Rubi 4.16.1.4 Results on Entire Integration Test Suite

Test results for the 175 integration problems in "Apostol Problems.m"

Test results for the 113 integration problems in "Moses Problems.m"

Test results for the 705 integration problems in "Timofeev Problems.m"

Problem 222: Result valid but suboptimal antiderivative.

$$\int \frac{\sqrt{1-x}\; x\; (1+x)^{2/3}}{-(1-x)^{5/6}\,(1+x)^{1/3}+(1-x)^{2/3}\sqrt{1+x}}\,dx$$

Optimal (type 3, 292 leaves, ? steps):

$$-\frac{1}{12}(1-3x)(1-x)^{2/3}(1+x)^{1/3}+\frac{1}{4}\sqrt{1-x}\;x\sqrt{1+x}-\frac{1}{4}(1-x)(3+x)+\frac{1}{12}(1-x)^{1/3}(1+x)^{2/3}(1+3x)+\frac{1}{12}(1-x)^{1/6}(1+x)^{5/6}(2+3x)-\frac{1}{12}(1-x)^{5/6}(1+x)^{1/6}(10+3x)+\frac{1}{6}\text{ArcTan}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right]-\frac{4\,\text{ArcTan}\left[\frac{(1-x)^{1/3}-2(1+x)^{1/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{5}{6}\text{ArcTan}\left[\frac{(1-x)^{1/3}-(1+x)^{1/3}}{(1-x)^{1/6}(1+x)^{1/6}}\right]+\frac{\text{ArcTanh}\left[\frac{\sqrt{3}\,(1-x)^{1/6}(1+x)^{1/6}}{(1-x)^{1/3}+(1+x)^{1/3}}\right]}{6\sqrt{3}}$$

Result (type 3, 522 leaves, 46 steps):

$$\frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}(1+x)^{1/6}+\frac{1}{6}(1-x)^{2/3}(1+x)^{1/3}-\frac{1}{4}(1-x)^{5/3}(1+x)^{1/3}+\frac{1}{3}(1-x)^{1/3}(1+x)^{2/3}-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3}+$$
$$\frac{5}{12}(1-x)^{1/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{7/6}(1+x)^{5/6}-\frac{1}{4}(1-x)^{5/6}(1+x)^{7/6}+\frac{1}{4}x\sqrt{1-x^2}+\frac{\text{ArcSin}[x]}{4}-\frac{2}{3}\text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right]+$$
$$\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1-x)^{1/3}}{\sqrt{3}\,(1+x)^{1/3}}\right]}{3\sqrt{3}}+\frac{1}{3}\text{ArcTan}\left[\sqrt{3}-\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{1}{3}\text{ArcTan}\left[\sqrt{3}+\frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right]-\frac{2\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{\sqrt{3}\,(1-x)^{1/3}}\right]}{3\sqrt{3}}-\frac{1}{9}\text{Log}[1-x]+$$
$$\frac{1}{9}\text{Log}[1+x]+\frac{1}{3}\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}-\frac{\sqrt{3}\,(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}+\frac{\text{Log}\left[1+\frac{(1-x)^{1/3}}{(1+x)^{1/3}}+\frac{\sqrt{3}\,(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}-\frac{1}{3}\text{Log}\left[1+\frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right]$$

Problem 226: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left((-1+x)^2(1+x)\right)^{1/3}}\,dx$$

Optimal (type 3, 67 leaves, ? steps):

$$\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)}{\left((-1+x)^2(1+x)\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2}\text{Log}[1+x]-\frac{3}{2}\text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2(1+x)\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2(1+x)^{1/3}}{3^{1/6}(3-3x)^{1/3}}\right]}{3^{1/6}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[-\frac{8}{3}(-1+x)\right]}{2\times 3^{2/3}\left(1-x-x^2+x^3\right)^{1/3}}-\frac{3^{1/3}(3-3x)^{2/3}(1+x)^{1/3}\,\text{Log}\left[1+\frac{3^{1/3}(1+x)^{1/3}}{(3-3x)^{1/3}}\right]}{2\left(1-x-x^2+x^3\right)^{1/3}}$$

Problem 228: Result valid but suboptimal antiderivative.

$$\int\frac{\left((-1+x)^2(1+x)\right)^{1/3}}{x^2}\,dx$$

Optimal (type 3, 150 leaves, ? steps):

$$-\frac{\left((-1+x)^2 (1+x)\right)^{1/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2 (-1+x)}{\left((-1+x)^2 (1+x)\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2 (-1+x)}{\left((-1+x)^2 (1+x)\right)^{1/3}}}{\sqrt{3}}\right]+$$
$$\frac{\text{Log}[x]}{6}-\frac{2}{3} \text{Log}[1+x]-\frac{3}{2} \text{Log}\left[1-\frac{-1+x}{\left((-1+x)^2 (1+x)\right)^{1/3}}\right]-\frac{1}{2} \text{Log}\left[1+\frac{-1+x}{\left((-1+x)^2 (1+x)\right)^{1/3}}\right]$$

Result (type 3, 404 leaves, 6 steps):

$$-\frac{\left(1-x-x^2+x^3\right)^{1/3}}{x}-\frac{3\times 3^{1/6} \left(1-x-x^2+x^3\right)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2 (3-3 x)^{1/3}}{3^{5/6} (1+x)^{1/3}}\right]}{(3-3 x)^{2/3} (1+x)^{1/3}}-$$
$$\frac{3^{1/6} \left(1-x-x^2+x^3\right)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2 (3-3 x)^{1/3}}{3^{5/6} (1+x)^{1/3}}\right]}{(3-3 x)^{2/3} (1+x)^{1/3}}+\frac{\left(1-x-x^2+x^3\right)^{1/3} \text{Log}[x]}{2\times 3^{1/3} (3-3 x)^{2/3} (1+x)^{1/3}}-\frac{3^{2/3} \left(1-x-x^2+x^3\right)^{1/3} \text{Log}\left[\frac{4 (1+x)}{3}\right]}{2 (3-3 x)^{2/3} (1+x)^{1/3}}-$$
$$\frac{3\times 3^{2/3} \left(1-x-x^2+x^3\right)^{1/3} \text{Log}\left[1+\frac{(3-3 x)^{1/3}}{3^{1/3} (1+x)^{1/3}}\right]}{2 (3-3 x)^{2/3} (1+x)^{1/3}}-\frac{3^{2/3} \left(1-x-x^2+x^3\right)^{1/3} \text{Log}\left[\left(\frac{2}{3}\right)^{2/3} (3-3 x)^{1/3}-\frac{2^{2/3} (1+x)^{1/3}}{3^{1/3}}\right]}{2 (3-3 x)^{2/3} (1+x)^{1/3}}$$

Problem 232: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\left(9+3 x-5 x^2+x^3\right)^{1/3}} \, dx$$

Optimal (type 3, 75 leaves, ? steps):

$$\sqrt{3}\ \text{ArcTan}\left[\frac{1+\frac{2 (-3+x)}{\left(9+3 x-5 x^2+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\frac{1}{2} \text{Log}[1+x]-\frac{3}{2} \text{Log}\left[1-\frac{-3+x}{\left(9+3 x-5 x^2+x^3\right)^{1/3}}\right]$$

Result (type 3, 188 leaves, 3 steps):

$$-\frac{(9-3 x)^{2/3} (1+x)^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2 (1+x)^{1/3}}{3^{1/6} (9-3 x)^{1/3}}\right]}{3^{1/6} \left(9+3 x-5 x^2+x^3\right)^{1/3}}-\frac{(9-3 x)^{2/3} (1+x)^{1/3} \text{Log}\left[-\frac{32}{3} (-3+x)\right]}{2\times 3^{2/3} \left(9+3 x-5 x^2+x^3\right)^{1/3}}-\frac{3^{1/3} (9-3 x)^{2/3} (1+x)^{1/3} \text{Log}\left[1+\frac{3^{1/3} (1+x)^{1/3}}{(9-3 x)^{1/3}}\right]}{2 \left(9+3 x-5 x^2+x^3\right)^{1/3}}$$

Problem 288: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x+\sqrt{1+x+x^2}} \, dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-x+\sqrt{1+x+x^2}-\frac{3}{2}\,\text{ArcSinh}\left[\frac{1+2x}{\sqrt{3}}\right]+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]$$

Result (type 3, 59 leaves, 3 steps):

$$\frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)}+2\,\text{Log}\left[x+\sqrt{1+x+x^2}\right]-\frac{3}{2}\,\text{Log}\left[1+2\left(x+\sqrt{1+x+x^2}\right)\right]$$

Problem 306: Result valid but suboptimal antiderivative.

$$\int\left(x\left(1-x^2\right)\right)^{1/3}\,dx$$

Optimal (type 3, 93 leaves, ? steps):

$$\frac{1}{2}\,x\left(x\left(1-x^2\right)\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{2x-\left(x\left(1-x^2\right)\right)^{1/3}}{\sqrt{3}\left(x\left(1-x^2\right)\right)^{1/3}}\right]}{2\sqrt{3}}+\frac{\text{Log}[x]}{12}-\frac{1}{4}\,\text{Log}\left[x+\left(x\left(1-x^2\right)\right)^{1/3}\right]$$

Result (type 3, 200 leaves, 12 steps):

$$\frac{1}{2}\,x\left(x-x^3\right)^{1/3}-\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{ArcTan}\left[\frac{1-\frac{2x^{2/3}}{\left(1-x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\sqrt{3}\left(x-x^3\right)^{2/3}}+\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{Log}\left[1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{12\left(x-x^3\right)^{2/3}}-\frac{x^{2/3}\left(1-x^2\right)^{2/3}\text{Log}\left[1+\frac{x^{2/3}}{\left(1-x^2\right)^{1/3}}\right]}{6\left(x-x^3\right)^{2/3}}$$

Problem 401: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Tan}[x]}{\left(-1+\sqrt{\text{Tan}[x]}\right)^2}\,dx$$

Optimal (type 3, 84 leaves, ? steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[\frac{1-\text{Tan}[x]}{\sqrt{2}\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{\text{ArcTanh}\left[\frac{1+\text{Tan}[x]}{\sqrt{2}\sqrt{\text{Tan}[x]}}\right]}{\sqrt{2}}+\frac{1}{2}\,\text{Log}[\text{Cos}[x]]+\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Result (type 3, 133 leaves, 19 steps):

$$-\frac{x}{2}+\frac{\text{ArcTan}\left[1-\sqrt{2}\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\sqrt{\text{Tan}[x]}\right]}{\sqrt{2}}+\frac{1}{2}\,\text{Log}[\text{Cos}[x]]+$$
$$\text{Log}\left[1-\sqrt{\text{Tan}[x]}\right]-\frac{\text{Log}\left[1-\sqrt{2}\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1+\sqrt{2}\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]}{2\sqrt{2}}+\frac{1}{1-\sqrt{\text{Tan}[x]}}$$

Problem 416: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Cos}[2x] - \sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\sqrt{2}\,\text{Log}\left[\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sec}[x]\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}\right]-\frac{\text{ArcSin}[\text{Cos}[x]-\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{ArcTanh}[\text{Sin}[x]]\,\text{Cos}[x]\,\sqrt{\text{Sin}[2x]}}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}-\frac{\text{Sin}[2x]}{\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}$$

Result (type 3, 234 leaves, 27 steps):

$$-2\,\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-\sqrt{2}\,\text{ArcSinh}[\text{Tan}[x]]\,\text{Cot}[x]\,\left(\text{Sec}[x]^2\right)^{3/2}\sqrt{\text{Cos}[x]\,\text{Sin}[x]}\,\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}-\frac{\sqrt{2}\,\text{ArcTan}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}+\frac{\sqrt{2}\,\text{ArcTan}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{\text{Tan}[x]}}-\frac{\text{Log}\left[1-\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}+\frac{\text{Log}\left[1+\sqrt{2}\,\sqrt{\text{Tan}[x]}+\text{Tan}[x]\right]\text{Sec}[x]^2\sqrt{\text{Cos}[x]^3\,\text{Sin}[x]}}{\sqrt{2}\,\sqrt{\text{Tan}[x]}}$$

Problem 447: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]^2\left(-\text{Cos}[2x]+2\,\text{Tan}[x]^2\right)}{\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}\,dx$$

Optimal (type 3, 100 leaves, ? steps):

$$2\,\text{ArcTanh}\left[\frac{\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]-\frac{11\,\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Tan}[x]}{\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}\right]}{4\sqrt{2}}+\frac{\text{Tan}[x]}{2\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{2\,\text{Tan}[x]^3}{3\left(\text{Tan}[x]\,\text{Tan}[2x]\right)^{3/2}}+\frac{3\,\text{Tan}[x]}{4\sqrt{\text{Tan}[x]\,\text{Tan}[2x]}}$$

Result (type 3, 208 leaves, 21 steps):

$$\frac{3\,\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Cot}[x]\left(1-\text{Tan}[x]^2\right)}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}+\frac{\text{Tan}[x]\left(1-\text{Tan}[x]^2\right)}{3\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}}-\frac{11\,\text{ArcTan}\left[\sqrt{-1+\text{Tan}[x]^2}\right]\text{Tan}[x]}{4\sqrt{2}\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}+\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-1+\text{Tan}[x]^2}}{\sqrt{2}}\right]\text{Tan}[x]}{\sqrt{\frac{\text{Tan}[x]^2}{1-\text{Tan}[x]^2}}\sqrt{-1+\text{Tan}[x]^2}}$$

Problem 455: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sin}[x]^6\,\text{Tan}[x]}{\text{Cos}[2x]^{3/4}}\,dx$$

Optimal (type 3, 102 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{1-\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}-\frac{\text{ArcTanh}\left[\frac{1+\sqrt{\text{Cos}[2x]}}{\sqrt{2}\,\text{Cos}[2x]^{1/4}}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+\frac{1}{36}\text{Cos}[2x]^{9/4}$$

Result (type 3, 154 leaves, 14 steps):

$$\frac{\text{ArcTan}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}\right]}{\sqrt{2}}+\frac{7}{4}\text{Cos}[2x]^{1/4}-\frac{1}{5}\text{Cos}[2x]^{5/4}+$$
$$\frac{1}{36}\text{Cos}[2x]^{9/4}+\frac{\text{Log}\left[1-\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}-\frac{\text{Log}\left[1+\sqrt{2}\,\text{Cos}[2x]^{1/4}+\sqrt{\text{Cos}[2x]}\right]}{2\sqrt{2}}$$

Problem 567: Result valid but suboptimal antiderivative.

$$\int e^{x/2}\,x^2\,\text{Cos}[x]^3\,dx$$

Optimal (type 3, 187 leaves, ? steps):

$$-\frac{132}{125}e^{x/2}\text{Cos}[x]+\frac{18}{25}e^{x/2}x\,\text{Cos}[x]+\frac{48}{185}e^{x/2}x^2\text{Cos}[x]+\frac{2}{37}e^{x/2}x^2\text{Cos}[x]^3-\frac{428\,e^{x/2}\text{Cos}[3x]}{50653}+\frac{70\,e^{x/2}x\,\text{Cos}[3x]}{1369}-$$
$$\frac{24}{125}e^{x/2}\text{Sin}[x]-\frac{24}{25}e^{x/2}x\,\text{Sin}[x]+\frac{96}{185}e^{x/2}x^2\text{Sin}[x]+\frac{12}{37}e^{x/2}x^2\text{Cos}[x]^2\text{Sin}[x]-\frac{792\,e^{x/2}\text{Sin}[3x]}{50653}-\frac{24\,e^{x/2}x\,\text{Sin}[3x]}{1369}$$

Result (type 3, 253 leaves, 31 steps):

$$-\frac{6687696\,e^{x/2}\text{Cos}[x]}{6331625}+\frac{24792\,e^{x/2}x\,\text{Cos}[x]}{34225}+\frac{48}{185}e^{x/2}x^2\text{Cos}[x]+\frac{16\,e^{x/2}\text{Cos}[x]^3}{50653}-\frac{8\,e^{x/2}x\,\text{Cos}[x]^3}{1369}+$$
$$\frac{2}{37}e^{x/2}x^2\text{Cos}[x]^3-\frac{432\,e^{x/2}\text{Cos}[3x]}{50653}+\frac{72\,e^{x/2}x\,\text{Cos}[3x]}{1369}-\frac{1218672\,e^{x/2}\text{Sin}[x]}{6331625}-\frac{32556\,e^{x/2}x\,\text{Sin}[x]}{34225}+\frac{96}{185}e^{x/2}x^2\text{Sin}[x]+$$
$$\frac{96\,e^{x/2}\text{Cos}[x]^2\text{Sin}[x]}{50653}-\frac{48\,e^{x/2}x\,\text{Cos}[x]^2\text{Sin}[x]}{1369}+\frac{12}{37}e^{x/2}x^2\text{Cos}[x]^2\text{Sin}[x]-\frac{816\,e^{x/2}\text{Sin}[3x]}{50653}-\frac{12\,e^{x/2}x\,\text{Sin}[3x]}{1369}$$

Problem 695: Result valid but suboptimal antiderivative.

$$\int \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right] dx$$

Optimal (type 3, 55 leaves, ? steps):

$$-\frac{\sqrt{2}\, a \sqrt{\frac{-a+x}{a+x}}}{\sqrt{\frac{a}{a+x}}} + (a+x)\, \text{ArcSin}\left[\sqrt{\frac{-a+x}{a+x}}\right]$$

Result (type 3, 118 leaves, 8 steps):

$$-\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}\,(a+x) + x\, \text{ArcSin}\left[\sqrt{-\frac{a-x}{a+x}}\right] - \frac{a\sqrt{\frac{a}{a+x}}\, \text{ArcTanh}\left[\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right]}{\sqrt{-\frac{a}{a+x}}}$$

Test results for the 50 integration problems in "Charlwood Problems.m"

Problem 3: Unable to integrate problem.

$$\int -\text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right] dx$$

Optimal (type 3, 69 leaves, ? steps):

$$\frac{\left(\sqrt{x} + 3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{4\sqrt{2}} - \left(\frac{3}{8}+x\right) \text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right]$$

Result (type 8, 60 leaves, 3 steps):

$$-x\, \text{ArcSin}\left[\sqrt{x} - \sqrt{1+x}\right] + \frac{\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}, x\right]}{2\sqrt{2}}$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int \text{Log}\left[1+x\sqrt{1+x^2}\right]\,dx$$

Optimal (type 3, 97 leaves, ? steps):

$$-2x+\sqrt{2\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\ \left(x+\sqrt{1+x^2}\right)\right]-\sqrt{2\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\ \left(x+\sqrt{1+x^2}\right)\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Result (type 3, 332 leaves, 32 steps):

$$-2x-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+x\,\text{Log}\left[1+x\sqrt{1+x^2}\right]$$

Problem 5: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\text{Cos}[x]^2}{\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\,dx$$

Optimal (type 3, 45 leaves, ? steps):

$$\frac{x}{3}+\frac{1}{3}\text{ArcTan}\left[\frac{\text{Cos}[x]\left(1+\text{Cos}[x]^2\right)\text{Sin}[x]}{1+\text{Cos}[x]^2\sqrt{1+\text{Cos}[x]^2+\text{Cos}[x]^4}}\right]$$

Result (type 4, 289 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{\text{Tan}[x]}{\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}\right]\text{Cos}[x]^2\sqrt{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}}{2\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}-$$

$$\frac{\left(1+\sqrt{3}\right)\text{Cos}[x]^2\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\left(\sqrt{3}+\text{Tan}[x]^2\right)\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}}{4\times 3^{1/4}\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}}+$$

$$\left(\left(2+\sqrt{3}\right)\text{Cos}[x]^2\,\text{EllipticPi}\left[\frac{1}{6}\left(3-2\sqrt{3}\right),2\,\text{ArcTan}\left[\frac{\text{Tan}[x]}{3^{1/4}}\right],\frac{1}{4}\left(2-\sqrt{3}\right)\right]\left(\sqrt{3}+\text{Tan}[x]^2\right)\sqrt{\frac{3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4}{\left(\sqrt{3}+\text{Tan}[x]^2\right)^2}}\right)\Big/$$

$$\left(4\times 3^{1/4}\sqrt{\text{Cos}[x]^4\left(3+3\,\text{Tan}[x]^2+\text{Tan}[x]^4\right)}\right)$$

Problem 12: Result unnecessarily involves imaginary or complex numbers.

$$\int \text{ArcTan}\left[x+\sqrt{1-x^2}\right]dx$$

Optimal (type 3, 141 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-$$

$$\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 269 leaves, 40 steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{\sqrt{3}}+\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+$$

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{\sqrt{3}}-\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]+x\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]-\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]}{\sqrt{1-x^2}} \, dx$$

Optimal (type 3, 152 leaves, ? steps):

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right]-\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{-1+2x^2}{\sqrt{3}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{4}\text{ArcTanh}\left[x\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Result (type 3, 286 leaves, 32 steps) :

$$-\frac{\text{ArcSin}[x]}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left[\frac{1-2x^2}{\sqrt{3}}\right]+\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]}{2\sqrt{3}}-\frac{1}{12}\left(3\,i-\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right]+\frac{\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]}{2\sqrt{3}}+\frac{1}{12}\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right]-\sqrt{1-x^2}\,\text{ArcTan}\left[x+\sqrt{1-x^2}\right]+\frac{1}{8}\text{Log}\left[1-x^2+x^4\right]$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Sec}[x]}{\sqrt{-1+\text{Sec}[x]^4}} \, dx$$

Optimal (type 3, 28 leaves, ? steps):

$$-\frac{\text{ArcTanh}\left[\frac{\text{Cos}[x]\,\text{Cot}[x]\,\sqrt{-1+\text{Sec}[x]^4}}{\sqrt{2}}\right]}{\sqrt{2}}$$

Result (type 3, 59 leaves, 5 steps) :

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{2}\,\text{Sin}[x]}{\sqrt{2\,\text{Sin}[x]^2-\text{Sin}[x]^4}}\right]\sqrt{1-\text{Cos}[x]^4}\,\text{Sec}[x]^2}{\sqrt{2}\,\sqrt{-1+\text{Sec}[x]^4}}$$

Test results for the 376 integration problems in "Stewart Problems.m"

Test results for the 284 integration problems in "Hearn Problems.m"

Problem 169: Unable to integrate problem.

$$\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Result (type 8, 69 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2}x+2x^2}x}{\left(-1+e^{x^2}x\right)^2},\,x\right]+2\,\text{CannotIntegrate}\left[\frac{e^{1-e^{x^2}x+2x^2}x^3}{\left(-1+e^{x^2}x\right)^2},\,x\right]$$

Problem 278: Unable to integrate problem.

