# Mathematica 11.3 Integration Test Results

### Test results for the 23 problems in "4.4.1.2 (d csc)^m (a+b cot)^n.m"

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Csc}[x]^3}{\mathrm{i} + \mathsf{Cot}[x]} \, \mathrm{d} x$$

Optimal (type 3, 12 leaves, 2 steps):

Result (type 3, 26 leaves):

$$- \, \mathsf{Csc} \, \big[ \, x \, \big] \, + \, \dot{\mathbb{1}} \, \left( \mathsf{Log} \, \Big[ \, \mathsf{Cos} \, \Big[ \, \frac{x}{2} \, \Big] \, \Big] \, - \, \mathsf{Log} \, \Big[ \, \mathsf{Sin} \, \Big[ \, \frac{x}{2} \, \Big] \, \Big] \, \right)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathsf{Csc}[x]^5}{\mathrm{i} + \mathsf{Cot}[x]} \, \mathrm{d} x$$

Optimal (type 3, 28 leaves, 3 steps):

$$\frac{1}{2} \pm \operatorname{ArcTanh}\left[\operatorname{Cos}\left[x\right]\right] + \frac{1}{2} \pm \operatorname{Cot}\left[x\right] \operatorname{Csc}\left[x\right] - \frac{\operatorname{Csc}\left[x\right]^{3}}{3}$$

Result (type 3, 67 leaves):

$$\begin{split} \frac{1}{24} & \text{ in } \mathsf{Csc}\left[x\right]^3 \left(8 \text{ in } + 9 \left(\mathsf{Log}\left[\mathsf{Cos}\left[\frac{x}{2}\right]\right] - \mathsf{Log}\left[\mathsf{Sin}\left[\frac{x}{2}\right]\right]\right) \mathsf{Sin}\left[x\right] + \\ & 6 \mathsf{Sin}\left[2 \text{ } x\right] - 3 \mathsf{ Log}\left[\mathsf{Cos}\left[\frac{x}{2}\right]\right] \mathsf{Sin}\left[3 \text{ } x\right] + 3 \mathsf{ Log}\left[\mathsf{Sin}\left[\frac{x}{2}\right]\right] \mathsf{Sin}\left[3 \text{ } x\right] \right) \end{split}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{Csc}[x]^7}{i + \operatorname{Cot}[x]} \, \mathrm{d}x$$

Optimal (type 3, 40 leaves, 4 steps):

$$\frac{3}{8} \, \, \dot{\mathbb{1}} \, \, \mathsf{ArcTanh} \, [\mathsf{Cos} \, [\, x \,] \, \,] \, + \, \frac{3}{8} \, \, \dot{\mathbb{1}} \, \, \mathsf{Cot} \, [\, x \,] \, \, \mathsf{Csc} \, [\, x \,] \, + \, \frac{1}{4} \, \, \dot{\mathbb{1}} \, \, \mathsf{Cot} \, [\, x \,] \, \, \mathsf{Csc} \, [\, x \,]^{\, 3} \, - \, \frac{\mathsf{Csc} \, [\, x \,]^{\, 5}}{5} \, + \, \frac{\mathsf{Csc} \, [\, x$$

Result (type 3, 99 leaves):

#### Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sin[x]^2}{a+b\cot[x]} dx$$

Optimal (type 3, 72 leaves, 7 steps):

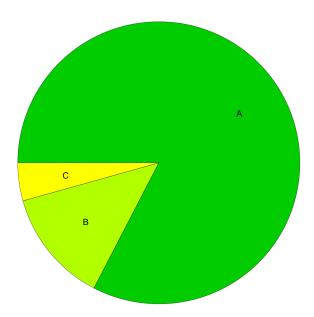
$$\frac{a\,\left(a^2+3\,b^2\right)\,x}{2\,\left(a^2+b^2\right)^2} - \frac{b^3\,Log\,[\,b\,Cos\,[\,x\,]\,\,+\,a\,Sin\,[\,x\,]\,\,]}{\left(a^2+b^2\right)^2} - \frac{\left(b+a\,Cot\,[\,x\,]\,\right)\,Sin\,[\,x\,]^{\,2}}{2\,\left(a^2+b^2\right)}$$

Result (type 3, 94 leaves):

$$\begin{split} &\frac{1}{4\,\left(a^2+b^2\right)^2} \left(2\,a^3\,x+6\,a\,b^2\,x-4\,\dot{\mathbb{1}}\,b^3\,x+4\,\dot{\mathbb{1}}\,b^3\,\text{ArcTan}\,[\text{Tan}\,[\,x\,]\,\,]\right. \\ &\left. b\,\left(a^2+b^2\right)\,\text{Cos}\,[\,2\,x\,]\,-2\,b^3\,\text{Log}\,[\,\left(b\,\text{Cos}\,[\,x\,]\,+a\,\text{Sin}\,[\,x\,]\,\right)^{\,2}\,\right] - a^3\,\text{Sin}\,[\,2\,x\,]\,-a\,b^2\,\text{Sin}\,[\,2\,x\,]\, \right) \end{split}$$

## **Summary of Integration Test Results**

#### 23 integration problems



- A 19 optimal antiderivatives
- B 3 more than twice size of optimal antiderivatives
- C 1 unnecessarily complex antiderivatives
- D 0 unable to integrate problems
- E 0 integration timeouts