,x^2\left[\operatorname{ArcCosh}[c\,x]^2 + \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \\ & 2\operatorname{ArcCosh}[c\,x]\,\operatorname{Log}\Big[1 + \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \\ & 4\,\mathrm{i}\operatorname{ArcSin}\Big[\frac{\sqrt{1 - \frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\operatorname{Log}\Big[1 + \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] + \\ & 4\,\mathrm{i}\operatorname{ArcSin}\Big[\frac{\sqrt{1 - \frac{\mathrm{i}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\operatorname{Log}\Big[1 - \frac{\mathrm{i}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big] - \\ & 2\operatorname{PolyLog}\Big[2, -\frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big]\Big] - \\ & 2\operatorname{PolyLog}\Big[2, \frac{\mathrm{i}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)\,\mathrm{e}^{-\operatorname{ArcCosh}(c\,x)}}{\sqrt{e}}\Big]\Big] + \\ & \operatorname{PolyLog}\Big[2, \frac{\mathrm{i}\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d} + \mathrm{e}\right)$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \, ArcCosh[c \, x]}{x^4 \, \left(d + e \, x^2\right)} \, \mathrm{d}x$$

### Optimal (type 4, 624 leaves, 28 steps):

$$\frac{b \ c \ \sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}}{6 \ d \ x^2} - \frac{a + b \ ArcCosh[c \ x]}{3 \ d \ x^3} + \frac{e \ (a + b \ ArcCosh[c \ x])}{d^2 \ x} + \frac{b \ c^3 \ ArcTan[\sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}]}{6 \ d} - \frac{e^{3/2} \ (a + b \ ArcCosh[c \ x]) \ Log[1 - \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ \sqrt{-c^2 \ d - e}}]}{2 \ (-d)^{5/2}} - \frac{e^{3/2} \ (a + b \ ArcCosh[c \ x]) \ Log[1 + \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ -\sqrt{-c^2 \ d - e}}]}{2 \ (-d)^{5/2}} + \frac{e^{3/2} \ (a + b \ ArcCosh[c \ x]) \ Log[1 + \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}]}{2 \ (-d)^{5/2}} - \frac{e^{3/2} \ (a + b \ ArcCosh[c \ x]) \ Log[1 + \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}]}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ -\sqrt{-c^2 \ d - e}}]}{2 \ (-d)^{5/2}} + \frac{b \ e^{3/2} \ PolyLog[2, \ \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ -\sqrt{-c^2 \ d - e}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}{2 \ (-d)^{5/2}} + \frac{b \ e^{3/2} \ PolyLog[2, \ \frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}{2 \ (-d)^{5/2}} - \frac{b \ e^{3/2} \ PolyLog[2, \ -\frac{\sqrt{e} \ e^{ArcCosh(c \ x)}}{c \sqrt{-d} \ +\sqrt{-c^2 \ d - e}}}}$$

#### Result (type 4, 972 leaves):

$$\begin{split} \frac{1}{12\,\mathsf{d}^{5/2}\,\mathsf{x}^3} \left( & -4\,\mathsf{a}\,\mathsf{d}^{3/2} + 12\,\mathsf{a}\,\sqrt{\mathsf{d}}\,\,\mathsf{e}\,\mathsf{x}^2 + 12\,\mathsf{a}\,\mathsf{e}^{3/2}\,\mathsf{x}^3\,\mathsf{ArcTan}\Big[\,\frac{\sqrt{\mathsf{e}}\,\,\mathsf{x}}{\sqrt{\mathsf{d}}}\,\Big] \,+ \\ & b \left( & 12\,\sqrt{\mathsf{d}}\,\,\mathsf{e}\,\mathsf{x}^2\,\left(\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,] \,+\,\mathsf{c}\,\,\mathsf{x}\,\,\mathsf{ArcTan}\Big[\,\frac{1}{\sqrt{-1+\mathsf{c}\,\,\mathsf{x}}}\,\sqrt{1+\mathsf{c}\,\,\mathsf{x}}\,\Big] \right) \,+ \\ & 2\,\mathsf{d}^{3/2}\left(\mathsf{c}\,\,\mathsf{x}\,\sqrt{-1+\mathsf{c}\,\,\mathsf{x}}\,\,\sqrt{1+\mathsf{c}\,\,\mathsf{x}}\,\,-\,2\,\mathsf{ArcCosh}\,[\,\mathsf{c}\,\,\mathsf{x}\,] \,-\,\mathsf{c}^3\,\,\mathsf{x}^3\,\mathsf{ArcTan}\Big[\,\frac{1}{\sqrt{-1+\mathsf{c}\,\,\mathsf{x}}\,\,\sqrt{1+\mathsf{c}\,\,\mathsf{x}}}\,\Big] \right) \,+ \end{split}$$

$$3 i e^{3/2} x^3 \left( ArcCosh[cx]^2 + \right)$$

$$8 \ \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt 2} \Big] \ \mathsf{ArcTanh} \Big[ \frac{\left( \mathsf{c} \ \sqrt{\mathsf{d}} \ + \ \verb"i" \sqrt{\mathsf{e}} \ \right) \ \mathsf{Tanh} \Big[ \frac{1}{2} \ \mathsf{ArcCosh} \, [\, \mathsf{c} \ \mathsf{x} \, ] \ \Big]}{\sqrt{\mathsf{c}^2 \ \mathsf{d} + \mathsf{e}}} \Big] \ + \frac{1}{2} \ \mathsf{ArcCosh} \Big[ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{x} \, ] \Big]}{\mathsf{deg} \Big[ \mathsf{c} \ \mathsf{x} \, ]} \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{c} \ \mathsf{c} \ \mathsf{x} \, ] \ \mathsf{deg} \Big[ \mathsf{c} \ \mathsf{c}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\mathrm{i}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\right)\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\,\,$$

$$4 \; \text{$\stackrel{1}{\text{a}}$ ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{$\stackrel{1}{\text{c}}} \, \text{$\sqrt{d}}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{$\text{Log} \Big[ 1 - \frac{\text{$\stackrel{1}{\text{c}}} \, \Big( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \Big) \, \, e^{-\text{ArcCosh} \, [\, c \, \, x \, ]}}{\sqrt{e}} \, \Big] \; + \frac{1}{\sqrt{e}} \; + \frac{1}{\sqrt{e}} \; \left[ \frac{1}{\sqrt{e}} \, \frac{1}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,\,\frac{\mathrm{i}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,$$

$$4\,\,\dot{\mathbb{1}}\,\,\text{ArcSin}\,\Big[\,\frac{\sqrt{\,1+\frac{\dot{\mathbb{1}}\,c\,\sqrt{d}\,}{\sqrt{e}}\,}}{\sqrt{2}}\,\Big]\,\,\text{Log}\,\Big[\,1+\frac{\dot{\mathbb{1}}\,\,\Big(\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\Big)\,\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,-$$

$$2 \, \text{PolyLog} \Big[ 2 \text{,} \quad \frac{\text{i} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, \text{e}^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] \, - \\$$

$$8 \ \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \ \mathsf{ArcTanh} \Big[ \frac{\left( \mathsf{c} \ \sqrt{\mathsf{d}} \ - \ \verb"i" \sqrt{\mathsf{e}} \ \right) \ \mathsf{Tanh} \Big[ \frac{1}{2} \ \mathsf{ArcCosh} \, [\, \mathsf{c} \ \mathsf{x} \, ] \, \Big]}{\sqrt{\mathsf{c}^2 \ \mathsf{d} + \mathsf{e}}} \Big] \ + \\ = \frac{1}{2} \left( \mathsf{c} \ \mathsf{v} \, \mathsf{d} \, - \ \mathsf{e} \, \mathsf{e$$

$$2\operatorname{ArcCosh}[c \ x] \ \operatorname{Log}\left[1 + \frac{i \left(-c \sqrt{d} + \sqrt{c^2 \ d + e}\right) e^{-\operatorname{ArcCosh}[c \ x]}}{\sqrt{e}}\right] -$$

$$4 \, \, \mathtt{i} \, \, \text{ArcSin} \Big[ \, \frac{\sqrt{1 - \frac{\mathtt{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \Big[ 1 + \frac{\mathtt{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{\mathrm{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\dot{\mathbb{1}}\,\,\left(c\,\,\sqrt{d}\,\,+\,\sqrt{c^2\,d+e}\,\,\right)\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,+\,\,\frac{1}{2}\,\left(\frac{1}{2}\,\,\left(\frac{1$$

$$4 \, \text{in ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{ic } \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 - \frac{\text{ii} \, \Big( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \Big) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - 2 \, \text{PolyLog} \Big[ 2 \text{, } - \frac{\text{ii} \, \Big( - c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \Big) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - 2 \, \text{PolyLog} \Big[ 2 \text{, } \frac{\text{ii} \, \Big( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \Big) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] \Big] \Big| \Big| \Big|$$

### Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \, \left(a + b \, ArcCosh \left[\, c \, x \, \right]\,\right)}{\left(d + e \, x^2\right)^2} \, \mathrm{d}x$$

### Optimal (type 4, 562 leaves, 24 steps):

$$\frac{d \left(a + b \operatorname{ArcCosh}[c \ x]\right)}{2 \ e^{2} \left(d + e \ x^{2}\right)} - \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right)^{2}}{2 \ b \ e^{2}} - \\ \frac{b \ c \ \sqrt{d} \ \sqrt{-1 + c^{2} \ x^{2}} \ \operatorname{ArcTanh}\left[\frac{\sqrt{c^{2} \ d + e} \ x}{\sqrt{d} \ \sqrt{-1 + c^{2} x^{2}}}\right]}{\sqrt{d} \ \sqrt{-1 + c \ x}} + \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right) \ \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} - \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right) \ \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} - \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right) \ \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right) \ \operatorname{Log}\left[1 - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} - \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{\sqrt{e} \ e^{\operatorname{ArcCosh}[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{b}{c} + \frac{b}{c}}\right]}{2 \ e^{2}} + \frac{b \ \operatorname{PolyLog}\left[2, - \frac{b}{c} + \frac$$

### Result (type 4, 1108 leaves):

$$\frac{1}{4 \, e^2} \left[ \frac{2 \, a \, d}{d + e \, x^2} + 2 \, a \, \text{Log} \left[ d + e \, x^2 \right] + b \, \left[ \frac{\sqrt{d} \, \, \text{ArcCosh} \left[ c \, x \right]}{\sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e} \, \, x} + \frac{\sqrt{d} \, \, \text{ArcCosh} \left[ c \, x \right]}{\sqrt{d} \, + \dot{\mathbb{1}} \, \sqrt{e} \, \, x} + 2 \, \text{ArcCosh} \left[ c \, x \right]^2 + 2 \, \left[ \frac{d}{d} + e \, x^2 \right] + 2 \, \left[ \frac{d}{d} + e \,$$

$$8 \, \, \dot{\mathbb{1}} \, \operatorname{ArcSin} \Big[ \frac{\sqrt{1 - \frac{\dot{\mathbb{1}} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \operatorname{ArcTanh} \Big[ \, \frac{\left(c \, \sqrt{d} \, - \dot{\mathbb{1}} \, \sqrt{e} \, \right) \, \operatorname{Tanh} \Big[ \, \frac{1}{2} \, \operatorname{ArcCosh} \left[ \, c \, \, x \, \right] \, \Big]}{\sqrt{c^2 \, d + e}} \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, \Big[ \, c \, \, x \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, \Big] \, + \frac{1}{2} \, \operatorname{ArcCosh} \Big[ \, c \, \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big] \, \Big[ \, c \, x \, \Big[ \, x \, \Big[ \, c \, x \, \Big[ \, x \, \Big[$$

$$8 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ ArcTanh} \Big[ \frac{\left( \text{c} \sqrt{d} + \text{i} \sqrt{e} \right) \text{ Tanh} \Big[ \frac{1}{2} \text{ ArcCosh} \{ \text{c} \, \text{x} \} \Big]}{\sqrt{c^2 \, d + e}} \Big] + \\ 2 \text{ ArcCosh} \Big[ \text{c} \, \text{x} \Big] \text{ Log} \Big[ 1 - \frac{\text{i} \left( - \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] - \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 - \frac{\text{i} \left( - \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] - \\ 2 \text{ ArcCosh} \Big[ \text{c} \, \text{x} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( - \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 2 \text{ ArcCosh} \Big[ \text{c} \, \text{x} \Big] \text{ Log} \Big[ 1 - \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 - \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 2 \text{ ArcCosh} \Big[ \text{c} \, \text{x} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{i} \text{c} \sqrt{d}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( \text{c} \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} \{ \text{c} \, \text{x} \}}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 + \frac$$

2 PolyLog[2, 
$$\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-ArcCosh[c x]}}{\sqrt{e}}$$
]

### Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{x (d + e x^2)^2} dx$$

Optimal (type 4, 581 leaves, 29 steps):

$$\frac{a + b \operatorname{ArcCosh}[c \, x]}{2 \, d \, (d + e \, x^2)} = \frac{b \, c \, \sqrt{-1 + c^2 \, x^2} \, \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 \, d + e}}{\sqrt{d} \, \sqrt{-1 + c^2 \, x^2}}\right]}{2 \, d^{3/2} \, \sqrt{c^2 \, d + e} \, \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}} = \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \, \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \, \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \, \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} + \frac{\left(a + b \operatorname{ArcCosh}[c \, x]\right) \, \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{b \, \operatorname{PolyLog}\left[2, \, -\frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{b \, \operatorname{PolyLog}\left[2, \, -\frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{b \, \operatorname{PolyLog}\left[2, \, -\frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{b \, \operatorname{PolyLog}\left[2, \, -\frac{e^2 \, \operatorname{ArcCosh}[c \, x]}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2} = \frac{b \, \operatorname{PolyLog}\left[2, \, -\frac{e^2 \, \operatorname{ArcCosh}[c \, x]}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{2 \, d^2}$$

### Result (type 4, 1146 leaves):

$$\frac{a}{2 \ d^2 + 2 \ d \ e \ x^2} + \frac{a \ Log \left[ \, x \, \right]}{d^2} - \frac{a \ Log \left[ \, d + e \ x^2 \, \right]}{2 \ d^2} + \\$$

$$\frac{1}{4\,d^2}\,b\,\left(\frac{\sqrt{d}\,\operatorname{ArcCosh}\,[\,c\,\,x\,]}{\sqrt{d}\,\,-\,\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,x}\,+\,\,\frac{\sqrt{d}\,\operatorname{ArcCosh}\,[\,c\,\,x\,]}{\sqrt{d}\,\,+\,\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,x}\,-\,8\,\,\dot{\mathbb{1}}\,\operatorname{ArcSin}\,\Big[\,\frac{\sqrt{1-\frac{\dot{\mathbb{1}}\,c\,\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\right)$$

$$\begin{array}{l} \text{ArcTanh} \Big[ \frac{\left( c \sqrt{d} - i \sqrt{e} \right) \, \text{Tanh} \big[ \frac{1}{2} \, \text{ArcCosh} [c \, x] \, \big]}{\sqrt{c^2 \, d + e}} \, \Big] - 8 \, i \, \text{ArcSin} \big[ \frac{\sqrt{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \big] }{\sqrt{2}} \Big] \\ \text{ArcTanh} \Big[ \frac{\left( c \sqrt{d} + i \sqrt{e} \right) \, \text{Tanh} \big[ \frac{1}{2} \, \text{ArcCosh} [c \, x] \, \big]}{\sqrt{c^2 \, d + e}} \Big] + 4 \, \text{ArcCosh} [c \, x] \, \text{Log} \big[ 1 + e^{-2 \, \text{ArcCosh} [c \, x]} \, \big] - 2 \, \text{ArcCosh} [c \, x] \, \text{Log} \big[ 1 - \frac{i \left( - c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \big] + \\ 4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( - c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] + \\ 4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( - c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ 2 \, \text{ArcCosh} [c \, x] \, \text{Log} \Big[ 1 - \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ 4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 - \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ 2 \, \text{ArcCosh} [c \, x] \, \text{Log} \Big[ 1 + \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ 4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ 4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ \frac{i \left( c \sqrt{d} \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \right] \, \text{Log} \Big[ 1 + \frac{i \left( c \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] - \\ \frac{i \left( c \sqrt{d} \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \right] \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] - \\ \frac{i \left( c \sqrt{d} \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] - \\ \frac{i \left( c \sqrt{d} \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] - \\ \frac{i \, c \sqrt{d} \, \text{Log} \Big[ \frac{1 + \frac{i \, c \sqrt{d}}{\sqrt{e}}}{\sqrt{e}} \Big] \, \text{Log} \Big[ \frac{1 + \frac{i$$