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Optimal (type 3, 94 leaves, ? steps):

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2\left(-1+2x^2\right)}-\text{ArcTanh}\left[\frac{x\,(2+x)\left(7-x+27x^2+33x^3\right)}{\left(2+37x^2+31x^3\right)\sqrt{1+2x^2+4x^3+x^4}}\right]$$

Result (type 8, 354 leaves, 10 steps):

$$\frac{9}{4}\text{CannotIntegrate}\left[\frac{1}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{4}\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}-2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]+\text{CannotIntegrate}\left[\frac{x}{\sqrt{1+2x^2+4x^3+x^4}},x\right]+\frac{1}{2}\text{CannotIntegrate}\left[\frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{4}\text{CannotIntegrate}\left[\frac{1}{\left(\sqrt{2}+2x\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{8}\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{1}{8}\left(15+\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{13}{8}\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{1}{8}\left(15-\sqrt{2}\right)\text{CannotIntegrate}\left[\frac{1}{\left(1+\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}},x\right]-\frac{17}{2}\text{CannotIntegrate}\left[\frac{x}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}},x\right]$$

Problem 279: Unable to integrate problem.

$$\int\frac{(1+2y)\sqrt{1-5y-5y^2}}{y\,(1+y)\,(2+y)\sqrt{1-y-y^2}}\,dy$$

Optimal (type 3, 142 leaves, ? steps):

$$-\frac{1}{4}\text{ArcTanh}\left[\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right]-\frac{1}{2}\text{ArcTanh}\left[\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right]+\frac{9}{4}\text{ArcTanh}\left[\frac{(11+7y)\sqrt{1-5y-5y^2}}{3\,(7+5y)\sqrt{1-y-y^2}}\right]$$

Result (type 8, 115 leaves, 2 steps):

$$\frac{1}{2}\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}},y\right]+\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}},y\right]-\frac{3}{2}\text{CannotIntegrate}\left[\frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}},y\right]$$

Problem 281: Unable to integrate problem.

$$\int\left(\sqrt{9-4\sqrt{2}}\,x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)dx$$

Optimal (type 4, 4030 leaves, ? steps):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\,x^2-\sqrt{2}\left(-\frac{1}{3}\sqrt{1+4x+2x^2+x^4}+\frac{1}{3}(1+x)\sqrt{1+4x+2x^2+x^4}+\right.$$

$$\frac{4\,i\,\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{1+4x+2x^2+x^4}}{4\times2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x}-$$

$$\Bigg(8\times2^{2/3}\sqrt{\frac{3}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}$$

$$\sqrt{\left(\left(i\left(-19899+3445\sqrt{33}+\left(-26+6\sqrt{33}\right)^{2/3}\left(-2574+466\sqrt{33}\right)+\left(-26+6\sqrt{33}\right)^{1/3}\left(-19899+3445\sqrt{33}\right)+\left(59697-10335\sqrt{33}\right)x\right)\right)\Big/\right.}$$

$$\left(\left(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\right.$$

$$\left.\left.\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\right)$$

$$\sqrt{1+4x+2x^2+x^4}\ \text{EllipticE}\Big[\text{ArcSin}\Big[\Big(\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\right.}$$

$$\left.4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\Big)\Big/\left(\sqrt{\frac{39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{39-13\,i\sqrt{3}+9\,i\sqrt{11}-9\sqrt{33}+4\left(3+i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}}\right.$$

$$\left.\sqrt{\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}\right)\Big],$$

$$\left.\frac{4\left(21+7\,i\sqrt{3}-3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3-i\sqrt{3}-3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{4\left(21-7\,i\sqrt{3}+3\,i\sqrt{11}-3\sqrt{33}\right)+\left(3+i\sqrt{3}+3\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\Big]\right)\Big/$$

$$\Bigg(\left(4\times2^{2/3}-\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}+3\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$

$$\sqrt{\left(\left(i\left(1+x\right)\right)\Big/\left(\left(104-24\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)\right.\right.}$$

$$\left.\left.\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)\right)\right)$$

$$\sqrt{\left(26-6\sqrt{33}+\left(-13+13\,i\sqrt{3}-9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}$$

$$\left.\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}\right)+$$

$$\Big(\Big(2^{1/3}\left(13-13\,i\sqrt{3}+9\,i\sqrt{11}-3\sqrt{33}\right)+4\times 2^{2/3}\left(1+i\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{1/3}+20\left(-13+3\sqrt{33}\right)^{2/3}\Big)$$

$$\left(4\times 2^{2/3}\left(i+\sqrt{3}\right)+8\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)\sqrt{\frac{52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}}{-13+3\sqrt{33}+4\left(-26+6\sqrt{33}\right)^{1/3}}}$$

$$\sqrt{\left(\frac{1}{1+x}\left(-8\,i\left(-13+3\sqrt{33}\right)+\left(-43\,i-13\sqrt{3}+9\sqrt{11}+5\,i\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(2\,i+4\sqrt{3}-2\,i\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+\left(8\,i\left(-13+3\sqrt{33}\right)+\left(13\,i-13\sqrt{3}+9\sqrt{11}-3\,i\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)x\right)\right)}$$

$$\sqrt{1+4x+2x^2+x^4}\ \text{EllipticF}\Big[\text{ArcSin}\Big[\Big(\sqrt{52-12\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}}$$

$$\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}\Big)\Big/$$

$$\left(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\sqrt{1+x}\right)\Big],$$

$$\frac{4\left(21\,i-7\sqrt{3}+3\sqrt{11}-3\,i\sqrt{33}\right)+\left(3\,i+\sqrt{3}+3\sqrt{11}+3\,i\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{-56\sqrt{3}+24\sqrt{11}+2\left(\sqrt{3}+3\sqrt{11}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\Big]\Big)\Big/$$

$$\Big(3\times 2^{2/3}\times 3^{3/4}\left(-13+3\sqrt{33}\right)^{1/3}\sqrt{39+13\,i\sqrt{3}-9\,i\sqrt{11}-9\sqrt{33}+4\left(3-i\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\sqrt{1+x}$$

$$\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)-2\,i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6\,i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$

$$\sqrt{\left(26-6\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}+6\left(-13+3\sqrt{33}\right)x\right)}$$

$$\sqrt{\left(\left(8\left(-13+3\sqrt{33}\right)-\left(5-3\,i\sqrt{3}+3\,i\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+\left(-26+6\sqrt{33}\right)^{1/3}\left(-41+15\,i\sqrt{3}-3\,i\sqrt{11}+7\sqrt{33}\right)+\left(104-24\sqrt{33}+\left(-13-13\,i\sqrt{3}+9\,i\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+4\,i\left(i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)x\right)\Big/\left(\left(-39-13\,i\sqrt{3}+9\,i\sqrt{11}+9\sqrt{33}+4\,i\left(3\,i+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)}\Big)+$$

$$\Bigg(\left(4\times 2^{2/3}+2\left(-13+3\sqrt{33}\right)^{1/3}-2^{1/3}\left(-13+3\sqrt{33}\right)^{2/3}\right)\left(4\times 2^{2/3}\left(\mathrm{i}+\sqrt{3}\right)-4\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)$$

$$\left(4\times 2^{2/3}\left(-\mathrm{i}+\sqrt{3}\right)+4\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)$$

$$\sqrt{\left(\left(-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big/\left(104-24\sqrt{33}+\left(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)\right)}\sqrt{1+x}$$

$$\sqrt{\left(\left(104-24\sqrt{33}+2\left(1+14\,\mathrm{i}\sqrt{3}-6\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-7-\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\right)\Big/\left(\left(-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)}$$

$$\sqrt{\left(\left(104-24\sqrt{33}+2\left(1-14\,\mathrm{i}\sqrt{3}+6\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-7+\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{2/3}+2\left(-52+12\sqrt{33}+2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}-4\left(-26+6\sqrt{33}\right)^{2/3}\right)x\right)\Big/\left(\left(-39-13\,\mathrm{i}\sqrt{3}+9\,\mathrm{i}\sqrt{11}+9\sqrt{33}+4\,\mathrm{i}\left(3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)(1+x)\right)\right)}\sqrt{1+4x+2x^2+x^4}$$

$$\text{EllipticPi}\Bigg[\frac{2^{1/3}\left(4\times 2^{1/3}\left(-3\,\mathrm{i}+\sqrt{3}\right)+\left(3\,\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}\right)}{4\times 2^{2/3}\left(-\mathrm{i}+\sqrt{3}\right)-8\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}},$$

$$\text{ArcSin}\Bigg[\left(\sqrt{13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x}\right)\Big/$$

$$\left(2^{1/6}\sqrt{3}\left(-13+3\sqrt{33}\right)^{2/3}\sqrt{\left(\left(-39+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+9\sqrt{33}-4\,\mathrm{i}\left(-3\,\mathrm{i}+\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{1/3}\right)\Big/\left(104-24\sqrt{33}+\left(-13+13\,\mathrm{i}\sqrt{3}-9\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}+\left(-4-4\,\mathrm{i}\sqrt{3}\right)\left(-26+6\sqrt{33}\right)^{2/3}\right)\right)}\sqrt{1+x}\right)\Bigg],$$

$$\frac{4\left(21-7\,\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}-3\sqrt{33}\right)+\left(3+\mathrm{i}\sqrt{3}+3\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}{4\left(21+7\,\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}-3\sqrt{33}\right)+\left(3-\mathrm{i}\sqrt{3}-3\,\mathrm{i}\sqrt{11}+3\sqrt{33}\right)\left(-26+6\sqrt{33}\right)^{1/3}}\Bigg]\Bigg)\Big/$$

$$\Bigg(2^{1/6}\sqrt{3}\left(4\times 2^{2/3}\left(\mathrm{i}+\sqrt{3}\right)+2\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(-\mathrm{i}+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}-6\,\mathrm{i}\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$

$$\left(4\times 2^{2/3}\left(-i+\sqrt{3}\right)-2i\left(-13+3\sqrt{33}\right)^{1/3}+2^{1/3}\left(i+\sqrt{3}\right)\left(-13+3\sqrt{33}\right)^{2/3}+6i\left(-13+3\sqrt{33}\right)^{1/3}x\right)$$

$$\left.\sqrt{13-3\sqrt{33}-2^{1/3}\left(-13+3\sqrt{33}\right)^{4/3}+4\left(-26+6\sqrt{33}\right)^{2/3}+\left(-39+9\sqrt{33}\right)x}\,\right)\right)$$

Result (type 8, 47 leaves, 1 step):

$$\frac{1}{2}\sqrt{9-4\sqrt{2}}\;x^2-\sqrt{2}\;\text{CannotIntegrate}\left[\sqrt{1+4x+2x^2+x^4},\,x\right]$$

Problem 284: Unable to integrate problem.

$$\int\frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Optimal (type 3, 71 leaves, ? steps):

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\text{Log}\left[1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right]-\left(-1+\sqrt{2}\right)\text{Log}\left[-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right]\right)$$

Result (type 8, 248 leaves, 5 steps):

$$2\,\text{CannotIntegrate}\left[\frac{1}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},\,x\right]+4\,\text{CannotIntegrate}\left[\frac{x}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},\,x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{x^2}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},\,x\right]+12\,\text{CannotIntegrate}\left[\frac{x^7}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},\,x\right]+$$
$$10\,\text{CannotIntegrate}\left[\frac{x^8}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}},\,x\right]+\frac{1}{2}\text{Log}\left[1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right]$$

Test results for the 9 integration problems in "Jeffrey Problems.m"

Problem 2: Result valid but suboptimal antiderivative.

$$\int\frac{1+\text{Cos}[x]+2\,\text{Sin}[x]}{3+\text{Cos}[x]^2+2\,\text{Sin}[x]-2\,\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]$$

Result (type 3, 38 leaves, 43 steps):

$$-\text{ArcTan}\left[\frac{2\,\text{Cos}[x]-\text{Sin}[x]}{2+\text{Sin}[x]}\right]+\text{Cot}\left[\frac{x}{2}\right]-\frac{\text{Sin}[x]}{1-\text{Cos}[x]}$$

Problem 3: Result valid but suboptimal antiderivative.

$$\int\frac{2+\text{Cos}[x]+5\,\text{Sin}[x]}{4\,\text{Cos}[x]-2\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[x]-2\,\text{Sin}[x]^2}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1-3\,\text{Cos}[x]+\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 42 leaves, 25 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]-\text{Log}\left[1+\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 4: Result valid but suboptimal antiderivative.

$$\int\frac{3+7\,\text{Cos}[x]+2\,\text{Sin}[x]}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-5\,\text{Sin}[x]-\text{Cos}[x]\,\text{Sin}[x]}\,dx$$

Optimal (type 3, 19 leaves, ? steps):

$$-\text{Log}[1+\text{Cos}[x]-2\,\text{Sin}[x]]+\text{Log}[3+\text{Cos}[x]+\text{Sin}[x]]$$

Result (type 3, 31 leaves, 32 steps):

$$-\text{Log}\left[1-2\,\text{Tan}\left[\frac{x}{2}\right]\right]+\text{Log}\left[2+\text{Tan}\left[\frac{x}{2}\right]+\text{Tan}\left[\frac{x}{2}\right]^2\right]$$

Problem 5: Unable to integrate problem.

$$\int\frac{-1+4\,\text{Cos}[x]+5\,\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 43 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{\text{Sin}[x]}{3+\text{Cos}[x]}\right]-2\,\text{ArcTan}\left[\frac{3\,\text{Sin}[x]+7\,\text{Cos}[x]\,\text{Sin}[x]}{1+2\,\text{Cos}[x]+5\,\text{Cos}[x]^2}\right]$$

Result (type 8, 79 leaves, 2 steps):

$$\text{CannotIntegrate}\left[\frac{1}{1+4\,\text{Cos}[x]+3\,\text{Cos}[x]^2-4\,\text{Cos}[x]^3},\,x\right]+$$
$$4\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},\,x\right]+5\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1-4\,\text{Cos}[x]-3\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},\,x\right]$$

Problem 6: Unable to integrate problem.

$$\int\frac{-5+2\,\text{Cos}[x]+7\,\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3}\,dx$$

Optimal (type 3, 25 leaves, ? steps):

$$x-2\,\text{ArcTan}\left[\frac{2\,\text{Cos}[x]\,\text{Sin}[x]}{1-\text{Cos}[x]+2\,\text{Cos}[x]^2}\right]$$

Result (type 8, 81 leaves, 2 steps):

$$-5\,\text{CannotIntegrate}\left[\frac{1}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},\,x\right]+$$
$$2\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},\,x\right]+7\,\text{CannotIntegrate}\left[\frac{\text{Cos}[x]^2}{-1+2\,\text{Cos}[x]-9\,\text{Cos}[x]^2+4\,\text{Cos}[x]^3},\,x\right]$$

Test results for the 7 integration problems in "Hebisch Problems.m"

Problem 2: Unable to integrate problem.

$$\int\frac{e^{\frac{x}{2+x^2}}\left(2-x^2\right)}{2\,x+x^3}\,dx$$

Optimal (type 4, 10 leaves, ? steps):

$$\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 76 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}-x},\,x\right]+\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x},\,x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i\sqrt{2}+x},\,x\right]$$

Problem 3: Unable to integrate problem.

$$\int \frac{e^{\frac{x}{2+x^2}} \left(2+2 x+3 x^2-x^3+2 x^4\right)}{2 x+x^3} \, dx$$

Optimal (type 4, 28 leaves, ? steps):

$$e^{\frac{x}{2+x^2}} \left(2+x^2\right)+\text{ExpIntegralEi}\left[\frac{x}{2+x^2}\right]$$

Result (type 8, 131 leaves, 5 steps) :

$$-\text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}},x\right]+\left(1+i \sqrt{2}\right) \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}-x},x\right]+ \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{x},x\right]+2 \text{CannotIntegrate}\left[e^{\frac{x}{2+x^2}} x,x\right]-\left(1-i \sqrt{2}\right) \text{CannotIntegrate}\left[\frac{e^{\frac{x}{2+x^2}}}{i \sqrt{2}+x},x\right]$$

Problem 5: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{-1+x^2}} \left(1-3 x-x^2+x^3\right)}{1-x-x^2+x^3} \, dx$$

Optimal (type 3, 13 leaves, ? steps):

$$e^{\frac{1}{-1+x^2}} (1+x)$$

Result (type 8, 75 leaves, 6 steps) :

$$\text{CannotIntegrate}\left[e^{\frac{1}{-1+x^2}},x\right]+\frac{1}{2} \text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1-x},x\right]-\text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{(-1+x)^2},x\right]+\frac{1}{2} \text{CannotIntegrate}\left[\frac{e^{\frac{1}{-1+x^2}}}{1+x},x\right]$$

Problem 7: Unable to integrate problem.

$$\int \frac{e^{x+\frac{1}{\text{Log}[x]}} \left(-1+(1+x) \text{Log}[x]^2\right)}{\text{Log}[x]^2} \, dx$$

Optimal (type 3, 10 leaves, ? steps):

$$e^{x+\frac{1}{\text{Log}[x]}} x$$

Result (type 8, 40 leaves, 2 steps):

$$\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}},x\right]+\text{CannotIntegrate}\left[e^{x+\frac{1}{\text{Log}[x]}}\,x,x\right]-\text{CannotIntegrate}\left[\frac{e^{x+\frac{1}{\text{Log}[x]}}}{\text{Log}[x]^2},x\right]$$

Test results for the 8 integration problems in "Wester Problems.m"

Test results for the 93 integration problems in "Welz Problems.m"

Problem 9: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x^2}\,\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 3, 365 leaves, 18 steps):

$$\frac{2(1-2x)\sqrt{x}}{5(1+x-x^2)}-\frac{2(1-2x)\sqrt{-1+x^2}}{5(1+x-x^2)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]+\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{2}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 10: Result valid but suboptimal antiderivative.

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2 \sqrt{-1+x^2}}\,dx$$

Optimal (type 3, 220 leaves, ? steps):

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\text{ArcTan}\left[\frac{\sqrt{-2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-\left(1-\sqrt{5}\right)x}\right]-\frac{1}{25}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{1}{2}\sqrt{-2+2\sqrt{5}}\,\sqrt{x}\right]-\frac{1}{50}\sqrt{110+50\sqrt{5}}\,\text{ArcTanh}\left[\frac{\sqrt{2+2\sqrt{5}}\,\sqrt{-1+x^2}}{2-x-\sqrt{5}\,x}\right]$$

Result (type 3, 541 leaves, 25 steps):

$$\frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,\sqrt{x}\right]-\frac{1}{5}\sqrt{\frac{1}{10}\left(-11+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,\sqrt{x}\right]-\frac{1}{5}\sqrt{\frac{1}{5}\left(-2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\,\text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\,\sqrt{-1+x^2}}\right]$$

Problem 37: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(1+x)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 121 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{2\,(1-x)+2^{2/3}\,(1-x^3)^{1/3}}{2^{2/3}\,\sqrt{3}\,(1-x^3)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[1-x]}{4\times 2^{1/3}}-\frac{\text{Log}[1+x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 97 leaves, 1 step):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[(1-x)\,(1+x)^2\right]}{4\times 2^{1/3}}+\frac{3\,\text{Log}\left[-1+x+2^{2/3}\,(1-x^3)^{1/3}\right]}{4\times 2^{1/3}}$$

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\,(2-3\,x+x^2)^{1/3}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{1/3}\,(2-x)}{\sqrt{3}\,(2-3\,x+x^2)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}[2-x]}{4\times 2^{1/3}}-\frac{\text{Log}[x]}{2\times 2^{1/3}}+\frac{3\,\text{Log}\left[2-x-2^{2/3}\,(2-3\,x+x^2)^{1/3}\right]}{4\times 2^{1/3}}$$

Result (type 3, 176 leaves, 2 steps):

$$-\frac{\sqrt{3}\,(-2+x)^{1/3}\,(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}-\frac{2^{1/3}\,(-2+x)^{2/3}}{\sqrt{3}\,(-1+x)^{1/3}}\right]}{2\times 2^{1/3}\,(2-3\,x+x^2)^{1/3}}+\frac{3\,(-2+x)^{1/3}\,(-1+x)^{1/3}\,\text{Log}\left[-\frac{(-2+x)^{2/3}}{2^{1/3}}-2^{1/3}\,(-1+x)^{1/3}\right]}{4\times 2^{1/3}\,(2-3\,x+x^2)^{1/3}}-\frac{(-2+x)^{1/3}\,(-1+x)^{1/3}\,\text{Log}[x]}{2\times 2^{1/3}\,(2-3\,x+x^2)^{1/3}}$$

Problem 40: Result valid but suboptimal antiderivative.

$$\int \frac{1}{(-5+7\,x-3\,x^2+x^3)^{1/3}}\,dx$$

Optimal (type 3, 81 leaves, ? steps):

$$\frac{1}{2}\,\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2\,(-1+x)}{\sqrt{3}\,(-5+7\,x-3\,x^2+x^3)^{1/3}}\right]+\frac{1}{4}\,\text{Log}[1-x]-\frac{3}{4}\,\text{Log}\left[1-x+(-5+7\,x-3\,x^2+x^3)^{1/3}\right]$$

Result (type 3, 131 leaves, 5 steps):

$$\frac{\sqrt{3}\left(4+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{\left(4+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(4(-1+x)+(-1+x)^3\right)^{1/3}}-\frac{3\left(4+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\left(4+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]}{4\left(4(-1+x)+(-1+x)^3\right)^{1/3}}$$

Problem 41: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left(x\left(-q+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\left(x\left(-q+x^2\right)\right)^{1/3}}\right]+\frac{\text{Log}[x]}{4}-\frac{3}{4}\text{Log}\left[-x+\left(x\left(-q+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 117 leaves, 5 steps):

$$\frac{\sqrt{3}\,x^{1/3}\left(-q+x^2\right)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2x^{2/3}}{\left(-q+x^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-q\,x+x^3\right)^{1/3}}-\frac{3\,x^{1/3}\left(-q+x^2\right)^{1/3}\,\text{Log}\left[x^{2/3}-\left(-q+x^2\right)^{1/3}\right]}{4\left(-q\,x+x^3\right)^{1/3}}$$

Problem 42: Result valid but suboptimal antiderivative.

$$\int\frac{1}{\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 79 leaves, ? steps):

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}}\right]+\frac{1}{4}\text{Log}[1-x]-\frac{3}{4}\text{Log}\left[1-x+\left((-1+x)\left(q-2x+x^2\right)\right)^{1/3}\right]$$

Result (type 3, 145 leaves, 5 steps):

$$\frac{\sqrt{3}\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2(-1+x)^{2/3}}{\left(-1+q+(-1+x)^2\right)^{1/3}}}{\sqrt{3}}\right]}{2\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}}-\frac{3\left(-1+q+(-1+x)^2\right)^{1/3}(-1+x)^{1/3}\,\text{Log}\left[-\left(-1+q+(-1+x)^2\right)^{1/3}+(-1+x)^{2/3}\right]}{4\left(-(1-q)(-1+x)+(-1+x)^3\right)^{1/3}}$$

Problem 43: Unable to integrate problem.

$$\int \frac{1}{x\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\,dx$$

Optimal (type 3, 118 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2q^{1/3}(-1+x)}{\sqrt{3}\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}}\right]}{2q^{1/3}}+\frac{\text{Log}[1-x]}{4q^{1/3}}+\frac{\text{Log}[x]}{2q^{1/3}}-\frac{3\,\text{Log}\left[-q^{1/3}(-1+x)+\left((-1+x)\left(q-2qx+x^2\right)\right)^{1/3}\right]}{4q^{1/3}}$$

Result (type 8, 677 leaves, 2 steps):

$$\frac{1}{3\left(-q+3qx+(-1-2q)x^2+x^3\right)^{1/3}}\left(-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{1/3}}+3x\right)^{1/3}$$

$$\left(-1+5q-4q^2+\frac{(1-4q)^2(1-q)^2}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}}+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}+\right.$$

$$\left.\frac{3\left(1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}\right)\left(\frac{1}{3}(-1-2q)+x\right)}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{1/3}}+9\left(\frac{1}{3}(-1-2q)+x\right)^2\right)^{1/3}$$

$$\text{Unintegrable}\left[3\Bigg/\left(x\left(-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{2/3}}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{-(-1+q)^3q}\right)^{1/3}}+3x\right)^{1/3}\right.\right.$$

$$\left(-1+5q-4q^2+\frac{(1-4q)^2(1-q)^2}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}}+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}+\right.$$

$$\left.\left.\left.9\left(\frac{1}{3}(-1-2q)+x\right)^2+\frac{\left(1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{2/3}\right)(-1-2q+3x)}{\left(1+6q-15q^2+8q^3+3\sqrt{3}\sqrt{(1-q)^3q}\right)^{1/3}}\right)^{1/3}\right),x\right]$$

Problem 44: Unable to integrate problem.

$$\int \frac{2-(1+k)\,x}{\left((1-x)\,x\,(1-k\,x)\right)^{1/3}\,(1-(1+k)\,x)}\,dx$$

Optimal (type 3, 111 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,k^{1/3}\,x}{((1-x)\,x\,(1-k\,x))^{1/3}}}{\sqrt{3}}\right]}{k^{1/3}}+\frac{\text{Log}[x]}{2\,k^{1/3}}+\frac{\text{Log}[1-(1+k)\,x]}{2\,k^{1/3}}-\frac{3\,\text{Log}\left[-k^{1/3}\,x+((1-x)\,x\,(1-k\,x))^{1/3}\right]}{2\,k^{1/3}}$$

Result (type 8, 139 leaves, 3 steps):

$$\frac{3\,(1-x)^{1/3}\,x\,(1-k\,x)^{1/3}\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},x,k\,x\right]}{2\,((1-x)\,x\,(1-k\,x))^{1/3}}+\frac{(1-x)^{1/3}\,x^{1/3}\,(1-k\,x)^{1/3}\,\text{CannotIntegrate}\left[\frac{1}{(1-x)^{1/3}\,x^{1/3}\,(1+(-1-k)\,x)\,(1-k\,x)^{1/3}},x\right]}{((1-x)\,x\,(1-k\,x))^{1/3}}$$

Problem 45: Unable to integrate problem.

$$\int \frac{1-k\,x}{(1+(-2+k)\,x)\,((1-x)\,x\,(1-k\,x))^{2/3}}\,dx$$

Optimal (type 3, 176 leaves, ? steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2^{1/3}\,(1-k\,x)}{(1-k)^{1/3}\,((1-x)\,x\,(1-k\,x))^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}[1-(2-k)\,x]}{2^{2/3}\,(1-k)^{1/3}}+\frac{\text{Log}[1-k\,x]}{2\times 2^{2/3}\,(1-k)^{1/3}}-\frac{3\,\text{Log}\left[-1+k\,x+2^{2/3}\,(1-k)^{1/3}\,((1-x)\,x\,(1-k\,x))^{1/3}\right]}{2\times 2^{2/3}\,(1-k)^{1/3}}$$

Result (type 8, 78 leaves, 1 step):

$$\frac{(1-x)^{2/3}\,x^{2/3}\,(1-k\,x)^{2/3}\,\text{CannotIntegrate}\left[\frac{(1-k\,x)^{1/3}}{(1-x)^{2/3}\,x^{2/3}\,(1+(-2+k)\,x)},x\right]}{((1-x)\,x\,(1-k\,x))^{2/3}}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,x+c\,x^2}{(1-x+x^2)\,(1-x^3)^{1/3}}\,dx$$

Optimal (type 3, 326 leaves, ? steps):

$$-\frac{1}{6}c\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]+\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-2\,\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]\right)+$$

$$\frac{(a-b-2c)\left(-2\sqrt{3}\,\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]-3\,\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]\right)}{12\times 2^{1/3}}+$$

$$\frac{(a+b)\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\cdot 2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]+\text{Log}\left[3-6x+6x^2-3x^3\right]-3\,\text{Log}\left[-2^{1/3}(-1+x)+(1-x^3)^{1/3}\right]\right)}{4\times 2^{1/3}}-$$

$$\frac{(a-b-2c)\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]-3\,\text{Log}\left[2^{1/3}x+(1-x^3)^{1/3}\right]\right)}{12\times 2^{1/3}}$$

Result (type 3, 576 leaves, 7 steps):

$$-\frac{c\,\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\left(2a+b-i\sqrt{3}\,b-(1+i\sqrt{3})\,c\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}(1-i\sqrt{3}+2x)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(2a+b+i\sqrt{3}\,b-c+i\sqrt{3}\,c\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}(1+i\sqrt{3}+2x)}{(1-x^3)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+\frac{\left(3ib-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{12\times 2^{1/3}\left(i+\sqrt{3}\right)}+$$

$$\frac{\left(3ib+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{12\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$

$$\frac{1}{2}c\,\text{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{\left(3ib-\sqrt{3}\left(2a+b-c-i\sqrt{3}\,c\right)\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-$$

$$\frac{\left(3ib+\sqrt{3}\left(2a+b-c+i\sqrt{3}\,c\right)\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}(1-x^3)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 52: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\text{ArcTan}\left[\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right]$$

Result (type 4, 204 leaves, 9 steps):

$$\frac{2\,i\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2}{1-i\,a}\right]}{\sqrt{-(a-x)\left(1+x^2\right)}}+\frac{4\sqrt{1+a^2}\sqrt{\frac{a-x}{i+a}}\sqrt{1+x^2}\,\text{EllipticPi}\left[\frac{2}{1-i\left(a-\sqrt{1+a^2}\right)},\text{ArcSin}\left[\frac{\sqrt{1-i\,x}}{\sqrt{2}}\right],\frac{2}{1-i\,a}\right]}{\left(1-i\left(a-\sqrt{1+a^2}\right)\right)\sqrt{-(a-x)\left(1+x^2\right)}}$$

Problem 55: Result valid but suboptimal antiderivative.

$$\int \frac{1}{x\left(4-6x+3x^2\right)^{1/3}}\,dx$$

Optimal (type 3, 88 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{-2+x-2\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}}{\sqrt{3}\,(-2+x)}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[\frac{-4+2x+2\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}}{x}\right]}{2\times 2^{2/3}}$$

Result (type 3, 97 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\left(4-6x+3x^2\right)^{1/3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}[x]}{2\times 2^{2/3}}+\frac{\text{Log}\left[6-3x-3\times 2^{1/3}\left(4-6x+3x^2\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Problem 59: Unable to integrate problem.

$$\int \frac{\left(1-x^3\right)^{1/3}}{1-x+x^2}\,dx$$

Optimal (type 3, 280 leaves, ? steps):

$$\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{1+\frac{2\,2^{1/3}(-1+x)}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}(1-x^3)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[-3\,(-1+x)\,(1-x+x^2)\right]}{2\times 2^{2/3}}+$$
$$\frac{\text{Log}\left[2^{1/3}-(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[-2^{1/3}(-1+x)+(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}+\frac{1}{2}\text{Log}\left[x+(1-x^3)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}x+(1-x^3)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 79 leaves, 2 steps):

$$\frac{2\,i\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{1+i\sqrt{3}-2x},x\right]}{\sqrt{3}}+\frac{2\,i\,\text{CannotIntegrate}\left[\frac{(1-x^3)^{1/3}}{-1+i\sqrt{3}+2x},x\right]}{\sqrt{3}}$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-2+a+x}{(-a+x)\sqrt{(2-a)\,a\,x+(-1-2\,a+a^2)\,x^2+x^3}}\,dx$$

Optimal (type 1, 1 leaves, ? steps):

0

Result (type 4, 529 leaves, 5 steps):

$$\frac{2\,(1-a)\,\sqrt{x}\,\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}\;\text{ArcTan}\left[\frac{\sqrt{-1+2\,a-a^2}\,\sqrt{x}}{\sqrt{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}}\right]}{a\,\sqrt{-1+2\,a-a^2}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}}+$$

$$\left(((2-a)\,a)^{3/4}\,\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/$$

$$\left(a\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)+\left((2-a)\,\left(1-\sqrt{(2-a)\,a}\right)\sqrt{x}\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)\sqrt{\frac{(2-a)\,a-(1+2\,a-a^2)\,x+x^2}{(2-a)\,a\,\left(1+\frac{x}{\sqrt{(2-a)\,a}}\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\,\sqrt{(2-a)\,a}},\,2\,\text{ArcTan}\left[\frac{\sqrt{x}}{((2-a)\,a)^{1/4}}\right],\,\frac{1}{4}\left(2+\frac{1+2\,a-a^2}{\sqrt{(2-a)\,a}}\right)\right]\right)\Big/\left(((2-a)\,a)^{3/4}\,\sqrt{(2-a)\,a\,x-(1+2\,a-a^2)\,x^2+x^3}\right)$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{-a+(-1+2\,a)\,x}{(-a+x)\,\sqrt{a^2\,x-(-1+2\,a+a^2)\,x^2+(-1+2\,a)\,x^3}}\,dx$$

Optimal (type 3, 46 leaves, ? steps):

$$\text{Log}\left[\frac{-a^2+2\,a\,x+x^2-2\left(x+\sqrt{(1-x)\,x\,(a^2+x-2\,a\,x)}\right)}{(a-x)^2}\right]$$

Result (type 4, 180 leaves, 7 steps) :

$$-\frac{2\,(1-2\,a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{x}\right],\,-\frac{1-2\,a}{a^2}\right]}{\sqrt{a^2\,x+(1-2\,a-a^2)\,x^2-(1-2\,a)\,x^3}}+\frac{4\,(1-a)\,\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2\,a)\,x}{a^2}}\;\text{EllipticPi}\left[\frac{1}{a},\,\text{ArcSin}\left[\sqrt{x}\right],\,-\frac{1-2\,a}{a^2}\right]}{\sqrt{a^2\,x+(1-2\,a-a^2)\,x^2-(1-2\,a)\,x^3}}$$

Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+x}{\left(1-x+x^2\right)\left(1-x^3\right)^{1/3}}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 4 steps):

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$
$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$
$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Problem 93: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(1+x)^2}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 135 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{2^{1/3}}$$

Result (type 3, 409 leaves, 5 steps):

$$-\frac{\left(3-i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1-i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(3+i\sqrt{3}\right)\text{ArcTan}\left[\frac{2-\frac{2^{1/3}\left(1+i\sqrt{3}+2x\right)}{\left(1-x^3\right)^{1/3}}}{2\sqrt{3}}\right]}{2\times 2^{1/3}\left(i-\sqrt{3}\right)}+$$
$$\frac{\left(i-\sqrt{3}\right)\text{Log}\left[-\left(1-i\sqrt{3}-2x\right)^2\left(1-i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}+\frac{\left(i+\sqrt{3}\right)\text{Log}\left[-\left(1+i\sqrt{3}-2x\right)^2\left(1+i\sqrt{3}+2x\right)\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}-$$
$$\frac{3\left(i-\sqrt{3}\right)\text{Log}\left[1-i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i+\sqrt{3}\right)}-\frac{3\left(i+\sqrt{3}\right)\text{Log}\left[1+i\sqrt{3}+2x+2\times 2^{2/3}\left(1-x^3\right)^{1/3}\right]}{4\times 2^{1/3}\left(i-\sqrt{3}\right)}$$

Test results for the 14 integration problems in "Bronstein Problems.m"

Problem 6: Result valid but suboptimal antiderivative.

$$\int \frac{x}{\sqrt{-71-96x+10x^2+x^4}}\,dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\frac{1}{8}\text{Log}\left[10001+3124x^2-1408x^3+54x^4-128x^5+20x^6+x^8+\sqrt{-71-96x+10x^2+x^4}\left(-781+528x-27x^2+80x^3-15x^4-x^6\right)\right]$$

Result (type 3, 76 leaves, 1 step):

$$\frac{1}{8}\text{Log}\left[10001+3124x^2-1408x^3+54x^4-128x^5+20x^6+x^8+\sqrt{-71-96x+10x^2+x^4}\left(781-528x+27x^2-80x^3+15x^4+x^6\right)\right]$$

Problem 12: Unable to integrate problem.

$$\int \frac{x^2+2x\,\text{Log}[x]+\text{Log}[x]^2+(1+x)\sqrt{x+\text{Log}[x]}}{x^3+2x^2\,\text{Log}[x]+x\,\text{Log}[x]^2}\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\text{Log}[x]-\frac{2}{\sqrt{x+\text{Log}[x]}}$$

Result (type 8, 65 leaves, 3 steps):

$$\text{CannotIntegrate}\left[\frac{1}{(x+\text{Log}[x])^{3/2}},x\right]-\text{CannotIntegrate}\left[\frac{1}{\text{Log}[x]\ (x+\text{Log}[x])^{3/2}},x\right]-$$
$$\text{CannotIntegrate}\left[\frac{1}{\text{Log}[x]^2\sqrt{x+\text{Log}[x]}},x\right]+\text{CannotIntegrate}\left[\frac{\sqrt{x+\text{Log}[x]}}{x\,\text{Log}[x]^2},x\right]+\text{Log}[x]$$

Test results for the 35 integration problems in "Bondarenko Problems.m"

Problem 21: Result valid but suboptimal antiderivative.

$$\int\frac{1}{(\text{Cos}[x]+\text{Cos}[3x])^5}\,dx$$

Optimal (type 3, 108 leaves, ? steps):

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]+\frac{1483\,\text{ArcTanh}\left[\sqrt{2}\ \text{Sin}[x]\right]}{512\sqrt{2}}+\frac{\text{Sin}[x]}{32\left(1-2\,\text{Sin}[x]^2\right)^4}-$$
$$\frac{17\,\text{Sin}[x]}{192\left(1-2\,\text{Sin}[x]^2\right)^3}+\frac{203\,\text{Sin}[x]}{768\left(1-2\,\text{Sin}[x]^2\right)^2}-\frac{437\,\text{Sin}[x]}{512\left(1-2\,\text{Sin}[x]^2\right)}-\frac{43}{256}\text{Sec}[x]\ \text{Tan}[x]-\frac{1}{128}\text{Sec}[x]^3\,\text{Tan}[x]$$

Result (type 3, 786 leaves, 45 steps) :

$$-\frac{523}{256}\text{ArcTanh}[\text{Sin}[x]]-\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-$$
$$\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]-\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+\frac{1483\,\text{Log}\left[2-\sqrt{2}+\text{Cos}[x]-\sqrt{2}\,\text{Cos}[x]-\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}+$$
$$\frac{1483\,\text{Log}\left[2+\sqrt{2}+\text{Cos}[x]+\sqrt{2}\,\text{Cos}[x]+\text{Sin}[x]+\sqrt{2}\,\text{Sin}[x]\right]}{2048\sqrt{2}}-\frac{1}{128\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^4}+\frac{1}{64\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^3}-\frac{47}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)^2}+\frac{45}{256\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}+$$
$$\frac{1}{128\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^4}-\frac{1}{64\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^3}+\frac{47}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)^2}-\frac{45}{256\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}-\frac{7-17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{119\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-$$
$$\frac{11\left(1+3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{1-43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}-\frac{65\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{451\left(1+\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-$$
$$\frac{89+15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1-2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}+\frac{7+17\,\text{Tan}\left[\frac{x}{2}\right]}{4\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^4}+\frac{11\left(1-3\,\text{Tan}\left[\frac{x}{2}\right]\right)}{12\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}-\frac{119\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{48\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^3}+$$
$$\frac{65\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{384\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{1+43\,\text{Tan}\left[\frac{x}{2}\right]}{32\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)^2}+\frac{89-15\,\text{Tan}\left[\frac{x}{2}\right]}{64\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}-\frac{451\left(1-\text{Tan}\left[\frac{x}{2}\right]\right)}{512\left(1+2\,\text{Tan}\left[\frac{x}{2}\right]-\text{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 24: Result valid but suboptimal antiderivative.

$$\int\frac{\text{Tanh}[x]}{\sqrt{e^x+e^{2x}}}\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$2\,e^{-x}\sqrt{e^x+e^{2x}}-\frac{\text{ArcTan}\left[\frac{i-(1-2\,i)\,e^x}{2\sqrt{1+i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1+i}}+\frac{\text{ArcTan}\left[\frac{i+(1+2\,i)\,e^x}{2\sqrt{1-i}\,\sqrt{e^x+e^{2x}}}\right]}{\sqrt{1-i}}$$

Result (type 3, 147 leaves, 11 steps):

$$\frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}}-\frac{(1-i)^{3/2}\sqrt{e^x}\,\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1-i}\,\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}-\frac{(1+i)^{3/2}\sqrt{e^x}\,\sqrt{1+e^x}\,\text{ArcTanh}\left[\frac{\sqrt{1+i}\,\sqrt{e^x}}{\sqrt{1+e^x}}\right]}{\sqrt{e^x+e^{2x}}}$$

Problem 26: Result valid but suboptimal antiderivative.

$$\int \mathrm{Log}\left[x^2+\sqrt{1-x^2}\right] dx$$

Optimal (type 3, 185 leaves, ? steps):

$$-2x-\mathrm{ArcSin}[x]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+$$
$$\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+x\,\mathrm{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Result (type 3, 349 leaves, 31 steps):

$$-2x-\mathrm{ArcSin}[x]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+$$
$$2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\mathrm{ArcTan}\left[\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]+\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right]-$$
$$2\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]-\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\,\mathrm{ArcTanh}\left[\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right]+x\,\mathrm{Log}\left[x^2+\sqrt{1-x^2}\right]$$

Test results for the 1917 integration problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 369: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,x^m}{2\,(a+b\,x)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x}}\right)dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b x}}$$

Result (type 5, 92 leaves, 5 steps):

$$\frac{x^m \left(-\frac{b x}{a}\right)^{-m} \text{Hypergeometric2F1}\left[-\frac{1}{2},-m,\frac{1}{2},1+\frac{b x}{a}\right]}{\sqrt{a+b x}}-\frac{2 m x^m \left(-\frac{b x}{a}\right)^{-m} \sqrt{a+b x}\, \text{Hypergeometric2F1}\left[\frac{1}{2},1-m,\frac{3}{2},1+\frac{b x}{a}\right]}{a}$$

Test results for the 3189 integration problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 945: Result valid but suboptimal antiderivative.

$$\int (e x)^m (a-b x)^{2+n} (a+b x)^n \, dx$$

Optimal (type 5, 211 leaves, ? steps):

$$-\frac{(e x)^{1+m} (a-b x)^{1+n} (a+b x)^{1+n}}{e (3+m+2 n)}+\frac{2 a^2 (2+m+n) (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2 x^2}{a^2}\right]}{e (1+m) (3+m+2 n)}-$$
$$\frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)}$$

Result (type 5, 238 leaves, 11 steps):

$$\frac{a^2 (e x)^{1+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{1+m}{2},-n,\frac{3+m}{2},\frac{b^2 x^2}{a^2}\right]}{e (1+m)}-$$
$$\frac{2 a b (e x)^{2+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{2+m}{2},-n,\frac{4+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^2 (2+m)}+$$
$$\frac{b^2 (e x)^{3+m} (a-b x)^n (a+b x)^n \left(1-\frac{b^2 x^2}{a^2}\right)^{-n} \text{Hypergeometric2F1}\left[\frac{3+m}{2},-n,\frac{5+m}{2},\frac{b^2 x^2}{a^2}\right]}{e^3 (3+m)}$$

Test results for the 159 integration problems in "1.1.1.4 (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Test results for the 34 integration problems in "1.1.1.5 P(x) (a+b x)^m (c+d x)^n.m"

Test results for the 78 integration problems in "1.1.1.6 P(x) (a+b x)^m (c+d x)^n (e+f x)^p.m"

Test results for the 35 integration problems in "1.1.1.7 P(x) (a+b x)^m (c+d x)^n (e+f x)^p (g+h x)^q.m"

Test results for the 1071 integration problems in "1.1.2.2 (c x)^m (a+b x^2)^p.m"

Problem 662: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{a\,(2+m)\,x^{1+m}}{\sqrt{a+b\,x^2}}+\frac{b\,(3+m)\,x^{3+m}}{\sqrt{a+b\,x^2}}\right)dx$$

Optimal (type 3, 17 leaves, ? steps):

$$x^{2+m}\sqrt{a+b\,x^2}$$

Result (type 5, 127 leaves, 5 steps):

$$\frac{a\,x^{2+m}\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{b\,x^2}{a}\right]}{\sqrt{a+b\,x^2}}+\frac{b\,(3+m)\,x^{4+m}\sqrt{1+\frac{b\,x^2}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{4+m}{2},\frac{6+m}{2},-\frac{b\,x^2}{a}\right]}{(4+m)\sqrt{a+b\,x^2}}$$

Problem 664: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b\,x^{1+m}}{\left(a+b\,x^2\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^2}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^2}}$$

Result (type 5, 123 leaves, 5 steps):

$$\frac{x^m \sqrt{1+\frac{b x^2}{a}}\ \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{2},\frac{2+m}{2},-\frac{b x^2}{a}\right]}{\sqrt{a+b x^2}}-\frac{b x^{2+m} \sqrt{1+\frac{b x^2}{a}}\ \text{Hypergeometric2F1}\left[\frac{3}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{b x^2}{a}\right]}{a (2+m) \sqrt{a+b x^2}}$$

Test results for the 346 integration problems in "1.1.2.3 (a+b x^2)^p (c+d x^2)^q.m"

Problem 298: Result unnecessarily involves higher level functions.