 $2 \operatorname{PolyLog} \left[ 2, - e^{-2 \operatorname{ArcCosh}[c \, x]} \right] + 2 \operatorname{PolyLog} \left[ 2, - \frac{i \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\operatorname{ArcCosh}[c \, x]}}{2} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right] + c \operatorname{PolyLog} \left[ -c \, \sqrt{d} + \sqrt$ 

$$2 \, \mathsf{PolyLog} \Big[ 2 \text{,} \quad \frac{\mathbb{i} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\mathsf{ArcCosh} [\, c \, x \,]}}{\sqrt{e}} \Big] \, + \\ 2 \, \mathsf{PolyLog} \Big[ 2 \text{,} \quad -\frac{\mathbb{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\mathsf{ArcCosh} [\, c \, x \,]}}{\sqrt{e}} \Big] \, + \\ 2 \, \mathsf{PolyLog} \Big[ 2 \text{,} \quad \frac{\mathbb{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{-\mathsf{ArcCosh} [\, c \, x \,]}}{\sqrt{e}} \Big]$$

# Problem 41: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\! \frac{a+b\, ArcCosh\, [\, c\,\, x\,]}{x^3\, \left(d+e\, x^2\right)^2}\, \mathrm{d}x$$

Optimal (type 4, 616 leaves, 31 steps):

$$\frac{b \ c \ \sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}}{2 \ d^{2} \ x} - \frac{a + b \ ArcCosh[c \ x]}{2 \ d^{2} \ x^{2}} - \frac{e \ \left(a + b \ ArcCosh[c \ x]\right)}{2 \ d^{2} \ \left(d + e \ x^{2}\right)} + \\ \frac{b \ c \ e \ \sqrt{-1 + c^{2} \ x^{2}} \ ArcTanh\left[\frac{\sqrt{c^{2} \ d + e} \ x}{\sqrt{d} \ \sqrt{-1 + c^{2} \ x^{2}}}\right]}{2 \ d^{5/2} \ \sqrt{c^{2} \ d + e} \ \sqrt{-1 + c \ x} \ \sqrt{1 + c \ x}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 - \frac{\sqrt{e} \ e^{ArcCosh[c \ x]}}{c \ \sqrt{-d} - \sqrt{-c^{2} \ d - e}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 - \frac{\sqrt{e} \ e^{ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 - \frac{\sqrt{e} \ e^{ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 - \frac{\sqrt{e} \ e^{ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right]}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}{c \ \sqrt{-d} + \sqrt{-c^{2} \ d - e}}}\right)}{d^{3}} + \frac{e \ \left(a + b \ ArcCosh[c \ x]\right) \ Log\left[1 + \frac{e^{2} \ ArcCosh[c \ x]}{c \ \sqrt{-d} + \sqrt{-c$$

Result (type 4, 1237 leaves):

$$-\frac{a}{2\,d^2\,x^2}-\frac{a\,e}{2\,d^2\,\left(d+e\,x^2\right)}-\frac{2\,a\,e\,Log\,[\,x\,]}{d^3}+\frac{a\,e\,Log\,\left[\,d+e\,x^2\,\right]}{d^3}+$$

$$b \left( \frac{\frac{\text{ArcCosh[c\,x]}}{\text{c x}\,\sqrt{-1 + \text{c x}}} \,\sqrt{1 + \text{c x}} \,- \text{ArcCosh[c\,x]}}{2\,d^2\,x^2} + \frac{\frac{\text{c Log}\left[\frac{2\,e\left[\text{i}\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,x - \text{i}\,\sqrt{-c^2\,d - e}}{\,c\,\sqrt{-c^2\,d - e}}\,\left(\sqrt{d}\,\,+ \text{i}\,\sqrt{e}\,\,x\right)\right]}{\sqrt{-c^2\,d - e}}\right)}{2\,d^2\,x^2} + \frac{1}{2\,d^2\,x^2} + \frac{2\,d^{5/2}}{2\,d^{5/2}} + \frac{1}{2\,d^{5/2}} + \frac{1}{2\,d^{5/2$$

 $e \left( \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \right] \; \left( \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \right] \; + \; 2 \; \mathsf{Log} \left[ \mathsf{1} \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \right]} \; \right] \right) \; - \; \mathsf{PolyLog} \left[ \mathsf{2} \; \text{,} \; - \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \right]} \; \right] \right) \; + \; \mathsf{PolyLog} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; + \; \mathsf{e}^{-2 \, \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right]} \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x} \; \right] \; + \; \mathsf{e}^{-2 \, \mathsf{arcCosh} \left[ \mathsf{c} \; \mathsf{x}$ 

$$\frac{1}{2 d^3} e \left[ ArcCosh [cx]^2 + \right]$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{c} \; \mathsf{d} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \Big] \; + \; \mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e} \; \mathsf{e}} \; \mathsf{e} \; \mathsf{e}} \; \mathsf{e} \; \mathsf{e}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\text{i}\,\,\left(-\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\text{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,-\,\,$$

$$4 \, \, \text{i ArcSin} \Big[ \, \frac{\sqrt{1 + \frac{\text{i c} \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \Big[ 1 - \frac{\text{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{1}{\sqrt{e}} \, \, \text{Im} \Big[ \frac{1}{\sqrt{e}} \, \frac{\text{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, + \frac{1}{\sqrt{e}} \, \, \text{Im} \Big[ \frac{1}{\sqrt{e}} \, \frac{1}{\sqrt{e}}$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\dot{\mathbb{1}}\,\,\left(\,c\,\,\sqrt{\,d\,}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\,\right)\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{\,e\,}}\,\Big]\,\,+\,$$

$$4\,\,\dot{\mathbb{1}}\,\,\text{ArcSin}\Big[\,\frac{\sqrt{\,1+\frac{\dot{\mathbb{1}}\,c\,\sqrt{d}\,}{\sqrt{e}}\,}}{\sqrt{2}}\,\Big]\,\,\text{Log}\Big[\,1+\frac{\dot{\mathbb{1}}\,\,\Big(c\,\,\sqrt{d}\,\,+\,\sqrt{c^2\,d+e}\,\,\Big)\,\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-$$

$$2 \, \text{PolyLog} \Big[ 2 \text{,} \quad \frac{\text{i} \left( -c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \text{e}^{-\text{ArcCosh} \, [\, c \, x \, ]}}{\sqrt{e}} \, \Big] \, - \\$$

$$2 \, \text{PolyLog} \, \Big[ \, 2 \, \text{,} \, - \, \frac{ \text{i} \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \text{e}^{-\text{ArcCosh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \Big] \, \right) \, + \, \frac{1}{2 \, d^3} \, e \, \left( \text{ArcCosh} \, [\, c \, \, x \,]^{\, 2} \, + \, \frac{1}{2 \, d^3} \, e^{-\frac{1}{2} \, d$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; - \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; - \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{c} \; \mathsf{d} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}}{\mathsf{d} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{e}} \; \mathsf{d} \; \mathsf{e}} \;$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{c^2\,d\,+\,e}\,\,\right)\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\,$$

$$4\,\, \text{\^{1} ArcSin} \Big[\, \frac{\sqrt{1-\frac{\text{\^{1}}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\, \Big]\,\, Log \, \Big[\, 1\,+\,\, \frac{\,\, \text{\^{1}}\,\, \left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,d\,+\,e}\,\,\right)\,\,e^{-ArcCosh\,[\,c\,\,x\,]}}{\sqrt{e}}\, \Big]\,\,+\,\, \frac{\,\, \text{\^{1}}\,\, \left(-\,c\,\,\sqrt{\,c\,\,a}\,\,+\,\sqrt{\,c^{\,2}\,d\,+\,e}\,\,\right)\,\,e^{-ArcCosh\,[\,c\,\,x\,]}}{\sqrt{e}}\, \Big]\,\,+\,\, \frac{\,\, \text{\^{1}}\,\, \left(-\,c\,\,\sqrt{\,c\,\,a}\,\,+\,\sqrt{\,c^{\,2}\,d\,+\,e}\,\,\right)\,\,e^{-ArcCosh\,[\,c\,\,x\,]}}{\sqrt{e}}\, \Big]\,\,+\,\, \frac{\,\, \text{\^{1}}\,\, \left(-\,c\,\,\sqrt{\,c\,\,a}\,\,+\,\,e^{-ArcCosh\,[\,c\,\,x\,]}\right)\,\,+\,\, \frac{\,\, \text{\^{1}}\,\, \left(-\,c\,\,\sqrt{\,c\,\,a}\,\,+\,\,e^{-ArcCosh\,[\,c\,\,x\,]}\right)}{\sqrt{e}}\, \Big]\,\,+\,\, \frac{\,\, \text{\^{1}}\,\,$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\dot{\mathbb{1}}\,\,\left(c\,\,\sqrt{d}\,\,+\,\sqrt{c^2\,d\,+\,e}\,\,\right)\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,+\,$$

$$4 \; \text{$\stackrel{1}{\text{a}}$ ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{$\stackrel{1}{\text{c}}} \; \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{$\text{Log} \Big[ 1 - \frac{\text{$\stackrel{1}{\text{c}}} \; \left( c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}{\sqrt{e}} \; \Big] \; - }$$

$$2 \, \text{PolyLog} \left[ 2, - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] -$$

2 PolyLog[2, 
$$\frac{i \left(c \sqrt{d} + \sqrt{c^2 d + e}\right) e^{-ArcCosh[c x]}}{\sqrt{e}}$$
]

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \, \left( a + b \, \text{ArcCosh} \left[ \, c \, \, x \, \right] \, \right)}{\left( \, d + e \, \, x^2 \, \right)^2} \, \, \text{d} \, x$$

Optimal (type 4, 839 leaves, 49 steps):

$$\frac{a\,x}{e^2} - \frac{b\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{c\,e^2} + \frac{b\,x\,\text{ArcCosh}[c\,x]}{e^2} - \frac{d\,\left(a+b\,\text{ArcCosh}[c\,x)\right)}{4\,e^{5/2}\left(\sqrt{-d}\,-\sqrt{e}\,x\right)} + \frac{d\,\left(a+b\,\text{ArcCosh}[c\,x]\right)}{4\,e^{5/2}\left(\sqrt{-d}\,+\sqrt{e}\,x\right)} + \frac{b\,c\,d\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}\,+\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,\sqrt{c\,\sqrt{-d}\,+\sqrt{e}}\,\sqrt{-1+c\,x}} - \frac{b\,c\,d\,\text{ArcTanh}\left[\frac{\sqrt{c\,\sqrt{-d}\,+\sqrt{e}}\,\sqrt{1+c\,x}}{\sqrt{2}\,\sqrt{-d}\,-\sqrt{e}}\,\sqrt{-1+c\,x}}\right]}{2\,\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\sqrt{-1+c\,x}} + \frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} + \frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} + \frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCosh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} - \frac{4\,e^{5/2}}{2\,d\,e^{\frac{3}{2}\,\sqrt{-d}\,\sqrt{-c^2\,d-e}}} + \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} - \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} + \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} - \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} + \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} - \frac{4\,e^{5/2}}{4\,e^{5/2}} - \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right]}{4\,e^{5/2}} - \frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2\,,\,-\frac{\sqrt{e}\,e^{\text{ArcCosh}[c\,x)}}{c\,\sqrt{-d}\,+\sqrt{-c^2\,d-e}}\right$$

### Result (type 4, 1185 leaves):

$$\frac{1}{8 \, e^{5/2}} \left[ 8 \, a \, \sqrt{e} \, x + \frac{4 \, a \, d \, \sqrt{e} \, x}{d + e \, x^2} \right. -$$

$$12 \text{ a } \sqrt{d} \text{ ArcTan} \left[ \frac{\sqrt{e} \text{ x}}{\sqrt{d}} \right] + b \left[ \frac{8 \sqrt{e} \left( -\sqrt{\frac{-1+c \text{ x}}{1+c \text{ x}}} \right) \left( 1+c \text{ x} \right) + c \text{ x ArcCosh} \left[ c \text{ x} \right] \right)}{c} + \frac{12 \text{ a} \sqrt{d} \text{ ArcTan} \left[ \frac{\sqrt{e} \text{ x}}{\sqrt{d}} \right] + b \left[ \frac{\sqrt{e} \text{ x}}{\sqrt{e}} \right] + b \left[ \frac{e} \text{ x}}{\sqrt{e}} \right] + b \left[ \frac{e} \text{ x}}{\sqrt{e}} \right] + b \left[ \frac{e} \text{ x}}{\sqrt{e}} \right] + b \left[$$

$$2\,d\,\left(\frac{\text{ArcCosh}\,[\,c\,\,x\,]}{-\,\dot{\mathbb{1}}\,\,\sqrt{d}\,\,+\,\sqrt{e}\,\,\,x}\,+\,\frac{c\,\,\text{Log}\,\Big[\,\frac{2\,e\,\Big(\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{-c^2\,d-e}\,\,\,\sqrt{-1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\Big)}{c\,\,\sqrt{-\,c^2\,d-e}\,\,\,\Big(\,\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,\,x\Big)}\,\Big]}\,+\,\frac{c\,\,\text{Log}\,\Big[\,\frac{2\,e\,\Big(\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{-c^2\,d-e}\,\,\,\sqrt{-1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\Big)}{c\,\,\sqrt{-\,c^2\,d-e}\,\,\,\Big(\,\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,\,x\Big)}}\,\Big]}\,+\,\frac{c\,\,\text{Log}\,\Big[\,\frac{2\,e\,\Big(\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{-c^2\,d-e}\,\,\,\sqrt{-1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\Big)}{c\,\,\sqrt{-c^2\,d-e}\,\,\,\Big(\,\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,\,x\Big)}}\,\Big]}\,+\,\frac{c\,\,\text{Log}\,\Big[\,\frac{2\,e\,\Big(\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{-c^2\,d-e}\,\,\,\sqrt{-1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\Big)}{c\,\,\sqrt{-c^2\,d-e}\,\,\,\Big(\,\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,\,x\Big)}}\,\Big]}\,+\,\frac{c\,\,\text{Log}\,\Big[\,\frac{2\,e\,\Big(\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+c^2\,\sqrt{d}\,\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,x\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,}\Big)}{c\,\,\sqrt{-c^2\,d-e}\,\,\,\Big(\,\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,x\,\,\,\sqrt{e}\,\,x\,\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}\,\,\,\sqrt{1+c\,\,x}$$

$$2\,d\,\left(\frac{ArcCosh\,[\,c\,\,x]}{\frac{1}{i}\,\,\sqrt{d}\,\,+\sqrt{e}\,\,x}\,+\,\frac{c\,\,Log\,\big[\,\frac{2\,e\,\left(-\sqrt{e}\,\,-\,i\,\,c^{2}\,\sqrt{d}\,\,x+\sqrt{-c^{2}\,d-e}\,\,\sqrt{-1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\right)}{c\,\,\sqrt{-c^{2}\,d-e}\,\,\left(\,i\,\,\sqrt{d}\,\,+\sqrt{e}\,\,x\right)}\,}{\sqrt{-c^{2}\,d-e}}\,\right)\\ -\,3\,\,\dot{\mathbb{1}}\,\,\sqrt{d}\,\,\left(ArcCosh\,[\,c\,\,x\,]^{\,2}\,+\,\sqrt{e}\,\,x\right)$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(c \; \sqrt{d} \; + \; \verb"i" \; \sqrt{e} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; c \; x \; \right] \; \right]}{\sqrt{c^2 \; d + e}} \; \Big] \; + \\$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\mathrm{i}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\,\,$$

$$4 \, \, \text{i} \, \, \text{ArcSin} \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \Big[ 1 - \frac{\text{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] \, + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, \, x \, \, \, x \, \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, \, \, x \, \, \, \, \, \right]} \, \, e^{$$

$$4\,\, \text{$\stackrel{1}{\text{ArcSin}}$} \Big[ \, \frac{\sqrt{1 + \frac{\text{$\stackrel{1}{\text{c}}}\,\, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \,\, \text{Log} \Big[ 1 + \frac{\text{$\stackrel{1}{\text{c}}}\,\, \left( c\,\, \sqrt{d} \,\, + \, \sqrt{c^2\,d + e} \,\, \right) \,\, \text{$e^{-\text{ArcCosh}}[\,c\,\,x\,]$}}{\sqrt{e}} \, \Big] \,\, - \frac{1}{\sqrt{e}} \,\, \left( -\frac{1}{\sqrt{e}} \,\, \frac{1}{\sqrt{e}} \,\, \frac$$

$$2 \, \text{PolyLog} \, \Big[ \, 2 \, , \, \, \frac{ \, \mathbb{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [ \, c \, \, x \, ]} \, }{ \sqrt{e} } \, \Big] \, - \,$$

$$8 \ \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \ \mathsf{ArcTanh} \Big[ \ \frac{\left( \mathsf{c} \ \sqrt{\mathsf{d}} \ - \ \verb"i" \sqrt{\mathsf{e}} \ \right) \ \mathsf{Tanh} \left[ \ \frac{1}{2} \ \mathsf{ArcCosh} \left[ \mathsf{c} \ \mathsf{x} \right] \ \right]}{\sqrt{\mathsf{c}^2 \ \mathsf{d} + \mathsf{e}}} \Big] \ + \\ = \frac{1}{2} \ \mathsf{ArcTanh} \Big[ \ \frac{\mathsf{d} \ \mathsf{c} \ \mathsf{d} \ \mathsf{d}}{\mathsf{d} \ \mathsf{d}} + \frac{\mathsf{d} \ \mathsf{d}}{\mathsf{d}} + \frac{\mathsf{d}}{\mathsf{d}} + \mathsf{d}}{\mathsf{d}} \Big] \ + \\ = \frac{1}{2} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big] \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} \ \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} + \mathsf{d} + \mathsf{d} + \mathsf{d}} \Big] \Big] \ \mathsf{d} \ \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} \ \mathsf{d} + \mathsf{d} + \mathsf{d}} \Big[ \ \mathsf{d} + \mathsf{d}$$

$$2\, \text{ArcCosh}\, [\, c\, \, x\, ] \, \, \, \text{Log}\, \Big[\, 1\, +\, \frac{\, \mathbb{i}\, \, \left(-\, c\, \, \sqrt{\,d\,}\, \, +\, \sqrt{\,c^{\,2}\, \,d\, +\, e\,}\, \right) \, \, \mathbb{e}^{-\text{ArcCosh}\, [\, c\, \, x\, ]}}{\sqrt{\,e\,}} \, \Big] \, \, -\, \\$$

$$4 \text{ in ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{in } c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{in } \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] + \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] + \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] + \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - 2 \text{ PolyLog} \Big[ 2 \text{, } -\frac{\text{in } \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - 2 \text{ PolyLog} \Big[ 2 \text{, } -\frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \times]}}{\sqrt{e}} \Big] - \frac{\text{in } \left( c \sqrt{d} + \sqrt{c$$

# Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \left(a + b \operatorname{ArcCosh}[c x]\right)}{\left(d + e x^2\right)^2} \, dx$$

Optimal (type 4, 792 leaves, 46 steps):

$$\frac{a + b \operatorname{ArcCosh}[c \, x]}{4 \, e^{3/2} \left( \sqrt{-d} - \sqrt{e} \, \, x \right)} - \frac{a + b \operatorname{ArcCosh}[c \, x]}{4 \, e^{3/2} \left( \sqrt{-d} + \sqrt{e} \, \, x \right)} - \frac{a + b \operatorname{ArcCosh}[c \, x]}{4 \, e^{3/2} \left( \sqrt{-d} + \sqrt{e} \, \, x \right)} - \frac{a + b \operatorname{ArcCosh}[c \, x]}{2 \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{-1 + c \, x}} + \frac{2 \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} \, e^{3/2}}{2 \sqrt{c \sqrt{-d} - \sqrt{e}} \sqrt{c \sqrt{-d} + \sqrt{e}} \, \sqrt{-1 + c \, x}} + \frac{(a + b \operatorname{ArcCosh}[c \, x]) \operatorname{Log} \left[ 1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} - \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} - \frac{(a + b \operatorname{ArcCosh}[c \, x]) \operatorname{Log} \left[ 1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} - \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{(a + b \operatorname{ArcCosh}[c \, x]) \operatorname{Log} \left[ 1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} - \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} - \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} - \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} - \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac{b \operatorname{PolyLog} \left[ 2, - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \sqrt{-d} + \sqrt{-c^2 \, d - e}}} \right]}{4 \sqrt{-d} \, e^{3/2}} + \frac$$

### Result (type 4, 1130 leaves):

$$\frac{1}{8 \ e^{3/2}} \left( -\frac{4 \ a \ \sqrt{e} \ x}{d + e \ x^2} + \frac{4 \ a \ ArcTan \left[ \frac{\sqrt{e} \ x}{\sqrt{d}} \right]}{\sqrt{d}} + \right.$$

$$\frac{2 \, c \, \text{Log} \left[ \, \frac{2 \, e \, \left( -\sqrt{e} \, -\text{i} \, c^2 \, \sqrt{d} \, \, x + \sqrt{-c^2 \, d - e} \, \, \sqrt{-1 + c \, x} \, \, \sqrt{1 + c \, x} \, \, \right)}{c \, \sqrt{-c^2 \, d - e} \, \left( \text{i} \, \sqrt{d} \, + \sqrt{e} \, \, x \right)} \, + \, \frac{1}{\sqrt{d}} \, \, \text{ii} \, \left[ \text{ArcCosh} \, [\, c \, \, x \, ] \, ^2 \, + \right. \right.$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \mathsf{arcTanh} \left[ \; \frac{\mathsf{d} \; \mathsf{v} \; \mathsf{v}$$

$$2\, \operatorname{ArcCosh} \left[\, c \; x \, \right] \; Log \left[\, 1 \, - \, \frac{\, \mathrm{i} \; \left(\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right) \; \operatorname{e}^{-\operatorname{ArcCosh} \left[\, c \; x \, \right]}}{\sqrt{\,e \;}} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,d \;} \, + \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,c^{2} \; d \, + \, e \;} \, \right] \; - \, \left[\, - \, c \; \sqrt{\,c^{2} \; d \, + \, e \;}$$

$$4 \; \text{$\stackrel{1}{\text{a}}$ ArcSin} \Big[ \frac{\sqrt{1 + \frac{\text{$\stackrel{1}{\text{c}}$ c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{$\text{Log} \Big[ 1 - \frac{\text{$\stackrel{1}{\text{c}}$ } \left( - c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}{\sqrt{e}} \Big] \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]}} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; \text{$\text{cosh} \left[ c \; x \right]} \; + \frac{1}{\sqrt{e}} \; + \frac{1}{\sqrt{$$

$$4\,\,\dot{\mathbb{1}}\,\text{ArcSin}\Big[\,\frac{\sqrt{1+\frac{\dot{\mathbb{1}}\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\,\Big]\,\,\text{Log}\,\Big[\,1+\frac{\dot{\mathbb{1}}\,\,\Big(c\,\sqrt{d}\,\,+\,\sqrt{c^2\,d+e}\,\,\Big)\,\,\,e^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\frac{1}{2}\,\,$$

$$2 \, \text{PolyLog} \, \Big[ \, 2 \, , \, \, \frac{ \, \mathbb{i} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [ \, c \, \, x \, ]} }{\sqrt{e}} \, \Big] \, - \, \\$$

$$2 \, \text{PolyLog} \left[ 2 \text{, } - \frac{\dot{\mathbb{I}} \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \, \right] \right) - \frac{1}{\sqrt{d}} \, \, \dot{\mathbb{I}} \left( \text{ArcCosh} \left[ c \, x \right]^2 + \right) = \frac{1}{\sqrt{d}} \, \dot{\mathbb{I}} \left( \frac{1}{\sqrt{d}} \, + \sqrt{c^2 \, d + e} \, \right) + \left($$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCosh}[c x]}{\left(d + e x^{2}\right)^{2}} dx$$

Optimal (type 4, 804 leaves, 26 steps):

$$\frac{\mathsf{a} + \mathsf{b} \, \mathsf{ArcCosh} \, [\mathsf{c} \, \mathsf{x}]}{\mathsf{4} \, \mathsf{d} \, \sqrt{\mathsf{e}} \, \left( \sqrt{-\mathsf{d}} - \sqrt{\mathsf{e}} \, \mathsf{x} \right)} + \frac{\mathsf{a} + \mathsf{b} \, \mathsf{ArcCosh} \, [\mathsf{c} \, \mathsf{x}]}{\mathsf{4} \, \mathsf{d} \, \sqrt{\mathsf{e}} \, \left( \sqrt{-\mathsf{d}} + \sqrt{\mathsf{e}} \, \mathsf{x} \right)} + \frac{\mathsf{a} + \mathsf{b} \, \mathsf{ArcCosh} \, [\mathsf{c} \, \mathsf{x}]}{\mathsf{2} \, \mathsf{d} \, \sqrt{\mathsf{c} \, \sqrt{-\mathsf{d}} - \sqrt{\mathsf{e}}} \, \sqrt{-\mathsf{d} + \sqrt{\mathsf{e}}} \, \sqrt{-\mathsf{d} + \sqrt{\mathsf{e}}} \, \sqrt{\mathsf{e}}} \\ = \frac{\mathsf{b} \, \mathsf{c} \, \mathsf{ArcTanh} \left[ \frac{\sqrt{\mathsf{c} \, \sqrt{-\mathsf{d}} + \sqrt{\mathsf{e}}} \, \sqrt{\mathsf{d} + \sqrt{\mathsf{e}}} \, \sqrt{\mathsf{d}} + \sqrt{\mathsf{e}}}{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \right]}{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} + \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ = \frac{\mathsf{b} \, \mathsf{c} \, \mathsf{ArcTanh} \left[ \frac{\sqrt{\mathsf{c} \, \sqrt{-\mathsf{d}} + \sqrt{\mathsf{e}}} \, \sqrt{-\mathsf{d}} + \sqrt{\mathsf{e}}}{\mathsf{d} \, \mathsf{d}} \right]}{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} + \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ = \frac{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}}{\mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ = \frac{\mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}}{\mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e}} \\ \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{d} \, \mathsf{e} \, \mathsf{d} \, \mathsf{e} \, \mathsf{e} \, \mathsf{d} \, \mathsf{e} \,$$

#### Result (type 4, 1126 leaves):

$$\frac{1}{2}\left[\frac{a\,x}{d^2+d\,e\,x^2} + \frac{a\,\text{ArcTan}\Big[\frac{\sqrt{e}\,x}{\sqrt{d}}\Big]}{d^{3/2}\,\sqrt{e}} + \frac{1}{2\,d^{3/2}\,\sqrt{e}}\,b\,\left[\frac{\sqrt{d}\,\,\text{ArcCosh}[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x} + \frac{\sqrt{d}\,\,\text{ArcCosh}[c\,x]}{i\,\sqrt{d}\,+\sqrt{e}\,\,x} + \frac{\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x} + \frac{\sqrt{d}\,\,x}{i\,\sqrt{d}\,\,x$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\, ArcCosh\, [\, c\,\, x\,]}{x^2\, \left(d+e\, x^2\right)^2}\, \mathrm{d} x$$

### Optimal (type 4, 846 leaves, 49 steps):

$$-\frac{a + b \operatorname{ArcCosh}[c \, x]}{d^2 \, x} + \frac{\sqrt{e} \, \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{4 \, d^2 \, \left(\sqrt{-d} - \sqrt{e} \, \, x\right)} - \frac{\sqrt{e} \, \left(a + b \operatorname{ArcCosh}[c \, x]\right)}{4 \, d^2 \, \left(\sqrt{-d} + \sqrt{e} \, \, x\right)} + \frac{b \, c \operatorname{ArcTan}\left[\sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}\right]}{d^2} - \frac{b \, c \, \sqrt{e} \, \operatorname{ArcTanh}\left[\frac{\sqrt{c \, \sqrt{-d} - \sqrt{e}} \, \sqrt{-1 + c \, x}}{\sqrt{c \, \sqrt{-d} + \sqrt{e}} \, \sqrt{-1 + c \, x}}\right]}{2 \, d^2 \, \sqrt{c \, \sqrt{-d} - \sqrt{e}} \, \sqrt{-1 + c \, x}} + \frac{b \, c \, \sqrt{e} \, \operatorname{ArcTanh}\left[\frac{\sqrt{c \, \sqrt{-d} + \sqrt{e}} \, \sqrt{-1 + c \, x}}{\sqrt{c \, \sqrt{-d} - \sqrt{e}} \, \sqrt{-1 + c \, x}}\right]}{2 \, d^2 \, \sqrt{c \, \sqrt{-d} - \sqrt{e}} \, \sqrt{-1 + c \, x}}} - \frac{3 \, \sqrt{e} \, \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, \sqrt{e} \, \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, \sqrt{e} \, \left(a + b \operatorname{ArcCosh}[c \, x]\right) \operatorname{Log}\left[1 - \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \left[a + b \operatorname{ArcCosh}[c \, x]\right] \operatorname{Log}\left[1 + \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} - \sqrt{-c^2 \, d - e}}\right]}}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}}\right]}{4 \, \left(-d\right)^{5/2}} + \frac{3 \, b \, \sqrt{e} \, \operatorname{PolyLog}\left[2, \, \frac{\sqrt{e} \, e^{\operatorname{ArcCosh}[c \, x]}}{c \, \sqrt{-d} + \sqrt{-c^2 \, d - e}}\right]}{4$$