$$\int \frac{\left(1-2 x^2\right)^m}{\sqrt{1-x^2}} \, dx$$

Optimal (type 5, 62 leaves, ? steps):

$$-\frac{2^{-2-m} \sqrt{x^2} \left(2-4 x^2\right)^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\left(1-2 x^2\right)^2\right]}{(1+m) x}$$

Result (type 6, 23 leaves, 1 step):

$$x \,\text{AppellF1}\left[\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2 x^2,x^2\right]$$

Test results for the 1156 integration problems in "1.1.2.4 (e x)^m (a+b x^2)^p (c+d x^2)^q.m"

Test results for the 115 integration problems in "1.1.2.5 (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 51 integration problems in "1.1.2.6 (g x)^m (a+b x^2)^p (c+d x^2)^q (e+f x^2)^r.m"

Test results for the 174 integration problems in "1.1.2.8 P(x) (c x)^m (a+b x^2)^p.m"

Test results for the 3071 integration problems in "1.1.3.2 (c x)^m (a+b x^n)^p.m"

Problem 2679: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\frac{b\,n\,x^{-1+m+n}}{2\left(a+b\,x^n\right)^{3/2}}+\frac{m\,x^{-1+m}}{\sqrt{a+b\,x^n}}\right)dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b\,x^n}{a}\right]}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n}\sqrt{1+\frac{b\,x^n}{a}}\;\text{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b\,x^n}{a}\right]}{2\,a\,(m+n)\sqrt{a+b\,x^n}}$$

Problem 2690: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(\frac{6\,a\,x^2}{b\,(4+m)\sqrt{a+b\,x^{-2+m}}}+\frac{x^m}{\sqrt{a+b\,x^{-2+m}}}\right)dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2 x^3 \sqrt{a+b x^{-2+m}}}{b (4+m)}$$

Result (type 5, 160 leaves, 5 steps):

$$\frac{2 a x^3 \sqrt{1+\frac{b x^{-2+m}}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{3}{2-m},-\frac{1+m}{2-m},-\frac{b x^{-2+m}}{a}\right]}{b (4+m) \sqrt{a+b x^{-2+m}}}+\frac{x^{1+m} \sqrt{1+\frac{b x^{-2+m}}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},-\frac{1+m}{2-m},\frac{1-2 m}{2-m},-\frac{b x^{-2+m}}{a}\right]}{(1+m) \sqrt{a+b x^{-2+m}}}$$

Problem 2692: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\frac{b n x^{-1+m+n}}{2 \left(a+b x^n\right)^{3/2}}+\frac{m x^{-1+m}}{\sqrt{a+b x^n}}\right) dx$$

Optimal (type 3, 15 leaves, ? steps):

$$\frac{x^m}{\sqrt{a+b x^n}}$$

Result (type 5, 126 leaves, 5 steps):

$$\frac{x^m \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{b x^n}{a}\right]}{\sqrt{a+b x^n}}-\frac{b n x^{m+n} \sqrt{1+\frac{b x^n}{a}} \text{Hypergeometric2F1}\left[\frac{3}{2},\frac{m+n}{n},2+\frac{m}{n},-\frac{b x^n}{a}\right]}{2 a (m+n) \sqrt{a+b x^n}}$$

Test results for the 286 integration problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Test results for the 913 integration problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 455: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2x\left(4c+dx^3\right)}{81cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]}{81\times 3^{1/4}cd^{7/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}}$$

Result (type 6, 66 leaves, 2 steps):

$$\frac{x^7\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left[\frac{7}{3},2,\frac{3}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right]}{448c^3\sqrt{c+dx^3}}$$

Test results for the 46 integration problems in "1.1.3.6 (g x)^m (a+b x^n)^p (c+d x^n)^q (e+f x^n)^r.m"

Test results for the 594 integration problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Test results for the 454 integration problems in "1.1.4.2 (c x)^m (a x^j+b x^n)^p.m"

Test results for the 298 integration problems in "1.1.4.3 (e x)^m (a x^j+b x^k)^p (c+d x^n)^q.m"

Test results for the 143 integration problems in "1.2.1.1 (a+b x+c x^2)^p.m"

Test results for the 2590 integration problems in "1.2.1.2 (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 2646 integration problems in "1.2.1.3 (d+e x)^m (f+g x) (a+b x+c x^2)^p.m"

Test results for the 958 integration problems in "1.2.1.4 (d+e x)^m (f+g x)^n (a+b x+c x^2)^p.m"

Problem 833: Result valid but suboptimal antiderivative.

$$\int\frac{\sqrt{-1+x}\,\sqrt{1+x}}{1+x-x^2}\,dx$$

Optimal (type 3, 91 leaves, ? steps):

$$-\text{ArcCosh}[x]+\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\frac{\sqrt{1+x}}{\sqrt{-2+\sqrt{5}}\ \sqrt{-1+x}}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\frac{\sqrt{1+x}}{\sqrt{2+\sqrt{5}}\ \sqrt{-1+x}}\right]$$

Result (type 3, 191 leaves, 9 steps):

$$\frac{\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTan}\left[\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(-1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-$$

$$\frac{\sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTanh}\left[\frac{x}{\sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}-\frac{\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \sqrt{-1+x}\ \sqrt{1+x}\ \text{ArcTanh}\left[\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\ \sqrt{-1+x^2}}\right]}{\sqrt{-1+x^2}}$$

Test results for the 123 integration problems in "1.2.1.5 (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 143 integration problems in "1.2.1.6 (g+h x)^m (a+b x+c x^2)^p (d+e x+f x^2)^q.m"

Test results for the 400 integration problems in "1.2.1.9 P(x) (d+e x)^m (a+b x+c x^2)^p.m"

Test results for the 1126 integration problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 integration problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Test results for the 413 integration problems in "1.2.2.4 (f x)^m (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Problem 374: Result valid but suboptimal antiderivative.

$$\int \frac{\left(d+e\,x^2\right)^{3/2}}{x^2\left(a+b\,x^2+c\,x^4\right)}\,dx$$

Optimal (type 3, 260 leaves, ? steps):

$$-\frac{d\sqrt{d+e x^2}}{a x}-\frac{\left(2 c d-\left(b-\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \left(b-\sqrt{b^2-4 a c}\right)^{3/2}}+\frac{\left(2 c d-\left(b+\sqrt{b^2-4 a c}\right) e\right)^{3/2} \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{\sqrt{b^2-4 a c}\, \left(b+\sqrt{b^2-4 a c}\right)^{3/2}}$$

Result (type 3, 432 leaves, 16 steps):

$$-\frac{d\sqrt{d+e x^2}}{a x}-\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, \left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b-\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b-\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{2 a \sqrt{b-\sqrt{b^2-4 a c}}}-$$

$$\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, \left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTan}\left[\frac{\sqrt{2 c d-\left(b+\sqrt{b^2-4 a c}\right) e}\, x}{\sqrt{b+\sqrt{b^2-4 a c}}\, \sqrt{d+e x^2}}\right]}{2 a \sqrt{b+\sqrt{b^2-4 a c}}}+\frac{d\sqrt{e}\, \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{a}-$$

$$\frac{\sqrt{e}\left(d-\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{2 a}-\frac{\sqrt{e}\left(d+\frac{b d-2 a e}{\sqrt{b^2-4 a c}}\right) \text{ArcTanh}\left[\frac{\sqrt{e}\, x}{\sqrt{d+e x^2}}\right]}{2 a}$$

Test results for the 111 integration problems in "1.2.2.5 P(x) (a+b x^2+c x^4)^p.m"

Test results for the 145 integration problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Test results for the 42 integration problems in "1.2.2.7 P(x) (d+e x^2)^q (a+b x^2+c x^4)^p.m"

Test results for the 4 integration problems in "1.2.2.8 P(x) (d+e x)^q (a+b x^2+c x^4)^p.m"

Test results for the 664 integration problems in "1.2.3.2 (d x)^m (a+b x^n+c x^(2 n))^p.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int x^8 \left(a^2+2 a b x^3+b^2 x^6\right)^{3/2} \, dx$$

Optimal (type 2, 119 leaves, ? steps):

$$\frac{a^2 \left(a+b x^3\right)^3 \sqrt{a^2+2 a b x^3+b^2 x^6}}{12 b^3}-\frac{2 a \left(a+b x^3\right)^4 \sqrt{a^2+2 a b x^3+b^2 x^6}}{15 b^3}+\frac{\left(a+b x^3\right)^5 \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 b^3}$$

Result (type 2, 167 leaves, 4 steps):

$$\frac{a^3 x^9 \sqrt{a^2+2 a b x^3+b^2 x^6}}{9 \left(a+b x^3\right)}+\frac{a^2 b x^{12} \sqrt{a^2+2 a b x^3+b^2 x^6}}{4 \left(a+b x^3\right)}+\frac{a b^2 x^{15} \sqrt{a^2+2 a b x^3+b^2 x^6}}{5 \left(a+b x^3\right)}+\frac{b^3 x^{18} \sqrt{a^2+2 a b x^3+b^2 x^6}}{18 \left(a+b x^3\right)}$$

Problem 478: Result unnecessarily involves higher level functions.

$$\int \left(\frac{\left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{x^2}-\frac{2 b^3 (1-2 p) (1-p) p \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{3 a^3 x}\right) dx$$

Optimal (type 3, 146 leaves, ? steps):

$$-\frac{\left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a x}+\frac{b (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^2 x^{2/3}}-\frac{b^2 (1-2 p) (1-p) \left(a+b x^{1/3}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p}{a^3 x^{1/3}}$$

Result (type 5, 162 leaves, 7 steps):

$$\frac{1}{a^3 (1+2p)} 2 b^3 (1-2p) (1-p) p \left(1+\frac{b x^{1/3}}{a}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p \text{Hypergeometric2F1}\left[1, 1+2p, 2 (1+p), 1+\frac{b x^{1/3}}{a}\right] +$$

$$\frac{3 b^3 \left(1+\frac{b x^{1/3}}{a}\right) \left(a^2+2 a b x^{1/3}+b^2 x^{2/3}\right)^p \text{Hypergeometric2F1}\left[4, 1+2p, 2 (1+p), 1+\frac{b x^{1/3}}{a}\right]}{a^3 (1+2p)}$$

Test results for the 96 integration problems in "1.2.3.3 (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 156 integration problems in "1.2.3.4 (f x)^m (d+e x^n)^q (a+b x^n+c x^(2 n))^p.m"

Test results for the 17 integration problems in "1.2.3.5 P(x) (d x)^m (a+b x^n+c x^(2 n))^p.m"

Test results for the 140 integration problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Test results for the 494 integration problems in "1.3.1 Rational functions.m"

Problem 174: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(b x^{1+p} \left(b x+c x^3\right)^p+2 c x^{3+p} \left(b x+c x^3\right)^p\right) dx$$

Optimal (type 3, 27 leaves, ? steps):

$$\frac{x^{1+p} \left(b x+c x^3\right)^{1+p}}{2 (1+p)}$$

Result (type 5, 116 leaves, 7 steps):

$$\frac{b x^{2+p} \left(1+\frac{c x^2}{b}\right)^{-p} \left(b x+c x^3\right)^p \text{Hypergeometric2F1}\left[-p, 1+p, 2+p, -\frac{c x^2}{b}\right]}{2 (1+p)}+\frac{c x^{4+p} \left(1+\frac{c x^2}{b}\right)^{-p} \left(b x+c x^3\right)^p \text{Hypergeometric2F1}\left[-p, 2+p, 3+p, -\frac{c x^2}{b}\right]}{2+p}$$

Problem 221: Result valid but suboptimal antiderivative.

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves, 3 steps):

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Problem 222: Result valid but suboptimal antiderivative.

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$

Optimal (type 1, 33 leaves, ? steps):

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

Result (type 1, 96 leaves, 2 steps):

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Problem 329: Result valid but suboptimal antiderivative.

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$$

Optimal (type 3, 31 leaves, ? steps):

$$\text{Log}[1-x]-\frac{1}{2}\text{Log}[3-x]+\frac{3}{2}\text{Log}[1+x]-2\,\text{Log}[3+x]$$

Result (type 3, 41 leaves, 11 steps):

$$-\frac{3}{2}\,\text{ArcTanh}\left[\frac{x}{3}\right]+\frac{\text{ArcTanh}[x]}{2}+\frac{5}{4}\,\text{Log}\left[1-x^2\right]-\frac{5}{4}\,\text{Log}\left[9-x^2\right]$$

Problem 393: Unable to integrate problem.

$$\int\frac{\left(1+x^2\right)^2}{a\,x^6+b\,\left(1+x^2\right)^3}\,dx$$

Optimal (type 3, 168 leaves, ? steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{a^{1/3}+b^{1/3}}\,x}{b^{1/6}}\right]}{3\,\sqrt{a^{1/3}+b^{1/3}}\,b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-(-1)^{1/3}\,a^{1/3}+b^{1/3}}\,x}{b^{1/6}}\right]}{3\,\sqrt{-(-1)^{1/3}\,a^{1/3}+b^{1/3}}\,b^{5/6}}+\frac{\text{ArcTan}\left[\frac{\sqrt{(-1)^{2/3}\,a^{1/3}+b^{1/3}}\,x}{b^{1/6}}\right]}{3\,\sqrt{(-1)^{2/3}\,a^{1/3}+b^{1/3}}\,b^{5/6}}$$

Result (type 8, 68 leaves, 5 steps):

$$\text{CannotIntegrate}\left[\frac{1}{a\,x^6+b\,\left(1+x^2\right)^3},\,x\right]+2\,\text{CannotIntegrate}\left[\frac{x^2}{a\,x^6+b\,\left(1+x^2\right)^3},\,x\right]+\text{CannotIntegrate}\left[\frac{x^4}{a\,x^6+b\,\left(1+x^2\right)^3},\,x\right]$$

Problem 493: Unable to integrate problem.

$$\int\left(\frac{3\,\left(-47+228\,x+120\,x^2+19\,x^3\right)}{\left(3+x+x^4\right)^4}+\frac{42-320\,x-75\,x^2-8\,x^3}{\left(3+x+x^4\right)^3}+\frac{30\,x}{\left(3+x+x^4\right)^2}\right)dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3\,x+5\,x^2+x^4-5\,x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 121 leaves, 7 steps):

$$-\frac{19}{4\,\left(3+x+x^4\right)^3}+\frac{1}{\left(3+x+x^4\right)^2}-\frac{621}{4}\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},\,x\right]+$$
$$684\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},\,x\right]+360\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},\,x\right]+44\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},\,x\right]-$$
$$320\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},\,x\right]-75\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^3},\,x\right]+30\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^2},\,x\right]$$

Problem 494: Unable to integrate problem.

$$\int \left(\frac{-3+10\,x+4\,x^3-30\,x^5}{\left(3+x+x^4\right)^3}-\frac{3\,\left(1+4\,x^3\right)\,\left(2-3\,x+5\,x^2+x^4-5\,x^6\right)}{\left(3+x+x^4\right)^4}\right)\,dx$$

Optimal (type 1, 27 leaves, ? steps):

$$\frac{2-3\,x+5\,x^2+x^4-5\,x^6}{\left(3+x+x^4\right)^3}$$

Result (type 8, 177 leaves, 13 steps):

$$\frac{7}{2\,\left(3+x+x^4\right)^3}-\frac{63\,x}{22\,\left(3+x+x^4\right)^3}-\frac{12\,x^2}{\left(3+x+x^4\right)^3}-\frac{5\,x^3}{\left(3+x+x^4\right)^3}+\frac{3\,x^4}{2\,\left(3+x+x^4\right)^3}-\frac{10\,x^6}{\left(3+x+x^4\right)^3}-\frac{1}{2\,\left(3+x+x^4\right)^2}+\frac{5\,x^2}{\left(3+x+x^4\right)^2}+\frac{144}{11}\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^4},\,x\right]+\frac{828}{11}\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^4},\,x\right]+18\,\text{CannotIntegrate}\left[\frac{x^2}{\left(3+x+x^4\right)^4},\,x\right]-4\,\text{CannotIntegrate}\left[\frac{1}{\left(3+x+x^4\right)^3},\,x\right]-20\,\text{CannotIntegrate}\left[\frac{x}{\left(3+x+x^4\right)^3},\,x\right]$$

Test results for the 886 integration problems in "1.3.2 Algebraic functions.m"

Problem 234: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\frac{\sqrt{a\,x^{2\,n}}}{\sqrt{1+x^n}}+\frac{2\,x^{-n}\,\sqrt{a\,x^{2\,n}}}{(2+n)\,\sqrt{1+x^n}}\right)\,dx$$

Optimal (type 3, 34 leaves, ? steps):

$$\frac{2\,x^{1-n}\,\sqrt{a\,x^{2\,n}}\,\sqrt{1+x^n}}{2+n}$$

Result (type 5, 80 leaves, 5 steps):

$$\frac{x\,\sqrt{a\,x^{2\,n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,1+\frac{1}{n},\,2+\frac{1}{n},\,-x^n\right]}{1+n}+\frac{2\,x^{1-n}\,\sqrt{a\,x^{2\,n}}\,\text{Hypergeometric2F1}\left[\frac{1}{2},\,\frac{1}{n},\,1+\frac{1}{n},\,-x^n\right]}{2+n}$$

Problem 454: Unable to integrate problem.

$$\int \frac{1}{x^2} (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n \left(-a d+(b d m+a e n) x+(c d+b e+a f+2 c d m+b e m+b e n+2 a f n) x^2+(2 c e+2 b f+2 a g+2 c e m+b f m+c e n+2 b f n+3 a g n) x^3+(3 c f+3 b g+2 c f m+b g m+2 c f n+3 b g n) x^4+c g (4+2 m+3 n) x^5\right) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a+b x+c x^2)^{1+m} (d+e x+f x^2+g x^3)^{1+n}}{x}$$

Result (type 8, 306 leaves, 2 steps):

$$(c (d+2 d m)+b e (1+m+n)+a f (1+2 n)) \text{CannotIntegrate}\left[(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]-a d \text{CannotIntegrate}\left[\frac{(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n}{x^2}, x\right]+(b d m+a e n) \text{CannotIntegrate}\left[\frac{(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n}{x}, x\right]+(c e (2+2 m+n)+b f (2+m+2 n)+a g (2+3 n)) \text{CannotIntegrate}\left[x (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]+(c f (3+2 m+2 n)+b g (3+m+3 n)) \text{CannotIntegrate}\left[x^2 (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]+c g (4+2 m+3 n) \text{CannotIntegrate}\left[x^3 (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]$$

Problem 455: Unable to integrate problem.

$$\int \frac{1}{x^3} (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n \left(-2 a d+(-b d-a e+b d m+a e n) x+(2 c d m+b e m+b e n+2 a f n) x^2+(c e+b f+a g+2 c e m+b f m+c e n+2 b f n+3 a g n) x^3+(2 c f+2 b g+2 c f m+b g m+2 c f n+3 b g n) x^4+c g (3+2 m+3 n) x^5\right) dx$$

Optimal (type 3, 37 leaves, ? steps):

$$\frac{(a+b x+c x^2)^{1+m} (d+e x+f x^2+g x^3)^{1+n}}{x^2}$$

Result (type 8, 305 leaves, 2 steps):

$$(c e (1+2 m+n)+b f (1+m+2 n)+a g (1+3 n)) \text{CannotIntegrate}\left[(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]-2 a d \text{CannotIntegrate}\left[\frac{(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n}{x^3}, x\right]-(b d (1-m)+a e (1-n)) \text{CannotIntegrate}\left[\frac{(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n}{x^2}, x\right]+(2 c d m+2 a f n+b e (m+n)) \text{CannotIntegrate}\left[\frac{(a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n}{x}, x\right]+(2 c f (1+m+n)+b g (2+m+3 n)) \text{CannotIntegrate}\left[x (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]+c g (3+2 m+3 n) \text{CannotIntegrate}\left[x^2 (a+b x+c x^2)^m (d+e x+f x^2+g x^3)^n, x\right]$$

Problem 798: Result unnecessarily involves higher level functions.

$$\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right) dx$$

Optimal (type 2, 35 leaves, ? steps):

$$-\frac{\left(1-x^6\right)^{2/3}}{5 x^5}+\frac{1}{5} x \left(1-x^6\right)^{2/3}$$

Result (type 5, 36 leaves, 3 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^6\right]}{5 x^5}+x\,\text{Hypergeometric2F1}\left[-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^6\right]$$

Problem 857: Unable to integrate problem.

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Optimal (type 3, 66 leaves, ? steps):

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}-\frac{3\,\text{ArcSin}\left[\sqrt{x}-\sqrt{1+x}\right]}{2\sqrt{2}}$$

Result (type 8, 31 leaves, 1 step):

$$\text{CannotIntegrate}\left[\frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}},x\right]$$

Problem 858: Result valid but suboptimal antiderivative.

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$$

Optimal (type 3, 78 leaves, ? steps):

$$-\sqrt{2\left(1+\sqrt{5}\right)}\;\text{ArcTan}\left[\sqrt{-2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]+\sqrt{2\left(-1+\sqrt{5}\right)}\;\text{ArcTanh}\left[\sqrt{2+\sqrt{5}}\,\left(x+\sqrt{1+x^2}\right)\right]$$

Result (type 3, 319 leaves, 25 steps):

$$-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-$$

$$\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\ \text{ArcTan}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ \sqrt{1+x^2}\right]-2\sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]+$$

$$\sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{-1+\sqrt{5}}}\ x\right]-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\ \text{ArcTanh}\left[\sqrt{\frac{2}{1+\sqrt{5}}}\ \sqrt{1+x^2}\right]$$

Problem 878: Unable to integrate problem.

$$\int\frac{1-x^2}{\left(1-x+x^2\right)\left(1-x^3\right)^{2/3}}\,dx$$

Optimal (type 3, 103 leaves, ? steps):

$$\frac{\sqrt{3}\ \text{ArcTan}\left[\frac{1-\frac{2\cdot 2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}}-\frac{\text{Log}\left[1+2\left(1-x\right)^3-x^3\right]}{2\times 2^{2/3}}+\frac{3\,\text{Log}\left[2^{1/3}\left(1-x\right)+\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 8, 103 leaves, 5 steps):

$$-\left(1+i\sqrt{3}\right)\ \text{CannotIntegrate}\left[\frac{1}{\left(-1-i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},\ x\right]-$$

$$\left(1-i\sqrt{3}\right)\ \text{CannotIntegrate}\left[\frac{1}{\left(-1+i\sqrt{3}+2x\right)\left(1-x^3\right)^{2/3}},\ x\right]-x\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right]$$

Problem 879: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2}{\sqrt{-1+x^4}\left(1+x^4\right)}\,dx$$

Optimal (type 3, 49 leaves, ? steps):

$$-\frac{1}{4}\text{ArcTan}\left[\frac{1+x^2}{x\sqrt{-1+x^4}}\right]-\frac{1}{4}\text{ArcTanh}\left[\frac{1-x^2}{x\sqrt{-1+x^4}}\right]$$

Result (type 3, 47 leaves, 9 steps):

$$\left(-\frac{1}{8}-\frac{i}{8}\right) \text{ArcTan}\left[\frac{(1+i)\,x}{\sqrt{-1+x^4}}\right]+\left(\frac{1}{8}+\frac{i}{8}\right) \text{ArcTanh}\left[\frac{(1+i)\,x}{\sqrt{-1+x^4}}\right]$$

Test results for the 98 integration problems in "2.1 u (F^(c (a+b x)))^n.m"

Test results for the 93 integration problems in "2.2 (c+d x)^m (F^(g (e+f x)))^n (a+b (F^(g (e+f x)))^n)^p.m"

Test results for the 774 integration problems in "2.3 Exponential functions.m"

Problem 692: Unable to integrate problem.

$$\int e^{x^x}\, x^{2x}\,(1+\text{Log}[x])\, dx$$

Optimal (type 3, 11 leaves, ? steps):

$$e^{x^x}\left(-1+x^x\right)$$

Result (type 8, 29 leaves, 2 steps):

$$\text{CannotIntegrate}\left[e^{x^x}\, x^{2x},\, x\right]+\text{CannotIntegrate}\left[e^{x^x}\, x^{2x}\,\text{Log}[x],\, x\right]$$

Problem 694: Unable to integrate problem.

$$\int x^{-2-\frac{1}{x}}\,(1-\text{Log}[x])\, dx$$

Optimal (type 3, 9 leaves, ? steps):

$$-x^{-1/x}$$

Result (type 8, 28 leaves, 2 steps):

$$\text{CannotIntegrate}\left[x^{-2-\frac{1}{x}},\, x\right]-\text{CannotIntegrate}\left[x^{-2-\frac{1}{x}}\,\text{Log}[x],\, x\right]$$

Test results for the 193 integration problems in "3.1.2 (d x)^m (a+b log(c x^n))^p.m"

Test results for the 456 integration problems in "3.1.4 (f x)^m (d+e x^r)^q (a+b log(c x^n))^p.m"

Test results for the 249 integration problems in "3.1.5 u (a+b log(c x^n))^p.m"

Test results for the 314 integration problems in "3.2.1 (f+g x)^m (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

Test results for the 263 integration problems in "3.2.2 (f+g x)^m (h+i x)^q (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

Test results for the 108 integration problems in "3.2.3 u log(e (f (a+b x)^p (c+d x)^q)^r)^s.m"

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}\left[e\left(f\,(a+b\,x)^{p}\,(c+d\,x)^{q}\right)^{r}\right]^{2}}{g+h\,x}\,dx$$

Optimal (type 4, 1471 leaves, ? steps):

$$\frac{p\,q\,r^2\,\text{Log}\left[-\frac{b\,c-a\,d}{d\,(a+b\,x)}\right]\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]^2}{h}+\frac{p^2\,r^2\,\text{Log}[a+b\,x]^2\,\text{Log}[g+h\,x]}{h}+\frac{2\,p\,q\,r^2\,\text{Log}[a+b\,x]\,\text{Log}[c+d\,x]\,\text{Log}[g+h\,x]}{h}+$$
$$\frac{q^2\,r^2\,\text{Log}[c+d\,x]^2\,\text{Log}[g+h\,x]}{h}-\frac{2\,p\,r\,\text{Log}[a+b\,x]\,\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\,\text{Log}[g+h\,x]}{h}-$$
$$\frac{2\,q\,r\,\text{Log}[c+d\,x]\,\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\,\text{Log}[g+h\,x]}{h}+\frac{\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]^2\,\text{Log}[g+h\,x]}{h}-$$
$$\frac{p^2\,r^2\,\text{Log}[a+b\,x]^2\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}-\frac{2\,p\,q\,r^2\,\text{Log}[a+b\,x]\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+\frac{p\,q\,r^2\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]^2\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}-$$
$$\frac{2\,p\,q\,r^2\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+\frac{p\,q\,r^2\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]^2\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+$$
$$\frac{2\,p\,r\,\text{Log}[a+b\,x]\,\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}-\frac{2\,p\,q\,r^2\,\text{Log}[a+b\,x]\,\text{Log}[c+d\,x]\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}-$$
$$\frac{q^2\,r^2\,\text{Log}[c+d\,x]^2\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{Log}[a+b\,x]\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}-\frac{p\,q\,r^2\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]^2\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}+$$
$$\frac{2\,p\,q\,r^2\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,q\,r\,\text{Log}[c+d\,x]\,\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}-$$
$$\frac{p\,q\,r^2\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]^2\,\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]}{h}-\frac{2\,p\,r\,\left(q\,r\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]-\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{PolyLog}\left[2,-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]}{h}+$$
$$\frac{2\,q\,r\,\left(p\,r\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]+\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{PolyLog}\left[2,-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{h}-$$
$$\frac{2\,p\,q\,r^2\,\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2,\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]}{h}-\frac{2\,p^2\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]}{h}-\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]}{h}-$$
$$\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]}{h}-\frac{2\,q^2\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]}{h}-\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,\frac{b\,(c+d\,x)}{d\,(a+b\,x)}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,\frac{(b\,g-a\,h)\,(c+d\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]}{h}$$

Result (type 4, 2096 leaves, 29 steps) :

$$-\frac{\text{Log}\left[(a+b\,x)^{p\,r}\right]^2\,\text{Log}[g+h\,x]}{h}-\frac{2\,p\,q\,r^2\,\text{Log}\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]\,\text{Log}[c+d\,x]\,\text{Log}[g+h\,x]}{h}-\frac{2\,p\,q\,r^2\,\text{Log}[a+b\,x]\,\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]\,\text{Log}[g+h\,x]}{h}+$$
$$\frac{2\,q\,r\,\left(p\,r\,\text{Log}[a+b\,x]-\text{Log}\left[(a+b\,x)^{p\,r}\right]\right)\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\text{Log}[g+h\,x]}{h}+\frac{2\,p\,r\,\text{Log}\left[-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]\,\left(q\,r\,\text{Log}[c+d\,x]-\text{Log}\left[(c+d\,x)^{q\,r}\right]\right)\,\text{Log}[g+h\,x]}{h}-$$
$$\frac{\text{Log}\left[(c+d\,x)^{q\,r}\right]^2\,\text{Log}[g+h\,x]}{h}+\frac{1}{h}2\,p\,r\,\text{Log}\left[-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]\,\left(\text{Log}\left[(a+b\,x)^{p\,r}\right]+\text{Log}\left[(c+d\,x)^{q\,r}\right]-\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{Log}[g+h\,x]+$$

$$\frac{1}{h}2\,q\,r\,\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\,\left(\text{Log}\left[(a+b\,x)^{p\,r}\right]+\text{Log}\left[(c+d\,x)^{q\,r}\right]-\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{Log}[g+h\,x]+$$

$$\frac{\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]^2\,\text{Log}[g+h\,x]}{h}+\frac{\text{Log}\left[(a+b\,x)^{p\,r}\right]^2\,\text{Log}\left[\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+\frac{\text{Log}\left[(c+d\,x)^{q\,r}\right]^2\,\text{Log}\left[\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}-$$

$$\frac{p\,q\,r^2\,\left(\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]+\text{Log}\left[\frac{b\,g-a\,h}{b\,(g+h\,x)}\right]-\text{Log}\left[\frac{(b\,g-a\,h)\,(c+d\,x)}{(b\,c-a\,d)\,(g+h\,x)}\right]\right)\,\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]^2}{h}+$$

$$\frac{p\,q\,r^2\,\left(\text{Log}\left[\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]-\text{Log}\left[-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]\right)\,\left(\text{Log}[a+b\,x]+\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\right)^2}{h}-$$

$$\frac{p\,q\,r^2\,\left(\text{Log}\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]+\text{Log}\left[\frac{d\,g-c\,h}{d\,(g+h\,x)}\right]-\text{Log}\left[-\frac{(d\,g-c\,h)\,(a+b\,x)}{(b\,c-a\,d)\,(g+h\,x)}\right]\right)\,\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]^2}{h}+$$

$$\frac{p\,q\,r^2\,\left(\text{Log}\left[-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]-\text{Log}\left[-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]\right)\,\left(\text{Log}[c+d\,x]+\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]\right)^2}{h}-\frac{2\,p\,q\,r^2\,\left(\text{Log}[g+h\,x]-\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\right)\,\text{PolyLog}\left[2,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{h}+$$

$$\frac{2\,p\,r\,\text{Log}\left[(a+b\,x)^{p\,r}\right]\,\text{PolyLog}\left[2,-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]}{h}-\frac{2\,p\,q\,r^2\,\left(\text{Log}[g+h\,x]-\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]\right)\,\text{PolyLog}\left[2,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{h}+$$

$$\frac{2\,q\,r\,\text{Log}\left[(c+d\,x)^{q\,r}\right]\,\text{PolyLog}\left[2,-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2,\frac{h\,(a+b\,x)}{b\,(g+h\,x)}\right]}{h}-$$

$$\frac{2\,p\,q\,r^2\,\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\,\text{PolyLog}\left[2,-\frac{(d\,g-c\,h)\,(a+b\,x)}{(b\,c-a\,d)\,(g+h\,x)}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]\,\text{PolyLog}\left[2,\frac{h\,(c+d\,x)}{d\,(g+h\,x)}\right]}{h}-$$

$$\frac{2\,p\,q\,r^2\,\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]\,\text{PolyLog}\left[2,\frac{(b\,g-a\,h)\,(c+d\,x)}{(b\,c-a\,d)\,(g+h\,x)}\right]}{h}+\frac{2\,p\,r\,\left(q\,r\,\text{Log}[c+d\,x]-\text{Log}\left[(c+d\,x)^{q\,r}\right]\right)\,\text{PolyLog}\left[2,\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+$$

$$\frac{2\,p\,r\,\left(\text{Log}\left[(a+b\,x)^{p\,r}\right]+\text{Log}\left[(c+d\,x)^{q\,r}\right]-\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{PolyLog}\left[2,\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}-$$

$$\frac{2\,p\,q\,r^2\,\left(\text{Log}[c+d\,x]+\text{Log}\left[\frac{(b\,c-a\,d)\,(g+h\,x)}{(b\,g-a\,h)\,(c+d\,x)}\right]\right)\,\text{PolyLog}\left[2,\frac{b\,(g+h\,x)}{b\,g-a\,h}\right]}{h}+\frac{2\,q\,r\,\left(p\,r\,\text{Log}[a+b\,x]-\text{Log}\left[(a+b\,x)^{p\,r}\right]\right)\,\text{PolyLog}\left[2,\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}+$$

$$\frac{2\,q\,r\,\left(\text{Log}\left[(a+b\,x)^{p\,r}\right]+\text{Log}\left[(c+d\,x)^{q\,r}\right]-\text{Log}\left[e\,\left(f\,(a+b\,x)^p\,(c+d\,x)^q\right)^r\right]\right)\,\text{PolyLog}\left[2,\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}-$$

$$\frac{2\,p\,q\,r^2\,\left(\text{Log}[a+b\,x]+\text{Log}\left[-\frac{(b\,c-a\,d)\,(g+h\,x)}{(d\,g-c\,h)\,(a+b\,x)}\right]\right)\,\text{PolyLog}\left[2,\frac{d\,(g+h\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,-\frac{d\,(a+b\,x)}{b\,c-a\,d}\right]}{h}-\frac{2\,p^2\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(a+b\,x)}{b\,g-a\,h}\right]}{h}+$$

$$\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,\frac{b\,(c+d\,x)}{b\,c-a\,d}\right]}{h}-\frac{2\,q^2\,r^2\,\text{PolyLog}\left[3,-\frac{h\,(c+d\,x)}{d\,g-c\,h}\right]}{h}+\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,\frac{h\,(a+b\,x)}{b\,(g+h\,x)}\right]}{h}-\frac{2\,p\,q\,r^2\,\text{PolyLog}\left[3,-\frac{(d\,g-c\,h)\,(a+b\,x)}{(b\,c-a\,d)\,(g+h\,x)}\right]}{h}+$$

$$\frac{2 p q r^2 \text{PolyLog}\left[3,\frac{h (c+d x)}{d (g+h x)}\right]}{h}-\frac{2 p q r^2 \text{PolyLog}\left[3,\frac{(b g-a h) (c+d x)}{(b c-a d) (g+h x)}\right]}{h}+\frac{2 p q r^2 \text{PolyLog}\left[3,\frac{b (g+h x)}{b g-a h}\right]}{h}+\frac{2 p q r^2 \text{PolyLog}\left[3,\frac{d (g+h x)}{d g-c h}\right]}{h}$$

Problem 74: Unable to integrate problem.

$$\int \left(\frac{1}{(c+d x) (-a+c+(-b+d) x) \text{Log}\left[\frac{a+b x}{c+d x}\right]}+\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(a+b x) (c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}\right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(b c-a d) \text{Log}\left[\frac{a+b x}{c+d x}\right]}$$

Result (type 8, 152 leaves, 3 steps):

$$\frac{b \text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(a+b x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2},x\right]}{b c-a d}-\frac{d \text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2},x\right]}{b c-a d}+\text{Unintegrable}\left[\frac{1}{(c+d x) (-a+c+(-b+d) x) \text{Log}\left[\frac{a+b x}{c+d x}\right]},x\right]$$

Problem 75: Unable to integrate problem.

$$\int \left(-\frac{1}{(a+b x) (a-c+(b-d) x) \text{Log}\left[\frac{a+b x}{c+d x}\right]}+\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(a+b x) (c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}\right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(b c-a d) \text{Log}\left[\frac{a+b x}{c+d x}\right]}$$

Result (type 8, 154 leaves, 3 steps):

$$\frac{b \text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(a+b x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2},x\right]}{b c-a d}-\frac{d \text{CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2},x\right]}{b c-a d}-\text{Unintegrable}\left[\frac{1}{(a+b x) (a-c+(b-d) x) \text{Log}\left[\frac{a+b x}{c+d x}\right]},x\right]$$

Test results for the 547 integration problems in "3.3 u (a+b log(c (d+e x)^n))^p.m"

Problem 370: Unable to integrate problem.

$$\int \frac{\text{Log}[f x^m] \left(a+b \text{Log}[c (d+e x)^n]\right)^2}{x} \, dx$$

Optimal (type 4, 823 leaves, ? steps):

$$\frac{1}{2} m \text{Log}[x]^2 \left(a-b n \text{Log}[d+e x]+b \text{Log}[c (d+e x)^n]\right)^2+\text{Log}[x] \left(-m \text{Log}[x]+\text{Log}[f x^m]\right) \left(a-b n \text{Log}[d+e x]+b \text{Log}[c (d+e x)^n]\right)^2+$$
$$2 b n \left(-m \text{Log}[x]+\text{Log}[f x^m]\right) \left(a-b n \text{Log}[d+e x]+b \text{Log}[c (d+e x)^n]\right) \left(\text{Log}[x] \left(\text{Log}[d+e x]-\text{Log}\left[1+\frac{e x}{d}\right]\right)-\text{PolyLog}\left[2,-\frac{e x}{d}\right]\right)+$$
$$2 b m n \left(a-b n \text{Log}[d+e x]+b \text{Log}[c (d+e x)^n]\right) \left(\frac{1}{2} \text{Log}[x]^2 \left(\text{Log}[d+e x]-\text{Log}\left[1+\frac{e x}{d}\right]\right)-\text{Log}[x] \text{PolyLog}\left[2,-\frac{e x}{d}\right]+\text{PolyLog}\left[3,-\frac{e x}{d}\right]\right)-$$
$$b^2 n^2 \left(m \text{Log}[x]-\text{Log}[f x^m]\right) \left(\text{Log}\left[-\frac{e x}{d}\right] \text{Log}[d+e x]^2+2 \text{Log}[d+e x] \text{PolyLog}\left[2,1+\frac{e x}{d}\right]-2 \text{PolyLog}\left[3,1+\frac{e x}{d}\right]\right)+$$
$$\frac{1}{12} b^2 m n^2 \left(\text{Log}\left[-\frac{e x}{d}\right]^4+6 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d+e x}\right]^2-4 \left(\text{Log}\left[-\frac{e x}{d}\right]+\text{Log}\left[\frac{d}{d+e x}\right]\right) \text{Log}\left[-\frac{e x}{d+e x}\right]^3+\right.$$
$$\text{Log}\left[-\frac{e x}{d+e x}\right]^4+6 \text{Log}[x]^2 \text{Log}[d+e x]^2+4 \left(2 \text{Log}\left[-\frac{e x}{d}\right]^3-3 \text{Log}[x]^2 \text{Log}[d+e x]\right) \text{Log}\left[1+\frac{e x}{d}\right]+$$
$$6 \left(\text{Log}[x]-\text{Log}\left[-\frac{e x}{d}\right]\right) \left(\text{Log}[x]+3 \text{Log}\left[-\frac{e x}{d}\right]\right) \text{Log}\left[1+\frac{e x}{d}\right]^2-4 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d+e x}\right] \left(\text{Log}\left[-\frac{e x}{d}\right]+3 \text{Log}\left[1+\frac{e x}{d}\right]\right)+$$
$$12 \left(\text{Log}\left[-\frac{e x}{d}\right]^2-2 \text{Log}\left[-\frac{e x}{d}\right] \left(\text{Log}\left[-\frac{e x}{d+e x}\right]+\text{Log}\left[1+\frac{e x}{d}\right]\right)+2 \text{Log}[x] \left(-\text{Log}[d+e x]+\text{Log}\left[1+\frac{e x}{d}\right]\right)\right) \text{PolyLog}\left[2,-\frac{e x}{d}\right]-$$
$$12 \text{Log}\left[-\frac{e x}{d+e x}\right]^2 \text{PolyLog}\left[2,\frac{e x}{d+e x}\right]+12 \left(\text{Log}\left[-\frac{e x}{d}\right]-\text{Log}\left[-\frac{e x}{d+e x}\right]\right)^2 \text{PolyLog}\left[2,1+\frac{e x}{d}\right]+24 \left(\text{Log}[x]-\text{Log}\left[-\frac{e x}{d}\right]\right)$$
$$\text{Log}\left[1+\frac{e x}{d}\right] \text{PolyLog}\left[2,1+\frac{e x}{d}\right]+24 \left(\text{Log}\left[-\frac{e x}{d+e x}\right]+\text{Log}[d+e x]\right) \text{PolyLog}\left[3,-\frac{e x}{d}\right]+24 \text{Log}\left[-\frac{e x}{d+e x}\right] \text{PolyLog}\left[3,\frac{e x}{d+e x}\right]+$$
$$\left.24 \left(-\text{Log}[x]+\text{Log}\left[-\frac{e x}{d+e x}\right]\right) \text{PolyLog}\left[3,1+\frac{e x}{d}\right]-24 \left(\text{PolyLog}\left[4,-\frac{e x}{d}\right]+\text{PolyLog}\left[4,\frac{e x}{d+e x}\right]-\text{PolyLog}\left[4,1+\frac{e x}{d}\right]\right)\right)$$

Result (type 8, 72 leaves, 1 step):

$$\frac{\text{Log}[f x^m]^2 \left(a+b \text{Log}[c (d+e x)^n]\right)^2}{2 m}-\frac{b e n \,\text{Unintegrable}\left[\frac{\text{Log}[f x^m]^2 \left(a+b \text{Log}[c (d+e x)^n]\right)}{d+e x},x\right]}{m}$$

Problem 374: Unable to integrate problem.

$$\int \frac{\text{Log}[x] \text{Log}[a+b x]^2}{x} \, dx$$

Optimal (type 4, 519 leaves, ? steps):

$$\frac{1}{12}\left(\text{Log}\left[-\frac{b x}{a}\right]^4+6 \text{Log}\left[-\frac{b x}{a}\right]^2 \text{Log}\left[-\frac{b x}{a+b x}\right]^2-4 \left(\text{Log}\left[-\frac{b x}{a}\right]+\text{Log}\left[\frac{a}{a+b x}\right]\right) \text{Log}\left[-\frac{b x}{a+b x}\right]^3+\right.$$
$$\text{Log}\left[-\frac{b x}{a+b x}\right]^4+6 \text{Log}[x]^2 \text{Log}[a+b x]^2+4 \left(2 \text{Log}\left[-\frac{b x}{a}\right]^3-3 \text{Log}[x]^2 \text{Log}[a+b x]\right) \text{Log}\left[1+\frac{b x}{a}\right]+$$
$$6 \left(\text{Log}[x]-\text{Log}\left[-\frac{b x}{a}\right]\right) \left(\text{Log}[x]+3 \text{Log}\left[-\frac{b x}{a}\right]\right) \text{Log}\left[1+\frac{b x}{a}\right]^2-4 \text{Log}\left[-\frac{b x}{a}\right]^2 \text{Log}\left[-\frac{b x}{a+b x}\right] \left(\text{Log}\left[-\frac{b x}{a}\right]+3 \text{Log}\left[1+\frac{b x}{a}\right]\right)+$$
$$12 \left(\text{Log}\left[-\frac{b x}{a}\right]^2-2 \text{Log}\left[-\frac{b x}{a}\right] \left(\text{Log}\left[-\frac{b x}{a+b x}\right]+\text{Log}\left[1+\frac{b x}{a}\right]\right)+2 \text{Log}[x] \left(-\text{Log}[a+b x]+\text{Log}\left[1+\frac{b x}{a}\right]\right)\right) \text{PolyLog}\left[2,-\frac{b x}{a}\right]-$$
$$12 \text{Log}\left[-\frac{b x}{a+b x}\right]^2 \text{PolyLog}\left[2,\frac{b x}{a+b x}\right]+12 \left(\text{Log}\left[-\frac{b x}{a}\right]-\text{Log}\left[-\frac{b x}{a+b x}\right]\right)^2 \text{PolyLog}\left[2,1+\frac{b x}{a}\right]+$$
$$24 \left(\text{Log}[x]-\text{Log}\left[-\frac{b x}{a}\right]\right) \text{Log}\left[1+\frac{b x}{a}\right] \text{PolyLog}\left[2,1+\frac{b x}{a}\right]+24 \left(\text{Log}\left[-\frac{b x}{a+b x}\right]+\text{Log}[a+b x]\right) \text{PolyLog}\left[3,-\frac{b x}{a}\right]+$$
$$24 \text{Log}\left[-\frac{b x}{a+b x}\right] \text{PolyLog}\left[3,\frac{b x}{a+b x}\right]+24 \left(-\text{Log}[x]+\text{Log}\left[-\frac{b x}{a+b x}\right]\right) \text{PolyLog}\left[3,1+\frac{b x}{a}\right]-$$
$$\left.24 \left(\text{PolyLog}\left[4,-\frac{b x}{a}\right]+\text{PolyLog}\left[4,\frac{b x}{a+b x}\right]-\text{PolyLog}\left[4,1+\frac{b x}{a}\right]\right)\right)$$