#### Result (type 4, 1203 leaves):

$$\frac{1}{8\,d^{5/2}} \left[ -\frac{8\,a\,\sqrt{d}}{x} - \frac{4\,a\,\sqrt{d}\,e\,x}{d+e\,x^2} - 12\,a\,\sqrt{e}\,\operatorname{ArcTan}\Big[\,\frac{\sqrt{e}\,\,x}{\sqrt{d}}\,\Big] \right. \\ \left. -\frac{8\,\sqrt{d}\,\left(\operatorname{ArcCosh}\left[\,c\,x\right] + c\,x\,\operatorname{ArcTan}\Big[\,\frac{1}{\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}\,\Big]\,\right)}{x} - \frac{1}{2}\left[ -\frac{1}{2}\left[ -\frac{1}{2}$$

$$2\,\sqrt{d}\,\,\sqrt{e}\,\,\left[\frac{\text{ArcCosh}\,[\,c\,\,x\,]}{-\,\dot{\mathbb{1}}\,\,\sqrt{d}\,\,+\,\sqrt{e}\,\,x}\,+\,\frac{c\,\,\text{Log}\,[\,\frac{2\,e\,\left(\dot{\mathbb{1}}\,\,\sqrt{e}\,\,+\,c^{2}\,\sqrt{d}\,\,x\,-\,\dot{\mathbb{1}}\,\,\sqrt{-c^{2}\,d-e}\,\,\sqrt{-1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\sqrt{1+c\,\,x}\,\,\right)}{c\,\,\sqrt{-c^{2}\,d-e}\,\,\left(\sqrt{d}\,\,+\,\dot{\mathbb{1}}\,\,\sqrt{e}\,\,x\right)}\,\,\right]}{\sqrt{-\,c^{2}\,d-e}}\,+\,2\,\,\sqrt{d}\,\,\sqrt{e}$$

$$\left(-\frac{ArcCosh\left[c\;x\right]}{\frac{i}{\sqrt{d}\;+\sqrt{e}\;x}}-\frac{c\;Log\left[\frac{2\,e\left(-\sqrt{e}\;-i\;c^2\;\sqrt{d}\;x+\sqrt{-c^2\;d-e}\;\sqrt{-1+c\;x}\;\sqrt{1+c\;x}\right)}{c\;\sqrt{-c^2\;d-e}\;\left(i\;\sqrt{d}\;+\sqrt{e}\;x\right)}}{\sqrt{-c^2\;d-e}}\right]}{\sqrt{-c^2\;d-e}}\right)-3\;i\;\sqrt{e}\;\left(ArcCosh\left[c\;x\right]^2+\frac{1}{2}\left(-\sqrt{e}\;a^2+\sqrt{$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 + \frac{\verb"i" c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; + \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \mathsf{c} \; \mathsf{x} \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{d} \; \mathsf{d$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,-\,\,\frac{\mathrm{i}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e}\,\,\right)\,\,\,\mathrm{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\,\,$$

$$4 \, \, \text{i} \, \, \text{ArcSin} \Big[ \, \frac{\sqrt{1 + \frac{\text{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \, \, \text{Log} \Big[ 1 - \frac{\text{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] \, + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \Big] + \frac{1}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]}}{\sqrt{e}} \, \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, x \, \, \, x \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, \, x \, \, \, x \, \, \, \right]} \, \, e^{-\text{ArcCosh} \left[ \, c \, \, x \, \, \, \, x \, \, \, \, \, \right]} \, \, e^{$$

$$2\, \operatorname{ArcCosh} \left[\, c \,\, x \,\right] \,\, \operatorname{Log} \left[\, 1 \,+\, \frac{\,\,\dot{\mathbb{I}} \,\, \left(\, c \,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{e}^{-\operatorname{ArcCosh} \left[\, c \,\, x \,\right]}}{\sqrt{\,e\,}} \,\, \right] \,\, + \,\, \left(\, c \,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \operatorname{e}^{-\operatorname{ArcCosh} \left[\, c \,\, x \,\right]} \,\, \left(\, c \,\, \sqrt{\,d\,} \,\, + \, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{\,c^{\,2} \,\, d \,+\, e\,}\,\,\right) \,\, \left(\, c \,\, \sqrt{$$

$$4\,\, \text{$\stackrel{1}{\text{ArcSin}}$} \Big[ \, \frac{\sqrt{1 + \frac{\text{$\stackrel{1}{\text{c}}}\,\, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \, \Big] \,\, \text{Log} \Big[ 1 + \frac{\text{$\stackrel{1}{\text{c}}}\,\, \left( c\,\, \sqrt{d} \,\, + \, \sqrt{c^2\,d + e} \,\, \right) \,\, \text{$e^{-\text{ArcCosh}}[\,c\,\,x\,]$}}{\sqrt{e}} \, \Big] \,\, - \frac{1}{\sqrt{e}} \,\, \left( -\frac{1}{\sqrt{e}} \,\, \frac{1}{\sqrt{e}} \,\, \frac$$

$$2 \, \text{PolyLog} \, \Big[ \, 2 \, , \, \, \frac{ \, \mathbb{i} \, \left( - \, c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [ \, c \, \, x \, ]} \, }{ \sqrt{e} } \, \Big] \, - \,$$

$$2\,\text{PolyLog}\!\left[2\text{,}\,-\frac{\dot{\mathbb{I}}\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,\,\text{e}^{-\text{ArcCosh}\left[c\,x\right]}}{\sqrt{e}}\,\right] + 3\,\,\dot{\mathbb{I}}\,\sqrt{e}\,\left[\text{ArcCosh}\left[c\,x\right]^2 + \right]$$

$$8 \ \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \ \mathsf{ArcTanh} \Big[ \frac{\left( \mathsf{c} \ \sqrt{\mathsf{d}} \ - \ \verb"i" \sqrt{\mathsf{e}} \ \right) \ \mathsf{Tanh} \left[ \frac{1}{2} \ \mathsf{ArcCosh} \left[ \mathsf{c} \ \mathsf{x} \right] \ \right]}{\sqrt{\mathsf{c}^2 \ \mathsf{d} + \mathsf{e}}} \Big] \ + \\ = \frac{1}{2} \left( \mathsf{arcTanh} \left[ \frac{\mathsf{d} \ \mathsf{d} \ \mathsf$$

$$2\, \text{ArcCosh}\, [\, c\, \, x\, ] \, \, \, \text{Log}\, \Big[\, 1\, +\, \frac{\, \mathbb{i}\, \, \left(-\, c\, \, \sqrt{\,d\,}\, \, +\, \sqrt{\,c^{\,2}\, \,d\, +\, e\,}\, \right) \, \, \mathbb{e}^{-\text{ArcCosh}\, [\, c\, \, x\, ]}}{\sqrt{\,e\,}} \, \Big] \, \, -\, \\$$

$$4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i c} \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 + \frac{\text{i} \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] + \\ 2 \text{ ArcCosh}[c \, x] \text{ Log} \Big[ 1 - \frac{\text{i} \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] + \\ 4 \text{ i ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{i} \, c \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \text{ Log} \Big[ 1 - \frac{\text{i} \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ 2 \text{ PolyLog} \Big[ 2 \text{, } - \frac{\text{i} \left( -c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ 2 \text{ PolyLog} \Big[ 2 \text{, } - \frac{\text{i} \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] - \\ \\ \frac{\text{i} \left( c \sqrt{d} + \sqrt{c^2 d + e} \right) e^{-\text{ArcCosh}[c \, x]}}{\sqrt{e}} \Big] \right]$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 \, \left(a + b \, \text{ArcCosh} \left[\, c \, \, x \, \right]\,\right)}{\left(d + e \, x^2\right)^3} \, \text{d} x$$

Optimal (type 4, 737 leaves, 29 steps):

$$\frac{b \ c \ d \ x \ \left(1-c^2 \, x^2\right)}{8 \ e^2 \ \left(c^2 \ d+e\right) \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x} \ \left(d+e \ x^2\right)} - \frac{d^2 \ \left(a+b \ Arc Cosh \left[c \ x\right]\right)}{4 \ e^3 \ \left(d+e \ x^2\right)^2} + \\ \frac{d \ \left(a+b \ Arc Cosh \left[c \ x\right]\right)}{e^3 \ \left(d+e \ x^2\right)} - \frac{\left(a+b \ Arc Cosh \left[c \ x\right]\right)^2}{2 \ b \ e^3} - \frac{b \ c \ \sqrt{d} \ \sqrt{-1+c^2 \ x^2} \ Arc Tanh \left[\frac{\sqrt{c^2 \ d+e} \ x}{\sqrt{d} \ \sqrt{-1+c^2 \ x^2}}\right]}{e^3 \ \sqrt{c^2 \ d+e} \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x}} + \\ \frac{b \ c \ \sqrt{d} \ \left(2 \ c^2 \ d+e\right) \ \sqrt{-1+c^2 \ x^2} \ Arc Tanh \left[\frac{\sqrt{c^2 \ d+e} \ x}{\sqrt{d} \ \sqrt{-1+c^2 \ x^2}}\right]}{8 \ e^3 \ \left(c^2 \ d+e\right)^{3/2} \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x}} + \\ \frac{\left(a+b \ Arc Cosh \left[c \ x\right]\right) \ Log \left[1-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{\left(a+b \ Arc Cosh \left[c \ x\right]\right) \ Log \left[1+\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-c^2 \ d-e}}}\right]}{2 \ e^3} + \\ \frac{b \ Poly Log \left[2,-\frac{\sqrt{e} \ e^{Arc Cosh \left[c \ x\right]}}{c \ \sqrt{-d} \ \sqrt{-$$

#### Result (type 4, 1564 leaves):

$$-\,\frac{a\,d^{2}}{4\,e^{3}\,\left(d+e\,x^{2}\right)^{\,2}}\,+\,\frac{a\,d}{e^{3}\,\left(d+e\,x^{2}\right)}\,+\,\frac{a\,Log\left[\,d+e\,x^{2}\,\right]}{2\,e^{3}}\,+\,$$

$$b = \frac{1}{16 \, e^3} \, 7 \, \, \dot{\mathbb{I}} \, \sqrt{d} \, \left( \frac{ \frac{ \text{ArcCosh} \left[ c \, x \right] }{ - \, \dot{\mathbb{I}} \, \sqrt{d} \, + \sqrt{e} \, \, x } + \frac{ c \, Log \left[ \frac{ 2 \, e \, \left( \dot{\mathbb{I}} \, \sqrt{e} \, + c^2 \, \sqrt{d} \, \, x - \dot{\mathbb{I}} \, \sqrt{-c^2 \, d - e} \, \, \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x} \, \right) }{ \sqrt{-c^2 \, d - e} \, \left( \sqrt{d} \, + \dot{\mathbb{I}} \, \sqrt{e} \, \, x \right) } \, \right] - \frac{1}{16 \, e^3} \, \left( \frac{1}{16 \, e^3} \, \, \right) \, \left( \frac{1}{16 \, e^3} \, \, \left( \frac{1}{16 \, e^3} \, \, \right) \, \left( \frac{1}{16 \, e^3}$$

$$7 \text{ is } \sqrt{d} \left( -\frac{\underset{i \sqrt{d} + \sqrt{e} \ x}{\text{ArcCosh[c x]}}}{\underset{i \sqrt{d} + \sqrt{e} \ x}{\text{c} \log \left[ \frac{2e \left[ -\sqrt{e - i} \ c^2 \sqrt{d} \ x + \sqrt{-c^2 d - e} \ \sqrt{-1 + c \, x} \ \sqrt{1 + c \, x} \right]}{c \sqrt{-c^2 d - e} \left[ \underset{i \sqrt{d} + \sqrt{e} \ x}{\text{c} \sqrt{-c^2 d - e}} \right]} \right] } \right)$$

$$\begin{split} &\frac{1}{16\,e^{5/2}}d\left[\frac{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{(c^2\,d+e)\,\left(-i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)} - \frac{ArcCosh(c\,x)}{\sqrt{e}\,\left(-i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)^2} + \\ &\left[c^3\,\sqrt{d}\,\left(Log[4] + Log[\left[e\,\sqrt{c^2\,d+e}\,\left(-i\,\sqrt{e}\,-c^2\,\sqrt{d}\,\,x + \sqrt{c^2\,d+e}\,\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\right)\right]\right)\right/ \\ &\left. \left(c^3\,\left(d+i\,\sqrt{d}\,\,\sqrt{e}\,\,x\right)\right)\right]\right)\right)\right/\left(\sqrt{e}\,\left(c^2\,d+e\right)^{3/2}\right)\right] - \\ &\frac{1}{16\,e^{5/2}}d\left[\frac{c\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}{(c^2\,d+e)\,\left(i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)} - \frac{ArcCosh[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}\,+\sqrt{e}\,\,x\right)^2} - \\ &\left[c^3\,\sqrt{d}\,\left(Log[4] + Log\left[\left[e\,\sqrt{c^2\,d+e}\,\left(-i\,\sqrt{e}\,+c^2\,\sqrt{d}\,x + \sqrt{c^2\,d+e}\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\right)\right]\right)\right/ \\ &\left. \left(c^3\,\left(d-i\,\sqrt{d}\,\,\sqrt{e}\,\,x\right)\right)\right]\right)\right)\right/\left(\sqrt{e}\,\left(c^2\,d+e\right)^{3/2}\right) + \frac{1}{4\,e^3} \\ &\left[ArcCosh[c\,x]^2 + 8\,i\,ArcSin\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,ArcTanh\left[\frac{\left(c\,\sqrt{d}\,+i\,\sqrt{e}\,\right)\,Tanh\left[\frac{1}{2}\,ArcCosh(c\,x)\right]}{\sqrt{e^2\,d+e}}\right] + \\ &2ArcCosh[c\,x]\,Log\left[1-\frac{i\,\left(-c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}}\right] - \\ &2ArcCosh[c\,x]\,Log\left[1+\frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}}\right] + \\ &2ArcCosh[c\,x]\,Log\left[1+\frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}}\right] + \\ &4\,i\,ArcSin\left[\frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]Log\left[1+\frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}}\right] - \\ &2\,PolyLog\left[2,-\frac{i\,\left(c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}}\right]}{\sqrt{e}}\right] + \frac{1}{4\,e^3} \end{aligned}$$

$$\left[ \operatorname{ArcCosh}[c\,x]^2 + 8\,i\,\operatorname{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big] \operatorname{ArcTanh}\Big[\frac{\left(c\,\sqrt{d}-i\,\sqrt{e}\right)\,\operatorname{Tanh}\Big[\frac{1}{2}\operatorname{ArcCosh}[c\,x]}{\sqrt{c^2\,d+e}}\Big] + \\ 2\operatorname{ArcCosh}[c\,x]\operatorname{Log}\Big[1 + \frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ 4\,i\,\operatorname{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\operatorname{Log}\Big[1 + \frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] + \\ 2\operatorname{ArcCosh}[c\,x]\operatorname{Log}\Big[1 - \frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] + \\ 4\,i\,\operatorname{ArcSin}\Big[\frac{\sqrt{1-\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\operatorname{Log}\Big[1 - \frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ 2\operatorname{PolyLog}\Big[2, -\frac{i\,\left(-c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ 2\operatorname{PolyLog}\Big[2, \frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ \\ 2\operatorname{PolyLog}\Big[2, \frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ \\ \operatorname{PolyLog}\Big[2, \frac{i\,\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\ \\ \\ \operatorname{PolyLog}\Big[2, \frac{i\,\left(c\,\sqrt{d}+\sqrt{d^2\,d+e}\right)\,e^{-\operatorname{ArcCosh}[c\,x]}}{\sqrt{e}}\Big] - \\$$

Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\!\frac{a+b\,ArcCosh\,[\,c\,\,x\,]}{x\,\left(d+e\,x^2\right)^3}\,\mathrm{d}x$$

Optimal (type 4, 755 leaves, 34 steps):

$$\frac{b \, c \, e \, x \, \left(1-c^2 \, x^2\right)}{8 \, d^2 \, \left(c^2 \, d+e\right) \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x} \, \left(d+e \, x^2\right)} + \frac{a + b \, ArcCosh[c \, x]}{4 \, d \, \left(d+e \, x^2\right)^2} + \frac{a + b \, ArcCosh[c \, x]}{2 \, d^2 \, \left(d+e \, x^2\right)}$$

$$\frac{b \, c \, \sqrt{-1+c^2 \, x^2} \, ArcTanh\left[\frac{\sqrt{c^2 \, d+e} \, x}{\sqrt{d \, \sqrt{-1+c^2 \, x^2}}}\right]}{2 \, d^{5/2} \, \sqrt{c^2 \, d+e} \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}} - \frac{b \, c \, \left(2 \, c^2 \, d+e\right) \, \sqrt{-1+c^2 \, x^2} \, ArcTanh\left[\frac{\sqrt{c^2 \, d+e} \, x}{\sqrt{d \, \sqrt{-1+c^2 \, x^2}}}\right]}{2 \, d^{5/2} \, \sqrt{c^2 \, d+e} \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}} - \frac{8 \, d^{5/2} \, \left(c^2 \, d+e\right)^{3/2} \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}}{\sqrt{1+c \, x}} - \frac{\left(a + b \, ArcCosh[c \, x]\right) \, Log\left[1-\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, \sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{2 \, d^3}{2 \, d^3}$$

$$\frac{\left(a + b \, ArcCosh[c \, x]\right) \, Log\left[1-\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, -\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{\sqrt{e} \, e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} + \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^{ArcCosh[c \, x)}}{c \, \sqrt{-d} \, +\sqrt{-c^2 \, d-e}}\right]}{2 \, d^3} - \frac{b \, PolyLog\left[2, -\frac{e^$$

## Result (type 4, 1613 leaves):

$$\frac{a}{4 \ d \ \left(d + e \ x^2\right)^2} + \frac{a}{2 \ d^2 \ \left(d + e \ x^2\right)} + \frac{a \ Log \left[\,x\,\right]}{d^3} - \frac{a \ Log \left[\,d + e \ x^2\,\right]}{2 \ d^3} + \frac{a \ Log \left[\,x \,\right]}{2 \ d^3} + \frac{a \ Log \left[\,x \,\right]}$$

$$b = \frac{\int i \left[ \frac{ArcCosh[c\,x]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x} + \frac{c\,Log\Big[\frac{2\,e\,\Big(i\,\sqrt{e}\,+c^2\,\sqrt{d}\,\,x-i\,\sqrt{-c^2\,d-e}\,\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\Big)}{c\,\sqrt{-c^2\,d-e}\,\,\Big(\sqrt{d}\,+i\,\sqrt{e}\,\,x\Big)} \right]}{\sqrt{-c^2\,d-e}} - \frac{16\,d^{5/2}$$

$$\sqrt{e} \left[ \frac{c\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{(c^2\,d+e)} \left( -i\,\sqrt{d}\,+\sqrt{e}\,\,x \right) - \frac{ArcCosh[c\,x]}{\sqrt{e}\,\left( -i\,\sqrt{d}\,+\sqrt{e}\,\,x \right)^2} + \\ \left( c^3\,\sqrt{d}\,\left( Log[4] + Log\left[ \left[ e\,\sqrt{c^2\,d+e}\,\left( -i\,\sqrt{e}\,-c^2\,\sqrt{d}\,\,x + \sqrt{c^2\,d+e}\,\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\right) \right] \right) \right) \right/ \\ \left( c^3\,\left( d+i\,\sqrt{d}\,\sqrt{e}\,\,x \right) \right) \right] \right) \right) \left/ \left( \sqrt{e}\,\left( c^2\,d+e \right)^{3/2} \right) \right| + \\ \frac{1}{16\,d^2} \sqrt{e} \left( \frac{c\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}{\left( c^2\,d+e \right)\,\left( i\,\sqrt{d}\,+\sqrt{e}\,\,x \right) - \frac{ArcCosh[c\,x]}{\sqrt{e}\,\left( i\,\sqrt{d}\,+\sqrt{e}\,\,x \right)^2} - \\ \left( c^3\,\sqrt{d}\,\left( Log[4] + Log\left[ \left[ e\,\sqrt{c^2\,d+e}\,\left( -i\,\sqrt{e}\,+c^2\,\sqrt{d}\,\,x + \sqrt{c^2\,d+e}\,\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\right) \right] \right) \right/ \\ \left( c^3\,\left( d-i\,\sqrt{d}\,\,\sqrt{e}\,\,x \right) \right) \right] \right) \right) \left/ \left( \sqrt{e}\,\left( c^2\,d+e \right)^{3/2} \right) + \frac{1}{2\,d^3} \\ \left( ArcCosh[c\,x]\,\left( ArcCosh[c\,x] + 2\,Log\left[ 1+e^{-2\,ArcCosh[c\,x]} \right] \right) - PolyLog[2,\,-e^{-2\,ArcCosh[c\,x]}] \right) - \\ \frac{1}{4\,d^3} \\ \left( ArcCosh[c\,x]\,\left( ArcCosh[c\,x] + 2\,Log\left[ 1+e^{-2\,ArcCosh[c\,x]} \right] \right) - PolyLog[2,\,-e^{-2\,ArcCosh[c\,x]}] \right) - \\ 2\,ArcCosh[c\,x]\,Log\left[ 1 - \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] - \\ 4\,i\,ArcSon\left[ \frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] Log\left[ 1 - \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] + \\ 2\,ArcCosh[c\,x]\,Log\left[ 1 + \frac{i\,\left( c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] + \\ 4\,i\,ArcSon\left[ \frac{\sqrt{1+\frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] Log\left[ 1 + \frac{i\,\left( c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] - \\ 2\,PolyLog\left[ 2,\, \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] - \\ 2\,PolyLog\left[ 2,\, \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] - \\ \\ 2\,PolyLog\left[ 2,\, \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-ArcCosh[c\,x]}}{\sqrt{e}} \right] - \\ \\ \frac{1}{2}\,PolyLog\left[ 2,\, \frac{i\,\left( -c\,\sqrt{d}\,+\sqrt{c^2\,d+e}\,\right)\,e^{-$$

$$2 \, \text{PolyLog} \Big[ 2, \, -\frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left( c \, x \right)}}{\sqrt{e}} \Big] - \frac{1}{4 \, d^3}$$
 
$$\left[ \text{ArcCosh} \left[ c \, x \right]^2 + 8 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{ArcTanh} \Big[ \frac{\left( c \, \sqrt{d} - i \, \sqrt{e} \, \right) \, \text{Tanh} \Big[ \frac{i}{2} \, \text{ArcCosh} \left[ c \, x \right]}{\sqrt{c^2 \, d + e}} \Big] + \frac{2 \, \text{ArcCosh} \left[ c \, x \right] \, \text{Log} \Big[ 1 + \frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] + \frac{2 \, \text{ArcCosh} \Big[ c \, x \Big]}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 - \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, -\frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] - \frac{2 \, \text{PolyLog} \Big[ 2, \, \frac{i$$

Problem 50: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b\, \text{ArcCosh}\, [\, c\,\, x\,]}{x^3\, \left(d+e\, x^2\right)^3}\, \, \text{d}\, x$$

Optimal (type 4, 815 leaves, 36 steps):

## Result (type 4, 1670 leaves):

$$-\frac{a}{2\,d^3\,x^2}\,-\frac{a\,e}{4\,d^2\,\left(d+e\,x^2\right)^2}\,-\frac{a\,e}{d^3\,\left(d+e\,x^2\right)}\,-\frac{3\,a\,e\,Log\,[\,x\,]}{d^4}\,+\,\frac{3\,a\,e\,Log\,[\,d+e\,x^2\,]}{2\,d^4}\,+\,\frac{3\,a\,e\,$$

$$b \left( \frac{\frac{\text{ArcCosh[c\,x]}}{\text{c x}\,\sqrt{-1+c\,x}}\,\sqrt{\frac{1+c\,x}{-1+c\,x}}\,-\frac{\text{ArcCosh[c\,x]}}{\text{c}\,\sqrt{-c^2\,d-e}}\,+\,\frac{\frac{\text{c}\,\text{Log}\left[\frac{2\,e\left[i\,\sqrt{e}\,+c^2\,\sqrt{d}\,\,x-i\,\sqrt{-c^2\,d-e}}{c\,\sqrt{-c^2\,d-e}}\,\sqrt{\frac{-1+c\,x}{v}}\,\sqrt{\frac{1+c\,x}{v}}\right]}{\sqrt{-c^2\,d-e}}\right)}{2\,d^3\,x^2}\,+\,\frac{\frac{\text{c}\,\text{Log}\left[\frac{2\,e\left[i\,\sqrt{e}\,+c^2\,\sqrt{d}\,\,x-i\,\sqrt{-c^2\,d-e}}{\sqrt{-c^2\,d-e}}\,\sqrt{\frac{-1+c\,x}{v}}\,\sqrt{\frac{1+c\,x}{v}}\right]}{\sqrt{-c^2\,d-e}}\right]}{16\,d^{7/2}}$$

$$9 \ \ \dot{\mathbb{1}} \ e \ \left( - \frac{\frac{\mathsf{ArcCosh} \left[ c \ x \right]}{\mathsf{i} \ \sqrt{\mathsf{d}} \ \mathsf{i} \ \sqrt{\mathsf{e}} \ \mathsf{x}} - \frac{\mathsf{c} \ \mathsf{Log} \left[ \frac{^{2e \left[ -\sqrt{\mathsf{e}} \ -\mathsf{i} \ c^2 \ \sqrt{\mathsf{d}} \ \ \mathsf{x} + \sqrt{-\mathsf{c}^2 \ \mathsf{d} - \mathsf{e}} \ \sqrt{-\mathsf{1} + \mathsf{c} \ \mathsf{x}} \ \sqrt{\mathsf{1} + \mathsf{c} \ \mathsf{x}} \ \right)}}{\sqrt{-\mathsf{c}^2 \ \mathsf{d} - \mathsf{e}}} \right] } \right] \\ - \frac{1}{16 \ \mathsf{d}^{7/2}}$$

$$\begin{split} e^{3/2} \left( \frac{c\sqrt{-1+cx}}{\langle c^2d + e \rangle} \frac{1+cx}{\sqrt{1+cx}} - \frac{ArcCosh[c\,x]}{\sqrt{e} \left(-i\,\sqrt{d} + \sqrt{e}\,x\right)^2} + \\ \left( c^3\,\sqrt{d} \left( log[4] + log \left[ \left( e\,\sqrt{c^2\,d + e} \, \left( -i\,\sqrt{e} + c^2\,\sqrt{d}\,x + \sqrt{c^2\,d + e} \, \sqrt{-1+c\,x}\,\sqrt{1+c\,x} \, \right) \right) \right) \right) \right/ \left( c^3\left( d + i\,\sqrt{d}\,\sqrt{e}\,x \right) \right) \right] \right) \right) / \left( \sqrt{e} \left( c^2\,d + e \right)^{3/2} \right) - \\ \frac{1}{16\,d^2} e^{3/2} \left( \frac{c\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}{\left( c^2\,d + e \right) \left( i\,\sqrt{d} + \sqrt{e}\,x \right)} - \frac{ArcCosh[c\,x]}{\sqrt{e} \left( i\,\sqrt{d} + \sqrt{e}\,x \right)^2} - \\ \left( c^3\,\sqrt{d} \left( log[4] + log \left[ \left( e\,\sqrt{c^2\,d + e} \, \left( -i\,\sqrt{e} + c^2\,\sqrt{d}\,x + \sqrt{c^2\,d + e}\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x} \, \right) \right) \right) \right) \right) \\ \left( c^3\left( d - i\,\sqrt{d}\,\sqrt{e}\,x \right) \right) \right] \right) \right) / \left( \sqrt{e} \left( c^2\,d + e \right)^{3/2} \right) - \frac{1}{2\,d^4} \\ 3\,e\, \left( ArcCosh[c\,x] \left( ArcCosh[c\,x] + 2\,log[1 + e^{-2ArcCosh[c\,x]}] \right) - Polylog[2, -e^{-2ArcCosh[c\,x]}] \right) + \\ \frac{1}{4\,d^4} 3\,e\, \left( ArcCosh[c\,x] + 2\,log[1 + \frac{i\,(-c\,\sqrt{d} + \sqrt{c^2\,d + e})\,e^{-ArcCosh(c\,x)}}{\sqrt{c^2\,d + e}} \right) - \\ 2\,ArcCosh[c\,x] \log[1 - \frac{i\,\left( -c\,\sqrt{d} + \sqrt{c^2\,d + e} \right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right) + \\ 2\,ArcCosh[c\,x] \log[1 + \frac{i\,(c\,\sqrt{d} + \sqrt{c^2\,d + e})\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right) + \\ 2\,ArcCosh[c\,x] log[1 + \frac{i\,(c\,\sqrt{d} + \sqrt{c^2\,d + e})\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right) + \\ 4\,i\,ArcSin[\frac{\sqrt{1 + \frac{i\,c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}] log[1 + \frac{i\,\left( c\,\sqrt{d} + \sqrt{c^2\,d + e} \right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right] - \\ 2\,Polylog[2, \frac{i\,\left( -c\,\sqrt{d} + \sqrt{c^2\,d + e} \right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right) - \\ 2\,Polylog[2, \frac{i\,\left( -c\,\sqrt{d} + \sqrt{c^2\,d + e} \right)\,e^{-ArcCosh(c\,x)}}{\sqrt{e}} \right) - \\ \end{array}$$

$$2 \, \text{PolyLog} \, \left[ \, 2 \, , \, - \, \frac{\dot{\mathbb{1}} \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, \, \mathbb{e}^{-\text{ArcCosh} \, [\, c \, \, x \,]}}{\sqrt{e}} \, \right] \, + \, \frac{1}{4 \, d^4} \, 3 \, e \, \left( \text{ArcCosh} \, [\, c \, \, x \,]^{\, 2} \, + \right) \, d^4 \,$$

$$8 \; \verb"iArcSin" \Big[ \frac{\sqrt{1 - \frac{\verb"i" c \sqrt d}{\sqrt e}}}{\sqrt{2}} \Big] \; \mathsf{ArcTanh} \Big[ \; \frac{\left(\mathsf{c} \; \sqrt{\mathsf{d}} \; - \; \verb"i" \; \sqrt{\mathsf{e}} \; \right) \; \mathsf{Tanh} \left[ \; \frac{1}{2} \; \mathsf{ArcCosh} \left[ \; \mathsf{c} \; \mathsf{x} \; \right] \; \right]}{\sqrt{\mathsf{c}^2 \; \mathsf{d} + \mathsf{e}}} \; \Big] \; + \; \mathsf{ArcTanh} \Big[ \; \frac{\mathsf{d} \; \mathsf{d} \; \mathsf$$

$$2\,\text{ArcCosh}\,[\,c\,\,x\,]\,\,\text{Log}\,\Big[\,1\,+\,\,\frac{\dot{\mathbb{1}}\,\,\left(-\,c\,\,\sqrt{d}\,\,+\,\sqrt{\,c^{\,2}\,\,d\,+\,e\,}\,\right)\,\,\,\mathbb{e}^{-\text{ArcCosh}\,[\,c\,\,x\,]}}{\sqrt{e}}\,\Big]\,\,-\,$$

$$4 \; \text{$\stackrel{1}{\text{a}}$ ArcSin} \Big[ \frac{\sqrt{1 - \frac{\text{$\stackrel{1}{\text{c}}} \; \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \; \text{$\text{Log} \Big[ 1 - \frac{\text{$\stackrel{1}{\text{c}}} \; \left( c \; \sqrt{d} \; + \sqrt{c^2 \; d + e} \; \right) \; e^{-\text{ArcCosh} \left[ c \; x \right]}}{\sqrt{e}} \; \Big] \; - \frac{1}{\sqrt{e}} \; \left( \frac{1}{\sqrt{e}} \right) \; \left( \frac{1}$$

$$2 \, \text{PolyLog} \left[ 2, - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \right] - \frac{\dot{\mathbb{I}} \left( -c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}}$$

2 PolyLog[2, 
$$\frac{i\left(c\sqrt{d} + \sqrt{c^2 d + e}\right) e^{-ArcCosh[c x]}}{\sqrt{e}}$$
]

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \, \left( a + b \, \text{ArcCosh} \left[ \, c \, \, x \, \right] \, \right)}{\left( d + e \, \, x^2 \right)^3} \, \, \text{d} x$$

Optimal (type 4, 1224 leaves, 80 steps):

$$\frac{b \ c \ \sqrt{-d} \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x}}{16 \ e^2 \ (c^2 \ d + e) \ (\sqrt{-d} \ - \sqrt{e} \ x)} - \frac{b \ c \ \sqrt{-d} \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x}}{16 \ e^2 \ (c^2 \ d + e) \ (\sqrt{-d} + \sqrt{e} \ x)} - \frac{\sqrt{-d} \ (a + b \ ArcCosh[c \ x])}{16 \ e^{5/2} \ (\sqrt{-d} \ - \sqrt{e} \ x)} + \frac{\sqrt{-d} \ (a + b \ ArcCosh[c \ x])}{16 \ e^{5/2} \ (\sqrt{-d} + \sqrt{e} \ x)^2} - \frac{b \ c^3 \ d \ ArcTosh[c \ x])}{16 \ e^{5/2} \ (\sqrt{-d} \ - \sqrt{e} \ x)} + \frac{\sqrt{-d} \ (a + b \ ArcCosh[c \ x])}{16 \ e^{5/2} \ (\sqrt{-d} + \sqrt{e} \ x)^2} - \frac{b \ c^3 \ d \ ArcTosh[c \ x])}{16 \ e^{5/2} \ (\sqrt{-d} + \sqrt{e} \ x)^2} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{3/2} \ e^{5/2}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{3/2} \ e^{5/2}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{3/2} \ e^{5/2}} + \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ ArcTosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ a + b \ ArcCosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ a + b \ ArcCosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ a + b \ ArcCosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ a + b \ ArcCosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e} \ y)^{-1+cx}} - \frac{b \ c^3 \ d \ a + b \ ArcCosh[c \ x]}{8 \ (c \ \sqrt{-d} \ - \sqrt{e$$

#### Result (type 4, 1594 leaves):

$$\frac{\text{a d } x}{4 \, e^2 \, \left(d + e \, x^2\right)^2} - \frac{5 \, \text{a } x}{8 \, e^2 \, \left(d + e \, x^2\right)} + \frac{3 \, \text{a ArcTan} \left[\frac{\sqrt{e} \, \, x}{\sqrt{d}}\right]}{8 \, \sqrt{d} \, e^{5/2}} + \\ \left[5 \, \left(\frac{\frac{\text{ArcCosh} \left[c \, x\right]}{-i \, \sqrt{d} + \sqrt{e} \, x} + \frac{c \, \text{Log} \left[\frac{2 \, e \, \left(i \, \sqrt{e} \, + c^2 \, \sqrt{d} \, \, x - i \, \sqrt{-c^2 \, d - e} \, \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}\right)}{\sqrt{-c^2 \, d - e}}\right]}{\sqrt{-c^2 \, d - e}} \right]} \right]} \right] + \frac{16 \, e^{5/2}}$$

$$5 = \frac{ \operatorname{ArcCosh(cx)}_{1 \sqrt{d} + \sqrt{e} \times x} - \frac{\operatorname{cLog}[\frac{2e}{r\sqrt{d} + r\sqrt{d} + x\sqrt{d} +$$

$$2 \, \text{PolyLog} \Big[ 2, -\frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \{ c \, x \}}}{\sqrt{e}} \Big] \\ -\frac{1}{32 \, \sqrt{d} \, e^{5/2}} \, 3 \, i \left[ \text{ArcCosh} \left[ c \, x \right]^2 + 8 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \\ -\frac{1}{\sqrt{c^2 \, d + e}} \right] \\ -\frac{1}{\sqrt{c^2 \, d + e}} \Big[ \frac{\left( c \, \sqrt{d} - i \, \sqrt{e} \, \right) \, \text{Tanh} \Big[ \frac{1}{2} \, \text{ArcCosh} \left[ c \, x \right]}{\sqrt{2}} \Big] + \\ -2 \, \text{ArcCosh} \Big[ c \, x \Big] \, \text{Log} \Big[ 1 + \frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \Big] - \\ -\frac{4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \Big] + \\ -\frac{4 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 - \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \Big] - \\ -2 \, \text{PolyLog} \Big[ 2, -\frac{i \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \Big] - \\ -2 \, \text{PolyLog} \Big[ 2, \frac{i \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}}} \Big] \right]$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 \, \left(a + b \, \text{ArcCosh} \left[\, c \, \, x \, \right]\,\right)}{\left(d + e \, x^2\right)^3} \, \text{d} x$$

## Optimal (type 4, 1234 leaves, 62 steps):

$$\begin{array}{c} b\, c\, \sqrt{-1+c\, x} \, \sqrt{1+c\, x} & b\, c\, \sqrt{-1+c\, x} \, \sqrt{1+c\, x} \\ \hline 16\, \sqrt{-d}\, e\, \left(c^2\, d+e\right) \, \left(\sqrt{-d}\, -\sqrt{e}\, \, x\right) & 16\, \sqrt{-d}\, e\, \left(c^2\, d+e\right) \, \left(\sqrt{-d}\, +\sqrt{e}\, \, x\right) \\ \hline a+b\, ArcCosh[c\, x] \\ \hline 16\, \sqrt{-d}\, e^{3/2} \, \left(\sqrt{-d}\, -\sqrt{e}\, \, x\right)^2 & -\frac{a+b\, ArcCosh[c\, x]}{16\, d\, e^{3/2} \, \left(\sqrt{-d}\, -\sqrt{e}\, \, x\right)} & +\frac{a+b\, ArcCosh[c\, x]}{16\, \sqrt{-d}\, e^{3/2} \, \left(\sqrt{-d}\, +\sqrt{e}\, \, x\right)^2} \\ \hline a+b\, ArcCosh[c\, x] \\ \hline 16\, d\, e^{3/2} \, \left(\sqrt{-d}\, +\sqrt{e}\, \, x\right) & +\frac{b\, c^3\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, -\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, \sqrt{e}}\, \sqrt{-1+c\, x}}\, \right]}{8\, \left(c\, \sqrt{-d}\, -\sqrt{e}\, \right)^{3/2} \, \left(c\, \sqrt{-d}\, +\sqrt{e}\, \right)^{3/2} \, e^{3/2}} \\ \hline b\, c\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, -\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{-1+c\, x}}\, \right] & b\, c^3\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, -\sqrt{e}}\, \sqrt{-1+c\, x}}\, \right]}{8\, d\, \sqrt{c\, \sqrt{-d}\, -\sqrt{e}}\, \sqrt{-1+c\, x}} & a+b\, ArcCosh[c\, x]\, \left(c\, \sqrt{-d}\, +\sqrt{e}\, \frac{\sqrt{1+c\, x}}{\sqrt{-1+c\, x}}\, \right) \\ \hline b\, c\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{-1+c\, x}}\, \right]} & a+b\, ArcCosh[c\, x]\, \left(c\, \sqrt{-d}\, +\sqrt{e}\, \frac{\sqrt{1+c\, x}}{\sqrt{-1+c\, x}}\, \right) \\ \hline b\, c\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{-1+c\, x}}\, \right]} & a+b\, ArcCosh[c\, x]\, \left(c\, \sqrt{-d}\, +\sqrt{e}\, \frac{\sqrt{1+c\, x}}{\sqrt{-1+c\, x}}\, \right) \\ \hline b\, c\, ArcTanh[\, \frac{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{1+c\, x}}{\sqrt{c\, \sqrt{-d}\, +\sqrt{e}}\, \sqrt{-1+c\, x}}\, \right]} & a+b\, ArcCosh[c\, x]\, \left(c\, \sqrt{-d}\, +\sqrt{e}\, \frac{\sqrt{1+c\, x}}{\sqrt{-1+c\, x}}\, \right) \\ \hline a\, d\, \sqrt{c\, \sqrt{-d}\, -\sqrt{e}\, \sqrt{-d}\, +\sqrt{e}\, \sqrt{-d}\, \sqrt{-1+c\, x}}} & a+b\, ArcCosh[c\, x]\, \right) \, Log\left[1-\frac{\sqrt{e}\, e^{ArcCosh(c\, x)}}{c\, \sqrt{-d}\, -\sqrt{-c^2\, d-e}}\, +\sqrt{e}\, \sqrt{-d}\, \sqrt{-c^2\, d-e}}\, \right]} \\ \hline a\, (a+b\, ArcCosh[c\, x]\, )\, Log\left[1+\frac{\sqrt{e}\, e^{ArcCosh(c\, x)}}{c\, \sqrt{-d}\, -\sqrt{-c^2\, d-e}}\, -\sqrt{e}\, e^{ArcCosh(c\, x)}}\, \right] \\ \hline a\, (a+b\, ArcCosh[c\, x]\, )\, Log\left[1+\frac{\sqrt{e}\, e^{ArcCosh(c\, x)}}{c\, \sqrt{-d}\, -\sqrt{-c^2\, d-e}}\, -\sqrt{e}\, e^{ArcCosh(c\, x)}}\, \right]} \\ \hline a\, (a+b\, ArcCosh[c\, x]\, )\, Log\left[1+\frac{\sqrt{e}\, e^{ArcCosh(c\, x)}}{c\, \sqrt{-d}\, -\sqrt{-c^2\, d-e}}\, -\sqrt{e}\, e^{ArcCosh(c\, x)}}\, \right] \\ \hline a\, (a+b\, ArcCosh[c\, x]\, )\, Log\left[1+\frac{\sqrt{e}\, e^{ArcCosh(c\, x)}}{c\, \sqrt{-d}\, -\sqrt{-c^2\, d-e}}\, -\sqrt{e}\, e^{ArcCosh(c\, x)}}\,$$

## Result (type 4, 1602 leaves):

$$-\frac{a\,x}{4\,e\,\left(\text{d}+e\,x^2\right)^2} + \frac{a\,x}{8\,d\,e\,\left(\text{d}+e\,x^2\right)} + \frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,\,x}{\sqrt{d}}\right]}{8\,d^{3/2}\,e^{3/2}} + \\ \\ b\,\left[\frac{\frac{\text{ArcCosh}\left[\text{c}\,x\right]}{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}}{\frac{-i\,\sqrt{d}\,+\sqrt{e}\,\,x}{+\sqrt{e}\,\,x}} + \frac{c\,\text{Log}\left[\frac{2\,e\,\left(\text{i}\,\sqrt{e}\,+c^2\,\sqrt{d}\,\,x-\text{i}\,\sqrt{-c^2\,d-e}}\,\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}\,\,\right)}{c\,\sqrt{-c^2\,d-e}}\right]}{\sqrt{-c^2\,d-e}} - \\ \\ 16\,d\,e^{3/2}$$

$$\frac{-\frac{\text{ArcCosh}(c \times x)}{s \sqrt{d} \cdot \sqrt{e} \times x}}{s \sqrt{d} \cdot \sqrt{e} \times x} - \frac{c \log \left[\frac{2^{2} \left|\sqrt{x} + e^{\sqrt{d} \cdot x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + x\right|}{\sqrt{c^{2} d - e}} - \frac{1}{16 \sqrt{d} \cdot e} \right]}{16 \sqrt{d} \cdot e}$$

$$i \left( \frac{c \sqrt{-1 + c \times} \sqrt{1 + c \times}}{(c^{2} d + e) \left( -i \sqrt{d} + \sqrt{e} \times x \right)} - \frac{\text{ArcCosh}(c \times x)}{\sqrt{e} \left( -i \sqrt{d} + \sqrt{e} \times x \right)^{2}} + \left( c^{3} \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \left( e^{\sqrt{c^{2} d + e}} \left( -i \sqrt{e} - c^{2} \sqrt{d} \times x + \sqrt{c^{2} d + e}} \sqrt{-1 + c \times} \sqrt{1 + c \times} \times 1 \right) \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right)^{3/2} \right) \right) + \frac{1}{16 \sqrt{d}} e^{i} \left( \frac{c \sqrt{-1 + c \times} \sqrt{1 + c \times}}{\left( c^{2} d + e \right) \left( i \sqrt{d} + \sqrt{e} \times x \right)} - \frac{\text{ArcCosh}(c \times x)}{\sqrt{e} \left( i \sqrt{d} + \sqrt{e} \times x \right)^{2}} - \left( c^{3} \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \left( e^{\sqrt{c^{2} d + e}} \left( -i \sqrt{e} - c^{2} \sqrt{d} \times x + \sqrt{c^{2} d + e}} \sqrt{-1 + c \times} \sqrt{1 + c \times} \right) \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right)^{3/2} \right) + \frac{1}{32 \frac{d^{3/2}}{d^{3/2}}} i \left( \frac{\text{ArcCosh}(c \times x)^{2} + \sqrt{c^{2} d + e}}{\sqrt{e}} \right) + \frac{1}{32 \frac{d^{3/2}}{d^{3/2}}} i \left( \frac{\text{ArcCosh}(c \times x)^{2} + \sqrt{c^{2} d + e}}{\sqrt{e}} \right) + \frac{1}{\sqrt{e}} i \left( \frac{\sqrt{d} + \sqrt{c^{2} d + e}}{\sqrt{e}} \right) e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) + \frac{1}{32 \frac{d^{3/2}}{d^{3/2}}} i \left( \frac{\text{ArcCosh}(c \times x)^{2} + \sqrt{c^{2} d + e}}{\sqrt{e}} \right) e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) \frac{1}{2} e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) e^{-\text{ArcCosh}(c \times x)} \right) / \left( \sqrt{e} \left( c^{2} d + e \right) e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e \right) e^{-\text{ArcCosh}(c \times x)} \right) \right) / \left( \sqrt{e} \left( c^{2} d + e$$

$$2 \, \text{PolyLog} \Big[ 2 \text{, } -\frac{i \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} [c \, x]}}{\sqrt{e}} \Big] \\ -\frac{1}{32 \, d^{3/2} \, e^{3/2}} \, i \, \left[ \text{ArcCosh} \, [c \, x]^2 + 8 \, i \, \text{ArcSin} \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \Big] \\ -\frac{1}{\sqrt{c^2 \, d + e}} \left[ \frac{\left( c \, \sqrt{d} \, - i \, \sqrt{e} \, \right) \, \text{Tanh} \Big[ \frac{1}{2} \, \text{ArcCosh} \, [c \, x] \, \Big]}{\sqrt{c^2 \, d + e}} \Big] + \\ 2 \, \text{ArcCosh} \, [c \, x] \, \text{Log} \Big[ 1 + \frac{i \, \left( - c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}{\sqrt{e}} \Big] - \\ 4 \, i \, \text{ArcSin} \, \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 + \frac{i \, \left( - c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}{\sqrt{e}} \Big] + \\ 4 \, i \, \text{ArcSin} \, \Big[ \frac{\sqrt{1 - \frac{i \, c \, \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \Big] \, \text{Log} \Big[ 1 - \frac{i \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}{\sqrt{e}} \Big] - \\ 2 \, \text{PolyLog} \, \Big[ 2 \text{, } -\frac{i \, \left( - c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}{\sqrt{e}} \Big] - \\ 2 \, \text{PolyLog} \, \Big[ 2 \text{, } \frac{i \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}{\sqrt{e}} \Big] - \\ 2 \, \text{PolyLog} \, \Big[ 2 \text{, } \frac{i \, \left( c \, \sqrt{d} \, + \sqrt{c^2 \, d + e} \, \right) \, e^{-\text{ArcCosh} \, [c \, x]}}}{\sqrt{e}} \Big] \Big]$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \, ArcCosh \, [\, c \, \, x \,]}{\left(d + e \, x^2\right)^3} \, \mathrm{d}x$$