Result (type 8, 40 leaves, 1 step):

$$\frac{1}{2} \text{Log}[x]^2 \text{Log}[a+b x]^2-b \text{Unintegrable}\left[\frac{\text{Log}[x]^2 \text{Log}[a+b x]}{a+b x},x\right]$$

Test results for the 641 integration problems in "3.4 u (a+b log(c (d+e x^m)^n))^p.m"

Test results for the 314 integration problems in "3.5 Logarithm functions.m"

Test results for the 538 integration problems in "4.1.0 (a sin)^m (b trg)^n.m"

Test results for the 72 integration problems in "4.1.1.1 (a+b sin)^n.m"

Test results for the 653 integration problems in "4.1.1.2 (g cos)^p (a+b sin)^m.m"

Problem 648: Result valid but suboptimal antiderivative.

$$\int (e \text{Cos}[c+d x])^{-3-m} (a+b \text{Sin}[c+d x])^m \, dx$$

Optimal (type 5, 311 leaves, ? steps):

$$\frac{(e\,\mathrm{Cos}[c+d\,x])^{-m}\,\mathrm{Sec}[c+d\,x]^4\,(-1+\mathrm{Sin}[c+d\,x])\,(1+\mathrm{Sin}[c+d\,x])\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}}{(a-b)\,d\,e^3\,(2+m)}+\frac{1}{(a-b)^2\,d\,e^3\,m\,(2+m)}$$
$$(-2\,b+a\,(2+m))\,(e\,\mathrm{Cos}[c+d\,x])^{-m}\,\mathrm{Sec}[c+d\,x]^4\,(-1+\mathrm{Sin}[c+d\,x])\,(1+\mathrm{Sin}[c+d\,x])^2\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}-$$
$$\frac{1}{(a-b)^3\,d\,e^3\,m\,(1+m)}\left(-b^2+a^2\,(1+m)\right)\,(e\,\mathrm{Cos}[c+d\,x])^{-m}\,\mathrm{Hypergeometric2F1}\left[\frac{m}{2},\,1+m,\,2+m,\,-\frac{2\,(a+b\,\mathrm{Sin}[c+d\,x])}{(a-b)\,(-1+\mathrm{Sin}[c+d\,x])}\right]$$
$$\mathrm{Sec}[c+d\,x]^4\,(1+\mathrm{Sin}[c+d\,x])^3\,\left(\frac{(a+b)\,(1+\mathrm{Sin}[c+d\,x])}{(a-b)\,(-1+\mathrm{Sin}[c+d\,x])}\right)^{\frac{1}{2}(-2+m)}\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}$$

Result (type 5, 420 leaves, 5 steps):

$$-\frac{(e\,\mathrm{Cos}[c+d\,x])^{-2-m}\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}}{(a-b)\,d\,e\,(2+m)}-$$
$$\left(b\,(e\,\mathrm{Cos}[c+d\,x])^{-2-m}\,\mathrm{Hypergeometric2F1}\left[1+m,\,\frac{2+m}{2},\,2+m,\,\frac{2\,(a+b\,\mathrm{Sin}[c+d\,x])}{(a+b)\,(1+\mathrm{Sin}[c+d\,x])}\right]\,(1-\mathrm{Sin}[c+d\,x])\,\left(-\frac{(a-b)\,(1-\mathrm{Sin}[c+d\,x])}{(a+b)\,(1+\mathrm{Sin}[c+d\,x])}\right)^{m/2}\right.$$
$$\left.(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}\right)\Big/\left(\left(a^2-b^2\right)\,d\,e\,(1+m)\,(2+m)\right)+\frac{a\,(e\,\mathrm{Cos}[c+d\,x])^{-2-m}\,(1+\mathrm{Sin}[c+d\,x])\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}}{\left(a^2-b^2\right)\,d\,e\,(2+m)}+$$
$$\left(2^{-m/2}\,a\,(a+b+a\,m)\,(e\,\mathrm{Cos}[c+d\,x])^{-2-m}\,\mathrm{Hypergeometric2F1}\left[-\frac{m}{2},\,\frac{2+m}{2},\,\frac{2-m}{2},\,\frac{(a-b)\,(1-\mathrm{Sin}[c+d\,x])}{2\,(a+b\,\mathrm{Sin}[c+d\,x])}\right]\right.$$
$$\left.(1-\mathrm{Sin}[c+d\,x])\,\left(\frac{(a+b)\,(1+\mathrm{Sin}[c+d\,x])}{a+b\,\mathrm{Sin}[c+d\,x]}\right)^{\frac{2+m}{2}}\,(a+b\,\mathrm{Sin}[c+d\,x])^{1+m}\right)\Big/\left((a-b)\,(a+b)^2\,d\,e\,m\,(2+m)\right)$$

Test results for the 208 integration problems in "4.1.1.3 (g tan)^p (a+b sin)^m.m"

Test results for the 837 integration problems in "4.1.2.1 (a+b sin)^m (c+d sin)^n.m"

Test results for the 1563 integration problems in "4.1.2.2 (g cos)^p (a+b sin)^m (c+d sin)^n.m"

Problem 1480: Unable to integrate problem.

$$\int \frac{\mathrm{Sec}[e+f\,x]^4\,(a+b\,\mathrm{Sin}[e+f\,x])^{5/2}}{\sqrt{d\,\mathrm{Sin}[e+f\,x]}}\,dx$$

Optimal (type 4, 366 leaves, ? steps):

$$\frac{5\,a\,\mathrm{Sec}[e+f\,x]\,(b+a\,\mathrm{Sin}[e+f\,x])\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}{6\,f\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}}+\frac{\mathrm{Sec}[e+f\,x]^3\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,(a+b\,\mathrm{Sin}[e+f\,x])^{5/2}}{3\,d\,f}-$$

$$\frac{5\,a\,(a+b)^{3/2}\,\sqrt{-\frac{a\,(-1+\mathrm{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{a\,(1+\mathrm{Csc}[e+f\,x])}{a-b}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{d}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}}\right],-\frac{a+b}{a-b}\right]\,\mathrm{Tan}[e+f\,x]}{6\,\sqrt{d}\,f}-$$

$$\left(5\,a\,b\,(a+b)\,\sqrt{-\frac{a\,(-1+\mathrm{Csc}[e+f\,x])}{a+b}}\,\sqrt{\frac{b+a\,\mathrm{Csc}[e+f\,x]}{-a+b}}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\sqrt{-\frac{b+a\,\mathrm{Csc}[e+f\,x]}{a-b}}\right],\frac{-a+b}{a+b}\right]\,(1+\mathrm{Sin}[e+f\,x])\,\mathrm{Tan}[e+f\,x]\right)\Big/$$

$$\left(6\,f\,\sqrt{\frac{a\,(1+\mathrm{Csc}[e+f\,x])}{a-b}}\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}\right)$$

Result (type 8, 87 leaves, 1 step) :

$$\frac{\mathrm{Sec}[e+f\,x]^3\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,(a+b\,\mathrm{Sin}[e+f\,x])^{5/2}}{3\,d\,f}+\frac{5}{6}\,a\,\mathrm{Unintegrable}\left[\frac{\mathrm{Sec}[e+f\,x]^2\,(a+b\,\mathrm{Sin}[e+f\,x])^{3/2}}{\sqrt{d\,\mathrm{Sin}[e+f\,x]}},x\right]$$

Problem 1515: Unable to integrate problem.

$$\int\frac{\mathrm{Sec}[e+f\,x]^6\,(a+b\,\mathrm{Sin}[e+f\,x])^{9/2}}{\sqrt{d\,\mathrm{Sin}[e+f\,x]}}\,dx$$

Optimal (type 4, 502 leaves, ? steps):

$$-\frac{3\,a\,b\,\left(-2\,a^2+b^2\right)\,\mathrm{Cos}[e+f\,x]\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}{5\,f\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}}+$$

$$\frac{\mathrm{Sec}[e+f\,x]^5\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{9/2}}{5\,d\,f}-\frac{1}{20\,d\,f}3\,a\,\mathrm{Sec}[e+f\,x]^3\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}$$

$$\left(-a\,\left(7\,a^2+b^2\right)+2\,b\,\left(-7\,a^2+b^2\right)\,\mathrm{Sin}[e+f\,x]+5\,a\,\left(a^2-b^2\right)\,\mathrm{Sin}[e+f\,x]^2+\left(8\,a^2\,b-4\,b^3\right)\,\mathrm{Sin}[e+f\,x]^3\right)-\frac{1}{20\,\sqrt{d}\,f}3\,a\,\left(a+b\right)^{3/2}\,\left(5\,a^2+3\,a\,b-4\,b^2\right)$$

$$\sqrt{-\frac{a\,\left(-1+\mathrm{Csc}[e+f\,x]\right)}{a+b}}\,\sqrt{\frac{a\,\left(1+\mathrm{Csc}[e+f\,x]\right)}{a-b}}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{d}\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}{\sqrt{a+b}\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}}\right],-\frac{a+b}{a-b}\right]\,\mathrm{Tan}[e+f\,x]-$$

$$\frac{1}{5\,d\,f\,\sqrt{a+b\,\mathrm{Sin}[e+f\,x]}}3\,b\,\left(2\,a^4-3\,a^2\,b^2+b^4\right)\,\sqrt{-\frac{a\,\left(-1+\mathrm{Csc}[e+f\,x]\right)}{a+b}}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\sqrt{-\frac{b+a\,\mathrm{Csc}[e+f\,x]}{a-b}}\right],1-\frac{2\,a}{a+b}\right]$$

$$\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,\sqrt{-\frac{a\,\mathrm{Csc}[e+f\,x]^2\,\left(1+\mathrm{Sin}[e+f\,x]\right)\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)}{\left(a-b\right)^2}}\,\mathrm{Tan}[e+f\,x]$$

Result (type 8, 87 leaves, 1 step):

$$\frac{\mathrm{Sec}[e+f\,x]^5\,\sqrt{d\,\mathrm{Sin}[e+f\,x]}\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{9/2}}{5\,d\,f}+\frac{9}{10}\,a\,\mathrm{Unintegrable}\left[\frac{\mathrm{Sec}[e+f\,x]^4\,\left(a+b\,\mathrm{Sin}[e+f\,x]\right)^{7/2}}{\sqrt{d\,\mathrm{Sin}[e+f\,x]}},x\right]$$

Test results for the 51 integration problems in "4.1.2.3 (g sin)^p (a+b sin)^m (c+d sin)^n.m"

Test results for the 358 integration problems in "4.1.3.1 (a+b sin)^m (c+d sin)^n (A+B sin).m"

Test results for the 19 integration problems in "4.1.4.1 (a+b sin)^m (A+B sin+C sin^2).m"

Test results for the 34 integration problems in "4.1.4.2 (a+b sin)^m (c+d sin)^n (A+B sin+C sin^2).m"

Test results for the 594 integration problems in "4.1.7 (d trig)^m (a+b (c sin)^n)^p.m"

Test results for the 9 integration problems in "4.1.8 (a+b sin)^m (c+d trig)^n.m"

Test results for the 19 integration problems in "4.1.9 trig^m (a+b sin^n+c sin^(2 n))^p.m"

Test results for the 348 integration problems in "4.1.10 (c+d x)^m (a+b sin)^n.m"

Test results for the 113 integration problems in "4.1.11 (e x)^m (a+b x^n)^p sin.m"

Test results for the 357 integration problems in "4.1.12 (e x)^m (a+b sin(c+d x^n))^p.m"

Test results for the 36 integration problems in "4.1.13 (d+e x)^m sin(a+b x+c x^2)^n.m"

Test results for the 294 integration problems in "4.2.0 (a cos)^m (b trg)^n.m"

Test results for the 62 integration problems in "4.2.1.1 (a+b cos)^n.m"

Test results for the 88 integration problems in "4.2.1.2 (g sin)^p (a+b cos)^m.m"

Test results for the 22 integration problems in "4.2.1.3 (g tan)^p (a+b cos)^m.m"

Test results for the 932 integration problems in "4.2.2.1 (a+b cos)^m (c+d cos)^n.m"

Test results for the 4 integration problems in "4.2.2.2 (g sin)^p (a+b cos)^m (c+d cos)^n.m"

Test results for the 1 integration problems in "4.2.2.3 (g cos)^p (a+b cos)^m (c+d cos)^n.m"

Test results for the 644 integration problems in "4.2.3.1 (a+b cos)^m (c+d cos)^n (A+B cos).m"

Test results for the 393 integration problems in "4.2.4.1 (a+b cos)^m (A+B cos+C cos^2).m"

Test results for the 1541 integration problems in "4.2.4.2 (a+b cos)^m (c+d cos)^n (A+B cos+C cos^2).m"

Test results for the 98 integration problems in "4.2.7 (d trig)^m (a+b (c cos)^n)^p.m"

Test results for the 21 integration problems in "4.2.8 (a+b cos)^m (c+d trig)^n.m"

Test results for the 20 integration problems in "4.2.9 trig^m (a+b cos^n+c cos^(2 n))^p.m"

Test results for the 189 integration problems in "4.2.10 (c+d x)^m (a+b cos)^n.m"

Test results for the 99 integration problems in "4.2.12 (e x)^m (a+b cos(c+d x^n))^p.m"

Test results for the 34 integration problems in "4.2.13 (d+e x)^m cos(a+b x+c x^2)^n.m"

Test results for the 387 integration problems in "4.3.0 (a trg)^m (b tan)^n.m"

Test results for the 700 integration problems in "4.3.1.2 (d sec)^m (a+b tan)^n.m"

Test results for the 91 integration problems in "4.3.1.3 (d sin)^m (a+b tan)^n.m"

Test results for the 1328 integration problems in "4.3.2.1 (a+b tan)^m (c+d tan)^n.m"

Test results for the 855 integration problems in "4.3.3.1 (a+b tan)^m (c+d tan)^n (A+B tan).m"

Test results for the 171 integration problems in "4.3.4.2 (a+b tan)^m (c+d tan)^n (A+B tan+C tan^2).m"

Test results for the 499 integration problems in "4.3.7 (d trig)^m (a+b (c tan)^n)^p.m"

Test results for the 51 integration problems in "4.3.9 trig^m (a+b tan^n+c tan^(2 n))^p.m"

Test results for the 63 integration problems in "4.3.10 (c+d x)^m (a+b tan)^n.m"

Problem 17: Unable to integrate problem.

$$\int \left(\frac{x^2}{\sqrt{\text{Tan}\left[a+b\,x^2\right]}} + \frac{\sqrt{\text{Tan}\left[a+b\,x^2\right]}}{b} + x^2\,\text{Tan}\left[a+b\,x^2\right]^{3/2} \right) dx$$

Optimal (type 3, 17 leaves, ? steps):

$$\frac{x\,\sqrt{\text{Tan}\left[a+b\,x^2\right]}}{b}$$

Result (type 8, 55 leaves, 1 step) :

$$\text{Unintegrable}\left[\frac{x^2}{\sqrt{\text{Tan}\left[a+b\,x^2\right]}},\,x\right] + \frac{\text{Unintegrable}\left[\sqrt{\text{Tan}\left[a+b\,x^2\right]},\,x\right]}{b} + \text{Unintegrable}\left[x^2\,\text{Tan}\left[a+b\,x^2\right]^{3/2},\,x\right]$$

Test results for the 66 integration problems in "4.3.11 (e x)^m (a+b tan(c+d x^n))^p.m"

Test results for the 52 integration problems in "4.4.0 (a trg)^m (b cot)^n.m"

Test results for the 23 integration problems in "4.4.1.2 (d csc)^m (a+b cot)^n.m"

Test results for the 19 integration problems in "4.4.1.3 (d cos)^m (a+b cot)^n.m"

Test results for the 106 integration problems in "4.4.2.1 (a+b cot)^m (c+d cot)^n.m"

Test results for the 64 integration problems in "4.4.7 (d trig)^m (a+b (c cot)^n)^p.m"

Test results for the 32 integration problems in "4.4.9 trig^m (a+b cot^n+c cot^(2 n))^p.m"

Test results for the 61 integration problems in "4.4.10 (c+d x)^m (a+b cot)^n.m"

Test results for the 299 integration problems in "4.5.0 (a sec)^m (b trg)^n.m"

Test results for the 879 integration problems in "4.5.1.2 (d sec)^n (a+b sec)^m.m"

Problem 286: Result unnecessarily involves higher level functions.

$$\int \text{Sec}[c+d x]^{5/3} \left(a+a \text{Sec}[c+d x]\right)^{2/3} dx$$

Optimal (type 5, 327 leaves, ? steps):

$$-\frac{3\,a\,\mathrm{Sec}[c+d\,x]^{5/3}\,\mathrm{Sin}[c+d\,x]}{2\,d\,\left(a\,\left(1+\mathrm{Sec}[c+d\,x]\right)\right)^{1/3}}+\frac{9\,\mathrm{Sec}[c+d\,x]^{2/3}\,\left(a\,\left(1+\mathrm{Sec}[c+d\,x]\right)\right)^{2/3}\,\mathrm{Sin}[c+d\,x]}{4\,d}-\frac{9\,\left(a\,\left(1+\mathrm{Sec}[c+d\,x]\right)\right)^{2/3}\,\mathrm{Tan}[c+d\,x]}{4\,d\,\left(\frac{1}{1+\mathrm{Cos}[c+d\,x]}\right)^{1/3}\,\left(1+\mathrm{Sec}[c+d\,x]\right)^{7/3}}+$$
$$\left(\mathrm{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{3},\frac{5}{4},\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]^4\right]\,\left(\mathrm{Cos}[c+d\,x]\,\mathrm{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^4\right)^{1/3}\,\left(a\,\left(1+\mathrm{Sec}[c+d\,x]\right)\right)^{2/3}\,\mathrm{Tan}[c+d\,x]\right)\Big/$$
$$\left(8\,d\,\left(\frac{1}{1+\mathrm{Cos}[c+d\,x]}\right)^{1/3}\,\left(1+\mathrm{Sec}[c+d\,x]\right)^{4/3}\right)-$$
$$\left(5\,\mathrm{Hypergeometric2F1}\left[\frac{1}{3},\frac{3}{4},\frac{7}{4},\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]^4\right]\,\left(\mathrm{Cos}[c+d\,x]\,\mathrm{Sec}\left[\frac{1}{2}\,(c+d\,x)\right]^4\right)^{1/3}\,\left(a\,\left(1+\mathrm{Sec}[c+d\,x]\right)\right)^{2/3}\,\mathrm{Tan}[c+d\,x]^3\right)\Big/$$
$$\left(8\,d\,\left(\frac{1}{1+\mathrm{Cos}[c+d\,x]}\right)^{1/3}\,\left(1+\mathrm{Sec}[c+d\,x]\right)^{10/3}\right)$$

Result (type 6, 79 leaves, 3 steps) :

$$\frac{1}{d\,\left(1+\mathrm{Sec}[c+d\,x]\right)^{7/6}}2\times 2^{1/6}\,\mathrm{AppellF1}\left[\frac{1}{2},-\frac{2}{3},-\frac{1}{6},\frac{3}{2},1-\mathrm{Sec}[c+d\,x],\frac{1}{2}\,\left(1-\mathrm{Sec}[c+d\,x]\right)\right]\,\left(a+a\,\mathrm{Sec}[c+d\,x]\right)^{2/3}\,\mathrm{Tan}[c+d\,x]$$

Test results for the 306 integration problems in "4.5.1.3 (d sin)^n (a+b sec)^m.m"

Test results for the 365 integration problems in "4.5.1.4 (d tan)^n (a+b sec)^m.m"

Problem 207: Result valid but suboptimal antiderivative.

$$\int\frac{\mathrm{Tan}[e+f\,x]^2}{\left(a+a\,\mathrm{Sec}[e+f\,x]\right)^{9/2}}\,dx$$

Optimal (type 3, 177 leaves, ? steps):

$$-\frac{2\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\,\mathrm{Tan}[e+f\,x]}{\sqrt{a+a\,\mathrm{Sec}[e+f\,x]}}\right]}{a^{9/2}\,f}+\frac{91\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\,\mathrm{Tan}[e+f\,x]}{\sqrt{2}\,\sqrt{a+a\,\mathrm{Sec}[e+f\,x]}}\right]}{32\,\sqrt{2}\,a^{9/2}\,f}+$$
$$\frac{\mathrm{Tan}[e+f\,x]}{3\,a\,f\,\left(a+a\,\mathrm{Sec}[e+f\,x]\right)^{7/2}}+\frac{11\,\mathrm{Tan}[e+f\,x]}{24\,a^2\,f\,\left(a+a\,\mathrm{Sec}[e+f\,x]\right)^{5/2}}+\frac{27\,\mathrm{Tan}[e+f\,x]}{32\,a^3\,f\,\left(a+a\,\mathrm{Sec}[e+f\,x]\right)^{3/2}}$$

Result (type 3, 227 leaves, 7 steps) :

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Tan}[e+f\,x]}{\sqrt{a+a\,\text{Sec}[e+f\,x]}}\right]}{a^{9/2}\,f}+\frac{91\,\text{ArcTan}\left[\frac{\sqrt{a}\,\text{Tan}[e+f\,x]}{\sqrt{2}\,\sqrt{a+a\,\text{Sec}[e+f\,x]}}\right]}{32\,\sqrt{2}\,a^{9/2}\,f}+\frac{27\,\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^2\,\text{Sin}[e+f\,x]}{64\,a^4\,f\,\sqrt{a+a\,\text{Sec}[e+f\,x]}}+$$
$$\frac{11\,\text{Cos}[e+f\,x]\,\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^4\,\text{Sin}[e+f\,x]}{96\,a^4\,f\,\sqrt{a+a\,\text{Sec}[e+f\,x]}}+\frac{\text{Cos}[e+f\,x]^2\,\text{Sec}\left[\frac{1}{2}\,(e+f\,x)\right]^6\,\text{Sin}[e+f\,x]}{24\,a^4\,f\,\sqrt{a+a\,\text{Sec}[e+f\,x]}}$$

Test results for the 241 integration problems in "4.5.2.1 (a+b sec)^m (c+d sec)^n.m"

Test results for the 286 integration problems in "4.5.2.3 (g sec)^p (a+b sec)^m (c+d sec)^n.m"

Test results for the 634 integration problems in "4.5.3.1 (a+b sec)^m (d sec)^n (A+B sec).m"

Test results for the 70 integration problems in "4.5.4.1 (a+b sec)^m (A+B sec+C sec^2).m"

Test results for the 1373 integration problems in "4.5.4.2 (a+b sec)^m (d sec)^n (A+B sec+C sec^2).m"

Test results for the 471 integration problems in "4.5.7 (d trig)^m (a+b (c sec)^n)^p.m"

Test results for the 46 integration problems in "4.5.10 (c+d x)^m (a+b sec)^n.m"

Test results for the 83 integration problems in "4.5.11 (e x)^m (a+b sec(c+d x^n))^p.m"

Test results for the 70 integration problems in "4.6.0 (a csc)^m (b trg)^n.m"

Test results for the 59 integration problems in "4.6.1.2 (d csc)^n (a+b csc)^m.m"

Test results for the 16 integration problems in "4.6.1.3 (d cos)^n (a+b csc)^m.m"

Test results for the 23 integration problems in "4.6.1.4 (d cot)^n (a+b csc)^m.m"

Test results for the 24 integration problems in "4.6.3.1 (a+b csc)^m (d csc)^n (A+B csc).m"

Test results for the 1 integration problems in "4.6.4.2 (a+b csc)^m (d csc)^n (A+B csc+C csc^2).m"

Test results for the 27 integration problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.m"

Test results for the 84 integration problems in "4.6.11 (e x)^m (a+b csc(c+d x^n))^p.m"

Test results for the 254 integration problems in "4.7.1 (c trig)^m (d trig)^n.m"

Test results for the 294 integration problems in "4.7.2 trig^m (a trig+b trig)^n.m"

Problem 15: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^3}{(a \cos[x] + b \sin[x])^2} \, dx$$

Optimal (type 3, 107 leaves, ? steps):

$$\frac{6 a^2 b \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2}}+\frac{3 a \left(a^2-b^2\right)+a \left(a^2+b^2\right) \operatorname{Cos}[2 x]-b \left(a^2+b^2\right) \operatorname{Sin}[2 x]}{2 \left(a^2+b^2\right)^2 (a \operatorname{Cos}[x]+b \operatorname{Sin}[x])}$$

Result (type 3, 283 leaves, 19 steps):

$$-\frac{3 a^2 \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[x]-a \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{b \left(a^2+b^2\right)^{3/2}}-\frac{2 a^2 b \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2}}+\frac{2 a^2 \left(3 a^2+b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b \left(a^2+b^2\right)^{5/2}}-\frac{\operatorname{Cos}[x]}{b^2}+$$
$$\frac{3 a^2 \operatorname{Cos}[x]}{b^2 \left(a^2+b^2\right)}-\frac{2 a \operatorname{Sin}[x]}{b^3}+\frac{3 a^3 \operatorname{Sin}[x]}{b^3 \left(a^2+b^2\right)}-\frac{2 a^3 \operatorname{Cos}\left[\frac{x}{2}\right]^2 \left(2 a b+\left(a^2-b^2\right) \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{b^3 \left(a^2+b^2\right)^2}+\frac{2 a^2 \left(a+b \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{\left(a^2+b^2\right)^2 \left(a+2 b \operatorname{Tan}\left[\frac{x}{2}\right]-a \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 23: Result valid but suboptimal antiderivative.

$$\int \frac{\sin[x]^2}{(a \cos[x] + b \sin[x])^3} \, dx$$

Optimal (type 3, 92 leaves, ? steps):

$$-\frac{\left(a^2-2 b^2\right) \operatorname{ArcTanh}\left[\frac{-b+a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2}}+\frac{a \left(3 a b \operatorname{Cos}[x]+\left(a^2+4 b^2\right) \operatorname{Sin}[x]\right)}{2 \left(a^2+b^2\right)^2 (a \operatorname{Cos}[x]+b \operatorname{Sin}[x])^2}$$

Result (type 3, 300 leaves, 13 steps) :

$$\frac{2 a^2 \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[x]-a \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{b^2 \left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[x]-a \operatorname{Sin}[x]}{\sqrt{a^2+b^2}}\right]}{b^2 \sqrt{a^2+b^2}}-\frac{a^2 \left(2 a^2-b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b^2 \left(a^2+b^2\right)^{5/2}}+$$
$$\frac{2 a}{b \left(a^2+b^2\right) (a \operatorname{Cos}[x]+b \operatorname{Sin}[x])}+\frac{2 \left(a b+\left(a^2+2 b^2\right) \operatorname{Tan}\left[\frac{x}{2}\right]\right)}{a \left(a^2+b^2\right) \left(a+2 b \operatorname{Tan}\left[\frac{x}{2}\right]-a \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)^2}-\frac{4 a^4+3 a^2 b^2+2 b^4+a b \left(5 a^2+2 b^2\right) \operatorname{Tan}\left[\frac{x}{2}\right]}{a b \left(a^2+b^2\right)^2 \left(a+2 b \operatorname{Tan}\left[\frac{x}{2}\right]-a \operatorname{Tan}\left[\frac{x}{2}\right]^2\right)}$$

Problem 123: Result valid but suboptimal antiderivative.

$$\int \frac{\operatorname{Cos}[c+d x]^3}{(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^2} \, dx$$

Optimal (type 3, 138 leaves, ? steps):

$$-\frac{3 a b^2 \operatorname{ArcTanh}\left[\frac{b \operatorname{Cos}[c+d x]-a \operatorname{Sin}[c+d x]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2} d}+\frac{2 a b \operatorname{Cos}[c+d x]}{\left(a^2+b^2\right)^2 d}+\frac{\left(a^2-b^2\right) \operatorname{Sin}[c+d x]}{\left(a^2+b^2\right)^2 d}-\frac{b^3}{\left(a^2+b^2\right)^2 d (a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])}$$

Result (type 3, 231 leaves, 11 steps) :

$$\frac{2 b^4 \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a \left(a^2+b^2\right)^{5/2} d}-\frac{2 b^2 \left(3 a^2+b^2\right) \operatorname{ArcTanh}\left[\frac{b-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a \left(a^2+b^2\right)^{5/2} d}+$$
$$\frac{2 \left(2 a b+\left(a^2-b^2\right) \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{\left(a^2+b^2\right)^2 d \left(1+\operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}-\frac{2 b^3 \left(a+b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a \left(a^2+b^2\right)^2 d \left(a+2 b \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]-a \operatorname{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}$$

Problem 131: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{\operatorname{Cos}[c+d x]^4}{(a \operatorname{Cos}[c+d x]+b \operatorname{Sin}[c+d x])^3} \, dx$$

Optimal (type 3, 211 leaves, ? steps):

$$\frac{1}{2d}\left(-\frac{6 b^2 \left(-4 a^2+b^2\right) \text{ArcTanh}\left[\frac{-b+a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{7/2}}-\frac{2 b \left(-3 a^2+b^2\right) \text{Cos}[c+d x]}{\left(a^2+b^2\right)^3}+\frac{2 a \left(a^2-3 b^2\right) \text{Sin}[c+d x]}{\left(a^2+b^2\right)^3}+\frac{b^4 \text{Sin}[c+d x]}{a (a-i b)^2 (a+i b)^2 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}-\frac{b^3 \left(8 a^2+b^2\right)}{a \left(a^2+b^2\right)^3 (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])}\right)$$

Result (type 3, 492 leaves, 15 steps) :

$$-\frac{3 b^4 \left(a^2+2 b^2\right) \text{ArcTanh}\left[\frac{b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 \left(a^2+b^2\right)^{7/2} d}+\frac{4 b^4 \left(3 a^2+2 b^2\right) \text{ArcTanh}\left[\frac{b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 \left(a^2+b^2\right)^{7/2} d}-\frac{2 b^2 \left(6 a^4+3 a^2 b^2+b^4\right) \text{ArcTanh}\left[\frac{b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{a^2 \left(a^2+b^2\right)^{7/2} d}+\frac{2 \left(b \left(3 a^2-b^2\right)+a \left(a^2-3 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{\left(a^2+b^2\right)^3 d \left(1+\text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}+\frac{2 b^4 \left(a b+\left(a^2+2 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^3 \left(a^2+b^2\right)^2 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)^2}-\frac{3 b^4 \left(a^2+2 b^2\right) \left(b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^3 \left(a^2+b^2\right)^3 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}-\frac{4 b^3 \left(2 a^4-b^4+a b \left(3 a^2+2 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^3 \left(a^2+b^2\right)^3 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}$$

Problem 133: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Cos}[c+d x]^2}{(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3} \, dx$$

Optimal (type 3, 119 leaves, ? steps):

$$\frac{\left(2 a^2-b^2\right) \text{ArcTanh}\left[\frac{-b+a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2} d}-\frac{b \left(\left(4 a^2+b^2\right) \text{Cos}[c+d x]+3 a b \text{Sin}[c+d x]\right)}{2 \left(a^2+b^2\right)^2 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^2}$$

Result (type 3, 225 leaves, 6 steps) :

$$-\frac{\left(2 a^2-b^2\right) \text{ArcTanh}\left[\frac{b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{5/2} d}+\frac{2 b^2 \left(a b+\left(a^2+2 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^3 \left(a^2+b^2\right) d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)^2}-\frac{b \left(4 a^4+3 a^2 b^2+2 b^4+a b \left(5 a^2+2 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^3 \left(a^2+b^2\right)^2 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}$$

Problem 142: Rubi result verified and simpler than optimal antiderivative.

$$\int \frac{\text{Cos}[c+d x]^3}{(a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^4} \, dx$$

Optimal (type 3, 166 leaves, ? steps):

$$\frac{a \left(2 a^2-3 b^2\right) \text{ArcTanh}\left[\frac{-b+a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{7/2} d}+\frac{-3 \left(3 a^4 b-a^2 b^3+b^5\right) \text{Cos}[2 (c+d x)]+\frac{1}{2} b \left(-9 a^2+b^2\right) \left(2 \left(a^2+b^2\right)+3 a b \text{Sin}[2 (c+d x)]\right)}{6 (a-i b)^3 (a+i b)^3 d (a \text{Cos}[c+d x]+b \text{Sin}[c+d x])^3}$$

Result (type 3, 362 leaves, 7 steps):

$$-\frac{a \left(2 a^2-3 b^2\right) \text{ArcTanh}\left[\frac{b-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]}{\sqrt{a^2+b^2}}\right]}{\left(a^2+b^2\right)^{7/2} d}-\frac{8 b^3 \left(a \left(a^2+2 b^2\right)+b \left(3 a^2+4 b^2\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{3 a^5 \left(a^2+b^2\right) d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)^3}+\frac{2 b^2 \left(b \left(15 a^4+18 a^2 b^2+8 b^4\right)+a \left(9 a^4+30 a^2 b^2+16 b^4\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{3 a^5 \left(a^2+b^2\right)^2 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)^2}-\frac{b \left(6 a^6+9 a^4 b^2+12 a^2 b^4+4 b^6+a b \left(9 a^4+6 a^2 b^2+2 b^4\right) \text{Tan}\left[\frac{1}{2} (c+d x)\right]\right)}{a^4 \left(a^2+b^2\right)^3 d \left(a+2 b \text{Tan}\left[\frac{1}{2} (c+d x)\right]-a \text{Tan}\left[\frac{1}{2} (c+d x)\right]^2\right)}$$

Test results for the 397 integration problems in "4.7.3 (c+d x)^m trig^n trig^p.m"

Test results for the 9 integration problems in "4.7.4 x^m (a+b trig^n)^p.m"

Test results for the 250 integration problems in "4.7.5 x^m trig(a+b log(c x^n))^p.m"

Problem 179: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\left(1+b^2 n^2\right) \operatorname{Sec}\left[a+b \operatorname{Log}\left[c x^n\right]\right]+2 b^2 n^2 \operatorname{Sec}\left[a+b \operatorname{Log}\left[c x^n\right]\right]^3\right) dx$$

Optimal (type 3, 41 leaves, ? steps):

$$-x \operatorname{Sec}\left[a+b \operatorname{Log}\left[c x^n\right]\right]+b n x \operatorname{Sec}\left[a+b \operatorname{Log}\left[c x^n\right]\right] \operatorname{Tan}\left[a+b \operatorname{Log}\left[c x^n\right]\right]$$

Result (type 5, 175 leaves, 7 steps):

$$-2 e^{i a} (1-i b n) x \left(c x^n\right)^{i b} \text{Hypergeometric2F1}\left[1, \frac{1}{2}\left(1-\frac{i}{b n}\right), \frac{1}{2}\left(3-\frac{i}{b n}\right), -e^{2 i a} \left(c x^n\right)^{2 i b}\right]+\frac{16 b^2 e^{3 i a} n^2 x \left(c x^n\right)^{3 i b} \text{Hypergeometric2F1}\left[3, \frac{1}{2}\left(3-\frac{i}{b n}\right), \frac{1}{2}\left(5-\frac{i}{b n}\right), -e^{2 i a} \left(c x^n\right)^{2 i b}\right]}{1+3 i b n}$$

Problem 180: Result unnecessarily involves higher level functions.

$$\int x^m \operatorname{Sec}\left[a+2 \operatorname{Log}\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]^3 dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m} \operatorname{Sec}\left[a+2 \operatorname{Log}\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]}{2 (1+m)}+\frac{x^{1+m} \operatorname{Sec}\left[a+2 \operatorname{Log}\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right] \operatorname{Tan}\left[a+2 \operatorname{Log}\left[c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right]\right]}{2 \sqrt{-(1+m)^2}}$$

Result (type 5, 146 leaves, 3 steps):

$$\left(8 e^{3 i a} x^{1+m} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^{6 i} \text{Hypergeometric2F1}\left[3, \frac{1}{2}\left(3-\frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), \frac{1}{2}\left(5-\frac{i (1+m)}{\sqrt{-(1+m)^2}}\right), -e^{2 i a} \left(c x^{\frac{1}{2} \sqrt{-(1+m)^2}}\right)^{4 i}\right]\right) \Big/ \left(1-i \left(i m-3 \sqrt{-(1+m)^2}\right)\right)$$

Problem 221: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(-\left(1+b^2 n^2\right) \operatorname{Csc}\left[a+b \operatorname{Log}\left[c x^n\right]\right]+2 b^2 n^2 \operatorname{Csc}\left[a+b \operatorname{Log}\left[c x^n\right]\right]^3\right) dx$$

Optimal (type 3, 42 leaves, ? steps):

$-x\,\mathrm{Csc}[a+b\,\mathrm{Log}[c\,x^n]]-b\,n\,x\,\mathrm{Cot}[a+b\,\mathrm{Log}[c\,x^n]]\,\mathrm{Csc}[a+b\,\mathrm{Log}[c\,x^n]]$

Result (type 5, 172 leaves, 7 steps):

$$2\,e^{i\,a}\,(i+b\,n)\,x\,(c\,x^n)^{i\,b}\,\mathrm{Hypergeometric2F1}\left[1,\frac{1}{2}\left(1-\frac{i}{b\,n}\right),\frac{1}{2}\left(3-\frac{i}{b\,n}\right),e^{2\,i\,a}\,(c\,x^n)^{2\,i\,b}\right]-\frac{16\,b^2\,e^{3\,i\,a}\,n^2\,x\,(c\,x^n)^{3\,i\,b}\,\mathrm{Hypergeometric2F1}\left[3,\frac{1}{2}\left(3-\frac{i}{b\,n}\right),\frac{1}{2}\left(5-\frac{i}{b\,n}\right),e^{2\,i\,a}\,(c\,x^n)^{2\,i\,b}\right]}{i-3\,b\,n}$$

Problem 222: Result unnecessarily involves higher level functions.

$$\int x^m\,\mathrm{Csc}\left[a+2\,\mathrm{Log}\left[c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right]\right]^3\,dx$$

Optimal (type 3, 110 leaves, ? steps):

$$\frac{x^{1+m}\,\mathrm{Csc}\left[a+2\,\mathrm{Log}\left[c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right]\right]}{2\,(1+m)}-\frac{x^{1+m}\,\mathrm{Cot}\left[a+2\,\mathrm{Log}\left[c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right]\right]\,\mathrm{Csc}\left[a+2\,\mathrm{Log}\left[c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right]\right]}{2\,\sqrt{-(1+m)^2}}$$

Result (type 5, 142 leaves, 3 steps):

$$-\frac{1}{i+i\,m-3\,\sqrt{-(1+m)^2}}8\,e^{3\,i\,a}\,x^{1+m}\left(c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{6\,i}\,\mathrm{Hypergeometric2F1}\left[3,\frac{1}{2}\left(3-\frac{i\,(1+m)}{\sqrt{-(1+m)^2}}\right),\frac{1}{2}\left(5-\frac{i\,(1+m)}{\sqrt{-(1+m)^2}}\right),e^{2\,i\,a}\left(c\,x^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)^{4\,i}\right]$$

Test results for the 142 integration problems in "4.7.6 f^(a+b x+c x^2) trig(d+e x+f x^2)^n.m"

Problem 28: Unable to integrate problem.

$$\int F^{c\,(a+b\,x)}\,(f\,x)^m\,\mathrm{Sin}[d+e\,x]\,dx$$

Optimal (type 4, 139 leaves, ? steps):

$$-\frac{e^{-i\,d}\,F^{a\,c}\,(f\,x)^m\,\mathrm{Gamma}[1+m,x\,(i\,e-b\,c\,\mathrm{Log}[F])]\,(x\,(i\,e-b\,c\,\mathrm{Log}[F]))^{-m}}{2\,(e+i\,b\,c\,\mathrm{Log}[F])}-\frac{e^{i\,d}\,F^{a\,c}\,(f\,x)^m\,\mathrm{Gamma}[1+m,-x\,(i\,e+b\,c\,\mathrm{Log}[F])]\,(-x\,(i\,e+b\,c\,\mathrm{Log}[F]))^{-m}}{2\,(e-i\,b\,c\,\mathrm{Log}[F])}$$

Result (type 8, 24 leaves, 1 step):

$\text{CannotIntegrate}\left[F^{a\,c+b\,c\,x}\,(f\,x)^m\,\text{Sin}[d+e\,x],\,x\right]$

Problem 32: Unable to integrate problem.

$$\int f\,F^{c\,(a+b\,x)}\,(f\,x)^m\,\left(e\,x\,\text{Cos}[d+e\,x]+(1+m+b\,c\,x\,\text{Log}[F])\,\text{Sin}[d+e\,x]\right)\,dx$$

Optimal (type 3, 23 leaves, ? steps):

$f\,F^{c\,(a+b\,x)}\,x\,(f\,x)^m\,\text{Sin}[d+e\,x]$

Result (type 8, 89 leaves, 6 steps):

$e\,\text{CannotIntegrate}\left[F^{a\,c+b\,c\,x}\,(f\,x)^{1+m}\,\text{Cos}[d+e\,x],\,x\right]+$
$f\,(1+m)\,\text{CannotIntegrate}\left[F^{a\,c+b\,c\,x}\,(f\,x)^m\,\text{Sin}[d+e\,x],\,x\right]+b\,c\,\text{CannotIntegrate}\left[F^{a\,c+b\,c\,x}\,(f\,x)^{1+m}\,\text{Sin}[d+e\,x],\,x\right]\,\text{Log}[F]$

Test results for the 950 integration problems in "4.7.7 Trig functions.m"

Problem 759: Result valid but suboptimal antiderivative.

$$\int \left(\text{Cos}[x]^{12}\,\text{Sin}[x]^{10}-\text{Cos}[x]^{10}\,\text{Sin}[x]^{12}\right)\,dx$$

Optimal (type 3, 12 leaves, ? steps):

$\frac{1}{11}\,\text{Cos}[x]^{11}\,\text{Sin}[x]^{11}$

Result (type 3, 129 leaves, 25 steps):

$\frac{3\,\text{Cos}[x]^{11}\,\text{Sin}[x]}{5632}-\frac{3\,\text{Cos}[x]^{13}\,\text{Sin}[x]}{5632}+\frac{1}{512}\,\text{Cos}[x]^{11}\,\text{Sin}[x]^3-\frac{7\,\text{Cos}[x]^{13}\,\text{Sin}[x]^3}{2816}+\frac{7\,\text{Cos}[x]^{11}\,\text{Sin}[x]^5}{1280}-\frac{7}{880}\,\text{Cos}[x]^{13}\,\text{Sin}[x]^5+$
$\frac{1}{80}\,\text{Cos}[x]^{11}\,\text{Sin}[x]^7-\frac{9}{440}\,\text{Cos}[x]^{13}\,\text{Sin}[x]^7+\frac{1}{40}\,\text{Cos}[x]^{11}\,\text{Sin}[x]^9-\frac{1}{22}\,\text{Cos}[x]^{13}\,\text{Sin}[x]^9+\frac{1}{22}\,\text{Cos}[x]^{11}\,\text{Sin}[x]^{11}$

Problem 796: Unable to integrate problem.

$$\int e^{\text{Sin}[x]}\,\text{Sec}[x]^2\,\left(x\,\text{Cos}[x]^3-\text{Sin}[x]\right)\,dx$$

Optimal (type 3, 13 leaves, ? steps):

$e^{\text{Sin}[x]}\,(-1+x\,\text{Cos}[x])\,\text{Sec}[x]$

Result (type 8, 24 leaves, 2 steps):

$\text{CannotIntegrate}\left[e^{\text{Sin}[x]}\, x\, \text{Cos}[x], x\right] - \text{CannotIntegrate}\left[e^{\text{Sin}[x]}\, \text{Sec}[x]\, \text{Tan}[x], x\right]$

Problem 858: Result valid but suboptimal antiderivative.

$$\int \frac{1}{\text{Cos}[x]^{3/2} \sqrt{3\,\text{Cos}[x] + \text{Sin}[x]}} \, dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{2\sqrt{3\,\text{Cos}[x] + \text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}$$

Result (type 3, 88 leaves, 5 steps):

$$\frac{2\,\text{Cos}\left[\frac{x}{2}\right]^2 \left(3 + 2\,\text{Tan}\left[\frac{x}{2}\right] - 3\,\text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{\text{Cos}\left[\frac{x}{2}\right]^2 \left(3 + 2\,\text{Tan}\left[\frac{x}{2}\right] - 3\,\text{Tan}\left[\frac{x}{2}\right]^2\right)} \sqrt{\text{Cos}\left[\frac{x}{2}\right]^2 \left(1 - \text{Tan}\left[\frac{x}{2}\right]^2\right)}}$$

Problem 860: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Cos}[x] + \text{Sin}[x]}{\sqrt{1 + \text{Sin}[2\,x]}} \, dx$$