Optimal (type 4, 1234 leaves, 34 steps):

$$\frac{b\,c\,\sqrt{-1+c\,x}\,\,\sqrt{1+c\,x}}{16\,\left(-d\right)^{3/2}\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}\,-\sqrt{e}\,\,x\right)} - \frac{16\,\left(-d\right)^{3/2}\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}\,+\sqrt{e}\,\,x\right)}{16\,\left(-d\right)^{3/2}\,\left(c^2\,d+e\right)\,\left(\sqrt{-d}\,+\sqrt{e}\,\,x\right)} - \frac{3\,\left(a+b\,ArcCosh\left[c\,x\right]\right)}{16\,d^2\,\sqrt{e}\,\left(\sqrt{-d}\,-\sqrt{e}\,\,x\right)} + \frac{a+b\,ArcCosh\left[c\,x\right]}{16\,\left(-d\right)^{3/2}\,\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,\,x\right)^2} + \frac{3\,\left(a+b\,ArcCosh\left[c\,x\right]\right)}{16\,d^2\,\sqrt{e}\,\left(\sqrt{-d}\,+\sqrt{e}\,\,x\right)} - \frac{b\,c^3\,ArcTanh\left[\frac{\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}\,+\sqrt{e}}\,\,\sqrt{-1+c\,x}}\right]}{8\,d\,\left(c\,\sqrt{-d}\,-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}\,+\sqrt{e}\right)^{3/2}\,\sqrt{e}} + \frac{3\,b\,c\,ArcTanh\left[\frac{\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}}{\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}}\right]}{8\,d^2\,\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}} + \frac{b\,c^3\,ArcTanh\left[\frac{\sqrt{c\,\sqrt{-d}\,+\sqrt{e}}\,\,\sqrt{1+c\,x}}{\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}}\right]}{8\,d^2\,\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}} + \frac{8\,d\,\left(c\,\sqrt{-d}\,-\sqrt{e}\right)^{3/2}\,\left(c\,\sqrt{-d}\,+\sqrt{e}\right)^{3/2}\,\sqrt{e}}{8\,d^2\,\sqrt{c\,\sqrt{-d}\,-\sqrt{e}}\,\,\sqrt{-1+c\,x}}} + \frac{3\,\left(a+b\,ArcCosh\left[c\,x\right]\right)\,Log\left[1-\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{3\,\left(a+b\,ArcCosh\left[c\,x\right]\right)\,Log\left[1+\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{3\,b\,PolyLog\left[2,\,\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{3\,b\,PolyLog\left[2,\,\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{3\,b\,PolyLog\left[2,\,\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{3\,b\,PolyLog\left[2,\,\frac{\sqrt{e}\,\,e^{MrcCosh\left[c\,x\right]}}{c\,\sqrt{-d}\,-\sqrt{-c^2\,d-e}}\right]}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{16\,\left(-d\right)^{5/2}\,\sqrt{e}}{16\,\left(-d\right)^{5/2}\,\sqrt{e}} - \frac{16\,\left(-d\right)^{5/2}\,\sqrt{e}}$$

### Result (type 4, 1593 leaves):

$$\frac{a \, x}{4 \, d \, \left(d + e \, x^2\right)^2} + \frac{3 \, a \, x}{8 \, d^2 \, \left(d + e \, x^2\right)} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{8 \, d^{5/2} \, \sqrt{e}} + \\ \left(3 \, \left(\frac{\frac{\text{ArcCosh}\left[c \, x\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}}{+ \frac{c \, \text{Log}\left[\frac{2e \left(i \, \sqrt{e} + c^2 \, \sqrt{d} \, x - i \, \sqrt{-c^2 \, d - e} \, \sqrt{-1 + c \, x} \, \sqrt{1 + c \, x}\,\right)}{c \, \sqrt{-c^2 \, d - e}}\right)}{\sqrt{-c^2 \, d - e}}\right) \\ b \, \frac{16 \, d^2 \, \sqrt{e}}{-i \, \sqrt{d} + \sqrt{e} \, x} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{d}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{d} + \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}\right]}{-i \, \sqrt{e} \, \sqrt{e} \, x}} + \frac{3 \, a \, \text{ArcTan}\left[\frac{\sqrt{e} \, x}{\sqrt{e}}$$

$$3 \frac{ - \frac{\text{ArcCosh}(cx)}{1 \sqrt{d} \cdot \sqrt{c} \cdot x} - \frac{\text{c} \log \left| \frac{x_0 \left[ \sqrt{x} + x_1 \sqrt{x} \sqrt{x} + x_2 \sqrt{x} - x_3 \sqrt{x} + x_4 \sqrt{x} \right]}{\sqrt{c^2 d + e}} \right)}{\sqrt{c^2 d + e}} + \frac{1}{16 \, d^{3/2}}$$

$$i \left( \frac{c \sqrt{-1 + c \cdot x} \sqrt{1 + c \cdot x}}{\left( c^2 d + e \right) \left( -i \sqrt{d} + \sqrt{e} \cdot x \right)} - \frac{\text{ArcCosh}(c \cdot x)}{\sqrt{e} \left( -i \sqrt{d} + \sqrt{e} \cdot x \right)^2} + \left( c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \left( e \sqrt{c^2 d} + e \right) \left( -i \sqrt{e} - c^2 \sqrt{d} \cdot x + \sqrt{c^2 d} + e \right) \sqrt{-1 + c \cdot x} \sqrt{1 + c \cdot x} \right) \right] / \left( \sqrt{e} \left( c^2 d + e \right)^{3/2} \right) \right) - \frac{1}{16 \, d^{3/2}} \cdot \left( \frac{c \sqrt{-1 + c \cdot x} \sqrt{1 + c \cdot x}}{\left( c^2 d + e \right) \left( i \sqrt{d} + \sqrt{e} \cdot x \right)} - \frac{\text{ArcCosh}(c \cdot x)}{\sqrt{e} \left( i \sqrt{d} + \sqrt{e} \cdot x \right)} - \frac{\text{ArcCosh}(c \cdot x)}{\sqrt{e} \left( i \sqrt{d} + \sqrt{e} \cdot x \right)^2} \right) - \left( c^3 \sqrt{d} \left( \text{Log}[4] + \text{Log}\left[ \left( e \sqrt{c^2 d + e} \left( -i \sqrt{e} + c^2 \sqrt{d} \cdot x + \sqrt{c^2 d + e} \sqrt{-1 + c \cdot x} \sqrt{1 + c \cdot x} \right) \right) \right) / \left( \sqrt{e} \left( c^2 d + e \right)^{3/2} \right) \right) + \frac{1}{32 \, d^{5/2} \sqrt{e}} \, 3 \, i \left[ \text{ArcCosh}[c \cdot x] \right] + \frac{1}{\sqrt{e}} \right) \left( \sqrt{e} \sqrt{e^2 d + e} \right) \left( e^{-ArcCosh}[c \cdot x] \right) \right] + \frac{1}{\sqrt{e}} \left( e^{-ArcCosh}[c \cdot x] \right) - \frac{1}{\sqrt{e}} \left( e^{-ArcCosh}[c \cdot x] \right) \right) - \frac{1}{\sqrt{e}} \left( e^{-ArcCosh}[c \cdot x] \right) - \frac{1}{\sqrt{e}} \left( e^{-ArcCosh$$

$$2 \, \text{PolyLog} \Big[ 2, -\frac{\mathrm{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \text{ArcCosh} \left[ c \, x \right]}}{\sqrt{e}} \Big] \\ -\frac{1}{32 \, d^{5/2} \, \sqrt{e}} \, 3 \, \mathrm{i} \left[ \frac{1}{4} \, \operatorname{ArcCosh} \left[ c \, x \right]^2 + 8 \, \mathrm{i} \, \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}{\sqrt{2}} \right] \\ -\frac{\mathrm{ArcTanh} \left[ \frac{\left( c \, \sqrt{d} - \mathrm{i} \, \sqrt{e} \right) \, \operatorname{Tanh} \left[ \frac{1}{2} \, \operatorname{ArcCosh} \left[ c \, x \right] \, \right]}{\sqrt{c^2 \, d + e}} \right] + \\ 2 \, \operatorname{ArcCosh} \left[ c \, x \right] \, \operatorname{Log} \left[ 1 + \frac{\mathrm{i} \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] - \\ 4 \, \mathrm{i} \, \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \right] \, \operatorname{Log} \left[ 1 + \frac{\mathrm{i} \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] + \\ 4 \, \mathrm{i} \, \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{\mathrm{i} \, c \, \sqrt{d}}{\sqrt{e}}}}}{\sqrt{2}} \right] \, \operatorname{Log} \left[ 1 - \frac{\mathrm{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] - \\ 2 \, \operatorname{PolyLog} \left[ 2, - \frac{\mathrm{i} \left( - c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] - \\ 2 \, \operatorname{PolyLog} \left[ 2, \frac{\mathrm{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] - \\ 2 \, \operatorname{PolyLog} \left[ 2, \frac{\mathrm{i} \left( c \, \sqrt{d} + \sqrt{c^2 \, d + e} \right) \, e^{- \operatorname{ArcCosh} \left[ c \, x \right)}}{\sqrt{e}} \right] \right]$$

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m \left(d + e x^2\right)^3 \left(a + b \operatorname{ArcCosh}[c x]\right) dx$$

#### Optimal (type 5, 518 leaves, 7 steps):

## Result (type 6, 3413 leaves):

$$\frac{\text{a d}^3 \, x^{1+m}}{1+m} + \frac{3 \, \text{a d}^2 \, e \, x^{2+m}}{3+m} + \frac{3 \, \text{a d}^2 \, x^{5+m}}{5+m} + \frac{a \, e^3 \, x^{7+m}}{7+m} + \frac{1}{c} \, \text{b d}^3 \, x^m \, (\text{c x})^{-m}$$

$$\left( -\frac{1}{1+m} 12 \, (\text{c x})^m \left( \left( \sqrt{-1+\text{c x}} \, \sqrt{1+\text{c x}} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right/$$

$$\left( 6 \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] + \text{AppellF1} \left[ \frac{3}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{5}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right)$$

$$\left( -\frac{1+\text{c x}}{1+\text{c x}} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right/$$

$$\left( 6 \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] + \left( -1+\text{c x} \right) \, \left( 4 \, \text{m AppellF1} \left[ \frac{3}{2}, \, 1-\text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right) \right)$$

$$\frac{5}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] - \text{AppellF1} \left[ \frac{3}{2}, \, -\text{m}, \, \frac{3}{2}, \, \frac{5}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right)$$

$$\frac{(\text{c x})^{1+m} \, \text{ArcCosh} \left[ \text{c x} \right]}{1+\text{m}} \right) + \frac{1}{\text{c}} \, 3 \, \text{b} \, d^2 \, \text{e} \, x^{2+m} \, \left( \text{c x} \right)^{-2-m} \left( -\frac{1}{3+m} \, 4 \, \left( \text{c x} \right)^m \right)$$

$$\left( \left[ 3 \, \sqrt{-1+\text{c x}} \, \sqrt{1+\text{c x}} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-\text{c x}, \, \frac{1}{2} \, \left( 1-\text{c x} \right) \, \right] \right) \right) \right)$$

$$\left(6 \, \mathsf{AppelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \frac{3}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] + \\ \left( 1 + \mathsf{cx} \right) \left( 4 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, 1 \, \, \mathsf{m}, \, \frac{1}{2}, \, \frac{5}{2}, \, 1 \, \, \mathsf{cx}, \, \frac{1}{2} \left( 1 \, \, \, \mathsf{cx} \right) \right] + \\ \left( 1 + \mathsf{cx} \right) \left( 4 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{5}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) - \\ \left( 3 \, \sqrt{\frac{-1 + \mathsf{cx}}{1 + \mathsf{cx}}} \, \, \mathsf{AppelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] + \left( -1 + \mathsf{cx} \right) \left( 4 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{5}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) + \\ \left( 6 \, \mathsf{AppelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] + \left( -1 + \mathsf{cx} \right) \left( 4 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{5}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) + \\ \left( -1 + \mathsf{cx} \right)^{3/2} \, \sqrt{1 + \mathsf{cx}} \, \left( \left[ 5 \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{5}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) \right) \\ \left( 30 \, \mathsf{AppelIF1} \left[ \frac{3}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{7}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] + \mathsf{AppelIF1} \left[ \frac{5}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{7}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) \\ \left( 4 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{5}{2}, \, 1 \, \mathsf{m}, \, \frac{1}{2}, \, \frac{7}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right) \right] + \\ \left( 7 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{5}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{7}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right) \right) \right) \\ \left( 2 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{7}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{9}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) \right) \\ \left( 2 \, \mathsf{m} \, \mathsf{AppelIF1} \left[ \frac{7}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{9}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) \right) \\ \left( 2 \, \mathsf{appelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right] \right) \right) \right) \\ \left( 2 \, \mathsf{appelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \mathsf{cx}, \, \frac{1}{2} \left( 1 - \mathsf{cx} \right) \right) \right] \right) \right) \\ \left( 2 \, \mathsf{appelIF1} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \,$$

$$\left( 60 \ (\text{cx})^m \left( -1 + \text{cx} \right)^{3/2} \sqrt{1 + \text{cx}} \ \text{AppellFI} \left[ \frac{3}{2}, -\text{m}, -\frac{1}{2}, \frac{5}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 30 \ \text{AppellFI} \left[ \frac{3}{2}, -\text{m}, -\frac{1}{2}, \frac{5}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] + 3 \left( -1 + \text{cx} \right) \left( 4 \ \text{m AppellFI} \left[ \frac{5}{2}, 1 - \text{m}, -\frac{1}{2}, \frac{7}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( -\frac{1}{2}, \frac{7}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] + \text{AppellFI} \left[ \frac{5}{2}, -\text{m}, -\frac{1}{2}, \frac{7}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 252 \ (\text{cx})^m \left( -1 + \text{cx} \right)^{5/2} \sqrt{1 + \text{cx}} \ \text{AppellFI} \left[ \frac{5}{2}, -\text{m}, -\frac{1}{2}, \frac{7}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{AppellFI} \left[ \frac{5}{2}, -\text{m}, -\frac{1}{2}, \frac{7}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{AppellFI} \left[ \frac{5}{2}, -\text{m}, -\frac{1}{2}, \frac{9}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{M AppellFI} \left[ \frac{7}{2}, -\text{m}, -\frac{1}{2}, \frac{9}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{M AppellFI} \left[ \frac{7}{2}, -\text{m}, -\frac{1}{2}, \frac{9}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{M AppellFI} \left[ \frac{7}{2}, -\text{m}, -\frac{1}{2}, \frac{9}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{M AppellFI} \left[ \frac{7}{2}, -\text{m}, -\frac{1}{2}, \frac{9}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 70 \ \text{M AppellFI} \left[ \frac{9}{2}, -\text{m}, -\frac{1}{2}, \frac{11}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 90 \ \text{M AppellFI} \left[ \frac{9}{2}, -\text{m}, -\frac{1}{2}, \frac{11}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 90 \ \text{M AppellFI} \left[ \frac{11}{2}, -\text{m}, -\frac{1}{2}, \frac{13}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 110 \ \text{M AppellFI} \left[ \frac{11}{2}, -\text{m}, -\frac{1}{2}, \frac{13}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 111 \ \text{M AppellFI} \left[ \frac{13}{2}, -\text{m}, -\frac{1}{2}, \frac{15}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 13 \ \text{M AppellFI} \left[ \frac{13}{2}, -\text{m}, -\frac{1}{2}, \frac{15}{2}, 1 - \text{cx}, \frac{1}{2} \left( 1 - \text{cx} \right) \right] \right) \right)$$

$$\left( 13 \ \text{M AppellFI} \left[ \frac{15}{2}, -\text{m}, -$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^{m} (d + e x^{2})^{2} (a + b \operatorname{ArcCosh}[c x]) dx$$

Optimal (type 5, 323 leaves, 6 steps):

$$\frac{b \ e \ \left(2 \ c^2 \ d \ (5+m)^2 + e \ \left(12+7 \ m+m^2\right)\right) \ x^{2+m} \ \left(1-c^2 \ x^2\right)}{c^3 \ \left(3+m\right)^2 \ \left(5+m\right)^2 \sqrt{-1+c \ x} \ \sqrt{1+c \ x}} + \frac{b \ e^2 \ x^{4+m} \ \left(1-c^2 \ x^2\right)}{c \ \left(5+m\right)^2 \sqrt{-1+c \ x} \ \sqrt{1+c \ x}} + \frac{d^2 \ x^{1+m} \ \left(a+b \ Arc Cosh \left[c \ x\right]\right)}{1+m} + \frac{2 \ d \ e \ x^{3+m} \ \left(a+b \ Arc Cosh \left[c \ x\right]\right)}{3+m} + \frac{e^2 \ x^{5+m} \ \left(a+b \ Arc Cosh \left[c \ x\right]\right)}{5+m} - \frac{b \ \left(c^4 \ d^2 \ \left(3+m\right) \ \left(5+m\right)}{1+m} + \frac{e \ \left(2+m\right) \ \left(2 \ c^2 \ d \ \left(5+m\right)^2 + e \ \left(12+7 \ m+m^2\right)\right)}{\left(3+m\right) \ \left(5+m\right)} \right) x^{2+m} \sqrt{1-c^2 \ x^2}$$

$$\text{Hypergeometric 2F1} \left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, c^2 \ x^2\right] \left/ \left(c^3 \ \left(2+m\right) \ \left(3+m\right) \ \left(5+m\right) \ \sqrt{-1+c \ x} \ \sqrt{1+c \ x}\right) \right.$$

Result (type 6, 2064 leaves):

$$\frac{\text{a d}^2 \, x^{1 \cdot m}}{1 + \text{m}} + \frac{2 \, \text{a d} \, \text{e} \, x^{3 \cdot m}}{3 + \text{m}} + \frac{\text{a e}^2 \, x^{5 \cdot m}}{5 + \text{m}} + \frac{1}{\text{c}} \, \text{b d}^2 \, x^m \, (\text{c} \, x)^{-m}$$

$$\left( -\frac{1}{1 + \text{m}} 12 \, (\text{c} \, x)^m \left[ \left( \sqrt{-1 + \text{c} \, x} \, \sqrt{1 + \text{c} \, x} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] + \left( -1 + \text{c} \, x \right) \, \left( 4 \, \text{m AppellF1} \left[ \frac{3}{2}, \, 1 - \text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] + \text{AppellF1} \left[ \frac{3}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{5}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] \right) \right)$$

$$\left( \sqrt{\frac{-1 + \text{c} \, x}{1 + \text{c} \, x}} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] \right) \right) \right)$$

$$\left( 6 \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] + \left( -1 + \text{c} \, x \right) \, \left( 4 \, \text{m AppellF1} \left[ \frac{3}{2}, \, 1 - \text{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] \right) \right) \right)$$

$$\frac{5}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] - \text{AppellF1} \left[ \frac{3}{2}, \, -\text{m}, \, \frac{3}{2}, \, \frac{5}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] \right) \right) \right)$$

$$\frac{\left( \text{c} \, x \right)^{1 + \text{m}} \, \text{ArcCosh} \left[ \text{c} \, x \right]}{1 + \text{m}} \right) + \frac{1}{\text{c}} \, 2 \, \text{b} \, \text{d} \, e \, x^{2 + m} \, \left( \text{c} \, x \right)^{-2 - m} \left( -\frac{1}{3 + m} \, 4 \, \left( \text{c} \, x \right)^m \right) \right) \right) \right)$$

$$\left( \left[ 3 \, \sqrt{-1 + \text{c} \, x} \, \sqrt{1 + \text{c} \, x} \, \text{AppellF1} \left[ \frac{1}{2}, \, -\text{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1 - \text{c} \, x, \, \frac{1}{2} \, \left( 1 - \text{c} \, x \right) \, \right] \right) \right) \right) \right)$$

$$\left( 40 \; (c \; x)^m \; \left( -1 + c \; x \right)^{3/2} \sqrt{1 + c \; x} \; \text{AppellF1} \left[ \frac{3}{2}, \; -m, \; -\frac{1}{2}, \; \frac{5}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) / \\ \left( 30 \; \text{AppellF1} \left[ \frac{3}{2}, \; -m, \; -\frac{1}{2}, \; \frac{5}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] + 3 \; \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{5}{2}, \; 1 - m, \; -\frac{1}{2}, \; \frac{7}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) + \\ \left( 112 \; (c \; x)^m \; \left( -1 + c \; x \right)^{5/2} \sqrt{1 + c \; x} \; \text{AppellF1} \left[ \frac{5}{2}, \; -m, \; -\frac{1}{2}, \; \frac{7}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) / \\ \left( 70 \; \text{AppellF1} \left[ \frac{5}{2}, \; -m, \; -\frac{1}{2}, \; \frac{7}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] + 5 \; \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{7}{2}, \; 1 - m, \; -\frac{1}{2}, \; \frac{9}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) + \\ \left( 108 \; (c \; x)^m \; \left( -1 + c \; x \right)^{7/2} \sqrt{1 + c \; x} \; \text{AppellF1} \left[ \frac{7}{2}, \; -m, \; -\frac{1}{2}, \; \frac{9}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) + \\ \left( 7 \; \left( 18 \; \text{AppellF1} \left[ \frac{7}{2}, \; -m, \; -\frac{1}{2}, \; \frac{9}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) + \\ \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{9}{2}, \; -m, \; -\frac{1}{2}, \; \frac{11}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) + \\ \left( 44 \; (c \; x)^m \; \left( -1 + c \; x \right)^{9/2} \sqrt{1 + c \; x} \; \text{AppellF1} \left[ \frac{9}{2}, \; -m, \; -\frac{1}{2}, \; \frac{11}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) \right) + \\ \left( 9 \; \left( 22 \; \text{AppellF1} \left[ \frac{9}{2}, \; -m, \; -\frac{1}{2}, \; \frac{11}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) \right) + \\ \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{11}{2}, \; 1 - m, \; -\frac{1}{2}, \; \frac{13}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) \right) + \\ \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{11}{2}, \; 1 - m, \; -\frac{1}{2}, \; \frac{13}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) \right) + \\ \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{11}{2}, \; 1 - m, \; -\frac{1}{2}, \; \frac{13}{2}, \; 1 - c \; x, \; \frac{1}{2} \; \left( 1 - c \; x \right) \right] \right) \right) \right) + \\ \left( -1 + c \; x \right) \; \left( 4 \; \text{m AppellF1} \left[ \frac{11}{2}, \; 1 - m$$

Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^m \left(d + e x^2\right) \left(a + b \operatorname{ArcCosh}\left[c x\right]\right) dx$$

Optimal (type 5, 178 leaves, 5 steps):

$$-\frac{b \, e \, x^{2+m} \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}}{c \, \left(3+m\right)^2} + \frac{d \, x^{1+m} \, \left(a+b \, ArcCosh\left[c \, x\right]\right)}{1+m} + \frac{e \, x^{3+m} \, \left(a+b \, ArcCosh\left[c \, x\right]\right)}{3+m} - \\ \left(b \, \left(e \, \left(1+m\right) \, \left(2+m\right) + c^2 \, d \, \left(3+m\right)^2\right) \, x^{2+m} \, \sqrt{1-c^2 \, x^2} \, \, \text{Hypergeometric2F1} \left[\frac{1}{2}, \, \frac{2+m}{2}, \, \frac{4+m}{2}, \, c^2 \, x^2\right]\right) \right/ \\ \left(c \, \left(1+m\right) \, \left(2+m\right) \, \left(3+m\right)^2 \, \sqrt{-1+c \, x} \, \sqrt{1+c \, x}\right)$$

Result (type 6, 1035 leaves):

$$\frac{a\;d\;x^{1+m}}{1+m}\;+\;\frac{a\;e\;x^{3+m}}{3\;+\;m}\;+\;\frac{1}{c}b\;d\;x^{m}\;\left(\;c\;x\;\right)^{\;-m}$$

$$\left[ -\frac{1}{1+m} 12 \; (c\,x)^n \left[ \left[ \sqrt{-1+c\,x} \; \sqrt{1+c\,x} \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] + \left( -1+c\,x \right) \left( 4 \; \mathsf{mAppellFI} \left[ \frac{3}{2}, \, 1-\mathsf{m}, \, -\frac{1}{2}, \, \frac{5}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right) \right]$$

$$\left[ \sqrt{\frac{-1+c\,x}{1+c\,x}} \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] + \left( -1+c\,x \right) \left( 4 \; \mathsf{mAppellFI} \left[ \frac{3}{2}, \, 1-\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, \frac{1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right) \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, -\mathsf{m}, \, -\frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{3}{2}, \, -\mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, \mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, \mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[ \frac{1}{2}, \, \mathsf{m}, \, \frac{1}{2}, \, \frac{3}{2}, \, 1-c\,x, \, \frac{1}{2} \; (1-c\,x) \right] \right] \right]$$

$$\left[ \left( 6 \; \mathsf{AppellFI} \left[$$

$$5 \left(-1 + c \, x\right) \left(4 \, \text{m AppellF1}\left[\frac{7}{2}, \, 1 - \text{m}, \, -\frac{1}{2}, \, \frac{9}{2}, \, 1 - c \, x, \, \frac{1}{2} \left(1 - c \, x\right)\,\right] + \\ \text{AppellF1}\left[\frac{7}{2}, \, -\text{m}, \, \frac{1}{2}, \, \frac{9}{2}, \, 1 - c \, x, \, \frac{1}{2} \left(1 - c \, x\right)\,\right]\right)\right) \right) + \frac{(c \, x)^{3 + m} \, \text{ArcCosh}\left[c \, x\right]}{3 + m}$$

# Problem 64: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcCosh}[c \ x]\right)^{2}}{d + e \ x^{2}} \, dx$$

# Optimal (type 4, 763 leaves, 22 steps):

#### Result (type 8, 22 leaves):

$$\int \frac{\left(a + b \, ArcCosh\left[\,c \,\, x\,\right]\,\right)^{\,2}}{d + e \,\, x^{2}} \, \mathrm{d} x$$

# Problem 73: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\text{d} + \text{e } x^2\right) \; \left(\text{a} + \text{b ArcCosh}\left[\text{c } x\right]\right)^2} \, \text{d} x$$

Optimal (type 8, 23 leaves, 0 steps):

$$Int \left[ \frac{1}{\left(d+e\;x^2\right)\; \left(a+b\;ArcCosh\left[c\;x\right]\right)^2}\text{, }x\right]$$

Result (type 1, 1 leaves):

???

# Problem 74: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(d+e\,x^2\right)^2\,\left(a+b\,ArcCosh\left[c\,x\right]\right)^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

Int 
$$\left[\frac{1}{\left(d+e\,x^2\right)^2\left(a+b\,ArcCosh\left[c\,x\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

# Problem 108: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\mathsf{d} + \mathsf{e} \; \mathsf{x}^2\right)^{3/2} \, \left(\mathsf{a} + \mathsf{b} \, \mathsf{ArcCosh} \left[\mathsf{c} \; \mathsf{x}\right]\right)^2} \, \mathsf{d} \mathsf{x}$$

Optimal (type 8, 25 leaves, 0 steps):

Int 
$$\left[\frac{1}{\left(d+e\,x^2\right)^{3/2}\left(a+b\,ArcCosh\left[c\,x\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

# Problem 109: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{\left(\text{d} + \text{e} \; \text{x}^2\right)^{5/2} \, \left(\text{a} + \text{b} \, \text{ArcCosh} \left[\text{c} \; \text{x}\right]\right)^2} \, \text{d} \text{x}$$

Optimal (type 8, 25 leaves, 0 steps):

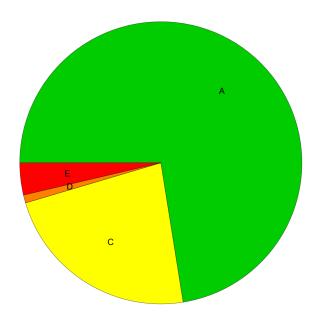
Int 
$$\left[\frac{1}{\left(d+e\,x^2\right)^{5/2}\left(a+b\,ArcCosh\left[c\,x\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

# **Summary of Integration Test Results**

# 109 integration problems



- A 79 optimal antiderivatives
- B 0 more than twice size of optimal antiderivatives
- C 25 unnecessarily complex antiderivatives
- D 1 unable to integrate problems
- E 4 integration timeouts