Optimal (type 3, 19 leaves, ? steps):

$$\frac{x\sqrt{1 + \text{Sin}[2\,x]}}{\text{Cos}[x] + \text{Sin}[x]}$$

Result (type 3, 72 leaves, 17 steps):

$$\frac{2\,\text{ArcTan}\left[\text{Tan}\left[\frac{x}{2}\right]\right] \text{Cos}\left[\frac{x}{2}\right]^2 \left(1 + 2\,\text{Tan}\left[\frac{x}{2}\right] - \text{Tan}\left[\frac{x}{2}\right]^2\right)}{\sqrt{\text{Cos}\left[\frac{x}{2}\right]^4 \left(1 + 2\,\text{Tan}\left[\frac{x}{2}\right] - \text{Tan}\left[\frac{x}{2}\right]^2\right)^2}}$$

Problem 912: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Cos}[x] + \text{Sin}[x]}{\sqrt{\text{Cos}[x]} \sqrt{\text{Sin}[x]}} \, dx$$

Optimal (type 3, 57 leaves, ? steps):

$$-\sqrt{2}\,\text{ArcTan}\left[1-\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}\right]+\sqrt{2}\,\text{ArcTan}\left[1+\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}\right]$$

Result (type 3, 243 leaves, 22 steps):

$$\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\,\sqrt{\text{Cos}[x]}}{\sqrt{\text{Sin}[x]}}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\,\sqrt{\text{Cos}[x]}}{\sqrt{\text{Sin}[x]}}\right]}{\sqrt{2}}-\frac{\text{ArcTan}\left[1-\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[1+\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}\right]}{\sqrt{2}}-\frac{\text{Log}\left[1+\text{Cot}[x]-\frac{\sqrt{2}\,\sqrt{\text{Cos}[x]}}{\sqrt{\text{Sin}[x]}}\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1+\text{Cot}[x]+\frac{\sqrt{2}\,\sqrt{\text{Cos}[x]}}{\sqrt{\text{Sin}[x]}}\right]}{2\sqrt{2}}+\frac{\text{Log}\left[1-\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}+\text{Tan}[x]\right]}{2\sqrt{2}}-\frac{\text{Log}\left[1+\frac{\sqrt{2}\,\sqrt{\text{Sin}[x]}}{\sqrt{\text{Cos}[x]}}+\text{Tan}[x]\right]}{2\sqrt{2}}$$

Problem 914: Unable to integrate problem.

$$\int\left(10\,x^9\,\text{Cos}\left[x^5\,\text{Log}[x]\right]-x^{10}\left(x^4+5\,x^4\,\text{Log}[x]\right)\text{Sin}\left[x^5\,\text{Log}[x]\right]\right)dx$$

Optimal (type 3, 11 leaves, ? steps):

$$x^{10}\,\text{Cos}\left[x^5\,\text{Log}[x]\right]$$

Result (type 8, 48 leaves, 4 steps):

$$10\,\text{CannotIntegrate}\left[x^9\,\text{Cos}\left[x^5\,\text{Log}[x]\right],x\right]-\text{CannotIntegrate}\left[x^{14}\,\text{Sin}\left[x^5\,\text{Log}[x]\right],x\right]-5\,\text{CannotIntegrate}\left[x^{14}\,\text{Log}[x]\,\text{Sin}\left[x^5\,\text{Log}[x]\right],x\right]$$

Problem 931: Unable to integrate problem.

$$\int\left(\frac{x^4}{b\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}+\frac{x^2\,\text{Cos}[a+b\,x]}{\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}+\frac{4\,x\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}{3\,b}\right)dx$$

Optimal (type 3, 26 leaves, ? steps):

$$\frac{2\,x^2\sqrt{x^3+3\,\text{Sin}[a+b\,x]}}{3\,b}$$

Result (type 8, 82 leaves, 1 step):

$$\frac{\text{CannotIntegrate}\left[\frac{x^4}{\sqrt{x^3+3\,\text{Sin}[a+b\,x]}},x\right]}{b}+\text{CannotIntegrate}\left[\frac{x^2\,\text{Cos}[a+b\,x]}{\sqrt{x^3+3\,\text{Sin}[a+b\,x]}},x\right]+\frac{4\,\text{CannotIntegrate}\left[x\sqrt{x^3+3\,\text{Sin}[a+b\,x]},x\right]}{3\,b}$$

Problem 933: Unable to integrate problem.

$$\int \frac{\text{Cos}[x] + \text{Sin}[x]}{e^{-x} + \text{Sin}[x]} \, dx$$

Optimal (type 3, 9 leaves, ? steps):

$$\text{Log}\left[1 + e^x \,\text{Sin}[x]\right]$$

Result (type 8, 36 leaves, 5 steps) :

$$x - \text{CannotIntegrate}\left[\frac{1}{1 + e^x \,\text{Sin}[x]}, x\right] - \text{CannotIntegrate}\left[\frac{\text{Cot}[x]}{1 + e^x \,\text{Sin}[x]}, x\right] + \text{Log}[\text{Sin}[x]]$$

Test results for the 227 integration problems in "5.1.2 (d x)^m (a+b arcsin(c x))^n.m"

Test results for the 595 integration problems in "5.1.4a (f x)^m (d-c^2 d x^2)^p (a+b arcsin(c x))^n.m"

Test results for the 108 integration problems in "5.1.4b (f x)^m (d+e x^2)^p (a+b arcsin(c x))^n.m"

Test results for the 474 integration problems in "5.1.5 Inverse sine functions.m"

Problem 474: Unable to integrate problem.

$$\int \frac{\sqrt{1 - x^2} + x\,\text{ArcSin}[x]}{\text{ArcSin}[x] - x^2\,\text{ArcSin}[x]} \, dx$$

Optimal (type 3, 16 leaves, ? steps):

$$-\frac{1}{2}\,\text{Log}\left[1 - x^2\right] + \text{Log}[\text{ArcSin}[x]]$$

Result (type 8, 32 leaves, 1 step) :

$$\text{Unintegrable}\left[\frac{\sqrt{1 - x^2} + x\,\text{ArcSin}[x]}{\left(1 - x^2\right)\,\text{ArcSin}[x]}, x\right]$$

Test results for the 227 integration problems in "5.2.2 (d x)^m (a+b arccos(c x))^n.m"

Test results for the 151 integration problems in "5.2.5 Inverse cosine functions.m"

Test results for the 166 integration problems in "5.3.2 (d x)^m (a+b arctan(c x^n))^p.m"

Test results for the 31 integration problems in "5.3.3 (d+e x)^m (a+b arctan(c x^n))^p.m"

Test results for the 1301 integration problems in "5.3.4 u (a+b arctan(c x))^p.m"

Problem 1137: Result valid but suboptimal antiderivative.

$$\int x^3 \left(d+e x^2\right)^3 (a+b \operatorname{ArcTan}[c x]) \, dx$$

Optimal (type 3, 240 leaves, ? steps):

$$\frac{b\left(10 c^6 d^3-20 c^4 d^2 e+15 c^2 d e^2-4 e^3\right) x}{40 c^9}-\frac{b\left(10 c^6 d^3-20 c^4 d^2 e+15 c^2 d e^2-4 e^3\right) x^3}{120 c^7}-\frac{b e\left(20 c^4 d^2-15 c^2 d e+4 e^2\right) x^5}{200 c^5}-\frac{b\left(15 c^2 d-4 e\right) e^2 x^7}{280 c^3}-\frac{b e^3 x^9}{90 c}+\frac{b\left(c^2 d-e\right)^4\left(c^2 d+4 e\right) \operatorname{ArcTan}[c x]}{40 c^{10} e^2}-\frac{d\left(d+e x^2\right)^4 (a+b \operatorname{ArcTan}[c x])}{8 e^2}+\frac{\left(d+e x^2\right)^5 (a+b \operatorname{ArcTan}[c x])}{10 e^2}$$

Result (type 3, 285 leaves, 8 steps) :

$$\frac{b\left(325 c^8 d^4+1815 c^6 d^3 e-4977 c^4 d^2 e^2+4305 c^2 d e^3-1260 e^4\right) x}{12600 c^9 e}+\frac{b\left(5 c^6 d^3+750 c^4 d^2 e-1071 c^2 d e^2+420 e^3\right) x\left(d+e x^2\right)}{12600 c^7 e}-\frac{b\left(25 c^4 d^2-135 c^2 d e+84 e^2\right) x\left(d+e x^2\right)^2}{4200 c^5 e}-\frac{b\left(23 c^2 d-36 e\right) x\left(d+e x^2\right)^3}{2520 c^3 e}-\frac{b x\left(d+e x^2\right)^4}{90 c e}+\frac{b\left(c^2 d-e\right)^4\left(c^2 d+4 e\right) \operatorname{ArcTan}[c x]}{40 c^{10} e^2}-\frac{d\left(d+e x^2\right)^4 (a+b \operatorname{ArcTan}[c x])}{8 e^2}+\frac{\left(d+e x^2\right)^5 (a+b \operatorname{ArcTan}[c x])}{10 e^2}$$

Test results for the 70 integration problems in "5.3.5 u (a+b arctan(c+d x))^p.m"

Test results for the 385 integration problems in "5.3.6 Exponentials of inverse tangent.m"

Test results for the 153 integration problems in "5.3.7 Inverse tangent functions.m"

Test results for the 234 integration problems in "5.4.1 Inverse cotangent functions.m"

Test results for the 12 integration problems in "5.4.2 Exponentials of inverse cotangent.m"

Test results for the 174 integration problems in "5.5.1 u (a+b arcsec(c x))^n.m"

Test results for the 50 integration problems in "5.5.2 Inverse secant functions.m"

Test results for the 178 integration problems in "5.6.1 u (a+b arccsc(c x))^n.m"

Test results for the 49 integration problems in "5.6.2 Inverse cosecant functions.m"

Test results for the 502 integration problems in "6.1.1 (c+d x)^m (a+b sinh)^n.m"

Test results for the 102 integration problems in "6.1.3 (e x)^m (a+b sinh(c+d x^n))^p.m"

Test results for the 33 integration problems in "6.1.4 (d+e x)^m sinh(a+b x+c x^2)^n.m"

Test results for the 525 integration problems in "6.1.7 hyper^m (a+b sinh^n)^p.m"

Test results for the 369 integration problems in "6.1.5 Hyperbolic sine functions.m"

Test results for the 183 integration problems in "6.2.1 (c+d x)^m (a+b cosh)^n.m"

Test results for the 111 integration problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.m"

Test results for the 68 integration problems in "6.2.3 (e x)^m (a+b cosh(c+d x^n))^p.m"

Test results for the 33 integration problems in "6.2.4 (d+e x)^m cosh(a+b x+c x^2)^n.m"

Test results for the 85 integration problems in "6.2.7 hyper^m (a+b cosh^n)^p.m"

Test results for the 336 integration problems in "6.2.5 Hyperbolic cosine functions.m"

Test results for the 77 integration problems in "6.3.1 (c+d x)^m (a+b tanh)^n.m"

Test results for the 263 integration problems in "6.3.7 (d hyper)^m (a+b (c tanh)^n)^p.m"

Test results for the 204 integration problems in "6.3.2 Hyperbolic tangent functions.m"

Test results for the 61 integration problems in "6.4.1 (c+d x)^m (a+b coth)^n.m"

Test results for the 53 integration problems in "6.4.7 (d hyper)^m (a+b (c coth)^n)^p.m"

Test results for the 181 integration problems in "6.4.2 Hyperbolic cotangent functions.m"

Test results for the 16 integration problems in "6.5.1 (c+d x)^m (a+b sech)^n.m"

Test results for the 84 integration problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

Test results for the 220 integration problems in "6.5.7 (d hyper)^m (a+b (c sech)^n)^p.m"

Test results for the 201 integration problems in "6.5.3 Hyperbolic secant functions.m"

Problem 186: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(\left(1-b^2 n^2\right) \text{Sech}\left[a+b \text{Log}\left[c x^n\right]\right]+2 b^2 n^2 \text{Sech}\left[a+b \text{Log}\left[c x^n\right]\right]^3\right) dx$$

Optimal (type 3, 40 leaves, ? steps):

$x\,\mathrm{Sech}[a+b\,\mathrm{Log}[c\,x^n]]+b\,n\,x\,\mathrm{Sech}[a+b\,\mathrm{Log}[c\,x^n]]\,\mathrm{Tanh}[a+b\,\mathrm{Log}[c\,x^n]]$

Result (type 5, 139 leaves, 9 steps):

$$2\,e^a\,(1-b\,n)\,x\,(c\,x^n)^b\,\mathrm{Hypergeometric2F1}\left[1,\frac{b+\frac{1}{n}}{2\,b},\frac{1}{2}\left(3+\frac{1}{b\,n}\right),-e^{2\,a}\,(c\,x^n)^{2\,b}\right]+\frac{16\,b^2\,e^{3\,a}\,n^2\,x\,(c\,x^n)^{3\,b}\,\mathrm{Hypergeometric2F1}\left[3,\frac{3\,b+\frac{1}{n}}{2\,b},\frac{1}{2}\left(5+\frac{1}{b\,n}\right),-e^{2\,a}\,(c\,x^n)^{2\,b}\right]}{1+3\,b\,n}$$

Test results for the 29 integration problems in "6.6.1 (c+d x)^m (a+b csch)^n.m"

Test results for the 83 integration problems in "6.6.2 (e x)^m (a+b csch(c+d x^n))^p.m"

Test results for the 27 integration problems in "6.6.7 (d hyper)^m (a+b (c csch)^n)^p.m"

Test results for the 175 integration problems in "6.6.3 Hyperbolic cosecant functions.m"

Problem 160: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\left(-\left(1-b^2\,n^2\right)\mathrm{Csch}[a+b\,\mathrm{Log}[c\,x^n]]+2\,b^2\,n^2\,\mathrm{Csch}[a+b\,\mathrm{Log}[c\,x^n]]^3\right)dx$$

Optimal (type 3, 42 leaves, ? steps):

$-x\,\mathrm{Csch}[a+b\,\mathrm{Log}[c\,x^n]]-b\,n\,x\,\mathrm{Coth}[a+b\,\mathrm{Log}[c\,x^n]]\,\mathrm{Csch}[a+b\,\mathrm{Log}[c\,x^n]]$

Result (type 5, 137 leaves, 9 steps):

$$2\,e^a\,(1-b\,n)\,x\,(c\,x^n)^b\,\mathrm{Hypergeometric2F1}\left[1,\frac{b+\frac{1}{n}}{2\,b},\frac{1}{2}\left(3+\frac{1}{b\,n}\right),e^{2\,a}\,(c\,x^n)^{2\,b}\right]-\frac{16\,b^2\,e^{3\,a}\,n^2\,x\,(c\,x^n)^{3\,b}\,\mathrm{Hypergeometric2F1}\left[3,\frac{3\,b+\frac{1}{n}}{2\,b},\frac{1}{2}\left(5+\frac{1}{b\,n}\right),e^{2\,a}\,(c\,x^n)^{2\,b}\right]}{1+3\,b\,n}$$

Test results for the 1059 integration problems in "6.7.1 Hyperbolic functions.m"

Test results for the 156 integration problems in "7.1.2 (d x)^m (a+b arcsinh(c x))^n.m"

Test results for the 541 integration problems in "7.1.4a (f x)^m (d+c^2 d x^2)^p (a+b arcsinh(c x))^n.m"

Test results for the 58 integration problems in "7.1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n.m"

Test results for the 371 integration problems in "7.1.5 Inverse hyperbolic sine functions.m"

Test results for the 166 integration problems in "7.2.2 (d x)^m (a+b arccosh(c x))^n.m"

Test results for the 453 integration problems in "7.2.4a (f x)^m (d-c^2 d x^2)^p (a+b arccosh(c x))^n.m"

Test results for the 109 integration problems in "7.2.4b (f x)^m (d+e x^2)^p (a+b arccosh(c x))^n.m"

Test results for the 293 integration problems in "7.2.5 Inverse hyperbolic cosine functions.m"

Problem 61: Unable to integrate problem.

$$\int \frac{\left(d - c^2 d x^2\right)^{3/2} \left(a + b \operatorname{ArcCosh}[c x]\right)}{f + g x} \, dx$$

Optimal (type 4, 1270 leaves, ? steps):

$$-\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}}{g^3}+\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{a\,d\,(2+3\,c\,x-2\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{6\,g}+\frac{b\,c\,d\,x\,(-12-9\,c\,x+4\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{36\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}-$$
$$\frac{a\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{2\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,(2+3\,c\,x-2\,c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{6\,g}-\frac{b\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]^2}{4\,g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$
$$\frac{d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}-\frac{2\,a\,d\,(c\,f-g)^{3/2}\,(c\,f+g)^{3/2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\left[\frac{\sqrt{c\,f+g}\,\sqrt{1+c\,x}}{\sqrt{c\,f-g}\,\sqrt{-1+c\,x}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\left[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\left[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\right]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Result (type 8, 1150 leaves, 28 steps) :

$$\frac{b\,c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}}{g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{b\,c^2\,d\,(c\,f-g)\,x^2\,\sqrt{d-c^2\,d\,x^2}}{4\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}}{g^3\,(1-c\,x)\,(1+c\,x)}-$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]}{g^3}+\frac{c\,d\,(c\,f-g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])}{2\,g^2}-$$
$$\frac{d\,(c\,f-g)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{4\,b\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+\frac{c\,d\,(c\,f-g)\,(c\,f+g)\,x\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,g^3\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{d\,(c\,f-g)^2\,(c\,f+g)^2\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+\frac{d\,(c\,f-g)\,(c\,f+g)\,(1-c^2\,x^2)\,\sqrt{d-c^2\,d\,x^2}\,(a+b\,\text{ArcCosh}[c\,x])^2}{2\,b\,c\,g^2\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}\,(f+g\,x)}+$$
$$\frac{a\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcTanh}\Big[\frac{g+c^2\,f\,x}{\sqrt{c^2\,f^2-g^2}\,\sqrt{-1+c^2\,x^2}}\Big]}{g^4\,(1-c\,x)\,(1+c\,x)}-$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{ArcCosh}[c\,x]\,\text{Log}\Big[1+\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\Big[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f-\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}+$$
$$\frac{b\,d\,(c\,f-g)\,(c\,f+g)\,\sqrt{c^2\,f^2-g^2}\,\sqrt{d-c^2\,d\,x^2}\,\text{PolyLog}\Big[2,-\frac{e^{\text{ArcCosh}[c\,x]}\,g}{c\,f+\sqrt{c^2\,f^2-g^2}}\Big]}{g^4\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}-$$
$$\frac{c\,d\,\sqrt{d-c^2\,d\,x^2}\,\text{Unintegrable}\big[(-1+c\,x)^{3/2}\,\sqrt{1+c\,x}\,(a+b\,\text{ArcCosh}[c\,x]),\,x\big]}{g\,\sqrt{-1+c\,x}\,\sqrt{1+c\,x}}$$

Test results for the 243 integration problems in "7.3.2 (d x)^m (a+b arctanh(c x^n))^p.m"

Test results for the 49 integration problems in "7.3.3 (d+e x)^m (a+b arctanh(c x^n))^p.m"

Test results for the 538 integration problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Test results for the 62 integration problems in "7.3.5 u (a+b arctanh(c+d x))^p.m"

Test results for the 1378 integration problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.m"

Test results for the 361 integration problems in "7.3.7 Inverse hyperbolic tangent functions.m"

Test results for the 300 integration problems in "7.4.1 Inverse hyperbolic cotangent functions.m"

Test results for the 935 integration problems in "7.4.2 Exponentials of inverse hyperbolic cotangent functions.m"

Test results for the 190 integration problems in "7.5.1 u (a+b arcsech(c x))^n.m"

Test results for the 100 integration problems in "7.5.2 Inverse hyperbolic secant functions.m"

Test results for the 178 integration problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Test results for the 71 integration problems in "7.6.2 Inverse hyperbolic cosecant functions.m"

Test results for the 311 integration problems in "8.1 Error functions.m"

Test results for the 218 integration problems in "8.2 Fresnel integral functions.m"

Test results for the 208 integration problems in "8.3 Exponential integral functions.m"

Test results for the 136 integration problems in "8.4 Trig integral functions.m"

Test results for the 136 integration problems in "8.5 Hyperbolic integral functions.m"

Test results for the 233 integration problems in "8.6 Gamma functions.m"

Test results for the 14 integration problems in "8.7 Zeta function.m"

Test results for the 198 integration problems in "8.8 Polylogarithm function.m"

Test results for the 398 integration problems in "8.9 Product logarithm function.m"

Test results for the 97 integration problems in "8.10 Formal derivatives.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int (g[x]\, f'[x] + f[x]\, g'[x])\, dx$$

Optimal (type 9, 5 leaves, ? steps):

f[x] g[x]

Result (type 9, 19 leaves, 1 step):

CannotIntegrate[g[x] f'[x], x] + CannotIntegrate[f[x] g'[x], x]

Problem 43: Result valid but suboptimal antiderivative.

$$\int \left(\text{Cos}[x]\, g\left[e^x\right] f'[\text{Sin}[x]] + e^x f[\text{Sin}[x]]\, g'\left[e^x\right]\right) dx$$

Optimal (type 9, 8 leaves, ? steps):

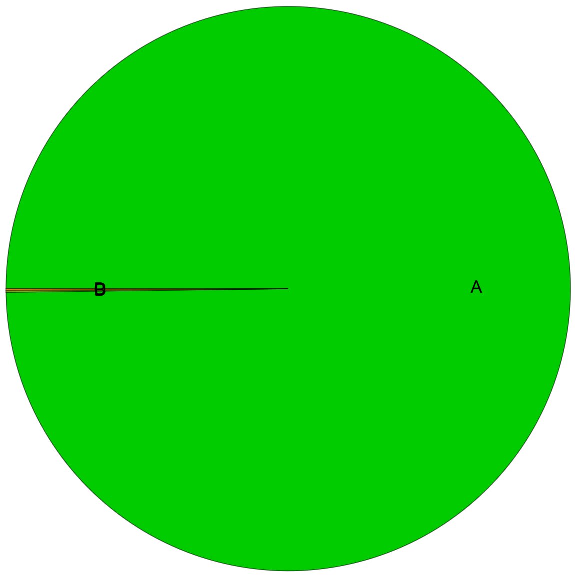
$f[\text{Sin}[x]]\, g\left[e^x\right]$

Result (type 9, 30 leaves, 1 step):

$\text{CannotIntegrate}\left[\text{Cos}[x]\, g\left[e^x\right] f'[\text{Sin}[x]], x\right] + \text{CannotIntegrate}\left[e^x f[\text{Sin}[x]]\, g'\left[e^x\right], x\right]$

Summary of Entire Integration Test Suite results

72 254 integration problems



A - 72 110 optimal antiderivatives

B - 51 valid but suboptimal antiderivatives

C - 29 unnecessarily complex antiderivatives

D - 64 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives